# Computer algebra independent integration tests

## 1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/1.1.1.2-a+b-x-^m-c+d-x-^n

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# Contents

# Chapter 1

# Introduction

## Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 1603 ]. This is test number [ 1 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 100.00 ( 1603 ) | 0.00 ( 0 ) |
| Mathematica | 100.00 ( 1603 ) | 0.00 ( 0 ) |
| Fricas | 100.00 ( 1603 ) | 0.00 ( 0 ) |
| Maple | 95.57 ( 1532 ) | 4.43 ( 71 ) |
| Maxima | 82.84 ( 1328 ) | 17.16 ( 275 ) |
| Giac | 79.60 ( 1276 ) | 20.40 ( 327 ) |
| Mupad | 77.42 ( 1241 ) | 22.58 ( 362 ) |
| Sympy | 71.62 ( 1148 ) | % 28.38 ( 455 ) |
| IntegrateAlgebraic | 63.38 ( 1016 ) | 36.62 ( 587 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

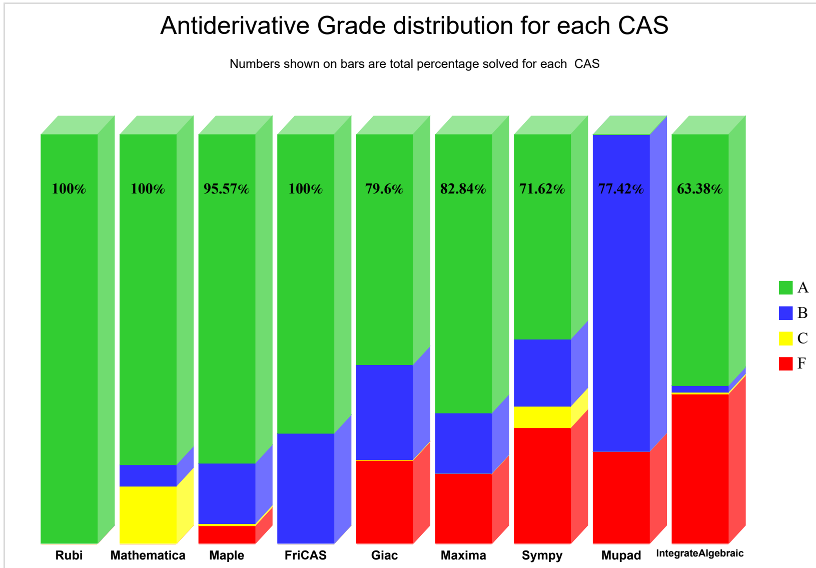
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

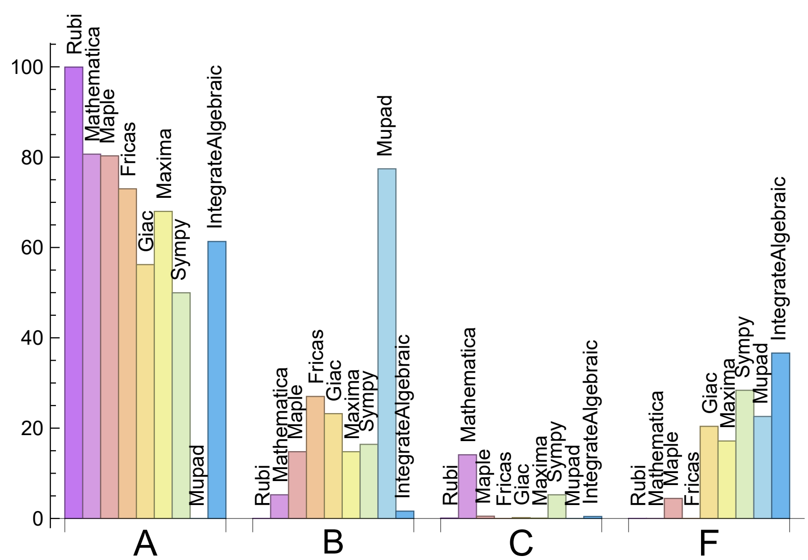
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 99.94 | 0.00 | 0.06 | 0.00 |
| Mathematica | 80.66 | 5.24 | 14.10 | 0.00 |
| Maple | 80.29 | 14.78 | 0.50 | 4.43 |
| Fricas | 72.99 | 27.01 | 0.00 | 0.00 |
| Maxima | 68.00 | 14.78 | 0.06 | 17.16 |
| IntegrateAlgebraic | 61.32 | 1.62 | 0.44 | 36.62 |
| Giac | 56.21 | 23.21 | 0.19 | 20.40 |
| Sympy | 49.97 | 16.41 | 5.24 | 28.38 |
| Mupad | N/A | 77.42 | 0.00 | 22.58 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the CAS systems for each grade level.




The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 % | 0.00 % | 0.00 % |
| Mathematica | 0 | 0.00 % | 0.00 % | 0.00 % |
| Maple | 71 | 100.00 % | 0.00 % | 0.00 % |
| Fricas | 0 | 0.00 % | 0.00 % | 0.00 % |
| IntegrateAlgebraic | 587 | 100.00 % | 0.00 % | 0.00 % |
| Giac | 327 | 63.00 % | 15.60 % | 21.41 % |
| Maxima | 275 | 55.64 % | 0.00 % | 44.36 % |
| Sympy | 455 | 62.20 % | 37.36 % | 0.44 % |
| Mupad | 362 | 100.00 % | 0.00 % | 0.00 % |

Table 1.4: Failure statistics for each CAS

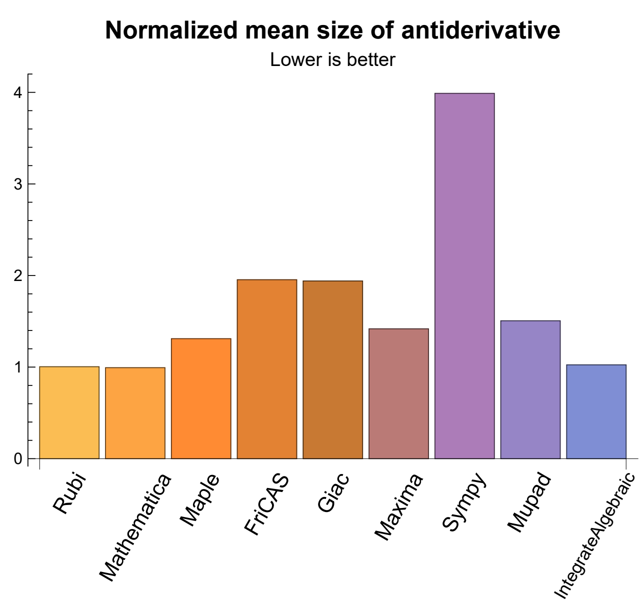
## 1.3 Performance

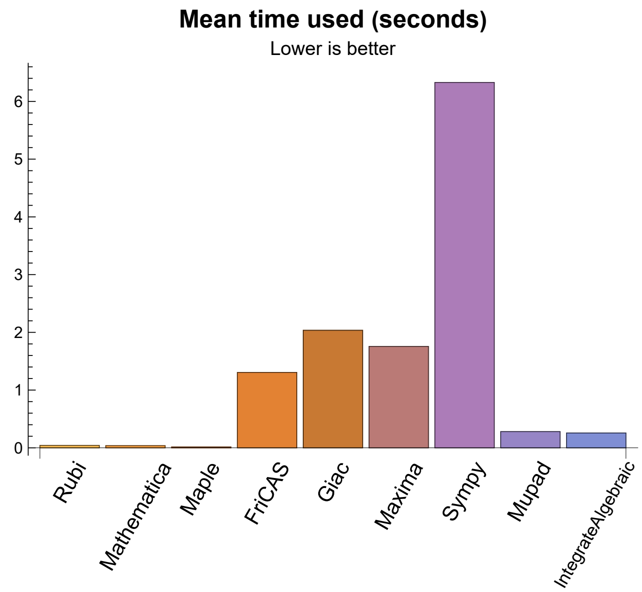
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.04 | 75.73 | 1.00 | 63.00 | 1.00 |
| Mathematica | 0.04 | 73.35 | 0.99 | 41.00 | 0.85 |
| Maple | 0.02 | 106.43 | 1.31 | 53.00 | 0.93 |
| Maxima | 1.76 | 107.90 | 1.42 | 56.00 | 0.99 |
| Fricas | 1.31 | 202.55 | 1.95 | 77.00 | 1.35 |
| Sympy | 6.33 | 271.67 | 3.99 | 92.00 | 1.67 |
| Giac | 2.04 | 159.13 | 1.94 | 65.00 | 1.07 |
| Mupad | 0.28 | 123.92 | 1.51 | 52.00 | 0.97 |
| IntegrateAlgebraic | 0.26 | 77.16 | 1.02 | 57.00 | 0.95 |

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

**IntegrateAlgebraic** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {}

**IntegrateAlgebraic** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
           leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

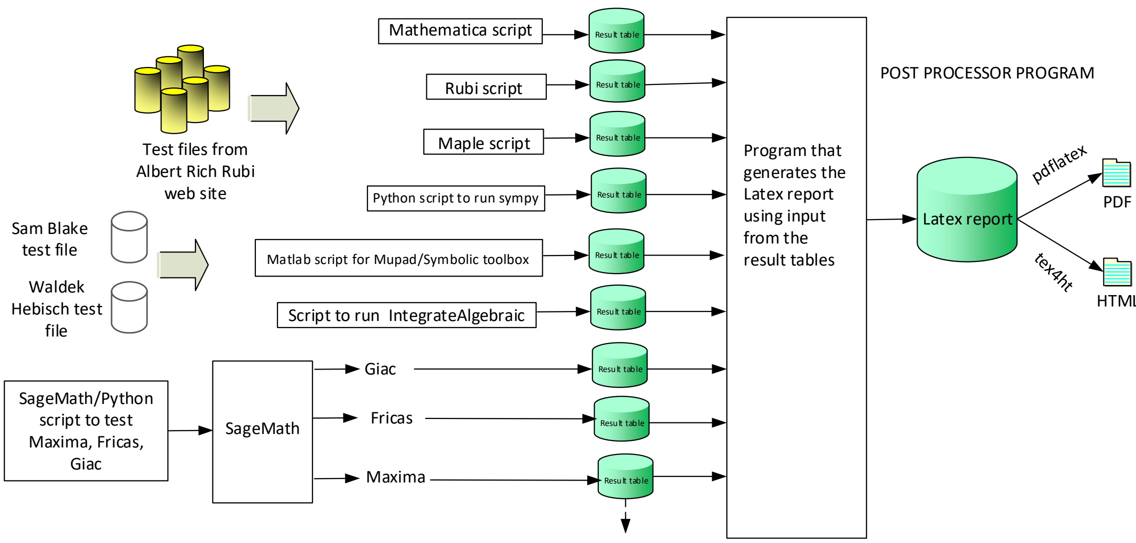
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.




**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    *The following field present only in Rubi and Mathematica Tables*

13. integer. 1 if result was verified or 0 if not verified.

    *The following fields present only in Rubi Tables*

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

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May 11, 2021

# Chapter 2

# detailed summary tables of results

## Local contents

## 2.1 List of integrals sorted by grade for each CAS

### Local contents

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928,

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1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111,
1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128,
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1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179,
1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196,
1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213,
1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230,
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1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264,
1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281,
1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298,
1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315,
1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332,
1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349,
1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366,
1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383,
1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400,
1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417,
1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434,
1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451,
1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468,
1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485,
1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502,
1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519,
1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536,
1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553,
1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570,
1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587,
1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { }

C grade: { 369 }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 298, 300, 301, 302, 303, 304, 307, 309, 310, 311, 312, 313, 314, 315, 316, 317, 322, 325, 326, 328, 330, 331, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 351, 352, 353, 354, 355, 359, 360, 368, 369, 370, 371, 372, 373, 374, 375, 378, 379, 380, 381, 382, 385, 386, 387, 388, 389, 392, 393, 394, 395, 396, 399, 400, 401, 402, 406, 407, 408, 409, 410, 413, 414, 415, 416, 420, 421, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 458, 459, 464, 469, 470, 471, 472, 478, 479, 484, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 527, 528, 529, 530, 533, 534, 535, 536, 539, 540, 541, 542, 545, 546, 547, 548, 551, 552, 553, 554, 557, 558, 559, 560, 563, 564, 565, 566, 569, 570, 571, 572, 573, 574, 575, 576, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 598, 599, 600, 601, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 637, 638, 639, 640, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 675, 677, 679, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1104, 1105, 1106, 1107, 1110, 1111, 1112, 1115, 1116, 1117, 1120, 1121, 1122, 1124, 1125, 1126, 1127, 1128, 1129, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1163, 1164, 1165, 1166, 1176, 1177, 1180, 1181, 1184, 1190, 1191, 1192, 1205, 1209, 1210, 1211, 1212, 1213, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1258, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1289, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1302, 1305, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1318, 1319, 1320, 1321, 1322, 1323, 1328, 1329, 1330, 1331, 1332, 1333, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1419, 1423, 1424, 1425, 1426, 1427, 1429, 1430, 1431, 1432, 1435, 1437, 1438, 1439, 1440, 1441, 1443, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1456, 1457, 1458, 1459, 1463, 1464, 1465, 1466, 1470, 1471, 1472, 1473, 1477, 1478, 1479, 1480, 1486, 1487, 1488, 1489, 1493, 1494, 1495, 1496, 1500, 1501, 1502, 1503, 1506, 1507, 1508, 1509, 1515, 1516, 1517, 1518, 1522, 1523, 1524, 1525, 1529, 1530, 1531, 1532, 1536, 1537, 1538, 1539, 1543, 1544, 1545, 1546, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 212, 226, 227, 243, 244, 1130, 1140, 1152, 1153, 1162, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1178, 1179, 1182, 1183, 1185, 1186, 1187, 1188, 1189, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1256, 1257, 1259, 1260, 1417, 1418, 1420, 1421, 1422, 1428, 1433, 1434, 1436, 1442, 1444, 1452 }

C grade: { 290, 291, 297, 299, 305, 306, 308, 318, 319, 320, 321, 323, 324, 327, 329, 332, 333, 341, 342, 348, 349, 350, 356, 357, 358, 361, 362, 363, 364, 365, 366, 367, 376, 377, 383, 384, 390, 391, 397, 398, 403, 404, 405, 411, 412, 417, 418, 419, 422, 423, 453, 454, 455, 456, 457, 460, 461, 462, 463, 465, 466, 467, 468, 473, 474, 475, 476, 477, 480, 481, 482, 483, 485, 486, 487, 488, 525, 526, 531, 532, 537, 538, 543, 544, 549, 550, 555, 556, 561, 562, 567, 568, 577, 578, 584, 596, 597, 602, 616, 617, 623, 635, 636, 641, 674, 676, 678, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 1012, 1013, 1026, 1027, 1028, 1047, 1048, 1049, 1057, 1058, 1059, 1101, 1102, 1103, 1108, 1109, 1113, 1114, 1118, 1119, 1123, 1277, 1278, 1279, 1280, 1288, 1290, 1291, 1292, 1300, 1301, 1303, 1304, 1306, 1315, 1316, 1317, 1324, 1325, 1326, 1327, 1334, 1335, 1336, 1337, 1369, 1370, 1379, 1380, 1381, 1396, 1397, 1398, 1406, 1407, 1408, 1453, 1454, 1455, 1460, 1461, 1462, 1467, 1468, 1469, 1474, 1475, 1476, 1481, 1482, 1483, 1484, 1485, 1490, 1491, 1492, 1497, 1498, 1499, 1504, 1505, 1510, 1511, 1512, 1513, 1514, 1519, 1520, 1521, 1526, 1527, 1528, 1533, 1534, 1535, 1540, 1541, 1542, 1547, 1548, 1549 }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 580, 581, 582, 583, 586, 587, 588, 589, 590, 591, 592, 594, 595, 596, 599, 600, 601, 604, 605, 606, 607, 608, 609, 610, 612, 613, 614, 615, 616, 618, 619, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 633, 634, 637, 638, 639, 640, 642, 643, 644, 645, 646, 647, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1011, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1070, 1071, 1072, 1073, 1074, 1075, 1080, 1081, 1082, 1083, 1084, 1088, 1089, 1090, 1091, 1092, 1094, 1095, 1096, 1097, 1098, 1099, 1104, 1105, 1106, 1107, 1110, 1111, 1112, 1115, 1116, 1117, 1120, 1121, 1122, 1125, 1126, 1127, 1128, 1129, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1142, 1143, 1144, 1145, 1146, 1147, 1149, 1150, 1151, 1157, 1158, 1164, 1165, 1166, 1176, 1205, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1240, 1241, 1242, 1243, 1244, 1245, 1250, 1251, 1252, 1253, 1254, 1255, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1282, 1283, 1284, 1285, 1286, 1289, 1290, 1291, 1292, 1294, 1295, 1296, 1297, 1298, 1302, 1303, 1304, 1305, 1306, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1348, 1349, 1350, 1351, 1352, 1353, 1358, 1360, 1361, 1362, 1363, 1364, 1371, 1372, 1373, 1374, 1382, 1383, 1384, 1385, 1389, 1391, 1392, 1393, 1394, 1395, 1400, 1401, 1402, 1403, 1404, 1410, 1411, 1412, 1413, 1414, 1424, 1435, 1438, 1456, 1457, 1458, 1459, 1463, 1464, 1465, 1466, 1470, 1471, 1472, 1473, 1477, 1478, 1479, 1480, 1486, 1487, 1488, 1489, 1493, 1494, 1495, 1496, 1500, 1501, 1502, 1503, 1506, 1507, 1508, 1509, 1515, 1516, 1517, 1518, 1522, 1523, 1524, 1525, 1529, 1530, 1531, 1532, 1536, 1537, 1538, 1539, 1543, 1544, 1545, 1546, 1550, 1551, 1552, 1553, 1554, 1557, 1560, 1561, 1563, 1564, 1565, 1566, 1569, 1570, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 199, 212, 213, 226, 227, 228, 243, 244, 442, 516, 517, 543, 544, 572, 578, 584, 593, 597, 602, 611, 617, 623, 632, 635, 636, 641, 648, 649, 699, 700, 701, 947, 948, 949, 965, 984, 997, 998, 999, 1000, 1009, 1010, 1012, 1013, 1022, 1036, 1040, 1041, 1049, 1050, 1060, 1068, 1069, 1076, 1077, 1078, 1079, 1085, 1086, 1087, 1093, 1100, 1124, 1130, 1131, 1140, 1141, 1148, 1152, 1153, 1154, 1155, 1156, 1159, 1160, 1161, 1162, 1163, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1237, 1238, 1239, 1246, 1247, 1248, 1249, 1256, 1257, 1258, 1259, 1260, 1261, 1269, 1281, 1287, 1288, 1293, 1299, 1300, 1301, 1307, 1318, 1327, 1328, 1347, 1354, 1355, 1356, 1357, 1365, 1366, 1367, 1368, 1375, 1376, 1377, 1378, 1386, 1387, 1388, 1390, 1405, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1436, 1437, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1555, 1556, 1558, 1559, 1562, 1567, 1568, 1571, 1572 }

C grade: { 1102, 1108, 1113, 1114, 1118, 1119, 1123, 1481 }

F grade: { 369, 501, 531, 532, 555, 556, 579, 585, 598, 603, 1101, 1103, 1109, 1359, 1369, 1370, 1379, 1380, 1381, 1396, 1397, 1398, 1399, 1406, 1407, 1408, 1409, 1453, 1454, 1455, 1460, 1461, 1462, 1467, 1468, 1469, 1474, 1475, 1476, 1482, 1483, 1484, 1485, 1490, 1491, 1492, 1497, 1498, 1499, 1504, 1505, 1510, 1511, 1512, 1513, 1514, 1519, 1520, 1521, 1526, 1527, 1528, 1533, 1534, 1535, 1540, 1541, 1542, 1547, 1548, 1549 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 217, 218, 219, 220, 221, 222, 223, 224, 225, 230, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 525, 526, 527, 528, 529, 530, 531, 532, 537, 538, 539, 540, 541, 542, 543, 544, 549, 550, 551, 552, 553, 554, 555, 556, 562, 563, 565, 566, 567, 568, 569, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 711, 714, 715, 716, 717, 722, 723, 724, 725, 726, 730, 731, 732, 733, 734, 735, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 770, 771, 772, 773, 774, 778, 779, 780, 781, 786, 787, 789, 790, 791, 792, 793, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 815, 816, 817, 818, 819, 820, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 839, 840, 841, 842, 843, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 924, 931, 932, 933, 934, 939, 940, 941, 942, 952, 953, 954, 955, 956, 959, 960, 961, 962, 963, 964, 966, 967, 969, 970, 971, 972, 973, 974, 975, 977, 978, 979, 980, 981, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1057, 1058, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1129, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1142, 1143, 1144, 1145, 1146, 1147, 1149, 1150, 1154, 1158, 1159, 1160, 1161, 1176, 1205, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1238, 1239, 1240, 1241, 1242, 1243, 1248, 1249, 1250, 1251, 1252, 1265, 1269, 1270, 1271, 1272, 1273, 1274, 1281, 1282, 1283, 1284, 1285, 1286, 1293, 1294, 1295, 1296, 1297, 1298, 1305, 1306, 1308, 1309, 1310, 1311, 1312, 1318, 1319, 1320, 1321, 1322, 1323, 1328, 1329, 1330, 1331, 1332, 1333, 1338, 1339, 1341, 1342, 1343, 1344, 1345, 1346, 1348, 1349, 1350, 1351, 1424, 1426, 1427, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1438, 1440, 1441, 1443, 1447, 1450, 1451, 1556, 1557, 1559, 1560, 1561, 1573, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 68, 73, 82, 83, 90, 105, 115, 116, 132, 133, 146, 147, 148, 186, 199, 212, 213, 214, 215, 216, 226, 227, 228, 229, 231, 232, 233, 243, 244, 489, 490, 491, 492, 505, 506, 507, 508, 521, 522, 523, 524, 533, 534, 535, 536, 545, 546, 547, 548, 557, 558, 559, 560, 561, 564, 570, 571, 572, 608, 609, 610, 611, 648, 788, 794, 813, 821, 838, 844, 845, 846, 875, 876, 877, 930, 935, 936, 937, 938, 943, 944, 945, 946, 947, 948, 949, 950, 951, 957, 965, 968, 976, 984, 1001, 1002, 1003, 1004, 1005, 1013, 1014, 1015, 1016, 1017, 1018, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1055, 1056, 1059, 1060, 1061, 1128, 1130, 1131, 1140, 1141, 1148, 1151, 1152, 1153, 1155, 1156, 1157, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1236, 1237, 1244, 1245, 1246, 1247, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1266, 1267, 1268, 1307, 1340, 1347, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1425, 1428, 1437, 1439, 1442, 1444, 1446, 1449, 1554, 1555, 1558 }

C grade: { 958 }

F grade: { 708, 709, 710, 712, 713, 718, 719, 720, 721, 727, 728, 729, 736, 737, 766, 767, 768, 769, 775, 776, 777, 782, 783, 784, 785, 814, 920, 921, 922, 923, 925, 926, 927, 928, 929, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1275, 1276, 1277, 1278, 1279, 1280, 1287, 1288, 1289, 1290, 1291, 1292, 1299, 1300, 1301, 1302, 1303, 1304, 1313, 1314, 1315, 1316, 1317, 1324, 1325, 1326, 1327, 1334, 1335, 1336, 1337, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1445, 1448, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1574, 1575, 1576 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 204, 205, 206, 207, 208, 209, 210, 211, 222, 223, 230, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 412, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 688, 696, 698, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 939, 940, 941, 942, 952, 953, 954, 955, 956, 958, 959, 960, 961, 962, 963, 964, 966, 967, 969, 970, 971, 972, 973, 974, 975, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1088, 1089, 1090, 1091, 1092, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1125, 1126, 1127, 1128, 1129, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1142, 1143, 1144, 1145, 1146, 1147, 1149, 1150, 1158, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1240, 1241, 1242, 1243, 1250, 1251, 1252, 1271, 1272, 1273, 1274, 1275, 1276, 1287, 1288, 1299, 1300, 1301, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1319, 1320, 1321, 1322, 1323, 1324, 1329, 1330, 1331, 1332, 1343, 1344, 1345, 1346, 1348, 1349, 1350, 1351, 1354, 1355, 1356, 1357, 1358, 1365, 1366, 1367, 1368, 1369, 1376, 1377, 1378, 1379, 1386, 1387, 1388, 1389, 1391, 1397, 1398, 1400, 1417, 1418, 1419, 1420, 1422, 1424, 1426, 1427, 1429, 1435, 1436, 1438, 1440, 1441, 1443, 1446, 1450, 1453, 1463, 1470, 1475, 1477, 1481, 1493, 1500, 1506, 1510, 1536, 1543, 1550, 1554, 1557, 1560, 1561, 1564, 1565, 1569, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 37, 38, 42, 68, 73, 82, 83, 90, 105, 106, 115, 116, 132, 133, 134, 146, 147, 148, 186, 199, 201, 202, 203, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 238, 243, 244, 295, 303, 314, 315, 316, 355, 356, 388, 410, 411, 418, 442, 586, 604, 625, 643, 648, 684, 685, 686, 687, 689, 690, 691, 692, 693, 694, 695, 697, 699, 700, 701, 930, 935, 936, 937, 938, 943, 944, 945, 946, 947, 948, 949, 950, 951, 957, 965, 968, 976, 999, 1000, 1001, 1013, 1014, 1015, 1029, 1030, 1031, 1032, 1040, 1041, 1050, 1060, 1061, 1087, 1093, 1101, 1124, 1130, 1131, 1140, 1141, 1148, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1236, 1237, 1238, 1239, 1244, 1245, 1246, 1247, 1248, 1249, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1302, 1303, 1304, 1314, 1315, 1316, 1317, 1318, 1325, 1326, 1327, 1328, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1347, 1352, 1353, 1359, 1360, 1361, 1362, 1363, 1364, 1370, 1371, 1372, 1373, 1374, 1375, 1380, 1381, 1382, 1383, 1384, 1385, 1390, 1392, 1393, 1394, 1395, 1396, 1399, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1421, 1423, 1425, 1428, 1430, 1431, 1432, 1433, 1434, 1437, 1439, 1442, 1444, 1445, 1447, 1448, 1449, 1451, 1452, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1464, 1465, 1466, 1467, 1468, 1469, 1471, 1472, 1473, 1474, 1476, 1478, 1479, 1480, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1494, 1495, 1496, 1497, 1498, 1499, 1501, 1502, 1503, 1504, 1505, 1507, 1508, 1509, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1537, 1538, 1539, 1540, 1541, 1542, 1544, 1545, 1546, 1547, 1548, 1549, 1551, 1552, 1553, 1555, 1556, 1558, 1559, 1562, 1563, 1566, 1567, 1568, 1570, 1571, 1572 }

C grade: { }

F grade: { }

### 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 218, 219, 220, 221, 222, 223, 224, 225, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 287, 289, 290, 291, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 330, 333, 338, 339, 340, 341, 342, 346, 347, 349, 350, 352, 353, 354, 355, 359, 361, 363, 364, 370, 374, 381, 387, 388, 395, 402, 409, 415, 416, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 495, 497, 498, 499, 500, 501, 503, 505, 506, 507, 508, 509, 511, 513, 514, 515, 516, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 569, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 608, 609, 610, 611, 612, 613, 616, 617, 618, 619, 620, 625, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 666, 667, 668, 674, 675, 676, 677, 678, 679, 680, 681, 684, 685, 686, 687, 699, 700, 701, 702, 703, 704, 705, 706, 707, 712, 713, 714, 715, 716, 717, 718, 721, 722, 723, 724, 725, 726, 727, 728, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 743, 744, 745, 746, 749, 750, 751, 752, 753, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 770, 771, 772, 773, 774, 775, 778, 779, 780, 781, 782, 783, 786, 787, 793, 799, 800, 801, 805, 806, 807, 808, 809, 855, 863, 871, 877, 924, 931, 932, 933, 934, 939, 940, 941, 942, 946, 953, 954, 955, 956, 959, 960, 961, 962, 963, 964, 966, 969, 970, 971, 973, 974, 975, 977, 980, 981, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 1000, 1001, 1006, 1007, 1011, 1012, 1020, 1021, 1024, 1025, 1026, 1035, 1036, 1037, 1038, 1039, 1042, 1047, 1048, 1049, 1051, 1052, 1057, 1061, 1062, 1063, 1087, 1091, 1092, 1093, 1094, 1096, 1098, 1128, 1129, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1142, 1143, 1145, 1146, 1147, 1149, 1158, 1159, 1160, 1161, 1229, 1230, 1231, 1232, 1233, 1237, 1238, 1239, 1240, 1241, 1242, 1248, 1249, 1250, 1251, 1270, 1271, 1272, 1273, 1274, 1275, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1350, 1351, 1419, 1424, 1426, 1438, 1440, 1446, 1447, 1449, 1450, 1451, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 55, 68, 73, 82, 83, 90, 91, 104, 105, 106, 115, 116, 131, 132, 133, 134, 146, 147, 148, 186, 187, 199, 201, 212, 213, 214, 215, 216, 217, 226, 227, 228, 229, 230, 231, 232, 233, 234, 243, 244, 284, 285, 286, 288, 292, 293, 297, 325, 329, 334, 335, 336, 337, 343, 344, 345, 348, 351, 356, 357, 358, 360, 367, 371, 372, 373, 378, 379, 380, 385, 386, 392, 393, 394, 406, 407, 408, 413, 414, 494, 496, 502, 504, 510, 512, 518, 520, 575, 576, 582, 583, 584, 585, 586, 587, 588, 589, 601, 602, 603, 604, 606, 607, 614, 615, 621, 622, 623, 624, 626, 627, 628, 640, 641, 642, 643, 645, 646, 766, 776, 784, 788, 792, 794, 798, 804, 930, 935, 936, 937, 938, 943, 944, 945, 947, 948, 949, 950, 951, 952, 957, 965, 967, 968, 972, 976, 978, 979, 984, 997, 998, 999, 1002, 1003, 1004, 1005, 1008, 1009, 1010, 1013, 1014, 1015, 1016, 1017, 1022, 1023, 1027, 1028, 1029, 1030, 1031, 1040, 1041, 1050, 1053, 1054, 1055, 1056, 1064, 1065, 1066, 1086, 1088, 1095, 1130, 1131, 1139, 1140, 1141, 1144, 1148, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1228, 1234, 1235, 1236, 1243, 1244, 1245, 1246, 1247, 1252, 1253, 1254, 1255, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1276, 1554, 1573 }

C grade: { 328, 331, 332, 362, 368, 369, 375, 376, 377, 382, 383, 384, 389, 390, 391, 396, 397, 398, 399, 400, 401, 403, 404, 405, 410, 411, 412, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 517, 532, 544, 556, 568, 605, 644, 661, 662, 663, 664, 665, 669, 670, 671, 672, 673, 698, 958, 1043, 1044, 1045, 1046, 1058, 1059, 1060, 1071, 1072, 1073, 1074, 1079, 1080, 1081, 1082, 1097, 1099, 1100, 1349, 1421, 1428, 1431, 1433, 1435, 1442, 1444, 1481 }

F grade: { 365, 366, 438, 445, 682, 683, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 708, 709, 710, 711, 719, 720, 729, 741, 742, 747, 748, 754, 767, 768, 769, 777, 785, 789, 790, 791, 795, 796, 797, 802, 803, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 856, 857, 858, 859, 860, 861, 862, 864, 865, 866, 867, 868, 869, 870, 872, 873, 874, 875, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 925, 926, 927, 928, 929, 1018, 1019, 1032, 1033, 1034, 1067, 1068, 1069, 1070, 1075, 1076, 1077, 1078, 1083, 1084, 1085, 1089, 1090, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1256, 1257, 1258, 1259, 1277, 1278, 1279, 1280, 1288, 1289, 1290, 1291, 1292, 1300, 1301, 1302, 1303, 1304, 1314, 1315, 1316, 1317, 1325, 1326, 1327, 1335, 1336, 1337, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1420, 1422, 1423, 1425, 1427, 1429, 1430, 1432, 1434, 1436, 1437, 1439, 1441, 1443, 1445, 1448, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1574, 1575, 1576 }

### 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 287, 288, 289, 290, 291, 296, 297, 298, 299, 304, 305, 306, 307, 308, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 370, 374, 375, 376, 377, 381, 382, 383, 384, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 495, 496, 503, 504, 511, 512, 519, 520, 574, 575, 576, 577, 583, 595, 613, 614, 615, 616, 634, 647, 648, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 703, 706, 707, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 746, 747, 748, 754, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 794, 795, 796, 802, 810, 811, 812, 813, 814, 818, 819, 820, 821, 822, 827, 828, 829, 830, 831, 832, 835, 836, 837, 838, 839, 840, 843, 844, 845, 846, 847, 848, 851, 852, 853, 854, 855, 859, 860, 861, 862, 863, 867, 868, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 884, 888, 889, 931, 932, 933, 934, 935, 936, 937, 939, 940, 941, 942, 943, 944, 945, 951, 952, 953, 955, 956, 957, 959, 960, 961, 962, 963, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 995, 997, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1008, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1054, 1055, 1056, 1057, 1058, 1067, 1071, 1086, 1087, 1091, 1092, 1093, 1094, 1095, 1129, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1142, 1143, 1144, 1145, 1146, 1147, 1149, 1150, 1151, 1157, 1158, 1160, 1161, 1164, 1165, 1166, 1176, 1205, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1240, 1241, 1242, 1243, 1244, 1248, 1249, 1250, 1251, 1252, 1262, 1263, 1264, 1265, 1274, 1275, 1276, 1277, 1278, 1287, 1288, 1289, 1290, 1291, 1299, 1300, 1301, 1302, 1303, 1305, 1306, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1320, 1321, 1322, 1323, 1324, 1325, 1330, 1331, 1332, 1333, 1334, 1345, 1346, 1348, 1349, 1350, 1351, 1352, 1353, 1358, 1386, 1387, 1388, 1389, 1390, 1397, 1398, 1399, 1400, 1410, 1416, 1417, 1418, 1419, 1420, 1422, 1423, 1424, 1425, 1426, 1427, 1429, 1430, 1431, 1432, 1433, 1434, 1437, 1438, 1439, 1440, 1441, 1443, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1561, 1573, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 37, 38, 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 212, 226, 227, 243, 244, 284, 285, 286, 292, 293, 294, 295, 300, 301, 302, 303, 309, 310, 311, 312, 313, 314, 315, 316, 371, 372, 373, 378, 379, 380, 385, 386, 387, 388, 435, 442, 494, 502, 510, 518, 573, 578, 579, 580, 581, 582, 584, 585, 586, 587, 588, 589, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 612, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 699, 700, 701, 702, 704, 705, 742, 869, 870, 881, 882, 883, 885, 886, 890, 920, 922, 930, 938, 946, 947, 948, 949, 950, 954, 964, 984, 994, 996, 998, 1006, 1007, 1009, 1019, 1020, 1021, 1022, 1023, 1024, 1041, 1049, 1050, 1051, 1052, 1053, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1068, 1069, 1070, 1072, 1073, 1074, 1075, 1080, 1081, 1082, 1083, 1084, 1085, 1088, 1089, 1090, 1096, 1097, 1098, 1099, 1128, 1130, 1131, 1140, 1141, 1148, 1152, 1153, 1154, 1155, 1156, 1159, 1162, 1163, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1236, 1237, 1238, 1239, 1245, 1246, 1247, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1304, 1307, 1317, 1318, 1319, 1326, 1327, 1328, 1329, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1347, 1354, 1355, 1356, 1357, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1391, 1392, 1393, 1394, 1395, 1396, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1411, 1412, 1413, 1414, 1415, 1421, 1428, 1442, 1444, 1554, 1555, 1556, 1557, 1558, 1559, 1560 }

C grade: { 958, 1435, 1436 }

F grade: { 368, 369, 489, 490, 491, 492, 493, 497, 498, 499, 500, 501, 505, 506, 507, 508, 509, 513, 514, 515, 516, 517, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 590, 591, 592, 593, 608, 609, 610, 611, 629, 630, 631, 632, 649, 708, 709, 710, 711, 712, 713, 743, 744, 745, 749, 750, 751, 752, 753, 755, 756, 757, 758, 759, 760, 761, 791, 792, 793, 797, 798, 799, 800, 801, 803, 804, 805, 806, 807, 808, 809, 815, 816, 817, 823, 824, 825, 826, 833, 834, 841, 842, 849, 850, 856, 857, 858, 864, 865, 866, 887, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 921, 923, 924, 925, 926, 927, 928, 929, 1076, 1077, 1078, 1079, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1574, 1575, 1576 }

### 2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 491, 492, 494, 495, 496, 499, 500, 502, 503, 504, 507, 508, 510, 511, 512, 515, 516, 518, 519, 520, 571, 572, 573, 574, 575, 576, 580, 581, 582, 583, 586, 587, 588, 589, 592, 593, 594, 595, 599, 600, 601, 604, 605, 606, 607, 610, 611, 612, 613, 614, 615, 619, 620, 621, 622, 625, 626, 627, 628, 631, 632, 633, 634, 638, 639, 640, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 717, 718, 721, 727, 728, 735, 736, 737, 739, 740, 741, 742, 743, 744, 745, 747, 748, 749, 750, 751, 752, 753, 755, 756, 757, 758, 759, 760, 761, 792, 793, 798, 799, 800, 801, 804, 805, 806, 807, 808, 809, 855, 863, 871, 877, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 998, 999, 1001, 1002, 1003, 1004, 1005, 1014, 1015, 1016, 1017, 1018, 1029, 1030, 1031, 1032, 1033, 1034, 1036, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1051, 1052, 1053, 1054, 1055, 1056, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1070, 1071, 1072, 1073, 1074, 1075, 1078, 1079, 1080, 1081, 1082, 1083, 1086, 1087, 1088, 1089, 1090, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1104, 1105, 1106, 1107, 1120, 1121, 1122, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1357, 1358, 1360, 1361, 1362, 1363, 1364, 1371, 1372, 1373, 1374, 1382, 1383, 1384, 1385, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1400, 1401, 1402, 1403, 1404, 1405, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1456, 1457, 1458, 1459, 1470, 1471, 1472, 1473, 1486, 1487, 1488, 1489, 1500, 1501, 1502, 1503, 1515, 1516, 1517, 1518, 1522, 1523, 1524, 1525, 1529, 1530, 1531, 1532, 1536, 1537, 1538, 1539, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

C grade: { }

F grade: { 369, 489, 490, 493, 497, 498, 501, 505, 506, 509, 513, 514, 517, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 577, 578, 579, 584, 585, 590, 591, 596, 597, 598, 602, 603, 608, 609, 616, 617, 618, 623, 624, 629, 630, 635, 636, 637, 641, 642, 714, 715, 716, 719, 720, 722, 723, 724, 725, 726, 729, 730, 731, 732, 733, 734, 738, 746, 754, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 794, 795, 796, 797, 802, 803, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 856, 857, 858, 859, 860, 861, 862, 864, 865, 866, 867, 868, 869, 870, 872, 873, 874, 875, 876, 878, 879, 880, 994, 995, 996, 997, 1000, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1035, 1037, 1038, 1039, 1047, 1048, 1049, 1050, 1057, 1058, 1059, 1060, 1068, 1069, 1076, 1077, 1084, 1085, 1091, 1101, 1102, 1103, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1123, 1354, 1355, 1356, 1359, 1365, 1366, 1367, 1368, 1369, 1370, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1386, 1387, 1388, 1396, 1397, 1398, 1399, 1406, 1407, 1408, 1409, 1453, 1454, 1455, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1519, 1520, 1521, 1526, 1527, 1528, 1533, 1534, 1535, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549 }

### 2.1.9 IntegrateAlgebraic

A grade: { 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 42, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 994, 995, 996, 997, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1010, 1011, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1023, 1024, 1025, 1026, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1091, 1092, 1094, 1095, 1096, 1097, 1100, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1422, 1423, 1425, 1426, 1427, 1429, 1430, 1432, 1436, 1440, 1441, 1443, 1445, 1446, 1447, 1448, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1542, 1543, 1544, 1545, 1546, 1547, 1549, 1550, 1551, 1552, 1553, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 315, 998, 1009, 1022, 1041, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1093, 1269, 1307, 1318, 1328, 1421, 1428, 1431, 1433, 1434, 1437, 1439, 1442, 1444 }

C grade: { 999, 1012, 1027, 1040, 1049, 1059, 1449 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 34, 35, 36, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 368, 369, 428, 435, 442, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 1098, 1099, 1101, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1424, 1435, 1438, 1541, 1548, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 1 | 1 | 1 | 2 | 1 | 1 | 0 | 1 | 1 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 2.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.005 | 0.420 | 1.406 | 0.008 | 1.087 | 0.041 | 0.000 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 1 | 1 | 1 | 2 | 1 | 1 | 0 | 1 | 1 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 2.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.002 | 0.415 | 1.443 | 0.023 | 1.067 | 0.005 | 0.000 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 3 | 3 | 3 | 4 | 3 | 3 | 2 | 3 | 3 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 1.00 | 1.00 | 0.67 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.000 | 0.413 | 1.448 | 0.016 | 1.037 | 0.006 | 0.000 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 3 | 3 | 3 | 4 | 3 | 3 | 3 | 3 | 3 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.000 | 0.419 | 1.257 | 0.015 | 1.114 | 0.004 | 0.000 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 5 | 5 | 5 | 4 | 3 | 3 | 5 | 3 | 3 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.60 | 0.60 | 1.00 | 0.60 | 0.60 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.000 | 0.422 | 1.295 | 0.015 | 1.158 | 0.008 | 0.000 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 3 | 3 | 3 | 4 | 3 | 3 | 2 | 3 | 3 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 1.00 | 1.00 | 0.67 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.001 | 0.000 | 0.000 | 0.434 | 1.458 | 0.015 | 0.836 | 0.002 | 0.000 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 3 | 3 | 3 | 4 | 3 | 3 | 2 | 3 | 3 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.33 | 1.00 | 1.00 | 0.67 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.000 | 0.436 | 0.679 | 0.016 | 0.747 | 0.002 | 0.000 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 4 | 4 | 4 | 5 | 4 | 4 | 3 | 4 | 4 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.25 | 1.00 | 1.00 | 0.75 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.000 | 0.426 | 0.861 | 0.016 | 1.248 | 0.002 | 0.000 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 12 | 11 | 18 | 10 | 11 | 11 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.79 | 1.29 | 0.71 | 0.79 | 0.79 | 0.00 |
| time (sec) | N/A | 0.008 | 0.000 | 0.000 | 0.445 | 1.203 | 0.055 | 1.075 | 0.003 | 0.000 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 3 | 5 | 5 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 0.43 | 0.71 | 0.71 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.001 | 0.421 | 1.310 | 0.055 | 1.182 | 0.118 | 0.000 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 3 | 5 | 5 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 0.43 | 0.71 | 0.71 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.002 | 0.416 | 1.137 | 0.054 | 1.183 | 0.022 | 0.000 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 3 | 5 | 5 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 0.43 | 0.71 | 0.71 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.000 | 0.420 | 1.130 | 0.055 | 1.095 | 0.010 | 0.000 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 3 | 5 | 5 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 0.43 | 0.71 | 0.71 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.001 | 0.415 | 1.105 | 0.017 | 0.953 | 0.012 | 0.000 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 1 | 1 | 1 | 2 | 1 | 1 | 0 | 1 | 1 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 2.00 | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.000 | 0.442 | 1.287 | 0.015 | 1.002 | 0.002 | 0.000 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 3 | 2 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.50 | 1.00 | 1.00 | 1.00 | 1.50 | 1.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.002 | 0.435 | 0.995 | 0.060 | 0.803 | 0.036 | 0.000 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 5 | 5 | 5 | 6 | 5 | 5 | 3 | 5 | 5 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 1.00 | 1.00 | 0.60 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.001 | 0.000 | 0.000 | 0.432 | 1.793 | 0.058 | 0.849 | 0.035 | 0.000 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 7 | 5 | 5 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 1.00 | 0.71 | 0.71 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.001 | 0.489 | 1.336 | 0.058 | 0.841 | 0.014 | 0.000 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 7 | 5 | 5 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 1.00 | 0.71 | 0.71 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.000 | 0.426 | 1.263 | 0.059 | 1.132 | 0.012 | 0.000 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 7 | 5 | 5 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 1.00 | 0.71 | 0.71 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.000 | 0.416 | 1.311 | 0.060 | 1.290 | 0.070 | 0.000 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 6 | 5 | 5 | 7 | 5 | 5 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.56 | 0.56 | 0.78 | 0.56 | 0.56 | 1.00 |
| time (sec) | N/A | 0.001 | 0.001 | 0.017 | 0.428 | 0.863 | 0.068 | 0.900 | 0.077 | 0.013 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 6 | 5 | 5 | 7 | 5 | 5 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.56 | 0.56 | 0.78 | 0.56 | 0.56 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.002 | 0.415 | 1.547 | 0.059 | 0.982 | 0.031 | 0.003 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 6 | 5 | 5 | 7 | 5 | 5 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.56 | 0.56 | 0.78 | 0.56 | 0.56 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.002 | 0.425 | 0.932 | 0.062 | 1.042 | 0.029 | 0.003 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 5 | 5 | 5 | 7 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.003 | 0.419 | 0.771 | 0.059 | 1.127 | 0.031 | 0.003 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 7 | 5 | 5 | 7 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 1.00 | 0.71 | 0.71 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.002 | 0.698 | 1.427 | 0.061 | 1.010 | 0.031 | 0.003 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 6 | 5 | 5 | 8 | 5 | 5 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.56 | 0.56 | 0.89 | 0.56 | 0.56 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.001 | 0.616 | 1.362 | 0.061 | 0.924 | 0.034 | 0.003 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 6 | 5 | 5 | 7 | 5 | 5 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.56 | 0.56 | 0.78 | 0.56 | 0.56 | 1.00 |
| time (sec) | N/A | 0.001 | 0.001 | 0.003 | 0.559 | 1.265 | 0.060 | 1.051 | 0.073 | 0.003 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 6 | 5 | 5 | 7 | 5 | 5 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.56 | 0.56 | 0.78 | 0.56 | 0.56 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.003 | 0.555 | 1.470 | 0.061 | 1.047 | 0.066 | 0.003 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 6 | 5 | 5 | 7 | 5 | 5 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.56 | 0.56 | 0.78 | 0.56 | 0.56 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.001 | 0.505 | 0.699 | 0.062 | 0.913 | 0.065 | 0.003 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 6 | 5 | 5 | 7 | 5 | 5 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.56 | 0.56 | 0.78 | 0.56 | 0.56 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.002 | 0.480 | 1.391 | 0.061 | 0.848 | 0.065 | 0.003 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 6 | 5 | 5 | 7 | 5 | 5 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.56 | 0.56 | 0.78 | 0.56 | 0.56 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.001 | 0.456 | 1.350 | 0.061 | 1.136 | 0.040 | 0.003 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 5 | 5 | 5 | 7 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.002 | 0.521 | 0.814 | 0.060 | 0.949 | 0.067 | 0.003 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 7 | 5 | 5 | 7 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 1.00 | 0.71 | 0.71 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.003 | 0.513 | 1.424 | 0.060 | 1.119 | 0.073 | 0.003 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 6 | 5 | 5 | 8 | 5 | 5 | 9 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.56 | 0.56 | 0.89 | 0.56 | 0.56 | 1.00 |
| time (sec) | N/A | 0.000 | 0.001 | 0.001 | 0.491 | 1.106 | 0.060 | 0.923 | 0.051 | 0.003 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 12 | 11 | 10 | 12 | 11 | 20 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 1.00 | 0.91 | 1.09 | 1.00 | 1.82 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.003 | 0.638 | 1.686 | 0.063 | 0.948 | 0.345 | 0.003 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 12 | 13 | 16 | 12 | 17 | 16 | 12 | 0 |
| N.S. | 1 | 1.00 | 0.75 | 0.81 | 1.00 | 0.75 | 1.06 | 1.00 | 0.75 | 0.00 |
| time (sec) | N/A | 0.003 | 0.002 | 0.003 | 0.537 | 1.371 | 0.064 | 1.037 | 0.183 | 0.011 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 22 | 21 | 21 | 19 | 22 | 21 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.91 | 0.91 | 0.83 | 0.96 | 0.91 | 0.00 |
| time (sec) | N/A | 0.014 | 0.004 | 0.002 | 0.665 | 1.446 | 0.087 | 1.071 | 0.143 | 0.001 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 20 | 19 | 104 | 270 | 444 | 93 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.83 | 4.52 | 11.74 | 19.30 | 4.04 | 1.00 |
| time (sec) | N/A | 0.012 | 0.020 | 0.003 | 0.558 | 1.484 | 71.801 | 1.635 | 0.183 | 0.009 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 20 | 19 | 59 | 156 | 195 | 45 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.83 | 2.57 | 6.78 | 8.48 | 1.96 | 1.00 |
| time (sec) | N/A | 0.011 | 0.014 | 0.002 | 0.885 | 0.869 | 5.270 | 1.043 | 0.170 | 0.011 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 20 | 19 | 19 | 82 | 19 | 19 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.83 | 0.83 | 3.57 | 0.83 | 0.83 | 1.00 |
| time (sec) | N/A | 0.011 | 0.010 | 0.003 | 0.738 | 1.403 | 0.444 | 1.070 | 0.075 | 0.010 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 20 | 19 | 19 | 31 | 19 | 19 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.90 | 0.90 | 1.48 | 0.90 | 0.90 | 1.00 |
| time (sec) | N/A | 0.010 | 0.008 | 0.004 | 0.920 | 1.073 | 1.782 | 1.165 | 0.106 | 0.010 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 20 | 19 | 34 | 58 | 19 | 19 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.90 | 1.62 | 2.76 | 0.90 | 0.90 | 1.00 |
| time (sec) | N/A | 0.011 | 0.008 | 0.002 | 1.003 | 1.572 | 1.795 | 1.645 | 0.135 | 0.013 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 20 | 19 | 68 | 102 | 19 | 19 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.83 | 2.96 | 4.43 | 0.83 | 0.83 | 1.00 |
| time (sec) | N/A | 0.011 | 0.011 | 0.003 | 0.848 | 1.406 | 6.786 | 1.317 | 0.179 | 0.011 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 14 | 13 | 13 | 12 | 13 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.76 | 0.76 | 0.71 | 0.76 | 0.76 | 0.00 |
| time (sec) | N/A | 0.008 | 0.001 | 0.000 | 0.895 | 1.341 | 0.063 | 1.216 | 0.020 | 0.000 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 14 | 13 | 13 | 12 | 13 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.76 | 0.76 | 0.71 | 0.76 | 0.76 | 0.00 |
| time (sec) | N/A | 0.007 | 0.001 | 0.000 | 0.861 | 1.452 | 0.063 | 1.738 | 0.019 | 0.000 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 14 | 13 | 13 | 12 | 13 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.76 | 0.76 | 0.71 | 0.76 | 0.76 | 0.00 |
| time (sec) | N/A | 0.006 | 0.001 | 0.001 | 0.831 | 1.255 | 0.064 | 1.172 | 0.019 | 0.000 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 12 | 11 | 10 | 10 | 8 | 10 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.83 | 0.83 | 0.67 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.002 | 0.000 | 0.002 | 0.887 | 1.360 | 0.060 | 1.248 | 0.017 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 8 | 9 | 8 | 8 | 7 | 9 | 8 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.12 | 1.00 | 1.00 | 0.88 | 1.12 | 1.00 | 0.00 |
| time (sec) | N/A | 0.003 | 0.001 | 0.008 | 0.843 | 1.264 | 0.090 | 1.057 | 0.017 | 0.000 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 12 | 11 | 13 | 7 | 12 | 11 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 1.00 | 1.18 | 0.64 | 1.09 | 1.00 | 0.00 |
| time (sec) | N/A | 0.004 | 0.002 | 0.018 | 0.871 | 1.408 | 0.111 | 1.048 | 0.033 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 15 | 14 | 11 | 11 | 12 | 11 | 11 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.82 | 0.65 | 0.65 | 0.71 | 0.65 | 0.65 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.005 | 1.125 | 1.316 | 0.110 | 1.223 | 0.023 | 0.000 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 14 | 13 | 13 | 14 | 13 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.76 | 0.76 | 0.82 | 0.76 | 0.76 | 0.00 |
| time (sec) | N/A | 0.005 | 0.002 | 0.004 | 1.097 | 1.376 | 0.131 | 1.383 | 0.026 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 14 | 13 | 13 | 14 | 13 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.76 | 0.76 | 0.82 | 0.76 | 0.76 | 0.00 |
| time (sec) | N/A | 0.005 | 0.002 | 0.006 | 1.051 | 1.466 | 0.163 | 1.679 | 0.027 | 0.000 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 25 | 24 | 24 | 26 | 24 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.80 | 0.80 | 0.87 | 0.80 | 0.80 | 0.00 |
| time (sec) | N/A | 0.012 | 0.002 | 0.002 | 1.195 | 0.749 | 0.072 | 1.611 | 0.077 | 0.000 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 25 | 24 | 24 | 24 | 24 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.80 | 0.80 | 0.80 | 0.80 | 0.80 | 0.00 |
| time (sec) | N/A | 0.010 | 0.002 | 0.000 | 1.051 | 1.130 | 0.073 | 1.023 | 0.031 | 0.000 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 25 | 24 | 24 | 26 | 24 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.80 | 0.80 | 0.87 | 0.80 | 0.80 | 0.00 |
| time (sec) | N/A | 0.009 | 0.001 | 0.002 | 0.989 | 1.152 | 0.067 | 1.414 | 0.030 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 20 | 20 | 19 | 12 | 20 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 1.43 | 1.43 | 1.36 | 0.86 | 1.43 | 0.00 |
| time (sec) | N/A | 0.001 | 0.001 | 0.001 | 1.078 | 1.218 | 0.071 | 1.163 | 0.029 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 21 | 20 | 20 | 20 | 21 | 20 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.91 | 0.91 | 0.91 | 0.95 | 0.91 | 0.00 |
| time (sec) | N/A | 0.006 | 0.001 | 0.001 | 1.115 | 0.841 | 0.108 | 1.350 | 0.029 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 21 | 20 | 24 | 17 | 21 | 20 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.05 | 1.00 | 1.20 | 0.85 | 1.05 | 1.00 | 0.00 |
| time (sec) | N/A | 0.008 | 0.001 | 0.006 | 1.103 | 1.366 | 0.126 | 1.092 | 0.066 | 0.000 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 24 | 23 | 21 | 26 | 22 | 22 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.88 | 1.08 | 0.92 | 0.92 | 0.96 | 0.00 |
| time (sec) | N/A | 0.008 | 0.003 | 0.007 | 1.179 | 0.909 | 0.167 | 1.179 | 0.044 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 26 | 25 | 22 | 22 | 24 | 22 | 22 | 0 |
| N.S. | 1 | 1.00 | 1.53 | 1.47 | 1.29 | 1.29 | 1.41 | 1.29 | 1.29 | 0.00 |
| time (sec) | N/A | 0.002 | 0.006 | 0.006 | 1.354 | 1.294 | 0.178 | 1.479 | 0.035 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 25 | 24 | 24 | 26 | 24 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.80 | 0.80 | 0.87 | 0.80 | 0.80 | 0.00 |
| time (sec) | N/A | 0.008 | 0.003 | 0.005 | 1.344 | 0.885 | 0.187 | 1.116 | 0.035 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 25 | 24 | 24 | 26 | 24 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.80 | 0.80 | 0.87 | 0.80 | 0.80 | 0.00 |
| time (sec) | N/A | 0.008 | 0.007 | 0.005 | 1.329 | 1.434 | 0.190 | 1.191 | 0.035 | 0.000 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 25 | 24 | 24 | 26 | 24 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.80 | 0.80 | 0.87 | 0.80 | 0.80 | 0.00 |
| time (sec) | N/A | 0.008 | 0.003 | 0.005 | 1.347 | 0.719 | 0.198 | 1.069 | 0.034 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 25 | 24 | 24 | 26 | 24 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.80 | 0.80 | 0.87 | 0.80 | 0.80 | 0.00 |
| time (sec) | N/A | 0.008 | 0.006 | 0.006 | 1.351 | 1.363 | 0.214 | 1.199 | 0.036 | 0.000 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 43 | 36 | 35 | 35 | 37 | 35 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.81 | 0.81 | 0.86 | 0.81 | 0.81 | 0.00 |
| time (sec) | N/A | 0.018 | 0.002 | 0.002 | 1.320 | 1.074 | 0.077 | 1.218 | 0.042 | 0.000 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 43 | 36 | 35 | 35 | 37 | 35 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.81 | 0.81 | 0.86 | 0.81 | 0.81 | 0.00 |
| time (sec) | N/A | 0.016 | 0.002 | 0.001 | 1.354 | 0.686 | 0.074 | 1.272 | 0.041 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 43 | 36 | 35 | 35 | 39 | 35 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.81 | 0.81 | 0.91 | 0.81 | 0.81 | 0.00 |
| time (sec) | N/A | 0.015 | 0.002 | 0.000 | 1.337 | 0.951 | 0.072 | 1.221 | 0.039 | 0.000 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 40 | 35 | 34 | 34 | 36 | 34 | 34 | 0 |
| N.S. | 1 | 1.00 | 1.33 | 1.17 | 1.13 | 1.13 | 1.20 | 1.13 | 1.13 | 0.00 |
| time (sec) | N/A | 0.009 | 0.002 | 0.001 | 1.378 | 0.695 | 0.074 | 1.206 | 0.040 | 0.000 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 31 | 31 | 32 | 12 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 2.21 | 2.21 | 2.29 | 0.86 | 2.21 | 0.00 |
| time (sec) | N/A | 0.002 | 0.002 | 0.001 | 1.386 | 1.341 | 0.074 | 1.364 | 0.041 | 0.000 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 35 | 32 | 31 | 31 | 34 | 32 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.91 | 0.89 | 0.89 | 0.97 | 0.91 | 0.89 | 0.00 |
| time (sec) | N/A | 0.011 | 0.003 | 0.003 | 1.343 | 1.334 | 0.122 | 0.949 | 0.035 | 0.001 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 34 | 33 | 32 | 36 | 31 | 33 | 32 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.97 | 0.94 | 1.06 | 0.91 | 0.97 | 0.94 | 0.00 |
| time (sec) | N/A | 0.013 | 0.004 | 0.006 | 1.293 | 1.542 | 0.130 | 1.357 | 0.035 | 0.000 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 33 | 32 | 30 | 37 | 32 | 31 | 32 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.97 | 0.91 | 1.12 | 0.97 | 0.94 | 0.97 | 0.00 |
| time (sec) | N/A | 0.013 | 0.005 | 0.004 | 1.344 | 1.328 | 0.187 | 1.177 | 0.030 | 0.000 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 37 | 34 | 34 | 37 | 36 | 35 | 34 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.92 | 1.00 | 0.97 | 0.95 | 0.92 | 0.00 |
| time (sec) | N/A | 0.012 | 0.004 | 0.006 | 1.305 | 1.006 | 0.235 | 1.141 | 0.071 | 0.000 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 39 | 36 | 33 | 33 | 36 | 33 | 33 | 0 |
| N.S. | 1 | 1.00 | 2.29 | 2.12 | 1.94 | 1.94 | 2.12 | 1.94 | 1.94 | 0.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.006 | 1.334 | 1.297 | 0.256 | 1.286 | 0.026 | 0.000 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 41 | 36 | 35 | 35 | 37 | 35 | 34 | 0 |
| N.S. | 1 | 1.00 | 1.14 | 1.00 | 0.97 | 0.97 | 1.03 | 0.97 | 0.94 | 0.00 |
| time (sec) | N/A | 0.005 | 0.006 | 0.005 | 1.351 | 1.478 | 0.247 | 1.133 | 0.027 | 0.000 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 43 | 36 | 35 | 35 | 37 | 35 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.81 | 0.81 | 0.86 | 0.81 | 0.81 | 0.00 |
| time (sec) | N/A | 0.013 | 0.004 | 0.006 | 1.355 | 0.974 | 0.336 | 1.152 | 0.025 | 0.000 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 43 | 36 | 35 | 35 | 37 | 35 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.81 | 0.81 | 0.86 | 0.81 | 0.81 | 0.00 |
| time (sec) | N/A | 0.013 | 0.003 | 0.004 | 1.355 | 1.026 | 0.290 | 1.123 | 0.026 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 66 | 57 | 56 | 56 | 63 | 56 | 56 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.85 | 0.85 | 0.95 | 0.85 | 0.85 | 0.00 |
| time (sec) | N/A | 0.032 | 0.003 | 0.006 | 1.383 | 0.841 | 0.089 | 0.940 | 0.025 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 69 | 58 | 57 | 57 | 65 | 57 | 57 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.83 | 0.83 | 0.94 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.028 | 0.002 | 0.001 | 1.362 | 1.287 | 0.082 | 1.205 | 0.023 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 69 | 58 | 57 | 57 | 66 | 57 | 57 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.83 | 0.83 | 0.96 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.025 | 0.002 | 0.000 | 1.469 | 1.327 | 0.079 | 1.095 | 0.024 | 0.000 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 66 | 57 | 56 | 56 | 63 | 56 | 56 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 0.89 | 0.88 | 0.88 | 0.98 | 0.88 | 0.88 | 0.00 |
| time (sec) | N/A | 0.026 | 0.002 | 0.001 | 1.397 | 1.278 | 0.100 | 1.047 | 0.024 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 67 | 58 | 57 | 57 | 65 | 57 | 57 | 0 |
| N.S. | 1 | 1.00 | 1.43 | 1.23 | 1.21 | 1.21 | 1.38 | 1.21 | 1.21 | 0.00 |
| time (sec) | N/A | 0.021 | 0.002 | 0.002 | 1.322 | 1.406 | 0.077 | 1.715 | 0.024 | 0.000 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 67 | 58 | 57 | 57 | 65 | 57 | 57 | 0 |
| N.S. | 1 | 1.00 | 2.23 | 1.93 | 1.90 | 1.90 | 2.17 | 1.90 | 1.90 | 0.00 |
| time (sec) | N/A | 0.008 | 0.002 | 0.000 | 1.311 | 0.797 | 0.084 | 0.930 | 0.023 | 0.000 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 53 | 53 | 60 | 12 | 53 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 3.79 | 3.79 | 4.29 | 0.86 | 3.79 | 0.00 |
| time (sec) | N/A | 0.002 | 0.002 | 0.002 | 1.367 | 1.208 | 0.081 | 1.202 | 0.024 | 0.000 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 59 | 54 | 53 | 53 | 60 | 54 | 53 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.90 | 0.90 | 1.02 | 0.92 | 0.90 | 0.00 |
| time (sec) | N/A | 0.018 | 0.003 | 0.003 | 1.372 | 1.437 | 0.155 | 1.116 | 0.029 | 0.000 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 58 | 55 | 54 | 59 | 56 | 55 | 54 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.93 | 1.02 | 0.97 | 0.95 | 0.93 | 0.00 |
| time (sec) | N/A | 0.021 | 0.005 | 0.007 | 1.396 | 0.969 | 0.175 | 1.275 | 0.029 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 60 | 55 | 53 | 59 | 60 | 54 | 55 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.88 | 0.98 | 1.00 | 0.90 | 0.92 | 0.00 |
| time (sec) | N/A | 0.022 | 0.005 | 0.005 | 1.362 | 0.747 | 0.204 | 1.187 | 0.029 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 60 | 55 | 55 | 59 | 60 | 56 | 55 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.92 | 0.98 | 1.00 | 0.93 | 0.92 | 0.00 |
| time (sec) | N/A | 0.022 | 0.004 | 0.006 | 1.407 | 1.732 | 0.258 | 0.995 | 0.039 | 0.000 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 57 | 54 | 54 | 59 | 58 | 55 | 54 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.95 | 1.04 | 1.02 | 0.96 | 0.95 | 0.00 |
| time (sec) | N/A | 0.020 | 0.005 | 0.007 | 1.365 | 0.807 | 0.289 | 1.380 | 0.077 | 0.000 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 61 | 56 | 56 | 59 | 60 | 57 | 56 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.92 | 0.97 | 0.98 | 0.93 | 0.92 | 0.00 |
| time (sec) | N/A | 0.021 | 0.004 | 0.007 | 1.348 | 1.934 | 0.357 | 1.368 | 0.041 | 0.000 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 65 | 58 | 55 | 55 | 60 | 55 | 55 | 0 |
| N.S. | 1 | 1.00 | 3.82 | 3.41 | 3.24 | 3.24 | 3.53 | 3.24 | 3.24 | 0.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.004 | 1.299 | 1.136 | 0.371 | 0.941 | 0.039 | 0.000 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 67 | 58 | 57 | 57 | 61 | 57 | 57 | 0 |
| N.S. | 1 | 1.00 | 1.86 | 1.61 | 1.58 | 1.58 | 1.69 | 1.58 | 1.58 | 0.00 |
| time (sec) | N/A | 0.005 | 0.004 | 0.006 | 1.356 | 1.371 | 0.414 | 1.116 | 0.073 | 0.000 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 67 | 58 | 57 | 57 | 61 | 57 | 57 | 0 |
| N.S. | 1 | 1.00 | 1.20 | 1.04 | 1.02 | 1.02 | 1.09 | 1.02 | 1.02 | 0.00 |
| time (sec) | N/A | 0.010 | 0.004 | 0.006 | 1.395 | 1.767 | 0.431 | 1.100 | 0.039 | 0.000 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 67 | 58 | 57 | 57 | 61 | 57 | 56 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.85 | 0.85 | 0.91 | 0.85 | 0.84 | 0.00 |
| time (sec) | N/A | 0.022 | 0.006 | 0.003 | 1.333 | 1.101 | 0.450 | 1.207 | 0.084 | 0.000 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 69 | 58 | 57 | 57 | 61 | 57 | 57 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.83 | 0.83 | 0.88 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.021 | 0.004 | 0.007 | 1.374 | 0.979 | 0.490 | 1.003 | 0.082 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 69 | 58 | 57 | 57 | 61 | 57 | 57 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.83 | 0.83 | 0.88 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.021 | 0.004 | 0.006 | 1.314 | 0.835 | 0.582 | 1.381 | 0.042 | 0.000 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 67 | 58 | 57 | 57 | 61 | 57 | 56 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.85 | 0.85 | 0.91 | 0.85 | 0.84 | 0.00 |
| time (sec) | N/A | 0.021 | 0.004 | 0.006 | 1.364 | 0.730 | 0.616 | 1.233 | 0.040 | 0.000 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 67 | 58 | 57 | 57 | 61 | 57 | 56 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.85 | 0.85 | 0.91 | 0.85 | 0.84 | 0.00 |
| time (sec) | N/A | 0.022 | 0.004 | 0.004 | 1.378 | 1.185 | 0.625 | 1.116 | 0.038 | 0.000 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 95 | 80 | 79 | 79 | 94 | 79 | 79 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.83 | 0.83 | 0.99 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.048 | 0.003 | 0.001 | 1.400 | 1.286 | 0.098 | 0.952 | 0.153 | 0.000 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 95 | 80 | 79 | 79 | 92 | 79 | 79 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.83 | 0.83 | 0.97 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.040 | 0.002 | 0.001 | 1.299 | 1.343 | 0.093 | 0.992 | 0.074 | 0.000 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 95 | 80 | 79 | 79 | 94 | 79 | 79 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.83 | 0.83 | 0.99 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.039 | 0.002 | 0.001 | 1.292 | 1.325 | 0.107 | 1.009 | 0.068 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 96 | 92 | 79 | 78 | 78 | 90 | 78 | 78 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.82 | 0.81 | 0.81 | 0.94 | 0.81 | 0.81 | 0.00 |
| time (sec) | N/A | 0.040 | 0.002 | 0.000 | 1.389 | 1.344 | 0.106 | 1.083 | 0.060 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 93 | 80 | 79 | 79 | 92 | 79 | 79 | 0 |
| N.S. | 1 | 1.00 | 1.15 | 0.99 | 0.98 | 0.98 | 1.14 | 0.98 | 0.98 | 0.00 |
| time (sec) | N/A | 0.036 | 0.002 | 0.001 | 1.351 | 1.257 | 0.099 | 1.127 | 0.062 | 0.000 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 93 | 80 | 79 | 79 | 92 | 79 | 79 | 0 |
| N.S. | 1 | 1.00 | 1.45 | 1.25 | 1.23 | 1.23 | 1.44 | 1.23 | 1.23 | 0.00 |
| time (sec) | N/A | 0.030 | 0.002 | 0.001 | 1.341 | 1.327 | 0.089 | 1.221 | 0.104 | 0.000 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 93 | 80 | 79 | 79 | 92 | 79 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.98 | 1.70 | 1.68 | 1.68 | 1.96 | 1.68 | 0.66 | 0.00 |
| time (sec) | N/A | 0.024 | 0.002 | 0.001 | 1.327 | 1.111 | 0.093 | 0.911 | 0.121 | 0.000 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 91 | 80 | 79 | 79 | 90 | 79 | 25 | 0 |
| N.S. | 1 | 1.00 | 3.03 | 2.67 | 2.63 | 2.63 | 3.00 | 2.63 | 0.83 | 0.00 |
| time (sec) | N/A | 0.008 | 0.002 | 0.001 | 1.349 | 0.996 | 0.090 | 0.918 | 0.115 | 0.000 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 75 | 83 | 12 | 75 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 5.36 | 5.93 | 0.86 | 5.36 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.000 | 1.389 | 0.891 | 0.082 | 1.062 | 0.060 | 0.000 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 87 | 76 | 75 | 75 | 88 | 76 | 75 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.86 | 0.86 | 1.01 | 0.87 | 0.86 | 0.00 |
| time (sec) | N/A | 0.027 | 0.003 | 0.003 | 1.338 | 1.383 | 0.187 | 1.301 | 0.072 | 0.000 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 86 | 77 | 76 | 81 | 85 | 77 | 76 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.88 | 0.94 | 0.99 | 0.90 | 0.88 | 0.00 |
| time (sec) | N/A | 0.032 | 0.004 | 0.007 | 1.357 | 1.314 | 0.201 | 1.123 | 0.055 | 0.000 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 84 | 77 | 75 | 81 | 85 | 76 | 77 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.89 | 0.96 | 1.01 | 0.90 | 0.92 | 0.00 |
| time (sec) | N/A | 0.033 | 0.004 | 0.007 | 1.377 | 1.618 | 0.252 | 1.060 | 0.051 | 0.000 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 86 | 77 | 77 | 81 | 87 | 78 | 77 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.90 | 0.94 | 1.01 | 0.91 | 0.90 | 0.00 |
| time (sec) | N/A | 0.032 | 0.004 | 0.007 | 1.329 | 1.330 | 0.308 | 1.091 | 0.051 | 0.000 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 86 | 77 | 77 | 81 | 85 | 78 | 77 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.90 | 0.94 | 0.99 | 0.91 | 0.90 | 0.00 |
| time (sec) | N/A | 0.032 | 0.004 | 0.008 | 1.397 | 1.518 | 0.328 | 1.042 | 0.090 | 0.000 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 84 | 77 | 77 | 81 | 83 | 78 | 77 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.92 | 0.96 | 0.99 | 0.93 | 0.92 | 0.00 |
| time (sec) | N/A | 0.032 | 0.004 | 0.006 | 1.382 | 1.642 | 0.449 | 1.014 | 0.106 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 85 | 76 | 76 | 81 | 82 | 77 | 81 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 0.89 | 0.95 | 0.96 | 0.91 | 0.95 | 0.00 |
| time (sec) | N/A | 0.031 | 0.005 | 0.008 | 1.384 | 1.404 | 0.595 | 0.995 | 0.110 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 89 | 78 | 78 | 81 | 83 | 79 | 78 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.88 | 0.91 | 0.93 | 0.89 | 0.88 | 0.00 |
| time (sec) | N/A | 0.033 | 0.004 | 0.007 | 1.363 | 1.332 | 0.627 | 1.274 | 0.068 | 0.000 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 87 | 80 | 77 | 77 | 83 | 77 | 77 | 0 |
| N.S. | 1 | 1.00 | 5.12 | 4.71 | 4.53 | 4.53 | 4.88 | 4.53 | 4.53 | 0.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.005 | 1.295 | 1.407 | 0.604 | 0.923 | 0.067 | 0.000 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 91 | 80 | 79 | 79 | 85 | 79 | 23 | 0 |
| N.S. | 1 | 1.00 | 2.53 | 2.22 | 2.19 | 2.19 | 2.36 | 2.19 | 0.64 | 0.00 |
| time (sec) | N/A | 0.005 | 0.004 | 0.006 | 1.314 | 1.210 | 0.729 | 1.007 | 0.093 | 0.000 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 93 | 80 | 79 | 79 | 85 | 79 | 79 | 0 |
| N.S. | 1 | 1.00 | 1.66 | 1.43 | 1.41 | 1.41 | 1.52 | 1.41 | 1.41 | 0.00 |
| time (sec) | N/A | 0.010 | 0.004 | 0.005 | 1.382 | 1.323 | 0.762 | 1.064 | 0.109 | 0.000 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 93 | 80 | 79 | 79 | 85 | 79 | 79 | 0 |
| N.S. | 1 | 1.00 | 1.22 | 1.05 | 1.04 | 1.04 | 1.12 | 1.04 | 1.04 | 0.00 |
| time (sec) | N/A | 0.016 | 0.004 | 0.006 | 1.364 | 0.941 | 0.747 | 1.149 | 0.106 | 0.000 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 96 | 93 | 80 | 79 | 79 | 85 | 79 | 79 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.83 | 0.82 | 0.82 | 0.89 | 0.82 | 0.82 | 0.00 |
| time (sec) | N/A | 0.026 | 0.004 | 0.006 | 1.445 | 1.125 | 0.795 | 1.147 | 0.066 | 0.000 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 93 | 80 | 79 | 79 | 85 | 79 | 78 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.85 | 0.85 | 0.91 | 0.85 | 0.84 | 0.00 |
| time (sec) | N/A | 0.033 | 0.006 | 0.005 | 1.368 | 0.863 | 0.782 | 1.039 | 0.066 | 0.000 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 95 | 80 | 79 | 79 | 85 | 79 | 79 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.83 | 0.83 | 0.89 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.030 | 0.004 | 0.006 | 1.343 | 0.823 | 0.878 | 0.890 | 0.070 | 0.000 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 95 | 80 | 79 | 79 | 85 | 79 | 79 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.83 | 0.83 | 0.89 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.030 | 0.004 | 0.005 | 1.390 | 1.428 | 0.878 | 1.002 | 0.110 | 0.000 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 132 | 132 | 132 | 113 | 112 | 112 | 133 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.85 | 0.85 | 1.01 | 0.85 | 0.85 | 0.00 |
| time (sec) | N/A | 0.076 | 0.003 | 0.001 | 1.326 | 1.229 | 0.108 | 0.978 | 0.150 | 0.000 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 132 | 132 | 132 | 113 | 112 | 112 | 131 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.85 | 0.85 | 0.99 | 0.85 | 0.85 | 0.00 |
| time (sec) | N/A | 0.059 | 0.004 | 0.001 | 1.340 | 1.052 | 0.111 | 0.987 | 0.084 | 0.000 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 132 | 132 | 132 | 113 | 112 | 112 | 133 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.85 | 0.85 | 1.01 | 0.85 | 0.85 | 0.00 |
| time (sec) | N/A | 0.060 | 0.003 | 0.000 | 1.369 | 0.740 | 0.104 | 1.119 | 0.125 | 0.000 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 147 | 147 | 125 | 112 | 111 | 111 | 126 | 111 | 111 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.76 | 0.76 | 0.76 | 0.86 | 0.76 | 0.76 | 0.00 |
| time (sec) | N/A | 0.064 | 0.003 | 0.001 | 1.365 | 1.087 | 0.113 | 1.013 | 0.087 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 132 | 132 | 130 | 113 | 112 | 112 | 131 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.86 | 0.85 | 0.85 | 0.99 | 0.85 | 0.85 | 0.00 |
| time (sec) | N/A | 0.057 | 0.003 | 0.001 | 1.337 | 1.074 | 0.106 | 0.836 | 0.082 | 0.000 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 126 | 113 | 112 | 112 | 128 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 1.12 | 1.01 | 1.00 | 1.00 | 1.14 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.052 | 0.003 | 0.000 | 1.384 | 0.973 | 0.102 | 1.102 | 0.121 | 0.000 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 132 | 113 | 112 | 112 | 133 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 1.35 | 1.15 | 1.14 | 1.14 | 1.36 | 1.14 | 1.14 | 0.00 |
| time (sec) | N/A | 0.045 | 0.003 | 0.000 | 1.365 | 0.717 | 0.102 | 1.143 | 0.123 | 0.000 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 130 | 113 | 112 | 112 | 131 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 1.60 | 1.40 | 1.38 | 1.38 | 1.62 | 1.38 | 1.38 | 0.00 |
| time (sec) | N/A | 0.039 | 0.003 | 0.001 | 1.292 | 1.217 | 0.109 | 1.055 | 0.120 | 0.000 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 128 | 113 | 112 | 112 | 129 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 2.00 | 1.77 | 1.75 | 1.75 | 2.02 | 1.75 | 1.75 | 0.00 |
| time (sec) | N/A | 0.036 | 0.003 | 0.002 | 1.358 | 1.135 | 0.118 | 1.087 | 0.118 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 126 | 113 | 112 | 112 | 128 | 112 | 31 | 0 |
| N.S. | 1 | 1.00 | 2.68 | 2.40 | 2.38 | 2.38 | 2.72 | 2.38 | 0.66 | 0.00 |
| time (sec) | N/A | 0.030 | 0.003 | 0.002 | 1.374 | 1.120 | 0.107 | 1.178 | 0.070 | 0.000 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 128 | 113 | 112 | 112 | 129 | 112 | 25 | 0 |
| N.S. | 1 | 1.00 | 4.27 | 3.77 | 3.73 | 3.73 | 4.30 | 3.73 | 0.83 | 0.00 |
| time (sec) | N/A | 0.008 | 0.003 | 0.001 | 1.345 | 1.009 | 0.110 | 1.444 | 0.095 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 108 | 114 | 12 | 108 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 7.71 | 8.14 | 0.86 | 7.71 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.000 | 1.335 | 1.258 | 0.115 | 1.164 | 0.112 | 0.000 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 122 | 122 | 122 | 109 | 108 | 108 | 126 | 109 | 108 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 0.89 | 0.89 | 1.03 | 0.89 | 0.89 | 0.00 |
| time (sec) | N/A | 0.043 | 0.004 | 0.004 | 1.388 | 1.557 | 0.257 | 1.076 | 0.079 | 0.000 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 115 | 110 | 109 | 114 | 117 | 110 | 109 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.95 | 0.99 | 1.02 | 0.96 | 0.95 | 0.00 |
| time (sec) | N/A | 0.047 | 0.010 | 0.007 | 1.352 | 0.839 | 0.266 | 0.960 | 0.115 | 0.000 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 119 | 110 | 108 | 114 | 122 | 109 | 110 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.91 | 0.96 | 1.03 | 0.92 | 0.92 | 0.00 |
| time (sec) | N/A | 0.049 | 0.005 | 0.006 | 1.438 | 1.414 | 0.310 | 0.928 | 0.071 | 0.000 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 115 | 110 | 108 | 114 | 119 | 109 | 110 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.94 | 0.99 | 1.03 | 0.95 | 0.96 | 0.00 |
| time (sec) | N/A | 0.047 | 0.010 | 0.007 | 1.368 | 1.134 | 0.338 | 1.139 | 0.062 | 0.000 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 119 | 110 | 110 | 114 | 121 | 111 | 110 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.92 | 0.96 | 1.02 | 0.93 | 0.92 | 0.00 |
| time (sec) | N/A | 0.049 | 0.008 | 0.008 | 1.269 | 1.565 | 0.437 | 1.118 | 0.098 | 0.000 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 117 | 110 | 110 | 114 | 121 | 111 | 110 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.94 | 0.97 | 1.03 | 0.95 | 0.94 | 0.00 |
| time (sec) | N/A | 0.052 | 0.010 | 0.006 | 1.399 | 1.313 | 0.573 | 1.145 | 0.099 | 0.000 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 119 | 110 | 110 | 114 | 122 | 111 | 110 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.92 | 0.96 | 1.03 | 0.93 | 0.92 | 0.00 |
| time (sec) | N/A | 0.049 | 0.005 | 0.008 | 1.367 | 1.201 | 0.566 | 1.240 | 0.055 | 0.000 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 115 | 110 | 110 | 114 | 119 | 111 | 110 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.96 | 0.99 | 1.03 | 0.97 | 0.96 | 0.00 |
| time (sec) | N/A | 0.050 | 0.010 | 0.009 | 1.298 | 1.308 | 0.695 | 1.100 | 0.095 | 0.000 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 119 | 110 | 110 | 114 | 119 | 111 | 110 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.92 | 0.96 | 1.00 | 0.93 | 0.92 | 0.00 |
| time (sec) | N/A | 0.049 | 0.005 | 0.009 | 1.422 | 1.143 | 0.765 | 1.156 | 0.068 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 114 | 109 | 109 | 114 | 117 | 110 | 114 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.96 | 1.00 | 1.03 | 0.96 | 1.00 | 0.00 |
| time (sec) | N/A | 0.053 | 0.006 | 0.008 | 1.398 | 1.208 | 0.825 | 1.211 | 0.076 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 124 | 124 | 124 | 111 | 111 | 114 | 119 | 112 | 111 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.90 | 0.92 | 0.96 | 0.90 | 0.90 | 0.00 |
| time (sec) | N/A | 0.048 | 0.005 | 0.009 | 1.400 | 1.425 | 1.013 | 1.128 | 0.074 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 114 | 113 | 110 | 110 | 119 | 110 | 110 | 0 |
| N.S. | 1 | 1.00 | 6.71 | 6.65 | 6.47 | 6.47 | 7.00 | 6.47 | 6.47 | 0.00 |
| time (sec) | N/A | 0.002 | 0.010 | 0.006 | 1.335 | 1.265 | 1.026 | 1.064 | 0.134 | 0.000 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 128 | 113 | 112 | 112 | 121 | 112 | 23 | 0 |
| N.S. | 1 | 1.00 | 3.56 | 3.14 | 3.11 | 3.11 | 3.36 | 3.11 | 0.64 | 0.00 |
| time (sec) | N/A | 0.005 | 0.004 | 0.004 | 1.393 | 1.335 | 0.992 | 0.963 | 0.096 | 0.000 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 126 | 113 | 112 | 112 | 121 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 2.25 | 2.02 | 2.00 | 2.00 | 2.16 | 2.00 | 2.00 | 0.00 |
| time (sec) | N/A | 0.010 | 0.009 | 0.005 | 1.373 | 1.392 | 1.223 | 1.031 | 0.132 | 0.000 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 128 | 113 | 112 | 112 | 121 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 1.68 | 1.49 | 1.47 | 1.47 | 1.59 | 1.47 | 1.47 | 0.00 |
| time (sec) | N/A | 0.017 | 0.008 | 0.007 | 1.285 | 1.286 | 1.099 | 0.891 | 0.094 | 0.000 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 96 | 130 | 113 | 112 | 112 | 121 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 1.35 | 1.18 | 1.17 | 1.17 | 1.26 | 1.17 | 1.17 | 0.00 |
| time (sec) | N/A | 0.025 | 0.010 | 0.006 | 1.361 | 1.339 | 1.253 | 1.517 | 0.130 | 0.000 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 116 | 116 | 132 | 113 | 112 | 112 | 121 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 1.14 | 0.97 | 0.97 | 0.97 | 1.04 | 0.97 | 0.97 | 0.00 |
| time (sec) | N/A | 0.035 | 0.005 | 0.006 | 1.388 | 0.769 | 1.268 | 1.101 | 0.135 | 0.000 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 126 | 113 | 112 | 112 | 121 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.83 | 0.82 | 0.82 | 0.89 | 0.82 | 0.82 | 0.00 |
| time (sec) | N/A | 0.049 | 0.010 | 0.007 | 1.402 | 1.139 | 1.327 | 1.153 | 0.134 | 0.000 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 130 | 113 | 112 | 112 | 121 | 112 | 112 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.86 | 0.86 | 0.93 | 0.86 | 0.86 | 0.00 |
| time (sec) | N/A | 0.047 | 0.004 | 0.007 | 1.362 | 0.765 | 1.350 | 0.975 | 0.099 | 0.000 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 126 | 126 | 126 | 113 | 112 | 112 | 121 | 112 | 111 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.89 | 0.89 | 0.96 | 0.89 | 0.88 | 0.00 |
| time (sec) | N/A | 0.049 | 0.007 | 0.007 | 1.396 | 1.342 | 1.424 | 1.110 | 0.137 | 0.000 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 14 | 13 | 13 | 12 | 12 | 13 | 11 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.87 | 0.87 | 0.80 | 0.80 | 0.87 | 0.73 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.002 | 1.360 | 1.486 | 0.068 | 1.092 | 0.021 | 0.000 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 19 | 18 | 18 | 27 | 22 | 17 | 16 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 0.90 | 0.90 | 1.35 | 1.10 | 0.85 | 0.80 | 0.00 |
| time (sec) | N/A | 0.004 | 0.001 | 0.000 | 1.374 | 2.366 | 0.077 | 1.204 | 0.075 | 0.000 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 70 | 63 | 64 | 63 | 61 | 65 | 62 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.91 | 0.90 | 0.87 | 0.93 | 0.89 | 0.00 |
| time (sec) | N/A | 0.034 | 0.004 | 0.003 | 1.394 | 1.219 | 0.163 | 1.003 | 0.080 | 0.000 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 57 | 52 | 52 | 52 | 49 | 53 | 51 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.91 | 0.91 | 0.91 | 0.86 | 0.93 | 0.89 | 0.00 |
| time (sec) | N/A | 0.023 | 0.004 | 0.003 | 1.325 | 1.211 | 0.152 | 1.353 | 0.100 | 0.000 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 44 | 41 | 42 | 41 | 37 | 43 | 40 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.95 | 0.93 | 0.84 | 0.98 | 0.91 | 0.00 |
| time (sec) | N/A | 0.020 | 0.004 | 0.003 | 1.237 | 1.168 | 0.144 | 1.050 | 0.038 | 0.000 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 30 | 29 | 29 | 26 | 30 | 29 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.97 | 0.94 | 0.94 | 0.84 | 0.97 | 0.94 | 0.00 |
| time (sec) | N/A | 0.014 | 0.003 | 0.003 | 1.374 | 1.282 | 0.130 | 0.938 | 0.039 | 0.000 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 19 | 18 | 17 | 14 | 19 | 18 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.00 | 0.94 | 0.78 | 1.06 | 1.00 | 0.00 |
| time (sec) | N/A | 0.009 | 0.003 | 0.001 | 1.309 | 1.143 | 0.122 | 1.019 | 0.076 | 0.000 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 10 | 10 | 11 | 10 | 10 | 7 | 11 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.10 | 1.00 | 1.00 | 0.70 | 1.10 | 1.00 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.000 | 1.297 | 1.414 | 0.070 | 0.922 | 0.021 | 0.000 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 19 | 18 | 16 | 10 | 20 | 15 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.00 | 0.89 | 0.56 | 1.11 | 0.83 | 0.00 |
| time (sec) | N/A | 0.004 | 0.004 | 0.005 | 1.395 | 1.246 | 0.153 | 1.104 | 0.085 | 0.000 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 28 | 29 | 28 | 26 | 19 | 30 | 25 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.04 | 1.00 | 0.93 | 0.68 | 1.07 | 0.89 | 0.00 |
| time (sec) | N/A | 0.013 | 0.005 | 0.008 | 1.300 | 1.397 | 0.195 | 0.968 | 0.051 | 0.000 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 42 | 41 | 40 | 41 | 31 | 45 | 38 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.98 | 0.95 | 0.98 | 0.74 | 1.07 | 0.90 | 0.00 |
| time (sec) | N/A | 0.018 | 0.005 | 0.007 | 1.337 | 1.301 | 0.217 | 1.021 | 0.058 | 0.000 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 56 | 53 | 51 | 54 | 44 | 56 | 48 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.91 | 0.96 | 0.79 | 1.00 | 0.86 | 0.00 |
| time (sec) | N/A | 0.021 | 0.005 | 0.007 | 1.353 | 1.220 | 0.242 | 1.070 | 0.105 | 0.000 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 68 | 63 | 62 | 65 | 56 | 67 | 60 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.91 | 0.96 | 0.82 | 0.99 | 0.88 | 0.00 |
| time (sec) | N/A | 0.035 | 0.005 | 0.007 | 1.365 | 1.221 | 0.275 | 1.060 | 0.065 | 0.000 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 77 | 78 | 82 | 96 | 78 | 103 | 83 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 0.96 | 1.01 | 1.19 | 0.96 | 1.27 | 1.02 | 0.00 |
| time (sec) | N/A | 0.059 | 0.024 | 0.009 | 1.332 | 1.310 | 0.275 | 1.140 | 0.140 | 0.000 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 66 | 67 | 70 | 85 | 71 | 90 | 72 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.93 | 0.97 | 1.18 | 0.99 | 1.25 | 1.00 | 0.00 |
| time (sec) | N/A | 0.042 | 0.017 | 0.007 | 1.327 | 1.171 | 0.252 | 1.222 | 0.070 | 0.000 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 54 | 57 | 59 | 73 | 54 | 79 | 62 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.98 | 1.02 | 1.26 | 0.93 | 1.36 | 1.07 | 0.00 |
| time (sec) | N/A | 0.033 | 0.019 | 0.009 | 1.367 | 1.205 | 0.215 | 1.071 | 0.067 | 0.000 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 43 | 45 | 47 | 62 | 44 | 66 | 50 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.98 | 1.02 | 1.35 | 0.96 | 1.43 | 1.09 | 0.00 |
| time (sec) | N/A | 0.027 | 0.013 | 0.006 | 1.357 | 1.216 | 0.204 | 1.139 | 0.079 | 0.000 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 29 | 34 | 36 | 47 | 31 | 50 | 36 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.03 | 1.09 | 1.42 | 0.94 | 1.52 | 1.09 | 0.00 |
| time (sec) | N/A | 0.018 | 0.013 | 0.006 | 1.341 | 1.277 | 0.173 | 1.147 | 0.081 | 0.000 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 20 | 24 | 26 | 28 | 20 | 42 | 23 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 1.04 | 1.13 | 1.22 | 0.87 | 1.83 | 1.00 | 0.00 |
| time (sec) | N/A | 0.012 | 0.007 | 0.007 | 1.320 | 1.218 | 0.169 | 1.049 | 0.036 | 0.000 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 12 | 13 | 12 | 13 | 10 | 12 | 12 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.08 | 0.83 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.002 | 0.002 | 0.000 | 1.376 | 1.177 | 0.149 | 1.175 | 0.029 | 0.000 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 24 | 30 | 28 | 39 | 22 | 38 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.03 | 0.97 | 1.34 | 0.76 | 1.31 | 0.90 | 0.00 |
| time (sec) | N/A | 0.014 | 0.012 | 0.008 | 1.302 | 1.285 | 0.222 | 1.000 | 0.122 | 0.000 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 35 | 43 | 45 | 63 | 37 | 52 | 45 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.02 | 1.07 | 1.50 | 0.88 | 1.24 | 1.07 | 0.00 |
| time (sec) | N/A | 0.021 | 0.039 | 0.008 | 1.393 | 1.108 | 0.303 | 1.217 | 0.120 | 0.000 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 53 | 57 | 64 | 86 | 54 | 74 | 57 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.98 | 1.10 | 1.48 | 0.93 | 1.28 | 0.98 | 0.00 |
| time (sec) | N/A | 0.028 | 0.050 | 0.010 | 1.405 | 0.554 | 0.312 | 1.130 | 0.110 | 0.001 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 66 | 68 | 73 | 95 | 66 | 90 | 69 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.99 | 1.06 | 1.38 | 0.96 | 1.30 | 1.00 | 0.00 |
| time (sec) | N/A | 0.035 | 0.056 | 0.009 | 1.410 | 0.933 | 0.336 | 1.015 | 0.080 | 0.000 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 79 | 79 | 86 | 108 | 80 | 104 | 79 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.94 | 1.02 | 1.29 | 0.95 | 1.24 | 0.94 | 0.00 |
| time (sec) | N/A | 0.043 | 0.043 | 0.012 | 1.340 | 1.179 | 0.394 | 0.988 | 0.116 | 0.000 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 99 | 99 | 89 | 94 | 103 | 129 | 109 | 95 | 91 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.95 | 1.04 | 1.30 | 1.10 | 0.96 | 0.92 | 0.00 |
| time (sec) | N/A | 0.070 | 0.027 | 0.008 | 1.386 | 1.038 | 0.532 | 0.891 | 0.234 | 0.000 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 77 | 83 | 91 | 117 | 92 | 83 | 78 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.97 | 1.06 | 1.36 | 1.07 | 0.97 | 0.91 | 0.00 |
| time (sec) | N/A | 0.053 | 0.025 | 0.007 | 1.395 | 1.112 | 0.405 | 1.153 | 0.158 | 0.000 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 77 | 77 | 67 | 72 | 81 | 107 | 85 | 73 | 67 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.94 | 1.05 | 1.39 | 1.10 | 0.95 | 0.87 | 0.00 |
| time (sec) | N/A | 0.046 | 0.023 | 0.007 | 1.357 | 1.117 | 0.359 | 1.120 | 0.124 | 0.001 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 55 | 61 | 69 | 95 | 70 | 61 | 54 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.95 | 1.08 | 1.48 | 1.09 | 0.95 | 0.84 | 0.00 |
| time (sec) | N/A | 0.036 | 0.018 | 0.007 | 1.332 | 0.884 | 0.339 | 0.947 | 0.078 | 0.000 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 40 | 49 | 57 | 83 | 58 | 44 | 43 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 0.98 | 1.14 | 1.66 | 1.16 | 0.88 | 0.86 | 0.00 |
| time (sec) | N/A | 0.026 | 0.040 | 0.007 | 1.315 | 0.838 | 0.310 | 0.964 | 0.146 | 0.000 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 33 | 40 | 48 | 61 | 46 | 37 | 46 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 0.98 | 1.17 | 1.49 | 1.12 | 0.90 | 1.12 | 0.00 |
| time (sec) | N/A | 0.020 | 0.013 | 0.006 | 1.341 | 0.655 | 0.250 | 1.080 | 0.093 | 0.000 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 20 | 27 | 32 | 32 | 32 | 18 | 32 | 0 |
| N.S. | 1 | 1.00 | 1.18 | 1.59 | 1.88 | 1.88 | 1.88 | 1.06 | 1.88 | 0.00 |
| time (sec) | N/A | 0.002 | 0.005 | 0.003 | 1.392 | 1.208 | 0.200 | 1.142 | 0.072 | 0.000 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 24 | 26 | 12 | 26 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 1.71 | 1.86 | 0.86 | 1.86 | 0.00 |
| time (sec) | N/A | 0.001 | 0.002 | 0.000 | 1.287 | 1.013 | 0.209 | 0.948 | 0.068 | 0.000 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 37 | 42 | 51 | 80 | 46 | 43 | 43 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.98 | 1.19 | 1.86 | 1.07 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.019 | 0.029 | 0.008 | 1.349 | 0.868 | 0.349 | 1.031 | 0.100 | 0.000 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 53 | 56 | 69 | 109 | 66 | 60 | 63 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.98 | 1.21 | 1.91 | 1.16 | 1.05 | 1.11 | 0.00 |
| time (sec) | N/A | 0.029 | 0.049 | 0.009 | 1.318 | 1.146 | 0.404 | 1.074 | 0.114 | 0.000 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 68 | 73 | 86 | 130 | 78 | 73 | 79 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.96 | 1.13 | 1.71 | 1.03 | 0.96 | 1.04 | 0.00 |
| time (sec) | N/A | 0.036 | 0.052 | 0.010 | 1.371 | 1.182 | 0.406 | 1.386 | 0.119 | 0.000 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 79 | 84 | 97 | 141 | 92 | 86 | 91 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.94 | 1.09 | 1.58 | 1.03 | 0.97 | 1.02 | 0.00 |
| time (sec) | N/A | 0.048 | 0.068 | 0.012 | 1.456 | 1.096 | 0.480 | 0.938 | 0.128 | 0.000 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 90 | 94 | 108 | 152 | 102 | 97 | 101 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.97 | 1.11 | 1.57 | 1.05 | 1.00 | 1.04 | 0.00 |
| time (sec) | N/A | 0.052 | 0.057 | 0.012 | 1.348 | 0.946 | 0.476 | 1.008 | 0.092 | 0.001 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 101 | 109 | 125 | 162 | 131 | 106 | 103 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.96 | 1.10 | 1.42 | 1.15 | 0.93 | 0.90 | 0.00 |
| time (sec) | N/A | 0.085 | 0.036 | 0.008 | 1.435 | 0.863 | 0.524 | 1.110 | 0.370 | 0.000 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 105 | 105 | 90 | 98 | 114 | 151 | 119 | 95 | 90 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.93 | 1.09 | 1.44 | 1.13 | 0.90 | 0.86 | 0.00 |
| time (sec) | N/A | 0.070 | 0.028 | 0.008 | 1.395 | 1.040 | 0.483 | 1.013 | 0.221 | 0.000 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 90 | 87 | 102 | 139 | 107 | 83 | 79 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.97 | 1.13 | 1.54 | 1.19 | 0.92 | 0.88 | 0.00 |
| time (sec) | N/A | 0.057 | 0.022 | 0.009 | 1.422 | 0.973 | 0.484 | 1.110 | 0.153 | 0.000 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 68 | 76 | 91 | 129 | 94 | 72 | 66 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.94 | 1.12 | 1.59 | 1.16 | 0.89 | 0.81 | 0.00 |
| time (sec) | N/A | 0.048 | 0.024 | 0.008 | 1.458 | 0.792 | 0.461 | 0.873 | 0.125 | 0.000 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 51 | 64 | 79 | 116 | 82 | 55 | 55 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 0.98 | 1.22 | 1.78 | 1.26 | 0.85 | 0.85 | 0.00 |
| time (sec) | N/A | 0.036 | 0.045 | 0.008 | 1.405 | 0.880 | 0.398 | 0.864 | 0.173 | 0.000 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 44 | 55 | 70 | 94 | 70 | 46 | 45 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 0.95 | 1.21 | 1.62 | 1.21 | 0.79 | 0.78 | 0.00 |
| time (sec) | N/A | 0.030 | 0.016 | 0.006 | 1.391 | 1.104 | 0.306 | 1.032 | 0.070 | 0.000 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 31 | 41 | 54 | 54 | 56 | 29 | 56 | 0 |
| N.S. | 1 | 1.00 | 1.82 | 2.41 | 3.18 | 3.18 | 3.29 | 1.71 | 3.29 | 0.00 |
| time (sec) | N/A | 0.002 | 0.011 | 0.005 | 1.405 | 1.042 | 0.297 | 1.269 | 0.085 | 0.000 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 20 | 27 | 43 | 43 | 44 | 18 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.67 | 0.90 | 1.43 | 1.43 | 1.47 | 0.60 | 1.47 | 0.00 |
| time (sec) | N/A | 0.013 | 0.005 | 0.005 | 1.374 | 0.772 | 0.317 | 1.008 | 0.072 | 0.000 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 35 | 37 | 12 | 37 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 2.50 | 2.64 | 0.86 | 2.64 | 0.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.002 | 1.379 | 1.139 | 0.265 | 1.021 | 0.077 | 0.000 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 48 | 54 | 73 | 124 | 70 | 54 | 60 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.95 | 1.28 | 2.18 | 1.23 | 0.95 | 1.05 | 0.00 |
| time (sec) | N/A | 0.028 | 0.034 | 0.007 | 1.466 | 1.071 | 0.444 | 1.024 | 0.128 | 0.000 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 64 | 69 | 91 | 153 | 90 | 71 | 85 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.99 | 1.30 | 2.19 | 1.29 | 1.01 | 1.21 | 0.00 |
| time (sec) | N/A | 0.039 | 0.059 | 0.009 | 1.401 | 1.099 | 0.460 | 1.160 | 0.084 | 0.000 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 79 | 88 | 108 | 174 | 104 | 86 | 101 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.95 | 1.16 | 1.87 | 1.12 | 0.92 | 1.09 | 0.00 |
| time (sec) | N/A | 0.048 | 0.057 | 0.012 | 1.413 | 0.847 | 0.557 | 0.953 | 0.136 | 0.000 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 102 | 102 | 88 | 99 | 117 | 183 | 114 | 93 | 113 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.97 | 1.15 | 1.79 | 1.12 | 0.91 | 1.11 | 0.00 |
| time (sec) | N/A | 0.053 | 0.055 | 0.011 | 1.405 | 0.743 | 0.529 | 0.987 | 0.104 | 0.000 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 101 | 110 | 130 | 196 | 128 | 108 | 123 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.94 | 1.11 | 1.68 | 1.09 | 0.92 | 1.05 | 0.00 |
| time (sec) | N/A | 0.070 | 0.068 | 0.011 | 1.390 | 1.132 | 0.582 | 1.000 | 0.173 | 0.000 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 150 | 150 | 139 | 143 | 180 | 250 | 190 | 128 | 126 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.95 | 1.20 | 1.67 | 1.27 | 0.85 | 0.84 | 0.00 |
| time (sec) | N/A | 0.135 | 0.027 | 0.011 | 1.472 | 1.033 | 0.931 | 1.207 | 1.086 | 0.000 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 139 | 139 | 128 | 132 | 169 | 239 | 180 | 117 | 115 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.95 | 1.22 | 1.72 | 1.29 | 0.84 | 0.83 | 0.00 |
| time (sec) | N/A | 0.114 | 0.032 | 0.010 | 1.587 | 1.117 | 0.914 | 1.475 | 0.553 | 0.000 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 128 | 128 | 104 | 121 | 157 | 228 | 165 | 105 | 102 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 0.95 | 1.23 | 1.78 | 1.29 | 0.82 | 0.80 | 0.00 |
| time (sec) | N/A | 0.089 | 0.048 | 0.009 | 1.564 | 0.928 | 0.843 | 1.069 | 0.179 | 0.000 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 118 | 118 | 104 | 109 | 145 | 215 | 153 | 88 | 91 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.92 | 1.23 | 1.82 | 1.30 | 0.75 | 0.77 | 0.00 |
| time (sec) | N/A | 0.071 | 0.029 | 0.009 | 1.490 | 1.033 | 0.820 | 0.916 | 0.336 | 0.000 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 77 | 100 | 136 | 193 | 141 | 79 | 81 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.92 | 1.25 | 1.77 | 1.29 | 0.72 | 0.74 | 0.00 |
| time (sec) | N/A | 0.064 | 0.024 | 0.007 | 1.448 | 1.078 | 0.642 | 1.164 | 0.106 | 0.000 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 64 | 87 | 120 | 120 | 128 | 62 | 72 | 0 |
| N.S. | 1 | 1.00 | 3.76 | 5.12 | 7.06 | 7.06 | 7.53 | 3.65 | 4.24 | 0.00 |
| time (sec) | N/A | 0.002 | 0.011 | 0.005 | 1.434 | 0.949 | 0.583 | 1.083 | 0.122 | 0.000 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 53 | 72 | 109 | 109 | 116 | 51 | 22 | 0 |
| N.S. | 1 | 1.00 | 1.51 | 2.06 | 3.11 | 3.11 | 3.31 | 1.46 | 0.63 | 0.00 |
| time (sec) | N/A | 0.005 | 0.010 | 0.006 | 1.452 | 0.862 | 0.573 | 1.143 | 0.072 | 0.001 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 64 | 42 | 57 | 98 | 98 | 104 | 40 | 48 | 0 |
| N.S. | 1 | 1.23 | 0.81 | 1.10 | 1.88 | 1.88 | 2.00 | 0.77 | 0.92 | 0.00 |
| time (sec) | N/A | 0.029 | 0.010 | 0.005 | 1.401 | 1.120 | 0.559 | 1.001 | 0.068 | 0.000 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 31 | 42 | 87 | 87 | 92 | 29 | 31 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 0.89 | 1.85 | 1.85 | 1.96 | 0.62 | 0.66 | 0.00 |
| time (sec) | N/A | 0.021 | 0.008 | 0.004 | 1.451 | 0.885 | 0.547 | 0.997 | 0.080 | 0.000 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 20 | 27 | 76 | 76 | 80 | 18 | 18 | 0 |
| N.S. | 1 | 1.00 | 0.67 | 0.90 | 2.53 | 2.53 | 2.67 | 0.60 | 0.60 | 0.00 |
| time (sec) | N/A | 0.013 | 0.006 | 0.007 | 1.373 | 0.882 | 0.503 | 1.392 | 0.100 | 0.000 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 68 | 73 | 12 | 70 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 4.86 | 5.21 | 0.86 | 5.00 | 0.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.001 | 1.411 | 1.198 | 0.464 | 1.091 | 0.064 | 0.000 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 99 | 99 | 81 | 90 | 139 | 256 | 141 | 87 | 102 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.91 | 1.40 | 2.59 | 1.42 | 0.88 | 1.03 | 0.00 |
| time (sec) | N/A | 0.049 | 0.063 | 0.010 | 1.512 | 0.655 | 0.681 | 1.014 | 0.455 | 0.000 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 97 | 108 | 157 | 285 | 162 | 104 | 151 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.92 | 1.34 | 2.44 | 1.38 | 0.89 | 1.29 | 0.00 |
| time (sec) | N/A | 0.073 | 0.094 | 0.010 | 1.590 | 1.322 | 0.797 | 1.048 | 0.186 | 0.000 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 144 | 144 | 112 | 133 | 174 | 306 | 175 | 119 | 167 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 0.92 | 1.21 | 2.12 | 1.22 | 0.83 | 1.16 | 0.00 |
| time (sec) | N/A | 0.089 | 0.073 | 0.012 | 1.548 | 1.326 | 0.843 | 1.273 | 0.209 | 0.000 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 157 | 157 | 123 | 144 | 185 | 317 | 187 | 130 | 179 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 0.92 | 1.18 | 2.02 | 1.19 | 0.83 | 1.14 | 0.00 |
| time (sec) | N/A | 0.103 | 0.092 | 0.013 | 1.592 | 1.289 | 0.992 | 1.254 | 0.311 | 0.000 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 186 | 186 | 161 | 177 | 234 | 338 | 250 | 149 | 151 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.95 | 1.26 | 1.82 | 1.34 | 0.80 | 0.81 | 0.00 |
| time (sec) | N/A | 0.177 | 0.047 | 0.012 | 1.730 | 1.199 | 1.545 | 1.069 | 0.979 | 0.000 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 177 | 177 | 150 | 166 | 223 | 327 | 236 | 138 | 138 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.94 | 1.26 | 1.85 | 1.33 | 0.78 | 0.78 | 0.00 |
| time (sec) | N/A | 0.142 | 0.029 | 0.010 | 1.651 | 0.681 | 1.477 | 1.696 | 0.227 | 0.000 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 159 | 159 | 137 | 154 | 211 | 314 | 224 | 121 | 127 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.97 | 1.33 | 1.97 | 1.41 | 0.76 | 0.80 | 0.00 |
| time (sec) | N/A | 0.121 | 0.033 | 0.011 | 1.660 | 1.417 | 1.326 | 1.287 | 0.943 | 0.000 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 154 | 154 | 111 | 145 | 202 | 292 | 212 | 112 | 117 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 0.94 | 1.31 | 1.90 | 1.38 | 0.73 | 0.76 | 0.00 |
| time (sec) | N/A | 0.108 | 0.034 | 0.007 | 1.601 | 1.211 | 1.105 | 1.216 | 0.187 | 0.000 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 97 | 131 | 186 | 186 | 199 | 95 | 107 | 0 |
| N.S. | 1 | 1.00 | 5.71 | 7.71 | 10.94 | 10.94 | 11.71 | 5.59 | 6.29 | 0.00 |
| time (sec) | N/A | 0.002 | 0.018 | 0.006 | 1.509 | 1.078 | 0.980 | 1.231 | 0.141 | 0.000 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 86 | 117 | 175 | 175 | 187 | 84 | 22 | 0 |
| N.S. | 1 | 1.00 | 2.46 | 3.34 | 5.00 | 5.00 | 5.34 | 2.40 | 0.63 | 0.00 |
| time (sec) | N/A | 0.005 | 0.014 | 0.006 | 1.592 | 0.880 | 0.999 | 0.962 | 0.126 | 0.000 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 75 | 102 | 164 | 164 | 175 | 73 | 85 | 0 |
| N.S. | 1 | 1.00 | 1.44 | 1.96 | 3.15 | 3.15 | 3.37 | 1.40 | 1.63 | 0.00 |
| time (sec) | N/A | 0.010 | 0.016 | 0.007 | 1.493 | 0.867 | 0.913 | 1.013 | 0.140 | 0.000 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 64 | 86 | 153 | 153 | 163 | 62 | 71 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.25 | 2.22 | 2.22 | 2.36 | 0.90 | 1.03 | 0.00 |
| time (sec) | N/A | 0.016 | 0.015 | 0.004 | 1.538 | 0.976 | 0.840 | 0.899 | 0.078 | 0.000 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 53 | 72 | 142 | 142 | 151 | 51 | 61 | 0 |
| N.S. | 1 | 1.00 | 0.65 | 0.89 | 1.75 | 1.75 | 1.86 | 0.63 | 0.75 | 0.00 |
| time (sec) | N/A | 0.040 | 0.015 | 0.006 | 1.455 | 1.134 | 0.794 | 1.219 | 0.077 | 0.000 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 42 | 57 | 131 | 131 | 139 | 40 | 48 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 0.89 | 2.05 | 2.05 | 2.17 | 0.62 | 0.75 | 0.00 |
| time (sec) | N/A | 0.030 | 0.010 | 0.005 | 1.475 | 0.806 | 0.698 | 1.125 | 0.127 | 0.000 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 31 | 42 | 120 | 120 | 128 | 29 | 31 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 0.89 | 2.55 | 2.55 | 2.72 | 0.62 | 0.66 | 0.00 |
| time (sec) | N/A | 0.021 | 0.011 | 0.005 | 1.416 | 1.174 | 0.679 | 1.043 | 0.150 | 0.000 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 20 | 27 | 109 | 109 | 116 | 18 | 18 | 0 |
| N.S. | 1 | 1.00 | 0.67 | 0.90 | 3.63 | 3.63 | 3.87 | 0.60 | 0.60 | 0.00 |
| time (sec) | N/A | 0.014 | 0.006 | 0.004 | 1.421 | 1.043 | 0.723 | 1.045 | 0.067 | 0.000 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 101 | 109 | 12 | 103 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 7.21 | 7.79 | 0.86 | 7.36 | 0.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.001 | 1.336 | 1.102 | 0.671 | 1.034 | 0.141 | 0.000 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 141 | 141 | 127 | 126 | 205 | 388 | 212 | 120 | 145 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.89 | 1.45 | 2.75 | 1.50 | 0.85 | 1.03 | 0.00 |
| time (sec) | N/A | 0.075 | 0.102 | 0.012 | 1.755 | 1.076 | 1.023 | 1.048 | 0.762 | 0.000 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 158 | 158 | 130 | 147 | 223 | 417 | 233 | 137 | 217 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.93 | 1.41 | 2.64 | 1.47 | 0.87 | 1.37 | 0.00 |
| time (sec) | N/A | 0.125 | 0.127 | 0.014 | 1.705 | 1.173 | 1.148 | 1.014 | 0.395 | 0.000 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 191 | 191 | 145 | 178 | 240 | 438 | 246 | 152 | 233 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 0.93 | 1.26 | 2.29 | 1.29 | 0.80 | 1.22 | 0.00 |
| time (sec) | N/A | 0.141 | 0.111 | 0.015 | 1.712 | 0.610 | 1.176 | 1.102 | 0.438 | 0.000 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 198 | 198 | 156 | 189 | 251 | 449 | 258 | 163 | 245 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.95 | 1.27 | 2.27 | 1.30 | 0.82 | 1.24 | 0.00 |
| time (sec) | N/A | 0.165 | 0.124 | 0.013 | 1.720 | 1.257 | 1.241 | 1.349 | 0.586 | 0.000 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 141 | 141 | 141 | 132 | 132 | 136 | 143 | 133 | 132 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.94 | 0.96 | 1.01 | 0.94 | 0.94 | 0.00 |
| time (sec) | N/A | 0.079 | 0.011 | 0.010 | 1.372 | 1.036 | 0.914 | 1.139 | 0.081 | 0.000 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 132 | 132 | 132 | 121 | 121 | 125 | 131 | 122 | 121 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.92 | 0.95 | 0.99 | 0.92 | 0.92 | 0.00 |
| time (sec) | N/A | 0.070 | 0.005 | 0.010 | 1.373 | 1.076 | 0.848 | 1.277 | 0.092 | 0.000 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 114 | 109 | 109 | 114 | 117 | 110 | 114 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.96 | 1.00 | 1.03 | 0.96 | 1.00 | 0.00 |
| time (sec) | N/A | 0.058 | 0.006 | 0.000 | 1.432 | 1.046 | 0.865 | 1.056 | 0.002 | 0.000 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 109 | 100 | 100 | 103 | 107 | 101 | 100 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.92 | 0.94 | 0.98 | 0.93 | 0.92 | 0.00 |
| time (sec) | N/A | 0.052 | 0.005 | 0.008 | 1.335 | 0.948 | 0.786 | 1.040 | 0.082 | 0.000 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 96 | 91 | 88 | 88 | 95 | 88 | 88 | 0 |
| N.S. | 1 | 1.00 | 5.65 | 5.35 | 5.18 | 5.18 | 5.59 | 5.18 | 5.18 | 0.00 |
| time (sec) | N/A | 0.002 | 0.009 | 0.008 | 1.337 | 0.777 | 0.727 | 1.168 | 0.090 | 0.000 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 91 | 80 | 79 | 79 | 85 | 79 | 23 | 0 |
| N.S. | 1 | 1.00 | 2.53 | 2.22 | 2.19 | 2.19 | 2.36 | 2.19 | 0.64 | 0.00 |
| time (sec) | N/A | 0.006 | 0.004 | 0.000 | 1.357 | 0.864 | 0.705 | 1.037 | 0.002 | 0.000 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 80 | 69 | 68 | 68 | 73 | 68 | 68 | 0 |
| N.S. | 1 | 1.00 | 1.43 | 1.23 | 1.21 | 1.21 | 1.30 | 1.21 | 1.21 | 0.00 |
| time (sec) | N/A | 0.010 | 0.009 | 0.007 | 1.368 | 1.132 | 0.563 | 1.018 | 0.098 | 0.000 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 67 | 58 | 57 | 57 | 61 | 57 | 56 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.87 | 0.85 | 0.85 | 0.91 | 0.85 | 0.84 | 0.00 |
| time (sec) | N/A | 0.025 | 0.007 | 0.000 | 1.362 | 1.112 | 0.493 | 1.011 | 0.002 | 0.000 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 56 | 47 | 46 | 46 | 49 | 46 | 46 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.82 | 0.82 | 0.88 | 0.82 | 0.82 | 0.00 |
| time (sec) | N/A | 0.019 | 0.008 | 0.006 | 1.335 | 0.808 | 0.497 | 1.246 | 0.035 | 0.000 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 43 | 36 | 35 | 35 | 37 | 35 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.81 | 0.81 | 0.86 | 0.81 | 0.81 | 0.00 |
| time (sec) | N/A | 0.014 | 0.004 | 0.004 | 1.346 | 0.992 | 0.379 | 1.086 | 0.032 | 0.000 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 25 | 24 | 24 | 26 | 24 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.80 | 0.80 | 0.87 | 0.80 | 0.80 | 0.00 |
| time (sec) | N/A | 0.009 | 0.007 | 0.005 | 1.341 | 1.086 | 0.270 | 0.873 | 0.036 | 0.000 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 14 | 13 | 13 | 14 | 13 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.76 | 0.76 | 0.82 | 0.76 | 0.76 | 0.00 |
| time (sec) | N/A | 0.005 | 0.002 | 0.005 | 1.303 | 0.834 | 0.210 | 0.995 | 0.029 | 0.000 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 7 | 7 | 7 | 6 | 5 | 5 | 7 | 5 | 5 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.71 | 0.71 | 1.00 | 0.71 | 0.71 | 0.00 |
| time (sec) | N/A | 0.000 | 0.000 | 0.002 | 1.318 | 0.942 | 0.074 | 1.044 | 0.020 | 0.000 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 134 | 134 | 134 | 119 | 117 | 120 | 116 | 122 | 114 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 0.87 | 0.90 | 0.87 | 0.91 | 0.85 | 0.00 |
| time (sec) | N/A | 0.061 | 0.006 | 0.010 | 1.349 | 0.865 | 0.410 | 1.161 | 0.127 | 0.000 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 146 | 146 | 134 | 135 | 141 | 163 | 139 | 180 | 135 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.92 | 0.97 | 1.12 | 0.95 | 1.23 | 0.92 | 0.00 |
| time (sec) | N/A | 0.090 | 0.094 | 0.011 | 1.398 | 0.741 | 0.613 | 1.284 | 0.083 | 0.000 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 163 | 163 | 145 | 150 | 163 | 207 | 163 | 152 | 157 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.92 | 1.00 | 1.27 | 1.00 | 0.93 | 0.96 | 0.00 |
| time (sec) | N/A | 0.108 | 0.108 | 0.014 | 1.432 | 1.100 | 0.678 | 1.128 | 0.228 | 0.000 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 14 | 13 | 13 | 12 | 15 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.76 | 0.76 | 0.71 | 0.88 | 0.59 | 0.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.006 | 1.342 | 0.801 | 0.117 | 1.147 | 0.170 | 0.000 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 14 | 13 | 13 | 12 | 15 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.76 | 0.76 | 0.71 | 0.88 | 0.59 | 0.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.001 | 1.295 | 1.159 | 0.120 | 1.006 | 0.143 | 0.000 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 24 | 19 | 18 | 21 | 20 | 20 | 18 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 0.75 | 0.88 | 0.83 | 0.83 | 0.75 | 0.00 |
| time (sec) | N/A | 0.008 | 0.003 | 0.007 | 1.381 | 0.591 | 0.139 | 1.013 | 0.054 | 0.000 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 24 | 23 | 28 | 26 | 25 | 18 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.74 | 0.90 | 0.84 | 0.81 | 0.58 | 0.00 |
| time (sec) | N/A | 0.010 | 0.003 | 0.008 | 1.294 | 0.953 | 0.147 | 1.072 | 0.037 | 0.000 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 38 | 29 | 28 | 33 | 31 | 30 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.76 | 0.74 | 0.87 | 0.82 | 0.79 | 0.63 | 0.00 |
| time (sec) | N/A | 0.012 | 0.003 | 0.006 | 1.353 | 0.570 | 0.162 | 1.028 | 0.088 | 0.000 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 45 | 34 | 33 | 38 | 36 | 35 | 28 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.76 | 0.73 | 0.84 | 0.80 | 0.78 | 0.62 | 0.00 |
| time (sec) | N/A | 0.012 | 0.003 | 0.006 | 1.325 | 0.743 | 0.166 | 1.140 | 0.040 | 0.000 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 26 | 23 | 22 | 32 | 19 | 25 | 20 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.82 | 0.79 | 1.14 | 0.68 | 0.89 | 0.71 | 0.00 |
| time (sec) | N/A | 0.009 | 0.013 | 0.007 | 1.367 | 0.854 | 0.136 | 1.114 | 0.055 | 0.000 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 31 | 28 | 31 | 48 | 31 | 40 | 34 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.80 | 0.89 | 1.37 | 0.89 | 1.14 | 0.97 | 0.00 |
| time (sec) | N/A | 0.010 | 0.013 | 0.008 | 1.387 | 1.191 | 0.148 | 0.951 | 0.089 | 0.000 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 36 | 33 | 38 | 59 | 36 | 51 | 31 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.79 | 0.90 | 1.40 | 0.86 | 1.21 | 0.74 | 0.00 |
| time (sec) | N/A | 0.014 | 0.013 | 0.009 | 1.315 | 1.071 | 0.157 | 0.937 | 0.042 | 0.000 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 44 | 38 | 43 | 64 | 41 | 60 | 37 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.78 | 0.88 | 1.31 | 0.84 | 1.22 | 0.76 | 0.00 |
| time (sec) | N/A | 0.016 | 0.030 | 0.006 | 1.308 | 1.191 | 0.169 | 1.232 | 0.093 | 0.000 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 56 | 43 | 48 | 69 | 46 | 69 | 41 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.86 | 1.23 | 0.82 | 1.23 | 0.73 | 0.00 |
| time (sec) | N/A | 0.020 | 0.010 | 0.009 | 1.357 | 0.941 | 0.175 | 1.154 | 0.095 | 0.000 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 29 | 32 | 30 | 50 | 27 | 27 | 29 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 0.82 | 0.77 | 1.28 | 0.69 | 0.69 | 0.74 | 0.00 |
| time (sec) | N/A | 0.012 | 0.020 | 0.007 | 1.389 | 1.081 | 0.170 | 1.034 | 0.125 | 0.000 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 39 | 37 | 41 | 68 | 41 | 37 | 35 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.80 | 0.89 | 1.48 | 0.89 | 0.80 | 0.76 | 0.00 |
| time (sec) | N/A | 0.015 | 0.021 | 0.009 | 1.391 | 0.923 | 0.180 | 0.894 | 0.092 | 0.000 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 44 | 42 | 48 | 79 | 46 | 43 | 41 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.79 | 0.91 | 1.49 | 0.87 | 0.81 | 0.77 | 0.00 |
| time (sec) | N/A | 0.016 | 0.025 | 0.009 | 1.384 | 0.948 | 0.181 | 1.068 | 0.094 | 0.000 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 49 | 47 | 53 | 84 | 51 | 47 | 47 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.78 | 0.88 | 1.40 | 0.85 | 0.78 | 0.78 | 0.00 |
| time (sec) | N/A | 0.021 | 0.019 | 0.008 | 1.380 | 0.753 | 0.207 | 1.291 | 0.046 | 0.000 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 54 | 52 | 58 | 89 | 56 | 52 | 51 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 0.78 | 0.87 | 1.33 | 0.84 | 0.78 | 0.76 | 0.00 |
| time (sec) | N/A | 0.022 | 0.022 | 0.010 | 1.385 | 0.986 | 0.216 | 1.133 | 0.047 | 0.000 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 10 | 9 | 6 | 6 | 7 | 7 | 6 | 0 |
| N.S. | 1 | 1.00 | 1.25 | 1.12 | 0.75 | 0.75 | 0.88 | 0.88 | 0.75 | 0.00 |
| time (sec) | N/A | 0.001 | 0.001 | 0.001 | 1.337 | 1.123 | 0.074 | 0.902 | 0.151 | 0.000 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 10 | 10 | 9 | 8 | 8 | 8 | 9 | 6 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.80 | 0.80 | 0.80 | 0.90 | 0.60 | 0.00 |
| time (sec) | N/A | 0.001 | 0.001 | 0.002 | 1.339 | 0.982 | 0.071 | 1.146 | 0.077 | 0.000 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 16 | 13 | 12 | 10 | 14 | 13 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.14 | 0.93 | 0.86 | 0.71 | 1.00 | 0.93 | 0.71 | 0.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.002 | 1.251 | 0.785 | 0.079 | 1.285 | 0.105 | 0.000 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 17 | 16 | 20 | 17 | 17 | 16 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.80 | 1.00 | 0.85 | 0.85 | 0.80 | 0.00 |
| time (sec) | N/A | 0.007 | 0.005 | 0.000 | 1.372 | 1.156 | 0.081 | 1.102 | 0.108 | 0.000 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 19 | 18 | 21 | 19 | 19 | 14 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.82 | 0.95 | 0.86 | 0.86 | 0.64 | 0.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.001 | 1.322 | 0.889 | 0.079 | 0.931 | 0.052 | 0.001 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 19 | 18 | 23 | 19 | 19 | 16 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.82 | 1.05 | 0.86 | 0.86 | 0.73 | 0.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.001 | 1.308 | 0.837 | 0.080 | 1.091 | 0.057 | 0.001 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 19 | 18 | 24 | 19 | 19 | 16 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.86 | 1.14 | 0.90 | 0.90 | 0.76 | 0.00 |
| time (sec) | N/A | 0.004 | 0.007 | 0.000 | 1.327 | 0.871 | 0.092 | 1.040 | 0.152 | 0.000 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 22 | 19 | 18 | 24 | 20 | 19 | 14 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 0.95 | 0.90 | 1.20 | 1.00 | 0.95 | 0.70 | 0.00 |
| time (sec) | N/A | 0.004 | 0.009 | 0.001 | 1.310 | 1.013 | 0.088 | 1.079 | 0.175 | 0.001 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 12 | 11 | 11 | 8 | 13 | 9 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 1.00 | 1.00 | 0.73 | 1.18 | 0.82 | 0.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.004 | 1.305 | 0.767 | 0.135 | 0.992 | 0.097 | 0.000 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 12 | 12 | 11 | 11 | 8 | 13 | 9 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.92 | 0.92 | 0.67 | 1.08 | 0.75 | 0.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.006 | 1.361 | 0.619 | 0.125 | 1.021 | 0.036 | 0.000 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 20 | 19 | 21 | 14 | 21 | 16 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.05 | 1.00 | 1.11 | 0.74 | 1.11 | 0.84 | 0.00 |
| time (sec) | N/A | 0.008 | 0.004 | 0.007 | 1.318 | 0.932 | 0.177 | 0.916 | 0.043 | 0.000 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 18 | 17 | 21 | 14 | 19 | 14 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.94 | 1.17 | 0.78 | 1.06 | 0.78 | 0.00 |
| time (sec) | N/A | 0.010 | 0.003 | 0.006 | 1.333 | 0.732 | 0.191 | 1.144 | 0.029 | 0.001 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 15 | 14 | 15 | 10 | 15 | 20 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.00 | 1.07 | 0.71 | 1.07 | 1.43 | 0.00 |
| time (sec) | N/A | 0.009 | 0.005 | 0.001 | 1.325 | 0.880 | 0.148 | 1.251 | 0.036 | 0.000 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 46 | 43 | 56 | 53 | 1742 | 116 | 56 | 59 |
| N.S. | 1 | 1.00 | 0.64 | 0.60 | 0.78 | 0.74 | 24.19 | 1.61 | 0.78 | 0.82 |
| time (sec) | N/A | 0.018 | 0.023 | 0.015 | 1.250 | 1.358 | 2.910 | 0.948 | 0.050 | 0.062 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 35 | 32 | 41 | 42 | 666 | 93 | 37 | 45 |
| N.S. | 1 | 1.00 | 0.66 | 0.60 | 0.77 | 0.79 | 12.57 | 1.75 | 0.70 | 0.85 |
| time (sec) | N/A | 0.013 | 0.016 | 0.004 | 1.407 | 1.047 | 2.039 | 1.032 | 0.046 | 0.019 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 24 | 21 | 26 | 30 | 202 | 66 | 25 | 35 |
| N.S. | 1 | 1.00 | 0.71 | 0.62 | 0.76 | 0.88 | 5.94 | 1.94 | 0.74 | 1.03 |
| time (sec) | N/A | 0.008 | 0.011 | 0.003 | 1.315 | 1.140 | 1.393 | 1.284 | 0.028 | 0.012 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 12 | 12 | 12 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 1.00 |
| time (sec) | N/A | 0.001 | 0.004 | 0.003 | 1.336 | 1.114 | 0.074 | 1.058 | 0.021 | 0.006 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 35 | 28 | 42 | 73 | 68 | 32 | 27 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 1.20 | 2.09 | 1.94 | 0.91 | 0.77 | 1.00 |
| time (sec) | N/A | 0.010 | 0.009 | 0.013 | 2.919 | 1.114 | 1.602 | 1.131 | 0.094 | 0.032 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 47 | 37 | 47 | 93 | 44 | 41 | 31 | 39 |
| N.S. | 1 | 1.00 | 1.21 | 0.95 | 1.21 | 2.38 | 1.13 | 1.05 | 0.79 | 1.00 |
| time (sec) | N/A | 0.011 | 0.027 | 0.012 | 2.869 | 1.227 | 2.221 | 0.964 | 0.053 | 0.052 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 35 | 53 | 88 | 119 | 97 | 66 | 48 | 55 |
| N.S. | 1 | 1.00 | 0.54 | 0.82 | 1.35 | 1.83 | 1.49 | 1.02 | 0.74 | 0.85 |
| time (sec) | N/A | 0.018 | 0.007 | 0.012 | 3.027 | 0.980 | 4.015 | 1.136 | 0.068 | 0.085 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 35 | 65 | 121 | 145 | 122 | 84 | 66 | 71 |
| N.S. | 1 | 1.00 | 0.40 | 0.75 | 1.39 | 1.67 | 1.40 | 0.97 | 0.76 | 0.82 |
| time (sec) | N/A | 0.027 | 0.007 | 0.013 | 3.020 | 0.982 | 6.678 | 1.229 | 0.107 | 0.113 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 46 | 43 | 56 | 64 | 1742 | 193 | 56 | 59 |
| N.S. | 1 | 1.00 | 0.64 | 0.60 | 0.78 | 0.89 | 24.19 | 2.68 | 0.78 | 0.82 |
| time (sec) | N/A | 0.019 | 0.024 | 0.005 | 1.348 | 1.234 | 3.199 | 1.108 | 0.048 | 0.022 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 35 | 32 | 41 | 53 | 733 | 156 | 37 | 45 |
| N.S. | 1 | 1.00 | 0.66 | 0.60 | 0.77 | 1.00 | 13.83 | 2.94 | 0.70 | 0.85 |
| time (sec) | N/A | 0.013 | 0.017 | 0.006 | 1.347 | 0.751 | 2.175 | 0.936 | 0.041 | 0.020 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 24 | 21 | 26 | 41 | 80 | 119 | 25 | 46 |
| N.S. | 1 | 1.00 | 0.71 | 0.62 | 0.76 | 1.21 | 2.35 | 3.50 | 0.74 | 1.35 |
| time (sec) | N/A | 0.008 | 0.013 | 0.002 | 1.347 | 0.809 | 0.740 | 1.240 | 0.026 | 0.013 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 28 | 12 | 58 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 1.75 | 0.75 | 3.62 | 0.75 | 1.00 |
| time (sec) | N/A | 0.001 | 0.005 | 0.001 | 1.287 | 0.614 | 0.072 | 1.126 | 0.016 | 0.006 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 44 | 38 | 52 | 88 | 71 | 44 | 37 | 50 |
| N.S. | 1 | 1.00 | 0.90 | 0.78 | 1.06 | 1.80 | 1.45 | 0.90 | 0.76 | 1.02 |
| time (sec) | N/A | 0.014 | 0.021 | 0.007 | 2.937 | 1.137 | 2.295 | 1.077 | 0.042 | 0.026 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 33 | 47 | 58 | 102 | 92 | 56 | 42 | 49 |
| N.S. | 1 | 1.00 | 0.65 | 0.92 | 1.14 | 2.00 | 1.80 | 1.10 | 0.82 | 0.96 |
| time (sec) | N/A | 0.015 | 0.007 | 0.011 | 2.875 | 1.042 | 2.664 | 1.043 | 0.100 | 0.061 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 68 | 51 | 86 | 124 | 76 | 64 | 46 | 56 |
| N.S. | 1 | 1.00 | 1.10 | 0.82 | 1.39 | 2.00 | 1.23 | 1.03 | 0.74 | 0.90 |
| time (sec) | N/A | 0.016 | 0.034 | 0.012 | 2.984 | 1.118 | 3.283 | 1.287 | 0.057 | 0.102 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 35 | 63 | 119 | 145 | 124 | 84 | 64 | 71 |
| N.S. | 1 | 1.00 | 0.42 | 0.75 | 1.42 | 1.73 | 1.48 | 1.00 | 0.76 | 0.85 |
| time (sec) | N/A | 0.022 | 0.009 | 0.012 | 3.047 | 1.006 | 5.837 | 1.161 | 0.102 | 0.116 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 46 | 43 | 56 | 75 | 146 | 281 | 56 | 51 |
| N.S. | 1 | 1.00 | 0.64 | 0.60 | 0.78 | 1.04 | 2.03 | 3.90 | 0.78 | 0.71 |
| time (sec) | N/A | 0.017 | 0.027 | 0.006 | 1.347 | 1.003 | 4.469 | 1.158 | 0.050 | 0.024 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 35 | 32 | 41 | 64 | 124 | 233 | 37 | 39 |
| N.S. | 1 | 1.00 | 0.66 | 0.60 | 0.77 | 1.21 | 2.34 | 4.40 | 0.70 | 0.74 |
| time (sec) | N/A | 0.012 | 0.020 | 0.005 | 1.366 | 1.167 | 3.770 | 1.090 | 0.044 | 0.023 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 24 | 21 | 26 | 52 | 102 | 182 | 25 | 57 |
| N.S. | 1 | 1.00 | 0.71 | 0.62 | 0.76 | 1.53 | 3.00 | 5.35 | 0.74 | 1.68 |
| time (sec) | N/A | 0.009 | 0.016 | 0.003 | 1.371 | 1.103 | 2.599 | 1.152 | 0.028 | 0.015 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 39 | 12 | 95 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 2.44 | 0.75 | 5.94 | 0.75 | 1.00 |
| time (sec) | N/A | 0.001 | 0.006 | 0.002 | 1.341 | 0.980 | 0.077 | 0.983 | 0.018 | 0.007 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 58 | 50 | 64 | 114 | 97 | 56 | 52 | 66 |
| N.S. | 1 | 1.00 | 0.89 | 0.77 | 0.98 | 1.75 | 1.49 | 0.86 | 0.80 | 1.02 |
| time (sec) | N/A | 0.021 | 0.048 | 0.007 | 3.017 | 1.155 | 4.117 | 1.027 | 0.053 | 0.029 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 33 | 61 | 71 | 126 | 99 | 74 | 58 | 64 |
| N.S. | 1 | 1.00 | 0.50 | 0.92 | 1.08 | 1.91 | 1.50 | 1.12 | 0.88 | 0.97 |
| time (sec) | N/A | 0.020 | 0.009 | 0.010 | 2.945 | 0.803 | 3.747 | 1.088 | 0.115 | 0.074 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 78 | 78 | 35 | 61 | 101 | 133 | 126 | 80 | 64 | 68 |
| N.S. | 1 | 1.00 | 0.45 | 0.78 | 1.29 | 1.71 | 1.62 | 1.03 | 0.82 | 0.87 |
| time (sec) | N/A | 0.021 | 0.009 | 0.010 | 2.940 | 1.037 | 4.300 | 1.120 | 0.053 | 0.111 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 79 | 63 | 115 | 146 | 104 | 79 | 64 | 68 |
| N.S. | 1 | 1.00 | 0.98 | 0.78 | 1.42 | 1.80 | 1.28 | 0.98 | 0.79 | 0.84 |
| time (sec) | N/A | 0.023 | 0.038 | 0.010 | 2.985 | 0.995 | 5.157 | 1.130 | 0.046 | 0.140 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 35 | 75 | 144 | 167 | 155 | 99 | 79 | 83 |
| N.S. | 1 | 1.00 | 0.34 | 0.73 | 1.40 | 1.62 | 1.50 | 0.96 | 0.77 | 0.81 |
| time (sec) | N/A | 0.031 | 0.009 | 0.012 | 2.934 | 0.958 | 8.359 | 1.094 | 0.111 | 0.154 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 146 | 146 | 90 | 87 | 116 | 141 | 279 | 781 | 116 | 115 |
| N.S. | 1 | 1.00 | 0.62 | 0.60 | 0.79 | 0.97 | 1.91 | 5.35 | 0.79 | 0.79 |
| time (sec) | N/A | 0.042 | 0.055 | 0.006 | 1.334 | 1.020 | 40.296 | 1.140 | 0.036 | 0.038 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 127 | 127 | 79 | 76 | 101 | 130 | 257 | 709 | 101 | 101 |
| N.S. | 1 | 1.00 | 0.62 | 0.60 | 0.80 | 1.02 | 2.02 | 5.58 | 0.80 | 0.80 |
| time (sec) | N/A | 0.035 | 0.042 | 0.007 | 1.343 | 0.727 | 36.609 | 1.324 | 0.031 | 0.034 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 68 | 65 | 86 | 119 | 235 | 637 | 86 | 75 |
| N.S. | 1 | 1.00 | 0.62 | 0.59 | 0.78 | 1.08 | 2.14 | 5.79 | 0.78 | 0.68 |
| time (sec) | N/A | 0.032 | 0.038 | 0.006 | 1.347 | 0.911 | 28.760 | 1.054 | 0.027 | 0.031 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 57 | 54 | 71 | 108 | 212 | 565 | 71 | 63 |
| N.S. | 1 | 1.00 | 0.63 | 0.59 | 0.78 | 1.19 | 2.33 | 6.21 | 0.78 | 0.69 |
| time (sec) | N/A | 0.023 | 0.032 | 0.006 | 1.359 | 1.074 | 25.700 | 1.122 | 0.025 | 0.028 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 46 | 43 | 56 | 97 | 190 | 493 | 56 | 51 |
| N.S. | 1 | 1.00 | 0.64 | 0.60 | 0.78 | 1.35 | 2.64 | 6.85 | 0.78 | 0.71 |
| time (sec) | N/A | 0.018 | 0.030 | 0.006 | 1.293 | 0.955 | 20.346 | 1.102 | 0.045 | 0.024 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 35 | 32 | 41 | 86 | 168 | 421 | 36 | 39 |
| N.S. | 1 | 1.00 | 0.66 | 0.60 | 0.77 | 1.62 | 3.17 | 7.94 | 0.68 | 0.74 |
| time (sec) | N/A | 0.013 | 0.022 | 0.004 | 1.341 | 0.956 | 16.856 | 1.058 | 0.040 | 0.023 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 24 | 21 | 26 | 74 | 146 | 347 | 25 | 79 |
| N.S. | 1 | 1.00 | 0.71 | 0.62 | 0.76 | 2.18 | 4.29 | 10.21 | 0.74 | 2.32 |
| time (sec) | N/A | 0.008 | 0.017 | 0.003 | 1.356 | 0.718 | 14.851 | 1.269 | 0.030 | 0.015 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 61 | 12 | 229 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 3.81 | 0.75 | 14.31 | 0.75 | 1.00 |
| time (sec) | N/A | 0.002 | 0.007 | 0.003 | 1.336 | 1.046 | 0.083 | 0.991 | 0.018 | 0.006 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 78 | 74 | 88 | 158 | 148 | 80 | 76 | 94 |
| N.S. | 1 | 1.00 | 0.80 | 0.76 | 0.91 | 1.63 | 1.53 | 0.82 | 0.78 | 0.97 |
| time (sec) | N/A | 0.034 | 0.085 | 0.007 | 2.942 | 1.180 | 11.102 | 1.227 | 0.037 | 0.031 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 33 | 84 | 97 | 172 | 150 | 104 | 84 | 88 |
| N.S. | 1 | 1.00 | 0.34 | 0.86 | 0.99 | 1.76 | 1.53 | 1.06 | 0.86 | 0.90 |
| time (sec) | N/A | 0.034 | 0.011 | 0.010 | 2.968 | 0.857 | 9.922 | 1.427 | 0.043 | 0.070 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 35 | 86 | 131 | 180 | 184 | 112 | 117 | 92 |
| N.S. | 1 | 1.00 | 0.31 | 0.75 | 1.15 | 1.58 | 1.61 | 0.98 | 1.03 | 0.81 |
| time (sec) | N/A | 0.035 | 0.011 | 0.011 | 2.952 | 1.165 | 8.989 | 1.104 | 0.047 | 0.111 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 35 | 87 | 145 | 178 | 184 | 112 | 131 | 92 |
| N.S. | 1 | 1.00 | 0.31 | 0.76 | 1.27 | 1.56 | 1.61 | 0.98 | 1.15 | 0.81 |
| time (sec) | N/A | 0.036 | 0.011 | 0.013 | 2.939 | 0.927 | 7.912 | 1.101 | 0.121 | 0.138 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 116 | 116 | 35 | 85 | 155 | 177 | 182 | 110 | 94 | 92 |
| N.S. | 1 | 1.00 | 0.30 | 0.73 | 1.34 | 1.53 | 1.57 | 0.95 | 0.81 | 0.79 |
| time (sec) | N/A | 0.037 | 0.011 | 0.013 | 2.965 | 0.975 | 8.547 | 1.215 | 0.062 | 0.168 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 101 | 87 | 169 | 190 | 158 | 109 | 94 | 92 |
| N.S. | 1 | 1.00 | 0.85 | 0.73 | 1.42 | 1.60 | 1.33 | 0.92 | 0.79 | 0.77 |
| time (sec) | N/A | 0.039 | 0.044 | 0.013 | 2.965 | 0.901 | 10.250 | 1.190 | 0.119 | 0.203 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 141 | 141 | 35 | 99 | 198 | 211 | 209 | 129 | 109 | 107 |
| N.S. | 1 | 1.00 | 0.25 | 0.70 | 1.40 | 1.50 | 1.48 | 0.91 | 0.77 | 0.76 |
| time (sec) | N/A | 0.051 | 0.011 | 0.014 | 3.034 | 1.269 | 15.689 | 1.042 | 0.133 | 0.234 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 163 | 163 | 35 | 111 | 229 | 233 | 236 | 144 | 124 | 119 |
| N.S. | 1 | 1.00 | 0.21 | 0.68 | 1.40 | 1.43 | 1.45 | 0.88 | 0.76 | 0.73 |
| time (sec) | N/A | 0.068 | 0.011 | 0.014 | 3.083 | 1.415 | 22.200 | 0.951 | 0.126 | 0.256 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 39 | 32 | 31 | 78 | 148 | 31 | 31 | 39 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.79 | 2.00 | 3.79 | 0.79 | 0.79 | 1.00 |
| time (sec) | N/A | 0.011 | 0.011 | 0.007 | 2.939 | 1.210 | 1.737 | 1.136 | 0.094 | 0.021 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 52 | 35 | 34 | 98 | 121 | 41 | 34 | 42 |
| N.S. | 1 | 1.00 | 1.24 | 0.83 | 0.81 | 2.33 | 2.88 | 0.98 | 0.81 | 1.00 |
| time (sec) | N/A | 0.010 | 0.024 | 0.012 | 2.942 | 0.999 | 2.140 | 1.081 | 0.097 | 0.043 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 38 | 55 | 83 | 124 | 207 | 66 | 54 | 60 |
| N.S. | 1 | 1.00 | 0.54 | 0.77 | 1.17 | 1.75 | 2.92 | 0.93 | 0.76 | 0.85 |
| time (sec) | N/A | 0.016 | 0.009 | 0.010 | 2.964 | 1.091 | 4.161 | 1.081 | 0.103 | 0.077 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 55 | 55 | 48 | 44 | 43 | 93 | 187 | 43 | 43 | 58 |
| N.S. | 1 | 1.00 | 0.87 | 0.80 | 0.78 | 1.69 | 3.40 | 0.78 | 0.78 | 1.05 |
| time (sec) | N/A | 0.015 | 0.024 | 0.007 | 3.018 | 0.914 | 2.463 | 1.058 | 0.041 | 0.028 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 36 | 48 | 47 | 105 | 197 | 58 | 47 | 55 |
| N.S. | 1 | 1.00 | 0.63 | 0.84 | 0.82 | 1.84 | 3.46 | 1.02 | 0.82 | 0.96 |
| time (sec) | N/A | 0.014 | 0.009 | 0.011 | 3.001 | 1.043 | 2.837 | 0.959 | 0.045 | 0.054 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 72 | 53 | 80 | 129 | 190 | 66 | 52 | 62 |
| N.S. | 1 | 1.00 | 1.06 | 0.78 | 1.18 | 1.90 | 2.79 | 0.97 | 0.76 | 0.91 |
| time (sec) | N/A | 0.015 | 0.035 | 0.012 | 3.021 | 1.039 | 3.302 | 0.952 | 0.097 | 0.086 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 60 | 58 | 57 | 119 | 240 | 57 | 57 | 74 |
| N.S. | 1 | 1.00 | 0.82 | 0.79 | 0.78 | 1.63 | 3.29 | 0.78 | 0.78 | 1.01 |
| time (sec) | N/A | 0.019 | 0.033 | 0.006 | 2.918 | 1.162 | 4.241 | 1.218 | 0.043 | 0.029 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 36 | 64 | 63 | 131 | 245 | 75 | 63 | 72 |
| N.S. | 1 | 1.00 | 0.49 | 0.86 | 0.85 | 1.77 | 3.31 | 1.01 | 0.85 | 0.97 |
| time (sec) | N/A | 0.020 | 0.011 | 0.012 | 3.049 | 0.978 | 3.686 | 1.020 | 0.103 | 0.055 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 38 | 70 | 97 | 139 | 267 | 83 | 69 | 76 |
| N.S. | 1 | 1.00 | 0.44 | 0.81 | 1.13 | 1.62 | 3.10 | 0.97 | 0.80 | 0.88 |
| time (sec) | N/A | 0.022 | 0.010 | 0.013 | 2.924 | 0.898 | 3.927 | 1.037 | 0.095 | 0.094 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 57 | 54 | 71 | 53 | 3755 | 61 | 71 | 73 |
| N.S. | 1 | 1.00 | 0.64 | 0.61 | 0.80 | 0.60 | 42.19 | 0.69 | 0.80 | 0.82 |
| time (sec) | N/A | 0.022 | 0.028 | 0.006 | 1.345 | 0.719 | 4.839 | 1.040 | 0.024 | 0.025 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 46 | 43 | 56 | 42 | 1640 | 49 | 56 | 59 |
| N.S. | 1 | 1.00 | 0.68 | 0.63 | 0.82 | 0.62 | 24.12 | 0.72 | 0.82 | 0.87 |
| time (sec) | N/A | 0.018 | 0.026 | 0.003 | 1.254 | 1.040 | 2.703 | 1.207 | 0.046 | 0.022 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 35 | 32 | 41 | 31 | 600 | 37 | 37 | 45 |
| N.S. | 1 | 1.00 | 0.69 | 0.63 | 0.80 | 0.61 | 11.76 | 0.73 | 0.73 | 0.88 |
| time (sec) | N/A | 0.012 | 0.016 | 0.005 | 1.323 | 0.707 | 1.767 | 1.342 | 0.038 | 0.020 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 23 | 21 | 26 | 19 | 162 | 23 | 25 | 24 |
| N.S. | 1 | 1.00 | 0.72 | 0.66 | 0.81 | 0.59 | 5.06 | 0.72 | 0.78 | 0.75 |
| time (sec) | N/A | 0.008 | 0.011 | 0.003 | 1.320 | 0.949 | 1.160 | 1.006 | 0.027 | 0.013 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 12 | 10 | 12 | 12 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 0.86 | 0.71 | 0.86 | 0.86 | 1.00 |
| time (sec) | N/A | 0.001 | 0.003 | 0.002 | 1.298 | 0.938 | 0.066 | 0.985 | 0.018 | 0.006 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 18 | 32 | 56 | 24 | 21 | 17 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 0.78 | 1.39 | 2.43 | 1.04 | 0.91 | 0.74 | 1.00 |
| time (sec) | N/A | 0.007 | 0.004 | 0.006 | 2.936 | 1.117 | 1.108 | 1.244 | 0.055 | 0.018 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 47 | 40 | 60 | 93 | 44 | 47 | 33 | 41 |
| N.S. | 1 | 1.00 | 1.15 | 0.98 | 1.46 | 2.27 | 1.07 | 1.15 | 0.80 | 1.00 |
| time (sec) | N/A | 0.011 | 0.065 | 0.006 | 2.999 | 0.860 | 2.302 | 1.069 | 0.111 | 0.053 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 33 | 66 | 92 | 123 | 102 | 69 | 51 | 63 |
| N.S. | 1 | 1.00 | 0.49 | 0.97 | 1.35 | 1.81 | 1.50 | 1.01 | 0.75 | 0.93 |
| time (sec) | N/A | 0.017 | 0.006 | 0.006 | 2.917 | 1.124 | 4.366 | 1.195 | 0.061 | 0.078 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 33 | 90 | 121 | 145 | 129 | 84 | 69 | 71 |
| N.S. | 1 | 1.00 | 0.37 | 1.00 | 1.34 | 1.61 | 1.43 | 0.93 | 0.77 | 0.79 |
| time (sec) | N/A | 0.025 | 0.006 | 0.008 | 2.916 | 0.907 | 7.022 | 0.908 | 0.052 | 0.074 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 57 | 54 | 71 | 63 | 3606 | 77 | 71 | 63 |
| N.S. | 1 | 1.00 | 0.67 | 0.64 | 0.84 | 0.74 | 42.42 | 0.91 | 0.84 | 0.74 |
| time (sec) | N/A | 0.024 | 0.028 | 0.006 | 1.329 | 0.949 | 4.806 | 1.252 | 0.028 | 0.025 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 45 | 42 | 56 | 51 | 1538 | 61 | 56 | 49 |
| N.S. | 1 | 1.00 | 0.68 | 0.64 | 0.85 | 0.77 | 23.30 | 0.92 | 0.85 | 0.74 |
| time (sec) | N/A | 0.017 | 0.020 | 0.006 | 1.375 | 0.953 | 2.935 | 1.102 | 0.048 | 0.024 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 34 | 32 | 41 | 40 | 534 | 46 | 35 | 37 |
| N.S. | 1 | 1.00 | 0.69 | 0.65 | 0.84 | 0.82 | 10.90 | 0.94 | 0.71 | 0.76 |
| time (sec) | N/A | 0.013 | 0.016 | 0.006 | 1.290 | 0.843 | 1.827 | 1.145 | 0.040 | 0.023 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 21 | 20 | 26 | 29 | 37 | 29 | 19 | 21 |
| N.S. | 1 | 1.00 | 0.70 | 0.67 | 0.87 | 0.97 | 1.23 | 0.97 | 0.63 | 0.70 |
| time (sec) | N/A | 0.008 | 0.010 | 0.003 | 1.324 | 0.911 | 0.669 | 0.995 | 0.087 | 0.011 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 20 | 12 | 12 | 12 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 1.43 | 0.86 | 0.86 | 0.86 | 1.00 |
| time (sec) | N/A | 0.001 | 0.004 | 0.002 | 1.341 | 0.742 | 0.070 | 1.238 | 0.019 | 0.008 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 30 | 31 | 45 | 110 | 146 | 37 | 30 | 38 |
| N.S. | 1 | 1.00 | 0.79 | 0.82 | 1.18 | 2.89 | 3.84 | 0.97 | 0.79 | 1.00 |
| time (sec) | N/A | 0.011 | 0.005 | 0.009 | 2.926 | 1.028 | 1.866 | 1.070 | 0.043 | 0.028 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 59 | 31 | 55 | 76 | 151 | 73 | 64 | 60 | 52 |
| N.S. | 1 | 1.04 | 0.54 | 0.96 | 1.33 | 2.65 | 1.28 | 1.12 | 1.05 | 0.91 |
| time (sec) | N/A | 0.017 | 0.007 | 0.011 | 3.012 | 1.170 | 3.411 | 0.905 | 0.122 | 0.066 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 85 | 33 | 67 | 108 | 189 | 107 | 80 | 90 | 71 |
| N.S. | 1 | 0.98 | 0.38 | 0.77 | 1.24 | 2.17 | 1.23 | 0.92 | 1.03 | 0.82 |
| time (sec) | N/A | 0.023 | 0.006 | 0.013 | 3.063 | 1.074 | 5.983 | 1.047 | 0.059 | 0.099 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 57 | 54 | 71 | 74 | 3456 | 75 | 68 | 63 |
| N.S. | 1 | 1.00 | 0.66 | 0.62 | 0.82 | 0.85 | 39.72 | 0.86 | 0.78 | 0.72 |
| time (sec) | N/A | 0.022 | 0.028 | 0.005 | 1.349 | 0.995 | 4.575 | 1.081 | 0.049 | 0.029 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 45 | 43 | 56 | 62 | 163 | 59 | 47 | 47 |
| N.S. | 1 | 1.00 | 0.66 | 0.63 | 0.82 | 0.91 | 2.40 | 0.87 | 0.69 | 0.69 |
| time (sec) | N/A | 0.017 | 0.024 | 0.006 | 1.253 | 0.964 | 1.200 | 1.032 | 0.043 | 0.024 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 35 | 32 | 41 | 52 | 121 | 39 | 35 | 39 |
| N.S. | 1 | 1.00 | 0.71 | 0.65 | 0.84 | 1.06 | 2.47 | 0.80 | 0.71 | 0.80 |
| time (sec) | N/A | 0.013 | 0.017 | 0.004 | 1.328 | 1.269 | 1.275 | 0.940 | 0.084 | 0.024 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 24 | 21 | 26 | 41 | 80 | 20 | 20 | 24 |
| N.S. | 1 | 1.00 | 0.75 | 0.66 | 0.81 | 1.28 | 2.50 | 0.62 | 0.62 | 0.75 |
| time (sec) | N/A | 0.008 | 0.012 | 0.003 | 1.338 | 0.988 | 1.128 | 1.055 | 0.033 | 0.013 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 31 | 14 | 12 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 1.94 | 0.88 | 0.75 | 0.75 | 1.00 |
| time (sec) | N/A | 0.001 | 0.005 | 0.002 | 1.307 | 0.912 | 0.075 | 1.038 | 0.018 | 0.007 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 32 | 43 | 53 | 177 | 697 | 45 | 42 | 49 |
| N.S. | 1 | 1.00 | 0.59 | 0.80 | 0.98 | 3.28 | 12.91 | 0.83 | 0.78 | 0.91 |
| time (sec) | N/A | 0.015 | 0.006 | 0.010 | 2.912 | 1.016 | 2.993 | 1.003 | 0.050 | 0.041 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 80 | 33 | 67 | 89 | 221 | 818 | 65 | 73 | 67 |
| N.S. | 1 | 1.08 | 0.45 | 0.91 | 1.20 | 2.99 | 11.05 | 0.88 | 0.99 | 0.91 |
| time (sec) | N/A | 0.024 | 0.006 | 0.012 | 3.032 | 1.133 | 5.596 | 1.027 | 0.109 | 0.071 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 106 | 106 | 35 | 80 | 123 | 255 | 464 | 93 | 105 | 83 |
| N.S. | 1 | 1.00 | 0.33 | 0.75 | 1.16 | 2.41 | 4.38 | 0.88 | 0.99 | 0.78 |
| time (sec) | N/A | 0.034 | 0.007 | 0.015 | 3.001 | 1.152 | 8.571 | 0.994 | 0.120 | 0.112 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 25 | 20 | 19 | 58 | 54 | 19 | 19 | 25 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.76 | 2.32 | 2.16 | 0.76 | 0.76 | 1.00 |
| time (sec) | N/A | 0.006 | 0.005 | 0.004 | 2.991 | 1.400 | 1.226 | 0.986 | 0.048 | 0.019 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 53 | 37 | 46 | 97 | 121 | 43 | 36 | 44 |
| N.S. | 1 | 1.00 | 1.20 | 0.84 | 1.05 | 2.20 | 2.75 | 0.98 | 0.82 | 1.00 |
| time (sec) | N/A | 0.011 | 0.065 | 0.007 | 2.976 | 0.908 | 2.458 | 0.886 | 0.041 | 0.043 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 36 | 59 | 86 | 128 | 216 | 68 | 57 | 69 |
| N.S. | 1 | 1.00 | 0.49 | 0.80 | 1.16 | 1.73 | 2.92 | 0.92 | 0.77 | 0.93 |
| time (sec) | N/A | 0.017 | 0.006 | 0.007 | 2.957 | 1.364 | 4.199 | 0.965 | 0.045 | 0.066 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 33 | 35 | 34 | 124 | 478 | 34 | 34 | 42 |
| N.S. | 1 | 1.00 | 0.79 | 0.83 | 0.81 | 2.95 | 11.38 | 0.81 | 0.81 | 1.00 |
| time (sec) | N/A | 0.010 | 0.007 | 0.008 | 2.972 | 1.265 | 2.199 | 1.016 | 0.095 | 0.028 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 65 | 34 | 54 | 67 | 164 | 156 | 64 | 52 | 59 |
| N.S. | 1 | 1.05 | 0.55 | 0.87 | 1.08 | 2.65 | 2.52 | 1.03 | 0.84 | 0.95 |
| time (sec) | N/A | 0.016 | 0.007 | 0.014 | 3.022 | 0.870 | 3.499 | 1.013 | 0.063 | 0.059 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 93 | 36 | 75 | 104 | 198 | 226 | 81 | 101 | 79 |
| N.S. | 1 | 0.98 | 0.38 | 0.79 | 1.09 | 2.08 | 2.38 | 0.85 | 1.06 | 0.83 |
| time (sec) | N/A | 0.023 | 0.007 | 0.014 | 2.926 | 0.894 | 5.567 | 1.006 | 0.132 | 0.100 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 35 | 49 | 42 | 182 | 0 | 42 | 48 | 55 |
| N.S. | 1 | 1.00 | 0.58 | 0.82 | 0.70 | 3.03 | 0.00 | 0.70 | 0.80 | 0.92 |
| time (sec) | N/A | 0.015 | 0.008 | 0.010 | 2.938 | 0.847 | 0.000 | 1.057 | 0.092 | 0.041 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 88 | 36 | 68 | 82 | 226 | 0 | 66 | 70 | 75 |
| N.S. | 1 | 1.09 | 0.44 | 0.84 | 1.01 | 2.79 | 0.00 | 0.81 | 0.86 | 0.93 |
| time (sec) | N/A | 0.023 | 0.009 | 0.013 | 2.928 | 0.827 | 0.000 | 1.004 | 0.118 | 0.067 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 116 | 116 | 38 | 92 | 121 | 260 | 1108 | 97 | 117 | 93 |
| N.S. | 1 | 1.00 | 0.33 | 0.79 | 1.04 | 2.24 | 9.55 | 0.84 | 1.01 | 0.80 |
| time (sec) | N/A | 0.033 | 0.007 | 0.015 | 2.875 | 0.963 | 11.219 | 0.902 | 0.071 | 0.119 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 12 | 11 | 14 | 78 | 0 | 11 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.85 | 1.08 | 6.00 | 0.00 | 0.85 | 0.00 |
| time (sec) | N/A | 0.006 | 0.041 | 0.006 | 1.875 | 0.907 | 84.074 | 0.000 | 0.413 | 0.254 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | F | A | A | C | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 92 | 13 | 0 | 11 | 11 | 73 | 0 | -1 | 0 |
| N.S. | 1 | 7.08 | 1.00 | 0.00 | 0.85 | 0.85 | 5.62 | 0.00 | -0.08 | 0.00 |
| time (sec) | N/A | 0.045 | 0.015 | 0.085 | 1.862 | 0.943 | 5.344 | 0.000 | 0.000 | 0.593 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 18 | 32 | 56 | 24 | 21 | 17 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 0.78 | 1.39 | 2.43 | 1.04 | 0.91 | 0.74 | 1.00 |
| time (sec) | N/A | 0.008 | 0.004 | 0.000 | 2.991 | 1.114 | 1.091 | 0.847 | 0.002 | 0.023 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 46 | 43 | 56 | 53 | 1742 | 117 | 56 | 51 |
| N.S. | 1 | 1.00 | 0.64 | 0.60 | 0.78 | 0.74 | 24.19 | 1.62 | 0.78 | 0.71 |
| time (sec) | N/A | 0.019 | 0.024 | 0.005 | 1.385 | 0.835 | 2.833 | 0.867 | 0.052 | 0.023 |

| Problem 372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 35 | 32 | 41 | 42 | 666 | 92 | 37 | 39 |
| N.S. | 1 | 1.00 | 0.66 | 0.60 | 0.77 | 0.79 | 12.57 | 1.74 | 0.70 | 0.74 |
| time (sec) | N/A | 0.012 | 0.017 | 0.006 | 1.304 | 0.861 | 1.856 | 0.834 | 0.038 | 0.022 |

| Problem 373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 24 | 21 | 26 | 30 | 202 | 67 | 25 | 35 |
| N.S. | 1 | 1.00 | 0.71 | 0.62 | 0.76 | 0.88 | 5.94 | 1.97 | 0.74 | 1.03 |
| time (sec) | N/A | 0.008 | 0.012 | 0.003 | 1.293 | 0.911 | 1.201 | 1.038 | 0.028 | 0.013 |

| Problem 374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 12 | 12 | 12 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 1.00 |
| time (sec) | N/A | 0.001 | 0.004 | 0.003 | 1.257 | 0.959 | 0.065 | 0.892 | 0.017 | 0.006 |

| Problem 375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 113 | 85 | 86 | 91 | 180 | 87 | 107 | 116 |
| N.S. | 1 | 1.00 | 1.24 | 0.93 | 0.95 | 1.00 | 1.98 | 0.96 | 1.18 | 1.27 |
| time (sec) | N/A | 0.047 | 0.047 | 0.007 | 3.078 | 1.081 | 2.018 | 2.374 | 0.121 | 0.067 |

| Problem 376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 33 | 92 | 93 | 139 | 643 | 105 | 117 | 125 |
| N.S. | 1 | 1.00 | 0.34 | 0.95 | 0.96 | 1.43 | 6.63 | 1.08 | 1.21 | 1.29 |
| time (sec) | N/A | 0.033 | 0.006 | 0.008 | 2.959 | 0.756 | 2.185 | 2.555 | 0.067 | 0.157 |

| Problem 377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 127 | 127 | 35 | 113 | 139 | 187 | 2266 | 128 | 196 | 145 |
| N.S. | 1 | 1.00 | 0.28 | 0.89 | 1.09 | 1.47 | 17.84 | 1.01 | 1.54 | 1.14 |
| time (sec) | N/A | 0.049 | 0.007 | 0.011 | 2.986 | 0.979 | 2.577 | 2.426 | 0.230 | 0.215 |

| Problem 378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 46 | 43 | 56 | 53 | 1742 | 117 | 56 | 51 |
| N.S. | 1 | 1.00 | 0.64 | 0.60 | 0.78 | 0.74 | 24.19 | 1.62 | 0.78 | 0.71 |
| time (sec) | N/A | 0.018 | 0.025 | 0.004 | 1.364 | 0.821 | 3.073 | 1.125 | 0.045 | 0.022 |

| Problem 379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 35 | 32 | 41 | 42 | 666 | 92 | 37 | 39 |
| N.S. | 1 | 1.00 | 0.66 | 0.60 | 0.77 | 0.79 | 12.57 | 1.74 | 0.70 | 0.74 |
| time (sec) | N/A | 0.013 | 0.017 | 0.006 | 1.349 | 0.654 | 1.938 | 0.825 | 0.041 | 0.022 |

| Problem 380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 24 | 21 | 26 | 31 | 202 | 68 | 25 | 35 |
| N.S. | 1 | 1.00 | 0.71 | 0.62 | 0.76 | 0.91 | 5.94 | 2.00 | 0.74 | 1.03 |
| time (sec) | N/A | 0.008 | 0.012 | 0.003 | 1.350 | 0.819 | 1.276 | 0.892 | 0.028 | 0.014 |

| Problem 381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 12 | 12 | 12 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 1.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.003 | 1.286 | 0.841 | 0.065 | 1.144 | 0.018 | 0.006 |

| Problem 382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 86 | 84 | 85 | 110 | 182 | 86 | 117 | 117 |
| N.S. | 1 | 1.00 | 0.93 | 0.91 | 0.92 | 1.20 | 1.98 | 0.93 | 1.27 | 1.27 |
| time (sec) | N/A | 0.032 | 0.039 | 0.003 | 2.971 | 0.922 | 2.056 | 2.200 | 0.113 | 0.063 |

| Problem 383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 94 | 94 | 33 | 92 | 93 | 252 | 643 | 106 | 127 | 125 |
| N.S. | 1 | 1.00 | 0.35 | 0.98 | 0.99 | 2.68 | 6.84 | 1.13 | 1.35 | 1.33 |
| time (sec) | N/A | 0.033 | 0.007 | 0.011 | 2.982 | 0.995 | 2.238 | 2.292 | 0.114 | 0.156 |

| Problem 384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 127 | 127 | 35 | 113 | 139 | 350 | 2266 | 129 | 194 | 147 |
| N.S. | 1 | 1.00 | 0.28 | 0.89 | 1.09 | 2.76 | 17.84 | 1.02 | 1.53 | 1.16 |
| time (sec) | N/A | 0.047 | 0.008 | 0.011 | 2.983 | 0.938 | 2.655 | 2.470 | 0.329 | 0.248 |

| Problem 385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 46 | 43 | 56 | 64 | 1844 | 193 | 56 | 51 |
| N.S. | 1 | 1.00 | 0.64 | 0.60 | 0.78 | 0.89 | 25.61 | 2.68 | 0.78 | 0.71 |
| time (sec) | N/A | 0.018 | 0.025 | 0.006 | 1.357 | 0.917 | 3.178 | 1.223 | 0.047 | 0.024 |

| Problem 386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 35 | 32 | 41 | 53 | 733 | 157 | 37 | 39 |
| N.S. | 1 | 1.00 | 0.66 | 0.60 | 0.77 | 1.00 | 13.83 | 2.96 | 0.70 | 0.74 |
| time (sec) | N/A | 0.013 | 0.018 | 0.004 | 1.303 | 0.771 | 2.160 | 0.984 | 0.042 | 0.023 |

| Problem 387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 24 | 21 | 26 | 41 | 80 | 118 | 25 | 24 |
| N.S. | 1 | 1.00 | 0.71 | 0.62 | 0.76 | 1.21 | 2.35 | 3.47 | 0.74 | 0.71 |
| time (sec) | N/A | 0.009 | 0.013 | 0.003 | 1.286 | 1.175 | 1.493 | 1.018 | 0.027 | 0.014 |

| Problem 388 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 28 | 12 | 58 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 1.75 | 0.75 | 3.62 | 0.75 | 1.00 |
| time (sec) | N/A | 0.002 | 0.005 | 0.002 | 1.330 | 0.864 | 0.066 | 1.249 | 0.018 | 0.007 |

| Problem 389 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 105 | 105 | 130 | 95 | 96 | 98 | 209 | 97 | 123 | 131 |
| N.S. | 1 | 1.00 | 1.24 | 0.90 | 0.91 | 0.93 | 1.99 | 0.92 | 1.17 | 1.25 |
| time (sec) | N/A | 0.041 | 0.032 | 0.006 | 3.026 | 0.660 | 2.388 | 2.025 | 0.060 | 0.064 |

| Problem 390 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 107 | 107 | 33 | 103 | 104 | 111 | 719 | 119 | 131 | 135 |
| N.S. | 1 | 1.00 | 0.31 | 0.96 | 0.97 | 1.04 | 6.72 | 1.11 | 1.22 | 1.26 |
| time (sec) | N/A | 0.042 | 0.007 | 0.011 | 3.028 | 0.715 | 2.580 | 2.348 | 0.073 | 0.185 |

| Problem 391 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 124 | 124 | 35 | 111 | 136 | 162 | 2266 | 127 | 174 | 146 |
| N.S. | 1 | 1.00 | 0.28 | 0.90 | 1.10 | 1.31 | 18.27 | 1.02 | 1.40 | 1.18 |
| time (sec) | N/A | 0.044 | 0.008 | 0.012 | 3.055 | 0.985 | 2.738 | 1.940 | 0.122 | 0.227 |

| Problem 392 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 46 | 43 | 56 | 42 | 1640 | 49 | 56 | 51 |
| N.S. | 1 | 1.00 | 0.64 | 0.60 | 0.78 | 0.58 | 22.78 | 0.68 | 0.78 | 0.71 |
| time (sec) | N/A | 0.018 | 0.028 | 0.005 | 1.315 | 0.869 | 2.777 | 0.903 | 0.043 | 0.025 |

| Problem 393 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 35 | 32 | 41 | 31 | 600 | 37 | 37 | 45 |
| N.S. | 1 | 1.00 | 0.66 | 0.60 | 0.77 | 0.58 | 11.32 | 0.70 | 0.70 | 0.85 |
| time (sec) | N/A | 0.012 | 0.020 | 0.005 | 1.376 | 0.757 | 1.770 | 0.908 | 0.038 | 0.022 |

| Problem 394 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 24 | 21 | 26 | 20 | 162 | 25 | 25 | 24 |
| N.S. | 1 | 1.00 | 0.71 | 0.62 | 0.76 | 0.59 | 4.76 | 0.74 | 0.74 | 0.71 |
| time (sec) | N/A | 0.008 | 0.012 | 0.003 | 1.297 | 0.846 | 1.157 | 0.900 | 0.029 | 0.014 |

| Problem 395 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 12 | 12 | 12 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 1.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.002 | 1.317 | 0.989 | 0.064 | 0.973 | 0.016 | 0.007 |

| Problem 396 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 79 | 79 | 66 | 75 | 76 | 213 | 155 | 77 | 99 | 104 |
| N.S. | 1 | 1.00 | 0.84 | 0.95 | 0.96 | 2.70 | 1.96 | 0.97 | 1.25 | 1.32 |
| time (sec) | N/A | 0.025 | 0.013 | 0.004 | 2.932 | 0.870 | 1.881 | 2.371 | 0.086 | 0.055 |

| Problem 397 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 33 | 95 | 106 | 306 | 831 | 109 | 130 | 128 |
| N.S. | 1 | 1.00 | 0.33 | 0.95 | 1.06 | 3.06 | 8.31 | 1.09 | 1.30 | 1.28 |
| time (sec) | N/A | 0.033 | 0.006 | 0.006 | 2.995 | 0.894 | 2.204 | 2.445 | 0.137 | 0.160 |

| Problem 398 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 35 | 117 | 142 | 296 | 2730 | 130 | 182 | 149 |
| N.S. | 1 | 1.00 | 0.27 | 0.90 | 1.09 | 2.28 | 21.00 | 1.00 | 1.40 | 1.15 |
| time (sec) | N/A | 0.048 | 0.007 | 0.006 | 3.028 | 0.955 | 2.586 | 2.236 | 0.226 | 0.153 |

| Problem 399 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 80 | 80 | 48 | 45 | 64 | 44 | 4974 | 57 | 64 | 59 |
| N.S. | 1 | 1.00 | 0.60 | 0.56 | 0.80 | 0.55 | 62.18 | 0.71 | 0.80 | 0.74 |
| time (sec) | N/A | 0.019 | 0.027 | 0.004 | 1.339 | 0.901 | 2.977 | 1.069 | 0.048 | 0.024 |

| Problem 400 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 37 | 34 | 47 | 33 | 1326 | 43 | 43 | 51 |
| N.S. | 1 | 1.00 | 0.63 | 0.58 | 0.80 | 0.56 | 22.47 | 0.73 | 0.73 | 0.86 |
| time (sec) | N/A | 0.013 | 0.021 | 0.005 | 1.330 | 0.939 | 1.897 | 1.071 | 0.040 | 0.022 |

| Problem 401 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 26 | 23 | 30 | 22 | 486 | 29 | 29 | 26 |
| N.S. | 1 | 1.00 | 0.68 | 0.61 | 0.79 | 0.58 | 12.79 | 0.76 | 0.76 | 0.68 |
| time (sec) | N/A | 0.009 | 0.012 | 0.003 | 1.340 | 0.906 | 1.261 | 0.979 | 0.029 | 0.014 |

| Problem 402 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 15 | 14 | 14 | 12 | 14 | 14 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.78 | 0.78 | 0.67 | 0.78 | 0.78 | 1.00 |
| time (sec) | N/A | 0.001 | 0.004 | 0.002 | 1.317 | 0.710 | 0.066 | 1.017 | 0.020 | 0.007 |

| Problem 403 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 35 | 83 | 86 | 285 | 160 | 112 | 117 | 113 |
| N.S. | 1 | 1.00 | 0.43 | 1.01 | 1.05 | 3.48 | 1.95 | 1.37 | 1.43 | 1.38 |
| time (sec) | N/A | 0.033 | 0.011 | 0.007 | 3.007 | 0.942 | 1.878 | 2.509 | 0.095 | 0.055 |

| Problem 404 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 36 | 103 | 116 | 328 | 838 | 144 | 133 | 136 |
| N.S. | 1 | 1.00 | 0.35 | 1.00 | 1.13 | 3.18 | 8.14 | 1.40 | 1.29 | 1.32 |
| time (sec) | N/A | 0.034 | 0.006 | 0.007 | 2.989 | 1.061 | 2.236 | 2.429 | 0.175 | 0.139 |

| Problem 405 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 38 | 128 | 159 | 374 | 2744 | 167 | 216 | 160 |
| N.S. | 1 | 1.00 | 0.28 | 0.94 | 1.17 | 2.75 | 20.18 | 1.23 | 1.59 | 1.18 |
| time (sec) | N/A | 0.044 | 0.006 | 0.008 | 3.093 | 0.656 | 2.615 | 2.242 | 0.220 | 0.128 |

| Problem 406 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 46 | 43 | 56 | 42 | 1640 | 49 | 56 | 51 |
| N.S. | 1 | 1.00 | 0.66 | 0.61 | 0.80 | 0.60 | 23.43 | 0.70 | 0.80 | 0.73 |
| time (sec) | N/A | 0.018 | 0.024 | 0.006 | 1.345 | 0.528 | 2.787 | 1.026 | 0.045 | 0.026 |

| Problem 407 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 35 | 32 | 41 | 31 | 600 | 37 | 37 | 45 |
| N.S. | 1 | 1.00 | 0.69 | 0.63 | 0.80 | 0.61 | 11.76 | 0.73 | 0.73 | 0.88 |
| time (sec) | N/A | 0.012 | 0.017 | 0.004 | 1.314 | 0.829 | 1.817 | 0.897 | 0.041 | 0.023 |

| Problem 408 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 23 | 21 | 26 | 19 | 162 | 23 | 25 | 24 |
| N.S. | 1 | 1.00 | 0.72 | 0.66 | 0.81 | 0.59 | 5.06 | 0.72 | 0.78 | 0.75 |
| time (sec) | N/A | 0.007 | 0.011 | 0.003 | 1.335 | 0.904 | 1.190 | 0.891 | 0.029 | 0.011 |

| Problem 409 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 12 | 10 | 12 | 12 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 0.86 | 0.71 | 0.86 | 0.86 | 1.00 |
| time (sec) | N/A | 0.001 | 0.003 | 0.003 | 1.295 | 0.886 | 0.063 | 0.976 | 0.017 | 0.007 |

| Problem 410 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 80 | 80 | 93 | 76 | 77 | 115 | 150 | 78 | 95 | 105 |
| N.S. | 1 | 1.00 | 1.16 | 0.95 | 0.96 | 1.44 | 1.88 | 0.98 | 1.19 | 1.31 |
| time (sec) | N/A | 0.024 | 0.026 | 0.005 | 2.874 | 0.998 | 1.930 | 2.317 | 0.166 | 0.061 |

| Problem 411 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 31 | 95 | 106 | 166 | 830 | 108 | 122 | 128 |
| N.S. | 1 | 1.00 | 0.32 | 0.97 | 1.08 | 1.69 | 8.47 | 1.10 | 1.24 | 1.31 |
| time (sec) | N/A | 0.033 | 0.005 | 0.007 | 2.951 | 0.692 | 2.266 | 2.373 | 0.130 | 0.155 |

| Problem 412 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 33 | 117 | 142 | 162 | 2728 | 130 | 175 | 149 |
| N.S. | 1 | 1.00 | 0.25 | 0.90 | 1.09 | 1.25 | 20.98 | 1.00 | 1.35 | 1.15 |
| time (sec) | N/A | 0.048 | 0.006 | 0.009 | 3.043 | 0.816 | 2.727 | 2.049 | 0.131 | 0.124 |

| Problem 413 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 46 | 43 | 56 | 52 | 1538 | 62 | 56 | 51 |
| N.S. | 1 | 1.00 | 0.66 | 0.61 | 0.80 | 0.74 | 21.97 | 0.89 | 0.80 | 0.73 |
| time (sec) | N/A | 0.018 | 0.022 | 0.004 | 1.358 | 0.859 | 2.891 | 1.103 | 0.053 | 0.026 |

| Problem 414 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 34 | 32 | 41 | 40 | 534 | 46 | 35 | 37 |
| N.S. | 1 | 1.00 | 0.69 | 0.65 | 0.84 | 0.82 | 10.90 | 0.94 | 0.71 | 0.76 |
| time (sec) | N/A | 0.013 | 0.016 | 0.006 | 1.327 | 0.830 | 1.882 | 0.996 | 0.042 | 0.026 |

| Problem 415 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 23 | 20 | 26 | 29 | 41 | 30 | 20 | 23 |
| N.S. | 1 | 1.00 | 0.72 | 0.62 | 0.81 | 0.91 | 1.28 | 0.94 | 0.62 | 0.72 |
| time (sec) | N/A | 0.009 | 0.011 | 0.003 | 1.303 | 0.830 | 0.724 | 0.934 | 0.030 | 0.012 |

| Problem 416 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 20 | 12 | 12 | 12 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 1.43 | 0.86 | 0.86 | 0.86 | 1.00 |
| time (sec) | N/A | 0.001 | 0.004 | 0.002 | 1.353 | 1.055 | 0.067 | 0.764 | 0.021 | 0.009 |

| Problem 417 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 30 | 87 | 88 | 285 | 184 | 89 | 114 | 118 |
| N.S. | 1 | 1.00 | 0.32 | 0.94 | 0.95 | 3.06 | 1.98 | 0.96 | 1.23 | 1.27 |
| time (sec) | N/A | 0.033 | 0.005 | 0.009 | 3.032 | 0.955 | 2.215 | 2.377 | 0.056 | 0.075 |

| Problem 418 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 115 | 31 | 108 | 122 | 407 | 857 | 120 | 173 | 138 |
| N.S. | 1 | 1.02 | 0.27 | 0.96 | 1.08 | 3.60 | 7.58 | 1.06 | 1.53 | 1.22 |
| time (sec) | N/A | 0.044 | 0.006 | 0.013 | 2.993 | 0.829 | 2.525 | 2.399 | 0.068 | 0.168 |

| Problem 419 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 149 | 147 | 33 | 131 | 158 | 407 | 2793 | 140 | 221 | 161 |
| N.S. | 1 | 0.99 | 0.22 | 0.88 | 1.06 | 2.73 | 18.74 | 0.94 | 1.48 | 1.08 |
| time (sec) | N/A | 0.058 | 0.006 | 0.014 | 2.960 | 1.047 | 3.185 | 2.590 | 0.131 | 0.264 |

| Problem 420 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 66 | 87 | 86 | 88 | 138 | 87 | 105 | 102 |
| N.S. | 1 | 1.00 | 0.93 | 1.23 | 1.21 | 1.24 | 1.94 | 1.23 | 1.48 | 1.44 |
| time (sec) | N/A | 0.031 | 0.021 | 0.011 | 3.089 | 0.971 | 2.134 | 1.032 | 0.101 | 0.048 |

| Problem 421 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 68 | 91 | 90 | 92 | 136 | 91 | 108 | 106 |
| N.S. | 1 | 1.00 | 0.93 | 1.25 | 1.23 | 1.26 | 1.86 | 1.25 | 1.48 | 1.45 |
| time (sec) | N/A | 0.031 | 0.022 | 0.004 | 2.995 | 0.865 | 1.889 | 0.767 | 0.128 | 0.050 |

| Problem 422 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 41 | 97 | 94 | 93 | 134 | 95 | 112 | 111 |
| N.S. | 1 | 1.00 | 0.55 | 1.31 | 1.27 | 1.26 | 1.81 | 1.28 | 1.51 | 1.50 |
| time (sec) | N/A | 0.032 | 0.013 | 0.009 | 2.903 | 0.828 | 1.824 | 1.068 | 0.108 | 0.050 |

| Problem 423 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 41 | 101 | 98 | 97 | 139 | 99 | 115 | 115 |
| N.S. | 1 | 1.00 | 0.54 | 1.33 | 1.29 | 1.28 | 1.83 | 1.30 | 1.51 | 1.51 |
| time (sec) | N/A | 0.028 | 0.012 | 0.004 | 2.922 | 0.865 | 1.827 | 1.003 | 0.072 | 0.075 |

| Problem 424 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 95 | 88 | 87 | 86 | 134 | 88 | 101 | 103 |
| N.S. | 1 | 1.00 | 1.32 | 1.22 | 1.21 | 1.19 | 1.86 | 1.22 | 1.40 | 1.43 |
| time (sec) | N/A | 0.024 | 0.025 | 0.009 | 2.877 | 0.756 | 1.857 | 1.003 | 0.141 | 0.052 |

| Problem 425 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 99 | 92 | 91 | 90 | 136 | 92 | 104 | 107 |
| N.S. | 1 | 1.00 | 1.34 | 1.24 | 1.23 | 1.22 | 1.84 | 1.24 | 1.41 | 1.45 |
| time (sec) | N/A | 0.025 | 0.025 | 0.003 | 2.978 | 0.961 | 1.921 | 1.086 | 0.107 | 0.048 |

| Problem 426 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 108 | 96 | 93 | 95 | 134 | 94 | 107 | 110 |
| N.S. | 1 | 1.00 | 1.46 | 1.30 | 1.26 | 1.28 | 1.81 | 1.27 | 1.45 | 1.49 |
| time (sec) | N/A | 0.023 | 0.032 | 0.008 | 2.951 | 0.890 | 1.983 | 0.988 | 0.156 | 0.052 |

| Problem 427 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 112 | 100 | 97 | 99 | 133 | 98 | 110 | 114 |
| N.S. | 1 | 1.00 | 1.47 | 1.32 | 1.28 | 1.30 | 1.75 | 1.29 | 1.45 | 1.50 |
| time (sec) | N/A | 0.023 | 0.031 | 0.003 | 2.889 | 1.153 | 1.895 | 1.074 | 0.160 | 0.055 |

| Problem 428 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 22 | 31 | 25 | 33 | 87 | 43 | 30 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.24 | 1.00 | 1.32 | 3.48 | 1.72 | 1.20 | 0.00 |
| time (sec) | N/A | 0.008 | 0.015 | 0.003 | 1.339 | 0.959 | 0.299 | 1.006 | 0.313 | 0.014 |

| Problem 429 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 17 | 14 | 13 | 18 | 19 | 13 | 13 | 21 |
| N.S. | 1 | 1.00 | 0.81 | 0.67 | 0.62 | 0.86 | 0.90 | 0.62 | 0.62 | 1.00 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 1.334 | 0.846 | 1.589 | 0.828 | 0.091 | 0.011 |

| Problem 430 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 17 | 14 | 13 | 18 | 19 | 13 | 13 | 21 |
| N.S. | 1 | 1.00 | 0.81 | 0.67 | 0.62 | 0.86 | 0.90 | 0.62 | 0.62 | 1.00 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 1.293 | 1.006 | 0.547 | 0.880 | 0.027 | 0.009 |

| Problem 431 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 17 | 14 | 13 | 16 | 19 | 13 | 13 | 21 |
| N.S. | 1 | 1.00 | 0.81 | 0.67 | 0.62 | 0.76 | 0.90 | 0.62 | 0.62 | 1.00 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 1.338 | 0.992 | 1.621 | 0.859 | 0.025 | 0.008 |

| Problem 432 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 16 | 13 | 13 | 12 | 17 | 13 | 12 | 20 |
| N.S. | 1 | 1.00 | 0.84 | 0.68 | 0.68 | 0.63 | 0.89 | 0.68 | 0.63 | 1.05 |
| time (sec) | N/A | 0.004 | 0.005 | 0.004 | 1.316 | 0.921 | 0.156 | 1.081 | 0.025 | 0.008 |

| Problem 433 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 14 | 12 | 13 | 12 | 15 | 13 | 11 | 14 |
| N.S. | 1 | 1.00 | 0.82 | 0.71 | 0.76 | 0.71 | 0.88 | 0.76 | 0.65 | 0.82 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 1.328 | 0.548 | 0.350 | 0.920 | 0.028 | 0.010 |

| Problem 434 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 15 | 12 | 11 | 11 | 19 | 11 | 13 | 15 |
| N.S. | 1 | 1.00 | 0.79 | 0.63 | 0.58 | 0.58 | 1.00 | 0.58 | 0.68 | 0.79 |
| time (sec) | N/A | 0.004 | 0.005 | 0.002 | 1.362 | 0.828 | 0.562 | 0.966 | 0.027 | 0.012 |

| Problem 435 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 38 | 87 | 43 | 85 | 299 | 117 | 93 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 2.02 | 1.00 | 1.98 | 6.95 | 2.72 | 2.16 | 0.00 |
| time (sec) | N/A | 0.014 | 0.033 | 0.005 | 1.357 | 0.776 | 0.535 | 1.102 | 0.416 | 0.022 |

| Problem 436 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 28 | 25 | 24 | 29 | 34 | 24 | 24 | 34 |
| N.S. | 1 | 1.00 | 0.78 | 0.69 | 0.67 | 0.81 | 0.94 | 0.67 | 0.67 | 0.94 |
| time (sec) | N/A | 0.007 | 0.008 | 0.003 | 1.272 | 0.708 | 2.597 | 1.156 | 0.103 | 0.014 |

| Problem 437 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 28 | 25 | 24 | 29 | 34 | 24 | 24 | 34 |
| N.S. | 1 | 1.00 | 0.78 | 0.69 | 0.67 | 0.81 | 0.94 | 0.67 | 0.67 | 0.94 |
| time (sec) | N/A | 0.007 | 0.007 | 0.004 | 1.318 | 0.882 | 1.031 | 1.118 | 0.035 | 0.013 |

| Problem 438 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 28 | 25 | 24 | 27 | 0 | 24 | 24 | 34 |
| N.S. | 1 | 1.00 | 0.78 | 0.69 | 0.67 | 0.75 | 0.00 | 0.67 | 0.67 | 0.94 |
| time (sec) | N/A | 0.007 | 0.007 | 0.004 | 1.294 | 0.810 | 0.000 | 0.899 | 0.042 | 0.011 |

| Problem 439 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 28 | 25 | 24 | 24 | 32 | 24 | 24 | 34 |
| N.S. | 1 | 1.00 | 0.82 | 0.74 | 0.71 | 0.71 | 0.94 | 0.71 | 0.71 | 1.00 |
| time (sec) | N/A | 0.007 | 0.007 | 0.004 | 1.381 | 0.839 | 0.262 | 0.917 | 0.037 | 0.011 |

| Problem 440 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 27 | 25 | 24 | 23 | 31 | 24 | 24 | 27 |
| N.S. | 1 | 1.00 | 0.84 | 0.78 | 0.75 | 0.72 | 0.97 | 0.75 | 0.75 | 0.84 |
| time (sec) | N/A | 0.007 | 0.008 | 0.004 | 1.318 | 0.891 | 0.430 | 1.016 | 0.035 | 0.014 |

| Problem 441 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 26 | 23 | 23 | 24 | 31 | 23 | 24 | 28 |
| N.S. | 1 | 1.00 | 0.81 | 0.72 | 0.72 | 0.75 | 0.97 | 0.72 | 0.75 | 0.88 |
| time (sec) | N/A | 0.007 | 0.009 | 0.004 | 1.289 | 0.805 | 0.593 | 1.002 | 0.030 | 0.017 |

| Problem 442 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 54 | 170 | 61 | 157 | 663 | 224 | 167 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 2.79 | 1.00 | 2.57 | 10.87 | 3.67 | 2.74 | 0.00 |
| time (sec) | N/A | 0.020 | 0.032 | 0.004 | 1.358 | 0.892 | 0.884 | 1.027 | 0.389 | 0.023 |

| Problem 443 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 39 | 36 | 35 | 40 | 49 | 35 | 35 | 47 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.69 | 0.78 | 0.96 | 0.69 | 0.69 | 0.92 |
| time (sec) | N/A | 0.011 | 0.010 | 0.004 | 1.343 | 0.810 | 3.881 | 0.990 | 0.045 | 0.015 |

| Problem 444 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 39 | 36 | 35 | 40 | 49 | 35 | 35 | 47 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.69 | 0.78 | 0.96 | 0.69 | 0.69 | 0.92 |
| time (sec) | N/A | 0.012 | 0.010 | 0.004 | 1.339 | 0.884 | 1.703 | 0.991 | 0.046 | 0.015 |

| Problem 445 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 39 | 36 | 35 | 38 | 0 | 35 | 35 | 47 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.69 | 0.75 | 0.00 | 0.69 | 0.69 | 0.92 |
| time (sec) | N/A | 0.011 | 0.010 | 0.005 | 1.256 | 0.822 | 0.000 | 1.082 | 0.042 | 0.013 |

| Problem 446 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 39 | 36 | 35 | 35 | 46 | 35 | 35 | 47 |
| N.S. | 1 | 1.00 | 0.83 | 0.77 | 0.74 | 0.74 | 0.98 | 0.74 | 0.74 | 1.00 |
| time (sec) | N/A | 0.011 | 0.010 | 0.004 | 1.352 | 0.768 | 0.445 | 0.939 | 0.043 | 0.013 |

| Problem 447 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 38 | 36 | 35 | 34 | 44 | 35 | 35 | 38 |
| N.S. | 1 | 1.00 | 0.84 | 0.80 | 0.78 | 0.76 | 0.98 | 0.78 | 0.78 | 0.84 |
| time (sec) | N/A | 0.011 | 0.011 | 0.005 | 1.356 | 0.801 | 0.639 | 1.032 | 0.047 | 0.016 |

| Problem 448 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 38 | 34 | 34 | 34 | 46 | 34 | 35 | 38 |
| N.S. | 1 | 1.00 | 0.81 | 0.72 | 0.72 | 0.72 | 0.98 | 0.72 | 0.74 | 0.81 |
| time (sec) | N/A | 0.011 | 0.012 | 0.005 | 1.368 | 0.904 | 0.777 | 1.065 | 0.039 | 0.019 |

| Problem 449 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 61 | 54 | 54 | 132 | 121 | 59 | 48 | 67 |
| N.S. | 1 | 1.00 | 0.90 | 0.79 | 0.79 | 1.94 | 1.78 | 0.87 | 0.71 | 0.99 |
| time (sec) | N/A | 0.032 | 0.026 | 0.009 | 3.010 | 0.966 | 7.295 | 0.875 | 0.057 | 0.047 |

| Problem 450 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 49 | 43 | 42 | 103 | 105 | 45 | 37 | 53 |
| N.S. | 1 | 1.00 | 0.92 | 0.81 | 0.79 | 1.94 | 1.98 | 0.85 | 0.70 | 1.00 |
| time (sec) | N/A | 0.017 | 0.019 | 0.007 | 2.941 | 0.703 | 1.926 | 1.238 | 0.052 | 0.037 |

| Problem 451 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 40 | 32 | 31 | 85 | 92 | 31 | 28 | 40 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.78 | 2.12 | 2.30 | 0.78 | 0.70 | 1.00 |
| time (sec) | N/A | 0.012 | 0.010 | 0.006 | 2.971 | 0.982 | 0.724 | 0.989 | 0.041 | 0.021 |

| Problem 452 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 29 | 19 | 18 | 68 | 94 | 18 | 19 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 0.66 | 0.62 | 2.34 | 3.24 | 0.62 | 0.66 | 1.00 |
| time (sec) | N/A | 0.008 | 0.005 | 0.006 | 2.926 | 0.907 | 1.291 | 0.947 | 0.044 | 0.016 |

| Problem 453 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 25 | 32 | 31 | 93 | 102 | 31 | 28 | 40 |
| N.S. | 1 | 1.00 | 0.62 | 0.80 | 0.78 | 2.32 | 2.55 | 0.78 | 0.70 | 1.00 |
| time (sec) | N/A | 0.013 | 0.005 | 0.008 | 2.883 | 0.954 | 2.773 | 0.997 | 0.044 | 0.027 |

| Problem 454 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 27 | 43 | 41 | 118 | 121 | 41 | 38 | 48 |
| N.S. | 1 | 1.00 | 0.51 | 0.81 | 0.77 | 2.23 | 2.28 | 0.77 | 0.72 | 0.91 |
| time (sec) | N/A | 0.018 | 0.005 | 0.011 | 3.017 | 0.886 | 7.834 | 1.157 | 0.102 | 0.039 |

| Problem 455 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 27 | 54 | 52 | 144 | 139 | 52 | 49 | 61 |
| N.S. | 1 | 1.00 | 0.40 | 0.79 | 0.76 | 2.12 | 2.04 | 0.76 | 0.72 | 0.90 |
| time (sec) | N/A | 0.023 | 0.006 | 0.010 | 2.915 | 1.040 | 24.823 | 0.994 | 0.110 | 0.048 |

| Problem 456 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 27 | 61 | 63 | 161 | 479 | 65 | 58 | 74 |
| N.S. | 1 | 1.00 | 0.39 | 0.87 | 0.90 | 2.30 | 6.84 | 0.93 | 0.83 | 1.06 |
| time (sec) | N/A | 0.022 | 0.005 | 0.011 | 2.964 | 0.925 | 24.566 | 0.928 | 0.114 | 0.078 |

| Problem 457 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 27 | 47 | 49 | 134 | 411 | 46 | 46 | 58 |
| N.S. | 1 | 1.00 | 0.47 | 0.82 | 0.86 | 2.35 | 7.21 | 0.81 | 0.81 | 1.02 |
| time (sec) | N/A | 0.017 | 0.004 | 0.011 | 2.907 | 0.826 | 9.176 | 0.956 | 0.124 | 0.072 |

| Problem 458 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 46 | 37 | 37 | 115 | 337 | 36 | 34 | 46 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.80 | 2.50 | 7.33 | 0.78 | 0.74 | 1.00 |
| time (sec) | N/A | 0.013 | 0.020 | 0.010 | 2.936 | 0.893 | 4.445 | 0.901 | 0.040 | 0.059 |

| Problem 459 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 45 | 36 | 35 | 116 | 328 | 35 | 33 | 45 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.78 | 2.58 | 7.29 | 0.78 | 0.73 | 1.00 |
| time (sec) | N/A | 0.013 | 0.017 | 0.009 | 2.925 | 0.873 | 7.527 | 0.894 | 0.095 | 0.054 |

| Problem 460 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 25 | 48 | 51 | 147 | 434 | 49 | 48 | 54 |
| N.S. | 1 | 1.00 | 0.45 | 0.86 | 0.91 | 2.62 | 7.75 | 0.88 | 0.86 | 0.96 |
| time (sec) | N/A | 0.017 | 0.005 | 0.013 | 2.943 | 0.957 | 17.734 | 1.006 | 0.124 | 0.073 |

| Problem 461 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 27 | 60 | 64 | 184 | 507 | 58 | 58 | 68 |
| N.S. | 1 | 1.00 | 0.39 | 0.87 | 0.93 | 2.67 | 7.35 | 0.84 | 0.84 | 0.99 |
| time (sec) | N/A | 0.022 | 0.005 | 0.015 | 2.882 | 1.005 | 50.524 | 0.957 | 0.150 | 0.076 |

| Problem 462 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 27 | 79 | 86 | 227 | 906 | 77 | 81 | 89 |
| N.S. | 1 | 1.00 | 0.28 | 0.83 | 0.91 | 2.39 | 9.54 | 0.81 | 0.85 | 0.94 |
| time (sec) | N/A | 0.033 | 0.005 | 0.015 | 3.067 | 0.909 | 135.242 | 1.027 | 0.122 | 0.132 |

| Problem 463 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 27 | 66 | 73 | 200 | 816 | 59 | 69 | 76 |
| N.S. | 1 | 1.00 | 0.33 | 0.80 | 0.89 | 2.44 | 9.95 | 0.72 | 0.84 | 0.93 |
| time (sec) | N/A | 0.023 | 0.005 | 0.015 | 2.941 | 0.593 | 53.287 | 0.953 | 0.143 | 0.119 |

| Problem 464 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 59 | 50 | 61 | 185 | 726 | 47 | 58 | 63 |
| N.S. | 1 | 1.00 | 0.84 | 0.71 | 0.87 | 2.64 | 10.37 | 0.67 | 0.83 | 0.90 |
| time (sec) | N/A | 0.019 | 0.035 | 0.013 | 2.960 | 1.009 | 29.374 | 0.908 | 0.131 | 0.117 |

| Problem 465 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 27 | 52 | 64 | 186 | 721 | 52 | 56 | 60 |
| N.S. | 1 | 1.00 | 0.37 | 0.71 | 0.88 | 2.55 | 9.88 | 0.71 | 0.77 | 0.82 |
| time (sec) | N/A | 0.019 | 0.005 | 0.009 | 3.012 | 0.919 | 15.275 | 1.066 | 0.125 | 0.107 |

| Problem 466 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 25 | 53 | 60 | 186 | 712 | 47 | 57 | 63 |
| N.S. | 1 | 1.00 | 0.36 | 0.76 | 0.86 | 2.66 | 10.17 | 0.67 | 0.81 | 0.90 |
| time (sec) | N/A | 0.019 | 0.005 | 0.009 | 2.956 | 0.984 | 25.687 | 0.862 | 0.126 | 0.077 |

| Problem 467 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 25 | 66 | 73 | 214 | 865 | 59 | 70 | 70 |
| N.S. | 1 | 1.00 | 0.30 | 0.80 | 0.89 | 2.61 | 10.55 | 0.72 | 0.85 | 0.85 |
| time (sec) | N/A | 0.025 | 0.005 | 0.015 | 2.992 | 0.719 | 54.354 | 1.142 | 0.149 | 0.118 |

| Problem 468 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 27 | 79 | 86 | 250 | 962 | 71 | 80 | 81 |
| N.S. | 1 | 1.00 | 0.28 | 0.83 | 0.91 | 2.63 | 10.13 | 0.75 | 0.84 | 0.85 |
| time (sec) | N/A | 0.029 | 0.005 | 0.016 | 2.981 | 0.843 | 138.083 | 1.101 | 0.155 | 0.120 |

| Problem 469 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 61 | 54 | 70 | 131 | 116 | 61 | 51 | 67 |
| N.S. | 1 | 1.00 | 0.90 | 0.79 | 1.03 | 1.93 | 1.71 | 0.90 | 0.75 | 0.99 |
| time (sec) | N/A | 0.024 | 0.026 | 0.006 | 2.942 | 1.084 | 7.100 | 1.034 | 0.147 | 0.048 |

| Problem 470 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 49 | 43 | 58 | 103 | 100 | 47 | 37 | 53 |
| N.S. | 1 | 1.00 | 0.92 | 0.81 | 1.09 | 1.94 | 1.89 | 0.89 | 0.70 | 1.00 |
| time (sec) | N/A | 0.018 | 0.019 | 0.008 | 2.920 | 0.669 | 1.870 | 0.940 | 0.114 | 0.037 |

| Problem 471 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 40 | 32 | 47 | 83 | 87 | 33 | 28 | 40 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 1.18 | 2.08 | 2.18 | 0.82 | 0.70 | 1.00 |
| time (sec) | N/A | 0.014 | 0.010 | 0.004 | 3.075 | 0.758 | 0.708 | 1.045 | 0.112 | 0.025 |

| Problem 472 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 29 | 19 | 34 | 67 | 88 | 20 | 19 | 29 |
| N.S. | 1 | 1.00 | 1.00 | 0.66 | 1.17 | 2.31 | 3.03 | 0.69 | 0.66 | 1.00 |
| time (sec) | N/A | 0.011 | 0.006 | 0.005 | 3.026 | 0.980 | 1.253 | 1.005 | 0.125 | 0.018 |

| Problem 473 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 24 | 32 | 47 | 91 | 94 | 33 | 28 | 40 |
| N.S. | 1 | 1.00 | 0.60 | 0.80 | 1.18 | 2.28 | 2.35 | 0.82 | 0.70 | 1.00 |
| time (sec) | N/A | 0.014 | 0.004 | 0.007 | 2.867 | 0.698 | 2.757 | 1.019 | 0.056 | 0.027 |

| Problem 474 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 26 | 43 | 55 | 113 | 112 | 41 | 37 | 48 |
| N.S. | 1 | 1.00 | 0.49 | 0.81 | 1.04 | 2.13 | 2.11 | 0.77 | 0.70 | 0.91 |
| time (sec) | N/A | 0.018 | 0.005 | 0.010 | 2.960 | 1.061 | 7.665 | 0.981 | 0.122 | 0.044 |

| Problem 475 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 26 | 54 | 68 | 143 | 131 | 54 | 48 | 61 |
| N.S. | 1 | 1.00 | 0.38 | 0.79 | 1.00 | 2.10 | 1.93 | 0.79 | 0.71 | 0.90 |
| time (sec) | N/A | 0.022 | 0.005 | 0.010 | 2.975 | 0.898 | 24.442 | 0.968 | 0.129 | 0.047 |

| Problem 476 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 26 | 61 | 81 | 167 | 444 | 69 | 61 | 76 |
| N.S. | 1 | 1.00 | 0.37 | 0.87 | 1.16 | 2.39 | 6.34 | 0.99 | 0.87 | 1.09 |
| time (sec) | N/A | 0.025 | 0.005 | 0.012 | 3.057 | 0.699 | 24.751 | 0.984 | 0.071 | 0.075 |

| Problem 477 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 26 | 49 | 68 | 138 | 381 | 51 | 47 | 56 |
| N.S. | 1 | 1.00 | 0.46 | 0.86 | 1.19 | 2.42 | 6.68 | 0.89 | 0.82 | 0.98 |
| time (sec) | N/A | 0.018 | 0.005 | 0.011 | 2.919 | 0.576 | 9.127 | 0.978 | 0.115 | 0.065 |

| Problem 478 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 61 | 40 | 56 | 123 | 311 | 40 | 35 | 49 |
| N.S. | 1 | 1.00 | 1.30 | 0.85 | 1.19 | 2.62 | 6.62 | 0.85 | 0.74 | 1.04 |
| time (sec) | N/A | 0.015 | 0.016 | 0.009 | 2.988 | 0.839 | 4.428 | 1.064 | 0.112 | 0.057 |

| Problem 479 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 46 | 39 | 56 | 122 | 303 | 41 | 34 | 46 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 1.22 | 2.65 | 6.59 | 0.89 | 0.74 | 1.00 |
| time (sec) | N/A | 0.014 | 0.020 | 0.008 | 2.970 | 0.577 | 7.413 | 0.883 | 0.055 | 0.052 |

| Problem 480 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 24 | 49 | 69 | 151 | 403 | 52 | 49 | 55 |
| N.S. | 1 | 1.00 | 0.42 | 0.86 | 1.21 | 2.65 | 7.07 | 0.91 | 0.86 | 0.96 |
| time (sec) | N/A | 0.018 | 0.006 | 0.013 | 3.005 | 0.848 | 17.544 | 1.006 | 0.073 | 0.064 |

| Problem 481 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 26 | 60 | 82 | 187 | 471 | 61 | 60 | 69 |
| N.S. | 1 | 1.00 | 0.37 | 0.86 | 1.17 | 2.67 | 6.73 | 0.87 | 0.86 | 0.99 |
| time (sec) | N/A | 0.022 | 0.006 | 0.016 | 2.998 | 0.998 | 50.249 | 0.930 | 0.138 | 0.080 |

| Problem 482 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 26 | 70 | 103 | 227 | 840 | 81 | 83 | 91 |
| N.S. | 1 | 1.00 | 0.27 | 0.72 | 1.06 | 2.34 | 8.66 | 0.84 | 0.86 | 0.94 |
| time (sec) | N/A | 0.030 | 0.006 | 0.015 | 2.992 | 0.786 | 136.150 | 1.033 | 0.141 | 0.124 |

| Problem 483 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 26 | 58 | 90 | 199 | 756 | 63 | 69 | 78 |
| N.S. | 1 | 1.00 | 0.31 | 0.69 | 1.07 | 2.37 | 9.00 | 0.75 | 0.82 | 0.93 |
| time (sec) | N/A | 0.026 | 0.005 | 0.012 | 2.996 | 0.651 | 53.453 | 0.951 | 0.065 | 0.119 |

| Problem 484 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 60 | 52 | 78 | 186 | 673 | 51 | 58 | 65 |
| N.S. | 1 | 1.00 | 0.83 | 0.72 | 1.08 | 2.58 | 9.35 | 0.71 | 0.81 | 0.90 |
| time (sec) | N/A | 0.021 | 0.036 | 0.013 | 2.989 | 0.708 | 29.248 | 0.916 | 0.143 | 0.117 |

| Problem 485 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 75 | 75 | 26 | 54 | 80 | 183 | 668 | 55 | 57 | 60 |
| N.S. | 1 | 1.00 | 0.35 | 0.72 | 1.07 | 2.44 | 8.91 | 0.73 | 0.76 | 0.80 |
| time (sec) | N/A | 0.020 | 0.005 | 0.010 | 2.848 | 1.046 | 15.189 | 0.933 | 0.137 | 0.099 |

| Problem 486 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 24 | 63 | 77 | 185 | 660 | 51 | 58 | 64 |
| N.S. | 1 | 1.00 | 0.33 | 0.88 | 1.07 | 2.57 | 9.17 | 0.71 | 0.81 | 0.89 |
| time (sec) | N/A | 0.020 | 0.005 | 0.009 | 2.884 | 1.003 | 25.674 | 1.155 | 0.135 | 0.086 |

| Problem 487 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 24 | 58 | 90 | 213 | 802 | 63 | 69 | 71 |
| N.S. | 1 | 1.00 | 0.29 | 0.69 | 1.07 | 2.54 | 9.55 | 0.75 | 0.82 | 0.85 |
| time (sec) | N/A | 0.024 | 0.005 | 0.015 | 2.930 | 0.971 | 54.560 | 1.046 | 0.155 | 0.110 |

| Problem 488 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 26 | 69 | 103 | 249 | 892 | 73 | 80 | 82 |
| N.S. | 1 | 1.00 | 0.27 | 0.71 | 1.06 | 2.57 | 9.20 | 0.75 | 0.82 | 0.85 |
| time (sec) | N/A | 0.030 | 0.004 | 0.017 | 2.950 | 1.014 | 138.884 | 1.042 | 0.165 | 0.122 |

| Problem 489 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 122 | 122 | 96 | 120 | 178 | 162 | 153 | 0 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.79 | 0.98 | 1.46 | 1.33 | 1.25 | 0.00 | -0.01 | 0.78 |
| time (sec) | N/A | 0.042 | 0.185 | 0.009 | 2.950 | 0.948 | 11.697 | 0.000 | 0.000 | 0.145 |

| Problem 490 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 85 | 102 | 146 | 141 | 122 | 0 | -1 | 82 |
| N.S. | 1 | 1.00 | 0.87 | 1.04 | 1.49 | 1.44 | 1.24 | 0.00 | -0.01 | 0.84 |
| time (sec) | N/A | 0.031 | 0.107 | 0.005 | 3.044 | 0.537 | 6.383 | 0.000 | 0.000 | 0.095 |

| Problem 491 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 72 | 81 | 108 | 114 | 97 | 0 | 52 | 68 |
| N.S. | 1 | 1.00 | 0.97 | 1.09 | 1.46 | 1.54 | 1.31 | 0.00 | 0.70 | 0.92 |
| time (sec) | N/A | 0.023 | 0.110 | 0.006 | 2.962 | 0.906 | 3.570 | 0.000 | 0.150 | 0.065 |

| Problem 492 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 62 | 62 | 70 | 93 | 42 | 0 | 41 | 47 |
| N.S. | 1 | 1.00 | 1.41 | 1.41 | 1.59 | 2.11 | 0.95 | 0.00 | 0.93 | 1.07 |
| time (sec) | N/A | 0.017 | 0.086 | 0.004 | 2.992 | 0.947 | 1.916 | 0.000 | 0.683 | 0.058 |

| Problem 493 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 64 | 61 | 54 | 89 | 68 | 0 | -1 | 47 |
| N.S. | 1 | 1.00 | 1.42 | 1.36 | 1.20 | 1.98 | 1.51 | 0.00 | -0.02 | 1.04 |
| time (sec) | N/A | 0.017 | 0.095 | 0.037 | 3.004 | 0.929 | 1.561 | 0.000 | 0.000 | 0.071 |

| Problem 494 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 16 | 15 | 15 | 41 | 33 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.76 | 0.71 | 0.71 | 1.95 | 1.57 | 1.00 | 1.00 |
| time (sec) | N/A | 0.002 | 0.007 | 0.004 | 1.357 | 1.006 | 1.461 | 1.327 | 0.237 | 0.020 |

| Problem 495 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 29 | 24 | 31 | 34 | 65 | 50 | 32 | 40 |
| N.S. | 1 | 1.00 | 0.66 | 0.55 | 0.70 | 0.77 | 1.48 | 1.14 | 0.73 | 0.91 |
| time (sec) | N/A | 0.005 | 0.009 | 0.004 | 1.287 | 0.694 | 4.875 | 1.102 | 0.255 | 0.084 |

| Problem 496 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 40 | 35 | 46 | 45 | 347 | 66 | 43 | 51 |
| N.S. | 1 | 1.00 | 0.59 | 0.51 | 0.68 | 0.66 | 5.10 | 0.97 | 0.63 | 0.75 |
| time (sec) | N/A | 0.010 | 0.011 | 0.005 | 1.354 | 0.760 | 13.772 | 0.974 | 0.264 | 0.092 |

| Problem 497 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 127 | 127 | 98 | 127 | 170 | 164 | 323 | 0 | -1 | 104 |
| N.S. | 1 | 1.00 | 0.77 | 1.00 | 1.34 | 1.29 | 2.54 | 0.00 | -0.01 | 0.82 |
| time (sec) | N/A | 0.042 | 0.141 | 0.010 | 3.010 | 0.950 | 11.648 | 0.000 | 0.000 | 0.135 |

| Problem 498 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 102 | 102 | 87 | 108 | 135 | 142 | 260 | 0 | -1 | 91 |
| N.S. | 1 | 1.00 | 0.85 | 1.06 | 1.32 | 1.39 | 2.55 | 0.00 | -0.01 | 0.89 |
| time (sec) | N/A | 0.030 | 0.117 | 0.006 | 2.990 | 1.022 | 6.325 | 0.000 | 0.000 | 0.116 |

| Problem 499 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 77 | 77 | 75 | 86 | 95 | 118 | 207 | 0 | 58 | 78 |
| N.S. | 1 | 1.00 | 0.97 | 1.12 | 1.23 | 1.53 | 2.69 | 0.00 | 0.75 | 1.01 |
| time (sec) | N/A | 0.024 | 0.097 | 0.006 | 2.974 | 0.904 | 3.595 | 0.000 | 0.083 | 0.088 |

| Problem 500 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 65 | 66 | 52 | 94 | 119 | 0 | 43 | 55 |
| N.S. | 1 | 1.00 | 1.41 | 1.43 | 1.13 | 2.04 | 2.59 | 0.00 | 0.93 | 1.20 |
| time (sec) | N/A | 0.017 | 0.090 | 0.004 | 2.899 | 0.982 | 1.959 | 0.000 | 0.593 | 0.070 |

| Problem 501 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 69 | 0 | 35 | 91 | 148 | 0 | -1 | 53 |
| N.S. | 1 | 1.00 | 1.47 | 0.00 | 0.74 | 1.94 | 3.15 | 0.00 | -0.02 | 1.13 |
| time (sec) | N/A | 0.016 | 0.060 | 0.032 | 2.928 | 0.747 | 1.701 | 0.000 | 0.000 | 0.087 |

| Problem 502 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 17 | 16 | 23 | 88 | 42 | 21 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.73 | 1.05 | 4.00 | 1.91 | 0.95 | 1.00 |
| time (sec) | N/A | 0.002 | 0.007 | 0.004 | 1.315 | 1.023 | 1.549 | 1.396 | 0.243 | 0.023 |

| Problem 503 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 30 | 25 | 33 | 34 | 241 | 61 | 32 | 40 |
| N.S. | 1 | 1.00 | 0.65 | 0.54 | 0.72 | 0.74 | 5.24 | 1.33 | 0.70 | 0.87 |
| time (sec) | N/A | 0.005 | 0.009 | 0.005 | 1.336 | 0.865 | 5.006 | 1.384 | 0.253 | 0.104 |

| Problem 504 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 41 | 36 | 49 | 46 | 707 | 79 | 43 | 52 |
| N.S. | 1 | 1.00 | 0.58 | 0.51 | 0.69 | 0.65 | 9.96 | 1.11 | 0.61 | 0.73 |
| time (sec) | N/A | 0.010 | 0.011 | 0.005 | 1.351 | 0.957 | 26.813 | 1.336 | 0.269 | 0.114 |

| Problem 505 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 108 | 108 | 70 | 108 | 163 | 140 | 117 | 0 | -1 | 85 |
| N.S. | 1 | 1.00 | 0.65 | 1.00 | 1.51 | 1.30 | 1.08 | 0.00 | -0.01 | 0.79 |
| time (sec) | N/A | 0.032 | 0.047 | 0.008 | 3.030 | 0.917 | 10.124 | 0.000 | 0.000 | 0.096 |

| Problem 506 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 58 | 93 | 134 | 121 | 90 | 0 | -1 | 72 |
| N.S. | 1 | 1.00 | 0.69 | 1.11 | 1.60 | 1.44 | 1.07 | 0.00 | -0.01 | 0.86 |
| time (sec) | N/A | 0.020 | 0.034 | 0.005 | 3.016 | 0.918 | 5.223 | 0.000 | 0.000 | 0.088 |

| Problem 507 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 51 | 75 | 98 | 101 | 71 | 0 | 46 | 59 |
| N.S. | 1 | 1.00 | 0.80 | 1.17 | 1.53 | 1.58 | 1.11 | 0.00 | 0.72 | 0.92 |
| time (sec) | N/A | 0.015 | 0.027 | 0.004 | 2.906 | 1.136 | 2.912 | 0.000 | 0.098 | 0.055 |

| Problem 508 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 40 | 58 | 68 | 86 | 37 | 0 | 40 | 46 |
| N.S. | 1 | 1.00 | 1.00 | 1.45 | 1.70 | 2.15 | 0.92 | 0.00 | 1.00 | 1.15 |
| time (sec) | N/A | 0.008 | 0.013 | 0.004 | 2.959 | 1.198 | 1.648 | 0.000 | 0.617 | 0.054 |

| Problem 509 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 41 | 59 | 54 | 87 | 48 | 0 | -1 | 47 |
| N.S. | 1 | 1.00 | 1.00 | 1.44 | 1.32 | 2.12 | 1.17 | 0.00 | -0.02 | 1.15 |
| time (sec) | N/A | 0.009 | 0.012 | 0.024 | 2.951 | 1.025 | 1.429 | 0.000 | 0.000 | 0.066 |

| Problem 510 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 13 | 12 | 12 | 37 | 29 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 0.72 | 0.67 | 0.67 | 2.06 | 1.61 | 1.00 | 1.00 |
| time (sec) | N/A | 0.001 | 0.005 | 0.003 | 1.314 | 0.781 | 1.451 | 1.152 | 0.208 | 0.023 |

| Problem 511 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 23 | 18 | 26 | 25 | 56 | 42 | 26 | 31 |
| N.S. | 1 | 1.00 | 0.61 | 0.47 | 0.68 | 0.66 | 1.47 | 1.11 | 0.68 | 0.82 |
| time (sec) | N/A | 0.004 | 0.007 | 0.004 | 1.317 | 1.038 | 4.717 | 1.092 | 0.215 | 0.071 |

| Problem 512 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 32 | 27 | 41 | 34 | 270 | 55 | 34 | 40 |
| N.S. | 1 | 1.00 | 0.54 | 0.46 | 0.69 | 0.58 | 4.58 | 0.93 | 0.58 | 0.68 |
| time (sec) | N/A | 0.008 | 0.010 | 0.003 | 1.274 | 1.253 | 13.796 | 1.111 | 0.224 | 0.076 |

| Problem 513 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 71 | 116 | 147 | 141 | 252 | 0 | -1 | 94 |
| N.S. | 1 | 1.00 | 0.63 | 1.04 | 1.31 | 1.26 | 2.25 | 0.00 | -0.01 | 0.84 |
| time (sec) | N/A | 0.029 | 0.044 | 0.009 | 2.932 | 1.043 | 9.916 | 0.000 | 0.000 | 0.129 |

| Problem 514 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 60 | 100 | 117 | 125 | 196 | 0 | -1 | 81 |
| N.S. | 1 | 1.00 | 0.69 | 1.15 | 1.34 | 1.44 | 2.25 | 0.00 | -0.01 | 0.93 |
| time (sec) | N/A | 0.023 | 0.035 | 0.005 | 2.976 | 1.030 | 5.294 | 0.000 | 0.000 | 0.110 |

| Problem 515 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 51 | 81 | 81 | 107 | 156 | 0 | 53 | 70 |
| N.S. | 1 | 1.00 | 0.78 | 1.25 | 1.25 | 1.65 | 2.40 | 0.00 | 0.82 | 1.08 |
| time (sec) | N/A | 0.015 | 0.029 | 0.005 | 2.953 | 1.198 | 2.942 | 0.000 | 0.104 | 0.076 |

| Problem 516 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 41 | 63 | 49 | 89 | 121 | 0 | 42 | 55 |
| N.S. | 1 | 1.00 | 1.00 | 1.54 | 1.20 | 2.17 | 2.95 | 0.00 | 1.02 | 1.34 |
| time (sec) | N/A | 0.008 | 0.014 | 0.003 | 2.912 | 1.218 | 1.711 | 0.000 | 0.562 | 0.064 |

| Problem 517 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | C | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 42 | 90 | 35 | 90 | 136 | 0 | -1 | 53 |
| N.S. | 1 | 1.00 | 1.00 | 2.14 | 0.83 | 2.14 | 3.24 | 0.00 | -0.02 | 1.26 |
| time (sec) | N/A | 0.009 | 0.014 | 0.036 | 3.013 | 1.235 | 1.580 | 0.000 | 0.000 | 0.077 |

| Problem 518 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 14 | 13 | 18 | 82 | 35 | 18 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.74 | 0.68 | 0.95 | 4.32 | 1.84 | 0.95 | 1.26 |
| time (sec) | N/A | 0.001 | 0.005 | 0.003 | 1.285 | 1.292 | 1.506 | 1.116 | 0.224 | 0.072 |

| Problem 519 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 24 | 19 | 28 | 25 | 194 | 48 | 26 | 31 |
| N.S. | 1 | 1.00 | 0.60 | 0.48 | 0.70 | 0.62 | 4.85 | 1.20 | 0.65 | 0.78 |
| time (sec) | N/A | 0.004 | 0.008 | 0.004 | 1.334 | 0.667 | 4.909 | 0.853 | 0.219 | 0.084 |

| Problem 520 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 33 | 28 | 44 | 35 | 554 | 61 | 34 | 41 |
| N.S. | 1 | 1.00 | 0.53 | 0.45 | 0.71 | 0.56 | 8.94 | 0.98 | 0.55 | 0.66 |
| time (sec) | N/A | 0.008 | 0.010 | 0.003 | 1.312 | 0.904 | 24.598 | 0.975 | 0.224 | 0.091 |

| Problem 521 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 143 | 143 | 107 | 138 | 212 | 184 | 178 | 0 | -1 | 108 |
| N.S. | 1 | 1.00 | 0.75 | 0.97 | 1.48 | 1.29 | 1.24 | 0.00 | -0.01 | 0.76 |
| time (sec) | N/A | 0.051 | 0.208 | 0.007 | 2.964 | 1.096 | 17.714 | 0.000 | 0.000 | 0.144 |

| Problem 522 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 96 | 120 | 178 | 163 | 153 | 0 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.81 | 1.01 | 1.50 | 1.37 | 1.29 | 0.00 | -0.01 | 0.80 |
| time (sec) | N/A | 0.038 | 0.125 | 0.006 | 2.961 | 0.698 | 9.279 | 0.000 | 0.000 | 0.101 |

| Problem 523 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 85 | 96 | 144 | 140 | 124 | 0 | -1 | 82 |
| N.S. | 1 | 1.00 | 0.89 | 1.01 | 1.52 | 1.47 | 1.31 | 0.00 | -0.01 | 0.86 |
| time (sec) | N/A | 0.029 | 0.115 | 0.006 | 3.022 | 0.718 | 5.590 | 0.000 | 0.000 | 0.106 |

| Problem 524 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 69 | 78 | 107 | 119 | 75 | 0 | -1 | 66 |
| N.S. | 1 | 1.00 | 0.97 | 1.10 | 1.51 | 1.68 | 1.06 | 0.00 | -0.01 | 0.93 |
| time (sec) | N/A | 0.022 | 0.098 | 0.006 | 2.985 | 1.104 | 3.173 | 0.000 | 0.000 | 0.093 |

| Problem 525 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 46 | 71 | 84 | 109 | 92 | 0 | -1 | 54 |
| N.S. | 1 | 1.00 | 0.73 | 1.13 | 1.33 | 1.73 | 1.46 | 0.00 | -0.02 | 0.86 |
| time (sec) | N/A | 0.021 | 0.011 | 0.018 | 2.987 | 1.331 | 2.722 | 0.000 | 0.000 | 0.134 |

| Problem 526 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 48 | 67 | 67 | 109 | 71 | 0 | -1 | 55 |
| N.S. | 1 | 1.00 | 0.75 | 1.05 | 1.05 | 1.70 | 1.11 | 0.00 | -0.02 | 0.86 |
| time (sec) | N/A | 0.021 | 0.010 | 0.019 | 2.926 | 0.905 | 3.036 | 0.000 | 0.000 | 0.130 |

| Problem 527 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 149 | 149 | 110 | 146 | 207 | 185 | 376 | 0 | -1 | 117 |
| N.S. | 1 | 1.00 | 0.74 | 0.98 | 1.39 | 1.24 | 2.52 | 0.00 | -0.01 | 0.79 |
| time (sec) | N/A | 0.053 | 0.171 | 0.007 | 3.115 | 1.090 | 17.693 | 0.000 | 0.000 | 0.177 |

| Problem 528 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 124 | 124 | 99 | 127 | 170 | 163 | 323 | 0 | -1 | 104 |
| N.S. | 1 | 1.00 | 0.80 | 1.02 | 1.37 | 1.31 | 2.60 | 0.00 | -0.01 | 0.84 |
| time (sec) | N/A | 0.041 | 0.145 | 0.006 | 2.968 | 1.116 | 9.060 | 0.000 | 0.000 | 0.134 |

| Problem 529 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 99 | 99 | 87 | 102 | 133 | 141 | 264 | 0 | -1 | 91 |
| N.S. | 1 | 1.00 | 0.88 | 1.03 | 1.34 | 1.42 | 2.67 | 0.00 | -0.01 | 0.92 |
| time (sec) | N/A | 0.031 | 0.121 | 0.007 | 2.990 | 1.165 | 5.537 | 0.000 | 0.000 | 0.124 |

| Problem 530 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 71 | 83 | 93 | 119 | 190 | 0 | -1 | 75 |
| N.S. | 1 | 1.00 | 0.96 | 1.12 | 1.26 | 1.61 | 2.57 | 0.00 | -0.01 | 1.01 |
| time (sec) | N/A | 0.021 | 0.116 | 0.005 | 2.896 | 1.425 | 3.212 | 0.000 | 0.000 | 0.099 |

| Problem 531 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 47 | 0 | 68 | 109 | 197 | 0 | -1 | 61 |
| N.S. | 1 | 1.00 | 0.71 | 0.00 | 1.03 | 1.65 | 2.98 | 0.00 | -0.02 | 0.92 |
| time (sec) | N/A | 0.020 | 0.011 | 0.025 | 2.846 | 1.150 | 2.881 | 0.000 | 0.000 | 0.129 |

| Problem 532 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | A | A | C | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 49 | 0 | 49 | 115 | 187 | 0 | -1 | 62 |
| N.S. | 1 | 1.00 | 0.73 | 0.00 | 0.73 | 1.72 | 2.79 | 0.00 | -0.01 | 0.93 |
| time (sec) | N/A | 0.022 | 0.011 | 0.029 | 2.859 | 1.620 | 3.241 | 0.000 | 0.000 | 0.139 |

| Problem 533 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 126 | 126 | 78 | 123 | 194 | 156 | 136 | 0 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.62 | 0.98 | 1.54 | 1.24 | 1.08 | 0.00 | -0.01 | 0.75 |
| time (sec) | N/A | 0.034 | 0.050 | 0.006 | 2.986 | 1.182 | 15.671 | 0.000 | 0.000 | 0.125 |

| Problem 534 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 105 | 105 | 70 | 108 | 163 | 137 | 117 | 0 | -1 | 84 |
| N.S. | 1 | 1.00 | 0.67 | 1.03 | 1.55 | 1.30 | 1.11 | 0.00 | -0.01 | 0.80 |
| time (sec) | N/A | 0.026 | 0.032 | 0.004 | 3.043 | 1.308 | 7.858 | 0.000 | 0.000 | 0.086 |

| Problem 535 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 60 | 87 | 132 | 124 | 92 | 0 | -1 | 72 |
| N.S. | 1 | 1.00 | 0.73 | 1.06 | 1.61 | 1.51 | 1.12 | 0.00 | -0.01 | 0.88 |
| time (sec) | N/A | 0.016 | 0.033 | 0.004 | 2.993 | 1.409 | 4.809 | 0.000 | 0.000 | 0.088 |

| Problem 536 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 48 | 72 | 98 | 105 | 76 | 0 | -1 | 59 |
| N.S. | 1 | 1.00 | 0.79 | 1.18 | 1.61 | 1.72 | 1.25 | 0.00 | -0.02 | 0.97 |
| time (sec) | N/A | 0.011 | 0.026 | 0.005 | 2.895 | 1.513 | 2.816 | 0.000 | 0.000 | 0.077 |

| Problem 537 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 28 | 72 | 81 | 99 | 73 | 0 | -1 | 51 |
| N.S. | 1 | 1.00 | 0.48 | 1.24 | 1.40 | 1.71 | 1.26 | 0.00 | -0.02 | 0.88 |
| time (sec) | N/A | 0.012 | 0.005 | 0.018 | 2.914 | 1.228 | 2.442 | 0.000 | 0.000 | 0.100 |

| Problem 538 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 30 | 73 | 67 | 108 | 70 | 0 | -1 | 55 |
| N.S. | 1 | 1.00 | 0.50 | 1.22 | 1.12 | 1.80 | 1.17 | 0.00 | -0.02 | 0.92 |
| time (sec) | N/A | 0.014 | 0.006 | 0.017 | 2.972 | 1.307 | 2.812 | 0.000 | 0.000 | 0.107 |

| Problem 539 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 131 | 131 | 79 | 132 | 179 | 157 | 291 | 0 | -1 | 104 |
| N.S. | 1 | 1.00 | 0.60 | 1.01 | 1.37 | 1.20 | 2.22 | 0.00 | -0.01 | 0.79 |
| time (sec) | N/A | 0.040 | 0.052 | 0.006 | 2.943 | 0.906 | 15.279 | 0.000 | 0.000 | 0.161 |

| Problem 540 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 70 | 116 | 147 | 139 | 252 | 0 | -1 | 94 |
| N.S. | 1 | 1.00 | 0.64 | 1.06 | 1.35 | 1.28 | 2.31 | 0.00 | -0.01 | 0.86 |
| time (sec) | N/A | 0.028 | 0.045 | 0.006 | 2.925 | 1.120 | 7.726 | 0.000 | 0.000 | 0.116 |

| Problem 541 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 60 | 94 | 115 | 125 | 199 | 0 | -1 | 81 |
| N.S. | 1 | 1.00 | 0.71 | 1.12 | 1.37 | 1.49 | 2.37 | 0.00 | -0.01 | 0.96 |
| time (sec) | N/A | 0.016 | 0.042 | 0.004 | 3.042 | 1.308 | 4.780 | 0.000 | 0.000 | 0.117 |

| Problem 542 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 49 | 78 | 79 | 107 | 167 | 0 | -1 | 69 |
| N.S. | 1 | 1.00 | 0.78 | 1.24 | 1.25 | 1.70 | 2.65 | 0.00 | -0.02 | 1.10 |
| time (sec) | N/A | 0.012 | 0.029 | 0.004 | 2.984 | 1.471 | 2.863 | 0.000 | 0.000 | 0.102 |

| Problem 543 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 28 | 97 | 63 | 101 | 160 | 0 | -1 | 58 |
| N.S. | 1 | 1.00 | 0.47 | 1.62 | 1.05 | 1.68 | 2.67 | 0.00 | -0.02 | 0.97 |
| time (sec) | N/A | 0.012 | 0.005 | 0.020 | 2.989 | 0.745 | 2.493 | 0.000 | 0.000 | 0.121 |

| Problem 544 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | C | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 30 | 98 | 49 | 111 | 182 | 0 | -1 | 62 |
| N.S. | 1 | 1.00 | 0.48 | 1.58 | 0.79 | 1.79 | 2.94 | 0.00 | -0.02 | 1.00 |
| time (sec) | N/A | 0.013 | 0.006 | 0.022 | 3.004 | 1.153 | 2.923 | 0.000 | 0.000 | 0.133 |

| Problem 545 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 164 | 164 | 118 | 156 | 244 | 206 | 207 | 0 | -1 | 121 |
| N.S. | 1 | 1.00 | 0.72 | 0.95 | 1.49 | 1.26 | 1.26 | 0.00 | -0.01 | 0.74 |
| time (sec) | N/A | 0.063 | 0.244 | 0.005 | 2.989 | 0.782 | 25.936 | 0.000 | 0.000 | 0.123 |

| Problem 546 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 140 | 107 | 138 | 212 | 185 | 180 | 0 | -1 | 108 |
| N.S. | 1 | 1.00 | 0.76 | 0.99 | 1.51 | 1.32 | 1.29 | 0.00 | -0.01 | 0.77 |
| time (sec) | N/A | 0.048 | 0.138 | 0.006 | 2.979 | 1.346 | 16.411 | 0.000 | 0.000 | 0.149 |

| Problem 547 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 116 | 116 | 96 | 111 | 176 | 162 | 155 | 0 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.83 | 0.96 | 1.52 | 1.40 | 1.34 | 0.00 | -0.01 | 0.82 |
| time (sec) | N/A | 0.038 | 0.199 | 0.006 | 2.981 | 1.454 | 9.860 | 0.000 | 0.000 | 0.127 |

| Problem 548 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 80 | 93 | 141 | 141 | 102 | 0 | -1 | 79 |
| N.S. | 1 | 1.00 | 0.87 | 1.01 | 1.53 | 1.53 | 1.11 | 0.00 | -0.01 | 0.86 |
| time (sec) | N/A | 0.028 | 0.111 | 0.006 | 2.991 | 1.509 | 6.229 | 0.000 | 0.000 | 0.111 |

| Problem 549 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 48 | 84 | 125 | 137 | 126 | 0 | -1 | 73 |
| N.S. | 1 | 1.00 | 0.54 | 0.94 | 1.40 | 1.54 | 1.42 | 0.00 | -0.01 | 0.82 |
| time (sec) | N/A | 0.028 | 0.012 | 0.018 | 2.977 | 1.387 | 6.148 | 0.000 | 0.000 | 0.135 |

| Problem 550 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 50 | 82 | 100 | 138 | 99 | 0 | -1 | 69 |
| N.S. | 1 | 1.00 | 0.58 | 0.95 | 1.16 | 1.60 | 1.15 | 0.00 | -0.01 | 0.80 |
| time (sec) | N/A | 0.026 | 0.011 | 0.020 | 2.949 | 1.530 | 5.622 | 0.000 | 0.000 | 0.155 |

| Problem 551 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 171 | 171 | 120 | 165 | 242 | 208 | 435 | 0 | -1 | 130 |
| N.S. | 1 | 1.00 | 0.70 | 0.96 | 1.42 | 1.22 | 2.54 | 0.00 | -0.01 | 0.76 |
| time (sec) | N/A | 0.061 | 0.191 | 0.007 | 2.864 | 1.399 | 25.960 | 0.000 | 0.000 | 0.305 |

| Problem 552 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 146 | 146 | 109 | 146 | 207 | 186 | 379 | 0 | -1 | 117 |
| N.S. | 1 | 1.00 | 0.75 | 1.00 | 1.42 | 1.27 | 2.60 | 0.00 | -0.01 | 0.80 |
| time (sec) | N/A | 0.051 | 0.149 | 0.006 | 3.008 | 1.135 | 16.398 | 0.000 | 0.000 | 0.180 |

| Problem 553 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 121 | 121 | 98 | 118 | 168 | 164 | 326 | 0 | -1 | 104 |
| N.S. | 1 | 1.00 | 0.81 | 0.98 | 1.39 | 1.36 | 2.69 | 0.00 | -0.01 | 0.86 |
| time (sec) | N/A | 0.038 | 0.132 | 0.007 | 3.046 | 1.245 | 9.807 | 0.000 | 0.000 | 0.158 |

| Problem 554 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 96 | 82 | 99 | 130 | 142 | 246 | 0 | -1 | 88 |
| N.S. | 1 | 1.00 | 0.85 | 1.03 | 1.35 | 1.48 | 2.56 | 0.00 | -0.01 | 0.92 |
| time (sec) | N/A | 0.030 | 0.123 | 0.005 | 2.930 | 0.691 | 6.228 | 0.000 | 0.000 | 0.133 |

| Problem 555 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 49 | 0 | 112 | 137 | 267 | 0 | -1 | 79 |
| N.S. | 1 | 1.00 | 0.53 | 0.00 | 1.20 | 1.47 | 2.87 | 0.00 | -0.01 | 0.85 |
| time (sec) | N/A | 0.028 | 0.012 | 0.030 | 2.992 | 1.302 | 6.222 | 0.000 | 0.000 | 0.161 |

| Problem 556 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | A | A | C | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 51 | 0 | 84 | 139 | 245 | 0 | -1 | 76 |
| N.S. | 1 | 1.00 | 0.57 | 0.00 | 0.93 | 1.54 | 2.72 | 0.00 | -0.01 | 0.84 |
| time (sec) | N/A | 0.030 | 0.012 | 0.028 | 3.000 | 1.327 | 5.845 | 0.000 | 0.000 | 0.187 |

| Problem 557 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 144 | 144 | 86 | 138 | 223 | 172 | 158 | 0 | -1 | 105 |
| N.S. | 1 | 1.00 | 0.60 | 0.96 | 1.55 | 1.19 | 1.10 | 0.00 | -0.01 | 0.73 |
| time (sec) | N/A | 0.045 | 0.059 | 0.005 | 2.977 | 1.291 | 22.960 | 0.000 | 0.000 | 0.106 |

| Problem 558 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 78 | 123 | 194 | 155 | 138 | 0 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.63 | 1.00 | 1.58 | 1.26 | 1.12 | 0.00 | -0.01 | 0.77 |
| time (sec) | N/A | 0.028 | 0.048 | 0.004 | 2.943 | 1.316 | 14.423 | 0.000 | 0.000 | 0.129 |

| Problem 559 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 102 | 102 | 70 | 99 | 161 | 140 | 119 | 0 | -1 | 85 |
| N.S. | 1 | 1.00 | 0.69 | 0.97 | 1.58 | 1.37 | 1.17 | 0.00 | -0.01 | 0.83 |
| time (sec) | N/A | 0.022 | 0.041 | 0.006 | 2.986 | 1.428 | 8.611 | 0.000 | 0.000 | 0.105 |

| Problem 560 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 79 | 79 | 57 | 84 | 129 | 123 | 97 | 0 | -1 | 70 |
| N.S. | 1 | 1.00 | 0.72 | 1.06 | 1.63 | 1.56 | 1.23 | 0.00 | -0.01 | 0.89 |
| time (sec) | N/A | 0.016 | 0.032 | 0.004 | 3.028 | 1.242 | 5.458 | 0.000 | 0.000 | 0.101 |

| Problem 561 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 79 | 79 | 28 | 81 | 113 | 116 | 94 | 0 | -1 | 62 |
| N.S. | 1 | 1.00 | 0.35 | 1.03 | 1.43 | 1.47 | 1.19 | 0.00 | -0.01 | 0.78 |
| time (sec) | N/A | 0.016 | 0.007 | 0.019 | 2.943 | 1.236 | 5.602 | 0.000 | 0.000 | 0.117 |

| Problem 562 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 30 | 82 | 96 | 123 | 88 | 0 | -1 | 63 |
| N.S. | 1 | 1.00 | 0.37 | 1.01 | 1.19 | 1.52 | 1.09 | 0.00 | -0.01 | 0.78 |
| time (sec) | N/A | 0.017 | 0.006 | 0.019 | 2.945 | 1.445 | 5.152 | 0.000 | 0.000 | 0.129 |

| Problem 563 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 150 | 150 | 87 | 148 | 209 | 173 | 337 | 0 | -1 | 114 |
| N.S. | 1 | 1.00 | 0.58 | 0.99 | 1.39 | 1.15 | 2.25 | 0.00 | -0.01 | 0.76 |
| time (sec) | N/A | 0.045 | 0.059 | 0.004 | 2.960 | 1.280 | 22.838 | 0.000 | 0.000 | 0.150 |

| Problem 564 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 128 | 128 | 79 | 132 | 179 | 157 | 294 | 0 | -1 | 104 |
| N.S. | 1 | 1.00 | 0.62 | 1.03 | 1.40 | 1.23 | 2.30 | 0.00 | -0.01 | 0.81 |
| time (sec) | N/A | 0.029 | 0.046 | 0.006 | 3.013 | 1.317 | 14.298 | 0.000 | 0.000 | 0.185 |

| Problem 565 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 106 | 106 | 71 | 107 | 145 | 141 | 255 | 0 | -1 | 94 |
| N.S. | 1 | 1.00 | 0.67 | 1.01 | 1.37 | 1.33 | 2.41 | 0.00 | -0.01 | 0.89 |
| time (sec) | N/A | 0.023 | 0.041 | 0.005 | 2.887 | 1.438 | 8.592 | 0.000 | 0.000 | 0.151 |

| Problem 566 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 58 | 91 | 112 | 125 | 209 | 0 | -1 | 79 |
| N.S. | 1 | 1.00 | 0.71 | 1.11 | 1.37 | 1.52 | 2.55 | 0.00 | -0.01 | 0.96 |
| time (sec) | N/A | 0.017 | 0.034 | 0.005 | 2.976 | 1.261 | 5.522 | 0.000 | 0.000 | 0.137 |

| Problem 567 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 28 | 106 | 96 | 117 | 202 | 0 | -1 | 68 |
| N.S. | 1 | 1.00 | 0.34 | 1.29 | 1.17 | 1.43 | 2.46 | 0.00 | -0.01 | 0.83 |
| time (sec) | N/A | 0.017 | 0.006 | 0.019 | 2.923 | 1.321 | 5.619 | 0.000 | 0.000 | 0.152 |

| Problem 568 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 30 | 107 | 79 | 126 | 221 | 0 | -1 | 70 |
| N.S. | 1 | 1.00 | 0.36 | 1.27 | 0.94 | 1.50 | 2.63 | 0.00 | -0.01 | 0.83 |
| time (sec) | N/A | 0.017 | 0.007 | 0.020 | 2.888 | 0.860 | 5.346 | 0.000 | 0.000 | 0.168 |

| Problem 569 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 85 | 102 | 146 | 140 | 128 | 0 | -1 | 82 |
| N.S. | 1 | 1.00 | 0.84 | 1.01 | 1.45 | 1.39 | 1.27 | 0.00 | -0.01 | 0.81 |
| time (sec) | N/A | 0.030 | 0.161 | 0.006 | 3.014 | 1.332 | 8.521 | 0.000 | 0.000 | 0.098 |

| Problem 570 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 77 | 77 | 85 | 84 | 112 | 119 | 100 | 0 | -1 | 69 |
| N.S. | 1 | 1.00 | 1.10 | 1.09 | 1.45 | 1.55 | 1.30 | 0.00 | -0.01 | 0.90 |
| time (sec) | N/A | 0.022 | 0.050 | 0.006 | 2.867 | 1.151 | 4.301 | 0.000 | 0.000 | 0.083 |

| Problem 571 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 68 | 65 | 73 | 91 | 44 | 0 | 44 | 49 |
| N.S. | 1 | 1.00 | 1.42 | 1.35 | 1.52 | 1.90 | 0.92 | 0.00 | 0.92 | 1.02 |
| time (sec) | N/A | 0.016 | 0.039 | 0.004 | 2.913 | 1.471 | 2.187 | 0.000 | 0.548 | 0.065 |

| Problem 572 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 50 | 48 | 41 | 57 | 22 | 0 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.79 | 1.71 | 1.46 | 2.04 | 0.79 | 0.00 | 1.07 | 1.07 |
| time (sec) | N/A | 0.013 | 0.013 | 0.004 | 2.953 | 1.037 | 1.097 | 0.000 | 0.030 | 0.040 |

| Problem 573 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 16 | 15 | 15 | 19 | 33 | 15 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.79 | 0.79 | 1.00 | 1.74 | 0.79 | 1.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.004 | 1.342 | 0.946 | 0.913 | 2.050 | 0.349 | 0.020 |

| Problem 574 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 27 | 22 | 31 | 23 | 42 | 50 | 25 | 29 |
| N.S. | 1 | 1.00 | 0.61 | 0.50 | 0.70 | 0.52 | 0.95 | 1.14 | 0.57 | 0.66 |
| time (sec) | N/A | 0.005 | 0.007 | 0.004 | 1.299 | 1.150 | 1.920 | 1.902 | 0.342 | 0.075 |

| Problem 575 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 40 | 35 | 46 | 34 | 287 | 66 | 36 | 40 |
| N.S. | 1 | 1.00 | 0.59 | 0.51 | 0.68 | 0.50 | 4.22 | 0.97 | 0.53 | 0.59 |
| time (sec) | N/A | 0.010 | 0.009 | 0.005 | 1.337 | 1.210 | 6.249 | 1.669 | 0.353 | 0.089 |

| Problem 576 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 51 | 46 | 61 | 45 | 488 | 82 | 47 | 51 |
| N.S. | 1 | 1.00 | 0.55 | 0.50 | 0.66 | 0.49 | 5.30 | 0.89 | 0.51 | 0.55 |
| time (sec) | N/A | 0.016 | 0.012 | 0.004 | 1.304 | 1.211 | 16.137 | 1.412 | 0.382 | 0.096 |

| Problem 577 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 96 | 50 | 119 | 131 | 175 | 105 | 131 | -1 | 82 |
| N.S. | 1 | 1.00 | 0.52 | 1.24 | 1.36 | 1.82 | 1.09 | 1.36 | -0.01 | 0.85 |
| time (sec) | N/A | 0.029 | 0.010 | 0.035 | 2.924 | 1.413 | 8.139 | 92.143 | 0.000 | 0.136 |

| Problem 578 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 50 | 106 | 92 | 145 | 71 | 115 | -1 | 61 |
| N.S. | 1 | 1.00 | 0.74 | 1.56 | 1.35 | 2.13 | 1.04 | 1.69 | -0.01 | 0.90 |
| time (sec) | N/A | 0.022 | 0.010 | 0.030 | 3.019 | 1.392 | 3.678 | 93.123 | 0.000 | 0.126 |

| Problem 579 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 64 | 0 | 57 | 119 | 46 | 85 | -1 | 50 |
| N.S. | 1 | 1.00 | 1.33 | 0.00 | 1.19 | 2.48 | 0.96 | 1.77 | -0.02 | 1.04 |
| time (sec) | N/A | 0.016 | 0.069 | 0.035 | 2.981 | 1.326 | 1.780 | 94.858 | 0.000 | 0.084 |

| Problem 580 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 16 | 15 | 22 | 17 | 45 | 22 | 19 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.79 | 1.16 | 0.89 | 2.37 | 1.16 | 1.00 |
| time (sec) | N/A | 0.002 | 0.005 | 0.005 | 1.321 | 1.006 | 0.879 | 1.109 | 0.326 | 0.021 |

| Problem 581 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 25 | 22 | 32 | 34 | 41 | 82 | 39 | 25 |
| N.S. | 1 | 1.00 | 0.64 | 0.56 | 0.82 | 0.87 | 1.05 | 2.10 | 1.00 | 0.64 |
| time (sec) | N/A | 0.005 | 0.008 | 0.006 | 1.320 | 0.889 | 1.598 | 1.052 | 0.386 | 0.076 |

| Problem 582 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 38 | 33 | 50 | 49 | 219 | 98 | 46 | 40 |
| N.S. | 1 | 1.00 | 0.60 | 0.52 | 0.79 | 0.78 | 3.48 | 1.56 | 0.73 | 0.63 |
| time (sec) | N/A | 0.010 | 0.009 | 0.005 | 1.310 | 1.351 | 3.983 | 1.201 | 0.413 | 0.108 |

| Problem 583 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 49 | 44 | 64 | 58 | 348 | 121 | 58 | 49 |
| N.S. | 1 | 1.00 | 0.56 | 0.51 | 0.74 | 0.67 | 4.00 | 1.39 | 0.67 | 0.56 |
| time (sec) | N/A | 0.017 | 0.010 | 0.005 | 1.309 | 1.184 | 11.154 | 1.213 | 0.426 | 0.121 |

| Problem 584 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 50 | 147 | 109 | 214 | 396 | 197 | -1 | 78 |
| N.S. | 1 | 1.00 | 0.55 | 1.62 | 1.20 | 2.35 | 4.35 | 2.16 | -0.01 | 0.86 |
| time (sec) | N/A | 0.028 | 0.010 | 0.049 | 2.939 | 1.420 | 7.610 | 92.464 | 0.000 | 0.153 |

| Problem 585 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 80 | 0 | 69 | 186 | 328 | 165 | -1 | 64 |
| N.S. | 1 | 1.00 | 1.16 | 0.00 | 1.00 | 2.70 | 4.75 | 2.39 | -0.01 | 0.93 |
| time (sec) | N/A | 0.022 | 0.121 | 0.032 | 2.826 | 0.724 | 4.027 | 105.595 | 0.000 | 0.138 |

| Problem 586 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 16 | 15 | 33 | 42 | 86 | 36 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 0.76 | 0.71 | 1.57 | 2.00 | 4.10 | 1.71 | 1.00 |
| time (sec) | N/A | 0.002 | 0.005 | 0.003 | 1.328 | 1.243 | 1.427 | 1.631 | 0.242 | 0.024 |

| Problem 587 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 29 | 24 | 27 | 43 | 92 | 81 | 54 | 29 |
| N.S. | 1 | 1.00 | 0.67 | 0.56 | 0.63 | 1.00 | 2.14 | 1.88 | 1.26 | 0.67 |
| time (sec) | N/A | 0.005 | 0.009 | 0.005 | 1.344 | 1.494 | 1.896 | 1.500 | 0.400 | 0.082 |

| Problem 588 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 40 | 35 | 46 | 58 | 153 | 159 | 71 | 40 |
| N.S. | 1 | 1.00 | 0.62 | 0.55 | 0.72 | 0.91 | 2.39 | 2.48 | 1.11 | 0.62 |
| time (sec) | N/A | 0.009 | 0.011 | 0.007 | 1.340 | 1.370 | 3.972 | 1.632 | 0.420 | 0.104 |

| Problem 589 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 49 | 44 | 64 | 71 | 337 | 175 | 88 | 51 |
| N.S. | 1 | 1.00 | 0.58 | 0.52 | 0.76 | 0.85 | 4.01 | 2.08 | 1.05 | 0.61 |
| time (sec) | N/A | 0.015 | 0.014 | 0.005 | 1.268 | 1.040 | 7.061 | 2.238 | 0.471 | 0.114 |

| Problem 590 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 105 | 105 | 88 | 108 | 135 | 141 | 270 | 0 | -1 | 91 |
| N.S. | 1 | 1.00 | 0.84 | 1.03 | 1.29 | 1.34 | 2.57 | 0.00 | -0.01 | 0.87 |
| time (sec) | N/A | 0.030 | 0.146 | 0.007 | 2.964 | 1.346 | 8.432 | 0.000 | 0.000 | 0.118 |

| Problem 591 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 80 | 80 | 86 | 89 | 98 | 119 | 214 | 0 | -1 | 78 |
| N.S. | 1 | 1.00 | 1.08 | 1.11 | 1.22 | 1.49 | 2.68 | 0.00 | -0.01 | 0.98 |
| time (sec) | N/A | 0.023 | 0.048 | 0.006 | 2.960 | 0.772 | 4.296 | 0.000 | 0.000 | 0.100 |

| Problem 592 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 71 | 70 | 56 | 93 | 121 | 0 | 47 | 59 |
| N.S. | 1 | 1.00 | 1.42 | 1.40 | 1.12 | 1.86 | 2.42 | 0.00 | 0.94 | 1.18 |
| time (sec) | N/A | 0.017 | 0.042 | 0.006 | 3.004 | 1.213 | 2.278 | 0.000 | 0.517 | 0.075 |

| Problem 593 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 52 | 51 | 21 | 57 | 54 | 0 | 27 | 38 |
| N.S. | 1 | 1.00 | 1.79 | 1.76 | 0.72 | 1.97 | 1.86 | 0.00 | 0.93 | 1.31 |
| time (sec) | N/A | 0.013 | 0.014 | 0.006 | 2.883 | 0.919 | 1.153 | 0.000 | 0.031 | 0.059 |

| Problem 594 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 17 | 16 | 16 | 46 | 35 | 16 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.80 | 0.80 | 2.30 | 1.75 | 0.80 | 1.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.005 | 1.341 | 0.690 | 0.969 | 1.277 | 0.401 | 0.021 |

| Problem 595 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 28 | 23 | 32 | 22 | 177 | 54 | 26 | 28 |
| N.S. | 1 | 1.00 | 0.61 | 0.50 | 0.70 | 0.48 | 3.85 | 1.17 | 0.57 | 0.61 |
| time (sec) | N/A | 0.005 | 0.007 | 0.004 | 1.302 | 1.771 | 2.055 | 1.453 | 0.350 | 0.107 |

| Problem 596 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 51 | 127 | 118 | 181 | 224 | 154 | -1 | 100 |
| N.S. | 1 | 1.00 | 0.51 | 1.27 | 1.18 | 1.81 | 2.24 | 1.54 | -0.01 | 1.00 |
| time (sec) | N/A | 0.030 | 0.010 | 0.040 | 2.922 | 1.372 | 8.030 | 98.066 | 0.000 | 0.168 |

| Problem 597 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 51 | 114 | 75 | 152 | 155 | 130 | -1 | 81 |
| N.S. | 1 | 1.00 | 0.72 | 1.61 | 1.06 | 2.14 | 2.18 | 1.83 | -0.01 | 1.14 |
| time (sec) | N/A | 0.022 | 0.010 | 0.030 | 2.952 | 1.304 | 3.704 | 111.127 | 0.000 | 0.142 |

| Problem 598 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 66 | 0 | 38 | 128 | 102 | 98 | -1 | 68 |
| N.S. | 1 | 1.00 | 1.32 | 0.00 | 0.76 | 2.56 | 2.04 | 1.96 | -0.02 | 1.36 |
| time (sec) | N/A | 0.016 | 0.075 | 0.030 | 3.012 | 1.207 | 1.891 | 113.038 | 0.000 | 0.109 |

| Problem 599 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 17 | 16 | 25 | 44 | 53 | 24 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.80 | 1.25 | 2.20 | 2.65 | 1.20 | 1.00 |
| time (sec) | N/A | 0.002 | 0.005 | 0.003 | 1.297 | 1.031 | 0.943 | 1.374 | 0.339 | 0.024 |

| Problem 600 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 26 | 23 | 34 | 38 | 112 | 94 | 42 | 37 |
| N.S. | 1 | 1.00 | 0.63 | 0.56 | 0.83 | 0.93 | 2.73 | 2.29 | 1.02 | 0.90 |
| time (sec) | N/A | 0.005 | 0.008 | 0.003 | 1.305 | 0.963 | 1.678 | 1.441 | 0.400 | 0.103 |

| Problem 601 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 39 | 34 | 52 | 51 | 452 | 112 | 48 | 50 |
| N.S. | 1 | 1.00 | 0.59 | 0.52 | 0.79 | 0.77 | 6.85 | 1.70 | 0.73 | 0.76 |
| time (sec) | N/A | 0.010 | 0.009 | 0.005 | 1.347 | 1.201 | 4.640 | 1.478 | 0.432 | 0.147 |

| Problem 602 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 51 | 160 | 94 | 215 | 971 | 221 | -1 | 96 |
| N.S. | 1 | 1.00 | 0.54 | 1.68 | 0.99 | 2.26 | 10.22 | 2.33 | -0.01 | 1.01 |
| time (sec) | N/A | 0.029 | 0.012 | 0.043 | 3.018 | 1.353 | 8.483 | 112.518 | 0.000 | 0.195 |

| Problem 603 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 82 | 0 | 52 | 188 | 833 | 194 | -1 | 82 |
| N.S. | 1 | 1.00 | 1.14 | 0.00 | 0.72 | 2.61 | 11.57 | 2.69 | -0.01 | 1.14 |
| time (sec) | N/A | 0.022 | 0.171 | 0.031 | 2.940 | 1.562 | 4.496 | 110.078 | 0.000 | 0.173 |

| Problem 604 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 17 | 16 | 34 | 95 | 102 | 37 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.73 | 1.55 | 4.32 | 4.64 | 1.68 | 1.00 |
| time (sec) | N/A | 0.002 | 0.005 | 0.005 | 1.364 | 1.106 | 1.507 | 1.633 | 0.252 | 0.028 |

| Problem 605 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 30 | 25 | 30 | 44 | 211 | 96 | 56 | 30 |
| N.S. | 1 | 1.00 | 0.67 | 0.56 | 0.67 | 0.98 | 4.69 | 2.13 | 1.24 | 0.67 |
| time (sec) | N/A | 0.005 | 0.009 | 0.003 | 1.344 | 1.251 | 1.996 | 1.426 | 0.406 | 0.117 |

| Problem 606 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 41 | 36 | 50 | 59 | 314 | 189 | 73 | 41 |
| N.S. | 1 | 1.00 | 0.61 | 0.54 | 0.75 | 0.88 | 4.69 | 2.82 | 1.09 | 0.61 |
| time (sec) | N/A | 0.010 | 0.012 | 0.005 | 1.290 | 1.194 | 4.265 | 1.613 | 0.441 | 0.139 |

| Problem 607 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 50 | 45 | 68 | 70 | 688 | 207 | 92 | 50 |
| N.S. | 1 | 1.00 | 0.57 | 0.51 | 0.77 | 0.80 | 7.82 | 2.35 | 1.05 | 0.57 |
| time (sec) | N/A | 0.016 | 0.014 | 0.005 | 1.329 | 1.234 | 13.450 | 1.697 | 0.475 | 0.147 |

| Problem 608 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 60 | 93 | 134 | 124 | 95 | 0 | -1 | 73 |
| N.S. | 1 | 1.00 | 0.68 | 1.06 | 1.52 | 1.41 | 1.08 | 0.00 | -0.01 | 0.83 |
| time (sec) | N/A | 0.022 | 0.040 | 0.005 | 2.975 | 1.149 | 7.397 | 0.000 | 0.000 | 0.090 |

| Problem 609 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 51 | 78 | 102 | 105 | 75 | 0 | -1 | 62 |
| N.S. | 1 | 1.00 | 0.76 | 1.16 | 1.52 | 1.57 | 1.12 | 0.00 | -0.01 | 0.93 |
| time (sec) | N/A | 0.014 | 0.028 | 0.005 | 2.858 | 0.996 | 3.671 | 0.000 | 0.000 | 0.076 |

| Problem 610 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 43 | 62 | 70 | 87 | 54 | 0 | 43 | 49 |
| N.S. | 1 | 1.00 | 1.00 | 1.44 | 1.63 | 2.02 | 1.26 | 0.00 | 1.00 | 1.14 |
| time (sec) | N/A | 0.009 | 0.016 | 0.004 | 2.919 | 1.022 | 1.934 | 0.000 | 0.585 | 0.060 |

| Problem 611 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 24 | 46 | 41 | 55 | 24 | 0 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 1.92 | 1.71 | 2.29 | 1.00 | 0.00 | 1.25 | 1.25 |
| time (sec) | N/A | 0.006 | 0.004 | 0.004 | 2.959 | 1.278 | 1.022 | 0.000 | 0.035 | 0.040 |

| Problem 612 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 12 | 15 | 29 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 0.75 | 0.94 | 1.81 | 0.75 | 1.00 |
| time (sec) | N/A | 0.001 | 0.004 | 0.003 | 1.355 | 1.282 | 0.879 | 1.108 | 0.330 | 0.019 |

| Problem 613 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 23 | 18 | 26 | 17 | 34 | 42 | 17 | 23 |
| N.S. | 1 | 1.00 | 0.61 | 0.47 | 0.68 | 0.45 | 0.89 | 1.11 | 0.45 | 0.61 |
| time (sec) | N/A | 0.004 | 0.006 | 0.005 | 1.315 | 1.108 | 1.873 | 1.087 | 0.316 | 0.068 |

| Problem 614 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 32 | 27 | 41 | 26 | 224 | 55 | 26 | 32 |
| N.S. | 1 | 1.00 | 0.54 | 0.46 | 0.69 | 0.44 | 3.80 | 0.93 | 0.44 | 0.54 |
| time (sec) | N/A | 0.007 | 0.008 | 0.006 | 1.311 | 0.870 | 6.104 | 1.065 | 0.323 | 0.073 |

| Problem 615 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 80 | 80 | 40 | 35 | 56 | 34 | 374 | 68 | 33 | 40 |
| N.S. | 1 | 1.00 | 0.50 | 0.44 | 0.70 | 0.42 | 4.68 | 0.85 | 0.41 | 0.50 |
| time (sec) | N/A | 0.012 | 0.009 | 0.006 | 1.295 | 0.809 | 15.993 | 1.025 | 0.333 | 0.079 |

| Problem 616 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 30 | 106 | 119 | 152 | 80 | 119 | -1 | 72 |
| N.S. | 1 | 1.00 | 0.35 | 1.23 | 1.38 | 1.77 | 0.93 | 1.38 | -0.01 | 0.84 |
| time (sec) | N/A | 0.020 | 0.006 | 0.028 | 3.055 | 1.254 | 7.099 | 11.125 | 0.000 | 0.118 |

| Problem 617 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 30 | 100 | 90 | 134 | 58 | 106 | -1 | 59 |
| N.S. | 1 | 1.00 | 0.48 | 1.59 | 1.43 | 2.13 | 0.92 | 1.68 | -0.02 | 0.94 |
| time (sec) | N/A | 0.014 | 0.006 | 0.024 | 2.988 | 1.333 | 3.065 | 10.155 | 0.000 | 0.104 |

| Problem 618 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 44 | 48 | 57 | 117 | 41 | 82 | -1 | 50 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 1.30 | 2.66 | 0.93 | 1.86 | -0.02 | 1.14 |
| time (sec) | N/A | 0.009 | 0.029 | 0.112 | 2.896 | 1.294 | 1.565 | 10.610 | 0.000 | 0.075 |

| Problem 619 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 12 | 11 | 11 | 15 | 44 | 11 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.73 | 0.73 | 1.00 | 2.93 | 0.73 | 1.00 |
| time (sec) | N/A | 0.001 | 0.003 | 0.004 | 1.340 | 1.319 | 0.862 | 1.216 | 0.307 | 0.021 |

| Problem 620 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 21 | 18 | 26 | 28 | 34 | 74 | 17 | 21 |
| N.S. | 1 | 1.00 | 0.66 | 0.56 | 0.81 | 0.88 | 1.06 | 2.31 | 0.53 | 0.66 |
| time (sec) | N/A | 0.003 | 0.007 | 0.005 | 1.335 | 1.003 | 1.543 | 1.123 | 0.348 | 0.064 |

| Problem 621 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 32 | 27 | 41 | 39 | 170 | 86 | 37 | 32 |
| N.S. | 1 | 1.00 | 0.60 | 0.51 | 0.77 | 0.74 | 3.21 | 1.62 | 0.70 | 0.60 |
| time (sec) | N/A | 0.007 | 0.008 | 0.004 | 1.372 | 0.644 | 3.857 | 1.232 | 0.380 | 0.094 |

| Problem 622 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 39 | 35 | 56 | 47 | 269 | 107 | 46 | 39 |
| N.S. | 1 | 1.00 | 0.53 | 0.47 | 0.76 | 0.64 | 3.64 | 1.45 | 0.62 | 0.53 |
| time (sec) | N/A | 0.014 | 0.008 | 0.004 | 1.303 | 1.221 | 10.971 | 1.110 | 0.426 | 0.100 |

| Problem 623 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 30 | 136 | 105 | 186 | 308 | 182 | -1 | 73 |
| N.S. | 1 | 1.00 | 0.35 | 1.58 | 1.22 | 2.16 | 3.58 | 2.12 | -0.01 | 0.85 |
| time (sec) | N/A | 0.020 | 0.007 | 0.036 | 3.003 | 1.350 | 6.617 | 10.823 | 0.000 | 0.132 |

| Problem 624 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 52 | 55 | 69 | 171 | 257 | 154 | -1 | 63 |
| N.S. | 1 | 1.00 | 0.80 | 0.85 | 1.06 | 2.63 | 3.95 | 2.37 | -0.02 | 0.97 |
| time (sec) | N/A | 0.014 | 0.069 | 0.040 | 2.971 | 1.288 | 3.578 | 10.772 | 0.000 | 0.120 |

| Problem 625 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 13 | 12 | 27 | 27 | 82 | 12 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 0.72 | 0.67 | 1.50 | 1.50 | 4.56 | 0.67 | 1.00 |
| time (sec) | N/A | 0.001 | 0.004 | 0.005 | 1.328 | 1.185 | 1.404 | 1.224 | 0.252 | 0.021 |

| Problem 626 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 23 | 18 | 24 | 32 | 75 | 79 | 42 | 23 |
| N.S. | 1 | 1.00 | 0.62 | 0.49 | 0.65 | 0.86 | 2.03 | 2.14 | 1.14 | 0.62 |
| time (sec) | N/A | 0.003 | 0.006 | 0.005 | 1.318 | 0.814 | 1.842 | 1.215 | 0.357 | 0.068 |

| Problem 627 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 55 | 55 | 32 | 27 | 40 | 45 | 117 | 145 | 57 | 32 |
| N.S. | 1 | 1.00 | 0.58 | 0.49 | 0.73 | 0.82 | 2.13 | 2.64 | 1.04 | 0.58 |
| time (sec) | N/A | 0.006 | 0.008 | 0.004 | 1.401 | 1.474 | 3.889 | 1.362 | 0.378 | 0.095 |

| Problem 628 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 40 | 35 | 55 | 55 | 257 | 158 | 71 | 40 |
| N.S. | 1 | 1.00 | 0.56 | 0.49 | 0.77 | 0.77 | 3.62 | 2.23 | 1.00 | 0.56 |
| time (sec) | N/A | 0.009 | 0.010 | 0.005 | 1.276 | 1.049 | 6.853 | 1.272 | 0.418 | 0.097 |

| Problem 629 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 61 | 100 | 117 | 125 | 206 | 0 | -1 | 82 |
| N.S. | 1 | 1.00 | 0.67 | 1.10 | 1.29 | 1.37 | 2.26 | 0.00 | -0.01 | 0.90 |
| time (sec) | N/A | 0.021 | 0.041 | 0.005 | 3.033 | 1.440 | 7.511 | 0.000 | 0.000 | 0.105 |

| Problem 630 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 52 | 84 | 85 | 107 | 163 | 0 | -1 | 72 |
| N.S. | 1 | 1.00 | 0.75 | 1.22 | 1.23 | 1.55 | 2.36 | 0.00 | -0.01 | 1.04 |
| time (sec) | N/A | 0.015 | 0.029 | 0.004 | 2.927 | 1.344 | 3.591 | 0.000 | 0.000 | 0.091 |

| Problem 631 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 45 | 67 | 52 | 90 | 121 | 0 | 46 | 59 |
| N.S. | 1 | 1.00 | 1.00 | 1.49 | 1.16 | 2.00 | 2.69 | 0.00 | 1.02 | 1.31 |
| time (sec) | N/A | 0.010 | 0.016 | 0.006 | 2.928 | 1.280 | 1.966 | 0.000 | 0.520 | 0.069 |

| Problem 632 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 24 | 50 | 21 | 56 | 58 | 0 | 27 | 38 |
| N.S. | 1 | 1.00 | 1.00 | 2.08 | 0.88 | 2.33 | 2.42 | 0.00 | 1.12 | 1.58 |
| time (sec) | N/A | 0.007 | 0.004 | 0.004 | 2.962 | 1.294 | 1.081 | 0.000 | 0.032 | 0.056 |

| Problem 633 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 14 | 13 | 13 | 39 | 30 | 13 | 17 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.76 | 0.76 | 2.29 | 1.76 | 0.76 | 1.00 |
| time (sec) | N/A | 0.001 | 0.004 | 0.004 | 1.351 | 1.130 | 0.934 | 1.283 | 0.310 | 0.020 |

| Problem 634 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 24 | 19 | 28 | 18 | 139 | 43 | 19 | 25 |
| N.S. | 1 | 1.00 | 0.60 | 0.48 | 0.70 | 0.45 | 3.48 | 1.08 | 0.48 | 0.62 |
| time (sec) | N/A | 0.004 | 0.006 | 0.003 | 1.346 | 0.874 | 1.957 | 1.117 | 0.292 | 0.092 |

| Problem 635 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 30 | 138 | 101 | 155 | 173 | 136 | -1 | 88 |
| N.S. | 1 | 1.00 | 0.34 | 1.55 | 1.13 | 1.74 | 1.94 | 1.53 | -0.01 | 0.99 |
| time (sec) | N/A | 0.021 | 0.006 | 0.028 | 2.991 | 1.226 | 7.026 | 10.853 | 0.000 | 0.170 |

| Problem 636 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 30 | 133 | 71 | 138 | 128 | 119 | -1 | 75 |
| N.S. | 1 | 1.00 | 0.46 | 2.05 | 1.09 | 2.12 | 1.97 | 1.83 | -0.02 | 1.15 |
| time (sec) | N/A | 0.014 | 0.006 | 0.028 | 2.995 | 1.303 | 3.199 | 10.407 | 0.000 | 0.152 |

| Problem 637 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 45 | 67 | 38 | 122 | 92 | 92 | -1 | 66 |
| N.S. | 1 | 1.00 | 1.00 | 1.49 | 0.84 | 2.71 | 2.04 | 2.04 | -0.02 | 1.47 |
| time (sec) | N/A | 0.009 | 0.038 | 0.046 | 2.982 | 1.042 | 1.693 | 10.063 | 0.000 | 0.118 |

| Problem 638 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 20 | 39 | 50 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 1.25 | 2.44 | 3.12 | 0.75 | 1.00 |
| time (sec) | N/A | 0.001 | 0.004 | 0.005 | 1.273 | 1.158 | 0.931 | 1.026 | 0.299 | 0.025 |

| Problem 639 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 21 | 18 | 28 | 29 | 90 | 83 | 27 | 29 |
| N.S. | 1 | 1.00 | 0.62 | 0.53 | 0.82 | 0.85 | 2.65 | 2.44 | 0.79 | 0.85 |
| time (sec) | N/A | 0.003 | 0.007 | 0.004 | 1.259 | 1.132 | 1.606 | 1.086 | 0.321 | 0.088 |

| Problem 640 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 33 | 28 | 44 | 40 | 354 | 96 | 38 | 40 |
| N.S. | 1 | 1.00 | 0.59 | 0.50 | 0.79 | 0.71 | 6.32 | 1.71 | 0.68 | 0.71 |
| time (sec) | N/A | 0.008 | 0.008 | 0.006 | 1.281 | 1.265 | 4.293 | 1.168 | 0.362 | 0.125 |

| Problem 641 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 30 | 168 | 86 | 187 | 753 | 200 | -1 | 89 |
| N.S. | 1 | 1.00 | 0.34 | 1.89 | 0.97 | 2.10 | 8.46 | 2.25 | -0.01 | 1.00 |
| time (sec) | N/A | 0.022 | 0.007 | 0.038 | 2.990 | 1.137 | 6.803 | 10.724 | 0.000 | 0.186 |

| Problem 642 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 53 | 73 | 50 | 173 | 649 | 178 | -1 | 79 |
| N.S. | 1 | 1.00 | 0.79 | 1.09 | 0.75 | 2.58 | 9.69 | 2.66 | -0.01 | 1.18 |
| time (sec) | N/A | 0.015 | 0.056 | 0.041 | 2.919 | 1.391 | 3.700 | 10.628 | 0.000 | 0.168 |

| Problem 643 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 14 | 13 | 28 | 65 | 95 | 13 | 26 |
| N.S. | 1 | 1.00 | 1.00 | 0.74 | 0.68 | 1.47 | 3.42 | 5.00 | 0.68 | 1.37 |
| time (sec) | N/A | 0.001 | 0.005 | 0.003 | 1.382 | 1.034 | 1.458 | 1.252 | 0.232 | 0.106 |

| Problem 644 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 24 | 19 | 25 | 33 | 177 | 90 | 45 | 31 |
| N.S. | 1 | 1.00 | 0.62 | 0.49 | 0.64 | 0.85 | 4.54 | 2.31 | 1.15 | 0.79 |
| time (sec) | N/A | 0.004 | 0.008 | 0.005 | 1.302 | 1.310 | 1.908 | 1.100 | 0.357 | 0.100 |

| Problem 645 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 33 | 28 | 42 | 46 | 243 | 170 | 59 | 40 |
| N.S. | 1 | 1.00 | 0.57 | 0.48 | 0.72 | 0.79 | 4.19 | 2.93 | 1.02 | 0.69 |
| time (sec) | N/A | 0.006 | 0.010 | 0.004 | 1.352 | 1.184 | 3.995 | 1.122 | 0.367 | 0.120 |

| Problem 646 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 75 | 75 | 41 | 36 | 58 | 56 | 529 | 183 | 73 | 48 |
| N.S. | 1 | 1.00 | 0.55 | 0.48 | 0.77 | 0.75 | 7.05 | 2.44 | 0.97 | 0.64 |
| time (sec) | N/A | 0.010 | 0.013 | 0.004 | 1.351 | 1.099 | 12.398 | 1.250 | 0.438 | 0.122 |

| Problem 647 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 25 | 41 | 37 | 27 | 54 | 17 | 31 | 39 |
| N.S. | 1 | 1.00 | 0.93 | 1.52 | 1.37 | 1.00 | 2.00 | 0.63 | 1.15 | 1.44 |
| time (sec) | N/A | 0.005 | 0.009 | 0.006 | 3.004 | 1.516 | 1.646 | 1.174 | 0.570 | 0.076 |

| Problem 648 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 12 | 27 | 14 | 14 | 20 | 6 | 16 | 8 |
| N.S. | 1 | 1.00 | 1.50 | 3.38 | 1.75 | 1.75 | 2.50 | 0.75 | 2.00 | 1.00 |
| time (sec) | N/A | 0.003 | 0.008 | 0.004 | 2.951 | 1.255 | 0.971 | 1.095 | 0.051 | 0.037 |

| Problem 649 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 48 | 21 | 57 | 42 | 0 | 23 | 38 |
| N.S. | 1 | 1.00 | 1.00 | 2.53 | 1.11 | 3.00 | 2.21 | 0.00 | 1.21 | 2.00 |
| time (sec) | N/A | 0.005 | 0.005 | 0.007 | 2.889 | 1.162 | 1.060 | 0.000 | 0.125 | 0.054 |

| Problem 650 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 17 | 14 | 13 | 18 | 19 | 13 | 13 | 21 |
| N.S. | 1 | 1.00 | 0.81 | 0.67 | 0.62 | 0.86 | 0.90 | 0.62 | 0.62 | 1.00 |
| time (sec) | N/A | 0.004 | 0.005 | 0.001 | 1.296 | 1.253 | 2.009 | 0.977 | 0.028 | 0.010 |

| Problem 651 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 17 | 14 | 13 | 18 | 19 | 13 | 13 | 21 |
| N.S. | 1 | 1.00 | 0.81 | 0.67 | 0.62 | 0.86 | 0.90 | 0.62 | 0.62 | 1.00 |
| time (sec) | N/A | 0.004 | 0.005 | 0.002 | 1.351 | 1.236 | 1.304 | 1.045 | 0.025 | 0.010 |

| Problem 652 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 17 | 14 | 13 | 16 | 19 | 13 | 13 | 21 |
| N.S. | 1 | 1.00 | 0.81 | 0.67 | 0.62 | 0.76 | 0.90 | 0.62 | 0.62 | 1.00 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 1.348 | 1.238 | 0.446 | 0.988 | 0.024 | 0.009 |

| Problem 653 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 17 | 14 | 13 | 16 | 19 | 13 | 13 | 21 |
| N.S. | 1 | 1.00 | 0.81 | 0.67 | 0.62 | 0.76 | 0.90 | 0.62 | 0.62 | 1.00 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 1.368 | 1.166 | 1.520 | 1.027 | 0.025 | 0.009 |

| Problem 654 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 17 | 14 | 13 | 13 | 19 | 13 | 13 | 21 |
| N.S. | 1 | 1.00 | 0.81 | 0.67 | 0.62 | 0.62 | 0.90 | 0.62 | 0.62 | 1.00 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 1.307 | 1.303 | 1.659 | 1.121 | 0.024 | 0.009 |

| Problem 655 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 16 | 13 | 13 | 12 | 17 | 13 | 12 | 20 |
| N.S. | 1 | 1.00 | 0.84 | 0.68 | 0.68 | 0.63 | 0.89 | 0.68 | 0.63 | 1.05 |
| time (sec) | N/A | 0.003 | 0.005 | 0.002 | 1.283 | 1.134 | 1.494 | 1.116 | 0.023 | 0.009 |

| Problem 656 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 16 | 14 | 13 | 12 | 17 | 13 | 13 | 16 |
| N.S. | 1 | 1.00 | 0.84 | 0.74 | 0.68 | 0.63 | 0.89 | 0.68 | 0.68 | 0.84 |
| time (sec) | N/A | 0.004 | 0.005 | 0.001 | 1.336 | 0.996 | 0.387 | 1.126 | 0.027 | 0.011 |

| Problem 657 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 12 | 13 | 13 | 17 | 13 | 13 | 17 |
| N.S. | 1 | 1.00 | 1.00 | 0.63 | 0.68 | 0.68 | 0.89 | 0.68 | 0.68 | 0.89 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 1.299 | 1.167 | 0.451 | 1.101 | 0.026 | 0.012 |

| Problem 658 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 28 | 25 | 24 | 29 | 34 | 24 | 24 | 28 |
| N.S. | 1 | 1.00 | 0.78 | 0.69 | 0.67 | 0.81 | 0.94 | 0.67 | 0.67 | 0.78 |
| time (sec) | N/A | 0.007 | 0.008 | 0.004 | 1.312 | 1.302 | 3.748 | 1.044 | 0.045 | 0.013 |

| Problem 659 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 28 | 25 | 24 | 29 | 34 | 24 | 24 | 28 |
| N.S. | 1 | 1.00 | 0.78 | 0.69 | 0.67 | 0.81 | 0.94 | 0.67 | 0.67 | 0.78 |
| time (sec) | N/A | 0.007 | 0.008 | 0.005 | 1.324 | 1.018 | 2.646 | 1.059 | 0.038 | 0.013 |

| Problem 660 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 28 | 25 | 24 | 27 | 34 | 24 | 24 | 34 |
| N.S. | 1 | 1.00 | 0.78 | 0.69 | 0.67 | 0.75 | 0.94 | 0.67 | 0.67 | 0.94 |
| time (sec) | N/A | 0.007 | 0.007 | 0.003 | 1.320 | 1.101 | 1.062 | 1.050 | 0.038 | 0.013 |

| Problem 661 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 28 | 25 | 24 | 27 | 2633 | 24 | 24 | 34 |
| N.S. | 1 | 1.00 | 0.78 | 0.69 | 0.67 | 0.75 | 73.14 | 0.67 | 0.67 | 0.94 |
| time (sec) | N/A | 0.007 | 0.007 | 0.006 | 1.277 | 1.147 | 2.218 | 1.036 | 0.035 | 0.012 |

| Problem 662 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 28 | 25 | 24 | 24 | 1765 | 24 | 24 | 34 |
| N.S. | 1 | 1.00 | 0.78 | 0.69 | 0.67 | 0.67 | 49.03 | 0.67 | 0.67 | 0.94 |
| time (sec) | N/A | 0.008 | 0.008 | 0.005 | 1.318 | 1.104 | 2.045 | 1.106 | 0.038 | 0.012 |

| Problem 663 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 28 | 25 | 24 | 24 | 1741 | 24 | 24 | 34 |
| N.S. | 1 | 1.00 | 0.82 | 0.74 | 0.71 | 0.71 | 51.21 | 0.71 | 0.71 | 1.00 |
| time (sec) | N/A | 0.007 | 0.008 | 0.006 | 1.380 | 1.331 | 2.075 | 0.921 | 0.034 | 0.013 |

| Problem 664 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 27 | 25 | 24 | 23 | 1826 | 24 | 24 | 27 |
| N.S. | 1 | 1.00 | 0.84 | 0.78 | 0.75 | 0.72 | 57.06 | 0.75 | 0.75 | 0.84 |
| time (sec) | N/A | 0.007 | 0.008 | 0.003 | 1.355 | 1.287 | 2.090 | 1.175 | 0.037 | 0.015 |

| Problem 665 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 27 | 25 | 24 | 23 | 1957 | 24 | 24 | 27 |
| N.S. | 1 | 1.00 | 0.79 | 0.74 | 0.71 | 0.68 | 57.56 | 0.71 | 0.71 | 0.79 |
| time (sec) | N/A | 0.007 | 0.008 | 0.004 | 1.351 | 1.207 | 2.063 | 1.221 | 0.036 | 0.015 |

| Problem 666 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 39 | 36 | 35 | 40 | 49 | 35 | 35 | 47 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.69 | 0.78 | 0.96 | 0.69 | 0.69 | 0.92 |
| time (sec) | N/A | 0.010 | 0.011 | 0.005 | 1.346 | 1.254 | 6.393 | 1.060 | 0.043 | 0.015 |

| Problem 667 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 39 | 36 | 35 | 40 | 49 | 35 | 35 | 47 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.69 | 0.78 | 0.96 | 0.69 | 0.69 | 0.92 |
| time (sec) | N/A | 0.011 | 0.010 | 0.005 | 1.289 | 1.213 | 4.551 | 1.061 | 0.044 | 0.014 |

| Problem 668 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 39 | 36 | 35 | 38 | 49 | 35 | 35 | 47 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.69 | 0.75 | 0.96 | 0.69 | 0.69 | 0.92 |
| time (sec) | N/A | 0.011 | 0.010 | 0.006 | 1.337 | 1.396 | 2.187 | 1.066 | 0.044 | 0.014 |

| Problem 669 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 39 | 36 | 35 | 38 | 5012 | 35 | 35 | 47 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.69 | 0.75 | 98.27 | 0.69 | 0.69 | 0.92 |
| time (sec) | N/A | 0.011 | 0.010 | 0.005 | 1.349 | 1.178 | 3.223 | 1.161 | 0.048 | 0.014 |

| Problem 670 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 39 | 36 | 35 | 35 | 6246 | 35 | 35 | 47 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.69 | 0.69 | 122.47 | 0.69 | 0.69 | 0.92 |
| time (sec) | N/A | 0.010 | 0.010 | 0.005 | 1.327 | 0.776 | 3.191 | 0.859 | 0.044 | 0.014 |

| Problem 671 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 39 | 36 | 35 | 35 | 6667 | 35 | 35 | 47 |
| N.S. | 1 | 1.00 | 0.80 | 0.73 | 0.71 | 0.71 | 136.06 | 0.71 | 0.71 | 0.96 |
| time (sec) | N/A | 0.011 | 0.010 | 0.006 | 1.293 | 1.300 | 3.171 | 0.941 | 0.044 | 0.015 |

| Problem 672 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 39 | 36 | 35 | 35 | 4004 | 35 | 35 | 39 |
| N.S. | 1 | 1.00 | 0.80 | 0.73 | 0.71 | 0.71 | 81.71 | 0.71 | 0.71 | 0.80 |
| time (sec) | N/A | 0.011 | 0.011 | 0.004 | 1.323 | 0.976 | 3.261 | 1.072 | 0.044 | 0.018 |

| Problem 673 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 39 | 36 | 35 | 35 | 3964 | 35 | 35 | 39 |
| N.S. | 1 | 1.00 | 0.80 | 0.73 | 0.71 | 0.71 | 80.90 | 0.71 | 0.71 | 0.80 |
| time (sec) | N/A | 0.011 | 0.011 | 0.005 | 1.361 | 1.313 | 3.238 | 1.167 | 0.043 | 0.018 |

| Problem 674 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 125 | 125 | 38 | 122 | 130 | 147 | 241 | 138 | 151 | 150 |
| N.S. | 1 | 1.00 | 0.30 | 0.98 | 1.04 | 1.18 | 1.93 | 1.10 | 1.21 | 1.20 |
| time (sec) | N/A | 0.070 | 0.010 | 0.008 | 3.003 | 0.851 | 47.121 | 1.037 | 0.243 | 0.095 |

| Problem 675 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 140 | 121 | 128 | 116 | 240 | 136 | 126 | 144 |
| N.S. | 1 | 1.00 | 1.14 | 0.98 | 1.04 | 0.94 | 1.95 | 1.11 | 1.02 | 1.17 |
| time (sec) | N/A | 0.056 | 0.058 | 0.006 | 3.051 | 1.419 | 25.855 | 1.208 | 0.068 | 0.086 |

| Problem 676 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 111 | 111 | 29 | 107 | 114 | 128 | 228 | 118 | 130 | 136 |
| N.S. | 1 | 1.00 | 0.26 | 0.96 | 1.03 | 1.15 | 2.05 | 1.06 | 1.17 | 1.23 |
| time (sec) | N/A | 0.040 | 0.009 | 0.007 | 2.905 | 1.453 | 9.084 | 1.043 | 0.150 | 0.079 |

| Problem 677 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 126 | 108 | 115 | 114 | 219 | 119 | 126 | 135 |
| N.S. | 1 | 1.00 | 1.16 | 0.99 | 1.06 | 1.05 | 2.01 | 1.09 | 1.16 | 1.24 |
| time (sec) | N/A | 0.039 | 0.026 | 0.007 | 2.926 | 1.276 | 6.098 | 1.147 | 0.074 | 0.077 |

| Problem 678 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 27 | 96 | 103 | 313 | 212 | 118 | 120 | 126 |
| N.S. | 1 | 1.00 | 0.27 | 0.96 | 1.03 | 3.13 | 2.12 | 1.18 | 1.20 | 1.26 |
| time (sec) | N/A | 0.028 | 0.007 | 0.005 | 2.833 | 1.298 | 7.412 | 1.171 | 0.113 | 0.062 |

| Problem 679 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 103 | 95 | 102 | 307 | 212 | 117 | 110 | 125 |
| N.S. | 1 | 1.00 | 1.03 | 0.95 | 1.02 | 3.07 | 2.12 | 1.17 | 1.10 | 1.25 |
| time (sec) | N/A | 0.028 | 0.024 | 0.006 | 2.963 | 1.487 | 11.348 | 1.175 | 0.206 | 0.071 |

| Problem 680 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 25 | 104 | 111 | 113 | 218 | 125 | 124 | 134 |
| N.S. | 1 | 1.00 | 0.23 | 0.95 | 1.02 | 1.04 | 2.00 | 1.15 | 1.14 | 1.23 |
| time (sec) | N/A | 0.038 | 0.005 | 0.009 | 2.963 | 1.239 | 25.292 | 1.211 | 0.147 | 0.091 |

| Problem 681 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 111 | 111 | 27 | 105 | 112 | 147 | 221 | 120 | 138 | 137 |
| N.S. | 1 | 1.00 | 0.24 | 0.95 | 1.01 | 1.32 | 1.99 | 1.08 | 1.24 | 1.23 |
| time (sec) | N/A | 0.039 | 0.005 | 0.008 | 2.994 | 1.425 | 34.964 | 1.139 | 0.072 | 0.089 |

| Problem 682 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 129 | 129 | 27 | 123 | 133 | 162 | 0 | 135 | 150 | 159 |
| N.S. | 1 | 1.00 | 0.21 | 0.95 | 1.03 | 1.26 | 0.00 | 1.05 | 1.16 | 1.23 |
| time (sec) | N/A | 0.047 | 0.005 | 0.011 | 2.959 | 1.349 | 0.000 | 1.052 | 0.265 | 0.183 |

| Problem 683 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 125 | 125 | 27 | 123 | 133 | 147 | 0 | 135 | 142 | 156 |
| N.S. | 1 | 1.00 | 0.22 | 0.98 | 1.06 | 1.18 | 0.00 | 1.08 | 1.14 | 1.25 |
| time (sec) | N/A | 0.047 | 0.005 | 0.013 | 2.932 | 1.396 | 0.000 | 1.069 | 0.152 | 0.184 |

| Problem 684 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 27 | 112 | 120 | 394 | 787 | 136 | 142 | 145 |
| N.S. | 1 | 1.00 | 0.23 | 0.97 | 1.04 | 3.43 | 6.84 | 1.18 | 1.23 | 1.26 |
| time (sec) | N/A | 0.038 | 0.004 | 0.010 | 2.994 | 1.316 | 106.983 | 1.199 | 0.236 | 0.163 |

| Problem 685 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 27 | 112 | 120 | 389 | 607 | 136 | 120 | 145 |
| N.S. | 1 | 1.00 | 0.23 | 0.96 | 1.03 | 3.32 | 5.19 | 1.16 | 1.03 | 1.24 |
| time (sec) | N/A | 0.038 | 0.005 | 0.011 | 3.026 | 1.253 | 71.720 | 1.129 | 0.063 | 0.168 |

| Problem 686 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 116 | 116 | 27 | 120 | 127 | 396 | 774 | 132 | 144 | 144 |
| N.S. | 1 | 1.00 | 0.23 | 1.03 | 1.09 | 3.41 | 6.67 | 1.14 | 1.24 | 1.24 |
| time (sec) | N/A | 0.041 | 0.005 | 0.009 | 2.964 | 1.438 | 79.660 | 1.111 | 0.358 | 0.157 |

| Problem 687 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 113 | 25 | 120 | 127 | 387 | 590 | 132 | 134 | 144 |
| N.S. | 1 | 1.00 | 0.22 | 1.06 | 1.12 | 3.42 | 5.22 | 1.17 | 1.19 | 1.27 |
| time (sec) | N/A | 0.040 | 0.005 | 0.008 | 3.009 | 1.193 | 115.985 | 1.035 | 0.223 | 0.156 |

| Problem 688 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 124 | 124 | 25 | 121 | 132 | 156 | 0 | 145 | 151 | 152 |
| N.S. | 1 | 1.00 | 0.20 | 0.98 | 1.06 | 1.26 | 0.00 | 1.17 | 1.22 | 1.23 |
| time (sec) | N/A | 0.049 | 0.005 | 0.012 | 3.036 | 1.009 | 0.000 | 0.975 | 0.152 | 0.174 |

| Problem 689 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 128 | 128 | 27 | 121 | 132 | 189 | 0 | 137 | 166 | 155 |
| N.S. | 1 | 1.00 | 0.21 | 0.95 | 1.03 | 1.48 | 0.00 | 1.07 | 1.30 | 1.21 |
| time (sec) | N/A | 0.050 | 0.005 | 0.014 | 2.943 | 0.650 | 0.000 | 1.006 | 0.165 | 0.181 |

| Problem 690 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 140 | 27 | 124 | 143 | 506 | 0 | 146 | 165 | 161 |
| N.S. | 1 | 1.00 | 0.19 | 0.89 | 1.02 | 3.61 | 0.00 | 1.04 | 1.18 | 1.15 |
| time (sec) | N/A | 0.049 | 0.005 | 0.013 | 2.952 | 1.028 | 0.000 | 1.058 | 0.172 | 0.261 |

| Problem 691 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 140 | 27 | 124 | 143 | 503 | 0 | 146 | 139 | 161 |
| N.S. | 1 | 1.00 | 0.19 | 0.89 | 1.02 | 3.59 | 0.00 | 1.04 | 0.99 | 1.15 |
| time (sec) | N/A | 0.054 | 0.004 | 0.012 | 2.879 | 1.115 | 0.000 | 1.125 | 0.066 | 0.252 |

| Problem 692 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 143 | 143 | 27 | 132 | 153 | 508 | 0 | 149 | 172 | 164 |
| N.S. | 1 | 1.00 | 0.19 | 0.92 | 1.07 | 3.55 | 0.00 | 1.04 | 1.20 | 1.15 |
| time (sec) | N/A | 0.050 | 0.005 | 0.013 | 2.922 | 0.989 | 0.000 | 1.064 | 0.265 | 0.239 |

| Problem 693 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 143 | 143 | 27 | 132 | 152 | 501 | 0 | 148 | 146 | 163 |
| N.S. | 1 | 1.00 | 0.19 | 0.92 | 1.06 | 3.50 | 0.00 | 1.03 | 1.02 | 1.14 |
| time (sec) | N/A | 0.052 | 0.005 | 0.013 | 2.959 | 1.207 | 0.000 | 1.110 | 0.235 | 0.236 |

| Problem 694 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 140 | 27 | 136 | 151 | 510 | 0 | 143 | 167 | 157 |
| N.S. | 1 | 1.00 | 0.19 | 0.97 | 1.08 | 3.64 | 0.00 | 1.02 | 1.19 | 1.12 |
| time (sec) | N/A | 0.053 | 0.005 | 0.007 | 2.963 | 0.886 | 0.000 | 1.064 | 0.192 | 0.158 |

| Problem 695 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 140 | 25 | 136 | 151 | 499 | 0 | 143 | 157 | 157 |
| N.S. | 1 | 1.00 | 0.18 | 0.97 | 1.08 | 3.56 | 0.00 | 1.02 | 1.12 | 1.12 |
| time (sec) | N/A | 0.050 | 0.004 | 0.007 | 2.997 | 1.365 | 0.000 | 0.999 | 0.241 | 0.152 |

| Problem 696 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 152 | 152 | 25 | 139 | 154 | 211 | 0 | 155 | 174 | 168 |
| N.S. | 1 | 1.00 | 0.16 | 0.91 | 1.01 | 1.39 | 0.00 | 1.02 | 1.14 | 1.11 |
| time (sec) | N/A | 0.060 | 0.005 | 0.017 | 2.989 | 1.286 | 0.000 | 1.163 | 0.087 | 0.272 |

| Problem 697 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 152 | 152 | 27 | 139 | 154 | 244 | 0 | 150 | 182 | 168 |
| N.S. | 1 | 1.00 | 0.18 | 0.91 | 1.01 | 1.61 | 0.00 | 0.99 | 1.20 | 1.11 |
| time (sec) | N/A | 0.061 | 0.005 | 0.017 | 2.991 | 0.668 | 0.000 | 1.080 | 0.173 | 0.270 |

| Problem 698 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 58 | 62 | 61 | 82 | 243 | 64 | 46 | 58 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.05 | 1.41 | 4.19 | 1.10 | 0.79 | 1.00 |
| time (sec) | N/A | 0.021 | 0.017 | 0.010 | 2.909 | 0.961 | 2.352 | 1.169 | 0.068 | 0.085 |

| Problem 699 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 187 | 187 | 166 | 1535 | 187 | 1277 | 9996 | 1925 | 1274 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 8.21 | 1.00 | 6.83 | 53.45 | 10.29 | 6.81 | 0.00 |
| time (sec) | N/A | 0.085 | 0.111 | 0.007 | 1.387 | 1.168 | 6.929 | 1.204 | 1.370 | 0.180 |

| Problem 700 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 133 | 133 | 118 | 782 | 133 | 665 | 4257 | 992 | 683 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 5.88 | 1.00 | 5.00 | 32.01 | 7.46 | 5.14 | 0.00 |
| time (sec) | N/A | 0.050 | 0.068 | 0.006 | 1.366 | 1.347 | 3.337 | 1.396 | 0.777 | 0.051 |

| Problem 701 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 54 | 170 | 61 | 157 | 663 | 224 | 167 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 2.79 | 1.00 | 2.57 | 10.87 | 3.67 | 2.74 | 0.00 |
| time (sec) | N/A | 0.018 | 0.031 | 0.000 | 1.299 | 1.035 | 0.917 | 1.067 | 0.443 | 0.000 |

| Problem 702 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 38 | 87 | 43 | 85 | 299 | 117 | 93 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 2.02 | 1.00 | 1.98 | 6.95 | 2.72 | 2.16 | 0.00 |
| time (sec) | N/A | 0.013 | 0.031 | 0.000 | 1.323 | 1.167 | 0.546 | 1.044 | 0.370 | 0.000 |

| Problem 703 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 22 | 31 | 25 | 33 | 87 | 43 | 30 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.24 | 1.00 | 1.32 | 3.48 | 1.72 | 1.20 | 0.00 |
| time (sec) | N/A | 0.006 | 0.014 | 0.000 | 1.354 | 0.792 | 0.308 | 1.021 | 0.304 | 0.000 |

| Problem 704 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 67 | 126 | 101 | 143 | 1318 | 226 | 176 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 1.52 | 1.22 | 1.72 | 15.88 | 2.72 | 2.12 | 0.00 |
| time (sec) | N/A | 0.031 | 0.054 | 0.007 | 1.364 | 1.286 | 2.326 | 1.212 | 0.534 | 0.026 |

| Problem 705 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 57 | 73 | 68 | 96 | 597 | 140 | 192 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 1.22 | 1.13 | 1.60 | 9.95 | 2.33 | 3.20 | 0.00 |
| time (sec) | N/A | 0.019 | 0.028 | 0.006 | 1.374 | 1.077 | 1.315 | 1.074 | 0.557 | 0.024 |

| Problem 706 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 33 | 36 | 42 | 53 | 201 | 76 | 94 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.92 | 1.08 | 1.36 | 5.15 | 1.95 | 2.41 | 0.00 |
| time (sec) | N/A | 0.012 | 0.017 | 0.002 | 1.311 | 1.144 | 0.702 | 1.051 | 0.378 | 0.020 |

| Problem 707 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 17 | 19 | 18 | 20 | 20 | 18 | 18 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 1.06 | 1.00 | 1.11 | 1.11 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.003 | 0.009 | 0.003 | 1.296 | 1.231 | 0.066 | 1.137 | 0.198 | 0.013 |

| Problem 708 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 64 | 77 | 0 | 104 | 0 | 0 | 136 | 0 |
| N.S. | 1 | 1.00 | 0.58 | 0.70 | 0.00 | 0.95 | 0.00 | 0.00 | 1.24 | 0.00 |
| time (sec) | N/A | 0.036 | 0.026 | 0.006 | 0.000 | 0.993 | 0.000 | 0.000 | 0.523 | 0.026 |

| Problem 709 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 39 | 44 | 0 | 64 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 1.00 | 0.61 | 0.69 | 0.00 | 1.00 | 0.00 | 0.00 | 1.25 | 0.00 |
| time (sec) | N/A | 0.009 | 0.016 | 0.005 | 0.000 | 1.226 | 0.000 | 0.000 | 0.447 | 0.026 |

| Problem 710 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 25 | 29 | 0 | 33 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.04 | 0.00 | 1.18 | 0.00 | 0.00 | 1.04 | 0.00 |
| time (sec) | N/A | 0.003 | 0.006 | 0.003 | 0.000 | 0.973 | 0.000 | 0.000 | 0.349 | 0.026 |

| Problem 711 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 20 | 22 | 32 | 0 | 0 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.05 | 1.16 | 1.68 | 0.00 | 0.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.003 | 0.005 | 0.003 | 1.302 | 0.952 | 0.000 | 0.000 | 0.504 | 0.035 |

| Problem 712 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 40 | 41 | 0 | 64 | 323 | 0 | 86 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.71 | 0.00 | 1.10 | 5.57 | 0.00 | 1.48 | 0.00 |
| time (sec) | N/A | 0.013 | 0.014 | 0.005 | 0.000 | 1.147 | 94.815 | 0.000 | 0.498 | 0.025 |

| Problem 713 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 40 | 41 | 0 | 64 | 323 | 0 | 86 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.71 | 0.00 | 1.10 | 5.57 | 0.00 | 1.48 | 0.00 |
| time (sec) | N/A | 0.011 | 0.002 | 0.000 | 0.000 | 1.021 | 94.488 | 0.000 | 0.002 | 0.041 |

| Problem 714 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 24 | 21 | 33 | 22 | 36 | 22 | -1 | 24 |
| N.S. | 1 | 1.00 | 0.69 | 0.60 | 0.94 | 0.63 | 1.03 | 0.63 | -0.03 | 0.69 |
| time (sec) | N/A | 0.011 | 0.005 | 0.006 | 1.334 | 0.718 | 0.435 | 0.886 | 0.000 | 0.020 |

| Problem 715 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 24 | 21 | 31 | 22 | 36 | 22 | -1 | 24 |
| N.S. | 1 | 1.00 | 0.69 | 0.60 | 0.89 | 0.63 | 1.03 | 0.63 | -0.03 | 0.69 |
| time (sec) | N/A | 0.010 | 0.004 | 0.002 | 1.370 | 0.970 | 0.339 | 1.031 | 0.000 | 0.020 |

| Problem 716 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 24 | 21 | 28 | 22 | 36 | 22 | -1 | 24 |
| N.S. | 1 | 1.00 | 0.69 | 0.60 | 0.80 | 0.63 | 1.03 | 0.63 | -0.03 | 0.69 |
| time (sec) | N/A | 0.009 | 0.004 | 0.004 | 1.348 | 0.972 | 0.268 | 0.854 | 0.000 | 0.018 |

| Problem 717 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 22 | 19 | 25 | 20 | 34 | 22 | 20 | 22 |
| N.S. | 1 | 1.00 | 0.67 | 0.58 | 0.76 | 0.61 | 1.03 | 0.67 | 0.61 | 0.67 |
| time (sec) | N/A | 0.008 | 0.003 | 0.010 | 1.316 | 0.947 | 0.224 | 1.047 | 0.542 | 0.017 |

| Problem 718 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 24 | 17 | 0 | 16 | 29 | 17 | 14 | 20 |
| N.S. | 1 | 1.00 | 0.89 | 0.63 | 0.00 | 0.59 | 1.07 | 0.63 | 0.52 | 0.74 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 0.000 | 0.920 | 0.228 | 1.090 | 0.192 | 0.016 |

| Problem 719 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 20 | 20 | 0 | 19 | 0 | 17 | -1 | 19 |
| N.S. | 1 | 1.00 | 0.71 | 0.71 | 0.00 | 0.68 | 0.00 | 0.61 | -0.04 | 0.68 |
| time (sec) | N/A | 0.005 | 0.005 | 0.022 | 0.000 | 1.272 | 0.000 | 0.925 | 0.000 | 0.019 |

| Problem 720 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 20 | 21 | 0 | 20 | 0 | 20 | -1 | 24 |
| N.S. | 1 | 1.00 | 0.62 | 0.66 | 0.00 | 0.62 | 0.00 | 0.62 | -0.03 | 0.75 |
| time (sec) | N/A | 0.007 | 0.006 | 0.008 | 0.000 | 1.227 | 0.000 | 1.099 | 0.000 | 0.022 |

| Problem 721 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 22 | 19 | 0 | 18 | 36 | 19 | 28 | 24 |
| N.S. | 1 | 1.00 | 0.85 | 0.73 | 0.00 | 0.69 | 1.38 | 0.73 | 1.08 | 0.92 |
| time (sec) | N/A | 0.004 | 0.004 | 0.005 | 0.000 | 1.063 | 0.507 | 0.971 | 0.144 | 0.021 |

| Problem 722 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 24 | 21 | 33 | 24 | 36 | 22 | -1 | 24 |
| N.S. | 1 | 1.00 | 0.65 | 0.57 | 0.89 | 0.65 | 0.97 | 0.59 | -0.03 | 0.65 |
| time (sec) | N/A | 0.013 | 0.006 | 0.003 | 1.335 | 0.908 | 1.161 | 1.173 | 0.000 | 0.021 |

| Problem 723 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 24 | 21 | 31 | 24 | 36 | 22 | -1 | 24 |
| N.S. | 1 | 1.00 | 0.65 | 0.57 | 0.84 | 0.65 | 0.97 | 0.59 | -0.03 | 0.65 |
| time (sec) | N/A | 0.012 | 0.006 | 0.005 | 1.339 | 1.085 | 0.909 | 0.929 | 0.000 | 0.020 |

| Problem 724 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 24 | 21 | 28 | 24 | 36 | 22 | -1 | 24 |
| N.S. | 1 | 1.00 | 0.65 | 0.57 | 0.76 | 0.65 | 0.97 | 0.59 | -0.03 | 0.65 |
| time (sec) | N/A | 0.011 | 0.005 | 0.003 | 1.353 | 0.856 | 0.728 | 0.993 | 0.000 | 0.019 |

| Problem 725 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 22 | 19 | 25 | 24 | 34 | 22 | -1 | 22 |
| N.S. | 1 | 1.00 | 0.59 | 0.51 | 0.68 | 0.65 | 0.92 | 0.59 | -0.03 | 0.59 |
| time (sec) | N/A | 0.010 | 0.005 | 0.003 | 1.297 | 0.826 | 0.558 | 0.878 | 0.000 | 0.019 |

| Problem 726 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 25 | 18 | 22 | 24 | 31 | 22 | -1 | 21 |
| N.S. | 1 | 1.00 | 0.68 | 0.49 | 0.59 | 0.65 | 0.84 | 0.59 | -0.03 | 0.57 |
| time (sec) | N/A | 0.009 | 0.002 | 0.005 | 1.336 | 1.087 | 0.577 | 1.090 | 0.000 | 0.019 |

| Problem 727 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 23 | 21 | 0 | 22 | 31 | 22 | 20 | 24 |
| N.S. | 1 | 1.00 | 0.66 | 0.60 | 0.00 | 0.63 | 0.89 | 0.63 | 0.57 | 0.69 |
| time (sec) | N/A | 0.008 | 0.002 | 0.003 | 0.000 | 1.010 | 0.572 | 1.082 | 0.267 | 0.020 |

| Problem 728 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 21 | 20 | 0 | 18 | 32 | 17 | 14 | 23 |
| N.S. | 1 | 1.00 | 0.72 | 0.69 | 0.00 | 0.62 | 1.10 | 0.59 | 0.48 | 0.79 |
| time (sec) | N/A | 0.004 | 0.003 | 0.003 | 0.000 | 1.042 | 0.741 | 0.989 | 0.222 | 0.021 |

| Problem 729 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 21 | 20 | 0 | 21 | 0 | 17 | -1 | 23 |
| N.S. | 1 | 1.00 | 0.70 | 0.67 | 0.00 | 0.70 | 0.00 | 0.57 | -0.03 | 0.77 |
| time (sec) | N/A | 0.005 | 0.005 | 0.003 | 0.000 | 0.879 | 0.000 | 0.958 | 0.000 | 0.022 |

| Problem 730 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 24 | 21 | 33 | 28 | 36 | 28 | -1 | 24 |
| N.S. | 1 | 1.00 | 0.59 | 0.51 | 0.80 | 0.68 | 0.88 | 0.68 | -0.02 | 0.59 |
| time (sec) | N/A | 0.016 | 0.007 | 0.003 | 1.244 | 1.218 | 2.543 | 0.966 | 0.000 | 0.025 |

| Problem 731 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 24 | 21 | 31 | 28 | 36 | 28 | -1 | 24 |
| N.S. | 1 | 1.00 | 0.59 | 0.51 | 0.76 | 0.68 | 0.88 | 0.68 | -0.02 | 0.59 |
| time (sec) | N/A | 0.015 | 0.007 | 0.003 | 1.305 | 0.594 | 2.097 | 0.801 | 0.000 | 0.022 |

| Problem 732 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 24 | 21 | 28 | 28 | 36 | 28 | -1 | 24 |
| N.S. | 1 | 1.00 | 0.59 | 0.51 | 0.68 | 0.68 | 0.88 | 0.68 | -0.02 | 0.59 |
| time (sec) | N/A | 0.013 | 0.007 | 0.003 | 1.351 | 0.981 | 1.742 | 1.027 | 0.000 | 0.022 |

| Problem 733 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 22 | 19 | 25 | 28 | 34 | 28 | -1 | 22 |
| N.S. | 1 | 1.00 | 0.54 | 0.46 | 0.61 | 0.68 | 0.83 | 0.68 | -0.02 | 0.54 |
| time (sec) | N/A | 0.013 | 0.006 | 0.005 | 1.341 | 0.987 | 1.410 | 0.948 | 0.000 | 0.020 |

| Problem 734 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 25 | 18 | 22 | 28 | 31 | 28 | -1 | 21 |
| N.S. | 1 | 1.00 | 0.61 | 0.44 | 0.54 | 0.68 | 0.76 | 0.68 | -0.02 | 0.51 |
| time (sec) | N/A | 0.012 | 0.003 | 0.002 | 1.402 | 1.212 | 1.429 | 1.074 | 0.000 | 0.023 |

| Problem 735 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 23 | 21 | 24 | 28 | 31 | 28 | 25 | 24 |
| N.S. | 1 | 1.00 | 0.56 | 0.51 | 0.59 | 0.68 | 0.76 | 0.68 | 0.61 | 0.59 |
| time (sec) | N/A | 0.011 | 0.003 | 0.003 | 1.283 | 0.791 | 1.529 | 1.103 | 0.284 | 0.023 |

| Problem 736 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 27 | 21 | 0 | 28 | 34 | 28 | 25 | 24 |
| N.S. | 1 | 1.00 | 0.66 | 0.51 | 0.00 | 0.68 | 0.83 | 0.68 | 0.61 | 0.59 |
| time (sec) | N/A | 0.010 | 0.003 | 0.003 | 0.000 | 1.076 | 1.578 | 0.994 | 0.265 | 0.022 |

| Problem 737 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 25 | 21 | 0 | 26 | 36 | 28 | 20 | 24 |
| N.S. | 1 | 1.00 | 0.64 | 0.54 | 0.00 | 0.67 | 0.92 | 0.72 | 0.51 | 0.62 |
| time (sec) | N/A | 0.008 | 0.002 | 0.004 | 0.000 | 0.774 | 1.623 | 1.139 | 0.260 | 0.023 |

| Problem 738 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 24 | 21 | 33 | 25 | 36 | 26 | -1 | 27 |
| N.S. | 1 | 1.00 | 0.69 | 0.60 | 0.94 | 0.71 | 1.03 | 0.74 | -0.03 | 0.77 |
| time (sec) | N/A | 0.009 | 0.005 | 0.003 | 1.403 | 1.263 | 0.606 | 1.305 | 0.000 | 0.021 |

| Problem 739 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 24 | 21 | 26 | 23 | 36 | 24 | 23 | 25 |
| N.S. | 1 | 1.00 | 0.69 | 0.60 | 0.74 | 0.66 | 1.03 | 0.69 | 0.66 | 0.71 |
| time (sec) | N/A | 0.008 | 0.004 | 0.003 | 1.301 | 1.017 | 0.522 | 1.158 | 0.251 | 0.020 |

| Problem 740 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 23 | 20 | 22 | 19 | 34 | 22 | 19 | 23 |
| N.S. | 1 | 1.00 | 0.72 | 0.62 | 0.69 | 0.59 | 1.06 | 0.69 | 0.59 | 0.72 |
| time (sec) | N/A | 0.004 | 0.002 | 0.003 | 1.301 | 1.180 | 0.462 | 1.105 | 0.221 | 0.020 |

| Problem 741 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 19 | 18 | 20 | 22 | 0 | 35 | 17 | 26 |
| N.S. | 1 | 1.00 | 0.66 | 0.62 | 0.69 | 0.76 | 0.00 | 1.21 | 0.59 | 0.90 |
| time (sec) | N/A | 0.005 | 0.002 | 0.004 | 1.325 | 1.017 | 0.000 | 1.321 | 0.513 | 0.021 |

| Problem 742 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 23 | 18 | 17 | 23 | 0 | 47 | 22 | 30 |
| N.S. | 1 | 1.00 | 0.85 | 0.67 | 0.63 | 0.85 | 0.00 | 1.74 | 0.81 | 1.11 |
| time (sec) | N/A | 0.006 | 0.006 | 0.004 | 1.326 | 1.006 | 0.000 | 0.984 | 1.222 | 0.022 |

| Problem 743 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 23 | 19 | 19 | 21 | 31 | 0 | 25 | 27 |
| N.S. | 1 | 1.00 | 0.88 | 0.73 | 0.73 | 0.81 | 1.19 | 0.00 | 0.96 | 1.04 |
| time (sec) | N/A | 0.004 | 0.006 | 0.003 | 1.251 | 1.028 | 0.544 | 0.000 | 0.157 | 0.022 |

| Problem 744 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 22 | 21 | 19 | 23 | 36 | 0 | 26 | 27 |
| N.S. | 1 | 1.00 | 0.63 | 0.60 | 0.54 | 0.66 | 1.03 | 0.00 | 0.74 | 0.77 |
| time (sec) | N/A | 0.007 | 0.006 | 0.005 | 1.279 | 0.899 | 0.636 | 0.000 | 0.149 | 0.023 |

| Problem 745 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 24 | 21 | 19 | 23 | 37 | 0 | 26 | 27 |
| N.S. | 1 | 1.00 | 0.69 | 0.60 | 0.54 | 0.66 | 1.06 | 0.00 | 0.74 | 0.77 |
| time (sec) | N/A | 0.007 | 0.005 | 0.003 | 1.296 | 1.099 | 0.810 | 0.000 | 0.149 | 0.022 |

| Problem 746 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 23 | 20 | 32 | 19 | 34 | 25 | -1 | 23 |
| N.S. | 1 | 1.00 | 0.61 | 0.53 | 0.84 | 0.50 | 0.89 | 0.66 | -0.03 | 0.61 |
| time (sec) | N/A | 0.006 | 0.004 | 0.003 | 1.307 | 1.045 | 0.635 | 1.014 | 0.000 | 0.024 |

| Problem 747 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 21 | 20 | 23 | 22 | 0 | 40 | 30 | 23 |
| N.S. | 1 | 1.00 | 0.60 | 0.57 | 0.66 | 0.63 | 0.00 | 1.14 | 0.86 | 0.66 |
| time (sec) | N/A | 0.005 | 0.004 | 0.004 | 1.346 | 1.112 | 0.000 | 0.986 | 0.320 | 0.024 |

| Problem 748 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 22 | 21 | 21 | 23 | 0 | 47 | 28 | 24 |
| N.S. | 1 | 1.00 | 0.67 | 0.64 | 0.64 | 0.70 | 0.00 | 1.42 | 0.85 | 0.73 |
| time (sec) | N/A | 0.007 | 0.003 | 0.003 | 1.312 | 1.070 | 0.000 | 1.224 | 0.250 | 0.025 |

| Problem 749 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 22 | 17 | 23 | 21 | 34 | 0 | 25 | 20 |
| N.S. | 1 | 1.00 | 0.76 | 0.59 | 0.79 | 0.72 | 1.17 | 0.00 | 0.86 | 0.69 |
| time (sec) | N/A | 0.004 | 0.003 | 0.003 | 1.346 | 0.705 | 0.538 | 0.000 | 0.149 | 0.022 |

| Problem 750 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 25 | 18 | 19 | 23 | 32 | 0 | 26 | 21 |
| N.S. | 1 | 1.00 | 0.61 | 0.44 | 0.46 | 0.56 | 0.78 | 0.00 | 0.63 | 0.51 |
| time (sec) | N/A | 0.008 | 0.008 | 0.003 | 1.318 | 1.761 | 0.632 | 0.000 | 0.158 | 0.022 |

| Problem 751 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 27 | 21 | 19 | 23 | 32 | 0 | 26 | 24 |
| N.S. | 1 | 1.00 | 0.66 | 0.51 | 0.46 | 0.56 | 0.78 | 0.00 | 0.63 | 0.59 |
| time (sec) | N/A | 0.007 | 0.008 | 0.004 | 1.338 | 1.071 | 0.769 | 0.000 | 0.151 | 0.023 |

| Problem 752 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 22 | 21 | 19 | 23 | 36 | 0 | 26 | 24 |
| N.S. | 1 | 1.00 | 0.54 | 0.51 | 0.46 | 0.56 | 0.88 | 0.00 | 0.63 | 0.59 |
| time (sec) | N/A | 0.008 | 0.007 | 0.004 | 1.278 | 0.958 | 0.934 | 0.000 | 0.149 | 0.024 |

| Problem 753 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 24 | 21 | 19 | 23 | 37 | 0 | 26 | 24 |
| N.S. | 1 | 1.00 | 0.59 | 0.51 | 0.46 | 0.56 | 0.90 | 0.00 | 0.63 | 0.59 |
| time (sec) | N/A | 0.007 | 0.006 | 0.004 | 1.348 | 1.276 | 1.164 | 0.000 | 0.151 | 0.024 |

| Problem 754 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 22 | 21 | 24 | 23 | 0 | 47 | -1 | 24 |
| N.S. | 1 | 1.00 | 0.67 | 0.64 | 0.73 | 0.70 | 0.00 | 1.42 | -0.03 | 0.73 |
| time (sec) | N/A | 0.008 | 0.005 | 0.004 | 1.429 | 0.789 | 0.000 | 1.015 | 0.000 | 0.029 |

| Problem 755 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 24 | 19 | 26 | 21 | 36 | 0 | 25 | 22 |
| N.S. | 1 | 1.00 | 0.83 | 0.66 | 0.90 | 0.72 | 1.24 | 0.00 | 0.86 | 0.76 |
| time (sec) | N/A | 0.005 | 0.006 | 0.003 | 1.342 | 1.050 | 0.931 | 0.000 | 0.153 | 0.025 |

| Problem 756 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 24 | 21 | 23 | 23 | 37 | 0 | 26 | 24 |
| N.S. | 1 | 1.00 | 0.59 | 0.51 | 0.56 | 0.56 | 0.90 | 0.00 | 0.63 | 0.59 |
| time (sec) | N/A | 0.008 | 0.004 | 0.003 | 1.387 | 1.093 | 0.918 | 0.000 | 0.151 | 0.024 |

| Problem 757 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 27 | 19 | 23 | 23 | 36 | 0 | 26 | 22 |
| N.S. | 1 | 1.00 | 0.66 | 0.46 | 0.56 | 0.56 | 0.88 | 0.00 | 0.63 | 0.54 |
| time (sec) | N/A | 0.007 | 0.003 | 0.003 | 1.374 | 0.999 | 0.916 | 0.000 | 0.160 | 0.022 |

| Problem 758 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 27 | 18 | 19 | 23 | 32 | 0 | 26 | 21 |
| N.S. | 1 | 1.00 | 0.66 | 0.44 | 0.46 | 0.56 | 0.78 | 0.00 | 0.63 | 0.51 |
| time (sec) | N/A | 0.008 | 0.008 | 0.003 | 1.334 | 0.743 | 1.121 | 0.000 | 0.158 | 0.024 |

| Problem 759 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 27 | 21 | 19 | 23 | 32 | 0 | 26 | 24 |
| N.S. | 1 | 1.00 | 0.66 | 0.51 | 0.46 | 0.56 | 0.78 | 0.00 | 0.63 | 0.59 |
| time (sec) | N/A | 0.008 | 0.007 | 0.004 | 1.316 | 1.030 | 1.341 | 0.000 | 0.162 | 0.026 |

| Problem 760 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 22 | 21 | 19 | 23 | 36 | 0 | 26 | 24 |
| N.S. | 1 | 1.00 | 0.54 | 0.51 | 0.46 | 0.56 | 0.88 | 0.00 | 0.63 | 0.59 |
| time (sec) | N/A | 0.009 | 0.008 | 0.005 | 1.370 | 1.098 | 1.644 | 0.000 | 0.159 | 0.025 |

| Problem 761 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 24 | 21 | 19 | 23 | 37 | 0 | 26 | 24 |
| N.S. | 1 | 1.00 | 0.59 | 0.51 | 0.46 | 0.56 | 0.90 | 0.00 | 0.63 | 0.59 |
| time (sec) | N/A | 0.008 | 0.007 | 0.002 | 1.342 | 0.827 | 1.968 | 0.000 | 0.162 | 0.027 |

| Problem 762 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 35 | 32 | 54 | 33 | 60 | 35 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.61 | 0.56 | 0.95 | 0.58 | 1.05 | 0.61 | -0.02 | 0.61 |
| time (sec) | N/A | 0.015 | 0.006 | 0.004 | 1.395 | 0.860 | 0.587 | 1.078 | 0.000 | 0.027 |

| Problem 763 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 35 | 32 | 52 | 33 | 61 | 35 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.61 | 0.56 | 0.91 | 0.58 | 1.07 | 0.61 | -0.02 | 0.61 |
| time (sec) | N/A | 0.015 | 0.006 | 0.005 | 1.348 | 0.752 | 0.463 | 1.042 | 0.000 | 0.024 |

| Problem 764 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 35 | 32 | 49 | 33 | 60 | 35 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.61 | 0.56 | 0.86 | 0.58 | 1.05 | 0.61 | -0.02 | 0.61 |
| time (sec) | N/A | 0.013 | 0.006 | 0.004 | 1.311 | 1.105 | 0.368 | 1.110 | 0.000 | 0.024 |

| Problem 765 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 55 | 55 | 33 | 30 | 44 | 31 | 60 | 35 | -1 | 33 |
| N.S. | 1 | 1.00 | 0.60 | 0.55 | 0.80 | 0.56 | 1.09 | 0.64 | -0.02 | 0.60 |
| time (sec) | N/A | 0.014 | 0.007 | 0.004 | 1.352 | 1.117 | 0.297 | 0.948 | 0.000 | 0.025 |

| Problem 766 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 25 | 28 | 0 | 27 | 51 | 29 | -1 | 31 |
| N.S. | 1 | 1.00 | 0.96 | 1.08 | 0.00 | 1.04 | 1.96 | 1.12 | -0.04 | 1.19 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 0.000 | 1.078 | 0.297 | 0.972 | 0.000 | 0.025 |

| Problem 767 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 33 | 33 | 0 | 32 | 0 | 32 | -1 | 34 |
| N.S. | 1 | 1.00 | 0.67 | 0.67 | 0.00 | 0.65 | 0.00 | 0.65 | -0.02 | 0.69 |
| time (sec) | N/A | 0.009 | 0.009 | 0.007 | 0.000 | 0.762 | 0.000 | 1.081 | 0.000 | 0.031 |

| Problem 768 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 31 | 32 | 0 | 31 | 0 | 31 | -1 | 37 |
| N.S. | 1 | 1.00 | 0.63 | 0.65 | 0.00 | 0.63 | 0.00 | 0.63 | -0.02 | 0.76 |
| time (sec) | N/A | 0.011 | 0.011 | 0.010 | 0.000 | 1.212 | 0.000 | 1.015 | 0.000 | 0.034 |

| Problem 769 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 36 | 34 | 0 | 33 | 0 | 35 | -1 | 38 |
| N.S. | 1 | 1.00 | 0.67 | 0.63 | 0.00 | 0.61 | 0.00 | 0.65 | -0.02 | 0.70 |
| time (sec) | N/A | 0.012 | 0.010 | 0.010 | 0.000 | 1.356 | 0.000 | 0.987 | 0.000 | 0.041 |

| Problem 770 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 35 | 32 | 54 | 36 | 60 | 35 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.58 | 0.53 | 0.90 | 0.60 | 1.00 | 0.58 | -0.02 | 0.58 |
| time (sec) | N/A | 0.019 | 0.008 | 0.006 | 1.345 | 0.843 | 1.498 | 1.086 | 0.000 | 0.028 |

| Problem 771 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 35 | 32 | 52 | 36 | 61 | 35 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.58 | 0.53 | 0.87 | 0.60 | 1.02 | 0.58 | -0.02 | 0.58 |
| time (sec) | N/A | 0.017 | 0.008 | 0.005 | 1.329 | 1.140 | 1.225 | 1.085 | 0.000 | 0.029 |

| Problem 772 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 35 | 32 | 49 | 36 | 60 | 35 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.58 | 0.53 | 0.82 | 0.60 | 1.00 | 0.58 | -0.02 | 0.58 |
| time (sec) | N/A | 0.016 | 0.007 | 0.005 | 1.312 | 1.208 | 0.971 | 1.185 | 0.000 | 0.025 |

| Problem 773 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 33 | 30 | 44 | 36 | 60 | 35 | -1 | 33 |
| N.S. | 1 | 1.00 | 0.55 | 0.50 | 0.73 | 0.60 | 1.00 | 0.58 | -0.02 | 0.55 |
| time (sec) | N/A | 0.015 | 0.007 | 0.006 | 1.359 | 0.934 | 0.770 | 1.129 | 0.000 | 0.026 |

| Problem 774 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 36 | 29 | 40 | 36 | 54 | 35 | -1 | 32 |
| N.S. | 1 | 1.00 | 0.60 | 0.48 | 0.67 | 0.60 | 0.90 | 0.58 | -0.02 | 0.53 |
| time (sec) | N/A | 0.015 | 0.004 | 0.003 | 1.296 | 1.069 | 0.793 | 0.951 | 0.000 | 0.028 |

| Problem 775 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 34 | 32 | 0 | 34 | 54 | 35 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.59 | 0.55 | 0.00 | 0.59 | 0.93 | 0.60 | -0.02 | 0.60 |
| time (sec) | N/A | 0.013 | 0.004 | 0.004 | 0.000 | 1.064 | 0.805 | 1.026 | 0.000 | 0.029 |

| Problem 776 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 26 | 31 | 0 | 30 | 51 | 29 | -1 | 34 |
| N.S. | 1 | 1.00 | 0.96 | 1.15 | 0.00 | 1.11 | 1.89 | 1.07 | -0.04 | 1.26 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 0.000 | 1.061 | 0.939 | 1.224 | 0.000 | 0.027 |

| Problem 777 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 34 | 33 | 0 | 35 | 0 | 32 | -1 | 37 |
| N.S. | 1 | 1.00 | 0.65 | 0.63 | 0.00 | 0.67 | 0.00 | 0.62 | -0.02 | 0.71 |
| time (sec) | N/A | 0.010 | 0.009 | 0.005 | 0.000 | 1.133 | 0.000 | 1.198 | 0.000 | 0.033 |

| Problem 778 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 35 | 32 | 49 | 42 | 60 | 44 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.53 | 0.48 | 0.74 | 0.64 | 0.91 | 0.67 | -0.02 | 0.53 |
| time (sec) | N/A | 0.019 | 0.008 | 0.004 | 1.344 | 0.943 | 2.186 | 1.221 | 0.000 | 0.027 |

| Problem 779 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 33 | 30 | 44 | 42 | 60 | 44 | -1 | 33 |
| N.S. | 1 | 1.00 | 0.50 | 0.45 | 0.67 | 0.64 | 0.91 | 0.67 | -0.02 | 0.50 |
| time (sec) | N/A | 0.017 | 0.008 | 0.005 | 1.317 | 1.016 | 1.802 | 1.048 | 0.000 | 0.026 |

| Problem 780 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 36 | 29 | 40 | 42 | 54 | 44 | -1 | 32 |
| N.S. | 1 | 1.00 | 0.55 | 0.44 | 0.61 | 0.64 | 0.82 | 0.67 | -0.02 | 0.48 |
| time (sec) | N/A | 0.016 | 0.004 | 0.004 | 1.281 | 0.820 | 1.823 | 0.956 | 0.000 | 0.027 |

| Problem 781 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 34 | 32 | 40 | 42 | 54 | 44 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.52 | 0.48 | 0.61 | 0.64 | 0.82 | 0.67 | -0.02 | 0.53 |
| time (sec) | N/A | 0.015 | 0.005 | 0.005 | 1.375 | 1.110 | 1.841 | 0.991 | 0.000 | 0.028 |

| Problem 782 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 38 | 32 | 0 | 42 | 54 | 44 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.58 | 0.48 | 0.00 | 0.64 | 0.82 | 0.67 | -0.02 | 0.53 |
| time (sec) | N/A | 0.014 | 0.004 | 0.004 | 0.000 | 1.191 | 1.948 | 0.942 | 0.000 | 0.029 |

| Problem 783 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 36 | 32 | 0 | 40 | 60 | 44 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.56 | 0.50 | 0.00 | 0.62 | 0.94 | 0.69 | -0.02 | 0.55 |
| time (sec) | N/A | 0.015 | 0.003 | 0.005 | 0.000 | 1.183 | 2.007 | 1.053 | 0.000 | 0.031 |

| Problem 784 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 26 | 31 | 0 | 36 | 56 | 41 | -1 | 34 |
| N.S. | 1 | 1.00 | 0.90 | 1.07 | 0.00 | 1.24 | 1.93 | 1.41 | -0.03 | 1.17 |
| time (sec) | N/A | 0.004 | 0.006 | 0.002 | 0.000 | 0.974 | 2.034 | 1.116 | 0.000 | 0.030 |

| Problem 785 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 35 | 33 | 0 | 41 | 0 | 41 | -1 | 37 |
| N.S. | 1 | 1.00 | 0.60 | 0.57 | 0.00 | 0.71 | 0.00 | 0.71 | -0.02 | 0.64 |
| time (sec) | N/A | 0.011 | 0.010 | 0.006 | 0.000 | 1.088 | 0.000 | 1.082 | 0.000 | 0.034 |

| Problem 786 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 35 | 32 | 54 | 36 | 60 | 41 | -1 | 38 |
| N.S. | 1 | 1.00 | 0.61 | 0.56 | 0.95 | 0.63 | 1.05 | 0.72 | -0.02 | 0.67 |
| time (sec) | N/A | 0.013 | 0.006 | 0.004 | 1.345 | 1.240 | 0.788 | 0.943 | 0.000 | 0.028 |

| Problem 787 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 35 | 32 | 47 | 34 | 61 | 38 | -1 | 36 |
| N.S. | 1 | 1.00 | 0.61 | 0.56 | 0.82 | 0.60 | 1.07 | 0.67 | -0.02 | 0.63 |
| time (sec) | N/A | 0.012 | 0.005 | 0.002 | 1.322 | 1.068 | 0.647 | 1.252 | 0.000 | 0.028 |

| Problem 788 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 24 | 31 | 42 | 30 | 56 | 36 | -1 | 34 |
| N.S. | 1 | 1.00 | 1.00 | 1.29 | 1.75 | 1.25 | 2.33 | 1.50 | -0.04 | 1.42 |
| time (sec) | N/A | 0.003 | 0.002 | 0.003 | 1.370 | 1.157 | 0.540 | 1.147 | 0.000 | 0.025 |

| Problem 789 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 32 | 31 | 35 | 35 | 0 | 50 | -1 | 40 |
| N.S. | 1 | 1.00 | 0.62 | 0.60 | 0.67 | 0.67 | 0.00 | 0.96 | -0.02 | 0.77 |
| time (sec) | N/A | 0.010 | 0.003 | 0.005 | 1.348 | 0.991 | 0.000 | 1.118 | 0.000 | 0.028 |

| Problem 790 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 34 | 29 | 35 | 34 | 0 | 65 | -1 | 43 |
| N.S. | 1 | 1.00 | 0.72 | 0.62 | 0.74 | 0.72 | 0.00 | 1.38 | -0.02 | 0.91 |
| time (sec) | N/A | 0.011 | 0.009 | 0.003 | 1.357 | 1.032 | 0.000 | 0.943 | 0.000 | 0.031 |

| Problem 791 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 35 | 34 | 31 | 36 | 0 | 0 | -1 | 44 |
| N.S. | 1 | 1.00 | 0.71 | 0.69 | 0.63 | 0.73 | 0.00 | 0.00 | -0.02 | 0.90 |
| time (sec) | N/A | 0.011 | 0.008 | 0.007 | 1.289 | 1.091 | 0.000 | 0.000 | 0.000 | 0.035 |

| Problem 792 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 33 | 30 | 33 | 32 | 53 | 0 | 33 | 38 |
| N.S. | 1 | 1.00 | 1.27 | 1.15 | 1.27 | 1.23 | 2.04 | 0.00 | 1.27 | 1.46 |
| time (sec) | N/A | 0.004 | 0.010 | 0.005 | 1.330 | 1.067 | 0.662 | 0.000 | 0.183 | 0.030 |

| Problem 793 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 35 | 32 | 33 | 34 | 61 | 0 | 42 | 38 |
| N.S. | 1 | 1.00 | 0.61 | 0.56 | 0.58 | 0.60 | 1.07 | 0.00 | 0.74 | 0.67 |
| time (sec) | N/A | 0.013 | 0.007 | 0.006 | 1.326 | 1.255 | 0.819 | 0.000 | 0.187 | 0.031 |

| Problem 794 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 26 | 31 | 52 | 30 | 56 | 39 | -1 | 34 |
| N.S. | 1 | 1.00 | 0.96 | 1.15 | 1.93 | 1.11 | 2.07 | 1.44 | -0.04 | 1.26 |
| time (sec) | N/A | 0.004 | 0.004 | 0.003 | 1.356 | 1.064 | 0.804 | 1.054 | 0.000 | 0.028 |

| Problem 795 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 34 | 33 | 45 | 35 | 0 | 55 | -1 | 39 |
| N.S. | 1 | 1.00 | 0.56 | 0.54 | 0.74 | 0.57 | 0.00 | 0.90 | -0.02 | 0.64 |
| time (sec) | N/A | 0.011 | 0.007 | 0.005 | 1.331 | 1.178 | 0.000 | 1.096 | 0.000 | 0.032 |

| Problem 796 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 33 | 32 | 42 | 34 | 0 | 69 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.59 | 0.57 | 0.75 | 0.61 | 0.00 | 1.23 | -0.02 | 0.62 |
| time (sec) | N/A | 0.012 | 0.005 | 0.004 | 1.379 | 1.198 | 0.000 | 1.083 | 0.000 | 0.034 |

| Problem 797 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 34 | 32 | 35 | 36 | 0 | 0 | -1 | 38 |
| N.S. | 1 | 1.00 | 0.59 | 0.55 | 0.60 | 0.62 | 0.00 | 0.00 | -0.02 | 0.66 |
| time (sec) | N/A | 0.012 | 0.004 | 0.004 | 1.327 | 1.040 | 0.000 | 0.000 | 0.000 | 0.036 |

| Problem 798 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 36 | 27 | 37 | 32 | 53 | 0 | 33 | 32 |
| N.S. | 1 | 1.00 | 1.24 | 0.93 | 1.28 | 1.10 | 1.83 | 0.00 | 1.14 | 1.10 |
| time (sec) | N/A | 0.004 | 0.012 | 0.005 | 1.361 | 0.804 | 0.667 | 0.000 | 0.191 | 0.028 |

| Problem 799 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 38 | 32 | 33 | 34 | 56 | 0 | 42 | 35 |
| N.S. | 1 | 1.00 | 0.58 | 0.48 | 0.50 | 0.52 | 0.85 | 0.00 | 0.64 | 0.53 |
| time (sec) | N/A | 0.013 | 0.009 | 0.006 | 1.310 | 0.989 | 0.812 | 0.000 | 0.193 | 0.030 |

| Problem 800 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 33 | 32 | 33 | 34 | 56 | 0 | 42 | 35 |
| N.S. | 1 | 1.00 | 0.50 | 0.48 | 0.50 | 0.52 | 0.85 | 0.00 | 0.64 | 0.53 |
| time (sec) | N/A | 0.013 | 0.012 | 0.005 | 1.358 | 1.284 | 0.982 | 0.000 | 0.197 | 0.032 |

| Problem 801 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 35 | 32 | 33 | 34 | 61 | 0 | 42 | 35 |
| N.S. | 1 | 1.00 | 0.53 | 0.48 | 0.50 | 0.52 | 0.92 | 0.00 | 0.64 | 0.53 |
| time (sec) | N/A | 0.014 | 0.009 | 0.005 | 1.293 | 1.146 | 1.183 | 0.000 | 0.175 | 0.032 |

| Problem 802 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 33 | 32 | 45 | 34 | 0 | 65 | -1 | 35 |
| N.S. | 1 | 1.00 | 0.59 | 0.57 | 0.80 | 0.61 | 0.00 | 1.16 | -0.02 | 0.62 |
| time (sec) | N/A | 0.013 | 0.008 | 0.005 | 1.451 | 0.968 | 0.000 | 1.066 | 0.000 | 0.037 |

| Problem 803 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 36 | 34 | 38 | 36 | 0 | 0 | -1 | 40 |
| N.S. | 1 | 1.00 | 0.62 | 0.59 | 0.66 | 0.62 | 0.00 | 0.00 | -0.02 | 0.69 |
| time (sec) | N/A | 0.012 | 0.009 | 0.006 | 1.398 | 0.842 | 0.000 | 0.000 | 0.000 | 0.037 |

| Problem 804 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 35 | 30 | 44 | 32 | 58 | 0 | 33 | 33 |
| N.S. | 1 | 1.00 | 1.21 | 1.03 | 1.52 | 1.10 | 2.00 | 0.00 | 1.14 | 1.14 |
| time (sec) | N/A | 0.004 | 0.007 | 0.004 | 1.377 | 0.859 | 0.957 | 0.000 | 0.180 | 0.030 |

| Problem 805 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 38 | 30 | 37 | 34 | 61 | 0 | 42 | 33 |
| N.S. | 1 | 1.00 | 0.58 | 0.45 | 0.56 | 0.52 | 0.92 | 0.00 | 0.64 | 0.50 |
| time (sec) | N/A | 0.013 | 0.004 | 0.005 | 1.357 | 0.830 | 0.963 | 0.000 | 0.175 | 0.031 |

| Problem 806 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 38 | 29 | 37 | 34 | 56 | 0 | 42 | 32 |
| N.S. | 1 | 1.00 | 0.58 | 0.44 | 0.56 | 0.52 | 0.85 | 0.00 | 0.64 | 0.48 |
| time (sec) | N/A | 0.012 | 0.012 | 0.006 | 1.309 | 1.489 | 1.154 | 0.000 | 0.176 | 0.032 |

| Problem 807 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 38 | 32 | 33 | 34 | 56 | 0 | 42 | 35 |
| N.S. | 1 | 1.00 | 0.58 | 0.48 | 0.50 | 0.52 | 0.85 | 0.00 | 0.64 | 0.53 |
| time (sec) | N/A | 0.013 | 0.009 | 0.004 | 1.345 | 0.773 | 1.399 | 0.000 | 0.180 | 0.032 |

| Problem 808 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 33 | 32 | 33 | 34 | 56 | 0 | 42 | 35 |
| N.S. | 1 | 1.00 | 0.50 | 0.48 | 0.50 | 0.52 | 0.85 | 0.00 | 0.64 | 0.53 |
| time (sec) | N/A | 0.013 | 0.014 | 0.005 | 1.369 | 0.739 | 1.704 | 0.000 | 0.182 | 0.034 |

| Problem 809 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 35 | 32 | 33 | 34 | 61 | 0 | 42 | 35 |
| N.S. | 1 | 1.00 | 0.53 | 0.48 | 0.50 | 0.52 | 0.92 | 0.00 | 0.64 | 0.53 |
| time (sec) | N/A | 0.013 | 0.009 | 0.006 | 1.352 | 1.201 | 2.029 | 0.000 | 0.180 | 0.034 |

| Problem 810 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 102 | 102 | 63 | 63 | 128 | 62 | 0 | 81 | -1 | 64 |
| N.S. | 1 | 1.00 | 0.62 | 0.62 | 1.25 | 0.61 | 0.00 | 0.79 | -0.01 | 0.63 |
| time (sec) | N/A | 0.036 | 0.018 | 0.006 | 1.570 | 0.968 | 0.000 | 1.106 | 0.000 | 0.055 |

| Problem 811 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 80 | 80 | 52 | 52 | 110 | 51 | 0 | 69 | -1 | 54 |
| N.S. | 1 | 1.00 | 0.65 | 0.65 | 1.38 | 0.64 | 0.00 | 0.86 | -0.01 | 0.68 |
| time (sec) | N/A | 0.025 | 0.015 | 0.008 | 1.561 | 1.073 | 0.000 | 0.944 | 0.000 | 0.048 |

| Problem 812 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 40 | 40 | 91 | 39 | 0 | 54 | -1 | 41 |
| N.S. | 1 | 1.00 | 0.69 | 0.69 | 1.57 | 0.67 | 0.00 | 0.93 | -0.02 | 0.71 |
| time (sec) | N/A | 0.018 | 0.011 | 0.007 | 1.494 | 1.189 | 0.000 | 1.015 | 0.000 | 0.036 |

| Problem 813 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 28 | 29 | 74 | 27 | 0 | 37 | -1 | 29 |
| N.S. | 1 | 1.00 | 0.74 | 0.76 | 1.95 | 0.71 | 0.00 | 0.97 | -0.03 | 0.76 |
| time (sec) | N/A | 0.012 | 0.007 | 0.005 | 1.451 | 1.041 | 0.000 | 0.962 | 0.000 | 0.026 |

| Problem 814 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 21 | 21 | 0 | 20 | 0 | 28 | -1 | 22 |
| N.S. | 1 | 1.00 | 0.95 | 0.95 | 0.00 | 0.91 | 0.00 | 1.27 | -0.05 | 1.00 |
| time (sec) | N/A | 0.003 | 0.004 | 0.004 | 0.000 | 1.171 | 0.000 | 0.949 | 0.000 | 0.019 |

| Problem 815 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 26 | 26 | 24 | 64 | 0 | 0 | -1 | 37 |
| N.S. | 1 | 1.00 | 0.62 | 0.62 | 0.57 | 1.52 | 0.00 | 0.00 | -0.02 | 0.88 |
| time (sec) | N/A | 0.007 | 0.007 | 0.008 | 1.376 | 1.420 | 0.000 | 0.000 | 0.000 | 0.037 |

| Problem 816 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 32 | 33 | 37 | 31 | 0 | 0 | -1 | 44 |
| N.S. | 1 | 1.00 | 0.52 | 0.54 | 0.61 | 0.51 | 0.00 | 0.00 | -0.02 | 0.72 |
| time (sec) | N/A | 0.017 | 0.011 | 0.011 | 1.384 | 0.768 | 0.000 | 0.000 | 0.000 | 0.039 |

| Problem 817 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 53 | 51 | 52 | 44 | 0 | 0 | -1 | 58 |
| N.S. | 1 | 1.00 | 0.63 | 0.61 | 0.62 | 0.52 | 0.00 | 0.00 | -0.01 | 0.69 |
| time (sec) | N/A | 0.023 | 0.015 | 0.010 | 1.361 | 1.154 | 0.000 | 0.000 | 0.000 | 0.050 |

| Problem 818 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 107 | 107 | 64 | 63 | 124 | 67 | 0 | 81 | -1 | 67 |
| N.S. | 1 | 1.00 | 0.60 | 0.59 | 1.16 | 0.63 | 0.00 | 0.76 | -0.01 | 0.63 |
| time (sec) | N/A | 0.032 | 0.013 | 0.006 | 1.619 | 1.037 | 0.000 | 0.998 | 0.000 | 0.046 |

| Problem 819 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 53 | 52 | 109 | 55 | 0 | 69 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.63 | 0.62 | 1.30 | 0.65 | 0.00 | 0.82 | -0.01 | 0.68 |
| time (sec) | N/A | 0.025 | 0.011 | 0.005 | 1.541 | 1.305 | 0.000 | 1.155 | 0.000 | 0.044 |

| Problem 820 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 42 | 40 | 93 | 42 | 0 | 54 | -1 | 44 |
| N.S. | 1 | 1.00 | 0.69 | 0.66 | 1.52 | 0.69 | 0.00 | 0.89 | -0.02 | 0.72 |
| time (sec) | N/A | 0.019 | 0.005 | 0.005 | 1.482 | 1.421 | 0.000 | 1.119 | 0.000 | 0.039 |

| Problem 821 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 30 | 29 | 75 | 29 | 0 | 37 | -1 | 33 |
| N.S. | 1 | 1.00 | 0.75 | 0.72 | 1.88 | 0.72 | 0.00 | 0.92 | -0.02 | 0.82 |
| time (sec) | N/A | 0.012 | 0.004 | 0.003 | 1.479 | 0.835 | 0.000 | 0.998 | 0.000 | 0.029 |

| Problem 822 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 22 | 21 | 13 | 21 | 0 | 28 | -1 | 22 |
| N.S. | 1 | 1.00 | 0.96 | 0.91 | 0.57 | 0.91 | 0.00 | 1.22 | -0.04 | 0.96 |
| time (sec) | N/A | 0.004 | 0.003 | 0.004 | 1.357 | 1.045 | 0.000 | 1.137 | 0.000 | 0.023 |

| Problem 823 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 27 | 26 | 24 | 66 | 0 | 0 | -1 | 37 |
| N.S. | 1 | 1.00 | 0.61 | 0.59 | 0.55 | 1.50 | 0.00 | 0.00 | -0.02 | 0.84 |
| time (sec) | N/A | 0.007 | 0.008 | 0.005 | 1.380 | 0.793 | 0.000 | 0.000 | 0.000 | 0.035 |

| Problem 824 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 34 | 33 | 37 | 33 | 0 | 0 | -1 | 44 |
| N.S. | 1 | 1.00 | 0.53 | 0.52 | 0.58 | 0.52 | 0.00 | 0.00 | -0.02 | 0.69 |
| time (sec) | N/A | 0.016 | 0.011 | 0.006 | 1.356 | 1.105 | 0.000 | 0.000 | 0.000 | 0.041 |

| Problem 825 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 53 | 51 | 52 | 47 | 0 | 0 | -1 | 58 |
| N.S. | 1 | 1.00 | 0.60 | 0.58 | 0.59 | 0.53 | 0.00 | 0.00 | -0.01 | 0.66 |
| time (sec) | N/A | 0.024 | 0.015 | 0.005 | 1.446 | 0.913 | 0.000 | 0.000 | 0.000 | 0.050 |

| Problem 826 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 65 | 62 | 66 | 59 | 0 | 0 | -1 | 69 |
| N.S. | 1 | 1.00 | 0.58 | 0.55 | 0.59 | 0.53 | 0.00 | 0.00 | -0.01 | 0.62 |
| time (sec) | N/A | 0.031 | 0.026 | 0.012 | 1.449 | 1.009 | 0.000 | 0.000 | 0.000 | 0.069 |

| Problem 827 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 142 | 142 | 76 | 74 | 146 | 91 | 0 | 116 | -1 | 79 |
| N.S. | 1 | 1.00 | 0.54 | 0.52 | 1.03 | 0.64 | 0.00 | 0.82 | -0.01 | 0.56 |
| time (sec) | N/A | 0.045 | 0.022 | 0.006 | 1.584 | 1.343 | 0.000 | 1.184 | 0.000 | 0.062 |

| Problem 828 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 65 | 63 | 130 | 77 | 0 | 99 | -1 | 67 |
| N.S. | 1 | 1.00 | 0.56 | 0.54 | 1.11 | 0.66 | 0.00 | 0.85 | -0.01 | 0.57 |
| time (sec) | N/A | 0.051 | 0.006 | 0.006 | 1.602 | 1.190 | 0.000 | 1.011 | 0.000 | 0.052 |

| Problem 829 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 54 | 52 | 114 | 63 | 0 | 84 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.59 | 0.57 | 1.24 | 0.68 | 0.00 | 0.91 | -0.01 | 0.62 |
| time (sec) | N/A | 0.030 | 0.005 | 0.004 | 1.562 | 0.868 | 0.000 | 0.960 | 0.000 | 0.048 |

| Problem 830 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 42 | 40 | 97 | 48 | 0 | 66 | -1 | 44 |
| N.S. | 1 | 1.00 | 0.63 | 0.60 | 1.45 | 0.72 | 0.00 | 0.99 | -0.01 | 0.66 |
| time (sec) | N/A | 0.020 | 0.005 | 0.006 | 1.496 | 1.227 | 0.000 | 1.134 | 0.000 | 0.039 |

| Problem 831 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 30 | 29 | 77 | 33 | 0 | 46 | -1 | 33 |
| N.S. | 1 | 1.00 | 0.68 | 0.66 | 1.75 | 0.75 | 0.00 | 1.05 | -0.02 | 0.75 |
| time (sec) | N/A | 0.012 | 0.004 | 0.004 | 1.512 | 0.676 | 0.000 | 1.043 | 0.000 | 0.032 |

| Problem 832 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 22 | 21 | 13 | 23 | 0 | 34 | -1 | 22 |
| N.S. | 1 | 1.00 | 0.88 | 0.84 | 0.52 | 0.92 | 0.00 | 1.36 | -0.04 | 0.88 |
| time (sec) | N/A | 0.004 | 0.004 | 0.003 | 1.359 | 1.360 | 0.000 | 1.102 | 0.000 | 0.024 |

| Problem 833 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 28 | 27 | 24 | 70 | 0 | 0 | -1 | 37 |
| N.S. | 1 | 1.00 | 0.58 | 0.56 | 0.50 | 1.46 | 0.00 | 0.00 | -0.02 | 0.77 |
| time (sec) | N/A | 0.008 | 0.009 | 0.006 | 1.352 | 1.536 | 0.000 | 0.000 | 0.000 | 0.037 |

| Problem 834 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 34 | 33 | 37 | 37 | 0 | 0 | -1 | 44 |
| N.S. | 1 | 1.00 | 0.49 | 0.47 | 0.53 | 0.53 | 0.00 | 0.00 | -0.01 | 0.63 |
| time (sec) | N/A | 0.017 | 0.012 | 0.006 | 1.394 | 0.936 | 0.000 | 0.000 | 0.000 | 0.042 |

| Problem 835 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 51 | 50 | 142 | 54 | 0 | 81 | -1 | 60 |
| N.S. | 1 | 1.00 | 0.61 | 0.60 | 1.71 | 0.65 | 0.00 | 0.98 | -0.01 | 0.72 |
| time (sec) | N/A | 0.024 | 0.011 | 0.006 | 1.519 | 1.395 | 0.000 | 1.153 | 0.000 | 0.046 |

| Problem 836 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 39 | 38 | 100 | 42 | 0 | 67 | -1 | 47 |
| N.S. | 1 | 1.00 | 0.64 | 0.62 | 1.64 | 0.69 | 0.00 | 1.10 | -0.02 | 0.77 |
| time (sec) | N/A | 0.017 | 0.012 | 0.004 | 1.494 | 1.351 | 0.000 | 1.087 | 0.000 | 0.040 |

| Problem 837 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 27 | 27 | 64 | 30 | 0 | 51 | -1 | 36 |
| N.S. | 1 | 1.00 | 0.69 | 0.69 | 1.64 | 0.77 | 0.00 | 1.31 | -0.03 | 0.92 |
| time (sec) | N/A | 0.012 | 0.007 | 0.003 | 1.472 | 0.947 | 0.000 | 1.187 | 0.000 | 0.030 |

| Problem 838 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 19 | 46 | 23 | 0 | 36 | -1 | 25 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 2.30 | 1.15 | 0.00 | 1.80 | -0.05 | 1.25 |
| time (sec) | N/A | 0.004 | 0.002 | 0.005 | 1.459 | 0.630 | 0.000 | 0.901 | 0.000 | 0.019 |

| Problem 839 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 25 | 24 | 35 | 70 | 0 | 59 | -1 | 43 |
| N.S. | 1 | 1.00 | 0.66 | 0.63 | 0.92 | 1.84 | 0.00 | 1.55 | -0.03 | 1.13 |
| time (sec) | N/A | 0.007 | 0.004 | 0.005 | 1.461 | 0.945 | 0.000 | 0.966 | 0.000 | 0.033 |

| Problem 840 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 36 | 30 | 37 | 34 | 0 | 91 | -1 | 53 |
| N.S. | 1 | 1.00 | 0.67 | 0.56 | 0.69 | 0.63 | 0.00 | 1.69 | -0.02 | 0.98 |
| time (sec) | N/A | 0.016 | 0.011 | 0.006 | 1.370 | 0.772 | 0.000 | 1.194 | 0.000 | 0.039 |

| Problem 841 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 77 | 77 | 52 | 51 | 55 | 47 | 0 | 0 | -1 | 67 |
| N.S. | 1 | 1.00 | 0.68 | 0.66 | 0.71 | 0.61 | 0.00 | 0.00 | -0.01 | 0.87 |
| time (sec) | N/A | 0.029 | 0.012 | 0.005 | 1.375 | 0.949 | 0.000 | 0.000 | 0.000 | 0.052 |

| Problem 842 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 63 | 62 | 69 | 58 | 0 | 0 | -1 | 78 |
| N.S. | 1 | 1.00 | 0.63 | 0.62 | 0.69 | 0.58 | 0.00 | 0.00 | -0.01 | 0.78 |
| time (sec) | N/A | 0.026 | 0.012 | 0.005 | 1.396 | 0.840 | 0.000 | 0.000 | 0.000 | 0.058 |

| Problem 843 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 53 | 52 | 162 | 54 | 0 | 86 | -1 | 59 |
| N.S. | 1 | 1.00 | 0.56 | 0.55 | 1.71 | 0.57 | 0.00 | 0.91 | -0.01 | 0.62 |
| time (sec) | N/A | 0.027 | 0.011 | 0.006 | 1.790 | 1.146 | 0.000 | 1.181 | 0.000 | 0.048 |

| Problem 844 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 41 | 40 | 140 | 42 | 0 | 71 | -1 | 46 |
| N.S. | 1 | 1.00 | 0.59 | 0.57 | 2.00 | 0.60 | 0.00 | 1.01 | -0.01 | 0.66 |
| time (sec) | N/A | 0.020 | 0.008 | 0.005 | 1.660 | 0.830 | 0.000 | 0.958 | 0.000 | 0.042 |

| Problem 845 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 29 | 29 | 116 | 30 | 0 | 55 | -1 | 33 |
| N.S. | 1 | 1.00 | 0.64 | 0.64 | 2.58 | 0.67 | 0.00 | 1.22 | -0.02 | 0.73 |
| time (sec) | N/A | 0.013 | 0.007 | 0.004 | 1.602 | 1.111 | 0.000 | 1.094 | 0.000 | 0.035 |

| Problem 846 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 22 | 21 | 74 | 23 | 0 | 36 | -1 | 22 |
| N.S. | 1 | 1.00 | 0.96 | 0.91 | 3.22 | 1.00 | 0.00 | 1.57 | -0.04 | 0.96 |
| time (sec) | N/A | 0.004 | 0.004 | 0.002 | 1.604 | 1.088 | 0.000 | 0.970 | 0.000 | 0.025 |

| Problem 847 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 44 | 44 | 27 | 26 | 35 | 70 | 0 | 63 | -1 | 37 |
| N.S. | 1 | 1.00 | 0.61 | 0.59 | 0.80 | 1.59 | 0.00 | 1.43 | -0.02 | 0.84 |
| time (sec) | N/A | 0.008 | 0.007 | 0.005 | 1.430 | 1.291 | 0.000 | 1.047 | 0.000 | 0.036 |

| Problem 848 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 35 | 33 | 51 | 34 | 0 | 91 | -1 | 44 |
| N.S. | 1 | 1.00 | 0.56 | 0.52 | 0.81 | 0.54 | 0.00 | 1.44 | -0.02 | 0.70 |
| time (sec) | N/A | 0.017 | 0.006 | 0.006 | 1.481 | 0.946 | 0.000 | 1.080 | 0.000 | 0.039 |

| Problem 849 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 51 | 49 | 65 | 47 | 0 | 0 | -1 | 58 |
| N.S. | 1 | 1.00 | 0.57 | 0.55 | 0.73 | 0.53 | 0.00 | 0.00 | -0.01 | 0.65 |
| time (sec) | N/A | 0.022 | 0.007 | 0.006 | 1.538 | 1.160 | 0.000 | 0.000 | 0.000 | 0.053 |

| Problem 850 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 66 | 59 | 69 | 58 | 0 | 0 | -1 | 66 |
| N.S. | 1 | 1.00 | 0.57 | 0.51 | 0.60 | 0.50 | 0.00 | 0.00 | -0.01 | 0.57 |
| time (sec) | N/A | 0.028 | 0.015 | 0.006 | 1.359 | 1.048 | 0.000 | 0.000 | 0.000 | 0.055 |

| Problem 851 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 106 | 106 | 81 | 88 | 135 | 83 | 0 | 96 | -1 | 85 |
| N.S. | 1 | 1.00 | 0.76 | 0.83 | 1.27 | 0.78 | 0.00 | 0.91 | -0.01 | 0.80 |
| time (sec) | N/A | 0.039 | 0.028 | 0.012 | 1.516 | 1.144 | 0.000 | 0.997 | 0.000 | 0.080 |

| Problem 852 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 70 | 76 | 118 | 72 | 0 | 80 | -1 | 74 |
| N.S. | 1 | 1.00 | 0.82 | 0.89 | 1.39 | 0.85 | 0.00 | 0.94 | -0.01 | 0.87 |
| time (sec) | N/A | 0.032 | 0.021 | 0.012 | 1.545 | 0.846 | 0.000 | 1.057 | 0.000 | 0.064 |

| Problem 853 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 53 | 62 | 96 | 57 | 0 | 58 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.82 | 0.95 | 1.48 | 0.88 | 0.00 | 0.89 | -0.02 | 0.88 |
| time (sec) | N/A | 0.022 | 0.020 | 0.010 | 1.515 | 1.016 | 0.000 | 0.920 | 0.000 | 0.056 |

| Problem 854 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 36 | 41 | 79 | 38 | 0 | 46 | -1 | 39 |
| N.S. | 1 | 1.00 | 0.77 | 0.87 | 1.68 | 0.81 | 0.00 | 0.98 | -0.02 | 0.83 |
| time (sec) | N/A | 0.016 | 0.012 | 0.008 | 1.467 | 0.669 | 0.000 | 1.061 | 0.000 | 0.038 |

| Problem 855 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 23 | 23 | 16 | 23 | 39 | 29 | 22 | 24 |
| N.S. | 1 | 1.00 | 0.96 | 0.96 | 0.67 | 0.96 | 1.62 | 1.21 | 0.92 | 1.00 |
| time (sec) | N/A | 0.004 | 0.007 | 0.004 | 1.361 | 1.252 | 0.816 | 1.016 | 0.165 | 0.022 |

| Problem 856 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 45 | 52 | 38 | 42 | 0 | 0 | -1 | 48 |
| N.S. | 1 | 1.00 | 0.69 | 0.80 | 0.58 | 0.65 | 0.00 | 0.00 | -0.02 | 0.74 |
| time (sec) | N/A | 0.018 | 0.017 | 0.011 | 1.318 | 1.054 | 0.000 | 0.000 | 0.000 | 0.048 |

| Problem 857 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 57 | 74 | 58 | 60 | 0 | 0 | -1 | 59 |
| N.S. | 1 | 1.00 | 0.66 | 0.85 | 0.67 | 0.69 | 0.00 | 0.00 | -0.01 | 0.68 |
| time (sec) | N/A | 0.027 | 0.030 | 0.013 | 1.397 | 1.044 | 0.000 | 0.000 | 0.000 | 0.068 |

| Problem 858 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 82 | 95 | 79 | 77 | 0 | 0 | -1 | 77 |
| N.S. | 1 | 1.00 | 0.73 | 0.85 | 0.71 | 0.69 | 0.00 | 0.00 | -0.01 | 0.69 |
| time (sec) | N/A | 0.036 | 0.024 | 0.016 | 1.420 | 1.162 | 0.000 | 0.000 | 0.000 | 0.087 |

| Problem 859 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 111 | 111 | 82 | 88 | 132 | 91 | 0 | 96 | -1 | 85 |
| N.S. | 1 | 1.00 | 0.74 | 0.79 | 1.19 | 0.82 | 0.00 | 0.86 | -0.01 | 0.77 |
| time (sec) | N/A | 0.037 | 0.020 | 0.007 | 1.625 | 1.095 | 0.000 | 1.174 | 0.000 | 0.070 |

| Problem 860 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 71 | 76 | 115 | 79 | 0 | 80 | -1 | 74 |
| N.S. | 1 | 1.00 | 0.80 | 0.85 | 1.29 | 0.89 | 0.00 | 0.90 | -0.01 | 0.83 |
| time (sec) | N/A | 0.029 | 0.017 | 0.005 | 1.553 | 1.100 | 0.000 | 0.947 | 0.000 | 0.060 |

| Problem 861 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 55 | 62 | 98 | 63 | 0 | 58 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.81 | 0.91 | 1.44 | 0.93 | 0.00 | 0.85 | -0.01 | 0.84 |
| time (sec) | N/A | 0.020 | 0.007 | 0.006 | 1.424 | 1.263 | 0.000 | 0.956 | 0.000 | 0.053 |

| Problem 862 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 38 | 41 | 80 | 43 | 0 | 46 | -1 | 39 |
| N.S. | 1 | 1.00 | 0.78 | 0.84 | 1.63 | 0.88 | 0.00 | 0.94 | -0.02 | 0.80 |
| time (sec) | N/A | 0.015 | 0.009 | 0.004 | 1.480 | 1.413 | 0.000 | 0.961 | 0.000 | 0.041 |

| Problem 863 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 24 | 23 | 16 | 24 | 44 | 29 | 24 | 24 |
| N.S. | 1 | 1.00 | 0.96 | 0.92 | 0.64 | 0.96 | 1.76 | 1.16 | 0.96 | 0.96 |
| time (sec) | N/A | 0.004 | 0.006 | 0.003 | 1.335 | 1.094 | 2.226 | 1.143 | 0.148 | 0.027 |

| Problem 864 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 46 | 52 | 38 | 47 | 0 | 0 | -1 | 48 |
| N.S. | 1 | 1.00 | 0.68 | 0.76 | 0.56 | 0.69 | 0.00 | 0.00 | -0.01 | 0.71 |
| time (sec) | N/A | 0.018 | 0.017 | 0.006 | 1.397 | 1.119 | 0.000 | 0.000 | 0.000 | 0.047 |

| Problem 865 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 59 | 74 | 58 | 65 | 0 | 0 | -1 | 59 |
| N.S. | 1 | 1.00 | 0.65 | 0.81 | 0.64 | 0.71 | 0.00 | 0.00 | -0.01 | 0.65 |
| time (sec) | N/A | 0.024 | 0.022 | 0.007 | 1.435 | 1.162 | 0.000 | 0.000 | 0.000 | 0.060 |

| Problem 866 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 82 | 95 | 79 | 82 | 0 | 0 | -1 | 77 |
| N.S. | 1 | 1.00 | 0.70 | 0.81 | 0.68 | 0.70 | 0.00 | 0.00 | -0.01 | 0.66 |
| time (sec) | N/A | 0.032 | 0.022 | 0.006 | 1.332 | 0.928 | 0.000 | 0.000 | 0.000 | 0.072 |

| Problem 867 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 107 | 107 | 80 | 86 | 168 | 85 | 0 | 155 | -1 | 91 |
| N.S. | 1 | 1.00 | 0.75 | 0.80 | 1.57 | 0.79 | 0.00 | 1.45 | -0.01 | 0.85 |
| time (sec) | N/A | 0.033 | 0.015 | 0.006 | 1.558 | 0.969 | 0.000 | 1.131 | 0.000 | 0.069 |

| Problem 868 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 69 | 74 | 129 | 74 | 0 | 143 | -1 | 80 |
| N.S. | 1 | 1.00 | 0.80 | 0.86 | 1.50 | 0.86 | 0.00 | 1.66 | -0.01 | 0.93 |
| time (sec) | N/A | 0.026 | 0.012 | 0.005 | 1.555 | 0.986 | 0.000 | 1.041 | 0.000 | 0.064 |

| Problem 869 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 52 | 60 | 88 | 59 | 0 | 127 | -1 | 63 |
| N.S. | 1 | 1.00 | 0.81 | 0.94 | 1.38 | 0.92 | 0.00 | 1.98 | -0.02 | 0.98 |
| time (sec) | N/A | 0.020 | 0.012 | 0.006 | 1.450 | 1.051 | 0.000 | 1.133 | 0.000 | 0.057 |

| Problem 870 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 35 | 39 | 68 | 40 | 0 | 89 | -1 | 45 |
| N.S. | 1 | 1.00 | 0.81 | 0.91 | 1.58 | 0.93 | 0.00 | 2.07 | -0.02 | 1.05 |
| time (sec) | N/A | 0.013 | 0.009 | 0.004 | 1.411 | 1.200 | 0.000 | 1.149 | 0.000 | 0.041 |

| Problem 871 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 21 | 21 | 25 | 85 | 38 | 25 | 27 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.95 | 1.14 | 3.86 | 1.73 | 1.14 | 1.23 |
| time (sec) | N/A | 0.003 | 0.003 | 0.003 | 1.453 | 1.078 | 1.169 | 1.159 | 0.156 | 0.027 |

| Problem 872 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 44 | 50 | 61 | 44 | 0 | 86 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.75 | 0.85 | 1.03 | 0.75 | 0.00 | 1.46 | -0.02 | 0.97 |
| time (sec) | N/A | 0.016 | 0.005 | 0.006 | 1.460 | 0.994 | 0.000 | 1.084 | 0.000 | 0.050 |

| Problem 873 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 78 | 78 | 60 | 71 | 57 | 62 | 0 | 126 | -1 | 68 |
| N.S. | 1 | 1.00 | 0.77 | 0.91 | 0.73 | 0.79 | 0.00 | 1.62 | -0.01 | 0.87 |
| time (sec) | N/A | 0.022 | 0.026 | 0.006 | 1.432 | 0.952 | 0.000 | 1.142 | 0.000 | 0.067 |

| Problem 874 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 81 | 95 | 76 | 79 | 0 | 152 | -1 | 86 |
| N.S. | 1 | 1.00 | 0.79 | 0.92 | 0.74 | 0.77 | 0.00 | 1.48 | -0.01 | 0.83 |
| time (sec) | N/A | 0.032 | 0.018 | 0.006 | 1.407 | 1.284 | 0.000 | 1.222 | 0.000 | 0.073 |

| Problem 875 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 54 | 62 | 149 | 63 | 0 | 127 | -1 | 59 |
| N.S. | 1 | 1.00 | 0.74 | 0.85 | 2.04 | 0.86 | 0.00 | 1.74 | -0.01 | 0.81 |
| time (sec) | N/A | 0.021 | 0.015 | 0.005 | 1.611 | 0.998 | 0.000 | 1.264 | 0.000 | 0.058 |

| Problem 876 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 37 | 41 | 108 | 44 | 0 | 89 | -1 | 39 |
| N.S. | 1 | 1.00 | 0.76 | 0.84 | 2.20 | 0.90 | 0.00 | 1.82 | -0.02 | 0.80 |
| time (sec) | N/A | 0.014 | 0.011 | 0.003 | 1.557 | 1.116 | 0.000 | 1.025 | 0.000 | 0.046 |

| Problem 877 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 24 | 23 | 47 | 29 | 90 | 38 | 25 | 24 |
| N.S. | 1 | 1.00 | 0.96 | 0.92 | 1.88 | 1.16 | 3.60 | 1.52 | 1.00 | 0.96 |
| time (sec) | N/A | 0.004 | 0.006 | 0.003 | 1.483 | 0.958 | 1.940 | 1.164 | 0.167 | 0.029 |

| Problem 878 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 46 | 52 | 82 | 48 | 0 | 83 | -1 | 48 |
| N.S. | 1 | 1.00 | 0.68 | 0.76 | 1.21 | 0.71 | 0.00 | 1.22 | -0.01 | 0.71 |
| time (sec) | N/A | 0.017 | 0.014 | 0.006 | 1.520 | 1.019 | 0.000 | 1.057 | 0.000 | 0.052 |

| Problem 879 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 59 | 74 | 79 | 66 | 0 | 137 | -1 | 61 |
| N.S. | 1 | 1.00 | 0.66 | 0.82 | 0.88 | 0.73 | 0.00 | 1.52 | -0.01 | 0.68 |
| time (sec) | N/A | 0.024 | 0.011 | 0.005 | 1.412 | 1.209 | 0.000 | 1.109 | 0.000 | 0.059 |

| Problem 880 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 118 | 118 | 80 | 93 | 98 | 83 | 0 | 152 | -1 | 77 |
| N.S. | 1 | 1.00 | 0.68 | 0.79 | 0.83 | 0.70 | 0.00 | 1.29 | -0.01 | 0.65 |
| time (sec) | N/A | 0.032 | 0.009 | 0.007 | 1.463 | 0.978 | 0.000 | 1.027 | 0.000 | 0.067 |

| Problem 881 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 131 | 131 | 97 | 136 | 116 | 153 | 0 | 300 | 214 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 1.04 | 0.89 | 1.17 | 0.00 | 2.29 | 1.63 | 0.00 |
| time (sec) | N/A | 0.037 | 0.068 | 0.007 | 1.464 | 1.243 | 0.000 | 1.111 | 0.348 | 0.215 |

| Problem 882 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 96 | 68 | 83 | 80 | 106 | 0 | 200 | 142 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.86 | 0.83 | 1.10 | 0.00 | 2.08 | 1.48 | 0.00 |
| time (sec) | N/A | 0.029 | 0.047 | 0.006 | 1.413 | 1.564 | 0.000 | 0.961 | 0.252 | 0.199 |

| Problem 883 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 44 | 46 | 51 | 63 | 0 | 119 | 85 | 0 |
| N.S. | 1 | 1.00 | 0.70 | 0.73 | 0.81 | 1.00 | 0.00 | 1.89 | 1.35 | 0.00 |
| time (sec) | N/A | 0.017 | 0.030 | 0.003 | 1.450 | 0.786 | 0.000 | 1.144 | 0.222 | 0.189 |

| Problem 884 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 29 | 29 | 28 | 30 | 0 | 42 | 31 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.97 | 0.93 | 1.00 | 0.00 | 1.40 | 1.03 | 0.00 |
| time (sec) | N/A | 0.006 | 0.014 | 0.002 | 1.395 | 0.986 | 0.000 | 1.011 | 0.228 | 0.190 |

| Problem 885 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 169 | 169 | 132 | 199 | 157 | 233 | 0 | 426 | 307 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 1.18 | 0.93 | 1.38 | 0.00 | 2.52 | 1.82 | 0.00 |
| time (sec) | N/A | 0.055 | 0.077 | 0.009 | 1.437 | 1.340 | 0.000 | 1.196 | 0.414 | 0.223 |

| Problem 886 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 135 | 135 | 98 | 136 | 116 | 164 | 0 | 300 | 219 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 1.01 | 0.86 | 1.21 | 0.00 | 2.22 | 1.62 | 0.00 |
| time (sec) | N/A | 0.039 | 0.055 | 0.007 | 1.441 | 1.055 | 0.000 | 1.141 | 0.320 | 0.214 |

| Problem 887 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 99 | 99 | 70 | 83 | 80 | 113 | 0 | 0 | 146 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.84 | 0.81 | 1.14 | 0.00 | 0.00 | 1.47 | 0.00 |
| time (sec) | N/A | 0.028 | 0.031 | 0.006 | 1.426 | 1.015 | 0.000 | 0.000 | 0.261 | 0.223 |

| Problem 888 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 46 | 46 | 51 | 68 | 0 | 119 | 88 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.71 | 0.78 | 1.05 | 0.00 | 1.83 | 1.35 | 0.00 |
| time (sec) | N/A | 0.018 | 0.008 | 0.004 | 1.426 | 0.835 | 0.000 | 0.910 | 0.231 | 0.218 |

| Problem 889 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 30 | 29 | 28 | 33 | 0 | 42 | 45 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.94 | 0.90 | 1.06 | 0.00 | 1.35 | 1.45 | 0.00 |
| time (sec) | N/A | 0.006 | 0.015 | 0.001 | 1.423 | 1.013 | 0.000 | 1.087 | 0.229 | 0.210 |

| Problem 890 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 217 | 217 | 172 | 280 | 203 | 352 | 0 | 640 | 424 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 1.29 | 0.94 | 1.62 | 0.00 | 2.95 | 1.95 | 0.00 |
| time (sec) | N/A | 0.074 | 0.108 | 0.009 | 1.529 | 0.885 | 0.000 | 1.063 | 0.496 | 0.232 |

| Problem 891 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 179 | 179 | 133 | 199 | 157 | 265 | 0 | 0 | 319 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 1.11 | 0.88 | 1.48 | 0.00 | 0.00 | 1.78 | 0.00 |
| time (sec) | N/A | 0.051 | 0.022 | 0.007 | 1.452 | 1.152 | 0.000 | 0.000 | 0.379 | 0.245 |

| Problem 892 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 143 | 143 | 99 | 136 | 116 | 186 | 0 | 0 | 229 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.95 | 0.81 | 1.30 | 0.00 | 0.00 | 1.60 | 0.00 |
| time (sec) | N/A | 0.042 | 0.018 | 0.007 | 1.454 | 1.146 | 0.000 | 0.000 | 0.317 | 0.236 |

| Problem 893 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 105 | 105 | 70 | 83 | 80 | 127 | 0 | 0 | 154 | 0 |
| N.S. | 1 | 1.00 | 0.67 | 0.79 | 0.76 | 1.21 | 0.00 | 0.00 | 1.47 | 0.00 |
| time (sec) | N/A | 0.029 | 0.045 | 0.006 | 1.361 | 1.045 | 0.000 | 0.000 | 0.265 | 0.245 |

| Problem 894 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 46 | 46 | 51 | 76 | 0 | 0 | 94 | 0 |
| N.S. | 1 | 1.00 | 0.67 | 0.67 | 0.74 | 1.10 | 0.00 | 0.00 | 1.36 | 0.00 |
| time (sec) | N/A | 0.019 | 0.009 | 0.002 | 1.453 | 1.052 | 0.000 | 0.000 | 0.239 | 0.234 |

| Problem 895 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 31 | 29 | 28 | 37 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.88 | 0.85 | 1.12 | 0.00 | 0.00 | 1.48 | 0.00 |
| time (sec) | N/A | 0.006 | 0.017 | 0.002 | 1.388 | 0.838 | 0.000 | 0.000 | 0.231 | 0.226 |

| Problem 896 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 96 | 134 | 104 | 158 | 0 | 0 | 186 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 1.09 | 0.85 | 1.28 | 0.00 | 0.00 | 1.51 | 0.00 |
| time (sec) | N/A | 0.037 | 0.040 | 0.006 | 1.449 | 1.056 | 0.000 | 0.000 | 0.369 | 0.224 |

| Problem 897 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 67 | 81 | 83 | 110 | 0 | 0 | 121 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 0.90 | 0.92 | 1.22 | 0.00 | 0.00 | 1.34 | 0.00 |
| time (sec) | N/A | 0.026 | 0.029 | 0.004 | 1.464 | 1.328 | 0.000 | 0.000 | 0.295 | 0.219 |

| Problem 898 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 43 | 44 | 45 | 66 | 0 | 0 | 71 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 0.75 | 0.76 | 1.12 | 0.00 | 0.00 | 1.20 | 0.00 |
| time (sec) | N/A | 0.016 | 0.021 | 0.003 | 1.487 | 0.957 | 0.000 | 0.000 | 0.277 | 0.203 |

| Problem 899 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 28 | 27 | 31 | 33 | 0 | 0 | 36 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 1.11 | 1.18 | 0.00 | 0.00 | 1.29 | 0.00 |
| time (sec) | N/A | 0.005 | 0.011 | 0.002 | 1.437 | 1.690 | 0.000 | 0.000 | 0.220 | 0.191 |

| Problem 900 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 135 | 135 | 98 | 136 | 104 | 168 | 0 | 0 | 201 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 1.01 | 0.77 | 1.24 | 0.00 | 0.00 | 1.49 | 0.00 |
| time (sec) | N/A | 0.044 | 0.041 | 0.007 | 1.464 | 0.989 | 0.000 | 0.000 | 0.404 | 0.254 |

| Problem 901 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 99 | 99 | 69 | 83 | 83 | 118 | 0 | 0 | 133 | 0 |
| N.S. | 1 | 1.00 | 0.70 | 0.84 | 0.84 | 1.19 | 0.00 | 0.00 | 1.34 | 0.00 |
| time (sec) | N/A | 0.031 | 0.034 | 0.005 | 1.455 | 1.165 | 0.000 | 0.000 | 0.313 | 0.248 |

| Problem 902 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 45 | 46 | 45 | 72 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.71 | 0.69 | 1.11 | 0.00 | 0.00 | 1.23 | 0.00 |
| time (sec) | N/A | 0.019 | 0.023 | 0.003 | 1.467 | 1.586 | 0.000 | 0.000 | 0.289 | 0.242 |

| Problem 903 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 30 | 29 | 31 | 37 | 0 | 0 | 42 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.94 | 1.00 | 1.19 | 0.00 | 0.00 | 1.35 | 0.00 |
| time (sec) | N/A | 0.007 | 0.014 | 0.002 | 1.441 | 0.912 | 0.000 | 0.000 | 0.229 | 0.226 |

| Problem 904 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 135 | 135 | 99 | 136 | 104 | 168 | 0 | 0 | 201 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 1.01 | 0.77 | 1.24 | 0.00 | 0.00 | 1.49 | 0.00 |
| time (sec) | N/A | 0.046 | 0.037 | 0.006 | 1.472 | 0.951 | 0.000 | 0.000 | 0.410 | 0.271 |

| Problem 905 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 99 | 99 | 70 | 83 | 83 | 118 | 0 | 0 | 133 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.84 | 0.84 | 1.19 | 0.00 | 0.00 | 1.34 | 0.00 |
| time (sec) | N/A | 0.031 | 0.030 | 0.006 | 1.451 | 1.874 | 0.000 | 0.000 | 0.355 | 0.263 |

| Problem 906 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 46 | 46 | 45 | 72 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.71 | 0.69 | 1.11 | 0.00 | 0.00 | 1.23 | 0.00 |
| time (sec) | N/A | 0.020 | 0.020 | 0.003 | 1.407 | 0.919 | 0.000 | 0.000 | 0.285 | 0.248 |

| Problem 907 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 29 | 31 | 37 | 0 | 0 | 42 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.00 | 1.19 | 0.00 | 0.00 | 1.35 | 0.00 |
| time (sec) | N/A | 0.007 | 0.012 | 0.003 | 1.453 | 1.488 | 0.000 | 0.000 | 0.230 | 0.243 |

| Problem 908 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 38 | 40 | 39 | 58 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.58 | 0.62 | 0.60 | 0.89 | 0.00 | 0.00 | 0.68 | 0.00 |
| time (sec) | N/A | 0.030 | 0.027 | 0.003 | 1.518 | 1.412 | 0.000 | 0.000 | 0.266 | 0.462 |

| Problem 909 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 38 | 40 | 39 | 50 | 0 | 0 | 42 | 0 |
| N.S. | 1 | 1.00 | 0.62 | 0.66 | 0.64 | 0.82 | 0.00 | 0.00 | 0.69 | 0.00 |
| time (sec) | N/A | 0.029 | 0.024 | 0.002 | 1.554 | 1.494 | 0.000 | 0.000 | 0.237 | 0.392 |

| Problem 910 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 38 | 40 | 39 | 44 | 0 | 0 | 39 | 0 |
| N.S. | 1 | 1.00 | 0.64 | 0.68 | 0.66 | 0.75 | 0.00 | 0.00 | 0.66 | 0.00 |
| time (sec) | N/A | 0.027 | 0.020 | 0.003 | 1.520 | 1.520 | 0.000 | 0.000 | 0.212 | 0.350 |

| Problem 911 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 33 | 32 | 32 | 36 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.67 | 0.67 | 0.75 | 0.00 | 0.00 | 0.62 | 0.00 |
| time (sec) | N/A | 0.019 | 0.015 | 0.001 | 1.482 | 0.751 | 0.000 | 0.000 | 0.211 | 0.396 |

| Problem 912 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 38 | 40 | 39 | 53 | 0 | 0 | 48 | 0 |
| N.S. | 1 | 1.00 | 0.58 | 0.62 | 0.60 | 0.82 | 0.00 | 0.00 | 0.74 | 0.00 |
| time (sec) | N/A | 0.036 | 0.025 | 0.003 | 1.506 | 1.060 | 0.000 | 0.000 | 0.259 | 0.385 |

| Problem 913 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 38 | 40 | 39 | 53 | 0 | 0 | 47 | 0 |
| N.S. | 1 | 1.00 | 0.57 | 0.60 | 0.58 | 0.79 | 0.00 | 0.00 | 0.70 | 0.00 |
| time (sec) | N/A | 0.037 | 0.027 | 0.002 | 1.517 | 1.209 | 0.000 | 0.000 | 0.275 | 0.601 |

| Problem 914 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 48 | 95 | 64 | 123 | 0 | 0 | 127 | 0 |
| N.S. | 1 | 1.00 | 0.47 | 0.92 | 0.62 | 1.19 | 0.00 | 0.00 | 1.23 | 0.00 |
| time (sec) | N/A | 0.049 | 0.051 | 0.004 | 1.600 | 1.276 | 0.000 | 0.000 | 0.312 | 0.854 |

| Problem 915 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 48 | 95 | 64 | 105 | 0 | 0 | 121 | 0 |
| N.S. | 1 | 1.00 | 0.49 | 0.98 | 0.66 | 1.08 | 0.00 | 0.00 | 1.25 | 0.00 |
| time (sec) | N/A | 0.045 | 0.045 | 0.005 | 1.551 | 1.420 | 0.000 | 0.000 | 0.279 | 0.621 |

| Problem 916 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 94 | 94 | 72 | 95 | 64 | 94 | 0 | 0 | 116 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 1.01 | 0.68 | 1.00 | 0.00 | 0.00 | 1.23 | 0.00 |
| time (sec) | N/A | 0.041 | 0.043 | 0.004 | 1.525 | 1.546 | 0.000 | 0.000 | 0.264 | 0.549 |

| Problem 917 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 62 | 79 | 57 | 85 | 0 | 0 | 62 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 0.98 | 0.70 | 1.05 | 0.00 | 0.00 | 0.77 | 0.00 |
| time (sec) | N/A | 0.036 | 0.034 | 0.005 | 1.635 | 1.185 | 0.000 | 0.000 | 0.258 | 0.622 |

| Problem 918 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 62 | 83 | 59 | 92 | 0 | 0 | 66 | 0 |
| N.S. | 1 | 1.00 | 0.67 | 0.89 | 0.63 | 0.99 | 0.00 | 0.00 | 0.71 | 0.00 |
| time (sec) | N/A | 0.045 | 0.039 | 0.006 | 1.583 | 1.308 | 0.000 | 0.000 | 0.319 | 0.688 |

| Problem 919 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 105 | 105 | 72 | 95 | 64 | 106 | 0 | 0 | 82 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.90 | 0.61 | 1.01 | 0.00 | 0.00 | 0.78 | 0.00 |
| time (sec) | N/A | 0.054 | 0.046 | 0.006 | 1.569 | 1.415 | 0.000 | 0.000 | 0.339 | 0.868 |

| Problem 920 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 32 | 32 | 0 | 40 | 0 | 74 | 33 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.97 | 0.00 | 1.21 | 0.00 | 2.24 | 1.00 | 0.00 |
| time (sec) | N/A | 0.010 | 0.017 | 0.004 | 0.000 | 1.223 | 0.000 | 1.261 | 0.266 | 0.088 |

| Problem 921 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 34 | 33 | 0 | 40 | 0 | 0 | 34 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 1.03 | 0.00 | 1.25 | 0.00 | 0.00 | 1.06 | 0.00 |
| time (sec) | N/A | 0.009 | 0.019 | 0.003 | 0.000 | 1.227 | 0.000 | 0.000 | 0.235 | 0.083 |

| Problem 922 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 32 | 32 | 0 | 38 | 0 | 72 | 33 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.97 | 0.00 | 1.15 | 0.00 | 2.18 | 1.00 | 0.00 |
| time (sec) | N/A | 0.010 | 0.015 | 0.004 | 0.000 | 1.508 | 0.000 | 1.070 | 0.217 | 0.078 |

| Problem 923 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 28 | 31 | 0 | 36 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.03 | 0.00 | 1.20 | 0.00 | 0.00 | 1.07 | 0.00 |
| time (sec) | N/A | 0.008 | 0.015 | 0.004 | 0.000 | 1.366 | 0.000 | 0.000 | 0.200 | 0.072 |

| Problem 924 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 26 | 25 | 27 | 31 | 264 | 0 | 26 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 1.04 | 1.19 | 10.15 | 0.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.006 | 0.007 | 0.003 | 1.446 | 1.181 | 59.694 | 0.000 | 0.262 | 0.080 |

| Problem 925 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 32 | 38 | 0 | 37 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 1.15 | 0.00 | 1.12 | 0.00 | 0.00 | 0.97 | 0.00 |
| time (sec) | N/A | 0.011 | 0.010 | 0.003 | 0.000 | 1.170 | 0.000 | 0.000 | 0.240 | 0.064 |

| Problem 926 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 32 | 32 | 0 | 37 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.91 | 0.00 | 1.06 | 0.00 | 0.00 | 1.43 | 0.00 |
| time (sec) | N/A | 0.011 | 0.010 | 0.003 | 0.000 | 0.881 | 0.000 | 0.000 | 0.248 | 0.073 |

| Problem 927 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 32 | 33 | 0 | 37 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 1.00 | 0.00 | 1.12 | 0.00 | 0.00 | 1.55 | 0.00 |
| time (sec) | N/A | 0.010 | 0.010 | 0.002 | 0.000 | 1.320 | 0.000 | 0.000 | 0.251 | 0.081 |

| Problem 928 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 38 | 39 | 0 | 49 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.03 | 0.00 | 1.29 | 0.00 | 0.00 | 1.32 | 0.00 |
| time (sec) | N/A | 0.010 | 0.029 | 0.003 | 0.000 | 1.422 | 0.000 | 0.000 | 0.338 | 0.111 |

| Problem 929 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 39 | 40 | 0 | 57 | 0 | 0 | 39 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.03 | 0.00 | 1.46 | 0.00 | 0.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.010 | 0.015 | 0.003 | 0.000 | 1.357 | 0.000 | 0.000 | 0.265 | 0.118 |

| Problem 930 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 16 | 31 | 31 | 34 | 31 | 27 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.82 | 1.82 | 2.00 | 1.82 | 1.59 | 0.00 |
| time (sec) | N/A | 0.004 | 0.002 | 0.002 | 1.367 | 1.189 | 0.121 | 1.086 | 0.050 | 0.000 |

| Problem 931 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 19 | 18 | 20 | 20 | 20 | 20 | 16 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.78 | 0.87 | 0.87 | 0.87 | 0.87 | 0.70 | 0.00 |
| time (sec) | N/A | 0.006 | 0.001 | 0.001 | 1.239 | 1.255 | 0.119 | 1.194 | 0.034 | 0.000 |

| Problem 932 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 8 | 9 | 8 | 8 | 7 | 8 | 8 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.12 | 1.00 | 1.00 | 0.88 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.002 | 0.000 | 0.001 | 1.270 | 1.353 | 0.107 | 1.004 | 0.010 | 0.000 |

| Problem 933 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 14 | 13 | 13 | 19 | 14 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.00 | 1.46 | 1.08 | 1.00 | 0.00 |
| time (sec) | N/A | 0.003 | 0.002 | 0.001 | 1.310 | 1.218 | 0.112 | 1.073 | 0.048 | 0.000 |

| Problem 934 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 16 | 19 | 19 | 19 | 15 | 15 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.27 | 1.27 | 1.27 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.003 | 0.004 | 0.000 | 1.338 | 1.344 | 0.177 | 0.988 | 0.037 | 0.000 |

| Problem 935 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 16 | 47 | 47 | 53 | 15 | 49 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 2.76 | 2.76 | 3.12 | 0.88 | 2.88 | 0.00 |
| time (sec) | N/A | 0.003 | 0.005 | 0.001 | 1.307 | 1.329 | 0.288 | 1.187 | 0.153 | 0.001 |

| Problem 936 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 16 | 61 | 61 | 68 | 15 | 63 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 3.59 | 3.59 | 4.00 | 0.88 | 3.71 | 0.00 |
| time (sec) | N/A | 0.003 | 0.006 | 0.002 | 1.354 | 1.358 | 0.363 | 0.905 | 0.064 | 0.001 |

| Problem 937 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 16 | 75 | 75 | 83 | 15 | 77 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 4.41 | 4.41 | 4.88 | 0.88 | 4.53 | 0.00 |
| time (sec) | N/A | 0.003 | 0.006 | 0.002 | 1.332 | 0.879 | 0.422 | 0.962 | 0.054 | 0.000 |

| Problem 938 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 16 | 35 | 35 | 34 | 35 | 27 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 2.06 | 2.06 | 2.00 | 2.06 | 1.59 | 0.00 |
| time (sec) | N/A | 0.004 | 0.003 | 0.002 | 1.372 | 1.251 | 0.130 | 1.007 | 0.156 | 0.000 |

| Problem 939 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 19 | 18 | 21 | 21 | 20 | 21 | 16 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.78 | 0.91 | 0.91 | 0.87 | 0.91 | 0.70 | 0.00 |
| time (sec) | N/A | 0.005 | 0.001 | 0.001 | 1.315 | 1.256 | 0.118 | 1.037 | 0.030 | 0.000 |

| Problem 940 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 8 | 9 | 8 | 8 | 7 | 8 | 8 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.12 | 1.00 | 1.00 | 0.88 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.001 | 0.000 | 0.000 | 1.327 | 1.188 | 0.104 | 1.050 | 0.009 | 0.000 |

| Problem 941 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 14 | 13 | 13 | 17 | 14 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.00 | 1.31 | 1.08 | 1.00 | 0.00 |
| time (sec) | N/A | 0.003 | 0.002 | 0.001 | 1.367 | 1.274 | 0.104 | 0.890 | 0.142 | 0.001 |

| Problem 942 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 14 | 16 | 16 | 12 | 13 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.23 | 1.23 | 0.92 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.003 | 0.004 | 0.002 | 1.381 | 1.199 | 0.154 | 1.001 | 0.040 | 0.000 |

| Problem 943 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 36 | 36 | 44 | 12 | 38 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 2.57 | 2.57 | 3.14 | 0.86 | 2.71 | 0.00 |
| time (sec) | N/A | 0.003 | 0.005 | 0.001 | 1.344 | 1.286 | 0.280 | 1.136 | 0.047 | 0.000 |

| Problem 944 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 14 | 59 | 59 | 68 | 20 | 61 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 3.93 | 3.93 | 4.53 | 1.33 | 4.07 | 0.00 |
| time (sec) | N/A | 0.003 | 0.006 | 0.001 | 1.346 | 1.308 | 0.362 | 0.978 | 0.050 | 0.000 |

| Problem 945 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 16 | 75 | 75 | 83 | 15 | 77 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 4.41 | 4.41 | 4.88 | 0.88 | 4.53 | 0.00 |
| time (sec) | N/A | 0.003 | 0.006 | 0.000 | 1.360 | 1.364 | 0.419 | 0.922 | 0.173 | 0.000 |

| Problem 946 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 25 | 27 | 649 | 80 | 212 | 141 | 107 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 1.12 | 27.04 | 3.33 | 8.83 | 5.88 | 4.46 | 0.00 |
| time (sec) | N/A | 0.009 | 0.020 | 0.002 | 1.809 | 0.999 | 2.293 | 1.132 | 0.326 | 0.069 |

| Problem 947 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 114 | 113 | 113 | 124 | 113 | 113 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 6.71 | 6.65 | 6.65 | 7.29 | 6.65 | 6.65 | 0.00 |
| time (sec) | N/A | 0.004 | 0.002 | 0.001 | 1.313 | 1.247 | 0.099 | 0.957 | 0.051 | 0.000 |

| Problem 948 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 100 | 99 | 99 | 110 | 99 | 99 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 5.88 | 5.82 | 5.82 | 6.47 | 5.82 | 5.82 | 0.00 |
| time (sec) | N/A | 0.004 | 0.002 | 0.003 | 1.359 | 0.982 | 0.095 | 0.986 | 0.041 | 0.000 |

| Problem 949 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 72 | 71 | 71 | 78 | 71 | 71 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 4.80 | 4.73 | 4.73 | 5.20 | 4.73 | 4.73 | 0.00 |
| time (sec) | N/A | 0.003 | 0.002 | 0.002 | 1.353 | 1.275 | 0.084 | 1.093 | 0.030 | 0.000 |

| Problem 950 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 16 | 48 | 48 | 51 | 48 | 57 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 2.82 | 2.82 | 3.00 | 2.82 | 3.35 | 0.00 |
| time (sec) | N/A | 0.004 | 0.002 | 0.000 | 1.351 | 1.103 | 0.102 | 0.893 | 0.025 | 0.000 |

| Problem 951 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 16 | 37 | 37 | 46 | 18 | 43 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 2.18 | 2.18 | 2.71 | 1.06 | 2.53 | 0.00 |
| time (sec) | N/A | 0.004 | 0.001 | 0.002 | 1.309 | 1.183 | 0.107 | 0.978 | 0.048 | 0.000 |

| Problem 952 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 16 | 26 | 26 | 29 | 26 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.53 | 1.53 | 1.71 | 1.53 | 1.41 | 0.00 |
| time (sec) | N/A | 0.004 | 0.001 | 0.000 | 1.305 | 0.709 | 0.109 | 1.002 | 0.036 | 0.000 |

| Problem 953 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 16 | 15 | 15 | 15 | 15 | 15 | 13 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.83 | 0.83 | 0.83 | 0.83 | 0.83 | 0.72 | 0.00 |
| time (sec) | N/A | 0.005 | 0.001 | 0.001 | 1.380 | 1.358 | 0.109 | 1.211 | 0.024 | 0.000 |

| Problem 954 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 5 | 5 | 5 | 6 | 5 | 5 | 3 | 15 | 5 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 1.00 | 1.00 | 0.60 | 3.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.001 | 0.000 | 0.002 | 1.399 | 1.219 | 0.107 | 1.111 | 0.008 | 0.000 |

| Problem 955 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 14 | 13 | 13 | 17 | 14 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.00 | 1.31 | 1.08 | 1.00 | 0.00 |
| time (sec) | N/A | 0.004 | 0.002 | 0.002 | 1.311 | 1.329 | 0.123 | 0.955 | 0.040 | 0.000 |

| Problem 956 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 16 | 19 | 19 | 17 | 15 | 19 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.27 | 1.27 | 1.13 | 1.00 | 1.27 | 0.00 |
| time (sec) | N/A | 0.004 | 0.002 | 0.000 | 1.328 | 1.216 | 0.200 | 1.090 | 0.046 | 0.001 |

| Problem 957 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 16 | 33 | 33 | 36 | 15 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 1.94 | 1.94 | 2.12 | 0.88 | 2.06 | 0.00 |
| time (sec) | N/A | 0.004 | 0.004 | 0.001 | 1.332 | 1.362 | 0.260 | 1.047 | 0.146 | 0.001 |

| Problem 958 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | C | A | C | C | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 28 | 23 | 6 | 1 | 53 | 11 | 35 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.21 | 0.04 | 1.89 | 0.39 | 1.25 | 0.00 |
| time (sec) | N/A | 0.003 | 0.006 | 0.003 | 2.992 | 1.267 | 1.461 | 1.008 | 0.222 | 1.781 |

| Problem 959 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 40 | 45 | 44 | 44 | 44 | 44 | 44 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 1.18 | 1.16 | 1.16 | 1.16 | 1.16 | 1.16 | 0.00 |
| time (sec) | N/A | 0.012 | 0.003 | 0.002 | 1.228 | 1.290 | 0.080 | 0.998 | 0.160 | 0.000 |

| Problem 960 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 42 | 45 | 44 | 44 | 46 | 44 | 44 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 1.18 | 1.16 | 1.16 | 1.21 | 1.16 | 1.16 | 0.00 |
| time (sec) | N/A | 0.017 | 0.002 | 0.002 | 1.313 | 0.729 | 0.074 | 1.037 | 0.048 | 0.000 |

| Problem 961 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 17 | 16 | 16 | 15 | 16 | 18 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.89 | 0.89 | 0.83 | 0.89 | 1.00 | 0.00 |
| time (sec) | N/A | 0.004 | 0.001 | 0.001 | 1.351 | 1.034 | 0.063 | 1.010 | 0.023 | 0.000 |

| Problem 962 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 12 | 11 | 10 | 10 | 8 | 10 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.83 | 0.83 | 0.67 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.002 | 0.000 | 0.000 | 1.336 | 1.068 | 0.058 | 0.859 | 0.017 | 0.000 |

| Problem 963 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 25 | 24 | 23 | 17 | 25 | 23 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 1.04 | 1.00 | 0.74 | 1.09 | 1.00 | 0.00 |
| time (sec) | N/A | 0.012 | 0.005 | 0.003 | 1.308 | 1.305 | 0.140 | 0.924 | 0.046 | 0.000 |

| Problem 964 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 28 | 35 | 37 | 39 | 29 | 81 | 37 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.09 | 1.16 | 1.22 | 0.91 | 2.53 | 1.16 | 0.00 |
| time (sec) | N/A | 0.017 | 0.017 | 0.006 | 1.308 | 1.247 | 0.187 | 1.114 | 0.052 | 0.000 |

| Problem 965 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 33 | 30 | 30 | 27 | 14 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 2.54 | 2.31 | 2.31 | 2.08 | 1.08 | 1.00 | 0.00 |
| time (sec) | N/A | 0.002 | 0.008 | 0.005 | 1.310 | 0.679 | 0.241 | 0.892 | 0.148 | 0.000 |

| Problem 966 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 25 | 35 | 54 | 54 | 56 | 23 | 54 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 0.92 | 1.42 | 1.42 | 1.47 | 0.61 | 1.42 | 0.00 |
| time (sec) | N/A | 0.019 | 0.011 | 0.007 | 1.337 | 1.242 | 0.307 | 0.979 | 0.049 | 0.000 |

| Problem 967 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 24 | 35 | 67 | 67 | 73 | 40 | 67 | 0 |
| N.S. | 1 | 1.00 | 0.63 | 0.92 | 1.76 | 1.76 | 1.92 | 1.05 | 1.76 | 0.00 |
| time (sec) | N/A | 0.019 | 0.010 | 0.004 | 1.349 | 1.408 | 0.392 | 0.953 | 0.167 | 0.001 |

| Problem 968 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 27 | 35 | 84 | 84 | 88 | 25 | 82 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.92 | 2.21 | 2.21 | 2.32 | 0.66 | 2.16 | 0.00 |
| time (sec) | N/A | 0.019 | 0.013 | 0.004 | 1.312 | 1.215 | 0.463 | 1.035 | 0.075 | 0.001 |

| Problem 969 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 68 | 73 | 72 | 72 | 78 | 72 | 72 | 0 |
| N.S. | 1 | 1.00 | 1.19 | 1.28 | 1.26 | 1.26 | 1.37 | 1.26 | 1.26 | 0.00 |
| time (sec) | N/A | 0.030 | 0.003 | 0.001 | 1.293 | 1.140 | 0.086 | 1.041 | 0.032 | 0.000 |

| Problem 970 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 38 | 35 | 34 | 34 | 36 | 34 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.89 | 0.89 | 0.95 | 0.89 | 0.82 | 0.00 |
| time (sec) | N/A | 0.017 | 0.002 | 0.001 | 1.338 | 0.486 | 0.078 | 1.105 | 0.040 | 0.000 |

| Problem 971 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 40 | 37 | 36 | 36 | 39 | 36 | 36 | 0 |
| N.S. | 1 | 1.00 | 1.25 | 1.16 | 1.12 | 1.12 | 1.22 | 1.12 | 1.12 | 0.00 |
| time (sec) | N/A | 0.015 | 0.002 | 0.002 | 1.318 | 1.129 | 0.074 | 1.020 | 0.049 | 0.000 |

| Problem 972 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 20 | 20 | 19 | 12 | 20 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 1.43 | 1.43 | 1.36 | 0.86 | 1.43 | 0.00 |
| time (sec) | N/A | 0.001 | 0.001 | 0.002 | 1.270 | 1.134 | 0.064 | 0.967 | 0.027 | 0.000 |

| Problem 973 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 37 | 37 | 35 | 34 | 31 | 46 | 34 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.86 | 0.81 | 0.79 | 0.72 | 1.07 | 0.79 | 0.00 |
| time (sec) | N/A | 0.014 | 0.007 | 0.003 | 1.289 | 1.226 | 0.165 | 0.875 | 0.047 | 0.000 |

| Problem 974 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 35 | 44 | 46 | 57 | 39 | 79 | 46 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.07 | 1.12 | 1.39 | 0.95 | 1.93 | 1.12 | 0.00 |
| time (sec) | N/A | 0.022 | 0.029 | 0.007 | 1.378 | 0.602 | 0.199 | 1.083 | 0.149 | 0.001 |

| Problem 975 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 33 | 56 | 61 | 69 | 54 | 46 | 59 | 0 |
| N.S. | 1 | 1.00 | 0.63 | 1.08 | 1.17 | 1.33 | 1.04 | 0.88 | 1.13 | 0.00 |
| time (sec) | N/A | 0.028 | 0.024 | 0.007 | 1.333 | 1.028 | 0.306 | 1.072 | 0.172 | 0.001 |

| Problem 976 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 31 | 52 | 60 | 60 | 61 | 29 | 58 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 1.86 | 2.14 | 2.14 | 2.18 | 1.04 | 2.07 | 0.00 |
| time (sec) | N/A | 0.005 | 0.018 | 0.004 | 1.333 | 1.295 | 0.351 | 0.984 | 0.046 | 0.001 |

| Problem 977 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 35 | 51 | 78 | 78 | 85 | 64 | 76 | 0 |
| N.S. | 1 | 1.00 | 0.62 | 0.91 | 1.39 | 1.39 | 1.52 | 1.14 | 1.36 | 0.00 |
| time (sec) | N/A | 0.024 | 0.012 | 0.005 | 1.380 | 1.156 | 0.439 | 1.070 | 0.052 | 0.001 |

| Problem 978 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 38 | 52 | 95 | 95 | 100 | 36 | 91 | 0 |
| N.S. | 1 | 1.00 | 0.67 | 0.91 | 1.67 | 1.67 | 1.75 | 0.63 | 1.60 | 0.00 |
| time (sec) | N/A | 0.025 | 0.018 | 0.004 | 1.386 | 1.191 | 0.505 | 0.846 | 0.187 | 0.001 |

| Problem 979 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 37 | 52 | 108 | 108 | 117 | 36 | 104 | 0 |
| N.S. | 1 | 1.00 | 0.63 | 0.88 | 1.83 | 1.83 | 1.98 | 0.61 | 1.76 | 0.00 |
| time (sec) | N/A | 0.029 | 0.015 | 0.006 | 1.420 | 0.990 | 0.600 | 0.934 | 0.108 | 0.001 |

| Problem 980 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 42 | 49 | 48 | 52 | 49 | 59 | 48 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.80 | 0.79 | 0.85 | 0.80 | 0.97 | 0.79 | 0.00 |
| time (sec) | N/A | 0.021 | 0.006 | 0.003 | 1.342 | 1.372 | 0.181 | 1.148 | 0.049 | 0.000 |

| Problem 981 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 31 | 35 | 34 | 38 | 34 | 45 | 32 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 0.81 | 0.79 | 0.88 | 0.79 | 1.05 | 0.74 | 0.00 |
| time (sec) | N/A | 0.014 | 0.005 | 0.003 | 1.342 | 1.228 | 0.153 | 0.871 | 0.148 | 0.000 |

| Problem 982 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 19 | 18 | 20 | 15 | 19 | 18 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.00 | 1.11 | 0.83 | 1.06 | 1.00 | 0.00 |
| time (sec) | N/A | 0.010 | 0.003 | 0.003 | 1.339 | 1.348 | 0.122 | 1.013 | 0.039 | 0.000 |

| Problem 983 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 10 | 10 | 11 | 10 | 10 | 7 | 11 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.10 | 1.00 | 1.00 | 0.70 | 1.10 | 1.00 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.000 | 1.326 | 1.157 | 0.066 | 1.055 | 0.019 | 0.000 |

| Problem 984 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 38 | 37 | 28 | 22 | 39 | 17 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 2.24 | 2.18 | 1.65 | 1.29 | 2.29 | 1.00 | 0.00 |
| time (sec) | N/A | 0.009 | 0.006 | 0.006 | 1.401 | 1.228 | 0.171 | 1.195 | 0.170 | 0.000 |

| Problem 985 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 53 | 58 | 60 | 60 | 48 | 53 | 42 | 0 |
| N.S. | 1 | 1.00 | 1.26 | 1.38 | 1.43 | 1.43 | 1.14 | 1.26 | 1.00 | 0.00 |
| time (sec) | N/A | 0.029 | 0.015 | 0.007 | 1.256 | 1.410 | 0.283 | 1.030 | 0.071 | 0.000 |

| Problem 986 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 65 | 78 | 82 | 98 | 71 | 69 | 64 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 1.24 | 1.30 | 1.56 | 1.13 | 1.10 | 1.02 | 0.00 |
| time (sec) | N/A | 0.039 | 0.022 | 0.007 | 1.348 | 1.489 | 0.381 | 0.866 | 0.078 | 0.000 |

| Problem 987 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 46 | 53 | 53 | 79 | 51 | 80 | 52 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 0.98 | 0.98 | 1.46 | 0.94 | 1.48 | 0.96 | 0.00 |
| time (sec) | N/A | 0.031 | 0.020 | 0.007 | 1.294 | 0.767 | 0.246 | 1.119 | 0.055 | 0.000 |

| Problem 988 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 33 | 40 | 40 | 61 | 36 | 59 | 39 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.03 | 1.03 | 1.56 | 0.92 | 1.51 | 1.00 | 0.00 |
| time (sec) | N/A | 0.021 | 0.017 | 0.007 | 1.362 | 1.254 | 0.194 | 1.183 | 0.168 | 0.000 |

| Problem 989 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 23 | 28 | 28 | 33 | 24 | 54 | 27 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.04 | 1.04 | 1.22 | 0.89 | 2.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.013 | 0.009 | 0.005 | 1.338 | 1.188 | 0.172 | 1.022 | 0.043 | 0.000 |

| Problem 990 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 12 | 13 | 12 | 13 | 10 | 12 | 12 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.08 | 0.83 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.001 | 0.002 | 0.000 | 1.372 | 1.278 | 0.137 | 1.106 | 0.024 | 0.000 |

| Problem 991 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 50 | 56 | 55 | 51 | 44 | 44 | 37 | 0 |
| N.S. | 1 | 1.00 | 1.22 | 1.37 | 1.34 | 1.24 | 1.07 | 1.07 | 0.90 | 0.00 |
| time (sec) | N/A | 0.028 | 0.016 | 0.007 | 1.376 | 1.011 | 0.284 | 0.953 | 0.177 | 0.001 |

| Problem 992 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 74 | 76 | 64 | 76 | 49 | 83 | 46 | 0 |
| N.S. | 1 | 1.00 | 1.61 | 1.65 | 1.39 | 1.65 | 1.07 | 1.80 | 1.00 | 0.00 |
| time (sec) | N/A | 0.017 | 0.024 | 0.010 | 1.315 | 1.154 | 0.273 | 1.066 | 0.181 | 0.001 |

| Problem 993 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 68 | 96 | 108 | 146 | 104 | 81 | 86 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 1.16 | 1.30 | 1.76 | 1.25 | 0.98 | 1.04 | 0.00 |
| time (sec) | N/A | 0.050 | 0.043 | 0.011 | 1.355 | 1.172 | 0.512 | 0.961 | 0.098 | 0.001 |

| Problem 994 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 108 | 108 | 60 | 113 | 68 | 62 | 289 | 185 | -1 | 128 |
| N.S. | 1 | 1.00 | 0.56 | 1.05 | 0.63 | 0.57 | 2.68 | 1.71 | -0.01 | 1.19 |
| time (sec) | N/A | 0.021 | 0.052 | 0.008 | 3.005 | 0.761 | 48.589 | 1.273 | 0.000 | 0.091 |

| Problem 995 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 56 | 99 | 54 | 57 | 253 | 115 | -1 | 114 |
| N.S. | 1 | 1.00 | 0.64 | 1.12 | 0.61 | 0.65 | 2.88 | 1.31 | -0.01 | 1.30 |
| time (sec) | N/A | 0.015 | 0.051 | 0.005 | 2.965 | 1.318 | 21.067 | 1.061 | 0.000 | 0.083 |

| Problem 996 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 50 | 85 | 40 | 52 | 218 | 101 | -1 | 100 |
| N.S. | 1 | 1.00 | 0.74 | 1.25 | 0.59 | 0.76 | 3.21 | 1.49 | -0.01 | 1.47 |
| time (sec) | N/A | 0.011 | 0.041 | 0.006 | 2.953 | 1.358 | 9.031 | 1.293 | 0.000 | 0.078 |

| Problem 997 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 44 | 71 | 28 | 47 | 168 | 50 | -1 | 82 |
| N.S. | 1 | 1.00 | 0.92 | 1.48 | 0.58 | 0.98 | 3.50 | 1.04 | -0.02 | 1.71 |
| time (sec) | N/A | 0.006 | 0.038 | 0.005 | 2.902 | 1.356 | 4.494 | 1.023 | 0.000 | 0.073 |

| Problem 998 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 20 | 57 | 17 | 38 | 133 | 42 | 37 | 73 |
| N.S. | 1 | 1.00 | 0.71 | 2.04 | 0.61 | 1.36 | 4.75 | 1.50 | 1.32 | 2.61 |
| time (sec) | N/A | 0.004 | 0.006 | 0.005 | 2.978 | 0.881 | 2.728 | 1.036 | 0.205 | 0.059 |

| Problem 999 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | A | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 32 | 42 | 14 | 37 | 100 | 28 | 14 | 45 |
| N.S. | 1 | 1.00 | 1.52 | 2.00 | 0.67 | 1.76 | 4.76 | 1.33 | 0.67 | 2.14 |
| time (sec) | N/A | 0.004 | 0.007 | 0.006 | 2.901 | 1.114 | 1.843 | 1.014 | 0.145 | 0.087 |

| Problem 1000 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 36 | 64 | 21 | 48 | 71 | 33 | -1 | 39 |
| N.S. | 1 | 1.00 | 1.57 | 2.78 | 0.91 | 2.09 | 3.09 | 1.43 | -0.04 | 1.70 |
| time (sec) | N/A | 0.004 | 0.013 | 0.030 | 2.954 | 1.275 | 1.619 | 1.064 | 0.000 | 0.045 |

| Problem 1001 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 15 | 38 | 33 | 61 | 19 | 34 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 0.75 | 1.90 | 1.65 | 3.05 | 0.95 | 1.70 | 1.00 |
| time (sec) | N/A | 0.002 | 0.005 | 0.003 | 1.261 | 1.254 | 1.674 | 0.937 | 0.270 | 0.067 |

| Problem 1002 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 23 | 18 | 64 | 53 | 173 | 22 | 50 | 34 |
| N.S. | 1 | 1.00 | 0.56 | 0.44 | 1.56 | 1.29 | 4.22 | 0.54 | 1.22 | 0.83 |
| time (sec) | N/A | 0.004 | 0.009 | 0.003 | 1.397 | 0.920 | 6.547 | 1.046 | 0.244 | 0.056 |

| Problem 1003 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 30 | 25 | 95 | 70 | 568 | 29 | 64 | 48 |
| N.S. | 1 | 1.00 | 0.49 | 0.41 | 1.56 | 1.15 | 9.31 | 0.48 | 1.05 | 0.79 |
| time (sec) | N/A | 0.008 | 0.012 | 0.003 | 1.272 | 1.172 | 19.923 | 1.224 | 0.269 | 0.066 |

| Problem 1004 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 35 | 30 | 131 | 85 | 1562 | 35 | 80 | 77 |
| N.S. | 1 | 1.00 | 0.43 | 0.37 | 1.62 | 1.05 | 19.28 | 0.43 | 0.99 | 0.95 |
| time (sec) | N/A | 0.013 | 0.014 | 0.003 | 1.348 | 1.308 | 53.780 | 1.118 | 0.282 | 0.069 |

| Problem 1005 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 40 | 35 | 172 | 100 | 3650 | 42 | 94 | 95 |
| N.S. | 1 | 1.00 | 0.40 | 0.35 | 1.70 | 0.99 | 36.14 | 0.42 | 0.93 | 0.94 |
| time (sec) | N/A | 0.019 | 0.017 | 0.004 | 1.322 | 0.988 | 135.085 | 1.359 | 0.292 | 0.074 |

| Problem 1006 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 66 | 127 | 66 | 67 | 325 | 237 | -1 | 169 |
| N.S. | 1 | 1.00 | 0.61 | 1.17 | 0.61 | 0.61 | 2.98 | 2.17 | -0.01 | 1.55 |
| time (sec) | N/A | 0.020 | 0.058 | 0.005 | 3.027 | 1.159 | 75.198 | 1.256 | 0.000 | 0.155 |

| Problem 1007 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 61 | 113 | 52 | 62 | 289 | 185 | -1 | 151 |
| N.S. | 1 | 1.00 | 0.69 | 1.27 | 0.58 | 0.70 | 3.25 | 2.08 | -0.01 | 1.70 |
| time (sec) | N/A | 0.013 | 0.055 | 0.005 | 2.904 | 1.154 | 32.987 | 1.330 | 0.000 | 0.132 |

| Problem 1008 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 55 | 99 | 40 | 57 | 250 | 91 | -1 | 133 |
| N.S. | 1 | 1.00 | 0.80 | 1.43 | 0.58 | 0.83 | 3.62 | 1.32 | -0.01 | 1.93 |
| time (sec) | N/A | 0.008 | 0.043 | 0.005 | 2.977 | 1.224 | 15.248 | 1.164 | 0.000 | 0.112 |

| Problem 1009 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 29 | 85 | 29 | 46 | 214 | 101 | -1 | 115 |
| N.S. | 1 | 1.00 | 0.59 | 1.73 | 0.59 | 0.94 | 4.37 | 2.06 | -0.02 | 2.35 |
| time (sec) | N/A | 0.007 | 0.011 | 0.005 | 2.911 | 1.189 | 7.461 | 1.116 | 0.000 | 0.096 |

| Problem 1010 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 44 | 71 | 28 | 47 | 165 | 66 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.92 | 1.48 | 0.58 | 0.98 | 3.44 | 1.38 | -0.02 | 1.98 |
| time (sec) | N/A | 0.006 | 0.033 | 0.006 | 2.967 | 1.280 | 4.820 | 0.920 | 0.000 | 0.073 |

| Problem 1011 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 37 | 57 | 28 | 40 | 136 | 31 | -1 | 68 |
| N.S. | 1 | 1.00 | 0.79 | 1.21 | 0.60 | 0.85 | 2.89 | 0.66 | -0.02 | 1.45 |
| time (sec) | N/A | 0.007 | 0.012 | 0.004 | 3.088 | 1.239 | 3.275 | 1.172 | 0.000 | 0.060 |

| Problem 1012 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | A | A | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 35 | 72 | 42 | 52 | 100 | 35 | -1 | 59 |
| N.S. | 1 | 1.00 | 0.85 | 1.76 | 1.02 | 1.27 | 2.44 | 0.85 | -0.02 | 1.44 |
| time (sec) | N/A | 0.007 | 0.006 | 0.016 | 2.993 | 0.725 | 2.920 | 1.190 | 0.000 | 0.145 |

| Problem 1013 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | B | B | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 37 | 76 | 66 | 71 | 500 | 38 | -1 | 55 |
| N.S. | 1 | 1.00 | 0.90 | 1.85 | 1.61 | 1.73 | 12.20 | 0.93 | -0.02 | 1.34 |
| time (sec) | N/A | 0.005 | 0.006 | 0.020 | 2.973 | 1.311 | 3.698 | 1.019 | 0.000 | 0.062 |

| Problem 1014 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 15 | 94 | 52 | 88 | 19 | 50 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 0.75 | 4.70 | 2.60 | 4.40 | 0.95 | 2.50 | 1.00 |
| time (sec) | N/A | 0.002 | 0.006 | 0.003 | 1.284 | 0.996 | 6.256 | 1.057 | 0.252 | 0.065 |

| Problem 1015 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 23 | 18 | 131 | 69 | 228 | 22 | 64 | 34 |
| N.S. | 1 | 1.00 | 0.56 | 0.44 | 3.20 | 1.68 | 5.56 | 0.54 | 1.56 | 0.83 |
| time (sec) | N/A | 0.004 | 0.010 | 0.003 | 1.384 | 1.148 | 19.081 | 1.127 | 0.268 | 0.065 |

| Problem 1016 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 30 | 25 | 172 | 86 | 677 | 29 | 80 | 48 |
| N.S. | 1 | 1.00 | 0.49 | 0.41 | 2.82 | 1.41 | 11.10 | 0.48 | 1.31 | 0.79 |
| time (sec) | N/A | 0.008 | 0.012 | 0.003 | 1.365 | 0.666 | 51.652 | 1.106 | 0.319 | 0.070 |

| Problem 1017 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 35 | 30 | 218 | 101 | 1753 | 35 | 94 | 62 |
| N.S. | 1 | 1.00 | 0.43 | 0.37 | 2.69 | 1.25 | 21.64 | 0.43 | 1.16 | 0.77 |
| time (sec) | N/A | 0.013 | 0.015 | 0.002 | 1.392 | 1.532 | 132.965 | 1.167 | 0.310 | 0.076 |

| Problem 1018 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 40 | 35 | 269 | 116 | 0 | 42 | 110 | 76 |
| N.S. | 1 | 1.00 | 0.40 | 0.35 | 2.66 | 1.15 | 0.00 | 0.42 | 1.09 | 0.75 |
| time (sec) | N/A | 0.019 | 0.017 | 0.003 | 1.380 | 1.225 | 0.000 | 1.231 | 0.328 | 0.086 |

| Problem 1019 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 75 | 155 | 78 | 77 | 0 | 323 | -1 | 205 |
| N.S. | 1 | 1.00 | 0.58 | 1.19 | 0.60 | 0.59 | 0.00 | 2.48 | -0.01 | 1.58 |
| time (sec) | N/A | 0.025 | 0.069 | 0.006 | 3.012 | 0.765 | 0.000 | 1.400 | 0.000 | 0.193 |

| Problem 1020 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 70 | 141 | 64 | 72 | 360 | 296 | -1 | 187 |
| N.S. | 1 | 1.00 | 0.64 | 1.28 | 0.58 | 0.65 | 3.27 | 2.69 | -0.01 | 1.70 |
| time (sec) | N/A | 0.019 | 0.061 | 0.005 | 2.970 | 1.155 | 117.569 | 1.493 | 0.000 | 0.163 |

| Problem 1021 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 66 | 127 | 52 | 67 | 321 | 143 | -1 | 169 |
| N.S. | 1 | 1.00 | 0.73 | 1.41 | 0.58 | 0.74 | 3.57 | 1.59 | -0.01 | 1.88 |
| time (sec) | N/A | 0.012 | 0.063 | 0.005 | 3.006 | 1.319 | 53.580 | 1.143 | 0.000 | 0.138 |

| Problem 1022 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 34 | 113 | 41 | 51 | 286 | 185 | -1 | 151 |
| N.S. | 1 | 1.00 | 0.49 | 1.61 | 0.59 | 0.73 | 4.09 | 2.64 | -0.01 | 2.16 |
| time (sec) | N/A | 0.011 | 0.014 | 0.004 | 3.103 | 1.292 | 25.761 | 1.308 | 0.000 | 0.120 |

| Problem 1023 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 55 | 99 | 40 | 57 | 246 | 114 | -1 | 133 |
| N.S. | 1 | 1.00 | 0.80 | 1.43 | 0.58 | 0.83 | 3.57 | 1.65 | -0.01 | 1.93 |
| time (sec) | N/A | 0.008 | 0.045 | 0.005 | 3.049 | 1.064 | 16.531 | 1.181 | 0.000 | 0.104 |

| Problem 1024 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 50 | 85 | 40 | 52 | 214 | 101 | -1 | 100 |
| N.S. | 1 | 1.00 | 0.74 | 1.25 | 0.59 | 0.76 | 3.15 | 1.49 | -0.01 | 1.47 |
| time (sec) | N/A | 0.009 | 0.037 | 0.005 | 3.060 | 1.129 | 9.886 | 1.045 | 0.000 | 0.077 |

| Problem 1025 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 44 | 71 | 42 | 47 | 172 | 39 | -1 | 84 |
| N.S. | 1 | 1.00 | 0.66 | 1.06 | 0.63 | 0.70 | 2.57 | 0.58 | -0.01 | 1.25 |
| time (sec) | N/A | 0.010 | 0.023 | 0.005 | 2.934 | 1.502 | 7.503 | 1.053 | 0.000 | 0.067 |

| Problem 1026 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 35 | 77 | 56 | 58 | 139 | 42 | -1 | 81 |
| N.S. | 1 | 1.00 | 0.54 | 1.18 | 0.86 | 0.89 | 2.14 | 0.65 | -0.02 | 1.25 |
| time (sec) | N/A | 0.010 | 0.007 | 0.018 | 2.971 | 1.113 | 7.762 | 1.007 | 0.000 | 0.084 |

| Problem 1027 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | A | B | A | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 37 | 84 | 99 | 75 | 576 | 44 | -1 | 73 |
| N.S. | 1 | 1.00 | 0.59 | 1.33 | 1.57 | 1.19 | 9.14 | 0.70 | -0.02 | 1.16 |
| time (sec) | N/A | 0.010 | 0.008 | 0.020 | 2.971 | 0.705 | 7.473 | 0.977 | 0.000 | 0.183 |

| Problem 1028 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 37 | 84 | 160 | 91 | 1608 | 44 | -1 | 69 |
| N.S. | 1 | 1.00 | 0.59 | 1.33 | 2.54 | 1.44 | 25.52 | 0.70 | -0.02 | 1.10 |
| time (sec) | N/A | 0.007 | 0.009 | 0.022 | 3.075 | 1.181 | 11.213 | 0.982 | 0.000 | 0.069 |

| Problem 1029 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 15 | 171 | 66 | 116 | 19 | 64 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 0.75 | 8.55 | 3.30 | 5.80 | 0.95 | 3.20 | 1.00 |
| time (sec) | N/A | 0.002 | 0.008 | 0.003 | 1.355 | 1.300 | 19.494 | 1.108 | 0.279 | 0.060 |

| Problem 1030 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 23 | 18 | 218 | 83 | 282 | 22 | 80 | 34 |
| N.S. | 1 | 1.00 | 0.56 | 0.44 | 5.32 | 2.02 | 6.88 | 0.54 | 1.95 | 0.83 |
| time (sec) | N/A | 0.004 | 0.011 | 0.005 | 1.403 | 1.061 | 53.145 | 1.227 | 0.303 | 0.070 |

| Problem 1031 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 30 | 25 | 269 | 100 | 785 | 29 | 94 | 48 |
| N.S. | 1 | 1.00 | 0.49 | 0.41 | 4.41 | 1.64 | 12.87 | 0.48 | 1.54 | 0.79 |
| time (sec) | N/A | 0.008 | 0.016 | 0.003 | 1.415 | 0.973 | 133.938 | 1.268 | 0.310 | 0.078 |

| Problem 1032 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 35 | 30 | 325 | 115 | 0 | 35 | 110 | 62 |
| N.S. | 1 | 1.00 | 0.43 | 0.37 | 4.01 | 1.42 | 0.00 | 0.43 | 1.36 | 0.77 |
| time (sec) | N/A | 0.013 | 0.017 | 0.002 | 1.456 | 1.309 | 0.000 | 0.990 | 0.315 | 0.081 |

| Problem 1033 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 40 | 35 | 386 | 130 | 0 | 42 | 124 | 76 |
| N.S. | 1 | 1.00 | 0.40 | 0.35 | 3.82 | 1.29 | 0.00 | 0.42 | 1.23 | 0.75 |
| time (sec) | N/A | 0.019 | 0.021 | 0.003 | 1.393 | 1.361 | 0.000 | 0.877 | 0.352 | 0.085 |

| Problem 1034 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 121 | 121 | 45 | 40 | 452 | 145 | 0 | 48 | 140 | 90 |
| N.S. | 1 | 1.00 | 0.37 | 0.33 | 3.74 | 1.20 | 0.00 | 0.40 | 1.16 | 0.74 |
| time (sec) | N/A | 0.025 | 0.022 | 0.003 | 1.384 | 1.162 | 0.000 | 0.858 | 0.370 | 0.093 |

| Problem 1035 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 47 | 98 | 42 | 55 | 75 | 42 | -1 | 86 |
| N.S. | 1 | 1.00 | 0.73 | 1.53 | 0.66 | 0.86 | 1.17 | 0.66 | -0.02 | 1.34 |
| time (sec) | N/A | 0.012 | 0.039 | 0.013 | 2.989 | 1.272 | 33.751 | 0.699 | 0.000 | 0.081 |

| Problem 1036 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 91 | 118 | 42 | 48 | 76 | 34 | 55 | 72 |
| N.S. | 1 | 1.00 | 1.47 | 1.90 | 0.68 | 0.77 | 1.23 | 0.55 | 0.89 | 1.16 |
| time (sec) | N/A | 0.028 | 0.102 | 0.013 | 3.030 | 1.323 | 7.083 | 0.702 | 0.151 | 0.292 |

| Problem 1037 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 61 | 85 | 56 | 52 | 201 | 101 | -1 | 100 |
| N.S. | 1 | 1.00 | 0.70 | 0.98 | 0.64 | 0.60 | 2.31 | 1.16 | -0.01 | 1.15 |
| time (sec) | N/A | 0.018 | 0.025 | 0.006 | 2.993 | 1.286 | 14.676 | 0.758 | 0.000 | 0.070 |

| Problem 1038 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 54 | 71 | 42 | 47 | 175 | 69 | -1 | 84 |
| N.S. | 1 | 1.00 | 0.81 | 1.06 | 0.63 | 0.70 | 2.61 | 1.03 | -0.01 | 1.25 |
| time (sec) | N/A | 0.011 | 0.026 | 0.005 | 2.901 | 1.264 | 5.643 | 0.703 | 0.000 | 0.067 |

| Problem 1039 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 47 | 57 | 28 | 40 | 139 | 44 | -1 | 67 |
| N.S. | 1 | 1.00 | 1.00 | 1.21 | 0.60 | 0.85 | 2.96 | 0.94 | -0.02 | 1.43 |
| time (sec) | N/A | 0.007 | 0.020 | 0.003 | 3.023 | 1.401 | 2.586 | 0.697 | 0.000 | 0.071 |

| Problem 1040 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | A | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 30 | 41 | 12 | 36 | 100 | 27 | 12 | 44 |
| N.S. | 1 | 1.00 | 1.50 | 2.05 | 0.60 | 1.80 | 5.00 | 1.35 | 0.60 | 2.20 |
| time (sec) | N/A | 0.003 | 0.013 | 0.004 | 3.101 | 1.281 | 1.547 | 0.650 | 0.119 | 0.088 |

| Problem 1041 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 2 | 2 | 2 | 27 | 2 | 22 | 41 | 13 | 22 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 13.50 | 1.00 | 11.00 | 20.50 | 6.50 | 11.00 | 10.00 |
| time (sec) | N/A | 0.001 | 0.005 | 0.004 | 2.946 | 0.831 | 1.035 | 0.654 | 0.078 | 0.038 |

| Problem 1042 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 14 | 16 | 23 | 29 | 19 | 13 | 17 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.94 | 1.35 | 1.71 | 1.12 | 0.76 | 1.00 |
| time (sec) | N/A | 0.003 | 0.003 | 0.002 | 2.983 | 1.148 | 0.937 | 0.677 | 0.283 | 0.018 |

| Problem 1043 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 23 | 18 | 38 | 39 | 139 | 22 | 43 | 33 |
| N.S. | 1 | 1.00 | 0.56 | 0.44 | 0.93 | 0.95 | 3.39 | 0.54 | 1.05 | 0.80 |
| time (sec) | N/A | 0.004 | 0.007 | 0.003 | 3.120 | 1.198 | 2.246 | 0.643 | 0.309 | 0.056 |

| Problem 1044 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 30 | 25 | 64 | 56 | 332 | 29 | 55 | 48 |
| N.S. | 1 | 1.00 | 0.49 | 0.41 | 1.05 | 0.92 | 5.44 | 0.48 | 0.90 | 0.79 |
| time (sec) | N/A | 0.008 | 0.009 | 0.004 | 3.026 | 1.180 | 7.892 | 0.690 | 0.324 | 0.059 |

| Problem 1045 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 35 | 30 | 95 | 71 | 595 | 35 | 67 | 62 |
| N.S. | 1 | 1.00 | 0.43 | 0.37 | 1.17 | 0.88 | 7.35 | 0.43 | 0.83 | 0.77 |
| time (sec) | N/A | 0.014 | 0.011 | 0.004 | 3.029 | 1.283 | 22.133 | 0.664 | 0.345 | 0.067 |

| Problem 1046 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 40 | 35 | 131 | 86 | 933 | 42 | 80 | 76 |
| N.S. | 1 | 1.00 | 0.40 | 0.35 | 1.30 | 0.85 | 9.24 | 0.42 | 0.79 | 0.75 |
| time (sec) | N/A | 0.018 | 0.012 | 0.004 | 3.085 | 1.224 | 58.395 | 0.672 | 0.363 | 0.070 |

| Problem 1047 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 37 | 84 | 70 | 65 | 207 | 81 | -1 | 98 |
| N.S. | 1 | 1.00 | 0.44 | 0.99 | 0.82 | 0.76 | 2.44 | 0.95 | -0.01 | 1.15 |
| time (sec) | N/A | 0.016 | 0.013 | 0.017 | 2.839 | 1.038 | 17.475 | 0.748 | 0.000 | 0.101 |

| Problem 1048 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 37 | 77 | 56 | 58 | 168 | 73 | -1 | 82 |
| N.S. | 1 | 1.00 | 0.57 | 1.18 | 0.86 | 0.89 | 2.58 | 1.12 | -0.02 | 1.26 |
| time (sec) | N/A | 0.011 | 0.011 | 0.017 | 2.987 | 1.324 | 6.987 | 0.795 | 0.000 | 0.087 |

| Problem 1049 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | A | B | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 37 | 71 | 41 | 53 | 133 | 70 | -1 | 49 |
| N.S. | 1 | 1.00 | 0.90 | 1.73 | 1.00 | 1.29 | 3.24 | 1.71 | -0.02 | 1.20 |
| time (sec) | N/A | 0.007 | 0.010 | 0.016 | 2.862 | 1.390 | 2.485 | 0.726 | 0.000 | 0.132 |

| Problem 1050 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 34 | 67 | 21 | 50 | 104 | 55 | -1 | 39 |
| N.S. | 1 | 1.00 | 1.48 | 2.91 | 0.91 | 2.17 | 4.52 | 2.39 | -0.04 | 1.70 |
| time (sec) | N/A | 0.003 | 0.036 | 0.015 | 2.863 | 0.769 | 1.538 | 0.691 | 0.000 | 0.044 |

| Problem 1051 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 15 | 16 | 23 | 29 | 43 | 14 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.89 | 1.28 | 1.61 | 2.39 | 0.78 | 1.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.004 | 2.940 | 1.022 | 1.199 | 0.651 | 0.360 | 0.018 |

| Problem 1052 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 13 | 15 | 11 | 22 | 65 | 62 | 14 | 34 |
| N.S. | 1 | 1.00 | 0.72 | 0.83 | 0.61 | 1.22 | 3.61 | 3.44 | 0.78 | 1.89 |
| time (sec) | N/A | 0.002 | 0.003 | 0.002 | 1.338 | 1.243 | 1.857 | 0.719 | 0.305 | 0.059 |

| Problem 1053 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 30 | 25 | 40 | 54 | 158 | 67 | 42 | 48 |
| N.S. | 1 | 1.00 | 0.71 | 0.60 | 0.95 | 1.29 | 3.76 | 1.60 | 1.00 | 1.14 |
| time (sec) | N/A | 0.005 | 0.007 | 0.003 | 1.416 | 1.225 | 5.279 | 0.701 | 0.322 | 0.066 |

| Problem 1054 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 33 | 28 | 79 | 59 | 282 | 73 | 55 | 62 |
| N.S. | 1 | 1.00 | 0.53 | 0.45 | 1.27 | 0.95 | 4.55 | 1.18 | 0.89 | 1.00 |
| time (sec) | N/A | 0.008 | 0.009 | 0.003 | 1.346 | 1.037 | 16.833 | 0.664 | 0.337 | 0.077 |

| Problem 1055 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 40 | 35 | 134 | 86 | 423 | 79 | 68 | 76 |
| N.S. | 1 | 1.00 | 0.49 | 0.43 | 1.63 | 1.05 | 5.16 | 0.96 | 0.83 | 0.93 |
| time (sec) | N/A | 0.013 | 0.010 | 0.004 | 1.375 | 1.268 | 44.937 | 0.695 | 0.355 | 0.076 |

| Problem 1056 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 102 | 102 | 45 | 40 | 201 | 91 | 592 | 85 | 80 | 90 |
| N.S. | 1 | 1.00 | 0.44 | 0.39 | 1.97 | 0.89 | 5.80 | 0.83 | 0.78 | 0.88 |
| time (sec) | N/A | 0.020 | 0.012 | 0.005 | 1.367 | 1.248 | 113.605 | 0.676 | 0.363 | 0.082 |

| Problem 1057 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 37 | 89 | 125 | 85 | 250 | 127 | -1 | 131 |
| N.S. | 1 | 1.00 | 0.36 | 0.86 | 1.21 | 0.83 | 2.43 | 1.23 | -0.01 | 1.27 |
| time (sec) | N/A | 0.021 | 0.015 | 0.022 | 3.009 | 1.499 | 45.202 | 0.889 | 0.000 | 0.104 |

| Problem 1058 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | C | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 37 | 84 | 111 | 81 | 207 | 119 | -1 | 113 |
| N.S. | 1 | 1.00 | 0.43 | 0.97 | 1.28 | 0.93 | 2.38 | 1.37 | -0.01 | 1.30 |
| time (sec) | N/A | 0.015 | 0.012 | 0.020 | 3.000 | 1.270 | 17.501 | 0.800 | 0.000 | 0.090 |

| Problem 1059 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | A | C | B | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 37 | 79 | 98 | 75 | 160 | 115 | -1 | 61 |
| N.S. | 1 | 1.00 | 0.59 | 1.25 | 1.56 | 1.19 | 2.54 | 1.83 | -0.02 | 0.97 |
| time (sec) | N/A | 0.011 | 0.011 | 0.021 | 2.965 | 1.259 | 6.461 | 0.752 | 0.000 | 0.171 |

| Problem 1060 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | C | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 49 | 73 | 66 | 71 | 126 | 102 | -1 | 54 |
| N.S. | 1 | 1.00 | 1.20 | 1.78 | 1.61 | 1.73 | 3.07 | 2.49 | -0.02 | 1.32 |
| time (sec) | N/A | 0.006 | 0.057 | 0.018 | 3.011 | 1.191 | 3.284 | 0.703 | 0.000 | 0.056 |

| Problem 1061 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 15 | 38 | 37 | 65 | 89 | 32 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 0.75 | 1.90 | 1.85 | 3.25 | 4.45 | 1.60 | 1.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.002 | 1.324 | 1.305 | 1.694 | 0.708 | 0.263 | 0.068 |

| Problem 1062 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 23 | 18 | 38 | 38 | 65 | 89 | 33 | 33 |
| N.S. | 1 | 1.00 | 0.56 | 0.44 | 0.93 | 0.93 | 1.59 | 2.17 | 0.80 | 0.80 |
| time (sec) | N/A | 0.004 | 0.005 | 0.003 | 2.934 | 1.346 | 2.346 | 0.672 | 0.310 | 0.052 |

| Problem 1063 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 30 | 25 | 38 | 49 | 165 | 108 | 48 | 48 |
| N.S. | 1 | 1.00 | 0.52 | 0.43 | 0.66 | 0.84 | 2.84 | 1.86 | 0.83 | 0.83 |
| time (sec) | N/A | 0.008 | 0.006 | 0.003 | 1.351 | 1.202 | 5.397 | 0.678 | 0.338 | 0.066 |

| Problem 1064 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 23 | 23 | 25 | 35 | 279 | 113 | 41 | 62 |
| N.S. | 1 | 1.00 | 0.53 | 0.53 | 0.58 | 0.81 | 6.49 | 2.63 | 0.95 | 1.44 |
| time (sec) | N/A | 0.005 | 0.007 | 0.002 | 1.391 | 1.525 | 9.607 | 0.683 | 0.365 | 0.073 |

| Problem 1065 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 40 | 35 | 52 | 84 | 423 | 119 | 75 | 76 |
| N.S. | 1 | 1.00 | 0.63 | 0.56 | 0.83 | 1.33 | 6.71 | 1.89 | 1.19 | 1.21 |
| time (sec) | N/A | 0.008 | 0.011 | 0.001 | 1.394 | 1.269 | 27.605 | 0.710 | 0.377 | 0.076 |

| Problem 1066 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 45 | 40 | 91 | 101 | 592 | 125 | 86 | 90 |
| N.S. | 1 | 1.00 | 0.54 | 0.48 | 1.10 | 1.22 | 7.13 | 1.51 | 1.04 | 1.08 |
| time (sec) | N/A | 0.014 | 0.013 | 0.002 | 1.344 | 1.334 | 71.007 | 0.693 | 0.413 | 0.081 |

| Problem 1067 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 50 | 45 | 146 | 114 | 0 | 131 | 99 | 104 |
| N.S. | 1 | 1.00 | 0.49 | 0.44 | 1.42 | 1.11 | 0.00 | 1.27 | 0.96 | 1.01 |
| time (sec) | N/A | 0.019 | 0.014 | 0.003 | 1.421 | 0.875 | 0.000 | 0.737 | 0.423 | 0.087 |

| Problem 1068 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 126 | 126 | 114 | 193 | 72 | 201 | 0 | 679 | -1 | 206 |
| N.S. | 1 | 1.00 | 0.90 | 1.53 | 0.57 | 1.60 | 0.00 | 5.39 | -0.01 | 1.63 |
| time (sec) | N/A | 0.055 | 0.100 | 0.011 | 3.025 | 1.606 | 0.000 | 1.574 | 0.000 | 0.380 |

| Problem 1069 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 96 | 104 | 143 | 50 | 155 | 0 | 403 | -1 | 160 |
| N.S. | 1 | 1.00 | 1.08 | 1.49 | 0.52 | 1.61 | 0.00 | 4.20 | -0.01 | 1.67 |
| time (sec) | N/A | 0.037 | 0.082 | 0.005 | 3.092 | 1.400 | 0.000 | 1.253 | 0.000 | 0.277 |

| Problem 1070 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 69 | 98 | 28 | 127 | 0 | 173 | 59 | 105 |
| N.S. | 1 | 1.00 | 1.03 | 1.46 | 0.42 | 1.90 | 0.00 | 2.58 | 0.88 | 1.57 |
| time (sec) | N/A | 0.029 | 0.061 | 0.006 | 3.083 | 1.602 | 0.000 | 0.902 | 0.300 | 0.187 |

| Problem 1071 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 47 | 57 | 8 | 101 | 85 | 49 | 44 | 43 |
| N.S. | 1 | 1.00 | 1.09 | 1.33 | 0.19 | 2.35 | 1.98 | 1.14 | 1.02 | 1.00 |
| time (sec) | N/A | 0.024 | 0.017 | 0.005 | 2.991 | 1.644 | 3.953 | 0.764 | 0.175 | 0.082 |

| Problem 1072 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 27 | 25 | 21 | 39 | 82 | 116 | 23 | 47 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.78 | 1.44 | 3.04 | 4.30 | 0.85 | 1.74 |
| time (sec) | N/A | 0.003 | 0.017 | 0.003 | 1.315 | 1.484 | 4.444 | 0.701 | 0.391 | 0.103 |

| Problem 1073 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 42 | 32 | 45 | 57 | 82 | 237 | 62 | 93 |
| N.S. | 1 | 1.00 | 0.69 | 0.52 | 0.74 | 0.93 | 1.34 | 3.89 | 1.02 | 1.52 |
| time (sec) | N/A | 0.010 | 0.028 | 0.002 | 1.347 | 1.084 | 13.690 | 0.807 | 0.415 | 0.118 |

| Problem 1074 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 49 | 37 | 67 | 74 | 85 | 333 | 50 | 141 |
| N.S. | 1 | 1.00 | 0.54 | 0.41 | 0.74 | 0.81 | 0.93 | 3.66 | 0.55 | 1.55 |
| time (sec) | N/A | 0.018 | 0.036 | 0.003 | 1.358 | 1.123 | 55.151 | 1.065 | 0.442 | 0.125 |

| Problem 1075 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 121 | 121 | 54 | 42 | 89 | 89 | 0 | 437 | 66 | 187 |
| N.S. | 1 | 1.00 | 0.45 | 0.35 | 0.74 | 0.74 | 0.00 | 3.61 | 0.55 | 1.55 |
| time (sec) | N/A | 0.028 | 0.040 | 0.003 | 1.401 | 1.261 | 0.000 | 1.563 | 0.478 | 0.135 |

| Problem 1076 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 135 | 135 | 120 | 243 | 89 | 232 | 0 | 0 | -1 | 215 |
| N.S. | 1 | 1.00 | 0.89 | 1.80 | 0.66 | 1.72 | 0.00 | 0.00 | -0.01 | 1.59 |
| time (sec) | N/A | 0.052 | 0.150 | 0.012 | 3.121 | 1.165 | 0.000 | 0.000 | 0.000 | 0.328 |

| Problem 1077 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 102 | 102 | 109 | 185 | 63 | 193 | 0 | 0 | -1 | 169 |
| N.S. | 1 | 1.00 | 1.07 | 1.81 | 0.62 | 1.89 | 0.00 | 0.00 | -0.01 | 1.66 |
| time (sec) | N/A | 0.036 | 0.127 | 0.006 | 3.024 | 1.426 | 0.000 | 0.000 | 0.000 | 0.240 |

| Problem 1078 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 95 | 127 | 39 | 159 | 0 | 0 | 72 | 114 |
| N.S. | 1 | 1.00 | 1.40 | 1.87 | 0.57 | 2.34 | 0.00 | 0.00 | 1.06 | 1.68 |
| time (sec) | N/A | 0.027 | 0.123 | 0.006 | 3.086 | 1.401 | 0.000 | 0.000 | 0.203 | 0.161 |

| Problem 1079 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | C | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 48 | 71 | 14 | 108 | 90 | 0 | 53 | 39 |
| N.S. | 1 | 1.00 | 1.26 | 1.87 | 0.37 | 2.84 | 2.37 | 0.00 | 1.39 | 1.03 |
| time (sec) | N/A | 0.020 | 0.014 | 0.005 | 2.933 | 0.828 | 4.688 | 0.000 | 0.177 | 0.082 |

| Problem 1080 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 29 | 30 | 25 | 45 | 94 | 115 | 26 | 55 |
| N.S. | 1 | 1.00 | 0.97 | 1.00 | 0.83 | 1.50 | 3.13 | 3.83 | 0.87 | 1.83 |
| time (sec) | N/A | 0.004 | 0.014 | 0.003 | 1.400 | 0.957 | 5.179 | 1.861 | 0.497 | 0.118 |

| Problem 1081 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 46 | 45 | 53 | 72 | 94 | 251 | 80 | 101 |
| N.S. | 1 | 1.00 | 0.69 | 0.67 | 0.79 | 1.07 | 1.40 | 3.75 | 1.19 | 1.51 |
| time (sec) | N/A | 0.011 | 0.025 | 0.003 | 1.426 | 1.230 | 15.849 | 2.380 | 0.584 | 0.129 |

| Problem 1082 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 57 | 56 | 79 | 98 | 97 | 366 | 111 | 149 |
| N.S. | 1 | 1.00 | 0.57 | 0.56 | 0.79 | 0.98 | 0.97 | 3.66 | 1.11 | 1.49 |
| time (sec) | N/A | 0.020 | 0.031 | 0.004 | 1.322 | 1.403 | 59.496 | 2.573 | 0.650 | 0.141 |

| Problem 1083 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 133 | 133 | 76 | 67 | 105 | 122 | 0 | 487 | 170 | 195 |
| N.S. | 1 | 1.00 | 0.57 | 0.50 | 0.79 | 0.92 | 0.00 | 3.66 | 1.28 | 1.47 |
| time (sec) | N/A | 0.034 | 0.041 | 0.003 | 1.293 | 1.602 | 0.000 | 3.296 | 0.714 | 0.148 |

| Problem 1084 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 44 | 134 | 46 | 65 | 0 | 227 | -1 | 229 |
| N.S. | 1 | 1.00 | 0.44 | 1.34 | 0.46 | 0.65 | 0.00 | 2.27 | -0.01 | 2.29 |
| time (sec) | N/A | 0.017 | 0.034 | 0.009 | 2.864 | 1.210 | 0.000 | 1.243 | 0.000 | 0.967 |

| Problem 1085 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 39 | 102 | 34 | 60 | 0 | 125 | -1 | 179 |
| N.S. | 1 | 1.00 | 0.53 | 1.38 | 0.46 | 0.81 | 0.00 | 1.69 | -0.01 | 2.42 |
| time (sec) | N/A | 0.010 | 0.035 | 0.005 | 2.859 | 0.868 | 0.000 | 0.984 | 0.000 | 0.841 |

| Problem 1086 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 30 | 70 | 22 | 52 | 187 | 55 | 44 | 122 |
| N.S. | 1 | 1.00 | 0.70 | 1.63 | 0.51 | 1.21 | 4.35 | 1.28 | 1.02 | 2.84 |
| time (sec) | N/A | 0.006 | 0.010 | 0.004 | 3.068 | 1.135 | 4.742 | 1.069 | 0.256 | 0.677 |

| Problem 1087 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 37 | 9 | 28 | 41 | 15 | 40 | 36 |
| N.S. | 1 | 1.00 | 1.00 | 2.85 | 0.69 | 2.15 | 3.15 | 1.15 | 3.08 | 2.77 |
| time (sec) | N/A | 0.003 | 0.016 | 0.003 | 2.943 | 1.093 | 3.347 | 0.885 | 0.051 | 0.588 |

| Problem 1088 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 16 | 28 | 12 | 26 | 156 | 71 | 24 | 80 |
| N.S. | 1 | 1.00 | 0.57 | 1.00 | 0.43 | 0.93 | 5.57 | 2.54 | 0.86 | 2.86 |
| time (sec) | N/A | 0.002 | 0.018 | 0.004 | 1.358 | 1.392 | 85.283 | 1.062 | 0.461 | 0.732 |

| Problem 1089 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 37 | 35 | 25 | 39 | 0 | 128 | 49 | 334 |
| N.S. | 1 | 1.00 | 0.65 | 0.61 | 0.44 | 0.68 | 0.00 | 2.25 | 0.86 | 5.86 |
| time (sec) | N/A | 0.006 | 0.025 | 0.003 | 1.277 | 1.314 | 0.000 | 1.017 | 0.311 | 0.848 |

| Problem 1090 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 42 | 40 | 37 | 49 | 0 | 181 | 66 | 616 |
| N.S. | 1 | 1.00 | 0.49 | 0.47 | 0.44 | 0.58 | 0.00 | 2.13 | 0.78 | 7.25 |
| time (sec) | N/A | 0.011 | 0.032 | 0.003 | 1.315 | 1.257 | 0.000 | 1.021 | 0.452 | 1.250 |

| Problem 1091 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 80 | 89 | 67 | 62 | 199 | 101 | -1 | 115 |
| N.S. | 1 | 1.00 | 0.88 | 0.98 | 0.74 | 0.68 | 2.19 | 1.11 | -0.01 | 1.26 |
| time (sec) | N/A | 0.022 | 0.047 | 0.009 | 2.968 | 1.095 | 7.482 | 0.879 | 0.000 | 0.093 |

| Problem 1092 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 69 | 61 | 38 | 52 | 124 | 42 | 41 | 78 |
| N.S. | 1 | 1.00 | 1.35 | 1.20 | 0.75 | 1.02 | 2.43 | 0.82 | 0.80 | 1.53 |
| time (sec) | N/A | 0.010 | 0.023 | 0.006 | 2.964 | 1.289 | 3.010 | 1.019 | 0.207 | 0.058 |

| Problem 1093 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 12 | 31 | 6 | 32 | 26 | 8 | 31 | 20 |
| N.S. | 1 | 1.00 | 1.50 | 3.88 | 0.75 | 4.00 | 3.25 | 1.00 | 3.88 | 2.50 |
| time (sec) | N/A | 0.004 | 0.010 | 0.004 | 3.001 | 1.354 | 1.607 | 1.019 | 0.178 | 0.035 |

| Problem 1094 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 21 | 20 | 30 | 29 | 100 | 53 | 32 | 31 |
| N.S. | 1 | 1.00 | 0.57 | 0.54 | 0.81 | 0.78 | 2.70 | 1.43 | 0.86 | 0.84 |
| time (sec) | N/A | 0.004 | 0.006 | 0.004 | 1.318 | 1.267 | 2.302 | 0.848 | 0.255 | 0.047 |

| Problem 1095 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 79 | 79 | 33 | 30 | 59 | 49 | 282 | 97 | 69 | 61 |
| N.S. | 1 | 1.00 | 0.42 | 0.38 | 0.75 | 0.62 | 3.57 | 1.23 | 0.87 | 0.77 |
| time (sec) | N/A | 0.013 | 0.013 | 0.004 | 1.337 | 1.222 | 9.849 | 1.096 | 0.370 | 0.058 |

| Problem 1096 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 16 | 16 | 12 | 22 | 73 | 62 | 22 | 34 |
| N.S. | 1 | 1.00 | 0.76 | 0.76 | 0.57 | 1.05 | 3.48 | 2.95 | 1.05 | 1.62 |
| time (sec) | N/A | 0.002 | 0.005 | 0.003 | 1.367 | 1.555 | 1.800 | 0.896 | 0.364 | 0.059 |

| Problem 1097 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 19 | 19 | 15 | 29 | 73 | 82 | 26 | 43 |
| N.S. | 1 | 1.00 | 0.79 | 0.79 | 0.62 | 1.21 | 3.04 | 3.42 | 1.08 | 1.79 |
| time (sec) | N/A | 0.003 | 0.007 | 0.003 | 1.371 | 1.530 | 5.162 | 1.098 | 0.463 | 0.084 |

| Problem 1098 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 21 | 19 | 12 | 22 | 90 | 71 | 22 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 0.73 | 0.46 | 0.85 | 3.46 | 2.73 | 0.85 | 0.00 |
| time (sec) | N/A | 0.002 | 0.013 | 0.002 | 1.332 | 1.289 | 20.446 | 1.044 | 0.374 | 0.132 |

| Problem 1099 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 19 | 24 | 15 | 29 | 83 | 91 | 26 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 0.83 | 0.52 | 1.00 | 2.86 | 3.14 | 0.90 | 0.00 |
| time (sec) | N/A | 0.003 | 0.018 | 0.003 | 1.254 | 1.211 | 31.496 | 1.144 | 0.321 | 0.245 |

| Problem 1100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | C | F(-1) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 39 | 76 | 39 | 108 | 88 | 0 | 56 | 39 |
| N.S. | 1 | 1.00 | 1.00 | 1.95 | 1.00 | 2.77 | 2.26 | 0.00 | 1.44 | 1.00 |
| time (sec) | N/A | 0.024 | 0.020 | 0.010 | 1.429 | 1.272 | 4.766 | 0.000 | 0.218 | 0.081 |

| Problem 1101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 241 | 241 | 42 | 0 | 0 | 505 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.17 | 0.00 | 0.00 | 2.10 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.254 | 0.021 | 0.072 | 0.000 | 1.407 | 0.000 | 0.000 | 0.000 | 0.132 |

| Problem 1102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 256 | 256 | 70 | 477 | 0 | 194 | 0 | 0 | -1 | 126 |
| N.S. | 1 | 1.00 | 0.27 | 1.86 | 0.00 | 0.76 | 0.00 | 0.00 | -0.00 | 0.49 |
| time (sec) | N/A | 0.173 | 0.021 | 2.284 | 0.000 | 1.486 | 0.000 | 0.000 | 0.000 | 0.470 |

| Problem 1103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 233 | 233 | 68 | 0 | 0 | 227 | 0 | 0 | -1 | 83 |
| N.S. | 1 | 1.00 | 0.29 | 0.00 | 0.00 | 0.97 | 0.00 | 0.00 | -0.00 | 0.36 |
| time (sec) | N/A | 0.131 | 0.023 | 0.076 | 0.000 | 1.499 | 0.000 | 0.000 | 0.000 | 0.197 |

| Problem 1104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 33 | 31 | 0 | 32 | 0 | 0 | 38 | 33 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.00 | 0.97 | 0.00 | 0.00 | 1.15 | 1.00 |
| time (sec) | N/A | 0.003 | 0.014 | 0.046 | 0.000 | 1.713 | 0.000 | 0.000 | 0.549 | 0.112 |

| Problem 1105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 45 | 44 | 0 | 44 | 0 | 0 | 46 | 55 |
| N.S. | 1 | 1.00 | 0.67 | 0.66 | 0.00 | 0.66 | 0.00 | 0.00 | 0.69 | 0.82 |
| time (sec) | N/A | 0.009 | 0.021 | 0.048 | 0.000 | 1.570 | 0.000 | 0.000 | 0.666 | 0.125 |

| Problem 1106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 52 | 50 | 0 | 58 | 0 | 0 | 51 | 77 |
| N.S. | 1 | 1.00 | 0.52 | 0.50 | 0.00 | 0.58 | 0.00 | 0.00 | 0.51 | 0.77 |
| time (sec) | N/A | 0.018 | 0.025 | 0.054 | 0.000 | 1.417 | 0.000 | 0.000 | 0.752 | 0.129 |

| Problem 1107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 133 | 133 | 57 | 55 | 0 | 70 | 0 | 0 | 57 | 99 |
| N.S. | 1 | 1.00 | 0.43 | 0.41 | 0.00 | 0.53 | 0.00 | 0.00 | 0.43 | 0.74 |
| time (sec) | N/A | 0.028 | 0.029 | 0.062 | 0.000 | 1.472 | 0.000 | 0.000 | 0.794 | 0.124 |

| Problem 1108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 256 | 256 | 70 | 464 | 0 | 204 | 0 | 0 | -1 | 128 |
| N.S. | 1 | 1.00 | 0.27 | 1.81 | 0.00 | 0.80 | 0.00 | 0.00 | -0.00 | 0.50 |
| time (sec) | N/A | 0.158 | 0.025 | 2.142 | 0.000 | 1.475 | 0.000 | 0.000 | 0.000 | 0.601 |

| Problem 1109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 233 | 233 | 70 | 0 | 0 | 227 | 0 | 0 | -1 | 83 |
| N.S. | 1 | 1.00 | 0.30 | 0.00 | 0.00 | 0.97 | 0.00 | 0.00 | -0.00 | 0.36 |
| time (sec) | N/A | 0.135 | 0.023 | 0.060 | 0.000 | 1.411 | 0.000 | 0.000 | 0.000 | 0.153 |

| Problem 1110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 31 | 0 | 31 | 0 | 0 | -1 | 31 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 0.00 | 0.00 | -0.03 | 1.00 |
| time (sec) | N/A | 0.003 | 0.014 | 0.042 | 0.000 | 1.416 | 0.000 | 0.000 | 0.000 | 0.062 |

| Problem 1111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 45 | 44 | 0 | 44 | 0 | 0 | -1 | 54 |
| N.S. | 1 | 1.00 | 0.67 | 0.66 | 0.00 | 0.66 | 0.00 | 0.00 | -0.01 | 0.81 |
| time (sec) | N/A | 0.010 | 0.020 | 0.053 | 0.000 | 1.161 | 0.000 | 0.000 | 0.000 | 0.114 |

| Problem 1112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 52 | 50 | 0 | 58 | 0 | 0 | -1 | 77 |
| N.S. | 1 | 1.00 | 0.52 | 0.50 | 0.00 | 0.58 | 0.00 | 0.00 | -0.01 | 0.77 |
| time (sec) | N/A | 0.018 | 0.024 | 0.056 | 0.000 | 1.400 | 0.000 | 0.000 | 0.000 | 0.117 |

| Problem 1113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 291 | 291 | 70 | 469 | 0 | 244 | 0 | 0 | -1 | 150 |
| N.S. | 1 | 1.00 | 0.24 | 1.61 | 0.00 | 0.84 | 0.00 | 0.00 | -0.00 | 0.52 |
| time (sec) | N/A | 0.181 | 0.028 | 2.013 | 0.000 | 1.425 | 0.000 | 0.000 | 0.000 | 0.901 |

| Problem 1114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 266 | 266 | 70 | 459 | 0 | 307 | 0 | 0 | -1 | 116 |
| N.S. | 1 | 1.00 | 0.26 | 1.73 | 0.00 | 1.15 | 0.00 | 0.00 | -0.00 | 0.44 |
| time (sec) | N/A | 0.140 | 0.022 | 1.882 | 0.000 | 0.844 | 0.000 | 0.000 | 0.000 | 0.192 |

| Problem 1115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 33 | 31 | 0 | 32 | 0 | 0 | -1 | 33 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.00 | 0.97 | 0.00 | 0.00 | -0.03 | 1.00 |
| time (sec) | N/A | 0.003 | 0.012 | 0.044 | 0.000 | 1.509 | 0.000 | 0.000 | 0.000 | 0.085 |

| Problem 1116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 38 | 33 | 0 | 36 | 0 | 0 | -1 | 55 |
| N.S. | 1 | 1.00 | 0.58 | 0.51 | 0.00 | 0.55 | 0.00 | 0.00 | -0.02 | 0.85 |
| time (sec) | N/A | 0.009 | 0.017 | 0.049 | 0.000 | 1.099 | 0.000 | 0.000 | 0.000 | 0.105 |

| Problem 1117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 50 | 44 | 0 | 58 | 0 | 0 | -1 | 77 |
| N.S. | 1 | 1.00 | 0.50 | 0.44 | 0.00 | 0.58 | 0.00 | 0.00 | -0.01 | 0.77 |
| time (sec) | N/A | 0.017 | 0.023 | 0.058 | 0.000 | 1.514 | 0.000 | 0.000 | 0.000 | 0.109 |

| Problem 1118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 287 | 287 | 70 | 481 | 0 | 240 | 0 | 0 | -1 | 146 |
| N.S. | 1 | 1.00 | 0.24 | 1.68 | 0.00 | 0.84 | 0.00 | 0.00 | -0.00 | 0.51 |
| time (sec) | N/A | 0.178 | 0.029 | 2.037 | 0.000 | 1.282 | 0.000 | 0.000 | 0.000 | 0.709 |

| Problem 1119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 264 | 264 | 70 | 476 | 0 | 303 | 0 | 0 | -1 | 114 |
| N.S. | 1 | 1.00 | 0.27 | 1.80 | 0.00 | 1.15 | 0.00 | 0.00 | -0.00 | 0.43 |
| time (sec) | N/A | 0.139 | 0.024 | 2.008 | 0.000 | 0.935 | 0.000 | 0.000 | 0.000 | 0.193 |

| Problem 1120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 31 | 0 | 31 | 0 | 0 | 27 | 31 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.87 | 1.00 |
| time (sec) | N/A | 0.003 | 0.012 | 0.041 | 0.000 | 0.970 | 0.000 | 0.000 | 1.161 | 0.066 |

| Problem 1121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 38 | 33 | 0 | 36 | 0 | 0 | 40 | 55 |
| N.S. | 1 | 1.00 | 0.57 | 0.49 | 0.00 | 0.54 | 0.00 | 0.00 | 0.60 | 0.82 |
| time (sec) | N/A | 0.009 | 0.018 | 0.049 | 0.000 | 1.496 | 0.000 | 0.000 | 0.597 | 0.129 |

| Problem 1122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 50 | 44 | 0 | 58 | 0 | 0 | 46 | 77 |
| N.S. | 1 | 1.00 | 0.50 | 0.44 | 0.00 | 0.58 | 0.00 | 0.00 | 0.46 | 0.77 |
| time (sec) | N/A | 0.018 | 0.022 | 0.057 | 0.000 | 0.880 | 0.000 | 0.000 | 0.764 | 0.135 |

| Problem 1123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 297 | 297 | 70 | 490 | 0 | 351 | 0 | 0 | -1 | 137 |
| N.S. | 1 | 1.00 | 0.24 | 1.65 | 0.00 | 1.18 | 0.00 | 0.00 | -0.00 | 0.46 |
| time (sec) | N/A | 0.140 | 0.025 | 0.059 | 0.000 | 1.486 | 0.000 | 0.000 | 0.000 | 0.219 |

| Problem 1124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 33 | 50 | 0 | 45 | 0 | 0 | 38 | 33 |
| N.S. | 1 | 1.00 | 1.00 | 1.52 | 0.00 | 1.36 | 0.00 | 0.00 | 1.15 | 1.00 |
| time (sec) | N/A | 0.003 | 0.012 | 0.044 | 0.000 | 1.429 | 0.000 | 0.000 | 0.546 | 0.109 |

| Problem 1125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 45 | 44 | 0 | 44 | 0 | 0 | 38 | 54 |
| N.S. | 1 | 1.00 | 0.67 | 0.66 | 0.00 | 0.66 | 0.00 | 0.00 | 0.57 | 0.81 |
| time (sec) | N/A | 0.010 | 0.020 | 0.045 | 0.000 | 1.361 | 0.000 | 0.000 | 0.633 | 0.141 |

| Problem 1126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 50 | 44 | 0 | 58 | 0 | 0 | 45 | 77 |
| N.S. | 1 | 1.00 | 0.50 | 0.44 | 0.00 | 0.58 | 0.00 | 0.00 | 0.45 | 0.77 |
| time (sec) | N/A | 0.018 | 0.023 | 0.055 | 0.000 | 1.266 | 0.000 | 0.000 | 0.535 | 0.139 |

| Problem 1127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 133 | 133 | 57 | 56 | 0 | 54 | 0 | 0 | 56 | 99 |
| N.S. | 1 | 1.00 | 0.43 | 0.42 | 0.00 | 0.41 | 0.00 | 0.00 | 0.42 | 0.74 |
| time (sec) | N/A | 0.029 | 0.030 | 0.065 | 0.000 | 1.490 | 0.000 | 0.000 | 0.687 | 0.140 |

| Problem 1128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 77 | 103 | 167 | 128 | 819 | 256 | 133 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.24 | 2.01 | 1.54 | 9.87 | 3.08 | 1.60 | 0.00 |
| time (sec) | N/A | 0.029 | 0.043 | 0.007 | 1.530 | 1.245 | 1.295 | 1.145 | 0.493 | 0.100 |

| Problem 1129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 43 | 47 | 81 | 58 | 245 | 103 | 66 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 0.89 | 1.53 | 1.09 | 4.62 | 1.94 | 1.25 | 0.00 |
| time (sec) | N/A | 0.017 | 0.021 | 0.003 | 1.396 | 1.344 | 0.699 | 1.050 | 0.319 | 0.081 |

| Problem 1130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 84 | 97 | 96 | 97 | 100 | 97 | 88 | 0 |
| N.S. | 1 | 1.00 | 2.21 | 2.55 | 2.53 | 2.55 | 2.63 | 2.55 | 2.32 | 0.00 |
| time (sec) | N/A | 0.015 | 0.019 | 0.002 | 1.383 | 1.203 | 0.083 | 1.032 | 0.190 | 0.000 |

| Problem 1131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 67 | 73 | 69 | 72 | 73 | 72 | 65 | 0 |
| N.S. | 1 | 1.00 | 1.76 | 1.92 | 1.82 | 1.89 | 1.92 | 1.89 | 1.71 | 0.00 |
| time (sec) | N/A | 0.012 | 0.011 | 0.000 | 1.406 | 1.411 | 0.078 | 1.139 | 0.159 | 0.000 |

| Problem 1132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 46 | 49 | 48 | 49 | 49 | 49 | 47 | 0 |
| N.S. | 1 | 1.00 | 1.21 | 1.29 | 1.26 | 1.29 | 1.29 | 1.29 | 1.24 | 0.00 |
| time (sec) | N/A | 0.027 | 0.008 | 0.000 | 1.300 | 1.728 | 0.071 | 0.993 | 0.047 | 0.000 |

| Problem 1133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 28 | 25 | 24 | 26 | 26 | 26 | 25 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 0.86 | 0.93 | 0.93 | 0.93 | 0.89 | 0.00 |
| time (sec) | N/A | 0.015 | 0.005 | 0.001 | 1.350 | 1.558 | 0.060 | 0.994 | 0.035 | 0.000 |

| Problem 1134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 12 | 11 | 10 | 10 | 8 | 10 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.83 | 0.83 | 0.67 | 0.83 | 0.83 | 0.00 |
| time (sec) | N/A | 0.002 | 0.000 | 0.000 | 1.350 | 1.457 | 0.055 | 0.903 | 0.019 | 0.000 |

| Problem 1135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 25 | 32 | 25 | 24 | 20 | 26 | 26 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.28 | 1.00 | 0.96 | 0.80 | 1.04 | 1.04 | 0.00 |
| time (sec) | N/A | 0.017 | 0.008 | 0.004 | 1.251 | 1.698 | 0.150 | 0.955 | 0.049 | 0.000 |

| Problem 1136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 31 | 39 | 35 | 39 | 27 | 57 | 31 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 1.22 | 1.09 | 1.22 | 0.84 | 1.78 | 0.97 | 0.00 |
| time (sec) | N/A | 0.019 | 0.012 | 0.005 | 1.339 | 1.682 | 0.187 | 1.020 | 0.171 | 0.000 |

| Problem 1137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 26 | 35 | 38 | 38 | 39 | 24 | 39 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.25 | 1.36 | 1.36 | 1.39 | 0.86 | 1.39 | 0.00 |
| time (sec) | N/A | 0.004 | 0.009 | 0.006 | 1.362 | 1.748 | 0.260 | 1.062 | 0.159 | 0.000 |

| Problem 1138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 27 | 35 | 50 | 50 | 53 | 25 | 52 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.92 | 1.32 | 1.32 | 1.39 | 0.66 | 1.37 | 0.00 |
| time (sec) | N/A | 0.020 | 0.009 | 0.006 | 1.315 | 1.754 | 0.338 | 0.749 | 0.165 | 0.000 |

| Problem 1139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 27 | 35 | 61 | 61 | 65 | 41 | 63 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.92 | 1.61 | 1.61 | 1.71 | 1.08 | 1.66 | 0.00 |
| time (sec) | N/A | 0.020 | 0.010 | 0.004 | 1.385 | 1.202 | 0.428 | 0.924 | 0.041 | 0.000 |

| Problem 1140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 148 | 163 | 156 | 170 | 168 | 170 | 144 | 0 |
| N.S. | 1 | 1.00 | 2.28 | 2.51 | 2.40 | 2.62 | 2.58 | 2.62 | 2.22 | 0.00 |
| time (sec) | N/A | 0.088 | 0.027 | 0.002 | 1.362 | 1.105 | 0.097 | 1.132 | 0.066 | 0.000 |

| Problem 1141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 122 | 125 | 124 | 130 | 133 | 130 | 115 | 0 |
| N.S. | 1 | 1.00 | 1.88 | 1.92 | 1.91 | 2.00 | 2.05 | 2.00 | 1.77 | 0.00 |
| time (sec) | N/A | 0.064 | 0.015 | 0.001 | 1.344 | 1.565 | 0.091 | 1.132 | 0.050 | 0.000 |

| Problem 1142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 79 | 87 | 81 | 89 | 87 | 89 | 74 | 0 |
| N.S. | 1 | 1.00 | 1.22 | 1.34 | 1.25 | 1.37 | 1.34 | 1.37 | 1.14 | 0.00 |
| time (sec) | N/A | 0.045 | 0.011 | 0.001 | 1.339 | 1.333 | 0.080 | 0.795 | 0.166 | 0.000 |

| Problem 1143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 47 | 49 | 48 | 49 | 49 | 49 | 47 | 0 |
| N.S. | 1 | 1.00 | 1.24 | 1.29 | 1.26 | 1.29 | 1.29 | 1.29 | 1.24 | 0.00 |
| time (sec) | N/A | 0.027 | 0.010 | 0.001 | 1.306 | 1.183 | 0.071 | 1.027 | 0.044 | 0.000 |

| Problem 1144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 20 | 20 | 19 | 12 | 20 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 1.43 | 1.43 | 1.36 | 0.86 | 1.43 | 0.00 |
| time (sec) | N/A | 0.002 | 0.002 | 0.000 | 1.314 | 1.249 | 0.061 | 0.963 | 0.028 | 0.000 |

| Problem 1145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 49 | 49 | 43 | 74 | 61 | 63 | 44 | 60 | 62 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.51 | 1.24 | 1.29 | 0.90 | 1.22 | 1.27 | 0.00 |
| time (sec) | N/A | 0.019 | 0.018 | 0.003 | 1.356 | 0.839 | 0.224 | 1.196 | 0.190 | 0.000 |

| Problem 1146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 47 | 86 | 67 | 92 | 60 | 98 | 71 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.69 | 1.31 | 1.80 | 1.18 | 1.92 | 1.39 | 0.00 |
| time (sec) | N/A | 0.035 | 0.037 | 0.009 | 1.372 | 1.420 | 0.337 | 0.937 | 0.200 | 0.000 |

| Problem 1147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 49 | 92 | 79 | 99 | 80 | 68 | 77 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.56 | 1.34 | 1.68 | 1.36 | 1.15 | 1.31 | 0.00 |
| time (sec) | N/A | 0.035 | 0.025 | 0.006 | 1.300 | 1.144 | 0.454 | 1.031 | 0.197 | 0.000 |

| Problem 1148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 53 | 70 | 84 | 84 | 88 | 59 | 80 | 0 |
| N.S. | 1 | 1.00 | 1.89 | 2.50 | 3.00 | 3.00 | 3.14 | 2.11 | 2.86 | 0.00 |
| time (sec) | N/A | 0.004 | 0.022 | 0.006 | 1.374 | 1.391 | 0.597 | 0.960 | 0.037 | 0.000 |

| Problem 1149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 56 | 71 | 98 | 98 | 104 | 96 | 39 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.09 | 1.51 | 1.51 | 1.60 | 1.48 | 0.60 | 0.00 |
| time (sec) | N/A | 0.033 | 0.019 | 0.006 | 1.329 | 1.490 | 0.764 | 1.126 | 0.193 | 0.000 |

| Problem 1150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 57 | 71 | 109 | 109 | 116 | 61 | 107 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.09 | 1.68 | 1.68 | 1.78 | 0.94 | 1.65 | 0.00 |
| time (sec) | N/A | 0.033 | 0.024 | 0.007 | 1.350 | 1.337 | 0.956 | 0.870 | 0.202 | 0.000 |

| Problem 1151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 58 | 71 | 120 | 120 | 128 | 61 | 118 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.09 | 1.85 | 1.85 | 1.97 | 0.94 | 1.82 | 0.00 |
| time (sec) | N/A | 0.033 | 0.020 | 0.005 | 1.391 | 1.334 | 1.159 | 1.036 | 0.089 | 0.000 |

| Problem 1152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 235 | 281 | 277 | 303 | 308 | 303 | 261 | 0 |
| N.S. | 1 | 1.00 | 2.55 | 3.05 | 3.01 | 3.29 | 3.35 | 3.29 | 2.84 | 0.00 |
| time (sec) | N/A | 0.157 | 0.075 | 0.000 | 1.380 | 1.137 | 0.116 | 1.011 | 0.240 | 0.000 |

| Problem 1153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 217 | 229 | 225 | 245 | 243 | 245 | 208 | 0 |
| N.S. | 1 | 1.00 | 2.36 | 2.49 | 2.45 | 2.66 | 2.64 | 2.66 | 2.26 | 0.00 |
| time (sec) | N/A | 0.114 | 0.029 | 0.001 | 1.318 | 1.198 | 0.106 | 1.116 | 0.214 | 0.000 |

| Problem 1154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 161 | 177 | 167 | 188 | 190 | 188 | 152 | 0 |
| N.S. | 1 | 1.00 | 1.75 | 1.92 | 1.82 | 2.04 | 2.07 | 2.04 | 1.65 | 0.00 |
| time (sec) | N/A | 0.083 | 0.019 | 0.002 | 1.340 | 1.091 | 0.097 | 0.973 | 0.056 | 0.000 |

| Problem 1155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 122 | 125 | 124 | 130 | 133 | 130 | 115 | 0 |
| N.S. | 1 | 1.00 | 1.88 | 1.92 | 1.91 | 2.00 | 2.05 | 2.00 | 1.77 | 0.00 |
| time (sec) | N/A | 0.064 | 0.014 | 0.001 | 1.340 | 1.269 | 0.087 | 1.039 | 0.046 | 0.000 |

| Problem 1156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 67 | 73 | 69 | 72 | 73 | 72 | 65 | 0 |
| N.S. | 1 | 1.00 | 1.76 | 1.92 | 1.82 | 1.89 | 1.92 | 1.89 | 1.71 | 0.00 |
| time (sec) | N/A | 0.015 | 0.008 | 0.000 | 1.390 | 1.087 | 0.077 | 0.877 | 0.032 | 0.000 |

| Problem 1157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 31 | 31 | 32 | 12 | 31 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 2.21 | 2.21 | 2.29 | 0.86 | 2.21 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.000 | 1.346 | 1.182 | 0.065 | 0.997 | 0.037 | 0.000 |

| Problem 1158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 74 | 133 | 114 | 116 | 83 | 115 | 118 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 1.82 | 1.56 | 1.59 | 1.14 | 1.58 | 1.62 | 0.00 |
| time (sec) | N/A | 0.028 | 0.031 | 0.004 | 1.327 | 1.787 | 0.303 | 1.031 | 0.201 | 0.000 |

| Problem 1159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 75 | 75 | 72 | 149 | 118 | 173 | 102 | 167 | 123 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 1.99 | 1.57 | 2.31 | 1.36 | 2.23 | 1.64 | 0.00 |
| time (sec) | N/A | 0.055 | 0.051 | 0.008 | 1.317 | 1.416 | 0.503 | 0.991 | 0.215 | 0.000 |

| Problem 1160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 78 | 78 | 114 | 160 | 125 | 188 | 128 | 112 | 130 | 0 |
| N.S. | 1 | 1.00 | 1.46 | 2.05 | 1.60 | 2.41 | 1.64 | 1.44 | 1.67 | 0.00 |
| time (sec) | N/A | 0.052 | 0.042 | 0.009 | 1.373 | 1.343 | 0.821 | 0.952 | 0.821 | 0.000 |

| Problem 1161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 80 | 166 | 142 | 176 | 148 | 118 | 138 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.93 | 1.65 | 2.05 | 1.72 | 1.37 | 1.60 | 0.00 |
| time (sec) | N/A | 0.050 | 0.041 | 0.006 | 1.353 | 1.510 | 1.126 | 0.944 | 0.254 | 0.001 |

| Problem 1162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 91 | 122 | 143 | 143 | 155 | 159 | 135 | 0 |
| N.S. | 1 | 1.00 | 3.25 | 4.36 | 5.11 | 5.11 | 5.54 | 5.68 | 4.82 | 0.00 |
| time (sec) | N/A | 0.003 | 0.031 | 0.007 | 1.392 | 1.542 | 1.497 | 0.965 | 0.072 | 0.000 |

| Problem 1163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 97 | 121 | 160 | 160 | 172 | 114 | 39 | 0 |
| N.S. | 1 | 1.00 | 1.67 | 2.09 | 2.76 | 2.76 | 2.97 | 1.97 | 0.67 | 0.00 |
| time (sec) | N/A | 0.010 | 0.035 | 0.006 | 1.467 | 1.647 | 1.961 | 1.001 | 0.082 | 0.000 |

| Problem 1164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 97 | 122 | 171 | 171 | 184 | 114 | 165 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 1.33 | 1.86 | 1.86 | 2.00 | 1.24 | 1.79 | 0.00 |
| time (sec) | N/A | 0.050 | 0.031 | 0.006 | 1.439 | 1.487 | 2.536 | 0.985 | 0.222 | 0.000 |

| Problem 1165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 97 | 122 | 182 | 182 | 196 | 114 | 176 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 1.33 | 1.98 | 1.98 | 2.13 | 1.24 | 1.91 | 0.00 |
| time (sec) | N/A | 0.049 | 0.032 | 0.006 | 1.445 | 1.457 | 3.123 | 0.947 | 0.114 | 0.000 |

| Problem 1166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 97 | 122 | 193 | 193 | 207 | 114 | 187 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 1.33 | 2.10 | 2.10 | 2.25 | 1.24 | 2.03 | 0.00 |
| time (sec) | N/A | 0.046 | 0.036 | 0.006 | 1.522 | 1.437 | 3.954 | 0.864 | 0.232 | 0.000 |

| Problem 1167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 200 | 200 | 993 | 1033 | 1023 | 1175 | 1163 | 1175 | 997 | 0 |
| N.S. | 1 | 1.00 | 4.96 | 5.16 | 5.12 | 5.88 | 5.82 | 5.88 | 4.98 | 0.00 |
| time (sec) | N/A | 0.676 | 0.147 | 0.002 | 1.431 | 0.865 | 0.232 | 1.046 | 0.554 | 0.000 |

| Problem 1168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 200 | 200 | 897 | 925 | 921 | 1050 | 1046 | 1050 | 892 | 0 |
| N.S. | 1 | 1.00 | 4.48 | 4.62 | 4.60 | 5.25 | 5.23 | 5.25 | 4.46 | 0.00 |
| time (sec) | N/A | 0.574 | 0.111 | 0.002 | 1.396 | 1.282 | 0.214 | 1.006 | 0.357 | 0.000 |

| Problem 1169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 200 | 200 | 785 | 817 | 807 | 924 | 935 | 924 | 781 | 0 |
| N.S. | 1 | 1.00 | 3.92 | 4.08 | 4.04 | 4.62 | 4.68 | 4.62 | 3.90 | 0.00 |
| time (sec) | N/A | 0.454 | 0.086 | 0.003 | 1.335 | 1.250 | 0.194 | 1.009 | 0.402 | 0.000 |

| Problem 1170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 173 | 173 | 684 | 709 | 706 | 798 | 796 | 798 | 683 | 0 |
| N.S. | 1 | 1.00 | 3.95 | 4.10 | 4.08 | 4.61 | 4.60 | 4.61 | 3.95 | 0.00 |
| time (sec) | N/A | 0.435 | 0.081 | 0.001 | 1.317 | 1.201 | 0.178 | 0.953 | 0.256 | 0.000 |

| Problem 1171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 144 | 144 | 574 | 601 | 594 | 670 | 673 | 670 | 570 | 0 |
| N.S. | 1 | 1.00 | 3.99 | 4.17 | 4.12 | 4.65 | 4.67 | 4.65 | 3.96 | 0.00 |
| time (sec) | N/A | 0.360 | 0.076 | 0.002 | 1.384 | 1.092 | 0.164 | 1.279 | 0.214 | 0.000 |

| Problem 1172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 473 | 493 | 489 | 546 | 549 | 546 | 470 | 0 |
| N.S. | 1 | 1.00 | 3.97 | 4.14 | 4.11 | 4.59 | 4.61 | 4.59 | 3.95 | 0.00 |
| time (sec) | N/A | 0.279 | 0.054 | 0.001 | 1.479 | 1.307 | 0.146 | 1.215 | 0.313 | 0.000 |

| Problem 1173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 360 | 385 | 376 | 420 | 427 | 420 | 356 | 0 |
| N.S. | 1 | 1.00 | 3.91 | 4.18 | 4.09 | 4.57 | 4.64 | 4.57 | 3.87 | 0.00 |
| time (sec) | N/A | 0.218 | 0.042 | 0.002 | 1.373 | 1.253 | 0.131 | 1.259 | 0.273 | 0.000 |

| Problem 1174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 261 | 277 | 273 | 294 | 303 | 294 | 249 | 0 |
| N.S. | 1 | 1.00 | 4.02 | 4.26 | 4.20 | 4.52 | 4.66 | 4.52 | 3.83 | 0.00 |
| time (sec) | N/A | 0.159 | 0.030 | 0.001 | 1.363 | 1.206 | 0.116 | 1.240 | 0.107 | 0.000 |

| Problem 1175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 151 | 169 | 163 | 169 | 178 | 169 | 143 | 0 |
| N.S. | 1 | 1.00 | 3.97 | 4.45 | 4.29 | 4.45 | 4.68 | 4.45 | 3.76 | 0.00 |
| time (sec) | N/A | 0.016 | 0.015 | 0.000 | 1.376 | 1.566 | 0.100 | 1.297 | 0.079 | 0.000 |

| Problem 1176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 75 | 83 | 12 | 75 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 5.36 | 5.93 | 0.86 | 5.36 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.000 | 1.315 | 1.285 | 0.076 | 1.298 | 0.057 | 0.000 |

| Problem 1177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 169 | 169 | 304 | 539 | 460 | 462 | 408 | 497 | 509 | 0 |
| N.S. | 1 | 1.00 | 1.80 | 3.19 | 2.72 | 2.73 | 2.41 | 2.94 | 3.01 | 0.00 |
| time (sec) | N/A | 0.073 | 0.148 | 0.007 | 1.395 | 1.171 | 0.802 | 1.298 | 0.217 | 0.000 |

| Problem 1178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 187 | 187 | 388 | 571 | 467 | 632 | 428 | 567 | 841 | 0 |
| N.S. | 1 | 1.00 | 2.07 | 3.05 | 2.50 | 3.38 | 2.29 | 3.03 | 4.50 | 0.00 |
| time (sec) | N/A | 0.235 | 0.123 | 0.011 | 1.403 | 1.580 | 1.445 | 1.281 | 0.241 | 0.000 |

| Problem 1179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 185 | 185 | 389 | 599 | 473 | 703 | 447 | 477 | 690 | 0 |
| N.S. | 1 | 1.00 | 2.10 | 3.24 | 2.56 | 3.80 | 2.42 | 2.58 | 3.73 | 0.00 |
| time (sec) | N/A | 0.217 | 0.132 | 0.014 | 1.538 | 1.202 | 2.949 | 1.265 | 0.266 | 0.000 |

| Problem 1180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 187 | 187 | 199 | 622 | 484 | 739 | 474 | 470 | 559 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 3.33 | 2.59 | 3.95 | 2.53 | 2.51 | 2.99 | 0.00 |
| time (sec) | N/A | 0.213 | 0.109 | 0.014 | 1.628 | 1.509 | 6.124 | 1.318 | 0.289 | 0.000 |

| Problem 1181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 187 | 187 | 173 | 641 | 494 | 754 | 500 | 660 | 512 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 3.43 | 2.64 | 4.03 | 2.67 | 3.53 | 2.74 | 0.00 |
| time (sec) | N/A | 0.198 | 0.112 | 0.015 | 1.734 | 1.575 | 22.438 | 1.287 | 0.772 | 0.000 |

| Problem 1182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 181 | 181 | 389 | 656 | 504 | 732 | 524 | 463 | 508 | 0 |
| N.S. | 1 | 1.00 | 2.15 | 3.62 | 2.78 | 4.04 | 2.90 | 2.56 | 2.81 | 0.00 |
| time (sec) | N/A | 0.191 | 0.153 | 0.014 | 1.817 | 1.399 | 97.193 | 1.381 | 0.339 | 0.000 |

| Problem 1183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 186 | 186 | 390 | 666 | 516 | 692 | 0 | 459 | 517 | 0 |
| N.S. | 1 | 1.00 | 2.10 | 3.58 | 2.77 | 3.72 | 0.00 | 2.47 | 2.78 | 0.00 |
| time (sec) | N/A | 0.172 | 0.204 | 0.013 | 1.799 | 1.513 | 0.000 | 1.298 | 0.369 | 0.000 |

| Problem 1184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 194 | 194 | 308 | 672 | 534 | 624 | 0 | 466 | 461 | 0 |
| N.S. | 1 | 1.00 | 1.59 | 3.46 | 2.75 | 3.22 | 0.00 | 2.40 | 2.38 | 0.00 |
| time (sec) | N/A | 0.156 | 0.163 | 0.010 | 1.648 | 1.416 | 0.000 | 1.281 | 0.353 | 0.000 |

| Problem 1185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 353 | 464 | 509 | 509 | 0 | 489 | 571 | 0 |
| N.S. | 1 | 1.00 | 12.61 | 16.57 | 18.18 | 18.18 | 0.00 | 17.46 | 20.39 | 0.00 |
| time (sec) | N/A | 0.003 | 0.125 | 0.008 | 1.650 | 1.395 | 0.000 | 1.290 | 0.169 | 0.000 |

| Problem 1186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 367 | 464 | 548 | 548 | 0 | 496 | 39 | 0 |
| N.S. | 1 | 1.00 | 6.33 | 8.00 | 9.45 | 9.45 | 0.00 | 8.55 | 0.67 | 0.00 |
| time (sec) | N/A | 0.009 | 0.126 | 0.009 | 1.709 | 1.410 | 0.000 | 1.272 | 0.146 | 0.000 |

| Problem 1187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 371 | 464 | 559 | 559 | 0 | 496 | 600 | 0 |
| N.S. | 1 | 1.00 | 4.17 | 5.21 | 6.28 | 6.28 | 0.00 | 5.57 | 6.74 | 0.00 |
| time (sec) | N/A | 0.022 | 0.123 | 0.008 | 1.731 | 1.496 | 0.000 | 1.311 | 0.446 | 0.000 |

| Problem 1188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 120 | 120 | 369 | 464 | 570 | 570 | 0 | 496 | 548 | 0 |
| N.S. | 1 | 1.00 | 3.08 | 3.87 | 4.75 | 4.75 | 0.00 | 4.13 | 4.57 | 0.00 |
| time (sec) | N/A | 0.033 | 0.122 | 0.005 | 1.812 | 0.711 | 0.000 | 1.303 | 0.518 | 0.000 |

| Problem 1189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 151 | 151 | 371 | 464 | 581 | 581 | 0 | 496 | 559 | 0 |
| N.S. | 1 | 1.00 | 2.46 | 3.07 | 3.85 | 3.85 | 0.00 | 3.28 | 3.70 | 0.00 |
| time (sec) | N/A | 0.047 | 0.129 | 0.008 | 1.757 | 1.417 | 0.000 | 1.264 | 0.229 | 0.000 |

| Problem 1190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 198 | 198 | 369 | 463 | 592 | 592 | 0 | 496 | 570 | 0 |
| N.S. | 1 | 1.00 | 1.86 | 2.34 | 2.99 | 2.99 | 0.00 | 2.51 | 2.88 | 0.00 |
| time (sec) | N/A | 0.153 | 0.127 | 0.009 | 1.745 | 1.229 | 0.000 | 1.237 | 0.399 | 0.000 |

| Problem 1191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 200 | 200 | 371 | 464 | 603 | 603 | 0 | 496 | 581 | 0 |
| N.S. | 1 | 1.00 | 1.86 | 2.32 | 3.02 | 3.02 | 0.00 | 2.48 | 2.90 | 0.00 |
| time (sec) | N/A | 0.143 | 0.127 | 0.008 | 1.835 | 1.353 | 0.000 | 1.285 | 1.238 | 0.000 |

| Problem 1192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 200 | 200 | 371 | 464 | 614 | 614 | 0 | 496 | 592 | 0 |
| N.S. | 1 | 1.00 | 1.86 | 2.32 | 3.07 | 3.07 | 0.00 | 2.48 | 2.96 | 0.00 |
| time (sec) | N/A | 0.140 | 0.128 | 0.006 | 1.911 | 1.099 | 0.000 | 1.288 | 2.196 | 0.001 |

| Problem 1193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 275 | 275 | 1817 | 1891 | 1877 | 2186 | 2088 | 2186 | 1847 | 0 |
| N.S. | 1 | 1.00 | 6.61 | 6.88 | 6.83 | 7.95 | 7.59 | 7.95 | 6.72 | 0.00 |
| time (sec) | N/A | 1.465 | 0.290 | 0.003 | 1.546 | 1.332 | 0.372 | 1.314 | 0.984 | 0.000 |

| Problem 1194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 279 | 279 | 1702 | 1741 | 1740 | 2010 | 1965 | 2010 | 1702 | 0 |
| N.S. | 1 | 1.00 | 6.10 | 6.24 | 6.24 | 7.20 | 7.04 | 7.20 | 6.10 | 0.00 |
| time (sec) | N/A | 1.275 | 0.226 | 0.004 | 1.575 | 1.112 | 0.344 | 1.352 | 1.026 | 0.000 |

| Problem 1195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 279 | 279 | 1539 | 1591 | 1581 | 1833 | 1775 | 1833 | 1549 | 0 |
| N.S. | 1 | 1.00 | 5.52 | 5.70 | 5.67 | 6.57 | 6.36 | 6.57 | 5.55 | 0.00 |
| time (sec) | N/A | 1.112 | 0.173 | 0.002 | 1.555 | 0.946 | 0.313 | 1.343 | 0.689 | 0.000 |

| Problem 1196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 250 | 250 | 1397 | 1441 | 1437 | 1656 | 1598 | 1656 | 1404 | 0 |
| N.S. | 1 | 1.00 | 5.59 | 5.76 | 5.75 | 6.62 | 6.39 | 6.62 | 5.62 | 0.00 |
| time (sec) | N/A | 1.042 | 0.185 | 0.003 | 1.508 | 1.166 | 0.296 | 1.298 | 0.788 | 0.000 |

| Problem 1197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 225 | 225 | 1241 | 1291 | 1283 | 1478 | 1428 | 1478 | 1253 | 0 |
| N.S. | 1 | 1.00 | 5.52 | 5.74 | 5.70 | 6.57 | 6.35 | 6.57 | 5.57 | 0.00 |
| time (sec) | N/A | 0.899 | 0.161 | 0.002 | 1.512 | 1.171 | 0.273 | 1.290 | 0.707 | 0.000 |

| Problem 1198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 200 | 200 | 1105 | 1141 | 1135 | 1302 | 1280 | 1302 | 1106 | 0 |
| N.S. | 1 | 1.00 | 5.52 | 5.70 | 5.68 | 6.51 | 6.40 | 6.51 | 5.53 | 0.00 |
| time (sec) | N/A | 0.766 | 0.136 | 0.001 | 1.520 | 1.082 | 0.246 | 1.260 | 0.613 | 0.000 |

| Problem 1199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 170 | 170 | 939 | 991 | 977 | 1124 | 1088 | 1124 | 953 | 0 |
| N.S. | 1 | 1.00 | 5.52 | 5.83 | 5.75 | 6.61 | 6.40 | 6.61 | 5.61 | 0.00 |
| time (sec) | N/A | 0.673 | 0.124 | 0.001 | 1.439 | 1.065 | 0.227 | 1.296 | 0.532 | 0.000 |

| Problem 1200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 146 | 146 | 811 | 841 | 835 | 948 | 940 | 948 | 806 | 0 |
| N.S. | 1 | 1.00 | 5.55 | 5.76 | 5.72 | 6.49 | 6.44 | 6.49 | 5.52 | 0.00 |
| time (sec) | N/A | 0.529 | 0.088 | 0.003 | 1.465 | 0.776 | 0.209 | 1.316 | 0.341 | 0.000 |

| Problem 1201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 660 | 691 | 686 | 771 | 748 | 771 | 664 | 0 |
| N.S. | 1 | 1.00 | 5.55 | 5.81 | 5.76 | 6.48 | 6.29 | 6.48 | 5.58 | 0.00 |
| time (sec) | N/A | 0.437 | 0.079 | 0.002 | 1.423 | 1.100 | 0.183 | 1.270 | 0.426 | 0.000 |

| Problem 1202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 511 | 541 | 535 | 594 | 586 | 594 | 495 | 0 |
| N.S. | 1 | 1.00 | 5.55 | 5.88 | 5.82 | 6.46 | 6.37 | 6.46 | 5.38 | 0.00 |
| time (sec) | N/A | 0.349 | 0.065 | 0.002 | 1.324 | 1.054 | 0.162 | 1.275 | 0.231 | 0.000 |

| Problem 1203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 358 | 391 | 384 | 417 | 415 | 417 | 348 | 0 |
| N.S. | 1 | 1.00 | 5.51 | 6.02 | 5.91 | 6.42 | 6.38 | 6.42 | 5.35 | 0.00 |
| time (sec) | N/A | 0.251 | 0.046 | 0.001 | 1.332 | 0.944 | 0.145 | 1.263 | 0.320 | 0.000 |

| Problem 1204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 220 | 241 | 240 | 241 | 248 | 241 | 208 | 0 |
| N.S. | 1 | 1.00 | 5.79 | 6.34 | 6.32 | 6.34 | 6.53 | 6.34 | 5.47 | 0.00 |
| time (sec) | N/A | 0.016 | 0.029 | 0.001 | 1.434 | 1.158 | 0.119 | 1.265 | 0.129 | 0.000 |

| Problem 1205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 108 | 114 | 12 | 108 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 7.71 | 8.14 | 0.86 | 7.71 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.001 | 1.371 | 0.985 | 0.088 | 1.278 | 0.080 | 0.000 |

| Problem 1206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 241 | 241 | 591 | 1022 | 866 | 868 | 799 | 961 | 979 | 0 |
| N.S. | 1 | 1.00 | 2.45 | 4.24 | 3.59 | 3.60 | 3.32 | 3.99 | 4.06 | 0.00 |
| time (sec) | N/A | 0.098 | 0.339 | 0.009 | 1.523 | 1.282 | 1.415 | 1.345 | 0.130 | 0.000 |

| Problem 1207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 258 | 258 | 708 | 1066 | 874 | 1124 | 816 | 1012 | 3475 | 0 |
| N.S. | 1 | 1.00 | 2.74 | 4.13 | 3.39 | 4.36 | 3.16 | 3.92 | 13.47 | 0.00 |
| time (sec) | N/A | 0.473 | 0.240 | 0.016 | 1.395 | 1.307 | 2.653 | 1.265 | 0.352 | 0.000 |

| Problem 1208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 262 | 262 | 708 | 1105 | 881 | 1233 | 843 | 924 | 3299 | 0 |
| N.S. | 1 | 1.00 | 2.70 | 4.22 | 3.36 | 4.71 | 3.22 | 3.53 | 12.59 | 0.00 |
| time (sec) | N/A | 0.444 | 0.241 | 0.018 | 1.621 | 1.262 | 5.671 | 1.252 | 0.378 | 0.000 |

| Problem 1209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 258 | 258 | 427 | 1141 | 891 | 1316 | 867 | 907 | 2219 | 0 |
| N.S. | 1 | 1.00 | 1.66 | 4.42 | 3.45 | 5.10 | 3.36 | 3.52 | 8.60 | 0.00 |
| time (sec) | N/A | 0.441 | 0.179 | 0.022 | 1.730 | 1.247 | 32.528 | 1.261 | 0.385 | 0.000 |

| Problem 1210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 262 | 262 | 359 | 1172 | 903 | 1365 | 0 | 1168 | 1494 | 0 |
| N.S. | 1 | 1.00 | 1.37 | 4.47 | 3.45 | 5.21 | 0.00 | 4.46 | 5.70 | 0.00 |
| time (sec) | N/A | 0.422 | 0.196 | 0.020 | 1.932 | 1.255 | 0.000 | 1.379 | 0.381 | 0.000 |

| Problem 1211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 260 | 260 | 305 | 1199 | 912 | 1395 | 0 | 883 | 1141 | 0 |
| N.S. | 1 | 1.00 | 1.17 | 4.61 | 3.51 | 5.37 | 0.00 | 3.40 | 4.39 | 0.00 |
| time (sec) | N/A | 0.421 | 0.214 | 0.023 | 2.253 | 1.270 | 0.000 | 1.339 | 0.396 | 0.000 |

| Problem 1212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 262 | 262 | 265 | 1222 | 925 | 1386 | 0 | 878 | 997 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 4.66 | 3.53 | 5.29 | 0.00 | 3.35 | 3.81 | 0.00 |
| time (sec) | N/A | 0.387 | 0.219 | 0.021 | 2.451 | 1.228 | 0.000 | 1.290 | 0.422 | 0.000 |

| Problem 1213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 258 | 258 | 239 | 1241 | 934 | 1362 | 0 | 872 | 950 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 4.81 | 3.62 | 5.28 | 0.00 | 3.38 | 3.68 | 0.00 |
| time (sec) | N/A | 0.365 | 0.249 | 0.022 | 2.444 | 1.398 | 0.000 | 1.258 | 0.428 | 0.000 |

| Problem 1214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 258 | 258 | 712 | 1256 | 945 | 1296 | 0 | 871 | 946 | 0 |
| N.S. | 1 | 1.00 | 2.76 | 4.87 | 3.66 | 5.02 | 0.00 | 3.38 | 3.67 | 0.00 |
| time (sec) | N/A | 0.341 | 0.317 | 0.019 | 2.584 | 1.387 | 0.000 | 1.292 | 0.263 | 0.000 |

| Problem 1215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 257 | 257 | 708 | 1266 | 957 | 1216 | 0 | 867 | 955 | 0 |
| N.S. | 1 | 1.00 | 2.75 | 4.93 | 3.72 | 4.73 | 0.00 | 3.37 | 3.72 | 0.00 |
| time (sec) | N/A | 0.311 | 0.424 | 0.018 | 2.362 | 1.228 | 0.000 | 1.248 | 0.502 | 0.000 |

| Problem 1216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 271 | 271 | 591 | 1271 | 975 | 1107 | 0 | 874 | 866 | 0 |
| N.S. | 1 | 1.00 | 2.18 | 4.69 | 3.60 | 4.08 | 0.00 | 3.23 | 3.20 | 0.00 |
| time (sec) | N/A | 0.287 | 0.361 | 0.013 | 2.014 | 1.220 | 0.000 | 1.357 | 0.555 | 0.000 |

| Problem 1217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 665 | 866 | 920 | 920 | 0 | 951 | 1066 | 0 |
| N.S. | 1 | 1.00 | 23.75 | 30.93 | 32.86 | 32.86 | 0.00 | 33.96 | 38.07 | 0.00 |
| time (sec) | N/A | 0.003 | 0.285 | 0.007 | 2.128 | 1.349 | 0.000 | 1.355 | 0.458 | 0.000 |

| Problem 1218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 684 | 867 | 986 | 986 | 0 | 961 | 39 | 0 |
| N.S. | 1 | 1.00 | 11.79 | 14.95 | 17.00 | 17.00 | 0.00 | 16.57 | 0.67 | 0.00 |
| time (sec) | N/A | 0.010 | 0.281 | 0.009 | 2.167 | 1.291 | 0.000 | 1.322 | 0.394 | 0.000 |

| Problem 1219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 690 | 867 | 997 | 997 | 0 | 961 | 1098 | 0 |
| N.S. | 1 | 1.00 | 7.75 | 9.74 | 11.20 | 11.20 | 0.00 | 10.80 | 12.34 | 0.00 |
| time (sec) | N/A | 0.020 | 0.294 | 0.009 | 2.208 | 1.258 | 0.000 | 1.283 | 0.475 | 0.000 |

| Problem 1220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 120 | 120 | 692 | 867 | 1008 | 1008 | 0 | 961 | 1109 | 0 |
| N.S. | 1 | 1.00 | 5.77 | 7.22 | 8.40 | 8.40 | 0.00 | 8.01 | 9.24 | 0.00 |
| time (sec) | N/A | 0.030 | 0.287 | 0.009 | 2.160 | 1.316 | 0.000 | 1.393 | 1.295 | 0.000 |

| Problem 1221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 151 | 151 | 690 | 867 | 1019 | 1019 | 0 | 961 | 1120 | 0 |
| N.S. | 1 | 1.00 | 4.57 | 5.74 | 6.75 | 6.75 | 0.00 | 6.36 | 7.42 | 0.00 |
| time (sec) | N/A | 0.044 | 0.288 | 0.009 | 2.257 | 1.123 | 0.000 | 1.319 | 2.279 | 0.001 |

| Problem 1222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 182 | 182 | 694 | 867 | 1030 | 1030 | 0 | 961 | 1131 | 0 |
| N.S. | 1 | 1.00 | 3.81 | 4.76 | 5.66 | 5.66 | 0.00 | 5.28 | 6.21 | 0.00 |
| time (sec) | N/A | 0.063 | 0.280 | 0.009 | 2.266 | 1.261 | 0.000 | 1.311 | 0.584 | 0.000 |

| Problem 1223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 213 | 213 | 690 | 867 | 1041 | 1041 | 0 | 961 | 1142 | 0 |
| N.S. | 1 | 1.00 | 3.24 | 4.07 | 4.89 | 4.89 | 0.00 | 4.51 | 5.36 | 0.00 |
| time (sec) | N/A | 0.079 | 0.323 | 0.010 | 2.293 | 1.314 | 0.000 | 1.322 | 0.658 | 0.000 |

| Problem 1224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 244 | 244 | 694 | 867 | 1052 | 1052 | 0 | 961 | 1153 | 0 |
| N.S. | 1 | 1.00 | 2.84 | 3.55 | 4.31 | 4.31 | 0.00 | 3.94 | 4.73 | 0.00 |
| time (sec) | N/A | 0.105 | 0.279 | 0.009 | 2.545 | 1.191 | 0.000 | 1.403 | 12.020 | 0.001 |

| Problem 1225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 273 | 273 | 692 | 866 | 1063 | 1063 | 0 | 961 | 1164 | 0 |
| N.S. | 1 | 1.00 | 2.53 | 3.17 | 3.89 | 3.89 | 0.00 | 3.52 | 4.26 | 0.00 |
| time (sec) | N/A | 0.284 | 0.282 | 0.009 | 2.454 | 1.303 | 0.000 | 1.310 | 25.721 | 0.001 |

| Problem 1226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 279 | 279 | 692 | 867 | 1074 | 1074 | 0 | 961 | 1175 | 0 |
| N.S. | 1 | 1.00 | 2.48 | 3.11 | 3.85 | 3.85 | 0.00 | 3.44 | 4.21 | 0.00 |
| time (sec) | N/A | 0.272 | 0.288 | 0.009 | 2.509 | 1.233 | 0.000 | 1.308 | 0.799 | 0.000 |

| Problem 1227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 279 | 279 | 692 | 867 | 1085 | 1085 | 0 | 961 | 1186 | 0 |
| N.S. | 1 | 1.00 | 2.48 | 3.11 | 3.89 | 3.89 | 0.00 | 3.44 | 4.25 | 0.00 |
| time (sec) | N/A | 0.271 | 0.301 | 0.008 | 2.457 | 1.286 | 0.000 | 1.301 | 1.036 | 0.001 |

| Problem 1228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 122 | 122 | 167 | 302 | 258 | 259 | 209 | 273 | 280 | 0 |
| N.S. | 1 | 1.00 | 1.37 | 2.48 | 2.11 | 2.12 | 1.71 | 2.24 | 2.30 | 0.00 |
| time (sec) | N/A | 0.053 | 0.072 | 0.005 | 1.350 | 1.090 | 0.500 | 1.260 | 0.074 | 0.000 |

| Problem 1229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 115 | 209 | 177 | 179 | 136 | 184 | 189 | 0 |
| N.S. | 1 | 1.00 | 1.17 | 2.13 | 1.81 | 1.83 | 1.39 | 1.88 | 1.93 | 0.00 |
| time (sec) | N/A | 0.038 | 0.044 | 0.005 | 1.405 | 1.193 | 0.392 | 1.228 | 0.218 | 0.000 |

| Problem 1230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 74 | 133 | 114 | 115 | 83 | 116 | 118 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.80 | 1.54 | 1.55 | 1.12 | 1.57 | 1.59 | 0.00 |
| time (sec) | N/A | 0.030 | 0.028 | 0.004 | 1.301 | 0.667 | 0.299 | 1.200 | 0.065 | 0.000 |

| Problem 1231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 43 | 74 | 60 | 62 | 44 | 60 | 62 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.48 | 1.20 | 1.24 | 0.88 | 1.20 | 1.24 | 0.00 |
| time (sec) | N/A | 0.021 | 0.017 | 0.003 | 1.354 | 1.483 | 0.219 | 1.210 | 0.225 | 0.000 |

| Problem 1232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 25 | 32 | 26 | 25 | 20 | 27 | 25 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 1.23 | 1.00 | 0.96 | 0.77 | 1.04 | 0.96 | 0.00 |
| time (sec) | N/A | 0.018 | 0.008 | 0.003 | 1.319 | 0.929 | 0.148 | 1.200 | 0.201 | 0.000 |

| Problem 1233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 10 | 10 | 11 | 10 | 10 | 7 | 11 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.10 | 1.00 | 1.00 | 0.70 | 1.10 | 1.00 | 0.00 |
| time (sec) | N/A | 0.001 | 0.001 | 0.001 | 1.304 | 0.901 | 0.062 | 1.261 | 0.022 | 0.000 |

| Problem 1234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 26 | 37 | 36 | 26 | 128 | 46 | 25 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 1.03 | 1.00 | 0.72 | 3.56 | 1.28 | 0.69 | 0.00 |
| time (sec) | N/A | 0.008 | 0.013 | 0.006 | 1.360 | 1.331 | 0.328 | 1.243 | 0.258 | 0.000 |

| Problem 1235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 53 | 57 | 92 | 93 | 233 | 78 | 46 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.00 | 1.61 | 1.63 | 4.09 | 1.37 | 0.81 | 0.00 |
| time (sec) | N/A | 0.032 | 0.025 | 0.008 | 1.363 | 1.265 | 0.682 | 1.316 | 0.142 | 0.001 |

| Problem 1236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 67 | 81 | 202 | 242 | 381 | 165 | 182 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.99 | 2.46 | 2.95 | 4.65 | 2.01 | 2.22 | 0.00 |
| time (sec) | N/A | 0.045 | 0.066 | 0.009 | 1.411 | 1.364 | 1.064 | 1.294 | 0.161 | 0.000 |

| Problem 1237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 228 | 326 | 264 | 373 | 231 | 339 | 327 | 0 |
| N.S. | 1 | 1.00 | 1.75 | 2.51 | 2.03 | 2.87 | 1.78 | 2.61 | 2.52 | 0.00 |
| time (sec) | N/A | 0.139 | 0.075 | 0.010 | 1.398 | 1.211 | 0.886 | 1.267 | 0.245 | 0.000 |

| Problem 1238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 165 | 230 | 183 | 267 | 155 | 245 | 203 | 0 |
| N.S. | 1 | 1.00 | 1.59 | 2.21 | 1.76 | 2.57 | 1.49 | 2.36 | 1.95 | 0.00 |
| time (sec) | N/A | 0.100 | 0.057 | 0.009 | 1.357 | 0.903 | 0.680 | 1.265 | 0.073 | 0.000 |

| Problem 1239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 75 | 75 | 114 | 149 | 117 | 172 | 102 | 166 | 123 | 0 |
| N.S. | 1 | 1.00 | 1.52 | 1.99 | 1.56 | 2.29 | 1.36 | 2.21 | 1.64 | 0.00 |
| time (sec) | N/A | 0.062 | 0.036 | 0.009 | 1.357 | 1.816 | 0.506 | 1.264 | 0.077 | 0.000 |

| Problem 1240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 47 | 86 | 67 | 92 | 60 | 98 | 71 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.69 | 1.31 | 1.80 | 1.18 | 1.92 | 1.39 | 0.00 |
| time (sec) | N/A | 0.039 | 0.037 | 0.007 | 1.351 | 1.158 | 0.338 | 1.246 | 0.236 | 0.000 |

| Problem 1241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 39 | 34 | 37 | 27 | 57 | 32 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.26 | 1.10 | 1.19 | 0.87 | 1.84 | 1.03 | 0.00 |
| time (sec) | N/A | 0.021 | 0.011 | 0.006 | 1.337 | 1.310 | 0.185 | 1.262 | 0.041 | 0.000 |

| Problem 1242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 12 | 13 | 12 | 13 | 10 | 12 | 12 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.08 | 0.83 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.001 | 1.285 | 1.150 | 0.128 | 1.229 | 0.187 | 0.000 |

| Problem 1243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 53 | 58 | 90 | 92 | 233 | 77 | 47 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 1.04 | 1.61 | 1.64 | 4.16 | 1.38 | 0.84 | 0.00 |
| time (sec) | N/A | 0.032 | 0.029 | 0.008 | 1.321 | 1.207 | 0.684 | 1.342 | 0.292 | 0.000 |

| Problem 1244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 66 | 82 | 208 | 241 | 406 | 153 | 74 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 1.01 | 2.57 | 2.98 | 5.01 | 1.89 | 0.91 | 0.00 |
| time (sec) | N/A | 0.050 | 0.071 | 0.008 | 1.429 | 1.238 | 1.113 | 1.207 | 0.334 | 0.000 |

| Problem 1245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 98 | 109 | 386 | 494 | 634 | 216 | 330 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 1.00 | 3.54 | 4.53 | 5.82 | 1.98 | 3.03 | 0.00 |
| time (sec) | N/A | 0.076 | 0.074 | 0.013 | 1.555 | 1.234 | 1.717 | 1.347 | 0.396 | 0.000 |

| Problem 1246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 158 | 158 | 303 | 464 | 364 | 548 | 340 | 362 | 441 | 0 |
| N.S. | 1 | 1.00 | 1.92 | 2.94 | 2.30 | 3.47 | 2.15 | 2.29 | 2.79 | 0.00 |
| time (sec) | N/A | 0.202 | 0.113 | 0.011 | 1.474 | 1.259 | 2.154 | 1.276 | 0.274 | 0.000 |

| Problem 1247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 133 | 133 | 230 | 346 | 271 | 416 | 258 | 264 | 291 | 0 |
| N.S. | 1 | 1.00 | 1.73 | 2.60 | 2.04 | 3.13 | 1.94 | 1.98 | 2.19 | 0.00 |
| time (sec) | N/A | 0.121 | 0.073 | 0.009 | 1.476 | 1.205 | 1.650 | 1.288 | 0.099 | 0.000 |

| Problem 1248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 167 | 245 | 191 | 291 | 185 | 183 | 196 | 0 |
| N.S. | 1 | 1.00 | 1.62 | 2.38 | 1.85 | 2.83 | 1.80 | 1.78 | 1.90 | 0.00 |
| time (sec) | N/A | 0.087 | 0.056 | 0.008 | 1.386 | 0.771 | 1.246 | 1.354 | 0.099 | 0.000 |

| Problem 1249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 78 | 78 | 114 | 160 | 125 | 188 | 128 | 112 | 130 | 0 |
| N.S. | 1 | 1.00 | 1.46 | 2.05 | 1.60 | 2.41 | 1.64 | 1.44 | 1.67 | 0.00 |
| time (sec) | N/A | 0.055 | 0.039 | 0.009 | 1.340 | 1.403 | 0.827 | 1.276 | 0.109 | 0.000 |

| Problem 1250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 48 | 92 | 80 | 100 | 80 | 69 | 77 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 1.56 | 1.36 | 1.69 | 1.36 | 1.17 | 1.31 | 0.00 |
| time (sec) | N/A | 0.038 | 0.025 | 0.006 | 1.327 | 1.437 | 0.450 | 1.299 | 0.228 | 0.000 |

| Problem 1251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 26 | 35 | 38 | 38 | 39 | 24 | 39 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.25 | 1.36 | 1.36 | 1.39 | 0.86 | 1.39 | 0.00 |
| time (sec) | N/A | 0.003 | 0.009 | 0.004 | 1.359 | 0.987 | 0.263 | 1.251 | 0.029 | 0.000 |

| Problem 1252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 24 | 26 | 12 | 26 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 1.71 | 1.86 | 0.86 | 1.86 | 0.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.000 | 1.335 | 1.238 | 0.182 | 1.203 | 0.024 | 0.000 |

| Problem 1253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 67 | 81 | 202 | 242 | 381 | 165 | 183 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.99 | 2.46 | 2.95 | 4.65 | 2.01 | 2.23 | 0.00 |
| time (sec) | N/A | 0.046 | 0.053 | 0.010 | 1.452 | 1.197 | 1.074 | 1.360 | 0.299 | 0.000 |

| Problem 1254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 97 | 108 | 386 | 495 | 632 | 217 | 329 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.98 | 3.51 | 4.50 | 5.75 | 1.97 | 2.99 | 0.00 |
| time (sec) | N/A | 0.074 | 0.105 | 0.013 | 1.594 | 1.611 | 1.720 | 1.265 | 0.395 | 0.000 |

| Problem 1255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 143 | 143 | 128 | 140 | 594 | 760 | 881 | 345 | 542 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.98 | 4.15 | 5.31 | 6.16 | 2.41 | 3.79 | 0.00 |
| time (sec) | N/A | 0.102 | 0.116 | 0.012 | 1.551 | 1.182 | 2.419 | 1.277 | 0.526 | 0.001 |

| Problem 1256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 232 | 232 | 584 | 1035 | 786 | 1093 | 0 | 723 | 784 | 0 |
| N.S. | 1 | 1.00 | 2.52 | 4.46 | 3.39 | 4.71 | 0.00 | 3.12 | 3.38 | 0.00 |
| time (sec) | N/A | 0.357 | 0.271 | 0.017 | 2.198 | 0.956 | 0.000 | 1.321 | 0.257 | 0.000 |

| Problem 1257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 209 | 209 | 474 | 845 | 649 | 852 | 0 | 581 | 649 | 0 |
| N.S. | 1 | 1.00 | 2.27 | 4.04 | 3.11 | 4.08 | 0.00 | 2.78 | 3.11 | 0.00 |
| time (sec) | N/A | 0.278 | 0.204 | 0.013 | 1.959 | 0.972 | 0.000 | 1.268 | 0.430 | 0.000 |

| Problem 1258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 194 | 194 | 308 | 672 | 535 | 625 | 0 | 467 | 460 | 0 |
| N.S. | 1 | 1.00 | 1.59 | 3.46 | 2.76 | 3.22 | 0.00 | 2.41 | 2.37 | 0.00 |
| time (sec) | N/A | 0.209 | 0.162 | 0.009 | 1.662 | 1.120 | 0.000 | 1.298 | 0.383 | 0.000 |

| Problem 1259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 271 | 357 | 398 | 398 | 0 | 369 | 378 | 0 |
| N.S. | 1 | 1.00 | 9.68 | 12.75 | 14.21 | 14.21 | 0.00 | 13.18 | 13.50 | 0.00 |
| time (sec) | N/A | 0.003 | 0.090 | 0.007 | 1.606 | 1.380 | 0.000 | 1.282 | 0.146 | 0.000 |

| Problem 1260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 205 | 265 | 326 | 326 | 354 | 271 | 39 | 0 |
| N.S. | 1 | 1.00 | 3.53 | 4.57 | 5.62 | 5.62 | 6.10 | 4.67 | 0.67 | 0.00 |
| time (sec) | N/A | 0.011 | 0.060 | 0.008 | 1.579 | 1.312 | 54.908 | 1.317 | 0.277 | 0.000 |

| Problem 1261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 144 | 186 | 247 | 247 | 267 | 184 | 237 | 0 |
| N.S. | 1 | 1.00 | 1.62 | 2.09 | 2.78 | 2.78 | 3.00 | 2.07 | 2.66 | 0.00 |
| time (sec) | N/A | 0.019 | 0.048 | 0.006 | 1.517 | 1.064 | 9.645 | 1.294 | 0.110 | 0.000 |

| Problem 1262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 94 | 122 | 182 | 182 | 196 | 114 | 176 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 1.33 | 1.98 | 1.98 | 2.13 | 1.24 | 1.91 | 0.00 |
| time (sec) | N/A | 0.057 | 0.029 | 0.007 | 1.469 | 0.957 | 3.101 | 1.240 | 0.099 | 0.000 |

| Problem 1263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 55 | 71 | 131 | 131 | 139 | 61 | 129 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.09 | 2.02 | 2.02 | 2.14 | 0.94 | 1.98 | 0.00 |
| time (sec) | N/A | 0.040 | 0.024 | 0.005 | 1.456 | 1.256 | 1.391 | 1.222 | 0.086 | 0.000 |

| Problem 1264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 27 | 35 | 94 | 94 | 100 | 25 | 96 | 0 |
| N.S. | 1 | 1.00 | 0.71 | 0.92 | 2.47 | 2.47 | 2.63 | 0.66 | 2.53 | 0.00 |
| time (sec) | N/A | 0.022 | 0.010 | 0.006 | 1.380 | 1.203 | 0.731 | 1.309 | 0.226 | 0.000 |

| Problem 1265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 79 | 85 | 12 | 81 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 5.64 | 6.07 | 0.86 | 5.79 | 0.00 |
| time (sec) | N/A | 0.002 | 0.003 | 0.000 | 1.365 | 1.172 | 0.456 | 1.257 | 0.223 | 0.000 |

| Problem 1266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 202 | 202 | 196 | 192 | 1418 | 1589 | 1776 | 703 | 1299 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.95 | 7.02 | 7.87 | 8.79 | 3.48 | 6.43 | 0.00 |
| time (sec) | N/A | 0.169 | 0.097 | 0.016 | 2.975 | 1.545 | 4.488 | 1.333 | 0.873 | 0.000 |

| Problem 1267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 231 | 231 | 213 | 223 | 1881 | 2264 | 2336 | 714 | 1738 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.97 | 8.14 | 9.80 | 10.11 | 3.09 | 7.52 | 0.00 |
| time (sec) | N/A | 0.270 | 0.243 | 0.020 | 3.883 | 1.461 | 7.745 | 1.370 | 1.386 | 0.000 |

| Problem 1268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 276 | 276 | 254 | 265 | 2399 | 3016 | 2917 | 1029 | 2224 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.96 | 8.69 | 10.93 | 10.57 | 3.73 | 8.06 | 0.00 |
| time (sec) | N/A | 0.356 | 0.205 | 0.020 | 5.223 | 1.588 | 20.658 | 1.493 | 1.911 | 0.000 |

| Problem 1269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 156 | 156 | 123 | 273 | 259 | 338 | 314 | 641 | 137 | 315 |
| N.S. | 1 | 1.00 | 0.79 | 1.75 | 1.66 | 2.17 | 2.01 | 4.11 | 0.88 | 2.02 |
| time (sec) | N/A | 0.063 | 0.145 | 0.008 | 1.425 | 1.215 | 5.117 | 1.378 | 0.081 | 0.096 |

| Problem 1270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 129 | 129 | 101 | 186 | 181 | 245 | 223 | 470 | 112 | 213 |
| N.S. | 1 | 1.00 | 0.78 | 1.44 | 1.40 | 1.90 | 1.73 | 3.64 | 0.87 | 1.65 |
| time (sec) | N/A | 0.052 | 0.095 | 0.007 | 1.365 | 1.692 | 4.195 | 1.249 | 0.225 | 0.069 |

| Problem 1271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 79 | 116 | 118 | 164 | 146 | 322 | 87 | 132 |
| N.S. | 1 | 1.00 | 0.79 | 1.16 | 1.18 | 1.64 | 1.46 | 3.22 | 0.87 | 1.32 |
| time (sec) | N/A | 0.036 | 0.063 | 0.005 | 1.374 | 1.164 | 3.336 | 1.275 | 0.065 | 0.052 |

| Problem 1272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 61 | 63 | 68 | 99 | 85 | 200 | 68 | 72 |
| N.S. | 1 | 1.00 | 0.86 | 0.89 | 0.96 | 1.39 | 1.20 | 2.82 | 0.96 | 1.01 |
| time (sec) | N/A | 0.025 | 0.038 | 0.007 | 1.350 | 1.394 | 2.695 | 1.291 | 0.240 | 0.038 |

| Problem 1273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 30 | 27 | 33 | 46 | 36 | 100 | 29 | 33 |
| N.S. | 1 | 1.00 | 0.71 | 0.64 | 0.79 | 1.10 | 0.86 | 2.38 | 0.69 | 0.79 |
| time (sec) | N/A | 0.014 | 0.019 | 0.004 | 1.294 | 1.432 | 2.123 | 1.372 | 0.043 | 0.021 |

| Problem 1274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 12 | 12 | 12 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 0.75 | 0.75 | 0.75 | 0.75 | 1.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.002 | 1.350 | 1.278 | 0.060 | 1.342 | 0.021 | 0.006 |

| Problem 1275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 62 | 92 | 0 | 143 | 61 | 62 | 50 | 72 |
| N.S. | 1 | 1.00 | 1.00 | 1.48 | 0.00 | 2.31 | 0.98 | 1.00 | 0.81 | 1.16 |
| time (sec) | N/A | 0.053 | 0.038 | 0.011 | 0.000 | 1.362 | 4.389 | 1.265 | 0.066 | 0.069 |

| Problem 1276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 69 | 64 | 0 | 232 | 573 | 72 | 61 | 91 |
| N.S. | 1 | 1.00 | 0.99 | 0.91 | 0.00 | 3.31 | 8.19 | 1.03 | 0.87 | 1.30 |
| time (sec) | N/A | 0.030 | 0.081 | 0.013 | 0.000 | 1.435 | 58.580 | 1.380 | 0.242 | 0.215 |

| Problem 1277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 52 | 111 | 0 | 456 | 0 | 126 | 135 | 125 |
| N.S. | 1 | 1.00 | 0.47 | 1.01 | 0.00 | 4.15 | 0.00 | 1.15 | 1.23 | 1.14 |
| time (sec) | N/A | 0.078 | 0.016 | 0.013 | 0.000 | 1.059 | 0.000 | 1.331 | 0.302 | 0.423 |

| Problem 1278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 146 | 146 | 52 | 170 | 0 | 785 | 0 | 207 | 207 | 176 |
| N.S. | 1 | 1.00 | 0.36 | 1.16 | 0.00 | 5.38 | 0.00 | 1.42 | 1.42 | 1.21 |
| time (sec) | N/A | 0.098 | 0.014 | 0.015 | 0.000 | 1.092 | 0.000 | 1.350 | 0.374 | 0.753 |

| Problem 1279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 182 | 182 | 52 | 248 | 0 | 1176 | 0 | 311 | 297 | 226 |
| N.S. | 1 | 1.00 | 0.29 | 1.36 | 0.00 | 6.46 | 0.00 | 1.71 | 1.63 | 1.24 |
| time (sec) | N/A | 0.123 | 0.015 | 0.016 | 0.000 | 0.886 | 0.000 | 1.386 | 0.217 | 1.067 |

| Problem 1280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 218 | 218 | 52 | 337 | 0 | 1673 | 0 | 432 | 401 | 317 |
| N.S. | 1 | 1.00 | 0.24 | 1.55 | 0.00 | 7.67 | 0.00 | 1.98 | 1.84 | 1.45 |
| time (sec) | N/A | 0.151 | 0.016 | 0.018 | 0.000 | 1.345 | 0.000 | 1.473 | 0.490 | 1.515 |

| Problem 1281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 158 | 158 | 123 | 273 | 259 | 418 | 763 | 1084 | 137 | 315 |
| N.S. | 1 | 1.00 | 0.78 | 1.73 | 1.64 | 2.65 | 4.83 | 6.86 | 0.87 | 1.99 |
| time (sec) | N/A | 0.053 | 0.146 | 0.007 | 1.346 | 1.366 | 26.419 | 1.532 | 0.244 | 0.099 |

| Problem 1282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 129 | 129 | 101 | 186 | 181 | 311 | 559 | 807 | 112 | 213 |
| N.S. | 1 | 1.00 | 0.78 | 1.44 | 1.40 | 2.41 | 4.33 | 6.26 | 0.87 | 1.65 |
| time (sec) | N/A | 0.042 | 0.097 | 0.007 | 1.355 | 1.071 | 20.089 | 1.403 | 0.242 | 0.075 |

| Problem 1283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 79 | 116 | 118 | 216 | 386 | 566 | 87 | 132 |
| N.S. | 1 | 1.00 | 0.79 | 1.16 | 1.18 | 2.16 | 3.86 | 5.66 | 0.87 | 1.32 |
| time (sec) | N/A | 0.034 | 0.069 | 0.006 | 1.359 | 0.777 | 14.380 | 1.311 | 0.249 | 0.058 |

| Problem 1284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 61 | 63 | 68 | 137 | 240 | 360 | 68 | 72 |
| N.S. | 1 | 1.00 | 0.86 | 0.89 | 0.96 | 1.93 | 3.38 | 5.07 | 0.96 | 1.01 |
| time (sec) | N/A | 0.024 | 0.043 | 0.006 | 1.378 | 1.239 | 9.606 | 1.413 | 0.059 | 0.039 |

| Problem 1285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 30 | 27 | 33 | 69 | 146 | 192 | 29 | 33 |
| N.S. | 1 | 1.00 | 0.71 | 0.64 | 0.79 | 1.64 | 3.48 | 4.57 | 0.69 | 0.79 |
| time (sec) | N/A | 0.014 | 0.021 | 0.003 | 1.374 | 1.270 | 0.666 | 1.227 | 0.215 | 0.024 |

| Problem 1286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 28 | 12 | 58 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 1.75 | 0.75 | 3.62 | 0.75 | 1.00 |
| time (sec) | N/A | 0.001 | 0.005 | 0.002 | 1.359 | 1.196 | 0.061 | 1.244 | 0.017 | 0.007 |

| Problem 1287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 77 | 167 | 0 | 188 | 82 | 105 | 93 | 90 |
| N.S. | 1 | 1.00 | 0.90 | 1.94 | 0.00 | 2.19 | 0.95 | 1.22 | 1.08 | 1.05 |
| time (sec) | N/A | 0.046 | 0.073 | 0.009 | 0.000 | 1.144 | 14.700 | 1.287 | 0.075 | 0.124 |

| Problem 1288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 50 | 148 | 0 | 210 | 0 | 113 | 109 | 107 |
| N.S. | 1 | 1.00 | 0.59 | 1.74 | 0.00 | 2.47 | 0.00 | 1.33 | 1.28 | 1.26 |
| time (sec) | N/A | 0.038 | 0.014 | 0.013 | 0.000 | 1.736 | 0.000 | 1.299 | 0.105 | 0.257 |

| Problem 1289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 90 | 121 | 0 | 383 | 0 | 108 | 135 | 116 |
| N.S. | 1 | 1.00 | 0.90 | 1.21 | 0.00 | 3.83 | 0.00 | 1.08 | 1.35 | 1.16 |
| time (sec) | N/A | 0.048 | 0.100 | 0.013 | 0.000 | 1.005 | 0.000 | 1.357 | 0.284 | 0.379 |

| Problem 1290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 52 | 163 | 0 | 666 | 0 | 185 | 209 | 166 |
| N.S. | 1 | 1.00 | 0.38 | 1.20 | 0.00 | 4.90 | 0.00 | 1.36 | 1.54 | 1.22 |
| time (sec) | N/A | 0.057 | 0.017 | 0.015 | 0.000 | 1.375 | 0.000 | 1.400 | 0.338 | 0.620 |

| Problem 1291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 172 | 172 | 52 | 222 | 0 | 1043 | 0 | 285 | 296 | 226 |
| N.S. | 1 | 1.00 | 0.30 | 1.29 | 0.00 | 6.06 | 0.00 | 1.66 | 1.72 | 1.31 |
| time (sec) | N/A | 0.074 | 0.016 | 0.017 | 0.000 | 1.545 | 0.000 | 1.467 | 0.371 | 1.008 |

| Problem 1292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 208 | 208 | 52 | 300 | 0 | 1492 | 0 | 410 | 398 | 317 |
| N.S. | 1 | 1.00 | 0.25 | 1.44 | 0.00 | 7.17 | 0.00 | 1.97 | 1.91 | 1.52 |
| time (sec) | N/A | 0.093 | 0.017 | 0.017 | 0.000 | 1.308 | 0.000 | 1.392 | 0.473 | 1.984 |

| Problem 1293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 158 | 158 | 123 | 273 | 259 | 497 | 1292 | 1599 | 137 | 315 |
| N.S. | 1 | 1.00 | 0.78 | 1.73 | 1.64 | 3.15 | 8.18 | 10.12 | 0.87 | 1.99 |
| time (sec) | N/A | 0.051 | 0.114 | 0.005 | 1.362 | 1.287 | 43.079 | 1.522 | 0.267 | 0.107 |

| Problem 1294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 129 | 129 | 101 | 186 | 181 | 377 | 960 | 1204 | 112 | 213 |
| N.S. | 1 | 1.00 | 0.78 | 1.44 | 1.40 | 2.92 | 7.44 | 9.33 | 0.87 | 1.65 |
| time (sec) | N/A | 0.043 | 0.110 | 0.007 | 1.332 | 1.379 | 33.636 | 1.447 | 0.234 | 0.078 |

| Problem 1295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 79 | 116 | 118 | 268 | 549 | 857 | 87 | 132 |
| N.S. | 1 | 1.00 | 0.79 | 1.16 | 1.18 | 2.68 | 5.49 | 8.57 | 0.87 | 1.32 |
| time (sec) | N/A | 0.031 | 0.072 | 0.007 | 1.402 | 1.541 | 4.609 | 1.579 | 0.076 | 0.060 |

| Problem 1296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 61 | 63 | 68 | 174 | 355 | 558 | 68 | 72 |
| N.S. | 1 | 1.00 | 0.86 | 0.89 | 0.96 | 2.45 | 5.00 | 7.86 | 0.96 | 1.01 |
| time (sec) | N/A | 0.023 | 0.046 | 0.007 | 1.381 | 1.231 | 3.579 | 1.764 | 0.067 | 0.044 |

| Problem 1297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 30 | 27 | 33 | 93 | 194 | 306 | 29 | 33 |
| N.S. | 1 | 1.00 | 0.71 | 0.64 | 0.79 | 2.21 | 4.62 | 7.29 | 0.69 | 0.79 |
| time (sec) | N/A | 0.015 | 0.024 | 0.003 | 1.395 | 1.215 | 2.363 | 1.615 | 0.045 | 0.027 |

| Problem 1298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 39 | 12 | 95 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 2.44 | 0.75 | 5.94 | 0.75 | 1.00 |
| time (sec) | N/A | 0.001 | 0.006 | 0.002 | 1.296 | 1.215 | 0.065 | 1.781 | 0.021 | 0.007 |

| Problem 1299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 105 | 263 | 0 | 290 | 121 | 171 | 130 | 130 |
| N.S. | 1 | 1.00 | 0.94 | 2.35 | 0.00 | 2.59 | 1.08 | 1.53 | 1.16 | 1.16 |
| time (sec) | N/A | 0.058 | 0.155 | 0.008 | 0.000 | 1.247 | 27.012 | 1.598 | 0.083 | 0.118 |

| Problem 1300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 50 | 258 | 0 | 330 | 0 | 181 | 161 | 187 |
| N.S. | 1 | 1.00 | 0.45 | 2.35 | 0.00 | 3.00 | 0.00 | 1.65 | 1.46 | 1.70 |
| time (sec) | N/A | 0.055 | 0.015 | 0.014 | 0.000 | 1.252 | 0.000 | 1.277 | 0.124 | 0.367 |

| Problem 1301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 52 | 238 | 0 | 344 | 0 | 171 | 199 | 155 |
| N.S. | 1 | 1.00 | 0.44 | 2.00 | 0.00 | 2.89 | 0.00 | 1.44 | 1.67 | 1.30 |
| time (sec) | N/A | 0.049 | 0.018 | 0.016 | 0.000 | 1.565 | 0.000 | 1.242 | 0.160 | 0.479 |

| Problem 1302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 126 | 126 | 119 | 204 | 0 | 563 | 0 | 161 | 222 | 155 |
| N.S. | 1 | 1.00 | 0.94 | 1.62 | 0.00 | 4.47 | 0.00 | 1.28 | 1.76 | 1.23 |
| time (sec) | N/A | 0.050 | 0.152 | 0.015 | 0.000 | 1.197 | 0.000 | 0.985 | 0.365 | 0.619 |

| Problem 1303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 162 | 162 | 52 | 246 | 0 | 894 | 0 | 259 | 309 | 226 |
| N.S. | 1 | 1.00 | 0.32 | 1.52 | 0.00 | 5.52 | 0.00 | 1.60 | 1.91 | 1.40 |
| time (sec) | N/A | 0.070 | 0.018 | 0.015 | 0.000 | 1.419 | 0.000 | 1.091 | 0.412 | 1.076 |

| Problem 1304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 198 | 198 | 52 | 305 | 0 | 1337 | 0 | 380 | 411 | 307 |
| N.S. | 1 | 1.00 | 0.26 | 1.54 | 0.00 | 6.75 | 0.00 | 1.92 | 2.08 | 1.55 |
| time (sec) | N/A | 0.092 | 0.018 | 0.018 | 0.000 | 1.491 | 0.000 | 1.251 | 0.503 | 1.429 |

| Problem 1305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 51 | 30 | 29 | 33 | 104 | 29 | 29 | 35 |
| N.S. | 1 | 1.00 | 1.46 | 0.86 | 0.83 | 0.94 | 2.97 | 0.83 | 0.83 | 1.00 |
| time (sec) | N/A | 0.008 | 0.030 | 0.010 | 2.970 | 1.372 | 1.498 | 0.962 | 0.062 | 0.046 |

| Problem 1306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 56 | 56 | 28 | 40 | 43 | 46 | 167 | 37 | 45 | 43 |
| N.S. | 1 | 1.00 | 0.50 | 0.71 | 0.77 | 0.82 | 2.98 | 0.66 | 0.80 | 0.77 |
| time (sec) | N/A | 0.013 | 0.005 | 0.010 | 3.033 | 1.122 | 2.609 | 1.036 | 0.041 | 0.061 |

| Problem 1307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 154 | 154 | 123 | 273 | 283 | 261 | 728 | 283 | 137 | 315 |
| N.S. | 1 | 1.00 | 0.80 | 1.77 | 1.84 | 1.69 | 4.73 | 1.84 | 0.89 | 2.05 |
| time (sec) | N/A | 0.051 | 0.088 | 0.007 | 1.383 | 1.263 | 79.908 | 1.066 | 0.069 | 0.099 |

| Problem 1308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 127 | 127 | 101 | 186 | 204 | 182 | 532 | 204 | 112 | 213 |
| N.S. | 1 | 1.00 | 0.80 | 1.46 | 1.61 | 1.43 | 4.19 | 1.61 | 0.88 | 1.68 |
| time (sec) | N/A | 0.041 | 0.088 | 0.006 | 1.381 | 1.180 | 56.898 | 0.959 | 0.241 | 0.068 |

| Problem 1309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 96 | 79 | 116 | 137 | 115 | 366 | 137 | 87 | 132 |
| N.S. | 1 | 1.00 | 0.82 | 1.21 | 1.43 | 1.20 | 3.81 | 1.43 | 0.91 | 1.38 |
| time (sec) | N/A | 0.031 | 0.056 | 0.006 | 1.378 | 0.936 | 37.064 | 1.010 | 0.264 | 0.052 |

| Problem 1310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 60 | 63 | 82 | 64 | 231 | 82 | 68 | 72 |
| N.S. | 1 | 1.00 | 0.87 | 0.91 | 1.19 | 0.93 | 3.35 | 1.19 | 0.99 | 1.04 |
| time (sec) | N/A | 0.021 | 0.035 | 0.005 | 1.369 | 1.461 | 20.940 | 1.099 | 0.067 | 0.038 |

| Problem 1311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 29 | 26 | 39 | 25 | 121 | 39 | 28 | 32 |
| N.S. | 1 | 1.00 | 0.72 | 0.65 | 0.98 | 0.62 | 3.02 | 0.98 | 0.70 | 0.80 |
| time (sec) | N/A | 0.013 | 0.017 | 0.003 | 1.345 | 1.060 | 4.778 | 0.884 | 0.048 | 0.025 |

| Problem 1312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 12 | 10 | 12 | 12 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 0.86 | 0.71 | 0.86 | 0.86 | 1.00 |
| time (sec) | N/A | 0.001 | 0.003 | 0.001 | 1.298 | 1.224 | 0.063 | 0.909 | 0.021 | 0.006 |

| Problem 1313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 47 | 37 | 0 | 119 | 44 | 38 | 38 | 57 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 0.00 | 2.53 | 0.94 | 0.81 | 0.81 | 1.21 |
| time (sec) | N/A | 0.020 | 0.015 | 0.006 | 0.000 | 1.204 | 5.409 | 0.879 | 0.273 | 0.047 |

| Problem 1314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 76 | 77 | 0 | 280 | 0 | 87 | 74 | 98 |
| N.S. | 1 | 1.00 | 1.00 | 1.01 | 0.00 | 3.68 | 0.00 | 1.14 | 0.97 | 1.29 |
| time (sec) | N/A | 0.027 | 0.071 | 0.010 | 0.000 | 1.294 | 0.000 | 1.030 | 0.094 | 0.204 |

| Problem 1315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 50 | 115 | 0 | 549 | 0 | 148 | 142 | 124 |
| N.S. | 1 | 1.00 | 0.44 | 1.01 | 0.00 | 4.82 | 0.00 | 1.30 | 1.25 | 1.09 |
| time (sec) | N/A | 0.037 | 0.011 | 0.009 | 0.000 | 1.574 | 0.000 | 0.933 | 0.332 | 0.226 |

| Problem 1316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 147 | 147 | 50 | 147 | 0 | 884 | 0 | 231 | 218 | 173 |
| N.S. | 1 | 1.00 | 0.34 | 1.00 | 0.00 | 6.01 | 0.00 | 1.57 | 1.48 | 1.18 |
| time (sec) | N/A | 0.050 | 0.011 | 0.008 | 0.000 | 1.383 | 0.000 | 1.020 | 0.395 | 0.273 |

| Problem 1317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 180 | 180 | 50 | 179 | 0 | 1325 | 0 | 331 | 307 | 223 |
| N.S. | 1 | 1.00 | 0.28 | 0.99 | 0.00 | 7.36 | 0.00 | 1.84 | 1.71 | 1.24 |
| time (sec) | N/A | 0.065 | 0.012 | 0.010 | 0.000 | 1.171 | 0.000 | 1.134 | 0.455 | 0.440 |

| Problem 1318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 152 | 152 | 123 | 273 | 267 | 271 | 243 | 350 | 192 | 315 |
| N.S. | 1 | 1.00 | 0.81 | 1.80 | 1.76 | 1.78 | 1.60 | 2.30 | 1.26 | 2.07 |
| time (sec) | N/A | 0.049 | 0.120 | 0.006 | 1.556 | 1.092 | 47.937 | 1.008 | 0.077 | 0.071 |

| Problem 1319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 101 | 186 | 189 | 192 | 168 | 240 | 153 | 213 |
| N.S. | 1 | 1.00 | 0.82 | 1.51 | 1.54 | 1.56 | 1.37 | 1.95 | 1.24 | 1.73 |
| time (sec) | N/A | 0.037 | 0.080 | 0.007 | 1.353 | 1.544 | 32.865 | 1.082 | 0.056 | 0.069 |

| Problem 1320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 94 | 94 | 78 | 116 | 125 | 124 | 109 | 152 | 114 | 131 |
| N.S. | 1 | 1.00 | 0.83 | 1.23 | 1.33 | 1.32 | 1.16 | 1.62 | 1.21 | 1.39 |
| time (sec) | N/A | 0.030 | 0.056 | 0.006 | 1.381 | 1.261 | 21.510 | 1.049 | 0.082 | 0.054 |

| Problem 1321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 59 | 63 | 75 | 73 | 65 | 84 | 67 | 71 |
| N.S. | 1 | 1.00 | 0.88 | 0.94 | 1.12 | 1.09 | 0.97 | 1.25 | 1.00 | 1.06 |
| time (sec) | N/A | 0.021 | 0.031 | 0.005 | 1.300 | 1.134 | 13.292 | 1.041 | 0.264 | 0.041 |

| Problem 1322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 27 | 26 | 37 | 35 | 60 | 34 | 25 | 29 |
| N.S. | 1 | 1.00 | 0.71 | 0.68 | 0.97 | 0.92 | 1.58 | 0.89 | 0.66 | 0.76 |
| time (sec) | N/A | 0.014 | 0.019 | 0.003 | 1.332 | 1.129 | 0.614 | 1.027 | 0.054 | 0.026 |

| Problem 1323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 20 | 12 | 12 | 12 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 1.43 | 0.86 | 0.86 | 0.86 | 1.00 |
| time (sec) | N/A | 0.002 | 0.004 | 0.003 | 1.331 | 1.205 | 0.064 | 1.020 | 0.024 | 0.009 |

| Problem 1324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 46 | 68 | 0 | 214 | 60 | 69 | 57 | 79 |
| N.S. | 1 | 1.00 | 0.67 | 0.99 | 0.00 | 3.10 | 0.87 | 1.00 | 0.83 | 1.14 |
| time (sec) | N/A | 0.027 | 0.010 | 0.010 | 0.000 | 1.428 | 11.487 | 0.935 | 0.270 | 0.087 |

| Problem 1325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 99 | 99 | 48 | 101 | 0 | 423 | 0 | 143 | 123 | 115 |
| N.S. | 1 | 1.00 | 0.48 | 1.02 | 0.00 | 4.27 | 0.00 | 1.44 | 1.24 | 1.16 |
| time (sec) | N/A | 0.038 | 0.013 | 0.014 | 0.000 | 1.318 | 0.000 | 1.025 | 0.186 | 0.288 |

| Problem 1326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 140 | 50 | 179 | 0 | 782 | 0 | 234 | 205 | 163 |
| N.S. | 1 | 1.00 | 0.36 | 1.28 | 0.00 | 5.59 | 0.00 | 1.67 | 1.46 | 1.16 |
| time (sec) | N/A | 0.052 | 0.013 | 0.018 | 0.000 | 1.278 | 0.000 | 1.030 | 0.444 | 0.495 |

| Problem 1327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 173 | 173 | 50 | 292 | 0 | 1204 | 0 | 326 | 294 | 223 |
| N.S. | 1 | 1.00 | 0.29 | 1.69 | 0.00 | 6.96 | 0.00 | 1.88 | 1.70 | 1.29 |
| time (sec) | N/A | 0.066 | 0.015 | 0.022 | 0.000 | 1.531 | 0.000 | 1.312 | 0.541 | 0.677 |

| Problem 1328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 152 | 152 | 123 | 273 | 265 | 283 | 196 | 335 | 229 | 315 |
| N.S. | 1 | 1.00 | 0.81 | 1.80 | 1.74 | 1.86 | 1.29 | 2.20 | 1.51 | 2.07 |
| time (sec) | N/A | 0.048 | 0.119 | 0.007 | 1.380 | 1.357 | 59.926 | 0.913 | 0.083 | 0.074 |

| Problem 1329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 125 | 125 | 101 | 186 | 187 | 203 | 136 | 229 | 175 | 213 |
| N.S. | 1 | 1.00 | 0.81 | 1.49 | 1.50 | 1.62 | 1.09 | 1.83 | 1.40 | 1.70 |
| time (sec) | N/A | 0.039 | 0.084 | 0.007 | 1.462 | 1.130 | 43.586 | 1.016 | 0.302 | 0.076 |

| Problem 1330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 96 | 76 | 115 | 122 | 136 | 461 | 141 | 128 | 130 |
| N.S. | 1 | 1.00 | 0.79 | 1.20 | 1.27 | 1.42 | 4.80 | 1.47 | 1.33 | 1.35 |
| time (sec) | N/A | 0.031 | 0.058 | 0.007 | 1.374 | 1.183 | 1.436 | 0.995 | 0.088 | 0.067 |

| Problem 1331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 62 | 62 | 72 | 85 | 265 | 72 | 68 | 72 |
| N.S. | 1 | 1.00 | 0.93 | 0.93 | 1.07 | 1.27 | 3.96 | 1.07 | 1.01 | 1.07 |
| time (sec) | N/A | 0.021 | 0.035 | 0.007 | 1.394 | 1.242 | 1.265 | 1.105 | 0.072 | 0.051 |

| Problem 1332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 29 | 26 | 28 | 46 | 124 | 28 | 29 | 32 |
| N.S. | 1 | 1.00 | 0.72 | 0.65 | 0.70 | 1.15 | 3.10 | 0.70 | 0.72 | 0.80 |
| time (sec) | N/A | 0.014 | 0.020 | 0.003 | 1.279 | 1.113 | 1.122 | 1.021 | 0.246 | 0.030 |

| Problem 1333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 31 | 14 | 12 | 12 | 16 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 1.94 | 0.88 | 0.75 | 0.75 | 1.00 |
| time (sec) | N/A | 0.001 | 0.004 | 0.004 | 1.363 | 1.319 | 0.066 | 0.958 | 0.026 | 0.009 |

| Problem 1334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 48 | 90 | 0 | 398 | 83 | 113 | 100 | 97 |
| N.S. | 1 | 1.00 | 0.52 | 0.97 | 0.00 | 4.28 | 0.89 | 1.22 | 1.08 | 1.04 |
| time (sec) | N/A | 0.039 | 0.010 | 0.013 | 0.000 | 1.440 | 13.575 | 1.156 | 0.333 | 0.133 |

| Problem 1335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 124 | 124 | 50 | 125 | 0 | 782 | 0 | 216 | 161 | 157 |
| N.S. | 1 | 1.00 | 0.40 | 1.01 | 0.00 | 6.31 | 0.00 | 1.74 | 1.30 | 1.27 |
| time (sec) | N/A | 0.050 | 0.015 | 0.017 | 0.000 | 1.424 | 0.000 | 1.111 | 0.377 | 0.356 |

| Problem 1336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 167 | 167 | 52 | 206 | 0 | 1226 | 0 | 298 | 243 | 223 |
| N.S. | 1 | 1.00 | 0.31 | 1.23 | 0.00 | 7.34 | 0.00 | 1.78 | 1.46 | 1.34 |
| time (sec) | N/A | 0.062 | 0.016 | 0.019 | 0.000 | 1.361 | 0.000 | 1.198 | 0.285 | 0.600 |

| Problem 1337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 200 | 200 | 52 | 319 | 0 | 1840 | 0 | 432 | 334 | 304 |
| N.S. | 1 | 1.00 | 0.26 | 1.60 | 0.00 | 9.20 | 0.00 | 2.16 | 1.67 | 1.52 |
| time (sec) | N/A | 0.134 | 0.019 | 0.023 | 0.000 | 1.434 | 0.000 | 1.199 | 0.641 | 0.866 |

| Problem 1338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 25 | 23 | 18 | 95 | 66 | 637 | 17 | 22 |
| N.S. | 1 | 1.00 | 1.14 | 1.05 | 0.82 | 4.32 | 3.00 | 28.95 | 0.77 | 1.00 |
| time (sec) | N/A | 0.005 | 0.020 | 0.004 | 1.373 | 1.136 | 1.215 | 1.262 | 0.050 | 0.052 |

| Problem 1339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 25 | 23 | 18 | 75 | 66 | 495 | 17 | 22 |
| N.S. | 1 | 1.00 | 1.14 | 1.05 | 0.82 | 3.41 | 3.00 | 22.50 | 0.77 | 1.00 |
| time (sec) | N/A | 0.005 | 0.012 | 0.003 | 1.364 | 1.159 | 1.063 | 1.274 | 0.034 | 0.043 |

| Problem 1340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 25 | 23 | 374 | 67 | 73 | 374 | 17 | 22 |
| N.S. | 1 | 1.00 | 1.14 | 1.05 | 17.00 | 3.05 | 3.32 | 17.00 | 0.77 | 1.00 |
| time (sec) | N/A | 0.004 | 0.014 | 0.003 | 1.460 | 0.827 | 1.542 | 0.973 | 0.028 | 0.048 |

| Problem 1341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 25 | 23 | 18 | 56 | 73 | 266 | 17 | 22 |
| N.S. | 1 | 1.00 | 1.14 | 1.05 | 0.82 | 2.55 | 3.32 | 12.09 | 0.77 | 1.00 |
| time (sec) | N/A | 0.004 | 0.015 | 0.003 | 1.431 | 1.401 | 1.646 | 1.176 | 0.028 | 0.056 |

| Problem 1342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 25 | 23 | 18 | 45 | 73 | 178 | 17 | 22 |
| N.S. | 1 | 1.00 | 1.14 | 1.05 | 0.82 | 2.05 | 3.32 | 8.09 | 0.77 | 1.00 |
| time (sec) | N/A | 0.004 | 0.014 | 0.002 | 1.381 | 1.435 | 1.615 | 0.956 | 0.028 | 0.057 |

| Problem 1343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 25 | 23 | 18 | 34 | 80 | 106 | 17 | 22 |
| N.S. | 1 | 1.00 | 1.14 | 1.05 | 0.82 | 1.55 | 3.64 | 4.82 | 0.77 | 1.00 |
| time (sec) | N/A | 0.004 | 0.014 | 0.004 | 1.411 | 1.258 | 4.108 | 0.987 | 0.028 | 0.056 |

| Problem 1344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 26 | 23 | 18 | 23 | 53 | 54 | 17 | 22 |
| N.S. | 1 | 1.00 | 1.18 | 1.05 | 0.82 | 1.05 | 2.41 | 2.45 | 0.77 | 1.00 |
| time (sec) | N/A | 0.004 | 0.014 | 0.003 | 1.393 | 1.311 | 8.308 | 1.070 | 0.029 | 0.056 |

| Problem 1345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 24 | 23 | 18 | 18 | 29 | 18 | 17 | 20 |
| N.S. | 1 | 1.00 | 1.20 | 1.15 | 0.90 | 0.90 | 1.45 | 0.90 | 0.85 | 1.00 |
| time (sec) | N/A | 0.004 | 0.008 | 0.003 | 1.383 | 1.189 | 15.476 | 1.028 | 0.030 | 0.058 |

| Problem 1346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 24 | 23 | 18 | 29 | 48 | 18 | 17 | 27 |
| N.S. | 1 | 1.00 | 1.20 | 1.15 | 0.90 | 1.45 | 2.40 | 0.90 | 0.85 | 1.35 |
| time (sec) | N/A | 0.005 | 0.010 | 0.002 | 1.340 | 1.099 | 39.429 | 0.811 | 0.030 | 0.058 |

| Problem 1347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 22 | 21 | 21 | 27 | 22 | 10 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 1.57 | 1.50 | 1.50 | 1.93 | 1.57 | 0.71 | 1.00 |
| time (sec) | N/A | 0.004 | 0.003 | 0.008 | 1.350 | 1.215 | 0.659 | 0.870 | 0.054 | 0.016 |

| Problem 1348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 25 | 25 | 25 | 19 | 18 | 18 | 61 | 18 | 15 | 25 |
| N.S. | 1 | 1.00 | 1.00 | 0.76 | 0.72 | 0.72 | 2.44 | 0.72 | 0.60 | 1.00 |
| time (sec) | N/A | 0.009 | 0.009 | 0.006 | 3.003 | 1.047 | 1.124 | 0.980 | 0.060 | 0.030 |

| Problem 1349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 104 | 84 | 86 | 86 | 170 | 87 | 104 | 115 |
| N.S. | 1 | 1.00 | 1.24 | 1.00 | 1.02 | 1.02 | 2.02 | 1.04 | 1.24 | 1.37 |
| time (sec) | N/A | 0.038 | 0.048 | 0.006 | 3.003 | 1.333 | 2.257 | 1.045 | 0.068 | 0.121 |

| Problem 1350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 18 | 15 | 19 | 19 | 114 | 26 | 14 | 22 |
| N.S. | 1 | 1.00 | 0.67 | 0.56 | 0.70 | 0.70 | 4.22 | 0.96 | 0.52 | 0.81 |
| time (sec) | N/A | 0.005 | 0.009 | 0.003 | 1.349 | 1.240 | 1.093 | 0.945 | 0.255 | 0.014 |

| Problem 1351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 38 | 38 | 23 | 20 | 28 | 24 | 146 | 38 | 21 | 31 |
| N.S. | 1 | 1.00 | 0.61 | 0.53 | 0.74 | 0.63 | 3.84 | 1.00 | 0.55 | 0.82 |
| time (sec) | N/A | 0.007 | 0.010 | 0.003 | 1.360 | 1.322 | 1.518 | 0.860 | 0.046 | 0.016 |

| Problem 1352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 139 | 139 | 106 | 161 | 0 | 570 | 0 | 196 | 204 | 191 |
| N.S. | 1 | 1.00 | 0.76 | 1.16 | 0.00 | 4.10 | 0.00 | 1.41 | 1.47 | 1.37 |
| time (sec) | N/A | 0.108 | 0.076 | 0.008 | 0.000 | 1.265 | 0.000 | 1.100 | 0.211 | 0.231 |

| Problem 1353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 140 | 140 | 154 | 160 | 0 | 900 | 0 | 207 | 206 | 190 |
| N.S. | 1 | 1.00 | 1.10 | 1.14 | 0.00 | 6.43 | 0.00 | 1.48 | 1.47 | 1.36 |
| time (sec) | N/A | 0.074 | 0.075 | 0.007 | 0.000 | 1.583 | 0.000 | 0.959 | 0.369 | 0.213 |

| Problem 1354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 230 | 230 | 194 | 858 | 0 | 702 | 0 | 1107 | -1 | 198 |
| N.S. | 1 | 1.00 | 0.84 | 3.73 | 0.00 | 3.05 | 0.00 | 4.81 | -0.00 | 0.86 |
| time (sec) | N/A | 0.160 | 1.465 | 0.012 | 0.000 | 1.736 | 0.000 | 1.838 | 0.000 | 0.311 |

| Problem 1355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 192 | 192 | 190 | 645 | 0 | 540 | 0 | 726 | -1 | 176 |
| N.S. | 1 | 1.00 | 0.99 | 3.36 | 0.00 | 2.81 | 0.00 | 3.78 | -0.01 | 0.92 |
| time (sec) | N/A | 0.095 | 0.598 | 0.008 | 0.000 | 1.421 | 0.000 | 1.639 | 0.000 | 0.294 |

| Problem 1356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 154 | 154 | 151 | 460 | 0 | 410 | 0 | 438 | -1 | 154 |
| N.S. | 1 | 1.00 | 0.98 | 2.99 | 0.00 | 2.66 | 0.00 | 2.84 | -0.01 | 1.00 |
| time (sec) | N/A | 0.072 | 0.435 | 0.008 | 0.000 | 1.369 | 0.000 | 1.377 | 0.000 | 0.261 |

| Problem 1357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 116 | 116 | 118 | 305 | 0 | 300 | 0 | 232 | 88 | 129 |
| N.S. | 1 | 1.00 | 1.02 | 2.63 | 0.00 | 2.59 | 0.00 | 2.00 | 0.76 | 1.11 |
| time (sec) | N/A | 0.055 | 0.277 | 0.007 | 0.000 | 1.503 | 0.000 | 1.254 | 0.136 | 0.207 |

| Problem 1358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 117 | 107 | 0 | 236 | 0 | 93 | 260 | 104 |
| N.S. | 1 | 1.00 | 1.62 | 1.49 | 0.00 | 3.28 | 0.00 | 1.29 | 3.61 | 1.44 |
| time (sec) | N/A | 0.038 | 0.104 | 0.006 | 0.000 | 1.322 | 0.000 | 1.095 | 4.005 | 0.306 |

| Problem 1359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 99 | 0 | 0 | 241 | 0 | 131 | -1 | 66 |
| N.S. | 1 | 1.00 | 1.50 | 0.00 | 0.00 | 3.65 | 0.00 | 1.98 | -0.02 | 1.00 |
| time (sec) | N/A | 0.033 | 0.227 | 0.082 | 0.000 | 1.477 | 0.000 | 1.192 | 0.000 | 0.100 |

| Problem 1360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 65 | 0 | 152 | 27 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 2.03 | 0.00 | 4.75 | 0.84 | 1.00 |
| time (sec) | N/A | 0.003 | 0.012 | 0.005 | 0.000 | 1.530 | 0.000 | 1.433 | 0.717 | 0.038 |

| Problem 1361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 175 | 0 | 447 | 127 | 57 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 2.65 | 0.00 | 6.77 | 1.92 | 0.86 |
| time (sec) | N/A | 0.009 | 0.020 | 0.006 | 0.000 | 2.318 | 0.000 | 1.444 | 0.822 | 0.101 |

| Problem 1362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 337 | 0 | 689 | 203 | 83 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 3.34 | 0.00 | 6.82 | 2.01 | 0.82 |
| time (sec) | N/A | 0.016 | 0.039 | 0.009 | 0.000 | 3.931 | 0.000 | 1.582 | 0.966 | 0.104 |

| Problem 1363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 532 | 0 | 989 | 292 | 109 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 3.91 | 0.00 | 7.27 | 2.15 | 0.80 |
| time (sec) | N/A | 0.029 | 0.054 | 0.011 | 0.000 | 13.035 | 0.000 | 2.041 | 1.184 | 0.111 |

| Problem 1364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 171 | 171 | 170 | 256 | 0 | 781 | 0 | 1345 | 397 | 117 |
| N.S. | 1 | 1.00 | 0.99 | 1.50 | 0.00 | 4.57 | 0.00 | 7.87 | 2.32 | 0.68 |
| time (sec) | N/A | 0.041 | 0.077 | 0.013 | 0.000 | 27.264 | 0.000 | 2.383 | 1.432 | 0.141 |

| Problem 1365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 227 | 227 | 187 | 853 | 0 | 702 | 0 | 1740 | -1 | 197 |
| N.S. | 1 | 1.00 | 0.82 | 3.76 | 0.00 | 3.09 | 0.00 | 7.67 | -0.00 | 0.87 |
| time (sec) | N/A | 0.130 | 1.780 | 0.006 | 0.000 | 1.522 | 0.000 | 2.297 | 0.000 | 0.413 |

| Problem 1366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 189 | 189 | 193 | 640 | 0 | 534 | 0 | 1071 | -1 | 176 |
| N.S. | 1 | 1.00 | 1.02 | 3.39 | 0.00 | 2.83 | 0.00 | 5.67 | -0.01 | 0.93 |
| time (sec) | N/A | 0.090 | 0.567 | 0.007 | 0.000 | 1.352 | 0.000 | 1.892 | 0.000 | 0.343 |

| Problem 1367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 151 | 151 | 152 | 459 | 0 | 410 | 0 | 576 | -1 | 153 |
| N.S. | 1 | 1.00 | 1.01 | 3.04 | 0.00 | 2.72 | 0.00 | 3.81 | -0.01 | 1.01 |
| time (sec) | N/A | 0.068 | 0.445 | 0.006 | 0.000 | 1.479 | 0.000 | 1.606 | 0.000 | 0.239 |

| Problem 1368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 113 | 109 | 308 | 0 | 306 | 0 | 233 | -1 | 134 |
| N.S. | 1 | 1.00 | 0.96 | 2.73 | 0.00 | 2.71 | 0.00 | 2.06 | -0.01 | 1.19 |
| time (sec) | N/A | 0.051 | 0.291 | 0.007 | 0.000 | 1.024 | 0.000 | 1.246 | 0.000 | 0.176 |

| Problem 1369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 71 | 0 | 0 | 311 | 0 | 204 | -1 | 159 |
| N.S. | 1 | 1.00 | 0.72 | 0.00 | 0.00 | 3.17 | 0.00 | 2.08 | -0.01 | 1.62 |
| time (sec) | N/A | 0.047 | 0.053 | 0.079 | 0.000 | 1.392 | 0.000 | 1.499 | 0.000 | 0.557 |

| Problem 1370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 73 | 0 | 0 | 325 | 0 | 455 | -1 | 85 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 3.53 | 0.00 | 4.95 | -0.01 | 0.92 |
| time (sec) | N/A | 0.040 | 0.050 | 0.079 | 0.000 | 2.061 | 0.000 | 1.744 | 0.000 | 0.140 |

| Problem 1371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 104 | 0 | 374 | 27 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 3.25 | 0.00 | 11.69 | 0.84 | 1.00 |
| time (sec) | N/A | 0.003 | 0.015 | 0.005 | 0.000 | 2.261 | 0.000 | 1.695 | 0.796 | 0.055 |

| Problem 1372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 235 | 0 | 1024 | 178 | 51 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 3.56 | 0.00 | 15.52 | 2.70 | 0.77 |
| time (sec) | N/A | 0.008 | 0.025 | 0.005 | 0.000 | 3.925 | 0.000 | 2.122 | 0.926 | 0.122 |

| Problem 1373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 426 | 0 | 1394 | 268 | 73 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 4.22 | 0.00 | 13.80 | 2.65 | 0.72 |
| time (sec) | N/A | 0.016 | 0.045 | 0.007 | 0.000 | 13.562 | 0.000 | 2.937 | 1.113 | 0.136 |

| Problem 1374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 649 | 0 | 1823 | 376 | 95 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 4.77 | 0.00 | 13.40 | 2.76 | 0.70 |
| time (sec) | N/A | 0.028 | 0.064 | 0.011 | 0.000 | 28.364 | 0.000 | 3.282 | 1.333 | 0.150 |

| Problem 1375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 262 | 262 | 209 | 1089 | 0 | 882 | 0 | 3120 | -1 | 220 |
| N.S. | 1 | 1.00 | 0.80 | 4.16 | 0.00 | 3.37 | 0.00 | 11.91 | -0.00 | 0.84 |
| time (sec) | N/A | 0.149 | 2.529 | 0.008 | 0.000 | 1.425 | 0.000 | 3.533 | 0.000 | 0.504 |

| Problem 1376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 224 | 224 | 187 | 848 | 0 | 702 | 0 | 1962 | -1 | 198 |
| N.S. | 1 | 1.00 | 0.83 | 3.79 | 0.00 | 3.13 | 0.00 | 8.76 | -0.00 | 0.88 |
| time (sec) | N/A | 0.119 | 1.468 | 0.008 | 0.000 | 0.894 | 0.000 | 2.326 | 0.000 | 0.429 |

| Problem 1377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 186 | 186 | 191 | 641 | 0 | 540 | 0 | 1083 | -1 | 175 |
| N.S. | 1 | 1.00 | 1.03 | 3.45 | 0.00 | 2.90 | 0.00 | 5.82 | -0.01 | 0.94 |
| time (sec) | N/A | 0.086 | 0.580 | 0.008 | 0.000 | 1.301 | 0.000 | 1.926 | 0.000 | 0.285 |

| Problem 1378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 148 | 148 | 139 | 465 | 0 | 412 | 0 | 446 | -1 | 160 |
| N.S. | 1 | 1.00 | 0.94 | 3.14 | 0.00 | 2.78 | 0.00 | 3.01 | -0.01 | 1.08 |
| time (sec) | N/A | 0.066 | 0.407 | 0.008 | 0.000 | 1.250 | 0.000 | 1.431 | 0.000 | 0.198 |

| Problem 1379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 138 | 138 | 71 | 0 | 0 | 439 | 0 | 287 | -1 | 151 |
| N.S. | 1 | 1.00 | 0.51 | 0.00 | 0.00 | 3.18 | 0.00 | 2.08 | -0.01 | 1.09 |
| time (sec) | N/A | 0.063 | 0.068 | 0.083 | 0.000 | 1.423 | 0.000 | 2.018 | 0.000 | 0.281 |

| Problem 1380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 128 | 128 | 73 | 0 | 0 | 475 | 0 | 650 | -1 | 227 |
| N.S. | 1 | 1.00 | 0.57 | 0.00 | 0.00 | 3.71 | 0.00 | 5.08 | -0.01 | 1.77 |
| time (sec) | N/A | 0.062 | 0.062 | 0.080 | 0.000 | 1.933 | 0.000 | 2.186 | 0.000 | 0.992 |

| Problem 1381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 120 | 120 | 73 | 0 | 0 | 463 | 0 | 1025 | -1 | 119 |
| N.S. | 1 | 1.00 | 0.61 | 0.00 | 0.00 | 3.86 | 0.00 | 8.54 | -0.01 | 0.99 |
| time (sec) | N/A | 0.052 | 0.070 | 0.082 | 0.000 | 2.685 | 0.000 | 2.477 | 0.000 | 0.141 |

| Problem 1382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 138 | 0 | 706 | 27 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 4.31 | 0.00 | 22.06 | 0.84 | 1.00 |
| time (sec) | N/A | 0.003 | 0.018 | 0.003 | 0.000 | 3.847 | 0.000 | 2.541 | 0.971 | 0.057 |

| Problem 1383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 295 | 0 | 1826 | 229 | 57 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 4.47 | 0.00 | 27.67 | 3.47 | 0.86 |
| time (sec) | N/A | 0.009 | 0.031 | 0.005 | 0.000 | 13.617 | 0.000 | 3.803 | 1.140 | 0.124 |

| Problem 1384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 513 | 0 | 2316 | 333 | 73 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 5.08 | 0.00 | 22.93 | 3.30 | 0.72 |
| time (sec) | N/A | 0.016 | 0.052 | 0.007 | 0.000 | 31.126 | 0.000 | 4.592 | 1.355 | 0.165 |

| Problem 1385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 765 | 0 | 2868 | 459 | 95 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 5.62 | 0.00 | 21.09 | 3.38 | 0.70 |
| time (sec) | N/A | 0.028 | 0.074 | 0.010 | 0.000 | 55.759 | 0.000 | 6.088 | 1.615 | 0.148 |

| Problem 1386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 183 | 183 | 189 | 650 | 0 | 542 | 0 | 268 | -1 | 172 |
| N.S. | 1 | 1.00 | 1.03 | 3.55 | 0.00 | 2.96 | 0.00 | 1.46 | -0.01 | 0.94 |
| time (sec) | N/A | 0.097 | 0.656 | 0.007 | 0.000 | 1.099 | 0.000 | 1.229 | 0.000 | 0.198 |

| Problem 1387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 148 | 148 | 150 | 465 | 0 | 412 | 0 | 198 | -1 | 160 |
| N.S. | 1 | 1.00 | 1.01 | 3.14 | 0.00 | 2.78 | 0.00 | 1.34 | -0.01 | 1.08 |
| time (sec) | N/A | 0.072 | 0.524 | 0.009 | 0.000 | 1.017 | 0.000 | 1.270 | 0.000 | 0.212 |

| Problem 1388 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 113 | 119 | 308 | 0 | 306 | 0 | 139 | -1 | 134 |
| N.S. | 1 | 1.00 | 1.05 | 2.73 | 0.00 | 2.71 | 0.00 | 1.23 | -0.01 | 1.19 |
| time (sec) | N/A | 0.050 | 0.365 | 0.007 | 0.000 | 1.362 | 0.000 | 0.945 | 0.000 | 0.191 |

| Problem 1389 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 103 | 107 | 0 | 235 | 0 | 97 | 261 | 106 |
| N.S. | 1 | 1.00 | 1.41 | 1.47 | 0.00 | 3.22 | 0.00 | 1.33 | 3.58 | 1.45 |
| time (sec) | N/A | 0.035 | 0.268 | 0.006 | 0.000 | 0.768 | 0.000 | 1.115 | 3.803 | 0.287 |

| Problem 1390 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 77 | 76 | 0 | 178 | 0 | 50 | 45 | 42 |
| N.S. | 1 | 1.00 | 1.83 | 1.81 | 0.00 | 4.24 | 0.00 | 1.19 | 1.07 | 1.00 |
| time (sec) | N/A | 0.026 | 0.065 | 0.007 | 0.000 | 1.110 | 0.000 | 1.097 | 0.288 | 0.088 |

| Problem 1391 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 27 | 0 | 42 | 0 | 66 | 26 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 1.40 | 0.00 | 2.20 | 0.87 | 1.00 |
| time (sec) | N/A | 0.003 | 0.009 | 0.006 | 0.000 | 0.916 | 0.000 | 1.043 | 0.732 | 0.033 |

| Problem 1392 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 118 | 0 | 121 | 71 | 56 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 1.79 | 0.00 | 1.83 | 1.08 | 0.85 |
| time (sec) | N/A | 0.008 | 0.017 | 0.004 | 0.000 | 0.916 | 0.000 | 1.080 | 0.894 | 0.100 |

| Problem 1393 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 75 | 105 | 0 | 251 | 0 | 227 | 133 | 83 |
| N.S. | 1 | 1.00 | 0.74 | 1.04 | 0.00 | 2.49 | 0.00 | 2.25 | 1.32 | 0.82 |
| time (sec) | N/A | 0.016 | 0.031 | 0.007 | 0.000 | 1.315 | 0.000 | 1.270 | 1.005 | 0.104 |

| Problem 1394 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 116 | 171 | 0 | 419 | 0 | 386 | 209 | 109 |
| N.S. | 1 | 1.00 | 0.85 | 1.26 | 0.00 | 3.08 | 0.00 | 2.84 | 1.54 | 0.80 |
| time (sec) | N/A | 0.028 | 0.047 | 0.008 | 0.000 | 2.891 | 0.000 | 1.468 | 1.192 | 0.111 |

| Problem 1395 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 171 | 171 | 168 | 256 | 0 | 638 | 0 | 596 | 303 | 135 |
| N.S. | 1 | 1.00 | 0.98 | 1.50 | 0.00 | 3.73 | 0.00 | 3.49 | 1.77 | 0.79 |
| time (sec) | N/A | 0.041 | 0.066 | 0.012 | 0.000 | 11.400 | 0.000 | 1.568 | 1.370 | 0.114 |

| Problem 1396 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 174 | 174 | 73 | 0 | 0 | 603 | 0 | 279 | -1 | 173 |
| N.S. | 1 | 1.00 | 0.42 | 0.00 | 0.00 | 3.47 | 0.00 | 1.60 | -0.01 | 0.99 |
| time (sec) | N/A | 0.089 | 0.067 | 0.079 | 0.000 | 1.732 | 0.000 | 1.385 | 0.000 | 0.337 |

| Problem 1397 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 138 | 138 | 73 | 0 | 0 | 441 | 0 | 201 | -1 | 151 |
| N.S. | 1 | 1.00 | 0.53 | 0.00 | 0.00 | 3.20 | 0.00 | 1.46 | -0.01 | 1.09 |
| time (sec) | N/A | 0.067 | 0.055 | 0.082 | 0.000 | 1.505 | 0.000 | 1.260 | 0.000 | 0.243 |

| Problem 1398 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 73 | 0 | 0 | 311 | 0 | 137 | -1 | 120 |
| N.S. | 1 | 1.00 | 0.74 | 0.00 | 0.00 | 3.17 | 0.00 | 1.40 | -0.01 | 1.22 |
| time (sec) | N/A | 0.047 | 0.048 | 0.087 | 0.000 | 1.048 | 0.000 | 1.273 | 0.000 | 0.563 |

| Problem 1399 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 95 | 0 | 0 | 241 | 0 | 96 | -1 | 66 |
| N.S. | 1 | 1.00 | 1.44 | 0.00 | 0.00 | 3.65 | 0.00 | 1.45 | -0.02 | 1.00 |
| time (sec) | N/A | 0.032 | 0.349 | 0.088 | 0.000 | 1.731 | 0.000 | 1.223 | 0.000 | 0.102 |

| Problem 1400 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 27 | 0 | 42 | 0 | 47 | 26 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 1.40 | 0.00 | 1.57 | 0.87 | 1.00 |
| time (sec) | N/A | 0.003 | 0.008 | 0.005 | 0.000 | 1.291 | 0.000 | 1.038 | 0.741 | 0.033 |

| Problem 1401 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 42 | 52 | 0 | 125 | 0 | 142 | 71 | 46 |
| N.S. | 1 | 1.00 | 0.68 | 0.84 | 0.00 | 2.02 | 0.00 | 2.29 | 1.15 | 0.74 |
| time (sec) | N/A | 0.009 | 0.017 | 0.004 | 0.000 | 1.554 | 0.000 | 1.222 | 0.858 | 0.112 |

| Problem 1402 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 75 | 105 | 0 | 273 | 0 | 368 | 141 | 73 |
| N.S. | 1 | 1.00 | 0.74 | 1.04 | 0.00 | 2.70 | 0.00 | 3.64 | 1.40 | 0.72 |
| time (sec) | N/A | 0.018 | 0.030 | 0.009 | 0.000 | 1.919 | 0.000 | 1.544 | 1.064 | 0.125 |

| Problem 1403 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 114 | 170 | 0 | 455 | 0 | 830 | 227 | 93 |
| N.S. | 1 | 1.00 | 0.84 | 1.25 | 0.00 | 3.35 | 0.00 | 6.10 | 1.67 | 0.68 |
| time (sec) | N/A | 0.027 | 0.042 | 0.010 | 0.000 | 2.452 | 0.000 | 2.459 | 1.312 | 0.134 |

| Problem 1404 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 171 | 171 | 166 | 256 | 0 | 689 | 0 | 1518 | 337 | 117 |
| N.S. | 1 | 1.00 | 0.97 | 1.50 | 0.00 | 4.03 | 0.00 | 8.88 | 1.97 | 0.68 |
| time (sec) | N/A | 0.043 | 0.061 | 0.012 | 0.000 | 8.186 | 0.000 | 4.771 | 1.500 | 0.141 |

| Problem 1405 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 206 | 206 | 226 | 356 | 0 | 955 | 0 | 2438 | 454 | 139 |
| N.S. | 1 | 1.00 | 1.10 | 1.73 | 0.00 | 4.64 | 0.00 | 11.83 | 2.20 | 0.67 |
| time (sec) | N/A | 0.057 | 0.080 | 0.013 | 0.000 | 15.843 | 0.000 | 8.708 | 1.959 | 0.154 |

| Problem 1406 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 204 | 204 | 73 | 0 | 0 | 879 | 0 | 500 | -1 | 194 |
| N.S. | 1 | 1.00 | 0.36 | 0.00 | 0.00 | 4.31 | 0.00 | 2.45 | -0.00 | 0.95 |
| time (sec) | N/A | 0.109 | 0.097 | 0.083 | 0.000 | 2.899 | 0.000 | 2.418 | 0.000 | 0.338 |

| Problem 1407 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 170 | 170 | 73 | 0 | 0 | 657 | 0 | 380 | -1 | 172 |
| N.S. | 1 | 1.00 | 0.43 | 0.00 | 0.00 | 3.86 | 0.00 | 2.24 | -0.01 | 1.01 |
| time (sec) | N/A | 0.079 | 0.082 | 0.088 | 0.000 | 2.244 | 0.000 | 2.141 | 0.000 | 0.275 |

| Problem 1408 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 128 | 128 | 73 | 0 | 0 | 475 | 0 | 276 | -1 | 166 |
| N.S. | 1 | 1.00 | 0.57 | 0.00 | 0.00 | 3.71 | 0.00 | 2.16 | -0.01 | 1.30 |
| time (sec) | N/A | 0.058 | 0.078 | 0.082 | 0.000 | 1.624 | 0.000 | 1.972 | 0.000 | 0.978 |

| Problem 1409 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 111 | 0 | 0 | 325 | 0 | 181 | -1 | 85 |
| N.S. | 1 | 1.00 | 1.21 | 0.00 | 0.00 | 3.53 | 0.00 | 1.97 | -0.01 | 0.92 |
| time (sec) | N/A | 0.040 | 0.550 | 0.086 | 0.000 | 1.487 | 0.000 | 1.434 | 0.000 | 0.136 |

| Problem 1410 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 65 | 0 | 51 | 130 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 2.03 | 0.00 | 1.59 | 4.06 | 1.00 |
| time (sec) | N/A | 0.003 | 0.010 | 0.005 | 0.000 | 1.146 | 0.000 | 1.147 | 0.561 | 0.037 |

| Problem 1411 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 53 | 0 | 118 | 0 | 126 | 127 | 57 |
| N.S. | 1 | 1.00 | 0.70 | 0.80 | 0.00 | 1.79 | 0.00 | 1.91 | 1.92 | 0.86 |
| time (sec) | N/A | 0.008 | 0.015 | 0.004 | 0.000 | 1.317 | 0.000 | 1.019 | 0.897 | 0.094 |

| Problem 1412 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 78 | 104 | 0 | 273 | 0 | 373 | 132 | 73 |
| N.S. | 1 | 1.00 | 0.80 | 1.06 | 0.00 | 2.79 | 0.00 | 3.81 | 1.35 | 0.74 |
| time (sec) | N/A | 0.017 | 0.031 | 0.007 | 0.000 | 2.079 | 0.000 | 1.480 | 1.030 | 0.123 |

| Problem 1413 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 135 | 135 | 118 | 169 | 0 | 447 | 0 | 670 | 224 | 92 |
| N.S. | 1 | 1.00 | 0.87 | 1.25 | 0.00 | 3.31 | 0.00 | 4.96 | 1.66 | 0.68 |
| time (sec) | N/A | 0.028 | 0.052 | 0.010 | 0.000 | 2.418 | 0.000 | 2.063 | 1.289 | 0.138 |

| Problem 1414 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 172 | 172 | 170 | 256 | 0 | 715 | 0 | 1203 | 346 | 117 |
| N.S. | 1 | 1.00 | 0.99 | 1.49 | 0.00 | 4.16 | 0.00 | 6.99 | 2.01 | 0.68 |
| time (sec) | N/A | 0.044 | 0.067 | 0.010 | 0.000 | 8.263 | 0.000 | 3.918 | 1.531 | 0.150 |

| Problem 1415 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 207 | 207 | 233 | 356 | 0 | 999 | 0 | 1964 | 478 | 139 |
| N.S. | 1 | 1.00 | 1.13 | 1.72 | 0.00 | 4.83 | 0.00 | 9.49 | 2.31 | 0.67 |
| time (sec) | N/A | 0.061 | 0.082 | 0.015 | 0.000 | 19.331 | 0.000 | 7.760 | 1.908 | 0.165 |

| Problem 1416 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 19 | 86 | 48 | 31 | 0 | 24 | 50 | 28 |
| N.S. | 1 | 1.00 | 1.00 | 4.53 | 2.53 | 1.63 | 0.00 | 1.26 | 2.63 | 1.47 |
| time (sec) | N/A | 0.006 | 0.010 | 0.009 | 1.361 | 1.049 | 0.000 | 1.018 | 0.308 | 0.052 |

| Problem 1417 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 39 | 66 | 33 | 27 | 0 | 23 | 47 | 25 |
| N.S. | 1 | 1.00 | 2.05 | 3.47 | 1.74 | 1.42 | 0.00 | 1.21 | 2.47 | 1.32 |
| time (sec) | N/A | 0.005 | 0.012 | 0.009 | 1.366 | 0.811 | 0.000 | 1.055 | 0.339 | 0.051 |

| Problem 1418 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 39 | 66 | 33 | 27 | 0 | 23 | 43 | 25 |
| N.S. | 1 | 1.00 | 2.05 | 3.47 | 1.74 | 1.42 | 0.00 | 1.21 | 2.26 | 1.32 |
| time (sec) | N/A | 0.005 | 0.011 | 0.008 | 1.389 | 0.876 | 0.000 | 0.965 | 0.326 | 0.053 |

| Problem 1419 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 34 | 60 | 32 | 25 | 15 | 21 | 33 | 25 |
| N.S. | 1 | 1.00 | 2.00 | 3.53 | 1.88 | 1.47 | 0.88 | 1.24 | 1.94 | 1.47 |
| time (sec) | N/A | 0.004 | 0.010 | 0.007 | 1.330 | 0.952 | 1.265 | 0.936 | 0.309 | 0.041 |

| Problem 1420 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 39 | 64 | 33 | 27 | 0 | 23 | 44 | 25 |
| N.S. | 1 | 1.00 | 2.05 | 3.37 | 1.74 | 1.42 | 0.00 | 1.21 | 2.32 | 1.32 |
| time (sec) | N/A | 0.005 | 0.011 | 0.008 | 1.316 | 0.966 | 0.000 | 1.032 | 0.318 | 0.051 |

| Problem 1421 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | C | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 25 | 57 | 26 | 26 | 75 | 23 | 50 | 25 |
| N.S. | 1 | 1.00 | 2.27 | 5.18 | 2.36 | 2.36 | 6.82 | 2.09 | 4.55 | 2.27 |
| time (sec) | N/A | 0.003 | 0.005 | 0.007 | 1.329 | 1.122 | 4.199 | 1.075 | 0.304 | 0.051 |

| Problem 1422 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 39 | 66 | 33 | 27 | 0 | 23 | 46 | 25 |
| N.S. | 1 | 1.00 | 2.05 | 3.47 | 1.74 | 1.42 | 0.00 | 1.21 | 2.42 | 1.32 |
| time (sec) | N/A | 0.005 | 0.011 | 0.009 | 1.375 | 0.729 | 0.000 | 0.995 | 0.292 | 0.050 |

| Problem 1423 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 66 | 33 | 28 | 0 | 23 | 47 | 25 |
| N.S. | 1 | 1.00 | 1.00 | 4.40 | 2.20 | 1.87 | 0.00 | 1.53 | 3.13 | 1.67 |
| time (sec) | N/A | 0.004 | 0.004 | 0.007 | 1.380 | 1.131 | 0.000 | 1.012 | 0.293 | 0.053 |

| Problem 1424 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 10 | 10 | 11 | 10 | 10 | 7 | 11 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.10 | 1.00 | 1.00 | 0.70 | 1.10 | 1.00 | 0.00 |
| time (sec) | N/A | 0.001 | 0.001 | 0.001 | 1.319 | 0.996 | 0.062 | 0.952 | 0.257 | 0.000 |

| Problem 1425 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 66 | 33 | 28 | 0 | 23 | 43 | 25 |
| N.S. | 1 | 1.00 | 1.00 | 4.40 | 2.20 | 1.87 | 0.00 | 1.53 | 2.87 | 1.67 |
| time (sec) | N/A | 0.004 | 0.004 | 0.006 | 1.386 | 0.952 | 0.000 | 0.925 | 0.286 | 0.053 |

| Problem 1426 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 19 | 19 | 36 | 58 | 32 | 25 | 20 | 21 | 37 | 25 |
| N.S. | 1 | 1.00 | 1.89 | 3.05 | 1.68 | 1.32 | 1.05 | 1.11 | 1.95 | 1.32 |
| time (sec) | N/A | 0.005 | 0.009 | 0.006 | 1.382 | 0.984 | 1.349 | 1.067 | 0.280 | 0.042 |

| Problem 1427 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 41 | 65 | 30 | 28 | 0 | 23 | 44 | 25 |
| N.S. | 1 | 1.00 | 1.95 | 3.10 | 1.43 | 1.33 | 0.00 | 1.10 | 2.10 | 1.19 |
| time (sec) | N/A | 0.006 | 0.009 | 0.007 | 1.682 | 1.077 | 0.000 | 1.023 | 0.286 | 0.054 |

| Problem 1428 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | C | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 25 | 57 | 26 | 26 | 75 | 23 | 50 | 25 |
| N.S. | 1 | 1.00 | 2.27 | 5.18 | 2.36 | 2.36 | 6.82 | 2.09 | 4.55 | 2.27 |
| time (sec) | N/A | 0.002 | 0.003 | 0.000 | 1.361 | 0.866 | 4.282 | 0.959 | 0.002 | 0.001 |

| Problem 1429 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 41 | 66 | 33 | 28 | 0 | 23 | 50 | 25 |
| N.S. | 1 | 1.00 | 1.95 | 3.14 | 1.57 | 1.33 | 0.00 | 1.10 | 2.38 | 1.19 |
| time (sec) | N/A | 0.006 | 0.011 | 0.007 | 1.242 | 1.047 | 0.000 | 0.958 | 0.283 | 0.052 |

| Problem 1430 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 22 | 65 | 21 | 44 | 0 | 18 | 44 | 26 |
| N.S. | 1 | 1.00 | 1.38 | 4.06 | 1.31 | 2.75 | 0.00 | 1.12 | 2.75 | 1.62 |
| time (sec) | N/A | 0.014 | 0.013 | 0.012 | 2.990 | 0.827 | 0.000 | 1.049 | 0.084 | 0.053 |

| Problem 1431 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | C | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 56 | 9 | 31 | 76 | 15 | 44 | 26 |
| N.S. | 1 | 1.00 | 1.00 | 5.09 | 0.82 | 2.82 | 6.91 | 1.36 | 4.00 | 2.36 |
| time (sec) | N/A | 0.003 | 0.009 | 0.007 | 3.033 | 0.751 | 4.440 | 0.907 | 0.078 | 0.051 |

| Problem 1432 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 22 | 66 | 19 | 43 | 0 | 18 | 40 | 26 |
| N.S. | 1 | 1.00 | 1.38 | 4.12 | 1.19 | 2.69 | 0.00 | 1.12 | 2.50 | 1.62 |
| time (sec) | N/A | 0.013 | 0.013 | 0.010 | 3.008 | 1.067 | 0.000 | 1.050 | 0.320 | 0.052 |

| Problem 1433 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | B | C | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 10 | 51 | 58 | 18 | 26 | 24 | 18 | 34 | 24 |
| N.S. | 1 | 1.00 | 5.10 | 5.80 | 1.80 | 2.60 | 2.40 | 1.80 | 3.40 | 2.40 |
| time (sec) | N/A | 0.010 | 0.014 | 0.004 | 3.140 | 1.122 | 1.277 | 0.934 | 0.293 | 0.041 |

| Problem 1434 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | B | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 49 | 66 | 21 | 44 | 0 | 13 | 41 | 26 |
| N.S. | 1 | 1.00 | 4.45 | 6.00 | 1.91 | 4.00 | 0.00 | 1.18 | 3.73 | 2.36 |
| time (sec) | N/A | 0.010 | 0.015 | 0.009 | 3.014 | 1.088 | 0.000 | 1.172 | 0.303 | 0.048 |

| Problem 1435 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | C | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 28 | 26 | 16 | 1 | 53 | 12 | 47 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.90 | 0.55 | 0.03 | 1.83 | 0.41 | 1.62 | 0.00 |
| time (sec) | N/A | 0.004 | 0.009 | 0.004 | 1.355 | 0.990 | 1.980 | 0.968 | 0.069 | 0.079 |

| Problem 1436 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | A | F | C | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 53 | 66 | 21 | 44 | 0 | 23 | 47 | 26 |
| N.S. | 1 | 1.00 | 2.04 | 2.54 | 0.81 | 1.69 | 0.00 | 0.88 | 1.81 | 1.00 |
| time (sec) | N/A | 0.014 | 0.016 | 0.010 | 3.012 | 0.815 | 0.000 | 1.071 | 0.297 | 0.050 |

| Problem 1437 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 70 | 33 | 30 | 0 | 25 | 49 | 59 |
| N.S. | 1 | 1.00 | 1.00 | 4.38 | 2.06 | 1.88 | 0.00 | 1.56 | 3.06 | 3.69 |
| time (sec) | N/A | 0.005 | 0.007 | 0.009 | 1.357 | 1.069 | 0.000 | 1.037 | 0.312 | 0.060 |

| Problem 1438 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 12 | 13 | 11 | 11 | 8 | 12 | 11 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 0.92 | 0.92 | 0.67 | 1.00 | 0.92 | 0.00 |
| time (sec) | N/A | 0.001 | 0.001 | 0.001 | 1.423 | 0.673 | 0.066 | 1.102 | 0.034 | 0.000 |

| Problem 1439 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 70 | 33 | 30 | 0 | 25 | 45 | 59 |
| N.S. | 1 | 1.00 | 1.00 | 4.38 | 2.06 | 1.88 | 0.00 | 1.56 | 2.81 | 3.69 |
| time (sec) | N/A | 0.005 | 0.007 | 0.008 | 1.327 | 0.912 | 0.000 | 0.853 | 0.311 | 0.055 |

| Problem 1440 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 37 | 64 | 32 | 27 | 53 | 23 | 39 | 27 |
| N.S. | 1 | 1.00 | 1.85 | 3.20 | 1.60 | 1.35 | 2.65 | 1.15 | 1.95 | 1.35 |
| time (sec) | N/A | 0.005 | 0.008 | 0.007 | 1.356 | 0.703 | 1.337 | 1.025 | 0.282 | 0.041 |

| Problem 1441 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 40 | 70 | 33 | 30 | 0 | 25 | 46 | 27 |
| N.S. | 1 | 1.00 | 1.82 | 3.18 | 1.50 | 1.36 | 0.00 | 1.14 | 2.09 | 1.23 |
| time (sec) | N/A | 0.007 | 0.011 | 0.007 | 1.383 | 0.766 | 0.000 | 1.095 | 0.285 | 0.056 |

| Problem 1442 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | C | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 12 | 12 | 27 | 61 | 26 | 28 | 78 | 25 | 52 | 27 |
| N.S. | 1 | 1.00 | 2.25 | 5.08 | 2.17 | 2.33 | 6.50 | 2.08 | 4.33 | 2.25 |
| time (sec) | N/A | 0.003 | 0.005 | 0.006 | 1.274 | 1.018 | 4.625 | 1.104 | 0.294 | 0.052 |

| Problem 1443 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 40 | 69 | 30 | 30 | 0 | 25 | 52 | 27 |
| N.S. | 1 | 1.00 | 1.82 | 3.14 | 1.36 | 1.36 | 0.00 | 1.14 | 2.36 | 1.23 |
| time (sec) | N/A | 0.007 | 0.011 | 0.007 | 1.369 | 1.118 | 0.000 | 1.277 | 0.292 | 0.054 |

| Problem 1444 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | C | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 25 | 57 | 26 | 26 | 75 | 23 | 40 | 25 |
| N.S. | 1 | 1.00 | 2.27 | 5.18 | 2.36 | 2.36 | 6.82 | 2.09 | 3.64 | 2.27 |
| time (sec) | N/A | 0.002 | 0.005 | 0.006 | 1.377 | 0.839 | 4.209 | 1.040 | 0.324 | 0.053 |

| Problem 1445 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 41 | 100 | 0 | 175 | 0 | 57 | 66 | 57 |
| N.S. | 1 | 1.00 | 0.95 | 2.33 | 0.00 | 4.07 | 0.00 | 1.33 | 1.53 | 1.33 |
| time (sec) | N/A | 0.015 | 0.025 | 0.018 | 0.000 | 0.899 | 0.000 | 0.972 | 0.495 | 0.115 |

| Problem 1446 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 31 | 48 | 41 | 26 | 44 | 23 | 30 | 30 |
| N.S. | 1 | 1.00 | 1.41 | 2.18 | 1.86 | 1.18 | 2.00 | 1.05 | 1.36 | 1.36 |
| time (sec) | N/A | 0.004 | 0.009 | 0.007 | 2.869 | 0.976 | 1.029 | 0.923 | 0.439 | 0.042 |

| Problem 1447 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 26 | 57 | 28 | 46 | 58 | 30 | 43 | 35 |
| N.S. | 1 | 1.00 | 1.00 | 2.19 | 1.08 | 1.77 | 2.23 | 1.15 | 1.65 | 1.35 |
| time (sec) | N/A | 0.008 | 0.009 | 0.007 | 3.041 | 1.074 | 1.092 | 1.260 | 0.122 | 0.073 |

| Problem 1448 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 67 | 118 | 0 | 176 | 0 | 58 | 63 | 61 |
| N.S. | 1 | 1.00 | 1.60 | 2.81 | 0.00 | 4.19 | 0.00 | 1.38 | 1.50 | 1.45 |
| time (sec) | N/A | 0.016 | 0.050 | 0.043 | 0.000 | 0.847 | 0.000 | 1.079 | 0.515 | 0.120 |

| Problem 1449 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | A | B | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 10 | 14 | 27 | 14 | 14 | 26 | 8 | 16 | 24 |
| N.S. | 1 | 1.00 | 1.40 | 2.70 | 1.40 | 1.40 | 2.60 | 0.80 | 1.60 | 2.40 |
| time (sec) | N/A | 0.007 | 0.010 | 0.005 | 2.993 | 0.982 | 0.992 | 1.099 | 0.290 | 0.041 |

| Problem 1450 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 31 | 21 | 21 | 44 | 13 | 27 | 38 |
| N.S. | 1 | 1.00 | 1.00 | 1.55 | 1.05 | 1.05 | 2.20 | 0.65 | 1.35 | 1.90 |
| time (sec) | N/A | 0.007 | 0.004 | 0.006 | 3.119 | 1.072 | 1.003 | 1.085 | 0.295 | 0.070 |

| Problem 1451 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 26 | 26 | 45 | 39 | 11 | 44 | 58 | 21 | 40 | 36 |
| N.S. | 1 | 1.00 | 1.73 | 1.50 | 0.42 | 1.69 | 2.23 | 0.81 | 1.54 | 1.38 |
| time (sec) | N/A | 0.009 | 0.012 | 0.006 | 3.000 | 0.772 | 1.069 | 0.934 | 0.080 | 0.055 |

| Problem 1452 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 103 | 84 | 0 | 185 | 0 | 54 | 44 | 43 |
| N.S. | 1 | 1.00 | 2.40 | 1.95 | 0.00 | 4.30 | 0.00 | 1.26 | 1.02 | 1.00 |
| time (sec) | N/A | 0.028 | 0.081 | 0.009 | 0.000 | 0.728 | 0.000 | 1.162 | 0.343 | 0.093 |

| Problem 1453 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 219 | 219 | 73 | 0 | 0 | 717 | 0 | 0 | -1 | 294 |
| N.S. | 1 | 1.00 | 0.33 | 0.00 | 0.00 | 3.27 | 0.00 | 0.00 | -0.00 | 1.34 |
| time (sec) | N/A | 0.088 | 0.034 | 0.060 | 0.000 | 0.885 | 0.000 | 0.000 | 0.000 | 0.495 |

| Problem 1454 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 172 | 172 | 73 | 0 | 0 | 596 | 0 | 0 | -1 | 300 |
| N.S. | 1 | 1.00 | 0.42 | 0.00 | 0.00 | 3.47 | 0.00 | 0.00 | -0.01 | 1.74 |
| time (sec) | N/A | 0.046 | 0.025 | 0.033 | 0.000 | 0.978 | 0.000 | 0.000 | 0.000 | 7.149 |

| Problem 1455 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 149 | 149 | 71 | 0 | 0 | 233 | 0 | 0 | -1 | 200 |
| N.S. | 1 | 1.00 | 0.48 | 0.00 | 0.00 | 1.56 | 0.00 | 0.00 | -0.01 | 1.34 |
| time (sec) | N/A | 0.033 | 0.028 | 0.084 | 0.000 | 1.065 | 0.000 | 0.000 | 0.000 | 0.161 |

| Problem 1456 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 65 | 0 | 0 | 92 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 2.03 | 0.00 | 0.00 | 2.88 | 1.00 |
| time (sec) | N/A | 0.003 | 0.013 | 0.005 | 0.000 | 0.992 | 0.000 | 0.000 | 0.713 | 0.040 |

| Problem 1457 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 175 | 0 | 0 | 127 | 57 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 2.65 | 0.00 | 0.00 | 1.92 | 0.86 |
| time (sec) | N/A | 0.009 | 0.023 | 0.006 | 0.000 | 1.078 | 0.000 | 0.000 | 1.032 | 0.106 |

| Problem 1458 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 337 | 0 | 0 | 203 | 73 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 3.34 | 0.00 | 0.00 | 2.01 | 0.72 |
| time (sec) | N/A | 0.017 | 0.040 | 0.008 | 0.000 | 0.868 | 0.000 | 0.000 | 1.017 | 0.121 |

| Problem 1459 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 533 | 0 | 0 | 293 | 95 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 3.92 | 0.00 | 0.00 | 2.15 | 0.70 |
| time (sec) | N/A | 0.028 | 0.058 | 0.009 | 0.000 | 1.110 | 0.000 | 0.000 | 1.152 | 0.131 |

| Problem 1460 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 216 | 216 | 73 | 0 | 0 | 740 | 0 | 0 | -1 | 292 |
| N.S. | 1 | 1.00 | 0.34 | 0.00 | 0.00 | 3.43 | 0.00 | 0.00 | -0.00 | 1.35 |
| time (sec) | N/A | 0.090 | 0.030 | 0.063 | 0.000 | 1.072 | 0.000 | 0.000 | 0.000 | 0.497 |

| Problem 1461 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 171 | 171 | 73 | 0 | 0 | 618 | 0 | 0 | -1 | 297 |
| N.S. | 1 | 1.00 | 0.43 | 0.00 | 0.00 | 3.61 | 0.00 | 0.00 | -0.01 | 1.74 |
| time (sec) | N/A | 0.041 | 0.029 | 0.035 | 0.000 | 1.129 | 0.000 | 0.000 | 0.000 | 7.305 |

| Problem 1462 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 126 | 126 | 71 | 0 | 0 | 519 | 0 | 0 | -1 | 177 |
| N.S. | 1 | 1.00 | 0.56 | 0.00 | 0.00 | 4.12 | 0.00 | 0.00 | -0.01 | 1.40 |
| time (sec) | N/A | 0.014 | 0.029 | 0.086 | 0.000 | 0.715 | 0.000 | 0.000 | 0.000 | 0.140 |

| Problem 1463 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 42 | 0 | 0 | -1 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 1.31 | 0.00 | 0.00 | -0.03 | 1.00 |
| time (sec) | N/A | 0.003 | 0.012 | 0.006 | 0.000 | 1.320 | 0.000 | 0.000 | 0.000 | 0.040 |

| Problem 1464 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 118 | 0 | 0 | -1 | 51 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 1.79 | 0.00 | 0.00 | -0.02 | 0.77 |
| time (sec) | N/A | 0.009 | 0.017 | 0.004 | 0.000 | 1.397 | 0.000 | 0.000 | 0.000 | 0.159 |

| Problem 1465 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 251 | 0 | 0 | -1 | 73 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 2.49 | 0.00 | 0.00 | -0.01 | 0.72 |
| time (sec) | N/A | 0.018 | 0.034 | 0.007 | 0.000 | 1.588 | 0.000 | 0.000 | 0.000 | 0.173 |

| Problem 1466 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 420 | 0 | 0 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 3.09 | 0.00 | 0.00 | -0.01 | 0.70 |
| time (sec) | N/A | 0.028 | 0.048 | 0.009 | 0.000 | 1.407 | 0.000 | 0.000 | 0.000 | 0.177 |

| Problem 1467 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 216 | 216 | 73 | 0 | 0 | 741 | 0 | 0 | -1 | 285 |
| N.S. | 1 | 1.00 | 0.34 | 0.00 | 0.00 | 3.43 | 0.00 | 0.00 | -0.00 | 1.32 |
| time (sec) | N/A | 0.080 | 0.035 | 0.038 | 0.000 | 1.578 | 0.000 | 0.000 | 0.000 | 0.297 |

| Problem 1468 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 169 | 169 | 73 | 0 | 0 | 619 | 0 | 0 | -1 | 298 |
| N.S. | 1 | 1.00 | 0.43 | 0.00 | 0.00 | 3.66 | 0.00 | 0.00 | -0.01 | 1.76 |
| time (sec) | N/A | 0.041 | 0.027 | 0.033 | 0.000 | 1.529 | 0.000 | 0.000 | 0.000 | 7.896 |

| Problem 1469 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 126 | 126 | 73 | 0 | 0 | 521 | 0 | 0 | -1 | 177 |
| N.S. | 1 | 1.00 | 0.58 | 0.00 | 0.00 | 4.13 | 0.00 | 0.00 | -0.01 | 1.40 |
| time (sec) | N/A | 0.015 | 0.030 | 0.089 | 0.000 | 1.236 | 0.000 | 0.000 | 0.000 | 0.168 |

| Problem 1470 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 27 | 0 | 42 | 0 | 0 | 26 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 1.40 | 0.00 | 0.00 | 0.87 | 1.00 |
| time (sec) | N/A | 0.003 | 0.009 | 0.005 | 0.000 | 0.738 | 0.000 | 0.000 | 0.828 | 0.050 |

| Problem 1471 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 118 | 0 | 0 | 71 | 56 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 1.79 | 0.00 | 0.00 | 1.08 | 0.85 |
| time (sec) | N/A | 0.010 | 0.017 | 0.004 | 0.000 | 2.109 | 0.000 | 0.000 | 0.977 | 0.116 |

| Problem 1472 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 75 | 105 | 0 | 251 | 0 | 0 | 133 | 83 |
| N.S. | 1 | 1.00 | 0.74 | 1.04 | 0.00 | 2.49 | 0.00 | 0.00 | 1.32 | 0.82 |
| time (sec) | N/A | 0.017 | 0.033 | 0.007 | 0.000 | 1.108 | 0.000 | 0.000 | 1.508 | 0.115 |

| Problem 1473 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 116 | 171 | 0 | 419 | 0 | 0 | 209 | 95 |
| N.S. | 1 | 1.00 | 0.85 | 1.26 | 0.00 | 3.08 | 0.00 | 0.00 | 1.54 | 0.70 |
| time (sec) | N/A | 0.030 | 0.049 | 0.010 | 0.000 | 0.832 | 0.000 | 0.000 | 1.273 | 0.139 |

| Problem 1474 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 241 | 241 | 73 | 0 | 0 | 423 | 0 | 0 | -1 | 318 |
| N.S. | 1 | 1.00 | 0.30 | 0.00 | 0.00 | 1.76 | 0.00 | 0.00 | -0.00 | 1.32 |
| time (sec) | N/A | 0.107 | 0.062 | 0.098 | 0.000 | 1.042 | 0.000 | 0.000 | 0.000 | 0.500 |

| Problem 1475 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 195 | 195 | 73 | 0 | 0 | 306 | 0 | 0 | -1 | 326 |
| N.S. | 1 | 1.00 | 0.37 | 0.00 | 0.00 | 1.57 | 0.00 | 0.00 | -0.01 | 1.67 |
| time (sec) | N/A | 0.071 | 0.048 | 0.099 | 0.000 | 1.222 | 0.000 | 0.000 | 0.000 | 11.017 |

| Problem 1476 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 149 | 149 | 73 | 0 | 0 | 233 | 0 | 0 | -1 | 200 |
| N.S. | 1 | 1.00 | 0.49 | 0.00 | 0.00 | 1.56 | 0.00 | 0.00 | -0.01 | 1.34 |
| time (sec) | N/A | 0.031 | 0.044 | 0.082 | 0.000 | 1.458 | 0.000 | 0.000 | 0.000 | 0.160 |

| Problem 1477 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 27 | 0 | 42 | 0 | 0 | -1 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 1.40 | 0.00 | 0.00 | -0.03 | 1.00 |
| time (sec) | N/A | 0.003 | 0.007 | 0.004 | 0.000 | 0.760 | 0.000 | 0.000 | 0.000 | 0.050 |

| Problem 1478 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 45 | 53 | 0 | 126 | 0 | 0 | -1 | 49 |
| N.S. | 1 | 1.00 | 0.68 | 0.80 | 0.00 | 1.91 | 0.00 | 0.00 | -0.02 | 0.74 |
| time (sec) | N/A | 0.009 | 0.020 | 0.005 | 0.000 | 1.212 | 0.000 | 0.000 | 0.000 | 0.112 |

| Problem 1479 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 75 | 105 | 0 | 273 | 0 | 0 | -1 | 73 |
| N.S. | 1 | 1.00 | 0.74 | 1.04 | 0.00 | 2.70 | 0.00 | 0.00 | -0.01 | 0.72 |
| time (sec) | N/A | 0.018 | 0.031 | 0.007 | 0.000 | 1.106 | 0.000 | 0.000 | 0.000 | 0.120 |

| Problem 1480 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 116 | 171 | 0 | 456 | 0 | 0 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.85 | 1.26 | 0.00 | 3.35 | 0.00 | 0.00 | -0.01 | 0.70 |
| time (sec) | N/A | 0.030 | 0.047 | 0.009 | 0.000 | 1.521 | 0.000 | 0.000 | 0.000 | 0.134 |

| Problem 1481 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | C | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 77 | 77 | 48 | 573 | 0 | 107 | 39 | 0 | -1 | 113 |
| N.S. | 1 | 1.00 | 0.62 | 7.44 | 0.00 | 1.39 | 0.51 | 0.00 | -0.01 | 1.47 |
| time (sec) | N/A | 0.014 | 0.019 | 0.377 | 0.000 | 1.352 | 2.548 | 0.000 | 0.000 | 0.218 |

| Problem 1482 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 205 | 205 | 73 | 0 | 0 | 2151 | 0 | 0 | -1 | 218 |
| N.S. | 1 | 1.00 | 0.36 | 0.00 | 0.00 | 10.49 | 0.00 | 0.00 | -0.00 | 1.06 |
| time (sec) | N/A | 0.139 | 0.059 | 0.080 | 0.000 | 1.557 | 0.000 | 0.000 | 0.000 | 0.532 |

| Problem 1483 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 167 | 167 | 73 | 0 | 0 | 1468 | 0 | 0 | -1 | 189 |
| N.S. | 1 | 1.00 | 0.44 | 0.00 | 0.00 | 8.79 | 0.00 | 0.00 | -0.01 | 1.13 |
| time (sec) | N/A | 0.102 | 0.043 | 0.045 | 0.000 | 1.342 | 0.000 | 0.000 | 0.000 | 0.382 |

| Problem 1484 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 152 | 152 | 71 | 0 | 0 | 857 | 0 | 0 | -1 | 244 |
| N.S. | 1 | 1.00 | 0.47 | 0.00 | 0.00 | 5.64 | 0.00 | 0.00 | -0.01 | 1.61 |
| time (sec) | N/A | 0.098 | 0.047 | 0.105 | 0.000 | 1.040 | 0.000 | 0.000 | 0.000 | 13.905 |

| Problem 1485 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 134 | 134 | 73 | 0 | 0 | 368 | 0 | 0 | -1 | 134 |
| N.S. | 1 | 1.00 | 0.54 | 0.00 | 0.00 | 2.75 | 0.00 | 0.00 | -0.01 | 1.00 |
| time (sec) | N/A | 0.088 | 0.053 | 0.092 | 0.000 | 1.170 | 0.000 | 0.000 | 0.000 | 0.209 |

| Problem 1486 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 104 | 0 | 0 | 99 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 3.25 | 0.00 | 0.00 | 3.09 | 1.00 |
| time (sec) | N/A | 0.003 | 0.015 | 0.004 | 0.000 | 1.034 | 0.000 | 0.000 | 0.810 | 0.055 |

| Problem 1487 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 235 | 0 | 0 | 178 | 51 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 3.56 | 0.00 | 0.00 | 2.70 | 0.77 |
| time (sec) | N/A | 0.009 | 0.027 | 0.006 | 0.000 | 1.236 | 0.000 | 0.000 | 0.955 | 0.175 |

| Problem 1488 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 426 | 0 | 0 | 268 | 73 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 4.22 | 0.00 | 0.00 | 2.65 | 0.72 |
| time (sec) | N/A | 0.017 | 0.049 | 0.006 | 0.000 | 1.523 | 0.000 | 0.000 | 1.126 | 0.182 |

| Problem 1489 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 649 | 0 | 0 | 376 | 95 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 4.77 | 0.00 | 0.00 | 2.76 | 0.70 |
| time (sec) | N/A | 0.029 | 0.067 | 0.013 | 0.000 | 1.656 | 0.000 | 0.000 | 1.360 | 0.194 |

| Problem 1490 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 167 | 167 | 73 | 0 | 0 | 1468 | 0 | 0 | -1 | 189 |
| N.S. | 1 | 1.00 | 0.44 | 0.00 | 0.00 | 8.79 | 0.00 | 0.00 | -0.01 | 1.13 |
| time (sec) | N/A | 0.102 | 0.031 | 0.077 | 0.000 | 1.148 | 0.000 | 0.000 | 0.000 | 0.401 |

| Problem 1491 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 127 | 127 | 73 | 0 | 0 | 814 | 0 | 0 | -1 | 176 |
| N.S. | 1 | 1.00 | 0.57 | 0.00 | 0.00 | 6.41 | 0.00 | 0.00 | -0.01 | 1.39 |
| time (sec) | N/A | 0.074 | 0.026 | 0.042 | 0.000 | 1.133 | 0.000 | 0.000 | 0.000 | 7.844 |

| Problem 1492 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 71 | 0 | 0 | 234 | 0 | 0 | -1 | 85 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 2.75 | 0.00 | 0.00 | -0.01 | 1.00 |
| time (sec) | N/A | 0.058 | 0.029 | 0.092 | 0.000 | 0.747 | 0.000 | 0.000 | 0.000 | 0.122 |

| Problem 1493 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 42 | 0 | 0 | -1 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 1.31 | 0.00 | 0.00 | -0.03 | 1.00 |
| time (sec) | N/A | 0.003 | 0.012 | 0.005 | 0.000 | 1.096 | 0.000 | 0.000 | 0.000 | 0.040 |

| Problem 1494 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 118 | 0 | 0 | -1 | 51 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 1.79 | 0.00 | 0.00 | -0.02 | 0.77 |
| time (sec) | N/A | 0.009 | 0.017 | 0.005 | 0.000 | 1.300 | 0.000 | 0.000 | 0.000 | 0.159 |

| Problem 1495 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 252 | 0 | 0 | -1 | 73 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 2.50 | 0.00 | 0.00 | -0.01 | 0.72 |
| time (sec) | N/A | 0.017 | 0.033 | 0.009 | 0.000 | 2.549 | 0.000 | 0.000 | 0.000 | 0.165 |

| Problem 1496 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 419 | 0 | 0 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 3.08 | 0.00 | 0.00 | -0.01 | 0.70 |
| time (sec) | N/A | 0.031 | 0.049 | 0.010 | 0.000 | 5.098 | 0.000 | 0.000 | 0.000 | 0.178 |

| Problem 1497 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 167 | 167 | 73 | 0 | 0 | 1457 | 0 | 0 | -1 | 182 |
| N.S. | 1 | 1.00 | 0.44 | 0.00 | 0.00 | 8.72 | 0.00 | 0.00 | -0.01 | 1.09 |
| time (sec) | N/A | 0.105 | 0.035 | 0.046 | 0.000 | 1.026 | 0.000 | 0.000 | 0.000 | 0.273 |

| Problem 1498 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 127 | 127 | 73 | 0 | 0 | 808 | 0 | 0 | -1 | 176 |
| N.S. | 1 | 1.00 | 0.57 | 0.00 | 0.00 | 6.36 | 0.00 | 0.00 | -0.01 | 1.39 |
| time (sec) | N/A | 0.081 | 0.028 | 0.042 | 0.000 | 1.416 | 0.000 | 0.000 | 0.000 | 8.418 |

| Problem 1499 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 73 | 0 | 0 | 234 | 0 | 0 | -1 | 85 |
| N.S. | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 2.75 | 0.00 | 0.00 | -0.01 | 1.00 |
| time (sec) | N/A | 0.067 | 0.031 | 0.092 | 0.000 | 1.238 | 0.000 | 0.000 | 0.000 | 0.116 |

| Problem 1500 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 27 | 0 | 42 | 0 | 0 | 26 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 1.40 | 0.00 | 0.00 | 0.87 | 1.00 |
| time (sec) | N/A | 0.003 | 0.007 | 0.005 | 0.000 | 1.171 | 0.000 | 0.000 | 0.706 | 0.051 |

| Problem 1501 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 118 | 0 | 0 | 71 | 56 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 1.79 | 0.00 | 0.00 | 1.08 | 0.85 |
| time (sec) | N/A | 0.009 | 0.015 | 0.005 | 0.000 | 1.214 | 0.000 | 0.000 | 0.868 | 0.115 |

| Problem 1502 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 75 | 105 | 0 | 251 | 0 | 0 | 133 | 83 |
| N.S. | 1 | 1.00 | 0.74 | 1.04 | 0.00 | 2.49 | 0.00 | 0.00 | 1.32 | 0.82 |
| time (sec) | N/A | 0.017 | 0.033 | 0.007 | 0.000 | 0.877 | 0.000 | 0.000 | 1.020 | 0.118 |

| Problem 1503 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 116 | 171 | 0 | 419 | 0 | 0 | 209 | 109 |
| N.S. | 1 | 1.00 | 0.85 | 1.26 | 0.00 | 3.08 | 0.00 | 0.00 | 1.54 | 0.80 |
| time (sec) | N/A | 0.028 | 0.048 | 0.010 | 0.000 | 1.317 | 0.000 | 0.000 | 1.260 | 0.134 |

| Problem 1504 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 152 | 152 | 73 | 0 | 0 | 857 | 0 | 0 | -1 | 200 |
| N.S. | 1 | 1.00 | 0.48 | 0.00 | 0.00 | 5.64 | 0.00 | 0.00 | -0.01 | 1.32 |
| time (sec) | N/A | 0.087 | 0.050 | 0.106 | 0.000 | 1.217 | 0.000 | 0.000 | 0.000 | 14.156 |

| Problem 1505 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 108 | 108 | 73 | 0 | 0 | 273 | 0 | 0 | -1 | 108 |
| N.S. | 1 | 1.00 | 0.68 | 0.00 | 0.00 | 2.53 | 0.00 | 0.00 | -0.01 | 1.00 |
| time (sec) | N/A | 0.065 | 0.044 | 0.089 | 0.000 | 1.716 | 0.000 | 0.000 | 0.000 | 0.125 |

| Problem 1506 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 27 | 0 | 42 | 0 | 0 | -1 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 1.40 | 0.00 | 0.00 | -0.03 | 1.00 |
| time (sec) | N/A | 0.003 | 0.008 | 0.005 | 0.000 | 0.772 | 0.000 | 0.000 | 0.000 | 0.050 |

| Problem 1507 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 45 | 53 | 0 | 126 | 0 | 0 | -1 | 49 |
| N.S. | 1 | 1.00 | 0.68 | 0.80 | 0.00 | 1.91 | 0.00 | 0.00 | -0.02 | 0.74 |
| time (sec) | N/A | 0.008 | 0.017 | 0.006 | 0.000 | 1.167 | 0.000 | 0.000 | 0.000 | 0.112 |

| Problem 1508 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 76 | 105 | 0 | 273 | 0 | 0 | -1 | 73 |
| N.S. | 1 | 1.00 | 0.75 | 1.04 | 0.00 | 2.70 | 0.00 | 0.00 | -0.01 | 0.72 |
| time (sec) | N/A | 0.017 | 0.033 | 0.007 | 0.000 | 1.386 | 0.000 | 0.000 | 0.000 | 0.128 |

| Problem 1509 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 116 | 171 | 0 | 457 | 0 | 0 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.85 | 1.26 | 0.00 | 3.36 | 0.00 | 0.00 | -0.01 | 0.70 |
| time (sec) | N/A | 0.027 | 0.043 | 0.010 | 0.000 | 2.577 | 0.000 | 0.000 | 0.000 | 0.132 |

| Problem 1510 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 279 | 279 | 65 | 0 | 0 | 247 | 0 | 0 | -1 | 176 |
| N.S. | 1 | 1.00 | 0.23 | 0.00 | 0.00 | 0.89 | 0.00 | 0.00 | -0.00 | 0.63 |
| time (sec) | N/A | 0.304 | 0.037 | 0.088 | 0.000 | 1.107 | 0.000 | 0.000 | 0.000 | 0.220 |

| Problem 1511 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 193 | 193 | 42 | 0 | 0 | 448 | 0 | 0 | -1 | 123 |
| N.S. | 1 | 1.00 | 0.22 | 0.00 | 0.00 | 2.32 | 0.00 | 0.00 | -0.01 | 0.64 |
| time (sec) | N/A | 0.138 | 0.010 | 0.082 | 0.000 | 0.920 | 0.000 | 0.000 | 0.000 | 0.121 |

| Problem 1512 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 427 | 427 | 73 | 0 | 0 | 5633 | 0 | 0 | -1 | 364 |
| N.S. | 1 | 1.00 | 0.17 | 0.00 | 0.00 | 13.19 | 0.00 | 0.00 | -0.00 | 0.85 |
| time (sec) | N/A | 0.643 | 0.053 | 0.105 | 0.000 | 1.537 | 0.000 | 0.000 | 0.000 | 0.770 |

| Problem 1513 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 378 | 378 | 73 | 0 | 0 | 3025 | 0 | 0 | -1 | 385 |
| N.S. | 1 | 1.00 | 0.19 | 0.00 | 0.00 | 8.00 | 0.00 | 0.00 | -0.00 | 1.02 |
| time (sec) | N/A | 0.497 | 0.041 | 0.052 | 0.000 | 1.395 | 0.000 | 0.000 | 0.000 | 12.199 |

| Problem 1514 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 332 | 332 | 73 | 0 | 0 | 663 | 0 | 0 | -1 | 256 |
| N.S. | 1 | 1.00 | 0.22 | 0.00 | 0.00 | 2.00 | 0.00 | 0.00 | -0.00 | 0.77 |
| time (sec) | N/A | 0.487 | 0.056 | 0.097 | 0.000 | 1.002 | 0.000 | 0.000 | 0.000 | 0.238 |

| Problem 1515 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 65 | 0 | 0 | 130 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 2.03 | 0.00 | 0.00 | 4.06 | 1.00 |
| time (sec) | N/A | 0.003 | 0.015 | 0.006 | 0.000 | 1.169 | 0.000 | 0.000 | 0.565 | 0.042 |

| Problem 1516 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 175 | 0 | 0 | 137 | 51 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 2.65 | 0.00 | 0.00 | 2.08 | 0.77 |
| time (sec) | N/A | 0.009 | 0.034 | 0.006 | 0.000 | 0.930 | 0.000 | 0.000 | 0.750 | 0.155 |

| Problem 1517 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 338 | 0 | 0 | 213 | 73 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 3.35 | 0.00 | 0.00 | 2.11 | 0.72 |
| time (sec) | N/A | 0.021 | 0.057 | 0.009 | 0.000 | 0.753 | 0.000 | 0.000 | 0.949 | 0.165 |

| Problem 1518 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 533 | 0 | 0 | 302 | 95 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 3.92 | 0.00 | 0.00 | 2.22 | 0.70 |
| time (sec) | N/A | 0.034 | 0.082 | 0.011 | 0.000 | 1.105 | 0.000 | 0.000 | 1.147 | 0.173 |

| Problem 1519 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 427 | 427 | 73 | 0 | 0 | 5633 | 0 | 0 | -1 | 365 |
| N.S. | 1 | 1.00 | 0.17 | 0.00 | 0.00 | 13.19 | 0.00 | 0.00 | -0.00 | 0.85 |
| time (sec) | N/A | 0.642 | 0.047 | 0.098 | 0.000 | 1.471 | 0.000 | 0.000 | 0.000 | 0.746 |

| Problem 1520 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 378 | 378 | 73 | 0 | 0 | 2997 | 0 | 0 | -1 | 385 |
| N.S. | 1 | 1.00 | 0.19 | 0.00 | 0.00 | 7.93 | 0.00 | 0.00 | -0.00 | 1.02 |
| time (sec) | N/A | 0.557 | 0.037 | 0.050 | 0.000 | 1.254 | 0.000 | 0.000 | 0.000 | 17.905 |

| Problem 1521 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 334 | 334 | 73 | 0 | 0 | 755 | 0 | 0 | -1 | 258 |
| N.S. | 1 | 1.00 | 0.22 | 0.00 | 0.00 | 2.26 | 0.00 | 0.00 | -0.00 | 0.77 |
| time (sec) | N/A | 0.561 | 0.072 | 0.092 | 0.000 | 1.175 | 0.000 | 0.000 | 0.000 | 0.330 |

| Problem 1522 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 65 | 0 | 0 | 130 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 2.03 | 0.00 | 0.00 | 4.06 | 1.00 |
| time (sec) | N/A | 0.003 | 0.011 | 0.004 | 0.000 | 0.771 | 0.000 | 0.000 | 0.587 | 0.054 |

| Problem 1523 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 175 | 0 | 0 | 137 | 51 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 2.65 | 0.00 | 0.00 | 2.08 | 0.77 |
| time (sec) | N/A | 0.009 | 0.032 | 0.005 | 0.000 | 1.139 | 0.000 | 0.000 | 0.744 | 0.171 |

| Problem 1524 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 338 | 0 | 0 | 214 | 73 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 3.35 | 0.00 | 0.00 | 2.12 | 0.72 |
| time (sec) | N/A | 0.020 | 0.055 | 0.008 | 0.000 | 1.156 | 0.000 | 0.000 | 0.944 | 0.183 |

| Problem 1525 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 533 | 0 | 0 | 303 | 95 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 3.92 | 0.00 | 0.00 | 2.23 | 0.70 |
| time (sec) | N/A | 0.033 | 0.089 | 0.010 | 0.000 | 1.179 | 0.000 | 0.000 | 1.158 | 0.196 |

| Problem 1526 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 424 | 424 | 73 | 0 | 0 | 5633 | 0 | 0 | -1 | 363 |
| N.S. | 1 | 1.00 | 0.17 | 0.00 | 0.00 | 13.29 | 0.00 | 0.00 | -0.00 | 0.86 |
| time (sec) | N/A | 0.552 | 0.039 | 0.058 | 0.000 | 1.557 | 0.000 | 0.000 | 0.000 | 0.779 |

| Problem 1527 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 403 | 403 | 73 | 0 | 0 | 3084 | 0 | 0 | -1 | 417 |
| N.S. | 1 | 1.00 | 0.18 | 0.00 | 0.00 | 7.65 | 0.00 | 0.00 | -0.00 | 1.03 |
| time (sec) | N/A | 0.526 | 0.066 | 0.136 | 0.000 | 1.431 | 0.000 | 0.000 | 0.000 | 23.487 |

| Problem 1528 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F(-1) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 358 | 358 | 73 | 0 | 0 | 855 | 0 | 0 | -1 | 282 |
| N.S. | 1 | 1.00 | 0.20 | 0.00 | 0.00 | 2.39 | 0.00 | 0.00 | -0.00 | 0.79 |
| time (sec) | N/A | 0.501 | 0.085 | 0.096 | 0.000 | 1.463 | 0.000 | 0.000 | 0.000 | 0.334 |

| Problem 1529 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 104 | 0 | 0 | 199 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 3.25 | 0.00 | 0.00 | 6.22 | 1.00 |
| time (sec) | N/A | 0.003 | 0.023 | 0.006 | 0.000 | 1.335 | 0.000 | 0.000 | 0.756 | 0.055 |

| Problem 1530 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 235 | 0 | 0 | 189 | 51 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 3.56 | 0.00 | 0.00 | 2.86 | 0.77 |
| time (sec) | N/A | 0.009 | 0.036 | 0.004 | 0.000 | 1.343 | 0.000 | 0.000 | 0.907 | 0.178 |

| Problem 1531 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 427 | 0 | 0 | 278 | 73 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 4.23 | 0.00 | 0.00 | 2.75 | 0.72 |
| time (sec) | N/A | 0.018 | 0.060 | 0.008 | 0.000 | 1.370 | 0.000 | 0.000 | 1.144 | 0.202 |

| Problem 1532 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 649 | 0 | 0 | 385 | 95 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 4.77 | 0.00 | 0.00 | 2.83 | 0.70 |
| time (sec) | N/A | 0.030 | 0.094 | 0.010 | 0.000 | 1.394 | 0.000 | 0.000 | 1.429 | 0.230 |

| Problem 1533 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 424 | 424 | 73 | 0 | 0 | 5633 | 0 | 0 | -1 | 363 |
| N.S. | 1 | 1.00 | 0.17 | 0.00 | 0.00 | 13.29 | 0.00 | 0.00 | -0.00 | 0.86 |
| time (sec) | N/A | 0.610 | 0.054 | 0.056 | 0.000 | 2.014 | 0.000 | 0.000 | 0.000 | 0.791 |

| Problem 1534 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 378 | 378 | 73 | 0 | 0 | 3025 | 0 | 0 | -1 | 388 |
| N.S. | 1 | 1.00 | 0.19 | 0.00 | 0.00 | 8.00 | 0.00 | 0.00 | -0.00 | 1.03 |
| time (sec) | N/A | 0.560 | 0.036 | 0.051 | 0.000 | 1.850 | 0.000 | 0.000 | 0.000 | 16.568 |

| Problem 1535 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 309 | 309 | 73 | 0 | 0 | 620 | 0 | 0 | -1 | 233 |
| N.S. | 1 | 1.00 | 0.24 | 0.00 | 0.00 | 2.01 | 0.00 | 0.00 | -0.00 | 0.75 |
| time (sec) | N/A | 0.509 | 0.047 | 0.098 | 0.000 | 0.932 | 0.000 | 0.000 | 0.000 | 0.217 |

| Problem 1536 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 32 | 27 | 0 | 42 | 0 | 0 | 27 | 32 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.00 | 1.31 | 0.00 | 0.00 | 0.84 | 1.00 |
| time (sec) | N/A | 0.003 | 0.013 | 0.004 | 0.000 | 0.700 | 0.000 | 0.000 | 0.763 | 0.041 |

| Problem 1537 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 54 | 0 | 118 | 0 | 0 | 127 | 51 |
| N.S. | 1 | 1.00 | 0.70 | 0.82 | 0.00 | 1.79 | 0.00 | 0.00 | 1.92 | 0.77 |
| time (sec) | N/A | 0.010 | 0.027 | 0.005 | 0.000 | 1.381 | 0.000 | 0.000 | 0.860 | 0.106 |

| Problem 1538 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 252 | 0 | 0 | 203 | 73 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 2.50 | 0.00 | 0.00 | 2.01 | 0.72 |
| time (sec) | N/A | 0.020 | 0.045 | 0.008 | 0.000 | 1.372 | 0.000 | 0.000 | 1.033 | 0.117 |

| Problem 1539 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 420 | 0 | 0 | 292 | 95 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 3.09 | 0.00 | 0.00 | 2.15 | 0.70 |
| time (sec) | N/A | 0.030 | 0.068 | 0.010 | 0.000 | 1.437 | 0.000 | 0.000 | 1.200 | 0.123 |

| Problem 1540 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 424 | 424 | 71 | 0 | 0 | 5591 | 0 | 0 | -1 | 356 |
| N.S. | 1 | 1.00 | 0.17 | 0.00 | 0.00 | 13.19 | 0.00 | 0.00 | -0.00 | 0.84 |
| time (sec) | N/A | 0.549 | 0.056 | 0.059 | 0.000 | 2.183 | 0.000 | 0.000 | 0.000 | 0.428 |

| Problem 1541 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 378 | 378 | 71 | 0 | 0 | 2997 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.19 | 0.00 | 0.00 | 7.93 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.479 | 0.034 | 0.055 | 0.000 | 1.834 | 0.000 | 0.000 | 0.000 | 106.342 |

| Problem 1542 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 309 | 309 | 71 | 0 | 0 | 620 | 0 | 0 | -1 | 233 |
| N.S. | 1 | 1.00 | 0.23 | 0.00 | 0.00 | 2.01 | 0.00 | 0.00 | -0.00 | 0.75 |
| time (sec) | N/A | 0.444 | 0.042 | 0.099 | 0.000 | 1.678 | 0.000 | 0.000 | 0.000 | 0.224 |

| Problem 1543 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 27 | 0 | 42 | 0 | 0 | -1 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 1.40 | 0.00 | 0.00 | -0.03 | 1.00 |
| time (sec) | N/A | 0.003 | 0.010 | 0.004 | 0.000 | 1.350 | 0.000 | 0.000 | 0.000 | 0.054 |

| Problem 1544 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 46 | 53 | 0 | 118 | 0 | 0 | -1 | 57 |
| N.S. | 1 | 1.00 | 0.70 | 0.80 | 0.00 | 1.79 | 0.00 | 0.00 | -0.02 | 0.86 |
| time (sec) | N/A | 0.009 | 0.016 | 0.006 | 0.000 | 1.441 | 0.000 | 0.000 | 0.000 | 0.118 |

| Problem 1545 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 77 | 105 | 0 | 252 | 0 | 0 | -1 | 83 |
| N.S. | 1 | 1.00 | 0.76 | 1.04 | 0.00 | 2.50 | 0.00 | 0.00 | -0.01 | 0.82 |
| time (sec) | N/A | 0.018 | 0.036 | 0.008 | 0.000 | 1.238 | 0.000 | 0.000 | 0.000 | 0.125 |

| Problem 1546 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 118 | 171 | 0 | 420 | 0 | 0 | -1 | 109 |
| N.S. | 1 | 1.00 | 0.87 | 1.26 | 0.00 | 3.09 | 0.00 | 0.00 | -0.01 | 0.80 |
| time (sec) | N/A | 0.031 | 0.060 | 0.010 | 0.000 | 1.114 | 0.000 | 0.000 | 0.000 | 0.132 |

| Problem 1547 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F(-1) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 449 | 449 | 71 | 0 | 0 | 5690 | 0 | 0 | -1 | 389 |
| N.S. | 1 | 1.00 | 0.16 | 0.00 | 0.00 | 12.67 | 0.00 | 0.00 | -0.00 | 0.87 |
| time (sec) | N/A | 0.660 | 0.079 | 0.124 | 0.000 | 2.225 | 0.000 | 0.000 | 0.000 | 0.811 |

| Problem 1548 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 403 | 403 | 71 | 0 | 0 | 3084 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.18 | 0.00 | 0.00 | 7.65 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.596 | 0.055 | 0.126 | 0.000 | 2.008 | 0.000 | 0.000 | 0.000 | 156.135 |

| Problem 1549 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 332 | 332 | 71 | 0 | 0 | 663 | 0 | 0 | -1 | 256 |
| N.S. | 1 | 1.00 | 0.21 | 0.00 | 0.00 | 2.00 | 0.00 | 0.00 | -0.00 | 0.77 |
| time (sec) | N/A | 0.541 | 0.031 | 0.095 | 0.000 | 1.582 | 0.000 | 0.000 | 0.000 | 0.259 |

| Problem 1550 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 30 | 30 | 30 | 27 | 0 | 42 | 0 | 0 | 26 | 30 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.00 | 1.40 | 0.00 | 0.00 | 0.87 | 1.00 |
| time (sec) | N/A | 0.003 | 0.010 | 0.006 | 0.000 | 1.434 | 0.000 | 0.000 | 0.680 | 0.051 |

| Problem 1551 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 45 | 53 | 0 | 126 | 0 | 0 | 72 | 49 |
| N.S. | 1 | 1.00 | 0.70 | 0.83 | 0.00 | 1.97 | 0.00 | 0.00 | 1.12 | 0.77 |
| time (sec) | N/A | 0.011 | 0.027 | 0.007 | 0.000 | 1.330 | 0.000 | 0.000 | 0.834 | 0.123 |

| Problem 1552 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 77 | 105 | 0 | 273 | 0 | 0 | 132 | 73 |
| N.S. | 1 | 1.00 | 0.79 | 1.07 | 0.00 | 2.79 | 0.00 | 0.00 | 1.35 | 0.74 |
| time (sec) | N/A | 0.020 | 0.040 | 0.007 | 0.000 | 1.569 | 0.000 | 0.000 | 0.960 | 0.134 |

| Problem 1553 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 134 | 134 | 118 | 171 | 0 | 457 | 0 | 0 | 209 | 95 |
| N.S. | 1 | 1.00 | 0.88 | 1.28 | 0.00 | 3.41 | 0.00 | 0.00 | 1.56 | 0.71 |
| time (sec) | N/A | 0.033 | 0.063 | 0.012 | 0.000 | 1.489 | 0.000 | 0.000 | 1.146 | 0.141 |

| Problem 1554 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 12 | 106 | 17 | 20 | 23 | 11 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 9.64 | 1.55 | 1.82 | 2.09 | 1.00 | 0.00 |
| time (sec) | N/A | 0.002 | 0.013 | 0.003 | 1.123 | 1.189 | 0.269 | 0.996 | 0.462 | 0.050 |

| Problem 1555 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 94 | 389 | 246 | 497 | 4058 | 833 | 478 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 3.54 | 2.24 | 4.52 | 36.89 | 7.57 | 4.35 | 0.00 |
| time (sec) | N/A | 0.055 | 0.107 | 0.009 | 1.166 | 1.257 | 4.666 | 1.036 | 0.942 | 0.049 |

| Problem 1556 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 78 | 78 | 67 | 159 | 138 | 235 | 1506 | 385 | 226 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 2.04 | 1.77 | 3.01 | 19.31 | 4.94 | 2.90 | 0.00 |
| time (sec) | N/A | 0.032 | 0.096 | 0.008 | 1.147 | 1.181 | 2.137 | 0.970 | 0.655 | 0.044 |

| Problem 1557 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 41 | 49 | 63 | 83 | 377 | 132 | 88 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.07 | 1.37 | 1.80 | 8.20 | 2.87 | 1.91 | 0.00 |
| time (sec) | N/A | 0.018 | 0.033 | 0.003 | 1.063 | 0.726 | 0.856 | 0.859 | 0.484 | 0.034 |

| Problem 1558 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 111 | 111 | 95 | 386 | 246 | 496 | 4058 | 833 | 478 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 3.48 | 2.22 | 4.47 | 36.56 | 7.50 | 4.31 | 0.00 |
| time (sec) | N/A | 0.057 | 0.110 | 0.009 | 1.295 | 1.035 | 4.439 | 0.979 | 0.913 | 0.047 |

| Problem 1559 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 78 | 78 | 67 | 159 | 138 | 237 | 1506 | 385 | 226 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 2.04 | 1.77 | 3.04 | 19.31 | 4.94 | 2.90 | 0.00 |
| time (sec) | N/A | 0.032 | 0.096 | 0.008 | 1.168 | 1.352 | 2.089 | 0.925 | 0.618 | 0.045 |

| Problem 1560 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 41 | 46 | 63 | 83 | 377 | 132 | 88 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.98 | 1.34 | 1.77 | 8.02 | 2.81 | 1.87 | 0.00 |
| time (sec) | N/A | 0.018 | 0.037 | 0.003 | 1.169 | 1.281 | 0.822 | 0.999 | 0.494 | 0.034 |

| Problem 1561 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 17 | 19 | 18 | 20 | 20 | 18 | 18 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 1.06 | 1.00 | 1.11 | 1.11 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.003 | 0.011 | 0.003 | 1.116 | 1.254 | 0.063 | 1.018 | 0.379 | 0.013 |

| Problem 1562 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 143 | 143 | 112 | 322 | 0 | 512 | 0 | 0 | 528 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 2.25 | 0.00 | 3.58 | 0.00 | 0.00 | 3.69 | 0.00 |
| time (sec) | N/A | 0.063 | 0.097 | 0.007 | 0.000 | 1.434 | 0.000 | 0.000 | 1.096 | 0.048 |

| Problem 1563 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 59 | 127 | 0 | 206 | 0 | 0 | 220 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 1.48 | 0.00 | 2.40 | 0.00 | 0.00 | 2.56 | 0.00 |
| time (sec) | N/A | 0.012 | 0.039 | 0.006 | 0.000 | 1.384 | 0.000 | 0.000 | 0.767 | 0.050 |

| Problem 1564 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-2) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 36 | 45 | 0 | 60 | 0 | 0 | 102 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.15 | 0.00 | 1.54 | 0.00 | 0.00 | 2.62 | 0.00 |
| time (sec) | N/A | 0.004 | 0.017 | 0.003 | 0.000 | 1.323 | 0.000 | 0.000 | 0.559 | 0.047 |

| Problem 1565 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-2) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 38 | 41 | 0 | 59 | 0 | 0 | 97 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 1.11 | 0.00 | 1.59 | 0.00 | 0.00 | 2.62 | 0.00 |
| time (sec) | N/A | 0.006 | 0.021 | 0.004 | 0.000 | 1.331 | 0.000 | 0.000 | 0.532 | 0.047 |

| Problem 1566 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 80 | 80 | 60 | 123 | 0 | 207 | 0 | 0 | 214 | 0 |
| N.S. | 1 | 1.00 | 0.75 | 1.54 | 0.00 | 2.59 | 0.00 | 0.00 | 2.68 | 0.00 |
| time (sec) | N/A | 0.020 | 0.033 | 0.004 | 0.000 | 1.277 | 0.000 | 0.000 | 0.736 | 0.046 |

| Problem 1567 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 131 | 131 | 113 | 318 | 0 | 509 | 0 | 0 | 525 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 2.43 | 0.00 | 3.89 | 0.00 | 0.00 | 4.01 | 0.00 |
| time (sec) | N/A | 0.046 | 0.084 | 0.008 | 0.000 | 1.015 | 0.000 | 0.000 | 0.992 | 0.048 |

| Problem 1568 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 186 | 186 | 195 | 661 | 0 | 959 | 0 | 0 | 944 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 3.55 | 0.00 | 5.16 | 0.00 | 0.00 | 5.08 | 0.00 |
| time (sec) | N/A | 0.086 | 0.124 | 0.010 | 0.000 | 1.405 | 0.000 | 0.000 | 1.642 | 0.046 |

| Problem 1569 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 36 | 36 | 36 | 42 | 0 | 58 | 0 | 0 | 98 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.17 | 0.00 | 1.61 | 0.00 | 0.00 | 2.72 | 0.00 |
| time (sec) | N/A | 0.005 | 0.013 | 0.005 | 0.000 | 1.323 | 0.000 | 0.000 | 0.556 | 0.047 |

| Problem 1570 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 79 | 79 | 59 | 124 | 0 | 205 | 0 | 0 | 214 | 0 |
| N.S. | 1 | 1.00 | 0.75 | 1.57 | 0.00 | 2.59 | 0.00 | 0.00 | 2.71 | 0.00 |
| time (sec) | N/A | 0.017 | 0.037 | 0.005 | 0.000 | 1.292 | 0.000 | 0.000 | 0.745 | 0.047 |

| Problem 1571 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 112 | 319 | 0 | 507 | 0 | 0 | 528 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 2.45 | 0.00 | 3.90 | 0.00 | 0.00 | 4.06 | 0.00 |
| time (sec) | N/A | 0.037 | 0.092 | 0.007 | 0.000 | 1.298 | 0.000 | 0.000 | 1.023 | 0.046 |

| Problem 1572 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 185 | 185 | 195 | 662 | 0 | 954 | 0 | 0 | 945 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 3.58 | 0.00 | 5.16 | 0.00 | 0.00 | 5.11 | 0.00 |
| time (sec) | N/A | 0.062 | 0.129 | 0.009 | 0.000 | 0.925 | 0.000 | 0.000 | 1.609 | 0.046 |

| Problem 1573 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 53 | 57 | 92 | 93 | 233 | 78 | 46 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.00 | 1.61 | 1.63 | 4.09 | 1.37 | 0.81 | 0.00 |
| time (sec) | N/A | 0.030 | 0.030 | 0.008 | 1.162 | 1.494 | 0.694 | 0.923 | 0.439 | 0.001 |

| Problem 1574 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 54 | 57 | 0 | 85 | 0 | 0 | 81 | 0 |
| N.S. | 1 | 1.00 | 0.57 | 0.60 | 0.00 | 0.89 | 0.00 | 0.00 | 0.85 | 0.00 |
| time (sec) | N/A | 0.036 | 0.074 | 0.006 | 0.000 | 1.317 | 0.000 | 0.000 | 1.037 | 0.221 |

| Problem 1575 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 46 | 66 | 0 | 84 | 0 | 0 | 119 | 0 |
| N.S. | 1 | 1.00 | 0.47 | 0.68 | 0.00 | 0.87 | 0.00 | 0.00 | 1.23 | 0.00 |
| time (sec) | N/A | 0.022 | 0.040 | 0.004 | 0.000 | 1.392 | 0.000 | 0.000 | 2.138 | 0.365 |

| Problem 1576 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 46 | 66 | 0 | 84 | 0 | 0 | 142 | 0 |
| N.S. | 1 | 1.00 | 0.47 | 0.68 | 0.00 | 0.87 | 0.00 | 0.00 | 1.46 | 0.00 |
| time (sec) | N/A | 0.018 | 0.066 | 0.005 | 0.000 | 1.397 | 0.000 | 0.000 | 0.850 | 0.139 |

| Problem 1577 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 28 | 23 | 22 | 22 | 22 | 22 | 22 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.82 | 0.79 | 0.79 | 0.79 | 0.79 | 0.79 | 0.00 |
| time (sec) | N/A | 0.005 | 0.000 | 0.000 | 1.073 | 1.167 | 0.060 | 1.009 | 0.037 | 0.000 |

| Problem 1578 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 12 | 11 | 11 | 8 | 11 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.73 | 0.73 | 0.53 | 0.73 | 0.67 | 0.00 |
| time (sec) | N/A | 0.002 | 0.000 | 0.000 | 1.082 | 1.036 | 0.054 | 1.084 | 0.021 | 0.000 |

| Problem 1579 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 10 | 9 | 9 | 5 | 9 | 8 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.91 | 0.82 | 0.82 | 0.45 | 0.82 | 0.73 | 0.00 |
| time (sec) | N/A | 0.002 | 0.000 | 0.000 | 0.962 | 0.965 | 0.054 | 0.941 | 0.017 | 0.000 |

| Problem 1580 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 9 | 9 | 9 | 10 | 9 | 9 | 7 | 9 | 7 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.11 | 1.00 | 1.00 | 0.78 | 1.00 | 0.78 | 0.00 |
| time (sec) | N/A | 0.001 | 0.000 | 0.000 | 1.015 | 0.967 | 0.053 | 0.961 | 0.029 | 0.000 |

| Problem 1581 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 13 | 12 | 12 | 10 | 12 | 12 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 0.86 | 0.71 | 0.86 | 0.86 | 0.00 |
| time (sec) | N/A | 0.002 | 0.000 | 0.000 | 1.000 | 0.713 | 0.055 | 1.059 | 0.020 | 0.000 |

| Problem 1582 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 10 | 9 | 9 | 8 | 9 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.91 | 0.82 | 0.82 | 0.73 | 0.82 | 0.91 | 0.00 |
| time (sec) | N/A | 0.001 | 0.000 | 0.001 | 1.097 | 0.718 | 0.054 | 1.104 | 0.019 | 0.000 |

| Problem 1583 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 12 | 11 | 11 | 8 | 11 | 8 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.73 | 0.73 | 0.53 | 0.73 | 0.53 | 0.00 |
| time (sec) | N/A | 0.002 | 0.000 | 0.001 | 1.054 | 1.016 | 0.055 | 0.886 | 0.021 | 0.000 |

| Problem 1584 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 18 | 18 | 18 | 15 | 14 | 14 | 15 | 14 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.78 | 0.78 | 0.83 | 0.78 | 0.72 | 0.00 |
| time (sec) | N/A | 0.003 | 0.000 | 0.000 | 0.971 | 0.493 | 0.057 | 0.848 | 0.024 | 0.000 |

| Problem 1585 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 17 | 16 | 16 | 12 | 16 | 15 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.80 | 0.80 | 0.60 | 0.80 | 0.75 | 0.00 |
| time (sec) | N/A | 0.003 | 0.000 | 0.000 | 0.973 | 1.094 | 0.054 | 1.098 | 0.026 | 0.000 |

| Problem 1586 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 13 | 12 | 12 | 10 | 12 | 12 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.81 | 0.75 | 0.75 | 0.62 | 0.75 | 0.75 | 0.00 |
| time (sec) | N/A | 0.002 | 0.000 | 0.000 | 0.984 | 0.993 | 0.056 | 0.959 | 0.023 | 0.000 |

| Problem 1587 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 14 | 13 | 13 | 12 | 13 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.00 | 1.00 | 0.92 | 1.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.002 | 0.000 | 0.000 | 1.145 | 0.530 | 0.057 | 1.005 | 0.028 | 0.000 |

| Problem 1588 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 21 | 20 | 27 | 20 | 21 | 20 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.91 | 1.23 | 0.91 | 0.95 | 0.91 | 0.00 |
| time (sec) | N/A | 0.004 | 0.007 | 0.002 | 1.034 | 1.200 | 0.160 | 0.848 | 0.037 | 0.001 |

| Problem 1589 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 22 | 17 | 16 | 17 | 15 | 16 | 17 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.73 | 0.77 | 0.68 | 0.73 | 0.77 | 0.00 |
| time (sec) | N/A | 0.003 | 0.001 | 0.001 | 1.084 | 1.277 | 0.081 | 0.809 | 0.029 | 0.000 |

| Problem 1590 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 14 | 13 | 17 | 14 | 14 | 11 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.87 | 1.13 | 0.93 | 0.93 | 0.73 | 0.00 |
| time (sec) | N/A | 0.002 | 0.002 | 0.001 | 1.041 | 1.228 | 0.085 | 1.082 | 0.029 | 0.000 |

| Problem 1591 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 10 | 10 | 10 | 11 | 10 | 11 | 7 | 11 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.10 | 1.00 | 1.10 | 0.70 | 1.10 | 1.00 | 0.00 |
| time (sec) | N/A | 0.002 | 0.002 | 0.002 | 1.026 | 1.018 | 0.078 | 1.119 | 0.033 | 0.000 |

| Problem 1592 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 12 | 11 | 12 | 10 | 11 | 12 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.80 | 0.73 | 0.80 | 0.67 | 0.73 | 0.80 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.000 | 0.965 | 1.306 | 0.083 | 1.075 | 0.025 | 0.000 |

| Problem 1593 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 11 | 11 | 11 | 10 | 9 | 9 | 8 | 10 | 9 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.91 | 0.82 | 0.82 | 0.73 | 0.91 | 0.82 | 0.00 |
| time (sec) | N/A | 0.001 | 0.001 | 0.001 | 0.919 | 1.193 | 0.076 | 0.888 | 0.027 | 0.000 |

| Problem 1594 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 12 | 11 | 10 | 8 | 11 | 10 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.85 | 0.77 | 0.62 | 0.85 | 0.77 | 0.00 |
| time (sec) | N/A | 0.002 | 0.002 | 0.001 | 0.966 | 1.231 | 0.076 | 0.902 | 0.031 | 0.000 |

| Problem 1595 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 12 | 11 | 11 | 10 | 12 | 11 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 0.85 | 0.85 | 0.77 | 0.92 | 0.85 | 0.00 |
| time (sec) | N/A | 0.002 | 0.001 | 0.000 | 1.077 | 1.286 | 0.076 | 1.014 | 0.026 | 0.000 |

| Problem 1596 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 17 | 12 | 11 | 11 | 12 | 11 | 11 | 17 |
| N.S. | 1 | 1.00 | 1.00 | 0.71 | 0.65 | 0.65 | 0.71 | 0.65 | 0.65 | 1.00 |
| time (sec) | N/A | 0.002 | 0.002 | 0.001 | 1.066 | 1.276 | 0.059 | 1.063 | 0.030 | 0.023 |

| Problem 1597 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 13 | 10 | 9 | 9 | 10 | 9 | 8 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.69 | 0.69 | 0.77 | 0.69 | 0.62 | 1.00 |
| time (sec) | N/A | 0.002 | 0.002 | 0.000 | 1.001 | 1.198 | 0.057 | 0.855 | 0.024 | 0.012 |

| Problem 1598 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 14 | 11 | 9 | 10 | 12 | 9 | 10 | 19 |
| N.S. | 1 | 1.00 | 0.93 | 0.73 | 0.60 | 0.67 | 0.80 | 0.60 | 0.67 | 1.27 |
| time (sec) | N/A | 0.001 | 0.007 | 0.003 | 1.113 | 0.966 | 0.061 | 1.077 | 0.021 | 0.008 |

| Problem 1599 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 14 | 13 | 18 | 14 | 14 | 15 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.87 | 1.20 | 0.93 | 0.93 | 1.00 | 1.47 |
| time (sec) | N/A | 0.002 | 0.014 | 0.002 | 1.015 | 1.251 | 0.061 | 0.955 | 0.291 | 0.016 |

| Problem 1600 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 17 | 17 | 14 | 11 | 11 | 10 | 14 | 11 | 12 | 14 |
| N.S. | 1 | 1.00 | 0.82 | 0.65 | 0.65 | 0.59 | 0.82 | 0.65 | 0.71 | 0.82 |
| time (sec) | N/A | 0.002 | 0.005 | 0.003 | 1.003 | 1.200 | 0.059 | 1.138 | 0.027 | 0.011 |

| Problem 1601 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 10 | 9 | 11 | 14 | 12 | 11 | 8 | 10 |
| N.S. | 1 | 1.00 | 0.67 | 0.60 | 0.73 | 0.93 | 0.80 | 0.73 | 0.53 | 0.67 |
| time (sec) | N/A | 0.002 | 0.004 | 0.003 | 1.007 | 1.225 | 0.061 | 0.892 | 0.027 | 0.010 |

| Problem 1602 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 24 | 24 | 24 | 17 | 16 | 14 | 19 | 16 | 15 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.71 | 0.67 | 0.58 | 0.79 | 0.67 | 0.62 | 1.00 |
| time (sec) | N/A | 0.002 | 0.005 | 0.001 | 1.046 | 1.192 | 0.062 | 1.089 | 0.029 | 0.009 |

| Problem 1603 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 23 | 23 | 23 | 16 | 15 | 19 | 20 | 16 | 17 | 26 |
| N.S. | 1 | 1.00 | 1.00 | 0.70 | 0.65 | 0.83 | 0.87 | 0.70 | 0.74 | 1.13 |
| time (sec) | N/A | 0.003 | 0.015 | 0.001 | 1.034 | 1.272 | 0.062 | 0.824 | 0.281 | 0.014 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1] had the largest ratio of [1.000]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 1 | 1 | 1.00 | 1 | 1.000 |
| 2 | A | 1 | 1 | 1.00 | 1 | 1.000 |
| 3 | A | 1 | 1 | 1.00 | 1 | 1.000 |
| 4 | A | 1 | 1 | 1.00 | 1 | 1.000 |
| 5 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 6 | A | 1 | 1 | 1.00 | 1 | 1.000 |
| 7 | A | 1 | 1 | 1.00 | 1 | 1.000 |
| 8 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 9 | A | 1 | 1 | 1.00 | 13 | 0.077 |
| 10 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 11 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 12 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 13 | A | 1 | 1 | 1.00 | 1 | 1.000 |
| 14 | A | 1 | 1 | 1.00 | 1 | 1.000 |
| 15 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 16 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 17 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 18 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 19 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 20 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 21 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 22 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 23 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 24 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 25 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 26 | A | 1 | 1 | 1.00 | 5 | 0.200 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 27 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 28 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 29 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 30 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 31 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 32 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 33 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 34 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 35 | A | 1 | 1 | 1.00 | 5 | 0.200 |
| 36 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 37 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 38 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 39 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 40 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 41 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 42 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 43 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 44 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 45 | A | 2 | 1 | 1.00 | 7 | 0.143 |
| 46 | A | 1 | 0 | 1.00 | 5 | 0.000 |
| 47 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 48 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 49 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 50 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 51 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 52 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 53 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 54 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 55 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 56 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 57 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 58 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 59 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 60 | A | 2 | 1 | 1.00 | 11 | 0.091 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 61 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 62 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 63 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 64 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 65 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 66 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 67 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 68 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 69 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 70 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 71 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 72 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 73 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 74 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 75 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 76 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 77 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 78 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 79 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 80 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 81 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 82 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 83 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 84 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 85 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 86 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 87 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 88 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 89 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 90 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 91 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 92 | A | 3 | 2 | 1.00 | 11 | 0.182 |
| 93 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 94 | A | 2 | 1 | 1.00 | 11 | 0.091 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 95 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 96 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 97 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 98 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 99 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 100 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 101 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 102 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 103 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 104 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 105 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 106 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 107 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 108 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 109 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 110 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 111 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 112 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 113 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 114 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 115 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 116 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 117 | A | 3 | 2 | 1.00 | 11 | 0.182 |
| 118 | A | 4 | 2 | 1.00 | 11 | 0.182 |
| 119 | A | 5 | 2 | 1.00 | 11 | 0.182 |
| 120 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 121 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 122 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 123 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 124 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 125 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 126 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 127 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 128 | A | 2 | 1 | 1.00 | 11 | 0.091 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 129 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 130 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 131 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 132 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 133 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 134 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 135 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 136 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 137 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 138 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 139 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 140 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 141 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 142 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 143 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 144 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 145 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 146 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 147 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 148 | A | 3 | 2 | 1.00 | 11 | 0.182 |
| 149 | A | 4 | 2 | 1.00 | 11 | 0.182 |
| 150 | A | 5 | 2 | 1.00 | 11 | 0.182 |
| 151 | A | 6 | 2 | 1.00 | 11 | 0.182 |
| 152 | A | 7 | 2 | 1.00 | 11 | 0.182 |
| 153 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 154 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 155 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 156 | A | 1 | 1 | 1.00 | 12 | 0.083 |
| 157 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 158 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 159 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 160 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 161 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 162 | A | 1 | 1 | 1.00 | 7 | 0.143 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 163 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 164 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 165 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 166 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 167 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 168 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 169 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 170 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 171 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 172 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 173 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 174 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 175 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 176 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 177 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 178 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 179 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 180 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 181 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 182 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 183 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 184 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 185 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 186 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 187 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 188 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 189 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 190 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 191 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 192 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 193 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 194 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 195 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 196 | A | 2 | 1 | 1.00 | 11 | 0.091 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 197 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 198 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 199 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 200 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 201 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 202 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 203 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 204 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 205 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 206 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 207 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 208 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 209 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 210 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 211 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 212 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 213 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 214 | A | 2 | 1 | 1.23 | 11 | 0.091 |
| 215 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 216 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 217 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 218 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 219 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 220 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 221 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 222 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 223 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 224 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 225 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 226 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 227 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 228 | A | 3 | 2 | 1.00 | 11 | 0.182 |
| 229 | A | 4 | 2 | 1.00 | 11 | 0.182 |
| 230 | A | 2 | 1 | 1.00 | 11 | 0.091 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 231 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 232 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 233 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 234 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 235 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 236 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 237 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 238 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 239 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 240 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 241 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 242 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 243 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 244 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 245 | A | 3 | 2 | 1.00 | 11 | 0.182 |
| 246 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 247 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 248 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 249 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 250 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 251 | A | 1 | 1 | 1.00 | 3 | 0.333 |
| 252 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 253 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 254 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 255 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 256 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 257 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 258 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 259 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 260 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 261 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 262 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 263 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 264 | A | 2 | 1 | 1.00 | 11 | 0.091 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 265 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 266 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 267 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 268 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 269 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 270 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 271 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 272 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 273 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 274 | A | 1 | 1 | 1.00 | 13 | 0.077 |
| 275 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 276 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 277 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 278 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 279 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 280 | A | 3 | 3 | 1.00 | 11 | 0.273 |
| 281 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 282 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 283 | A | 3 | 1 | 1.00 | 17 | 0.059 |
| 284 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 285 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 286 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 287 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 288 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 289 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 290 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 291 | A | 5 | 4 | 1.00 | 13 | 0.308 |
| 292 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 293 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 294 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 295 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 296 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 297 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 298 | A | 4 | 3 | 1.00 | 13 | 0.231 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 299 | A | 5 | 4 | 1.00 | 13 | 0.308 |
| 300 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 301 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 302 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 303 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 304 | A | 5 | 3 | 1.00 | 13 | 0.231 |
| 305 | A | 5 | 4 | 1.00 | 13 | 0.308 |
| 306 | A | 5 | 4 | 1.00 | 13 | 0.308 |
| 307 | A | 5 | 3 | 1.00 | 13 | 0.231 |
| 308 | A | 6 | 4 | 1.00 | 13 | 0.308 |
| 309 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 310 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 311 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 312 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 313 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 314 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 315 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 316 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 317 | A | 7 | 3 | 1.00 | 13 | 0.231 |
| 318 | A | 7 | 4 | 1.00 | 13 | 0.308 |
| 319 | A | 7 | 4 | 1.00 | 13 | 0.308 |
| 320 | A | 7 | 4 | 1.00 | 13 | 0.308 |
| 321 | A | 7 | 4 | 1.00 | 13 | 0.308 |
| 322 | A | 7 | 3 | 1.00 | 13 | 0.231 |
| 323 | A | 8 | 4 | 1.00 | 13 | 0.308 |
| 324 | A | 9 | 4 | 1.00 | 13 | 0.308 |
| 325 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 326 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 327 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 328 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 329 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 330 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 331 | A | 5 | 3 | 1.00 | 15 | 0.200 |
| 332 | A | 5 | 4 | 1.00 | 15 | 0.267 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 333 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 334 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 335 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 336 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 337 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 338 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 339 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 340 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 341 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 342 | A | 5 | 3 | 1.00 | 13 | 0.231 |
| 343 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 344 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 345 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 346 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 347 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 348 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 349 | A | 4 | 3 | 1.04 | 13 | 0.231 |
| 350 | A | 5 | 3 | 0.98 | 13 | 0.231 |
| 351 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 352 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 353 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 354 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 355 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 356 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 357 | A | 5 | 3 | 1.08 | 13 | 0.231 |
| 358 | A | 6 | 3 | 1.00 | 13 | 0.231 |
| 359 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 360 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 361 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 362 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 363 | A | 4 | 3 | 1.05 | 15 | 0.200 |
| 364 | A | 5 | 3 | 0.98 | 15 | 0.200 |
| 365 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 366 | A | 5 | 3 | 1.09 | 15 | 0.200 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 367 | A | 6 | 3 | 1.00 | 15 | 0.200 |
| 368 | A | 2 | 2 | 1.00 | 31 | 0.065 |
| 369 | C | 5 | 2 | 7.08 | 34 | 0.059 |
| 370 | A | 3 | 3 | 1.00 | 29 | 0.103 |
| 371 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 372 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 373 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 374 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 375 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 376 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 377 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 378 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 379 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 380 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 381 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 382 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 383 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 384 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 385 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 386 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 387 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 388 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 389 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 390 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 391 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 392 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 393 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 394 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 395 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 396 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 397 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 398 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 399 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 400 | A | 2 | 1 | 1.00 | 15 | 0.067 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 401 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 402 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 403 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 404 | A | 5 | 5 | 1.00 | 15 | 0.333 |
| 405 | A | 6 | 5 | 1.00 | 15 | 0.333 |
| 406 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 407 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 408 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 409 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 410 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 411 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 412 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 413 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 414 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 415 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 416 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 417 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 418 | A | 6 | 5 | 1.02 | 13 | 0.385 |
| 419 | A | 7 | 5 | 0.99 | 13 | 0.385 |
| 420 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 421 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 422 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 423 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 424 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 425 | A | 4 | 4 | 1.00 | 18 | 0.222 |
| 426 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 427 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 428 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 429 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 430 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 431 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 432 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 433 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 434 | A | 2 | 1 | 1.00 | 11 | 0.091 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 435 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 436 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 437 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 438 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 439 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 440 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 441 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 442 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 443 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 444 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 445 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 446 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 447 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 448 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 449 | A | 5 | 3 | 1.00 | 13 | 0.231 |
| 450 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 451 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 452 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 453 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 454 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 455 | A | 5 | 3 | 1.00 | 13 | 0.231 |
| 456 | A | 5 | 4 | 1.00 | 13 | 0.308 |
| 457 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 458 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 459 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 460 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 461 | A | 5 | 3 | 1.00 | 13 | 0.231 |
| 462 | A | 6 | 4 | 1.00 | 13 | 0.308 |
| 463 | A | 5 | 4 | 1.00 | 13 | 0.308 |
| 464 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 465 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 466 | A | 4 | 3 | 1.00 | 13 | 0.231 |
| 467 | A | 5 | 3 | 1.00 | 13 | 0.231 |
| 468 | A | 6 | 3 | 1.00 | 13 | 0.231 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 469 | A | 5 | 3 | 1.00 | 15 | 0.200 |
| 470 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 471 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 472 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 473 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 474 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 475 | A | 5 | 3 | 1.00 | 15 | 0.200 |
| 476 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 477 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 478 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 479 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 480 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 481 | A | 5 | 3 | 1.00 | 15 | 0.200 |
| 482 | A | 6 | 4 | 1.00 | 15 | 0.267 |
| 483 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 484 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 485 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 486 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 487 | A | 5 | 3 | 1.00 | 15 | 0.200 |
| 488 | A | 6 | 3 | 1.00 | 15 | 0.200 |
| 489 | A | 7 | 4 | 1.00 | 15 | 0.267 |
| 490 | A | 6 | 4 | 1.00 | 15 | 0.267 |
| 491 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 492 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 493 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 494 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 495 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 496 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 497 | A | 7 | 4 | 1.00 | 16 | 0.250 |
| 498 | A | 6 | 4 | 1.00 | 16 | 0.250 |
| 499 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 500 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 501 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 502 | A | 1 | 1 | 1.00 | 16 | 0.062 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 503 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 504 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 505 | A | 6 | 3 | 1.00 | 15 | 0.200 |
| 506 | A | 5 | 3 | 1.00 | 15 | 0.200 |
| 507 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 508 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 509 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 510 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 511 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 512 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 513 | A | 6 | 3 | 1.00 | 16 | 0.188 |
| 514 | A | 5 | 3 | 1.00 | 16 | 0.188 |
| 515 | A | 4 | 3 | 1.00 | 16 | 0.188 |
| 516 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 517 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 518 | A | 1 | 1 | 1.00 | 16 | 0.062 |
| 519 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 520 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 521 | A | 8 | 4 | 1.00 | 15 | 0.267 |
| 522 | A | 7 | 4 | 1.00 | 15 | 0.267 |
| 523 | A | 6 | 4 | 1.00 | 15 | 0.267 |
| 524 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 525 | A | 5 | 5 | 1.00 | 15 | 0.333 |
| 526 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 527 | A | 8 | 4 | 1.00 | 16 | 0.250 |
| 528 | A | 7 | 4 | 1.00 | 16 | 0.250 |
| 529 | A | 6 | 4 | 1.00 | 16 | 0.250 |
| 530 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 531 | A | 5 | 5 | 1.00 | 16 | 0.312 |
| 532 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 533 | A | 7 | 3 | 1.00 | 15 | 0.200 |
| 534 | A | 6 | 3 | 1.00 | 15 | 0.200 |
| 535 | A | 5 | 3 | 1.00 | 15 | 0.200 |
| 536 | A | 4 | 3 | 1.00 | 15 | 0.200 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 537 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 538 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 539 | A | 7 | 3 | 1.00 | 16 | 0.188 |
| 540 | A | 6 | 3 | 1.00 | 16 | 0.188 |
| 541 | A | 5 | 3 | 1.00 | 16 | 0.188 |
| 542 | A | 4 | 3 | 1.00 | 16 | 0.188 |
| 543 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 544 | A | 4 | 3 | 1.00 | 16 | 0.188 |
| 545 | A | 9 | 4 | 1.00 | 15 | 0.267 |
| 546 | A | 8 | 4 | 1.00 | 15 | 0.267 |
| 547 | A | 7 | 4 | 1.00 | 15 | 0.267 |
| 548 | A | 6 | 4 | 1.00 | 15 | 0.267 |
| 549 | A | 6 | 5 | 1.00 | 15 | 0.333 |
| 550 | A | 6 | 5 | 1.00 | 15 | 0.333 |
| 551 | A | 9 | 4 | 1.00 | 16 | 0.250 |
| 552 | A | 8 | 4 | 1.00 | 16 | 0.250 |
| 553 | A | 7 | 4 | 1.00 | 16 | 0.250 |
| 554 | A | 6 | 4 | 1.00 | 16 | 0.250 |
| 555 | A | 6 | 5 | 1.00 | 16 | 0.312 |
| 556 | A | 6 | 5 | 1.00 | 16 | 0.312 |
| 557 | A | 8 | 3 | 1.00 | 15 | 0.200 |
| 558 | A | 7 | 3 | 1.00 | 15 | 0.200 |
| 559 | A | 6 | 3 | 1.00 | 15 | 0.200 |
| 560 | A | 5 | 3 | 1.00 | 15 | 0.200 |
| 561 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 562 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 563 | A | 8 | 3 | 1.00 | 16 | 0.188 |
| 564 | A | 7 | 3 | 1.00 | 16 | 0.188 |
| 565 | A | 6 | 3 | 1.00 | 16 | 0.188 |
| 566 | A | 5 | 3 | 1.00 | 16 | 0.188 |
| 567 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 568 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 569 | A | 6 | 4 | 1.00 | 15 | 0.267 |
| 570 | A | 5 | 4 | 1.00 | 15 | 0.267 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 571 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 572 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 573 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 574 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 575 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 576 | A | 4 | 2 | 1.00 | 15 | 0.133 |
| 577 | A | 6 | 5 | 1.00 | 15 | 0.333 |
| 578 | A | 5 | 5 | 1.00 | 15 | 0.333 |
| 579 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 580 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 581 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 582 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 583 | A | 4 | 2 | 1.00 | 15 | 0.133 |
| 584 | A | 6 | 5 | 1.00 | 15 | 0.333 |
| 585 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 586 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 587 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 588 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 589 | A | 4 | 2 | 1.00 | 15 | 0.133 |
| 590 | A | 6 | 4 | 1.00 | 16 | 0.250 |
| 591 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 592 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 593 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 594 | A | 1 | 1 | 1.00 | 16 | 0.062 |
| 595 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 596 | A | 6 | 5 | 1.00 | 16 | 0.312 |
| 597 | A | 5 | 5 | 1.00 | 16 | 0.312 |
| 598 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 599 | A | 1 | 1 | 1.00 | 16 | 0.062 |
| 600 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 601 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 602 | A | 6 | 5 | 1.00 | 16 | 0.312 |
| 603 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 604 | A | 1 | 1 | 1.00 | 16 | 0.062 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 605 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 606 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 607 | A | 4 | 2 | 1.00 | 16 | 0.125 |
| 608 | A | 5 | 3 | 1.00 | 15 | 0.200 |
| 609 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 610 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 611 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 612 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 613 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 614 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 615 | A | 4 | 2 | 1.00 | 15 | 0.133 |
| 616 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 617 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 618 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 619 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 620 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 621 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 622 | A | 4 | 2 | 1.00 | 15 | 0.133 |
| 623 | A | 5 | 4 | 1.00 | 15 | 0.267 |
| 624 | A | 4 | 3 | 1.00 | 15 | 0.200 |
| 625 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 626 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 627 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 628 | A | 4 | 2 | 1.00 | 15 | 0.133 |
| 629 | A | 5 | 3 | 1.00 | 16 | 0.188 |
| 630 | A | 4 | 3 | 1.00 | 16 | 0.188 |
| 631 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 632 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 633 | A | 1 | 1 | 1.00 | 16 | 0.062 |
| 634 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 635 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 636 | A | 4 | 4 | 1.00 | 16 | 0.250 |
| 637 | A | 3 | 3 | 1.00 | 16 | 0.188 |
| 638 | A | 1 | 1 | 1.00 | 16 | 0.062 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 639 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 640 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 641 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 642 | A | 4 | 3 | 1.00 | 16 | 0.188 |
| 643 | A | 1 | 1 | 1.00 | 16 | 0.062 |
| 644 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 645 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 646 | A | 4 | 2 | 1.00 | 16 | 0.125 |
| 647 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 648 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 649 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 650 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 651 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 652 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 653 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 654 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 655 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 656 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 657 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 658 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 659 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 660 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 661 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 662 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 663 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 664 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 665 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 666 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 667 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 668 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 669 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 670 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 671 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 672 | A | 2 | 1 | 1.00 | 13 | 0.077 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 673 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 674 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 675 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 676 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 677 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 678 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 679 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 680 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 681 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 682 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 683 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 684 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 685 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 686 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 687 | A | 5 | 5 | 1.00 | 13 | 0.385 |
| 688 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 689 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 690 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 691 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 692 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 693 | A | 6 | 6 | 1.00 | 13 | 0.462 |
| 694 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 695 | A | 6 | 5 | 1.00 | 13 | 0.385 |
| 696 | A | 7 | 5 | 1.00 | 13 | 0.385 |
| 697 | A | 7 | 5 | 1.00 | 13 | 0.385 |
| 698 | A | 5 | 5 | 1.00 | 15 | 0.333 |
| 699 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 700 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 701 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 702 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 703 | A | 2 | 1 | 1.00 | 9 | 0.111 |
| 704 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 705 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 706 | A | 2 | 1 | 1.00 | 9 | 0.111 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 707 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 708 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 709 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 710 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 711 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 712 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 713 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 714 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 715 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 716 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 717 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 718 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 719 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 720 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 721 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 722 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 723 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 724 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 725 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 726 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 727 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 728 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 729 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 730 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 731 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 732 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 733 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 734 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 735 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 736 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 737 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 738 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 739 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 740 | A | 2 | 1 | 1.00 | 16 | 0.062 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 741 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 742 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 743 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 744 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 745 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 746 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 747 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 748 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 749 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 750 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 751 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 752 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 753 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 754 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 755 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 756 | A | 3 | 2 | 1.00 | 16 | 0.125 |
| 757 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 758 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 759 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 760 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 761 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 762 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 763 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 764 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 765 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 766 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 767 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 768 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 769 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 770 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 771 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 772 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 773 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 774 | A | 3 | 2 | 1.00 | 20 | 0.100 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 775 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 776 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 777 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 778 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 779 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 780 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 781 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 782 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 783 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 784 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 785 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 786 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 787 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 788 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 789 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 790 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 791 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 792 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 793 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 794 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 795 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 796 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 797 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 798 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 799 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 800 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 801 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 802 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 803 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 804 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 805 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 806 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 807 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 808 | A | 3 | 2 | 1.00 | 20 | 0.100 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 809 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 810 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 811 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 812 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 813 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 814 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 815 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 816 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 817 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 818 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 819 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 820 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 821 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 822 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 823 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 824 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 825 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 826 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 827 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 828 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 829 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 830 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 831 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 832 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 833 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 834 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 835 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 836 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 837 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 838 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 839 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 840 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 841 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 842 | A | 3 | 2 | 1.00 | 20 | 0.100 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 843 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 844 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 845 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 846 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 847 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 848 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 849 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 850 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 851 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 852 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 853 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 854 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 855 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 856 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 857 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 858 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 859 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 860 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 861 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 862 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 863 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 864 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 865 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 866 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 867 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 868 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 869 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 870 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 871 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 872 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 873 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 874 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 875 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 876 | A | 3 | 2 | 1.00 | 20 | 0.100 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 877 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 878 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 879 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 880 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 881 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 882 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 883 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 884 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 885 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 886 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 887 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 888 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 889 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 890 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 891 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 892 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 893 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 894 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 895 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 896 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 897 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 898 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 899 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 900 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 901 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 902 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 903 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 904 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 905 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 906 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 907 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 908 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 909 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 910 | A | 4 | 3 | 1.00 | 20 | 0.150 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 911 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 912 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 913 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 914 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 915 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 916 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 917 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 918 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 919 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 920 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 921 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 922 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 923 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 924 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 925 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 926 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 927 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 928 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 929 | A | 3 | 3 | 1.00 | 27 | 0.111 |
| 930 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 931 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 932 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 933 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 934 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 935 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 936 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 937 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 938 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 939 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 940 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 941 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 942 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 943 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 944 | A | 2 | 2 | 1.00 | 20 | 0.100 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 945 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 946 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 947 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 948 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 949 | A | 2 | 2 | 1.00 | 16 | 0.125 |
| 950 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 951 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 952 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 953 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 954 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 955 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 956 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 957 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 958 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 959 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 960 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 961 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 962 | A | 1 | 0 | 1.00 | 5 | 0.000 |
| 963 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 964 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 965 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 966 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 967 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 968 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 969 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 970 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 971 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 972 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 973 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 974 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 975 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 976 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 977 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 978 | A | 2 | 1 | 1.00 | 19 | 0.053 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 979 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 980 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 981 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 982 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 983 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 984 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 985 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 986 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 987 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 988 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 989 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 990 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 991 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 992 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 993 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 994 | A | 7 | 4 | 1.00 | 17 | 0.235 |
| 995 | A | 6 | 4 | 1.00 | 17 | 0.235 |
| 996 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 997 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 998 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 999 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 1000 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 1001 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 1002 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1003 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 1004 | A | 4 | 2 | 1.00 | 17 | 0.118 |
| 1005 | A | 5 | 2 | 1.00 | 17 | 0.118 |
| 1006 | A | 7 | 4 | 1.00 | 17 | 0.235 |
| 1007 | A | 6 | 4 | 1.00 | 17 | 0.235 |
| 1008 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1009 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 1010 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 1011 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 1012 | A | 4 | 4 | 1.00 | 17 | 0.235 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 1013 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 1014 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 1015 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1016 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 1017 | A | 4 | 2 | 1.00 | 17 | 0.118 |
| 1018 | A | 5 | 2 | 1.00 | 17 | 0.118 |
| 1019 | A | 8 | 4 | 1.00 | 17 | 0.235 |
| 1020 | A | 7 | 4 | 1.00 | 17 | 0.235 |
| 1021 | A | 6 | 4 | 1.00 | 17 | 0.235 |
| 1022 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 1023 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1024 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1025 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 1026 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1027 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1028 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 1029 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 1030 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1031 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 1032 | A | 4 | 2 | 1.00 | 17 | 0.118 |
| 1033 | A | 5 | 2 | 1.00 | 17 | 0.118 |
| 1034 | A | 6 | 2 | 1.00 | 17 | 0.118 |
| 1035 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 1036 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 1037 | A | 6 | 3 | 1.00 | 17 | 0.176 |
| 1038 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 1039 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 1040 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 1041 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1042 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 1043 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1044 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 1045 | A | 4 | 2 | 1.00 | 17 | 0.118 |
| 1046 | A | 5 | 2 | 1.00 | 17 | 0.118 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1047 | A | 6 | 4 | 1.00 | 17 | 0.235 |
| 1048 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1049 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 1050 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 1051 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 1052 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 1053 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1054 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 1055 | A | 4 | 2 | 1.00 | 17 | 0.118 |
| 1056 | A | 5 | 2 | 1.00 | 17 | 0.118 |
| 1057 | A | 7 | 4 | 1.00 | 17 | 0.235 |
| 1058 | A | 6 | 4 | 1.00 | 17 | 0.235 |
| 1059 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1060 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 1061 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 1062 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1063 | A | 3 | 2 | 1.00 | 17 | 0.118 |
| 1064 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1065 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 1066 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 1067 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 1068 | A | 6 | 4 | 1.00 | 20 | 0.200 |
| 1069 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 1070 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 1071 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 1072 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 1073 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1074 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 1075 | A | 4 | 2 | 1.00 | 20 | 0.100 |
| 1076 | A | 6 | 4 | 1.00 | 23 | 0.174 |
| 1077 | A | 5 | 4 | 1.00 | 23 | 0.174 |
| 1078 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 1079 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 1080 | A | 1 | 1 | 1.00 | 23 | 0.043 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1081 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1082 | A | 3 | 2 | 1.00 | 23 | 0.087 |
| 1083 | A | 4 | 2 | 1.00 | 23 | 0.087 |
| 1084 | A | 5 | 3 | 1.00 | 19 | 0.158 |
| 1085 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 1086 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 1087 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1088 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1089 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1090 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1091 | A | 7 | 4 | 1.00 | 17 | 0.235 |
| 1092 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1093 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 1094 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1095 | A | 4 | 2 | 1.00 | 17 | 0.118 |
| 1096 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 1097 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 1098 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 1099 | A | 1 | 1 | 1.00 | 20 | 0.050 |
| 1100 | A | 3 | 3 | 1.00 | 23 | 0.130 |
| 1101 | A | 11 | 8 | 1.00 | 20 | 0.400 |
| 1102 | A | 12 | 9 | 1.00 | 25 | 0.360 |
| 1103 | A | 11 | 8 | 1.00 | 25 | 0.320 |
| 1104 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 1105 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 1106 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 1107 | A | 4 | 2 | 1.00 | 25 | 0.080 |
| 1108 | A | 12 | 9 | 1.00 | 25 | 0.360 |
| 1109 | A | 11 | 8 | 1.00 | 25 | 0.320 |
| 1110 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 1111 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 1112 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 1113 | A | 13 | 10 | 1.00 | 25 | 0.400 |
| 1114 | A | 12 | 9 | 1.00 | 25 | 0.360 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1115 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 1116 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 1117 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 1118 | A | 13 | 10 | 1.00 | 25 | 0.400 |
| 1119 | A | 12 | 9 | 1.00 | 25 | 0.360 |
| 1120 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 1121 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 1122 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 1123 | A | 13 | 9 | 1.00 | 25 | 0.360 |
| 1124 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 1125 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 1126 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 1127 | A | 4 | 2 | 1.00 | 25 | 0.080 |
| 1128 | A | 2 | 1 | 1.00 | 19 | 0.053 |
| 1129 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1130 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1131 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1132 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1133 | A | 2 | 1 | 1.00 | 11 | 0.091 |
| 1134 | A | 1 | 0 | 1.00 | 5 | 0.000 |
| 1135 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1136 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1137 | A | 1 | 1 | 1.00 | 13 | 0.077 |
| 1138 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1139 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1140 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1141 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1142 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1143 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1144 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 1145 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1146 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1147 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1148 | A | 1 | 1 | 1.00 | 15 | 0.067 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1149 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1150 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1151 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1152 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1153 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1154 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1155 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1156 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1157 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 1158 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1159 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1160 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1161 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1162 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 1163 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 1164 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1165 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1166 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1167 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1168 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1169 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1170 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1171 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1172 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1173 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1174 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1175 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1176 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 1177 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1178 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1179 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1180 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1181 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1182 | A | 2 | 1 | 1.00 | 15 | 0.067 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 1183 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1184 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1185 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 1186 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 1187 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 1188 | A | 4 | 2 | 1.00 | 15 | 0.133 |
| 1189 | A | 5 | 2 | 1.00 | 15 | 0.133 |
| 1190 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1191 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1192 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1193 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1194 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1195 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1196 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1197 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1198 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1199 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1200 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1201 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1202 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1203 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1204 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1205 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 1206 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1207 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1208 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1209 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1210 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1211 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1212 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1213 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1214 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1215 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1216 | A | 2 | 1 | 1.00 | 15 | 0.067 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1217 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 1218 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 1219 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 1220 | A | 4 | 2 | 1.00 | 15 | 0.133 |
| 1221 | A | 5 | 2 | 1.00 | 15 | 0.133 |
| 1222 | A | 6 | 2 | 1.00 | 15 | 0.133 |
| 1223 | A | 7 | 2 | 1.00 | 15 | 0.133 |
| 1224 | A | 8 | 2 | 1.00 | 15 | 0.133 |
| 1225 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1226 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1227 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1228 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1229 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1230 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1231 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1232 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1233 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 1234 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 1235 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1236 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1237 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1238 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1239 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1240 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1241 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1242 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 1243 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1244 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1245 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1246 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1247 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1248 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1249 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1250 | A | 2 | 1 | 1.00 | 15 | 0.067 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 1251 | A | 1 | 1 | 1.00 | 13 | 0.077 |
| 1252 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 1253 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1254 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1255 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1256 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1257 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1258 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1259 | A | 1 | 1 | 1.00 | 15 | 0.067 |
| 1260 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 1261 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 1262 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1263 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1264 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1265 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 1266 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1267 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1268 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1269 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1270 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1271 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1272 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1273 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1274 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 1275 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 1276 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 1277 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 1278 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1279 | A | 6 | 4 | 1.00 | 17 | 0.235 |
| 1280 | A | 7 | 4 | 1.00 | 17 | 0.235 |
| 1281 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1282 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1283 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1284 | A | 2 | 1 | 1.00 | 17 | 0.059 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1285 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1286 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 1287 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 1288 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 1289 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 1290 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1291 | A | 6 | 4 | 1.00 | 17 | 0.235 |
| 1292 | A | 7 | 4 | 1.00 | 17 | 0.235 |
| 1293 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1294 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1295 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1296 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1297 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1298 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 1299 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 1300 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1301 | A | 5 | 4 | 1.00 | 17 | 0.235 |
| 1302 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 1303 | A | 6 | 4 | 1.00 | 17 | 0.235 |
| 1304 | A | 7 | 4 | 1.00 | 17 | 0.235 |
| 1305 | A | 3 | 3 | 1.00 | 13 | 0.231 |
| 1306 | A | 4 | 4 | 1.00 | 13 | 0.308 |
| 1307 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1308 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1309 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1310 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1311 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1312 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 1313 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1314 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 1315 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 1316 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 1317 | A | 6 | 3 | 1.00 | 17 | 0.176 |
| 1318 | A | 2 | 1 | 1.00 | 17 | 0.059 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 1319 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1320 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1321 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1322 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1323 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 1324 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 1325 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 1326 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 1327 | A | 6 | 3 | 1.00 | 17 | 0.176 |
| 1328 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1329 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1330 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1331 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 1332 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1333 | A | 1 | 1 | 1.00 | 9 | 0.111 |
| 1334 | A | 4 | 3 | 1.00 | 17 | 0.176 |
| 1335 | A | 5 | 3 | 1.00 | 17 | 0.176 |
| 1336 | A | 6 | 3 | 1.00 | 17 | 0.176 |
| 1337 | A | 7 | 3 | 1.00 | 17 | 0.176 |
| 1338 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1339 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1340 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1341 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1342 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1343 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1344 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1345 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1346 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1347 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 1348 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1349 | A | 5 | 5 | 1.00 | 15 | 0.333 |
| 1350 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1351 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1352 | A | 4 | 4 | 1.00 | 17 | 0.235 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1353 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 1354 | A | 8 | 4 | 1.00 | 19 | 0.210 |
| 1355 | A | 7 | 4 | 1.00 | 19 | 0.210 |
| 1356 | A | 6 | 4 | 1.00 | 19 | 0.210 |
| 1357 | A | 5 | 4 | 1.00 | 19 | 0.210 |
| 1358 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 1359 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 1360 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1361 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1362 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1363 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1364 | A | 5 | 2 | 1.00 | 19 | 0.105 |
| 1365 | A | 8 | 4 | 1.00 | 19 | 0.210 |
| 1366 | A | 7 | 4 | 1.00 | 19 | 0.210 |
| 1367 | A | 6 | 4 | 1.00 | 19 | 0.210 |
| 1368 | A | 5 | 4 | 1.00 | 19 | 0.210 |
| 1369 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 1370 | A | 5 | 4 | 1.00 | 19 | 0.210 |
| 1371 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1372 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1373 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1374 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1375 | A | 9 | 4 | 1.00 | 19 | 0.210 |
| 1376 | A | 8 | 4 | 1.00 | 19 | 0.210 |
| 1377 | A | 7 | 4 | 1.00 | 19 | 0.210 |
| 1378 | A | 6 | 4 | 1.00 | 19 | 0.210 |
| 1379 | A | 6 | 5 | 1.00 | 19 | 0.263 |
| 1380 | A | 6 | 5 | 1.00 | 19 | 0.263 |
| 1381 | A | 6 | 4 | 1.00 | 19 | 0.210 |
| 1382 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1383 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1384 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1385 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1386 | A | 7 | 4 | 1.00 | 19 | 0.210 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1387 | A | 6 | 4 | 1.00 | 19 | 0.210 |
| 1388 | A | 5 | 4 | 1.00 | 19 | 0.210 |
| 1389 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 1390 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 1391 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1392 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1393 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1394 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1395 | A | 5 | 2 | 1.00 | 19 | 0.105 |
| 1396 | A | 7 | 5 | 1.00 | 19 | 0.263 |
| 1397 | A | 6 | 5 | 1.00 | 19 | 0.263 |
| 1398 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 1399 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 1400 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1401 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1402 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1403 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1404 | A | 5 | 2 | 1.00 | 19 | 0.105 |
| 1405 | A | 6 | 2 | 1.00 | 19 | 0.105 |
| 1406 | A | 8 | 5 | 1.00 | 19 | 0.263 |
| 1407 | A | 7 | 5 | 1.00 | 19 | 0.263 |
| 1408 | A | 6 | 5 | 1.00 | 19 | 0.263 |
| 1409 | A | 5 | 4 | 1.00 | 19 | 0.210 |
| 1410 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1411 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1412 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1413 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1414 | A | 5 | 2 | 1.00 | 19 | 0.105 |
| 1415 | A | 6 | 2 | 1.00 | 19 | 0.105 |
| 1416 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1417 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1418 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1419 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1420 | A | 2 | 2 | 1.00 | 19 | 0.105 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 1421 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1422 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1423 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1424 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 1425 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1426 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 1427 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1428 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1429 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1430 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 1431 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1432 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 1433 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 1434 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 1435 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1436 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 1437 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1438 | A | 1 | 1 | 1.00 | 8 | 0.125 |
| 1439 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1440 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1441 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1442 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 1443 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1444 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1445 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 1446 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 1447 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1448 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 1449 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 1450 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 1451 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1452 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 1453 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1454 | A | 2 | 2 | 1.00 | 19 | 0.105 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1455 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1456 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1457 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1458 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1459 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1460 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1461 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1462 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1463 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1464 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1465 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1466 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1467 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1468 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1469 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1470 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1471 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1472 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1473 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1474 | A | 4 | 3 | 1.00 | 19 | 0.158 |
| 1475 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 1476 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1477 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1478 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1479 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1480 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1481 | A | 2 | 2 | 1.00 | 15 | 0.133 |
| 1482 | A | 8 | 6 | 1.00 | 19 | 0.316 |
| 1483 | A | 7 | 6 | 1.00 | 19 | 0.316 |
| 1484 | A | 7 | 7 | 1.00 | 19 | 0.368 |
| 1485 | A | 7 | 6 | 1.00 | 19 | 0.316 |
| 1486 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1487 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1488 | A | 3 | 2 | 1.00 | 19 | 0.105 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 1489 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1490 | A | 7 | 6 | 1.00 | 19 | 0.316 |
| 1491 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 1492 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 1493 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1494 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1495 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1496 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1497 | A | 7 | 6 | 1.00 | 19 | 0.316 |
| 1498 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 1499 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 1500 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1501 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1502 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1503 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1504 | A | 7 | 7 | 1.00 | 19 | 0.368 |
| 1505 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 1506 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1507 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1508 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1509 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1510 | A | 11 | 8 | 1.00 | 20 | 0.400 |
| 1511 | A | 11 | 8 | 1.00 | 20 | 0.400 |
| 1512 | A | 14 | 9 | 1.00 | 19 | 0.474 |
| 1513 | A | 13 | 9 | 1.00 | 19 | 0.474 |
| 1514 | A | 13 | 9 | 1.00 | 19 | 0.474 |
| 1515 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1516 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1517 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1518 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1519 | A | 14 | 9 | 1.00 | 19 | 0.474 |
| 1520 | A | 13 | 9 | 1.00 | 19 | 0.474 |
| 1521 | A | 13 | 9 | 1.00 | 19 | 0.474 |
| 1522 | A | 1 | 1 | 1.00 | 19 | 0.053 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1523 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1524 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1525 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1526 | A | 14 | 9 | 1.00 | 19 | 0.474 |
| 1527 | A | 14 | 10 | 1.00 | 19 | 0.526 |
| 1528 | A | 14 | 9 | 1.00 | 19 | 0.474 |
| 1529 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1530 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1531 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1532 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1533 | A | 14 | 9 | 1.00 | 19 | 0.474 |
| 1534 | A | 13 | 9 | 1.00 | 19 | 0.474 |
| 1535 | A | 12 | 8 | 1.00 | 19 | 0.421 |
| 1536 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1537 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1538 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1539 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1540 | A | 14 | 9 | 1.00 | 19 | 0.474 |
| 1541 | A | 13 | 9 | 1.00 | 19 | 0.474 |
| 1542 | A | 12 | 8 | 1.00 | 19 | 0.421 |
| 1543 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1544 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1545 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1546 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1547 | A | 15 | 10 | 1.00 | 19 | 0.526 |
| 1548 | A | 14 | 10 | 1.00 | 19 | 0.526 |
| 1549 | A | 13 | 9 | 1.00 | 19 | 0.474 |
| 1550 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1551 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1552 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1553 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1554 | A | 1 | 1 | 1.00 | 16 | 0.062 |
| 1555 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1556 | A | 2 | 1 | 1.00 | 15 | 0.067 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 1557 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1558 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1559 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 1560 | A | 2 | 1 | 1.00 | 13 | 0.077 |
| 1561 | A | 1 | 1 | 1.00 | 7 | 0.143 |
| 1562 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1563 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1564 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1565 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1566 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1567 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1568 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1569 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1570 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1571 | A | 3 | 2 | 1.00 | 19 | 0.105 |
| 1572 | A | 4 | 2 | 1.00 | 19 | 0.105 |
| 1573 | A | 3 | 2 | 1.00 | 24 | 0.083 |
| 1574 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 1575 | A | 2 | 2 | 1.00 | 44 | 0.045 |
| 1576 | A | 2 | 2 | 1.00 | 51 | 0.039 |
| 1577 | A | 1 | 0 | 1.00 | 15 | 0.000 |
| 1578 | A | 1 | 0 | 1.00 | 9 | 0.000 |
| 1579 | A | 1 | 0 | 1.00 | 5 | 0.000 |
| 1580 | A | 1 | 0 | 1.00 | 5 | 0.000 |
| 1581 | A | 1 | 0 | 1.00 | 9 | 0.000 |
| 1582 | A | 1 | 0 | 1.00 | 9 | 0.000 |
| 1583 | A | 1 | 0 | 1.00 | 15 | 0.000 |
| 1584 | A | 1 | 0 | 1.00 | 10 | 0.000 |
| 1585 | A | 1 | 0 | 1.00 | 10 | 0.000 |
| 1586 | A | 1 | 0 | 1.00 | 12 | 0.000 |
| 1587 | A | 1 | 0 | 1.00 | 15 | 0.000 |
| 1588 | A | 1 | 0 | 1.00 | 17 | 0.000 |
| 1589 | A | 1 | 0 | 1.00 | 8 | 0.000 |
| 1590 | A | 1 | 0 | 1.00 | 10 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1591 | A | 1 | 0 | 1.00 | 11 | 0.000 |
| 1592 | A | 1 | 0 | 1.00 | 11 | 0.000 |
| 1593 | A | 1 | 0 | 1.00 | 6 | 0.000 |
| 1594 | A | 1 | 0 | 1.00 | 11 | 0.000 |
| 1595 | A | 1 | 0 | 1.00 | 10 | 0.000 |
| 1596 | A | 1 | 0 | 1.00 | 11 | 0.000 |
| 1597 | A | 1 | 0 | 1.00 | 7 | 0.000 |
| 1598 | A | 1 | 0 | 1.00 | 17 | 0.000 |
| 1599 | A | 1 | 0 | 1.00 | 18 | 0.000 |
| 1600 | A | 1 | 0 | 1.00 | 11 | 0.000 |
| 1601 | A | 1 | 0 | 1.00 | 15 | 0.000 |
| 1602 | A | 1 | 0 | 1.00 | 18 | 0.000 |
| 1603 | A | 1 | 0 | 1.00 | 20 | 0.000 |

# Chapter 3

# Listing of integrals

## Local contents

## 3.1 $\int 0 \, dx$

**Optimal**. Leaf size=1

$$0$$

**Rubi [A]** time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$0$$

Antiderivative was successfully verified.

```
[In] Int[0,x]
```

```
[Out] 0
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 0 \, dx = 0$$

**Mathematica [A]** time = 0.00, size = 1, normalized size = 1.00

$$0$$

Antiderivative was successfully verified.

```
[In] Integrate[0,x]
```

```
[Out] 0
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int 0 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[0,x]
```

```
[Out] IntegrateAlgebraic[0, x]
```

**fricas [A]** time = 1.41, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="fricas")
[Out] 0
```

**giac [A]** time = 1.09, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="giac")
[Out] 0
```

**maple [A]** time = 0.00, size = 2, normalized size = 2.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(0,x)
[Out] 0
```

**maxima [A]** time = 0.42, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="maxima")
[Out] 0
```

**mupad [B]** time = 0.04, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(0,x)
```

```
[Out] 0
```

**sympy** [A] time = 0.01, size = 0, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x)
```

```
[Out] 0
```

## 3.2 $\int 1 \, dx$

**Optimal**. Leaf size=1

$$x$$

**Rubi [A]** time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[1,x]
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 1 \, dx = x$$

**Mathematica [A]** time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[1,x]
[Out] x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int 1 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1,x]
```

```
[Out] IntegrateAlgebraic[1, x]
```

**fricas [A]** time = 1.44, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="fricas")
[Out] x
```

**giac [A]** time = 1.07, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="giac")
[Out] x
```

**maple [A]** time = 0.00, size = 2, normalized size = 2.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
[Out] x
```

**maxima [A]** time = 0.41, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="maxima")
[Out] x
```

**mupad [B]** time = 0.01, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

**sympy** [**A**]     time = 0.02, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

## 3.3 $\int 5\,dx$

**Optimal**. Leaf size=3

$$5x$$

**Rubi** [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$5x$$

Antiderivative was successfully verified.

```
[In] Int[5,x]
```

```
[Out] 5*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 5\,dx = 5x$$

**Mathematica** [A] time = 0.00, size = 3, normalized size = 1.00

$$5x$$

Antiderivative was successfully verified.

```
[In] Integrate[5,x]
```

```
[Out] 5*x
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int 5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[5,x]
```

[Out] IntegrateAlgebraic[5, x]

**fricas [A]** time = 1.45, size = 3, normalized size = 1.00

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(5,x, algorithm="fricas")

[Out] 5*x

**giac [A]** time = 1.04, size = 3, normalized size = 1.00

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(5,x, algorithm="giac")

[Out] 5*x

**maple [A]** time = 0.00, size = 4, normalized size = 1.33

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(5,x)

[Out] 5*x

**maxima [A]** time = 0.41, size = 3, normalized size = 1.00

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(5,x, algorithm="maxima")

[Out] 5*x

**mupad [B]** time = 0.01, size = 3, normalized size = 1.00

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(5,x)

```
[Out] 5*x
```

**sympy [A]** time = 0.02, size = 2, normalized size = 0.67

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x)
[Out] 5*x
```

## 3.4 $\int -2\,dx$

**Optimal**. Leaf size=3

$$-2x$$

**Rubi** [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$-2x$$

Antiderivative was successfully verified.

```
[In] Int[-2,x]
[Out] -2*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int -2\,dx = -2x$$

**Mathematica** [A] time = 0.00, size = 3, normalized size = 1.00

$$-2x$$

Antiderivative was successfully verified.

```
[In] Integrate[-2,x]
[Out] -2*x
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-2,x]
```

```
[Out] IntegrateAlgebraic[-2, x]
```

**fricas [A]** time = 1.26, size = 3, normalized size = 1.00

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="fricas")
[Out] -2*x
```

**giac [A]** time = 1.11, size = 3, normalized size = 1.00

$$-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="giac")
[Out] -2*x
```

**maple [A]** time = 0.00, size = 4, normalized size = 1.33

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2,x)
[Out] -2*x
```

**maxima [A]** time = 0.42, size = 3, normalized size = 1.00

$$-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="maxima")
[Out] -2*x
```

**mupad [B]** time = 0.00, size = 3, normalized size = 1.00

$$-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2,x)
```

```
[Out] -2*x
```

**sympy** [**A**] time = 0.02, size = 3, normalized size = 1.00

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x)
```

```
[Out] -2*x
```

## 3.5 $\int -\frac{3}{2}\, dx$

**Optimal**. Leaf size=5

$$-\frac{3x}{2}$$

**Rubi** [A] time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {8}

$$-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[-3/2,x]
[Out] (-3*x)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int -\frac{3}{2}\, dx = -\frac{3x}{2}$$

**Mathematica** [A] time = 0.00, size = 5, normalized size = 1.00

$$-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/2,x]
[Out] (-3*x)/2
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{3}{2}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-3/2,x]
[Out] IntegrateAlgebraic[-3/2, x]
```

**fricas [A]** time = 1.29, size = 3, normalized size = 0.60

$$-\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="fricas")
[Out] -3/2*x
```

**giac [A]** time = 1.16, size = 3, normalized size = 0.60

$$-\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="giac")
[Out] -3/2*x
```

**maple [A]** time = 0.00, size = 4, normalized size = 0.80

$$-\frac{3x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/2,x)
[Out] -3/2*x
```

**maxima [A]** time = 0.42, size = 3, normalized size = 0.60

$$-\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="maxima")
[Out] -3/2*x
```

**mupad [B]** time = 0.01, size = 3, normalized size = 0.60

$$-\frac{3\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/2,x)
```

```
[Out] -(3*x)/2
```

**sympy** [**A**]   time = 0.01, size = 5, normalized size = 1.00

$$-\frac{3x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x)
```

```
[Out] -3*x/2
```

## 3.6 $\int \pi \, dx$

**Optimal**. Leaf size=3

$$\pi x$$

**Rubi** [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$\pi x$$

Antiderivative was successfully verified.

```
[In] Int[Pi,x]
```

```
[Out] Pi*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \pi \, dx = \pi x$$

**Mathematica** [A] time = 0.00, size = 3, normalized size = 1.00

$$\pi x$$

Antiderivative was successfully verified.

```
[In] Integrate[Pi,x]
```

```
[Out] Pi*x
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \pi \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Pi,x]
```

[Out] IntegrateAlgebraic[Pi, x]

**fricas [A]** time = 1.46, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(pi,x, algorithm="fricas")

[Out] pi*x

**giac [A]** time = 0.84, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(pi,x, algorithm="giac")

[Out] pi*x

**maple [A]** time = 0.00, size = 4, normalized size = 1.33

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(Pi,x)

[Out] Pi*x

**maxima [A]** time = 0.43, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(pi,x, algorithm="maxima")

[Out] pi*x

**mupad [B]** time = 0.00, size = 3, normalized size = 1.00

$$\Pi x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(Pi,x)

```
[Out] Pi*x
```

**sympy** [**A**] time = 0.01, size = 2, normalized size = 0.67

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x)
```

```
[Out] pi*x
```

## 3.7 $\int a\,dx$

**Optimal**. Leaf size=3

$$ax$$

**Rubi** [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$ax$$

Antiderivative was successfully verified.

```
[In] Int[a,x]
```

```
[Out] a*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int a\,dx = ax$$

**Mathematica** [A] time = 0.00, size = 3, normalized size = 1.00

$$ax$$

Antiderivative was successfully verified.

```
[In] Integrate[a,x]
```

```
[Out] a*x
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int a\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a,x]
```

[Out] IntegrateAlgebraic[a, x]

**fricas** [A] time = 0.68, size = 3, normalized size = 1.00

$$xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a,x, algorithm="fricas")

[Out] x*a

**giac** [A] time = 0.75, size = 3, normalized size = 1.00

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a,x, algorithm="giac")

[Out] a*x

**maple** [A] time = 0.00, size = 4, normalized size = 1.33

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a,x)

[Out] a*x

**maxima** [A] time = 0.44, size = 3, normalized size = 1.00

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a,x, algorithm="maxima")

[Out] a*x

**mupad** [B] time = 0.00, size = 3, normalized size = 1.00

$$a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a,x)

```
[Out] a*x
```

**sympy [A]** time = 0.02, size = 2, normalized size = 0.67

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x)
```

```
[Out] a*x
```

## 3.8 $\int 3a\,dx$

**Optimal**. Leaf size=4

$$3ax$$

**Rubi** [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {8}

$$3ax$$

Antiderivative was successfully verified.

```
[In] Int[3*a,x]
```

```
[Out] 3*a*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 3a\,dx = 3ax$$

**Mathematica** [A] time = 0.00, size = 4, normalized size = 1.00

$$3ax$$

Antiderivative was successfully verified.

```
[In] Integrate[3*a,x]
```

```
[Out] 3*a*x
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int 3a\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[3*a,x]
```

[Out] IntegrateAlgebraic[3*a, x]

**fricas** [A] time = 0.86, size = 4, normalized size = 1.00

$$3xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*a,x, algorithm="fricas")

[Out] 3*x*a

**giac** [A] time = 1.25, size = 4, normalized size = 1.00

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*a,x, algorithm="giac")

[Out] 3*a*x

**maple** [A] time = 0.00, size = 5, normalized size = 1.25

$$3ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(3*a,x)

[Out] 3*a*x

**maxima** [A] time = 0.43, size = 4, normalized size = 1.00

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*a,x, algorithm="maxima")

[Out] 3*a*x

**mupad** [B] time = 0.00, size = 4, normalized size = 1.00

$$3\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(3*a,x)

```
[Out] 3*a*x
```

**sympy [A]** time = 0.02, size = 3, normalized size = 0.75

$$3ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x)
```

```
[Out] 3*a*x
```

## 3.9 $\int \frac{\pi}{\sqrt{16-e^2}} \, dx$

**Optimal**. Leaf size=14

$$\frac{\pi x}{\sqrt{16-e^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {8}

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Pi/Sqrt[16 - E^2],x]
[Out] (Pi*x)/Sqrt[16 - E^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\pi}{\sqrt{16-e^2}} \, dx = \frac{\pi x}{\sqrt{16-e^2}}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Pi/Sqrt[16 - E^2],x]
[Out] (Pi*x)/Sqrt[16 - E^2]
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\pi}{\sqrt{16-e^2}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Pi/Sqrt[16 - E^2],x]
```

```
[Out] IntegrateAlgebraic[Pi/Sqrt[16 - E^2], x]
```

**fricas [A]** time = 1.20, size = 18, normalized size = 1.29

$$-\frac{\pi x\sqrt{-e^2+16}}{e^2-16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="fricas")
```

```
[Out] -pi*x*sqrt(-e^2 + 16)/(e^2 - 16)
```

**giac [A]** time = 1.08, size = 11, normalized size = 0.79

$$\frac{\pi x}{\sqrt{-e^2+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="giac")
```

```
[Out] pi*x/sqrt(-e^2 + 16)
```

**maple [A]** time = 0.00, size = 12, normalized size = 0.86

$$\frac{\pi x}{\sqrt{16-\mathrm{e}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/(16-exp(2))^(1/2),x)
```

```
[Out] Pi*x/(16-exp(2))^(1/2)
```

**maxima [A]** time = 0.45, size = 11, normalized size = 0.79

$$\frac{\pi x}{\sqrt{-e^2+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="maxima")
```

```
[Out] pi*x/sqrt(-e^2 + 16)
```

**mupad [B]** time = 0.00, size = 11, normalized size = 0.79

$$\frac{\Pi\,x}{\sqrt{16-\mathrm{e}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/(16 - exp(2))^(1/2),x)
[Out] (Pi*x)/(16 - exp(2))^(1/2)
```

**sympy [A]** time = 0.06, size = 10, normalized size = 0.71

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))**(1/2),x)
[Out] pi*x/sqrt(16 - exp(2))
```

## 3.10 $\int x^{100}\,dx$

**Optimal**. Leaf size=7

$$\frac{x^{101}}{101}$$

**Rubi** [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^{101}}{101}$$

Antiderivative was successfully verified.

```
[In] Int[x^100,x]
```

```
[Out] x^101/101
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{100}\,dx = \frac{x^{101}}{101}$$

**Mathematica** [A] time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^{101}}{101}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^100,x]
```

```
[Out] x^101/101
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{100}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^100,x]
```

```
[Out] IntegrateAlgebraic[x^100, x]
```

**fricas [A]** time = 1.31, size = 5, normalized size = 0.71

$$\frac{1}{101}x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="fricas")
```

```
[Out] 1/101*x^101
```

**giac [A]** time = 1.18, size = 5, normalized size = 0.71

$$\frac{1}{101}\,x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="giac")
```

```
[Out] 1/101*x^101
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^100,x)
```

```
[Out] 1/101*x^101
```

**maxima [A]** time = 0.42, size = 5, normalized size = 0.71

$$\frac{1}{101}\,x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="maxima")
```

```
[Out] 1/101*x^101
```

**mupad** [B] time = 0.12, size = 5, normalized size = 0.71

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^100,x)
```

```
[Out] x^101/101
```

**sympy** [A] time = 0.06, size = 3, normalized size = 0.43

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**100,x)
```

```
[Out] x**101/101
```

## 3.11 $\int x^3\,dx$

**Optimal**. Leaf size=7

$$\frac{x^4}{4}$$

**Rubi** [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3,x]
```

```
[Out] x^4/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3\,dx = \frac{x^4}{4}$$

**Mathematica** [A] time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3,x]
```

```
[Out] x^4/4
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3,x]
```

```
[Out] IntegrateAlgebraic[x^3, x]
```

**fricas** [A] time = 1.14, size = 5, normalized size = 0.71

$$\frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4
```

**giac** [A] time = 1.18, size = 5, normalized size = 0.71

$$\frac{1}{4}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4
```

**maple** [A] time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3,x)
```

```
[Out] 1/4*x^4
```

**maxima** [A] time = 0.42, size = 5, normalized size = 0.71

$$\frac{1}{4}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4
```

**mupad** [B] time = 0.02, size = 5, normalized size = 0.71

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3,x)
[Out] x^4/4
```

**sympy** [A] time = 0.05, size = 3, normalized size = 0.43

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3,x)
[Out] x**4/4
```

## 3.12 $\int x^2\,dx$

**Optimal**. Leaf size=7

$$\frac{x^3}{3}$$

**Rubi** [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {30}

$$\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2,x]
```

```
[Out] x^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\,dx = \frac{x^3}{3}$$

**Mathematica** [A] time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2,x]
```

```
[Out] x^3/3
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2,x]
```

```
[Out] IntegrateAlgebraic[x^2, x]
```

**fricas [A]** time = 1.13, size = 5, normalized size = 0.71

$$\frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3
```

**giac [A]** time = 1.10, size = 5, normalized size = 0.71

$$\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2,x)
```

```
[Out] 1/3*x^3
```

**maxima [A]** time = 0.42, size = 5, normalized size = 0.71

$$\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3
```

**mupad [B]** time = 0.01, size = 5, normalized size = 0.71

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2,x)
[Out] x^3/3
```

**sympy [A]** time = 0.05, size = 3, normalized size = 0.43

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2,x)
[Out] x**3/3
```

## 3.13 $\int x\,dx$

**Optimal**. Leaf size=7

$$\frac{x^2}{2}$$

**Rubi [A]** time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {30}

$$\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x,x]

[Out] x^2/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\,dx = \frac{x^2}{2}$$

**Mathematica [A]** time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x,x]

[Out] x^2/2

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x,x]
```

```
[Out] IntegrateAlgebraic[x, x]
```

**fricas [A]** time = 1.10, size = 5, normalized size = 0.71

$$\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2
```

**giac [A]** time = 0.95, size = 5, normalized size = 0.71

$$\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="giac")
```

```
[Out] 1/2*x^2
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x,x)
```

```
[Out] 1/2*x^2
```

**maxima [A]** time = 0.42, size = 5, normalized size = 0.71

$$\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2
```

**mupad [B]** time = 0.01, size = 5, normalized size = 0.71

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x,x)
```

```
[Out] x^2/2
```

**sympy [A]** time = 0.02, size = 3, normalized size = 0.43

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x)
```

```
[Out] x**2/2
```

## 3.14 $\int 1 \, dx$

**Optimal**. Leaf size=1

$$x$$

**Rubi [A]** time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[1,x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 1 \, dx = x$$

**Mathematica [A]** time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[1,x]
```

```
[Out] x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int 1 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1,x]
```

```
[Out] IntegrateAlgebraic[1, x]
```

**fricas [A]** time = 1.29, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="fricas")
[Out] x
```

**giac [A]** time = 1.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="giac")
[Out] x
```

**maple [A]** time = 0.00, size = 2, normalized size = 2.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
[Out] x
```

**maxima [A]** time = 0.44, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="maxima")
[Out] x
```

**mupad [B]** time = 0.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

**sympy** [**A**] time = 0.01, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

## 3.15 $\int \frac{1}{x} \, dx$

**Optimal**. Leaf size=2

$$\log(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {29}

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x} \, dx = \log(x)$$

**Mathematica [A]** time = 0.00, size = 2, normalized size = 1.00

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
```

```
[Out] Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1),x]
```

```
[Out] IntegrateAlgebraic[x^(-1), x]
```

**fricas [A]** time = 1.00, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
[Out] log(x)
```

**giac [A]** time = 0.80, size = 3, normalized size = 1.50

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
[Out] log(abs(x))
```

**maple [A]** time = 0.00, size = 3, normalized size = 1.50

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
[Out] ln(x)
```

**maxima [A]** time = 0.44, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
[Out] log(x)
```

**mupad [B]** time = 0.04, size = 2, normalized size = 1.00

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] log(x)
```

**sympy [A]** time = 0.06, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

## 3.16 $\int \frac{1}{x^2} \, dx$

**Optimal**. Leaf size=5

$$-\frac{1}{x}$$

**Rubi** [A] time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2),x]
```

```
[Out] -x^(-1)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2} \, dx = -\frac{1}{x}$$

**Mathematica** [A] time = 0.00, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2),x]
```

```
[Out] -x^(-1)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-2),x]
```

```
[Out] IntegrateAlgebraic[x^(-2), x]
```

**fricas [A]** time = 1.79, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="fricas")
```

```
[Out] -1/x
```

**giac [A]** time = 0.85, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="giac")
```

```
[Out] -1/x
```

**maple [A]** time = 0.00, size = 6, normalized size = 1.20

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2,x)
```

```
[Out] -1/x
```

**maxima [A]** time = 0.43, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="maxima")
```

```
[Out] -1/x
```

**mupad** [**B**] time = 0.03, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2,x)
[Out] -1/x
```

**sympy** [**A**] time = 0.06, size = 3, normalized size = 0.60

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2,x)
[Out] -1/x
```

## 3.17 $\int \frac{1}{x^3} \, dx$

**Optimal**. Leaf size=7

$$-\frac{1}{2x^2}$$

**Rubi** [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3),x]
```

```
[Out] -1/(2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3} \, dx = -\frac{1}{2x^2}$$

**Mathematica** [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3),x]
```

```
[Out] -1/2*1/x^2
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-3),x]
[Out] IntegrateAlgebraic[x^(-3), x]
```

**fricas [A]** time = 1.34, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="fricas")
[Out] -1/2/x^2
```

**giac [A]** time = 0.84, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="giac")
[Out] -1/2/x^2
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.86

$$-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3,x)
[Out] -1/2/x^2
```

**maxima [A]** time = 0.49, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="maxima")
[Out] -1/2/x^2
```

**mupad [B]** time = 0.01, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3,x)
```

```
[Out] -1/(2*x^2)
```

**sympy [A]** time = 0.06, size = 7, normalized size = 1.00

$$-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3,x)
```

```
[Out] -1/(2*x**2)
```

## 3.18 $\int \frac{1}{x^4} dx$

**Optimal**. Leaf size=7

$$-\frac{1}{3x^3}$$

**Rubi [A]** time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4),x]
[Out] -1/(3*x^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4} dx = -\frac{1}{3x^3}$$

**Mathematica [A]** time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4),x]
[Out] -1/3*1/x^3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-4),x]
```

```
[Out] IntegrateAlgebraic[x^(-4), x]
```

**fricas [A]** time = 1.26, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="fricas")
```

```
[Out] -1/3/x^3
```

**giac [A]** time = 1.13, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="giac")
```

```
[Out] -1/3/x^3
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.86

$$-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4,x)
```

```
[Out] -1/3/x^3
```

**maxima [A]** time = 0.43, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="maxima")
```

```
[Out] -1/3/x^3
```

**mupad** [B] time = 0.01, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4,x)
[Out] -1/(3*x^3)
```

**sympy** [A] time = 0.06, size = 7, normalized size = 1.00

$$-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4,x)
[Out] -1/(3*x**3)
```

## 3.19 $\int \frac{1}{x^{100}} \, dx$

**Optimal**. Leaf size=7

$$-\frac{1}{99x^{99}}$$

**Rubi [A]** time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{99x^{99}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-100),x]
```

```
[Out] -1/(99*x^99)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{100}} \, dx = -\frac{1}{99x^{99}}$$

**Mathematica [A]** time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{99x^{99}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-100),x]
```

```
[Out] -1/99*1/x^99
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{100}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-100),x]
```

```
[Out] IntegrateAlgebraic[x^(-100), x]
```

**fricas [A]** time = 1.31, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="fricas")
```

```
[Out] -1/99/x^99
```

**giac [A]** time = 1.29, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="giac")
```

```
[Out] -1/99/x^99
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.86

$$-\frac{1}{99x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^100,x)
```

```
[Out] -1/99/x^99
```

**maxima [A]** time = 0.42, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="maxima")
```

```
[Out] -1/99/x^99
```

**mupad** [B] time = 0.07, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^100,x)
```

```
[Out] -1/(99*x^99)
```

**sympy** [A] time = 0.06, size = 7, normalized size = 1.00

$$-\frac{1}{99x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**100,x)
```

```
[Out] -1/(99*x**99)
```

## 3.20 $\int x^{5/2}\,dx$

**Optimal**. Leaf size=9

$$\frac{2x^{7/2}}{7}$$

**Rubi** [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2),x]
```

```
[Out] (2*x^(7/2))/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/2}\,dx = \frac{2x^{7/2}}{7}$$

**Mathematica** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2),x]
```

```
[Out] (2*x^(7/2))/7
```

**IntegrateAlgebraic** [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2),x]
```

```
[Out] (2*x^(7/2))/7
```

**fricas [A]** time = 0.86, size = 5, normalized size = 0.56

$$\frac{2}{7}\,x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*x^(7/2)
```

**giac [A]** time = 0.90, size = 5, normalized size = 0.56

$$\frac{2}{7}\,x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="giac")
```

```
[Out] 2/7*x^(7/2)
```

**maple [A]** time = 0.02, size = 6, normalized size = 0.67

$$\frac{2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2),x)
```

```
[Out] 2/7*x^(7/2)
```

**maxima [A]** time = 0.43, size = 5, normalized size = 0.56

$$\frac{2}{7}\,x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*x^(7/2)
```

**mupad [B]** time = 0.08, size = 5, normalized size = 0.56

$$\frac{2\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2),x)
[Out] (2*x^(7/2))/7
```

**sympy [A]** time = 0.07, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2),x)
[Out] 2*x**(7/2)/7
```

## 3.21 $\int x^{3/2}\,dx$

**Optimal**. Leaf size=9

$$\frac{2x^{5/2}}{5}$$

**Rubi** [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2),x]
[Out] (2*x^(5/2))/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\,dx = \frac{2x^{5/2}}{5}$$

**Mathematica** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2),x]
[Out] (2*x^(5/2))/5
```

**IntegrateAlgebraic** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2),x]
```

```
[Out] (2*x^(5/2))/5
```

**fricas [A]** time = 1.55, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*x^(5/2)
```

**giac [A]** time = 0.98, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.67

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x)
```

```
[Out] 2/5*x^(5/2)
```

**maxima [A]** time = 0.42, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)
```

**mupad [B]** time = 0.03, size = 5, normalized size = 0.56

$$\frac{2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x)
[Out] (2*x^(5/2))/5
```

**sympy [A]** time = 0.06, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2),x)
[Out] 2*x**(5/2)/5
```

## 3.22 $\int \sqrt{x}\,dx$

**Optimal**. Leaf size=9

$$\frac{2x^{3/2}}{3}$$

**Rubi** [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{x}\,dx = \frac{2x^{3/2}}{3}$$

**Mathematica** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3
```

**IntegrateAlgebraic** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3
```

**fricas [A]** time = 0.93, size = 5, normalized size = 0.56

$$\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2)
```

**giac [A]** time = 1.04, size = 5, normalized size = 0.56

$$\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.67

$$\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)
```

**maxima [A]** time = 0.42, size = 5, normalized size = 0.56

$$\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)
```

**mupad [B]** time = 0.03, size = 5, normalized size = 0.56

$$\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2),x)
[Out] (2*x^(3/2))/3
```

**sympy [A]** time = 0.06, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2),x)
[Out] 2*x**(3/2)/3
```

## 3.23 $\int \frac{1}{\sqrt{x}} \, dx$

**Optimal**. Leaf size=7

$$2\sqrt{x}$$

**Rubi [A]** time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}} \, dx = 2\sqrt{x}$$

**Mathematica [A]** time = 0.00, size = 7, normalized size = 1.00

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]
```

**IntegrateAlgebraic [A]** time = 0.00, size = 7, normalized size = 1.00

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]
```

**fricas [A]** time = 0.77, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)
```

**giac [A]** time = 1.13, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.86

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2),x)
```

```
[Out] 2*x^(1/2)
```

**maxima [A]** time = 0.42, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)
```

**mupad [B]** time = 0.03, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2),x)
```

```
[Out] 2*x^(1/2)
```

**sympy** [A] time = 0.06, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2),x)
```

```
[Out] 2*sqrt(x)
```

## 3.24 $\int \frac{1}{x^{3/2}}\,dx$

**Optimal**. Leaf size=7

$$-\frac{2}{\sqrt{x}}$$

**Rubi** [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3/2),x]
```

```
[Out] -2/Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}}\,dx = -\frac{2}{\sqrt{x}}$$

**Mathematica** [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3/2),x]
```

```
[Out] -2/Sqrt[x]
```

**IntegrateAlgebraic** [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-3/2),x]
[Out] -2/Sqrt[x]
```

**fricas [A]** time = 1.43, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="fricas")
[Out] -2/sqrt(x)
```

**giac [A]** time = 1.01, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="giac")
[Out] -2/sqrt(x)
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.86

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2),x)
[Out] -2/x^(1/2)
```

**maxima [A]** time = 0.70, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="maxima")
[Out] -2/sqrt(x)
```

**mupad [B]** time = 0.03, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2),x)
```

```
[Out] -2/x^(1/2)
```

**sympy [A]** time = 0.06, size = 7, normalized size = 1.00

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2),x)
```

```
[Out] -2/sqrt(x)
```

## 3.25 $\int \frac{1}{x^{5/2}} \, dx$

**Optimal**. Leaf size=9

$$-\frac{2}{3x^{3/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5/2),x]
```

```
[Out] -2/(3*x^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}} \, dx = -\frac{2}{3x^{3/2}}$$

**Mathematica [A]** time = 0.00, size = 9, normalized size = 1.00

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5/2),x]
```

```
[Out] -2/(3*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.00, size = 9, normalized size = 1.00

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-5/2),x]
[Out] -2/(3*x^(3/2))
```

**fricas** [A] time = 1.36, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="fricas")
[Out] -2/3/x^(3/2)
```

**giac** [A] time = 0.92, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="giac")
[Out] -2/3/x^(3/2)
```

**maple** [A] time = 0.00, size = 6, normalized size = 0.67

$$-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2),x)
[Out] -2/3/x^(3/2)
```

**maxima** [A] time = 0.62, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/x^(3/2)
```

**mupad [B]** time = 0.03, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2),x)
[Out] -2/(3*x^(3/2))
```

**sympy [A]** time = 0.06, size = 8, normalized size = 0.89

$$-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2),x)
[Out] -2/(3*x**(3/2))
```

## 3.26 $\int x^{5/3}\,dx$

**Optimal**. Leaf size=9

$$\frac{3x^{8/3}}{8}$$

**Rubi** [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3),x]
```

```
[Out] (3*x^(8/3))/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/3}\,dx = \frac{3x^{8/3}}{8}$$

**Mathematica** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3),x]
```

```
[Out] (3*x^(8/3))/8
```

**IntegrateAlgebraic** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3),x]
```

```
[Out] (3*x^(8/3))/8
```

**fricas [A]** time = 1.26, size = 5, normalized size = 0.56

$$\frac{3}{8}\,x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/8*x^(8/3)
```

**giac [A]** time = 1.05, size = 5, normalized size = 0.56

$$\frac{3}{8}\,x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="giac")
```

```
[Out] 3/8*x^(8/3)
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.67

$$\frac{3x^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3),x)
```

```
[Out] 3/8*x^(8/3)
```

**maxima [A]** time = 0.56, size = 5, normalized size = 0.56

$$\frac{3}{8}\,x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/8*x^(8/3)
```

**mupad [B]** time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{8/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3),x)
[Out] (3*x^(8/3))/8
```

**sympy [A]** time = 0.06, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3),x)
[Out] 3*x**(8/3)/8
```

## 3.27 $\int x^{4/3}\,dx$

**Optimal**. Leaf size=9

$$\frac{3x^{7/3}}{7}$$

**Rubi** [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3),x]
```

```
[Out] (3*x^(7/3))/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{4/3}\,dx = \frac{3x^{7/3}}{7}$$

**Mathematica** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3),x]
```

```
[Out] (3*x^(7/3))/7
```

**IntegrateAlgebraic** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3),x]
```

```
[Out] (3*x^(7/3))/7
```

**fricas [A]** time = 1.47, size = 5, normalized size = 0.56

$$\frac{3}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7*x^(7/3)
```

**giac [A]** time = 1.05, size = 5, normalized size = 0.56

$$\frac{3}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="giac")
```

```
[Out] 3/7*x^(7/3)
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.67

$$\frac{3x^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3),x)
```

```
[Out] 3/7*x^(7/3)
```

**maxima [A]** time = 0.56, size = 5, normalized size = 0.56

$$\frac{3}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/7*x^(7/3)
```

**mupad [B]** time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{7/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3),x)
[Out] (3*x^(7/3))/7
```

**sympy [A]** time = 0.06, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3),x)
[Out] 3*x**(7/3)/7
```

## 3.28 $\int x^{2/3}\,dx$

**Optimal**. Leaf size=9

$$\frac{3x^{5/3}}{5}$$

**Rubi** [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3),x]
```

```
[Out] (3*x^(5/3))/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{2/3}\,dx=\frac{3x^{5/3}}{5}$$

**Mathematica** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3),x]
```

```
[Out] (3*x^(5/3))/5
```

**IntegrateAlgebraic** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3),x]
```

```
[Out] (3*x^(5/3))/5
```

**fricas [A]** time = 0.70, size = 5, normalized size = 0.56

$$\frac{3}{5}\, x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*x^(5/3)
```

**giac [A]** time = 0.91, size = 5, normalized size = 0.56

$$\frac{3}{5}\, x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="giac")
```

```
[Out] 3/5*x^(5/3)
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.67

$$\frac{3x^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3),x)
```

```
[Out] 3/5*x^(5/3)
```

**maxima [A]** time = 0.51, size = 5, normalized size = 0.56

$$\frac{3}{5}\, x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/5*x^(5/3)
```

**mupad [B]** time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3),x)
[Out] (3*x^(5/3))/5
```

**sympy [A]** time = 0.06, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3),x)
[Out] 3*x**(5/3)/5
```

## 3.29 $\int \sqrt[3]{x}\, dx$

**Optimal**. Leaf size=9

$$\frac{3x^{4/3}}{4}$$

**Rubi** [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3),x]
[Out] (3*x^(4/3))/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{x}\, dx = \frac{3x^{4/3}}{4}$$

**Mathematica** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3),x]
[Out] (3*x^(4/3))/4
```

**IntegrateAlgebraic** [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3),x]
```

```
[Out] (3*x^(4/3))/4
```

**fricas [A]** time = 1.39, size = 5, normalized size = 0.56

$$\frac{3}{4}\,x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*x^(4/3)
```

**giac [A]** time = 0.85, size = 5, normalized size = 0.56

$$\frac{3}{4}\,x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*x^(4/3)
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.67

$$\frac{3x^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3),x)
```

```
[Out] 3/4*x^(4/3)
```

**maxima [A]** time = 0.48, size = 5, normalized size = 0.56

$$\frac{3}{4}\,x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*x^(4/3)
```

**mupad [B]** time = 0.06, size = 5, normalized size = 0.56

$$\frac{3\,x^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3),x)
[Out] (3*x^(4/3))/4
```

**sympy [A]** time = 0.06, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3),x)
[Out] 3*x**(4/3)/4
```

## 3.30 $\int \frac{1}{\sqrt[3]{x}}\,dx$

**Optimal**. Leaf size=9

$$\frac{3x^{2/3}}{2}$$

**Rubi [A]** time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1/3),x]
```

```
[Out] (3*x^(2/3))/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}}\,dx = \frac{3x^{2/3}}{2}$$

**Mathematica [A]** time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1/3),x]
```

```
[Out] (3*x^(2/3))/2
```

**IntegrateAlgebraic [A]** time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1/3),x]
```

```
[Out] (3*x^(2/3))/2
```

**fricas** [A]  time = 1.35, size = 5, normalized size = 0.56

$$\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3)
```

**giac** [A]  time = 1.14, size = 5, normalized size = 0.56

$$\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3)
```

**maple** [A]  time = 0.00, size = 6, normalized size = 0.67

$$\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3),x)
```

```
[Out] 3/2*x^(2/3)
```

**maxima** [A]  time = 0.46, size = 5, normalized size = 0.56

$$\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3)
```

**mupad** [B]  time = 0.04, size = 5, normalized size = 0.56

$$\frac{3\,x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3),x)
```

```
[Out] (3*x^(2/3))/2
```

**sympy** [A]  time = 0.06, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3),x)
```

```
[Out] 3*x**(2/3)/2
```

## 3.31 $\int \frac{1}{x^{2/3}} \, dx$

**Optimal**. Leaf size=7

$$3\sqrt[3]{x}$$

**Rubi** [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2/3),x]
[Out] 3*x^(1/3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{2/3}} \, dx = 3\sqrt[3]{x}$$

**Mathematica** [A] time = 0.00, size = 7, normalized size = 1.00

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2/3),x]
[Out] 3*x^(1/3)
```

**IntegrateAlgebraic** [A] time = 0.00, size = 7, normalized size = 1.00

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-2/3),x]
```

```
[Out] 3*x^(1/3)
```

**fricas [A]** time = 0.81, size = 5, normalized size = 0.71

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3*x^(1/3)
```

**giac [A]** time = 0.95, size = 5, normalized size = 0.71

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="giac")
```

```
[Out] 3*x^(1/3)
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.86

$$3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3),x)
```

```
[Out] 3*x^(1/3)
```

**maxima [A]** time = 0.52, size = 5, normalized size = 0.71

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3*x^(1/3)
```

**mupad [B]** time = 0.07, size = 5, normalized size = 0.71

$$3\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3),x)
```

```
[Out] 3*x^(1/3)
```

**sympy** [**A**] time = 0.06, size = 5, normalized size = 0.71

$$3\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3),x)
```

```
[Out] 3*x**(1/3)
```

## 3.32 $\int \frac{1}{x^{4/3}} \, dx$

**Optimal**. Leaf size=7

$$-\frac{3}{\sqrt[3]{x}}$$

**Rubi** [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4/3),x]
```

```
[Out] -3/x^(1/3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{4/3}} \, dx = -\frac{3}{\sqrt[3]{x}}$$

**Mathematica** [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4/3),x]
```

```
[Out] -3/x^(1/3)
```

**IntegrateAlgebraic** [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-4/3),x]
```

```
[Out] -3/x^(1/3)
```

**fricas** [A]    time = 1.42, size = 5, normalized size = 0.71

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="fricas")
```

```
[Out] -3/x^(1/3)
```

**giac** [A]    time = 1.12, size = 5, normalized size = 0.71

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="giac")
```

```
[Out] -3/x^(1/3)
```

**maple** [A]    time = 0.00, size = 6, normalized size = 0.86

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3),x)
```

```
[Out] -3/x^(1/3)
```

**maxima** [A]    time = 0.51, size = 5, normalized size = 0.71

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="maxima")
```

```
[Out] -3/x^(1/3)
```

**mupad [B]** time = 0.07, size = 5, normalized size = 0.71

$$-\frac{3}{x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3),x)
[Out] -3/x^(1/3)
```

**sympy [A]** time = 0.06, size = 7, normalized size = 1.00

$$-\frac{3}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3),x)
[Out] -3/x**(1/3)
```

## 3.33 $\int \frac{1}{x^{5/3}} dx$

**Optimal**. Leaf size=9

$$-\frac{3}{2x^{2/3}}$$

**Rubi** [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5/3),x]
[Out] -3/(2*x^(2/3))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/3}} dx = -\frac{3}{2x^{2/3}}$$

**Mathematica** [A] time = 0.00, size = 9, normalized size = 1.00

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5/3),x]
[Out] -3/(2*x^(2/3))
```

**IntegrateAlgebraic** [A] time = 0.00, size = 9, normalized size = 1.00

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-5/3),x]
```

```
[Out] -3/(2*x^(2/3))
```

**fricas** [A] time = 1.11, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="fricas")
```

```
[Out] -3/2/x^(2/3)
```

**giac** [A] time = 0.92, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="giac")
```

```
[Out] -3/2/x^(2/3)
```

**maple** [A] time = 0.00, size = 6, normalized size = 0.67

$$-\frac{3}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3),x)
```

```
[Out] -3/2/x^(2/3)
```

**maxima** [A] time = 0.49, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="maxima")
```

```
[Out] -3/2/x^(2/3)
```

**mupad [B]** time = 0.05, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3),x)
```

```
[Out] -3/(2*x^(2/3))
```

**sympy [A]** time = 0.06, size = 8, normalized size = 0.89

$$-\frac{3}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3),x)
```

```
[Out] -3/(2*x**(2/3))
```

## 3.34 $\int x^n \, dx$

**Optimal**. Leaf size=11

$$\frac{x^{n+1}}{n+1}$$

**Rubi** [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^n \, dx = \frac{x^{1+n}}{1+n}$$

**Mathematica** [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^n, x]
```

**fricas** **[A]** time = 1.69, size = 10, normalized size = 0.91

$$\frac{xx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="fricas")
```

```
[Out] x*x^n/(n + 1)
```

**giac** **[A]** time = 0.95, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="giac")
```

```
[Out] x^(n + 1)/(n + 1)
```

**maple** **[A]** time = 0.00, size = 12, normalized size = 1.09

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x)
```

```
[Out] x^(1+n)/(1+n)
```

**maxima** **[A]** time = 0.64, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="maxima")
```

```
[Out] x^(n + 1)/(n + 1)
```

**mupad [B]** time = 0.35, size = 20, normalized size = 1.82

$$\begin{cases} \ln(x) & \text{if } n = -1 \\ \frac{x^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x)
[Out] piecewise(n == -1, log(x), n ~= -1, x^(n + 1)/(n + 1))
```

**sympy [A]** time = 0.06, size = 12, normalized size = 1.09

$$\begin{cases} \frac{x^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n,x)
[Out] Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

## 3.35 $\int (bx)^n \, dx$

**Optimal**. Leaf size=16

$$\frac{(bx)^{n+1}}{b(n+1)}$$

**Rubi** [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {32}

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^n,x]
```

```
[Out] (b*x)^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (bx)^n \, dx = \frac{(bx)^{1+n}}{b(1+n)}$$

**Mathematica** [A] time = 0.00, size = 12, normalized size = 0.75

$$\frac{x(bx)^n}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^n,x]
```

```
[Out] (x*(b*x)^n)/(1 + n)
```

**IntegrateAlgebraic** [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (bx)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(b*x)^n, x]
```

**fricas [A]** time = 1.37, size = 12, normalized size = 0.75

$$\frac{(bx)^n x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="fricas")
```

```
[Out] (b*x)^n*x/(n + 1)
```

**giac [A]** time = 1.04, size = 16, normalized size = 1.00

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="giac")
```

```
[Out] (b*x)^(n + 1)/(b*(n + 1))
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$\frac{x\,(bx)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^n,x)
```

```
[Out] x/(n+1)*(b*x)^n
```

**maxima [A]** time = 0.54, size = 16, normalized size = 1.00

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="maxima")
```

```
[Out] (b*x)^(n + 1)/(b*(n + 1))
```

**mupad [B]** time = 0.18, size = 12, normalized size = 0.75

$$\frac{x\,(b\,x)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^n,x)
```

```
[Out] (x*(b*x)^n)/(n + 1)
```

**sympy [A]** time = 0.06, size = 17, normalized size = 1.06

$$\frac{\begin{cases} \frac{(bx)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**n,x)
```

```
[Out] Piecewise(((b*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(b*x), True))/b
```

## 3.36 $\int \frac{1}{\sqrt{-a}+e(c+dx)}\,dx$

**Optimal**. Leaf size=23

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

**Rubi** [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {33, 31}

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

```
[Out] Log[Sqrt[-a] + c*e + d*e*x]/(d*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a}+e(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+ex}\,dx, x, c+dx\right)}{d}\\ &= \frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

```
[Out] Log[Sqrt[-a] + c*e + d*e*x]/(d*e)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a} + e(c + dx)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

```
[Out] IntegrateAlgebraic[(Sqrt[-a] + e*(c + d*x))^(-1), x]
```

**fricas [A]** time = 1.45, size = 21, normalized size = 0.91

$$\frac{\log\left(dex + ce + \sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] log(d*e*x + c*e + sqrt(-a))/(d*e)
```

**giac [A]** time = 1.07, size = 22, normalized size = 0.96

$$\frac{e^{(-1)} \log\left(\left|(dx + c)e + \sqrt{-a}\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] e^(-1)*log(abs((d*x + c)*e + sqrt(-a)))/d
```

**maple [A]** time = 0.00, size = 22, normalized size = 0.96

$$\frac{\ln\left(dex + ce + \sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*(d*x+c)+(-a)^(1/2)),x)
```

```
[Out] ln(c*e+d*e*x+(-a)^(1/2))/d/e
```

**maxima [A]** time = 0.67, size = 21, normalized size = 0.91

$$\frac{\log\left((dx+c)e+\sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="maxima")
[Out] log((d*x + c)*e + sqrt(-a))/(d*e)
```

**mupad [B]** time = 0.14, size = 21, normalized size = 0.91

$$\frac{\ln\left(\sqrt{-a}+c\,e+d\,e\,x\right)}{d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-a)^(1/2) + e*(c + d*x)),x)
[Out] log((-a)^(1/2) + c*e + d*e*x)/(d*e)
```

**sympy [A]** time = 0.09, size = 19, normalized size = 0.83

$$\frac{\log\left(ce+dex+\sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)**(1/2)),x)
[Out] log(c*e + d*e*x + sqrt(-a))/(d*e)
```

## 3.37 $\int (c+d(a+bx))^{5/2}\,dx$

**Optimal**. Leaf size=23

$$\frac{2(d(a+bx)+c)^{7/2}}{7bd}$$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(7/2))/(7*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (c+d(a+bx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (c+dx)^{5/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{7/2}}{7bd}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 23, normalized size = 1.00

$$\frac{2(d(a+bx)+c)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(7/2))/(7*b*d)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(ad+bdx+c)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + a*d + b*d*x)^(7/2))/(7*b*d)
```

**fricas [B]** time = 1.48, size = 104, normalized size = 4.52

$$\frac{2\left(b^3d^3x^3+a^3d^3+3\,a^2cd^2+3\,ac^2d+c^3+3\left(ab^2d^3+b^2cd^2\right)x^2+3\left(a^2bd^3+2\,abcd^2+bc^2d\right)x\right)\sqrt{bdx+ad+c}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*d^3*x^3 + a^3*d^3 + 3*a^2*c*d^2 + 3*a*c^2*d + c^3 + 3*(a*b^2*d^3 +
 b^2*c*d^2)*x^2 + 3*(a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d)*x)*sqrt(b*d*x + a*d
 + c)/(b*d)
```

**giac [B]** time = 1.64, size = 444, normalized size = 19.30

$$\frac{2\left(35\,(bdx+ad+c)^{\frac{3}{2}}a^2d^2-35\left(3\,\sqrt{bdx+ad+c}\,ad-(bdx+ad+c)^{\frac{3}{2}}+3\,\sqrt{bdx+ad+c}\,c\right)a^2d^2-21\,(bdx+ad+c)^{\frac{5}{2}}ad+70\,(bdx+ad+c)^{\frac{3}{2}}acd-70\left(3\,\sqrt{bdx+ad+c}\,ad-(bdx+ad+c)^{\frac{3}{2}}+3\,\sqrt{bdx+ad+c}\,c\right)acd+5\,(bdx+ad+c)^{\frac{7}{2}}-21\,(bdx+ad+c)^{\frac{5}{2}}c+35\,(bdx+ad+c)^{\frac{3}{2}}c^2-35\left(3\,\sqrt{bdx+ad+c}\,ad-(bdx+ad+c)^{\frac{3}{2}}+3\,\sqrt{bdx+ad+c}\,c\right)c^2+7\left(15\,\sqrt{bdx+ad+c}\,a^2d^2-10\,(bdx+ad+c)^{\frac{3}{2}}ad+30\,\sqrt{bdx+ad+c}\,acd+3\,(bdx+ad+c)^{\frac{5}{2}}-10\,(bdx+ad+c)^{\frac{3}{2}}c+15\,\sqrt{bdx+ad+c}\,c^2\right)ad+7\left(15\,\sqrt{bdx+ad+c}\,a^2d^2-10\,(bdx+ad+c)^{\frac{3}{2}}ad+30\,\sqrt{bdx+ad+c}\,acd+3\,(bdx+ad+c)^{\frac{5}{2}}-10\,(bdx+ad+c)^{\frac{3}{2}}c+15\,\sqrt{bdx+ad+c}\,c^2\right)c\right)}{35\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*(b*d*x + a*d + c)^(3/2)*a^2*d^2 - 35*(3*sqrt(b*d*x + a*d + c)*a*d
- (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a^2*d^2 - 21*(b*d*x
+ a*d + c)^(5/2)*a*d + 70*(b*d*x + a*d + c)^(3/2)*a*c*d - 70*(3*sqrt(b*d*x
+ a*d + c)*a*d - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a*c*d
 + 5*(b*d*x + a*d + c)^(7/2) - 21*(b*d*x + a*d + c)^(5/2)*c + 35*(b*d*x + a
*d + c)^(3/2)*c^2 - 35*(3*sqrt(b*d*x + a*d + c)*a*d - (b*d*x + a*d + c)^(3/
2) + 3*sqrt(b*d*x + a*d + c)*c)*c^2 + 7*(15*sqrt(b*d*x + a*d + c)*a^2*d^2 -
 10*(b*d*x + a*d + c)^(3/2)*a*d + 30*sqrt(b*d*x + a*d + c)*a*c*d + 3*(b*d*x
 + a*d + c)^(5/2) - 10*(b*d*x + a*d + c)^(3/2)*c + 15*sqrt(b*d*x + a*d + c)
*c^2)*a*d + 7*(15*sqrt(b*d*x + a*d + c)*a^2*d^2 - 10*(b*d*x + a*d + c)^(3/2
)*a*d + 30*sqrt(b*d*x + a*d + c)*a*c*d + 3*(b*d*x + a*d + c)^(5/2) - 10*(b*
d*x + a*d + c)^(3/2)*c + 15*sqrt(b*d*x + a*d + c)*c^2)*c)/(b*d)
```

**maple [A]** time = 0.00, size = 20, normalized size = 0.87

$$\frac{2 (bdx + ad + c)^{\frac{7}{2}}}{7bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(5/2),x)
```

```
[Out] 2/7*(b*d*x+a*d+c)^(7/2)/d/b
```

**maxima [A]** time = 0.56, size = 19, normalized size = 0.83

$$\frac{2 ((bx + a)d + c)^{\frac{7}{2}}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*((b*x + a)*d + c)^(7/2)/(b*d)
```

**mupad [B]** time = 0.18, size = 93, normalized size = 4.04

$$\frac{6\,x\,\sqrt{c+d\,\left(a+b\,x\right)}\,{\left(c+a\,d\right)}^2}{7}+\frac{2\,\sqrt{c+d\,\left(a+b\,x\right)}\,{\left(c+a\,d\right)}^3}{7\,b\,d}+\frac{2\,b^2\,d^2\,x^3\,\sqrt{c+d\,\left(a+b\,x\right)}}{7}+\frac{6\,b\,d\,x^2\,\sqrt{c+d\,\left(a+b\,x\right)}\,\left(c+a\,d\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(5/2),x)
```

```
[Out] (6*x*(c + d*(a + b*x))^(1/2)*(c + a*d)^2)/7 + (2*(c + d*(a + b*x))^(1/2)*(c
 + a*d)^3)/(7*b*d) + (2*b^2*d^2*x^3*(c + d*(a + b*x))^(1/2))/7 + (6*b*d*x^2
*(c + d*(a + b*x))^(1/2)*(c + a*d))/7
```

**sympy [A]** time = 71.80, size = 270, normalized size = 11.74

$$\begin{cases} c^{\frac{5}{2}} x & \text{for}\: b = 0 \wedge d = 0 \\ x \left(ad + c\right)^{\frac{5}{2}} & \text{for}\: b = 0 \\ c^{\frac{5}{2}} x & \text{for}\: d = 0 \\ \frac{2a^3d^2\sqrt{ad+bdx+c}}{7b} + \frac{6a^2d^2x\sqrt{ad+bdx+c}}{7} + \frac{6a^2cd\sqrt{ad+bdx+c}}{7b} + \frac{6abd^2x^2\sqrt{ad+bdx+c}}{7} + \frac{12acdx\sqrt{ad+bdx+c}}{7} + \frac{6ac^2\sqrt{ad+bdx+c}}{7b} + \frac{2b^2d^2x^3\sqrt{ad+bdx+c}}{7} + \frac{6bcdx^2\sqrt{ad+bdx+c}}{7} + \frac{6c^2x\sqrt{ad+bdx+c}}{7} + \frac{2c^3\sqrt{ad+bdx+c}}{7bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((c**(5/2)*x, Eq(b, 0) & Eq(d, 0)), (x*(a*d + c)**(5/2), Eq(b, 0))
, (c**(5/2)*x, Eq(d, 0)), (2*a**3*d**2*sqrt(a*d + b*d*x + c)/(7*b) + 6*a**2
```

```
*d**2*x*sqrt(a*d + b*d*x + c)/7 + 6*a**2*c*d*sqrt(a*d + b*d*x + c)/(7*b) +
6*a*b*d**2*x**2*sqrt(a*d + b*d*x + c)/7 + 12*a*c*d*x*sqrt(a*d + b*d*x + c)/
7 + 6*a*c**2*sqrt(a*d + b*d*x + c)/(7*b) + 2*b**2*d**2*x**3*sqrt(a*d + b*d*
x + c)/7 + 6*b*c*d*x**2*sqrt(a*d + b*d*x + c)/7 + 6*c**2*x*sqrt(a*d + b*d*x
 + c)/7 + 2*c**3*sqrt(a*d + b*d*x + c)/(7*b*d), True))
```

## 3.38 $\int (c+d(a+bx))^{3/2}\,dx$

**Optimal**. Leaf size=23

$$\frac{2(d(a+bx)+c)^{5/2}}{5bd}$$

**Rubi** [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(5/2))/(5*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (c+d(a+bx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (c+dx)^{3/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{5/2}}{5bd}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(d(a+bx)+c)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(5/2))/(5*b*d)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(ad+bdx+c)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + a*d + b*d*x)^(5/2))/(5*b*d)
```

**fricas [B]** time = 0.87, size = 59, normalized size = 2.57

$$\frac{2\left(b^2d^2x^2+a^2d^2+2\,acd+c^2+2\left(abd^2+bcd\right)x\right)\sqrt{bdx+ad+c}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*d^2*x^2 + a^2*d^2 + 2*a*c*d + c^2 + 2*(a*b*d^2 + b*c*d)*x)*sqrt(b*
d*x + a*d + c)/(b*d)
```

**giac [B]** time = 1.04, size = 195, normalized size = 8.48

$$\frac{2\left(30\,\sqrt{bdx+ad+c}\,a^2d^2-10\,(bdx+ad+c)^{\frac{3}{2}}ad+60\,\sqrt{bdx+ad+c}\,acd-10\left(3\,\sqrt{bdx+ad+c}\,ad-(bdx+ad+c)^{\frac{3}{2}}+3\,\sqrt{bdx+ad+c}\,c\right)ad+3\,(bdx+ad+c)^{\frac{5}{2}}-10\,(bdx+ad+c)^{\frac{3}{2}}c+30\,\sqrt{bdx+ad+c}\,c^2-10\left(3\,\sqrt{bdx+ad+c}\,ad-(bdx+ad+c)^{\frac{3}{2}}+3\,\sqrt{bdx+ad+c}\,c\right)c\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(30*sqrt(b*d*x + a*d + c)*a^2*d^2 - 10*(b*d*x + a*d + c)^(3/2)*a*d + 6
0*sqrt(b*d*x + a*d + c)*a*c*d - 10*(3*sqrt(b*d*x + a*d + c)*a*d - (b*d*x +
a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a*d + 3*(b*d*x + a*d + c)^(5/2)
 - 10*(b*d*x + a*d + c)^(3/2)*c + 30*sqrt(b*d*x + a*d + c)*c^2 - 10*(3*sqrt
(b*d*x + a*d + c)*a*d - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c
)*c)/(b*d)
```

**maple [A]** time = 0.00, size = 20, normalized size = 0.87

$$\frac{2\,(bdx+ad+c)^{\frac{5}{2}}}{5bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(3/2),x)
```

```
[Out] 2/5*(b*d*x+a*d+c)^(5/2)/d/b
```

**maxima** [A] time = 0.88, size = 19, normalized size = 0.83

$$\frac{2\left((bx+a)d+c\right)^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*((b*x + a)*d + c)^(5/2)/(b*d)
```

**mupad** [B] time = 0.17, size = 45, normalized size = 1.96

$$\sqrt{c+d\,\left(a+b\,x\right)}\,\left(x\,\left(\frac{4\,c}{5}+\frac{4\,a\,d}{5}\right)+\frac{2\,{\left(c+a\,d\right)}^2}{5\,b\,d}+\frac{2\,b\,d\,x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(3/2),x)
```

```
[Out] (c + d*(a + b*x))^(1/2)*(x*((4*c)/5 + (4*a*d)/5) + (2*(c + a*d)^2)/(5*b*d)
+ (2*b*d*x^2)/5)
```

**sympy** [A] time = 5.27, size = 156, normalized size = 6.78

$$\begin{cases} c^{\frac{3}{2}}x & \text{for}\: b = 0 \wedge d = 0 \\ x\left(ad+c\right)^{\frac{3}{2}} & \text{for}\: b = 0 \\ c^{\frac{3}{2}}x & \text{for}\: d = 0 \\ \frac{2a^{2}d\sqrt{ad+bdx+c}}{5b} + \frac{4adx\sqrt{ad+bdx+c}}{5} + \frac{4ac\sqrt{ad+bdx+c}}{5b} + \frac{2bdx^{2}\sqrt{ad+bdx+c}}{5} + \frac{4cx\sqrt{ad+bdx+c}}{5} + \frac{2c^{2}\sqrt{ad+bdx+c}}{5bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((c**(3/2)*x, Eq(b, 0) & Eq(d, 0)), (x*(a*d + c)**(3/2), Eq(b, 0))
, (c**(3/2)*x, Eq(d, 0)), (2*a**2*d*sqrt(a*d + b*d*x + c)/(5*b) + 4*a*d*x*s
qrt(a*d + b*d*x + c)/5 + 4*a*c*sqrt(a*d + b*d*x + c)/(5*b) + 2*b*d*x**2*sqr
t(a*d + b*d*x + c)/5 + 4*c*x*sqrt(a*d + b*d*x + c)/5 + 2*c**2*sqrt(a*d + b*
d*x + c)/(5*b*d), True))
```

## 3.39 $\int \sqrt{c+d(a+bx)}\,dx$

**Optimal**. Leaf size=23

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c+d(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{c+dx}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{3/2}}{3bd}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*(a + b*x)],x]
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(ad+bdx+c)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*(a + b*x)],x]
[Out] (2*(c + a*d + b*d*x)^(3/2))/(3*b*d)
```

**fricas** [A] time = 1.40, size = 19, normalized size = 0.83

$$\frac{2\,(bdx+ad+c)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*d*x + a*d + c)^(3/2)/(b*d)
```

**giac** [A] time = 1.07, size = 19, normalized size = 0.83

$$\frac{2\,(bdx+ad+c)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="giac")
[Out] 2/3*(b*d*x + a*d + c)^(3/2)/(b*d)
```

**maple** [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{2\,(bdx+ad+c)^{\frac{3}{2}}}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(1/2),x)
[Out] 2/3*(b*d*x+a*d+c)^(3/2)/d/b
```

**maxima** [A] time = 0.74, size = 19, normalized size = 0.83

$$\frac{2\left((bx+a)d+c\right)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 2/3*((b*x + a)*d + c)^(3/2)/(b*d)
```

**mupad** [B] time = 0.08, size = 19, normalized size = 0.83

$$\frac{2\left(c+d\,\left(a+b\,x\right)\right)^{3/2}}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(1/2),x)
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

**sympy** [A] time = 0.44, size = 82, normalized size = 3.57

$$\begin{cases} \sqrt{c}\,x & \text{for}\: b = 0 \wedge d = 0 \\ x\sqrt{ad+c} & \text{for}\: b = 0 \\ \sqrt{c}\,x & \text{for}\: d = 0 \\ \frac{2a\sqrt{ad+bdx+c}}{3b} + \frac{2x\sqrt{ad+bdx+c}}{3} + \frac{2c\sqrt{ad+bdx+c}}{3bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(1/2),x)
[Out] Piecewise((sqrt(c)*x, Eq(b, 0) & Eq(d, 0)), (x*sqrt(a*d + c), Eq(b, 0)), (s
qrt(c)*x, Eq(d, 0)), (2*a*sqrt(a*d + b*d*x + c)/(3*b) + 2*x*sqrt(a*d + b*d*
x + c)/3 + 2*c*sqrt(a*d + b*d*x + c)/(3*b*d), True))
```

## 3.40 $\int \frac{1}{\sqrt{c+d(a+bx)}}\,dx$

**Optimal**. Leaf size=21

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

**Rubi [A]** time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*Sqrt[c + d*(a + b*x)])/(b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c+d(a+bx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}}\,dx, x, a+bx\right)}{b}\\ &= \frac{2\sqrt{c+d(a+bx)}}{bd}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 21, normalized size = 1.00

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*Sqrt[c + d*(a + b*x)])/(b*d)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 21, normalized size = 1.00

$$\frac{2\sqrt{ad+bdx+c}}{bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*Sqrt[c + a*d + b*d*x])/(b*d)
```

**fricas [A]** time = 1.07, size = 19, normalized size = 0.90

$$\frac{2\sqrt{bdx+ad+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*d*x + a*d + c)/(b*d)
```

**giac [A]** time = 1.17, size = 19, normalized size = 0.90

$$\frac{2\sqrt{bdx+ad+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*d*x + a*d + c)/(b*d)
```

**maple [A]** time = 0.00, size = 20, normalized size = 0.95

$$\frac{2\sqrt{bdx+ad+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(1/2),x)
```

```
[Out] 2*(b*d*x+a*d+c)^(1/2)/d/b
```

**maxima [A]** time = 0.92, size = 19, normalized size = 0.90

$$\frac{2\sqrt{(bx+a)d+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt((b*x + a)*d + c)/(b*d)
```

**mupad [B]** time = 0.11, size = 19, normalized size = 0.90

$$\frac{2\sqrt{c+d\ (a+b\,x)}}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(1/2),x)
```

```
[Out] (2*(c + d*(a + b*x))^(1/2))/(b*d)
```

**sympy [A]** time = 1.78, size = 31, normalized size = 1.48

$$\begin{cases} \frac{x}{\sqrt{ad+c}} & \text{for}\ b=0 \\ \frac{x}{\sqrt{c}} & \text{for}\ d=0 \\ \frac{2\sqrt{c+d(a+bx)}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((x/sqrt(a*d + c), Eq(b, 0)), (x/sqrt(c), Eq(d, 0)), (2*sqrt(c + d
*(a + b*x))/(b*d), True))
```

## 3.41 $\int \frac{1}{(c+d(a+bx))^{3/2}}\,dx$

**Optimal**. Leaf size=21

$$-\frac{2}{bd\sqrt{d(a+bx)+c}}$$

**Rubi** [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$-\frac{2}{bd\sqrt{d(a+bx)+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + d*(a + b*x)])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+d(a+bx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(c+dx)^{3/2}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{2}{bd\sqrt{c+d(a+bx)}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{2}{bd\sqrt{d(a+bx)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + d*(a + b*x)])
```

**IntegrateAlgebraic** [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{2}{bd\sqrt{ad+bdx+c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + a*d + b*d*x])
```

**fricas** [A] time = 1.57, size = 34, normalized size = 1.62

$$-\frac{2\sqrt{bdx+ad+c}}{b^2d^2x+abd^2+bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*d*x + a*d + c)/(b^2*d^2*x + a*b*d^2 + b*c*d)
```

**giac** [A] time = 1.65, size = 19, normalized size = 0.90

$$-\frac{2}{\sqrt{bdx+ad+c}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(b*d*x + a*d + c)*b*d)
```

**maple** [A] time = 0.00, size = 20, normalized size = 0.95

$$-\frac{2}{\sqrt{bdx+ad+c}\;bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(3/2),x)
```

```
[Out] -2/(b*d*x+a*d+c)^(1/2)/d/b
```

**maxima [A]** time = 1.00, size = 19, normalized size = 0.90

$$-\frac{2}{\sqrt{(bx+a)d+c}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt((b*x + a)*d + c)*b*d)
```

**mupad [B]** time = 0.13, size = 19, normalized size = 0.90

$$-\frac{2}{b\,d\,\sqrt{c+d\,\left(a+b\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(3/2),x)
[Out] -2/(b*d*(c + d*(a + b*x))^(1/2))
```

**sympy [A]** time = 1.79, size = 58, normalized size = 2.76

$$\begin{cases} \frac{x}{c^{\frac{3}{2}}} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{x}{\left(ad + c\right)^{\frac{3}{2}}} & \text{for}\: b = 0 \\ \frac{x}{c^{\frac{3}{2}}} & \text{for}\: d = 0 \\ - \frac{2 \sqrt{ad + bdx + c}}{abd^{2} + b^{2} d^{2} x + bcd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(3/2),x)
[Out] Piecewise((x/c**(3/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(3/2), Eq(b, 0))
, (x/c**(3/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(a*b*d**2 + b**2*d**2*x
 + b*c*d), True))
```

## 3.42 $\int \frac{1}{(c+d(a+bx))^{5/2}}\,dx$

**Optimal**. Leaf size=23

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(-5/2),x]
```

```
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+d(a+bx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(c+dx)^{5/2}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{2}{3bd(c+d(a+bx))^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 23, normalized size = 1.00

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(-5/2),x]
```

```
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.01, size = 23, normalized size = 1.00

$$-\frac{2}{3bd(ad+bdx+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*(a + b*x))^(-5/2),x]
```

```
[Out] -2/(3*b*d*(c + a*d + b*d*x)^(3/2))
```

**fricas [B]** time = 1.41, size = 68, normalized size = 2.96

$$-\frac{2\sqrt{bdx+ad+c}}{3\left(b^3d^3x^2+a^2bd^3+2\,abcd^2+bc^2d+2\left(ab^2d^3+b^2cd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*d*x + a*d + c)/(b^3*d^3*x^2 + a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d
 + 2*(a*b^2*d^3 + b^2*c*d^2)*x)
```

**giac [A]** time = 1.32, size = 19, normalized size = 0.83

$$-\frac{2}{3\,(bdx+ad+c)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((b*d*x + a*d + c)^(3/2)*b*d)
```

**maple [A]** time = 0.00, size = 20, normalized size = 0.87

$$-\frac{2}{3\,(bdx+ad+c)^{\frac{3}{2}}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(5/2),x)
[Out] -2/3/(b*d*x+a*d+c)^(3/2)/d/b
```

**maxima** [A] time = 0.85, size = 19, normalized size = 0.83

$$-\frac{2}{3\left((bx+a)d+c\right)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="maxima")
[Out] -2/3/(((b*x + a)*d + c)^(3/2)*b*d)
```

**mupad** [B] time = 0.18, size = 19, normalized size = 0.83

$$-\frac{2}{3\,b\,d\,\left(c+d\,\left(a+b\,x\right)\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(5/2),x)
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

**sympy** [A] time = 6.79, size = 102, normalized size = 4.43

$$\begin{cases} \frac{x}{c^{\frac{5}{2}}} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{x}{(ad+c)^{\frac{5}{2}}} & \text{for}\: b = 0 \\ \frac{x}{c^{\frac{5}{2}}} & \text{for}\: d = 0 \\ -\frac{2\sqrt{ad+bdx+c}}{3a^2bd^3+6ab^2d^3x+6abcd^2+3b^3d^3x^2+6b^2cd^2x+3bc^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(5/2),x)
[Out] Piecewise((x/c**(5/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(5/2), Eq(b, 0))
, (x/c**(5/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(3*a**2*b*d**3 + 6*a*b*
*2*d**3*x + 6*a*b*c*d**2 + 3*b**3*d**3*x**2 + 6*b**2*c*d**2*x + 3*b*c**2*d)
, True))
```

## 3.43 $\int x^3(a + bx)\,dx$

**Optimal**. Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

**Rubi [A]** time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a + bx)\,dx &= \int \left(ax^3 + bx^4\right)\,dx\\ &= \frac{ax^4}{4} + \frac{bx^5}{5}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x),x]
[Out] IntegrateAlgebraic[x^3*(a + b*x), x]
```

**fricas [A]** time = 1.34, size = 13, normalized size = 0.76

$$\frac{1}{5}x^5b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="fricas")
[Out] 1/5*x^5*b + 1/4*x^4*a
```

**giac [A]** time = 1.22, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="giac")
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{5}b\,x^5 + \frac{1}{4}a\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a),x)
[Out] 1/4*a*x^4+1/5*b*x^5
```

**maxima [A]** time = 0.89, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

**mupad [B]** time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^4\,\left(5\,a+4\,b\,x\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x),x)
```

```
[Out] (x^4*(5*a + 4*b*x))/20
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^{4}}{4} + \frac{bx^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a),x)
```

```
[Out] a*x**4/4 + b*x**5/5
```

## 3.44 $\int x^2(a+bx)\,dx$

**Optimal**. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

**Rubi [A]** time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x),x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)\,dx &= \int \left(ax^2+bx^3\right)\,dx\\ &= \frac{ax^3}{3}+\frac{bx^4}{4}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x),x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x),x]
[Out] IntegrateAlgebraic[x^2*(a + b*x), x]
```

**fricas** [A] time = 1.45, size = 13, normalized size = 0.76

$$\frac{1}{4}x^4b+\frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="fricas")
[Out] 1/4*x^4*b + 1/3*x^3*a
```

**giac** [A] time = 1.74, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="giac")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

**maple** [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{4}b\,x^4+\frac{1}{3}a\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a),x)
[Out] 1/3*a*x^3+1/4*b*x^4
```

**maxima** [A] time = 0.86, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

**mupad [B]** time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^3\,\left(4\,a+3\,b\,x\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x),x)
```

```
[Out] (x^3*(4*a + 3*b*x))/12
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a),x)
```

```
[Out] a*x**3/3 + b*x**4/4
```

## 3.45 $\int x(a+bx)\,dx$

**Optimal**. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

**Rubi [A]** time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {43}

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x),x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)\,dx&=\int \left(ax+bx^2\right)\,dx\\&=\frac{ax^2}{2}+\frac{bx^3}{3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x),x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x),x]
[Out] IntegrateAlgebraic[x*(a + b*x), x]
```

**fricas [A]** time = 1.26, size = 13, normalized size = 0.76

$$\frac{1}{3}x^3b+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="fricas")
[Out] 1/3*x^3*b + 1/2*x^2*a
```

**giac [A]** time = 1.17, size = 13, normalized size = 0.76

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="giac")
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{3}b\,x^3+\frac{1}{2}a\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a),x)
[Out] 1/2*a*x^2+1/3*b*x^3
```

**maxima [A]** time = 0.83, size = 13, normalized size = 0.76

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="maxima")
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

**mupad [B]** time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^2\ (3\,a+2\,b\,x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x),x)
[Out] (x^2*(3*a + 2*b*x))/6
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x)
[Out] a*x**2/2 + b*x**3/3
```

## 3.46 $\int (a+bx)\,dx$

**Optimal**. Leaf size=12

$$ax+\frac{bx^2}{2}$$

**Rubi [A]** time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x,x]
[Out] a*x + (b*x^2)/2
```

Rubi steps

$$\int (a+bx)\,dx=ax+\frac{bx^2}{2}$$

**Mathematica [A]** time = 0.00, size = 12, normalized size = 1.00

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x,x]
[Out] a*x + (b*x^2)/2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + b*x,x]
[Out] IntegrateAlgebraic[a + b*x, x]
```

**fricas** [A]   time = 1.36, size = 10, normalized size = 0.83

$$\frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="fricas")
[Out] 1/2*x^2*b + x*a
```

**giac** [A]   time = 1.25, size = 10, normalized size = 0.83

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="giac")
[Out] 1/2*b*x^2 + a*x
```

**maple** [A]   time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2}b\,x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a,x)
[Out] a*x+1/2*b*x^2
```

**maxima** [A]   time = 0.89, size = 10, normalized size = 0.83

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="maxima")
[Out] 1/2*b*x^2 + a*x
```

**mupad** [B]   time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^2}{2} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x,x)
```

```
[Out] a*x + (b*x^2)/2
```

**sympy** **[A]** time = 0.06, size = 8, normalized size = 0.67

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] a*x + b*x**2/2
```

## 3.47 $\int \frac{a+bx}{x}\,dx$

**Optimal**. Leaf size=8

$$a\log(x) + bx$$

**Rubi [A]** time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$a\log(x) + bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x,x]
```

```
[Out] b*x + a*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x}\,dx &= \int \left(b+\frac{a}{x}\right)dx\\ &= bx + a\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 8, normalized size = 1.00

$$a\log(x) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x,x]
```

```
[Out] b*x + a*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/x, x]
```

**fricas [A]** time = 1.26, size = 8, normalized size = 1.00

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

**giac [A]** time = 1.06, size = 9, normalized size = 1.12

$$bx + a\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="giac")
```

```
[Out] b*x + a*log(abs(x))
```

**maple [A]** time = 0.01, size = 9, normalized size = 1.12

$$a\ln(x) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x,x)
```

```
[Out] b*x+a*ln(x)
```

**maxima [A]** time = 0.84, size = 8, normalized size = 1.00

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="maxima")
```

```
[Out] b*x + a*log(x)
```

**mupad [B]** time = 0.02, size = 8, normalized size = 1.00

$$b\,x + a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x,x)
```

```
[Out] b*x + a*log(x)
```

**sympy [A]**   time = 0.09, size = 7, normalized size = 0.88

$$a\log(x) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x)
```

```
[Out] a*log(x) + b*x
```

## 3.48 $\int \frac{a+bx}{x^2} \, dx$

**Optimal**. Leaf size=11

$$b\log(x) - \frac{a}{x}$$

**Rubi** [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$b\log(x) - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^2,x]
```

```
[Out] -(a/x) + b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^2} \, dx &= \int \left(\frac{a}{x^2}+\frac{b}{x}\right) \, dx \\ &= -\frac{a}{x}+b\log(x) \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 11, normalized size = 1.00

$$b\log(x) - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^2,x]
```

```
[Out] -(a/x) + b*Log[x]
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x^2,x]
[Out] IntegrateAlgebraic[(a + b*x)/x^2, x]
```

**fricas** [A] time = 1.41, size = 13, normalized size = 1.18

$$\frac{bx\log(x)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="fricas")
[Out] (b*x*log(x) - a)/x
```

**giac** [A] time = 1.05, size = 12, normalized size = 1.09

$$b\log\left(|x|\right)-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="giac")
[Out] b*log(abs(x)) - a/x
```

**maple** [A] time = 0.02, size = 12, normalized size = 1.09

$$b\ln(x)-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2,x)
[Out] -a/x+b*ln(x)
```

**maxima** [A] time = 0.87, size = 11, normalized size = 1.00

$$b\log(x)-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - a/x
```

**mupad [B]** time = 0.03, size = 11, normalized size = 1.00

$$b\ln(x) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^2,x)
```

```
[Out] b*log(x) - a/x
```

**sympy [A]** time = 0.11, size = 7, normalized size = 0.64

$$-\frac{a}{x} + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2,x)
```

```
[Out] -a/x + b*log(x)
```

## 3.49 $\int \frac{a+bx}{x^3} \, dx$

**Optimal**. Leaf size=17

$$-\frac{(a+bx)^2}{2ax^2}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {37}

$$-\frac{(a+bx)^2}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^3,x]
[Out] -(a + b*x)^2/(2*a*x^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx}{x^3} \, dx = -\frac{(a+bx)^2}{2ax^2}$$

**Mathematica [A]** time = 0.00, size = 15, normalized size = 0.88

$$-\frac{a}{2x^2} - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^3,x]
[Out] -1/2*a/x^2 - b/x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/x^3, x]
```

**fricas [A]** time = 1.32, size = 11, normalized size = 0.65

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

**giac [A]** time = 1.22, size = 11, normalized size = 0.65

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{x}-\frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3,x)
```

```
[Out] -b/x-1/2*a/x^2
```

**maxima [A]** time = 1.12, size = 11, normalized size = 0.65

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

**mupad [B]** time = 0.02, size = 11, normalized size = 0.65

$$-\frac{a+2\,b\,x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^3,x)
```

```
[Out] -(a + 2*b*x)/(2*x^2)
```

**sympy [A]** time = 0.11, size = 12, normalized size = 0.71

$$\frac{-a-2bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3,x)
```

```
[Out] (-a - 2*b*x)/(2*x**2)
```

## 3.50 $\int \frac{a+bx}{x^4} \, dx$

**Optimal**. Leaf size=17

$$-\frac{a}{3x^3} - \frac{b}{2x^2}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{a}{3x^3} - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^4,x]
```

```
[Out] -a/(3*x^3) - b/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^4} \, dx &= \int \left(\frac{a}{x^4} + \frac{b}{x^3}\right) \, dx \\ &= -\frac{a}{3x^3} - \frac{b}{2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{3x^3} - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/(2*x^2)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x^4,x]
[Out] IntegrateAlgebraic[(a + b*x)/x^4, x]
```

**fricas** [A] time = 1.38, size = 13, normalized size = 0.76

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="fricas")
[Out] -1/6*(3*b*x + 2*a)/x^3
```

**giac** [A] time = 1.38, size = 13, normalized size = 0.76

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="giac")
[Out] -1/6*(3*b*x + 2*a)/x^3
```

**maple** [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{2x^2}-\frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4,x)
[Out] -1/3*a/x^3-1/2*b/x^2
```

**maxima** [A] time = 1.10, size = 13, normalized size = 0.76

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

**mupad [B]** time = 0.03, size = 13, normalized size = 0.76

$$-\frac{2\,a+3\,b\,x}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^4,x)
```

```
[Out] -(2*a + 3*b*x)/(6*x^3)
```

**sympy [A]** time = 0.13, size = 14, normalized size = 0.82

$$\frac{-2a-3bx}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4,x)
```

```
[Out] (-2*a - 3*b*x)/(6*x**3)
```

## 3.51 $\int \frac{a+bx}{x^5} \, dx$

**Optimal**. Leaf size=17

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^5} \, dx &= \int \left(\frac{a}{x^5}+\frac{b}{x^4}\right) dx\\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(3*x^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x^5,x]
[Out] IntegrateAlgebraic[(a + b*x)/x^5, x]
```

**fricas [A]** time = 1.47, size = 13, normalized size = 0.76

$$-\frac{4\,bx+3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="fricas")
[Out] -1/12*(4*b*x + 3*a)/x^4
```

**giac [A]** time = 1.68, size = 13, normalized size = 0.76

$$-\frac{4\,bx+3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="giac")
[Out] -1/12*(4*b*x + 3*a)/x^4
```

**maple [A]** time = 0.01, size = 14, normalized size = 0.82

$$-\frac{b}{3x^3}-\frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^5,x)
[Out] -1/4*a/x^4-1/3*b/x^3
```

**maxima [A]** time = 1.05, size = 13, normalized size = 0.76

$$-\frac{4\,bx+3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*b*x + 3*a)/x^4
```

**mupad [B]** time = 0.03, size = 13, normalized size = 0.76

$$-\frac{3\,a+4\,b\,x}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^5,x)
```

```
[Out] -(3*a + 4*b*x)/(12*x^4)
```

**sympy [A]** time = 0.16, size = 14, normalized size = 0.82

$$\frac{-3a-4bx}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**5,x)
```

```
[Out] (-3*a - 4*b*x)/(12*x**4)
```

## 3.52 $\int x^3(a + bx)^2\,dx$

**Optimal**. Leaf size=30

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*x)^2,x]

[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^2\,dx &= \int \left(a^2x^3 + 2abx^4 + b^2x^5\right) dx\\ &= \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*x)^2,x]

[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x)^2, x]
```

**fricas [A]** time = 0.75, size = 24, normalized size = 0.80

$$\frac{1}{6}x^6b^2 + \frac{2}{5}x^5ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^2 + 2/5*x^5*b*a + 1/4*x^4*a^2
```

**giac [A]** time = 1.61, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{6}b^2x^6 + \frac{2}{5}ab\,x^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2,x)
```

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

**maxima [A]** time = 1.20, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

**mupad [B]** time = 0.08, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^6)/6 + (2*a*b*x^5)/5
```

**sympy [A]** time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^{2} x^{4}}{4}+\frac{2 a b x^{5}}{5}+\frac{b^{2} x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

## 3.53 $\int x^2(a+bx)^2\,dx$

**Optimal**. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^2\,dx&=\int\left(a^2x^2+2abx^3+b^2x^4\right)dx\\&=\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^2,x]
[Out] IntegrateAlgebraic[x^2*(a + b*x)^2, x]
```

**fricas [A]** time = 1.13, size = 24, normalized size = 0.80

$$\frac{1}{5}x^5b^2 + \frac{1}{2}x^4ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="fricas")
[Out] 1/5*x^5*b^2 + 1/2*x^4*b*a + 1/3*x^3*a^2
```

**giac [A]** time = 1.02, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="giac")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{5}b^2x^5 + \frac{1}{2}ab\,x^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2,x)
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

**maxima [A]** time = 1.05, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

**mupad [B]** time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

**sympy [A]** time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^{2} x^{3}}{3} + \frac{a b x^{4}}{2} + \frac{b^{2} x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

## 3.54 $\int x(a+bx)^2\,dx$

**Optimal**. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^2\,dx&=\int\left(a^2x+2abx^2+b^2x^3\right)\,dx\\&=\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^2,x]
[Out] IntegrateAlgebraic[x*(a + b*x)^2, x]
```

**fricas [A]** time = 1.15, size = 24, normalized size = 0.80

$$\frac{1}{4}x^4b^2+\frac{2}{3}x^3ba+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="fricas")
[Out] 1/4*x^4*b^2 + 2/3*x^3*b*a + 1/2*x^2*a^2
```

**giac [A]** time = 1.41, size = 24, normalized size = 0.80

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="giac")
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{4}b^2x^4+\frac{2}{3}ab\,x^3+\frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2,x)
[Out] 1/2*a^2*x^2+2/3*a*b*x^3+1/4*b^2*x^4
```

**maxima [A]** time = 0.99, size = 24, normalized size = 0.80

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

**mupad [B]** time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^2}{2}+\frac{2\,a\,b\,x^3}{3}+\frac{b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^4)/4 + (2*a*b*x^3)/3
```

**sympy [A]** time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^{2}x^{2}}{2}+\frac{2abx^{3}}{3}+\frac{b^{2}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2,x)
```

```
[Out] a**2*x**2/2 + 2*a*b*x**3/3 + b**2*x**4/4
```

## 3.55 $\int (a+bx)^2\,dx$

**Optimal**. Leaf size=14

$$\frac{(a+bx)^3}{3b}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2,x]
[Out] (a + b*x)^3/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^2\,dx = \frac{(a+bx)^3}{3b}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2,x]
[Out] (a + b*x)^3/(3*b)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2, x]
```

**fricas [A]** time = 1.22, size = 20, normalized size = 1.43

$$\frac{1}{3}x^3b^2 + x^2ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*b^2 + x^2*b*a + x*a^2
```

**giac [A]** time = 1.16, size = 12, normalized size = 0.86

$$\frac{(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2,x)
```

```
[Out] 1/3*(b*x+a)^3/b
```

**maxima [A]** time = 1.08, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

**mupad** [**B**]   time = 0.03, size = 20, normalized size = 1.43

$$a^2\,x + a\,b\,x^2 + \frac{b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2,x)
[Out] a^2*x + (b^2*x^3)/3 + a*b*x^2
```

**sympy** [**B**]   time = 0.07, size = 19, normalized size = 1.36

$$a^2x + abx^2 + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2,x)
[Out] a**2*x + a*b*x**2 + b**2*x**3/3
```

## 3.56 $\int \frac{(a+bx)^2}{x}\,dx$

**Optimal**. Leaf size=22

$$a^2\log(x) + 2abx + \frac{b^2x^2}{2}$$

**Rubi [A]** time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$a^2\log(x) + 2abx + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x,x]
```

```
[Out] 2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x}\,dx &= \int \left(2ab + \frac{a^2}{x} + b^2x\right)dx\\ &= 2abx + \frac{b^2x^2}{2} + a^2\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 22, normalized size = 1.00

$$a^2\log(x) + 2abx + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x,x]
```

```
[Out] 2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/x, x]
```

**fricas [A]** time = 0.84, size = 20, normalized size = 0.91

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="fricas")
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

**giac [A]** time = 1.35, size = 21, normalized size = 0.95

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="giac")
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(abs(x))
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.95

$$\frac{b^2x^2}{2} + a^2\ln(x) + 2abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x,x)
[Out] 2*a*b*x+1/2*b^2*x^2+a^2*ln(x)
```

**maxima [A]** time = 1.11, size = 20, normalized size = 0.91

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="maxima")
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

**mupad [B]** time = 0.03, size = 20, normalized size = 0.91

$$a^2\ \ln(x)+\frac{b^2\,x^2}{2}+2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x,x)
[Out] a^2*log(x) + (b^2*x^2)/2 + 2*a*b*x
```

**sympy [A]** time = 0.11, size = 20, normalized size = 0.91

$$a^2\log{(x)}+2abx+\frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x,x)
[Out] a**2*log(x) + 2*a*b*x + b**2*x**2/2
```

## 3.57 $\int \frac{(a+bx)^2}{x^2} \, dx$

**Optimal**. Leaf size=20

$$-\frac{a^2}{x}+2ab\log(x)+b^2x$$

**Rubi [A]** time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{x}+2ab\log(x)+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^2,x]
```

```
[Out] -(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^2}\,dx &= \int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx\\ &= -\frac{a^2}{x}+b^2x+2ab\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 20, normalized size = 1.00

$$-\frac{a^2}{x}+2ab\log(x)+b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^2,x]
```

```
[Out] -(a^2/x) + b^2*x + 2*a*b*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^2,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/x^2, x]
```

**fricas [A]** time = 1.37, size = 24, normalized size = 1.20

$$\frac{b^2x^2 + 2\,abx\log(x) - a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)/x
```

**giac [A]** time = 1.09, size = 21, normalized size = 1.05

$$b^2x + 2\,ab\log\left(|x|\right) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="giac")
[Out] b^2*x + 2*a*b*log(abs(x)) - a^2/x
```

**maple [A]** time = 0.01, size = 21, normalized size = 1.05

$$2ab\ln(x) + b^2x - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2,x)
[Out] -a^2/x+b^2*x+2*a*b*ln(x)
```

**maxima [A]** time = 1.10, size = 20, normalized size = 1.00

$$b^2x + 2\,ab\log(x) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="maxima")
[Out] b^2*x + 2*a*b*log(x) - a^2/x
```

**mupad [B]** time = 0.07, size = 20, normalized size = 1.00

$$b^2\,x-\frac{a^2}{x}+2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^2,x)
[Out] b^2*x - a^2/x + 2*a*b*log(x)
```

**sympy [A]** time = 0.13, size = 17, normalized size = 0.85

$$-\frac{a^2}{x}+2ab\log{(x)}+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2,x)
[Out] -a**2/x + 2*a*b*log(x) + b**2*x
```

## 3.58 $\int \frac{(a+bx)^2}{x^3} \, dx$

**Optimal**. Leaf size=24

$$-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) - (2*a*b)/x + b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^2}{x^3} \, dx &= \int \left( \frac{a^2}{x^3} + \frac{2ab}{x^2} + \frac{b^2}{x} \right) dx \\ &= -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x) \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^3,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/x^3, x]
```

**fricas [A]** time = 0.91, size = 26, normalized size = 1.08

$$\frac{2\,b^2x^2\log(x) - 4\,abx - a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

**giac [A]** time = 1.18, size = 22, normalized size = 0.92

$$b^2\log\left(|x|\right) - \frac{4\,abx + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="giac")
[Out] b^2*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^2
```

**maple [A]** time = 0.01, size = 23, normalized size = 0.96

$$b^2\ln(x) - \frac{2ab}{x} - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3,x)
[Out] -1/2*a^2/x^2-2*a*b/x+b^2*ln(x)
```

**maxima [A]** time = 1.18, size = 21, normalized size = 0.88

$$b^2\log(x) - \frac{4\,abx + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] b^2*log(x) - 1/2*(4*a*b*x + a^2)/x^2
```

**mupad [B]** time = 0.04, size = 23, normalized size = 0.96

$$b^2\ \ln(x) - \frac{\frac{a^2}{2} + 2\,b\,x\,a}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^3,x)
```

```
[Out] b^2*log(x) - (a^2/2 + 2*a*b*x)/x^2
```

**sympy [A]** time = 0.17, size = 22, normalized size = 0.92

$$b^2 \log\left(x\right) + \frac{-a^2 - 4abx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3,x)
```

```
[Out] b**2*log(x) + (-a**2 - 4*a*b*x)/(2*x**2)
```

## 3.59 $\int \frac{(a+bx)^2}{x^4} \, dx$

**Optimal**. Leaf size=17

$$-\frac{(a+bx)^3}{3ax^3}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^3}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^4,x]
```

```
[Out] -(a + b*x)^3/(3*a*x^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4} \, dx = -\frac{(a+bx)^3}{3ax^3}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 1.53

$$-\frac{a^2}{3x^3} - \frac{ab}{x^2} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (a*b)/x^2 - b^2/x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^4,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/x^4, x]
```

**fricas [A]** time = 1.29, size = 22, normalized size = 1.29

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="fricas")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

**giac [A]** time = 1.48, size = 22, normalized size = 1.29

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="giac")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

**maple [A]** time = 0.01, size = 25, normalized size = 1.47

$$-\frac{b^2}{x}-\frac{ab}{x^2}-\frac{a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4,x)
[Out] -a*b/x^2-b^2/x-1/3*a^2/x^3
```

**maxima [A]** time = 1.35, size = 22, normalized size = 1.29

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

**mupad [B]** time = 0.04, size = 22, normalized size = 1.29

$$-\frac{\frac{a^2}{3}+a\,b\,x+b^2\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^4,x)
```

```
[Out] -(a^2/3 + b^2*x^2 + a*b*x)/x^3
```

**sympy [A]** time = 0.18, size = 24, normalized size = 1.41

$$\frac{-a^2-3abx-3b^2x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4,x)
```

```
[Out] (-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*x**3)
```

## 3.60 $\int \frac{(a+bx)^2}{x^5} \, dx$

**Optimal**. Leaf size=30

$$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^5}\,dx &= \int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx\\ &= -\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^5,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/x^5, x]
```

**fricas [A]** time = 0.88, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

**giac [A]** time = 1.12, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="giac")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{2x^2}-\frac{2ab}{3x^3}-\frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^5,x)
[Out] -1/4*a^2/x^4-2/3*a*b/x^3-1/2*b^2/x^2
```

**maxima [A]** time = 1.34, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

**mupad [B]** time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{4}+\frac{2\,a\,b\,x}{3}+\frac{b^2\,x^2}{2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^5,x)
```

```
[Out] -(a^2/4 + (b^2*x^2)/2 + (2*a*b*x)/3)/x^4
```

**sympy [A]** time = 0.19, size = 26, normalized size = 0.87

$$\frac{-3a^2-8abx-6b^2x^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**5,x)
```

```
[Out] (-3*a**2 - 8*a*b*x - 6*b**2*x**2)/(12*x**4)
```

## 3.61 $\int \frac{(a+bx)^2}{x^6} dx$

**Optimal**. Leaf size=30

$$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^6}\,dx &= \int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right) dx\\ &= -\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.00

$$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^6,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/x^6, x]
```

**fricas [A]** time = 1.43, size = 24, normalized size = 0.80

$$-\frac{10\,b^2x^2+15\,abx+6\,a^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="fricas")
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

**giac [A]** time = 1.19, size = 24, normalized size = 0.80

$$-\frac{10\,b^2x^2+15\,abx+6\,a^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="giac")
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{3x^3}-\frac{ab}{2x^4}-\frac{a^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^6,x)
[Out] -1/5*a^2/x^5-1/2*a*b/x^4-1/3*b^2/x^3
```

**maxima [A]** time = 1.33, size = 24, normalized size = 0.80

$$-\frac{10\,b^2x^2+15\,abx+6\,a^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

**mupad** [B] time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{5}+\frac{a\,b\,x}{2}+\frac{b^2\,x^2}{3}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^6,x)
```

```
[Out] -(a^2/5 + (b^2*x^2)/3 + (a*b*x)/2)/x^5
```

**sympy** [A] time = 0.19, size = 26, normalized size = 0.87

$$\frac{-6a^2-15abx-10b^2x^2}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**6,x)
```

```
[Out] (-6*a**2 - 15*a*b*x - 10*b**2*x**2)/(30*x**5)
```

## 3.62 $\int \frac{(a+bx)^2}{x^7}\,dx$

**Optimal**. Leaf size=30

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

**Rubi** [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^7}\,dx &= \int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx\\ &= -\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^7,x]
```

```
[Out] -1/6*a^2/x^6 - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^7,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/x^7, x]
```

**fricas [A]** time = 0.72, size = 24, normalized size = 0.80

$$-\frac{15\,b^2x^2+24\,abx+10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="fricas")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

**giac [A]** time = 1.07, size = 24, normalized size = 0.80

$$-\frac{15\,b^2x^2+24\,abx+10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="giac")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{4x^4}-\frac{2ab}{5x^5}-\frac{a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^7,x)
[Out] -1/6*a^2/x^6-2/5*a*b/x^5-1/4*b^2/x^4
```

**maxima [A]** time = 1.35, size = 24, normalized size = 0.80

$$-\frac{15\,b^2x^2+24\,abx+10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

**mupad [B]** time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{6}+\frac{2\,a\,b\,x}{5}+\frac{b^2\,x^2}{4}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^7,x)
```

```
[Out] -(a^2/6 + (b^2*x^2)/4 + (2*a*b*x)/5)/x^6
```

**sympy [A]** time = 0.20, size = 26, normalized size = 0.87

$$\frac{-10a^2-24abx-15b^2x^2}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**7,x)
```

```
[Out] (-10*a**2 - 24*a*b*x - 15*b**2*x**2)/(60*x**6)
```

## 3.63 $\int \frac{(a+bx)^2}{x^8} dx$

**Optimal**. Leaf size=30

$$-\frac{a^2}{7x^7} - \frac{ab}{3x^6} - \frac{b^2}{5x^5}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{7x^7} - \frac{ab}{3x^6} - \frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^8,x]
[Out] -a^2/(7*x^7) - (a*b)/(3*x^6) - b^2/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^8} dx &= \int \left(\frac{a^2}{x^8} + \frac{2ab}{x^7} + \frac{b^2}{x^6}\right) dx \\ &= -\frac{a^2}{7x^7} - \frac{ab}{3x^6} - \frac{b^2}{5x^5}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.00

$$-\frac{a^2}{7x^7} - \frac{ab}{3x^6} - \frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^8,x]
[Out] -1/7*a^2/x^7 - (a*b)/(3*x^6) - b^2/(5*x^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^8,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/x^8, x]
```

**fricas [A]** time = 1.36, size = 24, normalized size = 0.80

$$-\frac{21\,b^2x^2+35\,abx+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="fricas")
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

**giac [A]** time = 1.20, size = 24, normalized size = 0.80

$$-\frac{21\,b^2x^2+35\,abx+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="giac")
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

**maple [A]** time = 0.01, size = 25, normalized size = 0.83

$$-\frac{b^2}{5x^5}-\frac{ab}{3x^6}-\frac{a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^8,x)
[Out] -1/7*a^2/x^7-1/3*a*b/x^6-1/5*b^2/x^5
```

**maxima [A]** time = 1.35, size = 24, normalized size = 0.80

$$-\frac{21\,b^2x^2+35\,abx+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

**mupad [B]** time = 0.04, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{7}+\frac{a\,b\,x}{3}+\frac{b^2\,x^2}{5}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^8,x)
```

```
[Out] -(a^2/7 + (b^2*x^2)/5 + (a*b*x)/3)/x^7
```

**sympy [A]** time = 0.21, size = 26, normalized size = 0.87

$$\frac{-15a^2-35abx-21b^2x^2}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**8,x)
```

```
[Out] (-15*a**2 - 35*a*b*x - 21*b**2*x**2)/(105*x**7)
```

## 3.64 $\int x^4(a+bx)^3\,dx$

**Optimal**. Leaf size=43

$$\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

**Rubi [A]** time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{1}{2}a^2bx^6+\frac{a^3x^5}{5}+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^3\,dx&=\int\left(a^3x^4+3a^2bx^5+3ab^2x^6+b^3x^7\right)dx\\&=\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x)^3,x]
[Out] IntegrateAlgebraic[x^4*(a + b*x)^3, x]
```

**fricas [A]** time = 1.07, size = 35, normalized size = 0.81

$$\frac{1}{8}x^8b^3 + \frac{3}{7}x^7b^2a + \frac{1}{2}x^6ba^2 + \frac{1}{5}x^5a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="fricas")
[Out] 1/8*x^8*b^3 + 3/7*x^7*b^2*a + 1/2*x^6*b*a^2 + 1/5*x^5*a^3
```

**giac [A]** time = 1.22, size = 35, normalized size = 0.81

$$\frac{1}{8}\,b^3x^8 + \frac{3}{7}\,ab^2x^7 + \frac{1}{2}\,a^2bx^6 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="giac")
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{8}b^3x^8 + \frac{3}{7}a\,b^2x^7 + \frac{1}{2}a^2b\,x^6 + \frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^3,x)
[Out] 1/5*a^3*x^5+1/2*a^2*b*x^6+3/7*a*b^2*x^7+1/8*b^3*x^8
```

**maxima [A]** time = 1.32, size = 35, normalized size = 0.81

$$\frac{1}{8}\,b^3x^8 + \frac{3}{7}\,ab^2x^7 + \frac{1}{2}\,a^2bx^6 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

**mupad [B]** time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^5}{5}+\frac{a^2\,b\,x^6}{2}+\frac{3\,a\,b^2\,x^7}{7}+\frac{b^3\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^3,x)
```

```
[Out] (a^3*x^5)/5 + (b^3*x^8)/8 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7
```

**sympy [A]** time = 0.08, size = 37, normalized size = 0.86

$$\frac{a^{3} x^{5}}{5}+\frac{a^{2} b x^{6}}{2}+\frac{3 a b^{2} x^{7}}{7}+\frac{b^{3} x^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**3,x)
```

```
[Out] a**3*x**5/5 + a**2*b*x**6/2 + 3*a*b**2*x**7/7 + b**3*x**8/8
```

## 3.65 $\int x^3(a+bx)^3\,dx$

**Optimal**. Leaf size=43

$$\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

**Rubi [A]** time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{5}a^2bx^5+\frac{a^3x^4}{4}+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^3\,dx &= \int \left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right)dx\\ &= \frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^3,x]
[Out] IntegrateAlgebraic[x^3*(a + b*x)^3, x]
```

**fricas [A]** time = 0.69, size = 35, normalized size = 0.81

$$\frac{1}{7}x^7b^3 + \frac{1}{2}x^6b^2a + \frac{3}{5}x^5ba^2 + \frac{1}{4}x^4a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="fricas")
[Out] 1/7*x^7*b^3 + 1/2*x^6*b^2*a + 3/5*x^5*b*a^2 + 1/4*x^4*a^3
```

**giac [A]** time = 1.27, size = 35, normalized size = 0.81

$$\frac{1}{7}\,b^3x^7 + \frac{1}{2}\,ab^2x^6 + \frac{3}{5}\,a^2bx^5 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="giac")
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{7}b^3x^7 + \frac{1}{2}a\,b^2x^6 + \frac{3}{5}a^2b\,x^5 + \frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^3,x)
[Out] 1/4*a^3*x^4+3/5*a^2*b*x^5+1/2*a*b^2*x^6+1/7*b^3*x^7
```

**maxima [A]** time = 1.35, size = 35, normalized size = 0.81

$$\frac{1}{7}\,b^3x^7 + \frac{1}{2}\,ab^2x^6 + \frac{3}{5}\,a^2bx^5 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

**mupad [B]** time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^4}{4}+\frac{3\,a^2\,b\,x^5}{5}+\frac{a\,b^2\,x^6}{2}+\frac{b^3\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^7)/7 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2
```

**sympy [A]** time = 0.07, size = 37, normalized size = 0.86

$$\frac{a^{3}x^{4}}{4}+\frac{3a^{2}bx^{5}}{5}+\frac{ab^{2}x^{6}}{2}+\frac{b^{3}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**3,x)
```

```
[Out] a**3*x**4/4 + 3*a**2*b*x**5/5 + a*b**2*x**6/2 + b**3*x**7/7
```

## 3.66 $\int x^2(a+bx)^3\,dx$

**Optimal**. Leaf size=43

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

**Rubi [A]** time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{4}a^2bx^4+\frac{a^3x^3}{3}+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^3,x]
[Out] (a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^3\,dx&=\int\left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right)dx\\&=\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^3,x]
[Out] (a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^3,x]
[Out] IntegrateAlgebraic[x^2*(a + b*x)^3, x]
```

**fricas [A]** time = 0.95, size = 35, normalized size = 0.81

$$\frac{1}{6}x^6b^3+\frac{3}{5}x^5b^2a+\frac{3}{4}x^4ba^2+\frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="fricas")
[Out] 1/6*x^6*b^3 + 3/5*x^5*b^2*a + 3/4*x^4*b*a^2 + 1/3*x^3*a^3
```

**giac [A]** time = 1.22, size = 35, normalized size = 0.81

$$\frac{1}{6}\,b^3x^6+\frac{3}{5}\,ab^2x^5+\frac{3}{4}\,a^2bx^4+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="giac")
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{6}b^3x^6+\frac{3}{5}a\,b^2x^5+\frac{3}{4}a^2b\,x^4+\frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^3,x)
[Out] 1/3*a^3*x^3+3/4*a^2*b*x^4+3/5*a*b^2*x^5+1/6*b^3*x^6
```

**maxima [A]** time = 1.34, size = 35, normalized size = 0.81

$$\frac{1}{6}\,b^3x^6+\frac{3}{5}\,ab^2x^5+\frac{3}{4}\,a^2bx^4+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

**mupad [B]** time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^3}{3}+\frac{3\,a^2\,b\,x^4}{4}+\frac{3\,a\,b^2\,x^5}{5}+\frac{b^3\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^6)/6 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5
```

**sympy [A]** time = 0.07, size = 39, normalized size = 0.91

$$\frac{a^{3}x^{3}}{3}+\frac{3a^{2}bx^{4}}{4}+\frac{3ab^{2}x^{5}}{5}+\frac{b^{3}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**4/4 + 3*a*b**2*x**5/5 + b**3*x**6/6
```

## 3.67 $\int x(a+bx)^3\,dx$

**Optimal**. Leaf size=30

$$\frac{(a+bx)^5}{5b^2}-\frac{a(a+bx)^4}{4b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^5}{5b^2}-\frac{a(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^3,x]
```

```
[Out] -(a*(a + b*x)^4)/(4*b^2) + (a + b*x)^5/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^3\,dx&=\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx\\&=-\frac{a(a+bx)^4}{4b^2}+\frac{(a+bx)^5}{5b^2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 40, normalized size = 1.33

$$\frac{a^3x^2}{2}+a^2bx^3+\frac{3}{4}ab^2x^4+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^3,x]
```

```
[Out] (a^3*x^2)/2 + a^2*b*x^3 + (3*a*b^2*x^4)/4 + (b^3*x^5)/5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^3,x]
[Out] IntegrateAlgebraic[x*(a + b*x)^3, x]
```

**fricas [A]** time = 0.70, size = 34, normalized size = 1.13

$$\frac{1}{5}x^5b^3 + \frac{3}{4}x^4b^2a + x^3ba^2 + \frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="fricas")
[Out] 1/5*x^5*b^3 + 3/4*x^4*b^2*a + x^3*b*a^2 + 1/2*x^2*a^3
```

**giac [A]** time = 1.21, size = 34, normalized size = 1.13

$$\frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 + \frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="giac")
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

**maple [A]** time = 0.00, size = 35, normalized size = 1.17

$$\frac{1}{5}b^3x^5 + \frac{3}{4}a\,b^2x^4 + a^2b\,x^3 + \frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^3,x)
[Out] 1/5*b^3*x^5+3/4*a*b^2*x^4+a^2*b*x^3+1/2*a^3*x^2
```

**maxima [A]** time = 1.38, size = 34, normalized size = 1.13

$$\frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 + \frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="maxima")
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

**mupad [B]** time = 0.04, size = 34, normalized size = 1.13

$$\frac{a^3\,x^2}{2}+a^2\,b\,x^3+\frac{3\,a\,b^2\,x^4}{4}+\frac{b^3\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^3,x)
[Out] (a^3*x^2)/2 + (b^3*x^5)/5 + a^2*b*x^3 + (3*a*b^2*x^4)/4
```

**sympy [A]** time = 0.07, size = 36, normalized size = 1.20

$$\frac{a^3x^2}{2}+a^2bx^3+\frac{3ab^2x^4}{4}+\frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**3,x)
[Out] a**3*x**2/2 + a**2*b*x**3 + 3*a*b**2*x**4/4 + b**3*x**5/5
```

## 3.68 $\int (a+bx)^3 \, dx$

**Optimal**. Leaf size=14

$$\frac{(a+bx)^4}{4b}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3,x]
```

```
[Out] (a + b*x)^4/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^3 \, dx = \frac{(a+bx)^4}{4b}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3,x]
```

```
[Out] (a + b*x)^4/(4*b)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3, x]
```

**fricas [B]** time = 1.34, size = 31, normalized size = 2.21

$$\frac{1}{4}x^4b^3 + x^3b^2a + \frac{3}{2}x^2ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b^3 + x^3*b^2*a + 3/2*x^2*b*a^2 + x*a^3
```

**giac [A]** time = 1.36, size = 12, normalized size = 0.86

$$\frac{(bx+a)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b*x + a)^4/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx+a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3,x)
```

```
[Out] 1/4*(b*x+a)^4/b
```

**maxima [B]** time = 1.39, size = 31, normalized size = 2.21

$$\frac{1}{4}\,b^3x^4 + ab^2x^3 + \frac{3}{2}\,a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

**mupad [B]** time = 0.04, size = 31, normalized size = 2.21

$$a^3\,x+\frac{3\,a^2\,b\,x^2}{2}+a\,b^2\,x^3+\frac{b^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3,x)
```

```
[Out] a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3
```

**sympy [B]** time = 0.07, size = 32, normalized size = 2.29

$$a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

## 3.69 $\int \frac{(a+bx)^3}{x} \, dx$

**Optimal**. Leaf size=35

$$a^3\log(x) + 3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$3a^2bx + a^3\log(x) + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x} \, dx &= \int \left(3a^2b + \frac{a^3}{x} + 3ab^2x + b^3x^2\right) dx \\ &= 3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3} + a^3\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 35, normalized size = 1.00

$$a^3\log(x) + 3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x,x]
[Out] IntegrateAlgebraic[(a + b*x)^3/x, x]
```

**fricas [A]** time = 1.33, size = 31, normalized size = 0.89

$$\frac{1}{3}\,b^3x^3+\frac{3}{2}\,ab^2x^2+3\,a^2bx+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="fricas")
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

**giac [A]** time = 0.95, size = 32, normalized size = 0.91

$$\frac{1}{3}\,b^3x^3+\frac{3}{2}\,ab^2x^2+3\,a^2bx+a^3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="giac")
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(abs(x))
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.91

$$\frac{b^3x^3}{3}+\frac{3a\,b^2x^2}{2}+a^3\ln(x)+3a^2bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x,x)
[Out] 3*a^2*b*x+3/2*a*b^2*x^2+1/3*b^3*x^3+a^3*ln(x)
```

**maxima [A]** time = 1.34, size = 31, normalized size = 0.89

$$\frac{1}{3}\,b^3x^3+\frac{3}{2}\,ab^2x^2+3\,a^2bx+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="maxima")
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

**mupad [B]** time = 0.03, size = 31, normalized size = 0.89

$$a^3\ \ln(x)+\frac{b^3\,x^3}{3}+\frac{3\,a\,b^2\,x^2}{2}+3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x,x)
[Out] a^3*log(x) + (b^3*x^3)/3 + (3*a*b^2*x^2)/2 + 3*a^2*b*x
```

**sympy [A]** time = 0.12, size = 34, normalized size = 0.97

$$a^3\log{(x)}+3a^2bx+\frac{3ab^2x^2}{2}+\frac{b^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x,x)
[Out] a**3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3
```

## 3.70 $\int \frac{(a+bx)^3}{x^2}\,dx$

**Optimal**. Leaf size=34

$$-\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2}$$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$3a^2b\log(x)-\frac{a^3}{x}+3ab^2x+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^2}\,dx &= \int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right)dx\\ &= -\frac{a^3}{x}+3ab^2x+\frac{b^3x^2}{2}+3a^2b\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 34, normalized size = 1.00

$$-\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^2,x]
[Out] IntegrateAlgebraic[(a + b*x)^3/x^2, x]
```

**fricas [A]** time = 1.54, size = 36, normalized size = 1.06

$$\frac{b^3x^3 + 6\,ab^2x^2 + 6\,a^2bx\log(x) - 2\,a^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="fricas")
[Out] 1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

**giac [A]** time = 1.36, size = 33, normalized size = 0.97

$$\frac{1}{2}\,b^3x^2 + 3\,ab^2x + 3\,a^2b\log\left(|x|\right) - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="giac")
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(abs(x)) - a^3/x
```

**maple [A]** time = 0.01, size = 33, normalized size = 0.97

$$\frac{b^3x^2}{2} + 3a^2b\ln(x) + 3a\,b^2x - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^2,x)
[Out] -a^3/x+3*a*b^2*x+1/2*b^3*x^2+3*a^2*b*ln(x)
```

**maxima [A]** time = 1.29, size = 32, normalized size = 0.94

$$\frac{1}{2}\,b^3x^2 + 3\,ab^2x + 3\,a^2b\log(x) - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="maxima")
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(x) - a^3/x
```

**mupad [B]** time = 0.03, size = 32, normalized size = 0.94

$$\frac{b^3\,x^2}{2}-\frac{a^3}{x}+3\,a^2\,b\,\ln(x)+3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^2,x)
[Out] (b^3*x^2)/2 - a^3/x + 3*a^2*b*log(x) + 3*a*b^2*x
```

**sympy [A]** time = 0.13, size = 31, normalized size = 0.91

$$-\frac{a^3}{x}+3a^2b\log{(x)}+3ab^2x+\frac{b^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**2,x)
[Out] -a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2
```

## 3.71 $\int \frac{(a+bx)^3}{x^3} dx$

**Optimal**. Leaf size=33

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x$$

**Rubi [A]** time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{x}-\frac{a^3}{2x^2}+3ab^2\log(x)+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^3} dx &= \int \left(b^3+\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}\right) dx\\ &= -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+b^3x+3ab^2\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 1.00

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^3,x]
```

```
[Out] -1/2*a^3/x^2 - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^3,x]
[Out] IntegrateAlgebraic[(a + b*x)^3/x^3, x]
```

**fricas [A]** time = 1.33, size = 37, normalized size = 1.12

$$\frac{2\,b^3x^3 + 6\,ab^2x^2\log(x) - 6\,a^2bx - a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

**giac [A]** time = 1.18, size = 31, normalized size = 0.94

$$b^3x + 3\,ab^2\log(|x|) - \frac{6\,a^2bx + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="giac")
[Out] b^3*x + 3*a*b^2*log(abs(x)) - 1/2*(6*a^2*b*x + a^3)/x^2
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.97

$$3a\,b^2\ln(x) + b^3x - \frac{3a^2b}{x} - \frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^3,x)
[Out] -1/2*a^3/x^2-3*a^2*b/x+b^3*x+3*a*b^2*ln(x)
```

**maxima [A]** time = 1.34, size = 30, normalized size = 0.91

$$b^3x + 3\,ab^2\log(x) - \frac{6\,a^2bx + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] b^3*x + 3*a*b^2*log(x) - 1/2*(6*a^2*b*x + a^3)/x^2
```

**mupad [B]** time = 0.03, size = 32, normalized size = 0.97

$$b^3\,x-\frac{\frac{a^3}{2}+3\,b\,x\,a^2}{x^2}+3\,a\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^3,x)
```

```
[Out] b^3*x - (a^3/2 + 3*a^2*b*x)/x^2 + 3*a*b^2*log(x)
```

**sympy [A]** time = 0.19, size = 32, normalized size = 0.97

$$3ab^2\log{(x)}+b^3x+\frac{-a^3-6a^2bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**3,x)
```

```
[Out] 3*a*b**2*log(x) + b**3*x + (-a**3 - 6*a**2*b*x)/(2*x**2)
```

## 3.72 $\int \frac{(a+bx)^3}{x^4}\,dx$

**Optimal**. Leaf size=37

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{2x^2}-\frac{a^3}{3x^3}-\frac{3ab^2}{x}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^4}\,dx &= \int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right)dx\\ &= -\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 37, normalized size = 1.00

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/x^4, x]
```

**fricas** [A] time = 1.01, size = 37, normalized size = 1.00

$$\frac{6\,b^3x^3\log(x) - 18\,ab^2x^2 - 9\,a^2bx - 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

**giac** [A] time = 1.14, size = 35, normalized size = 0.95

$$b^3\log\left(|x|\right) - \frac{18\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x)) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

**maple** [A] time = 0.01, size = 34, normalized size = 0.92

$$b^3\ln(x) - \frac{3a\,b^2}{x} - \frac{3a^2b}{2x^2} - \frac{a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^4,x)
```

```
[Out] -1/3*a^3/x^3-3/2*a^2*b/x^2-3*a*b^2/x+b^3*ln(x)
```

**maxima** [A] time = 1.30, size = 34, normalized size = 0.92

$$b^3\log(x) - \frac{18\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="maxima")
[Out] b^3*log(x) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

**mupad [B]** time = 0.07, size = 34, normalized size = 0.92

$$b^3\ \ln(x) - \frac{\frac{a^3}{3} + \frac{3\,a^2\,b\,x}{2} + 3\,a\,b^2\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^4,x)
[Out] b^3*log(x) - (a^3/3 + 3*a*b^2*x^2 + (3*a^2*b*x)/2)/x^3
```

**sympy [A]** time = 0.24, size = 36, normalized size = 0.97

$$b^3 \log{(x)} + \frac{-2a^3 - 9a^2bx - 18ab^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**4,x)
[Out] b**3*log(x) + (-2*a**3 - 9*a**2*b*x - 18*a*b**2*x**2)/(6*x**3)
```

## 3.73 $\int \frac{(a+bx)^3}{x^5} \, dx$

**Optimal**. Leaf size=17

$$-\frac{(a+bx)^4}{4ax^4}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^4}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^5,x]
```

```
[Out] -(a + b*x)^4/(4*a*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^3}{x^5} \, dx = -\frac{(a+bx)^4}{4ax^4}$$

**Mathematica [B]** time = 0.00, size = 39, normalized size = 2.29

$$-\frac{a^3}{4x^4} - \frac{a^2b}{x^3} - \frac{3ab^2}{2x^2} - \frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^5,x]
```

```
[Out] -1/4*a^3/x^4 - (a^2*b)/x^3 - (3*a*b^2)/(2*x^2) - b^3/x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^5,x]
[Out] IntegrateAlgebraic[(a + b*x)^3/x^5, x]
```

**fricas [B]** time = 1.30, size = 33, normalized size = 1.94

$$-\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="fricas")
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

**giac [B]** time = 1.29, size = 33, normalized size = 1.94

$$-\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="giac")
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

**maple [B]** time = 0.01, size = 36, normalized size = 2.12

$$-\frac{b^3}{x}-\frac{3a\,b^2}{2x^2}-\frac{a^2b}{x^3}-\frac{a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^5,x)
[Out] -a^2*b/x^3-1/4*a^3/x^4-b^3/x-3/2*a*b^2/x^2
```

**maxima [B]** time = 1.33, size = 33, normalized size = 1.94

$$-\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="maxima")
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

**mupad [B]** time = 0.03, size = 33, normalized size = 1.94

$$-\frac{\frac{a^3}{4}+a^2\,b\,x+\frac{3\,a\,b^2\,x^2}{2}+b^3\,x^3}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^5,x)
[Out] -(a^3/4 + b^3*x^3 + (3*a*b^2*x^2)/2 + a^2*b*x)/x^4
```

**sympy [B]** time = 0.26, size = 36, normalized size = 2.12

$$\frac{-a^3-4a^2bx-6ab^2x^2-4b^3x^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**5,x)
[Out] (-a**3 - 4*a**2*b*x - 6*a*b**2*x**2 - 4*b**3*x**3)/(4*x**4)
```

## 3.74 $\int \frac{(a+bx)^3}{x^6} \, dx$

**Optimal**. Leaf size=36

$$\frac{b(a+bx)^4}{20a^2x^4} - \frac{(a+bx)^4}{5ax^5}$$

**Rubi** [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^4}{20a^2x^4} - \frac{(a+bx)^4}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^6,x]
```

```
[Out] -(a + b*x)^4/(5*a*x^5) + (b*(a + b*x)^4)/(20*a^2*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{x^6} \, dx &= -\frac{(a+bx)^4}{5ax^5} - \frac{b \int \frac{(a+bx)^3}{x^5} \, dx}{5a} \\ &= -\frac{(a+bx)^4}{5ax^5} + \frac{b(a+bx)^4}{20a^2x^4} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 41, normalized size = 1.14

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{4x^4}-\frac{ab^2}{x^3}-\frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^6,x]
[Out] -1/5*a^3/x^5 - (3*a^2*b)/(4*x^4) - (a*b^2)/x^3 - b^3/(2*x^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^6,x]
[Out] IntegrateAlgebraic[(a + b*x)^3/x^6, x]
```

**fricas [A]** time = 1.48, size = 35, normalized size = 0.97

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="fricas")
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

**giac [A]** time = 1.13, size = 35, normalized size = 0.97

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="giac")
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

**maple [A]** time = 0.00, size = 36, normalized size = 1.00

$$-\frac{b^3}{2x^2}-\frac{a\,b^2}{x^3}-\frac{3a^2b}{4x^4}-\frac{a^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^6,x)
```

```
[Out] -1/5*a^3/x^5-a*b^2/x^3-3/4*a^2*b/x^4-1/2*b^3/x^2
```

**maxima** [A] time = 1.35, size = 35, normalized size = 0.97

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

**mupad** [B] time = 0.03, size = 34, normalized size = 0.94

$$-\frac{\frac{a^3}{5}+\frac{3\,a^2\,b\,x}{4}+a\,b^2\,x^2+\frac{b^3\,x^3}{2}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^6,x)
```

```
[Out] -(a^3/5 + (b^3*x^3)/2 + a*b^2*x^2 + (3*a^2*b*x)/4)/x^5
```

**sympy** [A] time = 0.25, size = 37, normalized size = 1.03

$$\frac{-4a^3-15a^2bx-20ab^2x^2-10b^3x^3}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**6,x)
```

```
[Out] (-4*a**3 - 15*a**2*b*x - 20*a*b**2*x**2 - 10*b**3*x**3)/(20*x**5)
```

## 3.75 $\int \frac{(a+bx)^3}{x^7} dx$

**Optimal**. Leaf size=43

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

**Rubi [A]** time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{6x^6}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^7} dx &= \int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right) dx\\ &= -\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^7,x]
```

```
[Out] -1/6*a^3/x^6 - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/x^7, x]
```

**fricas [A]** time = 0.97, size = 35, normalized size = 0.81

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

**giac [A]** time = 1.15, size = 35, normalized size = 0.81

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

**maple [A]** time = 0.01, size = 36, normalized size = 0.84

$$-\frac{b^3}{3x^3}-\frac{3a\,b^2}{4x^4}-\frac{3a^2b}{5x^5}-\frac{a^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^7,x)
```

```
[Out] -1/6*a^3/x^6-3/5*a^2*b/x^5-3/4*a*b^2/x^4-1/3*b^3/x^3
```

**maxima [A]** time = 1.35, size = 35, normalized size = 0.81

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

**mupad [B]** time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3}{6}+\frac{3\,a^2\,b\,x}{5}+\frac{3\,a\,b^2\,x^2}{4}+\frac{b^3\,x^3}{3}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^7,x)
```

```
[Out] -(a^3/6 + (b^3*x^3)/3 + (3*a*b^2*x^2)/4 + (3*a^2*b*x)/5)/x^6
```

**sympy [A]** time = 0.34, size = 37, normalized size = 0.86

$$\frac{-10a^3-36a^2bx-45ab^2x^2-20b^3x^3}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**7,x)
```

```
[Out] (-10*a**3 - 36*a**2*b*x - 45*a*b**2*x**2 - 20*b**3*x**3)/(60*x**6)
```

## 3.76 $\int \frac{(a+bx)^3}{x^8}\,dx$

**Optimal**. Leaf size=43

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

**Rubi** [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2b}{2x^6}-\frac{a^3}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^8,x]
```

```
[Out] -a^3/(7*x^7) - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^8}\,dx &= \int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right)dx\\ &= -\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^8,x]
```

```
[Out] -1/7*a^3/x^7 - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/x^8, x]
```

**fricas [A]** time = 1.03, size = 35, normalized size = 0.81

$$-\frac{35\,b^3x^3 + 84\,ab^2x^2 + 70\,a^2bx + 20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

**giac [A]** time = 1.12, size = 35, normalized size = 0.81

$$-\frac{35\,b^3x^3 + 84\,ab^2x^2 + 70\,a^2bx + 20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.84

$$-\frac{b^3}{4x^4} - \frac{3a\,b^2}{5x^5} - \frac{a^2b}{2x^6} - \frac{a^3}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^8,x)
```

```
[Out] -1/7*a^3/x^7-1/2*a^2*b/x^6-3/5*a*b^2/x^5-1/4*b^3/x^4
```

**maxima [A]** time = 1.35, size = 35, normalized size = 0.81

$$-\frac{35\,b^3x^3 + 84\,ab^2x^2 + 70\,a^2bx + 20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

**mupad [B]** time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3}{7}+\frac{a^2\,b\,x}{2}+\frac{3\,a\,b^2\,x^2}{5}+\frac{b^3\,x^3}{4}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^8,x)
```

```
[Out] -(a^3/7 + (b^3*x^3)/4 + (3*a*b^2*x^2)/5 + (a^2*b*x)/2)/x^7
```

**sympy [A]** time = 0.29, size = 37, normalized size = 0.86

$$\frac{-20a^3-70a^2bx-84ab^2x^2-35b^3x^3}{140x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**8,x)
```

```
[Out] (-20*a**3 - 70*a**2*b*x - 84*a*b**2*x**2 - 35*b**3*x**3)/(140*x**7)
```

## 3.77 $\int x^6(a+bx)^5\,dx$

**Optimal**. Leaf size=66

$$\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

**Rubi [A]** time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$a^2b^3x^{10}+\frac{10}{9}a^3b^2x^9+\frac{5}{8}a^4bx^8+\frac{a^5x^7}{7}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b^
4*x^11)/11 + (b^5*x^12)/12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^5\,dx&=\int\left(a^5x^6+5a^4bx^7+10a^3b^2x^8+10a^2b^3x^9+5ab^4x^{10}+b^5x^{11}\right)dx\\&=\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 66, normalized size = 1.00

$$\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b^
4*x^11)/11 + (b^5*x^12)/12
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^6(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x^6*(a + b*x)^5, x]
```

**fricas [A]** time = 0.84, size = 56, normalized size = 0.85

$$\frac{1}{12}x^{12}b^5+\frac{5}{11}x^{11}b^4a+x^{10}b^3a^2+\frac{10}{9}x^9b^2a^3+\frac{5}{8}x^8ba^4+\frac{1}{7}x^7a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^5 + 5/11*x^11*b^4*a + x^10*b^3*a^2 + 10/9*x^9*b^2*a^3 + 5/8*x^8
*b*a^4 + 1/7*x^7*a^5
```

**giac [A]** time = 0.94, size = 56, normalized size = 0.85

$$\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

**maple [A]** time = 0.01, size = 57, normalized size = 0.86

$$\frac{1}{12}b^5x^{12}+\frac{5}{11}a\,b^4x^{11}+a^2b^3x^{10}+\frac{10}{9}a^3b^2x^9+\frac{5}{8}a^4b\,x^8+\frac{1}{7}a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^5,x)
```

```
[Out] 1/7*a^5*x^7+5/8*a^4*b*x^8+10/9*a^3*b^2*x^9+a^2*b^3*x^10+5/11*a*b^4*x^11+1/1
2*b^5*x^12
```

**maxima** [A] time = 1.38, size = 56, normalized size = 0.85

$$\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

**mupad** [B] time = 0.02, size = 56, normalized size = 0.85

$$\frac{a^5\,x^7}{7}+\frac{5\,a^4\,b\,x^8}{8}+\frac{10\,a^3\,b^2\,x^9}{9}+a^2\,b^3\,x^{10}+\frac{5\,a\,b^4\,x^{11}}{11}+\frac{b^5\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^5,x)
```

```
[Out] (a^5*x^7)/7 + (b^5*x^12)/12 + (5*a^4*b*x^8)/8 + (5*a*b^4*x^11)/11 + (10*a^3
*b^2*x^9)/9 + a^2*b^3*x^10
```

**sympy** [A] time = 0.09, size = 63, normalized size = 0.95

$$\frac{a^5x^7}{7}+\frac{5a^4bx^8}{8}+\frac{10a^3b^2x^9}{9}+a^2b^3x^{10}+\frac{5ab^4x^{11}}{11}+\frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**5,x)
```

```
[Out] a**5*x**7/7 + 5*a**4*b*x**8/8 + 10*a**3*b**2*x**9/9 + a**2*b**3*x**10 + 5*a
*b**4*x**11/11 + b**5*x**12/12
```

## 3.78 $\int x^5(a+bx)^5\,dx$

**Optimal**. Leaf size=69

$$\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

**Rubi [A]** time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{10}{9}a^2b^3x^9+\frac{5}{4}a^3b^2x^8+\frac{5}{7}a^4bx^7+\frac{a^5x^6}{6}+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a
*b^4*x^10)/2 + (b^5*x^11)/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^5\,dx&=\int\left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)dx\\&=\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a
*b^4*x^10)/2 + (b^5*x^11)/11
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^5(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x^5*(a + b*x)^5, x]
```

**fricas [A]** time = 1.29, size = 57, normalized size = 0.83

$$\frac{1}{11}x^{11}b^5+\frac{1}{2}x^{10}b^4a+\frac{10}{9}x^9b^3a^2+\frac{5}{4}x^8b^2a^3+\frac{5}{7}x^7ba^4+\frac{1}{6}x^6a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^5 + 1/2*x^10*b^4*a + 10/9*x^9*b^3*a^2 + 5/4*x^8*b^2*a^3 + 5/7*x
^7*b*a^4 + 1/6*x^6*a^5
```

**giac [A]** time = 1.20, size = 57, normalized size = 0.83

$$\frac{1}{11}\,b^5x^{11}+\frac{1}{2}\,ab^4x^{10}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{4}\,a^3b^2x^8+\frac{5}{7}\,a^4bx^7+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

**maple [A]** time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{11}b^5x^{11}+\frac{1}{2}a\,b^4x^{10}+\frac{10}{9}a^2b^3x^9+\frac{5}{4}a^3b^2x^8+\frac{5}{7}a^4b\,x^7+\frac{1}{6}a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^5,x)
```

```
[Out] 1/6*a^5*x^6+5/7*a^4*b*x^7+5/4*a^3*b^2*x^8+10/9*a^2*b^3*x^9+1/2*a*b^4*x^10+1
/11*b^5*x^11
```

**maxima** [A] time = 1.36, size = 57, normalized size = 0.83

$$\frac{1}{11}\,b^5x^{11} + \frac{1}{2}\,ab^4x^{10} + \frac{10}{9}\,a^2b^3x^9 + \frac{5}{4}\,a^3b^2x^8 + \frac{5}{7}\,a^4bx^7 + \frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

**mupad** [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^6}{6} + \frac{5\,a^4\,b\,x^7}{7} + \frac{5\,a^3\,b^2\,x^8}{4} + \frac{10\,a^2\,b^3\,x^9}{9} + \frac{a\,b^4\,x^{10}}{2} + \frac{b^5\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^5,x)
```

```
[Out] (a^5*x^6)/6 + (b^5*x^11)/11 + (5*a^4*b*x^7)/7 + (a*b^4*x^10)/2 + (5*a^3*b^2
*x^8)/4 + (10*a^2*b^3*x^9)/9
```

**sympy** [A] time = 0.08, size = 65, normalized size = 0.94

$$\frac{a^5x^6}{6} + \frac{5a^4bx^7}{7} + \frac{5a^3b^2x^8}{4} + \frac{10a^2b^3x^9}{9} + \frac{ab^4x^{10}}{2} + \frac{b^5x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**5,x)
```

```
[Out] a**5*x**6/6 + 5*a**4*b*x**7/7 + 5*a**3*b**2*x**8/4 + 10*a**2*b**3*x**9/9 +
a*b**4*x**10/2 + b**5*x**11/11
```

## 3.79 $\int x^4(a+bx)^5\,dx$

**Optimal**. Leaf size=69

$$\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

**Rubi [A]** time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5}{4}a^2b^3x^8+\frac{10}{7}a^3b^2x^7+\frac{5}{6}a^4bx^6+\frac{a^5x^5}{5}+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (5
*a*b^4*x^9)/9 + (b^5*x^10)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^5\,dx &= \int \left(a^5x^4+5a^4bx^5+10a^3b^2x^6+10a^2b^3x^7+5ab^4x^8+b^5x^9\right)dx\\ &= \frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (5
*a*b^4*x^9)/9 + (b^5*x^10)/10
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x)^5, x]
```

**fricas [A]** time = 1.33, size = 57, normalized size = 0.83

$$\frac{1}{10}x^{10}b^5+\frac{5}{9}x^9b^4a+\frac{5}{4}x^8b^3a^2+\frac{10}{7}x^7b^2a^3+\frac{5}{6}x^6ba^4+\frac{1}{5}x^5a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^5 + 5/9*x^9*b^4*a + 5/4*x^8*b^3*a^2 + 10/7*x^7*b^2*a^3 + 5/6*x^
6*b*a^4 + 1/5*x^5*a^5
```

**giac [A]** time = 1.10, size = 57, normalized size = 0.83

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

**maple [A]** time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{10}b^5x^{10}+\frac{5}{9}a\,b^4x^9+\frac{5}{4}a^2b^3x^8+\frac{10}{7}a^3b^2x^7+\frac{5}{6}a^4b\,x^6+\frac{1}{5}a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^5,x)
```

```
[Out] 1/5*a^5*x^5+5/6*a^4*b*x^6+10/7*a^3*b^2*x^7+5/4*a^2*b^3*x^8+5/9*a*b^4*x^9+1/
10*b^5*x^10
```

**maxima** [A] time = 1.47, size = 57, normalized size = 0.83

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

**mupad** [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^5}{5}+\frac{5\,a^4\,b\,x^6}{6}+\frac{10\,a^3\,b^2\,x^7}{7}+\frac{5\,a^2\,b^3\,x^8}{4}+\frac{5\,a\,b^4\,x^9}{9}+\frac{b^5\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^5,x)
```

```
[Out] (a^5*x^5)/5 + (b^5*x^10)/10 + (5*a^4*b*x^6)/6 + (5*a*b^4*x^9)/9 + (10*a^3*b
^2*x^7)/7 + (5*a^2*b^3*x^8)/4
```

**sympy** [A] time = 0.08, size = 66, normalized size = 0.96

$$\frac{a^5x^5}{5}+\frac{5a^4bx^6}{6}+\frac{10a^3b^2x^7}{7}+\frac{5a^2b^3x^8}{4}+\frac{5ab^4x^9}{9}+\frac{b^5x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**5,x)
```

```
[Out] a**5*x**5/5 + 5*a**4*b*x**6/6 + 10*a**3*b**2*x**7/7 + 5*a**2*b**3*x**8/4 +
5*a*b**4*x**9/9 + b**5*x**10/10
```

## 3.80 $\int x^3(a+bx)^5\,dx$

**Optimal**. Leaf size=64

$$-\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}+\frac{(a+bx)^9}{9b^4}-\frac{3a(a+bx)^8}{8b^4}$$

**Rubi [A]** time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2(a+bx)^7}{7b^4}-\frac{a^3(a+bx)^6}{6b^4}+\frac{(a+bx)^9}{9b^4}-\frac{3a(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^5,x]
```

```
[Out] -(a^3*(a + b*x)^6)/(6*b^4) + (3*a^2*(a + b*x)^7)/(7*b^4) - (3*a*(a + b*x)^8
)/(8*b^4) + (a + b*x)^9/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^5\,dx &= \int\left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}-\frac{3a(a+bx)^8}{8b^4}+\frac{(a+bx)^9}{9b^4}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 66, normalized size = 1.03

$$\frac{a^5x^4}{4}+a^4bx^5+\frac{5}{3}a^3b^2x^6+\frac{10}{7}a^2b^3x^7+\frac{5}{8}ab^4x^8+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^5,x]
```

```
[Out] (a^5*x^4)/4 + a^4*b*x^5 + (5*a^3*b^2*x^6)/3 + (10*a^2*b^3*x^7)/7 + (5*a*b^4
*x^8)/8 + (b^5*x^9)/9
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x)^5, x]
```

**fricas [A]** time = 1.28, size = 56, normalized size = 0.88

$$\frac{1}{9}x^9b^5+\frac{5}{8}x^8b^4a+\frac{10}{7}x^7b^3a^2+\frac{5}{3}x^6b^2a^3+x^5ba^4+\frac{1}{4}x^4a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^5 + 5/8*x^8*b^4*a + 10/7*x^7*b^3*a^2 + 5/3*x^6*b^2*a^3 + x^5*b*a^
4 + 1/4*x^4*a^5
```

**giac [A]** time = 1.05, size = 56, normalized size = 0.88

$$\frac{1}{9}\,b^5x^9+\frac{5}{8}\,ab^4x^8+\frac{10}{7}\,a^2b^3x^7+\frac{5}{3}\,a^3b^2x^6+a^4bx^5+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

**maple [A]** time = 0.00, size = 57, normalized size = 0.89

$$\frac{1}{9}b^5x^9+\frac{5}{8}a\,b^4x^8+\frac{10}{7}a^2b^3x^7+\frac{5}{3}a^3b^2x^6+a^4b\,x^5+\frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^5,x)
```

```
[Out] 1/9*b^5*x^9+5/8*a*b^4*x^8+10/7*a^2*b^3*x^7+5/3*a^3*b^2*x^6+a^4*b*x^5+1/4*a^
5*x^4
```

**maxima** [A] time = 1.40, size = 56, normalized size = 0.88

$$\frac{1}{9}\,b^5x^9+\frac{5}{8}\,ab^4x^8+\frac{10}{7}\,a^2b^3x^7+\frac{5}{3}\,a^3b^2x^6+a^4bx^5+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

**mupad** [B] time = 0.02, size = 56, normalized size = 0.88

$$\frac{a^5\,x^4}{4}+a^4\,b\,x^5+\frac{5\,a^3\,b^2\,x^6}{3}+\frac{10\,a^2\,b^3\,x^7}{7}+\frac{5\,a\,b^4\,x^8}{8}+\frac{b^5\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^5,x)
```

```
[Out] (a^5*x^4)/4 + (b^5*x^9)/9 + a^4*b*x^5 + (5*a*b^4*x^8)/8 + (5*a^3*b^2*x^6)/3
 + (10*a^2*b^3*x^7)/7
```

**sympy** [A] time = 0.10, size = 63, normalized size = 0.98

$$\frac{a^5x^4}{4}+a^4bx^5+\frac{5a^3b^2x^6}{3}+\frac{10a^2b^3x^7}{7}+\frac{5ab^4x^8}{8}+\frac{b^5x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**5,x)
```

```
[Out] a**5*x**4/4 + a**4*b*x**5 + 5*a**3*b**2*x**6/3 + 10*a**2*b**3*x**7/7 + 5*a*
b**4*x**8/8 + b**5*x**9/9
```

## 3.81 $\int x^2(a + bx)^5 \, dx$

**Optimal**. Leaf size=47

$$\frac{a^2(a + bx)^6}{6b^3} + \frac{(a + bx)^8}{8b^3} - \frac{2a(a + bx)^7}{7b^3}$$

**Rubi [A]** time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a + bx)^6}{6b^3} + \frac{(a + bx)^8}{8b^3} - \frac{2a(a + bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^5,x]
```

```
[Out] (a^2*(a + b*x)^6)/(6*b^3) - (2*a*(a + b*x)^7)/(7*b^3) + (a + b*x)^8/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int x^2(a + bx)^5 \, dx &= \int \left( \frac{a^2(a + bx)^5}{b^2} - \frac{2a(a + bx)^6}{b^2} + \frac{(a + bx)^7}{b^2} \right) dx \\ &= \frac{a^2(a + bx)^6}{6b^3} - \frac{2a(a + bx)^7}{7b^3} + \frac{(a + bx)^8}{8b^3} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 67, normalized size = 1.43

$$\frac{a^5x^3}{3} + \frac{5}{4}a^4bx^4 + 2a^3b^2x^5 + \frac{5}{3}a^2b^3x^6 + \frac{5}{7}ab^4x^7 + \frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^5,x]
```

```
[Out] (a^5*x^3)/3 + (5*a^4*b*x^4)/4 + 2*a^3*b^2*x^5 + (5*a^2*b^3*x^6)/3 + (5*a*b^
4*x^7)/7 + (b^5*x^8)/8
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x)^5, x]
```

**fricas [A]** time = 1.41, size = 57, normalized size = 1.21

$$\frac{1}{8}x^8b^5+\frac{5}{7}x^7b^4a+\frac{5}{3}x^6b^3a^2+2x^5b^2a^3+\frac{5}{4}x^4ba^4+\frac{1}{3}x^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^5 + 5/7*x^7*b^4*a + 5/3*x^6*b^3*a^2 + 2*x^5*b^2*a^3 + 5/4*x^4*b*a
^4 + 1/3*x^3*a^5
```

**giac [A]** time = 1.71, size = 57, normalized size = 1.21

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

**maple [A]** time = 0.00, size = 58, normalized size = 1.23

$$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^5,x)
```

```
[Out] 1/8*b^5*x^8+5/7*a*b^4*x^7+5/3*a^2*b^3*x^6+2*a^3*b^2*x^5+5/4*a^4*b*x^4+1/3*a
^5*x^3
```

**maxima** [A] time = 1.32, size = 57, normalized size = 1.21

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

**mupad** [B] time = 0.02, size = 57, normalized size = 1.21

$$\frac{a^5\,x^3}{3}+\frac{5\,a^4\,b\,x^4}{4}+2\,a^3\,b^2\,x^5+\frac{5\,a^2\,b^3\,x^6}{3}+\frac{5\,a\,b^4\,x^7}{7}+\frac{b^5\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^5,x)
```

```
[Out] (a^5*x^3)/3 + (b^5*x^8)/8 + (5*a^4*b*x^4)/4 + (5*a*b^4*x^7)/7 + 2*a^3*b^2*x
^5 + (5*a^2*b^3*x^6)/3
```

**sympy** [A] time = 0.08, size = 65, normalized size = 1.38

$$\frac{a^5x^3}{3}+\frac{5a^4bx^4}{4}+2a^3b^2x^5+\frac{5a^2b^3x^6}{3}+\frac{5ab^4x^7}{7}+\frac{b^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**5,x)
```

```
[Out] a**5*x**3/3 + 5*a**4*b*x**4/4 + 2*a**3*b**2*x**5 + 5*a**2*b**3*x**6/3 + 5*a
*b**4*x**7/7 + b**5*x**8/8
```

## 3.82 $\int x(a+bx)^5\,dx$

**Optimal**. Leaf size=30

$$\frac{(a+bx)^7}{7b^2}-\frac{a(a+bx)^6}{6b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^7}{7b^2}-\frac{a(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^5,x]
```

```
[Out] -(a*(a + b*x)^6)/(6*b^2) + (a + b*x)^7/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^5\,dx &= \int\left(-\frac{a(a+bx)^5}{b}+\frac{(a+bx)^6}{b}\right)dx\\ &= -\frac{a(a+bx)^6}{6b^2}+\frac{(a+bx)^7}{7b^2}\end{aligned}$$

**Mathematica [B]** time = 0.00, size = 67, normalized size = 2.23

$$\frac{a^5x^2}{2}+\frac{5}{3}a^4bx^3+\frac{5}{2}a^3b^2x^4+2a^2b^3x^5+\frac{5}{6}ab^4x^6+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^5,x]
```

```
[Out] (a^5*x^2)/2 + (5*a^4*b*x^3)/3 + (5*a^3*b^2*x^4)/2 + 2*a^2*b^3*x^5 + (5*a*b^
4*x^6)/6 + (b^5*x^7)/7
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x)^5, x]
```

**fricas [B]** time = 0.80, size = 57, normalized size = 1.90

$$\frac{1}{7}x^7b^5+\frac{5}{6}x^6b^4a+2x^5b^3a^2+\frac{5}{2}x^4b^2a^3+\frac{5}{3}x^3ba^4+\frac{1}{2}x^2a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^5 + 5/6*x^6*b^4*a + 2*x^5*b^3*a^2 + 5/2*x^4*b^2*a^3 + 5/3*x^3*b*a
^4 + 1/2*x^2*a^5
```

**giac [B]** time = 0.93, size = 57, normalized size = 1.90

$$\frac{1}{7}\,b^5x^7+\frac{5}{6}\,ab^4x^6+2\,a^2b^3x^5+\frac{5}{2}\,a^3b^2x^4+\frac{5}{3}\,a^4bx^3+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

**maple [B]** time = 0.00, size = 58, normalized size = 1.93

$$\frac{1}{7}b^5x^7+\frac{5}{6}a\,b^4x^6+2a^2b^3x^5+\frac{5}{2}a^3b^2x^4+\frac{5}{3}a^4b\,x^3+\frac{1}{2}a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^5,x)
```

```
[Out] 1/7*b^5*x^7+5/6*a*b^4*x^6+2*a^2*b^3*x^5+5/2*a^3*b^2*x^4+5/3*a^4*b*x^3+1/2*a
^5*x^2
```

**maxima [B]** time = 1.31, size = 57, normalized size = 1.90

$$\frac{1}{7}\,b^5x^7+\frac{5}{6}\,ab^4x^6+2\,a^2b^3x^5+\frac{5}{2}\,a^3b^2x^4+\frac{5}{3}\,a^4bx^3+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

**mupad [B]** time = 0.02, size = 57, normalized size = 1.90

$$\frac{a^5\,x^2}{2}+\frac{5\,a^4\,b\,x^3}{3}+\frac{5\,a^3\,b^2\,x^4}{2}+2\,a^2\,b^3\,x^5+\frac{5\,a\,b^4\,x^6}{6}+\frac{b^5\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^5,x)
```

```
[Out] (a^5*x^2)/2 + (b^5*x^7)/7 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^6)/6 + (5*a^3*b^2*
x^4)/2 + 2*a^2*b^3*x^5
```

**sympy [B]** time = 0.08, size = 65, normalized size = 2.17

$$\frac{a^5x^2}{2}+\frac{5a^4bx^3}{3}+\frac{5a^3b^2x^4}{2}+2a^2b^3x^5+\frac{5ab^4x^6}{6}+\frac{b^5x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**5,x)
```

```
[Out] a**5*x**2/2 + 5*a**4*b*x**3/3 + 5*a**3*b**2*x**4/2 + 2*a**2*b**3*x**5 + 5*a
*b**4*x**6/6 + b**5*x**7/7
```

## 3.83 $\int (a+bx)^5\,dx$

**Optimal**. Leaf size=14

$$\frac{(a+bx)^6}{6b}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5,x]
```

```
[Out] (a + b*x)^6/(6*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^5\,dx = \frac{(a+bx)^6}{6b}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5,x]
```

```
[Out] (a + b*x)^6/(6*b)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5, x]
```

**fricas [B]** time = 1.21, size = 53, normalized size = 3.79

$$\frac{1}{6}x^6b^5 + x^5b^4a + \frac{5}{2}x^4b^3a^2 + \frac{10}{3}x^3b^2a^3 + \frac{5}{2}x^2ba^4 + xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^5 + x^5*b^4*a + 5/2*x^4*b^3*a^2 + 10/3*x^3*b^2*a^3 + 5/2*x^2*b*a^
4 + x*a^5
```

**giac [A]** time = 1.20, size = 12, normalized size = 0.86

$$\frac{(bx+a)^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/6*(b*x + a)^6/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx+a)^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5,x)
```

```
[Out] 1/6*(b*x+a)^6/b
```

**maxima [B]** time = 1.37, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

**mupad [B]** time = 0.02, size = 53, normalized size = 3.79

$$a^5\,x+\frac{5\,a^4\,b\,x^2}{2}+\frac{10\,a^3\,b^2\,x^3}{3}+\frac{5\,a^2\,b^3\,x^4}{2}+a\,b^4\,x^5+\frac{b^5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5,x)
```

```
[Out] a^5*x + (b^5*x^6)/6 + (5*a^4*b*x^2)/2 + a*b^4*x^5 + (10*a^3*b^2*x^3)/3 + (5
*a^2*b^3*x^4)/2
```

**sympy [B]** time = 0.08, size = 60, normalized size = 4.29

$$a^{5}x+\frac{5a^{4}bx^{2}}{2}+\frac{10a^{3}b^{2}x^{3}}{3}+\frac{5a^{2}b^{3}x^{4}}{2}+ab^{4}x^{5}+\frac{b^{5}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b**
4*x**5 + b**5*x**6/6
```

## 3.84 $\int \frac{(a+bx)^5}{x} \, dx$

**Optimal**. Leaf size=59

$$a^5\log(x)+5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}$$

**Rubi [A]** time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+5a^4bx+a^5\log(x)+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x,x]
```

```
[Out] 5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5
)/5 + a^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x}\,dx &= \int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)dx\\ &= 5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}+a^5\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 59, normalized size = 1.00

$$a^5\log(x)+5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x,x]
```

```
[Out] 5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5
)/5 + a^5*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x, x]
```

**fricas [A]** time = 1.44, size = 53, normalized size = 0.90

$$\frac{1}{5}\,b^5x^5 + \frac{5}{4}\,ab^4x^4 + \frac{10}{3}\,a^2b^3x^3 + 5\,a^3b^2x^2 + 5\,a^4bx + a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(x)
```

**giac [A]** time = 1.12, size = 54, normalized size = 0.92

$$\frac{1}{5}\,b^5x^5 + \frac{5}{4}\,ab^4x^4 + \frac{10}{3}\,a^2b^3x^3 + 5\,a^3b^2x^2 + 5\,a^4bx + a^5\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(abs(x))
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.92

$$\frac{b^5x^5}{5} + \frac{5a\,b^4x^4}{4} + \frac{10a^2b^3x^3}{3} + 5a^3b^2x^2 + a^5\ln(x) + 5a^4bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x,x)
```

```
[Out] 5*a^4*b*x+5*a^3*b^2*x^2+10/3*a^2*b^3*x^3+5/4*a*b^4*x^4+1/5*b^5*x^5+a^5*ln(x
)
```

**maxima [A]** time = 1.37, size = 53, normalized size = 0.90

$$\frac{1}{5}\,b^5x^5+\frac{5}{4}\,ab^4x^4+\frac{10}{3}\,a^2b^3x^3+5\,a^3b^2x^2+5\,a^4bx+a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(x)
```

**mupad [B]** time = 0.03, size = 53, normalized size = 0.90

$$a^5\,\ln(x)+\frac{b^5\,x^5}{5}+\frac{5\,a\,b^4\,x^4}{4}+5\,a^3\,b^2\,x^2+\frac{10\,a^2\,b^3\,x^3}{3}+5\,a^4\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x,x)
```

```
[Out] a^5*log(x) + (b^5*x^5)/5 + (5*a*b^4*x^4)/4 + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^
3)/3 + 5*a^4*b*x
```

**sympy [A]** time = 0.15, size = 60, normalized size = 1.02

$$a^5\log{(x)}+5a^4bx+5a^3b^2x^2+\frac{10a^2b^3x^3}{3}+\frac{5ab^4x^4}{4}+\frac{b^5x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x,x)
```

```
[Out] a**5*log(x) + 5*a**4*b*x + 5*a**3*b**2*x**2 + 10*a**2*b**3*x**3/3 + 5*a*b**
4*x**4/4 + b**5*x**5/5
```

## 3.85 $\int \frac{(a+bx)^5}{x^2} dx$

**Optimal**. Leaf size=58

$$-\frac{a^5}{x} + 5a^4b\log(x) + 10a^3b^2x + 5a^2b^3x^2 + \frac{5}{3}ab^4x^3 + \frac{b^5x^4}{4}$$

**Rubi** [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$5a^2b^3x^2 + 10a^3b^2x + 5a^4b\log(x) - \frac{a^5}{x} + \frac{5}{3}ab^4x^3 + \frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^2,x]
```

```
[Out] -(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5
*a^4*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{x^2} dx &= \int \left(10a^3b^2 + \frac{a^5}{x^2} + \frac{5a^4b}{x} + 10a^2b^3x + 5ab^4x^2 + b^5x^3\right) dx \\ &= -\frac{a^5}{x} + 10a^3b^2x + 5a^2b^3x^2 + \frac{5}{3}ab^4x^3 + \frac{b^5x^4}{4} + 5a^4b\log(x) \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 58, normalized size = 1.00

$$-\frac{a^5}{x} + 5a^4b\log(x) + 10a^3b^2x + 5a^2b^3x^2 + \frac{5}{3}ab^4x^3 + \frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^2,x]
```

```
[Out] -(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5
*a^4*b*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^2, x]
```

**fricas [A]** time = 0.97, size = 59, normalized size = 1.02

$$\frac{3\,b^5x^5 + 20\,ab^4x^4 + 60\,a^2b^3x^3 + 120\,a^3b^2x^2 + 60\,a^4bx\log(x) - 12\,a^5}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^5 + 20*a*b^4*x^4 + 60*a^2*b^3*x^3 + 120*a^3*b^2*x^2 + 60*a^4*
b*x*log(x) - 12*a^5)/x
```

**giac [A]** time = 1.28, size = 55, normalized size = 0.95

$$\frac{1}{4}\,b^5x^4 + \frac{5}{3}\,ab^4x^3 + 5\,a^2b^3x^2 + 10\,a^3b^2x + 5\,a^4b\log\left(|x|\right) - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(ab
s(x)) - a^5/x
```

**maple [A]** time = 0.01, size = 55, normalized size = 0.95

$$\frac{b^5x^4}{4} + \frac{5a\,b^4x^3}{3} + 5a^2b^3x^2 + 5a^4b\ln(x) + 10a^3b^2x - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^2,x)
```

```
[Out] -a^5/x+10*a^3*b^2*x+5*a^2*b^3*x^2+5/3*a*b^4*x^3+1/4*b^5*x^4+5*a^4*b*ln(x)
```

**maxima [A]** time = 1.40, size = 54, normalized size = 0.93

$$\frac{1}{4}\,b^5x^4+\frac{5}{3}\,ab^4x^3+5\,a^2b^3x^2+10\,a^3b^2x+5\,a^4b\log(x)-\frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(x)
 - a^5/x
```

**mupad [B]** time = 0.03, size = 54, normalized size = 0.93

$$\frac{b^5\,x^4}{4}-\frac{a^5}{x}+10\,a^3\,b^2\,x+\frac{5\,a\,b^4\,x^3}{3}+5\,a^4\,b\,\ln(x)+5\,a^2\,b^3\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^2,x)
```

```
[Out] (b^5*x^4)/4 - a^5/x + 10*a^3*b^2*x + (5*a*b^4*x^3)/3 + 5*a^4*b*log(x) + 5*a
^2*b^3*x^2
```

**sympy [A]** time = 0.18, size = 56, normalized size = 0.97

$$-\frac{a^5}{x}+5a^4b\log{(x)}+10a^3b^2x+5a^2b^3x^2+\frac{5ab^4x^3}{3}+\frac{b^5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**2,x)
```

```
[Out] -a**5/x + 5*a**4*b*log(x) + 10*a**3*b**2*x + 5*a**2*b**3*x**2 + 5*a*b**4*x*
*3/3 + b**5*x**4/4
```

## 3.86 $\int \frac{(a+bx)^5}{x^3} \, dx$

**Optimal**. Leaf size=60

$$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^3b^2\log(x)+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}$$

**Rubi [A]** time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$10a^2b^3x+10a^3b^2\log(x)-\frac{5a^4b}{x}-\frac{a^5}{2x^2}+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^3,x]
```

```
[Out] -a^5/(2*x^2) - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3 +
 10*a^3*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^3} \, dx &= \int \left(10a^2b^3+\frac{a^5}{x^3}+\frac{5a^4b}{x^2}+\frac{10a^3b^2}{x}+5ab^4x+b^5x^2\right) \, dx\\ &= -\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}+10a^3b^2\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^3b^2\log(x)+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3 +
 10*a^3*b^2*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^3, x]
```

**fricas [A]** time = 0.75, size = 59, normalized size = 0.98

$$\frac{2\,b^5x^5+15\,ab^4x^4+60\,a^2b^3x^3+60\,a^3b^2x^2\log(x)-30\,a^4bx-3\,a^5}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*x^5 + 15*a*b^4*x^4 + 60*a^2*b^3*x^3 + 60*a^3*b^2*x^2*log(x) - 30
*a^4*b*x - 3*a^5)/x^2
```

**giac [A]** time = 1.19, size = 54, normalized size = 0.90

$$\frac{1}{3}\,b^5x^3+\frac{5}{2}\,ab^4x^2+10\,a^2b^3x+10\,a^3b^2\log\left(|x|\right)-\frac{10\,a^4bx+a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(abs(x)) - 1/2*(
10*a^4*b*x + a^5)/x^2
```

**maple [A]** time = 0.00, size = 55, normalized size = 0.92

$$\frac{b^5x^3}{3}+\frac{5a\,b^4x^2}{2}+10a^3b^2\ln(x)+10a^2b^3x-\frac{5a^4b}{x}-\frac{a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^3,x)
```

```
[Out] -1/2*a^5/x^2-5*a^4*b/x+10*a^2*b^3*x+5/2*a*b^4*x^2+1/3*b^5*x^3+10*a^3*b^2*ln
(x)
```

**maxima [A]** time = 1.36, size = 53, normalized size = 0.88

$$\frac{1}{3}b^5x^3 + \frac{5}{2}ab^4x^2 + 10\,a^2b^3x + 10\,a^3b^2\log(x) - \frac{10\,a^4bx + a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(x) - 1/2*(10*a^
4*b*x + a^5)/x^2
```

**mupad [B]** time = 0.03, size = 55, normalized size = 0.92

$$\frac{b^5\,x^3}{3} - \frac{\frac{a^5}{2} + 5\,b\,x\,a^4}{x^2} + 10\,a^2\,b^3\,x + \frac{5\,a\,b^4\,x^2}{2} + 10\,a^3\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^3,x)
```

```
[Out] (b^5*x^3)/3 - (a^5/2 + 5*a^4*b*x)/x^2 + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + 10
*a^3*b^2*log(x)
```

**sympy [A]** time = 0.20, size = 60, normalized size = 1.00

$$10a^3b^2\log(x) + 10a^2b^3x + \frac{5ab^4x^2}{2} + \frac{b^5x^3}{3} + \frac{-a^5 - 10a^4bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**3,x)
```

```
[Out] 10*a**3*b**2*log(x) + 10*a**2*b**3*x + 5*a*b**4*x**2/2 + b**5*x**3/3 + (-a*
*5 - 10*a**4*b*x)/(2*x**2)
```

## 3.87 $\int \frac{(a+bx)^5}{x^4} \, dx$

**Optimal**. Leaf size=60

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+10a^2b^3\log(x)+5ab^4x+\frac{b^5x^2}{2}$$

**Rubi [A]** time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{x}+10a^2b^3\log(x)-\frac{5a^4b}{2x^2}-\frac{a^5}{3x^3}+5ab^4x+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^4,x]
```

```
[Out] -a^5/(3*x^3) - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2
 + 10*a^2*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^4}\,dx &= \int \left(5ab^4+\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+b^5x\right)dx\\ &= -\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5ab^4x+\frac{b^5x^2}{2}+10a^2b^3\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+10a^2b^3\log(x)+5ab^4x+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^4,x]
```

```
[Out] -1/3*a^5/x^3 - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2
 + 10*a^2*b^3*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^4, x]
```

**fricas [A]** time = 1.73, size = 59, normalized size = 0.98

$$\frac{3\,b^5x^5 + 30\,ab^4x^4 + 60\,a^2b^3x^3\log(x) - 60\,a^3b^2x^2 - 15\,a^4bx - 2\,a^5}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*x^5 + 30*a*b^4*x^4 + 60*a^2*b^3*x^3*log(x) - 60*a^3*b^2*x^2 - 15
*a^4*b*x - 2*a^5)/x^3
```

**giac [A]** time = 1.00, size = 56, normalized size = 0.93

$$\frac{1}{2}\,b^5x^2 + 5\,ab^4x + 10\,a^2b^3\log\left(|x|\right) - \frac{60\,a^3b^2x^2 + 15\,a^4bx + 2\,a^5}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(abs(x)) - 1/6*(60*a^3*b^2*x^2 + 15
*a^4*b*x + 2*a^5)/x^3
```

**maple [A]** time = 0.01, size = 55, normalized size = 0.92

$$\frac{b^5x^2}{2} + 10a^2b^3\ln(x) + 5a\,b^4x - \frac{10a^3b^2}{x} - \frac{5a^4b}{2x^2} - \frac{a^5}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^4,x)
```

```
[Out] -1/3*a^5/x^3-5/2*a^4*b/x^2-10*a^3*b^2/x+5*a*b^4*x+1/2*b^5*x^2+10*a^2*b^3*ln
(x)
```

**maxima [A]** time = 1.41, size = 55, normalized size = 0.92

$$\frac{1}{2}\,b^5x^2 + 5\,ab^4x + 10\,a^2b^3\log(x) - \frac{60\,a^3b^2x^2 + 15\,a^4bx + 2\,a^5}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(x) - 1/6*(60*a^3*b^2*x^2 + 15*a^4*
b*x + 2*a^5)/x^3
```

**mupad [B]** time = 0.04, size = 55, normalized size = 0.92

$$\frac{b^5\,x^2}{2} - \frac{\frac{a^5}{3} + \frac{5\,a^4\,b\,x}{2} + 10\,a^3\,b^2\,x^2}{x^3} + 10\,a^2\,b^3\,\ln(x) + 5\,a\,b^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^4,x)
```

```
[Out] (b^5*x^2)/2 - (a^5/3 + 10*a^3*b^2*x^2 + (5*a^4*b*x)/2)/x^3 + 10*a^2*b^3*log
(x) + 5*a*b^4*x
```

**sympy [A]** time = 0.26, size = 60, normalized size = 1.00

$$10a^2b^3\log{(x)} + 5ab^4x + \frac{b^5x^2}{2} + \frac{-2a^5 - 15a^4bx - 60a^3b^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**4,x)
```

```
[Out] 10*a**2*b**3*log(x) + 5*a*b**4*x + b**5*x**2/2 + (-2*a**5 - 15*a**4*b*x - 6
0*a**3*b**2*x**2)/(6*x**3)
```

## 3.88 $\int \frac{(a+bx)^5}{x^5} \, dx$

**Optimal**. Leaf size=57

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+5ab^4\log(x)+b^5x$$

**Rubi [A]** time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}-\frac{5a^4b}{3x^3}-\frac{a^5}{4x^4}+5ab^4\log(x)+b^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^5,x]
```

```
[Out] -a^5/(4*x^4) - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*x
 + 5*a*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{x^5}\,dx &= \int \left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right)dx\\ &= -\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+b^5x+5ab^4\log(x) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 57, normalized size = 1.00

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+5ab^4\log(x)+b^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^5,x]
```

```
[Out] -1/4*a^5/x^4 - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*x
 + 5*a*b^4*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^5, x]
```

**fricas [A]** time = 0.81, size = 59, normalized size = 1.04

$$\frac{12\,b^5x^5+60\,ab^4x^4\log(x)-120\,a^2b^3x^3-60\,a^3b^2x^2-20\,a^4bx-3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^5*x^5 + 60*a*b^4*x^4*log(x) - 120*a^2*b^3*x^3 - 60*a^3*b^2*x^2 -
 20*a^4*b*x - 3*a^5)/x^4
```

**giac [A]** time = 1.38, size = 55, normalized size = 0.96

$$b^5x+5\,ab^4\log\left(|x|\right)-\frac{120\,a^2b^3x^3+60\,a^3b^2x^2+20\,a^4bx+3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="giac")
```

```
[Out] b^5*x + 5*a*b^4*log(abs(x)) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*a
^4*b*x + 3*a^5)/x^4
```

**maple [A]** time = 0.01, size = 54, normalized size = 0.95

$$5a\,b^4\ln(x)+b^5x-\frac{10a^2b^3}{x}-\frac{5a^3b^2}{x^2}-\frac{5a^4b}{3x^3}-\frac{a^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^5,x)
```

[Out] -1/4*a^5/x^4-5/3*a^4*b/x^3-5*a^3*b^2/x^2-10*a^2*b^3/x+b^5*x+5*a*b^4*ln(x)

**maxima** [A] time = 1.37, size = 54, normalized size = 0.95

$$b^5x + 5\,ab^4\log(x) - \frac{120\,a^2b^3x^3 + 60\,a^3b^2x^2 + 20\,a^4bx + 3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5/x^5,x, algorithm="maxima")

[Out] b^5*x + 5*a*b^4*log(x) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*a^4*b*x + 3*a^5)/x^4

**mupad** [B] time = 0.08, size = 54, normalized size = 0.95

$$b^5\,x - \frac{\frac{a^5}{4} + \frac{5\,a^4\,b\,x}{3} + 5\,a^3\,b^2\,x^2 + 10\,a^2\,b^3\,x^3}{x^4} + 5\,a\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^5/x^5,x)

[Out] b^5*x - (a^5/4 + 5*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + (5*a^4*b*x)/3)/x^4 + 5*a*b^4*log(x)

**sympy** [A] time = 0.29, size = 58, normalized size = 1.02

$$5ab^4\log{(x)} + b^5x + \frac{-3a^5 - 20a^4bx - 60a^3b^2x^2 - 120a^2b^3x^3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**5/x**5,x)

[Out] 5*a*b**4*log(x) + b**5*x + (-3*a**5 - 20*a**4*b*x - 60*a**3*b**2*x**2 - 120*a**2*b**3*x**3)/(12*x**4)

## 3.89 $\int \frac{(a+bx)^5}{x^6} \, dx$

**Optimal**. Leaf size=61

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5a^4b}{4x^4}-\frac{a^5}{5x^5}-\frac{5ab^4}{x}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^6,x]

[Out] -a^5/(5*x^5) - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 -
 (5*a*b^4)/x + b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^6}\,dx &= \int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx\\ &= -\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 61, normalized size = 1.00

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^6,x]
```

```
[Out] -1/5*a^5/x^5 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 -
 (5*a*b^4)/x + b^5*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^6, x]
```

**fricas [A]** time = 1.93, size = 59, normalized size = 0.97

$$\frac{60\,b^5x^5\log(x)-300\,ab^4x^4-300\,a^2b^3x^3-200\,a^3b^2x^2-75\,a^4bx-12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^5*x^5*log(x) - 300*a*b^4*x^4 - 300*a^2*b^3*x^3 - 200*a^3*b^2*x^2
 - 75*a^4*b*x - 12*a^5)/x^5
```

**giac [A]** time = 1.37, size = 57, normalized size = 0.93

$$b^5\log\left(|x|\right)-\frac{300\,ab^4x^4+300\,a^2b^3x^3+200\,a^3b^2x^2+75\,a^4bx+12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="giac")
```

```
[Out] b^5*log(abs(x)) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2 +
 75*a^4*b*x + 12*a^5)/x^5
```

**maple [A]** time = 0.01, size = 56, normalized size = 0.92

$$b^5\ln(x)-\frac{5a\,b^4}{x}-\frac{5a^2b^3}{x^2}-\frac{10a^3b^2}{3x^3}-\frac{5a^4b}{4x^4}-\frac{a^5}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^6,x)
```

```
[Out] -1/5*a^5/x^5-5/4*a^4*b/x^4-10/3*a^3*b^2/x^3-5*a^2*b^3/x^2-5*a*b^4/x+b^5*ln(
x)
```

**maxima [A]** time = 1.35, size = 56, normalized size = 0.92

$$b^5 \log(x) - \frac{300\,ab^4x^4 + 300\,a^2b^3x^3 + 200\,a^3b^2x^2 + 75\,a^4bx + 12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="maxima")
```

```
[Out] b^5*log(x) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2 + 75*a
^4*b*x + 12*a^5)/x^5
```

**mupad [B]** time = 0.04, size = 56, normalized size = 0.92

$$b^5\,\ln(x) - \frac{\frac{a^5}{5} + \frac{5\,a^4\,b\,x}{4} + \frac{10\,a^3\,b^2\,x^2}{3} + 5\,a^2\,b^3\,x^3 + 5\,a\,b^4\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^6,x)
```

```
[Out] b^5*log(x) - (a^5/5 + 5*a*b^4*x^4 + (10*a^3*b^2*x^2)/3 + 5*a^2*b^3*x^3 + (5
*a^4*b*x)/4)/x^5
```

**sympy [A]** time = 0.36, size = 60, normalized size = 0.98

$$b^5 \log{(x)} + \frac{-12a^5 - 75a^4bx - 200a^3b^2x^2 - 300a^2b^3x^3 - 300ab^4x^4}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**6,x)
```

```
[Out] b**5*log(x) + (-12*a**5 - 75*a**4*b*x - 200*a**3*b**2*x**2 - 300*a**2*b**3*
x**3 - 300*a*b**4*x**4)/(60*x**5)
```

## 3.90 $\int \frac{(a+bx)^5}{x^7} dx$

**Optimal**. Leaf size=17

$$-\frac{(a+bx)^6}{6ax^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.091, Rules used = {37}

$$-\frac{(a+bx)^6}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^7,x]
```

```
[Out] -(a + b*x)^6/(6*a*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^5}{x^7} dx = -\frac{(a+bx)^6}{6ax^6}$$

**Mathematica [B]** time = 0.00, size = 65, normalized size = 3.82

$$-\frac{a^5}{6x^6} - \frac{a^4b}{x^5} - \frac{5a^3b^2}{2x^4} - \frac{10a^2b^3}{3x^3} - \frac{5ab^4}{2x^2} - \frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^7,x]
```

```
[Out] -1/6*a^5/x^6 - (a^4*b)/x^5 - (5*a^3*b^2)/(2*x^4) - (10*a^2*b^3)/(3*x^3) - (
5*a*b^4)/(2*x^2) - b^5/x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^7, x]
```

**fricas [B]** time = 1.14, size = 55, normalized size = 3.24

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

**giac [B]** time = 0.94, size = 55, normalized size = 3.24

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

**maple [B]** time = 0.00, size = 58, normalized size = 3.41

$$-\frac{b^5}{x}-\frac{5a\,b^4}{2x^2}-\frac{10a^2b^3}{3x^3}-\frac{5a^3b^2}{2x^4}-\frac{a^4b}{x^5}-\frac{a^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^7,x)
```

```
[Out] -1/6*a^5/x^6-a^4*b/x^5-5/2*a^3*b^2/x^4-b^5/x-10/3*a^2*b^3/x^3-5/2*a*b^4/x^2
```

**maxima [B]** time = 1.30, size = 55, normalized size = 3.24

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

**mupad [B]** time = 0.04, size = 55, normalized size = 3.24

$$-\frac{\frac{a^5}{6}+a^4\,b\,x+\frac{5\,a^3\,b^2\,x^2}{2}+\frac{10\,a^2\,b^3\,x^3}{3}+\frac{5\,a\,b^4\,x^4}{2}+b^5\,x^5}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^7,x)
```

```
[Out] -(a^5/6 + b^5*x^5 + (5*a*b^4*x^4)/2 + (5*a^3*b^2*x^2)/2 + (10*a^2*b^3*x^3)/
3 + a^4*b*x)/x^6
```

**sympy [B]** time = 0.37, size = 60, normalized size = 3.53

$$\frac{-a^5-6a^4bx-15a^3b^2x^2-20a^2b^3x^3-15ab^4x^4-6b^5x^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**7,x)
```

```
[Out] (-a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*x**
4 - 6*b**5*x**5)/(6*x**6)
```

## 3.91 $\int \frac{(a+bx)^5}{x^8} dx$

**Optimal**. Leaf size=36

$$\frac{b(a+bx)^6}{42a^2x^6} - \frac{(a+bx)^6}{7ax^7}$$

**Rubi [A]** time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^6}{42a^2x^6} - \frac{(a+bx)^6}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^8,x]
```

```
[Out] -(a + b*x)^6/(7*a*x^7) + (b*(a + b*x)^6)/(42*a^2*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^8} dx &= -\frac{(a+bx)^6}{7ax^7} - \frac{b\int \frac{(a+bx)^5}{x^7} dx}{7a} \\ &= -\frac{(a+bx)^6}{7ax^7} + \frac{b(a+bx)^6}{42a^2x^6}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 67, normalized size = 1.86

$$-\frac{a^5}{7x^7}-\frac{5a^4b}{6x^6}-\frac{2a^3b^2}{x^5}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{3x^3}-\frac{b^5}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^8,x]
```

```
[Out] -1/7*a^5/x^7 - (5*a^4*b)/(6*x^6) - (2*a^3*b^2)/x^5 - (5*a^2*b^3)/(2*x^4) -
(5*a*b^4)/(3*x^3) - b^5/(2*x^2)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx)^5}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^8, x]
```

**fricas** [A] time = 1.37, size = 57, normalized size = 1.58

$$-\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

**giac** [A] time = 1.12, size = 57, normalized size = 1.58

$$-\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="giac")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

**maple** [A] time = 0.01, size = 58, normalized size = 1.61

$$-\frac{b^5}{2x^2}-\frac{5a\,b^4}{3x^3}-\frac{5a^2b^3}{2x^4}-\frac{2a^3b^2}{x^5}-\frac{5a^4b}{6x^6}-\frac{a^5}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^8,x)
```

```
[Out] -2*a^3*b^2/x^5-1/7*a^5/x^7-5/2*a^2*b^3/x^4-5/6*a^4*b/x^6-5/3*a*b^4/x^3-1/2*
b^5/x^2
```

**maxima** [A] time = 1.36, size = 57, normalized size = 1.58

$$-\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="maxima")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

**mupad** [B] time = 0.07, size = 57, normalized size = 1.58

$$-\frac{\frac{a^5}{7}+\frac{5\,a^4\,b\,x}{6}+2\,a^3\,b^2\,x^2+\frac{5\,a^2\,b^3\,x^3}{2}+\frac{5\,a\,b^4\,x^4}{3}+\frac{b^5\,x^5}{2}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^8,x)
```

```
[Out] -(a^5/7 + (b^5*x^5)/2 + (5*a*b^4*x^4)/3 + 2*a^3*b^2*x^2 + (5*a^2*b^3*x^3)/2
 + (5*a^4*b*x)/6)/x^7
```

**sympy** [B] time = 0.41, size = 61, normalized size = 1.69

$$\frac{-6a^5-35a^4bx-84a^3b^2x^2-105a^2b^3x^3-70ab^4x^4-21b^5x^5}{42x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**8,x)
```

```
[Out] (-6*a**5 - 35*a**4*b*x - 84*a**3*b**2*x**2 - 105*a**2*b**3*x**3 - 70*a*b**4
*x**4 - 21*b**5*x**5)/(42*x**7)
```

## 3.92 $\int \frac{(a+bx)^5}{x^9} dx$

**Optimal**. Leaf size=56

$$-\frac{b^2(a+bx)^6}{168a^3x^6}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{(a+bx)^6}{8ax^8}$$

**Rubi [A]** time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^6}{168a^3x^6}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{(a+bx)^6}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^9,x]
```

```
[Out] -(a + b*x)^6/(8*a*x^8) + (b*(a + b*x)^6)/(28*a^2*x^7) - (b^2*(a + b*x)^6)/(
168*a^3*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^5}{x^9}\,dx = -\frac{(a+bx)^6}{8ax^8} - \frac{b\int \frac{(a+bx)^5}{x^8}\,dx}{4a}$$
$$= -\frac{(a+bx)^6}{8ax^8} + \frac{b(a+bx)^6}{28a^2x^7} + \frac{b^2\int \frac{(a+bx)^5}{x^7}\,dx}{28a^2}$$
$$= -\frac{(a+bx)^6}{8ax^8} + \frac{b(a+bx)^6}{28a^2x^7} - \frac{b^2(a+bx)^6}{168a^3x^6}$$

**Mathematica** [A] time = 0.00, size = 67, normalized size = 1.20

$$-\frac{a^5}{8x^8} - \frac{5a^4b}{7x^7} - \frac{5a^3b^2}{3x^6} - \frac{2a^2b^3}{x^5} - \frac{5ab^4}{4x^4} - \frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^9,x]
```

```
[Out] -1/8*a^5/x^8 - (5*a^4*b)/(7*x^7) - (5*a^3*b^2)/(3*x^6) - (2*a^2*b^3)/x^5 -
(5*a*b^4)/(4*x^4) - b^5/(3*x^3)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^9,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^9, x]
```

**fricas** [A] time = 1.77, size = 57, normalized size = 1.02

$$-\frac{56\,b^5x^5 + 210\,ab^4x^4 + 336\,a^2b^3x^3 + 280\,a^3b^2x^2 + 120\,a^4bx + 21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="fricas")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

**giac** [A] time = 1.10, size = 57, normalized size = 1.02

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="giac")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

**maple** [A] time = 0.01, size = 58, normalized size = 1.04

$$-\frac{b^5}{3x^3}-\frac{5a\,b^4}{4x^4}-\frac{2a^2b^3}{x^5}-\frac{5a^3b^2}{3x^6}-\frac{5a^4b}{7x^7}-\frac{a^5}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^9,x)
```

```
[Out] -5/7*a^4*b/x^7-1/8*a^5/x^8-5/4*a*b^4/x^4-5/3*a^3*b^2/x^6-1/3*b^5/x^3-2*a^2*
b^3/x^5
```

**maxima** [A] time = 1.40, size = 57, normalized size = 1.02

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="maxima")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

**mupad** [B] time = 0.04, size = 57, normalized size = 1.02

$$-\frac{\frac{a^5}{8}+\frac{5\,a^4\,b\,x}{7}+\frac{5\,a^3\,b^2\,x^2}{3}+2\,a^2\,b^3\,x^3+\frac{5\,a\,b^4\,x^4}{4}+\frac{b^5\,x^5}{3}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^9,x)
```

```
[Out] -(a^5/8 + (b^5*x^5)/3 + (5*a*b^4*x^4)/4 + (5*a^3*b^2*x^2)/3 + 2*a^2*b^3*x^3
 + (5*a^4*b*x)/7)/x^8
```

**sympy** [A] time = 0.43, size = 61, normalized size = 1.09

$$\frac{-21a^5 - 120a^4bx - 280a^3b^2x^2 - 336a^2b^3x^3 - 210ab^4x^4 - 56b^5x^5}{168x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**9,x)
```

```
[Out] (-21*a**5 - 120*a**4*b*x - 280*a**3*b**2*x**2 - 336*a**2*b**3*x**3 - 210*a*
b**4*x**4 - 56*b**5*x**5)/(168*x**8)
```

## 3.93 $\int \frac{(a+bx)^5}{x^{10}} \, dx$

**Optimal**. Leaf size=67

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

**Rubi [A]** time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{10}} \, dx &= \int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right) dx\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 67, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^10, x]
```

**fricas [A]** time = 1.10, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

**giac [A]** time = 1.21, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

**maple [A]** time = 0.00, size = 58, normalized size = 0.87

$$-\frac{b^5}{4x^4}-\frac{a\,b^4}{x^5}-\frac{5a^2b^3}{3x^6}-\frac{10a^3b^2}{7x^7}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^10,x)
```

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

**maxima [A]** time = 1.33, size = 57, normalized size = 0.85

$$-\frac{126 b^5 x^5 + 504 a b^4 x^4 + 840 a^2 b^3 x^3 + 720 a^3 b^2 x^2 + 315 a^4 b x + 56 a^5}{504 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

**mupad [B]** time = 0.08, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{9} + \frac{5 a^4 b x}{8} + \frac{10 a^3 b^2 x^2}{7} + \frac{5 a^2 b^3 x^3}{3} + a b^4 x^4 + \frac{b^5 x^5}{4}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^10,x)
```

```
[Out] -(a^5/9 + (b^5*x^5)/4 + a*b^4*x^4 + (10*a^3*b^2*x^2)/7 + (5*a^2*b^3*x^3)/3
+ (5*a^4*b*x)/8)/x^9
```

**sympy [A]** time = 0.45, size = 61, normalized size = 0.91

$$\frac{-56a^5 - 315a^4bx - 720a^3b^2x^2 - 840a^2b^3x^3 - 504ab^4x^4 - 126b^5x^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**10,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504*a*
b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

## 3.94 $\int \frac{(a+bx)^5}{x^{11}} \, dx$

**Optimal**. Leaf size=69

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

**Rubi** [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5a^4b}{9x^9}-\frac{a^5}{10x^{10}}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^11,x]
```

```
[Out] -a^5/(10*x^10) - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^5}{x^{11}} \, dx &= \int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right) dx\\
&= -\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^11,x]
```

```
[Out] -1/10*a^5/x^10 - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^11,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^11, x]
```

**fricas [A]** time = 0.98, size = 57, normalized size = 0.83

$$-\frac{252\,b^5x^5+1050\,ab^4x^4+1800\,a^2b^3x^3+1575\,a^3b^2x^2+700\,a^4bx+126\,a^5}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

**giac [A]** time = 1.00, size = 57, normalized size = 0.83

$$-\frac{252\,b^5x^5+1050\,ab^4x^4+1800\,a^2b^3x^3+1575\,a^3b^2x^2+700\,a^4bx+126\,a^5}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="giac")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

**maple [A]** time = 0.01, size = 58, normalized size = 0.84

$$-\frac{b^5}{5x^5}-\frac{5a\,b^4}{6x^6}-\frac{10a^2b^3}{7x^7}-\frac{5a^3b^2}{4x^8}-\frac{5a^4b}{9x^9}-\frac{a^5}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^11,x)
```

```
[Out] -1/10*a^5/x^10-5/9*a^4*b/x^9-5/4*a^3*b^2/x^8-10/7*a^2*b^3/x^7-5/6*a*b^4/x^6
-1/5*b^5/x^5
```

**maxima [A]** time = 1.37, size = 57, normalized size = 0.83

$$-\frac{252\,b^5x^5+1050\,ab^4x^4+1800\,a^2b^3x^3+1575\,a^3b^2x^2+700\,a^4bx+126\,a^5}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="maxima")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

**mupad [B]** time = 0.08, size = 57, normalized size = 0.83

$$-\frac{\frac{a^5}{10}+\frac{5\,a^4\,b\,x}{9}+\frac{5\,a^3\,b^2\,x^2}{4}+\frac{10\,a^2\,b^3\,x^3}{7}+\frac{5\,a\,b^4\,x^4}{6}+\frac{b^5\,x^5}{5}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^11,x)
```

```
[Out] -(a^5/10 + (b^5*x^5)/5 + (5*a*b^4*x^4)/6 + (5*a^3*b^2*x^2)/4 + (10*a^2*b^3*
x^3)/7 + (5*a^4*b*x)/9)/x^10
```

**sympy [A]** time = 0.49, size = 61, normalized size = 0.88

$$\frac{-126a^5-700a^4bx-1575a^3b^2x^2-1800a^2b^3x^3-1050ab^4x^4-252b^5x^5}{1260x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**11,x)
```

```
[Out] (-126*a**5 - 700*a**4*b*x - 1575*a**3*b**2*x**2 - 1800*a**2*b**3*x**3 - 105
0*a*b**4*x**4 - 252*b**5*x**5)/(1260*x**10)
```

## 3.95 $\int \frac{(a+bx)^5}{x^{12}} \, dx$

**Optimal**. Leaf size=69

$$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

**Rubi [A]** time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{a^4b}{2x^{10}}-\frac{a^5}{11x^{11}}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^12,x]
```

```
[Out] -a^5/(11*x^11) - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*x
^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{12}} \, dx &= \int \left(\frac{a^5}{x^{12}}+\frac{5a^4b}{x^{11}}+\frac{10a^3b^2}{x^{10}}+\frac{10a^2b^3}{x^9}+\frac{5ab^4}{x^8}+\frac{b^5}{x^7}\right) dx\\ &= -\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^12,x]
```

```
[Out] -1/11*a^5/x^11 - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*x
^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^12,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^12, x]
```

**fricas [A]** time = 0.83, size = 57, normalized size = 0.83

$$-\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="fricas")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

**giac [A]** time = 1.38, size = 57, normalized size = 0.83

$$-\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="giac")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

**maple [A]** time = 0.01, size = 58, normalized size = 0.84

$$-\frac{b^5}{6x^6}-\frac{5a\,b^4}{7x^7}-\frac{5a^2b^3}{4x^8}-\frac{10a^3b^2}{9x^9}-\frac{a^4b}{2x^{10}}-\frac{a^5}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^12,x)
```

```
[Out] -1/11*a^5/x^11-1/2*a^4*b/x^10-10/9*a^3*b^2/x^9-5/4*a^2*b^3/x^8-5/7*a*b^4/x^
7-1/6*b^5/x^6
```

**maxima** [A] time = 1.31, size = 57, normalized size = 0.83

$$-\frac{462 b^5 x^5 + 1980 a b^4 x^4 + 3465 a^2 b^3 x^3 + 3080 a^3 b^2 x^2 + 1386 a^4 b x + 252 a^5}{2772 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="maxima")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

**mupad** [B] time = 0.04, size = 57, normalized size = 0.83

$$-\frac{\frac{a^5}{11} + \frac{a^4 b x}{2} + \frac{10 a^3 b^2 x^2}{9} + \frac{5 a^2 b^3 x^3}{4} + \frac{5 a b^4 x^4}{7} + \frac{b^5 x^5}{6}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^12,x)
```

```
[Out] -(a^5/11 + (b^5*x^5)/6 + (5*a*b^4*x^4)/7 + (10*a^3*b^2*x^2)/9 + (5*a^2*b^3*
x^3)/4 + (a^4*b*x)/2)/x^11
```

**sympy** [A] time = 0.58, size = 61, normalized size = 0.88

$$\frac{-252a^5 - 1386a^4bx - 3080a^3b^2x^2 - 3465a^2b^3x^3 - 1980ab^4x^4 - 462b^5x^5}{2772x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**12,x)
```

```
[Out] (-252*a**5 - 1386*a**4*b*x - 3080*a**3*b**2*x**2 - 3465*a**2*b**3*x**3 - 19
80*a*b**4*x**4 - 462*b**5*x**5)/(2772*x**11)
```

## 3.96 $\int \frac{(a+bx)^5}{x^{13}} \, dx$

**Optimal**. Leaf size=67

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

**Rubi** [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{12x^{12}}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^13,x]
```

```
[Out] -a^5/(12*x^12) - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^9
) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{13}} \, dx &= \int \left(\frac{a^5}{x^{13}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{11}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^9}+\frac{b^5}{x^8}\right) dx\\ &= -\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^13,x]
```

```
[Out] -1/12*a^5/x^12 - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^9
) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^13,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^13, x]
```

**fricas [A]** time = 0.73, size = 57, normalized size = 0.85

$$-\frac{792\,b^5x^5+3465\,ab^4x^4+6160\,a^2b^3x^3+5544\,a^3b^2x^2+2520\,a^4bx+462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="fricas")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

**giac [A]** time = 1.23, size = 57, normalized size = 0.85

$$-\frac{792\,b^5x^5+3465\,ab^4x^4+6160\,a^2b^3x^3+5544\,a^3b^2x^2+2520\,a^4bx+462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="giac")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

**maple [A]** time = 0.01, size = 58, normalized size = 0.87

$$-\frac{b^5}{7x^7}-\frac{5a\,b^4}{8x^8}-\frac{10a^2b^3}{9x^9}-\frac{a^3b^2}{x^{10}}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^13,x)
```

```
[Out] -1/12*a^5/x^12-5/11*a^4*b/x^11-a^3*b^2/x^10-10/9*a^2*b^3/x^9-5/8*a*b^4/x^8-
1/7*b^5/x^7
```

**maxima [A]** time = 1.36, size = 57, normalized size = 0.85

$$-\frac{792\,b^5x^5 + 3465\,ab^4x^4 + 6160\,a^2b^3x^3 + 5544\,a^3b^2x^2 + 2520\,a^4bx + 462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="maxima")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

**mupad [B]** time = 0.04, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{12} + \frac{5\,a^4\,b\,x}{11} + a^3\,b^2\,x^2 + \frac{10\,a^2\,b^3\,x^3}{9} + \frac{5\,a\,b^4\,x^4}{8} + \frac{b^5\,x^5}{7}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^13,x)
```

```
[Out] -(a^5/12 + (b^5*x^5)/7 + (5*a*b^4*x^4)/8 + a^3*b^2*x^2 + (10*a^2*b^3*x^3)/9
 + (5*a^4*b*x)/11)/x^12
```

**sympy [A]** time = 0.62, size = 61, normalized size = 0.91

$$\frac{-462a^5 - 2520a^4bx - 5544a^3b^2x^2 - 6160a^2b^3x^3 - 3465ab^4x^4 - 792b^5x^5}{5544x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**13,x)
```

```
[Out] (-462*a**5 - 2520*a**4*b*x - 5544*a**3*b**2*x**2 - 6160*a**2*b**3*x**3 - 34
65*a*b**4*x**4 - 792*b**5*x**5)/(5544*x**12)
```

## 3.97 $\int \frac{(a+bx)^5}{x^{14}} dx$

**Optimal**. Leaf size=67

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

**Rubi** [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5a^4b}{12x^{12}}-\frac{a^5}{13x^{13}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^14,x]
```

```
[Out] -a^5/(13*x^13) - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/x
^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{14}}\,dx &= \int \left(\frac{a^5}{x^{14}}+\frac{5a^4b}{x^{13}}+\frac{10a^3b^2}{x^{12}}+\frac{10a^2b^3}{x^{11}}+\frac{5ab^4}{x^{10}}+\frac{b^5}{x^9}\right)dx\\ &= -\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^14,x]
```

```
[Out] -1/13*a^5/x^13 - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/x
^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^14,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^14, x]
```

**fricas [A]** time = 1.19, size = 57, normalized size = 0.85

$$-\frac{1287\,b^5x^5+5720\,ab^4x^4+10296\,a^2b^3x^3+9360\,a^3b^2x^2+4290\,a^4bx+792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

**giac [A]** time = 1.12, size = 57, normalized size = 0.85

$$-\frac{1287\,b^5x^5+5720\,ab^4x^4+10296\,a^2b^3x^3+9360\,a^3b^2x^2+4290\,a^4bx+792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

**maple [A]** time = 0.00, size = 58, normalized size = 0.87

$$-\frac{b^5}{8x^8}-\frac{5a\,b^4}{9x^9}-\frac{a^2b^3}{x^{10}}-\frac{10a^3b^2}{11x^{11}}-\frac{5a^4b}{12x^{12}}-\frac{a^5}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^14,x)
```

```
[Out] -1/13*a^5/x^13-5/12*a^4*b/x^12-10/11*a^3*b^2/x^11-a^2*b^3/x^10-5/9*a*b^4/x^
9-1/8*b^5/x^8
```

**maxima [A]** time = 1.38, size = 57, normalized size = 0.85

$$-\frac{1287\,b^5x^5 + 5720\,ab^4x^4 + 10296\,a^2b^3x^3 + 9360\,a^3b^2x^2 + 4290\,a^4bx + 792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

**mupad [B]** time = 0.04, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{13} + \frac{5\,a^4\,b\,x}{12} + \frac{10\,a^3\,b^2\,x^2}{11} + a^2\,b^3\,x^3 + \frac{5\,a\,b^4\,x^4}{9} + \frac{b^5\,x^5}{8}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^14,x)
```

```
[Out] -(a^5/13 + (b^5*x^5)/8 + (5*a*b^4*x^4)/9 + (10*a^3*b^2*x^2)/11 + a^2*b^3*x^
3 + (5*a^4*b*x)/12)/x^13
```

**sympy [A]** time = 0.62, size = 61, normalized size = 0.91

$$\frac{-792a^5 - 4290a^4bx - 9360a^3b^2x^2 - 10296a^2b^3x^3 - 5720ab^4x^4 - 1287b^5x^5}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**14,x)
```

```
[Out] (-792*a**5 - 4290*a**4*b*x - 9360*a**3*b**2*x**2 - 10296*a**2*b**3*x**3 - 5
720*a*b**4*x**4 - 1287*b**5*x**5)/(10296*x**13)
```

## 3.98 $\int x^8(a+bx)^7\,dx$

**Optimal**. Leaf size=95

$$\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

**Rubi [A]** time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{2}a^2b^5x^{14}+\frac{35}{13}a^3b^4x^{13}+\frac{35}{12}a^4b^3x^{12}+\frac{21}{11}a^5b^2x^{11}+\frac{7}{10}a^6bx^{10}+\frac{a^7x^9}{9}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x)^7,x]
```

```
[Out] (a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/
12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7*x
^16)/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^8(a+bx)^7\,dx&=\int\left(a^7x^8+7a^6bx^9+21a^5b^2x^{10}+35a^4b^3x^{11}+35a^3b^4x^{12}+21a^2b^5x^{13}+7ab^6x^{14}+b^7x^{15}\right)dx\\&=\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x)^7,x]
```

```
[Out] (a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/
12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7*x
^16)/16
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^8(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^8*(a + b*x)^7, x]
```

**fricas [A]** time = 1.29, size = 79, normalized size = 0.83

$$\frac{1}{16}x^{16}b^7+\frac{7}{15}x^{15}b^6a+\frac{3}{2}x^{14}b^5a^2+\frac{35}{13}x^{13}b^4a^3+\frac{35}{12}x^{12}b^3a^4+\frac{21}{11}x^{11}b^2a^5+\frac{7}{10}x^{10}ba^6+\frac{1}{9}x^9a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^7 + 7/15*x^15*b^6*a + 3/2*x^14*b^5*a^2 + 35/13*x^13*b^4*a^3 + 3
5/12*x^12*b^3*a^4 + 21/11*x^11*b^2*a^5 + 7/10*x^10*b*a^6 + 1/9*x^9*a^7
```

**giac [A]** time = 0.95, size = 79, normalized size = 0.83

$$\frac{1}{16}\,b^7x^{16}+\frac{7}{15}\,ab^6x^{15}+\frac{3}{2}\,a^2b^5x^{14}+\frac{35}{13}\,a^3b^4x^{13}+\frac{35}{12}\,a^4b^3x^{12}+\frac{21}{11}\,a^5b^2x^{11}+\frac{7}{10}\,a^6bx^{10}+\frac{1}{9}\,a^7x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

**maple [A]** time = 0.00, size = 80, normalized size = 0.84

$$\frac{1}{16}b^7x^{16}+\frac{7}{15}a\,b^6x^{15}+\frac{3}{2}a^2b^5x^{14}+\frac{35}{13}a^3b^4x^{13}+\frac{35}{12}a^4b^3x^{12}+\frac{21}{11}a^5b^2x^{11}+\frac{7}{10}a^6b\,x^{10}+\frac{1}{9}a^7x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^7,x)
```

```
[Out] 1/9*a^7*x^9+7/10*a^6*b*x^10+21/11*a^5*b^2*x^11+35/12*a^4*b^3*x^12+35/13*a^3
*b^4*x^13+3/2*a^2*b^5*x^14+7/15*a*b^6*x^15+1/16*b^7*x^16
```

**maxima [A]** time = 1.40, size = 79, normalized size = 0.83

$$\frac{1}{16}\,b^7x^{16}+\frac{7}{15}\,ab^6x^{15}+\frac{3}{2}\,a^2b^5x^{14}+\frac{35}{13}\,a^3b^4x^{13}+\frac{35}{12}\,a^4b^3x^{12}+\frac{21}{11}\,a^5b^2x^{11}+\frac{7}{10}\,a^6bx^{10}+\frac{1}{9}\,a^7x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

**mupad [B]** time = 0.15, size = 79, normalized size = 0.83

$$\frac{a^7\,x^9}{9}+\frac{7\,a^6\,b\,x^{10}}{10}+\frac{21\,a^5\,b^2\,x^{11}}{11}+\frac{35\,a^4\,b^3\,x^{12}}{12}+\frac{35\,a^3\,b^4\,x^{13}}{13}+\frac{3\,a^2\,b^5\,x^{14}}{2}+\frac{7\,a\,b^6\,x^{15}}{15}+\frac{b^7\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x)^7,x)
```

```
[Out] (a^7*x^9)/9 + (b^7*x^16)/16 + (7*a^6*b*x^10)/10 + (7*a*b^6*x^15)/15 + (21*a
^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*
x^14)/2
```

**sympy [A]** time = 0.10, size = 94, normalized size = 0.99

$$\frac{a^7x^9}{9}+\frac{7a^6bx^{10}}{10}+\frac{21a^5b^2x^{11}}{11}+\frac{35a^4b^3x^{12}}{12}+\frac{35a^3b^4x^{13}}{13}+\frac{3a^2b^5x^{14}}{2}+\frac{7ab^6x^{15}}{15}+\frac{b^7x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**7,x)
```

```
[Out] a**7*x**9/9 + 7*a**6*b*x**10/10 + 21*a**5*b**2*x**11/11 + 35*a**4*b**3*x**1
2/12 + 35*a**3*b**4*x**13/13 + 3*a**2*b**5*x**14/2 + 7*a*b**6*x**15/15 + b*
*7*x**16/16
```

## 3.99 $\int x^7(a+bx)^7\,dx$

**Optimal**. Leaf size=95

$$\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

**Rubi** [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21}{13}a^2b^5x^{13}+\frac{35}{12}a^3b^4x^{12}+\frac{35}{11}a^4b^3x^{11}+\frac{21}{10}a^5b^2x^{10}+\frac{7}{9}a^6bx^9+\frac{a^7x^8}{8}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^7,x]
```

```
[Out] (a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/11
 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^15
)/15
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx)^7\,dx &= \int \left(a^7x^7+7a^6bx^8+21a^5b^2x^9+35a^4b^3x^{10}+35a^3b^4x^{11}+21a^2b^5x^{12}+7ab^6x^{13}+b^7x^{14}\right)dx\\ &= \frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^7,x]
```

```
[Out] (a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/11
 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^15
)/15
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^7(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^7*(a + b*x)^7, x]
```

**fricas [A]** time = 1.34, size = 79, normalized size = 0.83

$$\frac{1}{15}x^{15}b^7+\frac{1}{2}x^{14}b^6a+\frac{21}{13}x^{13}b^5a^2+\frac{35}{12}x^{12}b^4a^3+\frac{35}{11}x^{11}b^3a^4+\frac{21}{10}x^{10}b^2a^5+\frac{7}{9}x^9ba^6+\frac{1}{8}x^8a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^7 + 1/2*x^14*b^6*a + 21/13*x^13*b^5*a^2 + 35/12*x^12*b^4*a^3 +
35/11*x^11*b^3*a^4 + 21/10*x^10*b^2*a^5 + 7/9*x^9*b*a^6 + 1/8*x^8*a^7
```

**giac [A]** time = 0.99, size = 79, normalized size = 0.83

$$\frac{1}{15}\,b^7x^{15}+\frac{1}{2}\,ab^6x^{14}+\frac{21}{13}\,a^2b^5x^{13}+\frac{35}{12}\,a^3b^4x^{12}+\frac{35}{11}\,a^4b^3x^{11}+\frac{21}{10}\,a^5b^2x^{10}+\frac{7}{9}\,a^6bx^9+\frac{1}{8}\,a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

**maple [A]** time = 0.00, size = 80, normalized size = 0.84

$$\frac{1}{15}b^7x^{15}+\frac{1}{2}a\,b^6x^{14}+\frac{21}{13}a^2b^5x^{13}+\frac{35}{12}a^3b^4x^{12}+\frac{35}{11}a^4b^3x^{11}+\frac{21}{10}a^5b^2x^{10}+\frac{7}{9}a^6b\,x^9+\frac{1}{8}a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^7,x)
```

```
[Out] 1/8*a^7*x^8+7/9*a^6*b*x^9+21/10*a^5*b^2*x^10+35/11*a^4*b^3*x^11+35/12*a^3*b
^4*x^12+21/13*a^2*b^5*x^13+1/2*a*b^6*x^14+1/15*b^7*x^15
```

**maxima** [A] time = 1.30, size = 79, normalized size = 0.83

$$\frac{1}{15}\,b^{7}x^{15}+\frac{1}{2}\,ab^{6}x^{14}+\frac{21}{13}\,a^{2}b^{5}x^{13}+\frac{35}{12}\,a^{3}b^{4}x^{12}+\frac{35}{11}\,a^{4}b^{3}x^{11}+\frac{21}{10}\,a^{5}b^{2}x^{10}+\frac{7}{9}\,a^{6}bx^{9}+\frac{1}{8}\,a^{7}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

**mupad** [B] time = 0.07, size = 79, normalized size = 0.83

$$\frac{a^7\,x^8}{8}+\frac{7\,a^6\,b\,x^9}{9}+\frac{21\,a^5\,b^2\,x^{10}}{10}+\frac{35\,a^4\,b^3\,x^{11}}{11}+\frac{35\,a^3\,b^4\,x^{12}}{12}+\frac{21\,a^2\,b^5\,x^{13}}{13}+\frac{a\,b^6\,x^{14}}{2}+\frac{b^7\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x)^7,x)
```

```
[Out] (a^7*x^8)/8 + (b^7*x^15)/15 + (7*a^6*b*x^9)/9 + (a*b^6*x^14)/2 + (21*a^5*b^
2*x^10)/10 + (35*a^4*b^3*x^11)/11 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13
)/13
```

**sympy** [A] time = 0.09, size = 92, normalized size = 0.97

$$\frac{a^{7}x^{8}}{8}+\frac{7a^{6}bx^{9}}{9}+\frac{21a^{5}b^{2}x^{10}}{10}+\frac{35a^{4}b^{3}x^{11}}{11}+\frac{35a^{3}b^{4}x^{12}}{12}+\frac{21a^{2}b^{5}x^{13}}{13}+\frac{ab^{6}x^{14}}{2}+\frac{b^{7}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**7,x)
```

```
[Out] a**7*x**8/8 + 7*a**6*b*x**9/9 + 21*a**5*b**2*x**10/10 + 35*a**4*b**3*x**11/
11 + 35*a**3*b**4*x**12/12 + 21*a**2*b**5*x**13/13 + a*b**6*x**14/2 + b**7*
x**15/15
```

## 3.100 $\int x^6(a+bx)^7\,dx$

**Optimal**. Leaf size=95

$$\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

**Rubi** [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{7}{4}a^2b^5x^{12}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{2}a^4b^3x^{10}+\frac{7}{3}a^5b^2x^9+\frac{7}{8}a^6bx^8+\frac{a^7x^7}{7}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^7,x]
```

```
[Out] (a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (3
5*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^7\,dx&=\int\left(a^7x^6+7a^6bx^7+21a^5b^2x^8+35a^4b^3x^9+35a^3b^4x^{10}+21a^2b^5x^{11}+7ab^6x^{12}+b^7x^{13}\right)dx\\&=\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^7,x]
```

```
[Out] (a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (3
5*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/14
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^6(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^6*(a + b*x)^7, x]
```

**fricas [A]** time = 1.32, size = 79, normalized size = 0.83

$$\frac{1}{14}x^{14}b^7+\frac{7}{13}x^{13}b^6a+\frac{7}{4}x^{12}b^5a^2+\frac{35}{11}x^{11}b^4a^3+\frac{7}{2}x^{10}b^3a^4+\frac{7}{3}x^9b^2a^5+\frac{7}{8}x^8ba^6+\frac{1}{7}x^7a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^7 + 7/13*x^13*b^6*a + 7/4*x^12*b^5*a^2 + 35/11*x^11*b^4*a^3 + 7
/2*x^10*b^3*a^4 + 7/3*x^9*b^2*a^5 + 7/8*x^8*b*a^6 + 1/7*x^7*a^7
```

**giac [A]** time = 1.01, size = 79, normalized size = 0.83

$$\frac{1}{14}\,b^7x^{14}+\frac{7}{13}\,ab^6x^{13}+\frac{7}{4}\,a^2b^5x^{12}+\frac{35}{11}\,a^3b^4x^{11}+\frac{7}{2}\,a^4b^3x^{10}+\frac{7}{3}\,a^5b^2x^9+\frac{7}{8}\,a^6bx^8+\frac{1}{7}\,a^7x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

**maple [A]** time = 0.00, size = 80, normalized size = 0.84

$$\frac{1}{14}b^7x^{14}+\frac{7}{13}a\,b^6x^{13}+\frac{7}{4}a^2b^5x^{12}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{2}a^4b^3x^{10}+\frac{7}{3}a^5b^2x^9+\frac{7}{8}a^6b\,x^8+\frac{1}{7}a^7x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^7,x)
```

```
[Out] 1/7*a^7*x^7+7/8*a^6*b*x^8+7/3*a^5*b^2*x^9+7/2*a^4*b^3*x^10+35/11*a^3*b^4*x^
11+7/4*a^2*b^5*x^12+7/13*a*b^6*x^13+1/14*b^7*x^14
```

**maxima [A]** time = 1.29, size = 79, normalized size = 0.83

$$\frac{1}{14}\,b^7x^{14}+\frac{7}{13}\,ab^6x^{13}+\frac{7}{4}\,a^2b^5x^{12}+\frac{35}{11}\,a^3b^4x^{11}+\frac{7}{2}\,a^4b^3x^{10}+\frac{7}{3}\,a^5b^2x^9+\frac{7}{8}\,a^6bx^8+\frac{1}{7}\,a^7x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

**mupad [B]** time = 0.07, size = 79, normalized size = 0.83

$$\frac{a^7\,x^7}{7}+\frac{7\,a^6\,b\,x^8}{8}+\frac{7\,a^5\,b^2\,x^9}{3}+\frac{7\,a^4\,b^3\,x^{10}}{2}+\frac{35\,a^3\,b^4\,x^{11}}{11}+\frac{7\,a^2\,b^5\,x^{12}}{4}+\frac{7\,a\,b^6\,x^{13}}{13}+\frac{b^7\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^7,x)
```

```
[Out] (a^7*x^7)/7 + (b^7*x^14)/14 + (7*a^6*b*x^8)/8 + (7*a*b^6*x^13)/13 + (7*a^5*
b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (35*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4
```

**sympy [A]** time = 0.11, size = 94, normalized size = 0.99

$$\frac{a^7x^7}{7}+\frac{7a^6bx^8}{8}+\frac{7a^5b^2x^9}{3}+\frac{7a^4b^3x^{10}}{2}+\frac{35a^3b^4x^{11}}{11}+\frac{7a^2b^5x^{12}}{4}+\frac{7ab^6x^{13}}{13}+\frac{b^7x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**7,x)
```

```
[Out] a**7*x**7/7 + 7*a**6*b*x**8/8 + 7*a**5*b**2*x**9/3 + 7*a**4*b**3*x**10/2 +
35*a**3*b**4*x**11/11 + 7*a**2*b**5*x**12/4 + 7*a*b**6*x**13/13 + b**7*x**1
4/14
```

## 3.101 $\int x^5(a+bx)^7\,dx$

**Optimal**. Leaf size=96

$$-\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}+\frac{(a+bx)^{13}}{13b^6}-\frac{5a(a+bx)^{12}}{12b^6}$$

**Rubi [A]** time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^5(a+bx)^8}{8b^6}+\frac{(a+bx)^{13}}{13b^6}-\frac{5a(a+bx)^{12}}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^7,x]
```

```
[Out] -(a^5*(a + b*x)^8)/(8*b^6) + (5*a^4*(a + b*x)^9)/(9*b^6) - (a^3*(a + b*x)^1
0)/b^6 + (10*a^2*(a + b*x)^11)/(11*b^6) - (5*a*(a + b*x)^12)/(12*b^6) + (a
+ b*x)^13/(13*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^7\,dx &= \int\left(-\frac{a^5(a+bx)^7}{b^5}+\frac{5a^4(a+bx)^8}{b^5}-\frac{10a^3(a+bx)^9}{b^5}+\frac{10a^2(a+bx)^{10}}{b^5}-\frac{5a(a+bx)^{11}}{b^5}+\frac{(a+bx)^{12}}{b^5}\right)dx\\ &= -\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{5a(a+bx)^{12}}{12b^6}+\frac{(a+bx)^{13}}{13b^6}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 92, normalized size = 0.96

$$\frac{a^7x^6}{6}+a^6bx^7+\frac{21}{8}a^5b^2x^8+\frac{35}{9}a^4b^3x^9+\frac{7}{2}a^3b^4x^{10}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{12}ab^6x^{12}+\frac{b^7x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^7,x]
```

```
[Out] (a^7*x^6)/6 + a^6*b*x^7 + (21*a^5*b^2*x^8)/8 + (35*a^4*b^3*x^9)/9 + (7*a^3*
b^4*x^10)/2 + (21*a^2*b^5*x^11)/11 + (7*a*b^6*x^12)/12 + (b^7*x^13)/13
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^5(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^5*(a + b*x)^7, x]
```

**fricas [A]** time = 1.34, size = 78, normalized size = 0.81

$$\frac{1}{13}x^{13}b^7 + \frac{7}{12}x^{12}b^6a + \frac{21}{11}x^{11}b^5a^2 + \frac{7}{2}x^{10}b^4a^3 + \frac{35}{9}x^9b^3a^4 + \frac{21}{8}x^8b^2a^5 + x^7ba^6 + \frac{1}{6}x^6a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^7 + 7/12*x^12*b^6*a + 21/11*x^11*b^5*a^2 + 7/2*x^10*b^4*a^3 + 3
5/9*x^9*b^3*a^4 + 21/8*x^8*b^2*a^5 + x^7*b*a^6 + 1/6*x^6*a^7
```

**giac [A]** time = 1.08, size = 78, normalized size = 0.81

$$\frac{1}{13}\,b^7x^{13} + \frac{7}{12}\,ab^6x^{12} + \frac{21}{11}\,a^2b^5x^{11} + \frac{7}{2}\,a^3b^4x^{10} + \frac{35}{9}\,a^4b^3x^9 + \frac{21}{8}\,a^5b^2x^8 + a^6bx^7 + \frac{1}{6}\,a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

**maple [A]** time = 0.00, size = 79, normalized size = 0.82

$$\frac{1}{13}b^7x^{13} + \frac{7}{12}a\,b^6x^{12} + \frac{21}{11}a^2b^5x^{11} + \frac{7}{2}a^3b^4x^{10} + \frac{35}{9}a^4b^3x^9 + \frac{21}{8}a^5b^2x^8 + a^6b\,x^7 + \frac{1}{6}a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^7,x)
```

```
[Out] 1/13*b^7*x^13+7/12*a*b^6*x^12+21/11*a^2*b^5*x^11+7/2*a^3*b^4*x^10+35/9*a^4*
b^3*x^9+21/8*a^5*b^2*x^8+a^6*b*x^7+1/6*a^7*x^6
```

**maxima [A]** time = 1.39, size = 78, normalized size = 0.81

$$\frac{1}{13}\,b^7x^{13}+\frac{7}{12}\,ab^6x^{12}+\frac{21}{11}\,a^2b^5x^{11}+\frac{7}{2}\,a^3b^4x^{10}+\frac{35}{9}\,a^4b^3x^9+\frac{21}{8}\,a^5b^2x^8+a^6bx^7+\frac{1}{6}\,a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

**mupad [B]** time = 0.06, size = 78, normalized size = 0.81

$$\frac{a^7\,x^6}{6}+a^6\,b\,x^7+\frac{21\,a^5\,b^2\,x^8}{8}+\frac{35\,a^4\,b^3\,x^9}{9}+\frac{7\,a^3\,b^4\,x^{10}}{2}+\frac{21\,a^2\,b^5\,x^{11}}{11}+\frac{7\,a\,b^6\,x^{12}}{12}+\frac{b^7\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^7,x)
```

```
[Out] (a^7*x^6)/6 + (b^7*x^13)/13 + a^6*b*x^7 + (7*a*b^6*x^12)/12 + (21*a^5*b^2*x
^8)/8 + (35*a^4*b^3*x^9)/9 + (7*a^3*b^4*x^10)/2 + (21*a^2*b^5*x^11)/11
```

**sympy [A]** time = 0.11, size = 90, normalized size = 0.94

$$\frac{a^7x^6}{6}+a^6bx^7+\frac{21a^5b^2x^8}{8}+\frac{35a^4b^3x^9}{9}+\frac{7a^3b^4x^{10}}{2}+\frac{21a^2b^5x^{11}}{11}+\frac{7ab^6x^{12}}{12}+\frac{b^7x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**7,x)
```

```
[Out] a**7*x**6/6 + a**6*b*x**7 + 21*a**5*b**2*x**8/8 + 35*a**4*b**3*x**9/9 + 7*a
**3*b**4*x**10/2 + 21*a**2*b**5*x**11/11 + 7*a*b**6*x**12/12 + b**7*x**13/1
3
```

## 3.102 $\int x^4(a+bx)^7\,dx$

**Optimal**. Leaf size=81

$$\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}+\frac{(a+bx)^{12}}{12b^5}-\frac{4a(a+bx)^{11}}{11b^5}$$

**Rubi** [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{a^4(a+bx)^8}{8b^5}+\frac{(a+bx)^{12}}{12b^5}-\frac{4a(a+bx)^{11}}{11b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^7,x]
```

```
[Out] (a^4*(a + b*x)^8)/(8*b^5) - (4*a^3*(a + b*x)^9)/(9*b^5) + (3*a^2*(a + b*x)^
10)/(5*b^5) - (4*a*(a + b*x)^11)/(11*b^5) + (a + b*x)^12/(12*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^7\,dx&=\int\left(\frac{a^4(a+bx)^7}{b^4}-\frac{4a^3(a+bx)^8}{b^4}+\frac{6a^2(a+bx)^9}{b^4}-\frac{4a(a+bx)^{10}}{b^4}+\frac{(a+bx)^{11}}{b^4}\right)dx\\&=\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{12}}{12b^5}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 93, normalized size = 1.15

$$\frac{a^7x^5}{5}+\frac{7}{6}a^6bx^6+3a^5b^2x^7+\frac{35}{8}a^4b^3x^8+\frac{35}{9}a^3b^4x^9+\frac{21}{10}a^2b^5x^{10}+\frac{7}{11}ab^6x^{11}+\frac{b^7x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^7,x]
```

```
[Out] (a^7*x^5)/5 + (7*a^6*b*x^6)/6 + 3*a^5*b^2*x^7 + (35*a^4*b^3*x^8)/8 + (35*a^
3*b^4*x^9)/9 + (21*a^2*b^5*x^10)/10 + (7*a*b^6*x^11)/11 + (b^7*x^12)/12
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x)^7, x]
```

**fricas [A]** time = 1.26, size = 79, normalized size = 0.98

$$\frac{1}{12}x^{12}b^7+\frac{7}{11}x^{11}b^6a+\frac{21}{10}x^{10}b^5a^2+\frac{35}{9}x^9b^4a^3+\frac{35}{8}x^8b^3a^4+3x^7b^2a^5+\frac{7}{6}x^6ba^6+\frac{1}{5}x^5a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^7 + 7/11*x^11*b^6*a + 21/10*x^10*b^5*a^2 + 35/9*x^9*b^4*a^3 + 3
5/8*x^8*b^3*a^4 + 3*x^7*b^2*a^5 + 7/6*x^6*b*a^6 + 1/5*x^5*a^7
```

**giac [A]** time = 1.13, size = 79, normalized size = 0.98

$$\frac{1}{12}\,b^7x^{12}+\frac{7}{11}\,ab^6x^{11}+\frac{21}{10}\,a^2b^5x^{10}+\frac{35}{9}\,a^3b^4x^9+\frac{35}{8}\,a^4b^3x^8+3\,a^5b^2x^7+\frac{7}{6}\,a^6bx^6+\frac{1}{5}\,a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

**maple [A]** time = 0.00, size = 80, normalized size = 0.99

$$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^7,x)
```

```
[Out] 1/12*b^7*x^12+7/11*a*b^6*x^11+21/10*a^2*b^5*x^10+35/9*a^3*b^4*x^9+35/8*a^4*
b^3*x^8+3*a^5*b^2*x^7+7/6*a^6*b*x^6+1/5*a^7*x^5
```

**maxima** [A] time = 1.35, size = 79, normalized size = 0.98

$$\frac{1}{12}\,b^{7}x^{12}+\frac{7}{11}\,ab^{6}x^{11}+\frac{21}{10}\,a^{2}b^{5}x^{10}+\frac{35}{9}\,a^{3}b^{4}x^{9}+\frac{35}{8}\,a^{4}b^{3}x^{8}+3\,a^{5}b^{2}x^{7}+\frac{7}{6}\,a^{6}bx^{6}+\frac{1}{5}\,a^{7}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

**mupad** [B] time = 0.06, size = 79, normalized size = 0.98

$$\frac{a^7\,x^5}{5}+\frac{7\,a^6\,b\,x^6}{6}+3\,a^5\,b^2\,x^7+\frac{35\,a^4\,b^3\,x^8}{8}+\frac{35\,a^3\,b^4\,x^9}{9}+\frac{21\,a^2\,b^5\,x^{10}}{10}+\frac{7\,a\,b^6\,x^{11}}{11}+\frac{b^7\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^7,x)
```

```
[Out] (a^7*x^5)/5 + (b^7*x^12)/12 + (7*a^6*b*x^6)/6 + (7*a*b^6*x^11)/11 + 3*a^5*b
^2*x^7 + (35*a^4*b^3*x^8)/8 + (35*a^3*b^4*x^9)/9 + (21*a^2*b^5*x^10)/10
```

**sympy** [A] time = 0.10, size = 92, normalized size = 1.14

$$\frac{a^{7}x^{5}}{5}+\frac{7a^{6}bx^{6}}{6}+3a^{5}b^{2}x^{7}+\frac{35a^{4}b^{3}x^{8}}{8}+\frac{35a^{3}b^{4}x^{9}}{9}+\frac{21a^{2}b^{5}x^{10}}{10}+\frac{7ab^{6}x^{11}}{11}+\frac{b^{7}x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**7,x)
```

```
[Out] a**7*x**5/5 + 7*a**6*b*x**6/6 + 3*a**5*b**2*x**7 + 35*a**4*b**3*x**8/8 + 35
*a**3*b**4*x**9/9 + 21*a**2*b**5*x**10/10 + 7*a*b**6*x**11/11 + b**7*x**12/
12
```

## 3.103 $\int x^3(a+bx)^7\,dx$

**Optimal**. Leaf size=64

$$-\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}+\frac{(a+bx)^{11}}{11b^4}-\frac{3a(a+bx)^{10}}{10b^4}$$

**Rubi [A]** time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^9}{3b^4}-\frac{a^3(a+bx)^8}{8b^4}+\frac{(a+bx)^{11}}{11b^4}-\frac{3a(a+bx)^{10}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^7,x]
```

```
[Out] -(a^3*(a + b*x)^8)/(8*b^4) + (a^2*(a + b*x)^9)/(3*b^4) - (3*a*(a + b*x)^10)
/(10*b^4) + (a + b*x)^11/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^7\,dx &= \int \left(-\frac{a^3(a+bx)^7}{b^3}+\frac{3a^2(a+bx)^8}{b^3}-\frac{3a(a+bx)^9}{b^3}+\frac{(a+bx)^{10}}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}-\frac{3a(a+bx)^{10}}{10b^4}+\frac{(a+bx)^{11}}{11b^4}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 93, normalized size = 1.45

$$\frac{a^7x^4}{4}+\frac{7}{5}a^6bx^5+\frac{7}{2}a^5b^2x^6+5a^4b^3x^7+\frac{35}{8}a^3b^4x^8+\frac{7}{3}a^2b^5x^9+\frac{7}{10}ab^6x^{10}+\frac{b^7x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^7,x]
```

[Out] (a^7*x^4)/4 + (7*a^6*b*x^5)/5 + (7*a^5*b^2*x^6)/2 + 5*a^4*b^3*x^7 + (35*a^3*b^4*x^8)/8 + (7*a^2*b^5*x^9)/3 + (7*a*b^6*x^10)/10 + (b^7*x^11)/11

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^7\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3*(a + b*x)^7,x]

[Out] IntegrateAlgebraic[x^3*(a + b*x)^7, x]

**fricas [A]** time = 1.33, size = 79, normalized size = 1.23

$$\frac{1}{11}x^{11}b^7+\frac{7}{10}x^{10}b^6a+\frac{7}{3}x^9b^5a^2+\frac{35}{8}x^8b^4a^3+5x^7b^3a^4+\frac{7}{2}x^6b^2a^5+\frac{7}{5}x^5ba^6+\frac{1}{4}x^4a^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x+a)^7,x, algorithm="fricas")

[Out] 1/11*x^11*b^7 + 7/10*x^10*b^6*a + 7/3*x^9*b^5*a^2 + 35/8*x^8*b^4*a^3 + 5*x^7*b^3*a^4 + 7/2*x^6*b^2*a^5 + 7/5*x^5*b*a^6 + 1/4*x^4*a^7

**giac [A]** time = 1.22, size = 79, normalized size = 1.23

$$\frac{1}{11}\,b^7x^{11}+\frac{7}{10}\,ab^6x^{10}+\frac{7}{3}\,a^2b^5x^9+\frac{35}{8}\,a^3b^4x^8+5\,a^4b^3x^7+\frac{7}{2}\,a^5b^2x^6+\frac{7}{5}\,a^6bx^5+\frac{1}{4}\,a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x+a)^7,x, algorithm="giac")

[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4

**maple [A]** time = 0.00, size = 80, normalized size = 1.25

$$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x+a)^7,x)

[Out] 1/11*b^7*x^11+7/10*a*b^6*x^10+7/3*a^2*b^5*x^9+35/8*a^3*b^4*x^8+5*a^4*b^3*x^7+7/2*a^5*b^2*x^6+7/5*a^6*b*x^5+1/4*a^7*x^4

**maxima** [A] time = 1.34, size = 79, normalized size = 1.23

$$\frac{1}{11}\,b^{7}x^{11}+\frac{7}{10}\,ab^{6}x^{10}+\frac{7}{3}\,a^{2}b^{5}x^{9}+\frac{35}{8}\,a^{3}b^{4}x^{8}+5\,a^{4}b^{3}x^{7}+\frac{7}{2}\,a^{5}b^{2}x^{6}+\frac{7}{5}\,a^{6}bx^{5}+\frac{1}{4}\,a^{7}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^
4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

**mupad** [B] time = 0.10, size = 79, normalized size = 1.23

$$\frac{a^7\,x^4}{4}+\frac{7\,a^6\,b\,x^5}{5}+\frac{7\,a^5\,b^2\,x^6}{2}+5\,a^4\,b^3\,x^7+\frac{35\,a^3\,b^4\,x^8}{8}+\frac{7\,a^2\,b^5\,x^9}{3}+\frac{7\,a\,b^6\,x^{10}}{10}+\frac{b^7\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^7,x)
```

```
[Out] (a^7*x^4)/4 + (b^7*x^11)/11 + (7*a^6*b*x^5)/5 + (7*a*b^6*x^10)/10 + (7*a^5*
b^2*x^6)/2 + 5*a^4*b^3*x^7 + (35*a^3*b^4*x^8)/8 + (7*a^2*b^5*x^9)/3
```

**sympy** [A] time = 0.09, size = 92, normalized size = 1.44

$$\frac{a^{7}x^{4}}{4}+\frac{7a^{6}bx^{5}}{5}+\frac{7a^{5}b^{2}x^{6}}{2}+5a^{4}b^{3}x^{7}+\frac{35a^{3}b^{4}x^{8}}{8}+\frac{7a^{2}b^{5}x^{9}}{3}+\frac{7ab^{6}x^{10}}{10}+\frac{b^{7}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**7,x)
```

```
[Out] a**7*x**4/4 + 7*a**6*b*x**5/5 + 7*a**5*b**2*x**6/2 + 5*a**4*b**3*x**7 + 35*
a**3*b**4*x**8/8 + 7*a**2*b**5*x**9/3 + 7*a*b**6*x**10/10 + b**7*x**11/11
```

## 3.104 $\int x^2(a+bx)^7\,dx$

**Optimal**. Leaf size=47

$$\frac{a^2(a+bx)^8}{8b^3}+\frac{(a+bx)^{10}}{10b^3}-\frac{2a(a+bx)^9}{9b^3}$$

**Rubi [A]** time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^8}{8b^3}+\frac{(a+bx)^{10}}{10b^3}-\frac{2a(a+bx)^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^7,x]
```

```
[Out] (a^2*(a + b*x)^8)/(8*b^3) - (2*a*(a + b*x)^9)/(9*b^3) + (a + b*x)^10/(10*b^
3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^7\,dx&=\int\left(\frac{a^2(a+bx)^7}{b^2}-\frac{2a(a+bx)^8}{b^2}+\frac{(a+bx)^9}{b^2}\right)dx\\&=\frac{a^2(a+bx)^8}{8b^3}-\frac{2a(a+bx)^9}{9b^3}+\frac{(a+bx)^{10}}{10b^3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 93, normalized size = 1.98

$$\frac{a^7x^3}{3}+\frac{7}{4}a^6bx^4+\frac{21}{5}a^5b^2x^5+\frac{35}{6}a^4b^3x^6+5a^3b^4x^7+\frac{21}{8}a^2b^5x^8+\frac{7}{9}ab^6x^9+\frac{b^7x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^7,x]
```

```
[Out] (a^7*x^3)/3 + (7*a^6*b*x^4)/4 + (21*a^5*b^2*x^5)/5 + (35*a^4*b^3*x^6)/6 + 5
*a^3*b^4*x^7 + (21*a^2*b^5*x^8)/8 + (7*a*b^6*x^9)/9 + (b^7*x^10)/10
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x)^7, x]
```

**fricas [A]** time = 1.11, size = 79, normalized size = 1.68

$$\frac{1}{10}x^{10}b^7+\frac{7}{9}x^9b^6a+\frac{21}{8}x^8b^5a^2+5x^7b^4a^3+\frac{35}{6}x^6b^3a^4+\frac{21}{5}x^5b^2a^5+\frac{7}{4}x^4ba^6+\frac{1}{3}x^3a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^7 + 7/9*x^9*b^6*a + 21/8*x^8*b^5*a^2 + 5*x^7*b^4*a^3 + 35/6*x^6
*b^3*a^4 + 21/5*x^5*b^2*a^5 + 7/4*x^4*b*a^6 + 1/3*x^3*a^7
```

**giac [A]** time = 0.91, size = 79, normalized size = 1.68

$$\frac{1}{10}\,b^7x^{10}+\frac{7}{9}\,ab^6x^9+\frac{21}{8}\,a^2b^5x^8+5\,a^3b^4x^7+\frac{35}{6}\,a^4b^3x^6+\frac{21}{5}\,a^5b^2x^5+\frac{7}{4}\,a^6bx^4+\frac{1}{3}\,a^7x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

**maple [A]** time = 0.00, size = 80, normalized size = 1.70

$$\frac{1}{10}b^7x^{10}+\frac{7}{9}a\,b^6x^9+\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6b\,x^4+\frac{1}{3}a^7x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^7,x)
```

```
[Out] 1/10*b^7*x^10+7/9*a*b^6*x^9+21/8*a^2*b^5*x^8+5*a^3*b^4*x^7+35/6*a^4*b^3*x^6
+21/5*a^5*b^2*x^5+7/4*a^6*b*x^4+1/3*a^7*x^3
```

**maxima** [A] time = 1.33, size = 79, normalized size = 1.68

$$\frac{1}{10}\,b^7x^{10}+\frac{7}{9}\,ab^6x^9+\frac{21}{8}\,a^2b^5x^8+5\,a^3b^4x^7+\frac{35}{6}\,a^4b^3x^6+\frac{21}{5}\,a^5b^2x^5+\frac{7}{4}\,a^6bx^4+\frac{1}{3}\,a^7x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

**mupad** [B] time = 0.12, size = 31, normalized size = 0.66

$$\frac{(a+b\,x)^8\,\left(8\,a^2-64\,a\,b\,x+288\,b^2\,x^2\right)}{2880\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^8*(8*a^2 + 288*b^2*x^2 - 64*a*b*x))/(2880*b^3)
```

**sympy** [B] time = 0.09, size = 92, normalized size = 1.96

$$\frac{a^7x^3}{3}+\frac{7a^6bx^4}{4}+\frac{21a^5b^2x^5}{5}+\frac{35a^4b^3x^6}{6}+5a^3b^4x^7+\frac{21a^2b^5x^8}{8}+\frac{7ab^6x^9}{9}+\frac{b^7x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**7,x)
```

```
[Out] a**7*x**3/3 + 7*a**6*b*x**4/4 + 21*a**5*b**2*x**5/5 + 35*a**4*b**3*x**6/6 +
 5*a**3*b**4*x**7 + 21*a**2*b**5*x**8/8 + 7*a*b**6*x**9/9 + b**7*x**10/10
```

## 3.105 $\int x(a+bx)^7\,dx$

**Optimal**. Leaf size=30

$$\frac{(a+bx)^9}{9b^2}-\frac{a(a+bx)^8}{8b^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^9}{9b^2}-\frac{a(a+bx)^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^7,x]
```

```
[Out] -(a*(a + b*x)^8)/(8*b^2) + (a + b*x)^9/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^7\,dx &= \int \left(-\frac{a(a+bx)^7}{b}+\frac{(a+bx)^8}{b}\right)dx\\ &= -\frac{a(a+bx)^8}{8b^2}+\frac{(a+bx)^9}{9b^2}\end{aligned}$$

**Mathematica** [B] time = 0.00, size = 91, normalized size = 3.03

$$\frac{a^7x^2}{2}+\frac{7}{3}a^6bx^3+\frac{21}{4}a^5b^2x^4+7a^4b^3x^5+\frac{35}{6}a^3b^4x^6+3a^2b^5x^7+\frac{7}{8}ab^6x^8+\frac{b^7x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^7,x]
```

```
[Out] (a^7*x^2)/2 + (7*a^6*b*x^3)/3 + (21*a^5*b^2*x^4)/4 + 7*a^4*b^3*x^5 + (35*a^
3*b^4*x^6)/6 + 3*a^2*b^5*x^7 + (7*a*b^6*x^8)/8 + (b^7*x^9)/9
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x)^7, x]
```

**fricas [B]** time = 1.00, size = 79, normalized size = 2.63

$$\frac{1}{9}x^9b^7+\frac{7}{8}x^8b^6a+3x^7b^5a^2+\frac{35}{6}x^6b^4a^3+7x^5b^3a^4+\frac{21}{4}x^4b^2a^5+\frac{7}{3}x^3ba^6+\frac{1}{2}x^2a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^7 + 7/8*x^8*b^6*a + 3*x^7*b^5*a^2 + 35/6*x^6*b^4*a^3 + 7*x^5*b^3*
a^4 + 21/4*x^4*b^2*a^5 + 7/3*x^3*b*a^6 + 1/2*x^2*a^7
```

**giac [B]** time = 0.92, size = 79, normalized size = 2.63

$$\frac{1}{9}\,b^7x^9+\frac{7}{8}\,ab^6x^8+3\,a^2b^5x^7+\frac{35}{6}\,a^3b^4x^6+7\,a^4b^3x^5+\frac{21}{4}\,a^5b^2x^4+\frac{7}{3}\,a^6bx^3+\frac{1}{2}\,a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*
x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

**maple [B]** time = 0.00, size = 80, normalized size = 2.67

$$\frac{1}{9}b^7x^9+\frac{7}{8}a\,b^6x^8+3a^2b^5x^7+\frac{35}{6}a^3b^4x^6+7a^4b^3x^5+\frac{21}{4}a^5b^2x^4+\frac{7}{3}a^6b\,x^3+\frac{1}{2}a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^7,x)
```

```
[Out] 1/9*b^7*x^9+7/8*a*b^6*x^8+3*a^2*b^5*x^7+35/6*a^3*b^4*x^6+7*a^4*b^3*x^5+21/4
*a^5*b^2*x^4+7/3*a^6*b*x^3+1/2*a^7*x^2
```

**maxima [B]** time = 1.35, size = 79, normalized size = 2.63

$$\frac{1}{9}\,b^7x^9+\frac{7}{8}\,ab^6x^8+3\,a^2b^5x^7+\frac{35}{6}\,a^3b^4x^6+7\,a^4b^3x^5+\frac{21}{4}\,a^5b^2x^4+\frac{7}{3}\,a^6bx^3+\frac{1}{2}\,a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*
x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

**mupad [B]** time = 0.12, size = 25, normalized size = 0.83

$$-\frac{2\left(\frac{a\,(a+b\,x)^8}{16}-\frac{(a+b\,x)^9}{18}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^7,x)
```

```
[Out] -(2*((a*(a + b*x)^8)/16 - (a + b*x)^9/18))/b^2
```

**sympy [B]** time = 0.09, size = 90, normalized size = 3.00

$$\frac{a^7x^2}{2}+\frac{7a^6bx^3}{3}+\frac{21a^5b^2x^4}{4}+7a^4b^3x^5+\frac{35a^3b^4x^6}{6}+3a^2b^5x^7+\frac{7ab^6x^8}{8}+\frac{b^7x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**7,x)
```

```
[Out] a**7*x**2/2 + 7*a**6*b*x**3/3 + 21*a**5*b**2*x**4/4 + 7*a**4*b**3*x**5 + 35
*a**3*b**4*x**6/6 + 3*a**2*b**5*x**7 + 7*a*b**6*x**8/8 + b**7*x**9/9
```

## 3.106 $\int (a+bx)^7 \, dx$

**Optimal**. Leaf size=14

$$\frac{(a+bx)^8}{8b}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7,x]
```

```
[Out] (a + b*x)^8/(8*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^7 \, dx = \frac{(a+bx)^8}{8b}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7,x]
```

```
[Out] (a + b*x)^8/(8*b)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^7 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7, x]
```

**fricas [B]** time = 0.89, size = 75, normalized size = 5.36

$$\frac{1}{8}x^8b^7 + x^7b^6a + \frac{7}{2}x^6b^5a^2 + 7x^5b^4a^3 + \frac{35}{4}x^4b^3a^4 + 7x^3b^2a^5 + \frac{7}{2}x^2ba^6 + xa^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^7 + x^7*b^6*a + 7/2*x^6*b^5*a^2 + 7*x^5*b^4*a^3 + 35/4*x^4*b^3*a^
4 + 7*x^3*b^2*a^5 + 7/2*x^2*b*a^6 + x*a^7
```

**giac [A]** time = 1.06, size = 12, normalized size = 0.86

$$\frac{(bx+a)^8}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(b*x + a)^8/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx+a)^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7,x)
```

```
[Out] 1/8*(b*x+a)^8/b
```

**maxima [A]** time = 1.39, size = 12, normalized size = 0.86

$$\frac{(bx+a)^8}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x + a)^8/b
```

**mupad [B]** time = 0.06, size = 75, normalized size = 5.36

$$a^7\,x+\frac{7\,a^6\,b\,x^2}{2}+7\,a^5\,b^2\,x^3+\frac{35\,a^4\,b^3\,x^4}{4}+7\,a^3\,b^4\,x^5+\frac{7\,a^2\,b^5\,x^6}{2}+a\,b^6\,x^7+\frac{b^7\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7,x)
```

```
[Out] a^7*x + (b^7*x^8)/8 + (7*a^6*b*x^2)/2 + a*b^6*x^7 + 7*a^5*b^2*x^3 + (35*a^4
*b^3*x^4)/4 + 7*a^3*b^4*x^5 + (7*a^2*b^5*x^6)/2
```

**sympy [B]** time = 0.08, size = 83, normalized size = 5.93

$$a^7x+\frac{7a^6bx^2}{2}+7a^5b^2x^3+\frac{35a^4b^3x^4}{4}+7a^3b^4x^5+\frac{7a^2b^5x^6}{2}+ab^6x^7+\frac{b^7x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7,x)
```

```
[Out] a**7*x + 7*a**6*b*x**2/2 + 7*a**5*b**2*x**3 + 35*a**4*b**3*x**4/4 + 7*a**3*
b**4*x**5 + 7*a**2*b**5*x**6/2 + a*b**6*x**7 + b**7*x**8/8
```

## 3.107 $\int \frac{(a+bx)^7}{x} dx$

**Optimal**. Leaf size=87

$$a^7 \log(x) + 7a^6bx + \frac{21}{2}a^5b^2x^2 + \frac{35}{3}a^4b^3x^3 + \frac{35}{4}a^3b^4x^4 + \frac{21}{5}a^2b^5x^5 + \frac{7}{6}ab^6x^6 + \frac{b^7x^7}{7}$$

**Rubi** [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21}{2}a^5b^2x^2 + \frac{35}{3}a^4b^3x^3 + \frac{35}{4}a^3b^4x^4 + \frac{21}{5}a^2b^5x^5 + 7a^6bx + a^7 \log(x) + \frac{7}{6}ab^6x^6 + \frac{b^7x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x,x]
```

```
[Out] 7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
(21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^7}{x} dx &= \int \left(7a^6b + \frac{a^7}{x} + 21a^5b^2x + 35a^4b^3x^2 + 35a^3b^4x^3 + 21a^2b^5x^4 + 7ab^6x^5 + b^7x^6\right) dx \\ &= 7a^6bx + \frac{21}{2}a^5b^2x^2 + \frac{35}{3}a^4b^3x^3 + \frac{35}{4}a^3b^4x^4 + \frac{21}{5}a^2b^5x^5 + \frac{7}{6}ab^6x^6 + \frac{b^7x^7}{7} + a^7 \log(x) \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 87, normalized size = 1.00

$$a^7 \log(x) + 7a^6bx + \frac{21}{2}a^5b^2x^2 + \frac{35}{3}a^4b^3x^3 + \frac{35}{4}a^3b^4x^4 + \frac{21}{5}a^2b^5x^5 + \frac{7}{6}ab^6x^6 + \frac{b^7x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x,x]
```

```
[Out] 7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
(21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x, x]
```

**fricas [A]** time = 1.38, size = 75, normalized size = 0.86

$$\frac{1}{7}\,b^7x^7+\frac{7}{6}\,ab^6x^6+\frac{21}{5}\,a^2b^5x^5+\frac{35}{4}\,a^3b^4x^4+\frac{35}{3}\,a^4b^3x^3+\frac{21}{2}\,a^5b^2x^2+7\,a^6bx+a^7\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="fricas")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

**giac [A]** time = 1.30, size = 76, normalized size = 0.87

$$\frac{1}{7}\,b^7x^7+\frac{7}{6}\,ab^6x^6+\frac{21}{5}\,a^2b^5x^5+\frac{35}{4}\,a^3b^4x^4+\frac{35}{3}\,a^4b^3x^3+\frac{21}{2}\,a^5b^2x^2+7\,a^6bx+a^7\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="giac")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(abs(x))
```

**maple [A]** time = 0.00, size = 76, normalized size = 0.87

$$\frac{b^7x^7}{7}+\frac{7a\,b^6x^6}{6}+\frac{21a^2b^5x^5}{5}+\frac{35a^3b^4x^4}{4}+\frac{35a^4b^3x^3}{3}+\frac{21a^5b^2x^2}{2}+a^7\ln(x)+7a^6bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x,x)
```

```
[Out] 7*a^6*b*x+21/2*a^5*b^2*x^2+35/3*a^4*b^3*x^3+35/4*a^3*b^4*x^4+21/5*a^2*b^5*x
^5+7/6*a*b^6*x^6+1/7*b^7*x^7+a^7*ln(x)
```

**maxima [A]** time = 1.34, size = 75, normalized size = 0.86

$$\frac{1}{7}\,b^7x^7+\frac{7}{6}\,ab^6x^6+\frac{21}{5}\,a^2b^5x^5+\frac{35}{4}\,a^3b^4x^4+\frac{35}{3}\,a^4b^3x^3+\frac{21}{2}\,a^5b^2x^2+7\,a^6bx+a^7\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="maxima")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

**mupad [B]** time = 0.07, size = 75, normalized size = 0.86

$$a^7\,\ln(x)+\frac{b^7\,x^7}{7}+\frac{7\,a\,b^6\,x^6}{6}+\frac{21\,a^5\,b^2\,x^2}{2}+\frac{35\,a^4\,b^3\,x^3}{3}+\frac{35\,a^3\,b^4\,x^4}{4}+\frac{21\,a^2\,b^5\,x^5}{5}+7\,a^6\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x,x)
```

```
[Out] a^7*log(x) + (b^7*x^7)/7 + (7*a*b^6*x^6)/6 + (21*a^5*b^2*x^2)/2 + (35*a^4*b
^3*x^3)/3 + (35*a^3*b^4*x^4)/4 + (21*a^2*b^5*x^5)/5 + 7*a^6*b*x
```

**sympy [A]** time = 0.19, size = 88, normalized size = 1.01

$$a^7\log(x)+7a^6bx+\frac{21a^5b^2x^2}{2}+\frac{35a^4b^3x^3}{3}+\frac{35a^3b^4x^4}{4}+\frac{21a^2b^5x^5}{5}+\frac{7ab^6x^6}{6}+\frac{b^7x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x,x)
```

```
[Out] a**7*log(x) + 7*a**6*b*x + 21*a**5*b**2*x**2/2 + 35*a**4*b**3*x**3/3 + 35*a
**3*b**4*x**4/4 + 21*a**2*b**5*x**5/5 + 7*a*b**6*x**6/6 + b**7*x**7/7
```

## 3.108 $\int \frac{(a+bx)^7}{x^2}\,dx$

**Optimal**. Leaf size=86

$$-\frac{a^7}{x}+7a^6b\log(x)+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}$$

**Rubi [A]** time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+21a^5b^2x+7a^6b\log(x)-\frac{a^7}{x}+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^2,x]
```

```
[Out] -(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2
*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^2}\,dx &= \int \left(21a^5b^2+\frac{a^7}{x^2}+\frac{7a^6b}{x}+35a^4b^3x+35a^3b^4x^2+21a^2b^5x^3+7ab^6x^4+b^7x^5\right)dx\\ &= -\frac{a^7}{x}+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}+7a^6b\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{x}+7a^6b\log(x)+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^2,x]
```

```
[Out] -(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2
*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^2, x]
```

**fricas [A]** time = 1.31, size = 81, normalized size = 0.94

$$\frac{10\,b^7x^7 + 84\,ab^6x^6 + 315\,a^2b^5x^5 + 700\,a^3b^4x^4 + 1050\,a^4b^3x^3 + 1260\,a^5b^2x^2 + 420\,a^6bx\log(x) - 60\,a^7}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^7*x^7 + 84*a*b^6*x^6 + 315*a^2*b^5*x^5 + 700*a^3*b^4*x^4 + 1050*
a^4*b^3*x^3 + 1260*a^5*b^2*x^2 + 420*a^6*b*x*log(x) - 60*a^7)/x
```

**giac [A]** time = 1.12, size = 77, normalized size = 0.90

$$\frac{1}{6}\,b^7x^6 + \frac{7}{5}\,ab^6x^5 + \frac{21}{4}\,a^2b^5x^4 + \frac{35}{3}\,a^3b^4x^3 + \frac{35}{2}\,a^4b^3x^2 + 21\,a^5b^2x + 7\,a^6b\log\left(|x|\right) - \frac{a^7}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="giac")
```

```
[Out] 1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a^
4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(abs(x)) - a^7/x
```

**maple [A]** time = 0.01, size = 77, normalized size = 0.90

$$\frac{b^7x^6}{6} + \frac{7a\,b^6x^5}{5} + \frac{21a^2b^5x^4}{4} + \frac{35a^3b^4x^3}{3} + \frac{35a^4b^3x^2}{2} + 7a^6b\ln(x) + 21a^5b^2x - \frac{a^7}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^2,x)
```

[Out] -a^7/x+21*a^5*b^2*x+35/2*a^4*b^3*x^2+35/3*a^3*b^4*x^3+21/4*a^2*b^5*x^4+7/5*a*b^6*x^5+1/6*b^7*x^6+7*a^6*b*ln(x)

**maxima [A]** time = 1.36, size = 76, normalized size = 0.88

$$\frac{1}{6}\,b^{7}x^{6}+\frac{7}{5}\,ab^{6}x^{5}+\frac{21}{4}\,a^{2}b^{5}x^{4}+\frac{35}{3}\,a^{3}b^{4}x^{3}+\frac{35}{2}\,a^{4}b^{3}x^{2}+21\,a^{5}b^{2}x+7\,a^{6}b\log\left(x\right)-\frac{a^{7}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^7/x^2,x, algorithm="maxima")

[Out] 1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a^4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(x) - a^7/x

**mupad [B]** time = 0.05, size = 76, normalized size = 0.88

$$\frac{b^{7}\,x^{6}}{6}-\frac{a^{7}}{x}+21\,a^{5}\,b^{2}\,x+\frac{7\,a\,b^{6}\,x^{5}}{5}+7\,a^{6}\,b\,\ln\left(x\right)+\frac{35\,a^{4}\,b^{3}\,x^{2}}{2}+\frac{35\,a^{3}\,b^{4}\,x^{3}}{3}+\frac{21\,a^{2}\,b^{5}\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^7/x^2,x)

[Out] (b^7*x^6)/6 - a^7/x + 21*a^5*b^2*x + (7*a*b^6*x^5)/5 + 7*a^6*b*log(x) + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2*b^5*x^4)/4

**sympy [A]** time = 0.20, size = 85, normalized size = 0.99

$$-\frac{a^{7}}{x}+7a^{6}b\log\left(x\right)+21a^{5}b^{2}x+\frac{35a^{4}b^{3}x^{2}}{2}+\frac{35a^{3}b^{4}x^{3}}{3}+\frac{21a^{2}b^{5}x^{4}}{4}+\frac{7ab^{6}x^{5}}{5}+\frac{b^{7}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**7/x**2,x)

[Out] -a**7/x + 7*a**6*b*log(x) + 21*a**5*b**2*x + 35*a**4*b**3*x**2/2 + 35*a**3*b**4*x**3/3 + 21*a**2*b**5*x**4/4 + 7*a*b**6*x**5/5 + b**7*x**6/6

## 3.109 $\int \frac{(a+bx)^7}{x^3} \, dx$

**Optimal**. Leaf size=84

$$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+21a^5b^2\log(x)+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}$$

**Rubi [A]** time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+35a^4b^3x+21a^5b^2\log(x)-\frac{7a^6b}{x}-\frac{a^7}{2x^2}+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^3,x]
```

```
[Out] -a^7/(2*x^2) - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5*
x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^3}\,dx &= \int \left(35a^4b^3+\frac{a^7}{x^3}+\frac{7a^6b}{x^2}+\frac{21a^5b^2}{x}+35a^3b^4x+21a^2b^5x^2+7ab^6x^3+b^7x^4\right)dx\\ &= -\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}+21a^5b^2\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 84, normalized size = 1.00

$$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+21a^5b^2\log(x)+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^3,x]
```

```
[Out] -1/2*a^7/x^2 - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5*
x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^3, x]
```

**fricas [A]** time = 1.62, size = 81, normalized size = 0.96

$$\frac{4\,b^7x^7+35\,ab^6x^6+140\,a^2b^5x^5+350\,a^3b^4x^4+700\,a^4b^3x^3+420\,a^5b^2x^2\log(x)-140\,a^6bx-10\,a^7}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*x^7 + 35*a*b^6*x^6 + 140*a^2*b^5*x^5 + 350*a^3*b^4*x^4 + 700*a^
4*b^3*x^3 + 420*a^5*b^2*x^2*log(x) - 140*a^6*b*x - 10*a^7)/x^2
```

**giac [A]** time = 1.06, size = 76, normalized size = 0.90

$$\frac{1}{5}\,b^7x^5+\frac{7}{4}\,ab^6x^4+7\,a^2b^5x^3+\frac{35}{2}\,a^3b^4x^2+35\,a^4b^3x+21\,a^5b^2\log\left(|x|\right)-\frac{14\,a^6bx+a^7}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="giac")
```

```
[Out] 1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^3
*x + 21*a^5*b^2*log(abs(x)) - 1/2*(14*a^6*b*x + a^7)/x^2
```

**maple [A]** time = 0.01, size = 77, normalized size = 0.92

$$\frac{b^7x^5}{5}+\frac{7a\,b^6x^4}{4}+7a^2b^5x^3+\frac{35a^3b^4x^2}{2}+21a^5b^2\ln(x)+35a^4b^3x-\frac{7a^6b}{x}-\frac{a^7}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^3,x)
```

```
[Out] -1/2*a^7/x^2-7*a^6*b/x+35*a^4*b^3*x+35/2*a^3*b^4*x^2+7*a^2*b^5*x^3+7/4*a*b^
6*x^4+1/5*b^7*x^5+21*a^5*b^2*ln(x)
```

**maxima [A]** time = 1.38, size = 75, normalized size = 0.89

$$\frac{1}{5}\,b^7x^5+\frac{7}{4}\,ab^6x^4+7\,a^2b^5x^3+\frac{35}{2}\,a^3b^4x^2+35\,a^4b^3x+21\,a^5b^2\log(x)-\frac{14\,a^6bx+a^7}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="maxima")
```

```
[Out] 1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^3
*x + 21*a^5*b^2*log(x) - 1/2*(14*a^6*b*x + a^7)/x^2
```

**mupad [B]** time = 0.05, size = 77, normalized size = 0.92

$$\frac{b^7\,x^5}{5}-\frac{\frac{a^7}{2}+7\,b\,x\,a^6}{x^2}+35\,a^4\,b^3\,x+\frac{7\,a\,b^6\,x^4}{4}+\frac{35\,a^3\,b^4\,x^2}{2}+7\,a^2\,b^5\,x^3+21\,a^5\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^3,x)
```

```
[Out] (b^7*x^5)/5 - (a^7/2 + 7*a^6*b*x)/x^2 + 35*a^4*b^3*x + (7*a*b^6*x^4)/4 + (3
5*a^3*b^4*x^2)/2 + 7*a^2*b^5*x^3 + 21*a^5*b^2*log(x)
```

**sympy [A]** time = 0.25, size = 85, normalized size = 1.01

$$21a^5b^2\log(x)+35a^4b^3x+\frac{35a^3b^4x^2}{2}+7a^2b^5x^3+\frac{7ab^6x^4}{4}+\frac{b^7x^5}{5}+\frac{-a^7-14a^6bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**3,x)
```

```
[Out] 21*a**5*b**2*log(x) + 35*a**4*b**3*x + 35*a**3*b**4*x**2/2 + 7*a**2*b**5*x*
*3 + 7*a*b**6*x**4/4 + b**7*x**5/5 + (-a**7 - 14*a**6*b*x)/(2*x**2)
```

# 3.110 $\int \frac{(a+bx)^7}{x^4} \, dx$

**Optimal**. Leaf size=86

$$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^4b^3\log(x)+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

**Rubi [A]** time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21}{2}a^2b^5x^2-\frac{21a^5b^2}{x}+35a^3b^4x+35a^4b^3\log(x)-\frac{7a^6b}{2x^2}-\frac{a^7}{3x^3}+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^4,x]
```

```
[Out] -a^7/(3*x^3) - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2*
b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^4}\,dx &= \int \left(35a^3b^4+\frac{a^7}{x^4}+\frac{7a^6b}{x^3}+\frac{21a^5b^2}{x^2}+\frac{35a^4b^3}{x}+21a^2b^5x+7ab^6x^2+b^7x^3\right)dx\\ &= -\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}+35a^4b^3\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^4b^3\log(x)+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^4,x]
```

```
[Out] -1/3*a^7/x^3 - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2*
b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^4, x]
```

**fricas [A]** time = 1.33, size = 81, normalized size = 0.94

$$\frac{3\,b^7x^7+28\,ab^6x^6+126\,a^2b^5x^5+420\,a^3b^4x^4+420\,a^4b^3x^3\log(x)-252\,a^5b^2x^2-42\,a^6bx-4\,a^7}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*x^7 + 28*a*b^6*x^6 + 126*a^2*b^5*x^5 + 420*a^3*b^4*x^4 + 420*a^
4*b^3*x^3*log(x) - 252*a^5*b^2*x^2 - 42*a^6*b*x - 4*a^7)/x^3
```

**giac [A]** time = 1.09, size = 78, normalized size = 0.91

$$\frac{1}{4}\,b^7x^4+\frac{7}{3}\,ab^6x^3+\frac{21}{2}\,a^2b^5x^2+35\,a^3b^4x+35\,a^4b^3\log\left(|x|\right)-\frac{126\,a^5b^2x^2+21\,a^6bx+2\,a^7}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="giac")
```

```
[Out] 1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3*
log(abs(x)) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

**maple [A]** time = 0.01, size = 77, normalized size = 0.90

$$\frac{b^7x^4}{4}+\frac{7a\,b^6x^3}{3}+\frac{21a^2b^5x^2}{2}+35a^4b^3\ln(x)+35a^3b^4x-\frac{21a^5b^2}{x}-\frac{7a^6b}{2x^2}-\frac{a^7}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^4,x)
```

```
[Out] -1/3*a^7/x^3-7/2*a^6*b/x^2-21*a^5*b^2/x+35*a^3*b^4*x+21/2*a^2*b^5*x^2+7/3*a
*b^6*x^3+1/4*b^7*x^4+35*a^4*b^3*ln(x)
```

**maxima [A]** time = 1.33, size = 77, normalized size = 0.90

$$\frac{1}{4}\,b^{7}x^{4}+\frac{7}{3}\,ab^{6}x^{3}+\frac{21}{2}\,a^{2}b^{5}x^{2}+35\,a^{3}b^{4}x+35\,a^{4}b^{3}\log(x)-\frac{126\,a^{5}b^{2}x^{2}+21\,a^{6}bx+2\,a^{7}}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3*
log(x) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

**mupad [B]** time = 0.05, size = 77, normalized size = 0.90

$$\frac{b^{7}\,x^{4}}{4}-\frac{\frac{a^{7}}{3}+\frac{7\,a^{6}\,b\,x}{2}+21\,a^{5}\,b^{2}\,x^{2}}{x^{3}}+35\,a^{3}\,b^{4}\,x+\frac{7\,a\,b^{6}\,x^{3}}{3}+\frac{21\,a^{2}\,b^{5}\,x^{2}}{2}+35\,a^{4}\,b^{3}\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^4,x)
```

```
[Out] (b^7*x^4)/4 - (a^7/3 + 21*a^5*b^2*x^2 + (7*a^6*b*x)/2)/x^3 + 35*a^3*b^4*x +
 (7*a*b^6*x^3)/3 + (21*a^2*b^5*x^2)/2 + 35*a^4*b^3*log(x)
```

**sympy [A]** time = 0.31, size = 87, normalized size = 1.01

$$35a^{4}b^{3}\log\left(x\right)+35a^{3}b^{4}x+\frac{21a^{2}b^{5}x^{2}}{2}+\frac{7ab^{6}x^{3}}{3}+\frac{b^{7}x^{4}}{4}+\frac{-2a^{7}-21a^{6}bx-126a^{5}b^{2}x^{2}}{6x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**4,x)
```

```
[Out] 35*a**4*b**3*log(x) + 35*a**3*b**4*x + 21*a**2*b**5*x**2/2 + 7*a*b**6*x**3/
3 + b**7*x**4/4 + (-2*a**7 - 21*a**6*b*x - 126*a**5*b**2*x**2)/(6*x**3)
```

## 3.111 $\int \frac{(a+bx)^7}{x^5} dx$

**Optimal**. Leaf size=86

$$-\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+35a^3b^4\log(x)+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}$$

**Rubi [A]** time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+35a^3b^4\log(x)-\frac{7a^6b}{3x^3}-\frac{a^7}{4x^4}+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^5,x]
```

```
[Out] -a^7/(4*x^4) - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^5}\,dx &= \int \left(21a^2b^5+\frac{a^7}{x^5}+\frac{7a^6b}{x^4}+\frac{21a^5b^2}{x^3}+\frac{35a^4b^3}{x^2}+\frac{35a^3b^4}{x}+7ab^6x+b^7x^2\right)dx\\ &= -\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}+35a^3b^4\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+35a^3b^4\log(x)+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^5,x]
```

```
[Out] -1/4*a^7/x^4 - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^5, x]
```

**fricas [A]** time = 1.52, size = 81, normalized size = 0.94

$$\frac{4\,b^7x^7+42\,ab^6x^6+252\,a^2b^5x^5+420\,a^3b^4x^4\log(x)-420\,a^4b^3x^3-126\,a^5b^2x^2-28\,a^6bx-3\,a^7}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^7*x^7 + 42*a*b^6*x^6 + 252*a^2*b^5*x^5 + 420*a^3*b^4*x^4*log(x) -
 420*a^4*b^3*x^3 - 126*a^5*b^2*x^2 - 28*a^6*b*x - 3*a^7)/x^4
```

**giac [A]** time = 1.04, size = 78, normalized size = 0.91

$$\frac{1}{3}\,b^7x^3+\frac{7}{2}\,ab^6x^2+21\,a^2b^5x+35\,a^3b^4\log\left(|x|\right)-\frac{420\,a^4b^3x^3+126\,a^5b^2x^2+28\,a^6bx+3\,a^7}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="giac")
```

```
[Out] 1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(abs(x)) - 1/12*
(420*a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4
```

**maple [A]** time = 0.01, size = 77, normalized size = 0.90

$$\frac{b^7x^3}{3}+\frac{7a\,b^6x^2}{2}+35a^3b^4\ln(x)+21a^2b^5x-\frac{35a^4b^3}{x}-\frac{21a^5b^2}{2x^2}-\frac{7a^6b}{3x^3}-\frac{a^7}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^5,x)
```

[Out] -1/4*a^7/x^4-7/3*a^6*b/x^3-21/2*a^5*b^2/x^2-35*a^4*b^3/x+21*a^2*b^5*x+7/2*a*b^6*x^2+1/3*b^7*x^3+35*a^3*b^4*ln(x)

**maxima [A]** time = 1.40, size = 77, normalized size = 0.90

$$\frac{1}{3}\,b^7x^3+\frac{7}{2}\,ab^6x^2+21\,a^2b^5x+35\,a^3b^4\log(x)-\frac{420\,a^4b^3x^3+126\,a^5b^2x^2+28\,a^6bx+3\,a^7}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^7/x^5,x, algorithm="maxima")

[Out] 1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(x) - 1/12*(420*a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4

**mupad [B]** time = 0.09, size = 77, normalized size = 0.90

$$\frac{b^7\,x^3}{3}-\frac{\frac{a^7}{4}+\frac{7\,a^6\,b\,x}{3}+\frac{21\,a^5\,b^2\,x^2}{2}+35\,a^4\,b^3\,x^3}{x^4}+21\,a^2\,b^5\,x+\frac{7\,a\,b^6\,x^2}{2}+35\,a^3\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^7/x^5,x)

[Out] (b^7*x^3)/3 - (a^7/4 + (21*a^5*b^2*x^2)/2 + 35*a^4*b^3*x^3 + (7*a^6*b*x)/3)/x^4 + 21*a^2*b^5*x + (7*a*b^6*x^2)/2 + 35*a^3*b^4*log(x)

**sympy [A]** time = 0.33, size = 85, normalized size = 0.99

$$35a^3b^4\log{(x)}+21a^2b^5x+\frac{7ab^6x^2}{2}+\frac{b^7x^3}{3}+\frac{-3a^7-28a^6bx-126a^5b^2x^2-420a^4b^3x^3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**7/x**5,x)

[Out] 35*a**3*b**4*log(x) + 21*a**2*b**5*x + 7*a*b**6*x**2/2 + b**7*x**3/3 + (-3*a**7 - 28*a**6*b*x - 126*a**5*b**2*x**2 - 420*a**4*b**3*x**3)/(12*x**4)

## 3.112 $\int \frac{(a+bx)^7}{x^6}\,dx$

**Optimal**. Leaf size=84

$$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)+7ab^6x+\frac{b^7x^2}{2}$$

**Rubi** [A] time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)-\frac{7a^6b}{4x^4}-\frac{a^7}{5x^5}+7ab^6x+\frac{b^7x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^6,x]
```

```
[Out] -a^7/(5*x^5) - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2) -
 (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^6}\,dx &= \int \left(7ab^6+\frac{a^7}{x^6}+\frac{7a^6b}{x^5}+\frac{21a^5b^2}{x^4}+\frac{35a^4b^3}{x^3}+\frac{35a^3b^4}{x^2}+\frac{21a^2b^5}{x}+b^7x\right)dx\\ &= -\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7ab^6x+\frac{b^7x^2}{2}+21a^2b^5\log(x)\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 84, normalized size = 1.00

$$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)+7ab^6x+\frac{b^7x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^6,x]
```

```
[Out] -1/5*a^7/x^5 - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2) -
 (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^6, x]
```

**fricas [A]** time = 1.64, size = 81, normalized size = 0.96

$$\frac{10\,b^7x^7+140\,ab^6x^6+420\,a^2b^5x^5\log(x)-700\,a^3b^4x^4-350\,a^4b^3x^3-140\,a^5b^2x^2-35\,a^6bx-4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^7*x^7 + 140*a*b^6*x^6 + 420*a^2*b^5*x^5*log(x) - 700*a^3*b^4*x^4
 - 350*a^4*b^3*x^3 - 140*a^5*b^2*x^2 - 35*a^6*b*x - 4*a^7)/x^5
```

**giac [A]** time = 1.01, size = 78, normalized size = 0.93

$$\frac{1}{2}\,b^7x^2+7\,ab^6x+21\,a^2b^5\log\left(|x|\right)-\frac{700\,a^3b^4x^4+350\,a^4b^3x^3+140\,a^5b^2x^2+35\,a^6bx+4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="giac")
```

```
[Out] 1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(abs(x)) - 1/20*(700*a^3*b^4*x^4 +
350*a^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

**maple [A]** time = 0.01, size = 77, normalized size = 0.92

$$\frac{b^7x^2}{2}+21a^2b^5\ln(x)+7a\,b^6x-\frac{35a^3b^4}{x}-\frac{35a^4b^3}{2x^2}-\frac{7a^5b^2}{x^3}-\frac{7a^6b}{4x^4}-\frac{a^7}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^6,x)
```

```
[Out] -1/5*a^7/x^5-7/4*a^6*b/x^4-7*a^5*b^2/x^3-35/2*a^4*b^3/x^2-35*a^3*b^4/x+7*a*
b^6*x+1/2*b^7*x^2+21*a^2*b^5*ln(x)
```

**maxima [A]** time = 1.38, size = 77, normalized size = 0.92

$$\frac{1}{2}\,b^{7}x^{2}+7\,ab^{6}x+21\,a^{2}b^{5}\log(x)-\frac{700\,a^{3}b^{4}x^{4}+350\,a^{4}b^{3}x^{3}+140\,a^{5}b^{2}x^{2}+35\,a^{6}bx+4\,a^{7}}{20\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="maxima")
```

```
[Out] 1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(x) - 1/20*(700*a^3*b^4*x^4 + 350*a
^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

**mupad [B]** time = 0.11, size = 77, normalized size = 0.92

$$\frac{b^7\,x^2}{2}-\frac{\frac{a^7}{5}+\frac{7\,a^6\,b\,x}{4}+7\,a^5\,b^2\,x^2+\frac{35\,a^4\,b^3\,x^3}{2}+35\,a^3\,b^4\,x^4}{x^5}+21\,a^2\,b^5\,\ln(x)+7\,a\,b^6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^6,x)
```

```
[Out] (b^7*x^2)/2 - (a^7/5 + 7*a^5*b^2*x^2 + (35*a^4*b^3*x^3)/2 + 35*a^3*b^4*x^4
+ (7*a^6*b*x)/4)/x^5 + 21*a^2*b^5*log(x) + 7*a*b^6*x
```

**sympy [A]** time = 0.45, size = 83, normalized size = 0.99

$$21a^{2}b^{5}\log\left(x\right)+7ab^{6}x+\frac{b^{7}x^{2}}{2}+\frac{-4a^{7}-35a^{6}bx-140a^{5}b^{2}x^{2}-350a^{4}b^{3}x^{3}-700a^{3}b^{4}x^{4}}{20x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**6,x)
```

```
[Out] 21*a**2*b**5*log(x) + 7*a*b**6*x + b**7*x**2/2 + (-4*a**7 - 35*a**6*b*x - 1
40*a**5*b**2*x**2 - 350*a**4*b**3*x**3 - 700*a**3*b**4*x**4)/(20*x**5)
```

## 3.113 $\int \frac{(a+bx)^7}{x^7} \, dx$

**Optimal**. Leaf size=85

$$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+7ab^6\log(x)+b^7x$$

**Rubi [A]** time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}-\frac{7a^6b}{5x^5}-\frac{a^7}{6x^6}+7ab^6\log(x)+b^7x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^7,x]
```

```
[Out] -a^7/(6*x^6) - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*x
^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^7}\,dx &= \int \left(b^7+\frac{a^7}{x^7}+\frac{7a^6b}{x^6}+\frac{21a^5b^2}{x^5}+\frac{35a^4b^3}{x^4}+\frac{35a^3b^4}{x^3}+\frac{21a^2b^5}{x^2}+\frac{7ab^6}{x}\right)dx\\ &= -\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7ab^6\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 85, normalized size = 1.00

$$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+7ab^6\log(x)+b^7x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^7,x]
```

```
[Out] -1/6*a^7/x^6 - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*x
^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^7, x]
```

**fricas [A]** time = 1.40, size = 81, normalized size = 0.95

$$\frac{60\,b^7x^7 + 420\,ab^6x^6\log(x) - 1260\,a^2b^5x^5 - 1050\,a^3b^4x^4 - 700\,a^4b^3x^3 - 315\,a^5b^2x^2 - 84\,a^6bx - 10\,a^7}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*x^7 + 420*a*b^6*x^6*log(x) - 1260*a^2*b^5*x^5 - 1050*a^3*b^4*x
^4 - 700*a^4*b^3*x^3 - 315*a^5*b^2*x^2 - 84*a^6*b*x - 10*a^7)/x^6
```

**giac [A]** time = 0.99, size = 77, normalized size = 0.91

$$b^7x + 7\,ab^6\log\left(|x|\right) - \frac{1260\,a^2b^5x^5 + 1050\,a^3b^4x^4 + 700\,a^4b^3x^3 + 315\,a^5b^2x^2 + 84\,a^6bx + 10\,a^7}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="giac")
```

```
[Out] b^7*x + 7*a*b^6*log(abs(x)) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 + 7
00*a^4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

**maple [A]** time = 0.01, size = 76, normalized size = 0.89

$$7a\,b^6\ln(x) + b^7x - \frac{21a^2b^5}{x} - \frac{35a^3b^4}{2x^2} - \frac{35a^4b^3}{3x^3} - \frac{21a^5b^2}{4x^4} - \frac{7a^6b}{5x^5} - \frac{a^7}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^7,x)
```

```
[Out] -1/6*a^7/x^6-7/5*a^6*b/x^5-21/4*a^5*b^2/x^4-35/3*a^4*b^3/x^3-35/2*a^3*b^4/x
^2-21*a^2*b^5/x+b^7*x+7*a*b^6*ln(x)
```

**maxima [A]** time = 1.38, size = 76, normalized size = 0.89

$$b^7x + 7\,ab^6\log(x) - \frac{1260\,a^2b^5x^5 + 1050\,a^3b^4x^4 + 700\,a^4b^3x^3 + 315\,a^5b^2x^2 + 84\,a^6bx + 10\,a^7}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="maxima")
```

```
[Out] b^7*x + 7*a*b^6*log(x) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 + 700*a^
4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

**mupad [B]** time = 0.11, size = 81, normalized size = 0.95

$$-\frac{10\,a^7 - 60\,b^7\,x^7 + 315\,a^5\,b^2\,x^2 + 700\,a^4\,b^3\,x^3 + 1050\,a^3\,b^4\,x^4 + 1260\,a^2\,b^5\,x^5 + 84\,a^6\,b\,x - 420\,a\,b^6\,x^6\,\ln(x)}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^7,x)
```

```
[Out] -(10*a^7 - 60*b^7*x^7 + 315*a^5*b^2*x^2 + 700*a^4*b^3*x^3 + 1050*a^3*b^4*x^
4 + 1260*a^2*b^5*x^5 + 84*a^6*b*x - 420*a*b^6*x^6*log(x))/(60*x^6)
```

**sympy [A]** time = 0.60, size = 82, normalized size = 0.96

$$7ab^6\log(x) + b^7x + \frac{-10a^7 - 84a^6bx - 315a^5b^2x^2 - 700a^4b^3x^3 - 1050a^3b^4x^4 - 1260a^2b^5x^5}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**7,x)
```

```
[Out] 7*a*b**6*log(x) + b**7*x + (-10*a**7 - 84*a**6*b*x - 315*a**5*b**2*x**2 - 7
00*a**4*b**3*x**3 - 1050*a**3*b**4*x**4 - 1260*a**2*b**5*x**5)/(60*x**6)
```

## 3.114 $\int \frac{(a+bx)^7}{x^8} \, dx$

**Optimal**. Leaf size=89

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

**Rubi** [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7a^6b}{6x^6}-\frac{a^7}{7x^7}-\frac{7ab^6}{x}+b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^8,x]
```

```
[Out] -a^7/(7*x^7) - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*x
^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log[x
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^8}\,dx &= \int \left(\frac{a^7}{x^8}+\frac{7a^6b}{x^7}+\frac{21a^5b^2}{x^6}+\frac{35a^4b^3}{x^5}+\frac{35a^3b^4}{x^4}+\frac{21a^2b^5}{x^3}+\frac{7ab^6}{x^2}+\frac{b^7}{x}\right)dx\\ &= -\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 89, normalized size = 1.00

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^8,x]
```

```
[Out] -1/7*a^7/x^7 - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*x
^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log[x
]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^8, x]
```

**fricas [A]** time = 1.33, size = 81, normalized size = 0.91

$$\frac{420\,b^7x^7\log(x) - 2940\,ab^6x^6 - 4410\,a^2b^5x^5 - 4900\,a^3b^4x^4 - 3675\,a^4b^3x^3 - 1764\,a^5b^2x^2 - 490\,a^6bx - 60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="fricas")
```

```
[Out] 1/420*(420*b^7*x^7*log(x) - 2940*a*b^6*x^6 - 4410*a^2*b^5*x^5 - 4900*a^3*b^
4*x^4 - 3675*a^4*b^3*x^3 - 1764*a^5*b^2*x^2 - 490*a^6*b*x - 60*a^7)/x^7
```

**giac [A]** time = 1.27, size = 79, normalized size = 0.89

$$b^7\log(|x|) - \frac{2940\,ab^6x^6 + 4410\,a^2b^5x^5 + 4900\,a^3b^4x^4 + 3675\,a^4b^3x^3 + 1764\,a^5b^2x^2 + 490\,a^6bx + 60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="giac")
```

```
[Out] b^7*log(abs(x)) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*x
^4 + 3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

**maple [A]** time = 0.01, size = 78, normalized size = 0.88

$$b^7\ln(x) - \frac{7a\,b^6}{x} - \frac{21a^2b^5}{2x^2} - \frac{35a^3b^4}{3x^3} - \frac{35a^4b^3}{4x^4} - \frac{21a^5b^2}{5x^5} - \frac{7a^6b}{6x^6} - \frac{a^7}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^8,x)
```

```
[Out] -1/7*a^7/x^7-7/6*a^6*b/x^6-21/5*a^5*b^2/x^5-35/4*a^4*b^3/x^4-35/3*a^3*b^4/x
^3-21/2*a^2*b^5/x^2-7*a*b^6/x+b^7*ln(x)
```

**maxima [A]** time = 1.36, size = 78, normalized size = 0.88

$$b^7\log(x) - \frac{2940\,ab^6x^6 + 4410\,a^2b^5x^5 + 4900\,a^3b^4x^4 + 3675\,a^4b^3x^3 + 1764\,a^5b^2x^2 + 490\,a^6bx + 60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="maxima")
```

```
[Out] b^7*log(x) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*x^4 +
3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

**mupad [B]** time = 0.07, size = 78, normalized size = 0.88

$$b^7\,\ln(x) - \frac{\frac{a^7}{7} + \frac{7\,a^6\,b\,x}{6} + \frac{21\,a^5\,b^2\,x^2}{5} + \frac{35\,a^4\,b^3\,x^3}{4} + \frac{35\,a^3\,b^4\,x^4}{3} + \frac{21\,a^2\,b^5\,x^5}{2} + 7\,a\,b^6\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^8,x)
```

```
[Out] b^7*log(x) - (a^7/7 + 7*a*b^6*x^6 + (21*a^5*b^2*x^2)/5 + (35*a^4*b^3*x^3)/4
 + (35*a^3*b^4*x^4)/3 + (21*a^2*b^5*x^5)/2 + (7*a^6*b*x)/6)/x^7
```

**sympy [A]** time = 0.63, size = 83, normalized size = 0.93

$$b^7\log(x) + \frac{-60a^7 - 490a^6bx - 1764a^5b^2x^2 - 3675a^4b^3x^3 - 4900a^3b^4x^4 - 4410a^2b^5x^5 - 2940ab^6x^6}{420x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**8,x)
```

```
[Out] b**7*log(x) + (-60*a**7 - 490*a**6*b*x - 1764*a**5*b**2*x**2 - 3675*a**4*b*
*3*x**3 - 4900*a**3*b**4*x**4 - 4410*a**2*b**5*x**5 - 2940*a*b**6*x**6)/(42
0*x**7)
```

## 3.115 $\int \frac{(a+bx)^7}{x^9} dx$

**Optimal**. Leaf size=17

$$-\frac{(a+bx)^8}{8ax^8}$$

**Rubi** [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^8}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^9,x]
```

```
[Out] -(a + b*x)^8/(8*a*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^9} dx = -\frac{(a+bx)^8}{8ax^8}$$

**Mathematica** [B] time = 0.00, size = 87, normalized size = 5.12

$$-\frac{a^7}{8x^8}-\frac{a^6b}{x^7}-\frac{7a^5b^2}{2x^6}-\frac{7a^4b^3}{x^5}-\frac{35a^3b^4}{4x^4}-\frac{7a^2b^5}{x^3}-\frac{7ab^6}{2x^2}-\frac{b^7}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^9,x]
```

```
[Out] -1/8*a^7/x^8 - (a^6*b)/x^7 - (7*a^5*b^2)/(2*x^6) - (7*a^4*b^3)/x^5 - (35*a^
3*b^4)/(4*x^4) - (7*a^2*b^5)/x^3 - (7*a*b^6)/(2*x^2) - b^7/x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^9,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^9, x]
```

**fricas [B]** time = 1.41, size = 77, normalized size = 4.53

$$-\frac{8\,b^7x^7+28\,ab^6x^6+56\,a^2b^5x^5+70\,a^3b^4x^4+56\,a^4b^3x^3+28\,a^5b^2x^2+8\,a^6bx+a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

**giac [B]** time = 0.92, size = 77, normalized size = 4.53

$$-\frac{8\,b^7x^7+28\,ab^6x^6+56\,a^2b^5x^5+70\,a^3b^4x^4+56\,a^4b^3x^3+28\,a^5b^2x^2+8\,a^6bx+a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

**maple [B]** time = 0.00, size = 80, normalized size = 4.71

$$-\frac{b^7}{x}-\frac{7a\,b^6}{2x^2}-\frac{7a^2b^5}{x^3}-\frac{35a^3b^4}{4x^4}-\frac{7a^4b^3}{x^5}-\frac{7a^5b^2}{2x^6}-\frac{a^6b}{x^7}-\frac{a^7}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^9,x)
```

```
[Out] -7*a^4*b^3/x^5-1/8*a^7/x^8-7/2*a^5*b^2/x^6-35/4*a^3*b^4/x^4-7*a^2*b^5/x^3-b
^7/x-a^6*b/x^7-7/2*a*b^6/x^2
```

**maxima [B]** time = 1.29, size = 77, normalized size = 4.53

$$-\frac{8\,b^7x^7 + 28\,ab^6x^6 + 56\,a^2b^5x^5 + 70\,a^3b^4x^4 + 56\,a^4b^3x^3 + 28\,a^5b^2x^2 + 8\,a^6bx + a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x, algorithm="maxima")
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

**mupad [B]** time = 0.07, size = 77, normalized size = 4.53

$$-\frac{\frac{a^7}{8} + a^6\,b\,x + \frac{7\,a^5\,b^2\,x^2}{2} + 7\,a^4\,b^3\,x^3 + \frac{35\,a^3\,b^4\,x^4}{4} + 7\,a^2\,b^5\,x^5 + \frac{7\,a\,b^6\,x^6}{2} + b^7\,x^7}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^9,x)
```

```
[Out] -(a^7/8 + b^7*x^7 + (7*a*b^6*x^6)/2 + (7*a^5*b^2*x^2)/2 + 7*a^4*b^3*x^3 + (
35*a^3*b^4*x^4)/4 + 7*a^2*b^5*x^5 + a^6*b*x)/x^8
```

**sympy [B]** time = 0.60, size = 83, normalized size = 4.88

$$\frac{-a^7 - 8a^6bx - 28a^5b^2x^2 - 56a^4b^3x^3 - 70a^3b^4x^4 - 56a^2b^5x^5 - 28ab^6x^6 - 8b^7x^7}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**9,x)
```

```
[Out] (-a**7 - 8*a**6*b*x - 28*a**5*b**2*x**2 - 56*a**4*b**3*x**3 - 70*a**3*b**4*
x**4 - 56*a**2*b**5*x**5 - 28*a*b**6*x**6 - 8*b**7*x**7)/(8*x**8)
```

## 3.116 $\int \frac{(a+bx)^7}{x^{10}} dx$

**Optimal**. Leaf size=36

$$\frac{b(a+bx)^8}{72a^2x^8} - \frac{(a+bx)^8}{9ax^9}$$

**Rubi [A]** time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^8}{72a^2x^8} - \frac{(a+bx)^8}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^10,x]
```

```
[Out] -(a + b*x)^8/(9*a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{10}} dx &= -\frac{(a+bx)^8}{9ax^9} - \frac{b\int \frac{(a+bx)^7}{x^9} dx}{9a}\\ &= -\frac{(a+bx)^8}{9ax^9} + \frac{b(a+bx)^8}{72a^2x^8}\end{aligned}$$

**Mathematica [B]** time = 0.00, size = 91, normalized size = 2.53

$$-\frac{a^7}{9x^9}-\frac{7a^6b}{8x^8}-\frac{3a^5b^2}{x^7}-\frac{35a^4b^3}{6x^6}-\frac{7a^3b^4}{x^5}-\frac{21a^2b^5}{4x^4}-\frac{7ab^6}{3x^3}-\frac{b^7}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*a^7/x^9 - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6) -
 (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^10, x]
```

**fricas [B]** time = 1.21, size = 79, normalized size = 2.19

$$-\frac{36\,b^7x^7+168\,ab^6x^6+378\,a^2b^5x^5+504\,a^3b^4x^4+420\,a^4b^3x^3+216\,a^5b^2x^2+63\,a^6bx+8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

**giac [B]** time = 1.01, size = 79, normalized size = 2.19

$$-\frac{36\,b^7x^7+168\,ab^6x^6+378\,a^2b^5x^5+504\,a^3b^4x^4+420\,a^4b^3x^3+216\,a^5b^2x^2+63\,a^6bx+8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

**maple [B]** time = 0.01, size = 80, normalized size = 2.22

$$-\frac{b^{7}}{2x^{2}}-\frac{7a\,b^{6}}{3x^{3}}-\frac{21a^{2}b^{5}}{4x^{4}}-\frac{7a^{3}b^{4}}{x^{5}}-\frac{35a^{4}b^{3}}{6x^{6}}-\frac{3a^{5}b^{2}}{x^{7}}-\frac{7a^{6}b}{8x^{8}}-\frac{a^{7}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^10,x)
```

```
[Out] -7*a^3*b^4/x^5-35/6*a^4*b^3/x^6-1/2*b^7/x^2-7/8*a^6*b/x^8-3*a^5*b^2/x^7-21/
4*a^2*b^5/x^4-7/3*a*b^6/x^3-1/9*a^7/x^9
```

**maxima [B]** time = 1.31, size = 79, normalized size = 2.19

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

**mupad [B]** time = 0.09, size = 23, normalized size = 0.64

$$-\frac{(8\,a-b\,x)\,\left(a+b\,x\right)^{8}}{72\,a^{2}\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^10,x)
```

```
[Out] -((8*a - b*x)*(a + b*x)^8)/(72*a^2*x^9)
```

**sympy [B]** time = 0.73, size = 85, normalized size = 2.36

$$\frac{-8a^{7}-63a^{6}bx-216a^{5}b^{2}x^{2}-420a^{4}b^{3}x^{3}-504a^{3}b^{4}x^{4}-378a^{2}b^{5}x^{5}-168ab^{6}x^{6}-36b^{7}x^{7}}{72x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**10,x)
```

```
[Out] (-8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a**3
*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x**9)
```

## 3.117 $\int \frac{(a+bx)^7}{x^{11}} \, dx$

**Optimal**. Leaf size=56

$$-\frac{b^2(a+bx)^8}{360a^3x^8}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{(a+bx)^8}{10ax^{10}}$$

**Rubi** [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^8}{360a^3x^8}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{(a+bx)^8}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^11,x]
```

```
[Out] -(a + b*x)^8/(10*a*x^10) + (b*(a + b*x)^8)/(45*a^2*x^9) - (b^2*(a + b*x)^8)
/(360*a^3*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^{11}}\,dx = -\frac{(a+bx)^8}{10ax^{10}} - \frac{b\int \frac{(a+bx)^7}{x^{10}}\,dx}{5a}$$
$$= -\frac{(a+bx)^8}{10ax^{10}} + \frac{b(a+bx)^8}{45a^2x^9} + \frac{b^2\int \frac{(a+bx)^7}{x^9}\,dx}{45a^2}$$
$$= -\frac{(a+bx)^8}{10ax^{10}} + \frac{b(a+bx)^8}{45a^2x^9} - \frac{b^2(a+bx)^8}{360a^3x^8}$$

**Mathematica [A]** time = 0.00, size = 93, normalized size = 1.66

$$-\frac{a^7}{10x^{10}} - \frac{7a^6b}{9x^9} - \frac{21a^5b^2}{8x^8} - \frac{5a^4b^3}{x^7} - \frac{35a^3b^4}{6x^6} - \frac{21a^2b^5}{5x^5} - \frac{7ab^6}{4x^4} - \frac{b^7}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^11,x]
```

```
[Out] -1/10*a^7/x^10 - (7*a^6*b)/(9*x^9) - (21*a^5*b^2)/(8*x^8) - (5*a^4*b^3)/x^7
 - (35*a^3*b^4)/(6*x^6) - (21*a^2*b^5)/(5*x^5) - (7*a*b^6)/(4*x^4) - b^7/(3
*x^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^11,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^11, x]
```

**fricas [A]** time = 1.32, size = 79, normalized size = 1.41

$$-\frac{120\,b^7x^7 + 630\,ab^6x^6 + 1512\,a^2b^5x^5 + 2100\,a^3b^4x^4 + 1800\,a^4b^3x^3 + 945\,a^5b^2x^2 + 280\,a^6bx + 36\,a^7}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="fricas")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

**giac** [A] time = 1.06, size = 79, normalized size = 1.41

$$-\frac{120\,b^{7}x^{7}+630\,ab^{6}x^{6}+1512\,a^{2}b^{5}x^{5}+2100\,a^{3}b^{4}x^{4}+1800\,a^{4}b^{3}x^{3}+945\,a^{5}b^{2}x^{2}+280\,a^{6}bx+36\,a^{7}}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="giac")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

**maple** [A] time = 0.00, size = 80, normalized size = 1.43

$$-\frac{b^{7}}{3x^{3}}-\frac{7a\,b^{6}}{4x^{4}}-\frac{21a^{2}b^{5}}{5x^{5}}-\frac{35a^{3}b^{4}}{6x^{6}}-\frac{5a^{4}b^{3}}{x^{7}}-\frac{21a^{5}b^{2}}{8x^{8}}-\frac{7a^{6}b}{9x^{9}}-\frac{a^{7}}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^11,x)
```

```
[Out] -21/5*a^2*b^5/x^5-21/8*a^5*b^2/x^8-35/6*a^3*b^4/x^6-1/10*a^7/x^10-5*a^4*b^3
/x^7-7/4*a*b^6/x^4-1/3*b^7/x^3-7/9*a^6*b/x^9
```

**maxima** [A] time = 1.38, size = 79, normalized size = 1.41

$$-\frac{120\,b^{7}x^{7}+630\,ab^{6}x^{6}+1512\,a^{2}b^{5}x^{5}+2100\,a^{3}b^{4}x^{4}+1800\,a^{4}b^{3}x^{3}+945\,a^{5}b^{2}x^{2}+280\,a^{6}bx+36\,a^{7}}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="maxima")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

**mupad** [B] time = 0.11, size = 79, normalized size = 1.41

$$-\frac{\frac{a^{7}}{10}+\frac{7\,a^{6}\,b\,x}{9}+\frac{21\,a^{5}\,b^{2}\,x^{2}}{8}+5\,a^{4}\,b^{3}\,x^{3}+\frac{35\,a^{3}\,b^{4}\,x^{4}}{6}+\frac{21\,a^{2}\,b^{5}\,x^{5}}{5}+\frac{7\,a\,b^{6}\,x^{6}}{4}+\frac{b^{7}\,x^{7}}{3}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^11,x)
```

```
[Out] -(a^7/10 + (b^7*x^7)/3 + (7*a*b^6*x^6)/4 + (21*a^5*b^2*x^2)/8 + 5*a^4*b^3*x
^3 + (35*a^3*b^4*x^4)/6 + (21*a^2*b^5*x^5)/5 + (7*a^6*b*x)/9)/x^10
```

**sympy [A]** time = 0.76, size = 85, normalized size = 1.52

$$\frac{-36a^7 - 280a^6bx - 945a^5b^2x^2 - 1800a^4b^3x^3 - 2100a^3b^4x^4 - 1512a^2b^5x^5 - 630ab^6x^6 - 120b^7x^7}{360x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**11,x)
```

```
[Out] (-36*a**7 - 280*a**6*b*x - 945*a**5*b**2*x**2 - 1800*a**4*b**3*x**3 - 2100*
a**3*b**4*x**4 - 1512*a**2*b**5*x**5 - 630*a*b**6*x**6 - 120*b**7*x**7)/(36
0*x**10)
```

## 3.118 $\int \frac{(a+bx)^7}{x^{12}} dx$

**Optimal**. Leaf size=76

$$\frac{b^3(a+bx)^8}{1320a^4x^8} - \frac{b^2(a+bx)^8}{165a^3x^9} + \frac{3b(a+bx)^8}{110a^2x^{10}} - \frac{(a+bx)^8}{11ax^{11}}$$

**Rubi [A]** time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b^3(a+bx)^8}{1320a^4x^8} - \frac{b^2(a+bx)^8}{165a^3x^9} + \frac{3b(a+bx)^8}{110a^2x^{10}} - \frac{(a+bx)^8}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^12,x]
```

```
[Out] -(a + b*x)^8/(11*a*x^11) + (3*b*(a + b*x)^8)/(110*a^2*x^10) - (b^2*(a + b*x
)^8)/(165*a^3*x^9) + (b^3*(a + b*x)^8)/(1320*a^4*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^{12}}\,dx = -\frac{(a+bx)^8}{11ax^{11}} - \frac{(3b)\int \frac{(a+bx)^7}{x^{11}}\,dx}{11a}$$
$$= -\frac{(a+bx)^8}{11ax^{11}} + \frac{3b(a+bx)^8}{110a^2x^{10}} + \frac{\left(3b^2\right)\int \frac{(a+bx)^7}{x^{10}}\,dx}{55a^2}$$
$$= -\frac{(a+bx)^8}{11ax^{11}} + \frac{3b(a+bx)^8}{110a^2x^{10}} - \frac{b^2(a+bx)^8}{165a^3x^9} - \frac{b^3\int \frac{(a+bx)^7}{x^9}\,dx}{165a^3}$$
$$= -\frac{(a+bx)^8}{11ax^{11}} + \frac{3b(a+bx)^8}{110a^2x^{10}} - \frac{b^2(a+bx)^8}{165a^3x^9} + \frac{b^3(a+bx)^8}{1320a^4x^8}$$

**Mathematica [A]** time = 0.00, size = 93, normalized size = 1.22

$$-\frac{a^7}{11x^{11}} - \frac{7a^6b}{10x^{10}} - \frac{7a^5b^2}{3x^9} - \frac{35a^4b^3}{8x^8} - \frac{5a^3b^4}{x^7} - \frac{7a^2b^5}{2x^6} - \frac{7ab^6}{5x^5} - \frac{b^7}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^12,x]
```

```
[Out] -1/11*a^7/x^11 - (7*a^6*b)/(10*x^10) - (7*a^5*b^2)/(3*x^9) - (35*a^4*b^3)/(
8*x^8) - (5*a^3*b^4)/x^7 - (7*a^2*b^5)/(2*x^6) - (7*a*b^6)/(5*x^5) - b^7/(4
*x^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^12,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^12, x]
```

**fricas [A]** time = 0.94, size = 79, normalized size = 1.04

$$-\frac{330\,b^7x^7 + 1848\,ab^6x^6 + 4620\,a^2b^5x^5 + 6600\,a^3b^4x^4 + 5775\,a^4b^3x^3 + 3080\,a^5b^2x^2 + 924\,a^6bx + 120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

**giac [A]** time = 1.15, size = 79, normalized size = 1.04

$$-\frac{330\,b^7x^7+1848\,ab^6x^6+4620\,a^2b^5x^5+6600\,a^3b^4x^4+5775\,a^4b^3x^3+3080\,a^5b^2x^2+924\,a^6bx+120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="giac")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

**maple [A]** time = 0.01, size = 80, normalized size = 1.05

$$-\frac{b^7}{4x^4}-\frac{7a\,b^6}{5x^5}-\frac{7a^2b^5}{2x^6}-\frac{5a^3b^4}{x^7}-\frac{35a^4b^3}{8x^8}-\frac{7a^5b^2}{3x^9}-\frac{7a^6b}{10x^{10}}-\frac{a^7}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^12,x)
```

```
[Out] -7/5*a*b^6/x^5-7/3*a^5*b^2/x^9-5*a^3*b^4/x^7-7/2*a^2*b^5/x^6-1/4*b^7/x^4-7/
10*a^6*b/x^10-1/11*a^7/x^11-35/8*a^4*b^3/x^8
```

**maxima [A]** time = 1.36, size = 79, normalized size = 1.04

$$-\frac{330\,b^7x^7+1848\,ab^6x^6+4620\,a^2b^5x^5+6600\,a^3b^4x^4+5775\,a^4b^3x^3+3080\,a^5b^2x^2+924\,a^6bx+120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

**mupad [B]** time = 0.11, size = 79, normalized size = 1.04

$$-\frac{\frac{a^7}{11}+\frac{7\,a^6\,b\,x}{10}+\frac{7\,a^5\,b^2\,x^2}{3}+\frac{35\,a^4\,b^3\,x^3}{8}+5\,a^3\,b^4\,x^4+\frac{7\,a^2\,b^5\,x^5}{2}+\frac{7\,a\,b^6\,x^6}{5}+\frac{b^7\,x^7}{4}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^12,x)
```

```
[Out] -(a^7/11 + (b^7*x^7)/4 + (7*a*b^6*x^6)/5 + (7*a^5*b^2*x^2)/3 + (35*a^4*b^3*
x^3)/8 + 5*a^3*b^4*x^4 + (7*a^2*b^5*x^5)/2 + (7*a^6*b*x)/10)/x^11
```

**sympy** [**A**] time = 0.75, size = 85, normalized size = 1.12

$$\frac{-120a^7 - 924a^6bx - 3080a^5b^2x^2 - 5775a^4b^3x^3 - 6600a^3b^4x^4 - 4620a^2b^5x^5 - 1848ab^6x^6 - 330b^7x^7}{1320x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**12,x)
```

```
[Out] (-120*a**7 - 924*a**6*b*x - 3080*a**5*b**2*x**2 - 5775*a**4*b**3*x**3 - 660
0*a**3*b**4*x**4 - 4620*a**2*b**5*x**5 - 1848*a*b**6*x**6 - 330*b**7*x**7)/
(1320*x**11)
```

## 3.119 $\int \frac{(a+bx)^7}{x^{13}} \, dx$

**Optimal**. Leaf size=96

$$-\frac{b^4(a+bx)^8}{3960a^5x^8}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{(a+bx)^8}{12ax^{12}}$$

**Rubi [A]** time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^4(a+bx)^8}{3960a^5x^8}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{(a+bx)^8}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^13,x]
```

```
[Out] -(a + b*x)^8/(12*a*x^12) + (b*(a + b*x)^8)/(33*a^2*x^11) - (b^2*(a + b*x)^8
)/(110*a^3*x^10) + (b^3*(a + b*x)^8)/(495*a^4*x^9) - (b^4*(a + b*x)^8)/(396
0*a^5*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^{13}}\,dx = -\frac{(a+bx)^8}{12ax^{12}} - \frac{b\int \frac{(a+bx)^7}{x^{12}}\,dx}{3a}$$
$$= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} + \frac{b^2\int \frac{(a+bx)^7}{x^{11}}\,dx}{11a^2}$$
$$= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} - \frac{b^3\int \frac{(a+bx)^7}{x^{10}}\,dx}{55a^3}$$
$$= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} + \frac{b^3(a+bx)^8}{495a^4x^9} + \frac{b^4\int \frac{(a+bx)^7}{x^9}\,dx}{495a^4}$$
$$= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} + \frac{b^3(a+bx)^8}{495a^4x^9} - \frac{b^4(a+bx)^8}{3960a^5x^8}$$

**Mathematica [A]** time = 0.00, size = 93, normalized size = 0.97

$$-\frac{a^7}{12x^{12}} - \frac{7a^6b}{11x^{11}} - \frac{21a^5b^2}{10x^{10}} - \frac{35a^4b^3}{9x^9} - \frac{35a^3b^4}{8x^8} - \frac{3a^2b^5}{x^7} - \frac{7ab^6}{6x^6} - \frac{b^7}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^13,x]
```

```
[Out] -1/12*a^7/x^12 - (7*a^6*b)/(11*x^11) - (21*a^5*b^2)/(10*x^10) - (35*a^4*b^3
)/(9*x^9) - (35*a^3*b^4)/(8*x^8) - (3*a^2*b^5)/x^7 - (7*a*b^6)/(6*x^6) - b^
7/(5*x^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^13,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^13, x]
```

**fricas [A]** time = 1.13, size = 79, normalized size = 0.82

$$-\frac{792\,b^7x^7 + 4620\,ab^6x^6 + 11880\,a^2b^5x^5 + 17325\,a^3b^4x^4 + 15400\,a^4b^3x^3 + 8316\,a^5b^2x^2 + 2520\,a^6bx + 330\,a^7}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="fricas")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

**giac** [A] time = 1.15, size = 79, normalized size = 0.82

$$-\frac{792\,b^{7}x^{7}+4620\,ab^{6}x^{6}+11880\,a^{2}b^{5}x^{5}+17325\,a^{3}b^{4}x^{4}+15400\,a^{4}b^{3}x^{3}+8316\,a^{5}b^{2}x^{2}+2520\,a^{6}bx+330\,a^{7}}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="giac")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

**maple** [A] time = 0.01, size = 80, normalized size = 0.83

$$-\frac{b^{7}}{5x^{5}}-\frac{7a\,b^{6}}{6x^{6}}-\frac{3a^{2}b^{5}}{x^{7}}-\frac{35a^{3}b^{4}}{8x^{8}}-\frac{35a^{4}b^{3}}{9x^{9}}-\frac{21a^{5}b^{2}}{10x^{10}}-\frac{7a^{6}b}{11x^{11}}-\frac{a^{7}}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^13,x)
```

```
[Out] -1/5*b^7/x^5-7/6*a*b^6/x^6-35/8*a^3*b^4/x^8-7/11*a^6*b/x^11-3*a^2*b^5/x^7-3
5/9*a^4*b^3/x^9-21/10*a^5*b^2/x^10-1/12*a^7/x^12
```

**maxima** [A] time = 1.45, size = 79, normalized size = 0.82

$$-\frac{792\,b^{7}x^{7}+4620\,ab^{6}x^{6}+11880\,a^{2}b^{5}x^{5}+17325\,a^{3}b^{4}x^{4}+15400\,a^{4}b^{3}x^{3}+8316\,a^{5}b^{2}x^{2}+2520\,a^{6}bx+330\,a^{7}}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="maxima")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

**mupad** [B] time = 0.07, size = 79, normalized size = 0.82

$$-\frac{\frac{a^{7}}{12}+\frac{7\,a^{6}\,b\,x}{11}+\frac{21\,a^{5}\,b^{2}\,x^{2}}{10}+\frac{35\,a^{4}\,b^{3}\,x^{3}}{9}+\frac{35\,a^{3}\,b^{4}\,x^{4}}{8}+3\,a^{2}\,b^{5}\,x^{5}+\frac{7\,a\,b^{6}\,x^{6}}{6}+\frac{b^{7}\,x^{7}}{5}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^13,x)
```

```
[Out] -(a^7/12 + (b^7*x^7)/5 + (7*a*b^6*x^6)/6 + (21*a^5*b^2*x^2)/10 + (35*a^4*b^
3*x^3)/9 + (35*a^3*b^4*x^4)/8 + 3*a^2*b^5*x^5 + (7*a^6*b*x)/11)/x^12
```

**sympy [A]** time = 0.80, size = 85, normalized size = 0.89

$$\frac{-330a^7 - 2520a^6bx - 8316a^5b^2x^2 - 15400a^4b^3x^3 - 17325a^3b^4x^4 - 11880a^2b^5x^5 - 4620ab^6x^6 - 792b^7x^7}{3960x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**13,x)
```

```
[Out] (-330*a**7 - 2520*a**6*b*x - 8316*a**5*b**2*x**2 - 15400*a**4*b**3*x**3 - 1
7325*a**3*b**4*x**4 - 11880*a**2*b**5*x**5 - 4620*a*b**6*x**6 - 792*b**7*x*
*7)/(3960*x**12)
```

## 3.120 $\int \frac{(a+bx)^7}{x^{14}}\,dx$

**Optimal**. Leaf size=93

$$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

**Rubi** [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{7a^6b}{12x^{12}}-\frac{a^7}{13x^{13}}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^14,x]
```

```
[Out] -a^7/(13*x^13) - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3)
/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b^7
/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{14}}\,dx &= \int \left(\frac{a^7}{x^{14}}+\frac{7a^6b}{x^{13}}+\frac{21a^5b^2}{x^{12}}+\frac{35a^4b^3}{x^{11}}+\frac{35a^3b^4}{x^{10}}+\frac{21a^2b^5}{x^9}+\frac{7ab^6}{x^8}+\frac{b^7}{x^7}\right)dx\\ &= -\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 93, normalized size = 1.00

$$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^14,x]
```

```
[Out] -1/13*a^7/x^13 - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3)
/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b^7
/(6*x^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^14,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^14, x]
```

**fricas [A]** time = 0.86, size = 79, normalized size = 0.85

$$-\frac{1716\,b^7x^7 + 10296\,ab^6x^6 + 27027\,a^2b^5x^5 + 40040\,a^3b^4x^4 + 36036\,a^4b^3x^3 + 19656\,a^5b^2x^2 + 6006\,a^6bx + 792\,a^7}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

**giac [A]** time = 1.04, size = 79, normalized size = 0.85

$$-\frac{1716\,b^7x^7 + 10296\,ab^6x^6 + 27027\,a^2b^5x^5 + 40040\,a^3b^4x^4 + 36036\,a^4b^3x^3 + 19656\,a^5b^2x^2 + 6006\,a^6bx + 792\,a^7}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

**maple [A]** time = 0.00, size = 80, normalized size = 0.86

$$-\frac{b^7}{6x^6} - \frac{a\,b^6}{x^7} - \frac{21a^2b^5}{8x^8} - \frac{35a^3b^4}{9x^9} - \frac{7a^4b^3}{2x^{10}} - \frac{21a^5b^2}{11x^{11}} - \frac{7a^6b}{12x^{12}} - \frac{a^7}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^14,x)
```

```
[Out] -1/13*a^7/x^13-7/12*a^6*b/x^12-21/11*a^5*b^2/x^11-7/2*a^4*b^3/x^10-35/9*a^3
*b^4/x^9-21/8*a^2*b^5/x^8-a*b^6/x^7-1/6*b^7/x^6
```

**maxima [A]** time = 1.37, size = 79, normalized size = 0.85

$$-\frac{1716\,b^7x^7 + 10296\,ab^6x^6 + 27027\,a^2b^5x^5 + 40040\,a^3b^4x^4 + 36036\,a^4b^3x^3 + 19656\,a^5b^2x^2 + 6006\,a^6bx + 792\,a^7}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

**mupad [B]** time = 0.07, size = 78, normalized size = 0.84

$$-\frac{\frac{a^7}{13} + \frac{7\,a^6\,b\,x}{12} + \frac{21\,a^5\,b^2\,x^2}{11} + \frac{7\,a^4\,b^3\,x^3}{2} + \frac{35\,a^3\,b^4\,x^4}{9} + \frac{21\,a^2\,b^5\,x^5}{8} + a\,b^6\,x^6 + \frac{b^7\,x^7}{6}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^14,x)
```

```
[Out] -(a^7/13 + (b^7*x^7)/6 + a*b^6*x^6 + (21*a^5*b^2*x^2)/11 + (7*a^4*b^3*x^3)/
2 + (35*a^3*b^4*x^4)/9 + (21*a^2*b^5*x^5)/8 + (7*a^6*b*x)/12)/x^13
```

**sympy [A]** time = 0.78, size = 85, normalized size = 0.91

$$\frac{-792a^7 - 6006a^6bx - 19656a^5b^2x^2 - 36036a^4b^3x^3 - 40040a^3b^4x^4 - 27027a^2b^5x^5 - 10296ab^6x^6 - 1716b^7x^7}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**14,x)
```

```
[Out] (-792*a**7 - 6006*a**6*b*x - 19656*a**5*b**2*x**2 - 36036*a**4*b**3*x**3 -
40040*a**3*b**4*x**4 - 27027*a**2*b**5*x**5 - 10296*a*b**6*x**6 - 1716*b**7
*x**7)/(10296*x**13)
```

## 3.121 $\int \frac{(a+bx)^7}{x^{15}}\,dx$

**Optimal**. Leaf size=95

$$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

**Rubi** [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7a^6b}{13x^{13}}-\frac{a^7}{14x^{14}}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^15,x]
```

```
[Out] -a^7/(14*x^14) - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)/
(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8)
- b^7/(7*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{15}}\,dx &= \int \left(\frac{a^7}{x^{15}}+\frac{7a^6b}{x^{14}}+\frac{21a^5b^2}{x^{13}}+\frac{35a^4b^3}{x^{12}}+\frac{35a^3b^4}{x^{11}}+\frac{21a^2b^5}{x^{10}}+\frac{7ab^6}{x^9}+\frac{b^7}{x^8}\right)dx\\ &= -\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 95, normalized size = 1.00

$$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^15,x]
```

```
[Out] -1/14*a^7/x^14 - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)/
(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8)
- b^7/(7*x^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^15,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^15, x]
```

**fricas [A]** time = 0.82, size = 79, normalized size = 0.83

$$-\frac{3432\,b^{7}x^{7}+21021\,ab^{6}x^{6}+56056\,a^{2}b^{5}x^{5}+84084\,a^{3}b^{4}x^{4}+76440\,a^{4}b^{3}x^{3}+42042\,a^{5}b^{2}x^{2}+12936\,a^{6}bx+1716\,a^{7}}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="fricas")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

**giac [A]** time = 0.89, size = 79, normalized size = 0.83

$$-\frac{3432\,b^{7}x^{7}+21021\,ab^{6}x^{6}+56056\,a^{2}b^{5}x^{5}+84084\,a^{3}b^{4}x^{4}+76440\,a^{4}b^{3}x^{3}+42042\,a^{5}b^{2}x^{2}+12936\,a^{6}bx+1716\,a^{7}}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="giac")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

**maple [A]** time = 0.01, size = 80, normalized size = 0.84

$$-\frac{b^{7}}{7x^{7}}-\frac{7a\,b^{6}}{8x^{8}}-\frac{7a^{2}b^{5}}{3x^{9}}-\frac{7a^{3}b^{4}}{2x^{10}}-\frac{35a^{4}b^{3}}{11x^{11}}-\frac{7a^{5}b^{2}}{4x^{12}}-\frac{7a^{6}b}{13x^{13}}-\frac{a^{7}}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^15,x)
```

```
[Out] -1/14*a^7/x^14-7/13*a^6*b/x^13-7/4*a^5*b^2/x^12-35/11*a^4*b^3/x^11-7/2*a^3*
b^4/x^10-7/3*a^2*b^5/x^9-7/8*a*b^6/x^8-1/7*b^7/x^7
```

**maxima** [A] time = 1.34, size = 79, normalized size = 0.83

$$-\frac{3432\,b^7x^7+21021\,ab^6x^6+56056\,a^2b^5x^5+84084\,a^3b^4x^4+76440\,a^4b^3x^3+42042\,a^5b^2x^2+12936\,a^6bx+1716\,a^7}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="maxima")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

**mupad** [B] time = 0.07, size = 79, normalized size = 0.83

$$-\frac{\frac{a^7}{14}+\frac{7\,a^6\,b\,x}{13}+\frac{7\,a^5\,b^2\,x^2}{4}+\frac{35\,a^4\,b^3\,x^3}{11}+\frac{7\,a^3\,b^4\,x^4}{2}+\frac{7\,a^2\,b^5\,x^5}{3}+\frac{7\,a\,b^6\,x^6}{8}+\frac{b^7\,x^7}{7}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^15,x)
```

```
[Out] -(a^7/14 + (b^7*x^7)/7 + (7*a*b^6*x^6)/8 + (7*a^5*b^2*x^2)/4 + (35*a^4*b^3*
x^3)/11 + (7*a^3*b^4*x^4)/2 + (7*a^2*b^5*x^5)/3 + (7*a^6*b*x)/13)/x^14
```

**sympy** [A] time = 0.88, size = 85, normalized size = 0.89

$$\frac{-1716a^7-12936a^6bx-42042a^5b^2x^2-76440a^4b^3x^3-84084a^3b^4x^4-56056a^2b^5x^5-21021ab^6x^6-3432b^7x^7}{24024x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**15,x)
```

```
[Out] (-1716*a**7 - 12936*a**6*b*x - 42042*a**5*b**2*x**2 - 76440*a**4*b**3*x**3
- 84084*a**3*b**4*x**4 - 56056*a**2*b**5*x**5 - 21021*a*b**6*x**6 - 3432*b*
*7*x**7)/(24024*x**14)
```

## 3.122 $\int \frac{(a+bx)^7}{x^{16}} dx$

**Optimal**. Leaf size=95

$$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

**Rubi** [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{a^6b}{2x^{14}}-\frac{a^7}{15x^{15}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^16,x]
```

```
[Out] -a^7/(15*x^15) - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/(
12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9*x
^9) - b^7/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{16}}\,dx &= \int \left(\frac{a^7}{x^{16}}+\frac{7a^6b}{x^{15}}+\frac{21a^5b^2}{x^{14}}+\frac{35a^4b^3}{x^{13}}+\frac{35a^3b^4}{x^{12}}+\frac{21a^2b^5}{x^{11}}+\frac{7ab^6}{x^{10}}+\frac{b^7}{x^9}\right)dx\\ &= -\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 95, normalized size = 1.00

$$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^16,x]
```

```
[Out] -1/15*a^7/x^15 - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/(
12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9*x
^9) - b^7/(8*x^8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^16,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^16, x]
```

**fricas [A]** time = 1.43, size = 79, normalized size = 0.83

$$-\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6bx+3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="fricas")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

**giac [A]** time = 1.00, size = 79, normalized size = 0.83

$$-\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6bx+3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="giac")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

**maple [A]** time = 0.00, size = 80, normalized size = 0.84

$$-\frac{b^7}{8x^8}-\frac{7a\,b^6}{9x^9}-\frac{21a^2b^5}{10x^{10}}-\frac{35a^3b^4}{11x^{11}}-\frac{35a^4b^3}{12x^{12}}-\frac{21a^5b^2}{13x^{13}}-\frac{a^6b}{2x^{14}}-\frac{a^7}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^16,x)
```

```
[Out] -1/15*a^7/x^15-1/2*a^6*b/x^14-21/13*a^5*b^2/x^13-35/12*a^4*b^3/x^12-35/11*a
^3*b^4/x^11-21/10*a^2*b^5/x^10-7/9*a*b^6/x^9-1/8*b^7/x^8
```

**maxima [A]** time = 1.39, size = 79, normalized size = 0.83

$$-\frac{6435\,b^7x^7 + 40040\,ab^6x^6 + 108108\,a^2b^5x^5 + 163800\,a^3b^4x^4 + 150150\,a^4b^3x^3 + 83160\,a^5b^2x^2 + 25740\,a^6bx + 3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="maxima")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

**mupad [B]** time = 0.11, size = 79, normalized size = 0.83

$$-\frac{\frac{a^7}{15} + \frac{a^6\,b\,x}{2} + \frac{21\,a^5\,b^2\,x^2}{13} + \frac{35\,a^4\,b^3\,x^3}{12} + \frac{35\,a^3\,b^4\,x^4}{11} + \frac{21\,a^2\,b^5\,x^5}{10} + \frac{7\,a\,b^6\,x^6}{9} + \frac{b^7\,x^7}{8}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^16,x)
```

```
[Out] -(a^7/15 + (b^7*x^7)/8 + (7*a*b^6*x^6)/9 + (21*a^5*b^2*x^2)/13 + (35*a^4*b^
3*x^3)/12 + (35*a^3*b^4*x^4)/11 + (21*a^2*b^5*x^5)/10 + (a^6*b*x)/2)/x^15
```

**sympy [A]** time = 0.88, size = 85, normalized size = 0.89

$$\frac{-3432a^7 - 25740a^6bx - 83160a^5b^2x^2 - 150150a^4b^3x^3 - 163800a^3b^4x^4 - 108108a^2b^5x^5 - 40040ab^6x^6 - 6435b^7x^7}{51480x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**16,x)
```

```
[Out] (-3432*a**7 - 25740*a**6*b*x - 83160*a**5*b**2*x**2 - 150150*a**4*b**3*x**3
 - 163800*a**3*b**4*x**4 - 108108*a**2*b**5*x**5 - 40040*a*b**6*x**6 - 6435
*b**7*x**7)/(51480*x**15)
```

## 3.123 $\int x^{11}(a+bx)^{10}\,dx$

**Optimal**. Leaf size=132

$$\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab^9$$

**Rubi [A]** time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{9}{4}a^2b^8x^{20}+\frac{120}{19}a^3b^7x^{19}+\frac{35}{3}a^4b^6x^{18}+\frac{252}{17}a^5b^5x^{17}+\frac{105}{8}a^6b^4x^{16}+8a^7b^3x^{15}+\frac{45}{14}a^8b^2x^{14}+\frac{10}{13}a^9bx^{13}+\frac{a^{10}x^{12}}{12}+\frac{10}{21}ab^9x^{21}+\frac{b^{10}x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x)^10,x]
```

```
[Out] (a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15
 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (12
0*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^22)/
22
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{11}(a+bx)^{10}\,dx=\int \left(a^{10}x^{11}+10a^9bx^{12}+45a^8b^2x^{13}+120a^7b^3x^{14}+210a^6b^4x^{15}+252a^5b^5x^{16}+210a^4b^6x^{1}\right.$$

$$=\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+$$

**Mathematica [A]** time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab^9x^{21}+\frac{b^{10}x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x)^10,x]
```

```
[Out] (a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15
 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (12
0*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^22)/
22
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^11*(a + b*x)^10, x]
```

**fricas [A]** time = 1.23, size = 112, normalized size = 0.85

$$\frac{1}{22}x^{22}b^{10}+\frac{10}{21}x^{21}b^{9}a+\frac{9}{4}x^{20}b^{8}a^{2}+\frac{120}{19}x^{19}b^{7}a^{3}+\frac{35}{3}x^{18}b^{6}a^{4}+\frac{252}{17}x^{17}b^{5}a^{5}+\frac{105}{8}x^{16}b^{4}a^{6}+8x^{15}b^{3}a^{7}+\frac{45}{14}x^{14}b^{2}a^{8}+\frac{10}{13}x^{13}ba^{9}+\frac{1}{12}x^{12}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^10 + 10/21*x^21*b^9*a + 9/4*x^20*b^8*a^2 + 120/19*x^19*b^7*a^3
+ 35/3*x^18*b^6*a^4 + 252/17*x^17*b^5*a^5 + 105/8*x^16*b^4*a^6 + 8*x^15*b^3
*a^7 + 45/14*x^14*b^2*a^8 + 10/13*x^13*b*a^9 + 1/12*x^12*a^10
```

**giac [A]** time = 0.98, size = 112, normalized size = 0.85

$$\frac{1}{22}b^{10}x^{22}+\frac{10}{21}ab^{9}x^{21}+\frac{9}{4}a^{2}b^{8}x^{20}+\frac{120}{19}a^{3}b^{7}x^{19}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{105}{8}a^{6}b^{4}x^{16}+8a^{7}b^{3}x^{15}+\frac{45}{14}a^{8}b^{2}x^{14}+\frac{10}{13}a^{9}bx^{13}+\frac{1}{12}a^{10}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

**maple [A]** time = 0.00, size = 113, normalized size = 0.86

$$\frac{1}{22}b^{10}x^{22}+\frac{10}{21}a\,b^{9}x^{21}+\frac{9}{4}a^{2}b^{8}x^{20}+\frac{120}{19}a^{3}b^{7}x^{19}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{105}{8}a^{6}b^{4}x^{16}+8a^{7}b^{3}x^{15}+\frac{45}{14}a^{8}b^{2}x^{14}+\frac{10}{13}a^{9}b\,x^{13}+\frac{1}{12}a^{10}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x+a)^10,x)
```

```
[Out] 1/12*a^10*x^12+10/13*a^9*b*x^13+45/14*a^8*b^2*x^14+8*a^7*b^3*x^15+105/8*a^6
*b^4*x^16+252/17*a^5*b^5*x^17+35/3*a^4*b^6*x^18+120/19*a^3*b^7*x^19+9/4*a^2
*b^8*x^20+10/21*a*b^9*x^21+1/22*b^10*x^22
```

**maxima [A]** time = 1.33, size = 112, normalized size = 0.85

$$\frac{1}{22}\,b^{10}x^{22}+\frac{10}{21}\,ab^{9}x^{21}+\frac{9}{4}\,a^{2}b^{8}x^{20}+\frac{120}{19}\,a^{3}b^{7}x^{19}+\frac{35}{3}\,a^{4}b^{6}x^{18}+\frac{252}{17}\,a^{5}b^{5}x^{17}+\frac{105}{8}\,a^{6}b^{4}x^{16}+8\,a^{7}b^{3}x^{15}+\frac{45}{14}\,a^{8}b^{2}x^{14}+\frac{10}{13}\,a^{9}bx^{13}+\frac{1}{12}\,a^{10}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

**mupad [B]** time = 0.15, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{12}}{12}+\frac{10\,a^9\,b\,x^{13}}{13}+\frac{45\,a^8\,b^2\,x^{14}}{14}+8\,a^7\,b^3\,x^{15}+\frac{105\,a^6\,b^4\,x^{16}}{8}+\frac{252\,a^5\,b^5\,x^{17}}{17}+\frac{35\,a^4\,b^6\,x^{18}}{3}+\frac{120\,a^3\,b^7\,x^{19}}{19}+\frac{9\,a^2\,b^8\,x^{20}}{4}+\frac{10\,a\,b^9\,x^{21}}{21}+\frac{b^{10}\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x)^10,x)
```

```
[Out] (a^10*x^12)/12 + (b^10*x^22)/22 + (10*a^9*b*x^13)/13 + (10*a*b^9*x^21)/21 +
 (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^
5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (120*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)
/4
```

**sympy [A]** time = 0.11, size = 133, normalized size = 1.01

$$\frac{a^{10}x^{12}}{12}+\frac{10a^{9}bx^{13}}{13}+\frac{45a^{8}b^{2}x^{14}}{14}+8a^{7}b^{3}x^{15}+\frac{105a^{6}b^{4}x^{16}}{8}+\frac{252a^{5}b^{5}x^{17}}{17}+\frac{35a^{4}b^{6}x^{18}}{3}+\frac{120a^{3}b^{7}x^{19}}{19}+\frac{9a^{2}b^{8}x^{20}}{4}+\frac{10ab^{9}x^{21}}{21}+\frac{b^{10}x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x+a)**10,x)
```

```
[Out] a**10*x**12/12 + 10*a**9*b*x**13/13 + 45*a**8*b**2*x**14/14 + 8*a**7*b**3*x
**15 + 105*a**6*b**4*x**16/8 + 252*a**5*b**5*x**17/17 + 35*a**4*b**6*x**18/
3 + 120*a**3*b**7*x**19/19 + 9*a**2*b**8*x**20/4 + 10*a*b**9*x**21/21 + b**
10*x**22/22
```

## 3.124 $\int x^{10}(a+bx)^{10}\,dx$

**Optimal**. Leaf size=132

$$\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x^{20}$$

**Rubi [A]** time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{45}{19}a^2b^8x^{19}+\frac{20}{3}a^3b^7x^{18}+\frac{210}{17}a^4b^6x^{17}+\frac{63}{4}a^5b^5x^{16}+14a^6b^4x^{15}+\frac{60}{7}a^7b^3x^{14}+\frac{45}{13}a^8b^2x^{13}+\frac{5}{6}a^9bx^{12}+\frac{a^{10}x^{11}}{11}+\frac{1}{2}ab^9x^{20}+\frac{b^{10}x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a + b*x)^10,x]
```

```
[Out] (a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14
)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20*a
^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{10}(a+bx)^{10}\,dx = \int \left(a^{10}x^{10}+10a^9bx^{11}+45a^8b^2x^{12}+120a^7b^3x^{13}+210a^6b^4x^{14}+252a^5b^5x^{15}+210a^4b^6x^{16}\right.$$
$$= \frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}$$

**Mathematica [A]** time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x^{20}+\frac{b^{10}x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a + b*x)^10,x]
```

```
[Out] (a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14
)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20*a
^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{10}(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^10*(a + b*x)^10, x]
```

**fricas [A]** time = 1.05, size = 112, normalized size = 0.85

$$\frac{1}{21}x^{21}b^{10}+\frac{1}{2}x^{20}b^{9}a+\frac{45}{19}x^{19}b^{8}a^{2}+\frac{20}{3}x^{18}b^{7}a^{3}+\frac{210}{17}x^{17}b^{6}a^{4}+\frac{63}{4}x^{16}b^{5}a^{5}+14x^{15}b^{4}a^{6}+\frac{60}{7}x^{14}b^{3}a^{7}+\frac{45}{13}x^{13}b^{2}a^{8}+\frac{5}{6}x^{12}ba^{9}+\frac{1}{11}x^{11}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^10 + 1/2*x^20*b^9*a + 45/19*x^19*b^8*a^2 + 20/3*x^18*b^7*a^3 +
210/17*x^17*b^6*a^4 + 63/4*x^16*b^5*a^5 + 14*x^15*b^4*a^6 + 60/7*x^14*b^3*a
^7 + 45/13*x^13*b^2*a^8 + 5/6*x^12*b*a^9 + 1/11*x^11*a^10
```

**giac [A]** time = 0.99, size = 112, normalized size = 0.85

$$\frac{1}{21}\,b^{10}x^{21}+\frac{1}{2}\,ab^{9}x^{20}+\frac{45}{19}\,a^{2}b^{8}x^{19}+\frac{20}{3}\,a^{3}b^{7}x^{18}+\frac{210}{17}\,a^{4}b^{6}x^{17}+\frac{63}{4}\,a^{5}b^{5}x^{16}+14\,a^{6}b^{4}x^{15}+\frac{60}{7}\,a^{7}b^{3}x^{14}+\frac{45}{13}\,a^{8}b^{2}x^{13}+\frac{5}{6}\,a^{9}bx^{12}+\frac{1}{11}\,a^{10}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^
14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

**maple [A]** time = 0.00, size = 113, normalized size = 0.86

$$\frac{1}{21}b^{10}x^{21}+\frac{1}{2}a\,b^{9}x^{20}+\frac{45}{19}a^{2}b^{8}x^{19}+\frac{20}{3}a^{3}b^{7}x^{18}+\frac{210}{17}a^{4}b^{6}x^{17}+\frac{63}{4}a^{5}b^{5}x^{16}+14a^{6}b^{4}x^{15}+\frac{60}{7}a^{7}b^{3}x^{14}+\frac{45}{13}a^{8}b^{2}x^{13}+\frac{5}{6}a^{9}b\,x^{12}+\frac{1}{11}a^{10}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b*x+a)^10,x)
```

```
[Out] 1/11*a^10*x^11+5/6*a^9*b*x^12+45/13*a^8*b^2*x^13+60/7*a^7*b^3*x^14+14*a^6*b
^4*x^15+63/4*a^5*b^5*x^16+210/17*a^4*b^6*x^17+20/3*a^3*b^7*x^18+45/19*a^2*b
^8*x^19+1/2*a*b^9*x^20+1/21*b^10*x^21
```

**maxima [A]** time = 1.34, size = 112, normalized size = 0.85

$$\frac{1}{21}\,b^{10}x^{21}+\frac{1}{2}\,ab^{9}x^{20}+\frac{45}{19}\,a^{2}b^{8}x^{19}+\frac{20}{3}\,a^{3}b^{7}x^{18}+\frac{210}{17}\,a^{4}b^{6}x^{17}+\frac{63}{4}\,a^{5}b^{5}x^{16}+14\,a^{6}b^{4}x^{15}+\frac{60}{7}\,a^{7}b^{3}x^{14}+\frac{45}{13}\,a^{8}b^{2}x^{13}+\frac{5}{6}\,a^{9}bx^{12}+\frac{1}{11}\,a^{10}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^
14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

**mupad [B]** time = 0.08, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{11}}{11}+\frac{5\,a^{9}\,b\,x^{12}}{6}+\frac{45\,a^{8}\,b^{2}\,x^{13}}{13}+\frac{60\,a^{7}\,b^{3}\,x^{14}}{7}+14\,a^{6}\,b^{4}\,x^{15}+\frac{63\,a^{5}\,b^{5}\,x^{16}}{4}+\frac{210\,a^{4}\,b^{6}\,x^{17}}{17}+\frac{20\,a^{3}\,b^{7}\,x^{18}}{3}+\frac{45\,a^{2}\,b^{8}\,x^{19}}{19}+\frac{a\,b^{9}\,x^{20}}{2}+\frac{b^{10}\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b*x)^10,x)
```

```
[Out] (a^10*x^11)/11 + (b^10*x^21)/21 + (5*a^9*b*x^12)/6 + (a*b^9*x^20)/2 + (45*a
^8*b^2*x^13)/13 + (60*a^7*b^3*x^14)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)
/4 + (210*a^4*b^6*x^17)/17 + (20*a^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19
```

**sympy [A]** time = 0.11, size = 131, normalized size = 0.99

$$\frac{a^{10}x^{11}}{11}+\frac{5a^{9}bx^{12}}{6}+\frac{45a^{8}b^{2}x^{13}}{13}+\frac{60a^{7}b^{3}x^{14}}{7}+14a^{6}b^{4}x^{15}+\frac{63a^{5}b^{5}x^{16}}{4}+\frac{210a^{4}b^{6}x^{17}}{17}+\frac{20a^{3}b^{7}x^{18}}{3}+\frac{45a^{2}b^{8}x^{19}}{19}+\frac{ab^{9}x^{20}}{2}+\frac{b^{10}x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b*x+a)**10,x)
```

```
[Out] a**10*x**11/11 + 5*a**9*b*x**12/6 + 45*a**8*b**2*x**13/13 + 60*a**7*b**3*x*
*14/7 + 14*a**6*b**4*x**15 + 63*a**5*b**5*x**16/4 + 210*a**4*b**6*x**17/17
+ 20*a**3*b**7*x**18/3 + 45*a**2*b**8*x**19/19 + a*b**9*x**20/2 + b**10*x**
21/21
```

## 3.125 $\int x^9(a+bx)^{10}\,dx$

**Optimal**. Leaf size=132

$$\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}ab$$

**Rubi [A]** time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5}{2}a^2b^8x^{18}+\frac{120}{17}a^3b^7x^{17}+\frac{105}{8}a^4b^6x^{16}+\frac{84}{5}a^5b^5x^{15}+15a^6b^4x^{14}+\frac{120}{13}a^7b^3x^{13}+\frac{15}{4}a^8b^2x^{12}+\frac{10}{11}a^9bx^{11}+\frac{a^{10}x^{10}}{10}+\frac{10}{19}ab^9x^{19}+\frac{b^{10}x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x)^10,x]
```

```
[Out] (a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^
13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (12
0*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^20)/
20
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^9(a+bx)^{10}\,dx &= \int \left(a^{10}x^9+10a^9bx^{10}+45a^8b^2x^{11}+120a^7b^3x^{12}+210a^6b^4x^{13}+252a^5b^5x^{14}+210a^4b^6x^{15}\right.\\ &= \frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}ab^9x^{19}+\frac{b^{10}x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x)^10,x]
```

```
[Out] (a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^
13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (12
0*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^20)/
20
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^9(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^9*(a + b*x)^10, x]
```

**fricas [A]** time = 0.74, size = 112, normalized size = 0.85

$$\frac{1}{20}x^{20}b^{10}+\frac{10}{19}x^{19}b^9a+\frac{5}{2}x^{18}b^8a^2+\frac{120}{17}x^{17}b^7a^3+\frac{105}{8}x^{16}b^6a^4+\frac{84}{5}x^{15}b^5a^5+15x^{14}b^4a^6+\frac{120}{13}x^{13}b^3a^7+\frac{15}{4}x^{12}b^2a^8+\frac{10}{11}x^{11}ba^9+\frac{1}{10}x^{10}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^10 + 10/19*x^19*b^9*a + 5/2*x^18*b^8*a^2 + 120/17*x^17*b^7*a^3
+ 105/8*x^16*b^6*a^4 + 84/5*x^15*b^5*a^5 + 15*x^14*b^4*a^6 + 120/13*x^13*b^
3*a^7 + 15/4*x^12*b^2*a^8 + 10/11*x^11*b*a^9 + 1/10*x^10*a^10
```

**giac [A]** time = 1.12, size = 112, normalized size = 0.85

$$\frac{1}{20}b^{10}x^{20}+\frac{10}{19}ab^9x^{19}+\frac{5}{2}a^2b^8x^{18}+\frac{120}{17}a^3b^7x^{17}+\frac{105}{8}a^4b^6x^{16}+\frac{84}{5}a^5b^5x^{15}+15a^6b^4x^{14}+\frac{120}{13}a^7b^3x^{13}+\frac{15}{4}a^8b^2x^{12}+\frac{10}{11}a^9bx^{11}+\frac{1}{10}a^{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

**maple [A]** time = 0.00, size = 113, normalized size = 0.86

$$\frac{1}{20}b^{10}x^{20}+\frac{10}{19}a\,b^9x^{19}+\frac{5}{2}a^2b^8x^{18}+\frac{120}{17}a^3b^7x^{17}+\frac{105}{8}a^4b^6x^{16}+\frac{84}{5}a^5b^5x^{15}+15a^6b^4x^{14}+\frac{120}{13}a^7b^3x^{13}+\frac{15}{4}a^8b^2x^{12}+\frac{10}{11}a^9b\,x^{11}+\frac{1}{10}a^{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x+a)^10,x)
```

```
[Out] 1/10*a^10*x^10+10/11*a^9*b*x^11+15/4*a^8*b^2*x^12+120/13*a^7*b^3*x^13+15*a^
6*b^4*x^14+84/5*a^5*b^5*x^15+105/8*a^4*b^6*x^16+120/17*a^3*b^7*x^17+5/2*a^2
*b^8*x^18+10/19*a*b^9*x^19+1/20*b^10*x^20
```

**maxima** [A] time = 1.37, size = 112, normalized size = 0.85

$$\frac{1}{20}\,b^{10}x^{20}+\frac{10}{19}\,ab^{9}x^{19}+\frac{5}{2}\,a^{2}b^{8}x^{18}+\frac{120}{17}\,a^{3}b^{7}x^{17}+\frac{105}{8}\,a^{4}b^{6}x^{16}+\frac{84}{5}\,a^{5}b^{5}x^{15}+15\,a^{6}b^{4}x^{14}+\frac{120}{13}\,a^{7}b^{3}x^{13}+\frac{15}{4}\,a^{8}b^{2}x^{12}+\frac{10}{11}\,a^{9}bx^{11}+\frac{1}{10}\,a^{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

**mupad** [B] time = 0.12, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{10}}{10}+\frac{10\,a^9\,b\,x^{11}}{11}+\frac{15\,a^8\,b^2\,x^{12}}{4}+\frac{120\,a^7\,b^3\,x^{13}}{13}+15\,a^6\,b^4\,x^{14}+\frac{84\,a^5\,b^5\,x^{15}}{5}+\frac{105\,a^4\,b^6\,x^{16}}{8}+\frac{120\,a^3\,b^7\,x^{17}}{17}+\frac{5\,a^2\,b^8\,x^{18}}{2}+\frac{10\,a\,b^9\,x^{19}}{19}+\frac{b^{10}\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x)^10,x)
```

```
[Out] (a^10*x^10)/10 + (b^10*x^20)/20 + (10*a^9*b*x^11)/11 + (10*a*b^9*x^19)/19 +
 (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^
5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (120*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)
/2
```

**sympy** [A] time = 0.10, size = 133, normalized size = 1.01

$$\frac{a^{10}x^{10}}{10}+\frac{10a^{9}bx^{11}}{11}+\frac{15a^{8}b^{2}x^{12}}{4}+\frac{120a^{7}b^{3}x^{13}}{13}+15a^{6}b^{4}x^{14}+\frac{84a^{5}b^{5}x^{15}}{5}+\frac{105a^{4}b^{6}x^{16}}{8}+\frac{120a^{3}b^{7}x^{17}}{17}+\frac{5a^{2}b^{8}x^{18}}{2}+\frac{10ab^{9}x^{19}}{19}+\frac{b^{10}x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x+a)**10,x)
```

```
[Out] a**10*x**10/10 + 10*a**9*b*x**11/11 + 15*a**8*b**2*x**12/4 + 120*a**7*b**3*
x**13/13 + 15*a**6*b**4*x**14 + 84*a**5*b**5*x**15/5 + 105*a**4*b**6*x**16/
8 + 120*a**3*b**7*x**17/17 + 5*a**2*b**8*x**18/2 + 10*a*b**9*x**19/19 + b**
10*x**20/20
```

## 3.126 $\int x^8(a+bx)^{10}\,dx$

**Optimal**. Leaf size=147

$$\frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{28a^2(a+bx)^{17}}{17b^9}+\frac{(a$$

**Rubi [A]** time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{28a^2(a+bx)^{17}}{17b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{a^8(a+bx)^{11}}{11b^9}+\frac{(a+bx)^{19}}{19b^9}-\frac{4a(a+bx)^{18}}{9b^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x)^10,x]
```

```
[Out] (a^8*(a + b*x)^11)/(11*b^9) - (2*a^7*(a + b*x)^12)/(3*b^9) + (28*a^6*(a + b
*x)^13)/(13*b^9) - (4*a^5*(a + b*x)^14)/b^9 + (14*a^4*(a + b*x)^15)/(3*b^9)
 - (7*a^3*(a + b*x)^16)/(2*b^9) + (28*a^2*(a + b*x)^17)/(17*b^9) - (4*a*(a
+ b*x)^18)/(9*b^9) + (a + b*x)^19/(19*b^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^8(a+bx)^{10}\,dx &= \int\left(\frac{a^8(a+bx)^{10}}{b^8}-\frac{8a^7(a+bx)^{11}}{b^8}+\frac{28a^6(a+bx)^{12}}{b^8}-\frac{56a^5(a+bx)^{13}}{b^8}+\frac{70a^4(a+bx)^{14}}{b^8}-56\right.\\ &= \frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7a^3(a+}{2b}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 125, normalized size = 0.85

$$\frac{a^{10}x^9}{9}+a^9bx^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+\frac{45}{17}a^2b^8x^{17}+\frac{5}{9}ab^9x^{18}+\frac{b^{10}x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x)^10,x]
```

```
[Out] (a^10*x^9)/9 + a^9*b*x^10 + (45*a^8*b^2*x^11)/11 + 10*a^7*b^3*x^12 + (210*a
^6*b^4*x^13)/13 + 18*a^5*b^5*x^14 + 14*a^4*b^6*x^15 + (15*a^3*b^7*x^16)/2 +
 (45*a^2*b^8*x^17)/17 + (5*a*b^9*x^18)/9 + (b^10*x^19)/19
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^8(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^8*(a + b*x)^10, x]
```

**fricas [A]** time = 1.09, size = 111, normalized size = 0.76

$$\frac{1}{19}x^{19}b^{10}+\frac{5}{9}x^{18}b^9a+\frac{45}{17}x^{17}b^8a^2+\frac{15}{2}x^{16}b^7a^3+14x^{15}b^6a^4+18x^{14}b^5a^5+\frac{210}{13}x^{13}b^4a^6+10x^{12}b^3a^7+\frac{45}{11}x^{11}b^2a^8+x^{10}ba^9+\frac{1}{9}x^9a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^10 + 5/9*x^18*b^9*a + 45/17*x^17*b^8*a^2 + 15/2*x^16*b^7*a^3 +
14*x^15*b^6*a^4 + 18*x^14*b^5*a^5 + 210/13*x^13*b^4*a^6 + 10*x^12*b^3*a^7 +
 45/11*x^11*b^2*a^8 + x^10*b*a^9 + 1/9*x^9*a^10
```

**giac [A]** time = 1.01, size = 111, normalized size = 0.76

$$\frac{1}{19}\,b^{10}x^{19}+\frac{5}{9}\,ab^9x^{18}+\frac{45}{17}\,a^2b^8x^{17}+\frac{15}{2}\,a^3b^7x^{16}+14\,a^4b^6x^{15}+18\,a^5b^5x^{14}+\frac{210}{13}\,a^6b^4x^{13}+10\,a^7b^3x^{12}+\frac{45}{11}\,a^8b^2x^{11}+a^9bx^{10}+\frac{1}{9}\,a^{10}x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

**maple [A]** time = 0.00, size = 112, normalized size = 0.76

$$\frac{1}{19}b^{10}x^{19}+\frac{5}{9}a\,b^9x^{18}+\frac{45}{17}a^2b^8x^{17}+\frac{15}{2}a^3b^7x^{16}+14a^4b^6x^{15}+18a^5b^5x^{14}+\frac{210}{13}a^6b^4x^{13}+10a^7b^3x^{12}+\frac{45}{11}a^8b^2x^{11}+a^9b\,x^{10}+\frac{1}{9}a^{10}x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^10,x)
```

```
[Out] 1/19*b^10*x^19+5/9*a*b^9*x^18+45/17*a^2*b^8*x^17+15/2*a^3*b^7*x^16+14*a^4*b
^6*x^15+18*a^5*b^5*x^14+210/13*a^6*b^4*x^13+10*a^7*b^3*x^12+45/11*a^8*b^2*x
^11+a^9*b*x^10+1/9*a^10*x^9
```

**maxima** [A] time = 1.36, size = 111, normalized size = 0.76

$$\frac{1}{19}b^{10}x^{19}+\frac{5}{9}ab^9x^{18}+\frac{45}{17}a^2b^8x^{17}+\frac{15}{2}a^3b^7x^{16}+14a^4b^6x^{15}+18a^5b^5x^{14}+\frac{210}{13}a^6b^4x^{13}+10a^7b^3x^{12}+\frac{45}{11}a^8b^2x^{11}+a^9bx^{10}+\frac{1}{9}a^{10}x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

**mupad** [B] time = 0.09, size = 111, normalized size = 0.76

$$\frac{a^{10}x^9}{9}+a^9bx^{10}+\frac{45a^8b^2x^{11}}{11}+10a^7b^3x^{12}+\frac{210a^6b^4x^{13}}{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15a^3b^7x^{16}}{2}+\frac{45a^2b^8x^{17}}{17}+\frac{5ab^9x^{18}}{9}+\frac{b^{10}x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x)^10,x)
```

```
[Out] (a^10*x^9)/9 + (b^10*x^19)/19 + a^9*b*x^10 + (5*a*b^9*x^18)/9 + (45*a^8*b^2
*x^11)/11 + 10*a^7*b^3*x^12 + (210*a^6*b^4*x^13)/13 + 18*a^5*b^5*x^14 + 14*
a^4*b^6*x^15 + (15*a^3*b^7*x^16)/2 + (45*a^2*b^8*x^17)/17
```

**sympy** [A] time = 0.11, size = 126, normalized size = 0.86

$$\frac{a^{10}x^9}{9}+a^9bx^{10}+\frac{45a^8b^2x^{11}}{11}+10a^7b^3x^{12}+\frac{210a^6b^4x^{13}}{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15a^3b^7x^{16}}{2}+\frac{45a^2b^8x^{17}}{17}+\frac{5ab^9x^{18}}{9}+\frac{b^{10}x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**10,x)
```

```
[Out] a**10*x**9/9 + a**9*b*x**10 + 45*a**8*b**2*x**11/11 + 10*a**7*b**3*x**12 +
210*a**6*b**4*x**13/13 + 18*a**5*b**5*x**14 + 14*a**4*b**6*x**15 + 15*a**3*
b**7*x**16/2 + 45*a**2*b**8*x**17/17 + 5*a*b**9*x**18/9 + b**10*x**19/19
```

## 3.127 $\int x^7(a+bx)^{10}\,dx$

**Optimal**. Leaf size=132

$$-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{21a^2(a+bx)^{16}}{16b^8}+\frac{(a+bx)^{18}}{18b^8}-\frac{7a(\cdots}{\cdots}$$

**Rubi [A]** time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21a^2(a+bx)^{16}}{16b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{(a+bx)^{18}}{18b^8}-\frac{7a(a+bx)^{17}}{17b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^10,x]
```

```
[Out] -(a^7*(a + b*x)^11)/(11*b^8) + (7*a^6*(a + b*x)^12)/(12*b^8) - (21*a^5*(a +
 b*x)^13)/(13*b^8) + (5*a^4*(a + b*x)^14)/(2*b^8) - (7*a^3*(a + b*x)^15)/(3
*b^8) + (21*a^2*(a + b*x)^16)/(16*b^8) - (7*a*(a + b*x)^17)/(17*b^8) + (a +
 b*x)^18/(18*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^7(a+bx)^{10}\,dx=\int\left(-\frac{a^7(a+bx)^{10}}{b^7}+\frac{7a^6(a+bx)^{11}}{b^7}-\frac{21a^5(a+bx)^{12}}{b^7}+\frac{35a^4(a+bx)^{13}}{b^7}-\frac{35a^3(a+bx)^{14}}{b^7}+\cdots\right.$$
$$=-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{21a^2(\cdots}{1\cdots}$$

**Mathematica [A]** time = 0.00, size = 130, normalized size = 0.98

$$\frac{a^{10}x^8}{8}+\frac{10}{9}a^9bx^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x^{15}+\frac{45}{16}a^2b^8x^{16}+\frac{10}{17}ab^9x^{17}+\frac{b^{10}x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^10,x]
```

```
[Out] (a^10*x^8)/8 + (10*a^9*b*x^9)/9 + (9*a^8*b^2*x^10)/2 + (120*a^7*b^3*x^11)/1
1 + (35*a^6*b^4*x^12)/2 + (252*a^5*b^5*x^13)/13 + 15*a^4*b^6*x^14 + 8*a^3*b
^7*x^15 + (45*a^2*b^8*x^16)/16 + (10*a*b^9*x^17)/17 + (b^10*x^18)/18
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^7(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^7*(a + b*x)^10, x]
```

**fricas [A]** time = 1.07, size = 112, normalized size = 0.85

$$\frac{1}{18}x^{18}b^{10}+\frac{10}{17}x^{17}b^9a+\frac{45}{16}x^{16}b^8a^2+8x^{15}b^7a^3+15x^{14}b^6a^4+\frac{252}{13}x^{13}b^5a^5+\frac{35}{2}x^{12}b^4a^6+\frac{120}{11}x^{11}b^3a^7+\frac{9}{2}x^{10}b^2a^8+\frac{10}{9}x^9ba^9+\frac{1}{8}x^8a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^10 + 10/17*x^17*b^9*a + 45/16*x^16*b^8*a^2 + 8*x^15*b^7*a^3 + 1
5*x^14*b^6*a^4 + 252/13*x^13*b^5*a^5 + 35/2*x^12*b^4*a^6 + 120/11*x^11*b^3*
a^7 + 9/2*x^10*b^2*a^8 + 10/9*x^9*b*a^9 + 1/8*x^8*a^10
```

**giac [A]** time = 0.84, size = 112, normalized size = 0.85

$$\frac{1}{18}b^{10}x^{18}+\frac{10}{17}ab^9x^{17}+\frac{45}{16}a^2b^8x^{16}+8a^3b^7x^{15}+15a^4b^6x^{14}+\frac{252}{13}a^5b^5x^{13}+\frac{35}{2}a^6b^4x^{12}+\frac{120}{11}a^7b^3x^{11}+\frac{9}{2}a^8b^2x^{10}+\frac{10}{9}a^9bx^9+\frac{1}{8}a^{10}x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 1
5*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x
^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

**maple [A]** time = 0.00, size = 113, normalized size = 0.86

$$\frac{1}{18}b^{10}x^{18}+\frac{10}{17}a\,b^9x^{17}+\frac{45}{16}a^2b^8x^{16}+8a^3b^7x^{15}+15a^4b^6x^{14}+\frac{252}{13}a^5b^5x^{13}+\frac{35}{2}a^6b^4x^{12}+\frac{120}{11}a^7b^3x^{11}+\frac{9}{2}a^8b^2x^{10}+\frac{10}{9}a^9b\,x^9+\frac{1}{8}a^{10}x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^10,x)
```

```
[Out] 1/18*b^10*x^18+10/17*a*b^9*x^17+45/16*a^2*b^8*x^16+8*a^3*b^7*x^15+15*a^4*b^
6*x^14+252/13*a^5*b^5*x^13+35/2*a^6*b^4*x^12+120/11*a^7*b^3*x^11+9/2*a^8*b^
2*x^10+10/9*a^9*b*x^9+1/8*a^10*x^8
```

**maxima [A]** time = 1.34, size = 112, normalized size = 0.85

$$\frac{1}{18}b^{10}x^{18}+\frac{10}{17}ab^9x^{17}+\frac{45}{16}a^2b^8x^{16}+8a^3b^7x^{15}+15a^4b^6x^{14}+\frac{252}{13}a^5b^5x^{13}+\frac{35}{2}a^6b^4x^{12}+\frac{120}{11}a^7b^3x^{11}+\frac{9}{2}a^8b^2x^{10}+\frac{10}{9}a^9bx^9+\frac{1}{8}a^{10}x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 1
5*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x
^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

**mupad [B]** time = 0.08, size = 112, normalized size = 0.85

$$\frac{a^{10}x^8}{8}+\frac{10a^9bx^9}{9}+\frac{9a^8b^2x^{10}}{2}+\frac{120a^7b^3x^{11}}{11}+\frac{35a^6b^4x^{12}}{2}+\frac{252a^5b^5x^{13}}{13}+15a^4b^6x^{14}+8a^3b^7x^{15}+\frac{45a^2b^8x^{16}}{16}+\frac{10ab^9x^{17}}{17}+\frac{b^{10}x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x)^10,x)
```

```
[Out] (a^10*x^8)/8 + (b^10*x^18)/18 + (10*a^9*b*x^9)/9 + (10*a*b^9*x^17)/17 + (9*
a^8*b^2*x^10)/2 + (120*a^7*b^3*x^11)/11 + (35*a^6*b^4*x^12)/2 + (252*a^5*b^
5*x^13)/13 + 15*a^4*b^6*x^14 + 8*a^3*b^7*x^15 + (45*a^2*b^8*x^16)/16
```

**sympy [A]** time = 0.11, size = 131, normalized size = 0.99

$$\frac{a^{10}x^8}{8}+\frac{10a^9bx^9}{9}+\frac{9a^8b^2x^{10}}{2}+\frac{120a^7b^3x^{11}}{11}+\frac{35a^6b^4x^{12}}{2}+\frac{252a^5b^5x^{13}}{13}+15a^4b^6x^{14}+8a^3b^7x^{15}+\frac{45a^2b^8x^{16}}{16}+\frac{10ab^9x^{17}}{17}+\frac{b^{10}x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**10,x)
```

```
[Out] a**10*x**8/8 + 10*a**9*b*x**9/9 + 9*a**8*b**2*x**10/2 + 120*a**7*b**3*x**11
/11 + 35*a**6*b**4*x**12/2 + 252*a**5*b**5*x**13/13 + 15*a**4*b**6*x**14 +
8*a**3*b**7*x**15 + 45*a**2*b**8*x**16/16 + 10*a*b**9*x**17/17 + b**10*x**1
8/18
```

## 3.128 $\int x^6(a+bx)^{10}\,dx$

**Optimal**. Leaf size=112

$$\frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}+\frac{(a+bx)^{17}}{17b^7}-\frac{3a(a+bx)^{16}}{8b^7}$$

**Rubi [A]** time = 0.05, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{15}}{b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{a^6(a+bx)^{11}}{11b^7}+\frac{(a+bx)^{17}}{17b^7}-\frac{3a(a+bx)^{16}}{8b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^10,x]
```

```
[Out] (a^6*(a + b*x)^11)/(11*b^7) - (a^5*(a + b*x)^12)/(2*b^7) + (15*a^4*(a + b*x
)^13)/(13*b^7) - (10*a^3*(a + b*x)^14)/(7*b^7) + (a^2*(a + b*x)^15)/b^7 - (
3*a*(a + b*x)^16)/(8*b^7) + (a + b*x)^17/(17*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^{10}\,dx &= \int \left(\frac{a^6(a+bx)^{10}}{b^6}-\frac{6a^5(a+bx)^{11}}{b^6}+\frac{15a^4(a+bx)^{12}}{b^6}-\frac{20a^3(a+bx)^{13}}{b^6}+\frac{15a^2(a+bx)^{14}}{b^6}-\frac{6a\\ &= \frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}-\frac{3a(a+bx}{8b^7}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 126, normalized size = 1.12

$$\frac{a^{10}x^7}{7}+\frac{5}{4}a^9bx^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a^2b^8x^{15}+\frac{5}{8}ab^9x^{16}+\frac{b^{10}x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^10,x]
```

```
[Out] (a^10*x^7)/7 + (5*a^9*b*x^8)/4 + 5*a^8*b^2*x^9 + 12*a^7*b^3*x^10 + (210*a^6
*b^4*x^11)/11 + 21*a^5*b^5*x^12 + (210*a^4*b^6*x^13)/13 + (60*a^3*b^7*x^14)
/7 + 3*a^2*b^8*x^15 + (5*a*b^9*x^16)/8 + (b^10*x^17)/17
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^6(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^6*(a + b*x)^10, x]
```

**fricas [A]** time = 0.97, size = 112, normalized size = 1.00

$$\frac{1}{17}x^{17}b^{10}+\frac{5}{8}x^{16}b^9a+3x^{15}b^8a^2+\frac{60}{7}x^{14}b^7a^3+\frac{210}{13}x^{13}b^6a^4+21x^{12}b^5a^5+\frac{210}{11}x^{11}b^4a^6+12x^{10}b^3a^7+5x^9b^2a^8+\frac{5}{4}x^8ba^9+\frac{1}{7}x^7a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^10 + 5/8*x^16*b^9*a + 3*x^15*b^8*a^2 + 60/7*x^14*b^7*a^3 + 210/
13*x^13*b^6*a^4 + 21*x^12*b^5*a^5 + 210/11*x^11*b^4*a^6 + 12*x^10*b^3*a^7 +
 5*x^9*b^2*a^8 + 5/4*x^8*b*a^9 + 1/7*x^7*a^10
```

**giac [A]** time = 1.10, size = 112, normalized size = 1.00

$$\frac{1}{17}b^{10}x^{17}+\frac{5}{8}ab^9x^{16}+3a^2b^8x^{15}+\frac{60}{7}a^3b^7x^{14}+\frac{210}{13}a^4b^6x^{13}+21a^5b^5x^{12}+\frac{210}{11}a^6b^4x^{11}+12a^7b^3x^{10}+5a^8b^2x^9+\frac{5}{4}a^9bx^8+\frac{1}{7}a^{10}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

**maple [A]** time = 0.00, size = 113, normalized size = 1.01

$$\frac{1}{17}b^{10}x^{17}+\frac{5}{8}a\,b^9x^{16}+3a^2b^8x^{15}+\frac{60}{7}a^3b^7x^{14}+\frac{210}{13}a^4b^6x^{13}+21a^5b^5x^{12}+\frac{210}{11}a^6b^4x^{11}+12a^7b^3x^{10}+5a^8b^2x^9+\frac{5}{4}a^9b\,x^8+\frac{1}{7}a^{10}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^10,x)
```

```
[Out] 1/17*b^10*x^17+5/8*a*b^9*x^16+3*a^2*b^8*x^15+60/7*a^3*b^7*x^14+210/13*a^4*b
^6*x^13+21*a^5*b^5*x^12+210/11*a^6*b^4*x^11+12*a^7*b^3*x^10+5*a^8*b^2*x^9+5
/4*a^9*b*x^8+1/7*a^10*x^7
```

**maxima [A]** time = 1.38, size = 112, normalized size = 1.00

$$\frac{1}{17}\,b^{10}x^{17}+\frac{5}{8}\,ab^{9}x^{16}+3\,a^{2}b^{8}x^{15}+\frac{60}{7}\,a^{3}b^{7}x^{14}+\frac{210}{13}\,a^{4}b^{6}x^{13}+21\,a^{5}b^{5}x^{12}+\frac{210}{11}\,a^{6}b^{4}x^{11}+12\,a^{7}b^{3}x^{10}+5\,a^{8}b^{2}x^{9}+\frac{5}{4}\,a^{9}bx^{8}+\frac{1}{7}\,a^{10}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

**mupad [B]** time = 0.12, size = 112, normalized size = 1.00

$$\frac{a^{10}\,x^7}{7}+\frac{5\,a^9\,b\,x^8}{4}+5\,a^8\,b^2\,x^9+12\,a^7\,b^3\,x^{10}+\frac{210\,a^6\,b^4\,x^{11}}{11}+21\,a^5\,b^5\,x^{12}+\frac{210\,a^4\,b^6\,x^{13}}{13}+\frac{60\,a^3\,b^7\,x^{14}}{7}+3\,a^2\,b^8\,x^{15}+\frac{5\,a\,b^9\,x^{16}}{8}+\frac{b^{10}\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^10,x)
```

```
[Out] (a^10*x^7)/7 + (b^10*x^17)/17 + (5*a^9*b*x^8)/4 + (5*a*b^9*x^16)/8 + 5*a^8*
b^2*x^9 + 12*a^7*b^3*x^10 + (210*a^6*b^4*x^11)/11 + 21*a^5*b^5*x^12 + (210*
a^4*b^6*x^13)/13 + (60*a^3*b^7*x^14)/7 + 3*a^2*b^8*x^15
```

**sympy [A]** time = 0.10, size = 128, normalized size = 1.14

$$\frac{a^{10}x^{7}}{7}+\frac{5a^{9}bx^{8}}{4}+5a^{8}b^{2}x^{9}+12a^{7}b^{3}x^{10}+\frac{210a^{6}b^{4}x^{11}}{11}+21a^{5}b^{5}x^{12}+\frac{210a^{4}b^{6}x^{13}}{13}+\frac{60a^{3}b^{7}x^{14}}{7}+3a^{2}b^{8}x^{15}+\frac{5ab^{9}x^{16}}{8}+\frac{b^{10}x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**10,x)
```

```
[Out] a**10*x**7/7 + 5*a**9*b*x**8/4 + 5*a**8*b**2*x**9 + 12*a**7*b**3*x**10 + 21
0*a**6*b**4*x**11/11 + 21*a**5*b**5*x**12 + 210*a**4*b**6*x**13/13 + 60*a**
3*b**7*x**14/7 + 3*a**2*b**8*x**15 + 5*a*b**9*x**16/8 + b**10*x**17/17
```

## 3.129 $\int x^5(a+bx)^{10}\,dx$

**Optimal**. Leaf size=98

$$-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}+\frac{(a+bx)^{16}}{16b^6}-\frac{a(a+bx)^{15}}{3b^6}$$

**Rubi [A]** time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{(a+bx)^{16}}{16b^6}-\frac{a(a+bx)^{15}}{3b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^10,x]
```

```
[Out] -(a^5*(a + b*x)^11)/(11*b^6) + (5*a^4*(a + b*x)^12)/(12*b^6) - (10*a^3*(a +
 b*x)^13)/(13*b^6) + (5*a^2*(a + b*x)^14)/(7*b^6) - (a*(a + b*x)^15)/(3*b^6
) + (a + b*x)^16/(16*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^{10}\,dx &= \int\left(-\frac{a^5(a+bx)^{10}}{b^5}+\frac{5a^4(a+bx)^{11}}{b^5}-\frac{10a^3(a+bx)^{12}}{b^5}+\frac{10a^2(a+bx)^{13}}{b^5}-\frac{5a(a+bx)^{14}}{b^5}+\right.\\ &= -\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{a(a+bx)^{15}}{3b^6}+\frac{(a+bx)}{16b^6}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 132, normalized size = 1.35

$$\frac{a^{10}x^6}{6}+\frac{10}{7}a^9bx^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x^{13}+\frac{45}{14}a^2b^8x^{14}+\frac{2}{3}ab^9x^{15}+\frac{b^{10}x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^10,x]
```

```
[Out] (a^10*x^6)/6 + (10*a^9*b*x^7)/7 + (45*a^8*b^2*x^8)/8 + (40*a^7*b^3*x^9)/3 +
 21*a^6*b^4*x^10 + (252*a^5*b^5*x^11)/11 + (35*a^4*b^6*x^12)/2 + (120*a^3*b
^7*x^13)/13 + (45*a^2*b^8*x^14)/14 + (2*a*b^9*x^15)/3 + (b^10*x^16)/16
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^5(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^5*(a + b*x)^10, x]
```

**fricas [A]** time = 0.72, size = 112, normalized size = 1.14

$$\frac{1}{16}x^{16}b^{10} + \frac{2}{3}x^{15}b^{9}a + \frac{45}{14}x^{14}b^{8}a^{2} + \frac{120}{13}x^{13}b^{7}a^{3} + \frac{35}{2}x^{12}b^{6}a^{4} + \frac{252}{11}x^{11}b^{5}a^{5} + 21x^{10}b^{4}a^{6} + \frac{40}{3}x^{9}b^{3}a^{7} + \frac{45}{8}x^{8}b^{2}a^{8} + \frac{10}{7}x^{7}ba^{9} + \frac{1}{6}x^{6}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^10 + 2/3*x^15*b^9*a + 45/14*x^14*b^8*a^2 + 120/13*x^13*b^7*a^3
+ 35/2*x^12*b^6*a^4 + 252/11*x^11*b^5*a^5 + 21*x^10*b^4*a^6 + 40/3*x^9*b^3*
a^7 + 45/8*x^8*b^2*a^8 + 10/7*x^7*b*a^9 + 1/6*x^6*a^10
```

**giac [A]** time = 1.14, size = 112, normalized size = 1.14

$$\frac{1}{16}\,b^{10}x^{16} + \frac{2}{3}\,ab^{9}x^{15} + \frac{45}{14}\,a^{2}b^{8}x^{14} + \frac{120}{13}\,a^{3}b^{7}x^{13} + \frac{35}{2}\,a^{4}b^{6}x^{12} + \frac{252}{11}\,a^{5}b^{5}x^{11} + 21\,a^{6}b^{4}x^{10} + \frac{40}{3}\,a^{7}b^{3}x^{9} + \frac{45}{8}\,a^{8}b^{2}x^{8} + \frac{10}{7}\,a^{9}bx^{7} + \frac{1}{6}\,a^{10}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

**maple [A]** time = 0.00, size = 113, normalized size = 1.15

$$\frac{1}{16}b^{10}x^{16} + \frac{2}{3}a\,b^{9}x^{15} + \frac{45}{14}a^{2}b^{8}x^{14} + \frac{120}{13}a^{3}b^{7}x^{13} + \frac{35}{2}a^{4}b^{6}x^{12} + \frac{252}{11}a^{5}b^{5}x^{11} + 21a^{6}b^{4}x^{10} + \frac{40}{3}a^{7}b^{3}x^{9} + \frac{45}{8}a^{8}b^{2}x^{8} + \frac{10}{7}a^{9}b\,x^{7} + \frac{1}{6}a^{10}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^10,x)
```

```
[Out] 1/16*b^10*x^16+2/3*a*b^9*x^15+45/14*a^2*b^8*x^14+120/13*a^3*b^7*x^13+35/2*a
^4*b^6*x^12+252/11*a^5*b^5*x^11+21*a^6*b^4*x^10+40/3*a^7*b^3*x^9+45/8*a^8*b
^2*x^8+10/7*a^9*b*x^7+1/6*a^10*x^6
```

**maxima [A]** time = 1.36, size = 112, normalized size = 1.14

$$\frac{1}{16}\,b^{10}x^{16}+\frac{2}{3}\,ab^{9}x^{15}+\frac{45}{14}\,a^{2}b^{8}x^{14}+\frac{120}{13}\,a^{3}b^{7}x^{13}+\frac{35}{2}\,a^{4}b^{6}x^{12}+\frac{252}{11}\,a^{5}b^{5}x^{11}+21\,a^{6}b^{4}x^{10}+\frac{40}{3}\,a^{7}b^{3}x^{9}+\frac{45}{8}\,a^{8}b^{2}x^{8}+\frac{10}{7}\,a^{9}bx^{7}+\frac{1}{6}\,a^{10}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

**mupad [B]** time = 0.12, size = 112, normalized size = 1.14

$$\frac{a^{10}\,x^{6}}{6}+\frac{10\,a^{9}\,b\,x^{7}}{7}+\frac{45\,a^{8}\,b^{2}\,x^{8}}{8}+\frac{40\,a^{7}\,b^{3}\,x^{9}}{3}+21\,a^{6}\,b^{4}\,x^{10}+\frac{252\,a^{5}\,b^{5}\,x^{11}}{11}+\frac{35\,a^{4}\,b^{6}\,x^{12}}{2}+\frac{120\,a^{3}\,b^{7}\,x^{13}}{13}+\frac{45\,a^{2}\,b^{8}\,x^{14}}{14}+\frac{2\,a\,b^{9}\,x^{15}}{3}+\frac{b^{10}\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^10,x)
```

```
[Out] (a^10*x^6)/6 + (b^10*x^16)/16 + (10*a^9*b*x^7)/7 + (2*a*b^9*x^15)/3 + (45*a
^8*b^2*x^8)/8 + (40*a^7*b^3*x^9)/3 + 21*a^6*b^4*x^10 + (252*a^5*b^5*x^11)/1
1 + (35*a^4*b^6*x^12)/2 + (120*a^3*b^7*x^13)/13 + (45*a^2*b^8*x^14)/14
```

**sympy [A]** time = 0.10, size = 133, normalized size = 1.36

$$\frac{a^{10}x^{6}}{6}+\frac{10a^{9}bx^{7}}{7}+\frac{45a^{8}b^{2}x^{8}}{8}+\frac{40a^{7}b^{3}x^{9}}{3}+21a^{6}b^{4}x^{10}+\frac{252a^{5}b^{5}x^{11}}{11}+\frac{35a^{4}b^{6}x^{12}}{2}+\frac{120a^{3}b^{7}x^{13}}{13}+\frac{45a^{2}b^{8}x^{14}}{14}+\frac{2ab^{9}x^{15}}{3}+\frac{b^{10}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**10,x)
```

```
[Out] a**10*x**6/6 + 10*a**9*b*x**7/7 + 45*a**8*b**2*x**8/8 + 40*a**7*b**3*x**9/3
 + 21*a**6*b**4*x**10 + 252*a**5*b**5*x**11/11 + 35*a**4*b**6*x**12/2 + 120
*a**3*b**7*x**13/13 + 45*a**2*b**8*x**14/14 + 2*a*b**9*x**15/3 + b**10*x**1
6/16
```

## 3.130 $\int x^4(a+bx)^{10}\,dx$

**Optimal**. Leaf size=81

$$\frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}+\frac{(a+bx)^{15}}{15b^5}-\frac{2a(a+bx)^{14}}{7b^5}$$

**Rubi [A]** time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{6a^2(a+bx)^{13}}{13b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{a^4(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{15}}{15b^5}-\frac{2a(a+bx)^{14}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^10,x]
```

```
[Out] (a^4*(a + b*x)^11)/(11*b^5) - (a^3*(a + b*x)^12)/(3*b^5) + (6*a^2*(a + b*x)
^13)/(13*b^5) - (2*a*(a + b*x)^14)/(7*b^5) + (a + b*x)^15/(15*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^{10}\,dx &= \int\left(\frac{a^4(a+bx)^{10}}{b^4}-\frac{4a^3(a+bx)^{11}}{b^4}+\frac{6a^2(a+bx)^{12}}{b^4}-\frac{4a(a+bx)^{13}}{b^4}+\frac{(a+bx)^{14}}{b^4}\right)dx\\ &= \frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}-\frac{2a(a+bx)^{14}}{7b^5}+\frac{(a+bx)^{15}}{15b^5}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 130, normalized size = 1.60

$$\frac{a^{10}x^5}{5}+\frac{5}{3}a^9bx^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}a^2b^8x^{13}+\frac{5}{7}ab^9x^{14}+\frac{b^{10}x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^10,x]
```

```
[Out] (a^10*x^5)/5 + (5*a^9*b*x^6)/3 + (45*a^8*b^2*x^7)/7 + 15*a^7*b^3*x^8 + (70*
a^6*b^4*x^9)/3 + (126*a^5*b^5*x^10)/5 + (210*a^4*b^6*x^11)/11 + 10*a^3*b^7*
x^12 + (45*a^2*b^8*x^13)/13 + (5*a*b^9*x^14)/7 + (b^10*x^15)/15
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x)^10, x]
```

**fricas [A]** time = 1.22, size = 112, normalized size = 1.38

$$\frac{1}{15}x^{15}b^{10} + \frac{5}{7}x^{14}b^9a + \frac{45}{13}x^{13}b^8a^2 + 10x^{12}b^7a^3 + \frac{210}{11}x^{11}b^6a^4 + \frac{126}{5}x^{10}b^5a^5 + \frac{70}{3}x^9b^4a^6 + 15x^8b^3a^7 + \frac{45}{7}x^7b^2a^8 + \frac{5}{3}x^6ba^9 + \frac{1}{5}x^5a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^10 + 5/7*x^14*b^9*a + 45/13*x^13*b^8*a^2 + 10*x^12*b^7*a^3 + 21
0/11*x^11*b^6*a^4 + 126/5*x^10*b^5*a^5 + 70/3*x^9*b^4*a^6 + 15*x^8*b^3*a^7
+ 45/7*x^7*b^2*a^8 + 5/3*x^6*b*a^9 + 1/5*x^5*a^10
```

**giac [A]** time = 1.06, size = 112, normalized size = 1.38

$$\frac{1}{15}\,b^{10}x^{15} + \frac{5}{7}\,ab^9x^{14} + \frac{45}{13}\,a^2b^8x^{13} + 10\,a^3b^7x^{12} + \frac{210}{11}\,a^4b^6x^{11} + \frac{126}{5}\,a^5b^5x^{10} + \frac{70}{3}\,a^6b^4x^9 + 15\,a^7b^3x^8 + \frac{45}{7}\,a^8b^2x^7 + \frac{5}{3}\,a^9bx^6 + \frac{1}{5}\,a^{10}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

**maple [A]** time = 0.00, size = 113, normalized size = 1.40

$$\frac{1}{15}b^{10}x^{15} + \frac{5}{7}a\,b^9x^{14} + \frac{45}{13}a^2b^8x^{13} + 10a^3b^7x^{12} + \frac{210}{11}a^4b^6x^{11} + \frac{126}{5}a^5b^5x^{10} + \frac{70}{3}a^6b^4x^9 + 15a^7b^3x^8 + \frac{45}{7}a^8b^2x^7 + \frac{5}{3}a^9b\,x^6 + \frac{1}{5}a^{10}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^10,x)
```

```
[Out] 1/15*b^10*x^15+5/7*a*b^9*x^14+45/13*a^2*b^8*x^13+10*a^3*b^7*x^12+210/11*a^4
*b^6*x^11+126/5*a^5*b^5*x^10+70/3*a^6*b^4*x^9+15*a^7*b^3*x^8+45/7*a^8*b^2*x
^7+5/3*a^9*b*x^6+1/5*a^10*x^5
```

**maxima** [A] time = 1.29, size = 112, normalized size = 1.38

$$\frac{1}{15}\,b^{10}x^{15} + \frac{5}{7}\,ab^{9}x^{14} + \frac{45}{13}\,a^{2}b^{8}x^{13} + 10\,a^{3}b^{7}x^{12} + \frac{210}{11}\,a^{4}b^{6}x^{11} + \frac{126}{5}\,a^{5}b^{5}x^{10} + \frac{70}{3}\,a^{6}b^{4}x^{9} + 15\,a^{7}b^{3}x^{8} + \frac{45}{7}\,a^{8}b^{2}x^{7} + \frac{5}{3}\,a^{9}bx^{6} + \frac{1}{5}\,a^{10}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

**mupad** [B] time = 0.12, size = 112, normalized size = 1.38

$$\frac{a^{10}\,x^5}{5} + \frac{5\,a^9\,b\,x^6}{3} + \frac{45\,a^8\,b^2\,x^7}{7} + 15\,a^7\,b^3\,x^8 + \frac{70\,a^6\,b^4\,x^9}{3} + \frac{126\,a^5\,b^5\,x^{10}}{5} + \frac{210\,a^4\,b^6\,x^{11}}{11} + 10\,a^3\,b^7\,x^{12} + \frac{45\,a^2\,b^8\,x^{13}}{13} + \frac{5\,a\,b^9\,x^{14}}{7} + \frac{b^{10}\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^10,x)
```

```
[Out] (a^10*x^5)/5 + (b^10*x^15)/15 + (5*a^9*b*x^6)/3 + (5*a*b^9*x^14)/7 + (45*a^
8*b^2*x^7)/7 + 15*a^7*b^3*x^8 + (70*a^6*b^4*x^9)/3 + (126*a^5*b^5*x^10)/5 +
 (210*a^4*b^6*x^11)/11 + 10*a^3*b^7*x^12 + (45*a^2*b^8*x^13)/13
```

**sympy** [A] time = 0.11, size = 131, normalized size = 1.62

$$\frac{a^{10}x^{5}}{5} + \frac{5a^{9}bx^{6}}{3} + \frac{45a^{8}b^{2}x^{7}}{7} + 15a^{7}b^{3}x^{8} + \frac{70a^{6}b^{4}x^{9}}{3} + \frac{126a^{5}b^{5}x^{10}}{5} + \frac{210a^{4}b^{6}x^{11}}{11} + 10a^{3}b^{7}x^{12} + \frac{45a^{2}b^{8}x^{13}}{13} + \frac{5ab^{9}x^{14}}{7} + \frac{b^{10}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**10,x)
```

```
[Out] a**10*x**5/5 + 5*a**9*b*x**6/3 + 45*a**8*b**2*x**7/7 + 15*a**7*b**3*x**8 +
70*a**6*b**4*x**9/3 + 126*a**5*b**5*x**10/5 + 210*a**4*b**6*x**11/11 + 10*a
**3*b**7*x**12 + 45*a**2*b**8*x**13/13 + 5*a*b**9*x**14/7 + b**10*x**15/15
```

## 3.131 $\int x^3(a+bx)^{10}\,dx$

**Optimal**. Leaf size=64

$$-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}+\frac{(a+bx)^{14}}{14b^4}-\frac{3a(a+bx)^{13}}{13b^4}$$

**Rubi [A]** time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{12}}{4b^4}-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{(a+bx)^{14}}{14b^4}-\frac{3a(a+bx)^{13}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^10,x]
```

```
[Out] -(a^3*(a + b*x)^11)/(11*b^4) + (a^2*(a + b*x)^12)/(4*b^4) - (3*a*(a + b*x)^
13)/(13*b^4) + (a + b*x)^14/(14*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{10}\,dx &= \int\left(-\frac{a^3(a+bx)^{10}}{b^3}+\frac{3a^2(a+bx)^{11}}{b^3}-\frac{3a(a+bx)^{12}}{b^3}+\frac{(a+bx)^{13}}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}-\frac{3a(a+bx)^{13}}{13b^4}+\frac{(a+bx)^{14}}{14b^4}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 128, normalized size = 2.00

$$\frac{a^{10}x^4}{4}+2a^9bx^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{11}+\frac{15}{4}a^2b^8x^{12}+\frac{10}{13}ab^9x^{13}+\frac{b^{10}x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^10,x]
```

```
[Out] (a^10*x^4)/4 + 2*a^9*b*x^5 + (15*a^8*b^2*x^6)/2 + (120*a^7*b^3*x^7)/7 + (10
5*a^6*b^4*x^8)/4 + 28*a^5*b^5*x^9 + 21*a^4*b^6*x^10 + (120*a^3*b^7*x^11)/11
 + (15*a^2*b^8*x^12)/4 + (10*a*b^9*x^13)/13 + (b^10*x^14)/14
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x)^10, x]
```

**fricas [A]** time = 1.14, size = 112, normalized size = 1.75

$$\frac{1}{14}x^{14}b^{10}+\frac{10}{13}x^{13}b^{9}a+\frac{15}{4}x^{12}b^{8}a^{2}+\frac{120}{11}x^{11}b^{7}a^{3}+21x^{10}b^{6}a^{4}+28x^{9}b^{5}a^{5}+\frac{105}{4}x^{8}b^{4}a^{6}+\frac{120}{7}x^{7}b^{3}a^{7}+\frac{15}{2}x^{6}b^{2}a^{8}+2x^{5}ba^{9}+\frac{1}{4}x^{4}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^10 + 10/13*x^13*b^9*a + 15/4*x^12*b^8*a^2 + 120/11*x^11*b^7*a^3
 + 21*x^10*b^6*a^4 + 28*x^9*b^5*a^5 + 105/4*x^8*b^4*a^6 + 120/7*x^7*b^3*a^7
 + 15/2*x^6*b^2*a^8 + 2*x^5*b*a^9 + 1/4*x^4*a^10
```

**giac [A]** time = 1.09, size = 112, normalized size = 1.75

$$\frac{1}{14}\,b^{10}x^{14}+\frac{10}{13}\,ab^{9}x^{13}+\frac{15}{4}\,a^{2}b^{8}x^{12}+\frac{120}{11}\,a^{3}b^{7}x^{11}+21\,a^{4}b^{6}x^{10}+28\,a^{5}b^{5}x^{9}+\frac{105}{4}\,a^{6}b^{4}x^{8}+\frac{120}{7}\,a^{7}b^{3}x^{7}+\frac{15}{2}\,a^{8}b^{2}x^{6}+2\,a^{9}bx^{5}+\frac{1}{4}\,a^{10}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

**maple [A]** time = 0.00, size = 113, normalized size = 1.77

$$\frac{1}{14}b^{10}x^{14}+\frac{10}{13}a\,b^{9}x^{13}+\frac{15}{4}a^{2}b^{8}x^{12}+\frac{120}{11}a^{3}b^{7}x^{11}+21a^{4}b^{6}x^{10}+28a^{5}b^{5}x^{9}+\frac{105}{4}a^{6}b^{4}x^{8}+\frac{120}{7}a^{7}b^{3}x^{7}+\frac{15}{2}a^{8}b^{2}x^{6}+2a^{9}b\,x^{5}+\frac{1}{4}a^{10}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^10,x)
```

```
[Out] 1/14*b^10*x^14+10/13*a*b^9*x^13+15/4*a^2*b^8*x^12+120/11*a^3*b^7*x^11+21*a^
4*b^6*x^10+28*a^5*b^5*x^9+105/4*a^6*b^4*x^8+120/7*a^7*b^3*x^7+15/2*a^8*b^2*
x^6+2*a^9*b*x^5+1/4*a^10*x^4
```

**maxima [A]** time = 1.36, size = 112, normalized size = 1.75

$$\frac{1}{14}b^{10}x^{14}+\frac{10}{13}ab^9x^{13}+\frac{15}{4}a^2b^8x^{12}+\frac{120}{11}a^3b^7x^{11}+21a^4b^6x^{10}+28a^5b^5x^9+\frac{105}{4}a^6b^4x^8+\frac{120}{7}a^7b^3x^7+\frac{15}{2}a^8b^2x^6+2a^9bx^5+\frac{1}{4}a^{10}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

**mupad [B]** time = 0.12, size = 112, normalized size = 1.75

$$\frac{a^{10}x^4}{4}+2a^9bx^5+\frac{15a^8b^2x^6}{2}+\frac{120a^7b^3x^7}{7}+\frac{105a^6b^4x^8}{4}+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120a^3b^7x^{11}}{11}+\frac{15a^2b^8x^{12}}{4}+\frac{10ab^9x^{13}}{13}+\frac{b^{10}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^10,x)
```

```
[Out] (a^10*x^4)/4 + (b^10*x^14)/14 + 2*a^9*b*x^5 + (10*a*b^9*x^13)/13 + (15*a^8*
b^2*x^6)/2 + (120*a^7*b^3*x^7)/7 + (105*a^6*b^4*x^8)/4 + 28*a^5*b^5*x^9 + 2
1*a^4*b^6*x^10 + (120*a^3*b^7*x^11)/11 + (15*a^2*b^8*x^12)/4
```

**sympy [B]** time = 0.12, size = 129, normalized size = 2.02

$$\frac{a^{10}x^4}{4}+2a^9bx^5+\frac{15a^8b^2x^6}{2}+\frac{120a^7b^3x^7}{7}+\frac{105a^6b^4x^8}{4}+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120a^3b^7x^{11}}{11}+\frac{15a^2b^8x^{12}}{4}+\frac{10ab^9x^{13}}{13}+\frac{b^{10}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**10,x)
```

```
[Out] a**10*x**4/4 + 2*a**9*b*x**5 + 15*a**8*b**2*x**6/2 + 120*a**7*b**3*x**7/7 +
 105*a**6*b**4*x**8/4 + 28*a**5*b**5*x**9 + 21*a**4*b**6*x**10 + 120*a**3*b
**7*x**11/11 + 15*a**2*b**8*x**12/4 + 10*a*b**9*x**13/13 + b**10*x**14/14
```

## 3.132 $\int x^2(a+bx)^{10}\,dx$

**Optimal**. Leaf size=47

$$\frac{a^2(a+bx)^{11}}{11b^3}+\frac{(a+bx)^{13}}{13b^3}-\frac{a(a+bx)^{12}}{6b^3}$$

**Rubi [A]** time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{11}}{11b^3}+\frac{(a+bx)^{13}}{13b^3}-\frac{a(a+bx)^{12}}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^10,x]
```

```
[Out] (a^2*(a + b*x)^11)/(11*b^3) - (a*(a + b*x)^12)/(6*b^3) + (a + b*x)^13/(13*b
^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{10}\,dx &= \int\left(\frac{a^2(a+bx)^{10}}{b^2}-\frac{2a(a+bx)^{11}}{b^2}+\frac{(a+bx)^{12}}{b^2}\right)dx\\ &= \frac{a^2(a+bx)^{11}}{11b^3}-\frac{a(a+bx)^{12}}{6b^3}+\frac{(a+bx)^{13}}{13b^3}\end{aligned}$$

**Mathematica [B]** time = 0.00, size = 126, normalized size = 2.68

$$\frac{a^{10}x^3}{3}+\frac{5}{2}a^9bx^4+9a^8b^2x^5+20a^7b^3x^6+30a^6b^4x^7+\frac{63}{2}a^5b^5x^8+\frac{70}{3}a^4b^6x^9+12a^3b^7x^{10}+\frac{45}{11}a^2b^8x^{11}+\frac{5}{6}ab^9x^{12}+\frac{b^{10}x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^10,x]
```

```
[Out] (a^10*x^3)/3 + (5*a^9*b*x^4)/2 + 9*a^8*b^2*x^5 + 20*a^7*b^3*x^6 + 30*a^6*b^
4*x^7 + (63*a^5*b^5*x^8)/2 + (70*a^4*b^6*x^9)/3 + 12*a^3*b^7*x^10 + (45*a^2
*b^8*x^11)/11 + (5*a*b^9*x^12)/6 + (b^10*x^13)/13
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x)^10, x]
```

**fricas [B]** time = 1.12, size = 112, normalized size = 2.38

$$\frac{1}{13}x^{13}b^{10}+\frac{5}{6}x^{12}b^{9}a+\frac{45}{11}x^{11}b^{8}a^{2}+12x^{10}b^{7}a^{3}+\frac{70}{3}x^{9}b^{6}a^{4}+\frac{63}{2}x^{8}b^{5}a^{5}+30x^{7}b^{4}a^{6}+20x^{6}b^{3}a^{7}+9x^{5}b^{2}a^{8}+\frac{5}{2}x^{4}ba^{9}+\frac{1}{3}x^{3}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^10 + 5/6*x^12*b^9*a + 45/11*x^11*b^8*a^2 + 12*x^10*b^7*a^3 + 70
/3*x^9*b^6*a^4 + 63/2*x^8*b^5*a^5 + 30*x^7*b^4*a^6 + 20*x^6*b^3*a^7 + 9*x^5
*b^2*a^8 + 5/2*x^4*b*a^9 + 1/3*x^3*a^10
```

**giac [B]** time = 1.18, size = 112, normalized size = 2.38

$$\frac{1}{13}\,b^{10}x^{13}+\frac{5}{6}\,ab^{9}x^{12}+\frac{45}{11}\,a^{2}b^{8}x^{11}+12\,a^{3}b^{7}x^{10}+\frac{70}{3}\,a^{4}b^{6}x^{9}+\frac{63}{2}\,a^{5}b^{5}x^{8}+30\,a^{6}b^{4}x^{7}+20\,a^{7}b^{3}x^{6}+9\,a^{8}b^{2}x^{5}+\frac{5}{2}\,a^{9}bx^{4}+\frac{1}{3}\,a^{10}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

**maple [B]** time = 0.00, size = 113, normalized size = 2.40

$$\frac{1}{13}b^{10}x^{13}+\frac{5}{6}a\,b^{9}x^{12}+\frac{45}{11}a^{2}b^{8}x^{11}+12a^{3}b^{7}x^{10}+\frac{70}{3}a^{4}b^{6}x^{9}+\frac{63}{2}a^{5}b^{5}x^{8}+30a^{6}b^{4}x^{7}+20a^{7}b^{3}x^{6}+9a^{8}b^{2}x^{5}+\frac{5}{2}a^{9}b\,x^{4}+\frac{1}{3}a^{10}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^10,x)
```

```
[Out] 1/13*b^10*x^13+5/6*a*b^9*x^12+45/11*a^2*b^8*x^11+12*a^3*b^7*x^10+70/3*a^4*b
^6*x^9+63/2*a^5*b^5*x^8+30*a^6*b^4*x^7+20*a^7*b^3*x^6+9*a^8*b^2*x^5+5/2*a^9
*b*x^4+1/3*a^10*x^3
```

**maxima [B]** time = 1.37, size = 112, normalized size = 2.38

$$\frac{1}{13}\,b^{10}x^{13}+\frac{5}{6}\,ab^{9}x^{12}+\frac{45}{11}\,a^{2}b^{8}x^{11}+12\,a^{3}b^{7}x^{10}+\frac{70}{3}\,a^{4}b^{6}x^{9}+\frac{63}{2}\,a^{5}b^{5}x^{8}+30\,a^{6}b^{4}x^{7}+20\,a^{7}b^{3}x^{6}+9\,a^{8}b^{2}x^{5}+\frac{5}{2}\,a^{9}bx^{4}+\frac{1}{3}\,a^{10}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

**mupad [B]** time = 0.07, size = 31, normalized size = 0.66

$$\frac{(a+b\,x)^{11}\,\left(8\,a^2-88\,a\,b\,x+528\,b^2\,x^2\right)}{6864\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^10,x)
```

```
[Out] ((a + b*x)^11*(8*a^2 + 528*b^2*x^2 - 88*a*b*x))/(6864*b^3)
```

**sympy [B]** time = 0.11, size = 128, normalized size = 2.72

$$\frac{a^{10}x^{3}}{3}+\frac{5a^{9}bx^{4}}{2}+9a^{8}b^{2}x^{5}+20a^{7}b^{3}x^{6}+30a^{6}b^{4}x^{7}+\frac{63a^{5}b^{5}x^{8}}{2}+\frac{70a^{4}b^{6}x^{9}}{3}+12a^{3}b^{7}x^{10}+\frac{45a^{2}b^{8}x^{11}}{11}+\frac{5ab^{9}x^{12}}{6}+\frac{b^{10}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**10,x)
```

```
[Out] a**10*x**3/3 + 5*a**9*b*x**4/2 + 9*a**8*b**2*x**5 + 20*a**7*b**3*x**6 + 30*
a**6*b**4*x**7 + 63*a**5*b**5*x**8/2 + 70*a**4*b**6*x**9/3 + 12*a**3*b**7*x
**10 + 45*a**2*b**8*x**11/11 + 5*a*b**9*x**12/6 + b**10*x**13/13
```

## 3.133 $\int x(a+bx)^{10}\,dx$

**Optimal**. Leaf size=30

$$\frac{(a+bx)^{12}}{12b^2}-\frac{a(a+bx)^{11}}{11b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^{12}}{12b^2}-\frac{a(a+bx)^{11}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^10,x]
```

```
[Out] -(a*(a + b*x)^11)/(11*b^2) + (a + b*x)^12/(12*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{10}\,dx&=\int\left(-\frac{a(a+bx)^{10}}{b}+\frac{(a+bx)^{11}}{b}\right)dx\\&=-\frac{a(a+bx)^{11}}{11b^2}+\frac{(a+bx)^{12}}{12b^2}\end{aligned}$$

**Mathematica [B]** time = 0.00, size = 128, normalized size = 4.27

$$\frac{a^{10}x^2}{2}+\frac{10}{3}a^9bx^3+\frac{45}{4}a^8b^2x^4+24a^7b^3x^5+35a^6b^4x^6+36a^5b^5x^7+\frac{105}{4}a^4b^6x^8+\frac{40}{3}a^3b^7x^9+\frac{9}{2}a^2b^8x^{10}+\frac{10}{11}ab^9x^{11}+\frac{b^{10}x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^10,x]
```

```
[Out] (a^10*x^2)/2 + (10*a^9*b*x^3)/3 + (45*a^8*b^2*x^4)/4 + 24*a^7*b^3*x^5 + 35*
a^6*b^4*x^6 + 36*a^5*b^5*x^7 + (105*a^4*b^6*x^8)/4 + (40*a^3*b^7*x^9)/3 + (
9*a^2*b^8*x^10)/2 + (10*a*b^9*x^11)/11 + (b^10*x^12)/12
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x)^10, x]
```

**fricas [B]** time = 1.01, size = 112, normalized size = 3.73

$$\frac{1}{12}x^{12}b^{10}+\frac{10}{11}x^{11}b^{9}a+\frac{9}{2}x^{10}b^{8}a^{2}+\frac{40}{3}x^{9}b^{7}a^{3}+\frac{105}{4}x^{8}b^{6}a^{4}+36x^{7}b^{5}a^{5}+35x^{6}b^{4}a^{6}+24x^{5}b^{3}a^{7}+\frac{45}{4}x^{4}b^{2}a^{8}+\frac{10}{3}x^{3}ba^{9}+\frac{1}{2}x^{2}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^10 + 10/11*x^11*b^9*a + 9/2*x^10*b^8*a^2 + 40/3*x^9*b^7*a^3 + 1
05/4*x^8*b^6*a^4 + 36*x^7*b^5*a^5 + 35*x^6*b^4*a^6 + 24*x^5*b^3*a^7 + 45/4*
x^4*b^2*a^8 + 10/3*x^3*b*a^9 + 1/2*x^2*a^10
```

**giac [B]** time = 1.44, size = 112, normalized size = 3.73

$$\frac{1}{12}\,b^{10}x^{12}+\frac{10}{11}\,ab^{9}x^{11}+\frac{9}{2}\,a^{2}b^{8}x^{10}+\frac{40}{3}\,a^{3}b^{7}x^{9}+\frac{105}{4}\,a^{4}b^{6}x^{8}+36\,a^{5}b^{5}x^{7}+35\,a^{6}b^{4}x^{6}+24\,a^{7}b^{3}x^{5}+\frac{45}{4}\,a^{8}b^{2}x^{4}+\frac{10}{3}\,a^{9}bx^{3}+\frac{1}{2}\,a^{10}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

**maple [B]** time = 0.00, size = 113, normalized size = 3.77

$$\frac{1}{12}b^{10}x^{12}+\frac{10}{11}a\,b^{9}x^{11}+\frac{9}{2}a^{2}b^{8}x^{10}+\frac{40}{3}a^{3}b^{7}x^{9}+\frac{105}{4}a^{4}b^{6}x^{8}+36a^{5}b^{5}x^{7}+35a^{6}b^{4}x^{6}+24a^{7}b^{3}x^{5}+\frac{45}{4}a^{8}b^{2}x^{4}+\frac{10}{3}a^{9}b\,x^{3}+\frac{1}{2}a^{10}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^10,x)
```

```
[Out] 1/12*b^10*x^12+10/11*a*b^9*x^11+9/2*a^2*b^8*x^10+40/3*a^3*b^7*x^9+105/4*a^4
*b^6*x^8+36*a^5*b^5*x^7+35*a^6*b^4*x^6+24*a^7*b^3*x^5+45/4*a^8*b^2*x^4+10/3
*a^9*b*x^3+1/2*a^10*x^2
```

**maxima [B]** time = 1.35, size = 112, normalized size = 3.73

$$\frac{1}{12}\,b^{10}x^{12}+\frac{10}{11}\,ab^{9}x^{11}+\frac{9}{2}\,a^{2}b^{8}x^{10}+\frac{40}{3}\,a^{3}b^{7}x^{9}+\frac{105}{4}\,a^{4}b^{6}x^{8}+36\,a^{5}b^{5}x^{7}+35\,a^{6}b^{4}x^{6}+24\,a^{7}b^{3}x^{5}+\frac{45}{4}\,a^{8}b^{2}x^{4}+\frac{10}{3}\,a^{9}bx^{3}+\frac{1}{2}\,a^{10}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

**mupad [B]** time = 0.09, size = 25, normalized size = 0.83

$$-\frac{2\left(\frac{a\,(a+b\,x)^{11}}{22}-\frac{(a+b\,x)^{12}}{24}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^10,x)
```

```
[Out] -(2*((a*(a + b*x)^11)/22 - (a + b*x)^12/24))/b^2
```

**sympy [B]** time = 0.11, size = 129, normalized size = 4.30

$$\frac{a^{10}x^2}{2}+\frac{10a^9bx^3}{3}+\frac{45a^8b^2x^4}{4}+24a^7b^3x^5+35a^6b^4x^6+36a^5b^5x^7+\frac{105a^4b^6x^8}{4}+\frac{40a^3b^7x^9}{3}+\frac{9a^2b^8x^{10}}{2}+\frac{10ab^9x^{11}}{11}+\frac{b^{10}x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**10,x)
```

```
[Out] a**10*x**2/2 + 10*a**9*b*x**3/3 + 45*a**8*b**2*x**4/4 + 24*a**7*b**3*x**5 +
 35*a**6*b**4*x**6 + 36*a**5*b**5*x**7 + 105*a**4*b**6*x**8/4 + 40*a**3*b**
7*x**9/3 + 9*a**2*b**8*x**10/2 + 10*a*b**9*x**11/11 + b**10*x**12/12
```

## 3.134 $\int (a+bx)^{10}\,dx$

**Optimal**. Leaf size=14

$$\frac{(a+bx)^{11}}{11b}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{10}\,dx = \frac{(a+bx)^{11}}{11b}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10, x]
```

**fricas [B]** time = 1.26, size = 108, normalized size = 7.71

$$\frac{1}{11}x^{11}b^{10} + x^{10}b^9a + 5x^9b^8a^2 + 15x^8b^7a^3 + 30x^7b^6a^4 + 42x^6b^5a^5 + 42x^5b^4a^6 + 30x^4b^3a^7 + 15x^3b^2a^8 + 5x^2ba^9 + xa^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^10 + x^10*b^9*a + 5*x^9*b^8*a^2 + 15*x^8*b^7*a^3 + 30*x^7*b^6*a
^4 + 42*x^6*b^5*a^5 + 42*x^5*b^4*a^6 + 30*x^4*b^3*a^7 + 15*x^3*b^2*a^8 + 5*
x^2*b*a^9 + x*a^10
```

**giac [A]** time = 1.16, size = 12, normalized size = 0.86

$$\frac{(bx + a)^{11}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/11*(b*x + a)^11/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx + a)^{11}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10,x)
```

```
[Out] 1/11*(b*x+a)^11/b
```

**maxima [A]** time = 1.33, size = 12, normalized size = 0.86

$$\frac{(bx + a)^{11}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/11*(b*x + a)^11/b
```

**mupad [B]** time = 0.11, size = 108, normalized size = 7.71

$$a^{10}\,x+5\,a^9\,b\,x^2+15\,a^8\,b^2\,x^3+30\,a^7\,b^3\,x^4+42\,a^6\,b^4\,x^5+42\,a^5\,b^5\,x^6+30\,a^4\,b^6\,x^7+15\,a^3\,b^7\,x^8+5\,a^2\,b^8\,x^9+a\,b^9\,x^{10}+\frac{b^{10}\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10,x)
```

```
[Out] a^10*x + (b^10*x^11)/11 + 5*a^9*b*x^2 + a*b^9*x^10 + 15*a^8*b^2*x^3 + 30*a^
7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7*x
^8 + 5*a^2*b^8*x^9
```

**sympy [B]** time = 0.11, size = 114, normalized size = 8.14

$$a^{10}x+5a^9bx^2+15a^8b^2x^3+30a^7b^3x^4+42a^6b^4x^5+42a^5b^5x^6+30a^4b^6x^7+15a^3b^7x^8+5a^2b^8x^9+ab^9x^{10}+\frac{b^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10,x)
```

```
[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*b
**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*a*
*2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

## 3.135 $\int \frac{(a+bx)^{10}}{x}\,dx$

**Optimal**. Leaf size=122

$$a^{10}\log(x)+10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+\frac{10}{9}ab^9x^9+$$

**Rubi** [A] time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+10a^9bx+a^{10}\log(x)+\frac{10}{9}ab^9x^9+\frac{b^{10}x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x,x]
```

```
[Out] 10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (2
52*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)
/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x}\,dx = \int \left(10a^9b+\frac{a^{10}}{x}+45a^8b^2x+120a^7b^3x^2+210a^6b^4x^3+252a^5b^5x^4+210a^4b^6x^5+120a^3b^7x^6\right.$$

$$= 10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2$$

**Mathematica** [A] time = 0.00, size = 122, normalized size = 1.00

$$a^{10}\log(x)+10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+\frac{10}{9}ab^9x^9+\frac{b^{10}x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x,x]
```

```
[Out] 10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (2
52*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)
/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x, x]
```

**fricas [A]** time = 1.56, size = 108, normalized size = 0.89

$$\frac{1}{10}\,b^{10}x^{10}+\frac{10}{9}\,ab^{9}x^{9}+\frac{45}{8}\,a^{2}b^{8}x^{8}+\frac{120}{7}\,a^{3}b^{7}x^{7}+35\,a^{4}b^{6}x^{6}+\frac{252}{5}\,a^{5}b^{5}x^{5}+\frac{105}{2}\,a^{6}b^{4}x^{4}+40\,a^{7}b^{3}x^{3}+\frac{45}{2}\,a^{8}b^{2}x^{2}+10\,a^{9}bx+a^{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

**giac [A]** time = 1.08, size = 109, normalized size = 0.89

$$\frac{1}{10}\,b^{10}x^{10}+\frac{10}{9}\,ab^{9}x^{9}+\frac{45}{8}\,a^{2}b^{8}x^{8}+\frac{120}{7}\,a^{3}b^{7}x^{7}+35\,a^{4}b^{6}x^{6}+\frac{252}{5}\,a^{5}b^{5}x^{5}+\frac{105}{2}\,a^{6}b^{4}x^{4}+40\,a^{7}b^{3}x^{3}+\frac{45}{2}\,a^{8}b^{2}x^{2}+10\,a^{9}bx+a^{10}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="giac")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(abs(x))
```

**maple [A]** time = 0.00, size = 109, normalized size = 0.89

$$\frac{b^{10}x^{10}}{10}+\frac{10a\,b^{9}x^{9}}{9}+\frac{45a^{2}b^{8}x^{8}}{8}+\frac{120a^{3}b^{7}x^{7}}{7}+35a^{4}b^{6}x^{6}+\frac{252a^{5}b^{5}x^{5}}{5}+\frac{105a^{6}b^{4}x^{4}}{2}+40a^{7}b^{3}x^{3}+\frac{45a^{8}b^{2}x^{2}}{2}+a^{10}\ln(x)+10a^{9}bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x,x)
```

```
[Out] 10*a^9*b*x+45/2*a^8*b^2*x^2+40*a^7*b^3*x^3+105/2*a^6*b^4*x^4+252/5*a^5*b^5*
x^5+35*a^4*b^6*x^6+120/7*a^3*b^7*x^7+45/8*a^2*b^8*x^8+10/9*a*b^9*x^9+1/10*b
^10*x^10+a^10*ln(x)
```

**maxima** [A] time = 1.39, size = 108, normalized size = 0.89

$$\frac{1}{10}\,b^{10}x^{10} + \frac{10}{9}\,ab^{9}x^{9} + \frac{45}{8}\,a^{2}b^{8}x^{8} + \frac{120}{7}\,a^{3}b^{7}x^{7} + 35\,a^{4}b^{6}x^{6} + \frac{252}{5}\,a^{5}b^{5}x^{5} + \frac{105}{2}\,a^{6}b^{4}x^{4} + 40\,a^{7}b^{3}x^{3} + \frac{45}{2}\,a^{8}b^{2}x^{2} + 10\,a^{9}bx + a^{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

**mupad** [B] time = 0.08, size = 108, normalized size = 0.89

$$a^{10}\,\ln(x) + \frac{b^{10}\,x^{10}}{10} + \frac{10\,a\,b^{9}\,x^{9}}{9} + \frac{45\,a^{8}\,b^{2}\,x^{2}}{2} + 40\,a^{7}\,b^{3}\,x^{3} + \frac{105\,a^{6}\,b^{4}\,x^{4}}{2} + \frac{252\,a^{5}\,b^{5}\,x^{5}}{5} + 35\,a^{4}\,b^{6}\,x^{6} + \frac{120\,a^{3}\,b^{7}\,x^{7}}{7} + \frac{45\,a^{2}\,b^{8}\,x^{8}}{8} + 10\,a^{9}\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x,x)
```

```
[Out] a^10*log(x) + (b^10*x^10)/10 + (10*a*b^9*x^9)/9 + (45*a^8*b^2*x^2)/2 + 40*a
^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (252*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (
120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)/8 + 10*a^9*b*x
```

**sympy** [A] time = 0.26, size = 126, normalized size = 1.03

$$a^{10}\log{(x)} + 10a^{9}bx + \frac{45a^{8}b^{2}x^{2}}{2} + 40a^{7}b^{3}x^{3} + \frac{105a^{6}b^{4}x^{4}}{2} + \frac{252a^{5}b^{5}x^{5}}{5} + 35a^{4}b^{6}x^{6} + \frac{120a^{3}b^{7}x^{7}}{7} + \frac{45a^{2}b^{8}x^{8}}{8} + \frac{10ab^{9}x^{9}}{9} + \frac{b^{10}x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x,x)
```

```
[Out] a**10*log(x) + 10*a**9*b*x + 45*a**8*b**2*x**2/2 + 40*a**7*b**3*x**3 + 105*
a**6*b**4*x**4/2 + 252*a**5*b**5*x**5/5 + 35*a**4*b**6*x**6 + 120*a**3*b**7
*x**7/7 + 45*a**2*b**8*x**8/8 + 10*a*b**9*x**9/9 + b**10*x**10/10
```

## 3.136 $\int \frac{(a+bx)^{10}}{x^2}\,dx$

**Optimal**. Leaf size=115

$$-\frac{a^{10}}{x}+10a^9b\log(x)+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

**Rubi [A]** time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+45a^8b^2x+10a^9b\log(x)-\frac{a^{10}}{x}+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^2,x]
```

```
[Out] -(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4
 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4 +
 (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^2}\,dx &= \int \left(45a^8b^2+\frac{a^{10}}{x^2}+\frac{10a^9b}{x}+120a^7b^3x+210a^6b^4x^2+252a^5b^5x^3+210a^4b^6x^4+120a^3b^7x^5+\right.\\ &= -\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}a\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{x}+10a^9b\log(x)+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^2,x]
```

```
[Out] -(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4
 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4 +
 (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^2, x]
```

**fricas [A]** time = 0.84, size = 114, normalized size = 0.99

$$\frac{28\,b^{10}x^{10}+315\,ab^{9}x^{9}+1620\,a^{2}b^{8}x^{8}+5040\,a^{3}b^{7}x^{7}+10584\,a^{4}b^{6}x^{6}+15876\,a^{5}b^{5}x^{5}+17640\,a^{6}b^{4}x^{4}+15120\,a^{7}b^{3}x^{3}+11340\,a^{8}b^{2}x^{2}+2520\,a^{9}bx\log(x)-252\,a^{10}}{252\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^10*x^10 + 315*a*b^9*x^9 + 1620*a^2*b^8*x^8 + 5040*a^3*b^7*x^7 +
 10584*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 17640*a^6*b^4*x^4 + 15120*a^7*b^3*
x^3 + 11340*a^8*b^2*x^2 + 2520*a^9*b*x*log(x) - 252*a^10)/x
```

**giac [A]** time = 0.96, size = 110, normalized size = 0.96

$$\frac{1}{9}\,b^{10}x^{9}+\frac{5}{4}\,ab^{9}x^{8}+\frac{45}{7}\,a^{2}b^{8}x^{7}+20\,a^{3}b^{7}x^{6}+42\,a^{4}b^{6}x^{5}+63\,a^{5}b^{5}x^{4}+70\,a^{6}b^{4}x^{3}+60\,a^{7}b^{3}x^{2}+45\,a^{8}b^{2}x+10\,a^{9}b\log\left(|x|\right)-\frac{a^{10}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*b
^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x +
10*a^9*b*log(abs(x)) - a^10/x
```

**maple [A]** time = 0.01, size = 110, normalized size = 0.96

$$\frac{b^{10}x^{9}}{9}+\frac{5a\,b^{9}x^{8}}{4}+\frac{45a^{2}b^{8}x^{7}}{7}+20a^{3}b^{7}x^{6}+42a^{4}b^{6}x^{5}+63a^{5}b^{5}x^{4}+70a^{6}b^{4}x^{3}+60a^{7}b^{3}x^{2}+10a^{9}b\ln(x)+45a^{8}b^{2}x-\frac{a^{10}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^2,x)
```

```
[Out] -a^10/x+45*a^8*b^2*x+60*a^7*b^3*x^2+70*a^6*b^4*x^3+63*a^5*b^5*x^4+42*a^4*b^
6*x^5+20*a^3*b^7*x^6+45/7*a^2*b^8*x^7+5/4*a*b^9*x^8+1/9*b^10*x^9+10*a^9*b*l
n(x)
```

**maxima [A]** time = 1.35, size = 109, normalized size = 0.95

$$\frac{1}{9}\,b^{10}x^{9}+\frac{5}{4}\,ab^{9}x^{8}+\frac{45}{7}\,a^{2}b^{8}x^{7}+20\,a^{3}b^{7}x^{6}+42\,a^{4}b^{6}x^{5}+63\,a^{5}b^{5}x^{4}+70\,a^{6}b^{4}x^{3}+60\,a^{7}b^{3}x^{2}+45\,a^{8}b^{2}x+10\,a^{9}b\log(x)-\frac{a^{10}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*b
^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x +
10*a^9*b*log(x) - a^10/x
```

**mupad [B]** time = 0.12, size = 109, normalized size = 0.95

$$\frac{b^{10}\,x^{9}}{9}-\frac{a^{10}}{x}+45\,a^{8}\,b^{2}\,x+\frac{5\,a\,b^{9}\,x^{8}}{4}+10\,a^{9}\,b\,\ln(x)+60\,a^{7}\,b^{3}\,x^{2}+70\,a^{6}\,b^{4}\,x^{3}+63\,a^{5}\,b^{5}\,x^{4}+42\,a^{4}\,b^{6}\,x^{5}+20\,a^{3}\,b^{7}\,x^{6}+\frac{45\,a^{2}\,b^{8}\,x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^2,x)
```

```
[Out] (b^10*x^9)/9 - a^10/x + 45*a^8*b^2*x + (5*a*b^9*x^8)/4 + 10*a^9*b*log(x) +
60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4 + 42*a^4*b^6*x^5 + 20*a^3*
b^7*x^6 + (45*a^2*b^8*x^7)/7
```

**sympy [A]** time = 0.27, size = 117, normalized size = 1.02

$$-\frac{a^{10}}{x}+10a^{9}b\log{(x)}+45a^{8}b^{2}x+60a^{7}b^{3}x^{2}+70a^{6}b^{4}x^{3}+63a^{5}b^{5}x^{4}+42a^{4}b^{6}x^{5}+20a^{3}b^{7}x^{6}+\frac{45a^{2}b^{8}x^{7}}{7}+\frac{5ab^{9}x^{8}}{4}+\frac{b^{10}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**2,x)
```

```
[Out] -a**10/x + 10*a**9*b*log(x) + 45*a**8*b**2*x + 60*a**7*b**3*x**2 + 70*a**6*
b**4*x**3 + 63*a**5*b**5*x**4 + 42*a**4*b**6*x**5 + 20*a**3*b**7*x**6 + 45*
a**2*b**8*x**7/7 + 5*a*b**9*x**8/4 + b**10*x**9/9
```

## 3.137 $\int \frac{(a+bx)^{10}}{x^3}\,dx$

**Optimal**. Leaf size=119

$$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+45a^8b^2\log(x)+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+\frac{10}{7}ab^9x^7+\frac{b^{10}}{8}$$

**Rubi [A]** time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+120a^7b^3x+45a^8b^2\log(x)-\frac{10a^9b}{x}-\frac{a^{10}}{2x^2}+\frac{10}{7}ab^9x^7+\frac{b^{10}x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^3,x]
```

```
[Out] -a^10/(2*x^2) - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^5
*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*b^
9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^3}\,dx=\int \left(120a^7b^3+\frac{a^{10}}{x^3}+\frac{10a^9b}{x^2}+\frac{45a^8b^2}{x}+210a^6b^4x+252a^5b^5x^2+210a^4b^6x^3+120a^3b^7x^4+$$
$$=-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+$$

**Mathematica [A]** time = 0.01, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+45a^8b^2\log(x)+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+\frac{10}{7}ab^9x^7+\frac{b^{10}x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^3,x]
```

```
[Out] -1/2*a^10/x^2 - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^5
*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*b^
9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^3, x]
```

**fricas [A]** time = 1.41, size = 114, normalized size = 0.96

$$\frac{7\,b^{10}x^{10}+80\,ab^{9}x^{9}+420\,a^{2}b^{8}x^{8}+1344\,a^{3}b^{7}x^{7}+2940\,a^{4}b^{6}x^{6}+4704\,a^{5}b^{5}x^{5}+5880\,a^{6}b^{4}x^{4}+6720\,a^{7}b^{3}x^{3}+2520\,a^{8}b^{2}x^{2}\log(x)-560\,a^{9}bx-28\,a^{10}}{56\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^10*x^10 + 80*a*b^9*x^9 + 420*a^2*b^8*x^8 + 1344*a^3*b^7*x^7 + 294
0*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 5880*a^6*b^4*x^4 + 6720*a^7*b^3*x^3 + 25
20*a^8*b^2*x^2*log(x) - 560*a^9*b*x - 28*a^10)/x^2
```

**giac [A]** time = 0.93, size = 109, normalized size = 0.92

$$\frac{1}{8}\,b^{10}x^{8}+\frac{10}{7}\,ab^{9}x^{7}+\frac{15}{2}\,a^{2}b^{8}x^{6}+24\,a^{3}b^{7}x^{5}+\frac{105}{2}\,a^{4}b^{6}x^{4}+84\,a^{5}b^{5}x^{3}+105\,a^{6}b^{4}x^{2}+120\,a^{7}b^{3}x+45\,a^{8}b^{2}\log\left(|x|\right)-\frac{20\,a^{9}bx+a^{10}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*a
^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^2*
log(abs(x)) - 1/2*(20*a^9*b*x + a^10)/x^2
```

**maple [A]** time = 0.01, size = 110, normalized size = 0.92

$$\frac{b^{10}x^{8}}{8}+\frac{10a\,b^{9}x^{7}}{7}+\frac{15a^{2}b^{8}x^{6}}{2}+24a^{3}b^{7}x^{5}+\frac{105a^{4}b^{6}x^{4}}{2}+84a^{5}b^{5}x^{3}+105a^{6}b^{4}x^{2}+45a^{8}b^{2}\ln(x)+120a^{7}b^{3}x-\frac{10a^{9}b}{x}-\frac{a^{10}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^3,x)
```

```
[Out] -1/2*a^10/x^2-10*a^9*b/x+120*a^7*b^3*x+105*a^6*b^4*x^2+84*a^5*b^5*x^3+105/2
*a^4*b^6*x^4+24*a^3*b^7*x^5+15/2*a^2*b^8*x^6+10/7*a*b^9*x^7+1/8*b^10*x^8+45
*a^8*b^2*ln(x)
```

**maxima [A]** time = 1.44, size = 108, normalized size = 0.91

$$\frac{1}{8}\, b^{10} x^{8} + \frac{10}{7}\, a b^{9} x^{7} + \frac{15}{2}\, a^{2} b^{8} x^{6} + 24\, a^{3} b^{7} x^{5} + \frac{105}{2}\, a^{4} b^{6} x^{4} + 84\, a^{5} b^{5} x^{3} + 105\, a^{6} b^{4} x^{2} + 120\, a^{7} b^{3} x + 45\, a^{8} b^{2} \log\left(x\right) - \frac{20\, a^{9} b x + a^{10}}{2\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*a
^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^2*
log(x) - 1/2*(20*a^9*b*x + a^10)/x^2
```

**mupad [B]** time = 0.07, size = 110, normalized size = 0.92

$$\frac{b^{10}\, x^8}{8} - \frac{\frac{a^{10}}{2} + 10\, b\, x\, a^9}{x^2} + 120\, a^7\, b^3\, x + \frac{10\, a\, b^9\, x^7}{7} + 105\, a^6\, b^4\, x^2 + 84\, a^5\, b^5\, x^3 + \frac{105\, a^4\, b^6\, x^4}{2} + 24\, a^3\, b^7\, x^5 + \frac{15\, a^2\, b^8\, x^6}{2} + 45\, a^8\, b^2\, \ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^3,x)
```

```
[Out] (b^10*x^8)/8 - (a^10/2 + 10*a^9*b*x)/x^2 + 120*a^7*b^3*x + (10*a*b^9*x^7)/7
 + 105*a^6*b^4*x^2 + 84*a^5*b^5*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5
+ (15*a^2*b^8*x^6)/2 + 45*a^8*b^2*log(x)
```

**sympy [A]** time = 0.31, size = 122, normalized size = 1.03

$$45 a^{8} b^{2} \log{\left(x \right)} + 120 a^{7} b^{3} x + 105 a^{6} b^{4} x^{2} + 84 a^{5} b^{5} x^{3} + \frac{105 a^{4} b^{6} x^{4}}{2} + 24 a^{3} b^{7} x^{5} + \frac{15 a^{2} b^{8} x^{6}}{2} + \frac{10 a b^{9} x^{7}}{7} + \frac{b^{10} x^{8}}{8} + \frac{- a^{10} - 20 a^{9} b x}{2 x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**3,x)
```

```
[Out] 45*a**8*b**2*log(x) + 120*a**7*b**3*x + 105*a**6*b**4*x**2 + 84*a**5*b**5*x
**3 + 105*a**4*b**6*x**4/2 + 24*a**3*b**7*x**5 + 15*a**2*b**8*x**6/2 + 10*a
*b**9*x**7/7 + b**10*x**8/8 + (-a**10 - 20*a**9*b*x)/(2*x**2)
```

## 3.138 $\int \frac{(a+bx)^{10}}{x^4} \, dx$

**Optimal**. Leaf size=115

$$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+120a^7b^3\log(x)+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

**Rubi** [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5-\frac{45a^8b^2}{x}+210a^6b^4x+120a^7b^3\log(x)-\frac{5a^9b}{x^2}-\frac{a^{10}}{3x^3}+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^4,x]
```

```
[Out] -a^10/(3*x^3) - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b^
5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3 +
 (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^4}\,dx &= \int \left(210a^6b^4+\frac{a^{10}}{x^4}+\frac{10a^9b}{x^3}+\frac{45a^8b^2}{x^2}+\frac{120a^7b^3}{x}+252a^5b^5x+210a^4b^6x^2+120a^3b^7x^3+45a\right.\\ &= -\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+120a^7b^3\log(x)+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^4,x]
```

```
[Out] -1/3*a^10/x^3 - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b^
5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3 +
 (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^4, x]
```

**fricas [A]** time = 1.13, size = 114, normalized size = 0.99

$$\frac{3\,b^{10}x^{10}+35\,ab^{9}x^{9}+189\,a^{2}b^{8}x^{8}+630\,a^{3}b^{7}x^{7}+1470\,a^{4}b^{6}x^{6}+2646\,a^{5}b^{5}x^{5}+4410\,a^{6}b^{4}x^{4}+2520\,a^{7}b^{3}x^{3}\log(x)-945\,a^{8}b^{2}x^{2}-105\,a^{9}bx-7\,a^{10}}{21\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^10*x^10 + 35*a*b^9*x^9 + 189*a^2*b^8*x^8 + 630*a^3*b^7*x^7 + 1470
*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 4410*a^6*b^4*x^4 + 2520*a^7*b^3*x^3*log(x
) - 945*a^8*b^2*x^2 - 105*a^9*b*x - 7*a^10)/x^3
```

**giac [A]** time = 1.14, size = 109, normalized size = 0.95

$$\frac{1}{7}\,b^{10}x^{7}+\frac{5}{3}\,ab^{9}x^{6}+9\,a^{2}b^{8}x^{5}+30\,a^{3}b^{7}x^{4}+70\,a^{4}b^{6}x^{3}+126\,a^{5}b^{5}x^{2}+210\,a^{6}b^{4}x+120\,a^{7}b^{3}\log\left(|x|\right)-\frac{135\,a^{8}b^{2}x^{2}+15\,a^{9}bx+a^{10}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6*
x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(abs(x)) - 1/3*(135*
a^8*b^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

**maple [A]** time = 0.01, size = 110, normalized size = 0.96

$$\frac{b^{10}x^{7}}{7}+\frac{5a\,b^{9}x^{6}}{3}+9a^{2}b^{8}x^{5}+30a^{3}b^{7}x^{4}+70a^{4}b^{6}x^{3}+126a^{5}b^{5}x^{2}+120a^{7}b^{3}\ln(x)+210a^{6}b^{4}x-\frac{45a^{8}b^{2}}{x}-\frac{5a^{9}b}{x^{2}}-\frac{a^{10}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^4,x)
```

```
[Out] -1/3*a^10/x^3-5*a^9*b/x^2-45*a^8*b^2/x+210*a^6*b^4*x+126*a^5*b^5*x^2+70*a^4
*b^6*x^3+30*a^3*b^7*x^4+9*a^2*b^8*x^5+5/3*a*b^9*x^6+1/7*b^10*x^7+120*a^7*b^
3*ln(x)
```

**maxima [A]** time = 1.37, size = 108, normalized size = 0.94

$$\frac{1}{7}b^{10}x^7 + \frac{5}{3}ab^9x^6 + 9a^2b^8x^5 + 30a^3b^7x^4 + 70a^4b^6x^3 + 126a^5b^5x^2 + 210a^6b^4x + 120a^7b^3\log(x) - \frac{135a^8b^2x^2 + 15a^9bx + a^{10}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6*
x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(x) - 1/3*(135*a^8*b
^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

**mupad [B]** time = 0.06, size = 110, normalized size = 0.96

$$\frac{b^{10}x^7}{7} - \frac{\frac{a^{10}}{3} + 5a^9bx + 45a^8b^2x^2}{x^3} + 210a^6b^4x + \frac{5ab^9x^6}{3} + 126a^5b^5x^2 + 70a^4b^6x^3 + 30a^3b^7x^4 + 9a^2b^8x^5 + 120a^7b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^4,x)
```

```
[Out] (b^10*x^7)/7 - (a^10/3 + 45*a^8*b^2*x^2 + 5*a^9*b*x)/x^3 + 210*a^6*b^4*x +
(5*a*b^9*x^6)/3 + 126*a^5*b^5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2
*b^8*x^5 + 120*a^7*b^3*log(x)
```

**sympy [A]** time = 0.34, size = 119, normalized size = 1.03

$$120a^7b^3\log(x) + 210a^6b^4x + 126a^5b^5x^2 + 70a^4b^6x^3 + 30a^3b^7x^4 + 9a^2b^8x^5 + \frac{5ab^9x^6}{3} + \frac{b^{10}x^7}{7} + \frac{-a^{10} - 15a^9bx - 135a^8b^2x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**4,x)
```

```
[Out] 120*a**7*b**3*log(x) + 210*a**6*b**4*x + 126*a**5*b**5*x**2 + 70*a**4*b**6*
x**3 + 30*a**3*b**7*x**4 + 9*a**2*b**8*x**5 + 5*a*b**9*x**6/3 + b**10*x**7/
7 + (-a**10 - 15*a**9*b*x - 135*a**8*b**2*x**2)/(3*x**3)
```

## 3.139 $\int \frac{(a+bx)^{10}}{x^5}\,dx$

**Optimal**. Leaf size=119

$$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+210a^6b^4\log(x)+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9x^5+\frac{b^{10}x^6}{6}$$

**Rubi** [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{2x^2}+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4-\frac{120a^7b^3}{x}+252a^5b^5x+210a^6b^4\log(x)-\frac{10a^9b}{3x^3}-\frac{a^{10}}{4x^4}+2ab^9x^5+\frac{b^{10}x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^5,x]
```

```
[Out] -a^10/(4*x^4) - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/x
 + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4 +
2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^5}\,dx = \int \left(252a^5b^5+\frac{a^{10}}{x^5}+\frac{10a^9b}{x^4}+\frac{45a^8b^2}{x^3}+\frac{120a^7b^3}{x^2}+\frac{210a^6b^4}{x}+210a^4b^6x+120a^3b^7x^2+45a^2\right.$$
$$=-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9$$

**Mathematica** [A] time = 0.01, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+210a^6b^4\log(x)+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9x^5+\frac{b^{10}x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^5,x]
```

```
[Out] -1/4*a^10/x^4 - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/x
 + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4 +
2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^5, x]
```

**fricas [A]** time = 1.56, size = 114, normalized size = 0.96

$$\frac{2\,b^{10}x^{10}+24\,ab^{9}x^{9}+135\,a^{2}b^{8}x^{8}+480\,a^{3}b^{7}x^{7}+1260\,a^{4}b^{6}x^{6}+3024\,a^{5}b^{5}x^{5}+2520\,a^{6}b^{4}x^{4}\log(x)-1440\,a^{7}b^{3}x^{3}-270\,a^{8}b^{2}x^{2}-40\,a^{9}bx-3\,a^{10}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^10*x^10 + 24*a*b^9*x^9 + 135*a^2*b^8*x^8 + 480*a^3*b^7*x^7 + 1260
*a^4*b^6*x^6 + 3024*a^5*b^5*x^5 + 2520*a^6*b^4*x^4*log(x) - 1440*a^7*b^3*x^
3 - 270*a^8*b^2*x^2 - 40*a^9*b*x - 3*a^10)/x^4
```

**giac [A]** time = 1.12, size = 111, normalized size = 0.93

$$\frac{1}{6}\,b^{10}x^{6}+2\,ab^{9}x^{5}+\frac{45}{4}\,a^{2}b^{8}x^{4}+40\,a^{3}b^{7}x^{3}+105\,a^{4}b^{6}x^{2}+252\,a^{5}b^{5}x+210\,a^{6}b^{4}\log\left(|x|\right)-\frac{1440\,a^{7}b^{3}x^{3}+270\,a^{8}b^{2}x^{2}+40\,a^{9}bx+3\,a^{10}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b^
6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(abs(x)) - 1/12*(1440*a^7*b^3*x^3 +
270*a^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

**maple [A]** time = 0.01, size = 110, normalized size = 0.92

$$\frac{b^{10}x^{6}}{6}+2a\,b^{9}x^{5}+\frac{45a^{2}b^{8}x^{4}}{4}+40a^{3}b^{7}x^{3}+105a^{4}b^{6}x^{2}+210a^{6}b^{4}\ln(x)+252a^{5}b^{5}x-\frac{120a^{7}b^{3}}{x}-\frac{45a^{8}b^{2}}{2x^{2}}-\frac{10a^{9}b}{3x^{3}}-\frac{a^{10}}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^5,x)
```

```
[Out] -1/4*a^10/x^4-10/3*a^9*b/x^3-45/2*a^8*b^2/x^2-120*a^7*b^3/x+252*a^5*b^5*x+1
05*a^4*b^6*x^2+40*a^3*b^7*x^3+45/4*a^2*b^8*x^4+2*a*b^9*x^5+1/6*b^10*x^6+210
*a^6*b^4*ln(x)
```

**maxima [A]** time = 1.27, size = 110, normalized size = 0.92

$$\frac{1}{6}b^{10}x^6 + 2ab^9x^5 + \frac{45}{4}a^2b^8x^4 + 40a^3b^7x^3 + 105a^4b^6x^2 + 252a^5b^5x + 210a^6b^4\log(x) - \frac{1440a^7b^3x^3 + 270a^8b^2x^2 + 40a^9bx + 3a^{10}}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b^
6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(x) - 1/12*(1440*a^7*b^3*x^3 + 270*a
^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

**mupad [B]** time = 0.10, size = 110, normalized size = 0.92

$$\frac{b^{10}x^6}{6} - \frac{\frac{a^{10}}{4} + \frac{10a^9bx}{3} + \frac{45a^8b^2x^2}{2} + 120a^7b^3x^3}{x^4} + 252a^5b^5x + 2ab^9x^5 + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45a^2b^8x^4}{4} + 210a^6b^4\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^5,x)
```

```
[Out] (b^10*x^6)/6 - (a^10/4 + (45*a^8*b^2*x^2)/2 + 120*a^7*b^3*x^3 + (10*a^9*b*x
)/3)/x^4 + 252*a^5*b^5*x + 2*a*b^9*x^5 + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 +
 (45*a^2*b^8*x^4)/4 + 210*a^6*b^4*log(x)
```

**sympy [A]** time = 0.44, size = 121, normalized size = 1.02

$$210a^6b^4\log(x) + 252a^5b^5x + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45a^2b^8x^4}{4} + 2ab^9x^5 + \frac{b^{10}x^6}{6} + \frac{-3a^{10} - 40a^9bx - 270a^8b^2x^2 - 1440a^7b^3x^3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**5,x)
```

```
[Out] 210*a**6*b**4*log(x) + 252*a**5*b**5*x + 105*a**4*b**6*x**2 + 40*a**3*b**7*
x**3 + 45*a**2*b**8*x**4/4 + 2*a*b**9*x**5 + b**10*x**6/6 + (-3*a**10 - 40*
a**9*b*x - 270*a**8*b**2*x**2 - 1440*a**7*b**3*x**3)/(12*x**4)
```

## 3.140 $\int \frac{(a+bx)^{10}}{x^6}\,dx$

**Optimal**. Leaf size=117

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+252a^5b^5\log(x)+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

**Rubi** [A] time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}+60a^3b^7x^2+15a^2b^8x^3-\frac{210a^6b^4}{x}+210a^4b^6x+252a^5b^5\log(x)-\frac{5a^9b}{2x^4}-\frac{a^{10}}{5x^5}+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^6,x]
```

```
[Out] -a^10/(5*x^5) - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 - (
210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b^9
*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^6}\,dx &= \int \left(210a^4b^6+\frac{a^{10}}{x^6}+\frac{10a^9b}{x^5}+\frac{45a^8b^2}{x^4}+\frac{120a^7b^3}{x^3}+\frac{210a^6b^4}{x^2}+\frac{252a^5b^5}{x}+120a^3b^7x+45a^2b^8\right.\\ &= -\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 117, normalized size = 1.00

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+252a^5b^5\log(x)+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^6,x]
```

```
[Out] -1/5*a^10/x^5 - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 - (
210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b^9
*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^6, x]
```

**fricas [A]** time = 1.31, size = 114, normalized size = 0.97

$$\frac{2\,b^{10}x^{10} + 25\,ab^{9}x^{9} + 150\,a^{2}b^{8}x^{8} + 600\,a^{3}b^{7}x^{7} + 2100\,a^{4}b^{6}x^{6} + 2520\,a^{5}b^{5}x^{5}\log(x) - 2100\,a^{6}b^{4}x^{4} - 600\,a^{7}b^{3}x^{3} - 150\,a^{8}b^{2}x^{2} - 25\,a^{9}bx - 2\,a^{10}}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^10*x^10 + 25*a*b^9*x^9 + 150*a^2*b^8*x^8 + 600*a^3*b^7*x^7 + 2100
*a^4*b^6*x^6 + 2520*a^5*b^5*x^5*log(x) - 2100*a^6*b^4*x^4 - 600*a^7*b^3*x^3
 - 150*a^8*b^2*x^2 - 25*a^9*b*x - 2*a^10)/x^5
```

**giac [A]** time = 1.14, size = 111, normalized size = 0.95

$$\frac{1}{5}\,b^{10}x^{5} + \frac{5}{2}\,ab^{9}x^{4} + 15\,a^{2}b^{8}x^{3} + 60\,a^{3}b^{7}x^{2} + 210\,a^{4}b^{6}x + 252\,a^{5}b^{5}\log\left(|x|\right) - \frac{2100\,a^{6}b^{4}x^{4} + 600\,a^{7}b^{3}x^{3} + 150\,a^{8}b^{2}x^{2} + 25\,a^{9}bx + 2\,a^{10}}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b^
6*x + 252*a^5*b^5*log(abs(x)) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3 +
150*a^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

**maple [A]** time = 0.01, size = 110, normalized size = 0.94

$$\frac{b^{10}x^{5}}{5} + \frac{5a\,b^{9}x^{4}}{2} + 15a^{2}b^{8}x^{3} + 60a^{3}b^{7}x^{2} + 252a^{5}b^{5}\ln(x) + 210a^{4}b^{6}x - \frac{210a^{6}b^{4}}{x} - \frac{60a^{7}b^{3}}{x^{2}} - \frac{15a^{8}b^{2}}{x^{3}} - \frac{5a^{9}b}{2x^{4}} - \frac{a^{10}}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^6,x)
```

```
[Out] -1/5*a^10/x^5-5/2*a^9*b/x^4-15*a^8*b^2/x^3-60*a^7*b^3/x^2-210*a^6*b^4/x+210
*a^4*b^6*x+60*a^3*b^7*x^2+15*a^2*b^8*x^3+5/2*a*b^9*x^4+1/5*b^10*x^5+252*a^5
*b^5*ln(x)
```

**maxima [A]** time = 1.40, size = 110, normalized size = 0.94

$$\frac{1}{5}\,b^{10}x^5+\frac{5}{2}\,ab^9x^4+15\,a^2b^8x^3+60\,a^3b^7x^2+210\,a^4b^6x+252\,a^5b^5\log(x)-\frac{2100\,a^6b^4x^4+600\,a^7b^3x^3+150\,a^8b^2x^2+25\,a^9bx+2\,a^{10}}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b^
6*x + 252*a^5*b^5*log(x) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3 + 150*a
^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

**mupad [B]** time = 0.10, size = 110, normalized size = 0.94

$$\frac{b^{10}\,x^5}{5}-\frac{\frac{a^{10}}{5}+\frac{5\,a^9\,b\,x}{2}+15\,a^8\,b^2\,x^2+60\,a^7\,b^3\,x^3+210\,a^6\,b^4\,x^4}{x^5}+210\,a^4\,b^6\,x+\frac{5\,a\,b^9\,x^4}{2}+60\,a^3\,b^7\,x^2+15\,a^2\,b^8\,x^3+252\,a^5\,b^5\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^6,x)
```

```
[Out] (b^10*x^5)/5 - (a^10/5 + 15*a^8*b^2*x^2 + 60*a^7*b^3*x^3 + 210*a^6*b^4*x^4
+ (5*a^9*b*x)/2)/x^5 + 210*a^4*b^6*x + (5*a*b^9*x^4)/2 + 60*a^3*b^7*x^2 + 1
5*a^2*b^8*x^3 + 252*a^5*b^5*log(x)
```

**sympy [A]** time = 0.57, size = 121, normalized size = 1.03

$$252a^5b^5\log{(x)}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5ab^9x^4}{2}+\frac{b^{10}x^5}{5}+\frac{-2a^{10}-25a^9bx-150a^8b^2x^2-600a^7b^3x^3-2100a^6b^4x^4}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**6,x)
```

```
[Out] 252*a**5*b**5*log(x) + 210*a**4*b**6*x + 60*a**3*b**7*x**2 + 15*a**2*b**8*x
**3 + 5*a*b**9*x**4/2 + b**10*x**5/5 + (-2*a**10 - 25*a**9*b*x - 150*a**8*b
**2*x**2 - 600*a**7*b**3*x**3 - 2100*a**6*b**4*x**4)/(10*x**5)
```

## 3.141 $\int \frac{(a+bx)^{10}}{x^7}\,dx$

**Optimal**. Leaf size=119

$$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+210a^4b^6\log(x)+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

**Rubi** [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}+\frac{45}{2}a^2b^8x^2-\frac{252a^5b^5}{x}+120a^3b^7x+210a^4b^6\log(x)-\frac{2a^9b}{x^5}-\frac{a^{10}}{6x^6}+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^7,x]
```

```
[Out] -a^10/(6*x^6) - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 - (
105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 + (
10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^7}\,dx &= \int \left(120a^3b^7+\frac{a^{10}}{x^7}+\frac{10a^9b}{x^6}+\frac{45a^8b^2}{x^5}+\frac{120a^7b^3}{x^4}+\frac{210a^6b^4}{x^3}+\frac{252a^5b^5}{x^2}+\frac{210a^4b^6}{x}+45a^2b^8\right.\\ &= -\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+210a^4b^6\log(x)+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^7,x]
```

```
[Out] -1/6*a^10/x^6 - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 - (
105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 + (
10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^7, x]
```

**fricas [A]** time = 1.20, size = 114, normalized size = 0.96

$$\frac{3\,b^{10}x^{10} + 40\,ab^9x^9 + 270\,a^2b^8x^8 + 1440\,a^3b^7x^7 + 2520\,a^4b^6x^6\log(x) - 3024\,a^5b^5x^5 - 1260\,a^6b^4x^4 - 480\,a^7b^3x^3 - 135\,a^8b^2x^2 - 24\,a^9bx - 2\,a^{10}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*x^10 + 40*a*b^9*x^9 + 270*a^2*b^8*x^8 + 1440*a^3*b^7*x^7 + 252
0*a^4*b^6*x^6*log(x) - 3024*a^5*b^5*x^5 - 1260*a^6*b^4*x^4 - 480*a^7*b^3*x^
3 - 135*a^8*b^2*x^2 - 24*a^9*b*x - 2*a^10)/x^6
```

**giac [A]** time = 1.24, size = 111, normalized size = 0.93

$$\frac{1}{4}\,b^{10}x^4 + \frac{10}{3}\,ab^9x^3 + \frac{45}{2}\,a^2b^8x^2 + 120\,a^3b^7x + 210\,a^4b^6\log\left(|x|\right) - \frac{3024\,a^5b^5x^5 + 1260\,a^6b^4x^4 + 480\,a^7b^3x^3 + 135\,a^8b^2x^2 + 24\,a^9bx + 2\,a^{10}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4*
b^6*log(abs(x)) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3*x
^3 + 135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

**maple [A]** time = 0.01, size = 110, normalized size = 0.92

$$\frac{b^{10}x^4}{4} + \frac{10a\,b^9x^3}{3} + \frac{45a^2b^8x^2}{2} + 210a^4b^6\ln(x) + 120a^3b^7x - \frac{252a^5b^5}{x} - \frac{105a^6b^4}{x^2} - \frac{40a^7b^3}{x^3} - \frac{45a^8b^2}{4x^4} - \frac{2a^9b}{x^5} - \frac{a^{10}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^7,x)
```

```
[Out] -1/6*a^10/x^6-2*a^9*b/x^5-45/4*a^8*b^2/x^4-40*a^7*b^3/x^3-105*a^6*b^4/x^2-2
52*a^5*b^5/x+120*a^3*b^7*x+45/2*a^2*b^8*x^2+10/3*a*b^9*x^3+1/4*b^10*x^4+210
*a^4*b^6*ln(x)
```

**maxima [A]** time = 1.37, size = 110, normalized size = 0.92

$$\frac{1}{4}\,b^{10}x^4+\frac{10}{3}\,ab^9x^3+\frac{45}{2}\,a^2b^8x^2+120\,a^3b^7x+210\,a^4b^6\log(x)-\frac{3024\,a^5b^5x^5+1260\,a^6b^4x^4+480\,a^7b^3x^3+135\,a^8b^2x^2+24\,a^9bx+2\,a^{10}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4*
b^6*log(x) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3*x^3 +
135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

**mupad [B]** time = 0.05, size = 110, normalized size = 0.92

$$\frac{b^{10}\,x^4}{4}-\frac{\frac{a^{10}}{6}+2\,a^9\,b\,x+\frac{45\,a^8\,b^2\,x^2}{4}+40\,a^7\,b^3\,x^3+105\,a^6\,b^4\,x^4+252\,a^5\,b^5\,x^5}{x^6}+120\,a^3\,b^7\,x+\frac{10\,a\,b^9\,x^3}{3}+\frac{45\,a^2\,b^8\,x^2}{2}+210\,a^4\,b^6\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^7,x)
```

```
[Out] (b^10*x^4)/4 - (a^10/6 + (45*a^8*b^2*x^2)/4 + 40*a^7*b^3*x^3 + 105*a^6*b^4*
x^4 + 252*a^5*b^5*x^5 + 2*a^9*b*x)/x^6 + 120*a^3*b^7*x + (10*a*b^9*x^3)/3 +
 (45*a^2*b^8*x^2)/2 + 210*a^4*b^6*log(x)
```

**sympy [A]** time = 0.57, size = 122, normalized size = 1.03

$$210a^4b^6\log(x)+120a^3b^7x+\frac{45a^2b^8x^2}{2}+\frac{10ab^9x^3}{3}+\frac{b^{10}x^4}{4}+\frac{-2a^{10}-24a^9bx-135a^8b^2x^2-480a^7b^3x^3-1260a^6b^4x^4-3024a^5b^5x^5}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**7,x)
```

```
[Out] 210*a**4*b**6*log(x) + 120*a**3*b**7*x + 45*a**2*b**8*x**2/2 + 10*a*b**9*x*
*3/3 + b**10*x**4/4 + (-2*a**10 - 24*a**9*b*x - 135*a**8*b**2*x**2 - 480*a*
*7*b**3*x**3 - 1260*a**6*b**4*x**4 - 3024*a**5*b**5*x**5)/(12*x**6)
```

## 3.142 $\int \frac{(a+bx)^{10}}{x^8} dx$

**Optimal**. Leaf size=115

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+120a^3b^7\log(x)+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}$$

**Rubi [A]** time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+120a^3b^7\log(x)-\frac{5a^9b}{3x^6}-\frac{a^{10}}{7x^7}+5ab^9x^2+\frac{b^{10}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^8,x]
```

```
[Out] -a^10/(7*x^7) - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (7
0*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a*b
^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^8} dx &= \int \left(45a^2b^8+\frac{a^{10}}{x^8}+\frac{10a^9b}{x^7}+\frac{45a^8b^2}{x^6}+\frac{120a^7b^3}{x^5}+\frac{210a^6b^4}{x^4}+\frac{252a^5b^5}{x^3}+\frac{210a^4b^6}{x^2}+\frac{120a^3b^7}{x}\right.\\ &= -\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+120a^3b^7\log(x)+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^8,x]
```

```
[Out] -1/7*a^10/x^7 - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (7
0*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a*b
^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^8, x]
```

**fricas [A]** time = 1.31, size = 114, normalized size = 0.99

$$\frac{7\,b^{10}x^{10} + 105\,ab^{9}x^{9} + 945\,a^{2}b^{8}x^{8} + 2520\,a^{3}b^{7}x^{7}\log(x) - 4410\,a^{4}b^{6}x^{6} - 2646\,a^{5}b^{5}x^{5} - 1470\,a^{6}b^{4}x^{4} - 630\,a^{7}b^{3}x^{3} - 189\,a^{8}b^{2}x^{2} - 35\,a^{9}bx - 3\,a^{10}}{21\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^10*x^10 + 105*a*b^9*x^9 + 945*a^2*b^8*x^8 + 2520*a^3*b^7*x^7*log(
x) - 4410*a^4*b^6*x^6 - 2646*a^5*b^5*x^5 - 1470*a^6*b^4*x^4 - 630*a^7*b^3*x
^3 - 189*a^8*b^2*x^2 - 35*a^9*b*x - 3*a^10)/x^7
```

**giac [A]** time = 1.10, size = 111, normalized size = 0.97

$$\frac{1}{3}\,b^{10}x^{3} + 5\,ab^{9}x^{2} + 45\,a^{2}b^{8}x + 120\,a^{3}b^{7}\log\left(|x|\right) - \frac{4410\,a^{4}b^{6}x^{6} + 2646\,a^{5}b^{5}x^{5} + 1470\,a^{6}b^{4}x^{4} + 630\,a^{7}b^{3}x^{3} + 189\,a^{8}b^{2}x^{2} + 35\,a^{9}bx + 3\,a^{10}}{21\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(abs(x)) - 1/21*
(4410*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3 +
 189*a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

**maple [A]** time = 0.01, size = 110, normalized size = 0.96

$$\frac{b^{10}x^{3}}{3} + 5a\,b^{9}x^{2} + 120a^{3}b^{7}\ln(x) + 45a^{2}b^{8}x - \frac{210a^{4}b^{6}}{x} - \frac{126a^{5}b^{5}}{x^{2}} - \frac{70a^{6}b^{4}}{x^{3}} - \frac{30a^{7}b^{3}}{x^{4}} - \frac{9a^{8}b^{2}}{x^{5}} - \frac{5a^{9}b}{3x^{6}} - \frac{a^{10}}{7x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^8,x)
```

```
[Out] -1/7*a^10/x^7-5/3*a^9*b/x^6-9*a^8*b^2/x^5-30*a^7*b^3/x^4-70*a^6*b^4/x^3-126
*a^5*b^5/x^2-210*a^4*b^6/x+45*a^2*b^8*x+5*a*b^9*x^2+1/3*b^10*x^3+120*a^3*b^
7*ln(x)
```

**maxima** [A] time = 1.30, size = 110, normalized size = 0.96

$$\frac{1}{3}b^{10}x^3 + 5\,ab^9x^2 + 45\,a^2b^8x + 120\,a^3b^7\log(x) - \frac{4410\,a^4b^6x^6 + 2646\,a^5b^5x^5 + 1470\,a^6b^4x^4 + 630\,a^7b^3x^3 + 189\,a^8b^2x^2 + 35\,a^9bx + 3\,a^{10}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(x) - 1/21*(4410
*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3 + 189*
a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

**mupad** [B] time = 0.10, size = 110, normalized size = 0.96

$$\frac{b^{10}\,x^3}{3} - \frac{\frac{a^{10}}{7} + \frac{5\,a^9\,b\,x}{3} + 9\,a^8\,b^2\,x^2 + 30\,a^7\,b^3\,x^3 + 70\,a^6\,b^4\,x^4 + 126\,a^5\,b^5\,x^5 + 210\,a^4\,b^6\,x^6}{x^7} + 45\,a^2\,b^8\,x + 5\,a\,b^9\,x^2 + 120\,a^3\,b^7\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^8,x)
```

```
[Out] (b^10*x^3)/3 - (a^10/7 + 9*a^8*b^2*x^2 + 30*a^7*b^3*x^3 + 70*a^6*b^4*x^4 +
126*a^5*b^5*x^5 + 210*a^4*b^6*x^6 + (5*a^9*b*x)/3)/x^7 + 45*a^2*b^8*x + 5*a
*b^9*x^2 + 120*a^3*b^7*log(x)
```

**sympy** [A] time = 0.70, size = 119, normalized size = 1.03

$$120a^3b^7\log(x) + 45a^2b^8x + 5ab^9x^2 + \frac{b^{10}x^3}{3} + \frac{-3a^{10} - 35a^9bx - 189a^8b^2x^2 - 630a^7b^3x^3 - 1470a^6b^4x^4 - 2646a^5b^5x^5 - 4410a^4b^6x^6}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**8,x)
```

```
[Out] 120*a**3*b**7*log(x) + 45*a**2*b**8*x + 5*a*b**9*x**2 + b**10*x**3/3 + (-3*
a**10 - 35*a**9*b*x - 189*a**8*b**2*x**2 - 630*a**7*b**3*x**3 - 1470*a**6*b
**4*x**4 - 2646*a**5*b**5*x**5 - 4410*a**4*b**6*x**6)/(21*x**7)
```

## 3.143 $\int \frac{(a+bx)^{10}}{x^9} \, dx$

**Optimal**. Leaf size=119

$$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)+10ab^9x+\frac{b^{10}x^2}{2}$$

**Rubi** [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)-\frac{10a^9b}{7x^7}-\frac{a^{10}}{8x^8}+10ab^9x+\frac{b^{10}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^9,x]
```

```
[Out] -a^10/(8*x^8) - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x^
5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a^3
*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^9}\,dx &= \int \left(10ab^9+\frac{a^{10}}{x^9}+\frac{10a^9b}{x^8}+\frac{45a^8b^2}{x^7}+\frac{120a^7b^3}{x^6}+\frac{210a^6b^4}{x^5}+\frac{252a^5b^5}{x^4}+\frac{210a^4b^6}{x^3}+\frac{120a^3b^7}{x^2}\right.\\ &= -\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10ab^9x+\frac{b^1}{}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)+10ab^9x+\frac{b^{10}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^9,x]
```

```
[Out] -1/8*a^10/x^8 - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x^
5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a^3
*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^9,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^9, x]
```

**fricas [A]** time = 1.14, size = 114, normalized size = 0.96

$$\frac{28\,b^{10}x^{10}+560\,ab^{9}x^{9}+2520\,a^{2}b^{8}x^{8}\log(x)-6720\,a^{3}b^{7}x^{7}-5880\,a^{4}b^{6}x^{6}-4704\,a^{5}b^{5}x^{5}-2940\,a^{6}b^{4}x^{4}-1344\,a^{7}b^{3}x^{3}-420\,a^{8}b^{2}x^{2}-80\,a^{9}bx-7\,a^{10}}{56\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="fricas")
```

```
[Out] 1/56*(28*b^10*x^10 + 560*a*b^9*x^9 + 2520*a^2*b^8*x^8*log(x) - 6720*a^3*b^7
*x^7 - 5880*a^4*b^6*x^6 - 4704*a^5*b^5*x^5 - 2940*a^6*b^4*x^4 - 1344*a^7*b^
3*x^3 - 420*a^8*b^2*x^2 - 80*a^9*b*x - 7*a^10)/x^8
```

**giac [A]** time = 1.16, size = 111, normalized size = 0.93

$$\frac{1}{2}\,b^{10}x^{2}+10\,ab^{9}x+45\,a^{2}b^{8}\log\left(|x|\right)-\frac{6720\,a^{3}b^{7}x^{7}+5880\,a^{4}b^{6}x^{6}+4704\,a^{5}b^{5}x^{5}+2940\,a^{6}b^{4}x^{4}+1344\,a^{7}b^{3}x^{3}+420\,a^{8}b^{2}x^{2}+80\,a^{9}bx+7\,a^{10}}{56\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(abs(x)) - 1/56*(6720*a^3*b^7*x^7
 + 5880*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*x^
3 + 420*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

**maple [A]** time = 0.01, size = 110, normalized size = 0.92

$$\frac{b^{10}x^{2}}{2}+45a^{2}b^{8}\ln(x)+10a\,b^{9}x-\frac{120a^{3}b^{7}}{x}-\frac{105a^{4}b^{6}}{x^{2}}-\frac{84a^{5}b^{5}}{x^{3}}-\frac{105a^{6}b^{4}}{2x^{4}}-\frac{24a^{7}b^{3}}{x^{5}}-\frac{15a^{8}b^{2}}{2x^{6}}-\frac{10a^{9}b}{7x^{7}}-\frac{a^{10}}{8x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^9,x)
```

```
[Out] -1/8*a^10/x^8-10/7*a^9*b/x^7-15/2*a^8*b^2/x^6-24*a^7*b^3/x^5-105/2*a^6*b^4/
x^4-84*a^5*b^5/x^3-105*a^4*b^6/x^2-120*a^3*b^7/x+10*a*b^9*x+1/2*b^10*x^2+45
*a^2*b^8*ln(x)
```

**maxima** [A] time = 1.42, size = 110, normalized size = 0.92

$$\frac{1}{2}\,b^{10}x^2 + 10\,ab^9x + 45\,a^2b^8\log(x) - \frac{6720\,a^3b^7x^7 + 5880\,a^4b^6x^6 + 4704\,a^5b^5x^5 + 2940\,a^6b^4x^4 + 1344\,a^7b^3x^3 + 420\,a^8b^2x^2 + 80\,a^9bx + 7\,a^{10}}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(x) - 1/56*(6720*a^3*b^7*x^7 + 58
80*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*x^3 + 4
20*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

**mupad** [B] time = 0.07, size = 110, normalized size = 0.92

$$\frac{b^{10}\,x^2}{2} - \frac{\frac{a^{10}}{8} + \frac{10\,a^9\,b\,x}{7} + \frac{15\,a^8\,b^2\,x^2}{2} + 24\,a^7\,b^3\,x^3 + \frac{105\,a^6\,b^4\,x^4}{2} + 84\,a^5\,b^5\,x^5 + 105\,a^4\,b^6\,x^6 + 120\,a^3\,b^7\,x^7}{x^8} + 45\,a^2\,b^8\,\ln(x) + 10\,a\,b^9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^9,x)
```

```
[Out] (b^10*x^2)/2 - (a^10/8 + (15*a^8*b^2*x^2)/2 + 24*a^7*b^3*x^3 + (105*a^6*b^4
*x^4)/2 + 84*a^5*b^5*x^5 + 105*a^4*b^6*x^6 + 120*a^3*b^7*x^7 + (10*a^9*b*x)
/7)/x^8 + 45*a^2*b^8*log(x) + 10*a*b^9*x
```

**sympy** [A] time = 0.76, size = 119, normalized size = 1.00

$$45a^2b^8\log(x) + 10ab^9x + \frac{b^{10}x^2}{2} + \frac{-7a^{10} - 80a^9bx - 420a^8b^2x^2 - 1344a^7b^3x^3 - 2940a^6b^4x^4 - 4704a^5b^5x^5 - 5880a^4b^6x^6 - 6720a^3b^7x^7}{56x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**9,x)
```

```
[Out] 45*a**2*b**8*log(x) + 10*a*b**9*x + b**10*x**2/2 + (-7*a**10 - 80*a**9*b*x
- 420*a**8*b**2*x**2 - 1344*a**7*b**3*x**3 - 2940*a**6*b**4*x**4 - 4704*a**
5*b**5*x**5 - 5880*a**4*b**6*x**6 - 6720*a**3*b**7*x**7)/(56*x**8)
```

## 3.144 $\int \frac{(a+bx)^{10}}{x^{10}}\,dx$

**Optimal**. Leaf size=114

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

**Rubi [A]** time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}-\frac{5a^9b}{4x^8}-\frac{a^{10}}{9x^9}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^10,x]
```

```
[Out] -a^10/(9*x^9) - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{10}}\,dx &= \int \left(b^{10}+\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}+\cdots\right.\\ &= -\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+\cdots\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 114, normalized size = 1.00

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^10, x]
```

**fricas [A]** time = 1.21, size = 114, normalized size = 1.00

$$\frac{252\,b^{10}x^{10}+2520\,ab^{9}x^{9}\log(x)-11340\,a^{2}b^{8}x^{8}-15120\,a^{3}b^{7}x^{7}-17640\,a^{4}b^{6}x^{6}-15876\,a^{5}b^{5}x^{5}-10584\,a^{6}b^{4}x^{4}-5040\,a^{7}b^{3}x^{3}-1620\,a^{8}b^{2}x^{2}-315\,a^{9}bx-28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a^
3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 504
0*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

**giac [A]** time = 1.21, size = 110, normalized size = 0.96

$$b^{10}x+10\,ab^{9}\log\left(|x|\right)-\frac{11340\,a^{2}b^{8}x^{8}+15120\,a^{3}b^{7}x^{7}+17640\,a^{4}b^{6}x^{6}+15876\,a^{5}b^{5}x^{5}+10584\,a^{6}b^{4}x^{4}+5040\,a^{7}b^{3}x^{3}+1620\,a^{8}b^{2}x^{2}+315\,a^{9}bx+28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="giac")
```

```
[Out] b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^
7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^
3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

**maple [A]** time = 0.01, size = 109, normalized size = 0.96

$$10a\,b^{9}\ln(x)+b^{10}x-\frac{45a^{2}b^{8}}{x}-\frac{60a^{3}b^{7}}{x^{2}}-\frac{70a^{4}b^{6}}{x^{3}}-\frac{63a^{5}b^{5}}{x^{4}}-\frac{42a^{6}b^{4}}{x^{5}}-\frac{20a^{7}b^{3}}{x^{6}}-\frac{45a^{8}b^{2}}{7x^{7}}-\frac{5a^{9}b}{4x^{8}}-\frac{a^{10}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^10,x)
```

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

**maxima [A]** time = 1.40, size = 109, normalized size = 0.96

$$b^{10}x + 10\,ab^{9}\log(x) - \frac{11340\,a^{2}b^{8}x^{8} + 15120\,a^{3}b^{7}x^{7} + 17640\,a^{4}b^{6}x^{6} + 15876\,a^{5}b^{5}x^{5} + 10584\,a^{6}b^{4}x^{4} + 5040\,a^{7}b^{3}x^{3} + 1620\,a^{8}b^{2}x^{2} + 315\,a^{9}bx + 28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

```
[Out] b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 + 1
7640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x^3
 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

**mupad [B]** time = 0.08, size = 114, normalized size = 1.00

$$-\frac{\frac{a^{10}}{9} - b^{10}\,x^{10} + \frac{45\,a^{8}\,b^{2}\,x^{2}}{7} + 20\,a^{7}\,b^{3}\,x^{3} + 42\,a^{6}\,b^{4}\,x^{4} + 63\,a^{5}\,b^{5}\,x^{5} + 70\,a^{4}\,b^{6}\,x^{6} + 60\,a^{3}\,b^{7}\,x^{7} + 45\,a^{2}\,b^{8}\,x^{8} + \frac{5\,a^{9}\,b\,x}{4} - 10\,a\,b^{9}\,x^{9}\,\ln(x)}{x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^10,x)
```

```
[Out] -(a^10/9 - b^10*x^10 + (45*a^8*b^2*x^2)/7 + 20*a^7*b^3*x^3 + 42*a^6*b^4*x^4
 + 63*a^5*b^5*x^5 + 70*a^4*b^6*x^6 + 60*a^3*b^7*x^7 + 45*a^2*b^8*x^8 + (5*a
^9*b*x)/4 - 10*a*b^9*x^9*log(x))/x^9
```

**sympy [A]** time = 0.82, size = 117, normalized size = 1.03

$$10ab^{9}\log{(x)} + b^{10}x + \frac{-28a^{10} - 315a^{9}bx - 1620a^{8}b^{2}x^{2} - 5040a^{7}b^{3}x^{3} - 10584a^{6}b^{4}x^{4} - 15876a^{5}b^{5}x^{5} - 17640a^{4}b^{6}x^{6} - 15120a^{3}b^{7}x^{7} - 11340a^{2}b^{8}x^{8}}{252x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**10,x)
```

```
[Out] 10*a*b**9*log(x) + b**10*x + (-28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x**
2 - 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 176
40*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(252*x**9)
```

## 3.145 $\int \frac{(a+bx)^{10}}{x^{11}}\,dx$

**Optimal**. Leaf size=124

$$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log(x)$$

**Rubi** [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10a^9b}{9x^9}-\frac{a^{10}}{10x^{10}}-\frac{10ab^9}{x}+b^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^11,x]
```

```
[Out] -a^10/(10*x^10) - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)
/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4)
 - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{11}}\,dx = \int \left(\frac{a^{10}}{x^{11}}+\frac{10a^9b}{x^{10}}+\frac{45a^8b^2}{x^9}+\frac{120a^7b^3}{x^8}+\frac{210a^6b^4}{x^7}+\frac{252a^5b^5}{x^6}+\frac{210a^4b^6}{x^5}+\frac{120a^3b^7}{x^4}+\frac{45a^2b}{x^3}\right.$$
$$=-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-$$

**Mathematica** [A] time = 0.00, size = 124, normalized size = 1.00

$$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^11,x]
```

```
[Out] -1/10*a^10/x^10 - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)
/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4)
 - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^11,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^11, x]
```

**fricas [A]** time = 1.43, size = 114, normalized size = 0.92

$$\frac{2520\,b^{10}x^{10}\log(x)-25200\,ab^{9}x^{9}-56700\,a^{2}b^{8}x^{8}-100800\,a^{3}b^{7}x^{7}-132300\,a^{4}b^{6}x^{6}-127008\,a^{5}b^{5}x^{5}-88200\,a^{6}b^{4}x^{4}-43200\,a^{7}b^{3}x^{3}-14175\,a^{8}b^{2}x^{2}-2800\,a^{9}bx-252\,a^{10}}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^10*x^10*log(x) - 25200*a*b^9*x^9 - 56700*a^2*b^8*x^8 - 10080
0*a^3*b^7*x^7 - 132300*a^4*b^6*x^6 - 127008*a^5*b^5*x^5 - 88200*a^6*b^4*x^4
 - 43200*a^7*b^3*x^3 - 14175*a^8*b^2*x^2 - 2800*a^9*b*x - 252*a^10)/x^10
```

**giac [A]** time = 1.13, size = 112, normalized size = 0.90

$$b^{10}\log(|x|)-\frac{25200\,ab^{9}x^{9}+56700\,a^{2}b^{8}x^{8}+100800\,a^{3}b^{7}x^{7}+132300\,a^{4}b^{6}x^{6}+127008\,a^{5}b^{5}x^{5}+88200\,a^{6}b^{4}x^{4}+43200\,a^{7}b^{3}x^{3}+14175\,a^{8}b^{2}x^{2}+2800\,a^{9}bx+252\,a^{10}}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="giac")
```

```
[Out] b^10*log(abs(x)) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^3
*b^7*x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 + 43
200*a^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10
```

**maple [A]** time = 0.01, size = 111, normalized size = 0.90

$$b^{10}\ln(x)-\frac{10a\,b^{9}}{x}-\frac{45a^{2}b^{8}}{2x^{2}}-\frac{40a^{3}b^{7}}{x^{3}}-\frac{105a^{4}b^{6}}{2x^{4}}-\frac{252a^{5}b^{5}}{5x^{5}}-\frac{35a^{6}b^{4}}{x^{6}}-\frac{120a^{7}b^{3}}{7x^{7}}-\frac{45a^{8}b^{2}}{8x^{8}}-\frac{10a^{9}b}{9x^{9}}-\frac{a^{10}}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^11,x)
```

```
[Out] -1/10*a^10/x^10-10/9*a^9*b/x^9-45/8*a^8*b^2/x^8-120/7*a^7*b^3/x^7-35*a^6*b^
4/x^6-252/5*a^5*b^5/x^5-105/2*a^4*b^6/x^4-40*a^3*b^7/x^3-45/2*a^2*b^8/x^2-1
0*a*b^9/x+b^10*ln(x)
```

**maxima [A]** time = 1.40, size = 111, normalized size = 0.90

$$b^{10}\log(x) - \frac{25200\,ab^9x^9 + 56700\,a^2b^8x^8 + 100800\,a^3b^7x^7 + 132300\,a^4b^6x^6 + 127008\,a^5b^5x^5 + 88200\,a^6b^4x^4 + 43200\,a^7b^3x^3 + 14175\,a^8b^2x^2 + 2800\,a^9bx + 252\,a^{10}}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="maxima")
```

```
[Out] b^10*log(x) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^3*b^7*
x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 + 43200*a
^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10
```

**mupad [B]** time = 0.07, size = 111, normalized size = 0.90

$$b^{10}\,\ln(x) - \frac{\frac{a^{10}}{10} + \frac{10\,a^9\,b\,x}{9} + \frac{45\,a^8\,b^2\,x^2}{8} + \frac{120\,a^7\,b^3\,x^3}{7} + 35\,a^6\,b^4\,x^4 + \frac{252\,a^5\,b^5\,x^5}{5} + \frac{105\,a^4\,b^6\,x^6}{2} + 40\,a^3\,b^7\,x^7 + \frac{45\,a^2\,b^8\,x^8}{2} + 10\,a\,b^9\,x^9}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^11,x)
```

```
[Out] b^10*log(x) - (a^10/10 + 10*a*b^9*x^9 + (45*a^8*b^2*x^2)/8 + (120*a^7*b^3*x
^3)/7 + 35*a^6*b^4*x^4 + (252*a^5*b^5*x^5)/5 + (105*a^4*b^6*x^6)/2 + 40*a^3
*b^7*x^7 + (45*a^2*b^8*x^8)/2 + (10*a^9*b*x)/9)/x^10
```

**sympy [A]** time = 1.01, size = 119, normalized size = 0.96

$$b^{10}\log(x) + \frac{-252a^{10} - 2800a^9bx - 14175a^8b^2x^2 - 43200a^7b^3x^3 - 88200a^6b^4x^4 - 127008a^5b^5x^5 - 132300a^4b^6x^6 - 100800a^3b^7x^7 - 56700a^2b^8x^8 - 25200ab^9x^9}{2520x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**11,x)
```

```
[Out] b**10*log(x) + (-252*a**10 - 2800*a**9*b*x - 14175*a**8*b**2*x**2 - 43200*a
**7*b**3*x**3 - 88200*a**6*b**4*x**4 - 127008*a**5*b**5*x**5 - 132300*a**4*
b**6*x**6 - 100800*a**3*b**7*x**7 - 56700*a**2*b**8*x**8 - 25200*a*b**9*x**
9)/(2520*x**10)
```

## 3.146 $\int \frac{(a+bx)^{10}}{x^{12}} dx$

**Optimal**. Leaf size=17

$$-\frac{(a+bx)^{11}}{11ax^{11}}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^{11}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^12,x]
```

```
[Out] -(a + b*x)^11/(11*a*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{12}} dx = -\frac{(a+bx)^{11}}{11ax^{11}}$$

**Mathematica [B]** time = 0.01, size = 114, normalized size = 6.71

$$-\frac{a^{10}}{11x^{11}}-\frac{a^9b}{x^{10}}-\frac{5a^8b^2}{x^9}-\frac{15a^7b^3}{x^8}-\frac{30a^6b^4}{x^7}-\frac{42a^5b^5}{x^6}-\frac{42a^4b^6}{x^5}-\frac{30a^3b^7}{x^4}-\frac{15a^2b^8}{x^3}-\frac{5ab^9}{x^2}-\frac{b^{10}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^12,x]
```

```
[Out] -1/11*a^10/x^11 - (a^9*b)/x^10 - (5*a^8*b^2)/x^9 - (15*a^7*b^3)/x^8 - (30*a
^6*b^4)/x^7 - (42*a^5*b^5)/x^6 - (42*a^4*b^6)/x^5 - (30*a^3*b^7)/x^4 - (15*
a^2*b^8)/x^3 - (5*a*b^9)/x^2 - b^10/x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^12,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^12, x]
```

**fricas [B]** time = 1.26, size = 110, normalized size = 6.47

$$-\frac{11\,b^{10}x^{10}+55\,ab^{9}x^{9}+165\,a^{2}b^{8}x^{8}+330\,a^{3}b^{7}x^{7}+462\,a^{4}b^{6}x^{6}+462\,a^{5}b^{5}x^{5}+330\,a^{6}b^{4}x^{4}+165\,a^{7}b^{3}x^{3}+55\,a^{8}b^{2}x^{2}+11\,a^{9}bx+a^{10}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="fricas")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

**giac [B]** time = 1.06, size = 110, normalized size = 6.47

$$-\frac{11\,b^{10}x^{10}+55\,ab^{9}x^{9}+165\,a^{2}b^{8}x^{8}+330\,a^{3}b^{7}x^{7}+462\,a^{4}b^{6}x^{6}+462\,a^{5}b^{5}x^{5}+330\,a^{6}b^{4}x^{4}+165\,a^{7}b^{3}x^{3}+55\,a^{8}b^{2}x^{2}+11\,a^{9}bx+a^{10}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="giac")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

**maple [B]** time = 0.01, size = 113, normalized size = 6.65

$$-\frac{b^{10}}{x}-\frac{5a\,b^{9}}{x^{2}}-\frac{15a^{2}b^{8}}{x^{3}}-\frac{30a^{3}b^{7}}{x^{4}}-\frac{42a^{4}b^{6}}{x^{5}}-\frac{42a^{5}b^{5}}{x^{6}}-\frac{30a^{6}b^{4}}{x^{7}}-\frac{15a^{7}b^{3}}{x^{8}}-\frac{5a^{8}b^{2}}{x^{9}}-\frac{a^{9}b}{x^{10}}-\frac{a^{10}}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^12,x)
```

```
[Out] -1/11*a^10/x^11-42*a^5*b^5/x^6-5*a*b^9/x^2-5*a^8*b^2/x^9-15*a^2*b^8/x^3-30*
a^3*b^7/x^4-42*a^4*b^6/x^5-b^10/x-a^9*b/x^10-30*a^6*b^4/x^7-15*a^7*b^3/x^8
```

**maxima** [B] time = 1.33, size = 110, normalized size = 6.47

$$-\frac{11\,b^{10}x^{10}+55\,ab^{9}x^{9}+165\,a^{2}b^{8}x^{8}+330\,a^{3}b^{7}x^{7}+462\,a^{4}b^{6}x^{6}+462\,a^{5}b^{5}x^{5}+330\,a^{6}b^{4}x^{4}+165\,a^{7}b^{3}x^{3}+55\,a^{8}b^{2}x^{2}+11\,a^{9}bx+a^{10}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="maxima")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

**mupad** [B] time = 0.13, size = 110, normalized size = 6.47

$$-\frac{\frac{a^{10}}{11}+a^{9}\,b\,x+5\,a^{8}\,b^{2}\,x^{2}+15\,a^{7}\,b^{3}\,x^{3}+30\,a^{6}\,b^{4}\,x^{4}+42\,a^{5}\,b^{5}\,x^{5}+42\,a^{4}\,b^{6}\,x^{6}+30\,a^{3}\,b^{7}\,x^{7}+15\,a^{2}\,b^{8}\,x^{8}+5\,a\,b^{9}\,x^{9}+b^{10}\,x^{10}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^12,x)
```

```
[Out] -(a^10/11 + b^10*x^10 + 5*a*b^9*x^9 + 5*a^8*b^2*x^2 + 15*a^7*b^3*x^3 + 30*a
^6*b^4*x^4 + 42*a^5*b^5*x^5 + 42*a^4*b^6*x^6 + 30*a^3*b^7*x^7 + 15*a^2*b^8*
x^8 + a^9*b*x)/x^11
```

**sympy** [B] time = 1.03, size = 119, normalized size = 7.00

$$\frac{-a^{10}-11a^{9}bx-55a^{8}b^{2}x^{2}-165a^{7}b^{3}x^{3}-330a^{6}b^{4}x^{4}-462a^{5}b^{5}x^{5}-462a^{4}b^{6}x^{6}-330a^{3}b^{7}x^{7}-165a^{2}b^{8}x^{8}-55ab^{9}x^{9}-11b^{10}x^{10}}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**12,x)
```

```
[Out] (-a**10 - 11*a**9*b*x - 55*a**8*b**2*x**2 - 165*a**7*b**3*x**3 - 330*a**6*b
**4*x**4 - 462*a**5*b**5*x**5 - 462*a**4*b**6*x**6 - 330*a**3*b**7*x**7 - 1
65*a**2*b**8*x**8 - 55*a*b**9*x**9 - 11*b**10*x**10)/(11*x**11)
```

## 3.147 $\int \frac{(a+bx)^{10}}{x^{13}}\,dx$

**Optimal**. Leaf size=36

$$\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}$$

**Rubi [A]** time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^13,x]
```

```
[Out] -(a + b*x)^11/(12*a*x^12) + (b*(a + b*x)^11)/(132*a^2*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{13}}\,dx &= -\frac{(a+bx)^{11}}{12ax^{12}}-\frac{b\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{12a}\\ &= -\frac{(a+bx)^{11}}{12ax^{12}}+\frac{b(a+bx)^{11}}{132a^2x^{11}}\end{aligned}$$

**Mathematica [B]** time = 0.00, size = 128, normalized size = 3.56

$$-\frac{a^{10}}{12x^{12}}-\frac{10a^9b}{11x^{11}}-\frac{9a^8b^2}{2x^{10}}-\frac{40a^7b^3}{3x^9}-\frac{105a^6b^4}{4x^8}-\frac{36a^5b^5}{x^7}-\frac{35a^4b^6}{x^6}-\frac{24a^3b^7}{x^5}-\frac{45a^2b^8}{4x^4}-\frac{10ab^9}{3x^3}-\frac{b^{10}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^13,x]
```

```
[Out] -1/12*a^10/x^12 - (10*a^9*b)/(11*x^11) - (9*a^8*b^2)/(2*x^10) - (40*a^7*b^3
)/(3*x^9) - (105*a^6*b^4)/(4*x^8) - (36*a^5*b^5)/x^7 - (35*a^4*b^6)/x^6 - (
24*a^3*b^7)/x^5 - (45*a^2*b^8)/(4*x^4) - (10*a*b^9)/(3*x^3) - b^10/(2*x^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^13,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^13, x]
```

**fricas [B]** time = 1.34, size = 112, normalized size = 3.11

$$-\frac{66\,b^{10}x^{10}+440\,ab^9x^9+1485\,a^2b^8x^8+3168\,a^3b^7x^7+4620\,a^4b^6x^6+4752\,a^5b^5x^5+3465\,a^6b^4x^4+1760\,a^7b^3x^3+594\,a^8b^2x^2+120\,a^9bx+11\,a^{10}}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="fricas")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

**giac [B]** time = 0.96, size = 112, normalized size = 3.11

$$-\frac{66\,b^{10}x^{10}+440\,ab^9x^9+1485\,a^2b^8x^8+3168\,a^3b^7x^7+4620\,a^4b^6x^6+4752\,a^5b^5x^5+3465\,a^6b^4x^4+1760\,a^7b^3x^3+594\,a^8b^2x^2+120\,a^9bx+11\,a^{10}}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="giac")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

**maple [B]** time = 0.00, size = 113, normalized size = 3.14

$$-\frac{b^{10}}{2x^2}-\frac{10a\,b^9}{3x^3}-\frac{45a^2b^8}{4x^4}-\frac{24a^3b^7}{x^5}-\frac{35a^4b^6}{x^6}-\frac{36a^5b^5}{x^7}-\frac{105a^6b^4}{4x^8}-\frac{40a^7b^3}{3x^9}-\frac{9a^8b^2}{2x^{10}}-\frac{10a^9b}{11x^{11}}-\frac{a^{10}}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^13,x)
```

```
[Out] -35*a^4*b^6/x^6-10/11*a^9*b/x^11-1/12*a^10/x^12-40/3*a^7*b^3/x^9-10/3*a*b^9
/x^3-45/4*a^2*b^8/x^4-1/2*b^10/x^2-24*a^3*b^7/x^5-9/2*a^8*b^2/x^10-36*a^5*b
^5/x^7-105/4*a^6*b^4/x^8
```

**maxima [B]** time = 1.39, size = 112, normalized size = 3.11

$$-\frac{66\,b^{10}x^{10}+440\,ab^9x^9+1485\,a^2b^8x^8+3168\,a^3b^7x^7+4620\,a^4b^6x^6+4752\,a^5b^5x^5+3465\,a^6b^4x^4+1760\,a^7b^3x^3+594\,a^8b^2x^2+120\,a^9bx+11\,a^{10}}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="maxima")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

**mupad [B]** time = 0.10, size = 23, normalized size = 0.64

$$-\frac{(11\,a-b\,x)\,\left(a+b\,x\right)^{11}}{132\,a^2\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^13,x)
```

```
[Out] -((11*a - b*x)*(a + b*x)^11)/(132*a^2*x^12)
```

**sympy [B]** time = 0.99, size = 121, normalized size = 3.36

$$\frac{-11a^{10}-120a^9bx-594a^8b^2x^2-1760a^7b^3x^3-3465a^6b^4x^4-4752a^5b^5x^5-4620a^4b^6x^6-3168a^3b^7x^7-1485a^2b^8x^8-440ab^9x^9-66b^{10}x^{10}}{132x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**13,x)
```

```
[Out] (-11*a**10 - 120*a**9*b*x - 594*a**8*b**2*x**2 - 1760*a**7*b**3*x**3 - 3465
*a**6*b**4*x**4 - 4752*a**5*b**5*x**5 - 4620*a**4*b**6*x**6 - 3168*a**3*b**
7*x**7 - 1485*a**2*b**8*x**8 - 440*a*b**9*x**9 - 66*b**10*x**10)/(132*x**12
)
```

## 3.148 $\int \frac{(a+bx)^{10}}{x^{14}}\,dx$

**Optimal**. Leaf size=56

$$-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{(a+bx)^{11}}{13ax^{13}}$$

**Rubi [A]** time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{(a+bx)^{11}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^14,x]
```

```
[Out] -(a + b*x)^11/(13*a*x^13) + (b*(a + b*x)^11)/(78*a^2*x^12) - (b^2*(a + b*x)
^11)/(858*a^3*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx = -\frac{(a+bx)^{11}}{13ax^{13}} - \frac{(2b)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{13a}$$
$$= -\frac{(a+bx)^{11}}{13ax^{13}} + \frac{b(a+bx)^{11}}{78a^2x^{12}} + \frac{b^2\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{78a^2}$$
$$= -\frac{(a+bx)^{11}}{13ax^{13}} + \frac{b(a+bx)^{11}}{78a^2x^{12}} - \frac{b^2(a+bx)^{11}}{858a^3x^{11}}$$

**Mathematica [B]** time = 0.01, size = 126, normalized size = 2.25

$$-\frac{a^{10}}{13x^{13}} - \frac{5a^9b}{6x^{12}} - \frac{45a^8b^2}{11x^{11}} - \frac{12a^7b^3}{x^{10}} - \frac{70a^6b^4}{3x^9} - \frac{63a^5b^5}{2x^8} - \frac{30a^4b^6}{x^7} - \frac{20a^3b^7}{x^6} - \frac{9a^2b^8}{x^5} - \frac{5ab^9}{2x^4} - \frac{b^{10}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^14,x]
```

```
[Out] -1/13*a^10/x^13 - (5*a^9*b)/(6*x^12) - (45*a^8*b^2)/(11*x^11) - (12*a^7*b^3
)/x^10 - (70*a^6*b^4)/(3*x^9) - (63*a^5*b^5)/(2*x^8) - (30*a^4*b^6)/x^7 - (
20*a^3*b^7)/x^6 - (9*a^2*b^8)/x^5 - (5*a*b^9)/(2*x^4) - b^10/(3*x^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^14,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^14, x]
```

**fricas [B]** time = 1.39, size = 112, normalized size = 2.00

$$-\frac{286\,b^{10}x^{10} + 2145\,ab^9x^9 + 7722\,a^2b^8x^8 + 17160\,a^3b^7x^7 + 25740\,a^4b^6x^6 + 27027\,a^5b^5x^5 + 20020\,a^6b^4x^4 + 10296\,a^7b^3x^3 + 3510\,a^8b^2x^2 + 715\,a^9bx + 66\,a^{10}}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="fricas")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

**giac [B]** time = 1.03, size = 112, normalized size = 2.00

$$-\frac{286\,b^{10}x^{10}+2145\,ab^{9}x^{9}+7722\,a^{2}b^{8}x^{8}+17160\,a^{3}b^{7}x^{7}+25740\,a^{4}b^{6}x^{6}+27027\,a^{5}b^{5}x^{5}+20020\,a^{6}b^{4}x^{4}+10296\,a^{7}b^{3}x^{3}+3510\,a^{8}b^{2}x^{2}+715\,a^{9}bx+66\,a^{10}}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="giac")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

**maple [B]** time = 0.00, size = 113, normalized size = 2.02

$$-\frac{b^{10}}{3x^{3}}-\frac{5a\,b^{9}}{2x^{4}}-\frac{9a^{2}b^{8}}{x^{5}}-\frac{20a^{3}b^{7}}{x^{6}}-\frac{30a^{4}b^{6}}{x^{7}}-\frac{63a^{5}b^{5}}{2x^{8}}-\frac{70a^{6}b^{4}}{3x^{9}}-\frac{12a^{7}b^{3}}{x^{10}}-\frac{45a^{8}b^{2}}{11x^{11}}-\frac{5a^{9}b}{6x^{12}}-\frac{a^{10}}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^14,x)
```

```
[Out] -20*a^3*b^7/x^6-1/13*a^10/x^13-70/3*a^6*b^4/x^9-9*a^2*b^8/x^5-1/3*b^10/x^3-
45/11*a^8*b^2/x^11-5/2*a*b^9/x^4-12*a^7*b^3/x^10-5/6*a^9*b/x^12-30*a^4*b^6/
x^7-63/2*a^5*b^5/x^8
```

**maxima [B]** time = 1.37, size = 112, normalized size = 2.00

$$-\frac{286\,b^{10}x^{10}+2145\,ab^{9}x^{9}+7722\,a^{2}b^{8}x^{8}+17160\,a^{3}b^{7}x^{7}+25740\,a^{4}b^{6}x^{6}+27027\,a^{5}b^{5}x^{5}+20020\,a^{6}b^{4}x^{4}+10296\,a^{7}b^{3}x^{3}+3510\,a^{8}b^{2}x^{2}+715\,a^{9}bx+66\,a^{10}}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="maxima")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

**mupad [B]** time = 0.13, size = 112, normalized size = 2.00

$$-\frac{\frac{a^{10}}{13}+\frac{5\,a^{9}\,b\,x}{6}+\frac{45\,a^{8}\,b^{2}\,x^{2}}{11}+12\,a^{7}\,b^{3}\,x^{3}+\frac{70\,a^{6}\,b^{4}\,x^{4}}{3}+\frac{63\,a^{5}\,b^{5}\,x^{5}}{2}+30\,a^{4}\,b^{6}\,x^{6}+20\,a^{3}\,b^{7}\,x^{7}+9\,a^{2}\,b^{8}\,x^{8}+\frac{5\,a\,b^{9}\,x^{9}}{2}+\frac{b^{10}\,x^{10}}{3}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^14,x)
```

```
[Out] -(a^10/13 + (b^10*x^10)/3 + (5*a*b^9*x^9)/2 + (45*a^8*b^2*x^2)/11 + 12*a^7*
b^3*x^3 + (70*a^6*b^4*x^4)/3 + (63*a^5*b^5*x^5)/2 + 30*a^4*b^6*x^6 + 20*a^3
*b^7*x^7 + 9*a^2*b^8*x^8 + (5*a^9*b*x)/6)/x^13
```

**sympy [B]** time = 1.22, size = 121, normalized size = 2.16

$$\frac{-66a^{10} - 715a^9bx - 3510a^8b^2x^2 - 10296a^7b^3x^3 - 20020a^6b^4x^4 - 27027a^5b^5x^5 - 25740a^4b^6x^6 - 17160a^3b^7x^7 - 7722a^2b^8x^8 - 2145ab^9x^9 - 286b^{10}x^{10}}{858x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**14,x)
```

```
[Out] (-66*a**10 - 715*a**9*b*x - 3510*a**8*b**2*x**2 - 10296*a**7*b**3*x**3 - 20
020*a**6*b**4*x**4 - 27027*a**5*b**5*x**5 - 25740*a**4*b**6*x**6 - 17160*a*
*3*b**7*x**7 - 7722*a**2*b**8*x**8 - 2145*a*b**9*x**9 - 286*b**10*x**10)/(8
58*x**13)
```

## 3.149 $\int \frac{(a+bx)^{10}}{x^{15}}\,dx$

**Optimal**. Leaf size=76

$$\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{(a+bx)^{11}}{14ax^{14}}$$

**Rubi [A]** time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{(a+bx)^{11}}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^15,x]
```

```
[Out] -(a + b*x)^11/(14*a*x^14) + (3*b*(a + b*x)^11)/(182*a^2*x^13) - (b^2*(a + b
*x)^11)/(364*a^3*x^12) + (b^3*(a + b*x)^11)/(4004*a^4*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{15}}\,dx &= -\frac{(a+bx)^{11}}{14ax^{14}} - \frac{(3b)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{14a} \\ &= -\frac{(a+bx)^{11}}{14ax^{14}} + \frac{3b(a+bx)^{11}}{182a^2x^{13}} + \frac{\left(3b^2\right)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{91a^2} \\ &= -\frac{(a+bx)^{11}}{14ax^{14}} + \frac{3b(a+bx)^{11}}{182a^2x^{13}} - \frac{b^2(a+bx)^{11}}{364a^3x^{12}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{364a^3} \\ &= -\frac{(a+bx)^{11}}{14ax^{14}} + \frac{3b(a+bx)^{11}}{182a^2x^{13}} - \frac{b^2(a+bx)^{11}}{364a^3x^{12}} + \frac{b^3(a+bx)^{11}}{4004a^4x^{11}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 128, normalized size = 1.68

$$-\frac{a^{10}}{14x^{14}} - \frac{10a^9b}{13x^{13}} - \frac{15a^8b^2}{4x^{12}} - \frac{120a^7b^3}{11x^{11}} - \frac{21a^6b^4}{x^{10}} - \frac{28a^5b^5}{x^9} - \frac{105a^4b^6}{4x^8} - \frac{120a^3b^7}{7x^7} - \frac{15a^2b^8}{2x^6} - \frac{2ab^9}{x^5} - \frac{b^{10}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^15,x]
```

```
[Out] -1/14*a^10/x^14 - (10*a^9*b)/(13*x^13) - (15*a^8*b^2)/(4*x^12) - (120*a^7*b
^3)/(11*x^11) - (21*a^6*b^4)/x^10 - (28*a^5*b^5)/x^9 - (105*a^4*b^6)/(4*x^8
) - (120*a^3*b^7)/(7*x^7) - (15*a^2*b^8)/(2*x^6) - (2*a*b^9)/x^5 - b^10/(4*
x^4)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^15,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^15, x]
```

**fricas** [A] time = 1.29, size = 112, normalized size = 1.47

$$-\frac{1001\,b^{10}x^{10} + 8008\,ab^9x^9 + 30030\,a^2b^8x^8 + 68640\,a^3b^7x^7 + 105105\,a^4b^6x^6 + 112112\,a^5b^5x^5 + 84084\,a^6b^4x^4 + 43680\,a^7b^3x^3 + 15015\,a^8b^2x^2 + 3080\,a^9bx + 286\,a^{10}}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="fricas")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

**giac [A]** time = 0.89, size = 112, normalized size = 1.47

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^{9}x^{9}+30030\,a^{2}b^{8}x^{8}+68640\,a^{3}b^{7}x^{7}+105105\,a^{4}b^{6}x^{6}+112112\,a^{5}b^{5}x^{5}+84084\,a^{6}b^{4}x^{4}+43680\,a^{7}b^{3}x^{3}+15015\,a^{8}b^{2}x^{2}+3080\,a^{9}bx+286\,a^{10}}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="giac")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

**maple [A]** time = 0.01, size = 113, normalized size = 1.49

$$-\frac{b^{10}}{4x^{4}}-\frac{2a\,b^{9}}{x^{5}}-\frac{15a^{2}b^{8}}{2x^{6}}-\frac{120a^{3}b^{7}}{7x^{7}}-\frac{105a^{4}b^{6}}{4x^{8}}-\frac{28a^{5}b^{5}}{x^{9}}-\frac{21a^{6}b^{4}}{x^{10}}-\frac{120a^{7}b^{3}}{11x^{11}}-\frac{15a^{8}b^{2}}{4x^{12}}-\frac{10a^{9}b}{13x^{13}}-\frac{a^{10}}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^15,x)
```

```
[Out] -15/4*a^8*b^2/x^12-15/2*a^2*b^8/x^6-10/13*a^9*b/x^13-28*a^5*b^5/x^9-1/4*b^1
0/x^4-120/11*a^7*b^3/x^11-2*a*b^9/x^5-1/14*a^10/x^14-21*a^6*b^4/x^10-120/7*
a^3*b^7/x^7-105/4*a^4*b^6/x^8
```

**maxima [A]** time = 1.28, size = 112, normalized size = 1.47

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^{9}x^{9}+30030\,a^{2}b^{8}x^{8}+68640\,a^{3}b^{7}x^{7}+105105\,a^{4}b^{6}x^{6}+112112\,a^{5}b^{5}x^{5}+84084\,a^{6}b^{4}x^{4}+43680\,a^{7}b^{3}x^{3}+15015\,a^{8}b^{2}x^{2}+3080\,a^{9}bx+286\,a^{10}}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="maxima")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

**mupad [B]** time = 0.09, size = 112, normalized size = 1.47

$$-\frac{\frac{a^{10}}{14}+\frac{10\,a^9\,b\,x}{13}+\frac{15\,a^8\,b^2\,x^2}{4}+\frac{120\,a^7\,b^3\,x^3}{11}+21\,a^6\,b^4\,x^4+28\,a^5\,b^5\,x^5+\frac{105\,a^4\,b^6\,x^6}{4}+\frac{120\,a^3\,b^7\,x^7}{7}+\frac{15\,a^2\,b^8\,x^8}{2}+2\,a\,b^9\,x^9+\frac{b^{10}\,x^{10}}{4}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^15,x)
```

```
[Out] -(a^10/14 + (b^10*x^10)/4 + 2*a*b^9*x^9 + (15*a^8*b^2*x^2)/4 + (120*a^7*b^3
*x^3)/11 + 21*a^6*b^4*x^4 + 28*a^5*b^5*x^5 + (105*a^4*b^6*x^6)/4 + (120*a^3
*b^7*x^7)/7 + (15*a^2*b^8*x^8)/2 + (10*a^9*b*x)/13)/x^14
```

**sympy [A]** time = 1.10, size = 121, normalized size = 1.59

$$\frac{-286a^{10} - 3080a^9bx - 15015a^8b^2x^2 - 43680a^7b^3x^3 - 84084a^6b^4x^4 - 112112a^5b^5x^5 - 105105a^4b^6x^6 - 68640a^3b^7x^7 - 30030a^2b^8x^8 - 8008ab^9x^9 - 1001b^{10}x^{10}}{4004x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**15,x)
```

```
[Out] (-286*a**10 - 3080*a**9*b*x - 15015*a**8*b**2*x**2 - 43680*a**7*b**3*x**3 -
 84084*a**6*b**4*x**4 - 112112*a**5*b**5*x**5 - 105105*a**4*b**6*x**6 - 686
40*a**3*b**7*x**7 - 30030*a**2*b**8*x**8 - 8008*a*b**9*x**9 - 1001*b**10*x*
*10)/(4004*x**14)
```

## 3.150 $\int \frac{(a+bx)^{10}}{x^{16}} dx$

**Optimal**. Leaf size=96

$$-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{(a+bx)^{11}}{15ax^{15}}$$

**Rubi [A]** time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{(a+bx)^{11}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^16,x]
```

```
[Out] -(a + b*x)^11/(15*a*x^15) + (2*b*(a + b*x)^11)/(105*a^2*x^14) - (2*b^2*(a +
 b*x)^11)/(455*a^3*x^13) + (b^3*(a + b*x)^11)/(1365*a^4*x^12) - (b^4*(a + b
*x)^11)/(15015*a^5*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{10}}{x^{16}}\,dx &= -\frac{(a+bx)^{11}}{15ax^{15}} - \frac{(4b)\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{15a} \\
&= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} + \frac{\left(2b^2\right)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{35a^2} \\
&= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} - \frac{\left(4b^3\right)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{455a^3} \\
&= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} + \frac{b^3(a+bx)^{11}}{1365a^4x^{12}} + \frac{b^4\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{1365a^4} \\
&= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} + \frac{b^3(a+bx)^{11}}{1365a^4x^{12}} - \frac{b^4(a+bx)^{11}}{15015a^5x^{11}}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 130, normalized size = 1.35

$$-\frac{a^{10}}{15x^{15}} - \frac{5a^9b}{7x^{14}} - \frac{45a^8b^2}{13x^{13}} - \frac{10a^7b^3}{x^{12}} - \frac{210a^6b^4}{11x^{11}} - \frac{126a^5b^5}{5x^{10}} - \frac{70a^4b^6}{3x^9} - \frac{15a^3b^7}{x^8} - \frac{45a^2b^8}{7x^7} - \frac{5ab^9}{3x^6} - \frac{b^{10}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^16,x]
```

```
[Out] -1/15*a^10/x^15 - (5*a^9*b)/(7*x^14) - (45*a^8*b^2)/(13*x^13) - (10*a^7*b^3
)/x^12 - (210*a^6*b^4)/(11*x^11) - (126*a^5*b^5)/(5*x^10) - (70*a^4*b^6)/(3
*x^9) - (15*a^3*b^7)/x^8 - (45*a^2*b^8)/(7*x^7) - (5*a*b^9)/(3*x^6) - b^10/
(5*x^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^16,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^16, x]
```

**fricas [A]** time = 1.34, size = 112, normalized size = 1.17

$$-\frac{3003\,b^{10}x^{10} + 25025\,ab^9x^9 + 96525\,a^2b^8x^8 + 225225\,a^3b^7x^7 + 350350\,a^4b^6x^6 + 378378\,a^5b^5x^5 + 286650\,a^6b^4x^4 + 150150\,a^7b^3x^3 + 51975\,a^8b^2x^2 + 10725\,a^9bx + 1001\,a^{10}}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

**giac** [A] time = 1.52, size = 112, normalized size = 1.17

$$-\frac{3003\,b^{10}x^{10}+25025\,ab^{9}x^{9}+96525\,a^{2}b^{8}x^{8}+225225\,a^{3}b^{7}x^{7}+350350\,a^{4}b^{6}x^{6}+378378\,a^{5}b^{5}x^{5}+286650\,a^{6}b^{4}x^{4}+150150\,a^{7}b^{3}x^{3}+51975\,a^{8}b^{2}x^{2}+10725\,a^{9}bx+1001\,a^{10}}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="giac")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

**maple** [A] time = 0.01, size = 113, normalized size = 1.18

$$-\frac{b^{10}}{5x^{5}}-\frac{5a\,b^{9}}{3x^{6}}-\frac{45a^{2}b^{8}}{7x^{7}}-\frac{15a^{3}b^{7}}{x^{8}}-\frac{70a^{4}b^{6}}{3x^{9}}-\frac{126a^{5}b^{5}}{5x^{10}}-\frac{210a^{6}b^{4}}{11x^{11}}-\frac{10a^{7}b^{3}}{x^{12}}-\frac{45a^{8}b^{2}}{13x^{13}}-\frac{5a^{9}b}{7x^{14}}-\frac{a^{10}}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^16,x)
```

```
[Out] -5/3*a*b^9/x^6-210/11*a^6*b^4/x^11-70/3*a^4*b^6/x^9-1/5*b^10/x^5-45/13*a^8*
b^2/x^13-5/7*a^9*b/x^14-10*a^7*b^3/x^12-126/5*a^5*b^5/x^10-1/15*a^10/x^15-4
5/7*a^2*b^8/x^7-15*a^3*b^7/x^8
```

**maxima** [A] time = 1.36, size = 112, normalized size = 1.17

$$-\frac{3003\,b^{10}x^{10}+25025\,ab^{9}x^{9}+96525\,a^{2}b^{8}x^{8}+225225\,a^{3}b^{7}x^{7}+350350\,a^{4}b^{6}x^{6}+378378\,a^{5}b^{5}x^{5}+286650\,a^{6}b^{4}x^{4}+150150\,a^{7}b^{3}x^{3}+51975\,a^{8}b^{2}x^{2}+10725\,a^{9}bx+1001\,a^{10}}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

**mupad** [B] time = 0.13, size = 112, normalized size = 1.17

$$-\frac{\frac{a^{10}}{15}+\frac{5\,a^{9}\,b\,x}{7}+\frac{45\,a^{8}\,b^{2}\,x^{2}}{13}+10\,a^{7}\,b^{3}\,x^{3}+\frac{210\,a^{6}\,b^{4}\,x^{4}}{11}+\frac{126\,a^{5}\,b^{5}\,x^{5}}{5}+\frac{70\,a^{4}\,b^{6}\,x^{6}}{3}+15\,a^{3}\,b^{7}\,x^{7}+\frac{45\,a^{2}\,b^{8}\,x^{8}}{7}+\frac{5\,a\,b^{9}\,x^{9}}{3}+\frac{b^{10}\,x^{10}}{5}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^16,x)
```

```
[Out] -(a^10/15 + (b^10*x^10)/5 + (5*a*b^9*x^9)/3 + (45*a^8*b^2*x^2)/13 + 10*a^7*
b^3*x^3 + (210*a^6*b^4*x^4)/11 + (126*a^5*b^5*x^5)/5 + (70*a^4*b^6*x^6)/3 +
 15*a^3*b^7*x^7 + (45*a^2*b^8*x^8)/7 + (5*a^9*b*x)/7)/x^15
```

**sympy** [A] time = 1.25, size = 121, normalized size = 1.26

$$\frac{-1001a^{10} - 10725a^9bx - 51975a^8b^2x^2 - 150150a^7b^3x^3 - 286650a^6b^4x^4 - 378378a^5b^5x^5 - 350350a^4b^6x^6 - 225225a^3b^7x^7 - 96525a^2b^8x^8 - 25025ab^9x^9 - 3003b^{10}x^{10}}{15015x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**16,x)
```

```
[Out] (-1001*a**10 - 10725*a**9*b*x - 51975*a**8*b**2*x**2 - 150150*a**7*b**3*x**
3 - 286650*a**6*b**4*x**4 - 378378*a**5*b**5*x**5 - 350350*a**4*b**6*x**6 -
 225225*a**3*b**7*x**7 - 96525*a**2*b**8*x**8 - 25025*a*b**9*x**9 - 3003*b*
*10*x**10)/(15015*x**15)
```

## 3.151 $\int \frac{(a+bx)^{10}}{x^{17}}\,dx$

**Optimal**. Leaf size=116

$$\frac{b^5(a+bx)^{11}}{48048a^6x^{11}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{(a+bx)^{11}}{16ax^{16}}$$

**Rubi** [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b^5(a+bx)^{11}}{48048a^6x^{11}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{(a+bx)^{11}}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^17,x]
```

```
[Out] -(a + b*x)^11/(16*a*x^16) + (b*(a + b*x)^11)/(48*a^2*x^15) - (b^2*(a + b*x)
^11)/(168*a^3*x^14) + (b^3*(a + b*x)^11)/(728*a^4*x^13) - (b^4*(a + b*x)^11
)/(4368*a^5*x^12) + (b^5*(a + b*x)^11)/(48048*a^6*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{10}}{x^{17}}\,dx &= -\frac{(a+bx)^{11}}{16ax^{16}} - \frac{(5b)\int \frac{(a+bx)^{10}}{x^{16}}\,dx}{16a} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} + \frac{b^2\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{12a^2} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{56a^3} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} + \frac{b^4\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{364a^4} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} - \frac{b^4(a+bx)^{11}}{4368a^5x^{12}} - \frac{b^5\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{4368a^5} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} - \frac{b^4(a+bx)^{11}}{4368a^5x^{12}} + \frac{b^5(a+bx)^{11}}{48048a^6x^{11}}
\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 132, normalized size = 1.14

$$-\frac{a^{10}}{16x^{16}} - \frac{2a^9b}{3x^{15}} - \frac{45a^8b^2}{14x^{14}} - \frac{120a^7b^3}{13x^{13}} - \frac{35a^6b^4}{2x^{12}} - \frac{252a^5b^5}{11x^{11}} - \frac{21a^4b^6}{x^{10}} - \frac{40a^3b^7}{3x^9} - \frac{45a^2b^8}{8x^8} - \frac{10ab^9}{7x^7} - \frac{b^{10}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^17,x]
```

```
[Out] -1/16*a^10/x^16 - (2*a^9*b)/(3*x^15) - (45*a^8*b^2)/(14*x^14) - (120*a^7*b^
3)/(13*x^13) - (35*a^6*b^4)/(2*x^12) - (252*a^5*b^5)/(11*x^11) - (21*a^4*b^
6)/x^10 - (40*a^3*b^7)/(3*x^9) - (45*a^2*b^8)/(8*x^8) - (10*a*b^9)/(7*x^7)
- b^10/(6*x^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{17}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^17,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^17, x]
```

**fricas [A]** time = 0.77, size = 112, normalized size = 0.97

$$-\frac{8008\,b^{10}x^{10} + 68640\,ab^9x^9 + 270270\,a^2b^8x^8 + 640640\,a^3b^7x^7 + 1009008\,a^4b^6x^6 + 1100736\,a^5b^5x^5 + 840840\,a^6b^4x^4 + 443520\,a^7b^3x^3 + 154440\,a^8b^2x^2 + 32032\,a^9bx + 3003\,a^{10}}{48048\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="fricas")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

**giac [A]** time = 1.10, size = 112, normalized size = 0.97

$$-\frac{8008 \, b^{10} x^{10} + 68640 \, a b^{9} x^{9} + 270270 \, a^{2} b^{8} x^{8} + 640640 \, a^{3} b^{7} x^{7} + 1009008 \, a^{4} b^{6} x^{6} + 1100736 \, a^{5} b^{5} x^{5} + 840840 \, a^{6} b^{4} x^{4} + 443520 \, a^{7} b^{3} x^{3} + 154440 \, a^{8} b^{2} x^{2} + 32032 \, a^{9} b x + 3003 \, a^{10}}{48048 \, x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="giac")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

**maple [A]** time = 0.01, size = 113, normalized size = 0.97

$$-\frac{b^{10}}{6x^{6}}-\frac{10a\,b^{9}}{7x^{7}}-\frac{45a^{2}b^{8}}{8x^{8}}-\frac{40a^{3}b^{7}}{3x^{9}}-\frac{21a^{4}b^{6}}{x^{10}}-\frac{252a^{5}b^{5}}{11x^{11}}-\frac{35a^{6}b^{4}}{2x^{12}}-\frac{120a^{7}b^{3}}{13x^{13}}-\frac{45a^{8}b^{2}}{14x^{14}}-\frac{2a^{9}b}{3x^{15}}-\frac{a^{10}}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^17,x)
```

```
[Out] -1/6*b^10/x^6-120/13*a^7*b^3/x^13-40/3*a^3*b^7/x^9-252/11*a^5*b^5/x^11-45/1
4*a^8*b^2/x^14-1/16*a^10/x^16-21*a^4*b^6/x^10-2/3*a^9*b/x^15-10/7*a*b^9/x^7
-45/8*a^2*b^8/x^8-35/2*a^6*b^4/x^12
```

**maxima [A]** time = 1.39, size = 112, normalized size = 0.97

$$-\frac{8008 \, b^{10} x^{10} + 68640 \, a b^{9} x^{9} + 270270 \, a^{2} b^{8} x^{8} + 640640 \, a^{3} b^{7} x^{7} + 1009008 \, a^{4} b^{6} x^{6} + 1100736 \, a^{5} b^{5} x^{5} + 840840 \, a^{6} b^{4} x^{4} + 443520 \, a^{7} b^{3} x^{3} + 154440 \, a^{8} b^{2} x^{2} + 32032 \, a^{9} b x + 3003 \, a^{10}}{48048 \, x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="maxima")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

**mupad [B]** time = 0.13, size = 112, normalized size = 0.97

$$-\frac{\frac{a^{10}}{16}+\frac{2\,a^9\,b\,x}{3}+\frac{45\,a^8\,b^2\,x^2}{14}+\frac{120\,a^7\,b^3\,x^3}{13}+\frac{35\,a^6\,b^4\,x^4}{2}+\frac{252\,a^5\,b^5\,x^5}{11}+21\,a^4\,b^6\,x^6+\frac{40\,a^3\,b^7\,x^7}{3}+\frac{45\,a^2\,b^8\,x^8}{8}+\frac{10\,a\,b^9\,x^9}{7}+\frac{b^{10}\,x^{10}}{6}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^17,x)
```

```
[Out] -(a^10/16 + (b^10*x^10)/6 + (10*a*b^9*x^9)/7 + (45*a^8*b^2*x^2)/14 + (120*a
^7*b^3*x^3)/13 + (35*a^6*b^4*x^4)/2 + (252*a^5*b^5*x^5)/11 + 21*a^4*b^6*x^6
 + (40*a^3*b^7*x^7)/3 + (45*a^2*b^8*x^8)/8 + (2*a^9*b*x)/3)/x^16
```

**sympy [A]** time = 1.27, size = 121, normalized size = 1.04

$$\frac{-3003a^{10} - 32032a^{9}bx - 154440a^{8}b^{2}x^{2} - 443520a^{7}b^{3}x^{3} - 840840a^{6}b^{4}x^{4} - 1100736a^{5}b^{5}x^{5} - 1009008a^{4}b^{6}x^{6} - 640640a^{3}b^{7}x^{7} - 270270a^{2}b^{8}x^{8} - 68640ab^{9}x^{9} - 8008b^{10}x^{10}}{48048x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**17,x)
```

```
[Out] (-3003*a**10 - 32032*a**9*b*x - 154440*a**8*b**2*x**2 - 443520*a**7*b**3*x*
*3 - 840840*a**6*b**4*x**4 - 1100736*a**5*b**5*x**5 - 1009008*a**4*b**6*x**
6 - 640640*a**3*b**7*x**7 - 270270*a**2*b**8*x**8 - 68640*a*b**9*x**9 - 800
8*b**10*x**10)/(48048*x**16)
```

## 3.152 $\int \frac{(a+bx)^{10}}{x^{18}} \, dx$

**Optimal**. Leaf size=136

$$-\frac{b^6(a+bx)^{11}}{136136a^7x^{11}}+\frac{b^5(a+bx)^{11}}{12376a^6x^{12}}-\frac{3b^4(a+bx)^{11}}{6188a^5x^{13}}+\frac{b^3(a+bx)^{11}}{476a^4x^{14}}-\frac{b^2(a+bx)^{11}}{136a^3x^{15}}+\frac{3b(a+bx)^{11}}{136a^2x^{16}}-\frac{(a+bx)^{11}}{17ax^{17}}$$

**Rubi** [A] time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^6(a+bx)^{11}}{136136a^7x^{11}}+\frac{b^5(a+bx)^{11}}{12376a^6x^{12}}-\frac{3b^4(a+bx)^{11}}{6188a^5x^{13}}+\frac{b^3(a+bx)^{11}}{476a^4x^{14}}-\frac{b^2(a+bx)^{11}}{136a^3x^{15}}+\frac{3b(a+bx)^{11}}{136a^2x^{16}}-\frac{(a+bx)^{11}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^18,x]
```

```
[Out] -(a + b*x)^11/(17*a*x^17) + (3*b*(a + b*x)^11)/(136*a^2*x^16) - (b^2*(a + b
*x)^11)/(136*a^3*x^15) + (b^3*(a + b*x)^11)/(476*a^4*x^14) - (3*b^4*(a + b*
x)^11)/(6188*a^5*x^13) + (b^5*(a + b*x)^11)/(12376*a^6*x^12) - (b^6*(a + b*
x)^11)/(136136*a^7*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx = -\frac{(a+bx)^{11}}{17ax^{17}} - \frac{(6b)\int \frac{(a+bx)^{10}}{x^{17}}\,dx}{17a}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} + \frac{\left(15b^2\right)\int \frac{(a+bx)^{10}}{x^{16}}\,dx}{136a^2}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{34a^3}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} + \frac{\left(3b^4\right)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{476a^4}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} - \frac{\left(3b^5\right)\int \frac{(a+bx)^{10}}{x^{13}}}{3094a^5}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} + \frac{b^5(a+bx)^{11}}{12376a^6x^{12}} +$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} + \frac{b^5(a+bx)^{11}}{12376a^6x^{12}} -$$

**Mathematica [A]** time = 0.01, size = 126, normalized size = 0.93

$$-\frac{a^{10}}{17x^{17}} - \frac{5a^9b}{8x^{16}} - \frac{3a^8b^2}{x^{15}} - \frac{60a^7b^3}{7x^{14}} - \frac{210a^6b^4}{13x^{13}} - \frac{21a^5b^5}{x^{12}} - \frac{210a^4b^6}{11x^{11}} - \frac{12a^3b^7}{x^{10}} - \frac{5a^2b^8}{x^9} - \frac{5ab^9}{4x^8} - \frac{b^{10}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^18,x]
```

```
[Out] -1/17*a^10/x^17 - (5*a^9*b)/(8*x^16) - (3*a^8*b^2)/x^15 - (60*a^7*b^3)/(7*x
^14) - (210*a^6*b^4)/(13*x^13) - (21*a^5*b^5)/x^12 - (210*a^4*b^6)/(11*x^11
) - (12*a^3*b^7)/x^10 - (5*a^2*b^8)/x^9 - (5*a*b^9)/(4*x^8) - b^10/(7*x^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^18,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^18, x]
```

**fricas [A]** time = 1.14, size = 112, normalized size = 0.82

$$-\frac{19448\,b^{10}x^{10}+170170\,ab^{9}x^{9}+680680\,a^{2}b^{8}x^{8}+1633632\,a^{3}b^{7}x^{7}+2598960\,a^{4}b^{6}x^{6}+2858856\,a^{5}b^{5}x^{5}+2199120\,a^{6}b^{4}x^{4}+1166880\,a^{7}b^{3}x^{3}+408408\,a^{8}b^{2}x^{2}+85085\,a^{9}bx+8008\,a^{10}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="fricas")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

**giac [A]** time = 1.15, size = 112, normalized size = 0.82

$$-\frac{19448\,b^{10}x^{10}+170170\,ab^{9}x^{9}+680680\,a^{2}b^{8}x^{8}+1633632\,a^{3}b^{7}x^{7}+2598960\,a^{4}b^{6}x^{6}+2858856\,a^{5}b^{5}x^{5}+2199120\,a^{6}b^{4}x^{4}+1166880\,a^{7}b^{3}x^{3}+408408\,a^{8}b^{2}x^{2}+85085\,a^{9}bx+8008\,a^{10}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="giac")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

**maple [A]** time = 0.01, size = 113, normalized size = 0.83

$$-\frac{b^{10}}{7x^{7}}-\frac{5a\,b^{9}}{4x^{8}}-\frac{5a^{2}b^{8}}{x^{9}}-\frac{12a^{3}b^{7}}{x^{10}}-\frac{210a^{4}b^{6}}{11x^{11}}-\frac{21a^{5}b^{5}}{x^{12}}-\frac{210a^{6}b^{4}}{13x^{13}}-\frac{60a^{7}b^{3}}{7x^{14}}-\frac{3a^{8}b^{2}}{x^{15}}-\frac{5a^{9}b}{8x^{16}}-\frac{a^{10}}{17x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^18,x)
```

```
[Out] -3*a^8*b^2/x^15-12*a^3*b^7/x^10-5*a^2*b^8/x^9-21*a^5*b^5/x^12-1/17*a^10/x^1
7-5/8*a^9*b/x^16-60/7*a^7*b^3/x^14-210/13*a^6*b^4/x^13-1/7*b^10/x^7-5/4*a*b
^9/x^8-210/11*a^4*b^6/x^11
```

**maxima [A]** time = 1.40, size = 112, normalized size = 0.82

$$-\frac{19448\,b^{10}x^{10}+170170\,ab^{9}x^{9}+680680\,a^{2}b^{8}x^{8}+1633632\,a^{3}b^{7}x^{7}+2598960\,a^{4}b^{6}x^{6}+2858856\,a^{5}b^{5}x^{5}+2199120\,a^{6}b^{4}x^{4}+1166880\,a^{7}b^{3}x^{3}+408408\,a^{8}b^{2}x^{2}+85085\,a^{9}bx+8008\,a^{10}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="maxima")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

**mupad [B]** time = 0.13, size = 112, normalized size = 0.82

$$-\frac{\frac{a^{10}}{17}+\frac{5\,a^9\,b\,x}{8}+3\,a^8\,b^2\,x^2+\frac{60\,a^7\,b^3\,x^3}{7}+\frac{210\,a^6\,b^4\,x^4}{13}+21\,a^5\,b^5\,x^5+\frac{210\,a^4\,b^6\,x^6}{11}+12\,a^3\,b^7\,x^7+5\,a^2\,b^8\,x^8+\frac{5\,a\,b^9\,x^9}{4}+\frac{b^{10}\,x^{10}}{7}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^18,x)
```

```
[Out] -(a^10/17 + (b^10*x^10)/7 + (5*a*b^9*x^9)/4 + 3*a^8*b^2*x^2 + (60*a^7*b^3*x
^3)/7 + (210*a^6*b^4*x^4)/13 + 21*a^5*b^5*x^5 + (210*a^4*b^6*x^6)/11 + 12*a
^3*b^7*x^7 + 5*a^2*b^8*x^8 + (5*a^9*b*x)/8)/x^17
```

**sympy [A]** time = 1.33, size = 121, normalized size = 0.89

$$\frac{-8008a^{10}-85085a^9bx-408408a^8b^2x^2-1166880a^7b^3x^3-2199120a^6b^4x^4-2858856a^5b^5x^5-2598960a^4b^6x^6-1633632a^3b^7x^7-680680a^2b^8x^8-170170ab^9x^9-19448b^{10}x^{10}}{136136x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**18,x)
```

```
[Out] (-8008*a**10 - 85085*a**9*b*x - 408408*a**8*b**2*x**2 - 1166880*a**7*b**3*x
**3 - 2199120*a**6*b**4*x**4 - 2858856*a**5*b**5*x**5 - 2598960*a**4*b**6*x
**6 - 1633632*a**3*b**7*x**7 - 680680*a**2*b**8*x**8 - 170170*a*b**9*x**9 -
 19448*b**10*x**10)/(136136*x**17)
```

# 3.153 $\int \frac{(a+bx)^{10}}{x^{19}}\,dx$

**Optimal**. Leaf size=130

$$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

**Rubi [A]** time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10a^9b}{17x^{17}}-\frac{a^{10}}{18x^{18}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^19,x]
```

```
[Out] -a^10/(18*x^18) - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b^
3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x^1
2) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9) -
b^10/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{19}}\,dx = \int \left(\frac{a^{10}}{x^{19}}+\frac{10a^9b}{x^{18}}+\frac{45a^8b^2}{x^{17}}+\frac{120a^7b^3}{x^{16}}+\frac{210a^6b^4}{x^{15}}+\frac{252a^5b^5}{x^{14}}+\frac{210a^4b^6}{x^{13}}+\frac{120a^3b^7}{x^{12}}+\frac{45a^2b^8}{x^{11}}\right.$$
$$= -\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab}{9x^9}$$

**Mathematica [A]** time = 0.00, size = 130, normalized size = 1.00

$$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^19,x]
```

```
[Out] -1/18*a^10/x^18 - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b^
3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x^1
2) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9) -
b^10/(8*x^8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{19}}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^19,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^19, x]
```

**fricas [A]** time = 0.76, size = 112, normalized size = 0.86

$$-\frac{43758\, b^{10} x^{10} + 388960\, a b^{9} x^{9} + 1575288\, a^{2} b^{8} x^{8} + 3818880\, a^{3} b^{7} x^{7} + 6126120\, a^{4} b^{6} x^{6} + 6785856\, a^{5} b^{5} x^{5} + 5250960\, a^{6} b^{4} x^{4} + 2800512\, a^{7} b^{3} x^{3} + 984555\, a^{8} b^{2} x^{2} + 205920\, a^{9} b x + 19448\, a^{10}}{350064\, x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="fricas")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

**giac [A]** time = 0.98, size = 112, normalized size = 0.86

$$-\frac{43758\, b^{10} x^{10} + 388960\, a b^{9} x^{9} + 1575288\, a^{2} b^{8} x^{8} + 3818880\, a^{3} b^{7} x^{7} + 6126120\, a^{4} b^{6} x^{6} + 6785856\, a^{5} b^{5} x^{5} + 5250960\, a^{6} b^{4} x^{4} + 2800512\, a^{7} b^{3} x^{3} + 984555\, a^{8} b^{2} x^{2} + 205920\, a^{9} b x + 19448\, a^{10}}{350064\, x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="giac")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

**maple [A]** time = 0.01, size = 113, normalized size = 0.87

$$-\frac{b^{10}}{8x^{8}} - \frac{10a\, b^{9}}{9x^{9}} - \frac{9a^{2}b^{8}}{2x^{10}} - \frac{120a^{3}b^{7}}{11x^{11}} - \frac{35a^{4}b^{6}}{2x^{12}} - \frac{252a^{5}b^{5}}{13x^{13}} - \frac{15a^{6}b^{4}}{x^{14}} - \frac{8a^{7}b^{3}}{x^{15}} - \frac{45a^{8}b^{2}}{16x^{16}} - \frac{10a^{9}b}{17x^{17}} - \frac{a^{10}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^19,x)
```

```
[Out] -1/18*a^10/x^18-10/17*a^9*b/x^17-45/16*a^8*b^2/x^16-8*a^7*b^3/x^15-15*a^6*b
^4/x^14-252/13*a^5*b^5/x^13-35/2*a^4*b^6/x^12-120/11*a^3*b^7/x^11-9/2*a^2*b
^8/x^10-10/9*a*b^9/x^9-1/8*b^10/x^8
```

**maxima [A]** time = 1.36, size = 112, normalized size = 0.86

$$-\frac{43758\,b^{10}x^{10}+388960\,ab^{9}x^{9}+1575288\,a^{2}b^{8}x^{8}+3818880\,a^{3}b^{7}x^{7}+6126120\,a^{4}b^{6}x^{6}+6785856\,a^{5}b^{5}x^{5}+5250960\,a^{6}b^{4}x^{4}+2800512\,a^{7}b^{3}x^{3}+984555\,a^{8}b^{2}x^{2}+205920\,a^{9}bx+19448\,a^{10}}{350064\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="maxima")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

**mupad [B]** time = 0.10, size = 112, normalized size = 0.86

$$-\frac{\frac{a^{10}}{18}+\frac{10\,a^9\,b\,x}{17}+\frac{45\,a^8\,b^2\,x^2}{16}+8\,a^7\,b^3\,x^3+15\,a^6\,b^4\,x^4+\frac{252\,a^5\,b^5\,x^5}{13}+\frac{35\,a^4\,b^6\,x^6}{2}+\frac{120\,a^3\,b^7\,x^7}{11}+\frac{9\,a^2\,b^8\,x^8}{2}+\frac{10\,a\,b^9\,x^9}{9}+\frac{b^{10}\,x^{10}}{8}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^19,x)
```

```
[Out] -(a^10/18 + (b^10*x^10)/8 + (10*a*b^9*x^9)/9 + (45*a^8*b^2*x^2)/16 + 8*a^7*
b^3*x^3 + 15*a^6*b^4*x^4 + (252*a^5*b^5*x^5)/13 + (35*a^4*b^6*x^6)/2 + (120
*a^3*b^7*x^7)/11 + (9*a^2*b^8*x^8)/2 + (10*a^9*b*x)/17)/x^18
```

**sympy [A]** time = 1.35, size = 121, normalized size = 0.93

$$\frac{-19448a^{10}-205920a^{9}bx-984555a^{8}b^{2}x^{2}-2800512a^{7}b^{3}x^{3}-5250960a^{6}b^{4}x^{4}-6785856a^{5}b^{5}x^{5}-6126120a^{4}b^{6}x^{6}-3818880a^{3}b^{7}x^{7}-1575288a^{2}b^{8}x^{8}-388960ab^{9}x^{9}-43758b^{10}x^{10}}{350064x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**19,x)
```

```
[Out] (-19448*a**10 - 205920*a**9*b*x - 984555*a**8*b**2*x**2 - 2800512*a**7*b**3
*x**3 - 5250960*a**6*b**4*x**4 - 6785856*a**5*b**5*x**5 - 6126120*a**4*b**6
*x**6 - 3818880*a**3*b**7*x**7 - 1575288*a**2*b**8*x**8 - 388960*a*b**9*x**
9 - 43758*b**10*x**10)/(350064*x**18)
```

## 3.154 $\int \frac{(a+bx)^{10}}{x^{20}} \, dx$

**Optimal**. Leaf size=126

$$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

**Rubi** [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{5a^9b}{9x^{18}}-\frac{a^{10}}{19x^{19}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^20,x]
```

```
[Out] -a^10/(19*x^19) - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^3
)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^13
) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*x^9
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{20}} \, dx = \int \left(\frac{a^{10}}{x^{20}}+\frac{10a^9b}{x^{19}}+\frac{45a^8b^2}{x^{18}}+\frac{120a^7b^3}{x^{17}}+\frac{210a^6b^4}{x^{16}}+\frac{252a^5b^5}{x^{15}}+\frac{210a^4b^6}{x^{14}}+\frac{120a^3b^7}{x^{13}}+\frac{45a^2b}{x^{12}}\right.$$

$$= -\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}$$

**Mathematica** [A] time = 0.01, size = 126, normalized size = 1.00

$$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^20,x]
```

```
[Out] -1/19*a^10/x^19 - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^3
)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^13
) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*x^9
)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{20}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^20,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^20, x]
```

**fricas [A]** time = 1.34, size = 112, normalized size = 0.89

$$-\frac{92378\,b^{10}x^{10}+831402\,ab^{9}x^{9}+3401190\,a^{2}b^{8}x^{8}+8314020\,a^{3}b^{7}x^{7}+13430340\,a^{4}b^{6}x^{6}+14965236\,a^{5}b^{5}x^{5}+11639628\,a^{6}b^{4}x^{4}+6235515\,a^{7}b^{3}x^{3}+2200770\,a^{8}b^{2}x^{2}+461890\,a^{9}bx+43758\,a^{10}}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="fricas")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

**giac [A]** time = 1.11, size = 112, normalized size = 0.89

$$-\frac{92378\,b^{10}x^{10}+831402\,ab^{9}x^{9}+3401190\,a^{2}b^{8}x^{8}+8314020\,a^{3}b^{7}x^{7}+13430340\,a^{4}b^{6}x^{6}+14965236\,a^{5}b^{5}x^{5}+11639628\,a^{6}b^{4}x^{4}+6235515\,a^{7}b^{3}x^{3}+2200770\,a^{8}b^{2}x^{2}+461890\,a^{9}bx+43758\,a^{10}}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="giac")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

**maple [A]** time = 0.01, size = 113, normalized size = 0.90

$$-\frac{b^{10}}{9x^{9}}-\frac{a\,b^{9}}{x^{10}}-\frac{45a^{2}b^{8}}{11x^{11}}-\frac{10a^{3}b^{7}}{x^{12}}-\frac{210a^{4}b^{6}}{13x^{13}}-\frac{18a^{5}b^{5}}{x^{14}}-\frac{14a^{6}b^{4}}{x^{15}}-\frac{15a^{7}b^{3}}{2x^{16}}-\frac{45a^{8}b^{2}}{17x^{17}}-\frac{5a^{9}b}{9x^{18}}-\frac{a^{10}}{19x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^20,x)
```

```
[Out] -1/19*a^10/x^19-5/9*a^9*b/x^18-45/17*a^8*b^2/x^17-15/2*a^7*b^3/x^16-14*a^6*
b^4/x^15-18*a^5*b^5/x^14-210/13*a^4*b^6/x^13-10*a^3*b^7/x^12-45/11*a^2*b^8/
x^11-a*b^9/x^10-1/9*b^10/x^9
```

**maxima [A]** time = 1.40, size = 112, normalized size = 0.89

$$-\frac{92378\,b^{10}x^{10} + 831402\,ab^{9}x^{9} + 3401190\,a^{2}b^{8}x^{8} + 8314020\,a^{3}b^{7}x^{7} + 13430340\,a^{4}b^{6}x^{6} + 14965236\,a^{5}b^{5}x^{5} + 11639628\,a^{6}b^{4}x^{4} + 6235515\,a^{7}b^{3}x^{3} + 2200770\,a^{8}b^{2}x^{2} + 461890\,a^{9}bx + 43758\,a^{10}}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="maxima")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

**mupad [B]** time = 0.14, size = 111, normalized size = 0.88

$$-\frac{\frac{a^{10}}{19} + \frac{5\,a^{9}\,b\,x}{9} + \frac{45\,a^{8}\,b^{2}\,x^{2}}{17} + \frac{15\,a^{7}\,b^{3}\,x^{3}}{2} + 14\,a^{6}\,b^{4}\,x^{4} + 18\,a^{5}\,b^{5}\,x^{5} + \frac{210\,a^{4}\,b^{6}\,x^{6}}{13} + 10\,a^{3}\,b^{7}\,x^{7} + \frac{45\,a^{2}\,b^{8}\,x^{8}}{11} + a\,b^{9}\,x^{9} + \frac{b^{10}\,x^{10}}{9}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^20,x)
```

```
[Out] -(a^10/19 + (b^10*x^10)/9 + a*b^9*x^9 + (45*a^8*b^2*x^2)/17 + (15*a^7*b^3*x
^3)/2 + 14*a^6*b^4*x^4 + 18*a^5*b^5*x^5 + (210*a^4*b^6*x^6)/13 + 10*a^3*b^7
*x^7 + (45*a^2*b^8*x^8)/11 + (5*a^9*b*x)/9)/x^19
```

**sympy [A]** time = 1.42, size = 121, normalized size = 0.96

$$\frac{-43758a^{10} - 461890a^{9}bx - 2200770a^{8}b^{2}x^{2} - 6235515a^{7}b^{3}x^{3} - 11639628a^{6}b^{4}x^{4} - 14965236a^{5}b^{5}x^{5} - 13430340a^{4}b^{6}x^{6} - 8314020a^{3}b^{7}x^{7} - 3401190a^{2}b^{8}x^{8} - 831402ab^{9}x^{9} - 92378b^{10}x^{10}}{831402x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**20,x)
```

```
[Out] (-43758*a**10 - 461890*a**9*b*x - 2200770*a**8*b**2*x**2 - 6235515*a**7*b**
3*x**3 - 11639628*a**6*b**4*x**4 - 14965236*a**5*b**5*x**5 - 13430340*a**4*
b**6*x**6 - 8314020*a**3*b**7*x**7 - 3401190*a**2*b**8*x**8 - 831402*a*b**9
*x**9 - 92378*b**10*x**10)/(831402*x**19)
```

## 3.155 $\int c(a+bx)\,dx$

**Optimal**. Leaf size=15

$$\frac{c(a+bx)^2}{2b}$$

**Rubi** [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {9}

$$\frac{c(a+bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[c*(a + b*x),x]
```

```
[Out] (c*(a + b*x)^2)/(2*b)
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[(a*(b + c*x)^2)/(2*c), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int c(a+bx)\,dx = \frac{c(a+bx)^2}{2b}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 0.93

$$c\left(ax+\frac{bx^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[c*(a + b*x),x]
```

```
[Out] c*(a*x + (b*x^2)/2)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int c(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[c*(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[c*(a + b*x), x]
```

**fricas** [A]   time = 1.49, size = 12, normalized size = 0.80

$$\frac{1}{2}x^2cb + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + x*c*a
```

**giac** [A]   time = 1.09, size = 13, normalized size = 0.87

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c
```

**maple** [A]   time = 0.00, size = 13, normalized size = 0.87

$$\left(\frac{1}{2}b\,x^2 + ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*(b*x+a),x)
```

```
[Out] c*(1/2*b*x^2+a*x)
```

**maxima** [A]   time = 1.36, size = 13, normalized size = 0.87

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c
```

**mupad [B]** time = 0.02, size = 11, normalized size = 0.73

$$\frac{c\,x\,\left(2\,a+b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*(a + b*x),x)
```

```
[Out] (c*x*(2*a + b*x))/2
```

**sympy [A]** time = 0.07, size = 12, normalized size = 0.80

$$acx + \frac{bcx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x)
```

```
[Out] a*c*x + b*c*x**2/2
```

## 3.156 $\int \frac{(c+d)(a+bx)}{e} dx$

**Optimal**. Leaf size=20

$$\frac{(c+d)(a+bx)^2}{2be}$$

**Rubi** [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {9}

$$\frac{(c+d)(a+bx)^2}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d)*(a + b*x))/e,x]
```

```
[Out] ((c + d)*(a + b*x)^2)/(2*b*e)
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[(a*(b + c*x)^2)/(2*c), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(c+d)(a+bx)}{e} dx = \frac{(c+d)(a+bx)^2}{2be}$$

**Mathematica** [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{(c+d)\left(ax+\frac{bx^2}{2}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d)*(a + b*x))/e,x]
```

```
[Out] ((c + d)*(a*x + (b*x^2)/2))/e
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d)(a+bx)}{e} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d)*(a + b*x))/e,x]
```

```
[Out] IntegrateAlgebraic[((c + d)*(a + b*x))/e, x]
```

**fricas [A]** time = 2.37, size = 27, normalized size = 1.35

$$\frac{(bc + bd)x^2 + 2\,(ac + ad)x}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="fricas")
```

```
[Out] 1/2*((b*c + b*d)*x^2 + 2*(a*c + a*d)*x)/e
```

**giac [A]** time = 1.20, size = 17, normalized size = 0.85

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)(c + d)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*(c + d)*e^(-1)
```

**maple [A]** time = 0.00, size = 18, normalized size = 0.90

$$\frac{(c + d)\left(\frac{1}{2}b\,x^2 + ax\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d)*(b*x+a)/e,x)
```

```
[Out] (c+d)/e*(1/2*b*x^2+a*x)
```

**maxima [A]** time = 1.37, size = 18, normalized size = 0.90

$$\frac{\left(bx^2 + 2\,ax\right)(c + d)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*(c + d)/e
```

**mupad [B]** time = 0.07, size = 16, normalized size = 0.80

$$\frac{x\,\left(c+d\right)\,\left(2\,a+b\,x\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d)*(a + b*x))/e,x)
[Out] (x*(c + d)*(2*a + b*x))/(2*e)
```

**sympy [A]** time = 0.08, size = 22, normalized size = 1.10

$$\frac{x^{2}\left(bc+bd\right)}{2e}+\frac{x\left(ac+ad\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x)
[Out] x**2*(b*c + b*d)/(2*e) + x*(a*c + a*d)/e
```

## 3.157 $\int \frac{x^5}{a+bx}\,dx$

**Optimal**. Leaf size=70

$$-\frac{a^5\log(a+bx)}{b^6}+\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

**Rubi [A]** time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}+\frac{a^4x}{b^5}-\frac{a^5\log(a+bx)}{b^6}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^5
/(5*b) - (a^5*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a+bx}\,dx &= \int \left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right)dx\\ &= \frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\log(a+bx)}{b^6}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 70, normalized size = 1.00

$$-\frac{a^5\log(a+bx)}{b^6}+\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^5
/(5*b) - (a^5*Log[a + b*x])/b^6
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x), x]
```

**fricas [A]** time = 1.22, size = 63, normalized size = 0.90

$$\frac{12\,b^5x^5 - 15\,ab^4x^4 + 20\,a^2b^3x^3 - 30\,a^3b^2x^2 + 60\,a^4bx - 60\,a^5\log\left(bx+a\right)}{60\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*x^5 - 15*a*b^4*x^4 + 20*a^2*b^3*x^3 - 30*a^3*b^2*x^2 + 60*a^4*
b*x - 60*a^5*log(b*x + a))/b^6
```

**giac [A]** time = 1.00, size = 65, normalized size = 0.93

$$-\frac{a^5\log\left(|bx+a|\right)}{b^6} + \frac{12\,b^4x^5 - 15\,ab^3x^4 + 20\,a^2b^2x^3 - 30\,a^3bx^2 + 60\,a^4x}{60\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] -a^5*log(abs(b*x + a))/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*x
^3 - 30*a^3*b*x^2 + 60*a^4*x)/b^5
```

**maple [A]** time = 0.00, size = 63, normalized size = 0.90

$$\frac{x^5}{5b} - \frac{a\,x^4}{4b^2} + \frac{a^2x^3}{3b^3} - \frac{a^3x^2}{2b^4} - \frac{a^5\ln\left(bx+a\right)}{b^6} + \frac{a^4x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a),x)
```

[Out] a^4*x/b^5-1/2*a^3*x^2/b^4+1/3*a^2*x^3/b^3-1/4*a*x^4/b^2+1/5*x^5/b-a^5*ln(b*x+a)/b^6

**maxima [A]** time = 1.39, size = 64, normalized size = 0.91

$$-\frac{a^5 \log (bx+a)}{b^6} + \frac{12\, b^4 x^5 - 15\, ab^3 x^4 + 20\, a^2 b^2 x^3 - 30\, a^3 b x^2 + 60\, a^4 x}{60\, b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x+a),x, algorithm="maxima")

[Out] -a^5*log(b*x + a)/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*x^3 - 30*a^3*b*x^2 + 60*a^4*x)/b^5

**mupad [B]** time = 0.08, size = 62, normalized size = 0.89

$$\frac{x^5}{5\,b} - \frac{a^5\,\ln (a+b\,x)}{b^6} - \frac{a\,x^4}{4\,b^2} + \frac{a^4\,x}{b^5} + \frac{a^2\,x^3}{3\,b^3} - \frac{a^3\,x^2}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a + b*x),x)

[Out] x^5/(5*b) - (a^5*log(a + b*x))/b^6 - (a*x^4)/(4*b^2) + (a^4*x)/b^5 + (a^2*x^3)/(3*b^3) - (a^3*x^2)/(2*b^4)

**sympy [A]** time = 0.16, size = 61, normalized size = 0.87

$$-\frac{a^5 \log (a+bx)}{b^6} + \frac{a^4 x}{b^5} - \frac{a^3 x^2}{2b^4} + \frac{a^2 x^3}{3b^3} - \frac{a x^4}{4b^2} + \frac{x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x+a),x)

[Out] -a**5*log(a + b*x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x**3/(3*b**3) - a*x**4/(4*b**2) + x**5/(5*b)

## 3.158 $\int \frac{x^4}{a+bx}\,dx$

**Optimal**. Leaf size=57

$$\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

**Rubi** [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^2}{2b^3}-\frac{a^3x}{b^4}+\frac{a^4\log(a+bx)}{b^5}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a+bx}\,dx &= \int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx\\ &= -\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 57, normalized size = 1.00

$$\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x), x]
```

**fricas [A]** time = 1.21, size = 52, normalized size = 0.91

$$\frac{3\,b^4x^4 - 4\,ab^3x^3 + 6\,a^2b^2x^2 - 12\,a^3bx + 12\,a^4\log\left(bx+a\right)}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))/b^5
```

**giac [A]** time = 1.35, size = 53, normalized size = 0.93

$$\frac{a^4\log\left(|bx+a|\right)}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 1
2*a^3*x)/b^4
```

**maple [A]** time = 0.00, size = 52, normalized size = 0.91

$$\frac{x^4}{4b} - \frac{a\,x^3}{3b^2} + \frac{a^2x^2}{2b^3} + \frac{a^4\ln\left(bx+a\right)}{b^5} - \frac{a^3x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a),x)
```

```
[Out] -a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

**maxima [A]** time = 1.33, size = 52, normalized size = 0.91

$$\frac{a^4 \log(bx+a)}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3
*x)/b^4
```

**mupad [B]** time = 0.10, size = 51, normalized size = 0.89

$$\frac{x^4}{4\,b} + \frac{a^4\,\ln(a+b\,x)}{b^5} - \frac{a\,x^3}{3\,b^2} - \frac{a^3\,x}{b^4} + \frac{a^2\,x^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x),x)
```

```
[Out] x^4/(4*b) + (a^4*log(a + b*x))/b^5 - (a*x^3)/(3*b^2) - (a^3*x)/b^4 + (a^2*x
^2)/(2*b^3)
```

**sympy [A]** time = 0.15, size = 49, normalized size = 0.86

$$\frac{a^4 \log(a+bx)}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a),x)
```

```
[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2)
 + x**4/(4*b)
```

## 3.159 $\int \frac{x^3}{a+bx}\,dx$

**Optimal**. Leaf size=44

$$-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

**Rubi** [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x}{b^3}-\frac{a^3\log(a+bx)}{b^4}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+bx}\,dx&=\int\left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx\\&=\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 44, normalized size = 1.00

$$-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x),x]
[Out] IntegrateAlgebraic[x^3/(a + b*x), x]
```

**fricas [A]** time = 1.17, size = 41, normalized size = 0.93

$$\frac{2\,b^3x^3 - 3\,ab^2x^2 + 6\,a^2bx - 6\,a^3\log\left(bx + a\right)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="fricas")
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

**giac [A]** time = 1.05, size = 43, normalized size = 0.98

$$-\frac{a^3\log\left(|bx + a|\right)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="giac")
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

**maple [A]** time = 0.00, size = 41, normalized size = 0.93

$$\frac{x^3}{3b} - \frac{a\,x^2}{2b^2} - \frac{a^3\ln\left(bx + a\right)}{b^4} + \frac{a^2x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a),x)
[Out] a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

**maxima [A]** time = 1.24, size = 42, normalized size = 0.95

$$-\frac{a^3\log\left(bx + a\right)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="maxima")
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

**mupad [B]** time = 0.04, size = 40, normalized size = 0.91

$$\frac{x^3}{3\,b}-\frac{a^3\,\ln\left(a+b\,x\right)}{b^4}-\frac{a\,x^2}{2\,b^2}+\frac{a^2\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x),x)
[Out] x^3/(3*b) - (a^3*log(a + b*x))/b^4 - (a*x^2)/(2*b^2) + (a^2*x)/b^3
```

**sympy [A]** time = 0.14, size = 37, normalized size = 0.84

$$-\frac{a^3\log\left(a+bx\right)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a),x)
[Out] -a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

## 3.160 $\int \frac{x^2}{a+bx}\,dx$

**Optimal**. Leaf size=31

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

**Rubi [A]** time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+bx}\,dx &= \int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx\\ &= -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 31, normalized size = 1.00

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x),x]
[Out] IntegrateAlgebraic[x^2/(a + b*x), x]
```

**fricas [A]** time = 1.28, size = 29, normalized size = 0.94

$$\frac{b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

**giac [A]** time = 0.94, size = 30, normalized size = 0.97

$$\frac{a^2\log\left(|bx+a|\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="giac")
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

**maple [A]** time = 0.00, size = 30, normalized size = 0.97

$$\frac{x^2}{2b}+\frac{a^2\ln\left(bx+a\right)}{b^3}-\frac{ax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a),x)
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

**maxima [A]** time = 1.37, size = 29, normalized size = 0.94

$$\frac{a^2\log\left(bx+a\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

**mupad [B]** time = 0.04, size = 29, normalized size = 0.94

$$\frac{2\,a^2\,\ln\left(a+b\,x\right)+b^2\,x^2-2\,a\,b\,x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x),x)
```

```
[Out] (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

**sympy [A]** time = 0.13, size = 26, normalized size = 0.84

$$\frac{a^2\log\left(a+bx\right)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

## 3.161 $\int \frac{x}{a+bx} dx$

**Optimal**. Leaf size=18

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx}\,dx &= \int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx\\ &= \frac{x}{b}-\frac{a\log(a+bx)}{b^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x),x]
[Out] IntegrateAlgebraic[x/(a + b*x), x]
```

**fricas** [A] time = 1.14, size = 17, normalized size = 0.94

$$\frac{bx - a\log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="fricas")
[Out] (b*x - a*log(b*x + a))/b^2
```

**giac** [A] time = 1.02, size = 19, normalized size = 1.06

$$\frac{x}{b} - \frac{a\log\left(|bx + a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="giac")
[Out] x/b - a*log(abs(b*x + a))/b^2
```

**maple** [A] time = 0.00, size = 19, normalized size = 1.06

$$-\frac{a\ln\left(bx + a\right)}{b^2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a),x)
[Out] x/b-a*ln(b*x+a)/b^2
```

**maxima** [A] time = 1.31, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a\log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="maxima")
```

```
[Out] x/b - a*log(b*x + a)/b^2
```

**mupad [B]** time = 0.08, size = 18, normalized size = 1.00

$$-\frac{a\,\ln\left(a+b\,x\right)-b\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x),x)
```

```
[Out] -(a*log(a + b*x) - b*x)/b^2
```

**sympy [A]** time = 0.12, size = 14, normalized size = 0.78

$$-\frac{a\log\left(a+bx\right)}{b^2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

## 3.162 $\int \frac{1}{a+bx}\,dx$

**Optimal**. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx}\,dx = \frac{\log(a+bx)}{b}$$

**Mathematica [A]** time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-1), x]
```

**fricas** [A] time = 1.41, size = 10, normalized size = 1.00

$$\frac{\log\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

**giac** [A] time = 0.92, size = 11, normalized size = 1.10

$$\frac{\log\left(|bx + a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

**maple** [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\ln\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x)
```

```
[Out] ln(b*x+a)/b
```

**maxima** [A] time = 1.30, size = 10, normalized size = 1.00

$$\frac{\log\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

**mupad** [B] time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x),x)
```

```
[Out] log(a + b*x)/b
```

**sympy** [A] time = 0.07, size = 7, normalized size = 0.70

$$\frac{\log\left(a+bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(a + b*x)/b
```

## 3.163 $\int \frac{1}{x(a+bx)}\,dx$

**Optimal**. Leaf size=18

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

**Rubi** [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)}\,dx &= \frac{\int \frac{1}{x}\,dx}{a}-\frac{b\int \frac{1}{a+bx}\,dx}{a}\\ &= \frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)),x]
[Out] Log[x]/a - Log[a + b*x]/a
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)),x]
[Out] IntegrateAlgebraic[1/(x*(a + b*x)), x]
```

**fricas** [A] time = 1.25, size = 16, normalized size = 0.89

$$-\frac{\log(bx+a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="fricas")
[Out] -(log(b*x + a) - log(x))/a
```

**giac** [A] time = 1.10, size = 20, normalized size = 1.11

$$-\frac{\log(|bx+a|)}{a}+\frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="giac")
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

**maple** [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a),x)
```

```
[Out] ln(x)/a-ln(b*x+a)/a
```

**maxima [A]** time = 1.39, size = 18, normalized size = 1.00

$$-\frac{\log\left(bx+a\right)}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/a + log(x)/a
```

**mupad [B]** time = 0.09, size = 15, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)),x)
```

```
[Out] -(2*atanh((2*b*x)/a + 1))/a
```

**sympy [A]** time = 0.15, size = 10, normalized size = 0.56

$$\frac{\log\left(x\right)-\log\left(\frac{a}{b}+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

## 3.164 $\int \frac{1}{x^2(a+bx)}\,dx$

**Optimal**. Leaf size=28

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)),x]
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)}\,dx &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 28, normalized size = 1.00

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)),x]
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)), x]
```

**fricas** [A] time = 1.40, size = 26, normalized size = 0.93

$$\frac{bx\log(bx+a) - bx\log(x) - a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

**giac** [A] time = 0.97, size = 30, normalized size = 1.07

$$\frac{b\log(|bx+a|)}{a^2} - \frac{b\log(|x|)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

**maple** [A] time = 0.01, size = 29, normalized size = 1.04

$$-\frac{b\ln(x)}{a^2} + \frac{b\ln(bx+a)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a),x)
```

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

**maxima** [A] time = 1.30, size = 28, normalized size = 1.00

$$\frac{b\log(bx+a)}{a^2} - \frac{b\log(x)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="maxima")
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

**mupad [B]** time = 0.05, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)),x)
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

**sympy [A]** time = 0.20, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a),x)
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

## 3.165 $\int \frac{1}{x^3(a+bx)} dx$

**Optimal**. Leaf size=42

$$\frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx)}{a^3} + \frac{b}{a^2 x} - \frac{1}{2ax^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx)}{a^3} + \frac{b}{a^2 x} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3(a+bx)} dx &= \int \left(\frac{1}{ax^3} - \frac{b}{a^2x^2} + \frac{b^2}{a^3x} - \frac{b^3}{a^3(a+bx)}\right) dx \\ &= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b^2\log(x)}{a^3} - \frac{b^2\log(a+bx)}{a^3} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 42, normalized size = 1.00

$$\frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx)}{a^3} + \frac{b}{a^2 x} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)),x]
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)), x]
```

**fricas [A]** time = 1.30, size = 41, normalized size = 0.98

$$-\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log(x)-2\,abx+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="fricas")
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

**giac [A]** time = 1.02, size = 45, normalized size = 1.07

$$-\frac{b^2\log\left(|bx+a|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="giac")
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

**maple [A]** time = 0.01, size = 41, normalized size = 0.98

$$\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(bx+a\right)}{a^3}+\frac{b}{a^2x}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a),x)
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

**maxima** [A] time = 1.34, size = 40, normalized size = 0.95

$$-\frac{b^2 \log\left(bx+a\right)}{a^3}+\frac{b^2 \log(x)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

**mupad** [B] time = 0.06, size = 38, normalized size = 0.90

$$-\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)),x)
```

```
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

**sympy** [A] time = 0.22, size = 31, normalized size = 0.74

$$\frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

## 3.166 $\int \frac{1}{x^4(a+bx)}\,dx$

**Optimal**. Leaf size=56

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

**Rubi [A]** time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a
+ b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)}\,dx &= \int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 56, normalized size = 1.00

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)), x]
```

**fricas [A]** time = 1.22, size = 54, normalized size = 0.96

$$\frac{6\,b^3x^3\log\left(bx+a\right)-6\,b^3x^3\log(x)-6\,ab^2x^2+3\,a^2bx-2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
2*a^3)/(a^4*x^3)
```

**giac [A]** time = 1.07, size = 56, normalized size = 1.00

$$\frac{b^3\log\left(|bx+a|\right)}{a^4}-\frac{b^3\log\left(|x|\right)}{a^4}-\frac{6\,ab^2x^2-3\,a^2bx+2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2*
b*x + 2*a^3)/(a^4*x^3)
```

**maple [A]** time = 0.01, size = 53, normalized size = 0.95

$$-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln\left(bx+a\right)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a),x)
```

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

**maxima [A]** time = 1.35, size = 51, normalized size = 0.91

$$\frac{b^3 \log(bx+a)}{a^4} - \frac{b^3 \log(x)}{a^4} - \frac{6\,b^2x^2 - 3\,abx + 2\,a^2}{6\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/(
a^3*x^3)
```

**mupad [B]** time = 0.10, size = 48, normalized size = 0.86

$$\frac{2\,b^3 \operatorname{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4} - \frac{\frac{a^3}{3} - \frac{a^2\,b\,x}{2} + a\,b^2\,x^2}{a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)),x)
```

```
[Out] (2*b^3*atanh((2*b*x)/a + 1))/a^4 - (a^3/3 + a*b^2*x^2 - (a^2*b*x)/2)/(a^4*x
^3)
```

**sympy [A]** time = 0.24, size = 44, normalized size = 0.79

$$\frac{-2a^2 + 3abx - 6b^2x^2}{6a^3x^3} + \frac{b^3\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a),x)
```

```
[Out] (-2*a**2 + 3*a*b*x - 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b +
 x))/a**4
```

## 3.167 $\int \frac{1}{x^5(a+bx)}\,dx$

**Optimal**. Leaf size=68

$$\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}+\frac{b^3}{a^4x}-\frac{b^2}{2a^3x^2}+\frac{b}{3a^2x^3}-\frac{1}{4ax^4}$$

**Rubi** [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}+\frac{b}{3a^2x^3}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)),x]
```

```
[Out] -1/(4*a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[x])
/a^5 - (b^4*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)}\,dx &= \int \left(\frac{1}{ax^5}-\frac{b}{a^2x^4}+\frac{b^2}{a^3x^3}-\frac{b^3}{a^4x^2}+\frac{b^4}{a^5x}-\frac{b^5}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{4ax^4}+\frac{b}{3a^2x^3}-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 68, normalized size = 1.00

$$\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}+\frac{b^3}{a^4x}-\frac{b^2}{2a^3x^2}+\frac{b}{3a^2x^3}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[x
])/a^5 - (b^4*Log[a + b*x])/a^5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x)), x]
```

**fricas [A]** time = 1.22, size = 65, normalized size = 0.96

$$-\frac{12\,b^4x^4\log\left(bx+a\right)-12\,b^4x^4\log(x)-12\,ab^3x^3+6\,a^2b^2x^2-4\,a^3bx+3\,a^4}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(12*b^4*x^4*log(b*x + a) - 12*b^4*x^4*log(x) - 12*a*b^3*x^3 + 6*a^2*b
^2*x^2 - 4*a^3*b*x + 3*a^4)/(a^5*x^4)
```

**giac [A]** time = 1.06, size = 67, normalized size = 0.99

$$-\frac{b^4\log\left(|bx+a|\right)}{a^5}+\frac{b^4\log\left(|x|\right)}{a^5}+\frac{12\,ab^3x^3-6\,a^2b^2x^2+4\,a^3bx-3\,a^4}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] -b^4*log(abs(b*x + a))/a^5 + b^4*log(abs(x))/a^5 + 1/12*(12*a*b^3*x^3 - 6*a
^2*b^2*x^2 + 4*a^3*b*x - 3*a^4)/(a^5*x^4)
```

**maple [A]** time = 0.01, size = 63, normalized size = 0.93

$$\frac{b^4\ln(x)}{a^5}-\frac{b^4\ln\left(bx+a\right)}{a^5}+\frac{b^3}{a^4x}-\frac{b^2}{2a^3x^2}+\frac{b}{3a^2x^3}-\frac{1}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a),x)
```

```
[Out] -1/4/a/x^4+1/3*b/a^2/x^3-1/2*b^2/a^3/x^2+b^3/a^4/x+b^4*ln(x)/a^5-b^4*ln(b*x
+a)/a^5
```

**maxima [A]** time = 1.36, size = 62, normalized size = 0.91

$$-\frac{b^4 \log(bx+a)}{a^5}+\frac{b^4 \log(x)}{a^5}+\frac{12\,b^3x^3-6\,ab^2x^2+4\,a^2bx-3\,a^3}{12\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^4*log(b*x + a)/a^5 + b^4*log(x)/a^5 + 1/12*(12*b^3*x^3 - 6*a*b^2*x^2 + 4
*a^2*b*x - 3*a^3)/(a^4*x^4)
```

**mupad [B]** time = 0.07, size = 60, normalized size = 0.88

$$-\frac{\frac{a^4}{4}-\frac{a^3\,b\,x}{3}+\frac{a^2\,b^2\,x^2}{2}-a\,b^3\,x^3}{a^5\,x^4}-\frac{2\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)),x)
```

```
[Out] - (a^4/4 - a*b^3*x^3 + (a^2*b^2*x^2)/2 - (a^3*b*x)/3)/(a^5*x^4) - (2*b^4*at
anh((2*b*x)/a + 1))/a^5
```

**sympy [A]** time = 0.28, size = 56, normalized size = 0.82

$$\frac{-3a^3+4a^2bx-6ab^2x^2+12b^3x^3}{12a^4x^4}+\frac{b^4\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a),x)
```

```
[Out] (-3*a**3 + 4*a**2*b*x - 6*a*b**2*x**2 + 12*b**3*x**3)/(12*a**4*x**4) + b**4
*(log(x) - log(a/b + x))/a**5
```

# 3.168 $\int \frac{x^6}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=81

$$-\frac{a^6}{b^7(a+bx)}-\frac{6a^5\log(a+bx)}{b^7}+\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}$$

**Rubi** [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{a^6}{b^7(a+bx)}+\frac{5a^4x}{b^6}-\frac{6a^5\log(a+bx)}{b^7}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^2,x]
```

```
[Out] (5*a^4*x)/b^6 - (2*a^3*x^2)/b^5 + (a^2*x^3)/b^4 - (a*x^4)/(2*b^3) + x^5/(5*
b^2) - a^6/(b^7*(a + b*x)) - (6*a^5*Log[a + b*x])/b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^2}\,dx &= \int \left(\frac{5a^4}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{b^4}-\frac{2ax^3}{b^3}+\frac{x^4}{b^2}+\frac{a^6}{b^6(a+bx)^2}-\frac{6a^5}{b^6(a+bx)}\right)dx\\ &= \frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}-\frac{a^6}{b^7(a+bx)}-\frac{6a^5\log(a+bx)}{b^7}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 77, normalized size = 0.95

$$\frac{-\frac{10a^6}{a+bx}-60a^5\log(a+bx)+50a^4bx-20a^3b^2x^2+10a^2b^3x^3-5ab^4x^4+2b^5x^5}{10b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^2,x]
```

```
[Out] (50*a^4*b*x - 20*a^3*b^2*x^2 + 10*a^2*b^3*x^3 - 5*a*b^4*x^4 + 2*b^5*x^5 - (
10*a^6)/(a + b*x) - 60*a^5*Log[a + b*x])/(10*b^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x)^2, x]
```

**fricas [A]** time = 1.31, size = 96, normalized size = 1.19

$$\frac{2\,b^{6}x^{6}-3\,ab^{5}x^{5}+5\,a^{2}b^{4}x^{4}-10\,a^{3}b^{3}x^{3}+30\,a^{4}b^{2}x^{2}+50\,a^{5}bx-10\,a^{6}-60\left(a^{5}bx+a^{6}\right)\log\left(bx+a\right)}{10\left(b^{8}x+ab^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^6*x^6 - 3*a*b^5*x^5 + 5*a^2*b^4*x^4 - 10*a^3*b^3*x^3 + 30*a^4*b^2
*x^2 + 50*a^5*b*x - 10*a^6 - 60*(a^5*b*x + a^6)*log(b*x + a))/(b^8*x + a*b^
7)
```

**giac [A]** time = 1.14, size = 103, normalized size = 1.27

$$-\frac{(bx+a)^{5}\left(\frac{15\,a}{bx+a}-\frac{50\,a^{2}}{(bx+a)^{2}}+\frac{100\,a^{3}}{(bx+a)^{3}}-\frac{150\,a^{4}}{(bx+a)^{4}}-2\right)}{10\,b^{7}}+\frac{6\,a^{5}\log\left(\frac{|bx+a|}{(bx+a)^{2}|b|}\right)}{b^{7}}-\frac{a^{6}}{(bx+a)b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/10*(b*x + a)^5*(15*a/(b*x + a) - 50*a^2/(b*x + a)^2 + 100*a^3/(b*x + a)^
3 - 150*a^4/(b*x + a)^4 - 2)/b^7 + 6*a^5*log(abs(b*x + a)/((b*x + a)^2*abs(
b)))/b^7 - a^6/((b*x + a)*b^7)
```

**maple [A]** time = 0.01, size = 78, normalized size = 0.96

$$\frac{x^{5}}{5b^{2}}-\frac{a\,x^{4}}{2b^{3}}+\frac{a^{2}x^{3}}{b^{4}}-\frac{2a^{3}x^{2}}{b^{5}}-\frac{a^{6}}{(bx+a)\,b^{7}}-\frac{6a^{5}\ln(bx+a)}{b^{7}}+\frac{5a^{4}x}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^2,x)
```

```
[Out] 5*a^4*x/b^6-2*a^3*x^2/b^5+a^2*x^3/b^4-1/2*a*x^4/b^3+1/5*x^5/b^2-a^6/b^7/(b*
x+a)-6*a^5*ln(b*x+a)/b^7
```

**maxima [A]** time = 1.33, size = 82, normalized size = 1.01

$$-\frac{a^6}{b^8x + ab^7} - \frac{6\,a^5\log\left(bx + a\right)}{b^7} + \frac{2\,b^4x^5 - 5\,ab^3x^4 + 10\,a^2b^2x^3 - 20\,a^3bx^2 + 50\,a^4x}{10\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^6/(b^8*x + a*b^7) - 6*a^5*log(b*x + a)/b^7 + 1/10*(2*b^4*x^5 - 5*a*b^3*x
^4 + 10*a^2*b^2*x^3 - 20*a^3*b*x^2 + 50*a^4*x)/b^6
```

**mupad [B]** time = 0.14, size = 83, normalized size = 1.02

$$\frac{x^5}{5\,b^2} - \frac{6\,a^5\,\ln\left(a + b\,x\right)}{b^7} - \frac{a\,x^4}{2\,b^3} + \frac{5\,a^4\,x}{b^6} + \frac{a^2\,x^3}{b^4} - \frac{2\,a^3\,x^2}{b^5} - \frac{a^6}{b\,\left(x\,b^7 + a\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^2,x)
```

```
[Out] x^5/(5*b^2) - (6*a^5*log(a + b*x))/b^7 - (a*x^4)/(2*b^3) + (5*a^4*x)/b^6 +
(a^2*x^3)/b^4 - (2*a^3*x^2)/b^5 - a^6/(b*(a*b^6 + b^7*x))
```

**sympy [A]** time = 0.28, size = 78, normalized size = 0.96

$$-\frac{a^6}{ab^7 + b^8x} - \frac{6a^5\log\left(a + bx\right)}{b^7} + \frac{5a^4x}{b^6} - \frac{2a^3x^2}{b^5} + \frac{a^2x^3}{b^4} - \frac{ax^4}{2b^3} + \frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**2,x)
```

```
[Out] -a**6/(a*b**7 + b**8*x) - 6*a**5*log(a + b*x)/b**7 + 5*a**4*x/b**6 - 2*a**3
*x**2/b**5 + a**2*x**3/b**4 - a*x**4/(2*b**3) + x**5/(5*b**2)
```

## 3.169 $\int \frac{x^5}{(a+bx)^2} \, dx$

**Optimal**. Leaf size=72

$$\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}$$

**Rubi** [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2x^2}{2b^4}+\frac{a^5}{b^6(a+bx)}-\frac{4a^3x}{b^5}+\frac{5a^4\log(a+bx)}{b^6}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^2,x]
```

```
[Out] (-4*a^3*x)/b^5 + (3*a^2*x^2)/(2*b^4) - (2*a*x^3)/(3*b^3) + x^4/(4*b^2) + a^
5/(b^6*(a + b*x)) + (5*a^4*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^2}\,dx &= \int \left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a^5}{b^5(a+bx)^2}+\frac{5a^4}{b^5(a+bx)}\right)dx\\ &= -\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}+\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 66, normalized size = 0.92

$$\frac{\frac{12a^5}{a+bx}+60a^4\log(a+bx)-48a^3bx+18a^2b^2x^2-8ab^3x^3+3b^4x^4}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^2,x]
```

```
[Out] (-48*a^3*b*x + 18*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4 + (12*a^5)/(a + b*x
) + 60*a^4*Log[a + b*x])/(12*b^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x)^2, x]
```

**fricas [A]** time = 1.17, size = 85, normalized size = 1.18

$$\frac{3\,b^5x^5-5\,ab^4x^4+10\,a^2b^3x^3-30\,a^3b^2x^2-48\,a^4bx+12\,a^5+60\,\left(a^4bx+a^5\right)\log\left(bx+a\right)}{12\,\left(b^7x+ab^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^5 - 5*a*b^4*x^4 + 10*a^2*b^3*x^3 - 30*a^3*b^2*x^2 - 48*a^4*b*
x + 12*a^5 + 60*(a^4*b*x + a^5)*log(b*x + a))/(b^7*x + a*b^6)
```

**giac [A]** time = 1.22, size = 90, normalized size = 1.25

$$-\frac{(bx+a)^4\left(\frac{20\,a}{bx+a}-\frac{60\,a^2}{(bx+a)^2}+\frac{120\,a^3}{(bx+a)^3}-3\right)}{12\,b^6}-\frac{5\,a^4\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^6}+\frac{a^5}{(bx+a)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/12*(b*x + a)^4*(20*a/(b*x + a) - 60*a^2/(b*x + a)^2 + 120*a^3/(b*x + a)^
3 - 3)/b^6 - 5*a^4*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^6 + a^5/((b*x +
 a)*b^6)
```

**maple [A]** time = 0.01, size = 67, normalized size = 0.93

$$\frac{x^4}{4b^2}-\frac{2a\,x^3}{3b^3}+\frac{3a^2x^2}{2b^4}+\frac{a^5}{(bx+a)\,b^6}+\frac{5a^4\ln\left(bx+a\right)}{b^6}-\frac{4a^3x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^2,x)
```

```
[Out] -4*a^3*x/b^5+3/2*a^2*x^2/b^4-2/3*a*x^3/b^3+1/4*x^4/b^2+a^5/b^6/(b*x+a)+5*a^
4*ln(b*x+a)/b^6
```

**maxima [A]** time = 1.33, size = 70, normalized size = 0.97

$$\frac{a^5}{b^7x + ab^6} + \frac{5\,a^4\log\left(bx + a\right)}{b^6} + \frac{3\,b^3x^4 - 8\,ab^2x^3 + 18\,a^2bx^2 - 48\,a^3x}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^5/(b^7*x + a*b^6) + 5*a^4*log(b*x + a)/b^6 + 1/12*(3*b^3*x^4 - 8*a*b^2*x^
3 + 18*a^2*b*x^2 - 48*a^3*x)/b^5
```

**mupad [B]** time = 0.07, size = 72, normalized size = 1.00

$$\frac{x^4}{4\,b^2} + \frac{5\,a^4\,\ln\left(a + b\,x\right)}{b^6} - \frac{2\,a\,x^3}{3\,b^3} - \frac{4\,a^3\,x}{b^5} + \frac{3\,a^2\,x^2}{2\,b^4} + \frac{a^5}{b\,\left(x\,b^6 + a\,b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^2,x)
```

```
[Out] x^4/(4*b^2) + (5*a^4*log(a + b*x))/b^6 - (2*a*x^3)/(3*b^3) - (4*a^3*x)/b^5
+ (3*a^2*x^2)/(2*b^4) + a^5/(b*(a*b^5 + b^6*x))
```

**sympy [A]** time = 0.25, size = 71, normalized size = 0.99

$$\frac{a^5}{ab^6 + b^7x} + \frac{5a^4\log\left(a + bx\right)}{b^6} - \frac{4a^3x}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{2ax^3}{3b^3} + \frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**2,x)
```

```
[Out] a**5/(a*b**6 + b**7*x) + 5*a**4*log(a + b*x)/b**6 - 4*a**3*x/b**5 + 3*a**2*
x**2/(2*b**4) - 2*a*x**3/(3*b**3) + x**4/(4*b**2)
```

## 3.170 $\int \frac{x^4}{(a+bx)^2} \, dx$

**Optimal**. Leaf size=58

$$-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

**Rubi [A]** time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{b^5(a+bx)}+\frac{3a^2x}{b^4}-\frac{4a^3\log(a+bx)}{b^5}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^2,x]
```

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^2}\,dx &= \int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx\\ &= \frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 54, normalized size = 0.93

$$\frac{-\frac{3a^4}{a+bx}-12a^3\log(a+bx)+9a^2bx-3ab^2x^2+b^3x^3}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^2,x]
```

```
[Out] (9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*x
])/(3*b^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x)^2, x]
```

**fricas [A]** time = 1.20, size = 73, normalized size = 1.26

$$\frac{b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4 - 12\left(a^3bx + a^4\right)\log\left(bx + a\right)}{3\left(b^6x + ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

**giac [A]** time = 1.07, size = 79, normalized size = 1.36

$$-\frac{(bx+a)^3\left(\frac{6\,a}{bx+a} - \frac{18\,a^2}{(bx+a)^2} - 1\right)}{3\,b^5} + \frac{4\,a^3\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^5} - \frac{a^4}{(bx+a)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(b*x + a)^3*(6*a/(b*x + a) - 18*a^2/(b*x + a)^2 - 1)/b^5 + 4*a^3*log(a
bs(b*x + a)/((b*x + a)^2*abs(b)))/b^5 - a^4/((b*x + a)*b^5)
```

**maple [A]** time = 0.01, size = 57, normalized size = 0.98

$$\frac{x^3}{3b^2} - \frac{a\,x^2}{b^3} - \frac{a^4}{(bx+a)\,b^5} - \frac{4a^3\ln\left(bx+a\right)}{b^5} + \frac{3a^2x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^2,x)
```

```
[Out] 3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

**maxima [A]** time = 1.37, size = 59, normalized size = 1.02

$$-\frac{a^4}{b^6x+ab^5}-\frac{4\,a^3\log\left(bx+a\right)}{b^5}+\frac{b^2x^3-3\,abx^2+9\,a^2x}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
9*a^2*x)/b^4
```

**mupad [B]** time = 0.07, size = 62, normalized size = 1.07

$$\frac{x^3}{3\,b^2}-\frac{4\,a^3\,\ln\left(a+b\,x\right)}{b^5}-\frac{a\,x^2}{b^3}+\frac{3\,a^2\,x}{b^4}-\frac{a^4}{b\,\left(x\,b^5+a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^2,x)
```

```
[Out] x^3/(3*b^2) - (4*a^3*log(a + b*x))/b^5 - (a*x^2)/b^3 + (3*a^2*x)/b^4 - a^4/
(b*(a*b^4 + b^5*x))
```

**sympy [A]** time = 0.22, size = 54, normalized size = 0.93

$$-\frac{a^4}{ab^5+b^6x}-\frac{4a^3\log\left(a+bx\right)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**2,x)
```

```
[Out] -a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**2
/b**3 + x**3/(3*b**2)
```

## 3.171 $\int \frac{x^3}{(a+bx)^2} \, dx$

**Optimal**. Leaf size=46

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^2,x]
```

```
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^2}\,dx &= \int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx\\ &= -\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 43, normalized size = 0.93

$$\frac{\frac{2a^3}{a+bx}+6a^2\log(a+bx)-4abx+b^2x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^2,x]
```

```
[Out] (-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x)^2, x]
```

**fricas [A]** time = 1.22, size = 62, normalized size = 1.35

$$\frac{b^3x^3 - 3\,ab^2x^2 - 4\,a^2bx + 2\,a^3 + 6\left(a^2bx + a^3\right)\log\left(bx + a\right)}{2\left(b^5x + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

**giac [A]** time = 1.14, size = 66, normalized size = 1.43

$$-\frac{(bx+a)^2\left(\frac{6\,a}{bx+a}-1\right)}{2\,b^4} - \frac{3\,a^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4} + \frac{a^3}{(bx+a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*x + a)^2*(6*a/(b*x + a) - 1)/b^4 - 3*a^2*log(abs(b*x + a)/((b*x + a
)^2*abs(b)))/b^4 + a^3/((b*x + a)*b^4)
```

**maple [A]** time = 0.01, size = 45, normalized size = 0.98

$$\frac{x^2}{2b^2} + \frac{a^3}{(bx+a)\,b^4} + \frac{3a^2\ln\left(bx+a\right)}{b^4} - \frac{2ax}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^2,x)
```

```
[Out] -2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

**maxima [A]** time = 1.36, size = 47, normalized size = 1.02

$$\frac{a^3}{b^5x+ab^4}+\frac{3\,a^2\log\left(bx+a\right)}{b^4}+\frac{bx^2-4\,ax}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

**mupad [B]** time = 0.08, size = 50, normalized size = 1.09

$$\frac{x^2}{2\,b^2}+\frac{3\,a^2\,\ln\left(a+b\,x\right)}{b^4}+\frac{a^3}{b\,\left(x\,b^4+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^2,x)
```

```
[Out] x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)/
b^3
```

**sympy [A]** time = 0.20, size = 44, normalized size = 0.96

$$\frac{a^3}{ab^4+b^5x}+\frac{3a^2\log\left(a+bx\right)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**2,x)
```

```
[Out] a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b*
*2)
```

## 3.172 $\int \frac{x^2}{(a+bx)^2} \, dx$

**Optimal**. Leaf size=33

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^2,x]
```

```
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^2}\,dx &= \int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx\\ &= \frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 0.88

$$\frac{-\frac{a^2}{a+bx}-2a\log(a+bx)+bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^2,x]
```

[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(a + b*x)^2,x]

[Out] IntegrateAlgebraic[x^2/(a + b*x)^2, x]

**fricas [A]** time = 1.28, size = 47, normalized size = 1.42

$$\frac{b^2x^2 + abx - a^2 - 2\left(abx + a^2\right)\log\left(bx + a\right)}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x+a)^2,x, algorithm="fricas")

[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)

**giac [A]** time = 1.15, size = 50, normalized size = 1.52

$$\frac{2\,a\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3} + \frac{bx+a}{b^3} - \frac{a^2}{(bx+a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x+a)^2,x, algorithm="giac")

[Out] 2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^3 + (b*x + a)/b^3 - a^2/((b*x + a)*b^3)

**maple [A]** time = 0.01, size = 34, normalized size = 1.03

$$-\frac{a^2}{(bx+a)\,b^3} - \frac{2a\ln\left(bx+a\right)}{b^3} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x+a)^2,x)

[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3

**maxima** [A] time = 1.34, size = 36, normalized size = 1.09

$$-\frac{a^2}{b^4x + ab^3} + \frac{x}{b^2} - \frac{2\,a\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="maxima")
[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

**mupad** [B] time = 0.08, size = 36, normalized size = 1.09

$$\frac{x}{b^2} - \frac{a^2}{x\,b^4 + a\,b^3} - \frac{2\,a\,\ln\left(a + b\,x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^2,x)
[Out] x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

**sympy** [A] time = 0.17, size = 31, normalized size = 0.94

$$-\frac{a^2}{ab^3 + b^4x} - \frac{2a\log\left(a + bx\right)}{b^3} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**2,x)
[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

## 3.173 $\int \frac{x}{(a+bx)^2} \, dx$

**Optimal**. Leaf size=23

$$\frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^2,x]
```

```
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^2} \, dx &= \int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right) dx \\ &= \frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 20, normalized size = 0.87

$$\frac{\frac{a}{a+bx} + \log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^2,x]
```

```
[Out] (a/(a + b*x) + Log[a + b*x])/b^2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x)^2,x]
[Out] IntegrateAlgebraic[x/(a + b*x)^2, x]
```

**fricas [A]** time = 1.22, size = 28, normalized size = 1.22

$$\frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="fricas")
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

**giac [A]** time = 1.05, size = 42, normalized size = 1.83

$$-\frac{\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{a}{(bx+a)b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="giac")
[Out] -(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b))/b
```

**maple [A]** time = 0.01, size = 24, normalized size = 1.04

$$\frac{a}{(bx+a)\,b^2}+\frac{\ln(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^2,x)
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

**maxima [A]** time = 1.32, size = 26, normalized size = 1.13

$$\frac{a}{b^3x+ab^2}+\frac{\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

**mupad [B]** time = 0.04, size = 23, normalized size = 1.00

$$\frac{\ln(a+b\,x)}{b^2}+\frac{a}{b^2\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^2,x)
```

```
[Out] log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

**sympy [A]** time = 0.17, size = 20, normalized size = 0.87

$$\frac{a}{ab^2+b^3x}+\frac{\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**2,x)
```

```
[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

## 3.174 $\int \frac{1}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

**Rubi** [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{b(a+bx)}$$

**Mathematica** [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-2), x]
```

**fricas [A]** time = 1.18, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

**giac [A]** time = 1.18, size = 12, normalized size = 1.00

$$-\frac{1}{(bx + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

**maple [A]** time = 0.00, size = 13, normalized size = 1.08

$$-\frac{1}{(bx + a)\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x)
```

```
[Out] -1/b/(b*x+a)
```

**maxima [A]** time = 1.38, size = 12, normalized size = 1.00

$$-\frac{1}{(bx + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

**mupad [B]** time = 0.03, size = 12, normalized size = 1.00

$$-\frac{1}{b\ (a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^2,x)
[Out] -1/(b*(a + b*x))
```

**sympy [A]** time = 0.15, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
[Out] -1/(a*b + b**2*x)
```

## 3.175 $\int \frac{1}{x(a+bx)^2} \, dx$

**Optimal**. Leaf size=29

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

**Rubi** [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^2),x]
```

```
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^2} \, dx &= \int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) dx\\ &= \frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.83

$$\frac{\frac{a}{a+bx}-\log(a+bx)+\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^2),x]
```

```
[Out] (a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^2),x]
[Out] IntegrateAlgebraic[1/(x*(a + b*x)^2), x]
```

**fricas [A]** time = 1.28, size = 39, normalized size = 1.34

$$-\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="fricas")
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

**giac [A]** time = 1.00, size = 38, normalized size = 1.31

$$b\left(\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^2b}+\frac{1}{(bx+a)ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="giac")
[Out] b*(log(abs(-a/(b*x + a) + 1))/(a^2*b) + 1/((b*x + a)*a*b))
```

**maple [A]** time = 0.01, size = 30, normalized size = 1.03

$$\frac{1}{(bx+a)\,a}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^2,x)
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

**maxima [A]** time = 1.30, size = 28, normalized size = 0.97

$$\frac{1}{abx+a^2}-\frac{\log(bx+a)}{a^2}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

**mupad [B]** time = 0.12, size = 26, normalized size = 0.90

$$\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^2),x)
```

```
[Out] 1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2
```

**sympy [A]** time = 0.22, size = 22, normalized size = 0.76

$$\frac{1}{a^2+abx}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**2,x)
```

```
[Out] 1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

## 3.176 $\int \frac{1}{x^2(a+bx)^2}\,dx$

**Optimal**. Leaf size=42

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

**Rubi [A]** time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{b}{a+bx}+\frac{1}{x}\right)-2b\log(a+bx)+2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)^2), x]
```

**fricas [A]** time = 1.11, size = 63, normalized size = 1.50

$$-\frac{2\,abx + a^2 - 2\left(b^2x^2 + abx\right)\log\left(bx + a\right) + 2\left(b^2x^2 + abx\right)\log(x)}{a^3bx^2 + a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

**giac [A]** time = 1.22, size = 52, normalized size = 1.24

$$-\frac{2\,b\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3} - \frac{b}{(bx+a)a^2} + \frac{b}{a^3\left(\frac{a}{bx+a}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*b*log(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(a/(b*x +
a) - 1))
```

**maple [A]** time = 0.01, size = 43, normalized size = 1.02

$$-\frac{b}{\left(bx+a\right)a^2} - \frac{2b\ln(x)}{a^3} + \frac{2b\ln\left(bx+a\right)}{a^3} - \frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2,x)
```

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

**maxima [A]** time = 1.39, size = 45, normalized size = 1.07

$$-\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log\left(bx+a\right)}{a^3}-\frac{2\,b\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

**mupad [B]** time = 0.12, size = 45, normalized size = 1.07

$$\frac{2\,b\,\ln\left(\frac{a+b\,x}{x}\right)}{a^3}-\frac{1}{a\,x\,\left(a+b\,x\right)}-\frac{2\,b}{a^2\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^2),x)
```

```
[Out] (2*b*log((a + b*x)/x))/a^3 - 1/(a*x*(a + b*x)) - (2*b)/(a^2*(a + b*x))
```

**sympy [A]** time = 0.30, size = 37, normalized size = 0.88

$$\frac{-a-2bx}{a^3x+a^2bx^2}+\frac{2b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2,x)
```

```
[Out] (-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

## 3.177 $\int \frac{1}{x^3(a+bx)^2}\,dx$

**Optimal**. Leaf size=58

$$\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^2),x]
```

```
[Out] -1/(2*a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4 -
 (3*b^2*Log[a + b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 53, normalized size = 0.91

$$\frac{a\left(\frac{2b^2}{a+bx}-\frac{a}{x^2}+\frac{4b}{x}\right)-6b^2\log(a+bx)+6b^2\log(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^2),x]
```

```
[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
b*x])/(2*a^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)^2), x]
```

**fricas [A]** time = 0.55, size = 86, normalized size = 1.48

$$\frac{6\,ab^2x^2+3\,a^2bx-a^3-6\left(b^3x^3+ab^2x^2\right)\log\left(bx+a\right)+6\left(b^3x^3+ab^2x^2\right)\log(x)}{2\left(a^4bx^3+a^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

**giac [A]** time = 1.13, size = 74, normalized size = 1.28

$$\frac{3\,b^2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^4}+\frac{b^2}{(bx+a)a^3}-\frac{\frac{6\,ab^2}{bx+a}-5\,b^2}{2\,a^4\left(\frac{a}{bx+a}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3*b^2*log(abs(-a/(b*x + a) + 1))/a^4 + b^2/((b*x + a)*a^3) - 1/2*(6*a*b^2/(
b*x + a) - 5*b^2)/(a^4*(a/(b*x + a) - 1)^2)
```

**maple [A]** time = 0.01, size = 57, normalized size = 0.98

$$\frac{b^2}{(bx+a)\,a^3}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln\left(bx+a\right)}{a^4}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^2,x)
```

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

**maxima** [A] time = 1.40, size = 64, normalized size = 1.10

$$\frac{6\,b^2x^2+3\,abx-a^2}{2\left(a^3bx^3+a^4x^2\right)}-\frac{3\,b^2\log\left(bx+a\right)}{a^4}+\frac{3\,b^2\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

**mupad** [B] time = 0.11, size = 57, normalized size = 0.98

$$\frac{\frac{3\,b^2\,x^2}{a^3}-\frac{1}{2\,a}+\frac{3\,b\,x}{2\,a^2}}{b\,x^3+a\,x^2}-\frac{6\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^2),x)
```

```
[Out] ((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*atan
h((2*b*x)/a + 1))/a^4
```

**sympy** [A] time = 0.31, size = 54, normalized size = 0.93

$$\frac{-a^2+3abx+6b^2x^2}{2a^4x^2+2a^3bx^3}+\frac{3b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(log
(x) - log(a/b + x))/a**4
```

## 3.178 $\int \frac{1}{x^4(a+bx)^2}\,dx$

**Optimal**. Leaf size=69

$$-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

**Rubi [A]** time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^2),x]
```

```
[Out] -1/(3*a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4*b
^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 66, normalized size = 0.96

$$-\frac{\frac{a\left(a^3-2a^2bx+6ab^2x^2+12b^3x^3\right)}{x^3(a+bx)}-12b^3\log(a+bx)+12b^3\log(x)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^2),x]
```

```
[Out] -1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 12
*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)^2), x]
```

**fricas [A]** time = 0.93, size = 95, normalized size = 1.38

$$-\frac{12\,ab^3x^3 + 6\,a^2b^2x^2 - 2\,a^3bx + a^4 - 12\left(b^4x^4 + ab^3x^3\right)\log\left(bx+a\right) + 12\left(b^4x^4 + ab^3x^3\right)\log(x)}{3\left(a^5bx^4 + a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3*
x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

**giac [A]** time = 1.02, size = 90, normalized size = 1.30

$$-\frac{4\,b^3\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^5} - \frac{b^3}{(bx+a)a^4} - \frac{\frac{30\,ab^3}{bx+a} - \frac{18\,a^2b^3}{(bx+a)^2} - 13\,b^3}{3\,a^5\left(\frac{a}{bx+a}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4*b^3*log(abs(-a/(b*x + a) + 1))/a^5 - b^3/((b*x + a)*a^4) - 1/3*(30*a*b^3
/(b*x + a) - 18*a^2*b^3/(b*x + a)^2 - 13*b^3)/(a^5*(a/(b*x + a) - 1)^3)
```

**maple [A]** time = 0.01, size = 68, normalized size = 0.99

$$-\frac{b^3}{(bx+a)\,a^4} - \frac{4b^3\ln(x)}{a^5} + \frac{4b^3\ln\left(bx+a\right)}{a^5} - \frac{3b^2}{a^4x} + \frac{b}{a^3x^2} - \frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^2,x)
```

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

**maxima [A]** time = 1.41, size = 73, normalized size = 1.06

$$-\frac{12\,b^3x^3 + 6\,ab^2x^2 - 2\,a^2bx + a^3}{3\,\left(a^4bx^4 + a^5x^3\right)} + \frac{4\,b^3\log\left(bx + a\right)}{a^5} - \frac{4\,b^3\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) + 4
*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

**mupad [B]** time = 0.08, size = 69, normalized size = 1.00

$$\frac{8\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5} - \frac{\frac{1}{3\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}-\frac{2\,b\,x}{3\,a^2}}{b\,x^4+a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^2),x)
```

```
[Out] (8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3)
/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

**sympy [A]** time = 0.34, size = 66, normalized size = 0.96

$$\frac{-a^3 + 2a^2bx - 6ab^2x^2 - 12b^3x^3}{3a^5x^3 + 3a^4bx^4} + \frac{4b^3\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**2,x)
```

```
[Out] (-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*b
*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

## 3.179 $\int \frac{1}{x^5(a+bx)^2}\,dx$

**Optimal**. Leaf size=84

$$\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

**Rubi [A]** time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{3b^2}{2a^4x^2}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^2),x]
```

```
[Out] -1/(4*a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x)
+ b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 79, normalized size = 0.94

$$\frac{\frac{a\left(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4\right)}{x^4(a+bx)}-60b^4\log(a+bx)+60b^4\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^2),x]
```

```
[Out] ((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^4
*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x)^2), x]
```

**fricas [A]** time = 1.18, size = 108, normalized size = 1.29

$$\frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5-60\,\left(b^5x^5+ab^4x^4\right)\log\left(bx+a\right)+60\,\left(b^5x^5+ab^4x^4\right)\log(x)}{12\,\left(a^6bx^5+a^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/(a
^6*b*x^5 + a^7*x^4)
```

**giac [A]** time = 0.99, size = 104, normalized size = 1.24

$$\frac{5\,b^4\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^6}+\frac{b^4}{(bx+a)a^5}-\frac{\frac{260\,ab^4}{bx+a}-\frac{300\,a^2b^4}{(bx+a)^2}+\frac{120\,a^3b^4}{(bx+a)^3}-77\,b^4}{12\,a^6\left(\frac{a}{bx+a}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*b^4*log(abs(-a/(b*x + a) + 1))/a^6 + b^4/((b*x + a)*a^5) - 1/12*(260*a*b^
4/(b*x + a) - 300*a^2*b^4/(b*x + a)^2 + 120*a^3*b^4/(b*x + a)^3 - 77*b^4)/(
a^6*(a/(b*x + a) - 1)^4)
```

**maple [A]** time = 0.01, size = 79, normalized size = 0.94

$$\frac{b^4}{\left(bx+a\right)a^5}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln\left(bx+a\right)}{a^6}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^2,x)
```

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

**maxima [A]** time = 1.34, size = 86, normalized size = 1.02

$$\frac{60\,b^4x^4+30\,ab^3x^3-10\,a^2b^2x^2+5\,a^3bx-3\,a^4}{12\left(a^5bx^5+a^6x^4\right)}-\frac{5\,b^4\log\left(bx+a\right)}{a^6}+\frac{5\,b^4\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5*
b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

**mupad [B]** time = 0.12, size = 79, normalized size = 0.94

$$\frac{\frac{5\,b^3\,x^3}{2\,a^4}-\frac{5\,b^2\,x^2}{6\,a^3}-\frac{1}{4\,a}+\frac{5\,b^4\,x^4}{a^5}+\frac{5\,b\,x}{12\,a^2}}{b\,x^5+a\,x^4}-\frac{10\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^2),x)
```

```
[Out] ((5*b^3*x^3)/(2*a^4) - (5*b^2*x^2)/(6*a^3) - 1/(4*a) + (5*b^4*x^4)/a^5 + (5
*b*x)/(12*a^2))/(a*x^4 + b*x^5) - (10*b^4*atanh((2*b*x)/a + 1))/a^6
```

**sympy [A]** time = 0.39, size = 80, normalized size = 0.95

$$\frac{-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4}{12a^6x^4+12a^5bx^5}+\frac{5b^4\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)/
(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

## 3.180 $\int \frac{x^7}{(a+bx)^3} dx$

**Optimal**. Leaf size=99

$$\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}-\frac{21a^5\log(a+bx)}{b^8}+\frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}$$

**Rubi** [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}+\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}+\frac{15a^4x}{b^7}-\frac{21a^5\log(a+bx)}{b^8}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^3,x]
```

```
[Out] (15*a^4*x)/b^7 - (5*a^3*x^2)/b^6 + (2*a^2*x^3)/b^5 - (3*a*x^4)/(4*b^4) + x^
5/(5*b^3) + a^7/(2*b^8*(a + b*x)^2) - (7*a^6)/(b^8*(a + b*x)) - (21*a^5*Log
[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^3}\,dx &= \int \left(\frac{15a^4}{b^7}-\frac{10a^3x}{b^6}+\frac{6a^2x^2}{b^5}-\frac{3ax^3}{b^4}+\frac{x^4}{b^3}-\frac{a^7}{b^7(a+bx)^3}+\frac{7a^6}{b^7(a+bx)^2}-\frac{21a^5}{b^7(a+bx)}\right)dx\\ &= \frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}+\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}-\frac{21a^5\log(a+bx)}{b^8}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 89, normalized size = 0.90

$$\frac{\frac{10a^7}{(a+bx)^2}-\frac{140a^6}{a+bx}-420a^5\log(a+bx)+300a^4bx-100a^3b^2x^2+40a^2b^3x^3-15ab^4x^4+4b^5x^5}{20b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^3,x]
```

```
[Out] (300*a^4*b*x - 100*a^3*b^2*x^2 + 40*a^2*b^3*x^3 - 15*a*b^4*x^4 + 4*b^5*x^5
+ (10*a^7)/(a + b*x)^2 - (140*a^6)/(a + b*x) - 420*a^5*Log[a + b*x])/(20*b^
8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x)^3, x]
```

**fricas [A]** time = 1.04, size = 129, normalized size = 1.30

$$\frac{4\,b^{7}x^{7}-7\,ab^{6}x^{6}+14\,a^{2}b^{5}x^{5}-35\,a^{3}b^{4}x^{4}+140\,a^{4}b^{3}x^{3}+500\,a^{5}b^{2}x^{2}+160\,a^{6}bx-130\,a^{7}-420\left(a^{5}b^{2}x^{2}+2\,a^{6}bx+a^{7}\right)\log\left(bx+a\right)}{20\left(b^{10}x^{2}+2\,ab^{9}x+a^{2}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*x^7 - 7*a*b^6*x^6 + 14*a^2*b^5*x^5 - 35*a^3*b^4*x^4 + 140*a^4*b
^3*x^3 + 500*a^5*b^2*x^2 + 160*a^6*b*x - 130*a^7 - 420*(a^5*b^2*x^2 + 2*a^6
*b*x + a^7)*log(b*x + a))/(b^10*x^2 + 2*a*b^9*x + a^2*b^8)
```

**giac [A]** time = 0.89, size = 95, normalized size = 0.96

$$-\frac{21\,a^{5}\log\left(|bx+a|\right)}{b^{8}}-\frac{14\,a^{6}bx+13\,a^{7}}{2\,(bx+a)^{2}b^{8}}+\frac{4\,b^{12}x^{5}-15\,ab^{11}x^{4}+40\,a^{2}b^{10}x^{3}-100\,a^{3}b^{9}x^{2}+300\,a^{4}b^{8}x}{20\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -21*a^5*log(abs(b*x + a))/b^8 - 1/2*(14*a^6*b*x + 13*a^7)/((b*x + a)^2*b^8)
 + 1/20*(4*b^12*x^5 - 15*a*b^11*x^4 + 40*a^2*b^10*x^3 - 100*a^3*b^9*x^2 + 3
00*a^4*b^8*x)/b^15
```

**maple [A]** time = 0.01, size = 94, normalized size = 0.95

$$\frac{x^{5}}{5b^{3}}-\frac{3a\,x^{4}}{4b^{4}}+\frac{2a^{2}x^{3}}{b^{5}}+\frac{a^{7}}{2\left(bx+a\right)^{2}b^{8}}-\frac{5a^{3}x^{2}}{b^{6}}-\frac{7a^{6}}{\left(bx+a\right)b^{8}}-\frac{21a^{5}\ln\left(bx+a\right)}{b^{8}}+\frac{15a^{4}x}{b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^3,x)
```

```
[Out] 15*a^4*x/b^7-5*a^3*x^2/b^6+2*a^2*x^3/b^5-3/4*a*x^4/b^4+1/5*x^5/b^3+1/2*a^7/
b^8/(b*x+a)^2-7*a^6/b^8/(b*x+a)-21*a^5*ln(b*x+a)/b^8
```

**maxima [A]** time = 1.39, size = 103, normalized size = 1.04

$$-\frac{14\,a^{6}bx+13\,a^{7}}{2\,\left(b^{10}x^{2}+2\,ab^{9}x+a^{2}b^{8}\right)}-\frac{21\,a^{5}\log\left(bx+a\right)}{b^{8}}+\frac{4\,b^{4}x^{5}-15\,ab^{3}x^{4}+40\,a^{2}b^{2}x^{3}-100\,a^{3}bx^{2}+300\,a^{4}x}{20\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(14*a^6*b*x + 13*a^7)/(b^10*x^2 + 2*a*b^9*x + a^2*b^8) - 21*a^5*log(b*
x + a)/b^8 + 1/20*(4*b^4*x^5 - 15*a*b^3*x^4 + 40*a^2*b^2*x^3 - 100*a^3*b*x^
2 + 300*a^4*x)/b^7
```

**mupad [B]** time = 0.23, size = 91, normalized size = 0.92

$$-\frac{\frac{7\,a\,{\left(a+b\,x\right)}^4}{4}-\frac{{\left(a+b\,x\right)}^5}{5}-7\,a^2\,{\left(a+b\,x\right)}^3+\frac{35\,a^3\,{\left(a+b\,x\right)}^2}{2}+\frac{7\,a^6}{a+b\,x}-\frac{a^7}{2\,{\left(a+b\,x\right)}^2}+21\,a^5\,\ln\left(a+b\,x\right)-35\,a^4\,b\,x}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^3,x)
```

```
[Out] -((7*a*(a + b*x)^4)/4 - (a + b*x)^5/5 - 7*a^2*(a + b*x)^3 + (35*a^3*(a + b*
x)^2)/2 + (7*a^6)/(a + b*x) - a^7/(2*(a + b*x)^2) + 21*a^5*log(a + b*x) - 3
5*a^4*b*x)/b^8
```

**sympy [A]** time = 0.53, size = 109, normalized size = 1.10

$$-\frac{21a^{5}\log\left(a+bx\right)}{b^{8}}+\frac{15a^{4}x}{b^{7}}-\frac{5a^{3}x^{2}}{b^{6}}+\frac{2a^{2}x^{3}}{b^{5}}-\frac{3ax^{4}}{4b^{4}}+\frac{-13a^{7}-14a^{6}bx}{2a^{2}b^{8}+4ab^{9}x+2b^{10}x^{2}}+\frac{x^{5}}{5b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**3,x)
```

```
[Out] -21*a**5*log(a + b*x)/b**8 + 15*a**4*x/b**7 - 5*a**3*x**2/b**6 + 2*a**2*x**
3/b**5 - 3*a*x**4/(4*b**4) + (-13*a**7 - 14*a**6*b*x)/(2*a**2*b**8 + 4*a*b*
*9*x + 2*b**10*x**2) + x**5/(5*b**3)
```

## 3.181 $\int \frac{x^6}{(a+bx)^3} \, dx$

**Optimal**. Leaf size=86

$$-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}+\frac{15a^4\log(a+bx)}{b^7}-\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}$$

**Rubi** [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2x^2}{b^5}-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}-\frac{10a^3x}{b^6}+\frac{15a^4\log(a+bx)}{b^7}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^3,x]
```

```
[Out] (-10*a^3*x)/b^6 + (3*a^2*x^2)/b^5 - (a*x^3)/b^4 + x^4/(4*b^3) - a^6/(2*b^7*
(a + b*x)^2) + (6*a^5)/(b^7*(a + b*x)) + (15*a^4*Log[a + b*x])/b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^3}\,dx &= \int \left(-\frac{10a^3}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{b^4}+\frac{x^3}{b^3}+\frac{a^6}{b^6(a+bx)^3}-\frac{6a^5}{b^6(a+bx)^2}+\frac{15a^4}{b^6(a+bx)}\right)dx\\ &= -\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}+\frac{15a^4\log(a+bx)}{b^7}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 77, normalized size = 0.90

$$\frac{-\frac{2a^6}{(a+bx)^2}+\frac{24a^5}{a+bx}+60a^4\log(a+bx)-40a^3bx+12a^2b^2x^2-4ab^3x^3+b^4x^4}{4b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^3,x]
```

```
[Out] (-40*a^3*b*x + 12*a^2*b^2*x^2 - 4*a*b^3*x^3 + b^4*x^4 - (2*a^6)/(a + b*x)^2
 + (24*a^5)/(a + b*x) + 60*a^4*Log[a + b*x])/(4*b^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x)^3, x]
```

**fricas [A]** time = 1.11, size = 117, normalized size = 1.36

$$\frac{b^6x^6 - 2\,ab^5x^5 + 5\,a^2b^4x^4 - 20\,a^3b^3x^3 - 68\,a^4b^2x^2 - 16\,a^5bx + 22\,a^6 + 60\left(a^4b^2x^2 + 2\,a^5bx + a^6\right)\log\left(bx+a\right)}{4\left(b^9x^2 + 2\,ab^8x + a^2b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^6*x^6 - 2*a*b^5*x^5 + 5*a^2*b^4*x^4 - 20*a^3*b^3*x^3 - 68*a^4*b^2*x^
2 - 16*a^5*b*x + 22*a^6 + 60*(a^4*b^2*x^2 + 2*a^5*b*x + a^6)*log(b*x + a))/
(b^9*x^2 + 2*a*b^8*x + a^2*b^7)
```

**giac [A]** time = 1.15, size = 83, normalized size = 0.97

$$\frac{15\,a^4\log\left(|bx+a|\right)}{b^7} + \frac{12\,a^5bx + 11\,a^6}{2\,(bx+a)^2b^7} + \frac{b^9x^4 - 4\,ab^8x^3 + 12\,a^2b^7x^2 - 40\,a^3b^6x}{4\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 15*a^4*log(abs(b*x + a))/b^7 + 1/2*(12*a^5*b*x + 11*a^6)/((b*x + a)^2*b^7)
+ 1/4*(b^9*x^4 - 4*a*b^8*x^3 + 12*a^2*b^7*x^2 - 40*a^3*b^6*x)/b^12
```

**maple [A]** time = 0.01, size = 83, normalized size = 0.97

$$\frac{x^4}{4b^3} - \frac{a\,x^3}{b^4} - \frac{a^6}{2\,(bx+a)^2\,b^7} + \frac{3a^2x^2}{b^5} + \frac{6a^5}{(bx+a)\,b^7} + \frac{15a^4\ln\left(bx+a\right)}{b^7} - \frac{10a^3x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^3,x)
```

```
[Out] -10*a^3*x/b^6+3*a^2*x^2/b^5-a*x^3/b^4+1/4*x^4/b^3-1/2*a^6/b^7/(b*x+a)^2+6*a
^5/b^7/(b*x+a)+15*a^4*ln(b*x+a)/b^7
```

**maxima [A]** time = 1.40, size = 91, normalized size = 1.06

$$\frac{12\,a^5bx+11\,a^6}{2\left(b^9x^2+2\,ab^8x+a^2b^7\right)}+\frac{15\,a^4\log\left(bx+a\right)}{b^7}+\frac{b^3x^4-4\,ab^2x^3+12\,a^2bx^2-40\,a^3x}{4\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*a^5*b*x + 11*a^6)/(b^9*x^2 + 2*a*b^8*x + a^2*b^7) + 15*a^4*log(b*x
+ a)/b^7 + 1/4*(b^3*x^4 - 4*a*b^2*x^3 + 12*a^2*b*x^2 - 40*a^3*x)/b^6
```

**mupad [B]** time = 0.16, size = 78, normalized size = 0.91

$$\frac{\frac{(a+b\,x)^4}{4}-2\,a\,(a+b\,x)^3+\frac{15\,a^2\,(a+b\,x)^2}{2}+\frac{6\,a^5}{a+b\,x}-\frac{a^6}{2\,(a+b\,x)^2}+15\,a^4\,\ln\left(a+b\,x\right)-20\,a^3\,b\,x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^3,x)
```

```
[Out] ((a + b*x)^4/4 - 2*a*(a + b*x)^3 + (15*a^2*(a + b*x)^2)/2 + (6*a^5)/(a + b*
x) - a^6/(2*(a + b*x)^2) + 15*a^4*log(a + b*x) - 20*a^3*b*x)/b^7
```

**sympy [A]** time = 0.40, size = 92, normalized size = 1.07

$$\frac{15a^4\log\left(a+bx\right)}{b^7}-\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{11a^6+12a^5bx}{2a^2b^7+4ab^8x+2b^9x^2}+\frac{x^4}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**3,x)
```

```
[Out] 15*a**4*log(a + b*x)/b**7 - 10*a**3*x/b**6 + 3*a**2*x**2/b**5 - a*x**3/b**4
 + (11*a**6 + 12*a**5*b*x)/(2*a**2*b**7 + 4*a*b**8*x + 2*b**9*x**2) + x**4/
(4*b**3)
```

## 3.182 $\int \frac{x^5}{(a+bx)^3}\,dx$

**Optimal**. Leaf size=77

$$\frac{a^5}{2b^6(a+bx)^2}-\frac{5a^4}{b^6(a+bx)}-\frac{10a^3\log(a+bx)}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^3}{3b^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^5}{2b^6(a+bx)^2}-\frac{5a^4}{b^6(a+bx)}+\frac{6a^2x}{b^5}-\frac{10a^3\log(a+bx)}{b^6}-\frac{3ax^2}{2b^4}+\frac{x^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^3,x]
```

```
[Out] (6*a^2*x)/b^5 - (3*a*x^2)/(2*b^4) + x^3/(3*b^3) + a^5/(2*b^6*(a + b*x)^2) -
 (5*a^4)/(b^6*(a + b*x)) - (10*a^3*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^3}\,dx &= \int \left(\frac{6a^2}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{b^3}-\frac{a^5}{b^5(a+bx)^3}+\frac{5a^4}{b^5(a+bx)^2}-\frac{10a^3}{b^5(a+bx)}\right)dx\\ &= \frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}+\frac{x^3}{3b^3}+\frac{a^5}{2b^6(a+bx)^2}-\frac{5a^4}{b^6(a+bx)}-\frac{10a^3\log(a+bx)}{b^6}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 67, normalized size = 0.87

$$\frac{\frac{3a^5}{(a+bx)^2}-\frac{30a^4}{a+bx}-60a^3\log(a+bx)+36a^2bx-9ab^2x^2+2b^3x^3}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^3,x]
```

```
[Out] (36*a^2*b*x - 9*a*b^2*x^2 + 2*b^3*x^3 + (3*a^5)/(a + b*x)^2 - (30*a^4)/(a +
 b*x) - 60*a^3*Log[a + b*x])/(6*b^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x)^3, x]
```

**fricas [A]** time = 1.12, size = 107, normalized size = 1.39

$$\frac{2\,b^5x^5 - 5\,ab^4x^4 + 20\,a^2b^3x^3 + 63\,a^3b^2x^2 + 6\,a^4bx - 27\,a^5 - 60\left(a^3b^2x^2 + 2\,a^4bx + a^5\right)\log\left(bx+a\right)}{6\left(b^8x^2 + 2\,ab^7x + a^2b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*x^5 - 5*a*b^4*x^4 + 20*a^2*b^3*x^3 + 63*a^3*b^2*x^2 + 6*a^4*b*x
- 27*a^5 - 60*(a^3*b^2*x^2 + 2*a^4*b*x + a^5)*log(b*x + a))/(b^8*x^2 + 2*a*
b^7*x + a^2*b^6)
```

**giac [A]** time = 1.12, size = 73, normalized size = 0.95

$$-\frac{10\,a^3\log\left(|bx+a|\right)}{b^6} - \frac{10\,a^4bx + 9\,a^5}{2\left(bx+a\right)^2b^6} + \frac{2\,b^6x^3 - 9\,ab^5x^2 + 36\,a^2b^4x}{6\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -10*a^3*log(abs(b*x + a))/b^6 - 1/2*(10*a^4*b*x + 9*a^5)/((b*x + a)^2*b^6)
+ 1/6*(2*b^6*x^3 - 9*a*b^5*x^2 + 36*a^2*b^4*x)/b^9
```

**maple [A]** time = 0.01, size = 72, normalized size = 0.94

$$\frac{x^3}{3b^3} + \frac{a^5}{2\left(bx+a\right)^2b^6} - \frac{3a\,x^2}{2b^4} - \frac{5a^4}{\left(bx+a\right)b^6} - \frac{10a^3\ln\left(bx+a\right)}{b^6} + \frac{6a^2x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^3,x)
```

```
[Out] 6*a^2*x/b^5-3/2*a*x^2/b^4+1/3*x^3/b^3+1/2*a^5/b^6/(b*x+a)^2-5*a^4/b^6/(b*x+
a)-10*a^3*ln(b*x+a)/b^6
```

**maxima [A]** time = 1.36, size = 81, normalized size = 1.05

$$-\frac{10\,a^4bx+9\,a^5}{2\left(b^8x^2+2\,ab^7x+a^2b^6\right)}-\frac{10\,a^3\log\left(bx+a\right)}{b^6}+\frac{2\,b^2x^3-9\,abx^2+36\,a^2x}{6\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(10*a^4*b*x + 9*a^5)/(b^8*x^2 + 2*a*b^7*x + a^2*b^6) - 10*a^3*log(b*x
+ a)/b^6 + 1/6*(2*b^2*x^3 - 9*a*b*x^2 + 36*a^2*x)/b^5
```

**mupad [B]** time = 0.12, size = 67, normalized size = 0.87

$$-\frac{\frac{5\,a\,{\left(a+b\,x\right)}^2}{2}-\frac{{\left(a+b\,x\right)}^3}{3}+\frac{5\,a^4}{a+b\,x}-\frac{a^5}{2\,{\left(a+b\,x\right)}^2}+10\,a^3\,\ln\left(a+b\,x\right)-10\,a^2\,b\,x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^3,x)
```

```
[Out] -((5*a*(a + b*x)^2)/2 - (a + b*x)^3/3 + (5*a^4)/(a + b*x) - a^5/(2*(a + b*x
)^2) + 10*a^3*log(a + b*x) - 10*a^2*b*x)/b^6
```

**sympy [A]** time = 0.36, size = 85, normalized size = 1.10

$$-\frac{10a^3\log\left(a+bx\right)}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}+\frac{-9a^5-10a^4bx}{2a^2b^6+4ab^7x+2b^8x^2}+\frac{x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**3,x)
```

```
[Out] -10*a**3*log(a + b*x)/b**6 + 6*a**2*x/b**5 - 3*a*x**2/(2*b**4) + (-9*a**5 -
 10*a**4*b*x)/(2*a**2*b**6 + 4*a*b**7*x + 2*b**8*x**2) + x**3/(3*b**3)
```

## 3.183 $\int \frac{x^4}{(a+bx)^3} \, dx$

**Optimal**. Leaf size=64

$$-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{2b^3}$$

**Rubi** [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^3,x]
```

```
[Out] (-3*a*x)/b^4 + x^2/(2*b^3) - a^4/(2*b^5*(a + b*x)^2) + (4*a^3)/(b^5*(a + b*
x)) + (6*a^2*Log[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^3}\,dx &= \int \left(-\frac{3a}{b^4}+\frac{x}{b^3}+\frac{a^4}{b^4(a+bx)^3}-\frac{4a^3}{b^4(a+bx)^2}+\frac{6a^2}{b^4(a+bx)}\right)dx\\ &= -\frac{3ax}{b^4}+\frac{x^2}{2b^3}-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 55, normalized size = 0.86

$$\frac{-\frac{a^4}{(a+bx)^2}+\frac{8a^3}{a+bx}+12a^2\log(a+bx)-6abx+b^2x^2}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^3,x]
```

```
[Out] (-6*a*b*x + b^2*x^2 - a^4/(a + b*x)^2 + (8*a^3)/(a + b*x) + 12*a^2*Log[a +
b*x])/(2*b^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x)^3, x]
```

**fricas [A]** time = 0.88, size = 95, normalized size = 1.48

$$\frac{b^4x^4 - 4\,ab^3x^3 - 11\,a^2b^2x^2 + 2\,a^3bx + 7\,a^4 + 12\left(a^2b^2x^2 + 2\,a^3bx + a^4\right)\log\left(bx + a\right)}{2\left(b^7x^2 + 2\,ab^6x + a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*b
^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

**giac [A]** time = 0.95, size = 61, normalized size = 0.95

$$\frac{6\,a^2\log\left(|bx + a|\right)}{b^5} + \frac{b^3x^2 - 6\,ab^2x}{2\,b^6} + \frac{8\,a^3bx + 7\,a^4}{2\,(bx + a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 6*a^2*log(abs(b*x + a))/b^5 + 1/2*(b^3*x^2 - 6*a*b^2*x)/b^6 + 1/2*(8*a^3*b*
x + 7*a^4)/((b*x + a)^2*b^5)
```

**maple [A]** time = 0.01, size = 61, normalized size = 0.95

$$-\frac{a^4}{2\,(bx + a)^2\,b^5} + \frac{x^2}{2b^3} + \frac{4a^3}{(bx + a)\,b^5} + \frac{6a^2\ln\left(bx + a\right)}{b^5} - \frac{3ax}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^3,x)
```

```
[Out] -3*a*x/b^4+1/2*x^2/b^3-1/2*a^4/b^5/(b*x+a)^2+4*a^3/b^5/(b*x+a)+6*a^2*ln(b*x
+a)/b^5
```

**maxima [A]** time = 1.33, size = 69, normalized size = 1.08

$$\frac{8\,a^3bx + 7\,a^4}{2\left(b^7x^2 + 2\,ab^6x + a^2b^5\right)} + \frac{6\,a^2\log\left(bx + a\right)}{b^5} + \frac{bx^2 - 6\,ax}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a^3*b*x + 7*a^4)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5) + 6*a^2*log(b*x + a
)/b^5 + 1/2*(b*x^2 - 6*a*x)/b^4
```

**mupad [B]** time = 0.08, size = 54, normalized size = 0.84

$$\frac{\frac{(a+b\,x)^2}{2} + \frac{4\,a^3}{a+b\,x} - \frac{a^4}{2\,(a+b\,x)^2} + 6\,a^2\,\ln\left(a + b\,x\right) - 4\,a\,b\,x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^3,x)
```

```
[Out] ((a + b*x)^2/2 + (4*a^3)/(a + b*x) - a^4/(2*(a + b*x)^2) + 6*a^2*log(a + b*
x) - 4*a*b*x)/b^5
```

**sympy [A]** time = 0.34, size = 70, normalized size = 1.09

$$\frac{6a^2\log\left(a + bx\right)}{b^5} - \frac{3ax}{b^4} + \frac{7a^4 + 8a^3bx}{2a^2b^5 + 4ab^6x + 2b^7x^2} + \frac{x^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**3,x)
```

```
[Out] 6*a**2*log(a + b*x)/b**5 - 3*a*x/b**4 + (7*a**4 + 8*a**3*b*x)/(2*a**2*b**5
+ 4*a*b**6*x + 2*b**7*x**2) + x**2/(2*b**3)
```

## 3.184 $\int \frac{x^3}{(a+bx)^3}\,dx$

**Optimal**. Leaf size=50

$$\frac{a^3}{2b^4(a+bx)^2}-\frac{3a^2}{b^4(a+bx)}-\frac{3a\log(a+bx)}{b^4}+\frac{x}{b^3}$$

**Rubi** [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{2b^4(a+bx)^2}-\frac{3a^2}{b^4(a+bx)}-\frac{3a\log(a+bx)}{b^4}+\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^3,x]
```

```
[Out] x/b^3 + a^3/(2*b^4*(a + b*x)^2) - (3*a^2)/(b^4*(a + b*x)) - (3*a*Log[a + b*
x]])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^3}\,dx &= \int \left(\frac{1}{b^3}-\frac{a^3}{b^3(a+bx)^3}+\frac{3a^2}{b^3(a+bx)^2}-\frac{3a}{b^3(a+bx)}\right)dx\\ &= \frac{x}{b^3}+\frac{a^3}{2b^4(a+bx)^2}-\frac{3a^2}{b^4(a+bx)}-\frac{3a\log(a+bx)}{b^4}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 40, normalized size = 0.80

$$-\frac{\frac{a^2(5a+6bx)}{(a+bx)^2}+6a\log(a+bx)-2bx}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^3,x]
```

```
[Out] -1/2*(-2*b*x + (a^2*(5*a + 6*b*x))/(a + b*x)^2 + 6*a*Log[a + b*x])/b^4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x)^3, x]
```

**fricas [A]** time = 0.84, size = 83, normalized size = 1.66

$$\frac{2\,b^3x^3+4\,ab^2x^2-4\,a^2bx-5\,a^3-6\left(ab^2x^2+2\,a^2bx+a^3\right)\log\left(bx+a\right)}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

**giac [A]** time = 0.96, size = 44, normalized size = 0.88

$$\frac{x}{b^3}-\frac{3\,a\log\left(|bx+a|\right)}{b^4}-\frac{6\,a^2bx+5\,a^3}{2\,(bx+a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] x/b^3 - 3*a*log(abs(b*x + a))/b^4 - 1/2*(6*a^2*b*x + 5*a^3)/((b*x + a)^2*b^
4)
```

**maple [A]** time = 0.01, size = 49, normalized size = 0.98

$$\frac{a^3}{2\,(bx+a)^2\,b^4}-\frac{3a^2}{(bx+a)\,b^4}-\frac{3a\ln\left(bx+a\right)}{b^4}+\frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^3,x)
```

```
[Out] x/b^3+1/2*a^3/b^4/(b*x+a)^2-3*a^2/b^4/(b*x+a)-3*a*ln(b*x+a)/b^4
```

**maxima [A]** time = 1.31, size = 57, normalized size = 1.14

$$-\frac{6\,a^2bx+5\,a^3}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)}+\frac{x}{b^3}-\frac{3\,a\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*a^2*b*x + 5*a^3)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) + x/b^3 - 3*a*log(
b*x + a)/b^4
```

**mupad [B]** time = 0.15, size = 43, normalized size = 0.86

$$-\frac{3\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{3\,a^2}{a+b\,x}-\frac{a^3}{2\,{\left(a+b\,x\right)}^2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^3,x)
```

```
[Out] -(3*a*log(a + b*x) - b*x + (3*a^2)/(a + b*x) - a^3/(2*(a + b*x)^2))/b^4
```

**sympy [A]** time = 0.31, size = 58, normalized size = 1.16

$$-\frac{3a\log\left(a+bx\right)}{b^4}+\frac{-5a^3-6a^2bx}{2a^2b^4+4ab^5x+2b^6x^2}+\frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**3,x)
```

```
[Out] -3*a*log(a + b*x)/b**4 + (-5*a**3 - 6*a**2*b*x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + x/b**3
```

## 3.185 $\int \frac{x^2}{(a+bx)^3}\,dx$

**Optimal**. Leaf size=41

$$-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

**Rubi** [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^3,x]
```

```
[Out] -a^2/(2*b^3*(a + b*x)^2) + (2*a)/(b^3*(a + b*x)) + Log[a + b*x]/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^3}\,dx&=\int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx\\&=-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.80

$$\frac{\frac{a(3a+4bx)}{(a+bx)^2}+2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^3,x]
```

```
[Out] ((a*(3*a + 4*b*x))/(a + b*x)^2 + 2*Log[a + b*x])/(2*b^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x)^3, x]
```

**fricas [A]** time = 0.65, size = 61, normalized size = 1.49

$$\frac{4\,abx+3\,a^2+2\left(b^2x^2+2\,abx+a^2\right)\log\left(bx+a\right)}{2\left(b^5x^2+2\,ab^4x+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3)
```

**giac [A]** time = 1.08, size = 37, normalized size = 0.90

$$\frac{\log\left(|bx+a|\right)}{b^3}+\frac{4\,ax+\frac{3\,a^2}{b}}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^3 + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*b^2)
```

**maple [A]** time = 0.01, size = 40, normalized size = 0.98

$$-\frac{a^2}{2\,(bx+a)^2\,b^3}+\frac{2a}{(bx+a)\,b^3}+\frac{\ln\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^3,x)
```

```
[Out] -1/2*a^2/b^3/(b*x+a)^2+2*a/b^3/(b*x+a)+ln(b*x+a)/b^3
```

**maxima [A]** time = 1.34, size = 48, normalized size = 1.17

$$\frac{4\,abx + 3\,a^2}{2\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)} + \frac{\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) + log(b*x + a)/b^3
```

**mupad [B]** time = 0.09, size = 46, normalized size = 1.12

$$\frac{\ln\left(a+b\,x\right)}{b^3}+\frac{\frac{3\,a^2}{2\,b^3}+\frac{2\,a\,x}{b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^3,x)
```

```
[Out] log(a + b*x)/b^3 + ((3*a^2)/(2*b^3) + (2*a*x)/b^2)/(a^2 + b^2*x^2 + 2*a*b*x
)
```

**sympy [A]** time = 0.25, size = 46, normalized size = 1.12

$$\frac{3a^2 + 4abx}{2a^2b^3 + 4ab^4x + 2b^5x^2} + \frac{\log\left(a + bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**3,x)
```

```
[Out] (3*a**2 + 4*a*b*x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + log(a + b*x)/
b**3
```

## 3.186 $\int \frac{x}{(a+bx)^3} dx$

**Optimal**. Leaf size=17

$$\frac{x^2}{2a(a+bx)^2}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {37}

$$\frac{x^2}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^3,x]
```

```
[Out] x^2/(2*a*(a + b*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x}{(a+bx)^3} dx = \frac{x^2}{2a(a+bx)^2}$$

**Mathematica [A]** time = 0.01, size = 20, normalized size = 1.18

$$-\frac{a+2bx}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^3,x]
```

```
[Out] -1/2*(a + 2*b*x)/(b^2*(a + b*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x)^3,x]
[Out] IntegrateAlgebraic[x/(a + b*x)^3, x]
```

**fricas [B]** time = 1.21, size = 32, normalized size = 1.88

$$-\frac{2\,bx+a}{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="fricas")
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

**giac [A]** time = 1.14, size = 18, normalized size = 1.06

$$-\frac{2\,bx+a}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="giac")
[Out] -1/2*(2*b*x + a)/((b*x + a)^2*b^2)
```

**maple [A]** time = 0.00, size = 27, normalized size = 1.59

$$\frac{a}{2\,(bx+a)^2\,b^2}-\frac{1}{(bx+a)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^3,x)
[Out] 1/2*a/b^2/(b*x+a)^2-1/b^2/(b*x+a)
```

**maxima [B]** time = 1.39, size = 32, normalized size = 1.88

$$-\frac{2\,bx+a}{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

**mupad [B]** time = 0.07, size = 32, normalized size = 1.88

$$-\frac{\frac{a}{2\,b^2}+\frac{x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^3,x)
```

```
[Out] -(a/(2*b^2) + x/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

**sympy [B]** time = 0.20, size = 32, normalized size = 1.88

$$\frac{-a-2bx}{2a^2b^2+4ab^3x+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**3,x)
```

```
[Out] (-a - 2*b*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

## 3.187 $\int \frac{1}{(a+bx)^3} \, dx$

**Optimal**. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3),x]
```

```
[Out] -1/(2*b*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3} \, dx = -\frac{1}{2b(a+bx)^2}$$

**Mathematica [A]** time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3),x]
```

```
[Out] -1/2*1/(b*(a + b*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-3), x]
```

**fricas [A]** time = 1.01, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(b^3x^2 + 2\,ab^2x + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

**giac [A]** time = 0.95, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(bx + a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{2\,(bx + a)^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3,x)
```

```
[Out] -1/2/b/(b*x+a)^2
```

**maxima [A]** time = 1.29, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(bx + a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

**mupad [B]** time = 0.07, size = 26, normalized size = 1.86

$$-\frac{1}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^3,x)
```

```
[Out] -1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)
```

**sympy [B]** time = 0.21, size = 26, normalized size = 1.86

$$-\frac{1}{2a^2b+4ab^2x+2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3,x)
```

```
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

## 3.188 $\int \frac{1}{x(a+bx)^3} dx$

**Optimal**. Leaf size=43

$$-\frac{\log(a+bx)}{a^3}+\frac{\log(x)}{a^3}+\frac{1}{a^2(a+bx)}+\frac{1}{2a(a+bx)^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^2(a+bx)}-\frac{\log(a+bx)}{a^3}+\frac{\log(x)}{a^3}+\frac{1}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^3),x]
```

```
[Out] 1/(2*a*(a + b*x)^2) + 1/(a^2*(a + b*x)) + Log[x]/a^3 - Log[a + b*x]/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx\\ &= \frac{1}{2a(a+bx)^2}+\frac{1}{a^2(a+bx)}+\frac{\log(x)}{a^3}-\frac{\log(a+bx)}{a^3}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 37, normalized size = 0.86

$$\frac{\frac{a(3a+2bx)}{(a+bx)^2}-2\log(a+bx)+2\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^3),x]
```

```
[Out] ((a*(3*a + 2*b*x))/(a + b*x)^2 + 2*Log[x] - 2*Log[a + b*x])/(2*a^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^3),x]
[Out] IntegrateAlgebraic[1/(x*(a + b*x)^3), x]
```

**fricas [A]** time = 0.87, size = 80, normalized size = 1.86

$$\frac{2\,abx+3\,a^2-2\left(b^2x^2+2\,abx+a^2\right)\log\left(bx+a\right)+2\left(b^2x^2+2\,abx+a^2\right)\log(x)}{2\left(a^3b^2x^2+2\,a^4bx+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x^
2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

**giac [A]** time = 1.03, size = 43, normalized size = 1.00

$$-\frac{\log\left(|bx+a|\right)}{a^3}+\frac{\log\left(|x|\right)}{a^3}+\frac{2\,abx+3\,a^2}{2\left(bx+a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="giac")
[Out] -log(abs(b*x + a))/a^3 + log(abs(x))/a^3 + 1/2*(2*a*b*x + 3*a^2)/((b*x + a)
^2*a^3)
```

**maple [A]** time = 0.01, size = 42, normalized size = 0.98

$$\frac{1}{2\left(bx+a\right)^2a}+\frac{1}{\left(bx+a\right)a^2}+\frac{\ln(x)}{a^3}-\frac{\ln\left(bx+a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^3,x)
[Out] 1/2/a/(b*x+a)^2+1/a^2/(b*x+a)+ln(x)/a^3-ln(b*x+a)/a^3
```

**maxima** [A] time = 1.35, size = 51, normalized size = 1.19

$$\frac{2\,bx+3\,a}{2\left(a^2b^2x^2+2\,a^3bx+a^4\right)}-\frac{\log\left(bx+a\right)}{a^3}+\frac{\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) - log(b*x + a)/a^3 + log(
x)/a^3
```

**mupad** [B] time = 0.10, size = 43, normalized size = 1.00

$$\frac{\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a}+\frac{1}{2\,a\,\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^3),x)
```

```
[Out] (1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2)
```

**sympy** [A] time = 0.35, size = 46, normalized size = 1.07

$$\frac{3a+2bx}{2a^4+4a^3bx+2a^2b^2x^2}+\frac{\log\left(x\right)-\log\left(\frac{a}{b}+x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**3,x)
```

```
[Out] (3*a + 2*b*x)/(2*a**4 + 4*a**3*b*x + 2*a**2*b**2*x**2) + (log(x) - log(a/b
+ x))/a**3
```

## 3.189 $\int \frac{1}{x^2(a+bx)^3} dx$

**Optimal**. Leaf size=57

$$-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}-\frac{2b}{a^3(a+bx)}-\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}$$

**Rubi [A]** time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{2b}{a^3(a+bx)}-\frac{b}{2a^2(a+bx)^2}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}-\frac{1}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^3),x]
```

```
[Out] -(1/(a^3*x)) - b/(2*a^2*(a + b*x)^2) - (2*b)/(a^3*(a + b*x)) - (3*b*Log[x])
/a^4 + (3*b*Log[a + b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^2}-\frac{3b}{a^4x}+\frac{b^2}{a^2(a+bx)^3}+\frac{2b^2}{a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}-\frac{2b}{a^3(a+bx)}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 53, normalized size = 0.93

$$-\frac{\frac{a\left(2a^2+9abx+6b^2x^2\right)}{x(a+bx)^2}-6b\log(a+bx)+6b\log(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^3),x]
```

```
[Out] -1/2*((a*(2*a^2 + 9*a*b*x + 6*b^2*x^2))/(x*(a + b*x)^2) + 6*b*Log[x] - 6*b*
Log[a + b*x])/a^4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)^3), x]
```

**fricas [A]** time = 1.15, size = 109, normalized size = 1.91

$$-\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3-6\,\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\log\left(bx+a\right)+6\,\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\log(x)}{2\,\left(a^4b^2x^3+2\,a^5bx^2+a^6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3 +
2*a^5*b*x^2 + a^6*x)
```

**giac [A]** time = 1.07, size = 60, normalized size = 1.05

$$\frac{3\,b\log\left(|bx+a|\right)}{a^4}-\frac{3\,b\log\left(|x|\right)}{a^4}-\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3}{2\,(bx+a)^2a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3*b*log(abs(b*x + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^2 + 9*a^2*
b*x + 2*a^3)/((b*x + a)^2*a^4*x)
```

**maple [A]** time = 0.01, size = 56, normalized size = 0.98

$$-\frac{b}{2\,(bx+a)^2\,a^2}-\frac{2b}{(bx+a)\,a^3}-\frac{3b\ln(x)}{a^4}+\frac{3b\ln\left(bx+a\right)}{a^4}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^3,x)
```

```
[Out] -1/a^3/x-1/2*b/a^2/(b*x+a)^2-2*b/a^3/(b*x+a)-3*b*ln(x)/a^4+3*b*ln(b*x+a)/a^
4
```

**maxima [A]** time = 1.32, size = 69, normalized size = 1.21

$$-\frac{6\,b^2x^2+9\,abx+2\,a^2}{2\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}+\frac{3\,b\log\left(bx+a\right)}{a^4}-\frac{3\,b\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b^2*x^2 + 9*a*b*x + 2*a^2)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x) + 3*
b*log(b*x + a)/a^4 - 3*b*log(x)/a^4
```

**mupad [B]** time = 0.11, size = 63, normalized size = 1.11

$$\frac{6\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}-\frac{\frac{1}{a}+\frac{3\,b^2\,x^2}{a^3}+\frac{9\,b\,x}{2\,a^2}}{a^2\,x+2\,a\,b\,x^2+b^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^3),x)
```

```
[Out] (6*b*atanh((2*b*x)/a + 1))/a^4 - (1/a + (3*b^2*x^2)/a^3 + (9*b*x)/(2*a^2))/
(a^2*x + b^2*x^3 + 2*a*b*x^2)
```

**sympy [A]** time = 0.40, size = 66, normalized size = 1.16

$$\frac{-2a^2-9abx-6b^2x^2}{2a^5x+4a^4bx^2+2a^3b^2x^3}+\frac{3b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**3,x)
```

```
[Out] (-2*a**2 - 9*a*b*x - 6*b**2*x**2)/(2*a**5*x + 4*a**4*b*x**2 + 2*a**3*b**2*x
**3) + 3*b*(-log(x) + log(a/b + x))/a**4
```

## 3.190 $\int \frac{1}{x^3(a+bx)^3} dx$

**Optimal**. Leaf size=76

$$\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}+\frac{3b^2}{a^4(a+bx)}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}-\frac{1}{2a^3x^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{3b^2}{a^4(a+bx)}+\frac{b^2}{2a^3(a+bx)^2}+\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}+\frac{3b}{a^4x}-\frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/(2*a^3*x^2) + (3*b)/(a^4*x) + b^2/(2*a^3*(a + b*x)^2) + (3*b^2)/(a^4*(a
+ b*x)) + (6*b^2*Log[x])/a^5 - (6*b^2*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^3}\,dx&=\int \left(\frac{1}{a^3x^3}-\frac{3b}{a^4x^2}+\frac{6b^2}{a^5x}-\frac{b^3}{a^3(a+bx)^3}-\frac{3b^3}{a^4(a+bx)^2}-\frac{6b^3}{a^5(a+bx)}\right)dx\\&=-\frac{1}{2a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}+\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 68, normalized size = 0.89

$$\frac{\frac{a\left(-a^3+4a^2bx+18ab^2x^2+12b^3x^3\right)}{x^2(a+bx)^2}-12b^2\log(a+bx)+12b^2\log(x)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^3),x]
```

```
[Out] ((a*(-a^3 + 4*a^2*b*x + 18*a*b^2*x^2 + 12*b^3*x^3))/(x^2*(a + b*x)^2) + 12*
b^2*Log[x] - 12*b^2*Log[a + b*x])/(2*a^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)^3), x]
```

**fricas [A]** time = 1.18, size = 130, normalized size = 1.71

$$\frac{12\,ab^3x^3+18\,a^2b^2x^2+4\,a^3bx-a^4-12\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\log\left(bx+a\right)+12\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\log(x)}{2\left(a^5b^2x^4+2\,a^6bx^3+a^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b^
3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2
)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

**giac [A]** time = 1.39, size = 73, normalized size = 0.96

$$-\frac{6\,b^2\log\left(|bx+a|\right)}{a^5}+\frac{6\,b^2\log\left(|x|\right)}{a^5}+\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\left(bx^2+ax\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -6*b^2*log(abs(b*x + a))/a^5 + 6*b^2*log(abs(x))/a^5 + 1/2*(12*b^3*x^3 + 18
*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x)^2*a^4)
```

**maple [A]** time = 0.01, size = 73, normalized size = 0.96

$$\frac{b^2}{2\left(bx+a\right)^2a^3}+\frac{3b^2}{\left(bx+a\right)a^4}+\frac{6b^2\ln(x)}{a^5}-\frac{6b^2\ln\left(bx+a\right)}{a^5}+\frac{3b}{a^4x}-\frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^3,x)
```

```
[Out] -1/2/a^3/x^2+3*b/a^4/x+1/2*b^2/a^3/(b*x+a)^2+3*b^2/a^4/(b*x+a)+6*b^2*ln(x)/
a^5-6*b^2*ln(b*x+a)/a^5
```

**maxima [A]** time = 1.37, size = 86, normalized size = 1.13

$$\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\,\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}-\frac{6\,b^2\log\left(bx+a\right)}{a^5}+\frac{6\,b^2\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/(a^4*b^2*x^4 + 2*a^5*b*x^
3 + a^6*x^2) - 6*b^2*log(b*x + a)/a^5 + 6*b^2*log(x)/a^5
```

**mupad [B]** time = 0.12, size = 79, normalized size = 1.04

$$\frac{\frac{9\,b^2\,x^2}{a^3}-\frac{1}{2\,a}+\frac{6\,b^3\,x^3}{a^4}+\frac{2\,b\,x}{a^2}}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4}-\frac{12\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^3),x)
```

```
[Out] ((9*b^2*x^2)/a^3 - 1/(2*a) + (6*b^3*x^3)/a^4 + (2*b*x)/a^2)/(a^2*x^2 + b^2*
x^4 + 2*a*b*x^3) - (12*b^2*atanh((2*b*x)/a + 1))/a^5
```

**sympy [A]** time = 0.41, size = 78, normalized size = 1.03

$$\frac{-a^3+4a^2bx+18ab^2x^2+12b^3x^3}{2a^6x^2+4a^5bx^3+2a^4b^2x^4}+\frac{6b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**3,x)
```

```
[Out] (-a**3 + 4*a**2*b*x + 18*a*b**2*x**2 + 12*b**3*x**3)/(2*a**6*x**2 + 4*a**5*
b*x**3 + 2*a**4*b**2*x**4) + 6*b**2*(log(x) - log(a/b + x))/a**5
```

## 3.191 $\int \frac{1}{x^4(a+bx)^3} \, dx$

**Optimal**. Leaf size=89

$$-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}-\frac{4b^3}{a^5(a+bx)}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}+\frac{3b}{2a^4x^2}-\frac{1}{3a^3x^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{4b^3}{a^5(a+bx)}-\frac{b^3}{2a^4(a+bx)^2}-\frac{6b^2}{a^5x}-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}+\frac{3b}{2a^4x^2}-\frac{1}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^3),x]
```

```
[Out] -1/(3*a^3*x^3) + (3*b)/(2*a^4*x^2) - (6*b^2)/(a^5*x) - b^3/(2*a^4*(a + b*x)
^2) - (4*b^3)/(a^5*(a + b*x)) - (10*b^3*Log[x])/a^6 + (10*b^3*Log[a + b*x])
/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^3} \, dx &= \int \left(\frac{1}{a^3x^4}-\frac{3b}{a^4x^3}+\frac{6b^2}{a^5x^2}-\frac{10b^3}{a^6x}+\frac{b^4}{a^4(a+bx)^3}+\frac{4b^4}{a^5(a+bx)^2}+\frac{10b^4}{a^6(a+bx)}\right) dx\\ &= -\frac{1}{3a^3x^3}+\frac{3b}{2a^4x^2}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}-\frac{4b^3}{a^5(a+bx)}-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 79, normalized size = 0.89

$$-\frac{\frac{a\left(2a^4-5a^3bx+20a^2b^2x^2+90ab^3x^3+60b^4x^4\right)}{x^3(a+bx)^2}-60b^3\log(a+bx)+60b^3\log(x)}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^3),x]
```

```
[Out] -1/6*((a*(2*a^4 - 5*a^3*b*x + 20*a^2*b^2*x^2 + 90*a*b^3*x^3 + 60*b^4*x^4))/
(x^3*(a + b*x)^2) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])/a^6
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)^3), x]
```

**fricas [A]** time = 1.10, size = 141, normalized size = 1.58

$$-\frac{60\,ab^4x^4+90\,a^2b^3x^3+20\,a^3b^2x^2-5\,a^4bx+2\,a^5-60\left(b^5x^5+2\,ab^4x^4+a^2b^3x^3\right)\log\left(bx+a\right)+60\left(b^5x^5+2\,ab^4x^4+a^2b^3x^3\right)\log(x)}{6\left(a^6b^2x^5+2\,a^7bx^4+a^8x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(60*a*b^4*x^4 + 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5 -
60*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*log(b*x + a) + 60*(b^5*x^5 + 2*a*b
^4*x^4 + a^2*b^3*x^3)*log(x))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3)
```

**giac [A]** time = 0.94, size = 86, normalized size = 0.97

$$\frac{10\,b^3\log\left(|bx+a|\right)}{a^6}-\frac{10\,b^3\log\left(|x|\right)}{a^6}-\frac{60\,ab^4x^4+90\,a^2b^3x^3+20\,a^3b^2x^2-5\,a^4bx+2\,a^5}{6\,(bx+a)^2a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 10*b^3*log(abs(b*x + a))/a^6 - 10*b^3*log(abs(x))/a^6 - 1/6*(60*a*b^4*x^4 +
 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5)/((b*x + a)^2*a^6*x^3)
```

**maple [A]** time = 0.01, size = 84, normalized size = 0.94

$$-\frac{b^3}{2\,(bx+a)^2\,a^4}-\frac{4b^3}{(bx+a)\,a^5}-\frac{10b^3\ln(x)}{a^6}+\frac{10b^3\ln\left(bx+a\right)}{a^6}-\frac{6b^2}{a^5x}+\frac{3b}{2a^4x^2}-\frac{1}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^3,x)
```

```
[Out] -1/3/a^3/x^3+3/2*b/a^4/x^2-6*b^2/a^5/x-1/2*b^3/a^4/(b*x+a)^2-4*b^3/a^5/(b*x
+a)-10*b^3*ln(x)/a^6+10*b^3*ln(b*x+a)/a^6
```

**maxima [A]** time = 1.46, size = 97, normalized size = 1.09

$$-\frac{60\,b^{4}x^{4}+90\,ab^{3}x^{3}+20\,a^{2}b^{2}x^{2}-5\,a^{3}bx+2\,a^{4}}{6\left(a^{5}b^{2}x^{5}+2\,a^{6}bx^{4}+a^{7}x^{3}\right)}+\frac{10\,b^{3}\log\left(bx+a\right)}{a^{6}}-\frac{10\,b^{3}\log(x)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(60*b^4*x^4 + 90*a*b^3*x^3 + 20*a^2*b^2*x^2 - 5*a^3*b*x + 2*a^4)/(a^5*
b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3) + 10*b^3*log(b*x + a)/a^6 - 10*b^3*log(x)/
a^6
```

**mupad [B]** time = 0.13, size = 91, normalized size = 1.02

$$\frac{20\,b^{3}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{6}}-\frac{\frac{1}{3\,a}+\frac{10\,b^{2}\,x^{2}}{3\,a^{3}}+\frac{15\,b^{3}\,x^{3}}{a^{4}}+\frac{10\,b^{4}\,x^{4}}{a^{5}}-\frac{5\,b\,x}{6\,a^{2}}}{a^{2}\,x^{3}+2\,a\,b\,x^{4}+b^{2}\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^3),x)
```

```
[Out] (20*b^3*atanh((2*b*x)/a + 1))/a^6 - (1/(3*a) + (10*b^2*x^2)/(3*a^3) + (15*b
^3*x^3)/a^4 + (10*b^4*x^4)/a^5 - (5*b*x)/(6*a^2))/(a^2*x^3 + b^2*x^5 + 2*a*
b*x^4)
```

**sympy [A]** time = 0.48, size = 92, normalized size = 1.03

$$\frac{-2a^{4}+5a^{3}bx-20a^{2}b^{2}x^{2}-90ab^{3}x^{3}-60b^{4}x^{4}}{6a^{7}x^{3}+12a^{6}bx^{4}+6a^{5}b^{2}x^{5}}+\frac{10b^{3}\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**3,x)
```

```
[Out] (-2*a**4 + 5*a**3*b*x - 20*a**2*b**2*x**2 - 90*a*b**3*x**3 - 60*b**4*x**4)/
(6*a**7*x**3 + 12*a**6*b*x**4 + 6*a**5*b**2*x**5) + 10*b**3*(-log(x) + log(
a/b + x))/a**6
```

## 3.192 $\int \frac{1}{x^5(a+bx)^3}\,dx$

**Optimal**. Leaf size=97

$$\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}+\frac{5b^4}{a^6(a+bx)}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}-\frac{3b^2}{a^5x^2}+\frac{b}{a^4x^3}-\frac{1}{4a^3x^4}$$

**Rubi** [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{3b^2}{a^5x^2}+\frac{5b^4}{a^6(a+bx)}+\frac{b^4}{2a^5(a+bx)^2}+\frac{10b^3}{a^6x}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}+\frac{b}{a^4x^3}-\frac{1}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^3),x]
```

```
[Out] -1/(4*a^3*x^4) + b/(a^4*x^3) - (3*b^2)/(a^5*x^2) + (10*b^3)/(a^6*x) + b^4/(
2*a^5*(a + b*x)^2) + (5*b^4)/(a^6*(a + b*x)) + (15*b^4*Log[x])/a^7 - (15*b^
4*Log[a + b*x])/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^5}-\frac{3b}{a^4x^4}+\frac{6b^2}{a^5x^3}-\frac{10b^3}{a^6x^2}+\frac{15b^4}{a^7x}-\frac{b^5}{a^5(a+bx)^3}-\frac{5b^5}{a^6(a+bx)^2}-\frac{15b^5}{a^7(a+bx)}\right)dx\\ &= -\frac{1}{4a^3x^4}+\frac{b}{a^4x^3}-\frac{3b^2}{a^5x^2}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}+\frac{5b^4}{a^6(a+bx)}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 90, normalized size = 0.93

$$\frac{\frac{a\left(-a^5+2a^4bx-5a^3b^2x^2+20a^2b^3x^3+90ab^4x^4+60b^5x^5\right)}{x^4(a+bx)^2}-60b^4\log(a+bx)+60b^4\log(x)}{4a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^3),x]
```

```
[Out] ((a*(-a^5 + 2*a^4*b*x - 5*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 90*a*b^4*x^4 + 60*
b^5*x^5))/(x^4*(a + b*x)^2) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(4*a^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x)^3), x]
```

**fricas [A]** time = 0.95, size = 152, normalized size = 1.57

$$\frac{60\,ab^5x^5+90\,a^2b^4x^4+20\,a^3b^3x^3-5\,a^4b^2x^2+2\,a^5bx-a^6-60\left(b^6x^6+2\,ab^5x^5+a^2b^4x^4\right)\log\left(bx+a\right)+60\left(b^6x^6+2\,ab^5x^5+a^2b^4x^4\right)\log(x)}{4\left(a^7b^2x^6+2\,a^8bx^5+a^9x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(60*a*b^5*x^5 + 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^5
*b*x - a^6 - 60*(b^6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(b*x + a) + 60*(b^
6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(x))/(a^7*b^2*x^6 + 2*a^8*b*x^5 + a^9
*x^4)
```

**giac [A]** time = 1.01, size = 97, normalized size = 1.00

$$-\frac{15\,b^4\log\left(|bx+a|\right)}{a^7}+\frac{15\,b^4\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^5+90\,a^2b^4x^4+20\,a^3b^3x^3-5\,a^4b^2x^2+2\,a^5bx-a^6}{4\,(bx+a)^2a^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15*b^4*log(abs(b*x + a))/a^7 + 15*b^4*log(abs(x))/a^7 + 1/4*(60*a*b^5*x^5
+ 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^5*b*x - a^6)/((b*x
+ a)^2*a^7*x^4)
```

**maple [A]** time = 0.01, size = 94, normalized size = 0.97

$$\frac{b^4}{2\,(bx+a)^2\,a^5}+\frac{5b^4}{(bx+a)\,a^6}+\frac{15b^4\ln(x)}{a^7}-\frac{15b^4\ln\left(bx+a\right)}{a^7}+\frac{10b^3}{a^6x}-\frac{3b^2}{a^5x^2}+\frac{b}{a^4x^3}-\frac{1}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^3,x)
```

```
[Out] -1/4/a^3/x^4+b/a^4/x^3-3*b^2/a^5/x^2+10*b^3/a^6/x+1/2*b^4/a^5/(b*x+a)^2+5*b
^4/a^6/(b*x+a)+15*b^4*ln(x)/a^7-15*b^4*ln(b*x+a)/a^7
```

**maxima [A]** time = 1.35, size = 108, normalized size = 1.11

$$\frac{60\,b^5x^5+90\,ab^4x^4+20\,a^2b^3x^3-5\,a^3b^2x^2+2\,a^4bx-a^5}{4\left(a^6b^2x^6+2\,a^7bx^5+a^8x^4\right)}-\frac{15\,b^4\log\left(bx+a\right)}{a^7}+\frac{15\,b^4\log(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(60*b^5*x^5 + 90*a*b^4*x^4 + 20*a^2*b^3*x^3 - 5*a^3*b^2*x^2 + 2*a^4*b*x
 - a^5)/(a^6*b^2*x^6 + 2*a^7*b*x^5 + a^8*x^4) - 15*b^4*log(b*x + a)/a^7 + 1
5*b^4*log(x)/a^7
```

**mupad [B]** time = 0.09, size = 101, normalized size = 1.04

$$\frac{\frac{5\,b^3\,x^3}{a^4}-\frac{5\,b^2\,x^2}{4\,a^3}-\frac{1}{4\,a}+\frac{45\,b^4\,x^4}{2\,a^5}+\frac{15\,b^5\,x^5}{a^6}+\frac{b\,x}{2\,a^2}}{a^2\,x^4+2\,a\,b\,x^5+b^2\,x^6}-\frac{30\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^3),x)
```

```
[Out] ((5*b^3*x^3)/a^4 - (5*b^2*x^2)/(4*a^3) - 1/(4*a) + (45*b^4*x^4)/(2*a^5) + (
15*b^5*x^5)/a^6 + (b*x)/(2*a^2))/(a^2*x^4 + b^2*x^6 + 2*a*b*x^5) - (30*b^4*
atanh((2*b*x)/a + 1))/a^7
```

**sympy [A]** time = 0.48, size = 102, normalized size = 1.05

$$\frac{-a^5+2a^4bx-5a^3b^2x^2+20a^2b^3x^3+90ab^4x^4+60b^5x^5}{4a^8x^4+8a^7bx^5+4a^6b^2x^6}+\frac{15b^4\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**3,x)
```

```
[Out] (-a**5 + 2*a**4*b*x - 5*a**3*b**2*x**2 + 20*a**2*b**3*x**3 + 90*a*b**4*x**4
 + 60*b**5*x**5)/(4*a**8*x**4 + 8*a**7*b*x**5 + 4*a**6*b**2*x**6) + 15*b**4
*(log(x) - log(a/b + x))/a**7
```

## 3.193 $\int \frac{x^8}{(a+bx)^4}\,dx$

**Optimal**. Leaf size=114

$$-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-\frac{56a^5\log(a+bx)}{b^9}+\frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}$$

**Rubi** [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}+\frac{35a^4x}{b^8}-\frac{56a^5\log(a+bx)}{b^9}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^4,x]
```

```
[Out] (35*a^4*x)/b^8 - (10*a^3*x^2)/b^7 + (10*a^2*x^3)/(3*b^6) - (a*x^4)/b^5 + x^
5/(5*b^4) - a^8/(3*b^9*(a + b*x)^3) + (4*a^7)/(b^9*(a + b*x)^2) - (28*a^6)/
(b^9*(a + b*x)) - (56*a^5*Log[a + b*x])/b^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{(a+bx)^4}\,dx &= \int \left(\frac{35a^4}{b^8}-\frac{20a^3x}{b^7}+\frac{10a^2x^2}{b^6}-\frac{4ax^3}{b^5}+\frac{x^4}{b^4}+\frac{a^8}{b^8(a+bx)^4}-\frac{8a^7}{b^8(a+bx)^3}+\frac{28a^6}{b^8(a+bx)^2}-\frac{56a}{b^8(a+}\right.\\ &= \frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-\frac{56a^5\log}{b}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 101, normalized size = 0.89

$$\frac{-\frac{5a^8}{(a+bx)^3}+\frac{60a^7}{(a+bx)^2}-\frac{420a^6}{a+bx}-840a^5\log(a+bx)+525a^4bx-150a^3b^2x^2+50a^2b^3x^3-15ab^4x^4+3b^5x^5}{15b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^4,x]
```

```
[Out] (525*a^4*b*x - 150*a^3*b^2*x^2 + 50*a^2*b^3*x^3 - 15*a*b^4*x^4 + 3*b^5*x^5
- (5*a^8)/(a + b*x)^3 + (60*a^7)/(a + b*x)^2 - (420*a^6)/(a + b*x) - 840*a^
5*Log[a + b*x])/(15*b^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a + b*x)^4, x]
```

**fricas [A]** time = 0.86, size = 162, normalized size = 1.42

$$\frac{3\,b^{8}x^{8}-6\,ab^{7}x^{7}+14\,a^{2}b^{6}x^{6}-42\,a^{3}b^{5}x^{5}+210\,a^{4}b^{4}x^{4}+1175\,a^{5}b^{3}x^{3}+1005\,a^{6}b^{2}x^{2}-255\,a^{7}bx-365\,a^{8}-840\left(a^{5}b^{3}x^{3}+3\,a^{6}b^{2}x^{2}+3\,a^{7}bx+a^{8}\right)\log\left(bx+a\right)}{15\left(b^{12}x^{3}+3\,ab^{11}x^{2}+3\,a^{2}b^{10}x+a^{3}b^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^8*x^8 - 6*a*b^7*x^7 + 14*a^2*b^6*x^6 - 42*a^3*b^5*x^5 + 210*a^4*b
^4*x^4 + 1175*a^5*b^3*x^3 + 1005*a^6*b^2*x^2 - 255*a^7*b*x - 365*a^8 - 840*
(a^5*b^3*x^3 + 3*a^6*b^2*x^2 + 3*a^7*b*x + a^8)*log(b*x + a))/(b^12*x^3 + 3
*a*b^11*x^2 + 3*a^2*b^10*x + a^3*b^9)
```

**giac [A]** time = 1.11, size = 106, normalized size = 0.93

$$-\frac{56\,a^{5}\log\left(|bx+a|\right)}{b^{9}}-\frac{84\,a^{6}b^{2}x^{2}+156\,a^{7}bx+73\,a^{8}}{3\,(bx+a)^{3}b^{9}}+\frac{3\,b^{16}x^{5}-15\,ab^{15}x^{4}+50\,a^{2}b^{14}x^{3}-150\,a^{3}b^{13}x^{2}+525\,a^{4}b^{12}x}{15\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -56*a^5*log(abs(b*x + a))/b^9 - 1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8)
/((b*x + a)^3*b^9) + 1/15*(3*b^16*x^5 - 15*a*b^15*x^4 + 50*a^2*b^14*x^3 - 1
50*a^3*b^13*x^2 + 525*a^4*b^12*x)/b^20
```

**maple [A]** time = 0.01, size = 109, normalized size = 0.96

$$\frac{x^{5}}{5b^{4}}-\frac{a\,x^{4}}{b^{5}}-\frac{a^{8}}{3\,(bx+a)^{3}\,b^{9}}+\frac{10a^{2}x^{3}}{3b^{6}}+\frac{4a^{7}}{(bx+a)^{2}\,b^{9}}-\frac{10a^{3}x^{2}}{b^{7}}-\frac{28a^{6}}{(bx+a)\,b^{9}}-\frac{56a^{5}\ln\left(bx+a\right)}{b^{9}}+\frac{35a^{4}x}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^4,x)
```

```
[Out] 35*a^4*x/b^8-10*a^3*x^2/b^7+10/3*a^2*x^3/b^6-a*x^4/b^5+1/5*x^5/b^4-1/3*a^8/
b^9/(b*x+a)^3+4*a^7/b^9/(b*x+a)^2-28*a^6/b^9/(b*x+a)-56*a^5*ln(b*x+a)/b^9
```

**maxima [A]** time = 1.44, size = 125, normalized size = 1.10

$$-\frac{84\,a^6b^2x^2+156\,a^7bx+73\,a^8}{3\left(b^{12}x^3+3\,ab^{11}x^2+3\,a^2b^{10}x+a^3b^9\right)}-\frac{56\,a^5\log\left(bx+a\right)}{b^9}+\frac{3\,b^4x^5-15\,ab^3x^4+50\,a^2b^2x^3-150\,a^3bx^2+525\,a^4x}{15\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8)/(b^12*x^3 + 3*a*b^11*x^2 + 3*a
^2*b^10*x + a^3*b^9) - 56*a^5*log(b*x + a)/b^9 + 1/15*(3*b^4*x^5 - 15*a*b^3
*x^4 + 50*a^2*b^2*x^3 - 150*a^3*b*x^2 + 525*a^4*x)/b^8
```

**mupad [B]** time = 0.37, size = 103, normalized size = 0.90

$$-\frac{2\,a\,\left(a+b\,x\right)^4-\frac{\left(a+b\,x\right)^5}{5}-\frac{28\,a^2\,\left(a+b\,x\right)^3}{3}+28\,a^3\,\left(a+b\,x\right)^2+\frac{28\,a^6}{a+b\,x}-\frac{4\,a^7}{\left(a+b\,x\right)^2}+\frac{a^8}{3\,\left(a+b\,x\right)^3}+56\,a^5\,\ln\left(a+b\,x\right)-70\,a^4\,b\,x}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x)^4,x)
```

```
[Out] -(2*a*(a + b*x)^4 - (a + b*x)^5/5 - (28*a^2*(a + b*x)^3)/3 + 28*a^3*(a + b*
x)^2 + (28*a^6)/(a + b*x) - (4*a^7)/(a + b*x)^2 + a^8/(3*(a + b*x)^3) + 56*
a^5*log(a + b*x) - 70*a^4*b*x)/b^9
```

**sympy [A]** time = 0.52, size = 131, normalized size = 1.15

$$-\frac{56a^5\log\left(a+bx\right)}{b^9}+\frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{-73a^8-156a^7bx-84a^6b^2x^2}{3a^3b^9+9a^2b^{10}x+9ab^{11}x^2+3b^{12}x^3}+\frac{x^5}{5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**4,x)
```

```
[Out] -56*a**5*log(a + b*x)/b**9 + 35*a**4*x/b**8 - 10*a**3*x**2/b**7 + 10*a**2*x
**3/(3*b**6) - a*x**4/b**5 + (-73*a**8 - 156*a**7*b*x - 84*a**6*b**2*x**2)/
(3*a**3*b**9 + 9*a**2*b**10*x + 9*a*b**11*x**2 + 3*b**12*x**3) + x**5/(5*b*
*4)
```

## 3.194 $\int \frac{x^7}{(a+bx)^4} dx$

**Optimal**. Leaf size=105

$$\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+bx)}{b^8}-\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}$$

**Rubi** [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5a^2x^2}{b^6}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}-\frac{20a^3x}{b^7}+\frac{35a^4\log(a+bx)}{b^8}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^4,x]
```

```
[Out] (-20*a^3*x)/b^7 + (5*a^2*x^2)/b^6 - (4*a*x^3)/(3*b^5) + x^4/(4*b^4) + a^7/(
3*b^8*(a + b*x)^3) - (7*a^6)/(2*b^8*(a + b*x)^2) + (21*a^5)/(b^8*(a + b*x))
 + (35*a^4*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^7}{(a+bx)^4}dx=\int\left(-\frac{20a^3}{b^7}+\frac{10a^2x}{b^6}-\frac{4ax^2}{b^5}+\frac{x^3}{b^4}-\frac{a^7}{b^7(a+bx)^4}+\frac{7a^6}{b^7(a+bx)^3}-\frac{21a^5}{b^7(a+bx)^2}+\frac{35a^4}{b^7(a+bx)}\right)d$$
$$=-\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+}{b^8}$$

**Mathematica** [A] time = 0.03, size = 90, normalized size = 0.86

$$\frac{\frac{4a^7}{(a+bx)^3}-\frac{42a^6}{(a+bx)^2}+\frac{252a^5}{a+bx}+420a^4\log(a+bx)-240a^3bx+60a^2b^2x^2-16ab^3x^3+3b^4x^4}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^4,x]
```

```
[Out] (-240*a^3*b*x + 60*a^2*b^2*x^2 - 16*a*b^3*x^3 + 3*b^4*x^4 + (4*a^7)/(a + b*
x)^3 - (42*a^6)/(a + b*x)^2 + (252*a^5)/(a + b*x) + 420*a^4*Log[a + b*x])/(
12*b^8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x)^4, x]
```

**fricas [A]** time = 1.04, size = 151, normalized size = 1.44

$$\frac{3\,b^7x^7-7\,ab^6x^6+21\,a^2b^5x^5-105\,a^3b^4x^4-556\,a^4b^3x^3-408\,a^5b^2x^2+222\,a^6bx+214\,a^7+420\left(a^4b^3x^3+3\,a^5b^2x^2+3\,a^6bx+a^7\right)\log\left(bx+a\right)}{12\left(b^{11}x^3+3\,ab^{10}x^2+3\,a^2b^9x+a^3b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*x^7 - 7*a*b^6*x^6 + 21*a^2*b^5*x^5 - 105*a^3*b^4*x^4 - 556*a^4*
b^3*x^3 - 408*a^5*b^2*x^2 + 222*a^6*b*x + 214*a^7 + 420*(a^4*b^3*x^3 + 3*a^
5*b^2*x^2 + 3*a^6*b*x + a^7)*log(b*x + a))/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2
*b^9*x + a^3*b^8)
```

**giac [A]** time = 1.01, size = 95, normalized size = 0.90

$$\frac{35\,a^4\log\left(|bx+a|\right)}{b^8}+\frac{126\,a^5b^2x^2+231\,a^6bx+107\,a^7}{6\,(bx+a)^3b^8}+\frac{3\,b^{12}x^4-16\,ab^{11}x^3+60\,a^2b^{10}x^2-240\,a^3b^9x}{12\,b^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 35*a^4*log(abs(b*x + a))/b^8 + 1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^7
)/((b*x + a)^3*b^8) + 1/12*(3*b^12*x^4 - 16*a*b^11*x^3 + 60*a^2*b^10*x^2 -
240*a^3*b^9*x)/b^16
```

**maple [A]** time = 0.01, size = 98, normalized size = 0.93

$$\frac{x^4}{4b^4}+\frac{a^7}{3\,(bx+a)^3\,b^8}-\frac{4a\,x^3}{3b^5}-\frac{7a^6}{2\,(bx+a)^2\,b^8}+\frac{5a^2x^2}{b^6}+\frac{21a^5}{(bx+a)\,b^8}+\frac{35a^4\ln\left(bx+a\right)}{b^8}-\frac{20a^3x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^4,x)
```

```
[Out] -20*a^3*x/b^7+5*a^2*x^2/b^6-4/3*a*x^3/b^5+1/4*x^4/b^4+1/3*a^7/b^8/(b*x+a)^3
-7/2*a^6/b^8/(b*x+a)^2+21*a^5/b^8/(b*x+a)+35*a^4*ln(b*x+a)/b^8
```

**maxima [A]** time = 1.39, size = 114, normalized size = 1.09

$$\frac{126\,a^5b^2x^2+231\,a^6bx+107\,a^7}{6\left(b^{11}x^3+3\,ab^{10}x^2+3\,a^2b^9x+a^3b^8\right)}+\frac{35\,a^4\log\left(bx+a\right)}{b^8}+\frac{3\,b^3x^4-16\,ab^2x^3+60\,a^2bx^2-240\,a^3x}{12\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^7)/(b^11*x^3 + 3*a*b^10*x^2 + 3*
a^2*b^9*x + a^3*b^8) + 35*a^4*log(b*x + a)/b^8 + 1/12*(3*b^3*x^4 - 16*a*b^2
*x^3 + 60*a^2*b*x^2 - 240*a^3*x)/b^7
```

**mupad [B]** time = 0.22, size = 90, normalized size = 0.86

$$\frac{\frac{(a+b\,x)^4}{4}-\frac{7\,a\,(a+b\,x)^3}{3}+\frac{21\,a^2\,(a+b\,x)^2}{2}+\frac{21\,a^5}{a+b\,x}-\frac{7\,a^6}{2\,(a+b\,x)^2}+\frac{a^7}{3\,(a+b\,x)^3}+35\,a^4\,\ln\left(a+b\,x\right)-35\,a^3\,b\,x}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^4,x)
```

```
[Out] ((a + b*x)^4/4 - (7*a*(a + b*x)^3)/3 + (21*a^2*(a + b*x)^2)/2 + (21*a^5)/(a
 + b*x) - (7*a^6)/(2*(a + b*x)^2) + a^7/(3*(a + b*x)^3) + 35*a^4*log(a + b*
x) - 35*a^3*b*x)/b^8
```

**sympy [A]** time = 0.48, size = 119, normalized size = 1.13

$$\frac{35a^4\log\left(a+bx\right)}{b^8}-\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{107a^7+231a^6bx+126a^5b^2x^2}{6a^3b^8+18a^2b^9x+18ab^{10}x^2+6b^{11}x^3}+\frac{x^4}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**4,x)
```

```
[Out] 35*a**4*log(a + b*x)/b**8 - 20*a**3*x/b**7 + 5*a**2*x**2/b**6 - 4*a*x**3/(3
*b**5) + (107*a**7 + 231*a**6*b*x + 126*a**5*b**2*x**2)/(6*a**3*b**8 + 18*a
**2*b**9*x + 18*a*b**10*x**2 + 6*b**11*x**3) + x**4/(4*b**4)
```

## 3.195 $\int \frac{x^6}{(a+bx)^4} dx$

**Optimal**. Leaf size=90

$$-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}+\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}$$

**Rubi** [A] time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}+\frac{10a^2x}{b^6}-\frac{20a^3\log(a+bx)}{b^7}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^4,x]
```

```
[Out] (10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (3
*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x])/
b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^4}\,dx &= \int \left(\frac{10a^2}{b^6}-\frac{4ax}{b^5}+\frac{x^2}{b^4}+\frac{a^6}{b^6(a+bx)^4}-\frac{6a^5}{b^6(a+bx)^3}+\frac{15a^4}{b^6(a+bx)^2}-\frac{20a^3}{b^6(a+bx)}\right)dx\\ &= \frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 90, normalized size = 1.00

$$-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}+\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^4,x]
```

```
[Out] (10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (3
*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x])/
b^7
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x)^4, x]
```

**fricas [A]** time = 0.97, size = 139, normalized size = 1.54

$$\frac{b^6x^6 - 3\,ab^5x^5 + 15\,a^2b^4x^4 + 73\,a^3b^3x^3 + 39\,a^4b^2x^2 - 51\,a^5bx - 37\,a^6 - 60\left(a^3b^3x^3 + 3\,a^4b^2x^2 + 3\,a^5bx + a^6\right)\log\left(bx+a\right)}{3\left(b^{10}x^3 + 3\,ab^9x^2 + 3\,a^2b^8x + a^3b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^6*x^6 - 3*a*b^5*x^5 + 15*a^2*b^4*x^4 + 73*a^3*b^3*x^3 + 39*a^4*b^2*x
^2 - 51*a^5*b*x - 37*a^6 - 60*(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*x + a^
6)*log(b*x + a))/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2*b^8*x + a^3*b^7)
```

**giac [A]** time = 1.11, size = 83, normalized size = 0.92

$$-\frac{20\,a^3\log\left(|bx+a|\right)}{b^7} - \frac{45\,a^4b^2x^2 + 81\,a^5bx + 37\,a^6}{3\,(bx+a)^3b^7} + \frac{b^8x^3 - 6\,ab^7x^2 + 30\,a^2b^6x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -20*a^3*log(abs(b*x + a))/b^7 - 1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)/
((b*x + a)^3*b^7) + 1/3*(b^8*x^3 - 6*a*b^7*x^2 + 30*a^2*b^6*x)/b^12
```

**maple [A]** time = 0.01, size = 87, normalized size = 0.97

$$-\frac{a^6}{3\,(bx+a)^3\,b^7} + \frac{x^3}{3b^4} + \frac{3a^5}{(bx+a)^2\,b^7} - \frac{2a\,x^2}{b^5} - \frac{15a^4}{(bx+a)\,b^7} - \frac{20a^3\ln\left(bx+a\right)}{b^7} + \frac{10a^2x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^4,x)
```

```
[Out] 10*a^2*x/b^6-2*a*x^2/b^5+1/3*x^3/b^4-1/3*a^6/b^7/(b*x+a)^3+3*a^5/b^7/(b*x+a
)^2-15*a^4/b^7/(b*x+a)-20*a^3*ln(b*x+a)/b^7
```

**maxima** [A] time = 1.42, size = 102, normalized size = 1.13

$$-\frac{45\,a^4b^2x^2+81\,a^5bx+37\,a^6}{3\left(b^{10}x^3+3\,ab^9x^2+3\,a^2b^8x+a^3b^7\right)}-\frac{20\,a^3\log\left(bx+a\right)}{b^7}+\frac{b^2x^3-6\,abx^2+30\,a^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2
*b^8*x + a^3*b^7) - 20*a^3*log(b*x + a)/b^7 + 1/3*(b^2*x^3 - 6*a*b*x^2 + 30
*a^2*x)/b^6
```

**mupad** [B] time = 0.15, size = 79, normalized size = 0.88

$$-\frac{3\,a\,(a+b\,x)^2-\frac{(a+b\,x)^3}{3}+\frac{15\,a^4}{a+b\,x}-\frac{3\,a^5}{(a+b\,x)^2}+\frac{a^6}{3\,(a+b\,x)^3}+20\,a^3\,\ln(a+b\,x)-15\,a^2\,b\,x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^4,x)
```

```
[Out] -(3*a*(a + b*x)^2 - (a + b*x)^3/3 + (15*a^4)/(a + b*x) - (3*a^5)/(a + b*x)^
2 + a^6/(3*(a + b*x)^3) + 20*a^3*log(a + b*x) - 15*a^2*b*x)/b^7
```

**sympy** [A] time = 0.48, size = 107, normalized size = 1.19

$$-\frac{20a^3\log\left(a+bx\right)}{b^7}+\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{-37a^6-81a^5bx-45a^4b^2x^2}{3a^3b^7+9a^2b^8x+9ab^9x^2+3b^{10}x^3}+\frac{x^3}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**4,x)
```

```
[Out] -20*a**3*log(a + b*x)/b**7 + 10*a**2*x/b**6 - 2*a*x**2/b**5 + (-37*a**6 - 8
1*a**5*b*x - 45*a**4*b**2*x**2)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**
2 + 3*b**10*x**3) + x**3/(3*b**4)
```

## 3.196 $\int \frac{x^5}{(a+bx)^4}\,dx$

**Optimal**. Leaf size=81

$$\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}-\frac{4ax}{b^5}+\frac{x^2}{2b^4}$$

**Rubi [A]** time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}-\frac{4ax}{b^5}+\frac{x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^4,x]
```

```
[Out] (-4*a*x)/b^5 + x^2/(2*b^4) + a^5/(3*b^6*(a + b*x)^3) - (5*a^4)/(2*b^6*(a +
b*x)^2) + (10*a^3)/(b^6*(a + b*x)) + (10*a^2*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^4}\,dx &= \int \left(-\frac{4a}{b^5}+\frac{x}{b^4}-\frac{a^5}{b^5(a+bx)^4}+\frac{5a^4}{b^5(a+bx)^3}-\frac{10a^3}{b^5(a+bx)^2}+\frac{10a^2}{b^5(a+bx)}\right)dx\\ &= -\frac{4ax}{b^5}+\frac{x^2}{2b^4}+\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 68, normalized size = 0.84

$$\frac{\frac{2a^5}{(a+bx)^3}-\frac{15a^4}{(a+bx)^2}+\frac{60a^3}{a+bx}+60a^2\log(a+bx)-24abx+3b^2x^2}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^4,x]
```

```
[Out] (-24*a*b*x + 3*b^2*x^2 + (2*a^5)/(a + b*x)^3 - (15*a^4)/(a + b*x)^2 + (60*a
^3)/(a + b*x) + 60*a^2*Log[a + b*x])/(6*b^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x)^4, x]
```

**fricas [A]** time = 0.79, size = 129, normalized size = 1.59

$$\frac{3\,b^5x^5 - 15\,ab^4x^4 - 63\,a^2b^3x^3 - 9\,a^3b^2x^2 + 81\,a^4bx + 47\,a^5 + 60\left(a^2b^3x^3 + 3\,a^3b^2x^2 + 3\,a^4bx + a^5\right)\log\left(bx+a\right)}{6\left(b^9x^3 + 3\,ab^8x^2 + 3\,a^2b^7x + a^3b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*x^5 - 15*a*b^4*x^4 - 63*a^2*b^3*x^3 - 9*a^3*b^2*x^2 + 81*a^4*b*x
 + 47*a^5 + 60*(a^2*b^3*x^3 + 3*a^3*b^2*x^2 + 3*a^4*b*x + a^5)*log(b*x + a)
)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6)
```

**giac [A]** time = 0.87, size = 72, normalized size = 0.89

$$\frac{10\,a^2\log\left(|bx+a|\right)}{b^6} + \frac{b^4x^2 - 8\,ab^3x}{2\,b^8} + \frac{60\,a^3b^2x^2 + 105\,a^4bx + 47\,a^5}{6\,(bx+a)^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 10*a^2*log(abs(b*x + a))/b^6 + 1/2*(b^4*x^2 - 8*a*b^3*x)/b^8 + 1/6*(60*a^3*
b^2*x^2 + 105*a^4*b*x + 47*a^5)/((b*x + a)^3*b^6)
```

**maple [A]** time = 0.01, size = 76, normalized size = 0.94

$$\frac{a^5}{3\left(bx+a\right)^3b^6} - \frac{5a^4}{2\left(bx+a\right)^2b^6} + \frac{x^2}{2b^4} + \frac{10a^3}{\left(bx+a\right)b^6} + \frac{10a^2\ln\left(bx+a\right)}{b^6} - \frac{4ax}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^4,x)
```

```
[Out] -4*a*x/b^5+1/2*x^2/b^4+1/3*a^5/b^6/(b*x+a)^3-5/2*a^4/b^6/(b*x+a)^2+10*a^3/b
^6/(b*x+a)+10*a^2*ln(b*x+a)/b^6
```

**maxima [A]** time = 1.46, size = 91, normalized size = 1.12

$$\frac{60\,a^3b^2x^2 + 105\,a^4bx + 47\,a^5}{6\left(b^9x^3 + 3\,ab^8x^2 + 3\,a^2b^7x + a^3b^6\right)} + \frac{10\,a^2\log\left(bx + a\right)}{b^6} + \frac{bx^2 - 8\,ax}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*a^3*b^2*x^2 + 105*a^4*b*x + 47*a^5)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*
b^7*x + a^3*b^6) + 10*a^2*log(b*x + a)/b^6 + 1/2*(b*x^2 - 8*a*x)/b^5
```

**mupad [B]** time = 0.12, size = 66, normalized size = 0.81

$$\frac{\frac{(a+b\,x)^2}{2} + \frac{10\,a^3}{a+b\,x} - \frac{5\,a^4}{2\,(a+b\,x)^2} + \frac{a^5}{3\,(a+b\,x)^3} + 10\,a^2\,\ln\left(a + b\,x\right) - 5\,a\,b\,x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^4,x)
```

```
[Out] ((a + b*x)^2/2 + (10*a^3)/(a + b*x) - (5*a^4)/(2*(a + b*x)^2) + a^5/(3*(a +
 b*x)^3) + 10*a^2*log(a + b*x) - 5*a*b*x)/b^6
```

**sympy [A]** time = 0.46, size = 94, normalized size = 1.16

$$\frac{10a^2\log\left(a + bx\right)}{b^6} - \frac{4ax}{b^5} + \frac{47a^5 + 105a^4bx + 60a^3b^2x^2}{6a^3b^6 + 18a^2b^7x + 18ab^8x^2 + 6b^9x^3} + \frac{x^2}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**4,x)
```

```
[Out] 10*a**2*log(a + b*x)/b**6 - 4*a*x/b**5 + (47*a**5 + 105*a**4*b*x + 60*a**3*
b**2*x**2)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) +
x**2/(2*b**4)
```

## 3.197 $\int \frac{x^4}{(a+bx)^4} dx$

**Optimal**. Leaf size=65

$$-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}+\frac{x}{b^4}$$

**Rubi** [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}+\frac{x}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^4,x]
```

```
[Out] x/b^4 - a^4/(3*b^5*(a + b*x)^3) + (2*a^3)/(b^5*(a + b*x)^2) - (6*a^2)/(b^5*
(a + b*x)) - (4*a*Log[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^4} dx &= \int \left(\frac{1}{b^4}+\frac{a^4}{b^4(a+bx)^4}-\frac{4a^3}{b^4(a+bx)^3}+\frac{6a^2}{b^4(a+bx)^2}-\frac{4a}{b^4(a+bx)}\right) dx\\ &= \frac{x}{b^4}-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 51, normalized size = 0.78

$$-\frac{\frac{a^2(13a^2+30abx+18b^2x^2)}{(a+bx)^3}+12a\log(a+bx)-3bx}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^4,x]
```

```
[Out] -1/3*(-3*b*x + (a^2*(13*a^2 + 30*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 12*a*Lo
g[a + b*x])/b^5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x)^4, x]
```

**fricas [A]** time = 0.88, size = 116, normalized size = 1.78

$$\frac{3\,b^4x^4+9\,ab^3x^3-9\,a^2b^2x^2-27\,a^3bx-13\,a^4-12\left(ab^3x^3+3\,a^2b^2x^2+3\,a^3bx+a^4\right)\log\left(bx+a\right)}{3\left(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*x^4 + 9*a*b^3*x^3 - 9*a^2*b^2*x^2 - 27*a^3*b*x - 13*a^4 - 12*(a*
b^3*x^3 + 3*a^2*b^2*x^2 + 3*a^3*b*x + a^4)*log(b*x + a))/(b^8*x^3 + 3*a*b^7
*x^2 + 3*a^2*b^6*x + a^3*b^5)
```

**giac [A]** time = 0.86, size = 55, normalized size = 0.85

$$\frac{x}{b^4}-\frac{4\,a\log\left(|bx+a|\right)}{b^5}-\frac{18\,a^2b^2x^2+30\,a^3bx+13\,a^4}{3\left(bx+a\right)^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] x/b^4 - 4*a*log(abs(b*x + a))/b^5 - 1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*a
^4)/((b*x + a)^3*b^5)
```

**maple [A]** time = 0.01, size = 64, normalized size = 0.98

$$-\frac{a^4}{3\left(bx+a\right)^3b^5}+\frac{2a^3}{\left(bx+a\right)^2b^5}-\frac{6a^2}{\left(bx+a\right)b^5}-\frac{4a\ln\left(bx+a\right)}{b^5}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x+a)^4,x)

[Out] x/b^4-1/3*a^4/b^5/(b*x+a)^3+2*a^3/b^5/(b*x+a)^2-6*a^2/b^5/(b*x+a)-4*a*ln(b*x+a)/b^5

**maxima** [A] time = 1.41, size = 79, normalized size = 1.22

$$-\frac{18\,a^2b^2x^2+30\,a^3bx+13\,a^4}{3\left(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5\right)}+\frac{x}{b^4}-\frac{4\,a\log\left(bx+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x+a)^4,x, algorithm="maxima")

[Out] -1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*a^4)/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*b^6*x + a^3*b^5) + x/b^4 - 4*a*log(b*x + a)/b^5

**mupad** [B] time = 0.17, size = 55, normalized size = 0.85

$$-\frac{4\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{6\,a^2}{a+b\,x}-\frac{2\,a^3}{{\left(a+b\,x\right)}^2}+\frac{a^4}{3\,{\left(a+b\,x\right)}^3}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a + b*x)^4,x)

[Out] -(4*a*log(a + b*x) - b*x + (6*a^2)/(a + b*x) - (2*a^3)/(a + b*x)^2 + a^4/(3*(a + b*x)^3))/b^5

**sympy** [A] time = 0.40, size = 82, normalized size = 1.26

$$-\frac{4a\log\left(a+bx\right)}{b^5}+\frac{-13a^4-30a^3bx-18a^2b^2x^2}{3a^3b^5+9a^2b^6x+9ab^7x^2+3b^8x^3}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x+a)**4,x)

[Out] -4*a*log(a + b*x)/b**5 + (-13*a**4 - 30*a**3*b*x - 18*a**2*b**2*x**2)/(3*a**3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + x/b**4

## 3.198 $\int \frac{x^3}{(a+bx)^4} dx$

**Optimal**. Leaf size=58

$$\frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4}$$

**Rubi** [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^4,x]
```

```
[Out] a^3/(3*b^4*(a + b*x)^3) - (3*a^2)/(2*b^4*(a + b*x)^2) + (3*a)/(b^4*(a + b*x
)) + Log[a + b*x]/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a+bx)^4} dx &= \int \left(-\frac{a^3}{b^3(a+bx)^4} + \frac{3a^2}{b^3(a+bx)^3} - \frac{3a}{b^3(a+bx)^2} + \frac{1}{b^3(a+bx)}\right) dx \\ &= \frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 44, normalized size = 0.76

$$\frac{\frac{a(11a^2+27abx+18b^2x^2)}{(a+bx)^3} + 6\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^4,x]
```

```
[Out] ((a*(11*a^2 + 27*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 6*Log[a + b*x])/(6*b^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x)^4, x]
```

**fricas [A]** time = 1.10, size = 94, normalized size = 1.62

$$\frac{18\,ab^2x^2+27\,a^2bx+11\,a^3+6\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\log\left(bx+a\right)}{6\left(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

**giac [A]** time = 1.03, size = 46, normalized size = 0.79

$$\frac{\log\left(|bx+a|\right)}{b^4}+\frac{18\,abx^2+27\,a^2x+\frac{11\,a^3}{b}}{6\left(bx+a\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^4 + 1/6*(18*a*b*x^2 + 27*a^2*x + 11*a^3/b)/((b*x + a)^3
*b^3)
```

**maple [A]** time = 0.01, size = 55, normalized size = 0.95

$$\frac{a^3}{3\left(bx+a\right)^3b^4}-\frac{3a^2}{2\left(bx+a\right)^2b^4}+\frac{3a}{\left(bx+a\right)b^4}+\frac{\ln\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^4,x)
```

[Out] 1/3*a^3/b^4/(b*x+a)^3-3/2*a^2/b^4/(b*x+a)^2+3*a/b^4/(b*x+a)+ln(b*x+a)/b^4

**maxima** [A] time = 1.39, size = 70, normalized size = 1.21

$$\frac{18\,ab^2x^2 + 27\,a^2bx + 11\,a^3}{6\left(b^7x^3 + 3\,ab^6x^2 + 3\,a^2b^5x + a^3b^4\right)} + \frac{\log\left(bx + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x+a)^4,x, algorithm="maxima")

[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3)/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4) + log(b*x + a)/b^4

**mupad** [B] time = 0.07, size = 45, normalized size = 0.78

$$\frac{\ln\left(a + b\,x\right) + \frac{3\,a}{a+b\,x} - \frac{3\,a^2}{2\,\left(a+b\,x\right)^2} + \frac{a^3}{3\,\left(a+b\,x\right)^3}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b*x)^4,x)

[Out] (log(a + b*x) + (3*a)/(a + b*x) - (3*a^2)/(2*(a + b*x)^2) + a^3/(3*(a + b*x)^3))/b^4

**sympy** [A] time = 0.31, size = 70, normalized size = 1.21

$$\frac{11a^3 + 27a^2bx + 18ab^2x^2}{6a^3b^4 + 18a^2b^5x + 18ab^6x^2 + 6b^7x^3} + \frac{\log\left(a + bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x+a)**4,x)

[Out] (11*a**3 + 27*a**2*b*x + 18*a*b**2*x**2)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + log(a + b*x)/b**4

## 3.199 $\int \frac{x^2}{(a+bx)^4} \, dx$

**Optimal**. Leaf size=17

$$\frac{x^3}{3a(a+bx)^3}$$

**Rubi** [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$\frac{x^3}{3a(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^4,x]
```

```
[Out] x^3/(3*a*(a + b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^2}{(a+bx)^4} \, dx = \frac{x^3}{3a(a+bx)^3}$$

**Mathematica** [A] time = 0.01, size = 31, normalized size = 1.82

$$-\frac{a^2+3abx+3b^2x^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^4,x]
```

```
[Out] -1/3*(a^2 + 3*a*b*x + 3*b^2*x^2)/(b^3*(a + b*x)^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^4,x]
[Out] IntegrateAlgebraic[x^2/(a + b*x)^4, x]
```

**fricas [B]** time = 1.04, size = 54, normalized size = 3.18

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="fricas")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

**giac [A]** time = 1.27, size = 29, normalized size = 1.71

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="giac")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^3*b^3)
```

**maple [B]** time = 0.00, size = 41, normalized size = 2.41

$$-\frac{a^2}{3\,(bx+a)^3\,b^3}+\frac{a}{(bx+a)^2\,b^3}-\frac{1}{(bx+a)\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^4,x)
[Out] a/b^3/(b*x+a)^2-1/3*a^2/b^3/(b*x+a)^3-1/b^3/(b*x+a)
```

**maxima [B]** time = 1.40, size = 54, normalized size = 3.18

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

**mupad [B]** time = 0.09, size = 56, normalized size = 3.29

$$-\frac{a^2+3\,a\,b\,x+3\,b^2\,x^2}{3\,a^3\,b^3+9\,a^2\,b^4\,x+9\,a\,b^5\,x^2+3\,b^6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^4,x)
```

```
[Out] -(a^2 + 3*b^2*x^2 + 3*a*b*x)/(3*a^3*b^3 + 3*b^6*x^3 + 9*a^2*b^4*x + 9*a*b^5
*x^2)
```

**sympy [B]** time = 0.30, size = 56, normalized size = 3.29

$$\frac{-a^2-3abx-3b^2x^2}{3a^3b^3+9a^2b^4x+9ab^5x^2+3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**4,x)
```

```
[Out] (-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**
2 + 3*b**6*x**3)
```

## 3.200 $\int \frac{x}{(a+bx)^4} \, dx$

**Optimal**. Leaf size=30

$$\frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^4,x]
```

```
[Out] a/(3*b^2*(a + b*x)^3) - 1/(2*b^2*(a + b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^4} \, dx &= \int \left(-\frac{a}{b(a+bx)^4}+\frac{1}{b(a+bx)^3}\right) dx \\ &= \frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 20, normalized size = 0.67

$$-\frac{a+3bx}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^4,x]
```

```
[Out] -1/6*(a + 3*b*x)/(b^2*(a + b*x)^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx)^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x)^4,x]
[Out] IntegrateAlgebraic[x/(a + b*x)^4, x]
```

**fricas [A]** time = 0.77, size = 43, normalized size = 1.43

$$-\frac{3\,bx+a}{6\,\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="fricas")
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

**giac [A]** time = 1.01, size = 18, normalized size = 0.60

$$-\frac{3\,bx+a}{6\,(bx+a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="giac")
[Out] -1/6*(3*b*x + a)/((b*x + a)^3*b^2)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.90

$$\frac{a}{3\,(bx+a)^3\,b^2}-\frac{1}{2\,(bx+a)^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^4,x)
[Out] 1/3*a/b^2/(b*x+a)^3-1/2/b^2/(b*x+a)^2
```

**maxima [A]** time = 1.37, size = 43, normalized size = 1.43

$$-\frac{3\,bx+a}{6\,\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

**mupad [B]** time = 0.07, size = 44, normalized size = 1.47

$$-\frac{\frac{a}{6\,b^2}+\frac{x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^4,x)
```

```
[Out] -(a/(6*b^2) + x/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

**sympy [A]** time = 0.32, size = 44, normalized size = 1.47

$$\frac{-a-3bx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**4,x)
```

```
[Out] (-a - 3*b*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6*b**5*x**3)
```

## 3.201 $\int \frac{1}{(a+bx)^4}\,dx$

**Optimal**. Leaf size=14

$$-\frac{1}{3b(a+bx)^3}$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4),x]
```

```
[Out] -1/(3*b*(a + b*x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4}\,dx = -\frac{1}{3b(a+bx)^3}$$

**Mathematica [A]** time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4),x]
```

```
[Out] -1/3*1/(b*(a + b*x)^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-4), x]
```

**fricas [B]** time = 1.14, size = 35, normalized size = 2.50

$$-\frac{1}{3\left(b^{4}x^{3}+3\,ab^{3}x^{2}+3\,a^{2}b^{2}x+a^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

**giac [A]** time = 1.02, size = 12, normalized size = 0.86

$$-\frac{1}{3\,(bx+a)^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3/((b*x + a)^3*b)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{3\,(bx+a)^{3}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4,x)
```

```
[Out] -1/3/b/(b*x+a)^3
```

**maxima [A]** time = 1.38, size = 12, normalized size = 0.86

$$-\frac{1}{3\,(bx+a)^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3/((b*x + a)^3*b)
```

**mupad [B]** time = 0.08, size = 37, normalized size = 2.64

$$-\frac{1}{3\,a^3\,b+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2+3\,b^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^4,x)
```

```
[Out] -1/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2)
```

**sympy [B]** time = 0.27, size = 37, normalized size = 2.64

$$-\frac{1}{3a^3b+9a^2b^2x+9ab^3x^2+3b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4,x)
```

```
[Out] -1/(3*a**3*b + 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**4*x**3)
```

## 3.202 $\int \frac{1}{x(a+bx)^4}\,dx$

**Optimal**. Leaf size=57

$$-\frac{\log(a+bx)}{a^4}+\frac{\log(x)}{a^4}+\frac{1}{a^3(a+bx)}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{3a(a+bx)^3}$$

**Rubi** [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^3(a+bx)}+\frac{1}{2a^2(a+bx)^2}-\frac{\log(a+bx)}{a^4}+\frac{\log(x)}{a^4}+\frac{1}{3a(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^4),x]
```

```
[Out] 1/(3*a*(a + b*x)^3) + 1/(2*a^2*(a + b*x)^2) + 1/(a^3*(a + b*x)) + Log[x]/a^
4 - Log[a + b*x]/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x}-\frac{b}{a(a+bx)^4}-\frac{b}{a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{b}{a^4(a+bx)}\right)dx\\ &= \frac{1}{3a(a+bx)^3}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{a^3(a+bx)}+\frac{\log(x)}{a^4}-\frac{\log(a+bx)}{a^4}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 48, normalized size = 0.84

$$\frac{\frac{a\left(11a^2+15abx+6b^2x^2\right)}{(a+bx)^3}-6\log(a+bx)+6\log(x)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^4),x]
```

```
[Out] ((a*(11*a^2 + 15*a*b*x + 6*b^2*x^2))/(a + b*x)^3 + 6*Log[x] - 6*Log[a + b*x
])/(6*a^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^4),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x)^4), x]
```

**fricas [B]** time = 1.07, size = 124, normalized size = 2.18

$$\frac{6\,ab^2x^2+15\,a^2bx+11\,a^3-6\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\log\left(bx+a\right)+6\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\log(x)}{6\left(a^4b^3x^3+3\,a^5b^2x^2+3\,a^6bx+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*b^2*x^2 + 15*a^2*b*x + 11*a^3 - 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b
*x + a^3)*log(b*x + a) + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(x)
)/(a^4*b^3*x^3 + 3*a^5*b^2*x^2 + 3*a^6*b*x + a^7)
```

**giac [A]** time = 1.02, size = 54, normalized size = 0.95

$$-\frac{\log\left(|bx+a|\right)}{a^4}+\frac{\log\left(|x|\right)}{a^4}+\frac{6\,ab^2x^2+15\,a^2bx+11\,a^3}{6\left(bx+a\right)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^4 + log(abs(x))/a^4 + 1/6*(6*a*b^2*x^2 + 15*a^2*b*x +
11*a^3)/((b*x + a)^3*a^4)
```

**maple [A]** time = 0.01, size = 54, normalized size = 0.95

$$\frac{1}{3\left(bx+a\right)^3a}+\frac{1}{2\left(bx+a\right)^2a^2}+\frac{1}{\left(bx+a\right)a^3}+\frac{\ln(x)}{a^4}-\frac{\ln\left(bx+a\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^4,x)
```

```
[Out] 1/3/a/(b*x+a)^3+1/2/a^2/(b*x+a)^2+1/a^3/(b*x+a)+ln(x)/a^4-ln(b*x+a)/a^4
```

**maxima** [A] time = 1.47, size = 73, normalized size = 1.28

$$\frac{6\,b^2x^2+15\,abx+11\,a^2}{6\left(a^3b^3x^3+3\,a^4b^2x^2+3\,a^5bx+a^6\right)}-\frac{\log\left(bx+a\right)}{a^4}+\frac{\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*b^2*x^2 + 15*a*b*x + 11*a^2)/(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*
x + a^6) - log(b*x + a)/a^4 + log(x)/a^4
```

**mupad** [B] time = 0.13, size = 60, normalized size = 1.05

$$\frac{\frac{\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a}+\frac{1}{2\,a\,(a+b\,x)^2}}{a}+\frac{1}{3\,a\,(a+b\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^4),x)
```

```
[Out] ((1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2))/a + 1/(3
*a*(a + b*x)^3)
```

**sympy** [A] time = 0.44, size = 70, normalized size = 1.23

$$\frac{11a^2+15abx+6b^2x^2}{6a^6+18a^5bx+18a^4b^2x^2+6a^3b^3x^3}+\frac{\log\left(x\right)-\log\left(\frac{a}{b}+x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**4,x)
```

```
[Out] (11*a**2 + 15*a*b*x + 6*b**2*x**2)/(6*a**6 + 18*a**5*b*x + 18*a**4*b**2*x**
2 + 6*a**3*b**3*x**3) + (log(x) - log(a/b + x))/a**4
```

## 3.203 $\int \frac{1}{x^2(a+bx)^4}\,dx$

**Optimal**. Leaf size=70

$$-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}-\frac{3b}{a^4(a+bx)}-\frac{1}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{b}{3a^2(a+bx)^3}$$

**Rubi [A]** time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{3b}{a^4(a+bx)}-\frac{b}{a^3(a+bx)^2}-\frac{b}{3a^2(a+bx)^3}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}-\frac{1}{a^4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -(1/(a^4*x)) - b/(3*a^2*(a + b*x)^3) - b/(a^3*(a + b*x)^2) - (3*b)/(a^4*(a
+ b*x)) - (4*b*Log[x])/a^5 + (4*b*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^2}-\frac{4b}{a^5x}+\frac{b^2}{a^2(a+bx)^4}+\frac{2b^2}{a^3(a+bx)^3}+\frac{3b^2}{a^4(a+bx)^2}+\frac{4b^2}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{a^4x}-\frac{b}{3a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{3b}{a^4(a+bx)}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 64, normalized size = 0.91

$$-\frac{\frac{a\left(3a^3+22a^2bx+30ab^2x^2+12b^3x^3\right)}{x(a+bx)^3}-12b\log(a+bx)+12b\log(x)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -1/3*((a*(3*a^3 + 22*a^2*b*x + 30*a*b^2*x^2 + 12*b^3*x^3))/(x*(a + b*x)^3)
+ 12*b*Log[x] - 12*b*Log[a + b*x])/a^5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^4),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)^4), x]
```

**fricas [B]** time = 1.10, size = 153, normalized size = 2.19

$$-\frac{12\,ab^3x^3+30\,a^2b^2x^2+22\,a^3bx+3\,a^4-12\,\left(b^4x^4+3\,ab^3x^3+3\,a^2b^2x^2+a^3bx\right)\log\left(bx+a\right)+12\,\left(b^4x^4+3\,ab^3x^3+3\,a^2b^2x^2+a^3bx\right)\log\left(x\right)}{3\,\left(a^5b^3x^4+3\,a^6b^2x^3+3\,a^7bx^2+a^8x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 30*a^2*b^2*x^2 + 22*a^3*b*x + 3*a^4 - 12*(b^4*x^4 + 3*
a*b^3*x^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(b*x + a) + 12*(b^4*x^4 + 3*a*b^3*x
^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(x))/(a^5*b^3*x^4 + 3*a^6*b^2*x^3 + 3*a^7*
b*x^2 + a^8*x)
```

**giac [A]** time = 1.16, size = 71, normalized size = 1.01

$$\frac{4\,b\log\left(|bx+a|\right)}{a^5}-\frac{4\,b\log\left(|x|\right)}{a^5}-\frac{12\,ab^3x^3+30\,a^2b^2x^2+22\,a^3bx+3\,a^4}{3\,\left(bx+a\right)^3a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 4*b*log(abs(b*x + a))/a^5 - 4*b*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 30*a^
2*b^2*x^2 + 22*a^3*b*x + 3*a^4)/((b*x + a)^3*a^5*x)
```

**maple [A]** time = 0.01, size = 69, normalized size = 0.99

$$-\frac{b}{3\left(bx+a\right)^3a^2}-\frac{b}{\left(bx+a\right)^2a^3}-\frac{3b}{\left(bx+a\right)a^4}-\frac{4b\ln(x)}{a^5}+\frac{4b\ln\left(bx+a\right)}{a^5}-\frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^4,x)
```

```
[Out] -1/a^4/x-1/3*b/a^2/(b*x+a)^3-b/a^3/(b*x+a)^2-3*b/a^4/(b*x+a)-4*b*ln(x)/a^5+
4*b*ln(b*x+a)/a^5
```

**maxima** [A] time = 1.40, size = 91, normalized size = 1.30

$$-\frac{12\,b^3x^3 + 30\,ab^2x^2 + 22\,a^2bx + 3\,a^3}{3\left(a^4b^3x^4 + 3\,a^5b^2x^3 + 3\,a^6bx^2 + a^7x\right)} + \frac{4\,b\log\left(bx + a\right)}{a^5} - \frac{4\,b\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 30*a*b^2*x^2 + 22*a^2*b*x + 3*a^3)/(a^4*b^3*x^4 + 3*a^5*
b^2*x^3 + 3*a^6*b*x^2 + a^7*x) + 4*b*log(b*x + a)/a^5 - 4*b*log(x)/a^5
```

**mupad** [B] time = 0.08, size = 85, normalized size = 1.21

$$\frac{8\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}-\frac{\frac{1}{a}+\frac{10\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}+\frac{22\,b\,x}{3\,a^2}}{a^3\,x+3\,a^2\,b\,x^2+3\,a\,b^2\,x^3+b^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^4),x)
```

```
[Out] (8*b*atanh((2*b*x)/a + 1))/a^5 - (1/a + (10*b^2*x^2)/a^3 + (4*b^3*x^3)/a^4
+ (22*b*x)/(3*a^2))/(a^3*x + b^3*x^4 + 3*a^2*b*x^2 + 3*a*b^2*x^3)
```

**sympy** [A] time = 0.46, size = 90, normalized size = 1.29

$$\frac{-3a^3 - 22a^2bx - 30ab^2x^2 - 12b^3x^3}{3a^7x + 9a^6bx^2 + 9a^5b^2x^3 + 3a^4b^3x^4} + \frac{4b\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**4,x)
```

```
[Out] (-3*a**3 - 22*a**2*b*x - 30*a*b**2*x**2 - 12*b**3*x**3)/(3*a**7*x + 9*a**6*
b*x**2 + 9*a**5*b**2*x**3 + 3*a**4*b**3*x**4) + 4*b*(-log(x) + log(a/b + x)
)/a**5
```

## 3.204 $\int \frac{1}{x^3(a+bx)^4}\,dx$

**Optimal**. Leaf size=93

$$\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}+\frac{6b^2}{a^5(a+bx)}+\frac{4b}{a^5x}+\frac{3b^2}{2a^4(a+bx)^2}-\frac{1}{2a^4x^2}+\frac{b^2}{3a^3(a+bx)^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{6b^2}{a^5(a+bx)}+\frac{3b^2}{2a^4(a+bx)^2}+\frac{b^2}{3a^3(a+bx)^3}+\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}+\frac{4b}{a^5x}-\frac{1}{2a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^4),x]
```

```
[Out] -1/(2*a^4*x^2) + (4*b)/(a^5*x) + b^2/(3*a^3*(a + b*x)^3) + (3*b^2)/(2*a^4*(
a + b*x)^2) + (6*b^2)/(a^5*(a + b*x)) + (10*b^2*Log[x])/a^6 - (10*b^2*Log[a
 + b*x])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^3}-\frac{4b}{a^5x^2}+\frac{10b^2}{a^6x}-\frac{b^3}{a^3(a+bx)^4}-\frac{3b^3}{a^4(a+bx)^3}-\frac{6b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{2a^4x^2}+\frac{4b}{a^5x}+\frac{b^2}{3a^3(a+bx)^3}+\frac{3b^2}{2a^4(a+bx)^2}+\frac{6b^2}{a^5(a+bx)}+\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 79, normalized size = 0.85

$$\frac{\frac{a\left(-3a^4+15a^3bx+110a^2b^2x^2+150ab^3x^3+60b^4x^4\right)}{x^2(a+bx)^3}-60b^2\log(a+bx)+60b^2\log(x)}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^4),x]
```

```
[Out] ((a*(-3*a^4 + 15*a^3*b*x + 110*a^2*b^2*x^2 + 150*a*b^3*x^3 + 60*b^4*x^4))/(
x^2*(a + b*x)^3) + 60*b^2*Log[x] - 60*b^2*Log[a + b*x])/(6*a^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^4),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)^4), x]
```

**fricas [A]** time = 0.85, size = 174, normalized size = 1.87

$$\frac{60\,ab^4x^4+150\,a^2b^3x^3+110\,a^3b^2x^2+15\,a^4bx-3\,a^5-60\left(b^5x^5+3\,ab^4x^4+3\,a^2b^3x^3+a^3b^2x^2\right)\log\left(bx+a\right)+60\left(b^5x^5+3\,ab^4x^4+3\,a^2b^3x^3+a^3b^2x^2\right)\log\left(x\right)}{6\left(a^6b^3x^5+3\,a^7b^2x^4+3\,a^8bx^3+a^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*a*b^4*x^4 + 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5
- 60*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(b*x + a) + 6
0*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(x))/(a^6*b^3*x^
5 + 3*a^7*b^2*x^4 + 3*a^8*b*x^3 + a^9*x^2)
```

**giac [A]** time = 0.95, size = 86, normalized size = 0.92

$$-\frac{10\,b^2\log\left(|bx+a|\right)}{a^6}+\frac{10\,b^2\log\left(|x|\right)}{a^6}+\frac{60\,ab^4x^4+150\,a^2b^3x^3+110\,a^3b^2x^2+15\,a^4bx-3\,a^5}{6\left(bx+a\right)^3a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -10*b^2*log(abs(b*x + a))/a^6 + 10*b^2*log(abs(x))/a^6 + 1/6*(60*a*b^4*x^4
+ 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5)/((b*x + a)^3*a^6*
x^2)
```

**maple [A]** time = 0.01, size = 88, normalized size = 0.95

$$\frac{b^2}{3\left(bx+a\right)^3a^3}+\frac{3b^2}{2\left(bx+a\right)^2a^4}+\frac{6b^2}{\left(bx+a\right)a^5}+\frac{10b^2\ln(x)}{a^6}-\frac{10b^2\ln\left(bx+a\right)}{a^6}+\frac{4b}{a^5x}-\frac{1}{2a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^4,x)
```

```
[Out] -1/2/a^4/x^2+4*b/a^5/x+1/3*b^2/a^3/(b*x+a)^3+3/2*b^2/a^4/(b*x+a)^2+6*b^2/a^
5/(b*x+a)+10*b^2*ln(x)/a^6-10*b^2*ln(b*x+a)/a^6
```

**maxima [A]** time = 1.41, size = 108, normalized size = 1.16

$$\frac{60\,b^4x^4 + 150\,ab^3x^3 + 110\,a^2b^2x^2 + 15\,a^3bx - 3\,a^4}{6\left(a^5b^3x^5 + 3\,a^6b^2x^4 + 3\,a^7bx^3 + a^8x^2\right)} - \frac{10\,b^2\log\left(bx + a\right)}{a^6} + \frac{10\,b^2\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*b^4*x^4 + 150*a*b^3*x^3 + 110*a^2*b^2*x^2 + 15*a^3*b*x - 3*a^4)/(a^
5*b^3*x^5 + 3*a^6*b^2*x^4 + 3*a^7*b*x^3 + a^8*x^2) - 10*b^2*log(b*x + a)/a^
6 + 10*b^2*log(x)/a^6
```

**mupad [B]** time = 0.14, size = 101, normalized size = 1.09

$$\frac{\frac{55\,b^2\,x^2}{3\,a^3} - \frac{1}{2\,a} + \frac{25\,b^3\,x^3}{a^4} + \frac{10\,b^4\,x^4}{a^5} + \frac{5\,b\,x}{2\,a^2}}{a^3\,x^2 + 3\,a^2\,b\,x^3 + 3\,a\,b^2\,x^4 + b^3\,x^5} - \frac{20\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^4),x)
```

```
[Out] ((55*b^2*x^2)/(3*a^3) - 1/(2*a) + (25*b^3*x^3)/a^4 + (10*b^4*x^4)/a^5 + (5*
b*x)/(2*a^2))/(a^3*x^2 + b^3*x^5 + 3*a^2*b*x^3 + 3*a*b^2*x^4) - (20*b^2*ata
nh((2*b*x)/a + 1))/a^6
```

**sympy [A]** time = 0.56, size = 104, normalized size = 1.12

$$\frac{-3a^4 + 15a^3bx + 110a^2b^2x^2 + 150ab^3x^3 + 60b^4x^4}{6a^8x^2 + 18a^7bx^3 + 18a^6b^2x^4 + 6a^5b^3x^5} + \frac{10b^2\left(\log\left(x\right) - \log\left(\frac{a}{b} + x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**4,x)
```

```
[Out] (-3*a**4 + 15*a**3*b*x + 110*a**2*b**2*x**2 + 150*a*b**3*x**3 + 60*b**4*x**
4)/(6*a**8*x**2 + 18*a**7*b*x**3 + 18*a**6*b**2*x**4 + 6*a**5*b**3*x**5) +
10*b**2*(log(x) - log(a/b + x))/a**6
```

## 3.205 $\int \frac{1}{x^4(a+bx)^4} \, dx$

**Optimal**. Leaf size=102

$$-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}-\frac{10b^3}{a^6(a+bx)}-\frac{10b^2}{a^6x}-\frac{2b^3}{a^5(a+bx)^2}+\frac{2b}{a^5x^2}-\frac{b^3}{3a^4(a+bx)^3}-\frac{1}{3a^4x^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{10b^3}{a^6(a+bx)}-\frac{2b^3}{a^5(a+bx)^2}-\frac{b^3}{3a^4(a+bx)^3}-\frac{10b^2}{a^6x}-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}+\frac{2b}{a^5x^2}-\frac{1}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^4),x]
```

```
[Out] -1/(3*a^4*x^3) + (2*b)/(a^5*x^2) - (10*b^2)/(a^6*x) - b^3/(3*a^4*(a + b*x)^
3) - (2*b^3)/(a^5*(a + b*x)^2) - (10*b^3)/(a^6*(a + b*x)) - (20*b^3*Log[x])
/a^7 + (20*b^3*Log[a + b*x])/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^4}-\frac{4b}{a^5x^3}+\frac{10b^2}{a^6x^2}-\frac{20b^3}{a^7x}+\frac{b^4}{a^4(a+bx)^4}+\frac{4b^4}{a^5(a+bx)^3}+\frac{10b^4}{a^6(a+bx)^2}+\frac{20b^4}{a^7(a+bx)}\right) dx\\ &= -\frac{1}{3a^4x^3}+\frac{2b}{a^5x^2}-\frac{10b^2}{a^6x}-\frac{b^3}{3a^4(a+bx)^3}-\frac{2b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 88, normalized size = 0.86

$$-\frac{\frac{a\left(a^5-3a^4bx+15a^3b^2x^2+110a^2b^3x^3+150ab^4x^4+60b^5x^5\right)}{x^3(a+bx)^3}-60b^3\log(a+bx)+60b^3\log(x)}{3a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^4),x]
```

```
[Out] -1/3*((a*(a^5 - 3*a^4*b*x + 15*a^3*b^2*x^2 + 110*a^2*b^3*x^3 + 150*a*b^4*x^
4 + 60*b^5*x^5))/(x^3*(a + b*x)^3) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])/a
^7
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x)^4),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)^4), x]
```

**fricas [A]** time = 0.74, size = 183, normalized size = 1.79

$$-\frac{60\,ab^5x^5+150\,a^2b^4x^4+110\,a^3b^3x^3+15\,a^4b^2x^2-3\,a^5bx+a^6-60\,\left(b^6x^6+3\,ab^5x^5+3\,a^2b^4x^4+a^3b^3x^3\right)\log\left(bx+a\right)+60\,\left(b^6x^6+3\,ab^5x^5+3\,a^2b^4x^4+a^3b^3x^3\right)\log(x)}{3\,\left(a^7b^3x^6+3\,a^8b^2x^5+3\,a^9bx^4+a^{10}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(60*a*b^5*x^5 + 150*a^2*b^4*x^4 + 110*a^3*b^3*x^3 + 15*a^4*b^2*x^2 - 3
*a^5*b*x + a^6 - 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)*l
og(b*x + a) + 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)*log(
x))/(a^7*b^3*x^6 + 3*a^8*b^2*x^5 + 3*a^9*b*x^4 + a^10*x^3)
```

**giac [A]** time = 0.99, size = 93, normalized size = 0.91

$$\frac{20\,b^3\log\left(|bx+a|\right)}{a^7}-\frac{20\,b^3\log\left(|x|\right)}{a^7}-\frac{60\,b^5x^5+150\,ab^4x^4+110\,a^2b^3x^3+15\,a^3b^2x^2-3\,a^4bx+a^5}{3\,\left(bx^2+ax\right)^3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 20*b^3*log(abs(b*x + a))/a^7 - 20*b^3*log(abs(x))/a^7 - 1/3*(60*b^5*x^5 + 1
50*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^4*b*x + a^5)/((b*x^2
+ a*x)^3*a^6)
```

**maple [A]** time = 0.01, size = 99, normalized size = 0.97

$$-\frac{b^3}{3\left(bx+a\right)^3a^4}-\frac{2b^3}{\left(bx+a\right)^2a^5}-\frac{10b^3}{\left(bx+a\right)a^6}-\frac{20b^3\ln(x)}{a^7}+\frac{20b^3\ln\left(bx+a\right)}{a^7}-\frac{10b^2}{a^6x}+\frac{2b}{a^5x^2}-\frac{1}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^4,x)
```

```
[Out] -1/3/a^4/x^3+2*b/a^5/x^2-10*b^2/a^6/x-1/3*b^3/a^4/(b*x+a)^3-2*b^3/a^5/(b*x+
a)^2-10*b^3/a^6/(b*x+a)-20*b^3*ln(x)/a^7+20*b^3*ln(b*x+a)/a^7
```

**maxima [A]** time = 1.40, size = 117, normalized size = 1.15

$$-\frac{60\,b^5x^5+150\,ab^4x^4+110\,a^2b^3x^3+15\,a^3b^2x^2-3\,a^4bx+a^5}{3\left(a^6b^3x^6+3\,a^7b^2x^5+3\,a^8bx^4+a^9x^3\right)}+\frac{20\,b^3\log\left(bx+a\right)}{a^7}-\frac{20\,b^3\log(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(60*b^5*x^5 + 150*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^4
*b*x + a^5)/(a^6*b^3*x^6 + 3*a^7*b^2*x^5 + 3*a^8*b*x^4 + a^9*x^3) + 20*b^3*
log(b*x + a)/a^7 - 20*b^3*log(x)/a^7
```

**mupad [B]** time = 0.10, size = 113, normalized size = 1.11

$$\frac{40\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^7}-\frac{\frac{1}{3\,a}+\frac{5\,b^2\,x^2}{a^3}+\frac{110\,b^3\,x^3}{3\,a^4}+\frac{50\,b^4\,x^4}{a^5}+\frac{20\,b^5\,x^5}{a^6}-\frac{b\,x}{a^2}}{a^3\,x^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^5+b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^4),x)
```

```
[Out] (40*b^3*atanh((2*b*x)/a + 1))/a^7 - (1/(3*a) + (5*b^2*x^2)/a^3 + (110*b^3*x
^3)/(3*a^4) + (50*b^4*x^4)/a^5 + (20*b^5*x^5)/a^6 - (b*x)/a^2)/(a^3*x^3 + b
^3*x^6 + 3*a^2*b*x^4 + 3*a*b^2*x^5)
```

**sympy [A]** time = 0.53, size = 114, normalized size = 1.12

$$\frac{-a^5+3a^4bx-15a^3b^2x^2-110a^2b^3x^3-150ab^4x^4-60b^5x^5}{3a^9x^3+9a^8bx^4+9a^7b^2x^5+3a^6b^3x^6}+\frac{20b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**4,x)
```

```
[Out] (-a**5 + 3*a**4*b*x - 15*a**3*b**2*x**2 - 110*a**2*b**3*x**3 - 150*a*b**4*x
**4 - 60*b**5*x**5)/(3*a**9*x**3 + 9*a**8*b*x**4 + 9*a**7*b**2*x**5 + 3*a**
6*b**3*x**6) + 20*b**3*(-log(x) + log(a/b + x))/a**7
```

## 3.206 $\int \frac{1}{x^5(a+bx)^4} \, dx$

**Optimal**. Leaf size=117

$$\frac{35b^4\log(x)}{a^8} - \frac{35b^4\log(a+bx)}{a^8} + \frac{15b^4}{a^7(a+bx)} + \frac{20b^3}{a^7x} + \frac{5b^4}{2a^6(a+bx)^2} - \frac{5b^2}{a^6x^2} + \frac{b^4}{3a^5(a+bx)^3} + \frac{4b}{3a^5x^3} - \frac{1}{4a^4x^4}$$

**Rubi** [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{5b^2}{a^6x^2} + \frac{15b^4}{a^7(a+bx)} + \frac{5b^4}{2a^6(a+bx)^2} + \frac{b^4}{3a^5(a+bx)^3} + \frac{20b^3}{a^7x} + \frac{35b^4\log(x)}{a^8} - \frac{35b^4\log(a+bx)}{a^8} + \frac{4b}{3a^5x^3} - \frac{1}{4a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^4),x]
```

```
[Out] -1/(4*a^4*x^4) + (4*b)/(3*a^5*x^3) - (5*b^2)/(a^6*x^2) + (20*b^3)/(a^7*x) +
 b^4/(3*a^5*(a + b*x)^3) + (5*b^4)/(2*a^6*(a + b*x)^2) + (15*b^4)/(a^7*(a +
 b*x)) + (35*b^4*Log[x])/a^8 - (35*b^4*Log[a + b*x])/a^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^5(a+bx)^4} \, dx = \int \left(\frac{1}{a^4x^5} - \frac{4b}{a^5x^4} + \frac{10b^2}{a^6x^3} - \frac{20b^3}{a^7x^2} + \frac{35b^4}{a^8x} - \frac{b^5}{a^5(a+bx)^4} - \frac{5b^5}{a^6(a+bx)^3} - \frac{15b^5}{a^7(a+bx)^2} - \frac{35}{a^8(a}\right.$$
$$= -\frac{1}{4a^4x^4} + \frac{4b}{3a^5x^3} - \frac{5b^2}{a^6x^2} + \frac{20b^3}{a^7x} + \frac{b^4}{3a^5(a+bx)^3} + \frac{5b^4}{2a^6(a+bx)^2} + \frac{15b^4}{a^7(a+bx)} + \frac{35b^4\log(x}{a^8}$$

**Mathematica** [A] time = 0.07, size = 101, normalized size = 0.86

$$\frac{\frac{a\left(-3a^6+7a^5bx-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+1050ab^5x^5+420b^6x^6\right)}{x^4(a+bx)^3} - 420b^4\log(a+bx) + 420b^4\log(x)}{12a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^4),x]
```

```
[Out] ((a*(-3*a^6 + 7*a^5*b*x - 21*a^4*b^2*x^2 + 105*a^3*b^3*x^3 + 770*a^2*b^4*x^
4 + 1050*a*b^5*x^5 + 420*b^6*x^6))/(x^4*(a + b*x)^3) + 420*b^4*Log[x] - 420
*b^4*Log[a + b*x])/(12*a^8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x)^4),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x)^4), x]
```

**fricas [A]** time = 1.13, size = 196, normalized size = 1.68

$$\frac{420\,ab^6x^6+1050\,a^2b^5x^5+770\,a^3b^4x^4+105\,a^4b^3x^3-21\,a^5b^2x^2+7\,a^6bx-3\,a^7-420\left(b^7x^7+3\,ab^6x^6+3\,a^2b^5x^5+a^3b^4x^4\right)\log\left(bx+a\right)+420\left(b^7x^7+3\,ab^6x^6+3\,a^2b^5x^5+a^3b^4x^4\right)\log(x)}{12\left(a^8b^3x^7+3\,a^9b^2x^6+3\,a^{10}bx^5+a^{11}x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(420*a*b^6*x^6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3
- 21*a^5*b^2*x^2 + 7*a^6*b*x - 3*a^7 - 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2*b
^5*x^5 + a^3*b^4*x^4)*log(b*x + a) + 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2*b^5
*x^5 + a^3*b^4*x^4)*log(x))/(a^8*b^3*x^7 + 3*a^9*b^2*x^6 + 3*a^10*b*x^5 + a
^11*x^4)
```

**giac [A]** time = 1.00, size = 108, normalized size = 0.92

$$-\frac{35\,b^4\log\left(|bx+a|\right)}{a^8}+\frac{35\,b^4\log\left(|x|\right)}{a^8}+\frac{420\,ab^6x^6+1050\,a^2b^5x^5+770\,a^3b^4x^4+105\,a^4b^3x^3-21\,a^5b^2x^2+7\,a^6bx-3\,a^7}{12\,(bx+a)^3a^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -35*b^4*log(abs(b*x + a))/a^8 + 35*b^4*log(abs(x))/a^8 + 1/12*(420*a*b^6*x^
6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3 - 21*a^5*b^2*x^2 +
 7*a^6*b*x - 3*a^7)/((b*x + a)^3*a^8*x^4)
```

**maple [A]** time = 0.01, size = 110, normalized size = 0.94

$$\frac{b^4}{3\left(bx+a\right)^3a^5}+\frac{5b^4}{2\left(bx+a\right)^2a^6}+\frac{15b^4}{\left(bx+a\right)a^7}+\frac{35b^4\ln(x)}{a^8}-\frac{35b^4\ln\left(bx+a\right)}{a^8}+\frac{20b^3}{a^7x}-\frac{5b^2}{a^6x^2}+\frac{4b}{3a^5x^3}-\frac{1}{4a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^4,x)
```

```
[Out] -1/4/a^4/x^4+4/3*b/a^5/x^3-5*b^2/a^6/x^2+20*b^3/a^7/x+1/3*b^4/a^5/(b*x+a)^3
+5/2*b^4/a^6/(b*x+a)^2+15*b^4/a^7/(b*x+a)+35*b^4*ln(x)/a^8-35*b^4*ln(b*x+a)
/a^8
```

**maxima [A]** time = 1.39, size = 130, normalized size = 1.11

$$\frac{420\,b^6x^6+1050\,ab^5x^5+770\,a^2b^4x^4+105\,a^3b^3x^3-21\,a^4b^2x^2+7\,a^5bx-3\,a^6}{12\left(a^7b^3x^7+3\,a^8b^2x^6+3\,a^9bx^5+a^{10}x^4\right)}-\frac{35\,b^4\log\left(bx+a\right)}{a^8}+\frac{35\,b^4\log(x)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(420*b^6*x^6 + 1050*a*b^5*x^5 + 770*a^2*b^4*x^4 + 105*a^3*b^3*x^3 - 21
*a^4*b^2*x^2 + 7*a^5*b*x - 3*a^6)/(a^7*b^3*x^7 + 3*a^8*b^2*x^6 + 3*a^9*b*x^
5 + a^10*x^4) - 35*b^4*log(b*x + a)/a^8 + 35*b^4*log(x)/a^8
```

**mupad [B]** time = 0.17, size = 123, normalized size = 1.05

$$\frac{\frac{35\,b^3\,x^3}{4\,a^4}-\frac{7\,b^2\,x^2}{4\,a^3}-\frac{1}{4\,a}+\frac{385\,b^4\,x^4}{6\,a^5}+\frac{175\,b^5\,x^5}{2\,a^6}+\frac{35\,b^6\,x^6}{a^7}+\frac{7\,b\,x}{12\,a^2}}{a^3\,x^4+3\,a^2\,b\,x^5+3\,a\,b^2\,x^6+b^3\,x^7}-\frac{70\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^4),x)
```

```
[Out] ((35*b^3*x^3)/(4*a^4) - (7*b^2*x^2)/(4*a^3) - 1/(4*a) + (385*b^4*x^4)/(6*a^
5) + (175*b^5*x^5)/(2*a^6) + (35*b^6*x^6)/a^7 + (7*b*x)/(12*a^2))/(a^3*x^4
+ b^3*x^7 + 3*a^2*b*x^5 + 3*a*b^2*x^6) - (70*b^4*atanh((2*b*x)/a + 1))/a^8
```

**sympy [A]** time = 0.58, size = 128, normalized size = 1.09

$$\frac{-3a^6+7a^5bx-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+1050ab^5x^5+420b^6x^6}{12a^{10}x^4+36a^9bx^5+36a^8b^2x^6+12a^7b^3x^7}+\frac{35b^4\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**4,x)
```

```
[Out] (-3*a**6 + 7*a**5*b*x - 21*a**4*b**2*x**2 + 105*a**3*b**3*x**3 + 770*a**2*b
**4*x**4 + 1050*a*b**5*x**5 + 420*b**6*x**6)/(12*a**10*x**4 + 36*a**9*b*x**
5 + 36*a**8*b**2*x**6 + 12*a**7*b**3*x**7) + 35*b**4*(log(x) - log(a/b + x)
)/a**8
```

## 3.207 $\int \frac{x^{10}}{(a+bx)^7} \, dx$

**Optimal**. Leaf size=150

$$-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\frac{40a^7}{b^{11}(a+bx)^3}-\frac{105a^6}{b^{11}(a+bx)^2}+\frac{252a^5}{b^{11}(a+bx)}+\frac{210a^4\log(a+bx)}{b^{11}}-\frac{84a^3}{b^{10}}$$

**Rubi** [A] time = 0.14, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{14a^2x^2}{b^9}-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\frac{40a^7}{b^{11}(a+bx)^3}-\frac{105a^6}{b^{11}(a+bx)^2}+\frac{252a^5}{b^{11}(a+bx)}-\frac{84a^3x}{b^{10}}+\frac{210a^4\log(a+bx)}{b^{11}}-\frac{7ax^3}{3b^8}+\frac{x^4}{4b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x)^7,x]
```

```
[Out] (-84*a^3*x)/b^10 + (14*a^2*x^2)/b^9 - (7*a*x^3)/(3*b^8) + x^4/(4*b^7) - a^1
0/(6*b^11*(a + b*x)^6) + (2*a^9)/(b^11*(a + b*x)^5) - (45*a^8)/(4*b^11*(a +
 b*x)^4) + (40*a^7)/(b^11*(a + b*x)^3) - (105*a^6)/(b^11*(a + b*x)^2) + (25
2*a^5)/(b^11*(a + b*x)) + (210*a^4*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{10}}{(a+bx)^7}\,dx = \int \left(-\frac{84a^3}{b^{10}}+\frac{28a^2x}{b^9}-\frac{7ax^2}{b^8}+\frac{x^3}{b^7}+\frac{a^{10}}{b^{10}(a+bx)^7}-\frac{10a^9}{b^{10}(a+bx)^6}+\frac{45a^8}{b^{10}(a+bx)^5}-\frac{120a^7}{b^{10}(a+bx)^4}\right.$$

$$=-\frac{84a^3x}{b^{10}}+\frac{14a^2x^2}{b^9}-\frac{7ax^3}{3b^8}+\frac{x^4}{4b^7}-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\frac{40a^7}{b^{11}(a+bx}$$

**Mathematica** [A] time = 0.03, size = 139, normalized size = 0.93

$$\frac{2131a^{10}+10266a^9bx+18105a^8b^2x^2+11540a^7b^3x^3-3945a^6b^4x^4-9138a^5b^5x^5-4043a^4b^6x^6+2520a^4(a+bx)^6\log(a+bx)-360a^3b^7x^7+45a^2b^8x^8-10ab^9x^9+3b^{10}x^{10}}{12b^{11}(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x)^7,x]
```

```
[Out] (2131*a^10 + 10266*a^9*b*x + 18105*a^8*b^2*x^2 + 11540*a^7*b^3*x^3 - 3945*a
^6*b^4*x^4 - 9138*a^5*b^5*x^5 - 4043*a^4*b^6*x^6 - 360*a^3*b^7*x^7 + 45*a^2
*b^8*x^8 - 10*a*b^9*x^9 + 3*b^10*x^10 + 2520*a^4*(a + b*x)^6*Log[a + b*x])/
(12*b^11*(a + b*x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^10/(a + b*x)^7, x]
```

**fricas [A]** time = 1.03, size = 250, normalized size = 1.67

$$\frac{3\,b^{10}x^{10}-10\,ab^{9}x^{9}+45\,a^{2}b^{8}x^{8}-360\,a^{3}b^{7}x^{7}-4043\,a^{4}b^{6}x^{6}-9138\,a^{5}b^{5}x^{5}-3945\,a^{6}b^{4}x^{4}+11540\,a^{7}b^{3}x^{3}+18105\,a^{8}b^{2}x^{2}+10266\,a^{9}bx+2131\,a^{10}+2520\left(a^{4}b^{6}x^{6}+6\,a^{5}b^{5}x^{5}+15\,a^{6}b^{4}x^{4}+20\,a^{7}b^{3}x^{3}+15\,a^{8}b^{2}x^{2}+6\,a^{9}bx+a^{10}\right)\log\left(bx+a\right)}{12\left(b^{17}x^{6}+6\,ab^{16}x^{5}+15\,a^{2}b^{15}x^{4}+20\,a^{3}b^{14}x^{3}+15\,a^{4}b^{13}x^{2}+6\,a^{5}b^{12}x+a^{6}b^{11}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*x^10 - 10*a*b^9*x^9 + 45*a^2*b^8*x^8 - 360*a^3*b^7*x^7 - 4043*
a^4*b^6*x^6 - 9138*a^5*b^5*x^5 - 3945*a^6*b^4*x^4 + 11540*a^7*b^3*x^3 + 181
05*a^8*b^2*x^2 + 10266*a^9*b*x + 2131*a^10 + 2520*(a^4*b^6*x^6 + 6*a^5*b^5*
x^5 + 15*a^6*b^4*x^4 + 20*a^7*b^3*x^3 + 15*a^8*b^2*x^2 + 6*a^9*b*x + a^10)*
log(b*x + a))/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15*x^4 + 20*a^3*b^14*x^3
+ 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11)
```

**giac [A]** time = 1.21, size = 128, normalized size = 0.85

$$\frac{210\,a^{4}\log\left(|bx+a|\right)}{b^{11}}+\frac{3024\,a^{5}b^{5}x^{5}+13860\,a^{6}b^{4}x^{4}+25680\,a^{7}b^{3}x^{3}+23985\,a^{8}b^{2}x^{2}+11274\,a^{9}bx+2131\,a^{10}}{12\left(bx+a\right)^{6}b^{11}}+\frac{3\,b^{21}x^{4}-28\,ab^{20}x^{3}+168\,a^{2}b^{19}x^{2}-1008\,a^{3}b^{18}x}{12\,b^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 210*a^4*log(abs(b*x + a))/b^11 + 1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^4
 + 25680*a^7*b^3*x^3 + 23985*a^8*b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/((b*x
 + a)^6*b^11) + 1/12*(3*b^21*x^4 - 28*a*b^20*x^3 + 168*a^2*b^19*x^2 - 1008*
a^3*b^18*x)/b^28
```

**maple [A]** time = 0.01, size = 143, normalized size = 0.95

$$-\frac{a^{10}}{6\left(bx+a\right)^{6}b^{11}}+\frac{2a^{9}}{\left(bx+a\right)^{5}b^{11}}-\frac{45a^{8}}{4\left(bx+a\right)^{4}b^{11}}+\frac{x^{4}}{4b^{7}}+\frac{40a^{7}}{\left(bx+a\right)^{3}b^{11}}-\frac{7a\,x^{3}}{3b^{8}}-\frac{105a^{6}}{\left(bx+a\right)^{2}b^{11}}+\frac{14a^{2}x^{2}}{b^{9}}+\frac{252a^{5}}{\left(bx+a\right)b^{11}}+\frac{210a^{4}\ln\left(bx+a\right)}{b^{11}}-\frac{84a^{3}x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x+a)^7,x)
```

```
[Out] -84*a^3*x/b^10+14*a^2*x^2/b^9-7/3*a*x^3/b^8+1/4*x^4/b^7-1/6*a^10/b^11/(b*x+
a)^6+2*a^9/b^11/(b*x+a)^5-45/4*a^8/b^11/(b*x+a)^4+40*a^7/b^11/(b*x+a)^3-105
*a^6/b^11/(b*x+a)^2+252*a^5/b^11/(b*x+a)+210*a^4*ln(b*x+a)/b^11
```

**maxima [A]** time = 1.47, size = 180, normalized size = 1.20

$$\frac{3024\,a^5b^5x^5+13860\,a^6b^4x^4+25680\,a^7b^3x^3+23985\,a^8b^2x^2+11274\,a^9bx+2131\,a^{10}}{12\left(b^{17}x^6+6\,ab^{16}x^5+15\,a^2b^{15}x^4+20\,a^3b^{14}x^3+15\,a^4b^{13}x^2+6\,a^5b^{12}x+a^6b^{11}\right)}+\frac{210\,a^4\log\left(bx+a\right)}{b^{11}}+\frac{3\,b^3x^4-28\,ab^2x^3+168\,a^2bx^2-1008\,a^3x}{12\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^4 + 25680*a^7*b^3*x^3 + 23985*a^8*
b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15
*x^4 + 20*a^3*b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11) + 210*a
^4*log(b*x + a)/b^11 + 1/12*(3*b^3*x^4 - 28*a*b^2*x^3 + 168*a^2*b*x^2 - 100
8*a^3*x)/b^10
```

**mupad [B]** time = 1.09, size = 126, normalized size = 0.84

$$\frac{\frac{(a+b\,x)^4}{4}-\frac{10\,a\,(a+b\,x)^3}{3}+\frac{45\,a^2\,(a+b\,x)^2}{2}+\frac{252\,a^5}{a+b\,x}-\frac{105\,a^6}{(a+b\,x)^2}+\frac{40\,a^7}{(a+b\,x)^3}-\frac{45\,a^8}{4\,(a+b\,x)^4}+\frac{2\,a^9}{(a+b\,x)^5}-\frac{a^{10}}{6\,(a+b\,x)^6}+210\,a^4\,\ln\left(a+b\,x\right)-120\,a^3\,b\,x}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^4/4 - (10*a*(a + b*x)^3)/3 + (45*a^2*(a + b*x)^2)/2 + (252*a^5)/
(a + b*x) - (105*a^6)/(a + b*x)^2 + (40*a^7)/(a + b*x)^3 - (45*a^8)/(4*(a +
 b*x)^4) + (2*a^9)/(a + b*x)^5 - a^10/(6*(a + b*x)^6) + 210*a^4*log(a + b*x
) - 120*a^3*b*x)/b^11
```

**sympy [A]** time = 0.93, size = 190, normalized size = 1.27

$$\frac{210a^4\log\left(a+bx\right)}{b^{11}}-\frac{84a^3x}{b^{10}}+\frac{14a^2x^2}{b^9}-\frac{7ax^3}{3b^8}+\frac{2131a^{10}+11274a^9bx+23985a^8b^2x^2+25680a^7b^3x^3+13860a^6b^4x^4+3024a^5b^5x^5}{12a^6b^{11}+72a^5b^{12}x+180a^4b^{13}x^2+240a^3b^{14}x^3+180a^2b^{15}x^4+72ab^{16}x^5+12b^{17}x^6}+\frac{x^4}{4b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x+a)**7,x)
```

```
[Out] 210*a**4*log(a + b*x)/b**11 - 84*a**3*x/b**10 + 14*a**2*x**2/b**9 - 7*a*x**
3/(3*b**8) + (2131*a**10 + 11274*a**9*b*x + 23985*a**8*b**2*x**2 + 25680*a*
*7*b**3*x**3 + 13860*a**6*b**4*x**4 + 3024*a**5*b**5*x**5)/(12*a**6*b**11 +
 72*a**5*b**12*x + 180*a**4*b**13*x**2 + 240*a**3*b**14*x**3 + 180*a**2*b**
15*x**4 + 72*a*b**16*x**5 + 12*b**17*x**6) + x**4/(4*b**7)
```

## 3.208 $\int \frac{x^9}{(a+bx)^7}\,dx$

**Optimal**. Leaf size=139

$$\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\frac{126a^4}{b^{10}(a+bx)}-\frac{84a^3\log(a+bx)}{b^{10}}+\frac{28a^2x}{b^9}$$

**Rubi [A]** time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\frac{126a^4}{b^{10}(a+bx)}+\frac{28a^2x}{b^9}-\frac{84a^3\log(a+bx)}{b^{10}}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x)^7,x]
```

```
[Out] (28*a^2*x)/b^9 - (7*a*x^2)/(2*b^8) + x^3/(3*b^7) + a^9/(6*b^10*(a + b*x)^6)
 - (9*a^8)/(5*b^10*(a + b*x)^5) + (9*a^7)/(b^10*(a + b*x)^4) - (28*a^6)/(b^
10*(a + b*x)^3) + (63*a^5)/(b^10*(a + b*x)^2) - (126*a^4)/(b^10*(a + b*x))
- (84*a^3*Log[a + b*x])/b^10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a+bx)^7}\,dx &= \int \left(\frac{28a^2}{b^9}-\frac{7ax}{b^8}+\frac{x^2}{b^7}-\frac{a^9}{b^9(a+bx)^7}+\frac{9a^8}{b^9(a+bx)^6}-\frac{36a^7}{b^9(a+bx)^5}+\frac{84a^6}{b^9(a+bx)^4}-\frac{126a^5}{b^9(a+bx)^3}\right.\\ &= \frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}+\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63}{b^{10}(a}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 128, normalized size = 0.92

$$-\frac{2509a^9+12534a^8bx+23775a^7b^2x^2+19100a^6b^3x^3+1725a^5b^4x^4-6870a^4b^5x^5-3665a^3b^6x^6+2520a^3(a+bx)^6\log(a+bx)-360a^2b^7x^7+45ab^8x^8-10b^9x^9}{30b^{10}(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x)^7,x]
```

```
[Out] -1/30*(2509*a^9 + 12534*a^8*b*x + 23775*a^7*b^2*x^2 + 19100*a^6*b^3*x^3 + 1
725*a^5*b^4*x^4 - 6870*a^4*b^5*x^5 - 3665*a^3*b^6*x^6 - 360*a^2*b^7*x^7 + 4
5*a*b^8*x^8 - 10*b^9*x^9 + 2520*a^3*(a + b*x)^6*Log[a + b*x])/(b^10*(a + b*
x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^9/(a + b*x)^7, x]
```

**fricas [A]** time = 1.12, size = 239, normalized size = 1.72

$$\frac{10\,b^9x^9-45\,ab^8x^8+360\,a^2b^7x^7+3665\,a^3b^6x^6+6870\,a^4b^5x^5-1725\,a^5b^4x^4-19100\,a^6b^3x^3-23775\,a^7b^2x^2-12534\,a^8bx-2509\,a^9-2520\left(a^3b^6x^6+6\,a^4b^5x^5+15\,a^5b^4x^4+20\,a^6b^3x^3+15\,a^7b^2x^2+6\,a^8bx+a^9\right)\log\left(bx+a\right)}{30\left(b^{16}x^6+6\,ab^{15}x^5+15\,a^2b^{14}x^4+20\,a^3b^{13}x^3+15\,a^4b^{12}x^2+6\,a^5b^{11}x+a^6b^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^9*x^9 - 45*a*b^8*x^8 + 360*a^2*b^7*x^7 + 3665*a^3*b^6*x^6 + 6870
*a^4*b^5*x^5 - 1725*a^5*b^4*x^4 - 19100*a^6*b^3*x^3 - 23775*a^7*b^2*x^2 - 1
2534*a^8*b*x - 2509*a^9 - 2520*(a^3*b^6*x^6 + 6*a^4*b^5*x^5 + 15*a^5*b^4*x^
4 + 20*a^6*b^3*x^3 + 15*a^7*b^2*x^2 + 6*a^8*b*x + a^9)*log(b*x + a))/(b^16*
x^6 + 6*a*b^15*x^5 + 15*a^2*b^14*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x^2 +
6*a^5*b^11*x + a^6*b^10)
```

**giac [A]** time = 1.48, size = 117, normalized size = 0.84

$$-\frac{84\,a^3\log\left(|bx+a|\right)}{b^{10}}-\frac{3780\,a^4b^5x^5+17010\,a^5b^4x^4+31080\,a^6b^3x^3+28710\,a^7b^2x^2+13374\,a^8bx+2509\,a^9}{30\,(bx+a)^6b^{10}}+\frac{2\,b^{14}x^3-21\,ab^{13}x^2+168\,a^2b^{12}x}{6\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -84*a^3*log(abs(b*x + a))/b^10 - 1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^4
 + 31080*a^6*b^3*x^3 + 28710*a^7*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/((b*x
+ a)^6*b^10) + 1/6*(2*b^14*x^3 - 21*a*b^13*x^2 + 168*a^2*b^12*x)/b^21
```

**maple [A]** time = 0.01, size = 132, normalized size = 0.95

$$\frac{a^9}{6\,(bx+a)^6\,b^{10}}-\frac{9a^8}{5\,(bx+a)^5\,b^{10}}+\frac{9a^7}{(bx+a)^4\,b^{10}}-\frac{28a^6}{(bx+a)^3\,b^{10}}+\frac{x^3}{3b^7}+\frac{63a^5}{(bx+a)^2\,b^{10}}-\frac{7a\,x^2}{2b^8}-\frac{126a^4}{(bx+a)\,b^{10}}-\frac{84a^3\ln\left(bx+a\right)}{b^{10}}+\frac{28a^2x}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x+a)^7,x)
```

```
[Out] 28*a^2*x/b^9-7/2*a*x^2/b^8+1/3*x^3/b^7+1/6*a^9/b^10/(b*x+a)^6-9/5*a^8/b^10/
(b*x+a)^5+9*a^7/b^10/(b*x+a)^4-28*a^6/b^10/(b*x+a)^3+63*a^5/b^10/(b*x+a)^2-
126*a^4/b^10/(b*x+a)-84*a^3*ln(b*x+a)/b^10
```

**maxima [A]** time = 1.59, size = 169, normalized size = 1.22

$$-\frac{3780\,a^4b^5x^5+17010\,a^5b^4x^4+31080\,a^6b^3x^3+28710\,a^7b^2x^2+13374\,a^8bx+2509\,a^9}{30\left(b^{16}x^6+6\,ab^{15}x^5+15\,a^2b^{14}x^4+20\,a^3b^{13}x^3+15\,a^4b^{12}x^2+6\,a^5b^{11}x+a^6b^{10}\right)}-\frac{84\,a^3\log\left(bx+a\right)}{b^{10}}+\frac{2\,b^2x^3-21\,abx^2+168\,a^2x}{6\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^4 + 31080*a^6*b^3*x^3 + 28710*a^7
*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/(b^16*x^6 + 6*a*b^15*x^5 + 15*a^2*b^14
*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x^2 + 6*a^5*b^11*x + a^6*b^10) - 84*a^
3*log(b*x + a)/b^10 + 1/6*(2*b^2*x^3 - 21*a*b*x^2 + 168*a^2*x)/b^9
```

**mupad [B]** time = 0.55, size = 115, normalized size = 0.83

$$-\frac{\frac{9\,a\,{\left(a+b\,x\right)}^2}{2}-\frac{{\left(a+b\,x\right)}^3}{3}+\frac{126\,a^4}{a+b\,x}-\frac{63\,a^5}{{\left(a+b\,x\right)}^2}+\frac{28\,a^6}{{\left(a+b\,x\right)}^3}-\frac{9\,a^7}{{\left(a+b\,x\right)}^4}+\frac{9\,a^8}{5\,{\left(a+b\,x\right)}^5}-\frac{a^9}{6\,{\left(a+b\,x\right)}^6}+84\,a^3\,\ln\left(a+b\,x\right)-36\,a^2\,b\,x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x)^7,x)
```

```
[Out] -((9*a*(a + b*x)^2)/2 - (a + b*x)^3/3 + (126*a^4)/(a + b*x) - (63*a^5)/(a +
 b*x)^2 + (28*a^6)/(a + b*x)^3 - (9*a^7)/(a + b*x)^4 + (9*a^8)/(5*(a + b*x)
^5) - a^9/(6*(a + b*x)^6) + 84*a^3*log(a + b*x) - 36*a^2*b*x)/b^10
```

**sympy [A]** time = 0.91, size = 180, normalized size = 1.29

$$-\frac{84a^3\log\left(a+bx\right)}{b^{10}}+\frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{-2509a^9-13374a^8bx-28710a^7b^2x^2-31080a^6b^3x^3-17010a^5b^4x^4-3780a^4b^5x^5}{30a^6b^{10}+180a^5b^{11}x+450a^4b^{12}x^2+600a^3b^{13}x^3+450a^2b^{14}x^4+180ab^{15}x^5+30b^{16}x^6}+\frac{x^3}{3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x+a)**7,x)
```

```
[Out] -84*a**3*log(a + b*x)/b**10 + 28*a**2*x/b**9 - 7*a*x**2/(2*b**8) + (-2509*a
**9 - 13374*a**8*b*x - 28710*a**7*b**2*x**2 - 31080*a**6*b**3*x**3 - 17010*
a**5*b**4*x**4 - 3780*a**4*b**5*x**5)/(30*a**6*b**10 + 180*a**5*b**11*x + 4
50*a**4*b**12*x**2 + 600*a**3*b**13*x**3 + 450*a**2*b**14*x**4 + 180*a*b**1
5*x**5 + 30*b**16*x**6) + x**3/(3*b**7)
```

## 3.209 $\int \frac{x^8}{(a+bx)^7}\,dx$

**Optimal**. Leaf size=128

$$-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+bx)}+\frac{28a^2\log(a+bx)}{b^9}-\frac{7ax}{b^8}+\frac{x^2}{2b^7}$$

**Rubi [A]** time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+bx)}+\frac{28a^2\log(a+bx)}{b^9}-\frac{7ax}{b^8}+\frac{x^2}{2b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^7,x]
```

```
[Out] (-7*a*x)/b^8 + x^2/(2*b^7) - a^8/(6*b^9*(a + b*x)^6) + (8*a^7)/(5*b^9*(a +
b*x)^5) - (7*a^6)/(b^9*(a + b*x)^4) + (56*a^5)/(3*b^9*(a + b*x)^3) - (35*a^
4)/(b^9*(a + b*x)^2) + (56*a^3)/(b^9*(a + b*x)) + (28*a^2*Log[a + b*x])/b^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^8}{(a+bx)^7}\,dx = \int \left(-\frac{7a}{b^8}+\frac{x}{b^7}+\frac{a^8}{b^8(a+bx)^7}-\frac{8a^7}{b^8(a+bx)^6}+\frac{28a^6}{b^8(a+bx)^5}-\frac{56a^5}{b^8(a+bx)^4}+\frac{70a^4}{b^8(a+bx)^3}-\frac{56}{b^8(a}\right.$$

$$= -\frac{7ax}{b^8}+\frac{x^2}{2b^7}-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{5}{b^9(}$$

**Mathematica [A]** time = 0.05, size = 104, normalized size = 0.81

$$\frac{-\frac{5a^8}{(a+bx)^6}+\frac{48a^7}{(a+bx)^5}-\frac{210a^6}{(a+bx)^4}+\frac{560a^5}{(a+bx)^3}-\frac{1050a^4}{(a+bx)^2}+\frac{1680a^3}{a+bx}+840a^2\log(a+bx)-210abx+15b^2x^2}{30b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^7,x]
```

```
[Out] (-210*a*b*x + 15*b^2*x^2 - (5*a^8)/(a + b*x)^6 + (48*a^7)/(a + b*x)^5 - (21
0*a^6)/(a + b*x)^4 + (560*a^5)/(a + b*x)^3 - (1050*a^4)/(a + b*x)^2 + (1680
*a^3)/(a + b*x) + 840*a^2*Log[a + b*x])/(30*b^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a + b*x)^7, x]
```

**fricas [A]** time = 0.93, size = 228, normalized size = 1.78

$$\frac{15\,b^8x^8-120\,ab^7x^7-1035\,a^2b^6x^6-1170\,a^3b^5x^5+3375\,a^4b^4x^4+10100\,a^5b^3x^3+10725\,a^6b^2x^2+5298\,a^7bx+1023\,a^8+840\left(a^2b^6x^6+6\,a^3b^5x^5+15\,a^4b^4x^4+20\,a^5b^3x^3+15\,a^6b^2x^2+6\,a^7bx+a^8\right)\log\left(bx+a\right)}{30\left(b^{15}x^6+6\,ab^{14}x^5+15\,a^2b^{13}x^4+20\,a^3b^{12}x^3+15\,a^4b^{11}x^2+6\,a^5b^{10}x+a^6b^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(15*b^8*x^8 - 120*a*b^7*x^7 - 1035*a^2*b^6*x^6 - 1170*a^3*b^5*x^5 + 33
75*a^4*b^4*x^4 + 10100*a^5*b^3*x^3 + 10725*a^6*b^2*x^2 + 5298*a^7*b*x + 102
3*a^8 + 840*(a^2*b^6*x^6 + 6*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 20*a^5*b^3*x^3
+ 15*a^6*b^2*x^2 + 6*a^7*b*x + a^8)*log(b*x + a))/(b^15*x^6 + 6*a*b^14*x^5
+ 15*a^2*b^13*x^4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x + a^6*
b^9)
```

**giac [A]** time = 1.07, size = 105, normalized size = 0.82

$$\frac{28\,a^2\log\left(|bx+a|\right)}{b^9}+\frac{b^7x^2-14\,ab^6x}{2\,b^{14}}+\frac{1680\,a^3b^5x^5+7350\,a^4b^4x^4+13160\,a^5b^3x^3+11970\,a^6b^2x^2+5508\,a^7bx+1023\,a^8}{30\,(bx+a)^6b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 28*a^2*log(abs(b*x + a))/b^9 + 1/2*(b^7*x^2 - 14*a*b^6*x)/b^14 + 1/30*(1680
*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*b^2*x^2 + 5
508*a^7*b*x + 1023*a^8)/((b*x + a)^6*b^9)
```

**maple [A]** time = 0.01, size = 121, normalized size = 0.95

$$-\frac{a^8}{6\,(bx+a)^6\,b^9}+\frac{8a^7}{5\,(bx+a)^5\,b^9}-\frac{7a^6}{(bx+a)^4\,b^9}+\frac{56a^5}{3\,(bx+a)^3\,b^9}-\frac{35a^4}{(bx+a)^2\,b^9}+\frac{x^2}{2b^7}+\frac{56a^3}{(bx+a)\,b^9}+\frac{28a^2\ln\left(bx+a\right)}{b^9}-\frac{7ax}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^7,x)
```

```
[Out] -7*a*x/b^8+1/2*x^2/b^7-1/6*a^8/b^9/(b*x+a)^6+8/5*a^7/b^9/(b*x+a)^5-7*a^6/b^
9/(b*x+a)^4+56/3*a^5/b^9/(b*x+a)^3-35*a^4/b^9/(b*x+a)^2+56*a^3/b^9/(b*x+a)+
28*a^2*ln(b*x+a)/b^9
```

**maxima [A]** time = 1.56, size = 157, normalized size = 1.23

$$\frac{1680 a^3b^5x^5 + 7350 a^4b^4x^4 + 13160 a^5b^3x^3 + 11970 a^6b^2x^2 + 5508 a^7bx + 1023 a^8}{30 \left(b^{15}x^6 + 6 ab^{14}x^5 + 15 a^2b^{13}x^4 + 20 a^3b^{12}x^3 + 15 a^4b^{11}x^2 + 6 a^5b^{10}x + a^6b^9\right)} + \frac{28 a^2 \log (bx + a)}{b^9} + \frac{bx^2 - 14 ax}{2 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/30*(1680*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*b
^2*x^2 + 5508*a^7*b*x + 1023*a^8)/(b^15*x^6 + 6*a*b^14*x^5 + 15*a^2*b^13*x^
4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x + a^6*b^9) + 28*a^2*lo
g(b*x + a)/b^9 + 1/2*(b*x^2 - 14*a*x)/b^8
```

**mupad [B]** time = 0.18, size = 102, normalized size = 0.80

$$\frac{\frac{(a+b\,x)^2}{2} + \frac{56\,a^3}{a+b\,x} - \frac{35\,a^4}{(a+b\,x)^2} + \frac{56\,a^5}{3\,(a+b\,x)^3} - \frac{7\,a^6}{(a+b\,x)^4} + \frac{8\,a^7}{5\,(a+b\,x)^5} - \frac{a^8}{6\,(a+b\,x)^6} + 28\,a^2\,\ln(a+b\,x) - 8\,a\,b\,x}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^2/2 + (56*a^3)/(a + b*x) - (35*a^4)/(a + b*x)^2 + (56*a^5)/(3*(a
 + b*x)^3) - (7*a^6)/(a + b*x)^4 + (8*a^7)/(5*(a + b*x)^5) - a^8/(6*(a + b*
x)^6) + 28*a^2*log(a + b*x) - 8*a*b*x)/b^9
```

**sympy [A]** time = 0.84, size = 165, normalized size = 1.29

$$\frac{28a^2 \log (a + bx)}{b^9} - \frac{7ax}{b^8} + \frac{1023a^8 + 5508a^7bx + 11970a^6b^2x^2 + 13160a^5b^3x^3 + 7350a^4b^4x^4 + 1680a^3b^5x^5}{30a^6b^9 + 180a^5b^{10}x + 450a^4b^{11}x^2 + 600a^3b^{12}x^3 + 450a^2b^{13}x^4 + 180ab^{14}x^5 + 30b^{15}x^6} + \frac{x^2}{2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**7,x)
```

```
[Out] 28*a**2*log(a + b*x)/b**9 - 7*a*x/b**8 + (1023*a**8 + 5508*a**7*b*x + 11970
*a**6*b**2*x**2 + 13160*a**5*b**3*x**3 + 7350*a**4*b**4*x**4 + 1680*a**3*b*
*5*x**5)/(30*a**6*b**9 + 180*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a**3*
b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**14*x**5 + 30*b**15*x**6) + x**2
/(2*b**7)
```

## 3.210 $\int \frac{x^7}{(a+bx)^7} dx$

**Optimal**. Leaf size=118

$$\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log(a+bx)}{b^8}+\frac{x}{b^7}$$

**Rubi** [A] time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log(a+bx)}{b^8}+\frac{x}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^7,x]
```

```
[Out] x/b^7 + a^7/(6*b^8*(a + b*x)^6) - (7*a^6)/(5*b^8*(a + b*x)^5) + (21*a^5)/(4
*b^8*(a + b*x)^4) - (35*a^4)/(3*b^8*(a + b*x)^3) + (35*a^3)/(2*b^8*(a + b*x
)^2) - (21*a^2)/(b^8*(a + b*x)) - (7*a*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^7}{(a+bx)^7} dx = \int \left(\frac{1}{b^7}-\frac{a^7}{b^7(a+bx)^7}+\frac{7a^6}{b^7(a+bx)^6}-\frac{21a^5}{b^7(a+bx)^5}+\frac{35a^4}{b^7(a+bx)^4}-\frac{35a^3}{b^7(a+bx)^3}+\frac{21a^2}{b^7(a+bx)^2}$$
$$=\frac{x}{b^7}+\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx}$$

**Mathematica** [A] time = 0.03, size = 104, normalized size = 0.88

$$-\frac{669a^7+3594a^6bx+7725a^5b^2x^2+8200a^4b^3x^3+4050a^3b^4x^4+360a^2b^5x^5-360ab^6x^6+420a(a+bx)^6\log(a+bx)-60b^7x^7}{60b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^7,x]
```

```
[Out] -1/60*(669*a^7 + 3594*a^6*b*x + 7725*a^5*b^2*x^2 + 8200*a^4*b^3*x^3 + 4050*
a^3*b^4*x^4 + 360*a^2*b^5*x^5 - 360*a*b^6*x^6 - 60*b^7*x^7 + 420*a*(a + b*x
)^6*Log[a + b*x])/(b^8*(a + b*x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x)^7, x]
```

**fricas [A]** time = 1.03, size = 215, normalized size = 1.82

$$\frac{60\,b^7x^7+360\,ab^6x^6-360\,a^2b^5x^5-4050\,a^3b^4x^4-8200\,a^4b^3x^3-7725\,a^5b^2x^2-3594\,a^6bx-669\,a^7-420\left(ab^6x^6+6\,a^2b^5x^5+15\,a^3b^4x^4+20\,a^4b^3x^3+15\,a^5b^2x^2+6\,a^6bx+a^7\right)\log\left(bx+a\right)}{60\left(b^{14}x^6+6\,ab^{13}x^5+15\,a^2b^{12}x^4+20\,a^3b^{11}x^3+15\,a^4b^{10}x^2+6\,a^5b^9x+a^6b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*x^7 + 360*a*b^6*x^6 - 360*a^2*b^5*x^5 - 4050*a^3*b^4*x^4 - 820
0*a^4*b^3*x^3 - 7725*a^5*b^2*x^2 - 3594*a^6*b*x - 669*a^7 - 420*(a*b^6*x^6
+ 6*a^2*b^5*x^5 + 15*a^3*b^4*x^4 + 20*a^4*b^3*x^3 + 15*a^5*b^2*x^2 + 6*a^6*
b*x + a^7)*log(b*x + a))/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^4 + 20*a^
3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8)
```

**giac [A]** time = 0.92, size = 88, normalized size = 0.75

$$\frac{x}{b^7}-\frac{7\,a\log\left(|bx+a|\right)}{b^8}-\frac{1260\,a^2b^5x^5+5250\,a^3b^4x^4+9100\,a^4b^3x^3+8085\,a^5b^2x^2+3654\,a^6bx+669\,a^7}{60\left(bx+a\right)^6b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] x/b^7 - 7*a*log(abs(b*x + a))/b^8 - 1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*x
^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b^2*x^2 + 3654*a^6*b*x + 669*a^7)/((b*x +
a)^6*b^8)
```

**maple [A]** time = 0.01, size = 109, normalized size = 0.92

$$\frac{a^7}{6\left(bx+a\right)^6b^8}-\frac{7a^6}{5\left(bx+a\right)^5b^8}+\frac{21a^5}{4\left(bx+a\right)^4b^8}-\frac{35a^4}{3\left(bx+a\right)^3b^8}+\frac{35a^3}{2\left(bx+a\right)^2b^8}-\frac{21a^2}{\left(bx+a\right)b^8}-\frac{7a\ln\left(bx+a\right)}{b^8}+\frac{x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^7,x)
```

```
[Out] x/b^7+1/6*a^7/b^8/(b*x+a)^6-7/5*a^6/b^8/(b*x+a)^5+21/4*a^5/b^8/(b*x+a)^4-35
/3*a^4/b^8/(b*x+a)^3+35/2*a^3/b^8/(b*x+a)^2-21*a^2/b^8/(b*x+a)-7*a*ln(b*x+a
)/b^8
```

**maxima [A]** time = 1.49, size = 145, normalized size = 1.23

$$-\frac{1260\,a^{2}b^{5}x^{5}+5250\,a^{3}b^{4}x^{4}+9100\,a^{4}b^{3}x^{3}+8085\,a^{5}b^{2}x^{2}+3654\,a^{6}bx+669\,a^{7}}{60\left(b^{14}x^{6}+6\,ab^{13}x^{5}+15\,a^{2}b^{12}x^{4}+20\,a^{3}b^{11}x^{3}+15\,a^{4}b^{10}x^{2}+6\,a^{5}b^{9}x+a^{6}b^{8}\right)}+\frac{x}{b^{7}}-\frac{7\,a\log\left(bx+a\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*x^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b^
2*x^2 + 3654*a^6*b*x + 669*a^7)/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^4
+ 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8) + x/b^7 - 7*a*
log(b*x + a)/b^8
```

**mupad [B]** time = 0.34, size = 91, normalized size = 0.77

$$-\frac{7\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{21\,a^{2}}{a+b\,x}-\frac{35\,a^{3}}{2\,{\left(a+b\,x\right)}^{2}}+\frac{35\,a^{4}}{3\,{\left(a+b\,x\right)}^{3}}-\frac{21\,a^{5}}{4\,{\left(a+b\,x\right)}^{4}}+\frac{7\,a^{6}}{5\,{\left(a+b\,x\right)}^{5}}-\frac{a^{7}}{6\,{\left(a+b\,x\right)}^{6}}}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^7,x)
```

```
[Out] -(7*a*log(a + b*x) - b*x + (21*a^2)/(a + b*x) - (35*a^3)/(2*(a + b*x)^2) +
(35*a^4)/(3*(a + b*x)^3) - (21*a^5)/(4*(a + b*x)^4) + (7*a^6)/(5*(a + b*x)^
5) - a^7/(6*(a + b*x)^6))/b^8
```

**sympy [A]** time = 0.82, size = 153, normalized size = 1.30

$$-\frac{7a\log\left(a+bx\right)}{b^{8}}+\frac{-669a^{7}-3654a^{6}bx-8085a^{5}b^{2}x^{2}-9100a^{4}b^{3}x^{3}-5250a^{3}b^{4}x^{4}-1260a^{2}b^{5}x^{5}}{60a^{6}b^{8}+360a^{5}b^{9}x+900a^{4}b^{10}x^{2}+1200a^{3}b^{11}x^{3}+900a^{2}b^{12}x^{4}+360ab^{13}x^{5}+60b^{14}x^{6}}+\frac{x}{b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**7,x)
```

```
[Out] -7*a*log(a + b*x)/b**8 + (-669*a**7 - 3654*a**6*b*x - 8085*a**5*b**2*x**2 -
 9100*a**4*b**3*x**3 - 5250*a**3*b**4*x**4 - 1260*a**2*b**5*x**5)/(60*a**6*
b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a
**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) + x/b**7
```

## 3.211 $\int \frac{x^6}{(a+bx)^7} dx$

**Optimal**. Leaf size=109

$$-\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a+bx)}{b^7}$$

**Rubi [A]** time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a+bx)}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^7,x]
```

```
[Out] -a^6/(6*b^7*(a + b*x)^6) + (6*a^5)/(5*b^7*(a + b*x)^5) - (15*a^4)/(4*b^7*(a
 + b*x)^4) + (20*a^3)/(3*b^7*(a + b*x)^3) - (15*a^2)/(2*b^7*(a + b*x)^2) +
(6*a)/(b^7*(a + b*x)) + Log[a + b*x]/b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^7}\,dx &= \int \left(\frac{a^6}{b^6(a+bx)^7}-\frac{6a^5}{b^6(a+bx)^6}+\frac{15a^4}{b^6(a+bx)^5}-\frac{20a^3}{b^6(a+bx)^4}+\frac{15a^2}{b^6(a+bx)^3}-\frac{6a}{b^6(a+bx)^2}+\frac{}{b^6(a}\right.\\ &= -\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{l}{}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 77, normalized size = 0.71

$$\frac{\frac{a\left(147a^5+822a^4bx+1875a^3b^2x^2+2200a^2b^3x^3+1350ab^4x^4+360b^5x^5\right)}{(a+bx)^6}+60\log(a+bx)}{60b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^7,x]
```

```
[Out] ((a*(147*a^5 + 822*a^4*b*x + 1875*a^3*b^2*x^2 + 2200*a^2*b^3*x^3 + 1350*a*b
^4*x^4 + 360*b^5*x^5))/(a + b*x)^6 + 60*Log[a + b*x])/(60*b^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x)^7, x]
```

**fricas [A]** time = 1.08, size = 193, normalized size = 1.77

$$\frac{360\,ab^5x^5+1350\,a^2b^4x^4+2200\,a^3b^3x^3+1875\,a^4b^2x^2+822\,a^5bx+147\,a^6+60\left(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6\right)\log\left(bx+a\right)}{60\left(b^{13}x^6+6\,ab^{12}x^5+15\,a^2b^{11}x^4+20\,a^3b^{10}x^3+15\,a^4b^9x^2+6\,a^5b^8x+a^6b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x^
2 + 822*a^5*b*x + 147*a^6 + 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20
*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a))/(b^13*x^6 +
6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 20*a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a^5*b
^8*x + a^6*b^7)
```

**giac [A]** time = 1.16, size = 79, normalized size = 0.72

$$\frac{\log\left(|bx+a|\right)}{b^7}+\frac{360\,ab^4x^5+1350\,a^2b^3x^4+2200\,a^3b^2x^3+1875\,a^4bx^2+822\,a^5x+\frac{147\,a^6}{b}}{60\,(bx+a)^6b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^7 + 1/60*(360*a*b^4*x^5 + 1350*a^2*b^3*x^4 + 2200*a^3*b
^2*x^3 + 1875*a^4*b*x^2 + 822*a^5*x + 147*a^6/b)/((b*x + a)^6*b^6)
```

**maple [A]** time = 0.01, size = 100, normalized size = 0.92

$$-\frac{a^6}{6\,(bx+a)^6\,b^7}+\frac{6a^5}{5\,(bx+a)^5\,b^7}-\frac{15a^4}{4\,(bx+a)^4\,b^7}+\frac{20a^3}{3\,(bx+a)^3\,b^7}-\frac{15a^2}{2\,(bx+a)^2\,b^7}+\frac{6a}{(bx+a)\,b^7}+\frac{\ln\left(bx+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^7,x)
```

```
[Out] -1/6*a^6/b^7/(b*x+a)^6+6/5*a^5/b^7/(b*x+a)^5-15/4*a^4/b^7/(b*x+a)^4+20/3*a^
3/b^7/(b*x+a)^3-15/2*a^2/b^7/(b*x+a)^2+6*a/b^7/(b*x+a)+ln(b*x+a)/b^7
```

**maxima [A]** time = 1.45, size = 136, normalized size = 1.25

$$\frac{360\,ab^5x^5 + 1350\,a^2b^4x^4 + 2200\,a^3b^3x^3 + 1875\,a^4b^2x^2 + 822\,a^5bx + 147\,a^6}{60\left(b^{13}x^6 + 6\,ab^{12}x^5 + 15\,a^2b^{11}x^4 + 20\,a^3b^{10}x^3 + 15\,a^4b^9x^2 + 6\,a^5b^8x + a^6b^7\right)} + \frac{\log\left(bx + a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x^
2 + 822*a^5*b*x + 147*a^6)/(b^13*x^6 + 6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 20*
a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a^5*b^8*x + a^6*b^7) + log(b*x + a)/b^7
```

**mupad [B]** time = 0.11, size = 81, normalized size = 0.74

$$\frac{\ln\left(a + b\,x\right) + \frac{6\,a}{a+b\,x} - \frac{15\,a^2}{2\,{\left(a+b\,x\right)}^2} + \frac{20\,a^3}{3\,{\left(a+b\,x\right)}^3} - \frac{15\,a^4}{4\,{\left(a+b\,x\right)}^4} + \frac{6\,a^5}{5\,{\left(a+b\,x\right)}^5} - \frac{a^6}{6\,{\left(a+b\,x\right)}^6}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^7,x)
```

```
[Out] (log(a + b*x) + (6*a)/(a + b*x) - (15*a^2)/(2*(a + b*x)^2) + (20*a^3)/(3*(a
 + b*x)^3) - (15*a^4)/(4*(a + b*x)^4) + (6*a^5)/(5*(a + b*x)^5) - a^6/(6*(a
 + b*x)^6))/b^7
```

**sympy [A]** time = 0.64, size = 141, normalized size = 1.29

$$\frac{147a^6 + 822a^5bx + 1875a^4b^2x^2 + 2200a^3b^3x^3 + 1350a^2b^4x^4 + 360ab^5x^5}{60a^6b^7 + 360a^5b^8x + 900a^4b^9x^2 + 1200a^3b^{10}x^3 + 900a^2b^{11}x^4 + 360ab^{12}x^5 + 60b^{13}x^6} + \frac{\log\left(a + bx\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**7,x)
```

```
[Out] (147*a**6 + 822*a**5*b*x + 1875*a**4*b**2*x**2 + 2200*a**3*b**3*x**3 + 1350
*a**2*b**4*x**4 + 360*a*b**5*x**5)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a*
*4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**
5 + 60*b**13*x**6) + log(a + b*x)/b**7
```

## 3.212 $\int \frac{x^5}{(a+bx)^7} dx$

**Optimal**. Leaf size=17

$$\frac{x^6}{6a(a+bx)^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$\frac{x^6}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^7,x]
```

```
[Out] x^6/(6*a*(a + b*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^5}{(a+bx)^7} dx = \frac{x^6}{6a(a+bx)^6}$$

**Mathematica [B]** time = 0.01, size = 64, normalized size = 3.76

$$-\frac{a^5+6a^4bx+15a^3b^2x^2+20a^2b^3x^3+15ab^4x^4+6b^5x^5}{6b^6(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^7,x]
```

```
[Out] -1/6*(a^5 + 6*a^4*b*x + 15*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 15*a*b^4*x^4 + 6*
b^5*x^5)/(b^6*(a + b*x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x)^7, x]
```

**fricas [B]** time = 0.95, size = 120, normalized size = 7.06

$$-\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\left(b^{12}x^6 + 6\,ab^{11}x^5 + 15\,a^2b^{10}x^4 + 20\,a^3b^9x^3 + 15\,a^4b^8x^2 + 6\,a^5b^7x + a^6b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15*a
^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

**giac [B]** time = 1.08, size = 62, normalized size = 3.65

$$-\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\,(bx+a)^6b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/((b*x + a)^6*b^6)
```

**maple [B]** time = 0.00, size = 87, normalized size = 5.12

$$\frac{a^5}{6\,(bx+a)^6\,b^6} - \frac{a^4}{(bx+a)^5\,b^6} + \frac{5a^3}{2\,(bx+a)^4\,b^6} - \frac{10a^2}{3\,(bx+a)^3\,b^6} + \frac{5a}{2\,(bx+a)^2\,b^6} - \frac{1}{(bx+a)\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^7,x)
```

```
[Out] 1/6*a^5/b^6/(b*x+a)^6+5/2*a/b^6/(b*x+a)^2-a^4/b^6/(b*x+a)^5+5/2*a^3/b^6/(b*
x+a)^4-10/3*a^2/b^6/(b*x+a)^3-1/b^6/(b*x+a)
```

**maxima** [B] time = 1.43, size = 120, normalized size = 7.06

$$-\frac{6\,b^{5}x^{5}+15\,ab^{4}x^{4}+20\,a^{2}b^{3}x^{3}+15\,a^{3}b^{2}x^{2}+6\,a^{4}bx+a^{5}}{6\left(b^{12}x^{6}+6\,ab^{11}x^{5}+15\,a^{2}b^{10}x^{4}+20\,a^{3}b^{9}x^{3}+15\,a^{4}b^{8}x^{2}+6\,a^{5}b^{7}x+a^{6}b^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15*a
^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

**mupad** [B] time = 0.12, size = 72, normalized size = 4.24

$$\frac{\frac{5\,a}{2\,(a+b\,x)^2}-\frac{1}{a+b\,x}-\frac{10\,a^2}{3\,(a+b\,x)^3}+\frac{5\,a^3}{2\,(a+b\,x)^4}-\frac{a^4}{(a+b\,x)^5}+\frac{a^5}{6\,(a+b\,x)^6}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^7,x)
```

```
[Out] ((5*a)/(2*(a + b*x)^2) - 1/(a + b*x) - (10*a^2)/(3*(a + b*x)^3) + (5*a^3)/(
2*(a + b*x)^4) - a^4/(a + b*x)^5 + a^5/(6*(a + b*x)^6))/b^6
```

**sympy** [B] time = 0.58, size = 128, normalized size = 7.53

$$\frac{-a^{5}-6a^{4}bx-15a^{3}b^{2}x^{2}-20a^{2}b^{3}x^{3}-15ab^{4}x^{4}-6b^{5}x^{5}}{6a^{6}b^{6}+36a^{5}b^{7}x+90a^{4}b^{8}x^{2}+120a^{3}b^{9}x^{3}+90a^{2}b^{10}x^{4}+36ab^{11}x^{5}+6b^{12}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**7,x)
```

```
[Out] (-a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*x**
4 - 6*b**5*x**5)/(6*a**6*b**6 + 36*a**5*b**7*x + 90*a**4*b**8*x**2 + 120*a*
*3*b**9*x**3 + 90*a**2*b**10*x**4 + 36*a*b**11*x**5 + 6*b**12*x**6)
```

## 3.213 $\int \frac{x^4}{(a+bx)^7} \, dx$

**Optimal**. Leaf size=35

$$\frac{x^5}{30a^2(a+bx)^5} + \frac{x^5}{6a(a+bx)^6}$$

**Rubi** [A] time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{x^5}{30a^2(a+bx)^5} + \frac{x^5}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^7,x]
```

```
[Out] x^5/(6*a*(a + b*x)^6) + x^5/(30*a^2*(a + b*x)^5)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(a+bx)^7} \, dx &= \frac{x^5}{6a(a+bx)^6} + \frac{\int \frac{x^4}{(a+bx)^6} \, dx}{6a} \\ &= \frac{x^5}{6a(a+bx)^6} + \frac{x^5}{30a^2(a+bx)^5} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 53, normalized size = 1.51

$$-\frac{a^4+6a^3bx+15a^2b^2x^2+20ab^3x^3+15b^4x^4}{30b^5(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^7,x]
```

```
[Out] -1/30*(a^4 + 6*a^3*b*x + 15*a^2*b^2*x^2 + 20*a*b^3*x^3 + 15*b^4*x^4)/(b^5*(
a + b*x)^6)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x)^7, x]
```

**fricas** [B] time = 0.86, size = 109, normalized size = 3.11

$$-\frac{15\,b^4x^4+20\,ab^3x^3+15\,a^2b^2x^2+6\,a^3bx+a^4}{30\left(b^{11}x^6+6\,ab^{10}x^5+15\,a^2b^9x^4+20\,a^3b^8x^3+15\,a^4b^7x^2+6\,a^5b^6x+a^6b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11*
x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a
^5*b^6*x + a^6*b^5)
```

**giac** [A] time = 1.14, size = 51, normalized size = 1.46

$$-\frac{15\,b^4x^4+20\,ab^3x^3+15\,a^2b^2x^2+6\,a^3bx+a^4}{30\,(bx+a)^6b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/((b*x
+ a)^6*b^5)
```

**maple [B]** time = 0.01, size = 72, normalized size = 2.06

$$-\frac{a^{4}}{6\left(bx+a\right)^{6}b^{5}}+\frac{4a^{3}}{5\left(bx+a\right)^{5}b^{5}}-\frac{3a^{2}}{2\left(bx+a\right)^{4}b^{5}}+\frac{4a}{3\left(bx+a\right)^{3}b^{5}}-\frac{1}{2\left(bx+a\right)^{2}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^7,x)
```

```
[Out] -1/6*a^4/b^5/(b*x+a)^6-1/2/b^5/(b*x+a)^2-3/2*a^2/b^5/(b*x+a)^4+4/5*a^3/b^5/
(b*x+a)^5+4/3*a/b^5/(b*x+a)^3
```

**maxima [B]** time = 1.45, size = 109, normalized size = 3.11

$$-\frac{15\,b^{4}x^{4}+20\,ab^{3}x^{3}+15\,a^{2}b^{2}x^{2}+6\,a^{3}bx+a^{4}}{30\left(b^{11}x^{6}+6\,ab^{10}x^{5}+15\,a^{2}b^{9}x^{4}+20\,a^{3}b^{8}x^{3}+15\,a^{4}b^{7}x^{2}+6\,a^{5}b^{6}x+a^{6}b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11*
x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a
^5*b^6*x + a^6*b^5)
```

**mupad [B]** time = 0.07, size = 22, normalized size = 0.63

$$\frac{x^5\,\left(6\,a+b\,x\right)}{30\,a^2\,{\left(a+b\,x\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^7,x)
```

```
[Out] (x^5*(6*a + b*x))/(30*a^2*(a + b*x)^6)
```

**sympy [B]** time = 0.57, size = 116, normalized size = 3.31

$$\frac{-a^{4}-6a^{3}bx-15a^{2}b^{2}x^{2}-20ab^{3}x^{3}-15b^{4}x^{4}}{30a^{6}b^{5}+180a^{5}b^{6}x+450a^{4}b^{7}x^{2}+600a^{3}b^{8}x^{3}+450a^{2}b^{9}x^{4}+180ab^{10}x^{5}+30b^{11}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**7,x)
```

```
[Out] (-a**4 - 6*a**3*b*x - 15*a**2*b**2*x**2 - 20*a*b**3*x**3 - 15*b**4*x**4)/(3
0*a**6*b**5 + 180*a**5*b**6*x + 450*a**4*b**7*x**2 + 600*a**3*b**8*x**3 + 4
50*a**2*b**9*x**4 + 180*a*b**10*x**5 + 30*b**11*x**6)
```

## 3.214 $\int \frac{x^3}{(a+bx)^7} dx$

**Optimal**. Leaf size=52

$$\frac{x^4}{60a^3(a+bx)^4}+\frac{x^4}{15a^2(a+bx)^5}+\frac{x^4}{6a(a+bx)^6}$$

**Rubi** [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{6b^4(a+bx)^6}-\frac{3a^2}{5b^4(a+bx)^5}+\frac{3a}{4b^4(a+bx)^4}-\frac{1}{3b^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^7,x]
```

```
[Out] a^3/(6*b^4*(a + b*x)^6) - (3*a^2)/(5*b^4*(a + b*x)^5) + (3*a)/(4*b^4*(a + b
*x)^4) - 1/(3*b^4*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^7}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^7}+\frac{3a^2}{b^3(a+bx)^6}-\frac{3a}{b^3(a+bx)^5}+\frac{1}{b^3(a+bx)^4}\right)dx\\ &= \frac{a^3}{6b^4(a+bx)^6}-\frac{3a^2}{5b^4(a+bx)^5}+\frac{3a}{4b^4(a+bx)^4}-\frac{1}{3b^4(a+bx)^3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 42, normalized size = 0.81

$$-\frac{a^3+6a^2bx+15ab^2x^2+20b^3x^3}{60b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^7,x]
[Out] -1/60*(a^3 + 6*a^2*b*x + 15*a*b^2*x^2 + 20*b^3*x^3)/(b^4*(a + b*x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^7,x]
[Out] IntegrateAlgebraic[x^3/(a + b*x)^7, x]
```

**fricas [B]** time = 1.12, size = 98, normalized size = 1.88

$$-\frac{20\,b^3x^3+15\,ab^2x^2+6\,a^2bx+a^3}{60\left(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="fricas")
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^5
 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4
)
```

**giac [A]** time = 1.00, size = 40, normalized size = 0.77

$$-\frac{20\,b^3x^3+15\,ab^2x^2+6\,a^2bx+a^3}{60\,(bx+a)^6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="giac")
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/((b*x + a)^6*b^4)
```

**maple [A]** time = 0.00, size = 57, normalized size = 1.10

$$\frac{a^3}{6\,(bx+a)^6\,b^4}-\frac{3a^2}{5\,(bx+a)^5\,b^4}+\frac{3a}{4\,(bx+a)^4\,b^4}-\frac{1}{3\,(bx+a)^3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^7,x)
```

[Out] 1/6*a^3/b^4/(b*x+a)^6-3/5*a^2/b^4/(b*x+a)^5+3/4*a/b^4/(b*x+a)^4-1/3/b^4/(b*x+a)^3

**maxima [B]** time = 1.40, size = 98, normalized size = 1.88

$$-\frac{20\,b^{3}x^{3}+15\,ab^{2}x^{2}+6\,a^{2}bx+a^{3}}{60\,\left(b^{10}x^{6}+6\,ab^{9}x^{5}+15\,a^{2}b^{8}x^{4}+20\,a^{3}b^{7}x^{3}+15\,a^{4}b^{6}x^{2}+6\,a^{5}b^{5}x+a^{6}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x+a)^7,x, algorithm="maxima")

[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4
)

**mupad [B]** time = 0.07, size = 48, normalized size = 0.92

$$\frac{\frac{3\,a}{4\,{\left(a+b\,x\right)}^4}-\frac{1}{3\,{\left(a+b\,x\right)}^3}-\frac{3\,a^2}{5\,{\left(a+b\,x\right)}^5}+\frac{a^3}{6\,{\left(a+b\,x\right)}^6}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b*x)^7,x)

[Out] ((3*a)/(4*(a + b*x)^4) - 1/(3*(a + b*x)^3) - (3*a^2)/(5*(a + b*x)^5) + a^3/(6*(a + b*x)^6))/b^4

**sympy [B]** time = 0.56, size = 104, normalized size = 2.00

$$\frac{-a^{3}-6a^{2}bx-15ab^{2}x^{2}-20b^{3}x^{3}}{60a^{6}b^{4}+360a^{5}b^{5}x+900a^{4}b^{6}x^{2}+1200a^{3}b^{7}x^{3}+900a^{2}b^{8}x^{4}+360ab^{9}x^{5}+60b^{10}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x+a)**7,x)

[Out] (-a**3 - 6*a**2*b*x - 15*a*b**2*x**2 - 20*b**3*x**3)/(60*a**6*b**4 + 360*a**5*b**5*x + 900*a**4*b**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 + 360*a*b**9*x**5 + 60*b**10*x**6)

## 3.215 $\int \frac{x^2}{(a+bx)^7} dx$

**Optimal**. Leaf size=47

$$-\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}$$

**Rubi** [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^7,x]
```

```
[Out] -a^2/(6*b^3*(a + b*x)^6) + (2*a)/(5*b^3*(a + b*x)^5) - 1/(4*b^3*(a + b*x)^4
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^7} dx &= \int \left(\frac{a^2}{b^2(a+bx)^7}-\frac{2a}{b^2(a+bx)^6}+\frac{1}{b^2(a+bx)^5}\right) dx \\ &= -\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 31, normalized size = 0.66

$$-\frac{a^2+6abx+15b^2x^2}{60b^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^2 + 6*a*b*x + 15*b^2*x^2)/(b^3*(a + b*x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x)^7, x]
```

**fricas [B]** time = 0.88, size = 87, normalized size = 1.85

$$-\frac{15\,b^2x^2+6\,abx+a^2}{60\left(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
+ 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

**giac [A]** time = 1.00, size = 29, normalized size = 0.62

$$-\frac{15\,b^2x^2+6\,abx+a^2}{60\,(bx+a)^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/((b*x + a)^6*b^3)
```

**maple [A]** time = 0.00, size = 42, normalized size = 0.89

$$-\frac{a^2}{6\,(bx+a)^6\,b^3}+\frac{2a}{5\,(bx+a)^5\,b^3}-\frac{1}{4\,(bx+a)^4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^7,x)
```

```
[Out] -1/6*a^2/b^3/(b*x+a)^6+2/5*a/b^3/(b*x+a)^5-1/4/b^3/(b*x+a)^4
```

**maxima [B]** time = 1.45, size = 87, normalized size = 1.85

$$-\frac{15\,b^2x^2+6\,abx+a^2}{60\,\left(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
+ 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

**mupad [B]** time = 0.08, size = 31, normalized size = 0.66

$$-\frac{8\,a^2+48\,a\,b\,x+120\,b^2\,x^2}{480\,b^3\,{\left(a+b\,x\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^7,x)
```

```
[Out] -(8*a^2 + 120*b^2*x^2 + 48*a*b*x)/(480*b^3*(a + b*x)^6)
```

**sympy [B]** time = 0.55, size = 92, normalized size = 1.96

$$\frac{-a^2-6abx-15b^2x^2}{60a^6b^3+360a^5b^4x+900a^4b^5x^2+1200a^3b^6x^3+900a^2b^7x^4+360ab^8x^5+60b^9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**7,x)
```

```
[Out] (-a**2 - 6*a*b*x - 15*b**2*x**2)/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4
*b**5*x**2 + 1200*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 6
0*b**9*x**6)
```

## 3.216 $\int \frac{x}{(a+bx)^7} \, dx$

**Optimal**. Leaf size=30

$$\frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^7,x]
```

```
[Out] a/(6*b^2*(a + b*x)^6) - 1/(5*b^2*(a + b*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^7} \, dx &= \int \left(-\frac{a}{b(a+bx)^7} + \frac{1}{b(a+bx)^6}\right) dx \\ &= \frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 20, normalized size = 0.67

$$-\frac{a+6bx}{30b^2(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^7,x]
```

```
[Out] -1/30*(a + 6*b*x)/(b^2*(a + b*x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x)^7, x]
```

**fricas [B]** time = 0.88, size = 76, normalized size = 2.53

$$-\frac{6\,bx + a}{30\,\left(b^8x^6 + 6\,ab^7x^5 + 15\,a^2b^6x^4 + 20\,a^3b^5x^3 + 15\,a^4b^4x^2 + 6\,a^5b^3x + a^6b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3
+ 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)
```

**giac [A]** time = 1.39, size = 18, normalized size = 0.60

$$-\frac{6\,bx + a}{30\,(bx + a)^6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/30*(6*b*x + a)/((b*x + a)^6*b^2)
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.90

$$\frac{a}{6\,(bx + a)^6\,b^2} - \frac{1}{5\,(bx + a)^5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^7,x)
```

```
[Out] 1/6*a/b^2/(b*x+a)^6-1/5/b^2/(b*x+a)^5
```

**maxima [B]** time = 1.37, size = 76, normalized size = 2.53

$$-\frac{6\,bx + a}{30\,\left(b^8x^6 + 6\,ab^7x^5 + 15\,a^2b^6x^4 + 20\,a^3b^5x^3 + 15\,a^4b^4x^2 + 6\,a^5b^3x + a^6b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3
+ 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)
```

**mupad [B]** time = 0.10, size = 18, normalized size = 0.60

$$-\frac{a+6\,b\,x}{30\,b^2\,{\left(a+b\,x\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^7,x)
```

```
[Out] -(a + 6*b*x)/(30*b^2*(a + b*x)^6)
```

**sympy [B]** time = 0.50, size = 80, normalized size = 2.67

$$\frac{-a-6bx}{30a^6b^2+180a^5b^3x+450a^4b^4x^2+600a^3b^5x^3+450a^2b^6x^4+180ab^7x^5+30b^8x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**7,x)
```

```
[Out] (-a - 6*b*x)/(30*a**6*b**2 + 180*a**5*b**3*x + 450*a**4*b**4*x**2 + 600*a**
3*b**5*x**3 + 450*a**2*b**6*x**4 + 180*a*b**7*x**5 + 30*b**8*x**6)
```

## 3.217 $\int \frac{1}{(a+bx)^7} dx$

**Optimal**. Leaf size=14

$$-\frac{1}{6b(a+bx)^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{6b(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-7),x]
```

```
[Out] -1/(6*b*(a + b*x)^6)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^7} dx = -\frac{1}{6b(a+bx)^6}$$

**Mathematica [A]** time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{6b(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-7),x]
```

```
[Out] -1/6*1/(b*(a + b*x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-7),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-7), x]
```

**fricas [B]** time = 1.20, size = 68, normalized size = 4.86

$$-\frac{1}{6\left(b^7x^6 + 6\,ab^6x^5 + 15\,a^2b^5x^4 + 20\,a^3b^4x^3 + 15\,a^4b^3x^2 + 6\,a^5b^2x + a^6b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/6/(b^7*x^6 + 6*a*b^6*x^5 + 15*a^2*b^5*x^4 + 20*a^3*b^4*x^3 + 15*a^4*b^3*
x^2 + 6*a^5*b^2*x + a^6*b)
```

**giac [A]** time = 1.09, size = 12, normalized size = 0.86

$$-\frac{1}{6\,(bx + a)^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/6/((b*x + a)^6*b)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{6\,(bx + a)^6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^7,x)
```

```
[Out] -1/6/b/(b*x+a)^6
```

**maxima [A]** time = 1.41, size = 12, normalized size = 0.86

$$-\frac{1}{6\,(bx + a)^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/6/((b*x + a)^6*b)
```

**mupad [B]** time = 0.06, size = 70, normalized size = 5.00

$$-\frac{1}{6\,a^6\,b+36\,a^5\,b^2\,x+90\,a^4\,b^3\,x^2+120\,a^3\,b^4\,x^3+90\,a^2\,b^5\,x^4+36\,a\,b^6\,x^5+6\,b^7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^7,x)
```

```
[Out] -1/(6*a^6*b + 6*b^7*x^6 + 36*a^5*b^2*x + 36*a*b^6*x^5 + 90*a^4*b^3*x^2 + 12
0*a^3*b^4*x^3 + 90*a^2*b^5*x^4)
```

**sympy [B]** time = 0.46, size = 73, normalized size = 5.21

$$-\frac{1}{6a^6b+36a^5b^2x+90a^4b^3x^2+120a^3b^4x^3+90a^2b^5x^4+36ab^6x^5+6b^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**7,x)
```

```
[Out] -1/(6*a**6*b + 36*a**5*b**2*x + 90*a**4*b**3*x**2 + 120*a**3*b**4*x**3 + 90
*a**2*b**5*x**4 + 36*a*b**6*x**5 + 6*b**7*x**6)
```

## 3.218 $\int \frac{1}{x(a+bx)^7} dx$

**Optimal**. Leaf size=99

$$-\frac{\log(a+bx)}{a^7}+\frac{\log(x)}{a^7}+\frac{1}{a^6(a+bx)}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{6a(a+bx)^6}$$

**Rubi** [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^6(a+bx)}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{5a^2(a+bx)^5}-\frac{\log(a+bx)}{a^7}+\frac{\log(x)}{a^7}+\frac{1}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^7),x]
```

```
[Out] 1/(6*a*(a + b*x)^6) + 1/(5*a^2*(a + b*x)^5) + 1/(4*a^3*(a + b*x)^4) + 1/(3*
a^4*(a + b*x)^3) + 1/(2*a^5*(a + b*x)^2) + 1/(a^6*(a + b*x)) + Log[x]/a^7 -
 Log[a + b*x]/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^7}\,dx &= \int \left(\frac{1}{a^7x}-\frac{b}{a(a+bx)^7}-\frac{b}{a^2(a+bx)^6}-\frac{b}{a^3(a+bx)^5}-\frac{b}{a^4(a+bx)^4}-\frac{b}{a^5(a+bx)^3}-\frac{b}{a^6(a+bx)\right.\\ &= \frac{1}{6a(a+bx)^6}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{a^6(a+bx)}+\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 81, normalized size = 0.82

$$\frac{\frac{a\left(147a^5+522a^4bx+855a^3b^2x^2+740a^2b^3x^3+330ab^4x^4+60b^5x^5\right)}{(a+bx)^6}-60\log(a+bx)+60\log(x)}{60a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^7),x]
```

```
[Out] ((a*(147*a^5 + 522*a^4*b*x + 855*a^3*b^2*x^2 + 740*a^2*b^3*x^3 + 330*a*b^4*
x^4 + 60*b^5*x^5))/(a + b*x)^6 + 60*Log[x] - 60*Log[a + b*x])/(60*a^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^7),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x)^7), x]
```

**fricas [B]** time = 0.66, size = 256, normalized size = 2.59

$$\frac{60\,ab^5x^5+330\,a^2b^4x^4+740\,a^3b^3x^3+855\,a^4b^2x^2+522\,a^5bx+147\,a^6-60\left(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6\right)\log\left(bx+a\right)+60\left(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6\right)\log\left(x\right)}{60\left(a^7b^6x^6+6\,a^8b^5x^5+15\,a^9b^4x^4+20\,a^{10}b^3x^3+15\,a^{11}b^2x^2+6\,a^{12}bx+a^{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a*b^5*x^5 + 330*a^2*b^4*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 +
522*a^5*b*x + 147*a^6 - 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3
*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a) + 60*(b^6*x^6 + 6
*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x +
 a^6)*log(x))/(a^7*b^6*x^6 + 6*a^8*b^5*x^5 + 15*a^9*b^4*x^4 + 20*a^10*b^3*x
^3 + 15*a^11*b^2*x^2 + 6*a^12*b*x + a^13)
```

**giac [A]** time = 1.01, size = 87, normalized size = 0.88

$$-\frac{\log\left(|bx+a|\right)}{a^7}+\frac{\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^5+330\,a^2b^4x^4+740\,a^3b^3x^3+855\,a^4b^2x^2+522\,a^5bx+147\,a^6}{60\left(bx+a\right)^6a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^7 + log(abs(x))/a^7 + 1/60*(60*a*b^5*x^5 + 330*a^2*b^4
*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 + 522*a^5*b*x + 147*a^6)/((b*x + a
)^6*a^7)
```

**maple [A]** time = 0.01, size = 90, normalized size = 0.91

$$\frac{1}{6\left(bx+a\right)^6a}+\frac{1}{5\left(bx+a\right)^5a^2}+\frac{1}{4\left(bx+a\right)^4a^3}+\frac{1}{3\left(bx+a\right)^3a^4}+\frac{1}{2\left(bx+a\right)^2a^5}+\frac{1}{\left(bx+a\right)a^6}+\frac{\ln(x)}{a^7}-\frac{\ln\left(bx+a\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^7,x)
```

```
[Out] 1/6/a/(b*x+a)^6+1/5/a^2/(b*x+a)^5+1/4/a^3/(b*x+a)^4+1/3/a^4/(b*x+a)^3+1/2/a
^5/(b*x+a)^2+1/a^6/(b*x+a)+ln(x)/a^7-ln(b*x+a)/a^7
```

**maxima [A]** time = 1.51, size = 139, normalized size = 1.40

$$\frac{60\,b^5x^5 + 330\,ab^4x^4 + 740\,a^2b^3x^3 + 855\,a^3b^2x^2 + 522\,a^4bx + 147\,a^5}{60\left(a^6b^6x^6 + 6\,a^7b^5x^5 + 15\,a^8b^4x^4 + 20\,a^9b^3x^3 + 15\,a^{10}b^2x^2 + 6\,a^{11}bx + a^{12}\right)} - \frac{\log(bx+a)}{a^7} + \frac{\log(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(60*b^5*x^5 + 330*a*b^4*x^4 + 740*a^2*b^3*x^3 + 855*a^3*b^2*x^2 + 522*
a^4*b*x + 147*a^5)/(a^6*b^6*x^6 + 6*a^7*b^5*x^5 + 15*a^8*b^4*x^4 + 20*a^9*b
^3*x^3 + 15*a^10*b^2*x^2 + 6*a^11*b*x + a^12) - log(b*x + a)/a^7 + log(x)/a
^7
```

**mupad [B]** time = 0.45, size = 102, normalized size = 1.03

$$-\frac{\ln\left(\frac{a+b\,x}{x}\right) - \frac{15\,b^2\,x^2}{2\,(a+b\,x)^2} + \frac{20\,b^3\,x^3}{3\,(a+b\,x)^3} - \frac{15\,b^4\,x^4}{4\,(a+b\,x)^4} + \frac{6\,b^5\,x^5}{5\,(a+b\,x)^5} - \frac{b^6\,x^6}{6\,(a+b\,x)^6} + \frac{6\,b\,x}{a+b\,x}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^7),x)
```

```
[Out] -(log((a + b*x)/x) - (15*b^2*x^2)/(2*(a + b*x)^2) + (20*b^3*x^3)/(3*(a + b*
x)^3) - (15*b^4*x^4)/(4*(a + b*x)^4) + (6*b^5*x^5)/(5*(a + b*x)^5) - (b^6*x
^6)/(6*(a + b*x)^6) + (6*b*x)/(a + b*x))/a^7
```

**sympy [A]** time = 0.68, size = 141, normalized size = 1.42

$$\frac{147a^5 + 522a^4bx + 855a^3b^2x^2 + 740a^2b^3x^3 + 330ab^4x^4 + 60b^5x^5}{60a^{12} + 360a^{11}bx + 900a^{10}b^2x^2 + 1200a^9b^3x^3 + 900a^8b^4x^4 + 360a^7b^5x^5 + 60a^6b^6x^6} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**7,x)
```

```
[Out] (147*a**5 + 522*a**4*b*x + 855*a**3*b**2*x**2 + 740*a**2*b**3*x**3 + 330*a*
b**4*x**4 + 60*b**5*x**5)/(60*a**12 + 360*a**11*b*x + 900*a**10*b**2*x**2 +
 1200*a**9*b**3*x**3 + 900*a**8*b**4*x**4 + 360*a**7*b**5*x**5 + 60*a**6*b*
*6*x**6) + (log(x) - log(a/b + x))/a**7
```

## 3.219 $\int \frac{1}{x^2(a+bx)^7}\,dx$

**Optimal**. Leaf size=117

$$-\frac{7b\log(x)}{a^8}+\frac{7b\log(a+bx)}{a^8}-\frac{6b}{a^7(a+bx)}-\frac{1}{a^7x}-\frac{5b}{2a^6(a+bx)^2}-\frac{4b}{3a^5(a+bx)^3}-\frac{3b}{4a^4(a+bx)^4}-\frac{2b}{5a^3(a+bx)^5}-\frac{b}{6a^2(a+}$$

**Rubi [A]** time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{6b}{a^7(a+bx)}-\frac{5b}{2a^6(a+bx)^2}-\frac{4b}{3a^5(a+bx)^3}-\frac{3b}{4a^4(a+bx)^4}-\frac{2b}{5a^3(a+bx)^5}-\frac{b}{6a^2(a+bx)^6}-\frac{7b\log(x)}{a^8}+\frac{7b\log(a+bx)}{a^8}-\frac{1}{a^7x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^7),x]
```

```
[Out] -(1/(a^7*x)) - b/(6*a^2*(a + b*x)^6) - (2*b)/(5*a^3*(a + b*x)^5) - (3*b)/(4
*a^4*(a + b*x)^4) - (4*b)/(3*a^5*(a + b*x)^3) - (5*b)/(2*a^6*(a + b*x)^2) -
 (6*b)/(a^7*(a + b*x)) - (7*b*Log[x])/a^8 + (7*b*Log[a + b*x])/a^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^7}\,dx &= \int \left(\frac{1}{a^7x^2}-\frac{7b}{a^8x}+\frac{b^2}{a^2(a+bx)^7}+\frac{2b^2}{a^3(a+bx)^6}+\frac{3b^2}{a^4(a+bx)^5}+\frac{4b^2}{a^5(a+bx)^4}+\frac{5b^2}{a^6(a+bx)^3}+\frac{}{a}\right.\\ &= -\frac{1}{a^7x}-\frac{b}{6a^2(a+bx)^6}-\frac{2b}{5a^3(a+bx)^5}-\frac{3b}{4a^4(a+bx)^4}-\frac{4b}{3a^5(a+bx)^3}-\frac{5b}{2a^6(a+bx)^2}-\frac{6b}{a^7(a+}\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 97, normalized size = 0.83

$$-\frac{\frac{a\left(60a^6+1029a^5bx+3654a^4b^2x^2+5985a^3b^3x^3+5180a^2b^4x^4+2310ab^5x^5+420b^6x^6\right)}{x(a+bx)^6}-420b\log(a+bx)+420b\log(x)}{60a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^7),x]
```

```
[Out] -1/60*((a*(60*a^6 + 1029*a^5*b*x + 3654*a^4*b^2*x^2 + 5985*a^3*b^3*x^3 + 51
80*a^2*b^4*x^4 + 2310*a*b^5*x^5 + 420*b^6*x^6))/(x*(a + b*x)^6) + 420*b*Log
[x] - 420*b*Log[a + b*x])/a^8
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^7),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)^7), x]
```

**fricas [B]** time = 1.32, size = 285, normalized size = 2.44

$$-\frac{420\,ab^6x^6+2310\,a^2b^5x^5+5180\,a^3b^4x^4+5985\,a^4b^3x^3+3654\,a^5b^2x^2+1029\,a^6bx+60\,a^7-420\left(b^7x^7+6\,ab^6x^6+15\,a^2b^5x^5+20\,a^3b^4x^4+15\,a^4b^3x^3+6\,a^5b^2x^2+a^6bx\right)\log\left(bx+a\right)+420\left(b^7x^7+6\,ab^6x^6+15\,a^2b^5x^5+20\,a^3b^4x^4+15\,a^4b^3x^3+6\,a^5b^2x^2+a^6bx\right)\log\left(x\right)}{60\left(a^8b^6x^7+6\,a^9b^5x^6+15\,a^{10}b^4x^5+20\,a^{11}b^3x^4+15\,a^{12}b^2x^3+6\,a^{13}bx^2+a^{14}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(420*a*b^6*x^6 + 2310*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*x
^3 + 3654*a^5*b^2*x^2 + 1029*a^6*b*x + 60*a^7 - 420*(b^7*x^7 + 6*a*b^6*x^6
+ 15*a^2*b^5*x^5 + 20*a^3*b^4*x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6*b*
x)*log(b*x + a) + 420*(b^7*x^7 + 6*a*b^6*x^6 + 15*a^2*b^5*x^5 + 20*a^3*b^4*
x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6*b*x)*log(x))/(a^8*b^6*x^7 + 6*a^
9*b^5*x^6 + 15*a^10*b^4*x^5 + 20*a^11*b^3*x^4 + 15*a^12*b^2*x^3 + 6*a^13*b*
x^2 + a^14*x)
```

**giac [A]** time = 1.05, size = 104, normalized size = 0.89

$$\frac{7\,b\log\left(|bx+a|\right)}{a^8}-\frac{7\,b\log\left(|x|\right)}{a^8}-\frac{420\,ab^6x^6+2310\,a^2b^5x^5+5180\,a^3b^4x^4+5985\,a^4b^3x^3+3654\,a^5b^2x^2+1029\,a^6bx+60\,a^7}{60\,(bx+a)^6a^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 7*b*log(abs(b*x + a))/a^8 - 7*b*log(abs(x))/a^8 - 1/60*(420*a*b^6*x^6 + 231
0*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*x^3 + 3654*a^5*b^2*x^2 + 10
29*a^6*b*x + 60*a^7)/((b*x + a)^6*a^8*x)
```

**maple [A]** time = 0.01, size = 108, normalized size = 0.92

$$-\frac{b}{6\,(bx+a)^6\,a^2}-\frac{2b}{5\,(bx+a)^5\,a^3}-\frac{3b}{4\,(bx+a)^4\,a^4}-\frac{4b}{3\,(bx+a)^3\,a^5}-\frac{5b}{2\,(bx+a)^2\,a^6}-\frac{6b}{(bx+a)\,a^7}-\frac{7b\ln(x)}{a^8}+\frac{7b\ln(bx+a)}{a^8}-\frac{1}{a^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^7,x)
```

```
[Out] -1/a^7/x-1/6*b/a^2/(b*x+a)^6-2/5*b/a^3/(b*x+a)^5-3/4*b/a^4/(b*x+a)^4-4/3*b/
a^5/(b*x+a)^3-5/2*b/a^6/(b*x+a)^2-6*b/a^7/(b*x+a)-7*b*ln(x)/a^8+7*b*ln(b*x+
a)/a^8
```

**maxima [A]** time = 1.59, size = 157, normalized size = 1.34

$$-\frac{420\,b^6x^6+2310\,ab^5x^5+5180\,a^2b^4x^4+5985\,a^3b^3x^3+3654\,a^4b^2x^2+1029\,a^5bx+60\,a^6}{60\left(a^7b^6x^7+6\,a^8b^5x^6+15\,a^9b^4x^5+20\,a^{10}b^3x^4+15\,a^{11}b^2x^3+6\,a^{12}bx^2+a^{13}x\right)}+\frac{7\,b\log\left(bx+a\right)}{a^8}-\frac{7\,b\log(x)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(420*b^6*x^6 + 2310*a*b^5*x^5 + 5180*a^2*b^4*x^4 + 5985*a^3*b^3*x^3 +
 3654*a^4*b^2*x^2 + 1029*a^5*b*x + 60*a^6)/(a^7*b^6*x^7 + 6*a^8*b^5*x^6 + 1
5*a^9*b^4*x^5 + 20*a^10*b^3*x^4 + 15*a^11*b^2*x^3 + 6*a^12*b*x^2 + a^13*x)
+ 7*b*log(b*x + a)/a^8 - 7*b*log(x)/a^8
```

**mupad [B]** time = 0.19, size = 151, normalized size = 1.29

$$\frac{14\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^8}-\frac{\frac{1}{a}+\frac{609\,b^2\,x^2}{10\,a^3}+\frac{399\,b^3\,x^3}{4\,a^4}+\frac{259\,b^4\,x^4}{3\,a^5}+\frac{77\,b^5\,x^5}{2\,a^6}+\frac{7\,b^6\,x^6}{a^7}+\frac{343\,b\,x}{20\,a^2}}{a^6\,x+6\,a^5\,b\,x^2+15\,a^4\,b^2\,x^3+20\,a^3\,b^3\,x^4+15\,a^2\,b^4\,x^5+6\,a\,b^5\,x^6+b^6\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^7),x)
```

```
[Out] (14*b*atanh((2*b*x)/a + 1))/a^8 - (1/a + (609*b^2*x^2)/(10*a^3) + (399*b^3*
x^3)/(4*a^4) + (259*b^4*x^4)/(3*a^5) + (77*b^5*x^5)/(2*a^6) + (7*b^6*x^6)/a
^7 + (343*b*x)/(20*a^2))/(a^6*x + b^6*x^7 + 6*a^5*b*x^2 + 6*a*b^5*x^6 + 15*
a^4*b^2*x^3 + 20*a^3*b^3*x^4 + 15*a^2*b^4*x^5)
```

**sympy [A]** time = 0.80, size = 162, normalized size = 1.38

$$\frac{-60a^6-1029a^5bx-3654a^4b^2x^2-5985a^3b^3x^3-5180a^2b^4x^4-2310ab^5x^5-420b^6x^6}{60a^{13}x+360a^{12}bx^2+900a^{11}b^2x^3+1200a^{10}b^3x^4+900a^9b^4x^5+360a^8b^5x^6+60a^7b^6x^7}+\frac{7b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**7,x)
```

```
[Out] (-60*a**6 - 1029*a**5*b*x - 3654*a**4*b**2*x**2 - 5985*a**3*b**3*x**3 - 518
0*a**2*b**4*x**4 - 2310*a*b**5*x**5 - 420*b**6*x**6)/(60*a**13*x + 360*a**1
2*b*x**2 + 900*a**11*b**2*x**3 + 1200*a**10*b**3*x**4 + 900*a**9*b**4*x**5
+ 360*a**8*b**5*x**6 + 60*a**7*b**6*x**7) + 7*b*(-log(x) + log(a/b + x))/a*
*8
```

## 3.220 $\int \frac{1}{x^3(a+bx)^7} \, dx$

**Optimal**. Leaf size=144

$$\frac{28b^2\log(x)}{a^9}-\frac{28b^2\log(a+bx)}{a^9}+\frac{21b^2}{a^8(a+bx)}+\frac{7b}{a^8x}+\frac{15b^2}{2a^7(a+bx)^2}-\frac{1}{2a^7x^2}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{3b^2}{5a^4(a+b}$$

**Rubi** [A] time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{21b^2}{a^8(a+bx)}+\frac{15b^2}{2a^7(a+bx)^2}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{b^2}{6a^3(a+bx)^6}+\frac{28b^2\log(x)}{a^9}-\frac{28b^2\log(a+bx)}{a^9}+\frac{7b}{a^8x}-\frac{1}{2a^7x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^7),x]
```

```
[Out] -1/(2*a^7*x^2) + (7*b)/(a^8*x) + b^2/(6*a^3*(a + b*x)^6) + (3*b^2)/(5*a^4*(
a + b*x)^5) + (3*b^2)/(2*a^5*(a + b*x)^4) + (10*b^2)/(3*a^6*(a + b*x)^3) +
(15*b^2)/(2*a^7*(a + b*x)^2) + (21*b^2)/(a^8*(a + b*x)) + (28*b^2*Log[x])/a
^9 - (28*b^2*Log[a + b*x])/a^9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^7}\,dx &= \int \left(\frac{1}{a^7x^3}-\frac{7b}{a^8x^2}+\frac{28b^2}{a^9x}-\frac{b^3}{a^3(a+bx)^7}-\frac{3b^3}{a^4(a+bx)^6}-\frac{6b^3}{a^5(a+bx)^5}-\frac{10b^3}{a^6(a+bx)^4}-\frac{15}{a^7(a+}\right.\\ &= -\frac{1}{2a^7x^2}+\frac{7b}{a^8x}+\frac{b^2}{6a^3(a+bx)^6}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{15b^2}{2a^7(a+bx}\end{aligned}$$

**Mathematica** [A] time = 0.07, size = 112, normalized size = 0.78

$$\frac{\frac{a\left(-15a^7+120a^6bx+2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+4620ab^6x^6+840b^7x^7\right)}{x^2(a+bx)^6}-840b^2\log(a+bx)+840b^2\log(x)}{30a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^7),x]
```

```
[Out] ((a*(-15*a^7 + 120*a^6*b*x + 2058*a^5*b^2*x^2 + 7308*a^4*b^3*x^3 + 11970*a^
3*b^4*x^4 + 10360*a^2*b^5*x^5 + 4620*a*b^6*x^6 + 840*b^7*x^7))/(x^2*(a + b*
x)^6) + 840*b^2*Log[x] - 840*b^2*Log[a + b*x])/(30*a^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)^7} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^7),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)^7), x]
```

**fricas [B]** time = 1.33, size = 306, normalized size = 2.12

$$\frac{840\,ab^7x^7 + 4620\,a^2b^6x^6 + 10360\,a^3b^5x^5 + 11970\,a^4b^4x^4 + 7308\,a^5b^3x^3 + 2058\,a^6b^2x^2 + 120\,a^7bx - 15\,a^8 - 840\left(b^8x^8 + 6\,ab^7x^7 + 15\,a^2b^6x^6 + 20\,a^3b^5x^5 + 15\,a^4b^4x^4 + 6\,a^5b^3x^3 + a^6b^2x^2\right)\log(bx+a) + 840\left(b^8x^8 + 6\,ab^7x^7 + 15\,a^2b^6x^6 + 20\,a^3b^5x^5 + 15\,a^4b^4x^4 + 6\,a^5b^3x^3 + a^6b^2x^2\right)\log(x)}{30\left(a^9b^6x^8 + 6\,a^{10}b^5x^7 + 15\,a^{11}b^4x^6 + 20\,a^{12}b^3x^5 + 15\,a^{13}b^2x^4 + 6\,a^{14}bx^3 + a^{15}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(840*a*b^7*x^7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4*
x^4 + 7308*a^5*b^3*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8 - 840*(b^8
*x^8 + 6*a*b^7*x^7 + 15*a^2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 6*a
^5*b^3*x^3 + a^6*b^2*x^2)*log(b*x + a) + 840*(b^8*x^8 + 6*a*b^7*x^7 + 15*a^
2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 6*a^5*b^3*x^3 + a^6*b^2*x^2)*
log(x))/(a^9*b^6*x^8 + 6*a^10*b^5*x^7 + 15*a^11*b^4*x^6 + 20*a^12*b^3*x^5 +
 15*a^13*b^2*x^4 + 6*a^14*b*x^3 + a^15*x^2)
```

**giac [A]** time = 1.27, size = 119, normalized size = 0.83

$$-\frac{28\,b^2\log(|bx+a|)}{a^9} + \frac{28\,b^2\log(|x|)}{a^9} + \frac{840\,ab^7x^7 + 4620\,a^2b^6x^6 + 10360\,a^3b^5x^5 + 11970\,a^4b^4x^4 + 7308\,a^5b^3x^3 + 2058\,a^6b^2x^2 + 120\,a^7bx - 15\,a^8}{30\,(bx+a)^6a^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -28*b^2*log(abs(b*x + a))/a^9 + 28*b^2*log(abs(x))/a^9 + 1/30*(840*a*b^7*x^
7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4*x^4 + 7308*a^5*b^3
*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8)/((b*x + a)^6*a^9*x^2)
```

**maple** [A] time = 0.01, size = 133, normalized size = 0.92

$$\frac{b^2}{6\,(bx+a)^6\,a^3}+\frac{3b^2}{5\,(bx+a)^5\,a^4}+\frac{3b^2}{2\,(bx+a)^4\,a^5}+\frac{10b^2}{3\,(bx+a)^3\,a^6}+\frac{15b^2}{2\,(bx+a)^2\,a^7}+\frac{21b^2}{(bx+a)\,a^8}+\frac{28b^2\ln(x)}{a^9}-\frac{28b^2\ln(bx+a)}{a^9}+\frac{7b}{a^8x}-\frac{1}{2a^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^7,x)
```

```
[Out] -1/2/a^7/x^2+7*b/a^8/x+1/6*b^2/a^3/(b*x+a)^6+3/5*b^2/a^4/(b*x+a)^5+3/2*b^2/
a^5/(b*x+a)^4+10/3*b^2/a^6/(b*x+a)^3+15/2*b^2/a^7/(b*x+a)^2+21*b^2/a^8/(b*x
+a)+28*b^2*ln(x)/a^9-28*b^2*ln(b*x+a)/a^9
```

**maxima** [A] time = 1.55, size = 174, normalized size = 1.21

$$\frac{840\,b^7x^7+4620\,ab^6x^6+10360\,a^2b^5x^5+11970\,a^3b^4x^4+7308\,a^4b^3x^3+2058\,a^5b^2x^2+120\,a^6bx-15\,a^7}{30\left(a^8b^6x^8+6\,a^9b^5x^7+15\,a^{10}b^4x^6+20\,a^{11}b^3x^5+15\,a^{12}b^2x^4+6\,a^{13}bx^3+a^{14}x^2\right)}-\frac{28\,b^2\log(bx+a)}{a^9}+\frac{28\,b^2\log(x)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/30*(840*b^7*x^7 + 4620*a*b^6*x^6 + 10360*a^2*b^5*x^5 + 11970*a^3*b^4*x^4
+ 7308*a^4*b^3*x^3 + 2058*a^5*b^2*x^2 + 120*a^6*b*x - 15*a^7)/(a^8*b^6*x^8
+ 6*a^9*b^5*x^7 + 15*a^10*b^4*x^6 + 20*a^11*b^3*x^5 + 15*a^12*b^2*x^4 + 6*a
^13*b*x^3 + a^14*x^2) - 28*b^2*log(b*x + a)/a^9 + 28*b^2*log(x)/a^9
```

**mupad** [B] time = 0.21, size = 167, normalized size = 1.16

$$\frac{\frac{343\,b^2\,x^2}{5\,a^3}-\frac{1}{2\,a}+\frac{1218\,b^3\,x^3}{5\,a^4}+\frac{399\,b^4\,x^4}{a^5}+\frac{1036\,b^5\,x^5}{3\,a^6}+\frac{154\,b^6\,x^6}{a^7}+\frac{28\,b^7\,x^7}{a^8}+\frac{4\,b\,x}{a^2}}{a^6\,x^2+6\,a^5\,b\,x^3+15\,a^4\,b^2\,x^4+20\,a^3\,b^3\,x^5+15\,a^2\,b^4\,x^6+6\,a\,b^5\,x^7+b^6\,x^8}-\frac{56\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^7),x)
```

```
[Out] ((343*b^2*x^2)/(5*a^3) - 1/(2*a) + (1218*b^3*x^3)/(5*a^4) + (399*b^4*x^4)/a
^5 + (1036*b^5*x^5)/(3*a^6) + (154*b^6*x^6)/a^7 + (28*b^7*x^7)/a^8 + (4*b*x
)/a^2)/(a^6*x^2 + b^6*x^8 + 6*a^5*b*x^3 + 6*a*b^5*x^7 + 15*a^4*b^2*x^4 + 20
*a^3*b^3*x^5 + 15*a^2*b^4*x^6) - (56*b^2*atanh((2*b*x)/a + 1))/a^9
```

**sympy** [A] time = 0.84, size = 175, normalized size = 1.22

$$\frac{-15a^7+120a^6bx+2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+4620ab^6x^6+840b^7x^7}{30a^{14}x^2+180a^{13}bx^3+450a^{12}b^2x^4+600a^{11}b^3x^5+450a^{10}b^4x^6+180a^9b^5x^7+30a^8b^6x^8}+\frac{28b^2\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**7,x)
```

```
[Out] (-15*a**7 + 120*a**6*b*x + 2058*a**5*b**2*x**2 + 7308*a**4*b**3*x**3 + 1197
0*a**3*b**4*x**4 + 10360*a**2*b**5*x**5 + 4620*a*b**6*x**6 + 840*b**7*x**7)
/(30*a**14*x**2 + 180*a**13*b*x**3 + 450*a**12*b**2*x**4 + 600*a**11*b**3*x
**5 + 450*a**10*b**4*x**6 + 180*a**9*b**5*x**7 + 30*a**8*b**6*x**8) + 28*b*
*2*(log(x) - log(a/b + x))/a**9
```

## 3.221 $\int \frac{1}{x^4(a+bx)^7} dx$

**Optimal**. Leaf size=157

$$-\frac{84b^3\log(x)}{a^{10}}+\frac{84b^3\log(a+bx)}{a^{10}}-\frac{56b^3}{a^9(a+bx)}-\frac{28b^2}{a^9x}-\frac{35b^3}{2a^8(a+bx)^2}+\frac{7b}{2a^8x^2}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{1}{3a^7x^3}-\frac{5b^3}{2a^6(a+bx)^4}-$$

**Rubi [A]** time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{56b^3}{a^9(a+bx)}-\frac{35b^3}{2a^8(a+bx)^2}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{b^3}{6a^4(a+bx)^6}-\frac{28b^2}{a^9x}-\frac{84b^3\log(x)}{a^{10}}+\frac{84b^3\log(a+bx)}{a^{10}}+\frac{7b}{2a^8x^2}-\frac{1}{3a^7x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^7),x]
```

```
[Out] -1/(3*a^7*x^3) + (7*b)/(2*a^8*x^2) - (28*b^2)/(a^9*x) - b^3/(6*a^4*(a + b*x
)^6) - (4*b^3)/(5*a^5*(a + b*x)^5) - (5*b^3)/(2*a^6*(a + b*x)^4) - (20*b^3)
/(3*a^7*(a + b*x)^3) - (35*b^3)/(2*a^8*(a + b*x)^2) - (56*b^3)/(a^9*(a + b*
x)) - (84*b^3*Log[x])/a^10 + (84*b^3*Log[a + b*x])/a^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^4(a+bx)^7} dx = \int \left(\frac{1}{a^7x^4}-\frac{7b}{a^8x^3}+\frac{28b^2}{a^9x^2}-\frac{84b^3}{a^{10}x}+\frac{b^4}{a^4(a+bx)^7}+\frac{4b^4}{a^5(a+bx)^6}+\frac{10b^4}{a^6(a+bx)^5}+\frac{20b^4}{a^7(a+bx)^4}\right.$$

$$=-\frac{1}{3a^7x^3}+\frac{7b}{2a^8x^2}-\frac{28b^2}{a^9x}-\frac{b^3}{6a^4(a+bx)^6}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{20b^3}{3a^7(a+bx)^3}-$$

**Mathematica [A]** time = 0.09, size = 123, normalized size = 0.78

$$-\frac{\frac{a\left(10a^8-45a^7bx+360a^6b^2x^2+6174a^5b^3x^3+21924a^4b^4x^4+35910a^3b^5x^5+31080a^2b^6x^6+13860ab^7x^7+2520b^8x^8\right)}{x^3(a+bx)^6}-2520b^3\log(a+bx)+2520b^3\log(x)}{30a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^7),x]
```

```
[Out] -1/30*((a*(10*a^8 - 45*a^7*b*x + 360*a^6*b^2*x^2 + 6174*a^5*b^3*x^3 + 21924
*a^4*b^4*x^4 + 35910*a^3*b^5*x^5 + 31080*a^2*b^6*x^6 + 13860*a*b^7*x^7 + 25
20*b^8*x^8))/(x^3*(a + b*x)^6) + 2520*b^3*Log[x] - 2520*b^3*Log[a + b*x])/a
^10
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x)^7),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)^7), x]
```

**fricas [B]** time = 1.29, size = 317, normalized size = 2.02

$$-\frac{2520\,ab^8x^8+13860\,a^2b^7x^7+31080\,a^3b^6x^6+35910\,a^4b^5x^5+21924\,a^5b^4x^4+6174\,a^6b^3x^3+360\,a^7b^2x^2-45\,a^8bx+10\,a^9-2520\left(b^9x^9+6\,ab^8x^8+15\,a^2b^7x^7+20\,a^3b^6x^6+15\,a^4b^5x^5+6\,a^5b^4x^4+a^6b^3x^3\right)\log\left(bx+a\right)+2520\left(b^9x^9+6\,ab^8x^8+15\,a^2b^7x^7+20\,a^3b^6x^6+15\,a^4b^5x^5+6\,a^5b^4x^4+a^6b^3x^3\right)\log\left(x\right)}{30\left(a^{10}b^6x^9+6\,a^{11}b^5x^8+15\,a^{12}b^4x^7+20\,a^{13}b^3x^6+15\,a^{14}b^2x^5+6\,a^{15}bx^4+a^{16}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(2520*a*b^8*x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*b
^5*x^5 + 21924*a^5*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*b*
x + 10*a^9 - 2520*(b^9*x^9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x^6
+ 15*a^4*b^5*x^5 + 6*a^5*b^4*x^4 + a^6*b^3*x^3)*log(b*x + a) + 2520*(b^9*x^
9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x^6 + 15*a^4*b^5*x^5 + 6*a^5*
b^4*x^4 + a^6*b^3*x^3)*log(x))/(a^10*b^6*x^9 + 6*a^11*b^5*x^8 + 15*a^12*b^4
*x^7 + 20*a^13*b^3*x^6 + 15*a^14*b^2*x^5 + 6*a^15*b*x^4 + a^16*x^3)
```

**giac [A]** time = 1.25, size = 130, normalized size = 0.83

$$\frac{84\,b^3\log\left(|bx+a|\right)}{a^{10}}-\frac{84\,b^3\log\left(|x|\right)}{a^{10}}-\frac{2520\,ab^8x^8+13860\,a^2b^7x^7+31080\,a^3b^6x^6+35910\,a^4b^5x^5+21924\,a^5b^4x^4+6174\,a^6b^3x^3+360\,a^7b^2x^2-45\,a^8bx+10\,a^9}{30\,(bx+a)^6a^{10}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 84*b^3*log(abs(b*x + a))/a^10 - 84*b^3*log(abs(x))/a^10 - 1/30*(2520*a*b^8*
x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*b^5*x^5 + 21924*a^5
*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*b*x + 10*a^9)/((b*x
+ a)^6*a^10*x^3)
```

**maple** [A] time = 0.01, size = 144, normalized size = 0.92

$$-\frac{b^3}{6\,(bx+a)^6\,a^4}-\frac{4b^3}{5\,(bx+a)^5\,a^5}-\frac{5b^3}{2\,(bx+a)^4\,a^6}-\frac{20b^3}{3\,(bx+a)^3\,a^7}-\frac{35b^3}{2\,(bx+a)^2\,a^8}-\frac{56b^3}{(bx+a)\,a^9}-\frac{84b^3\ln(x)}{a^{10}}+\frac{84b^3\ln(bx+a)}{a^{10}}-\frac{28b^2}{a^9x}+\frac{7b}{2a^8x^2}-\frac{1}{3a^7x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^7,x)
```

```
[Out] -1/3/a^7/x^3+7/2*b/a^8/x^2-28*b^2/a^9/x-1/6*b^3/a^4/(b*x+a)^6-4/5*b^3/a^5/(
b*x+a)^5-5/2*b^3/a^6/(b*x+a)^4-20/3*b^3/a^7/(b*x+a)^3-35/2*b^3/a^8/(b*x+a)^
2-56*b^3/a^9/(b*x+a)-84*b^3*ln(x)/a^10+84*b^3*ln(b*x+a)/a^10
```

**maxima** [A] time = 1.59, size = 185, normalized size = 1.18

$$-\frac{2520\,b^8x^8+13860\,ab^7x^7+31080\,a^2b^6x^6+35910\,a^3b^5x^5+21924\,a^4b^4x^4+6174\,a^5b^3x^3+360\,a^6b^2x^2-45\,a^7bx+10\,a^8}{30\left(a^9b^6x^9+6\,a^{10}b^5x^8+15\,a^{11}b^4x^7+20\,a^{12}b^3x^6+15\,a^{13}b^2x^5+6\,a^{14}bx^4+a^{15}x^3\right)}+\frac{84\,b^3\log(bx+a)}{a^{10}}-\frac{84\,b^3\log(x)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(2520*b^8*x^8 + 13860*a*b^7*x^7 + 31080*a^2*b^6*x^6 + 35910*a^3*b^5*x
^5 + 21924*a^4*b^4*x^4 + 6174*a^5*b^3*x^3 + 360*a^6*b^2*x^2 - 45*a^7*b*x +
10*a^8)/(a^9*b^6*x^9 + 6*a^10*b^5*x^8 + 15*a^11*b^4*x^7 + 20*a^12*b^3*x^6 +
 15*a^13*b^2*x^5 + 6*a^14*b*x^4 + a^15*x^3) + 84*b^3*log(b*x + a)/a^10 - 84
*b^3*log(x)/a^10
```

**mupad** [B] time = 0.31, size = 179, normalized size = 1.14

$$\frac{168\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{10}}-\frac{\frac{1}{3\,a}+\frac{12\,b^2\,x^2}{a^3}+\frac{1029\,b^3\,x^3}{5\,a^4}+\frac{3654\,b^4\,x^4}{5\,a^5}+\frac{1197\,b^5\,x^5}{a^6}+\frac{1036\,b^6\,x^6}{a^7}+\frac{462\,b^7\,x^7}{a^8}+\frac{84\,b^8\,x^8}{a^9}-\frac{3\,b\,x}{2\,a^2}}{a^6\,x^3+6\,a^5\,b\,x^4+15\,a^4\,b^2\,x^5+20\,a^3\,b^3\,x^6+15\,a^2\,b^4\,x^7+6\,a\,b^5\,x^8+b^6\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^7),x)
```

```
[Out] (168*b^3*atanh((2*b*x)/a + 1))/a^10 - (1/(3*a) + (12*b^2*x^2)/a^3 + (1029*b
^3*x^3)/(5*a^4) + (3654*b^4*x^4)/(5*a^5) + (1197*b^5*x^5)/a^6 + (1036*b^6*x
^6)/a^7 + (462*b^7*x^7)/a^8 + (84*b^8*x^8)/a^9 - (3*b*x)/(2*a^2))/(a^6*x^3
+ b^6*x^9 + 6*a^5*b*x^4 + 6*a*b^5*x^8 + 15*a^4*b^2*x^5 + 20*a^3*b^3*x^6 + 1
5*a^2*b^4*x^7)
```

**sympy** [A] time = 0.99, size = 187, normalized size = 1.19

$$\frac{-10a^8+45a^7bx-360a^6b^2x^2-6174a^5b^3x^3-21924a^4b^4x^4-35910a^3b^5x^5-31080a^2b^6x^6-13860ab^7x^7-2520b^8x^8}{30a^{15}x^3+180a^{14}bx^4+450a^{13}b^2x^5+600a^{12}b^3x^6+450a^{11}b^4x^7+180a^{10}b^5x^8+30a^9b^6x^9}+\frac{84b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**7,x)
```

```
[Out] (-10*a**8 + 45*a**7*b*x - 360*a**6*b**2*x**2 - 6174*a**5*b**3*x**3 - 21924*
a**4*b**4*x**4 - 35910*a**3*b**5*x**5 - 31080*a**2*b**6*x**6 - 13860*a*b**7
*x**7 - 2520*b**8*x**8)/(30*a**15*x**3 + 180*a**14*b*x**4 + 450*a**13*b**2*
x**5 + 600*a**12*b**3*x**6 + 450*a**11*b**4*x**7 + 180*a**10*b**5*x**8 + 30
*a**9*b**6*x**9) + 84*b**3*(-log(x) + log(a/b + x))/a**10
```

## 3.222 $\int \frac{x^{12}}{(a+bx)^{10}}\,dx$

**Optimal**. Leaf size=186

$$-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a^8}{b^{13}(a+bx)^5}+\frac{198a^7}{b^{13}(a+bx)^4}-\frac{308a^6}{b^{13}(a+bx)^3}+\frac{39}{b^{13}(a}$$

**Rubi** [A] time = 0.18, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a^8}{b^{13}(a+bx)^5}+\frac{198a^7}{b^{13}(a+bx)^4}-\frac{308a^6}{b^{13}(a+bx)^3}+\frac{396a^5}{b^{13}(a+bx)^2}-\frac{495a^4}{b^{13}(a+bx)}+\frac{55a^2x}{b^{12}}-\frac{220a^3\log(a+bx)}{b^{13}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x)^10,x]
```

```
[Out] (55*a^2*x)/b^12 - (5*a*x^2)/b^11 + x^3/(3*b^10) - a^12/(9*b^13*(a + b*x)^9)
 + (3*a^11)/(2*b^13*(a + b*x)^8) - (66*a^10)/(7*b^13*(a + b*x)^7) + (110*a^
9)/(3*b^13*(a + b*x)^6) - (99*a^8)/(b^13*(a + b*x)^5) + (198*a^7)/(b^13*(a
+ b*x)^4) - (308*a^6)/(b^13*(a + b*x)^3) + (396*a^5)/(b^13*(a + b*x)^2) - (
495*a^4)/(b^13*(a + b*x)) - (220*a^3*Log[a + b*x])/b^13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx = \int \left(\frac{55a^2}{b^{12}}-\frac{10ax}{b^{11}}+\frac{x^2}{b^{10}}+\frac{a^{12}}{b^{12}(a+bx)^{10}}-\frac{12a^{11}}{b^{12}(a+bx)^9}+\frac{66a^{10}}{b^{12}(a+bx)^8}-\frac{220a^9}{b^{12}(a+bx)^7}+\frac{4}{b^{12}(}\right.$$
$$= \frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{}{b}$$

**Mathematica** [A] time = 0.05, size = 161, normalized size = 0.87

$$-\frac{35201a^{12}+289089a^{11}bx+1031616a^{10}b^2x^2+2074464a^9b^3x^3+2529576a^8b^4x^4+1831032a^7b^5x^5+638568a^6b^6x^6-58968a^5b^7x^7-139482a^4b^8x^8-43218a^3b^9x^9+27720a^3(a+bx)^9\log(a+bx)-2772a^2b^{10}x^{10}+252ab^{11}x^{11}-42b^{12}x^{12}}{126b^{13}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x)^10,x]
```

```
[Out] -1/126*(35201*a^12 + 289089*a^11*b*x + 1031616*a^10*b^2*x^2 + 2074464*a^9*b
^3*x^3 + 2529576*a^8*b^4*x^4 + 1831032*a^7*b^5*x^5 + 638568*a^6*b^6*x^6 - 5
8968*a^5*b^7*x^7 - 139482*a^4*b^8*x^8 - 43218*a^3*b^9*x^9 - 2772*a^2*b^10*x
^10 + 252*a*b^11*x^11 - 42*b^12*x^12 + 27720*a^3*(a + b*x)^9*Log[a + b*x])/
(b^13*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^12/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^12/(a + b*x)^10, x]
```

**fricas [A]** time = 1.20, size = 338, normalized size = 1.82

$$\frac{42 b^{12}x^{12} - 252 ab^{11}x^{11} + 2772 a^2b^{10}x^{10} + 43218 a^3b^9x^9 + 139482 a^4b^8x^8 + 58968 a^5b^7x^7 - 638568 a^6b^6x^6 - 1831032 a^7b^5x^5 - 2529576 a^8b^4x^4 - 2074464 a^9b^3x^3 - 1031616 a^{10}b^2x^2 - 289089 a^{11}bx - 35201 a^{12} - 27720 \left(a^3b^9x^9 + 9 a^4b^8x^8 + 36 a^5b^7x^7 + 84 a^6b^6x^6 + 126 a^7b^5x^5 + 126 a^8b^4x^4 + 84 a^9b^3x^3 + 36 a^{10}b^2x^2 + 9 a^{11}bx + a^{12}\right)\log\left(bx+a\right)}{126 \left(b^{22}x^9 + 9 ab^{21}x^8 + 36 a^2b^{20}x^7 + 84 a^3b^{19}x^6 + 126 a^4b^{18}x^5 + 126 a^5b^{17}x^4 + 84 a^6b^{16}x^3 + 36 a^7b^{15}x^2 + 9 a^8b^{14}x + a^9b^{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/126*(42*b^12*x^12 - 252*a*b^11*x^11 + 2772*a^2*b^10*x^10 + 43218*a^3*b^9*
x^9 + 139482*a^4*b^8*x^8 + 58968*a^5*b^7*x^7 - 638568*a^6*b^6*x^6 - 1831032
*a^7*b^5*x^5 - 2529576*a^8*b^4*x^4 - 2074464*a^9*b^3*x^3 - 1031616*a^10*b^2
*x^2 - 289089*a^11*b*x - 35201*a^12 - 27720*(a^3*b^9*x^9 + 9*a^4*b^8*x^8 +
36*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 126*a^7*b^5*x^5 + 126*a^8*b^4*x^4 + 84*a^
9*b^3*x^3 + 36*a^10*b^2*x^2 + 9*a^11*b*x + a^12)*log(b*x + a))/(b^22*x^9 +
9*a*b^21*x^8 + 36*a^2*b^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5 + 126*a
^5*b^17*x^4 + 84*a^6*b^16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^9*b^13)
```

**giac [A]** time = 1.07, size = 149, normalized size = 0.80

$$-\frac{220 a^3 \log\left(|bx+a|\right)}{b^{13}} - \frac{62370 a^4b^8x^8 + 449064 a^5b^7x^7 + 1435896 a^6b^6x^6 + 2652804 a^7b^5x^5 + 3089394 a^8b^4x^4 + 2318316 a^9b^3x^3 + 1093356 a^{10}b^2x^2 + 296019 a^{11}bx + 35201 a^{12}}{126 \left(bx+a\right)^9 b^{13}} + \frac{b^{20}x^3 - 15 ab^{19}x^2 + 165 a^2b^{18}x}{3 b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -220*a^3*log(abs(b*x + a))/b^13 - 1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^7
*x^7 + 1435896*a^6*b^6*x^6 + 2652804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 + 23
18316*a^9*b^3*x^3 + 1093356*a^10*b^2*x^2 + 296019*a^11*b*x + 35201*a^12)/((
b*x + a)^9*b^13) + 1/3*(b^20*x^3 - 15*a*b^19*x^2 + 165*a^2*b^18*x)/b^30
```

**maple [A]** time = 0.01, size = 177, normalized size = 0.95

$$-\frac{a^{12}}{9\left(bx+a\right)^{9}b^{13}}+\frac{3a^{11}}{2\left(bx+a\right)^{8}b^{13}}-\frac{66a^{10}}{7\left(bx+a\right)^{7}b^{13}}+\frac{110a^{9}}{3\left(bx+a\right)^{6}b^{13}}-\frac{99a^{8}}{\left(bx+a\right)^{5}b^{13}}+\frac{198a^{7}}{\left(bx+a\right)^{4}b^{13}}-\frac{308a^{6}}{\left(bx+a\right)^{3}b^{13}}+\frac{x^{3}}{3b^{10}}+\frac{396a^{5}}{\left(bx+a\right)^{2}b^{13}}-\frac{5a\,x^{2}}{b^{11}}-\frac{495a^{4}}{\left(bx+a\right)b^{13}}-\frac{220a^{3}\ln\left(bx+a\right)}{b^{13}}+\frac{55a^{2}x}{b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x+a)^10,x)
```

```
[Out] 55*a^2*x/b^12-5*a*x^2/b^11+1/3*x^3/b^10-1/9*a^12/b^13/(b*x+a)^9+3/2*a^11/b^
13/(b*x+a)^8-66/7*a^10/b^13/(b*x+a)^7+110/3*a^9/b^13/(b*x+a)^6-99*a^8/b^13/
(b*x+a)^5+198*a^7/b^13/(b*x+a)^4-308*a^6/b^13/(b*x+a)^3+396*a^5/b^13/(b*x+a
)^2-495*a^4/b^13/(b*x+a)-220*a^3*ln(b*x+a)/b^13
```

**maxima [A]** time = 1.73, size = 234, normalized size = 1.26

$$-\frac{62370\,a^{4}b^{8}x^{8}+449064\,a^{5}b^{7}x^{7}+1435896\,a^{6}b^{6}x^{6}+2652804\,a^{7}b^{5}x^{5}+3089394\,a^{8}b^{4}x^{4}+2318316\,a^{9}b^{3}x^{3}+1093356\,a^{10}b^{2}x^{2}+296019\,a^{11}bx+35201\,a^{12}}{126\left(b^{22}x^{9}+9\,ab^{21}x^{8}+36\,a^{2}b^{20}x^{7}+84\,a^{3}b^{19}x^{6}+126\,a^{4}b^{18}x^{5}+126\,a^{5}b^{17}x^{4}+84\,a^{6}b^{16}x^{3}+36\,a^{7}b^{15}x^{2}+9\,a^{8}b^{14}x+a^{9}b^{13}\right)}-\frac{220\,a^{3}\log\left(bx+a\right)}{b^{13}}+\frac{b^{2}x^{3}-15\,abx^{2}+165\,a^{2}x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^7*x^7 + 1435896*a^6*b^6*x^6 + 2652
804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 + 2318316*a^9*b^3*x^3 + 1093356*a^10*
b^2*x^2 + 296019*a^11*b*x + 35201*a^12)/(b^22*x^9 + 9*a*b^21*x^8 + 36*a^2*b
^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5 + 126*a^5*b^17*x^4 + 84*a^6*b^
16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^9*b^13) - 220*a^3*log(b*x + a)/
b^13 + 1/3*(b^2*x^3 - 15*a*b*x^2 + 165*a^2*x)/b^12
```

**mupad [B]** time = 0.98, size = 151, normalized size = 0.81

$$-\frac{6\,a\,{\left(a+b\,x\right)}^2-\frac{{\left(a+b\,x\right)}^3}{3}+\frac{495\,a^4}{a+b\,x}-\frac{396\,a^5}{{\left(a+b\,x\right)}^2}+\frac{308\,a^6}{{\left(a+b\,x\right)}^3}-\frac{198\,a^7}{{\left(a+b\,x\right)}^4}+\frac{99\,a^8}{{\left(a+b\,x\right)}^5}-\frac{110\,a^9}{3\,{\left(a+b\,x\right)}^6}+\frac{66\,a^{10}}{7\,{\left(a+b\,x\right)}^7}-\frac{3\,a^{11}}{2\,{\left(a+b\,x\right)}^8}+\frac{a^{12}}{9\,{\left(a+b\,x\right)}^9}+220\,a^3\,\ln\left(a+b\,x\right)-66\,a^2\,b\,x}{b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x)^10,x)
```

```
[Out] -(6*a*(a + b*x)^2 - (a + b*x)^3/3 + (495*a^4)/(a + b*x) - (396*a^5)/(a + b*
x)^2 + (308*a^6)/(a + b*x)^3 - (198*a^7)/(a + b*x)^4 + (99*a^8)/(a + b*x)^5
 - (110*a^9)/(3*(a + b*x)^6) + (66*a^10)/(7*(a + b*x)^7) - (3*a^11)/(2*(a +
 b*x)^8) + a^12/(9*(a + b*x)^9) + 220*a^3*log(a + b*x) - 66*a^2*b*x)/b^13
```

**sympy [A]** time = 1.54, size = 250, normalized size = 1.34

$$-\frac{220a^{3}\log\left(a+bx\right)}{b^{13}}+\frac{55a^{2}x}{b^{12}}-\frac{5ax^{2}}{b^{11}}+\frac{-35201a^{12}-296019a^{11}bx-1093356a^{10}b^{2}x^{2}-2318316a^{9}b^{3}x^{3}-3089394a^{8}b^{4}x^{4}-2652804a^{7}b^{5}x^{5}-1435896a^{6}b^{6}x^{6}-449064a^{5}b^{7}x^{7}-62370a^{4}b^{8}x^{8}}{126a^{9}b^{13}+1134a^{8}b^{14}x+4536a^{7}b^{15}x^{2}+10584a^{6}b^{16}x^{3}+15876a^{5}b^{17}x^{4}+15876a^{4}b^{18}x^{5}+10584a^{3}b^{19}x^{6}+4536a^{2}b^{20}x^{7}+1134ab^{21}x^{8}+126b^{22}x^{9}}+\frac{x^{3}}{3b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x+a)**10,x)
```

```
[Out] -220*a**3*log(a + b*x)/b**13 + 55*a**2*x/b**12 - 5*a*x**2/b**11 + (-35201*a
**12 - 296019*a**11*b*x - 1093356*a**10*b**2*x**2 - 2318316*a**9*b**3*x**3
- 3089394*a**8*b**4*x**4 - 2652804*a**7*b**5*x**5 - 1435896*a**6*b**6*x**6
- 449064*a**5*b**7*x**7 - 62370*a**4*b**8*x**8)/(126*a**9*b**13 + 1134*a**8
*b**14*x + 4536*a**7*b**15*x**2 + 10584*a**6*b**16*x**3 + 15876*a**5*b**17*
x**4 + 15876*a**4*b**18*x**5 + 10584*a**3*b**19*x**6 + 4536*a**2*b**20*x**7
 + 1134*a*b**21*x**8 + 126*b**22*x**9) + x**3/(3*b**10)
```

## 3.223 $\int \frac{x^{11}}{(a+bx)^{10}} \, dx$

**Optimal**. Leaf size=177

$$\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{231a^6}{2b^{12}(a+bx)^4}+\frac{154a^5}{b^{12}(a+bx)^3}-\frac{16}{b^{12}(a}$$

**Rubi** [A] time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{231a^6}{2b^{12}(a+bx)^4}+\frac{154a^5}{b^{12}(a+bx)^3}-\frac{165a^4}{b^{12}(a+bx)^2}+\frac{165a^3}{b^{12}(a+bx)}+\frac{55a^2\log(a+bx)}{b^{12}}-\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x)^10,x]
```

```
[Out] (-10*a*x)/b^11 + x^2/(2*b^10) + a^11/(9*b^12*(a + b*x)^9) - (11*a^10)/(8*b^
12*(a + b*x)^8) + (55*a^9)/(7*b^12*(a + b*x)^7) - (55*a^8)/(2*b^12*(a + b*x
)^6) + (66*a^7)/(b^12*(a + b*x)^5) - (231*a^6)/(2*b^12*(a + b*x)^4) + (154*
a^5)/(b^12*(a + b*x)^3) - (165*a^4)/(b^12*(a + b*x)^2) + (165*a^3)/(b^12*(a
 + b*x)) + (55*a^2*Log[a + b*x])/b^12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{(a+bx)^{10}} \, dx &= \int \left(-\frac{10a}{b^{11}}+\frac{x}{b^{10}}-\frac{a^{11}}{b^{11}(a+bx)^{10}}+\frac{11a^{10}}{b^{11}(a+bx)^9}-\frac{55a^9}{b^{11}(a+bx)^8}+\frac{165a^8}{b^{11}(a+bx)^7}-\frac{330a^7}{b^{11}(a+bx)}\right.\\ &= -\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}+\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+b}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 150, normalized size = 0.85

$$\frac{42131a^{11}+351459a^{10}bx+1281096a^9b^2x^2+2656584a^8b^3x^3+3402756a^7b^4x^4+2704212a^6b^5x^5+1220688a^5b^6x^6+190512a^4b^7x^7-77112a^3b^8x^8-36288a^2b^9x^9+27720a^2(a+bx)^9\log(a+bx)-2772ab^{10}x^{10}+252b^{11}x^{11}}{504b^{12}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x)^10,x]
```

```
[Out] (42131*a^11 + 351459*a^10*b*x + 1281096*a^9*b^2*x^2 + 2656584*a^8*b^3*x^3 +
 3402756*a^7*b^4*x^4 + 2704212*a^6*b^5*x^5 + 1220688*a^5*b^6*x^6 + 190512*a
^4*b^7*x^7 - 77112*a^3*b^8*x^8 - 36288*a^2*b^9*x^9 - 2772*a*b^10*x^10 + 252
*b^11*x^11 + 27720*a^2*(a + b*x)^9*Log[a + b*x])/(504*b^12*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^11/(a + b*x)^10, x]
```

**fricas [A]** time = 0.68, size = 327, normalized size = 1.85

$$\frac{252\,b^{11}x^{11} - 2772\,ab^{10}x^{10} - 36288\,a^2b^9x^9 - 77112\,a^3b^8x^8 + 190512\,a^4b^7x^7 + 1220688\,a^5b^6x^6 + 2704212\,a^6b^5x^5 + 3402756\,a^7b^4x^4 + 2656584\,a^8b^3x^3 + 1281096\,a^9b^2x^2 + 351459\,a^{10}bx + 42131\,a^{11} + 27720\left(a^2b^9x^9 + 9\,a^3b^8x^8 + 36\,a^4b^7x^7 + 84\,a^5b^6x^6 + 126\,a^6b^5x^5 + 126\,a^7b^4x^4 + 84\,a^8b^3x^3 + 36\,a^9b^2x^2 + 9\,a^{10}bx + a^{11}\right)\log\left(bx+a\right)}{504\left(b^{21}x^9 + 9\,ab^{20}x^8 + 36\,a^2b^{19}x^7 + 84\,a^3b^{18}x^6 + 126\,a^4b^{17}x^5 + 126\,a^5b^{16}x^4 + 84\,a^6b^{15}x^3 + 36\,a^7b^{14}x^2 + 9\,a^8b^{13}x + a^9b^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^11*x^11 - 2772*a*b^10*x^10 - 36288*a^2*b^9*x^9 - 77112*a^3*b^8
*x^8 + 190512*a^4*b^7*x^7 + 1220688*a^5*b^6*x^6 + 2704212*a^6*b^5*x^5 + 340
2756*a^7*b^4*x^4 + 2656584*a^8*b^3*x^3 + 1281096*a^9*b^2*x^2 + 351459*a^10*
b*x + 42131*a^11 + 27720*(a^2*b^9*x^9 + 9*a^3*b^8*x^8 + 36*a^4*b^7*x^7 + 84
*a^5*b^6*x^6 + 126*a^6*b^5*x^5 + 126*a^7*b^4*x^4 + 84*a^8*b^3*x^3 + 36*a^9*
b^2*x^2 + 9*a^10*b*x + a^11)*log(b*x + a))/(b^21*x^9 + 9*a*b^20*x^8 + 36*a^
2*b^19*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 + 84*a^6
*b^15*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12)
```

**giac [A]** time = 1.70, size = 138, normalized size = 0.78

$$\frac{55\,a^2\log\left(|bx+a|\right)}{b^{12}} + \frac{b^{10}x^2 - 20\,ab^9x}{2\,b^{20}} + \frac{83160\,a^3b^8x^8 + 582120\,a^4b^7x^7 + 1823976\,a^5b^6x^6 + 3318084\,a^6b^5x^5 + 3817044\,a^7b^4x^4 + 2835756\,a^8b^3x^3 + 1326204\,a^9b^2x^2 + 356499\,a^{10}bx + 42131\,a^{11}}{504\,(bx+a)^9b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 55*a^2*log(abs(b*x + a))/b^12 + 1/2*(b^10*x^2 - 20*a*b^9*x)/b^20 + 1/504*(8
3160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 3318084*a^6*b
^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*b^2*x^2 +
356499*a^10*b*x + 42131*a^11)/((b*x + a)^9*b^12)
```

**maple [A]** time = 0.01, size = 166, normalized size = 0.94

$$\frac{a^{11}}{9\left(bx+a\right)^{9} b^{12}}-\frac{11 a^{10}}{8\left(bx+a\right)^{8} b^{12}}+\frac{55 a^{9}}{7\left(bx+a\right)^{7} b^{12}}-\frac{55 a^{8}}{2\left(bx+a\right)^{6} b^{12}}+\frac{66 a^{7}}{\left(bx+a\right)^{5} b^{12}}-\frac{231 a^{6}}{2\left(bx+a\right)^{4} b^{12}}+\frac{154 a^{5}}{\left(bx+a\right)^{3} b^{12}}-\frac{165 a^{4}}{\left(bx+a\right)^{2} b^{12}}+\frac{x^{2}}{2 b^{10}}+\frac{165 a^{3}}{\left(bx+a\right) b^{12}}+\frac{55 a^{2} \ln\left(bx+a\right)}{b^{12}}-\frac{10 a x}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x+a)^10,x)
```

```
[Out] -10*a*x/b^11+1/2*x^2/b^10+1/9*a^11/b^12/(b*x+a)^9-11/8*a^10/b^12/(b*x+a)^8+
55/7*a^9/b^12/(b*x+a)^7-55/2*a^8/b^12/(b*x+a)^6+66*a^7/b^12/(b*x+a)^5-231/2
*a^6/b^12/(b*x+a)^4+154*a^5/b^12/(b*x+a)^3-165*a^4/b^12/(b*x+a)^2+165*a^3/b
^12/(b*x+a)+55*a^2*ln(b*x+a)/b^12
```

**maxima [A]** time = 1.65, size = 223, normalized size = 1.26

$$\frac{83160\, a^{3} b^{8} x^{8}+582120\, a^{4} b^{7} x^{7}+1823976\, a^{5} b^{6} x^{6}+3318084\, a^{6} b^{5} x^{5}+3817044\, a^{7} b^{4} x^{4}+2835756\, a^{8} b^{3} x^{3}+1326204\, a^{9} b^{2} x^{2}+356499\, a^{10} b x+42131\, a^{11}}{504\left(b^{21} x^{9}+9\, a b^{20} x^{8}+36\, a^{2} b^{19} x^{7}+84\, a^{3} b^{18} x^{6}+126\, a^{4} b^{17} x^{5}+126\, a^{5} b^{16} x^{4}+84\, a^{6} b^{15} x^{3}+36\, a^{7} b^{14} x^{2}+9\, a^{8} b^{13} x+a^{9} b^{12}\right)}+\frac{55\, a^{2} \log\left(bx+a\right)}{b^{12}}+\frac{b x^{2}-20\, a x}{2\, b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/504*(83160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 33180
84*a^6*b^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*b^
2*x^2 + 356499*a^10*b*x + 42131*a^11)/(b^21*x^9 + 9*a*b^20*x^8 + 36*a^2*b^1
9*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 + 84*a^6*b^15
*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12) + 55*a^2*log(b*x + a)/b^1
2 + 1/2*(b*x^2 - 20*a*x)/b^11
```

**mupad [B]** time = 0.23, size = 138, normalized size = 0.78

$$\frac{\frac{(a+b\,x)^2}{2}+\frac{165\,a^3}{a+b\,x}-\frac{165\,a^4}{(a+b\,x)^2}+\frac{154\,a^5}{(a+b\,x)^3}-\frac{231\,a^6}{2\,(a+b\,x)^4}+\frac{66\,a^7}{(a+b\,x)^5}-\frac{55\,a^8}{2\,(a+b\,x)^6}+\frac{55\,a^9}{7\,(a+b\,x)^7}-\frac{11\,a^{10}}{8\,(a+b\,x)^8}+\frac{a^{11}}{9\,(a+b\,x)^9}+55\,a^2\,\ln\left(a+b\,x\right)-11\,a\,b\,x}{b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x)^10,x)
```

```
[Out] ((a + b*x)^2/2 + (165*a^3)/(a + b*x) - (165*a^4)/(a + b*x)^2 + (154*a^5)/(a
 + b*x)^3 - (231*a^6)/(2*(a + b*x)^4) + (66*a^7)/(a + b*x)^5 - (55*a^8)/(2*
(a + b*x)^6) + (55*a^9)/(7*(a + b*x)^7) - (11*a^10)/(8*(a + b*x)^8) + a^11/
(9*(a + b*x)^9) + 55*a^2*log(a + b*x) - 11*a*b*x)/b^12
```

**sympy [A]** time = 1.48, size = 236, normalized size = 1.33

$$\frac{55a^{2}\log\left(a+bx\right)}{b^{12}}-\frac{10ax}{b^{11}}+\frac{42131a^{11}+356499a^{10}bx+1326204a^{9}b^{2}x^{2}+2835756a^{8}b^{3}x^{3}+3817044a^{7}b^{4}x^{4}+3318084a^{6}b^{5}x^{5}+1823976a^{5}b^{6}x^{6}+582120a^{4}b^{7}x^{7}+83160a^{3}b^{8}x^{8}}{504a^{9}b^{12}+4536a^{8}b^{13}x+18144a^{7}b^{14}x^{2}+42336a^{6}b^{15}x^{3}+63504a^{5}b^{16}x^{4}+63504a^{4}b^{17}x^{5}+42336a^{3}b^{18}x^{6}+18144a^{2}b^{19}x^{7}+4536ab^{20}x^{8}+504b^{21}x^{9}}+\frac{x^{2}}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x+a)**10,x)
```

```
[Out] 55*a**2*log(a + b*x)/b**12 - 10*a*x/b**11 + (42131*a**11 + 356499*a**10*b*x
 + 1326204*a**9*b**2*x**2 + 2835756*a**8*b**3*x**3 + 3817044*a**7*b**4*x**4
 + 3318084*a**6*b**5*x**5 + 1823976*a**5*b**6*x**6 + 582120*a**4*b**7*x**7
+ 83160*a**3*b**8*x**8)/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b*
*14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17
*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 +
 504*b**21*x**9) + x**2/(2*b**10)
```

## 3.224 $\int \frac{x^{10}}{(a+bx)^{10}} dx$

**Optimal**. Leaf size=159

$$-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)^4}-\frac{70a^4}{b^{11}(a+bx)^3}+\frac{60}{b^{11}(a}$$

**Rubi** [A] time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)^4}-\frac{70a^4}{b^{11}(a+bx)^3}+\frac{60a^3}{b^{11}(a+bx)^2}-\frac{45a^2}{b^{11}(a+bx)}-\frac{10a\log(a+bx)}{b^{11}}+\frac{x}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x)^10,x]
```

```
[Out] x/b^10 - a^10/(9*b^11*(a + b*x)^9) + (5*a^9)/(4*b^11*(a + b*x)^8) - (45*a^8
)/(7*b^11*(a + b*x)^7) + (20*a^7)/(b^11*(a + b*x)^6) - (42*a^6)/(b^11*(a +
b*x)^5) + (63*a^5)/(b^11*(a + b*x)^4) - (70*a^4)/(b^11*(a + b*x)^3) + (60*a
^3)/(b^11*(a + b*x)^2) - (45*a^2)/(b^11*(a + b*x)) - (10*a*Log[a + b*x])/b^
11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{(a+bx)^{10}} dx &= \int \left(\frac{1}{b^{10}}+\frac{a^{10}}{b^{10}(a+bx)^{10}}-\frac{10a^9}{b^{10}(a+bx)^9}+\frac{45a^8}{b^{10}(a+bx)^8}-\frac{120a^7}{b^{10}(a+bx)^7}+\frac{210a^6}{b^{10}(a+bx)^6}-\frac{2}{b^{10}(}\right.\\ &= \frac{x}{b^{10}}-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{6}{b^{11}(a}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 137, normalized size = 0.86

$$-\frac{4861a^{10}+41229a^9bx+153576a^8b^2x^2+328104a^7b^3x^3+439236a^6b^4x^4+375732a^5b^5x^5+197568a^4b^6x^6+54432a^3b^7x^7+2268a^2b^8x^8-2268ab^9x^9+2520a(a+bx)^9\log(a+bx)-252b^{10}x^{10}}{252b^{11}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x)^10,x]
```

```
[Out] -1/252*(4861*a^10 + 41229*a^9*b*x + 153576*a^8*b^2*x^2 + 328104*a^7*b^3*x^3
 + 439236*a^6*b^4*x^4 + 375732*a^5*b^5*x^5 + 197568*a^4*b^6*x^6 + 54432*a^3
*b^7*x^7 + 2268*a^2*b^8*x^8 - 2268*a*b^9*x^9 - 252*b^10*x^10 + 2520*a*(a +
b*x)^9*Log[a + b*x])/(b^11*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^10/(a + b*x)^10, x]
```

**fricas [B]** time = 1.42, size = 314, normalized size = 1.97

$$\frac{252\,b^{10}x^{10}+2268\,ab^{9}x^{9}-2268\,a^{2}b^{8}x^{8}-54432\,a^{3}b^{7}x^{7}-197568\,a^{4}b^{6}x^{6}-375732\,a^{5}b^{5}x^{5}-439236\,a^{6}b^{4}x^{4}-328104\,a^{7}b^{3}x^{3}-153576\,a^{8}b^{2}x^{2}-41229\,a^{9}bx-4861\,a^{10}-2520\left(ab^{9}x^{9}+9\,a^{2}b^{8}x^{8}+36\,a^{3}b^{7}x^{7}+84\,a^{4}b^{6}x^{6}+126\,a^{5}b^{5}x^{5}+126\,a^{6}b^{4}x^{4}+84\,a^{7}b^{3}x^{3}+36\,a^{8}b^{2}x^{2}+9\,a^{9}bx+a^{10}\right)\log\left(bx+a\right)}{252\left(b^{20}x^{9}+9\,ab^{19}x^{8}+36\,a^{2}b^{18}x^{7}+84\,a^{3}b^{17}x^{6}+126\,a^{4}b^{16}x^{5}+126\,a^{5}b^{15}x^{4}+84\,a^{6}b^{14}x^{3}+36\,a^{7}b^{13}x^{2}+9\,a^{8}b^{12}x+a^{9}b^{11}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2268*a*b^9*x^9 - 2268*a^2*b^8*x^8 - 54432*a^3*b^7*x^
7 - 197568*a^4*b^6*x^6 - 375732*a^5*b^5*x^5 - 439236*a^6*b^4*x^4 - 328104*a
^7*b^3*x^3 - 153576*a^8*b^2*x^2 - 41229*a^9*b*x - 4861*a^10 - 2520*(a*b^9*x
^9 + 9*a^2*b^8*x^8 + 36*a^3*b^7*x^7 + 84*a^4*b^6*x^6 + 126*a^5*b^5*x^5 + 12
6*a^6*b^4*x^4 + 84*a^7*b^3*x^3 + 36*a^8*b^2*x^2 + 9*a^9*b*x + a^10)*log(b*x
 + a))/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7 + 84*a^3*b^17*x^6 + 126*a
^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3 + 36*a^7*b^13*x^2 + 9*a^8*
b^12*x + a^9*b^11)
```

**giac [A]** time = 1.29, size = 121, normalized size = 0.76

$$\frac{x}{b^{10}}-\frac{10\,a\log\left(|bx+a|\right)}{b^{11}}-\frac{11340\,a^{2}b^{8}x^{8}+75600\,a^{3}b^{7}x^{7}+229320\,a^{4}b^{6}x^{6}+407484\,a^{5}b^{5}x^{5}+460404\,a^{6}b^{4}x^{4}+337176\,a^{7}b^{3}x^{3}+155844\,a^{8}b^{2}x^{2}+41481\,a^{9}bx+4861\,a^{10}}{252\,(bx+a)^{9}b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] x/b^10 - 10*a*log(abs(b*x + a))/b^11 - 1/252*(11340*a^2*b^8*x^8 + 75600*a^3
*b^7*x^7 + 229320*a^4*b^6*x^6 + 407484*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 + 3
37176*a^7*b^3*x^3 + 155844*a^8*b^2*x^2 + 41481*a^9*b*x + 4861*a^10)/((b*x +
 a)^9*b^11)
```

**maple** [A] time = 0.01, size = 154, normalized size = 0.97

$$-\frac{a^{10}}{9(bx+a)^9 b^{11}}+\frac{5a^9}{4(bx+a)^8 b^{11}}-\frac{45a^8}{7(bx+a)^7 b^{11}}+\frac{20a^7}{(bx+a)^6 b^{11}}-\frac{42a^6}{(bx+a)^5 b^{11}}+\frac{63a^5}{(bx+a)^4 b^{11}}-\frac{70a^4}{(bx+a)^3 b^{11}}+\frac{60a^3}{(bx+a)^2 b^{11}}-\frac{45a^2}{(bx+a) b^{11}}-\frac{10a\ln(bx+a)}{b^{11}}+\frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x+a)^10,x)
```

```
[Out] x/b^10-1/9*a^10/b^11/(b*x+a)^9+5/4*a^9/b^11/(b*x+a)^8-45/7*a^8/b^11/(b*x+a)
^7+20*a^7/b^11/(b*x+a)^6-42*a^6/b^11/(b*x+a)^5+63*a^5/b^11/(b*x+a)^4-70*a^4
/b^11/(b*x+a)^3+60*a^3/b^11/(b*x+a)^2-45*a^2/b^11/(b*x+a)-10*a*ln(b*x+a)/b^
11
```

**maxima** [A] time = 1.66, size = 211, normalized size = 1.33

$$-\frac{11340\,a^2b^8x^8+75600\,a^3b^7x^7+229320\,a^4b^6x^6+407484\,a^5b^5x^5+460404\,a^6b^4x^4+337176\,a^7b^3x^3+155844\,a^8b^2x^2+41481\,a^9bx+4861\,a^{10}}{252\left(b^{20}x^9+9\,ab^{19}x^8+36\,a^2b^{18}x^7+84\,a^3b^{17}x^6+126\,a^4b^{16}x^5+126\,a^5b^{15}x^4+84\,a^6b^{14}x^3+36\,a^7b^{13}x^2+9\,a^8b^{12}x+a^9b^{11}\right)}+\frac{x}{b^{10}}-\frac{10\,a\log(bx+a)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(11340*a^2*b^8*x^8 + 75600*a^3*b^7*x^7 + 229320*a^4*b^6*x^6 + 407484
*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 + 337176*a^7*b^3*x^3 + 155844*a^8*b^2*x^2
 + 41481*a^9*b*x + 4861*a^10)/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7 +
84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3 + 3
6*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11) + x/b^10 - 10*a*log(b*x + a)/b^11
```

**mupad** [B] time = 0.94, size = 127, normalized size = 0.80

$$-\frac{10\,a\,\ln(a+b\,x)-b\,x+\frac{45\,a^2}{a+b\,x}-\frac{60\,a^3}{(a+b\,x)^2}+\frac{70\,a^4}{(a+b\,x)^3}-\frac{63\,a^5}{(a+b\,x)^4}+\frac{42\,a^6}{(a+b\,x)^5}-\frac{20\,a^7}{(a+b\,x)^6}+\frac{45\,a^8}{7\,(a+b\,x)^7}-\frac{5\,a^9}{4\,(a+b\,x)^8}+\frac{a^{10}}{9\,(a+b\,x)^9}}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x)^10,x)
```

```
[Out] -(10*a*log(a + b*x) - b*x + (45*a^2)/(a + b*x) - (60*a^3)/(a + b*x)^2 + (70
*a^4)/(a + b*x)^3 - (63*a^5)/(a + b*x)^4 + (42*a^6)/(a + b*x)^5 - (20*a^7)/
(a + b*x)^6 + (45*a^8)/(7*(a + b*x)^7) - (5*a^9)/(4*(a + b*x)^8) + a^10/(9*
(a + b*x)^9))/b^11
```

**sympy** [A] time = 1.33, size = 224, normalized size = 1.41

$$-\frac{10a\log(a+bx)}{b^{11}}+\frac{-4861a^{10}-41481a^9bx-155844a^8b^2x^2-337176a^7b^3x^3-460404a^6b^4x^4-407484a^5b^5x^5-229320a^4b^6x^6-75600a^3b^7x^7-11340a^2b^8x^8}{252a^9b^{11}+2268a^8b^{12}x+9072a^7b^{13}x^2+21168a^6b^{14}x^3+31752a^5b^{15}x^4+31752a^4b^{16}x^5+21168a^3b^{17}x^6+9072a^2b^{18}x^7+2268ab^{19}x^8+252b^{20}x^9}+\frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x+a)**10,x)
```

```
[Out] -10*a*log(a + b*x)/b**11 + (-4861*a**10 - 41481*a**9*b*x - 155844*a**8*b**2
*x**2 - 337176*a**7*b**3*x**3 - 460404*a**6*b**4*x**4 - 407484*a**5*b**5*x*
*5 - 229320*a**4*b**6*x**6 - 75600*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(
252*a**9*b**11 + 2268*a**8*b**12*x + 9072*a**7*b**13*x**2 + 21168*a**6*b**1
4*x**3 + 31752*a**5*b**15*x**4 + 31752*a**4*b**16*x**5 + 21168*a**3*b**17*x
**6 + 9072*a**2*b**18*x**7 + 2268*a*b**19*x**8 + 252*b**20*x**9) + x/b**10
```

# 3.225 $\int \frac{x^9}{(a+bx)^{10}}\,dx$

**Optimal**. Leaf size=154

$$\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{28a^3}{b^{10}(a+bx)^3}-\frac{18}{b^{10}(a}$$

**Rubi [A]** time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{28a^3}{b^{10}(a+bx)^3}-\frac{18a^2}{b^{10}(a+bx)^2}+\frac{9a}{b^{10}(a+bx)}+\frac{\log(a+bx)}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x)^10,x]
```

```
[Out] a^9/(9*b^10*(a + b*x)^9) - (9*a^8)/(8*b^10*(a + b*x)^8) + (36*a^7)/(7*b^10*
(a + b*x)^7) - (14*a^6)/(b^10*(a + b*x)^6) + (126*a^5)/(5*b^10*(a + b*x)^5)
 - (63*a^4)/(2*b^10*(a + b*x)^4) + (28*a^3)/(b^10*(a + b*x)^3) - (18*a^2)/(
b^10*(a + b*x)^2) + (9*a)/(b^10*(a + b*x)) + Log[a + b*x]/b^10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a+bx)^{10}}\,dx &= \int \left(-\frac{a^9}{b^9(a+bx)^{10}}+\frac{9a^8}{b^9(a+bx)^9}-\frac{36a^7}{b^9(a+bx)^8}+\frac{84a^6}{b^9(a+bx)^7}-\frac{126a^5}{b^9(a+bx)^6}+\frac{126a^4}{b^9(a+bx)^5}-\right.\\ &= \frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+b}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 111, normalized size = 0.72

$$\frac{a\left(7129a^8+61641a^7bx+235224a^6b^2x^2+518616a^5b^3x^3+725004a^4b^4x^4+661500a^3b^5x^5+388080a^2b^6x^6+136080ab^7x^7+22680b^8x^8\right)}{2520b^{10}(a+bx)^9}+\frac{\log(a+bx)}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x)^10,x]
```

```
[Out] (a*(7129*a^8 + 61641*a^7*b*x + 235224*a^6*b^2*x^2 + 518616*a^5*b^3*x^3 + 72
5004*a^4*b^4*x^4 + 661500*a^3*b^5*x^5 + 388080*a^2*b^6*x^6 + 136080*a*b^7*x
^7 + 22680*b^8*x^8))/(2520*b^10*(a + b*x)^9) + Log[a + b*x]/b^10
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^9/(a + b*x)^10, x]
```

**fricas [B]** time = 1.21, size = 292, normalized size = 1.90

$$\frac{22680\,ab^{8}x^{8}+136080\,a^{2}b^{7}x^{7}+388080\,a^{3}b^{6}x^{6}+661500\,a^{4}b^{5}x^{5}+725004\,a^{5}b^{4}x^{4}+518616\,a^{6}b^{3}x^{3}+235224\,a^{7}b^{2}x^{2}+61641\,a^{8}bx+7129\,a^{9}+2520\left(b^{9}x^{9}+9\,ab^{8}x^{8}+36\,a^{2}b^{7}x^{7}+84\,a^{3}b^{6}x^{6}+126\,a^{4}b^{5}x^{5}+126\,a^{5}b^{4}x^{4}+84\,a^{6}b^{3}x^{3}+36\,a^{7}b^{2}x^{2}+9\,a^{8}bx+a^{9}\right)\log\left(bx+a\right)}{2520\left(b^{19}x^{9}+9\,ab^{18}x^{8}+36\,a^{2}b^{17}x^{7}+84\,a^{3}b^{16}x^{6}+126\,a^{4}b^{15}x^{5}+126\,a^{5}b^{14}x^{4}+84\,a^{6}b^{13}x^{3}+36\,a^{7}b^{12}x^{2}+9\,a^{8}b^{11}x+a^{9}b^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500*
a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^2
+ 61641*a^8*b*x + 7129*a^9 + 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^7 +
 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 + 36*a
^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a))/(b^19*x^9 + 9*a*b^18*x^8 + 36*a
^2*b^17*x^7 + 84*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 + 84*a^
6*b^13*x^3 + 36*a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10)
```

**giac [A]** time = 1.22, size = 112, normalized size = 0.73

$$\frac{\log\left(|bx+a|\right)}{b^{10}}+\frac{22680\,ab^{7}x^{8}+136080\,a^{2}b^{6}x^{7}+388080\,a^{3}b^{5}x^{6}+661500\,a^{4}b^{4}x^{5}+725004\,a^{5}b^{3}x^{4}+518616\,a^{6}b^{2}x^{3}+235224\,a^{7}bx^{2}+61641\,a^{8}x+\frac{7129\,a^{9}}{b}}{2520\,(bx+a)^{9}b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^10 + 1/2520*(22680*a*b^7*x^8 + 136080*a^2*b^6*x^7 + 388
080*a^3*b^5*x^6 + 661500*a^4*b^4*x^5 + 725004*a^5*b^3*x^4 + 518616*a^6*b^2*
x^3 + 235224*a^7*b*x^2 + 61641*a^8*x + 7129*a^9/b)/((b*x + a)^9*b^9)
```

**maple [A]** time = 0.01, size = 145, normalized size = 0.94

$$\frac{a^{9}}{9\,(bx+a)^{9}\,b^{10}}-\frac{9a^{8}}{8\,(bx+a)^{8}\,b^{10}}+\frac{36a^{7}}{7\,(bx+a)^{7}\,b^{10}}-\frac{14a^{6}}{(bx+a)^{6}\,b^{10}}+\frac{126a^{5}}{5\,(bx+a)^{5}\,b^{10}}-\frac{63a^{4}}{2\,(bx+a)^{4}\,b^{10}}+\frac{28a^{3}}{(bx+a)^{3}\,b^{10}}-\frac{18a^{2}}{(bx+a)^{2}\,b^{10}}+\frac{9a}{(bx+a)\,b^{10}}+\frac{\ln\left(bx+a\right)}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x+a)^10,x)
```

```
[Out] 1/9*a^9/b^10/(b*x+a)^9-9/8*a^8/b^10/(b*x+a)^8+36/7*a^7/b^10/(b*x+a)^7-14*a^
6/b^10/(b*x+a)^6+126/5*a^5/b^10/(b*x+a)^5-63/2*a^4/b^10/(b*x+a)^4+28*a^3/b^
10/(b*x+a)^3-18*a^2/b^10/(b*x+a)^2+9*a/b^10/(b*x+a)+ln(b*x+a)/b^10
```

**maxima** [A] time = 1.60, size = 202, normalized size = 1.31

$$\frac{22680\,ab^{8}x^{8} + 136080\,a^{2}b^{7}x^{7} + 388080\,a^{3}b^{6}x^{6} + 661500\,a^{4}b^{5}x^{5} + 725004\,a^{5}b^{4}x^{4} + 518616\,a^{6}b^{3}x^{3} + 235224\,a^{7}b^{2}x^{2} + 61641\,a^{8}bx + 7129\,a^{9}}{2520\left(b^{19}x^{9} + 9\,ab^{18}x^{8} + 36\,a^{2}b^{17}x^{7} + 84\,a^{3}b^{16}x^{6} + 126\,a^{4}b^{15}x^{5} + 126\,a^{5}b^{14}x^{4} + 84\,a^{6}b^{13}x^{3} + 36\,a^{7}b^{12}x^{2} + 9\,a^{8}b^{11}x + a^{9}b^{10}\right)} + \frac{\log\left(bx + a\right)}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500*
a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^2
+ 61641*a^8*b*x + 7129*a^9)/(b^19*x^9 + 9*a*b^18*x^8 + 36*a^2*b^17*x^7 + 84
*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 + 84*a^6*b^13*x^3 + 36*
a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10) + log(b*x + a)/b^10
```

**mupad** [B] time = 0.19, size = 117, normalized size = 0.76

$$\frac{\ln\left(a+b\,x\right)+\frac{9\,a}{a+b\,x}-\frac{18\,a^2}{{\left(a+b\,x\right)}^2}+\frac{28\,a^3}{{\left(a+b\,x\right)}^3}-\frac{63\,a^4}{2\,{\left(a+b\,x\right)}^4}+\frac{126\,a^5}{5\,{\left(a+b\,x\right)}^5}-\frac{14\,a^6}{{\left(a+b\,x\right)}^6}+\frac{36\,a^7}{7\,{\left(a+b\,x\right)}^7}-\frac{9\,a^8}{8\,{\left(a+b\,x\right)}^8}+\frac{a^9}{9\,{\left(a+b\,x\right)}^9}}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x)^10,x)
```

```
[Out] (log(a + b*x) + (9*a)/(a + b*x) - (18*a^2)/(a + b*x)^2 + (28*a^3)/(a + b*x)
^3 - (63*a^4)/(2*(a + b*x)^4) + (126*a^5)/(5*(a + b*x)^5) - (14*a^6)/(a + b
*x)^6 + (36*a^7)/(7*(a + b*x)^7) - (9*a^8)/(8*(a + b*x)^8) + a^9/(9*(a + b*
x)^9))/b^10
```

**sympy** [A] time = 1.11, size = 212, normalized size = 1.38

$$\frac{7129a^{9} + 61641a^{8}bx + 235224a^{7}b^{2}x^{2} + 518616a^{6}b^{3}x^{3} + 725004a^{5}b^{4}x^{4} + 661500a^{4}b^{5}x^{5} + 388080a^{3}b^{6}x^{6} + 136080a^{2}b^{7}x^{7} + 22680ab^{8}x^{8}}{2520a^{9}b^{10} + 22680a^{8}b^{11}x + 90720a^{7}b^{12}x^{2} + 211680a^{6}b^{13}x^{3} + 317520a^{5}b^{14}x^{4} + 317520a^{4}b^{15}x^{5} + 211680a^{3}b^{16}x^{6} + 90720a^{2}b^{17}x^{7} + 22680ab^{18}x^{8} + 2520b^{19}x^{9}} + \frac{\log\left(a + bx\right)}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x+a)**10,x)
```

```
[Out] (7129*a**9 + 61641*a**8*b*x + 235224*a**7*b**2*x**2 + 518616*a**6*b**3*x**3
 + 725004*a**5*b**4*x**4 + 661500*a**4*b**5*x**5 + 388080*a**3*b**6*x**6 +
136080*a**2*b**7*x**7 + 22680*a*b**8*x**8)/(2520*a**9*b**10 + 22680*a**8*b*
```

```
*11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*
x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x
**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + log(a + b*x)/b**10
```

## 3.226 $\int \frac{x^8}{(a+bx)^{10}} dx$

**Optimal**. Leaf size=17

$$\frac{x^9}{9a(a+bx)^9}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$\frac{x^9}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^10,x]
```

```
[Out] x^9/(9*a*(a + b*x)^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^8}{(a+bx)^{10}} dx = \frac{x^9}{9a(a+bx)^9}$$

**Mathematica [B]** time = 0.02, size = 97, normalized size = 5.71

$$-\frac{a^8+9a^7bx+36a^6b^2x^2+84a^5b^3x^3+126a^4b^4x^4+126a^3b^5x^5+84a^2b^6x^6+36ab^7x^7+9b^8x^8}{9b^9(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^10,x]
```

```
[Out] -1/9*(a^8 + 9*a^7*b*x + 36*a^6*b^2*x^2 + 84*a^5*b^3*x^3 + 126*a^4*b^4*x^4 +
 126*a^3*b^5*x^5 + 84*a^2*b^6*x^6 + 36*a*b^7*x^7 + 9*b^8*x^8)/(b^9*(a + b*x
)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a + b*x)^10, x]
```

**fricas [B]** time = 1.08, size = 186, normalized size = 10.94

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\left(b^{18}x^9+9\,ab^{17}x^8+36\,a^2b^{16}x^7+84\,a^3b^{15}x^6+126\,a^4b^{14}x^5+126\,a^5b^{13}x^4+84\,a^6b^{12}x^3+36\,a^7b^{11}x^2+9\,a^8b^{10}x+a^9b^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 + 9
*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126*a^
5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9)
```

**giac [B]** time = 1.23, size = 95, normalized size = 5.59

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,(bx+a)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/((b*x + a)^9*
b^9)
```

**maple [B]** time = 0.01, size = 131, normalized size = 7.71

$$-\frac{a^8}{9\,(bx+a)^9\,b^9}+\frac{a^7}{(bx+a)^8\,b^9}-\frac{4a^6}{(bx+a)^7\,b^9}+\frac{28a^5}{3\,(bx+a)^6\,b^9}-\frac{14a^4}{(bx+a)^5\,b^9}+\frac{14a^3}{(bx+a)^4\,b^9}-\frac{28a^2}{3\,(bx+a)^3\,b^9}+\frac{4a}{(bx+a)^2\,b^9}-\frac{1}{(bx+a)\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^10,x)
```

[Out] 28/3*a^5/b^9/(b*x+a)^6+4*a/b^9/(b*x+a)^2-14*a^4/b^9/(b*x+a)^5+a^7/b^9/(b*x+a)^8-28/3*a^2/b^9/(b*x+a)^3+14*a^3/b^9/(b*x+a)^4-1/9*a^8/b^9/(b*x+a)^9-4*a^6/b^9/(b*x+a)^7-1/b^9/(b*x+a)

**maxima [B]** time = 1.51, size = 186, normalized size = 10.94

$$-\frac{9\,b^{8}x^{8}+36\,ab^{7}x^{7}+84\,a^{2}b^{6}x^{6}+126\,a^{3}b^{5}x^{5}+126\,a^{4}b^{4}x^{4}+84\,a^{5}b^{3}x^{3}+36\,a^{6}b^{2}x^{2}+9\,a^{7}bx+a^{8}}{9\left(b^{18}x^{9}+9\,ab^{17}x^{8}+36\,a^{2}b^{16}x^{7}+84\,a^{3}b^{15}x^{6}+126\,a^{4}b^{14}x^{5}+126\,a^{5}b^{13}x^{4}+84\,a^{6}b^{12}x^{3}+36\,a^{7}b^{11}x^{2}+9\,a^{8}b^{10}x+a^{9}b^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b*x+a)^10,x, algorithm="maxima")

[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 + 9*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126*a^5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9)

**mupad [B]** time = 0.14, size = 107, normalized size = 6.29

$$-\frac{\frac{1}{a+b\,x}-\frac{4\,a}{{\left(a+b\,x\right)}^2}+\frac{28\,a^2}{3\,{\left(a+b\,x\right)}^3}-\frac{14\,a^3}{{\left(a+b\,x\right)}^4}+\frac{14\,a^4}{{\left(a+b\,x\right)}^5}-\frac{28\,a^5}{3\,{\left(a+b\,x\right)}^6}+\frac{4\,a^6}{{\left(a+b\,x\right)}^7}-\frac{a^7}{{\left(a+b\,x\right)}^8}+\frac{a^8}{9\,{\left(a+b\,x\right)}^9}}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(a + b*x)^10,x)

[Out] -(1/(a + b*x) - (4*a)/(a + b*x)^2 + (28*a^2)/(3*(a + b*x)^3) - (14*a^3)/(a + b*x)^4 + (14*a^4)/(a + b*x)^5 - (28*a^5)/(3*(a + b*x)^6) + (4*a^6)/(a + b*x)^7 - a^7/(a + b*x)^8 + a^8/(9*(a + b*x)^9))/b^9

**sympy [B]** time = 0.98, size = 199, normalized size = 11.71

$$\frac{-a^{8}-9a^{7}bx-36a^{6}b^{2}x^{2}-84a^{5}b^{3}x^{3}-126a^{4}b^{4}x^{4}-126a^{3}b^{5}x^{5}-84a^{2}b^{6}x^{6}-36ab^{7}x^{7}-9b^{8}x^{8}}{9a^{9}b^{9}+81a^{8}b^{10}x+324a^{7}b^{11}x^{2}+756a^{6}b^{12}x^{3}+1134a^{5}b^{13}x^{4}+1134a^{4}b^{14}x^{5}+756a^{3}b^{15}x^{6}+324a^{2}b^{16}x^{7}+81ab^{17}x^{8}+9b^{18}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(b*x+a)**10,x)

[Out] (-a**8 - 9*a**7*b*x - 36*a**6*b**2*x**2 - 84*a**5*b**3*x**3 - 126*a**4*b**4*x**4 - 126*a**3*b**5*x**5 - 84*a**2*b**6*x**6 - 36*a*b**7*x**7 - 9*b**8*x**8)/(9*a**9*b**9 + 81*a**8*b**10*x + 324*a**7*b**11*x**2 + 756*a**6*b**12*x**3 + 1134*a**5*b**13*x**4 + 1134*a**4*b**14*x**5 + 756*a**3*b**15*x**6 + 324*a**2*b**16*x**7 + 81*a*b**17*x**8 + 9*b**18*x**9)

## 3.227 $\int \frac{x^7}{(a+bx)^{10}}\,dx$

**Optimal**. Leaf size=35

$$\frac{x^8}{72a^2(a+bx)^8}+\frac{x^8}{9a(a+bx)^9}$$

**Rubi** [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{x^8}{72a^2(a+bx)^8}+\frac{x^8}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^10,x]
```

```
[Out] x^8/(9*a*(a + b*x)^9) + x^8/(72*a^2*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^{10}}\,dx &= \frac{x^8}{9a(a+bx)^9}+\frac{\int \frac{x^7}{(a+bx)^9}\,dx}{9a}\\ &= \frac{x^8}{9a(a+bx)^9}+\frac{x^8}{72a^2(a+bx)^8}\end{aligned}$$

**Mathematica [B]** time = 0.01, size = 86, normalized size = 2.46

$$-\frac{a^7+9a^6bx+36a^5b^2x^2+84a^4b^3x^3+126a^3b^4x^4+126a^2b^5x^5+84ab^6x^6+36b^7x^7}{72b^8(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^10,x]
```

```
[Out] -1/72*(a^7 + 9*a^6*b*x + 36*a^5*b^2*x^2 + 84*a^4*b^3*x^3 + 126*a^3*b^4*x^4
+ 126*a^2*b^5*x^5 + 84*a*b^6*x^6 + 36*b^7*x^7)/(b^8*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x)^10, x]
```

**fricas [B]** time = 0.88, size = 175, normalized size = 5.00

$$-\frac{36\,b^7x^7+84\,ab^6x^6+126\,a^2b^5x^5+126\,a^3b^4x^4+84\,a^4b^3x^3+36\,a^5b^2x^2+9\,a^6bx+a^7}{72\left(b^{17}x^9+9\,ab^{16}x^8+36\,a^2b^{15}x^7+84\,a^3b^{14}x^6+126\,a^4b^{13}x^5+126\,a^5b^{12}x^4+84\,a^6b^{11}x^3+36\,a^7b^{10}x^2+9\,a^8b^9x+a^9b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 + 3
6*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 + 84
*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

**giac [B]** time = 0.96, size = 84, normalized size = 2.40

$$-\frac{36\,b^7x^7+84\,ab^6x^6+126\,a^2b^5x^5+126\,a^3b^4x^4+84\,a^4b^3x^3+36\,a^5b^2x^2+9\,a^6bx+a^7}{72\,(bx+a)^9b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/((b*x + a)^9*b^8)
```

**maple [B]** time = 0.01, size = 117, normalized size = 3.34

$$\frac{a^{7}}{9 \left(bx+a\right)^{9} b^{8}}-\frac{7 a^{6}}{8 \left(bx+a\right)^{8} b^{8}}+\frac{3 a^{5}}{\left(bx+a\right)^{7} b^{8}}-\frac{35 a^{4}}{6 \left(bx+a\right)^{6} b^{8}}+\frac{7 a^{3}}{\left(bx+a\right)^{5} b^{8}}-\frac{21 a^{2}}{4 \left(bx+a\right)^{4} b^{8}}+\frac{7 a}{3 \left(bx+a\right)^{3} b^{8}}-\frac{1}{2 \left(bx+a\right)^{2} b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^10,x)
```

```
[Out] -35/6*a^4/b^8/(b*x+a)^6-1/2/b^8/(b*x+a)^2+3*a^5/b^8/(b*x+a)^7+7*a^3/b^8/(b*
x+a)^5-21/4*a^2/b^8/(b*x+a)^4-7/8*a^6/b^8/(b*x+a)^8+7/3*a/b^8/(b*x+a)^3+1/9
*a^7/b^8/(b*x+a)^9
```

**maxima [B]** time = 1.59, size = 175, normalized size = 5.00

$$-\frac{36 \, b^{7} x^{7} + 84 \, a b^{6} x^{6} + 126 \, a^{2} b^{5} x^{5} + 126 \, a^{3} b^{4} x^{4} + 84 \, a^{4} b^{3} x^{3} + 36 \, a^{5} b^{2} x^{2} + 9 \, a^{6} b x + a^{7}}{72 \, {\left(b^{17} x^{9} + 9 \, a b^{16} x^{8} + 36 \, a^{2} b^{15} x^{7} + 84 \, a^{3} b^{14} x^{6} + 126 \, a^{4} b^{13} x^{5} + 126 \, a^{5} b^{12} x^{4} + 84 \, a^{6} b^{11} x^{3} + 36 \, a^{7} b^{10} x^{2} + 9 \, a^{8} b^{9} x + a^{9} b^{8}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 + 3
6*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 + 84
*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

**mupad [B]** time = 0.13, size = 22, normalized size = 0.63

$$\frac{x^8\,\left(9\,a+b\,x\right)}{72\,a^2\,{\left(a+b\,x\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^10,x)
```

```
[Out] (x^8*(9*a + b*x))/(72*a^2*(a + b*x)^9)
```

**sympy [B]** time = 1.00, size = 187, normalized size = 5.34

$$\frac{- a^{7} - 9 a^{6} b x - 36 a^{5} b^{2} x^{2} - 84 a^{4} b^{3} x^{3} - 126 a^{3} b^{4} x^{4} - 126 a^{2} b^{5} x^{5} - 84 a b^{6} x^{6} - 36 b^{7} x^{7}}{72 a^{9} b^{8} + 648 a^{8} b^{9} x + 2592 a^{7} b^{10} x^{2} + 6048 a^{6} b^{11} x^{3} + 9072 a^{5} b^{12} x^{4} + 9072 a^{4} b^{13} x^{5} + 6048 a^{3} b^{14} x^{6} + 2592 a^{2} b^{15} x^{7} + 648 a b^{16} x^{8} + 72 b^{17} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**10,x)
```

```
[Out] (-a**7 - 9*a**6*b*x - 36*a**5*b**2*x**2 - 84*a**4*b**3*x**3 - 126*a**3*b**4
*x**4 - 126*a**2*b**5*x**5 - 84*a*b**6*x**6 - 36*b**7*x**7)/(72*a**9*b**8 +
```

```
 648*a**8*b**9*x + 2592*a**7*b**10*x**2 + 6048*a**6*b**11*x**3 + 9072*a**5*
b**12*x**4 + 9072*a**4*b**13*x**5 + 6048*a**3*b**14*x**6 + 2592*a**2*b**15*
x**7 + 648*a*b**16*x**8 + 72*b**17*x**9)
```

## 3.228 $\int \frac{x^6}{(a+bx)^{10}} \, dx$

**Optimal**. Leaf size=52

$$\frac{x^7}{252a^3(a+bx)^7} + \frac{x^7}{36a^2(a+bx)^8} + \frac{x^7}{9a(a+bx)^9}$$

**Rubi [A]** time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{x^7}{252a^3(a+bx)^7} + \frac{x^7}{36a^2(a+bx)^8} + \frac{x^7}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^10,x]
```

```
[Out] x^7/(9*a*(a + b*x)^9) + x^7/(36*a^2*(a + b*x)^8) + x^7/(252*a^3*(a + b*x)^7
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{x^6}{(a+bx)^{10}}\,dx = \frac{x^7}{9a(a+bx)^9} + \frac{2\int \frac{x^6}{(a+bx)^9}\,dx}{9a}$$
$$= \frac{x^7}{9a(a+bx)^9} + \frac{x^7}{36a^2(a+bx)^8} + \frac{\int \frac{x^6}{(a+bx)^8}\,dx}{36a^2}$$
$$= \frac{x^7}{9a(a+bx)^9} + \frac{x^7}{36a^2(a+bx)^8} + \frac{x^7}{252a^3(a+bx)^7}$$

**Mathematica** [A] time = 0.02, size = 75, normalized size = 1.44

$$-\frac{a^6 + 9a^5bx + 36a^4b^2x^2 + 84a^3b^3x^3 + 126a^2b^4x^4 + 126ab^5x^5 + 84b^6x^6}{252b^7(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^10,x]
```

```
[Out] -1/252*(a^6 + 9*a^5*b*x + 36*a^4*b^2*x^2 + 84*a^3*b^3*x^3 + 126*a^2*b^4*x^4
 + 126*a*b^5*x^5 + 84*b^6*x^6)/(b^7*(a + b*x)^9)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x)^10, x]
```

**fricas** [B] time = 0.87, size = 164, normalized size = 3.15

$$-\frac{84\,b^6x^6 + 126\,ab^5x^5 + 126\,a^2b^4x^4 + 84\,a^3b^3x^3 + 36\,a^4b^2x^2 + 9\,a^5bx + a^6}{252\,\left(b^{16}x^9 + 9\,ab^{15}x^8 + 36\,a^2b^{14}x^7 + 84\,a^3b^{13}x^6 + 126\,a^4b^{12}x^5 + 126\,a^5b^{11}x^4 + 84\,a^6b^{10}x^3 + 36\,a^7b^9x^2 + 9\,a^8b^8x + a^9b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7 +
```

```
84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3 +
36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

**giac [A]** time = 1.01, size = 73, normalized size = 1.40

$$-\frac{84\,b^6x^6+126\,ab^5x^5+126\,a^2b^4x^4+84\,a^3b^3x^3+36\,a^4b^2x^2+9\,a^5bx+a^6}{252\,(bx+a)^9b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/((b*x + a)^9*b^7)
```

**maple [B]** time = 0.01, size = 102, normalized size = 1.96

$$-\frac{a^6}{9\,(bx+a)^9\,b^7}+\frac{3a^5}{4\,(bx+a)^8\,b^7}-\frac{15a^4}{7\,(bx+a)^7\,b^7}+\frac{10a^3}{3\,(bx+a)^6\,b^7}-\frac{3a^2}{(bx+a)^5\,b^7}+\frac{3a}{2\,(bx+a)^4\,b^7}-\frac{1}{3\,(bx+a)^3\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^10,x)
```

```
[Out] 10/3*a^3/b^7/(b*x+a)^6-3*a^2/b^7/(b*x+a)^5+3/4*a^5/b^7/(b*x+a)^8+3/2*a/b^7/
(b*x+a)^4-1/3/b^7/(b*x+a)^3-1/9*a^6/b^7/(b*x+a)^9-15/7*a^4/b^7/(b*x+a)^7
```

**maxima [B]** time = 1.49, size = 164, normalized size = 3.15

$$-\frac{84\,b^6x^6+126\,ab^5x^5+126\,a^2b^4x^4+84\,a^3b^3x^3+36\,a^4b^2x^2+9\,a^5bx+a^6}{252\left(b^{16}x^9+9\,ab^{15}x^8+36\,a^2b^{14}x^7+84\,a^3b^{13}x^6+126\,a^4b^{12}x^5+126\,a^5b^{11}x^4+84\,a^6b^{10}x^3+36\,a^7b^9x^2+9\,a^8b^8x+a^9b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7 +
 84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3 +
36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

**mupad [B]** time = 0.14, size = 85, normalized size = 1.63

$$-\frac{\frac{1}{3\,(a+b\,x)^3}-\frac{3\,a}{2\,(a+b\,x)^4}+\frac{3\,a^2}{(a+b\,x)^5}-\frac{10\,a^3}{3\,(a+b\,x)^6}+\frac{15\,a^4}{7\,(a+b\,x)^7}-\frac{3\,a^5}{4\,(a+b\,x)^8}+\frac{a^6}{9\,(a+b\,x)^9}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^10,x)
```

```
[Out] -(1/(3*(a + b*x)^3) - (3*a)/(2*(a + b*x)^4) + (3*a^2)/(a + b*x)^5 - (10*a^3
)/(3*(a + b*x)^6) + (15*a^4)/(7*(a + b*x)^7) - (3*a^5)/(4*(a + b*x)^8) + a^
6/(9*(a + b*x)^9))/b^7
```

**sympy** [**B**] time = 0.91, size = 175, normalized size = 3.37

$$\frac{-a^6 - 9a^5bx - 36a^4b^2x^2 - 84a^3b^3x^3 - 126a^2b^4x^4 - 126ab^5x^5 - 84b^6x^6}{252a^9b^7 + 2268a^8b^8x + 9072a^7b^9x^2 + 21168a^6b^{10}x^3 + 31752a^5b^{11}x^4 + 31752a^4b^{12}x^5 + 21168a^3b^{13}x^6 + 9072a^2b^{14}x^7 + 2268ab^{15}x^8 + 252b^{16}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**10,x)
```

```
[Out] (-a**6 - 9*a**5*b*x - 36*a**4*b**2*x**2 - 84*a**3*b**3*x**3 - 126*a**2*b**4
*x**4 - 126*a*b**5*x**5 - 84*b**6*x**6)/(252*a**9*b**7 + 2268*a**8*b**8*x +
 9072*a**7*b**9*x**2 + 21168*a**6*b**10*x**3 + 31752*a**5*b**11*x**4 + 3175
2*a**4*b**12*x**5 + 21168*a**3*b**13*x**6 + 9072*a**2*b**14*x**7 + 2268*a*b
**15*x**8 + 252*b**16*x**9)
```

## 3.229 $\int \frac{x^5}{(a+bx)^{10}}\,dx$

**Optimal**. Leaf size=69

$$\frac{x^6}{504a^4(a+bx)^6}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{9a(a+bx)^9}$$

**Rubi** [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{x^6}{504a^4(a+bx)^6}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^10,x]
```

```
[Out] x^6/(9*a*(a + b*x)^9) + x^6/(24*a^2*(a + b*x)^8) + x^6/(84*a^3*(a + b*x)^7)
 + x^6/(504*a^4*(a + b*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{x^5}{(a+bx)^{10}}\,dx = \frac{x^6}{9a(a+bx)^9} + \frac{\int \frac{x^5}{(a+bx)^9}\,dx}{3a}$$
$$= \frac{x^6}{9a(a+bx)^9} + \frac{x^6}{24a^2(a+bx)^8} + \frac{\int \frac{x^5}{(a+bx)^8}\,dx}{12a^2}$$
$$= \frac{x^6}{9a(a+bx)^9} + \frac{x^6}{24a^2(a+bx)^8} + \frac{x^6}{84a^3(a+bx)^7} + \frac{\int \frac{x^5}{(a+bx)^7}\,dx}{84a^3}$$
$$= \frac{x^6}{9a(a+bx)^9} + \frac{x^6}{24a^2(a+bx)^8} + \frac{x^6}{84a^3(a+bx)^7} + \frac{x^6}{504a^4(a+bx)^6}$$

**Mathematica [A]** time = 0.01, size = 64, normalized size = 0.93

$$-\frac{a^5 + 9a^4bx + 36a^3b^2x^2 + 84a^2b^3x^3 + 126ab^4x^4 + 126b^5x^5}{504b^6(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^10,x]
```

```
[Out] -1/504*(a^5 + 9*a^4*b*x + 36*a^3*b^2*x^2 + 84*a^2*b^3*x^3 + 126*a*b^4*x^4 +
 126*b^5*x^5)/(b^6*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x)^10, x]
```

**fricas [B]** time = 0.98, size = 153, normalized size = 2.22

$$-\frac{126\,b^5x^5 + 126\,ab^4x^4 + 84\,a^2b^3x^3 + 36\,a^3b^2x^2 + 9\,a^4bx + a^5}{504\left(b^{15}x^9 + 9\,ab^{14}x^8 + 36\,a^2b^{13}x^7 + 84\,a^3b^{12}x^6 + 126\,a^4b^{11}x^5 + 126\,a^5b^{10}x^4 + 84\,a^6b^9x^3 + 36\,a^7b^8x^2 + 9\,a^8b^7x + a^9b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^6
+ 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2 + 9
*a^8*b^7*x + a^9*b^6)
```

**giac** [A] time = 0.90, size = 62, normalized size = 0.90

$$-\frac{126\,b^5x^5 + 126\,ab^4x^4 + 84\,a^2b^3x^3 + 36\,a^3b^2x^2 + 9\,a^4bx + a^5}{504\,(bx+a)^9b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/((b*x + a)^9*b^6)
```

**maple** [A] time = 0.00, size = 86, normalized size = 1.25

$$\frac{a^5}{9\,(bx+a)^9\,b^6} - \frac{5a^4}{8\,(bx+a)^8\,b^6} + \frac{10a^3}{7\,(bx+a)^7\,b^6} - \frac{5a^2}{3\,(bx+a)^6\,b^6} + \frac{a}{(bx+a)^5\,b^6} - \frac{1}{4\,(bx+a)^4\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^10,x)
```

```
[Out] -5/3*a^2/b^6/(b*x+a)^6-1/4/b^6/(b*x+a)^4+a/b^6/(b*x+a)^5-5/8*a^4/b^6/(b*x+a
)^8+10/7*a^3/b^6/(b*x+a)^7+1/9*a^5/b^6/(b*x+a)^9
```

**maxima** [B] time = 1.54, size = 153, normalized size = 2.22

$$-\frac{126\,b^5x^5 + 126\,ab^4x^4 + 84\,a^2b^3x^3 + 36\,a^3b^2x^2 + 9\,a^4bx + a^5}{504\,\left(b^{15}x^9 + 9\,ab^{14}x^8 + 36\,a^2b^{13}x^7 + 84\,a^3b^{12}x^6 + 126\,a^4b^{11}x^5 + 126\,a^5b^{10}x^4 + 84\,a^6b^9x^3 + 36\,a^7b^8x^2 + 9\,a^8b^7x + a^9b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^6
+ 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2 + 9
*a^8*b^7*x + a^9*b^6)
```

**mupad** [B] time = 0.08, size = 71, normalized size = 1.03

$$\frac{\frac{a}{(a+b\,x)^5} - \frac{1}{4\,(a+b\,x)^4} - \frac{5\,a^2}{3\,(a+b\,x)^6} + \frac{10\,a^3}{7\,(a+b\,x)^7} - \frac{5\,a^4}{8\,(a+b\,x)^8} + \frac{a^5}{9\,(a+b\,x)^9}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a + b*x)^10,x)

[Out] (a/(a + b*x)^5 - 1/(4*(a + b*x)^4) - (5*a^2)/(3*(a + b*x)^6) + (10*a^3)/(7*(a + b*x)^7) - (5*a^4)/(8*(a + b*x)^8) + a^5/(9*(a + b*x)^9))/b^6

**sympy [B]** time = 0.84, size = 163, normalized size = 2.36

$$\frac{-a^5 - 9a^4bx - 36a^3b^2x^2 - 84a^2b^3x^3 - 126ab^4x^4 - 126b^5x^5}{504a^9b^6 + 4536a^8b^7x + 18144a^7b^8x^2 + 42336a^6b^9x^3 + 63504a^5b^{10}x^4 + 63504a^4b^{11}x^5 + 42336a^3b^{12}x^6 + 18144a^2b^{13}x^7 + 4536ab^{14}x^8 + 504b^{15}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(b*x+a)**10,x)

[Out] (-a**5 - 9*a**4*b*x - 36*a**3*b**2*x**2 - 84*a**2*b**3*x**3 - 126*a*b**4*x**4 - 126*b**5*x**5)/(504*a**9*b**6 + 4536*a**8*b**7*x + 18144*a**7*b**8*x**2 + 42336*a**6*b**9*x**3 + 63504*a**5*b**10*x**4 + 63504*a**4*b**11*x**5 + 42336*a**3*b**12*x**6 + 18144*a**2*b**13*x**7 + 4536*a*b**14*x**8 + 504*b**15*x**9)

## 3.230 $\int \frac{x^4}{(a+bx)^{10}}\,dx$

**Optimal**. Leaf size=81

$$-\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

**Rubi** [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^10,x]
```

```
[Out] -a^4/(9*b^5*(a + b*x)^9) + a^3/(2*b^5*(a + b*x)^8) - (6*a^2)/(7*b^5*(a + b*
x)^7) + (2*a)/(3*b^5*(a + b*x)^6) - 1/(5*b^5*(a + b*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{10}}\,dx &= \int \left(\frac{a^4}{b^4(a+bx)^{10}}-\frac{4a^3}{b^4(a+bx)^9}+\frac{6a^2}{b^4(a+bx)^8}-\frac{4a}{b^4(a+bx)^7}+\frac{1}{b^4(a+bx)^6}\right)dx\\ &= -\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 53, normalized size = 0.65

$$-\frac{a^4+9a^3bx+36a^2b^2x^2+84ab^3x^3+126b^4x^4}{630b^5(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^10,x]
```

```
[Out] -1/630*(a^4 + 9*a^3*b*x + 36*a^2*b^2*x^2 + 84*a*b^3*x^3 + 126*b^4*x^4)/(b^5
*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x)^10, x]
```

**fricas [A]** time = 1.13, size = 142, normalized size = 1.75

$$-\frac{126\,b^4x^4+84\,ab^3x^3+36\,a^2b^2x^2+9\,a^3bx+a^4}{630\left(b^{14}x^9+9\,ab^{13}x^8+36\,a^2b^{12}x^7+84\,a^3b^{11}x^6+126\,a^4b^{10}x^5+126\,a^5b^9x^4+84\,a^6b^8x^3+36\,a^7b^7x^2+9\,a^8b^6x+a^9b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^1
4*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x^5
 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9*b^
5)
```

**giac [A]** time = 1.22, size = 51, normalized size = 0.63

$$-\frac{126\,b^4x^4+84\,ab^3x^3+36\,a^2b^2x^2+9\,a^3bx+a^4}{630\,(bx+a)^9b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/((b*
x + a)^9*b^5)
```

**maple [A]** time = 0.01, size = 72, normalized size = 0.89

$$-\frac{a^4}{9\,(bx+a)^9\,b^5}+\frac{a^3}{2\,(bx+a)^8\,b^5}-\frac{6a^2}{7\,(bx+a)^7\,b^5}+\frac{2a}{3\,(bx+a)^6\,b^5}-\frac{1}{5\,(bx+a)^5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^10,x)
```

```
[Out] -1/9*a^4/b^5/(b*x+a)^9+1/2*a^3/b^5/(b*x+a)^8-6/7*a^2/b^5/(b*x+a)^7+2/3*a/b^
5/(b*x+a)^6-1/5/b^5/(b*x+a)^5
```

**maxima [A]** time = 1.46, size = 142, normalized size = 1.75

$$-\frac{126\,b^4x^4+84\,ab^3x^3+36\,a^2b^2x^2+9\,a^3bx+a^4}{630\left(b^{14}x^9+9\,ab^{13}x^8+36\,a^2b^{12}x^7+84\,a^3b^{11}x^6+126\,a^4b^{10}x^5+126\,a^5b^9x^4+84\,a^6b^8x^3+36\,a^7b^7x^2+9\,a^8b^6x+a^9b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^1
4*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x^5
 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9*b^
5)
```

**mupad [B]** time = 0.08, size = 61, normalized size = 0.75

$$-\frac{\frac{1}{5\,{\left(a+b\,x\right)}^5}-\frac{2\,a}{3\,{\left(a+b\,x\right)}^6}+\frac{6\,a^2}{7\,{\left(a+b\,x\right)}^7}-\frac{a^3}{2\,{\left(a+b\,x\right)}^8}+\frac{a^4}{9\,{\left(a+b\,x\right)}^9}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^10,x)
```

```
[Out] -(1/(5*(a + b*x)^5) - (2*a)/(3*(a + b*x)^6) + (6*a^2)/(7*(a + b*x)^7) - a^3
/(2*(a + b*x)^8) + a^4/(9*(a + b*x)^9))/b^5
```

**sympy [B]** time = 0.79, size = 151, normalized size = 1.86

$$\frac{-a^4-9a^3bx-36a^2b^2x^2-84ab^3x^3-126b^4x^4}{630a^9b^5+5670a^8b^6x+22680a^7b^7x^2+52920a^6b^8x^3+79380a^5b^9x^4+79380a^4b^{10}x^5+52920a^3b^{11}x^6+22680a^2b^{12}x^7+5670ab^{13}x^8+630b^{14}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**10,x)
```

```
[Out] (-a**4 - 9*a**3*b*x - 36*a**2*b**2*x**2 - 84*a*b**3*x**3 - 126*b**4*x**4)/(
630*a**9*b**5 + 5670*a**8*b**6*x + 22680*a**7*b**7*x**2 + 52920*a**6*b**8*x
**3 + 79380*a**5*b**9*x**4 + 79380*a**4*b**10*x**5 + 52920*a**3*b**11*x**6
+ 22680*a**2*b**12*x**7 + 5670*a*b**13*x**8 + 630*b**14*x**9)
```

## 3.231 $\int \frac{x^3}{(a+bx)^{10}} dx$

**Optimal**. Leaf size=64

$$\frac{a^3}{9b^4(a+bx)^9}-\frac{3a^2}{8b^4(a+bx)^8}+\frac{3a}{7b^4(a+bx)^7}-\frac{1}{6b^4(a+bx)^6}$$

**Rubi** [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{9b^4(a+bx)^9}-\frac{3a^2}{8b^4(a+bx)^8}+\frac{3a}{7b^4(a+bx)^7}-\frac{1}{6b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^10,x]
```

```
[Out] a^3/(9*b^4*(a + b*x)^9) - (3*a^2)/(8*b^4*(a + b*x)^8) + (3*a)/(7*b^4*(a + b
*x)^7) - 1/(6*b^4*(a + b*x)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{10}} dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{10}}+\frac{3a^2}{b^3(a+bx)^9}-\frac{3a}{b^3(a+bx)^8}+\frac{1}{b^3(a+bx)^7}\right) dx\\ &= \frac{a^3}{9b^4(a+bx)^9}-\frac{3a^2}{8b^4(a+bx)^8}+\frac{3a}{7b^4(a+bx)^7}-\frac{1}{6b^4(a+bx)^6}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 42, normalized size = 0.66

$$-\frac{a^3+9a^2bx+36ab^2x^2+84b^3x^3}{504b^4(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^10,x]
[Out] -1/504*(a^3 + 9*a^2*b*x + 36*a*b^2*x^2 + 84*b^3*x^3)/(b^4*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^10,x]
[Out] IntegrateAlgebraic[x^3/(a + b*x)^10, x]
```

**fricas [B]** time = 0.81, size = 131, normalized size = 2.05

$$-\frac{84\,b^3x^3+36\,ab^2x^2+9\,a^2bx+a^3}{504\left(b^{13}x^9+9\,ab^{12}x^8+36\,a^2b^{11}x^7+84\,a^3b^{10}x^6+126\,a^4b^9x^5+126\,a^5b^8x^4+84\,a^6b^7x^3+36\,a^7b^6x^2+9\,a^8b^5x+a^9b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="fricas")
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*x
^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^4
+ 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)
```

**giac [A]** time = 1.12, size = 40, normalized size = 0.62

$$-\frac{84\,b^3x^3+36\,ab^2x^2+9\,a^2bx+a^3}{504\,(bx+a)^9b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="giac")
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/((b*x + a)^9*b^4)
```

**maple [A]** time = 0.00, size = 57, normalized size = 0.89

$$\frac{a^3}{9\,(bx+a)^9\,b^4}-\frac{3a^2}{8\,(bx+a)^8\,b^4}+\frac{3a}{7\,(bx+a)^7\,b^4}-\frac{1}{6\,(bx+a)^6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^10,x)
```

[Out] 1/9*a^3/b^4/(b*x+a)^9-3/8*a^2/b^4/(b*x+a)^8+3/7*a/b^4/(b*x+a)^7-1/6/b^4/(b*x+a)^6

**maxima [B]** time = 1.48, size = 131, normalized size = 2.05

$$-\frac{84\,b^3x^3+36\,ab^2x^2+9\,a^2bx+a^3}{504\,\left(b^{13}x^9+9\,ab^{12}x^8+36\,a^2b^{11}x^7+84\,a^3b^{10}x^6+126\,a^4b^9x^5+126\,a^5b^8x^4+84\,a^6b^7x^3+36\,a^7b^6x^2+9\,a^8b^5x+a^9b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x+a)^10,x, algorithm="maxima")

[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*x^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^4 + 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)

**mupad [B]** time = 0.13, size = 48, normalized size = 0.75

$$\frac{\frac{3\,a}{7\,{\left(a+b\,x\right)}^7}-\frac{1}{6\,{\left(a+b\,x\right)}^6}-\frac{3\,a^2}{8\,{\left(a+b\,x\right)}^8}+\frac{a^3}{9\,{\left(a+b\,x\right)}^9}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b*x)^10,x)

[Out] ((3*a)/(7*(a + b*x)^7) - 1/(6*(a + b*x)^6) - (3*a^2)/(8*(a + b*x)^8) + a^3/(9*(a + b*x)^9))/b^4

**sympy [B]** time = 0.70, size = 139, normalized size = 2.17

$$\frac{-a^3-9a^2bx-36ab^2x^2-84b^3x^3}{504a^9b^4+4536a^8b^5x+18144a^7b^6x^2+42336a^6b^7x^3+63504a^5b^8x^4+63504a^4b^9x^5+42336a^3b^{10}x^6+18144a^2b^{11}x^7+4536ab^{12}x^8+504b^{13}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x+a)**10,x)

[Out] (-a**3 - 9*a**2*b*x - 36*a*b**2*x**2 - 84*b**3*x**3)/(504*a**9*b**4 + 4536*a**8*b**5*x + 18144*a**7*b**6*x**2 + 42336*a**6*b**7*x**3 + 63504*a**5*b**8*x**4 + 63504*a**4*b**9*x**5 + 42336*a**3*b**10*x**6 + 18144*a**2*b**11*x**7 + 4536*a*b**12*x**8 + 504*b**13*x**9)

## 3.232 $\int \frac{x^2}{(a+bx)^{10}} \, dx$

**Optimal**. Leaf size=47

$$-\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}$$

**Rubi [A]** time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^10,x]
```

```
[Out] -a^2/(9*b^3*(a + b*x)^9) + a/(4*b^3*(a + b*x)^8) - 1/(7*b^3*(a + b*x)^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a+bx)^{10}} \, dx &= \int \left(\frac{a^2}{b^2(a+bx)^{10}}-\frac{2a}{b^2(a+bx)^9}+\frac{1}{b^2(a+bx)^8}\right) dx \\ &= -\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 0.66

$$-\frac{a^2+9abx+36b^2x^2}{252b^3(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^10,x]
```

```
[Out] -1/252*(a^2 + 9*a*b*x + 36*b^2*x^2)/(b^3*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x)^10, x]
```

**fricas [B]** time = 1.17, size = 120, normalized size = 2.55

$$-\frac{36\,b^2x^2+9\,abx+a^2}{252\left(b^{12}x^9+9\,ab^{11}x^8+36\,a^2b^{10}x^7+84\,a^3b^9x^6+126\,a^4b^8x^5+126\,a^5b^7x^4+84\,a^6b^6x^3+36\,a^7b^5x^2+9\,a^8b^4x+a^9b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10*
x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3 +
 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

**giac [A]** time = 1.04, size = 29, normalized size = 0.62

$$-\frac{36\,b^2x^2+9\,abx+a^2}{252\,(bx+a)^9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/((b*x + a)^9*b^3)
```

**maple [A]** time = 0.00, size = 42, normalized size = 0.89

$$-\frac{a^2}{9\,(bx+a)^9\,b^3}+\frac{a}{4\,(bx+a)^8\,b^3}-\frac{1}{7\,(bx+a)^7\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^10,x)
```

```
[Out] -1/9*a^2/b^3/(b*x+a)^9+1/4*a/b^3/(b*x+a)^8-1/7/b^3/(b*x+a)^7
```

**maxima [B]** time = 1.42, size = 120, normalized size = 2.55

$$-\frac{36\,b^{2}x^{2}+9\,abx+a^{2}}{252\left(b^{12}x^{9}+9\,ab^{11}x^{8}+36\,a^{2}b^{10}x^{7}+84\,a^{3}b^{9}x^{6}+126\,a^{4}b^{8}x^{5}+126\,a^{5}b^{7}x^{4}+84\,a^{6}b^{6}x^{3}+36\,a^{7}b^{5}x^{2}+9\,a^{8}b^{4}x+a^{9}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10*
x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3 +
 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

**mupad [B]** time = 0.15, size = 31, normalized size = 0.66

$$-\frac{8\,a^2+72\,a\,b\,x+288\,b^2\,x^2}{2016\,b^3\,{\left(a+b\,x\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^10,x)
```

```
[Out] -(8*a^2 + 288*b^2*x^2 + 72*a*b*x)/(2016*b^3*(a + b*x)^9)
```

**sympy [B]** time = 0.68, size = 128, normalized size = 2.72

$$\frac{-a^{2}-9abx-36b^{2}x^{2}}{252a^{9}b^{3}+2268a^{8}b^{4}x+9072a^{7}b^{5}x^{2}+21168a^{6}b^{6}x^{3}+31752a^{5}b^{7}x^{4}+31752a^{4}b^{8}x^{5}+21168a^{3}b^{9}x^{6}+9072a^{2}b^{10}x^{7}+2268ab^{11}x^{8}+252b^{12}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**10,x)
```

```
[Out] (-a**2 - 9*a*b*x - 36*b**2*x**2)/(252*a**9*b**3 + 2268*a**8*b**4*x + 9072*a
**7*b**5*x**2 + 21168*a**6*b**6*x**3 + 31752*a**5*b**7*x**4 + 31752*a**4*b*
*8*x**5 + 21168*a**3*b**9*x**6 + 9072*a**2*b**10*x**7 + 2268*a*b**11*x**8 +
 252*b**12*x**9)
```

## 3.233 $\int \frac{x}{(a+bx)^{10}}\,dx$

**Optimal**. Leaf size=30

$$\frac{a}{9b^2(a+bx)^9}-\frac{1}{8b^2(a+bx)^8}$$

**Rubi** [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{9b^2(a+bx)^9}-\frac{1}{8b^2(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^10,x]
```

```
[Out] a/(9*b^2*(a + b*x)^9) - 1/(8*b^2*(a + b*x)^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{10}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{10}}+\frac{1}{b(a+bx)^9}\right)dx\\ &= \frac{a}{9b^2(a+bx)^9}-\frac{1}{8b^2(a+bx)^8}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 20, normalized size = 0.67

$$-\frac{a+9bx}{72b^2(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^10,x]
```

```
[Out] -1/72*(a + 9*b*x)/(b^2*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x)^10, x]
```

**fricas [B]** time = 1.04, size = 109, normalized size = 3.63

$$-\frac{9\,bx+a}{72\,\left(b^{11}x^{9}+9\,ab^{10}x^{8}+36\,a^{2}b^{9}x^{7}+84\,a^{3}b^{8}x^{6}+126\,a^{4}b^{7}x^{5}+126\,a^{5}b^{6}x^{4}+84\,a^{6}b^{5}x^{3}+36\,a^{7}b^{4}x^{2}+9\,a^{8}b^{3}x+a^{9}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x^
6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 + 9
*a^8*b^3*x + a^9*b^2)
```

**giac [A]** time = 1.04, size = 18, normalized size = 0.60

$$-\frac{9\,bx+a}{72\,(bx+a)^{9}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(9*b*x + a)/((b*x + a)^9*b^2)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.90

$$\frac{a}{9\,(bx+a)^{9}\,b^{2}}-\frac{1}{8\,(bx+a)^{8}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^10,x)
```

```
[Out] 1/9*a/b^2/(b*x+a)^9-1/8/b^2/(b*x+a)^8
```

**maxima [B]** time = 1.42, size = 109, normalized size = 3.63

$$-\frac{9\,bx+a}{72\,\left(b^{11}x^{9}+9\,ab^{10}x^{8}+36\,a^{2}b^{9}x^{7}+84\,a^{3}b^{8}x^{6}+126\,a^{4}b^{7}x^{5}+126\,a^{5}b^{6}x^{4}+84\,a^{6}b^{5}x^{3}+36\,a^{7}b^{4}x^{2}+9\,a^{8}b^{3}x+a^{9}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x^
6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 + 9
*a^8*b^3*x + a^9*b^2)
```

**mupad [B]** time = 0.07, size = 18, normalized size = 0.60

$$-\frac{a+9\,b\,x}{72\,b^2\,{\left(a+b\,x\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^10,x)
```

```
[Out] -(a + 9*b*x)/(72*b^2*(a + b*x)^9)
```

**sympy [B]** time = 0.72, size = 116, normalized size = 3.87

$$\frac{-a-9bx}{72a^9b^2+648a^8b^3x+2592a^7b^4x^2+6048a^6b^5x^3+9072a^5b^6x^4+9072a^4b^7x^5+6048a^3b^8x^6+2592a^2b^9x^7+648ab^{10}x^8+72b^{11}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**10,x)
```

```
[Out] (-a - 9*b*x)/(72*a**9*b**2 + 648*a**8*b**3*x + 2592*a**7*b**4*x**2 + 6048*a
**6*b**5*x**3 + 9072*a**5*b**6*x**4 + 9072*a**4*b**7*x**5 + 6048*a**3*b**8*
x**6 + 2592*a**2*b**9*x**7 + 648*a*b**10*x**8 + 72*b**11*x**9)
```

## 3.234 $\int \frac{1}{(a+bx)^{10}}\,dx$

**Optimal**. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-10),x]
```

```
[Out] -1/(9*b*(a + b*x)^9)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{10}}\,dx = -\frac{1}{9b(a+bx)^9}$$

**Mathematica [A]** time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-10),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-10),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-10), x]
```

**fricas [B]** time = 1.10, size = 101, normalized size = 7.21

$$-\frac{1}{9\left(b^{10}x^9+9\,ab^9x^8+36\,a^2b^8x^7+84\,a^3b^7x^6+126\,a^4b^6x^5+126\,a^5b^5x^4+84\,a^6b^4x^3+36\,a^7b^3x^2+9\,a^8b^2x+a^9b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

**giac [A]** time = 1.03, size = 12, normalized size = 0.86

$$-\frac{1}{9\,(bx+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{9\,(bx+a)^9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^10,x)
```

```
[Out] -1/9/b/(b*x+a)^9
```

**maxima [A]** time = 1.34, size = 12, normalized size = 0.86

$$-\frac{1}{9\,(bx+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

**mupad [B]** time = 0.14, size = 103, normalized size = 7.36

$$-\frac{1}{9\,a^9\,b+81\,a^8\,b^2\,x+324\,a^7\,b^3\,x^2+756\,a^6\,b^4\,x^3+1134\,a^5\,b^5\,x^4+1134\,a^4\,b^6\,x^5+756\,a^3\,b^7\,x^6+324\,a^2\,b^8\,x^7+81\,a\,b^9\,x^8+9\,b^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^10,x)
```

```
[Out] -1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 +
756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 + 3
24*a^2*b^8*x^7)
```

**sympy [B]** time = 0.67, size = 109, normalized size = 7.79

$$-\frac{1}{9a^9b+81a^8b^2x+324a^7b^3x^2+756a^6b^4x^3+1134a^5b^5x^4+1134a^4b^6x^5+756a^3b^7x^6+324a^2b^8x^7+81ab^9x^8+9b^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**10,x)
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

## 3.235 $\int \frac{1}{x(a+bx)^{10}}\,dx$

**Optimal**. Leaf size=141

$$-\frac{\log(a+bx)}{a^{10}}+\frac{\log(x)}{a^{10}}+\frac{1}{a^9(a+bx)}+\frac{1}{2a^8(a+bx)^2}+\frac{1}{3a^7(a+bx)^3}+\frac{1}{4a^6(a+bx)^4}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{7a$$

**Rubi** [A] time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^9(a+bx)}+\frac{1}{2a^8(a+bx)^2}+\frac{1}{3a^7(a+bx)^3}+\frac{1}{4a^6(a+bx)^4}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{7a^3(a+bx)^7}+\frac{1}{8a^2(a+bx)^8}-\frac{\log(a+bx)}{a^{10}}+\frac{\log(x)}{a^{10}}+\frac{1}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^10),x]
```

```
[Out] 1/(9*a*(a + b*x)^9) + 1/(8*a^2*(a + b*x)^8) + 1/(7*a^3*(a + b*x)^7) + 1/(6*
a^4*(a + b*x)^6) + 1/(5*a^5*(a + b*x)^5) + 1/(4*a^6*(a + b*x)^4) + 1/(3*a^7
*(a + b*x)^3) + 1/(2*a^8*(a + b*x)^2) + 1/(a^9*(a + b*x)) + Log[x]/a^10 - L
og[a + b*x]/a^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^{10}}\,dx &= \int \left(\frac{1}{a^{10}x}-\frac{b}{a(a+bx)^{10}}-\frac{b}{a^2(a+bx)^9}-\frac{b}{a^3(a+bx)^8}-\frac{b}{a^4(a+bx)^7}-\frac{b}{a^5(a+bx)^6}-\frac{b}{a^6(a+\right.\\ &= \frac{1}{9a(a+bx)^9}+\frac{1}{8a^2(a+bx)^8}+\frac{1}{7a^3(a+bx)^7}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{4a^6(a+bx)^4}\end{aligned}$$

**Mathematica** [A] time = 0.10, size = 127, normalized size = 0.90

$$-\frac{\log(a+bx)}{a^{10}}+\frac{\log(x)}{a^{10}}+\frac{280a^8+315a^7(a+bx)+360a^6(a+bx)^2+420a^5(a+bx)^3+504a^4(a+bx)^4+630a^3(a+bx)^5+840a^2(a+bx)^6+1260a(a+bx)^7+2520(a+bx)^8}{2520a^9(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^10),x]
```

```
[Out] (280*a^8 + 315*a^7*(a + b*x) + 360*a^6*(a + b*x)^2 + 420*a^5*(a + b*x)^3 +
504*a^4*(a + b*x)^4 + 630*a^3*(a + b*x)^5 + 840*a^2*(a + b*x)^6 + 1260*a*(a
 + b*x)^7 + 2520*(a + b*x)^8)/(2520*a^9*(a + b*x)^9) + Log[x]/a^10 - Log[a
+ b*x]/a^10
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x)^10), x]
```

**fricas [B]** time = 1.08, size = 388, normalized size = 2.75

$$\frac{2520\,ab^{8}x^{8} + 21420\,a^{2}b^{7}x^{7} + 80220\,a^{3}b^{6}x^{6} + 173250\,a^{4}b^{5}x^{5} + 236754\,a^{5}b^{4}x^{4} + 210756\,a^{6}b^{3}x^{3} + 120564\,a^{7}b^{2}x^{2} + 41481\,a^{8}bx + 7129\,a^{9} - 2520\left(b^{9}x^{9} + 9\,ab^{8}x^{8} + 36\,a^{2}b^{7}x^{7} + 84\,a^{3}b^{6}x^{6} + 126\,a^{4}b^{5}x^{5} + 126\,a^{5}b^{4}x^{4} + 84\,a^{6}b^{3}x^{3} + 36\,a^{7}b^{2}x^{2} + 9\,a^{8}bx + a^{9}\right)\log\left(bx+a\right) + 2520\left(b^{9}x^{9} + 9\,ab^{8}x^{8} + 36\,a^{2}b^{7}x^{7} + 84\,a^{3}b^{6}x^{6} + 126\,a^{4}b^{5}x^{5} + 126\,a^{5}b^{4}x^{4} + 84\,a^{6}b^{3}x^{3} + 36\,a^{7}b^{2}x^{2} + 9\,a^{8}bx + a^{9}\right)\log\left(x\right)}{2520\left(a^{10}b^{9}x^{9} + 9\,a^{11}b^{8}x^{8} + 36\,a^{12}b^{7}x^{7} + 84\,a^{13}b^{6}x^{6} + 126\,a^{14}b^{5}x^{5} + 126\,a^{15}b^{4}x^{4} + 84\,a^{16}b^{3}x^{3} + 36\,a^{17}b^{2}x^{2} + 9\,a^{18}bx + a^{19}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*a*b^8*x^8 + 21420*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^4
*b^5*x^5 + 236754*a^5*b^4*x^4 + 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 + 4
1481*a^8*b*x + 7129*a^9 - 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^7 + 84
*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 + 36*a^7*
b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a) + 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*
a^2*b^7*x^7 + 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b
^3*x^3 + 36*a^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(x))/(a^10*b^9*x^9 + 9*a^11*b
^8*x^8 + 36*a^12*b^7*x^7 + 84*a^13*b^6*x^6 + 126*a^14*b^5*x^5 + 126*a^15*b^
4*x^4 + 84*a^16*b^3*x^3 + 36*a^17*b^2*x^2 + 9*a^18*b*x + a^19)
```

**giac [A]** time = 1.05, size = 120, normalized size = 0.85

$$-\frac{\log\left(|bx+a|\right)}{a^{10}} + \frac{\log\left(|x|\right)}{a^{10}} + \frac{2520\,ab^{8}x^{8} + 21420\,a^{2}b^{7}x^{7} + 80220\,a^{3}b^{6}x^{6} + 173250\,a^{4}b^{5}x^{5} + 236754\,a^{5}b^{4}x^{4} + 210756\,a^{6}b^{3}x^{3} + 120564\,a^{7}b^{2}x^{2} + 41481\,a^{8}bx + 7129\,a^{9}}{2520\,(bx+a)^{9}a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^10 + log(abs(x))/a^10 + 1/2520*(2520*a*b^8*x^8 + 21420
*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^4*b^5*x^5 + 236754*a^5*b^4*x^4
+ 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 + 41481*a^8*b*x + 7129*a^9)/((b*x
 + a)^9*a^10)
```

**maple** [A] time = 0.01, size = 126, normalized size = 0.89

$$\frac{1}{9\left(bx+a\right)^{9}a}+\frac{1}{8\left(bx+a\right)^{8}a^{2}}+\frac{1}{7\left(bx+a\right)^{7}a^{3}}+\frac{1}{6\left(bx+a\right)^{6}a^{4}}+\frac{1}{5\left(bx+a\right)^{5}a^{5}}+\frac{1}{4\left(bx+a\right)^{4}a^{6}}+\frac{1}{3\left(bx+a\right)^{3}a^{7}}+\frac{1}{2\left(bx+a\right)^{2}a^{8}}+\frac{1}{\left(bx+a\right)a^{9}}+\frac{\ln(x)}{a^{10}}-\frac{\ln\left(bx+a\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^10,x)
```

```
[Out] 1/9/a/(b*x+a)^9+1/8/a^2/(b*x+a)^8+1/7/a^3/(b*x+a)^7+1/6/a^4/(b*x+a)^6+1/5/a
^5/(b*x+a)^5+1/4/a^6/(b*x+a)^4+1/3/a^7/(b*x+a)^3+1/2/a^8/(b*x+a)^2+1/a^9/(b
*x+a)+ln(x)/a^10-ln(b*x+a)/a^10
```

**maxima** [A] time = 1.75, size = 205, normalized size = 1.45

$$\frac{2520\,b^{8}x^{8}+21420\,ab^{7}x^{7}+80220\,a^{2}b^{6}x^{6}+173250\,a^{3}b^{5}x^{5}+236754\,a^{4}b^{4}x^{4}+210756\,a^{5}b^{3}x^{3}+120564\,a^{6}b^{2}x^{2}+41481\,a^{7}bx+7129\,a^{8}}{2520\left(a^{9}b^{9}x^{9}+9\,a^{10}b^{8}x^{8}+36\,a^{11}b^{7}x^{7}+84\,a^{12}b^{6}x^{6}+126\,a^{13}b^{5}x^{5}+126\,a^{14}b^{4}x^{4}+84\,a^{15}b^{3}x^{3}+36\,a^{16}b^{2}x^{2}+9\,a^{17}bx+a^{18}\right)}-\frac{\log\left(bx+a\right)}{a^{10}}+\frac{\log(x)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2520*(2520*b^8*x^8 + 21420*a*b^7*x^7 + 80220*a^2*b^6*x^6 + 173250*a^3*b^5
*x^5 + 236754*a^4*b^4*x^4 + 210756*a^5*b^3*x^3 + 120564*a^6*b^2*x^2 + 41481
*a^7*b*x + 7129*a^8)/(a^9*b^9*x^9 + 9*a^10*b^8*x^8 + 36*a^11*b^7*x^7 + 84*a
^12*b^6*x^6 + 126*a^13*b^5*x^5 + 126*a^14*b^4*x^4 + 84*a^15*b^3*x^3 + 36*a^
16*b^2*x^2 + 9*a^17*b*x + a^18) - log(b*x + a)/a^10 + log(x)/a^10
```

**mupad** [B] time = 0.76, size = 145, normalized size = 1.03

$$\frac{1}{9\,a\,{\left(a+b\,x\right)}^9}-\frac{\ln\left(\frac{a+b\,x}{x}\right)-\frac{14\,b^2\,x^2}{{\left(a+b\,x\right)}^2}+\frac{56\,b^3\,x^3}{3\,{\left(a+b\,x\right)}^3}-\frac{35\,b^4\,x^4}{2\,{\left(a+b\,x\right)}^4}+\frac{56\,b^5\,x^5}{5\,{\left(a+b\,x\right)}^5}-\frac{14\,b^6\,x^6}{3\,{\left(a+b\,x\right)}^6}+\frac{8\,b^7\,x^7}{7\,{\left(a+b\,x\right)}^7}-\frac{b^8\,x^8}{8\,{\left(a+b\,x\right)}^8}+\frac{8\,b\,x}{a+b\,x}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^10),x)
```

```
[Out] 1/(9*a*(a + b*x)^9) - (log((a + b*x)/x) - (14*b^2*x^2)/(a + b*x)^2 + (56*b^
3*x^3)/(3*(a + b*x)^3) - (35*b^4*x^4)/(2*(a + b*x)^4) + (56*b^5*x^5)/(5*(a
+ b*x)^5) - (14*b^6*x^6)/(3*(a + b*x)^6) + (8*b^7*x^7)/(7*(a + b*x)^7) - (b
^8*x^8)/(8*(a + b*x)^8) + (8*b*x)/(a + b*x))/a^10
```

**sympy** [A] time = 1.02, size = 212, normalized size = 1.50

$$\frac{7129a^{8}+41481a^{7}bx+120564a^{6}b^{2}x^{2}+210756a^{5}b^{3}x^{3}+236754a^{4}b^{4}x^{4}+173250a^{3}b^{5}x^{5}+80220a^{2}b^{6}x^{6}+21420ab^{7}x^{7}+2520b^{8}x^{8}}{2520a^{18}+22680a^{17}bx+90720a^{16}b^{2}x^{2}+211680a^{15}b^{3}x^{3}+317520a^{14}b^{4}x^{4}+317520a^{13}b^{5}x^{5}+211680a^{12}b^{6}x^{6}+90720a^{11}b^{7}x^{7}+22680a^{10}b^{8}x^{8}+2520a^{9}b^{9}x^{9}}+\frac{\log\left(x\right)-\log\left(\frac{a}{b}+x\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**10,x)

[Out] (7129*a**8 + 41481*a**7*b*x + 120564*a**6*b**2*x**2 + 210756*a**5*b**3*x**3
 + 236754*a**4*b**4*x**4 + 173250*a**3*b**5*x**5 + 80220*a**2*b**6*x**6 + 2
1420*a*b**7*x**7 + 2520*b**8*x**8)/(2520*a**18 + 22680*a**17*b*x + 90720*a*
*16*b**2*x**2 + 211680*a**15*b**3*x**3 + 317520*a**14*b**4*x**4 + 317520*a*
*13*b**5*x**5 + 211680*a**12*b**6*x**6 + 90720*a**11*b**7*x**7 + 22680*a**1
0*b**8*x**8 + 2520*a**9*b**9*x**9) + (log(x) - log(a/b + x))/a**10
```

## 3.236 $\int \frac{1}{x^2(a+bx)^{10}} \, dx$

**Optimal**. Leaf size=158

$$-\frac{10b\log(x)}{a^{11}}+\frac{10b\log(a+bx)}{a^{11}}-\frac{9b}{a^{10}(a+bx)}-\frac{1}{a^{10}x}-\frac{4b}{a^9(a+bx)^2}-\frac{7b}{3a^8(a+bx)^3}-\frac{3b}{2a^7(a+bx)^4}-\frac{b}{a^6(a+bx)^5}-\frac{}{3a^5(}$$

**Rubi [A]** time = 0.12, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{9b}{a^{10}(a+bx)}-\frac{4b}{a^9(a+bx)^2}-\frac{7b}{3a^8(a+bx)^3}-\frac{3b}{2a^7(a+bx)^4}-\frac{b}{a^6(a+bx)^5}-\frac{2b}{3a^5(a+bx)^6}-\frac{3b}{7a^4(a+bx)^7}-\frac{b}{4a^3(a+bx)^8}-\frac{b}{9a^2(a+bx)^9}-\frac{10b\log(x)}{a^{11}}+\frac{10b\log(a+bx)}{a^{11}}-\frac{1}{a^{10}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^10),x]
```

```
[Out] -(1/(a^10*x)) - b/(9*a^2*(a + b*x)^9) - b/(4*a^3*(a + b*x)^8) - (3*b)/(7*a^
4*(a + b*x)^7) - (2*b)/(3*a^5*(a + b*x)^6) - b/(a^6*(a + b*x)^5) - (3*b)/(2
*a^7*(a + b*x)^4) - (7*b)/(3*a^8*(a + b*x)^3) - (4*b)/(a^9*(a + b*x)^2) - (
9*b)/(a^10*(a + b*x)) - (10*b*Log[x])/a^11 + (10*b*Log[a + b*x])/a^11
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^{10}} \, dx &= \int \left(\frac{1}{a^{10}x^2}-\frac{10b}{a^{11}x}+\frac{b^2}{a^2(a+bx)^{10}}+\frac{2b^2}{a^3(a+bx)^9}+\frac{3b^2}{a^4(a+bx)^8}+\frac{4b^2}{a^5(a+bx)^7}+\frac{5b^2}{a^6(a+bx)}\right.\\ &= -\frac{1}{a^{10}x}-\frac{b}{9a^2(a+bx)^9}-\frac{b}{4a^3(a+bx)^8}-\frac{3b}{7a^4(a+bx)^7}-\frac{2b}{3a^5(a+bx)^6}-\frac{b}{a^6(a+bx)^5}-\frac{}{2a^7(}\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 130, normalized size = 0.82

$$-\frac{\frac{a\left(252a^9+7129a^8bx+41481a^7b^2x^2+120564a^6b^3x^3+210756a^5b^4x^4+236754a^4b^5x^5+173250a^3b^6x^6+80220a^2b^7x^7+21420ab^8x^8+2520b^9x^9\right)}{x(a+bx)^9}-2520b\log(a+bx)+2520b\log(x)}{252a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^10),x]
```

```
[Out] -1/252*((a*(252*a^9 + 7129*a^8*b*x + 41481*a^7*b^2*x^2 + 120564*a^6*b^3*x^3
 + 210756*a^5*b^4*x^4 + 236754*a^4*b^5*x^5 + 173250*a^3*b^6*x^6 + 80220*a^2
*b^7*x^7 + 21420*a*b^8*x^8 + 2520*b^9*x^9))/(x*(a + b*x)^9) + 2520*b*Log[x]
 - 2520*b*Log[a + b*x])/a^11
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)^10), x]
```

**fricas [B]** time = 1.17, size = 417, normalized size = 2.64

$$-\frac{2520\,ab^{9}x^{9}+21420\,a^{2}b^{8}x^{8}+80220\,a^{3}b^{7}x^{7}+173250\,a^{4}b^{6}x^{6}+236754\,a^{5}b^{5}x^{5}+210756\,a^{6}b^{4}x^{4}+120564\,a^{7}b^{3}x^{3}+41481\,a^{8}b^{2}x^{2}+7129\,a^{9}bx+252\,a^{10}-2520\left(b^{10}x^{10}+9\,ab^{9}x^{9}+36\,a^{2}b^{8}x^{8}+84\,a^{3}b^{7}x^{7}+126\,a^{4}b^{6}x^{6}+126\,a^{5}b^{5}x^{5}+84\,a^{6}b^{4}x^{4}+36\,a^{7}b^{3}x^{3}+9\,a^{8}b^{2}x^{2}+a^{9}bx\right)\log\left(bx+a\right)+2520\left(b^{10}x^{10}+9\,ab^{9}x^{9}+36\,a^{2}b^{8}x^{8}+84\,a^{3}b^{7}x^{7}+126\,a^{4}b^{6}x^{6}+126\,a^{5}b^{5}x^{5}+84\,a^{6}b^{4}x^{4}+36\,a^{7}b^{3}x^{3}+9\,a^{8}b^{2}x^{2}+a^{9}bx\right)\log(x)}{252\left(a^{11}b^{9}x^{10}+9\,a^{12}b^{8}x^{9}+36\,a^{13}b^{7}x^{8}+84\,a^{14}b^{6}x^{7}+126\,a^{15}b^{5}x^{6}+126\,a^{16}b^{4}x^{5}+84\,a^{17}b^{3}x^{4}+36\,a^{18}b^{2}x^{3}+9\,a^{19}bx^{2}+a^{20}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(2520*a*b^9*x^9 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^4
*b^6*x^6 + 236754*a^5*b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 + 4
1481*a^8*b^2*x^2 + 7129*a^9*b*x + 252*a^10 - 2520*(b^10*x^10 + 9*a*b^9*x^9
+ 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4*b^6*x^6 + 126*a^5*b^5*x^5 + 84*
a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x^2 + a^9*b*x)*log(b*x + a) + 2520
*(b^10*x^10 + 9*a*b^9*x^9 + 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4*b^6*x
^6 + 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x^2 + a^
9*b*x)*log(x))/(a^11*b^9*x^10 + 9*a^12*b^8*x^9 + 36*a^13*b^7*x^8 + 84*a^14*
b^6*x^7 + 126*a^15*b^5*x^6 + 126*a^16*b^4*x^5 + 84*a^17*b^3*x^4 + 36*a^18*b
^2*x^3 + 9*a^19*b*x^2 + a^20*x)
```

**giac [A]** time = 1.01, size = 137, normalized size = 0.87

$$\frac{10\,b\log\left(|bx+a|\right)}{a^{11}}-\frac{10\,b\log\left(|x|\right)}{a^{11}}-\frac{2520\,ab^{9}x^{9}+21420\,a^{2}b^{8}x^{8}+80220\,a^{3}b^{7}x^{7}+173250\,a^{4}b^{6}x^{6}+236754\,a^{5}b^{5}x^{5}+210756\,a^{6}b^{4}x^{4}+120564\,a^{7}b^{3}x^{3}+41481\,a^{8}b^{2}x^{2}+7129\,a^{9}bx+252\,a^{10}}{252\,(bx+a)^{9}a^{11}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 10*b*log(abs(b*x + a))/a^11 - 10*b*log(abs(x))/a^11 - 1/252*(2520*a*b^9*x^9
 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^4*b^6*x^6 + 236754*a^5*
```

```
b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 + 41481*a^8*b^2*x^2 + 712
9*a^9*b*x + 252*a^10)/((b*x + a)^9*a^11*x)
```

**maple** [A] time = 0.01, size = 147, normalized size = 0.93

$$-\frac{b}{9\,(bx+a)^{9}\,a^{2}}-\frac{b}{4\,(bx+a)^{8}\,a^{3}}-\frac{3b}{7\,(bx+a)^{7}\,a^{4}}-\frac{2b}{3\,(bx+a)^{6}\,a^{5}}-\frac{b}{(bx+a)^{5}\,a^{6}}-\frac{3b}{2\,(bx+a)^{4}\,a^{7}}-\frac{7b}{3\,(bx+a)^{3}\,a^{8}}-\frac{4b}{(bx+a)^{2}\,a^{9}}-\frac{9b}{(bx+a)\,a^{10}}-\frac{10b\ln(x)}{a^{11}}+\frac{10b\ln(bx+a)}{a^{11}}-\frac{1}{a^{10}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^10,x)
```

```
[Out] -1/a^10/x-1/9*b/a^2/(b*x+a)^9-1/4*b/a^3/(b*x+a)^8-3/7*b/a^4/(b*x+a)^7-2/3*b
/a^5/(b*x+a)^6-b/a^6/(b*x+a)^5-3/2*b/a^7/(b*x+a)^4-7/3*b/a^8/(b*x+a)^3-4*b/
a^9/(b*x+a)^2-9*b/a^10/(b*x+a)-10*b*ln(x)/a^11+10*b*ln(b*x+a)/a^11
```

**maxima** [A] time = 1.71, size = 223, normalized size = 1.41

$$-\frac{2520\,b^{9}x^{9}+21420\,ab^{8}x^{8}+80220\,a^{2}b^{7}x^{7}+173250\,a^{3}b^{6}x^{6}+236754\,a^{4}b^{5}x^{5}+210756\,a^{5}b^{4}x^{4}+120564\,a^{6}b^{3}x^{3}+41481\,a^{7}b^{2}x^{2}+7129\,a^{8}bx+252\,a^{9}}{252\left(a^{10}b^{9}x^{10}+9\,a^{11}b^{8}x^{9}+36\,a^{12}b^{7}x^{8}+84\,a^{13}b^{6}x^{7}+126\,a^{14}b^{5}x^{6}+126\,a^{15}b^{4}x^{5}+84\,a^{16}b^{3}x^{4}+36\,a^{17}b^{2}x^{3}+9\,a^{18}bx^{2}+a^{19}x\right)}+\frac{10\,b\log(bx+a)}{a^{11}}-\frac{10\,b\log(x)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(2520*b^9*x^9 + 21420*a*b^8*x^8 + 80220*a^2*b^7*x^7 + 173250*a^3*b^6
*x^6 + 236754*a^4*b^5*x^5 + 210756*a^5*b^4*x^4 + 120564*a^6*b^3*x^3 + 41481
*a^7*b^2*x^2 + 7129*a^8*b*x + 252*a^9)/(a^10*b^9*x^10 + 9*a^11*b^8*x^9 + 36
*a^12*b^7*x^8 + 84*a^13*b^6*x^7 + 126*a^14*b^5*x^6 + 126*a^15*b^4*x^5 + 84*
a^16*b^3*x^4 + 36*a^17*b^2*x^3 + 9*a^18*b*x^2 + a^19*x) + 10*b*log(b*x + a)
/a^11 - 10*b*log(x)/a^11
```

**mupad** [B] time = 0.39, size = 217, normalized size = 1.37

$$\frac{20\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{11}}-\frac{\frac{1}{a}+\frac{4609\,b^2\,x^2}{28\,a^3}+\frac{3349\,b^3\,x^3}{7\,a^4}+\frac{2509\,b^4\,x^4}{3\,a^5}+\frac{1879\,b^5\,x^5}{2\,a^6}+\frac{1375\,b^6\,x^6}{2\,a^7}+\frac{955\,b^7\,x^7}{3\,a^8}+\frac{85\,b^8\,x^8}{a^9}+\frac{10\,b^9\,x^9}{a^{10}}+\frac{7129\,b\,x}{252\,a^2}}{a^9\,x+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^3+84\,a^6\,b^3\,x^4+126\,a^5\,b^4\,x^5+126\,a^4\,b^5\,x^6+84\,a^3\,b^6\,x^7+36\,a^2\,b^7\,x^8+9\,a\,b^8\,x^9+b^9\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^10),x)
```

```
[Out] (20*b*atanh((2*b*x)/a + 1))/a^11 - (1/a + (4609*b^2*x^2)/(28*a^3) + (3349*b
^3*x^3)/(7*a^4) + (2509*b^4*x^4)/(3*a^5) + (1879*b^5*x^5)/(2*a^6) + (1375*b
^6*x^6)/(2*a^7) + (955*b^7*x^7)/(3*a^8) + (85*b^8*x^8)/a^9 + (10*b^9*x^9)/a
^10 + (7129*b*x)/(252*a^2))/(a^9*x + b^9*x^10 + 9*a^8*b*x^2 + 9*a*b^8*x^9 +
 36*a^7*b^2*x^3 + 84*a^6*b^3*x^4 + 126*a^5*b^4*x^5 + 126*a^4*b^5*x^6 + 84*a
^3*b^6*x^7 + 36*a^2*b^7*x^8)
```

**sympy** [A] time = 1.15, size = 233, normalized size = 1.47

$$\frac{-252a^{9}-7129a^{8}bx-41481a^{7}b^{2}x^{2}-120564a^{6}b^{3}x^{3}-210756a^{5}b^{4}x^{4}-236754a^{4}b^{5}x^{5}-173250a^{3}b^{6}x^{6}-80220a^{2}b^{7}x^{7}-21420ab^{8}x^{8}-2520b^{9}x^{9}}{252a^{19}x+2268a^{18}bx^{2}+9072a^{17}b^{2}x^{3}+21168a^{16}b^{3}x^{4}+31752a^{15}b^{4}x^{5}+31752a^{14}b^{5}x^{6}+21168a^{13}b^{6}x^{7}+9072a^{12}b^{7}x^{8}+2268a^{11}b^{8}x^{9}+252a^{10}b^{9}x^{10}}+\frac{10b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**10,x)
```

```
[Out] (-252*a**9 - 7129*a**8*b*x - 41481*a**7*b**2*x**2 - 120564*a**6*b**3*x**3 -
 210756*a**5*b**4*x**4 - 236754*a**4*b**5*x**5 - 173250*a**3*b**6*x**6 - 80
220*a**2*b**7*x**7 - 21420*a*b**8*x**8 - 2520*b**9*x**9)/(252*a**19*x + 226
8*a**18*b*x**2 + 9072*a**17*b**2*x**3 + 21168*a**16*b**3*x**4 + 31752*a**15
*b**4*x**5 + 31752*a**14*b**5*x**6 + 21168*a**13*b**6*x**7 + 9072*a**12*b**
7*x**8 + 2268*a**11*b**8*x**9 + 252*a**10*b**9*x**10) + 10*b*(-log(x) + log
(a/b + x))/a**11
```

## 3.237 $\int \frac{1}{x^3(a+bx)^{10}}\,dx$

**Optimal**. Leaf size=191

$$\frac{55b^2\log(x)}{a^{12}}-\frac{55b^2\log(a+bx)}{a^{12}}+\frac{45b^2}{a^{11}(a+bx)}+\frac{10b}{a^{11}x}+\frac{18b^2}{a^{10}(a+bx)^2}-\frac{1}{2a^{10}x^2}+\frac{28b^2}{3a^9(a+bx)^3}+\frac{21b^2}{4a^8(a+bx)^4}+\frac{3b^2}{a^7(a+$$

**Rubi** [A] time = 0.14, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{45b^2}{a^{11}(a+bx)}+\frac{18b^2}{a^{10}(a+bx)^2}+\frac{28b^2}{3a^9(a+bx)^3}+\frac{21b^2}{4a^8(a+bx)^4}+\frac{3b^2}{a^7(a+bx)^5}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{3b^2}{8a^4(a+bx)^8}+\frac{b^2}{9a^3(a+bx)^9}+\frac{55b^2\log(x)}{a^{12}}-\frac{55b^2\log(a+bx)}{a^{12}}+\frac{10b}{a^{11}x}-\frac{1}{2a^{10}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^10),x]
```

```
[Out] -1/(2*a^10*x^2) + (10*b)/(a^11*x) + b^2/(9*a^3*(a + b*x)^9) + (3*b^2)/(8*a^
4*(a + b*x)^8) + (6*b^2)/(7*a^5*(a + b*x)^7) + (5*b^2)/(3*a^6*(a + b*x)^6)
+ (3*b^2)/(a^7*(a + b*x)^5) + (21*b^2)/(4*a^8*(a + b*x)^4) + (28*b^2)/(3*a^
9*(a + b*x)^3) + (18*b^2)/(a^10*(a + b*x)^2) + (45*b^2)/(a^11*(a + b*x)) +
(55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x])/a^12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^{10}}\,dx &= \int \left(\frac{1}{a^{10}x^3}-\frac{10b}{a^{11}x^2}+\frac{55b^2}{a^{12}x}-\frac{b^3}{a^3(a+bx)^{10}}-\frac{3b^3}{a^4(a+bx)^9}-\frac{6b^3}{a^5(a+bx)^8}-\frac{10b^3}{a^6(a+bx)^7}-\frac{}{a^7}\right.\\ &= -\frac{1}{2a^{10}x^2}+\frac{10b}{a^{11}x}+\frac{b^2}{9a^3(a+bx)^9}+\frac{3b^2}{8a^4(a+bx)^8}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{3b^2}{a^7(a+}\end{aligned}$$

**Mathematica** [A] time = 0.11, size = 145, normalized size = 0.76

$$\frac{\frac{a\left(-252a^{10}+2772a^9bx+78419a^8b^2x^2+456291a^7b^3x^3+1326204a^6b^4x^4+2318316a^5b^5x^5+2604294a^4b^6x^6+1905750a^3b^7x^7+882420a^2b^8x^8+235620ab^9x^9+27720b^{10}x^{10}\right)}{x^2(a+bx)^9}-27720b^2\log(a+bx)+27720b^2\log(x)}{504a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^10),x]
```

```
[Out] ((a*(-252*a^10 + 2772*a^9*b*x + 78419*a^8*b^2*x^2 + 456291*a^7*b^3*x^3 + 13
26204*a^6*b^4*x^4 + 2318316*a^5*b^5*x^5 + 2604294*a^4*b^6*x^6 + 1905750*a^3
*b^7*x^7 + 882420*a^2*b^8*x^8 + 235620*a*b^9*x^9 + 27720*b^10*x^10))/(x^2*(
a + b*x)^9) + 27720*b^2*Log[x] - 27720*b^2*Log[a + b*x])/(504*a^12)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)^10), x]
```

**fricas [B]** time = 0.61, size = 438, normalized size = 2.29

$$\frac{27720\,ab^{10}x^{10} + 235620\,a^2b^9x^9 + 882420\,a^3b^8x^8 + 1905750\,a^4b^7x^7 + 2604294\,a^5b^6x^6 + 2318316\,a^6b^5x^5 + 1326204\,a^7b^4x^4 + 456291\,a^8b^3x^3 + 78419\,a^9b^2x^2 + 2772\,a^{10}bx - 252\,a^{11} - 27720\left(b^{11}x^{11} + 9\,ab^{10}x^{10} + 36\,a^2b^9x^9 + 84\,a^3b^8x^8 + 126\,a^4b^7x^7 + 126\,a^5b^6x^6 + 84\,a^6b^5x^5 + 36\,a^7b^4x^4 + 9\,a^8b^3x^3 + a^9b^2x^2\right)\log\left(bx+a\right) + 27720\left(b^{11}x^{11} + 9\,ab^{10}x^{10} + 36\,a^2b^9x^9 + 84\,a^3b^8x^8 + 126\,a^4b^7x^7 + 126\,a^5b^6x^6 + 84\,a^6b^5x^5 + 36\,a^7b^4x^4 + 9\,a^8b^3x^3 + a^9b^2x^2\right)\log(x)}{504\left(a^{12}b^9x^{11} + 9\,a^{13}b^8x^{10} + 36\,a^{14}b^7x^9 + 84\,a^{15}b^6x^8 + 126\,a^{16}b^5x^7 + 126\,a^{17}b^4x^6 + 84\,a^{18}b^3x^5 + 36\,a^{19}b^2x^4 + 9\,a^{20}bx^3 + a^{21}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(27720*a*b^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 190575
0*a^4*b^7*x^7 + 2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b^4
*x^4 + 456291*a^8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11 -
27720*(b^11*x^11 + 9*a*b^10*x^10 + 36*a^2*b^9*x^9 + 84*a^3*b^8*x^8 + 126*a^
4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^6*b^5*x^5 + 36*a^7*b^4*x^4 + 9*a^8*b^3*x
^3 + a^9*b^2*x^2)*log(b*x + a) + 27720*(b^11*x^11 + 9*a*b^10*x^10 + 36*a^2*
b^9*x^9 + 84*a^3*b^8*x^8 + 126*a^4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^6*b^5*x
^5 + 36*a^7*b^4*x^4 + 9*a^8*b^3*x^3 + a^9*b^2*x^2)*log(x))/(a^12*b^9*x^11 +
 9*a^13*b^8*x^10 + 36*a^14*b^7*x^9 + 84*a^15*b^6*x^8 + 126*a^16*b^5*x^7 + 1
26*a^17*b^4*x^6 + 84*a^18*b^3*x^5 + 36*a^19*b^2*x^4 + 9*a^20*b*x^3 + a^21*x
^2)
```

**giac [A]** time = 1.10, size = 152, normalized size = 0.80

$$-\frac{55\,b^2\log\left(|bx+a|\right)}{a^{12}} + \frac{55\,b^2\log\left(|x|\right)}{a^{12}} + \frac{27720\,ab^{10}x^{10} + 235620\,a^2b^9x^9 + 882420\,a^3b^8x^8 + 1905750\,a^4b^7x^7 + 2604294\,a^5b^6x^6 + 2318316\,a^6b^5x^5 + 1326204\,a^7b^4x^4 + 456291\,a^8b^3x^3 + 78419\,a^9b^2x^2 + 2772\,a^{10}bx - 252\,a^{11}}{504\,(bx+a)^9a^{12}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -55*b^2*log(abs(b*x + a))/a^12 + 55*b^2*log(abs(x))/a^12 + 1/504*(27720*a*b
^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 1905750*a^4*b^7*x^7 +
2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b^4*x^4 + 456291*a^
8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11)/((b*x + a)^9*a^12
*x^2)
```

**maple** [A]  time = 0.02, size = 178, normalized size = 0.93

$$\frac{b^2}{9\left(bx+a\right)^9 a^3}+\frac{3b^2}{8\left(bx+a\right)^8 a^4}+\frac{6b^2}{7\left(bx+a\right)^7 a^5}+\frac{5b^2}{3\left(bx+a\right)^6 a^6}+\frac{3b^2}{\left(bx+a\right)^5 a^7}+\frac{21b^2}{4\left(bx+a\right)^4 a^8}+\frac{28b^2}{3\left(bx+a\right)^3 a^9}+\frac{18b^2}{\left(bx+a\right)^2 a^{10}}+\frac{45b^2}{\left(bx+a\right) a^{11}}+\frac{55b^2\ln(x)}{a^{12}}-\frac{55b^2\ln\left(bx+a\right)}{a^{12}}+\frac{10b}{a^{11}x}-\frac{1}{2a^{10}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^10,x)
```

```
[Out] -1/2/a^10/x^2+10*b/a^11/x+1/9*b^2/a^3/(b*x+a)^9+3/8*b^2/a^4/(b*x+a)^8+6/7*b
^2/a^5/(b*x+a)^7+5/3*b^2/a^6/(b*x+a)^6+3*b^2/a^7/(b*x+a)^5+21/4*b^2/a^8/(b*
x+a)^4+28/3*b^2/a^9/(b*x+a)^3+18*b^2/a^10/(b*x+a)^2+45*b^2/a^11/(b*x+a)+55*
b^2*ln(x)/a^12-55*b^2*ln(b*x+a)/a^12
```

**maxima** [A]  time = 1.71, size = 240, normalized size = 1.26

$$\frac{27720\,b^{10}x^{10}+235620\,ab^{9}x^{9}+882420\,a^{2}b^{8}x^{8}+1905750\,a^{3}b^{7}x^{7}+2604294\,a^{4}b^{6}x^{6}+2318316\,a^{5}b^{5}x^{5}+1326204\,a^{6}b^{4}x^{4}+456291\,a^{7}b^{3}x^{3}+78419\,a^{8}b^{2}x^{2}+2772\,a^{9}bx-252\,a^{10}}{504\left(a^{11}b^{9}x^{11}+9\,a^{12}b^{8}x^{10}+36\,a^{13}b^{7}x^{9}+84\,a^{14}b^{6}x^{8}+126\,a^{15}b^{5}x^{7}+126\,a^{16}b^{4}x^{6}+84\,a^{17}b^{3}x^{5}+36\,a^{18}b^{2}x^{4}+9\,a^{19}bx^{3}+a^{20}x^{2}\right)}-\frac{55\,b^{2}\log\left(bx+a\right)}{a^{12}}+\frac{55\,b^{2}\log\left(x\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/504*(27720*b^10*x^10 + 235620*a*b^9*x^9 + 882420*a^2*b^8*x^8 + 1905750*a^
3*b^7*x^7 + 2604294*a^4*b^6*x^6 + 2318316*a^5*b^5*x^5 + 1326204*a^6*b^4*x^4
 + 456291*a^7*b^3*x^3 + 78419*a^8*b^2*x^2 + 2772*a^9*b*x - 252*a^10)/(a^11*
b^9*x^11 + 9*a^12*b^8*x^10 + 36*a^13*b^7*x^9 + 84*a^14*b^6*x^8 + 126*a^15*b
^5*x^7 + 126*a^16*b^4*x^6 + 84*a^17*b^3*x^5 + 36*a^18*b^2*x^4 + 9*a^19*b*x^
3 + a^20*x^2) - 55*b^2*log(b*x + a)/a^12 + 55*b^2*log(x)/a^12
```

**mupad** [B]  time = 0.44, size = 233, normalized size = 1.22

$$\frac{\frac{78419\,b^2\,x^2}{504\,a^3}-\frac{1}{2\,a}+\frac{50699\,b^3\,x^3}{56\,a^4}+\frac{36839\,b^4\,x^4}{14\,a^5}+\frac{27599\,b^5\,x^5}{6\,a^6}+\frac{20669\,b^6\,x^6}{4\,a^7}+\frac{15125\,b^7\,x^7}{4\,a^8}+\frac{10505\,b^8\,x^8}{6\,a^9}+\frac{935\,b^9\,x^9}{2\,a^{10}}+\frac{55\,b^{10}\,x^{10}}{a^{11}}+\frac{11\,b\,x}{2\,a^2}}{a^9\,x^2+9\,a^8\,b\,x^3+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^5+126\,a^5\,b^4\,x^6+126\,a^4\,b^5\,x^7+84\,a^3\,b^6\,x^8+36\,a^2\,b^7\,x^9+9\,a\,b^8\,x^{10}+b^9\,x^{11}}-\frac{110\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^10),x)
```

```
[Out] ((78419*b^2*x^2)/(504*a^3) - 1/(2*a) + (50699*b^3*x^3)/(56*a^4) + (36839*b^
4*x^4)/(14*a^5) + (27599*b^5*x^5)/(6*a^6) + (20669*b^6*x^6)/(4*a^7) + (1512
5*b^7*x^7)/(4*a^8) + (10505*b^8*x^8)/(6*a^9) + (935*b^9*x^9)/(2*a^10) + (55
*b^10*x^10)/a^11 + (11*b*x)/(2*a^2))/(a^9*x^2 + b^9*x^11 + 9*a^8*b*x^3 + 9*
```

```
a*b^8*x^10 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^5 + 126*a^5*b^4*x^6 + 126*a^4*b^
5*x^7 + 84*a^3*b^6*x^8 + 36*a^2*b^7*x^9) - (110*b^2*atanh((2*b*x)/a + 1))/a
^12
```

**sympy [A]** time = 1.18, size = 246, normalized size = 1.29

$$\frac{-252a^{10}+2772a^9bx+78419a^8b^2x^2+456291a^7b^3x^3+1326204a^6b^4x^4+2318316a^5b^5x^5+2604294a^4b^6x^6+1905750a^3b^7x^7+882420a^2b^8x^8+235620ab^9x^9+27720b^{10}x^{10}}{504a^{20}x^2+4536a^{19}bx^3+18144a^{18}b^2x^4+42336a^{17}b^3x^5+63504a^{16}b^4x^6+63504a^{15}b^5x^7+42336a^{14}b^6x^8+18144a^{13}b^7x^9+4536a^{12}b^8x^{10}+504a^{11}b^9x^{11}}+\frac{55b^2\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**10,x)
```

```
[Out] (-252*a**10 + 2772*a**9*b*x + 78419*a**8*b**2*x**2 + 456291*a**7*b**3*x**3
+ 1326204*a**6*b**4*x**4 + 2318316*a**5*b**5*x**5 + 2604294*a**4*b**6*x**6
+ 1905750*a**3*b**7*x**7 + 882420*a**2*b**8*x**8 + 235620*a*b**9*x**9 + 277
20*b**10*x**10)/(504*a**20*x**2 + 4536*a**19*b*x**3 + 18144*a**18*b**2*x**4
 + 42336*a**17*b**3*x**5 + 63504*a**16*b**4*x**6 + 63504*a**15*b**5*x**7 +
42336*a**14*b**6*x**8 + 18144*a**13*b**7*x**9 + 4536*a**12*b**8*x**10 + 504
*a**11*b**9*x**11) + 55*b**2*(log(x) - log(a/b + x))/a**12
```

## 3.238 $\int \frac{1}{x^4(a+bx)^{10}}\,dx$

**Optimal**. Leaf size=198

$$-\frac{220b^3\log(x)}{a^{13}}+\frac{220b^3\log(a+bx)}{a^{13}}-\frac{165b^3}{a^{12}(a+bx)}-\frac{55b^2}{a^{12}x}-\frac{60b^3}{a^{11}(a+bx)^2}+\frac{5b}{a^{11}x^2}-\frac{28b^3}{a^{10}(a+bx)^3}-\frac{1}{3a^{10}x^3}-\frac{14b^3}{a^9(a+bx)^4}$$

**Rubi** [A] time = 0.16, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{165b^3}{a^{12}(a+bx)}-\frac{60b^3}{a^{11}(a+bx)^2}-\frac{28b^3}{a^{10}(a+bx)^3}-\frac{14b^3}{a^9(a+bx)^4}-\frac{7b^3}{a^8(a+bx)^5}-\frac{10b^3}{3a^7(a+bx)^6}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{b^3}{2a^5(a+bx)^8}-\frac{b^3}{9a^4(a+bx)^9}-\frac{55b^2}{a^{12}x}-\frac{220b^3\log(x)}{a^{13}}+\frac{220b^3\log(a+bx)}{a^{13}}+\frac{5b}{a^{11}x^2}-\frac{1}{3a^{10}x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^10),x]
```

```
[Out] -1/(3*a^10*x^3) + (5*b)/(a^11*x^2) - (55*b^2)/(a^12*x) - b^3/(9*a^4*(a + b*
x)^9) - b^3/(2*a^5*(a + b*x)^8) - (10*b^3)/(7*a^6*(a + b*x)^7) - (10*b^3)/(
3*a^7*(a + b*x)^6) - (7*b^3)/(a^8*(a + b*x)^5) - (14*b^3)/(a^9*(a + b*x)^4)
 - (28*b^3)/(a^10*(a + b*x)^3) - (60*b^3)/(a^11*(a + b*x)^2) - (165*b^3)/(a
^12*(a + b*x)) - (220*b^3*Log[x])/a^13 + (220*b^3*Log[a + b*x])/a^13
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx = \int \left(\frac{1}{a^{10}x^4}-\frac{10b}{a^{11}x^3}+\frac{55b^2}{a^{12}x^2}-\frac{220b^3}{a^{13}x}+\frac{b^4}{a^4(a+bx)^{10}}+\frac{4b^4}{a^5(a+bx)^9}+\frac{10b^4}{a^6(a+bx)^8}+\frac{20b^4}{a^7(a+\ldots}\right.$$

$$= -\frac{1}{3a^{10}x^3}+\frac{5b}{a^{11}x^2}-\frac{55b^2}{a^{12}x}-\frac{b^3}{9a^4(a+bx)^9}-\frac{b^3}{2a^5(a+bx)^8}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{10b^3}{3a^7(a+bx)^6}-\ldots$$

**Mathematica** [A] time = 0.12, size = 156, normalized size = 0.79

$$-\frac{-27720b^3\log(a+bx)+\frac{a\left(42a^{11}-252a^{10}bx+2772a^9b^2x^2+78419a^8b^3x^3+456291a^7b^4x^4+1326204a^6b^5x^5+2318316a^5b^6x^6+2604294a^4b^7x^7+1905750a^3b^8x^8+882420a^2b^9x^9+235620ab^{10}x^{10}+27720b^{11}x^{11}\right)}{x^3(a+bx)^9}+27720b^3\log(x)}{126a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^10),x]
```

```
[Out] -1/126*((a*(42*a^11 - 252*a^10*b*x + 2772*a^9*b^2*x^2 + 78419*a^8*b^3*x^3 +
 456291*a^7*b^4*x^4 + 1326204*a^6*b^5*x^5 + 2318316*a^5*b^6*x^6 + 2604294*a
^4*b^7*x^7 + 1905750*a^3*b^8*x^8 + 882420*a^2*b^9*x^9 + 235620*a*b^10*x^10
+ 27720*b^11*x^11))/(x^3*(a + b*x)^9) + 27720*b^3*Log[x] - 27720*b^3*Log[a
+ b*x])/a^13
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)^10), x]
```

**fricas [B]** time = 1.26, size = 449, normalized size = 2.27

$$-\frac{27720\,ab^{11}x^{11} + 235620\,a^{2}b^{10}x^{10} + 882420\,a^{3}b^{9}x^{9} + 1905750\,a^{4}b^{8}x^{8} + 2604294\,a^{5}b^{7}x^{7} + 2318316\,a^{6}b^{6}x^{6} + 1326204\,a^{7}b^{5}x^{5} + 456291\,a^{8}b^{4}x^{4} + 78419\,a^{9}b^{3}x^{3} + 2772\,a^{10}b^{2}x^{2} - 252\,a^{11}bx + 42\,a^{12} - 27720\left(b^{12}x^{12} + 9\,ab^{11}x^{11} + 36\,a^{2}b^{10}x^{10} + 84\,a^{3}b^{9}x^{9} + 126\,a^{4}b^{8}x^{8} + 126\,a^{5}b^{7}x^{7} + 84\,a^{6}b^{6}x^{6} + 36\,a^{7}b^{5}x^{5} + 9\,a^{8}b^{4}x^{4} + a^{9}b^{3}x^{3}\right)\log\left(bx+a\right) + 27720\left(b^{12}x^{12} + 9\,ab^{11}x^{11} + 36\,a^{2}b^{10}x^{10} + 84\,a^{3}b^{9}x^{9} + 126\,a^{4}b^{8}x^{8} + 126\,a^{5}b^{7}x^{7} + 84\,a^{6}b^{6}x^{6} + 36\,a^{7}b^{5}x^{5} + 9\,a^{8}b^{4}x^{4} + a^{9}b^{3}x^{3}\right)\log\left(x\right)}{126\left(a^{13}b^{9}x^{12} + 9\,a^{14}b^{8}x^{11} + 36\,a^{15}b^{7}x^{10} + 84\,a^{16}b^{6}x^{9} + 126\,a^{17}b^{5}x^{8} + 126\,a^{18}b^{4}x^{7} + 84\,a^{19}b^{3}x^{6} + 36\,a^{20}b^{2}x^{5} + 9\,a^{21}bx^{4} + a^{22}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/126*(27720*a*b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 190
5750*a^4*b^8*x^8 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^7*
b^5*x^5 + 456291*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 252*
a^11*b*x + 42*a^12 - 27720*(b^12*x^12 + 9*a*b^11*x^11 + 36*a^2*b^10*x^10 +
84*a^3*b^9*x^9 + 126*a^4*b^8*x^8 + 126*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 36*a^
7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*b^3*x^3)*log(b*x + a) + 27720*(b^12*x^12 +
9*a*b^11*x^11 + 36*a^2*b^10*x^10 + 84*a^3*b^9*x^9 + 126*a^4*b^8*x^8 + 126*a
^5*b^7*x^7 + 84*a^6*b^6*x^6 + 36*a^7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*b^3*x^3)
*log(x))/(a^13*b^9*x^12 + 9*a^14*b^8*x^11 + 36*a^15*b^7*x^10 + 84*a^16*b^6*
x^9 + 126*a^17*b^5*x^8 + 126*a^18*b^4*x^7 + 84*a^19*b^3*x^6 + 36*a^20*b^2*x
^5 + 9*a^21*b*x^4 + a^22*x^3)
```

**giac [A]** time = 1.35, size = 163, normalized size = 0.82

$$\frac{220\,b^{3}\log\left(|bx+a|\right)}{a^{13}} - \frac{220\,b^{3}\log\left(|x|\right)}{a^{13}} - \frac{27720\,ab^{11}x^{11} + 235620\,a^{2}b^{10}x^{10} + 882420\,a^{3}b^{9}x^{9} + 1905750\,a^{4}b^{8}x^{8} + 2604294\,a^{5}b^{7}x^{7} + 2318316\,a^{6}b^{6}x^{6} + 1326204\,a^{7}b^{5}x^{5} + 456291\,a^{8}b^{4}x^{4} + 78419\,a^{9}b^{3}x^{3} + 2772\,a^{10}b^{2}x^{2} - 252\,a^{11}bx + 42\,a^{12}}{126\,(bx+a)^{9}a^{13}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 220*b^3*log(abs(b*x + a))/a^13 - 220*b^3*log(abs(x))/a^13 - 1/126*(27720*a*
b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 1905750*a^4*b^8*x^8
 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^7*b^5*x^5 + 456291
*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 252*a^11*b*x + 42*a^
12)/((b*x + a)^9*a^13*x^3)
```

**maple** [A] time = 0.01, size = 189, normalized size = 0.95

$$-\frac{b^3}{9(bx+a)^9 a^4}-\frac{b^3}{2(bx+a)^8 a^5}-\frac{10b^3}{7(bx+a)^7 a^6}-\frac{10b^3}{3(bx+a)^6 a^7}-\frac{7b^3}{(bx+a)^5 a^8}-\frac{14b^3}{(bx+a)^4 a^9}-\frac{28b^3}{(bx+a)^3 a^{10}}-\frac{60b^3}{(bx+a)^2 a^{11}}-\frac{165b^3}{(bx+a)a^{12}}-\frac{220b^3\ln(x)}{a^{13}}+\frac{220b^3\ln(bx+a)}{a^{13}}-\frac{55b^2}{a^{12}x}+\frac{5b}{a^{11}x^2}-\frac{1}{3a^{10}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^10,x)
```

```
[Out] -1/3/a^10/x^3+5*b/a^11/x^2-55*b^2/a^12/x-1/9*b^3/a^4/(b*x+a)^9-1/2*b^3/a^5/
(b*x+a)^8-10/7*b^3/a^6/(b*x+a)^7-10/3*b^3/a^7/(b*x+a)^6-7*b^3/a^8/(b*x+a)^5
-14*b^3/a^9/(b*x+a)^4-28*b^3/a^10/(b*x+a)^3-60*b^3/a^11/(b*x+a)^2-165*b^3/a
^12/(b*x+a)-220*b^3*ln(x)/a^13+220*b^3*ln(b*x+a)/a^13
```

**maxima** [A] time = 1.72, size = 251, normalized size = 1.27

$$-\frac{27720\,b^{11}x^{11}+235620\,ab^{10}x^{10}+882420\,a^2b^9x^9+1905750\,a^3b^8x^8+2604294\,a^4b^7x^7+2318316\,a^5b^6x^6+1326204\,a^6b^5x^5+456291\,a^7b^4x^4+78419\,a^8b^3x^3+2772\,a^9b^2x^2-252\,a^{10}bx+42\,a^{11}}{126\left(a^{12}b^9x^{12}+9\,a^{13}b^8x^{11}+36\,a^{14}b^7x^{10}+84\,a^{15}b^6x^9+126\,a^{16}b^5x^8+126\,a^{17}b^4x^7+84\,a^{18}b^3x^6+36\,a^{19}b^2x^5+9\,a^{20}bx^4+a^{21}x^3\right)}+\frac{220\,b^3\log(bx+a)}{a^{13}}-\frac{220\,b^3\log(x)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/126*(27720*b^11*x^11 + 235620*a*b^10*x^10 + 882420*a^2*b^9*x^9 + 1905750
*a^3*b^8*x^8 + 2604294*a^4*b^7*x^7 + 2318316*a^5*b^6*x^6 + 1326204*a^6*b^5*
x^5 + 456291*a^7*b^4*x^4 + 78419*a^8*b^3*x^3 + 2772*a^9*b^2*x^2 - 252*a^10*
b*x + 42*a^11)/(a^12*b^9*x^12 + 9*a^13*b^8*x^11 + 36*a^14*b^7*x^10 + 84*a^1
5*b^6*x^9 + 126*a^16*b^5*x^8 + 126*a^17*b^4*x^7 + 84*a^18*b^3*x^6 + 36*a^19
*b^2*x^5 + 9*a^20*b*x^4 + a^21*x^3) + 220*b^3*log(b*x + a)/a^13 - 220*b^3*l
og(x)/a^13
```

**mupad** [B] time = 0.59, size = 245, normalized size = 1.24

$$\frac{440\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{13}}-\frac{\frac{1}{3\,a}+\frac{22\,b^2\,x^2}{a^3}+\frac{78419\,b^3\,x^3}{126\,a^4}+\frac{50699\,b^4\,x^4}{14\,a^5}+\frac{73678\,b^5\,x^5}{7\,a^6}+\frac{55198\,b^6\,x^6}{3\,a^7}+\frac{20669\,b^7\,x^7}{a^8}+\frac{15125\,b^8\,x^8}{a^9}+\frac{21010\,b^9\,x^9}{3\,a^{10}}+\frac{1870\,b^{10}\,x^{10}}{a^{11}}+\frac{220\,b^{11}\,x^{11}}{a^{12}}-\frac{2\,b\,x}{a^2}}{a^9\,x^3+9\,a^8\,b\,x^4+36\,a^7\,b^2\,x^5+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^7+126\,a^4\,b^5\,x^8+84\,a^3\,b^6\,x^9+36\,a^2\,b^7\,x^{10}+9\,a\,b^8\,x^{11}+b^9\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^10),x)
```

```
[Out] (440*b^3*atanh((2*b*x)/a + 1))/a^13 - (1/(3*a) + (22*b^2*x^2)/a^3 + (78419*
b^3*x^3)/(126*a^4) + (50699*b^4*x^4)/(14*a^5) + (73678*b^5*x^5)/(7*a^6) + (
55198*b^6*x^6)/(3*a^7) + (20669*b^7*x^7)/a^8 + (15125*b^8*x^8)/a^9 + (21010
```

```
*b^9*x^9)/(3*a^10) + (1870*b^10*x^10)/a^11 + (220*b^11*x^11)/a^12 - (2*b*x)
/a^2)/(a^9*x^3 + b^9*x^12 + 9*a^8*b*x^4 + 9*a*b^8*x^11 + 36*a^7*b^2*x^5 + 8
4*a^6*b^3*x^6 + 126*a^5*b^4*x^7 + 126*a^4*b^5*x^8 + 84*a^3*b^6*x^9 + 36*a^2
*b^7*x^10)
```

**sympy** [A] time = 1.24, size = 258, normalized size = 1.30

$$\frac{-42a^{11} + 252a^{10}bx - 2772a^9b^2x^2 - 78419a^8b^3x^3 - 456291a^7b^4x^4 - 1326204a^6b^5x^5 - 2318316a^5b^6x^6 - 2604294a^4b^7x^7 - 1905750a^3b^8x^8 - 882420a^2b^9x^9 - 235620ab^{10}x^{10} - 27720b^{11}x^{11}}{126a^{21}x^3 + 1134a^{20}bx^4 + 4536a^{19}b^2x^5 + 10584a^{18}b^3x^6 + 15876a^{17}b^4x^7 + 15876a^{16}b^5x^8 + 10584a^{15}b^6x^9 + 4536a^{14}b^7x^{10} + 1134a^{13}b^8x^{11} + 126a^{12}b^9x^{12}} + \frac{220b^3\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**10,x)
```

```
[Out] (-42*a**11 + 252*a**10*b*x - 2772*a**9*b**2*x**2 - 78419*a**8*b**3*x**3 - 4
56291*a**7*b**4*x**4 - 1326204*a**6*b**5*x**5 - 2318316*a**5*b**6*x**6 - 26
04294*a**4*b**7*x**7 - 1905750*a**3*b**8*x**8 - 882420*a**2*b**9*x**9 - 235
620*a*b**10*x**10 - 27720*b**11*x**11)/(126*a**21*x**3 + 1134*a**20*b*x**4
+ 4536*a**19*b**2*x**5 + 10584*a**18*b**3*x**6 + 15876*a**17*b**4*x**7 + 15
876*a**16*b**5*x**8 + 10584*a**15*b**6*x**9 + 4536*a**14*b**7*x**10 + 1134*
a**13*b**8*x**11 + 126*a**12*b**9*x**12) + 220*b**3*(-log(x) + log(a/b + x)
)/a**13
```

## 3.239 $\int \frac{(a+bx)^{12}}{x^{10}}\,dx$

**Optimal**. Leaf size=141

$$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+220a^3b^9\log(x)+66a^2b^{10}x+6$$

**Rubi [A]** time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+220a^3b^9\log(x)-\frac{3a^{11}b}{2x^8}-\frac{a^{12}}{9x^9}+6ab^{11}x^2+\frac{b^{12}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^12/x^10,x]
```

```
[Out] -a^12/(9*x^9) - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)/
(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396*a
^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3)/3
 + 220*a^3*b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx=\int \left(66a^2b^{10}+\frac{a^{12}}{x^{10}}+\frac{12a^{11}b}{x^9}+\frac{66a^{10}b^2}{x^8}+\frac{220a^9b^3}{x^7}+\frac{495a^8b^4}{x^6}+\frac{792a^7b^5}{x^5}+\frac{924a^6b^6}{x^4}+\frac{792a}{x}\right.$$

$$=-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}$$

**Mathematica [A]** time = 0.01, size = 141, normalized size = 1.00

$$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+220a^3b^9\log(x)+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^12/x^10,x]
```

```
[Out] -1/9*a^12/x^9 - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)/
(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396*a
^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3)/3
 + 220*a^3*b^9*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^12/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^12/x^10, x]
```

**fricas [A]** time = 1.04, size = 136, normalized size = 0.96

$$\frac{42\,b^{12}x^{12}+756\,ab^{11}x^{11}+8316\,a^{2}b^{10}x^{10}+27720\,a^{3}b^{9}x^{9}\log(x)-62370\,a^{4}b^{8}x^{8}-49896\,a^{5}b^{7}x^{7}-38808\,a^{6}b^{6}x^{6}-24948\,a^{7}b^{5}x^{5}-12474\,a^{8}b^{4}x^{4}-4620\,a^{9}b^{3}x^{3}-1188\,a^{10}b^{2}x^{2}-189\,a^{11}bx-14\,a^{12}}{126\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="fricas")
```

```
[Out] 1/126*(42*b^12*x^12 + 756*a*b^11*x^11 + 8316*a^2*b^10*x^10 + 27720*a^3*b^9*
x^9*log(x) - 62370*a^4*b^8*x^8 - 49896*a^5*b^7*x^7 - 38808*a^6*b^6*x^6 - 24
948*a^7*b^5*x^5 - 12474*a^8*b^4*x^4 - 4620*a^9*b^3*x^3 - 1188*a^10*b^2*x^2
- 189*a^11*b*x - 14*a^12)/x^9
```

**giac [A]** time = 1.14, size = 133, normalized size = 0.94

$$\frac{1}{3}\,b^{12}x^{3}+6\,ab^{11}x^{2}+66\,a^{2}b^{10}x+220\,a^{3}b^{9}\log\left(|x|\right)-\frac{62370\,a^{4}b^{8}x^{8}+49896\,a^{5}b^{7}x^{7}+38808\,a^{6}b^{6}x^{6}+24948\,a^{7}b^{5}x^{5}+12474\,a^{8}b^{4}x^{4}+4620\,a^{9}b^{3}x^{3}+1188\,a^{10}b^{2}x^{2}+189\,a^{11}bx+14\,a^{12}}{126\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="giac")
```

```
[Out] 1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(abs(x)) - 1/1
26*(62370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7*b
^5*x^5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*a^1
1*b*x + 14*a^12)/x^9
```

**maple [A]** time = 0.01, size = 132, normalized size = 0.94

$$\frac{b^{12}x^{3}}{3}+6a\,b^{11}x^{2}+220a^{3}b^{9}\ln(x)+66a^{2}b^{10}x-\frac{495a^{4}b^{8}}{x}-\frac{396a^{5}b^{7}}{x^{2}}-\frac{308a^{6}b^{6}}{x^{3}}-\frac{198a^{7}b^{5}}{x^{4}}-\frac{99a^{8}b^{4}}{x^{5}}-\frac{110a^{9}b^{3}}{3x^{6}}-\frac{66a^{10}b^{2}}{7x^{7}}-\frac{3a^{11}b}{2x^{8}}-\frac{a^{12}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^12/x^10,x)
```

```
[Out] -1/9*a^12/x^9-3/2*a^11*b/x^8-66/7*a^10*b^2/x^7-110/3*a^9*b^3/x^6-99*a^8*b^4
/x^5-198*a^7*b^5/x^4-308*a^6*b^6/x^3-396*a^5*b^7/x^2-495*a^4*b^8/x+66*a^2*b
^10*x+6*a*b^11*x^2+1/3*b^12*x^3+220*a^3*b^9*ln(x)
```

**maxima [A]** time = 1.37, size = 132, normalized size = 0.94

$$\frac{1}{3}\,b^{12}x^3+6\,ab^{11}x^2+66\,a^2b^{10}x+220\,a^3b^9\log(x)-\frac{62370\,a^4b^8x^8+49896\,a^5b^7x^7+38808\,a^6b^6x^6+24948\,a^7b^5x^5+12474\,a^8b^4x^4+4620\,a^9b^3x^3+1188\,a^{10}b^2x^2+189\,a^{11}bx+14\,a^{12}}{126\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(x) - 1/126*(6
2370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7*b^5*x^
5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*a^11*b*x
 + 14*a^12)/x^9
```

**mupad [B]** time = 0.08, size = 132, normalized size = 0.94

$$\frac{b^{12}\,x^3}{3}-\frac{\frac{a^{12}}{9}+\frac{3\,a^{11}\,b\,x}{2}+\frac{66\,a^{10}\,b^2\,x^2}{7}+\frac{110\,a^9\,b^3\,x^3}{3}+99\,a^8\,b^4\,x^4+198\,a^7\,b^5\,x^5+308\,a^6\,b^6\,x^6+396\,a^5\,b^7\,x^7+495\,a^4\,b^8\,x^8}{x^9}+66\,a^2\,b^{10}\,x+6\,a\,b^{11}\,x^2+220\,a^3\,b^9\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^12/x^10,x)
```

```
[Out] (b^12*x^3)/3 - (a^12/9 + (66*a^10*b^2*x^2)/7 + (110*a^9*b^3*x^3)/3 + 99*a^8
*b^4*x^4 + 198*a^7*b^5*x^5 + 308*a^6*b^6*x^6 + 396*a^5*b^7*x^7 + 495*a^4*b^
8*x^8 + (3*a^11*b*x)/2)/x^9 + 66*a^2*b^10*x + 6*a*b^11*x^2 + 220*a^3*b^9*lo
g(x)
```

**sympy [A]** time = 0.91, size = 143, normalized size = 1.01

$$220a^3b^9\log{(x)}+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}+\frac{-14a^{12}-189a^{11}bx-1188a^{10}b^2x^2-4620a^9b^3x^3-12474a^8b^4x^4-24948a^7b^5x^5-38808a^6b^6x^6-49896a^5b^7x^7-62370a^4b^8x^8}{126x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**12/x**10,x)
```

```
[Out] 220*a**3*b**9*log(x) + 66*a**2*b**10*x + 6*a*b**11*x**2 + b**12*x**3/3 + (-
14*a**12 - 189*a**11*b*x - 1188*a**10*b**2*x**2 - 4620*a**9*b**3*x**3 - 124
74*a**8*b**4*x**4 - 24948*a**7*b**5*x**5 - 38808*a**6*b**6*x**6 - 49896*a**
5*b**7*x**7 - 62370*a**4*b**8*x**8)/(126*x**9)
```

## 3.240 $\int \frac{(a+bx)^{11}}{x^{10}}\,dx$

**Optimal**. Leaf size=132

$$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+55a^2b^9\log(x)+11ab^{10}x+\frac{b^{11}x^2}{2}$$

**Rubi** [A] time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+55a^2b^9\log(x)-\frac{11a^{10}b}{8x^8}-\frac{a^{11}}{9x^9}+11ab^{10}x+\frac{b^{11}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^11/x^10,x]
```

```
[Out] -a^11/(9*x^9) - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/(
2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (16
5*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b^9*
Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{11}}{x^{10}}\,dx = \int \left(11ab^{10}+\frac{a^{11}}{x^{10}}+\frac{11a^{10}b}{x^9}+\frac{55a^9b^2}{x^8}+\frac{165a^8b^3}{x^7}+\frac{330a^7b^4}{x^6}+\frac{462a^6b^5}{x^5}+\frac{462a^5b^6}{x^4}+\frac{330a^4b^7}{x^3}\right.$$
$$= -\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+1$$

**Mathematica** [A] time = 0.01, size = 132, normalized size = 1.00

$$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+55a^2b^9\log(x)+11ab^{10}x+\frac{b^{11}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^11/x^10,x]
```

```
[Out] -1/9*a^11/x^9 - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/(
2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (16
5*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b^9*
Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{11}}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^11/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^11/x^10, x]
```

**fricas [A]** time = 1.08, size = 125, normalized size = 0.95

$$\frac{252\,b^{11}x^{11}+5544\,ab^{10}x^{10}+27720\,a^{2}b^{9}x^{9}\log(x)-83160\,a^{3}b^{8}x^{8}-83160\,a^{4}b^{7}x^{7}-77616\,a^{5}b^{6}x^{6}-58212\,a^{6}b^{5}x^{5}-33264\,a^{7}b^{4}x^{4}-13860\,a^{8}b^{3}x^{3}-3960\,a^{9}b^{2}x^{2}-693\,a^{10}bx-56\,a^{11}}{504\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^11*x^11 + 5544*a*b^10*x^10 + 27720*a^2*b^9*x^9*log(x) - 83160*
a^3*b^8*x^8 - 83160*a^4*b^7*x^7 - 77616*a^5*b^6*x^6 - 58212*a^6*b^5*x^5 - 3
3264*a^7*b^4*x^4 - 13860*a^8*b^3*x^3 - 3960*a^9*b^2*x^2 - 693*a^10*b*x - 56
*a^11)/x^9
```

**giac [A]** time = 1.28, size = 122, normalized size = 0.92

$$\frac{1}{2}\,b^{11}x^{2}+11\,ab^{10}x+55\,a^{2}b^{9}\log\left(|x|\right)-\frac{83160\,a^{3}b^{8}x^{8}+83160\,a^{4}b^{7}x^{7}+77616\,a^{5}b^{6}x^{6}+58212\,a^{6}b^{5}x^{5}+33264\,a^{7}b^{4}x^{4}+13860\,a^{8}b^{3}x^{3}+3960\,a^{9}b^{2}x^{2}+693\,a^{10}bx+56\,a^{11}}{504\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="giac")
```

```
[Out] 1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(abs(x)) - 1/504*(83160*a^3*b^8*
x^8 + 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a^7
*b^4*x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11)/x
^9
```

**maple [A]** time = 0.01, size = 121, normalized size = 0.92

$$\frac{b^{11}x^{2}}{2}+55a^{2}b^{9}\ln(x)+11a\,b^{10}x-\frac{165a^{3}b^{8}}{x}-\frac{165a^{4}b^{7}}{x^{2}}-\frac{154a^{5}b^{6}}{x^{3}}-\frac{231a^{6}b^{5}}{2x^{4}}-\frac{66a^{7}b^{4}}{x^{5}}-\frac{55a^{8}b^{3}}{2x^{6}}-\frac{55a^{9}b^{2}}{7x^{7}}-\frac{11a^{10}b}{8x^{8}}-\frac{a^{11}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^11/x^10,x)
```

```
[Out] -1/9*a^11/x^9-11/8*a^10*b/x^8-55/7*a^9*b^2/x^7-55/2*a^8*b^3/x^6-66*a^7*b^4/
x^5-231/2*a^6*b^5/x^4-154*a^5*b^6/x^3-165*a^4*b^7/x^2-165*a^3*b^8/x+11*a*b^
10*x+1/2*b^11*x^2+55*a^2*b^9*ln(x)
```

**maxima [A]** time = 1.37, size = 121, normalized size = 0.92

$$\frac{1}{2}\,b^{11}x^2+11\,ab^{10}x+55\,a^2b^9\log(x)-\frac{83160\,a^3b^8x^8+83160\,a^4b^7x^7+77616\,a^5b^6x^6+58212\,a^6b^5x^5+33264\,a^7b^4x^4+13860\,a^8b^3x^3+3960\,a^9b^2x^2+693\,a^{10}bx+56\,a^{11}}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="maxima")
```

```
[Out] 1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(x) - 1/504*(83160*a^3*b^8*x^8 +
 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a^7*b^4*
x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11)/x^9
```

**mupad [B]** time = 0.09, size = 121, normalized size = 0.92

$$\frac{b^{11}\,x^2}{2}-\frac{\frac{a^{11}}{9}+\frac{11\,a^{10}\,b\,x}{8}+\frac{55\,a^9\,b^2\,x^2}{7}+\frac{55\,a^8\,b^3\,x^3}{2}+66\,a^7\,b^4\,x^4+\frac{231\,a^6\,b^5\,x^5}{2}+154\,a^5\,b^6\,x^6+165\,a^4\,b^7\,x^7+165\,a^3\,b^8\,x^8}{x^9}+55\,a^2\,b^9\,\ln(x)+11\,a\,b^{10}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^11/x^10,x)
```

```
[Out] (b^11*x^2)/2 - (a^11/9 + (55*a^9*b^2*x^2)/7 + (55*a^8*b^3*x^3)/2 + 66*a^7*b
^4*x^4 + (231*a^6*b^5*x^5)/2 + 154*a^5*b^6*x^6 + 165*a^4*b^7*x^7 + 165*a^3*
b^8*x^8 + (11*a^10*b*x)/8)/x^9 + 55*a^2*b^9*log(x) + 11*a*b^10*x
```

**sympy [A]** time = 0.85, size = 131, normalized size = 0.99

$$55a^2b^9\log{(x)}+11ab^{10}x+\frac{b^{11}x^2}{2}+\frac{-56a^{11}-693a^{10}bx-3960a^9b^2x^2-13860a^8b^3x^3-33264a^7b^4x^4-58212a^6b^5x^5-77616a^5b^6x^6-83160a^4b^7x^7-83160a^3b^8x^8}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**11/x**10,x)
```

```
[Out] 55*a**2*b**9*log(x) + 11*a*b**10*x + b**11*x**2/2 + (-56*a**11 - 693*a**10*
b*x - 3960*a**9*b**2*x**2 - 13860*a**8*b**3*x**3 - 33264*a**7*b**4*x**4 - 5
8212*a**6*b**5*x**5 - 77616*a**5*b**6*x**6 - 83160*a**4*b**7*x**7 - 83160*a
**3*b**8*x**8)/(504*x**9)
```

## 3.241 $\int \frac{(a+bx)^{10}}{x^{10}}\,dx$

**Optimal**. Leaf size=114

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

**Rubi** [A] time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}-\frac{5a^9b}{4x^8}-\frac{a^{10}}{9x^9}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^10,x]
```

```
[Out] -a^10/(9*x^9) - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx = \int \left(b^{10}+\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}+\right.$$
$$= -\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+$$

**Mathematica** [A] time = 0.01, size = 114, normalized size = 1.00

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^10, x]
```

**fricas [A]** time = 1.05, size = 114, normalized size = 1.00

$$\frac{252\,b^{10}x^{10}+2520\,ab^{9}x^{9}\log(x)-11340\,a^{2}b^{8}x^{8}-15120\,a^{3}b^{7}x^{7}-17640\,a^{4}b^{6}x^{6}-15876\,a^{5}b^{5}x^{5}-10584\,a^{6}b^{4}x^{4}-5040\,a^{7}b^{3}x^{3}-1620\,a^{8}b^{2}x^{2}-315\,a^{9}bx-28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a^
3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 504
0*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

**giac [A]** time = 1.06, size = 110, normalized size = 0.96

$$b^{10}x+10\,ab^{9}\log\left(|x|\right)-\frac{11340\,a^{2}b^{8}x^{8}+15120\,a^{3}b^{7}x^{7}+17640\,a^{4}b^{6}x^{6}+15876\,a^{5}b^{5}x^{5}+10584\,a^{6}b^{4}x^{4}+5040\,a^{7}b^{3}x^{3}+1620\,a^{8}b^{2}x^{2}+315\,a^{9}bx+28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="giac")
```

```
[Out] b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^
7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^
3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

**maple [A]** time = 0.00, size = 109, normalized size = 0.96

$$10a\,b^{9}\ln(x)+b^{10}x-\frac{45a^{2}b^{8}}{x}-\frac{60a^{3}b^{7}}{x^{2}}-\frac{70a^{4}b^{6}}{x^{3}}-\frac{63a^{5}b^{5}}{x^{4}}-\frac{42a^{6}b^{4}}{x^{5}}-\frac{20a^{7}b^{3}}{x^{6}}-\frac{45a^{8}b^{2}}{7x^{7}}-\frac{5a^{9}b}{4x^{8}}-\frac{a^{10}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^10,x)
```

```
[Out] 10*a*b^9*ln(x)+b^10*x-45*a^2*b^8/x-60*a^3*b^7/x^2-70*a^4*b^6/x^3-63*a^5*b^5
/x^4-42*a^6*b^4/x^5-20*a^7*b^3/x^6-45/7*a^8*b^2/x^7-5/4*a^9*b/x^8-1/9*a^10/
x^9
```

**maxima [A]** time = 1.43, size = 109, normalized size = 0.96

$$b^{10}x + 10\,ab^9\log(x) - \frac{11340\,a^2b^8x^8 + 15120\,a^3b^7x^7 + 17640\,a^4b^6x^6 + 15876\,a^5b^5x^5 + 10584\,a^6b^4x^4 + 5040\,a^7b^3x^3 + 1620\,a^8b^2x^2 + 315\,a^9bx + 28\,a^{10}}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

```
[Out] b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 + 1
7640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x^3
 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

**mupad [B]** time = 0.00, size = 114, normalized size = 1.00

$$-\frac{\frac{a^{10}}{9} - b^{10}\,x^{10} + \frac{45\,a^8\,b^2\,x^2}{7} + 20\,a^7\,b^3\,x^3 + 42\,a^6\,b^4\,x^4 + 63\,a^5\,b^5\,x^5 + 70\,a^4\,b^6\,x^6 + 60\,a^3\,b^7\,x^7 + 45\,a^2\,b^8\,x^8 + \frac{5\,a^9\,b\,x}{4} - 10\,a\,b^9\,x^9\,\ln(x)}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^10,x)
```

```
[Out] -(a^10/9 - b^10*x^10 + (45*a^8*b^2*x^2)/7 + 20*a^7*b^3*x^3 + 42*a^6*b^4*x^4
 + 63*a^5*b^5*x^5 + 70*a^4*b^6*x^6 + 60*a^3*b^7*x^7 + 45*a^2*b^8*x^8 + (5*a
^9*b*x)/4 - 10*a*b^9*x^9*log(x))/x^9
```

**sympy [A]** time = 0.86, size = 117, normalized size = 1.03

$$10ab^9\log{(x)} + b^{10}x + \frac{-28a^{10} - 315a^9bx - 1620a^8b^2x^2 - 5040a^7b^3x^3 - 10584a^6b^4x^4 - 15876a^5b^5x^5 - 17640a^4b^6x^6 - 15120a^3b^7x^7 - 11340a^2b^8x^8}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**10,x)
```

```
[Out] 10*a*b**9*log(x) + b**10*x + (-28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x**
2 - 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 176
40*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(252*x**9)
```

## 3.242 $\int \frac{(a+bx)^9}{x^{10}} dx$

**Optimal**. Leaf size=109

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

**Rubi** [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9a^8b}{8x^8}-\frac{a^9}{9x^9}-\frac{9ab^8}{x}+b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9/x^10,x]
```

```
[Out] -a^9/(9*x^9) - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
- (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a^2
*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^9}{x^{10}}\,dx &= \int \left(\frac{a^9}{x^{10}}+\frac{9a^8b}{x^9}+\frac{36a^7b^2}{x^8}+\frac{84a^6b^3}{x^7}+\frac{126a^5b^4}{x^6}+\frac{126a^4b^5}{x^5}+\frac{84a^3b^6}{x^4}+\frac{36a^2b^7}{x^3}+\frac{9ab^8}{x^2}+\frac{b^9}{x}\right) d\\ &= -\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 109, normalized size = 1.00

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9/x^10,x]
```

```
[Out] -1/9*a^9/x^9 - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
- (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a^2
*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^9}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^9/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^9/x^10, x]
```

**fricas [A]** time = 0.95, size = 103, normalized size = 0.94

$$\frac{2520\,b^9x^9\log(x) - 22680\,ab^8x^8 - 45360\,a^2b^7x^7 - 70560\,a^3b^6x^6 - 79380\,a^4b^5x^5 - 63504\,a^5b^4x^4 - 35280\,a^6b^3x^3 - 12960\,a^7b^2x^2 - 2835\,a^8bx - 280\,a^9}{2520\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^9*x^9*log(x) - 22680*a*b^8*x^8 - 45360*a^2*b^7*x^7 - 70560*a
^3*b^6*x^6 - 79380*a^4*b^5*x^5 - 63504*a^5*b^4*x^4 - 35280*a^6*b^3*x^3 - 12
960*a^7*b^2*x^2 - 2835*a^8*b*x - 280*a^9)/x^9
```

**giac [A]** time = 1.04, size = 101, normalized size = 0.93

$$b^9\log(|x|) - \frac{22680\,ab^8x^8 + 45360\,a^2b^7x^7 + 70560\,a^3b^6x^6 + 79380\,a^4b^5x^5 + 63504\,a^5b^4x^4 + 35280\,a^6b^3x^3 + 12960\,a^7b^2x^2 + 2835\,a^8bx + 280\,a^9}{2520\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="giac")
```

```
[Out] b^9*log(abs(x)) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*b
^6*x^6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 12960*
a^7*b^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

**maple [A]** time = 0.01, size = 100, normalized size = 0.92

$$b^9\ln(x) - \frac{9a\,b^8}{x} - \frac{18a^2b^7}{x^2} - \frac{28a^3b^6}{x^3} - \frac{63a^4b^5}{2x^4} - \frac{126a^5b^4}{5x^5} - \frac{14a^6b^3}{x^6} - \frac{36a^7b^2}{7x^7} - \frac{9a^8b}{8x^8} - \frac{a^9}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9/x^10,x)
```

```
[Out] -1/9*a^9/x^9-9/8*a^8*b/x^8-36/7*a^7*b^2/x^7-14*a^6*b^3/x^6-126/5*a^5*b^4/x^
5-63/2*a^4*b^5/x^4-28*a^3*b^6/x^3-18*a^2*b^7/x^2-9*a*b^8/x+b^9*ln(x)
```

**maxima [A]** time = 1.34, size = 100, normalized size = 0.92

$$b^9 \log(x) - \frac{22680\, ab^8x^8 + 45360\, a^2b^7x^7 + 70560\, a^3b^6x^6 + 79380\, a^4b^5x^5 + 63504\, a^5b^4x^4 + 35280\, a^6b^3x^3 + 12960\, a^7b^2x^2 + 2835\, a^8bx + 280\, a^9}{2520\, x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="maxima")
```

```
[Out] b^9*log(x) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*b^6*x^
6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 12960*a^7*b
^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

**mupad [B]** time = 0.08, size = 100, normalized size = 0.92

$$b^9\, \ln(x) - \frac{\frac{a^9}{9} + \frac{9\, a^8\, b\, x}{8} + \frac{36\, a^7\, b^2\, x^2}{7} + 14\, a^6\, b^3\, x^3 + \frac{126\, a^5\, b^4\, x^4}{5} + \frac{63\, a^4\, b^5\, x^5}{2} + 28\, a^3\, b^6\, x^6 + 18\, a^2\, b^7\, x^7 + 9\, a\, b^8\, x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9/x^10,x)
```

```
[Out] b^9*log(x) - (a^9/9 + 9*a*b^8*x^8 + (36*a^7*b^2*x^2)/7 + 14*a^6*b^3*x^3 + (
126*a^5*b^4*x^4)/5 + (63*a^4*b^5*x^5)/2 + 28*a^3*b^6*x^6 + 18*a^2*b^7*x^7 +
 (9*a^8*b*x)/8)/x^9
```

**sympy [A]** time = 0.79, size = 107, normalized size = 0.98

$$b^9 \log{(x)} + \frac{-280a^9 - 2835a^8bx - 12960a^7b^2x^2 - 35280a^6b^3x^3 - 63504a^5b^4x^4 - 79380a^4b^5x^5 - 70560a^3b^6x^6 - 45360a^2b^7x^7 - 22680ab^8x^8}{2520x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9/x**10,x)
```

```
[Out] b**9*log(x) + (-280*a**9 - 2835*a**8*b*x - 12960*a**7*b**2*x**2 - 35280*a**
6*b**3*x**3 - 63504*a**5*b**4*x**4 - 79380*a**4*b**5*x**5 - 70560*a**3*b**6
*x**6 - 45360*a**2*b**7*x**7 - 22680*a*b**8*x**8)/(2520*x**9)
```

## 3.243 $\int \frac{(a+bx)^8}{x^{10}} dx$

**Optimal**. Leaf size=17

$$-\frac{(a+bx)^9}{9ax^9}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^9}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8/x^10,x]
```

```
[Out] -(a + b*x)^9/(9*a*x^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^8}{x^{10}} dx = -\frac{(a+bx)^9}{9ax^9}$$

**Mathematica [B]** time = 0.01, size = 96, normalized size = 5.65

$$-\frac{a^8}{9x^9} - \frac{a^7b}{x^8} - \frac{4a^6b^2}{x^7} - \frac{28a^5b^3}{3x^6} - \frac{14a^4b^4}{x^5} - \frac{14a^3b^5}{x^4} - \frac{28a^2b^6}{3x^3} - \frac{4ab^7}{x^2} - \frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8/x^10,x]
```

```
[Out] -1/9*a^8/x^9 - (a^7*b)/x^8 - (4*a^6*b^2)/x^7 - (28*a^5*b^3)/(3*x^6) - (14*a
^4*b^4)/x^5 - (14*a^3*b^5)/x^4 - (28*a^2*b^6)/(3*x^3) - (4*a*b^7)/x^2 - b^8
/x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^8}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^8/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^8/x^10, x]
```

**fricas [B]** time = 0.78, size = 88, normalized size = 5.18

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

**giac [B]** time = 1.17, size = 88, normalized size = 5.18

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="giac")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

**maple [B]** time = 0.01, size = 91, normalized size = 5.35

$$-\frac{b^8}{x}-\frac{4a\,b^7}{x^2}-\frac{28a^2b^6}{3x^3}-\frac{14a^3b^5}{x^4}-\frac{14a^4b^4}{x^5}-\frac{28a^5b^3}{3x^6}-\frac{4a^6b^2}{x^7}-\frac{a^7b}{x^8}-\frac{a^8}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/x^10,x)
```

```
[Out] -28/3*a^2*b^6/x^3-b^8/x-4*a*b^7/x^2-14*a^4*b^4/x^5-1/9*a^8/x^9-a^7*b/x^8-28
/3*a^5*b^3/x^6-14*a^3*b^5/x^4-4*a^6*b^2/x^7
```

**maxima** [**B**] time = 1.34, size = 88, normalized size = 5.18

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="maxima")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

**mupad** [**B**] time = 0.09, size = 88, normalized size = 5.18

$$-\frac{\frac{a^8}{9}+a^7\,b\,x+4\,a^6\,b^2\,x^2+\frac{28\,a^5\,b^3\,x^3}{3}+14\,a^4\,b^4\,x^4+14\,a^3\,b^5\,x^5+\frac{28\,a^2\,b^6\,x^6}{3}+4\,a\,b^7\,x^7+b^8\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8/x^10,x)
```

```
[Out] -(a^8/9 + b^8*x^8 + 4*a*b^7*x^7 + 4*a^6*b^2*x^2 + (28*a^5*b^3*x^3)/3 + 14*a
^4*b^4*x^4 + 14*a^3*b^5*x^5 + (28*a^2*b^6*x^6)/3 + a^7*b*x)/x^9
```

**sympy** [**B**] time = 0.73, size = 95, normalized size = 5.59

$$\frac{-a^8-9a^7bx-36a^6b^2x^2-84a^5b^3x^3-126a^4b^4x^4-126a^3b^5x^5-84a^2b^6x^6-36ab^7x^7-9b^8x^8}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8/x**10,x)
```

```
[Out] (-a**8 - 9*a**7*b*x - 36*a**6*b**2*x**2 - 84*a**5*b**3*x**3 - 126*a**4*b**4
*x**4 - 126*a**3*b**5*x**5 - 84*a**2*b**6*x**6 - 36*a*b**7*x**7 - 9*b**8*x*
*8)/(9*x**9)
```

## 3.244 $\int \frac{(a+bx)^7}{x^{10}}\,dx$

**Optimal**. Leaf size=36

$$\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}$$

**Rubi [A]** time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^10,x]
```

```
[Out] -(a + b*x)^8/(9*a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{10}}\,dx &= -\frac{(a+bx)^8}{9ax^9}-\frac{b\int \frac{(a+bx)^7}{x^9}\,dx}{9a}\\ &= -\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}\end{aligned}$$

**Mathematica [B]** time = 0.00, size = 91, normalized size = 2.53

$$-\frac{a^7}{9x^9}-\frac{7a^6b}{8x^8}-\frac{3a^5b^2}{x^7}-\frac{35a^4b^3}{6x^6}-\frac{7a^3b^4}{x^5}-\frac{21a^2b^5}{4x^4}-\frac{7ab^6}{3x^3}-\frac{b^7}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*a^7/x^9 - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6) -
 (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^10, x]
```

**fricas [B]** time = 0.86, size = 79, normalized size = 2.19

$$-\frac{36\,b^7x^7+168\,ab^6x^6+378\,a^2b^5x^5+504\,a^3b^4x^4+420\,a^4b^3x^3+216\,a^5b^2x^2+63\,a^6bx+8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

**giac [B]** time = 1.04, size = 79, normalized size = 2.19

$$-\frac{36\,b^7x^7+168\,ab^6x^6+378\,a^2b^5x^5+504\,a^3b^4x^4+420\,a^4b^3x^3+216\,a^5b^2x^2+63\,a^6bx+8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

**maple [B]** time = 0.00, size = 80, normalized size = 2.22

$$-\frac{b^{7}}{2x^{2}}-\frac{7a\,b^{6}}{3x^{3}}-\frac{21a^{2}b^{5}}{4x^{4}}-\frac{7a^{3}b^{4}}{x^{5}}-\frac{35a^{4}b^{3}}{6x^{6}}-\frac{3a^{5}b^{2}}{x^{7}}-\frac{7a^{6}b}{8x^{8}}-\frac{a^{7}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^10,x)
```

```
[Out] -1/2*b^7/x^2-7/3*a*b^6/x^3-21/4*a^2*b^5/x^4-7*a^3*b^4/x^5-35/6*a^4*b^3/x^6-
3*a^5*b^2/x^7-7/8*a^6*b/x^8-1/9*a^7/x^9
```

**maxima [B]** time = 1.36, size = 79, normalized size = 2.19

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

**mupad [B]** time = 0.00, size = 23, normalized size = 0.64

$$-\frac{(8\,a-b\,x)\,\left(a+b\,x\right)^{8}}{72\,a^{2}\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^10,x)
```

```
[Out] -((8*a - b*x)*(a + b*x)^8)/(72*a^2*x^9)
```

**sympy [B]** time = 0.70, size = 85, normalized size = 2.36

$$\frac{-8a^{7}-63a^{6}bx-216a^{5}b^{2}x^{2}-420a^{4}b^{3}x^{3}-504a^{3}b^{4}x^{4}-378a^{2}b^{5}x^{5}-168ab^{6}x^{6}-36b^{7}x^{7}}{72x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**10,x)
```

```
[Out] (-8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a**3
*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x**9)
```

## 3.245 $\int \frac{(a+bx)^6}{x^{10}} \, dx$

**Optimal**. Leaf size=56

$$-\frac{b^2(a+bx)^7}{252a^3x^7}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{(a+bx)^7}{9ax^9}$$

**Rubi [A]** time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^7}{252a^3x^7}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{(a+bx)^7}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/x^10,x]
```

```
[Out] -(a + b*x)^7/(9*a*x^9) + (b*(a + b*x)^7)/(36*a^2*x^8) - (b^2*(a + b*x)^7)/(
252*a^3*x^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^6}{x^{10}}\,dx = -\frac{(a+bx)^7}{9ax^9} - \frac{(2b)\int \frac{(a+bx)^6}{x^9}\,dx}{9a}$$
$$= -\frac{(a+bx)^7}{9ax^9} + \frac{b(a+bx)^7}{36a^2x^8} + \frac{b^2\int \frac{(a+bx)^6}{x^8}\,dx}{36a^2}$$
$$= -\frac{(a+bx)^7}{9ax^9} + \frac{b(a+bx)^7}{36a^2x^8} - \frac{b^2(a+bx)^7}{252a^3x^7}$$

**Mathematica** [A] time = 0.01, size = 80, normalized size = 1.43

$$-\frac{a^6}{9x^9} - \frac{3a^5b}{4x^8} - \frac{15a^4b^2}{7x^7} - \frac{10a^3b^3}{3x^6} - \frac{3a^2b^4}{x^5} - \frac{3ab^5}{2x^4} - \frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/x^10,x]
```

```
[Out] -1/9*a^6/x^9 - (3*a^5*b)/(4*x^8) - (15*a^4*b^2)/(7*x^7) - (10*a^3*b^3)/(3*x
^6) - (3*a^2*b^4)/x^5 - (3*a*b^5)/(2*x^4) - b^6/(3*x^3)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^6}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^6/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^6/x^10, x]
```

**fricas** [A] time = 1.13, size = 68, normalized size = 1.21

$$-\frac{84\,b^6x^6 + 378\,ab^5x^5 + 756\,a^2b^4x^4 + 840\,a^3b^3x^3 + 540\,a^4b^2x^2 + 189\,a^5bx + 28\,a^6}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="fricas")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

**giac** [A] time = 1.02, size = 68, normalized size = 1.21

$$-\frac{84\,b^{6}x^{6}+378\,ab^{5}x^{5}+756\,a^{2}b^{4}x^{4}+840\,a^{3}b^{3}x^{3}+540\,a^{4}b^{2}x^{2}+189\,a^{5}bx+28\,a^{6}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="giac")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

**maple** [A] time = 0.01, size = 69, normalized size = 1.23

$$-\frac{b^{6}}{3x^{3}}-\frac{3a\,b^{5}}{2x^{4}}-\frac{3a^{2}b^{4}}{x^{5}}-\frac{10a^{3}b^{3}}{3x^{6}}-\frac{15a^{4}b^{2}}{7x^{7}}-\frac{3a^{5}b}{4x^{8}}-\frac{a^{6}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/x^10,x)
```

```
[Out] -1/3*b^6/x^3-3*a^2*b^4/x^5-15/7*a^4*b^2/x^7-1/9*a^6/x^9-3/4*a^5*b/x^8-10/3*
a^3*b^3/x^6-3/2*a*b^5/x^4
```

**maxima** [A] time = 1.37, size = 68, normalized size = 1.21

$$-\frac{84\,b^{6}x^{6}+378\,ab^{5}x^{5}+756\,a^{2}b^{4}x^{4}+840\,a^{3}b^{3}x^{3}+540\,a^{4}b^{2}x^{2}+189\,a^{5}bx+28\,a^{6}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="maxima")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

**mupad** [B] time = 0.10, size = 68, normalized size = 1.21

$$-\frac{\frac{a^{6}}{9}+\frac{3\,a^{5}\,b\,x}{4}+\frac{15\,a^{4}\,b^{2}\,x^{2}}{7}+\frac{10\,a^{3}\,b^{3}\,x^{3}}{3}+3\,a^{2}\,b^{4}\,x^{4}+\frac{3\,a\,b^{5}\,x^{5}}{2}+\frac{b^{6}\,x^{6}}{3}}{x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6/x^10,x)
```

```
[Out] -(a^6/9 + (b^6*x^6)/3 + (3*a*b^5*x^5)/2 + (15*a^4*b^2*x^2)/7 + (10*a^3*b^3*
x^3)/3 + 3*a^2*b^4*x^4 + (3*a^5*b*x)/4)/x^9
```

**sympy [A]** time = 0.56, size = 73, normalized size = 1.30

$$\frac{-28a^6-189a^5bx-540a^4b^2x^2-840a^3b^3x^3-756a^2b^4x^4-378ab^5x^5-84b^6x^6}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/x**10,x)
```

```
[Out] (-28*a**6 - 189*a**5*b*x - 540*a**4*b**2*x**2 - 840*a**3*b**3*x**3 - 756*a*
*2*b**4*x**4 - 378*a*b**5*x**5 - 84*b**6*x**6)/(252*x**9)
```

## 3.246 $\int \frac{(a+bx)^5}{x^{10}}\,dx$

**Optimal**. Leaf size=67

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

**Rubi [A]** time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{10}}\,dx &= \int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 67, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^10, x]
```

**fricas [A]** time = 1.11, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

**giac [A]** time = 1.01, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

**maple [A]** time = 0.00, size = 58, normalized size = 0.87

$$-\frac{b^5}{4x^4}-\frac{a\,b^4}{x^5}-\frac{5a^2b^3}{3x^6}-\frac{10a^3b^2}{7x^7}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^10,x)
```

```
[Out] -1/4*b^5/x^4-a*b^4/x^5-5/3*a^2*b^3/x^6-10/7*a^3*b^2/x^7-5/8*a^4*b/x^8-1/9*a
^5/x^9
```

**maxima [A]** time = 1.36, size = 57, normalized size = 0.85

$$-\frac{126\, b^5 x^5 + 504\, ab^4 x^4 + 840\, a^2 b^3 x^3 + 720\, a^3 b^2 x^2 + 315\, a^4 bx + 56\, a^5}{504\, x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

**mupad [B]** time = 0.00, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{9} + \frac{5\, a^4\, b\, x}{8} + \frac{10\, a^3\, b^2\, x^2}{7} + \frac{5\, a^2\, b^3\, x^3}{3} + a\, b^4\, x^4 + \frac{b^5\, x^5}{4}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^10,x)
```

```
[Out] -(a^5/9 + (b^5*x^5)/4 + a*b^4*x^4 + (10*a^3*b^2*x^2)/7 + (5*a^2*b^3*x^3)/3
+ (5*a^4*b*x)/8)/x^9
```

**sympy [A]** time = 0.49, size = 61, normalized size = 0.91

$$\frac{-56a^5 - 315a^4bx - 720a^3b^2x^2 - 840a^2b^3x^3 - 504ab^4x^4 - 126b^5x^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**10,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504*a*
b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

## 3.247 $\int \frac{(a+bx)^4}{x^{10}}\,dx$

**Optimal**. Leaf size=56

$$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

**Rubi [A]** time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{6a^2b^2}{7x^7}-\frac{a^3b}{2x^8}-\frac{a^4}{9x^9}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/x^10,x]
```

```
[Out] -a^4/(9*x^9) - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) -
b^4/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{x^{10}}\,dx &= \int \left(\frac{a^4}{x^{10}}+\frac{4a^3b}{x^9}+\frac{6a^2b^2}{x^8}+\frac{4ab^3}{x^7}+\frac{b^4}{x^6}\right)dx\\ &= -\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/x^10,x]
```

[Out] -1/9*a^4/x^9 - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) - b^4/(5*x^5)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{x^{10}}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)^4/x^10,x]

[Out] IntegrateAlgebraic[(a + b*x)^4/x^10, x]

**fricas [A]** time = 0.81, size = 46, normalized size = 0.82

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^4/x^10,x, algorithm="fricas")

[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^4)/x^9

**giac [A]** time = 1.25, size = 46, normalized size = 0.82

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^4/x^10,x, algorithm="giac")

[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^4)/x^9

**maple [A]** time = 0.01, size = 47, normalized size = 0.84

$$-\frac{b^4}{5x^5}-\frac{2a\,b^3}{3x^6}-\frac{6a^2b^2}{7x^7}-\frac{a^3b}{2x^8}-\frac{a^4}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^4/x^10,x)

[Out] -1/9*a^4/x^9-1/2*a^3*b/x^8-6/7*a^2*b^2/x^7-2/3*a*b^3/x^6-1/5*b^4/x^5

**maxima** [A] time = 1.34, size = 46, normalized size = 0.82

$$-\frac{126\,b^4x^4 + 420\,ab^3x^3 + 540\,a^2b^2x^2 + 315\,a^3bx + 70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="maxima")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

**mupad** [B] time = 0.03, size = 46, normalized size = 0.82

$$-\frac{\frac{a^4}{9} + \frac{a^3\,b\,x}{2} + \frac{6\,a^2\,b^2\,x^2}{7} + \frac{2\,a\,b^3\,x^3}{3} + \frac{b^4\,x^4}{5}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/x^10,x)
```

```
[Out] -(a^4/9 + (b^4*x^4)/5 + (2*a*b^3*x^3)/3 + (6*a^2*b^2*x^2)/7 + (a^3*b*x)/2)/
x^9
```

**sympy** [A] time = 0.50, size = 49, normalized size = 0.88

$$\frac{-70a^4 - 315a^3bx - 540a^2b^2x^2 - 420ab^3x^3 - 126b^4x^4}{630x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/x**10,x)
```

```
[Out] (-70*a**4 - 315*a**3*b*x - 540*a**2*b**2*x**2 - 420*a*b**3*x**3 - 126*b**4*
x**4)/(630*x**9)
```

## 3.248 $\int \frac{(a+bx)^3}{x^{10}}\,dx$

**Optimal**. Leaf size=43

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

**Rubi [A]** time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{8x^8}-\frac{a^3}{9x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^10,x]
```

```
[Out] -a^3/(9*x^9) - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{10}}\,dx &= \int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^9}+\frac{3ab^2}{x^8}+\frac{b^3}{x^7}\right)dx\\ &= -\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^10,x]
```

```
[Out] -1/9*a^3/x^9 - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^10,x]
[Out] IntegrateAlgebraic[(a + b*x)^3/x^10, x]
```

**fricas [A]** time = 0.99, size = 35, normalized size = 0.81

$$-\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="fricas")
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

**giac [A]** time = 1.09, size = 35, normalized size = 0.81

$$-\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="giac")
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.84

$$-\frac{b^3}{6x^6}-\frac{3a\,b^2}{7x^7}-\frac{3a^2b}{8x^8}-\frac{a^3}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^10,x)
[Out] -1/9*a^3/x^9-3/8*a^2*b/x^8-3/7*a*b^2/x^7-1/6*b^3/x^6
```

**maxima [A]** time = 1.35, size = 35, normalized size = 0.81

$$-\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

**mupad [B]** time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3}{9}+\frac{3\,a^2\,b\,x}{8}+\frac{3\,a\,b^2\,x^2}{7}+\frac{b^3\,x^3}{6}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^10,x)
```

```
[Out] -(a^3/9 + (b^3*x^3)/6 + (3*a*b^2*x^2)/7 + (3*a^2*b*x)/8)/x^9
```

**sympy [A]** time = 0.38, size = 37, normalized size = 0.86

$$\frac{-56a^3-189a^2bx-216ab^2x^2-84b^3x^3}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**10,x)
```

```
[Out] (-56*a**3 - 189*a**2*b*x - 216*a*b**2*x**2 - 84*b**3*x**3)/(504*x**9)
```

## 3.249 $\int \frac{(a+bx)^2}{x^{10}}\,dx$

**Optimal**. Leaf size=30

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

**Rubi** [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^10,x]
```

```
[Out] -a^2/(9*x^9) - (a*b)/(4*x^8) - b^2/(7*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{10}}\,dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^9}+\frac{b^2}{x^8}\right)dx\\ &= -\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^10,x]
```

```
[Out] -1/9*a^2/x^9 - (a*b)/(4*x^8) - b^2/(7*x^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/x^10, x]
```

**fricas [A]** time = 1.09, size = 24, normalized size = 0.80

$$-\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

**giac [A]** time = 0.87, size = 24, normalized size = 0.80

$$-\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{7x^7}-\frac{ab}{4x^8}-\frac{a^2}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^10,x)
```

```
[Out] -1/9*a^2/x^9-1/4*a*b/x^8-1/7*b^2/x^7
```

**maxima [A]** time = 1.34, size = 24, normalized size = 0.80

$$-\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

**mupad [B]** time = 0.04, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{9}+\frac{a\,b\,x}{4}+\frac{b^2\,x^2}{7}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^10,x)
```

```
[Out] -(a^2/9 + (b^2*x^2)/7 + (a*b*x)/4)/x^9
```

**sympy [A]** time = 0.27, size = 26, normalized size = 0.87

$$\frac{-28a^2-63abx-36b^2x^2}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**10,x)
```

```
[Out] (-28*a**2 - 63*a*b*x - 36*b**2*x**2)/(252*x**9)
```

## 3.250 $\int \frac{a+bx}{x^{10}} dx$

**Optimal**. Leaf size=17

$$-\frac{a}{9x^9} - \frac{b}{8x^8}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{a}{9x^9} - \frac{b}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^10,x]
[Out] -a/(9*x^9) - b/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^{10}} dx &= \int \left(\frac{a}{x^{10}} + \frac{b}{x^9}\right) dx \\ &= -\frac{a}{9x^9} - \frac{b}{8x^8} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{9x^9} - \frac{b}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^10,x]
[Out] -1/9*a/x^9 - b/(8*x^8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x^10,x]
[Out] IntegrateAlgebraic[(a + b*x)/x^10, x]
```

**fricas [A]** time = 0.83, size = 13, normalized size = 0.76

$$-\frac{9\,bx+8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="fricas")
[Out] -1/72*(9*b*x + 8*a)/x^9
```

**giac [A]** time = 1.00, size = 13, normalized size = 0.76

$$-\frac{9\,bx+8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="giac")
[Out] -1/72*(9*b*x + 8*a)/x^9
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{8x^8}-\frac{a}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^10,x)
[Out] -1/9*a/x^9-1/8*b/x^8
```

**maxima [A]** time = 1.30, size = 13, normalized size = 0.76

$$-\frac{9\,bx+8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(9*b*x + 8*a)/x^9
```

**mupad** [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{8\,a+9\,b\,x}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^10,x)
```

```
[Out] -(8*a + 9*b*x)/(72*x^9)
```

**sympy** [A] time = 0.21, size = 14, normalized size = 0.82

$$\frac{-8a-9bx}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**10,x)
```

```
[Out] (-8*a - 9*b*x)/(72*x**9)
```

## 3.251 $\int \frac{1}{x^{10}} \, dx$

**Optimal**. Leaf size=7

$$-\frac{1}{9x^9}$$

**Rubi** [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-10),x]
[Out] -1/(9*x^9)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{10}} \, dx = -\frac{1}{9x^9}$$

**Mathematica** [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-10),x]
[Out] -1/9*1/x^9
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-10),x]
```

```
[Out] IntegrateAlgebraic[x^(-10), x]
```

**fricas [A]** time = 0.94, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="fricas")
```

```
[Out] -1/9/x^9
```

**giac [A]** time = 1.04, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="giac")
```

```
[Out] -1/9/x^9
```

**maple [A]** time = 0.00, size = 6, normalized size = 0.86

$$-\frac{1}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10,x)
```

```
[Out] -1/9/x^9
```

**maxima [A]** time = 1.32, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="maxima")
```

```
[Out] -1/9/x^9
```

**mupad [B]** time = 0.02, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10,x)
```

```
[Out] -1/(9*x^9)
```

**sympy [A]** time = 0.07, size = 7, normalized size = 1.00

$$-\frac{1}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10,x)
```

```
[Out] -1/(9*x**9)
```

## 3.252 $\int \frac{1}{x^{10}(a+bx)}\,dx$

**Optimal**. Leaf size=134

$$-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}-\frac{b^8}{a^9x}+\frac{b^7}{2a^8x^2}-\frac{b^6}{3a^7x^3}+\frac{b^5}{4a^6x^4}-\frac{b^4}{5a^5x^5}+\frac{b^3}{6a^4x^6}-\frac{b^2}{7a^3x^7}+\frac{b}{8a^2x^8}-\frac{1}{9ax^9}$$

**Rubi** [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{b^7}{2a^8x^2}-\frac{b^6}{3a^7x^3}+\frac{b^5}{4a^6x^4}-\frac{b^4}{5a^5x^5}+\frac{b^3}{6a^4x^6}-\frac{b^2}{7a^3x^7}-\frac{b^8}{a^9x}-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}+\frac{b}{8a^2x^8}-\frac{1}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)),x]
```

```
[Out] -1/(9*a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(5*a
^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^9*x)
 - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}(a+bx)}\,dx &= \int \left(\frac{1}{ax^{10}}-\frac{b}{a^2x^9}+\frac{b^2}{a^3x^8}-\frac{b^3}{a^4x^7}+\frac{b^4}{a^5x^6}-\frac{b^5}{a^6x^5}+\frac{b^6}{a^7x^4}-\frac{b^7}{a^8x^3}+\frac{b^8}{a^9x^2}-\frac{b^9}{a^{10}x}+\frac{b^{10}}{a^{10}(a+bx}\right.\\ &= -\frac{1}{9ax^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\log(x)}{a^{10}}+\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 134, normalized size = 1.00

$$-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}-\frac{b^8}{a^9x}+\frac{b^7}{2a^8x^2}-\frac{b^6}{3a^7x^3}+\frac{b^5}{4a^6x^4}-\frac{b^4}{5a^5x^5}+\frac{b^3}{6a^4x^6}-\frac{b^2}{7a^3x^7}+\frac{b}{8a^2x^8}-\frac{1}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)),x]
```

```
[Out] -1/9*1/(a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(5
*a^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^9*
x) - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^10*(a + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^10*(a + b*x)), x]
```

**fricas [A]** time = 0.86, size = 120, normalized size = 0.90

$$\frac{2520\,b^{9}x^{9}\log\left(bx+a\right)-2520\,b^{9}x^{9}\log\left(x\right)-2520\,ab^{8}x^{8}+1260\,a^{2}b^{7}x^{7}-840\,a^{3}b^{6}x^{6}+630\,a^{4}b^{5}x^{5}-504\,a^{5}b^{4}x^{4}+420\,a^{6}b^{3}x^{3}-360\,a^{7}b^{2}x^{2}+315\,a^{8}bx-280\,a^{9}}{2520\,a^{10}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^9*x^9*log(b*x + a) - 2520*b^9*x^9*log(x) - 2520*a*b^8*x^8 +
1260*a^2*b^7*x^7 - 840*a^3*b^6*x^6 + 630*a^4*b^5*x^5 - 504*a^5*b^4*x^4 + 42
0*a^6*b^3*x^3 - 360*a^7*b^2*x^2 + 315*a^8*b*x - 280*a^9)/(a^10*x^9)
```

**giac [A]** time = 1.16, size = 122, normalized size = 0.91

$$\frac{b^{9}\log\left(|bx+a|\right)}{a^{10}}-\frac{b^{9}\log\left(|x|\right)}{a^{10}}-\frac{2520\,ab^{8}x^{8}-1260\,a^{2}b^{7}x^{7}+840\,a^{3}b^{6}x^{6}-630\,a^{4}b^{5}x^{5}+504\,a^{5}b^{4}x^{4}-420\,a^{6}b^{3}x^{3}+360\,a^{7}b^{2}x^{2}-315\,a^{8}bx+280\,a^{9}}{2520\,a^{10}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="giac")
```

```
[Out] b^9*log(abs(b*x + a))/a^10 - b^9*log(abs(x))/a^10 - 1/2520*(2520*a*b^8*x^8
- 1260*a^2*b^7*x^7 + 840*a^3*b^6*x^6 - 630*a^4*b^5*x^5 + 504*a^5*b^4*x^4 -
420*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 315*a^8*b*x + 280*a^9)/(a^10*x^9)
```

**maple [A]** time = 0.01, size = 119, normalized size = 0.89

$$-\frac{b^{9}\ln\left(x\right)}{a^{10}}+\frac{b^{9}\ln\left(bx+a\right)}{a^{10}}-\frac{b^{8}}{a^{9}x}+\frac{b^{7}}{2a^{8}x^{2}}-\frac{b^{6}}{3a^{7}x^{3}}+\frac{b^{5}}{4a^{6}x^{4}}-\frac{b^{4}}{5a^{5}x^{5}}+\frac{b^{3}}{6a^{4}x^{6}}-\frac{b^{2}}{7a^{3}x^{7}}+\frac{b}{8a^{2}x^{8}}-\frac{1}{9a\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a),x)
```

```
[Out] -1/9/a/x^9+1/8*b/a^2/x^8-1/7*b^2/a^3/x^7+1/6*b^3/a^4/x^6-1/5*b^4/a^5/x^5+1/
4*b^5/a^6/x^4-1/3*b^6/a^7/x^3+1/2*b^7/a^8/x^2-b^8/a^9/x-b^9*ln(x)/a^10+b^9*
ln(b*x+a)/a^10
```

**maxima** [A] time = 1.35, size = 117, normalized size = 0.87

$$\frac{b^9\log(bx+a)}{a^{10}} - \frac{b^9\log(x)}{a^{10}} - \frac{2520\,b^8x^8 - 1260\,ab^7x^7 + 840\,a^2b^6x^6 - 630\,a^3b^5x^5 + 504\,a^4b^4x^4 - 420\,a^5b^3x^3 + 360\,a^6b^2x^2 - 315\,a^7bx + 280\,a^8}{2520\,a^9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^9*log(b*x + a)/a^10 - b^9*log(x)/a^10 - 1/2520*(2520*b^8*x^8 - 1260*a*b^7
*x^7 + 840*a^2*b^6*x^6 - 630*a^3*b^5*x^5 + 504*a^4*b^4*x^4 - 420*a^5*b^3*x^
3 + 360*a^6*b^2*x^2 - 315*a^7*b*x + 280*a^8)/(a^9*x^9)
```

**mupad** [B] time = 0.13, size = 114, normalized size = 0.85

$$-\frac{280\,a^9 + 2520\,a\,b^8\,x^8 - 5040\,b^9\,x^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right) + 360\,a^7\,b^2\,x^2 - 420\,a^6\,b^3\,x^3 + 504\,a^5\,b^4\,x^4 - 630\,a^4\,b^5\,x^5 + 840\,a^3\,b^6\,x^6 - 1260\,a^2\,b^7\,x^7 - 315\,a^8\,b\,x}{2520\,a^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x)),x)
```

```
[Out] -(280*a^9 + 2520*a*b^8*x^8 - 5040*b^9*x^9*atanh((2*b*x)/a + 1) + 360*a^7*b^
2*x^2 - 420*a^6*b^3*x^3 + 504*a^5*b^4*x^4 - 630*a^4*b^5*x^5 + 840*a^3*b^6*x
^6 - 1260*a^2*b^7*x^7 - 315*a^8*b*x)/(2520*a^10*x^9)
```

**sympy** [A] time = 0.41, size = 116, normalized size = 0.87

$$\frac{-280a^8 + 315a^7bx - 360a^6b^2x^2 + 420a^5b^3x^3 - 504a^4b^4x^4 + 630a^3b^5x^5 - 840a^2b^6x^6 + 1260ab^7x^7 - 2520b^8x^8}{2520a^9x^9} + \frac{b^9\left(-\log(x) + \log\left(\frac{a}{b}+x\right)\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a),x)
```

```
[Out] (-280*a**8 + 315*a**7*b*x - 360*a**6*b**2*x**2 + 420*a**5*b**3*x**3 - 504*a
**4*b**4*x**4 + 630*a**3*b**5*x**5 - 840*a**2*b**6*x**6 + 1260*a*b**7*x**7
- 2520*b**8*x**8)/(2520*a**9*x**9) + b**9*(-log(x) + log(a/b + x))/a**10
```

## 3.253 $\int \frac{1}{x^{10}(a+bx)^2} dx$

**Optimal**. Leaf size=146

$$-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a+bx)}{a^{11}}-\frac{b^9}{a^{10}(a+bx)}-\frac{9b^8}{a^{10}x}+\frac{4b^7}{a^9x^2}-\frac{7b^6}{3a^8x^3}+\frac{3b^5}{2a^7x^4}-\frac{b^4}{a^6x^5}+\frac{2b^3}{3a^5x^6}-\frac{3b^2}{7a^4x^7}+\frac{b}{4a^3x^8}-\frac{1}{9a^2x^9}$$

**Rubi [A]** time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{4b^7}{a^9x^2}-\frac{7b^6}{3a^8x^3}+\frac{3b^5}{2a^7x^4}-\frac{b^4}{a^6x^5}+\frac{2b^3}{3a^5x^6}-\frac{3b^2}{7a^4x^7}-\frac{b^9}{a^{10}(a+bx)}-\frac{9b^8}{a^{10}x}-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a+bx)}{a^{11}}+\frac{b}{4a^3x^8}-\frac{1}{9a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)^2),x]
```

```
[Out] -1/(9*a^2*x^9) + b/(4*a^3*x^8) - (3*b^2)/(7*a^4*x^7) + (2*b^3)/(3*a^5*x^6)
- b^4/(a^6*x^5) + (3*b^5)/(2*a^7*x^4) - (7*b^6)/(3*a^8*x^3) + (4*b^7)/(a^9*
x^2) - (9*b^8)/(a^10*x) - b^9/(a^10*(a + b*x)) - (10*b^9*Log[x])/a^11 + (10
*b^9*Log[a + b*x])/a^11
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^{10}(a+bx)^2} dx = \int \left(\frac{1}{a^2x^{10}}-\frac{2b}{a^3x^9}+\frac{3b^2}{a^4x^8}-\frac{4b^3}{a^5x^7}+\frac{5b^4}{a^6x^6}-\frac{6b^5}{a^7x^5}+\frac{7b^6}{a^8x^4}-\frac{8b^7}{a^9x^3}+\frac{9b^8}{a^{10}x^2}-\frac{10b^9}{a^{11}x}+\frac{b^{10}}{a^{10}(a+b\right.$$

$$= -\frac{1}{9a^2x^9}+\frac{b}{4a^3x^8}-\frac{3b^2}{7a^4x^7}+\frac{2b^3}{3a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3b^5}{2a^7x^4}-\frac{7b^6}{3a^8x^3}+\frac{4b^7}{a^9x^2}-\frac{9b^8}{a^{10}x}-\frac{b^9}{a^{10}(a+bx)}-$$

**Mathematica [A]** time = 0.09, size = 134, normalized size = 0.92

$$-\frac{\frac{a\left(28a^9-35a^8bx+45a^7b^2x^2-60a^6b^3x^3+84a^5b^4x^4-126a^4b^5x^5+210a^3b^6x^6-420a^2b^7x^7+1260ab^8x^8+2520b^9x^9\right)}{x^9(a+bx)}-2520b^9\log(a+bx)+2520b^9\log(x)}{252a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)^2),x]
```

```
[Out] -1/252*((a*(28*a^9 - 35*a^8*b*x + 45*a^7*b^2*x^2 - 60*a^6*b^3*x^3 + 84*a^5*
b^4*x^4 - 126*a^4*b^5*x^5 + 210*a^3*b^6*x^6 - 420*a^2*b^7*x^7 + 1260*a*b^8*
x^8 + 2520*b^9*x^9))/(x^9*(a + b*x)) + 2520*b^9*Log[x] - 2520*b^9*Log[a + b
*x])/a^11
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^10*(a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^10*(a + b*x)^2), x]
```

**fricas [A]** time = 0.74, size = 163, normalized size = 1.12

$$-\frac{2520\,ab^9x^9+1260\,a^2b^8x^8-420\,a^3b^7x^7+210\,a^4b^6x^6-126\,a^5b^5x^5+84\,a^6b^4x^4-60\,a^7b^3x^3+45\,a^8b^2x^2-35\,a^9bx+28\,a^{10}-2520\left(b^{10}x^{10}+ab^9x^9\right)\log\left(bx+a\right)+2520\left(b^{10}x^{10}+ab^9x^9\right)\log(x)}{252\left(a^{11}bx^{10}+a^{12}x^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/252*(2520*a*b^9*x^9 + 1260*a^2*b^8*x^8 - 420*a^3*b^7*x^7 + 210*a^4*b^6*x
^6 - 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 - 60*a^7*b^3*x^3 + 45*a^8*b^2*x^2 - 3
5*a^9*b*x + 28*a^10 - 2520*(b^10*x^10 + a*b^9*x^9)*log(b*x + a) + 2520*(b^1
0*x^10 + a*b^9*x^9)*log(x))/(a^11*b*x^10 + a^12*x^9)
```

**giac [A]** time = 1.28, size = 180, normalized size = 1.23

$$-\frac{10\,b^9\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^{11}}-\frac{b^9}{(bx+a)a^{10}}-\frac{\frac{41481\,ab^9}{bx+a}-\frac{155844\,a^2b^9}{(bx+a)^2}+\frac{337176\,a^3b^9}{(bx+a)^3}-\frac{460404\,a^4b^9}{(bx+a)^4}+\frac{407484\,a^5b^9}{(bx+a)^5}-\frac{229320\,a^6b^9}{(bx+a)^6}+\frac{75600\,a^7b^9}{(bx+a)^7}-\frac{11340\,a^8b^9}{(bx+a)^8}-4861\,b^9}{252\,a^{11}\left(\frac{a}{bx+a}-1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -10*b^9*log(abs(-a/(b*x + a) + 1))/a^11 - b^9/((b*x + a)*a^10) - 1/252*(414
81*a*b^9/(b*x + a) - 155844*a^2*b^9/(b*x + a)^2 + 337176*a^3*b^9/(b*x + a)^
3 - 460404*a^4*b^9/(b*x + a)^4 + 407484*a^5*b^9/(b*x + a)^5 - 229320*a^6*b^
9/(b*x + a)^6 + 75600*a^7*b^9/(b*x + a)^7 - 11340*a^8*b^9/(b*x + a)^8 - 486
1*b^9)/(a^11*(a/(b*x + a) - 1)^9)
```

**maple** [A] time = 0.01, size = 135, normalized size = 0.92

$$-\frac{b^9}{(bx+a)\,a^{10}}-\frac{10b^9\ln(x)}{a^{11}}+\frac{10b^9\ln(bx+a)}{a^{11}}-\frac{9b^8}{a^{10}x}+\frac{4b^7}{a^9x^2}-\frac{7b^6}{3a^8x^3}+\frac{3b^5}{2a^7x^4}-\frac{b^4}{a^6x^5}+\frac{2b^3}{3a^5x^6}-\frac{3b^2}{7a^4x^7}+\frac{b}{4a^3x^8}-\frac{1}{9a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a)^2,x)
```

```
[Out] -1/9/a^2/x^9+1/4*b/a^3/x^8-3/7*b^2/a^4/x^7+2/3*b^3/a^5/x^6-b^4/a^6/x^5+3/2*
b^5/a^7/x^4-7/3*b^6/a^8/x^3+4*b^7/a^9/x^2-9*b^8/a^10/x-b^9/a^10/(b*x+a)-10*
b^9*ln(x)/a^11+10*b^9*ln(b*x+a)/a^11
```

**maxima** [A] time = 1.40, size = 141, normalized size = 0.97

$$-\frac{2520\,b^9x^9+1260\,ab^8x^8-420\,a^2b^7x^7+210\,a^3b^6x^6-126\,a^4b^5x^5+84\,a^5b^4x^4-60\,a^6b^3x^3+45\,a^7b^2x^2-35\,a^8bx+28\,a^9}{252\left(a^{10}bx^{10}+a^{11}x^9\right)}+\frac{10\,b^9\log(bx+a)}{a^{11}}-\frac{10\,b^9\log(x)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/252*(2520*b^9*x^9 + 1260*a*b^8*x^8 - 420*a^2*b^7*x^7 + 210*a^3*b^6*x^6 -
 126*a^4*b^5*x^5 + 84*a^5*b^4*x^4 - 60*a^6*b^3*x^3 + 45*a^7*b^2*x^2 - 35*a^
8*b*x + 28*a^9)/(a^10*b*x^10 + a^11*x^9) + 10*b^9*log(b*x + a)/a^11 - 10*b^
9*log(x)/a^11
```

**mupad** [B] time = 0.08, size = 135, normalized size = 0.92

$$\frac{20\,b^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{11}}-\frac{\frac{1}{9\,a}+\frac{5\,b^2\,x^2}{28\,a^3}-\frac{5\,b^3\,x^3}{21\,a^4}+\frac{b^4\,x^4}{3\,a^5}-\frac{b^5\,x^5}{2\,a^6}+\frac{5\,b^6\,x^6}{6\,a^7}-\frac{5\,b^7\,x^7}{3\,a^8}+\frac{5\,b^8\,x^8}{a^9}+\frac{10\,b^9\,x^9}{a^{10}}-\frac{5\,b\,x}{36\,a^2}}{b\,x^{10}+a\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x)^2),x)
```

```
[Out] (20*b^9*atanh((2*b*x)/a + 1))/a^11 - (1/(9*a) + (5*b^2*x^2)/(28*a^3) - (5*b
^3*x^3)/(21*a^4) + (b^4*x^4)/(3*a^5) - (b^5*x^5)/(2*a^6) + (5*b^6*x^6)/(6*a
^7) - (5*b^7*x^7)/(3*a^8) + (5*b^8*x^8)/a^9 + (10*b^9*x^9)/a^10 - (5*b*x)/(
36*a^2))/(a*x^9 + b*x^10)
```

**sympy** [A] time = 0.61, size = 139, normalized size = 0.95

$$\frac{-28a^9+35a^8bx-45a^7b^2x^2+60a^6b^3x^3-84a^5b^4x^4+126a^4b^5x^5-210a^3b^6x^6+420a^2b^7x^7-1260ab^8x^8-2520b^9x^9}{252a^{11}x^9+252a^{10}bx^{10}}+\frac{10b^9\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a)**2,x)

[Out] (-28*a**9 + 35*a**8*b*x - 45*a**7*b**2*x**2 + 60*a**6*b**3*x**3 - 84*a**5*b
**4*x**4 + 126*a**4*b**5*x**5 - 210*a**3*b**6*x**6 + 420*a**2*b**7*x**7 - 1
260*a*b**8*x**8 - 2520*b**9*x**9)/(252*a**11*x**9 + 252*a**10*b*x**10) + 10
*b**9*(-log(x) + log(a/b + x))/a**11
```

## 3.254 $\int \frac{1}{x^{10}(a+bx)^3} \, dx$

**Optimal**. Leaf size=163

$$-\frac{55b^9\log(x)}{a^{12}}+\frac{55b^9\log(a+bx)}{a^{12}}-\frac{10b^9}{a^{11}(a+bx)}-\frac{45b^8}{a^{11}x}-\frac{b^9}{2a^{10}(a+bx)^2}+\frac{18b^7}{a^{10}x^2}-\frac{28b^6}{3a^9x^3}+\frac{21b^5}{4a^8x^4}-\frac{3b^4}{a^7x^5}+\frac{5b^3}{3a^6x^6}-\frac{6b^2}{7a^5x^7}$$

**Rubi** [A] time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{18b^7}{a^{10}x^2}-\frac{28b^6}{3a^9x^3}+\frac{21b^5}{4a^8x^4}-\frac{3b^4}{a^7x^5}+\frac{5b^3}{3a^6x^6}-\frac{6b^2}{7a^5x^7}-\frac{10b^9}{a^{11}(a+bx)}-\frac{b^9}{2a^{10}(a+bx)^2}-\frac{45b^8}{a^{11}x}-\frac{55b^9\log(x)}{a^{12}}+\frac{55b^9\log(a+bx)}{a^{12}}+\frac{3b}{8a^4x^8}-\frac{1}{9a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)^3),x]
```

```
[Out] -1/(9*a^3*x^9) + (3*b)/(8*a^4*x^8) - (6*b^2)/(7*a^5*x^7) + (5*b^3)/(3*a^6*x
^6) - (3*b^4)/(a^7*x^5) + (21*b^5)/(4*a^8*x^4) - (28*b^6)/(3*a^9*x^3) + (18
*b^7)/(a^10*x^2) - (45*b^8)/(a^11*x) - b^9/(2*a^10*(a + b*x)^2) - (10*b^9)/
(a^11*(a + b*x)) - (55*b^9*Log[x])/a^12 + (55*b^9*Log[a + b*x])/a^12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^{10}}-\frac{3b}{a^4x^9}+\frac{6b^2}{a^5x^8}-\frac{10b^3}{a^6x^7}+\frac{15b^4}{a^7x^6}-\frac{21b^5}{a^8x^5}+\frac{28b^6}{a^9x^4}-\frac{36b^7}{a^{10}x^3}+\frac{45b^8}{a^{11}x^2}-\frac{55b^9}{a^{12}x}+\frac{b^{10}}{a^{10}(a+\right.\\ &= -\frac{1}{9a^3x^9}+\frac{3b}{8a^4x^8}-\frac{6b^2}{7a^5x^7}+\frac{5b^3}{3a^6x^6}-\frac{3b^4}{a^7x^5}+\frac{21b^5}{4a^8x^4}-\frac{28b^6}{3a^9x^3}+\frac{18b^7}{a^{10}x^2}-\frac{45b^8}{a^{11}x}-\frac{b^9}{2a^{10}(a+bx)}\end{aligned}$$

**Mathematica** [A] time = 0.11, size = 145, normalized size = 0.89

$$-\frac{\frac{a\left(56a^{10}-77a^9bx+110a^8b^2x^2-165a^7b^3x^3+264a^6b^4x^4-462a^5b^5x^5+924a^4b^6x^6-2310a^3b^7x^7+9240a^2b^8x^8+41580ab^9x^9+27720b^{10}x^{10}\right)}{x^9(a+bx)^2}-27720b^9\log(a+bx)+27720b^9\log(x)}{504a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)^3),x]
```

```
[Out] -1/504*((a*(56*a^10 - 77*a^9*b*x + 110*a^8*b^2*x^2 - 165*a^7*b^3*x^3 + 264*
a^6*b^4*x^4 - 462*a^5*b^5*x^5 + 924*a^4*b^6*x^6 - 2310*a^3*b^7*x^7 + 9240*a
^2*b^8*x^8 + 41580*a*b^9*x^9 + 27720*b^10*x^10))/(x^9*(a + b*x)^2) + 27720*
b^9*Log[x] - 27720*b^9*Log[a + b*x])/a^12
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^10*(a + b*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^10*(a + b*x)^3), x]
```

**fricas [A]** time = 1.10, size = 207, normalized size = 1.27

$$-\frac{27720\,ab^{10}x^{10}+41580\,a^{2}b^{9}x^{9}+9240\,a^{3}b^{8}x^{8}-2310\,a^{4}b^{7}x^{7}+924\,a^{5}b^{6}x^{6}-462\,a^{6}b^{5}x^{5}+264\,a^{7}b^{4}x^{4}-165\,a^{8}b^{3}x^{3}+110\,a^{9}b^{2}x^{2}-77\,a^{10}bx+56\,a^{11}-27720\left(b^{11}x^{11}+2\,ab^{10}x^{10}+a^{2}b^{9}x^{9}\right)\log\left(bx+a\right)+27720\left(b^{11}x^{11}+2\,ab^{10}x^{10}+a^{2}b^{9}x^{9}\right)\log\left(x\right)}{504\left(a^{12}b^{2}x^{11}+2\,a^{13}bx^{10}+a^{14}x^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/504*(27720*a*b^10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^4
*b^7*x^7 + 924*a^5*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*b^
3*x^3 + 110*a^9*b^2*x^2 - 77*a^10*b*x + 56*a^11 - 27720*(b^11*x^11 + 2*a*b^
10*x^10 + a^2*b^9*x^9)*log(b*x + a) + 27720*(b^11*x^11 + 2*a*b^10*x^10 + a^
2*b^9*x^9)*log(x))/(a^12*b^2*x^11 + 2*a^13*b*x^10 + a^14*x^9)
```

**giac [A]** time = 1.13, size = 152, normalized size = 0.93

$$\frac{55\,b^{9}\log\left(|bx+a|\right)}{a^{12}}-\frac{55\,b^{9}\log\left(|x|\right)}{a^{12}}-\frac{27720\,ab^{10}x^{10}+41580\,a^{2}b^{9}x^{9}+9240\,a^{3}b^{8}x^{8}-2310\,a^{4}b^{7}x^{7}+924\,a^{5}b^{6}x^{6}-462\,a^{6}b^{5}x^{5}+264\,a^{7}b^{4}x^{4}-165\,a^{8}b^{3}x^{3}+110\,a^{9}b^{2}x^{2}-77\,a^{10}bx+56\,a^{11}}{504\left(bx+a\right)^{2}a^{12}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 55*b^9*log(abs(b*x + a))/a^12 - 55*b^9*log(abs(x))/a^12 - 1/504*(27720*a*b^
10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^4*b^7*x^7 + 924*a^5
*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*b^3*x^3 + 110*a^9*b^
2*x^2 - 77*a^10*b*x + 56*a^11)/((b*x + a)^2*a^12*x^9)
```

**maple [A]** time = 0.01, size = 150, normalized size = 0.92

$$-\frac{b^{9}}{2\left(bx+a\right)^{2}a^{10}}-\frac{10b^{9}}{\left(bx+a\right)a^{11}}-\frac{55b^{9}\ln\left(x\right)}{a^{12}}+\frac{55b^{9}\ln\left(bx+a\right)}{a^{12}}-\frac{45b^{8}}{a^{11}x}+\frac{18b^{7}}{a^{10}x^{2}}-\frac{28b^{6}}{3a^{9}x^{3}}+\frac{21b^{5}}{4a^{8}x^{4}}-\frac{3b^{4}}{a^{7}x^{5}}+\frac{5b^{3}}{3a^{6}x^{6}}-\frac{6b^{2}}{7a^{5}x^{7}}+\frac{3b}{8a^{4}x^{8}}-\frac{1}{9a^{3}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a)^3,x)
```

```
[Out] -1/9/a^3/x^9+3/8*b/a^4/x^8-6/7*b^2/a^5/x^7+5/3*b^3/a^6/x^6-3*b^4/a^7/x^5+21
/4*b^5/a^8/x^4-28/3*b^6/a^9/x^3+18*b^7/a^10/x^2-45*b^8/a^11/x-1/2*b^9/a^10/
(b*x+a)^2-10*b^9/a^11/(b*x+a)-55*b^9*ln(x)/a^12+55*b^9*ln(b*x+a)/a^12
```

**maxima [A]** time = 1.43, size = 163, normalized size = 1.00

$$-\frac{27720\,b^{10}x^{10}+41580\,ab^{9}x^{9}+9240\,a^{2}b^{8}x^{8}-2310\,a^{3}b^{7}x^{7}+924\,a^{4}b^{6}x^{6}-462\,a^{5}b^{5}x^{5}+264\,a^{6}b^{4}x^{4}-165\,a^{7}b^{3}x^{3}+110\,a^{8}b^{2}x^{2}-77\,a^{9}bx+56\,a^{10}}{504\left(a^{11}b^{2}x^{11}+2\,a^{12}bx^{10}+a^{13}x^{9}\right)}+\frac{55\,b^{9}\log\left(bx+a\right)}{a^{12}}-\frac{55\,b^{9}\log(x)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/504*(27720*b^10*x^10 + 41580*a*b^9*x^9 + 9240*a^2*b^8*x^8 - 2310*a^3*b^7
*x^7 + 924*a^4*b^6*x^6 - 462*a^5*b^5*x^5 + 264*a^6*b^4*x^4 - 165*a^7*b^3*x^
3 + 110*a^8*b^2*x^2 - 77*a^9*b*x + 56*a^10)/(a^11*b^2*x^11 + 2*a^12*b*x^10
+ a^13*x^9) + 55*b^9*log(b*x + a)/a^12 - 55*b^9*log(x)/a^12
```

**mupad [B]** time = 0.23, size = 157, normalized size = 0.96

$$\frac{110\,b^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{12}}-\frac{\frac{1}{9\,a}+\frac{55\,b^2\,x^2}{252\,a^3}-\frac{55\,b^3\,x^3}{168\,a^4}+\frac{11\,b^4\,x^4}{21\,a^5}-\frac{11\,b^5\,x^5}{12\,a^6}+\frac{11\,b^6\,x^6}{6\,a^7}-\frac{55\,b^7\,x^7}{12\,a^8}+\frac{55\,b^8\,x^8}{3\,a^9}+\frac{165\,b^9\,x^9}{2\,a^{10}}+\frac{55\,b^{10}\,x^{10}}{a^{11}}-\frac{11\,b\,x}{72\,a^2}}{a^2\,x^9+2\,a\,b\,x^{10}+b^2\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x)^3),x)
```

```
[Out] (110*b^9*atanh((2*b*x)/a + 1))/a^12 - (1/(9*a) + (55*b^2*x^2)/(252*a^3) - (
55*b^3*x^3)/(168*a^4) + (11*b^4*x^4)/(21*a^5) - (11*b^5*x^5)/(12*a^6) + (11
*b^6*x^6)/(6*a^7) - (55*b^7*x^7)/(12*a^8) + (55*b^8*x^8)/(3*a^9) + (165*b^9
*x^9)/(2*a^10) + (55*b^10*x^10)/a^11 - (11*b*x)/(72*a^2))/(a^2*x^9 + b^2*x^
11 + 2*a*b*x^10)
```

**sympy [A]** time = 0.68, size = 163, normalized size = 1.00

$$\frac{-56a^{10}+77a^{9}bx-110a^{8}b^{2}x^{2}+165a^{7}b^{3}x^{3}-264a^{6}b^{4}x^{4}+462a^{5}b^{5}x^{5}-924a^{4}b^{6}x^{6}+2310a^{3}b^{7}x^{7}-9240a^{2}b^{8}x^{8}-41580ab^{9}x^{9}-27720b^{10}x^{10}}{504a^{13}x^{9}+1008a^{12}bx^{10}+504a^{11}b^{2}x^{11}}+\frac{55b^{9}\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a)**3,x)
```

```
[Out] (-56*a**10 + 77*a**9*b*x - 110*a**8*b**2*x**2 + 165*a**7*b**3*x**3 - 264*a*
*6*b**4*x**4 + 462*a**5*b**5*x**5 - 924*a**4*b**6*x**6 + 2310*a**3*b**7*x**
7 - 9240*a**2*b**8*x**8 - 41580*a*b**9*x**9 - 27720*b**10*x**10)/(504*a**13
*x**9 + 1008*a**12*b*x**10 + 504*a**11*b**2*x**11) + 55*b**9*(-log(x) + log
(a/b + x))/a**12
```

## 3.255 $\int \frac{1}{x(2+3x)}\,dx$

**Optimal**. Leaf size=17

$$\frac{\log(x)}{2}-\frac{1}{2}\log(3x+2)$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\frac{\log(x)}{2}-\frac{1}{2}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + 3*x)),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(2+3x)}\,dx &= \frac{1}{2}\int \frac{1}{x}\,dx-\frac{3}{2}\int \frac{1}{2+3x}\,dx\\ &= \frac{\log(x)}{2}-\frac{1}{2}\log(2+3x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{\log(x)}{2}-\frac{1}{2}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + 3*x)),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x]/2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(2+3x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(2 + 3*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(2 + 3*x)), x]
```

**fricas [A]** time = 0.80, size = 13, normalized size = 0.76

$$-\frac{1}{2}\,\log\left(3\,x+2\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(3*x + 2) + 1/2*log(x)
```

**giac [A]** time = 1.15, size = 15, normalized size = 0.88

$$-\frac{1}{2}\,\log\left(|3\,x+2|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

**maple [A]** time = 0.01, size = 14, normalized size = 0.82

$$\frac{\ln(x)}{2}-\frac{\ln\left(3x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2+3*x),x)
```

```
[Out] 1/2*ln(x)-1/2*ln(2+3*x)
```

**maxima [A]** time = 1.34, size = 13, normalized size = 0.76

$$-\frac{1}{2}\,\log\left(3\,x+2\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="maxima")
[Out] -1/2*log(3*x + 2) + 1/2*log(x)
```

**mupad [B]** time = 0.17, size = 10, normalized size = 0.59

$$-\frac{\ln\left(\frac{2}{x}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x + 2)),x)
[Out] -log(2/x + 3)/2
```

**sympy [A]** time = 0.12, size = 12, normalized size = 0.71

$$\frac{\log\left(x\right)}{2}-\frac{\log\left(x+\frac{2}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x)
[Out] log(x)/2 - log(x + 2/3)/2
```

## 3.256 $\int \frac{1}{x(4+6x)}\,dx$

**Optimal**. Leaf size=17

$$\frac{\log(x)}{4}-\frac{1}{4}\log(3x+2)$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\frac{\log(x)}{4}-\frac{1}{4}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)),x]
```

```
[Out] Log[x]/4 - Log[2 + 3*x]/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)}\,dx&=\frac{1}{4}\int \frac{1}{x}\,dx-\frac{3}{2}\int \frac{1}{4+6x}\,dx\\&=\frac{\log(x)}{4}-\frac{1}{4}\log(2+3x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{\log(x)}{4}-\frac{1}{4}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)),x]
```

```
[Out] Log[x]/4 - Log[2 + 3*x]/4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(4+6x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(4 + 6*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(4 + 6*x)), x]
```

**fricas [A]** time = 1.16, size = 13, normalized size = 0.76

$$-\frac{1}{4}\,\log\left(3\,x+2\right)+\frac{1}{4}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(3*x + 2) + 1/4*log(x)
```

**giac [A]** time = 1.01, size = 15, normalized size = 0.88

$$-\frac{1}{4}\,\log\left(|3\,x+2|\right)+\frac{1}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(3*x + 2)) + 1/4*log(abs(x))
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.82

$$\frac{\ln(x)}{4}-\frac{\ln\left(3x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x),x)
```

```
[Out] 1/4*ln(x)-1/4*ln(3*x+2)
```

**maxima** [A] time = 1.29, size = 13, normalized size = 0.76

$$-\frac{1}{4}\,\log\left(3\,x+2\right)+\frac{1}{4}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(3*x + 2) + 1/4*log(x)
```

**mupad** [B] time = 0.14, size = 10, normalized size = 0.59

$$-\frac{\ln\left(\frac{4}{x}+6\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)),x)
```

```
[Out] -log(4/x + 6)/4
```

**sympy** [A] time = 0.12, size = 12, normalized size = 0.71

$$\frac{\log\left(x\right)}{4}-\frac{\log\left(x+\frac{2}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x)
```

```
[Out] log(x)/4 - log(x + 2/3)/4
```

## 3.257 $\int \frac{1}{x^2(4+6x)}\,dx$

**Optimal**. Leaf size=24

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

**Rubi [A]** time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)),x]
[Out] -1/(4*x) - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)}\,dx &= \int \left(\frac{1}{4x^2}-\frac{3}{8x}+\frac{9}{8(2+3x)}\right)dx\\ &= -\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(2+3x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 24, normalized size = 1.00

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)),x]
[Out] -1/4*1/x - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(4+6x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(4 + 6*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(4 + 6*x)), x]
```

**fricas [A]** time = 0.59, size = 21, normalized size = 0.88

$$\frac{3\,x\log\left(3\,x+2\right)-3\,x\log\left(x\right)-2}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="fricas")
```

```
[Out] 1/8*(3*x*log(3*x + 2) - 3*x*log(x) - 2)/x
```

**giac [A]** time = 1.01, size = 20, normalized size = 0.83

$$-\frac{1}{4\,x}+\frac{3}{8}\,\log\left(|3\,x+2|\right)-\frac{3}{8}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="giac")
```

```
[Out] -1/4/x + 3/8*log(abs(3*x + 2)) - 3/8*log(abs(x))
```

**maple [A]** time = 0.01, size = 19, normalized size = 0.79

$$-\frac{3\ln\left(x\right)}{8}+\frac{3\ln\left(3x+2\right)}{8}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x),x)
```

```
[Out] -1/4/x-3/8*ln(x)+3/8*ln(3*x+2)
```

**maxima [A]** time = 1.38, size = 18, normalized size = 0.75

$$-\frac{1}{4\,x}+\frac{3}{8}\,\log\left(3\,x+2\right)-\frac{3}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="maxima")
[Out] -1/4/x + 3/8*log(3*x + 2) - 3/8*log(x)
```

**mupad [B]** time = 0.05, size = 18, normalized size = 0.75

$$-\frac{3\,\ln\left(\frac{x}{6\,x+4}\right)}{8}-\frac{1}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)),x)
[Out] - (3*log(x/(6*x + 4)))/8 - 1/(4*x)
```

**sympy [A]** time = 0.14, size = 20, normalized size = 0.83

$$-\frac{3\log(x)}{8}+\frac{3\log\left(x+\frac{2}{3}\right)}{8}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x),x)
[Out] -3*log(x)/8 + 3*log(x + 2/3)/8 - 1/(4*x)
```

## 3.258 $\int \frac{1}{x^3(4+6x)}\,dx$

**Optimal**. Leaf size=31

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

**Rubi** [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)),x]
```

```
[Out] -1/(8*x^2) + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)}\,dx &= \int \left(\frac{1}{4x^3}-\frac{3}{8x^2}+\frac{9}{16x}-\frac{27}{16(2+3x)}\right)dx\\ &= -\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(2+3x)\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 31, normalized size = 1.00

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)),x]
```

```
[Out] -1/8*1/x^2 + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(4+6x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(4 + 6*x)),x]
[Out] IntegrateAlgebraic[1/(x^3*(4 + 6*x)), x]
```

**fricas [A]** time = 0.95, size = 28, normalized size = 0.90

$$-\frac{9\,x^2\log(3\,x+2)-9\,x^2\log(x)-6\,x+2}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="fricas")
[Out] -1/16*(9*x^2*log(3*x + 2) - 9*x^2*log(x) - 6*x + 2)/x^2
```

**giac [A]** time = 1.07, size = 25, normalized size = 0.81

$$\frac{3\,x-1}{8\,x^2}-\frac{9}{16}\,\log(|3\,x+2|)+\frac{9}{16}\,\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="giac")
[Out] 1/8*(3*x - 1)/x^2 - 9/16*log(abs(3*x + 2)) + 9/16*log(abs(x))
```

**maple [A]** time = 0.01, size = 24, normalized size = 0.77

$$\frac{9\ln(x)}{16}-\frac{9\ln(3x+2)}{16}+\frac{3}{8x}-\frac{1}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x),x)
[Out] -1/8/x^2+3/8/x+9/16*ln(x)-9/16*ln(3*x+2)
```

**maxima [A]** time = 1.29, size = 23, normalized size = 0.74

$$\frac{3\,x-1}{8\,x^2}-\frac{9}{16}\,\log(3\,x+2)+\frac{9}{16}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="maxima")
```

```
[Out] 1/8*(3*x - 1)/x^2 - 9/16*log(3*x + 2) + 9/16*log(x)
```

**mupad [B]** time = 0.04, size = 18, normalized size = 0.58

$$\frac{\frac{3\,x}{8}-\frac{1}{8}}{x^2}-\frac{9\,\mathrm{atanh}\left(3\,x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)),x)
```

```
[Out] ((3*x)/8 - 1/8)/x^2 - (9*atanh(3*x + 1))/8
```

**sympy [A]** time = 0.15, size = 26, normalized size = 0.84

$$\frac{9\log{(x)}}{16}-\frac{9\log{\left(x+\frac{2}{3}\right)}}{16}+\frac{3x-1}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x),x)
```

```
[Out] 9*log(x)/16 - 9*log(x + 2/3)/16 + (3*x - 1)/(8*x**2)
```

## 3.259 $\int \frac{1}{x^4(4+6x)}\,dx$

**Optimal**. Leaf size=38

$$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

**Rubi [A]** time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{3}{16x^2}-\frac{1}{12x^3}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)),x]
```

```
[Out] -1/(12*x^3) + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/32
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)}\,dx &= \int \left(\frac{1}{4x^4}-\frac{3}{8x^3}+\frac{9}{16x^2}-\frac{27}{32x}+\frac{81}{32(2+3x)}\right)dx\\ &= -\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 38, normalized size = 1.00

$$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)),x]
```

```
[Out] -1/12*1/x^3 + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/32
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(4+6x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(4 + 6*x)),x]
[Out] IntegrateAlgebraic[1/(x^4*(4 + 6*x)), x]
```

**fricas [A]** time = 0.57, size = 33, normalized size = 0.87

$$\frac{81\,x^3\log\left(3\,x+2\right)-81\,x^3\log(x)-54\,x^2+18\,x-8}{96\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="fricas")
[Out] 1/96*(81*x^3*log(3*x + 2) - 81*x^3*log(x) - 54*x^2 + 18*x - 8)/x^3
```

**giac [A]** time = 1.03, size = 30, normalized size = 0.79

$$-\frac{27\,x^2-9\,x+4}{48\,x^3}+\frac{27}{32}\,\log\left(|3\,x+2|\right)-\frac{27}{32}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="giac")
[Out] -1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(abs(3*x + 2)) - 27/32*log(abs(x))
```

**maple [A]** time = 0.01, size = 29, normalized size = 0.76

$$-\frac{27\ln(x)}{32}+\frac{27\ln\left(3x+2\right)}{32}-\frac{9}{16x}+\frac{3}{16x^2}-\frac{1}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x),x)
[Out] -1/12/x^3+3/16/x^2-9/16/x-27/32*ln(x)+27/32*ln(3*x+2)
```

**maxima [A]** time = 1.35, size = 28, normalized size = 0.74

$$-\frac{27\,x^2-9\,x+4}{48\,x^3}+\frac{27}{32}\,\log\left(3\,x+2\right)-\frac{27}{32}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="maxima")
```

```
[Out] -1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(3*x + 2) - 27/32*log(x)
```

**mupad [B]** time = 0.09, size = 24, normalized size = 0.63

$$\frac{27\,\mathrm{atanh}\left(3\,x+1\right)}{16}-\frac{\frac{9\,x^2}{16}-\frac{3\,x}{16}+\frac{1}{12}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)),x)
```

```
[Out] (27*atanh(3*x + 1))/16 - ((9*x^2)/16 - (3*x)/16 + 1/12)/x^3
```

**sympy [A]** time = 0.16, size = 31, normalized size = 0.82

$$-\frac{27\log\left(x\right)}{32}+\frac{27\log\left(x+\frac{2}{3}\right)}{32}+\frac{-27x^2+9x-4}{48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x),x)
```

```
[Out] -27*log(x)/32 + 27*log(x + 2/3)/32 + (-27*x**2 + 9*x - 4)/(48*x**3)
```

## 3.260 $\int \frac{1}{x^5(4+6x)}\,dx$

**Optimal**. Leaf size=45

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

**Rubi** [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{9}{32x^2}+\frac{1}{8x^3}-\frac{1}{16x^4}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)),x]
```

```
[Out] -1/(16*x^4) + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Log
[2 + 3*x])/64
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)}\,dx &= \int \left(\frac{1}{4x^5}-\frac{3}{8x^4}+\frac{9}{16x^3}-\frac{27}{32x^2}+\frac{81}{64x}-\frac{243}{64(2+3x)}\right)dx\\ &= -\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(2+3x)\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 45, normalized size = 1.00

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)),x]
```

```
[Out] -1/16*1/x^4 + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Log
[2 + 3*x])/64
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(4+6x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(4 + 6*x)),x]
[Out] IntegrateAlgebraic[1/(x^5*(4 + 6*x)), x]
```

**fricas [A]** time = 0.74, size = 38, normalized size = 0.84

$$-\frac{81\,x^4\log\left(3\,x+2\right)-81\,x^4\log(x)-54\,x^3+18\,x^2-8\,x+4}{64\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="fricas")
[Out] -1/64*(81*x^4*log(3*x + 2) - 81*x^4*log(x) - 54*x^3 + 18*x^2 - 8*x + 4)/x^4
```

**giac [A]** time = 1.14, size = 35, normalized size = 0.78

$$\frac{27\,x^3-9\,x^2+4\,x-2}{32\,x^4}-\frac{81}{64}\,\log\left(|3\,x+2|\right)+\frac{81}{64}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="giac")
[Out] 1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(abs(3*x + 2)) + 81/64*log(a
bs(x))
```

**maple [A]** time = 0.01, size = 34, normalized size = 0.76

$$\frac{81\ln(x)}{64}-\frac{81\ln\left(3x+2\right)}{64}+\frac{27}{32x}-\frac{9}{32x^2}+\frac{1}{8x^3}-\frac{1}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x),x)
[Out] -1/16/x^4+1/8/x^3-9/32/x^2+27/32/x+81/64*ln(x)-81/64*ln(3*x+2)
```

**maxima** [A] time = 1.32, size = 33, normalized size = 0.73

$$\frac{27\,x^3-9\,x^2+4\,x-2}{32\,x^4}-\frac{81}{64}\,\log\left(3\,x+2\right)+\frac{81}{64}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="maxima")
```

```
[Out] 1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(3*x + 2) + 81/64*log(x)
```

**mupad** [B] time = 0.04, size = 28, normalized size = 0.62

$$\frac{\frac{27\,x^3}{32}-\frac{9\,x^2}{32}+\frac{x}{8}-\frac{1}{16}}{x^4}-\frac{81\,\mathrm{atanh}\left(3\,x+1\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)),x)
```

```
[Out] (x/8 - (9*x^2)/32 + (27*x^3)/32 - 1/16)/x^4 - (81*atanh(3*x + 1))/32
```

**sympy** [A] time = 0.17, size = 36, normalized size = 0.80

$$\frac{81\log\left(x\right)}{64}-\frac{81\log\left(x+\frac{2}{3}\right)}{64}+\frac{27x^3-9x^2+4x-2}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x),x)
```

```
[Out] 81*log(x)/64 - 81*log(x + 2/3)/64 + (27*x**3 - 9*x**2 + 4*x - 2)/(32*x**4)
```

## 3.261 $\int \frac{1}{x(4+6x)^2} \, dx$

**Optimal**. Leaf size=28

$$\frac{1}{8(3x+2)} + \frac{\log(x)}{16} - \frac{1}{16}\log(3x+2)$$

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{8(3x+2)} + \frac{\log(x)}{16} - \frac{1}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)^2),x]
```

```
[Out] 1/(8*(2 + 3*x)) + Log[x]/16 - Log[2 + 3*x]/16
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)^2} \, dx &= \int \left(\frac{1}{16x} - \frac{3}{8(2+3x)^2} - \frac{3}{16(2+3x)}\right) dx \\ &= \frac{1}{8(2+3x)} + \frac{\log(x)}{16} - \frac{1}{16}\log(2+3x)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 0.93

$$\frac{1}{16}\left(\frac{2}{3x+2} + \log(-6x) - \log(6x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)^2),x]
```

```
[Out] (2/(2 + 3*x) + Log[-6*x] - Log[4 + 6*x])/16
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(4+6x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(4 + 6*x)^2),x]
[Out] IntegrateAlgebraic[1/(x*(4 + 6*x)^2), x]
```

**fricas [A]** time = 0.85, size = 32, normalized size = 1.14

$$-\frac{(3\,x+2)\log\left(3\,x+2\right)-(3\,x+2)\log\left(x\right)-2}{16\,(3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="fricas")
[Out] -1/16*((3*x + 2)*log(3*x + 2) - (3*x + 2)*log(x) - 2)/(3*x + 2)
```

**giac [A]** time = 1.11, size = 25, normalized size = 0.89

$$\frac{1}{8\,(3\,x+2)}+\frac{1}{16}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="giac")
[Out] 1/8/(3*x + 2) + 1/16*log(abs(-2/(3*x + 2) + 1))
```

**maple [A]** time = 0.01, size = 23, normalized size = 0.82

$$\frac{\ln(x)}{16}-\frac{\ln\left(3x+2\right)}{16}+\frac{1}{24x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x)^2,x)
[Out] 1/8/(3*x+2)+1/16*ln(x)-1/16*ln(3*x+2)
```

**maxima [A]** time = 1.37, size = 22, normalized size = 0.79

$$\frac{1}{8\,(3\,x+2)}-\frac{1}{16}\,\log\left(3\,x+2\right)+\frac{1}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="maxima")
[Out] 1/8/(3*x + 2) - 1/16*log(3*x + 2) + 1/16*log(x)
```

**mupad [B]** time = 0.06, size = 20, normalized size = 0.71

$$\frac{1}{8\,\left(3\,x+2\right)}-\frac{\ln\left(\frac{6\,x+4}{x}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)^2),x)
[Out] 1/(8*(3*x + 2)) - log((6*x + 4)/x)/16
```

**sympy [A]** time = 0.14, size = 19, normalized size = 0.68

$$\frac{\log\left(x\right)}{16}-\frac{\log\left(x+\frac{2}{3}\right)}{16}+\frac{1}{24x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)**2,x)
[Out] log(x)/16 - log(x + 2/3)/16 + 1/(24*x + 16)
```

## 3.262 $\int \frac{1}{x^2(4+6x)^2} dx$

**Optimal**. Leaf size=35

$$-\frac{1}{16x}-\frac{3}{16(3x+2)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(3x+2)$$

**Rubi** [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{16x}-\frac{3}{16(3x+2)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)^2),x]
```

```
[Out] -1/(16*x) - 3/(16*(2 + 3*x)) - (3*Log[x])/16 + (3*Log[2 + 3*x])/16
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^2}-\frac{3}{16x}+\frac{9}{16(2+3x)^2}+\frac{9}{16(2+3x)}\right)dx\\ &= -\frac{1}{16x}-\frac{3}{16(2+3x)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(2+3x)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 31, normalized size = 0.89

$$\frac{1}{16}\left(-\frac{1}{x}-\frac{3}{3x+2}-3\log(x)+3\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)^2),x]
```

```
[Out] (-x^(-1) - 3/(2 + 3*x) - 3*Log[x] + 3*Log[2 + 3*x])/16
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(4+6x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(4 + 6*x)^2),x]
[Out] IntegrateAlgebraic[1/(x^2*(4 + 6*x)^2), x]
```

**fricas [A]** time = 1.19, size = 48, normalized size = 1.37

$$\frac{3\left(3x^2+2x\right)\log\left(3x+2\right)-3\left(3x^2+2x\right)\log(x)-6x-2}{16\left(3x^2+2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="fricas")
[Out] 1/16*(3*(3*x^2 + 2*x)*log(3*x + 2) - 3*(3*x^2 + 2*x)*log(x) - 6*x - 2)/(3*x
^2 + 2*x)
```

**giac [A]** time = 0.95, size = 40, normalized size = 1.14

$$-\frac{3}{16\left(3x+2\right)}+\frac{3}{32\left(\frac{2}{3x+2}-1\right)}-\frac{3}{16}\log\left(\left|-\frac{2}{3x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="giac")
[Out] -3/16/(3*x + 2) + 3/32/(2/(3*x + 2) - 1) - 3/16*log(abs(-2/(3*x + 2) + 1))
```

**maple [A]** time = 0.01, size = 28, normalized size = 0.80

$$-\frac{3\ln(x)}{16}+\frac{3\ln\left(3x+2\right)}{16}-\frac{1}{16x}-\frac{3}{16\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x)^2,x)
[Out] -1/16/x-3/16/(3*x+2)-3/16*ln(x)+3/16*ln(3*x+2)
```

**maxima** [A] time = 1.39, size = 31, normalized size = 0.89

$$-\frac{3\,x+1}{8\left(3\,x^2+2\,x\right)}+\frac{3}{16}\,\log\left(3\,x+2\right)-\frac{3}{16}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x + 1)/(3*x^2 + 2*x) + 3/16*log(3*x + 2) - 3/16*log(x)
```

**mupad** [B] time = 0.09, size = 34, normalized size = 0.97

$$\frac{3\,\ln\left(\frac{6\,x+4}{x}\right)}{16}-\frac{3}{4\,\left(6\,x+4\right)}-\frac{1}{4\,x\,\left(6\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)^2),x)
```

```
[Out] (3*log((6*x + 4)/x))/16 - 3/(4*(6*x + 4)) - 1/(4*x*(6*x + 4))
```

**sympy** [A] time = 0.15, size = 31, normalized size = 0.89

$$\frac{-3x-1}{24x^2+16x}-\frac{3\log\left(x\right)}{16}+\frac{3\log\left(x+\frac{2}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x)**2,x)
```

```
[Out] (-3*x - 1)/(24*x**2 + 16*x) - 3*log(x)/16 + 3*log(x + 2/3)/16
```

## 3.263 $\int \frac{1}{x^3(4+6x)^2} \, dx$

**Optimal**. Leaf size=42

$$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(3x+2)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(3x+2)$$

**Rubi [A]** time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(3x+2)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)^2),x]
```

```
[Out] -1/(32*x^2) + 3/(16*x) + 9/(32*(2 + 3*x)) + (27*Log[x])/64 - (27*Log[2 + 3*
x])/64
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^3}-\frac{3}{16x^2}+\frac{27}{64x}-\frac{27}{32(2+3x)^2}-\frac{81}{64(2+3x)}\right)dx\\ &= -\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(2+3x)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(2+3x)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 36, normalized size = 0.86

$$\frac{1}{64}\left(-\frac{2}{x^2}+\frac{12}{x}+\frac{18}{3x+2}+27\log(x)-27\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)^2),x]
```

```
[Out] (-2/x^2 + 12/x + 18/(2 + 3*x) + 27*Log[x] - 27*Log[2 + 3*x])/64
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(4+6x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(4 + 6*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(4 + 6*x)^2), x]
```

**fricas [A]** time = 1.07, size = 59, normalized size = 1.40

$$\frac{54\,x^2-27\left(3\,x^3+2\,x^2\right)\log\left(3\,x+2\right)+27\left(3\,x^3+2\,x^2\right)\log(x)+18\,x-4}{64\left(3\,x^3+2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(54*x^2 - 27*(3*x^3 + 2*x^2)*log(3*x + 2) + 27*(3*x^3 + 2*x^2)*log(x)
+ 18*x - 4)/(3*x^3 + 2*x^2)
```

**giac [A]** time = 0.94, size = 51, normalized size = 1.21

$$\frac{9}{32\left(3\,x+2\right)}-\frac{9\left(\frac{12}{3\,x+2}-5\right)}{128\left(\frac{2}{3\,x+2}-1\right)^2}+\frac{27}{64}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] 9/32/(3*x + 2) - 9/128*(12/(3*x + 2) - 5)/(2/(3*x + 2) - 1)^2 + 27/64*log(a
bs(-2/(3*x + 2) + 1))
```

**maple [A]** time = 0.01, size = 33, normalized size = 0.79

$$\frac{27\ln(x)}{64}-\frac{27\ln\left(3x+2\right)}{64}+\frac{3}{16x}-\frac{1}{32x^2}+\frac{9}{32\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x)^2,x)
```

```
[Out] -1/32/x^2+3/16/x+9/32/(3*x+2)+27/64*ln(x)-27/64*ln(3*x+2)
```

**maxima [A]** time = 1.31, size = 38, normalized size = 0.90

$$\frac{27\,x^2+9\,x-2}{32\left(3\,x^3+2\,x^2\right)}-\frac{27}{64}\,\log\left(3\,x+2\right)+\frac{27}{64}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(27*x^2 + 9*x - 2)/(3*x^3 + 2*x^2) - 27/64*log(3*x + 2) + 27/64*log(x)
```

**mupad [B]** time = 0.04, size = 31, normalized size = 0.74

$$\frac{\frac{9\,x^2}{32}+\frac{3\,x}{32}-\frac{1}{48}}{x^3+\frac{2\,x^2}{3}}-\frac{27\,\mathrm{atanh}\left(3\,x+1\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)^2),x)
```

```
[Out] ((3*x)/32 + (9*x^2)/32 - 1/48)/((2*x^2)/3 + x^3) - (27*atanh(3*x + 1))/32
```

**sympy [A]** time = 0.16, size = 36, normalized size = 0.86

$$\frac{27\log\left(x\right)}{64}-\frac{27\log\left(x+\frac{2}{3}\right)}{64}+\frac{27x^2+9x-2}{96x^3+64x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x)**2,x)
```

```
[Out] 27*log(x)/64 - 27*log(x + 2/3)/64 + (27*x**2 + 9*x - 2)/(96*x**3 + 64*x**2)
```

## 3.264 $\int \frac{1}{x^4(4+6x)^2} \, dx$

**Optimal**. Leaf size=49

$$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(3x+2)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

**Rubi [A]** time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{3}{32x^2}-\frac{1}{48x^3}-\frac{27}{64x}-\frac{27}{64(3x+2)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)^2),x]
```

```
[Out] -1/(48*x^3) + 3/(32*x^2) - 27/(64*x) - 27/(64*(2 + 3*x)) - (27*Log[x])/32 +
 (27*Log[2 + 3*x])/32
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^4}-\frac{3}{16x^3}+\frac{27}{64x^2}-\frac{27}{32x}+\frac{81}{64(2+3x)^2}+\frac{81}{32(2+3x)}\right)dx\\ &= -\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(2+3x)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 44, normalized size = 0.90

$$\frac{1}{192}\left(-\frac{4\left(81x^3+27x^2-6x+2\right)}{x^3(3x+2)}-162\log(x)+162\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)^2),x]
```

```
[Out] ((-4*(2 - 6*x + 27*x^2 + 81*x^3))/(x^3*(2 + 3*x)) - 162*Log[x] + 162*Log[2
+ 3*x])/192
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(4+6x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(4 + 6*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(4 + 6*x)^2), x]
```

**fricas [A]** time = 1.19, size = 64, normalized size = 1.31

$$-\frac{162\,x^3+54\,x^2-81\left(3\,x^4+2\,x^3\right)\log\left(3\,x+2\right)+81\left(3\,x^4+2\,x^3\right)\log(x)-12\,x+4}{96\left(3\,x^4+2\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(162*x^3 + 54*x^2 - 81*(3*x^4 + 2*x^3)*log(3*x + 2) + 81*(3*x^4 + 2*x
^3)*log(x) - 12*x + 4)/(3*x^4 + 2*x^3)
```

**giac [A]** time = 1.23, size = 60, normalized size = 1.22

$$-\frac{27}{64\,(3\,x+2)}-\frac{9\left(\frac{60}{3\,x+2}-\frac{72}{(3\,x+2)^2}-13\right)}{128\left(\frac{2}{3\,x+2}-1\right)^3}-\frac{27}{32}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] -27/64/(3*x + 2) - 9/128*(60/(3*x + 2) - 72/(3*x + 2)^2 - 13)/(2/(3*x + 2)
- 1)^3 - 27/32*log(abs(-2/(3*x + 2) + 1))
```

**maple [A]** time = 0.01, size = 38, normalized size = 0.78

$$-\frac{27\ln(x)}{32}+\frac{27\ln(3x+2)}{32}-\frac{27}{64x}+\frac{3}{32x^2}-\frac{1}{48x^3}-\frac{27}{64\,(3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x)^2,x)
```

```
[Out] -1/48/x^3+3/32/x^2-27/64/x-27/64/(3*x+2)-27/32*ln(x)+27/32*ln(3*x+2)
```

**maxima [A]** time = 1.31, size = 43, normalized size = 0.88

$$-\frac{81\,x^3+27\,x^2-6\,x+2}{48\left(3\,x^4+2\,x^3\right)}+\frac{27}{32}\,\log\left(3\,x+2\right)-\frac{27}{32}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(81*x^3 + 27*x^2 - 6*x + 2)/(3*x^4 + 2*x^3) + 27/32*log(3*x + 2) - 27
/32*log(x)
```

**mupad [B]** time = 0.09, size = 37, normalized size = 0.76

$$\frac{27\,\mathrm{atanh}\left(3\,x+1\right)}{16}-\frac{\frac{9\,x^3}{16}+\frac{3\,x^2}{16}-\frac{x}{24}+\frac{1}{72}}{x^4+\frac{2\,x^3}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)^2),x)
```

```
[Out] (27*atanh(3*x + 1))/16 - ((3*x^2)/16 - x/24 + (9*x^3)/16 + 1/72)/((2*x^3)/3
 + x^4)
```

**sympy [A]** time = 0.17, size = 41, normalized size = 0.84

$$-\frac{27\log\left(x\right)}{32}+\frac{27\log\left(x+\frac{2}{3}\right)}{32}+\frac{-81x^3-27x^2+6x-2}{144x^4+96x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x)**2,x)
```

```
[Out] -27*log(x)/32 + 27*log(x + 2/3)/32 + (-81*x**3 - 27*x**2 + 6*x - 2)/(144*x*
*4 + 96*x**3)
```

## 3.265 $\int \frac{1}{x^5(4+6x)^2}\,dx$

**Optimal**. Leaf size=56

$$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

**Rubi [A]** time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{27}{128x^2}+\frac{1}{16x^3}-\frac{1}{64x^4}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)^2),x]
```

```
[Out] -1/(64*x^4) + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
(405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^5}-\frac{3}{16x^4}+\frac{27}{64x^3}-\frac{27}{32x^2}+\frac{405}{256x}-\frac{243}{128(2+3x)^2}-\frac{1215}{256(2+3x)}\right)dx\\ &= -\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(2+3x)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 56, normalized size = 1.00

$$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)^2),x]
```

```
[Out] -1/64*1/x^4 + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
(405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(4+6x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(4 + 6*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(4 + 6*x)^2), x]
```

**fricas [A]** time = 0.94, size = 69, normalized size = 1.23

$$\frac{810\,x^4+270\,x^3-60\,x^2-405\left(3\,x^5+2\,x^4\right)\log\left(3\,x+2\right)+405\left(3\,x^5+2\,x^4\right)\log(x)+20\,x-8}{256\left(3\,x^5+2\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/256*(810*x^4 + 270*x^3 - 60*x^2 - 405*(3*x^5 + 2*x^4)*log(3*x + 2) + 405*
(3*x^5 + 2*x^4)*log(x) + 20*x - 8)/(3*x^5 + 2*x^4)
```

**giac [A]** time = 1.15, size = 69, normalized size = 1.23

$$\frac{81}{128\left(3\,x+2\right)}-\frac{27\left(\frac{520}{3\,x+2}-\frac{1200}{(3\,x+2)^2}+\frac{960}{(3\,x+2)^3}-77\right)}{1024\left(\frac{2}{3\,x+2}-1\right)^4}+\frac{405}{256}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] 81/128/(3*x + 2) - 27/1024*(520/(3*x + 2) - 1200/(3*x + 2)^2 + 960/(3*x + 2
)^3 - 77)/(2/(3*x + 2) - 1)^4 + 405/256*log(abs(-2/(3*x + 2) + 1))
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.77

$$\frac{405\ln(x)}{256}-\frac{405\ln\left(3x+2\right)}{256}+\frac{27}{32x}-\frac{27}{128x^2}+\frac{1}{16x^3}-\frac{1}{64x^4}+\frac{81}{128\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x)^2,x)
```

```
[Out] -1/64/x^4+1/16/x^3-27/128/x^2+27/32/x+81/128/(3*x+2)+405/256*ln(x)-405/256*
ln(3*x+2)
```

**maxima** [A] time = 1.36, size = 48, normalized size = 0.86

$$\frac{405\,x^4+135\,x^3-30\,x^2+10\,x-4}{128\left(3\,x^5+2\,x^4\right)}-\frac{405}{256}\,\log\left(3\,x+2\right)+\frac{405}{256}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] 1/128*(405*x^4 + 135*x^3 - 30*x^2 + 10*x - 4)/(3*x^5 + 2*x^4) - 405/256*log
(3*x + 2) + 405/256*log(x)
```

**mupad** [B] time = 0.09, size = 41, normalized size = 0.73

$$\frac{\frac{135\,x^4}{128}+\frac{45\,x^3}{128}-\frac{5\,x^2}{64}+\frac{5\,x}{192}-\frac{1}{96}}{x^5+\frac{2\,x^4}{3}}-\frac{405\,\mathrm{atanh}\left(3\,x+1\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)^2),x)
```

```
[Out] ((5*x)/192 - (5*x^2)/64 + (45*x^3)/128 + (135*x^4)/128 - 1/96)/((2*x^4)/3 +
 x^5) - (405*atanh(3*x + 1))/128
```

**sympy** [A] time = 0.18, size = 46, normalized size = 0.82

$$\frac{405\log\left(x\right)}{256}-\frac{405\log\left(x+\frac{2}{3}\right)}{256}+\frac{405x^4+135x^3-30x^2+10x-4}{384x^5+256x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x)**2,x)
```

```
[Out] 405*log(x)/256 - 405*log(x + 2/3)/256 + (405*x**4 + 135*x**3 - 30*x**2 + 10
*x - 4)/(384*x**5 + 256*x**4)
```

## 3.266 $\int \frac{1}{x(4+6x)^3} \, dx$

**Optimal**. Leaf size=39

$$\frac{1}{32(3x+2)}+\frac{1}{32(3x+2)^2}+\frac{\log(x)}{64}-\frac{1}{64}\log(3x+2)$$

**Rubi** [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{32(3x+2)}+\frac{1}{32(3x+2)^2}+\frac{\log(x)}{64}-\frac{1}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)^3),x]
```

```
[Out] 1/(32*(2 + 3*x)^2) + 1/(32*(2 + 3*x)) + Log[x]/64 - Log[2 + 3*x]/64
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)^3} \, dx &= \int \left(\frac{1}{64x}-\frac{3}{16(2+3x)^3}-\frac{3}{32(2+3x)^2}-\frac{3}{64(2+3x)}\right) dx\\ &= \frac{1}{32(2+3x)^2}+\frac{1}{32(2+3x)}+\frac{\log(x)}{64}-\frac{1}{64}\log(2+3x)\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 29, normalized size = 0.74

$$\frac{1}{64}\left(\frac{6(x+1)}{(3x+2)^2}+\log(-6x)-\log(6x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)^3),x]
```

```
[Out] ((6*(1 + x))/(2 + 3*x)^2 + Log[-6*x] - Log[4 + 6*x])/64
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(4+6x)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(4 + 6*x)^3),x]
[Out] IntegrateAlgebraic[1/(x*(4 + 6*x)^3), x]
```

**fricas [A]** time = 1.08, size = 50, normalized size = 1.28

$$-\frac{\left(9\,x^2+12\,x+4\right)\log\left(3\,x+2\right)-\left(9\,x^2+12\,x+4\right)\log(x)-6\,x-6}{64\left(9\,x^2+12\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="fricas")
[Out] -1/64*((9*x^2 + 12*x + 4)*log(3*x + 2) - (9*x^2 + 12*x + 4)*log(x) - 6*x -
6)/(9*x^2 + 12*x + 4)
```

**giac [A]** time = 1.03, size = 27, normalized size = 0.69

$$\frac{3\,(x+1)}{32\,(3\,x+2)^2}-\frac{1}{64}\,\log\left(|3\,x+2|\right)+\frac{1}{64}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="giac")
[Out] 3/32*(x + 1)/(3*x + 2)^2 - 1/64*log(abs(3*x + 2)) + 1/64*log(abs(x))
```

**maple [A]** time = 0.01, size = 32, normalized size = 0.82

$$\frac{\ln(x)}{64}-\frac{\ln\left(3x+2\right)}{64}+\frac{1}{32\,(3x+2)^2}+\frac{1}{96x+64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x)^3,x)
[Out] 1/32/(3*x+2)^2+1/32/(3*x+2)+1/64*ln(x)-1/64*ln(3*x+2)
```

**maxima** [A] time = 1.39, size = 30, normalized size = 0.77

$$\frac{3\,(x+1)}{32\,\left(9\,x^2+12\,x+4\right)}-\frac{1}{64}\,\log\left(3\,x+2\right)+\frac{1}{64}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 3/32*(x + 1)/(9*x^2 + 12*x + 4) - 1/64*log(3*x + 2) + 1/64*log(x)
```

**mupad** [B] time = 0.13, size = 29, normalized size = 0.74

$$\frac{1}{32\,\left(3\,x+2\right)}-\frac{\ln\left(\frac{6\,x+4}{x}\right)}{64}+\frac{1}{8\,\left(6\,x+4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)^3),x)
```

```
[Out] 1/(32*(3*x + 2)) - log((6*x + 4)/x)/64 + 1/(8*(6*x + 4)^2)
```

**sympy** [A] time = 0.17, size = 27, normalized size = 0.69

$$\frac{3x+3}{288x^2+384x+128}+\frac{\log\left(x\right)}{64}-\frac{\log\left(x+\frac{2}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)**3,x)
```

```
[Out] (3*x + 3)/(288*x**2 + 384*x + 128) + log(x)/64 - log(x + 2/3)/64
```

## 3.267 $\int \frac{1}{x^2(4+6x)^3} \, dx$

**Optimal**. Leaf size=46

$$-\frac{1}{64x}-\frac{3}{32(3x+2)}-\frac{3}{64(3x+2)^2}-\frac{9\log(x)}{128}+\frac{9}{128}\log(3x+2)$$

**Rubi [A]** time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{64x}-\frac{3}{32(3x+2)}-\frac{3}{64(3x+2)^2}-\frac{9\log(x)}{128}+\frac{9}{128}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)^3),x]
```

```
[Out] -1/(64*x) - 3/(64*(2 + 3*x)^2) - 3/(32*(2 + 3*x)) - (9*Log[x])/128 + (9*Log
[2 + 3*x])/128
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)^3} \, dx &= \int \left(\frac{1}{64x^2}-\frac{9}{128x}+\frac{9}{32(2+3x)^3}+\frac{9}{32(2+3x)^2}+\frac{27}{128(2+3x)}\right) dx\\ &= -\frac{1}{64x}-\frac{3}{64(2+3x)^2}-\frac{3}{32(2+3x)}-\frac{9\log(x)}{128}+\frac{9}{128}\log(2+3x)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 39, normalized size = 0.85

$$\frac{1}{128}\left(-\frac{2\left(27x^2+27x+4\right)}{x(3x+2)^2}-9\log(x)+9\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)^3),x]
[Out] ((-2*(4 + 27*x + 27*x^2))/(x*(2 + 3*x)^2) - 9*Log[x] + 9*Log[2 + 3*x])/128
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(4+6x)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(4 + 6*x)^3),x]
[Out] IntegrateAlgebraic[1/(x^2*(4 + 6*x)^3), x]
```

**fricas [A]** time = 0.92, size = 68, normalized size = 1.48

$$-\frac{54\,x^2-9\left(9\,x^3+12\,x^2+4\,x\right)\log\left(3\,x+2\right)+9\left(9\,x^3+12\,x^2+4\,x\right)\log(x)+54\,x+8}{128\left(9\,x^3+12\,x^2+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="fricas")
[Out] -1/128*(54*x^2 - 9*(9*x^3 + 12*x^2 + 4*x)*log(3*x + 2) + 9*(9*x^3 + 12*x^2
+ 4*x)*log(x) + 54*x + 8)/(9*x^3 + 12*x^2 + 4*x)
```

**giac [A]** time = 0.89, size = 37, normalized size = 0.80

$$-\frac{27\,x^2+27\,x+4}{64\,(3\,x+2)^2x}+\frac{9}{128}\,\log\left(|3\,x+2|\right)-\frac{9}{128}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="giac")
[Out] -1/64*(27*x^2 + 27*x + 4)/((3*x + 2)^2*x) + 9/128*log(abs(3*x + 2)) - 9/128
*log(abs(x))
```

**maple [A]** time = 0.01, size = 37, normalized size = 0.80

$$-\frac{9\ln(x)}{128}+\frac{9\ln\left(3x+2\right)}{128}-\frac{1}{64x}-\frac{3}{64\left(3x+2\right)^2}-\frac{3}{32\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x)^3,x)
```

```
[Out] -1/64/x-3/64/(3*x+2)^2-3/32/(3*x+2)-9/128*ln(x)+9/128*ln(3*x+2)
```

**maxima [A]** time = 1.39, size = 41, normalized size = 0.89

$$-\frac{27\,x^2+27\,x+4}{64\,\left(9\,x^3+12\,x^2+4\,x\right)}+\frac{9}{128}\,\log\left(3\,x+2\right)-\frac{9}{128}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(27*x^2 + 27*x + 4)/(9*x^3 + 12*x^2 + 4*x) + 9/128*log(3*x + 2) - 9/1
28*log(x)
```

**mupad [B]** time = 0.09, size = 35, normalized size = 0.76

$$\frac{9\,\mathrm{atanh}\left(3\,x+1\right)}{64}-\frac{\frac{3\,x^2}{64}+\frac{3\,x}{64}+\frac{1}{144}}{x^3+\frac{4\,x^2}{3}+\frac{4\,x}{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)^3),x)
```

```
[Out] (9*atanh(3*x + 1))/64 - ((3*x)/64 + (3*x^2)/64 + 1/144)/((4*x)/9 + (4*x^2)/
3 + x^3)
```

**sympy [A]** time = 0.18, size = 41, normalized size = 0.89

$$\frac{-27x^2-27x-4}{576x^3+768x^2+256x}-\frac{9\log\left(x\right)}{128}+\frac{9\log\left(x+\frac{2}{3}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x)**3,x)
```

```
[Out] (-27*x**2 - 27*x - 4)/(576*x**3 + 768*x**2 + 256*x) - 9*log(x)/128 + 9*log(
x + 2/3)/128
```

## 3.268 $\int \frac{1}{x^3(4+6x)^3} dx$

**Optimal**. Leaf size=53

$$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{27}{128(3x+2)}+\frac{9}{128(3x+2)^2}+\frac{27\log(x)}{128}-\frac{27}{128}\log(3x+2)$$

**Rubi [A]** time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{27}{128(3x+2)}+\frac{9}{128(3x+2)^2}+\frac{27\log(x)}{128}-\frac{27}{128}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] -1/(128*x^2) + 9/(128*x) + 9/(128*(2 + 3*x)^2) + 27/(128*(2 + 3*x)) + (27*L
og[x])/128 - (27*Log[2 + 3*x])/128
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^3}-\frac{9}{128x^2}+\frac{27}{128x}-\frac{27}{64(2+3x)^3}-\frac{81}{128(2+3x)^2}-\frac{81}{128(2+3x)}\right)dx\\ &= -\frac{1}{128x^2}+\frac{9}{128x}+\frac{9}{128(2+3x)^2}+\frac{27}{128(2+3x)}+\frac{27\log(x)}{128}-\frac{27}{128}\log(2+3x)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 44, normalized size = 0.83

$$\frac{1}{128}\left(\frac{2\left(81x^3+81x^2+12x-2\right)}{x^2(3x+2)^2}+27\log(x)-27\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] ((2*(-2 + 12*x + 81*x^2 + 81*x^3))/(x^2*(2 + 3*x)^2) + 27*Log[x] - 27*Log[2
 + 3*x])/128
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(4+6x)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(4 + 6*x)^3), x]
```

**fricas [A]** time = 0.95, size = 79, normalized size = 1.49

$$\frac{162\,x^3+162\,x^2-27\left(9\,x^4+12\,x^3+4\,x^2\right)\log\left(3\,x+2\right)+27\left(9\,x^4+12\,x^3+4\,x^2\right)\log(x)+24\,x-4}{128\left(9\,x^4+12\,x^3+4\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(162*x^3 + 162*x^2 - 27*(9*x^4 + 12*x^3 + 4*x^2)*log(3*x + 2) + 27*(9
*x^4 + 12*x^3 + 4*x^2)*log(x) + 24*x - 4)/(9*x^4 + 12*x^3 + 4*x^2)
```

**giac [A]** time = 1.07, size = 43, normalized size = 0.81

$$\frac{81\,x^3+81\,x^2+12\,x-2}{64\left(3\,x^2+2\,x\right)^2}-\frac{27}{128}\,\log\left(|3\,x+2|\right)+\frac{27}{128}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] 1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(3*x^2 + 2*x)^2 - 27/128*log(abs(3*x + 2)
) + 27/128*log(abs(x))
```

**maple [A]** time = 0.01, size = 42, normalized size = 0.79

$$\frac{27\ln(x)}{128}-\frac{27\ln\left(3x+2\right)}{128}+\frac{9}{128x}-\frac{1}{128x^2}+\frac{9}{128\left(3x+2\right)^2}+\frac{27}{128\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x)^3,x)
```

```
[Out] -1/128/x^2+9/128/x+9/128/(3*x+2)^2+27/128/(3*x+2)+27/128*ln(x)-27/128*ln(3*
x+2)
```

**maxima [A]** time = 1.38, size = 48, normalized size = 0.91

$$\frac{81\,x^3+81\,x^2+12\,x-2}{64\,\left(9\,x^4+12\,x^3+4\,x^2\right)}-\frac{27}{128}\,\log\left(3\,x+2\right)+\frac{27}{128}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(9*x^4 + 12*x^3 + 4*x^2) - 27/128*log(3*x
 + 2) + 27/128*log(x)
```

**mupad [B]** time = 0.09, size = 41, normalized size = 0.77

$$\frac{\frac{9\,x^3}{64}+\frac{9\,x^2}{64}+\frac{x}{48}-\frac{1}{288}}{x^4+\frac{4\,x^3}{3}+\frac{4\,x^2}{9}}-\frac{27\,\mathrm{atanh}\left(3\,x+1\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)^3),x)
```

```
[Out] (x/48 + (9*x^2)/64 + (9*x^3)/64 - 1/288)/((4*x^2)/9 + (4*x^3)/3 + x^4) - (2
7*atanh(3*x + 1))/64
```

**sympy [A]** time = 0.18, size = 46, normalized size = 0.87

$$\frac{27\log\left(x\right)}{128}-\frac{27\log\left(x+\frac{2}{3}\right)}{128}+\frac{81x^3+81x^2+12x-2}{576x^4+768x^3+256x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x)**3,x)
```

```
[Out] 27*log(x)/128 - 27*log(x + 2/3)/128 + (81*x**3 + 81*x**2 + 12*x - 2)/(576*x
**4 + 768*x**3 + 256*x**2)
```

## 3.269 $\int \frac{1}{x^4(4+6x)^3} dx$

**Optimal**. Leaf size=60

$$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{64(3x+2)}-\frac{27}{256(3x+2)^2}-\frac{135\log(x)}{256}+\frac{135}{256}\log(3x+2)$$

**Rubi [A]** time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{9}{256x^2}-\frac{1}{192x^3}-\frac{27}{128x}-\frac{27}{64(3x+2)}-\frac{27}{256(3x+2)^2}-\frac{135\log(x)}{256}+\frac{135}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] -1/(192*x^3) + 9/(256*x^2) - 27/(128*x) - 27/(256*(2 + 3*x)^2) - 27/(64*(2
+ 3*x)) - (135*Log[x])/256 + (135*Log[2 + 3*x])/256
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^4}-\frac{9}{128x^3}+\frac{27}{128x^2}-\frac{135}{256x}+\frac{81}{128(2+3x)^3}+\frac{81}{64(2+3x)^2}+\frac{405}{256(2+3x)}\right)dx\\ &= -\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{256(2+3x)^2}-\frac{27}{64(2+3x)}-\frac{135\log(x)}{256}+\frac{135}{256}\log(2+3x)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 0.82

$$\frac{1}{768}\left(-\frac{2\left(1215x^4+1215x^3+180x^2-30x+8\right)}{x^3(3x+2)^2}-405\log(x)+405\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] ((-2*(8 - 30*x + 180*x^2 + 1215*x^3 + 1215*x^4))/(x^3*(2 + 3*x)^2) - 405*Lo
g[x] + 405*Log[2 + 3*x])/768
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(4+6x)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(4 + 6*x)^3), x]
```

**fricas [A]** time = 0.75, size = 84, normalized size = 1.40

$$-\frac{2430\,x^4+2430\,x^3+360\,x^2-405\left(9\,x^5+12\,x^4+4\,x^3\right)\log\left(3\,x+2\right)+405\left(9\,x^5+12\,x^4+4\,x^3\right)\log\left(x\right)-60\,x+16}{768\left(9\,x^5+12\,x^4+4\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/768*(2430*x^4 + 2430*x^3 + 360*x^2 - 405*(9*x^5 + 12*x^4 + 4*x^3)*log(3*
x + 2) + 405*(9*x^5 + 12*x^4 + 4*x^3)*log(x) - 60*x + 16)/(9*x^5 + 12*x^4 +
 4*x^3)
```

**giac [A]** time = 1.29, size = 47, normalized size = 0.78

$$-\frac{1215\,x^4+1215\,x^3+180\,x^2-30\,x+8}{384\left(3\,x+2\right)^2x^3}+\frac{135}{256}\,\log\left(|3\,x+2|\right)-\frac{135}{256}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] -1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/((3*x + 2)^2*x^3) + 135/2
56*log(abs(3*x + 2)) - 135/256*log(abs(x))
```

**maple [A]** time = 0.01, size = 47, normalized size = 0.78

$$-\frac{135\ln(x)}{256}+\frac{135\ln\left(3x+2\right)}{256}-\frac{27}{128x}+\frac{9}{256x^2}-\frac{1}{192x^3}-\frac{27}{256\left(3x+2\right)^2}-\frac{27}{64\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x)^3,x)
```

```
[Out] -1/192/x^3+9/256/x^2-27/128/x-27/256/(3*x+2)^2-27/64/(3*x+2)-135/256*ln(x)+
135/256*ln(3*x+2)
```

**maxima** [A] time = 1.38, size = 53, normalized size = 0.88

$$-\frac{1215\,x^4+1215\,x^3+180\,x^2-30\,x+8}{384\left(9\,x^5+12\,x^4+4\,x^3\right)}+\frac{135}{256}\,\log\left(3\,x+2\right)-\frac{135}{256}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] -1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/(9*x^5 + 12*x^4 + 4*x^3)
+ 135/256*log(3*x + 2) - 135/256*log(x)
```

**mupad** [B] time = 0.05, size = 47, normalized size = 0.78

$$\frac{135\,\mathrm{atanh}\left(3\,x+1\right)}{128}-\frac{\frac{45\,x^4}{128}+\frac{45\,x^3}{128}+\frac{5\,x^2}{96}-\frac{5\,x}{576}+\frac{1}{432}}{x^5+\frac{4\,x^4}{3}+\frac{4\,x^3}{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)^3),x)
```

```
[Out] (135*atanh(3*x + 1))/128 - ((5*x^2)/96 - (5*x)/576 + (45*x^3)/128 + (45*x^4
)/128 + 1/432)/((4*x^3)/9 + (4*x^4)/3 + x^5)
```

**sympy** [A] time = 0.21, size = 51, normalized size = 0.85

$$-\frac{135\log\left(x\right)}{256}+\frac{135\log\left(x+\frac{2}{3}\right)}{256}+\frac{-1215x^4-1215x^3-180x^2+30x-8}{3456x^5+4608x^4+1536x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x)**3,x)
```

```
[Out] -135*log(x)/256 + 135*log(x + 2/3)/256 + (-1215*x**4 - 1215*x**3 - 180*x**2
 + 30*x - 8)/(3456*x**5 + 4608*x**4 + 1536*x**3)
```

## 3.270 $\int \frac{1}{x^5(4+6x)^3} \, dx$

**Optimal**. Leaf size=67

$$-\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{405}{512(3x+2)}+\frac{81}{512(3x+2)^2}+\frac{1215\log(x)}{1024}-\frac{1215\log(3x+2)}{1024}$$

**Rubi** [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{27}{256x^2}+\frac{3}{128x^3}-\frac{1}{256x^4}+\frac{135}{256x}+\frac{405}{512(3x+2)}+\frac{81}{512(3x+2)^2}+\frac{1215\log(x)}{1024}-\frac{1215\log(3x+2)}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] -1/(256*x^4) + 3/(128*x^3) - 27/(256*x^2) + 135/(256*x) + 81/(512*(2 + 3*x)
^2) + 405/(512*(2 + 3*x)) + (1215*Log[x])/1024 - (1215*Log[2 + 3*x])/1024
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)^3} \, dx &= \int \left(\frac{1}{64x^5}-\frac{9}{128x^4}+\frac{27}{128x^3}-\frac{135}{256x^2}+\frac{1215}{1024x}-\frac{243}{256(2+3x)^3}-\frac{1215}{512(2+3x)^2}-\frac{3645}{1024(2+3x)}\right.\\ &= -\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{81}{512(2+3x)^2}+\frac{405}{512(2+3x)}+\frac{1215\log(x)}{1024}-\frac{1215\log(2+3x)}{1024}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 54, normalized size = 0.81

$$\frac{\frac{2(3645x^5+3645x^4+540x^3-90x^2+24x-8)}{x^4(3x+2)^2}+1215\log(x)-1215\log(3x+2)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] ((2*(-8 + 24*x - 90*x^2 + 540*x^3 + 3645*x^4 + 3645*x^5))/(x^4*(2 + 3*x)^2)
 + 1215*Log[x] - 1215*Log[2 + 3*x])/1024
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(4+6x)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(4 + 6*x)^3), x]
```

**fricas [A]** time = 0.99, size = 89, normalized size = 1.33

$$\frac{7290\,x^5+7290\,x^4+1080\,x^3-180\,x^2-1215\left(9\,x^6+12\,x^5+4\,x^4\right)\log\left(3\,x+2\right)+1215\left(9\,x^6+12\,x^5+4\,x^4\right)\log(x)+48\,x-16}{1024\left(9\,x^6+12\,x^5+4\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(7290*x^5 + 7290*x^4 + 1080*x^3 - 180*x^2 - 1215*(9*x^6 + 12*x^5 + 4
*x^4)*log(3*x + 2) + 1215*(9*x^6 + 12*x^5 + 4*x^4)*log(x) + 48*x - 16)/(9*x
^6 + 12*x^5 + 4*x^4)
```

**giac [A]** time = 1.13, size = 52, normalized size = 0.78

$$\frac{3645\,x^5+3645\,x^4+540\,x^3-90\,x^2+24\,x-8}{512\left(3\,x+2\right)^2x^4}-\frac{1215}{1024}\,\log\left(|3\,x+2|\right)+\frac{1215}{1024}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] 1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/((3*x + 2)^2*x^4)
 - 1215/1024*log(abs(3*x + 2)) + 1215/1024*log(abs(x))
```

**maple [A]** time = 0.01, size = 52, normalized size = 0.78

$$\frac{1215\ln(x)}{1024}-\frac{1215\ln\left(3x+2\right)}{1024}+\frac{135}{256x}-\frac{27}{256x^2}+\frac{3}{128x^3}-\frac{1}{256x^4}+\frac{81}{512\left(3x+2\right)^2}+\frac{405}{512\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x)^3,x)
```

```
[Out] -1/256/x^4+3/128/x^3-27/256/x^2+135/256/x+81/512/(3*x+2)^2+405/512/(3*x+2)+
1215/1024*ln(x)-1215/1024*ln(3*x+2)
```

**maxima [A]** time = 1.39, size = 58, normalized size = 0.87

$$\frac{3645\,x^5+3645\,x^4+540\,x^3-90\,x^2+24\,x-8}{512\,\left(9\,x^6+12\,x^5+4\,x^4\right)}-\frac{1215}{1024}\,\log\left(3\,x+2\right)+\frac{1215}{1024}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/(9*x^6 + 12*x^5 +
 4*x^4) - 1215/1024*log(3*x + 2) + 1215/1024*log(x)
```

**mupad [B]** time = 0.05, size = 51, normalized size = 0.76

$$\frac{\frac{405\,x^5}{512}+\frac{405\,x^4}{512}+\frac{15\,x^3}{128}-\frac{5\,x^2}{256}+\frac{x}{192}-\frac{1}{576}}{x^6+\frac{4\,x^5}{3}+\frac{4\,x^4}{9}}-\frac{1215\,\mathrm{atanh}\left(3\,x+1\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)^3),x)
```

```
[Out] (x/192 - (5*x^2)/256 + (15*x^3)/128 + (405*x^4)/512 + (405*x^5)/512 - 1/576
)/((4*x^4)/9 + (4*x^5)/3 + x^6) - (1215*atanh(3*x + 1))/512
```

**sympy [A]** time = 0.22, size = 56, normalized size = 0.84

$$\frac{1215\log\left(x\right)}{1024}-\frac{1215\log\left(x+\frac{2}{3}\right)}{1024}+\frac{3645x^5+3645x^4+540x^3-90x^2+24x-8}{4608x^6+6144x^5+2048x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x)**3,x)
```

```
[Out] 1215*log(x)/1024 - 1215*log(x + 2/3)/1024 + (3645*x**5 + 3645*x**4 + 540*x*
*3 - 90*x**2 + 24*x - 8)/(4608*x**6 + 6144*x**5 + 2048*x**4)
```

## 3.271 $\int \frac{1}{2+2x} \, dx$

**Optimal**. Leaf size=8

$$\frac{1}{2}\log(x+1)$$

**Rubi [A]** time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x)^(-1),x]
```

```
[Out] Log[1 + x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+2x} \, dx = \frac{1}{2}\log(1+x)$$

**Mathematica [A]** time = 0.00, size = 10, normalized size = 1.25

$$\frac{1}{2}\log(2x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x)^(-1),x]
```

```
[Out] Log[2 + 2*x]/2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+2x} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 2*x)^(-1),x]
[Out] IntegrateAlgebraic[(2 + 2*x)^(-1), x]
```

**fricas [A]** time = 1.12, size = 6, normalized size = 0.75

$$\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="fricas")
[Out] 1/2*log(x + 1)
```

**giac [A]** time = 0.90, size = 7, normalized size = 0.88

$$\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="giac")
[Out] 1/2*log(abs(x + 1))
```

**maple [A]** time = 0.00, size = 9, normalized size = 1.12

$$\frac{\ln\left(2x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+2*x),x)
[Out] 1/2*ln(2+2*x)
```

**maxima [A]** time = 1.34, size = 6, normalized size = 0.75

$$\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="maxima")
[Out] 1/2*log(x + 1)
```

**mupad** [B] time = 0.15, size = 6, normalized size = 0.75

$$\frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + 2),x)
```

```
[Out] log(x + 1)/2
```

**sympy** [A] time = 0.07, size = 7, normalized size = 0.88

$$\frac{\log(2x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x)
```

```
[Out] log(2*x + 2)/2
```

## 3.272 $\int \frac{1}{4-6x}\,dx$

**Optimal**. Leaf size=10

$$-\frac{1}{6}\log(2-3x)$$

**Rubi [A]** time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$-\frac{1}{6}\log(2-3x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 6*x)^(-1),x]
```

```
[Out] -Log[2 - 3*x]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{4-6x}\,dx = -\frac{1}{6}\log(2-3x)$$

**Mathematica [A]** time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{6}\log(4-6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 6*x)^(-1),x]
```

```
[Out] -1/6*Log[4 - 6*x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4-6x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 - 6*x)^(-1),x]
[Out] IntegrateAlgebraic[(4 - 6*x)^(-1), x]
```

**fricas [A]** time = 0.98, size = 8, normalized size = 0.80

$$-\frac{1}{6} \log(3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="fricas")
[Out] -1/6*log(3*x - 2)
```

**giac [A]** time = 1.15, size = 9, normalized size = 0.90

$$-\frac{1}{6} \log(|3x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="giac")
[Out] -1/6*log(abs(3*x - 2))
```

**maple [A]** time = 0.00, size = 9, normalized size = 0.90

$$-\frac{\ln(-6x+4)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-6*x),x)
[Out] -1/6*ln(4-6*x)
```

**maxima [A]** time = 1.34, size = 8, normalized size = 0.80

$$-\frac{1}{6} \log(3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="maxima")
[Out] -1/6*log(3*x - 2)
```

**mupad [B]** time = 0.08, size = 6, normalized size = 0.60

$$-\frac{\ln\left(x-\frac{2}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(6*x - 4),x)
[Out] -log(x - 2/3)/6
```

**sympy [A]** time = 0.07, size = 8, normalized size = 0.80

$$-\frac{\log(6x-4)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x)
[Out] -log(6*x - 4)/6
```

## 3.273 $\int \frac{1}{a+\sqrt{a}\,x} \, dx$

**Optimal**. Leaf size=14

$$\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {31}

$$\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Sqrt[a]*x)^(-1),x]
[Out] Log[Sqrt[a] + x]/Sqrt[a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+\sqrt{a}\,x} \, dx = \frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 1.14

$$\frac{\log\left(\sqrt{a}\,x+a\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Sqrt[a]*x)^(-1),x]
[Out] Log[a + Sqrt[a]*x]/Sqrt[a]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + \sqrt{a}\,x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + Sqrt[a]*x)^(-1),x]
[Out] IntegrateAlgebraic[(a + Sqrt[a]*x)^(-1), x]
```

**fricas [A]** time = 0.79, size = 10, normalized size = 0.71

$$\frac{\log\left(x + \sqrt{a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="fricas")
[Out] log(x + sqrt(a))/sqrt(a)
```

**giac [A]** time = 1.28, size = 13, normalized size = 0.93

$$\frac{\log\left(\left|\sqrt{a}\,x + a\right|\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="giac")
[Out] log(abs(sqrt(a)*x + a))/sqrt(a)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{\ln\left(\sqrt{a}\;x + a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x*a^(1/2)),x)
[Out] ln(a+x*a^(1/2))/a^(1/2)
```

**maxima [A]** time = 1.25, size = 12, normalized size = 0.86

$$\frac{\log\left(\sqrt{a}\,x + a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="maxima")
[Out] log(sqrt(a)*x + a)/sqrt(a)
```

**mupad [B]** time = 0.11, size = 10, normalized size = 0.71

$$\frac{\ln\left(x+\sqrt{a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a^(1/2)*x),x)
[Out] log(x + a^(1/2))/a^(1/2)
```

**sympy [A]** time = 0.08, size = 14, normalized size = 1.00

$$\frac{\log\left(\sqrt{a}\,x+a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a**(1/2)),x)
[Out] log(sqrt(a)*x + a)/sqrt(a)
```

## 3.274 $\int \frac{1}{a+\sqrt{-a}\,x}\,dx$

**Optimal**. Leaf size=20

$$\frac{\log\left(\sqrt{-a}\,x+a\right)}{\sqrt{-a}}$$

**Rubi** [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {31}

$$\frac{\log\left(\sqrt{-a}\,x+a\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+\sqrt{-a}\,x}\,dx=\frac{\log\left(a+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

**Mathematica** [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{\log\left(\sqrt{-a}\,x+a\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[-a]*x]/Sqrt[-a]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+\sqrt{-a}\,x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + Sqrt[-a]*x)^(-1),x]
[Out] IntegrateAlgebraic[(a + Sqrt[-a]*x)^(-1), x]
```

**fricas [A]** time = 1.16, size = 20, normalized size = 1.00

$$-\frac{\sqrt{-a}\,\log\left(x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="fricas")
[Out] -sqrt(-a)*log(x - sqrt(-a))/a
```

**giac [A]** time = 1.10, size = 17, normalized size = 0.85

$$\frac{\log\left(\left|\sqrt{-a}\,x+a\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="giac")
[Out] log(abs(sqrt(-a)*x + a))/sqrt(-a)
```

**maple [A]** time = 0.00, size = 17, normalized size = 0.85

$$\frac{\ln\left(a+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x*(-a)^(1/2)),x)
[Out] ln(a+x*(-a)^(1/2))/(-a)^(1/2)
```

**maxima [A]** time = 1.37, size = 16, normalized size = 0.80

$$\frac{\log\left(\sqrt{-a}\,x+a\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(-a)*x + a)/sqrt(-a)
```

**mupad [B]** time = 0.11, size = 16, normalized size = 0.80

$$\frac{\ln\left(x-\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (-a)^(1/2)*x),x)
```

```
[Out] log(x - (-a)^(1/2))/(-a)^(1/2)
```

**sympy [A]** time = 0.08, size = 17, normalized size = 0.85

$$\frac{\log\left(a+x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)**(1/2)),x)
```

```
[Out] log(a + x*sqrt(-a))/sqrt(-a)
```

## 3.275 $\int \frac{1}{a^2+\sqrt{-a}\,x}\,dx$

**Optimal**. Leaf size=22

$$\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + Sqrt[-a]*x)^(-1),x]
[Out] Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a^2+\sqrt{-a}\,x}\,dx=\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

**Mathematica [A]** time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + Sqrt[-a]*x)^(-1),x]
[Out] Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^2 + \sqrt{-a}\,x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + Sqrt[-a]*x)^(-1),x]
[Out] IntegrateAlgebraic[(a^2 + Sqrt[-a]*x)^(-1), x]
```

**fricas [A]** time = 0.89, size = 21, normalized size = 0.95

$$-\frac{\sqrt{-a}\,\log\left(-\sqrt{-a}\,a + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="fricas")
[Out] -sqrt(-a)*log(-sqrt(-a)*a + x)/a
```

**giac [A]** time = 0.93, size = 19, normalized size = 0.86

$$\frac{\log\left(\left|a^2 + \sqrt{-a}\,x\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="giac")
[Out] log(abs(a^2 + sqrt(-a)*x))/sqrt(-a)
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.86

$$\frac{\ln\left(a^2 + \sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+(-a)^(1/2)*x),x)
[Out] ln(a^2+(-a)^(1/2)*x)/(-a)^(1/2)
```

**maxima [A]** time = 1.32, size = 18, normalized size = 0.82

$$\frac{\log\left(a^2 + \sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(a^2 + sqrt(-a)*x)/sqrt(-a)
```

**mupad [B]** time = 0.05, size = 14, normalized size = 0.64

$$\frac{\ln\left(x+(-a)^{3/2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + (-a)^(1/2)*x),x)
```

```
[Out] log(x + (-a)^(3/2))/(-a)^(1/2)
```

**sympy [A]** time = 0.08, size = 19, normalized size = 0.86

$$\frac{\log\left(a^{2}+x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x*(-a)**(1/2)),x)
```

```
[Out] log(a**2 + x*sqrt(-a))/sqrt(-a)
```

## 3.276 $\int \frac{1}{a^3+\sqrt{-a}\,x}\,dx$

**Optimal**. Leaf size=22

$$\frac{\log\left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a^3+\sqrt{-a}\,x}\,dx=\frac{\log\left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

**Mathematica [A]** time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log\left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^3 + \sqrt{-a}\,x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^3 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a^3 + Sqrt[-a]*x)^(-1), x]
```

**fricas [A]** time = 0.84, size = 23, normalized size = 1.05

$$-\frac{\sqrt{-a}\,\log\left(-\sqrt{-a}\,a^2 + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(-sqrt(-a)*a^2 + x)/a
```

**giac [A]** time = 1.09, size = 19, normalized size = 0.86

$$\frac{\log\left(\left|a^3 + \sqrt{-a}\,x\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(a^3 + sqrt(-a)*x))/sqrt(-a)
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.86

$$\frac{\ln\left(a^3 + \sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3+(-a)^(1/2)*x),x)
```

```
[Out] ln(a^3+(-a)^(1/2)*x)/(-a)^(1/2)
```

**maxima [A]** time = 1.31, size = 18, normalized size = 0.82

$$\frac{\log\left(a^3 + \sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(a^3 + sqrt(-a)*x)/sqrt(-a)
```

**mupad** [B] time = 0.06, size = 16, normalized size = 0.73

$$\frac{\ln\left(x-(-a)^{5/2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + (-a)^(1/2)*x),x)
```

```
[Out] log(x - (-a)^(5/2))/(-a)^(1/2)
```

**sympy** [A] time = 0.08, size = 19, normalized size = 0.86

$$\frac{\log\left(a^3+x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**3+x*(-a)**(1/2)),x)
```

```
[Out] log(a**3 + x*sqrt(-a))/sqrt(-a)
```

## 3.277 $\int \frac{1}{\frac{1}{a}+\sqrt{-a}\,x}\,dx$

**Optimal**. Leaf size=21

$$\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

**Rubi [A]** time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 - (-a)^(3/2)*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\frac{1}{a}+\sqrt{-a}\,x}\,dx=\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

**Mathematica [A]** time = 0.01, size = 21, normalized size = 1.00

$$\frac{\log\left(\sqrt{-a}\,ax+1\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 + Sqrt[-a]*a*x]/Sqrt[-a]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{1}{a}+\sqrt{-a}\,x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a^(-1) + Sqrt[-a]*x)^(-1), x]
```

**fricas [A]** time = 0.87, size = 24, normalized size = 1.14

$$-\frac{\sqrt{-a}\,\log\left(a^{2}x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(a^2*x - sqrt(-a))/a
```

**giac [A]** time = 1.04, size = 19, normalized size = 0.90

$$\frac{\log\left(\left|\sqrt{-a}\,x+\frac{1}{a}\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(-a)*x + 1/a))/sqrt(-a)
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.90

$$\frac{\ln\left(\sqrt{-a}\,x+\frac{1}{a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a+(-a)^(1/2)*x),x)
```

```
[Out] ln(1/a+(-a)^(1/2)*x)/(-a)^(1/2)
```

**maxima [A]** time = 1.33, size = 18, normalized size = 0.86

$$\frac{\log\left(\sqrt{-a}\,x+\frac{1}{a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(-a)*x + 1/a)/sqrt(-a)
```

**mupad [B]** time = 0.15, size = 16, normalized size = 0.76

$$\frac{\ln\left(x-\frac{1}{(-a)^{3/2}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a + (-a)^(1/2)*x),x)
```

```
[Out] log(x - 1/(-a)^(3/2))/(-a)^(1/2)
```

**sympy [A]** time = 0.09, size = 19, normalized size = 0.90

$$\frac{\log\left(ax\sqrt{-a}+1\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)**(1/2)),x)
```

```
[Out] log(a*x*sqrt(-a) + 1)/sqrt(-a)
```

## 3.278 $\int \frac{1}{\frac{1}{a^2}+\sqrt{-a}\,x}\,dx$

**Optimal**. Leaf size=20

$$\frac{\log\left((-a)^{5/2}x+1\right)}{\sqrt{-a}}$$

**Rubi** [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left((-a)^{5/2}x+1\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-2) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 + (-a)^(5/2)*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\frac{1}{a^2}+\sqrt{-a}\,x}\,dx=\frac{\log\left(1+(-a)^{5/2}x\right)}{\sqrt{-a}}$$

**Mathematica** [A] time = 0.01, size = 22, normalized size = 1.10

$$\frac{\log\left(\frac{1}{a^2}+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-2) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^(-2) + Sqrt[-a]*x]/Sqrt[-a]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{1}{a^2} + \sqrt{-a}\, x}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(-2) + Sqrt[-a]*x)^(-1),x]
[Out] IntegrateAlgebraic[(a^(-2) + Sqrt[-a]*x)^(-1), x]
```

**fricas [A]** time = 1.01, size = 24, normalized size = 1.20

$$-\frac{\sqrt{-a}\,\log\left(a^3x - \sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="fricas")
[Out] -sqrt(-a)*log(a^3*x - sqrt(-a))/a
```

**giac [A]** time = 1.08, size = 19, normalized size = 0.95

$$\frac{\log\left(\left|\sqrt{-a}\, x + \frac{1}{a^2}\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="giac")
[Out] log(abs(sqrt(-a)*x + 1/a^2))/sqrt(-a)
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.95

$$\frac{\ln\left(\sqrt{-a}\, x + \frac{1}{a^2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a^2+(-a)^(1/2)*x),x)
[Out] ln(1/a^2+(-a)^(1/2)*x)/(-a)^(1/2)
```

**maxima** [A]    time = 1.31, size = 18, normalized size = 0.90

$$\frac{\log\left(\sqrt{-a}\,x+\frac{1}{a^{2}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(-a)*x + 1/a^2)/sqrt(-a)
```

**mupad** [B]    time = 0.18, size = 14, normalized size = 0.70

$$\frac{\ln\left(x+\frac{1}{(-a)^{5/2}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a^2 + (-a)^(1/2)*x),x)
```

```
[Out] log(x + 1/(-a)^(5/2))/(-a)^(1/2)
```

**sympy** [A]    time = 0.09, size = 20, normalized size = 1.00

$$\frac{\log\left(a^{2}x\sqrt{-a}+1\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a**2+x*(-a)**(1/2)),x)
```

```
[Out] log(a**2*x*sqrt(-a) + 1)/sqrt(-a)
```

## 3.279 $\int \frac{1}{x(1+bx)} dx$

**Optimal**. Leaf size=11

$$\log(x) - \log(bx + 1)$$

**Rubi [A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\log(x) - \log(bx + 1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x)),x]
```

```
[Out] Log[x] - Log[1 + b*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+bx)} dx &= -\left(b \int \frac{1}{1+bx} dx\right) + \int \frac{1}{x} dx \\ &= \log(x) - \log(1+bx)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 11, normalized size = 1.00

$$\log(x) - \log(bx + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x)),x]
[Out] Log[x] - Log[1 + b*x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1+bx)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + b*x)),x]
[Out] IntegrateAlgebraic[1/(x*(1 + b*x)), x]
```

**fricas [A]** time = 0.77, size = 11, normalized size = 1.00

$$-\log(bx+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="fricas")
[Out] -log(b*x + 1) + log(x)
```

**giac [A]** time = 0.99, size = 13, normalized size = 1.18

$$-\log(|bx+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="giac")
[Out] -log(abs(b*x + 1)) + log(abs(x))
```

**maple [A]** time = 0.00, size = 12, normalized size = 1.09

$$\ln(x) - \ln(bx+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+1),x)
[Out] ln(x)-ln(b*x+1)
```

**maxima [A]** time = 1.31, size = 11, normalized size = 1.00

$$-\log(bx+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="maxima")
```

```
[Out] -log(b*x + 1) + log(x)
```

**mupad [B]** time = 0.10, size = 9, normalized size = 0.82

$$-2\,\mathrm{atanh}\left(2\,b\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x + 1)),x)
```

```
[Out] -2*atanh(2*b*x + 1)
```

**sympy [A]** time = 0.14, size = 8, normalized size = 0.73

$$\log\left(x\right)-\log\left(x+\frac{1}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x)
```

```
[Out] log(x) - log(x + 1/b)
```

## 3.280 $\int \frac{1}{x(-1+bx)}\,dx$

**Optimal**. Leaf size=12

$$\log(1-bx)-\log(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\log(1-bx)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + b*x)),x]
```

```
[Out] -Log[x] + Log[1 - b*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(-1+bx)}\,dx &= b\int \frac{1}{-1+bx}\,dx-\int \frac{1}{x}\,dx\\ &= -\log(x)+\log(1-bx)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 12, normalized size = 1.00

$$\log(1-bx)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x)),x]
```

```
[Out] -Log[x] + Log[1 - b*x]
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(-1+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(-1 + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(-1 + b*x)), x]
```

**fricas** [A] time = 0.62, size = 11, normalized size = 0.92

$$\log(bx-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="fricas")
```

```
[Out] log(b*x - 1) - log(x)
```

**giac** [A] time = 1.02, size = 13, normalized size = 1.08

$$\log(|bx-1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="giac")
```

```
[Out] log(abs(b*x - 1)) - log(abs(x))
```

**maple** [A] time = 0.01, size = 12, normalized size = 1.00

$$-\ln(x) + \ln(bx-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-1),x)
```

```
[Out] ln(b*x-1)-ln(x)
```

**maxima** [A] time = 1.36, size = 11, normalized size = 0.92

$$\log(bx-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="maxima")
```

```
[Out] log(b*x - 1) - log(x)
```

**mupad [B]** time = 0.04, size = 9, normalized size = 0.75

$$-2\,\mathrm{atanh}\left(2\,b\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - 1)),x)
```

```
[Out] -2*atanh(2*b*x - 1)
```

**sympy [A]** time = 0.13, size = 8, normalized size = 0.67

$$-\log{\left(x \right)} + \log{\left(x - \frac{1}{b} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x)
```

```
[Out] -log(x) + log(x - 1/b)
```

## 3.281 $\int \frac{1}{x^2(1+bx)}\,dx$

**Optimal**. Leaf size=19

$$-b\log(x)+b\log(bx+1)-\frac{1}{x}$$

**Rubi [A]** time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-b\log(x)+b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(1+bx)}\,dx &= \int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right)dx\\ &= -\frac{1}{x}-b\log(x)+b\log(1+bx)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 19, normalized size = 1.00

$$-b\log(x)+b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(1+bx)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(1 + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(1 + b*x)), x]
```

**fricas [A]** time = 0.93, size = 21, normalized size = 1.11

$$\frac{bx\log(bx+1) - bx\log(x) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + 1) - b*x*log(x) - 1)/x
```

**giac [A]** time = 0.92, size = 21, normalized size = 1.11

$$b\log(|bx+1|) - b\log(|x|) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + 1)) - b*log(abs(x)) - 1/x
```

**maple [A]** time = 0.01, size = 20, normalized size = 1.05

$$-b\ln(x) + b\ln(bx+1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+1),x)
```

```
[Out] -1/x-b*ln(x)+b*ln(b*x+1)
```

**maxima [A]** time = 1.32, size = 19, normalized size = 1.00

$$b\log(bx+1) - b\log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="maxima")
```

```
[Out] b*log(b*x + 1) - b*log(x) - 1/x
```

**mupad [B]** time = 0.04, size = 16, normalized size = 0.84

$$2\,b\,\mathrm{atanh}\left(2\,b\,x+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x + 1)),x)
```

```
[Out] 2*b*atanh(2*b*x + 1) - 1/x
```

**sympy [A]** time = 0.18, size = 14, normalized size = 0.74

$$b\left(-\log\left(x\right)+\log\left(x+\frac{1}{b}\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+1),x)
```

```
[Out] b*(-log(x) + log(x + 1/b)) - 1/x
```

## 3.282 $\int \frac{1}{x^2(-1+bx)}\,dx$

**Optimal**. Leaf size=18

$$-b\log(x)+b\log(1-bx)+\frac{1}{x}$$

**Rubi [A]** time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-b\log(x)+b\log(1-bx)+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-1 + b*x)),x]
```

```
[Out] x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(-1+bx)}\,dx &= \int \left(-\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{-1+bx}\right)dx\\ &= \frac{1}{x}-b\log(x)+b\log(1-bx)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 18, normalized size = 1.00

$$-b\log(x)+b\log(1-bx)+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-1 + b*x)),x]
```

```
[Out] x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(-1+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(-1 + b*x)),x]
[Out] IntegrateAlgebraic[1/(x^2*(-1 + b*x)), x]
```

**fricas [A]** time = 0.73, size = 21, normalized size = 1.17

$$\frac{bx\log(bx-1) - bx\log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="fricas")
[Out] (b*x*log(b*x - 1) - b*x*log(x) + 1)/x
```

**giac [A]** time = 1.14, size = 19, normalized size = 1.06

$$b\log(|bx-1|) - b\log(|x|) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="giac")
[Out] b*log(abs(b*x - 1)) - b*log(abs(x)) + 1/x
```

**maple [A]** time = 0.01, size = 18, normalized size = 1.00

$$-b\ln(x) + b\ln(bx-1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-1),x)
[Out] b*ln(b*x-1)+1/x-b*ln(x)
```

**maxima [A]** time = 1.33, size = 17, normalized size = 0.94

$$b\log(bx-1) - b\log(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="maxima")
```

```
[Out] b*log(b*x - 1) - b*log(x) + 1/x
```

**mupad [B]** time = 0.03, size = 14, normalized size = 0.78

$$\frac{1}{x}-2\,b\,\mathrm{atanh}\left(2\,b\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - 1)),x)
```

```
[Out] 1/x - 2*b*atanh(2*b*x - 1)
```

**sympy [A]** time = 0.19, size = 14, normalized size = 0.78

$$b\left(-\log\left(x\right)+\log\left(x-\frac{1}{b}\right)\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-1),x)
```

```
[Out] b*(-log(x) + log(x - 1/b)) + 1/x
```

## 3.283 $\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) \, dx$

**Optimal**. Leaf size=14

$$b\log(bx+1)-\frac{1}{x}$$

**Rubi [A]** time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {44}

$$b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[b/x + 1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) + b*Log[1 + b*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) \, dx &= b\log(x)+\int \frac{1}{x^2(1+bx)} \, dx \\ &= b\log(x)+\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right) \, dx \\ &= -\frac{1}{x}+b\log(1+bx) \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 14, normalized size = 1.00

$$b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[b/x + 1/(x^2*(1 + b*x)),x]
[Out] -x^(-1) + b*Log[1 + b*x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x} + \frac{1}{x^2(1+bx)}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[b/x + 1/(x^2*(1 + b*x)),x]
[Out] IntegrateAlgebraic[b/x + 1/(x^2*(1 + b*x)), x]
```

**fricas [A]** time = 0.88, size = 15, normalized size = 1.07

$$\frac{bx\log\left(bx+1\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="fricas")
[Out] (b*x*log(b*x + 1) - 1)/x
```

**giac [A]** time = 1.25, size = 15, normalized size = 1.07

$$b\log\left(|bx+1|\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="giac")
[Out] b*log(abs(b*x + 1)) - 1/x
```

**maple [A]** time = 0.00, size = 15, normalized size = 1.07

$$b\ln\left(bx+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b/x+1/x^2/(b*x+1),x)
[Out] -1/x+b*ln(b*x+1)
```

**maxima** [A] time = 1.32, size = 14, normalized size = 1.00

$$b\log(bx+1)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="maxima")
```

```
[Out] b*log(b*x + 1) - 1/x
```

**mupad** [B] time = 0.04, size = 20, normalized size = 1.43

$$b\,\ln(x)+2\,b\,\mathrm{atanh}\left(2\,b\,x+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x + 1)) + b/x,x)
```

```
[Out] b*log(x) + 2*b*atanh(2*b*x + 1) - 1/x
```

**sympy** [A] time = 0.15, size = 10, normalized size = 0.71

$$b\log(bx+1)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x**2/(b*x+1),x)
```

```
[Out] b*log(b*x + 1) - 1/x
```

## 3.284 $\int x^3\sqrt{a+bx}\,dx$

**Optimal**. Leaf size=72

$$-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{9/2}}{9b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}$$

**Rubi** [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{2(a+bx)^{9/2}}{9b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x],x]
```

```
[Out] (-2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a + b*x)^(5/2))/(5*b^4) - (6*a*(
a + b*x)^(7/2))/(7*b^4) + (2*(a + b*x)^(9/2))/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+bx}\,dx&=\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{9/2}}{9b^4}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{3/2}\left(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3))/(315
*b^4)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 59, normalized size = 0.82

$$-\frac{2\left(105a^3(a+bx)^{3/2}-189a^2(a+bx)^{5/2}-35(a+bx)^{9/2}+135a(a+bx)^{7/2}\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[a + b*x],x]
```

```
[Out] (-2*(105*a^3*(a + b*x)^(3/2) - 189*a^2*(a + b*x)^(5/2) + 135*a*(a + b*x)^(7
/2) - 35*(a + b*x)^(9/2)))/(315*b^4)
```

**fricas [A]** time = 1.36, size = 53, normalized size = 0.74

$$\frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

**giac [B]** time = 0.95, size = 116, normalized size = 1.61

$$\frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a}{b^3}+\frac{35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4}{b^3}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2
 - 35*sqrt(b*x + a)*a^3)*a/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*
a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a
^4)/b^3)/b
```

**maple [A]** time = 0.02, size = 43, normalized size = 0.60

$$-\frac{2\,(bx+a)^{\frac{3}{2}}\left(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3\right)}{315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(1/2),x)
```

```
[Out] -2/315*(b*x+a)^(3/2)*(-35*b^3*x^3+30*a*b^2*x^2-24*a^2*b*x+16*a^3)/b^4
```

**maxima [A]** time = 1.25, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^4}-\frac{6\,(bx+a)^{\frac{7}{2}}a}{7\,b^4}+\frac{6\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^4}-\frac{2\,(bx+a)^{\frac{3}{2}}a^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^4 - 6/7*(b*x + a)^(7/2)*a/b^4 + 6/5*(b*x + a)^(5/2)*a
^2/b^4 - 2/3*(b*x + a)^(3/2)*a^3/b^4
```

**mupad [B]** time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{9/2}}{9\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{3/2}}{3\,b^4}+\frac{6\,a^2\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}-\frac{6\,a\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(9/2))/(9*b^4) - (2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a +
 b*x)^(5/2))/(5*b^4) - (6*a*(a + b*x)^(7/2))/(7*b^4)
```

**sympy [B]** time = 2.91, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(1/2),x)
```

```
[Out] -32*a**(49/2)*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a*
*18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b*
*9*x**5 + 315*a**14*b**10*x**6) + 32*a**(49/2)/(315*a**20*b**4 + 1890*a**19
*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**
4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) - 176*a**(47/2)*b*x*sqrt(1
 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300
*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*
b**10*x**6) + 192*a**(47/2)*b*x/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*
a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*
b**9*x**5 + 315*a**14*b**10*x**6) - 396*a**(45/2)*b**2*x**2*sqrt(1 + b*x/a)
```

```
/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b*
*7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**
6) + 480*a**(45/2)*b**2*x**2/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**
18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**
9*x**5 + 315*a**14*b**10*x**6) - 462*a**(43/2)*b**3*x**3*sqrt(1 + b*x/a)/(3
15*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*
x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6)
+ 640*a**(43/2)*b**3*x**3/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*
b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x
**5 + 315*a**14*b**10*x**6) - 210*a**(41/2)*b**4*x**4*sqrt(1 + b*x/a)/(315*
a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**
3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 4
80*a**(41/2)*b**4*x**4/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**
6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5
 + 315*a**14*b**10*x**6) + 378*a**(39/2)*b**5*x**5*sqrt(1 + b*x/a)/(315*a**
20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 +
 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 192*
a**(39/2)*b**5*x**5/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x
**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 +
315*a**14*b**10*x**6) + 1134*a**(37/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*a**20
*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4
725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 32*a**
(37/2)*b**6*x**6/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2
 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315
*a**14*b**10*x**6) + 1494*a**(35/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**20*b*
*4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725
*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 1098*a**(
33/2)*b**8*x**8*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*
a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*
b**9*x**5 + 315*a**14*b**10*x**6) + 430*a**(31/2)*b**9*x**9*sqrt(1 + b*x/a)
/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b*
*7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**
6) + 70*a**(29/2)*b**10*x**10*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*
b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4
 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6)
```

## 3.285 $\int x^2\sqrt{a+bx}\,dx$

**Optimal**. Leaf size=53

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

**Rubi** [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a + b
*x)^(7/2))/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx}\,dx&=\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x],x]
[Out] (2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 45, normalized size = 0.85

$$\frac{2\left(35a^2(a+bx)^{3/2}+15(a+bx)^{7/2}-42a(a+bx)^{5/2}\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a + b*x],x]
[Out] (2*(35*a^2*(a + b*x)^(3/2) - 42*a*(a + b*x)^(5/2) + 15*(a + b*x)^(7/2)))/(1
05*b^3)
```

**fricas [A]** time = 1.05, size = 42, normalized size = 0.79

$$\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

**giac [B]** time = 1.03, size = 93, normalized size = 1.75

$$\frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)}{b^2}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
a/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^
2 - 35*sqrt(b*x + a)*a^3)/b^2)/b
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.60

$$\frac{2\,(bx+a)^{\frac{3}{2}}\left(15b^2x^2-12abx+8a^2\right)}{105b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/2),x)
```

```
[Out] 2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

**maxima [A]** time = 1.41, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^3}-\frac{4\,(bx+a)^{\frac{5}{2}}a}{5\,b^3}+\frac{2\,(bx+a)^{\frac{3}{2}}a^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^3 - 4/5*(b*x + a)^(5/2)*a/b^3 + 2/3*(b*x + a)^(3/2)*a
^2/b^3
```

**mupad [B]** time = 0.05, size = 37, normalized size = 0.70

$$\frac{30\,{\left(a+b\,x\right)}^{7/2}-84\,a\,{\left(a+b\,x\right)}^{5/2}+70\,a^2\,{\left(a+b\,x\right)}^{3/2}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(1/2),x)
```

```
[Out] (30*(a + b*x)^(7/2) - 84*a*(a + b*x)^(5/2) + 70*a^2*(a + b*x)^(3/2))/(105*b
^3)
```

**sympy [B]** time = 2.04, size = 666, normalized size = 12.57

$$\frac{16a^{\frac{23}{2}}\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{16a^{\frac{23}{2}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{40a^{\frac{21}{2}}bx\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{48a^{\frac{21}{2}}bx}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{30a^{\frac{19}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{48a^{\frac{19}{2}}b^2x^2}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{40a^{\frac{17}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{16a^{\frac{17}{2}}b^3x^3}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{100a^{\frac{15}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{96a^{\frac{13}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{30a^{\frac{11}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b*
*5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*b**3 + 315*a**7*b**4
*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(21/2)*b*x*sqrt(1 + b
*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6
*x**3) - 48*a**(21/2)*b*x/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*
x**2 + 105*a**5*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(105*a*
*8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 48*a
**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 +
105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(1 + b*x/a)/(105*a**8*b**3
```

```
 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(17/2
)*b**3*x**3/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**
5*b**6*x**3) + 100*a**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315
*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a**(13/2)*b**5
*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2
 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1 + b*x/a)/(105*a**8*b
**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
```

## 3.286 $\int x\sqrt{a+bx}\,dx$

**Optimal**. Leaf size=34

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x],x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx}\,dx&=\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx\\&=-\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{3/2}(3bx-2a)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(-2*a + 3*b*x))/(15*b^2)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 35, normalized size = 1.03

$$-\frac{2\sqrt{a+bx}\left(2a^2-abx-3b^2x^2\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a + b*x],x]
[Out] (-2*Sqrt[a + b*x]*(2*a^2 - a*b*x - 3*b^2*x^2))/(15*b^2)
```

**fricas** [A] time = 1.14, size = 30, normalized size = 0.88

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

**giac** [B] time = 1.28, size = 66, normalized size = 1.94

$$\frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a}{b}+\frac{3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2}{b}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + (3*(b*x + a)^(5/2) - 10
*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{2\,(bx+a)^{\frac{3}{2}}\,(-3bx+2a)}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/2),x)
[Out] -2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

**maxima [A]** time = 1.31, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^2}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2
```

**mupad [B]** time = 0.03, size = 25, normalized size = 0.74

$$-\frac{10\,a\,(a+b\,x)^{3/2}-6\,(a+b\,x)^{5/2}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(1/2),x)
```

```
[Out] -(10*a*(a + b*x)^(3/2) - 6*(a + b*x)^(5/2))/(15*b^2)
```

**sympy [B]** time = 1.39, size = 202, normalized size = 5.94

$$-\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x}-\frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x}+\frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(9/2)/(15*a
**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15
*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 15*a*b**3*x) + 8*a**(5/2)*b**2*
x**2*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 6*a**(3/2)*b**3*x**3*sq
rt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x)
```

## 3.287 $\int \sqrt{a+bx}\,dx$

**Optimal**. Leaf size=16

$$\frac{2(a+bx)^{3/2}}{3b}$$

**Rubi [A]** time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+bx}\,dx=\frac{2(a+bx)^{3/2}}{3b}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x],x]
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

**fricas [A]** time = 1.11, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*x + a)^(3/2)/b
```

**giac [A]** time = 1.06, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/3*(b*x + a)^(3/2)/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2),x)
[Out] 2/3*(b*x+a)^(3/2)/b
```

**maxima [A]** time = 1.34, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

**sympy [A]** time = 0.07, size = 12, normalized size = 0.75

$$\frac{2\left(a+bx\right)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2),x)
```

```
[Out] 2*(a + b*x)**(3/2)/(3*b)
```

## 3.288 $\int \frac{\sqrt{a+bx}}{x} \, dx$

**Optimal**. Leaf size=35

$$2\sqrt{a+bx} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

**Rubi** [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$2\sqrt{a+bx} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x}\,dx = 2\sqrt{a+bx} + a\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= 2\sqrt{a+bx} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= 2\sqrt{a+bx} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

**Mathematica [A]** time = 0.01, size = 35, normalized size = 1.00

$$2\sqrt{a+bx} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x,x]
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 35, normalized size = 1.00

$$2\sqrt{a+bx} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x,x]
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

**fricas [A]** time = 1.11, size = 73, normalized size = 2.09

$$\left[\sqrt{a}\log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a} + 2a}{x}\right) + 2\sqrt{bx+a},\, 2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + 2\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

**giac** [A] time = 1.13, size = 32, normalized size = 0.91

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\,\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="giac")
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

**maple** [A] time = 0.01, size = 28, normalized size = 0.80

$$-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x,x)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

**maxima** [A] time = 2.92, size = 42, normalized size = 1.20

$$\sqrt{a}\,\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\,\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")
[Out] sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*sqrt(b
*x + a)
```

**mupad** [B] time = 0.09, size = 27, normalized size = 0.77

$$2\,\sqrt{a+b\,x}-2\,\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x,x)
[Out] 2*(a + b*x)^(1/2) - 2*a^(1/2)*atanh((a + b*x)^(1/2)/a^(1/2))
```

**sympy [B]** time = 1.60, size = 68, normalized size = 1.94

$$-2\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{2a}{\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{2\sqrt{b}\,\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/(
b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

## 3.289 $\int \frac{\sqrt{a+bx}}{x^2}\,dx$

**Optimal**. Leaf size=39

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Rubi [A]** time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{a+bx}}{x} + \frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= -\frac{\sqrt{a+bx}}{x} + \text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)$$
$$= -\frac{\sqrt{a+bx}}{x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Mathematica [A]** time = 0.03, size = 47, normalized size = 1.21

$$-\frac{bx\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{x\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^2,x]
[Out] -((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/(x*Sqrt[a +
b*x]))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 39, normalized size = 1.00

$$-\frac{\sqrt{a+bx}}{x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^2,x]
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

**fricas [A]** time = 1.23, size = 93, normalized size = 2.38

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}\,a}{2ax}, \frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

**giac [A]** time = 0.96, size = 41, normalized size = 1.05

$$\frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx+a}\,b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)/b
```

**maple [A]** time = 0.01, size = 37, normalized size = 0.95

$$2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{2bx}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^2,x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

**maxima [A]** time = 2.87, size = 47, normalized size = 1.21

$$\frac{b \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) - sq
rt(b*x + a)/x
```

**mupad [B]** time = 0.05, size = 31, normalized size = 0.79

$$-\frac{\sqrt{a+b\,x}}{x} - \frac{b\,\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^2,x)
```

```
[Out] - (a + b*x)^(1/2)/x - (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

**sympy [A]** time = 2.22, size = 44, normalized size = 1.13

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}-\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqr
t(a)
```

## 3.290 $\int \frac{\sqrt{a+bx}}{x^3}\,dx$

**Optimal**. Leaf size=65

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}$$

**Rubi** [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^3,x]
```

```
[Out] -Sqrt[a + b*x]/(2*x^2) - (b*Sqrt[a + b*x])/(4*a*x) + (b^2*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x^3}\,dx &= -\frac{\sqrt{a+bx}}{2x^2}+\frac{1}{4}b\int \frac{1}{x^2\sqrt{a+bx}}\,dx \\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}-\frac{b^2\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a} \\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{4a} \\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.54

$$-\frac{2b^2(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^3,x]
[Out] (-2*b^2*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/(3*a^3
)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 55, normalized size = 0.85

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx}\,(2a+bx)}{4ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^3,x]
```

```
[Out] -1/4*(Sqrt[a + b*x]*(2*a + b*x))/(a*x^2) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]])/(4*a^(3/2))
```

**fricas [A]** time = 0.98, size = 119, normalized size = 1.83

$$\left[\frac{\sqrt{a}\,b^2x^2\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\left(abx+2\,a^2\right)\sqrt{bx+a}}{8\,a^2x^2},-\frac{\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(abx+2\,a^2\right)\sqrt{bx+a}}{4\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(a*b
*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*
x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2)]
```

**giac [A]** time = 1.14, size = 66, normalized size = 1.02

$$-\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{(bx+a)^{\frac{3}{2}}b^3+\sqrt{bx+a}\,ab^3}{ab^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x + a)^(3/2)*b^
3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 53, normalized size = 0.82

$$2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}+\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{b^2x^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^3,x)
```

```
[Out] 2*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/x^2/b^2+1/8*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(3/2))
```

**maxima** [A] time = 3.03, size = 88, normalized size = 1.35

$$-\frac{b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}-\frac{(bx+a)^{\frac{3}{2}}b^2+\sqrt{bx+a}\,ab^2}{4\left((bx+a)^2a-2\,(bx+a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2) -
 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a - 2*(b*x +
a)*a^2 + a^3)
```

**mupad** [B] time = 0.07, size = 48, normalized size = 0.74

$$\frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{3/2}}-\frac{(a+b\,x)^{3/2}}{4\,a\,x^2}-\frac{\sqrt{a+b\,x}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^3,x)
```

```
[Out] (b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (a + b*x)^(3/2)/(4*a*x^2
) - (a + b*x)^(1/2)/(4*x^2)
```

**sympy** [A] time = 4.02, size = 97, normalized size = 1.49

$$-\frac{a}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{b^{\frac{3}{2}}}{4a\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**3,x)
```

```
[Out] -a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(a/(b
*x) + 1)) - b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**2*asinh(sqrt(a)/(
sqrt(b)*sqrt(x)))/(4*a**(3/2))
```

## 3.291 $\int \frac{\sqrt{a+bx}}{x^4}\,dx$

**Optimal**. Leaf size=87

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}-\frac{b\sqrt{a+bx}}{12ax^2}-\frac{\sqrt{a+bx}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^4,x]
```

```
[Out] -Sqrt[a + b*x]/(3*x^3) - (b*Sqrt[a + b*x])/(12*a*x^2) + (b^2*Sqrt[a + b*x])
/(8*a^2*x) - (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^4}\,dx &= -\frac{\sqrt{a+bx}}{3x^3}+\frac{1}{6}b\int \frac{1}{x^3\sqrt{a+bx}}\,dx\\ &= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}-\frac{b^2\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{8a}\\ &= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}+\frac{b^3\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a^2}\\ &= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{8a^2}\\ &= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.40

$$\frac{2b^3(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^4,x]
[Out] (2*b^3*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x)/a])/(3*a^4)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 71, normalized size = 0.82

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}-\frac{\sqrt{a+bx}\left(3a^2+8a(a+bx)-3(a+bx)^2\right)}{24a^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^4,x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(3*a^2 + 8*a*(a + b*x) - 3*(a + b*x)^2))/(a^2*x^3) - (
b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(5/2))
```

**fricas** [A] time = 0.98, size = 145, normalized size = 1.67

$$\left[\frac{3\,\sqrt{a}\,b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx+a}}{48\,a^3x^3}, \frac{3\,\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx+a}}{24\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x + a))/(a^3*x^3), 1/24*(3*sqrt(-a)
*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^
3)*sqrt(b*x + a))/(a^3*x^3)]
```

**giac** [A] time = 1.23, size = 84, normalized size = 0.97

$$\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\,(bx+a)^{\frac{5}{2}}b^4-8\,(bx+a)^{\frac{3}{2}}ab^4-3\,\sqrt{bx+a}\,a^2b^4}{a^2b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(5
/2)*b^4 - 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a^2*b^3*x^3))
/b
```

**maple** [A] time = 0.01, size = 65, normalized size = 0.75

$$2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}}+\frac{-\frac{(bx+a)^{\frac{3}{2}}}{6a}+\frac{(bx+a)^{\frac{5}{2}}}{16a^2}-\frac{\sqrt{bx+a}}{16}}{b^3x^3}\right)b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^4,x)
```

```
[Out] 2*b^3*((1/16/a^2*(b*x+a)^(5/2)-1/6*(b*x+a)^(3/2)/a-1/16*(b*x+a)^(1/2))/x^3/
b^3-1/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2))
```

**maxima [A]** time = 3.02, size = 121, normalized size = 1.39

$$\frac{b^3 \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}} + \frac{3\,(bx+a)^{\frac{5}{2}}b^3 - 8\,(bx+a)^{\frac{3}{2}}ab^3 - 3\,\sqrt{bx+a}\,a^2b^3}{24\left((bx+a)^3a^2 - 3\,(bx+a)^2a^3 + 3\,(bx+a)a^4 - a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) +
 1/24*(3*(b*x + a)^(5/2)*b^3 - 8*(b*x + a)^(3/2)*a*b^3 - 3*sqrt(b*x + a)*a^
2*b^3)/((b*x + a)^3*a^2 - 3*(b*x + a)^2*a^3 + 3*(b*x + a)*a^4 - a^5)
```

**mupad [B]** time = 0.11, size = 66, normalized size = 0.76

$$\frac{(a+b\,x)^{5/2}}{8\,a^2\,x^3} - \frac{(a+b\,x)^{3/2}}{3\,a\,x^3} - \frac{\sqrt{a+b\,x}}{8\,x^3} + \frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{8\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^4,x)
```

```
[Out] (a + b*x)^(5/2)/(8*a^2*x^3) - (a + b*x)^(3/2)/(3*a*x^3) - (a + b*x)^(1/2)/(
8*x^3) + (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^(5/2))
```

**sympy [A]** time = 6.68, size = 122, normalized size = 1.40

$$-\frac{a}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{5\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{b^{\frac{3}{2}}}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{b^{\frac{5}{2}}}{8a^2\sqrt{x}\,\sqrt{\frac{a}{bx}+1}} - \frac{b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**4,x)
```

```
[Out] -a/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 5*sqrt(b)/(12*x**(5/2)*sqrt(a/(
b*x) + 1)) + b**(3/2)/(24*a*x**(3/2)*sqrt(a/(b*x) + 1)) + b**(5/2)/(8*a**2*
sqrt(x)*sqrt(a/(b*x) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*a**(5
/2))
```

## 3.292 $\int x^3(a+bx)^{3/2}\,dx$

**Optimal**. Leaf size=72

$$-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{11/2}}{11b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}$$

**Rubi [A]** time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{11/2}}{11b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(7/2))/(7*b^4) - (2*a*(
a + b*x)^(9/2))/(3*b^4) + (2*(a + b*x)^(11/2))/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{3/2}\,dx&=\int\left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{11/2}}{11b^4}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{5/2}\left(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(11
55*b^4)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 59, normalized size = 0.82

$$-\frac{2\left(231a^3(a+bx)^{5/2}-495a^2(a+bx)^{7/2}-105(a+bx)^{11/2}+385a(a+bx)^{9/2}\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (-2*(231*a^3*(a + b*x)^(5/2) - 495*a^2*(a + b*x)^(7/2) + 385*a*(a + b*x)^(9
/2) - 105*(a + b*x)^(11/2)))/(1155*b^4)
```

**fricas [A]** time = 1.23, size = 64, normalized size = 0.89

$$\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

**giac [B]** time = 1.11, size = 193, normalized size = 2.68

$$\frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^2}{b^3}+\frac{22\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a}{b^3}+\frac{5\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)}{b^3}\right)}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a
^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^3 + 22*(35*(b*x + a)^(9/2) - 180*(b*x + a)
^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x
 + a)*a^4)*a/b^3 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*
x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 69
3*sqrt(b*x + a)*a^5)/b^3)/b
```

**maple [A]** time = 0.00, size = 43, normalized size = 0.60

$$-\frac{2\,(bx+a)^{\frac{5}{2}}\left(-105b^3x^3+70a\,b^2x^2-40a^2bx+16a^3\right)}{1155b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(3/2),x)
```

```
[Out] -2/1155*(b*x+a)^(5/2)*(-105*b^3*x^3+70*a*b^2*x^2-40*a^2*b*x+16*a^3)/b^4
```

**maxima [A]** time = 1.35, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b^4}-\frac{2\,(bx+a)^{\frac{9}{2}}a}{3\,b^4}+\frac{6\,(bx+a)^{\frac{7}{2}}a^2}{7\,b^4}-\frac{2\,(bx+a)^{\frac{5}{2}}a^3}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b^4 - 2/3*(b*x + a)^(9/2)*a/b^4 + 6/7*(b*x + a)^(7/2)
*a^2/b^4 - 2/5*(b*x + a)^(5/2)*a^3/b^4
```

**mupad [B]** time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}+\frac{6\,a^2\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}-\frac{2\,a\,{\left(a+b\,x\right)}^{9/2}}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b^4) - (2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a
 + b*x)^(7/2))/(7*b^4) - (2*a*(a + b*x)^(9/2))/(3*b^4)
```

**sympy [B]** time = 3.20, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(3/2),x)
```

```
[Out] -32*a**(51/2)*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*
a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**1
5*b**9*x**5 + 1155*a**14*b**10*x**6) + 32*a**(51/2)/(1155*a**20*b**4 + 6930
*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16
*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 176*a**(49/2)*
b*x*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6
*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**
5 + 1155*a**14*b**10*x**6) + 192*a**(49/2)*b*x/(1155*a**20*b**4 + 6930*a**1
9*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8
*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 396*a**(47/2)*b**2*
```

```
x**2*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**
6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x*
*5 + 1155*a**14*b**10*x**6) + 480*a**(47/2)*b**2*x**2/(1155*a**20*b**4 + 69
30*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**
16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 462*a**(45/2
)*b**3*x**3*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a*
*18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*
b**9*x**5 + 1155*a**14*b**10*x**6) + 640*a**(45/2)*b**3*x**3/(1155*a**20*b*
*4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17
325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 480*a
**(43/2)*b**4*x**4/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*
x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5
 + 1155*a**14*b**10*x**6) + 1848*a**(41/2)*b**5*x**5*sqrt(1 + b*x/a)/(1155*
a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x
**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6)
 + 192*a**(41/2)*b**5*x**5/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**
18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b
**9*x**5 + 1155*a**14*b**10*x**6) + 5544*a**(39/2)*b**6*x**6*sqrt(1 + b*x/a
)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**1
7*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**
10*x**6) + 32*a**(39/2)*b**6*x**6/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17
325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*
a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 8844*a**(37/2)*b**7*x**7*sqrt(1
+ b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 231
00*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a*
*14*b**10*x**6) + 8448*a**(35/2)*b**8*x**8*sqrt(1 + b*x/a)/(1155*a**20*b**4
 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 1732
5*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 4840*a*
*(33/2)*b**9*x**9*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17
325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*
a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 1540*a**(31/2)*b**10*x**10*sqrt(
1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 2
3100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*
a**14*b**10*x**6) + 210*a**(29/2)*b**11*x**11*sqrt(1 + b*x/a)/(1155*a**20*b
**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 1
7325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6)
```

## 3.293 $\int x^2(a+bx)^{3/2}\,dx$

**Optimal**. Leaf size=53

$$\frac{2a^2(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{9/2}}{9b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}$$

**Rubi [A]** time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{9/2}}{9b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(5/2))/(5*b^3) - (4*a*(a + b*x)^(7/2))/(7*b^3) + (2*(a + b
*x)^(9/2))/(9*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{3/2}\,dx&=\int\left(\frac{a^2(a+bx)^{3/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{9/2}}{9b^3}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{5/2}\left(8a^2-20abx+35b^2x^2\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 45, normalized size = 0.85

$$\frac{2\left(63a^2(a+bx)^{5/2}+35(a+bx)^{9/2}-90a(a+bx)^{7/2}\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*(63*a^2*(a + b*x)^(5/2) - 90*a*(a + b*x)^(7/2) + 35*(a + b*x)^(9/2)))/(3
15*b^3)
```

**fricas [A]** time = 0.75, size = 53, normalized size = 1.00

$$\frac{2\left(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4\right)\sqrt{bx+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

**giac [B]** time = 0.94, size = 156, normalized size = 2.94

$$\frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a^2}{b^2}+\frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a}{b^2}+\frac{35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4}{b^2}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*a^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2
)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(
7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x +
 a)*a^4)/b^2)/b
```

**maple [A]** time = 0.01, size = 32, normalized size = 0.60

$$\frac{2\,(bx+a)^{\frac{5}{2}}\left(35b^2x^2-20abx+8a^2\right)}{315b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(3/2),x)
```

```
[Out] 2/315*(b*x+a)^(5/2)*(35*b^2*x^2-20*a*b*x+8*a^2)/b^3
```

**maxima [A]** time = 1.35, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^3}-\frac{4\,(bx+a)^{\frac{7}{2}}a}{7\,b^3}+\frac{2\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^3 - 4/7*(b*x + a)^(7/2)*a/b^3 + 2/5*(b*x + a)^(5/2)*a
^2/b^3
```

**mupad [B]** time = 0.04, size = 37, normalized size = 0.70

$$\frac{70\,(a+b\,x)^{9/2}-180\,a\,(a+b\,x)^{7/2}+126\,a^2\,(a+b\,x)^{5/2}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(3/2),x)
```

```
[Out] (70*(a + b*x)^(9/2) - 180*a*(a + b*x)^(7/2) + 126*a^2*(a + b*x)^(5/2))/(315
*b^3)
```

**sympy [B]** time = 2.17, size = 733, normalized size = 13.83

$$\frac{16a^{\frac{25}{2}}\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{16a^{\frac{25}{2}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{40a^{\frac{23}{2}}bx\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{48a^{\frac{23}{2}}bx}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{30a^{\frac{21}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{48a^{\frac{21}{2}}b^2x^2}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{110a^{\frac{19}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{16a^{\frac{19}{2}}b^3x^3}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{380a^{\frac{17}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{516a^{\frac{15}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{310a^{\frac{13}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{70a^{\frac{11}{2}}b^7x^7\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(3/2),x)
```

```
[Out] 16*a**(25/2)*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b*
*5*x**2 + 315*a**5*b**6*x**3) - 16*a**(25/2)/(315*a**8*b**3 + 945*a**7*b**4
*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 40*a**(23/2)*b*x*sqrt(1 + b
*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6
*x**3) - 48*a**(23/2)*b*x/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*
x**2 + 315*a**5*b**6*x**3) + 30*a**(21/2)*b**2*x**2*sqrt(1 + b*x/a)/(315*a*
*8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) - 48*a
**(21/2)*b**2*x**2/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 +
315*a**5*b**6*x**3) + 110*a**(19/2)*b**3*x**3*sqrt(1 + b*x/a)/(315*a**8*b**
```

```
3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) - 16*a**(19/
2)*b**3*x**3/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a*
*5*b**6*x**3) + 380*a**(17/2)*b**4*x**4*sqrt(1 + b*x/a)/(315*a**8*b**3 + 94
5*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 516*a**(15/2)*b*
*5*x**5*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x*
*2 + 315*a**5*b**6*x**3) + 310*a**(13/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*a**
8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 70*a*
*(11/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a*
*6*b**5*x**2 + 315*a**5*b**6*x**3)
```

## 3.294 $\int x(a+bx)^{3/2}\,dx$

**Optimal**. Leaf size=34

$$\frac{2(a+bx)^{7/2}}{7b^2}-\frac{2a(a+bx)^{5/2}}{5b^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{7/2}}{7b^2}-\frac{2a(a+bx)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(3/2),x]
```

```
[Out] (-2*a*(a + b*x)^(5/2))/(5*b^2) + (2*(a + b*x)^(7/2))/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{3/2}\,dx &= \int\left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{5/2}}{5b^2}+\frac{2(a+bx)^{7/2}}{7b^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{5/2}(5bx-2a)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(-2*a + 5*b*x))/(35*b^2)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 46, normalized size = 1.35

$$-\frac{2\sqrt{a+bx}\,\left(2a^3-a^2bx-8ab^2x^2-5b^3x^3\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(3/2),x]
[Out] (-2*Sqrt[a + b*x]*(2*a^3 - a^2*b*x - 8*a*b^2*x^2 - 5*b^3*x^3))/(35*b^2)
```

**fricas [A]** time = 0.81, size = 41, normalized size = 1.21

$$\frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="fricas")
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

**giac [B]** time = 1.24, size = 119, normalized size = 3.50

$$\frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^2}{b}+\frac{14\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a}{b}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)}{b}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="giac")
[Out] 2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2/b + 14*(3*(b*x + a)^(5/
2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a/b + 3*(5*(b*x + a)^(7/2
) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/b
)/b
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.62

$$-\frac{2\,(bx+a)^{\frac{5}{2}}\,(-5bx+2a)}{35b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(3/2),x)
```

```
[Out] -2/35*(b*x+a)^(5/2)*(-5*b*x+2*a)/b^2
```

**maxima [A]** time = 1.35, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^2}-\frac{2\,(bx+a)^{\frac{5}{2}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^2 - 2/5*(b*x + a)^(5/2)*a/b^2
```

**mupad [B]** time = 0.03, size = 25, normalized size = 0.74

$$-\frac{14\,a\,(a+b\,x)^{5/2}-10\,(a+b\,x)^{7/2}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(3/2),x)
```

```
[Out] -(14*a*(a + b*x)^(5/2) - 10*(a + b*x)^(7/2))/(35*b^2)
```

**sympy [A]** time = 0.74, size = 80, normalized size = 2.35

$$\begin{cases} -\frac{4a^3\sqrt{a+bx}}{35b^2}+\frac{2a^2x\sqrt{a+bx}}{35b}+\frac{16ax^2\sqrt{a+bx}}{35}+\frac{2bx^3\sqrt{a+bx}}{7} & \text{for } b\neq 0 \\ \frac{a^{\frac{3}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-4*a**3*sqrt(a + b*x)/(35*b**2) + 2*a**2*x*sqrt(a + b*x)/(35*b)
+ 16*a*x**2*sqrt(a + b*x)/35 + 2*b*x**3*sqrt(a + b*x)/7, Ne(b, 0)), (a**(3/
2)*x**2/2, True))
```

## 3.295 $\int (a+bx)^{3/2}\,dx$

**Optimal**. Leaf size=16

$$\frac{2(a+bx)^{5/2}}{5b}$$

**Rubi** [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(3/2),x]

[Out] (2*(a + b*x)^(5/2))/(5*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{3/2}\,dx=\frac{2(a+bx)^{5/2}}{5b}$$

**Mathematica** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(3/2),x]

[Out] (2*(a + b*x)^(5/2))/(5*b)

**IntegrateAlgebraic** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b)
```

**fricas [B]** time = 0.61, size = 28, normalized size = 1.75

$$\frac{2\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

**giac [B]** time = 1.13, size = 58, normalized size = 3.62

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+30\,\sqrt{bx+a}\,a^{2}+10\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 30*sqrt(b*x + a)*a^2 + 10*
((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a)/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2),x)
```

```
[Out] 2/5*(b*x+a)^(5/2)/b
```

**maxima [A]** time = 1.29, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,(a+b\,x)^{5/2}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b)
```

**sympy [A]** time = 0.07, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2),x)
```

```
[Out] 2*(a + b*x)**(5/2)/(5*b)
```

# 3.296 $\int \frac{(a+bx)^{3/2}}{x} \, dx$

**Optimal**. Leaf size=49

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}$$

**Rubi** [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x,x]
```

```
[Out] 2*a*Sqrt[a + b*x] + (2*(a + b*x)^(3/2))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]
/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x}\,dx &= \frac{2}{3}(a+bx)^{3/2}+a\int \frac{\sqrt{a+bx}}{x}\,dx\\ &= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}+a^2\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}\\ &= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 44, normalized size = 0.90

$$\frac{2}{3}\sqrt{a+bx}\,(4a+bx)-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(4*a + b*x))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 50, normalized size = 1.02

$$\frac{2}{3}\left((a+bx)^{3/2}+3a\sqrt{a+bx}\right)-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x,x]
```

```
[Out] (2*(3*a*Sqrt[a + b*x] + (a + b*x)^(3/2)))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]]
```

**fricas [A]** time = 1.14, size = 88, normalized size = 1.80

$$\left[a^{\frac{3}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+\frac{2}{3}\,(bx+4a)\sqrt{bx+a}\,,2\,\sqrt{-a}\,a\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\frac{2}{3}\,(bx+4a)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(3/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/3*(b*x + 4*a)*sqr
t(b*x + a), 2*sqrt(-a)*a*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/3*(b*x + 4*a)
*sqrt(b*x + a)]
```

**giac [A]** time = 1.08, size = 44, normalized size = 0.90

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}+2\,\sqrt{bx+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/3*(b*x + a)^(3/2) + 2*sqr
t(b*x + a)*a
```

**maple [A]** time = 0.01, size = 38, normalized size = 0.78

$$-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\;a+\frac{2\,(bx+a)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x,x)
```

```
[Out] 2/3*(b*x+a)^(3/2)-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a*(b*x+a)^(1/2
)
```

**maxima [A]** time = 2.94, size = 52, normalized size = 1.06

$$a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}+2\,\sqrt{bx+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] a^(3/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/3*(b*x
 + a)^(3/2) + 2*sqrt(b*x + a)*a
```

**mupad [B]** time = 0.04, size = 37, normalized size = 0.76

$$2\,a\,\sqrt{a+b\,x}+\frac{2\,{\left(a+b\,x\right)}^{3/2}}{3}-2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x,x)
```

```
[Out] 2*a*(a + b*x)^(1/2) + (2*(a + b*x)^(3/2))/3 - 2*a^(3/2)*atanh((a + b*x)^(1/
2)/a^(1/2))
```

**sympy** [A] time = 2.29, size = 71, normalized size = 1.45

$$\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{3}+a^{\frac{3}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{2\sqrt{a}\,bx\sqrt{1+\frac{bx}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x,x)
```

```
[Out] 8*a**(3/2)*sqrt(1 + b*x/a)/3 + a**(3/2)*log(b*x/a) - 2*a**(3/2)*log(sqrt(1
+ b*x/a) + 1) + 2*sqrt(a)*b*x*sqrt(1 + b*x/a)/3
```

## 3.297 $\int \frac{(a+bx)^{3/2}}{x^2}\,dx$

**Optimal**. Leaf size=51

$$-\frac{(a+bx)^{3/2}}{x}+3b\sqrt{a+bx}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$-\frac{(a+bx)^{3/2}}{x}+3b\sqrt{a+bx}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[a + b*x] - (a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/S
qrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^2}\,dx &= -\frac{(a+bx)^{3/2}}{x}+\frac{1}{2}(3b)\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}+\frac{1}{2}(3ab)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}+(3a)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.65

$$\frac{2b(a+bx)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx}{a}+1\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^2,x]
[Out] (2*b*(a + b*x)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x)/a])/(5*a^2)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 49, normalized size = 0.96

$$\frac{\sqrt{a+bx}\,(2(a+bx)-3a)}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x]*(-3*a + 2*(a + b*x)))/x - 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]]
```

**fricas [A]** time = 1.04, size = 102, normalized size = 2.00

$$\left[\frac{3\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\,(2\,bx-a)\sqrt{bx+a}}{2\,x}, \frac{3\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(2\,bx-a)\sqrt{bx+a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b*x
 - a)*sqrt(b*x + a))/x, (3*sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) +
(2*b*x - a)*sqrt(b*x + a))/x]
```

**giac [A]** time = 1.04, size = 56, normalized size = 1.10

$$\frac{\frac{3\,ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx+a}\,b^2-\frac{\sqrt{bx+a}\,ab}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)*b^2 - sq
rt(b*x + a)*a*b/x)/b
```

**maple [A]** time = 0.01, size = 47, normalized size = 0.92

$$2\left(\left(-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)a+\sqrt{bx+a}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^2,x)
```

```
[Out] 2*b*((b*x+a)^(1/2)+a*(-1/2*(b*x+a)^(1/2)/b/x-3/2*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(1/2)))
```

**maxima [A]** time = 2.87, size = 58, normalized size = 1.14

$$\frac{3}{2}\sqrt{a}\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b-\frac{\sqrt{bx+a}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(a)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*
sqrt(b*x + a)*b - sqrt(b*x + a)*a/x
```

**mupad [B]** time = 0.10, size = 42, normalized size = 0.82

$$2\,b\,\sqrt{a+b\,x}-3\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)-\frac{a\,\sqrt{a+b\,x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^2,x)
```

```
[Out] 2*b*(a + b*x)^(1/2) - 3*a^(1/2)*b*atanh((a + b*x)^(1/2)/a^(1/2)) - (a*(a +
b*x)^(1/2))/x
```

**sympy [B]** time = 2.66, size = 92, normalized size = 1.80

$$-3\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{a^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{a\sqrt{b}}{\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**2,x)
```

```
[Out] -3*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqrt
(a/(b*x) + 1)) + a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(3/2)*sqrt(x)
/sqrt(a/(b*x) + 1)
```

## 3.298 $\int \frac{(a+bx)^{3/2}}{x^3} dx$

**Optimal**. Leaf size=62

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{(a+bx)^{3/2}}{2x^2} - \frac{3b\sqrt{a+bx}}{4x}$$

**Rubi [A]** time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{(a+bx)^{3/2}}{2x^2} - \frac{3b\sqrt{a+bx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] (-3*b*Sqrt[a + b*x])/(4*x) - (a + b*x)^(3/2)/(2*x^2) - (3*b^2*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x^3}\,dx &= -\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\int \frac{\sqrt{a+bx}}{x^2}\,dx\\ &= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\ &= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 68, normalized size = 1.10

$$-\frac{2a^2+3b^2x^2\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+7abx+5b^2x^2}{4x^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] -1/4*(2*a^2 + 7*a*b*x + 5*b^2*x^2 + 3*b^2*x^2*Sqrt[1 + (b*x)/a]*ArcTanh[Sqr
t[1 + (b*x)/a]])/(x^2*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.10, size = 56, normalized size = 0.90

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{\sqrt{a+bx}\,(5(a+bx)-3a)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] -1/4*(Sqrt[a + b*x]*(-3*a + 5*(a + b*x)))/x^2 - (3*b^2*ArcTanh[Sqrt[a + b*x
]/Sqrt[a]])/(4*Sqrt[a])
```

**fricas [A]** time = 1.12, size = 124, normalized size = 2.00

$$\left[\frac{3\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(5abx+2a^2\right)\sqrt{bx+a}}{8ax^2},\ \frac{3\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\left(5abx+2a^2\right)\sqrt{bx+a}}{4ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(5
*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sqrt
(b*x + a)*sqrt(-a)/a) - (5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2)]
```

**giac** [A] time = 1.29, size = 64, normalized size = 1.03

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^3-3\,\sqrt{bx+a}\,ab^3}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (5*(b*x + a)^(3/2)*b^3
 - 3*sqrt(b*x + a)*a*b^3)/(b^2*x^2))/b
```

**maple** [A] time = 0.01, size = 51, normalized size = 0.82

$$2\left(-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}+\frac{\frac{3\sqrt{bx+a}\,a}{8}-\frac{5(bx+a)^{\frac{3}{2}}}{8}}{b^2x^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^3,x)
```

```
[Out] 2*b^2*((-5/8*(b*x+a)^(3/2)+3/8*(b*x+a)^(1/2)*a)/x^2/b^2-3/8*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(1/2))
```

**maxima** [A] time = 2.98, size = 86, normalized size = 1.39

$$\frac{3\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,\sqrt{a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^2-3\,\sqrt{bx+a}\,ab^2}{4\left((bx+a)^2-2\,(bx+a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) -
1/4*(5*(b*x + a)^(3/2)*b^2 - 3*sqrt(b*x + a)*a*b^2)/((b*x + a)^2 - 2*(b*x +
 a)*a + a^2)
```

**mupad** [B] time = 0.06, size = 46, normalized size = 0.74

$$\frac{3\,a\,\sqrt{a+b\,x}}{4\,x^2}-\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,\sqrt{a}}-\frac{5\,{\left(a+b\,x\right)}^{3/2}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^3,x)
```

```
[Out] (3*a*(a + b*x)^(1/2))/(4*x^2) - (3*b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a
^(1/2)) - (5*(a + b*x)^(3/2))/(4*x^2)
```

**sympy** [A] time = 3.28, size = 76, normalized size = 1.23

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx}+1}}{2x^{\frac{3}{2}}}-\frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{4\sqrt{x}}-\frac{3b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**3,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x) + 1)/(2*x**(3/2)) - 5*b**(3/2)*sqrt(a/(b*x) + 1)/(4
*sqrt(x)) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a))
```

## 3.299 $\int \frac{(a+bx)^{3/2}}{x^4} dx$

**Optimal**. Leaf size=84

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}-\frac{b\sqrt{a+bx}}{4x^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{b\sqrt{a+bx}}{4x^2}-\frac{(a+bx)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^4,x]
```

```
[Out] -(b*Sqrt[a + b*x])/(4*x^2) - (b^2*Sqrt[a + b*x])/(8*a*x) - (a + b*x)^(3/2)/
(3*x^3) + (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x^4}\,dx &= -\frac{(a+bx)^{3/2}}{3x^3}+\frac{1}{2}b\int \frac{\sqrt{a+bx}}{x^3}\,dx\\ &= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{(a+bx)^{3/2}}{3x^3}+\frac{1}{8}b^2\int \frac{1}{x^2\sqrt{a+bx}}\,dx\\ &= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}-\frac{b^3\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a}\\ &= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}-\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{8a}\\ &= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.42

$$\frac{2b^3(a+bx)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^4,x]
[Out] (2*b^3*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x)/a])/(5*a^4)
```

**IntegrateAlgebraic [A]** time = 0.12, size = 71, normalized size = 0.85

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{\sqrt{a+bx}\left(3a^2-8a(a+bx)-3(a+bx)^2\right)}{24ax^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x^4,x]
```

```
[Out] (Sqrt[a + b*x]*(3*a^2 - 8*a*(a + b*x) - 3*(a + b*x)^2))/(24*a*x^3) + (b^3*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))
```

**fricas [A]** time = 1.01, size = 145, normalized size = 1.73

$$\left[\frac{3\,\sqrt{a}\,b^{3}x^{3}\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\left(3\,ab^{2}x^{2}+14\,a^{2}bx+8\,a^{3}\right)\sqrt{bx+a}}{48\,a^{2}x^{3}},-\frac{3\,\sqrt{-a}\,b^{3}x^{3}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(3\,ab^{2}x^{2}+14\,a^{2}bx+8\,a^{3}\right)\sqrt{bx+a}}{24\,a^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^2*x^3), -1/24*(3*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 + 14*a^2*b*x + 8
*a^3)*sqrt(b*x + a))/(a^2*x^3)]
```

**giac [A]** time = 1.16, size = 84, normalized size = 1.00

$$-\frac{\frac{3\,b^{4}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{3\,(bx+a)^{\frac{5}{2}}b^{4}+8\,(bx+a)^{\frac{3}{2}}ab^{4}-3\,\sqrt{bx+a}\,a^{2}b^{4}}{ab^{3}x^{3}}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (3*(b*x + a)^(5/
2)*b^4 + 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a*b^3*x^3))/b
```

**maple [A]** time = 0.01, size = 63, normalized size = 0.75

$$2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}+\frac{\frac{\sqrt{bx+a}\,a}{16}-\frac{(bx+a)^{\frac{5}{2}}}{16a}-\frac{(bx+a)^{\frac{3}{2}}}{6}}{b^{3}x^{3}}\right)b^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^4,x)
```

```
[Out] 2*b^3*((-1/16/a*(b*x+a)^(5/2)-1/6*(b*x+a)^(3/2)+1/16*(b*x+a)^(1/2)*a)/x^3/b
^3+1/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

**maxima** [A] time = 3.05, size = 119, normalized size = 1.42

$$-\frac{b^3 \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{3}{2}}} - \frac{3\,(bx+a)^{\frac{5}{2}}b^3 + 8\,(bx+a)^{\frac{3}{2}}ab^3 - 3\,\sqrt{bx+a}\,a^2b^3}{24\left((bx+a)^3a - 3\,(bx+a)^2a^2 + 3\,(bx+a)a^3 - a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
- 1/24*(3*(b*x + a)^(5/2)*b^3 + 8*(b*x + a)^(3/2)*a*b^3 - 3*sqrt(b*x + a)*a
^2*b^3)/((b*x + a)^3*a - 3*(b*x + a)^2*a^2 + 3*(b*x + a)*a^3 - a^4)
```

**mupad** [B] time = 0.10, size = 64, normalized size = 0.76

$$\frac{a\sqrt{a+b\,x}}{8\,x^3} - \frac{(a+b\,x)^{5/2}}{8\,a\,x^3} - \frac{(a+b\,x)^{3/2}}{3\,x^3} - \frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}}{8\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^4,x)
```

```
[Out] (a*(a + b*x)^(1/2))/(8*x^3) - (a + b*x)^(5/2)/(8*a*x^3) - (b^3*atan(((a + b
*x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^(3/2)) - (a + b*x)^(3/2)/(3*x^3)
```

**sympy** [A] time = 5.84, size = 124, normalized size = 1.48

$$-\frac{a^2}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{11a\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{17b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{b^{\frac{5}{2}}}{8a\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**4,x)
```

```
[Out] -a**2/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 11*a*sqrt(b)/(12*x**(5/2)*sq
rt(a/(b*x) + 1)) - 17*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) - b**(5/2)/(
8*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*a
**(3/2))
```

## 3.300 $\int x^3(a+bx)^{5/2}\,dx$

**Optimal**. Leaf size=72

$$-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{13/2}}{13b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}$$

**Rubi [A]** time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{13/2}}{13b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(a + b*x)^(9/2))/(3*b^4) - (6*a*(
a + b*x)^(11/2))/(11*b^4) + (2*(a + b*x)^(13/2))/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{5/2}\,dx&=\int\left(-\frac{a^3(a+bx)^{5/2}}{b^3}+\frac{3a^2(a+bx)^{7/2}}{b^3}-\frac{3a(a+bx)^{9/2}}{b^3}+\frac{(a+bx)^{11/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{13/2}}{13b^4}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{7/2}\left(-16a^3+56a^2bx-126ab^2x^2+231b^3x^3\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-16*a^3 + 56*a^2*b*x - 126*a*b^2*x^2 + 231*b^3*x^3))/(3
003*b^4)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 51, normalized size = 0.71

$$\frac{2(a+bx)^{7/2}\left(-429a^3+1001a^2(a+bx)-819a(a+bx)^2+231(a+bx)^3\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-429*a^3 + 1001*a^2*(a + b*x) - 819*a*(a + b*x)^2 + 231
*(a + b*x)^3))/(3003*b^4)
```

**fricas [A]** time = 1.00, size = 75, normalized size = 1.04

$$\frac{2\left(231\,b^6x^6+567\,ab^5x^5+371\,a^2b^4x^4+5\,a^3b^3x^3-6\,a^4b^2x^2+8\,a^5bx-16\,a^6\right)\sqrt{bx+a}}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*a
^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)/b^4
```

**giac [B]** time = 1.16, size = 281, normalized size = 3.90

$$\frac{2\left(\frac{429\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^3}{b^3}+\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a^2}{b^3}+\frac{65\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)a}{b^3}+\frac{5\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^2-8580\,(bx+a)^{\frac{7}{2}}a^3+9009\,(bx+a)^{\frac{5}{2}}a^4-6006\,(bx+a)^{\frac{3}{2}}a^5+3003\,\sqrt{bx+a}\,a^6\right)}{b^3}\right)}{15015\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15015*(429*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)
*a^2 - 35*sqrt(b*x + a)*a^3)*a^3/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x +
 a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(
b*x + a)*a^4)*a^2/b^3 + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 9
90*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^
4 - 693*sqrt(b*x + a)*a^5)*a/b^3 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)
^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x
 + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b^3)/b
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.60

$$-\frac{2\,(bx+a)^{\frac{7}{2}}\left(-231b^3x^3+126a\,b^2x^2-56a^2bx+16a^3\right)}{3003b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(5/2),x)
```

```
[Out] -2/3003*(b*x+a)^(7/2)*(-231*b^3*x^3+126*a*b^2*x^2-56*a^2*b*x+16*a^3)/b^4
```

**maxima [A]** time = 1.35, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{13}{2}}}{13\,b^4}-\frac{6\,(bx+a)^{\frac{11}{2}}a}{11\,b^4}+\frac{2\,(bx+a)^{\frac{9}{2}}a^2}{3\,b^4}-\frac{2\,(bx+a)^{\frac{7}{2}}a^3}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*x + a)^(13/2)/b^4 - 6/11*(b*x + a)^(11/2)*a/b^4 + 2/3*(b*x + a)^(9/
2)*a^2/b^4 - 2/7*(b*x + a)^(7/2)*a^3/b^4
```

**mupad [B]** time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,(a+b\,x)^{13/2}}{13\,b^4}-\frac{2\,a^3\,(a+b\,x)^{7/2}}{7\,b^4}+\frac{2\,a^2\,(a+b\,x)^{9/2}}{3\,b^4}-\frac{6\,a\,(a+b\,x)^{11/2}}{11\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(13/2))/(13*b^4) - (2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(a
 + b*x)^(9/2))/(3*b^4) - (6*a*(a + b*x)^(11/2))/(11*b^4)
```

**sympy [A]** time = 4.47, size = 146, normalized size = 2.03

$$\begin{cases} -\frac{32a^6\sqrt{a+bx}}{3003b^4}+\frac{16a^5x\sqrt{a+bx}}{3003b^3}-\frac{4a^4x^2\sqrt{a+bx}}{1001b^2}+\frac{10a^3x^3\sqrt{a+bx}}{3003b}+\frac{106a^2x^4\sqrt{a+bx}}{429}+\frac{54abx^5\sqrt{a+bx}}{143}+\frac{2b^2x^6\sqrt{a+bx}}{13} & \text{for } b\neq 0 \\ \frac{a^{\frac{5}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-32*a**6*sqrt(a + b*x)/(3003*b**4) + 16*a**5*x*sqrt(a + b*x)/(30
03*b**3) - 4*a**4*x**2*sqrt(a + b*x)/(1001*b**2) + 10*a**3*x**3*sqrt(a + b*
x)/(3003*b) + 106*a**2*x**4*sqrt(a + b*x)/429 + 54*a*b*x**5*sqrt(a + b*x)/1
43 + 2*b**2*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(5/2)*x**4/4, True))
```

## 3.301 $\int x^2(a+bx)^{5/2}\,dx$

**Optimal**. Leaf size=53

$$\frac{2a^2(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}$$

**Rubi [A]** time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(5/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(7/2))/(7*b^3) - (4*a*(a + b*x)^(9/2))/(9*b^3) + (2*(a + b
*x)^(11/2))/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{5/2}\,dx&=\int\left(\frac{a^2(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{7/2}}{b^2}+\frac{(a+bx)^{9/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}+\frac{2(a+bx)^{11/2}}{11b^3}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{7/2}\left(8a^2-28abx+63b^2x^2\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(5/2),x]
```

[Out] (2*(a + b*x)^(7/2)*(8*a^2 - 28*a*b*x + 63*b^2*x^2))/(693*b^3)

**IntegrateAlgebraic [A]** time = 0.02, size = 39, normalized size = 0.74

$$\frac{2(a+bx)^{7/2}\left(99a^2-154a(a+bx)+63(a+bx)^2\right)}{693b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(a + b*x)^(5/2),x]

[Out] (2*(a + b*x)^(7/2)*(99*a^2 - 154*a*(a + b*x) + 63*(a + b*x)^2))/(693*b^3)

**fricas [A]** time = 1.17, size = 64, normalized size = 1.21

$$\frac{2\left(63\,b^5x^5+161\,ab^4x^4+113\,a^2b^3x^3+3\,a^3b^2x^2-4\,a^4bx+8\,a^5\right)\sqrt{bx+a}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="fricas")

[Out] 2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^4*b*x + 8*a^5)*sqrt(b*x + a)/b^3

**giac [B]** time = 1.09, size = 233, normalized size = 4.40

$$\frac{2\left(\frac{231\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a^3}{b^2}+\frac{297\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^2}{b^2}+\frac{33\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a}{b^2}+\frac{5\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)}{b^2}\right)}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="giac")

[Out] 2/3465*(231*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^3/b^2 + 297*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^2 + 33*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a/b^2 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)/b^2)/b

**maple [A]** time = 0.00, size = 32, normalized size = 0.60

$$\frac{2\,(bx+a)^{\frac{7}{2}}\left(63b^2x^2-28abx+8a^2\right)}{693b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(5/2),x)
```

```
[Out] 2/693*(b*x+a)^(7/2)*(63*b^2*x^2-28*a*b*x+8*a^2)/b^3
```

**maxima [A]** time = 1.37, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b^3} - \frac{4\,(bx+a)^{\frac{9}{2}}a}{9\,b^3} + \frac{2\,(bx+a)^{\frac{7}{2}}a^2}{7\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b^3 - 4/9*(b*x + a)^(9/2)*a/b^3 + 2/7*(b*x + a)^(7/2)
*a^2/b^3
```

**mupad [B]** time = 0.04, size = 37, normalized size = 0.70

$$\frac{126\,(a+b\,x)^{11/2}-308\,a\,(a+b\,x)^{9/2}+198\,a^2\,(a+b\,x)^{7/2}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(5/2),x)
```

```
[Out] (126*(a + b*x)^(11/2) - 308*a*(a + b*x)^(9/2) + 198*a^2*(a + b*x)^(7/2))/(6
93*b^3)
```

**sympy [A]** time = 3.77, size = 124, normalized size = 2.34

$$\begin{cases} \frac{16a^5\sqrt{a+bx}}{693b^3} - \frac{8a^4x\sqrt{a+bx}}{693b^2} + \frac{2a^3x^2\sqrt{a+bx}}{231b} + \frac{226a^2x^3\sqrt{a+bx}}{693} + \frac{46abx^4\sqrt{a+bx}}{99} + \frac{2b^2x^5\sqrt{a+bx}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((16*a**5*sqrt(a + b*x)/(693*b**3) - 8*a**4*x*sqrt(a + b*x)/(693*b
**2) + 2*a**3*x**2*sqrt(a + b*x)/(231*b) + 226*a**2*x**3*sqrt(a + b*x)/693
+ 46*a*b*x**4*sqrt(a + b*x)/99 + 2*b**2*x**5*sqrt(a + b*x)/11, Ne(b, 0)), (
a**(5/2)*x**3/3, True))
```

## 3.302 $\int x(a+bx)^{5/2}\,dx$

**Optimal**. Leaf size=34

$$\frac{2(a+bx)^{9/2}}{9b^2}-\frac{2a(a+bx)^{7/2}}{7b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{9/2}}{9b^2}-\frac{2a(a+bx)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(5/2),x]
```

```
[Out] (-2*a*(a + b*x)^(7/2))/(7*b^2) + (2*(a + b*x)^(9/2))/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{5/2}\,dx &= \int \left(-\frac{a(a+bx)^{5/2}}{b}+\frac{(a+bx)^{7/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{7/2}}{7b^2}+\frac{2(a+bx)^{9/2}}{9b^2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{7/2}(7bx-2a)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-2*a + 7*b*x))/(63*b^2)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 57, normalized size = 1.68

$$-\frac{2\sqrt{a+bx}\,\left(2a^4-a^3bx-15a^2b^2x^2-19ab^3x^3-7b^4x^4\right)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(2*a^4 - a^3*b*x - 15*a^2*b^2*x^2 - 19*a*b^3*x^3 - 7*b^4*
x^4))/(63*b^2)
```

**fricas [A]** time = 1.10, size = 52, normalized size = 1.53

$$\frac{2\left(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4\right)\sqrt{bx+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*x
 + a)/b^2
```

**giac [B]** time = 1.15, size = 182, normalized size = 5.35

$$\frac{2\left(\frac{105\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^3}{b}+\frac{63\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a^2}{b}+\frac{27\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a}{b}+\frac{35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4}{b}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/315*(105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^3/b + 63*(3*(b*x + a)^(5
/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^2/b + 27*(5*(b*x + a)^
(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^
3)*a/b + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*
a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b)/b
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.62

$$-\frac{2\,(bx+a)^{\frac{7}{2}}\,(-7bx+2a)}{63b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(5/2),x)
```

```
[Out] -2/63*(b*x+a)^(7/2)*(-7*b*x+2*a)/b^2
```

**maxima [A]** time = 1.37, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^2}-\frac{2\,(bx+a)^{\frac{7}{2}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^2 - 2/7*(b*x + a)^(7/2)*a/b^2
```

**mupad [B]** time = 0.03, size = 25, normalized size = 0.74

$$-\frac{18\,a\,(a+b\,x)^{7/2}-14\,(a+b\,x)^{9/2}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(5/2),x)
```

```
[Out] -(18*a*(a + b*x)^(7/2) - 14*(a + b*x)^(9/2))/(63*b^2)
```

**sympy [A]** time = 2.60, size = 102, normalized size = 3.00

$$\begin{cases} -\frac{4a^4\sqrt{a+bx}}{63b^2}+\frac{2a^3x\sqrt{a+bx}}{63b}+\frac{10a^2x^2\sqrt{a+bx}}{21}+\frac{38abx^3\sqrt{a+bx}}{63}+\frac{2b^2x^4\sqrt{a+bx}}{9} & \text{for } b\neq 0\\ \frac{a^{\frac{5}{2}}x^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-4*a**4*sqrt(a + b*x)/(63*b**2) + 2*a**3*x*sqrt(a + b*x)/(63*b)
+ 10*a**2*x**2*sqrt(a + b*x)/21 + 38*a*b*x**3*sqrt(a + b*x)/63 + 2*b**2*x**
4*sqrt(a + b*x)/9, Ne(b, 0)), (a**(5/2)*x**2/2, True))
```

## 3.303 $\int (a+bx)^{5/2}\,dx$

**Optimal**. Leaf size=16

$$\frac{2(a+bx)^{7/2}}{7b}$$

**Rubi** [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{5/2}\,dx = \frac{2(a+bx)^{7/2}}{7b}$$

**Mathematica** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

**fricas [B]** time = 0.98, size = 39, normalized size = 2.44

$$\frac{2\left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3\right)\sqrt{bx+a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/b
```

**giac [B]** time = 0.98, size = 95, normalized size = 5.94

$$\frac{2\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 + 35\left((bx+a)^{\frac{3}{2}} - 3\,\sqrt{bx+a}\,a\right)a^2 + 7\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}\,a^2\right)a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 + 3
5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2 + 7*(3*(b*x + a)^(5/2) - 10*(b*
x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a)/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2),x)
```

```
[Out] 2/7*(b*x+a)^(7/2)/b
```

**maxima [A]** time = 1.34, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{7/2}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

**sympy [A]** time = 0.08, size = 12, normalized size = 0.75

$$\frac{2 \left(a + b x\right)^{\frac{7}{2}}}{7 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2),x)
```

```
[Out] 2*(a + b*x)**(7/2)/(7*b)
```

## 3.304 $\int \frac{(a+bx)^{5/2}}{x} \, dx$

**Optimal**. Leaf size=65

$$-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}$$

**Rubi** [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$2a^2\sqrt{a+bx}-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x,x]
```

```
[Out] 2*a^2*Sqrt[a + b*x] + (2*a*(a + b*x)^(3/2))/3 + (2*(a + b*x)^(5/2))/5 - 2*a
^(5/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x}\,dx &= \frac{2}{5}(a+bx)^{5/2} + a\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2} + a^2\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= 2a^2\sqrt{a+bx} + \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2} + a^3\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= 2a^2\sqrt{a+bx} + \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2} + \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b} \\
&= 2a^2\sqrt{a+bx} + \frac{2}{3}a(a+bx)^{3/2} + \frac{2}{5}(a+bx)^{5/2} - 2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

**Mathematica** [A] time = 0.05, size = 58, normalized size = 0.89

$$
-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + \frac{2}{3}a\sqrt{a+bx}\,(4a+bx) + \frac{2}{5}(a+bx)^{5/2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x,x]
```

```
[Out] (2*(a + b*x)^(5/2))/5 + (2*a*Sqrt[a + b*x]*(4*a + b*x))/3 - 2*a^(5/2)*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]]
```

**IntegrateAlgebraic** [A] time = 0.03, size = 66, normalized size = 1.02

$$
\frac{2}{15}\left(15a^2\sqrt{a+bx} + 3(a+bx)^{5/2} + 5a(a+bx)^{3/2}\right) - 2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x,x]
```

```
[Out] (2*(15*a^2*Sqrt[a + b*x] + 5*a*(a + b*x)^(3/2) + 3*(a + b*x)^(5/2)))/15 - 2
*a^(5/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

**fricas** [A] time = 1.16, size = 114, normalized size = 1.75

$$
\left[a^{\frac{5}{2}}\log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a} + 2a}{x}\right) + \frac{2}{15}\left(3b^2x^2 + 11abx + 23a^2\right)\sqrt{bx+a},\ 2\sqrt{-a}\,a^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \frac{2}{15}\left(3b^2x^2 + 11abx + 23a^2\right)\sqrt{bx+a}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(5/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/15*(3*b^2*x^2 + 1
1*a*b*x + 23*a^2)*sqrt(b*x + a), 2*sqrt(-a)*a^2*arctan(sqrt(b*x + a)*sqrt(-
a)/a) + 2/15*(3*b^2*x^2 + 11*a*b*x + 23*a^2)*sqrt(b*x + a)]
```

**giac [A]** time = 1.03, size = 56, normalized size = 0.86

$$\frac{2\,a^{3}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2}{5}\,(bx+a)^{\frac{5}{2}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}a+2\,\sqrt{bx+a}\,a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/5*(b*x + a)^(5/2) + 2/3*(
b*x + a)^(3/2)*a + 2*sqrt(b*x + a)*a^2
```

**maple [A]** time = 0.01, size = 50, normalized size = 0.77

$$-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\;a^{2}+\frac{2\,(bx+a)^{\frac{3}{2}}\,a}{3}+\frac{2\,(bx+a)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x,x)
```

```
[Out] 2/3*a*(b*x+a)^(3/2)+2/5*(b*x+a)^(5/2)-2*a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/
2))+2*a^2*(b*x+a)^(1/2)
```

**maxima [A]** time = 3.02, size = 64, normalized size = 0.98

$$a^{\frac{5}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{5}\,(bx+a)^{\frac{5}{2}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}a+2\,\sqrt{bx+a}\,a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] a^(5/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/5*(b*x
 + a)^(5/2) + 2/3*(b*x + a)^(3/2)*a + 2*sqrt(b*x + a)*a^2
```

**mupad [B]** time = 0.05, size = 52, normalized size = 0.80

$$\frac{2\,a\,{\left(a+b\,x\right)}^{3/2}}{3}+\frac{2\,{\left(a+b\,x\right)}^{5/2}}{5}+2\,a^2\,\sqrt{a+b\,x}+a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x,x)
```

```
[Out] (2*a*(a + b*x)^(3/2))/3 + (2*(a + b*x)^(5/2))/5 + 2*a^2*(a + b*x)^(1/2) + a
^(5/2)*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
```

**sympy [A]** time = 4.12, size = 97, normalized size = 1.49

$$\frac{46a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{15}+a^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{22a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}{15}+\frac{2\sqrt{a}\,b^2x^2\sqrt{1+\frac{bx}{a}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x,x)
```

```
[Out] 46*a**(5/2)*sqrt(1 + b*x/a)/15 + a**(5/2)*log(b*x/a) - 2*a**(5/2)*log(sqrt(
1 + b*x/a) + 1) + 22*a**(3/2)*b*x*sqrt(1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**2*
sqrt(1 + b*x/a)/5
```

## 3.305 $\int \frac{(a+bx)^{5/2}}{x^2}\,dx$

**Optimal**. Leaf size=66

$$-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{x}+\frac{5}{3}b(a+bx)^{3/2}+5ab\sqrt{a+bx}$$

**Rubi** [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{x}+\frac{5}{3}b(a+bx)^{3/2}+5ab\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] 5*a*b*Sqrt[a + b*x] + (5*b*(a + b*x)^(3/2))/3 - (a + b*x)^(5/2)/x - 5*a^(3/
2)*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^2}\,dx &= -\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}(5b)\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= \frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}(5ab)\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}\left(5a^2b\right)\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right) \\
&= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.50

$$\frac{2b(a+bx)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx}{a}+1\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] (2*b*(a + b*x)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + (b*x)/a])/(7*a^2)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 64, normalized size = 0.97

$$\frac{\sqrt{a+bx}\left(-15a^2+10a(a+bx)+2(a+bx)^2\right)}{3x}-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x]*(-15*a^2 + 10*a*(a + b*x) + 2*(a + b*x)^2))/(3*x) - 5*a^(3/2
)*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

**fricas [A]** time = 0.80, size = 126, normalized size = 1.91

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(2\,b^2x^2+14\,abx-3\,a^2\right)\sqrt{bx+a}}{6\,x},\ \frac{15\,\sqrt{-a}\,abx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(2\,b^2x^2+14\,abx-3\,a^2\right)\sqrt{bx+a}}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b^
2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x, 1/3*(15*sqrt(-a)*a*b*x*arctan(s
qrt(b*x + a)*sqrt(-a)/a) + (2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x]
```

**giac [A]** time = 1.09, size = 74, normalized size = 1.12

$$\frac{\frac{15\,a^2b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\,(bx+a)^{\frac{3}{2}}b^2+12\,\sqrt{bx+a}\,ab^2-\frac{3\,\sqrt{bx+a}\,a^2b}{x}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*a^2*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*(b*x + a)^(3/2)
*b^2 + 12*sqrt(b*x + a)*a*b^2 - 3*sqrt(b*x + a)*a^2*b/x)/b
```

**maple [A]** time = 0.01, size = 61, normalized size = 0.92

$$2\left(\left(-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)a^2+2\sqrt{bx+a}\;a+\frac{(bx+a)^{\frac{3}{2}}}{3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^2,x)
```

```
[Out] 2*b*(1/3*(b*x+a)^(3/2)+2*(b*x+a)^(1/2)*a+a^2*(-1/2*(b*x+a)^(1/2)/b/x-5/2*ar
ctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

**maxima [A]** time = 2.94, size = 71, normalized size = 1.08

$$\frac{5}{2}\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{bx+a}\,-\sqrt{a}}{\sqrt{bx+a}\,+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}b+4\,\sqrt{bx+a}\,ab-\frac{\sqrt{bx+a}\,a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/2*a^(3/2)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
3*(b*x + a)^(3/2)*b + 4*sqrt(b*x + a)*a*b - sqrt(b*x + a)*a^2/x
```

**mupad [B]** time = 0.11, size = 58, normalized size = 0.88

$$\frac{2\,b\,{\left(a+b\,x\right)}^{3/2}}{3}-\frac{a^2\,\sqrt{a+b\,x}}{x}+4\,a\,b\,\sqrt{a+b\,x}+a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^2,x)
```

```
[Out] (2*b*(a + b*x)^(3/2))/3 - (a^2*(a + b*x)^(1/2))/x + a^(3/2)*b*atan(((a + b*
x)^(1/2)*1i)/a^(1/2))*5i + 4*a*b*(a + b*x)^(1/2)
```

**sympy [A]** time = 3.75, size = 99, normalized size = 1.50

$$-\frac{a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{x}+\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{bx}{a}}}{3}+\frac{5a^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}-5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{2\sqrt{a}\,b^2x\sqrt{1+\frac{bx}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**2,x)
```

```
[Out] -a**(5/2)*sqrt(1 + b*x/a)/x + 14*a**(3/2)*b*sqrt(1 + b*x/a)/3 + 5*a**(3/2)*
b*log(b*x/a)/2 - 5*a**(3/2)*b*log(sqrt(1 + b*x/a) + 1) + 2*sqrt(a)*b**2*x*s
qrt(1 + b*x/a)/3
```

## 3.306 $\int \frac{(a+bx)^{5/2}}{x^3}\,dx$

**Optimal**. Leaf size=78

$$\frac{15}{4}b^2\sqrt{a+bx}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{2x^2}-\frac{5b(a+bx)^{3/2}}{4x}$$

**Rubi** [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$\frac{15}{4}b^2\sqrt{a+bx}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{2x^2}-\frac{5b(a+bx)^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] (15*b^2*Sqrt[a + b*x])/4 - (5*b*(a + b*x)^(3/2))/(4*x) - (a + b*x)^(5/2)/(2
*x^2) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^3}\,dx &= -\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{4}(5b)\int \frac{(a+bx)^{3/2}}{x^2}\,dx\\
&= -\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15ab^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{4}(15ab)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.45

$$-\frac{2b^2(a+bx)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{bx}{a}+1\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] (-2*b^2*(a + b*x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 + (b*x)/a])/(7*a^3
)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 68, normalized size = 0.87

$$\frac{\sqrt{a+bx}\,\left(15a^2-25a(a+bx)+8(a+bx)^2\right)}{4x^2}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x]*(15*a^2 - 25*a*(a + b*x) + 8*(a + b*x)^2))/(4*x^2) - (15*Sqr
t[a]*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

**fricas [A]** time = 1.04, size = 133, normalized size = 1.71

$$\left[\frac{15\,\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(8\,b^2x^2-9\,abx-2\,a^2\right)\sqrt{bx+a}}{8\,x^2}, \frac{15\,\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(8\,b^2x^2-9\,abx-2\,a^2\right)\sqrt{bx+a}}{4\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x + a))/x^2, 1/4*(15*sqrt(-a)*b^2*x^2*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x +
a))/x^2]
```

**giac [A]** time = 1.12, size = 80, normalized size = 1.03

$$\frac{\frac{15\,ab^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\,\sqrt{bx+a}\,b^3-\frac{9\,(bx+a)^{\frac{3}{2}}\,ab^3-7\,\sqrt{bx+a}\,a^2b^3}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(15*a*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*sqrt(b*x + a)*b^3
 - (9*(b*x + a)^(3/2)*a*b^3 - 7*sqrt(b*x + a)*a^2*b^3)/(b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 61, normalized size = 0.78

$$2\left(\left(-\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}+\frac{\frac{7\,\sqrt{bx+a}\,a}{8}-\frac{9(bx+a)^{\frac{3}{2}}}{8}}{b^2x^2}\right)a+\sqrt{bx+a}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*((b*x+a)^(1/2)+a*((-9/8*(b*x+a)^(3/2)+7/8*(b*x+a)^(1/2)*a)/x^2/b^2-15
/8*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

**maxima** [A] time = 2.94, size = 101, normalized size = 1.29

$$\frac{15}{8}\sqrt{a}\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b^2-\frac{9\,(bx+a)^{\frac{3}{2}}ab^2-7\sqrt{bx+a}\,a^2b^2}{4\left((bx+a)^2-2\,(bx+a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 15/8*sqrt(a)*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) +
 2*sqrt(b*x + a)*b^2 - 1/4*(9*(b*x + a)^(3/2)*a*b^2 - 7*sqrt(b*x + a)*a^2*b
^2)/((b*x + a)^2 - 2*(b*x + a)*a + a^2)
```

**mupad** [B] time = 0.05, size = 64, normalized size = 0.82

$$2\,b^2\sqrt{a+b\,x}+\frac{7\,a^2\sqrt{a+b\,x}}{4\,x^2}-\frac{9\,a\,(a+b\,x)^{3/2}}{4\,x^2}+\frac{\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)15\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(a + b*x)^(1/2) + (7*a^2*(a + b*x)^(1/2))/(4*x^2) + (a^(1/2)*b^2*atan
(((a + b*x)^(1/2)*1i)/a^(1/2))*15i)/4 - (9*a*(a + b*x)^(3/2))/(4*x^2)
```

**sympy** [A] time = 4.30, size = 126, normalized size = 1.62

$$-\frac{15\sqrt{a}\,b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4}-\frac{a^3}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{ab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**3,x)
```

```
[Out] -15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**3/(2*sqrt(b)*x**(5
/2)*sqrt(a/(b*x) + 1)) - 11*a**2*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) - a
*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(a/(b*x) +
 1)
```

## 3.307 $\int \frac{(a+bx)^{5/2}}{x^4} \, dx$

**Optimal**. Leaf size=81

$$-\frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{5b^2\sqrt{a+bx}}{8x}-\frac{(a+bx)^{5/2}}{3x^3}-\frac{5b(a+bx)^{3/2}}{12x^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x])/(8*x) - (5*b*(a + b*x)^(3/2))/(12*x^2) - (a + b*x)^(
5/2)/(3*x^3) - (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^4}\,dx &= -\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{6}(5b)\int \frac{(a+bx)^{3/2}}{x^3}\,dx\\
&= -\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{8}\left(5b^2\right)\int \frac{\sqrt{a+bx}}{x^2}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{16}\left(5b^3\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{8}\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\
&= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 79, normalized size = 0.98

$$-\frac{8a^3+34a^2bx+15b^3x^3\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+59ab^2x^2+33b^3x^3}{24x^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] -1/24*(8*a^3 + 34*a^2*b*x + 59*a*b^2*x^2 + 33*b^3*x^3 + 15*b^3*x^3*Sqrt[1 +
 (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/(x^3*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.14, size = 68, normalized size = 0.84

$$-\frac{\sqrt{a+bx}\,\left(15a^2-40a(a+bx)+33(a+bx)^2\right)}{24x^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(15*a^2 - 40*a*(a + b*x) + 33*(a + b*x)^2))/x^3 - (5*b
^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*Sqrt[a])
```

**fricas** [A] time = 0.99, size = 146, normalized size = 1.80

$$\left[\frac{15\,\sqrt{a}\,b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\left(33\,ab^2x^2+26\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{48\,ax^3},\frac{15\,\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-\left(33\,ab^2x^2+26\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{24\,ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a*x^3), 1/24*(15*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (33*a*b^2*x^2 + 26*a^2*b*x +
8*a^3)*sqrt(b*x + a))/(a*x^3)]
```

**giac** [A] time = 1.13, size = 79, normalized size = 0.98

$$\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{33\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+15\,\sqrt{bx+a}\,a^2b^4}{b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (33*(b*x + a)^(5/2)*
b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 15*sqrt(b*x + a)*a^2*b^4)/(b^3*x^3))/b
```

**maple** [A] time = 0.01, size = 63, normalized size = 0.78

$$2\left(-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}+\frac{-\frac{5\sqrt{bx+a}\,a^2}{16}+\frac{5(bx+a)^{\frac{3}{2}}a}{6}-\frac{11(bx+a)^{\frac{5}{2}}}{16}}{b^3x^3}\right)b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^4,x)
```

```
[Out] 2*b^3*((-11/16*(b*x+a)^(5/2)+5/6*(b*x+a)^(3/2)*a-5/16*(b*x+a)^(1/2)*a^2)/x^
3/b^3-5/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

**maxima** [A] time = 2.99, size = 115, normalized size = 1.42

$$\frac{5\,b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,\sqrt{a}}-\frac{33\,(bx+a)^{\frac{5}{2}}b^3-40\,(bx+a)^{\frac{3}{2}}ab^3+15\,\sqrt{bx+a}\,a^2b^3}{24\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) -
 1/24*(33*(b*x + a)^(5/2)*b^3 - 40*(b*x + a)^(3/2)*a*b^3 + 15*sqrt(b*x + a)
*a^2*b^3)/((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2 - a^3)
```

**mupad [B]** time = 0.05, size = 64, normalized size = 0.79

$$\frac{5\,a\,{\left(a+b\,x\right)}^{3/2}}{3\,x^3}-\frac{5\,a^2\,\sqrt{a+b\,x}}{8\,x^3}-\frac{11\,{\left(a+b\,x\right)}^{5/2}}{8\,x^3}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{8\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^4,x)
```

```
[Out] (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(8*a^(1/2)) - (5*a^2*(a + b*x)^
(1/2))/(8*x^3) - (11*(a + b*x)^(5/2))/(8*x^3) + (5*a*(a + b*x)^(3/2))/(3*x^
3)
```

**sympy [A]** time = 5.16, size = 104, normalized size = 1.28

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x^{\frac{5}{2}}}-\frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{12x^{\frac{3}{2}}}-\frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{8\sqrt{x}}-\frac{5b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**4,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x**(5/2)) - 13*a*b**(3/2)*sqrt(a/(b*x) +
 1)/(12*x**(3/2)) - 11*b**(5/2)*sqrt(a/(b*x) + 1)/(8*sqrt(x)) - 5*b**3*asin
h(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*sqrt(a))
```

## 3.308 $\int \frac{(a+bx)^{5/2}}{x^5} \, dx$

**Optimal**. Leaf size=103

$$\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}} - \frac{5b^3\sqrt{a+bx}}{64ax} - \frac{5b^2\sqrt{a+bx}}{32x^2} - \frac{(a+bx)^{5/2}}{4x^4} - \frac{5b(a+bx)^{3/2}}{24x^3}$$

**Rubi [A]** time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}} - \frac{5b^2\sqrt{a+bx}}{32x^2} - \frac{5b^3\sqrt{a+bx}}{64ax} - \frac{5b(a+bx)^{3/2}}{24x^3} - \frac{(a+bx)^{5/2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^5,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x])/(32*x^2) - (5*b^3*Sqrt[a + b*x])/(64*a*x) - (5*b*(a
+ b*x)^(3/2))/(24*x^3) - (a + b*x)^(5/2)/(4*x^4) + (5*b^4*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(64*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/2}}{x^5}\,dx &= -\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{8}(5b)\int \frac{(a+bx)^{3/2}}{x^4}\,dx\\
&= -\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{16}\left(5b^2\right)\int \frac{\sqrt{a+bx}}{x^3}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{64}\left(5b^3\right)\int \frac{1}{x^2\sqrt{a+bx}}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{\left(5b^4\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{128a}\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{\left(5b^3\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{\right.}{64a}\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.34

$$-\frac{2b^4(a+bx)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{bx}{a}+1\right)}{7a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^5,x]
```

```
[Out] (-2*b^4*(a + b*x)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, 1 + (b*x)/a])/(7*a^5
)
```

**IntegrateAlgebraic [A]** time = 0.15, size = 83, normalized size = 0.81

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}-\frac{\sqrt{a+bx}\,\left(15a^3-55a^2(a+bx)+73a(a+bx)^2+15(a+bx)^3\right)}{192ax^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^5,x]
```

```
[Out] -1/192*(Sqrt[a + b*x]*(15*a^3 - 55*a^2*(a + b*x) + 73*a*(a + b*x)^2 + 15*(a
 + b*x)^3))/(a*x^4) + (5*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(3/2))
```

**fricas [A]** time = 0.96, size = 167, normalized size = 1.62

$$\left[\frac{15\,\sqrt{a}\,b^4x^4\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\left(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48\,a^4\right)\sqrt{bx+a}}{384\,a^2x^4},-\frac{15\,\sqrt{-a}\,b^4x^4\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48\,a^4\right)\sqrt{bx+a}}{192\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*x^4*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^
2*x^4), -1/192*(15*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*
a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^2*x^4
)]
```

**giac [A]** time = 1.09, size = 99, normalized size = 0.96

$$-\frac{\frac{15\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{15\,(bx+a)^{\frac{7}{2}}b^5+73\,(bx+a)^{\frac{5}{2}}ab^5-55\,(bx+a)^{\frac{3}{2}}a^2b^5+15\,\sqrt{bx+a}\,a^3b^5}{ab^4x^4}}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/192*(15*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (15*(b*x + a)^
(7/2)*b^5 + 73*(b*x + a)^(5/2)*a*b^5 - 55*(b*x + a)^(3/2)*a^2*b^5 + 15*sqrt
(b*x + a)*a^3*b^5)/(a*b^4*x^4))/b
```

**maple [A]** time = 0.01, size = 75, normalized size = 0.73

$$2\left(\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128a^{\frac{3}{2}}}+\frac{-\frac{5\sqrt{bx+a}\,a^2}{128}+\frac{55(bx+a)^{\frac{3}{2}}a}{384}-\frac{5(bx+a)^{\frac{7}{2}}}{128a}-\frac{73(bx+a)^{\frac{5}{2}}}{384}}{b^4x^4}\right)b^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^5,x)
```

```
[Out] 2*b^4*((-5/128/a*(b*x+a)^(7/2)-73/384*(b*x+a)^(5/2)+55/384*(b*x+a)^(3/2)*a-
5/128*(b*x+a)^(1/2)*a^2)/x^4/b^4+5/128*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/
2))
```

**maxima [A]** time = 2.93, size = 144, normalized size = 1.40

$$-\frac{5\,b^4\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{128\,a^{\frac{3}{2}}}-\frac{15\,(bx+a)^{\frac{7}{2}}b^4+73\,(bx+a)^{\frac{5}{2}}ab^4-55\,(bx+a)^{\frac{3}{2}}a^2b^4+15\,\sqrt{bx+a}\,a^3b^4}{192\left((bx+a)^4a-4\,(bx+a)^3a^2+6\,(bx+a)^2a^3-4\,(bx+a)a^4+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] -5/128*b^4*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
 - 1/192*(15*(b*x + a)^(7/2)*b^4 + 73*(b*x + a)^(5/2)*a*b^4 - 55*(b*x + a)^
(3/2)*a^2*b^4 + 15*sqrt(b*x + a)*a^3*b^4)/((b*x + a)^4*a - 4*(b*x + a)^3*a^
2 + 6*(b*x + a)^2*a^3 - 4*(b*x + a)*a^4 + a^5)
```

**mupad [B]** time = 0.11, size = 79, normalized size = 0.77

$$\frac{55\,a\,(a+b\,x)^{3/2}}{192\,x^4}-\frac{5\,a^2\,\sqrt{a+b\,x}}{64\,x^4}-\frac{5\,(a+b\,x)^{7/2}}{64\,a\,x^4}-\frac{73\,(a+b\,x)^{5/2}}{192\,x^4}-\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{64\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^5,x)
```

```
[Out] (55*a*(a + b*x)^(3/2))/(192*x^4) - (5*a^2*(a + b*x)^(1/2))/(64*x^4) - (5*(a
 + b*x)^(7/2))/(64*a*x^4) - (b^4*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(64
*a^(3/2)) - (73*(a + b*x)^(5/2))/(192*x^4)
```

**sympy [A]** time = 8.36, size = 155, normalized size = 1.50

$$-\frac{a^3}{4\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{23a^2\sqrt{b}}{24x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{127ab^{\frac{3}{2}}}{96x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{133b^{\frac{5}{2}}}{192x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{7}{2}}}{64a\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{5b^4\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{64a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**5,x)
```

```
[Out] -a**3/(4*sqrt(b)*x**(9/2)*sqrt(a/(b*x) + 1)) - 23*a**2*sqrt(b)/(24*x**(7/2)
*sqrt(a/(b*x) + 1)) - 127*a*b**(3/2)/(96*x**(5/2)*sqrt(a/(b*x) + 1)) - 133*
b**(5/2)/(192*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(7/2)/(64*a*sqrt(x)*sqrt(a
/(b*x) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(64*a**(3/2))
```

## 3.309 $\int x^7(a+bx)^{9/2}\,dx$

**Optimal**. Leaf size=146

$$-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{2a^2(a+bx)^{21/2}}{b^8}+\frac{2(a}{}$$

**Rubi [A]** time = 0.04, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{21/2}}{b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{2(a+bx)^{25/2}}{25b^8}-\frac{14a(a+bx)^{23/2}}{23b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^6*(a + b*x)^(13/2))/(13*b^8) - (
14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4*(a + b*x)^(17/2))/(17*b^8) - (70
*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*(a + b*x)^(21/2))/b^8 - (14*a*(a +
 b*x)^(23/2))/(23*b^8) + (2*(a + b*x)^(25/2))/(25*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx)^{9/2}\,dx &= \int\left(-\frac{a^7(a+bx)^{9/2}}{b^7}+\frac{7a^6(a+bx)^{11/2}}{b^7}-\frac{21a^5(a+bx)^{13/2}}{b^7}+\frac{35a^4(a+bx)^{15/2}}{b^7}-\frac{35a^3(a+bx}{b^7}\right.\\ &= -\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{70a^3(a+bx}{19b^8}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 90, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(-2048a^7+11264a^6bx-36608a^5b^2x^2+91520a^4b^3x^3-194480a^3b^4x^4+369512a^2b^5x^5-646646ab^6x^6+1062347b^7x^7\right)}{26558675b^8}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*(a + b*x)^(9/2),x]

[Out] (2*(a + b*x)^(11/2)*(-2048*a^7 + 11264*a^6*b*x - 36608*a^5*b^2*x^2 + 91520*a^4*b^3*x^3 - 194480*a^3*b^4*x^4 + 369512*a^2*b^5*x^5 - 646646*a*b^6*x^6 + 1062347*b^7*x^7))/(26558675*b^8)

**IntegrateAlgebraic [A]** time = 0.04, size = 115, normalized size = 0.79

$$-\frac{2\left(2414425a^7(a+bx)^{11/2}-14300825a^6(a+bx)^{13/2}+37182145a^5(a+bx)^{15/2}-54679625a^4(a+bx)^{17/2}+48923875a^3(a+bx)^{19/2}-26558675a^2(a+bx)^{21/2}-1062347(a+bx)^{25/2}+8083075a(a+bx)^{23/2}\right)}{26558675b^8}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7*(a + b*x)^(9/2),x]

[Out] (-2*(2414425*a^7*(a + b*x)^(11/2) - 14300825*a^6*(a + b*x)^(13/2) + 37182145*a^5*(a + b*x)^(15/2) - 54679625*a^4*(a + b*x)^(17/2) + 48923875*a^3*(a + b*x)^(19/2) - 26558675*a^2*(a + b*x)^(21/2) + 8083075*a*(a + b*x)^(23/2) - 1062347*(a + b*x)^(25/2)))/(26558675*b^8)

**fricas [A]** time = 1.02, size = 141, normalized size = 0.97

$$\frac{2\left(1062347\,b^{12}x^{12}+4665089\,ab^{11}x^{11}+7759752\,a^2b^{10}x^{10}+5810090\,a^3b^9x^9+1659515\,a^4b^8x^8+429\,a^5b^7x^7-462\,a^6b^6x^6+504\,a^7b^5x^5-560\,a^8b^4x^4+640\,a^9b^3x^3-768\,a^{10}b^2x^2+1024\,a^{11}bx-2048\,a^{12}\right)\sqrt{bx+a}}{26558675\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="fricas")

[Out] 2/26558675*(1062347*b^12*x^12 + 4665089*a*b^11*x^11 + 7759752*a^2*b^10*x^10 + 5810090*a^3*b^9*x^9 + 1659515*a^4*b^8*x^8 + 429*a^5*b^7*x^7 - 462*a^6*b^6*x^6 + 504*a^7*b^5*x^5 - 560*a^8*b^4*x^4 + 640*a^9*b^3*x^3 - 768*a^10*b^2*x^2 + 1024*a^11*b*x - 2048*a^12)*sqrt(b*x + a)/b^8

**giac [B]** time = 1.14, size = 781, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="giac")

[Out] 2/1673196525*(260015*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^5/b^7 + 76475*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^4/b^7 + 72450*(12155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230

```
*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2
)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a^3/b^7 + 17
250*(46189*(b*x + a)^(21/2) - 510510*(b*x + a)^(19/2)*a + 2567565*(b*x + a)
^(17/2)*a^2 - 7759752*(b*x + a)^(15/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4
- 22221108*(b*x + a)^(11/2)*a^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(
b*x + a)^(7/2)*a^7 + 8729721*(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*
a^9 + 969969*sqrt(b*x + a)*a^10)*a^2/b^7 + 4125*(88179*(b*x + a)^(23/2) - 1
062347*(b*x + a)^(21/2)*a + 5870865*(b*x + a)^(19/2)*a^2 - 19684665*(b*x +
a)^(17/2)*a^3 + 44618574*(b*x + a)^(15/2)*a^4 - 72076158*(b*x + a)^(13/2)*a
^5 + 85180914*(b*x + a)^(11/2)*a^6 - 74364290*(b*x + a)^(9/2)*a^7 + 4780561
5*(b*x + a)^(7/2)*a^8 - 22309287*(b*x + a)^(5/2)*a^9 + 7436429*(b*x + a)^(3
/2)*a^10 - 2028117*sqrt(b*x + a)*a^11)*a/b^7 + 99*(676039*(b*x + a)^(25/2)
- 8817900*(b*x + a)^(23/2)*a + 53117350*(b*x + a)^(21/2)*a^2 - 195695500*(b
*x + a)^(19/2)*a^3 + 492116625*(b*x + a)^(17/2)*a^4 - 892371480*(b*x + a)^(
15/2)*a^5 + 1201269300*(b*x + a)^(13/2)*a^6 - 1216870200*(b*x + a)^(11/2)*a
^7 + 929553625*(b*x + a)^(9/2)*a^8 - 531173500*(b*x + a)^(7/2)*a^9 + 223092
870*(b*x + a)^(5/2)*a^10 - 67603900*(b*x + a)^(3/2)*a^11 + 16900975*sqrt(b*
x + a)*a^12)/b^7)/b
```

**maple [A]** time = 0.01, size = 87, normalized size = 0.60

$$-\frac{2 (bx+a)^{\frac{11}{2}} \left(-1062347 b^7 x^7+646646 a b^6 x^6-369512 a^2 b^5 x^5+194480 a^3 b^4 x^4-91520 a^4 b^3 x^3+36608 a^5 b^2 x^2-11264 a^6 b x+2048 a^7\right)}{26558675 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^(9/2),x)
```

```
[Out] -2/26558675*(b*x+a)^(11/2)*(-1062347*b^7*x^7+646646*a*b^6*x^6-369512*a^2*b^
5*x^5+194480*a^3*b^4*x^4-91520*a^4*b^3*x^3+36608*a^5*b^2*x^2-11264*a^6*b*x+
2048*a^7)/b^8
```

**maxima [A]** time = 1.33, size = 116, normalized size = 0.79

$$\frac{2 (bx+a)^{\frac{25}{2}}}{25 b^8}-\frac{14 (bx+a)^{\frac{23}{2}} a}{23 b^8}+\frac{2 (bx+a)^{\frac{21}{2}} a^2}{b^8}-\frac{70 (bx+a)^{\frac{19}{2}} a^3}{19 b^8}+\frac{70 (bx+a)^{\frac{17}{2}} a^4}{17 b^8}-\frac{14 (bx+a)^{\frac{15}{2}} a^5}{5 b^8}+\frac{14 (bx+a)^{\frac{13}{2}} a^6}{13 b^8}-\frac{2 (bx+a)^{\frac{11}{2}} a^7}{11 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/25*(b*x + a)^(25/2)/b^8 - 14/23*(b*x + a)^(23/2)*a/b^8 + 2*(b*x + a)^(21/
2)*a^2/b^8 - 70/19*(b*x + a)^(19/2)*a^3/b^8 + 70/17*(b*x + a)^(17/2)*a^4/b^
8 - 14/5*(b*x + a)^(15/2)*a^5/b^8 + 14/13*(b*x + a)^(13/2)*a^6/b^8 - 2/11*(
b*x + a)^(11/2)*a^7/b^8
```

**mupad [B]** time = 0.04, size = 116, normalized size = 0.79

$$\frac{2 (a+b x)^{25/2}}{25 b^8}-\frac{2 a^7 (a+b x)^{11/2}}{11 b^8}+\frac{14 a^6 (a+b x)^{13/2}}{13 b^8}-\frac{14 a^5 (a+b x)^{15/2}}{5 b^8}+\frac{70 a^4 (a+b x)^{17/2}}{17 b^8}-\frac{70 a^3 (a+b x)^{19/2}}{19 b^8}+\frac{2 a^2 (a+b x)^{21/2}}{b^8}-\frac{14 a (a+b x)^{23/2}}{23 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(25/2))/(25*b^8) - (2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^6
*(a + b*x)^(13/2))/(13*b^8) - (14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4*(
a + b*x)^(17/2))/(17*b^8) - (70*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*(a
+ b*x)^(21/2))/b^8 - (14*a*(a + b*x)^(23/2))/(23*b^8)
```

**sympy [A]** time = 40.30, size = 279, normalized size = 1.91

$$\begin{cases} -\frac{4096a^{12}\sqrt{a+bx}}{26558675b^8} + \frac{2048a^{11}x\sqrt{a+bx}}{26558675b^7} - \frac{1536a^{10}x^2\sqrt{a+bx}}{26558675b^6} + \frac{256a^9x^3\sqrt{a+bx}}{5311735b^5} - \frac{224a^8x^4\sqrt{a+bx}}{5311735b^4} + \frac{1008a^7x^5\sqrt{a+bx}}{26558675b^3} - \frac{84a^6x^6\sqrt{a+bx}}{2414425b^2} + \frac{6a^5x^7\sqrt{a+bx}}{185725b} + \frac{4642a^4x^8\sqrt{a+bx}}{37145} + \frac{956a^3bx^9\sqrt{a+bx}}{2185} + \frac{336a^2b^2x^{10}\sqrt{a+bx}}{575} + \frac{202ab^3x^{11}\sqrt{a+bx}}{575} + \frac{2b^4x^{12}\sqrt{a+bx}}{25} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-4096*a**12*sqrt(a + b*x)/(26558675*b**8) + 2048*a**11*x*sqrt(a
+ b*x)/(26558675*b**7) - 1536*a**10*x**2*sqrt(a + b*x)/(26558675*b**6) + 25
6*a**9*x**3*sqrt(a + b*x)/(5311735*b**5) - 224*a**8*x**4*sqrt(a + b*x)/(531
1735*b**4) + 1008*a**7*x**5*sqrt(a + b*x)/(26558675*b**3) - 84*a**6*x**6*sq
rt(a + b*x)/(2414425*b**2) + 6*a**5*x**7*sqrt(a + b*x)/(185725*b) + 4642*a*
*4*x**8*sqrt(a + b*x)/37145 + 956*a**3*b*x**9*sqrt(a + b*x)/2185 + 336*a**2
*b**2*x**10*sqrt(a + b*x)/575 + 202*a*b**3*x**11*sqrt(a + b*x)/575 + 2*b**4
*x**12*sqrt(a + b*x)/25, Ne(b, 0)), (a**(9/2)*x**8/8, True))
```

## 3.310 $\int x^6(a+bx)^{9/2}\,dx$

**Optimal**. Leaf size=127

$$\frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a+bx)^{19/2}}{19b^7}+\frac{2(a+bx)^{23/2}}{23b^7}-\frac{4a(a+b}{7b^7}$$

**Rubi** [A] time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{30a^2(a+bx)^{19/2}}{19b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^6(a+bx)^{11/2}}{11b^7}+\frac{2(a+bx)^{23/2}}{23b^7}-\frac{4a(a+bx)^{21/2}}{7b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^5*(a + b*x)^(13/2))/(13*b^7) + (2
*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a + b*x)^(17/2))/(17*b^7) + (30*a^2*(
a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*x)^(21/2))/(7*b^7) + (2*(a + b*x)^(
23/2))/(23*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^{9/2}\,dx &= \int \left(\frac{a^6(a+bx)^{9/2}}{b^6}-\frac{6a^5(a+bx)^{11/2}}{b^6}+\frac{15a^4(a+bx)^{13/2}}{b^6}-\frac{20a^3(a+bx)^{15/2}}{b^6}+\frac{15a^2(a+bx)^{1}}{b^6}\right.\\ &= \frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a+bx)^{19/}}{19b^7}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 79, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(1024a^6-5632a^5bx+18304a^4b^2x^2-45760a^3b^3x^3+97240a^2b^4x^4-184756ab^5x^5+323323b^6x^6\right)}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(1024*a^6 - 5632*a^5*b*x + 18304*a^4*b^2*x^2 - 45760*a^
3*b^3*x^3 + 97240*a^2*b^4*x^4 - 184756*a*b^5*x^5 + 323323*b^6*x^6))/(743642
9*b^7)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 101, normalized size = 0.80

$$\frac{2\left(676039a^6(a+bx)^{11/2}-3432198a^5(a+bx)^{13/2}+7436429a^4(a+bx)^{15/2}-8748740a^3(a+bx)^{17/2}+5870865a^2(a+bx)^{19/2}+323323(a+bx)^{23/2}-2124694a(a+bx)^{21/2}\right)}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*(676039*a^6*(a + b*x)^(11/2) - 3432198*a^5*(a + b*x)^(13/2) + 7436429*a^
4*(a + b*x)^(15/2) - 8748740*a^3*(a + b*x)^(17/2) + 5870865*a^2*(a + b*x)^(
19/2) - 2124694*a*(a + b*x)^(21/2) + 323323*(a + b*x)^(23/2)))/(7436429*b^7
)
```

**fricas [A]** time = 0.73, size = 130, normalized size = 1.02

$$\frac{2\left(323323\,b^{11}x^{11}+1431859\,ab^{10}x^{10}+2406690\,a^2b^9x^9+1826110\,a^3b^8x^8+530959\,a^4b^7x^7+231\,a^5b^6x^6-252\,a^6b^5x^5+280\,a^7b^4x^4-320\,a^8b^3x^3+384\,a^9b^2x^2-512\,a^{10}bx+1024\,a^{11}\right)\sqrt{bx+a}}{7436429\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/7436429*(323323*b^11*x^11 + 1431859*a*b^10*x^10 + 2406690*a^2*b^9*x^9 + 1
826110*a^3*b^8*x^8 + 530959*a^4*b^7*x^7 + 231*a^5*b^6*x^6 - 252*a^6*b^5*x^5
 + 280*a^7*b^4*x^4 - 320*a^8*b^3*x^3 + 384*a^9*b^2*x^2 - 512*a^10*b*x + 102
4*a^11)*sqrt(b*x + a)/b^7
```

**giac [B]** time = 1.32, size = 709, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/66927861*(22287*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b
*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6
006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^5/b^6 + 52003*(429*(b*x
 + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025
*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^
5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^4/b^6 + 6118*(643
5*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2
 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x +
```

```
a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 1
09395*sqrt(b*x + a)*a^8)*a^3/b^6 + 2898*(12155*(b*x + a)^(19/2) - 122265*(b
*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a
^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(
b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a
^8 - 230945*sqrt(b*x + a)*a^9)*a^2/b^6 + 345*(46189*(b*x + a)^(21/2) - 5105
10*(b*x + a)^(19/2)*a + 2567565*(b*x + a)^(17/2)*a^2 - 7759752*(b*x + a)^(1
5/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4 - 22221108*(b*x + a)^(11/2)*a^5 +
22632610*(b*x + a)^(9/2)*a^6 - 16628040*(b*x + a)^(7/2)*a^7 + 8729721*(b*x
+ a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*a^9 + 969969*sqrt(b*x + a)*a^10)*a
/b^6 + 33*(88179*(b*x + a)^(23/2) - 1062347*(b*x + a)^(21/2)*a + 5870865*(b
*x + a)^(19/2)*a^2 - 19684665*(b*x + a)^(17/2)*a^3 + 44618574*(b*x + a)^(15
/2)*a^4 - 72076158*(b*x + a)^(13/2)*a^5 + 85180914*(b*x + a)^(11/2)*a^6 - 7
4364290*(b*x + a)^(9/2)*a^7 + 47805615*(b*x + a)^(7/2)*a^8 - 22309287*(b*x
+ a)^(5/2)*a^9 + 7436429*(b*x + a)^(3/2)*a^10 - 2028117*sqrt(b*x + a)*a^11)
/b^6)/b
```

**maple [A]** time = 0.01, size = 76, normalized size = 0.60

$$\frac{2\left(bx+a\right)^{\frac{11}{2}}\left(323323x^{6}b^{6}-184756a\,x^{5}b^{5}+97240a^{2}x^{4}b^{4}-45760a^{3}x^{3}b^{3}+18304a^{4}x^{2}b^{2}-5632a^{5}xb+1024a^{6}\right)}{7436429b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^(9/2),x)
```

```
[Out] 2/7436429*(b*x+a)^(11/2)*(323323*b^6*x^6-184756*a*b^5*x^5+97240*a^2*b^4*x^4
-45760*a^3*b^3*x^3+18304*a^4*b^2*x^2-5632*a^5*b*x+1024*a^6)/b^7
```

**maxima [A]** time = 1.34, size = 101, normalized size = 0.80

$$\frac{2\left(bx+a\right)^{\frac{23}{2}}}{23\,b^{7}}-\frac{4\left(bx+a\right)^{\frac{21}{2}}a}{7\,b^{7}}+\frac{30\left(bx+a\right)^{\frac{19}{2}}a^{2}}{19\,b^{7}}-\frac{40\left(bx+a\right)^{\frac{17}{2}}a^{3}}{17\,b^{7}}+\frac{2\left(bx+a\right)^{\frac{15}{2}}a^{4}}{b^{7}}-\frac{12\left(bx+a\right)^{\frac{13}{2}}a^{5}}{13\,b^{7}}+\frac{2\left(bx+a\right)^{\frac{11}{2}}a^{6}}{11\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/23*(b*x + a)^(23/2)/b^7 - 4/7*(b*x + a)^(21/2)*a/b^7 + 30/19*(b*x + a)^(1
9/2)*a^2/b^7 - 40/17*(b*x + a)^(17/2)*a^3/b^7 + 2*(b*x + a)^(15/2)*a^4/b^7
- 12/13*(b*x + a)^(13/2)*a^5/b^7 + 2/11*(b*x + a)^(11/2)*a^6/b^7
```

**mupad [B]** time = 0.03, size = 101, normalized size = 0.80

$$\frac{2\,{\left(a+b\,x\right)}^{23/2}}{23\,b^7}+\frac{2\,a^6\,{\left(a+b\,x\right)}^{11/2}}{11\,b^7}-\frac{12\,a^5\,{\left(a+b\,x\right)}^{13/2}}{13\,b^7}+\frac{2\,a^4\,{\left(a+b\,x\right)}^{15/2}}{b^7}-\frac{40\,a^3\,{\left(a+b\,x\right)}^{17/2}}{17\,b^7}+\frac{30\,a^2\,{\left(a+b\,x\right)}^{19/2}}{19\,b^7}-\frac{4\,a\,{\left(a+b\,x\right)}^{21/2}}{7\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(23/2))/(23*b^7) + (2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^5
*(a + b*x)^(13/2))/(13*b^7) + (2*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a + b
*x)^(17/2))/(17*b^7) + (30*a^2*(a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*x)^
(21/2))/(7*b^7)
```

**sympy [A]** time = 36.61, size = 257, normalized size = 2.02

$$\begin{cases} \frac{2048a^{11}\sqrt{a+bx}}{7436429b^7} - \frac{1024a^{10}x\sqrt{a+bx}}{7436429b^6} + \frac{768a^9x^2\sqrt{a+bx}}{7436429b^5} - \frac{640a^8x^3\sqrt{a+bx}}{7436429b^4} + \frac{80a^7x^4\sqrt{a+bx}}{1062347b^3} - \frac{72a^6x^5\sqrt{a+bx}}{1062347b^2} + \frac{6a^5x^6\sqrt{a+bx}}{96577b} + \frac{7426a^4x^7\sqrt{a+bx}}{52003} + \frac{25540a^3bx^8\sqrt{a+bx}}{52003} + \frac{1980a^2b^2x^9\sqrt{a+bx}}{3059} + \frac{62ab^3x^{10}\sqrt{a+bx}}{161} + \frac{2b^4x^{11}\sqrt{a+bx}}{23} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^7}{7} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((2048*a**11*sqrt(a + b*x)/(7436429*b**7) - 1024*a**10*x*sqrt(a +
b*x)/(7436429*b**6) + 768*a**9*x**2*sqrt(a + b*x)/(7436429*b**5) - 640*a**8
*x**3*sqrt(a + b*x)/(7436429*b**4) + 80*a**7*x**4*sqrt(a + b*x)/(1062347*b*
*3) - 72*a**6*x**5*sqrt(a + b*x)/(1062347*b**2) + 6*a**5*x**6*sqrt(a + b*x)
/(96577*b) + 7426*a**4*x**7*sqrt(a + b*x)/52003 + 25540*a**3*b*x**8*sqrt(a
+ b*x)/52003 + 1980*a**2*b**2*x**9*sqrt(a + b*x)/3059 + 62*a*b**3*x**10*sqr
t(a + b*x)/161 + 2*b**4*x**11*sqrt(a + b*x)/23, Ne(b, 0)), (a**(9/2)*x**7/7
, True))
```

## 3.311 $\int x^5(a+bx)^{9/2}\,dx$

**Optimal**. Leaf size=110

$$-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}+\frac{2(a+bx)^{21/2}}{21b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}$$

**Rubi** [A] time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{2(a+bx)^{21/2}}{21b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^4*(a + b*x)^(13/2))/(13*b^6) - (
4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*(a + b*x)^(17/2))/(17*b^6) - (10*
a*(a + b*x)^(19/2))/(19*b^6) + (2*(a + b*x)^(21/2))/(21*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^{9/2}\,dx &= \int\left(-\frac{a^5(a+bx)^{9/2}}{b^5}+\frac{5a^4(a+bx)^{11/2}}{b^5}-\frac{10a^3(a+bx)^{13/2}}{b^5}+\frac{10a^2(a+bx)^{15/2}}{b^5}-\frac{5a(a+bx)^{17}}{b^5}\right.\\ &= -\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{10a(a+bx)^{19}}{19b^6}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 68, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(-256a^5+1408a^4bx-4576a^3b^2x^2+11440a^2b^3x^3-24310ab^4x^4+46189b^5x^5\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-256*a^5 + 1408*a^4*b*x - 4576*a^3*b^2*x^2 + 11440*a^2
*b^3*x^3 - 24310*a*b^4*x^4 + 46189*b^5*x^5))/(969969*b^6)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 75, normalized size = 0.68

$$\frac{2(a+bx)^{11/2}\left(-88179a^5+373065a^4(a+bx)-646646a^3(a+bx)^2+570570a^2(a+bx)^3-255255a(a+bx)^4+46189(a+bx)^5\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-88179*a^5 + 373065*a^4*(a + b*x) - 646646*a^3*(a + b*
x)^2 + 570570*a^2*(a + b*x)^3 - 255255*a*(a + b*x)^4 + 46189*(a + b*x)^5))/
(969969*b^6)
```

**fricas [A]** time = 0.91, size = 119, normalized size = 1.08

$$\frac{2\left(46189\,b^{10}x^{10}+206635\,ab^{9}x^{9}+351780\,a^{2}b^{8}x^{8}+271414\,a^{3}b^{7}x^{7}+80773\,a^{4}b^{6}x^{6}+63\,a^{5}b^{5}x^{5}-70\,a^{6}b^{4}x^{4}+80\,a^{7}b^{3}x^{3}-96\,a^{8}b^{2}x^{2}+128\,a^{9}bx-256\,a^{10}\right)\sqrt{bx+a}}{969969\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/969969*(46189*b^10*x^10 + 206635*a*b^9*x^9 + 351780*a^2*b^8*x^8 + 271414*
a^3*b^7*x^7 + 80773*a^4*b^6*x^6 + 63*a^5*b^5*x^5 - 70*a^6*b^4*x^4 + 80*a^7*
b^3*x^3 - 96*a^8*b^2*x^2 + 128*a^9*b*x - 256*a^10)*sqrt(b*x + a)/b^6
```

**giac [B]** time = 1.05, size = 637, normalized size = 5.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/2909907*(4199*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a
)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqr
t(b*x + a)*a^5)*a^5/b^5 + 4845*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2
)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^
(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^4/b^5 + 45
22*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)
*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x +
 a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^3/b^5
 + 266*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)
^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 87
```

```
5160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3
/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^2/b^5 + 63*(12155*(b*x + a)^(19/2) -
122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)
^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 +
2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a
)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a/b^5 + 3*(46189*(b*x + a)^(21/2) -
 510510*(b*x + a)^(19/2)*a + 2567565*(b*x + a)^(17/2)*a^2 - 7759752*(b*x +
a)^(15/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4 - 22221108*(b*x + a)^(11/2)*a
^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(b*x + a)^(7/2)*a^7 + 8729721*
(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*a^9 + 969969*sqrt(b*x + a)*a^
10)/b^5)/b
```

**maple [A]** time = 0.01, size = 65, normalized size = 0.59

$$-\frac{2\left(bx+a\right)^{\frac{11}{2}}\left(-46189b^{5}x^{5}+24310a\,b^{4}x^{4}-11440a^{2}b^{3}x^{3}+4576a^{3}b^{2}x^{2}-1408a^{4}bx+256a^{5}\right)}{969969b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^(9/2),x)
```

```
[Out] -2/969969*(b*x+a)^(11/2)*(-46189*b^5*x^5+24310*a*b^4*x^4-11440*a^2*b^3*x^3+
4576*a^3*b^2*x^2-1408*a^4*b*x+256*a^5)/b^6
```

**maxima [A]** time = 1.35, size = 86, normalized size = 0.78

$$\frac{2\left(bx+a\right)^{\frac{21}{2}}}{21\,b^{6}}-\frac{10\left(bx+a\right)^{\frac{19}{2}}a}{19\,b^{6}}+\frac{20\left(bx+a\right)^{\frac{17}{2}}a^{2}}{17\,b^{6}}-\frac{4\left(bx+a\right)^{\frac{15}{2}}a^{3}}{3\,b^{6}}+\frac{10\left(bx+a\right)^{\frac{13}{2}}a^{4}}{13\,b^{6}}-\frac{2\left(bx+a\right)^{\frac{11}{2}}a^{5}}{11\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/21*(b*x + a)^(21/2)/b^6 - 10/19*(b*x + a)^(19/2)*a/b^6 + 20/17*(b*x + a)^
(17/2)*a^2/b^6 - 4/3*(b*x + a)^(15/2)*a^3/b^6 + 10/13*(b*x + a)^(13/2)*a^4/
b^6 - 2/11*(b*x + a)^(11/2)*a^5/b^6
```

**mupad [B]** time = 0.03, size = 86, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{21/2}}{21\,b^6}-\frac{2\,a^5\,{\left(a+b\,x\right)}^{11/2}}{11\,b^6}+\frac{10\,a^4\,{\left(a+b\,x\right)}^{13/2}}{13\,b^6}-\frac{4\,a^3\,{\left(a+b\,x\right)}^{15/2}}{3\,b^6}+\frac{20\,a^2\,{\left(a+b\,x\right)}^{17/2}}{17\,b^6}-\frac{10\,a\,{\left(a+b\,x\right)}^{19/2}}{19\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^(9/2),x)
```

[Out] (2*(a + b*x)^(21/2))/(21*b^6) - (2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^4*(a + b*x)^(13/2))/(13*b^6) - (4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*(a + b*x)^(17/2))/(17*b^6) - (10*a*(a + b*x)^(19/2))/(19*b^6)

**sympy [A]** time = 28.76, size = 235, normalized size = 2.14

$$\begin{cases} -\frac{512a^{10}\sqrt{a+bx}}{969969b^6} + \frac{256a^9x\sqrt{a+bx}}{969969b^5} - \frac{64a^8x^2\sqrt{a+bx}}{323323b^4} + \frac{160a^7x^3\sqrt{a+bx}}{969969b^3} - \frac{20a^6x^4\sqrt{a+bx}}{138567b^2} + \frac{6a^5x^5\sqrt{a+bx}}{46189b} + \frac{2098a^4x^6\sqrt{a+bx}}{12597} + \frac{3796a^3bx^7\sqrt{a+bx}}{6783} + \frac{1640a^2b^2x^8\sqrt{a+bx}}{2261} + \frac{170ab^3x^9\sqrt{a+bx}}{399} + \frac{2b^4x^{10}\sqrt{a+bx}}{21} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(b*x+a)**(9/2),x)

[Out] Piecewise((-512*a**10*sqrt(a + b*x)/(969969*b**6) + 256*a**9*x*sqrt(a + b*x)/(969969*b**5) - 64*a**8*x**2*sqrt(a + b*x)/(323323*b**4) + 160*a**7*x**3*sqrt(a + b*x)/(969969*b**3) - 20*a**6*x**4*sqrt(a + b*x)/(138567*b**2) + 6*a**5*x**5*sqrt(a + b*x)/(46189*b) + 2098*a**4*x**6*sqrt(a + b*x)/12597 + 3796*a**3*b*x**7*sqrt(a + b*x)/6783 + 1640*a**2*b**2*x**8*sqrt(a + b*x)/2261 + 170*a*b**3*x**9*sqrt(a + b*x)/399 + 2*b**4*x**10*sqrt(a + b*x)/21, Ne(b, 0)), (a**(9/2)*x**6/6, True))

## 3.312 $\int x^4(a+bx)^{9/2}\,dx$

**Optimal**. Leaf size=91

$$\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}+\frac{2(a+bx)^{19/2}}{19b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}$$

**Rubi [A]** time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{2a^4(a+bx)^{11/2}}{11b^5}+\frac{2(a+bx)^{19/2}}{19b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3*(a + b*x)^(13/2))/(13*b^5) + (4*
a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a + b*x)^(17/2))/(17*b^5) + (2*(a + b
*x)^(19/2))/(19*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^{9/2}\,dx&=\int\left(\frac{a^4(a+bx)^{9/2}}{b^4}-\frac{4a^3(a+bx)^{11/2}}{b^4}+\frac{6a^2(a+bx)^{13/2}}{b^4}-\frac{4a(a+bx)^{15/2}}{b^4}+\frac{(a+bx)^{17/2}}{b^4}\right)dx\\&=\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}+\frac{2(a+bx)^{19/2}}{19b^5}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 57, normalized size = 0.63

$$\frac{2(a+bx)^{11/2}\left(128a^4-704a^3bx+2288a^2b^2x^2-5720ab^3x^3+12155b^4x^4\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(128*a^4 - 704*a^3*b*x + 2288*a^2*b^2*x^2 - 5720*a*b^3*
x^3 + 12155*b^4*x^4))/(230945*b^5)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 63, normalized size = 0.69

$$\frac{2(a+bx)^{11/2}\left(20995a^4-71060a^3(a+bx)+92378a^2(a+bx)^2-54340a(a+bx)^3+12155(a+bx)^4\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(20995*a^4 - 71060*a^3*(a + b*x) + 92378*a^2*(a + b*x)^
2 - 54340*a*(a + b*x)^3 + 12155*(a + b*x)^4))/(230945*b^5)
```

**fricas [A]** time = 1.07, size = 108, normalized size = 1.19

$$\frac{2\left(12155\,b^{9}x^{9}+55055\,ab^{8}x^{8}+95238\,a^{2}b^{7}x^{7}+75086\,a^{3}b^{6}x^{6}+23063\,a^{4}b^{5}x^{5}+35\,a^{5}b^{4}x^{4}-40\,a^{6}b^{3}x^{3}+48\,a^{7}b^{2}x^{2}-64\,a^{8}bx+128\,a^{9}\right)\sqrt{bx+a}}{230945\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/230945*(12155*b^9*x^9 + 55055*a*b^8*x^8 + 95238*a^2*b^7*x^7 + 75086*a^3*b
^6*x^6 + 23063*a^4*b^5*x^5 + 35*a^5*b^4*x^4 - 40*a^6*b^3*x^3 + 48*a^7*b^2*x
^2 - 64*a^8*b*x + 128*a^9)*sqrt(b*x + a)/b^5
```

**giac [B]** time = 1.12, size = 565, normalized size = 6.21

$$\frac{2\left(\frac{46189\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)a^5}{b^4}+\frac{104975\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)a^4}{b^4}+\frac{48450\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^2-8580(bx+a)^{\frac{7}{2}}a^3+9009(bx+a)^{\frac{5}{2}}a^4-6006(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)a^3}{b^4}+\frac{22610\left(429(bx+a)^{\frac{15}{2}}-3465(bx+a)^{\frac{13}{2}}a+12285(bx+a)^{\frac{11}{2}}a^2-25025(bx+a)^{\frac{9}{2}}a^3+32175(bx+a)^{\frac{7}{2}}a^4-27027(bx+a)^{\frac{5}{2}}a^5+15015(bx+a)^{\frac{3}{2}}a^6-6435\sqrt{bx+a}\,a^7\right)a^2}{b^4}+\frac{665\left(6435(bx+a)^{\frac{17}{2}}-58344(bx+a)^{\frac{15}{2}}a+235620(bx+a)^{\frac{13}{2}}a^2-556920(bx+a)^{\frac{11}{2}}a^3+850850(bx+a)^{\frac{9}{2}}a^4-875160(bx+a)^{\frac{7}{2}}a^5+612612(bx+a)^{\frac{5}{2}}a^6-291720(bx+a)^{\frac{3}{2}}a^7+109395\sqrt{bx+a}\,a^8\right)a}{b^4}+\frac{63\left(12155(bx+a)^{\frac{19}{2}}-122265(bx+a)^{\frac{17}{2}}a+554268(bx+a)^{\frac{15}{2}}a^2-1492260(bx+a)^{\frac{13}{2}}a^3+2645370(bx+a)^{\frac{11}{2}}a^4-3233230(bx+a)^{\frac{9}{2}}a^5+2771340(bx+a)^{\frac{7}{2}}a^6-1662804(bx+a)^{\frac{5}{2}}a^7+692835(bx+a)^{\frac{3}{2}}a^8-230945\sqrt{bx+a}\,a^9\right)}{b^4}\right)}{14549535\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/14549535*(46189*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x +
a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^5/b^4 + 1
04975*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^
2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)
*a^5)*a^4/b^4 + 48450*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 500
5*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4
 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^3/b^4 + 22610*(429*
(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 2
5025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2
)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^2/b^4 + 665*(
```

```
6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*
a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*
x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7
+ 109395*sqrt(b*x + a)*a^8)*a/b^4 + 63*(12155*(b*x + a)^(19/2) - 122265*(b*
x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^
3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b
*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^
8 - 230945*sqrt(b*x + a)*a^9)/b^4)/b
```

**maple** [A] time = 0.01, size = 54, normalized size = 0.59

$$\frac{2\,(bx+a)^{\frac{11}{2}}\left(12155x^4b^4-5720a\,x^3b^3+2288a^2x^2b^2-704a^3xb+128a^4\right)}{230945b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^(9/2),x)
```

```
[Out] 2/230945*(b*x+a)^(11/2)*(12155*b^4*x^4-5720*a*b^3*x^3+2288*a^2*b^2*x^2-704*
a^3*b*x+128*a^4)/b^5
```

**maxima** [A] time = 1.36, size = 71, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{19}{2}}}{19\,b^5}-\frac{8\,(bx+a)^{\frac{17}{2}}a}{17\,b^5}+\frac{4\,(bx+a)^{\frac{15}{2}}a^2}{5\,b^5}-\frac{8\,(bx+a)^{\frac{13}{2}}a^3}{13\,b^5}+\frac{2\,(bx+a)^{\frac{11}{2}}a^4}{11\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/19*(b*x + a)^(19/2)/b^5 - 8/17*(b*x + a)^(17/2)*a/b^5 + 4/5*(b*x + a)^(15
/2)*a^2/b^5 - 8/13*(b*x + a)^(13/2)*a^3/b^5 + 2/11*(b*x + a)^(11/2)*a^4/b^5
```

**mupad** [B] time = 0.02, size = 71, normalized size = 0.78

$$\frac{2\,(a+b\,x)^{19/2}}{19\,b^5}+\frac{2\,a^4\,(a+b\,x)^{11/2}}{11\,b^5}-\frac{8\,a^3\,(a+b\,x)^{13/2}}{13\,b^5}+\frac{4\,a^2\,(a+b\,x)^{15/2}}{5\,b^5}-\frac{8\,a\,(a+b\,x)^{17/2}}{17\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(19/2))/(19*b^5) + (2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3*
(a + b*x)^(13/2))/(13*b^5) + (4*a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a + b
*x)^(17/2))/(17*b^5)
```

**sympy [A]** time = 25.70, size = 212, normalized size = 2.33

$$\begin{cases} \frac{256a^9\sqrt{a+bx}}{230945b^5} - \frac{128a^8x\sqrt{a+bx}}{230945b^4} + \frac{96a^7x^2\sqrt{a+bx}}{230945b^3} - \frac{16a^6x^3\sqrt{a+bx}}{46189b^2} + \frac{14a^5x^4\sqrt{a+bx}}{46189b} + \frac{46126a^4x^5\sqrt{a+bx}}{230945} + \frac{13652a^3bx^6\sqrt{a+bx}}{20995} + \frac{1332a^2b^2x^7\sqrt{a+bx}}{1615} + \frac{154ab^3x^8\sqrt{a+bx}}{323} + \frac{2b^4x^9\sqrt{a+bx}}{19} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((256*a**9*sqrt(a + b*x)/(230945*b**5) - 128*a**8*x*sqrt(a + b*x)/
(230945*b**4) + 96*a**7*x**2*sqrt(a + b*x)/(230945*b**3) - 16*a**6*x**3*sqr
t(a + b*x)/(46189*b**2) + 14*a**5*x**4*sqrt(a + b*x)/(46189*b) + 46126*a**4
*x**5*sqrt(a + b*x)/230945 + 13652*a**3*b*x**6*sqrt(a + b*x)/20995 + 1332*a
**2*b**2*x**7*sqrt(a + b*x)/1615 + 154*a*b**3*x**8*sqrt(a + b*x)/323 + 2*b*
*4*x**9*sqrt(a + b*x)/19, Ne(b, 0)), (a**(9/2)*x**5/5, True))
```

# 3.313 $\int x^3(a+bx)^{9/2}\,dx$

**Optimal**. Leaf size=72

$$-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}+\frac{2(a+bx)^{17/2}}{17b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}$$

**Rubi** [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{17/2}}{17b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2*(a + b*x)^(13/2))/(13*b^4) - (2
*a*(a + b*x)^(15/2))/(5*b^4) + (2*(a + b*x)^(17/2))/(17*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{9/2}\,dx &= \int\left(-\frac{a^3(a+bx)^{9/2}}{b^3}+\frac{3a^2(a+bx)^{11/2}}{b^3}-\frac{3a(a+bx)^{13/2}}{b^3}+\frac{(a+bx)^{15/2}}{b^3}\right)dx\\ &= -\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}+\frac{2(a+bx)^{17/2}}{17b^4}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{11/2}\left(-16a^3+88a^2bx-286ab^2x^2+715b^3x^3\right)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-16*a^3 + 88*a^2*b*x - 286*a*b^2*x^2 + 715*b^3*x^3))/(
12155*b^4)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 51, normalized size = 0.71

$$\frac{2(a+bx)^{11/2}\left(-1105a^3+2805a^2(a+bx)-2431a(a+bx)^2+715(a+bx)^3\right)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-1105*a^3 + 2805*a^2*(a + b*x) - 2431*a*(a + b*x)^2 +
715*(a + b*x)^3))/(12155*b^4)
```

**fricas [A]** time = 0.96, size = 97, normalized size = 1.35

$$\frac{2\left(715\,b^8x^8+3289\,ab^7x^7+5808\,a^2b^6x^6+4714\,a^3b^5x^5+1515\,a^4b^4x^4+5\,a^5b^3x^3-6\,a^6b^2x^2+8\,a^7bx-16\,a^8\right)\sqrt{bx+a}}{12155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/12155*(715*b^8*x^8 + 3289*a*b^7*x^7 + 5808*a^2*b^6*x^6 + 4714*a^3*b^5*x^5
 + 1515*a^4*b^4*x^4 + 5*a^5*b^3*x^3 - 6*a^6*b^2*x^2 + 8*a^7*b*x - 16*a^8)*s
qrt(b*x + a)/b^4
```

**giac [B]** time = 1.10, size = 493, normalized size = 6.85

$$\frac{2\left(\frac{21879\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)a^5}{b^3}+\frac{12155\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)a^4}{b^3}+\frac{11050\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)a^3}{b^3}+\frac{2550\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^2-8580(bx+a)^{\frac{7}{2}}a^3+9009(bx+a)^{\frac{5}{2}}a^4-6006(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)a^2}{b^3}+\frac{595\left(429(bx+a)^{\frac{15}{2}}-3465(bx+a)^{\frac{13}{2}}a+12285(bx+a)^{\frac{11}{2}}a^2-25025(bx+a)^{\frac{9}{2}}a^3+32175(bx+a)^{\frac{7}{2}}a^4-27027(bx+a)^{\frac{5}{2}}a^5+15015(bx+a)^{\frac{3}{2}}a^6-6435\sqrt{bx+a}\,a^7\right)a}{b^3}+\frac{7\left(6435(bx+a)^{\frac{17}{2}}-58344(bx+a)^{\frac{15}{2}}a+235620(bx+a)^{\frac{13}{2}}a^2-556920(bx+a)^{\frac{11}{2}}a^3+850850(bx+a)^{\frac{9}{2}}a^4-875160(bx+a)^{\frac{7}{2}}a^5+612612(bx+a)^{\frac{5}{2}}a^6-291720(bx+a)^{\frac{3}{2}}a^7+109395\sqrt{bx+a}\,a^8\right)}{b^3}\right)}{765765\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/765765*(21879*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3
/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^5/b^3 + 12155*(35*(b*x + a)^(9/2) - 180*(
b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*
sqrt(b*x + a)*a^4)*a^4/b^3 + 11050*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/
2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^
(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^3/b^3 + 2550*(231*(b*x + a)^(13/2) - 1
638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^
3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a
)*a^6)*a^2/b^3 + 595*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 1228
5*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
```

```
 + a)*a^7)*a/b^3 + 7*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 23
5620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^
(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 29172
0*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)/b^3)/b
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.60

$$-\frac{2\left(bx+a\right)^{\frac{11}{2}}\left(-715b^{3}x^{3}+286a\,b^{2}x^{2}-88a^{2}bx+16a^{3}\right)}{12155b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(9/2),x)
```

```
[Out] -2/12155*(b*x+a)^(11/2)*(-715*b^3*x^3+286*a*b^2*x^2-88*a^2*b*x+16*a^3)/b^4
```

**maxima [A]** time = 1.29, size = 56, normalized size = 0.78

$$\frac{2\left(bx+a\right)^{\frac{17}{2}}}{17\,b^{4}}-\frac{2\left(bx+a\right)^{\frac{15}{2}}a}{5\,b^{4}}+\frac{6\left(bx+a\right)^{\frac{13}{2}}a^{2}}{13\,b^{4}}-\frac{2\left(bx+a\right)^{\frac{11}{2}}a^{3}}{11\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/17*(b*x + a)^(17/2)/b^4 - 2/5*(b*x + a)^(15/2)*a/b^4 + 6/13*(b*x + a)^(13
/2)*a^2/b^4 - 2/11*(b*x + a)^(11/2)*a^3/b^4
```

**mupad [B]** time = 0.04, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{17/2}}{17\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}+\frac{6\,a^2\,{\left(a+b\,x\right)}^{13/2}}{13\,b^4}-\frac{2\,a\,{\left(a+b\,x\right)}^{15/2}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(17/2))/(17*b^4) - (2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2*
(a + b*x)^(13/2))/(13*b^4) - (2*a*(a + b*x)^(15/2))/(5*b^4)
```

**sympy [A]** time = 20.35, size = 190, normalized size = 2.64

$$\begin{cases} -\frac{32a^{8}\sqrt{a+bx}}{12155b^{4}}+\frac{16a^{7}x\sqrt{a+bx}}{12155b^{3}}-\frac{12a^{6}x^{2}\sqrt{a+bx}}{12155b^{2}}+\frac{2a^{5}x^{3}\sqrt{a+bx}}{2431b}+\frac{606a^{4}x^{4}\sqrt{a+bx}}{2431}+\frac{9428a^{3}bx^{5}\sqrt{a+bx}}{12155}+\frac{1056a^{2}b^{2}x^{6}\sqrt{a+bx}}{1105}+\frac{46ab^{3}x^{7}\sqrt{a+bx}}{85}+\frac{2b^{4}x^{8}\sqrt{a+bx}}{17} & \text{for}\ b\neq 0\\ \frac{a^{\frac{9}{2}}x^{4}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-32*a**8*sqrt(a + b*x)/(12155*b**4) + 16*a**7*x*sqrt(a + b*x)/(1
2155*b**3) - 12*a**6*x**2*sqrt(a + b*x)/(12155*b**2) + 2*a**5*x**3*sqrt(a +
 b*x)/(2431*b) + 606*a**4*x**4*sqrt(a + b*x)/2431 + 9428*a**3*b*x**5*sqrt(a
 + b*x)/12155 + 1056*a**2*b**2*x**6*sqrt(a + b*x)/1105 + 46*a*b**3*x**7*sqr
t(a + b*x)/85 + 2*b**4*x**8*sqrt(a + b*x)/17, Ne(b, 0)), (a**(9/2)*x**4/4,
True))
```

## 3.314 $\int x^2(a+bx)^{9/2}\,dx$

**Optimal**. Leaf size=53

$$\frac{2a^2(a+bx)^{11/2}}{11b^3}+\frac{2(a+bx)^{15/2}}{15b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}$$

**Rubi [A]** time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{11/2}}{11b^3}+\frac{2(a+bx)^{15/2}}{15b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(11/2))/(11*b^3) - (4*a*(a + b*x)^(13/2))/(13*b^3) + (2*(a
 + b*x)^(15/2))/(15*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{9/2}\,dx&=\int\left(\frac{a^2(a+bx)^{9/2}}{b^2}-\frac{2a(a+bx)^{11/2}}{b^2}+\frac{(a+bx)^{13/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}+\frac{2(a+bx)^{15/2}}{15b^3}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{11/2}\left(8a^2-44abx+143b^2x^2\right)}{2145b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(8*a^2 - 44*a*b*x + 143*b^2*x^2))/(2145*b^3)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 39, normalized size = 0.74

$$\frac{2(a+bx)^{11/2}\left(195a^2-330a(a+bx)+143(a+bx)^2\right)}{2145b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(195*a^2 - 330*a*(a + b*x) + 143*(a + b*x)^2))/(2145*b^
3)
```

**fricas [B]** time = 0.96, size = 86, normalized size = 1.62

$$\frac{2\left(143\,b^7x^7+671\,ab^6x^6+1218\,a^2b^5x^5+1030\,a^3b^4x^4+355\,a^4b^3x^3+3\,a^5b^2x^2-4\,a^6bx+8\,a^7\right)\sqrt{bx+a}}{2145\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/2145*(143*b^7*x^7 + 671*a*b^6*x^6 + 1218*a^2*b^5*x^5 + 1030*a^3*b^4*x^4 +
 355*a^4*b^3*x^3 + 3*a^5*b^2*x^2 - 4*a^6*b*x + 8*a^7)*sqrt(b*x + a)/b^3
```

**giac [B]** time = 1.06, size = 421, normalized size = 7.94

$$\frac{2\left(\frac{3003\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)a^5}{b^2}+\frac{6435\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}a^3\right)a^4}{b^2}+\frac{1430\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}a^4\right)a^3}{b^2}+\frac{650\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}a^5\right)a^2}{b^2}+\frac{75\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^2-8580(bx+a)^{\frac{7}{2}}a^3+9009(bx+a)^{\frac{5}{2}}a^4-6006(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}a^6\right)a}{b^2}+\frac{7\left(429(bx+a)^{\frac{15}{2}}-3465(bx+a)^{\frac{13}{2}}a+12285(bx+a)^{\frac{11}{2}}a^2-25025(bx+a)^{\frac{9}{2}}a^3+32175(bx+a)^{\frac{7}{2}}a^4-27027(bx+a)^{\frac{5}{2}}a^5+15015(bx+a)^{\frac{3}{2}}a^6-6435\sqrt{bx+a}a^7\right)}{b^2}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/45045*(3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*
a^2)*a^5/b^2 + 6435*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a
)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^4/b^2 + 1430*(35*(b*x + a)^(9/2) - 18
0*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 3
15*sqrt(b*x + a)*a^4)*a^3/b^2 + 650*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9
/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)
^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2/b^2 + 75*(231*(b*x + a)^(13/2) - 16
38*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3
 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)
*a^6)*a/b^2 + 7*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*
x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 -
 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)
*a^7)/b^2)/b
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.60

$$\frac{2\,(bx+a)^{\frac{11}{2}}\left(143b^2x^2-44abx+8a^2\right)}{2145b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(9/2),x)
```

```
[Out] 2/2145*(b*x+a)^(11/2)*(143*b^2*x^2-44*a*b*x+8*a^2)/b^3
```

**maxima** [A] time = 1.34, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{15}{2}}}{15\,b^3}-\frac{4\,(bx+a)^{\frac{13}{2}}a}{13\,b^3}+\frac{2\,(bx+a)^{\frac{11}{2}}a^2}{11\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/15*(b*x + a)^(15/2)/b^3 - 4/13*(b*x + a)^(13/2)*a/b^3 + 2/11*(b*x + a)^(1
1/2)*a^2/b^3
```

**mupad** [B] time = 0.04, size = 36, normalized size = 0.68

$$\frac{\frac{2\,(a+b\,x)^{15/2}}{15}-\frac{4\,a\,(a+b\,x)^{13/2}}{13}+\frac{2\,a^2\,(a+b\,x)^{11/2}}{11}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(9/2),x)
```

```
[Out] ((2*(a + b*x)^(15/2))/15 - (4*a*(a + b*x)^(13/2))/13 + (2*a^2*(a + b*x)^(11
/2))/11)/b^3
```

**sympy** [A] time = 16.86, size = 168, normalized size = 3.17

$$\begin{cases} \frac{16a^7\sqrt{a+bx}}{2145b^3}-\frac{8a^6x\sqrt{a+bx}}{2145b^2}+\frac{2a^5x^2\sqrt{a+bx}}{715b}+\frac{142a^4x^3\sqrt{a+bx}}{429}+\frac{412a^3bx^4\sqrt{a+bx}}{429}+\frac{812a^2b^2x^5\sqrt{a+bx}}{715}+\frac{122ab^3x^6\sqrt{a+bx}}{195}+\frac{2b^4x^7\sqrt{a+bx}}{15} & \text{for } b\neq 0 \\ \frac{a^{\frac{9}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((16*a**7*sqrt(a + b*x)/(2145*b**3) - 8*a**6*x*sqrt(a + b*x)/(2145
*b**2) + 2*a**5*x**2*sqrt(a + b*x)/(715*b) + 142*a**4*x**3*sqrt(a + b*x)/42
9 + 412*a**3*b*x**4*sqrt(a + b*x)/429 + 812*a**2*b**2*x**5*sqrt(a + b*x)/71
5 + 122*a*b**3*x**6*sqrt(a + b*x)/195 + 2*b**4*x**7*sqrt(a + b*x)/15, Ne(b,
 0)), (a**(9/2)*x**3/3, True))
```

## 3.315 $\int x(a+bx)^{9/2}\,dx$

**Optimal**. Leaf size=34

$$\frac{2(a+bx)^{13/2}}{13b^2}-\frac{2a(a+bx)^{11/2}}{11b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{13/2}}{13b^2}-\frac{2a(a+bx)^{11/2}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a*(a + b*x)^(11/2))/(11*b^2) + (2*(a + b*x)^(13/2))/(13*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{9/2}\,dx&=\int\left(-\frac{a(a+bx)^{9/2}}{b}+\frac{(a+bx)^{11/2}}{b}\right)dx\\&=-\frac{2a(a+bx)^{11/2}}{11b^2}+\frac{2(a+bx)^{13/2}}{13b^2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{11/2}(11bx-2a)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-2*a + 11*b*x))/(143*b^2)
```

**IntegrateAlgebraic [B]** time = 0.01, size = 79, normalized size = 2.32

$$-\frac{2\sqrt{a+bx}\,\left(2a^6-a^5bx-35a^4b^2x^2-90a^3b^3x^3-100a^2b^4x^4-53ab^5x^5-11b^6x^6\right)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(9/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(2*a^6 - a^5*b*x - 35*a^4*b^2*x^2 - 90*a^3*b^3*x^3 - 100*
a^2*b^4*x^4 - 53*a*b^5*x^5 - 11*b^6*x^6))/(143*b^2)
```

**fricas [B]** time = 0.72, size = 74, normalized size = 2.18

$$\frac{2\left(11\,b^6x^6+53\,ab^5x^5+100\,a^2b^4x^4+90\,a^3b^3x^3+35\,a^4b^2x^2+a^5bx-2\,a^6\right)\sqrt{bx+a}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/143*(11*b^6*x^6 + 53*a*b^5*x^5 + 100*a^2*b^4*x^4 + 90*a^3*b^3*x^3 + 35*a^
4*b^2*x^2 + a^5*b*x - 2*a^6)*sqrt(b*x + a)/b^2
```

**giac [B]** time = 1.27, size = 347, normalized size = 10.21

$$\frac{2\left(\frac{3003\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^5}{b}+\frac{3003\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a^4}{b}+\frac{2574\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^3}{b}+\frac{286\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a^2}{b}+\frac{65\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)a}{b}+\frac{3\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^2-8580\,(bx+a)^{\frac{7}{2}}a^3+9009\,(bx+a)^{\frac{5}{2}}a^4-6006\,(bx+a)^{\frac{3}{2}}a^5+3003\,\sqrt{bx+a}\,a^6\right)}{b}\right)}{9009\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/9009*(3003*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^5/b + 3003*(3*(b*x + a
)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^4/b + 2574*(5*(b*x
 + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x +
 a)*a^3)*a^3/b + 286*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x
 + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^2/b +
65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 -
 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^
5)*a/b + 3*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)
^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*
x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b)/b
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.62

$$-\frac{2\,(bx+a)^{\frac{11}{2}}\,(-11bx+2a)}{143b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(9/2),x)
```

```
[Out] -2/143*(b*x+a)^(11/2)*(-11*b*x+2*a)/b^2
```

**maxima** [A] time = 1.36, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{13}{2}}}{13\,b^2}-\frac{2\,(bx+a)^{\frac{11}{2}}a}{11\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*x + a)^(13/2)/b^2 - 2/11*(b*x + a)^(11/2)*a/b^2
```

**mupad** [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{26\,a\,(a+b\,x)^{11/2}-22\,(a+b\,x)^{13/2}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(9/2),x)
```

```
[Out] -(26*a*(a + b*x)^(11/2) - 22*(a + b*x)^(13/2))/(143*b^2)
```

**sympy** [A] time = 14.85, size = 146, normalized size = 4.29

$$\begin{cases} -\frac{4a^6\sqrt{a+bx}}{143b^2}+\frac{2a^5x\sqrt{a+bx}}{143b}+\frac{70a^4x^2\sqrt{a+bx}}{143}+\frac{180a^3bx^3\sqrt{a+bx}}{143}+\frac{200a^2b^2x^4\sqrt{a+bx}}{143}+\frac{106ab^3x^5\sqrt{a+bx}}{143}+\frac{2b^4x^6\sqrt{a+bx}}{13} & \text{for}\ b\neq 0 \\ \frac{a^{\frac{9}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-4*a**6*sqrt(a + b*x)/(143*b**2) + 2*a**5*x*sqrt(a + b*x)/(143*b
) + 70*a**4*x**2*sqrt(a + b*x)/143 + 180*a**3*b*x**3*sqrt(a + b*x)/143 + 20
0*a**2*b**2*x**4*sqrt(a + b*x)/143 + 106*a*b**3*x**5*sqrt(a + b*x)/143 + 2*
b**4*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(9/2)*x**2/2, True))
```

## 3.316 $\int (a+bx)^{9/2}\,dx$

**Optimal**. Leaf size=16

$$\frac{2(a+bx)^{11/2}}{11b}$$

**Rubi** [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{9/2}\,dx=\frac{2(a+bx)^{11/2}}{11b}$$

**Mathematica** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

**fricas [B]** time = 1.05, size = 61, normalized size = 3.81

$$\frac{2\left(b^5x^5+5\,ab^4x^4+10\,a^2b^3x^3+10\,a^3b^2x^2+5\,a^4bx+a^5\right)\sqrt{bx+a}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/11*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x +
 a^5)*sqrt(b*x + a)/b
```

**giac [B]** time = 0.99, size = 229, normalized size = 14.31

$$\frac{2\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4+1155\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^4+462\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a^3+198\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^2+11\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/693*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^
2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 + 1155*((b*x + a)^(
3/2) - 3*sqrt(b*x + a)*a)*a^4 + 462*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)
*a + 15*sqrt(b*x + a)*a^2)*a^3 + 198*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2
)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2 + 11*(35*(b*x + a)
^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3
/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a)/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2),x)
```

```
[Out] 2/11*(b*x+a)^(11/2)/b
```

**maxima [A]** time = 1.34, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{11/2}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

**sympy [A]** time = 0.08, size = 12, normalized size = 0.75

$$\frac{2\left(a+bx\right)^{\frac{11}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2),x)
```

```
[Out] 2*(a + b*x)**(11/2)/(11*b)
```

## 3.317 $\int \frac{(a+bx)^{9/2}}{x} dx$

**Optimal**. Leaf size=97

$$-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a^4\sqrt{a+bx}+\frac{2}{3}a^3(a+bx)^{3/2}+\frac{2}{5}a^2(a+bx)^{5/2}+\frac{2}{7}a(a+bx)^{7/2}+\frac{2}{9}(a+bx)^{9/2}$$

**Rubi [A]** time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$2a^4\sqrt{a+bx}+\frac{2}{3}a^3(a+bx)^{3/2}+\frac{2}{5}a^2(a+bx)^{5/2}-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2}{7}a(a+bx)^{7/2}+\frac{2}{9}(a+bx)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x,x]
```

```
[Out] 2*a^4*Sqrt[a + b*x] + (2*a^3*(a + b*x)^(3/2))/3 + (2*a^2*(a + b*x)^(5/2))/5
 + (2*a*(a + b*x)^(7/2))/7 + (2*(a + b*x)^(9/2))/9 - 2*a^(9/2)*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x}\,dx &= \frac{2}{9}(a+bx)^{9/2} + a\int \frac{(a+bx)^{7/2}}{x}\,dx\\
&= \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^2\int \frac{(a+bx)^{5/2}}{x}\,dx\\
&= \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^3\int \frac{(a+bx)^{3/2}}{x}\,dx\\
&= \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^4\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^5\int \frac{1}{x\sqrt{a+}}\\
&= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + \frac{\left(2a^5\right)\text{Subst}}{}\\
&= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} - 2a^{9/2}\tanh^{-1}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 78, normalized size = 0.80

$$\frac{2}{315}\sqrt{a+bx}\left(563a^4+506a^3bx+408a^2b^2x^2+185ab^3x^3+35b^4x^4\right)-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(563*a^4 + 506*a^3*b*x + 408*a^2*b^2*x^2 + 185*a*b^3*x^3 +
 35*b^4*x^4))/315 - 2*a^(9/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 94, normalized size = 0.97

$$\frac{2}{315}\left(315a^4\sqrt{a+bx}+105a^3(a+bx)^{3/2}+63a^2(a+bx)^{5/2}+35(a+bx)^{9/2}+45a(a+bx)^{7/2}\right)-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x,x]
```

```
[Out] (2*(315*a^4*Sqrt[a + b*x] + 105*a^3*(a + b*x)^(3/2) + 63*a^2*(a + b*x)^(5/2
) + 45*a*(a + b*x)^(7/2) + 35*(a + b*x)^(9/2)))/315 - 2*a^(9/2)*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]]
```

**fricas [A]** time = 1.18, size = 158, normalized size = 1.63

$$\left[a^{\frac{9}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{315}\left(35b^4x^4+185ab^3x^3+408a^2b^2x^2+506a^3bx+563a^4\right)\sqrt{bx+a},\,2\sqrt{-a}a^4\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\frac{2}{315}\left(35b^4x^4+185ab^3x^3+408a^2b^2x^2+506a^3bx+563a^4\right)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(9/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/315*(35*b^4*x^4 +
 185*a*b^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a), 2*
sqrt(-a)*a^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/315*(35*b^4*x^4 + 185*a*b
^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a)]
```

**giac [A]** time = 1.23, size = 80, normalized size = 0.82

$$\frac{2a^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2}{9}(bx+a)^{\frac{9}{2}}+\frac{2}{7}(bx+a)^{\frac{7}{2}}a+\frac{2}{5}(bx+a)^{\frac{5}{2}}a^2+\frac{2}{3}(bx+a)^{\frac{3}{2}}a^3+2\sqrt{bx+a}\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/9*(b*x + a)^(9/2) + 2/7*(
b*x + a)^(7/2)*a + 2/5*(b*x + a)^(5/2)*a^2 + 2/3*(b*x + a)^(3/2)*a^3 + 2*sq
rt(b*x + a)*a^4
```

**maple [A]** time = 0.01, size = 74, normalized size = 0.76

$$-2a^{\frac{9}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\;a^4+\frac{2(bx+a)^{\frac{3}{2}}a^3}{3}+\frac{2(bx+a)^{\frac{5}{2}}a^2}{5}+\frac{2(bx+a)^{\frac{7}{2}}a}{7}+\frac{2(bx+a)^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x,x)
```

```
[Out] 2/3*a^3*(b*x+a)^(3/2)+2/5*a^2*(b*x+a)^(5/2)+2/7*a*(b*x+a)^(7/2)+2/9*(b*x+a)
^(9/2)-2*a^(9/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a^4*(b*x+a)^(1/2)
```

**maxima [A]** time = 2.94, size = 88, normalized size = 0.91

$$a^{\frac{9}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{9}(bx+a)^{\frac{9}{2}}+\frac{2}{7}(bx+a)^{\frac{7}{2}}a+\frac{2}{5}(bx+a)^{\frac{5}{2}}a^2+\frac{2}{3}(bx+a)^{\frac{3}{2}}a^3+2\sqrt{bx+a}\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="maxima")
```

```
[Out] a^(9/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/9*(b*x
 + a)^(9/2) + 2/7*(b*x + a)^(7/2)*a + 2/5*(b*x + a)^(5/2)*a^2 + 2/3*(b*x +
a)^(3/2)*a^3 + 2*sqrt(b*x + a)*a^4
```

**mupad [B]** time = 0.04, size = 76, normalized size = 0.78

$$\frac{2\,a\,(a+b\,x)^{7/2}}{7}+\frac{2\,(a+b\,x)^{9/2}}{9}+2\,a^4\,\sqrt{a+b\,x}+\frac{2\,a^3\,(a+b\,x)^{3/2}}{3}+\frac{2\,a^2\,(a+b\,x)^{5/2}}{5}+a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x,x)
```

```
[Out] (2*a*(a + b*x)^(7/2))/7 + (2*(a + b*x)^(9/2))/9 + 2*a^4*(a + b*x)^(1/2) + (
2*a^3*(a + b*x)^(3/2))/3 + (2*a^2*(a + b*x)^(5/2))/5 + a^(9/2)*atan(((a + b
*x)^(1/2)*1i)/a^(1/2))*2i
```

**sympy [A]** time = 11.10, size = 148, normalized size = 1.53

$$\frac{1126a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{315}+a^{\frac{9}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{1012a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{315}+\frac{272a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105}+\frac{74a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{63}+\frac{2\sqrt{a}\,b^4x^4\sqrt{1+\frac{bx}{a}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x,x)
```

```
[Out] 1126*a**(9/2)*sqrt(1 + b*x/a)/315 + a**(9/2)*log(b*x/a) - 2*a**(9/2)*log(sq
rt(1 + b*x/a) + 1) + 1012*a**(7/2)*b*x*sqrt(1 + b*x/a)/315 + 272*a**(5/2)*b
**2*x**2*sqrt(1 + b*x/a)/105 + 74*a**(3/2)*b**3*x**3*sqrt(1 + b*x/a)/63 + 2
*sqrt(a)*b**4*x**4*sqrt(1 + b*x/a)/9
```

## 3.318 $\int \frac{(a+bx)^{9/2}}{x^2}\,dx$

**Optimal**. Leaf size=98

$$-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{x}+\frac{9}{7}b(a+bx)^{7/2}+\frac{9}{5}ab(a+bx)^{5/2}$$

**Rubi [A]** time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$3a^2b(a+bx)^{3/2}+9a^3b\sqrt{a+bx}-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{9/2}}{x}+\frac{9}{7}b(a+bx)^{7/2}+\frac{9}{5}ab(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^2,x]
```

```
[Out] 9*a^3*b*Sqrt[a + b*x] + 3*a^2*b*(a + b*x)^(3/2) + (9*a*b*(a + b*x)^(5/2))/5
 + (9*b*(a + b*x)^(7/2))/7 - (a + b*x)^(9/2)/x - 9*a^(7/2)*b*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^2}\,dx &= -\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}(9b)\int \frac{(a+bx)^{7/2}}{x}\,dx\\
&= \frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}(9ab)\int \frac{(a+bx)^{5/2}}{x}\,dx\\
&= \frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}\left(9a^2b\right)\int \frac{(a+bx)^{3/2}}{x}\,dx\\
&= 3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}\left(9a^3b\right)\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= 9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}\left(9a^4b\right)\int\\
&= 9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\left(9a^4\right)\text{Subst}\\
&= 9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}-9a^{7/2}b\tanh^{-}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.34

$$\frac{2b(a+bx)^{11/2}\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^2,x]
```

```
[Out] (2*b*(a + b*x)^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, 1 + (b*x)/a])/(11*a^
2)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 88, normalized size = 0.90

$$\frac{\sqrt{a+bx}\left(-315a^4+210a^3(a+bx)+42a^2(a+bx)^2+18a(a+bx)^3+10(a+bx)^4\right)}{35x}-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x]*(-315*a^4 + 210*a^3*(a + b*x) + 42*a^2*(a + b*x)^2 + 18*a*(a
 + b*x)^3 + 10*(a + b*x)^4))/(35*x) - 9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x]/Sqr
t[a]]
```

**fricas [A]** time = 0.86, size = 172, normalized size = 1.76

$$\left[\frac{315\,a^{\frac{7}{2}}bx\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(10\,b^4x^4+58\,ab^3x^3+156\,a^2b^2x^2+388\,a^3bx-35\,a^4\right)\sqrt{bx+a}}{70\,x},\ \frac{315\,\sqrt{-a}\,a^3bx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(10\,b^4x^4+58\,ab^3x^3+156\,a^2b^2x^2+388\,a^3bx-35\,a^4\right)\sqrt{bx+a}}{35\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/70*(315*a^(7/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(10
*b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*x
+ a))/x, 1/35*(315*sqrt(-a)*a^3*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (10*
b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*x +
 a))/x]
```

**giac [A]** time = 1.43, size = 104, normalized size = 1.06

$$\frac{\frac{315\,a^4b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+10\,(bx+a)^{\frac{7}{2}}b^2+28\,(bx+a)^{\frac{5}{2}}ab^2+70\,(bx+a)^{\frac{3}{2}}a^2b^2+280\,\sqrt{bx+a}\,a^3b^2-\frac{35\,\sqrt{bx+a}\,a^4b}{x}}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/35*(315*a^4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 10*(b*x + a)^(7
/2)*b^2 + 28*(b*x + a)^(5/2)*a*b^2 + 70*(b*x + a)^(3/2)*a^2*b^2 + 280*sqrt(
b*x + a)*a^3*b^2 - 35*sqrt(b*x + a)*a^4*b/x)/b
```

**maple [A]** time = 0.01, size = 84, normalized size = 0.86

$$2\left(\left(-\frac{9\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)a^4+4\sqrt{bx+a}\,a^3+(bx+a)^{\frac{3}{2}}a^2+\frac{2\,(bx+a)^{\frac{5}{2}}a}{5}+\frac{(bx+a)^{\frac{7}{2}}}{7}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^2,x)
```

```
[Out] 2*b*(1/7*(b*x+a)^(7/2)+2/5*a*(b*x+a)^(5/2)+a^2*(b*x+a)^(3/2)+4*(b*x+a)^(1/2
)*a^3+a^4*(-1/2*(b*x+a)^(1/2)/b/x-9/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
)))
```

**maxima [A]** time = 2.97, size = 97, normalized size = 0.99

$$\frac{9}{2}\,a^{\frac{7}{2}}b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{7}\,(bx+a)^{\frac{7}{2}}b+\frac{4}{5}\,(bx+a)^{\frac{5}{2}}ab+2\,(bx+a)^{\frac{3}{2}}a^2b+8\,\sqrt{bx+a}\,a^3b-\frac{\sqrt{bx+a}\,a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="maxima")
```

```
[Out] 9/2*a^(7/2)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
7*(b*x + a)^(7/2)*b + 4/5*(b*x + a)^(5/2)*a*b + 2*(b*x + a)^(3/2)*a^2*b + 8
*sqrt(b*x + a)*a^3*b - sqrt(b*x + a)*a^4/x
```

**mupad [B]** time = 0.04, size = 84, normalized size = 0.86

$$\frac{2\,b\,(a+b\,x)^{7/2}}{7}-\frac{a^4\,\sqrt{a+b\,x}}{x}+\frac{4\,a\,b\,(a+b\,x)^{5/2}}{5}+8\,a^3\,b\,\sqrt{a+b\,x}+2\,a^2\,b\,(a+b\,x)^{3/2}+a^{7/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)9\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^2,x)
```

```
[Out] (2*b*(a + b*x)^(7/2))/7 - (a^4*(a + b*x)^(1/2))/x + a^(7/2)*b*atan(((a + b*
x)^(1/2)*1i)/a^(1/2))*9i + (4*a*b*(a + b*x)^(5/2))/5 + 8*a^3*b*(a + b*x)^(1
/2) + 2*a^2*b*(a + b*x)^(3/2)
```

**sympy [A]** time = 9.92, size = 150, normalized size = 1.53

$$-\frac{a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{x}+\frac{388a^{\frac{7}{2}}b\sqrt{1+\frac{bx}{a}}}{35}+\frac{9a^{\frac{7}{2}}b\log\left(\frac{bx}{a}\right)}{2}-9a^{\frac{7}{2}}b\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{156a^{\frac{5}{2}}b^2x\sqrt{1+\frac{bx}{a}}}{35}+\frac{58a^{\frac{3}{2}}b^3x^2\sqrt{1+\frac{bx}{a}}}{35}+\frac{2\sqrt{a}\,b^4x^3\sqrt{1+\frac{bx}{a}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**2,x)
```

```
[Out] -a**(9/2)*sqrt(1 + b*x/a)/x + 388*a**(7/2)*b*sqrt(1 + b*x/a)/35 + 9*a**(7/2
)*b*log(b*x/a)/2 - 9*a**(7/2)*b*log(sqrt(1 + b*x/a) + 1) + 156*a**(5/2)*b**
2*x*sqrt(1 + b*x/a)/35 + 58*a**(3/2)*b**3*x**2*sqrt(1 + b*x/a)/35 + 2*sqrt(
a)*b**4*x**3*sqrt(1 + b*x/a)/7
```

# 3.319 $\int \frac{(a+bx)^{9/2}}{x^3}\,dx$

**Optimal**. Leaf size=114

$$-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{63}{20}b^2(a+bx)^{5/2}+\frac{21}{4}ab^2(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{9b(a+bx)^{7/2}}{4x}$$

**Rubi** [A] time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$\frac{63}{4}a^2b^2\sqrt{a+bx}-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{63}{20}b^2(a+bx)^{5/2}+\frac{21}{4}ab^2(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{9b(a+bx)^{7/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (63*a^2*b^2*Sqrt[a + b*x])/4 + (21*a*b^2*(a + b*x)^(3/2))/4 + (63*b^2*(a +
b*x)^(5/2))/20 - (9*b*(a + b*x)^(7/2))/(4*x) - (a + b*x)^(9/2)/(2*x^2) - (6
3*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{9/2}}{x^3}\,dx &= -\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{4}(9b)\int \frac{(a+bx)^{7/2}}{x^2}\,dx\\
&= -\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63b^2\right)\int \frac{(a+bx)^{5/2}}{x}\,dx\\
&= \frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63ab^2\right)\int \frac{(a+bx)^{3/2}}{x}\,dx\\
&= \frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63a^2b^2\right)\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63a\right.\\
&= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{4}\left(63a\right.\\
&= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{63}{4}a^{5/2}\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.31

$$-\frac{2b^2(a+bx)^{11/2}\,{}_2F_1\left(3,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (-2*b^2*(a + b*x)^(11/2)*Hypergeometric2F1[3, 11/2, 13/2, 1 + (b*x)/a])/(11
*a^3)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 92, normalized size = 0.81

$$\frac{\sqrt{a+bx}\,\left(315a^4-525a^3(a+bx)+168a^2(a+bx)^2+24a(a+bx)^3+8(a+bx)^4\right)}{20x^2}-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x]*(315*a^4 - 525*a^3*(a + b*x) + 168*a^2*(a + b*x)^2 + 24*a*(a
 + b*x)^3 + 8*(a + b*x)^4))/(20*x^2) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x
]/Sqrt[a]])/4
```

**fricas [A]** time = 1.16, size = 180, normalized size = 1.58

$$\left[\frac{315\,a^{\frac{5}{2}}b^2x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,\left(8\,b^4x^4+56\,ab^3x^3+288\,a^2b^2x^2-85\,a^3bx-10\,a^4\right)\sqrt{bx+a}}{40\,x^2},\ \frac{315\,\sqrt{-a}\,a^2b^2x^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(8\,b^4x^4+56\,ab^3x^3+288\,a^2b^2x^2-85\,a^3bx-10\,a^4\right)\sqrt{bx+a}}{20\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/40*(315*a^(5/2)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2
*(8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sqrt(b*
x + a))/x^2, 1/20*(315*sqrt(-a)*a^2*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sqrt
(b*x + a))/x^2]
```

**giac [A]** time = 1.10, size = 112, normalized size = 0.98

$$\frac{\frac{315\,a^3b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\,(bx+a)^{\frac{5}{2}}b^3+40\,(bx+a)^{\frac{3}{2}}ab^3+240\,\sqrt{bx+a}\,a^2b^3-\frac{5\left(17\,(bx+a)^{\frac{3}{2}}a^3b^3-15\,\sqrt{bx+a}\,a^4b^3\right)}{b^2x^2}}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/20*(315*a^3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*(b*x + a)^(5/
2)*b^3 + 40*(b*x + a)^(3/2)*a*b^3 + 240*sqrt(b*x + a)*a^2*b^3 - 5*(17*(b*x
+ a)^(3/2)*a^3*b^3 - 15*sqrt(b*x + a)*a^4*b^3)/(b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 86, normalized size = 0.75

$$2\left(\left(-\frac{63\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}+\frac{\frac{15\sqrt{bx+a}\;a}{8}-\frac{17(bx+a)^{\frac{3}{2}}}{8}}{b^2x^2}\right)a^3+6\sqrt{bx+a}\;a^2+(bx+a)^{\frac{3}{2}}\,a+\frac{(bx+a)^{\frac{5}{2}}}{5}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^3,x)
```

```
[Out] 2*b^2*(1/5*(b*x+a)^(5/2)+(b*x+a)^(3/2)*a+6*(b*x+a)^(1/2)*a^2+a^3*((-17/8*(b
*x+a)^(3/2)+15/8*(b*x+a)^(1/2)*a)/x^2/b^2-63/8*arctanh((b*x+a)^(1/2)/a^(1/2
))/a^(1/2)))
```

**maxima** [A] time = 2.95, size = 131, normalized size = 1.15

$$\frac{63}{8}a^{\frac{5}{2}}b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{5}(bx+a)^{\frac{5}{2}}b^2+2(bx+a)^{\frac{3}{2}}ab^2+12\sqrt{bx+a}\,a^2b^2-\frac{17(bx+a)^{\frac{3}{2}}a^3b^2-15\sqrt{bx+a}\,a^4b^2}{4\left((bx+a)^2-2(bx+a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="maxima")
```

```
[Out] 63/8*a^(5/2)*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) +
 2/5*(b*x + a)^(5/2)*b^2 + 2*(b*x + a)^(3/2)*a*b^2 + 12*sqrt(b*x + a)*a^2*b
^2 - 1/4*(17*(b*x + a)^(3/2)*a^3*b^2 - 15*sqrt(b*x + a)*a^4*b^2)/((b*x + a)
^2 - 2*(b*x + a)*a + a^2)
```

**mupad** [B] time = 0.05, size = 117, normalized size = 1.03

$$\frac{2\,b^2\,{(a+b\,x)}^{5/2}}{5}+\frac{\frac{15\,a^4\,b^2\,\sqrt{a+b\,x}}{4}-\frac{17\,a^3\,b^2\,{(a+b\,x)}^{3/2}}{4}}{{(a+b\,x)}^2-2\,a\,(a+b\,x)+a^2}+12\,a^2\,b^2\,\sqrt{a+b\,x}+2\,a\,b^2\,{(a+b\,x)}^{3/2}+\frac{a^{5/2}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,63{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^3,x)
```

```
[Out] (2*b^2*(a + b*x)^(5/2))/5 + ((15*a^4*b^2*(a + b*x)^(1/2))/4 - (17*a^3*b^2*(
a + b*x)^(3/2))/4)/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + 12*a^2*b^2*(a + b*
x)^(1/2) + (a^(5/2)*b^2*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*63i)/4 + 2*a*b^2
*(a + b*x)^(3/2)
```

**sympy** [A] time = 8.99, size = 184, normalized size = 1.61

$$-\frac{63a^{\frac{5}{2}}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}-\frac{a^5}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{19a^4\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{203a^3b^{\frac{3}{2}}}{20\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{86a^2b^{\frac{5}{2}}\sqrt{x}}{5\sqrt{\frac{a}{bx}+1}}+\frac{16ab^{\frac{7}{2}}x^{\frac{3}{2}}}{5\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}x^{\frac{5}{2}}}{5\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**3,x)
```

```
[Out] -63*a**(5/2)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**5/(2*sqrt(b)*x**(
5/2)*sqrt(a/(b*x) + 1)) - 19*a**4*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) +
```

```
203*a**3*b**(3/2)/(20*sqrt(x)*sqrt(a/(b*x) + 1)) + 86*a**2*b**(5/2)*sqrt(x)
/(5*sqrt(a/(b*x) + 1)) + 16*a*b**(7/2)*x**(3/2)/(5*sqrt(a/(b*x) + 1)) + 2*b
**(9/2)*x**(5/2)/(5*sqrt(a/(b*x) + 1))
```

## 3.320 $\int \frac{(a+bx)^{9/2}}{x^4} dx$

**Optimal**. Leaf size=114

$$-\frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{35}{8}b^3(a+bx)^{3/2}+\frac{105}{8}ab^3\sqrt{a+bx}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{3b(a+bx)^{7/2}}{4x^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$-\frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{21b^2(a+bx)^{5/2}}{8x}+\frac{35}{8}b^3(a+bx)^{3/2}+\frac{105}{8}ab^3\sqrt{a+bx}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{3b(a+bx)^{7/2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^4,x]
```

```
[Out] (105*a*b^3*Sqrt[a + b*x])/8 + (35*b^3*(a + b*x)^(3/2))/8 - (21*b^2*(a + b*x
)^(5/2))/(8*x) - (3*b*(a + b*x)^(7/2))/(4*x^2) - (a + b*x)^(9/2)/(3*x^3) -
(105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^4}\,dx &= -\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{2}(3b)\int \frac{(a+bx)^{7/2}}{x^3}\,dx \\
&= -\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{8}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^2}\,dx \\
&= -\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{16}\left(105b^3\right)\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= \frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{16}\left(105ab^3\right)\int \frac{\sqrt{a+bx}}{x} \\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{16}\left(10\right. \\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{8}\left(105\right. \\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{105}{8}a^{3/}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.31

$$\frac{2b^3(a+bx)^{11/2}\,{}_2F_1\left(4,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^4,x]
```

```
[Out] (2*b^3*(a + b*x)^(11/2)*Hypergeometric2F1[4, 11/2, 13/2, 1 + (b*x)/a])/(11*
a^4)
```

**IntegrateAlgebraic [A]** time = 0.14, size = 92, normalized size = 0.81

$$\frac{\sqrt{a+bx}\left(-315a^4+840a^3(a+bx)-693a^2(a+bx)^2+144a(a+bx)^3+16(a+bx)^4\right)}{24x^3}-\frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^4,x]
```

```
[Out] (Sqrt[a + b*x]*(-315*a^4 + 840*a^3*(a + b*x) - 693*a^2*(a + b*x)^2 + 144*a*
(a + b*x)^3 + 16*(a + b*x)^4))/(24*x^3) - (105*a^(3/2)*b^3*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]])/8
```

**fricas [A]** time = 0.93, size = 178, normalized size = 1.56

$$\left[\frac{315\,a^{\frac{3}{2}}b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,\left(16\,b^4x^4+208\,ab^3x^3-165\,a^2b^2x^2-50\,a^3bx-8\,a^4\right)\sqrt{bx+a}}{48\,x^3},\ \frac{315\,\sqrt{-a}\,ab^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(16\,b^4x^4+208\,ab^3x^3-165\,a^2b^2x^2-50\,a^3bx-8\,a^4\right)\sqrt{bx+a}}{24\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(315*a^(3/2)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2
*(16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sqrt(b
*x + a))/x^3, 1/24*(315*sqrt(-a)*a*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a)
 + (16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sqrt
(b*x + a))/x^3]
```

**giac [A]** time = 1.10, size = 112, normalized size = 0.98

$$\frac{\frac{315\,a^2b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+16\,(bx+a)^{\frac{3}{2}}b^4+192\,\sqrt{bx+a}\,ab^4-\frac{165\,(bx+a)^{\frac{5}{2}}a^2b^4-280\,(bx+a)^{\frac{3}{2}}a^3b^4+123\,\sqrt{bx+a}\,a^4b^4}{b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(315*a^2*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 16*(b*x + a)^(3
/2)*b^4 + 192*sqrt(b*x + a)*a*b^4 - (165*(b*x + a)^(5/2)*a^2*b^4 - 280*(b*x
 + a)^(3/2)*a^3*b^4 + 123*sqrt(b*x + a)*a^4*b^4)/(b^3*x^3))/b
```

**maple [A]** time = 0.01, size = 87, normalized size = 0.76

$$2\left(\left(\left(-\frac{105\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}+\frac{-\frac{41\sqrt{bx+a}\,a^2}{16}+\frac{35(bx+a)^{\frac{3}{2}}a}{6}-\frac{55(bx+a)^{\frac{5}{2}}}{16}}{b^3x^3}\right)a^2+4\sqrt{bx+a}\,a+\frac{(bx+a)^{\frac{3}{2}}}{3}\right)b^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^4,x)
```

```
[Out] 2*b^3*(1/3*(b*x+a)^(3/2)+4*(b*x+a)^(1/2)*a+a^2*((-55/16*(b*x+a)^(5/2)+35/6*
(b*x+a)^(3/2)*a-41/16*(b*x+a)^(1/2)*a^2)/x^3/b^3-105/16*arctanh((b*x+a)^(1/
2)/a^(1/2))/a^(1/2)))
```

**maxima [A]** time = 2.94, size = 145, normalized size = 1.27

$$\frac{105}{16}\,a^{\frac{3}{2}}b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}b^3+8\,\sqrt{bx+a}\,ab^3-\frac{165\,(bx+a)^{\frac{5}{2}}a^2b^3-280\,(bx+a)^{\frac{3}{2}}a^3b^3+123\,\sqrt{bx+a}\,a^4b^3}{24\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="maxima")
```

```
[Out] 105/16*a^(3/2)*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))
 + 2/3*(b*x + a)^(3/2)*b^3 + 8*sqrt(b*x + a)*a*b^3 - 1/24*(165*(b*x + a)^(5
/2)*a^2*b^3 - 280*(b*x + a)^(3/2)*a^3*b^3 + 123*sqrt(b*x + a)*a^4*b^3)/((b*
x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2 - a^3)
```

**mupad [B]** time = 0.12, size = 131, normalized size = 1.15

$$\frac{2\,b^3\,{\left(a+b\,x\right)}^{3/2}}{3}+\frac{\frac{41\,a^4\,b^3\,\sqrt{a+b\,x}}{8}-\frac{35\,a^3\,b^3\,{\left(a+b\,x\right)}^{3/2}}{3}+\frac{55\,a^2\,b^3\,{\left(a+b\,x\right)}^{5/2}}{8}}{3\,a\,{\left(a+b\,x\right)}^2-3\,a^2\,\left(a+b\,x\right)-{\left(a+b\,x\right)}^3+a^3}+8\,a\,b^3\,\sqrt{a+b\,x}+\frac{a^{3/2}\,b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,105{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^4,x)
```

```
[Out] (2*b^3*(a + b*x)^(3/2))/3 + ((41*a^4*b^3*(a + b*x)^(1/2))/8 - (35*a^3*b^3*(
a + b*x)^(3/2))/3 + (55*a^2*b^3*(a + b*x)^(5/2))/8)/(3*a*(a + b*x)^2 - 3*a^
2*(a + b*x) - (a + b*x)^3 + a^3) + (a^(3/2)*b^3*atan(((a + b*x)^(1/2)*1i)/a
^(1/2))*105i)/8 + 8*a*b^3*(a + b*x)^(1/2)
```

**sympy [A]** time = 7.91, size = 184, normalized size = 1.61

$$-\frac{105a^{\frac{3}{2}}b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{8}-\frac{a^5}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{29a^4\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{215a^3b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{43a^2b^{\frac{5}{2}}}{24\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{28ab^{\frac{7}{2}}\sqrt{x}}{3\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**4,x)
```

```
[Out] -105*a**(3/2)*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/8 - a**5/(3*sqrt(b)*x**
(7/2)*sqrt(a/(b*x) + 1)) - 29*a**4*sqrt(b)/(12*x**(5/2)*sqrt(a/(b*x) + 1))
```

```
- 215*a**3*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) + 43*a**2*b**(5/2)/(24*
sqrt(x)*sqrt(a/(b*x) + 1)) + 28*a*b**(7/2)*sqrt(x)/(3*sqrt(a/(b*x) + 1)) +
2*b**(9/2)*x**(3/2)/(3*sqrt(a/(b*x) + 1))
```

# 3.321 $\int \frac{(a+bx)^{9/2}}{x^5}\,dx$

**Optimal**. Leaf size=116

$$\frac{315}{64}b^4\sqrt{a+bx}-\frac{315}{64}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{(a+bx)^{9/2}}{4x^4}-\frac{3b(a+bx)^{7/2}}{8x^3}$$

**Rubi** [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{105b^3(a+bx)^{3/2}}{64x}+\frac{315}{64}b^4\sqrt{a+bx}-\frac{315}{64}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{9/2}}{4x^4}-\frac{3b(a+bx)^{7/2}}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^5,x]
```

```
[Out] (315*b^4*Sqrt[a + b*x])/64 - (105*b^3*(a + b*x)^(3/2))/(64*x) - (21*b^2*(a
+ b*x)^(5/2))/(32*x^2) - (3*b*(a + b*x)^(7/2))/(8*x^3) - (a + b*x)^(9/2)/(4
*x^4) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/64
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^5}\,dx = -\frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{8}(9b)\int \frac{(a+bx)^{7/2}}{x^4}\,dx$$
$$= -\frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{16}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^3}\,dx$$
$$= -\frac{21b^2(a+bx)^{5/2}}{32x^2} - \frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{64}\left(105b^3\right)\int \frac{(a+bx)^{3/2}}{x^2}\,dx$$
$$= -\frac{105b^3(a+bx)^{3/2}}{64x} - \frac{21b^2(a+bx)^{5/2}}{32x^2} - \frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{128}\left(315b^4\right)\int \frac{\sqrt{a+bx}}{x}$$
$$= \frac{315}{64}b^4\sqrt{a+bx} - \frac{105b^3(a+bx)^{3/2}}{64x} - \frac{21b^2(a+bx)^{5/2}}{32x^2} - \frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{128}\left(31\right.$$
$$= \frac{315}{64}b^4\sqrt{a+bx} - \frac{105b^3(a+bx)^{3/2}}{64x} - \frac{21b^2(a+bx)^{5/2}}{32x^2} - \frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{64}\left(315\right.$$
$$= \frac{315}{64}b^4\sqrt{a+bx} - \frac{105b^3(a+bx)^{3/2}}{64x} - \frac{21b^2(a+bx)^{5/2}}{32x^2} - \frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} - \frac{315}{64}\sqrt{a}\,b$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.30

$$-\frac{2b^4(a+bx)^{11/2}\,{}_2F_1\left(5,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^5,x]
```

```
[Out] (-2*b^4*(a + b*x)^(11/2)*Hypergeometric2F1[5, 11/2, 13/2, 1 + (b*x)/a])/(11
*a^5)
```

**IntegrateAlgebraic [A]** time = 0.17, size = 92, normalized size = 0.79

$$\frac{\sqrt{a+bx}\,\left(315a^4-1155a^3(a+bx)+1533a^2(a+bx)^2-837a(a+bx)^3+128(a+bx)^4\right)}{64x^4}-\frac{315}{64}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^5,x]
```

```
[Out] (Sqrt[a + b*x]*(315*a^4 - 1155*a^3*(a + b*x) + 1533*a^2*(a + b*x)^2 - 837*a
*(a + b*x)^3 + 128*(a + b*x)^4))/(64*x^4) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])/64
```

**fricas [A]** time = 0.98, size = 177, normalized size = 1.53

$$\left[\frac{315\,\sqrt{a}\,b^4x^4\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,\left(128\,b^4x^4-325\,ab^3x^3-210\,a^2b^2x^2-88\,a^3bx-16\,a^4\right)\sqrt{bx+a}}{128\,x^4},\ \frac{315\,\sqrt{-a}\,b^4x^4\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(128\,b^4x^4-325\,ab^3x^3-210\,a^2b^2x^2-88\,a^3bx-16\,a^4\right)\sqrt{bx+a}}{64\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/128*(315*sqrt(a)*b^4*x^4*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
2*(128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)*sqr
t(b*x + a))/x^4, 1/64*(315*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)*s
qrt(b*x + a))/x^4]
```

**giac [A]** time = 1.22, size = 110, normalized size = 0.95

$$\frac{\frac{315\,ab^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+128\,\sqrt{bx+a}\,b^5-\frac{325\,(bx+a)^{\frac{7}{2}}ab^5-765\,(bx+a)^{\frac{5}{2}}a^2b^5+643\,(bx+a)^{\frac{3}{2}}a^3b^5-187\,\sqrt{bx+a}\,a^4b^5}{b^4x^4}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/64*(315*a*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 128*sqrt(b*x + a)
*b^5 - (325*(b*x + a)^(7/2)*a*b^5 - 765*(b*x + a)^(5/2)*a^2*b^5 + 643*(b*x
+ a)^(3/2)*a^3*b^5 - 187*sqrt(b*x + a)*a^4*b^5)/(b^4*x^4))/b
```

**maple [A]** time = 0.01, size = 85, normalized size = 0.73

$$2\left(\left(-\frac{315\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128\sqrt{a}}+\frac{\frac{187\sqrt{bx+a}\;a^3}{128}-\frac{643(bx+a)^{\frac{3}{2}}a^2}{128}+\frac{765(bx+a)^{\frac{5}{2}}a}{128}-\frac{325(bx+a)^{\frac{7}{2}}}{128}}{b^4x^4}\right)a+\sqrt{bx+a}\right)b^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^5,x)
```

```
[Out] 2*b^4*((b*x+a)^(1/2)+a*((-325/128*(b*x+a)^(7/2)+765/128*(b*x+a)^(5/2)*a-643
/128*(b*x+a)^(3/2)*a^2+187/128*(b*x+a)^(1/2)*a^3)/x^4/b^4-315/128*arctanh((
b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

**maxima [A]** time = 2.96, size = 155, normalized size = 1.34

$$\frac{315}{128}\sqrt{a}\,b^4\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b^4-\frac{325\,(bx+a)^{\frac{7}{2}}ab^4-765\,(bx+a)^{\frac{5}{2}}a^2b^4+643\,(bx+a)^{\frac{3}{2}}a^3b^4-187\sqrt{bx+a}\,a^4b^4}{64\left((bx+a)^4-4\,(bx+a)^3a+6\,(bx+a)^2a^2-4\,(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="maxima")
```

```
[Out] 315/128*sqrt(a)*b^4*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))
) + 2*sqrt(b*x + a)*b^4 - 1/64*(325*(b*x + a)^(7/2)*a*b^4 - 765*(b*x + a)^(
5/2)*a^2*b^4 + 643*(b*x + a)^(3/2)*a^3*b^4 - 187*sqrt(b*x + a)*a^4*b^4)/((b
*x + a)^4 - 4*(b*x + a)^3*a + 6*(b*x + a)^2*a^2 - 4*(b*x + a)*a^3 + a^4)
```

**mupad [B]** time = 0.06, size = 94, normalized size = 0.81

$$2\,b^4\sqrt{a+b\,x}+\frac{187\,a^4\sqrt{a+b\,x}}{64\,x^4}-\frac{643\,a^3\,(a+b\,x)^{3/2}}{64\,x^4}+\frac{765\,a^2\,(a+b\,x)^{5/2}}{64\,x^4}-\frac{325\,a\,(a+b\,x)^{7/2}}{64\,x^4}+\frac{\sqrt{a}\;b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\;1\mathrm{i}}{\sqrt{a}}\right)\,315\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^5,x)
```

```
[Out] 2*b^4*(a + b*x)^(1/2) + (187*a^4*(a + b*x)^(1/2))/(64*x^4) - (643*a^3*(a +
b*x)^(3/2))/(64*x^4) + (765*a^2*(a + b*x)^(5/2))/(64*x^4) + (a^(1/2)*b^4*at
an(((a + b*x)^(1/2)*1i)/a^(1/2))*315i)/64 - (325*a*(a + b*x)^(7/2))/(64*x^4
)
```

**sympy [A]** time = 8.55, size = 182, normalized size = 1.57

$$-\frac{315\sqrt{a}\,b^4\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64}-\frac{a^5}{4\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{13a^4\sqrt{b}}{8x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{149a^3b^{\frac{3}{2}}}{32x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{535a^2b^{\frac{5}{2}}}{64x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{197ab^{\frac{7}{2}}}{64\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**5,x)
```

```
[Out] -315*sqrt(a)*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/64 - a**5/(4*sqrt(b)*x**
(9/2)*sqrt(a/(b*x) + 1)) - 13*a**4*sqrt(b)/(8*x**(7/2)*sqrt(a/(b*x) + 1)) -
```

```
149*a**3*b**(3/2)/(32*x**(5/2)*sqrt(a/(b*x) + 1)) - 535*a**2*b**(5/2)/(64*
x**(3/2)*sqrt(a/(b*x) + 1)) - 197*a*b**(7/2)/(64*sqrt(x)*sqrt(a/(b*x) + 1))
 + 2*b**(9/2)*sqrt(x)/sqrt(a/(b*x) + 1)
```

## 3.322 $\int \frac{(a+bx)^{9/2}}{x^6} dx$

**Optimal**. Leaf size=119

$$-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}-\frac{63b^4\sqrt{a+bx}}{128x}-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{(a+bx)^{9/2}}{5x^5}-\frac{9b(a+bx)^{7/2}}{40x^4}$$

**Rubi [A]** time = 0.04, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{63b^4\sqrt{a+bx}}{128x}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}-\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] (-63*b^4*Sqrt[a + b*x])/(128*x) - (21*b^3*(a + b*x)^(3/2))/(64*x^2) - (21*b
^2*(a + b*x)^(5/2))/(80*x^3) - (9*b*(a + b*x)^(7/2))/(40*x^4) - (a + b*x)^(
9/2)/(5*x^5) - (63*b^5*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(128*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^6}\,dx &= -\frac{(a+bx)^{9/2}}{5x^5}+\frac{1}{10}(9b)\int \frac{(a+bx)^{7/2}}{x^5}\,dx\\
&= -\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}+\frac{1}{80}\left(63b^2\right)\int \frac{(a+bx)^{5/2}}{x^4}\,dx\\
&= -\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}+\frac{1}{32}\left(21b^3\right)\int \frac{(a+bx)^{3/2}}{x^3}\,dx\\
&= -\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}+\frac{1}{128}\left(63b^4\right)\int \frac{\sqrt{a+bx}}{x^2}\\
&= -\frac{63b^4\sqrt{a+bx}}{128x}-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}+\frac{1}{256}\left(63\right.\\
&= -\frac{63b^4\sqrt{a+bx}}{128x}-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}+\frac{1}{128}\left(63\right.\\
&= -\frac{63b^4\sqrt{a+bx}}{128x}-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}-\frac{63b^5\tan}{1}
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 101, normalized size = 0.85

$$-\frac{128a^5+784a^4bx+2024a^3b^2x^2+2858a^2b^3x^3+315b^5x^5\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+2455ab^4x^4+965b^5x^5}{640x^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] -1/640*(128*a^5 + 784*a^4*b*x + 2024*a^3*b^2*x^2 + 2858*a^2*b^3*x^3 + 2455*
a*b^4*x^4 + 965*b^5*x^5 + 315*b^5*x^5*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b
*x)/a]])/(x^5*Sqrt[a + b*x])
```

**IntegrateAlgebraic** [A] time = 0.20, size = 92, normalized size = 0.77

$$-\frac{\sqrt{a+bx}\left(315a^4-1470a^3(a+bx)+2688a^2(a+bx)^2-2370a(a+bx)^3+965(a+bx)^4\right)}{640x^5}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] -1/640*(Sqrt[a + b*x]*(315*a^4 - 1470*a^3*(a + b*x) + 2688*a^2*(a + b*x)^2
- 2370*a*(a + b*x)^3 + 965*(a + b*x)^4))/x^5 - (63*b^5*ArcTanh[Sqrt[a + b*x
]/Sqrt[a]])/(128*Sqrt[a])
```

**fricas [A]** time = 0.90, size = 190, normalized size = 1.60

$$\left[\frac{315\,\sqrt{a}\,b^{5}x^{5}\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,\left(965\,ab^{4}x^{4}+1490\,a^{2}b^{3}x^{3}+1368\,a^{3}b^{2}x^{2}+656\,a^{4}bx+128\,a^{5}\right)\sqrt{bx+a}}{1280\,ax^{5}},\ \frac{315\,\sqrt{-a}\,b^{5}x^{5}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-\left(965\,ab^{4}x^{4}+1490\,a^{2}b^{3}x^{3}+1368\,a^{3}b^{2}x^{2}+656\,a^{4}bx+128\,a^{5}\right)\sqrt{bx+a}}{640\,ax^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(315*sqrt(a)*b^5*x^5*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
 2*(965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 + 656*a^4*b*x + 128
*a^5)*sqrt(b*x + a))/(a*x^5), 1/640*(315*sqrt(-a)*b^5*x^5*arctan(sqrt(b*x +
 a)*sqrt(-a)/a) - (965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 + 65
6*a^4*b*x + 128*a^5)*sqrt(b*x + a))/(a*x^5)]
```

**giac [A]** time = 1.19, size = 109, normalized size = 0.92

$$\frac{\frac{315\,b^{6}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{965\,(bx+a)^{\frac{9}{2}}b^{6}-2370\,(bx+a)^{\frac{7}{2}}ab^{6}+2688\,(bx+a)^{\frac{5}{2}}a^{2}b^{6}-1470\,(bx+a)^{\frac{3}{2}}a^{3}b^{6}+315\,\sqrt{bx+a}\,a^{4}b^{6}}{b^{5}x^{5}}}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/640*(315*b^6*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (965*(b*x + a)^(9/
2)*b^6 - 2370*(b*x + a)^(7/2)*a*b^6 + 2688*(b*x + a)^(5/2)*a^2*b^6 - 1470*(
b*x + a)^(3/2)*a^3*b^6 + 315*sqrt(b*x + a)*a^4*b^6)/(b^5*x^5))/b
```

**maple [A]** time = 0.01, size = 87, normalized size = 0.73

$$2\left(-\frac{63\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{256\sqrt{a}}+\frac{-\frac{63\sqrt{bx+a}\,a^{4}}{256}+\frac{147(bx+a)^{\frac{3}{2}}a^{3}}{128}-\frac{21(bx+a)^{\frac{5}{2}}a^{2}}{10}+\frac{237(bx+a)^{\frac{7}{2}}a}{128}-\frac{193(bx+a)^{\frac{9}{2}}}{256}}{b^{5}x^{5}}\right)b^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^6,x)
```

```
[Out] 2*b^5*((-193/256*(b*x+a)^(9/2)+237/128*(b*x+a)^(7/2)*a-21/10*(b*x+a)^(5/2)*
a^2+147/128*(b*x+a)^(3/2)*a^3-63/256*(b*x+a)^(1/2)*a^4)/x^5/b^5-63/256*arct
anh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

**maxima** [A] time = 2.97, size = 169, normalized size = 1.42

$$\frac{63\,b^5\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{256\,\sqrt{a}}-\frac{965\,(bx+a)^{\frac{9}{2}}b^5-2370\,(bx+a)^{\frac{7}{2}}ab^5+2688\,(bx+a)^{\frac{5}{2}}a^2b^5-1470\,(bx+a)^{\frac{3}{2}}a^3b^5+315\,\sqrt{bx+a}\,a^4b^5}{640\left((bx+a)^5-5\,(bx+a)^4a+10\,(bx+a)^3a^2-10\,(bx+a)^2a^3+5\,(bx+a)a^4-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="maxima")
```

```
[Out] 63/256*b^5*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
 - 1/640*(965*(b*x + a)^(9/2)*b^5 - 2370*(b*x + a)^(7/2)*a*b^5 + 2688*(b*x
+ a)^(5/2)*a^2*b^5 - 1470*(b*x + a)^(3/2)*a^3*b^5 + 315*sqrt(b*x + a)*a^4*b
^5)/((b*x + a)^5 - 5*(b*x + a)^4*a + 10*(b*x + a)^3*a^2 - 10*(b*x + a)^2*a^
3 + 5*(b*x + a)*a^4 - a^5)
```

**mupad** [B] time = 0.12, size = 94, normalized size = 0.79

$$\frac{147\,a^3\,(a+b\,x)^{3/2}}{64\,x^5}-\frac{63\,a^4\,\sqrt{a+b\,x}}{128\,x^5}-\frac{193\,(a+b\,x)^{9/2}}{128\,x^5}-\frac{21\,a^2\,(a+b\,x)^{5/2}}{5\,x^5}+\frac{237\,a\,(a+b\,x)^{7/2}}{64\,x^5}+\frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,63\mathrm{i}}{128\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^6,x)
```

```
[Out] (147*a^3*(a + b*x)^(3/2))/(64*x^5) - (63*a^4*(a + b*x)^(1/2))/(128*x^5) - (
193*(a + b*x)^(9/2))/(128*x^5) - (21*a^2*(a + b*x)^(5/2))/(5*x^5) + (b^5*at
an(((a + b*x)^(1/2)*1i)/a^(1/2))*63i)/(128*a^(1/2)) + (237*a*(a + b*x)^(7/2
))/(64*x^5)
```

**sympy** [A] time = 10.25, size = 158, normalized size = 1.33

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^{\frac{9}{2}}}-\frac{41a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{40x^{\frac{7}{2}}}-\frac{171a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{80x^{\frac{5}{2}}}-\frac{149ab^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}{64x^{\frac{3}{2}}}-\frac{193b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{128\sqrt{x}}-\frac{63b^5\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{128\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**6,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**(9/2)) - 41*a**3*b**(3/2)*sqrt(a/(b*x
) + 1)/(40*x**(7/2)) - 171*a**2*b**(5/2)*sqrt(a/(b*x) + 1)/(80*x**(5/2)) -
149*a*b**(7/2)*sqrt(a/(b*x) + 1)/(64*x**(3/2)) - 193*b**(9/2)*sqrt(a/(b*x)
+ 1)/(128*sqrt(x)) - 63*b**5*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(128*sqrt(a))
```

## 3.323 $\int \frac{(a+bx)^{9/2}}{x^7}\,dx$

**Optimal**. Leaf size=141

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{(a+bx)^{9/2}}{6x^6}-\frac{3b(a+bx)^{7/2}}{20x^5}$$

**Rubi [A]** time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}-\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^7,x]
```

```
[Out] (-21*b^4*Sqrt[a + b*x])/(256*x^2) - (21*b^5*Sqrt[a + b*x])/(512*a*x) - (7*b
^3*(a + b*x)^(3/2))/(64*x^3) - (21*b^2*(a + b*x)^(5/2))/(160*x^4) - (3*b*(a
 + b*x)^(7/2))/(20*x^5) - (a + b*x)^(9/2)/(6*x^6) + (21*b^6*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]])/(512*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^7}\,dx &= -\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{4}(3b)\int \frac{(a+bx)^{7/2}}{x^6}\,dx \\
&= -\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{40}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^5}\,dx \\
&= -\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{64}\left(21b^3\right)\int \frac{(a+bx)^{3/2}}{x^4}\,dx \\
&= -\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{128}\left(21b^4\right)\int \frac{\sqrt{a+bx}}{x^3}\,d \\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{512}\Big(21b \\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+b}{6x} \\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+b}{6x} \\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+b}{6x}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.25

$$-\frac{2b^6(a+bx)^{11/2}\,{}_2F_1\left(\frac{11}{2},7;\frac{13}{2};\frac{bx}{a}+1\right)}{11a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^7,x]
```

```
[Out] (-2*b^6*(a + b*x)^(11/2)*Hypergeometric2F1[11/2, 7, 13/2, 1 + (b*x)/a])/(11
*a^7)
```

**IntegrateAlgebraic [A]** time = 0.23, size = 107, normalized size = 0.76

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}-\frac{\sqrt{a+bx}\,\left(315a^5-1785a^4(a+bx)+4158a^3(a+bx)^2-5058a^2(a+bx)^3+3335a(a+bx)^4+315(a+bx)^5\right)}{7680ax^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^7,x]
```

```
[Out] -1/7680*(Sqrt[a + b*x]*(315*a^5 - 1785*a^4*(a + b*x) + 4158*a^3*(a + b*x)^2
 - 5058*a^2*(a + b*x)^3 + 3335*a*(a + b*x)^4 + 315*(a + b*x)^5))/(a*x^6) +
(21*b^6*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(512*a^(3/2))
```

**fricas [A]** time = 1.27, size = 211, normalized size = 1.50

$$\left[\frac{315\sqrt{a}\,b^6x^6\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(315\,ab^5x^5+4910\,a^2b^4x^4+11432\,a^3b^3x^3+12144\,a^4b^2x^2+6272\,a^5bx+1280\,a^6\right)\sqrt{bx+a}}{15360\,a^2x^6},\ -\frac{315\sqrt{-a}\,b^6x^6\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(315\,ab^5x^5+4910\,a^2b^4x^4+11432\,a^3b^3x^3+12144\,a^4b^2x^2+6272\,a^5bx+1280\,a^6\right)\sqrt{bx+a}}{7680\,a^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/15360*(315*sqrt(a)*b^6*x^6*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
- 2*(315*a*b^5*x^5 + 4910*a^2*b^4*x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2*x
^2 + 6272*a^5*b*x + 1280*a^6)*sqrt(b*x + a))/(a^2*x^6), -1/7680*(315*sqrt(-
a)*b^6*x^6*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (315*a*b^5*x^5 + 4910*a^2*b^4
*x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2*x^2 + 6272*a^5*b*x + 1280*a^6)*sqr
t(b*x + a))/(a^2*x^6)]
```

**giac [A]** time = 1.04, size = 129, normalized size = 0.91

$$-\frac{\frac{315\,b^7\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{315\,(bx+a)^{\frac{11}{2}}b^7+3335\,(bx+a)^{\frac{9}{2}}ab^7-5058\,(bx+a)^{\frac{7}{2}}a^2b^7+4158\,(bx+a)^{\frac{5}{2}}a^3b^7-1785\,(bx+a)^{\frac{3}{2}}a^4b^7+315\,\sqrt{bx+a}\,a^5b^7}{ab^6x^6}}{7680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/7680*(315*b^7*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (315*(b*x +
a)^(11/2)*b^7 + 3335*(b*x + a)^(9/2)*a*b^7 - 5058*(b*x + a)^(7/2)*a^2*b^7 +
 4158*(b*x + a)^(5/2)*a^3*b^7 - 1785*(b*x + a)^(3/2)*a^4*b^7 + 315*sqrt(b*x
 + a)*a^5*b^7)/(a*b^6*x^6))/b
```

**maple [A]** time = 0.01, size = 99, normalized size = 0.70

$$2\left(\frac{21\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{1024a^{\frac{3}{2}}}+\frac{-\frac{21\sqrt{bx+a}\,a^4}{1024}+\frac{119(bx+a)^{\frac{3}{2}}a^3}{1024}-\frac{693(bx+a)^{\frac{5}{2}}a^2}{2560}+\frac{843(bx+a)^{\frac{7}{2}}a}{2560}-\frac{21(bx+a)^{\frac{11}{2}}}{1024a}-\frac{667(bx+a)^{\frac{9}{2}}}{3072}}{b^6x^6}\right)b^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^7,x)
```

```
[Out] 2*b^6*((-21/1024/a*(b*x+a)^(11/2)-667/3072*(b*x+a)^(9/2)+843/2560*(b*x+a)^(
7/2)*a-693/2560*(b*x+a)^(5/2)*a^2+119/1024*(b*x+a)^(3/2)*a^3-21/1024*(b*x+a
)^(1/2)*a^4)/x^6/b^6+21/1024*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

**maxima [A]** time = 3.03, size = 198, normalized size = 1.40

$$-\frac{21\,b^6\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{1024\,a^{\frac{3}{2}}}-\frac{315\,(bx+a)^{\frac{11}{2}}b^6+3335\,(bx+a)^{\frac{9}{2}}ab^6-5058\,(bx+a)^{\frac{7}{2}}a^2b^6+4158\,(bx+a)^{\frac{5}{2}}a^3b^6-1785\,(bx+a)^{\frac{3}{2}}a^4b^6+315\,\sqrt{bx+a}\,a^5b^6}{7680\left((bx+a)^6a-6\,(bx+a)^5a^2+15\,(bx+a)^4a^3-20\,(bx+a)^3a^4+15\,(bx+a)^2a^5-6\,(bx+a)a^6+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="maxima")
```

```
[Out] -21/1024*b^6*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/
2) - 1/7680*(315*(b*x + a)^(11/2)*b^6 + 3335*(b*x + a)^(9/2)*a*b^6 - 5058*(
b*x + a)^(7/2)*a^2*b^6 + 4158*(b*x + a)^(5/2)*a^3*b^6 - 1785*(b*x + a)^(3/2
)*a^4*b^6 + 315*sqrt(b*x + a)*a^5*b^6)/((b*x + a)^6*a - 6*(b*x + a)^5*a^2 +
 15*(b*x + a)^4*a^3 - 20*(b*x + a)^3*a^4 + 15*(b*x + a)^2*a^5 - 6*(b*x + a)
*a^6 + a^7)
```

**mupad [B]** time = 0.13, size = 109, normalized size = 0.77

$$\frac{119\,a^3\,(a+b\,x)^{3/2}}{512\,x^6}-\frac{21\,a^4\,\sqrt{a+b\,x}}{512\,x^6}-\frac{667\,(a+b\,x)^{9/2}}{1536\,x^6}-\frac{693\,a^2\,(a+b\,x)^{5/2}}{1280\,x^6}-\frac{21\,(a+b\,x)^{11/2}}{512\,a\,x^6}+\frac{843\,a\,(a+b\,x)^{7/2}}{1280\,x^6}-\frac{b^6\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,21\mathrm{i}}{512\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^7,x)
```

```
[Out] (119*a^3*(a + b*x)^(3/2))/(512*x^6) - (21*a^4*(a + b*x)^(1/2))/(512*x^6) -
(667*(a + b*x)^(9/2))/(1536*x^6) - (693*a^2*(a + b*x)^(5/2))/(1280*x^6) - (
21*(a + b*x)^(11/2))/(512*a*x^6) - (b^6*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*
21i)/(512*a^(3/2)) + (843*a*(a + b*x)^(7/2))/(1280*x^6)
```

**sympy [A]** time = 15.69, size = 209, normalized size = 1.48

$$-\frac{a^5}{6\sqrt{b}\,x^{\frac{13}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{59a^4\sqrt{b}}{60x^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1151a^3b^{\frac{3}{2}}}{480x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{2947a^2b^{\frac{5}{2}}}{960x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{8171ab^{\frac{7}{2}}}{3840x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1045b^{\frac{9}{2}}}{1536x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{21b^{\frac{11}{2}}}{512a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{21b^6\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{512a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**7,x)
```

```
[Out] -a**5/(6*sqrt(b)*x**(13/2)*sqrt(a/(b*x) + 1)) - 59*a**4*sqrt(b)/(60*x**(11/
2)*sqrt(a/(b*x) + 1)) - 1151*a**3*b**(3/2)/(480*x**(9/2)*sqrt(a/(b*x) + 1))
 - 2947*a**2*b**(5/2)/(960*x**(7/2)*sqrt(a/(b*x) + 1)) - 8171*a*b**(7/2)/(3
840*x**(5/2)*sqrt(a/(b*x) + 1)) - 1045*b**(9/2)/(1536*x**(3/2)*sqrt(a/(b*x)
 + 1)) - 21*b**(11/2)/(512*a*sqrt(x)*sqrt(a/(b*x) + 1)) + 21*b**6*asinh(sqr
t(a)/(sqrt(b)*sqrt(x)))/(512*a**(3/2))
```

# 3.324 $\int \frac{(a+bx)^{9/2}}{x^8}\,dx$

**Optimal**. Leaf size=163

$$-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^5\sqrt{a+bx}}{512ax^2}-\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{(a+bx)^{9/2}}{7x^7}-3-$$

**Rubi** [A] time = 0.07, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}}-\frac{3b^5\sqrt{a+bx}}{512ax^2}-\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] (-3*b^4*Sqrt[a + b*x])/(128*x^3) - (3*b^5*Sqrt[a + b*x])/(512*a*x^2) + (9*b
^6*Sqrt[a + b*x])/(1024*a^2*x) - (3*b^3*(a + b*x)^(3/2))/(64*x^4) - (3*b^2*
(a + b*x)^(5/2))/(40*x^5) - (3*b*(a + b*x)^(7/2))/(28*x^6) - (a + b*x)^(9/2
)/(7*x^7) - (9*b^7*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1024*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^8}\,dx = -\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{14}(9b)\int \frac{(a+bx)^{7/2}}{x^7}\,dx$$

$$= -\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{8}\left(3b^2\right)\int \frac{(a+bx)^{5/2}}{x^6}\,dx$$

$$= -\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{16}\left(3b^3\right)\int \frac{(a+bx)^{3/2}}{x^5}\,dx$$

$$= -\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{128}\left(9b^4\right)\int \frac{\sqrt{a+bx}}{x^4}\,dx$$

$$= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{256}\left(3b^5\right)\int$$

$$= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}$$

$$= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}$$

$$= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}$$

$$= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.21

$$\frac{2b^7(a+bx)^{11/2}\,{}_2F_1\left(\frac{11}{2},8;\frac{13}{2};\frac{bx}{a}+1\right)}{11a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] (2*b^7*(a + b*x)^(11/2)*Hypergeometric2F1[11/2, 8, 13/2, 1 + (b*x)/a])/(11*
a^8)
```

**IntegrateAlgebraic [A]** time = 0.26, size = 119, normalized size = 0.73

$$-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}}-\frac{\sqrt{a+bx}\left(315a^6-2100a^5(a+bx)+5943a^4(a+bx)^2-9216a^3(a+bx)^3+8393a^2(a+bx)^4+2100a(a+bx)^5-315(a+bx)^6\right)}{35840a^2x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] -1/35840*(Sqrt[a + b*x]*(315*a^6 - 2100*a^5*(a + b*x) + 5943*a^4*(a + b*x)^
2 - 9216*a^3*(a + b*x)^3 + 8393*a^2*(a + b*x)^4 + 2100*a*(a + b*x)^5 - 315*
(a + b*x)^6))/(a^2*x^7) - (9*b^7*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1024*a^(5
/2))
```

**fricas [A]** time = 1.42, size = 233, normalized size = 1.43

$$\left[\frac{315\sqrt{a}b^7x^7\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(315ab^6x^6-210a^2b^5x^5-14168a^3b^4x^4-39056a^4b^3x^3-44928a^5b^2x^2-24320a^6bx-5120a^7\right)\sqrt{bx+a}}{71680a^3x^7}, \frac{315\sqrt{-a}b^7x^7\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(315ab^6x^6-210a^2b^5x^5-14168a^3b^4x^4-39056a^4b^3x^3-44928a^5b^2x^2-24320a^6bx-5120a^7\right)\sqrt{bx+a}}{35840a^3x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/71680*(315*sqrt(a)*b^7*x^7*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(315*a*b^6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*x^
3 - 44928*a^5*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^7),
 1/35840*(315*sqrt(-a)*b^7*x^7*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (315*a*b^
6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*x^3 - 44928*a^5
*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^7)]
```

**giac [A]** time = 0.95, size = 144, normalized size = 0.88

$$\frac{\frac{315\,b^8\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{315\,(bx+a)^{\frac{13}{2}}b^8-2100\,(bx+a)^{\frac{11}{2}}ab^8-8393\,(bx+a)^{\frac{9}{2}}a^2b^8+9216\,(bx+a)^{\frac{7}{2}}a^3b^8-5943\,(bx+a)^{\frac{5}{2}}a^4b^8+2100\,(bx+a)^{\frac{3}{2}}a^5b^8-315\,\sqrt{bx+a}\,a^6b^8}{a^2b^7x^7}}{35840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/35840*(315*b^8*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (315*(b*x
+ a)^(13/2)*b^8 - 2100*(b*x + a)^(11/2)*a*b^8 - 8393*(b*x + a)^(9/2)*a^2*b^
8 + 9216*(b*x + a)^(7/2)*a^3*b^8 - 5943*(b*x + a)^(5/2)*a^4*b^8 + 2100*(b*x
 + a)^(3/2)*a^5*b^8 - 315*sqrt(b*x + a)*a^6*b^8)/(a^2*b^7*x^7))/b
```

**maple [A]** time = 0.01, size = 111, normalized size = 0.68

$$2\left(-\frac{9\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2048a^{\frac{5}{2}}}+\frac{-\frac{9\sqrt{bx+a}\;a^{4}}{2048}+\frac{15(bx+a)^{\frac{3}{2}}a^{3}}{512}-\frac{849(bx+a)^{\frac{5}{2}}a^{2}}{10240}+\frac{9(bx+a)^{\frac{7}{2}}a}{70}-\frac{15(bx+a)^{\frac{11}{2}}}{512a}+\frac{9(bx+a)^{\frac{13}{2}}}{2048a^{2}}-\frac{1199(bx+a)^{\frac{9}{2}}}{10240}}{b^{7}x^{7}}\right)b^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^8,x)
```

```
[Out] 2*b^7*((9/2048/a^2*(b*x+a)^(13/2)-15/512*(b*x+a)^(11/2)/a-1199/10240*(b*x+a
)^(9/2)+9/70*(b*x+a)^(7/2)*a-849/10240*(b*x+a)^(5/2)*a^2+15/512*(b*x+a)^(3/
2)*a^3-9/2048*(b*x+a)^(1/2)*a^4)/x^7/b^7-9/2048*arctanh((b*x+a)^(1/2)/a^(1/
2))/a^(5/2))
```

**maxima [A]** time = 3.08, size = 229, normalized size = 1.40

$$\frac{9\,b^{7}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2048\,a^{\frac{5}{2}}}+\frac{315\,(bx+a)^{\frac{13}{2}}b^{7}-2100\,(bx+a)^{\frac{11}{2}}ab^{7}-8393\,(bx+a)^{\frac{9}{2}}a^{2}b^{7}+9216\,(bx+a)^{\frac{7}{2}}a^{3}b^{7}-5943\,(bx+a)^{\frac{5}{2}}a^{4}b^{7}+2100\,(bx+a)^{\frac{3}{2}}a^{5}b^{7}-315\,\sqrt{bx+a}\,a^{6}b^{7}}{35840\left((bx+a)^{7}a^{2}-7\,(bx+a)^{6}a^{3}+21\,(bx+a)^{5}a^{4}-35\,(bx+a)^{4}a^{5}+35\,(bx+a)^{3}a^{6}-21\,(bx+a)^{2}a^{7}+7\,(bx+a)a^{8}-a^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="maxima")
```

```
[Out] 9/2048*b^7*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2)
 + 1/35840*(315*(b*x + a)^(13/2)*b^7 - 2100*(b*x + a)^(11/2)*a*b^7 - 8393*(
b*x + a)^(9/2)*a^2*b^7 + 9216*(b*x + a)^(7/2)*a^3*b^7 - 5943*(b*x + a)^(5/2
)*a^4*b^7 + 2100*(b*x + a)^(3/2)*a^5*b^7 - 315*sqrt(b*x + a)*a^6*b^7)/((b*x
 + a)^7*a^2 - 7*(b*x + a)^6*a^3 + 21*(b*x + a)^5*a^4 - 35*(b*x + a)^4*a^5 +
 35*(b*x + a)^3*a^6 - 21*(b*x + a)^2*a^7 + 7*(b*x + a)*a^8 - a^9)
```

**mupad [B]** time = 0.13, size = 124, normalized size = 0.76

$$\frac{15\,a^{3}\,(a+b\,x)^{3/2}}{256\,x^{7}}-\frac{9\,a^{4}\,\sqrt{a+b\,x}}{1024\,x^{7}}-\frac{1199\,(a+b\,x)^{9/2}}{5120\,x^{7}}-\frac{849\,a^{2}\,(a+b\,x)^{5/2}}{5120\,x^{7}}-\frac{15\,(a+b\,x)^{11/2}}{256\,a\,x^{7}}+\frac{9\,(a+b\,x)^{13/2}}{1024\,a^{2}\,x^{7}}+\frac{9\,a\,(a+b\,x)^{7/2}}{35\,x^{7}}+\frac{b^{7}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\;1\mathrm{i}}{\sqrt{a}}\right)\,9\mathrm{i}}{1024\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^8,x)
```

```
[Out] (15*a^3*(a + b*x)^(3/2))/(256*x^7) - (9*a^4*(a + b*x)^(1/2))/(1024*x^7) - (
1199*(a + b*x)^(9/2))/(5120*x^7) - (849*a^2*(a + b*x)^(5/2))/(5120*x^7) - (
15*(a + b*x)^(11/2))/(256*a*x^7) + (9*(a + b*x)^(13/2))/(1024*a^2*x^7) + (b
^7*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*9i)/(1024*a^(5/2)) + (9*a*(a + b*x)^(
7/2))/(35*x^7)
```

**sympy** [A] time = 22.20, size = 236, normalized size = 1.45

$$-\frac{a^{5}}{7\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{23a^{4}\sqrt{b}}{28x^{\frac{13}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{541a^{3}b^{\frac{3}{2}}}{280x^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5249a^{2}b^{\frac{5}{2}}}{2240x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{6653ab^{\frac{7}{2}}}{4480x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1027b^{\frac{9}{2}}}{2560x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{3b^{\frac{11}{2}}}{1024ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{9b^{\frac{13}{2}}}{1024a^{2}\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{9b^{7}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{1024a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**8,x)
```

```
[Out] -a**5/(7*sqrt(b)*x**(15/2)*sqrt(a/(b*x) + 1)) - 23*a**4*sqrt(b)/(28*x**(13/
2)*sqrt(a/(b*x) + 1)) - 541*a**3*b**(3/2)/(280*x**(11/2)*sqrt(a/(b*x) + 1))
 - 5249*a**2*b**(5/2)/(2240*x**(9/2)*sqrt(a/(b*x) + 1)) - 6653*a*b**(7/2)/(
4480*x**(7/2)*sqrt(a/(b*x) + 1)) - 1027*b**(9/2)/(2560*x**(5/2)*sqrt(a/(b*x
) + 1)) + 3*b**(11/2)/(1024*a*x**(3/2)*sqrt(a/(b*x) + 1)) + 9*b**(13/2)/(10
24*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 9*b**7*asinh(sqrt(a)/(sqrt(b)*sqrt(x))
)/(1024*a**(5/2))
```

## 3.325 $\int \frac{\sqrt{-a+bx}}{x} \, dx$

**Optimal**. Leaf size=39

$$2\sqrt{bx-a} - 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

**Rubi** [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 63, 205}

$$2\sqrt{bx-a} - 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x,x]
```

```
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx}}{x}\,dx = 2\sqrt{-a+bx} - a\int \frac{1}{x\sqrt{-a+bx}}\,dx$$
$$= 2\sqrt{-a+bx} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b}$$
$$= 2\sqrt{-a+bx} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

**Mathematica [A]** time = 0.01, size = 39, normalized size = 1.00

$$2\sqrt{bx-a} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x,x]
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

**IntegrateAlgebraic [A]** time = 0.02, size = 39, normalized size = 1.00

$$2\sqrt{bx-a} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-a + b*x]/x,x]
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

**fricas [A]** time = 1.21, size = 78, normalized size = 2.00

$$\left[\sqrt{-a}\log\left(\frac{bx - 2\sqrt{bx-a}\sqrt{-a} - 2a}{x}\right) + 2\sqrt{bx-a}, -2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + 2\sqrt{bx-a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="fricas")
[Out] [sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt(b*x - a),
-2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)]
```

**giac** [A] time = 1.14, size = 31, normalized size = 0.79

$$-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)
```

**maple** [A] time = 0.01, size = 32, normalized size = 0.82

$$-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x,x)
```

```
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x-a)^(1/2)
```

**maxima** [A] time = 2.94, size = 31, normalized size = 0.79

$$-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)
```

**mupad** [B] time = 0.09, size = 31, normalized size = 0.79

$$2\sqrt{b\,x-a}-2\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x,x)
```

```
[Out] 2*(b*x - a)^(1/2) - 2*a^(1/2)*atan((b*x - a)^(1/2)/a^(1/2))
```

**sympy [B]** time = 1.74, size = 148, normalized size = 3.79

$$\begin{cases} -2i\sqrt{a}\ \operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2ia}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}-1}} - \frac{2i\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ 2\sqrt{a}\ \operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) - \frac{2a}{\sqrt{b}\sqrt{x}\sqrt{-\frac{a}{bx}+1}} + \frac{2\sqrt{b}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x,x)
```

```
[Out] Piecewise((-2*I*sqrt(a)*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*I*a/(sqrt(b)*s
qrt(x)*sqrt(a/(b*x) - 1)) - 2*I*sqrt(b)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b
*x)) > 1), (2*sqrt(a)*asin(sqrt(a)/(sqrt(b)*sqrt(x))) - 2*a/(sqrt(b)*sqrt(x
)*sqrt(-a/(b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(-a/(b*x) + 1), True))
```

## 3.326 $\int \frac{\sqrt{-a+bx}}{x^2}\,dx$

**Optimal**. Leaf size=42

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

**Rubi [A]** time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 63, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx}}{x^2}\,dx = -\frac{\sqrt{-a+bx}}{x} + \frac{1}{2}b\int \frac{1}{x\sqrt{-a+bx}}\,dx$$
$$= -\frac{\sqrt{-a+bx}}{x} + \text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)$$
$$= -\frac{\sqrt{-a+bx}}{x} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Mathematica** [A] time = 0.02, size = 52, normalized size = 1.24

$$\frac{-bx\sqrt{1-\frac{bx}{a}}\tanh^{-1}\left(\sqrt{1-\frac{bx}{a}}\right)+a-bx}{x\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x^2,x]
```

```
[Out] (a - b*x - b*x*Sqrt[1 - (b*x)/a]*ArcTanh[Sqrt[1 - (b*x)/a]])/(x*Sqrt[-a + b
*x])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{bx-a}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

**fricas** [A] time = 1.00, size = 98, normalized size = 2.33

$$\left[-\frac{\sqrt{-a}\,bx\log\left(\frac{bx-2\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)+2\sqrt{bx-a}\,a}{2\,ax}, \frac{\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\sqrt{bx-a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt(
b*x - a)*a)/(a*x), (sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) - sqrt(b*x -
a)*a)/(a*x)]
```

**giac [A]** time = 1.08, size = 41, normalized size = 0.98

$$\frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{bx-a}\,b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - sqrt(b*x - a)*b/x)/b
```

**maple [A]** time = 0.01, size = 35, normalized size = 0.83

$$\frac{b \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{bx-a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x^2,x)
```

```
[Out] b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)-(b*x-a)^(1/2)/x
```

**maxima [A]** time = 2.94, size = 34, normalized size = 0.81

$$\frac{b \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{bx-a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] b*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - sqrt(b*x - a)/x
```

**mupad [B]** time = 0.10, size = 34, normalized size = 0.81

$$\frac{b \operatorname{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{b\,x-a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x^2,x)
```

```
[Out] (b*atan((b*x - a)^(1/2)/a^(1/2)))/a^(1/2) - (b*x - a)^(1/2)/x
```

**sympy** **[A]** time = 2.14, size = 121, normalized size = 2.88

$$\begin{cases} -\frac{ia}{\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{i\sqrt{b}}{\sqrt{x}\sqrt{\frac{a}{bx}-1}} + \frac{ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{\sqrt{x}} - \frac{b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x**2,x)
```

```
[Out] Piecewise((-I*a/(sqrt(b)*x**(3/2)*sqrt(a/(b*x) - 1)) + I*sqrt(b)/(sqrt(x)*s
qrt(a/(b*x) - 1)) + I*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), Abs(a/(b*
x)) > 1), (-sqrt(b)*sqrt(-a/(b*x) + 1)/sqrt(x) - b*asin(sqrt(a)/(sqrt(b)*sq
rt(x)))/sqrt(a), True))
```

## 3.327 $\int \frac{\sqrt{-a+bx}}{x^3} dx$

**Optimal**. Leaf size=71

$$\frac{b^2 \tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt{bx-a}}{2x^2} + \frac{b\sqrt{bx-a}}{4ax}$$

**Rubi** [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 51, 63, 205}

$$\frac{b^2 \tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt{bx-a}}{2x^2} + \frac{b\sqrt{bx-a}}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x^3,x]
```

```
[Out] -Sqrt[-a + b*x]/(2*x^2) + (b*Sqrt[-a + b*x])/(4*a*x) + (b^2*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-a+bx}}{x^3}\,dx &= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{1}{4}b\int \frac{1}{x^2\sqrt{-a+bx}}\,dx\\
&= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b^2\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a}\\
&= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{4a}\\
&= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 38, normalized size = 0.54

$$\frac{2b^2(bx-a)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{bx}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x^3,x]
[Out] (2*b^2*(-a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - (b*x)/a])/(3*a^3
)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 60, normalized size = 0.85

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{(2a-bx)\sqrt{bx-a}}{4ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-a + b*x]/x^3,x]
```

```
[Out] -1/4*((2*a - b*x)*Sqrt[-a + b*x])/(a*x^2) + (b^2*ArcTan[Sqrt[-a + b*x]/Sqrt
[a]])/(4*a^(3/2))
```

**fricas [A]** time = 1.09, size = 124, normalized size = 1.75

$$\left[-\frac{\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\left(abx-2\,a^2\right)\sqrt{bx-a}}{8\,a^2x^2},\ \frac{\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(abx-2\,a^2\right)\sqrt{bx-a}}{4\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*(
a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2), 1/4*(sqrt(a)*b^2*x^2*arctan(sqrt(b
*x - a)/sqrt(a)) + (a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2)]
```

**giac [A]** time = 1.08, size = 66, normalized size = 0.93

$$\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{(bx-a)^{\frac{3}{2}}b^3-\sqrt{bx-a}\,ab^3}{ab^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + ((b*x - a)^(3/2)*b^3 - sqr
t(b*x - a)*a*b^3)/(a*b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 55, normalized size = 0.77

$$\frac{b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}}-\frac{\sqrt{bx-a}}{4x^2}+\frac{(bx-a)^{\frac{3}{2}}}{4a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x^3,x)
```

```
[Out] 1/4/x^2/a*(b*x-a)^(3/2)-1/4*(b*x-a)^(1/2)/x^2+1/4*b^2*arctan((b*x-a)^(1/2)/
a^(1/2))/a^(3/2)
```

**maxima** [A] time = 2.96, size = 83, normalized size = 1.17

$$\frac{b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}}+\frac{(bx-a)^{\frac{3}{2}}b^2-\sqrt{bx-a}\,ab^2}{4\left((bx-a)^2a+2\,(bx-a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + 1/4*((b*x - a)^(3/2)*b^2 -
sqrt(b*x - a)*a*b^2)/((b*x - a)^2*a + 2*(b*x - a)*a^2 + a^3)
```

**mupad** [B] time = 0.10, size = 54, normalized size = 0.76

$$\frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{3/2}}-\frac{\sqrt{b\,x-a}}{4\,x^2}+\frac{(b\,x-a)^{3/2}}{4\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x^3,x)
```

```
[Out] (b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (b*x - a)^(1/2)/(4*x^2) +
 (b*x - a)^(3/2)/(4*a*x^2)
```

**sympy** [A] time = 4.16, size = 207, normalized size = 2.92

$$\begin{cases}-\frac{ia}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{3i\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{ib^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{\frac{a}{bx}-1}}+\frac{ib^2\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{for } \left|\frac{a}{bx}\right|>1\\ \frac{a}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{b^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{b^2\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x**3,x)
```

```
[Out] Piecewise((-I*a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(4*x**
(3/2)*sqrt(a/(b*x) - 1)) - I*b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) - 1)) + I*b
**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(3/2)), Abs(a/(b*x)) > 1), (a/(2
*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x)
 + 1)) + b**(3/2)/(4*a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b**2*asin(sqrt(a)/(sqr
t(b)*sqrt(x)))/(4*a**(3/2)), True))
```

## 3.328 $\int \frac{(-a+bx)^{3/2}}{x}\,dx$

**Optimal**. Leaf size=55

$$2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}+\frac{2}{3}(bx-a)^{3/2}$$

**Rubi [A]** time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 63, 205}

$$2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}+\frac{2}{3}(bx-a)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x,x]
```

```
[Out] -2*a*Sqrt[-a + b*x] + (2*(-a + b*x)^(3/2))/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b
*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(-a+bx)^{3/2}}{x}\,dx = \frac{2}{3}(-a+bx)^{3/2} - a\int \frac{\sqrt{-a+bx}}{x}\,dx$$
$$= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + a^2\int \frac{1}{x\sqrt{-a+bx}}\,dx$$
$$= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + \frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b}$$
$$= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + 2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

**Mathematica** [A] time = 0.02, size = 48, normalized size = 0.87

$$2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + \frac{2}{3}(bx-4a)\sqrt{bx-a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x,x]
[Out] (2*(-4*a + b*x)*Sqrt[-a + b*x])/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]
]
```

**IntegrateAlgebraic** [A] time = 0.03, size = 58, normalized size = 1.05

$$2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) - \frac{2}{3}\left(3a\sqrt{bx-a} - (bx-a)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(3/2)/x,x]
[Out] (-2*(3*a*Sqrt[-a + b*x] - (-a + b*x)^(3/2)))/3 + 2*a^(3/2)*ArcTan[Sqrt[-a +
 b*x]/Sqrt[a]]
```

**fricas** [A] time = 0.91, size = 93, normalized size = 1.69

$$\left[\sqrt{-a}\,a\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right) + \frac{2}{3}\sqrt{bx-a}\,(bx-4a),\, 2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + \frac{2}{3}\sqrt{bx-a}\,(bx-4a)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*a*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/3*sqrt(b*x -
a)*(b*x - 4*a), 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*sqrt(b*x - a)
*(b*x - 4*a)]
```

**giac [A]** time = 1.06, size = 43, normalized size = 0.78

$$2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}-2\,\sqrt{bx-a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2) - 2*sqrt(b*x
- a)*a
```

**maple [A]** time = 0.01, size = 44, normalized size = 0.80

$$2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2\sqrt{bx-a}\;a+\frac{2\,(bx-a)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x,x)
```

```
[Out] 2/3*(b*x-a)^(3/2)+2*a^(3/2)*arctan((b*x-a)^(1/2)/a^(1/2))-2*a*(b*x-a)^(1/2)
```

**maxima [A]** time = 3.02, size = 43, normalized size = 0.78

$$2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}-2\,\sqrt{bx-a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2) - 2*sqrt(b*x
- a)*a
```

**mupad [B]** time = 0.04, size = 43, normalized size = 0.78

$$2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)-2\,a\,\sqrt{b\,x-a}\,+\frac{2\,{\left(b\,x-a\right)}^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(3/2)/x,x)
```

```
[Out] 2*a^(3/2)*atan((b*x - a)^(1/2)/a^(1/2)) - 2*a*(b*x - a)^(1/2) + (2*(b*x - a
)^(3/2))/3
```

**sympy [C]** time = 2.46, size = 187, normalized size = 3.40

$$\begin{cases} -\frac{8a^{\frac{3}{2}}\sqrt{-1+\frac{bx}{a}}}{3} - ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right) + 2ia^{\frac{3}{2}}\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - 2a^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2\sqrt{a}\,bx\sqrt{-1+\frac{bx}{a}}}{3} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{8ia^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{3} - ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right) + 2ia^{\frac{3}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right) + \frac{2i\sqrt{a}\,bx\sqrt{1-\frac{bx}{a}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x,x)
```

```
[Out] Piecewise((-8*a**(3/2)*sqrt(-1 + b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a**
(3/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) - 2*a**(3/2)*asin(sqrt(a)/(sqrt(b)*sqrt(
x))) + 2*sqrt(a)*b*x*sqrt(-1 + b*x/a)/3, Abs(b*x/a) > 1), (-8*I*a**(3/2)*sq
rt(1 - b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a**(3/2)*log(sqrt(1 - b*x/a)
+ 1) + 2*I*sqrt(a)*b*x*sqrt(1 - b*x/a)/3, True))
```

## 3.329 $\int \frac{(-a+bx)^{3/2}}{x^2}\,dx$

**Optimal**. Leaf size=57

$$-\frac{(bx-a)^{3/2}}{x}+3b\sqrt{bx-a}-3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

**Rubi** [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 205}

$$-\frac{(bx-a)^{3/2}}{x}+3b\sqrt{bx-a}-3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[-a + b*x] - (-a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(-a+bx)^{3/2}}{x^2}\,dx &= -\frac{(-a+bx)^{3/2}}{x}+\frac{1}{2}(3b)\int \frac{\sqrt{-a+bx}}{x}\,dx\\ &= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-\frac{1}{2}(3ab)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\ &= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-(3a)\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\ &= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 36, normalized size = 0.63

$$\frac{2b(bx-a)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1-\frac{bx}{a}\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x^2,x]
[Out] (2*b*(-a + b*x)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 - (b*x)/a])/(5*a^2)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 55, normalized size = 0.96

$$\frac{\sqrt{bx-a}\,(2(bx-a)+3a)}{x}-3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(3/2)/x^2,x]
```

[Out] (Sqrt[-a + b*x]*(3*a + 2*(-a + b*x)))/x - 3*Sqrt[a]*b*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]]

**fricas [A]** time = 1.04, size = 105, normalized size = 1.84

$$\left[\frac{3\sqrt{-a}\,bx\log\left(\frac{bx-2\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)+2\,(2\,bx+a)\sqrt{bx-a}}{2\,x},-\frac{3\,\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-(2\,bx+a)\sqrt{bx-a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="fricas")

[Out] [1/2*(3*sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*(2*b
*x + a)*sqrt(b*x - a))/x, -(3*sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) - (
2*b*x + a)*sqrt(b*x - a))/x]

**giac [A]** time = 0.96, size = 58, normalized size = 1.02

$$-\frac{3\,\sqrt{a}\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2\,\sqrt{bx-a}\,b^2-\frac{\sqrt{bx-a}\,ab}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="giac")

[Out] -(3*sqrt(a)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) - 2*sqrt(b*x - a)*b^2 - sqrt(
b*x - a)*a*b/x)/b

**maple [A]** time = 0.01, size = 48, normalized size = 0.84

$$-3\sqrt{a}\;b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\;b+\frac{\sqrt{bx-a}\;a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x-a)^(3/2)/x^2,x)

[Out] 2*b*(b*x-a)^(1/2)+a*(b*x-a)^(1/2)/x-3*b*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/
2)

**maxima [A]** time = 3.00, size = 47, normalized size = 0.82

$$-3\,\sqrt{a}\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\,\sqrt{bx-a}\,b+\frac{\sqrt{bx-a}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -3*sqrt(a)*b*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)*b + sqrt(b*x -
 a)*a/x
```

**mupad [B]** time = 0.04, size = 47, normalized size = 0.82

$$2\,b\,\sqrt{b\,x-a}+\frac{a\,\sqrt{b\,x-a}}{x}-3\,\sqrt{a}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(3/2)/x^2,x)
```

```
[Out] 2*b*(b*x - a)^(1/2) + (a*(b*x - a)^(1/2))/x - 3*a^(1/2)*b*atan((b*x - a)^(1
/2)/a^(1/2))
```

**sympy [B]** time = 2.84, size = 197, normalized size = 3.46

$$\begin{cases} -3i\sqrt{a}\,b\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{ia^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{ia\sqrt{b}}{\sqrt{x}\,\sqrt{\frac{a}{bx}-1}}-\frac{2ib^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ 3\sqrt{a}\,b\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{a^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{a\sqrt{b}}{\sqrt{x}\,\sqrt{-\frac{a}{bx}+1}}+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x**2,x)
```

```
[Out] Piecewise((-3*I*sqrt(a)*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + I*a**2/(sqrt(b
)*x**(3/2)*sqrt(a/(b*x) - 1)) + I*a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) - 1)) - 2
*I*b**(3/2)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (3*sqrt(a)*b*asin
(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqrt(-a/(b*x) + 1)) -
a*sqrt(b)/(sqrt(x)*sqrt(-a/(b*x) + 1)) + 2*b**(3/2)*sqrt(x)/sqrt(-a/(b*x) +
 1), True))
```

## 3.330 $\int \frac{(-a+bx)^{3/2}}{x^3} \, dx$

**Optimal**. Leaf size=68

$$\frac{3b^2 \tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{(bx-a)^{3/2}}{2x^2} - \frac{3b\sqrt{bx-a}}{4x}$$

**Rubi [A]** time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 63, 205}

$$\frac{3b^2 \tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{(bx-a)^{3/2}}{2x^2} - \frac{3b\sqrt{bx-a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x^3,x]
```

```
[Out] (-3*b*Sqrt[-a + b*x])/(4*x) - (-a + b*x)^(3/2)/(2*x^2) + (3*b^2*ArcTan[Sqrt
[-a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(-a+bx)^{3/2}}{x^3}\,dx &= -\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\int \frac{\sqrt{-a+bx}}{x^2}\,dx\\ &= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\ &= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\ &= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 72, normalized size = 1.06

$$-\frac{2a^2+3b^2x^2\sqrt{1-\frac{bx}{a}}\,\tanh^{-1}\left(\sqrt{1-\frac{bx}{a}}\right)-7abx+5b^2x^2}{4x^2\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x^3,x]
```

```
[Out] -1/4*(2*a^2 - 7*a*b*x + 5*b^2*x^2 + 3*b^2*x^2*Sqrt[1 - (b*x)/a]*ArcTanh[Sqr
t[1 - (b*x)/a]])/(x^2*Sqrt[-a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.09, size = 62, normalized size = 0.91

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{\sqrt{bx-a}\,(5(bx-a)+3a)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(3/2)/x^3,x]
```

```
[Out] -1/4*(Sqrt[-a + b*x]*(3*a + 5*(-a + b*x)))/x^2 + (3*b^2*ArcTan[Sqrt[-a + b*
x]/Sqrt[a]])/(4*Sqrt[a])
```

**fricas [A]** time = 1.04, size = 129, normalized size = 1.90

$$\left[-\frac{3\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\,\sqrt{-a}-2a}{x}\right)+2\left(5\,abx-2\,a^2\right)\sqrt{bx-a}}{8\,ax^2},\ \frac{3\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\left(5\,abx-2\,a^2\right)\sqrt{bx-a}}{4\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2
*(5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2), 1/4*(3*sqrt(a)*b^2*x^2*arctan(sq
rt(b*x - a)/sqrt(a)) - (5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2)]
```

**giac [A]** time = 0.95, size = 66, normalized size = 0.97

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{5\,(bx-a)^{\frac{3}{2}}b^3+3\,\sqrt{bx-a}\,ab^3}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - (5*(b*x - a)^(3/2)*b^3 +
 3*sqrt(b*x - a)*a*b^3)/(b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 53, normalized size = 0.78

$$\frac{3b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{3\sqrt{bx-a}\;a}{4x^2}-\frac{5\,(bx-a)^{\frac{3}{2}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x^3,x)
```

```
[Out] -5/4*(b*x-a)^(3/2)/x^2-3/4/x^2*(b*x-a)^(1/2)*a+3/4*b^2*arctan((b*x-a)^(1/2)
/a^(1/2))/a^(1/2)
```

**maxima [A]** time = 3.02, size = 80, normalized size = 1.18

$$\frac{3\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,\sqrt{a}}-\frac{5\,(bx-a)^{\frac{3}{2}}b^2+3\,\sqrt{bx-a}\,ab^2}{4\left((bx-a)^2+2\,(bx-a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - 1/4*(5*(b*x - a)^(3/2)*b^2
+ 3*sqrt(b*x - a)*a*b^2)/((b*x - a)^2 + 2*(b*x - a)*a + a^2)
```

**mupad** [B] time = 0.10, size = 52, normalized size = 0.76

$$\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,\sqrt{a}}-\frac{5\,{\left(b\,x-a\right)}^{3/2}}{4\,x^2}-\frac{3\,a\,\sqrt{b\,x-a}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(3/2)/x^3,x)
```

```
[Out] (3*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(1/2)) - (5*(b*x - a)^(3/2))/(4*
x^2) - (3*a*(b*x - a)^(1/2))/(4*x^2)
```

**sympy** [A] time = 3.30, size = 190, normalized size = 2.79

$$\begin{cases} \frac{ia^2}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{7ia\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{5ib^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}-1}}+\frac{3ib^2\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ \frac{a\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{2x^{\frac{3}{2}}}-\frac{5b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{4\sqrt{x}}-\frac{3b^2\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x**3,x)
```

```
[Out] Piecewise((I*a**2/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) - 7*I*a*sqrt(b)/(4
*x**(3/2)*sqrt(a/(b*x) - 1)) + 5*I*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) - 1)) +
 3*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a)), Abs(a/(b*x)) > 1),
(a*sqrt(b)*sqrt(-a/(b*x) + 1)/(2*x**(3/2)) - 5*b**(3/2)*sqrt(-a/(b*x) + 1)/
(4*sqrt(x)) - 3*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a)), True))
```

## 3.331 $\int \frac{(-a+bx)^{5/2}}{x} \, dx$

**Optimal**. Leaf size=73

$$-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2a^2\sqrt{bx-a}-\frac{2}{3}a(bx-a)^{3/2}+\frac{2}{5}(bx-a)^{5/2}$$

**Rubi [A]** time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 63, 205}

$$2a^2\sqrt{bx-a}-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{2}{3}a(bx-a)^{3/2}+\frac{2}{5}(bx-a)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x,x]
```

```
[Out] 2*a^2*Sqrt[-a + b*x] - (2*a*(-a + b*x)^(3/2))/3 + (2*(-a + b*x)^(5/2))/5 -
2*a^(5/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{5/2}}{x}\,dx &= \frac{2}{5}(-a+bx)^{5/2} - a\int \frac{(-a+bx)^{3/2}}{x}\,dx \\
&= -\frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} + a^2\int \frac{\sqrt{-a+bx}}{x}\,dx \\
&= 2a^2\sqrt{-a+bx} - \frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} - a^3\int \frac{1}{x\sqrt{-a+bx}}\,dx \\
&= 2a^2\sqrt{-a+bx} - \frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} - \frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b} \\
&= 2a^2\sqrt{-a+bx} - \frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} - 2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

**Mathematica** [A] time = 0.03, size = 60, normalized size = 0.82

$$
\frac{2}{15}\sqrt{bx-a}\left(23a^2-11abx+3b^2x^2\right) - 2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[-a + b*x]*(23*a^2 - 11*a*b*x + 3*b^2*x^2))/15 - 2*a^(5/2)*ArcTan[Sq
rt[-a + b*x]/Sqrt[a]]
```

**IntegrateAlgebraic** [A] time = 0.03, size = 74, normalized size = 1.01

$$
\frac{2}{15}\left(15a^2\sqrt{bx-a}+3(bx-a)^{5/2}-5a(bx-a)^{3/2}\right) - 2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(5/2)/x,x]
```

```
[Out] (2*(15*a^2*Sqrt[-a + b*x] - 5*a*(-a + b*x)^(3/2) + 3*(-a + b*x)^(5/2)))/15
- 2*a^(5/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

**fricas** [A] time = 1.16, size = 119, normalized size = 1.63

$$
\left[\sqrt{-a}\,a^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+\frac{2}{15}\left(3b^2x^2-11abx+23a^2\right)\sqrt{bx-a},\ -2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{15}\left(3b^2x^2-11abx+23a^2\right)\sqrt{bx-a}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*a^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/15*(3*b^2*x
^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a), -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt
(a)) + 2/15*(3*b^2*x^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a)]
```

**giac [A]** time = 1.22, size = 57, normalized size = 0.78

$$-2\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{5}\,(bx-a)^{\frac{5}{2}}-\frac{2}{3}\,(bx-a)^{\frac{3}{2}}a+2\,\sqrt{bx-a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/5*(b*x - a)^(5/2) - 2/3*(b*x -
 a)^(3/2)*a + 2*sqrt(b*x - a)*a^2
```

**maple [A]** time = 0.01, size = 58, normalized size = 0.79

$$-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\;a^2-\frac{2\,(bx-a)^{\frac{3}{2}}\,a}{3}+\frac{2\,(bx-a)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x,x)
```

```
[Out] -2/3*a*(b*x-a)^(3/2)+2/5*(b*x-a)^(5/2)-2*a^(5/2)*arctan((b*x-a)^(1/2)/a^(1/
2))+2*a^2*(b*x-a)^(1/2)
```

**maxima [A]** time = 2.92, size = 57, normalized size = 0.78

$$-2\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{5}\,(bx-a)^{\frac{5}{2}}-\frac{2}{3}\,(bx-a)^{\frac{3}{2}}a+2\,\sqrt{bx-a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/5*(b*x - a)^(5/2) - 2/3*(b*x -
 a)^(3/2)*a + 2*sqrt(b*x - a)*a^2
```

**mupad [B]** time = 0.04, size = 57, normalized size = 0.78

$$\frac{2\,{\left(b\,x-a\right)}^{5/2}}{5}-\frac{2\,a\,{\left(b\,x-a\right)}^{3/2}}{3}-2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)+2\,a^2\,\sqrt{b\,x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x,x)
```

```
[Out] (2*(b*x - a)^(5/2))/5 - (2*a*(b*x - a)^(3/2))/3 - 2*a^(5/2)*atan((b*x - a)^
(1/2)/a^(1/2)) + 2*a^2*(b*x - a)^(1/2)
```

**sympy [C]** time = 4.24, size = 240, normalized size = 3.29

$$\begin{cases} \frac{46a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{15} + ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right) - 2ia^{\frac{5}{2}}\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + 2a^{\frac{5}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) - \frac{22a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}}{15} + \frac{2\sqrt{a}b^2x^2\sqrt{-1+\frac{bx}{a}}}{5} & \text{for}\ \left|\frac{bx}{a}\right| > 1 \\ \frac{46ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{15} + ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right) - 2ia^{\frac{5}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right) - \frac{22ia^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}}{15} + \frac{2i\sqrt{a}b^2x^2\sqrt{1-\frac{bx}{a}}}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x,x)
```

```
[Out] Piecewise((46*a**(5/2)*sqrt(-1 + b*x/a)/15 + I*a**(5/2)*log(b*x/a) - 2*I*a*
*(5/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) + 2*a**(5/2)*asin(sqrt(a)/(sqrt(b)*sqrt
(x))) - 22*a**(3/2)*b*x*sqrt(-1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**2*sqrt(-1 +
 b*x/a)/5, Abs(b*x/a) > 1), (46*I*a**(5/2)*sqrt(1 - b*x/a)/15 + I*a**(5/2)*
log(b*x/a) - 2*I*a**(5/2)*log(sqrt(1 - b*x/a) + 1) - 22*I*a**(3/2)*b*x*sqrt
(1 - b*x/a)/15 + 2*I*sqrt(a)*b**2*x**2*sqrt(1 - b*x/a)/5, True))
```

## 3.332 $\int \frac{(-a+bx)^{5/2}}{x^2} dx$

**Optimal**. Leaf size=74

$$5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{x}+\frac{5}{3}b(bx-a)^{3/2}-5ab\sqrt{bx-a}$$

**Rubi** [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 205}

$$5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{x}+\frac{5}{3}b(bx-a)^{3/2}-5ab\sqrt{bx-a}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] -5*a*b*Sqrt[-a + b*x] + (5*b*(-a + b*x)^(3/2))/3 - (-a + b*x)^(5/2)/x + 5*a
^(3/2)*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+bx)^{5/2}}{x^2}\,dx &= -\frac{(-a+bx)^{5/2}}{x}+\frac{1}{2}(5b)\int \frac{(-a+bx)^{3/2}}{x}\,dx\\
&= \frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}-\frac{1}{2}(5ab)\int \frac{\sqrt{-a+bx}}{x}\,dx\\
&= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+\frac{1}{2}\left(5a^2b\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+\left(5a^2\right)\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\
&= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

**Mathematica** [C] time = 0.01, size = 36, normalized size = 0.49

$$\frac{2b(bx-a)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{bx}{a}\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] (2*b*(-a + b*x)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 - (b*x)/a])/(7*a^2)
```

**IntegrateAlgebraic** [A] time = 0.06, size = 72, normalized size = 0.97

$$5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{\sqrt{bx-a}\left(-15a^2-10a(bx-a)+2(bx-a)^2\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[-a + b*x]*(-15*a^2 - 10*a*(-a + b*x) + 2*(-a + b*x)^2))/(3*x) + 5*a^(
3/2)*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

**fricas** [A] time = 0.98, size = 131, normalized size = 1.77

$$\left[\frac{15\sqrt{-a}\,abx\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(2b^2x^2-14abx-3a^2\right)\sqrt{bx-a}}{6x},\ \frac{15a^{\frac{3}{2}}bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(2b^2x^2-14abx-3a^2\right)\sqrt{bx-a}}{3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*sqrt(-a)*a*b*x*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*(
2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/x, 1/3*(15*a^(3/2)*b*x*arctan(
sqrt(b*x - a)/sqrt(a)) + (2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/x]
```

**giac** [A] time = 1.02, size = 75, normalized size = 1.01

$$\frac{15a^{\frac{3}{2}}b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2(bx-a)^{\frac{3}{2}}b^2-12\sqrt{bx-a}\,ab^2-\frac{3\sqrt{bx-a}\,a^2b}{x}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*a^(3/2)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) + 2*(b*x - a)^(3/2)*b^2 -
 12*sqrt(b*x - a)*a*b^2 - 3*sqrt(b*x - a)*a^2*b/x)/b
```

**maple** [A] time = 0.01, size = 64, normalized size = 0.86

$$5a^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-4\sqrt{bx-a}\;ab-\frac{\sqrt{bx-a}\;a^2}{x}+\frac{2(bx-a)^{\frac{3}{2}}b}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x^2,x)
```

```
[Out] 2/3*b*(b*x-a)^(3/2)-4*a*b*(b*x-a)^(1/2)-a^2*(b*x-a)^(1/2)/x+5*a^(3/2)*b*arc
tan((b*x-a)^(1/2)/a^(1/2))
```

**maxima** [A] time = 3.05, size = 63, normalized size = 0.85

$$5a^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}(bx-a)^{\frac{3}{2}}b-4\sqrt{bx-a}\,ab-\frac{\sqrt{bx-a}\,a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5*a^(3/2)*b*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2)*b - 4*sqrt(
b*x - a)*a*b - sqrt(b*x - a)*a^2/x
```

**mupad [B]** time = 0.10, size = 63, normalized size = 0.85

$$\frac{2\,b\,{\left(b\,x-a\right)}^{3/2}}{3}-\frac{a^2\,\sqrt{b\,x-a}}{x}+5\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)-4\,a\,b\,\sqrt{b\,x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x^2,x)
```

```
[Out] (2*b*(b*x - a)^(3/2))/3 - (a^2*(b*x - a)^(1/2))/x + 5*a^(3/2)*b*atan((b*x -
 a)^(1/2)/a^(1/2)) - 4*a*b*(b*x - a)^(1/2)
```

**sympy [C]** time = 3.69, size = 245, normalized size = 3.31

$$\begin{cases} -\frac{a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{x}-\frac{14a^{\frac{3}{2}}b\sqrt{-1+\frac{bx}{a}}}{3}-\frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}+5ia^{\frac{3}{2}}b\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-5a^{\frac{3}{2}}b\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2\sqrt{a}\,b^2x\sqrt{-1+\frac{bx}{a}}}{3} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{x}-\frac{14ia^{\frac{3}{2}}b\sqrt{1-\frac{bx}{a}}}{3}-\frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}+5ia^{\frac{3}{2}}b\log\left(\sqrt{1-\frac{bx}{a}}+1\right)+\frac{2i\sqrt{a}\,b^2x\sqrt{1-\frac{bx}{a}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x**2,x)
```

```
[Out] Piecewise((-a**(5/2)*sqrt(-1 + b*x/a)/x - 14*a**(3/2)*b*sqrt(-1 + b*x/a)/3
- 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*log(sqrt(b)*sqrt(x)/sqrt(a))
 - 5*a**(3/2)*b*asin(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*sqrt(a)*b**2*x*sqrt(-1
+ b*x/a)/3, Abs(b*x/a) > 1), (-I*a**(5/2)*sqrt(1 - b*x/a)/x - 14*I*a**(3/2)
*b*sqrt(1 - b*x/a)/3 - 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*log(sqr
t(1 - b*x/a) + 1) + 2*I*sqrt(a)*b**2*x*sqrt(1 - b*x/a)/3, True))
```

# 3.333 $\int \frac{(-a+bx)^{5/2}}{x^3} \, dx$

**Optimal**. Leaf size=86

$$\frac{15}{4}b^2\sqrt{bx-a} - \frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) - \frac{(bx-a)^{5/2}}{2x^2} - \frac{5b(bx-a)^{3/2}}{4x}$$

**Rubi [A]** time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 205}

$$\frac{15}{4}b^2\sqrt{bx-a} - \frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) - \frac{(bx-a)^{5/2}}{2x^2} - \frac{5b(bx-a)^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x^3,x]
```

```
[Out] (15*b^2*Sqrt[-a + b*x])/4 - (5*b*(-a + b*x)^(3/2))/(4*x) - (-a + b*x)^(5/2)
/(2*x^2) - (15*Sqrt[a]*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{5/2}}{x^3}\,dx &= -\frac{(-a+bx)^{5/2}}{2x^2}+\frac{1}{4}(5b)\int \frac{(-a+bx)^{3/2}}{x^2}\,dx \\
&= -\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{-a+bx}}{x}\,dx \\
&= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{1}{8}\left(15ab^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx \\
&= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{1}{4}(15ab)\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+\right. \\
&= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 38, normalized size = 0.44

$$
\frac{2b^2(bx-a)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{bx}{a}\right)}{7a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x^3,x]
```

```
[Out] (2*b^2*(-a + b*x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 - (b*x)/a])/(7*a^3
)
```

**IntegrateAlgebraic [A]** time = 0.09, size = 76, normalized size = 0.88

$$
\frac{\sqrt{bx-a}\left(15a^2+25a(bx-a)+8(bx-a)^2\right)}{4x^2}-\frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[-a + b*x]*(15*a^2 + 25*a*(-a + b*x) + 8*(-a + b*x)^2))/(4*x^2) - (15*
Sqrt[a]*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/4
```

**fricas [A]** time = 0.90, size = 139, normalized size = 1.62

$$\left[\frac{15\,\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)+2\left(8\,b^2x^2+9\,abx-2\,a^2\right)\sqrt{bx-a}}{8\,x^2}, -\frac{15\,\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\left(8\,b^2x^2+9\,abx-2\,a^2\right)\sqrt{bx-a}}{4\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2
*(8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x - a))/x^2, -1/4*(15*sqrt(a)*b^2*x^2
*arctan(sqrt(b*x - a)/sqrt(a)) - (8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x - a
))/x^2]
```

**giac [A]** time = 1.04, size = 83, normalized size = 0.97

$$-\frac{15\,\sqrt{a}\,b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-8\,\sqrt{bx-a}\,b^3-\frac{9\,(bx-a)^{\frac{3}{2}}ab^3+7\,\sqrt{bx-a}\,a^2b^3}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(15*sqrt(a)*b^3*arctan(sqrt(b*x - a)/sqrt(a)) - 8*sqrt(b*x - a)*b^3 -
(9*(b*x - a)^(3/2)*a*b^3 + 7*sqrt(b*x - a)*a^2*b^3)/(b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 70, normalized size = 0.81

$$-\frac{15\sqrt{a}\;b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4}+2\sqrt{bx-a}\;b^2+\frac{7\sqrt{bx-a}\;a^2}{4x^2}+\frac{9\,(bx-a)^{\frac{3}{2}}\,a}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(b*x-a)^(1/2)+9/4*a/x^2*(b*x-a)^(3/2)+7/4/x^2*(b*x-a)^(1/2)*a^2-15/4*
b^2*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)
```

**maxima [A]** time = 2.92, size = 97, normalized size = 1.13

$$-\frac{15}{4}\,\sqrt{a}\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\,\sqrt{bx-a}\,b^2+\frac{9\,(bx-a)^{\frac{3}{2}}ab^2+7\,\sqrt{bx-a}\,a^2b^2}{4\left((bx-a)^2+2\,(bx-a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -15/4*sqrt(a)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)*b^2 + 1/4
*(9*(b*x - a)^(3/2)*a*b^2 + 7*sqrt(b*x - a)*a^2*b^2)/((b*x - a)^2 + 2*(b*x
- a)*a + a^2)
```

**mupad [B]** time = 0.09, size = 69, normalized size = 0.80

$$2\,b^2\,\sqrt{b\,x-a}-\frac{15\,\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4}+\frac{9\,a\,{\left(b\,x-a\right)}^{3/2}}{4\,x^2}+\frac{7\,a^2\,\sqrt{b\,x-a}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(b*x - a)^(1/2) - (15*a^(1/2)*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/4 +
(9*a*(b*x - a)^(3/2))/(4*x^2) + (7*a^2*(b*x - a)^(1/2))/(4*x^2)
```

**sympy [A]** time = 3.93, size = 267, normalized size = 3.10

$$\begin{cases} -\frac{15i\sqrt{a}\,b^2\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4}-\frac{ia^3}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{11ia^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{iab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{2ib^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ \frac{15\sqrt{a}\,b^2\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4}+\frac{a^3}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{ab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x**3,x)
```

```
[Out] Piecewise((-15*I*sqrt(a)*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - I*a**3/(
2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 11*I*a**2*sqrt(b)/(4*x**(3/2)*sqrt(
a/(b*x) - 1)) - I*a*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) - 1)) - 2*I*b**(5/2)*s
qrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (15*sqrt(a)*b**2*asin(sqrt(a)/
(sqrt(b)*sqrt(x)))/4 + a**3/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - 11*a*
*2*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x) + 1)) + a*b**(3/2)/(4*sqrt(x)*sqrt(-a/
(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(-a/(b*x) + 1), True))
```

## 3.334 $\int \frac{x^4}{\sqrt{a+bx}} \, dx$

**Optimal**. Leaf size=89

$$\frac{2a^4\sqrt{a+bx}}{b^5} - \frac{8a^3(a+bx)^{3/2}}{3b^5} + \frac{12a^2(a+bx)^{5/2}}{5b^5} + \frac{2(a+bx)^{9/2}}{9b^5} - \frac{8a(a+bx)^{7/2}}{7b^5}$$

**Rubi** [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{12a^2(a+bx)^{5/2}}{5b^5} - \frac{8a^3(a+bx)^{3/2}}{3b^5} + \frac{2a^4\sqrt{a+bx}}{b^5} + \frac{2(a+bx)^{9/2}}{9b^5} - \frac{8a(a+bx)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*a^4*Sqrt[a + b*x])/b^5 - (8*a^3*(a + b*x)^(3/2))/(3*b^5) + (12*a^2*(a + b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2))/(7*b^5) + (2*(a + b*x)^(9/2))/(9*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+bx}} \, dx &= \int \left(\frac{a^4}{b^4\sqrt{a+bx}} - \frac{4a^3\sqrt{a+bx}}{b^4} + \frac{6a^2(a+bx)^{3/2}}{b^4} - \frac{4a(a+bx)^{5/2}}{b^4} + \frac{(a+bx)^{7/2}}{b^4}\right) dx\\ &= \frac{2a^4\sqrt{a+bx}}{b^5} - \frac{8a^3(a+bx)^{3/2}}{3b^5} + \frac{12a^2(a+bx)^{5/2}}{5b^5} - \frac{8a(a+bx)^{7/2}}{7b^5} + \frac{2(a+bx)^{9/2}}{9b^5}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 57, normalized size = 0.64

$$\frac{2\sqrt{a+bx}\left(128a^4 - 64a^3bx + 48a^2b^2x^2 - 40ab^3x^3 + 35b^4x^4\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(128*a^4 - 64*a^3*b*x + 48*a^2*b^2*x^2 - 40*a*b^3*x^3 + 35
*b^4*x^4))/(315*b^5)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 73, normalized size = 0.82

$$\frac{2\left(315a^4\sqrt{a+bx}-420a^3(a+bx)^{3/2}+378a^2(a+bx)^{5/2}+35(a+bx)^{9/2}-180a(a+bx)^{7/2}\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*(315*a^4*Sqrt[a + b*x] - 420*a^3*(a + b*x)^(3/2) + 378*a^2*(a + b*x)^(5/
2) - 180*a*(a + b*x)^(7/2) + 35*(a + b*x)^(9/2)))/(315*b^5)
```

**fricas [A]** time = 0.72, size = 53, normalized size = 0.60

$$\frac{2\left(35\,b^4x^4-40\,ab^3x^3+48\,a^2b^2x^2-64\,a^3bx+128\,a^4\right)\sqrt{bx+a}}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 - 40*a*b^3*x^3 + 48*a^2*b^2*x^2 - 64*a^3*b*x + 128*a^4)*s
qrt(b*x + a)/b^5
```

**giac [A]** time = 1.04, size = 61, normalized size = 0.69

$$\frac{2\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^5
```

**maple [A]** time = 0.01, size = 54, normalized size = 0.61

$$\frac{2\sqrt{bx+a}\,\left(35x^4b^4-40a\,x^3b^3+48a^2x^2b^2-64a^3xb+128a^4\right)}{315b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(1/2),x)
```

```
[Out] 2/315*(b*x+a)^(1/2)*(35*b^4*x^4-40*a*b^3*x^3+48*a^2*b^2*x^2-64*a^3*b*x+128*
a^4)/b^5
```

**maxima [A]** time = 1.34, size = 71, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^5}-\frac{8\,(bx+a)^{\frac{7}{2}}a}{7\,b^5}+\frac{12\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^5}-\frac{8\,(bx+a)^{\frac{3}{2}}a^3}{3\,b^5}+\frac{2\,\sqrt{bx+a}\,a^4}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^5 - 8/7*(b*x + a)^(7/2)*a/b^5 + 12/5*(b*x + a)^(5/2)*
a^2/b^5 - 8/3*(b*x + a)^(3/2)*a^3/b^5 + 2*sqrt(b*x + a)*a^4/b^5
```

**mupad [B]** time = 0.02, size = 71, normalized size = 0.80

$$\frac{2\,{\left(a+b\,x\right)}^{9/2}}{9\,b^5}+\frac{2\,a^4\,\sqrt{a+b\,x}}{b^5}-\frac{8\,a^3\,{\left(a+b\,x\right)}^{3/2}}{3\,b^5}+\frac{12\,a^2\,{\left(a+b\,x\right)}^{5/2}}{5\,b^5}-\frac{8\,a\,{\left(a+b\,x\right)}^{7/2}}{7\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(9/2))/(9*b^5) + (2*a^4*(a + b*x)^(1/2))/b^5 - (8*a^3*(a + b*x
)^(3/2))/(3*b^5) + (12*a^2*(a + b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2))
/(7*b^5)
```

**sympy [B]** time = 4.84, size = 3755, normalized size = 42.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(1/2),x)
```

```
[Out] 256*a**(89/2)*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a
**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**3
5*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**3
2*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 256*a**(89/
2)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*
b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
**14*x**9 + 315*a**30*b**15*x**10) + 2432*a**(87/2)*b*x*sqrt(1 + b*x/a)/(31
```

```
5*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8
*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*
x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x
**9 + 315*a**30*b**15*x**10) - 2560*a**(87/2)*b*x/(315*a**40*b**5 + 3150*a*
*39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b*
*9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**
12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*
x**10) + 10336*a**(85/2)*b**2*x**2*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a
**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b
**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b*
*12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15
*x**10) - 11520*a**(85/2)*b**2*x**2/(315*a**40*b**5 + 3150*a**39*b**6*x + 1
4175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 7938
0*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 1417
5*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 25840
*a**(83/2)*b**3*x**3*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x +
14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 793
80*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 141
75*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 3072
0*a**(83/2)*b**3*x**3/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**
7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*
x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*
x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 41990*a**(81/2)*b**
4*x**4*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b*
*7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10
*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13
*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 53760*a**(81/2)*b*
*4*x**4/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800
*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a
**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a*
*31*b**14*x**9 + 315*a**30*b**15*x**10) + 46252*a**(79/2)*b**5*x**5*sqrt(1
+ b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 3780
0*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*
a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a
**31*b**14*x**9 + 315*a**30*b**15*x**10) - 64512*a**(79/2)*b**5*x**5/(315*a
**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x*
*3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**
6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9
 + 315*a**30*b**15*x**10) + 35214*a**(77/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*
a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x
**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x*
*6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**
9 + 315*a**30*b**15*x**10) - 53760*a**(77/2)*b**6*x**6/(315*a**40*b**5 + 31
50*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**
36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**3
```

3*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b
**15*x**10) + 19632*a**(75/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3
150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a*
*36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**
33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*
b**15*x**10) - 30720*a**(75/2)*b**7*x**7/(315*a**40*b**5 + 3150*a**39*b**6*
x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 +
79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 +
14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) +
10860*a**(73/2)*b**8*x**8*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6
*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4
+ 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7
+ 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) -
11520*a**(73/2)*b**8*x**8/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**3
8*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b
**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b
**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 9160*a**(71/2)
*b**9*x**9*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**3
8*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b
**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b
**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 2560*a**(71/2)
*b**9*x**9/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37
800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 6615
0*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150
*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 8396*a**(69/2)*b**10*x**10*sqr
t(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 +
37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66
150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 31
50*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 256*a**(69/2)*b**10*x**10/(3
15*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**
8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11
*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*
x**9 + 315*a**30*b**15*x**10) + 5632*a**(67/2)*b**11*x**11*sqrt(1 + b*x/a)/
(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b
**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**
11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**1
4*x**9 + 315*a**30*b**15*x**10) + 2446*a**(65/2)*b**12*x**12*sqrt(1 + b*x/a
)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37
*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b
**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b*
*14*x**9 + 315*a**30*b**15*x**10) + 620*a**(63/2)*b**13*x**13*sqrt(1 + b*x/
a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*
b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
**14*x**9 + 315*a**30*b**15*x**10) + 70*a**(61/2)*b**14*x**14*sqrt(1 + b*x/

```
a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*
b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
**14*x**9 + 315*a**30*b**15*x**10)
```

## 3.335 $\int \frac{x^3}{\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=68

$$-\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{7/2}}{7b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}$$

**Rubi [A]** time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{3/2}}{b^4}-\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{7/2}}{7b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x],x]
```

```
[Out] (-2*a^3*Sqrt[a + b*x])/b^4 + (2*a^2*(a + b*x)^(3/2))/b^4 - (6*a*(a + b*x)^(
5/2))/(5*b^4) + (2*(a + b*x)^(7/2))/(7*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx}}\,dx &= \int \left(-\frac{a^3}{b^3\sqrt{a+bx}}+\frac{3a^2\sqrt{a+bx}}{b^3}-\frac{3a(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{5/2}}{b^3}\right)dx\\ &= -\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{7/2}}{7b^4}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 46, normalized size = 0.68

$$\frac{2\sqrt{a+bx}\left(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3\right)}{35b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35*b^4)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 59, normalized size = 0.87

$$-\frac{2\left(35a^3\sqrt{a+bx}-35a^2(a+bx)^{3/2}-5(a+bx)^{7/2}+21a(a+bx)^{5/2}\right)}{35b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a + b*x],x]
```

```
[Out] (-2*(35*a^3*Sqrt[a + b*x] - 35*a^2*(a + b*x)^(3/2) + 21*a*(a + b*x)^(5/2) -
 5*(a + b*x)^(7/2)))/(35*b^4)
```

**fricas [A]** time = 1.04, size = 42, normalized size = 0.62

$$\frac{2\left(5\,b^3x^3-6\,ab^2x^2+8\,a^2bx-16\,a^3\right)\sqrt{bx+a}}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x + a)/b^4
```

**giac [A]** time = 1.21, size = 49, normalized size = 0.72

$$\frac{2\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)/b^4
```

**maple [A]** time = 0.00, size = 43, normalized size = 0.63

$$-\frac{2\sqrt{bx+a}\;\left(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3\right)}{35b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)^(1/2)*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)/b^4
```

**maxima [A]** time = 1.25, size = 56, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^4}-\frac{6\,(bx+a)^{\frac{5}{2}}a}{5\,b^4}+\frac{2\,(bx+a)^{\frac{3}{2}}a^2}{b^4}-\frac{2\,\sqrt{bx+a}\,a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^4 - 6/5*(b*x + a)^(5/2)*a/b^4 + 2*(b*x + a)^(3/2)*a^2
/b^4 - 2*sqrt(b*x + a)*a^3/b^4
```

**mupad [B]** time = 0.05, size = 56, normalized size = 0.82

$$\frac{2\,(a+b\,x)^{7/2}}{7\,b^4}-\frac{2\,a^3\,\sqrt{a+b\,x}}{b^4}+\frac{2\,a^2\,(a+b\,x)^{3/2}}{b^4}-\frac{6\,a\,(a+b\,x)^{5/2}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b^4) - (2*a^3*(a + b*x)^(1/2))/b^4 + (2*a^2*(a + b*x
)^(3/2))/b^4 - (6*a*(a + b*x)^(5/2))/(5*b^4)
```

**sympy [B]** time = 2.70, size = 1640, normalized size = 24.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(1/2),x)
```

```
[Out] -32*a**(47/2)*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18
*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**
5 + 35*a**14*b**10*x**6) + 32*a**(47/2)/(35*a**20*b**4 + 210*a**19*b**5*x +
 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**1
5*b**9*x**5 + 35*a**14*b**10*x**6) - 176*a**(45/2)*b*x*sqrt(1 + b*x/a)/(35*
a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 192*a**
(45/2)*b*x/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a*
*17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*
x**6) - 396*a**(43/2)*b**2*x**2*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*
b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 +
210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 480*a**(43/2)*b**2*x**2/(35*a*
*20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 5
25*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 462*a**(4
```

```
1/2)*b**3*x**3*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**1
8*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x*
*5 + 35*a**14*b**10*x**6) + 640*a**(41/2)*b**3*x**3/(35*a**20*b**4 + 210*a*
*19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**
4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 280*a**(39/2)*b**4*x**4*sq
rt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700
*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**
10*x**6) + 480*a**(39/2)*b**4*x**4/(35*a**20*b**4 + 210*a**19*b**5*x + 525*
a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**
9*x**5 + 35*a**14*b**10*x**6) - 42*a**(37/2)*b**5*x**5*sqrt(1 + b*x/a)/(35*
a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 192*a**
(37/2)*b**5*x**5/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 +
700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*
b**10*x**6) + 84*a**(35/2)*b**6*x**6*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a
**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x*
*4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 32*a**(35/2)*b**6*x**6/(3
5*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3
 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 94*a*
*(33/2)*b**7*x**7*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a
**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9
*x**5 + 35*a**14*b**10*x**6) + 48*a**(31/2)*b**8*x**8*sqrt(1 + b*x/a)/(35*a
**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 10*a**(2
9/2)*b**9*x**9*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**1
8*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x*
*5 + 35*a**14*b**10*x**6)
```

## 3.336 $\int \frac{x^2}{\sqrt{a+bx}} \, dx$

**Optimal**. Leaf size=51

$$\frac{2a^2\sqrt{a+bx}}{b^3} + \frac{2(a+bx)^{5/2}}{5b^3} - \frac{4a(a+bx)^{3/2}}{3b^3}$$

**Rubi** [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2\sqrt{a+bx}}{b^3} + \frac{2(a+bx)^{5/2}}{5b^3} - \frac{4a(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5
/2))/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx}} \, dx &= \int \left(\frac{a^2}{b^2\sqrt{a+bx}} - \frac{2a\sqrt{a+bx}}{b^2} + \frac{(a+bx)^{3/2}}{b^2}\right) dx \\ &= \frac{2a^2\sqrt{a+bx}}{b^3} - \frac{4a(a+bx)^{3/2}}{3b^3} + \frac{2(a+bx)^{5/2}}{5b^3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 35, normalized size = 0.69

$$\frac{2\sqrt{a+bx}\left(8a^2-4abx+3b^2x^2\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 45, normalized size = 0.88

$$\frac{2\left(15a^2\sqrt{a+bx}+3(a+bx)^{5/2}-10a(a+bx)^{3/2}\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(15*a^2*Sqrt[a + b*x] - 10*a*(a + b*x)^(3/2) + 3*(a + b*x)^(5/2)))/(15*b
^3)
```

**fricas [A]** time = 0.71, size = 31, normalized size = 0.61

$$\frac{2\left(3\,b^2x^2-4\,abx+8\,a^2\right)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

**giac [A]** time = 1.34, size = 37, normalized size = 0.73

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.63

$$\frac{2\sqrt{bx+a}\,\left(3b^2x^2-4abx+8a^2\right)}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/15*(b*x+a)^(1/2)*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3
```

**maxima** [A] time = 1.32, size = 41, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^3}-\frac{4\,(bx+a)^{\frac{3}{2}}a}{3\,b^3}+\frac{2\,\sqrt{bx+a}\,a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x + a)*a^2/b
^3
```

**mupad** [B] time = 0.04, size = 37, normalized size = 0.73

$$\frac{6\,(a+b\,x)^{5/2}-20\,a\,(a+b\,x)^{3/2}+30\,a^2\,\sqrt{a+b\,x}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(1/2),x)
```

```
[Out] (6*(a + b*x)^(5/2) - 20*a*(a + b*x)^(3/2) + 30*a^2*(a + b*x)^(1/2))/(15*b^3
)
```

**sympy** [B] time = 1.77, size = 600, normalized size = 11.76

$$\frac{16a^{\frac{21}{2}}\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{16a^{\frac{21}{2}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{40a^{\frac{19}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{48a^{\frac{19}{2}}bx}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{30a^{\frac{17}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{48a^{\frac{17}{2}}b^2x^2}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{10a^{\frac{15}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{16a^{\frac{15}{2}}b^3x^3}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{10a^{\frac{13}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{6a^{\frac{11}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*
x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3 + 45*a**7*b**4*x + 4
5*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a**(19/2)*b*x*sqrt(1 + b*x/a)/(1
5*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*
a**(19/2)*b*x/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*
b**6*x**3) + 30*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7
*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2*x**2/(
15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10
*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a*
*6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(15/2)*b**3*x**3/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**(13/2)*b**4
*x**4*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 +
15*a**5*b**6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)
```

## 3.337 $\int \frac{x}{\sqrt{a+bx}} \, dx$

**Optimal**. Leaf size=32

$$\frac{2(a+bx)^{3/2}}{3b^2} - \frac{2a\sqrt{a+bx}}{b^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{3/2}}{3b^2} - \frac{2a\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}} \, dx &= \int \left(-\frac{a}{b\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b}\right) dx \\ &= -\frac{2a\sqrt{a+bx}}{b^2} + \frac{2(a+bx)^{3/2}}{3b^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.72

$$\frac{2(bx-2a)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x],x]
```

[Out] (2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)

**IntegrateAlgebraic [A]** time = 0.01, size = 24, normalized size = 0.75

$$-\frac{2(2a-bx)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/Sqrt[a + b*x],x]

[Out] (-2*(2*a - b*x)*Sqrt[a + b*x])/(3*b^2)

**fricas [A]** time = 0.95, size = 19, normalized size = 0.59

$$\frac{2\,\sqrt{bx+a}\,(bx-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2

**giac [A]** time = 1.01, size = 23, normalized size = 0.72

$$\frac{2\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x+a)^(1/2),x, algorithm="giac")

[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2

**maple [A]** time = 0.00, size = 21, normalized size = 0.66

$$-\frac{2\sqrt{bx+a}\,\left(-bx+2a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x+a)^(1/2),x)

[Out] -2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2

**maxima** [A] time = 1.32, size = 26, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^2} - \frac{2\,\sqrt{bx+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

**mupad** [B] time = 0.03, size = 25, normalized size = 0.78

$$-\frac{6\,a\,\sqrt{a+b\,x}\,-2\,(a+b\,x)^{3/2}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(1/2),x)
```

```
[Out] -(6*a*(a + b*x)^(1/2) - 2*(a + b*x)^(3/2))/(3*b^2)
```

**sympy** [B] time = 1.16, size = 162, normalized size = 5.06

$$-\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x} - \frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x} + \frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x} + \frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**2
*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**
3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*sqr
t(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)
```

## 3.338 $\int \frac{1}{\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=14

$$\frac{2\sqrt{a+bx}}{b}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

**IntegrateAlgebraic** [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

**fricas [A]** time = 0.94, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)/b
```

**giac [A]** time = 0.99, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2),x)
```

```
[Out] 2*(b*x+a)^(1/2)/b
```

**maxima [A]** time = 1.30, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.86

$$\frac{2\sqrt{a+b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(1/2))/b
```

**sympy [A]** time = 0.07, size = 10, normalized size = 0.71

$$\frac{2\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2),x)
```

```
[Out] 2*sqrt(a + b*x)/b
```

## 3.339 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

**IntegrateAlgebraic** [A] time = 0.02, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

**fricas** [A] time = 1.12, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

**giac** [A] time = 1.24, size = 21, normalized size = 0.91

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

**maple** [A] time = 0.01, size = 18, normalized size = 0.78

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

**maxima** [A] time = 2.94, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

**mupad** [B] time = 0.06, size = 17, normalized size = 0.74

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

**sympy** [A] time = 1.11, size = 24, normalized size = 1.04

$$-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

## 3.340 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=41

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

**Rubi** [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{ax} - \frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a}$$

$$= -\frac{\sqrt{a+bx}}{ax} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{a}$$

$$= -\frac{\sqrt{a+bx}}{ax} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

**Mathematica** [A] time = 0.06, size = 47, normalized size = 1.15

$$\frac{\sqrt{a+bx}\left(\frac{b\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{\sqrt{\frac{bx}{a}+1}} - \frac{a}{x}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x]),x]
[Out] (Sqrt[a + b*x]*(-(a/x) + (b*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[1 + (b*x)/a]))
/a^2
```

**IntegrateAlgebraic** [A] time = 0.05, size = 41, normalized size = 1.00

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + b*x]),x]
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

**fricas** [A] time = 0.86, size = 93, normalized size = 2.27

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) - 2\sqrt{bx+a}\,a}{2a^2x}, -\frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \sqrt{bx+a}\,a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

**giac** [A] time = 1.07, size = 47, normalized size = 1.15

$$-\frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{bx+a}\,b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

**maple** [A] time = 0.01, size = 40, normalized size = 0.98

$$2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{bx+a}}{2abx}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/a/x/b+1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

**maxima** [A] time = 3.00, size = 60, normalized size = 1.46

$$-\frac{\sqrt{bx+a}\,b}{(bx+a)a-a^2}-\frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x + a)*b/((b*x + a)*a - a^2) - 1/2*b*log((sqrt(b*x + a) - sqrt(a))/
(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
```

**mupad** [B] time = 0.11, size = 33, normalized size = 0.80

$$\frac{b\,\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\,x}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2) - (a + b*x)^(1/2)/(a*x)
```

**sympy [A]** time = 2.30, size = 44, normalized size = 1.07

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/a**(3/2)
```

## 3.341 $\int \frac{1}{x^3\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=68

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{\sqrt{a+bx}}{2ax^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{\sqrt{a+bx}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x]),x]
```

```
[Out] -Sqrt[a + b*x]/(2*a*x^2) + (3*b*Sqrt[a + b*x])/(4*a^2*x) - (3*b^2*ArcTanh[S
qrt[a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{2ax^2} - \frac{(3b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a}$$
$$= -\frac{\sqrt{a+bx}}{2ax^2} + \frac{3b\sqrt{a+bx}}{4a^2x} + \frac{\left(3b^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^2}$$
$$= -\frac{\sqrt{a+bx}}{2ax^2} + \frac{3b\sqrt{a+bx}}{4a^2x} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{4a^2}$$
$$= -\frac{\sqrt{a+bx}}{2ax^2} + \frac{3b\sqrt{a+bx}}{4a^2x} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.49

$$-\frac{2b^2\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x]),x]
[Out] (-2*b^2*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x)/a])/a^3
```

**IntegrateAlgebraic [A]** time = 0.08, size = 63, normalized size = 0.93

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}} - \frac{5a\sqrt{a+bx}-3(a+bx)^{3/2}}{4a^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*x]),x]
[Out] -1/4*(5*a*Sqrt[a + b*x] - 3*(a + b*x)^(3/2))/(a^2*x^2) - (3*b^2*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

**fricas [A]** time = 1.12, size = 123, normalized size = 1.81

$$\left[\frac{3\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(3abx-2a^2\right)\sqrt{bx+a}}{8a^3x^2}, \frac{3\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3abx-2a^2\right)\sqrt{bx+a}}{4a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(3
*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + (3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2)]
```

**giac [A]** time = 1.19, size = 69, normalized size = 1.01

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\,(bx+a)^{\frac{3}{2}}b^3-5\,\sqrt{bx+a}\,ab^3}{a^2b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/
2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 66, normalized size = 0.97

$$2\left(-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{bx+a}}{2abx}\right)}{4a}-\frac{\sqrt{bx+a}}{4a\,b^2x^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(1/2),x)
```

```
[Out] 2*b^2*(-1/4*(b*x+a)^(1/2)/a/x^2/b^2-3/4/a*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arc
tanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)))
```

**maxima [A]** time = 2.92, size = 92, normalized size = 1.35

$$\frac{3\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}+\frac{3\,(bx+a)^{\frac{3}{2}}b^2-5\,\sqrt{bx+a}\,ab^2}{4\left((bx+a)^2a^2-2\,(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) +
1/4*(3*(b*x + a)^(3/2)*b^2 - 5*sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a^2 - 2*(b
*x + a)*a^3 + a^4)
```

**mupad [B]** time = 0.06, size = 51, normalized size = 0.75

$$\frac{3(a+bx)^{3/2}}{4a^2x^2}-\frac{5\sqrt{a+bx}}{4ax^2}-\frac{3b^2\operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(1/2)),x)
```

```
[Out] (3*(a + b*x)^(3/2))/(4*a^2*x^2) - (5*(a + b*x)^(1/2))/(4*a*x^2) - (3*b^2*at
anh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(5/2))
```

**sympy [A]** time = 4.37, size = 102, normalized size = 1.50

$$-\frac{1}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(1/2),x)
```

```
[Out] -1/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(4*a*x**(3/2)*sqrt(a/(b
*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 3*b**2*asinh(sq
rt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2))
```

## 3.342 $\int \frac{1}{x^4\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=90

$$\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{5b^2\sqrt{a+bx}}{8a^3x}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{\sqrt{a+bx}}{3ax^3}$$

**Rubi [A]** time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{5b^2\sqrt{a+bx}}{8a^3x}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{\sqrt{a+bx}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x]),x]
```

```
[Out] -Sqrt[a + b*x]/(3*a*x^3) + (5*b*Sqrt[a + b*x])/(12*a^2*x^2) - (5*b^2*Sqrt[a
 + b*x])/(8*a^3*x) + (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{a+bx}}\,dx &= -\frac{\sqrt{a+bx}}{3ax^3} - \frac{(5b)\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{6a} \\
&= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} + \frac{\left(5b^2\right)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{8a^2} \\
&= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} - \frac{\left(5b^3\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a^3} \\
&= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{8a^3} \\
&= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} + \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.37

$$
\frac{2b^3\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{bx}{a}+1\right)}{a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x]),x]
```

```
[Out] (2*b^3*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x)/a])/a^4
```

**IntegrateAlgebraic [A]** time = 0.07, size = 71, normalized size = 0.79

$$
\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}} - \frac{\sqrt{a+bx}\left(33a^2-40a(a+bx)+15(a+bx)^2\right)}{24a^3x^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[a + b*x]),x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(33*a^2 - 40*a*(a + b*x) + 15*(a + b*x)^2))/(a^3*x^3)
+ (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(7/2))
```

**fricas** [A] time = 0.91, size = 145, normalized size = 1.61

$$\left[\frac{15\sqrt{a}\,b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)-2\left(15\,ab^2x^2-10\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{48\,a^4x^3},-\frac{15\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(15\,ab^2x^2-10\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{24\,a^4x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3), -1/24*(15*sqr
t(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 - 10*a^2*b*x
 + 8*a^3)*sqrt(b*x + a))/(a^4*x^3)]
```

**giac** [A] time = 0.91, size = 84, normalized size = 0.93

$$-\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{15\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+33\sqrt{bx+a}\,a^2b^4}{a^3b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a)
^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^3*
x^3))/b
```

**maple** [A] time = 0.01, size = 90, normalized size = 1.00

$$2\left(-\frac{5\left(-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{bx+a}}{2abx}\right)}{4a}-\frac{\sqrt{bx+a}}{4a\,b^2x^2}\right)}{6a}-\frac{\sqrt{bx+a}}{6a\,b^3x^3}\right)b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^(1/2),x)
```

```
[Out] 2*b^3*(-1/6*(b*x+a)^(1/2)/a/x^3/b^3-5/6/a*(-1/4*(b*x+a)^(1/2)/a/b^2/x^2-3/4
/a*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))))
```

**maxima [A]** time = 2.92, size = 121, normalized size = 1.34

$$-\frac{5\,b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{5}{2}}b^3-40\,(bx+a)^{\frac{3}{2}}ab^3+33\,\sqrt{bx+a}\,a^2b^3}{24\left((bx+a)^3a^3-3\,(bx+a)^2a^4+3\,(bx+a)a^5-a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(7/2)
- 1/24*(15*(b*x + a)^(5/2)*b^3 - 40*(b*x + a)^(3/2)*a*b^3 + 33*sqrt(b*x + a
)*a^2*b^3)/((b*x + a)^3*a^3 - 3*(b*x + a)^2*a^4 + 3*(b*x + a)*a^5 - a^6)
```

**mupad [B]** time = 0.05, size = 69, normalized size = 0.77

$$\frac{5\,(a+b\,x)^{3/2}}{3\,a^2\,x^3}-\frac{11\,\sqrt{a+b\,x}}{8\,a\,x^3}-\frac{5\,(a+b\,x)^{5/2}}{8\,a^3\,x^3}-\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{8\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^(1/2)),x)
```

```
[Out] (5*(a + b*x)^(3/2))/(3*a^2*x^3) - (11*(a + b*x)^(1/2))/(8*a*x^3) - (5*(a +
b*x)^(5/2))/(8*a^3*x^3) - (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(8*a^
(7/2))
```

**sympy [A]** time = 7.02, size = 129, normalized size = 1.43

$$-\frac{1}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{\sqrt{b}}{12ax^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{3}{2}}}{24a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{5}{2}}}{8a^3\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{5b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**(1/2),x)
```

```
[Out] -1/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(12*a*x**(5/2)*sqrt(a/(
b*x) + 1)) - 5*b**(3/2)/(24*a**2*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(5/2)/(
8*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) + 5*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/(8*a**(7/2))
```

## 3.343 $\int \frac{x^4}{(a+bx)^{3/2}} dx$

**Optimal**. Leaf size=85

$$-\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}$$

**Rubi** [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^4)/(b^5*Sqrt[a + b*x]) - (8*a^3*Sqrt[a + b*x])/b^5 + (4*a^2*(a + b*x)
^(3/2))/b^5 - (8*a*(a + b*x)^(5/2))/(5*b^5) + (2*(a + b*x)^(7/2))/(7*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{3/2}}\,dx &= \int \left(\frac{a^4}{b^4(a+bx)^{3/2}}-\frac{4a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{4a(a+bx)^{3/2}}{b^4}+\frac{(a+bx)^{5/2}}{b^4}\right)dx\\ &= -\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 57, normalized size = 0.67

$$\frac{2\left(-128a^4-64a^3bx+16a^2b^2x^2-8ab^3x^3+5b^4x^4\right)}{35b^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-128*a^4 - 64*a^3*b*x + 16*a^2*b^2*x^2 - 8*a*b^3*x^3 + 5*b^4*x^4))/(35*
b^5*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.03, size = 63, normalized size = 0.74

$$\frac{2\left(-35a^4-140a^3(a+bx)+70a^2(a+bx)^2-28a(a+bx)^3+5(a+bx)^4\right)}{35b^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-35*a^4 - 140*a^3*(a + b*x) + 70*a^2*(a + b*x)^2 - 28*a*(a + b*x)^3 + 5
*(a + b*x)^4))/(35*b^5*Sqrt[a + b*x])
```

**fricas [A]** time = 0.95, size = 63, normalized size = 0.74

$$\frac{2\left(5\,b^4x^4-8\,ab^3x^3+16\,a^2b^2x^2-64\,a^3bx-128\,a^4\right)\sqrt{bx+a}}{35\left(b^6x+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*x^4 - 8*a*b^3*x^3 + 16*a^2*b^2*x^2 - 64*a^3*b*x - 128*a^4)*sqrt
(b*x + a)/(b^6*x + a*b^5)
```

**giac [A]** time = 1.25, size = 77, normalized size = 0.91

$$-\frac{2\,a^4}{\sqrt{bx+a}\,b^5}+\frac{2\left(5\,(bx+a)^{\frac{7}{2}}b^{30}-28\,(bx+a)^{\frac{5}{2}}ab^{30}+70\,(bx+a)^{\frac{3}{2}}a^2b^{30}-140\,\sqrt{bx+a}\,a^3b^{30}\right)}{35\,b^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^4/(sqrt(b*x + a)*b^5) + 2/35*(5*(b*x + a)^(7/2)*b^30 - 28*(b*x + a)^(5
/2)*a*b^30 + 70*(b*x + a)^(3/2)*a^2*b^30 - 140*sqrt(b*x + a)*a^3*b^30)/b^35
```

**maple [A]** time = 0.01, size = 54, normalized size = 0.64

$$-\frac{2\left(-5x^4b^4+8a\,x^3b^3-16a^2x^2b^2+64a^3xb+128a^4\right)}{35\sqrt{bx+a}\;b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(3/2),x)
```

```
[Out] -2/35/(b*x+a)^(1/2)*(-5*b^4*x^4+8*a*b^3*x^3-16*a^2*b^2*x^2+64*a^3*b*x+128*a
^4)/b^5
```

**maxima [A]** time = 1.33, size = 71, normalized size = 0.84

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^5}-\frac{8\,(bx+a)^{\frac{5}{2}}a}{5\,b^5}+\frac{4\,(bx+a)^{\frac{3}{2}}a^2}{b^5}-\frac{8\,\sqrt{bx+a}\,a^3}{b^5}-\frac{2\,a^4}{\sqrt{bx+a}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^5 - 8/5*(b*x + a)^(5/2)*a/b^5 + 4*(b*x + a)^(3/2)*a^2
/b^5 - 8*sqrt(b*x + a)*a^3/b^5 - 2*a^4/(sqrt(b*x + a)*b^5)
```

**mupad [B]** time = 0.03, size = 71, normalized size = 0.84

$$\frac{2\,(a+b\,x)^{7/2}}{7\,b^5}-\frac{8\,a^3\,\sqrt{a+b\,x}}{b^5}+\frac{4\,a^2\,(a+b\,x)^{3/2}}{b^5}-\frac{2\,a^4}{b^5\,\sqrt{a+b\,x}}-\frac{8\,a\,(a+b\,x)^{5/2}}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b^5) - (8*a^3*(a + b*x)^(1/2))/b^5 + (4*a^2*(a + b*x
)^(3/2))/b^5 - (2*a^4)/(b^5*(a + b*x)^(1/2)) - (8*a*(a + b*x)^(5/2))/(5*b^5
)
```

**sympy [B]** time = 4.81, size = 3606, normalized size = 42.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(3/2),x)
```

```
[Out] -256*a**(87/2)*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**
38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**
10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*
x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 256*a**(87/2)/(35*a**
40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 +
7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200
*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30
*b**15*x**10) - 2432*a**(85/2)*b*x*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**
```

39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x
**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7
 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 2
560*a**(85/2)*b*x/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2
+ 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 735
0*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a*
*31*b**14*x**9 + 35*a**30*b**15*x**10) - 10336*a**(83/2)*b**2*x**2*sqrt(1 +
 b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**
37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b*
*11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*
x**9 + 35*a**30*b**15*x**10) + 11520*a**(83/2)*b**2*x**2/(35*a**40*b**5 + 3
50*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*
b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**1
2*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**1
0) - 25840*a**(81/2)*b**3*x**3*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b
**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4
+ 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1
575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 30720
*a**(81/2)*b**3*x**3/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x*
*2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 +
7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350
*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 41990*a**(79/2)*b**4*x**4*sqrt(
1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*
a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34
*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**
14*x**9 + 35*a**30*b**15*x**10) + 53760*a**(79/2)*b**4*x**4/(35*a**40*b**5
+ 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**
36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b
**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x
**10) - 46182*a**(77/2)*b**5*x**5*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**3
9*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x*
*4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7
+ 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 64
512*a**(77/2)*b**5*x**5/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7
*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5
 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 +
350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 34584*a**(75/2)*b**6*x**6*sq
rt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 42
00*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a*
*34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*
b**14*x**9 + 35*a**30*b**15*x**10) + 53760*a**(75/2)*b**6*x**6/(35*a**40*b*
*5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*
a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**3
3*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**1
5*x**10) - 17112*a**(73/2)*b**7*x**7*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a

```
**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9
*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x*
*7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) +
 30720*a**(73/2)*b**7*x**7/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b
**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x
**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8
 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 4980*a**(71/2)*b**8*x**8*
sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 +
4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*
a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**3
1*b**14*x**9 + 35*a**30*b**15*x**10) + 11520*a**(71/2)*b**8*x**8/(35*a**40*
b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 735
0*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a*
*33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b*
*15*x**10) - 340*a**(69/2)*b**9*x**9*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a
**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9
*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x*
*7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) +
 2560*a**(69/2)*b**9*x**9/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b*
*7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x*
*5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8
+ 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 424*a**(67/2)*b**10*x**10*
sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 +
4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*
a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**3
1*b**14*x**9 + 35*a**30*b**15*x**10) + 256*a**(67/2)*b**10*x**10/(35*a**40*
b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 735
0*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a*
*33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b*
*15*x**10) + 248*a**(65/2)*b**11*x**11*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350
*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b*
*9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*
x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10)
 + 74*a**(63/2)*b**12*x**12*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6
*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8
820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575
*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 10*a**(6
1/2)*b**13*x**13*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a
**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b
**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**1
3*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10)
```

# 3.344 $\int \frac{x^3}{(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=66

$$\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}$$

**Rubi** [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(3/2),x]
```

```
[Out] (2*a^3)/(b^4*Sqrt[a + b*x]) + (6*a^2*Sqrt[a + b*x])/b^4 - (2*a*(a + b*x)^(3
/2))/b^4 + (2*(a + b*x)^(5/2))/(5*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{3/2}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{3/2}}+\frac{3a^2}{b^3\sqrt{a+bx}}-\frac{3a\sqrt{a+bx}}{b^3}+\frac{(a+bx)^{3/2}}{b^3}\right)dx\\ &= \frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 45, normalized size = 0.68

$$\frac{2\left(16a^3+8a^2bx-2ab^2x^2+b^3x^3\right)}{5b^4\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(3/2),x]
```

```
[Out] (2*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.02, size = 49, normalized size = 0.74

$$\frac{2\left(5a^3+15a^2(a+bx)-5a(a+bx)^2+(a+bx)^3\right)}{5b^4\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^(3/2),x]
```

```
[Out] (2*(5*a^3 + 15*a^2*(a + b*x) - 5*a*(a + b*x)^2 + (a + b*x)^3))/(5*b^4*Sqrt[
a + b*x])
```

**fricas [A]** time = 0.95, size = 51, normalized size = 0.77

$$\frac{2\left(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3\right)\sqrt{bx+a}}{5\left(b^5x+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x + a)/(b^5*x + a*b
^4)
```

**giac [A]** time = 1.10, size = 61, normalized size = 0.92

$$\frac{2\,a^3}{\sqrt{bx+a}\,b^4}+\frac{2\left((bx+a)^{\frac{5}{2}}b^{16}-5\,(bx+a)^{\frac{3}{2}}ab^{16}+15\,\sqrt{bx+a}\,a^2b^{16}\right)}{5\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*a^3/(sqrt(b*x + a)*b^4) + 2/5*((b*x + a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a
*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/b^20
```

**maple [A]** time = 0.01, size = 42, normalized size = 0.64

$$\frac{\frac{2}{5}b^3x^3-\frac{4}{5}a\,b^2x^2+\frac{16}{5}a^2bx+\frac{32}{5}a^3}{b^4\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(3/2),x)
```

```
[Out] 2/5/(b*x+a)^(1/2)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)/b^4
```

**maxima [A]** time = 1.38, size = 56, normalized size = 0.85

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^4} - \frac{2\,(bx+a)^{\frac{3}{2}}a}{b^4} + \frac{6\,\sqrt{bx+a}\,a^2}{b^4} + \frac{2\,a^3}{\sqrt{bx+a}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^4 - 2*(b*x + a)^(3/2)*a/b^4 + 6*sqrt(b*x + a)*a^2/b^4
 + 2*a^3/(sqrt(b*x + a)*b^4)
```

**mupad [B]** time = 0.05, size = 56, normalized size = 0.85

$$\frac{2\,(a+b\,x)^{5/2}}{5\,b^4} + \frac{6\,a^2\,\sqrt{a+b\,x}}{b^4} + \frac{2\,a^3}{b^4\,\sqrt{a+b\,x}} - \frac{2\,a\,(a+b\,x)^{3/2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(1/2))/b^4 + (2*a^3)/(b^4*(a
 + b*x)^(1/2)) - (2*a*(a + b*x)^(3/2))/b^4
```

**sympy [B]** time = 2.94, size = 1538, normalized size = 23.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(3/2),x)
```

```
[Out] 32*a**(45/2)*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**
6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*
a**14*b**10*x**6) - 32*a**(45/2)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18
*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5
+ 5*a**14*b**10*x**6) + 176*a**(43/2)*b*x*sqrt(1 + b*x/a)/(5*a**20*b**4 + 3
0*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x
**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 192*a**(43/2)*b*x/(5*a**20
*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**
16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 396*a**(41/2)*b**
2*x**2*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2
```

```
 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*
b**10*x**6) - 480*a**(41/2)*b**2*x**2/(5*a**20*b**4 + 30*a**19*b**5*x + 75*
a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*
x**5 + 5*a**14*b**10*x**6) + 462*a**(39/2)*b**3*x**3*sqrt(1 + b*x/a)/(5*a**
20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a
**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 640*a**(39/2)*b
**3*x**3/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b
**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) +
290*a**(37/2)*b**4*x**4*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 7
5*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**
9*x**5 + 5*a**14*b**10*x**6) - 480*a**(37/2)*b**4*x**4/(5*a**20*b**4 + 30*a
**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4
 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 92*a**(35/2)*b**5*x**5*sqrt(1
 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*
b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) -
 192*a**(35/2)*b**5*x**5/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x*
*2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**1
4*b**10*x**6) + 16*a**(33/2)*b**6*x**6*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a
**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4
 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 32*a**(33/2)*b**6*x**6/(5*a**
20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a
**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 6*a**(31/2)*b**
7*x**7*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2
 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*
b**10*x**6) + 2*a**(29/2)*b**8*x**8*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**1
9*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 +
30*a**15*b**9*x**5 + 5*a**14*b**10*x**6)
```

## 3.345 $\int \frac{x^2}{(a+bx)^{3/2}} \, dx$

**Optimal**. Leaf size=49

$$-\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}$$

**Rubi [A]** time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^2)/(b^3*Sqrt[a + b*x]) - (4*a*Sqrt[a + b*x])/b^3 + (2*(a + b*x)^(3/2)
)/(3*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{3/2}} \, dx &= \int \left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right) dx\\ &= -\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 34, normalized size = 0.69

$$\frac{2\left(-8a^2-4abx+b^2x^2\right)}{3b^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[a + b*x])
```

**IntegrateAlgebraic** [A] time = 0.02, size = 37, normalized size = 0.76

$$\frac{2\left(-3a^2-6a(a+bx)+(a+bx)^2\right)}{3b^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-3*a^2 - 6*a*(a + b*x) + (a + b*x)^2))/(3*b^3*Sqrt[a + b*x])
```

**fricas** [A] time = 0.84, size = 40, normalized size = 0.82

$$\frac{2\left(b^2x^2-4\,abx-8\,a^2\right)\sqrt{bx+a}}{3\left(b^4x+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x + a)/(b^4*x + a*b^3)
```

**giac** [A] time = 1.14, size = 46, normalized size = 0.94

$$-\frac{2\,a^2}{\sqrt{bx+a}\,b^3}+\frac{2\left((bx+a)^{\frac{3}{2}}b^6-6\,\sqrt{bx+a}\,ab^6\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^2/(sqrt(b*x + a)*b^3) + 2/3*((b*x + a)^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b
^6)/b^9
```

**maple** [A] time = 0.01, size = 32, normalized size = 0.65

$$-\frac{2\left(-b^2x^2+4abx+8a^2\right)}{3\sqrt{bx+a}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(3/2),x)
```

```
[Out] -2/3/(b*x+a)^(1/2)*(-b^2*x^2+4*a*b*x+8*a^2)/b^3
```

**maxima [A]** time = 1.29, size = 41, normalized size = 0.84

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^3}-\frac{4\,\sqrt{bx+a}\,a}{b^3}-\frac{2\,a^2}{\sqrt{bx+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^3 - 4*sqrt(b*x + a)*a/b^3 - 2*a^2/(sqrt(b*x + a)*b^3)
```

**mupad [B]** time = 0.04, size = 35, normalized size = 0.71

$$-\frac{12\,a\,\left(a+b\,x\right)-2\,{\left(a+b\,x\right)}^2+6\,a^2}{3\,b^3\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(3/2),x)
```

```
[Out] -(12*a*(a + b*x) - 2*(a + b*x)^2 + 6*a^2)/(3*b^3*(a + b*x)^(1/2))
```

**sympy [B]** time = 1.83, size = 534, normalized size = 10.90

$$-\frac{16a^{\frac{19}{2}}\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}+\frac{16a^{\frac{19}{2}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}-\frac{40a^{\frac{17}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}+\frac{48a^{\frac{17}{2}}bx}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}-\frac{30a^{\frac{15}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}+\frac{48a^{\frac{15}{2}}b^2x^2}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}-\frac{4a^{\frac{13}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}+\frac{16a^{\frac{13}{2}}b^3x^3}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}+\frac{2a^{\frac{11}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(3/2),x)
```

```
[Out] -16*a**(19/2)*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x*
*2 + 3*a**5*b**6*x**3) + 16*a**(19/2)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6
*b**5*x**2 + 3*a**5*b**6*x**3) - 40*a**(17/2)*b*x*sqrt(1 + b*x/a)/(3*a**8*b
**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(17/2)*b
*x/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) - 30
*a**(15/2)*b**2*x**2*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*
b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(15/2)*b**2*x**2/(3*a**8*b**3 + 9*a**
7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) - 4*a**(13/2)*b**3*x**3*sqr
t(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*
x**3) + 16*a**(13/2)*b**3*x**3/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x
**2 + 3*a**5*b**6*x**3) + 2*a**(11/2)*b**4*x**4*sqrt(1 + b*x/a)/(3*a**8*b**
3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3)
```

## 3.346 $\int \frac{x}{(a+bx)^{3/2}}\, dx$

**Optimal**. Leaf size=30

$$\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(3/2),x]
```

```
[Out] (2*a)/(b^2*Sqrt[a + b*x]) + (2*Sqrt[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{3/2}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx\\ &= \frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 21, normalized size = 0.70

$$\frac{2(2a+bx)}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(3/2),x]
```

```
[Out] (2*(2*a + b*x))/(b^2*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.01, size = 21, normalized size = 0.70

$$\frac{2(2a+bx)}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x)^(3/2),x]
```

```
[Out] (2*(2*a + b*x))/(b^2*Sqrt[a + b*x])
```

**fricas [A]** time = 0.91, size = 29, normalized size = 0.97

$$\frac{2\,(bx+2\,a)\sqrt{bx+a}}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*x + 2*a)*sqrt(b*x + a)/(b^3*x + a*b^2)
```

**giac [A]** time = 1.00, size = 29, normalized size = 0.97

$$\frac{2\left(\frac{\sqrt{bx+a}}{b}+\frac{a}{\sqrt{bx+a}\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(b*x + a)/b + a/(sqrt(b*x + a)*b))/b
```

**maple [A]** time = 0.00, size = 20, normalized size = 0.67

$$\frac{2bx+4a}{b^2\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(3/2),x)
```

```
[Out] 2/(b*x+a)^(1/2)*(b*x+2*a)/b^2
```

**maxima** [A] time = 1.32, size = 26, normalized size = 0.87

$$\frac{2\sqrt{bx+a}}{b^2}+\frac{2a}{\sqrt{bx+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b^2 + 2*a/(sqrt(b*x + a)*b^2)
```

**mupad** [B] time = 0.09, size = 19, normalized size = 0.63

$$\frac{4\,a+2\,b\,x}{b^2\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(3/2),x)
```

```
[Out] (4*a + 2*b*x)/(b^2*(a + b*x)^(1/2))
```

**sympy** [A] time = 0.67, size = 37, normalized size = 1.23

$$\begin{cases} \frac{4a}{b^2\sqrt{a+bx}}+\frac{2x}{b\sqrt{a+bx}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(3/2),x)
```

```
[Out] Piecewise((4*a/(b**2*sqrt(a + b*x)) + 2*x/(b*sqrt(a + b*x)), Ne(b, 0)), (x*
*2/(2*a**(3/2)), True))
```

## 3.347 $\int \frac{1}{(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=14

$$-\frac{2}{b\sqrt{a+bx}}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}}\,dx = -\frac{2}{b\sqrt{a+bx}}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[a + b*x])
```

**IntegrateAlgebraic** [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(-3/2),x]
[Out] -2/(b*Sqrt[a + b*x])
```

**fricas [A]** time = 0.74, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{bx+a}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x + a)/(b^2*x + a*b)
```

**giac [A]** time = 1.24, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="giac")
[Out] -2/(sqrt(b*x + a)*b)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$-\frac{2}{\sqrt{bx+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2),x)
[Out] -2/b/(b*x+a)^(1/2)
```

**maxima [A]** time = 1.34, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(b*x + a)*b)
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.86

$$-\frac{2}{b\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(3/2),x)
[Out] -2/(b*(a + b*x)^(1/2))
```

**sympy [A]** time = 0.07, size = 12, normalized size = 0.86

$$-\frac{2}{b\sqrt{a+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2),x)
[Out] -2/(b*sqrt(a + b*x))
```

## 3.348 $\int \frac{1}{x(a+bx)^{3/2}} \, dx$

**Optimal**. Leaf size=38

$$\frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$\frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{3/2}}\,dx = \frac{2}{a\sqrt{a+bx}} + \frac{\int \frac{1}{x\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{a\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{ab}$$
$$= \frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.79

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx}{a}+1\right)}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(3/2)),x]
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x)/a])/(a*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.03, size = 38, normalized size = 1.00

$$\frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^(3/2)),x]
[Out] 2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

**fricas [A]** time = 1.03, size = 110, normalized size = 2.89

$$\left[\frac{(bx+a)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}\,a}{a^2bx+a^3}, \frac{2\left((bx+a)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,a\right)}{a^2bx+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b
*x + a)*a)/(a^2*b*x + a^3), 2*((b*x + a)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + sqrt(b*x + a)*a)/(a^2*b*x + a^3)]
```

**giac** [A] time = 1.07, size = 37, normalized size = 0.97

$$\frac{2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} + \frac{2}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(b*x + a)*a)
```

**maple** [A] time = 0.01, size = 31, normalized size = 0.82

$$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{bx+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(3/2),x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)+2/a/(b*x+a)^(1/2)
```

**maxima** [A] time = 2.93, size = 45, normalized size = 1.18

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2) + 2/(sqrt(
b*x + a)*a)
```

**mupad** [B] time = 0.04, size = 30, normalized size = 0.79

$$\frac{2}{a\,\sqrt{a+b\,x}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(3/2)),x)
```

```
[Out] 2/(a*(a + b*x)^(1/2)) - (2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2)
```

**sympy [B]** time = 1.87, size = 146, normalized size = 3.84

$$\frac{2a^3\sqrt{1+\frac{bx}{a}}}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{a^3\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{a^2bx\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^2bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x/a)/(a**(9/2) + a**(7/2)*b*x) + a**3*log(b*x/a)/(a**(9/2
) + a**(7/2)*b*x) - 2*a**3*log(sqrt(1 + b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*
x) + a**2*b*x*log(b*x/a)/(a**(9/2) + a**(7/2)*b*x) - 2*a**2*b*x*log(sqrt(1
+ b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*x)
```

## 3.349 $\int \frac{1}{x^2(a+bx)^{3/2}} dx$

**Optimal**. Leaf size=57

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3b}{a^2\sqrt{a+bx}}-\frac{1}{ax\sqrt{a+bx}}$$

**Rubi** [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{3\sqrt{a+bx}}{a^2x}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{ax\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x*Sqrt[a + b*x]) - (3*Sqrt[a + b*x])/(a^2*x) + (3*b*ArcTanh[Sqrt[a + b
*x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^{3/2}}\,dx &= \frac{2}{ax\sqrt{a+bx}}+\frac{3\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{a}\\ &= \frac{2}{ax\sqrt{a+bx}}-\frac{3\sqrt{a+bx}}{a^2x}-\frac{(3b)\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a^2}\\ &= \frac{2}{ax\sqrt{a+bx}}-\frac{3\sqrt{a+bx}}{a^2x}-\frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a^2}\\ &= \frac{2}{ax\sqrt{a+bx}}-\frac{3\sqrt{a+bx}}{a^2x}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 31, normalized size = 0.54

$$-\frac{2b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx}{a}+1\right)}{a^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x)/a])/(a^2*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.07, size = 52, normalized size = 0.91

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2a-3(a+bx)}{a^2x\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^(3/2)),x]
```

```
[Out] (2*a - 3*(a + b*x))/(a^2*x*Sqrt[a + b*x]) + (3*b*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]])/a^(5/2)
```

**fricas [A]** time = 1.17, size = 151, normalized size = 2.65

$$\left[\frac{3\left(b^2x^2+abx\right)\sqrt{a}\,\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(3abx+a^2\right)\sqrt{bx+a}}{2\left(a^3bx^2+a^4x\right)},-\frac{3\left(b^2x^2+abx\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3abx+a^2\right)\sqrt{bx+a}}{a^3bx^2+a^4x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + a*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)
/x) - 2*(3*a*b*x + a^2)*sqrt(b*x + a))/(a^3*b*x^2 + a^4*x), -(3*(b^2*x^2 +
a*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b*x + a^2)*sqrt(b*x
 + a))/(a^3*b*x^2 + a^4*x)]
```

**giac [A]** time = 0.90, size = 64, normalized size = 1.12

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}-\frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}\,a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - (3*(b*x + a)*b - 2*a*b
)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2)
```

**maple [A]** time = 0.01, size = 55, normalized size = 0.96

$$2\left(-\frac{1}{\sqrt{bx+a}\;a^{2}}-\frac{-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}+\frac{\sqrt{bx+a}}{2bx}}{a^{2}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(3/2),x)
```

```
[Out] 2*b*(-1/a^2/(b*x+a)^(1/2)-1/a^2*(1/2*(b*x+a)^(1/2)/b/x-3/2*arctanh((b*x+a)^
(1/2)/a^(1/2))/a^(1/2)))
```

**maxima [A]** time = 3.01, size = 76, normalized size = 1.33

$$-\frac{3\,(bx+a)b-2\,ab}{(bx+a)^{\frac{3}{2}}a^{2}-\sqrt{bx+a}\,a^{3}}-\frac{3\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -(3*(b*x + a)*b - 2*a*b)/((b*x + a)^(3/2)*a^2 - sqrt(b*x + a)*a^3) - 3/2*b*
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2)
```

**mupad [B]** time = 0.12, size = 60, normalized size = 1.05

$$\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,b}{a}-\frac{3\,b\,(a+b\,x)}{a^2}}{a\,\sqrt{a+b\,x}-{(a+b\,x)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(3/2)),x)
```

```
[Out] (3*b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*b)/a - (3*b*(a + b*x))/a
^2)/(a*(a + b*x)^(1/2) - (a + b*x)^(3/2))
```

**sympy [A]** time = 3.41, size = 73, normalized size = 1.28

$$-\frac{1}{a\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{a^2\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(3/2),x)
```

```
[Out] -1/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(a**2*sqrt(x)*sqrt(a/
(b*x) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2)
```

## 3.350 $\int \frac{1}{x^3(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=87

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15b^2}{4a^3\sqrt{a+bx}}+\frac{5b}{4a^2x\sqrt{a+bx}}-\frac{1}{2ax^2\sqrt{a+bx}}$$

**Rubi [A]** time = 0.02, antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{5\sqrt{a+bx}}{2a^2x^2}+\frac{15b\sqrt{a+bx}}{4a^3x}+\frac{2}{ax^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^2*Sqrt[a + b*x]) - (5*Sqrt[a + b*x])/(2*a^2*x^2) + (15*b*Sqrt[a + b*
x])/(4*a^3*x) - (15*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3(a+bx)^{3/2}}\,dx &= \frac{2}{ax^2\sqrt{a+bx}}+\frac{5\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{a}\\
&= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}-\frac{(15b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a^2}\\
&= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}+\frac{15b\sqrt{a+bx}}{4a^3x}+\frac{\left(15b^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^3}\\
&= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}+\frac{15b\sqrt{a+bx}}{4a^3x}+\frac{(15b)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{4a^3}\\
&= \frac{2}{ax^2\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{2a^2x^2}+\frac{15b\sqrt{a+bx}}{4a^3x}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.38

$$\frac{2b^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx}{a}+1\right)}{a^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(3/2)),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (b*x)/a])/(a^3*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.10, size = 71, normalized size = 0.82

$$\frac{8a^2-25a(a+bx)+15(a+bx)^2}{4a^3x^2\sqrt{a+bx}}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^(3/2)),x]
```

```
[Out] (8*a^2 - 25*a*(a + b*x) + 15*(a + b*x)^2)/(4*a^3*x^2*Sqrt[a + b*x]) - (15*b
^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

**fricas** [A] time = 1.07, size = 189, normalized size = 2.17

$$\left[\frac{15\left(b^{3}x^{3}+ab^{2}x^{2}\right)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(15ab^{2}x^{2}+5a^{2}bx-2a^{3}\right)\sqrt{bx+a}}{8\left(a^{4}bx^{3}+a^{5}x^{2}\right)},\frac{15\left(b^{3}x^{3}+ab^{2}x^{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(15ab^{2}x^{2}+5a^{2}bx-2a^{3}\right)\sqrt{bx+a}}{4\left(a^{4}bx^{3}+a^{5}x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) +
 2*a)/x) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 +
 a^5*x^2), 1/4*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 + a^
5*x^2)]
```

**giac** [A] time = 1.05, size = 80, normalized size = 0.92

$$\frac{15\,b^{2}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}\,a^{3}}+\frac{2\,b^{2}}{\sqrt{bx+a}\,a^{3}}+\frac{7\,(bx+a)^{\frac{3}{2}}b^{2}-9\sqrt{bx+a}\,ab^{2}}{4\,a^{3}b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*b^2/(sqrt(b*x +
a)*a^3) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)/(a^3*b^2*x^2)
```

**maple** [A] time = 0.01, size = 67, normalized size = 0.77

$$2\left(\frac{1}{\sqrt{bx+a}\;a^{3}}+\frac{-\frac{15\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}+\frac{-\frac{9\sqrt{bx+a}\;a}{8}+\frac{7(bx+a)^{\frac{3}{2}}}{8}}{b^{2}x^{2}}}{a^{3}}\right)b^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(3/2),x)
```

```
[Out] 2*b^2*(1/a^3/(b*x+a)^(1/2)+1/a^3*((7/8*(b*x+a)^(3/2)-9/8*(b*x+a)^(1/2)*a)/x
^2/b^2-15/8*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

**maxima** [A] time = 3.06, size = 108, normalized size = 1.24

$$\frac{15\,(bx+a)^{2}b^{2}-25\,(bx+a)ab^{2}+8\,a^{2}b^{2}}{4\left((bx+a)^{\frac{5}{2}}a^{3}-2\,(bx+a)^{\frac{3}{2}}a^{4}+\sqrt{bx+a}\,a^{5}\right)}+\frac{15\,b^{2}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(15*(b*x + a)^2*b^2 - 25*(b*x + a)*a*b^2 + 8*a^2*b^2)/((b*x + a)^(5/2)*
a^3 - 2*(b*x + a)^(3/2)*a^4 + sqrt(b*x + a)*a^5) + 15/8*b^2*log((sqrt(b*x +
 a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(7/2)
```

**mupad [B]** time = 0.06, size = 90, normalized size = 1.03

$$\frac{\frac{2\,b^2}{a}+\frac{15\,b^2\,{\left(a+b\,x\right)}^2}{4\,a^3}-\frac{25\,b^2\,\left(a+b\,x\right)}{4\,a^2}}{{\left(a+b\,x\right)}^{5/2}-2\,a\,{\left(a+b\,x\right)}^{3/2}+a^2\,\sqrt{a+b\,x}}-\frac{15\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(3/2)),x)
```

```
[Out] ((2*b^2)/a + (15*b^2*(a + b*x)^2)/(4*a^3) - (25*b^2*(a + b*x))/(4*a^2))/((a
 + b*x)^(5/2) - 2*a*(a + b*x)^(3/2) + a^2*(a + b*x)^(1/2)) - (15*b^2*atanh(
(a + b*x)^(1/2)/a^(1/2)))/(4*a^(7/2))
```

**sympy [A]** time = 5.98, size = 107, normalized size = 1.23

$$-\frac{1}{2a\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}-\frac{15b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(3/2),x)
```

```
[Out] -1/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + 5*sqrt(b)/(4*a**2*x**(3/2)*sq
rt(a/(b*x) + 1)) + 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) - 15*b**2
*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2))
```

## 3.351 $\int \frac{x^4}{(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=87

$$-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}$$

**Rubi** [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^4)/(3*b^5*(a + b*x)^(3/2)) + (8*a^3)/(b^5*Sqrt[a + b*x]) + (12*a^2*Sq
rt[a + b*x])/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5) + (2*(a + b*x)^(5/2))/(5*b
^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{5/2}}\,dx &= \int \left(\frac{a^4}{b^4(a+bx)^{5/2}}-\frac{4a^3}{b^4(a+bx)^{3/2}}+\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^4}+\frac{(a+bx)^{3/2}}{b^4}\right)dx\\ &= -\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 57, normalized size = 0.66

$$\frac{2\left(128a^4+192a^3bx+48a^2b^2x^2-8ab^3x^3+3b^4x^4\right)}{15b^5(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^(5/2),x]
```

```
[Out] (2*(128*a^4 + 192*a^3*b*x + 48*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4))/(15*
b^5*(a + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.03, size = 63, normalized size = 0.72

$$\frac{2\left(-5a^4+60a^3(a+bx)+90a^2(a+bx)^2-20a(a+bx)^3+3(a+bx)^4\right)}{15b^5(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^(5/2),x]
```

```
[Out] (2*(-5*a^4 + 60*a^3*(a + b*x) + 90*a^2*(a + b*x)^2 - 20*a*(a + b*x)^3 + 3*(
a + b*x)^4))/(15*b^5*(a + b*x)^(3/2))
```

**fricas [A]** time = 1.00, size = 74, normalized size = 0.85

$$\frac{2\left(3\,b^4x^4-8\,ab^3x^3+48\,a^2b^2x^2+192\,a^3bx+128\,a^4\right)\sqrt{bx+a}}{15\left(b^7x^2+2\,ab^6x+a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*x^4 - 8*a*b^3*x^3 + 48*a^2*b^2*x^2 + 192*a^3*b*x + 128*a^4)*sqr
t(b*x + a)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

**giac [A]** time = 1.08, size = 75, normalized size = 0.86

$$\frac{2\left(12\,(bx+a)a^3-a^4\right)}{3\,(bx+a)^{\frac{3}{2}}b^5}+\frac{2\left(3\,(bx+a)^{\frac{5}{2}}b^{20}-20\,(bx+a)^{\frac{3}{2}}ab^{20}+90\,\sqrt{bx+a}\,a^2b^{20}\right)}{15\,b^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*(b*x + a)*a^3 - a^4)/((b*x + a)^(3/2)*b^5) + 2/15*(3*(b*x + a)^(5/2
)*b^20 - 20*(b*x + a)^(3/2)*a*b^20 + 90*sqrt(b*x + a)*a^2*b^20)/b^25
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.62

$$\frac{\frac{2}{5}x^4b^4-\frac{16}{15}a\,x^3b^3+\frac{32}{5}a^2x^2b^2+\frac{128}{5}a^3xb+\frac{256}{15}a^4}{(bx+a)^{\frac{3}{2}}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(5/2),x)
```

```
[Out] 2/15/(b*x+a)^(3/2)*(3*b^4*x^4-8*a*b^3*x^3+48*a^2*b^2*x^2+192*a^3*b*x+128*a^
4)/b^5
```

**maxima [A]** time = 1.35, size = 71, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^5}-\frac{8\,(bx+a)^{\frac{3}{2}}a}{3\,b^5}+\frac{12\,\sqrt{bx+a}\,a^2}{b^5}+\frac{8\,a^3}{\sqrt{bx+a}\,b^5}-\frac{2\,a^4}{3\,(bx+a)^{\frac{3}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^5 - 8/3*(b*x + a)^(3/2)*a/b^5 + 12*sqrt(b*x + a)*a^2/
b^5 + 8*a^3/(sqrt(b*x + a)*b^5) - 2/3*a^4/((b*x + a)^(3/2)*b^5)
```

**mupad [B]** time = 0.05, size = 68, normalized size = 0.78

$$\frac{2\,(a+b\,x)^{5/2}}{5\,b^5}+\frac{8\,a^3\,\,(a+b\,x)-\frac{2\,a^4}{3}}{b^5\,(a+b\,x)^{3/2}}+\frac{12\,a^2\,\sqrt{a+b\,x}}{b^5}-\frac{8\,a\,(a+b\,x)^{3/2}}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b^5) + (8*a^3*(a + b*x) - (2*a^4)/3)/(b^5*(a + b*x)^
(3/2)) + (12*a^2*(a + b*x)^(1/2))/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5)
```

**sympy [B]** time = 4.57, size = 3456, normalized size = 39.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(5/2),x)
```

```
[Out] 256*a**(85/2)*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38
*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10
*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**
8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 256*a**(85/2)/(15*a**40*
b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150
*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**
33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**1
5*x**10) + 2432*a**(83/2)*b*x*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b*
```

```
*6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 +
3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675
*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 2560*a**
(83/2)*b*x/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a
**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*
b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14
*x**9 + 15*a**30*b**15*x**10) + 10336*a**(81/2)*b**2*x**2*sqrt(1 + b*x/a)/(
15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x*
*3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 +
 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a
**30*b**15*x**10) - 11520*a**(81/2)*b**2*x**2/(15*a**40*b**5 + 150*a**39*b*
*6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 +
3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675
*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 25840*a*
*(79/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a
**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b
**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13
*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 30720*a**(79/2)*b**3
*x**3/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*
b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11
*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9
 + 15*a**30*b**15*x**10) + 41990*a**(77/2)*b**4*x**4*sqrt(1 + b*x/a)/(15*a*
*40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 +
3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800
*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*
b**15*x**10) - 53760*a**(77/2)*b**4*x**4/(15*a**40*b**5 + 150*a**39*b**6*x
+ 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*
a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**3
2*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 46192*a**(75/
2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*
b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*
x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8
 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 64512*a**(75/2)*b**5*x**5
/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*
x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6
 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15
*a**30*b**15*x**10) + 34664*a**(73/2)*b**6*x**6*sqrt(1 + b*x/a)/(15*a**40*b
**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*
a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**3
3*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15
*x**10) - 53760*a**(73/2)*b**6*x**6/(15*a**40*b**5 + 150*a**39*b**6*x + 675
*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35
*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**
13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 17392*a**(71/2)*b*
*7*x**7*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*
```

x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 30720*a**(71/2)*b**7*x**7/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 5540*a**(69/2)*b**8*x**8*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 11520*a**(69/2)*b**8*x**8/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 1040*a**(67/2)*b**9*x**9*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 2560*a**(67/2)*b**9*x**9/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 136*a**(65/2)*b**10*x**10*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 256*a**(65/2)*b**10*x**10/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 32*a**(63/2)*b**11*x**11*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 6*a**(61/2)*b**12*x**12*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10)

## 3.352 $\int \frac{x^3}{(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=68

$$\frac{2a^3}{3b^4(a+bx)^{3/2}}-\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{6a\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{3/2}}{3b^4}$$

**Rubi** [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^3}{3b^4(a+bx)^{3/2}}-\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{6a\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{3/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*a^3)/(3*b^4*(a + b*x)^(3/2)) - (6*a^2)/(b^4*Sqrt[a + b*x]) - (6*a*Sqrt[a
 + b*x])/b^4 + (2*(a + b*x)^(3/2))/(3*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{5/2}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{5/2}}+\frac{3a^2}{b^3(a+bx)^{3/2}}-\frac{3a}{b^3\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^3}\right)dx\\ &= \frac{2a^3}{3b^4(a+bx)^{3/2}}-\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{6a\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{3/2}}{3b^4}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 45, normalized size = 0.66

$$\frac{2\left(-16a^3-24a^2bx-6ab^2x^2+b^3x^3\right)}{3b^4(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*(-16*a^3 - 24*a^2*b*x - 6*a*b^2*x^2 + b^3*x^3))/(3*b^4*(a + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 47, normalized size = 0.69

$$\frac{2\left(a^3-9a^2(a+bx)-9a(a+bx)^2+(a+bx)^3\right)}{3b^4(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*(a^3 - 9*a^2*(a + b*x) - 9*a*(a + b*x)^2 + (a + b*x)^3))/(3*b^4*(a + b*x
)^(3/2))
```

**fricas [A]** time = 0.96, size = 62, normalized size = 0.91

$$\frac{2\left(b^3x^3-6\,ab^2x^2-24\,a^2bx-16\,a^3\right)\sqrt{bx+a}}{3\left(b^6x^2+2\,ab^5x+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*sqrt(b*x + a)/(b^6*x^2 +
2*a*b^5*x + a^2*b^4)
```

**giac [A]** time = 1.03, size = 59, normalized size = 0.87

$$-\frac{2\left(9\,(bx+a)a^2-a^3\right)}{3\,(bx+a)^{\frac{3}{2}}b^4}+\frac{2\left((bx+a)^{\frac{3}{2}}b^8-9\,\sqrt{bx+a}\,ab^8\right)}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(9*(b*x + a)*a^2 - a^3)/((b*x + a)^(3/2)*b^4) + 2/3*((b*x + a)^(3/2)*b
^8 - 9*sqrt(b*x + a)*a*b^8)/b^12
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.63

$$-\frac{2\left(-b^3x^3+6a\,b^2x^2+24a^2bx+16a^3\right)}{3\,(bx+a)^{\frac{3}{2}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(5/2),x)
```

```
[Out] -2/3/(b*x+a)^(3/2)*(-b^3*x^3+6*a*b^2*x^2+24*a^2*b*x+16*a^3)/b^4
```

**maxima [A]** time = 1.25, size = 56, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^4}-\frac{6\,\sqrt{bx+a}\,a}{b^4}-\frac{6\,a^2}{\sqrt{bx+a}\,b^4}+\frac{2\,a^3}{3\,(bx+a)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^4 - 6*sqrt(b*x + a)*a/b^4 - 6*a^2/(sqrt(b*x + a)*b^4)
 + 2/3*a^3/((b*x + a)^(3/2)*b^4)
```

**mupad [B]** time = 0.04, size = 47, normalized size = 0.69

$$-\frac{18\,a\,(a+b\,x)^2+18\,a^2\,(a+b\,x)-2\,(a+b\,x)^3-2\,a^3}{3\,b^4\,(a+b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(5/2),x)
```

```
[Out] -(18*a*(a + b*x)^2 + 18*a^2*(a + b*x) - 2*(a + b*x)^3 - 2*a^3)/(3*b^4*(a +
b*x)^(3/2))
```

**sympy [A]** time = 1.20, size = 163, normalized size = 2.40

$$\begin{cases} -\frac{32a^3}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}}-\frac{48a^2bx}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}}-\frac{12ab^2x^2}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}}+\frac{2b^3x^3}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-32*a**3/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 48*
a**2*b*x/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 12*a*b**2*x**2
/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) + 2*b**3*x**3/(3*a*b**4*
sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)), Ne(b, 0)), (x**4/(4*a**(5/2)), Tru
e))
```

## 3.353 $\int \frac{x^2}{(a+bx)^{5/2}} \, dx$

**Optimal**. Leaf size=49

$$-\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}$$

**Rubi** [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^2)/(3*b^3*(a + b*x)^(3/2)) + (4*a)/(b^3*Sqrt[a + b*x]) + (2*Sqrt[a + 
b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, 
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{5/2}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{5/2}}-\frac{2a}{b^2(a+bx)^{3/2}}+\frac{1}{b^2\sqrt{a+bx}}\right)dx\\ &= -\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 35, normalized size = 0.71

$$\frac{2\left(8a^2+12abx+3b^2x^2\right)}{3b^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (2*(8*a^2 + 12*a*b*x + 3*b^2*x^2))/(3*b^3*(a + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 39, normalized size = 0.80

$$\frac{2\left(-a^2+6a(a+bx)+3(a+bx)^2\right)}{3b^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (2*(-a^2 + 6*a*(a + b*x) + 3*(a + b*x)^2))/(3*b^3*(a + b*x)^(3/2))
```

**fricas [A]** time = 1.27, size = 52, normalized size = 1.06

$$\frac{2\left(3\,b^{2}x^{2}+12\,abx+8\,a^{2}\right)\sqrt{bx+a}}{3\left(b^{5}x^{2}+2\,ab^{4}x+a^{2}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*sqrt(b*x + a)/(b^5*x^2 + 2*a*b^4*x + a^2
*b^3)
```

**giac [A]** time = 0.94, size = 39, normalized size = 0.80

$$\frac{2\sqrt{bx+a}}{b^{3}}+\frac{2\left(6\,(bx+a)a-a^{2}\right)}{3\,(bx+a)^{\frac{3}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 2/3*(6*(b*x + a)*a - a^2)/((b*x + a)^(3/2)*b^3)
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.65

$$\frac{2b^{2}x^{2}+8abx+\frac{16}{3}a^{2}}{b^{3}\,(bx+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(5/2),x)
```

```
[Out] 2/3/(b*x+a)^(3/2)*(3*b^2*x^2+12*a*b*x+8*a^2)/b^3
```

**maxima [A]** time = 1.33, size = 41, normalized size = 0.84

$$\frac{2\sqrt{bx+a}}{b^3}+\frac{4\,a}{\sqrt{bx+a}\,b^3}-\frac{2\,a^2}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 4*a/(sqrt(b*x + a)*b^3) - 2/3*a^2/((b*x + a)^(3/2)*b^
3)
```

**mupad [B]** time = 0.08, size = 35, normalized size = 0.71

$$\frac{6\,(a+b\,x)^2+12\,a\,(a+b\,x)-2\,a^2}{3\,b^3\,(a+b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(5/2),x)
```

```
[Out] (6*(a + b*x)^2 + 12*a*(a + b*x) - 2*a^2)/(3*b^3*(a + b*x)^(3/2))
```

**sympy [A]** time = 1.28, size = 121, normalized size = 2.47

$$\begin{cases}\frac{16a^2}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}}+\frac{24abx}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}}+\frac{6b^2x^2}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} & \text{for } b\neq 0\\ \frac{x^3}{3a^{\frac{5}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((16*a**2/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 24*a
*b*x/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 6*b**2*x**2/(3*a*b
**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)), Ne(b, 0)), (x**3/(3*a**(5/2)),
 True))
```

## 3.354 $\int \frac{x}{(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=32

$$\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}$$

**Rubi** [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(5/2),x]
```

```
[Out] (2*a)/(3*b^2*(a + b*x)^(3/2)) - 2/(b^2*Sqrt[a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{5/2}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)dx\\ &= \frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.75

$$-\frac{2(2a+3bx)}{3b^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(2*a + 3*b*x))/(3*b^2*(a + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.01, size = 24, normalized size = 0.75

$$-\frac{2(2a+3bx)}{3b^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(2*a + 3*b*x))/(3*b^2*(a + b*x)^(3/2))
```

**fricas [A]** time = 0.99, size = 41, normalized size = 1.28

$$-\frac{2\,(3\,bx+2\,a)\sqrt{bx+a}}{3\,\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*x + 2*a)*sqrt(b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

**giac [A]** time = 1.06, size = 20, normalized size = 0.62

$$-\frac{2\,(3\,bx+2\,a)}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*x + 2*a)/((b*x + a)^(3/2)*b^2)
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.66

$$-\frac{2\,(3bx+2a)}{3\,(bx+a)^{\frac{3}{2}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(5/2),x)
```

```
[Out] -2/3/(b*x+a)^(3/2)*(3*b*x+2*a)/b^2
```

**maxima** [A] time = 1.34, size = 26, normalized size = 0.81

$$-\frac{2}{\sqrt{bx+a}\,b^2}+\frac{2\,a}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b^2) + 2/3*a/((b*x + a)^(3/2)*b^2)
```

**mupad** [B] time = 0.03, size = 20, normalized size = 0.62

$$-\frac{4\,a+6\,b\,x}{3\,b^2\,(a+b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(5/2),x)
```

```
[Out] -(4*a + 6*b*x)/(3*b^2*(a + b*x)^(3/2))
```

**sympy** [A] time = 1.13, size = 80, normalized size = 2.50

$$\begin{cases} -\frac{4a}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}}-\frac{6bx}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-4*a/(3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)) - 6*b*x/(
3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)), Ne(b, 0)), (x**2/(2*a**(5
/2)), True))
```

## 3.355 $\int \frac{1}{(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=16

$$-\frac{2}{3b(a+bx)^{3/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-5/2),x]
```

```
[Out] -2/(3*b*(a + b*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}}\,dx = -\frac{2}{3b(a+bx)^{3/2}}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 1.00

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-5/2),x]
```

```
[Out] -2/(3*b*(a + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(-5/2),x]
[Out] -2/(3*b*(a + b*x)^(3/2))
```

**fricas [B]** time = 0.91, size = 31, normalized size = 1.94

$$-\frac{2\sqrt{bx+a}}{3\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] -2/3*sqrt(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

**giac [A]** time = 1.04, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="giac")
[Out] -2/3/((b*x + a)^(3/2)*b)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2),x)
[Out] -2/3/b/(b*x+a)^(3/2)
```

**maxima [A]** time = 1.31, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((b*x + a)^(3/2)*b)
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$-\frac{2}{3\,b\,{\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(5/2),x)
[Out] -2/(3*b*(a + b*x)^(3/2))
```

**sympy [A]** time = 0.07, size = 14, normalized size = 0.88

$$-\frac{2}{3b\left(a+bx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2),x)
[Out] -2/(3*b*(a + b*x)**(3/2))
```

# 3.356 $\int \frac{1}{x(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=54

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{a^2\sqrt{a+bx}}+\frac{2}{3a(a+bx)^{3/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$\frac{2}{a^2\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*(a + b*x)^(3/2)) + 2/(a^2*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx)^{5/2}}\,dx &= \frac{2}{3a(a+bx)^{3/2}}+\frac{\int \frac{1}{x(a+bx)^{3/2}}\,dx}{a}\\
&= \frac{2}{3a(a+bx)^{3/2}}+\frac{2}{a^2\sqrt{a+bx}}+\frac{\int \frac{1}{x\sqrt{a+bx}}\,dx}{a^2}\\
&= \frac{2}{3a(a+bx)^{3/2}}+\frac{2}{a^2\sqrt{a+bx}}+\frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a^2b}\\
&= \frac{2}{3a(a+bx)^{3/2}}+\frac{2}{a^2\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 32, normalized size = 0.59

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx}{a}+1\right)}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(5/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x)/a])/(3*a*(a + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.04, size = 49, normalized size = 0.91

$$\frac{2(3(a+bx)+a)}{3a^2(a+bx)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^(5/2)),x]
```

```
[Out] (2*(a + 3*(a + b*x)))/(3*a^2*(a + b*x)^(3/2)) - (2*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]])/a^(5/2)
```

**fricas [B]** time = 1.02, size = 177, normalized size = 3.28

$$\left[\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(3\,abx+4\,a^2\right)\sqrt{bx+a}}{3\left(a^3b^2x^2+2\,a^4bx+a^5\right)},\ \frac{2\left(3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(3\,abx+4\,a^2\right)\sqrt{bx+a}\right)}{3\left(a^3b^2x^2+2\,a^4bx+a^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a
) + 2*a)/x) + 2*(3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x +
 a^5), 2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(
-a)/a) + (3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)]
```

**giac [A]** time = 1.00, size = 45, normalized size = 0.83

$$\frac{2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\, a^2} + \frac{2\,(3\,bx + 4\,a)}{3\,(bx+a)^{\frac{3}{2}} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + 2/3*(3*b*x + 4*a)/((b*x +
 a)^(3/2)*a^2)
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.80

$$\frac{2}{3\,(bx+a)^{\frac{3}{2}}\, a} - \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2}{\sqrt{bx+a}\; a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(5/2),x)
```

```
[Out] 2/3/a/(b*x+a)^(3/2)-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)+2/(b*x+a)^(1/2
)/a^2
```

**maxima [A]** time = 2.91, size = 53, normalized size = 0.98

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\,(3\,bx + 4\,a)}{3\,(bx+a)^{\frac{3}{2}} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) + 2/3*(3*b
*x + 4*a)/((b*x + a)^(3/2)*a^2)
```

**mupad [B]** time = 0.05, size = 42, normalized size = 0.78

$$\frac{\frac{2\,(a+b\,x)}{a^2}+\frac{2}{3\,a}}{{(a+b\,x)}^{3/2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(5/2)),x)
```

```
[Out] ((2*(a + b*x))/a^2 + 2/(3*a))/(a + b*x)^(3/2) - (2*atanh((a + b*x)^(1/2)/a^
(1/2)))/a^(5/2)
```

**sympy [B]** time = 2.99, size = 697, normalized size = 12.91

$$\frac{8a^7\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{3a^7\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{6a^7\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{14a^6bx\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{9a^6bx\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{18a^6bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{6a^5b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{9a^5b^2x^2\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{18a^5b^2x^2\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{3a^4b^3x^3\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{6a^4b^3x^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(5/2),x)
```

```
[Out] 8*a**7*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x*
*2 + 3*a**(13/2)*b**3*x**3) + 3*a**7*log(b*x/a)/(3*a**(19/2) + 9*a**(17/2)*
b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*a**7*log(sqrt(1 +
b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(
13/2)*b**3*x**3) + 14*a**6*b*x*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b
*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 9*a**6*b*x*log(b*x/a)
/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*
x**3) - 18*a**6*b*x*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x
 + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**5*b**2*x**2*sqrt(1
 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13
/2)*b**3*x**3) + 9*a**5*b**2*x**2*log(b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x
 + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*a**5*b**2*x**2*log(s
qrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2
+ 3*a**(13/2)*b**3*x**3) + 3*a**4*b**3*x**3*log(b*x/a)/(3*a**(19/2) + 9*a**
(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*a**4*b**3*x
**3*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b
**2*x**2 + 3*a**(13/2)*b**3*x**3)
```

## 3.357 $\int \frac{1}{x^2(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=74

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{5b}{a^3\sqrt{a+bx}}-\frac{5b}{3a^2(a+bx)^{3/2}}-\frac{1}{ax(a+bx)^{3/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{5\sqrt{a+bx}}{a^3x}+\frac{10}{3a^2x\sqrt{a+bx}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2}{3ax(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x*(a + b*x)^(3/2)) + 10/(3*a^2*x*Sqrt[a + b*x]) - (5*Sqrt[a + b*x])/
(a^3*x) + (5*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2(a+bx)^{5/2}}\,dx &= \frac{2}{3ax(a+bx)^{3/2}}+\frac{5\int \frac{1}{x^2(a+bx)^{3/2}}\,dx}{3a}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}+\frac{5\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{a^2}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}-\frac{(5b)\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a^3}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}-\frac{5\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a^3}\\
&= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.45

$$
-\frac{2b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{bx}{a}+1\right)}{3a^2(a+bx)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-3/2, 2, -1/2, 1 + (b*x)/a])/(3*a^2*(a + b*x)^(3/2)
)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 67, normalized size = 0.91

$$
\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2a^2+10a(a+bx)-15(a+bx)^2}{3a^3x(a+bx)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] (2*a^2 + 10*a*(a + b*x) - 15*(a + b*x)^2)/(3*a^3*x*(a + b*x)^(3/2)) + (5*b*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

**fricas** [A] time = 1.13, size = 221, normalized size = 2.99

$$\left[\frac{15\left(b^{3}x^{3}+2\,ab^{2}x^{2}+a^{2}bx\right)\sqrt{a}\,\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\left(15\,ab^{2}x^{2}+20\,a^{2}bx+3\,a^{3}\right)\sqrt{bx+a}}{6\left(a^{4}b^{2}x^{3}+2\,a^{5}bx^{2}+a^{6}x\right)},-\frac{15\left(b^{3}x^{3}+2\,ab^{2}x^{2}+a^{2}bx\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(15\,ab^{2}x^{2}+20\,a^{2}bx+3\,a^{3}\right)\sqrt{bx+a}}{3\left(a^{4}b^{2}x^{3}+2\,a^{5}bx^{2}+a^{6}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x +
a)*sqrt(a) + 2*a)/x) - 2*(15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3)*sqrt(b*x + a))
/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x), -1/3*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2
*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 + 20*a^2*b*
x + 3*a^3)*sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)]
```

**giac** [A] time = 1.03, size = 65, normalized size = 0.88

$$-\frac{5\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{3}}-\frac{2\,(6\,(bx+a)b+ab)}{3\,(bx+a)^{\frac{3}{2}}a^{3}}-\frac{\sqrt{bx+a}}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - 2/3*(6*(b*x + a)*b + a
*b)/((b*x + a)^(3/2)*a^3) - sqrt(b*x + a)/(a^3*x)
```

**maple** [A] time = 0.01, size = 67, normalized size = 0.91

$$2\left(-\frac{1}{3\,(bx+a)^{\frac{3}{2}}\,a^{2}}-\frac{2}{\sqrt{bx+a}\,a^{3}}-\frac{-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\sqrt{bx+a}}{2bx}}{a^{3}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(5/2),x)
```

```
[Out] 2*b*(-1/3/a^2/(b*x+a)^(3/2)-2/(b*x+a)^(1/2)/a^3-1/a^3*(1/2*(b*x+a)^(1/2)/b/
x-5/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

**maxima** [A] time = 3.03, size = 89, normalized size = 1.20

$$-\frac{15\,(bx+a)^{2}b-10\,(bx+a)ab-2\,a^{2}b}{3\left((bx+a)^{\frac{5}{2}}a^{3}-(bx+a)^{\frac{3}{2}}a^{4}\right)}-\frac{5\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(15*(b*x + a)^2*b - 10*(b*x + a)*a*b - 2*a^2*b)/((b*x + a)^(5/2)*a^3 -
 (b*x + a)^(3/2)*a^4) - 5/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a)
+ sqrt(a)))/a^(7/2)
```

**mupad [B]** time = 0.11, size = 73, normalized size = 0.99

$$\frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{\frac{2\,b}{3\,a}+\frac{10\,b\,(a+b\,x)}{3\,a^2}-\frac{5\,b\,(a+b\,x)^2}{a^3}}{a\,(a+b\,x)^{3/2}-(a+b\,x)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(5/2)),x)
```

```
[Out] (5*b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(7/2) - ((2*b)/(3*a) + (10*b*(a + b*
x))/(3*a^2) - (5*b*(a + b*x)^2)/a^3)/(a*(a + b*x)^(3/2) - (a + b*x)^(5/2))
```

**sympy [B]** time = 5.60, size = 818, normalized size = 11.05

$$-\frac{6a^{17}\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{46a^{16}bx\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{15a^{16}bx\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{30a^{16}bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{70a^{15}b^2x^2\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{45a^{15}b^2x^2\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{90a^{15}b^2x^2\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{30a^{14}b^3x^3\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{45a^{14}b^3x^3\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{90a^{14}b^3x^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{15a^{13}b^4x^4\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{30a^{13}b^4x^4\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**17*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2
)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 46*a**16*b*x*sqrt(1 + b*x/a)/(6*a**(
39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x
**4) - 15*a**16*b*x*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a*
*(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**16*b*x*log(sqrt(1 + b*x/
a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a
**(33/2)*b**3*x**4) - 70*a**15*b**2*x**2*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 1
8*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 45*a
**15*b**2*x**2*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/
2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**15*b**2*x**2*log(sqrt(1 + b*x
/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*
a**(33/2)*b**3*x**4) - 30*a**14*b**3*x**3*sqrt(1 + b*x/a)/(6*a**(39/2)*x +
18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 45*
a**14*b**3*x**3*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35
/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**14*b**3*x**3*log(sqrt(1 + b*
x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6
*a**(33/2)*b**3*x**4) - 15*a**13*b**4*x**4*log(b*x/a)/(6*a**(39/2)*x + 18*a
**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**1
```

```
3*b**4*x**4*log(sqrt(1 + b*x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 +
 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4)
```

## 3.358 $\int \frac{1}{x^3(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=106

$$-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b^2}{4a^4\sqrt{a+bx}}+\frac{35b^2}{12a^3(a+bx)^{3/2}}+\frac{7b}{4a^2x(a+bx)^{3/2}}-\frac{1}{2ax^2(a+bx)^{3/2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{14}{3a^2x^2\sqrt{a+bx}}+\frac{35b\sqrt{a+bx}}{4a^4x}+\frac{2}{3ax^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^2*(a + b*x)^(3/2)) + 14/(3*a^2*x^2*Sqrt[a + b*x]) - (35*Sqrt[a + b
*x])/(6*a^3*x^2) + (35*b*Sqrt[a + b*x])/(4*a^4*x) - (35*b^2*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^{5/2}}\,dx = \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{7\int \frac{1}{x^3(a+bx)^{3/2}}\,dx}{3a}$$

$$= \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{a+bx}} + \frac{35\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{3a^2}$$

$$= \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{a+bx}} - \frac{35\sqrt{a+bx}}{6a^3x^2} - \frac{(35b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a^3}$$

$$= \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{a+bx}} - \frac{35\sqrt{a+bx}}{6a^3x^2} + \frac{35b\sqrt{a+bx}}{4a^4x} + \frac{\left(35b^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^4}$$

$$= \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{a+bx}} - \frac{35\sqrt{a+bx}}{6a^3x^2} + \frac{35b\sqrt{a+bx}}{4a^4x} + \frac{(35b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,d\right.}{4a^4}$$

$$= \frac{2}{3ax^2(a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{a+bx}} - \frac{35\sqrt{a+bx}}{6a^3x^2} + \frac{35b\sqrt{a+bx}}{4a^4x} - \frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.33

$$\frac{2b^2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\frac{bx}{a}+1\right)}{3a^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(5/2)),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[-3/2, 3, -1/2, 1 + (b*x)/a])/(3*a^3*(a + b*x)^(3/2
))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 83, normalized size = 0.78

$$\frac{8a^3+56a^2(a+bx)-175a(a+bx)^2+105(a+bx)^3}{12a^4x^2(a+bx)^{3/2}} - \frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^(5/2)),x]
```

[Out] (8*a^3 + 56*a^2*(a + b*x) - 175*a*(a + b*x)^2 + 105*(a + b*x)^3)/(12*a^4*x^2*(a + b*x)^(3/2)) - (35*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(9/2))

**fricas [A]** time = 1.15, size = 255, normalized size = 2.41

$$\left[\frac{105\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4\right)\sqrt{bx+a}}{24\left(a^5b^2x^4+2\,a^6bx^3+a^7x^2\right)},\frac{105\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4\right)\sqrt{bx+a}}{12\left(a^5b^2x^4+2\,a^6bx^3+a^7x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="fricas")

[Out] [1/24*(105*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2), 1/12*(105*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)]

**giac [A]** time = 0.99, size = 93, normalized size = 0.88

$$\frac{35\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^4}+\frac{2\left(9\,(bx+a)b^2+ab^2\right)}{3\,(bx+a)^{\frac{3}{2}}a^4}+\frac{11\,(bx+a)^{\frac{3}{2}}b^2-13\,\sqrt{bx+a}\,ab^2}{4\,a^4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="giac")

[Out] 35/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) + 2/3*(9*(b*x + a)*b^2 + a*b^2)/((b*x + a)^(3/2)*a^4) + 1/4*(11*(b*x + a)^(3/2)*b^2 - 13*sqrt(b*x + a)*a*b^2)/(a^4*b^2*x^2)

**maple [A]** time = 0.02, size = 80, normalized size = 0.75

$$2\left(\frac{1}{3\,(bx+a)^{\frac{3}{2}}\,a^3}+\frac{3}{\sqrt{bx+a}\;a^4}+\frac{-\frac{35\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\,\sqrt{a}}+\frac{-\frac{13\,\sqrt{bx+a}\;a}{8}+\frac{11(bx+a)^{\frac{3}{2}}}{8}}{b^2x^2}}{a^4}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b*x+a)^(5/2),x)

[Out] 2*b^2*(3/a^4/(b*x+a)^(1/2)+1/3/a^3/(b*x+a)^(3/2)+1/a^4*((11/8*(b*x+a)^(3/2)-13/8*(b*x+a)^(1/2)*a)/x^2/b^2-35/8*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
)

**maxima** [A] time = 3.00, size = 123, normalized size = 1.16

$$\frac{105\,(bx+a)^3b^2 - 175\,(bx+a)^2ab^2 + 56\,(bx+a)a^2b^2 + 8\,a^3b^2}{12\left((bx+a)^{\frac{7}{2}}a^4 - 2\,(bx+a)^{\frac{5}{2}}a^5 + (bx+a)^{\frac{3}{2}}a^6\right)} + \frac{35\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(105*(b*x + a)^3*b^2 - 175*(b*x + a)^2*a*b^2 + 56*(b*x + a)*a^2*b^2 +
8*a^3*b^2)/((b*x + a)^(7/2)*a^4 - 2*(b*x + a)^(5/2)*a^5 + (b*x + a)^(3/2)*a
^6) + 35/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(
9/2)
```

**mupad** [B] time = 0.12, size = 105, normalized size = 0.99

$$\frac{\frac{2\,b^2}{3\,a} - \frac{175\,b^2\,(a+b\,x)^2}{12\,a^3} + \frac{35\,b^2\,(a+b\,x)^3}{4\,a^4} + \frac{14\,b^2\,(a+b\,x)}{3\,a^2}}{(a+b\,x)^{7/2} - 2\,a\,(a+b\,x)^{5/2} + a^2\,(a+b\,x)^{3/2}} - \frac{35\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(5/2)),x)
```

```
[Out] ((2*b^2)/(3*a) - (175*b^2*(a + b*x)^2)/(12*a^3) + (35*b^2*(a + b*x)^3)/(4*a
^4) + (14*b^2*(a + b*x))/(3*a^2))/((a + b*x)^(7/2) - 2*a*(a + b*x)^(5/2) +
a^2*(a + b*x)^(3/2)) - (35*b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(9/2))
```

**sympy** [B] time = 8.57, size = 464, normalized size = 4.38

$$-\frac{6a^{\frac{89}{2}}b^{75}x^{75}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{21a^{\frac{87}{2}}b^{76}x^{76}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{140a^{\frac{85}{2}}b^{77}x^{77}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{105a^{\frac{83}{2}}b^{78}x^{78}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{105a^{42}b^{\frac{155}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{105a^{41}b^{\frac{157}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**(89/2)*b**75*x**75/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) +
 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 21*a**(87/2)*
b**76*x**76/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(
91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 140*a**(85/2)*b**77*x**77/
(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(15
3/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 105*a**(83/2)*b**78*x**78/(12*a**(93/2
)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157
/2)*sqrt(a/(b*x) + 1)) - 105*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*x) + 1)*
asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a
```

```
/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) - 105*a
**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) + 1)*asinh(sqrt(a)/(sqrt(b)*sqrt(x)
))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**
(153/2)*x**(157/2)*sqrt(a/(b*x) + 1))
```

## 3.359 $\int \frac{1}{x\sqrt{-a+bx}}\,dx$

**Optimal**. Leaf size=25

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Rubi [A]** time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-a+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*x]),x]
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

**IntegrateAlgebraic** [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-a + b*x]),x]
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

**fricas** [A] time = 1.40, size = 58, normalized size = 2.32

$$\left[-\frac{\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)}{a}, \frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x)/a, 2*arctan(sqrt(b
*x - a)/sqrt(a))/sqrt(a)]
```

**giac** [A] time = 0.99, size = 19, normalized size = 0.76

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a)
```

**maple [A]** time = 0.00, size = 20, normalized size = 0.80

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(1/2),x)
[Out] 2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
```

**maxima [A]** time = 2.99, size = 19, normalized size = 0.76

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="maxima")
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a)
```

**mupad [B]** time = 0.05, size = 19, normalized size = 0.76

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(1/2)),x)
[Out] (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(1/2)
```

**sympy [A]** time = 1.23, size = 54, normalized size = 2.16

$$\begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{2\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(1/2),x)

[Out] Piecewise((2*I*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), Abs(a/(b*x)) > 1),
 (-2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), True))
```

## 3.360 $\int \frac{1}{x^2\sqrt{-a+bx}}\,dx$

**Optimal**. Leaf size=44

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}$$

**Rubi [A]** time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-a + b*x]),x]
```

```
[Out] Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx = \frac{\sqrt{-a+bx}}{ax} + \frac{b\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a}$$
$$= \frac{\sqrt{-a+bx}}{ax} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{a}$$
$$= \frac{\sqrt{-a+bx}}{ax} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

**Mathematica** [A] time = 0.06, size = 53, normalized size = 1.20

$$\frac{b\sqrt{bx-a}\left(\frac{a}{bx} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{bx}{a}}\right)}{\sqrt{1-\frac{bx}{a}}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-a + b*x]),x]
[Out] (b*Sqrt[-a + b*x]*(a/(b*x) + ArcTanh[Sqrt[1 - (b*x)/a]]/Sqrt[1 - (b*x)/a]))
/a^2
```

**IntegrateAlgebraic** [A] time = 0.04, size = 44, normalized size = 1.00

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{bx-a}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[-a + b*x]),x]
[Out] Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

**fricas** [A] time = 0.91, size = 97, normalized size = 2.20

$$\left[-\frac{\sqrt{-a}\,bx\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\,\sqrt{bx-a}\,a}{2\,a^2x},\ \frac{\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\,a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*sqrt(
b*x - a)*a)/(a^2*x), (sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) + sqrt(b*x
- a)*a)/(a^2*x)]
```

**giac [A]** time = 0.89, size = 43, normalized size = 0.98

$$\frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{bx-a}\,b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + sqrt(b*x - a)*b/(a*x))/b
```

**maple [A]** time = 0.01, size = 37, normalized size = 0.84

$$\frac{b \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{bx-a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(1/2),x)
```

```
[Out] b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)+(b*x-a)^(1/2)/a/x
```

**maxima [A]** time = 2.98, size = 46, normalized size = 1.05

$$\frac{\sqrt{bx-a}\,b}{(bx-a)a + a^2} + \frac{b \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b*x - a)*b/((b*x - a)*a + a^2) + b*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/
2)
```

**mupad [B]** time = 0.04, size = 36, normalized size = 0.82

$$\frac{\sqrt{b\,x-a}}{a\,x} + \frac{b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(1/2)),x)
```

```
[Out] (b*x - a)^(1/2)/(a*x) + (b*atan((b*x - a)^(1/2)/a^(1/2)))/a^(3/2)
```

**sympy [B]** time = 2.46, size = 121, normalized size = 2.75

$$\begin{cases} \frac{i\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a\sqrt{x}} + \frac{ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{for}\ \left|\frac{a}{bx}\right| > 1 \\ -\frac{1}{\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{\sqrt{b}}{a\sqrt{x}\sqrt{-\frac{a}{bx}+1}} - \frac{b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((I*sqrt(b)*sqrt(a/(b*x) - 1)/(a*sqrt(x)) + I*b*acosh(sqrt(a)/(sqr
t(b)*sqrt(x)))/a**(3/2), Abs(a/(b*x)) > 1), (-1/(sqrt(b)*x**(3/2)*sqrt(-a/(
b*x) + 1)) + sqrt(b)/(a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b*asin(sqrt(a)/(sqrt(
b)*sqrt(x)))/a**(3/2), True))
```

## 3.361 $\int \frac{1}{x^3 \sqrt{-a+bx}} \, dx$

**Optimal**. Leaf size=74

$$\frac{3b^2 \tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{5/2}} + \frac{3b\sqrt{bx-a}}{4a^2x} + \frac{\sqrt{bx-a}}{2ax^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$\frac{3b^2 \tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{5/2}} + \frac{3b\sqrt{bx-a}}{4a^2x} + \frac{\sqrt{bx-a}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] Sqrt[-a + b*x]/(2*a*x^2) + (3*b*Sqrt[-a + b*x])/(4*a^2*x) + (3*b^2*ArcTan[S
qrt[-a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{-a+bx}}\,dx &= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{(3b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a}\\ &= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{\left(3b^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a^2}\\ &= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{4a^2}\\ &= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 36, normalized size = 0.49

$$\frac{2b^2\sqrt{bx-a}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] (2*b^2*Sqrt[-a + b*x]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (b*x)/a])/a^3
```

**IntegrateAlgebraic [A]** time = 0.07, size = 69, normalized size = 0.93

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3(bx-a)^{3/2}+5a\sqrt{bx-a}}{4a^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] (5*a*Sqrt[-a + b*x] + 3*(-a + b*x)^(3/2))/(4*a^2*x^2) + (3*b^2*ArcTan[Sqrt[
-a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

**fricas [A]** time = 1.36, size = 128, normalized size = 1.73

$$\left[-\frac{3\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)-2\left(3abx+2a^2\right)\sqrt{bx-a}}{8a^3x^2},\ \frac{3\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(3abx+2a^2\right)\sqrt{bx-a}}{4a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2
*(3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2), 1/4*(3*sqrt(a)*b^2*x^2*arctan(
sqrt(b*x - a)/sqrt(a)) + (3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2)]
```

**giac [A]** time = 0.97, size = 68, normalized size = 0.92

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{3\,(bx-a)^{\frac{3}{2}}b^3+5\,\sqrt{bx-a}\,ab^3}{a^2b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + (3*(b*x - a)^(3/2)*b^3 +
 5*sqrt(b*x - a)*a*b^3)/(a^2*b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 59, normalized size = 0.80

$$\frac{3b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}+\frac{3\sqrt{bx-a}\;b}{4a^2x}+\frac{\sqrt{bx-a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(1/2),x)
```

```
[Out] 3/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+1/2*(b*x-a)^(1/2)/a/x^2+3/4*b
*(b*x-a)^(1/2)/a^2/x
```

**maxima [A]** time = 2.96, size = 86, normalized size = 1.16

$$\frac{3\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{5}{2}}}+\frac{3\,(bx-a)^{\frac{3}{2}}b^2+5\,\sqrt{bx-a}\,ab^2}{4\left((bx-a)^2a^2+2\,(bx-a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 1/4*(3*(b*x - a)^(3/2)*b^2
+ 5*sqrt(b*x - a)*a*b^2)/((b*x - a)^2*a^2 + 2*(b*x - a)*a^3 + a^4)
```

**mupad [B]** time = 0.05, size = 57, normalized size = 0.77

$$\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{5/2}}+\frac{5\,\sqrt{b\,x-a}}{4\,a\,x^2}+\frac{3\,{\left(b\,x-a\right)}^{3/2}}{4\,a^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(1/2)),x)
```

```
[Out] (3*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(5/2)) + (5*(b*x - a)^(1/2))/(4*
a*x^2) + (3*(b*x - a)^(3/2))/(4*a^2*x^2)
```

**sympy [A]** time = 4.20, size = 216, normalized size = 2.92

$$\begin{cases} \frac{i}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{i\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{3ib^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}-1}}+\frac{3ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{1}{2\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{3b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((I/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + I*sqrt(b)/(4*a*x**(3/
2)*sqrt(a/(b*x) - 1)) - 3*I*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) - 1)) + 3
*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), Abs(a/(b*x)) > 1), (
-1/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - sqrt(b)/(4*a*x**(3/2)*sqrt(-a/
(b*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) - 3*b**2*asin(
sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), True))
```

## 3.362 $\int \frac{1}{x(-a+bx)^{3/2}} \, dx$

**Optimal**. Leaf size=42

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}$$

**Rubi [A]** time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(3/2)),x]
```

```
[Out] -2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = -\frac{2}{a\sqrt{-a+bx}} - \frac{\int \frac{1}{x\sqrt{-a+bx}}\,dx}{a}$$
$$= -\frac{2}{a\sqrt{-a+bx}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{ab}$$
$$= -\frac{2}{a\sqrt{-a+bx}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

**Mathematica** [C] time = 0.01, size = 33, normalized size = 0.79

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{bx}{a}\right)}{a\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (b*x)/a])/(a*Sqrt[-a + b*x])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 42, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2}{a\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a + b*x)^(3/2)),x]
```

```
[Out] -2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

**fricas** [A] time = 1.26, size = 124, normalized size = 2.95

$$\left[-\frac{(bx-a)\sqrt{-a}\,\log\left(\frac{bx+2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)+2\,\sqrt{bx-a}\,a}{a^2bx-a^3}, -\frac{2\left((bx-a)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\,a\right)}{a^2bx-a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - a)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqr
t(b*x - a)*a)/(a^2*b*x - a^3), -2*((b*x - a)*sqrt(a)*arctan(sqrt(b*x - a)/s
qrt(a)) + sqrt(b*x - a)*a)/(a^2*b*x - a^3)]
```

**giac** [A] time = 1.02, size = 34, normalized size = 0.81

$$-\frac{2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2}{\sqrt{bx-a}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) - 2/(sqrt(b*x - a)*a)
```

**maple** [A] time = 0.01, size = 35, normalized size = 0.83

$$-\frac{2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2}{\sqrt{bx-a}\; a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(3/2),x)
```

```
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)-2/a/(b*x-a)^(1/2)
```

**maxima** [A] time = 2.97, size = 34, normalized size = 0.81

$$-\frac{2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2}{\sqrt{bx-a}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) - 2/(sqrt(b*x - a)*a)
```

**mupad** [B] time = 0.10, size = 34, normalized size = 0.81

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2}{a\,\sqrt{b\,x-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(3/2)),x)
[Out] - (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(3/2) - 2/(a*(b*x - a)^(1/2))
```

**sympy [C]** time = 2.20, size = 478, normalized size = 11.38

$$\begin{cases} \frac{2ia^3\sqrt{-1+\frac{bx}{a}}}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx} - \frac{a^3\log\left(\frac{bx}{a}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx} + \frac{2a^3\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx} + \frac{2ia^3\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx} + \frac{a^2bx\log\left(\frac{bx}{a}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx} - \frac{2a^2bx\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx} - \frac{2ia^2bx\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{2a^3\sqrt{1-\frac{bx}{a}}}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx} + \frac{a^3\log\left(\frac{bx}{a}\right)}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx} - \frac{2a^3\log\left(\sqrt{1-\frac{bx}{a}}+1\right)}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx} - \frac{i\pi a^3}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx} - \frac{a^2bx\log\left(\frac{bx}{a}\right)}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx} + \frac{2a^2bx\log\left(\sqrt{1-\frac{bx}{a}}+1\right)}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx} + \frac{i\pi a^2bx}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(3/2),x)
[Out] Piecewise((2*I*a**3*sqrt(-1 + b*x/a)/(I*a**(9/2) - I*a**(7/2)*b*x) - a**3*l
og(b*x/a)/(I*a**(9/2) - I*a**(7/2)*b*x) + 2*a**3*log(sqrt(b)*sqrt(x)/sqrt(a
))/(I*a**(9/2) - I*a**(7/2)*b*x) + 2*I*a**3*asin(sqrt(a)/(sqrt(b)*sqrt(x)))
/(I*a**(9/2) - I*a**(7/2)*b*x) + a**2*b*x*log(b*x/a)/(I*a**(9/2) - I*a**(7/
2)*b*x) - 2*a**2*b*x*log(sqrt(b)*sqrt(x)/sqrt(a))/(I*a**(9/2) - I*a**(7/2)*
b*x) - 2*I*a**2*b*x*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(I*a**(9/2) - I*a**(7/2
)*b*x), Abs(b*x/a) > 1), (2*a**3*sqrt(1 - b*x/a)/(-I*a**(9/2) + I*a**(7/2)*
b*x) + a**3*log(b*x/a)/(-I*a**(9/2) + I*a**(7/2)*b*x) - 2*a**3*log(sqrt(1 -
 b*x/a) + 1)/(-I*a**(9/2) + I*a**(7/2)*b*x) - I*pi*a**3/(-I*a**(9/2) + I*a*
*(7/2)*b*x) - a**2*b*x*log(b*x/a)/(-I*a**(9/2) + I*a**(7/2)*b*x) + 2*a**2*b
*x*log(sqrt(1 - b*x/a) + 1)/(-I*a**(9/2) + I*a**(7/2)*b*x) + I*pi*a**2*b*x/
(-I*a**(9/2) + I*a**(7/2)*b*x), True))
```

# 3.363 $\int \frac{1}{x^2(-a+bx)^{3/2}} dx$

**Optimal**. Leaf size=62

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3b}{a^2\sqrt{bx-a}}+\frac{1}{ax\sqrt{bx-a}}$$

**Rubi [A]** time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$-\frac{3\sqrt{bx-a}}{a^2x}-\frac{3b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2}{ax\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(3/2)),x]
```

```
[Out] -2/(a*x*Sqrt[-a + b*x]) - (3*Sqrt[-a + b*x])/(a^2*x) - (3*b*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx &= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{a}\\
&= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\sqrt{-a+bx}}{a^2x}-\frac{(3b)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a^2}\\
&= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\sqrt{-a+bx}}{a^2x}-\frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{a^2}\\
&= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\sqrt{-a+bx}}{a^2x}-\frac{3b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 34, normalized size = 0.55

$$
-\frac{2b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1-\frac{bx}{a}\right)}{a^2\sqrt{bx-a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(3/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-1/2, 2, 1/2, 1 - (b*x)/a])/(a^2*Sqrt[-a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.06, size = 59, normalized size = 0.95

$$
-\frac{3b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3(bx-a)+2a}{a^2x\sqrt{bx-a}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(-a + b*x)^(3/2)),x]
```

```
[Out] -((2*a + 3*(-a + b*x))/(a^2*x*Sqrt[-a + b*x])) - (3*b*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]])/a^(5/2)
```

**fricas [A]** time = 0.87, size = 164, normalized size = 2.65

$$
\left[-\frac{3\left(b^2x^2-abx\right)\sqrt{-a}\,\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(3abx-a^2\right)\sqrt{bx-a}}{2\left(a^3bx^2-a^4x\right)},-\frac{3\left(b^2x^2-abx\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(3abx-a^2\right)\sqrt{bx-a}}{a^3bx^2-a^4x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(b^2*x^2 - a*b*x)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2
*a)/x) + 2*(3*a*b*x - a^2)*sqrt(b*x - a))/(a^3*b*x^2 - a^4*x), -(3*(b^2*x^2
 - a*b*x)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + (3*a*b*x - a^2)*sqrt(b*x
- a))/(a^3*b*x^2 - a^4*x)]
```

**giac [A]** time = 1.01, size = 64, normalized size = 1.03

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{3\,(bx-a)b+2\,ab}{\left((bx-a)^{\frac{3}{2}}+\sqrt{bx-a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) - (3*(b*x - a)*b + 2*a*b)/(((b*x
 - a)^(3/2) + sqrt(b*x - a)*a)*a^2)
```

**maple [A]** time = 0.01, size = 54, normalized size = 0.87

$$-\frac{3b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2b}{\sqrt{bx-a}\,a^2}-\frac{\sqrt{bx-a}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(3/2),x)
```

```
[Out] -2*b/a^2/(b*x-a)^(1/2)-1/a^2*(b*x-a)^(1/2)/x-3*b*arctan((b*x-a)^(1/2)/a^(1/
2))/a^(5/2)
```

**maxima [A]** time = 3.02, size = 67, normalized size = 1.08

$$-\frac{3\,(bx-a)b+2\,ab}{(bx-a)^{\frac{3}{2}}a^2+\sqrt{bx-a}\,a^3}-\frac{3\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] -(3*(b*x - a)*b + 2*a*b)/((b*x - a)^(3/2)*a^2 + sqrt(b*x - a)*a^3) - 3*b*ar
ctan(sqrt(b*x - a)/sqrt(a))/a^(5/2)
```

**mupad [B]** time = 0.06, size = 52, normalized size = 0.84

$$\frac{1}{a\,x\,\sqrt{b\,x-a}}-\frac{3\,b}{a^2\,\sqrt{b\,x-a}}-\frac{3\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(3/2)),x)
```

```
[Out] 1/(a*x*(b*x - a)^(1/2)) - (3*b)/(a^2*(b*x - a)^(1/2)) - (3*b*atan((b*x - a)
^(1/2)/a^(1/2)))/a^(5/2)
```

**sympy [A]** time = 3.50, size = 156, normalized size = 2.52

$$\begin{cases} -\frac{i}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{3i\sqrt{b}}{a^2\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{3ib\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{1}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{a^2\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{3b\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-I/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(a**2*sq
rt(x)*sqrt(a/(b*x) - 1)) - 3*I*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2),
 Abs(a/(b*x)) > 1), (1/(a*sqrt(b)*x**(3/2)*sqrt(-a/(b*x) + 1)) - 3*sqrt(b)/
(a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) + 3*b*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/a**
(5/2), True))
```

## 3.364 $\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=95

$$-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{15b^2}{4a^3\sqrt{bx-a}}+\frac{5b}{4a^2x\sqrt{bx-a}}+\frac{1}{2ax^2\sqrt{bx-a}}$$

**Rubi [A]** time = 0.02, antiderivative size = 93, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{5\sqrt{bx-a}}{2a^2x^2}-\frac{15b\sqrt{bx-a}}{4a^3x}-\frac{2}{ax^2\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(3/2)),x]
```

```
[Out] -2/(a*x^2*Sqrt[-a + b*x]) - (5*Sqrt[-a + b*x])/(2*a^2*x^2) - (15*b*Sqrt[-a
+ b*x])/(4*a^3*x) - (15*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx = -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\int \frac{1}{x^3\sqrt{-a+bx}}\,dx}{a}$$
$$= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{(15b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a^2}$$
$$= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{15b\sqrt{-a+bx}}{4a^3x} - \frac{\left(15b^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a^3}$$
$$= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{15b\sqrt{-a+bx}}{4a^3x} - \frac{(15b)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{4a^3}$$
$$= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{15b\sqrt{-a+bx}}{4a^3x} - \frac{15b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

**Mathematica [C]** time = 0.01, size = 36, normalized size = 0.38

$$-\frac{2b^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};1-\frac{bx}{a}\right)}{a^3\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(3/2)),x]
[Out] (-2*b^2*Hypergeometric2F1[-1/2, 3, 1/2, 1 - (b*x)/a])/(a^3*Sqrt[-a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.10, size = 79, normalized size = 0.83

$$-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{7/2}} - \frac{8a^2+25a(bx-a)+15(bx-a)^2}{4a^3x^2\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-a + b*x)^(3/2)),x]
[Out] -1/4*(8*a^2 + 25*a*(-a + b*x) + 15*(-a + b*x)^2)/(a^3*x^2*Sqrt[-a + b*x]) -
 (15*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

**fricas [A]** time = 0.89, size = 198, normalized size = 2.08

$$\left[-\frac{15\left(b^{3}x^{3}-ab^{2}x^{2}\right)\sqrt{-a}\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(15ab^{2}x^{2}-5a^{2}bx-2a^{3}\right)\sqrt{bx-a}}{8\left(a^{4}bx^{3}-a^{5}x^{2}\right)},-\frac{15\left(b^{3}x^{3}-ab^{2}x^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(15ab^{2}x^{2}-5a^{2}bx-2a^{3}\right)\sqrt{bx-a}}{4\left(a^{4}bx^{3}-a^{5}x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a
) - 2*a)/x) + 2*(15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*x^
3 - a^5*x^2), -1/4*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(a)*arctan(sqrt(b*x - a)/s
qrt(a)) + (15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*x^3 - a^
5*x^2)]
```

**giac [A]** time = 1.01, size = 81, normalized size = 0.85

$$-\frac{15\,b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{7}{2}}}-\frac{2\,b^{2}}{\sqrt{bx-a}\,a^{3}}-\frac{7\,(bx-a)^{\frac{3}{2}}b^{2}+9\,\sqrt{bx-a}\,ab^{2}}{4\,a^{3}b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="giac")
```

```
[Out] -15/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) - 2*b^2/(sqrt(b*x - a)*a^3)
 - 1/4*(7*(b*x - a)^(3/2)*b^2 + 9*sqrt(b*x - a)*a*b^2)/(a^3*b^2*x^2)
```

**maple [A]** time = 0.01, size = 75, normalized size = 0.79

$$-\frac{15b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}-\frac{2b^{2}}{\sqrt{bx-a}\;a^{3}}-\frac{9\sqrt{bx-a}}{4a^{2}x^{2}}-\frac{7\,(bx-a)^{\frac{3}{2}}}{4a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(3/2),x)
```

```
[Out] -2*b^2/a^3/(b*x-a)^(1/2)-7/4/a^3/x^2*(b*x-a)^(3/2)-9/4/a^2/x^2*(b*x-a)^(1/2
)-15/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)
```

**maxima [A]** time = 2.93, size = 104, normalized size = 1.09

$$-\frac{15\,(bx-a)^{2}b^{2}+25\,(bx-a)ab^{2}+8\,a^{2}b^{2}}{4\left((bx-a)^{\frac{5}{2}}a^{3}+2\,(bx-a)^{\frac{3}{2}}a^{4}+\sqrt{bx-a}\,a^{5}\right)}-\frac{15\,b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(15*(b*x - a)^2*b^2 + 25*(b*x - a)*a*b^2 + 8*a^2*b^2)/((b*x - a)^(5/2)
*a^3 + 2*(b*x - a)^(3/2)*a^4 + sqrt(b*x - a)*a^5) - 15/4*b^2*arctan(sqrt(b*
x - a)/sqrt(a))/a^(7/2)
```

**mupad [B]** time = 0.13, size = 101, normalized size = 1.06

$$-\frac{\frac{2\,b^2}{a}+\frac{15\,b^2\,{\left(a-b\,x\right)}^2}{4\,a^3}-\frac{25\,b^2\,\left(a-b\,x\right)}{4\,a^2}}{2\,a\,{\left(b\,x-a\right)}^{3/2}+{\left(b\,x-a\right)}^{5/2}+a^2\,\sqrt{b\,x-a}}-\frac{15\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(3/2)),x)
```

```
[Out] - ((2*b^2)/a + (15*b^2*(a - b*x)^2)/(4*a^3) - (25*b^2*(a - b*x))/(4*a^2))/(
2*a*(b*x - a)^(3/2) + (b*x - a)^(5/2) + a^2*(b*x - a)^(1/2)) - (15*b^2*atan
((b*x - a)^(1/2)/a^(1/2)))/(4*a^(7/2))
```

**sympy [A]** time = 5.57, size = 226, normalized size = 2.38

$$\begin{cases} -\frac{i}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{5i\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{15ib^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{15ib^2\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ \frac{1}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{15b^2\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-I/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) - 5*I*sqrt(b)/(4*a**
2*x**(3/2)*sqrt(a/(b*x) - 1)) + 15*I*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x)
- 1)) - 15*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), Abs(a/(b*x
)) > 1), (1/(2*a*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) + 5*sqrt(b)/(4*a**2*x
**(3/2)*sqrt(-a/(b*x) + 1)) - 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(-a/(b*x) + 1
)) + 15*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), True))
```

## 3.365 $\int \frac{1}{x(-a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=60

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{a^2\sqrt{bx-a}}-\frac{2}{3a(bx-a)^{3/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$\frac{2}{a^2\sqrt{bx-a}}+\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2}{3a(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(5/2)),x]
```

```
[Out] -2/(3*a*(-a + b*x)^(3/2)) + 2/(a^2*Sqrt[-a + b*x]) + (2*ArcTan[Sqrt[-a + b*
x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(-a+bx)^{5/2}}\,dx &= -\frac{2}{3a(-a+bx)^{3/2}}-\frac{\int \frac{1}{x(-a+bx)^{3/2}}\,dx}{a}\\ &= -\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{\int \frac{1}{x\sqrt{-a+bx}}\,dx}{a^2}\\ &= -\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{a^2b}\\ &= -\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.58

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{bx}{a}\right)}{3a(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, 1 - (b*x)/a])/(3*a*(-a + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.04, size = 55, normalized size = 0.92

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2(a-3(bx-a))}{3a^2(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a + b*x)^(5/2)),x]
```

```
[Out] (-2*(a - 3*(-a + b*x)))/(3*a^2*(-a + b*x)^(3/2)) + (2*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]])/a^(5/2)
```

**fricas [A]** time = 0.85, size = 182, normalized size = 3.03

$$\left[-\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\left(3\,abx-4\,a^2\right)\sqrt{bx-a}}{3\left(a^3b^2x^2-2\,a^4bx+a^5\right)},\ \frac{2\left(3\left(b^2x^2-2\,abx+a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(3\,abx-4\,a^2\right)\sqrt{bx-a}\right)}{3\left(a^3b^2x^2-2\,a^4bx+a^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt
(-a) - 2*a)/x) - 2*(3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*b*
x + a^5), 2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a)*arctan(sqrt(b*x - a)/sqr
t(a)) + (3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*b*x + a^5)]
```

**giac [A]** time = 1.06, size = 42, normalized size = 0.70

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx-4\,a)}{3\,(bx-a)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 2/3*(3*b*x - 4*a)/((b*x - a)^(3/2
)*a^2)
```

**maple [A]** time = 0.01, size = 49, normalized size = 0.82

$$-\frac{2}{3\,(bx-a)^{\frac{3}{2}}\,a}+\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{\sqrt{bx-a}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(5/2),x)
```

```
[Out] -2/3/a/(b*x-a)^(3/2)+2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x-a)^
(1/2)
```

**maxima [A]** time = 2.94, size = 42, normalized size = 0.70

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx-4\,a)}{3\,(bx-a)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 2/3*(3*b*x - 4*a)/((b*x - a)^(3/2
)*a^2)
```

**mupad [B]** time = 0.09, size = 48, normalized size = 0.80

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,\left(a-b\,x\right)}{a^2}+\frac{2}{3\,a}}{{\left(b\,x-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(5/2)),x)
```

```
[Out] (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(5/2) - ((2*(a - b*x))/a^2 + 2/(3*a))/(
b*x - a)^(3/2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(5/2),x)
```

```
[Out] Timed out
```

## 3.366 $\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=81

$$\frac{5b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{bx-a}}-\frac{5b}{3a^2(bx-a)^{3/2}}+\frac{1}{ax(bx-a)^{3/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$\frac{5\sqrt{bx-a}}{a^3x}+\frac{10}{3a^2x\sqrt{bx-a}}+\frac{5b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2}{3ax(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] -2/(3*a*x*(-a + b*x)^(3/2)) + 10/(3*a^2*x*Sqrt[-a + b*x]) + (5*Sqrt[-a + b*
x])/(a^3*x) + (5*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx &= -\frac{2}{3ax(-a+bx)^{3/2}} - \frac{5\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx}{3a} \\
&= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{a^2} \\
&= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\sqrt{-a+bx}}{a^3x} + \frac{(5b)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a^3} \\
&= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\sqrt{-a+bx}}{a^3x} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{a^3} \\
&= -\frac{2}{3ax(-a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{-a+bx}} + \frac{5\sqrt{-a+bx}}{a^3x} + \frac{5b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 36, normalized size = 0.44

$$-\frac{2b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};1-\frac{bx}{a}\right)}{3a^2(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-3/2, 2, -1/2, 1 - (b*x)/a])/(3*a^2*(-a + b*x)^(3/2
))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 75, normalized size = 0.93

$$\frac{5b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{7/2}} - \frac{2a^2-10a(bx-a)-15(bx-a)^2}{3a^3x(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] -1/3*(2*a^2 - 10*a*(-a + b*x) - 15*(-a + b*x)^2)/(a^3*x*(-a + b*x)^(3/2)) +
 (5*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(7/2)
```

**fricas** [A] time = 0.83, size = 226, normalized size = 2.79

$$\left[-\frac{15\left(b^{3}x^{3}-2\,ab^{2}x^{2}+a^{2}bx\right)\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\left(15\,ab^{2}x^{2}-20\,a^{2}bx+3\,a^{3}\right)\sqrt{bx-a}}{6\left(a^{4}b^{2}x^{3}-2\,a^{5}bx^{2}+a^{6}x\right)},\ \frac{15\left(b^{3}x^{3}-2\,ab^{2}x^{2}+a^{2}bx\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(15\,ab^{2}x^{2}-20\,a^{2}bx+3\,a^{3}\right)\sqrt{bx-a}}{3\left(a^{4}b^{2}x^{3}-2\,a^{5}bx^{2}+a^{6}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(b^3*x^3 - 2*a*b^2*x^2 + a^2*b*x)*sqrt(-a)*log((b*x - 2*sqrt(b*x
- a)*sqrt(-a) - 2*a)/x) - 2*(15*a*b^2*x^2 - 20*a^2*b*x + 3*a^3)*sqrt(b*x -
a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x), 1/3*(15*(b^3*x^3 - 2*a*b^2*x^2 + a
^2*b*x)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + (15*a*b^2*x^2 - 20*a^2*b*x
+ 3*a^3)*sqrt(b*x - a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x)]
```

**giac** [A] time = 1.00, size = 66, normalized size = 0.81

$$\frac{5\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{2\,(6\,(bx-a)b-ab)}{3\,(bx-a)^{\frac{3}{2}}a^{3}}+\frac{\sqrt{bx-a}}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 5*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) + 2/3*(6*(b*x - a)*b - a*b)/((b*x
 - a)^(3/2)*a^3) + sqrt(b*x - a)/(a^3*x)
```

**maple** [A] time = 0.01, size = 68, normalized size = 0.84

$$-\frac{2b}{3\,(bx-a)^{\frac{3}{2}}\,a^{2}}+\frac{5b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{4b}{\sqrt{bx-a}\,a^{3}}+\frac{\sqrt{bx-a}}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(5/2),x)
```

```
[Out] -2/3*b/a^2/(b*x-a)^(3/2)+4*b/a^3/(b*x-a)^(1/2)+1/a^3*(b*x-a)^(1/2)/x+5*b*ar
ctan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)
```

**maxima** [A] time = 2.93, size = 82, normalized size = 1.01

$$\frac{15\,(bx-a)^{2}b+10\,(bx-a)ab-2\,a^{2}b}{3\left((bx-a)^{\frac{5}{2}}a^{3}+(bx-a)^{\frac{3}{2}}a^{4}\right)}+\frac{5\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(15*(b*x - a)^2*b + 10*(b*x - a)*a*b - 2*a^2*b)/((b*x - a)^(5/2)*a^3 +
(b*x - a)^(3/2)*a^4) + 5*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2)
```

**mupad [B]** time = 0.12, size = 70, normalized size = 0.86

$$\frac{1}{a\,x\,{\left(b\,x-a\right)}^{3/2}}-\frac{20\,b}{3\,a^2\,{\left(b\,x-a\right)}^{3/2}}+\frac{5\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5\,b^2\,x}{a^3\,{\left(b\,x-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(5/2)),x)
```

```
[Out] 1/(a*x*(b*x - a)^(3/2)) - (20*b)/(3*a^2*(b*x - a)^(3/2)) + (5*b*atan((b*x -
 a)^(1/2)/a^(1/2)))/a^(7/2) + (5*b^2*x)/(a^3*(b*x - a)^(3/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(5/2),x)
```

```
[Out] Timed out
```

# 3.367 $\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=116

$$\frac{35b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b^2}{4a^4\sqrt{bx-a}}-\frac{35b^2}{12a^3(bx-a)^{3/2}}+\frac{7b}{4a^2x(bx-a)^{3/2}}+\frac{1}{2ax^2(bx-a)^{3/2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$\frac{35b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35\sqrt{bx-a}}{6a^3x^2}+\frac{14}{3a^2x^2\sqrt{bx-a}}+\frac{35b\sqrt{bx-a}}{4a^4x}-\frac{2}{3ax^2(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] -2/(3*a*x^2*(-a + b*x)^(3/2)) + 14/(3*a^2*x^2*Sqrt[-a + b*x]) + (35*Sqrt[-a
 + b*x])/(6*a^3*x^2) + (35*b*Sqrt[-a + b*x])/(4*a^4*x) + (35*b^2*ArcTan[Sqr
t[-a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx = -\frac{2}{3ax^2(-a+bx)^{3/2}} - \frac{7\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx}{3a}$$

$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\int \frac{1}{x^3\sqrt{-a+bx}}\,dx}{3a^2}$$

$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{(35b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a^3}$$

$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{\left(35b^2\right)\int \frac{1}{x\sqrt{-a+}}}{8a^4}$$

$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{(35b)\,\text{Subst}\Big(\int}{}$$

$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{35b^2\tan^{-1}\Big(\frac{\sqrt{-}}{}}{4a^{9/2}}$$

**Mathematica [C]** time = 0.01, size = 38, normalized size = 0.33

$$-\frac{2b^2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};1-\frac{bx}{a}\right)}{3a^3(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] (-2*b^2*Hypergeometric2F1[-3/2, 3, -1/2, 1 - (b*x)/a])/(3*a^3*(-a + b*x)^(3
/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 93, normalized size = 0.80

$$\frac{35b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{9/2}} - \frac{8a^3-56a^2(bx-a)-175a(bx-a)^2-105(bx-a)^3}{12a^4x^2(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] -1/12*(8*a^3 - 56*a^2*(-a + b*x) - 175*a*(-a + b*x)^2 - 105*(-a + b*x)^3)/(
a^4*x^2*(-a + b*x)^(3/2)) + (35*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(9
/2))
```

**fricas [A]** time = 0.96, size = 260, normalized size = 2.24

$$\left[-\frac{105\left(b^4x^4-2\,ab^3x^3+a^2b^2x^2\right)\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\left(105\,ab^3x^3-140\,a^2b^2x^2+21\,a^3bx+6\,a^4\right)\sqrt{bx-a}}{24\left(a^5b^2x^4-2\,a^6bx^3+a^7x^2\right)},\ \frac{105\left(b^4x^4-2\,ab^3x^3+a^2b^2x^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(105\,ab^3x^3-140\,a^2b^2x^2+21\,a^3bx+6\,a^4\right)\sqrt{bx-a}}{12\left(a^5b^2x^4-2\,a^6bx^3+a^7x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(105*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*log((b*x - 2*sqr
t(b*x - a)*sqrt(-a) - 2*a)/x) - 2*(105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a^3
*b*x + 6*a^4)*sqrt(b*x - a))/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2), 1/12*(1
05*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(
a)) + (105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a^3*b*x + 6*a^4)*sqrt(b*x - a))
/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2)]
```

**giac [A]** time = 0.90, size = 97, normalized size = 0.84

$$\frac{35\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{9}{2}}}+\frac{2\left(9\,(bx-a)b^2-ab^2\right)}{3\,(bx-a)^{\frac{3}{2}}a^4}+\frac{11\,(bx-a)^{\frac{3}{2}}b^2+13\,\sqrt{bx-a}\,ab^2}{4\,a^4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(9/2) + 2/3*(9*(b*x - a)*b^2 - a*b
^2)/((b*x - a)^(3/2)*a^4) + 1/4*(11*(b*x - a)^(3/2)*b^2 + 13*sqrt(b*x - a)*
a*b^2)/(a^4*b^2*x^2)
```

**maple [A]** time = 0.02, size = 92, normalized size = 0.79

$$-\frac{2b^2}{3\,(bx-a)^{\frac{3}{2}}\,a^3}+\frac{35b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{9}{2}}}+\frac{6b^2}{\sqrt{bx-a}\,a^4}+\frac{13\sqrt{bx-a}}{4a^3x^2}+\frac{11\,(bx-a)^{\frac{3}{2}}}{4a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(5/2),x)
```

```
[Out] -2/3*b^2/a^3/(b*x-a)^(3/2)+6*b^2/a^4/(b*x-a)^(1/2)+11/4/a^4/x^2*(b*x-a)^(3/
2)+13/4/a^3/x^2*(b*x-a)^(1/2)+35/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(9/2
)
```

**maxima** [A] time = 2.87, size = 121, normalized size = 1.04

$$\frac{105\,(bx-a)^3b^2+175\,(bx-a)^2ab^2+56\,(bx-a)a^2b^2-8\,a^3b^2}{12\left((bx-a)^{\frac{7}{2}}a^4+2\,(bx-a)^{\frac{5}{2}}a^5+(bx-a)^{\frac{3}{2}}a^6\right)}+\frac{35\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(105*(b*x - a)^3*b^2 + 175*(b*x - a)^2*a*b^2 + 56*(b*x - a)*a^2*b^2 -
8*a^3*b^2)/((b*x - a)^(7/2)*a^4 + 2*(b*x - a)^(5/2)*a^5 + (b*x - a)^(3/2)*a
^6) + 35/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(9/2)
```

**mupad** [B] time = 0.07, size = 117, normalized size = 1.01

$$\frac{35\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{9/2}}-\frac{\frac{2\,b^2}{3\,a}-\frac{175\,b^2\,(a-b\,x)^2}{12\,a^3}+\frac{35\,b^2\,(a-b\,x)^3}{4\,a^4}+\frac{14\,b^2\,(a-b\,x)}{3\,a^2}}{2\,a\,(b\,x-a)^{5/2}+(b\,x-a)^{7/2}+a^2\,(b\,x-a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(5/2)),x)
```

```
[Out] (35*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(9/2)) - ((2*b^2)/(3*a) - (175*
b^2*(a - b*x)^2)/(12*a^3) + (35*b^2*(a - b*x)^3)/(4*a^4) + (14*b^2*(a - b*x
))/(3*a^2))/(2*a*(b*x - a)^(5/2) + (b*x - a)^(7/2) + a^2*(b*x - a)^(3/2))
```

**sympy** [B] time = 11.22, size = 1108, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(5/2),x)
```

```
[Out] Piecewise((12*I*a**(89/2)*b**75*x**75/(24*a**(93/2)*b**(151/2)*x**(155/2)*s
qrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) +
42*I*a**(87/2)*b**76*x**76/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x)
 - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) - 280*I*a**(8
5/2)*b**77*x**77/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24
*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**(83/2)*b**78
*x**78/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)
*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**42*b**(155/2)*x**(155/
2)*sqrt(a/(b*x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2)*b**(151
/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(
a/(b*x) - 1)) - 105*pi*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*x) - 1)/(24*a*
```

```
*(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x
**(157/2)*sqrt(a/(b*x) - 1)) - 210*I*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*
x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2)*b**(151/2)*x**(155/2
)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1))
 + 105*pi*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) - 1)/(24*a**(93/2)*b**(1
51/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqr
t(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (-6*a**(89/2)*b**75*x**75/(12*a**(93/2)
*b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157
/2)*sqrt(-a/(b*x) + 1)) - 21*a**(87/2)*b**76*x**76/(12*a**(93/2)*b**(151/2)
*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a
/(b*x) + 1)) + 140*a**(85/2)*b**77*x**77/(12*a**(93/2)*b**(151/2)*x**(155/2
)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1
)) - 105*a**(83/2)*b**78*x**78/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(-a/
(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1)) - 105*a
**42*b**(155/2)*x**(155/2)*sqrt(-a/(b*x) + 1)*asin(sqrt(a)/(sqrt(b)*sqrt(x)
))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b*
*(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1)) + 105*a**41*b**(157/2)*x**(157/2)*s
qrt(-a/(b*x) + 1)*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*
x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/
(b*x) + 1)), True))
```

## 3.368 $\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=13

$$\frac{x^m}{\sqrt{a+bx}}$$

**Rubi** [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {12, 74}

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}}\,dx &= \frac{1}{2}\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{(a+bx)^{3/2}}\,dx\\ &= \frac{x^m}{\sqrt{a+bx}}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 13, normalized size = 1.00

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

**IntegrateAlgebraic [F]** time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+m}(2am + b(-1 + 2m)x)}{2(a + bx)^{3/2}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)
),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x
)^(3/2)), x]
```

**fricas [A]** time = 0.91, size = 14, normalized size = 1.08

$$\frac{xx^{m-1}}{\sqrt{bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="fri
cas")
```

```
[Out] x*x^(m - 1)/sqrt(b*x + a)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b(2m - 1)x + 2am)x^{m-1}}{2(bx + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/2*(b*(2*m - 1)*x + 2*a*m)*x^(m - 1)/(b*x + a)^(3/2), x)
```

**maple [A]** time = 0.01, size = 12, normalized size = 0.92

$$\frac{x^m}{\sqrt{bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x)
```

```
[Out] x^m/(b*x+a)^(1/2)
```

**maxima [A]** time = 1.87, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="max
ima")
```

```
[Out] x^m/sqrt(b*x + a)
```

**mupad [B]** time = 0.41, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*(2*a*m + b*x*(2*m - 1)))/(2*(a + b*x)^(3/2)),x)
```

```
[Out] x^m/(a + b*x)^(1/2)
```

**sympy [C]** time = 84.07, size = 78, normalized size = 6.00

$$\frac{mx^m\Gamma(m){}_2F_1\left(\begin{matrix}\frac{3}{2},m\\m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+1)}+\frac{bxx^m\,(2m-1)\,\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)**(3/2),x)
```

```
[Out] m*x**m*gamma(m)*hyper((3/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*g
amma(m + 1)) + b*x*x**m*(2*m - 1)*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,)
, b*x*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m + 2))
```

## 3.369 $\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right) dx$

**Optimal**. Leaf size=13

$$\frac{x^m}{\sqrt{a+bx}}$$

**Rubi [C]** time = 0.04, antiderivative size = 92, normalized size of antiderivative = 7.08, number of steps used = 5, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {67, 65}

$$\frac{x^m\left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{bx}{a}+1\right)}{\sqrt{a+bx}}-\frac{2mx^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[-(b*x^m)/(2*(a + b*x)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x],x]
```

```
[Out] (x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/((-((b*x)/a))^m*Sqrt[a
+ b*x]) - (2*m*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*
x)/a])/(a*(-((b*x)/a))^m)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rubi steps

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right)dx = -\left(\frac{1}{2}b\int \frac{x^m}{(a+bx)^{3/2}}dx\right)+m\int \frac{x^{-1+m}}{\sqrt{a+bx}}dx$$

$$= -\left(\frac{1}{2}\left(bx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{3/2}}dx\right)-\frac{\left(bmx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-1+m}}{\sqrt{a+bx}}dx}{a}$$

$$= \frac{x^m\left(-\frac{bx}{a}\right)^{-m}\ {}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{\sqrt{a+bx}}-\frac{2mx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},1-m\right.}{a}$$

**Mathematica** [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqrt[a + b*x],x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

**IntegrateAlgebraic** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqrt[a + b
*x],x]
```

```
[Out] Defer[IntegrateAlgebraic][-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqr
t[a + b*x], x]
```

**fricas** [A] time = 0.94, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] x^m/sqrt(b*x + a)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{mx^{m-1}}{\sqrt{bx+a}} - \frac{bx^m}{2\,(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="g
iac")
[Out] integrate(m*x^(m - 1)/sqrt(b*x + a) - 1/2*b*x^m/(b*x + a)^(3/2), x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int -\frac{b\,x^m}{2\,(bx+a)^{\frac{3}{2}}} + \frac{m\,x^{m-1}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)
[Out] int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)
```

**maxima [A]** time = 1.86, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="m
axima")
[Out] x^m/sqrt(b*x + a)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{m\,x^{m-1}}{\sqrt{a+b\,x}} - \frac{b\,x^m}{2\,(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^(m - 1))/(a + b*x)^(1/2) - (b*x^m)/(2*(a + b*x)^(3/2)),x)
[Out] int((m*x^(m - 1))/(a + b*x)^(1/2) - (b*x^m)/(2*(a + b*x)^(3/2)), x)
```

**sympy [C]** time = 5.34, size = 73, normalized size = 5.62

$$\frac{mx^m\Gamma(m)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m\\ m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+1)}-\frac{bxx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x**m/(b*x+a)**(3/2)+m*x**(-1+m)/(b*x+a)**(1/2),x)
```

```
[Out] m*x**m*gamma(m)*hyper((1/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*g
amma(m + 1)) - b*x*x**m*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,), b*x*exp_
polar(I*pi)/a)/(2*a**(3/2)*gamma(m + 2))
```

## 3.370 $\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {7, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx = \int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

**Mathematica** [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

**IntegrateAlgebraic** [A] time = 0.02, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

**fricas** [A] time = 1.11, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

**giac [A]** time = 0.85, size = 21, normalized size = 0.91

$$\frac{2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

**maple [A]** time = 0.00, size = 18, normalized size = 0.78

$$-\frac{2 \,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

**maxima [A]** time = 2.99, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

**mupad [B]** time = 0.00, size = 17, normalized size = 0.74

$$-\frac{2 \,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
```

```
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

**sympy** [**A**] time = 1.09, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

## 3.371 $\int x^3 \sqrt[3]{a+bx}\, dx$

**Optimal**. Leaf size=72

$$-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{13/3}}{13b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}$$

**Rubi** [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{7/3}}{7b^4}-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{3(a+bx)^{13/3}}{13b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(a + b*x)^(7/3))/(7*b^4) - (9*a*(
a + b*x)^(10/3))/(10*b^4) + (3*(a + b*x)^(13/3))/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3 \sqrt[3]{a+bx}\, dx &= \int \left(-\frac{a^3 \sqrt[3]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{4/3}}{b^3}-\frac{3a(a+bx)^{7/3}}{b^3}+\frac{(a+bx)^{10/3}}{b^3}\right) dx\\ &= -\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{13/3}}{13b^4}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{4/3}\left(-81a^3+108a^2bx-126ab^2x^2+140b^3x^3\right)}{1820b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(-81*a^3 + 108*a^2*b*x - 126*a*b^2*x^2 + 140*b^3*x^3))/(
1820*b^4)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 51, normalized size = 0.71

$$\frac{3(a+bx)^{4/3}\left(-455a^3+780a^2(a+bx)-546a(a+bx)^2+140(a+bx)^3\right)}{1820b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(-455*a^3 + 780*a^2*(a + b*x) - 546*a*(a + b*x)^2 + 140*
(a + b*x)^3))/(1820*b^4)
```

**fricas [A]** time = 0.84, size = 53, normalized size = 0.74

$$\frac{3\left(140\,b^4x^4+14\,ab^3x^3-18\,a^2b^2x^2+27\,a^3bx-81\,a^4\right)(bx+a)^{\frac{1}{3}}}{1820\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/1820*(140*b^4*x^4 + 14*a*b^3*x^3 - 18*a^2*b^2*x^2 + 27*a^3*b*x - 81*a^4)*
(b*x + a)^(1/3)/b^4
```

**giac [B]** time = 0.87, size = 117, normalized size = 1.62

$$\frac{3\left(\frac{13\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)a}{b^3}+\frac{4\left(35\,(bx+a)^{\frac{13}{3}}-182\,(bx+a)^{\frac{10}{3}}a+390\,(bx+a)^{\frac{7}{3}}a^2-455\,(bx+a)^{\frac{4}{3}}a^3+455\,(bx+a)^{\frac{1}{3}}a^4\right)}{b^3}\right)}{1820\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/1820*(13*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3
)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a/b^3 + 4*(35*(b*x + a)^(13/3) - 182*(b*x
+ a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455*(b*
x + a)^(1/3)*a^4)/b^3)/b
```

**maple [A]** time = 0.00, size = 43, normalized size = 0.60

$$-\frac{3\,(bx+a)^{\frac{4}{3}}\left(-140b^3x^3+126a\,b^2x^2-108a^2bx+81a^3\right)}{1820b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(1/3),x)
```

```
[Out] -3/1820*(b*x+a)^(4/3)*(-140*b^3*x^3+126*a*b^2*x^2-108*a^2*b*x+81*a^3)/b^4
```

**maxima [A]** time = 1.39, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{13}{3}}}{13\,b^4}-\frac{9\,(bx+a)^{\frac{10}{3}}a}{10\,b^4}+\frac{9\,(bx+a)^{\frac{7}{3}}a^2}{7\,b^4}-\frac{3\,(bx+a)^{\frac{4}{3}}a^3}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/13*(b*x + a)^(13/3)/b^4 - 9/10*(b*x + a)^(10/3)*a/b^4 + 9/7*(b*x + a)^(7/
3)*a^2/b^4 - 3/4*(b*x + a)^(4/3)*a^3/b^4
```

**mupad [B]** time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,(a+b\,x)^{13/3}}{13\,b^4}-\frac{3\,a^3\,(a+b\,x)^{4/3}}{4\,b^4}+\frac{9\,a^2\,(a+b\,x)^{7/3}}{7\,b^4}-\frac{9\,a\,(a+b\,x)^{10/3}}{10\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(13/3))/(13*b^4) - (3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(a
 + b*x)^(7/3))/(7*b^4) - (9*a*(a + b*x)^(10/3))/(10*b^4)
```

**sympy [B]** time = 2.83, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(1/3),x)
```

```
[Out] -243*a**(73/3)*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 2
7300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 1092
0*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 243*a**(73/3)/(1820*a**20*b**4
 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 273
00*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) - 1377*
a**(70/3)*b*x*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27
300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920
*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 1458*a**(70/3)*b*x/(1820*a**20*
b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 +
 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) - 3
```

```
213*a**(67/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b
**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x*
*4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 3645*a**(67/3)*b**2*x
**2/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a
**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14
*b**10*x**6) - 3927*a**(64/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(1820*a**20*b**4
 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 273
00*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 4860*
a**(64/3)*b**3*x**3/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**
6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x
**5 + 1820*a**14*b**10*x**6) - 2163*a**(61/3)*b**4*x**4*(1 + b*x/a)**(1/3)/
(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17
*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**
10*x**6) + 3645*a**(61/3)*b**4*x**4/(1820*a**20*b**4 + 10920*a**19*b**5*x +
 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10
920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 1827*a**(58/3)*b**5*x**5*(1
+ b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x*
*2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5
+ 1820*a**14*b**10*x**6) + 1458*a**(58/3)*b**5*x**5/(1820*a**20*b**4 + 1092
0*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**1
6*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 6573*a**(55/
3)*b**6*x**6*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 273
00*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*
a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 243*a**(55/3)*b**6*x**6/(1820*a*
*20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x*
*3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6)
 + 8787*a**(52/3)*b**7*x**7*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**
19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**
8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 6498*a**(49/3)*b*
*8*x**8*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a*
*18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15
*b**9*x**5 + 1820*a**14*b**10*x**6) + 2562*a**(46/3)*b**9*x**9*(1 + b*x/a)*
*(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 3640
0*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a*
*14*b**10*x**6) + 420*a**(43/3)*b**10*x**10*(1 + b*x/a)**(1/3)/(1820*a**20*
b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 +
 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6)
```

## 3.372 $\int x^2 \sqrt[3]{a+bx}\, dx$

**Optimal**. Leaf size=53

$$\frac{3a^2(a+bx)^{4/3}}{4b^3}+\frac{3(a+bx)^{10/3}}{10b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}$$

**Rubi** [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{4/3}}{4b^3}+\frac{3(a+bx)^{10/3}}{10b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(4/3))/(4*b^3) - (6*a*(a + b*x)^(7/3))/(7*b^3) + (3*(a + b
*x)^(10/3))/(10*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt[3]{a+bx}\,dx&=\int\left(\frac{a^2\sqrt[3]{a+bx}}{b^2}-\frac{2a(a+bx)^{4/3}}{b^2}+\frac{(a+bx)^{7/3}}{b^2}\right)dx\\&=\frac{3a^2(a+bx)^{4/3}}{4b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{10/3}}{10b^3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{4/3}\left(9a^2-12abx+14b^2x^2\right)}{140b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(1/3),x]
[Out] (3*(a + b*x)^(4/3)*(9*a^2 - 12*a*b*x + 14*b^2*x^2))/(140*b^3)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 39, normalized size = 0.74

$$\frac{3(a+bx)^{4/3}\left(35a^2-40a(a+bx)+14(a+bx)^2\right)}{140b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^(1/3),x]
[Out] (3*(a + b*x)^(4/3)*(35*a^2 - 40*a*(a + b*x) + 14*(a + b*x)^2))/(140*b^3)
```

**fricas [A]** time = 0.86, size = 42, normalized size = 0.79

$$\frac{3\left(14\,b^3x^3+2\,ab^2x^2-3\,a^2bx+9\,a^3\right)(bx+a)^{\frac{1}{3}}}{140\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="fricas")
[Out] 3/140*(14*b^3*x^3 + 2*a*b^2*x^2 - 3*a^2*b*x + 9*a^3)*(b*x + a)^(1/3)/b^3
```

**giac [B]** time = 0.83, size = 92, normalized size = 1.74

$$\frac{3\left(\frac{10\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)a}{b^2}+\frac{14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3}{b^2}\right)}{140\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="giac")
[Out] 3/140*(10*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2
)*a/b^2 + (14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)
*a^2 - 140*(b*x + a)^(1/3)*a^3)/b^2)/b
```

**maple [A]** time = 0.01, size = 32, normalized size = 0.60

$$\frac{3\,(bx+a)^{\frac{4}{3}}\left(14b^2x^2-12abx+9a^2\right)}{140b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/3),x)
```

```
[Out] 3/140*(b*x+a)^(4/3)*(14*b^2*x^2-12*a*b*x+9*a^2)/b^3
```

**maxima [A]** time = 1.30, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^3} - \frac{6\,(bx+a)^{\frac{7}{3}}a}{7\,b^3} + \frac{3\,(bx+a)^{\frac{4}{3}}a^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x + a)^(10/3)/b^3 - 6/7*(b*x + a)^(7/3)*a/b^3 + 3/4*(b*x + a)^(4/3)
*a^2/b^3
```

**mupad [B]** time = 0.04, size = 37, normalized size = 0.70

$$\frac{42\,(a+b\,x)^{10/3}-120\,a\,(a+b\,x)^{7/3}+105\,a^2\,(a+b\,x)^{4/3}}{140\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(1/3),x)
```

```
[Out] (42*(a + b*x)^(10/3) - 120*a*(a + b*x)^(7/3) + 105*a^2*(a + b*x)^(4/3))/(14
0*b^3)
```

**sympy [B]** time = 1.86, size = 666, normalized size = 12.57

$$\frac{27a^{\frac{34}{3}}\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} - \frac{27a^{\frac{34}{3}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{72a^{\frac{31}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} - \frac{81a^{\frac{31}{3}}bx}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{60a^{\frac{28}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} - \frac{81a^{\frac{28}{3}}b^2x^2}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{60a^{\frac{25}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} - \frac{27a^{\frac{25}{3}}b^3x^3}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{135a^{\frac{22}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{132a^{\frac{19}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3} + \frac{42a^{\frac{16}{3}}b^6x^6\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/3),x)
```

```
[Out] 27*a**(34/3)*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6
*b**5*x**2 + 140*a**5*b**6*x**3) - 27*a**(34/3)/(140*a**8*b**3 + 420*a**7*b
**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 72*a**(31/3)*b*x*(1 + b*
x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**
5*b**6*x**3) - 81*a**(31/3)*b*x/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6
*b**5*x**2 + 140*a**5*b**6*x**3) + 60*a**(28/3)*b**2*x**2*(1 + b*x/a)**(1/3
)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**
3) - 81*a**(28/3)*b**2*x**2/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**
5*x**2 + 140*a**5*b**6*x**3) + 60*a**(25/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(1
40*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) -
 27*a**(25/3)*b**3*x**3/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x*
```

```
*2 + 140*a**5*b**6*x**3) + 135*a**(22/3)*b**4*x**4*(1 + b*x/a)**(1/3)/(140*
a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 13
2*a**(19/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x +
 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 42*a**(16/3)*b**6*x**6*(1 + b*x
/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5
*b**6*x**3)
```

## 3.373 $\int x\sqrt[3]{a+bx}\,dx$

**Optimal**. Leaf size=34

$$\frac{3(a+bx)^{7/3}}{7b^2}-\frac{3a(a+bx)^{4/3}}{4b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{7/3}}{7b^2}-\frac{3a(a+bx)^{4/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x)^(4/3))/(4*b^2) + (3*(a + b*x)^(7/3))/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt[3]{a+bx}\,dx&=\int\left(-\frac{a\sqrt[3]{a+bx}}{b}+\frac{(a+bx)^{4/3}}{b}\right)dx\\&=-\frac{3a(a+bx)^{4/3}}{4b^2}+\frac{3(a+bx)^{7/3}}{7b^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{4/3}(4bx-3a)}{28b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(-3*a + 4*b*x))/(28*b^2)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 35, normalized size = 1.03

$$-\frac{3\sqrt[3]{a+bx}\left(3a^2-abx-4b^2x^2\right)}{28b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(1/3),x]
```

```
[Out] (-3*(a + b*x)^(1/3)*(3*a^2 - a*b*x - 4*b^2*x^2))/(28*b^2)
```

**fricas [A]** time = 0.91, size = 30, normalized size = 0.88

$$\frac{3\left(4\,b^2x^2+abx-3\,a^2\right)(bx+a)^{\frac{1}{3}}}{28\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/28*(4*b^2*x^2 + a*b*x - 3*a^2)*(b*x + a)^(1/3)/b^2
```

**giac [B]** time = 1.04, size = 67, normalized size = 1.97

$$\frac{3\left(\frac{7\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}a\right)a}{b}+\frac{2\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)}{b}\right)}{28\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/28*(7*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a/b + 2*(2*(b*x + a)^(7/3)
- 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)/b)/b
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.62

$$-\frac{3\,(bx+a)^{\frac{4}{3}}\,(-4bx+3a)}{28b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/3),x)
```

```
[Out] -3/28*(b*x+a)^(4/3)*(-4*b*x+3*a)/b^2
```

**maxima** [A] time = 1.29, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b^2}-\frac{3\,(bx+a)^{\frac{4}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b^2 - 3/4*(b*x + a)^(4/3)*a/b^2
```

**mupad** [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{21\,a\,(a+b\,x)^{4/3}-12\,(a+b\,x)^{7/3}}{28\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(1/3),x)
```

```
[Out] -(21*a*(a + b*x)^(4/3) - 12*(a + b*x)^(7/3))/(28*b^2)
```

**sympy** [B] time = 1.20, size = 202, normalized size = 5.94

$$-\frac{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{9a^{\frac{13}{3}}}{28a^2b^2+28ab^3x}-\frac{6a^{\frac{10}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{9a^{\frac{10}{3}}bx}{28a^2b^2+28ab^3x}+\frac{15a^{\frac{7}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{12a^{\frac{4}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/3),x)
```

```
[Out] -9*a**(13/3)*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 9*a**(13/3)/
(28*a**2*b**2 + 28*a*b**3*x) - 6*a**(10/3)*b*x*(1 + b*x/a)**(1/3)/(28*a**2*
b**2 + 28*a*b**3*x) + 9*a**(10/3)*b*x/(28*a**2*b**2 + 28*a*b**3*x) + 15*a**
(7/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 12*a**(4/
3)*b**3*x**3*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x)
```

## 3.374 $\int \sqrt[3]{a+bx}\,dx$

**Optimal**. Leaf size=16

$$\frac{3(a+bx)^{4/3}}{4b}$$

**Rubi [A]** time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{a+bx}\,dx=\frac{3(a+bx)^{4/3}}{4b}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

**fricas** [A]   time = 0.96, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b
```

**giac** [A]   time = 0.89, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b
```

**maple** [A]   time = 0.00, size = 13, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3),x)
```

```
[Out] 3/4*(b*x+a)^(4/3)/b
```

**maxima** [A]   time = 1.26, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,(a+b\,x)^{4/3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3),x)
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

**sympy [A]** time = 0.07, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{4}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3),x)
[Out] 3*(a + b*x)**(4/3)/(4*b)
```

## 3.375 $\int \frac{\sqrt[3]{a+bx}}{x} \, dx$

**Optimal**. Leaf size=91

$$3\sqrt[3]{a+bx}+\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)$$

**Rubi [A]** time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 57, 617, 204, 31}

$$3\sqrt[3]{a+bx}+\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(S
qrt[3]*a^(1/3))] - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3) - (a + b*x)^
(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx}}{x}\,dx &= 3\sqrt[3]{a+bx} + a\int \frac{1}{x(a+bx)^{2/3}}\,dx \\
&= 3\sqrt[3]{a+bx} - \frac{1}{2}\sqrt[3]{a}\log(x) - \frac{1}{2}\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right) - \frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{}{a}\right. \\
&= 3\sqrt[3]{a+bx} - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + \left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2}{}\right. \\
&= 3\sqrt[3]{a+bx} - \sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.05, size = 113, normalized size = 1.24

$$
-\frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)+3\sqrt[3]{a+bx}+\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3)
)/Sqrt[3]] + a^(1/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(1/3)*Log[a^(2/3)
+ a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

**IntegrateAlgebraic** [A] time = 0.07, size = 116, normalized size = 1.27

$$-\frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)+3\sqrt[3]{a+bx}+\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/
(Sqrt[3]*a^(1/3))] + a^(1/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(1/3)*Log[
a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

**fricas** [A] time = 1.08, size = 91, normalized size = 1.00

$$-\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)-\frac{1}{2}a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a)
/a) - 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
+ a^(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

**giac** [A] time = 2.37, size = 87, normalized size = 0.96

$$-\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{2}a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+3(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(1/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3*(b*x + a)^(1/3)
```

**maple** [A] time = 0.01, size = 85, normalized size = 0.93

$$-\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)+a^{\frac{1}{3}}\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)-\frac{a^{\frac{1}{3}}\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2}+3(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x,x)
```

```
[Out] 3*(b*x+a)^(1/3)+a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2*a^(1/3)*ln((b*x+a)^(2
/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-a^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^
(1/3)*(b*x+a)^(1/3)+1))
```

**maxima [A]** time = 3.08, size = 86, normalized size = 0.95

$$-\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

**mupad [B]** time = 0.12, size = 107, normalized size = 1.18

$$a^{1/3}\,\ln\left(9\,a\,(a+b\,x)^{1/3}-9\,a^{4/3}\right)+3\,(a+b\,x)^{1/3}+\frac{a^{1/3}\,\ln\left(9\,a\,(a+b\,x)^{1/3}-\frac{9\,a^{4/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{a^{1/3}\,\ln\left(9\,a\,(a+b\,x)^{1/3}+\frac{9\,a^{4/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x,x)
```

```
[Out] a^(1/3)*log(9*a*(a + b*x)^(1/3) - 9*a^(4/3)) + 3*(a + b*x)^(1/3) + (a^(1/3)
*log(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1)
)/2 - (a^(1/3)*log(9*a*(a + b*x)^(1/3) + (9*a^(4/3)*(3^(1/2)*1i + 1))/2)*(3
^(1/2)*1i + 1))/2
```

**sympy [C]** time = 2.02, size = 180, normalized size = 1.98

$$\frac{4\sqrt[3]{a}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+xe^{\frac{2i\pi}{3}}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+xe^{\frac{4i\pi}{3}}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x,x)
```

```
[Out] 4*a**(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(3*gamma(
7/3)) + 4*a**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pol
```

```
ar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(2*I*pi/3)
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)
/(3*gamma(7/3)) + 4*b**(1/3)*(a/b + x)**(1/3)*gamma(4/3)/gamma(7/3)
```

## 3.376 $\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx$

**Optimal**. Leaf size=97

$$-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{\sqrt[3]{a+bx}}{x}$$

**Rubi [A]** time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 57, 617, 204, 31}

$$-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{\sqrt[3]{a+bx}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x)^(1/3)/x) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/3) - (a +
b*x)^(1/3)])/(2*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = -\frac{\sqrt[3]{a+bx}}{x} + \frac{1}{3}b\int \frac{1}{x(a+bx)^{2/3}}\,dx$$

$$= -\frac{\sqrt[3]{a+bx}}{x} - \frac{b\log(x)}{6a^{2/3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2a^{2/3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a\right.}}{2\sqrt[3]{a}}$$

$$= -\frac{\sqrt[3]{a+bx}}{x} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{2/3}}$$

$$= -\frac{\sqrt[3]{a+bx}}{x} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.34

$$\frac{3b(a+bx)^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{bx}{a}+1\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] (3*b*(a + b*x)^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, 1 + (b*x)/a])/(4*a^2)
```

**IntegrateAlgebraic [A]** time = 0.16, size = 125, normalized size = 1.29

$$\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{6a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{\sqrt[3]{a+bx}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x)^(1/3)/x) - (b*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(2/3)) + (b*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(2/3)
) - (b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(6*a^(2/3)
)
```

**fricas [A]** time = 0.76, size = 139, normalized size = 1.43

$$-\frac{2\sqrt{3}\left(a^{2}\right)^{\frac{1}{6}}abx\arctan\left(\frac{\left(a^{2}\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}}a+2\sqrt{3}\left(a^{2}\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^{2}}\right)+\left(a^{2}\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}a+\left(a^{2}\right)^{\frac{1}{3}}a+\left(a^{2}\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\left(a^{2}\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}a-\left(a^{2}\right)^{\frac{2}{3}}\right)+6(bx+a)^{\frac{1}{3}}a^{2}}{6a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a^2)^(1/6)*a*b*x*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/
3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*b*x*log((b
*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2
/3)*b*x*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x + a)^(1/3)*a^2)/(a^2*
x)
```

**giac [A]** time = 2.55, size = 105, normalized size = 1.08

$$-\frac{\frac{2\sqrt{3}b^{2}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{b^{2}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2b^{2}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{6(bx+a)^{\frac{1}{3}}b}{x}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(2/3) + b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(2/3) - 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*x + a)^(1
/3)*b/x)/b
```

**maple** [A] time = 0.01, size = 92, normalized size = 0.95

$$-\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}+\frac{b \ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{b \ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{(bx+a)^{\frac{1}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x^2,x)
```

```
[Out] -(b*x+a)^(1/3)/x+1/3*b/a^(2/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/6*b/a^(2/3)*ln(
a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))-1/3*b/a^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

**maxima** [A] time = 2.96, size = 93, normalized size = 0.96

$$-\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}}-\frac{b \log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}}+\frac{b \log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{2}{3}}}-\frac{(bx+a)^{\frac{1}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/6*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2
/3) + 1/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(2/3) - (b*x + a)^(1/3)/x
```

**mupad** [B] time = 0.07, size = 117, normalized size = 1.21

$$\frac{b\, \ln\left(3\, b\, (a+b\, x)^{1/3}-3\, a^{1/3}\, b\right)}{3\, a^{2/3}}-\frac{(a+b\, x)^{1/3}}{x}-\frac{\ln\left(\frac{3\, a^{1/3}\, \left(b-\sqrt{3}\, b\, 1\mathrm{i}\right)}{2}+3\, b\, (a+b\, x)^{1/3}\right)\, \left(b-\sqrt{3}\, b\, 1\mathrm{i}\right)}{6\, a^{2/3}}-\frac{\ln\left(\frac{3\, a^{1/3}\, \left(b+\sqrt{3}\, b\, 1\mathrm{i}\right)}{2}+3\, b\, (a+b\, x)^{1/3}\right)\, \left(b+\sqrt{3}\, b\, 1\mathrm{i}\right)}{6\, a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x^2,x)
```

```
[Out] (b*log(3*b*(a + b*x)^(1/3) - 3*a^(1/3)*b))/(3*a^(2/3)) - (a + b*x)^(1/3)/x
- (log((3*a^(1/3)*(b - 3^(1/2)*b*1i))/2 + 3*b*(a + b*x)^(1/3))*(b - 3^(1/2)
*b*1i))/(6*a^(2/3)) - (log((3*a^(1/3)*(b + 3^(1/2)*b*1i))/2 + 3*b*(a + b*x)
^(1/3))*(b + 3^(1/2)*b*1i))/(6*a^(2/3))
```

**sympy** [C] time = 2.19, size = 643, normalized size = 6.63

$$\frac{4a^{\frac{7}{3}} b e^{\frac{2i\pi}{3}} \log\left(1-\frac{\sqrt[3]{b}\, \sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3 e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2 b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{4a^{\frac{7}{3}} b \log\left(1-\frac{\sqrt[3]{b}\, \sqrt[3]{\frac{a}{b}+x}\, e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3 e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2 b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{4a^{\frac{7}{3}} b e^{-\frac{2i\pi}{3}} \log\left(1-\frac{\sqrt[3]{b}\, \sqrt[3]{\frac{a}{b}+x}\, e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3 e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2 b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}} b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}} \log\left(1-\frac{\sqrt[3]{b}\, \sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3 e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2 b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}} b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\, \sqrt[3]{\frac{a}{b}+x}\, e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3 e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2 b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}} b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}} \log\left(1-\frac{\sqrt[3]{b}\, \sqrt[3]{\frac{a}{b}+x}\, e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3 e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2 b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{12a^2 b^{\frac{4}{3}}\sqrt[3]{\frac{a}{b}+x}\, e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}{9a^3 e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2 b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x**2,x)
```

```
[Out] 4*a**(7/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamm
a(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*
gamma(7/3)) + 4*a**(7/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*
pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b
 + x)*exp(2*I*pi/3)*gamma(7/3)) + 4*a**(7/3)*b*exp(-2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2
*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(
4/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3
))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**
(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(
7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b
+ x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a
**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*e
xp(2*I*pi/3)*gamma(7/3)) + 12*a**2*b**(4/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*
gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi
/3)*gamma(7/3))
```

## 3.377 $\int \frac{\sqrt[3]{a+bx}}{x^3} dx$

**Optimal**. Leaf size=127

$$\frac{b^2 \log(x)}{18a^{5/3}} - \frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{6a^{5/3}} + \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}} - \frac{\sqrt[3]{a+bx}}{2x^2} - \frac{b\sqrt[3]{a+bx}}{6ax}$$

**Rubi** [A] time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 57, 617, 204, 31}

$$\frac{b^2 \log(x)}{18a^{5/3}} - \frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{6a^{5/3}} + \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}} - \frac{\sqrt[3]{a+bx}}{2x^2} - \frac{b\sqrt[3]{a+bx}}{6ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] -(a + b*x)^(1/3)/(2*x^2) - (b*(a + b*x)^(1/3))/(6*a*x) + (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)) + (b^2*Log[
x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx &= -\frac{\sqrt[3]{a+bx}}{2x^2}+\frac{1}{6}b\int \frac{1}{x^2(a+bx)^{2/3}}\,dx\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}-\frac{b^2\int \frac{1}{x(a+bx)^{2/3}}\,dx}{9a}\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\log(x)}{18a^{5/3}}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{5/3}}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\ldots}\right.}{6\ldots}\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}-\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\ldots}{}\right.}{3a^{5/3}}\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}\end{aligned}$$

**Mathematica** [C]  time = 0.01, size = 35, normalized size = 0.28

$$-\frac{3b^2(a+bx)^{4/3}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};\frac{bx}{a}+1\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] (-3*b^2*(a + b*x)^(4/3)*Hypergeometric2F1[4/3, 3, 7/3, 1 + (b*x)/a])/(4*a^3
)
```

**IntegrateAlgebraic** [A]  time = 0.22, size = 145, normalized size = 1.14

$$-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{5/3}}+\frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{5/3}}+\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}\,(3a+bx)}{6ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] -1/6*((a + b*x)^(1/3)*(3*a + b*x))/(a*x^2) + (b^2*ArcTan[1/Sqrt[3] + (2*(a
+ b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)) - (b^2*Log[a^(1/3) -
(a + b*x)^(1/3)])/(9*a^(5/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3)
+ (a + b*x)^(2/3)])/(18*a^(5/3))
```

**fricas** [A]  time = 0.98, size = 187, normalized size = 1.47

$$\frac{2\sqrt{3}\,ab^2x^2\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\left(-a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)-3\left(a^2bx+3a^3\right)(bx+a)^{\frac{1}{3}}}{18a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*a*b^2*x^2*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(
1/3)*a - 2*sqrt(3)*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) +
 (-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)
*(b*x + a)^(1/3)) - 2*(-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (-a^2)^(
2/3)) - 3*(a^2*b*x + 3*a^3)*(b*x + a)^(1/3))/(a^3*x^2)
```

**giac** [A]  time = 2.43, size = 128, normalized size = 1.01

$$\frac{\frac{2\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left((bx+a)^{\frac{4}{3}}b^3+2(bx+a)^{\frac{1}{3}}ab^3\right)}{ab^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] 1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(5/3) + b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(5/3) - 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*((b*x + a)^(
4/3)*b^3 + 2*(b*x + a)^(1/3)*a*b^3)/(a*b^2*x^2))/b
```

**maple** [A]  time = 0.01, size = 113, normalized size = 0.89

$$\frac{\sqrt{3}\, b^{2} \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{5}{3}}}-\frac{b^{2} \ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{5}{3}}}+\frac{b^{2} \ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{18a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{1}{3}}}{3x^{2}}-\frac{(bx+a)^{\frac{4}{3}}}{6a\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x^3,x)
```

```
[Out] -1/6/x^2/a*(b*x+a)^(4/3)-1/3*(b*x+a)^(1/3)/x^2-1/9*b^2/a^(5/3)*ln(-a^(1/3)+
(b*x+a)^(1/3))+1/18*b^2/a^(5/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2
/3))+1/9*b^2/a^(5/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1)
)
```

**maxima** [A]  time = 2.99, size = 139, normalized size = 1.09

$$\frac{\sqrt{3} b^{2} \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{5}{3}}}+\frac{b^{2} \log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{5}{3}}}-\frac{b^{2} \log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{4}{3}} b^{2}+2\,(bx+a)^{\frac{1}{3}} ab^{2}}{6\left((bx+a)^{2} a-2\,(bx+a)a^{2}+a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(5/3) + 1/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(5/3) - 1/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(5/3) - 1/6*((b*x + a)^(
4/3)*b^2 + 2*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2*a - 2*(b*x + a)*a^2 + a^3)
```

**mupad** [B]  time = 0.23, size = 196, normalized size = 1.54

$$\frac{b^{2} \ln\left(\frac{b^{2}}{(-a)^{2/3}}-\frac{b^{2}\,(a+b\,x)^{1/3}}{a}\right)}{9\,(-a)^{5/3}}-\frac{\ln\left(\frac{b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}}{2\,(-a)^{2/3}}+\frac{b^{2}\,(a+b\,x)^{1/3}}{a}\right)\left(b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}\right)}{18\,(-a)^{5/3}}-\frac{\frac{b^{2}\,(a+b\,x)^{1/3}}{3}+\frac{b^{2}\,(a+b\,x)^{4/3}}{6\,a}}{(a+b\,x)^{2}-2\,a\,(a+b\,x)+a^{2}}+\frac{b^{2} \ln\left(\frac{b^{2}\,(a+b\,x)^{1/3}}{a}-\frac{b^{2}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{2/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,(-a)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x^3,x)
```

```
[Out] (b^2*log(b^2/(-a)^(2/3) - (b^2*(a + b*x)^(1/3))/a))/(9*(-a)^(5/3)) - (log((
3^(1/2)*b^2*1i + b^2)/(2*(-a)^(2/3)) + (b^2*(a + b*x)^(1/3))/a)*(3^(1/2)*b^
2*1i + b^2))/(18*(-a)^(5/3)) - ((b^2*(a + b*x)^(1/3))/3 + (b^2*(a + b*x)^(4
/3))/(6*a))/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + (b^2*log((b^2*(a + b*x)^(
1/3))/a - (b^2*((3^(1/2)*1i)/2 - 1/2))/(-a)^(2/3))*((3^(1/2)*1i)/2 - 1/2))/
(9*(-a)^(5/3))
```

**sympy [C]** time = 2.58, size = 2266, normalized size = 17.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x**3,x)
```

```
[Out] -4*a**(16/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))
*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27
*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a*
*7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)
+ 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b +
x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*exp(-2*I*pi/3)*log(1 - b
**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)
*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(
a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi
/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(
7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x
)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gam
ma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b
 + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3
)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*
(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*p
i/3)*gamma(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(2*I*pi/3)*log(1 - b**
(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/
3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)*
*2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma
(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*e
xp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*e
```

```
xp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3
)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b +
x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*g
amma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/
b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3
)*gamma(7/3)) + 4*a**(7/3)*b**5*(a/b + x)**3*exp(2*I*pi/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
) + 4*a**(7/3)*b**5*(a/b + x)**3*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**
6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*
pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 4*
a**(7/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3)
 - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2
*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7
/3)) - 12*a**5*b**(7/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**4*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi
/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(
2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) -
 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**3*b**(13/3)*(a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
)
```

## 3.378 $\int x^3(a+bx)^{2/3}\,dx$

**Optimal**. Leaf size=72

$$-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{14/3}}{14b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}$$

**Rubi** [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{14/3}}{14b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(a + b*x)^(8/3))/(8*b^4) - (9*a*(
a + b*x)^(11/3))/(11*b^4) + (3*(a + b*x)^(14/3))/(14*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{2/3}\,dx &= \int \left(-\frac{a^3(a+bx)^{2/3}}{b^3}+\frac{3a^2(a+bx)^{5/3}}{b^3}-\frac{3a(a+bx)^{8/3}}{b^3}+\frac{(a+bx)^{11/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}+\frac{3(a+bx)^{14/3}}{14b^4}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{5/3}\left(-81a^3+135a^2bx-180ab^2x^2+220b^3x^3\right)}{3080b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(-81*a^3 + 135*a^2*b*x - 180*a*b^2*x^2 + 220*b^3*x^3))/(
3080*b^4)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 51, normalized size = 0.71

$$\frac{3(a+bx)^{5/3}\left(-616a^3+1155a^2(a+bx)-840a(a+bx)^2+220(a+bx)^3\right)}{3080b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(-616*a^3 + 1155*a^2*(a + b*x) - 840*a*(a + b*x)^2 + 220
*(a + b*x)^3))/(3080*b^4)
```

**fricas [A]** time = 0.82, size = 53, normalized size = 0.74

$$\frac{3\left(220\,b^4x^4+40\,ab^3x^3-45\,a^2b^2x^2+54\,a^3bx-81\,a^4\right)(bx+a)^{\frac{2}{3}}}{3080\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/3080*(220*b^4*x^4 + 40*a*b^3*x^3 - 45*a^2*b^2*x^2 + 54*a^3*b*x - 81*a^4)*
(b*x + a)^(2/3)/b^4
```

**giac [B]** time = 1.12, size = 117, normalized size = 1.62

$$\frac{3\left(\frac{7\left(40\,(bx+a)^{\frac{11}{3}}-165\,(bx+a)^{\frac{8}{3}}a+264\,(bx+a)^{\frac{5}{3}}a^2-220\,(bx+a)^{\frac{2}{3}}a^3\right)a}{b^3}+\frac{2\left(110\,(bx+a)^{\frac{14}{3}}-560\,(bx+a)^{\frac{11}{3}}a+1155\,(bx+a)^{\frac{8}{3}}a^2-1232\,(bx+a)^{\frac{5}{3}}a^3+770\,(bx+a)^{\frac{2}{3}}a^4\right)}{b^3}\right)}{3080\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/3080*(7*(40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/3
)*a^2 - 220*(b*x + a)^(2/3)*a^3)*a/b^3 + 2*(110*(b*x + a)^(14/3) - 560*(b*x
 + a)^(11/3)*a + 1155*(b*x + a)^(8/3)*a^2 - 1232*(b*x + a)^(5/3)*a^3 + 770*
(b*x + a)^(2/3)*a^4)/b^3)/b
```

**maple [A]** time = 0.00, size = 43, normalized size = 0.60

$$-\frac{3\,(bx+a)^{\frac{5}{3}}\left(-220b^3x^3+180a\,b^2x^2-135a^2bx+81a^3\right)}{3080b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(2/3),x)
```

```
[Out] -3/3080*(b*x+a)^(5/3)*(-220*b^3*x^3+180*a*b^2*x^2-135*a^2*b*x+81*a^3)/b^4
```

**maxima [A]** time = 1.36, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{14}{3}}}{14\,b^4}-\frac{9\,(bx+a)^{\frac{11}{3}}\,a}{11\,b^4}+\frac{9\,(bx+a)^{\frac{8}{3}}\,a^2}{8\,b^4}-\frac{3\,(bx+a)^{\frac{5}{3}}\,a^3}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x + a)^(14/3)/b^4 - 9/11*(b*x + a)^(11/3)*a/b^4 + 9/8*(b*x + a)^(8/
3)*a^2/b^4 - 3/5*(b*x + a)^(5/3)*a^3/b^4
```

**mupad [B]** time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,{\left(a+b\,x\right)}^{14/3}}{14\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{5/3}}{5\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{8/3}}{8\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{11/3}}{11\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(2/3),x)
```

```
[Out] (3*(a + b*x)^(14/3))/(14*b^4) - (3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(a
 + b*x)^(8/3))/(8*b^4) - (9*a*(a + b*x)^(11/3))/(11*b^4)
```

**sympy [B]** time = 3.07, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(2/3),x)
```

```
[Out] -243*a**(74/3)*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 4
6200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 1848
0*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 243*a**(74/3)/(3080*a**20*b**4
 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 462
00*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) - 1296*
a**(71/3)*b*x*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46
200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480
*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 1458*a**(71/3)*b*x/(3080*a**20*
b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 +
 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) - 2
```

```
808*a**(68/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b
**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x*
*4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 3645*a**(68/3)*b**2*x
**2/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a
**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14
*b**10*x**6) - 3120*a**(65/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(3080*a**20*b**4
 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 462
00*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4860*
a**(65/3)*b**3*x**3/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**
6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x
**5 + 3080*a**14*b**10*x**6) - 1050*a**(62/3)*b**4*x**4*(1 + b*x/a)**(2/3)/
(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17
*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**
10*x**6) + 3645*a**(62/3)*b**4*x**4/(3080*a**20*b**4 + 18480*a**19*b**5*x +
 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18
480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4032*a**(59/3)*b**5*x**5*(1
+ b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x*
*2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5
+ 3080*a**14*b**10*x**6) + 1458*a**(59/3)*b**5*x**5/(3080*a**20*b**4 + 1848
0*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**1
6*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 11004*a**(56
/3)*b**6*x**6*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46
200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480
*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 243*a**(56/3)*b**6*x**6/(3080*a
**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x
**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6
) + 14352*a**(53/3)*b**7*x**7*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a
**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b
**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 10485*a**(50/3)
*b**8*x**8*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200
*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a*
*15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4080*a**(47/3)*b**9*x**9*(1 + b*x/
a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 6
1600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080
*a**14*b**10*x**6) + 660*a**(44/3)*b**10*x**10*(1 + b*x/a)**(2/3)/(3080*a**
20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**
3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6)
```

# 3.379 $\int x^2(a+bx)^{2/3}\,dx$

**Optimal**. Leaf size=53

$$\frac{3a^2(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{11/3}}{11b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}$$

**Rubi** [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{11/3}}{11b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(5/3))/(5*b^3) - (3*a*(a + b*x)^(8/3))/(4*b^3) + (3*(a + b
*x)^(11/3))/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{2/3}\,dx &= \int\left(\frac{a^2(a+bx)^{2/3}}{b^2}-\frac{2a(a+bx)^{5/3}}{b^2}+\frac{(a+bx)^{8/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{5/3}}{5b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}+\frac{3(a+bx)^{11/3}}{11b^3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{5/3}\left(9a^2-15abx+20b^2x^2\right)}{220b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(2/3),x]
```

[Out] (3*(a + b*x)^(5/3)*(9*a^2 - 15*a*b*x + 20*b^2*x^2))/(220*b^3)

**IntegrateAlgebraic [A]** time = 0.02, size = 39, normalized size = 0.74

$$\frac{3(a+bx)^{5/3}\left(44a^2-55a(a+bx)+20(a+bx)^2\right)}{220b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(a + b*x)^(2/3),x]

[Out] (3*(a + b*x)^(5/3)*(44*a^2 - 55*a*(a + b*x) + 20*(a + b*x)^2))/(220*b^3)

**fricas [A]** time = 0.65, size = 42, normalized size = 0.79

$$\frac{3\left(20\,b^3x^3+5\,ab^2x^2-6\,a^2bx+9\,a^3\right)(bx+a)^{\frac{2}{3}}}{220\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="fricas")

[Out] 3/220*(20*b^3*x^3 + 5*a*b^2*x^2 - 6*a^2*b*x + 9*a^3)*(b*x + a)^(2/3)/b^3

**giac [B]** time = 0.83, size = 92, normalized size = 1.74

$$\frac{3\left(\frac{11\left(5\,(bx+a)^{\frac{8}{3}}-16\,(bx+a)^{\frac{5}{3}}a+20\,(bx+a)^{\frac{2}{3}}a^2\right)a}{b^2}+\frac{40\,(bx+a)^{\frac{11}{3}}-165\,(bx+a)^{\frac{8}{3}}a+264\,(bx+a)^{\frac{5}{3}}a^2-220\,(bx+a)^{\frac{2}{3}}a^3}{b^2}\right)}{440\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="giac")

[Out] 3/440*(11*(5*(b*x + a)^(8/3) - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)*a/b^2 + (40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/3)*a^2 - 220*(b*x + a)^(2/3)*a^3)/b^2)/b

**maple [A]** time = 0.01, size = 32, normalized size = 0.60

$$\frac{3\,(bx+a)^{\frac{5}{3}}\left(20b^2x^2-15abx+9a^2\right)}{220b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(2/3),x)
```

```
[Out] 3/220*(b*x+a)^(5/3)*(20*b^2*x^2-15*a*b*x+9*a^2)/b^3
```

**maxima [A]**  time = 1.35, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{11}{3}}}{11\,b^3}-\frac{3\,(bx+a)^{\frac{8}{3}}a}{4\,b^3}+\frac{3\,(bx+a)^{\frac{5}{3}}a^2}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x + a)^(11/3)/b^3 - 3/4*(b*x + a)^(8/3)*a/b^3 + 3/5*(b*x + a)^(5/3)
*a^2/b^3
```

**mupad [B]**  time = 0.04, size = 37, normalized size = 0.70

$$\frac{60\,(a+b\,x)^{11/3}-165\,a\,(a+b\,x)^{8/3}+132\,a^2\,(a+b\,x)^{5/3}}{220\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(2/3),x)
```

```
[Out] (60*(a + b*x)^(11/3) - 165*a*(a + b*x)^(8/3) + 132*a^2*(a + b*x)^(5/3))/(22
0*b^3)
```

**sympy [B]**  time = 1.94, size = 666, normalized size = 12.57

$$\frac{27a^{\frac{35}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{27a^{\frac{35}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{63a^{\frac{32}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{81a^{\frac{32}{3}}bx}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{42a^{\frac{29}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{81a^{\frac{29}{3}}b^2x^2}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{78a^{\frac{26}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{27a^{\frac{26}{3}}b^3x^3}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{207a^{\frac{23}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{195a^{\frac{20}{3}}b^5x^5\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{60a^{\frac{17}{3}}b^6x^6\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(2/3),x)
```

```
[Out] 27*a**(35/3)*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6
*b**5*x**2 + 220*a**5*b**6*x**3) - 27*a**(35/3)/(220*a**8*b**3 + 660*a**7*b
**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 63*a**(32/3)*b*x*(1 + b*
x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**
5*b**6*x**3) - 81*a**(32/3)*b*x/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6
*b**5*x**2 + 220*a**5*b**6*x**3) + 42*a**(29/3)*b**2*x**2*(1 + b*x/a)**(2/3
)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**
3) - 81*a**(29/3)*b**2*x**2/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**
5*x**2 + 220*a**5*b**6*x**3) + 78*a**(26/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(2
20*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) -
 27*a**(26/3)*b**3*x**3/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x*
```

```
*2 + 220*a**5*b**6*x**3) + 207*a**(23/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(220*
a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 19
5*a**(20/3)*b**5*x**5*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x +
 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 60*a**(17/3)*b**6*x**6*(1 + b*x
/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5
*b**6*x**3)
```

## 3.380 $\int x(a+bx)^{2/3}\,dx$

**Optimal**. Leaf size=34

$$\frac{3(a+bx)^{8/3}}{8b^2}-\frac{3a(a+bx)^{5/3}}{5b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{8/3}}{8b^2}-\frac{3a(a+bx)^{5/3}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x)^(5/3))/(5*b^2) + (3*(a + b*x)^(8/3))/(8*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{2/3}\,dx&=\int\left(-\frac{a(a+bx)^{2/3}}{b}+\frac{(a+bx)^{5/3}}{b}\right)dx\\&=-\frac{3a(a+bx)^{5/3}}{5b^2}+\frac{3(a+bx)^{8/3}}{8b^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{5/3}(5bx-3a)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(-3*a + 5*b*x))/(40*b^2)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 35, normalized size = 1.03

$$-\frac{3(a+bx)^{2/3}\left(3a^2-2abx-5b^2x^2\right)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(2/3),x]
[Out] (-3*(a + b*x)^(2/3)*(3*a^2 - 2*a*b*x - 5*b^2*x^2))/(40*b^2)
```

**fricas [A]** time = 0.82, size = 31, normalized size = 0.91

$$\frac{3\left(5\,b^2x^2+2\,abx-3\,a^2\right)(bx+a)^{\frac{2}{3}}}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="fricas")
[Out] 3/40*(5*b^2*x^2 + 2*a*b*x - 3*a^2)*(b*x + a)^(2/3)/b^2
```

**giac [B]** time = 0.89, size = 68, normalized size = 2.00

$$\frac{3\left(\frac{4\left(2\,(bx+a)^{\frac{5}{3}}-5\,(bx+a)^{\frac{2}{3}}\,a\right)a}{b}+\frac{5\,(bx+a)^{\frac{8}{3}}-16\,(bx+a)^{\frac{5}{3}}\,a+20\,(bx+a)^{\frac{2}{3}}\,a^2}{b}\right)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="giac")
[Out] 3/40*(4*(2*(b*x + a)^(5/3) - 5*(b*x + a)^(2/3)*a)*a/b + (5*(b*x + a)^(8/3)
- 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)/b)/b
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.62

$$-\frac{3\,(bx+a)^{\frac{5}{3}}\,(-5bx+3a)}{40b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(2/3),x)
[Out] -3/40*(b*x+a)^(5/3)*(-5*b*x+3*a)/b^2
```

**maxima** [A] time = 1.35, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^2}-\frac{3\,(bx+a)^{\frac{5}{3}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^2 - 3/5*(b*x + a)^(5/3)*a/b^2
```

**mupad** [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{24\,a\,(a+b\,x)^{5/3}-15\,(a+b\,x)^{8/3}}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(2/3),x)
```

```
[Out] -(24*a*(a + b*x)^(5/3) - 15*(a + b*x)^(8/3))/(40*b^2)
```

**sympy** [B] time = 1.28, size = 202, normalized size = 5.94

$$-\frac{9a^{\frac{14}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{9a^{\frac{14}{3}}}{40a^2b^2+40ab^3x}-\frac{3a^{\frac{11}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{9a^{\frac{11}{3}}bx}{40a^2b^2+40ab^3x}+\frac{21a^{\frac{8}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{15a^{\frac{5}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(2/3),x)
```

```
[Out] -9*a**(14/3)*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 9*a**(14/3)/
(40*a**2*b**2 + 40*a*b**3*x) - 3*a**(11/3)*b*x*(1 + b*x/a)**(2/3)/(40*a**2*
b**2 + 40*a*b**3*x) + 9*a**(11/3)*b*x/(40*a**2*b**2 + 40*a*b**3*x) + 21*a**
(8/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 15*a**(5/
3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x)
```

## 3.381 $\int (a+bx)^{2/3}\,dx$

**Optimal**. Leaf size=16

$$\frac{3(a+bx)^{5/3}}{5b}$$

**Rubi [A]** time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{2/3}\,dx = \frac{3(a+bx)^{5/3}}{5b}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

**fricas** [A] time = 0.84, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b
```

**giac** [A] time = 1.14, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b
```

**maple** [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3),x)
```

```
[Out] 3/5*(b*x+a)^(5/3)/b
```

**maxima** [A] time = 1.29, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,{\left(a+b\,x\right)}^{5/3}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3),x)
```

```
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.75

$$\frac{3\left(a+bx\right)^{\frac{5}{3}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3),x)
```

```
[Out] 3*(a + b*x)**(5/3)/(5*b)
```

# 3.382 $\int \frac{(a+bx)^{2/3}}{x} \, dx$

**Optimal**. Leaf size=92

$$\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}(a+bx)^{2/3}$$

**Rubi [A]** time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 55, 617, 204, 31}

$$\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}(a+bx)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x)^(2/3))/2 + Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3)
)/(Sqrt[3]*a^(1/3))] - (a^(2/3)*Log[x])/2 + (3*a^(2/3)*Log[a^(1/3) - (a + b
*x)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{2/3}}{x}\,dx &= \frac{3}{2}(a+bx)^{2/3}+a\int \frac{1}{x\sqrt[3]{a+bx}}\,dx\\ &= \frac{3}{2}(a+bx)^{2/3}-\frac{1}{2}a^{2/3}\log(x)-\frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)+\frac{1}{2}(3a)\,\text{Subst}\left(\int\right.\\ &= \frac{3}{2}(a+bx)^{2/3}-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1\right.\\ &= \frac{3}{2}(a+bx)^{2/3}+\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 86, normalized size = 0.93

$$\frac{3}{2}\left(a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+(a+bx)^{2/3}\right)+\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-\frac{1}{2}a^{2/3}\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x,x]
```

```
[Out] Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - (a^(2/3
)*Log[x])/2 + (3*((a + b*x)^(2/3) + a^(2/3)*Log[a^(1/3) - (a + b*x)^(1/3)])
)/2
```

**IntegrateAlgebraic [A]** time = 0.06, size = 117, normalized size = 1.27

$$a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)+\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{2}(a+bx)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x)^(2/3))/2 + Sqrt[3]*a^(2/3)*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/
3))/(Sqrt[3]*a^(1/3))] + a^(2/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(2/3)*
Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

**fricas [A]** time = 0.92, size = 110, normalized size = 1.20

$$\sqrt{3}\left(a^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,a+2\sqrt{3}\left(a^2\right)^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}}{3\,a}\right)-\frac{1}{2}\left(a^2\right)^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)+\left(a^2\right)^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)+\frac{3}{2}(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(a^2)^(1/3)*(b*x + a)
^(1/3))/a) - 1/2*(a^2)^(1/3)*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^
(2/3)*(b*x + a)^(1/3)) + (a^2)^(1/3)*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) +
 3/2*(b*x + a)^(2/3)
```

**giac [A]** time = 2.20, size = 86, normalized size = 0.93

$$\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{2}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{2}\,(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="giac")
```

```
[Out] sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) -
 1/2*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^(
2/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/2*(b*x + a)^(2/3)
```

**maple [A]** time = 0.00, size = 84, normalized size = 0.91

$$\sqrt{3}\;a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\;\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)+a^{\frac{2}{3}}\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)-\frac{a^{\frac{2}{3}}\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\,a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2}+\frac{3\,(bx+a)^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x,x)
```

```
[Out] 3/2*(b*x+a)^(2/3)+a^(2/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/2*a^(2/3)*ln(a^(2/3)
+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))+a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
*(b*x+a)^(1/3)/a^(1/3)+1))
```

**maxima [A]** time = 2.97, size = 85, normalized size = 0.92

$$\sqrt{3}\, a^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\, a^{\frac{2}{3}} \log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{2}{3}} \log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}\,(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) -
 1/2*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^(
2/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/2*(b*x + a)^(2/3)
```

**mupad [B]** time = 0.11, size = 117, normalized size = 1.27

$$\frac{3\,(a+b\,x)^{2/3}}{2}+a^{2/3}\,\ln\left(9\,a^2\,(a+b\,x)^{1/3}-9\,a^{7/3}\right)+\frac{a^{2/3}\,\ln\left(9\,a^2\,(a+b\,x)^{1/3}-\frac{9\,a^{7/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{a^{2/3}\,\ln\left(9\,a^2\,(a+b\,x)^{1/3}-\frac{9\,a^{7/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x,x)
```

```
[Out] (3*(a + b*x)^(2/3))/2 + a^(2/3)*log(9*a^2*(a + b*x)^(1/3) - 9*a^(7/3)) + (a
^(2/3)*log(9*a^2*(a + b*x)^(1/3) - (9*a^(7/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/
2)*1i - 1))/2 - (a^(2/3)*log(9*a^2*(a + b*x)^(1/3) - (9*a^(7/3)*(3^(1/2)*1i
 + 1)^2)/4)*(3^(1/2)*1i + 1))/2
```

**sympy [C]** time = 2.06, size = 182, normalized size = 1.98

$$\frac{5a^{\frac{2}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5a^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5a^{\frac{2}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}{2\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x,x)
```

```
[Out] 5*a**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(3*gamma(
8/3)) + 5*a**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
```

```
r(2*I*pi/3)/a**(1/3))*gamma(5/3)/(3*gamma(8/3)) + 5*a**(2/3)*exp(-2*I*pi/3)
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)
/(3*gamma(8/3)) + 5*b**(2/3)*(a/b + x)**(2/3)*gamma(5/3)/(2*gamma(8/3))
```

## 3.383 $\int \frac{(a+bx)^{2/3}}{x^2} dx$

**Optimal**. Leaf size=94

$$-\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

**Rubi [A]** time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 55, 617, 204, 31}

$$-\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x^2,x]
```

```
[Out] -((a + b*x)^(2/3)/x) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(1/3) - (a
+ b*x)^(1/3)])/a^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{2/3}}{x^2}\,dx &= -\frac{(a+bx)^{2/3}}{x} + \frac{1}{3}(2b)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx \\
&= -\frac{(a+bx)^{2/3}}{x} - \frac{b\log(x)}{3\sqrt[3]{a}} + b\,\mathrm{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx}\right) - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,d\right.}{\sqrt[3]{a}} \\
&= -\frac{(a+bx)^{2/3}}{x} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}} \\
&= -\frac{(a+bx)^{2/3}}{x} + \frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.35

$$\frac{3b(a+bx)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{bx}{a}+1\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x^2,x]
```

```
[Out] (3*b*(a + b*x)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + (b*x)/a])/(5*a^2)
```

**IntegrateAlgebraic [A]** time = 0.16, size = 125, normalized size = 1.33

$$-\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3\sqrt[3]{a}}-\frac{(a+bx)^{2/3}}{x}+\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3\sqrt[3]{a}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3)/x^2,x]
```

```
[Out] -((a + b*x)^(2/3)/x) + (2*b*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(1/3)) + (2*b*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(
1/3)) - (b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*a^(
1/3))
```

**fricas [A]** time = 0.99, size = 252, normalized size = 2.68

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\,(bx+a)^{\frac{2}{3}}a}{3\,ax},\ \frac{6\sqrt{\frac{1}{3}}\,a^{\frac{2}{3}}bx\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\,(bx+a)^{\frac{2}{3}}a}{3\,ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(1/3)*a*b*x*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(b*x +
 a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x
+ a)^(1/3)*a^(2/3) + 3*a)/x) - a^(2/3)*b*x*log((b*x + a)^(2/3) + (b*x + a)^
(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3) - a^(1/3)) - 3
*(b*x + a)^(2/3)*a)/(a*x), 1/3*(6*sqrt(1/3)*a^(2/3)*b*x*arctan(sqrt(1/3)*(2
*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*b*x*log((b*x + a)^(2/3) + (b
*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3) - a^(1
/3)) - 3*(b*x + a)^(2/3)*a)/(a*x)]
```

**giac [A]** time = 2.29, size = 106, normalized size = 1.13

$$\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2\,b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}-\frac{3\,(bx+a)^{\frac{2}{3}}b}{x}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(1/3) - b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(1/3) + 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x + a)^(2/
3)*b/x)/b
```

**maple** [A] time = 0.01, size = 92, normalized size = 0.98

$$\frac{2\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}+\frac{2b\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{b\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{(bx+a)^{\frac{2}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x^2,x)
```

```
[Out] -(b*x+a)^(2/3)/x+2/3*b/a^(1/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/3*b/a^(1/3)*ln(
a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))+2/3*b*3^(1/2)/a^(1/3)*arctan(1
/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

**maxima** [A] time = 2.98, size = 93, normalized size = 0.99

$$\frac{2\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}}-\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{1}{3}}}+\frac{2\,b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}-\frac{(bx+a)^{\frac{2}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/
3) + 2/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(1/3) - (b*x + a)^(2/3)/x
```

**mupad** [B] time = 0.11, size = 127, normalized size = 1.35

$$\frac{2\,b\,\ln\left(4\,a^{1/3}\,b^2-4\,b^2\,(a+b\,x)^{1/3}\right)}{3\,a^{1/3}}-\frac{(a+b\,x)^{2/3}}{x}-\frac{\ln\left(a^{1/3}\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2-4\,b^2\,(a+b\,x)^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{1/3}}-\frac{\ln\left(a^{1/3}\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2-4\,b^2\,(a+b\,x)^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x^2,x)
```

```
[Out] (2*b*log(4*a^(1/3)*b^2 - 4*b^2*(a + b*x)^(1/3)))/(3*a^(1/3)) - (a + b*x)^(2
/3)/x - (log(a^(1/3)*(b - 3^(1/2)*b*1i)^2 - 4*b^2*(a + b*x)^(1/3))*(b - 3^(
```

```
1/2)*b*1i))/(3*a^(1/3)) - (log(a^(1/3)*(b + 3^(1/2)*b*1i)^2 - 4*b^2*(a + b*
x)^(1/3))*(b + 3^(1/2)*b*1i))/(3*a^(1/3))
```

**sympy [C]** time = 2.24, size = 643, normalized size = 6.84

$$\frac{10a^{\frac{8}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{10a^{\frac{8}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{10a^{\frac{8}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{15a^2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x**2,x)
```

```
[Out] 10*a**(8/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gam
ma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)
*gamma(8/3)) + 10*a**(8/3)*b*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/
3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3
) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*ex
p(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) - 10*
a**(5/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**
(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp
(2*I*pi/3)*gamma(8/3)) - 10*a**(5/3)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*
exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) - 1
0*a**(5/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(8/3)) + 15*a**2*b**(5/3)*(a/b + x)**(2/3)*exp(2*I*
pi/3)*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(
2*I*pi/3)*gamma(8/3))
```

## 3.384 $\int \frac{(a+bx)^{2/3}}{x^3}\,dx$

**Optimal**. Leaf size=127

$$\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}-\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}}-\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}$$

**Rubi** [A] time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 55, 617, 204, 31}

$$\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}-\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}}-\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] -(a + b*x)^(2/3)/(2*x^2) - (b*(a + b*x)^(2/3))/(3*a*x) - (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) + (b^2*Log[
x])/(18*a^(4/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{x^3}\,dx = -\frac{(a+bx)^{2/3}}{2x^2} + \frac{1}{3}b\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx$$

$$= -\frac{(a+bx)^{2/3}}{2x^2} - \frac{b(a+bx)^{2/3}}{3ax} - \frac{b^2\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a}$$

$$= -\frac{(a+bx)^{2/3}}{2x^2} - \frac{b(a+bx)^{2/3}}{3ax} + \frac{b^2\log(x)}{18a^{4/3}} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{6a^{4/3}} - \frac{b^2\,\text{Subst}\left(\int \frac{}{a^2}\right.}{}$$

$$= -\frac{(a+bx)^{2/3}}{2x^2} - \frac{b(a+bx)^{2/3}}{3ax} + \frac{b^2\log(x)}{18a^{4/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,\right.}{3a^{4/3}}$$

$$= -\frac{(a+bx)^{2/3}}{2x^2} - \frac{b(a+bx)^{2/3}}{3ax} - \frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}} + \frac{b^2\log(x)}{18a^{4/3}} - \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.28

$$-\frac{3b^2(a+bx)^{5/3}\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};\frac{bx}{a}+1\right)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] (-3*b^2*(a + b*x)^(5/3)*Hypergeometric2F1[5/3, 3, 8/3, 1 + (b*x)/a])/(5*a^3
)
```

**IntegrateAlgebraic [A]** time = 0.25, size = 147, normalized size = 1.16

$$-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{4/3}}+\frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{4/3}}-\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}}-\frac{(a+bx)^{2/3}(2(a+bx)+a)}{6ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] -1/6*((a + b*x)^(2/3)*(a + 2*(a + b*x)))/(a*x^2) - (b^2*ArcTan[1/Sqrt[3] +
(2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) - (b^2*Log[a^(1
/3) - (a + b*x)^(1/3)])/(9*a^(4/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^
(1/3) + (a + b*x)^(2/3)])/(18*a^(4/3))
```

**fricas [A]** time = 0.94, size = 350, normalized size = 2.76

$$\left[\frac{3\sqrt{\frac{1}{3}}ab^2x^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x}\right)+(-a)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-3\left(2abx+3a^2\right)(bx+a)^{\frac{2}{3}}}{18a^2x^2},\ -\frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+2(-a)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+3\left(2abx+3a^2\right)(bx+a)^{\frac{2}{3}}}{18a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*a*b^2*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2
*(b*x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(
1/3)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b^2*x^2*log((
b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b^
2*x^2*log((b*x + a)^(1/3) + (-a)^(1/3)) - 3*(2*a*b*x + 3*a^2)*(b*x + a)^(2/
3))/(a^2*x^2), -1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sqrt
(1/3)*(2*(b*x + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b^
2*x^2*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-
a)^(2/3)*b^2*x^2*log((b*x + a)^(1/3) + (-a)^(1/3)) + 3*(2*a*b*x + 3*a^2)*(b
*x + a)^(2/3))/(a^2*x^2)]
```

**giac [A]** time = 2.47, size = 129, normalized size = 1.02

$$-\frac{\frac{2\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3\left(2(bx+a)^{\frac{5}{3}}b^3+(bx+a)^{\frac{2}{3}}ab^3\right)}{ab^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(4/3) - b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(4/3) + 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3*(2*(b*x + a
)^(5/3)*b^3 + (b*x + a)^(2/3)*a*b^3)/(a*b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 113, normalized size = 0.89

$$-\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{4}{3}}}-\frac{b^2\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{4}{3}}}+\frac{b^2\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{18a^{\frac{4}{3}}}-\frac{(bx+a)^{\frac{2}{3}}}{6x^2}-\frac{(bx+a)^{\frac{5}{3}}}{3a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x^3,x)
```

```
[Out] -1/3/x^2/a*(b*x+a)^(5/3)-1/6*(b*x+a)^(2/3)/x^2-1/9*b^2/a^(4/3)*ln(-a^(1/3)+
(b*x+a)^(1/3))+1/18*b^2/a^(4/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2
/3))-1/9*b^2/a^(4/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1)
)
```

**maxima [A]** time = 2.98, size = 139, normalized size = 1.09

$$-\frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{4}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{4}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{4}{3}}}-\frac{2(bx+a)^{\frac{5}{3}}b^2+(bx+a)^{\frac{2}{3}}ab^2}{6\left((bx+a)^2a-2(bx+a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(4/3) + 1/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
```

```
/a^(4/3) - 1/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(4/3) - 1/6*(2*(b*x + a
)^(5/3)*b^2 + (b*x + a)^(2/3)*a*b^2)/((b*x + a)^2*a - 2*(b*x + a)*a^2 + a^3
)
```

**mupad [B]** time = 0.33, size = 194, normalized size = 1.53

$$\frac{(-1)^{1/3} b^2 \ln\left((a+bx)^{1/3} - (-1)^{2/3} a^{1/3}\right)}{9 a^{4/3}} - \frac{\frac{b^2 (a+bx)^{2/3}}{6} + \frac{b^2 (a+bx)^{5/3}}{3a}}{(a+bx)^2 - 2a(a+bx) + a^2} + \frac{(-1)^{1/3} b^2 \ln\left(\frac{b^4 (a+bx)^{1/3}}{9a^2} - \frac{(-1)^{2/3} b^4 \left(-\frac{1}{2} + \frac{\sqrt{3}\, 1i}{2}\right)^2}{9 a^{5/3}}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\, 1i}{2}\right)}{9 a^{4/3}} - \frac{(-1)^{1/3} b^2 \ln\left(\frac{b^4 (a+bx)^{1/3}}{9a^2} - \frac{(-1)^{2/3} b^4 \left(\frac{1}{2} + \frac{\sqrt{3}\, 1i}{2}\right)^2}{9 a^{5/3}}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\, 1i}{2}\right)}{9 a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x^3,x)
```

```
[Out] ((-1)^(1/3)*b^2*log((a + b*x)^(1/3) - (-1)^(2/3)*a^(1/3)))/(9*a^(4/3)) - ((
b^2*(a + b*x)^(2/3))/6 + (b^2*(a + b*x)^(5/3))/(3*a))/((a + b*x)^2 - 2*a*(a
 + b*x) + a^2) + ((-1)^(1/3)*b^2*log((b^4*(a + b*x)^(1/3))/(9*a^2) - ((-1)^
(2/3)*b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(9*a^(5/3)))*((3^(1/2)*1i)/2 - 1/2))/(9
*a^(4/3)) - ((-1)^(1/3)*b^2*log((b^4*(a + b*x)^(1/3))/(9*a^2) - ((-1)^(2/3)
*b^4*((3^(1/2)*1i)/2 + 1/2)^2)/(9*a^(5/3)))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(4
/3))
```

**sympy [C]** time = 2.65, size = 2266, normalized size = 17.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x**3,x)
```

```
[Out] -10*a**(17/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3)
)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) -
 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)*b**2*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3)
)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) -
 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)*b**2*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(
a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*exp(2*I
*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(
2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*
a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3
*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(54
*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8
```

```
/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a
/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a**7
*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) +
 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b +
x)**3*exp(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*exp(2*I*pi
/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(2*I
*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**
5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*ex
p(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(54
*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8
/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a
/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a
**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3
) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b
 + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*exp(2*I*
pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(2
*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a
**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*
exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(
a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a
**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3
) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b
 + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 15*a**5*b**(8/3)*(a/b + x)**(2/3)*exp(
2*I*pi/3)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b +
x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamm
a(8/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 15*a**4*b**(
11/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamm
a(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b
 + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)
*gamma(8/3)) + 30*a**3*b**(14/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(5/3)/
(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamm
a(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3
*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3))
```

## 3.385 $\int x^3(a+bx)^{4/3}\,dx$

**Optimal**. Leaf size=72

$$-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{16/3}}{16b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}$$

**Rubi [A]** time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{16/3}}{16b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(a + b*x)^(10/3))/(10*b^4) - (9*a
*(a + b*x)^(13/3))/(13*b^4) + (3*(a + b*x)^(16/3))/(16*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{4/3}\,dx&=\int\left(-\frac{a^3(a+bx)^{4/3}}{b^3}+\frac{3a^2(a+bx)^{7/3}}{b^3}-\frac{3a(a+bx)^{10/3}}{b^3}+\frac{(a+bx)^{13/3}}{b^3}\right)dx\\&=-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}+\frac{3(a+bx)^{16/3}}{16b^4}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{7/3}\left(-81a^3+189a^2bx-315ab^2x^2+455b^3x^3\right)}{7280b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-81*a^3 + 189*a^2*b*x - 315*a*b^2*x^2 + 455*b^3*x^3))/(
7280*b^4)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 51, normalized size = 0.71

$$\frac{3(a+bx)^{7/3}\left(-1040a^3+2184a^2(a+bx)-1680a(a+bx)^2+455(a+bx)^3\right)}{7280b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-1040*a^3 + 2184*a^2*(a + b*x) - 1680*a*(a + b*x)^2 + 4
55*(a + b*x)^3))/(7280*b^4)
```

**fricas [A]** time = 0.92, size = 64, normalized size = 0.89

$$\frac{3\left(455\,b^5x^5+595\,ab^4x^4+14\,a^2b^3x^3-18\,a^3b^2x^2+27\,a^4bx-81\,a^5\right)(bx+a)^{\frac{1}{3}}}{7280\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7280*(455*b^5*x^5 + 595*a*b^4*x^4 + 14*a^2*b^3*x^3 - 18*a^3*b^2*x^2 + 27*
a^4*b*x - 81*a^5)*(b*x + a)^(1/3)/b^4
```

**giac [B]** time = 1.22, size = 193, normalized size = 2.68

$$\frac{3\left(\frac{52\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)a^2}{b^3}+\frac{32\left(35\,(bx+a)^{\frac{13}{3}}-182\,(bx+a)^{\frac{10}{3}}a+390\,(bx+a)^{\frac{7}{3}}a^2-455\,(bx+a)^{\frac{4}{3}}a^3+455\,(bx+a)^{\frac{1}{3}}a^4\right)a}{b^3}+\frac{5\left(91\,(bx+a)^{\frac{16}{3}}-560\,(bx+a)^{\frac{13}{3}}a+1456\,(bx+a)^{\frac{10}{3}}a^2-2080\,(bx+a)^{\frac{7}{3}}a^3+1820\,(bx+a)^{\frac{4}{3}}a^4-1456\,(bx+a)^{\frac{1}{3}}a^5\right)}{b^3}\right)}{7280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/7280*(52*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3
)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a^2/b^3 + 32*(35*(b*x + a)^(13/3) - 182*(b
*x + a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455*
(b*x + a)^(1/3)*a^4)*a/b^3 + 5*(91*(b*x + a)^(16/3) - 560*(b*x + a)^(13/3)*
a + 1456*(b*x + a)^(10/3)*a^2 - 2080*(b*x + a)^(7/3)*a^3 + 1820*(b*x + a)^(
4/3)*a^4 - 1456*(b*x + a)^(1/3)*a^5)/b^3)/b
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.60

$$-\frac{3\,(bx+a)^{\frac{7}{3}}\left(-455b^3x^3+315a\,b^2x^2-189a^2bx+81a^3\right)}{7280b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(4/3),x)
```

```
[Out] -3/7280*(b*x+a)^(7/3)*(-455*b^3*x^3+315*a*b^2*x^2-189*a^2*b*x+81*a^3)/b^4
```

**maxima [A]** time = 1.36, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{16}{3}}}{16\,b^4}-\frac{9\,(bx+a)^{\frac{13}{3}}a}{13\,b^4}+\frac{9\,(bx+a)^{\frac{10}{3}}a^2}{10\,b^4}-\frac{3\,(bx+a)^{\frac{7}{3}}a^3}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x + a)^(16/3)/b^4 - 9/13*(b*x + a)^(13/3)*a/b^4 + 9/10*(b*x + a)^(1
0/3)*a^2/b^4 - 3/7*(b*x + a)^(7/3)*a^3/b^4
```

**mupad [B]** time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,(a+b\,x)^{16/3}}{16\,b^4}-\frac{3\,a^3\,(a+b\,x)^{7/3}}{7\,b^4}+\frac{9\,a^2\,(a+b\,x)^{10/3}}{10\,b^4}-\frac{9\,a\,(a+b\,x)^{13/3}}{13\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(16/3))/(16*b^4) - (3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(a
 + b*x)^(10/3))/(10*b^4) - (9*a*(a + b*x)^(13/3))/(13*b^4)
```

**sympy [B]** time = 3.18, size = 1844, normalized size = 25.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(4/3),x)
```

```
[Out] -243*a**(76/3)*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 1
09200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 4
3680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 243*a**(76/3)/(7280*a**20*b
**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3
+ 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) -
 1377*a**(73/3)*b*x*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*
x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**
4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1458*a**(73/3)*b*x/(72
80*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*
b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**
```

10*x**6) - 3213*a**(70/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 4
3680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 10920
0*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 3645*a
**(70/3)*b**2*x**2/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**
6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9
*x**5 + 7280*a**14*b**10*x**6) - 3927*a**(67/3)*b**3*x**3*(1 + b*x/a)**(1/3
)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a
**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**1
4*b**10*x**6) + 4860*a**(67/3)*b**3*x**3/(7280*a**20*b**4 + 43680*a**19*b**
5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x
**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) - 798*a**(64/3)*b**4*x
**4*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18
*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*
b**9*x**5 + 7280*a**14*b**10*x**6) + 3645*a**(64/3)*b**4*x**4/(7280*a**20*b
**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3
+ 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) +
 11382*a**(61/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**1
9*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b
**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1458*a**(61/3)*
b**5*x**5/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 +
145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7
280*a**14*b**10*x**6) + 35238*a**(58/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(7280*
a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**
7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*
x**6) + 243*a**(58/3)*b**6*x**6/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109
200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 436
80*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 56562*a**(55/3)*b**7*x**7*(1
+ b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x
**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x*
*5 + 7280*a**14*b**10*x**6) + 54273*a**(52/3)*b**8*x**8*(1 + b*x/a)**(1/3)/
(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**
17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*
b**10*x**6) + 31227*a**(49/3)*b**9*x**9*(1 + b*x/a)**(1/3)/(7280*a**20*b**4
 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 1
09200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 99
75*a**(46/3)*b**10*x**10*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*
b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**
8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1365*a**(43/3)*b*
*11*x**11*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200
*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*
a**15*b**9*x**5 + 7280*a**14*b**10*x**6)

## 3.386 $\int x^2(a+bx)^{4/3}\,dx$

**Optimal**. Leaf size=53

$$\frac{3a^2(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{13/3}}{13b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}$$

**Rubi** [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{13/3}}{13b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(4/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(7/3))/(7*b^3) - (3*a*(a + b*x)^(10/3))/(5*b^3) + (3*(a +
b*x)^(13/3))/(13*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{4/3}\,dx &= \int\left(\frac{a^2(a+bx)^{4/3}}{b^2}-\frac{2a(a+bx)^{7/3}}{b^2}+\frac{(a+bx)^{10/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}+\frac{3(a+bx)^{13/3}}{13b^3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{7/3}\left(9a^2-21abx+35b^2x^2\right)}{455b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(9*a^2 - 21*a*b*x + 35*b^2*x^2))/(455*b^3)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 39, normalized size = 0.74

$$\frac{3(a+bx)^{7/3}\left(65a^2-91a(a+bx)+35(a+bx)^2\right)}{455b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^(4/3),x]
[Out] (3*(a + b*x)^(7/3)*(65*a^2 - 91*a*(a + b*x) + 35*(a + b*x)^2))/(455*b^3)
```

**fricas [A]** time = 0.77, size = 53, normalized size = 1.00

$$\frac{3\left(35\,b^4x^4+49\,ab^3x^3+2\,a^2b^2x^2-3\,a^3bx+9\,a^4\right)(bx+a)^{\frac{1}{3}}}{455\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="fricas")
[Out] 3/455*(35*b^4*x^4 + 49*a*b^3*x^3 + 2*a^2*b^2*x^2 - 3*a^3*b*x + 9*a^4)*(b*x
+ a)^(1/3)/b^3
```

**giac [B]** time = 0.98, size = 157, normalized size = 2.96

$$\frac{3\left(\frac{65\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)a^2}{b^2}+\frac{13\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)a}{b^2}+\frac{2\left(35\,(bx+a)^{\frac{13}{3}}-182\,(bx+a)^{\frac{10}{3}}a+390\,(bx+a)^{\frac{7}{3}}a^2-455\,(bx+a)^{\frac{4}{3}}a^3+455\,(bx+a)^{\frac{1}{3}}a^4\right)}{b^2}\right)}{910\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="giac")
[Out] 3/910*(65*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2
)*a^2/b^2 + 13*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^
(4/3)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a/b^2 + 2*(35*(b*x + a)^(13/3) - 182*(
b*x + a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455
*(b*x + a)^(1/3)*a^4)/b^2)/b
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.60

$$\frac{3\,(bx+a)^{\frac{7}{3}}\left(35b^2x^2-21abx+9a^2\right)}{455b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(4/3),x)
```

```
[Out] 3/455*(b*x+a)^(7/3)*(35*b^2*x^2-21*a*b*x+9*a^2)/b^3
```

**maxima** [A] time = 1.30, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{13}{3}}}{13\,b^3} - \frac{3\,(bx+a)^{\frac{10}{3}}\,a}{5\,b^3} + \frac{3\,(bx+a)^{\frac{7}{3}}\,a^2}{7\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/13*(b*x + a)^(13/3)/b^3 - 3/5*(b*x + a)^(10/3)*a/b^3 + 3/7*(b*x + a)^(7/3
)*a^2/b^3
```

**mupad** [B] time = 0.04, size = 37, normalized size = 0.70

$$\frac{105\,(a+b\,x)^{13/3} - 273\,a\,(a+b\,x)^{10/3} + 195\,a^2\,(a+b\,x)^{7/3}}{455\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(4/3),x)
```

```
[Out] (105*(a + b*x)^(13/3) - 273*a*(a + b*x)^(10/3) + 195*a^2*(a + b*x)^(7/3))/(
455*b^3)
```

**sympy** [B] time = 2.16, size = 733, normalized size = 13.83

$$\frac{27a^{\frac{37}{3}}\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} - \frac{27a^{\frac{37}{3}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{72a^{\frac{34}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} - \frac{81a^{\frac{34}{3}}bx}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{60a^{\frac{31}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} - \frac{81a^{\frac{31}{3}}b^2x^2}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{165a^{\frac{28}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} - \frac{27a^{\frac{28}{3}}b^3x^3}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{555a^{\frac{25}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{762a^{\frac{22}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{462a^{\frac{19}{3}}b^6x^6\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3} + \frac{105a^{\frac{16}{3}}b^7x^7\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(4/3),x)
```

```
[Out] 27*a**(37/3)*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a*
*6*b**5*x**2 + 455*a**5*b**6*x**3) - 27*a**(37/3)/(455*a**8*b**3 + 1365*a**
7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 72*a**(34/3)*b*x*(1
+ b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 4
55*a**5*b**6*x**3) - 81*a**(34/3)*b*x/(455*a**8*b**3 + 1365*a**7*b**4*x + 1
365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 60*a**(31/3)*b**2*x**2*(1 + b*x/
a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**
5*b**6*x**3) - 81*a**(31/3)*b**2*x**2/(455*a**8*b**3 + 1365*a**7*b**4*x + 1
365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 165*a**(28/3)*b**3*x**3*(1 + b*x
/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a*
*5*b**6*x**3) - 27*a**(28/3)*b**3*x**3/(455*a**8*b**3 + 1365*a**7*b**4*x +
```

```
1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 555*a**(25/3)*b**4*x**4*(1 + b*
x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a
**5*b**6*x**3) + 762*a**(22/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(455*a**8*b**3
+ 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 462*a**(19
/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a
**6*b**5*x**2 + 455*a**5*b**6*x**3) + 105*a**(16/3)*b**7*x**7*(1 + b*x/a)**
(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b*
*6*x**3)
```

## 3.387 $\int x(a+bx)^{4/3}\,dx$

**Optimal**. Leaf size=34

$$\frac{3(a+bx)^{10/3}}{10b^2}-\frac{3a(a+bx)^{7/3}}{7b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{10/3}}{10b^2}-\frac{3a(a+bx)^{7/3}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x)^(7/3))/(7*b^2) + (3*(a + b*x)^(10/3))/(10*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{4/3}\,dx&=\int\left(-\frac{a(a+bx)^{4/3}}{b}+\frac{(a+bx)^{7/3}}{b}\right)dx\\&=-\frac{3a(a+bx)^{7/3}}{7b^2}+\frac{3(a+bx)^{10/3}}{10b^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{7/3}(7bx-3a)}{70b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-3*a + 7*b*x))/(70*b^2)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 24, normalized size = 0.71

$$-\frac{3(3a-7bx)(a+bx)^{7/3}}{70b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(4/3),x]
```

```
[Out] (-3*(3*a - 7*b*x)*(a + b*x)^(7/3))/(70*b^2)
```

**fricas [A]** time = 1.17, size = 41, normalized size = 1.21

$$\frac{3\left(7\,b^3x^3+11\,ab^2x^2+a^2bx-3\,a^3\right)(bx+a)^{\frac{1}{3}}}{70\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/70*(7*b^3*x^3 + 11*a*b^2*x^2 + a^2*b*x - 3*a^3)*(b*x + a)^(1/3)/b^2
```

**giac [B]** time = 1.02, size = 118, normalized size = 3.47

$$\frac{3\left(\frac{35\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}a\right)a^2}{b}+\frac{20\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)a}{b}+\frac{14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3}{b}\right)}{140\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/140*(35*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a^2/b + 20*(2*(b*x + a)^(
7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)*a/b + (14*(b*x + a)^(1
0/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)*a^2 - 140*(b*x + a)^(1/3)
*a^3)/b)/b
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.62

$$-\frac{3\,(bx+a)^{\frac{7}{3}}\,(-7bx+3a)}{70b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(4/3),x)
```

```
[Out] -3/70*(b*x+a)^(7/3)*(-7*b*x+3*a)/b^2
```

**maxima [A]** time = 1.29, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^2}-\frac{3\,(bx+a)^{\frac{7}{3}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="maxima")
[Out] 3/10*(b*x + a)^(10/3)/b^2 - 3/7*(b*x + a)^(7/3)*a/b^2
```

**mupad [B]** time = 0.03, size = 25, normalized size = 0.74

$$-\frac{30\,a\,(a+b\,x)^{7/3}-21\,(a+b\,x)^{10/3}}{70\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(4/3),x)
[Out] -(30*a*(a + b*x)^(7/3) - 21*(a + b*x)^(10/3))/(70*b^2)
```

**sympy [A]** time = 1.49, size = 80, normalized size = 2.35

$$\begin{cases} -\frac{9a^3\sqrt[3]{a+bx}}{70b^2}+\frac{3a^2x\sqrt[3]{a+bx}}{70b}+\frac{33ax^2\sqrt[3]{a+bx}}{70}+\frac{3bx^3\sqrt[3]{a+bx}}{10} & \text{for } b\neq 0 \\ \frac{a^{\frac{4}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(4/3),x)
[Out] Piecewise((-9*a**3*(a + b*x)**(1/3)/(70*b**2) + 3*a**2*x*(a + b*x)**(1/3)/(
70*b) + 33*a*x**2*(a + b*x)**(1/3)/70 + 3*b*x**3*(a + b*x)**(1/3)/10, Ne(b,
 0)), (a**(4/3)*x**2/2, True))
```

## 3.388 $\int (a+bx)^{4/3}\,dx$

**Optimal**. Leaf size=16

$$\frac{3(a+bx)^{7/3}}{7b}$$

**Rubi** [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{4/3}\,dx=\frac{3(a+bx)^{7/3}}{7b}$$

**Mathematica** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

**fricas [B]** time = 0.86, size = 28, normalized size = 1.75

$$\frac{3\left(b^2x^2 + 2\,abx + a^2\right)(bx + a)^{\frac{1}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7*(b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/3)/b
```

**giac [B]** time = 1.25, size = 58, normalized size = 3.62

$$\frac{3\left(2\,(bx + a)^{\frac{7}{3}} - 7\,(bx + a)^{\frac{4}{3}}a + 28\,(bx + a)^{\frac{1}{3}}a^2 + 7\left((bx + a)^{\frac{4}{3}} - 4\,(bx + a)^{\frac{1}{3}}a\right)a\right)}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/14*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 28*(b*x + a)^(1/3)*a^2 + 7*
((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a)/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$\frac{3\,(bx + a)^{\frac{7}{3}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3),x)
```

```
[Out] 3/7*(b*x+a)^(7/3)/b
```

**maxima [A]** time = 1.33, size = 12, normalized size = 0.75

$$\frac{3\,(bx + a)^{\frac{7}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\left(a+b\,x\right)^{7/3}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

**sympy [A]** time = 0.07, size = 12, normalized size = 0.75

$$\frac{3\left(a+bx\right)^{\frac{7}{3}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3),x)
```

```
[Out] 3*(a + b*x)**(7/3)/(7*b)
```

## 3.389 $\int \frac{(a+bx)^{4/3}}{x} \, dx$

**Optimal**. Leaf size=105

$$\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}$$

**Rubi** [A] time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 57, 617, 204, 31}

$$\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x,x]
```

```
[Out] 3*a*(a + b*x)^(1/3) + (3*(a + b*x)^(4/3))/4 - Sqrt[3]*a^(4/3)*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))] - (a^(4/3)*Log[x])/2 + (3*a^(4/3
)*Log[a^(1/3) - (a + b*x)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{4/3}}{x}\,dx &= \frac{3}{4}(a+bx)^{4/3} + a\int \frac{\sqrt[3]{a+bx}}{x}\,dx \\
&= 3a\sqrt[3]{a+bx} + \frac{3}{4}(a+bx)^{4/3} + a^2\int \frac{1}{x(a+bx)^{2/3}}\,dx \\
&= 3a\sqrt[3]{a+bx} + \frac{3}{4}(a+bx)^{4/3} - \frac{1}{2}a^{4/3}\log(x) - \frac{1}{2}\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right) - \frac{1}{2}\Big( \\
&= 3a\sqrt[3]{a+bx} + \frac{3}{4}(a+bx)^{4/3} - \frac{1}{2}a^{4/3}\log(x) + \frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + \left(3a^{4/3}\right)\text{Subst}\Big(\int \\
&= 3a\sqrt[3]{a+bx} + \frac{3}{4}(a+bx)^{4/3} - \sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}a^{4/3}\log(x) + \frac{3}{2}a^{4/3}\log\Big(\sqrt[3]{a} - \sqrt[3]{
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 130, normalized size = 1.24

$$\frac{1}{4}\left(4a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - 2a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right) - 4\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) + 15a\sqrt[3]{a+bx} + 3bx\sqrt[3]{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x,x]
```

```
[Out] (15*a*(a + b*x)^(1/3) + 3*b*x*(a + b*x)^(1/3) - 4*Sqrt[3]*a^(4/3)*ArcTan[(1
 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) - (a + b*x
)^(1/3)] - 2*a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3
)])/4
```

**IntegrateAlgebraic [A]** time = 0.06, size = 131, normalized size = 1.25

$$a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{4}\left((a+bx)^{4/3}+4a\sqrt[3]{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3)/x,x]
```

```
[Out] (3*(4*a*(a + b*x)^(1/3) + (a + b*x)^(4/3)))/4 - Sqrt[3]*a^(4/3)*ArcTan[1/Sq
rt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))] + a^(4/3)*Log[a^(1/3) - (a +
 b*x)^(1/3)] - (a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(
2/3)])/2
```

**fricas [A]** time = 0.66, size = 98, normalized size = 0.93

$$-\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{4}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}\,(bx+5\,a)(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a)
/a) - 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
+ a^(4/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/4*(b*x + 5*a)*(b*x + a)^(1/3)
```

**giac [A]** time = 2.02, size = 97, normalized size = 0.92

$$-\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{4}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{4}\,(bx+a)^{\frac{4}{3}}+3\,(bx+a)^{\frac{1}{3}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(4/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/4*(b*x + a)^(4/3) + 3*(b*x +
a)^(1/3)*a
```

**maple [A]** time = 0.01, size = 95, normalized size = 0.90

$$-\sqrt{3}\;a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\;\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)+a^{\frac{4}{3}}\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)-\frac{a^{\frac{4}{3}}\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\,a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2}+3\,(bx+a)^{\frac{1}{3}}\,a+\frac{3\,(bx+a)^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x,x)
```

```
[Out] 3/4*(b*x+a)^(4/3)+3*a*(b*x+a)^(1/3)+a^(4/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/2*
a^(4/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))-a^(4/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

**maxima [A]** time = 3.03, size = 96, normalized size = 0.91

$$-\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{4}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}\,(bx+a)^{\frac{4}{3}}+3\,(bx+a)^{\frac{1}{3}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(4/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/4*(b*x + a)^(4/3) + 3*(b*x + a)^(1
/3)*a
```

**mupad [B]** time = 0.06, size = 123, normalized size = 1.17

$$3\,a\,(a+b\,x)^{1/3}+\frac{3\,(a+b\,x)^{4/3}}{4}+a^{4/3}\,\ln\left(9\,a^2\,(a+b\,x)^{1/3}-9\,a^{7/3}\right)+\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-9\,a^2\,(a+b\,x)^{1/3}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+9\,a^2\,(a+b\,x)^{1/3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x,x)
```

```
[Out] 3*a*(a + b*x)^(1/3) + (3*(a + b*x)^(4/3))/4 + a^(4/3)*log(9*a^2*(a + b*x)^(
1/3) - 9*a^(7/3)) + (a^(4/3)*log((9*a^(7/3)*(3^(1/2)*1i - 1))/2 - 9*a^2*(a
+ b*x)^(1/3))*(3^(1/2)*1i - 1))/2 - (a^(4/3)*log((9*a^(7/3)*(3^(1/2)*1i + 1
))/2 + 9*a^2*(a + b*x)^(1/3))*(3^(1/2)*1i + 1))/2
```

**sympy [C]** time = 2.39, size = 209, normalized size = 1.99

$$\frac{7a^{\frac{4}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a^{\frac{4}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{7}{3}\right)}{\Gamma\left(\frac{10}{3}\right)}+\frac{7b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}{4\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x,x)
```

```
[Out] 7*a**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(7/3)/(3*gamma(
10/3)) + 7*a**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
```

```
lar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(3*gamma(10/3)) + 7*a**(4/3)*exp(2*I*pi/
3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/
3)/(3*gamma(10/3)) + 7*a*b**(1/3)*(a/b + x)**(1/3)*gamma(7/3)/gamma(10/3) +
 7*b**(4/3)*(a/b + x)**(4/3)*gamma(7/3)/(4*gamma(10/3))
```

## 3.390 $\int \frac{(a+bx)^{4/3}}{x^2} \, dx$

**Optimal**. Leaf size=107

$$-\frac{(a+bx)^{4/3}}{x}+4b\sqrt[3]{a+bx}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 50, 57, 617, 204, 31}

$$-\frac{(a+bx)^{4/3}}{x}+4b\sqrt[3]{a+bx}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x^2,x]
```

```
[Out] 4*b*(a + b*x)^(1/3) - (a + b*x)^(4/3)/x - (4*a^(1/3)*b*ArcTan[(a^(1/3) + 2*
(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (2*a^(1/3)*b*Log[x])/3 + 2*a
^(1/3)*b*Log[a^(1/3) - (a + b*x)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{x^2}\,dx = -\frac{(a+bx)^{4/3}}{x} + \frac{1}{3}(4b)\int \frac{\sqrt[3]{a+bx}}{x}\,dx$$

$$= 4b\sqrt[3]{a+bx} - \frac{(a+bx)^{4/3}}{x} + \frac{1}{3}(4ab)\int \frac{1}{x(a+bx)^{2/3}}\,dx$$

$$= 4b\sqrt[3]{a+bx} - \frac{(a+bx)^{4/3}}{x} - \frac{2}{3}\sqrt[3]{a}\,b\log(x) - \left(2\sqrt[3]{a}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right) - \left(2a\right.$$

$$= 4b\sqrt[3]{a+bx} - \frac{(a+bx)^{4/3}}{x} - \frac{2}{3}\sqrt[3]{a}\,b\log(x) + 2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + \left(4\sqrt[3]{a}\,b\right)\text{Subst}\left(\int\right.$$

$$= 4b\sqrt[3]{a+bx} - \frac{(a+bx)^{4/3}}{x} - \frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\sqrt[3]{a}\,b\log(x) + 2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a\right.$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.31

$$\frac{3b(a+bx)^{7/3}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{bx}{a}+1\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x^2,x]
```

```
[Out] (3*b*(a + b*x)^(7/3)*Hypergeometric2F1[2, 7/3, 10/3, 1 + (b*x)/a])/(7*a^2)
```

**IntegrateAlgebraic [A]** time = 0.19, size = 135, normalized size = 1.26

$$-\frac{2}{3}\sqrt[3]{a}\,b\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)+\frac{\sqrt[3]{a+bx}\,(3(a+bx)-4a)}{x}+\frac{4}{3}\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3)/x^2,x]
```

```
[Out] ((a + b*x)^(1/3)*(-4*a + 3*(a + b*x)))/x - (4*a^(1/3)*b*ArcTan[1/Sqrt[3] +
(2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] + (4*a^(1/3)*b*Log[a^(1/3)
- (a + b*x)^(1/3)])/3 - (2*a^(1/3)*b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3)
+ (a + b*x)^(2/3)])/3
```

**fricas [A]** time = 0.72, size = 111, normalized size = 1.04

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}bx\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3a}\right)+2\,a^{\frac{1}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\,(3\,bx-a)(bx+a)^{\frac{1}{3}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(3)*a^(1/3)*b*x*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) +
 sqrt(3)*a)/a) + 2*a^(1/3)*b*x*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3
) + a^(2/3)) - 4*a^(1/3)*b*x*log((b*x + a)^(1/3) - a^(1/3)) - 3*(3*b*x - a)
*(b*x + a)^(1/3))/x
```

**giac [A]** time = 2.35, size = 119, normalized size = 1.11

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)+2\,a^{\frac{1}{3}}b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)-9\,(bx+a)^{\frac{1}{3}}b^2+\frac{3\,(bx+a)^{\frac{1}{3}}ab}{x}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="giac")
```

```
[Out] -1/3*(4*sqrt(3)*a^(1/3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3)
)/a^(1/3)) + 2*a^(1/3)*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
a^(2/3)) - 4*a^(1/3)*b^2*log(abs((b*x + a)^(1/3) - a^(1/3))) - 9*(b*x + a)^
(1/3)*b^2 + 3*(b*x + a)^(1/3)*a*b/x)/b
```

**maple [A]** time = 0.01, size = 103, normalized size = 0.96

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3}+\frac{4a^{\frac{1}{3}}b\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3}-\frac{2a^{\frac{1}{3}}b\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{3}+3\,(bx+a)^{\frac{1}{3}}b-\frac{(bx+a)^{\frac{1}{3}}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x^2,x)
```

```
[Out] 3*b*(b*x+a)^(1/3)-a*(b*x+a)^(1/3)/x+4/3*b*a^(1/3)*ln(-a^(1/3)+(b*x+a)^(1/3)
)-2/3*b*a^(1/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))-4/3*b*a^(1/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

**maxima [A]** time = 3.03, size = 104, normalized size = 0.97

$$-\frac{4}{3}\sqrt{3}\,a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{2}{3}\,a^{\frac{1}{3}}b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{4}{3}\,a^{\frac{1}{3}}b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(bx+a)^{\frac{1}{3}}b-\frac{(bx+a)^{\frac{1}{3}}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*a^(1/3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(
1/3)) - 2/3*a^(1/3)*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/
3)) + 4/3*a^(1/3)*b*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)*b -
(b*x + a)^(1/3)*a/x
```

**mupad [B]** time = 0.07, size = 131, normalized size = 1.22

$$3\,b\,(a+b\,x)^{1/3}+\frac{4\,a^{1/3}\,b\,\ln\left(12\,a^{4/3}\,b-12\,a\,b\,(a+b\,x)^{1/3}\right)}{3}-\frac{a\,(a+b\,x)^{1/3}}{x}+\frac{2\,a^{1/3}\,b\,\ln\left(12\,a\,b\,(a+b\,x)^{1/3}-6\,a^{4/3}\,b\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}-\frac{2\,a^{1/3}\,b\,\ln\left(12\,a\,b\,(a+b\,x)^{1/3}+6\,a^{4/3}\,b\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x^2,x)
```

```
[Out] 3*b*(a + b*x)^(1/3) + (4*a^(1/3)*b*log(12*a^(4/3)*b - 12*a*b*(a + b*x)^(1/3
)))/3 - (a*(a + b*x)^(1/3))/x + (2*a^(1/3)*b*log(12*a*b*(a + b*x)^(1/3) - 6
*a^(4/3)*b*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/3 - (2*a^(1/3)*b*log(12*a*b*
(a + b*x)^(1/3) + 6*a^(4/3)*b*(3^(1/2)*1i + 1))*(3^(1/2)*1i + 1))/3
```

**sympy [C]** time = 2.58, size = 719, normalized size = 6.72

$$\frac{28a^{\frac{10}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{28a^{\frac{10}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{28a^{\frac{10}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{84a^3b^{\frac{4}{3}}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{63a^2b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x**2,x)
```

```
[Out] 28*a**(10/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*ga
mma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/
3)*gamma(10/3)) + 28*a**(10/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2
*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(10/3)*b*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(9*
a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/
3)) - 28*a**(7/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**
(1/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/
b + x)*exp(2*I*pi/3)*gamma(10/3)) - 28*a**(7/3)*b**2*(a/b + x)*log(1 - b**(
1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(
2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 28*
a**(7/3)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*ex
p_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) -
9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 84*a**3*b**(4/3)*(a/b + x)*
*(1/3)*exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*
b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 63*a**2*b**(7/3)*(a/b + x)**(4/3)*
exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(10/3))
```

# 3.391 $\int \frac{(a+bx)^{4/3}}{x^3} \, dx$

**Optimal**. Leaf size=124

$$-\frac{b^2 \log(x)}{9a^{2/3}}+\frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{2b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b\sqrt[3]{a+bx}}{3x}$$

**Rubi** [A] time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 57, 617, 204, 31}

$$-\frac{b^2 \log(x)}{9a^{2/3}}+\frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{2b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b\sqrt[3]{a+bx}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] (-2*b*(a + b*x)^(1/3))/(3*x) - (a + b*x)^(4/3)/(2*x^2) - (2*b^2*ArcTan[(a^(
1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b^2*Lo
g[x])/(9*a^(2/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{4/3}}{x^3}\,dx &= -\frac{(a+bx)^{4/3}}{2x^2}+\frac{1}{3}(2b)\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx\\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}+\frac{1}{9}\left(2b^2\right)\int \frac{1}{x(a+bx)^{2/3}}\,dx\\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{b^2\log(x)}{9a^{2/3}}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{b^2\operatorname{Subst}\left(\int \frac{}{a^{2/}}\right.}{}\\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}+\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{3a^{2/3}}\\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.28

$$-\frac{3b^2(a+bx)^{7/3}\,{}_2F_1\left(\frac{7}{3},3;\frac{10}{3};\frac{bx}{a}+1\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] (-3*b^2*(a + b*x)^(7/3)*Hypergeometric2F1[7/3, 3, 10/3, 1 + (b*x)/a])/(7*a^
3)
```

**IntegrateAlgebraic [A]** time = 0.23, size = 146, normalized size = 1.18

$$\frac{2b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{2/3}}-\frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9a^{2/3}}-\frac{2b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{\sqrt[3]{a+bx}\,(7(a+bx)-4a)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] -1/6*((a + b*x)^(1/3)*(-4*a + 7*(a + b*x)))/x^2 - (2*b^2*ArcTan[1/Sqrt[3] +
 (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) + (2*b^2*Log[a
^(1/3) - (a + b*x)^(1/3)])/(9*a^(2/3)) - (b^2*Log[a^(2/3) + a^(1/3)*(a + b*
x)^(1/3) + (a + b*x)^(2/3)])/(9*a^(2/3))
```

**fricas [A]** time = 0.98, size = 162, normalized size = 1.31

$$-\frac{4\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^2\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+2\left(a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-4\left(a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)+3\left(7a^2bx+3a^3\right)(bx+a)^{\frac{1}{3}}}{18a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/18*(4*sqrt(3)*(a^2)^(1/6)*a*b^2*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2
)^(1/3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 2*(a^2)^(2/3)*b^2
*x^2*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) -
 4*(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x
+ 3*a^3)*(b*x + a)^(1/3))/(a^2*x^2)
```

**giac [A]** time = 1.94, size = 127, normalized size = 1.02

$$-\frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{2b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{4b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{3\left(7(bx+a)^{\frac{4}{3}}b^3-4(bx+a)^{\frac{1}{3}}ab^3\right)}{b^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(2/3) + 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(2/3) - 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 3*(7*(b*x +
 a)^(4/3)*b^3 - 4*(b*x + a)^(1/3)*a*b^3)/(b^2*x^2))/b
```

**maple** [A] time = 0.01, size = 111, normalized size = 0.90

$$-\frac{2\sqrt{3}\, b^{2} \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}+\frac{2b^{2}\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{b^{2}\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\,(bx+a)^{\frac{1}{3}}\, a}{3x^{2}}-\frac{7\,(bx+a)^{\frac{4}{3}}}{6x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x^3,x)
```

```
[Out] -7/6*(b*x+a)^(4/3)/x^2+2/3/x^2*(b*x+a)^(1/3)*a+2/9*b^2/a^(2/3)*ln(-a^(1/3)+
(b*x+a)^(1/3))-1/9*b^2/a^(2/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/
3))-2/9*b^2/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

**maxima** [A] time = 3.06, size = 136, normalized size = 1.10

$$-\frac{2\,\sqrt{3}\, b^{2} \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{2}{3}}}-\frac{b^{2}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{2}{3}}}+\frac{2\,b^{2}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{2}{3}}}-\frac{7\,(bx+a)^{\frac{4}{3}}b^{2}-4\,(bx+a)^{\frac{1}{3}}ab^{2}}{6\left((bx+a)^{2}-2\,(bx+a)a+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(2/3) - 1/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(2/3) + 2/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(2/3) - 1/6*(7*(b*x + a)
^(4/3)*b^2 - 4*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2 - 2*(b*x + a)*a + a^2)
```

**mupad** [B] time = 0.12, size = 174, normalized size = 1.40

$$\frac{2\,b^{2}\,\ln\left(2\,b^{2}\,(a+b\,x)^{1/3}-2\,a^{1/3}\,b^{2}\right)}{9\,a^{2/3}}-\frac{\frac{7\,b^{2}\,(a+b\,x)^{4/3}}{6}-\frac{2\,a\,b^{2}\,(a+b\,x)^{1/3}}{3}}{(a+b\,x)^{2}-2\,a\,(a+b\,x)+a^{2}}-\frac{\ln\left(2\,b^{2}\,(a+b\,x)^{1/3}+a^{1/3}\,\left(b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}\right)\right)\,\left(b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}\right)}{9\,a^{2/3}}+\frac{b^{2}\,\ln\left(2\,b^{2}\,(a+b\,x)^{1/3}-9\,a^{1/3}\,b^{2}\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)}{a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x^3,x)
```

```
[Out] (2*b^2*log(2*b^2*(a + b*x)^(1/3) - 2*a^(1/3)*b^2))/(9*a^(2/3)) - ((7*b^2*(a
 + b*x)^(4/3))/6 - (2*a*b^2*(a + b*x)^(1/3))/3)/((a + b*x)^2 - 2*a*(a + b*x
) + a^2) - (log(2*b^2*(a + b*x)^(1/3) + a^(1/3)*(3^(1/2)*b^2*1i + b^2))*(3^
```

```
(1/2)*b^2*1i + b^2))/(9*a^(2/3)) + (b^2*log(2*b^2*(a + b*x)^(1/3) - 9*a^(1/
3)*b^2*((3^(1/2)*1i)/9 - 1/9))*((3^(1/2)*1i)/9 - 1/9))/a^(2/3)
```

**sympy [C]** time = 2.74, size = 2266, normalized size = 18.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x**3,x)
```

```
[Out] 28*a**(19/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))
*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3)
 - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*b**2
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)
/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*ga
mma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*
b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*b**2*exp(-2*I*p
i/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3
)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a
**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b +
 x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(7/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma
(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**
3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b + x)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(54
*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3
*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b + x)*exp(
-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*
gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*
I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3)
- 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*
(a/b + x)**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamm
a(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi
/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54
*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*(a/b
 + x)**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*ga
mma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*
pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) -
54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*(a
/b + x)**2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(
a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/
```

```
3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*
a**(10/3)*b**5*(a/b + x)**3*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*g
amma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*a**(
10/3)*b**5*(a/b + x)**3*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/
3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/
b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)
*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*a*
*(10/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*
exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3)
 - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)
**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gam
ma(10/3)) + 84*a**6*b**(7/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(7/3)/(54*
a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(1
0/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*
(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 231*a**5*b**(10/3)*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b
*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*p
i/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 1
47*a**4*b**(13/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(7/3)/(54*a**7*exp(2*
I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*
a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**
3*exp(2*I*pi/3)*gamma(10/3))
```

## 3.392 $\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx$

**Optimal**. Leaf size=72

$$-\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{11/3}}{11b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}$$

**Rubi** [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{3(a+bx)^{11/3}}{11b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(a + b*x)^(5/3))/(5*b^4) - (9*a*(
a + b*x)^(8/3))/(8*b^4) + (3*(a + b*x)^(11/3))/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx &= \int \left(-\frac{a^3}{b^3\sqrt[3]{a+bx}}+\frac{3a^2(a+bx)^{2/3}}{b^3}-\frac{3a(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{8/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{11/3}}{11b^4}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{2/3}\left(-81a^3+54a^2bx-45ab^2x^2+40b^3x^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-81*a^3 + 54*a^2*b*x - 45*a*b^2*x^2 + 40*b^3*x^3))/(440
*b^4)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 51, normalized size = 0.71

$$\frac{3(a+bx)^{2/3}\left(-220a^3+264a^2(a+bx)-165a(a+bx)^2+40(a+bx)^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-220*a^3 + 264*a^2*(a + b*x) - 165*a*(a + b*x)^2 + 40*(
a + b*x)^3))/(440*b^4)
```

**fricas [A]** time = 0.87, size = 42, normalized size = 0.58

$$\frac{3\left(40\,b^3x^3-45\,ab^2x^2+54\,a^2bx-81\,a^3\right)(bx+a)^{\frac{2}{3}}}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 - 45*a*b^2*x^2 + 54*a^2*b*x - 81*a^3)*(b*x + a)^(2/3)/b^4
```

**giac [A]** time = 0.90, size = 49, normalized size = 0.68

$$\frac{3\left(40\,(bx+a)^{\frac{11}{3}}-165\,(bx+a)^{\frac{8}{3}}a+264\,(bx+a)^{\frac{5}{3}}a^2-220\,(bx+a)^{\frac{2}{3}}a^3\right)}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/440*(40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/3)*a^
2 - 220*(b*x + a)^(2/3)*a^3)/b^4
```

**maple [A]** time = 0.00, size = 43, normalized size = 0.60

$$-\frac{3\,(bx+a)^{\frac{2}{3}}\left(-40b^3x^3+45a\,b^2x^2-54a^2bx+81a^3\right)}{440b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(1/3),x)
```

```
[Out] -3/440*(b*x+a)^(2/3)*(-40*b^3*x^3+45*a*b^2*x^2-54*a^2*b*x+81*a^3)/b^4
```

**maxima [A]** time = 1.31, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{11}{3}}}{11\,b^4}-\frac{9\,(bx+a)^{\frac{8}{3}}a}{8\,b^4}+\frac{9\,(bx+a)^{\frac{5}{3}}a^2}{5\,b^4}-\frac{3\,(bx+a)^{\frac{2}{3}}a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x + a)^(11/3)/b^4 - 9/8*(b*x + a)^(8/3)*a/b^4 + 9/5*(b*x + a)^(5/3)
*a^2/b^4 - 3/2*(b*x + a)^(2/3)*a^3/b^4
```

**mupad [B]** time = 0.04, size = 56, normalized size = 0.78

$$\frac{3\,(a+b\,x)^{11/3}}{11\,b^4}-\frac{3\,a^3\,(a+b\,x)^{2/3}}{2\,b^4}+\frac{9\,a^2\,(a+b\,x)^{5/3}}{5\,b^4}-\frac{9\,a\,(a+b\,x)^{8/3}}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(11/3))/(11*b^4) - (3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(a
 + b*x)^(5/3))/(5*b^4) - (9*a*(a + b*x)^(8/3))/(8*b^4)
```

**sympy [B]** time = 2.78, size = 1640, normalized size = 22.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(1/3),x)
```

```
[Out] -243*a**(71/3)*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 660
0*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**1
5*b**9*x**5 + 440*a**14*b**10*x**6) + 243*a**(71/3)/(440*a**20*b**4 + 2640*
a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**
8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 1296*a**(68/3)*b*x*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 1458*a**(68/3)*b*x/(440*a**20*b**4 + 2640*a**19*b**5
*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2
640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 2808*a**(65/3)*b**2*x**2*(1 +
 b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2 +
 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a
```

```
**14*b**10*x**6) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b**4 + 2640*a**19*b*
*5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 +
 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 3120*a**(62/3)*b**3*x**3*(1
 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2
 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440
*a**14*b**10*x**6) + 4860*a**(62/3)*b**3*x**3/(440*a**20*b**4 + 2640*a**19*
b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4
 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 1710*a**(59/3)*b**4*x**4*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4 + 2640*a**1
9*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x*
*4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 72*a**(56/3)*b**5*x**5*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 1458*a**(56/3)*b**5*x**5/(440*a**20*b**4 + 2640*a**1
9*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x*
*4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 1104*a**(53/3)*b**6*x**
6*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*
x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 +
 440*a**14*b**10*x**6) + 243*a**(53/3)*b**6*x**6/(440*a**20*b**4 + 2640*a**
19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x
**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 1152*a**(50/3)*b**7*x*
*7*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6
*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5
+ 440*a**14*b**10*x**6) + 585*a**(47/3)*b**8*x**8*(1 + b*x/a)**(2/3)/(440*a
**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3
 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 12
0*a**(44/3)*b**9*x**9*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*
x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 26
40*a**15*b**9*x**5 + 440*a**14*b**10*x**6)
```

## 3.393 $\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx$

**Optimal**. Leaf size=53

$$\frac{3a^2(a+bx)^{2/3}}{2b^3}+\frac{3(a+bx)^{8/3}}{8b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}$$

**Rubi [A]** time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{2/3}}{2b^3}+\frac{3(a+bx)^{8/3}}{8b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(2/3))/(2*b^3) - (6*a*(a + b*x)^(5/3))/(5*b^3) + (3*(a + b
*x)^(8/3))/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}}-\frac{2a(a+bx)^{2/3}}{b^2}+\frac{(a+bx)^{5/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{2/3}}{2b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{8/3}}{8b^3}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{2/3}\left(9a^2-6abx+5b^2x^2\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(9*a^2 - 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 45, normalized size = 0.85

$$\frac{3\left(20a^2(a+bx)^{2/3}+5(a+bx)^{8/3}-16a(a+bx)^{5/3}\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*(20*a^2*(a + b*x)^(2/3) - 16*a*(a + b*x)^(5/3) + 5*(a + b*x)^(8/3)))/(40
*b^3)
```

**fricas [A]** time = 0.76, size = 31, normalized size = 0.58

$$\frac{3\left(5\,b^2x^2-6\,abx+9\,a^2\right)(bx+a)^{\frac{2}{3}}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 - 6*a*b*x + 9*a^2)*(b*x + a)^(2/3)/b^3
```

**giac [A]** time = 0.91, size = 37, normalized size = 0.70

$$\frac{3\left(5\,(bx+a)^{\frac{8}{3}}-16\,(bx+a)^{\frac{5}{3}}a+20\,(bx+a)^{\frac{2}{3}}a^2\right)}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/40*(5*(b*x + a)^(8/3) - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)/b^
3
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.60

$$\frac{3\,(bx+a)^{\frac{2}{3}}\left(5b^2x^2-6abx+9a^2\right)}{40b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/3),x)
```

```
[Out] 3/40*(b*x+a)^(2/3)*(5*b^2*x^2-6*a*b*x+9*a^2)/b^3
```

**maxima [A]** time = 1.38, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^3}-\frac{6\,(bx+a)^{\frac{5}{3}}a}{5\,b^3}+\frac{3\,(bx+a)^{\frac{2}{3}}a^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^3 - 6/5*(b*x + a)^(5/3)*a/b^3 + 3/2*(b*x + a)^(2/3)*a
^2/b^3
```

**mupad [B]** time = 0.04, size = 37, normalized size = 0.70

$$\frac{15\,{\left(a+b\,x\right)}^{8/3}-48\,a\,{\left(a+b\,x\right)}^{5/3}+60\,a^2\,{\left(a+b\,x\right)}^{2/3}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(1/3),x)
```

```
[Out] (15*(a + b*x)^(8/3) - 48*a*(a + b*x)^(5/3) + 60*a^2*(a + b*x)^(2/3))/(40*b^
3)
```

**sympy [B]** time = 1.77, size = 600, normalized size = 11.32

$$\frac{27a^{\frac{32}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{27a^{\frac{32}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{63a^{\frac{29}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{81a^{\frac{29}{3}}bx}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{42a^{\frac{26}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{81a^{\frac{26}{3}}b^2x^2}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{18a^{\frac{23}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{27a^{\frac{23}{3}}b^3x^3}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{27a^{\frac{20}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{15a^{\frac{17}{3}}b^5x^5\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(1/3),x)
```

```
[Out] 27*a**(32/3)*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*
b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(32/3)/(40*a**8*b**3 + 120*a**7*b**4
*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a**(29/3)*b*x*(1 + b*x/a)
**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6
*x**3) - 81*a**(29/3)*b*x/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x
**2 + 40*a**5*b**6*x**3) + 42*a**(26/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a*
*8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a*
*(26/3)*b**2*x**2/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40
*a**5*b**6*x**3) + 18*a**(23/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**8*b**3
+ 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23/3)*
b**3*x**3/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b*
```

```
*6*x**3) + 27*a**(20/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a*
*7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 15*a**(17/3)*b**5*x**
5*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 +
 40*a**5*b**6*x**3)
```

## 3.394 $\int \frac{x}{\sqrt[3]{a+bx}}\,dx$

**Optimal**. Leaf size=34

$$\frac{3(a+bx)^{5/3}}{5b^2}-\frac{3a(a+bx)^{2/3}}{2b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{5/3}}{5b^2}-\frac{3a(a+bx)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x)^(2/3))/(2*b^2) + (3*(a + b*x)^(5/3))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{a+bx}}\,dx &= \int \left(-\frac{a}{b\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b}\right) dx\\ &= -\frac{3a(a+bx)^{2/3}}{2b^2}+\frac{3(a+bx)^{5/3}}{5b^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{2/3}(2bx-3a)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-3*a + 2*b*x))/(10*b^2)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 24, normalized size = 0.71

$$-\frac{3(3a-2bx)(a+bx)^{2/3}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x)^(1/3),x]
[Out] (-3*(3*a - 2*b*x)*(a + b*x)^(2/3))/(10*b^2)
```

**fricas** [A] time = 0.85, size = 20, normalized size = 0.59

$$\frac{3\,(2\,bx-3\,a)(bx+a)^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="fricas")
[Out] 3/10*(2*b*x - 3*a)*(b*x + a)^(2/3)/b^2
```

**giac** [A] time = 0.90, size = 25, normalized size = 0.74

$$\frac{3\left(2\,(bx+a)^{\frac{5}{3}}-5\,(bx+a)^{\frac{2}{3}}a\right)}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="giac")
[Out] 3/10*(2*(b*x + a)^(5/3) - 5*(b*x + a)^(2/3)*a)/b^2
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{3\,(bx+a)^{\frac{2}{3}}\,(-2bx+3a)}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/3),x)
[Out] -3/10*(b*x+a)^(2/3)*(-2*b*x+3*a)/b^2
```

**maxima** [A] time = 1.30, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b^2}-\frac{3\,(bx+a)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b^2 - 3/2*(b*x + a)^(2/3)*a/b^2
```

**mupad [B]** time = 0.03, size = 25, normalized size = 0.74

$$-\frac{15\,a\,{\left(a+b\,x\right)}^{2/3}-6\,{\left(a+b\,x\right)}^{5/3}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(1/3),x)
```

```
[Out] -(15*a*(a + b*x)^(2/3) - 6*(a + b*x)^(5/3))/(10*b^2)
```

**sympy [B]** time = 1.16, size = 162, normalized size = 4.76

$$-\frac{9a^{\frac{11}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}+\frac{9a^{\frac{11}{3}}}{10a^2b^2+10ab^3x}-\frac{3a^{\frac{8}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}+\frac{9a^{\frac{8}{3}}bx}{10a^2b^2+10ab^3x}+\frac{6a^{\frac{5}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/3),x)
```

```
[Out] -9*a**(11/3)*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x) + 9*a**(11/3)/
(10*a**2*b**2 + 10*a*b**3*x) - 3*a**(8/3)*b*x*(1 + b*x/a)**(2/3)/(10*a**2*b
**2 + 10*a*b**3*x) + 9*a**(8/3)*b*x/(10*a**2*b**2 + 10*a*b**3*x) + 6*a**(5/
3)*b**2*x**2*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x)
```

## 3.395 $\int \frac{1}{\sqrt[3]{a+bx}}\,dx$

**Optimal**. Leaf size=16

$$\frac{3(a+bx)^{2/3}}{2b}$$

**Rubi** [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{2/3}}{2b}$$

**Mathematica** [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(-1/3),x]
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

**fricas [A]** time = 0.99, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="fricas")
[Out] 3/2*(b*x + a)^(2/3)/b
```

**giac [A]** time = 0.97, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="giac")
[Out] 3/2*(b*x + a)^(2/3)/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3),x)
[Out] 3/2*(b*x+a)^(2/3)/b
```

**maxima [A]** time = 1.32, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,{\left(a+b\,x\right)}^{2/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(1/3),x)
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.75

$$\frac{3 \left(a + bx\right)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3),x)
[Out] 3*(a + b*x)**(2/3)/(2*b)
```

## 3.396 $\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$

**Optimal**. Leaf size=79

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

**Rubi [A]** time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3) -
 Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx}\right) - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} - \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

**Mathematica [A]** time = 0.01, size = 66, normalized size = 0.84

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + 2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) - \log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - Log[x] + 3*L
og[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 104, normalized size = 1.32

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2\sqrt[3]{a}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}} + \frac{\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3)
 + Log[a^(1/3) - (a + b*x)^(1/3)]/a^(1/3) - Log[a^(2/3) + a^(1/3)*(a + b*x)
^(1/3) + (a + b*x)^(2/3)]/(2*a^(1/3))
```

**fricas [A]** time = 0.87, size = 213, normalized size = 2.70

$$\left[\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a},\ \frac{2\,\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x + a)^(2/3)*a^
(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a
^(2/3) + 3*a)/x) - a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
a^(2/3)) + 2*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)))/a, 1/2*(2*sqrt(3)*a^(2
/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*log
((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x
+ a)^(1/3) - a^(1/3)))/a]
```

**giac [A]** time = 2.37, size = 77, normalized size = 0.97

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}}+\frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + log
(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3)
```

**maple [A]** time = 0.00, size = 75, normalized size = 0.95

$$\frac{\sqrt{3}\;\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}+\frac{\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\,a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/3),x)
```

```
[Out] 1/a^(1/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/2/a^(1/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a
^(1/3)+(b*x+a)^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a
^(1/3)+1))
```

**maxima** [A] time = 2.93, size = 76, normalized size = 0.96

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + log
((b*x + a)^(1/3) - a^(1/3))/a^(1/3)
```

**mupad** [B] time = 0.09, size = 99, normalized size = 1.25

$$\frac{\ln\left(9\,(a+b\,x)^{1/3}-9\,a^{1/3}\right)}{a^{1/3}}+\frac{\ln\left(9\,(a+b\,x)^{1/3}-\frac{9\,a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{1/3}}-\frac{\ln\left(9\,(a+b\,x)^{1/3}-\frac{9\,a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/3)),x)
```

```
[Out] log(9*(a + b*x)^(1/3) - 9*a^(1/3))/a^(1/3) + (log(9*(a + b*x)^(1/3) - (9*a^
(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a^(1/3)) - (log(9*(a + b*
x)^(1/3) - (9*a^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a^(1/3))
```

**sympy** [C] time = 1.88, size = 155, normalized size = 1.96

$$\frac{2\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/3),x)
```

```
[Out] 2*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(
5/3)) + 2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/
3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) + 2*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(3*a**(1
/3)*gamma(5/3))
```

## 3.397 $\int \frac{1}{x^2 \sqrt[3]{a+bx}} \, dx$

**Optimal**. Leaf size=100

$$\frac{b \log(x)}{6a^{4/3}} - \frac{b \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{2a^{4/3}} - \frac{b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\, a^{4/3}} - \frac{(a+bx)^{2/3}}{ax}$$

**Rubi [A]** time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$\frac{b \log(x)}{6a^{4/3}} - \frac{b \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{2a^{4/3}} - \frac{b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\, a^{4/3}} - \frac{(a+bx)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] -((a + b*x)^(2/3)/(a*x)) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) - (
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx = -\frac{(a+bx)^{2/3}}{ax}-\frac{b\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{3a}$$

$$= -\frac{(a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2a^{4/3}}-\frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\right.}{2a}$$

$$= -\frac{(a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{4/3}}$$

$$= -\frac{(a+bx)^{2/3}}{ax}-\frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.33

$$\frac{3b(a+bx)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{bx}{a}+1\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] (3*b*(a + b*x)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 + (b*x)/a])/(2*a^2)
```

**IntegrateAlgebraic [A]** time = 0.16, size = 128, normalized size = 1.28

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{4/3}}+\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{6a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}}-\frac{(a+bx)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] -((a + b*x)^(2/3)/(a*x)) - (b*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(4/3)) - (b*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(
4/3)) + (b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(6*a^(
4/3))
```

**fricas [A]** time = 0.89, size = 306, normalized size = 3.06

$$\left[\frac{3\sqrt{\frac{1}{3}}abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x}\right)+(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-6(bx+a)^{\frac{2}{3}}a}{6a^2x},\ -\frac{6\sqrt{\frac{1}{3}}abx\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+2(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+6(bx+a)^{\frac{2}{3}}a}{6a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2*(b*x
 + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/
a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b*x*log((b*x + a)^
(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*
x + a)^(1/3) + (-a)^(1/3)) - 6*(b*x + a)^(2/3)*a)/(a^2*x), -1/6*(6*sqrt(1/3
)*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) - (-a)^(1/3
))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x*log((b*x + a)^(2/3) - (b*x + a)^(1
/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)^(2/3)*b*x*log((b*x + a)^(1/3) + (-a)^
(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*x)]
```

**giac [A]** time = 2.45, size = 109, normalized size = 1.09

$$-\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{6(bx+a)^{\frac{2}{3}}b}{ax}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(4/3) - b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(4/3) + 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*x + a)^(2
/3)*b/(a*x))/b
```

**maple [A]** time = 0.01, size = 95, normalized size = 0.95

$$-\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{4}{3}}}-\frac{b\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}+\frac{b\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}-\frac{(bx+a)^{\frac{2}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/3),x)
```

```
[Out] -(b*x+a)^(2/3)/a/x-1/3*b/a^(4/3)*ln(-a^(1/3)+(b*x+a)^(1/3))+1/6*b/a^(4/3)*l
n(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))-1/3*b/a^(4/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

**maxima [A]** time = 3.00, size = 106, normalized size = 1.06

$$-\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}}-\frac{(bx+a)^{\frac{2}{3}}b}{(bx+a)a-a^2}+\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}}-\frac{b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(4/3) - (b*x + a)^(2/3)*b/((b*x + a)*a - a^2) + 1/6*b*log((b*x + a)^(2/3) +
 (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/3*b*log((b*x + a)^(1/3) - a
^(1/3))/a^(4/3)
```

**mupad [B]** time = 0.14, size = 130, normalized size = 1.30

$$-\frac{(a+b\,x)^{2/3}}{a\,x}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}-\frac{b\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{3\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(1/3)),x)
```

```
[Out] (log((b - 3^(1/2)*b*1i)^2/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b - 3^(
1/2)*b*1i))/(6*a^(4/3)) - (a + b*x)^(2/3)/(a*x) + (log((b + 3^(1/2)*b*1i)^2
/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b + 3^(1/2)*b*1i))/(6*a^(4/3)) -
 (b*log((a + b*x)^(1/3) - a^(1/3)))/(3*a^(4/3))
```

**sympy** **[C]** time = 2.20, size = 831, normalized size = 8.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/3),x)
```

```
[Out] -2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*p
i/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)
) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a
/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b*
*(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x
)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**
3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7
/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a*
*(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/
3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3
)*b**(10/3)*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*
I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) +
 6*a*b**3*(a/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)*
*(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3))
```

## 3.398 $\int \frac{1}{x^3 \sqrt[3]{a+bx}} \, dx$

**Optimal**. Leaf size=130

$$-\frac{b^2 \log(x)}{9a^{7/3}}+\frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}+\frac{2b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{2b(a+bx)^{2/3}}{3a^2x}-\frac{(a+bx)^{2/3}}{2ax^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$-\frac{b^2 \log(x)}{9a^{7/3}}+\frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}+\frac{2b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{2b(a+bx)^{2/3}}{3a^2x}-\frac{(a+bx)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] -(a + b*x)^(2/3)/(2*a*x^2) + (2*b*(a + b*x)^(2/3))/(3*a^2*x) + (2*b^2*ArcTa
n[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) - (
b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx &= -\frac{(a+bx)^{2/3}}{2ax^2} - \frac{(2b)\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{3a} \\
&= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} + \frac{\left(2b^2\right)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a^2} \\
&= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} - \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{3a^{7/3}} + \frac{b^2\,\mathrm{Subst}\left(\int \right.}{} \\
&= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}} - \frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\right.}{3a^{7/3}} \\
&= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} + \frac{2b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.27

$$-\frac{3b^2(a+bx)^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};\frac{bx}{a}+1\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] (-3*b^2*(a + b*x)^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, 1 + (b*x)/a])/(2*a^3
)
```

**IntegrateAlgebraic [A]** time = 0.15, size = 149, normalized size = 1.15

$$\frac{2b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{7/3}}-\frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9a^{7/3}}+\frac{2b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}a^{7/3}}-\frac{(a+bx)^{2/3}(7a-4(a+bx))}{6a^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] -1/6*((a + b*x)^(2/3)*(7*a - 4*(a + b*x)))/(a^2*x^2) + (2*b^2*ArcTan[1/Sqrt
[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) + (2*b^2*
Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(7/3)) - (b^2*Log[a^(2/3) + a^(1/3)*(a
 + b*x)^(1/3) + (a + b*x)^(2/3)])/(9*a^(7/3))
```

**fricas [A]** time = 0.96, size = 296, normalized size = 2.28

$$\left[\frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-2a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\left(4abx-3a^2\right)(bx+a)^{\frac{2}{3}}}{18a^3x^2},\ \frac{12\sqrt{\frac{1}{3}}a^{\frac{2}{3}}b^2x^2\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-2a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\left(4abx-3a^2\right)(bx+a)^{\frac{2}{3}}}{18a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(
b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*
(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) +
 (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3)
 - a^(1/3)) + 3*(4*a*b*x - 3*a^2)*(b*x + a)^(2/3))/(a^3*x^2), 1/18*(12*sqrt
(1/3)*a^(2/3)*b^2*x^2*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
)) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2
/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3) - a^(1/3)) + 3*(4*a*b*x - 3*a^
2)*(b*x + a)^(2/3))/(a^3*x^2)]
```

**giac [A]** time = 2.24, size = 130, normalized size = 1.00

$$\frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{2b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}+\frac{4b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}}+\frac{3\left(4(bx+a)^{\frac{5}{3}}b^3-7(bx+a)^{\frac{2}{3}}ab^3\right)}{a^2b^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(7/3) - 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(7/3) + 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4*(b*x +
a)^(5/3)*b^3 - 7*(b*x + a)^(2/3)*a*b^3)/(a^2*b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 117, normalized size = 0.90

$$\frac{2\sqrt{3}\, b^{2} \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{7}{3}}}+\frac{2b^{2} \ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}-\frac{b^{2} \ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\, a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2\,(bx+a)^{\frac{2}{3}}\, b}{3a^{2}x}-\frac{(bx+a)^{\frac{2}{3}}}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(1/3),x)
```

```
[Out] -1/2*(b*x+a)^(2/3)/a/x^2+2/3*b*(b*x+a)^(2/3)/a^2/x+2/9*b^2/a^(7/3)*ln(-a^(1
/3)+(b*x+a)^(1/3))-1/9*b^2/a^(7/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)
^(2/3))+2/9*b^2/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)
+1))
```

**maxima [A]** time = 3.03, size = 142, normalized size = 1.09

$$\frac{2\,\sqrt{3}\, b^{2} \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}}-\frac{b^{2} \log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{7}{3}}}+\frac{2\,b^{2} \log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{7}{3}}}+\frac{4\,(bx+a)^{\frac{5}{3}} b^{2}-7\,(bx+a)^{\frac{2}{3}} ab^{2}}{6\left((bx+a)^{2} a^{2}-2\,(bx+a)a^{3}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(7/3) - 1/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(7/3) + 2/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(7/3) + 1/6*(4*(b*x + a)^
(5/3)*b^2 - 7*(b*x + a)^(2/3)*a*b^2)/((b*x + a)^2*a^2 - 2*(b*x + a)*a^3 + a
^4)
```

**mupad [B]** time = 0.23, size = 182, normalized size = 1.40

$$\frac{2\,b^{2}\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{9\,a^{7/3}}-\frac{\frac{7\,b^{2}\,(a+b\,x)^{2/3}}{6\,a}-\frac{2\,b^{2}\,(a+b\,x)^{5/3}}{3\,a^{2}}}{(a+b\,x)^{2}-2\,a\,(a+b\,x)+a^{2}}-\frac{\ln\left(\frac{4\,b^{4}\,(a+b\,x)^{1/3}}{9\,a^{4}}-\frac{\left(b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}\right)^{2}}{9\,a^{11/3}}\right)\left(b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}\right)}{9\,a^{7/3}}+\frac{b^{2}\,\ln\left(\frac{4\,b^{4}\,(a+b\,x)^{1/3}}{9\,a^{4}}-\frac{9\,b^{4}\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^{2}}{a^{11/3}}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)}{a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(1/3)),x)
```

```
[Out] (2*b^2*log((a + b*x)^(1/3) - a^(1/3)))/(9*a^(7/3)) - ((7*b^2*(a + b*x)^(2/3
))/(6*a) - (2*b^2*(a + b*x)^(5/3))/(3*a^2))/((a + b*x)^2 - 2*a*(a + b*x) +
a^2) - (log((4*b^4*(a + b*x)^(1/3))/(9*a^4) - (3^(1/2)*b^2*1i + b^2)^2/(9*a
^(11/3)))*(3^(1/2)*b^2*1i + b^2))/(9*a^(7/3)) + (b^2*log((4*b^4*(a + b*x)^(
1/3))/(9*a^4) - (9*b^4*((3^(1/2)*1i)/9 - 1/9)^2)/a^(11/3))*((3^(1/2)*1i)/9
- 1/9))/a^(7/3)
```

**sympy [C]** time = 2.59, size = 2730, normalized size = 21.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(1/3),x)
```

```
[Out] 4*a**(14/3)*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I
*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5
/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**
4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b**(1
0/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b
**(10/3)*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 8
1*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(1
3/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(
a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*g
amma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*
a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)
*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**
(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(7/3)*
exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/
3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3)
- 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(
10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b +
 x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(
7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*
b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b
 + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3
)*exp(2*I*pi/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(2
```

```
*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b*
*(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b +
x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*ex
p(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*
gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(-2*I*pi/3)*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a*
*7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a
/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*p
i/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3
)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(
7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I
*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 4*a**(5/3)*b**(19/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3
)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3
)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3)
 - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/
3)*b**(19/3)*(a/b + x)**(13/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(
5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a*
*(5/3)*b**(19/3)*(a/b + x)**(13/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma
(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a
**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 21*a**4*b**4*(a
/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(
2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamm
a(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*
a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 33*a**3*b**5*(
a/b + x)**3*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**2*b**6*
(a/b + x)**4*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*ex
p(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 2
7*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
```

## 3.399 $\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx$

**Optimal**. Leaf size=80

$$\frac{3a^3(bx-a)^{2/3}}{2b^4}+\frac{9a^2(bx-a)^{5/3}}{5b^4}+\frac{3(bx-a)^{11/3}}{11b^4}+\frac{9a(bx-a)^{8/3}}{8b^4}$$

**Rubi** [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{9a^2(bx-a)^{5/3}}{5b^4}+\frac{3a^3(bx-a)^{2/3}}{2b^4}+\frac{3(bx-a)^{11/3}}{11b^4}+\frac{9a(bx-a)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a^3*(-a + b*x)^(2/3))/(2*b^4) + (9*a^2*(-a + b*x)^(5/3))/(5*b^4) + (9*a*
(-a + b*x)^(8/3))/(8*b^4) + (3*(-a + b*x)^(11/3))/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx &= \int \left(\frac{a^3}{b^3\sqrt[3]{-a+bx}}+\frac{3a^2(-a+bx)^{2/3}}{b^3}+\frac{3a(-a+bx)^{5/3}}{b^3}+\frac{(-a+bx)^{8/3}}{b^3}\right)dx\\ &= \frac{3a^3(-a+bx)^{2/3}}{2b^4}+\frac{9a^2(-a+bx)^{5/3}}{5b^4}+\frac{9a(-a+bx)^{8/3}}{8b^4}+\frac{3(-a+bx)^{11/3}}{11b^4}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 48, normalized size = 0.60

$$\frac{3(bx-a)^{2/3}\left(81a^3+54a^2bx+45ab^2x^2+40b^3x^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(81*a^3 + 54*a^2*b*x + 45*a*b^2*x^2 + 40*b^3*x^3))/(440
*b^4)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 59, normalized size = 0.74

$$\frac{3(bx-a)^{2/3}\left(220a^3+264a^2(bx-a)+165a(bx-a)^2+40(bx-a)^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(220*a^3 + 264*a^2*(-a + b*x) + 165*a*(-a + b*x)^2 + 40
*(-a + b*x)^3))/(440*b^4)
```

**fricas [A]** time = 0.90, size = 44, normalized size = 0.55

$$\frac{3\left(40\,b^3x^3+45\,ab^2x^2+54\,a^2bx+81\,a^3\right)(bx-a)^{\frac{2}{3}}}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 + 45*a*b^2*x^2 + 54*a^2*b*x + 81*a^3)*(b*x - a)^(2/3)/b^4
```

**giac [A]** time = 1.07, size = 57, normalized size = 0.71

$$\frac{3\left(40\,(bx-a)^{\frac{11}{3}}+165\,(bx-a)^{\frac{8}{3}}a+264\,(bx-a)^{\frac{5}{3}}a^2+220\,(bx-a)^{\frac{2}{3}}a^3\right)}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/440*(40*(b*x - a)^(11/3) + 165*(b*x - a)^(8/3)*a + 264*(b*x - a)^(5/3)*a^
2 + 220*(b*x - a)^(2/3)*a^3)/b^4
```

**maple [A]** time = 0.00, size = 45, normalized size = 0.56

$$\frac{3\left(40b^3x^3+45a\,b^2x^2+54a^2bx+81a^3\right)(bx-a)^{\frac{2}{3}}}{440b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x-a)^(1/3),x)
```

```
[Out] 3/440*(40*b^3*x^3+45*a*b^2*x^2+54*a^2*b*x+81*a^3)/b^4*(b*x-a)^(2/3)
```

**maxima [A]** time = 1.34, size = 64, normalized size = 0.80

$$\frac{3\,(bx-a)^{\frac{11}{3}}}{11\,b^4}+\frac{9\,(bx-a)^{\frac{8}{3}}a}{8\,b^4}+\frac{9\,(bx-a)^{\frac{5}{3}}a^2}{5\,b^4}+\frac{3\,(bx-a)^{\frac{2}{3}}a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x - a)^(11/3)/b^4 + 9/8*(b*x - a)^(8/3)*a/b^4 + 9/5*(b*x - a)^(5/3)
*a^2/b^4 + 3/2*(b*x - a)^(2/3)*a^3/b^4
```

**mupad [B]** time = 0.05, size = 64, normalized size = 0.80

$$\frac{3\,(b\,x-a)^{11/3}}{11\,b^4}+\frac{9\,a\,(b\,x-a)^{8/3}}{8\,b^4}+\frac{3\,a^3\,(b\,x-a)^{2/3}}{2\,b^4}+\frac{9\,a^2\,(b\,x-a)^{5/3}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x - a)^(1/3),x)
```

```
[Out] (3*(b*x - a)^(11/3))/(11*b^4) + (9*a*(b*x - a)^(8/3))/(8*b^4) + (3*a^3*(b*x
 - a)^(2/3))/(2*b^4) + (9*a^2*(b*x - a)^(5/3))/(5*b^4)
```

**sympy [C]** time = 2.98, size = 4974, normalized size = 62.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((243*a**(71/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*ex
p(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3
) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2
640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 243*a
**(71/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600
*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**1
6*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**1
0*x**6*exp(I*pi/3)) - 1296*a**(68/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(4
40*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6
*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4
*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(
I*pi/3)) - 1458*a**(68/3)*b*x/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5
```

```
*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*ex
p(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*p
i/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 2808*a**(65/3)*b**2*x**2*(-1 + b
*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*ex
p(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*p
i/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3)
 + 440*a**14*b**10*x**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b
**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(
I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/
3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) -
 3120*a**(62/3)*b**3*x**3*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*e
xp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/
3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) -
2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 4860
*a**(62/3)*b**3*x**3/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*
pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3)
 + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 44
0*a**14*b**10*x**6*exp(I*pi/3)) + 1710*a**(59/3)*b**4*x**4*(-1 + b*x/a)**(2
/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3)
 + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 66
00*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**
14*b**10*x**6*exp(I*pi/3)) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4*exp(I
*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) -
 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640
*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 72*a**(5
6/3)*b**5*x**5*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3)
 - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a
**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*
b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 1458*a**(56/3)*
b**5*x**5/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 660
0*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**
16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**
10*x**6*exp(I*pi/3)) - 1104*a**(53/3)*b**6*x**6*(-1 + b*x/a)**(2/3)*exp(I*p
i/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**
18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b*
*8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x*
*6*exp(I*pi/3)) + 243*a**(53/3)*b**6*x**6/(440*a**20*b**4*exp(I*pi/3) - 264
0*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*
b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*
x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 1152*a**(50/3)*b**7*
x**7*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**
19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*
x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*
exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 585*a**(47/3)*b**8*x**8*(
-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**
```

5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*e
xp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*
pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 120*a**(44/3)*b**9*x**9*(-1 + b
*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*ex
p(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*p
i/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3)
+ 440*a**14*b**10*x**6*exp(I*pi/3)), Abs(b*x/a) > 1), (-243*a**(71/3)*(1 -
b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) +
6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600
*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14
*b**10*x**6*exp(I*pi/3)) + 243*a**(71/3)/(440*a**20*b**4*exp(I*pi/3) - 2640
*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b
**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x
**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 1296*a**(68/3)*b*x*(1
- b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3
) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6
600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a*
*14*b**10*x**6*exp(I*pi/3)) - 1458*a**(68/3)*b*x/(440*a**20*b**4*exp(I*pi/3
) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800
*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**1
5*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 2808*a**(65/3
)*b**2*x**2*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**
5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*e
xp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*
pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*x**2/(440*a
**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**
2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp
(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi
/3)) + 3120*a**(62/3)*b**3*x**3*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi
/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 88
00*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a*
*15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 4860*a**(62
/3)*b**3*x**3/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) +
6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600
*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14
*b**10*x**6*exp(I*pi/3)) - 1710*a**(59/3)*b**4*x**4*(1 - b*x/a)**(2/3)/(440
*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x
**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*e
xp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*
pi/3)) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*
b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**
3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp
(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 72*a**(56/3)*b**5*x**5*(1 -
b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) +
6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600

```
*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14
*b**10*x**6*exp(I*pi/3)) - 1458*a**(56/3)*b**5*x**5/(440*a**20*b**4*exp(I*p
i/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8
800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a
**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 1104*a**(5
3/3)*b**6*x**6*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*
b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**
3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp
(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 243*a**(53/3)*b**6*x**6/(440
*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x
**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*e
xp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*
pi/3)) - 1152*a**(50/3)*b**7*x**7*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*
pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) -
8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*
a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 585*a**(4
7/3)*b**8*x**8*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*
b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**
3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp
(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 120*a**(44/3)*b**9*x**9*(1 -
 b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3)
+ 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 660
0*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**1
4*b**10*x**6*exp(I*pi/3)), True))
```

## 3.400 $\int \frac{x^2}{\sqrt[3]{-a+bx}} \, dx$

**Optimal**. Leaf size=59

$$\frac{3a^2(bx-a)^{2/3}}{2b^3} + \frac{3(bx-a)^{8/3}}{8b^3} + \frac{6a(bx-a)^{5/3}}{5b^3}$$

**Rubi [A]** time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3a^2(bx-a)^{2/3}}{2b^3} + \frac{3(bx-a)^{8/3}}{8b^3} + \frac{6a(bx-a)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(-a + b*x)^(2/3))/(2*b^3) + (6*a*(-a + b*x)^(5/3))/(5*b^3) + (3*(-a
+ b*x)^(8/3))/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt[3]{-a+bx}} \, dx &= \int \left(\frac{a^2}{b^2\sqrt[3]{-a+bx}} + \frac{2a(-a+bx)^{2/3}}{b^2} + \frac{(-a+bx)^{5/3}}{b^2}\right) dx \\
&= \frac{3a^2(-a+bx)^{2/3}}{2b^3} + \frac{6a(-a+bx)^{5/3}}{5b^3} + \frac{3(-a+bx)^{8/3}}{8b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 37, normalized size = 0.63

$$\frac{3(bx-a)^{2/3}\left(9a^2+6abx+5b^2x^2\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-a + b*x)^(1/3),x]
[Out] (3*(-a + b*x)^(2/3)*(9*a^2 + 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 51, normalized size = 0.86

$$\frac{3\left(20a^2(bx-a)^{2/3}+5(bx-a)^{8/3}+16a(bx-a)^{5/3}\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(-a + b*x)^(1/3),x]
[Out] (3*(20*a^2*(-a + b*x)^(2/3) + 16*a*(-a + b*x)^(5/3) + 5*(-a + b*x)^(8/3)))/
(40*b^3)
```

**fricas [A]** time = 0.94, size = 33, normalized size = 0.56

$$\frac{3\left(5\,b^2x^2+6\,abx+9\,a^2\right)(bx-a)^{\frac{2}{3}}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="fricas")
[Out] 3/40*(5*b^2*x^2 + 6*a*b*x + 9*a^2)*(b*x - a)^(2/3)/b^3
```

**giac [A]** time = 1.07, size = 43, normalized size = 0.73

$$\frac{3\left(5\,(bx-a)^{\frac{8}{3}}+16\,(bx-a)^{\frac{5}{3}}a+20\,(bx-a)^{\frac{2}{3}}a^2\right)}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="giac")
[Out] 3/40*(5*(b*x - a)^(8/3) + 16*(b*x - a)^(5/3)*a + 20*(b*x - a)^(2/3)*a^2)/b^
3
```

**maple [A]** time = 0.00, size = 34, normalized size = 0.58

$$\frac{3\left(5b^2x^2+6abx+9a^2\right)(bx-a)^{\frac{2}{3}}}{40b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x-a)^(1/3),x)
```

```
[Out] 3/40*(5*b^2*x^2+6*a*b*x+9*a^2)/b^3*(b*x-a)^(2/3)
```

**maxima** [A] time = 1.33, size = 47, normalized size = 0.80

$$\frac{3\,(bx-a)^{\frac{8}{3}}}{8\,b^3}+\frac{6\,(bx-a)^{\frac{5}{3}}a}{5\,b^3}+\frac{3\,(bx-a)^{\frac{2}{3}}a^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x - a)^(8/3)/b^3 + 6/5*(b*x - a)^(5/3)*a/b^3 + 3/2*(b*x - a)^(2/3)*a
^2/b^3
```

**mupad** [B] time = 0.04, size = 43, normalized size = 0.73

$$\frac{48\,a\,(b\,x-a)^{5/3}+15\,(b\,x-a)^{8/3}+60\,a^2\,(b\,x-a)^{2/3}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x - a)^(1/3),x)
```

```
[Out] (48*a*(b*x - a)^(5/3) + 15*(b*x - a)^(8/3) + 60*a^2*(b*x - a)^(2/3))/(40*b^
3)
```

**sympy** [C] time = 1.90, size = 1326, normalized size = 22.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((-27*a**(32/3)*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4
*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)/(
-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) +
 63*a**(29/3)*b*x*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 12
0*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x*exp(2*I*pi/3)/(-40
*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 42
*a**(26/3)*b**2*x**2*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x -
 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 81*a**(26/3)*b**2*x**2*exp(2*I*p
i/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x
**3) + 18*a**(23/3)*b**3*x**3*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7
*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23/3)*b**3*x**3*
exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a*
```

```
*5*b**6*x**3) - 27*a**(20/3)*b**4*x**4*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 +
 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 15*a**(17/3)*b
**5*x**5*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b*
*5*x**2 + 40*a**5*b**6*x**3), Abs(b*x/a) > 1), (-27*a**(32/3)*(1 - b*x/a)**
(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 +
 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*
b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a**(29/3)*b*x*(1 - b*
x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*
x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x*exp(2*I*pi/3)/(-40*a**8*b**3 +
 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 42*a**(26/3)*b
**2*x**2*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x
- 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 81*a**(26/3)*b**2*x**2*exp(2*I*
pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*
x**3) + 18*a**(23/3)*b**3*x**3*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b
**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23
/3)*b**3*x**3*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**
5*x**2 + 40*a**5*b**6*x**3) - 27*a**(20/3)*b**4*x**4*(1 - b*x/a)**(2/3)*exp
(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*
b**6*x**3) + 15*a**(17/3)*b**5*x**5*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a
**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3), True)
)
```

## 3.401 $\int \frac{x}{\sqrt[3]{-a+bx}} dx$

**Optimal**. Leaf size=38

$$\frac{3(bx-a)^{5/3}}{5b^2}+\frac{3a(bx-a)^{2/3}}{2b^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3(bx-a)^{5/3}}{5b^2}+\frac{3a(bx-a)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a*(-a + b*x)^(2/3))/(2*b^2) + (3*(-a + b*x)^(5/3))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{-a+bx}} dx &= \int \left(\frac{a}{b\sqrt[3]{-a+bx}}+\frac{(-a+bx)^{2/3}}{b}\right) dx\\ &= \frac{3a(-a+bx)^{2/3}}{2b^2}+\frac{3(-a+bx)^{5/3}}{5b^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 26, normalized size = 0.68

$$\frac{3(bx-a)^{2/3}(3a+2bx)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(3*a + 2*b*x))/(10*b^2)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 26, normalized size = 0.68

$$\frac{3(bx-a)^{2/3}(3a+2bx)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(-a + b*x)^(1/3),x]
[Out] (3*(-a + b*x)^(2/3)*(3*a + 2*b*x))/(10*b^2)
```

**fricas** [A] time = 0.91, size = 22, normalized size = 0.58

$$\frac{3\,(2\,bx+3\,a)(bx-a)^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="fricas")
[Out] 3/10*(2*b*x + 3*a)*(b*x - a)^(2/3)/b^2
```

**giac** [A] time = 0.98, size = 29, normalized size = 0.76

$$\frac{3\left(2\,(bx-a)^{\frac{5}{3}}+5\,(bx-a)^{\frac{2}{3}}a\right)}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="giac")
[Out] 3/10*(2*(b*x - a)^(5/3) + 5*(b*x - a)^(2/3)*a)/b^2
```

**maple** [A] time = 0.00, size = 23, normalized size = 0.61

$$\frac{3\,(2bx+3a)\,(bx-a)^{\frac{2}{3}}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x-a)^(1/3),x)
[Out] 3/10*(2*b*x+3*a)/b^2*(b*x-a)^(2/3)
```

**maxima** [A] time = 1.34, size = 30, normalized size = 0.79

$$\frac{3\,(bx-a)^{\frac{5}{3}}}{5\,b^2}+\frac{3\,(bx-a)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x - a)^(5/3)/b^2 + 3/2*(b*x - a)^(2/3)*a/b^2
```

**mupad [B]** time = 0.03, size = 29, normalized size = 0.76

$$\frac{15\,a\,(b\,x-a)^{2/3}+6\,(b\,x-a)^{5/3}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x - a)^(1/3),x)
```

```
[Out] (15*a*(b*x - a)^(2/3) + 6*(b*x - a)^(5/3))/(10*b^2)
```

**sympy [C]** time = 1.26, size = 486, normalized size = 12.79

$$\begin{cases} -\frac{9a^{\frac{11}{3}}\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{3a^{\frac{8}{3}}bx\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{6a^{\frac{5}{3}}b^2x^2\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ \frac{9a^{\frac{11}{3}}\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{3a^{\frac{8}{3}}bx\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{6a^{\frac{5}{3}}b^2x^2\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((-9*a**(11/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp(
I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**2*exp(I*pi/3)
 + 10*a*b**3*x*exp(I*pi/3)) + 3*a**(8/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*pi/3
)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*x/(-
10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 6*a**(5/3)*b**2*x**2*
(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*ex
p(I*pi/3)), Abs(b*x/a) > 1), (9*a**(11/3)*(1 - b*x/a)**(2/3)/(-10*a**2*b**2
*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**2*exp(I*
pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 3*a**(8/3)*b*x*(1 - b*x/a)**(2/3)/(-10*a
**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*x/(-10*a**2*
b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 6*a**(5/3)*b**2*x**2*(1 - b*x
/a)**(2/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)), True))
```

## 3.402 $\int \frac{1}{\sqrt[3]{-a+bx}} dx$

**Optimal**. Leaf size=18

$$\frac{3(bx-a)^{2/3}}{2b}$$

**Rubi** [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {32}

$$\frac{3(bx-a)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(-1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-a+bx}} dx = \frac{3(-a+bx)^{2/3}}{2b}$$

**Mathematica** [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3(bx-a)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(-1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{3(bx-a)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(-1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

**fricas** [A] time = 0.71, size = 14, normalized size = 0.78

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x - a)^(2/3)/b
```

**giac** [A] time = 1.02, size = 14, normalized size = 0.78

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*(b*x - a)^(2/3)/b
```

**maple** [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^(1/3),x)
```

```
[Out] 3/2*(b*x-a)^(2/3)/b
```

**maxima** [A] time = 1.32, size = 14, normalized size = 0.78

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x - a)^(2/3)/b
```

**mupad [B]** time = 0.02, size = 14, normalized size = 0.78

$$\frac{3\,{\left(b\,x-a\right)}^{2/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x - a)^(1/3),x)
[Out] (3*(b*x - a)^(2/3))/(2*b)
```

**sympy [A]** time = 0.07, size = 12, normalized size = 0.67

$$\frac{3 \left(-a + bx\right)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**(1/3),x)
[Out] 3*(-a + b*x)**(2/3)/(2*b)
```

## 3.403 $\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx$

**Optimal**. Leaf size=82

$$-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

**Rubi [A]** time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3
)) + Log[x]/(2*a^(1/3)) - (3*Log[a^(1/3) + (-a + b*x)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{-a+bx}\right) - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x, \sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}}$$

$$= \frac{\log(x)}{2\sqrt[3]{a}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{\log(x)}{2\sqrt[3]{a}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.43

$$\frac{3(bx-a)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-\frac{bx}{a}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(1/3)),x]
[Out] (3*(-a + b*x)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - (b*x)/a])/(2*a)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 113, normalized size = 1.38

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx-a}+(bx-a)^{2/3}\right)}{2\sqrt[3]{a}} - \frac{\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{\sqrt[3]{a}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a + b*x)^(1/3)),x]
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1
/3)) - Log[a^(1/3) + (-a + b*x)^(1/3)]/a^(1/3) + Log[a^(2/3) - a^(1/3)*(-a
+ b*x)^(1/3) + (-a + b*x)^(2/3)]/(2*a^(1/3))
```

**fricas** [A] time = 0.94, size = 285, normalized size = 3.48

$$\left[\frac{\sqrt{3}\,a\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3\,a}{x}\right)+(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)}{2\,a},\ \frac{2\,\sqrt{3}\,a\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*sqrt((-a)^(1/3)/a)*log((2*b*x + sqrt(3)*(2*(b*x - a)^(2/3)*
(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x
- a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a
)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b*x - a)^(1/3) - (-a)^
(1/3)))/a, 1/2*(2*sqrt(3)*a*sqrt(-(-a)^(1/3)/a)*arctan(1/3*sqrt(3)*(2*(b*x
- a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*log((b*x - a)^(2
/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b*x - a)
^(1/3) - (-a)^(1/3)))/a]
```

**giac** [A] time = 2.51, size = 112, normalized size = 1.37

$$-\frac{\sqrt{3}\,(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{a}+\frac{(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{2\,a}-\frac{(-a)^{\frac{2}{3}}\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*(-a)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a
)^(1/3))/a + 1/2*(-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3
) + (-a)^(2/3))/a - (-a)^(2/3)*log(abs((b*x - a)^(1/3) - (-a)^(1/3)))/a
```

**maple** [A] time = 0.01, size = 83, normalized size = 1.01

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}+\frac{\ln\left(a^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}\,a^{\frac{1}{3}}+(bx-a)^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(1/3),x)
```

```
[Out] -ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)+1/2/a^(1/3)*ln((b*x-a)^(2/3)-a^(1/3)*(b*
x-a)^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x-a)^(
1/3)-1))
```

**maxima [A]** time = 3.01, size = 86, normalized size = 1.05

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}+\frac{\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}}-\frac{\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a^(1/3) +
 1/2*log((b*x - a)^(2/3) - (b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) - log
((b*x - a)^(1/3) + a^(1/3))/a^(1/3)
```

**mupad [B]** time = 0.09, size = 117, normalized size = 1.43

$$\frac{\ln\left(9\,(b\,x-a)^{1/3}-9\,(-a)^{1/3}\right)}{(-a)^{1/3}}+\frac{\ln\left(9\,(b\,x-a)^{1/3}-\frac{9\,(-a)^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,(-a)^{1/3}}-\frac{\ln\left(9\,(b\,x-a)^{1/3}-\frac{9\,(-a)^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,(-a)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(1/3)),x)
```

```
[Out] log(9*(b*x - a)^(1/3) - 9*(-a)^(1/3))/(-a)^(1/3) + (log(9*(b*x - a)^(1/3) -
 (9*(-a)^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*(-a)^(1/3)) - (l
og(9*(b*x - a)^(1/3) - (9*(-a)^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1
))/(2*(-a)^(1/3))
```

**sympy [C]** time = 1.88, size = 160, normalized size = 1.95

$$-\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(1/3),x)
```

```
[Out] -2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(
1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*log(1 - b**(1/3)*(-a/b + x)**(
```

```
1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*exp(2
*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*g
amma(2/3)/(3*a**(1/3)*gamma(5/3))
```

## 3.404 $\int \frac{1}{x^2 \sqrt[3]{-a+bx}} dx$

**Optimal**. Leaf size=103

$$\frac{b \log(x)}{6a^{4/3}} - \frac{b \log\left(\sqrt[3]{bx-a} + \sqrt[3]{a}\right)}{2a^{4/3}} - \frac{b \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\, a^{4/3}} + \frac{(bx-a)^{2/3}}{ax}$$

**Rubi [A]** time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {51, 56, 617, 204, 31}

$$\frac{b \log(x)}{6a^{4/3}} - \frac{b \log\left(\sqrt[3]{bx-a} + \sqrt[3]{a}\right)}{2a^{4/3}} - \frac{b \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\, a^{4/3}} + \frac{(bx-a)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(a*x) - (b*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) + (-
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx &= \frac{(-a+bx)^{2/3}}{ax}+\frac{b\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx}{3a}\\
&= \frac{(-a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx,x,\sqrt[3]{-a+bx}\right)}{2a^{4/3}}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,x+x^2}\,dx\right.}{2a}\\
&= \frac{(-a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}\right)}{a^{4/3}}\\
&= \frac{(-a+bx)^{2/3}}{ax}-\frac{b\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 36, normalized size = 0.35

$$\frac{3b(bx-a)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};1-\frac{bx}{a}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(1/3)),x]
```

```
[Out] (3*b*(-a + b*x)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 - (b*x)/a])/(2*a^2)
```

**IntegrateAlgebraic [A]** time = 0.14, size = 136, normalized size = 1.32

$$-\frac{b\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{3a^{4/3}}+\frac{b\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{bx-a}+(bx-a)^{2/3}\right)}{6a^{4/3}}-\frac{b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{bx-a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{(bx-a)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(a*x) - (b*ArcTan[1/Sqrt[3] - (2*(-a + b*x)^(1/3))/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(4/3)) - (b*Log[a^(1/3) + (-a + b*x)^(1/3)])/(3*a^(
4/3)) + (b*Log[a^(2/3) - a^(1/3)*(-a + b*x)^(1/3) + (-a + b*x)^(2/3)])/(6*a
^(4/3))
```

**fricas [A]** time = 1.06, size = 328, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3a}{x}\right)+(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+6(bx-a)^{\frac{2}{3}}a}{6a^2x},\ \frac{6\sqrt{\frac{1}{3}}\,abx\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+6(bx-a)^{\frac{2}{3}}a}{6a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(2*(b*x
 - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/
a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*b*x*log((b*x - a)^
(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*
x - a)^(1/3) - (-a)^(1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x), 1/6*(6*sqrt(1/3)
*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x - a)^(1/3) + (-a)^(1/3)
)*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*b*x*log((b*x - a)^(2/3) + (b*x - a)^(1/
3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*x - a)^(1/3) - (-a)^(
1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x)]
```

**giac [A]** time = 2.43, size = 144, normalized size = 1.40

$$\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3(-a)^{\frac{1}{3}}}\right)}{(-a)^{\frac{1}{3}}a}-\frac{b^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{(-a)^{\frac{1}{3}}a}-\frac{2(-a)^{\frac{2}{3}}b^2\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{6(bx-a)^{\frac{2}{3}}b}{ax}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a)
^(1/3))/((-a)^(1/3)*a) - b^2*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/
3) + (-a)^(2/3))/((-a)^(1/3)*a) - 2*(-a)^(2/3)*b^2*log(abs((b*x - a)^(1/3)
- (-a)^(1/3)))/a^2 + 6*(b*x - a)^(2/3)*b/(a*x))/b
```

**maple [A]** time = 0.01, size = 103, normalized size = 1.00

$$\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{4}{3}}}-\frac{b\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}+\frac{b\ln\left(a^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx-a)^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}+\frac{(bx-a)^{\frac{2}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(1/3),x)
```

```
[Out] (b*x-a)^(2/3)/a/x-1/3*b*ln(a^(1/3)+(b*x-a)^(1/3))/a^(4/3)+1/6*b/a^(4/3)*ln(
a^(2/3)-(b*x-a)^(1/3)*a^(1/3)+(b*x-a)^(2/3))+1/3*b/a^(4/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2*(b*x-a)^(1/3)/a^(1/3)-1))
```

**maxima [A]** time = 2.99, size = 116, normalized size = 1.13

$$\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}}+\frac{(bx-a)^{\frac{2}{3}}b}{(bx-a)a+a^2}+\frac{b\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}}-\frac{b\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a^(
4/3) + (b*x - a)^(2/3)*b/((b*x - a)*a + a^2) + 1/6*b*log((b*x - a)^(2/3) -
(b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/3*b*log((b*x - a)^(1/3) + a^
(1/3))/a^(4/3)
```

**mupad [B]** time = 0.18, size = 133, normalized size = 1.29

$$\frac{(b\,x-a)^{2/3}}{a\,x}-\frac{b\,\ln\left((b\,x-a)^{1/3}+a^{1/3}\right)}{3\,a^{4/3}}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}+\frac{b^2\,(b\,x-a)^{1/3}}{a^2}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}+\frac{b^2\,(b\,x-a)^{1/3}}{a^2}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(1/3)),x)
```

```
[Out] (b*x - a)^(2/3)/(a*x) - (b*log((b*x - a)^(1/3) + a^(1/3)))/(3*a^(4/3)) + (l
og((b - 3^(1/2)*b*1i)^2/(4*a^(5/3)) + (b^2*(b*x - a)^(1/3))/a^2)*(b - 3^(1/
2)*b*1i))/(6*a^(4/3)) + (log((b + 3^(1/2)*b*1i)^2/(4*a^(5/3)) + (b^2*(b*x -
 a)^(1/3))/a^2)*(b + 3^(1/2)*b*1i))/(6*a^(4/3))
```

**sympy [C]** time = 2.24, size = 838, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(1/3),x)
```

```
[Out] -2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*e
xp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*ex
p(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**
(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/
3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)*
*(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3
))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) +
 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(2/3)*b
**(10/3)*(-a/b + x)**(7/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*p
i/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) -
2*a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(-a/b
 + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(
2*I*pi/3)*gamma(5/3)) - 2*a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*exp(-2*I*pi/
3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2
/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b*
*(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 6*a*b**3*(-a/b + x)**2
*exp(2*I*pi/3)*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3))
```

## 3.405 $\int \frac{1}{x^3 \sqrt[3]{-a+bx}} \, dx$

**Optimal**. Leaf size=136

$$\frac{b^2 \log(x)}{9a^{7/3}} - \frac{b^2 \log\left(\sqrt[3]{bx-a} + \sqrt[3]{a}\right)}{3a^{7/3}} - \frac{2b^2 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}} + \frac{2b(bx-a)^{2/3}}{3a^2x} + \frac{(bx-a)^{2/3}}{2ax^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {51, 56, 617, 204, 31}

$$\frac{b^2 \log(x)}{9a^{7/3}} - \frac{b^2 \log\left(\sqrt[3]{bx-a} + \sqrt[3]{a}\right)}{3a^{7/3}} - \frac{2b^2 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}} + \frac{2b(bx-a)^{2/3}}{3a^2x} + \frac{(bx-a)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(2*a*x^2) + (2*b*(-a + b*x)^(2/3))/(3*a^2*x) - (2*b^2*ArcT
an[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) +
 (b^2*Log[x])/(9*a^(7/3)) - (b^2*Log[a^(1/3) + (-a + b*x)^(1/3)])/(3*a^(7/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
```

```
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx = \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{(2b)\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx}{3a}$$

$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} + \frac{\left(2b^2\right)\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx}{9a^2}$$

$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} + \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x, \sqrt[3]{-a+bx}\right)}{3a^{7/3}} + \frac{b^2\,\text{Subs}}{}$$

$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} + \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{3a^{7/3}} + \frac{\left(2b^2\right)\text{Subst}\left(\int \right.}{}$$

$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} - \frac{2b^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}} + \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+\right.}}{3a^{7/3}}$$

**Mathematica [C]** time = 0.01, size = 38, normalized size = 0.28

$$\frac{3b^2(bx-a)^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};1-\frac{bx}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] (3*b^2*(-a + b*x)^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, 1 - (b*x)/a])/(2*a^3
)
```

**IntegrateAlgebraic [A]** time = 0.13, size = 160, normalized size = 1.18

$$-\frac{2b^2\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{9a^{7/3}}+\frac{b^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx-a}+(bx-a)^{2/3}\right)}{9a^{7/3}}-\frac{2b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{(bx-a)^{2/3}(4(bx-a)+7a)}{6a^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] ((-a + b*x)^(2/3)*(7*a + 4*(-a + b*x)))/(6*a^2*x^2) - (2*b^2*ArcTan[1/Sqrt[
3] - (2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) - (2*b^2*
Log[a^(1/3) + (-a + b*x)^(1/3)])/(9*a^(7/3)) + (b^2*Log[a^(2/3) - a^(1/3)*(
-a + b*x)^(1/3) + (-a + b*x)^(2/3)])/(9*a^(7/3))
```

**fricas [A]** time = 0.66, size = 374, normalized size = 2.75

$$\left[\frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3a}{x}\right)+2(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+3\left(4abx+3a^2\right)(bx-a)^{\frac{2}{3}}}{18a^3x^2},\ \frac{12\sqrt{\frac{1}{3}}ab^2x^2\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+2(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+3\left(4abx+3a^2\right)(bx-a)^{\frac{2}{3}}}{18a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(2
*(b*x - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(
1/3)/a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + 2*(-a)^(2/3)*b^2*x^2*log
((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 4*(-a)^(2/3)*
b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*x + 3*a^2)*(b*x - a)^(
2/3))/(a^3*x^2), 1/18*(12*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sq
rt(1/3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 2*(-a)^(2/3
)*b^2*x^2*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) -
4*(-a)^(2/3)*b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*x + 3*a^2
)*(b*x - a)^(2/3))/(a^3*x^2)]
```

**giac [A]** time = 2.24, size = 167, normalized size = 1.23

$$\frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3(-a)^{\frac{1}{3}}}\right)}{(-a)^{\frac{1}{3}}a^2}-\frac{2b^3\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{(-a)^{\frac{1}{3}}a^2}-\frac{4(-a)^{\frac{2}{3}}b^3\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a^3}+\frac{3\left(4(bx-a)^{\frac{5}{3}}b^3+7(bx-a)^{\frac{2}{3}}ab^3\right)}{a^2b^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a
)^(1/3))/((-a)^(1/3)*a^2) - 2*b^3*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3))/((-a)^(1/3)*a^2) - 4*(-a)^(2/3)*b^3*log(abs((b*x - a)
^(1/3) - (-a)^(1/3)))/a^3 + 3*(4*(b*x - a)^(5/3)*b^3 + 7*(b*x - a)^(2/3)*a*
b^3)/(a^2*b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 128, normalized size = 0.94

$$\frac{2\sqrt{3}\, b^{2} \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{9a^{\frac{7}{3}}}-\frac{2b^{2} \ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{b^{2} \ln\left(a^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}} a^{\frac{1}{3}}+(bx-a)^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2\,(bx-a)^{\frac{2}{3}}\, b}{3a^{2}x}+\frac{(bx-a)^{\frac{2}{3}}}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(1/3),x)
```

```
[Out] 1/2*(b*x-a)^(2/3)/a/x^2+2/3*b*(b*x-a)^(2/3)/a^2/x-2/9*b^2*ln(a^(1/3)+(b*x-a
)^(1/3))/a^(7/3)+1/9*b^2/a^(7/3)*ln(a^(2/3)-(b*x-a)^(1/3)*a^(1/3)+(b*x-a)^(
2/3))+2/9*b^2/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x-a)^(1/3)/a^(1/3)-1
))
```

**maxima [A]** time = 3.09, size = 159, normalized size = 1.17

$$\frac{2\,\sqrt{3}\, b^{2} \arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}}+\frac{b^{2} \log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{7}{3}}}-\frac{2\,b^{2} \log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{9\,a^{\frac{7}{3}}}+\frac{4\,(bx-a)^{\frac{5}{3}} b^{2}+7\,(bx-a)^{\frac{2}{3}} a b^{2}}{6\left((bx-a)^{2} a^{2}+2\,(bx-a) a^{3}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a
^(7/3) + 1/9*b^2*log((b*x - a)^(2/3) - (b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a
^(7/3) - 2/9*b^2*log((b*x - a)^(1/3) + a^(1/3))/a^(7/3) + 1/6*(4*(b*x - a)^
(5/3)*b^2 + 7*(b*x - a)^(2/3)*a*b^2)/((b*x - a)^2*a^2 + 2*(b*x - a)*a^3 + a
^4)
```

**mupad [B]** time = 0.22, size = 216, normalized size = 1.59

$$\frac{\frac{7\,b^{2}\,(b\,x-a)^{2/3}}{6\,a}+\frac{2\,b^{2}\,(b\,x-a)^{5/3}}{3\,a^{2}}}{(a-b\,x)^{2}-2\,a\,(a-b\,x)+a^{2}}-\frac{\ln\left(\frac{4\,b^{4}\,(b\,x-a)^{1/3}}{9\,a^{4}}-\frac{\left(b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}\right)^{2}}{9\,(-a)^{11/3}}\right)\left(b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}\right)}{9\,(-a)^{7/3}}+\frac{2\,b^{2}\,\ln\left(\frac{4\,b^{4}\,(b\,x-a)^{1/3}}{9\,a^{4}}-\frac{4\,b^{4}}{9\,(-a)^{11/3}}\right)}{9\,(-a)^{7/3}}+\frac{b^{2}\,\ln\left(\frac{4\,b^{4}\,(b\,x-a)^{1/3}}{9\,a^{4}}-\frac{9\,b^{4}\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^{2}}{(-a)^{11/3}}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)}{(-a)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(1/3)),x)
```

```
[Out] ((7*b^2*(b*x - a)^(2/3))/(6*a) + (2*b^2*(b*x - a)^(5/3))/(3*a^2))/((a - b*x
)^2 - 2*a*(a - b*x) + a^2) - (log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (3^(1/2
)*b^2*1i + b^2)^2/(9*(-a)^(11/3)))*(3^(1/2)*b^2*1i + b^2))/(9*(-a)^(7/3)) +
 (2*b^2*log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (4*b^4)/(9*(-a)^(11/3))))/(9*
(-a)^(7/3)) + (b^2*log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (9*b^4*((3^(1/2)*1
i)/9 - 1/9)^2)/(-a)^(11/3))*((3^(1/2)*1i)/9 - 1/9))/(-a)^(7/3)
```

**sympy [C]** time = 2.62, size = 2744, normalized size = 20.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(1/3),x)
```

```
[Out] -4*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)
*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)
*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3
)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3
) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(
14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)
**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I
*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamm
a(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4
*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/
b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-
a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/
3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*
pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*log(1 - b**(1/3)*(-a/b +
x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp
(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*
gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
 - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*
(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-
a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/
3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*
pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*exp(-2*I*pi/3)*log(1 - b*
*(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*
b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/
```

```
b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*
pi/3)*gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*log(1 - b**(1/
3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/
3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)
**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp
(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*
gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*log(1
- b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*
b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*
pi/3)*gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*exp(-2*I*pi/3)
*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3
)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b*
*(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b
 + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/
3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*log(
1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a
**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*
(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**
(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(
2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*exp(2*I*pi/
3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/
(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(
7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b +
 x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)
*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*exp(-2
*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*g
amma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81
*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/
3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b +
x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 21*a**4*b**4*(-a/b + x)**2*exp(2*I*p
i/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3
) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b
**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-
a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 33*a**3*b**5*(-a/b + x)**3*exp
(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*
a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13
/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 12*a**2*b**6*(-a/b + x)*
*4*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/
3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)
 + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*
b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
```

## 3.406 $\int \frac{x^3}{(a+bx)^{2/3}}\,dx$

**Optimal**. Leaf size=70

$$-\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}+\frac{3(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}$$

**Rubi [A]** time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{3(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a + b*x)^(4/3))/(4*b^4) - (9*a*(a +
b*x)^(7/3))/(7*b^4) + (3*(a + b*x)^(10/3))/(10*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{2/3}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{2/3}}+\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{7/3}}{b^3}\right)dx\\ &= -\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{10/3}}{10b^4}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 0.66

$$\frac{3\sqrt[3]{a+bx}\left(-81a^3+27a^2bx-18ab^2x^2+14b^3x^3\right)}{140b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(-81*a^3 + 27*a^2*b*x - 18*a*b^2*x^2 + 14*b^3*x^3))/(140
*b^4)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 51, normalized size = 0.73

$$\frac{3\sqrt[3]{a+bx}\left(-140a^3+105a^2(a+bx)-60a(a+bx)^2+14(a+bx)^3\right)}{140b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(-140*a^3 + 105*a^2*(a + b*x) - 60*a*(a + b*x)^2 + 14*(a
 + b*x)^3))/(140*b^4)
```

**fricas [A]** time = 0.53, size = 42, normalized size = 0.60

$$\frac{3\left(14\,b^3x^3-18\,ab^2x^2+27\,a^2bx-81\,a^3\right)(bx+a)^{\frac{1}{3}}}{140\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^3*x^3 - 18*a*b^2*x^2 + 27*a^2*b*x - 81*a^3)*(b*x + a)^(1/3)/b^4
```

**giac [A]** time = 1.03, size = 49, normalized size = 0.70

$$\frac{3\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)}{140\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/140*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)*a^2
 - 140*(b*x + a)^(1/3)*a^3)/b^4
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.61

$$-\frac{3\,(bx+a)^{\frac{1}{3}}\left(-14b^3x^3+18a\,b^2x^2-27a^2bx+81a^3\right)}{140b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(2/3),x)
```

```
[Out] -3/140*(b*x+a)^(1/3)*(-14*b^3*x^3+18*a*b^2*x^2-27*a^2*b*x+81*a^3)/b^4
```

**maxima [A]** time = 1.34, size = 56, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^4}-\frac{9\,(bx+a)^{\frac{7}{3}}a}{7\,b^4}+\frac{9\,(bx+a)^{\frac{4}{3}}a^2}{4\,b^4}-\frac{3\,(bx+a)^{\frac{1}{3}}a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x + a)^(10/3)/b^4 - 9/7*(b*x + a)^(7/3)*a/b^4 + 9/4*(b*x + a)^(4/3)
*a^2/b^4 - 3*(b*x + a)^(1/3)*a^3/b^4
```

**mupad [B]** time = 0.05, size = 56, normalized size = 0.80

$$\frac{3\,(a+b\,x)^{10/3}}{10\,b^4}-\frac{3\,a^3\,(a+b\,x)^{1/3}}{b^4}+\frac{9\,a^2\,(a+b\,x)^{4/3}}{4\,b^4}-\frac{9\,a\,(a+b\,x)^{7/3}}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(2/3),x)
```

```
[Out] (3*(a + b*x)^(10/3))/(10*b^4) - (3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a + b
*x)^(4/3))/(4*b^4) - (9*a*(a + b*x)^(7/3))/(7*b^4)
```

**sympy [B]** time = 2.79, size = 1640, normalized size = 23.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(2/3),x)
```

```
[Out] -243*a**(70/3)*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100
*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*
b**9*x**5 + 140*a**14*b**10*x**6) + 243*a**(70/3)/(140*a**20*b**4 + 840*a**
19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x
**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) - 1377*a**(67/3)*b*x*(1 +
 b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 +
2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**
14*b**10*x**6) + 1458*a**(67/3)*b*x/(140*a**20*b**4 + 840*a**19*b**5*x + 21
00*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**1
5*b**9*x**5 + 140*a**14*b**10*x**6) - 3213*a**(64/3)*b**2*x**2*(1 + b*x/a)*
*(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**
17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10
```

```
*x**6) + 3645*a**(64/3)*b**2*x**2/(140*a**20*b**4 + 840*a**19*b**5*x + 2100
*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*
b**9*x**5 + 140*a**14*b**10*x**6) - 3927*a**(61/3)*b**3*x**3*(1 + b*x/a)**(
1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17
*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x
**6) + 4860*a**(61/3)*b**3*x**3/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a
**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b*
*9*x**5 + 140*a**14*b**10*x**6) - 2583*a**(58/3)*b**4*x**4*(1 + b*x/a)**(1/
3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b
**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**
6) + 3645*a**(58/3)*b**4*x**4/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**
18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9
*x**5 + 140*a**14*b**10*x**6) - 693*a**(55/3)*b**5*x**5*(1 + b*x/a)**(1/3)/
(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7
*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6)
+ 1458*a**(55/3)*b**5*x**5/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*
b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x*
*5 + 140*a**14*b**10*x**6) + 273*a**(52/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(14
0*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x*
*3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) + 2
43*a**(52/3)*b**6*x**6/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6
*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 +
 140*a**14*b**10*x**6) + 387*a**(49/3)*b**7*x**7*(1 + b*x/a)**(1/3)/(140*a*
*20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 +
 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) + 198*a
**(46/3)*b**8*x**8*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x +
2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a*
*15*b**9*x**5 + 140*a**14*b**10*x**6) + 42*a**(43/3)*b**9*x**9*(1 + b*x/a)*
*(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**
17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10
*x**6)
```

# 3.407 $\int \frac{x^2}{(a+bx)^{2/3}}\,dx$

**Optimal**. Leaf size=51

$$\frac{3a^2\sqrt[3]{a+bx}}{b^3}+\frac{3(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}$$

**Rubi [A]** time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2\sqrt[3]{a+bx}}{b^3}+\frac{3(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(1/3))/b^3 - (3*a*(a + b*x)^(4/3))/(2*b^3) + (3*(a + b*x)^
(7/3))/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{2/3}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{2/3}}-\frac{2a\sqrt[3]{a+bx}}{b^2}+\frac{(a+bx)^{4/3}}{b^2}\right)dx\\ &= \frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}+\frac{3(a+bx)^{7/3}}{7b^3}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 35, normalized size = 0.69

$$\frac{3\sqrt[3]{a+bx}\left(9a^2-3abx+2b^2x^2\right)}{14b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*x)^(2/3),x]

[Out] (3*(a + b*x)^(1/3)*(9*a^2 - 3*a*b*x + 2*b^2*x^2))/(14*b^3)

**IntegrateAlgebraic [A]** time = 0.02, size = 45, normalized size = 0.88

$$\frac{3\left(14a^2\sqrt[3]{a+bx}+2(a+bx)^{7/3}-7a(a+bx)^{4/3}\right)}{14b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(a + b*x)^(2/3),x]

[Out] (3*(14*a^2*(a + b*x)^(1/3) - 7*a*(a + b*x)^(4/3) + 2*(a + b*x)^(7/3)))/(14*b^3)

**fricas [A]** time = 0.83, size = 31, normalized size = 0.61

$$\frac{3\left(2\,b^2x^2-3\,abx+9\,a^2\right)(bx+a)^{\frac{1}{3}}}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="fricas")

[Out] 3/14*(2*b^2*x^2 - 3*a*b*x + 9*a^2)*(b*x + a)^(1/3)/b^3

**giac [A]** time = 0.90, size = 37, normalized size = 0.73

$$\frac{3\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="giac")

[Out] 3/14*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)/b^3

**maple [A]** time = 0.00, size = 32, normalized size = 0.63

$$\frac{3\,(bx+a)^{\frac{1}{3}}\left(2b^2x^2-3abx+9a^2\right)}{14b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x+a)^(2/3),x)

```
[Out] 3/14*(b*x+a)^(1/3)*(2*b^2*x^2-3*a*b*x+9*a^2)/b^3
```

**maxima** [A] time = 1.31, size = 41, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b^3}-\frac{3\,(bx+a)^{\frac{4}{3}}a}{2\,b^3}+\frac{3\,(bx+a)^{\frac{1}{3}}a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b^3 - 3/2*(b*x + a)^(4/3)*a/b^3 + 3*(b*x + a)^(1/3)*a^2
/b^3
```

**mupad** [B] time = 0.04, size = 37, normalized size = 0.73

$$\frac{6\,(a+b\,x)^{7/3}-21\,a\,(a+b\,x)^{4/3}+42\,a^2\,(a+b\,x)^{1/3}}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(2/3),x)
```

```
[Out] (6*(a + b*x)^(7/3) - 21*a*(a + b*x)^(4/3) + 42*a^2*(a + b*x)^(1/3))/(14*b^3
)
```

**sympy** [B] time = 1.82, size = 600, normalized size = 11.76

$$\frac{27a^{\frac{31}{3}}\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{27a^{\frac{31}{3}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{72a^{\frac{28}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{81a^{\frac{28}{3}}bx}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{60a^{\frac{25}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{81a^{\frac{25}{3}}b^2x^2}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{18a^{\frac{22}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{27a^{\frac{22}{3}}b^3x^3}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{9a^{\frac{19}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{6a^{\frac{16}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(2/3),x)
```

```
[Out] 27*a**(31/3)*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b*
*5*x**2 + 14*a**5*b**6*x**3) - 27*a**(31/3)/(14*a**8*b**3 + 42*a**7*b**4*x
+ 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 72*a**(28/3)*b*x*(1 + b*x/a)**(1
/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3)
 - 81*a**(28/3)*b*x/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14
*a**5*b**6*x**3) + 60*a**(25/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(14*a**8*b**3
+ 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 81*a**(25/3)*b*
*2*x**2/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x
**3) + 18*a**(22/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b*
*4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 27*a**(22/3)*b**3*x**3/(14*
a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 9*a**
(19/3)*b**4*x**4*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**
6*b**5*x**2 + 14*a**5*b**6*x**3) + 6*a**(16/3)*b**5*x**5*(1 + b*x/a)**(1/3)
/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3)
```

## 3.408 $\int \frac{x}{(a+bx)^{2/3}}\,dx$

**Optimal**. Leaf size=32

$$\frac{3(a+bx)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx}}{b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x)^(1/3))/b^2 + (3*(a + b*x)^(4/3))/(4*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{2/3}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{b}\right)dx\\ &= -\frac{3a\sqrt[3]{a+bx}}{b^2}+\frac{3(a+bx)^{4/3}}{4b^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 23, normalized size = 0.72

$$\frac{3(bx-3a)\sqrt[3]{a+bx}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(2/3),x]
```

```
[Out] (3*(-3*a + b*x)*(a + b*x)^(1/3))/(4*b^2)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 24, normalized size = 0.75

$$-\frac{3(3a-bx)\sqrt[3]{a+bx}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x)^(2/3),x]
[Out] (-3*(3*a - b*x)*(a + b*x)^(1/3))/(4*b^2)
```

**fricas** [A] time = 0.90, size = 19, normalized size = 0.59

$$\frac{3\,(bx+a)^{\frac{1}{3}}(bx-3\,a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="fricas")
[Out] 3/4*(b*x + a)^(1/3)*(b*x - 3*a)/b^2
```

**giac** [A] time = 0.89, size = 23, normalized size = 0.72

$$\frac{3\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="giac")
[Out] 3/4*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)/b^2
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.66

$$-\frac{3\,(bx+a)^{\frac{1}{3}}\,(-bx+3a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(2/3),x)
[Out] -3/4*(b*x+a)^(1/3)*(-b*x+3*a)/b^2
```

**maxima** [A] time = 1.34, size = 26, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b^2}-\frac{3\,(bx+a)^{\frac{1}{3}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b^2 - 3*(b*x + a)^(1/3)*a/b^2
```

**mupad** [B] time = 0.03, size = 25, normalized size = 0.78

$$-\frac{12\,a\,(a+b\,x)^{1/3}-3\,(a+b\,x)^{4/3}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(2/3),x)
```

```
[Out] -(12*a*(a + b*x)^(1/3) - 3*(a + b*x)^(4/3))/(4*b^2)
```

**sympy** [B] time = 1.19, size = 162, normalized size = 5.06

$$-\frac{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}+\frac{9a^{\frac{10}{3}}}{4a^2b^2+4ab^3x}-\frac{6a^{\frac{7}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}+\frac{9a^{\frac{7}{3}}bx}{4a^2b^2+4ab^3x}+\frac{3a^{\frac{4}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(2/3),x)
```

```
[Out] -9*a**(10/3)*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x) + 9*a**(10/3)/(4
*a**2*b**2 + 4*a*b**3*x) - 6*a**(7/3)*b*x*(1 + b*x/a)**(1/3)/(4*a**2*b**2 +
 4*a*b**3*x) + 9*a**(7/3)*b*x/(4*a**2*b**2 + 4*a*b**3*x) + 3*a**(4/3)*b**2*
x**2*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x)
```

## 3.409 $\int \frac{1}{(a+bx)^{2/3}}\,dx$

**Optimal**. Leaf size=14

$$\frac{3\sqrt[3]{a+bx}}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}}{b}$$

**Mathematica [A]** time = 0.00, size = 14, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

**IntegrateAlgebraic [A]** time = 0.01, size = 14, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(-2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

**fricas [A]** time = 0.89, size = 12, normalized size = 0.86

$$\frac{3\left(bx+a\right)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

**giac [A]** time = 0.98, size = 12, normalized size = 0.86

$$\frac{3\left(bx+a\right)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{3\left(bx+a\right)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3),x)
```

```
[Out] 3*(b*x+a)^(1/3)/b
```

**maxima [A]** time = 1.30, size = 12, normalized size = 0.86

$$\frac{3\left(bx+a\right)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.86

$$\frac{3\,{\left(a+b\,x\right)}^{1/3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(2/3),x)
[Out] (3*(a + b*x)^(1/3))/b
```

**sympy [A]** time = 0.06, size = 10, normalized size = 0.71

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3),x)
[Out] 3*(a + b*x)**(1/3)/b
```

## 3.410 $\int \frac{1}{x(a+bx)^{2/3}}\,dx$

**Optimal**. Leaf size=80

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

**Rubi** [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {57, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(2/3)
) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx = -\frac{\log(x)}{2a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

**Mathematica [A]** time = 0.03, size = 93, normalized size = 1.16

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a
^(1/3) - (a + b*x)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*
x)^(2/3)])/a^(2/3)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 105, normalized size = 1.31

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{2/3}} - \frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2a^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(2/
3)) + Log[a^(1/3) - (a + b*x)^(1/3)]/a^(2/3) - Log[a^(2/3) + a^(1/3)*(a + b
*x)^(1/3) + (a + b*x)^(2/3)]/(2*a^(2/3))
```

**fricas [B]** time = 1.00, size = 115, normalized size = 1.44

$$-\frac{2\sqrt{3}\left(a^2\right)^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{3}\left(a^2\right)^{\frac{1}{6}}\left(\left(a^2\right)^{\frac{1}{3}}a+2\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+\left(a^2\right)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\left(a^2\right)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*a
 + 2*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*log((b*x + a)^(2/3)*a
+ (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2/3)*log((b*x + a
)^(1/3)*a - (a^2)^(2/3)))/a^2
```

**giac [A]** time = 2.32, size = 78, normalized size = 0.98

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}}+\frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + lo
g(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3)
```

**maple [A]** time = 0.00, size = 76, normalized size = 0.95

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{2}{3}}}+\frac{\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}-\frac{\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\,a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(2/3),x)
```

```
[Out] 1/a^(2/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/2/a^(2/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a
^(1/3)+(b*x+a)^(2/3))-1/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)
/a^(1/3)+1))
```

**maxima [A]** time = 2.87, size = 77, normalized size = 0.96

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{2}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + lo
g((b*x + a)^(1/3) - a^(1/3))/a^(2/3)
```

**mupad [B]** time = 0.17, size = 95, normalized size = 1.19

$$\frac{\ln\left(9\,(a+b\,x)^{1/3}-9\,a^{1/3}\right)}{a^{2/3}}+\frac{\ln\left(\frac{9\,a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-9\,(a+b\,x)^{1/3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}-\frac{\ln\left(\frac{9\,a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+9\,(a+b\,x)^{1/3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(2/3)),x)
```

```
[Out] log(9*(a + b*x)^(1/3) - 9*a^(1/3))/a^(2/3) + (log((9*a^(1/3)*(3^(1/2)*1i -
1))/2 - 9*(a + b*x)^(1/3))*(3^(1/2)*1i - 1))/(2*a^(2/3)) - (log((9*a^(1/3)*
(3^(1/2)*1i + 1))/2 + 9*(a + b*x)^(1/3))*(3^(1/2)*1i + 1))/(2*a^(2/3))
```

**sympy [C]** time = 1.93, size = 150, normalized size = 1.88

$$\frac{\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(2/3),x)
```

```
[Out] log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4/
3)) + exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/
```

```
a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b**(1/
3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*ga
mma(4/3))
```

## 3.411 $\int \frac{1}{x^2(a+bx)^{2/3}}\,dx$

**Optimal**. Leaf size=98

$$\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}}{ax}$$

**Rubi [A]** time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 57, 617, 204, 31}

$$\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(2/3)),x]
```

```
[Out] -((a + b*x)^(1/3)/(a*x)) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a^(1/3) -
 (a + b*x)^(1/3)])/a^(5/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(a+bx)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx}}{ax}-\frac{(2b)\int \frac{1}{x(a+bx)^{2/3}}\,dx}{3a}\\
&= -\frac{\sqrt[3]{a+bx}}{ax}+\frac{b\log(x)}{3a^{5/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{a^{5/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{\right.}{a^{4/3}}\\
&= -\frac{\sqrt[3]{a+bx}}{ax}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{5/3}}\\
&= -\frac{\sqrt[3]{a+bx}}{ax}+\frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 31, normalized size = 0.32

$$\frac{3b\sqrt[3]{a+bx}\;{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{bx}{a}+1\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(2/3)),x]
```

```
[Out] (3*b*(a + b*x)^(1/3)*Hypergeometric2F1[1/3, 2, 4/3, 1 + (b*x)/a])/a^2
```

**IntegrateAlgebraic [A]** time = 0.15, size = 128, normalized size = 1.31

$$-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{5/3}}+\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3a^{5/3}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^(2/3)),x]
```

```
[Out] -((a + b*x)^(1/3)/(a*x)) + (2*b*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) - (2*b*Log[a^(1/3) - (a + b*x)^(1/3)])/(3
*a^(5/3)) + (b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3
*a^(5/3))
```

**fricas [B]** time = 0.69, size = 166, normalized size = 1.69

$$\frac{2\sqrt{3}abx\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\left(-a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)-3(bx+a)^{\frac{1}{3}}a^2}{3a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*a*b*x*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*
a - 2*sqrt(3)*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) + (-a^
2)^(2/3)*b*x*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)*(b*x + a
)^(1/3)) - 2*(-a^2)^(2/3)*b*x*log((b*x + a)^(1/3)*a - (-a^2)^(2/3)) - 3*(b*
x + a)^(1/3)*a^2)/(a^3*x)
```

**giac [A]** time = 2.37, size = 108, normalized size = 1.10

$$\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3(bx+a)^{\frac{1}{3}}b}{ax}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(5/3) + b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(5/3) - 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x + a)^(1/
3)*b/(a*x))/b
```

**maple [A]** time = 0.01, size = 95, normalized size = 0.97

$$\frac{2\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{5}{3}}}-\frac{2b\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3a^{\frac{5}{3}}}+\frac{b\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{3a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{1}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(2/3),x)
```

```
[Out] -(b*x+a)^(1/3)/a/x-2/3*b/a^(5/3)*ln(-a^(1/3)+(b*x+a)^(1/3))+1/3*b/a^(5/3)*l
n(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))+2/3*b/a^(5/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

**maxima [A]** time = 2.95, size = 106, normalized size = 1.08

$$\frac{2\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{1}{3}}b}{(bx+a)a-a^{2}}+\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{5}{3}}}-\frac{2\,b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
5/3) - (b*x + a)^(1/3)*b/((b*x + a)*a - a^2) + 1/3*b*log((b*x + a)^(2/3) +
(b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 2/3*b*log((b*x + a)^(1/3) - a^
(1/3))/a^(5/3)
```

**mupad [B]** time = 0.13, size = 122, normalized size = 1.24

$$-\frac{(a+b\,x)^{1/3}}{a\,x}+\frac{\ln\left(\frac{3\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{a^{2/3}}+\frac{6\,b\,(a+b\,x)^{1/3}}{a}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{5/3}}+\frac{\ln\left(\frac{3\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{a^{2/3}}+\frac{6\,b\,(a+b\,x)^{1/3}}{a}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{5/3}}-\frac{2\,b\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{3\,a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(2/3)),x)
```

```
[Out] (log((3*(b - 3^(1/2)*b*1i))/a^(2/3) + (6*b*(a + b*x)^(1/3))/a)*(b - 3^(1/2)
*b*1i))/(3*a^(5/3)) - (a + b*x)^(1/3)/(a*x) + (log((3*(b + 3^(1/2)*b*1i))/a
^(2/3) + (6*b*(a + b*x)^(1/3))/a)*(b + 3^(1/2)*b*1i))/(3*a^(5/3)) - (2*b*lo
g((a + b*x)^(1/3) - a^(1/3)))/(3*a^(5/3))
```

**sympy [C]** time = 2.27, size = 830, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(2/3),x)
```

```
[Out] -2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*p
i/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)
) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*
exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*
exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*g
amma(4/3)) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(-2*I*pi/3)*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b*
*(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x
)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b + x)**(5/3)*e
xp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(9*a**3
*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b
+ x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b + x)**(5/3
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3
)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5
/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b +
 x)**(5/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*g
amma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 3*
a*b**2*(a/b + x)*exp(2*I*pi/3)*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)
*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*
gamma(4/3))
```

## 3.412 $\int \frac{1}{x^3(a+bx)^{2/3}}\,dx$

**Optimal**. Leaf size=130

$$-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 57, 617, 204, 31}

$$-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(2/3)),x]
```

```
[Out] -(a + b*x)^(1/3)/(2*a*x^2) + (5*b*(a + b*x)^(1/3))/(6*a^2*x) - (5*b^2*ArcTa
n[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) - (
5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(8
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3(a+bx)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx}}{2ax^2}-\frac{(5b)\int \frac{1}{x^2(a+bx)^{2/3}}\,dx}{6a}\\
&= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}+\frac{\left(5b^2\right)\int \frac{1}{x(a+bx)^{2/3}}\,dx}{9a^2}\\
&= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\log(x)}{18a^{8/3}}-\frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{8/3}}-\frac{\left(5b^2\right)\text{Su}}{}\\
&= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}+\frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-3-x}\right.}{3a}\\
&= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right.}{6a^{8/3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.25

$$
-\frac{3b^2\sqrt[3]{a+bx}\,{}_2F_1\left(\frac{1}{3},3;\frac{4}{3};\frac{bx}{a}+1\right)}{a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(2/3)),x]
```

```
[Out] (-3*b^2*(a + b*x)^(1/3)*Hypergeometric2F1[1/3, 3, 4/3, 1 + (b*x)/a])/a^3
```

**IntegrateAlgebraic [A]** time = 0.12, size = 149, normalized size = 1.15

$$\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{8/3}}-\frac{5b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}a^{8/3}}-\frac{\sqrt[3]{a+bx}\,(8a-5(a+bx))}{6a^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^(2/3)),x]
```

```
[Out] -1/6*((a + b*x)^(1/3)*(8*a - 5*(a + b*x)))/(a^2*x^2) - (5*b^2*ArcTan[1/Sqrt
[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (5*b^2*
Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(8/3)) - (5*b^2*Log[a^(2/3) + a^(1/3)*
(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*a^(8/3))
```

**fricas [A]** time = 0.82, size = 162, normalized size = 1.25

$$-\frac{10\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^2\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+5\left(a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-10\left(a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)-3\left(5a^2bx-3a^3\right)(bx+a)^{\frac{1}{3}}}{18a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/18*(10*sqrt(3)*(a^2)^(1/6)*a*b^2*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^
2)^(1/3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 5*(a^2)^(2/3)*b^
2*x^2*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3))
- 10*(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) - 3*(5*a^2*b*
x - 3*a^3)*(b*x + a)^(1/3))/(a^4*x^2)
```

**giac [A]** time = 2.05, size = 130, normalized size = 1.00

$$-\frac{\frac{10\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}}+\frac{5b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}}-\frac{10b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{8}{3}}}-\frac{3\left(5(bx+a)^{\frac{4}{3}}b^3-8(bx+a)^{\frac{1}{3}}ab^3\right)}{a^2b^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/18*(10*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(8/3) + 5*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3
```

```
))/a^(8/3) - 10*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(8/3) - 3*(5*(b*x
 + a)^(4/3)*b^3 - 8*(b*x + a)^(1/3)*a*b^3)/(a^2*b^2*x^2))/b
```

**maple [A]** time = 0.01, size = 117, normalized size = 0.90

$$-\frac{5\sqrt{3}\, b^{2} \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{8}{3}}}+\frac{5b^{2} \ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{8}{3}}}-\frac{5b^{2} \ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{18a^{\frac{8}{3}}}+\frac{5\,(bx+a)^{\frac{1}{3}}\, b}{6a^{2}x}-\frac{(bx+a)^{\frac{1}{3}}}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(2/3),x)
```

```
[Out] -1/2*(b*x+a)^(1/3)/a/x^2+5/6*b*(b*x+a)^(1/3)/a^2/x+5/9*b^2/a^(8/3)*ln(-a^(1
/3)+(b*x+a)^(1/3))-5/18*b^2/a^(8/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a
)^(2/3))-5/9*b^2/a^(8/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3
)+1))
```

**maxima [A]** time = 3.04, size = 142, normalized size = 1.09

$$-\frac{5\,\sqrt{3} b^{2} \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{8}{3}}}-\frac{5\,b^{2} \log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{8}{3}}}+\frac{5\,b^{2} \log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{8}{3}}}+\frac{5\,(bx+a)^{\frac{4}{3}} b^{2}-8\,(bx+a)^{\frac{1}{3}} ab^{2}}{6\left((bx+a)^{2} a^{2}-2\,(bx+a)a^{3}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -5/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(8/3) - 5/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(8/3) + 5/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(8/3) + 1/6*(5*(b*x + a
)^(4/3)*b^2 - 8*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2*a^2 - 2*(b*x + a)*a^3 +
 a^4)
```

**mupad [B]** time = 0.13, size = 175, normalized size = 1.35

$$\frac{5\,b^{2}\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{9\,a^{8/3}}-\frac{\frac{4\,b^{2}\,(a+b\,x)^{1/3}}{3\,a}-\frac{5\,b^{2}\,(a+b\,x)^{4/3}}{6\,a^{2}}}{(a+b\,x)^{2}-2\,a\,(a+b\,x)+a^{2}}+\frac{5\,b^{2}\,\ln\left(\frac{5\,b^{2}\,(a+b\,x)^{1/3}}{a^{2}}-\frac{5\,b^{2}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{5/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{8/3}}-\frac{5\,b^{2}\,\ln\left(\frac{5\,b^{2}\,(a+b\,x)^{1/3}}{a^{2}}+\frac{5\,b^{2}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{5/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(2/3)),x)
```

```
[Out] (5*b^2*log((a + b*x)^(1/3) - a^(1/3)))/(9*a^(8/3)) - ((4*b^2*(a + b*x)^(1/3
))/(3*a) - (5*b^2*(a + b*x)^(4/3))/(6*a^2))/((a + b*x)^2 - 2*a*(a + b*x) +
```

```
a^2) + (5*b^2*log((5*b^2*(a + b*x)^(1/3))/a^2 - (5*b^2*((3^(1/2)*1i)/2 - 1/
2))/a^(5/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3)) - (5*b^2*log((5*b^2*(a + b
*x)^(1/3))/a^2 + (5*b^2*((3^(1/2)*1i)/2 + 1/2))/a^(5/3))*((3^(1/2)*1i)/2 +
1/2))/(9*a^(8/3))
```

**sympy [C]** time = 2.73, size = 2728, normalized size = 20.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(2/3),x)
```

```
[Out] 10*a**(13/3)*b**(8/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I
*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(
4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**
4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)*b**(
8/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)
/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamm
a(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*
a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)
*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)*b**(8/3)*(a/b +
 x)**(2/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*
gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) +
162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(1
1/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(
a/b + x)**(5/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*g
amma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162
*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3
)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**
(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(a/b + x)**(5/3)*
log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/
(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(
5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x
)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp
(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(a/b + x)**(5/3)*exp(-2*I*p
i/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6
*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/
b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3
)*exp(2*I*pi/3)*gamma(4/3)) + 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*exp(2*
I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**
(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b +
x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp
```

```
(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*g
amma(4/3)) + 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b
+ x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*e
xp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*
gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3))
+ 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(
a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/
3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi
/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/
3)) - 10*a**(4/3)*b**(17/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*log(1 - b**(1/3
)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*
exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)
*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) -
 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 10*a**(4/3
)*b**(17/3)*(a/b + x)**(11/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2
*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi
/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3
) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b
**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 10*a**(4/3)*b**(17/3
)*(a/b + x)**(11/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2
*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamm
a(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a
**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 24*a**4*b**3*(a
/b + x)*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I
*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(
4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**
4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 39*a**3*b**4*(a/b
 + x)**2*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*
I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma
(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a*
*4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 15*a**2*b**5*(a/
b + x)**3*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2
*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamm
a(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a
**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3))
```

## 3.413 $\int \frac{x^3}{(a+bx)^{4/3}}\,dx$

**Optimal**. Leaf size=70

$$\frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}$$

**Rubi** [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*a^3)/(b^4*(a + b*x)^(1/3)) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) - (9*a*(a +
 b*x)^(5/3))/(5*b^4) + (3*(a + b*x)^(8/3))/(8*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{4/3}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{4/3}}+\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{(a+bx)^{5/3}}{b^3}\right)dx\\ &= \frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 0.66

$$\frac{3\left(81a^3+27a^2bx-9ab^2x^2+5b^3x^3\right)}{40b^4\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*(81*a^3 + 27*a^2*b*x - 9*a*b^2*x^2 + 5*b^3*x^3))/(40*b^4*(a + b*x)^(1/3)
)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 51, normalized size = 0.73

$$\frac{3\left(40a^3 + 60a^2(a + bx) - 24a(a + bx)^2 + 5(a + bx)^3\right)}{40b^4\sqrt[3]{a + bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*(40*a^3 + 60*a^2*(a + b*x) - 24*a*(a + b*x)^2 + 5*(a + b*x)^3))/(40*b^4*
(a + b*x)^(1/3))
```

**fricas [A]** time = 0.86, size = 52, normalized size = 0.74

$$\frac{3\left(5\,b^3x^3 - 9\,ab^2x^2 + 27\,a^2bx + 81\,a^3\right)(bx + a)^{\frac{2}{3}}}{40\left(b^5x + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^3*x^3 - 9*a*b^2*x^2 + 27*a^2*b*x + 81*a^3)*(b*x + a)^(2/3)/(b^5*x
 + a*b^4)
```

**giac [A]** time = 1.10, size = 62, normalized size = 0.89

$$\frac{3\,a^3}{(bx + a)^{\frac{1}{3}}b^4} + \frac{3\left(5\,(bx + a)^{\frac{8}{3}}b^{28} - 24\,(bx + a)^{\frac{5}{3}}ab^{28} + 60\,(bx + a)^{\frac{2}{3}}a^2b^{28}\right)}{40\,b^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3*a^3/((b*x + a)^(1/3)*b^4) + 3/40*(5*(b*x + a)^(8/3)*b^28 - 24*(b*x + a)^(
5/3)*a*b^28 + 60*(b*x + a)^(2/3)*a^2*b^28)/b^32
```

**maple [A]** time = 0.00, size = 43, normalized size = 0.61

$$\frac{\frac{3}{8}b^3x^3 - \frac{27}{40}a\,b^2x^2 + \frac{81}{40}a^2bx + \frac{243}{40}a^3}{(bx + a)^{\frac{1}{3}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(4/3),x)
```

```
[Out] 3/40/(b*x+a)^(1/3)*(5*b^3*x^3-9*a*b^2*x^2+27*a^2*b*x+81*a^3)/b^4
```

**maxima [A]** time = 1.36, size = 56, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^4}-\frac{9\,(bx+a)^{\frac{5}{3}}a}{5\,b^4}+\frac{9\,(bx+a)^{\frac{2}{3}}a^2}{2\,b^4}+\frac{3\,a^3}{(bx+a)^{\frac{1}{3}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^4 - 9/5*(b*x + a)^(5/3)*a/b^4 + 9/2*(b*x + a)^(2/3)*a
^2/b^4 + 3*a^3/((b*x + a)^(1/3)*b^4)
```

**mupad [B]** time = 0.05, size = 56, normalized size = 0.80

$$\frac{3\,(a+b\,x)^{8/3}}{8\,b^4}+\frac{9\,a^2\,(a+b\,x)^{2/3}}{2\,b^4}+\frac{3\,a^3}{b^4\,(a+b\,x)^{1/3}}-\frac{9\,a\,(a+b\,x)^{5/3}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(8/3))/(8*b^4) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) + (3*a^3)/(b^
4*(a + b*x)^(1/3)) - (9*a*(a + b*x)^(5/3))/(5*b^4)
```

**sympy [B]** time = 2.89, size = 1538, normalized size = 21.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(4/3),x)
```

```
[Out] 243*a**(68/3)*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a*
*18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*
x**5 + 40*a**14*b**10*x**6) - 243*a**(68/3)/(40*a**20*b**4 + 240*a**19*b**5
*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*
a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 1296*a**(65/3)*b*x*(1 + b*x/a)**(2
/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**
7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) -
 1458*a**(65/3)*b*x/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2
 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**
14*b**10*x**6) + 2808*a**(62/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**20*b**4
```

```
 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16
*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 3645*a**(62/3)*b*
*2*x**2/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17
*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**
6) + 3120*a**(59/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19
*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 +
 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 4860*a**(59/3)*b**3*x**3/(40*
a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 +
 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 1830*a*
*(56/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 60
0*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b
**9*x**5 + 40*a**14*b**10*x**6) - 3645*a**(56/3)*b**4*x**4/(40*a**20*b**4 +
 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b
**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 528*a**(53/3)*b**5*
x**5*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*
x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40
*a**14*b**10*x**6) - 1458*a**(53/3)*b**5*x**5/(40*a**20*b**4 + 240*a**19*b*
*5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 24
0*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 96*a**(50/3)*b**6*x**6*(1 + b*x/
a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**
17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x
**6) - 243*a**(50/3)*b**6*x**6/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**1
8*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x*
*5 + 40*a**14*b**10*x**6) + 48*a**(47/3)*b**7*x**7*(1 + b*x/a)**(2/3)/(40*a
**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 +
600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 15*a**(4
4/3)*b**8*x**8*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a
**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9
*x**5 + 40*a**14*b**10*x**6)
```

## 3.414 $\int \frac{x^2}{(a+bx)^{4/3}} dx$

**Optimal**. Leaf size=49

$$-\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}$$

**Rubi** [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(4/3),x]
```

```
[Out] (-3*a^2)/(b^3*(a + b*x)^(1/3)) - (3*a*(a + b*x)^(2/3))/b^3 + (3*(a + b*x)^(
5/3))/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{4/3}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{4/3}}-\frac{2a}{b^2\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b^2}\right)dx\\ &= -\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 34, normalized size = 0.69

$$\frac{3\left(-9a^2-3abx+b^2x^2\right)}{5b^3\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(4/3),x]
[Out] (3*(-9*a^2 - 3*a*b*x + b^2*x^2))/(5*b^3*(a + b*x)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.03, size = 37, normalized size = 0.76

$$\frac{3\left(-5a^2-5a(a+bx)+(a+bx)^2\right)}{5b^3\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^(4/3),x]
[Out] (3*(-5*a^2 - 5*a*(a + b*x) + (a + b*x)^2))/(5*b^3*(a + b*x)^(1/3))
```

**fricas [A]** time = 0.83, size = 40, normalized size = 0.82

$$\frac{3\left(b^2x^2-3\,abx-9\,a^2\right)(bx+a)^{\frac{2}{3}}}{5\left(b^4x+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="fricas")
[Out] 3/5*(b^2*x^2 - 3*a*b*x - 9*a^2)*(b*x + a)^(2/3)/(b^4*x + a*b^3)
```

**giac [A]** time = 1.00, size = 46, normalized size = 0.94

$$-\frac{3\,a^2}{(bx+a)^{\frac{1}{3}}b^3}+\frac{3\left((bx+a)^{\frac{5}{3}}b^{12}-5\,(bx+a)^{\frac{2}{3}}ab^{12}\right)}{5\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="giac")
[Out] -3*a^2/((b*x + a)^(1/3)*b^3) + 3/5*((b*x + a)^(5/3)*b^12 - 5*(b*x + a)^(2/3
)*a*b^12)/b^15
```

**maple [A]** time = 0.01, size = 32, normalized size = 0.65

$$-\frac{3\left(-b^2x^2+3abx+9a^2\right)}{5\,(bx+a)^{\frac{1}{3}}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(4/3),x)
```

```
[Out] -3/5/(b*x+a)^(1/3)*(-b^2*x^2+3*a*b*x+9*a^2)/b^3
```

**maxima [A]** time = 1.33, size = 41, normalized size = 0.84

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b^3} - \frac{3\,(bx+a)^{\frac{2}{3}}a}{b^3} - \frac{3\,a^2}{(bx+a)^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b^3 - 3*(b*x + a)^(2/3)*a/b^3 - 3*a^2/((b*x + a)^(1/3)*
b^3)
```

**mupad [B]** time = 0.04, size = 35, normalized size = 0.71

$$-\frac{15\,a\,\left(a+b\,x\right)-3\,{\left(a+b\,x\right)}^2+15\,a^2}{5\,b^3\,{\left(a+b\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(4/3),x)
```

```
[Out] -(15*a*(a + b*x) - 3*(a + b*x)^2 + 15*a^2)/(5*b^3*(a + b*x)^(1/3))
```

**sympy [B]** time = 1.88, size = 534, normalized size = 10.90

$$-\frac{27a^{\frac{29}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}+\frac{27a^{\frac{29}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}-\frac{63a^{\frac{26}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}+\frac{81a^{\frac{26}{3}}bx}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}-\frac{42a^{\frac{23}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}+\frac{81a^{\frac{23}{3}}b^2x^2}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}-\frac{3a^{\frac{20}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}+\frac{27a^{\frac{20}{3}}b^3x^3}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}+\frac{3a^{\frac{17}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(4/3),x)
```

```
[Out] -27*a**(29/3)*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b*
*5*x**2 + 5*a**5*b**6*x**3) + 27*a**(29/3)/(5*a**8*b**3 + 15*a**7*b**4*x +
15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) - 63*a**(26/3)*b*x*(1 + b*x/a)**(2/3)
/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 81
*a**(26/3)*b*x/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b
**6*x**3) - 42*a**(23/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**
7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 81*a**(23/3)*b**2*x**2/(
5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) - 3*a*
*(20/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**
6*b**5*x**2 + 5*a**5*b**6*x**3) + 27*a**(20/3)*b**3*x**3/(5*a**8*b**3 + 15*
a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 3*a**(17/3)*b**4*x**4
*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a
**5*b**6*x**3)
```

## 3.415 $\int \frac{x}{(a+bx)^{4/3}}\,dx$

**Optimal**. Leaf size=32

$$\frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(4/3),x]
```

```
[Out] (3*a)/(b^2*(a + b*x)^(1/3)) + (3*(a + b*x)^(2/3))/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{4/3}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{4/3}}+\frac{1}{b\sqrt[3]{a+bx}}\right)dx\\ &= \frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.72

$$\frac{3(3a+bx)}{2b^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(4/3),x]
```

```
[Out] (3*(3*a + b*x))/(2*b^2*(a + b*x)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.01, size = 23, normalized size = 0.72

$$\frac{3(3a+bx)}{2b^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x)^(4/3),x]
[Out] (3*(3*a + b*x))/(2*b^2*(a + b*x)^(1/3))
```

**fricas [A]** time = 0.83, size = 29, normalized size = 0.91

$$\frac{3\,(bx+3\,a)(bx+a)^{\frac{2}{3}}}{2\left(b^3x+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="fricas")
[Out] 3/2*(b*x + 3*a)*(b*x + a)^(2/3)/(b^3*x + a*b^2)
```

**giac [A]** time = 0.93, size = 30, normalized size = 0.94

$$\frac{3\left(\frac{(bx+a)^{\frac{2}{3}}}{b}+\frac{2\,a}{(bx+a)^{\frac{1}{3}}b}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="giac")
[Out] 3/2*((b*x + a)^(2/3)/b + 2*a/((b*x + a)^(1/3)*b))/b
```

**maple [A]** time = 0.00, size = 20, normalized size = 0.62

$$\frac{\frac{3bx}{2}+\frac{9a}{2}}{(bx+a)^{\frac{1}{3}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(4/3),x)
[Out] 3/2/(b*x+a)^(1/3)*(b*x+3*a)/b^2
```

**maxima** [A] time = 1.30, size = 26, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b^2}+\frac{3\,a}{(bx+a)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b^2 + 3*a/((b*x + a)^(1/3)*b^2)
```

**mupad** [B] time = 0.03, size = 20, normalized size = 0.62

$$\frac{9\,a+3\,b\,x}{2\,b^2\,(a+b\,x)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(4/3),x)
```

```
[Out] (9*a + 3*b*x)/(2*b^2*(a + b*x)^(1/3))
```

**sympy** [A] time = 0.72, size = 41, normalized size = 1.28

$$\begin{cases} \frac{9a}{2b^2\sqrt[3]{a+bx}}+\frac{3x}{2b\sqrt[3]{a+bx}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(4/3),x)
```

```
[Out] Piecewise((9*a/(2*b**2*(a + b*x)**(1/3)) + 3*x/(2*b*(a + b*x)**(1/3)), Ne(b
, 0)), (x**2/(2*a**(4/3)), True))
```

## 3.416 $\int \frac{1}{(a+bx)^{4/3}} dx$

**Optimal**. Leaf size=14

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4/3),x]
```

```
[Out] -3/(b*(a + b*x)^(1/3))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}} dx = -\frac{3}{b\sqrt[3]{a+bx}}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4/3),x]
```

```
[Out] -3/(b*(a + b*x)^(1/3))
```

**IntegrateAlgebraic** [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(-4/3),x]
[Out] -3/(b*(a + b*x)^(1/3))
```

**fricas [A]** time = 1.05, size = 20, normalized size = 1.43

$$-\frac{3 (bx+a)^{\frac{2}{3}}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="fricas")
[Out] -3*(b*x + a)^(2/3)/(b^2*x + a*b)
```

**giac [A]** time = 0.76, size = 12, normalized size = 0.86

$$-\frac{3}{(bx+a)^{\frac{1}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="giac")
[Out] -3/((b*x + a)^(1/3)*b)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$-\frac{3}{(bx+a)^{\frac{1}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3),x)
[Out] -3/b/(b*x+a)^(1/3)
```

**maxima [A]** time = 1.35, size = 12, normalized size = 0.86

$$-\frac{3}{(bx+a)^{\frac{1}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -3/((b*x + a)^(1/3)*b)
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.86

$$-\frac{3}{b\,{\left(a+b\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(4/3),x)
```

```
[Out] -3/(b*(a + b*x)^(1/3))
```

**sympy [A]** time = 0.07, size = 12, normalized size = 0.86

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3),x)
```

```
[Out] -3/(b*(a + b*x)**(1/3))
```

## 3.417 $\int \frac{1}{x(a+bx)^{4/3}}\,dx$

**Optimal**. Leaf size=93

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{a\sqrt[3]{a+bx}}$$

**Rubi [A]** time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{a\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*(a + b*x)^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt
[3]*a^(1/3))])/a^(4/3) - Log[x]/(2*a^(4/3)) + (3*Log[a^(1/3) - (a + b*x)^(1
/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^{4/3}}\,dx &= \frac{3}{a\sqrt[3]{a+bx}}+\frac{\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{a}\\ &= \frac{3}{a\sqrt[3]{a+bx}}-\frac{\log(x)}{2a^{4/3}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{a+bx}\right)}{2a}\\ &= \frac{3}{a\sqrt[3]{a+bx}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{4/3}}\\ &= \frac{3}{a\sqrt[3]{a+bx}}+\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.32

$$\frac{3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx}{a}+1\right)}{a\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(4/3)),x]
```

```
[Out] (3*Hypergeometric2F1[-1/3, 1, 2/3, 1 + (b*x)/a])/(a*(a + b*x)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 118, normalized size = 1.27

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{4/3}}-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{a^{4/3}}+\frac{3}{a\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*(a + b*x)^(1/3)) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sq
rt[3]*a^(1/3))])/a^(4/3) + Log[a^(1/3) - (a + b*x)^(1/3)]/a^(4/3) - Log[a^(
2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]/(2*a^(4/3))
```

**fricas [A]** time = 0.95, size = 285, normalized size = 3.06

$$\left[\frac{\sqrt{3}\left(abx+a^{2}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+6\,(bx+a)^{\frac{2}{3}}a}{2\left(a^{2}bx+a^{3}\right)},\ -\frac{(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\,(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{2\sqrt{3}\left(abx+a^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-6\,(bx+a)^{\frac{2}{3}}a}{2\left(a^{2}bx+a^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*(a*b*x + a^2)*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x +
 a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x
+ a)^(1/3)*a^(2/3) + 3*a)/x) - (b*x + a)*a^(2/3)*log((b*x + a)^(2/3) + (b*x
 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b*x + a)*a^(2/3)*log((b*x + a)^(1/3) -
a^(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*b*x + a^3), -1/2*((b*x + a)*a^(2/3)*lo
g((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*(b*x + a)*a^(2/3
)*log((b*x + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(a*b*x + a^2)*arctan(1/3*sqrt(
3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 6*(b*x + a)^(2/3)*a)/(a
^2*b*x + a^3)]
```

**giac [A]** time = 2.38, size = 89, normalized size = 0.96

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{4}{3}}}+\frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3}{(bx+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + log
(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/((b*x + a)^(1/3)*a)
```

**maple [A]** time = 0.01, size = 87, normalized size = 0.94

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{4}{3}}}+\frac{\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{a^{\frac{4}{3}}}-\frac{\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2a^{\frac{4}{3}}}+\frac{3}{(bx+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(4/3),x)
```

```
[Out] 3/a/(b*x+a)^(1/3)+1/a^(4/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/2/a^(4/3)*ln(a^(2/
3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))+1/a^(4/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

**maxima [A]** time = 3.03, size = 88, normalized size = 0.95

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{4}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{4}{3}}}+\frac{3}{(bx+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + log
((b*x + a)^(1/3) - a^(1/3))/a^(4/3) + 3/((b*x + a)^(1/3)*a)
```

**mupad [B]** time = 0.06, size = 114, normalized size = 1.23

$$\frac{\ln\left(9a(a+bx)^{1/3}-9a^{4/3}\right)}{a^{4/3}}+\frac{3}{a(a+bx)^{1/3}}+\frac{\ln\left(9a(a+bx)^{1/3}-\frac{9a^{4/3}(-1+\sqrt{3}\,1\mathrm{i})^2}{4}\right)(-1+\sqrt{3}\,1\mathrm{i})}{2a^{4/3}}-\frac{\ln\left(9a(a+bx)^{1/3}-\frac{9a^{4/3}(1+\sqrt{3}\,1\mathrm{i})^2}{4}\right)(1+\sqrt{3}\,1\mathrm{i})}{2a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(4/3)),x)
```

```
[Out] log(9*a*(a + b*x)^(1/3) - 9*a^(4/3))/a^(4/3) + 3/(a*(a + b*x)^(1/3)) + (log
(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/
```

```
(2*a^(4/3)) - (log(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i + 1)^2)/4)*
(3^(1/2)*1i + 1))/(2*a^(4/3))
```

**sympy [C]** time = 2.21, size = 184, normalized size = 1.98

$$-\frac{\Gamma\left(-\frac{1}{3}\right)}{a\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(4/3),x)
```

```
[Out] -gamma(-1/3)/(a*b**(1/3)*(a/b + x)**(1/3)*gamma(2/3)) - log(1 - b**(1/3)*(a
/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3)) - exp(2*I*pi/3
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/
3)/(3*a**(4/3)*gamma(2/3)) - exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/
3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3))
```

## 3.418 $\int \frac{1}{x^2(a+bx)^{4/3}}\,dx$

**Optimal**. Leaf size=113

$$\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{4b}{a^2\sqrt[3]{a+bx}}-\frac{1}{ax\sqrt[3]{a+bx}}$$

**Rubi [A]** time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$-\frac{4(a+bx)^{2/3}}{a^2x}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{3}{ax\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*x*(a + b*x)^(1/3)) - (4*(a + b*x)^(2/3))/(a^2*x) - (4*b*ArcTan[(a^(1/3
) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b*Log[x])
/(3*a^(7/3)) - (2*b*Log[a^(1/3) - (a + b*x)^(1/3)])/a^(7/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2(a+bx)^{4/3}}\,dx &= \frac{3}{ax\sqrt[3]{a+bx}} + \frac{4\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{a} \\
&= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} - \frac{(4b)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{3a^2} \\
&= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} + \frac{2b\log(x)}{3a^{7/3}} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{a^{7/3}} - \frac{(2b)\,\mathrm{Sub}
\\
&= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} + \frac{2b\log(x)}{3a^{7/3}} - \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}} + \frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\right.}{a^{7/3}} \\
&= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} - \frac{4b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{2b\log(x)}{3a^{7/3}} - \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 31, normalized size = 0.27

$$
-\frac{3b\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{bx}{a}+1\right)}{a^2\sqrt[3]{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(4/3)),x]
```

```
[Out] (-3*b*Hypergeometric2F1[-1/3, 2, 2/3, 1 + (b*x)/a])/(a^2*(a + b*x)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 138, normalized size = 1.22

$$-\frac{4b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}+\frac{2b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3a^{7/3}}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{3a-4(a+bx)}{a^2x\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^(4/3)),x]
```

```
[Out] (3*a - 4*(a + b*x))/(a^2*x*(a + b*x)^(1/3)) - (4*b*ArcTan[1/Sqrt[3] + (2*(a
 + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) - (4*b*Log[a^(1/3) - (
a + b*x)^(1/3)])/(3*a^(7/3)) + (2*b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) +
 (a + b*x)^(2/3)])/(3*a^(7/3))
```

**fricas [B]** time = 0.83, size = 407, normalized size = 3.60

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^2x^2+a^2bx\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x}\right)+2\left(b^2x^2+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4\left(b^2x^2+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-3\left(4abx+a^2\right)(bx+a)^{\frac{2}{3}}}{3\left(a^3bx^2+a^4x\right)},-\frac{12\sqrt{\frac{1}{3}}\left(ab^2x^2+a^2bx\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-2\left(b^2x^2+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+4\left(b^2x^2+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+3\left(4abx+a^2\right)(bx+a)^{\frac{2}{3}}}{3\left(a^3bx^2+a^4x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/3*(6*sqrt(1/3)*(a*b^2*x^2 + a^2*b*x)*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*s
qrt(1/3)*(2*(b*x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*
sqrt((-a)^(1/3)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + 2*(b^2*x^2 +
a*b*x)*(-a)^(2/3)*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(
2/3)) - 4*(b^2*x^2 + a*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3)) -
3*(4*a*b*x + a^2)*(b*x + a)^(2/3))/(a^3*b*x^2 + a^4*x), -1/3*(12*sqrt(1/3)*
(a*b^2*x^2 + a^2*b*x)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/
3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - 2*(b^2*x^2 + a*b*x)*(-a)^(2/3)*log(
(b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 4*(b^2*x^2 + a
*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3)) + 3*(4*a*b*x + a^2)*(b*x
 + a)^(2/3))/(a^3*b*x^2 + a^4*x)]
```

**giac [A]** time = 2.40, size = 120, normalized size = 1.06

$$-\frac{4\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{7}{3}}}+\frac{2b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{7}{3}}}-\frac{4b\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3a^{\frac{7}{3}}}-\frac{4(bx+a)b-3ab}{\left((bx+a)^{\frac{4}{3}}-(bx+a)^{\frac{1}{3}}a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(7/3) + 2/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(7
/3) - 4/3*b*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) - (4*(b*x + a)*b -
3*a*b)/(((b*x + a)^(4/3) - (b*x + a)^(1/3)*a)*a^2)
```

**maple** [A] time = 0.01, size = 108, normalized size = 0.96

$$-\frac{4\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{7}{3}}}-\frac{4b\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3a^{\frac{7}{3}}}+\frac{2b\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{3a^{\frac{7}{3}}}-\frac{3b}{(bx+a)^{\frac{1}{3}}a^{2}}-\frac{(bx+a)^{\frac{2}{3}}}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(4/3),x)
```

```
[Out] -3*b/a^2/(b*x+a)^(1/3)-1/a^2*(b*x+a)^(2/3)/x-4/3*b/a^(7/3)*ln(-a^(1/3)+(b*x
+a)^(1/3))+2/3*b/a^(7/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))-4/
3*b/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

**maxima** [A] time = 2.99, size = 122, normalized size = 1.08

$$-\frac{4\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,(bx+a)b-3\,ab}{(bx+a)^{\frac{4}{3}}a^{2}-(bx+a)^{\frac{1}{3}}a^{3}}+\frac{2\,b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(7/3) - (4*(b*x + a)*b - 3*a*b)/((b*x + a)^(4/3)*a^2 - (b*x + a)^(1/3)*a^3)
 + 2/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(7/3) -
 4/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(7/3)
```

**mupad** [B] time = 0.07, size = 173, normalized size = 1.53

$$-\frac{\frac{3\,b}{a}-\frac{4\,b\,(a+b\,x)}{a^2}}{a\,(a+b\,x)^{1/3}-(a+b\,x)^{4/3}}+\frac{\ln\left(a^{7/3}\left(2\,b-\sqrt{3}\,b\,2\mathrm{i}\right)^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)\left(2\,b-\sqrt{3}\,b\,2\mathrm{i}\right)}{3\,a^{7/3}}+\frac{\ln\left(a^{7/3}\left(2\,b+\sqrt{3}\,b\,2\mathrm{i}\right)^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)\left(2\,b+\sqrt{3}\,b\,2\mathrm{i}\right)}{3\,a^{7/3}}-\frac{4\,b\,\ln\left(16\,a^{7/3}\,b^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)}{3\,a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(4/3)),x)
```

```
[Out] (log(a^(7/3)*(2*b - 3^(1/2)*b*2i)^2 - 16*a^2*b^2*(a + b*x)^(1/3))*(2*b - 3^
(1/2)*b*2i))/(3*a^(7/3)) - ((3*b)/a - (4*b*(a + b*x))/a^2)/(a*(a + b*x)^(1/
```

```
3) - (a + b*x)^(4/3)) + (log(a^(7/3)*(2*b + 3^(1/2)*b*2i)^2 - 16*a^2*b^2*(a
 + b*x)^(1/3))*(2*b + 3^(1/2)*b*2i))/(3*a^(7/3)) - (4*b*log(16*a^(7/3)*b^2
- 16*a^2*b^2*(a + b*x)^(1/3)))/(3*a^(7/3))
```

**sympy** [C] time = 2.52, size = 857, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(4/3),x)
```

```
[Out] -9*a**(4/3)*b**(2/3)*exp(2*I*pi/3)*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/
3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*g
amma(2/3)) + 12*a**(1/3)*b**(5/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-9*a
**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)
**(4/3)*exp(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b +
x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*
pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*
(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a
/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*log(1 - b**(1/3)*(a/b + x)*
*(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(
1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)
*gamma(2/3)) + 4*b**2*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi
/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3)) +
4*b**2*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*ex
p_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(
2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/
3)) + 4*b**2*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4
*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)
*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3))
```

## 3.419 $\int \frac{1}{x^3(a+bx)^{4/3}} dx$

**Optimal**. Leaf size=149

$$-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}+\frac{14b^2}{3a^3\sqrt[3]{a+bx}}+\frac{7b}{6a^2x\sqrt[3]{a+bx}}-\frac{1}{2ax^2\sqrt[3]{a+bx}}$$

**Rubi** [A] time = 0.06, antiderivative size = 147, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}-\frac{7(a+bx)^{2/3}}{2a^2x^2}+\frac{14b(a+bx)^{2/3}}{3a^3x}+\frac{3}{ax^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*x^2*(a + b*x)^(1/3)) - (7*(a + b*x)^(2/3))/(2*a^2*x^2) + (14*b*(a + b*
x)^(2/3))/(3*a^3*x) + (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (7*b^2*Log[x])/(9*a^(10/3)) + (7*b^2*Log
[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx = \frac{3}{ax^2\sqrt[3]{a+bx}} + \frac{7\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx}{a}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} - \frac{(14b)\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{3a^2}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} + \frac{\left(14b^2\right)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a^3}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} - \frac{7b^2\log(x)}{9a^{10/3}} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a\right.}}{3a^{10/3}}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}} - \frac{(1}{}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} + \frac{14b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\,\mathrm{l}}{}$$

**Mathematica [C]** time = 0.01, size = 33, normalized size = 0.22

$$\frac{3b^2\,{}_2F_1\left(-\frac{1}{3},3;\frac{2}{3};\frac{bx}{a}+1\right)}{a^3\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b*x)^(4/3)),x]

[Out] (3*b^2*Hypergeometric2F1[-1/3, 3, 2/3, 1 + (b*x)/a])/(a^3*(a + b*x)^(1/3))

**IntegrateAlgebraic [A]** time = 0.26, size = 161, normalized size = 1.08

$$\frac{14b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{10/3}}-\frac{7b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9a^{10/3}}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}}+\frac{18a^2-49a(a+bx)+28(a+bx)^2}{6a^3x^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^(4/3)),x]

[Out] (18*a^2 - 49*a*(a + b*x) + 28*(a + b*x)^2)/(6*a^3*x^2*(a + b*x)^(1/3)) + (14*b^2*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) + (14*b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(10/3)) - (7*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(9*a^(10/3))

**fricas [A]** time = 1.05, size = 407, normalized size = 2.73

$$\left[\frac{42\sqrt{\frac{1}{3}}\left(ab^3x^3+a^2b^2x^2\right)\sqrt{-\frac{1}{a^{2/3}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{2/3}a^{2/3}-(bx+a)^{1/3}a-a^{4/3}\right)\sqrt{-\frac{1}{a^{2/3}}}-3(bx+a)^{1/3}a^{2/3}+3a}{x}\right)-14\left(b^3x^3+ab^2x^2\right)a^{2/3}\log\left((bx+a)^{2/3}+(bx+a)^{1/3}a^{1/3}+a^{2/3}\right)+28\left(b^3x^3+ab^2x^2\right)a^{2/3}\log\left((bx+a)^{1/3}-a^{1/3}\right)+3\left(28ab^2x^2+7a^2bx-3a^3\right)(bx+a)^{2/3}}{18\left(a^4bx^3+a^5x^2\right)},\ -\frac{14\left(b^3x^3+ab^2x^2\right)a^{2/3}\log\left((bx+a)^{2/3}+(bx+a)^{1/3}a^{1/3}+a^{2/3}\right)-28\left(b^3x^3+ab^2x^2\right)a^{2/3}\log\left((bx+a)^{1/3}-a^{1/3}\right)-\frac{84\sqrt{\frac{1}{3}}\left(ab^3x^3+a^2b^2x^2\right)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx+a)^{1/3}+a^{1/3}\right)}{a^{1/3}}\right)}{a^{1/3}}-3\left(28ab^2x^2+7a^2bx-3a^3\right)(bx+a)^{2/3}}{18\left(a^4bx^3+a^5x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="fricas")

[Out] [1/18*(42*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 14*(b^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 28*(b^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(28*a*b^2*x^2 + 7*a^2*b*x - 3*a^3)*(b*x + a)^(2/3))/(a^4*b*x^3 + a^5*x^2), -1/18*(14*(b^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*(b^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)) - 84*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(28*a*b^2*x^2 + 7*a^2*b*x - 3*a^3)*(b*x + a)^(2/3))/(a^4*b*x^3 + a^5*x^2)]

**giac [A]** time = 2.59, size = 140, normalized size = 0.94

$$\frac{14\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{10}{3}}}-\frac{7\,b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{10}{3}}}+\frac{14\,b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{10}{3}}}+\frac{3\,b^2}{(bx+a)^{\frac{1}{3}}a^3}+\frac{10\,(bx+a)^{\frac{5}{3}}b^2-13\,(bx+a)^{\frac{2}{3}}ab^2}{6\,a^3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 14/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(10/3) - 7/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(10/3) + 14/9*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(10/3) + 3*b^2/(
(b*x + a)^(1/3)*a^3) + 1/6*(10*(b*x + a)^(5/3)*b^2 - 13*(b*x + a)^(2/3)*a*b
^2)/(a^3*b^2*x^2)
```

**maple [A]** time = 0.01, size = 131, normalized size = 0.88

$$\frac{14\sqrt{3}\;b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{10}{3}}}+\frac{14b^2\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{10}{3}}}-\frac{7b^2\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\,a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{9a^{\frac{10}{3}}}+\frac{3b^2}{(bx+a)^{\frac{1}{3}}\,a^3}-\frac{13\,(bx+a)^{\frac{2}{3}}}{6a^2x^2}+\frac{5\,(bx+a)^{\frac{5}{3}}}{3a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(4/3),x)
```

```
[Out] 3*b^2/a^3/(b*x+a)^(1/3)+5/3/a^3/x^2*(b*x+a)^(5/3)-13/6/a^2/x^2*(b*x+a)^(2/3
)+14/9*b^2/a^(10/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-7/9*b^2/a^(10/3)*ln(a^(2/3)+
(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))+14/9*b^2/a^(10/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

**maxima [A]** time = 2.96, size = 158, normalized size = 1.06

$$\frac{14\,\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{10}{3}}}+\frac{28\,(bx+a)^2b^2-49\,(bx+a)ab^2+18\,a^2b^2}{6\left((bx+a)^{\frac{7}{3}}a^3-2\,(bx+a)^{\frac{4}{3}}a^4+(bx+a)^{\frac{1}{3}}a^5\right)}-\frac{7\,b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{10}{3}}}+\frac{14\,b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 14/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(10/3) + 1/6*(28*(b*x + a)^2*b^2 - 49*(b*x + a)*a*b^2 + 18*a^2*b^2)/((b*x
 + a)^(7/3)*a^3 - 2*(b*x + a)^(4/3)*a^4 + (b*x + a)^(1/3)*a^5) - 7/9*b^2*lo
g((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(10/3) + 14/9*b^2*
log((b*x + a)^(1/3) - a^(1/3))/a^(10/3)
```

**mupad [B]** time = 0.13, size = 221, normalized size = 1.48

$$\frac{\frac{3\,b^2}{a}+\frac{14\,b^2\,{\left(a+b\,x\right)}^2}{3\,a^3}-\frac{49\,b^2\,\left(a+b\,x\right)}{6\,a^2}}{{\left(a+b\,x\right)}^{7/3}-2\,a\,{\left(a+b\,x\right)}^{4/3}+a^2\,{\left(a+b\,x\right)}^{1/3}}+\frac{\ln\left(588\,a^3\,b^4\,{\left(a+b\,x\right)}^{1/3}-3\,a^{10/3}\,{\left(-7\,b^2+\sqrt{3}\,b^2\,7{}\mathrm{i}\right)}^2\right)\,\left(-7\,b^2+\sqrt{3}\,b^2\,7{}\mathrm{i}\right)}{9\,a^{10/3}}-\frac{\ln\left(588\,a^3\,b^4\,{\left(a+b\,x\right)}^{1/3}-3\,a^{10/3}\,{\left(7\,b^2+\sqrt{3}\,b^2\,7{}\mathrm{i}\right)}^2\right)\,\left(7\,b^2+\sqrt{3}\,b^2\,7{}\mathrm{i}\right)}{9\,a^{10/3}}+\frac{14\,b^2\,\ln\left(588\,a^3\,b^4\,{\left(a+b\,x\right)}^{1/3}-588\,a^{10/3}\,b^4\right)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(4/3)),x)
```

```
[Out] ((3*b^2)/a + (14*b^2*(a + b*x)^2)/(3*a^3) - (49*b^2*(a + b*x))/(6*a^2))/((a
 + b*x)^(7/3) - 2*a*(a + b*x)^(4/3) + a^2*(a + b*x)^(1/3)) + (log(588*a^3*b
^4*(a + b*x)^(1/3) - 3*a^(10/3)*(3^(1/2)*b^2*7i - 7*b^2)^2)*(3^(1/2)*b^2*7i
 - 7*b^2))/(9*a^(10/3)) - (log(588*a^3*b^4*(a + b*x)^(1/3) - 3*a^(10/3)*(3^
(1/2)*b^2*7i + 7*b^2)^2)*(3^(1/2)*b^2*7i + 7*b^2))/(9*a^(10/3)) + (14*b^2*l
og(588*a^3*b^4*(a + b*x)^(1/3) - 588*a^(10/3)*b^4))/(9*a^(10/3))
```

**sympy [C]** time = 3.19, size = 2793, normalized size = 18.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(4/3),x)
```

```
[Out] 54*a**(13/3)*b**(5/3)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(
1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi
/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/
3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 201*a*
*(10/3)*b**(8/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a/b +
x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 2
31*a**(7/3)*b**(11/3)*(a/b + x)**2*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)
*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2
/3)) - 84*a**(4/3)*b**(14/3)*(a/b + x)**3*exp(2*I*pi/3)*gamma(-1/3)/(-54*a*
*(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*ex
p(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*
gamma(2/3)) + 28*a**4*b**2*(a/b + x)**(1/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*
(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(
2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma
(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a
**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 28*a**4*b**2*(a
/b + x)**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2
*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3
)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) -
```

```
162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)
*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 28*a**4*b**2*(a/b + x)*
*(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamm
a(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(
19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)
*exp(2*I*pi/3)*gamma(2/3)) - 84*a**3*b**3*(a/b + x)**(4/3)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b +
 x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(
2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) -
84*a**3*b**3*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1
/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/
3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3)
 + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 84*a**3*
b**3*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)
/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(
2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(
16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a
/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*e
xp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a
**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b +
 x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*e
xp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)
*gamma(2/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*exp(-2*I*pi/3)*log(1 - b**(1/3
)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)
*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2
/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2
*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(
2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a*
*(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 28*a*b**5*(a/b +
 x)**(10/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma
(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(1
9/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b
+ x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*
exp(2*I*pi/3)*gamma(2/3)) - 28*a*b**5*(a/b + x)**(10/3)*exp(-2*I*pi/3)*log(
1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-5
4*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/
b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3
)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi
/3)*gamma(2/3)) - 28*a*b**5*(a/b + x)**(10/3)*log(1 - b**(1/3)*(a/b + x)**(
```

```
1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1
/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/
3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3
) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3))
```

## 3.420 $\int \frac{1}{x \sqrt[3]{a^3+b^3 x}} \, dx$

**Optimal**. Leaf size=71

$$\frac{3 \log \left(a-\sqrt[3]{a^3+b^3 x}\right)}{2 a}+\frac{\sqrt{3} \tan^{-1}\left(\frac{2 \sqrt[3]{a^3+b^3 x}+a}{\sqrt{3} a}\right)}{a}-\frac{\log (x)}{2 a}$$

**Rubi [A]** time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {55, 617, 204, 31}

$$\frac{3 \log \left(a-\sqrt[3]{a^3+b^3 x}\right)}{2 a}+\frac{\sqrt{3} \tan^{-1}\left(\frac{2 \sqrt[3]{a^3+b^3 x}+a}{\sqrt{3} a}\right)}{a}-\frac{\log (x)}{2 a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx = -\frac{\log(x)}{2a} + \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx, x, \sqrt[3]{a^3+b^3x}\right) - \frac{3\operatorname{Subst}\left(\int \frac{1}{a-x}\,dx, x, \sqrt[3]{a^3+b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a} + \frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a} - \frac{3\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a^3+b^3x}}{a}\right)}{a}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a} - \frac{\log(x)}{2a} + \frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}$$

**Mathematica [A]** time = 0.02, size = 66, normalized size = 0.93

$$\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}\,a}\right) - \log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - Log[x] + 3*Log
[a - (a^3 + b^3*x)^(1/3)])/(2*a)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 102, normalized size = 1.44

$$\frac{\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{a} + \frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}}{\sqrt{3}\,a}+\frac{1}{\sqrt{3}}\right)}{a} - \frac{\log\left(a\sqrt[3]{a^3+b^3x}+\left(a^3+b^3x\right)^{2/3}+a^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a + Log[a
 - (a^3 + b^3*x)^(1/3)]/a - Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 + b^3*x)
^(2/3)]/(2*a)
```

**fricas** [A] time = 0.97, size = 88, normalized size = 1.24

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)+2\log\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a) -
log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) + 2*log(-a + (b^3*x
+ a^3)^(1/3)))/a
```

**giac** [A] time = 1.03, size = 87, normalized size = 1.23

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(\left|-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2 +
 (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(abs(-a + (b^3*x + a^3
)^(1/3)))/a
```

**maple** [A] time = 0.01, size = 87, normalized size = 1.23

$$\frac{\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}+\frac{\ln\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{a}-\frac{\ln\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x+a^3)^(1/3),x)
```

```
[Out] 1/a*ln((b^3*x+a^3)^(1/3)-a)-1/2/a*ln((b^3*x+a^3)^(2/3)+(b^3*x+a^3)^(1/3)*a+
a^2)+arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))/a*3^(1/2))*3^(1/2)/a
```

**maxima [A]** time = 3.09, size = 86, normalized size = 1.21

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2 +
 (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(-a + (b^3*x + a^3)^(1
/3))/a
```

**mupad [B]** time = 0.10, size = 105, normalized size = 1.48

$$\frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}-9\,a\right)}{a}+\frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}-\frac{9\,a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}-\frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}-\frac{9\,a\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x + a^3)^(1/3)),x)
```

```
[Out] log(9*(b^3*x + a^3)^(1/3) - 9*a)/a + (log(9*(b^3*x + a^3)^(1/3) - (9*a*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a) - (log(9*(b^3*x + a^3)^(1/3) - (
9*a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a)
```

**sympy [C]** time = 2.13, size = 138, normalized size = 1.94

$$\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x+a**3)**(1/3),x)
```

```
[Out] exp(I*pi/3)*log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamm
a(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a**3
/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(2*I
*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

## 3.421 $\int \frac{1}{x \sqrt[3]{a^3-b^3 x}}\,dx$

**Optimal**. Leaf size=73

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}+\frac{\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}\,a}\right)}{a}-\frac{\log(x)}{2a}$$

**Rubi [A]** time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {55, 617, 204, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}+\frac{\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}\,a}\right)}{a}-\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx &= -\frac{\log(x)}{2a}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3-b^3x}\right)-\frac{3\,\text{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3-b^3x}\right)}{2a}\\ &= -\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}-\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3-b^3x}}{a}\right)}{a}\\ &= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 68, normalized size = 0.93

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)-\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - Log[x] + 3*Log
[a - (a^3 - b^3*x)^(1/3)])/(2*a)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 106, normalized size = 1.45

$$\frac{\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}+\frac{1}{\sqrt{3}}\right)}{a}-\frac{\log\left(a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}+a^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a + Log[a
 - (a^3 - b^3*x)^(1/3)]/a - Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)
^(2/3)]/(2*a)
```

**fricas** [A] time = 0.87, size = 92, normalized size = 1.26

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(-b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)+2\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a) -
 log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) + 2*log(-a + (-b^
3*x + a^3)^(1/3)))/a
```

**giac** [A] time = 0.77, size = 91, normalized size = 1.25

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(\left|-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(abs(-a + (-b^3*x +
 a^3)^(1/3)))/a
```

**maple** [A] time = 0.00, size = 91, normalized size = 1.25

$$\frac{\sqrt{3}\arctan\left(\frac{\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}+\frac{\ln\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{a}-\frac{\ln\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x+a^3)^(1/3),x)
```

```
[Out] 1/a*ln((-b^3*x+a^3)^(1/3)-a)-1/2/a*ln((-b^3*x+a^3)^(2/3)+(-b^3*x+a^3)^(1/3)
*a+a^2)+arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))/a*3^(1/2))*3^(1/2)/a
```

**maxima** [A] time = 2.99, size = 90, normalized size = 1.23

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(-a + (-b^3*x + a^3
)^(1/3))/a
```

**mupad** [B] time = 0.13, size = 108, normalized size = 1.48

$$\frac{\ln\left(9\left(a^3-b^3x\right)^{1/3}-9a\right)}{a}+\frac{\ln\left(9\left(a^3-b^3x\right)^{1/3}-\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a}-\frac{\ln\left(9\left(a^3-b^3x\right)^{1/3}-\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^3 - b^3*x)^(1/3)),x)
```

```
[Out] log(9*(a^3 - b^3*x)^(1/3) - 9*a)/a + (log(9*(a^3 - b^3*x)^(1/3) - (9*a*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a) - (log(9*(a^3 - b^3*x)^(1/3) - (
9*a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a)
```

**sympy** [C] time = 1.89, size = 136, normalized size = 1.86

$$-\frac{e^{-\frac{2i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x+a**3)**(1/3),x)
```

```
[Out] -exp(-2*I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*g
amma(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(I*pi)/(b*(-a**3
/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(5*I
*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

## 3.422 $\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx$

**Optimal**. Leaf size=74

$$-\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

**Rubi [A]** time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = \frac{\log(x)}{2a}+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx,x,\sqrt[3]{-a^3+b^3x}\right)-\frac{3\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt[3]{-a^3+b^3x}\right)}{2a}$$

$$=\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a}+\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}\right)}{a}$$

$$=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a}+\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a}$$

**Mathematica [C]** time = 0.01, size = 41, normalized size = 0.55

$$\frac{3\left(b^3x-a^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-\frac{b^3x}{a^3}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (3*(-a^3 + b^3*x)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - (b^3*x)/a^3])/(2
*a^3)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 111, normalized size = 1.50

$$-\frac{\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b^3x-a^3}}{\sqrt{3}\,a}\right)}{a}+\frac{\log\left(-a\sqrt[3]{b^3x-a^3}+\left(b^3x-a^3\right)^{2/3}+a^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a) - L
og[a + (-a^3 + b^3*x)^(1/3)]/a + Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a^3 +
 b^3*x)^(2/3)]/(2*a)
```

**fricas** [A] time = 0.83, size = 93, normalized size = 1.26

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)-2\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a) +
 log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) - 2*log(a + (b^3*x
- a^3)^(1/3)))/a
```

**giac** [A] time = 1.07, size = 95, normalized size = 1.28

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(\left|a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
- (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(abs(a + (b^3*x - a^3
)^(1/3)))/a
```

**maple** [A] time = 0.01, size = 97, normalized size = 1.31

$$\frac{\sqrt{3}\arctan\left(\frac{\left(-a+2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}-\frac{\ln\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}+\frac{\ln\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x-a^3)^(1/3),x)
```

```
[Out] 1/2/a*ln((b^3*x-a^3)^(2/3)-(b^3*x-a^3)^(1/3)*a+a^2)+1/a*3^(1/2)*arctan(1/3*
(2*(b^3*x-a^3)^(1/3)-a)*3^(1/2)/a)-ln(a+(b^3*x-a^3)^(1/3))/a
```

**maxima** [A] time = 2.90, size = 94, normalized size = 1.27

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^{2}-\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(a+\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
- (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(a + (b^3*x - a^3)^(1
/3))/a
```

**mupad** [B] time = 0.11, size = 112, normalized size = 1.51

$$-\frac{\ln\left(9\,a+9\,\left(b^{3}\,x-a^{3}\right)^{1/3}\right)}{a}-\frac{\ln\left(\frac{9\,a\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}+9\,\left(b^{3}\,x-a^{3}\right)^{1/3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}+\frac{\ln\left(\frac{9\,a\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}+9\,\left(b^{3}\,x-a^{3}\right)^{1/3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x - a^3)^(1/3)),x)
```

```
[Out] (log((9*a*(3^(1/2)*1i + 1)^2)/4 + 9*(b^3*x - a^3)^(1/3))*(3^(1/2)*1i + 1))/
(2*a) - (log((9*a*(3^(1/2)*1i - 1)^2)/4 + 9*(b^3*x - a^3)^(1/3))*(3^(1/2)*1
i - 1))/(2*a) - log(9*a + 9*(b^3*x - a^3)^(1/3))/a
```

**sympy** [C] time = 1.82, size = 134, normalized size = 1.81

$$-\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x-a**3)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gam
ma(-1/3)/(3*a*gamma(2/3)) + log(-a*exp_polar(I*pi)/(b*(-a**3/b**3 + x)**(1/
3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(5*I*pi
/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

## 3.423 $\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx$

**Optimal**. Leaf size=76

$$-\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

**Rubi [A]** time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = \frac{\log(x)}{2a}+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx,x,\sqrt[3]{-a^3-b^3x}\right)-\frac{3\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt[3]{-a^3-b^3x}\right)}{2a}$$

$$=\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a}+\frac{3\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right)}{a}$$

$$=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a}+\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a}$$

**Mathematica [C]** time = 0.01, size = 41, normalized size = 0.54

$$\frac{3\left(-a^3-b^3x\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{xb^3}{a^3}+1\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (3*(-a^3 - b^3*x)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 + (b^3*x)/a^3])/(2
*a^3)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 115, normalized size = 1.51

$$-\frac{\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log\left(-a\sqrt[3]{-a^3-b^3x}+\left(-a^3-b^3x\right)^{2/3}+a^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a) - L
og[a + (-a^3 - b^3*x)^(1/3)]/a + Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a^3 -
 b^3*x)^(2/3)]/(2*a)
```

**fricas [A]** time = 0.87, size = 97, normalized size = 1.28

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(-b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)-2\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
+ log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) - 2*log(a + (-b^
3*x - a^3)^(1/3)))/a
```

**giac [A]** time = 1.00, size = 99, normalized size = 1.30

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(\left|a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(abs(a + (-b^3*x -
 a^3)^(1/3)))/a
```

**maple [A]** time = 0.00, size = 101, normalized size = 1.33

$$\frac{\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}-\frac{\ln\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}+\frac{\ln\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x-a^3)^(1/3),x)
```

```
[Out] 1/2/a*ln((-b^3*x-a^3)^(2/3)-(-b^3*x-a^3)^(1/3)*a+a^2)+1/a*3^(1/2)*arctan(1/
3*(2*(-b^3*x-a^3)^(1/3)-a)*3^(1/2)/a)-ln(a+(-b^3*x-a^3)^(1/3))/a
```

**maxima** [A] time = 2.92, size = 98, normalized size = 1.29

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(a + (-b^3*x - a^3
)^(1/3))/a
```

**mupad** [B] time = 0.07, size = 115, normalized size = 1.51

$$-\frac{\ln\left(9\,a+9\left(-a^3-x\,b^3\right)^{1/3}\right)}{a}-\frac{\ln\left(\frac{9\,a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}+9\left(-a^3-x\,b^3\right)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}+\frac{\ln\left(\frac{9\,a\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}+9\left(-a^3-x\,b^3\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- b^3*x - a^3)^(1/3)),x)
```

```
[Out] (log((9*a*(3^(1/2)*1i + 1)^2)/4 + 9*(- b^3*x - a^3)^(1/3))*(3^(1/2)*1i + 1)
)/(2*a) - (log((9*a*(3^(1/2)*1i - 1)^2)/4 + 9*(- b^3*x - a^3)^(1/3))*(3^(1/
2)*1i - 1))/(2*a) - log(9*a + 9*(- b^3*x - a^3)^(1/3))/a
```

**sympy** [C] time = 1.83, size = 139, normalized size = 1.83

$$\frac{\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x-a**3)**(1/3),x)
```

```
[Out] log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a
*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a**3/b**3 + x)**(
1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) + exp(2*I*pi/3)*log(-a*exp_polar(2*
I*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

## 3.424 $\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx$

**Optimal**. Leaf size=72

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}-\frac{\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}\,a}\right)}{a^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {57, 617, 204, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}-\frac{\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}\,a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, \sqrt[3]{a^3+b^3x}\right)}{2a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx, x, \sqrt[3]{a^3+b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a^2} + \frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a^3+b^3x}}{a}\right)}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a^2} - \frac{\log(x)}{2a^2} + \frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}$$

**Mathematica** [A]   time = 0.03, size = 95, normalized size = 1.32

$$-\frac{-2\log\left(a-\sqrt[3]{a^3+b^3x}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)+\log\left(a\sqrt[3]{a^3+b^3x}+\left(a^3+b^3x\right)^{2/3}+a^2\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a -
 (a^3 + b^3*x)^(1/3)] + Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 + b^3*x)^(2/
3)])/a^2
```

**IntegrateAlgebraic** [A]   time = 0.05, size = 103, normalized size = 1.43

$$\frac{\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{a^2} - \frac{\log\left(a\sqrt[3]{a^3+b^3x}+\left(a^3+b^3x\right)^{2/3}+a^2\right)}{2a^2} - \frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}+\frac{1}{\sqrt{3}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) +
Log[a - (a^3 + b^3*x)^(1/3)]/a^2 - Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 +
 b^3*x)^(2/3)]/(2*a^2)
```

**fricas [A]** time = 0.76, size = 86, normalized size = 1.19

$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)-2\log\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a) +
 log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) - 2*log(-a + (b^3*x
 + a^3)^(1/3)))/a^2
```

**giac [A]** time = 1.00, size = 88, normalized size = 1.22

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(\left|-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (b^3*x
+ a^3)^(1/3)))/a^2
```

**maple [A]** time = 0.01, size = 88, normalized size = 1.22

$$-\frac{\sqrt{3}\arctan\left(\frac{\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}+\frac{\ln\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{a^2}-\frac{\ln\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x+a^3)^(2/3),x)
```

```
[Out] 1/a^2*ln(-a+(b^3*x+a^3)^(1/3))-1/2/a^2*ln(a^2+(b^3*x+a^3)^(1/3)*a+(b^3*x+a^
3)^(2/3))-arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))*3^(1/2)/a)*3^(1/2)/a^2
```

**maxima** [A] time = 2.88, size = 87, normalized size = 1.21

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^{2}}-\frac{\log\left(a^{2}+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a^{2}}+\frac{\log\left(-a+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(-a + (b^3*x + a^
3)^(1/3))/a^2
```

**mupad** [B] time = 0.14, size = 101, normalized size = 1.40

$$\frac{\ln\left(9a-9\left(a^{3}+x\,b^{3}\right)^{1/3}\right)}{a^{2}}+\frac{\ln\left(9\left(a^{3}+x\,b^{3}\right)^{1/3}-\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a^{2}}-\frac{\ln\left(9\left(a^{3}+x\,b^{3}\right)^{1/3}+\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x + a^3)^(2/3)),x)
```

```
[Out] log(9*a - 9*(b^3*x + a^3)^(1/3))/a^2 + (log(9*(b^3*x + a^3)^(1/3) - (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(b^3*x + a^3)^(1/3) +
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

**sympy** [C] time = 1.86, size = 134, normalized size = 1.86

$$\frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}\,e^{\frac{2i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}\,e^{\frac{4i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x+a**3)**(2/3),x)
```

```
[Out] log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(-2
*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3)/a)*gamma(1/3)
/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_p
olar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

## 3.425 $\int \frac{1}{x(a^3-b^3x)^{2/3}} \, dx$

**Optimal**. Leaf size=74

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {57, 617, 204, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2}-\frac{3\,\text{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3-b^3x}\right)}{2a^2}-\frac{3\,\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3-b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3-b^3x}}{a}\right)}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}$$

**Mathematica [A]** time = 0.02, size = 99, normalized size = 1.34

$$-\frac{-2\log\left(a-\sqrt[3]{a^3-b^3x}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)+\log\left(a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}+a^2\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a -
 (a^3 - b^3*x)^(1/3)] + Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)^(2/
3)])/a^2
```

**IntegrateAlgebraic [A]** time = 0.05, size = 107, normalized size = 1.45

$$\frac{\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{a^2}-\frac{\log\left(a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}+a^2\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}+\frac{1}{\sqrt{3}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) + Log[a - (a^3 - b^3*x)^(1/3)]/a^2 - Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)^(2/3)]/(2*a^2)

**fricas [A]** time = 0.96, size = 90, normalized size = 1.22

$$-\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,a+2\,\sqrt{3}\,\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}}{3\,a}\right)+\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)-2\,\log\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="fricas")

[Out] -1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a) + log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) - 2*log(-a + (-b^3*x + a^3)^(1/3)))/a^2

**giac [A]** time = 1.09, size = 92, normalized size = 1.24

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(a+2\,\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^{2}}-\frac{\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2\,a^{2}}+\frac{\log\left(\left|-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right|\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="giac")

[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (-b^3*x + a^3)^(1/3)))/a^2

**maple [A]** time = 0.00, size = 92, normalized size = 1.24

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}-\frac{\ln\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-b^3*x+a^3)^(2/3),x)

[Out] 1/a^2*ln(-a+(-b^3*x+a^3)^(1/3))-1/2/a^2*ln(a^2+(-b^3*x+a^3)^(1/3)*a+(-b^3*x+a^3)^(2/3))-arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))*3^(1/2)/a)*3^(1/2)/a^2

**maxima** [A] time = 2.98, size = 91, normalized size = 1.23

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^{2}}-\frac{\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a^{2}}+\frac{\log\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(-a + (-b^3*x
+ a^3)^(1/3))/a^2
```

**mupad** [B] time = 0.11, size = 104, normalized size = 1.41

$$\frac{\ln\left(9a-9\left(a^{3}-b^{3}x\right)^{1/3}\right)}{a^{2}}+\frac{\ln\left(9\left(a^{3}-b^{3}x\right)^{1/3}-\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a^{2}}-\frac{\ln\left(9\left(a^{3}-b^{3}x\right)^{1/3}+\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^3 - b^3*x)^(2/3)),x)
```

```
[Out] log(9*a - 9*(a^3 - b^3*x)^(1/3))/a^2 + (log(9*(a^3 - b^3*x)^(1/3) - (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(a^3 - b^3*x)^(1/3) +
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

**sympy** [C] time = 1.92, size = 136, normalized size = 1.84

$$\frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}\,e^{\frac{i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}\,e^{i\pi}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}\,e^{\frac{5i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x+a**3)**(2/3),x)
```

```
[Out] log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma(1/3)/(3*a**2*g
amma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi)/
a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(-a**3/b**3 + x
)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

## 3.426 $\int \frac{1}{x(-a^3+b^3x)^{2/3}}\,dx$

**Optimal**. Leaf size=74

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {58, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(-a^3+b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, \sqrt[3]{-a^3+b^3x}\right)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx, x, \sqrt[3]{-a^3+b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}\right)}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}$$

**Mathematica [A]** time = 0.03, size = 108, normalized size = 1.46

$$\frac{\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{a^2}-\frac{\log\left(-a\sqrt[3]{b^3x-a^3}+\left(b^3x-a^3\right)^{2/3}+a^2\right)}{2a^2}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b^3x-a^3}-a}{\sqrt{3}a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-a + 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2 + Log[a + (
-a^3 + b^3*x)^(1/3)]/a^2 - Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a^3 + b^3*x
)^(2/3)]/(2*a^2)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 110, normalized size = 1.49

$$\frac{\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{a^2}-\frac{\log\left(-a\sqrt[3]{b^3x-a^3}+\left(b^3x-a^3\right)^{2/3}+a^2\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) +
 Log[a + (-a^3 + b^3*x)^(1/3)]/a^2 - Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a
^3 + b^3*x)^(2/3)]/(2*a^2)
```

**fricas [A]** time = 0.89, size = 95, normalized size = 1.28

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)+2\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a) -
 log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) + 2*log(a + (b^3*x
- a^3)^(1/3)))/a^2
```

**giac [A]** time = 0.99, size = 94, normalized size = 1.27

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(\left|a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (b^3*x -
 a^3)^(1/3)))/a^2
```

**maple [A]** time = 0.01, size = 96, normalized size = 1.30

$$\frac{\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}+\frac{\ln\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}-\frac{\ln\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x-a^3)^(2/3),x)
```

```
[Out] -1/2/a^2*ln(a^2-(b^3*x-a^3)^(1/3)*a+(b^3*x-a^3)^(2/3))+1/a^2*3^(1/2)*arctan
(1/3*(-a+2*(b^3*x-a^3)^(1/3))*3^(1/2)/a)+ln(a+(b^3*x-a^3)^(1/3))/a^2
```

**maxima** [A] time = 2.95, size = 93, normalized size = 1.26

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(a + (b^3*x - a^3
)^(1/3))/a^2
```

**mupad** [B] time = 0.16, size = 107, normalized size = 1.45

$$\frac{\ln\left(9a+9\left(b^3x-a^3\right)^{1/3}\right)}{a^2}+\frac{\ln\left(9\left(b^3x-a^3\right)^{1/3}+\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}-\frac{\ln\left(9\left(b^3x-a^3\right)^{1/3}-\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x - a^3)^(2/3)),x)
```

```
[Out] log(9*a + 9*(b^3*x - a^3)^(1/3))/a^2 + (log(9*(b^3*x - a^3)^(1/3) + (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(b^3*x - a^3)^(1/3) -
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

**sympy** [C] time = 1.98, size = 134, normalized size = 1.81

$$-\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{\frac{i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{i\pi}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{\frac{5i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x-a**3)**(2/3),x)
```

```
[Out] -exp(-I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma(
1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi
)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3 + x
)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

## 3.427 $\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx$

**Optimal**. Leaf size=76

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {58, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, \sqrt[3]{-a^3-b^3x}\right)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx, x, \sqrt[3]{-a^3-b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right)}{a^2}$$

$$= -\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}$$

**Mathematica [A]** time = 0.03, size = 112, normalized size = 1.47

$$\frac{\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{a^2}-\frac{\log\left(-a\sqrt[3]{-a^3-b^3x}+\left(-a^3-b^3x\right)^{2/3}+a^2\right)}{2a^2}+\frac{\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{-a^3-b^3x}-a}{\sqrt{3}\,a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-a + 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2 + Log[a + (
-a^3 - b^3*x)^(1/3)]/a^2 - Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a^3 - b^3*x
)^(2/3)]/(2*a^2)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 114, normalized size = 1.50

$$\frac{\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{a^2}-\frac{\log\left(-a\sqrt[3]{-a^3-b^3x}+\left(-a^3-b^3x\right)^{2/3}+a^2\right)}{2a^2}-\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}\,a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) +
 Log[a + (-a^3 - b^3*x)^(1/3)]/a^2 - Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a
^3 - b^3*x)^(2/3)]/(2*a^2)
```

**fricas [A]** time = 1.15, size = 99, normalized size = 1.30

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(-b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)+2\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
- log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) + 2*log(a + (-b^
3*x - a^3)^(1/3)))/a^2
```

**giac [A]** time = 1.07, size = 98, normalized size = 1.29

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(\left|a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (-b^3
*x - a^3)^(1/3)))/a^2
```

**maple [A]** time = 0.00, size = 100, normalized size = 1.32

$$\frac{\sqrt{3}\arctan\left(\frac{\left(-a+2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}+\frac{\ln\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}-\frac{\ln\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x-a^3)^(2/3),x)
```

```
[Out] -1/2/a^2*ln(a^2-(-b^3*x-a^3)^(1/3)*a+(-b^3*x-a^3)^(2/3))+1/a^2*3^(1/2)*arct
an(1/3*(-a+2*(-b^3*x-a^3)^(1/3))*3^(1/2)/a)+ln(a+(-b^3*x-a^3)^(1/3))/a^2
```

**maxima** [A] time = 2.89, size = 97, normalized size = 1.28

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(a + (-b^3*x -
 a^3)^(1/3))/a^2
```

**mupad** [B] time = 0.16, size = 110, normalized size = 1.45

$$\frac{\ln\left(9a+9\left(-a^3-x\,b^3\right)^{1/3}\right)}{a^2}+\frac{\ln\left(9\left(-a^3-x\,b^3\right)^{1/3}+\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}-\frac{\ln\left(9\left(-a^3-x\,b^3\right)^{1/3}-\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- b^3*x - a^3)^(2/3)),x)
```

```
[Out] log(9*a + 9*(- b^3*x - a^3)^(1/3))/a^2 + (log(9*(- b^3*x - a^3)^(1/3) + (9*
a*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(- b^3*x - a^3)^(
1/3) - (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

**sympy** [C] time = 1.90, size = 133, normalized size = 1.75

$$\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}-\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{2i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{4i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x-a**3)**(2/3),x)
```

```
[Out] exp(-2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamma
(4/3)) - exp(-I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3)/
a)*gamma(1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(a**3/b**3 + x)**(1/3)*exp_po
lar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

## 3.428 $\int x^m(a+bx)\,dx$

**Optimal**. Leaf size=25

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

**Rubi [A]** time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)\,dx&=\int\left(ax^m+bx^{1+m}\right)\,dx\\&=\frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 22, normalized size = 0.88

$$x^{m+1}\left(\frac{a}{m+1}+\frac{bx}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x),x]
```

```
[Out] x^(1 + m)*(a/(1 + m) + (b*x)/(2 + m))
```

**IntegrateAlgebraic** [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x^m(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x),x]
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x), x]
```

**fricas** [A] time = 0.96, size = 33, normalized size = 1.32

$$\frac{\left((bm+b)x^2+(am+2\,a)x\right)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="fricas")
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

**giac** [A] time = 1.01, size = 43, normalized size = 1.72

$$\frac{bmx^2x^m+amxx^m+bx^2x^m+2\,axx^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="giac")
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

**maple** [A] time = 0.00, size = 31, normalized size = 1.24

$$\frac{(bmx+am+bx+2a)\,x^{m+1}}{(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a),x)
[Out] x^(1+m)*(b*m*x+a*m+b*x+2*a)/(2+m)/(1+m)
```

**maxima** [A] time = 1.34, size = 25, normalized size = 1.00

$$\frac{bx^{m+2}}{m+2}+\frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*x^(m + 2)/(m + 2) + a*x^(m + 1)/(m + 1)
```

**mupad** [B] time = 0.31, size = 30, normalized size = 1.20

$$\frac{x^{m+1}\,(2\,a+a\,m+b\,x+b\,m\,x)}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x),x)
```

```
[Out] (x^(m + 1)*(2*a + a*m + b*x + b*m*x))/(3*m + m^2 + 2)
```

**sympy** [A] time = 0.30, size = 87, normalized size = 3.48

$$\begin{cases} -\frac{a}{x}+b\log(x) & \text{for } m=-2\\ a\log(x)+bx & \text{for } m=-1\\ \frac{amxx^m}{m^2+3m+2}+\frac{2axx^m}{m^2+3m+2}+\frac{bmx^2x^m}{m^2+3m+2}+\frac{bx^2x^m}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a),x)
```

```
[Out] Piecewise((-a/x + b*log(x), Eq(m, -2)), (a*log(x) + b*x, Eq(m, -1)), (a*m*x
*x**m/(m**2 + 3*m + 2) + 2*a*x*x**m/(m**2 + 3*m + 2) + b*m*x**2*x**m/(m**2
+ 3*m + 2) + b*x**2*x**m/(m**2 + 3*m + 2), True))
```

## 3.429 $\int x^{5/2}(a+bx)\,dx$

**Optimal**. Leaf size=21

$$\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

**Rubi [A]** time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x),x]
[Out] (2*a*x^(7/2))/7 + (2*b*x^(9/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)\,dx&=\int\left(ax^{5/2}+bx^{7/2}\right)dx\\&=\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 0.81

$$\frac{2}{63}x^{7/2}(9a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x),x]
[Out] (2*x^(7/2)*(9*a + 7*b*x))/63
```

**IntegrateAlgebraic [A]** time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{63}\left(9ax^{7/2}+7bx^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x),x]
[Out] (2*(9*a*x^(7/2) + 7*b*x^(9/2)))/63
```

**fricas [A]** time = 0.85, size = 18, normalized size = 0.86

$$\frac{2}{63}\left(7\,bx^4+9\,ax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 2/63*(7*b*x^4 + 9*a*x^3)*sqrt(x)
```

**giac [A]** time = 0.83, size = 13, normalized size = 0.62

$$\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="giac")
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.67

$$\frac{2\,(7bx+9a)\,x^{\frac{7}{2}}}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a),x)
[Out] 2/63*x^(7/2)*(7*b*x+9*a)
```

**maxima [A]** time = 1.33, size = 13, normalized size = 0.62

$$\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

**mupad [B]** time = 0.09, size = 13, normalized size = 0.62

$$\frac{2\,x^{7/2}\,\left(9\,a+7\,b\,x\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x),x)
```

```
[Out] (2*x^(7/2)*(9*a + 7*b*x))/63
```

**sympy [A]** time = 1.59, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a),x)
```

```
[Out] 2*a*x**(7/2)/7 + 2*b*x**(9/2)/9
```

## 3.430 $\int x^{3/2}(a+bx)\,dx$

**Optimal**. Leaf size=21

$$\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}$$

**Rubi [A]** time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)\,dx&=\int\left(ax^{3/2}+bx^{5/2}\right)\,dx\\&=\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 0.81

$$\frac{2}{35}x^{5/2}(7a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x),x]
```

```
[Out] (2*x^(5/2)*(7*a + 5*b*x))/35
```

**IntegrateAlgebraic** [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{35}\left(7ax^{5/2}+5bx^{7/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x),x]
```

```
[Out] (2*(7*a*x^(5/2) + 5*b*x^(7/2)))/35
```

**fricas** [A] time = 1.01, size = 18, normalized size = 0.86

$$\frac{2}{35}\left(5\,bx^{3}+7\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b*x^3 + 7*a*x^2)*sqrt(x)
```

**giac** [A] time = 0.88, size = 13, normalized size = 0.62

$$\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

**maple** [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2\,(5bx+7a)\,x^{\frac{5}{2}}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a),x)
```

```
[Out] 2/35*x^(5/2)*(5*b*x+7*a)
```

**maxima** [A] time = 1.29, size = 13, normalized size = 0.62

$$\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

**mupad [B]** time = 0.03, size = 13, normalized size = 0.62

$$\frac{2\,x^{5/2}\,\left(7\,a+5\,b\,x\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x),x)
```

```
[Out] (2*x^(5/2)*(7*a + 5*b*x))/35
```

**sympy [A]** time = 0.55, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{5}{2}}}{5}+\frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(7/2)/7
```

## 3.431 $\int \sqrt{x}(a+bx)\,dx$

**Optimal**. Leaf size=21

$$\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

**Rubi [A]** time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x),x]

[Out] (2*a*x^(3/2))/3 + (2*b*x^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a+bx)\,dx &= \int \left(a\sqrt{x}+bx^{3/2}\right)dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 0.81

$$\frac{2}{15}x^{3/2}(5a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x),x]

[Out] (2*x^(3/2)*(5*a + 3*b*x))/15
```

**IntegrateAlgebraic** [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{15}\left(5ax^{3/2}+3bx^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x),x]
[Out] (2*(5*a*x^(3/2) + 3*b*x^(5/2)))/15
```

**fricas** [A] time = 0.99, size = 16, normalized size = 0.76

$$\frac{2}{15}\left(3\,bx^{2}+5\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b*x^2 + 5*a*x)*sqrt(x)
```

**giac** [A] time = 0.86, size = 13, normalized size = 0.62

$$\frac{2}{5}\,bx^{\frac{5}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="giac")
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

**maple** [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2\,(3bx+5a)\,x^{\frac{3}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*x^(1/2),x)
[Out] 2/15*x^(3/2)*(3*b*x+5*a)
```

**maxima** [A] time = 1.34, size = 13, normalized size = 0.62

$$\frac{2}{5}\,bx^{\frac{5}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

**mupad [B]** time = 0.02, size = 13, normalized size = 0.62

$$\frac{2\,x^{3/2}\,\left(5\,a+3\,b\,x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x),x)
```

```
[Out] (2*x^(3/2)*(5*a + 3*b*x))/15
```

**sympy [A]** time = 1.62, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x**(1/2),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(5/2)/5
```

## 3.432 $\int \frac{a+bx}{\sqrt{x}} \, dx$

**Optimal**. Leaf size=19

$$2a\sqrt{x} + \frac{2}{3}bx^{3/2}$$

**Rubi [A]** time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$2a\sqrt{x} + \frac{2}{3}bx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{x}} \, dx &= \int \left(\frac{a}{\sqrt{x}} + b\sqrt{x}\right) dx \\ &= 2a\sqrt{x} + \frac{2}{3}bx^{3/2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{x}\,(3a + bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(3*a + b*x))/3
```

**IntegrateAlgebraic** [A] time = 0.01, size = 20, normalized size = 1.05

$$\frac{2}{3}\left(3a\sqrt{x}+bx^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/Sqrt[x],x]
[Out] (2*(3*a*Sqrt[x] + b*x^(3/2)))/3
```

**fricas** [A] time = 0.92, size = 12, normalized size = 0.63

$$\frac{2}{3}(bx+3a)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*x + 3*a)*sqrt(x)
```

**giac** [A] time = 1.08, size = 13, normalized size = 0.68

$$\frac{2}{3}bx^{\frac{3}{2}}+2a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="giac")
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

**maple** [A] time = 0.00, size = 13, normalized size = 0.68

$$\frac{2(bx+3a)\sqrt{x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(1/2),x)
[Out] 2/3*x^(1/2)*(b*x+3*a)
```

**maxima** [A] time = 1.32, size = 13, normalized size = 0.68

$$\frac{2}{3}bx^{\frac{3}{2}}+2a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

**mupad [B]** time = 0.03, size = 12, normalized size = 0.63

$$\frac{2\sqrt{x}\,\left(3\,a+b\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(3*a + b*x))/3
```

**sympy [A]** time = 0.16, size = 17, normalized size = 0.89

$$2a\sqrt{x} + \frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(3/2)/3
```

## 3.433 $\int \frac{a+bx}{x^{3/2}}\,dx$

**Optimal**. Leaf size=17

$$2b\sqrt{x}-\frac{2a}{\sqrt{x}}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$2b\sqrt{x}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + 2*b*Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{3/2}}\,dx &= \int \left(\frac{a}{x^{3/2}}+\frac{b}{\sqrt{x}}\right)dx\\ &= -\frac{2a}{\sqrt{x}}+2b\sqrt{x}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 14, normalized size = 0.82

$$\frac{2(bx-a)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(3/2),x]
```

```
[Out] (2*(-a + b*x))/Sqrt[x]
```

**IntegrateAlgebraic [A]** time = 0.01, size = 14, normalized size = 0.82

$$\frac{2(bx-a)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(3/2),x]
[Out] (2*(-a + b*x))/Sqrt[x]
```

**fricas [A]** time = 0.55, size = 12, normalized size = 0.71

$$\frac{2\,(bx-a)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="fricas")
[Out] 2*(b*x - a)/sqrt(x)
```

**giac [A]** time = 0.92, size = 13, normalized size = 0.76

$$2\,b\sqrt{x}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="giac")
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

**maple [A]** time = 0.00, size = 12, normalized size = 0.71

$$-\frac{2\,(-bx+a)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(3/2),x)
[Out] -2*(-b*x+a)/x^(1/2)
```

**maxima [A]** time = 1.33, size = 13, normalized size = 0.76

$$2\,b\sqrt{x}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="maxima")
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

**mupad [B]** time = 0.03, size = 11, normalized size = 0.65

$$-\frac{2\,\left(a-b\,x\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(3/2),x)
[Out] -(2*(a - b*x))/x^(1/2)
```

**sympy [A]** time = 0.35, size = 15, normalized size = 0.88

$$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(3/2),x)
[Out] -2*a/sqrt(x) + 2*b*sqrt(x)
```

## 3.434 $\int \frac{a+bx}{x^{5/2}} \, dx$

**Optimal**. Leaf size=19

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}$$

**Rubi [A]** time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) - (2*b)/Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{5/2}} \, dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{x^{3/2}}\right) dx\\ &= -\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 15, normalized size = 0.79

$$-\frac{2(a+3bx)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(5/2),x]
```

```
[Out] (-2*(a + 3*b*x))/(3*x^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.01, size = 15, normalized size = 0.79

$$-\frac{2(a+3bx)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(5/2),x]
```

```
[Out] (-2*(a + 3*b*x))/(3*x^(3/2))
```

**fricas** [A] time = 0.83, size = 11, normalized size = 0.58

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

**giac** [A] time = 0.97, size = 11, normalized size = 0.58

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

**maple** [A] time = 0.00, size = 12, normalized size = 0.63

$$-\frac{2\,(3bx+a)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(5/2),x)
```

```
[Out] -2/3*(3*b*x+a)/x^(3/2)
```

**maxima** [A] time = 1.36, size = 11, normalized size = 0.58

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

**mupad [B]** time = 0.03, size = 13, normalized size = 0.68

$$-\frac{2\,a+6\,b\,x}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(5/2),x)
```

```
[Out] -(2*a + 6*b*x)/(3*x^(3/2))
```

**sympy [A]** time = 0.56, size = 19, normalized size = 1.00

$$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) - 2*b/sqrt(x)
```

## 3.435 $\int x^m (a+bx)^2 \, dx$

**Optimal**. Leaf size=43

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

**Rubi [A]** time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 +
m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^2\,dx &= \int \left(a^2x^m+2abx^{1+m}+b^2x^{2+m}\right)dx\\ &= \frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 38, normalized size = 0.88

$$x^{m+1}\left(\frac{a^2}{m+1}+\frac{2abx}{m+2}+\frac{b^2x^2}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

**IntegrateAlgebraic [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int x^m(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^2, x]
```

**fricas [A]** time = 0.78, size = 85, normalized size = 1.98

$$\frac{\left(\left(b^2m^2+3\,b^2m+2\,b^2\right)x^3+2\left(abm^2+4\,abm+3\,ab\right)x^2+\left(a^2m^2+5\,a^2m+6\,a^2\right)x\right)x^m}{m^3+6\,m^2+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2
*m^2 + 5*a^2*m + 6*a^2)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

**giac [B]** time = 1.10, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^3x^m+2\,abm^2x^2x^m+3\,b^2mx^3x^m+a^2m^2xx^m+8\,abmx^2x^m+2\,b^2x^3x^m+5\,a^2mxx^m+6\,abx^2x^m+6\,a^2xx^m}{m^3+6\,m^2+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^3*x^m + 2*a*b*m^2*x^2*x^m + 3*b^2*m*x^3*x^m + a^2*m^2*x*x^m + 8*
a*b*m*x^2*x^m + 2*b^2*x^3*x^m + 5*a^2*m*x*x^m + 6*a*b*x^2*x^m + 6*a^2*x*x^m
)/(m^3 + 6*m^2 + 11*m + 6)
```

**maple [A]** time = 0.00, size = 87, normalized size = 2.02

$$\frac{\left(b^2m^2x^2+2ab\,m^2x+3b^2m\,x^2+a^2m^2+8abmx+2b^2x^2+5a^2m+6abx+6a^2\right)x^{m+1}}{(m+3)\,(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^2,x)
```

```
[Out] x^(m+1)*(b^2*m^2*x^2+2*a*b*m^2*x+3*b^2*m*x^2+a^2*m^2+8*a*b*m*x+2*b^2*x^2+5*
a^2*m+6*a*b*x+6*a^2)/(3+m)/(m+2)/(m+1)
```

**maxima** [A] time = 1.36, size = 43, normalized size = 1.00

$$\frac{b^2 x^{m+3}}{m+3} + \frac{2\,abx^{m+2}}{m+2} + \frac{a^2 x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 3)/(m + 3) + 2*a*b*x^(m + 2)/(m + 2) + a^2*x^(m + 1)/(m + 1)
```

**mupad** [B] time = 0.42, size = 93, normalized size = 2.16

$$x^m \left( \frac{a^2\,x\,\left(m^2+5\,m+6\right)}{m^3+6\,m^2+11\,m+6} + \frac{b^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6} + \frac{2\,a\,b\,x^2\,\left(m^2+4\,m+3\right)}{m^3+6\,m^2+11\,m+6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^2,x)
```

```
[Out] x^m*((a^2*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (b^2*x^3*(3*m + m^2
 + 2))/(11*m + 6*m^2 + m^3 + 6) + (2*a*b*x^2*(4*m + m^2 + 3))/(11*m + 6*m^2
 + m^3 + 6))
```

**sympy** [A] time = 0.53, size = 299, normalized size = 6.95

$$\begin{cases} -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x) & \text{for } m = -3 \\ -\frac{a^2}{x} + 2ab\log(x) + b^2 x & \text{for } m = -2 \\ a^2 \log(x) + 2abx + \frac{b^2x^2}{2} & \text{for } m = -1 \\ \frac{a^2m^2xx^m}{m^3+6m^2+11m+6} + \frac{5a^2mxx^m}{m^3+6m^2+11m+6} + \frac{6a^2xx^m}{m^3+6m^2+11m+6} + \frac{2abm^2x^2x^m}{m^3+6m^2+11m+6} + \frac{8abmx^2x^m}{m^3+6m^2+11m+6} + \frac{6abx^2x^m}{m^3+6m^2+11m+6} + \frac{b^2m^2x^3x^m}{m^3+6m^2+11m+6} + \frac{3b^2mx^3x^m}{m^3+6m^2+11m+6} + \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(m, -3)), (-a**2/x + 2
*a*b*log(x) + b**2*x, Eq(m, -2)), (a**2*log(x) + 2*a*b*x + b**2*x**2/2, Eq(
m, -1)), (a**2*m**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*a**2*m*x*x**m/(m*
*3 + 6*m**2 + 11*m + 6) + 6*a**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*a*b*
m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*b*m*x**2*x**m/(m**3 + 6*m**
2 + 11*m + 6) + 6*a*b*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3
*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*b**2*m*x**3*x**m/(m**3 + 6*m**2 + 11*m
 + 6) + 2*b**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

## 3.436 $\int x^{5/2}(a+bx)^2\,dx$

**Optimal**. Leaf size=36

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

**Rubi [A]** time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(11/2))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)^2\,dx&=\int\left(a^2x^{5/2}+2abx^{7/2}+b^2x^{9/2}\right)dx\\&=\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{693}x^{7/2}\left(99a^2+154abx+63b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^2,x]
```

```
[Out] (2*x^(7/2)*(99*a^2 + 154*a*b*x + 63*b^2*x^2))/693
```

**IntegrateAlgebraic [A]** time = 0.01, size = 34, normalized size = 0.94

$$\frac{2}{693}\left(99a^2x^{7/2}+154abx^{9/2}+63b^2x^{11/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x)^2,x]
[Out] (2*(99*a^2*x^(7/2) + 154*a*b*x^(9/2) + 63*b^2*x^(11/2)))/693
```

**fricas [A]** time = 0.71, size = 29, normalized size = 0.81

$$\frac{2}{693}\left(63\,b^2x^5+154\,abx^4+99\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="fricas")
[Out] 2/693*(63*b^2*x^5 + 154*a*b*x^4 + 99*a^2*x^3)*sqrt(x)
```

**giac [A]** time = 1.16, size = 24, normalized size = 0.67

$$\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="giac")
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.69

$$\frac{2\left(63b^2x^2+154abx+99a^2\right)x^{\frac{7}{2}}}{693}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^2,x)
[Out] 2/693*x^(7/2)*(63*b^2*x^2+154*a*b*x+99*a^2)
```

**maxima [A]** time = 1.27, size = 24, normalized size = 0.67

$$\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

**mupad [B]** time = 0.10, size = 24, normalized size = 0.67

$$\frac{2\,x^{7/2}\,\left(99\,a^2+154\,a\,b\,x+63\,b^2\,x^2\right)}{693}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^2,x)
```

```
[Out] (2*x^(7/2)*(99*a^2 + 63*b^2*x^2 + 154*a*b*x))/693
```

**sympy [A]** time = 2.60, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(11/2)/11
```

## 3.437 $\int x^{3/2}(a+bx)^2\,dx$

**Optimal**. Leaf size=36

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

**Rubi [A]** time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^2,x]
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(9/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^2\,dx&=\int\left(a^2x^{3/2}+2abx^{5/2}+b^2x^{7/2}\right)dx\\&=\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{315}x^{5/2}\left(63a^2+90abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^2,x]
[Out] (2*x^(5/2)*(63*a^2 + 90*a*b*x + 35*b^2*x^2))/315
```

**IntegrateAlgebraic** [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{2}{315}\left(63a^2x^{5/2}+90abx^{7/2}+35b^2x^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x)^2,x]
[Out] (2*(63*a^2*x^(5/2) + 90*a*b*x^(7/2) + 35*b^2*x^(9/2)))/315
```

**fricas** [A] time = 0.88, size = 29, normalized size = 0.81

$$\frac{2}{315}\left(35\,b^2x^4+90\,abx^3+63\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="fricas")
[Out] 2/315*(35*b^2*x^4 + 90*a*b*x^3 + 63*a^2*x^2)*sqrt(x)
```

**giac** [A] time = 1.12, size = 24, normalized size = 0.67

$$\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="giac")
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{2\left(35b^2x^2+90abx+63a^2\right)x^{\frac{5}{2}}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^2,x)
[Out] 2/315*x^(5/2)*(35*b^2*x^2+90*a*b*x+63*a^2)
```

**maxima** [A] time = 1.32, size = 24, normalized size = 0.67

$$\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

**mupad [B]** time = 0.04, size = 24, normalized size = 0.67

$$\frac{2\,x^{5/2}\,\left(63\,a^2+90\,a\,b\,x+35\,b^2\,x^2\right)}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^2,x)
```

```
[Out] (2*x^(5/2)*(63*a^2 + 35*b^2*x^2 + 90*a*b*x))/315
```

**sympy [A]** time = 1.03, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(9/2)/9
```

## 3.438 $\int \sqrt{x}(a+bx)^2\,dx$

**Optimal**. Leaf size=36

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

**Rubi** [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(5/2))/5 + (2*b^2*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a+bx)^2\,dx &= \int \left(a^2\sqrt{x}+2abx^{3/2}+b^2x^{5/2}\right)dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{105}x^{3/2}\left(35a^2+42abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^2,x]
```

```
[Out] (2*x^(3/2)*(35*a^2 + 42*a*b*x + 15*b^2*x^2))/105
```

**IntegrateAlgebraic** [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{2}{105}\left(35a^2x^{3/2}+42abx^{5/2}+15b^2x^{7/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x)^2,x]
[Out] (2*(35*a^2*x^(3/2) + 42*a*b*x^(5/2) + 15*b^2*x^(7/2)))/105
```

**fricas** [A] time = 0.81, size = 27, normalized size = 0.75

$$\frac{2}{105}\left(15\,b^2x^3+42\,abx^2+35\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*b^2*x^3 + 42*a*b*x^2 + 35*a^2*x)*sqrt(x)
```

**giac** [A] time = 0.90, size = 24, normalized size = 0.67

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="giac")
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{2\left(15b^2x^2+42abx+35a^2\right)x^{\frac{3}{2}}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*x^(1/2),x)
[Out] 2/105*x^(3/2)*(15*b^2*x^2+42*a*b*x+35*a^2)
```

**maxima** [A] time = 1.29, size = 24, normalized size = 0.67

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

**mupad [B]** time = 0.04, size = 24, normalized size = 0.67

$$\frac{2\,x^{3/2}\,\left(35\,a^2+42\,a\,b\,x+15\,b^2\,x^2\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^2,x)
```

```
[Out] (2*x^(3/2)*(35*a^2 + 15*b^2*x^2 + 42*a*b*x))/105
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*x**(1/2),x)
```

```
[Out] Timed out
```

## 3.439 $\int \frac{(a+bx)^2}{\sqrt{x}} \, dx$

**Optimal**. Leaf size=34

$$2a^2\sqrt{x} + \frac{4}{3}abx^{3/2} + \frac{2}{5}b^2x^{5/2}$$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$2a^2\sqrt{x} + \frac{4}{3}abx^{3/2} + \frac{2}{5}b^2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[x],x]
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{\sqrt{x}} \, dx &= \int \left(\frac{a^2}{\sqrt{x}} + 2ab\sqrt{x} + b^2x^{3/2}\right) dx \\
&= 2a^2\sqrt{x} + \frac{4}{3}abx^{3/2} + \frac{2}{5}b^2x^{5/2}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 0.82

$$\frac{2}{15}\sqrt{x}\left(15a^2 + 10abx + 3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[x],x]
[Out] (2*Sqrt[x]*(15*a^2 + 10*a*b*x + 3*b^2*x^2))/15
```

**IntegrateAlgebraic [A]** time = 0.01, size = 34, normalized size = 1.00

$$\frac{2}{15}\left(15a^2\sqrt{x}+10abx^{3/2}+3b^2x^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/Sqrt[x],x]
[Out] (2*(15*a^2*Sqrt[x] + 10*a*b*x^(3/2) + 3*b^2*x^(5/2)))/15
```

**fricas [A]** time = 0.84, size = 24, normalized size = 0.71

$$\frac{2}{15}\left(3\,b^2x^2+10\,abx+15\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 + 10*a*b*x + 15*a^2)*sqrt(x)
```

**giac [A]** time = 0.92, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="giac")
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.74

$$\frac{2\left(3b^2x^2+10abx+15a^2\right)\sqrt{x}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(1/2),x)
[Out] 2/15*x^(1/2)*(3*b^2*x^2+10*a*b*x+15*a^2)
```

**maxima [A]** time = 1.38, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

**mupad [B]** time = 0.04, size = 24, normalized size = 0.71

$$\frac{2\sqrt{x}\,\left(15\,a^2+10\,a\,b\,x+3\,b^2\,x^2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(15*a^2 + 3*b^2*x^2 + 10*a*b*x))/15
```

**sympy [A]** time = 0.26, size = 32, normalized size = 0.94

$$2a^2\sqrt{x} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{2b^2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(5/2)/5
```

## 3.440 $\int \frac{(a+bx)^2}{x^{3/2}} \, dx$

**Optimal**. Leaf size=32

$$-\frac{2a^2}{\sqrt{x}} + 4ab\sqrt{x} + \frac{2}{3}b^2x^{3/2}$$

**Rubi** [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{\sqrt{x}} + 4ab\sqrt{x} + \frac{2}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + 4*a*b*Sqrt[x] + (2*b^2*x^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{3/2}} \, dx &= \int \left(\frac{a^2}{x^{3/2}} + \frac{2ab}{\sqrt{x}} + b^2\sqrt{x}\right) dx \\ &= -\frac{2a^2}{\sqrt{x}} + 4ab\sqrt{x} + \frac{2}{3}b^2x^{3/2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{2\left(-3a^2 + 6abx + b^2x^2\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(3/2),x]
```

```
[Out] (2*(-3*a^2 + 6*a*b*x + b^2*x^2))/(3*Sqrt[x])
```

**IntegrateAlgebraic** [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{2\left(-3a^2+6abx+b^2x^2\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^(3/2),x]
[Out] (2*(-3*a^2 + 6*a*b*x + b^2*x^2))/(3*Sqrt[x])
```

**fricas** [A] time = 0.89, size = 23, normalized size = 0.72

$$\frac{2\left(b^2x^2+6\,abx-3\,a^2\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="fricas")
[Out] 2/3*(b^2*x^2 + 6*a*b*x - 3*a^2)/sqrt(x)
```

**giac** [A] time = 1.02, size = 24, normalized size = 0.75

$$\frac{2}{3}\,b^2x^{\frac{3}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="giac")
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.78

$$-\frac{2\left(-b^2x^2-6abx+3a^2\right)}{3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(3/2),x)
[Out] -2/3*(-b^2*x^2-6*a*b*x+3*a^2)/x^(1/2)
```

**maxima** [A] time = 1.32, size = 24, normalized size = 0.75

$$\frac{2}{3}\,b^2x^{\frac{3}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

**mupad** [B] time = 0.03, size = 24, normalized size = 0.75

$$\frac{-6\,a^2 + 12\,a\,b\,x + 2\,b^2\,x^2}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(3/2),x)
```

```
[Out] (2*b^2*x^2 - 6*a^2 + 12*a*b*x)/(3*x^(1/2))
```

**sympy** [A] time = 0.43, size = 31, normalized size = 0.97

$$-\frac{2a^2}{\sqrt{x}} + 4ab\sqrt{x} + \frac{2b^2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(3/2),x)
```

```
[Out] -2*a**2/sqrt(x) + 4*a*b*sqrt(x) + 2*b**2*x**(3/2)/3
```

## 3.441 $\int \frac{(a+bx)^2}{x^{5/2}}\,dx$

**Optimal**. Leaf size=32

$$-\frac{2a^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$$

**Rubi [A]** time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) - (4*a*b)/Sqrt[x] + 2*b^2*Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{x^{3/2}}+\frac{b^2}{\sqrt{x}}\right)dx\\ &= -\frac{2a^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 0.81

$$-\frac{2\left(a^2+6abx-3b^2x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(5/2),x]
```

[Out] (-2*(a^2 + 6*a*b*x - 3*b^2*x^2))/(3*x^(3/2))

**IntegrateAlgebraic** [A] time = 0.02, size = 28, normalized size = 0.88

$$\frac{2\left(-a^2-6abx+3b^2x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x)^2/x^(5/2),x]

[Out] (2*(-a^2 - 6*a*b*x + 3*b^2*x^2))/(3*x^(3/2))

**fricas** [A] time = 0.81, size = 24, normalized size = 0.75

$$\frac{2\left(3\,b^2x^2-6\,abx-a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="fricas")

[Out] 2/3*(3*b^2*x^2 - 6*a*b*x - a^2)/x^(3/2)

**giac** [A] time = 1.00, size = 23, normalized size = 0.72

$$2\,b^2\sqrt{x}-\frac{2\left(6\,abx+a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="giac")

[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)

**maple** [A] time = 0.00, size = 23, normalized size = 0.72

$$-\frac{2\left(-3b^2x^2+6abx+a^2\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^2/x^(5/2),x)

[Out] -2/3*(-3*b^2*x^2+6*a*b*x+a^2)/x^(3/2)

**maxima** [A] time = 1.29, size = 23, normalized size = 0.72

$$2\,b^2\sqrt{x} - \frac{2\left(6\,abx + a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="maxima")
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

**mupad** [B] time = 0.03, size = 24, normalized size = 0.75

$$-\frac{2\,a^2 + 12\,a\,b\,x - 6\,b^2\,x^2}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(5/2),x)
[Out] -(2*a^2 - 6*b^2*x^2 + 12*a*b*x)/(3*x^(3/2))
```

**sympy** [A] time = 0.59, size = 31, normalized size = 0.97

$$-\frac{2a^2}{3x^{\frac{3}{2}}} - \frac{4ab}{\sqrt{x}} + 2b^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(5/2),x)
[Out] -2*a**2/(3*x**(3/2)) - 4*a*b/sqrt(x) + 2*b**2*sqrt(x)
```

## 3.442 $\int x^m (a+bx)^3 \, dx$

**Optimal**. Leaf size=61

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+2}}{m+2}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

**Rubi** [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^3\,dx &= \int \left(a^3x^m+3a^2bx^{1+m}+3ab^2x^{2+m}+b^3x^{3+m}\right)dx\\ &= \frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 54, normalized size = 0.89

$$x^{m+1}\left(\frac{a^3}{m+1}+\frac{3a^2bx}{m+2}+\frac{3ab^2x^2}{m+3}+\frac{b^3x^3}{m+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^3
*x^3)/(4 + m))
```

**IntegrateAlgebraic [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int x^m(a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^3, x]
```

**fricas [B]** time = 0.89, size = 157, normalized size = 2.57

$$\frac{\left(\left(b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3\right)x^4+3\left(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2\right)x^3+3\left(a^2bm^3+8\,a^2bm^2+19\,a^2bm+12\,a^2b\right)x^2+\left(a^3m^3+9\,a^3m^2+26\,a^3m+24\,a^3\right)x\right)x^m}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
+ 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*
a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*x^m/(m^4 + 10*m^3
 + 35*m^2 + 50*m + 24)
```

**giac [B]** time = 1.03, size = 224, normalized size = 3.67

$$\frac{b^3m^3x^4x^m+3\,ab^2m^3x^3x^m+6\,b^3m^2x^4x^m+3\,a^2bm^3x^2x^m+21\,ab^2m^2x^3x^m+11\,b^3mx^4x^m+a^3m^3xx^m+24\,a^2bm^2x^2x^m+42\,ab^2mx^3x^m+6\,b^3x^4x^m+9\,a^3m^2xx^m+57\,a^2bmx^2x^m+24\,ab^2x^3x^m+26\,a^3mxx^m+36\,a^2bx^2x^m+24\,a^3xx^m}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^4*x^m + 3*a*b^2*m^3*x^3*x^m + 6*b^3*m^2*x^4*x^m + 3*a^2*b*m^3*x^
2*x^m + 21*a*b^2*m^2*x^3*x^m + 11*b^3*m*x^4*x^m + a^3*m^3*x*x^m + 24*a^2*b*
m^2*x^2*x^m + 42*a*b^2*m*x^3*x^m + 6*b^3*x^4*x^m + 9*a^3*m^2*x*x^m + 57*a^2
*b*m*x^2*x^m + 24*a*b^2*x^3*x^m + 26*a^3*m*x*x^m + 36*a^2*b*x^2*x^m + 24*a^
3*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

**maple [B]** time = 0.00, size = 170, normalized size = 2.79

$$\frac{\left(b^3m^3x^3+3a\,b^2m^3x^2+6b^3m^2x^3+3a^2b\,m^3x+21a\,b^2m^2x^2+11b^3m\,x^3+a^3m^3+24a^2b\,m^2x+42a\,b^2m\,x^2+6b^3x^3+9a^3m^2+57a^2bmx+24a\,b^2x^2+26a^3m+36a^2bx+24a^3\right)x^{m+1}}{(m+4)\,(m+3)\,(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^3,x)
```

```
[Out] x^(m+1)*(b^3*m^3*x^3+3*a*b^2*m^3*x^2+6*b^3*m^2*x^3+3*a^2*b*m^3*x+21*a*b^2*m
^2*x^2+11*b^3*m*x^3+a^3*m^3+24*a^2*b*m^2*x+42*a*b^2*m*x^2+6*b^3*x^3+9*a^3*m
^2+57*a^2*b*m*x+24*a*b^2*x^2+26*a^3*m+36*a^2*b*x+24*a^3)/(4+m)/(m+3)/(m+2)/
(m+1)
```

**maxima [A]** time = 1.36, size = 61, normalized size = 1.00

$$\frac{b^3x^{m+4}}{m+4}+\frac{3\,ab^2x^{m+3}}{m+3}+\frac{3\,a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 4)/(m + 4) + 3*a*b^2*x^(m + 3)/(m + 3) + 3*a^2*b*x^(m + 2)/(m +
2) + a^3*x^(m + 1)/(m + 1)
```

**mupad [B]** time = 0.39, size = 167, normalized size = 2.74

$$x^m\left(\frac{a^3\,x\,\left(m^3+9\,m^2+26\,m+24\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{b^3\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a\,b^2\,x^3\,\left(m^3+7\,m^2+14\,m+8\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a^2\,b\,x^2\,\left(m^3+8\,m^2+19\,m+12\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^3,x)
```

```
[Out] x^m*((a^3*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (b^3*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) +
(3*a*b^2*x^3*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (3*a^2*b*x^2*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4))
```

**sympy [A]** time = 0.88, size = 663, normalized size = 10.87

$$\begin{cases} -\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x) & \text{for}\ m=-4\\ -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x & \text{for}\ m=-3\\ -\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2} & \text{for}\ m=-2\\ a^3\log(x)+3a^2bx+\frac{3ab^2x^2}{2}+\frac{b^3x^3}{3} & \text{for}\ m=-1\\ \frac{a^3m^3xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{9a^3m^2xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{26a^3mxx^m}{m^4+10m^3+35m^2+50m+24}+\frac{24a^3xx^m}{m^4+10m^3+35m^2+50m+24}+\frac{3a^2bm^3x^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{24a^2bm^2x^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{57a^2bmx^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{36a^2bx^2x^m}{m^4+10m^3+35m^2+50m+24}+\frac{3ab^2m^3x^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{21ab^2m^2x^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{42ab^2mx^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{24ab^2x^3x^m}{m^4+10m^3+35m^2+50m+24}+\frac{b^3m^3x^4x^m}{m^4+10m^3+35m^2+50m+24}+\frac{6b^3m^2x^4x^m}{m^4+10m^3+35m^2+50m+24}+\frac{11b^3mx^4x^m}{m^4+10m^3+35m^2+50m+24}+\frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x), E
q(m, -4)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*x, Eq(m, -
3)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq(m, -2)), (a*
*3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3, Eq(m, -1)), (a**3*m
**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*a**3*m**2*x*x**m/(m**
4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*a**3*m*x*x**m/(m**4 + 10*m**3 + 35*
```

```
m**2 + 50*m + 24) + 24*a**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
 3*a**2*b*m**3*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**2*b
*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 57*a**2*b*m*x**2*x
**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 36*a**2*b*x**2*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) + 3*a*b**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 21*a*b**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 5
0*m + 24) + 42*a*b**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*a*b**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + b**3*m**3*x**4
*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*m**2*x**4*x**m/(m**4
+ 10*m**3 + 35*m**2 + 50*m + 24) + 11*b**3*m*x**4*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24
), True))
```

## 3.443 $\int x^{5/2}(a+bx)^3\,dx$

**Optimal**. Leaf size=51

$$\frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

**Rubi** [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{3}a^2bx^{9/2}+\frac{2}{7}a^3x^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x^
(13/2))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)^3\,dx&=\int\left(a^3x^{5/2}+3a^2bx^{7/2}+3ab^2x^{9/2}+b^3x^{11/2}\right)dx\\&=\frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{2x^{7/2}\left(429a^3+1001a^2bx+819ab^2x^2+231b^3x^3\right)}{3003}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^3,x]
```

[Out] (2*x^(7/2)*(429*a^3 + 1001*a^2*b*x + 819*a*b^2*x^2 + 231*b^3*x^3))/3003

**IntegrateAlgebraic** [A] time = 0.01, size = 47, normalized size = 0.92

$$\frac{2\left(429a^3x^{7/2}+1001a^2bx^{9/2}+819ab^2x^{11/2}+231b^3x^{13/2}\right)}{3003}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(5/2)*(a + b*x)^3,x]

[Out] (2*(429*a^3*x^(7/2) + 1001*a^2*b*x^(9/2) + 819*a*b^2*x^(11/2) + 231*b^3*x^(13/2)))/3003

**fricas** [A] time = 0.81, size = 40, normalized size = 0.78

$$\frac{2}{3003}\left(231\,b^3x^6+819\,ab^2x^5+1001\,a^2bx^4+429\,a^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="fricas")

[Out] 2/3003*(231*b^3*x^6 + 819*a*b^2*x^5 + 1001*a^2*b*x^4 + 429*a^3*x^3)*sqrt(x)

**giac** [A] time = 0.99, size = 35, normalized size = 0.69

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="giac")

[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/2)

**maple** [A] time = 0.00, size = 36, normalized size = 0.71

$$\frac{2\left(231b^3x^3+819a\,b^2x^2+1001a^2bx+429a^3\right)x^{\frac{7}{2}}}{3003}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(b*x+a)^3,x)

[Out] 2/3003*x^(7/2)*(231*b^3*x^3+819*a*b^2*x^2+1001*a^2*b*x+429*a^3)

**maxima** [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{6}{11}\,ab^2x^{\frac{11}{2}} + \frac{2}{3}\,a^2bx^{\frac{9}{2}} + \frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/
2)
```

**mupad** [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{7/2}}{7} + \frac{2\,b^3\,x^{13/2}}{13} + \frac{2\,a^2\,b\,x^{9/2}}{3} + \frac{6\,a\,b^2\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^3,x)
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*b^3*x^(13/2))/13 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^
(11/2))/11
```

**sympy** [A] time = 3.88, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{7}{2}}}{7} + \frac{2a^2bx^{\frac{9}{2}}}{3} + \frac{6ab^2x^{\frac{11}{2}}}{11} + \frac{2b^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x*
*(13/2)/13
```

## 3.444 $\int x^{3/2}(a+bx)^3\,dx$

**Optimal**. Leaf size=51

$$\frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

**Rubi [A]** time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{5}a^3x^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(1
1/2))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^3\,dx &= \int \left(a^3x^{3/2}+3a^2bx^{5/2}+3ab^2x^{7/2}+b^3x^{9/2}\right)dx\\ &= \frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 39, normalized size = 0.76

$$\frac{2x^{5/2}\left(231a^3+495a^2bx+385ab^2x^2+105b^3x^3\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^3,x]
```

[Out] (2*x^(5/2)*(231*a^3 + 495*a^2*b*x + 385*a*b^2*x^2 + 105*b^3*x^3))/1155

**IntegrateAlgebraic [A]** time = 0.01, size = 47, normalized size = 0.92

$$\frac{2\left(231a^3x^{5/2}+495a^2bx^{7/2}+385ab^2x^{9/2}+105b^3x^{11/2}\right)}{1155}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(3/2)*(a + b*x)^3,x]

[Out] (2*(231*a^3*x^(5/2) + 495*a^2*b*x^(7/2) + 385*a*b^2*x^(9/2) + 105*b^3*x^(11
/2)))/1155

**fricas [A]** time = 0.88, size = 40, normalized size = 0.78

$$\frac{2}{1155}\left(105\,b^3x^5+385\,ab^2x^4+495\,a^2bx^3+231\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="fricas")

[Out] 2/1155*(105*b^3*x^5 + 385*a*b^2*x^4 + 495*a^2*b*x^3 + 231*a^3*x^2)*sqrt(x)

**giac [A]** time = 0.99, size = 35, normalized size = 0.69

$$\frac{2}{11}\,b^3x^{\frac{11}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="giac")

[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)

**maple [A]** time = 0.00, size = 36, normalized size = 0.71

$$\frac{2\left(105b^3x^3+385a\,b^2x^2+495a^2bx+231a^3\right)x^{\frac{5}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(b*x+a)^3,x)

[Out] 2/1155*x^(5/2)*(105*b^3*x^3+385*a*b^2*x^2+495*a^2*b*x+231*a^3)

**maxima [A]** time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{11}\,b^{3}x^{\frac{11}{2}}+\frac{2}{3}\,ab^{2}x^{\frac{9}{2}}+\frac{6}{7}\,a^{2}bx^{\frac{7}{2}}+\frac{2}{5}\,a^{3}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

**mupad [B]** time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,a^{3}\,x^{5/2}}{5}+\frac{2\,b^{3}\,x^{11/2}}{11}+\frac{6\,a^{2}\,b\,x^{7/2}}{7}+\frac{2\,a\,b^{2}\,x^{9/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^3,x)
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*b^3*x^(11/2))/11 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x^
(9/2))/3
```

**sympy [A]** time = 1.70, size = 49, normalized size = 0.96

$$\frac{2a^{3}x^{\frac{5}{2}}}{5}+\frac{6a^{2}bx^{\frac{7}{2}}}{7}+\frac{2ab^{2}x^{\frac{9}{2}}}{3}+\frac{2b^{3}x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 6*a**2*b*x**(7/2)/7 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**(
11/2)/11
```

## 3.445 $\int \sqrt{x}(a+bx)^3\,dx$

**Optimal**. Leaf size=51

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}$$

**Rubi** [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}a^3x^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(9
/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a+bx)^3\,dx &= \int \left(a^3\sqrt{x}+3a^2bx^{3/2}+3ab^2x^{5/2}+b^3x^{7/2}\right)dx\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{2}{315}x^{3/2}\left(105a^3+189a^2bx+135ab^2x^2+35b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^3,x]
```

```
[Out] (2*x^(3/2)*(105*a^3 + 189*a^2*b*x + 135*a*b^2*x^2 + 35*b^3*x^3))/315
```

**IntegrateAlgebraic [A]** time = 0.01, size = 47, normalized size = 0.92

$$\frac{2}{315}\left(105a^3x^{3/2}+189a^2bx^{5/2}+135ab^2x^{7/2}+35b^3x^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x)^3,x]
[Out] (2*(105*a^3*x^(3/2) + 189*a^2*b*x^(5/2) + 135*a*b^2*x^(7/2) + 35*b^3*x^(9/2
)))/315
```

**fricas [A]** time = 0.82, size = 38, normalized size = 0.75

$$\frac{2}{315}\left(35\,b^3x^4+135\,ab^2x^3+189\,a^2bx^2+105\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="fricas")
[Out] 2/315*(35*b^3*x^4 + 135*a*b^2*x^3 + 189*a^2*b*x^2 + 105*a^3*x)*sqrt(x)
```

**giac [A]** time = 1.08, size = 35, normalized size = 0.69

$$\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{6}{7}\,ab^2x^{\frac{7}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="giac")
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.71

$$\frac{2\left(35b^3x^3+135a\,b^2x^2+189a^2bx+105a^3\right)x^{\frac{3}{2}}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*x^(1/2),x)
[Out] 2/315*x^(3/2)*(35*b^3*x^3+135*a*b^2*x^2+189*a^2*b*x+105*a^3)
```

**maxima [A]** time = 1.26, size = 35, normalized size = 0.69

$$\frac{2}{9}\,b^{3}x^{\frac{9}{2}}+\frac{6}{7}\,ab^{2}x^{\frac{7}{2}}+\frac{6}{5}\,a^{2}bx^{\frac{5}{2}}+\frac{2}{3}\,a^{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="maxima")
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

**mupad [B]** time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,b^3\,x^{9/2}}{9}+\frac{6\,a^2\,b\,x^{5/2}}{5}+\frac{6\,a\,b^2\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^3,x)
[Out] (2*a^3*x^(3/2))/3 + (2*b^3*x^(9/2))/9 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(7
/2))/7
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*x**(1/2),x)
[Out] Timed out
```

# 3.446 $\int \frac{(a+bx)^3}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=47

$$2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}$$

**Rubi [A]** time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$2a^2bx^{3/2}+2a^3\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^3}{\sqrt{x}}+3a^2b\sqrt{x}+3ab^2x^{3/2}+b^3x^{5/2}\right)dx\\ &= 2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 39, normalized size = 0.83

$$\frac{2}{35}\sqrt{x}\left(35a^3+35a^2bx+21ab^2x^2+5b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/Sqrt[x],x]
```

[Out] (2*Sqrt[x]*(35*a^3 + 35*a^2*b*x + 21*a*b^2*x^2 + 5*b^3*x^3))/35

**IntegrateAlgebraic [A]** time = 0.01, size = 47, normalized size = 1.00

$$\frac{2}{35}\left(35a^3\sqrt{x}+35a^2bx^{3/2}+21ab^2x^{5/2}+5b^3x^{7/2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x)^3/Sqrt[x],x]

[Out] (2*(35*a^3*Sqrt[x] + 35*a^2*b*x^(3/2) + 21*a*b^2*x^(5/2) + 5*b^3*x^(7/2)))/
35

**fricas [A]** time = 0.77, size = 35, normalized size = 0.74

$$\frac{2}{35}\left(5\,b^3x^3+21\,ab^2x^2+35\,a^2bx+35\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="fricas")

[Out] 2/35*(5*b^3*x^3 + 21*a*b^2*x^2 + 35*a^2*b*x + 35*a^3)*sqrt(x)

**giac [A]** time = 0.94, size = 35, normalized size = 0.74

$$\frac{2}{7}\,b^3x^{\frac{7}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+2\,a^2bx^{\frac{3}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="giac")

[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)

**maple [A]** time = 0.00, size = 36, normalized size = 0.77

$$\frac{2\left(5b^3x^3+21a\,b^2x^2+35a^2bx+35a^3\right)\sqrt{x}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^3/x^(1/2),x)

[Out] 2/35*x^(1/2)*(5*b^3*x^3+21*a*b^2*x^2+35*a^2*b*x+35*a^3)

**maxima [A]** time = 1.35, size = 35, normalized size = 0.74

$$\frac{2}{7}\,b^3x^{\frac{7}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+2\,a^2bx^{\frac{3}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

**mupad [B]** time = 0.04, size = 35, normalized size = 0.74

$$2\,a^3\,\sqrt{x}+\frac{2\,b^3\,x^{7/2}}{7}+2\,a^2\,b\,x^{3/2}+\frac{6\,a\,b^2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(1/2),x)
```

```
[Out] 2*a^3*x^(1/2) + (2*b^3*x^(7/2))/7 + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5
```

**sympy [A]** time = 0.44, size = 46, normalized size = 0.98

$$2a^3\sqrt{x}+2a^2bx^{\frac{3}{2}}+\frac{6ab^2x^{\frac{5}{2}}}{5}+\frac{2b^3x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(7/2)/
7
```

## 3.447 $\int \frac{(a+bx)^3}{x^{3/2}} dx$

**Optimal**. Leaf size=45

$$-\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}$$

**Rubi** [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$6a^2b\sqrt{x}-\frac{2a^3}{\sqrt{x}}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 6*a^2*b*Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{3/2}}\,dx &= \int \left(\frac{a^3}{x^{3/2}}+\frac{3a^2b}{\sqrt{x}}+3ab^2\sqrt{x}+b^3x^{3/2}\right)dx\\ &= -\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 38, normalized size = 0.84

$$\frac{2\left(-5a^3+15a^2bx+5ab^2x^2+b^3x^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(3/2),x]
[Out] (2*(-5*a^3 + 15*a^2*b*x + 5*a*b^2*x^2 + b^3*x^3))/(5*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.02, size = 38, normalized size = 0.84

$$\frac{2\left(-5a^3+15a^2bx+5ab^2x^2+b^3x^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(3/2),x]
[Out] (2*(-5*a^3 + 15*a^2*b*x + 5*a*b^2*x^2 + b^3*x^3))/(5*Sqrt[x])
```

**fricas [A]** time = 0.80, size = 34, normalized size = 0.76

$$\frac{2\left(b^3x^3+5\,ab^2x^2+15\,a^2bx-5\,a^3\right)}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="fricas")
[Out] 2/5*(b^3*x^3 + 5*a*b^2*x^2 + 15*a^2*b*x - 5*a^3)/sqrt(x)
```

**giac [A]** time = 1.03, size = 35, normalized size = 0.78

$$\frac{2}{5}\,b^3x^{\frac{5}{2}}+2\,ab^2x^{\frac{3}{2}}+6\,a^2b\sqrt{x}-\frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="giac")
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.80

$$-\frac{2\left(-b^3x^3-5a\,b^2x^2-15a^2bx+5a^3\right)}{5\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(3/2),x)
[Out] -2/5*(-b^3*x^3-5*a*b^2*x^2-15*a^2*b*x+5*a^3)/x^(1/2)
```

**maxima** [A] time = 1.36, size = 35, normalized size = 0.78

$$\frac{2}{5}\,b^{3}x^{\frac{5}{2}}+2\,ab^{2}x^{\frac{3}{2}}+6\,a^{2}b\sqrt{x}-\frac{2\,a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

**mupad** [B] time = 0.05, size = 35, normalized size = 0.78

$$\frac{2\,b^{3}\,x^{5/2}}{5}-\frac{2\,a^{3}}{\sqrt{x}}+6\,a^{2}\,b\,\sqrt{x}+2\,a\,b^{2}\,x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(3/2),x)
```

```
[Out] (2*b^3*x^(5/2))/5 - (2*a^3)/x^(1/2) + 6*a^2*b*x^(1/2) + 2*a*b^2*x^(3/2)
```

**sympy** [A] time = 0.64, size = 44, normalized size = 0.98

$$-\frac{2a^{3}}{\sqrt{x}}+6a^{2}b\sqrt{x}+2ab^{2}x^{\frac{3}{2}}+\frac{2b^{3}x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 6*a**2*b*sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(5/2)/5
```

## 3.448 $\int \frac{(a+bx)^3}{x^{5/2}}\,dx$

**Optimal**. Leaf size=47

$$-\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}$$

**Rubi** [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{6a^2b}{\sqrt{x}}-\frac{2a^3}{3x^{3/2}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) - (6*a^2*b)/Sqrt[x] + 6*a*b^2*Sqrt[x] + (2*b^3*x^(3/2)
)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{5/2}}\,dx &= \int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{x^{3/2}}+\frac{3ab^2}{\sqrt{x}}+b^3\sqrt{x}\right)dx\\ &= -\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 38, normalized size = 0.81

$$\frac{2\left(-a^3-9a^2bx+9ab^2x^2+b^3x^3\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(5/2),x]
[Out] (2*(-a^3 - 9*a^2*b*x + 9*a*b^2*x^2 + b^3*x^3))/(3*x^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.02, size = 38, normalized size = 0.81

$$\frac{2\left(-a^3-9a^2bx+9ab^2x^2+b^3x^3\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(5/2),x]
[Out] (2*(-a^3 - 9*a^2*b*x + 9*a*b^2*x^2 + b^3*x^3))/(3*x^(3/2))
```

**fricas** [A] time = 0.90, size = 34, normalized size = 0.72

$$\frac{2\left(b^3x^3+9\,ab^2x^2-9\,a^2bx-a^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="fricas")
[Out] 2/3*(b^3*x^3 + 9*a*b^2*x^2 - 9*a^2*b*x - a^3)/x^(3/2)
```

**giac** [A] time = 1.07, size = 34, normalized size = 0.72

$$\frac{2}{3}\,b^3x^{\frac{3}{2}}+6\,ab^2\sqrt{x}-\frac{2\left(9\,a^2bx+a^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="giac")
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

**maple** [A] time = 0.00, size = 34, normalized size = 0.72

$$-\frac{2\left(-b^3x^3-9a\,b^2x^2+9a^2bx+a^3\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(5/2),x)
[Out] -2/3*(-b^3*x^3-9*a*b^2*x^2+9*a^2*b*x+a^3)/x^(3/2)
```

**maxima** [A] time = 1.37, size = 34, normalized size = 0.72

$$\frac{2}{3}\,b^3x^{\frac{3}{2}} + 6\,ab^2\sqrt{x} - \frac{2\left(9\,a^2bx + a^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

**mupad** [B] time = 0.04, size = 35, normalized size = 0.74

$$-\frac{2\,a^3+18\,a^2\,b\,x-18\,a\,b^2\,x^2-2\,b^3\,x^3}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(5/2),x)
```

```
[Out] -(2*a^3 - 2*b^3*x^3 - 18*a*b^2*x^2 + 18*a^2*b*x)/(3*x^(3/2))
```

**sympy** [A] time = 0.78, size = 46, normalized size = 0.98

$$-\frac{2a^3}{3x^{\frac{3}{2}}} - \frac{6a^2b}{\sqrt{x}} + 6ab^2\sqrt{x} + \frac{2b^3x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) - 6*a**2*b/sqrt(x) + 6*a*b**2*sqrt(x) + 2*b**3*x**(3/2
)/3
```

## 3.449 $\int \frac{x^{5/2}}{a+bx} dx$

**Optimal**. Leaf size=68

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{x}}{b^3}-\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

**Rubi [A]** time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 205}

$$\frac{2a^2\sqrt{x}}{b^3}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 - (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{a+bx}\,dx &= \frac{2x^{5/2}}{5b} - \frac{a\int \frac{x^{3/2}}{a+bx}\,dx}{b}\\
&= -\frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{a^2\int \frac{\sqrt{x}}{a+bx}\,dx}{b^2}\\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{a^3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{b^3}\\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^3}\\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 61, normalized size = 0.90

$$\frac{2\sqrt{x}\left(15a^2-5abx+3b^2x^2\right)}{15b^3} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(15*a^2 - 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTan[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 67, normalized size = 0.99

$$\frac{2\left(15a^2\sqrt{x}-5abx^{3/2}+3b^2x^{5/2}\right)}{15b^3} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x),x]
```

```
[Out] (2*(15*a^2*Sqrt[x] - 5*a*b*x^(3/2) + 3*b^2*x^(5/2)))/(15*b^3) - (2*a^(5/2)*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

**fricas** [A] time = 0.97, size = 132, normalized size = 1.94

$$\left[\frac{15\,a^2\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(3\,b^2x^2-5\,abx+15\,a^2\right)\sqrt{x}}{15\,b^3},\,-\frac{2\left(15\,a^2\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{\frac{a}{b}}}{a}\right)-\left(3\,b^2x^2-5\,abx+15\,a^2\right)\sqrt{x}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a))
+ 2*(3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3, -2/15*(15*a^2*sqrt(a/b)*ar
ctan(b*sqrt(x)*sqrt(a/b)/a) - (3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

**giac** [A] time = 0.87, size = 59, normalized size = 0.87

$$-\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^4x^{\frac{5}{2}}-5\,ab^3x^{\frac{3}{2}}+15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^4*x^(5/2) -
5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

**maple** [A] time = 0.01, size = 54, normalized size = 0.79

$$\frac{2x^{\frac{5}{2}}}{5b}-\frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3}-\frac{2a\,x^{\frac{3}{2}}}{3b^2}+\frac{2a^2\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a),x)
```

```
[Out] 2/5*x^(5/2)/b-2/3*a*x^(3/2)/b^2+2*a^2*x^(1/2)/b^3-2*a^3/b^3/(a*b)^(1/2)*arc
tan(x^(1/2)*b/(a*b)^(1/2))
```

**maxima** [A] time = 3.01, size = 54, normalized size = 0.79

$$-\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^2x^{\frac{5}{2}}-5\,abx^{\frac{3}{2}}+15\,a^2\sqrt{x}\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^2*x^(5/2) -
5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

**mupad [B]** time = 0.06, size = 48, normalized size = 0.71

$$\frac{2\,x^{5/2}}{5\,b}-\frac{2\,a\,x^{3/2}}{3\,b^2}+\frac{2\,a^2\,\sqrt{x}}{b^3}-\frac{2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x),x)
```

```
[Out] (2*x^(5/2))/(5*b) - (2*a*x^(3/2))/(3*b^2) + (2*a^2*x^(1/2))/b^3 - (2*a^(5/2
)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

**sympy [A]** time = 7.30, size = 121, normalized size = 1.78

$$\begin{cases} \frac{ia^{\frac{5}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^4\sqrt{\frac{1}{b}}}-\frac{ia^{\frac{5}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^4\sqrt{\frac{1}{b}}}+\frac{2a^2\sqrt{x}}{b^3}-\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{2x^{\frac{5}{2}}}{5b} & \text{for } b\neq 0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a),x)
```

```
[Out] Piecewise((I*a**(5/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b))
- I*a**(5/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b)) + 2*a**2*s
qrt(x)/b**3 - 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), Ne(b, 0)), (2*x**(7
/2)/(7*a), True))
```

## 3.450 $\int \frac{x^{3/2}}{a+bx}\,dx$

**Optimal**. Leaf size=53

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

**Rubi [A]** time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 205}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x),x]
```

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{a+bx}\,dx &= \frac{2x^{3/2}}{3b} - \frac{a\int \frac{\sqrt{x}}{a+bx}\,dx}{b}\\
&= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{a^2\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{b^2}\\
&= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^2}\\
&= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 0.92

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}} + \frac{2\sqrt{x}(bx-3a)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(-3*a + b*x))/(3*b^2) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/b^(5/2)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 53, normalized size = 1.00

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}} + \frac{2\left(bx^{3/2}-3a\sqrt{x}\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x),x]
```

```
[Out] (2*(-3*a*Sqrt[x] + b*x^(3/2)))/(3*b^2) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

**fricas [A]** time = 0.70, size = 103, normalized size = 1.94

$$\left[\frac{3\,a\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(bx-3\,a)\sqrt{x}}{3\,b^2}, \frac{2\left(3\,a\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(bx-3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*
(b*x - 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a)
+ (b*x - 3*a)*sqrt(x))/b^2]
```

**giac [A]** time = 1.24, size = 45, normalized size = 0.85

$$\frac{2\,a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{2\left(b^{2}x^{\frac{3}{2}}-3\,ab\sqrt{x}\right)}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b^2*x^(3/2) - 3*a*
b*sqrt(x))/b^3
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.81

$$\frac{2a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{2}}+\frac{2x^{\frac{3}{2}}}{3b}-\frac{2a\sqrt{x}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a),x)
```

```
[Out] 2/3*x^(3/2)/b-2*a*x^(1/2)/b^2+2*a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*
x^(1/2))
```

**maxima [A]** time = 2.94, size = 42, normalized size = 0.79

$$\frac{2\,a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{2\left(bx^{\frac{3}{2}}-3\,a\sqrt{x}\right)}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b*x^(3/2) - 3*a*sq
rt(x))/b^2
```

**mupad [B]** time = 0.05, size = 37, normalized size = 0.70

$$\frac{2\,x^{3/2}}{3\,b}-\frac{2\,a\,\sqrt{x}}{b^2}+\frac{2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x),x)
```

```
[Out] (2*x^(3/2))/(3*b) - (2*a*x^(1/2))/b^2 + (2*a^(3/2)*atan((b^(1/2)*x^(1/2))/a
^(1/2)))/b^(5/2)
```

**sympy [A]** time = 1.93, size = 105, normalized size = 1.98

$$\begin{cases} -\frac{ia^{\frac{3}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}}+\frac{ia^{\frac{3}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{3}{2}}}{3b} & \text{for}\ b\neq 0 \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a),x)
```

```
[Out] Piecewise((-I*a**(3/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b))
 + I*a**(3/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b)) - 2*a*sqr
t(x)/b**2 + 2*x**(3/2)/(3*b), Ne(b, 0)), (2*x**(5/2)/(5*a), True))
```

# 3.451 $\int \frac{\sqrt{x}}{a+bx} dx$

**Optimal**. Leaf size=40

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 205}

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a+bx}\,dx = \frac{2\sqrt{x}}{b} - \frac{a\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.02, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

**fricas** [A] time = 0.98, size = 85, normalized size = 2.12

$$\left[\frac{\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,\sqrt{x}}{b}, -\frac{2\left(\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*sqrt(x))
/b, -2*(sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - sqrt(x))/b]
```

**giac [A]** time = 0.99, size = 31, normalized size = 0.78

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

**maple [A]** time = 0.01, size = 32, normalized size = 0.80

$$-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a),x)
```

```
[Out] 2*x^(1/2)/b-2*a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 2.97, size = 31, normalized size = 0.78

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

**mupad [B]** time = 0.04, size = 28, normalized size = 0.70

$$\frac{2\,\sqrt{x}}{b}-\frac{2\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x),x)
```

```
[Out] (2*x^(1/2))/b - (2*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(3/2)
```

**sympy** [A] time = 0.72, size = 92, normalized size = 2.30

$$\begin{cases} \frac{i\sqrt{a}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}} - \frac{i\sqrt{a}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}} + \frac{2\sqrt{x}}{b} & \text{for } b \neq 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a),x)
```

```
[Out] Piecewise((I*sqrt(a)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) -
 I*sqrt(a)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) + 2*sqrt(x)/
b, Ne(b, 0)), (2*x**(3/2)/(3*a), True))
```

## 3.452 $\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx$

**Optimal**. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)),x]
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

**IntegrateAlgebraic** [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x)),x]
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

**fricas** [A] time = 0.91, size = 68, normalized size = 2.34

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(b*sqrt(x)))/(a*b)]
```

**giac** [A] time = 0.95, size = 18, normalized size = 0.62

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

**maple [A]** time = 0.01, size = 19, normalized size = 0.66

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/x^(1/2),x)
[Out] 2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 2.93, size = 18, normalized size = 0.62

$$\frac{2\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="maxima")
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

**mupad [B]** time = 0.04, size = 19, normalized size = 0.66

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)),x)
[Out] (2*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2))
```

**sympy [A]** time = 1.29, size = 94, normalized size = 3.24

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for}\: a = 0 \\ \frac{2\sqrt{x}}{a} & \text{for}\: b = 0 \\ -\frac{i\log\left(-i\sqrt{a}\,\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{\sqrt{a}\,b\sqrt{\frac{1}{b}}} + \frac{i\log\left(i\sqrt{a}\,\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{\sqrt{a}\,b\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (
2*sqrt(x)/a, Eq(b, 0)), (-I*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*
sqrt(1/b)) + I*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*sqrt(1/b)), Tr
ue))
```

## 3.453 $\int \frac{1}{x^{3/2}(a+bx)}\,dx$

**Optimal**. Leaf size=40

$$-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}$$

**Rubi** [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx)}\,dx = -\frac{2}{a\sqrt{x}} - \frac{b\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= -\frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

**Mathematica** [C] time = 0.00, size = 25, normalized size = 0.62

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx}{a}\right)}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)),x]
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x)/a)])/(a*Sqrt[x])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 40, normalized size = 1.00

$$-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x)),x]
[Out] -2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

**fricas** [A] time = 0.95, size = 93, normalized size = 2.32

$$\left[\frac{x\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\sqrt{x}}{ax}, \frac{2\left(x\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(x*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*sqrt(x
))/(a*x), 2*(x*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - sqrt(x))/(a*x)]
```

**giac** [A] time = 1.00, size = 31, normalized size = 0.78

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

**maple** [A] time = 0.01, size = 32, normalized size = 0.80

$$-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a),x)
```

```
[Out] -2/a/x^(1/2)-2/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima** [A] time = 2.88, size = 31, normalized size = 0.78

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

**mupad** [B] time = 0.04, size = 28, normalized size = 0.70

$$-\frac{2}{a\,\sqrt{x}}-\frac{2\,\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)),x)
[Out] - 2/(a*x^(1/2)) - (2*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(3/2)
```

**sympy [A]** time = 2.77, size = 102, normalized size = 2.55

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{a \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{2}{3 b x^{\frac{3}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{a \sqrt{x}} + \frac{i \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{a^{\frac{3}{2}} \sqrt{\frac{1}{b}}} - \frac{i \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{a^{\frac{3}{2}} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a),x)
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(a*sqrt(x)), Eq(b, 0)),
(-2/(3*b*x**(3/2)), Eq(a, 0)), (-2/(a*sqrt(x)) + I*log(-I*sqrt(a)*sqrt(1/b)
 + sqrt(x))/(a**(3/2)*sqrt(1/b)) - I*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a*
*(3/2)*sqrt(1/b)), True))
```

## 3.454 $\int \frac{1}{x^{5/2}(a+bx)}\,dx$

**Optimal**. Leaf size=53

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{3ax^{3/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(a+bx)}\,dx &= -\frac{2}{3ax^{3/2}}-\frac{b\int \frac{1}{x^{3/2}(a+bx)}\,dx}{a}\\ &= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b^2\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{a^2}\\ &= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\ &= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 27, normalized size = 0.51

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{bx}{a}\right)}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x)/a)])/(3*a*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.04, size = 48, normalized size = 0.91

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2(a-3bx)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x)),x]
```

```
[Out] (-2*(a - 3*b*x))/(3*a^2*x^(3/2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/a^(5/2)
```

**fricas [A]** time = 0.89, size = 118, normalized size = 2.23

$$\left[\frac{3\,bx^2\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,bx-a)\sqrt{x}}{3\,a^2x^2},\,-\frac{2\left(3\,bx^2\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx-a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a))
+ 2*(3*b*x - a)*sqrt(x))/(a^2*x^2), -2/3*(3*b*x^2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*sqrt(x))) - (3*b*x - a)*sqrt(x))/(a^2*x^2)]
```

**giac [A]** time = 1.16, size = 41, normalized size = 0.77

$$\frac{2\,b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}+\frac{2\,(3\,bx-a)}{3\,a^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.81

$$\frac{2b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^{2}}+\frac{2b}{a^{2}\sqrt{x}}-\frac{2}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a),x)
```

```
[Out] -2/3/a/x^(3/2)+2*b/a^2/x^(1/2)+2/a^2*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x^(1/2))
```

**maxima [A]** time = 3.02, size = 41, normalized size = 0.77

$$\frac{2\,b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}+\frac{2\,(3\,bx-a)}{3\,a^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

**mupad [B]** time = 0.10, size = 38, normalized size = 0.72

$$\frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2}{3\,a}-\frac{2\,b\,x}{a^2}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)),x)
```

```
[Out] (2*b^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(5/2) - (2/(3*a) - (2*b*x)/a^
2)/x^(3/2)
```

**sympy [A]** time = 7.83, size = 121, normalized size = 2.28

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{3 a x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{5 b x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{3 a x^{\frac{3}{2}}} + \frac{2 b}{a^{2} \sqrt{x}} - \frac{i b \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{a^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{i b \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{a^{\frac{5}{2}} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a*x**(3/2)), Eq(b, 0)
), (-2/(5*b*x**(5/2)), Eq(a, 0)), (-2/(3*a*x**(3/2)) + 2*b/(a**2*sqrt(x)) -
 I*b*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)) + I*b*log(I*s
qrt(a)*sqrt(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)), True))
```

## 3.455 $\int \frac{1}{x^{7/2}(a+bx)}\,dx$

**Optimal**. Leaf size=68

$$-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{3/2}}-\frac{2}{5ax^{5/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$-\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b}{3a^2x^{3/2}}-\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) - (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}(a+bx)}\,dx &= -\frac{2}{5ax^{5/2}}-\frac{b\int \frac{1}{x^{5/2}(a+bx)}\,dx}{a}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{b^2\int \frac{1}{x^{3/2}(a+bx)}\,dx}{a^2}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}-\frac{2b^2}{a^3\sqrt{x}}-\frac{b^3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{a^3}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}-\frac{2b^2}{a^3\sqrt{x}}-\frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a^3}\\
&= -\frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}-\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 27, normalized size = 0.40

$$
-\frac{2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\frac{bx}{a}\right)}{5ax^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x)),x]
[Out] (-2*Hypergeometric2F1[-5/2, 1, -3/2, -((b*x)/a)])/(5*a*x^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 61, normalized size = 0.90

$$
-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2\left(3a^2-5abx+15b^2x^2\right)}{15a^3x^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x)),x]
[Out] (-2*(3*a^2 - 5*a*b*x + 15*b^2*x^2))/(15*a^3*x^(5/2)) - (2*b^(5/2)*ArcTan[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

**fricas** [A] time = 1.04, size = 144, normalized size = 2.12

$$\left[\frac{15\,b^2x^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^2x^2-5\,abx+3\,a^2\right)\sqrt{x}}{15\,a^3x^3},\frac{2\left(15\,b^2x^3\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15\,b^2x^2-5\,abx+3\,a^2\right)\sqrt{x}\right)}{15\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x +
a)) - 2*(15*b^2*x^2 - 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3
*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 - 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

**giac** [A] time = 0.99, size = 52, normalized size = 0.76

$$-\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{2\left(15\,b^2x^2-5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

**maple** [A] time = 0.01, size = 54, normalized size = 0.79

$$-\frac{2b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^3}-\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{\frac{3}{2}}}-\frac{2}{5a\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a),x)
```

```
[Out] -2/5/a/x^(5/2)-2*b^2/a^3/x^(1/2)+2/3*b/a^2/x^(3/2)-2/a^3*b^3/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima** [A] time = 2.91, size = 52, normalized size = 0.76

$$-\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{2\left(15\,b^2x^2-5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

**mupad [B]** time = 0.11, size = 49, normalized size = 0.72

$$-\frac{\frac{2}{5\,a}+\frac{2\,b^2\,x^2}{a^3}-\frac{2\,b\,x}{3\,a^2}}{x^{5/2}}-\frac{2\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x)),x)
```

```
[Out] - (2/(5*a) + (2*b^2*x^2)/a^3 - (2*b*x)/(3*a^2))/x^(5/2) - (2*b^(5/2)*atan((
b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

**sympy [A]** time = 24.82, size = 139, normalized size = 2.04

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for}\: a = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for}\: b = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} + \frac{2b}{3a^2x^{\frac{3}{2}}} - \frac{2b^2}{a^3\sqrt{x}} + \frac{ib^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{7}{2}}\sqrt{\frac{1}{b}}} - \frac{ib^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{7}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/2)
) - 2*b**2/(a**3*sqrt(x)) + I*b**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a**
(7/2)*sqrt(1/b)) - I*b**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(7/2)*sqrt
(1/b)), True))
```

## 3.456 $\int \frac{x^{5/2}}{(a+bx)^2} dx$

**Optimal**. Leaf size=70

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{5a\sqrt{x}}{b^3}-\frac{x^{5/2}}{b(a+bx)}+\frac{5x^{3/2}}{3b^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 205}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{5a\sqrt{x}}{b^3}-\frac{x^{5/2}}{b(a+bx)}+\frac{5x^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^2,x]
```

```
[Out] (-5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) - x^(5/2)/(b*(a + b*x)) + (5*a^(3/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^2}\,dx &= -\frac{x^{5/2}}{b(a+bx)}+\frac{5\int \frac{x^{3/2}}{a+bx}\,dx}{2b}\\
&= \frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}-\frac{(5a)\int \frac{\sqrt{x}}{a+bx}\,dx}{2b^2}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{\left(5a^2\right)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2b^3}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.00, size = 27, normalized size = 0.39

$$
\frac{2x^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^2,x]
```

```
[Out] (2*x^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*x)/a)])/(7*a^2)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 74, normalized size = 1.06

$$
\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{-15a^2\sqrt{x}\,-10abx^{3/2}+2b^2x^{5/2}}{3b^3(a+bx)}
$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(5/2)/(a + b*x)^2,x]

[Out] (-15*a^2*Sqrt[x] - 10*a*b*x^(3/2) + 2*b^2*x^(5/2))/(3*b^3*(a + b*x)) + (5*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)

**fricas [A]** time = 0.92, size = 161, normalized size = 2.30

$$\left[\frac{15\left(abx+a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(2b^2x^2-10abx-15a^2\right)\sqrt{x}}{6\left(b^4x+ab^3\right)}, \frac{15\left(abx+a^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+\left(2b^2x^2-10abx-15a^2\right)\sqrt{x}}{3\left(b^4x+ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="fricas")

[Out] [1/6*(15*(a*b*x + a^2)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*(2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3), 1/3*(15*(a*b*x + a^2)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3)]

**giac [A]** time = 0.93, size = 65, normalized size = 0.93

$$\frac{5a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} - \frac{a^2\sqrt{x}}{(bx+a)b^3} + \frac{2\left(b^4x^{\frac{3}{2}} - 6ab^3\sqrt{x}\right)}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="giac")

[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - a^2*sqrt(x)/((b*x + a)*b^3) + 2/3*(b^4*x^(3/2) - 6*a*b^3*sqrt(x))/b^6

**maple [A]** time = 0.01, size = 61, normalized size = 0.87

$$\frac{5a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3} - \frac{a^2\sqrt{x}}{(bx+a)\,b^3} + \frac{2x^{\frac{3}{2}}}{3b^2} - \frac{4a\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(b*x+a)^2,x)

[Out] 2/3*x^(3/2)/b^2-4*a*x^(1/2)/b^3-1/b^3*a^2*x^(1/2)/(b*x+a)+5/b^3*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))

**maxima** [A] time = 2.96, size = 63, normalized size = 0.90

$$-\frac{a^2\sqrt{x}}{b^4x+ab^3}+\frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(bx^{\frac{3}{2}}-6\,a\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^2*sqrt(x)/(b^4*x + a*b^3) + 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)
*b^3) + 2/3*(b*x^(3/2) - 6*a*sqrt(x))/b^3
```

**mupad** [B] time = 0.11, size = 58, normalized size = 0.83

$$\frac{2\,x^{3/2}}{3\,b^2}-\frac{4\,a\,\sqrt{x}}{b^3}-\frac{a^2\,\sqrt{x}}{x\,b^4+a\,b^3}+\frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^2,x)
```

```
[Out] (2*x^(3/2))/(3*b^2) - (4*a*x^(1/2))/b^3 - (a^2*x^(1/2))/(a*b^3 + b^4*x) + (
5*a^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

**sympy** [A] time = 24.57, size = 479, normalized size = 6.84

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{7}{2}}}{7a^2} & \text{for}\: b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b^2} & \text{for}\: a = 0 \\ -\frac{30ia^{\frac{5}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}-\frac{20ia^{\frac{3}{2}}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}+\frac{4i\sqrt{a}b^3x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}+\frac{15a^3\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}-\frac{15a^3\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}+\frac{15a^2bx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}-\frac{15a^2bx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b,
0)), (2*x**(3/2)/(3*b**2), Eq(a, 0)), (-30*I*a**(5/2)*b*sqrt(x)*sqrt(1/b)/(
6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) - 20*I*a**(3/2)
*b**2*x**(3/2)*sqrt(1/b)/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*
sqrt(1/b)) + 4*I*sqrt(a)*b**3*x**(5/2)*sqrt(1/b)/(6*I*a**(3/2)*b**4*sqrt(1/
b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) + 15*a**3*log(-I*sqrt(a)*sqrt(1/b) + sqr
t(x))/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) - 15*a**
3*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqr
t(a)*b**5*x*sqrt(1/b)) + 15*a**2*b*x*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6
```

```
*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) - 15*a**2*b*x*lo
g(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)
*b**5*x*sqrt(1/b)), True))
```

## 3.457 $\int \frac{x^{3/2}}{(a+bx)^2} \, dx$

**Optimal**. Leaf size=57

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{x^{3/2}}{b(a+bx)}+\frac{3\sqrt{x}}{b^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 205}

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{x^{3/2}}{b(a+bx)}+\frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - x^(3/2)/(b*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(a+bx)^2}\,dx &= -\frac{x^{3/2}}{b(a+bx)}+\frac{3\int \frac{\sqrt{x}}{a+bx}\,dx}{2b}\\
&= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{(3a)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2b^2}\\
&= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{(3a)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.00, size = 27, normalized size = 0.47

$$\frac{2x^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{bx}{a}\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^2,x]
[Out] (2*x^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*x)/a)])/(5*a^2)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 58, normalized size = 1.02

$$\frac{3a\sqrt{x}+2bx^{3/2}}{b^2(a+bx)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*a*Sqrt[x] + 2*b*x^(3/2))/(b^2*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/b^(5/2)
```

**fricas [A]** time = 0.83, size = 134, normalized size = 2.35

$$\left[\frac{3\,(bx+a)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,bx+3\,a)\sqrt{x}}{2\left(b^3x+ab^2\right)},-\frac{3\,(bx+a)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{\frac{a}{b}}}{a}\right)-(2\,bx+3\,a)\sqrt{x}}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x + a)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
a)) + 2*(2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2), -(3*(b*x + a)*sqrt(a/b)*arc
tan(b*sqrt(x)*sqrt(a/b)/a) - (2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2)]
```

**giac [A]** time = 0.96, size = 46, normalized size = 0.81

$$-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{a\sqrt{x}}{(bx+a)b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + a*sqrt(x)/((b*x + a)*b^2
) + 2*sqrt(x)/b^2
```

**maple [A]** time = 0.01, size = 47, normalized size = 0.82

$$-\frac{3a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}+\frac{a\sqrt{x}}{(bx+a)\,b^2}+\frac{2\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^2,x)
```

```
[Out] 2*x^(1/2)/b^2+1/b^2*a*x^(1/2)/(b*x+a)-3/b^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1
/2)*b*x^(1/2))
```

**maxima [A]** time = 2.91, size = 49, normalized size = 0.86

$$\frac{a\sqrt{x}}{b^{3}x+ab^{2}}-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{2\,\sqrt{x}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*sqrt(x)/(b^3*x + a*b^2) - 3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
 + 2*sqrt(x)/b^2
```

**mupad [B]** time = 0.12, size = 46, normalized size = 0.81

$$\frac{2\,\sqrt{x}}{b^{2}}+\frac{a\,\sqrt{x}}{x\,b^{3}+a\,b^{2}}-\frac{3\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^2,x)
```

```
[Out] (2*x^(1/2))/b^2 + (a*x^(1/2))/(a*b^2 + b^3*x) - (3*a^(1/2)*atan((b^(1/2)*x^
(1/2))/a^(1/2)))/b^(5/2)
```

**sympy [A]** time = 9.18, size = 411, normalized size = 7.21

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{5}{2}}}{5a^{2}} & \text{for}\: b = 0 \\ \frac{2\sqrt{x}}{b^{2}} & \text{for}\: a = 0 \\ \frac{6ia^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}}+\frac{4i\sqrt{a}\,b^{2}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}}-\frac{3a^{2}\log\left(-i\sqrt{a}\,\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}}+\frac{3a^{2}\log\left(i\sqrt{a}\,\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}}-\frac{3abx\log\left(-i\sqrt{a}\,\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}}+\frac{3abx\log\left(i\sqrt{a}\,\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0
)), (2*sqrt(x)/b**2, Eq(a, 0)), (6*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(2*I*a**(
3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b)) + 4*I*sqrt(a)*b**2*x**(
3/2)*sqrt(1/b)/(2*I*a**(3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b))
 - 3*a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**3*sqrt(1/b)
+ 2*I*sqrt(a)*b**4*x*sqrt(1/b)) + 3*a**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))
/(2*I*a**(3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b)) - 3*a*b*x*log
(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)
*b**4*x*sqrt(1/b)) + 3*a*b*x*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/
2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b)), True))
```

# 3.458 $\int \frac{\sqrt{x}}{(a+bx)^2} \, dx$

**Optimal**. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}$$

**Rubi [A]** time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = -\frac{\sqrt{x}}{b(a+bx)} + \frac{\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2b}$$
$$= -\frac{\sqrt{x}}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}}{b(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} - \frac{\sqrt{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

**IntegrateAlgebraic** [A] time = 0.06, size = 46, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} - \frac{\sqrt{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

**fricas** [A] time = 0.89, size = 115, normalized size = 2.50

$$\left[-\frac{2\,ab\sqrt{x} + \sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)}{2\left(ab^3x + a^2b^2\right)}, -\frac{ab\sqrt{x} + \sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab^3x + a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a*b^3*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(b*x + a)*
arctan(sqrt(a*b)/(b*sqrt(x))))/(a*b^3*x + a^2*b^2)]
```

**giac** [A] time = 0.90, size = 36, normalized size = 0.78

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b} - \frac{\sqrt{x}}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - sqrt(x)/((b*x + a)*b)
```

**maple** [A] time = 0.01, size = 37, normalized size = 0.80

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} - \frac{\sqrt{x}}{(bx+a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^2,x)
```

```
[Out] -x^(1/2)/b/(b*x+a)+1/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima** [A] time = 2.94, size = 37, normalized size = 0.80

$$-\frac{\sqrt{x}}{b^2x+ab} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(b^2*x + a*b) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

**mupad** [B] time = 0.04, size = 34, normalized size = 0.74

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} - \frac{\sqrt{x}}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^2,x)
```

```
[Out] atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(1/2)*b^(3/2)) - x^(1/2)/(b*(a + b*x))
```

**sympy** [A] time = 4.45, size = 337, normalized size = 7.33

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for } b = 0 \\ -\frac{2}{b^2\sqrt{x}} & \text{for } a = 0 \\ -\frac{2i\sqrt{a}\,b\sqrt{x}\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} + \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} - \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} + \frac{bx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} - \frac{bx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0
)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (-2*I*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(2*I*a
**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*sqrt(1/b)) + a*log(-I*sqrt(a)*s
qrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*sqrt(
1/b)) - a*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**2*sqrt(1/b) +
 2*I*sqrt(a)*b**3*x*sqrt(1/b)) + b*x*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2
*I*a**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*sqrt(1/b)) - b*x*log(I*sqrt
(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*
sqrt(1/b)), True))
```

## 3.459 $\int \frac{1}{\sqrt{x}\,(a+bx)^2}\,dx$

**Optimal**. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}$$

**Rubi** [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx = \frac{\sqrt{x}}{a(a+bx)} + \frac{\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2a}$$
$$= \frac{\sqrt{x}}{a(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{\sqrt{x}}{a(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

**Mathematica** [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} + \frac{\sqrt{x}}{a^2+abx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^2),x]
[Out] Sqrt[x]/(a^2 + a*b*x) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} + \frac{\sqrt{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x)^2),x]
[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

**fricas** [A] time = 0.87, size = 116, normalized size = 2.58

$$\left[\frac{2\,ab\sqrt{x} - \sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)}{2\left(a^2b^2x + a^3b\right)}, \frac{ab\sqrt{x} - \sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{a^2b^2x + a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqrt
(x))/(b*x + a)))/(a^2*b^2*x + a^3*b), (a*b*sqrt(x) - sqrt(a*b)*(b*x + a)*ar
ctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b^2*x + a^3*b)]
```

**giac [A]** time = 0.89, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} + \frac{\sqrt{x}}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x)/((b*x + a)*a)
```

**maple [A]** time = 0.01, size = 36, normalized size = 0.80

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} + \frac{\sqrt{x}}{(bx+a)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/x^(1/2),x)
```

```
[Out] x^(1/2)/a/(b*x+a)+1/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 2.92, size = 35, normalized size = 0.78

$$\frac{\sqrt{x}}{abx+a^2} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/(a*b*x + a^2) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a)
```

**mupad [B]** time = 0.09, size = 33, normalized size = 0.73

$$\frac{\sqrt{x}}{a\,(a+b\,x)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^2),x)
```

```
[Out] x^(1/2)/(a*(a + b*x)) + atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(3/2)*b^(1/2))
```

**sympy** [A] time = 7.53, size = 328, normalized size = 7.29

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{a^2} & \text{for } b = 0 \\ -\frac{2}{3b^2x^{\frac{3}{2}}} & \text{for } a = 0 \\ \frac{2i\sqrt{a}\,b\sqrt{x}\sqrt{\frac{1}{b}}}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} + \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} - \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} + \frac{bx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} - \frac{bx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
(-2/(3*b**2*x**(3/2)), Eq(a, 0)), (2*I*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(2*I*a**
(5/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b)) + a*log(-I*sqrt(a)*sqrt(
1/b) + sqrt(x))/(2*I*a**(5/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b))
- a*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(5/2)*b*sqrt(1/b) + 2*I*a**(
3/2)*b**2*x*sqrt(1/b)) + b*x*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(5
/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b)) - b*x*log(I*sqrt(a)*sqrt(1
/b) + sqrt(x))/(2*I*a**(5/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b)),
True))
```

## 3.460 $\int \frac{1}{x^{3/2}(a+bx)^2}\,dx$

**Optimal**. Leaf size=56

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}$$

**Rubi [A]** time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a + b*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx)^2}\,dx = \frac{1}{a\sqrt{x}\,(a+bx)}+\frac{3\int \frac{1}{x^{3/2}(a+bx)}\,dx}{2a}$$
$$= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{(3b)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2a^2}$$
$$= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{(3b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}$$
$$= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

**Mathematica [C]** time = 0.00, size = 25, normalized size = 0.45

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{bx}{a}\right)}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, -((b*x)/a)])/(a^2*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.07, size = 54, normalized size = 0.96

$$\frac{-2a-3bx}{a^2\sqrt{x}\,(a+bx)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] (-2*a - 3*b*x)/(a^2*Sqrt[x]*(a + b*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

**fricas [A]** time = 0.96, size = 147, normalized size = 2.62

$$\left[\frac{3\left(bx^2+ax\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(3\,bx+2\,a)\sqrt{x}}{2\left(a^2bx^2+a^3x\right)},\ \frac{3\left(bx^2+ax\right)\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx+2\,a)\sqrt{x}}{a^2bx^2+a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 + a*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*
x + a)) - 2*(3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2 + a^3*x), (3*(b*x^2 + a*x)*sq
rt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2
 + a^3*x)]
```

**giac [A]** time = 1.01, size = 49, normalized size = 0.88

$$-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}-\frac{3\,bx+2\,a}{\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - (3*b*x + 2*a)/((b*x^(3/2
) + a*sqrt(x))*a^2)
```

**maple [A]** time = 0.01, size = 48, normalized size = 0.86

$$-\frac{3b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}-\frac{b\sqrt{x}}{(bx+a)\,a^2}-\frac{2}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^2,x)
```

```
[Out] -2/a^2/x^(1/2)-1/a^2*b*x^(1/2)/(b*x+a)-3/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(
1/2)*b*x^(1/2))
```

**maxima [A]** time = 2.94, size = 51, normalized size = 0.91

$$-\frac{3\,bx+2\,a}{a^2bx^{\frac{3}{2}}+a^3\sqrt{x}}-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(3*b*x + 2*a)/(a^2*b*x^(3/2) + a^3*sqrt(x)) - 3*b*arctan(b*sqrt(x)/sqrt(a*
b))/(sqrt(a*b)*a^2)
```

**mupad [B]** time = 0.12, size = 48, normalized size = 0.86

$$-\frac{\frac{2}{a}+\frac{3\,b\,x}{a^2}}{a\,\sqrt{x}\,+b\,x^{3/2}}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^2),x)
```

```
[Out] - (2/a + (3*b*x)/a^2)/(a*x^(1/2) + b*x^(3/2)) - (3*b^(1/2)*atan((b^(1/2)*x^
(1/2))/a^(1/2)))/a^(5/2)
```

**sympy [A]** time = 17.73, size = 434, normalized size = 7.75

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{5b^2x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ -\frac{2}{a^2\sqrt{x}} & \text{for}\: b = 0 \\ -\frac{4ia^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{2ia^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+2ia^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} - \frac{6i\sqrt{a}\,bx\sqrt{\frac{1}{b}}}{2ia^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+2ia^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} - \frac{3a\sqrt{x}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+2ia^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} + \frac{3a\sqrt{x}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+2ia^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} - \frac{3bx^{\frac{3}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+2ia^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} + \frac{3bx^{\frac{3}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+2ia^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**2*x**(5/2)), Eq(a,
 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-4*I*a**(3/2)*sqrt(1/b)/(2*I*a**(7/2)
*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b)) - 6*I*sqrt(a)*b*x*s
qrt(1/b)/(2*I*a**(7/2)*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b
)) - 3*a*sqrt(x)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(7/2)*sqrt(x)*
sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*a*sqrt(x)*log(I*sqrt(a)*
sqrt(1/b) + sqrt(x))/(2*I*a**(7/2)*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3
/2)*sqrt(1/b)) - 3*b*x**(3/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(
7/2)*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*b*x**(3/2)*
log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(7/2)*sqrt(x)*sqrt(1/b) + 2*I*a*
*(5/2)*b*x**(3/2)*sqrt(1/b)), True))
```

## 3.461 $\int \frac{1}{x^{5/2}(a+bx)^2} \, dx$

**Optimal**. Leaf size=69

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a+bx)}$$

**Rubi [A]** time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] -5/(3*a^2*x^(3/2)) + (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a + b*x)) + (5*b^(
3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(a+bx)^2}\,dx &= \frac{1}{ax^{3/2}(a+bx)} + \frac{5\int \frac{1}{x^{5/2}(a+bx)}\,dx}{2a} \\
&= -\frac{5}{3a^2x^{3/2}} + \frac{1}{ax^{3/2}(a+bx)} - \frac{(5b)\int \frac{1}{x^{3/2}(a+bx)}\,dx}{2a^2} \\
&= -\frac{5}{3a^2x^{3/2}} + \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a+bx)} + \frac{\left(5b^2\right)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2a^3} \\
&= -\frac{5}{3a^2x^{3/2}} + \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a+bx)} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3} \\
&= -\frac{5}{3a^2x^{3/2}} + \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a+bx)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

**Mathematica** [C] time = 0.01, size = 27, normalized size = 0.39

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\frac{bx}{a}\right)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 2, -1/2, -((b*x)/a)])/(3*a^2*x^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.08, size = 68, normalized size = 0.99

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{-2a^2+10abx+15b^2x^2}{3a^3x^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] (-2*a^2 + 10*a*b*x + 15*b^2*x^2)/(3*a^3*x^(3/2)*(a + b*x)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

**fricas** [A] time = 1.01, size = 184, normalized size = 2.67

$$\left[\frac{15\left(b^2x^3+abx^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\left(15b^2x^2+10abx-2a^2\right)\sqrt{x}}{6\left(a^3bx^3+a^4x^2\right)}, -\frac{15\left(b^2x^3+abx^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15b^2x^2+10abx-2a^2\right)\sqrt{x}}{3\left(a^3bx^3+a^4x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 + a*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) -
 a)/(b*x + a)) + 2*(15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^
4*x^2), -1/3*(15*(b^2*x^3 + a*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x
))) - (15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

**giac** [A] time = 0.96, size = 58, normalized size = 0.84

$$\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}+\frac{b^2\sqrt{x}}{(bx+a)a^3}+\frac{2\,(6\,bx-a)}{3\,a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + b^2*sqrt(x)/((b*x + a)*
a^3) + 2/3*(6*b*x - a)/(a^3*x^(3/2))
```

**maple** [A] time = 0.02, size = 60, normalized size = 0.87

$$\frac{5b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^3}+\frac{b^2\sqrt{x}}{(bx+a)\,a^3}+\frac{4b}{a^3\sqrt{x}}-\frac{2}{3a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^2,x)
```

```
[Out] -2/3/a^2/x^(3/2)+4*b/a^3/x^(1/2)+1/a^3*b^2*x^(1/2)/(b*x+a)+5/a^3*b^2/(a*b)^
(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima** [A] time = 2.88, size = 64, normalized size = 0.93

$$\frac{15\,b^2x^2+10\,abx-2\,a^2}{3\left(a^3bx^{\frac{5}{2}}+a^4x^{\frac{3}{2}}\right)}+\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(15*b^2*x^2 + 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) + a^4*x^(3/2)) + 5*b^2*a
rctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

**mupad [B]** time = 0.15, size = 58, normalized size = 0.84

$$\frac{\frac{5\,b^2\,x^2}{a^3}-\frac{2}{3\,a}+\frac{10\,b\,x}{3\,a^2}}{a\,x^{3/2}+b\,x^{5/2}}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^2),x)
```

```
[Out] ((5*b^2*x^2)/a^3 - 2/(3*a) + (10*b*x)/(3*a^2))/(a*x^(3/2) + b*x^(5/2)) + (5
*b^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

**sympy [A]** time = 50.52, size = 507, normalized size = 7.35

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{3 a^{2} x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{7 b^{2} x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ - \frac{4 i a^{\frac{5}{2}} \sqrt{\frac{1}{b}}}{6 i a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 6 i a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{20 i a^{\frac{3}{2}} b x \sqrt{\frac{1}{b}}}{6 i a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 6 i a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{30 i \sqrt{a} b^{2} x^{2} \sqrt{\frac{1}{b}}}{6 i a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 6 i a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{15 a b x^{\frac{3}{2}} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 i a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 6 i a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{15 a b x^{\frac{3}{2}} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 i a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 6 i a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{15 b^{2} x^{\frac{5}{2}} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 i a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 6 i a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{15 b^{2} x^{\frac{5}{2}} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 i a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 6 i a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a**2*x**(3/2)), Eq(b,
 0)), (-2/(7*b**2*x**(7/2)), Eq(a, 0)), (-4*I*a**(5/2)*sqrt(1/b)/(6*I*a**(9
/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 20*I*a**(3/2)
*b*x*sqrt(1/b)/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*s
qrt(1/b)) + 30*I*sqrt(a)*b**2*x**2*sqrt(1/b)/(6*I*a**(9/2)*x**(3/2)*sqrt(1/
b) + 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*a*b*x**(3/2)*log(-I*sqrt(a)*sq
rt(1/b) + sqrt(x))/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/
2)*sqrt(1/b)) - 15*a*b*x**(3/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**
(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*b**2*x**
(5/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b)
+ 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) - 15*b**2*x**(5/2)*log(I*sqrt(a)*sqrt(
1/b) + sqrt(x))/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*
sqrt(1/b)), True))
```

# 3.462 $\int \frac{x^{7/2}}{(a+bx)^3} \, dx$

**Optimal**. Leaf size=95

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}-\frac{35a\sqrt{x}}{4b^4}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{x^{7/2}}{2b(a+bx)^2}+\frac{35x^{3/2}}{12b^3}$$

**Rubi [A]** time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 205}

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{35a\sqrt{x}}{4b^4}-\frac{x^{7/2}}{2b(a+bx)^2}+\frac{35x^{3/2}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x)^3,x]
```

```
[Out] (-35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a + b*x)^2)
 - (7*x^(5/2))/(4*b^2*(a + b*x)) + (35*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{(a+bx)^3}\,dx &= -\frac{x^{7/2}}{2b(a+bx)^2}+\frac{7\int \frac{x^{5/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35\int \frac{x^{3/2}}{a+bx}\,dx}{8b^2}\\
&= \frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{(35a)\int \frac{\sqrt{x}}{a+bx}\,dx}{8b^3}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{\left(35a^2\right)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8b^4}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{\left(35a^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^4}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.00, size = 27, normalized size = 0.28

$$\frac{2x^{9/2}\,{}_2F_1\left(3,\frac{9}{2};\frac{11}{2};-\frac{bx}{a}\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x)^3,x]
```

```
[Out] (2*x^(9/2)*Hypergeometric2F1[3, 9/2, 11/2, -((b*x)/a)])/(9*a^3)
```

**IntegrateAlgebraic [A]** time = 0.13, size = 89, normalized size = 0.94

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}+\frac{-105a^3\sqrt{x}-175a^2bx^{3/2}-56ab^2x^{5/2}+8b^3x^{7/2}}{12b^4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(a + b*x)^3,x]
```

```
[Out] (-105*a^3*Sqrt[x] - 175*a^2*b*x^(3/2) - 56*a*b^2*x^(5/2) + 8*b^3*x^(7/2))/(
12*b^4*(a + b*x)^2) + (35*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(
9/2))
```

**fricas [A]** time = 0.91, size = 227, normalized size = 2.39

$$\left[\frac{105\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(8b^3x^3-56ab^2x^2-175a^2bx-105a^3\right)\sqrt{x}}{24\left(b^6x^2+2ab^5x+a^2b^4\right)},\frac{105\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+\left(8b^3x^3-56ab^2x^2-175a^2bx-105a^3\right)\sqrt{x}}{12\left(b^6x^2+2ab^5x+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*
sqrt(-a/b) - a)/(b*x + a)) + 2*(8*b^3*x^3 - 56*a*b^2*x^2 - 175*a^2*b*x - 10
5*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 + 2*a
^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*b^3*x^3 - 56*a*b
^2*x^2 - 175*a^2*b*x - 105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

**giac [A]** time = 1.03, size = 77, normalized size = 0.81

$$\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^4}-\frac{13\,a^2bx^{\frac{3}{2}}+11\,a^3\sqrt{x}}{4\,(bx+a)^2b^4}+\frac{2\left(b^6x^{\frac{3}{2}}-9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/4*(13*a^2*b*x^(3/2
) + 11*a^3*sqrt(x))/((b*x + a)^2*b^4) + 2/3*(b^6*x^(3/2) - 9*a*b^5*sqrt(x))
/b^9
```

**maple [A]** time = 0.02, size = 79, normalized size = 0.83

$$-\frac{13a^2x^{\frac{3}{2}}}{4\,(bx+a)^2\,b^3}-\frac{11a^3\sqrt{x}}{4\,(bx+a)^2\,b^4}+\frac{35a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;b^4}+\frac{2x^{\frac{3}{2}}}{3b^3}-\frac{6a\sqrt{x}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x+a)^3,x)
```

```
[Out] 2/3*x^(3/2)/b^3-6*a*x^(1/2)/b^4-13/4/b^3*a^2/(b*x+a)^2*x^(3/2)-11/4/b^4*a^3
/(b*x+a)^2*x^(1/2)+35/4/b^4*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 3.07, size = 86, normalized size = 0.91

$$-\frac{13\,a^{2}bx^{\frac{3}{2}}+11\,a^{3}\sqrt{x}}{4\left(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4}\right)}+\frac{35\,a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^{4}}+\frac{2\left(bx^{\frac{3}{2}}-9\,a\sqrt{x}\right)}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(13*a^2*b*x^(3/2) + 11*a^3*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) +
35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/3*(b*x^(3/2) - 9*a
*sqrt(x))/b^4
```

**mupad [B]** time = 0.12, size = 81, normalized size = 0.85

$$\frac{2\,x^{3/2}}{3\,b^3}-\frac{\frac{11\,a^3\,\sqrt{x}}{4}+\frac{13\,a^2\,b\,x^{3/2}}{4}}{a^2\,b^4+2\,a\,b^5\,x+b^6\,x^2}-\frac{6\,a\,\sqrt{x}}{b^4}+\frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x)^3,x)
```

```
[Out] (2*x^(3/2))/(3*b^3) - ((11*a^3*x^(1/2))/4 + (13*a^2*b*x^(3/2))/4)/(a^2*b^4
+ b^6*x^2 + 2*a*b^5*x) - (6*a*x^(1/2))/b^4 + (35*a^(3/2)*atan((b^(1/2)*x^(1
/2))/a^(1/2)))/(4*b^(9/2))
```

**sympy [A]** time = 135.24, size = 906, normalized size = 9.54

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 x^{\frac{9}{2}}}{9 a^{3}} & \text{for}\: b = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 b^{3}} & \text{for}\: a = 0 \\ - \frac{210 i a^{\frac{7}{2}} b \sqrt{x} \sqrt{\frac{1}{b}}}{24 i a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} + 48 i a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 i \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{350 i a^{\frac{5}{2}} b^{2} x^{\frac{3}{2}} \sqrt{\frac{1}{b}}}{24 i a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} + 48 i a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 i \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{112 i a^{\frac{3}{2}} b^{3} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}}{24 i a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} + 48 i a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 i \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{16 i \sqrt{a} b^{4} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}}{24 i a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} + 48 i a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 i \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{105 a^{4} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} + 48 i a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 i \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{105 a^{4} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} + 48 i a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 i \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{210 a^{3} b x \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} + 48 i a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 i \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{210 a^{3} b x \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} + 48 i a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 i \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{105 a^{2} b^{2} x^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} + 48 i a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 i \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{105 a^{2} b^{2} x^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} + 48 i a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 i \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*a**3), Eq(b,
0)), (2*x**(3/2)/(3*b**3), Eq(a, 0)), (-210*I*a**(7/2)*b*sqrt(x)*sqrt(1/b)/
(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(
```

```
a)*b**7*x**2*sqrt(1/b)) - 350*I*a**(5/2)*b**2*x**(3/2)*sqrt(1/b)/(24*I*a**(
5/2)*b**5*sqrt(1/b) + 48*I*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x*
*2*sqrt(1/b)) - 112*I*a**(3/2)*b**3*x**(5/2)*sqrt(1/b)/(24*I*a**(5/2)*b**5*
sqrt(1/b) + 48*I*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/
b)) + 16*I*sqrt(a)*b**4*x**(7/2)*sqrt(1/b)/(24*I*a**(5/2)*b**5*sqrt(1/b) +
48*I*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) + 105*a*
*4*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I
*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) - 105*a**4*l
og(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I*a**(
3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) + 210*a**3*b*x*lo
g(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I*a**(
3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) - 210*a**3*b*x*lo
g(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I*a**(3
/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) + 105*a**2*b**2*x*
*2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I
*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) - 105*a**2*b
**2*x**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) +
 48*I*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)), True))
```

## 3.463 $\int \frac{x^{5/2}}{(a+bx)^3} \, dx$

**Optimal**. Leaf size=82

$$-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{x^{5/2}}{2b(a+bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

**Rubi [A]** time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 205}

$$-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}-\frac{x^{5/2}}{2b(a+bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a + b*x)^2) - (5*x^(3/2))/(4*b^2*(a +
b*x)) - (15*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^3}\,dx &= -\frac{x^{5/2}}{2b(a+bx)^2}+\frac{5\int \frac{x^{3/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}+\frac{15\int \frac{\sqrt{x}}{a+bx}\,dx}{8b^2}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}
\end{aligned}$$

**Mathematica** [C] time = 0.00, size = 27, normalized size = 0.33

$$\frac{2x^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^3,x]
[Out] (2*x^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, -((b*x)/a)])/(7*a^3)
```

**IntegrateAlgebraic** [A] time = 0.12, size = 76, normalized size = 0.93

$$\frac{15a^2\sqrt{x}+25abx^{3/2}+8b^2x^{5/2}}{4b^3(a+bx)^2}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*a^2*Sqrt[x] + 25*a*b*x^(3/2) + 8*b^2*x^(5/2))/(4*b^3*(a + b*x)^2) - (15
*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

**fricas [A]** time = 0.59, size = 200, normalized size = 2.44

$$\left[\frac{15\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(8\,b^{2}x^{2}+25\,abx+15\,a^{2}\right)\sqrt{x}}{8\left(b^{5}x^{2}+2\,ab^{4}x+a^{2}b^{3}\right)},-\frac{15\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\left(8\,b^{2}x^{2}+25\,abx+15\,a^{2}\right)\sqrt{x}}{4\left(b^{5}x^{2}+2\,ab^{4}x+a^{2}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-
a/b) - a)/(b*x + a)) + 2*(8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a/b)*arctan
(b*sqrt(x)*sqrt(a/b)/a) - (8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3)]
```

**giac [A]** time = 0.95, size = 59, normalized size = 0.72

$$-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^{3}}+\frac{2\,\sqrt{x}}{b^{3}}+\frac{9\,abx^{\frac{3}{2}}+7\,a^{2}\sqrt{x}}{4\,(bx+a)^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3 + 1/4*(
9*a*b*x^(3/2) + 7*a^2*sqrt(x))/((b*x + a)^2*b^3)
```

**maple [A]** time = 0.02, size = 66, normalized size = 0.80

$$\frac{9a\,x^{\frac{3}{2}}}{4\,(bx+a)^{2}\,b^{2}}+\frac{7a^{2}\sqrt{x}}{4\,(bx+a)^{2}\,b^{3}}-\frac{15a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;b^{3}}+\frac{2\sqrt{x}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^3,x)
```

```
[Out] 2*x^(1/2)/b^3+9/4/b^2*a/(b*x+a)^2*x^(3/2)+7/4/b^3*a^2/(b*x+a)^2*x^(1/2)-15/
4/b^3*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 2.94, size = 73, normalized size = 0.89

$$\frac{9\,abx^{\frac{3}{2}} + 7\,a^2\sqrt{x}}{4\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)} - \frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^3} + \frac{2\,\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*a*b*x^(3/2) + 7*a^2*sqrt(x))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) - 15/4*
a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3
```

**mupad [B]** time = 0.14, size = 69, normalized size = 0.84

$$\frac{\frac{7\,a^2\,\sqrt{x}}{4} + \frac{9\,a\,b\,x^{3/2}}{4}}{a^2\,b^3 + 2\,a\,b^4\,x + b^5\,x^2} + \frac{2\,\sqrt{x}}{b^3} - \frac{15\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^3,x)
```

```
[Out] ((7*a^2*x^(1/2))/4 + (9*a*b*x^(3/2))/4)/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x) + (
2*x^(1/2))/b^3 - (15*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*b^(7/2))
```

**sympy [A]** time = 53.29, size = 816, normalized size = 9.95

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{7}{2}}}{7a^3} & \text{for}\: b = 0 \\ \frac{2\sqrt{x}}{b^3} & \text{for}\: a = 0 \\ \frac{30ia^{\frac{5}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} + \frac{50ia^{\frac{3}{2}}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} + \frac{16i\sqrt{a}b^3x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} - \frac{15a^3\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} + \frac{15a^3\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} - \frac{30a^2bx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} + \frac{30a^2bx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} - \frac{15ab^2x^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} + \frac{15ab^2x^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**3), Eq(b, 0
)), (2*sqrt(x)/b**3, Eq(a, 0)), (30*I*a**(5/2)*b*sqrt(x)*sqrt(1/b)/(8*I*a**
(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*x*sqrt(1/b) + 8*I*sqrt(a)*b**6*x*
*2*sqrt(1/b)) + 50*I*a**(3/2)*b**2*x**(3/2)*sqrt(1/b)/(8*I*a**(5/2)*b**4*sq
rt(1/b) + 16*I*a**(3/2)*b**5*x*sqrt(1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b))
 + 16*I*sqrt(a)*b**3*x**(5/2)*sqrt(1/b)/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I
*a**(3/2)*b**5*x*sqrt(1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) - 15*a**3*log
(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/
2)*b**5*x*sqrt(1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) + 15*a**3*log(I*sqrt
(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*
x*sqrt(1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) - 30*a**2*b*x*log(-I*sqrt(a)
```

```
*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*x*s
qrt(1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) + 30*a**2*b*x*log(I*sqrt(a)*sqr
t(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*x*sqrt(
1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) - 15*a*b**2*x**2*log(-I*sqrt(a)*sqr
t(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*x*sqrt(
1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) + 15*a*b**2*x**2*log(I*sqrt(a)*sqrt
(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*x*sqrt(1
/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)), True))
```

# 3.464 $\int \frac{x^{3/2}}{(a+bx)^3}\,dx$

**Optimal**. Leaf size=70

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}-\frac{3\sqrt{x}}{4b^2(a+bx)}-\frac{x^{3/2}}{2b(a+bx)^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 205}

$$-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}-\frac{x^{3/2}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] -x^(3/2)/(2*b*(a + b*x)^2) - (3*Sqrt[x])/(4*b^2*(a + b*x)) + (3*ArcTan[(Sqr
t[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(a+bx)^3}\,dx &= -\frac{x^{3/2}}{2b(a+bx)^2}+\frac{3\int \frac{\sqrt{x}}{(a+bx)^2}\,dx}{4b}\\ &= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8b^2}\\ &= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^2}\\ &= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 59, normalized size = 0.84

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}-\frac{\sqrt{x}\,(3a+5bx)}{4b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[x]*(3*a + 5*b*x))/(b^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 63, normalized size = 0.90

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}+\frac{-3a\sqrt{x}\,-5bx^{3/2}}{4b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] (-3*a*Sqrt[x] - 5*b*x^(3/2))/(4*b^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

**fricas [A]** time = 1.01, size = 185, normalized size = 2.64

$$\left[-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)+2\left(5\,ab^2x+3\,a^2b\right)\sqrt{x}}{8\left(ab^5x^2+2\,a^2b^4x+a^3b^3\right)},-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+\left(5\,ab^2x+3\,a^2b\right)\sqrt{x}}{4\left(ab^5x^2+2\,a^2b^4x+a^3b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) + 2*(5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^
4*x + a^3*b^3), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) + (5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*x +
a^3*b^3)]
```

**giac [A]** time = 0.91, size = 47, normalized size = 0.67

$$\frac{3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\, b^2} - \frac{5\, bx^{\frac{3}{2}} + 3\, a\sqrt{x}}{4\,(bx+a)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/4*(5*b*x^(3/2) + 3*a*sq
rt(x))/((b*x + a)^2*b^2)
```

**maple [A]** time = 0.01, size = 50, normalized size = 0.71

$$\frac{3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\; b^2} + \frac{-\frac{5x^{\frac{3}{2}}}{4b} - \frac{3a\sqrt{x}}{4b^2}}{(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^3,x)
```

```
[Out] 2*(-5/8/b*x^(3/2)-3/8*a/b^2*x^(1/2))/(b*x+a)^2+3/4/b^2/(a*b)^(1/2)*arctan(1
/(a*b)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 2.96, size = 61, normalized size = 0.87

$$-\frac{5\, bx^{\frac{3}{2}} + 3\, a\sqrt{x}}{4\left(b^4x^2 + 2\, ab^3x + a^2b^2\right)} + \frac{3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*b*x^(3/2) + 3*a*sqrt(x))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 3/4*arct
an(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

**mupad [B]** time = 0.13, size = 58, normalized size = 0.83

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,b^{5/2}}-\frac{\frac{5\,x^{3/2}}{4\,b}+\frac{3\,a\,\sqrt{x}}{4\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^3,x)
```

```
[Out] (3*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(1/2)*b^(5/2)) - ((5*x^(3/2))/(4*b
) + (3*a*x^(1/2))/(4*b^2))/(a^2 + b^2*x^2 + 2*a*b*x)
```

**sympy [A]** time = 29.37, size = 726, normalized size = 10.37

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 x^{\frac{5}{2}}}{5 a^{3}} & \text{for}\: b = 0 \\ - \frac{2}{b^{3} \sqrt{x}} & \text{for}\: a = 0 \\ - \frac{6 i a^{\frac{3}{2}} b \sqrt{x} \sqrt{\frac{1}{b}}}{8 i a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} + 16 i a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 i \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} - \frac{10 i \sqrt{a} b^{2} x^{\frac{3}{2}} \sqrt{\frac{1}{b}}}{8 i a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} + 16 i a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 i \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} + \frac{3 a^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} + 16 i a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 i \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} - \frac{3 a^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} + 16 i a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 i \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} + \frac{6 a b x \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} + 16 i a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 i \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} - \frac{6 a b x \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} + 16 i a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 i \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} + \frac{3 b^{2} x^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} + 16 i a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 i \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} - \frac{3 b^{2} x^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} + 16 i a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 i \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**3), Eq(b, 0
)), (-2/(b**3*sqrt(x)), Eq(a, 0)), (-6*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*I*
a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5
*x**2*sqrt(1/b)) - 10*I*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*I*a**(5/2)*b**3*
sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b
)) + 3*a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b
) + 16*I*a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*a
**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*
a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) + 6*a*b*x*log(
-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2
)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) - 6*a*b*x*log(I*sqrt(
a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x
*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) + 3*b**2*x**2*log(-I*sqrt(a)*
sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sq
rt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*b**2*x**2*log(I*sqrt(a)*sqrt
(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sqrt(1
/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)), True))
```

## 3.465 $\int \frac{\sqrt{x}}{(a+bx)^3} \, dx$

**Optimal**. Leaf size=73

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a+bx)}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a+bx)}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] -Sqrt[x]/(2*b*(a + b*x)^2) + Sqrt[x]/(4*a*b*(a + b*x)) + ArcTan[(Sqrt[b]*Sq
rt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{(a+bx)^3}\,dx &= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx}{4b}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8ab}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4ab}\\
&= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 27, normalized size = 0.37

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};-\frac{bx}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, -((b*x)/a)])/(3*a^3)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 60, normalized size = 0.82

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}-\frac{\sqrt{x}(a-bx)}{4ab(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[x]*(a - b*x))/(a*b*(a + b*x)^2) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]]/(4*a^(3/2)*b^(3/2))
```

**fricas [A]** time = 0.92, size = 186, normalized size = 2.55

$$\left[-\frac{\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)-2\left(ab^{2}x-a^{2}b\right)\sqrt{x}}{8\left(a^{2}b^{4}x^{2}+2\,a^{3}b^{3}x+a^{4}b^{2}\right)},-\frac{\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-\left(ab^{2}x-a^{2}b\right)\sqrt{x}}{4\left(a^{2}b^{4}x^{2}+2\,a^{3}b^{3}x+a^{4}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)) - 2*(a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x
+ a^4*b^2), -1/4*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)/(b*s
qrt(x))) - (a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x + a^4*b^2)
]
```

**giac [A]** time = 1.07, size = 52, normalized size = 0.71

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,ab}+\frac{bx^{\frac{3}{2}}-a\sqrt{x}}{4\,(bx+a)^{2}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(b*x^(3/2) - a*sqrt(x
))/((b*x + a)^2*a*b)
```

**maple [A]** time = 0.01, size = 52, normalized size = 0.71

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,ab}+\frac{\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^3,x)
```

```
[Out] 2*(1/8/a*x^(3/2)-1/8/b*x^(1/2))/(b*x+a)^2+1/4/b/a/(a*b)^(1/2)*arctan(1/(a*b
)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 3.01, size = 64, normalized size = 0.88

$$\frac{bx^{\frac{3}{2}} - a\sqrt{x}}{4\left(ab^3x^2 + 2\,a^2b^2x + a^3b\right)} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^(3/2) - a*sqrt(x))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/4*arctan(
b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

**mupad [B]** time = 0.13, size = 56, normalized size = 0.77

$$\frac{\frac{x^{3/2}}{4\,a} - \frac{\sqrt{x}}{4\,b}}{a^2 + 2\,a\,b\,x + b^2\,x^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^3,x)
```

```
[Out] (x^(3/2)/(4*a) - x^(1/2)/(4*b))/(a^2 + b^2*x^2 + 2*a*b*x) + atan((b^(1/2)*x
^(1/2))/a^(1/2))/(4*a^(3/2)*b^(3/2))
```

**sympy [A]** time = 15.27, size = 721, normalized size = 9.88

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a^3} & \text{for}\ b = 0 \\ -\frac{2}{3b^3x^{\frac{3}{2}}} & \text{for}\ a = 0 \\ -\frac{2ia^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8ia^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} + \frac{2i\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8ia^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} + \frac{a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} - \frac{a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} + \frac{2abx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} - \frac{2abx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} + \frac{b^2x^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} - \frac{b^2x^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**3), Eq(b,
0)), (-2/(3*b**3*x**(3/2)), Eq(a, 0)), (-2*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(
8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)
*b**4*x**2*sqrt(1/b)) + 2*I*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*I*a**(7/2)*b
**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqr
t(1/b)) + a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(
1/b) + 16*I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) -
 a**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*
I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) + 2*a*b*x*l
og(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(
```

```
5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) - 2*a*b*x*log(I*s
qrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b*
*3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) + b**2*x**2*log(-I*sqrt(
a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x
*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) - b**2*x**2*log(I*sqrt(a)*sq
rt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x*sqrt
(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)), True))
```

## 3.466 $\int \frac{1}{\sqrt{x}(a+bx)^3} dx$

**Optimal**. Leaf size=70

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{\sqrt{x}}{2a(a+bx)^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}+\frac{\sqrt{x}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x)^2) + (3*Sqrt[x])/(4*a^2*(a + b*x)) + (3*ArcTan[(Sqrt
[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}(a+bx)^3}\,dx &= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx}{4a}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8a^2}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4a^2}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}
\end{aligned}
$$

**Mathematica [C]** time = 0.00, size = 25, normalized size = 0.36

$$
\frac{2\sqrt{x}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{bx}{a}\right)}{a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] (2*Sqrt[x]*Hypergeometric2F1[1/2, 3, 3/2, -((b*x)/a)])/a^3
```

**IntegrateAlgebraic [A]** time = 0.08, size = 63, normalized size = 0.90

$$
\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}+\frac{5a\sqrt{x}+3bx^{3/2}}{4a^2(a+bx)^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] (5*a*Sqrt[x] + 3*b*x^(3/2))/(4*a^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

**fricas [A]** time = 0.98, size = 186, normalized size = 2.66

$$
\left[-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)-2\left(3\,ab^2x+5\,a^2b\right)\sqrt{x}}{8\left(a^3b^3x^2+2\,a^4b^2x+a^5b\right)},-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-\left(3\,ab^2x+5\,a^2b\right)\sqrt{x}}{4\left(a^3b^3x^2+2\,a^4b^2x+a^5b\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) - 2*(3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*
b^2*x + a^5*b), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) - (3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*b^2*x
+ a^5*b)]
```

**giac [A]** time = 0.86, size = 47, normalized size = 0.67

$$\frac{3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^2} + \frac{3\,bx^{\frac{3}{2}} + 5\,a\sqrt{x}}{4\,(bx+a)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/4*(3*b*x^(3/2) + 5*a*sq
rt(x))/((b*x + a)^2*a^2)
```

**maple [A]** time = 0.01, size = 53, normalized size = 0.76

$$\frac{\sqrt{x}}{2\,(bx+a)^2\,a} + \frac{3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;a^2} + \frac{3\sqrt{x}}{4\,(bx+a)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/x^(1/2),x)
```

```
[Out] 1/2*x^(1/2)/a/(b*x+a)^2+3/4*x^(1/2)/a^2/(b*x+a)+3/4/a^2/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 2.96, size = 60, normalized size = 0.86

$$\frac{3\,bx^{\frac{3}{2}} + 5\,a\sqrt{x}}{4\left(a^2b^2x^2 + 2\,a^3bx + a^4\right)} + \frac{3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*b*x^(3/2) + 5*a*sqrt(x))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 3/4*arcta
n(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

**mupad [B]** time = 0.13, size = 57, normalized size = 0.81

$$\frac{\frac{5\sqrt{x}}{4a}+\frac{3\,b\,x^{3/2}}{4\,a^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^3),x)
```

```
[Out] ((5*x^(1/2))/(4*a) + (3*b*x^(3/2))/(4*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (3*
atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(5/2)*b^(1/2))
```

**sympy [A]** time = 25.69, size = 712, normalized size = 10.17

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{5 b^{3} x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ \frac{2 \sqrt{x}}{a^{3}} & \text{for}\: b = 0 \\ \frac{10 i a^{\frac{3}{2}} b \sqrt{x} \sqrt{\frac{1}{b}}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} + \frac{6 i \sqrt{a} b^{2} x^{\frac{3}{2}} \sqrt{\frac{1}{b}}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} + \frac{3 a^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} - \frac{3 a^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} + \frac{6 a b x \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} - \frac{6 a b x \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} + \frac{3 b^{2} x^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} - \frac{3 b^{2} x^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**3*x**(5/2)), Eq(a,
 0)), (2*sqrt(x)/a**3, Eq(b, 0)), (10*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*I*a
**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x*
*2*sqrt(1/b)) + 6*I*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*I*a**(9/2)*b*sqrt(1/
b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3
*a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*
a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) - 3*a**2*log(
I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b*
*2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) + 6*a*b*x*log(-I*sqrt(a)
*sqrt(1/b) + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt
(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) - 6*a*b*x*log(I*sqrt(a)*sqrt(1/b)
 + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*
I*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3*b**2*x**2*log(-I*sqrt(a)*sqrt(1/b) + sq
rt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**
(5/2)*b**3*x**2*sqrt(1/b)) - 3*b**2*x**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))
/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*
b**3*x**2*sqrt(1/b)), True))
```

## 3.467 $\int \frac{1}{x^{3/2}(a+bx)^3}\,dx$

**Optimal**. Leaf size=82

$$-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{15}{4a^3\sqrt{x}}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^3),x]
```

```
[Out] -15/(4*a^3*Sqrt[x]) + 1/(2*a*Sqrt[x]*(a + b*x)^2) + 5/(4*a^2*Sqrt[x]*(a + b
*x)) - (15*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{3/2}(a+bx)^3}\,dx &= \frac{1}{2a\sqrt{x}\,(a+bx)^2} + \frac{5\int \frac{1}{x^{3/2}(a+bx)^2}\,dx}{4a} \\
&= \frac{1}{2a\sqrt{x}\,(a+bx)^2} + \frac{5}{4a^2\sqrt{x}\,(a+bx)} + \frac{15\int \frac{1}{x^{3/2}(a+bx)}\,dx}{8a^2} \\
&= -\frac{15}{4a^3\sqrt{x}} + \frac{1}{2a\sqrt{x}\,(a+bx)^2} + \frac{5}{4a^2\sqrt{x}\,(a+bx)} - \frac{(15b)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8a^3} \\
&= -\frac{15}{4a^3\sqrt{x}} + \frac{1}{2a\sqrt{x}\,(a+bx)^2} + \frac{5}{4a^2\sqrt{x}\,(a+bx)} - \frac{(15b)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^3} \\
&= -\frac{15}{4a^3\sqrt{x}} + \frac{1}{2a\sqrt{x}\,(a+bx)^2} + \frac{5}{4a^2\sqrt{x}\,(a+bx)} - \frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 25, normalized size = 0.30

$$
-\frac{2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};-\frac{bx}{a}\right)}{a^3\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 3, 1/2, -((b*x)/a)])/(a^3*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.12, size = 70, normalized size = 0.85

$$
\frac{-8a^2-25abx-15b^2x^2}{4a^3\sqrt{x}\,(a+bx)^2} - \frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x)^3),x]
```

```
[Out] (-8*a^2 - 25*a*b*x - 15*b^2*x^2)/(4*a^3*Sqrt[x]*(a + b*x)^2) - (15*Sqrt[b]*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

**fricas** [A] time = 0.72, size = 214, normalized size = 2.61

$$\left[\frac{15\left(b^2x^3+2\,abx^2+a^2x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^2x^2+25\,abx+8\,a^2\right)\sqrt{x}}{8\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)},\ \frac{15\left(b^2x^3+2\,abx^2+a^2x\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15\,b^2x^2+25\,abx+8\,a^2\right)\sqrt{x}}{4\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sq
rt(-b/a) - a)/(b*x + a)) - 2*(15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*
b^2*x^3 + 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(
b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

**giac** [A] time = 1.14, size = 59, normalized size = 0.72

$$-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^3}-\frac{2}{a^3\sqrt{x}}-\frac{7\,b^2x^{\frac{3}{2}}+9\,ab\sqrt{x}}{4\,(bx+a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/(a^3*sqrt(x)) - 1/4
*(7*b^2*x^(3/2) + 9*a*b*sqrt(x))/((b*x + a)^2*a^3)
```

**maple** [A] time = 0.02, size = 66, normalized size = 0.80

$$-\frac{7b^2x^{\frac{3}{2}}}{4\,(bx+a)^2\,a^3}-\frac{9b\sqrt{x}}{4\,(bx+a)^2\,a^2}-\frac{15b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;a^3}-\frac{2}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^3,x)
```

```
[Out] -2/a^3/x^(1/2)-7/4/a^3*b^2/(b*x+a)^2*x^(3/2)-9/4/a^2*b/(b*x+a)^2*x^(1/2)-15
/4/a^3*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima** [A] time = 2.99, size = 73, normalized size = 0.89

$$-\frac{15\,b^2x^2+25\,abx+8\,a^2}{4\left(a^3b^2x^{\frac{5}{2}}+2\,a^4bx^{\frac{3}{2}}+a^5\sqrt{x}\right)}-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(15*b^2*x^2 + 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) + 2*a^4*b*x^(3/2) + a
^5*sqrt(x)) - 15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

**mupad** [**B**] time = 0.15, size = 70, normalized size = 0.85

$$-\frac{\frac{2}{a}+\frac{15\,b^2\,x^2}{4\,a^3}+\frac{25\,b\,x}{4\,a^2}}{a^2\,\sqrt{x}+b^2\,x^{5/2}+2\,a\,b\,x^{3/2}}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^3),x)
```

```
[Out] - (2/a + (15*b^2*x^2)/(4*a^3) + (25*b*x)/(4*a^2))/(a^2*x^(1/2) + b^2*x^(5/2
) + 2*a*b*x^(3/2)) - (15*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(7/2
))
```

**sympy** [**A**] time = 54.35, size = 865, normalized size = 10.55

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{a^3\sqrt{x}} & \text{for } b = 0 \\ -\frac{2}{7b^3x^{\frac{7}{2}}} & \text{for } a = 0 \\ -\frac{16ia^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^2x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{50ia^{\frac{3}{2}}bx\sqrt{\frac{1}{b}}}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^2x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{30i\sqrt{a}b^2x^2\sqrt{\frac{1}{b}}}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^2x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{15a^2\sqrt{x}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^2x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} + \frac{15a^2\sqrt{x}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^2x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{30abx^{\frac{3}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^2x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} + \frac{30abx^{\frac{3}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^2x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{15b^2x^{\frac{5}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^2x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} + \frac{15b^2x^{\frac{5}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^2x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**3*sqrt(x)), Eq(b, 0)
), (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (-16*I*a**(5/2)*sqrt(1/b)/(8*I*a**(11/
2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b*
*2*x**(5/2)*sqrt(1/b)) - 50*I*a**(3/2)*b*x*sqrt(1/b)/(8*I*a**(11/2)*sqrt(x)
*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2
)*sqrt(1/b)) - 30*I*sqrt(a)*b**2*x**2*sqrt(1/b)/(8*I*a**(11/2)*sqrt(x)*sqrt
(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqr
t(1/b)) - 15*a**2*sqrt(x)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2
)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**
2*x**(5/2)*sqrt(1/b)) + 15*a**2*sqrt(x)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/
(8*I*a**(11/2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I
*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 30*a*b*x**(3/2)*log(-I*sqrt(a)*sqrt(1/
b) + sqrt(x))/(8*I*a**(11/2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*s
qrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) + 30*a*b*x**(3/2)*log(I*sq
rt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)
*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 15*b**2*x**
```

```
(5/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2)*sqrt(x)*sqrt(1/b)
+ 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)
) + 15*b**2*x**(5/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2)*sqrt
(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(
5/2)*sqrt(1/b)), True))
```

## 3.468 $\int \frac{1}{x^{5/2}(a+bx)^3}\,dx$

**Optimal**. Leaf size=95

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{35}{12a^3x^{3/2}}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

**Rubi [A]** time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35b}{4a^4\sqrt{x}}-\frac{35}{12a^3x^{3/2}}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^3),x]
```

```
[Out] -35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) + 1/(2*a*x^(3/2)*(a + b*x)^2)
 + 7/(4*a^2*x^(3/2)*(a + b*x)) + (35*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx)^3}\,dx = \frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7\int \frac{1}{x^{5/2}(a+bx)^2}\,dx}{4a}$$
$$= \frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35\int \frac{1}{x^{5/2}(a+bx)}\,dx}{8a^2}$$
$$= -\frac{35}{12a^3x^{3/2}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}-\frac{(35b)\int \frac{1}{x^{3/2}(a+bx)}\,dx}{8a^3}$$
$$= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{\left(35b^2\right)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8a^4}$$
$$= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{\left(35b^2\right)\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{\right.}{4a^4}$$
$$= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

**Mathematica [C]** time = 0.01, size = 27, normalized size = 0.28

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};-\frac{bx}{a}\right)}{3a^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^3),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 3, -1/2, -((b*x)/a)])/(3*a^3*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 81, normalized size = 0.85

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{-8a^3+56a^2bx+175ab^2x^2+105b^3x^3}{12a^4x^{3/2}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x)^3),x]
```

```
[Out] (-8*a^3 + 56*a^2*b*x + 175*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^(3/2)*(a + b*
x)^2) + (35*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(9/2))
```

**fricas** [A] time = 0.84, size = 250, normalized size = 2.63

$$\left[\frac{105\left(b^3x^4+2\,ab^2x^3+a^2bx^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\left(105\,b^3x^3+175\,ab^2x^2+56\,a^2bx-8\,a^3\right)\sqrt{x}}{24\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)},-\frac{105\left(b^3x^4+2\,ab^2x^3+a^2bx^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(105\,b^3x^3+175\,ab^2x^2+56\,a^2bx-8\,a^3\right)\sqrt{x}}{12\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sq
rt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*
b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2), -1/12*(105*(b^
3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x)))
- (105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4
+ 2*a^5*b*x^3 + a^6*x^2)]
```

**giac** [A] time = 1.10, size = 71, normalized size = 0.75

$$\frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^4}+\frac{2\,(9\,bx-a)}{3\,a^4x^{\frac{3}{2}}}+\frac{11\,b^3x^{\frac{3}{2}}+13\,ab^2\sqrt{x}}{4\,(bx+a)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/3*(9*b*x - a)/(a^4
*x^(3/2)) + 1/4*(11*b^3*x^(3/2) + 13*a*b^2*sqrt(x))/((b*x + a)^2*a^4)
```

**maple** [A] time = 0.02, size = 79, normalized size = 0.83

$$\frac{11b^3x^{\frac{3}{2}}}{4\,(bx+a)^2\,a^4}+\frac{13b^2\sqrt{x}}{4\,(bx+a)^2\,a^3}+\frac{35b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;a^4}+\frac{6b}{a^4\sqrt{x}}-\frac{2}{3a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^3,x)
```

```
[Out] -2/3/a^3/x^(3/2)+6*b/a^4/x^(1/2)+11/4/a^4*b^3/(b*x+a)^2*x^(3/2)+13/4/a^3*b^
2/(b*x+a)^2*x^(1/2)+35/4/a^4*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2)
)
```

**maxima** [A] time = 2.98, size = 86, normalized size = 0.91

$$\frac{105\,b^3x^3+175\,ab^2x^2+56\,a^2bx-8\,a^3}{12\left(a^4b^2x^{\frac{7}{2}}+2\,a^5bx^{\frac{5}{2}}+a^6x^{\frac{3}{2}}\right)}+\frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)/(a^4*b^2*x^(7/2) +
2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt
(a*b)*a^4)
```

**mupad [B]** time = 0.16, size = 80, normalized size = 0.84

$$\frac{\frac{175\,b^2\,x^2}{12\,a^3}-\frac{2}{3\,a}+\frac{35\,b^3\,x^3}{4\,a^4}+\frac{14\,b\,x}{3\,a^2}}{a^2\,x^{3/2}+b^2\,x^{7/2}+2\,a\,b\,x^{5/2}}+\frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^3),x)
```

```
[Out] ((175*b^2*x^2)/(12*a^3) - 2/(3*a) + (35*b^3*x^3)/(4*a^4) + (14*b*x)/(3*a^2)
)/(a^2*x^(3/2) + b^2*x^(7/2) + 2*a*b*x^(5/2)) + (35*b^(3/2)*atan((b^(1/2)*x
^(1/2))/a^(1/2)))/(4*a^(9/2))
```

**sympy [A]** time = 138.08, size = 962, normalized size = 10.13

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{3 a^{3} x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{9 b^{3} x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ - \frac{16 i a^{\frac{7}{2}} \sqrt{\frac{1}{b}}}{24 i a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 48 i a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 i a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} + \frac{112 i a^{\frac{5}{2}} b x \sqrt{\frac{1}{b}}}{24 i a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 48 i a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 i a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} + \frac{350 i a^{\frac{3}{2}} b^{2} x^{2} \sqrt{\frac{1}{b}}}{24 i a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 48 i a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 i a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} + \frac{210 i \sqrt{a} b^{3} x^{3} \sqrt{\frac{1}{b}}}{24 i a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 48 i a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 i a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} + \frac{105 a^{2} b x^{\frac{3}{2}} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 48 i a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 i a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} - \frac{105 a^{2} b x^{\frac{3}{2}} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 48 i a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 i a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} + \frac{210 a b^{2} x^{\frac{5}{2}} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 48 i a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 i a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} - \frac{210 a b^{2} x^{\frac{5}{2}} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 48 i a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 i a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} + \frac{105 b^{3} x^{\frac{7}{2}} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 48 i a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 i a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} - \frac{105 b^{3} x^{\frac{7}{2}} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 i a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 48 i a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 i a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a**3*x**(3/2)), Eq(b,
 0)), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (-16*I*a**(7/2)*sqrt(1/b)/(24*I*a**
(13/2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(
9/2)*b**2*x**(7/2)*sqrt(1/b)) + 112*I*a**(5/2)*b*x*sqrt(1/b)/(24*I*a**(13/2
)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)*
b**2*x**(7/2)*sqrt(1/b)) + 350*I*a**(3/2)*b**2*x**2*sqrt(1/b)/(24*I*a**(13/
2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)
*b**2*x**(7/2)*sqrt(1/b)) + 210*I*sqrt(a)*b**3*x**3*sqrt(1/b)/(24*I*a**(13/
2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)
*b**2*x**(7/2)*sqrt(1/b)) + 105*a**2*b*x**(3/2)*log(-I*sqrt(a)*sqrt(1/b) +
sqrt(x))/(24*I*a**(13/2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqr
t(1/b) + 24*I*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) - 105*a**2*b*x**(3/2)*log(I
*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(13/2)*x**(3/2)*sqrt(1/b) + 48*I*a**
(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) + 210*
a*b**2*x**(5/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(13/2)*x**(3/2
)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)*b**2*x**(
```

```
7/2)*sqrt(1/b)) - 210*a*b**2*x**(5/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2
4*I*a**(13/2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24
*I*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) + 105*b**3*x**(7/2)*log(-I*sqrt(a)*sqr
t(1/b) + sqrt(x))/(24*I*a**(13/2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**
(5/2)*sqrt(1/b) + 24*I*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) - 105*b**3*x**(7/2
)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(13/2)*x**(3/2)*sqrt(1/b) + 4
8*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)*b**2*x**(7/2)*sqrt(1/b))
, True))
```

## 3.469 $\int \frac{x^{5/2}}{-a+bx} \, dx$

**Optimal**. Leaf size=68

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

**Rubi [A]** time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 63, 208}

$$\frac{2a^2\sqrt{x}}{b^3}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 + (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{-a+bx}\,dx &= \frac{2x^{5/2}}{5b}+\frac{a\int \frac{x^{3/2}}{-a+bx}\,dx}{b}\\
&= \frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}+\frac{a^2\int \frac{\sqrt{x}}{-a+bx}\,dx}{b^2}\\
&= \frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}+\frac{a^3\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{b^3}\\
&= \frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}+\frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 61, normalized size = 0.90

$$\frac{2\sqrt{x}\left(15a^2+5abx+3b^2x^2\right)}{15b^3}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(15*a^2 + 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTanh[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 67, normalized size = 0.99

$$\frac{2\left(15a^2\sqrt{x}+5abx^{3/2}+3b^2x^{5/2}\right)}{15b^3}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*(15*a^2*Sqrt[x] + 5*a*b*x^(3/2) + 3*b^2*x^(5/2)))/(15*b^3) - (2*a^(5/2)*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

**fricas** [A] time = 1.08, size = 131, normalized size = 1.93

$$\left[\frac{15\,a^2\sqrt{\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(3\,b^2x^2+5\,abx+15\,a^2\right)\sqrt{x}}{15\,b^3},\ \frac{2\left(15\,a^2\sqrt{-\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\left(3\,b^2x^2+5\,abx+15\,a^2\right)\sqrt{x}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) +
2*(3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3, 2/15*(15*a^2*sqrt(-a/b)*arct
an(b*sqrt(x)*sqrt(-a/b)/a) + (3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

**giac** [A] time = 1.03, size = 61, normalized size = 0.90

$$\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^3}+\frac{2\left(3\,b^4x^{\frac{5}{2}}+5\,ab^3x^{\frac{3}{2}}+15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*a^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2/15*(3*b^4*x^(5/2) +
 5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

**maple** [A] time = 0.01, size = 54, normalized size = 0.79

$$-\frac{2a^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3}+\frac{\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{2ab\,x^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a),x)
```

```
[Out] 2/b^3*(1/5*b^2*x^(5/2)+1/3*a*b*x^(3/2)+a^2*x^(1/2))-2*a^3/b^3/(a*b)^(1/2)*a
rctanh(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima** [A] time = 2.94, size = 70, normalized size = 1.03

$$\frac{a^3\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^2x^{\frac{5}{2}}+5\,abx^{\frac{3}{2}}+15\,a^2\sqrt{x}\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a^3*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^3) +
2/15*(3*b^2*x^(5/2) + 5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

**mupad [B]** time = 0.15, size = 51, normalized size = 0.75

$$\frac{2\,x^{5/2}}{5\,b}+\frac{2\,a\,x^{3/2}}{3\,b^2}+\frac{2\,a^2\,\sqrt{x}}{b^3}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(5/2)/(a - b*x),x)
```

```
[Out] (2*x^(5/2))/(5*b) + (2*a*x^(3/2))/(3*b^2) + (2*a^2*x^(1/2))/b^3 + (a^(5/2)*
atan((b^(1/2)*x^(1/2)*1i)/a^(1/2))*2i)/b^(7/2)
```

**sympy [A]** time = 7.10, size = 116, normalized size = 1.71

$$\begin{cases} \frac{a^{\frac{5}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^4\sqrt{\frac{1}{b}}}-\frac{a^{\frac{5}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^4\sqrt{\frac{1}{b}}}+\frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{2x^{\frac{5}{2}}}{5b} & \text{for}\ b\neq 0 \\ -\frac{2x^{\frac{7}{2}}}{7a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a),x)
```

```
[Out] Piecewise((a**(5/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b)) - a*
*(5/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b)) + 2*a**2*sqrt(x)/b
**3 + 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), Ne(b, 0)), (-2*x**(7/2)/(7*
a), True))
```

## 3.470 $\int \frac{x^{3/2}}{-a+bx}\,dx$

**Optimal**. Leaf size=53

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

**Rubi [A]** time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 63, 208}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x),x]
```

```
[Out] (2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{-a+bx}\,dx &= \frac{2x^{3/2}}{3b} + \frac{a\int \frac{\sqrt{x}}{-a+bx}\,dx}{b} \\
&= \frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{a^2\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{b^2} \\
&= \frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 0.92

$$
\frac{2\sqrt{x}(3a+bx)}{3b^2} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(3*a + b*x))/(3*b^2) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/b^(5/2)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 53, normalized size = 1.00

$$
\frac{2\left(3a\sqrt{x}+bx^{3/2}\right)}{3b^2} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(-a + b*x),x]
```

```
[Out] (2*(3*a*Sqrt[x] + b*x^(3/2)))/(3*b^2) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

**fricas [A]** time = 0.67, size = 103, normalized size = 1.94

$$
\left[\frac{3\,a\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(bx+3\,a)\sqrt{x}}{3\,b^2}, \frac{2\left(3\,a\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(bx+3\,a)\sqrt{x}\right)}{3\,b^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*(b
*x + 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a)
+ (b*x + 3*a)*sqrt(x))/b^2]
```

**giac [A]** time = 0.94, size = 47, normalized size = 0.89

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^2}+\frac{2\left(b^2x^{\frac{3}{2}}+3\,ab\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) + 2/3*(b^2*x^(3/2) + 3*
a*b*sqrt(x))/b^3
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.81

$$-\frac{2a^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}+\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a),x)
```

```
[Out] 2/b^2*(1/3*b*x^(3/2)+a*x^(1/2))-2*a^2/b^2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)
*b*x^(1/2))
```

**maxima [A]** time = 2.92, size = 58, normalized size = 1.09

$$\frac{a^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(bx^{\frac{3}{2}}+3\,a\sqrt{x}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^2) +
2/3*(b*x^(3/2) + 3*a*sqrt(x))/b^2
```

**mupad [B]** time = 0.11, size = 37, normalized size = 0.70

$$\frac{2\,x^{3/2}}{3\,b}+\frac{2\,a\,\sqrt{x}}{b^2}-\frac{2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(3/2)/(a - b*x),x)
```

```
[Out] (2*x^(3/2))/(3*b) + (2*a*x^(1/2))/b^2 - (2*a^(3/2)*atanh((b^(1/2)*x^(1/2))/
a^(1/2)))/b^(5/2)
```

**sympy [A]** time = 1.87, size = 100, normalized size = 1.89

$$\begin{cases} \frac{a^{\frac{3}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}}-\frac{a^{\frac{3}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{3}{2}}}{3b} & \text{for } b\neq 0 \\ -\frac{2x^{\frac{5}{2}}}{5a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a),x)
```

```
[Out] Piecewise((a**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b)) - a*
*(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b)) + 2*a*sqrt(x)/b**2
 + 2*x**(3/2)/(3*b), Ne(b, 0)), (-2*x**(5/2)/(5*a), True))
```

## 3.471 $\int \frac{\sqrt{x}}{-a+bx}\,dx$

**Optimal**. Leaf size=40

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 63, 208}

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{-a+bx}\,dx = \frac{2\sqrt{x}}{b} + \frac{a\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(-a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

**fricas** [A] time = 0.76, size = 83, normalized size = 2.08

$$\left[\frac{\sqrt{\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,\sqrt{x}}{b}, \frac{2\left(\sqrt{-\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{-\frac{a}{b}}}{a}\right)+\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [(sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*sqrt(x))/b
, 2*(sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + sqrt(x))/b]
```

**giac [A]** time = 1.04, size = 33, normalized size = 0.82

$$\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) + 2*sqrt(x)/b
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.80

$$-\frac{2a\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a),x)
```

```
[Out] 2/b*x^(1/2)-2*a/b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 3.08, size = 47, normalized size = 1.18

$$\frac{a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b) + 2*sq
rt(x)/b
```

**mupad [B]** time = 0.11, size = 28, normalized size = 0.70

$$\frac{2\,\sqrt{x}}{b}-\frac{2\,\sqrt{a}\;\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/2)/(a - b*x),x)
```

```
[Out] (2*x^(1/2))/b - (2*a^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/b^(3/2)
```

**sympy [A]** time = 0.71, size = 87, normalized size = 2.18

$$\begin{cases} \frac{\sqrt{a}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}} - \frac{\sqrt{a}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}} + \frac{2\sqrt{x}}{b} & \text{for } b \neq 0 \\ -\frac{2x^{\frac{3}{2}}}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a),x)
```

```
[Out] Piecewise((sqrt(a)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) - sqr
t(a)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) + 2*sqrt(x)/b, Ne(b,
 0)), (-2*x**(3/2)/(3*a), True))
```

## 3.472 $\int \frac{1}{\sqrt{x}(-a+bx)}\,dx$

**Optimal**. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(-a+bx)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 29, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

**IntegrateAlgebraic** [A] time = 0.02, size = 29, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(-a + b*x)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

**fricas** [A] time = 0.98, size = 67, normalized size = 2.31

$$\left[\frac{\sqrt{ab}\log\left(\frac{bx+a-2\sqrt{ab}\sqrt{x}}{bx-a}\right)}{ab}, \frac{2\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt(x))/(b*x - a))/(a*b), 2*sqrt(-a*
b)*arctan(sqrt(-a*b)/(b*sqrt(x)))/(a*b)]
```

**giac** [A] time = 1.00, size = 20, normalized size = 0.69

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(-a*b))/sqrt(-a*b)
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.66

$$-\frac{2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)/x^(1/2),x)
[Out] -2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 3.03, size = 34, normalized size = 1.17

$$\frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="maxima")
[Out] log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/sqrt(a*b)
```

**mupad [B]** time = 0.13, size = 19, normalized size = 0.66

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/2)*(a - b*x)),x)
[Out] -(2*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2))
```

**sympy [A]** time = 1.25, size = 88, normalized size = 3.03

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for}\: a = 0 \\ -\frac{2\sqrt{x}}{a} & \text{for}\: b = 0 \\ \frac{\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{\sqrt{a}\,b\sqrt{\frac{1}{b}}} - \frac{\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{\sqrt{a}\,b\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (
-2*sqrt(x)/a, Eq(b, 0)), (log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*sqrt
(1/b)) - log(sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*sqrt(1/b)), True))
```

## 3.473 $\int \frac{1}{x^{3/2}(-a+bx)}\,dx$

**Optimal**. Leaf size=40

$$\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)),x]
```

```
[Out] 2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \frac{2}{a\sqrt{x}} + \frac{b\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{a}$$
$$= \frac{2}{a\sqrt{x}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

**Mathematica** [C]  time = 0.00, size = 24, normalized size = 0.60

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx}{a}\right)}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)),x]
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, (b*x)/a])/(a*Sqrt[x])
```

**IntegrateAlgebraic** [A]  time = 0.03, size = 40, normalized size = 1.00

$$\frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(-a + b*x)),x]
[Out] 2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

**fricas** [A]  time = 0.70, size = 91, normalized size = 2.28

$$\left[\frac{x\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\sqrt{x}}{ax}, \frac{2\left(x\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [(x*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) + 2*sqrt(x))
/(a*x), 2*(x*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + sqrt(x))/(a*x)]
```

**giac [A]** time = 1.02, size = 33, normalized size = 0.82

$$\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a}+\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) + 2/(a*sqrt(x))
```

**maple [A]** time = 0.01, size = 32, normalized size = 0.80

$$-\frac{2b\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a),x)
```

```
[Out] -2/a*b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))+2/a/x^(1/2)
```

**maxima [A]** time = 2.87, size = 47, normalized size = 1.18

$$\frac{b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] b*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a) + 2/(a
*sqrt(x))
```

**mupad [B]** time = 0.06, size = 28, normalized size = 0.70

$$\frac{2}{a\,\sqrt{x}}-\frac{2\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(3/2)*(a - b*x)),x)
```

```
[Out] 2/(a*x^(1/2)) - (2*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(3/2)
```

**sympy [A]** time = 2.76, size = 94, normalized size = 2.35

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a = 0 \\ \frac{2}{a\sqrt{x}} + \frac{\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{a^{\frac{3}{2}}\sqrt{\frac{1}{b}}} - \frac{\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{a^{\frac{3}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2/(a*sqrt(x)), Eq(b, 0)), (
-2/(3*b*x**(3/2)), Eq(a, 0)), (2/(a*sqrt(x)) + log(-sqrt(a)*sqrt(1/b) + sqr
t(x))/(a**(3/2)*sqrt(1/b)) - log(sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(3/2)*sqr
t(1/b)), True))
```

## 3.474 $\int \frac{1}{x^{5/2}(-a+bx)} dx$

**Optimal**. Leaf size=53

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2}{3ax^{3/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)),x]
```

```
[Out] 2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(-a+bx)}\,dx &= \frac{2}{3ax^{3/2}}+\frac{b\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{a}\\ &= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b^2\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{a^2}\\ &= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\ &= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

**Mathematica [C]** time = 0.00, size = 26, normalized size = 0.49

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx}{a}\right)}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)),x]
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, (b*x)/a])/(3*a*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.04, size = 48, normalized size = 0.91

$$\frac{2(a+3bx)}{3a^2x^{3/2}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(-a + b*x)),x]
[Out] (2*(a + 3*b*x))/(3*a^2*x^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/a^(5/2)
```

**fricas [A]** time = 1.06, size = 113, normalized size = 2.13

$$\left[\frac{3\,bx^2\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\,\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(3\,bx+a)\sqrt{x}}{3\,a^2x^2},\frac{2\left(3\,bx^2\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(3\,bx+a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*x^2*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) +
2*(3*b*x + a)*sqrt(x))/(a^2*x^2), 2/3*(3*b*x^2*sqrt(-b/a)*arctan(a*sqrt(-b/
a)/(b*sqrt(x))) + (3*b*x + a)*sqrt(x))/(a^2*x^2)]
```

**giac [A]** time = 0.98, size = 41, normalized size = 0.77

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a^2}+\frac{2\,(3\,bx+a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 2/3*(3*b*x + a)/(a^2*
x^(3/2))
```

**maple [A]** time = 0.01, size = 43, normalized size = 0.81

$$-\frac{2b^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}+\frac{2b}{a^2\sqrt{x}}+\frac{2}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a),x)
```

```
[Out] -2/a^2*b^2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))+2/3/a/x^(3/2)+2/a^2
*b/x^(1/2)
```

**maxima [A]** time = 2.96, size = 55, normalized size = 1.04

$$\frac{b^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,(3\,bx+a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] b^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2) +
2/3*(3*b*x + a)/(a^2*x^(3/2))
```

**mupad** [B] time = 0.12, size = 37, normalized size = 0.70

$$\frac{\frac{2}{3\,a}+\frac{2\,b\,x}{a^2}}{x^{3/2}}-\frac{2\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(5/2)*(a - b*x)),x)
```

```
[Out] (2/(3*a) + (2*b*x)/a^2)/x^(3/2) - (2*b^(3/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2
)))/a^(5/2)
```

**sympy** [A] time = 7.67, size = 112, normalized size = 2.11

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2}{3ax^{\frac{3}{2}}} & \text{for}\: b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\: a = 0 \\ \frac{2}{3ax^{\frac{3}{2}}} + \frac{2b}{a^2\sqrt{x}} + \frac{b\log\left(-\sqrt{a}\,\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{b\log\left(\sqrt{a}\,\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{5}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2/(3*a*x**(3/2)), Eq(b, 0))
, (-2/(5*b*x**(5/2)), Eq(a, 0)), (2/(3*a*x**(3/2)) + 2*b/(a**2*sqrt(x)) + b
*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)) - b*log(sqrt(a)*sqr
t(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)), True))
```

## 3.475 $\int \frac{1}{x^{7/2}(-a+bx)}\,dx$

**Optimal**. Leaf size=68

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{3/2}}+\frac{2}{5ax^{5/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(-a + b*x)),x]
```

```
[Out] 2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) + (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}(-a+bx)}\,dx &= \frac{2}{5ax^{5/2}}+\frac{b\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{a}\\
&= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{b^2\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{a^2}\\
&= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{b^3\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{a^3}\\
&= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^3}\\
&= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 26, normalized size = 0.38

$$\frac{2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{bx}{a}\right)}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(-a + b*x)),x]
[Out] (2*Hypergeometric2F1[-5/2, 1, -3/2, (b*x)/a])/(5*a*x^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 61, normalized size = 0.90

$$\frac{2\left(3a^2+5abx+15b^2x^2\right)}{15a^3x^{5/2}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(-a + b*x)),x]
[Out] (2*(3*a^2 + 5*a*b*x + 15*b^2*x^2))/(15*a^3*x^(5/2)) - (2*b^(5/2)*ArcTanh[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

**fricas** [A] time = 0.90, size = 143, normalized size = 2.10

$$\left[\frac{15\,b^2x^3\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\,\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\left(15\,b^2x^2+5\,abx+3\,a^2\right)\sqrt{x}}{15\,a^3x^3},\ \frac{2\left(15\,b^2x^3\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\left(15\,b^2x^2+5\,abx+3\,a^2\right)\sqrt{x}\right)}{15\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*x^3*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)
) + 2*(15*b^2*x^2 + 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3*s
qrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (15*b^2*x^2 + 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

**giac** [A] time = 0.97, size = 54, normalized size = 0.79

$$\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a^3}+\frac{2\left(15\,b^2x^2+5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*b^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/15*(15*b^2*x^2 + 5*
a*b*x + 3*a^2)/(a^3*x^(5/2))
```

**maple** [A] time = 0.01, size = 54, normalized size = 0.79

$$-\frac{2b^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^3}+\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{\frac{3}{2}}}+\frac{2}{5a\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x-a),x)
```

```
[Out] -2/a^3*b^3/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))+2/5/a/x^(5/2)+2/a^3
*b^2/x^(1/2)+2/3/a^2*b/x^(3/2)
```

**maxima** [A] time = 2.98, size = 68, normalized size = 1.00

$$\frac{b^3\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}+\frac{2\left(15\,b^2x^2+5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] b^3*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^3) +
2/15*(15*b^2*x^2 + 5*a*b*x + 3*a^2)/(a^3*x^(5/2))
```

**mupad [B]** time = 0.13, size = 48, normalized size = 0.71

$$\frac{\frac{2}{5\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{2\,b\,x}{3\,a^2}}{x^{5/2}}-\frac{2\,b^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(7/2)*(a - b*x)),x)
```

```
[Out] (2/(5*a) + (2*b^2*x^2)/a^3 + (2*b*x)/(3*a^2))/x^(5/2) - (2*b^(5/2)*atanh((b
^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

**sympy [A]** time = 24.44, size = 131, normalized size = 1.93

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{7 b x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ \frac{2}{5 a x^{\frac{5}{2}}} & \text{for}\: b = 0 \\ \frac{2}{5 a x^{\frac{5}{2}}} + \frac{2 b}{3 a^{2} x^{\frac{3}{2}}} + \frac{2 b^{2}}{a^{3} \sqrt{x}} + \frac{b^{2} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{a^{\frac{7}{2}} \sqrt{\frac{1}{b}}} - \frac{b^{2} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{a^{\frac{7}{2}} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (2/(5*a*x**(5/2)), Eq(b, 0)), (2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/2))
+ 2*b**2/(a**3*sqrt(x)) + b**2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(7/2)*
sqrt(1/b)) - b**2*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(7/2)*sqrt(1/b)), Tr
ue))
```

## 3.476 $\int \frac{x^{5/2}}{(-a+bx)^2} \, dx$

**Optimal**. Leaf size=70

$$-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{5a\sqrt{x}}{b^3}+\frac{x^{5/2}}{b(a-bx)}+\frac{5x^{3/2}}{3b^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{5a\sqrt{x}}{b^3}+\frac{x^{5/2}}{b(a-bx)}+\frac{5x^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x)^2,x]
```

```
[Out] (5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) + x^(5/2)/(b*(a - b*x)) - (5*a^(3/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(-a+bx)^2}\,dx &= \frac{x^{5/2}}{b(a-bx)} + \frac{5\int \frac{x^{3/2}}{-a+bx}\,dx}{2b} \\
&= \frac{5x^{3/2}}{3b^2} + \frac{x^{5/2}}{b(a-bx)} + \frac{(5a)\int \frac{\sqrt{x}}{-a+bx}\,dx}{2b^2} \\
&= \frac{5a\sqrt{x}}{b^3} + \frac{5x^{3/2}}{3b^2} + \frac{x^{5/2}}{b(a-bx)} + \frac{\left(5a^2\right)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2b^3} \\
&= \frac{5a\sqrt{x}}{b^3} + \frac{5x^{3/2}}{3b^2} + \frac{x^{5/2}}{b(a-bx)} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b^3} \\
&= \frac{5a\sqrt{x}}{b^3} + \frac{5x^{3/2}}{3b^2} + \frac{x^{5/2}}{b(a-bx)} - \frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 26, normalized size = 0.37

$$\frac{2x^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x)^2,x]
[Out] (2*x^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (b*x)/a])/(7*a^2)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 76, normalized size = 1.09

$$\frac{-15a^2\sqrt{x}+10abx^{3/2}+2b^2x^{5/2}}{3b^3(bx-a)} - \frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(-a + b*x)^2,x]
```

```
[Out] (-15*a^2*Sqrt[x] + 10*a*b*x^(3/2) + 2*b^2*x^(5/2))/(3*b^3*(-a + b*x)) - (5*
a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

**fricas [A]** time = 0.70, size = 167, normalized size = 2.39

$$\left[\frac{15\left(abx-a^{2}\right)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(2b^{2}x^{2}+10abx-15a^{2}\right)\sqrt{x}}{6\left(b^{4}x-ab^{3}\right)}, \frac{15\left(abx-a^{2}\right)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\left(2b^{2}x^{2}+10abx-15a^{2}\right)\sqrt{x}}{3\left(b^{4}x-ab^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x - a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x
 - a)) + 2*(2*b^2*x^2 + 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3), 1/3*(1
5*(a*b*x - a^2)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (2*b^2*x^2 + 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3)]
```

**giac [A]** time = 0.98, size = 69, normalized size = 0.99

$$\frac{5a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^{3}}-\frac{a^{2}\sqrt{x}}{(bx-a)b^{3}}+\frac{2\left(b^{4}x^{\frac{3}{2}}+6ab^{3}\sqrt{x}\right)}{3b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) - a^2*sqrt(x)/((b*x - a
)*b^3) + 2/3*(b^4*x^(3/2) + 6*a*b^3*sqrt(x))/b^6
```

**maple [A]** time = 0.01, size = 61, normalized size = 0.87

$$\frac{2\left(-\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}-\frac{\sqrt{x}}{2(bx-a)}\right)a^{2}}{b^{3}}+\frac{\frac{2bx^{\frac{3}{2}}}{3}+4a\sqrt{x}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a)^2,x)
```

```
[Out] 2/b^3*(1/3*b*x^(3/2)+2*a*x^(1/2))+2/b^3*a^2*(-1/2*x^(1/2)/(b*x-a)-5/2/(a*b)
^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2)))
```

**maxima** [A] time = 3.06, size = 81, normalized size = 1.16

$$-\frac{a^2\sqrt{x}}{b^4x-ab^3}+\frac{5\,a^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{2\left(bx^{\frac{3}{2}}+6\,a\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -a^2*sqrt(x)/(b^4*x - a*b^3) + 5/2*a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(
x) + sqrt(a*b)))/(sqrt(a*b)*b^3) + 2/3*(b*x^(3/2) + 6*a*sqrt(x))/b^3
```

**mupad** [B] time = 0.07, size = 61, normalized size = 0.87

$$\frac{2\,x^{3/2}}{3\,b^2}+\frac{4\,a\,\sqrt{x}}{b^3}+\frac{a^2\,\sqrt{x}}{a\,b^3-b^4\,x}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^2,x)
```

```
[Out] (2*x^(3/2))/(3*b^2) + (4*a*x^(1/2))/b^3 + (a^2*x^(1/2))/(a*b^3 - b^4*x) + (
a^(3/2)*atan((b^(1/2)*x^(1/2)*1i)/a^(1/2))*5i)/b^(7/2)
```

**sympy** [A] time = 24.75, size = 444, normalized size = 6.34

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 x^{\frac{7}{2}}}{7 a^{2}} & \text{for}\: b = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 b^{2}} & \text{for}\: a = 0 \\ - \frac{30 a^{\frac{5}{2}} b \sqrt{x} \sqrt{\frac{1}{b}}}{- 6 a^{\frac{3}{2}} b^{4} \sqrt{\frac{1}{b}} + 6 \sqrt{a} b^{5} x \sqrt{\frac{1}{b}}} + \frac{20 a^{\frac{3}{2}} b^{2} x^{\frac{3}{2}} \sqrt{\frac{1}{b}}}{- 6 a^{\frac{3}{2}} b^{4} \sqrt{\frac{1}{b}} + 6 \sqrt{a} b^{5} x \sqrt{\frac{1}{b}}} + \frac{4 \sqrt{a} b^{3} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}}{- 6 a^{\frac{3}{2}} b^{4} \sqrt{\frac{1}{b}} + 6 \sqrt{a} b^{5} x \sqrt{\frac{1}{b}}} - \frac{15 a^{3} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{- 6 a^{\frac{3}{2}} b^{4} \sqrt{\frac{1}{b}} + 6 \sqrt{a} b^{5} x \sqrt{\frac{1}{b}}} + \frac{15 a^{3} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{- 6 a^{\frac{3}{2}} b^{4} \sqrt{\frac{1}{b}} + 6 \sqrt{a} b^{5} x \sqrt{\frac{1}{b}}} + \frac{15 a^{2} b x \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{- 6 a^{\frac{3}{2}} b^{4} \sqrt{\frac{1}{b}} + 6 \sqrt{a} b^{5} x \sqrt{\frac{1}{b}}} - \frac{15 a^{2} b x \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{- 6 a^{\frac{3}{2}} b^{4} \sqrt{\frac{1}{b}} + 6 \sqrt{a} b^{5} x \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b,
0)), (2*x**(3/2)/(3*b**2), Eq(a, 0)), (-30*a**(5/2)*b*sqrt(x)*sqrt(1/b)/(-6
*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)) + 20*a**(3/2)*b**2*x
**(3/2)*sqrt(1/b)/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b))
 + 4*sqrt(a)*b**3*x**(5/2)*sqrt(1/b)/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a
)*b**5*x*sqrt(1/b)) - 15*a**3*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-6*a**(3/2
)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)) + 15*a**3*log(sqrt(a)*sqrt(1
/b) + sqrt(x))/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)) +
15*a**2*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-6*a**(3/2)*b**4*sqrt(1/b) +
 6*sqrt(a)*b**5*x*sqrt(1/b)) - 15*a**2*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))
/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)), True))
```

## 3.477 $\int \frac{x^{3/2}}{(-a+bx)^2}\,dx$

**Optimal**. Leaf size=57

$$-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x^{3/2}}{b(a-bx)}+\frac{3\sqrt{x}}{b^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x^{3/2}}{b(a-bx)}+\frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x)^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 + x^(3/2)/(b*(a - b*x)) - (3*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{(-a+bx)^2}\,dx &= \frac{x^{3/2}}{b(a-bx)}+\frac{3\int \frac{\sqrt{x}}{-a+bx}\,dx}{2b}\\
&= \frac{3\sqrt{x}}{b^2}+\frac{x^{3/2}}{b(a-bx)}+\frac{(3a)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2b^2}\\
&= \frac{3\sqrt{x}}{b^2}+\frac{x^{3/2}}{b(a-bx)}+\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= \frac{3\sqrt{x}}{b^2}+\frac{x^{3/2}}{b(a-bx)}-\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.00, size = 26, normalized size = 0.46

$$
\frac{2x^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx}{a}\right)}{5a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x)^2,x]
```

```
[Out] (2*x^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (b*x)/a])/(5*a^2)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 56, normalized size = 0.98

$$
\frac{\sqrt{x}\,(2bx-3a)}{b^2(bx-a)}-\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(3/2)/(-a + b*x)^2,x]

[Out] (Sqrt[x]*(-3*a + 2*b*x))/(b^2*(-a + b*x)) - (3*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(5/2)

**fricas [A]** time = 0.58, size = 138, normalized size = 2.42

$$\left[\frac{3\,(bx-a)\sqrt{\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(2\,bx-3\,a)\sqrt{x}}{2\,\left(b^3x-ab^2\right)},\frac{3\,(bx-a)\sqrt{-\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{-\frac{a}{b}}}{a}\right)+(2\,bx-3\,a)\sqrt{x}}{b^3x-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="fricas")

[Out] [1/2*(3*(b*x - a)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*(2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2), (3*(b*x - a)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2)]

**giac [A]** time = 0.98, size = 51, normalized size = 0.89

$$\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^2}-\frac{a\sqrt{x}}{(bx-a)b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="giac")

[Out] 3*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - a*sqrt(x)/((b*x - a)*b^2) + 2*sqrt(x)/b^2

**maple [A]** time = 0.01, size = 49, normalized size = 0.86

$$\frac{2\left(-\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}-\frac{\sqrt{x}}{2(bx-a)}\right)a}{b^2}+\frac{2\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(b*x-a)^2,x)

[Out] 2/b^2*x^(1/2)+2*a/b^2*(-1/2/(b*x-a)*x^(1/2)-3/2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2)))

**maxima** [A] time = 2.92, size = 68, normalized size = 1.19

$$-\frac{a\sqrt{x}}{b^3x-ab^2}+\frac{3\,a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -a*sqrt(x)/(b^3*x - a*b^2) + 3/2*a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) +
 sqrt(a*b)))/(sqrt(a*b)*b^2) + 2*sqrt(x)/b^2
```

**mupad** [B] time = 0.11, size = 47, normalized size = 0.82

$$\frac{2\,\sqrt{x}}{b^2}+\frac{a\,\sqrt{x}}{a\,b^2-b^3\,x}-\frac{3\,\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^2,x)
```

```
[Out] (2*x^(1/2))/b^2 + (a*x^(1/2))/(a*b^2 - b^3*x) - (3*a^(1/2)*atanh((b^(1/2)*x
^(1/2))/a^(1/2)))/b^(5/2)
```

**sympy** [A] time = 9.13, size = 381, normalized size = 6.68

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{5}{2}}}{5a^2} & \text{for}\: b = 0 \\ \frac{2\sqrt{x}}{b^2} & \text{for}\: a = 0 \\ -\frac{6a^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{4\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}-\frac{3a^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{3a^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{3abx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}-\frac{3abx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0
)), (2*sqrt(x)/b**2, Eq(a, 0)), (-6*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(-2*a**(3/
2)*b**3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) + 4*sqrt(a)*b**2*x**(3/2)*s
qrt(1/b)/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) - 3*a**2
*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(a)*
b**4*x*sqrt(1/b)) + 3*a**2*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b*
*3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) + 3*a*b*x*log(-sqrt(a)*sqrt(1/b)
 + sqrt(x))/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) - 3*a
*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(
a)*b**4*x*sqrt(1/b)), True))
```

## 3.478 $\int \frac{\sqrt{x}}{(-a+bx)^2} \, dx$

**Optimal**. Leaf size=47

$$\frac{\sqrt{x}}{b(a-bx)} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 63, 208}

$$\frac{\sqrt{x}}{b(a-bx)} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x)^2,x]
```

```
[Out] Sqrt[x]/(b*(a - b*x)) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2)
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx = \frac{\sqrt{x}}{b(a-bx)} + \frac{\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2b}$$
$$= \frac{\sqrt{x}}{b(a-bx)} + \frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{\sqrt{x}}{b(a-bx)} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

**Mathematica** [A] time = 0.02, size = 61, normalized size = 1.30

$$\frac{\sqrt{a}\sqrt{b}\sqrt{x} + (bx-a)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x)^2,x]
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[x] + (-a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(
Sqrt[a]*b^(3/2)*(a - b*x))
```

**IntegrateAlgebraic** [A] time = 0.06, size = 49, normalized size = 1.04

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}} - \frac{\sqrt{x}}{b(bx-a)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(-a + b*x)^2,x]
[Out] -(Sqrt[x]/(b*(-a + b*x))) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(
3/2))
```

**fricas** [A] time = 0.84, size = 123, normalized size = 2.62

$$\left[-\frac{2\,ab\sqrt{x} - \sqrt{ab}\,(bx-a)\log\left(\frac{bx+a-2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)}{2\left(ab^3x - a^2b^2\right)}, -\frac{ab\sqrt{x} - \sqrt{-ab}\,(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab^3x - a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a - 2*sqrt(a*b)*sqrt(
x))/(b*x - a)))/(a*b^3*x - a^2*b^2), -(a*b*sqrt(x) - sqrt(-a*b)*(b*x - a)*a
rctan(sqrt(-a*b)/(b*sqrt(x))))/(a*b^3*x - a^2*b^2)]
```

**giac [A]** time = 1.06, size = 40, normalized size = 0.85

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b} - \frac{\sqrt{x}}{(bx-a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) - sqrt(x)/((b*x - a)*b)
```

**maple [A]** time = 0.01, size = 40, normalized size = 0.85

$$-\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} - \frac{\sqrt{x}}{(bx-a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a)^2,x)
```

```
[Out] -1/b*x^(1/2)/(b*x-a)-1/b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 2.99, size = 56, normalized size = 1.19

$$-\frac{\sqrt{x}}{b^2x-ab} + \frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(b^2*x - a*b) + 1/2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(
a*b)))/(sqrt(a*b)*b)
```

**mupad [B]** time = 0.11, size = 35, normalized size = 0.74

$$\frac{\sqrt{x}}{b\,(a-b\,x)} - \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^2,x)
```

```
[Out] x^(1/2)/(b*(a - b*x)) - atanh((b^(1/2)*x^(1/2))/a^(1/2))/(a^(1/2)*b^(3/2))
```

**sympy [A]** time = 4.43, size = 311, normalized size = 6.62

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for}\: b = 0 \\ -\frac{2}{b^2\sqrt{x}} & \text{for}\: a = 0 \\ -\frac{2\sqrt{a}\,b\sqrt{x}\sqrt{\frac{1}{b}}}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} - \frac{a\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} + \frac{a\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} + \frac{bx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} - \frac{bx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0
)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (-2*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(-2*a**(
3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)) - a*log(-sqrt(a)*sqrt(1/b
) + sqrt(x))/(-2*a**(3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)) + a*
log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b*
*3*x*sqrt(1/b)) + b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**2*s
qrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)) - b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x
))/(-2*a**(3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)), True))
```

## 3.479 $\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx$

**Optimal**. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}$$

**Rubi [A]** time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx = \frac{\sqrt{x}}{a(a-bx)} - \frac{\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2a}$$
$$= \frac{\sqrt{x}}{a(a-bx)} - \frac{\operatorname{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{\sqrt{x}}{a(a-bx)} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} + \frac{\sqrt{x}}{a^2-abx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a^2 - a*b*x) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 46, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} + \frac{\sqrt{x}}{a(a-bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

**fricas [A]** time = 0.58, size = 122, normalized size = 2.65

$$\left[-\frac{2\,ab\sqrt{x} - \sqrt{ab}\,(bx-a)\log\left(\frac{bx+a+2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)}{2\left(a^2b^2x-a^3b\right)}, -\frac{ab\sqrt{x} + \sqrt{-ab}\,(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{a^2b^2x-a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a + 2*sqrt(a*b)*sqrt(
x))/(b*x - a)))/(a^2*b^2*x - a^3*b), -(a*b*sqrt(x) + sqrt(-a*b)*(b*x - a)*a
rctan(sqrt(-a*b)/(b*sqrt(x))))/(a^2*b^2*x - a^3*b)]
```

**giac [A]** time = 0.88, size = 41, normalized size = 0.89

$$-\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a}-\frac{\sqrt{x}}{(bx-a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) - sqrt(x)/((b*x - a)*a)
```

**maple [A]** time = 0.01, size = 39, normalized size = 0.85

$$\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}-\frac{\sqrt{x}}{(bx-a)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^2/x^(1/2),x)
```

```
[Out] -x^(1/2)/a/(b*x-a)+1/a/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima [A]** time = 2.97, size = 56, normalized size = 1.22

$$-\frac{\sqrt{x}}{abx-a^2}-\frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(a*b*x - a^2) - 1/2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(
a*b)))/(sqrt(a*b)*a)
```

**mupad [B]** time = 0.05, size = 34, normalized size = 0.74

$$\frac{\sqrt{x}}{a\,(a-b\,x)}+\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^2),x)
```

```
[Out] x^(1/2)/(a*(a - b*x)) + atanh((b^(1/2)*x^(1/2))/a^(1/2))/(a^(3/2)*b^(1/2))
```

**sympy [A]** time = 7.41, size = 303, normalized size = 6.59

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{a^2} & \text{for } b = 0 \\ -\frac{2}{3b^2x^{\frac{3}{2}}} & \text{for } a = 0 \\ -\frac{2\sqrt{a}\,b\sqrt{x}\sqrt{\frac{1}{b}}}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} + \frac{a\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} - \frac{a\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} - \frac{bx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} + \frac{bx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
(-2/(3*b**2*x**(3/2)), Eq(a, 0)), (-2*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(-2*a**(5
/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sqrt(1/b)) + a*log(-sqrt(a)*sqrt(1/b) +
 sqrt(x))/(-2*a**(5/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sqrt(1/b)) - a*log(s
qrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(5/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sq
rt(1/b)) - b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(5/2)*b*sqrt(1/b) +
 2*a**(3/2)*b**2*x*sqrt(1/b)) + b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a*
*(5/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sqrt(1/b)), True))
```

# 3.480 $\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx$

**Optimal**. Leaf size=57

$$\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}$$

**Rubi [A]** time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)^2),x]
```

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a - b*x)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx = \frac{1}{a\sqrt{x}\,(a-bx)} - \frac{3\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{2a}$$
$$= -\frac{3}{a^2\sqrt{x}} + \frac{1}{a\sqrt{x}\,(a-bx)} - \frac{(3b)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2a^2}$$
$$= -\frac{3}{a^2\sqrt{x}} + \frac{1}{a\sqrt{x}\,(a-bx)} - \frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a^2}$$
$$= -\frac{3}{a^2\sqrt{x}} + \frac{1}{a\sqrt{x}\,(a-bx)} + \frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

**Mathematica [C]** time = 0.01, size = 24, normalized size = 0.42

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx}{a}\right)}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)^2),x]
[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, (b*x)/a])/(a^2*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.06, size = 55, normalized size = 0.96

$$\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{3bx-2a}{a^2\sqrt{x}\,(a-bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(-a + b*x)^2),x]
[Out] (-2*a + 3*b*x)/(a^2*Sqrt[x]*(a - b*x)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/a^(5/2)
```

**fricas [A]** time = 0.85, size = 151, normalized size = 2.65

$$\left[\frac{3\left(bx^2-ax\right)\sqrt{\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\,\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2\,(3\,bx-2\,a)\sqrt{x}}{2\left(a^2bx^2-a^3x\right)}, -\frac{3\left(bx^2-ax\right)\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(3\,bx-2\,a)\sqrt{x}}{a^2bx^2-a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 - a*x)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x
- a)) - 2*(3*b*x - 2*a)*sqrt(x))/(a^2*b*x^2 - a^3*x), -(3*(b*x^2 - a*x)*sqr
t(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (3*b*x - 2*a)*sqrt(x))/(a^2*b*x^
2 - a^3*x)]
```

**giac [A]** time = 1.01, size = 52, normalized size = 0.91

$$-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a^2}-\frac{3\,bx-2\,a}{\left(bx^{\frac{3}{2}}-a\sqrt{x}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) - (3*b*x - 2*a)/((b*x^(3
/2) - a*sqrt(x))*a^2)
```

**maple [A]** time = 0.01, size = 49, normalized size = 0.86

$$-\frac{2\left(-\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}+\frac{\sqrt{x}}{2bx-2a}\right)b}{a^2}-\frac{2}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a)^2,x)
```

```
[Out] -2/a^2*b*(1/2/(b*x-a)*x^(1/2)-3/2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/
2)))-2/a^2/x^(1/2)
```

**maxima [A]** time = 3.00, size = 69, normalized size = 1.21

$$-\frac{3\,bx-2\,a}{a^2bx^{\frac{3}{2}}-a^3\sqrt{x}}-\frac{3\,b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

[Out] -(3*b*x - 2*a)/(a^2*b*x^(3/2) - a^3*sqrt(x)) - 3/2*b*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2)

**mupad [B]** time = 0.07, size = 49, normalized size = 0.86

$$\frac{3\sqrt{b}\,\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2}{a}-\frac{3\,b\,x}{a^2}}{a\,\sqrt{x}-b\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(3/2)*(a - b*x)^2),x)

[Out] (3*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(5/2) - (2/a - (3*b*x)/a^2)/(a*x^(1/2) - b*x^(3/2))

**sympy [A]** time = 17.54, size = 403, normalized size = 7.07

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{a^{2} \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{2}{5 b^{2} x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ - \frac{4 a^{\frac{3}{2}} \sqrt{\frac{1}{b}}}{2 a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 2 a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} + \frac{6 \sqrt{a} b x \sqrt{\frac{1}{b}}}{2 a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 2 a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} - \frac{3 a \sqrt{x} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{2 a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 2 a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} + \frac{3 a \sqrt{x} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{2 a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 2 a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} + \frac{3 b x^{\frac{3}{2}} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{2 a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 2 a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} - \frac{3 b x^{\frac{3}{2}} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{2 a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 2 a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(3/2)/(b*x-a)**2,x)

[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-2/(5*b**2*x**(5/2)), Eq(a, 0)), (-4*a**(3/2)*sqrt(1/b)/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 6*sqrt(a)*b*x*sqrt(1/b)/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)) - 3*a*sqrt(x)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*a*sqrt(x)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*b*x**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)) - 3*b*x**(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)), True))

## 3.481 $\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx$

**Optimal**. Leaf size=70

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a-bx)}$$

**Rubi** [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] -5/(3*a^2*x^(3/2)) - (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a - b*x)) + (5*b^(
3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx &= \frac{1}{ax^{3/2}(a-bx)} - \frac{5\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{2a}\\
&= -\frac{5}{3a^2x^{3/2}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{(5b)\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{2a^2}\\
&= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{\left(5b^2\right)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2a^3}\\
&= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3}\\
&= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} + \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

**Mathematica** [C] time = 0.01, size = 26, normalized size = 0.37

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{bx}{a}\right)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 2, -1/2, (b*x)/a])/(3*a^2*x^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.08, size = 69, normalized size = 0.99

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{-2a^2-10abx+15b^2x^2}{3a^3x^{3/2}(a-bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*a^2 - 10*a*b*x + 15*b^2*x^2)/(3*a^3*x^(3/2)*(a - b*x)) + (5*b^(3/2)*Arc
Tanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

**fricas** [A] time = 1.00, size = 187, normalized size = 2.67

$$\left[\frac{15\left(b^{2}x^{3}-abx^{2}\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2\left(15\,b^{2}x^{2}-10\,abx-2\,a^{2}\right)\sqrt{x}}{6\left(a^{3}bx^{3}-a^{4}x^{2}\right)},-\frac{15\left(b^{2}x^{3}-abx^{2}\right)\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\left(15\,b^{2}x^{2}-10\,abx-2\,a^{2}\right)\sqrt{x}}{3\left(a^{3}bx^{3}-a^{4}x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 - a*b*x^2)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) + a
)/(b*x - a)) - 2*(15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^4*
x^2), -1/3*(15*(b^2*x^3 - a*b*x^2)*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x
))) + (15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^4*x^2)]
```

**giac** [A] time = 0.93, size = 61, normalized size = 0.87

$$-\frac{5\,b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a^{3}}-\frac{b^{2}\sqrt{x}}{(bx-a)a^{3}}-\frac{2\,(6\,bx+a)}{3\,a^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] -5*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) - b^2*sqrt(x)/((b*x -
a)*a^3) - 2/3*(6*b*x + a)/(a^3*x^(3/2))
```

**maple** [A] time = 0.02, size = 60, normalized size = 0.86

$$-\frac{2\left(-\frac{5\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}+\frac{\sqrt{x}}{2bx-2a}\right)b^{2}}{a^{3}}-\frac{4b}{a^{3}\sqrt{x}}-\frac{2}{3a^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a)^2,x)
```

```
[Out] -2/a^3*b^2*(1/2/(b*x-a)*x^(1/2)-5/2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(
1/2)))-2/3/a^2/x^(3/2)-4/a^3*b/x^(1/2)
```

**maxima** [A] time = 3.00, size = 82, normalized size = 1.17

$$-\frac{15\,b^{2}x^{2}-10\,abx-2\,a^{2}}{3\left(a^{3}bx^{\frac{5}{2}}-a^{4}x^{\frac{3}{2}}\right)}-\frac{5\,b^{2}\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(15*b^2*x^2 - 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) - a^4*x^(3/2)) - 5/2*b^
2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^3)
```

**mupad [B]** time = 0.14, size = 60, normalized size = 0.86

$$\frac{5\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{\frac{2}{3\,a}-\frac{5\,b^2\,x^2}{a^3}+\frac{10\,b\,x}{3\,a^2}}{a\,x^{3/2}-b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^2),x)
```

```
[Out] (5*b^(3/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2) - (2/(3*a) - (5*b^2*x^
2)/a^3 + (10*b*x)/(3*a^2))/(a*x^(3/2) - b*x^(5/2))
```

**sympy [A]** time = 50.25, size = 471, normalized size = 6.73

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{7 b^{2} x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{3 a^{2} x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ - \frac{4 a^{\frac{5}{2}} \sqrt{\frac{1}{b}}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{20 a^{\frac{3}{2}} b x \sqrt{\frac{1}{b}}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{30 \sqrt{a} b^{2} x^{2} \sqrt{\frac{1}{b}}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{15 a b x^{\frac{3}{2}} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{15 a b x^{\frac{3}{2}} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{15 b^{2} x^{\frac{5}{2}} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{15 b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**2*x**(7/2)), Eq(a,
 0)), (-2/(3*a**2*x**(3/2)), Eq(b, 0)), (-4*a**(5/2)*sqrt(1/b)/(6*a**(9/2)*
x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)) - 20*a**(3/2)*b*x*sqr
t(1/b)/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)) +
30*sqrt(a)*b**2*x**2*sqrt(1/b)/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*
b*x**(5/2)*sqrt(1/b)) - 15*a*b*x**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(
6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*a*b*x
**(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6
*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*b**2*x**(5/2)*log(-sqrt(a)*sqrt(1/b) +
 sqrt(x))/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b))
 - 15*b**2*x**(5/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(6*a**(9/2)*x**(3/2)*s
qrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)), True))
```

## 3.482 $\int \frac{x^{7/2}}{(-a+bx)^3}\,dx$

**Optimal**. Leaf size=97

$$-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}+\frac{35a\sqrt{x}}{4b^4}+\frac{7x^{5/2}}{4b^2(a-bx)}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{35x^{3/2}}{12b^3}$$

**Rubi** [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{35a\sqrt{x}}{4b^4}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{35x^{3/2}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(-a + b*x)^3,x]
```

```
[Out] (35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a - b*x)^2)
+ (7*x^(5/2))/(4*b^2*(a - b*x)) - (35*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{(-a+bx)^3}\,dx &= -\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7\int \frac{x^{5/2}}{(-a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{35\int \frac{x^{3/2}}{-a+bx}\,dx}{8b^2}\\
&= \frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{(35a)\int \frac{\sqrt{x}}{-a+bx}\,dx}{8b^3}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{\left(35a^2\right)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8b^4}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{\left(35a^2\right)\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^4}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 26, normalized size = 0.27

$$
-\frac{2x^{9/2}\,{}_2F_1\left(3,\frac{9}{2};\frac{11}{2};\frac{bx}{a}\right)}{9a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(-a + b*x)^3,x]

[Out] (-2*x^(9/2)*Hypergeometric2F1[3, 9/2, 11/2, (b*x)/a])/(9*a^3)
```

**IntegrateAlgebraic [A]** time = 0.12, size = 91, normalized size = 0.94

$$\frac{105a^3\sqrt{x}-175a^2bx^{3/2}+56ab^2x^{5/2}+8b^3x^{7/2}}{12b^4(bx-a)^2}-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(-a + b*x)^3,x]
```

```
[Out] (105*a^3*Sqrt[x] - 175*a^2*b*x^(3/2) + 56*a*b^2*x^(5/2) + 8*b^3*x^(7/2))/(1
2*b^4*(-a + b*x)^2) - (35*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^
(9/2))
```

**fricas [A]** time = 0.79, size = 227, normalized size = 2.34

$$\left[\frac{105\left(ab^2x^2-2a^2bx+a^3\right)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(8b^3x^3+56ab^2x^2-175a^2bx+105a^3\right)\sqrt{x}}{24\left(b^6x^2-2ab^5x+a^2b^4\right)}, \frac{105\left(ab^2x^2-2a^2bx+a^3\right)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\left(8b^3x^3+56ab^2x^2-175a^2bx+105a^3\right)\sqrt{x}}{12\left(b^6x^2-2ab^5x+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*s
qrt(a/b) + a)/(b*x - a)) + 2*(8*b^3*x^3 + 56*a*b^2*x^2 - 175*a^2*b*x + 105*
a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 - 2*a^2
*b*x + a^3)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (8*b^3*x^3 + 56*a*b
^2*x^2 - 175*a^2*b*x + 105*a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4)]
```

**giac [A]** time = 1.03, size = 81, normalized size = 0.84

$$\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}\,b^4}-\frac{13\,a^2bx^{\frac{3}{2}}-11\,a^3\sqrt{x}}{4\,(bx-a)^2b^4}+\frac{2\left(b^6x^{\frac{3}{2}}+9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^4) - 1/4*(13*a^2*b*x^(3
/2) - 11*a^3*sqrt(x))/((b*x - a)^2*b^4) + 2/3*(b^6*x^(3/2) + 9*a*b^5*sqrt(x
))/b^9
```

**maple [A]** time = 0.02, size = 70, normalized size = 0.72

$$\frac{2\left(-\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}+\frac{-\frac{13bx^{\frac{3}{2}}}{8}+\frac{11a\sqrt{x}}{8}}{(bx-a)^2}\right)a^2}{b^4}+\frac{\frac{2bx^{\frac{3}{2}}}{3}+6a\sqrt{x}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x-a)^3,x)
```

```
[Out] 2/b^4*(1/3*b*x^(3/2)+3*a*x^(1/2))+2/b^4*a^2*((-13/8*b*x^(3/2)+11/8*a*x^(1/2
))/(b*x-a)^2-35/8/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2)))
```

**maxima [A]** time = 2.99, size = 103, normalized size = 1.06

$$-\frac{13\,a^{2}bx^{\frac{3}{2}}-11\,a^{3}\sqrt{x}}{4\left(b^{6}x^{2}-2\,ab^{5}x+a^{2}b^{4}\right)}+\frac{35\,a^{2}\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{4}}+\frac{2\left(bx^{\frac{3}{2}}+9\,a\sqrt{x}\right)}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(13*a^2*b*x^(3/2) - 11*a^3*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4) +
35/8*a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^
4) + 2/3*(b*x^(3/2) + 9*a*sqrt(x))/b^4
```

**mupad [B]** time = 0.14, size = 83, normalized size = 0.86

$$\frac{\frac{11\,a^3\,\sqrt{x}}{4}-\frac{13\,a^2\,b\,x^{3/2}}{4}}{a^2\,b^4-2\,a\,b^5\,x+b^6\,x^2}+\frac{2\,x^{3/2}}{3\,b^3}+\frac{6\,a\,\sqrt{x}}{b^4}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,35{}\mathrm{i}}{4\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(7/2)/(a - b*x)^3,x)
```

```
[Out] ((11*a^3*x^(1/2))/4 - (13*a^2*b*x^(3/2))/4)/(a^2*b^4 + b^6*x^2 - 2*a*b^5*x)
 + (2*x^(3/2))/(3*b^3) + (6*a*x^(1/2))/b^4 + (a^(3/2)*atan((b^(1/2)*x^(1/2)
*1i)/a^(1/2))*35i)/(4*b^(9/2))
```

**sympy [A]** time = 136.15, size = 840, normalized size = 8.66

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2 x^{\frac{9}{2}}}{9 a^{3}} & \text{for}\: b = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 b^{3}} & \text{for}\: a = 0 \\ \frac{210 a^{\frac{7}{2}} b \sqrt{x} \sqrt{\frac{1}{b}}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{350 a^{\frac{5}{2}} b^{2} x^{\frac{3}{2}} \sqrt{\frac{1}{b}}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{112 a^{\frac{3}{2}} b^{3} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{16 \sqrt{a} b^{4} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{105 a^{4} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{105 a^{4} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{210 a^{3} b x \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{210 a^{3} b x \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{105 a^{2} b^{2} x^{2} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{105 a^{2} b^{2} x^{2} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(9/2)/(9*a**3), Eq(b,
 0)), (2*x**(3/2)/(3*b**3), Eq(a, 0)), (210*a**(7/2)*b*sqrt(x)*sqrt(1/b)/(2
4*a**(5/2)*b**5*sqrt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*
```

```
x**2*sqrt(1/b)) - 350*a**(5/2)*b**2*x**(3/2)*sqrt(1/b)/(24*a**(5/2)*b**5*sq
rt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)) +
112*a**(3/2)*b**3*x**(5/2)*sqrt(1/b)/(24*a**(5/2)*b**5*sqrt(1/b) - 48*a**(3
/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)) + 16*sqrt(a)*b**4*x*
*(7/2)*sqrt(1/b)/(24*a**(5/2)*b**5*sqrt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b)
 + 24*sqrt(a)*b**7*x**2*sqrt(1/b)) + 105*a**4*log(-sqrt(a)*sqrt(1/b) + sqrt
(x))/(24*a**(5/2)*b**5*sqrt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a
)*b**7*x**2*sqrt(1/b)) - 105*a**4*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(
5/2)*b**5*sqrt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*s
qrt(1/b)) - 210*a**3*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(5/2)*b**
5*sqrt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)
) + 210*a**3*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(5/2)*b**5*sqrt(1/
b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)) + 105*a
**2*b**2*x**2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(5/2)*b**5*sqrt(1/b)
 - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)) - 105*a**
2*b**2*x**2*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(5/2)*b**5*sqrt(1/b) -
48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)), True))
```

## 3.483 $\int \frac{x^{5/2}}{(-a+bx)^3}\,dx$

**Optimal**. Leaf size=84

$$-\frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}+\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

**Rubi [A]** time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a - b*x)^2) + (5*x^(3/2))/(4*b^2*(a -
b*x)) - (15*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(-a+bx)^3}\,dx &= -\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5\int \frac{x^{3/2}}{(-a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{15\int \frac{\sqrt{x}}{-a+bx}\,dx}{8b^2}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{(15a)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{(15a)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.00, size = 26, normalized size = 0.31

$$-\frac{2x^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (-2*x^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, (b*x)/a])/(7*a^3)
```

**IntegrateAlgebraic [A]** time = 0.12, size = 78, normalized size = 0.93

$$\frac{15a^2\sqrt{x}-25abx^{3/2}+8b^2x^{5/2}}{4b^3(bx-a)^2}-\frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (15*a^2*Sqrt[x] - 25*a*b*x^(3/2) + 8*b^2*x^(5/2))/(4*b^3*(-a + b*x)^2) - (1
5*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

**fricas [A]** time = 0.65, size = 199, normalized size = 2.37

$$\left[\frac{15\left(b^{2}x^{2}-2\,abx+a^{2}\right)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(8\,b^{2}x^{2}-25\,abx+15\,a^{2}\right)\sqrt{x}}{8\left(b^{5}x^{2}-2\,ab^{4}x+a^{2}b^{3}\right)},\frac{15\left(b^{2}x^{2}-2\,abx+a^{2}\right)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\left(8\,b^{2}x^{2}-25\,abx+15\,a^{2}\right)\sqrt{x}}{4\left(b^{5}x^{2}-2\,ab^{4}x+a^{2}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/
b) + a)/(b*x - a)) + 2*(8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2 -
2*a*b^4*x + a^2*b^3), 1/4*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a/b)*arctan(b
*sqrt(x)*sqrt(-a/b)/a) + (8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
- 2*a*b^4*x + a^2*b^3)]
```

**giac [A]** time = 0.95, size = 63, normalized size = 0.75

$$\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}\,b^{3}}+\frac{2\sqrt{x}}{b^{3}}-\frac{9\,abx^{\frac{3}{2}}-7\,a^{2}\sqrt{x}}{4\,(bx-a)^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 15/4*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2*sqrt(x)/b^3 - 1/4*
(9*a*b*x^(3/2) - 7*a^2*sqrt(x))/((b*x - a)^2*b^3)
```

**maple [A]** time = 0.01, size = 58, normalized size = 0.69

$$\frac{2\left(-\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}+\frac{-\frac{9b\,x^{\frac{3}{2}}}{8}+\frac{7a\sqrt{x}}{8}}{(bx-a)^{2}}\right)a}{b^{3}}+\frac{2\sqrt{x}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a)^3,x)
```

```
[Out] 2/b^3*x^(1/2)+2/b^3*a*((-9/8*b*x^(3/2)+7/8*a*x^(1/2))/(b*x-a)^2-15/8/(a*b)^
(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2)))
```

**maxima [A]** time = 3.00, size = 90, normalized size = 1.07

$$-\frac{9\,abx^{\frac{3}{2}} - 7\,a^2\sqrt{x}}{4\left(b^5x^2 - 2\,ab^4x + a^2b^3\right)} + \frac{15\,a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3} + \frac{2\,\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(9*a*b*x^(3/2) - 7*a^2*sqrt(x))/(b^5*x^2 - 2*a*b^4*x + a^2*b^3) + 15/8
*a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^3) + 2
*sqrt(x)/b^3
```

**mupad [B]** time = 0.06, size = 69, normalized size = 0.82

$$\frac{\frac{7\,a^2\,\sqrt{x}}{4} - \frac{9\,a\,b\,x^{3/2}}{4}}{a^2\,b^3 - 2\,a\,b^4\,x + b^5\,x^2} + \frac{2\,\sqrt{x}}{b^3} - \frac{15\,\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(5/2)/(a - b*x)^3,x)
```

```
[Out] ((7*a^2*x^(1/2))/4 - (9*a*b*x^(3/2))/4)/(a^2*b^3 + b^5*x^2 - 2*a*b^4*x) + (
2*x^(1/2))/b^3 - (15*a^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*b^(7/2))
```

**sympy [A]** time = 53.45, size = 756, normalized size = 9.00

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{b^3} & \text{for}\: a = 0 \\ -\frac{2x^{\frac{7}{2}}}{7a^3} & \text{for}\: b = 0 \\ \frac{30a^{\frac{5}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}} - 16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}} + 8\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} - \frac{50a^{\frac{3}{2}}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}} - 16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}} + 8\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} + \frac{16\sqrt{a}b^3x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}} - 16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}} + 8\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} + \frac{15a^3\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}} - 16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}} + 8\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} - \frac{15a^3\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}} - 16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}} + 8\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} - \frac{30a^2bx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}} - 16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}} + 8\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} + \frac{30a^2bx\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}} - 16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}} + 8\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} + \frac{15ab^2x^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}} - 16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}} + 8\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} - \frac{15ab^2x^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}} - 16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}} + 8\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b**3, Eq(a, 0)), (
-2*x**(7/2)/(7*a**3), Eq(b, 0)), (30*a**(5/2)*b*sqrt(x)*sqrt(1/b)/(8*a**(5/
2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt
(1/b)) - 50*a**(3/2)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(5/2)*b**4*sqrt(1/b) - 1
6*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 16*sqrt(a)*b
**3*x**(5/2)*sqrt(1/b)/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt
(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 15*a**3*log(-sqrt(a)*sqrt(1/b) + s
```

```
qrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(
a)*b**6*x**2*sqrt(1/b)) - 15*a**3*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5
/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqr
t(1/b)) - 30*a**2*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sq
rt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 3
0*a**2*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16
*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 15*a*b**2*x**
2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2
)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) - 15*a*b**2*x**2*log(sq
rt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*
sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)), True))
```

## 3.484 $\int \frac{x^{3/2}}{(-a+bx)^3}\,dx$

**Optimal**. Leaf size=72

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}+\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{x^{3/2}}{2b(a-bx)^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 63, 208}

$$\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}-\frac{x^{3/2}}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] -x^(3/2)/(2*b*(a - b*x)^2) + (3*Sqrt[x])/(4*b^2*(a - b*x)) - (3*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(-a+bx)^3}\,dx &= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx}{4b}\\ &= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}+\frac{3\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8b^2}\\ &= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}+\frac{3\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^2}\\ &= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 0.83

$$\frac{\sqrt{x}\,(3a-5bx)}{4b^2(a-bx)^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(3*a - 5*b*x))/(4*b^2*(a - b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt[x])/
Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 65, normalized size = 0.90

$$\frac{3a\sqrt{x}\,-5bx^{3/2}}{4b^2(bx-a)^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] (3*a*Sqrt[x] - 5*b*x^(3/2))/(4*b^2*(-a + b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt
[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

**fricas [A]** time = 0.71, size = 186, normalized size = 2.58

$$\left[\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{ab}\,\log\left(\frac{bx+a-2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)-2\left(5\,ab^2x-3\,a^2b\right)\sqrt{x}}{8\left(ab^5x^2-2\,a^2b^4x+a^3b^3\right)},\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)-\left(5\,ab^2x-3\,a^2b\right)\sqrt{x}}{4\left(ab^5x^2-2\,a^2b^4x+a^3b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt
(x))/(b*x - a)) - 2*(5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4*x
 + a^3*b^3), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/
(b*sqrt(x))) - (5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4*x + a^
3*b^3)]
```

**giac** [A] time = 0.92, size = 51, normalized size = 0.71

$$\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}\,b^2}-\frac{5\,bx^{\frac{3}{2}}-3\,a\sqrt{x}}{4\,(bx-a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - 1/4*(5*b*x^(3/2) - 3*a*
sqrt(x))/((b*x - a)^2*b^2)
```

**maple** [A] time = 0.01, size = 52, normalized size = 0.72

$$-\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;b^2}+\frac{-\frac{5x^{\frac{3}{2}}}{4b}+\frac{3a\sqrt{x}}{4b^2}}{(bx-a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a)^3,x)
```

```
[Out] 2*(-5/8/b*x^(3/2)+3/8*a/b^2*x^(1/2))/(b*x-a)^2-3/4/b^2/(a*b)^(1/2)*arctanh(
1/(a*b)^(1/2)*b*x^(1/2))
```

**maxima** [A] time = 2.99, size = 78, normalized size = 1.08

$$-\frac{5\,bx^{\frac{3}{2}}-3\,a\sqrt{x}}{4\left(b^4x^2-2\,ab^3x+a^2b^2\right)}+\frac{3\,\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*b*x^(3/2) - 3*a*sqrt(x))/(b^4*x^2 - 2*a*b^3*x + a^2*b^2) + 3/8*log(
(b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^2)
```

**mupad [B]** time = 0.14, size = 58, normalized size = 0.81

$$-\frac{\frac{5\,x^{3/2}}{4\,b}-\frac{3\,a\,\sqrt{x}}{4\,b^2}}{a^2-2\,a\,b\,x+b^2\,x^2}-\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(3/2)/(a - b*x)^3,x)
```

```
[Out] - ((5*x^(3/2))/(4*b) - (3*a*x^(1/2))/(4*b^2))/(a^2 + b^2*x^2 - 2*a*b*x) - (
3*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(1/2)*b^(5/2))
```

**sympy [A]** time = 29.25, size = 673, normalized size = 9.35

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2 x^{\frac{5}{2}}}{5 a^{3}} & \text{for}\: b = 0 \\ - \frac{2}{b^{3} \sqrt{x}} & \text{for}\: a = 0 \\ \frac{6 a^{\frac{3}{2}} b \sqrt{x} \sqrt{\frac{1}{b}}}{8 a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} - 16 a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} - \frac{10 \sqrt{a} b^{2} x^{\frac{3}{2}} \sqrt{\frac{1}{b}}}{8 a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} - 16 a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} + \frac{3 a^{2} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} - 16 a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} - \frac{3 a^{2} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} - 16 a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} - \frac{6 a b x \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} - 16 a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} + \frac{6 a b x \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} - 16 a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} + \frac{3 b^{2} x^{2} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} - 16 a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} - \frac{3 b^{2} x^{2} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{5}{2}} b^{3} \sqrt{\frac{1}{b}} - 16 a^{\frac{3}{2}} b^{4} x \sqrt{\frac{1}{b}} + 8 \sqrt{a} b^{5} x^{2} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*x**(5/2)/(5*a**3), Eq(b,
0)), (-2/(b**3*sqrt(x)), Eq(a, 0)), (6*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*a**(
5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sq
rt(1/b)) - 10*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(5/2)*b**3*sqrt(1/b) -
16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) + 3*a**2*log(
-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4
*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*a**2*log(sqrt(a)*sqrt(1/b
) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*
sqrt(a)*b**5*x**2*sqrt(1/b)) - 6*a*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8
*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x*
*2*sqrt(1/b)) + 6*a*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*s
qrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) +
3*b**2*x**2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) -
16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*b**2*x**2
*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*
b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)), True))
```

## 3.485 $\int \frac{\sqrt{x}}{(-a+bx)^3} \, dx$

**Optimal**. Leaf size=75

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x)^3,x]
```

```
[Out] -Sqrt[x]/(2*b*(a - b*x)^2) + Sqrt[x]/(4*a*b*(a - b*x)) + ArcTanh[(Sqrt[b]*S
qrt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx &= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\int \frac{1}{\sqrt{x}\,(-a+bx)^2}\,dx}{4b}\\
&= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8ab}\\
&= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4ab}\\
&= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 26, normalized size = 0.35

$$-\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x)^3,x]
[Out] (-2*x^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (b*x)/a])/(3*a^3)
```

**IntegrateAlgebraic [A]** time = 0.10, size = 60, normalized size = 0.80

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}-\frac{\sqrt{x}\,(a+bx)}{4ab(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(-a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[x]*(a + b*x))/(a*b*(a - b*x)^2) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]]/(4*a^(3/2)*b^(3/2))
```

**fricas [A]** time = 1.05, size = 183, normalized size = 2.44

$$\left[\frac{\left(b^{2}x^{2}-2\,abx+a^{2}\right)\sqrt{ab}\,\log\left(\frac{bx+a+2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)-2\left(ab^{2}x+a^{2}b\right)\sqrt{x}}{8\left(a^{2}b^{4}x^{2}-2\,a^{3}b^{3}x+a^{4}b^{2}\right)},-\frac{\left(b^{2}x^{2}-2\,abx+a^{2}\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)+\left(ab^{2}x+a^{2}b\right)\sqrt{x}}{4\left(a^{2}b^{4}x^{2}-2\,a^{3}b^{3}x+a^{4}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a + 2*sqrt(a*b)*sqrt(x
))/(b*x - a)) - 2*(a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x + a
^4*b^2), -1/4*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/(b*sq
rt(x))) + (a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x + a^4*b^2)]
```

**giac [A]** time = 0.93, size = 55, normalized size = 0.73

$$-\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,ab}-\frac{bx^{\frac{3}{2}}+a\sqrt{x}}{4\,(bx-a)^{2}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] -1/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a*b) - 1/4*(b*x^(3/2) + a*sqr
t(x))/((b*x - a)^2*a*b)
```

**maple [A]** time = 0.01, size = 54, normalized size = 0.72

$$\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;ab}+\frac{-\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx-a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a)^3,x)
```

```
[Out] 2*(-1/8/a*x^(3/2)-1/8/b*x^(1/2))/(b*x-a)^2+1/4/b/a/(a*b)^(1/2)*arctanh(1/(a
*b)^(1/2)*b*x^(1/2))
```

**maxima** [A] time = 2.85, size = 80, normalized size = 1.07

$$-\frac{bx^{\frac{3}{2}} + a\sqrt{x}}{4\left(ab^3x^2 - 2\,a^2b^2x + a^3b\right)} - \frac{\log\left(\frac{b\sqrt{x} - \sqrt{ab}}{b\sqrt{x} + \sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^(3/2) + a*sqrt(x))/(a*b^3*x^2 - 2*a^2*b^2*x + a^3*b) - 1/8*log((b
*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a*b)
```

**mupad** [B] time = 0.14, size = 57, normalized size = 0.76

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{3/2}\,b^{3/2}} - \frac{\frac{x^{3/2}}{4\,a} + \frac{\sqrt{x}}{4\,b}}{a^2 - 2\,a\,b\,x + b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/2)/(a - b*x)^3,x)
```

```
[Out] atanh((b^(1/2)*x^(1/2))/a^(1/2))/(4*a^(3/2)*b^(3/2)) - (x^(3/2)/(4*a) + x^(
1/2)/(4*b))/(a^2 + b^2*x^2 - 2*a*b*x)
```

**sympy** [A] time = 15.19, size = 668, normalized size = 8.91

$$\begin{cases} \frac{\infty}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{3}{2}}}{3a^3} & \text{for}\: b = 0 \\ -\frac{2}{3b^3x^{\frac{3}{2}}} & \text{for}\: a = 0 \\ -\frac{2a^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}} - 16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}} + 8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} - \frac{2\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}} - 16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}} + 8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} - \frac{a^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}} - 16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}} + 8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} + \frac{a^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}} - 16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}} + 8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} + \frac{2abx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}} - 16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}} + 8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} - \frac{2abx\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}} - 16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}} + 8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} - \frac{b^2x^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}} - 16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}} + 8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} + \frac{b^2x^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}} - 16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}} + 8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(3/2)/(3*a**3), Eq(b,
 0)), (-2/(3*b**3*x**(3/2)), Eq(a, 0)), (-2*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8
*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x
**2*sqrt(1/b)) - 2*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(7/2)*b**2*sqrt(1/
b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)) - a**2*
log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*
b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)) + a**2*log(sqrt(a)*sqrt(
1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) +
 8*a**(3/2)*b**4*x**2*sqrt(1/b)) + 2*a*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x)
)/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b*
```

```
*4*x**2*sqrt(1/b)) - 2*a*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b
**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/
b)) - b**2*x**2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b
) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)) + b**2*x
**2*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/
2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)), True))
```

## 3.486 $\int \frac{1}{\sqrt{x}(-a+bx)^3}\,dx$

**Optimal**. Leaf size=72

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}-\frac{3\sqrt{x}}{4a^2(a-bx)}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$-\frac{3\sqrt{x}}{4a^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)^3),x]
```

```
[Out] -Sqrt[x]/(2*a*(a - b*x)^2) - (3*Sqrt[x])/(4*a^2*(a - b*x)) - (3*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(-a+bx)^3}\,dx = -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx}{4a}$$
$$= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} + \frac{3\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8a^2}$$
$$= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} + \frac{3\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^2}$$
$$= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

**Mathematica [C]** time = 0.01, size = 24, normalized size = 0.33

$$-\frac{2\sqrt{x}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)^3),x]
[Out] (-2*Sqrt[x]*Hypergeometric2F1[1/2, 3, 3/2, (b*x)/a])/a^3
```

**IntegrateAlgebraic [A]** time = 0.09, size = 64, normalized size = 0.89

$$\frac{3bx^{3/2}-5a\sqrt{x}}{4a^2(a-bx)^2} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(-a + b*x)^3),x]
[Out] (-5*a*Sqrt[x] + 3*b*x^(3/2))/(4*a^2*(a - b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt
[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

**fricas [A]** time = 1.00, size = 185, normalized size = 2.57

$$\left[\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{ab}\,\log\left(\frac{bx+a-2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)+2\left(3\,ab^2x-5\,a^2b\right)\sqrt{x}}{8\left(a^3b^3x^2-2\,a^4b^2x+a^5b\right)}, \frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)+\left(3\,ab^2x-5\,a^2b\right)\sqrt{x}}{4\left(a^3b^3x^2-2\,a^4b^2x+a^5b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="fricas")

[Out] [1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt(x))/(b*x - a)) + 2*(3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b^2*x + a^5*b), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/(b*sqrt(x))) + (3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b^2*x + a^5*b)]

**giac** [A] time = 1.15, size = 51, normalized size = 0.71

$$\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}\,a^2}+\frac{3\,bx^{\frac{3}{2}}-5\,a\sqrt{x}}{4\,(bx-a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="giac")

[Out] 3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 1/4*(3*b*x^(3/2) - 5*a*sqrt(x))/((b*x - a)^2*a^2)

**maple** [A] time = 0.01, size = 63, normalized size = 0.88

$$-\frac{\sqrt{x}}{2\,(bx-a)^2\,a}-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}-\frac{\sqrt{x}}{2(bx-a)a}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x-a)^3/x^(1/2),x)

[Out] -1/2*x^(1/2)/a/(b*x-a)^2-3/2/a*(-1/2/(b*x-a)/a*x^(1/2)+1/2/a/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2)))

**maxima** [A] time = 2.88, size = 77, normalized size = 1.07

$$\frac{3\,bx^{\frac{3}{2}}-5\,a\sqrt{x}}{4\left(a^2b^2x^2-2\,a^3bx+a^4\right)}+\frac{3\,\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="maxima")

```
[Out] 1/4*(3*b*x^(3/2) - 5*a*sqrt(x))/(a^2*b^2*x^2 - 2*a^3*b*x + a^4) + 3/8*log((
b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

**mupad [B]** time = 0.13, size = 58, normalized size = 0.81

$$-\frac{\frac{5\sqrt{x}}{4a}-\frac{3\,b\,x^{3/2}}{4\,a^2}}{a^2-2\,a\,b\,x+b^2\,x^2}-\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/2)*(a - b*x)^3),x)
```

```
[Out] - ((5*x^(1/2))/(4*a) - (3*b*x^(3/2))/(4*a^2))/(a^2 + b^2*x^2 - 2*a*b*x) - (
3*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(5/2)*b^(1/2))
```

**sympy [A]** time = 25.67, size = 660, normalized size = 9.17

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2\sqrt{x}}{a^3} & \text{for}\: b = 0 \\ -\frac{2}{5b^3x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ -\frac{10a^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} + \frac{6\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} + \frac{3a^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} - \frac{3a^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} - \frac{6abx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} + \frac{6abx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} + \frac{3b^2x^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} - \frac{3b^2x^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}}-16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}}+8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2*sqrt(x)/a**3, Eq(b, 0)),
 (-2/(5*b**3*x**(5/2)), Eq(a, 0)), (-10*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*a**
(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqr
t(1/b)) + 6*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(9/2)*b*sqrt(1/b) - 16*a*
*(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3*a**2*log(-sqr
t(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt
(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) - 3*a**2*log(sqrt(a)*sqrt(1/b) + sq
rt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*
b**3*x**2*sqrt(1/b)) - 6*a*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2
)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/
b)) + 6*a*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16
*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3*b**2*x**2*
log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**
2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) - 3*b**2*x**2*log(sqrt(a)*s
qrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b)
+ 8*a**(5/2)*b**3*x**2*sqrt(1/b)), True))
```

## 3.487 $\int \frac{1}{x^{3/2}(-a+bx)^3} dx$

**Optimal**. Leaf size=84

$$-\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15}{4a^3\sqrt{x}}-\frac{5}{4a^2\sqrt{x}\,(a-bx)}-\frac{1}{2a\sqrt{x}\,(a-bx)^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$-\frac{5}{4a^2\sqrt{x}\,(a-bx)}-\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15}{4a^3\sqrt{x}}-\frac{1}{2a\sqrt{x}\,(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)^3),x]
```

```
[Out] 15/(4*a^3*Sqrt[x]) - 1/(2*a*Sqrt[x]*(a - b*x)^2) - 5/(4*a^2*Sqrt[x]*(a - b*
x)) - (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx &= -\frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx}{4a}\\
&= -\frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} + \frac{15\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{8a^2}\\
&= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} + \frac{(15b)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8a^3}\\
&= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} + \frac{(15b)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^3}\\
&= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} - \frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 24, normalized size = 0.29

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx}{a}\right)}{a^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)^3),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 3, 1/2, (b*x)/a])/(a^3*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.11, size = 71, normalized size = 0.85

$$\frac{8a^2-25abx+15b^2x^2}{4a^3\sqrt{x}\,(a-bx)^2} - \frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(-a + b*x)^3),x]
```

```
[Out] (8*a^2 - 25*a*b*x + 15*b^2*x^2)/(4*a^3*Sqrt[x]*(a - b*x)^2) - (15*Sqrt[b]*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

**fricas** [A] time = 0.97, size = 213, normalized size = 2.54

$$\left[\frac{15\,\left(b^2x^3-2\,abx^2+a^2x\right)\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\,\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,\left(15\,b^2x^2-25\,abx+8\,a^2\right)\sqrt{x}}{8\,\left(a^3b^2x^3-2\,a^4bx^2+a^5x\right)},\ \frac{15\,\left(b^2x^3-2\,abx^2+a^2x\right)\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\left(15\,b^2x^2-25\,abx+8\,a^2\right)\sqrt{x}}{4\,\left(a^3b^2x^3-2\,a^4bx^2+a^5x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqr
t(b/a) + a)/(b*x - a)) + 2*(15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*b^
2*x^3 - 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt(-b
/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 - 2*a^4*b*x^2 + a^5*x)]
```

**giac** [A] time = 1.05, size = 63, normalized size = 0.75

$$\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,a^3}+\frac{2}{a^3\sqrt{x}}+\frac{7\,b^2x^{\frac{3}{2}}-9\,ab\sqrt{x}}{4\,(bx-a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 15/4*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/(a^3*sqrt(x)) + 1/
4*(7*b^2*x^(3/2) - 9*a*b*sqrt(x))/((b*x - a)^2*a^3)
```

**maple** [A] time = 0.02, size = 58, normalized size = 0.69

$$\frac{2\left(-\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}+\frac{\frac{7b\,x^{\frac{3}{2}}}{8}-\frac{9a\sqrt{x}}{8}}{(bx-a)^2}\right)b}{a^3}+\frac{2}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a)^3,x)
```

```
[Out] 2/a^3*b*((7/8*b*x^(3/2)-9/8*a*x^(1/2))/(b*x-a)^2-15/8/(a*b)^(1/2)*arctanh(1
/(a*b)^(1/2)*b*x^(1/2)))+2/a^3/x^(1/2)
```

**maxima** [A] time = 2.93, size = 90, normalized size = 1.07

$$\frac{15\,b^2x^2-25\,abx+8\,a^2}{4\left(a^3b^2x^{\frac{5}{2}}-2\,a^4bx^{\frac{3}{2}}+a^5\sqrt{x}\right)}+\frac{15\,b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(15*b^2*x^2 - 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) - 2*a^4*b*x^(3/2) + a^
5*sqrt(x)) + 15/8*b*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(s
qrt(a*b)*a^3)
```

**mupad [B]** time = 0.15, size = 69, normalized size = 0.82

$$\frac{\frac{2}{a}+\frac{15\,b^2\,x^2}{4\,a^3}-\frac{25\,b\,x}{4\,a^2}}{a^2\,\sqrt{x}+b^2\,x^{5/2}-2\,a\,b\,x^{3/2}}-\frac{15\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(3/2)*(a - b*x)^3),x)
```

```
[Out] (2/a + (15*b^2*x^2)/(4*a^3) - (25*b*x)/(4*a^2))/(a^2*x^(1/2) + b^2*x^(5/2)
- 2*a*b*x^(3/2)) - (15*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(7/2)
)
```

**sympy [A]** time = 54.56, size = 802, normalized size = 9.55

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2}{a^{3} \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{2}{7 b^{3} x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ \frac{16 a^{\frac{5}{2}} \sqrt{\frac{1}{b}}}{8 a^{\frac{11}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 16 a^{\frac{9}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 8 a^{\frac{7}{2}} b^{2} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{50 a^{\frac{3}{2}} b x \sqrt{\frac{1}{b}}}{8 a^{\frac{11}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 16 a^{\frac{9}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 8 a^{\frac{7}{2}} b^{2} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{30 \sqrt{a} b^{2} x^{2} \sqrt{\frac{1}{b}}}{8 a^{\frac{11}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 16 a^{\frac{9}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 8 a^{\frac{7}{2}} b^{2} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{15 a^{2} \sqrt{x} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{11}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 16 a^{\frac{9}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 8 a^{\frac{7}{2}} b^{2} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{15 a^{2} \sqrt{x} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{11}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 16 a^{\frac{9}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 8 a^{\frac{7}{2}} b^{2} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{30 a b x^{\frac{3}{2}} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{11}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 16 a^{\frac{9}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 8 a^{\frac{7}{2}} b^{2} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{30 a b x^{\frac{3}{2}} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{11}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 16 a^{\frac{9}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 8 a^{\frac{7}{2}} b^{2} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{15 b^{2} x^{\frac{5}{2}} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{11}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 16 a^{\frac{9}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 8 a^{\frac{7}{2}} b^{2} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{15 b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{\frac{11}{2}} \sqrt{x} \sqrt{\frac{1}{b}} - 16 a^{\frac{9}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 8 a^{\frac{7}{2}} b^{2} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (2/(a**3*sqrt(x)), Eq(b, 0))
, (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (16*a**(5/2)*sqrt(1/b)/(8*a**(11/2)*sqr
t(x)*sqrt(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2
)*sqrt(1/b)) - 50*a**(3/2)*b*x*sqrt(1/b)/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 1
6*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) + 30*
sqrt(a)*b**2*x**2*sqrt(1/b)/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 16*a**(9/2)*b*
x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) + 15*a**2*sqrt(x)*
log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 16*a**(9
/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 15*a**2*sq
rt(x)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 16*
a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 30*a*
b*x**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt(1/b)
 - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) +
 30*a*b*x**(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt
(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/
```

```
b)) + 15*b**2*x**(5/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(
x)*sqrt(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*
sqrt(1/b)) - 15*b**2*x**(5/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)
*sqrt(x)*sqrt(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**
(5/2)*sqrt(1/b)), True))
```

## 3.488 $\int \frac{1}{x^{5/2}(-a+bx)^3} dx$

**Optimal**. Leaf size=97

$$-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{35}{12a^3x^{3/2}}-\frac{7}{4a^2x^{3/2}(a-bx)}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{35b}{4a^4\sqrt{x}}+\frac{35}{12a^3x^{3/2}}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)^3),x]
```

```
[Out] 35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) - 1/(2*a*x^(3/2)*(a - b*x)^2)
- 7/(4*a^2*x^(3/2)*(a - b*x)) - (35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx = -\frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx}{4a}$$

$$= -\frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{35\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{8a^2}$$

$$= \frac{35}{12a^3x^{3/2}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{(35b)\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{8a^3}$$

$$= \frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{\left(35b^2\right)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8a^4}$$

$$= \frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x,\right.}{4a^4}$$

$$= \frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} - \frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

**Mathematica [C]** time = 0.00, size = 26, normalized size = 0.27

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\frac{bx}{a}\right)}{3a^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)^3),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 3, -1/2, (b*x)/a])/(3*a^3*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 82, normalized size = 0.85

$$\frac{8a^3+56a^2bx-175ab^2x^2+105b^3x^3}{12a^4x^{3/2}(a-bx)^2} - \frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(-a + b*x)^3),x]
```

```
[Out] (8*a^3 + 56*a^2*b*x - 175*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^(3/2)*(a - b*x
)^2) - (35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(9/2))
```

**fricas [A]** time = 1.01, size = 249, normalized size = 2.57

$$\left[\frac{105\left(b^{3}x^{4}-2\,ab^{2}x^{3}+a^{2}bx^{2}\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\left(105\,b^{3}x^{3}-175\,ab^{2}x^{2}+56\,a^{2}bx+8\,a^{3}\right)\sqrt{x}}{24\left(a^{4}b^{2}x^{4}-2\,a^{5}bx^{3}+a^{6}x^{2}\right)},\ \frac{105\left(b^{3}x^{4}-2\,ab^{2}x^{3}+a^{2}bx^{2}\right)\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\left(105\,b^{3}x^{3}-175\,ab^{2}x^{2}+56\,a^{2}bx+8\,a^{3}\right)\sqrt{x}}{12\left(a^{4}b^{2}x^{4}-2\,a^{5}bx^{3}+a^{6}x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*log((b*x - 2*a*sqr
t(x)*sqrt(b/a) + a)/(b*x - a)) + 2*(105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*
x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2), 1/12*(105*(b^3*x
^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) +
 (105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 -
 2*a^5*b*x^3 + a^6*x^2)]
```

**giac [A]** time = 1.04, size = 73, normalized size = 0.75

$$\frac{35\,b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}\,a^{4}}+\frac{2\,(9\,bx+a)}{3\,a^{4}x^{\frac{3}{2}}}+\frac{11\,b^{3}x^{\frac{3}{2}}-13\,ab^{2}\sqrt{x}}{4\,(bx-a)^{2}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^4) + 2/3*(9*b*x + a)/(a
^4*x^(3/2)) + 1/4*(11*b^3*x^(3/2) - 13*a*b^2*sqrt(x))/((b*x - a)^2*a^4)
```

**maple [A]** time = 0.02, size = 69, normalized size = 0.71

$$\frac{2\left(-\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}+\frac{\frac{11b\,x^{\frac{3}{2}}}{8}-\frac{13a\sqrt{x}}{8}}{(bx-a)^{2}}\right)b^{2}}{a^{4}}+\frac{6b}{a^{4}\sqrt{x}}+\frac{2}{3a^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a)^3,x)
```

```
[Out] 2/a^4*b^2*((11/8*b*x^(3/2)-13/8*a*x^(1/2))/(b*x-a)^2-35/8/(a*b)^(1/2)*arcta
nh(1/(a*b)^(1/2)*b*x^(1/2)))+2/3/a^3/x^(3/2)+6/a^4*b/x^(1/2)
```

**maxima [A]** time = 2.95, size = 103, normalized size = 1.06

$$\frac{105\,b^{3}x^{3}-175\,ab^{2}x^{2}+56\,a^{2}bx+8\,a^{3}}{12\left(a^{4}b^{2}x^{\frac{7}{2}}-2\,a^{5}bx^{\frac{5}{2}}+a^{6}x^{\frac{3}{2}}\right)}+\frac{35\,b^{2}\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*x + 8*a^3)/(a^4*b^2*x^(7/2) -
2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/8*b^2*log((b*sqrt(x) - sqrt(a*b))/(b*sq
rt(x) + sqrt(a*b)))/(sqrt(a*b)*a^4)
```

**mupad [B]** time = 0.17, size = 80, normalized size = 0.82

$$\frac{\frac{2}{3\,a}-\frac{175\,b^2\,x^2}{12\,a^3}+\frac{35\,b^3\,x^3}{4\,a^4}+\frac{14\,b\,x}{3\,a^2}}{a^2\,x^{3/2}+b^2\,x^{7/2}-2\,a\,b\,x^{5/2}}-\frac{35\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(5/2)*(a - b*x)^3),x)
```

```
[Out] (2/(3*a) - (175*b^2*x^2)/(12*a^3) + (35*b^3*x^3)/(4*a^4) + (14*b*x)/(3*a^2)
)/(a^2*x^(3/2) + b^2*x^(7/2) - 2*a*b*x^(5/2)) - (35*b^(3/2)*atanh((b^(1/2)*
x^(1/2))/a^(1/2)))/(4*a^(9/2))
```

**sympy [A]** time = 138.88, size = 892, normalized size = 9.20

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2}{3 a^{3} x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{9 b^{3} x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ \frac{16 a^{\frac{7}{2}} \sqrt{\frac{1}{b}}}{24 a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 48 a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} + \frac{112 a^{\frac{5}{2}} b x \sqrt{\frac{1}{b}}}{24 a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 48 a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} - \frac{350 a^{\frac{3}{2}} b^{2} x^{2} \sqrt{\frac{1}{b}}}{24 a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 48 a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} + \frac{210 \sqrt{a} b^{3} x^{3} \sqrt{\frac{1}{b}}}{24 a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 48 a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} + \frac{105 a^{2} b x^{\frac{3}{2}} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 48 a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} - \frac{105 a^{2} b x^{\frac{3}{2}} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 48 a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} - \frac{210 a b^{2} x^{\frac{5}{2}} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 48 a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} + \frac{210 a b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 48 a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} + \frac{105 b^{3} x^{\frac{7}{2}} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 48 a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} - \frac{105 b^{3} x^{\frac{7}{2}} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 48 a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (2/(3*a**3*x**(3/2)), Eq(b,
0)), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (16*a**(7/2)*sqrt(1/b)/(24*a**(13/2)
*x**(3/2)*sqrt(1/b) - 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*
x**(7/2)*sqrt(1/b)) + 112*a**(5/2)*b*x*sqrt(1/b)/(24*a**(13/2)*x**(3/2)*sqr
t(1/b) - 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt
(1/b)) - 350*a**(3/2)*b**2*x**2*sqrt(1/b)/(24*a**(13/2)*x**(3/2)*sqrt(1/b)
- 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(1/b))
+ 210*sqrt(a)*b**3*x**3*sqrt(1/b)/(24*a**(13/2)*x**(3/2)*sqrt(1/b) - 48*a**
(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) + 105*a*
*2*b*x**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(13/2)*x**(3/2)*sqrt
(1/b) - 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(
1/b)) - 105*a**2*b*x**(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(13/2)*
x**(3/2)*sqrt(1/b) - 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x
**(7/2)*sqrt(1/b)) - 210*a*b**2*x**(5/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/
(24*a**(13/2)*x**(3/2)*sqrt(1/b) - 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a
**(9/2)*b**2*x**(7/2)*sqrt(1/b)) + 210*a*b**2*x**(5/2)*log(sqrt(a)*sqrt(1/b
```

```
) + sqrt(x))/(24*a**(13/2)*x**(3/2)*sqrt(1/b) - 48*a**(11/2)*b*x**(5/2)*sqr
t(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) + 105*b**3*x**(7/2)*log(-sqrt
(a)*sqrt(1/b) + sqrt(x))/(24*a**(13/2)*x**(3/2)*sqrt(1/b) - 48*a**(11/2)*b*
x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) - 105*b**3*x**(7/
2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(13/2)*x**(3/2)*sqrt(1/b) - 48*a
**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)), True)
)
```

## 3.489 $\int x^{5/2}\sqrt{a+bx}\,dx$

**Optimal**. Leaf size=122

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

**Rubi** [A] time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[a + b*x],x]
```

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a + b*x])/(96*
b^2) + (a*x^(5/2)*Sqrt[a + b*x])/(24*b) + (x^(7/2)*Sqrt[a + b*x])/4 - (5*a^
4*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}\sqrt{a+bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{a+bx}+\frac{1}{8}a\int\frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$

$$= \frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{\left(5a^2\right)\int\frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{48b}$$

$$= -\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}+\frac{\left(5a^3\right)\int\frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{64b^2}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{\left(5a^4\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{128b^3}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{\left(5a^4\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+b}}\right.}{64b^3}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{\left(5a^4\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\right.}{64b^3}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}$$

**Mathematica [A]** time = 0.18, size = 96, normalized size = 0.79

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(15a^3-10a^2bx+8ab^2x^2+48b^3x^3\right)-\frac{15a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{192b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^3 - 10*a^2*b*x + 8*a*b^2*x^2 + 48*b^3
*x^3) - (15*a^(7/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))
/(192*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.14, size = 95, normalized size = 0.78

$$\frac{5a^4\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{64b^{7/2}}+\frac{\sqrt{a+bx}\left(15a^3\sqrt{x}-10a^2bx^{3/2}+8ab^2x^{5/2}+48b^3x^{7/2}\right)}{192b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(15*a^3*Sqrt[x] - 10*a^2*b*x^(3/2) + 8*a*b^2*x^(5/2) + 48*b^
3*x^(7/2)))/(192*b^3) + (5*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(64
*b^(7/2))
```

**fricas [A]** time = 0.95, size = 162, normalized size = 1.33

$$\left[\frac{15\,a^4\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\left(48\,b^4x^3+8\,ab^3x^2-10\,a^2b^2x+15\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{384\,b^4},\frac{15\,a^4\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(48\,b^4x^3+8\,ab^3x^2-10\,a^2b^2x+15\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{192\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(x)
)/b^4, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
(48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(x))
/b^4]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 120, normalized size = 0.98

$$\frac{(bx+a)^{\frac{3}{2}}x^{\frac{5}{2}}}{4b}-\frac{5\sqrt{(bx+a)x}\,a^4\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{128\sqrt{bx+a}\,b^{\frac{7}{2}}\sqrt{x}}-\frac{5\sqrt{bx+a}\,a^3\sqrt{x}}{64b^3}-\frac{5\,(bx+a)^{\frac{3}{2}}\,a\,x^{\frac{3}{2}}}{24b^2}+\frac{5\,(bx+a)^{\frac{3}{2}}\,a^2\sqrt{x}}{32b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(1/2),x)
```

```
[Out] 1/4/b*x^(5/2)*(b*x+a)^(3/2)-5/24*a/b^2*x^(3/2)*(b*x+a)^(3/2)+5/32*a^2/b^3*x
^(1/2)*(b*x+a)^(3/2)-5/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b^3-5/128*a^4/b^(7/2)*(
x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(
1/2))
```

**maxima [B]** time = 2.95, size = 178, normalized size = 1.46

$$\frac{5\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{bx+a}\,a^4b^3}{\sqrt{x}}+\frac{73\,(bx+a)^{\frac{3}{2}}\,a^4b^2}{x^{\frac{3}{2}}}-\frac{55\,(bx+a)^{\frac{5}{2}}\,a^4b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}\,a^4}{x^{\frac{7}{2}}}}{192\left(b^7-\frac{4\,(bx+a)b^6}{x}+\frac{6\,(bx+a)^2b^5}{x^2}-\frac{4\,(bx+a)^3b^4}{x^3}+\frac{(bx+a)^4b^3}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(7/2) + 1/192*(15*sqrt(b*x + a)*a^4*b^3/sqrt(x) + 73*(b*x + a)^(
3/2)*a^4*b^2/x^(3/2) - 55*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 15*(b*x + a)^(7/2
)*a^4/x^(7/2))/(b^7 - 4*(b*x + a)*b^6/x + 6*(b*x + a)^2*b^5/x^2 - 4*(b*x +
a)^3*b^4/x^3 + (b*x + a)^4*b^3/x^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{a+b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)*(a + b*x)^(1/2), x)
```

**sympy [A]** time = 11.70, size = 153, normalized size = 1.25

$$\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1+\frac{bx}{a}}}+\frac{7\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}}-\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}+\frac{bx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(1/2),x)
```

```
[Out] 5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 + b*x/a)) + 5*a**(5/2)*x**(3/2)/(192*b**
2*sqrt(1 + b*x/a)) - a**(3/2)*x**(5/2)/(96*b*sqrt(1 + b*x/a)) + 7*sqrt(a)*x
**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b*
*(7/2)) + b*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

# 3.490 $\int x^{3/2}\sqrt{a+bx}\,dx$

**Optimal**. Leaf size=98

$$\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}-\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

**Rubi [A]** time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a + b*x],x]
```

```
[Out] -(a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b^2) + (a*x^(3/2)*Sqrt[a + b*x])/(12*b) + (
x^(5/2)*Sqrt[a + b*x])/3 + (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
8*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}\sqrt{a+bx}\,dx &= \frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{1}{6}a\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx\\
&= \frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}-\frac{a^2\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{8b}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{16b^2}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{8b^2}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^2}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 85, normalized size = 0.87

$$\frac{\sqrt{a+bx}\left(\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}+\sqrt{b}\sqrt{x}\left(-3a^2+2abx+8b^2x^2\right)\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^2 + 2*a*b*x + 8*b^2*x^2) + (3*a^(5/2)
*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 82, normalized size = 0.84

$$\frac{\sqrt{a+bx}\left(-3a^2\sqrt{x}+2abx^{3/2}+8b^2x^{5/2}\right)}{24b^2}-\frac{a^3\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(-3*a^2*Sqrt[x] + 2*a*b*x^(3/2) + 8*b^2*x^(5/2)))/(24*b^2) -
 (a^3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*b^(5/2))
```

**fricas [A]** time = 0.54, size = 141, normalized size = 1.44

$$\left[\frac{3\,a^{3}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(8\,b^{3}x^{2}+2\,ab^{2}x-3\,a^{2}b\right)\sqrt{bx+a}\,\sqrt{x}}{48\,b^{3}},\,-\frac{3\,a^{3}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(8\,b^{3}x^{2}+2\,ab^{2}x-3\,a^{2}b\right)\sqrt{bx+a}\,\sqrt{x}}{24\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 2*a*b^2*x - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 2*a*b^2*x
 - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.00, size = 102, normalized size = 1.04

$$\frac{\sqrt{\left(bx+a\right)x}\;a^{3}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{16\sqrt{bx+a}\;b^{\frac{5}{2}}\sqrt{x}}+\frac{\sqrt{bx+a}\;a^{2}\sqrt{x}}{8b^{2}}+\frac{\left(bx+a\right)^{\frac{3}{2}}x^{\frac{3}{2}}}{3b}-\frac{\left(bx+a\right)^{\frac{3}{2}}a\sqrt{x}}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(1/2),x)
```

```
[Out] 1/3/b*x^(3/2)*(b*x+a)^(3/2)-1/4*a/b^2*x^(1/2)*(b*x+a)^(3/2)+1/8*a^2*x^(1/2)
*(b*x+a)^(1/2)/b^2+1/16*a^3/b^(5/2)*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima [B]** time = 3.04, size = 146, normalized size = 1.49

$$-\frac{a^{3}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{5}{2}}}-\frac{\frac{3\,\sqrt{bx+a}\,a^{3}b^{2}}{\sqrt{x}}+\frac{8\,(bx+a)^{\frac{3}{2}}a^{3}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{5}-\frac{3\,(bx+a)b^{4}}{x}+\frac{3\,(bx+a)^{2}b^{3}}{x^{2}}-\frac{(bx+a)^{3}b^{2}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(5/2) - 1/24*(3*sqrt(b*x + a)*a^3*b^2/sqrt(x) + 8*(b*x + a)^(3/2
)*a^3*b/x^(3/2) - 3*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^5 - 3*(b*x + a)*b^4/x +
 3*(b*x + a)^2*b^3/x^2 - (b*x + a)^3*b^2/x^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\sqrt{a+b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)*(a + b*x)^(1/2), x)
```

**sympy [A]** time = 6.38, size = 122, normalized size = 1.24

$$-\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1+\frac{bx}{a}}}+\frac{5\sqrt{a}\,x^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}}+\frac{a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{bx^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(1/2),x)
```

```
[Out] -a**(5/2)*sqrt(x)/(8*b**2*sqrt(1 + b*x/a)) - a**(3/2)*x**(3/2)/(24*b*sqrt(1
 + b*x/a)) + 5*sqrt(a)*x**(5/2)/(12*sqrt(1 + b*x/a)) + a**3*asinh(sqrt(b)*s
qrt(x)/sqrt(a))/(8*b**(5/2)) + b*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

# 3.491 $\int \sqrt{x}\sqrt{a+bx}\,dx$

**Optimal**. Leaf size=74

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a+bx}+\frac{a\sqrt{x}\sqrt{a+bx}}{4b}$$

**Rubi [A]** time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a+bx}+\frac{a\sqrt{x}\sqrt{a+bx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a + b*x],x]

[Out] (a*Sqrt[x]*Sqrt[a + b*x])/(4*b) + (x^(3/2)*Sqrt[a + b*x])/2 - (a^2*ArcTanh[
(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\sqrt{a+bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{a+bx} + \frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx\\
&= \frac{a\sqrt{x}\sqrt{a+bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a+bx} - \frac{a^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{8b}\\
&= \frac{a\sqrt{x}\sqrt{a+bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a+bx} - \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{4b}\\
&= \frac{a\sqrt{x}\sqrt{a+bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a+bx} - \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b}\\
&= \frac{a\sqrt{x}\sqrt{a+bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a+bx} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 72, normalized size = 0.97

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}(a+2bx) - \frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(a + 2*b*x) - (a^(3/2)*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(4*b^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 68, normalized size = 0.92

$$\frac{a^2\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{4b^{3/2}} + \frac{\sqrt{a+bx}\left(a\sqrt{x}+2bx^{3/2}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(a*Sqrt[x] + 2*b*x^(3/2)))/(4*b) + (a^2*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[a + b*x]])/(4*b^(3/2))
```

**fricas [A]** time = 0.91, size = 114, normalized size = 1.54

$$\left[\frac{a^{2}\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2\,b^{2}x+ab\right)\sqrt{bx+a}\sqrt{x}}{8\,b^{2}},\frac{a^{2}\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(2\,b^{2}x+ab\right)\sqrt{bx+a}\sqrt{x}}{4\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2*b
^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/4*(a^2*sqrt(-b)*arctan(sqrt(b*x +
 a)*sqrt(-b)/(b*sqrt(x))) + (2*b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 81, normalized size = 1.09

$$\frac{\sqrt{bx+a}\,x^{\frac{3}{2}}}{2}-\frac{\sqrt{(bx+a)\,x}\,a^{2}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8\sqrt{bx+a}\,b^{\frac{3}{2}}\sqrt{x}}+\frac{\sqrt{bx+a}\,a\sqrt{x}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+a)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+a)^(1/2)+1/4*a*x^(1/2)*(b*x+a)^(1/2)/b-1/8*a^2/b^(3/2)*((b
*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/
2))
```

**maxima [B]** time = 2.96, size = 108, normalized size = 1.46

$$\frac{a^{2}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{3}{2}}}+\frac{\frac{\sqrt{bx+a}\,a^{2}b}{\sqrt{x}}+\frac{(bx+a)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{3}-\frac{2\,(bx+a)b^{2}}{x}+\frac{(bx+a)^{2}b}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqr
t(x)))/b^(3/2) + 1/4*(sqrt(b*x + a)*a^2*b/sqrt(x) + (b*x + a)^(3/2)*a^2/x^(
3/2))/(b^3 - 2*(b*x + a)*b^2/x + (b*x + a)^2*b/x^2)
```

**mupad [B]** time = 0.15, size = 52, normalized size = 0.70

$$\sqrt{x}\,\left(\frac{x}{2}+\frac{a}{4\,b}\right)\,\sqrt{a+b\,x}-\frac{a^2\,\ln\left(a+2\,b\,x+2\,\sqrt{b}\,\sqrt{x}\,\sqrt{a+b\,x}\right)}{8\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 + a/(4*b))*(a + b*x)^(1/2) - (a^2*log(a + 2*b*x + 2*b^(1/2)*x^
(1/2)*(a + b*x)^(1/2)))/(8*b^(3/2))
```

**sympy [A]** time = 3.57, size = 97, normalized size = 1.31

$$\frac{a^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{1+\frac{bx}{a}}}+\frac{3\sqrt{a}\,x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}}-\frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+\frac{bx^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+a)**(1/2),x)
```

```
[Out] a**(3/2)*sqrt(x)/(4*b*sqrt(1 + b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 + b*x
/a)) - a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) + b*x**(5/2)/(2*sqr
t(a)*sqrt(1 + b*x/a))
```

# 3.492 $\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=44

$$\sqrt{x}\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\sqrt{x}\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b
]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx &= \sqrt{x}\sqrt{a+bx} + \frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx \\
&= \sqrt{x}\sqrt{a+bx} + a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right) \\
&= \sqrt{x}\sqrt{a+bx} + a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right) \\
&= \sqrt{x}\sqrt{a+bx} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 62, normalized size = 1.41

$$\frac{\frac{a^{3/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} + \sqrt{x}\,(a+bx)}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(a + b*x) + (a^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/Sqrt[b])/Sqrt[a + b*x]
```

**IntegrateAlgebraic [A]** time = 0.06, size = 47, normalized size = 1.07

$$\sqrt{x}\sqrt{a+bx} - \frac{a\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a + b*x] - (a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[b]
```

**fricas [A]** time = 0.95, size = 93, normalized size = 2.11

$$\left[\frac{a\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)+2\,\sqrt{bx+a}\,b\sqrt{x}}{2\,b},-\frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}\,b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b, -(a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)
)) - sqrt(b*x + a)*b*sqrt(x))/b]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.00, size = 62, normalized size = 1.41

$$\frac{\sqrt{(bx+a)\,x}\;a\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{bx+a}\;\sqrt{b}\;\sqrt{x}}+\sqrt{bx+a}\;\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)+1/2*a*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((b*x
+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

**maxima [B]** time = 2.99, size = 70, normalized size = 1.59

$$-\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,\sqrt{b}}-\frac{\sqrt{bx+a}\,a}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt
(x)))/sqrt(b) - sqrt(b*x + a)*a/((b - (b*x + a)/x)*sqrt(x))
```

**mupad [B]** time = 0.68, size = 41, normalized size = 0.93

$$\sqrt{x}\,\sqrt{a+b\,x}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a + b*x)^(1/2) + (2*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2) - a
^(1/2))))/b^(1/2)
```

**sympy [A]** time = 1.92, size = 42, normalized size = 0.95

$$\sqrt{a}\,\sqrt{x}\,\sqrt{1+\frac{bx}{a}}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*sqrt(1 + b*x/a) + a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

## 3.493 $\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx$

**Optimal**. Leaf size=45

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}$$

**Rubi [A]** time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b
*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + b\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + (2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + (2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + 2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 64, normalized size = 1.42

$$\frac{2\left(\sqrt{a}\,\sqrt{b}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)-\frac{a+bx}{\sqrt{x}}\right)}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(3/2),x]
```

```
[Out] (2*(-((a + b*x)/Sqrt[x]) + Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[
b]*Sqrt[x])/Sqrt[a]]))/Sqrt[a + b*x]
```

**IntegrateAlgebraic [A]** time = 0.07, size = 47, normalized size = 1.04

$$-\frac{2\sqrt{a+bx}}{\sqrt{x}} - 2\sqrt{b}\,\log\left(\sqrt{a+bx} - \sqrt{b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/Sqrt[x] - 2*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*
x]]
```

**fricas [A]** time = 0.93, size = 89, normalized size = 1.98

$$\left[\frac{\sqrt{b}x\log\left(2bx+2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\sqrt{bx+a}\sqrt{x}}{x},-\frac{2\left(\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(b*x +
 a)*sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
 sqrt(b*x + a)*sqrt(x))/x]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.04, size = 61, normalized size = 1.36

$$\frac{\sqrt{(bx+a)x}\,\sqrt{b}\,\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{bx+a}\,\sqrt{x}}-\frac{2\sqrt{bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(3/2),x)
```

```
[Out] -2*(b*x+a)^(1/2)/x^(1/2)+b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))*
((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
```

**maxima [A]** time = 3.00, size = 54, normalized size = 1.20

$$-\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\sqrt{bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x))) - 2*sqrt(b*x + a)/sqrt(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)/x^(3/2), x)
```

**sympy [A]** time = 1.56, size = 68, normalized size = 1.51

$$-\frac{2\sqrt{a}}{\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}+2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)-\frac{2b\sqrt{x}}{\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(3/2),x)
```

```
[Out] -2*sqrt(a)/(sqrt(x)*sqrt(1 + b*x/a)) + 2*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt
(a)) - 2*b*sqrt(x)/(sqrt(a)*sqrt(1 + b*x/a))
```

## 3.494 $\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx$

**Optimal**. Leaf size=21

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = -\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

**Mathematica** [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 21, normalized size = 1.00

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^(5/2),x]
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

**fricas [A]** time = 1.01, size = 15, normalized size = 0.71

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

**giac [B]** time = 1.33, size = 33, normalized size = 1.57

$$-\frac{2\,(bx+a)^{\frac{3}{2}}b^4}{3\,((bx+a)b-ab)^{\frac{3}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="giac")
[Out] -2/3*(b*x + a)^(3/2)*b^4/(((b*x + a)*b - a*b)^(3/2)*a*abs(b))
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.76

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(5/2),x)
[Out] -2/3*(b*x+a)^(3/2)/a/x^(3/2)
```

**maxima** [A] time = 1.36, size = 15, normalized size = 0.71

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

**mupad** [B] time = 0.24, size = 21, normalized size = 1.00

$$-\frac{\left(\frac{2\,b\,x}{3\,a}+\frac{2}{3}\right)\,\sqrt{a+b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(5/2),x)
```

```
[Out] -((((2*b*x)/(3*a) + 2/3)*(a + b*x)^(1/2))/x^(3/2)
```

**sympy** [B] time = 1.46, size = 41, normalized size = 1.95

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(5/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a)
```

## 3.495 $\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx$

**Optimal**. Leaf size=44

$$\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(5*a*x^(5/2)) + (4*b*(a + b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{5ax^{5/2}}-\frac{(2b)\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx}{5a}\\ &= -\frac{2(a+bx)^{3/2}}{5ax^{5/2}}+\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 29, normalized size = 0.66

$$-\frac{2(3a-2bx)(a+bx)^{3/2}}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (-2*(3*a - 2*b*x)*(a + b*x)^(3/2))/(15*a^2*x^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.08, size = 40, normalized size = 0.91

$$\frac{2\sqrt{a+bx}\left(-3a^2-abx+2b^2x^2\right)}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (2*Sqrt[a + b*x]*(-3*a^2 - a*b*x + 2*b^2*x^2))/(15*a^2*x^(5/2))
```

**fricas** [A] time = 0.69, size = 34, normalized size = 0.77

$$\frac{2\left(2\,b^2x^2-abx-3\,a^2\right)\sqrt{bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

**giac** [A] time = 1.10, size = 50, normalized size = 1.14

$$\frac{2\left(\frac{2\,(bx+a)b^5}{a^2}-\frac{5\,b^5}{a}\right)(bx+a)^{\frac{3}{2}}b}{15\,((bx+a)b-ab)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(2*(b*x + a)*b^5/a^2 - 5*b^5/a)*(b*x + a)^(3/2)*b/(((b*x + a)*b - a*b)
^(5/2)*abs(b))
```

**maple** [A] time = 0.00, size = 24, normalized size = 0.55

$$-\frac{2 \left(b x+a\right)^{\frac{3}{2}} \left(-2 b x+3 a\right)}{15 a^{2} x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(7/2),x)
```

```
[Out] -2/15*(b*x+a)^(3/2)*(-2*b*x+3*a)/x^(5/2)/a^2
```

**maxima** [A] time = 1.29, size = 31, normalized size = 0.70

$$\frac{2 \left(\frac{5 \left(b x+a\right)^{\frac{3}{2}} b}{x^{\frac{3}{2}}}-\frac{3 \left(b x+a\right)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15 \, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*(b*x + a)^(3/2)*b/x^(3/2) - 3*(b*x + a)^(5/2)/x^(5/2))/a^2
```

**mupad** [B] time = 0.25, size = 32, normalized size = 0.73

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2\,b\,x}{15\,a}-\frac{4\,b^2\,x^2}{15\,a^2}+\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(7/2),x)
```

```
[Out] -((a + b*x)^(1/2)*((2*b*x)/(15*a) - (4*b^2*x^2)/(15*a^2) + 2/5))/x^(5/2)
```

**sympy** [A] time = 4.88, size = 65, normalized size = 1.48

$$-\frac{2 \sqrt{b} \sqrt{\frac{a}{b x}+1}}{5 x^{2}}-\frac{2 b^{\frac{3}{2}} \sqrt{\frac{a}{b x}+1}}{15 a x}+\frac{4 b^{\frac{5}{2}} \sqrt{\frac{a}{b x}+1}}{15 a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(7/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(15*a*
x) + 4*b**(5/2)*sqrt(a/(b*x) + 1)/(15*a**2)
```

# 3.496 $\int \frac{\sqrt{a+bx}}{x^{9/2}} dx$

**Optimal**. Leaf size=68

$$-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(9/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(7*a*x^(7/2)) + (8*b*(a + b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a + b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx = -\frac{2(a+bx)^{3/2}}{7ax^{7/2}} - \frac{(4b)\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx}{7a}$$
$$= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}} + \frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}} + \frac{\left(8b^2\right)\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx}{35a^2}$$
$$= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}} + \frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}} - \frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 0.59

$$-\frac{2(a+bx)^{3/2}\left(15a^2-12abx+8b^2x^2\right)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(9/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(15*a^2 - 12*a*b*x + 8*b^2*x^2))/(105*a^3*x^(7/2))
```

**IntegrateAlgebraic** [A] time = 0.09, size = 51, normalized size = 0.75

$$-\frac{2\sqrt{a+bx}\left(15a^3+3a^2bx-4ab^2x^2+8b^3x^3\right)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^(9/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(15*a^3 + 3*a^2*b*x - 4*a*b^2*x^2 + 8*b^3*x^3))/(105*a^3*
x^(7/2))
```

**fricas** [A] time = 0.76, size = 45, normalized size = 0.66

$$-\frac{2\left(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3\right)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x^
(7/2))
```

**giac** [A] time = 0.97, size = 66, normalized size = 0.97

$$-\frac{2\left(\frac{35\,b^7}{a}+4\left(\frac{2\,(bx+a)b^7}{a^3}-\frac{7\,b^7}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{3}{2}}b}{105\left((bx+a)b-ab\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] -2/105*(35*b^7/a + 4*(2*(b*x + a)*b^7/a^3 - 7*b^7/a^2)*(b*x + a))*(b*x + a)
^(3/2)*b/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

**maple** [A] time = 0.00, size = 35, normalized size = 0.51

$$-\frac{2\,(bx+a)^{\frac{3}{2}}\left(8b^2x^2-12abx+15a^2\right)}{105a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(9/2),x)
```

```
[Out] -2/105*(b*x+a)^(3/2)*(8*b^2*x^2-12*a*b*x+15*a^2)/x^(7/2)/a^3
```

**maxima** [A] time = 1.35, size = 46, normalized size = 0.68

$$-\frac{2\left(\frac{35\,(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{42\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(35*(b*x + a)^(3/2)*b^2/x^(3/2) - 42*(b*x + a)^(5/2)*b/x^(5/2) + 15*
(b*x + a)^(7/2)/x^(7/2))/a^3
```

**mupad** [B] time = 0.26, size = 43, normalized size = 0.63

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{16\,b^3\,x^3}{105\,a^3}-\frac{8\,b^2\,x^2}{105\,a^2}+\frac{2\,b\,x}{35\,a}+\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(9/2),x)
```

```
[Out] -((a + b*x)^(1/2)*((16*b^3*x^3)/(105*a^3) - (8*b^2*x^2)/(105*a^2) + (2*b*x)
/(35*a) + 2/7))/x^(7/2)
```

**sympy [B]** time = 13.77, size = 347, normalized size = 5.10

$$-\frac{30a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{66a^4b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{34a^3b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{6a^2b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{24ab^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{16b^{\frac{19}{2}}x^5\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(9/2),x)
```

```
[Out] -30*a**5*b**(9/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**
4 + 105*a**3*b**6*x**5) - 66*a**4*b**(11/2)*x*sqrt(a/(b*x) + 1)/(105*a**5*b
**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*a**3*b**(13/2)*x**
2*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**
6*x**5) - 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210
*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*a*b**(17/2)*x**4*sqrt(a/(b*x) +
1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 16*b**(
19/2)*x**5*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105
*a**3*b**6*x**5)
```

## 3.497 $\int x^{5/2}\sqrt{a-bx}\,dx$

**Optimal**. Leaf size=127

$$\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

**Rubi** [A] time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}+\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[a - b*x],x]
```

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a - b*x])/(96
*b^2) - (a*x^(5/2)*Sqrt[a - b*x])/(24*b) + (x^(7/2)*Sqrt[a - b*x])/4 + (5*a
^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}\sqrt{a-bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{1}{8}a\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$

$$= -\frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{\left(5a^2\right)\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{48b}$$

$$= -\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2} - \frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{64b^2}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2} - \frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{\left(5a^4\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{128b^3}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2} - \frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{\left(5a^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\cdots}}\right.}{64b^3}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2} - \frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{\left(5a^4\right)\text{Subst}\left(\int \frac{1}{1+b\cdots}\right.}{64b^3}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2} - \frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}$$

**Mathematica [A]** time = 0.14, size = 98, normalized size = 0.77

$$\frac{\sqrt{a-bx}\left(\frac{15a^{7/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}} + \sqrt{b}\sqrt{x}\left(-15a^3-10a^2bx-8ab^2x^2+48b^3x^3\right)\right)}{192b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[a - b*x],x]

[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^3 - 10*a^2*b*x - 8*a*b^2*x^2 + 48*b^
3*x^3) + (15*a^(7/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))
/(192*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 104, normalized size = 0.82

$$\frac{5a^4\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{64b^4}+\frac{\sqrt{a-bx}\,\left(-15a^3\sqrt{x}-10a^2bx^{3/2}-8ab^2x^{5/2}+48b^3x^{7/2}\right)}{192b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^3*Sqrt[x] - 10*a^2*b*x^(3/2) - 8*a*b^2*x^(5/2) + 48*b
^3*x^(7/2)))/(192*b^3) + (5*a^4*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a -
 b*x]])/(64*b^4)
```

**fricas [A]** time = 0.95, size = 164, normalized size = 1.29

$$\left[-\frac{15\,a^4\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\left(48\,b^4x^3-8\,ab^3x^2-10\,a^2b^2x-15\,a^3b\right)\sqrt{-bx+a}\,\sqrt{x}}{384\,b^4},\,-\frac{15\,a^4\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(48\,b^4x^3-8\,ab^3x^2-10\,a^2b^2x-15\,a^3b\right)\sqrt{-bx+a}\,\sqrt{x}}{192\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) - 2*(48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a)*s
qrt(x))/b^4, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))
) - (48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a)*sqr
t(x))/b^4]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 127, normalized size = 1.00

$$-\frac{(-bx+a)^{\frac{3}{2}}\,x^{\frac{5}{2}}}{4b}+\frac{5\sqrt{(-bx+a)\,x}\;a^4\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{128\sqrt{-bx+a}\;b^{\frac{7}{2}}\sqrt{x}}+\frac{5\sqrt{-bx+a}\;a^3\sqrt{x}}{64b^3}-\frac{5\,(-bx+a)^{\frac{3}{2}}\,a\,x^{\frac{3}{2}}}{24b^2}-\frac{5\,(-bx+a)^{\frac{3}{2}}\,a^2\sqrt{x}}{32b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(1/2),x)
```

```
[Out] -1/4/b*x^(5/2)*(-b*x+a)^(3/2)-5/24*a/b^2*x^(3/2)*(-b*x+a)^(3/2)-5/32*a^2/b^
3*x^(1/2)*(-b*x+a)^(3/2)+5/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^3+5/128*a^4/b^(7
/2)*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-
b*x^2+a*x)^(1/2))
```

**maxima [A]** time = 3.01, size = 170, normalized size = 1.34

$$-\frac{5\,a^{4}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{64\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^{4}b^{3}}{\sqrt{x}}-\frac{73\,(-bx+a)^{\frac{3}{2}}a^{4}b^{2}}{x^{\frac{3}{2}}}-\frac{55\,(-bx+a)^{\frac{5}{2}}a^{4}b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+a)^{\frac{7}{2}}a^{4}}{x^{\frac{7}{2}}}}{192\left(b^{7}-\frac{4\,(bx-a)b^{6}}{x}+\frac{6\,(bx-a)^{2}b^{5}}{x^{2}}-\frac{4\,(bx-a)^{3}b^{4}}{x^{3}}+\frac{(bx-a)^{4}b^{3}}{x^{4}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/192*(15*sqrt
(-b*x + a)*a^4*b^3/sqrt(x) - 73*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) - 55*(-b*x
 + a)^(5/2)*a^4*b/x^(5/2) - 15*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^7 - 4*(b*x
- a)*b^6/x + 6*(b*x - a)^2*b^5/x^2 - 4*(b*x - a)^3*b^4/x^3 + (b*x - a)^4*b^
3/x^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{a-b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)*(a - b*x)^(1/2), x)
```

**sympy [A]** time = 11.65, size = 323, normalized size = 2.54

$$\begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b^{3}\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{-1+\frac{bx}{a}}}-\frac{7i\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{4}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}+\frac{ibx^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^{3}\sqrt{1-\frac{bx}{a}}}+\frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1-\frac{bx}{a}}}+\frac{7\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}}+\frac{5a^{4}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}-\frac{bx^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(5/2)*x
**(3/2)/(192*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(5/2)/(96*b*sqrt(-1 + b
*x/a)) - 7*I*sqrt(a)*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh(sqrt(b
)*sqrt(x)/sqrt(a))/(64*b**(7/2)) + I*b*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x/a)
), Abs(b*x/a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 - b*x/a)) + 5*a**(
5/2)*x**(3/2)/(192*b**2*sqrt(1 - b*x/a)) + a**(3/2)*x**(5/2)/(96*b*sqrt(1 -
 b*x/a)) + 7*sqrt(a)*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a**4*asin(sqrt(b)*sq
rt(x)/sqrt(a))/(64*b**(7/2)) - b*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a)), True
))
```

## 3.498 $\int x^{3/2}\sqrt{a-bx}\,dx$

**Optimal**. Leaf size=102

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

**Rubi [A]** time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] -(a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b^2) - (a*x^(3/2)*Sqrt[a - b*x])/(12*b) + (
x^(5/2)*Sqrt[a - b*x])/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8
*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{3/2}\sqrt{a-bx}\,dx=\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{1}{6}a\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx$$
$$=-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^2\int\frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{8b}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\int\frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{16b^2}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\,\text{Subst}\left(\int\frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{8b^2}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\,\text{Subst}\left(\int\frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^2}$$
$$=-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}$$

**Mathematica [A]** time = 0.12, size = 87, normalized size = 0.85

$$\frac{\sqrt{a-bx}\left(\frac{3a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}+\sqrt{b}\sqrt{x}\left(-3a^2-2abx+8b^2x^2\right)\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^2 - 2*a*b*x + 8*b^2*x^2) + (3*a^(5/2)
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(24*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 91, normalized size = 0.89

$$\frac{a^3\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{8b^3}+\frac{\sqrt{a-bx}\left(-3a^2\sqrt{x}-2abx^{3/2}+8b^2x^{5/2}\right)}{24b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-3*a^2*Sqrt[x] - 2*a*b*x^(3/2) + 8*b^2*x^(5/2)))/(24*b^2) +
 (a^3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*b^3)
```

**fricas [A]** time = 1.02, size = 142, normalized size = 1.39

$$\left[-\frac{3\,a^{3}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\left(8\,b^{3}x^{2}-2\,ab^{2}x-3\,a^{2}b\right)\sqrt{-bx+a}\,\sqrt{x}}{48\,b^{3}},\,-\frac{3\,a^{3}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(8\,b^{3}x^{2}-2\,ab^{2}x-3\,a^{2}b\right)\sqrt{-bx+a}\,\sqrt{x}}{24\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
- 2*(8*b^3*x^2 - 2*a*b^2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/24*(3
*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (8*b^3*x^2 - 2*a*b^
2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 108, normalized size = 1.06

$$\frac{\sqrt{(-bx+a)\,x}\;a^{3}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{16\sqrt{-bx+a}\;b^{\frac{5}{2}}\sqrt{x}}+\frac{\sqrt{-bx+a}\;a^{2}\sqrt{x}}{8b^{2}}-\frac{(-bx+a)^{\frac{3}{2}}\,x^{\frac{3}{2}}}{3b}-\frac{(-bx+a)^{\frac{3}{2}}\,a\sqrt{x}}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(1/2),x)
```

```
[Out] -1/3/b*x^(3/2)*(-b*x+a)^(3/2)-1/4*a/b^2*x^(1/2)*(-b*x+a)^(3/2)+1/8*a^2*x^(1
/2)*(-b*x+a)^(1/2)/b^2+1/16*a^3/b^(5/2)*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x
^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

**maxima** [A] time = 2.99, size = 135, normalized size = 1.32

$$-\frac{a^{3}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{8\,b^{\frac{5}{2}}}+\frac{\frac{3\,\sqrt{-bx+a}\,a^{3}b^{2}}{\sqrt{x}}-\frac{8\,(-bx+a)^{\frac{3}{2}}a^{3}b}{x^{\frac{3}{2}}}-\frac{3\,(-bx+a)^{\frac{5}{2}}a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{5}-\frac{3\,(bx-a)b^{4}}{x}+\frac{3\,(bx-a)^{2}b^{3}}{x^{2}}-\frac{(bx-a)^{3}b^{2}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/24*(3*sqrt(-b
*x + a)*a^3*b^2/sqrt(x) - 8*(-b*x + a)^(3/2)*a^3*b/x^(3/2) - 3*(-b*x + a)^(
5/2)*a^3/x^(5/2))/(b^5 - 3*(b*x - a)*b^4/x + 3*(b*x - a)^2*b^3/x^2 - (b*x -
 a)^3*b^2/x^3)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\sqrt{a-b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)*(a - b*x)^(1/2), x)
```

**sympy** [A] time = 6.33, size = 260, normalized size = 2.55

$$\begin{cases} \frac{ia^{\frac{5}{2}}\sqrt{x}}{8b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{-1+\frac{bx}{a}}}-\frac{5i\sqrt{a}\,x^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{3}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{ibx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1-\frac{bx}{a}}}+\frac{5\sqrt{a}\,x^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}}+\frac{a^{3}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{bx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*a**(5/2)*sqrt(x)/(8*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(3/
2)/(24*b*sqrt(-1 + b*x/a)) - 5*I*sqrt(a)*x**(5/2)/(12*sqrt(-1 + b*x/a)) - I
*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) + I*b*x**(7/2)/(3*sqrt(a)
*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(5/2)*sqrt(x)/(8*b**2*sqrt(1 - b*
x/a)) + a**(3/2)*x**(3/2)/(24*b*sqrt(1 - b*x/a)) + 5*sqrt(a)*x**(5/2)/(12*s
qrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) - b*x**(7
/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

## 3.499 $\int \sqrt{x}\sqrt{a-bx}\,dx$

**Optimal**. Leaf size=77

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a-bx}-\frac{a\sqrt{x}\sqrt{a-bx}}{4b}$$

**Rubi** [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a-bx}-\frac{a\sqrt{x}\sqrt{a-bx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a - b*x],x]
```

```
[Out] -(a*Sqrt[x]*Sqrt[a - b*x])/(4*b) + (x^(3/2)*Sqrt[a - b*x])/2 + (a^2*ArcTan[
(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\sqrt{a-bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{8b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{4b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 75, normalized size = 0.97

$$\frac{\sqrt{a-bx}\left(\frac{a^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}+\sqrt{b}\sqrt{x}\,(2bx-a)\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-a + 2*b*x) + (a^(3/2)*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(4*b^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 78, normalized size = 1.01

$$\frac{a^2\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^2}+\frac{\sqrt{a-bx}\left(2bx^{3/2}-a\sqrt{x}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-(a*Sqrt[x]) + 2*b*x^(3/2)))/(4*b) + (a^2*Sqrt[-b]*Log[-(Sq
rt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(4*b^2)
```

**fricas [A]** time = 0.90, size = 118, normalized size = 1.53

$$\left[-\frac{a^{2}\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\left(2\,b^{2}x-ab\right)\sqrt{-bx+a}\,\sqrt{x}}{8\,b^{2}},-\frac{a^{2}\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(2\,b^{2}x-ab\right)\sqrt{-bx+a}\,\sqrt{x}}{4\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2
*(2*b^2*x - a*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/4*(a^2*sqrt(b)*arctan(sqrt
(-b*x + a)/(sqrt(b)*sqrt(x))) - (2*b^2*x - a*b)*sqrt(-b*x + a)*sqrt(x))/b^2
]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 86, normalized size = 1.12

$$\frac{\sqrt{-bx+a}\;x^{\frac{3}{2}}}{2}+\frac{\sqrt{\left(-bx+a\right)x}\;a^{2}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{8\sqrt{-bx+a}\;b^{\frac{3}{2}}\sqrt{x}}-\frac{\sqrt{-bx+a}\;a\sqrt{x}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-b*x+a)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(-b*x+a)^(1/2)-1/4*a*x^(1/2)*(-b*x+a)^(1/2)/b+1/8*a^2/b^(3/2)*(
(-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1
/2)*b^(1/2))
```

**maxima [A]** time = 2.97, size = 95, normalized size = 1.23

$$-\frac{a^{2}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{3}{2}}}+\frac{\frac{\sqrt{-bx+a}\,a^{2}b}{\sqrt{x}}-\frac{\left(-bx+a\right)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{3}-\frac{2\,(bx-a)b^{2}}{x}+\frac{(bx-a)^{2}b}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/4*(sqrt(-b*x
+ a)*a^2*b/sqrt(x) - (-b*x + a)^(3/2)*a^2/x^(3/2))/(b^3 - 2*(b*x - a)*b^2/x
 + (b*x - a)^2*b/x^2)
```

**mupad [B]** time = 0.08, size = 58, normalized size = 0.75

$$\sqrt{x}\,\left(\frac{x}{2}-\frac{a}{4\,b}\right)\,\sqrt{a-b\,x}-\frac{a^2\,\ln\left(a-2\,b\,x+2\,\sqrt{-b}\,\sqrt{x}\,\sqrt{a-b\,x}\right)}{8\,{\left(-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a - b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - a/(4*b))*(a - b*x)^(1/2) - (a^2*log(a - 2*b*x + 2*(-b)^(1/2)
*x^(1/2)*(a - b*x)^(1/2)))/(8*(-b)^(3/2))
```

**sympy [A]** time = 3.60, size = 207, normalized size = 2.69

$$\begin{cases} \frac{ia^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{-1+\frac{bx}{a}}}-\frac{3i\sqrt{a}\,x^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}}-\frac{ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+\frac{ibx^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{a^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{1-\frac{bx}{a}}}+\frac{3\sqrt{a}\,x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}}+\frac{a^2\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}-\frac{bx^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*a**(3/2)*sqrt(x)/(4*b*sqrt(-1 + b*x/a)) - 3*I*sqrt(a)*x**(3/2)
/(4*sqrt(-1 + b*x/a)) - I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2))
+ I*b*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(3/2)*sq
rt(x)/(4*b*sqrt(1 - b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 - b*x/a)) + a**2
*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) - b*x**(5/2)/(2*sqrt(a)*sqrt(1
- b*x/a)), True))
```

## 3.500 $\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=46

$$\sqrt{x}\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

**Rubi** [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\sqrt{x}\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx &= \sqrt{x}\sqrt{a-bx}+\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx\\ &= \sqrt{x}\sqrt{a-bx}+a\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}\sqrt{a-bx}+a\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\ &= \sqrt{x}\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 65, normalized size = 1.41

$$\frac{\frac{a^{3/2}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}+\sqrt{x}(a-bx)}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(a - b*x) + (a^(3/2)*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqrt[x])/Sq
rt[a]])/Sqrt[b])/Sqrt[a - b*x]
```

**IntegrateAlgebraic [A]** time = 0.07, size = 55, normalized size = 1.20

$$\sqrt{x}\sqrt{a-bx}+\frac{a\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a - b*x] + (a*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]
])/b
```

**fricas [A]** time = 0.98, size = 94, normalized size = 2.04

$$\left[-\frac{a\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}\,+a\right)-2\,\sqrt{-bx+a}\,b\sqrt{x}}{2\,b},-\frac{a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+a}\,b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*s
qrt(-b*x + a)*b*sqrt(x))/b, -(a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt
(x))) - sqrt(-b*x + a)*b*sqrt(x))/b]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.00, size = 66, normalized size = 1.43

$$\frac{\sqrt{(-bx+a)\,x}\;a\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{2\sqrt{-bx+a}\;\sqrt{b}\;\sqrt{x}}+\sqrt{-bx+a}\;\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(-b*x+a)^(1/2)+1/2*a*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1
/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.90, size = 52, normalized size = 1.13

$$-\frac{a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}+\frac{\sqrt{-bx+a}\,a}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + sqrt(-b*x + a)*a/((b
- (b*x - a)/x)*sqrt(x))
```

**mupad [B]** time = 0.59, size = 43, normalized size = 0.93

$$\sqrt{x}\ \sqrt{a-b\,x}+\frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{b}\ \sqrt{x}}{\sqrt{a-b\,x}-\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a - b*x)^(1/2) + (2*a*atan((b^(1/2)*x^(1/2))/((a - b*x)^(1/2) - a^
(1/2))))/b^(1/2)
```

**sympy [A]** time = 1.96, size = 119, normalized size = 2.59

$$\begin{cases} -\frac{i\sqrt{a}\ \sqrt{x}}{\sqrt{-1+\frac{bx}{a}}}-\frac{ia\,\mathrm{acosh}\left(\frac{\sqrt{b}\ \sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{ibx^{\frac{3}{2}}}{\sqrt{a}\ \sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ \sqrt{a}\ \sqrt{x}\ \sqrt{1-\frac{bx}{a}}+\frac{a\,\mathrm{asin}\left(\frac{\sqrt{b}\ \sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(a)*sqrt(x)/sqrt(-1 + b*x/a) - I*a*acosh(sqrt(b)*sqrt(x)/
sqrt(a))/sqrt(b) + I*b*x**(3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1)
, (sqrt(a)*sqrt(x)*sqrt(1 - b*x/a) + a*asin(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b
), True))
```

## 3.501 $\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx$

**Optimal**. Leaf size=47

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}}-2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

**Rubi** [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 63, 217, 203}

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}}-2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - b\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx \\
&= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - (2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - (2b)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right) \\
&= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 69, normalized size = 1.47

$$-\frac{2\left(\sqrt{a}\,\sqrt{b}\,\sqrt{x}\,\sqrt{1-\frac{bx}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)+a-bx\right)}{\sqrt{x}\,\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(3/2),x]
```

```
[Out] (-2*(a - b*x + Sqrt[a]*Sqrt[b]*Sqrt[x]*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[a]]))/(Sqrt[x]*Sqrt[a - b*x])
```

**IntegrateAlgebraic [A]** time = 0.09, size = 53, normalized size = 1.13

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a -
b*x]]
```

**fricas** [A] time = 0.75, size = 91, normalized size = 1.94

$$\left[\frac{\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\sqrt{-bx+a}\,\sqrt{x}}{x},\frac{2\left(\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+a}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*sqrt(-
b*x + a)*sqrt(x))/x, 2*(sqrt(b)*x*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))
- sqrt(-b*x + a)*sqrt(x))/x]
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx+a}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(3/2),x)
```

```
[Out] int((-b*x+a)^(1/2)/x^(3/2),x)
```

**maxima** [A] time = 2.93, size = 35, normalized size = 0.74

$$2\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\,\sqrt{-bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a)/sqrt(
x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a-b\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((a - b*x)^(1/2)/x^(3/2), x)
```

**sympy [A]** time = 1.70, size = 148, normalized size = 3.15

$$\begin{cases} \frac{2i\sqrt{a}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}} + 2i\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - \frac{2ib\sqrt{x}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right| > 1 \\ -\frac{2\sqrt{a}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}} - 2\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + \frac{2b\sqrt{x}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*sqrt(a)/(sqrt(x)*sqrt(-1 + b*x/a)) + 2*I*sqrt(b)*acosh(sqrt(
b)*sqrt(x)/sqrt(a)) - 2*I*b*sqrt(x)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a)
> 1), (-2*sqrt(a)/(sqrt(x)*sqrt(1 - b*x/a)) - 2*sqrt(b)*asin(sqrt(b)*sqrt(x
)/sqrt(a)) + 2*b*sqrt(x)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

## 3.502 $\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx$

**Optimal**. Leaf size=22

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(5/2),x]
[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx = -\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

**Mathematica** [A] time = 0.01, size = 22, normalized size = 1.00

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(5/2),x]
[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 22, normalized size = 1.00

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a - b*x]/x^(5/2),x]
[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

**fricas [A]** time = 1.02, size = 23, normalized size = 1.05

$$\frac{2\,(bx-a)\sqrt{-bx+a}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] 2/3*(b*x - a)*sqrt(-b*x + a)/(a*x^(3/2))
```

**giac [B]** time = 1.40, size = 42, normalized size = 1.91

$$\frac{2\,(bx-a)\sqrt{-bx+a}\,b^4}{3\,((bx-a)b+ab)^{\frac{3}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="giac")
[Out] 2/3*(b*x - a)*sqrt(-b*x + a)*b^4/(((b*x - a)*b + a*b)^(3/2)*a*abs(b))
```

**maple [A]** time = 0.00, size = 17, normalized size = 0.77

$$-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(5/2),x)
[Out] -2/3*(-b*x+a)^(3/2)/a/x^(3/2)
```

**maxima** [A] time = 1.31, size = 16, normalized size = 0.73

$$-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(-b*x + a)^(3/2)/(a*x^(3/2))
```

**mupad** [B] time = 0.24, size = 21, normalized size = 0.95

$$\frac{\left(\frac{2\,b\,x}{3\,a}-\frac{2}{3}\right)\,\sqrt{a-b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(5/2),x)
```

```
[Out] (((2*b*x)/(3*a) - 2/3)*(a - b*x)^(1/2))/x^(3/2)
```

**sympy** [B] time = 1.55, size = 88, normalized size = 4.00

$$\begin{cases} -\frac{2\sqrt{b}\,\sqrt{\frac{a}{bx}-1}}{3x}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3a} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{2i\sqrt{b}\,\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{2ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 2*b**(3/2)*sqrt(a/(b*x) - 1
)/(3*a), Abs(a/(b*x)) > 1), (-2*I*sqrt(b)*sqrt(-a/(b*x) + 1)/(3*x) + 2*I*b*
*(3/2)*sqrt(-a/(b*x) + 1)/(3*a), True))
```

## 3.503 $\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx$

**Optimal**. Leaf size=46

$$-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(7/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(5*a*x^(5/2)) - (4*b*(a - b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{5ax^{5/2}}+\frac{(2b)\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx}{5a}\\ &= -\frac{2(a-bx)^{3/2}}{5ax^{5/2}}-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 0.65

$$-\frac{2(a-bx)^{3/2}(3a+2bx)}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(7/2),x]
[Out] (-2*(a - b*x)^(3/2)*(3*a + 2*b*x))/(15*a^2*x^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.10, size = 40, normalized size = 0.87

$$\frac{2\sqrt{a-bx}\left(-3a^2+abx+2b^2x^2\right)}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a - b*x]/x^(7/2),x]
[Out] (2*Sqrt[a - b*x]*(-3*a^2 + a*b*x + 2*b^2*x^2))/(15*a^2*x^(5/2))
```

**fricas** [A] time = 0.86, size = 34, normalized size = 0.74

$$\frac{2\left(2\,b^2x^2+abx-3\,a^2\right)\sqrt{-bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
[Out] 2/15*(2*b^2*x^2 + a*b*x - 3*a^2)*sqrt(-b*x + a)/(a^2*x^(5/2))
```

**giac** [A] time = 1.38, size = 61, normalized size = 1.33

$$\frac{2\left(\frac{2\,(bx-a)b^5}{a^2}+\frac{5\,b^5}{a}\right)(bx-a)\sqrt{-bx+a}\,b}{15\,((bx-a)b+ab)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="giac")
[Out] 2/15*(2*(b*x - a)*b^5/a^2 + 5*b^5/a)*(b*x - a)*sqrt(-b*x + a)*b/(((b*x - a)
*b + a*b)^(5/2)*abs(b))
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.54

$$-\frac{2\left(-bx+a\right)^{\frac{3}{2}}\left(2bx+3a\right)}{15a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(7/2),x)
[Out] -2/15*(-b*x+a)^(3/2)*(2*b*x+3*a)/x^(5/2)/a^2
```

**maxima** [A] time = 1.34, size = 33, normalized size = 0.72

$$-\frac{2\left(\frac{5\left(-bx+a\right)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{3\left(-bx+a\right)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
[Out] -2/15*(5*(-b*x + a)^(3/2)*b/x^(3/2) + 3*(-b*x + a)^(5/2)/x^(5/2))/a^2
```

**mupad** [B] time = 0.25, size = 32, normalized size = 0.70

$$\frac{\sqrt{a-b\,x}\,\left(\frac{4\,b^2\,x^2}{15\,a^2}+\frac{2\,b\,x}{15\,a}-\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(7/2),x)
[Out] ((a - b*x)^(1/2)*((4*b^2*x^2)/(15*a^2) + (2*b*x)/(15*a) - 2/5))/x^(5/2)
```

**sympy** [A] time = 5.01, size = 241, normalized size = 5.24

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{5x^{2}}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{15ax}+\frac{4b^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}{15a^{2}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ \frac{6ia^{3}b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{x\left(-15a^{3}bx+15a^{2}b^{2}x^{2}\right)}-\frac{8ia^{2}b^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}{-15a^{3}bx+15a^{2}b^{2}x^{2}}-\frac{2iab^{\frac{7}{2}}x\sqrt{-\frac{a}{bx}+1}}{-15a^{3}bx+15a^{2}b^{2}x^{2}}+\frac{4ib^{\frac{9}{2}}x^{2}\sqrt{-\frac{a}{bx}+1}}{-15a^{3}bx+15a^{2}b^{2}x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(7/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(5*x**2) + 2*b**(3/2)*sqrt(a/(b*x)
- 1)/(15*a*x) + 4*b**(5/2)*sqrt(a/(b*x) - 1)/(15*a**2), Abs(a/(b*x)) > 1),
(6*I*a**3*b**(3/2)*sqrt(-a/(b*x) + 1)/(x*(-15*a**3*b*x + 15*a**2*b**2*x**2)
) - 8*I*a**2*b**(5/2)*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2)
 - 2*I*a*b**(7/2)*x*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2) +
 4*I*b**(9/2)*x**2*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2), T
rue))
```

## 3.504 $\int \frac{\sqrt{a-bx}}{x^{9/2}} \, dx$

**Optimal**. Leaf size=71

$$-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(7*a*x^(7/2)) - (8*b*(a - b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a - b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx = -\frac{2(a-bx)^{3/2}}{7ax^{7/2}} + \frac{(4b)\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx}{7a}$$
$$= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}} - \frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}} + \frac{\left(8b^2\right)\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx}{35a^2}$$
$$= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}} - \frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}} - \frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}$$

**Mathematica** [A] time = 0.01, size = 41, normalized size = 0.58

$$-\frac{2(a-bx)^{3/2}\left(15a^2+12abx+8b^2x^2\right)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2)*(15*a^2 + 12*a*b*x + 8*b^2*x^2))/(105*a^3*x^(7/2))
```

**IntegrateAlgebraic** [A] time = 0.11, size = 52, normalized size = 0.73

$$\frac{2\sqrt{a-bx}\left(-15a^3+3a^2bx+4ab^2x^2+8b^3x^3\right)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (2*Sqrt[a - b*x]*(-15*a^3 + 3*a^2*b*x + 4*a*b^2*x^2 + 8*b^3*x^3))/(105*a^3*
x^(7/2))
```

**fricas** [A] time = 0.96, size = 46, normalized size = 0.65

$$\frac{2\left(8\,b^3x^3+4\,ab^2x^2+3\,a^2bx-15\,a^3\right)\sqrt{-bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(8*b^3*x^3 + 4*a*b^2*x^2 + 3*a^2*b*x - 15*a^3)*sqrt(-b*x + a)/(a^3*x^
(7/2))
```

**giac [A]** time = 1.34, size = 79, normalized size = 1.11

$$\frac{2\left(\frac{35\,b^7}{a}+4\left(\frac{2\,(bx-a)b^7}{a^3}+\frac{7\,b^7}{a^2}\right)(bx-a)\right)(bx-a)\sqrt{-bx+a}\,b}{105\,((bx-a)b+ab)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 2/105*(35*b^7/a + 4*(2*(b*x - a)*b^7/a^3 + 7*b^7/a^2)*(b*x - a))*(b*x - a)*
sqrt(-b*x + a)*b/(((b*x - a)*b + a*b)^(7/2)*abs(b))
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.51

$$-\frac{2\,(-bx+a)^{\frac{3}{2}}\left(8b^2x^2+12abx+15a^2\right)}{105a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(9/2),x)
```

```
[Out] -2/105*(-b*x+a)^(3/2)*(8*b^2*x^2+12*a*b*x+15*a^2)/x^(7/2)/a^3
```

**maxima [A]** time = 1.35, size = 49, normalized size = 0.69

$$-\frac{2\left(\frac{35\,(-bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{42\,(-bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(-bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(35*(-b*x + a)^(3/2)*b^2/x^(3/2) + 42*(-b*x + a)^(5/2)*b/x^(5/2) + 1
5*(-b*x + a)^(7/2)/x^(7/2))/a^3
```

**mupad [B]** time = 0.27, size = 43, normalized size = 0.61

$$\frac{\sqrt{a-b\,x}\,\left(\frac{8\,b^2\,x^2}{105\,a^2}+\frac{16\,b^3\,x^3}{105\,a^3}+\frac{2\,b\,x}{35\,a}-\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(9/2),x)
```

[Out] ((a - b*x)^(1/2)*((8*b^2*x^2)/(105*a^2) + (16*b^3*x^3)/(105*a^3) + (2*b*x)/(35*a) - 2/7))/x^(7/2)

**sympy [B]** time = 26.81, size = 707, normalized size = 9.96

$$\begin{cases} \frac{30a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{66a^4b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} + \frac{34a^3b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{6a^2b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} + \frac{24ab^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{16b^{\frac{19}{2}}x^5\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{30ia^5b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{66ia^4b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} + \frac{34ia^3b^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{6ia^2b^{\frac{15}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} + \frac{24iab^{\frac{17}{2}}x^4\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{16ib^{\frac{19}{2}}x^5\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+a)**(1/2)/x**(9/2),x)

[Out] Piecewise((30*a**5*b**(9/2)*sqrt(a/(b*x) - 1)/(-105*a**5*b**4*x**3 + 210*a*
*4*b**5*x**4 - 105*a**3*b**6*x**5) - 66*a**4*b**(11/2)*x*sqrt(a/(b*x) - 1)/
(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*x**5) + 34*a**3*b
**(13/2)*x**2*sqrt(a/(b*x) - 1)/(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 -
105*a**3*b**6*x**5) - 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) - 1)/(-105*a**5*b
**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*x**5) + 24*a*b**(17/2)*x**4*s
qrt(a/(b*x) - 1)/(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*
x**5) - 16*b**(19/2)*x**5*sqrt(a/(b*x) - 1)/(-105*a**5*b**4*x**3 + 210*a**4
*b**5*x**4 - 105*a**3*b**6*x**5), Abs(a/(b*x)) > 1), (30*I*a**5*b**(9/2)*sq
rt(-a/(b*x) + 1)/(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*
x**5) - 66*I*a**4*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(-105*a**5*b**4*x**3 + 210
*a**4*b**5*x**4 - 105*a**3*b**6*x**5) + 34*I*a**3*b**(13/2)*x**2*sqrt(-a/(b
*x) + 1)/(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*x**5) -
6*I*a**2*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(-105*a**5*b**4*x**3 + 210*a**4*
b**5*x**4 - 105*a**3*b**6*x**5) + 24*I*a*b**(17/2)*x**4*sqrt(-a/(b*x) + 1)/
(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*x**5) - 16*I*b**(
19/2)*x**5*sqrt(-a/(b*x) + 1)/(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 1
05*a**3*b**6*x**5), True))

## 3.505 $\int x^{5/2}\sqrt{2+bx}\,dx$

**Optimal**. Leaf size=108

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{24b^2}+\frac{1}{4}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{12b}$$

**Rubi** [A] time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{5x^{3/2}\sqrt{bx+2}}{24b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{4}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(24*b^2) + (x
^(5/2)*Sqrt[2 + b*x])/(12*b) + (x^(7/2)*Sqrt[2 + b*x])/4 - (5*ArcSinh[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}\sqrt{2+bx}\,dx &= \frac{1}{4}x^{7/2}\sqrt{2+bx}+\frac{1}{4}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx \\
&= \frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{12b} \\
&= -\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b^2} \\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{8b^3} \\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\cdot\right.}{4b^3} \\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.05, size = 70, normalized size = 0.65

$$
\frac{\sqrt{x}\sqrt{bx+2}\left(6b^3x^3+2b^2x^2-5bx+15\right)}{24b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 6*b^3*x^3))/(24*b^3) - (5*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.10, size = 85, normalized size = 0.79

$$
\frac{5\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{4b^{7/2}}+\frac{\sqrt{bx+2}\left(6b^3x^{7/2}+2b^2x^{5/2}-5bx^{3/2}+15\sqrt{x}\right)}{24b^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[2 + b*x]*(15*Sqrt[x] - 5*b*x^(3/2) + 2*b^2*x^(5/2) + 6*b^3*x^(7/2)))/
(24*b^3) + (5*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(7/2))
```

**fricas [A]** time = 0.92, size = 140, normalized size = 1.30

$$\left[\frac{\left(6\,b^{4}x^{3}+2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{24\,b^{4}},\ \frac{\left(6\,b^{4}x^{3}+2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{24\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15*
sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/24*((6*b^4*x^3
 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(s
qrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^4]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [59.8656459
874,25.8388736797]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
```

```
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [33.9285577983,15.451549686]Warning, choosing root of [1,0,%%%{
-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,
2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%
{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3
]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{
-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]
%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%
%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,
[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%
}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-
20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,
4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%
}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [54.7579903365,81.9516051291]Warning, cho
osing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-
8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%
%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,
[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%
%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4
,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3
]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+
%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3
,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%
}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{
24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,
1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+
%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [18.4052062202,51.
6443148847]1/b*(2*b*abs(b)/b^2*(2*(((90*b^11/1440/b^14*sqrt(b*x+2)*sqrt(b*x
+2)-750*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)+2445*b^11/1440/b^14)*sqrt(b
*x+2)*sqrt(b*x+2)-4185*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-35/8
/b^2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+4*abs(b)/b^2
*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sqrt(b*x+2)*sq
rt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/b/sqrt(b)*ln
(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))
```

**maple [A]** time = 0.01, size = 108, normalized size = 1.00

$$\frac{(bx+2)^{\frac{3}{2}} x^{\frac{5}{2}}}{4b}-\frac{5\,(bx+2)^{\frac{3}{2}} x^{\frac{3}{2}}}{12b^{2}}+\frac{5\,(bx+2)^{\frac{3}{2}} \sqrt{x}}{8b^{3}}-\frac{5\sqrt{bx+2}\,\sqrt{x}}{8b^{3}}-\frac{5\sqrt{(bx+2)\,x}\;\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{8\sqrt{bx+2}\;b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/4/b*x^(5/2)*(b*x+2)^(3/2)-5/12/b^2*x^(3/2)*(b*x+2)^(3/2)+5/8/b^3*x^(1/2)*
(b*x+2)^(3/2)-5/8*x^(1/2)*(b*x+2)^(1/2)/b^3-5/8/b^(7/2)*(x*(b*x+2))^(1/2)/(
b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [B]** time = 3.03, size = 163, normalized size = 1.51

$$\frac{\frac{15\sqrt{bx+2}\,b^3}{\sqrt{x}}+\frac{73\,(bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{55\,(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^7-\frac{4\,(bx+2)b^6}{x}+\frac{6\,(bx+2)^2b^5}{x^2}-\frac{4\,(bx+2)^3b^4}{x^3}+\frac{(bx+2)^4b^3}{x^4}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(b*x + 2)*b^3/sqrt(x) + 73*(b*x + 2)^(3/2)*b^2/x^(3/2) - 55*(b
*x + 2)^(5/2)*b/x^(5/2) + 15*(b*x + 2)^(7/2)/x^(7/2))/(b^7 - 4*(b*x + 2)*b^
6/x + 6*(b*x + 2)^2*b^5/x^2 - 4*(b*x + 2)^3*b^4/x^3 + (b*x + 2)^4*b^3/x^4)
+ 5/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(
x)))/b^(7/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{b\,x+2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(5/2)*(b*x + 2)^(1/2), x)
```

**sympy [A]** time = 10.12, size = 117, normalized size = 1.08

$$\frac{bx^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{7x^{\frac{7}{2}}}{12\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{24b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{24b^2\sqrt{bx+2}}+\frac{5\sqrt{x}}{4b^3\sqrt{bx+2}}-\frac{5\,\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(9/2)/(4*sqrt(b*x + 2)) + 7*x**(7/2)/(12*sqrt(b*x + 2)) - x**(5/2)/(24
*b*sqrt(b*x + 2)) + 5*x**(3/2)/(24*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(4*b**3*
sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2))
```

## 3.506 $\int x^{3/2}\sqrt{2+bx}\,dx$

**Optimal**. Leaf size=84

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{1}{3}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{6b}$$

**Rubi [A]** time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{1}{3}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[2 + b*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 + b*x])/(2*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(6*b) + (x^(5/2)
*Sqrt[2 + b*x])/3 + ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{3/2}\sqrt{2+bx}\,dx = \frac{1}{3}x^{5/2}\sqrt{2+bx} + \frac{1}{3}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$$
$$= \frac{x^{3/2}\sqrt{2+bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2+bx} - \frac{\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b}$$
$$= -\frac{\sqrt{x}\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2+bx} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b^2}$$
$$= -\frac{\sqrt{x}\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2+bx} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{\sqrt{x}\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2+bx} + \frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

**Mathematica [A]** time = 0.03, size = 58, normalized size = 0.69

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}} + \frac{\sqrt{x}\sqrt{bx+2}\left(2b^2x^2+bx-3\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 2*b^2*x^2))/(6*b^2) + ArcSinh[(Sqrt[b]*S
qrt[x])/Sqrt[2]]/b^(5/2)
```

**IntegrateAlgebraic [A]** time = 0.09, size = 72, normalized size = 0.86

$$\frac{\sqrt{bx+2}\left(2b^2x^{5/2}+bx^{3/2}-3\sqrt{x}\right)}{6b^2} - \frac{\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[2 + b*x]*(-3*Sqrt[x] + b*x^(3/2) + 2*b^2*x^(5/2)))/(6*b^2) - Log[-(Sq
rt[b]*Sqrt[x]) + Sqrt[2 + b*x]]/b^(5/2)
```

**fricas [A]** time = 0.92, size = 121, normalized size = 1.44

$$\left[\frac{\left(2\,b^3x^2+b^2x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b^3}, \frac{\left(2\,b^3x^2+b^2x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x +
 sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/6*((2*b^3*x^2 + b^2*x - 3*b)*sq
rt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x)))
)/b^3]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [59.8656459
874,25.8388736797]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [33.9285577983,15.451549686]Warning, choosing root of [1,0,%%%{
-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,
```

```
2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%
{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3
]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{
-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]
%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%
%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,
[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%
}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-
20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,
4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%
}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [54.7579903365,81.9516051291]Warning, cho
osing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-
8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%
%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,
[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%
%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4
,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3
]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+
%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3
,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%
}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{
24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,
1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+
%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [18.4052062202,51.
6443148847]1/b*(2*b*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-
78*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b
*(b*x+2)-2*b)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))
))+4*abs(b)/b^2/b*(2*(1/8*sqrt(b*x+2)*sqrt(b*x+2)-5/8)*sqrt(b*x+2)*sqrt(b*(
b*x+2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
)
```

**maple [A]** time = 0.00, size = 93, normalized size = 1.11

$$\frac{(bx+2)^{\frac{3}{2}} x^{\frac{3}{2}}}{3b}-\frac{(bx+2)^{\frac{3}{2}} \sqrt{x}}{2b^2}+\frac{\sqrt{bx+2}\,\sqrt{x}}{2b^2}+\frac{\sqrt{(bx+2)\,x}\;\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{2\sqrt{bx+2}\;b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/3/b*x^(3/2)*(b*x+2)^(3/2)-1/2/b^2*x^(1/2)*(b*x+2)^(3/2)+1/2*x^(1/2)*(b*x+
2)^(1/2)/b^2+1/2/b^(5/2)*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)
/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [B]** time = 3.02, size = 134, normalized size = 1.60

$$-\frac{\frac{3\sqrt{bx+2}\,b^2}{\sqrt{x}}+\frac{8\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^5-\frac{3\,(bx+2)b^4}{x}+\frac{3\,(bx+2)^2b^3}{x^2}-\frac{(bx+2)^3b^2}{x^3}\right)}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(b*x + 2)*b^2/sqrt(x) + 8*(b*x + 2)^(3/2)*b/x^(3/2) - 3*(b*x +
2)^(5/2)/x^(5/2))/(b^5 - 3*(b*x + 2)*b^4/x + 3*(b*x + 2)^2*b^3/x^2 - (b*x +
 2)^3*b^2/x^3) - 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt
(b*x + 2)/sqrt(x)))/b^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\sqrt{b\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(3/2)*(b*x + 2)^(1/2), x)
```

**sympy [A]** time = 5.22, size = 90, normalized size = 1.07

$$\frac{bx^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{5x^{\frac{5}{2}}}{6\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{6b\sqrt{bx+2}}-\frac{\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(7/2)/(3*sqrt(b*x + 2)) + 5*x**(5/2)/(6*sqrt(b*x + 2)) - x**(3/2)/(6*b
*sqrt(b*x + 2)) - sqrt(x)/(b**2*sqrt(b*x + 2)) + asinh(sqrt(2)*sqrt(b)*sqrt
(x)/2)/b**(5/2)
```

## 3.507 $\int \sqrt{x}\sqrt{2+bx}\,dx$

**Optimal**. Leaf size=64

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\sqrt{bx+2}}{2b}$$

**Rubi [A]** time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 - ArcSinh[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{x}\sqrt{2+bx}\,dx = \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} - \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b}$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} - \frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

**Mathematica** [A] time = 0.03, size = 51, normalized size = 0.80

$$\frac{\sqrt{x}(bx+1)\sqrt{bx+2}}{2b} - \frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*(1 + b*x)*Sqrt[2 + b*x])/(2*b) - ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]
]/b^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.06, size = 59, normalized size = 0.92

$$\frac{\log\left(\sqrt{bx+2} - \sqrt{b}\sqrt{x}\right)}{b^{3/2}} + \frac{\sqrt{bx+2}\left(bx^{3/2} + \sqrt{x}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[2 + b*x]*(Sqrt[x] + b*x^(3/2)))/(2*b) + Log[-(Sqrt[b]*Sqrt[x]) + Sqrt
[2 + b*x]]/b^(3/2)
```

**fricas** [A] time = 1.14, size = 101, normalized size = 1.58

$$\left[\frac{\left(b^2x+b\right)\sqrt{bx+2}\sqrt{x} + \sqrt{b}\log\left(bx - \sqrt{bx+2}\sqrt{b}\sqrt{x} + 1\right)}{2b^2}, \frac{\left(b^2x+b\right)\sqrt{bx+2}\sqrt{x} + 2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*s
qrt(b)*sqrt(x) + 1))/b^2, 1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + 2*sqrt(-
b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^2]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [59.8656459
874,25.8388736797]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [33.9285577983,15.451549686]Warning, choosing root of [1,0,%%%{
-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,
2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%
{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3
]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{
```

```
-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]
%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%
%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,
[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%
}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-
20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,
4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%
}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [54.7579903365,81.9516051291]Warning, cho
osing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-
8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%
%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,
[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%
%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4
,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3
]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+
%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3
,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%
}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{
24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,
1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+
%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [18.4052062202,51.
6443148847]1/b*(2*b*abs(b)/b^2/b*(2*(1/8*sqrt(b*x+2)*sqrt(b*x+2)-5/8)*sqrt(
b*x+2)*sqrt(b*(b*x+2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)
*sqrt(b*x+2))))+4*abs(b)/b^2*(1/2*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+2*b/2/sqr
t(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))
```

**maple [A]** time = 0.00, size = 75, normalized size = 1.17

$$\frac{\sqrt{bx+2}\, x^{\frac{3}{2}}}{2}+\frac{\sqrt{bx+2}\, \sqrt{x}}{2b}-\frac{\sqrt{(bx+2)\, x}\, \ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\, x^{2}+2x}\right)}{2\sqrt{bx+2}\, b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+2)^(1/2)+1/2*x^(1/2)*(b*x+2)^(1/2)/b-1/2/b^(3/2)*((b*x+2)*
x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [B]** time = 2.91, size = 98, normalized size = 1.53

$$\frac{\frac{\sqrt{bx+2}\, b}{\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{3}-\frac{2\,(bx+2)b^{2}}{x}+\frac{(bx+2)^{2}b}{x^{2}}}+\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\, b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(b*x + 2)*b/sqrt(x) + (b*x + 2)^(3/2)/x^(3/2))/(b^3 - 2*(b*x + 2)*b^2/
x + (b*x + 2)^2*b/x^2) + 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b
) + sqrt(b*x + 2)/sqrt(x)))/b^(3/2)
```

**mupad [B]** time = 0.10, size = 46, normalized size = 0.72

$$\sqrt{x}\left(\frac{x}{2}+\frac{1}{2\,b}\right)\sqrt{b\,x+2}-\frac{\ln\left(b\,x+\sqrt{b}\,\sqrt{x}\,\sqrt{b\,x+2}+1\right)}{2\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 + 1/(2*b))*(b*x + 2)^(1/2) - log(b*x + b^(1/2)*x^(1/2)*(b*x +
2)^(1/2) + 1)/(2*b^(3/2))
```

**sympy [A]** time = 2.91, size = 71, normalized size = 1.11

$$\frac{bx^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{3x^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{\sqrt{x}}{b\sqrt{bx+2}}-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(5/2)/(2*sqrt(b*x + 2)) + 3*x**(3/2)/(2*sqrt(b*x + 2)) + sqrt(x)/(b*sq
rt(b*x + 2)) - asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

## 3.508 $\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=40

$$\sqrt{x}\,\sqrt{bx+2}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Rubi** [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$\sqrt{x}\,\sqrt{bx+2}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{2+bx} + \int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx$$
$$= \sqrt{x}\,\sqrt{2+bx} + 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\,\sqrt{2+bx} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 1.00

$$\sqrt{x}\,\sqrt{bx+2} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

**IntegrateAlgebraic** [A] time = 0.05, size = 46, normalized size = 1.15

$$\sqrt{x}\,\sqrt{bx+2} - \frac{2\log\left(\sqrt{bx+2} - \sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[2 + b*x] - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

**fricas** [A] time = 1.20, size = 86, normalized size = 2.15

$$\left[\frac{\sqrt{bx+2}\,b\sqrt{x} + \sqrt{b}\,\log\left(bx + \sqrt{bx+2}\,\sqrt{b}\,\sqrt{x} + 1\right)}{b}, \frac{\sqrt{bx+2}\,b\sqrt{x} - 2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x)
 + 1))/b, (sqrt(b*x + 2)*b*sqrt(x) - 2*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-
b)/(b*sqrt(x))))/b]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]1/abs(b)*b^2/b*(1/b*sqrt(b*x+2)*sqr
t(b*(b*x+2)-2*b)-2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))
)
```

**maple** [A] time = 0.00, size = 58, normalized size = 1.45

$$\sqrt{bx+2}\,\sqrt{x}+\frac{\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(b*x+2)^(1/2)+((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^
(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2)
```

**maxima** [B] time = 2.96, size = 68, normalized size = 1.70

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{\sqrt{b}}-\frac{2\sqrt{bx+2}}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
sqrt(b) - 2*sqrt(b*x + 2)/((b - (b*x + 2)/x)*sqrt(x))
```

**mupad** [B] time = 0.62, size = 40, normalized size = 1.00

$$\sqrt{x}\,\sqrt{b\,x+2}-\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{b\,x+2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(b*x + 2)^(1/2) - (4*atanh((b^(1/2)*x^(1/2))/(2^(1/2) - (b*x + 2)^(
1/2))))/b^(1/2)
```

**sympy** [A] time = 1.65, size = 37, normalized size = 0.92

$$\sqrt{x}\,\sqrt{bx+2}+\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(b*x + 2) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

## 3.509 $\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx$

**Optimal**. Leaf size=41

$$2\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

**Rubi** [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 54, 215}

$$2\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+b\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\ &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+2\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 41, normalized size = 1.00

$$2\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(3/2),x]
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

**IntegrateAlgebraic** [A] time = 0.07, size = 47, normalized size = 1.15

$$-\frac{2\sqrt{bx+2}}{\sqrt{x}}-2\sqrt{b}\,\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + b*x]/x^(3/2),x]
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] - 2*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*
x]]
```

**fricas** [A] time = 1.02, size = 87, normalized size = 2.12

$$\left[\frac{\sqrt{b}\,x\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}\,+1\right)-2\,\sqrt{bx+2}\,\sqrt{x}}{x},-\frac{2\left(\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+2}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(b*x + 2)*
sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) + sqr
t(b*x + 2)*sqrt(x))/x]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]b/abs(b)*b^2/b*(-2*sqrt(b*x+2)*sqrt
(b*(b*x+2)-2*b)/(b*(b*x+2)-2*b)-2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b
)*sqrt(b*x+2))))
```

**maple** [A] time = 0.02, size = 59, normalized size = 1.44

$$\frac{\sqrt{(bx+2)\,x}\,\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}}-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(3/2),x)
```

```
[Out] -2*(b*x+2)^(1/2)/x^(1/2)+b^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))*((b*
x+2)*x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)
```

**maxima** [A] time = 2.95, size = 54, normalized size = 1.32

$$-\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sq
rt(x))) - 2*sqrt(b*x + 2)/sqrt(x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,x+2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(1/2)/x^(3/2), x)
```

**sympy** [A] time = 1.43, size = 48, normalized size = 1.17

$$-2\sqrt{b}\,\sqrt{1+\frac{2}{bx}}-\sqrt{b}\,\log\left(\frac{1}{bx}\right)+2\sqrt{b}\,\log\left(\sqrt{1+\frac{2}{bx}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(3/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(1 + 2/(b*x)) - sqrt(b)*log(1/(b*x)) + 2*sqrt(b)*log(sqrt(1
+ 2/(b*x)) + 1)
```

## 3.510 $\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx$

**Optimal**. Leaf size=18

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(5/2),x]
```

```
[Out] -(2 + b*x)^(3/2)/(3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx = -\frac{(2+bx)^{3/2}}{3x^{3/2}}$$

**Mathematica** [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(5/2),x]
```

```
[Out] -1/3*(2 + b*x)^(3/2)/x^(3/2)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 18, normalized size = 1.00

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + b*x]/x^(5/2),x]
[Out] -1/3*(2 + b*x)^(3/2)/x^(3/2)
```

**fricas [A]** time = 0.78, size = 12, normalized size = 0.67

$$-\frac{(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] -1/3*(b*x + 2)^(3/2)/x^(3/2)
```

**giac [B]** time = 1.15, size = 29, normalized size = 1.61

$$-\frac{(bx+2)^{\frac{3}{2}}b^4}{3\left((bx+2)b-2\,b\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="giac")
[Out] -1/3*(b*x + 2)^(3/2)*b^4/(((b*x + 2)*b - 2*b)^(3/2)*abs(b))
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.72

$$-\frac{(bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(5/2),x)
[Out] -1/3*(b*x+2)^(3/2)/x^(3/2)
```

**maxima [A]** time = 1.31, size = 12, normalized size = 0.67

$$-\frac{(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
[Out] -1/3*(b*x + 2)^(3/2)/x^(3/2)
```

**mupad [B]** time = 0.21, size = 18, normalized size = 1.00

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{b\,x}{3}+\frac{2}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(5/2),x)
[Out] -((b*x + 2)^(1/2)*((b*x)/3 + 2/3))/x^(3/2)
```

**sympy [B]** time = 1.45, size = 37, normalized size = 2.06

$$-\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-\frac{2\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(5/2),x)
[Out] -b**(3/2)*sqrt(1 + 2/(b*x))/3 - 2*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

## 3.511 $\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx$

**Optimal**. Leaf size=38

$$\frac{b(bx+2)^{3/2}}{15x^{3/2}}-\frac{(bx+2)^{3/2}}{5x^{5/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{b(bx+2)^{3/2}}{15x^{3/2}}-\frac{(bx+2)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(7/2),x]
```

```
[Out] -(2 + b*x)^(3/2)/(5*x^(5/2)) + (b*(2 + b*x)^(3/2))/(15*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx &= -\frac{(2+bx)^{3/2}}{5x^{5/2}}-\frac{1}{5}b\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx\\ &= -\frac{(2+bx)^{3/2}}{5x^{5/2}}+\frac{b(2+bx)^{3/2}}{15x^{3/2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.61

$$\frac{(bx-3)(bx+2)^{3/2}}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(7/2),x]
[Out] ((-3 + b*x)*(2 + b*x)^(3/2))/(15*x^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.07, size = 31, normalized size = 0.82

$$\frac{\sqrt{bx+2}\,\left(b^2x^2-bx-6\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + b*x]/x^(7/2),x]
[Out] (Sqrt[2 + b*x]*(-6 - b*x + b^2*x^2))/(15*x^(5/2))
```

**fricas** [A] time = 1.04, size = 25, normalized size = 0.66

$$\frac{\left(b^2x^2-bx-6\right)\sqrt{bx+2}}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
[Out] 1/15*(b^2*x^2 - b*x - 6)*sqrt(b*x + 2)/x^(5/2)
```

**giac** [A] time = 1.09, size = 42, normalized size = 1.11

$$\frac{\left((bx+2)b^5-5\,b^5\right)(bx+2)^{\frac{3}{2}}b}{15\,((bx+2)b-2\,b)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="giac")
[Out] 1/15*((b*x + 2)*b^5 - 5*b^5)*(b*x + 2)^(3/2)*b/(((b*x + 2)*b - 2*b)^(5/2)*a
bs(b))
```

**maple [A]** time = 0.00, size = 18, normalized size = 0.47

$$\frac{(bx+2)^{\frac{3}{2}}(bx-3)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(7/2),x)
```

```
[Out] 1/15*(b*x+2)^(3/2)*(b*x-3)/x^(5/2)
```

**maxima [A]** time = 1.32, size = 26, normalized size = 0.68

$$\frac{(bx+2)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{10\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/10*(b*x + 2)^(5/2)/x^(5/2)
```

**mupad [B]** time = 0.22, size = 26, normalized size = 0.68

$$-\frac{\sqrt{b\,x+2}\,\left(-\frac{b^2\,x^2}{15}+\frac{b\,x}{15}+\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(7/2),x)
```

```
[Out] -((b*x + 2)^(1/2)*((b*x)/15 - (b^2*x^2)/15 + 2/5))/x^(5/2)
```

**sympy [A]** time = 4.72, size = 56, normalized size = 1.47

$$\frac{b^{\frac{5}{2}}\sqrt{1+\frac{2}{bx}}}{15}-\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{15x}-\frac{2\sqrt{b}\,\sqrt{1+\frac{2}{bx}}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(7/2),x)
```

```
[Out] b**(5/2)*sqrt(1 + 2/(b*x))/15 - b**(3/2)*sqrt(1 + 2/(b*x))/(15*x) - 2*sqrt(
b)*sqrt(1 + 2/(b*x))/(5*x**2)
```

## 3.512 $\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx$

**Optimal**. Leaf size=59

$$-\frac{2b^2(bx+2)^{3/2}}{105x^{3/2}}+\frac{2b(bx+2)^{3/2}}{35x^{5/2}}-\frac{(bx+2)^{3/2}}{7x^{7/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2(bx+2)^{3/2}}{105x^{3/2}}+\frac{2b(bx+2)^{3/2}}{35x^{5/2}}-\frac{(bx+2)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(9/2),x]
```

```
[Out] -(2 + b*x)^(3/2)/(7*x^(7/2)) + (2*b*(2 + b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 + b*x)^(3/2))/(105*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx &= -\frac{(2+bx)^{3/2}}{7x^{7/2}} - \frac{1}{7}(2b)\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx \\ &= -\frac{(2+bx)^{3/2}}{7x^{7/2}} + \frac{2b(2+bx)^{3/2}}{35x^{5/2}} + \frac{1}{35}\left(2b^2\right)\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx \\ &= -\frac{(2+bx)^{3/2}}{7x^{7/2}} + \frac{2b(2+bx)^{3/2}}{35x^{5/2}} - \frac{2b^2(2+bx)^{3/2}}{105x^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 32, normalized size = 0.54

$$-\frac{(bx+2)^{3/2}\left(2b^2x^2-6bx+15\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(9/2),x]
```

```
[Out] -1/105*((2 + b*x)^(3/2)*(15 - 6*b*x + 2*b^2*x^2))/x^(7/2)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 40, normalized size = 0.68

$$\frac{\sqrt{bx+2}\left(-2b^3x^3+2b^2x^2-3bx-30\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + b*x]/x^(9/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(-30 - 3*b*x + 2*b^2*x^2 - 2*b^3*x^3))/(105*x^(7/2))
```

**fricas [A]** time = 1.25, size = 34, normalized size = 0.58

$$-\frac{\left(2\,b^3x^3-2\,b^2x^2+3\,bx+30\right)\sqrt{bx+2}}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x + 30)*sqrt(b*x + 2)/x^(7/2)
```

**giac [A]** time = 1.11, size = 55, normalized size = 0.93

$$-\frac{\left(35\,b^7+2\left((bx+2)b^7-7\,b^7\right)(bx+2)\right)(bx+2)^{\frac{3}{2}}b}{105\left((bx+2)b-2\,b\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(35*b^7 + 2*((b*x + 2)*b^7 - 7*b^7)*(b*x + 2))*(b*x + 2)^(3/2)*b/(((
b*x + 2)*b - 2*b)^(7/2)*abs(b))
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.46

$$-\frac{(bx+2)^{\frac{3}{2}}\left(2b^2x^2-6bx+15\right)}{105x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(9/2),x)
```

```
[Out] -1/105*(b*x+2)^(3/2)*(2*b^2*x^2-6*b*x+15)/x^(7/2)
```

**maxima [A]** time = 1.27, size = 41, normalized size = 0.69

$$-\frac{(bx+2)^{\frac{3}{2}}b^2}{12\,x^{\frac{3}{2}}}+\frac{(bx+2)^{\frac{5}{2}}b}{10\,x^{\frac{5}{2}}}-\frac{(bx+2)^{\frac{7}{2}}}{28\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b*x + 2)^(3/2)*b^2/x^(3/2) + 1/10*(b*x + 2)^(5/2)*b/x^(5/2) - 1/28*(
b*x + 2)^(7/2)/x^(7/2)
```

**mupad [B]** time = 0.22, size = 34, normalized size = 0.58

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^3\,x^3}{105}-\frac{2\,b^2\,x^2}{105}+\frac{b\,x}{35}+\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(9/2),x)
```

```
[Out] -((b*x + 2)^(1/2)*((b*x)/35 - (2*b^2*x^2)/105 + (2*b^3*x^3)/105 + 2/7))/x^(
7/2)
```

**sympy [B]** time = 13.80, size = 270, normalized size = 4.58

$$-\frac{2b^{\frac{19}{2}}x^5\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{6b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{3b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{34b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{132b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{120b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+2)**(1/2)/x**(9/2),x)

[Out] -2*b**(19/2)*x**5*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 6*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 3*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 34*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 132*b**(11/2)*x*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 120*b**(9/2)*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3)

## 3.513 $\int x^{5/2}\sqrt{2-bx}\,dx$

**Optimal**. Leaf size=112

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}+\frac{1}{4}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{12b}$$

**Rubi [A]** time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{4}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(24*b^2) - (
x^(5/2)*Sqrt[2 - b*x])/(12*b) + (x^(7/2)*Sqrt[2 - b*x])/4 + (5*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}\sqrt{2-bx}\,dx &= \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{1}{4}\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx \\
&= -\frac{x^{5/2}\sqrt{2-bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{12b} \\
&= -\frac{5x^{3/2}\sqrt{2-bx}}{24b^2} - \frac{x^{5/2}\sqrt{2-bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b^2} \\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{24b^2} - \frac{x^{5/2}\sqrt{2-bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{5\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{8b^3} \\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{24b^2} - \frac{x^{5/2}\sqrt{2-bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x,\right.}{4b^3} \\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{24b^2} - \frac{x^{5/2}\sqrt{2-bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 71, normalized size = 0.63

$$
\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}} + \frac{\sqrt{x}\sqrt{2-bx}\left(6b^3x^3-2b^2x^2-5bx-15\right)}{24b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x - 2*b^2*x^2 + 6*b^3*x^3))/(24*b^3) + (5
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 94, normalized size = 0.84

$$
\frac{5\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^4} + \frac{\sqrt{2-bx}\left(6b^3x^{7/2}-2b^2x^{5/2}-5bx^{3/2}-15\sqrt{x}\right)}{24b^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] - 5*b*x^(3/2) - 2*b^2*x^(5/2) + 6*b^3*x^(7/2)))
/(24*b^3) + (5*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*b^4)
```

**fricas [A]** time = 1.04, size = 141, normalized size = 1.26

$$\left[\frac{\left(6 b^{4} x^{3}-2 b^{3} x^{2}-5 b^{2} x-15 b\right) \sqrt{-b x+2} \sqrt{x}-15 \sqrt{-b} \log \left(-b x+\sqrt{-b x+2} \sqrt{-b} \sqrt{x}+1\right)}{24 b^{4}}, \frac{\left(6 b^{4} x^{3}-2 b^{3} x^{2}-5 b^{2} x-15 b\right) \sqrt{-b x+2} \sqrt{x}-30 \sqrt{b} \arctan \left(\frac{\sqrt{-b x+2}}{\sqrt{b} \sqrt{x}}\right)}{24 b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 15
*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^4, 1/24*((6*b^
4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arc
tan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^4]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-41.13435
40126,25.8388736797]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
```

```
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-67.0714422017,15.451549686]Warning, choosing root of [1,0,
%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[
2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}
+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,
[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}
+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-2
0,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2
]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}
+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[
3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}
+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{
4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,
0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+
%%%{16,[0,0]%%%}] at parameters values [-46.2420096635,81.9516051291]Warnin
g, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%
%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1
,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%
%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[
4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+
%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%
%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%
{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-82.594793
7798,51.6443148847]1/b*(2*b*abs(b)/b^2*(2*(((-90*b^11/1440/b^14*sqrt(-b*x+2
)*sqrt(-b*x+2)+750*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)-2445*b^11/1440
/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)+4185*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*
(-b*x+2)+2*b)-35/8/b^2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(
-b*x+2))))-4*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqrt(-b*x+2)-78*b^
5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*
(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*
x+2)))))
```

**maple [A]** time = 0.01, size = 116, normalized size = 1.04

$$-\frac{\left(-bx+2\right)^{\frac{3}{2}} x^{\frac{5}{2}}}{4b}-\frac{5\left(-bx+2\right)^{\frac{3}{2}} x^{\frac{3}{2}}}{12b^{2}}-\frac{5\left(-bx+2\right)^{\frac{3}{2}} \sqrt{x}}{8b^{3}}+\frac{5\sqrt{-bx+2}\,\sqrt{x}}{8b^{3}}+\frac{5\sqrt{\left(-bx+2\right)x}\;\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{8\sqrt{-bx+2}\;b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(1/2),x)
```

```
[Out] -1/4/b*x^(5/2)*(-b*x+2)^(3/2)-5/12/b^2*x^(3/2)*(-b*x+2)^(3/2)-5/8/b^3*x^(1/
2)*(-b*x+2)^(3/2)+5/8*x^(1/2)*(-b*x+2)^(1/2)/b^3+5/8/b^(7/2)*((-b*x+2)*x)^(
1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

**maxima [A]** time = 2.93, size = 147, normalized size = 1.31

$$\frac{\frac{15\sqrt{-bx+2}\,b^3}{\sqrt{x}}-\frac{73\,(-bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{55\,(-bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^7-\frac{4\,(bx-2)b^6}{x}+\frac{6\,(bx-2)^2b^5}{x^2}-\frac{4\,(bx-2)^3b^4}{x^3}+\frac{(bx-2)^4b^3}{x^4}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(-b*x + 2)*b^3/sqrt(x) - 73*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 55*
(-b*x + 2)^(5/2)*b/x^(5/2) - 15*(-b*x + 2)^(7/2)/x^(7/2))/(b^7 - 4*(b*x - 2
)*b^6/x + 6*(b*x - 2)^2*b^5/x^2 - 4*(b*x - 2)^3*b^4/x^3 + (b*x - 2)^4*b^3/x
^4) - 5/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\sqrt{2-b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)*(2 - b*x)^(1/2), x)
```

**sympy [A]** time = 9.92, size = 252, normalized size = 2.25

$$\begin{cases} \frac{ibx^{\frac{9}{2}}}{4\sqrt{bx-2}}-\frac{7ix^{\frac{7}{2}}}{12\sqrt{bx-2}}-\frac{ix^{\frac{5}{2}}}{24b\sqrt{bx-2}}-\frac{5ix^{\frac{3}{2}}}{24b^2\sqrt{bx-2}}+\frac{5i\sqrt{x}}{4b^3\sqrt{bx-2}}-\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ -\frac{bx^{\frac{9}{2}}}{4\sqrt{-bx+2}}+\frac{7x^{\frac{7}{2}}}{12\sqrt{-bx+2}}+\frac{x^{\frac{5}{2}}}{24b\sqrt{-bx+2}}+\frac{5x^{\frac{3}{2}}}{24b^2\sqrt{-bx+2}}-\frac{5\sqrt{x}}{4b^3\sqrt{-bx+2}}+\frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(9/2)/(4*sqrt(b*x - 2)) - 7*I*x**(7/2)/(12*sqrt(b*x - 2))
 - I*x**(5/2)/(24*b*sqrt(b*x - 2)) - 5*I*x**(3/2)/(24*b**2*sqrt(b*x - 2)) +
 5*I*sqrt(x)/(4*b**3*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/
(4*b**(7/2)), Abs(b*x)/2 > 1), (-b*x**(9/2)/(4*sqrt(-b*x + 2)) + 7*x**(7/2)
/(12*sqrt(-b*x + 2)) + x**(5/2)/(24*b*sqrt(-b*x + 2)) + 5*x**(3/2)/(24*b**2
*sqrt(-b*x + 2)) - 5*sqrt(x)/(4*b**3*sqrt(-b*x + 2)) + 5*asin(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(4*b**(7/2)), True))
```

# 3.514 $\int x^{3/2}\sqrt{2-bx}\,dx$

**Optimal**. Leaf size=87

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}+\frac{1}{3}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{6b}$$

**Rubi** [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{1}{3}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[2 - b*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(6*b) + (x^(5/2)
*Sqrt[2 - b*x])/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{3/2}\sqrt{2-bx}\,dx = \frac{1}{3}x^{5/2}\sqrt{2-bx} + \frac{1}{3}\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{x^{3/2}\sqrt{2-bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2-bx} + \frac{\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2-bx} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^2}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2-bx} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{6b} + \frac{1}{3}x^{5/2}\sqrt{2-bx} + \frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 0.69

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}} + \frac{\sqrt{x}\sqrt{2-bx}\left(2b^2x^2-bx-3\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-3 - b*x + 2*b^2*x^2))/(6*b^2) + ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[2]]/b^(5/2)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 81, normalized size = 0.93

$$\frac{\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^3} + \frac{\sqrt{2-bx}\left(2b^2x^{5/2}-bx^{3/2}-3\sqrt{x}\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[2 - b*x]*(-3*Sqrt[x] - b*x^(3/2) + 2*b^2*x^(5/2)))/(6*b^2) + (Sqrt[-b
]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^3
```

**fricas [A]** time = 1.03, size = 125, normalized size = 1.44

$$\left[\frac{\left(2\,b^3x^2-b^2x-3\,b\right)\sqrt{-bx+2}\,\sqrt{x}-3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6\,b^3}, \frac{\left(2\,b^3x^2-b^2x-3\,b\right)\sqrt{-bx+2}\,\sqrt{x}-6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="fricas")

[Out] [1/6*((2*b^3*x^2 - b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x) - 3*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, 1/6*((2*b^3*x^2 - b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x) - 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-41.1343540126,25.8388736797]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-67.0714422017,15.451549686]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[

```
2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}
+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,
[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}
+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-2
0,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2
]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}
+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[
3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}
+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{
4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,
0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+
%%%{16,[0,0]%%%}] at parameters values [-46.2420096635,81.9516051291]Warnin
g, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%
%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1
,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%
%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[
4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+
%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%
%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%
{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-82.594793
7798,51.6443148847]1/b*(2*b*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqr
t(-b*x+2)-78*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b^5/144/b^7)*sqrt(-
b*x+2)*sqrt(-b*(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sq
rt(-b)*sqrt(-b*x+2))))+4*abs(b)/b^2/b*(2*(1/8*sqrt(-b*x+2)*sqrt(-b*x+2)-5/8
)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)
+2*b)-sqrt(-b)*sqrt(-b*x+2)))))
```

**maple [A]** time = 0.00, size = 100, normalized size = 1.15

$$-\frac{(-bx+2)^{\frac{3}{2}}x^{\frac{3}{2}}}{3b}-\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{2b^2}+\frac{\sqrt{-bx+2}\,\sqrt{x}}{2b^2}+\frac{\sqrt{(-bx+2)\,x}\;\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^2+2x}}\right)}{2\sqrt{-bx+2}\,b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(1/2),x)
```

```
[Out] -1/3/b*x^(3/2)*(-b*x+2)^(3/2)-1/2/b^2*x^(1/2)*(-b*x+2)^(3/2)+1/2*x^(1/2)*(-
b*x+2)^(1/2)/b^2+1/2/b^(5/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arct
an((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima** [A] time = 2.98, size = 117, normalized size = 1.34

$$\frac{\frac{3\sqrt{-bx+2}\,b^2}{\sqrt{x}} - \frac{8(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} - \frac{3(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^5 - \frac{3(bx-2)b^4}{x} + \frac{3(bx-2)^2b^3}{x^2} - \frac{(bx-2)^3b^2}{x^3}\right)} - \frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(-b*x + 2)*b^2/sqrt(x) - 8*(-b*x + 2)^(3/2)*b/x^(3/2) - 3*(-b*x
+ 2)^(5/2)/x^(5/2))/(b^5 - 3*(b*x - 2)*b^4/x + 3*(b*x - 2)^2*b^3/x^2 - (b*x
 - 2)^3*b^2/x^3) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\sqrt{2-b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)*(2 - b*x)^(1/2), x)
```

**sympy** [A] time = 5.29, size = 196, normalized size = 2.25

$$\begin{cases} \frac{ibx^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{5ix^{\frac{5}{2}}}{6\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{6b\sqrt{bx-2}} + \frac{i\sqrt{x}}{b^2\sqrt{bx-2}} - \frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for } \frac{|bx|}{2} > 1 \\ -\frac{bx^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{5x^{\frac{5}{2}}}{6\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{6b\sqrt{-bx+2}} - \frac{\sqrt{x}}{b^2\sqrt{-bx+2}} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(7/2)/(3*sqrt(b*x - 2)) - 5*I*x**(5/2)/(6*sqrt(b*x - 2))
- I*x**(3/2)/(6*b*sqrt(b*x - 2)) + I*sqrt(x)/(b**2*sqrt(b*x - 2)) - I*acosh
(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2), Abs(b*x)/2 > 1), (-b*x**(7/2)/(3*sqrt
(-b*x + 2)) + 5*x**(5/2)/(6*sqrt(-b*x + 2)) + x**(3/2)/(6*b*sqrt(-b*x + 2))
 - sqrt(x)/(b**2*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
, True))
```

# 3.515 $\int \sqrt{x}\sqrt{2-bx}\,dx$

**Optimal**. Leaf size=65

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

**Rubi** [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[2 - b*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b) + (x^(3/2)*Sqrt[2 - b*x])/2 + ArcSin[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\sqrt{2-bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx \\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b} \\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b} \\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}
\end{aligned}
$$

**Mathematica** [A] time = 0.03, size = 51, normalized size = 0.78

$$
\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}} + \frac{\sqrt{x}\sqrt{2-bx}\,(bx-1)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-1 + b*x))/(2*b) + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]
]/b^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.08, size = 70, normalized size = 1.08

$$
\frac{\sqrt{-b}\,\log\left(\sqrt{2-bx} - \sqrt{-b}\sqrt{x}\right)}{b^2} + \frac{\sqrt{2-bx}\,\left(bx^{3/2} - \sqrt{x}\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[2 - b*x]*(-Sqrt[x] + b*x^(3/2)))/(2*b) + (Sqrt[-b]*Log[-(Sqrt[-b]*Sqr
t[x]) + Sqrt[2 - b*x]])/b^2
```

**fricas** [A] time = 1.20, size = 107, normalized size = 1.65

$$
\left[\frac{\left(b^2x-b\right)\sqrt{-bx+2}\,\sqrt{x} - \sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{2\,b^2}, \frac{\left(b^2x-b\right)\sqrt{-bx+2}\,\sqrt{x} - 2\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{2\,b^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="fricas")

[Out] [1/2*((b^2*x - b)*sqrt(-b*x + 2)*sqrt(x) - sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^2, 1/2*((b^2*x - b)*sqrt(-b*x + 2)*sqrt(x) - 2*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^2]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-41.1343540126,25.8388736797]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-67.0714422017,15.451549686]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}

```
+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-2
0,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2
]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}
+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[
3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}
+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{
4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,
0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+
%%%{16,[0,0]%%%}] at parameters values [-46.2420096635,81.9516051291]Warnin
g, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%
%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1
,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%
%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[
4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+
%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%
%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%
{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-82.594793
7798,51.6443148847]1/b*(-2*b*abs(b)/b^2/b*(2*(1/8*sqrt(-b*x+2)*sqrt(-b*x+2)
-5/8)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*ln(abs(sqrt(-b*(-b*
x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))-4*abs(b)/b^2*(1/2*sqrt(-b*x+2)*sqrt(-b*(
-b*x+2)+2*b)-2*b/2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x
+2)))))
```

**maple [A]** time = 0.00, size = 81, normalized size = 1.25

$$\frac{\sqrt{-bx+2}\,x^{\frac{3}{2}}}{2}-\frac{\sqrt{-bx+2}\,\sqrt{x}}{2b}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2\sqrt{-bx+2}\,b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-b*x+2)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(-b*x+2)^(1/2)-1/2*x^(1/2)*(-b*x+2)^(1/2)/b+1/2/b^(3/2)*((-b*x+
2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2
))
```

**maxima** [A] time = 2.95, size = 81, normalized size = 1.25

$$\frac{\frac{\sqrt{-bx+2}\,b}{\sqrt{x}} - \frac{(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^3 - \frac{2(bx-2)b^2}{x} + \frac{(bx-2)^2 b}{x^2}} - \frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(-b*x + 2)*b/sqrt(x) - (-b*x + 2)^(3/2)/x^(3/2))/(b^3 - 2*(b*x - 2)*b^
2/x + (b*x - 2)^2*b/x^2) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

**mupad** [B] time = 0.10, size = 53, normalized size = 0.82

$$\sqrt{x}\left(\frac{x}{2} - \frac{1}{2b}\right)\sqrt{2-bx} - \frac{\ln\left(\sqrt{-b}\,\sqrt{x}\,\sqrt{2-bx} - bx + 1\right)}{2(-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - 1/(2*b))*(2 - b*x)^(1/2) - log((-b)^(1/2)*x^(1/2)*(2 - b*x)^
(1/2) - b*x + 1)/(2*(-b)^(3/2))
```

**sympy** [A] time = 2.94, size = 156, normalized size = 2.40

$$\begin{cases} \frac{ibx^{\frac{5}{2}}}{2\sqrt{bx-2}} - \frac{3ix^{\frac{3}{2}}}{2\sqrt{bx-2}} + \frac{i\sqrt{x}}{b\sqrt{bx-2}} - \frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2} > 1 \\ -\frac{bx^{\frac{5}{2}}}{2\sqrt{-bx+2}} + \frac{3x^{\frac{3}{2}}}{2\sqrt{-bx+2}} - \frac{\sqrt{x}}{b\sqrt{-bx+2}} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(5/2)/(2*sqrt(b*x - 2)) - 3*I*x**(3/2)/(2*sqrt(b*x - 2))
+ I*sqrt(x)/(b*sqrt(b*x - 2)) - I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
, Abs(b*x)/2 > 1), (-b*x**(5/2)/(2*sqrt(-b*x + 2)) + 3*x**(3/2)/(2*sqrt(-b*
x + 2)) - sqrt(x)/(b*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(
3/2), True))
```

## 3.516 $\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=41

$$\sqrt{x}\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\sqrt{x}\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 - b*x] + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{2-bx} + \int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx$$
$$= \sqrt{x}\,\sqrt{2-bx} + 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\,\sqrt{2-bx} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Mathematica** [**A**] time = 0.01, size = 41, normalized size = 1.00

$$\sqrt{x}\,\sqrt{2-bx} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[2 - b*x] + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

**IntegrateAlgebraic** [**A**] time = 0.06, size = 55, normalized size = 1.34

$$\sqrt{x}\,\sqrt{2-bx} + \frac{2\sqrt{-b}\,\log\left(\sqrt{2-bx} - \sqrt{-b}\,\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[2 - b*x] + (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]
])/b
```

**fricas** [**A**] time = 1.22, size = 89, normalized size = 2.17

$$\left[\frac{\sqrt{-bx+2}\,b\sqrt{x} - \sqrt{-b}\,\log\left(-bx + \sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x} + 1\right)}{b}, \frac{\sqrt{-bx+2}\,b\sqrt{x} - 2\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b*x + 2)*b*sqrt(x) - sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sq
rt(x) + 1))/b, (sqrt(-b*x + 2)*b*sqrt(x) - 2*sqrt(b)*arctan(sqrt(-b*x + 2)/
(sqrt(b)*sqrt(x))))/b]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]1/abs(b)*b^2/b*(1/b*sqrt(-b*x+2
)*sqrt(-b*(-b*x+2)+2*b)+2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sq
rt(-b*x+2))))
```

**maple [B]** time = 0.00, size = 63, normalized size = 1.54

$$\sqrt{-bx+2}\,\sqrt{x}+\frac{\sqrt{(-bx+2)\,x}\;\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(-b*x+2)^(1/2)+((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*ar
ctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.91, size = 49, normalized size = 1.20

$$-\frac{2\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}+\frac{2\,\sqrt{-bx+2}}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + 2*sqrt(-b*x + 2)/((b
- (b*x - 2)/x)*sqrt(x))
```

**mupad [B]** time = 0.56, size = 42, normalized size = 1.02

$$\sqrt{x}\,\sqrt{2-b\,x}-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{2-b\,x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(2 - b*x)^(1/2) - (4*atan((b^(1/2)*x^(1/2))/(2^(1/2) - (2 - b*x)^(1
/2))))/b^(1/2)
```

**sympy [A]** time = 1.71, size = 121, normalized size = 2.95

$$\begin{cases} \frac{ibx^{\frac{3}{2}}}{\sqrt{bx-2}}-\frac{2i\sqrt{x}}{\sqrt{bx-2}}-\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx|}{2}>1 \\ -\frac{bx^{\frac{3}{2}}}{\sqrt{-bx+2}}+\frac{2\sqrt{x}}{\sqrt{-bx+2}}+\frac{2\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(3/2)/sqrt(b*x - 2) - 2*I*sqrt(x)/sqrt(b*x - 2) - 2*I*aco
sh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x)/2 > 1), (-b*x**(3/2)/sqrt(-
b*x + 2) + 2*sqrt(x)/sqrt(-b*x + 2) + 2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqr
t(b), True))
```

## 3.517 $\int \frac{\sqrt{2-bx}}{x^{3/2}}\,dx$

**Optimal**. Leaf size=42

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

**Rubi** [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{2-bx}}{\sqrt{x}} - b\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx\\ &= -\frac{2\sqrt{2-bx}}{\sqrt{x}} - (2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\sqrt{2-bx}}{\sqrt{x}} - 2\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 42, normalized size = 1.00

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}} - 2\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

**IntegrateAlgebraic** [A] time = 0.08, size = 53, normalized size = 1.26

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}} - 2\sqrt{-b}\,\log\left(\sqrt{2-bx} - \sqrt{-b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 -
b*x]]
```

**fricas** [A] time = 1.24, size = 90, normalized size = 2.14

$$\left[\frac{\sqrt{-b}\,x\log\left(-bx + \sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}\, + 1\right) - 2\,\sqrt{-bx+2}\,\sqrt{x}}{x}, \frac{2\left(\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right) - \sqrt{-bx+2}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - 2*sqrt(-b*x
+ 2)*sqrt(x))/x, 2*(sqrt(b)*x*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - sq
rt(-b*x + 2)*sqrt(x))/x]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]-b/abs(b)*b^2/b*(2*sqrt(-b*x+2)
*sqrt(-b*(-b*x+2)+2*b)/(-b*(-b*x+2)+2*b)+2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)
+2*b)-sqrt(-b)*sqrt(-b*x+2))))
```

**maple [B]** time = 0.04, size = 90, normalized size = 2.14

$$-\frac{\sqrt{(-bx+2)\,x}\,\sqrt{b}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^2+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}}+\frac{2\,(bx-2)\,\sqrt{(-bx+2)\,x}}{\sqrt{-\,(bx-2)\,x}\,\sqrt{-bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(3/2),x)
```

```
[Out] 2*(b*x-2)/(-x*(b*x-2))^(1/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)-b^(1
/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))*((-b*x+2)*x)^(1/2)/(-b*x+2)^
(1/2)/x^(1/2)
```

**maxima [A]** time = 3.01, size = 35, normalized size = 0.83

$$2\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\,\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + 2)/sqrt(
x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{2-b\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((2 - b*x)^(1/2)/x^(3/2), x)
```

**sympy [C]** time = 1.58, size = 136, normalized size = 3.24

$$\begin{cases}-2\sqrt{b}\,\sqrt{-1+\frac{2}{bx}}-i\sqrt{b}\,\log\left(\frac{1}{bx}\right)+2i\sqrt{b}\,\log\left(\frac{1}{\sqrt{b}\,\sqrt{x}}\right)-2\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right) & \text{for } \frac{2}{|bx|}>1\\ -2i\sqrt{b}\,\sqrt{1-\frac{2}{bx}}-i\sqrt{b}\,\log\left(\frac{1}{bx}\right)+2i\sqrt{b}\,\log\left(\sqrt{1-\frac{2}{bx}}+1\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(-1 + 2/(b*x)) - I*sqrt(b)*log(1/(b*x)) + 2*I*sqr
t(b)*log(1/(sqrt(b)*sqrt(x))) - 2*sqrt(b)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2),
2/Abs(b*x) > 1), (-2*I*sqrt(b)*sqrt(1 - 2/(b*x)) - I*sqrt(b)*log(1/(b*x)) +
 2*I*sqrt(b)*log(sqrt(1 - 2/(b*x)) + 1), True))
```

## 3.518 $\int \frac{\sqrt{2-bx}}{x^{5/2}} \, dx$

**Optimal**. Leaf size=19

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(5/2),x]
```

```
[Out] -(2 - b*x)^(3/2)/(3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{2-bx}}{x^{5/2}} \, dx = -\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

**Mathematica [A]** time = 0.01, size = 19, normalized size = 1.00

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(5/2),x]
```

```
[Out] -1/3*(2 - b*x)^(3/2)/x^(3/2)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 24, normalized size = 1.26

$$\frac{\sqrt{2-bx}\,(bx-2)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - b*x]/x^(5/2),x]
[Out] (Sqrt[2 - b*x]*(-2 + b*x))/(3*x^(3/2))
```

**fricas [A]** time = 1.29, size = 18, normalized size = 0.95

$$\frac{(bx-2)\sqrt{-bx+2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] 1/3*(b*x - 2)*sqrt(-b*x + 2)/x^(3/2)
```

**giac [B]** time = 1.12, size = 35, normalized size = 1.84

$$\frac{(bx-2)\sqrt{-bx+2}\,b^4}{3\,((bx-2)b+2\,b)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="giac")
[Out] 1/3*(b*x - 2)*sqrt(-b*x + 2)*b^4/(((b*x - 2)*b + 2*b)^(3/2)*abs(b))
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.74

$$-\frac{(-bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(5/2),x)
[Out] -1/3*(-b*x+2)^(3/2)/x^(3/2)
```

**maxima** [A] time = 1.28, size = 13, normalized size = 0.68

$$-\frac{(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-b*x + 2)^(3/2)/x^(3/2)
```

**mupad** [B] time = 0.22, size = 18, normalized size = 0.95

$$\frac{\sqrt{2-b\,x}\,\left(\frac{b\,x}{3}-\frac{2}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(5/2),x)
```

```
[Out] ((2 - b*x)^(1/2)*((b*x)/3 - 2/3))/x^(3/2)
```

**sympy** [B] time = 1.51, size = 82, normalized size = 4.32

$$\begin{cases} \frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3} - \frac{2\sqrt{b}\,\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for}\ \frac{2}{|bx|} > 1 \\ \frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3} - \frac{2i\sqrt{b}\,\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(5/2),x)
```

```
[Out] Piecewise((b**(3/2)*sqrt(-1 + 2/(b*x))/3 - 2*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*
x), 2/Abs(b*x) > 1), (I*b**(3/2)*sqrt(1 - 2/(b*x))/3 - 2*I*sqrt(b)*sqrt(1 -
 2/(b*x))/(3*x), True))
```

## 3.519 $\int \frac{\sqrt{2-bx}}{x^{7/2}}\,dx$

**Optimal**. Leaf size=40

$$-\frac{b(2-bx)^{3/2}}{15x^{3/2}}-\frac{(2-bx)^{3/2}}{5x^{5/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{b(2-bx)^{3/2}}{15x^{3/2}}-\frac{(2-bx)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(7/2),x]
```

```
[Out] -(2 - b*x)^(3/2)/(5*x^(5/2)) - (b*(2 - b*x)^(3/2))/(15*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{x^{7/2}}\,dx &= -\frac{(2-bx)^{3/2}}{5x^{5/2}}+\frac{1}{5}b\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx\\ &= -\frac{(2-bx)^{3/2}}{5x^{5/2}}-\frac{b(2-bx)^{3/2}}{15x^{3/2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.60

$$-\frac{(2-bx)^{3/2}(bx+3)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(7/2),x]
```

```
[Out] -1/15*((2 - b*x)^(3/2)*(3 + b*x))/x^(5/2)
```

**IntegrateAlgebraic** [A] time = 0.08, size = 31, normalized size = 0.78

$$\frac{\sqrt{2-bx}\,\left(b^2x^2+bx-6\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - b*x]/x^(7/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-6 + b*x + b^2*x^2))/(15*x^(5/2))
```

**fricas** [A] time = 0.67, size = 25, normalized size = 0.62

$$\frac{\left(b^2x^2+bx-6\right)\sqrt{-bx+2}}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(b^2*x^2 + b*x - 6)*sqrt(-b*x + 2)/x^(5/2)
```

**giac** [A] time = 0.85, size = 48, normalized size = 1.20

$$\frac{\left((bx-2)b^5+5\,b^5\right)(bx-2)\sqrt{-bx+2}\,b}{15\,((bx-2)b+2\,b)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((b*x - 2)*b^5 + 5*b^5)*(b*x - 2)*sqrt(-b*x + 2)*b/(((b*x - 2)*b + 2*b
)^(5/2)*abs(b))
```

**maple** [A] time = 0.00, size = 19, normalized size = 0.48

$$-\frac{(bx+3)(-bx+2)^{\frac{3}{2}}}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(7/2),x)
[Out] -1/15*(b*x+3)*(-b*x+2)^(3/2)/x^(5/2)
```

**maxima** [A] time = 1.33, size = 28, normalized size = 0.70

$$-\frac{(-bx+2)^{\frac{3}{2}}b}{6x^{\frac{3}{2}}}-\frac{(-bx+2)^{\frac{5}{2}}}{10x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
[Out] -1/6*(-b*x + 2)^(3/2)*b/x^(3/2) - 1/10*(-b*x + 2)^(5/2)/x^(5/2)
```

**mupad** [B] time = 0.22, size = 26, normalized size = 0.65

$$\frac{\sqrt{2-b\,x}\,\left(\frac{b^2\,x^2}{15}+\frac{b\,x}{15}-\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(7/2),x)
[Out] ((2 - b*x)^(1/2)*((b*x)/15 + (b^2*x^2)/15 - 2/5))/x^(5/2)
```

**sympy** [A] time = 4.91, size = 194, normalized size = 4.85

$$\begin{cases} \frac{b^{\frac{5}{2}}\sqrt{-1+\frac{2}{bx}}}{15}+\frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{15x}-\frac{2\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{5x^2} & \text{for}\ \frac{2}{|bx|}>1 \\ -\frac{ib^{\frac{9}{2}}x^2\sqrt{1-\frac{2}{bx}}}{-15b^2x^2+30bx}+\frac{ib^{\frac{7}{2}}x\sqrt{1-\frac{2}{bx}}}{-15b^2x^2+30bx}+\frac{8ib^{\frac{5}{2}}\sqrt{1-\frac{2}{bx}}}{-15b^2x^2+30bx}-\frac{12ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{x\left(-15b^2x^2+30bx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(7/2),x)
```

```
[Out] Piecewise((b**(5/2)*sqrt(-1 + 2/(b*x))/15 + b**(3/2)*sqrt(-1 + 2/(b*x))/(15
*x) - 2*sqrt(b)*sqrt(-1 + 2/(b*x))/(5*x**2), 2/Abs(b*x) > 1), (-I*b**(9/2)*
x**2*sqrt(1 - 2/(b*x))/(-15*b**2*x**2 + 30*b*x) + I*b**(7/2)*x*sqrt(1 - 2/(
b*x))/(-15*b**2*x**2 + 30*b*x) + 8*I*b**(5/2)*sqrt(1 - 2/(b*x))/(-15*b**2*x
**2 + 30*b*x) - 12*I*b**(3/2)*sqrt(1 - 2/(b*x))/(x*(-15*b**2*x**2 + 30*b*x)
), True))
```

## 3.520 $\int \frac{\sqrt{2-bx}}{x^{9/2}} \, dx$

**Optimal**. Leaf size=62

$$-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{(2-bx)^{3/2}}{7x^{7/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{(2-bx)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(9/2),x]
```

```
[Out] -(2 - b*x)^(3/2)/(7*x^(7/2)) - (2*b*(2 - b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 - b*x)^(3/2))/(105*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{x^{9/2}}\,dx &= -\frac{(2-bx)^{3/2}}{7x^{7/2}} + \frac{1}{7}(2b)\int \frac{\sqrt{2-bx}}{x^{7/2}}\,dx \\ &= -\frac{(2-bx)^{3/2}}{7x^{7/2}} - \frac{2b(2-bx)^{3/2}}{35x^{5/2}} + \frac{1}{35}\left(2b^2\right)\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx \\ &= -\frac{(2-bx)^{3/2}}{7x^{7/2}} - \frac{2b(2-bx)^{3/2}}{35x^{5/2}} - \frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.53

$$-\frac{(2-bx)^{3/2}\left(2b^2x^2+6bx+15\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(9/2),x]
[Out] -1/105*((2 - b*x)^(3/2)*(15 + 6*b*x + 2*b^2*x^2))/x^(7/2)
```

**IntegrateAlgebraic** [A] time = 0.09, size = 41, normalized size = 0.66

$$\frac{\sqrt{2-bx}\left(2b^3x^3+2b^2x^2+3bx-30\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - b*x]/x^(9/2),x]
[Out] (Sqrt[2 - b*x]*(-30 + 3*b*x + 2*b^2*x^2 + 2*b^3*x^3))/(105*x^(7/2))
```

**fricas** [A] time = 0.90, size = 35, normalized size = 0.56

$$\frac{\left(2\,b^3x^3+2\,b^2x^2+3\,bx-30\right)\sqrt{-bx+2}}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
[Out] 1/105*(2*b^3*x^3 + 2*b^2*x^2 + 3*b*x - 30)*sqrt(-b*x + 2)/x^(7/2)
```

**giac** [A] time = 0.98, size = 61, normalized size = 0.98

$$\frac{\left(35\,b^7+2\left((bx-2)b^7+7\,b^7\right)(bx-2)\right)(bx-2)\sqrt{-bx+2}\,b}{105\left((bx-2)b+2\,b\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(35*b^7 + 2*((b*x - 2)*b^7 + 7*b^7)*(b*x - 2))*(b*x - 2)*sqrt(-b*x +
2)*b/(((b*x - 2)*b + 2*b)^(7/2)*abs(b))
```

**maple [A]** time = 0.00, size = 28, normalized size = 0.45

$$-\frac{\left(2b^{2}x^{2}+6bx+15\right)\left(-bx+2\right)^{\frac{3}{2}}}{105x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(9/2),x)
```

```
[Out] -1/105*(2*b^2*x^2+6*b*x+15)*(-b*x+2)^(3/2)/x^(7/2)
```

**maxima [A]** time = 1.31, size = 44, normalized size = 0.71

$$-\frac{\left(-bx+2\right)^{\frac{3}{2}}b^{2}}{12\,x^{\frac{3}{2}}}-\frac{\left(-bx+2\right)^{\frac{5}{2}}b}{10\,x^{\frac{5}{2}}}-\frac{\left(-bx+2\right)^{\frac{7}{2}}}{28\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 1/10*(-b*x + 2)^(5/2)*b/x^(5/2) - 1/28
*(-b*x + 2)^(7/2)/x^(7/2)
```

**mupad [B]** time = 0.22, size = 34, normalized size = 0.55

$$\frac{\sqrt{2-b\,x}\,\left(\frac{2\,b^3\,x^3}{105}+\frac{2\,b^2\,x^2}{105}+\frac{b\,x}{35}-\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(9/2),x)
```

```
[Out] ((2 - b*x)^(1/2)*((b*x)/35 + (2*b^2*x^2)/105 + (2*b^3*x^3)/105 - 2/7))/x^(7
/2)
```

**sympy [B]** time = 24.60, size = 554, normalized size = 8.94

$$\begin{cases} -\frac{2b^{\frac{19}{2}}x^{5}\sqrt{-1+\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}}+\frac{6b^{\frac{17}{2}}x^{4}\sqrt{-1+\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}}-\frac{3b^{\frac{15}{2}}x^{3}\sqrt{-1+\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}}+\frac{34b^{\frac{13}{2}}x^{2}\sqrt{-1+\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}}-\frac{132b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}}+\frac{120b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}} & \text{for}\ \frac{2}{\left|bx\right|}>1 \\ -\frac{2ib^{\frac{19}{2}}x^{5}\sqrt{1-\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}}+\frac{6ib^{\frac{17}{2}}x^{4}\sqrt{1-\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}}-\frac{3ib^{\frac{15}{2}}x^{3}\sqrt{1-\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}}+\frac{34ib^{\frac{13}{2}}x^{2}\sqrt{1-\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}}-\frac{132ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}}+\frac{120ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{-105b^{6}x^{5}+420b^{5}x^{4}-420b^{4}x^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(9/2),x)
```

```
[Out] Piecewise((-2*b**(19/2)*x**5*sqrt(-1 + 2/(b*x))/(-105*b**6*x**5 + 420*b**5*
x**4 - 420*b**4*x**3) + 6*b**(17/2)*x**4*sqrt(-1 + 2/(b*x))/(-105*b**6*x**5
 + 420*b**5*x**4 - 420*b**4*x**3) - 3*b**(15/2)*x**3*sqrt(-1 + 2/(b*x))/(-1
05*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) + 34*b**(13/2)*x**2*sqrt(-1 +
 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) - 132*b**(11/2)*
x*sqrt(-1 + 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) + 120
*b**(9/2)*sqrt(-1 + 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**
3), 2/Abs(b*x) > 1), (-2*I*b**(19/2)*x**5*sqrt(1 - 2/(b*x))/(-105*b**6*x**5
 + 420*b**5*x**4 - 420*b**4*x**3) + 6*I*b**(17/2)*x**4*sqrt(1 - 2/(b*x))/(-
105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) - 3*I*b**(15/2)*x**3*sqrt(1
- 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) + 34*I*b**(13/2
)*x**2*sqrt(1 - 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) -
 132*I*b**(11/2)*x*sqrt(1 - 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*
b**4*x**3) + 120*I*b**(9/2)*sqrt(1 - 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x*
*4 - 420*b**4*x**3), True))
```

## 3.521 $\int x^{5/2}(a+bx)^{3/2}\,dx$

**Optimal**. Leaf size=143

$$-\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{7/2}}+\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

**Rubi** [A] time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{7/2}}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^(3/2),x]
```

```
[Out] (3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a + b*x])/(64*b
^2) + (a^2*x^(5/2)*Sqrt[a + b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a + b*x])/40 +
 (x^(7/2)*(a + b*x)^(3/2))/5 - (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]])/(128*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)^{3/2}\,dx &= \frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{1}{10}(3a)\int x^{5/2}\sqrt{a+bx}\,dx\\
&= \frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{1}{80}\left(3a^2\right)\int\frac{x^{5/2}}{\sqrt{a+bx}}\,dx\\
&= \frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}-\frac{a^3\int\frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{32b}\\
&= -\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{\left(3a^4\right)\int\frac{\sqrt{x}}{\sqrt{a+b}}}{128b^2}\\
&= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3}\\
&= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3}\\
&= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3}\\
&= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3}\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 107, normalized size = 0.75

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(15a^4-10a^3bx+8a^2b^2x^2+176ab^3x^3+128b^4x^4\right)-\frac{15a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{640b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^(3/2),x]
```

[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^4 - 10*a^3*b*x + 8*a^2*b^2*x^2 + 176*
a*b^3*x^3 + 128*b^4*x^4) - (15*a^(9/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/
Sqrt[1 + (b*x)/a]))/(640*b^(7/2))

**IntegrateAlgebraic [A]** time = 0.14, size = 108, normalized size = 0.76

$$\frac{3a^5\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{128b^{7/2}}+\frac{\sqrt{a+bx}\left(15a^4\sqrt{x}-10a^3bx^{3/2}+8a^2b^2x^{5/2}+176ab^3x^{7/2}+128b^4x^{9/2}\right)}{640b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(5/2)*(a + b*x)^(3/2),x]

[Out] (Sqrt[a + b*x]*(15*a^4*Sqrt[x] - 10*a^3*b*x^(3/2) + 8*a^2*b^2*x^(5/2) + 176
*a*b^3*x^(7/2) + 128*b^4*x^(9/2)))/(640*b^3) + (3*a^5*Log[-(Sqrt[b]*Sqrt[x]
) + Sqrt[a + b*x]])/(128*b^(7/2))

**fricas [A]** time = 1.10, size = 184, normalized size = 1.29

$$\left[\frac{15\,a^5\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\left(128\,b^5x^4+176\,ab^4x^3+8\,a^2b^3x^2-10\,a^3b^2x+15\,a^4b\right)\sqrt{bx+a}\sqrt{x}}{1280\,b^4},\ \frac{15\,a^5\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(128\,b^5x^4+176\,ab^4x^3+8\,a^2b^3x^2-10\,a^3b^2x+15\,a^4b\right)\sqrt{bx+a}\sqrt{x}}{640\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="fricas")

[Out] [1/1280*(15*a^5*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b^2*x + 15*a^4*b)*s
qrt(b*x + a)*sqrt(x))/b^4, 1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt
(-b)/(b*sqrt(x))) + (128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b
^2*x + 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^4]

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="giac")

[Out] Timed out

**maple [A]** time = 0.01, size = 138, normalized size = 0.97

$$-\frac{3\sqrt{(bx+a)\,x}\;a^5\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{256\sqrt{bx+a}\;b^{\frac{7}{2}}\sqrt{x}}-\frac{3\sqrt{bx+a}\;a^4\sqrt{x}}{128b^3}+\frac{(bx+a)^{\frac{5}{2}}\,x^{\frac{5}{2}}}{5b}-\frac{(bx+a)^{\frac{3}{2}}\,a^3\sqrt{x}}{64b^3}-\frac{(bx+a)^{\frac{5}{2}}\,a\,x^{\frac{3}{2}}}{8b^2}+\frac{(bx+a)^{\frac{5}{2}}\,a^2\sqrt{x}}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(3/2),x)
```

```
[Out] 1/5/b*x^(5/2)*(b*x+a)^(5/2)-1/8*a/b^2*x^(3/2)*(b*x+a)^(5/2)+1/16*a^2/b^3*x^
(1/2)*(b*x+a)^(5/2)-1/64*a^3/b^3*(b*x+a)^(3/2)*x^(1/2)-3/128*a^4*x^(1/2)*(b
*x+a)^(1/2)/b^3-3/256*a^5/b^(7/2)*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*l
n((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima [B]** time = 2.96, size = 212, normalized size = 1.48

$$\frac{3\,a^{5}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{256\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{bx+a}\,a^{5}b^{4}}{\sqrt{x}}-\frac{70\,(bx+a)^{\frac{3}{2}}a^{5}b^{3}}{x^{\frac{3}{2}}}-\frac{128\,(bx+a)^{\frac{5}{2}}a^{5}b^{2}}{x^{\frac{5}{2}}}+\frac{70\,(bx+a)^{\frac{7}{2}}a^{5}b}{x^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{9}{2}}a^{5}}{x^{\frac{9}{2}}}}{640\left(b^{8}-\frac{5\,(bx+a)b^{7}}{x}+\frac{10\,(bx+a)^{2}b^{6}}{x^{2}}-\frac{10\,(bx+a)^{3}b^{5}}{x^{3}}+\frac{5\,(bx+a)^{4}b^{4}}{x^{4}}-\frac{(bx+a)^{5}b^{3}}{x^{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/256*a^5*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(7/2) + 1/640*(15*sqrt(b*x + a)*a^5*b^4/sqrt(x) - 70*(b*x + a)^(
3/2)*a^5*b^3/x^(3/2) - 128*(b*x + a)^(5/2)*a^5*b^2/x^(5/2) + 70*(b*x + a)^(
7/2)*a^5*b/x^(7/2) - 15*(b*x + a)^(9/2)*a^5/x^(9/2))/(b^8 - 5*(b*x + a)*b^7
/x + 10*(b*x + a)^2*b^6/x^2 - 10*(b*x + a)^3*b^5/x^3 + 5*(b*x + a)^4*b^4/x^
4 - (b*x + a)^5*b^3/x^5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a+b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)*(a + b*x)^(3/2), x)
```

**sympy [A]** time = 17.71, size = 178, normalized size = 1.24

$$\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^{3}\sqrt{1+\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1+\frac{bx}{a}}}+\frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}}+\frac{19\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}}-\frac{3a^{5}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{b^{2}x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(3/2),x)
```

```
[Out] 3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 + b*x/a)) + a**(7/2)*x**(3/2)/(128*b**2
*sqrt(1 + b*x/a)) - a**(5/2)*x**(5/2)/(320*b*sqrt(1 + b*x/a)) + 23*a**(3/2)
*x**(7/2)/(80*sqrt(1 + b*x/a)) + 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1 + b*x/a))
 - 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x**(11/2)/(5
*sqrt(a)*sqrt(1 + b*x/a))
```

## 3.522 $\int x^{3/2}(a+bx)^{3/2}\,dx$

**Optimal**. Leaf size=119

$$\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}-\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

**Rubi** [A] time = 0.04, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^2) + (a^2*x^(3/2)*Sqrt[a + b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a + b*x])/8 + (x^(5/2)*(a + b*x)^(3/2))/4 + (3*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}(a+bx)^{3/2}\,dx &= \frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{1}{8}(3a)\int x^{3/2}\sqrt{a+bx}\,dx\\
&= \frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{1}{16}a^2\int\frac{x^{3/2}}{\sqrt{a+bx}}\,dx\\
&= \frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}-\frac{\left(3a^3\right)\int\frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{64b}\\
&= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{\left(3a^4\right)\int\frac{1}{\sqrt{x}\sqrt{a+}}}{128b^2}\\
&= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{\left(3a^4\right)\text{Subst}\Big(\int}{}\\
&= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{\left(3a^4\right)\text{Subst}\Big(\int}{}\\
&= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{3a^4\tanh^{-1}\Big(\frac{\sqrt{}}{\sqrt{}}}{64b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 96, normalized size = 0.81

$$\frac{\sqrt{a+bx}\left(\frac{3a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}+\sqrt{b}\sqrt{x}\left(-3a^3+2a^2bx+24ab^2x^2+16b^3x^3\right)\right)}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^3 + 2*a^2*b*x + 24*a*b^2*x^2 + 16*b^3
*x^3) + (3*a^(7/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/
(64*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.10, size = 95, normalized size = 0.80

$$\frac{\sqrt{a+bx}\left(-3a^3\sqrt{x}+2a^2bx^{3/2}+24ab^2x^{5/2}+16b^3x^{7/2}\right)}{64b^2}-\frac{3a^4\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-3*a^3*Sqrt[x] + 2*a^2*b*x^(3/2) + 24*a*b^2*x^(5/2) + 16*b^
3*x^(7/2)))/(64*b^2) - (3*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(64*
b^(5/2))
```

**fricas [A]** time = 0.70, size = 163, normalized size = 1.37

$$\left[\frac{3\,a^4\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{128\,b^3},-\frac{3\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{64\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*a^4*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/
b^3, -1/64*(3*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (16
*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/b^3
]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 120, normalized size = 1.01

$$\frac{3\sqrt{(bx+a)\,x}\;a^4\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{128\sqrt{bx+a}\;b^{\frac{5}{2}}\sqrt{x}}+\frac{3\sqrt{bx+a}\;a^3\sqrt{x}}{64b^2}+\frac{(bx+a)^{\frac{3}{2}}\,a^2\sqrt{x}}{32b^2}+\frac{(bx+a)^{\frac{5}{2}}\,x^{\frac{3}{2}}}{4b}-\frac{(bx+a)^{\frac{5}{2}}\,a\sqrt{x}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(3/2),x)
```

```
[Out] 1/4/b*x^(3/2)*(b*x+a)^(5/2)-1/8*a/b^2*x^(1/2)*(b*x+a)^(5/2)+1/32*a^2/b^2*(b
*x+a)^(3/2)*x^(1/2)+3/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b^2+3/128*a^4/b^(5/2)*((
b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1
/2))
```

**maxima [B]** time = 2.96, size = 178, normalized size = 1.50

$$-\frac{3\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{5}{2}}}-\frac{\frac{3\,\sqrt{bx+a}\,a^4b^3}{\sqrt{x}}-\frac{11\,(bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}-\frac{11\,(bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}+\frac{3\,(bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{64\left(b^6-\frac{4\,(bx+a)b^5}{x}+\frac{6\,(bx+a)^2b^4}{x^2}-\frac{4\,(bx+a)^3b^3}{x^3}+\frac{(bx+a)^4b^2}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(5/2) - 1/64*(3*sqrt(b*x + a)*a^4*b^3/sqrt(x) - 11*(b*x + a)^(3
/2)*a^4*b^2/x^(3/2) - 11*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 3*(b*x + a)^(7/2)*
a^4/x^(7/2))/(b^6 - 4*(b*x + a)*b^5/x + 6*(b*x + a)^2*b^4/x^2 - 4*(b*x + a)
^3*b^3/x^3 + (b*x + a)^4*b^2/x^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a+b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)*(a + b*x)^(3/2), x)
```

**sympy [A]** time = 9.28, size = 153, normalized size = 1.29

$$-\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1+\frac{bx}{a}}}+\frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1+\frac{bx}{a}}}+\frac{5\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{1+\frac{bx}{a}}}+\frac{3a^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}+\frac{b^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(3/2),x)
```

```
[Out] -3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 + b*x/a)) - a**(5/2)*x**(3/2)/(64*b*sqr
t(1 + b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 + b*x/a)) + 5*sqrt(a)*b*x**
(7/2)/(8*sqrt(1 + b*x/a)) + 3*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(5
/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

# 3.523 $\int \sqrt{x}\,(a+bx)^{3/2}\,dx$

**Optimal**. Leaf size=95

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}+\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

**Rubi** [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}+\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^(3/2),x]
```

```
[Out] (a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b) + (a*x^(3/2)*Sqrt[a + b*x])/4 + (x^(3/2)*
(a + b*x)^(3/2))/3 - (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8*b^(3
/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}(a+bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(a+bx)^{3/2}+\frac{1}{2}a\int \sqrt{x}\sqrt{a+bx}\,dx\\
&= \frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}+\frac{1}{8}a^2\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx\\
&= \frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{16b}\\
&= \frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{8b}\\
&= \frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{8b}\\
&= \frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 85, normalized size = 0.89

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(3a^2+14abx+8b^2x^2\right)-\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{24b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(3*a^2 + 14*a*b*x + 8*b^2*x^2) - (3*a^(5/2)
*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 82, normalized size = 0.86

$$\frac{a^3\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{8b^{3/2}}+\frac{\sqrt{a+bx}\left(3a^2\sqrt{x}+14abx^{3/2}+8b^2x^{5/2}\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(3*a^2*Sqrt[x] + 14*a*b*x^(3/2) + 8*b^2*x^(5/2)))/(24*b) + (
a^3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*b^(3/2))
```

**fricas [A]** time = 0.72, size = 140, normalized size = 1.47

$$\left[\frac{3\,a^3\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(8\,b^3x^2+14\,ab^2x+3\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{48\,b^2},\ \frac{3\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(8\,b^3x^2+14\,ab^2x+3\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 + 14*a*b^2*
x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 96, normalized size = 1.01

$$\frac{\sqrt{bx+a}\,a\,x^{\frac{3}{2}}}{4}-\frac{\sqrt{(bx+a)\,x}\,a^3\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{16\sqrt{bx+a}\,b^{\frac{3}{2}}\sqrt{x}}+\frac{\sqrt{bx+a}\,a^2\sqrt{x}}{8b}+\frac{(bx+a)^{\frac{3}{2}}\,x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*x^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*(b*x+a)^(3/2)+1/4*a*x^(3/2)*(b*x+a)^(1/2)+1/8*a^2*x^(1/2)*(b*x+
a)^(1/2)/b-1/16*a^3/b^(3/2)*((b*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((b*x
+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima [B]** time = 3.02, size = 144, normalized size = 1.52

$$\frac{a^3 \log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{3}{2}}}+\frac{\frac{3\,\sqrt{bx+a}\,a^3b^2}{\sqrt{x}}-\frac{8\,(bx+a)^{\frac{3}{2}}\,a^3b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}\,a^3}{x^{\frac{5}{2}}}}{24\left(b^4-\frac{3\,(bx+a)b^3}{x}+\frac{3\,(bx+a)^2b^2}{x^2}-\frac{(bx+a)^3b}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(3/2) + 1/24*(3*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 8*(b*x + a)^(3/2)
*a^3*b/x^(3/2) - 3*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^4 - 3*(b*x + a)*b^3/x +
3*(b*x + a)^2*b^2/x^2 - (b*x + a)^3*b/x^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a+b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(a + b*x)^(3/2), x)
```

**sympy [A]** time = 5.59, size = 124, normalized size = 1.31

$$\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1+\frac{bx}{a}}}+\frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1+\frac{bx}{a}}}+\frac{11\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}}-\frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{b^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*x**(1/2),x)
```

```
[Out] a**(5/2)*sqrt(x)/(8*b*sqrt(1 + b*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 +
b*x/a)) + 11*sqrt(a)*b*x**(5/2)/(12*sqrt(1 + b*x/a)) - a**3*asinh(sqrt(b)*s
qrt(x)/sqrt(a))/(8*b**(3/2)) + b**2*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

# 3.524 $\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=71

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}$$

**Rubi [A]** time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*a*Sqrt[x]*Sqrt[a + b*x])/4 + (Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a^2*ArcTan
h[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{4}(3a)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 69, normalized size = 0.97

$$\frac{1}{4}\sqrt{a+bx}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{\frac{bx}{a}+1}}+\sqrt{x}\,(5a+2bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[x]*(5*a + 2*b*x) + (3*a^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x)/a])))/4
```

**IntegrateAlgebraic [A]** time = 0.09, size = 66, normalized size = 0.93

$$\frac{1}{4}\sqrt{a+bx}\left(5a\sqrt{x}+2bx^{3/2}\right)-\frac{3a^2\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a + b*x]*(5*a*Sqrt[x] + 2*b*x^(3/2)))/4 - (3*a^2*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[a + b*x]])/(4*Sqrt[b])
```

**fricas [A]** time = 1.10, size = 119, normalized size = 1.68

$$\left[\frac{3\,a^2\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)+2\left(2\,b^2x+5\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{8\,b},-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(2\,b^2x+5\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b
]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 78, normalized size = 1.10

$$\frac{3\sqrt{(bx+a)\,x}\;a^2\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8\sqrt{bx+a}\;\sqrt{b}\;\sqrt{x}}+\frac{3\sqrt{bx+a}\;a\sqrt{x}}{4}+\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*x^(1/2)*(b*x+a)^(1/2)+3/8*a^2*((b*x+a)*x)^(
1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2
)
```

**maxima [B]** time = 2.99, size = 107, normalized size = 1.51

$$-\frac{3\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,\sqrt{b}}-\frac{\frac{3\,\sqrt{bx+a}\,a^2b}{\sqrt{x}}-\frac{5\,(bx+a)^{\frac{3}{2}}a^2}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2\,(bx+a)b}{x}+\frac{(bx+a)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/sqrt(b) - 1/4*(3*sqrt(b*x + a)*a^2*b/sqrt(x) - 5*(b*x + a)^(3/2)*a^
2/x^(3/2))/(b^2 - 2*(b*x + a)*b/x + (b*x + a)^2/x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^(1/2),x)
```

```
[Out] int((a + b*x)^(3/2)/x^(1/2), x)
```

**sympy [A]** time = 3.17, size = 75, normalized size = 1.06

$$\frac{5a^{\frac{3}{2}}\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}{4}+\frac{\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{2}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] 5*a**(3/2)*sqrt(x)*sqrt(1 + b*x/a)/4 + sqrt(a)*b*x**(3/2)*sqrt(1 + b*x/a)/2
 + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b))
```

## 3.525 $\int \frac{(a+bx)^{3/2}}{x^{3/2}} dx$

**Optimal**. Leaf size=63

$$-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{a+bx}+3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{a+bx}+3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] 3*b*Sqrt[x]*Sqrt[a + b*x] - (2*(a + b*x)^(3/2))/Sqrt[x] + 3*a*Sqrt[b]*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{\sqrt{x}}+(3b)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\
&= 3b\sqrt{x}\,\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+\frac{1}{2}(3ab)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\
&= 3b\sqrt{x}\,\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+(3ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= 3b\sqrt{x}\,\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+(3ab)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\
&= 3b\sqrt{x}\,\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}
$$

**Mathematica** [C]     time = 0.01, size = 46, normalized size = 0.73

$$
-\frac{2a\sqrt{a+bx}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}\,\sqrt{\frac{bx}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a + b*x]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x)/a)])/(Sqrt[x
]*Sqrt[1 + (b*x)/a])
```

**IntegrateAlgebraic** [A] time = 0.13, size = 54, normalized size = 0.86

$$\frac{(bx-2a)\sqrt{a+bx}}{\sqrt{x}}-3a\sqrt{b}\,\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] ((-2*a + b*x)*Sqrt[a + b*x])/Sqrt[x] - 3*a*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) +
 Sqrt[a + b*x]]
```

**fricas** [A] time = 1.33, size = 109, normalized size = 1.73

$$\left[\frac{3\,a\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\sqrt{bx+a}\,(bx-2\,a)\sqrt{x}}{2\,x},-\frac{3\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}\,(bx-2\,a)\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sq
rt(b*x + a)*(b*x - 2*a)*sqrt(x))/x, -(3*a*sqrt(-b)*x*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - sqrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x]
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A] time = 0.02, size = 71, normalized size = 1.13

$$\frac{3\sqrt{(bx+a)\,x}\;a\sqrt{b}\;\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{bx+a}\;\sqrt{x}}-\frac{\sqrt{bx+a}\;(-bx+2a)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(3/2),x)
```

```
[Out] -(b*x+a)^(1/2)*(-b*x+2*a)/x^(1/2)+3/2*a*b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x
^2+a*x)^(1/2))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
```

**maxima [A]** time = 2.99, size = 84, normalized size = 1.33

$$-\frac{3}{2}\,a\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}\,a}{\sqrt{x}}-\frac{\sqrt{bx+a}\,ab}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*a*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x +
 a)/sqrt(x))) - 2*sqrt(b*x + a)*a/sqrt(x) - sqrt(b*x + a)*a*b/((b - (b*x +
a)/x)*sqrt(x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^(3/2),x)
```

```
[Out] int((a + b*x)^(3/2)/x^(3/2), x)
```

**sympy [A]** time = 2.72, size = 92, normalized size = 1.46

$$-\frac{2a^{\frac{3}{2}}}{\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,b\sqrt{x}}{\sqrt{1+\frac{bx}{a}}}+3a\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(3/2),x)
```

```
[Out] -2*a**(3/2)/(sqrt(x)*sqrt(1 + b*x/a)) - sqrt(a)*b*sqrt(x)/sqrt(1 + b*x/a) +
 3*a*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt(1
 + b*x/a))
```

## 3.526 $\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx$

**Optimal**. Leaf size=64

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{a+bx}}{\sqrt{x}}$$

**Rubi [A]** time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*b*Sqrt[a + b*x])/Sqrt[x] - (2*(a + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{3x^{3/2}}+b\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+b^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+\left(2b^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 48, normalized size = 0.75

$$-\frac{2a\sqrt{a+bx}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a + b*x]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x)/a)])/(3*x^(
3/2)*Sqrt[1 + (b*x)/a])
```

**IntegrateAlgebraic [A]** time = 0.13, size = 55, normalized size = 0.86

$$-2b^{3/2}\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)-\frac{2\sqrt{a+bx}\,(a+4bx)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(a + 4*b*x))/(3*x^(3/2)) - 2*b^(3/2)*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[a + b*x]]
```

**fricas** [A] time = 0.90, size = 109, normalized size = 1.70

$$\left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,(4\,bx+a)\sqrt{bx+a}\,\sqrt{x}}{3\,x^2},-\frac{2\left(3\,\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(4\,bx+a)\sqrt{bx+a}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(4
*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*
x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2]
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A] time = 0.02, size = 67, normalized size = 1.05

$$\frac{\sqrt{(bx+a)\,x}\;b^{\frac{3}{2}}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{bx+a}\;\sqrt{x}}-\frac{2\sqrt{bx+a}\;(4bx+a)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(5/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(4*b*x+a)/x^(3/2)+b^(3/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+
a*x)^(1/2))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
```

**maxima** [A] time = 2.93, size = 67, normalized size = 1.05

$$-b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -b^(3/2)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x))) - 2*sqrt(b*x + a)*b/sqrt(x) - 2/3*(b*x + a)^(3/2)/x^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^(5/2),x)
```

```
[Out] int((a + b*x)^(3/2)/x^(5/2), x)
```

**sympy [A]** time = 3.04, size = 71, normalized size = 1.11

$$-\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x} - \frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3} - b^{\frac{3}{2}}\log\left(\frac{a}{bx}\right) + 2b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(5/2),x)
```

```
[Out] -2*a*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 8*b**(3/2)*sqrt(a/(b*x) + 1)/3 - b**
(3/2)*log(a/(b*x)) + 2*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1)
```

## 3.527 $\int x^{5/2}(a-bx)^{3/2}\,dx$

**Optimal**. Leaf size=149

$$\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{7/2}}-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

**Rubi** [A] time = 0.05, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}+\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{7/2}}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b^2) - (a^2*x^(5/2)*Sqrt[a - b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a - b*x])/40
+ (x^(7/2)*(a - b*x)^(3/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]])/(128*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}(a-bx)^{3/2}\,dx &= \frac{1}{5}x^{7/2}(a-bx)^{3/2}+\frac{1}{10}(3a)\int x^{5/2}\sqrt{a-bx}\,dx\\
&= \frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}+\frac{1}{80}\left(3a^2\right)\int\frac{x^{5/2}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}+\frac{a^3\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{32b}\\
&= -\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}+\frac{\left(3a^4\right)\int\frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{128b^2}\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}\\
&= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}
\end{aligned}
$$

**Mathematica [A]** time = 0.17, size = 110, normalized size = 0.74

$$
\frac{\sqrt{a-bx}\left(\frac{15a^{9/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}-\sqrt{b}\sqrt{x}\left(15a^4+10a^3bx+8a^2b^2x^2-176ab^3x^3+128b^4x^4\right)\right)}{640b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-(Sqrt[b]*Sqrt[x]*(15*a^4 + 10*a^3*b*x + 8*a^2*b^2*x^2 - 17
6*a*b^3*x^3 + 128*b^4*x^4)) + (15*a^(9/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]]
)/Sqrt[1 - (b*x)/a]))/(640*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.18, size = 117, normalized size = 0.79

$$\frac{3a^5\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{128b^4}+\frac{\sqrt{a-bx}\,\left(-15a^4\sqrt{x}-10a^3bx^{3/2}-8a^2b^2x^{5/2}+176ab^3x^{7/2}-128b^4x^{9/2}\right)}{640b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^4*Sqrt[x] - 10*a^3*b*x^(3/2) - 8*a^2*b^2*x^(5/2) + 17
6*a*b^3*x^(7/2) - 128*b^4*x^(9/2)))/(640*b^3) + (3*a^5*Sqrt[-b]*Log[-(Sqrt[
-b]*Sqrt[x]) + Sqrt[a - b*x]])/(128*b^4)
```

**fricas [A]** time = 1.09, size = 185, normalized size = 1.24

$$\left[-\frac{15\,a^5\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\left(128\,b^5x^4-176\,ab^4x^3+8\,a^2b^3x^2+10\,a^3b^2x+15\,a^4b\right)\sqrt{-bx+a}\,\sqrt{x}}{1280\,b^4},-\frac{15\,a^5\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(128\,b^5x^4-176\,ab^4x^3+8\,a^2b^3x^2+10\,a^3b^2x+15\,a^4b\right)\sqrt{-bx+a}\,\sqrt{x}}{640\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) + 2*(128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^3*x^2 + 10*a^3*b^2*x + 15*a^4
*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b*x +
a)/(sqrt(b)*sqrt(x))) + (128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^3*x^2 + 10*a
^3*b^2*x + 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 146, normalized size = 0.98

$$\frac{3\sqrt{(-bx+a)\,x}\;a^5\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{256\sqrt{-bx+a}\;b^{\frac{7}{2}}\sqrt{x}}+\frac{3\sqrt{-bx+a}\;a^4\sqrt{x}}{128b^3}-\frac{(-bx+a)^{\frac{5}{2}}\,x^{\frac{5}{2}}}{5b}+\frac{(-bx+a)^{\frac{3}{2}}\,a^3\sqrt{x}}{64b^3}-\frac{(-bx+a)^{\frac{5}{2}}\,a\,x^{\frac{3}{2}}}{8b^2}-\frac{(-bx+a)^{\frac{5}{2}}\,a^2\sqrt{x}}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(3/2),x)
```

```
[Out] -1/5/b*x^(5/2)*(-b*x+a)^(5/2)-1/8*a/b^2*x^(3/2)*(-b*x+a)^(5/2)-1/16*a^2/b^3
*x^(1/2)*(-b*x+a)^(5/2)+1/64*a^3/b^3*(-b*x+a)^(3/2)*x^(1/2)+3/128*a^4*x^(1/
2)*(-b*x+a)^(1/2)/b^3+3/256*a^5/b^(7/2)*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x
^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 3.12, size = 207, normalized size = 1.39

$$-\frac{3\,a^{5}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{128\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^{5}b^{4}}{\sqrt{x}}+\frac{70\,(-bx+a)^{\frac{3}{2}}a^{5}b^{3}}{x^{\frac{3}{2}}}-\frac{128\,(-bx+a)^{\frac{5}{2}}a^{5}b^{2}}{x^{\frac{5}{2}}}-\frac{70\,(-bx+a)^{\frac{7}{2}}a^{5}b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+a)^{\frac{9}{2}}a^{5}}{x^{\frac{9}{2}}}}{640\left(b^{8}-\frac{5\,(bx-a)b^{7}}{x}+\frac{10\,(bx-a)^{2}b^{6}}{x^{2}}-\frac{10\,(bx-a)^{3}b^{5}}{x^{3}}+\frac{5\,(bx-a)^{4}b^{4}}{x^{4}}-\frac{(bx-a)^{5}b^{3}}{x^{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/128*a^5*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/640*(15*sqr
t(-b*x + a)*a^5*b^4/sqrt(x) + 70*(-b*x + a)^(3/2)*a^5*b^3/x^(3/2) - 128*(-b
*x + a)^(5/2)*a^5*b^2/x^(5/2) - 70*(-b*x + a)^(7/2)*a^5*b/x^(7/2) - 15*(-b*
x + a)^(9/2)*a^5/x^(9/2))/(b^8 - 5*(b*x - a)*b^7/x + 10*(b*x - a)^2*b^6/x^2
 - 10*(b*x - a)^3*b^5/x^3 + 5*(b*x - a)^4*b^4/x^4 - (b*x - a)^5*b^3/x^5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(a-b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)*(a - b*x)^(3/2), x)
```

**sympy [A]** time = 17.69, size = 376, normalized size = 2.52

$$\begin{cases} \frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^{3}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{-1+\frac{bx}{a}}}-\frac{23ia^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}}+\frac{19i\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^{5}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}-\frac{ib^{2}x^{\frac{11}{2}}}{5\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^{3}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1-\frac{bx}{a}}}+\frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}-\frac{19\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}+\frac{3a^{5}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{b^{2}x^{\frac{11}{2}}}{5\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**3*sqrt(-1 + b*x/a)) - I*a**(7/2)*x*
*(3/2)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x**(5/2)/(320*b*sqrt(-1 + b
*x/a)) - 23*I*a**(3/2)*x**(7/2)/(80*sqrt(-1 + b*x/a)) + 19*I*sqrt(a)*b*x**(
9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b
**(7/2)) - I*b**2*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1),
(-3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/2)/(128*b*
*2*sqrt(1 - b*x/a)) + a**(5/2)*x**(5/2)/(320*b*sqrt(1 - b*x/a)) + 23*a**(3/
2)*x**(7/2)/(80*sqrt(1 - b*x/a)) - 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1 - b*x/a
)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x**(11/2)/(
5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

## 3.528 $\int x^{3/2}(a-bx)^{3/2}\,dx$

**Optimal**. Leaf size=124

$$\frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}-\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}+\frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^2) - (a^2*x^(3/2)*Sqrt[a - b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a - b*x])/8 + (x^(5/2)*(a - b*x)^(3/2))/4 + (3*a^4*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}(a-bx)^{3/2}\,dx &= \frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{1}{8}(3a)\int x^{3/2}\sqrt{a-bx}\,dx\\
&= \frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{1}{16}a^2\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{\left(3a^3\right)\int\frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{64b}\\
&= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{\left(3a^4\right)\int\frac{1}{\sqrt{x}\sqrt{a-}}}{128b^2}\\
&= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{\left(3a^4\right)\text{Subst}\Big(}{}\\
&= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{\left(3a^4\right)\text{Subst}\Big(}{}\\
&= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a\cdot}}\right.}{64b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 99, normalized size = 0.80

$$\frac{\sqrt{a-bx}\left(\frac{3a^{7/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}-\sqrt{b}\sqrt{x}\left(3a^3+2a^2bx-24ab^2x^2+16b^3x^3\right)\right)}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-(Sqrt[b]*Sqrt[x]*(3*a^3 + 2*a^2*b*x - 24*a*b^2*x^2 + 16*b^
3*x^3)) + (3*a^(7/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))
/(64*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 104, normalized size = 0.84

$$\frac{3a^4\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{64b^3}+\frac{\sqrt{a-bx}\,\left(-3a^3\sqrt{x}-2a^2bx^{3/2}+24ab^2x^{5/2}-16b^3x^{7/2}\right)}{64b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-3*a^3*Sqrt[x] - 2*a^2*b*x^(3/2) + 24*a*b^2*x^(5/2) - 16*b^
3*x^(7/2)))/(64*b^2) + (3*a^4*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b
*x]])/(64*b^3)
```

**fricas [A]** time = 1.12, size = 163, normalized size = 1.31

$$\left[-\frac{3\,a^4\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\left(16\,b^4x^3-24\,ab^3x^2+2\,a^2b^2x+3\,a^3b\right)\sqrt{-bx+a}\,\sqrt{x}}{128\,b^3},-\frac{3\,a^4\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(16\,b^4x^3-24\,ab^3x^2+2\,a^2b^2x+3\,a^3b\right)\sqrt{-bx+a}\,\sqrt{x}}{64\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(3*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x + a)*sqr
t(x))/b^3, -1/64*(3*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) +
(16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x + a)*sqrt(x))
/b^3]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 127, normalized size = 1.02

$$\frac{3\sqrt{(-bx+a)\,x}\;a^4\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{128\sqrt{-bx+a}\;b^{\frac{5}{2}}\sqrt{x}}+\frac{3\sqrt{-bx+a}\;a^3\sqrt{x}}{64b^2}+\frac{(-bx+a)^{\frac{3}{2}}\,a^2\sqrt{x}}{32b^2}-\frac{(-bx+a)^{\frac{5}{2}}\,x^{\frac{3}{2}}}{4b}-\frac{(-bx+a)^{\frac{5}{2}}\,a\sqrt{x}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(3/2),x)
```

```
[Out] -1/4/b*x^(3/2)*(-b*x+a)^(5/2)-1/8*a/b^2*x^(1/2)*(-b*x+a)^(5/2)+1/32*a^2/b^2
*(-b*x+a)^(3/2)*x^(1/2)+3/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^2+3/128*a^4/b^(5/
2)*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x
)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.97, size = 170, normalized size = 1.37

$$-\frac{3\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{64\,b^{\frac{5}{2}}}+\frac{\frac{3\,\sqrt{-bx+a}\,a^4b^3}{\sqrt{x}}+\frac{11\,(-bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}-\frac{11\,(-bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}-\frac{3\,(-bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{64\left(b^6-\frac{4\,(bx-a)b^5}{x}+\frac{6\,(bx-a)^2b^4}{x^2}-\frac{4\,(bx-a)^3b^3}{x^3}+\frac{(bx-a)^4b^2}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/64*(3*sqrt(-
b*x + a)*a^4*b^3/sqrt(x) + 11*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) - 11*(-b*x +
 a)^(5/2)*a^4*b/x^(5/2) - 3*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^6 - 4*(b*x - a
)*b^5/x + 6*(b*x - a)^2*b^4/x^2 - 4*(b*x - a)^3*b^3/x^3 + (b*x - a)^4*b^2/x
^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a-b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)*(a - b*x)^(3/2), x)
```

**sympy [A]** time = 9.06, size = 323, normalized size = 2.60

$$\begin{cases} \frac{3ia^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{-1+\frac{bx}{a}}}-\frac{13ia^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{-1+\frac{bx}{a}}}+\frac{5i\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}-\frac{ib^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1-\frac{bx}{a}}}+\frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1-\frac{bx}{a}}}-\frac{5\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{1-\frac{bx}{a}}}+\frac{3a^4\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}+\frac{b^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((3*I*a**(7/2)*sqrt(x)/(64*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x**
(3/2)/(64*b*sqrt(-1 + b*x/a)) - 13*I*a**(3/2)*x**(5/2)/(32*sqrt(-1 + b*x/a)
) + 5*I*sqrt(a)*b*x**(7/2)/(8*sqrt(-1 + b*x/a)) - 3*I*a**4*acosh(sqrt(b)*sq
rt(x)/sqrt(a))/(64*b**(5/2)) - I*b**2*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x/a))
, Abs(b*x/a) > 1), (-3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 - b*x/a)) + a**(5/2
)*x**(3/2)/(64*b*sqrt(1 - b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 - b*x/a
)) - 5*sqrt(a)*b*x**(7/2)/(8*sqrt(1 - b*x/a)) + 3*a**4*asin(sqrt(b)*sqrt(x)
/sqrt(a))/(64*b**(5/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a)), True))
```

# 3.529 $\int \sqrt{x}\,(a-bx)^{3/2}\,dx$

**Optimal**. Leaf size=99

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}-\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

**Rubi [A]** time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}-\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a - b*x)^(3/2),x]
```

```
[Out] -(a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b) + (a*x^(3/2)*Sqrt[a - b*x])/4 + (x^(3/2)
*(a - b*x)^(3/2))/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*b^(3
/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}(a-bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{1}{2}a\int \sqrt{x}\sqrt{a-bx}\,dx\\
&= \frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{1}{8}a^2\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{16b}\\
&= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{8b}\\
&= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{8b}\\
&= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 87, normalized size = 0.88

$$\frac{\sqrt{a-bx}\left(\frac{3a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}+\sqrt{b}\sqrt{x}\left(-3a^2+14abx-8b^2x^2\right)\right)}{24b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^2 + 14*a*b*x - 8*b^2*x^2) + (3*a^(5/2
)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(24*b^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 91, normalized size = 0.92

$$\frac{a^3\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{8b^2}+\frac{\sqrt{a-bx}\left(-3a^2\sqrt{x}+14abx^{3/2}-8b^2x^{5/2}\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-3*a^2*Sqrt[x] + 14*a*b*x^(3/2) - 8*b^2*x^(5/2)))/(24*b) +
(a^3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*b^2)
```

**fricas [A]** time = 1.16, size = 141, normalized size = 1.42

$$\left[-\frac{3\,a^{3}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\left(8\,b^{3}x^{2}-14\,ab^{2}x+3\,a^{2}b\right)\sqrt{-bx+a}\,\sqrt{x}}{48\,b^{2}},\,-\frac{3\,a^{3}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(8\,b^{3}x^{2}-14\,ab^{2}x+3\,a^{2}b\right)\sqrt{-bx+a}\,\sqrt{x}}{24\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*(8*b^3*x^2 - 14*a*b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/24*(
3*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (8*b^3*x^2 - 14*a*
b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 102, normalized size = 1.03

$$\frac{\sqrt{-bx+a}\;a\,x^{\frac{3}{2}}}{4}+\frac{\sqrt{\left(-bx+a\right)x}\;a^{3}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{16\sqrt{-bx+a}\;b^{\frac{3}{2}}\sqrt{x}}-\frac{\sqrt{-bx+a}\;a^{2}\sqrt{x}}{8b}+\frac{\left(-bx+a\right)^{\frac{3}{2}}x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)*x^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*(-b*x+a)^(3/2)+1/4*a*x^(3/2)*(-b*x+a)^(1/2)-1/8*a^2*x^(1/2)*(-b
*x+a)^(1/2)/b+1/16*a^3/b^(3/2)*((-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*ar
ctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

**maxima** [A] time = 2.99, size = 133, normalized size = 1.34

$$-\frac{a^3\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{8\,b^{\frac{3}{2}}}+\frac{\frac{3\sqrt{-bx+a}\,a^3b^2}{\sqrt{x}}+\frac{8\,(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}-\frac{3\,(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^4-\frac{3\,(bx-a)b^3}{x}+\frac{3\,(bx-a)^2b^2}{x^2}-\frac{(bx-a)^3b}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/24*(3*sqrt(-b
*x + a)*a^3*b^2/sqrt(x) + 8*(-b*x + a)^(3/2)*a^3*b/x^(3/2) - 3*(-b*x + a)^(
5/2)*a^3/x^(5/2))/(b^4 - 3*(b*x - a)*b^3/x + 3*(b*x - a)^2*b^2/x^2 - (b*x -
 a)^3*b/x^3)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{x}\,(a-b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(a - b*x)^(3/2), x)
```

**sympy** [A] time = 5.54, size = 264, normalized size = 2.67

$$\begin{cases}\frac{ia^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{-1+\frac{bx}{a}}}-\frac{17ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{-1+\frac{bx}{a}}}+\frac{11i\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}-\frac{ib^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1\\ -\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1-\frac{bx}{a}}}+\frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1-\frac{bx}{a}}}-\frac{11\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}}+\frac{a^3\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{b^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)*x**(1/2),x)
```

```
[Out] Piecewise((I*a**(5/2)*sqrt(x)/(8*b*sqrt(-1 + b*x/a)) - 17*I*a**(3/2)*x**(3/
2)/(24*sqrt(-1 + b*x/a)) + 11*I*sqrt(a)*b*x**(5/2)/(12*sqrt(-1 + b*x/a)) -
I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)) - I*b**2*x**(7/2)/(3*sqr
t(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(5/2)*sqrt(x)/(8*b*sqrt(1 - b
*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 - b*x/a)) - 11*sqrt(a)*b*x**(5/2)/
(12*sqrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)) + b*
*2*x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

## 3.530 $\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=74

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\sqrt{a-bx}+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}$$

**Rubi [A]** time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\sqrt{a-bx}+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*a*Sqrt[x]*Sqrt[a - b*x])/4 + (Sqrt[x]*(a - b*x)^(3/2))/2 + (3*a^2*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}(3a)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 71, normalized size = 0.96

$$\frac{1}{4}\sqrt{a-bx}\left(\frac{3a^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{1-\frac{bx}{a}}}+\sqrt{x}\,(5a-2bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[x]*(5*a - 2*b*x) + (3*a^(3/2)*ArcSin[(Sqrt[b]*Sqrt[x])
/Sqrt[a]])/(Sqrt[b]*Sqrt[1 - (b*x)/a])))/4
```

**IntegrateAlgebraic [A]** time = 0.10, size = 75, normalized size = 1.01

$$\frac{3a^2\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{4b}+\frac{1}{4}\sqrt{a-bx}\,\left(5a\sqrt{x}-2bx^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a - b*x]*(5*a*Sqrt[x] - 2*b*x^(3/2)))/4 + (3*a^2*Sqrt[-b]*Log[-(Sqrt[
-b]*Sqrt[x]) + Sqrt[a - b*x]])/(4*b)
```

**fricas [A]** time = 1.43, size = 119, normalized size = 1.61

$$\left[-\frac{3\,a^2\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,\left(2\,b^2x-5\,ab\right)\sqrt{-bx+a}\,\sqrt{x}}{8\,b},\,-\frac{3\,a^2\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(2\,b^2x-5\,ab\right)\sqrt{-bx+a}\,\sqrt{x}}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*(2*b^2*x - 5*a*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(b)*arctan(
sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x - 5*a*b)*sqrt(-b*x + a)*sqrt(x
))/b]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.00, size = 83, normalized size = 1.12

$$\frac{3\sqrt{(-bx+a)\,x}\;a^2\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{8\sqrt{-bx+a}\;\sqrt{b}\;\sqrt{x}}+\frac{3\sqrt{-bx+a}\;a\sqrt{x}}{4}+\frac{(-bx+a)^{\frac{3}{2}}\;\sqrt{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*x^(1/2)*(-b*x+a)^(1/2)+3/8*a^2*((-b*x+a)*x
)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2
)*b^(1/2))
```

**maxima [A]** time = 2.90, size = 93, normalized size = 1.26

$$-\frac{3\,a^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,\sqrt{b}}+\frac{\frac{3\,\sqrt{-bx+a}\,a^2b}{\sqrt{x}}+\frac{5\,(-bx+a)^{\frac{3}{2}}\,a^2}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2\,(bx-a)b}{x}+\frac{(bx-a)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/4*(3*sqrt(-b*
x + a)*a^2*b/sqrt(x) + 5*(-b*x + a)^(3/2)*a^2/x^(3/2))/(b^2 - 2*(b*x - a)*b
/x + (b*x - a)^2/x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(1/2),x)
```

```
[Out] int((a - b*x)^(3/2)/x^(1/2), x)
```

**sympy [A]** time = 3.21, size = 190, normalized size = 2.57

$$\begin{cases} -\frac{5ia^{\frac{3}{2}}\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}} + \frac{7i\sqrt{a}\,bx^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}} - \frac{3ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}} - \frac{ib^2x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{5a^{\frac{3}{2}}\sqrt{x}\sqrt{1-\frac{bx}{a}}}{4} - \frac{\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{2} + \frac{3a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((-5*I*a**(3/2)*sqrt(x)/(4*sqrt(-1 + b*x/a)) + 7*I*sqrt(a)*b*x**(3
/2)/(4*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(
b)) - I*b**2*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (5*a**
(3/2)*sqrt(x)*sqrt(1 - b*x/a)/4 - sqrt(a)*b*x**(3/2)*sqrt(1 - b*x/a)/2 + 3*
a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b)), True))
```

## 3.531 $\int \frac{(a-bx)^{3/2}}{x^{3/2}} \, dx$

**Optimal**. Leaf size=66

$$-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\sqrt{a-bx}-3a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\sqrt{a-bx}-3a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] -3*b*Sqrt[x]*Sqrt[a - b*x] - (2*(a - b*x)^(3/2))/Sqrt[x] - 3*a*Sqrt[b]*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{\sqrt{x}}-(3b)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= -3b\sqrt{x}\,\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-\frac{1}{2}(3ab)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= -3b\sqrt{x}\,\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-(3ab)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= -3b\sqrt{x}\,\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-(3ab)\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= -3b\sqrt{x}\,\sqrt{a-bx}-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 47, normalized size = 0.71

$$
-\frac{2a\sqrt{a-bx}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{a}\right)}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a - b*x]*Hypergeometric2F1[-3/2, -1/2, 1/2, (b*x)/a])/(Sqrt[x]*S
qrt[1 - (b*x)/a])
```

**IntegrateAlgebraic [A]** time = 0.13, size = 61, normalized size = 0.92

$$\frac{(-2a-bx)\sqrt{a-bx}}{\sqrt{x}}-3a\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] ((-2*a - b*x)*Sqrt[a - b*x])/Sqrt[x] - 3*a*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x])
 + Sqrt[a - b*x]]
```

**fricas [A]** time = 1.15, size = 109, normalized size = 1.65

$$\left[\frac{3\,a\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}\,+a\right)-2\,(bx+2\,a)\sqrt{-bx+a}\,\sqrt{x}}{2\,x},\,\frac{3\,a\sqrt{b}\,x\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(bx+2\,a)\sqrt{-bx+a}\,\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) -
2*(b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x, (3*a*sqrt(b)*x*arctan(sqrt(-b*x +
a)/(sqrt(b)*sqrt(x))) - (b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(3/2),x)
```

```
[Out] int((-b*x+a)^(3/2)/x^(3/2),x)
```

**maxima [A]** time = 2.85, size = 68, normalized size = 1.03

$$3\,a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\,\sqrt{-bx+a}\,a}{\sqrt{x}}-\frac{\sqrt{-bx+a}\,ab}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 3*a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a)*a/s
qrt(x) - sqrt(-b*x + a)*a*b/((b - (b*x - a)/x)*sqrt(x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a-b\,x)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(3/2),x)
```

```
[Out] int((a - b*x)^(3/2)/x^(3/2), x)
```

**sympy [A]** time = 2.88, size = 197, normalized size = 2.98

$$\begin{cases} \frac{2ia^{\frac{3}{2}}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}}-\frac{i\sqrt{a}\,b\sqrt{x}}{\sqrt{-1+\frac{bx}{a}}}+3ia\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)-\frac{ib^{2}x^{\frac{3}{2}}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{2a^{\frac{3}{2}}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}\,b\sqrt{x}}{\sqrt{1-\frac{bx}{a}}}-3a\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)+\frac{b^{2}x^{\frac{3}{2}}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*a**(3/2)/(sqrt(x)*sqrt(-1 + b*x/a)) - I*sqrt(a)*b*sqrt(x)/sq
rt(-1 + b*x/a) + 3*I*a*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a)) - I*b**2*x**(
3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*a**(3/2)/(sqrt(x)*sqr
t(1 - b*x/a)) + sqrt(a)*b*sqrt(x)/sqrt(1 - b*x/a) - 3*a*sqrt(b)*asin(sqrt(b
)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

## 3.532 $\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx$

**Optimal**. Leaf size=67

$$2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{a-bx}}{\sqrt{x}}$$

**Rubi [A]** time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 63, 217, 203}

$$2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{a-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[a - b*x])/Sqrt[x] - (2*(a - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{3x^{3/2}} - b\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right) \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right) \\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 49, normalized size = 0.73

$$-\frac{2a\sqrt{a-bx}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{a}\right)}{3x^{3/2}\sqrt{1-\frac{bx}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a - b*x]*Hypergeometric2F1[-3/2, -3/2, -1/2, (b*x)/a])/(3*x^(3/2
)*Sqrt[1 - (b*x)/a])
```

**IntegrateAlgebraic [A]** time = 0.14, size = 62, normalized size = 0.93

$$2\sqrt{-b}\,b\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)-\frac{2(a-4bx)\sqrt{a-bx}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*(a - 4*b*x)*Sqrt[a - b*x])/(3*x^(3/2)) + 2*Sqrt[-b]*b*Log[-(Sqrt[-b]*Sq
rt[x]) + Sqrt[a - b*x]]
```

**fricas [A]** time = 1.62, size = 115, normalized size = 1.72

$$\left[\frac{3\sqrt{-b}\,bx^2\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(4\,bx-a)\sqrt{-bx+a}\,\sqrt{x}}{3\,x^2},\,-\frac{2\left(3\,b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(4\,bx-a)\sqrt{-bx+a}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*(4*b*x - a)*sqrt(-b*x + a)*sqrt(x))/x^2, -2/3*(3*b^(3/2)*x^2*arctan(sqr
t(-b*x + a)/(sqrt(b)*sqrt(x))) - (4*b*x - a)*sqrt(-b*x + a)*sqrt(x))/x^2]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(5/2),x)
```

```
[Out] int((-b*x+a)^(3/2)/x^(5/2),x)
```

**maxima [A]** time = 2.86, size = 49, normalized size = 0.73

$$-2\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{2\,\sqrt{-bx+a}\,b}{\sqrt{x}}-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2*b^(3/2)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + 2*sqrt(-b*x + a)*b/sq
rt(x) - 2/3*(-b*x + a)^(3/2)/x^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(5/2),x)
```

```
[Out] int((a - b*x)^(3/2)/x^(5/2), x)
```

**sympy [C]** time = 3.24, size = 187, normalized size = 2.79

$$\begin{cases} -\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x} + \frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3} - 2ib^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right) + 2b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{2ia\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x} + \frac{8ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3} + ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right) - 2ib^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 8*b**(3/2)*sqrt(a/(b*x) -
 1)/3 - 2*I*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + I*b**(3/2)*log(a/(b*x
)) + 2*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)), Abs(a/(b*x)) > 1), (-2*I*a*s
qrt(b)*sqrt(-a/(b*x) + 1)/(3*x) + 8*I*b**(3/2)*sqrt(-a/(b*x) + 1)/3 + I*b**
(3/2)*log(a/(b*x)) - 2*I*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1), True))
```

## 3.533 $\int x^{5/2}(2+bx)^{3/2}\,dx$

**Optimal**. Leaf size=126

$$-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{3\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{x^{3/2}\sqrt{bx+2}}{8b^2}+\frac{1}{5}x^{7/2}(bx+2)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{20b}$$

**Rubi [A]** time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{x^{3/2}\sqrt{bx+2}}{8b^2}+\frac{3\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{5}x^{7/2}(bx+2)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 + b*x])/(8*b^2) + (x^(5
/2)*Sqrt[2 + b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 + b*x])/20 + (x^(7/2)*(2 + b*
x)^(3/2))/5 - (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{5/2}(2+bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(2+bx)^{3/2} + \frac{3}{5}\int x^{5/2}\sqrt{2+bx}\,dx$$

$$= \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)^{3/2} + \frac{3}{20}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx$$

$$= \frac{x^{5/2}\sqrt{2+bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)^{3/2} - \frac{\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{4b}$$

$$= -\frac{x^{3/2}\sqrt{2+bx}}{8b^2} + \frac{x^{5/2}\sqrt{2+bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)^{3/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b^2}$$

$$= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{x^{3/2}\sqrt{2+bx}}{8b^2} + \frac{x^{5/2}\sqrt{2+bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)^{3/2} - \frac{3\int}{}$$

$$= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{x^{3/2}\sqrt{2+bx}}{8b^2} + \frac{x^{5/2}\sqrt{2+bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)^{3/2} - \frac{3\,\mathrm{Su}}{}$$

$$= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{x^{3/2}\sqrt{2+bx}}{8b^2} + \frac{x^{5/2}\sqrt{2+bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2+bx} + \frac{1}{5}x^{7/2}(2+bx)^{3/2} - \frac{3\sin}{}$$

**Mathematica [A]** time = 0.05, size = 78, normalized size = 0.62

$$\frac{\sqrt{x}\sqrt{bx+2}\left(8b^4x^4+22b^3x^3+2b^2x^2-5bx+15\right)}{40b^3} - \frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 22*b^3*x^3 + 8*b^4*x^4))/(
40*b^3) - (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 95, normalized size = 0.75

$$\frac{3\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{4b^{7/2}} + \frac{\sqrt{bx+2}\left(8b^4x^{9/2}+22b^3x^{7/2}+2b^2x^{5/2}-5bx^{3/2}+15\sqrt{x}\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(15*Sqrt[x] - 5*b*x^(3/2) + 2*b^2*x^(5/2) + 22*b^3*x^(7/2) +
 8*b^4*x^(9/2)))/(40*b^3) + (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*
b^(7/2))
```

**fricas [A]** time = 1.18, size = 156, normalized size = 1.24

$$\left[\frac{\left(8\,b^{5}x^{4}+22\,b^{4}x^{3}+2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{40\,b^{4}}, \frac{\left(8\,b^{5}x^{4}+22\,b^{4}x^{3}+2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{40\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*
sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/4
0*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt
(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^4]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [82.7280518
371,8.05231268331]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
```

2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [64.3995612673,28.4266860783]Warning, choosing root of [1,0,%%%
{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2
,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%
%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,
3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%
{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2
]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+
%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6
,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%
%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{
-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1
,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%
%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{
16,[0,0]%%%}] at parameters values [39.1803401988,96.7771189027]Warning, ch
oosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{
-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%
%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24
,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%
%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{
4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,
3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}
+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[
3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%
{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1
,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}
+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [95.5969694792,66
.1769613782]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%
}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}
+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,
[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%
%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{
16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0
]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%
%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[
3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%
}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{
-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,

3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters v
alues [39.9828299829,94.1262030317]Warning, choosing root of [1,0,%%%{-4,[1
,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%
}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1
,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}
+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[
2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+
%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16
,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]
%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%
{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2
,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%
}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%
{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,
0]%%%}] at parameters values [88.2886286299,17.6881634681]1/b*(2*b^2*abs(b)
/b^2*(2*(((((5040*b^19/100800/b^23*sqrt(b*x+2)*sqrt(b*x+2)-51660*b^19/100800
/b^23)*sqrt(b*x+2)*sqrt(b*x+2)+215460*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*
x+2)-469350*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*x+2)+607950*b^19/100800/b^
23)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+63/8/b^3/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-
2*b)-sqrt(b)*sqrt(b*x+2))))+8*b*abs(b)/b^2*(2*(((90*b^11/1440/b^14*sqrt(b*x
+2)*sqrt(b*x+2)-750*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)+2445*b^11/1440/
b^14)*sqrt(b*x+2)*sqrt(b*x+2)-4185*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*(b*x+
2)-2*b)-35/8/b^2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+
8*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sq
rt(b*x+2)*sqrt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/
b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))

**maple [A]** time = 0.01, size = 123, normalized size = 0.98

$$\frac{(bx+2)^{\frac{5}{2}} x^{\frac{5}{2}}}{5b}-\frac{(bx+2)^{\frac{5}{2}} x^{\frac{3}{2}}}{4b^{2}}+\frac{(bx+2)^{\frac{5}{2}} \sqrt{x}}{4b^{3}}-\frac{(bx+2)^{\frac{3}{2}} \sqrt{x}}{8b^{3}}-\frac{3\sqrt{bx+2}\,\sqrt{x}}{8b^{3}}-\frac{3\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{8\sqrt{bx+2}\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(b*x+2)^(3/2),x)

[Out] 1/5/b*x^(5/2)*(b*x+2)^(5/2)-1/4/b^2*x^(3/2)*(b*x+2)^(5/2)+1/4/b^3*x^(1/2)*(
b*x+2)^(5/2)-1/8*(b*x+2)^(3/2)/b^3*x^(1/2)-3/8*(b*x+2)^(1/2)/b^3*x^(1/2)-3/
8*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(7/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2
+2*x)^(1/2))

**maxima [B]** time = 2.99, size = 194, normalized size = 1.54

$$\frac{\frac{15\sqrt{bx+2}\,b^4}{\sqrt{x}}-\frac{70\,(bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}-\frac{128\,(bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}+\frac{70\,(bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15\,(bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^8-\frac{5\,(bx+2)b^7}{x}+\frac{10\,(bx+2)^2b^6}{x^2}-\frac{10\,(bx+2)^3b^5}{x^3}+\frac{5\,(bx+2)^4b^4}{x^4}-\frac{(bx+2)^5b^3}{x^5}\right)}+\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(b*x + 2)*b^4/sqrt(x) - 70*(b*x + 2)^(3/2)*b^3/x^(3/2) - 128*(
b*x + 2)^(5/2)*b^2/x^(5/2) + 70*(b*x + 2)^(7/2)*b/x^(7/2) - 15*(b*x + 2)^(9
/2)/x^(9/2))/(b^8 - 5*(b*x + 2)*b^7/x + 10*(b*x + 2)^2*b^6/x^2 - 10*(b*x +
2)^3*b^5/x^3 + 5*(b*x + 2)^4*b^4/x^4 - (b*x + 2)^5*b^3/x^5) + 3/8*log(-(sqr
t(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(b\,x+2)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(5/2)*(b*x + 2)^(3/2), x)
```

**sympy [A]** time = 15.67, size = 136, normalized size = 1.08

$$\frac{b^2x^{\frac{11}{2}}}{5\sqrt{bx+2}}+\frac{19bx^{\frac{9}{2}}}{20\sqrt{bx+2}}+\frac{23x^{\frac{7}{2}}}{20\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{40b\sqrt{bx+2}}+\frac{x^{\frac{3}{2}}}{8b^2\sqrt{bx+2}}+\frac{3\sqrt{x}}{4b^3\sqrt{bx+2}}-\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(3/2),x)
```

```
[Out] b**2*x**(11/2)/(5*sqrt(b*x + 2)) + 19*b*x**(9/2)/(20*sqrt(b*x + 2)) + 23*x*
*(7/2)/(20*sqrt(b*x + 2)) - x**(5/2)/(40*b*sqrt(b*x + 2)) + x**(3/2)/(8*b**
2*sqrt(b*x + 2)) + 3*sqrt(x)/(4*b**3*sqrt(b*x + 2)) - 3*asinh(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(4*b**(7/2))
```

## 3.534 $\int x^{3/2}(2+bx)^{3/2}\,dx$

**Optimal**. Leaf size=105

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

**Rubi [A]** time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (3*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}(2+bx)^{3/2}\,dx &= \frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{3}{4}\int x^{3/2}\sqrt{2+bx}\,dx \\
&= \frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx \\
&= \frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}-\frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b} \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{3\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{8b^2} \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,d\right.}{4b^2} \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 70, normalized size = 0.67

$$
\frac{\sqrt{b}\sqrt{x}\sqrt{bx+2}\left(2b^3x^3+6b^2x^2+bx-3\right)+6\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 6*b^2*x^2 + 2*b^3*x^3) + 6*ArcSi
nh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(8*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 84, normalized size = 0.80

$$
\frac{\sqrt{bx+2}\left(2b^3x^{7/2}+6b^2x^{5/2}+bx^{3/2}-3\sqrt{x}\right)}{8b^2}-\frac{3\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{4b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(-3*Sqrt[x] + b*x^(3/2) + 6*b^2*x^(5/2) + 2*b^3*x^(7/2)))/(8
*b^2) - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(5/2))
```

**fricas [A]** time = 1.31, size = 137, normalized size = 1.30

$$\left[\frac{\left(2\,b^{4}x^{3}+6\,b^{3}x^{2}+b^{2}x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{8\,b^{3}},\ \frac{\left(2\,b^{4}x^{3}+6\,b^{3}x^{2}+b^{2}x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{8\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((2*b^4*x^3 + 6*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(
b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/8*((2*b^4*x^3 + 6*b
^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [82.7280518
371,8.05231268331]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
```

,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [64.3995612673,28.4266860783]Warning, choosing root of [1,0,%%%
{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2
,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%
%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,
3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%
{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2
]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+
%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6
,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%
%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{
-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1
,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%
%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{
16,[0,0]%%%}] at parameters values [39.1803401988,96.7771189027]Warning, ch
oosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{
-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%
%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24
,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%
%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{
4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,
3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}
+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[
3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%
{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1
,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}
+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [95.5969694792,66
.1769613782]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%
}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}
+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,
[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%
%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{
16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0
]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%
%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[
3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%
}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{
-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,
3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters v
alues [39.9828299829,94.1262030317]Warning, choosing root of [1,0,%%%{-4,[1
,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%

```
}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1
,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}
+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[
2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+
%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16
,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]
%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%
{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2
,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%
}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%
{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,
0]%%%}] at parameters values [88.2886286299,17.6881634681]1/b*(2*b^2*abs(b)
/b^2*(2*(((90*b^11/1440/b^14*sqrt(b*x+2)*sqrt(b*x+2)-750*b^11/1440/b^14)*sq
rt(b*x+2)*sqrt(b*x+2)+2445*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)-4185*b^1
1/1440/b^14)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-35/8/b^2/sqrt(b)*ln(abs(sqrt(b
*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+8*b*abs(b)/b^2*(2*((12*b^5/144/b^7*sqr
t(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*x+2)+198*b^5/144/b^
7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)
-sqrt(b)*sqrt(b*x+2))))+8*abs(b)/b^2/b*(2*(1/8*sqrt(b*x+2)*sqrt(b*x+2)-5/8)
*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-s
qrt(b)*sqrt(b*x+2)))))
```

**maple [A]** time = 0.00, size = 108, normalized size = 1.03

$$\frac{(bx+2)^{\frac{5}{2}} x^{\frac{3}{2}}}{4b}-\frac{(bx+2)^{\frac{5}{2}} \sqrt{x}}{4b^{2}}+\frac{(bx+2)^{\frac{3}{2}} \sqrt{x}}{8b^{2}}+\frac{3\sqrt{bx+2}\,\sqrt{x}}{8b^{2}}+\frac{3\sqrt{(bx+2)\,x}\ \ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{8\sqrt{bx+2}\;b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(3/2),x)
```

```
[Out] 1/4/b*x^(3/2)*(b*x+2)^(5/2)-1/4/b^2*x^(1/2)*(b*x+2)^(5/2)+1/8*(b*x+2)^(3/2)
/b^2*x^(1/2)+3/8*(b*x+2)^(1/2)/b^2*x^(1/2)+3/8*((b*x+2)*x)^(1/2)/(b*x+2)^(1
/2)/b^(5/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [B]** time = 3.04, size = 163, normalized size = 1.55

$$-\frac{\frac{3\sqrt{bx+2}\,b^{3}}{\sqrt{x}}-\frac{11\,(bx+2)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}-\frac{11\,(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{3\,(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{4\left(b^{6}-\frac{4\,(bx+2)b^{5}}{x}+\frac{6\,(bx+2)^{2}b^{4}}{x^{2}}-\frac{4\,(bx+2)^{3}b^{3}}{x^{3}}+\frac{(bx+2)^{4}b^{2}}{x^{4}}\right)}-\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(3*sqrt(b*x + 2)*b^3/sqrt(x) - 11*(b*x + 2)^(3/2)*b^2/x^(3/2) - 11*(b*
x + 2)^(5/2)*b/x^(5/2) + 3*(b*x + 2)^(7/2)/x^(7/2))/(b^6 - 4*(b*x + 2)*b^5/
x + 6*(b*x + 2)^2*b^4/x^2 - 4*(b*x + 2)^3*b^3/x^3 + (b*x + 2)^4*b^2/x^4) -
3/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)
))/b^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(b\,x+2)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(3/2)*(b*x + 2)^(3/2), x)
```

**sympy [A]** time = 7.86, size = 117, normalized size = 1.11

$$\frac{b^2x^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{5bx^{\frac{7}{2}}}{4\sqrt{bx+2}}+\frac{13x^{\frac{5}{2}}}{8\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}}-\frac{3\sqrt{x}}{4b^2\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(3/2),x)
```

```
[Out] b**2*x**(9/2)/(4*sqrt(b*x + 2)) + 5*b*x**(7/2)/(4*sqrt(b*x + 2)) + 13*x**(5
/2)/(8*sqrt(b*x + 2)) - x**(3/2)/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*sq
rt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

## 3.535 $\int \sqrt{x}(2+bx)^{3/2}\,dx$

**Optimal**. Leaf size=82

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\sqrt{bx+2}}{2b}$$

**Rubi** [A] time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 + (x^(3/2)*(2 + b
*x)^(3/2))/3 - ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{x}(2+bx)^{3/2}\,dx = \frac{1}{3}x^{3/2}(2+bx)^{3/2} + \int \sqrt{x}\sqrt{2+bx}\,dx$$
$$= \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} - \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b}$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{\sqrt{x}\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{3}x^{3/2}(2+bx)^{3/2} - \frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

**Mathematica [A]** time = 0.03, size = 60, normalized size = 0.73

$$\frac{\sqrt{x}\sqrt{bx+2}\left(2b^2x^2+7bx+3\right)}{6b} - \frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(3 + 7*b*x + 2*b^2*x^2))/(6*b) - ArcSinh[(Sqrt[b]*Sq
rt[x])/Sqrt[2]]/b^(3/2)
```

**IntegrateAlgebraic [A]** time = 0.09, size = 72, normalized size = 0.88

$$\frac{\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{3/2}} + \frac{\sqrt{bx+2}\left(2b^2x^{5/2}+7bx^{3/2}+3\sqrt{x}\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(3*Sqrt[x] + 7*b*x^(3/2) + 2*b^2*x^(5/2)))/(6*b) + Log[-(Sqr
t[b]*Sqrt[x]) + Sqrt[2 + b*x]]/b^(3/2)
```

**fricas [A]** time = 1.41, size = 124, normalized size = 1.51

$$\left[\frac{\left(2\,b^3x^2+7\,b^2x+3\,b\right)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b^2}, \frac{\left(2\,b^3x^2+7\,b^2x+3\,b\right)\sqrt{bx+2}\,\sqrt{x}+6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + 7*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x
 - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/6*((2*b^3*x^2 + 7*b^2*x + 3*b
)*sqrt(b*x + 2)*sqrt(x) + 6*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(
x))))/b^2]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [82.7280518
371,8.05231268331]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [64.3995612673,28.4266860783]Warning, choosing root of [1,0,%%%
{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2
```

,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%
%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,
3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%
{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2
]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+
%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6
,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%
%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{
-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1
,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%
%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{
16,[0,0]%%%}] at parameters values [39.1803401988,96.7771189027]Warning, ch
oosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{
-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%
%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24
,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%
%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{
4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,
3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}
+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[
3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%
{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1
,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}
+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [95.5969694792,66
.1769613782]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%
}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}
+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,
[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%
%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{
16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0
]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%
%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[
3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%
}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{
-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,
3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters v
alues [39.9828299829,94.1262030317]Warning, choosing root of [1,0,%%%{-4,[1
,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%
}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1
,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}
+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[
2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+
%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16

,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [88.2886286299,17.6881634681]1/b*(2*b^2*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+8*b*abs(b)/b^2/b*(2*(1/8*sqrt(b*x+2)*sqrt(b*x+2)-5/8)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+8*abs(b)/b^2*(1/2*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+2*b/2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))

**maple** [A] time = 0.00, size = 87, normalized size = 1.06

$$\frac{(bx+2)^{\frac{3}{2}} x^{\frac{3}{2}}}{3}+\frac{\sqrt{bx+2}\, x^{\frac{3}{2}}}{2}+\frac{\sqrt{bx+2}\, \sqrt{x}}{2b}-\frac{\sqrt{(bx+2)\,x}\, \ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\, b^{\frac{3}{2}} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+2)^(3/2)*x^(1/2),x)

[Out] 1/3*x^(3/2)*(b*x+2)^(3/2)+1/2*(b*x+2)^(1/2)*x^(3/2)+1/2*(b*x+2)^(1/2)/b*x^(1/2)-1/2*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(3/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))

**maxima** [B] time = 2.99, size = 132, normalized size = 1.61

$$\frac{\frac{3\,\sqrt{bx+2}\,b^{2}}{\sqrt{x}}-\frac{8\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{4}-\frac{3\,(bx+2)b^{3}}{x}+\frac{3\,(bx+2)^{2}b^{2}}{x^{2}}-\frac{(bx+2)^{3}b}{x^{3}}\right)}+\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="maxima")

[Out] 1/3*(3*sqrt(b*x + 2)*b^2/sqrt(x) - 8*(b*x + 2)^(3/2)*b/x^(3/2) - 3*(b*x + 2)^(5/2)/x^(5/2))/(b^4 - 3*(b*x + 2)*b^3/x + 3*(b*x + 2)^2*b^2/x^2 - (b*x + 2)^3*b/x^3) + 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(3/2)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,(b\,x+2)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(1/2)*(b*x + 2)^(3/2), x)
```

**sympy [A]** time = 4.81, size = 92, normalized size = 1.12

$$\frac{b^2x^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{11bx^{\frac{5}{2}}}{6\sqrt{bx+2}}+\frac{17x^{\frac{3}{2}}}{6\sqrt{bx+2}}+\frac{\sqrt{x}}{b\sqrt{bx+2}}-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)*x**(1/2),x)
```

```
[Out] b**2*x**(7/2)/(3*sqrt(b*x + 2)) + 11*b*x**(5/2)/(6*sqrt(b*x + 2)) + 17*x**(
3/2)/(6*sqrt(b*x + 2)) + sqrt(x)/(b*sqrt(b*x + 2)) - asinh(sqrt(2)*sqrt(b)*
sqrt(x)/2)/b**(3/2)
```

# 3.536 $\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=61

$$\frac{1}{2}\sqrt{x}\,(bx+2)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$\frac{1}{2}\sqrt{x}\,(bx+2)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/2 + (Sqrt[x]*(2 + b*x)^(3/2))/2 + (3*ArcSinh[(Sqr
t[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx = \frac{1}{2}\sqrt{x}\,(2+bx)^{3/2} + \frac{3}{2}\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx$$
$$= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx} + \frac{1}{2}\sqrt{x}\,(2+bx)^{3/2} + \frac{3}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx$$
$$= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx} + \frac{1}{2}\sqrt{x}\,(2+bx)^{3/2} + 3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx} + \frac{1}{2}\sqrt{x}\,(2+bx)^{3/2} + \frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Mathematica [A]** time = 0.03, size = 48, normalized size = 0.79

$$\frac{1}{2}\sqrt{x}\,\sqrt{bx+2}\,(bx+5) + \frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(5 + b*x))/2 + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
)/Sqrt[b]
```

**IntegrateAlgebraic [A]** time = 0.08, size = 59, normalized size = 0.97

$$\frac{1}{2}\sqrt{bx+2}\,\left(bx^{3/2}+5\sqrt{x}\right) - \frac{3\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[2 + b*x]*(5*Sqrt[x] + b*x^(3/2)))/2 - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqr
t[2 + b*x]])/Sqrt[b]
```

**fricas [A]** time = 1.51, size = 105, normalized size = 1.72

$$\left[\frac{\left(b^2x+5\,b\right)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{2\,b}, \frac{\left(b^2x+5\,b\right)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
2)*sqrt(b)*sqrt(x) + 1))/b, 1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) - 6*sq
rt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]1/abs(b)*b^2/b*(2*(1/4/b*sqrt(b*x+2
)*sqrt(b*x+2)+3/4/b)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-3/sqrt(b)*ln(abs(sqrt(
b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
```

**maple [A]** time = 0.00, size = 72, normalized size = 1.18

$$\frac{(bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{bx+2}\,\sqrt{x}}{2}+\frac{3\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(b*x+2)^(3/2)*x^(1/2)+3/2*(b*x+2)^(1/2)*x^(1/2)+3/2*((b*x+2)*x)^(1/2)/(
b*x+2)^(1/2)/b^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [B]** time = 2.89, size = 98, normalized size = 1.61

$$-\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,\sqrt{b}}-\frac{\frac{3\,\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{5\,(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{2}-\frac{2\,(bx+2)b}{x}+\frac{(bx+2)^{2}}{x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x
)))/sqrt(b) - (3*sqrt(b*x + 2)*b/sqrt(x) - 5*(b*x + 2)^(3/2)/x^(3/2))/(b^2
- 2*(b*x + 2)*b/x + (b*x + 2)^2/x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(1/2),x)
```

```
[Out] int((b*x + 2)^(3/2)/x^(1/2), x)
```

**sympy [A]** time = 2.82, size = 76, normalized size = 1.25

$$\frac{b^{2}x^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{7bx^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{5\sqrt{x}}{\sqrt{bx+2}}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] b**2*x**(5/2)/(2*sqrt(b*x + 2)) + 7*b*x**(3/2)/(2*sqrt(b*x + 2)) + 5*sqrt(x
)/sqrt(b*x + 2) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

## 3.537 $\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx$

**Optimal**. Leaf size=58

$$-\frac{2(bx+2)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\,\sqrt{bx+2}+6\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

**Rubi** [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$-\frac{2(bx+2)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\,\sqrt{bx+2}+6\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] 3*b*Sqrt[x]*Sqrt[2 + b*x] - (2*(2 + b*x)^(3/2))/Sqrt[x] + 6*Sqrt[b]*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(2+bx)^{3/2}}{\sqrt{x}} + (3b)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx \\
&= 3b\sqrt{x}\sqrt{2+bx} - \frac{2(2+bx)^{3/2}}{\sqrt{x}} + (3b)\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx \\
&= 3b\sqrt{x}\sqrt{2+bx} - \frac{2(2+bx)^{3/2}}{\sqrt{x}} + (6b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right) \\
&= 3b\sqrt{x}\sqrt{2+bx} - \frac{2(2+bx)^{3/2}}{\sqrt{x}} + 6\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.00, size = 28, normalized size = 0.48

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{2}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/x^(3/2),x]
[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -1/2*(b*x)])/Sqrt[x]
```

**IntegrateAlgebraic [A]** time = 0.10, size = 51, normalized size = 0.88

$$\frac{(bx-4)\sqrt{bx+2}}{\sqrt{x}} - 6\sqrt{b}\,\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(3/2)/x^(3/2),x]
[Out] ((-4 + b*x)*Sqrt[2 + b*x])/Sqrt[x] - 6*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqr
t[2 + b*x]]
```

**fricas [A]** time = 1.23, size = 99, normalized size = 1.71

$$\left[\frac{3\sqrt{b}\,x\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}\,+1\right)+\sqrt{bx+2}\,(bx-4)\sqrt{x}}{x},-\frac{6\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+2}\,(bx-4)\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + sqrt(b*x + 2)*
(b*x - 4)*sqrt(x))/x, -(6*sqrt(-b)*x*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(
x))) - sqrt(b*x + 2)*(b*x - 4)*sqrt(x))/x]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
```

```
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]b/abs(b)*b^2/b*(2*(1/2*sqrt(b*x+2)*
sqrt(b*x+2)-3)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)/(b*(b*x+2)-2*b)-6/sqrt(b)*ln
(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
```

**maple [A]** time = 0.02, size = 72, normalized size = 1.24

$$\frac{3\sqrt{(bx+2)\,x}\,\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}}+\frac{b^{2}x^{2}-2bx-8}{\sqrt{bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(3/2),x)
```

```
[Out] (b^2*x^2-2*b*x-8)/(b*x+2)^(1/2)/x^(1/2)+3*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)*b
^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [A]** time = 2.91, size = 81, normalized size = 1.40

$$-3\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{4\sqrt{bx+2}}{\sqrt{x}}-\frac{2\sqrt{bx+2}\,b}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -3*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x))) - 4*sqrt(b*x + 2)/sqrt(x) - 2*sqrt(b*x + 2)*b/((b - (b*x + 2)/x)*
sqrt(x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(3/2)/x^(3/2), x)
```

**sympy [A]** time = 2.44, size = 73, normalized size = 1.26

$$6\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)+\frac{b^{2}x^{\frac{3}{2}}}{\sqrt{bx+2}}-\frac{2b\sqrt{x}}{\sqrt{bx+2}}-\frac{8}{\sqrt{x}\,\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(3/2),x)
```

```
[Out] 6*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**2*x**(3/2)/sqrt(b*x + 2) -
2*b*sqrt(x)/sqrt(b*x + 2) - 8/(sqrt(x)*sqrt(b*x + 2))
```

## 3.538 $\int \frac{(2+bx)^{3/2}}{x^{5/2}} dx$

**Optimal**. Leaf size=60

$$2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{bx+2}}{\sqrt{x}}$$

**Rubi [A]** time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 54, 215}

$$2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*b*Sqrt[2 + b*x])/Sqrt[x] - (2*(2 + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(2+bx)^{3/2}}{x^{5/2}}\,dx = -\frac{2(2+bx)^{3/2}}{3x^{3/2}} + b\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx$$
$$= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx$$
$$= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.50

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{2}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/x^(5/2),x]
[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, -1/2*(b*x)])/(3*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 55, normalized size = 0.92

$$-2b^{3/2}\log\left(\sqrt{bx+2} - \sqrt{b}\sqrt{x}\right) - \frac{4\sqrt{bx+2}\,(2bx+1)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(3/2)/x^(5/2),x]
[Out] (-4*Sqrt[2 + b*x]*(1 + 2*b*x))/(3*x^(3/2)) - 2*b^(3/2)*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[2 + b*x]]
```

**fricas [A]** time = 1.31, size = 108, normalized size = 1.80

$$\left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-4\,(2\,bx+1)\sqrt{bx+2}\sqrt{x}}{3\,x^2}, -\frac{2\left(3\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)+2\,(2\,bx+1)\sqrt{bx+2}\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 4*(2*b*x
 + 1)*sqrt(b*x + 2)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))) + 2*(2*b*x + 1)*sqrt(b*x + 2)*sqrt(x))/x^2]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]1/abs(b)*b^2/b*(2*(-12*b^3/9*sqrt(b
*x+2)*sqrt(b*x+2)+18*b^3/9)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)/(b*(b*x+2)-2*b)
^2-2*b^2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
```

**maple [A]** time = 0.02, size = 73, normalized size = 1.22

$$\frac{\sqrt{(bx+2)\,x}\;b^{\frac{3}{2}}\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\;\sqrt{x}}-\frac{4\left(2b^{2}x^{2}+5bx+2\right)}{3\sqrt{bx+2}\;x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(5/2),x)
```

```
[Out] -4/3*(2*b^2*x^2+5*b*x+2)/x^(3/2)/(b*x+2)^(1/2)+b^(3/2)*ln((b*x+1)/b^(1/2)+(
b*x^2+2*x)^(1/2))*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)
```

**maxima [A]** time = 2.97, size = 67, normalized size = 1.12

$$-b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{2\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{2\,(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -b^(3/2)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sq
rt(x))) - 2*sqrt(b*x + 2)*b/sqrt(x) - 2/3*(b*x + 2)^(3/2)/x^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(5/2),x)
```

```
[Out] int((b*x + 2)^(3/2)/x^(5/2), x)
```

**sympy [A]** time = 2.81, size = 70, normalized size = 1.17

$$-\frac{8b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)+2b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)-\frac{4\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(5/2),x)
```

```
[Out] -8*b**(3/2)*sqrt(1 + 2/(b*x))/3 - b**(3/2)*log(1/(b*x)) + 2*b**(3/2)*log(sq
rt(1 + 2/(b*x)) + 1) - 4*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

# 3.539 $\int x^{5/2}(2-bx)^{3/2}\,dx$

**Optimal**. Leaf size=131

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{20b}$$

**Rubi [A]** time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 - b*x])/(8*b^2) - (x^(
5/2)*Sqrt[2 - b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 - b*x])/20 + (x^(7/2)*(2 - b
*x)^(3/2))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{5/2}(2-bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3}{5}\int x^{5/2}\sqrt{2-bx}\,dx$$
$$= \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3}{20}\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{4b}$$
$$= -\frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3\int}{}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3\,\mathrm{Su}}{}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3\sin}{}$$

**Mathematica [A]** time = 0.05, size = 79, normalized size = 0.60

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}} - \frac{\sqrt{x}\sqrt{2-bx}\left(8b^4x^4-22b^3x^3+2b^2x^2+5bx+15\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] -1/40*(Sqrt[x]*Sqrt[2 - b*x]*(15 + 5*b*x + 2*b^2*x^2 - 22*b^3*x^3 + 8*b^4*x
^4))/b^3 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.16, size = 104, normalized size = 0.79

$$\frac{3\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^4} + \frac{\sqrt{2-bx}\left(-8b^4x^{9/2}+22b^3x^{7/2}-2b^2x^{5/2}-5bx^{3/2}-15\sqrt{x}\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] - 5*b*x^(3/2) - 2*b^2*x^(5/2) + 22*b^3*x^(7/2)
- 8*b^4*x^(9/2)))/(40*b^3) + (3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 -
 b*x]])/(4*b^4)
```

**fricas [A]** time = 0.91, size = 157, normalized size = 1.20

$$\left[-\frac{\left(8\,b^{5}x^{4}-22\,b^{4}x^{3}+2\,b^{3}x^{2}+5\,b^{2}x+15\,b\right)\sqrt{-bx+2}\,\sqrt{x}+15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{40\,b^{4}},\ -\frac{\left(8\,b^{5}x^{4}-22\,b^{4}x^{3}+2\,b^{3}x^{2}+5\,b^{2}x+15\,b\right)\sqrt{-bx+2}\,\sqrt{x}+30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{40\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/40*((8*b^5*x^4 - 22*b^4*x^3 + 2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2
)*sqrt(x) + 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^
4, -1/40*((8*b^5*x^4 - 22*b^4*x^3 + 2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x +
 2)*sqrt(x) + 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^4]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-18.27194
81629,8.05231268331]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
```

%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-36.6004387327,28.4266860783]Warning, choosing root of [1,0
,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,
[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%
}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4
,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%
}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-
20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,
2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%
}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,
[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%
}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%
{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1
,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}
+%%%{16,[0,0]%%%}] at parameters values [-61.8196598012,96.7771189027]Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-5.403030
52077,66.1769613782]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%

%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-61.0171700171,94.1262030317]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-12.7113713701,17.6881634681]1/b*(-2*b^2*abs(b)/b^2*(2*(((((5040*b^19/100800/b^23*sqrt(-b*x+2)*sqrt(-b*x+2)-51660*b^19/100800/b^23)*sqrt(-b*x+2)*sqrt(-b*x+2)+215460*b^19/100800/b^23)*sqrt(-b*x+2)*sqrt(-b*x+2)-469350*b^19/100800/b^23)*sqrt(-b*x+2)*sqrt(-b*x+2)+607950*b^19/100800/b^23)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-63/8/b^3/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))+8*b*abs(b)/b^2*(2*(((-90*b^11/1440/b^14*sqrt(-b*x+2)*sqrt(-b*x+2)+750*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)-2445*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)+4185*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-35/8/b^2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))-8*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqrt(-b*x+2)-78*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2)))))

**maple [A]** time = 0.01, size = 132, normalized size = 1.01

$$-\frac{(-bx+2)^{\frac{5}{2}} x^{\frac{5}{2}}}{5b}-\frac{(-bx+2)^{\frac{5}{2}} x^{\frac{3}{2}}}{4b^{2}}-\frac{(-bx+2)^{\frac{5}{2}} \sqrt{x}}{4b^{3}}+\frac{(-bx+2)^{\frac{3}{2}} \sqrt{x}}{8b^{3}}+\frac{3\sqrt{-bx+2}\,\sqrt{x}}{8b^{3}}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{8\sqrt{-bx+2}\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(-b*x+2)^(3/2),x)

[Out] -1/5/b*x^(5/2)*(-b*x+2)^(5/2)-1/4/b^2*x^(3/2)*(-b*x+2)^(5/2)-1/4/b^3*x^(1/2)*(-b*x+2)^(5/2)+1/8*(-b*x+2)^(3/2)/b^3*x^(1/2)+3/8*(-b*x+2)^(1/2)/b^3*x^(1/2)+3/8*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(7/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))

**maxima** [A] time = 2.94, size = 179, normalized size = 1.37

$$\frac{\frac{15\sqrt{-bx+2}\,b^4}{\sqrt{x}}+\frac{70(-bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}-\frac{128(-bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}-\frac{70(-bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15(-bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^8-\frac{5(bx-2)b^7}{x}+\frac{10(bx-2)^2b^6}{x^2}-\frac{10(bx-2)^3b^5}{x^3}+\frac{5(bx-2)^4b^4}{x^4}-\frac{(bx-2)^5b^3}{x^5}\right)}-\frac{3\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(-b*x + 2)*b^4/sqrt(x) + 70*(-b*x + 2)^(3/2)*b^3/x^(3/2) - 128
*(-b*x + 2)^(5/2)*b^2/x^(5/2) - 70*(-b*x + 2)^(7/2)*b/x^(7/2) - 15*(-b*x +
2)^(9/2)/x^(9/2))/(b^8 - 5*(b*x - 2)*b^7/x + 10*(b*x - 2)^2*b^6/x^2 - 10*(b
*x - 2)^3*b^5/x^3 + 5*(b*x - 2)^4*b^4/x^4 - (b*x - 2)^5*b^3/x^5) - 3/4*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(2-b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)*(2 - b*x)^(3/2), x)
```

**sympy** [A] time = 15.28, size = 291, normalized size = 2.22

$$\begin{cases} -\frac{ib^2x^{\frac{11}{2}}}{5\sqrt{bx-2}}+\frac{19ibx^{\frac{9}{2}}}{20\sqrt{bx-2}}-\frac{23ix^{\frac{7}{2}}}{20\sqrt{bx-2}}-\frac{ix^{\frac{5}{2}}}{40b\sqrt{bx-2}}-\frac{ix^{\frac{3}{2}}}{8b^2\sqrt{bx-2}}+\frac{3i\sqrt{x}}{4b^3\sqrt{bx-2}}-\frac{3i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{for } \frac{|bx|}{2}>1 \\ \frac{b^2x^{\frac{11}{2}}}{5\sqrt{-bx+2}}-\frac{19bx^{\frac{9}{2}}}{20\sqrt{-bx+2}}+\frac{23x^{\frac{7}{2}}}{20\sqrt{-bx+2}}+\frac{x^{\frac{5}{2}}}{40b\sqrt{-bx+2}}+\frac{x^{\frac{3}{2}}}{8b^2\sqrt{-bx+2}}-\frac{3\sqrt{x}}{4b^3\sqrt{-bx+2}}+\frac{3\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(11/2)/(5*sqrt(b*x - 2)) + 19*I*b*x**(9/2)/(20*sqrt(b
*x - 2)) - 23*I*x**(7/2)/(20*sqrt(b*x - 2)) - I*x**(5/2)/(40*b*sqrt(b*x - 2
)) - I*x**(3/2)/(8*b**2*sqrt(b*x - 2)) + 3*I*sqrt(x)/(4*b**3*sqrt(b*x - 2))
 - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2)), Abs(b*x)/2 > 1), (b**
2*x**(11/2)/(5*sqrt(-b*x + 2)) - 19*b*x**(9/2)/(20*sqrt(-b*x + 2)) + 23*x**
(7/2)/(20*sqrt(-b*x + 2)) + x**(5/2)/(40*b*sqrt(-b*x + 2)) + x**(3/2)/(8*b*
*2*sqrt(-b*x + 2)) - 3*sqrt(x)/(4*b**3*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqr
t(b)*sqrt(x)/2)/(4*b**(7/2)), True))
```

## 3.540 $\int x^{3/2}(2-bx)^{3/2}\,dx$

**Optimal**. Leaf size=109

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

**Rubi** [A] time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (3*ArcSin[(Sqrt[b]*Sqrt
[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}(2-bx)^{3/2}\,dx &= \frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3}{4}\int x^{3/2}\sqrt{2-bx}\,dx \\
&= \frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx \\
&= -\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b} \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{8b^2} \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx\right.}{4b^2} \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 70, normalized size = 0.64

$$
\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{\sqrt{x}\sqrt{2-bx}\left(2b^3x^3-6b^2x^2+bx+3\right)}{8b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] -1/8*(Sqrt[x]*Sqrt[2 - b*x]*(3 + b*x - 6*b^2*x^2 + 2*b^3*x^3))/b^2 + (3*Arc
Sin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 94, normalized size = 0.86

$$
\frac{3\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^3}+\frac{\sqrt{2-bx}\left(-2b^3x^{7/2}+6b^2x^{5/2}-bx^{3/2}-3\sqrt{x}\right)}{8b^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-3*Sqrt[x] - b*x^(3/2) + 6*b^2*x^(5/2) - 2*b^3*x^(7/2)))/(8
*b^2) + (3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*b^3)
```

**fricas [A]** time = 1.12, size = 139, normalized size = 1.28

$$\left[-\frac{\left(2\,b^{4}x^{3}-6\,b^{3}x^{2}+b^{2}x+3\,b\right)\sqrt{-bx+2}\,\sqrt{x}+3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{8\,b^{3}},-\frac{\left(2\,b^{4}x^{3}-6\,b^{3}x^{2}+b^{2}x+3\,b\right)\sqrt{-bx+2}\,\sqrt{x}+6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*b^4*x^3 - 6*b^3*x^2 + b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqr
t(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, -1/8*((2*b^4*x^
3 - 6*b^3*x^2 + b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt
(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-18.27194
81629,8.05231268331]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
```

6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-36.6004387327,28.4266860783]Warning, choosing root of [1,0
,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,
[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%
}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4
,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%
}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-
20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,
2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%
}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,
[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%
}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%
{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1
,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}
+%%%{16,[0,0]%%%}] at parameters values [-61.8196598012,96.7771189027]Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-5.403030
52077,66.1769613782]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-61.0171700171,94.1262030317]Warning, choosing root of [1,0
,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,

```
[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%
}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4
,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%
}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-
20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,
2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%
}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,
[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%
}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%
{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1
,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}
+%%%{16,[0,0]%%%}] at parameters values [-12.7113713701,17.6881634681]1/b*(
-2*b^2*abs(b)/b^2*(2*(((-90*b^11/1440/b^14*sqrt(-b*x+2)*sqrt(-b*x+2)+750*b^
11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)-2445*b^11/1440/b^14)*sqrt(-b*x+2)*s
qrt(-b*x+2)+4185*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-35/8/b^
2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))+8*b*abs(b)
/b^2*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqrt(-b*x+2)-78*b^5/144/b^7)*sqrt(-b*
x+2)*sqrt(-b*x+2)+198*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-5/2/b
/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))+8*abs(b)/b^
2/b*(2*(1/8*sqrt(-b*x+2)*sqrt(-b*x+2)-5/8)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*
b)+6*b/4/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2)))))
```

**maple [A]** time = 0.01, size = 116, normalized size = 1.06

$$-\frac{(-bx+2)^{\frac{5}{2}} x^{\frac{3}{2}}}{4b}-\frac{(-bx+2)^{\frac{5}{2}} \sqrt{x}}{4b^{2}}+\frac{(-bx+2)^{\frac{3}{2}} \sqrt{x}}{8b^{2}}+\frac{3\sqrt{-bx+2}\,\sqrt{x}}{8b^{2}}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{8\sqrt{-bx+2}\,b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(3/2),x)
```

```
[Out] -1/4/b*x^(3/2)*(-b*x+2)^(5/2)-1/4/b^2*x^(1/2)*(-b*x+2)^(5/2)+1/8*(-b*x+2)^(
3/2)/b^2*x^(1/2)+3/8*(-b*x+2)^(1/2)/b^2*x^(1/2)+3/8*((-b*x+2)*x)^(1/2)/(-b*
x+2)^(1/2)/b^(5/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.92, size = 147, normalized size = 1.35

$$\frac{\frac{3\sqrt{-bx+2}\,b^{3}}{\sqrt{x}}+\frac{11\,(-bx+2)^{\frac{3}{2}} b^{2}}{x^{\frac{3}{2}}}-\frac{11\,(-bx+2)^{\frac{5}{2}} b}{x^{\frac{5}{2}}}-\frac{3\,(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{4\left(b^{6}-\frac{4\,(bx-2)b^{5}}{x}+\frac{6\,(bx-2)^{2} b^{4}}{x^{2}}-\frac{4\,(bx-2)^{3} b^{3}}{x^{3}}+\frac{(bx-2)^{4} b^{2}}{x^{4}}\right)}-\frac{3\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*sqrt(-b*x + 2)*b^3/sqrt(x) + 11*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 11*(-
b*x + 2)^(5/2)*b/x^(5/2) - 3*(-b*x + 2)^(7/2)/x^(7/2))/(b^6 - 4*(b*x - 2)*b
^5/x + 6*(b*x - 2)^2*b^4/x^2 - 4*(b*x - 2)^3*b^3/x^3 + (b*x - 2)^4*b^2/x^4)
 - 3/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(2-b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)*(2 - b*x)^(3/2), x)
```

**sympy [A]** time = 7.73, size = 252, normalized size = 2.31

$$\begin{cases} -\frac{ib^2x^{\frac{9}{2}}}{4\sqrt{bx-2}} + \frac{5ibx^{\frac{7}{2}}}{4\sqrt{bx-2}} - \frac{13ix^{\frac{5}{2}}}{8\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{8b\sqrt{bx-2}} + \frac{3i\sqrt{x}}{4b^2\sqrt{bx-2}} - \frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{for}\ \frac{|bx|}{2} > 1 \\ \frac{b^2x^{\frac{9}{2}}}{4\sqrt{-bx+2}} - \frac{5bx^{\frac{7}{2}}}{4\sqrt{-bx+2}} + \frac{13x^{\frac{5}{2}}}{8\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{8b\sqrt{-bx+2}} - \frac{3\sqrt{x}}{4b^2\sqrt{-bx+2}} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(9/2)/(4*sqrt(b*x - 2)) + 5*I*b*x**(7/2)/(4*sqrt(b*x
- 2)) - 13*I*x**(5/2)/(8*sqrt(b*x - 2)) - I*x**(3/2)/(8*b*sqrt(b*x - 2)) +
3*I*sqrt(x)/(4*b**2*sqrt(b*x - 2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(
4*b**(5/2)), Abs(b*x)/2 > 1), (b**2*x**(9/2)/(4*sqrt(-b*x + 2)) - 5*b*x**(7
/2)/(4*sqrt(-b*x + 2)) + 13*x**(5/2)/(8*sqrt(-b*x + 2)) + x**(3/2)/(8*b*sqr
t(-b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(b)*s
qrt(x)/2)/(4*b**(5/2)), True))
```

## 3.541 $\int \sqrt{x}(2-bx)^{3/2}\,dx$

**Optimal**. Leaf size=84

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

**Rubi [A]** time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b) + (x^(3/2)*Sqrt[2 - b*x])/2 + (x^(3/2)*(2 -
b*x)^(3/2))/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(2-bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(2-bx)^{3/2}+\int \sqrt{x}\sqrt{2-bx}\,dx\\ &= \frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx\\ &= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b}\\ &= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 60, normalized size = 0.71

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{2-bx}\left(2b^2x^2-7bx+3\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] -1/6*(Sqrt[x]*Sqrt[2 - b*x]*(3 - 7*b*x + 2*b^2*x^2))/b + ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[2]]/b^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.12, size = 81, normalized size = 0.96

$$\frac{\sqrt{2-bx}\left(-2b^2x^{5/2}+7bx^{3/2}-3\sqrt{x}\right)}{6b}+\frac{\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-3*Sqrt[x] + 7*b*x^(3/2) - 2*b^2*x^(5/2)))/(6*b) + (Sqrt[-b
]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^2
```

**fricas** [A] time = 1.31, size = 125, normalized size = 1.49

$$\left[-\frac{\left(2b^3x^2-7b^2x+3b\right)\sqrt{-bx+2}\sqrt{x}+3\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{6b^2},-\frac{\left(2b^3x^2-7b^2x+3b\right)\sqrt{-bx+2}\sqrt{x}+6\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{6b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="fricas")

[Out] [-1/6*((2*b^3*x^2 - 7*b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(
-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^2, -1/6*((2*b^3*x^2 - 7*b^2*
x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*
sqrt(x))))/b^2]

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-18.27194
81629,8.05231268331]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-36.6004387327,28.4266860783]Warning, choosing root of [1,0
,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,

[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%
}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4
,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%
}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-
20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,
2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%
}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,
[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%
}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%
{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1
,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}
+%%%{16,[0,0]%%%}] at parameters values [-61.8196598012,96.7771189027]Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-5.403030
52077,66.1769613782]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-61.0171700171,94.1262030317]Warning, choosing root of [1,0
,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,
[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%
}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4
,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%
}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-
20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,

```
2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%
}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,
[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%
}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%
{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1
,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}
+%%%{16,[0,0]%%%}] at parameters values [-12.7113713701,17.6881634681]1/b*(
-2*b^2*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqrt(-b*x+2)-78*b^5/144/
b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*(-b*x+
2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2)))
)-8*b*abs(b)/b^2/b*(2*(1/8*sqrt(-b*x+2)*sqrt(-b*x+2)-5/8)*sqrt(-b*x+2)*sqrt
(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt
(-b*x+2))))-8*abs(b)/b^2*(1/2*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-2*b/2/sqrt
(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2)))))
```

**maple [A]** time = 0.00, size = 94, normalized size = 1.12

$$\frac{(-bx+2)^{\frac{3}{2}} x^{\frac{3}{2}}}{3}+\frac{\sqrt{-bx+2}\, x^{\frac{3}{2}}}{2}-\frac{\sqrt{-bx+2}\, \sqrt{x}}{2b}+\frac{\sqrt{(-bx+2)\, x}\, \arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\, x^{2}+2x}}\right)}{2\sqrt{-bx+2}\, b^{\frac{3}{2}} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)*x^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*(-b*x+2)^(3/2)+1/2*(-b*x+2)^(1/2)*x^(3/2)-1/2*(-b*x+2)^(1/2)/b*
x^(1/2)+1/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(3/2)/x^(1/2)*arctan((x-1/b
)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 3.04, size = 115, normalized size = 1.37

$$\frac{\frac{3\sqrt{-bx+2}\, b^{2}}{\sqrt{x}}+\frac{8\,(-bx+2)^{\frac{3}{2}} b}{x^{\frac{3}{2}}}-\frac{3\,(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{4}-\frac{3\,(bx-2)b^{3}}{x}+\frac{3\,(bx-2)^{2} b^{2}}{x^{2}}-\frac{(bx-2)^{3} b}{x^{3}}\right)}-\frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(-b*x + 2)*b^2/sqrt(x) + 8*(-b*x + 2)^(3/2)*b/x^(3/2) - 3*(-b*x
+ 2)^(5/2)/x^(5/2))/(b^4 - 3*(b*x - 2)*b^3/x + 3*(b*x - 2)^2*b^2/x^2 - (b*x
 - 2)^3*b/x^3) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,(2-b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(2 - b*x)^(3/2), x)
```

**sympy [A]** time = 4.78, size = 199, normalized size = 2.37

$$\begin{cases} -\frac{ib^2x^{\frac{7}{2}}}{3\sqrt{bx-2}} + \frac{11ibx^{\frac{5}{2}}}{6\sqrt{bx-2}} - \frac{17ix^{\frac{3}{2}}}{6\sqrt{bx-2}} + \frac{i\sqrt{x}}{b\sqrt{bx-2}} - \frac{i\,\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2} > 1 \\ \frac{b^2x^{\frac{7}{2}}}{3\sqrt{-bx+2}} - \frac{11bx^{\frac{5}{2}}}{6\sqrt{-bx+2}} + \frac{17x^{\frac{3}{2}}}{6\sqrt{-bx+2}} - \frac{\sqrt{x}}{b\sqrt{-bx+2}} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)*x**(1/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(7/2)/(3*sqrt(b*x - 2)) + 11*I*b*x**(5/2)/(6*sqrt(b*x
 - 2)) - 17*I*x**(3/2)/(6*sqrt(b*x - 2)) + I*sqrt(x)/(b*sqrt(b*x - 2)) - I*
acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), Abs(b*x)/2 > 1), (b**2*x**(7/2)/
(3*sqrt(-b*x + 2)) - 11*b*x**(5/2)/(6*sqrt(-b*x + 2)) + 17*x**(3/2)/(6*sqrt
(-b*x + 2)) - sqrt(x)/(b*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/
b**(3/2), True))
```

## 3.542 $\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=63

$$\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 - b*x])/2 + (Sqrt[x]*(2 - b*x)^(3/2))/2 + (3*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx = \frac{1}{2}\sqrt{x}\,(2-bx)^{3/2} + \frac{3}{2}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx$$
$$= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx} + \frac{1}{2}\sqrt{x}\,(2-bx)^{3/2} + \frac{3}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx$$
$$= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx} + \frac{1}{2}\sqrt{x}\,(2-bx)^{3/2} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx} + \frac{1}{2}\sqrt{x}\,(2-bx)^{3/2} + \frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Mathematica** [A] time = 0.03, size = 49, normalized size = 0.78

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}} - \frac{1}{2}\sqrt{x}\,\sqrt{2-bx}\,(bx-5)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x]*(-5 + b*x)) + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[
2]])/Sqrt[b]
```

**IntegrateAlgebraic** [A] time = 0.10, size = 69, normalized size = 1.10

$$\frac{1}{2}\sqrt{2-bx}\,\left(5\sqrt{x} - bx^{3/2}\right) + \frac{3\sqrt{-b}\,\log\left(\sqrt{2-bx} - \sqrt{-b}\,\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[2 - b*x]*(5*Sqrt[x] - b*x^(3/2)))/2 + (3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt
[x]) + Sqrt[2 - b*x]])/b
```

**fricas** [A] time = 1.47, size = 107, normalized size = 1.70

$$\left[-\frac{\left(b^2x - 5\,b\right)\sqrt{-bx+2}\,\sqrt{x} + 3\,\sqrt{-b}\,\log\left(-bx + \sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x} + 1\right)}{2\,b}, -\frac{\left(b^2x - 5\,b\right)\sqrt{-bx+2}\,\sqrt{x} + 6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")

[Out] [-1/2*((b^2*x - 5*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b, -1/2*((b^2*x - 5*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.6438432182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-29.292030761,78.6493344628]1/abs(b)*b^2/b*(2*(1/4/b*sqrt(-b*x+2)*sqrt(-b*x+2)+3/4/b)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+3/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))

**maple [A]** time = 0.00, size = 78, normalized size = 1.24

$$\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{-bx+2}\,\sqrt{x}}{2}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(-b*x+2)^(3/2)*x^(1/2)+3/2*(-b*x+2)^(1/2)*x^(1/2)+3/2*((-b*x+2)*x)^(1/2
)/(-b*x+2)^(1/2)/b^(1/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.98, size = 79, normalized size = 1.25

$$-\frac{3\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}+\frac{\frac{3\,\sqrt{-bx+2}\,b}{\sqrt{x}}+\frac{5\,(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{2}-\frac{2\,(bx-2)b}{x}+\frac{(bx-2)^{2}}{x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + (3*sqrt(-b*x + 2)*b/s
qrt(x) + 5*(-b*x + 2)^(3/2)/x^(3/2))/(b^2 - 2*(b*x - 2)*b/x + (b*x - 2)^2/x
^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2-b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(1/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(1/2), x)
```

**sympy [A]** time = 2.86, size = 167, normalized size = 2.65

$$\begin{cases} -\frac{ib^{2}x^{\frac{5}{2}}}{2\sqrt{bx-2}}+\frac{7ibx^{\frac{3}{2}}}{2\sqrt{bx-2}}-\frac{5i\sqrt{x}}{\sqrt{bx-2}}-\frac{3i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx|}{2}>1 \\ \frac{b^{2}x^{\frac{5}{2}}}{2\sqrt{-bx+2}}-\frac{7bx^{\frac{3}{2}}}{2\sqrt{-bx+2}}+\frac{5\sqrt{x}}{\sqrt{-bx+2}}+\frac{3\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(5/2)/(2*sqrt(b*x - 2)) + 7*I*b*x**(3/2)/(2*sqrt(b*x
- 2)) - 5*I*sqrt(x)/sqrt(b*x - 2) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sq
rt(b), Abs(b*x)/2 > 1), (b**2*x**(5/2)/(2*sqrt(-b*x + 2)) - 7*b*x**(3/2)/(2
*sqrt(-b*x + 2)) + 5*sqrt(x)/sqrt(-b*x + 2) + 3*asin(sqrt(2)*sqrt(b)*sqrt(x
)/2)/sqrt(b), True))
```

## 3.543 $\int \frac{(2-bx)^{3/2}}{x^{3/2}} dx$

**Optimal**. Leaf size=60

$$-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\,\sqrt{2-bx}-6\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

**Rubi** [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\,\sqrt{2-bx}-6\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] -3*b*Sqrt[x]*Sqrt[2 - b*x] - (2*(2 - b*x)^(3/2))/Sqrt[x] - 6*Sqrt[b]*ArcSin
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(3b)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx \\
&= -3b\sqrt{x}\,\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(3b)\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx \\
&= -3b\sqrt{x}\,\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(6b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right) \\
&= -3b\sqrt{x}\,\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-6\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 28, normalized size = 0.47

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{2}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/x^(3/2),x]
[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -1/2, 1/2, (b*x)/2])/Sqrt[x]
```

**IntegrateAlgebraic [A]** time = 0.12, size = 58, normalized size = 0.97

$$\frac{(-bx-4)\sqrt{2-bx}}{\sqrt{x}}-6\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - b*x)^(3/2)/x^(3/2),x]
[Out] ((-4 - b*x)*Sqrt[2 - b*x])/Sqrt[x] - 6*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + S
qrt[2 - b*x]]
```

**fricas [A]** time = 0.74, size = 101, normalized size = 1.68

$$\left[\frac{3\sqrt{-b}\,x\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-(bx+4)\sqrt{-bx+2}\,\sqrt{x}}{x},\ \frac{6\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-(bx+4)\sqrt{-bx+2}\,\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - (b*x + 4)*
sqrt(-b*x + 2)*sqrt(x))/x, (6*sqrt(b)*x*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt
(x))) - (b*x + 4)*sqrt(-b*x + 2)*sqrt(x))/x]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
```

```
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]-b/abs(b)*b^2/b*(2*(-1/2*sqrt(-
b*x+2)*sqrt(-b*x+2)+3)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)/(-b*(-b*x+2)+2*b)
+6/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))
```

**maple [B]** time = 0.02, size = 97, normalized size = 1.62

$$-\frac{3\sqrt{(-bx+2)\,x}\,\sqrt{b}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^2+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}}+\frac{\left(b^2x^2+2bx-8\right)\sqrt{(-bx+2)\,x}}{\sqrt{-\left(bx-2\right)x}\,\sqrt{-bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(3/2),x)
```

```
[Out] (b^2*x^2+2*b*x-8)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1
/2)-3*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)*b^(1/2)/x^(1/2)*arctan((x-1/b)/(-b*
x^2+2*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.99, size = 63, normalized size = 1.05

$$6\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{4\sqrt{-bx+2}}{\sqrt{x}}-\frac{2\sqrt{-bx+2}\,b}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - 4*sqrt(-b*x + 2)/sqrt(
x) - 2*sqrt(-b*x + 2)*b/((b - (b*x - 2)/x)*sqrt(x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(2-b\,x\right)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(3/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(3/2), x)
```

**sympy [A]** time = 2.49, size = 160, normalized size = 2.67

$$\begin{cases} 6i\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right) - \frac{ib^2x^{\frac{3}{2}}}{\sqrt{bx-2}} - \frac{2ib\sqrt{x}}{\sqrt{bx-2}} + \frac{8i}{\sqrt{x}\,\sqrt{bx-2}} & \text{for } \frac{|bx|}{2} > 1 \\ -6\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right) + \frac{b^2x^{\frac{3}{2}}}{\sqrt{-bx+2}} + \frac{2b\sqrt{x}}{\sqrt{-bx+2}} - \frac{8}{\sqrt{x}\,\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(3/2),x)
```

```
[Out] Piecewise((6*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) - I*b**2*x**(3/2)/s
qrt(b*x - 2) - 2*I*b*sqrt(x)/sqrt(b*x - 2) + 8*I/(sqrt(x)*sqrt(b*x - 2)), A
bs(b*x)/2 > 1), (-6*sqrt(b)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**2*x**(3/2)
/sqrt(-b*x + 2) + 2*b*sqrt(x)/sqrt(-b*x + 2) - 8/(sqrt(x)*sqrt(-b*x + 2)),
True))
```

## 3.544 $\int \frac{(2-bx)^{3/2}}{x^{5/2}} \, dx$

**Optimal**. Leaf size=62

$$2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{2-bx}}{\sqrt{x}}$$

**Rubi [A]** time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[2 - b*x])/Sqrt[x] - (2*(2 - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(2-bx)^{3/2}}{3x^{3/2}} - b\int \frac{\sqrt{2-bx}}{x^{3/2}}\,dx\\ &= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\ &= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.48

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{2}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/x^(5/2),x]
[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (b*x)/2])/(3*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 62, normalized size = 1.00

$$\frac{4\sqrt{2-bx}\,(2bx-1)}{3x^{3/2}} + 2\sqrt{-b}\,b\log\left(\sqrt{2-bx} - \sqrt{-b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - b*x)^(3/2)/x^(5/2),x]
[Out] (4*Sqrt[2 - b*x]*(-1 + 2*b*x))/(3*x^(3/2)) + 2*Sqrt[-b]*b*Log[-(Sqrt[-b]*Sq
rt[x]) + Sqrt[2 - b*x]]
```

**fricas [A]** time = 1.15, size = 111, normalized size = 1.79

$$\left[\frac{3\sqrt{-b}\,bx^2\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+4\,(2\,bx-1)\sqrt{-bx+2}\,\sqrt{x}}{3\,x^2}, -\frac{2\left(3\,b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-2\,(2\,bx-1)\sqrt{-bx+2}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-b)*b*x^2*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + 4*
(2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x))/x^2, -2/3*(3*b^(3/2)*x^2*arctan(sqrt(-b
*x + 2)/(sqrt(b)*sqrt(x))) - 2*(2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x))/x^2]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]1/abs(b)*b^2/b*(2*(-12*b^3/9*sq
rt(-b*x+2)*sqrt(-b*x+2)+18*b^3/9)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)/(-b*(-
b*x+2)+2*b)^2+2*b^2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*
x+2))))
```

**maple [B]** time = 0.02, size = 98, normalized size = 1.58

$$\frac{\sqrt{(-bx+2)\,x}\;b^{\frac{3}{2}}\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{\sqrt{-bx+2}\;\sqrt{x}}-\frac{4\left(2b^{2}x^{2}-5bx+2\right)\sqrt{(-bx+2)\,x}}{3\sqrt{-(bx-2)\,x}\;\sqrt{-bx+2}\;x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(5/2),x)
```

```
[Out] -4/3*(2*b^2*x^2-5*b*x+2)/x^(3/2)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-b*
x+2)^(1/2)+b^(3/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))*((-b*x+2)*x)^
(1/2)/(-b*x+2)^(1/2)/x^(1/2)
```

**maxima [A]** time = 3.00, size = 49, normalized size = 0.79

$$-2\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{2\,\sqrt{-bx+2}\,b}{\sqrt{x}}-\frac{2\,(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2*b^(3/2)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + 2*sqrt(-b*x + 2)*b/sq
rt(x) - 2/3*(-b*x + 2)^(3/2)/x^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(2-b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(5/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(5/2), x)
```

**sympy [C]** time = 2.92, size = 182, normalized size = 2.94

$$\begin{cases}\frac{8b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3}+ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-2ib^{\frac{3}{2}}\log\left(\frac{1}{\sqrt{b}\,\sqrt{x}}\right)+2b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)-\frac{4\sqrt{b}\,\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for } \frac{2}{|bx|}>1\\ \frac{8ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}+ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-2ib^{\frac{3}{2}}\log\left(\sqrt{1-\frac{2}{bx}}+1\right)-\frac{4i\sqrt{b}\,\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(5/2),x)
```

```
[Out] Piecewise((8*b**(3/2)*sqrt(-1 + 2/(b*x))/3 + I*b**(3/2)*log(1/(b*x)) - 2*I*
b**(3/2)*log(1/(sqrt(b)*sqrt(x))) + 2*b**(3/2)*asin(sqrt(2)*sqrt(b)*sqrt(x)
/2) - 4*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*x), 2/Abs(b*x) > 1), (8*I*b**(3/2)*sq
rt(1 - 2/(b*x))/3 + I*b**(3/2)*log(1/(b*x)) - 2*I*b**(3/2)*log(sqrt(1 - 2/(
b*x)) + 1) - 4*I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

## 3.545 $\int x^{5/2}(a+bx)^{5/2}\,dx$

**Optimal**. Leaf size=164

$$-\frac{5a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{512b^{7/2}}+\frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}$$

**Rubi [A]** time = 0.06, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{512b^{7/2}}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (5*a^5*Sqrt[x]*Sqrt[a + b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a + b*x])/(76
8*b^2) + (a^3*x^(5/2)*Sqrt[a + b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a + b*x])/
32 + (a*x^(7/2)*(a + b*x)^(3/2))/12 + (x^(7/2)*(a + b*x)^(5/2))/6 - (5*a^6*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(512*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}(a+bx)^{5/2}\,dx = \frac{1}{6}x^{7/2}(a+bx)^{5/2} + \frac{1}{12}(5a)\int x^{5/2}(a+bx)^{3/2}\,dx$$

$$= \frac{1}{12}ax^{7/2}(a+bx)^{3/2} + \frac{1}{6}x^{7/2}(a+bx)^{5/2} + \frac{1}{8}a^2\int x^{5/2}\sqrt{a+bx}\,dx$$

$$= \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+bx)^{3/2} + \frac{1}{6}x^{7/2}(a+bx)^{5/2} + \frac{1}{64}a^3\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$

$$= \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+bx)^{3/2} + \frac{1}{6}x^{7/2}(a+bx)^{5/2} - \frac{\left(5a^4\right)\int \frac{x}{\sqrt{a}}}{384b}$$

$$= -\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+bx)^{3/2} + \frac{1}{6}x^{7/2}(a+$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+$$

**Mathematica [A]** time = 0.24, size = 118, normalized size = 0.72

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(15a^5-10a^4bx+8a^3b^2x^2+432a^2b^3x^3+640ab^4x^4+256b^5x^5\right)-\frac{15a^{11/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{1536b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^5 - 10*a^4*b*x + 8*a^3*b^2*x^2 + 432*
a^2*b^3*x^3 + 640*a*b^4*x^4 + 256*b^5*x^5) - (15*a^(11/2)*ArcSinh[(Sqrt[b]*
Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(1536*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 121, normalized size = 0.74

$$\frac{5a^6\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{512b^{7/2}}+\frac{\sqrt{a+bx}\left(15a^5\sqrt{x}-10a^4bx^{3/2}+8a^3b^2x^{5/2}+432a^2b^3x^{7/2}+640ab^4x^{9/2}+256b^5x^{11/2}\right)}{1536b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(15*a^5*Sqrt[x] - 10*a^4*b*x^(3/2) + 8*a^3*b^2*x^(5/2) + 432
*a^2*b^3*x^(7/2) + 640*a*b^4*x^(9/2) + 256*b^5*x^(11/2)))/(1536*b^3) + (5*a
^6*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(512*b^(7/2))
```

**fricas [A]** time = 0.78, size = 206, normalized size = 1.26

$$\left[\frac{15\,a^6\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(256\,b^6x^5+640\,ab^5x^4+432\,a^2b^4x^3+8\,a^3b^3x^2-10\,a^4b^2x+15\,a^5b\right)\sqrt{bx+a}\,\sqrt{x}}{3072\,b^4},\frac{15\,a^6\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(256\,b^6x^5+640\,ab^5x^4+432\,a^2b^4x^3+8\,a^3b^3x^2-10\,a^4b^2x+15\,a^5b\right)\sqrt{bx+a}\,\sqrt{x}}{1536\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3072*(15*a^6*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(256*b^6*x^5 + 640*a*b^5*x^4 + 432*a^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b
^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/1536*(15*a^6*sqrt(-b)*arctan
(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (256*b^6*x^5 + 640*a*b^5*x^4 + 432*a
^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x)
)/b^4]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.00, size = 156, normalized size = 0.95

$$-\frac{5\sqrt{(bx+a)\,x}\;a^6\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{1024\sqrt{bx+a}\;b^{\frac{7}{2}}\sqrt{x}}-\frac{5\sqrt{bx+a}\;a^5\sqrt{x}}{512b^3}-\frac{5\,(bx+a)^{\frac{3}{2}}\,a^4\sqrt{x}}{768b^3}+\frac{(bx+a)^{\frac{7}{2}}\,x^{\frac{5}{2}}}{6b}-\frac{(bx+a)^{\frac{5}{2}}\,a^3\sqrt{x}}{192b^3}-\frac{(bx+a)^{\frac{7}{2}}\,a\,x^{\frac{3}{2}}}{12b^2}+\frac{(bx+a)^{\frac{7}{2}}\,a^2\sqrt{x}}{32b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(5/2),x)
```

```
[Out] 1/6/b*x^(5/2)*(b*x+a)^(7/2)-1/12*a/b^2*x^(3/2)*(b*x+a)^(7/2)+1/32*a^2/b^3*x
^(1/2)*(b*x+a)^(7/2)-1/192*a^3/b^3*(b*x+a)^(5/2)*x^(1/2)-5/768*a^4/b^3*(b*x
+a)^(3/2)*x^(1/2)-5/512*a^5*x^(1/2)*(b*x+a)^(1/2)/b^3-5/1024*a^6/b^(7/2)*((
b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1
/2))
```

**maxima [B]** time = 2.99, size = 244, normalized size = 1.49

$$\frac{5\,a^6\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{1024\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{bx+a}\,a^6b^5}{\sqrt{x}}-\frac{85\,(bx+a)^{\frac{3}{2}}a^6b^4}{x^{\frac{3}{2}}}+\frac{198\,(bx+a)^{\frac{5}{2}}a^6b^3}{x^{\frac{5}{2}}}+\frac{198\,(bx+a)^{\frac{7}{2}}a^6b^2}{x^{\frac{7}{2}}}-\frac{85\,(bx+a)^{\frac{9}{2}}a^6b}{x^{\frac{9}{2}}}+\frac{15\,(bx+a)^{\frac{11}{2}}a^6}{x^{\frac{11}{2}}}}{1536\left(b^9-\frac{6\,(bx+a)b^8}{x}+\frac{15\,(bx+a)^2b^7}{x^2}-\frac{20\,(bx+a)^3b^6}{x^3}+\frac{15\,(bx+a)^4b^5}{x^4}-\frac{6\,(bx+a)^5b^4}{x^5}+\frac{(bx+a)^6b^3}{x^6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/1024*a^6*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(7/2) + 1/1536*(15*sqrt(b*x + a)*a^6*b^5/sqrt(x) - 85*(b*x + a)
^(3/2)*a^6*b^4/x^(3/2) + 198*(b*x + a)^(5/2)*a^6*b^3/x^(5/2) + 198*(b*x + a
)^(7/2)*a^6*b^2/x^(7/2) - 85*(b*x + a)^(9/2)*a^6*b/x^(9/2) + 15*(b*x + a)^(
11/2)*a^6/x^(11/2))/(b^9 - 6*(b*x + a)*b^8/x + 15*(b*x + a)^2*b^7/x^2 - 20*
(b*x + a)^3*b^6/x^3 + 15*(b*x + a)^4*b^5/x^4 - 6*(b*x + a)^5*b^4/x^5 + (b*x
 + a)^6*b^3/x^6)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)*(a + b*x)^(5/2), x)
```

**sympy [A]** time = 25.94, size = 207, normalized size = 1.26

$$\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1+\frac{bx}{a}}}+\frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1+\frac{bx}{a}}}+\frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1+\frac{bx}{a}}}+\frac{7\sqrt{a}\,b^2x^{\frac{11}{2}}}{12\sqrt{1+\frac{bx}{a}}}-\frac{5a^6\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}}+\frac{b^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(5/2),x)
```

```
[Out] 5*a**(11/2)*sqrt(x)/(512*b**3*sqrt(1 + b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536*
b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(5/2)/(768*b*sqrt(1 + b*x/a)) + 55*a**(
5/2)*x**(7/2)/(192*sqrt(1 + b*x/a)) + 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1 + b
*x/a)) + 7*sqrt(a)*b**2*x**(11/2)/(12*sqrt(1 + b*x/a)) - 5*a**6*asinh(sqrt(
b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + b**3*x**(13/2)/(6*sqrt(a)*sqrt(1 + b*x
/a))
```

# 3.546 $\int x^{3/2}(a+bx)^{5/2}\,dx$

**Optimal**. Leaf size=140

$$\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{5/2}}-\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{5/2}}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^(5/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^2) + (a^3*x^(3/2)*Sqrt[a + b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a + b*x])/16 + (a*x^(5/2)*(a + b*x)^(3/2))/8 + (x^(5
/2)*(a + b*x)^(5/2))/5 + (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
128*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{2}a\int x^{3/2}(a+bx)^{3/2}\,dx\\ &= \frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^{3/2}\sqrt{a+bx}\,dx\\ &= \frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{32}a^3\int\frac{x^{3/2}}{\sqrt{a+bx}}\,dx\\ &= \frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}-\frac{\left(3a^4\right)\int\frac{\sqrt{x}}{\sqrt{a+bx}}}{128b}\\ &= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+b\\ &= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+b\\ &= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+b\\ &= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+b\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 107, normalized size = 0.76

$$\frac{\sqrt{a+bx}\left(\frac{15a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}+\sqrt{b}\sqrt{x}\left(-15a^4+10a^3bx+248a^2b^2x^2+336ab^3x^3+128b^4x^4\right)\right)}{640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^4 + 10*a^3*b*x + 248*a^2*b^2*x^2 + 3
36*a*b^3*x^3 + 128*b^4*x^4) + (15*a^(9/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]
])/Sqrt[1 + (b*x)/a]))/(640*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.15, size = 108, normalized size = 0.77

$$\frac{\sqrt{a+bx}\left(-15a^4\sqrt{x}+10a^3bx^{3/2}+248a^2b^2x^{5/2}+336ab^3x^{7/2}+128b^4x^{9/2}\right)}{640b^2}-\frac{3a^5\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{128b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-15*a^4*Sqrt[x] + 10*a^3*b*x^(3/2) + 248*a^2*b^2*x^(5/2) +
336*a*b^3*x^(7/2) + 128*b^4*x^(9/2)))/(640*b^2) - (3*a^5*Log[-(Sqrt[b]*Sqrt
[x]) + Sqrt[a + b*x]])/(128*b^(5/2))
```

**fricas [A]** time = 1.35, size = 185, normalized size = 1.32

$$\left[\frac{15\,a^5\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(128\,b^5x^4+336\,ab^4x^3+248\,a^2b^3x^2+10\,a^3b^2x-15\,a^4b\right)\sqrt{bx+a}\,\sqrt{x}}{1280\,b^3},\ -\frac{15\,a^5\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(128\,b^5x^4+336\,ab^4x^3+248\,a^2b^3x^2+10\,a^3b^2x-15\,a^4b\right)\sqrt{bx+a}\,\sqrt{x}}{640\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/1280*(15*a^5*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*a^3*b^2*x - 15*a^4*b)
*sqrt(b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - (128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*
a^3*b^2*x - 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 138, normalized size = 0.99

$$\frac{3\sqrt{(bx+a)\,x}\;a^5\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{256\sqrt{bx+a}\;b^{\frac{5}{2}}\sqrt{x}}+\frac{3\sqrt{bx+a}\;a^4\sqrt{x}}{128b^2}+\frac{(bx+a)^{\frac{3}{2}}\,a^3\sqrt{x}}{64b^2}+\frac{(bx+a)^{\frac{5}{2}}\,a^2\sqrt{x}}{80b^2}+\frac{(bx+a)^{\frac{7}{2}}\,x^{\frac{3}{2}}}{5b}-\frac{3\,(bx+a)^{\frac{7}{2}}\,a\sqrt{x}}{40b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(5/2),x)
```

```
[Out] 1/5/b*x^(3/2)*(b*x+a)^(7/2)-3/40*a/b^2*x^(1/2)*(b*x+a)^(7/2)+1/80*a^2/b^2*(
b*x+a)^(5/2)*x^(1/2)+1/64*a^3/b^2*(b*x+a)^(3/2)*x^(1/2)+3/128*a^4*x^(1/2)*(
b*x+a)^(1/2)/b^2+3/256*a^5/b^(5/2)*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*
ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima** [B] time = 2.98, size = 212, normalized size = 1.51

$$-\frac{3\,a^5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{256\,b^{\frac{5}{2}}}-\frac{\frac{15\,\sqrt{bx+a}\,a^5b^4}{\sqrt{x}}-\frac{70\,(bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}}+\frac{128\,(bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}}+\frac{70\,(bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^7-\frac{5\,(bx+a)b^6}{x}+\frac{10\,(bx+a)^2b^5}{x^2}-\frac{10\,(bx+a)^3b^4}{x^3}+\frac{5\,(bx+a)^4b^3}{x^4}-\frac{(bx+a)^5b^2}{x^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/256*a^5*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(5/2) - 1/640*(15*sqrt(b*x + a)*a^5*b^4/sqrt(x) - 70*(b*x + a)^
(3/2)*a^5*b^3/x^(3/2) + 128*(b*x + a)^(5/2)*a^5*b^2/x^(5/2) + 70*(b*x + a)^
(7/2)*a^5*b/x^(7/2) - 15*(b*x + a)^(9/2)*a^5/x^(9/2))/(b^7 - 5*(b*x + a)*b^
6/x + 10*(b*x + a)^2*b^5/x^2 - 10*(b*x + a)^3*b^4/x^3 + 5*(b*x + a)^4*b^3/x
^4 - (b*x + a)^5*b^2/x^5)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(a + b*x)^(5/2), x)
```

**sympy** [A] time = 16.41, size = 180, normalized size = 1.29

$$-\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1+\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1+\frac{bx}{a}}}+\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}}+\frac{29\sqrt{a}\,b^2x^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}}+\frac{3a^5\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{b^3x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(5/2),x)
```

```
[Out] -3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(3/2)/(128*b*s
qrt(1 + b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 + b*x/a)) + 73*a**(3/2)
```

```
*b*x**(7/2)/(80*sqrt(1 + b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(40*sqrt(1 + b*
x/a)) + 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) + b**3*x**(11/
2)/(5*sqrt(a)*sqrt(1 + b*x/a))
```

## 3.547 $\int \sqrt{x}\,(a+bx)^{5/2}\,dx$

**Optimal**. Leaf size=116

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a + b*x])/32 + (
5*a*x^(3/2)*(a + b*x)^(3/2))/24 + (x^(3/2)*(a + b*x)^(5/2))/4 - (5*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{x}(a+bx)^{5/2}\,dx = \frac{1}{4}x^{3/2}(a+bx)^{5/2} + \frac{1}{8}(5a)\int \sqrt{x}(a+bx)^{3/2}\,dx$$
$$= \frac{5}{24}ax^{3/2}(a+bx)^{3/2} + \frac{1}{4}x^{3/2}(a+bx)^{5/2} + \frac{1}{16}\left(5a^2\right)\int \sqrt{x}\sqrt{a+bx}\,dx$$
$$= \frac{5}{32}a^2x^{3/2}\sqrt{a+bx} + \frac{5}{24}ax^{3/2}(a+bx)^{3/2} + \frac{1}{4}x^{3/2}(a+bx)^{5/2} + \frac{1}{64}\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx$$
$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a+bx} + \frac{5}{24}ax^{3/2}(a+bx)^{3/2} + \frac{1}{4}x^{3/2}(a+bx)^{5/2} - \frac{\left(5a^4\right)\int -}{12}$$
$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a+bx} + \frac{5}{24}ax^{3/2}(a+bx)^{3/2} + \frac{1}{4}x^{3/2}(a+bx)^{5/2} - \frac{\left(5a^4\right)\mathrm{Sub}}{}$$
$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a+bx} + \frac{5}{24}ax^{3/2}(a+bx)^{3/2} + \frac{1}{4}x^{3/2}(a+bx)^{5/2} - \frac{\left(5a^4\right)\mathrm{Sub}}{}$$
$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a+bx} + \frac{5}{24}ax^{3/2}(a+bx)^{3/2} + \frac{1}{4}x^{3/2}(a+bx)^{5/2} - \frac{5a^4\tanh^{}}{64}$$

**Mathematica [A]** time = 0.20, size = 96, normalized size = 0.83

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(15a^3+118a^2bx+136ab^2x^2+48b^3x^3\right)-\frac{15a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{192b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^3 + 118*a^2*b*x + 136*a*b^2*x^2 + 48*
b^3*x^3) - (15*a^(7/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a
]))/(192*b^(3/2))
```

**IntegrateAlgebraic** [A]  time = 0.13, size = 95, normalized size = 0.82

$$\frac{5a^4\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{64b^{3/2}}+\frac{\sqrt{a+bx}\left(15a^3\sqrt{x}+118a^2bx^{3/2}+136ab^2x^{5/2}+48b^3x^{7/2}\right)}{192b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(15*a^3*Sqrt[x] + 118*a^2*b*x^(3/2) + 136*a*b^2*x^(5/2) + 48
*b^3*x^(7/2)))/(192*b) + (5*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(6
4*b^(3/2))
```

**fricas** [A]  time = 1.45, size = 162, normalized size = 1.40

$$\left[\frac{15\,a^4\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{384\,b^2},\ \frac{15\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{192\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt
(x))/b^2, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))
 + (48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sq
rt(x))/b^2]
```

**giac** [F(-1)]  time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A]  time = 0.01, size = 111, normalized size = 0.96

$$\frac{5\sqrt{bx+a}\,a^2x^{\frac{3}{2}}}{32}-\frac{5\sqrt{(bx+a)\,x}\,a^4\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{128\sqrt{bx+a}\,b^{\frac{3}{2}}\sqrt{x}}+\frac{5\sqrt{bx+a}\,a^3\sqrt{x}}{64b}+\frac{5\,(bx+a)^{\frac{3}{2}}\,a\,x^{\frac{3}{2}}}{24}+\frac{(bx+a)^{\frac{5}{2}}\,x^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*x^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(b*x+a)^(5/2)+5/24*a*x^(3/2)*(b*x+a)^(3/2)+5/32*a^2*x^(3/2)*(b*
x+a)^(1/2)+5/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b-5/128*a^4/b^(3/2)*((b*x+a)*x)^(
1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima [B]** time = 2.98, size = 176, normalized size = 1.52

$$\frac{5\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{3}{2}}}+\frac{\frac{15\,\sqrt{bx+a}\,a^4b^3}{\sqrt{x}}-\frac{55\,(bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}+\frac{73\,(bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{192\left(b^5-\frac{4\,(bx+a)b^4}{x}+\frac{6\,(bx+a)^2b^3}{x^2}-\frac{4\,(bx+a)^3b^2}{x^3}+\frac{(bx+a)^4b}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 5/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(3/2) + 1/192*(15*sqrt(b*x + a)*a^4*b^3/sqrt(x) - 55*(b*x + a)^(
3/2)*a^4*b^2/x^(3/2) + 73*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 15*(b*x + a)^(7/2
)*a^4/x^(7/2))/(b^5 - 4*(b*x + a)*b^4/x + 6*(b*x + a)^2*b^3/x^2 - 4*(b*x +
a)^3*b^2/x^3 + (b*x + a)^4*b/x^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a+b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(1/2)*(a + b*x)^(5/2), x)
```

**sympy [A]** time = 9.86, size = 155, normalized size = 1.34

$$\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1+\frac{bx}{a}}}+\frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1+\frac{bx}{a}}}+\frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1+\frac{bx}{a}}}+\frac{23\sqrt{a}\,b^2x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}}-\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}+\frac{b^3x^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*x**(1/2),x)
```

```
[Out] 5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 + b*x/a)) + 133*a**(5/2)*x**(3/2)/(192*sqrt
(1 + b*x/a)) + 127*a**(3/2)*b*x**(5/2)/(96*sqrt(1 + b*x/a)) + 23*sqrt(a)*b*
*2*x**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(6
4*b**(3/2)) + b**3*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

## 3.548 $\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=92

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}+\frac{5}{8}a^2\sqrt{x}\sqrt{a+bx}+\frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}$$

**Rubi** [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{5}{8}a^2\sqrt{x}\sqrt{a+bx}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}+\frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/8 + (5*a*Sqrt[x]*(a + b*x)^(3/2))/12 + (Sqrt[
x]*(a + b*x)^(5/2))/3 + (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8
*Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{6}(5a)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx$$

$$= \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{8}\left(5a^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x,$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}$$

**Mathematica [A]** time = 0.11, size = 80, normalized size = 0.87

$$\frac{1}{24}\sqrt{a+bx}\left(\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx}{a}+1}} + \sqrt{x}\left(33a^2+26abx+8b^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[x]*(33*a^2 + 26*a*b*x + 8*b^2*x^2) + (15*a^(5/2)*ArcSi
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x)/a])))/24
```

**IntegrateAlgebraic [A]** time = 0.11, size = 79, normalized size = 0.86

$$\frac{1}{24}\sqrt{a+bx}\left(33a^2\sqrt{x} + 26abx^{3/2} + 8b^2x^{5/2}\right) - \frac{5a^3\log\left(\sqrt{a+bx} - \sqrt{b}\,\sqrt{x}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a + b*x]*(33*a^2*Sqrt[x] + 26*a*b*x^(3/2) + 8*b^2*x^(5/2)))/24 - (5*a
^3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*Sqrt[b])
```

**fricas [A]** time = 1.51, size = 141, normalized size = 1.53

$$\left[\frac{15\,a^{3}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(8\,b^{3}x^{2}+26\,ab^{2}x+33\,a^{2}b\right)\sqrt{bx+a}\,\sqrt{x}}{48\,b},\,-\frac{15\,a^{3}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(8\,b^{3}x^{2}+26\,ab^{2}x+33\,a^{2}b\right)\sqrt{bx+a}\,\sqrt{x}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b, -1/24*(15*a^3
*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 26*a*b^
2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 93, normalized size = 1.01

$$\frac{5\sqrt{(bx+a)\,x}\;a^{3}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{16\sqrt{bx+a}\;\sqrt{b}\;\sqrt{x}}+\frac{5\sqrt{bx+a}\;a^{2}\sqrt{x}}{8}+\frac{5\,(bx+a)^{\frac{3}{2}}\,a\sqrt{x}}{12}+\frac{(bx+a)^{\frac{5}{2}}\,\sqrt{x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(b*x+a)^(5/2)*x^(1/2)+5/12*a*(b*x+a)^(3/2)*x^(1/2)+5/8*a^2*x^(1/2)*(b*x
+a)^(1/2)+5/16*a^3*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((b*x+1/2*a)/b
^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

**maxima [B]** time = 2.99, size = 141, normalized size = 1.53

$$-\frac{5\,a^{3}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,\sqrt{b}}-\frac{\frac{15\,\sqrt{bx+a}\,a^{3}b^{2}}{\sqrt{x}}-\frac{40\,(bx+a)^{\frac{3}{2}}\,a^{3}b}{x^{\frac{3}{2}}}+\frac{33\,(bx+a)^{\frac{5}{2}}\,a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{3}-\frac{3\,(bx+a)b^{2}}{x}+\frac{3\,(bx+a)^{2}b}{x^{2}}-\frac{(bx+a)^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/sqrt(b) - 1/24*(15*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 40*(b*x + a)^(3
/2)*a^3*b/x^(3/2) + 33*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^3 - 3*(b*x + a)*b^2/
x + 3*(b*x + a)^2*b/x^2 - (b*x + a)^3/x^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(1/2),x)
```

```
[Out] int((a + b*x)^(5/2)/x^(1/2), x)
```

**sympy [A]** time = 6.23, size = 102, normalized size = 1.11

$$\frac{11a^{\frac{5}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{8}+\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{12}+\frac{\sqrt{a}\,b^{2}x^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{3}+\frac{5a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] 11*a**(5/2)*sqrt(x)*sqrt(1 + b*x/a)/8 + 13*a**(3/2)*b*x**(3/2)*sqrt(1 + b*x
/a)/12 + sqrt(a)*b**2*x**(5/2)*sqrt(1 + b*x/a)/3 + 5*a**3*asinh(sqrt(b)*sqr
t(x)/sqrt(a))/(8*sqrt(b))
```

## 3.549 $\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx$

**Optimal**. Leaf size=89

$$\frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}+\frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}$$

**Rubi** [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$\frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}+\frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (15*a*b*Sqrt[x]*Sqrt[a + b*x])/4 + (5*b*Sqrt[x]*(a + b*x)^(3/2))/2 - (2*(a
+ b*x)^(5/2))/Sqrt[x] + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
b*x]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(a+bx)^{5/2}}{\sqrt{x}}+(5b)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx\\
&= \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{4}(15ab)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\
&= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\
&= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\right.\\
&= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\right.\\
&= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 48, normalized size = 0.54

$$-\frac{2a^2\sqrt{a+bx}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}\,\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-2*a^2*Sqrt[a + b*x]*Hypergeometric2F1[-5/2, -1/2, 1/2, -((b*x)/a)])/(Sqrt
[x]*Sqrt[1 + (b*x)/a])
```

**IntegrateAlgebraic [A]** time = 0.13, size = 73, normalized size = 0.82

$$\frac{\sqrt{a+bx}\left(-8a^2+9abx+2b^2x^2\right)}{4\sqrt{x}}-\frac{15}{4}a^2\sqrt{b}\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-8*a^2 + 9*a*b*x + 2*b^2*x^2))/(4*Sqrt[x]) - (15*a^2*Sqrt[b
]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/4
```

**fricas [A]** time = 1.39, size = 137, normalized size = 1.54

$$\left[\frac{15\,a^2\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(2\,b^2x^2+9\,abx-8\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{8\,x},\,-\frac{15\,a^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(2\,b^2x^2+9\,abx-8\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{4\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x + a)*sqrt(x))/x, -1/4*(15*a^2*sqrt(
-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x^2 + 9*a*b*x - 8
*a^2)*sqrt(b*x + a)*sqrt(x))/x]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 84, normalized size = 0.94

$$\frac{15\sqrt{(bx+a)\,x}\;a^2\sqrt{b}\;\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8\sqrt{bx+a}\;\sqrt{x}}-\frac{\sqrt{bx+a}\;\left(-2b^2x^2-9abx+8a^2\right)}{4\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(3/2),x)
```

```
[Out] -1/4*(b*x+a)^(1/2)*(-2*b^2*x^2-9*a*b*x+8*a^2)/x^(1/2)+15/8*a^2*b^(1/2)*ln((
b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/
2)
```

**maxima [A]** time = 2.98, size = 125, normalized size = 1.40

$$-\frac{15}{8}\,a^2\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}\,a^2}{\sqrt{x}}-\frac{\frac{7\,\sqrt{bx+a}\,a^2b^2}{\sqrt{x}}-\frac{9\,(bx+a)^{\frac{3}{2}}a^2b}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2\,(bx+a)b}{x}+\frac{(bx+a)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -15/8*a^2*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*
x + a)/sqrt(x))) - 2*sqrt(b*x + a)*a^2/sqrt(x) - 1/4*(7*sqrt(b*x + a)*a^2*b
^2/sqrt(x) - 9*(b*x + a)^(3/2)*a^2*b/x^(3/2))/(b^2 - 2*(b*x + a)*b/x + (b*x
 + a)^2/x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(3/2),x)
```

```
[Out] int((a + b*x)^(5/2)/x^(3/2), x)
```

**sympy [A]** time = 6.15, size = 126, normalized size = 1.42

$$-\frac{2a^{\frac{5}{2}}}{\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1+\frac{bx}{a}}}+\frac{11\sqrt{a}\,b^2x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}}+\frac{15a^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4}+\frac{b^3x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(3/2),x)
```

```
[Out] -2*a**(5/2)/(sqrt(x)*sqrt(1 + b*x/a)) + a**(3/2)*b*sqrt(x)/(4*sqrt(1 + b*x/
a)) + 11*sqrt(a)*b**2*x**(3/2)/(4*sqrt(1 + b*x/a)) + 15*a**2*sqrt(b)*asinh(
sqrt(b)*sqrt(x)/sqrt(a))/4 + b**3*x**(5/2)/(2*sqrt(a)*sqrt(1 + b*x/a))
```

## 3.550 $\int \frac{(a+bx)^{5/2}}{x^{5/2}} dx$

**Optimal**. Leaf size=86

$$5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)+5b^2\sqrt{x}\sqrt{a+bx}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}$$

**Rubi** [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$5b^2\sqrt{x}\sqrt{a+bx}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a + b*x] - (10*b*(a + b*x)^(3/2))/(3*Sqrt[x]) - (2*(a +
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\frac{1}{3}(5b)\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx \\
&= -\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx \\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx \\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right) \\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right) \\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 50, normalized size = 0.58

$$-\frac{2a^2\sqrt{a+bx}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[a + b*x]*Hypergeometric2F1[-5/2, -3/2, -1/2, -((b*x)/a)])/(3*x
^(3/2)*Sqrt[1 + (b*x)/a])
```

**IntegrateAlgebraic [A]** time = 0.16, size = 69, normalized size = 0.80

$$\frac{\sqrt{a+bx}\left(-2a^2-14abx+3b^2x^2\right)}{3x^{3/2}}-5ab^{3/2}\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-2*a^2 - 14*a*b*x + 3*b^2*x^2))/(3*x^(3/2)) - 5*a*b^(3/2)*L
og[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
```

**fricas [A]** time = 1.53, size = 138, normalized size = 1.60

$$\left[\frac{15\,ab^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(3\,b^2x^2-14\,abx-2\,a^2\right)\sqrt{bx+a}\sqrt{x}}{6\,x^2}, -\frac{15\,a\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\left(3\,b^2x^2-14\,abx-2\,a^2\right)\sqrt{bx+a}\sqrt{x}}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(3*b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2, -1/3*(15*a*sqrt
(-b)*b*x^2*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 14*a*b
*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.02, size = 82, normalized size = 0.95

$$\frac{5\sqrt{(bx+a)\,x}\;a\,b^{\frac{3}{2}}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{bx+a}\;\sqrt{x}}-\frac{\sqrt{bx+a}\;\left(-3b^2x^2+14abx+2a^2\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(5/2),x)
```

```
[Out] -1/3*(b*x+a)^(1/2)*(-3*b^2*x^2+14*a*b*x+2*a^2)/x^(3/2)+5/2*a*b^(3/2)*ln((b*
x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
```

**maxima [A]** time = 2.95, size = 100, normalized size = 1.16

$$-\frac{5}{2}\,ab^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{4\sqrt{bx+a}\,ab}{\sqrt{x}}-\frac{\sqrt{bx+a}\,ab^{2}}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5/2*a*b^(3/2)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x +
 a)/sqrt(x))) - 4*sqrt(b*x + a)*a*b/sqrt(x) - sqrt(b*x + a)*a*b^2/((b - (b*
x + a)/x)*sqrt(x)) - 2/3*(b*x + a)^(3/2)*a/x^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\,x)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(5/2),x)
```

```
[Out] int((a + b*x)^(5/2)/x^(5/2), x)
```

**sympy [A]** time = 5.62, size = 99, normalized size = 1.15

$$-\frac{2a^{2}\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3}-\frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}+5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)+b^{\frac{5}{2}}x\sqrt{\frac{a}{bx}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(5/2),x)
```

```
[Out] -2*a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 14*a*b**(3/2)*sqrt(a/(b*x) + 1)/3
 - 5*a*b**(3/2)*log(a/(b*x))/2 + 5*a*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1) +
b**(5/2)*x*sqrt(a/(b*x) + 1)
```

# 3.551 $\int x^{5/2}(a-bx)^{5/2}\,dx$

**Optimal**. Leaf size=171

$$\frac{5a^6\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{512b^{7/2}}-\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x$$

**Rubi [A]** time = 0.06, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}+\frac{5a^6\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{512b^{7/2}}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a - b*x)^(5/2),x]
```

```
[Out] (-5*a^5*Sqrt[x]*Sqrt[a - b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a - b*x])/(7
68*b^2) - (a^3*x^(5/2)*Sqrt[a - b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a - b*x])
/32 + (a*x^(7/2)*(a - b*x)^(3/2))/12 + (x^(7/2)*(a - b*x)^(5/2))/6 + (5*a^6
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(512*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{5/2}(a-bx)^{5/2}\,dx &= \frac{1}{6}x^{7/2}(a-bx)^{5/2}+\frac{1}{12}(5a)\int x^{5/2}(a-bx)^{3/2}\,dx\\
&= \frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}+\frac{1}{8}a^2\int x^{5/2}\sqrt{a-bx}\,dx\\
&= \frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}+\frac{1}{64}a^3\int\frac{x^{5/2}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}+\frac{\left(5a^4\right)\int -}{384}\\
&= -\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-\\
&= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-\\
&= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-\\
&= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-\\
&= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 120, normalized size = 0.70

$$\frac{\sqrt{a-bx}\left(\frac{15a^{11/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}+\sqrt{b}\sqrt{x}\left(-15a^5-10a^4bx-8a^3b^2x^2+432a^2b^3x^3-640ab^4x^4+256b^5x^5\right)\right)}{1536b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^5 - 10*a^4*b*x - 8*a^3*b^2*x^2 + 432
*a^2*b^3*x^3 - 640*a*b^4*x^4 + 256*b^5*x^5) + (15*a^(11/2)*ArcSin[(Sqrt[b]*
Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(1536*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.30, size = 130, normalized size = 0.76

$$\frac{5a^6\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{512b^4}+\frac{\sqrt{a-bx}\left(-15a^5\sqrt{x}-10a^4bx^{3/2}-8a^3b^2x^{5/2}+432a^2b^3x^{7/2}-640ab^4x^{9/2}+256b^5x^{11/2}\right)}{1536b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^5*Sqrt[x] - 10*a^4*b*x^(3/2) - 8*a^3*b^2*x^(5/2) + 43
2*a^2*b^3*x^(7/2) - 640*a*b^4*x^(9/2) + 256*b^5*x^(11/2)))/(1536*b^3) + (5*
a^6*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(512*b^4)
```

**fricas [A]** time = 1.40, size = 208, normalized size = 1.22

$$\left[-\frac{15\,a^6\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(256\,b^6x^5-640\,ab^5x^4+432\,a^2b^4x^3-8\,a^3b^3x^2-10\,a^4b^2x-15\,a^5b\right)\sqrt{-bx+a}\sqrt{x}}{3072\,b^4},-\frac{15\,a^6\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\left(256\,b^6x^5-640\,ab^5x^4+432\,a^2b^4x^3-8\,a^3b^3x^2-10\,a^4b^2x-15\,a^5b\right)\sqrt{-bx+a}\sqrt{x}}{1536\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3072*(15*a^6*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(256*b^6*x^5 - 640*a*b^5*x^4 + 432*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 10*
a^4*b^2*x - 15*a^5*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/1536*(15*a^6*sqrt(b)*
arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (256*b^6*x^5 - 640*a*b^5*x^4 + 4
32*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 10*a^4*b^2*x - 15*a^5*b)*sqrt(-b*x + a)*sq
rt(x))/b^4]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 165, normalized size = 0.96

$$\frac{5\sqrt{(-bx+a)\,x}\;a^6\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{1024\sqrt{-bx+a}\;b^{\frac{7}{2}}\sqrt{x}}+\frac{5\sqrt{-bx+a}\;a^5\sqrt{x}}{512b^3}+\frac{5\,(-bx+a)^{\frac{3}{2}}\,a^4\sqrt{x}}{768b^3}-\frac{(-bx+a)^{\frac{7}{2}}\,x^{\frac{5}{2}}}{6b}+\frac{(-bx+a)^{\frac{5}{2}}\,a^3\sqrt{x}}{192b^3}-\frac{(-bx+a)^{\frac{7}{2}}\,a\,x^{\frac{3}{2}}}{12b^2}-\frac{(-bx+a)^{\frac{7}{2}}\,a^2\sqrt{x}}{32b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(5/2),x)
```

```
[Out] -1/6/b*x^(5/2)*(-b*x+a)^(7/2)-1/12*a/b^2*x^(3/2)*(-b*x+a)^(7/2)-1/32*a^2/b^
3*x^(1/2)*(-b*x+a)^(7/2)+1/192*a^3/b^3*(-b*x+a)^(5/2)*x^(1/2)+5/768*a^4/b^3
*(-b*x+a)^(3/2)*x^(1/2)+5/512*a^5*x^(1/2)*(-b*x+a)^(1/2)/b^3+5/1024*a^6/b^(
7/2)*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a
*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.86, size = 242, normalized size = 1.42

$$-\frac{5\,a^{6}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{512\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^{6}b^{5}}{\sqrt{x}}+\frac{85\,(-bx+a)^{\frac{3}{2}}a^{6}b^{4}}{x^{\frac{3}{2}}}+\frac{198\,(-bx+a)^{\frac{5}{2}}a^{6}b^{3}}{x^{\frac{5}{2}}}-\frac{198\,(-bx+a)^{\frac{7}{2}}a^{6}b^{2}}{x^{\frac{7}{2}}}-\frac{85\,(-bx+a)^{\frac{9}{2}}a^{6}b}{x^{\frac{9}{2}}}-\frac{15\,(-bx+a)^{\frac{11}{2}}a^{6}}{x^{\frac{11}{2}}}}{1536\left(b^{9}-\frac{6\,(bx-a)b^{8}}{x}+\frac{15\,(bx-a)^{2}b^{7}}{x^{2}}-\frac{20\,(bx-a)^{3}b^{6}}{x^{3}}+\frac{15\,(bx-a)^{4}b^{5}}{x^{4}}-\frac{6\,(bx-a)^{5}b^{4}}{x^{5}}+\frac{(bx-a)^{6}b^{3}}{x^{6}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -5/512*a^6*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/1536*(15*sq
rt(-b*x + a)*a^6*b^5/sqrt(x) + 85*(-b*x + a)^(3/2)*a^6*b^4/x^(3/2) + 198*(-
b*x + a)^(5/2)*a^6*b^3/x^(5/2) - 198*(-b*x + a)^(7/2)*a^6*b^2/x^(7/2) - 85*
(-b*x + a)^(9/2)*a^6*b/x^(9/2) - 15*(-b*x + a)^(11/2)*a^6/x^(11/2))/(b^9 -
6*(b*x - a)*b^8/x + 15*(b*x - a)^2*b^7/x^2 - 20*(b*x - a)^3*b^6/x^3 + 15*(b
*x - a)^4*b^5/x^4 - 6*(b*x - a)^5*b^4/x^5 + (b*x - a)^6*b^3/x^6)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a-b\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)*(a - b*x)^(5/2), x)
```

**sympy [A]** time = 25.96, size = 435, normalized size = 2.54

$$\begin{cases} \frac{5ia^{\frac{11}{2}}\sqrt{x}}{512b^{3}\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{-1+\frac{bx}{a}}}-\frac{55ia^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{-1+\frac{bx}{a}}}+\frac{67ia^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{-1+\frac{bx}{a}}}-\frac{7i\sqrt{a}\,b^{2}x^{\frac{11}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{6}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}}+\frac{ib^{3}x^{\frac{13}{2}}}{6\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^{3}\sqrt{1-\frac{bx}{a}}}+\frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1-\frac{bx}{a}}}+\frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1-\frac{bx}{a}}}-\frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1-\frac{bx}{a}}}+\frac{7\sqrt{a}\,b^{2}x^{\frac{11}{2}}}{12\sqrt{1-\frac{bx}{a}}}+\frac{5a^{6}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}}-\frac{b^{3}x^{\frac{13}{2}}}{6\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((5*I*a**(11/2)*sqrt(x)/(512*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(9/2)
*x**(3/2)/(1536*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x**(5/2)/(768*b*sqrt(-1
 + b*x/a)) - 55*I*a**(5/2)*x**(7/2)/(192*sqrt(-1 + b*x/a)) + 67*I*a**(3/2)*
b*x**(9/2)/(96*sqrt(-1 + b*x/a)) - 7*I*sqrt(a)*b**2*x**(11/2)/(12*sqrt(-1 +
 b*x/a)) - 5*I*a**6*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + I*b**3*
x**(13/2)/(6*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(11/2)*sqrt
(x)/(512*b**3*sqrt(1 - b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536*b**2*sqrt(1 - b*
x/a)) + a**(7/2)*x**(5/2)/(768*b*sqrt(1 - b*x/a)) + 55*a**(5/2)*x**(7/2)/(1
92*sqrt(1 - b*x/a)) - 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1 - b*x/a)) + 7*sqrt(
a)*b**2*x**(11/2)/(12*sqrt(1 - b*x/a)) + 5*a**6*asin(sqrt(b)*sqrt(x)/sqrt(a
))/(512*b**(7/2)) - b**3*x**(13/2)/(6*sqrt(a)*sqrt(1 - b*x/a)), True))
```

## 3.552 $\int x^{3/2}(a-bx)^{5/2}\,dx$

**Optimal**. Leaf size=146

$$\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{5/2}}-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}+\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{5/2}}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^2) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a - b*x])/16 + (a*x^(5/2)*(a - b*x)^(3/2))/8 + (x^(5
/2)*(a - b*x)^(5/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(1
28*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{3/2}(a-bx)^{5/2}\,dx = \frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{2}a\int x^{3/2}(a-bx)^{3/2}\,dx$$
$$= \frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^{3/2}\sqrt{a-bx}\,dx$$
$$= \frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{32}a^3\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx$$
$$= -\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{\left(3a^4\right)\int \frac{\sqrt{x}}{\sqrt{a-b}}}{128b}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx$$

**Mathematica [A]** time = 0.15, size = 109, normalized size = 0.75

$$\frac{\sqrt{a-bx}\left(\frac{15a^{9/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}+\sqrt{b}\sqrt{x}\left(-15a^4-10a^3bx+248a^2b^2x^2-336ab^3x^3+128b^4x^4\right)\right)}{640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^4 - 10*a^3*b*x + 248*a^2*b^2*x^2 - 3
36*a*b^3*x^3 + 128*b^4*x^4) + (15*a^(9/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]]
)/Sqrt[1 - (b*x)/a]))/(640*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.18, size = 117, normalized size = 0.80

$$\frac{3a^5\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{128b^3}+\frac{\sqrt{a-bx}\,\left(-15a^4\sqrt{x}-10a^3bx^{3/2}+248a^2b^2x^{5/2}-336ab^3x^{7/2}+128b^4x^{9/2}\right)}{640b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^4*Sqrt[x] - 10*a^3*b*x^(3/2) + 248*a^2*b^2*x^(5/2) -
336*a*b^3*x^(7/2) + 128*b^4*x^(9/2)))/(640*b^2) + (3*a^5*Sqrt[-b]*Log[-(Sqr
t[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(128*b^3)
```

**fricas [A]** time = 1.13, size = 186, normalized size = 1.27

$$\left[-\frac{15\,a^5\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\left(128\,b^5x^4-336\,ab^4x^3+248\,a^2b^3x^2-10\,a^3b^2x-15\,a^4b\right)\sqrt{-bx+a}\,\sqrt{x}}{1280\,b^3},\ -\frac{15\,a^5\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(128\,b^5x^4-336\,ab^4x^3+248\,a^2b^3x^2-10\,a^3b^2x-15\,a^4b\right)\sqrt{-bx+a}\,\sqrt{x}}{640\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^2*b^3*x^2 - 10*a^3*b^2*x - 15*a
^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b*x
+ a)/(sqrt(b)*sqrt(x))) - (128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^2*b^3*x^2 -
10*a^3*b^2*x - 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 146, normalized size = 1.00

$$\frac{3\sqrt{(-bx+a)\,x}\;a^5\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{256\sqrt{-bx+a}\;b^{\frac{5}{2}}\sqrt{x}}+\frac{3\sqrt{-bx+a}\;a^4\sqrt{x}}{128b^2}+\frac{(-bx+a)^{\frac{3}{2}}\,a^3\sqrt{x}}{64b^2}+\frac{(-bx+a)^{\frac{5}{2}}\,a^2\sqrt{x}}{80b^2}-\frac{(-bx+a)^{\frac{7}{2}}\,x^{\frac{3}{2}}}{5b}-\frac{3\,(-bx+a)^{\frac{7}{2}}\,a\sqrt{x}}{40b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(5/2),x)
```

```
[Out] -1/5/b*x^(3/2)*(-b*x+a)^(7/2)-3/40*a/b^2*x^(1/2)*(-b*x+a)^(7/2)+1/80*a^2/b^
2*(-b*x+a)^(5/2)*x^(1/2)+1/64*a^3/b^2*(-b*x+a)^(3/2)*x^(1/2)+3/128*a^4*x^(1
/2)*(-b*x+a)^(1/2)/b^2+3/256*a^5/b^(5/2)*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/
x^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 3.01, size = 207, normalized size = 1.42

$$-\frac{3\,a^{5}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{128\,b^{\frac{5}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^{5}b^{4}}{\sqrt{x}}+\frac{70\,(-bx+a)^{\frac{3}{2}}a^{5}b^{3}}{x^{\frac{3}{2}}}+\frac{128\,(-bx+a)^{\frac{5}{2}}a^{5}b^{2}}{x^{\frac{5}{2}}}-\frac{70\,(-bx+a)^{\frac{7}{2}}a^{5}b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+a)^{\frac{9}{2}}a^{5}}{x^{\frac{9}{2}}}}{640\left(b^{7}-\frac{5\,(bx-a)b^{6}}{x}+\frac{10\,(bx-a)^{2}b^{5}}{x^{2}}-\frac{10\,(bx-a)^{3}b^{4}}{x^{3}}+\frac{5\,(bx-a)^{4}b^{3}}{x^{4}}-\frac{(bx-a)^{5}b^{2}}{x^{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/128*a^5*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/640*(15*sqr
t(-b*x + a)*a^5*b^4/sqrt(x) + 70*(-b*x + a)^(3/2)*a^5*b^3/x^(3/2) + 128*(-b
*x + a)^(5/2)*a^5*b^2/x^(5/2) - 70*(-b*x + a)^(7/2)*a^5*b/x^(7/2) - 15*(-b*
x + a)^(9/2)*a^5/x^(9/2))/(b^7 - 5*(b*x - a)*b^6/x + 10*(b*x - a)^2*b^5/x^2
 - 10*(b*x - a)^3*b^4/x^3 + 5*(b*x - a)^4*b^3/x^4 - (b*x - a)^5*b^2/x^5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(a - b*x)^(5/2), x)
```

**sympy [A]** time = 16.40, size = 379, normalized size = 2.60

$$\begin{cases}\frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{-1+\frac{bx}{a}}}-\frac{129ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{-1+\frac{bx}{a}}}+\frac{73ia^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}}-\frac{29i\sqrt{a}\,b^{2}x^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^{5}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{ib^{3}x^{\frac{11}{2}}}{5\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1\\-\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1-\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1-\frac{bx}{a}}}-\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}+\frac{29\sqrt{a}\,b^{2}x^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}+\frac{3a^{5}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}-\frac{b^{3}x^{\frac{11}{2}}}{5\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x*
*(3/2)/(128*b*sqrt(-1 + b*x/a)) - 129*I*a**(5/2)*x**(5/2)/(320*sqrt(-1 + b*
x/a)) + 73*I*a**(3/2)*b*x**(7/2)/(80*sqrt(-1 + b*x/a)) - 29*I*sqrt(a)*b**2*
x**(9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(1
28*b**(5/2)) + I*b**3*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) >
1), (-3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/2)/(12
8*b*sqrt(1 - b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 - b*x/a)) - 73*a**
(3/2)*b*x**(7/2)/(80*sqrt(1 - b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(40*sqrt(1
 - b*x/a)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) - b**3*x**
(11/2)/(5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

## 3.553 $\int \sqrt{x}\,(a-bx)^{5/2}\,dx$

**Optimal**. Leaf size=121

$$\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a - b*x)^(5/2),x]
```

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a - b*x])/32 +
(5*a*x^(3/2)*(a - b*x)^(3/2))/24 + (x^(3/2)*(a - b*x)^(5/2))/4 + (5*a^4*Arc
Tan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\,(a-bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{1}{8}(5a)\int \sqrt{x}\,(a-bx)^{3/2}\,dx\\
&= \frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{1}{16}\left(5a^2\right)\int \sqrt{x}\,\sqrt{a-bx}\,dx\\
&= \frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{1}{64}\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx\\
&= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{\left(5a^4\right)\int \cdots}{1\cdots}\\
&= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{\left(5a^4\right)\mathrm{Su}\cdots}{\cdots}\\
&= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{\left(5a^4\right)\mathrm{Su}\cdots}{\cdots}\\
&= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{5a^4\tan^{-}\cdots}{64\cdots}
\end{aligned}
$$

**Mathematica [A]** time = 0.13, size = 98, normalized size = 0.81

$$
\frac{\sqrt{a-bx}\left(\frac{15a^{7/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}+\sqrt{b}\,\sqrt{x}\left(-15a^3+118a^2bx-136ab^2x^2+48b^3x^3\right)\right)}{192b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^3 + 118*a^2*b*x - 136*a*b^2*x^2 + 48
*b^3*x^3) + (15*a^(7/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a
]))/(192*b^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.16, size = 104, normalized size = 0.86

$$\frac{5a^4\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{64b^2}+\frac{\sqrt{a-bx}\,\left(-15a^3\sqrt{x}+118a^2bx^{3/2}-136ab^2x^{5/2}+48b^3x^{7/2}\right)}{192b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^3*Sqrt[x] + 118*a^2*b*x^(3/2) - 136*a*b^2*x^(5/2) + 4
8*b^3*x^(7/2)))/(192*b) + (5*a^4*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a
- b*x]])/(64*b^2)
```

**fricas [A]** time = 1.25, size = 164, normalized size = 1.36

$$\left[-\frac{15\,a^4\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\left(48\,b^4x^3-136\,ab^3x^2+118\,a^2b^2x-15\,a^3b\right)\sqrt{-bx+a}\,\sqrt{x}}{384\,b^2},\,-\frac{15\,a^4\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(48\,b^4x^3-136\,ab^3x^2+118\,a^2b^2x-15\,a^3b\right)\sqrt{-bx+a}\,\sqrt{x}}{192\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) - 2*(48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a
)*sqrt(x))/b^2, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(
x))) - (48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x +
a)*sqrt(x))/b^2]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 118, normalized size = 0.98

$$\frac{5\sqrt{-bx+a}\;a^2x^{\frac{3}{2}}}{32}+\frac{5\sqrt{\left(-bx+a\right)x}\;a^4\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{128\sqrt{-bx+a}\;b^{\frac{3}{2}}\sqrt{x}}-\frac{5\sqrt{-bx+a}\;a^3\sqrt{x}}{64b}+\frac{5\left(-bx+a\right)^{\frac{3}{2}}a\,x^{\frac{3}{2}}}{24}+\frac{\left(-bx+a\right)^{\frac{5}{2}}x^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)*x^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(-b*x+a)^(5/2)+5/24*a*x^(3/2)*(-b*x+a)^(3/2)+5/32*a^2*x^(3/2)*(
-b*x+a)^(1/2)-5/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b+5/128*a^4/b^(3/2)*((-b*x+a)
*x)^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1
/2))
```

**maxima [A]** time = 3.05, size = 168, normalized size = 1.39

$$-\frac{5\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{64\,b^{\frac{3}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^4b^3}{\sqrt{x}}+\frac{55\,(-bx+a)^{\frac{3}{2}}\,a^4b^2}{x^{\frac{3}{2}}}+\frac{73\,(-bx+a)^{\frac{5}{2}}\,a^4b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+a)^{\frac{7}{2}}\,a^4}{x^{\frac{7}{2}}}}{192\left(b^5-\frac{4\,(bx-a)b^4}{x}+\frac{6\,(bx-a)^2b^3}{x^2}-\frac{4\,(bx-a)^3b^2}{x^3}+\frac{(bx-a)^4b}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/192*(15*sqrt
(-b*x + a)*a^4*b^3/sqrt(x) + 55*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) + 73*(-b*x
 + a)^(5/2)*a^4*b/x^(5/2) - 15*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^5 - 4*(b*x
- a)*b^4/x + 6*(b*x - a)^2*b^3/x^2 - 4*(b*x - a)^3*b^2/x^3 + (b*x - a)^4*b/
x^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,(a-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a - b*x)^(5/2),x)
```

```
[Out] int(x^(1/2)*(a - b*x)^(5/2), x)
```

**sympy [A]** time = 9.81, size = 326, normalized size = 2.69

$$\begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{-1+\frac{bx}{a}}}-\frac{133ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{-1+\frac{bx}{a}}}+\frac{127ia^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{-1+\frac{bx}{a}}}-\frac{23i\sqrt{a}\,b^2x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}+\frac{ib^3x^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1-\frac{bx}{a}}}+\frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1-\frac{bx}{a}}}-\frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1-\frac{bx}{a}}}+\frac{23\sqrt{a}\,b^2x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}}+\frac{5a^4\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}-\frac{b^3x^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)*x**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b*sqrt(-1 + b*x/a)) - 133*I*a**(5/2)*x*
*(3/2)/(192*sqrt(-1 + b*x/a)) + 127*I*a**(3/2)*b*x**(5/2)/(96*sqrt(-1 + b*x
/a)) - 23*I*sqrt(a)*b**2*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh(sq
rt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) + I*b**3*x**(9/2)/(4*sqrt(a)*sqrt(-1 +
 b*x/a)), Abs(b*x/a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 - b*x/a)) + 13
3*a**(5/2)*x**(3/2)/(192*sqrt(1 - b*x/a)) - 127*a**(3/2)*b*x**(5/2)/(96*sqr
t(1 - b*x/a)) + 23*sqrt(a)*b**2*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a**4*asin
(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) - b**3*x**(9/2)/(4*sqrt(a)*sqrt(1 -
 b*x/a)), True))
```

# 3.554 $\int \frac{(a-bx)^{5/2}}{\sqrt{x}} dx$

**Optimal**. Leaf size=96

$$\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}+\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

**Rubi [A]** time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a - b*x])/8 + (5*a*Sqrt[x]*(a - b*x)^(3/2))/12 + (Sqrt[
x]*(a - b*x)^(5/2))/3 + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*
Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx &= \frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{6}(5a)\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx\\
&= \frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{8}\left(5a^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\right.\\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{8}\left(5a^3\right)\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\right.\\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 82, normalized size = 0.85

$$\frac{1}{24}\sqrt{a-bx}\left(\frac{15a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{1-\frac{bx}{a}}}+\sqrt{x}\,\left(33a^2-26abx+8b^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[x]*(33*a^2 - 26*a*b*x + 8*b^2*x^2) + (15*a^(5/2)*ArcSi
n[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 - (b*x)/a])))/24
```

**IntegrateAlgebraic [A]** time = 0.13, size = 88, normalized size = 0.92

$$\frac{5a^3\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{8b}+\frac{1}{24}\sqrt{a-bx}\,\left(33a^2\sqrt{x}-26abx^{3/2}+8b^2x^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a - b*x]*(33*a^2*Sqrt[x] - 26*a*b*x^(3/2) + 8*b^2*x^(5/2)))/24 + (5*a
^3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*b)
```

**fricas [A]** time = 0.69, size = 142, normalized size = 1.48

$$\left[-\frac{15\,a^{3}\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\left(8\,b^{3}x^{2}-26\,ab^{2}x+33\,a^{2}b\right)\sqrt{-bx+a}\,\sqrt{x}}{48\,b},\,-\frac{15\,a^{3}\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(8\,b^{3}x^{2}-26\,ab^{2}x+33\,a^{2}b\right)\sqrt{-bx+a}\,\sqrt{x}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 - 2*(8*b^3*x^2 - 26*a*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/24*(
15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (8*b^3*x^2 - 26*a
*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.00, size = 99, normalized size = 1.03

$$\frac{5\sqrt{(-bx+a)\,x}\;a^{3}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{16\sqrt{-bx+a}\;\sqrt{b}\;\sqrt{x}}+\frac{5\sqrt{-bx+a}\;a^{2}\sqrt{x}}{8}+\frac{5\,(-bx+a)^{\frac{3}{2}}\,a\sqrt{x}}{12}+\frac{(-bx+a)^{\frac{5}{2}}\,\sqrt{x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(-b*x+a)^(5/2)*x^(1/2)+5/12*a*(-b*x+a)^(3/2)*x^(1/2)+5/8*a^2*x^(1/2)*(-
b*x+a)^(1/2)+5/16*a^3*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arc
tan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.93, size = 130, normalized size = 1.35

$$-\frac{5\,a^3\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,\sqrt{b}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^3b^2}{\sqrt{x}}+\frac{40\,(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}+\frac{33\,(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^3-\frac{3\,(bx-a)b^2}{x}+\frac{3\,(bx-a)^2b}{x^2}-\frac{(bx-a)^3}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/24*(15*sqrt(-
b*x + a)*a^3*b^2/sqrt(x) + 40*(-b*x + a)^(3/2)*a^3*b/x^(3/2) + 33*(-b*x + a
)^(5/2)*a^3/x^(5/2))/(b^3 - 3*(b*x - a)*b^2/x + 3*(b*x - a)^2*b/x^2 - (b*x
- a)^3/x^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(1/2),x)
```

```
[Out] int((a - b*x)^(5/2)/x^(1/2), x)
```

**sympy [A]** time = 6.23, size = 246, normalized size = 2.56

$$\begin{cases} -\frac{11ia^{\frac{5}{2}}\sqrt{x}}{8\sqrt{-1+\frac{bx}{a}}}+\frac{59ia^{\frac{3}{2}}bx^{\frac{3}{2}}}{24\sqrt{-1+\frac{bx}{a}}}-\frac{17i\sqrt{a}\,b^2x^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{ib^3x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ \frac{11a^{\frac{5}{2}}\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}{8}-\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{12}+\frac{\sqrt{a}\,b^2x^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{3}+\frac{5a^3\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((-11*I*a**(5/2)*sqrt(x)/(8*sqrt(-1 + b*x/a)) + 59*I*a**(3/2)*b*x*
*(3/2)/(24*sqrt(-1 + b*x/a)) - 17*I*sqrt(a)*b**2*x**(5/2)/(12*sqrt(-1 + b*x
/a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)) + I*b**3*x**(7/2
)/(3*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (11*a**(5/2)*sqrt(x)*sqrt(
1 - b*x/a)/8 - 13*a**(3/2)*b*x**(3/2)*sqrt(1 - b*x/a)/12 + sqrt(a)*b**2*x**
(5/2)*sqrt(1 - b*x/a)/3 + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)),
 True))
```

## 3.555 $\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx$

**Optimal**. Leaf size=93

$$-\frac{15}{4}a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}$$

**Rubi** [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{15}{4}a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-15*a*b*Sqrt[x]*Sqrt[a - b*x])/4 - (5*b*Sqrt[x]*(a - b*x)^(3/2))/2 - (2*(a
 - b*x)^(5/2))/Sqrt[x] - (15*a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a -
b*x]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(a-bx)^{5/2}}{\sqrt{x}}-(5b)\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx\\
&= -\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}(15ab)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\right.\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\right.\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{15}{4}a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 49, normalized size = 0.53

$$-\frac{2a^2\sqrt{a-bx}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{a}\right)}{\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-2*a^2*Sqrt[a - b*x]*Hypergeometric2F1[-5/2, -1/2, 1/2, (b*x)/a])/(Sqrt[x]
*Sqrt[1 - (b*x)/a])
```

**IntegrateAlgebraic [A]** time = 0.16, size = 79, normalized size = 0.85

$$\frac{\sqrt{a-bx}\left(-8a^2-9abx+2b^2x^2\right)}{4\sqrt{x}}-\frac{15}{4}a^2\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-8*a^2 - 9*a*b*x + 2*b^2*x^2))/(4*Sqrt[x]) - (15*a^2*Sqrt[-
b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/4
```

**fricas [A]** time = 1.30, size = 137, normalized size = 1.47

$$\left[\frac{15\,a^2\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\left(2\,b^2x^2-9\,abx-8\,a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{8\,x},\ \frac{15\,a^2\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(2\,b^2x^2-9\,abx-8\,a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{4\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(2*b^2*x^2 - 9*a*b*x - 8*a^2)*sqrt(-b*x + a)*sqrt(x))/x, 1/4*(15*a^2*s
qrt(b)*x*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x^2 - 9*a*b*x -
8*a^2)*sqrt(-b*x + a)*sqrt(x))/x]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx+a\right)^{\frac{5}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(3/2),x)
```

```
[Out] int((-b*x+a)^(5/2)/x^(3/2),x)
```

**maxima [A]** time = 2.99, size = 112, normalized size = 1.20

$$\frac{15}{4}a^2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\frac{2\sqrt{-bx+a}\,a^2}{\sqrt{x}}-\frac{\frac{7\sqrt{-bx+a}\,a^2b^2}{\sqrt{x}}+\frac{9(-bx+a)^{\frac{3}{2}}a^2b}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2(bx-a)b}{x}+\frac{(bx-a)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 15/4*a^2*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a
)*a^2/sqrt(x) - 1/4*(7*sqrt(-b*x + a)*a^2*b^2/sqrt(x) + 9*(-b*x + a)^(3/2)*
a^2*b/x^(3/2))/(b^2 - 2*(b*x - a)*b/x + (b*x - a)^2/x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(3/2),x)
```

```
[Out] int((a - b*x)^(5/2)/x^(3/2), x)
```

**sympy [A]** time = 6.22, size = 267, normalized size = 2.87

$$\begin{cases} \frac{2ia^{\frac{5}{2}}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}}+\frac{ia^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}}-\frac{11i\sqrt{a}\,b^2x^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}}+\frac{15ia^2\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4}+\frac{ib^3x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{2a^{\frac{5}{2}}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1-\frac{bx}{a}}}+\frac{11\sqrt{a}\,b^2x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}}-\frac{15a^2\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4}-\frac{b^3x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*a**(5/2)/(sqrt(x)*sqrt(-1 + b*x/a)) + I*a**(3/2)*b*sqrt(x)/(
4*sqrt(-1 + b*x/a)) - 11*I*sqrt(a)*b**2*x**(3/2)/(4*sqrt(-1 + b*x/a)) + 15*
I*a**2*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/4 + I*b**3*x**(5/2)/(2*sqrt(a
```

```
)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*a**(5/2)/(sqrt(x)*sqrt(1 - b*x/a)
) - a**(3/2)*b*sqrt(x)/(4*sqrt(1 - b*x/a)) + 11*sqrt(a)*b**2*x**(3/2)/(4*sq
rt(1 - b*x/a)) - 15*a**2*sqrt(b)*asin(sqrt(b)*sqrt(x)/sqrt(a))/4 - b**3*x**
(5/2)/(2*sqrt(a)*sqrt(1 - b*x/a)), True))
```

# 3.556 $\int \frac{(a-bx)^{5/2}}{x^{5/2}} \, dx$

**Optimal**. Leaf size=90

$$5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)+5b^2\sqrt{x}\sqrt{a-bx}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}$$

**Rubi** [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$5b^2\sqrt{x}\sqrt{a-bx}+5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a - b*x] + (10*b*(a - b*x)^(3/2))/(3*Sqrt[x]) - (2*(a -
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x
]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(a-bx)^{5/2}}{3x^{3/2}}-\frac{1}{3}(5b)\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx\\
&= \frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 51, normalized size = 0.57

$$-\frac{2a^2\sqrt{a-bx}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{a}\right)}{3x^{3/2}\sqrt{1-\frac{bx}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[a - b*x]*Hypergeometric2F1[-5/2, -3/2, -1/2, (b*x)/a])/(3*x^(3
/2)*Sqrt[1 - (b*x)/a])
```

**IntegrateAlgebraic [A]** time = 0.19, size = 76, normalized size = 0.84

$$\frac{\sqrt{a-bx}\,\left(-2a^2+14abx+3b^2x^2\right)}{3x^{3/2}}+5a\sqrt{-b}\,b\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-2*a^2 + 14*a*b*x + 3*b^2*x^2))/(3*x^(3/2)) + 5*a*Sqrt[-b]*
b*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]]
```

**fricas [A]** time = 1.33, size = 139, normalized size = 1.54

$$\left[\frac{15\,a\sqrt{-b}\,bx^2\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\left(3\,b^2x^2+14\,abx-2\,a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{6\,x^2},-\frac{15\,ab^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(3\,b^2x^2+14\,abx-2\,a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) + 2*(3*b^2*x^2 + 14*a*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2, -1/3*(15
*a*b^(3/2)*x^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (3*b^2*x^2 + 14*a
*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int\frac{\left(-bx+a\right)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(5/2),x)
```

```
[Out] int((-b*x+a)^(5/2)/x^(5/2),x)
```

**maxima [A]** time = 3.00, size = 84, normalized size = 0.93

$$-5\,ab^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{4\,\sqrt{-bx+a}\,ab}{\sqrt{x}}+\frac{\sqrt{-bx+a}\,ab^{2}}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}-\frac{2\,(-bx+a)^{\frac{3}{2}}a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5*a*b^(3/2)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + 4*sqrt(-b*x + a)*a*
b/sqrt(x) + sqrt(-b*x + a)*a*b^2/((b - (b*x - a)/x)*sqrt(x)) - 2/3*(-b*x +
a)^(3/2)*a/x^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(5/2),x)
```

```
[Out] int((a - b*x)^(5/2)/x^(5/2), x)
```

**sympy [C]** time = 5.85, size = 245, normalized size = 2.72

$$\begin{cases} -\frac{2a^{2}\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x}+\frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3}-5iab^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}+5ab^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+b^{\frac{5}{2}}x\sqrt{\frac{a}{bx}-1} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{2ia^{2}\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{14iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3}+\frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}-5iab^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right)+ib^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 14*a*b**(3/2)*sqrt(a/(
b*x) - 1)/3 - 5*I*a*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + 5*I*a*b**(3/2
)*log(a/(b*x))/2 + 5*a*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)) + b**(5/2)*x*
sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (-2*I*a**2*sqrt(b)*sqrt(-a/(b*x) + 1)
/(3*x) + 14*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/3 + 5*I*a*b**(3/2)*log(a/(b*x))
/2 - 5*I*a*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1) + I*b**(5/2)*x*sqrt(-a/(b*x
) + 1), True))
```

## 3.557 $\int x^{5/2}(2+bx)^{5/2}\,dx$

**Optimal**. Leaf size=144

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{16b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{48b^2}+\frac{1}{6}x^{7/2}(bx+2)^{5/2}+\frac{1}{6}x^{7/2}(bx+2)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{24b}$$

**Rubi [A]** time = 0.05, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{5x^{3/2}\sqrt{bx+2}}{48b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{16b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{1}{6}x^{7/2}(bx+2)^{5/2}+\frac{1}{6}x^{7/2}(bx+2)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(48*b^2) + (
x^(5/2)*Sqrt[2 + b*x])/(24*b) + (x^(7/2)*Sqrt[2 + b*x])/8 + (x^(7/2)*(2 + b
*x)^(3/2))/6 + (x^(7/2)*(2 + b*x)^(5/2))/6 - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(2+bx)^{5/2}\,dx &= \frac{1}{6}x^{7/2}(2+bx)^{5/2} + \frac{5}{6}\int x^{5/2}(2+bx)^{3/2}\,dx \\ &= \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+bx)^{5/2} + \frac{1}{2}\int x^{5/2}\sqrt{2+bx}\,dx \\ &= \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+bx)^{5/2} + \frac{1}{8}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx \\ &= \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+bx)^{5/2} - \frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{24b} \\ &= -\frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+bx)^{5/2} + \\ &= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x \\ &= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x \\ &= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 86, normalized size = 0.60

$$\frac{\sqrt{x}\sqrt{bx+2}\left(8b^5x^5+40b^4x^4+54b^3x^3+2b^2x^2-5bx+15\right)}{48b^3} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 54*b^3*x^3 + 40*b^4*x^4 +
8*b^5*x^5))/(48*b^3) - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(8*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 105, normalized size = 0.73

$$\frac{5\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{8b^{7/2}} + \frac{\sqrt{bx+2}\left(8b^5x^{11/2}+40b^4x^{9/2}+54b^3x^{7/2}+2b^2x^{5/2}-5bx^{3/2}+15\sqrt{x}\right)}{48b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(15*Sqrt[x] - 5*b*x^(3/2) + 2*b^2*x^(5/2) + 54*b^3*x^(7/2) +
 40*b^4*x^(9/2) + 8*b^5*x^(11/2)))/(48*b^3) + (5*Log[-(Sqrt[b]*Sqrt[x]) + S
qrt[2 + b*x]])/(8*b^(7/2))
```

**fricas [A]** time = 1.29, size = 172, normalized size = 1.19

$$\left[\frac{\left(8\,b^{6}x^{5}+40\,b^{5}x^{4}+54\,b^{4}x^{3}+2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{48\,b^{4}},\frac{\left(8\,b^{6}x^{5}+40\,b^{5}x^{4}+54\,b^{4}x^{3}+2\,b^{3}x^{2}-5\,b^{2}x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{48\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*s
qrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) +
 1))/b^4, 1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x
+ 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(
b*sqrt(x))))/b^4]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [83.4865739
918,53.112478131]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,
0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+
%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2
]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%
%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2
,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}
```

```
+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32
,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%
%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%
{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,
2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}
+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-
8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at paramet
ers values [38.6973876911,89.629912049]Warning, choosing root of [1,0,%%%{-
4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2
]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{
8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]
%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-
64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%
%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%
%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[
4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}
+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-2
0,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4
]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}
+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16
,[0,0]%%%}] at parameters values [6.82230772497,55.0343274642]Warning, choo
sing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8
,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%
}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[
0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%
}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,
[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]
%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%
%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,
4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}
+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{2
4,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1
]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%
%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [53.4880634798,16.0
204098616]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+
%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[
2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%
%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3
,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}
+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16
,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%
%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%
{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,
1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+
%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-4
```

```
0,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]
%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters val
ues [46.2456374937,66.0382199469]Warning, choosing root of [1,0,%%%{-4,[1,1
]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+
%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0
]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%
%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,
2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%
%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[
0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%
%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-
20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3
]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+
%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1
,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]
%%%}] at parameters values [94.9264369817,51.8441526662]Warning, choosing r
oot of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]
%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{
28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%
%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{
4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%
%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%
%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,
[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}
+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6
,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0
]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+
%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32
,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [98.7121795234,4.66774101
928]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4
,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%
%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[
0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%
}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8
,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]
%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,
%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,
0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}
+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-4
0,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2
]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%
%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [9
0.2102860468,38.2197840363]1/b*(2*b^3*abs(b)/b^2*(2*(((((113400*b^29/272160
0/b^34*sqrt(b*x+2)*sqrt(b*x+2)-1383480*b^29/2721600/b^34)*sqrt(b*x+2)*sqrt(
b*x+2)+7093170*b^29/2721600/b^34)*sqrt(b*x+2)*sqrt(b*x+2)-19737270*b^29/272
```

```
1600/b^34)*sqrt(b*x+2)*sqrt(b*x+2)+32304825*b^29/2721600/b^34)*sqrt(b*x+2)*
sqrt(b*x+2)-33722325*b^29/2721600/b^34)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-231
/16/b^4/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+12*b^2*ab
s(b)/b^2*(2*(((((5040*b^19/100800/b^23*sqrt(b*x+2)*sqrt(b*x+2)-51660*b^19/10
0800/b^23)*sqrt(b*x+2)*sqrt(b*x+2)+215460*b^19/100800/b^23)*sqrt(b*x+2)*sqr
t(b*x+2)-469350*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*x+2)+607950*b^19/10080
0/b^23)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+63/8/b^3/sqrt(b)*ln(abs(sqrt(b*(b*x
+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+24*b*abs(b)/b^2*(2*((((90*b^11/1440/b^14*sqr
t(b*x+2)*sqrt(b*x+2)-750*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)+2445*b^11/
1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)-4185*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*
(b*x+2)-2*b)-35/8/b^2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2
))))+16*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b
^7)*sqrt(b*x+2)*sqrt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b
)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))
```

**maple [A]** time = 0.00, size = 138, normalized size = 0.96

$$\frac{(bx+2)^{\frac{7}{2}} x^{\frac{5}{2}}}{6b}-\frac{(bx+2)^{\frac{7}{2}} x^{\frac{3}{2}}}{6b^{2}}+\frac{(bx+2)^{\frac{7}{2}} \sqrt{x}}{8b^{3}}-\frac{(bx+2)^{\frac{5}{2}} \sqrt{x}}{24b^{3}}-\frac{5\,(bx+2)^{\frac{3}{2}} \sqrt{x}}{48b^{3}}-\frac{5\sqrt{bx+2}\,\sqrt{x}}{16b^{3}}-\frac{5\sqrt{(bx+2)\,x}\;\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{16\sqrt{bx+2}\;b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(5/2),x)
```

```
[Out] 1/6/b*x^(5/2)*(b*x+2)^(7/2)-1/6/b^2*x^(3/2)*(b*x+2)^(7/2)+1/8/b^3*x^(1/2)*(
b*x+2)^(7/2)-1/24*(b*x+2)^(5/2)/b^3*x^(1/2)-5/48*(b*x+2)^(3/2)/b^3*x^(1/2)-
5/16*(b*x+2)^(1/2)/b^3*x^(1/2)-5/16*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(7/2)
/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [B]** time = 2.98, size = 223, normalized size = 1.55

$$\frac{\frac{15\,\sqrt{bx+2}\,b^{5}}{\sqrt{x}}-\frac{85\,(bx+2)^{\frac{3}{2}} b^{4}}{x^{\frac{3}{2}}}+\frac{198\,(bx+2)^{\frac{5}{2}} b^{3}}{x^{\frac{5}{2}}}+\frac{198\,(bx+2)^{\frac{7}{2}} b^{2}}{x^{\frac{7}{2}}}-\frac{85\,(bx+2)^{\frac{9}{2}} b}{x^{\frac{9}{2}}}+\frac{15\,(bx+2)^{\frac{11}{2}}}{x^{\frac{11}{2}}}}{24\left(b^{9}-\frac{6\,(bx+2)b^{8}}{x}+\frac{15\,(bx+2)^{2}b^{7}}{x^{2}}-\frac{20\,(bx+2)^{3}b^{6}}{x^{3}}+\frac{15\,(bx+2)^{4}b^{5}}{x^{4}}-\frac{6\,(bx+2)^{5}b^{4}}{x^{5}}+\frac{(bx+2)^{6}b^{3}}{x^{6}}\right)}+\frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(15*sqrt(b*x + 2)*b^5/sqrt(x) - 85*(b*x + 2)^(3/2)*b^4/x^(3/2) + 198*(
b*x + 2)^(5/2)*b^3/x^(5/2) + 198*(b*x + 2)^(7/2)*b^2/x^(7/2) - 85*(b*x + 2)
^(9/2)*b/x^(9/2) + 15*(b*x + 2)^(11/2)/x^(11/2))/(b^9 - 6*(b*x + 2)*b^8/x +
 15*(b*x + 2)^2*b^7/x^2 - 20*(b*x + 2)^3*b^6/x^3 + 15*(b*x + 2)^4*b^5/x^4 -
 6*(b*x + 2)^5*b^4/x^5 + (b*x + 2)^6*b^3/x^6) + 5/16*log(-(sqrt(b) - sqrt(b
*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(b\,x+2)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(5/2)*(b*x + 2)^(5/2), x)
```

**sympy [A]** time = 22.96, size = 158, normalized size = 1.10

$$\frac{b^3x^{\frac{13}{2}}}{6\sqrt{bx+2}}+\frac{7b^2x^{\frac{11}{2}}}{6\sqrt{bx+2}}+\frac{67bx^{\frac{9}{2}}}{24\sqrt{bx+2}}+\frac{55x^{\frac{7}{2}}}{24\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{48b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{48b^2\sqrt{bx+2}}+\frac{5\sqrt{x}}{8b^3\sqrt{bx+2}}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(5/2),x)
```

```
[Out] b**3*x**(13/2)/(6*sqrt(b*x + 2)) + 7*b**2*x**(11/2)/(6*sqrt(b*x + 2)) + 67*
b*x**(9/2)/(24*sqrt(b*x + 2)) + 55*x**(7/2)/(24*sqrt(b*x + 2)) - x**(5/2)/(
48*b*sqrt(b*x + 2)) + 5*x**(3/2)/(48*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(8*b**
3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(8*b**(7/2))
```

# 3.558 $\int x^{3/2}(2+bx)^{5/2}\,dx$

**Optimal**. Leaf size=123

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

**Rubi** [A] time = 0.03, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (x^(5/2)*(2 + b*x)^(5/2
))/5 + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}(2+bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \int x^{3/2}(2+bx)^{3/2}\,dx \\
&= \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \frac{3}{4}\int x^{3/2}\sqrt{2+bx}\,dx \\
&= \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx \\
&= \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b} \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} +
\end{aligned}
$$

**Mathematica** [A] time = 0.05, size = 78, normalized size = 0.63

$$
\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}} + \frac{\sqrt{x}\sqrt{bx+2}\left(8b^4x^4+42b^3x^3+62b^2x^2+5bx-15\right)}{40b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-15 + 5*b*x + 62*b^2*x^2 + 42*b^3*x^3 + 8*b^4*x^4))
/(40*b^2) + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.13, size = 95, normalized size = 0.77

$$
\frac{\sqrt{bx+2}\left(8b^4x^{9/2}+42b^3x^{7/2}+62b^2x^{5/2}+5bx^{3/2}-15\sqrt{x}\right)}{40b^2} - \frac{3\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{4b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(-15*Sqrt[x] + 5*b*x^(3/2) + 62*b^2*x^(5/2) + 42*b^3*x^(7/2)
 + 8*b^4*x^(9/2)))/(40*b^2) - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(
4*b^(5/2))
```

**fricas [A]** time = 1.32, size = 155, normalized size = 1.26

$$\left[\frac{\left(8\,b^5x^4 + 42\,b^4x^3 + 62\,b^3x^2 + 5\,b^2x - 15\,b\right)\sqrt{bx+2}\,\sqrt{x} + 15\,\sqrt{b}\,\log\left(bx + \sqrt{bx+2}\,\sqrt{b}\,\sqrt{x} + 1\right)}{40\,b^3}, \frac{\left(8\,b^5x^4 + 42\,b^4x^3 + 62\,b^3x^2 + 5\,b^2x - 15\,b\right)\sqrt{bx+2}\,\sqrt{x} - 30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{40\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2)
*sqrt(x) + 15*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/
40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2)*sq
rt(x) - 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [83.4865739
918,53.112478131]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,
0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+
%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2
]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%
%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2
,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}
+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32
,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%
%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%
{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,
```

2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}
+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-
8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at paramet
ers values [38.6973876911,89.629912049]Warning, choosing root of [1,0,%%%{-
4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2
]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{
8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]
%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-
64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%
%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%
%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[
4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}
+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-2
0,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4
]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}
+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16
,[0,0]%%%}] at parameters values [6.82230772497,55.0343274642]Warning, choo
sing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8
,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%
}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[
0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%
}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,
[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]
%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%
%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,
4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}
+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{2
4,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1
]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%
%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [53.4880634798,16.0
204098616]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+
%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[
2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%
%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3
,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}
+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16
,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%
%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%
{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,
1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+
%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-4
0,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]
%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters val
ues [46.2456374937,66.0382199469]Warning, choosing root of [1,0,%%%{-4,[1,1
]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+

%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0
]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%
%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,
2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%
%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[
0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%
%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-
20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3
]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+
%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1
,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]
%%%}] at parameters values [94.9264369817,51.8441526662]Warning, choosing r
oot of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]
%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{
28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%
%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{
4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%
%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%
%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,
[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}
+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6
,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0
]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+
%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32
,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [98.7121795234,4.66774101
928]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4
,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%
%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[
0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%
}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8
,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]
%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,
%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,
0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}
+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-4
0,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2
]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%
%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [9
0.2102860468,38.2197840363]1/b*(2*b^3*abs(b)/b^2*(2*((((5040*b^19/100800/b^
23*sqrt(b*x+2)*sqrt(b*x+2)-51660*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*x+2)+
215460*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*x+2)-469350*b^19/100800/b^23)*s
qrt(b*x+2)*sqrt(b*x+2)+607950*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*(b*x+2)-
2*b)+63/8/b^3/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+12*
b^2*abs(b)/b^2*(2*(((90*b^11/1440/b^14*sqrt(b*x+2)*sqrt(b*x+2)-750*b^11/144
0/b^14)*sqrt(b*x+2)*sqrt(b*x+2)+2445*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2

```
)-4185*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-35/8/b^2/sqrt(b)*ln(
abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+24*b*abs(b)/b^2*(2*((12*b^5/
144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*x+2)+198
*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(
b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+16*abs(b)/b^2/b*(2*(1/8*sqrt(b*x+2)*sqrt
(b*x+2)-5/8)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqrt(b*(b
*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))
```

**maple [A]** time = 0.00, size = 123, normalized size = 1.00

$$\frac{(bx+2)^{\frac{7}{2}}x^{\frac{3}{2}}}{5b}-\frac{3(bx+2)^{\frac{7}{2}}\sqrt{x}}{20b^2}+\frac{(bx+2)^{\frac{5}{2}}\sqrt{x}}{20b^2}+\frac{(bx+2)^{\frac{3}{2}}\sqrt{x}}{8b^2}+\frac{3\sqrt{bx+2}\,\sqrt{x}}{8b^2}+\frac{3\sqrt{(bx+2)x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{bx^2+2x}\right)}{8\sqrt{bx+2}\,b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(5/2),x)
```

```
[Out] 1/5/b*x^(3/2)*(b*x+2)^(7/2)-3/20/b^2*x^(1/2)*(b*x+2)^(7/2)+1/20*(b*x+2)^(5/
2)/b^2*x^(1/2)+1/8*(b*x+2)^(3/2)/b^2*x^(1/2)+3/8*(b*x+2)^(1/2)/b^2*x^(1/2)+
3/8*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(5/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x
^2+2*x)^(1/2))
```

**maxima [B]** time = 2.94, size = 194, normalized size = 1.58

$$-\frac{\frac{15\sqrt{bx+2}\,b^4}{\sqrt{x}}-\frac{70(bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}+\frac{128(bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}+\frac{70(bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15(bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^7-\frac{5(bx+2)b^6}{x}+\frac{10(bx+2)^2b^5}{x^2}-\frac{10(bx+2)^3b^4}{x^3}+\frac{5(bx+2)^4b^3}{x^4}-\frac{(bx+2)^5b^2}{x^5}\right)}-\frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20*(15*sqrt(b*x + 2)*b^4/sqrt(x) - 70*(b*x + 2)^(3/2)*b^3/x^(3/2) + 128*
(b*x + 2)^(5/2)*b^2/x^(5/2) + 70*(b*x + 2)^(7/2)*b/x^(7/2) - 15*(b*x + 2)^(
9/2)/x^(9/2))/(b^7 - 5*(b*x + 2)*b^6/x + 10*(b*x + 2)^2*b^5/x^2 - 10*(b*x +
 2)^3*b^4/x^3 + 5*(b*x + 2)^4*b^3/x^4 - (b*x + 2)^5*b^2/x^5) - 3/8*log(-(sq
rt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(b\,x+2)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(3/2)*(b*x + 2)^(5/2), x)
```

**sympy [A]** time = 14.42, size = 138, normalized size = 1.12

$$\frac{b^3x^{\frac{11}{2}}}{5\sqrt{bx+2}}+\frac{29b^2x^{\frac{9}{2}}}{20\sqrt{bx+2}}+\frac{73bx^{\frac{7}{2}}}{20\sqrt{bx+2}}+\frac{129x^{\frac{5}{2}}}{40\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}}-\frac{3\sqrt{x}}{4b^2\sqrt{bx+2}}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(5/2),x)
```

```
[Out] b**3*x**(11/2)/(5*sqrt(b*x + 2)) + 29*b**2*x**(9/2)/(20*sqrt(b*x + 2)) + 73
*b*x**(7/2)/(20*sqrt(b*x + 2)) + 129*x**(5/2)/(40*sqrt(b*x + 2)) - x**(3/2)
/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(b*x + 2)) + 3*asinh(sqrt(2)*s
qrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

## 3.559 $\int \sqrt{x}\,(2+bx)^{5/2}\,dx$

**Optimal**. Leaf size=102

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(bx+2)^{5/2}+\frac{5}{12}x^{3/2}(bx+2)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{bx+2}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b}$$

**Rubi [A]** time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(bx+2)^{5/2}+\frac{5}{12}x^{3/2}(bx+2)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{bx+2}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 + b*x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 + b*x])/8 + (5*x^(3/2)*
(2 + b*x)^(3/2))/12 + (x^(3/2)*(2 + b*x)^(5/2))/4 - (5*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(2+bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(2+bx)^{5/2}+\frac{5}{4}\int \sqrt{x}(2+bx)^{3/2}\,dx\\ &= \frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}+\frac{5}{4}\int \sqrt{x}\sqrt{2+bx}\,dx\\ &= \frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}+\frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx\\ &= \frac{5\sqrt{x}\sqrt{2+bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}-\frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{8b}\\ &= \frac{5\sqrt{x}\sqrt{2+bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}-\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2+b}\dots}\right.}{4b}\\ &= \frac{5\sqrt{x}\sqrt{2+bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right.}{4b^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 70, normalized size = 0.69

$$\frac{\sqrt{x}\sqrt{bx+2}\left(6b^3x^3+34b^2x^2+59bx+15\right)}{24b}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 + 59*b*x + 34*b^2*x^2 + 6*b^3*x^3))/(24*b) - (5*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 85, normalized size = 0.83

$$\frac{5\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{4b^{3/2}}+\frac{\sqrt{bx+2}\left(6b^3x^{7/2}+34b^2x^{5/2}+59bx^{3/2}+15\sqrt{x}\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(15*Sqrt[x] + 59*b*x^(3/2) + 34*b^2*x^(5/2) + 6*b^3*x^(7/2))
)/(24*b) + (5*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(3/2))
```

**fricas [A]** time = 1.43, size = 140, normalized size = 1.37

$$\left[\frac{\left(6\,b^4x^3+34\,b^3x^2+59\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}\,+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}\,+1\right)}{24\,b^2},\frac{\left(6\,b^4x^3+34\,b^3x^2+59\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}\,+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 1
5*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/24*((6*b^4*x
^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arct
an(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^2]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [83.4865739
918,53.112478131]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,
0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+
%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2
]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%
%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2
,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}
+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32
,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%
%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%
{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,
2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}
+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-
8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at paramet
ers values [38.6973876911,89.629912049]Warning, choosing root of [1,0,%%%{-
4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2
```

]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{
8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]
%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-
64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%
%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%
%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[
4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}
+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-2
0,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4
]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}
+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16
,[0,0]%%%}] at parameters values [6.82230772497,55.0343274642]Warning, choo
sing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8
,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%
}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[
0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%
}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,
[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]
%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%
%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,
4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}
+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{2
4,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1
]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%
%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [53.4880634798,16.0
204098616]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+
%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[
2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%
%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3
,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}
+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16
,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%
%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%
{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,
1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+
%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-4
0,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]
%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters val
ues [46.2456374937,66.0382199469]Warning, choosing root of [1,0,%%%{-4,[1,1
]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+
%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0
]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%
%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,
2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%
%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[

```
0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%
%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-
20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3
]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+
%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1
,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]
%%%}] at parameters values [94.9264369817,51.8441526662]Warning, choosing r
oot of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]
%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{
28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%
%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{
4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%
%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%
%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,
[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}
+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6
,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0
]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+
%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32
,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [98.7121795234,4.66774101
928]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4
,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%
%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[
0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%
}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8
,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]
%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,
%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,
0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}
+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-4
0,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2
]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%
%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [9
0.2102860468,38.2197840363]1/b*(2*b^3*abs(b)/b^2*(2*(((90*b^11/1440/b^14*sq
rt(b*x+2)*sqrt(b*x+2)-750*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)+2445*b^11
/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)-4185*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b
*(b*x+2)-2*b)-35/8/b^2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+
2))))+12*b^2*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/
144/b^7)*sqrt(b*x+2)*sqrt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2
)-2*b)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+24*b
*abs(b)/b^2/b*(2*(1/8*sqrt(b*x+2)*sqrt(b*x+2)-5/8)*sqrt(b*x+2)*sqrt(b*(b*x+
2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+16*
abs(b)/b^2*(1/2*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+2*b/2/sqrt(b)*ln(abs(sqrt(b
*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))
```

**maple [A]** time = 0.01, size = 99, normalized size = 0.97

$$\frac{(bx+2)^{\frac{5}{2}} x^{\frac{3}{2}}}{4}+\frac{5 (bx+2)^{\frac{3}{2}} x^{\frac{3}{2}}}{12}+\frac{5\sqrt{bx+2}\, x^{\frac{3}{2}}}{8}+\frac{5\sqrt{bx+2}\,\sqrt{x}}{8b}-\frac{5\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{8\sqrt{bx+2}\, b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)*x^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(b*x+2)^(5/2)+5/12*(b*x+2)^(3/2)*x^(3/2)+5/8*(b*x+2)^(1/2)*x^(3
/2)+5/8*(b*x+2)^(1/2)/b*x^(1/2)-5/8*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(3/2)
/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [B]** time = 2.99, size = 161, normalized size = 1.58

$$\frac{\frac{15\sqrt{bx+2}\,b^{3}}{\sqrt{x}}-\frac{55\,(bx+2)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}+\frac{73\,(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^{5}-\frac{4\,(bx+2)b^{4}}{x}+\frac{6\,(bx+2)^{2}b^{3}}{x^{2}}-\frac{4\,(bx+2)^{3}b^{2}}{x^{3}}+\frac{(bx+2)^{4}b}{x^{4}}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(b*x + 2)*b^3/sqrt(x) - 55*(b*x + 2)^(3/2)*b^2/x^(3/2) + 73*(b
*x + 2)^(5/2)*b/x^(5/2) + 15*(b*x + 2)^(7/2)/x^(7/2))/(b^5 - 4*(b*x + 2)*b^
4/x + 6*(b*x + 2)^2*b^3/x^2 - 4*(b*x + 2)^3*b^2/x^3 + (b*x + 2)^4*b/x^4) +
5/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)
))/b^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(b\,x+2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(1/2)*(b*x + 2)^(5/2), x)
```

**sympy [A]** time = 8.61, size = 119, normalized size = 1.17

$$\frac{b^{3}x^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{23b^{2}x^{\frac{7}{2}}}{12\sqrt{bx+2}}+\frac{127bx^{\frac{5}{2}}}{24\sqrt{bx+2}}+\frac{133x^{\frac{3}{2}}}{24\sqrt{bx+2}}+\frac{5\sqrt{x}}{4b\sqrt{bx+2}}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)*x**(1/2),x)
```

```
[Out] b**3*x**(9/2)/(4*sqrt(b*x + 2)) + 23*b**2*x**(7/2)/(12*sqrt(b*x + 2)) + 127
*b*x**(5/2)/(24*sqrt(b*x + 2)) + 133*x**(3/2)/(24*sqrt(b*x + 2)) + 5*sqrt(x
)/(4*b*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(3/2))
```

## 3.560 $\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=79

$$\frac{1}{3}\sqrt{x}\,(bx+2)^{5/2}+\frac{5}{6}\sqrt{x}\,(bx+2)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$\frac{1}{3}\sqrt{x}\,(bx+2)^{5/2}+\frac{5}{6}\sqrt{x}\,(bx+2)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*Sqrt[x]*(2 + b*x)^(3/2))/6 + (Sqrt[x]*(2 +
 b*x)^(5/2))/3 + (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}\,(2+bx)^{5/2} + \frac{5}{3}\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx$$
$$= \frac{5}{6}\sqrt{x}\,(2+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(2+bx)^{5/2} + \frac{5}{2}\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx$$
$$= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx} + \frac{5}{6}\sqrt{x}\,(2+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(2+bx)^{5/2} + \frac{5}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx$$
$$= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx} + \frac{5}{6}\sqrt{x}\,(2+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(2+bx)^{5/2} + 5\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx} + \frac{5}{6}\sqrt{x}\,(2+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(2+bx)^{5/2} + \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Mathematica [A]** time = 0.03, size = 57, normalized size = 0.72

$$\frac{1}{6}\sqrt{x}\,\sqrt{bx+2}\,\left(2b^2x^2+13bx+33\right) + \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(33 + 13*b*x + 2*b^2*x^2))/6 + (5*ArcSinh[(Sqrt[b]*S
qrt[x])/Sqrt[2]])/Sqrt[b]
```

**IntegrateAlgebraic [A]** time = 0.10, size = 70, normalized size = 0.89

$$\frac{1}{6}\sqrt{bx+2}\,\left(2b^2x^{5/2}+13bx^{3/2}+33\sqrt{x}\right) - \frac{5\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[2 + b*x]*(33*Sqrt[x] + 13*b*x^(3/2) + 2*b^2*x^(5/2)))/6 - (5*Log[-(Sq
rt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

**fricas [A]** time = 1.24, size = 123, normalized size = 1.56

$$\left[\frac{\left(2\,b^3x^2+13\,b^2x+33\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b}, \frac{\left(2\,b^3x^2+13\,b^2x+33\,b\right)\sqrt{bx+2}\,\sqrt{x}-30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")

[Out] [1/6*((2*b^3*x^2 + 13*b^2*x + 33*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(
b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b, 1/6*((2*b^3*x^2 + 13*b^2*x + 3
3*b)*sqrt(b*x + 2)*sqrt(x) - 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*s
qrt(x))))/b]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]1/abs(b)*b^2/b*(2*((1/6/b*sqrt(b*x+
2)*sqrt(b*x+2)+5/12/b)*sqrt(b*x+2)*sqrt(b*x+2)+5/4/b)*sqrt(b*x+2)*sqrt(b*(b
*x+2)-2*b)-5/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))

**maple [A]** time = 0.00, size = 84, normalized size = 1.06

$$\frac{(bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5\,(bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{bx+2}\,\sqrt{x}}{2}+\frac{5\sqrt{(bx+2)\,x}\;\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(b*x+2)^(5/2)*x^(1/2)+5/6*(b*x+2)^(3/2)*x^(1/2)+5/2*(b*x+2)^(1/2)*x^(1/
2)+5/2*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(
b*x^2+2*x)^(1/2))
```

**maxima [B]** time = 3.03, size = 129, normalized size = 1.63

$$-\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,\sqrt{b}}-\frac{\frac{15\,\sqrt{bx+2}\,b^{2}}{\sqrt{x}}-\frac{40\,(bx+2)^{\frac{3}{2}}\,b}{x^{\frac{3}{2}}}+\frac{33\,(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{3}-\frac{3\,(bx+2)b^{2}}{x}+\frac{3\,(bx+2)^{2}b}{x^{2}}-\frac{(bx+2)^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x
)))/sqrt(b) - 1/3*(15*sqrt(b*x + 2)*b^2/sqrt(x) - 40*(b*x + 2)^(3/2)*b/x^(3
/2) + 33*(b*x + 2)^(5/2)/x^(5/2))/(b^3 - 3*(b*x + 2)*b^2/x + 3*(b*x + 2)^2*
b/x^2 - (b*x + 2)^3/x^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x+2)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(1/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(1/2), x)
```

**sympy [A]** time = 5.46, size = 97, normalized size = 1.23

$$\frac{b^{3}x^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{17b^{2}x^{\frac{5}{2}}}{6\sqrt{bx+2}}+\frac{59bx^{\frac{3}{2}}}{6\sqrt{bx+2}}+\frac{11\sqrt{x}}{\sqrt{bx+2}}+\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] b**3*x**(7/2)/(3*sqrt(b*x + 2)) + 17*b**2*x**(5/2)/(6*sqrt(b*x + 2)) + 59*b
*x**(3/2)/(6*sqrt(b*x + 2)) + 11*sqrt(x)/sqrt(b*x + 2) + 5*asinh(sqrt(2)*sq
rt(b)*sqrt(x)/2)/sqrt(b)
```

## 3.561 $\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx$

**Optimal**. Leaf size=79

$$-\frac{2(bx+2)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\,(bx+2)^{3/2}+\frac{15}{2}b\sqrt{x}\,\sqrt{bx+2}\,+15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

**Rubi** [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$-\frac{2(bx+2)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\,(bx+2)^{3/2}+\frac{15}{2}b\sqrt{x}\,\sqrt{bx+2}\,+15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (15*b*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*b*Sqrt[x]*(2 + b*x)^(3/2))/2 - (2*(2 +
b*x)^(5/2))/Sqrt[x] + 15*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(2+bx)^{5/2}}{\sqrt{x}} + (5b)\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx\\ &= \frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2} - \frac{2(2+bx)^{5/2}}{\sqrt{x}} + \frac{1}{2}(15b)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\ &= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx} + \frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2} - \frac{2(2+bx)^{5/2}}{\sqrt{x}} + \frac{1}{2}(15b)\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\ &= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx} + \frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2} - \frac{2(2+bx)^{5/2}}{\sqrt{x}} + (15b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx} + \frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2} - \frac{2(2+bx)^{5/2}}{\sqrt{x}} + 15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 28, normalized size = 0.35

$$-\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{2}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/x^(3/2),x]
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -1/2, 1/2, -1/2*(b*x)])/Sqrt[x]
```

**IntegrateAlgebraic [A]** time = 0.12, size = 62, normalized size = 0.78

$$\frac{\sqrt{bx+2}\,\left(b^2x^2+9bx-16\right)}{2\sqrt{x}} - 15\sqrt{b}\,\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(5/2)/x^(3/2),x]
[Out] (Sqrt[2 + b*x]*(-16 + 9*b*x + b^2*x^2))/(2*Sqrt[x]) - 15*Sqrt[b]*Log[-(Sqrt
[b]*Sqrt[x]) + Sqrt[2 + b*x]]
```

**fricas [A]** time = 1.24, size = 116, normalized size = 1.47

$$\left[\frac{15\sqrt{b}x\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\left(b^2x^2+9bx-16\right)\sqrt{bx+2}\sqrt{x}}{2x},-\frac{30\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)-\left(b^2x^2+9bx-16\right)\sqrt{bx+2}\sqrt{x}}{2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*x^2
+ 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))/x, -1/2*(30*sqrt(-b)*x*arctan(sqrt(b*x
 + 2)*sqrt(-b)/(b*sqrt(x))) - (b^2*x^2 + 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))
/x]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
```

```
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]b/abs(b)*b^2/b*(2*((1/4*sqrt(b*x+2)
*sqrt(b*x+2)+5/4)*sqrt(b*x+2)*sqrt(b*x+2)-15/2)*sqrt(b*x+2)*sqrt(b*(b*x+2)-
2*b)/(b*(b*x+2)-2*b)-15/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x
+2))))
```

**maple [A]** time = 0.02, size = 81, normalized size = 1.03

$$\frac{15\sqrt{(bx+2)\,x}\,\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}}+\frac{b^{3}x^{3}+11b^{2}x^{2}+2bx-32}{2\sqrt{bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(3/2),x)
```

```
[Out] 1/2*(b^3*x^3+11*b^2*x^2+2*b*x-32)/(b*x+2)^(1/2)/x^(1/2)+15/2*((b*x+2)*x)^(1
/2)/(b*x+2)^(1/2)*b^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [B]** time = 2.94, size = 113, normalized size = 1.43

$$-\frac{15}{2}\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{\frac{7\sqrt{bx+2}\,b^{2}}{\sqrt{x}}-\frac{9\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}}{b^{2}-\frac{2\,(bx+2)b}{x}+\frac{(bx+2)^{2}}{x^{2}}}-\frac{8\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -15/2*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x +
2)/sqrt(x))) - (7*sqrt(b*x + 2)*b^2/sqrt(x) - 9*(b*x + 2)^(3/2)*b/x^(3/2))/
(b^2 - 2*(b*x + 2)*b/x + (b*x + 2)^2/x^2) - 8*sqrt(b*x + 2)/sqrt(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x+2)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(3/2), x)
```

**sympy [A]** time = 5.60, size = 94, normalized size = 1.19

$$15\sqrt{b}\ \text{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)+\frac{b^3x^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{11b^2x^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{b\sqrt{x}}{\sqrt{bx+2}}-\frac{16}{\sqrt{x}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(3/2),x)
```

```
[Out] 15*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**3*x**(5/2)/(2*sqrt(b*x + 2
)) + 11*b**2*x**(3/2)/(2*sqrt(b*x + 2)) + b*sqrt(x)/sqrt(b*x + 2) - 16/(sqr
t(x)*sqrt(b*x + 2))
```

## 3.562 $\int \frac{(2+bx)^{5/2}}{x^{5/2}} dx$

**Optimal**. Leaf size=81

$$10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)+5b^2\sqrt{x}\sqrt{bx+2}-\frac{2(bx+2)^{5/2}}{3x^{3/2}}-\frac{10b(bx+2)^{3/2}}{3\sqrt{x}}$$

**Rubi [A]** time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$5b^2\sqrt{x}\sqrt{bx+2}+10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{5/2}}{3x^{3/2}}-\frac{10b(bx+2)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 + b*x] - (10*b*(2 + b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 +
b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\frac{1}{3}(5b)\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx\\
&= -\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\
&= 5b^2\sqrt{x}\,\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\
&= 5b^2\sqrt{x}\,\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(10b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= 5b^2\sqrt{x}\,\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.37

$$-\frac{8\sqrt{2}\;{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{2}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/x^(5/2),x]
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -3/2, -1/2, -1/2*(b*x)])/(3*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 63, normalized size = 0.78

$$\frac{\sqrt{bx+2}\left(3b^2x^2-28bx-8\right)}{3x^{3/2}}-10b^{3/2}\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(5/2)/x^(5/2),x]
[Out] (Sqrt[2 + b*x]*(-8 - 28*b*x + 3*b^2*x^2))/(3*x^(3/2)) - 10*b^(3/2)*Log[-(Sq
rt[b]*Sqrt[x]) + Sqrt[2 + b*x]]
```

**fricas [A]** time = 1.45, size = 123, normalized size = 1.52

$$\left[\frac{15\,b^{\frac{3}{2}}x^{2}\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}\,+1\right)+\left(3\,b^{2}x^{2}-28\,bx-8\right)\sqrt{bx+2}\,\sqrt{x}}{3\,x^{2}},\,-\frac{30\,\sqrt{-b}\,bx^{2}\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(3\,b^{2}x^{2}-28\,bx-8\right)\sqrt{bx+2}\,\sqrt{x}}{3\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (3*b^2*
x^2 - 28*b*x - 8)*sqrt(b*x + 2)*sqrt(x))/x^2, -1/3*(30*sqrt(-b)*b*x^2*arcta
n(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 28*b*x - 8)*sqrt(b*x +
 2)*sqrt(x))/x^2]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
```

```
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]1/abs(b)*b^2/b*(2*((9*b^4/18/b*sqrt
(b*x+2)*sqrt(b*x+2)-120*b^4/18/b)*sqrt(b*x+2)*sqrt(b*x+2)+180*b^4/18/b)*sqr
t(b*x+2)*sqrt(b*(b*x+2)-2*b)/(b*(b*x+2)-2*b)^2-10*b^2/sqrt(b)*ln(abs(sqrt(b
*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
```

**maple [A]** time = 0.02, size = 82, normalized size = 1.01

$$\frac{5\sqrt{(bx+2)\,x}\;b^{\frac{3}{2}}\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\;\sqrt{x}}+\frac{3b^{3}x^{3}-22b^{2}x^{2}-64bx-16}{3\sqrt{bx+2}\;x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(5/2),x)
```

```
[Out] 1/3*(3*b^3*x^3-22*b^2*x^2-64*b*x-16)/x^(3/2)/(b*x+2)^(1/2)+5*((b*x+2)*x)^(1
/2)/(b*x+2)^(1/2)*b^(3/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [A]** time = 2.94, size = 96, normalized size = 1.19

$$-5\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{8\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{2\sqrt{bx+2}\,b^{2}}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}-\frac{4\,(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5*b^(3/2)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x))) - 8*sqrt(b*x + 2)*b/sqrt(x) - 2*sqrt(b*x + 2)*b^2/((b - (b*x + 2)
/x)*sqrt(x)) - 4/3*(b*x + 2)^(3/2)/x^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x+2)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(5/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(5/2), x)
```

**sympy [A]** time = 5.15, size = 88, normalized size = 1.09

$$b^{\frac{5}{2}}x\sqrt{1+\frac{2}{bx}}-\frac{28b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-5b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)+10b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)-\frac{8\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(5/2),x)
```

```
[Out] b**(5/2)*x*sqrt(1 + 2/(b*x)) - 28*b**(3/2)*sqrt(1 + 2/(b*x))/3 - 5*b**(3/2)
*log(1/(b*x)) + 10*b**(3/2)*log(sqrt(1 + 2/(b*x)) + 1) - 8*sqrt(b)*sqrt(1 +
 2/(b*x))/(3*x)
```

# 3.563 $\int x^{5/2}(2-bx)^{5/2}\,dx$

**Optimal**. Leaf size=150

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}-\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{24b}$$

**Rubi [A]** time = 0.05, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(48*b^2) -
(x^(5/2)*Sqrt[2 - b*x])/(24*b) + (x^(7/2)*Sqrt[2 - b*x])/8 + (x^(7/2)*(2 -
b*x)^(3/2))/6 + (x^(7/2)*(2 - b*x)^(5/2))/6 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{5/2}(2-bx)^{5/2}\,dx = \frac{1}{6}x^{7/2}(2-bx)^{5/2} + \frac{5}{6}\int x^{5/2}(2-bx)^{3/2}\,dx$$
$$= \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x^{7/2}(2-bx)^{5/2} + \frac{1}{2}\int x^{5/2}\sqrt{2-bx}\,dx$$
$$= \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x^{7/2}(2-bx)^{5/2} + \frac{1}{8}\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{x^{5/2}\sqrt{2-bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x^{7/2}(2-bx)^{5/2} + \frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{24b}$$
$$= -\frac{5x^{3/2}\sqrt{2-bx}}{48b^2} - \frac{x^{5/2}\sqrt{2-bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x^{7/2}(2-bx)^{5/2} + \cdots$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{48b^2} - \frac{x^{5/2}\sqrt{2-bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x\cdots$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{48b^2} - \frac{x^{5/2}\sqrt{2-bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x\cdots$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{48b^2} - \frac{x^{5/2}\sqrt{2-bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x\cdots$$

**Mathematica [A]** time = 0.06, size = 87, normalized size = 0.58

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}} + \frac{\sqrt{x}\sqrt{2-bx}\left(8b^5x^5 - 40b^4x^4 + 54b^3x^3 - 2b^2x^2 - 5bx - 15\right)}{48b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x - 2*b^2*x^2 + 54*b^3*x^3 - 40*b^4*x^4 +
 8*b^5*x^5))/(48*b^3) + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(8*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.15, size = 114, normalized size = 0.76

$$\frac{5\sqrt{-b}\log\left(\sqrt{2-bx} - \sqrt{-b}\sqrt{x}\right)}{8b^4} + \frac{\sqrt{2-bx}\left(8b^5x^{11/2} - 40b^4x^{9/2} + 54b^3x^{7/2} - 2b^2x^{5/2} - 5bx^{3/2} - 15\sqrt{x}\right)}{48b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] - 5*b*x^(3/2) - 2*b^2*x^(5/2) + 54*b^3*x^(7/2)
- 40*b^4*x^(9/2) + 8*b^5*x^(11/2)))/(48*b^3) + (5*Sqrt[-b]*Log[-(Sqrt[-b]*S
qrt[x]) + Sqrt[2 - b*x]])/(8*b^4)
```

**fricas [A]** time = 1.28, size = 173, normalized size = 1.15

$$\left[\frac{\left(8\,b^{6}x^{5}-40\,b^{5}x^{4}+54\,b^{4}x^{3}-2\,b^{3}x^{2}-5\,b^{2}x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{48\,b^{4}},\frac{\left(8\,b^{6}x^{5}-40\,b^{5}x^{4}+54\,b^{4}x^{3}-2\,b^{3}x^{2}-5\,b^{2}x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{48\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*((8*b^6*x^5 - 40*b^5*x^4 + 54*b^4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*s
qrt(-b*x + 2)*sqrt(x) - 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt
(x) + 1))/b^4, 1/48*((8*b^6*x^5 - 40*b^5*x^4 + 54*b^4*x^3 - 2*b^3*x^2 - 5*b
^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqr
t(b)*sqrt(x))))/b^4]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.00, size = 148, normalized size = 0.99

$$-\frac{\left(-bx+2\right)^{\frac{7}{2}}x^{\frac{5}{2}}}{6b}-\frac{\left(-bx+2\right)^{\frac{7}{2}}x^{\frac{3}{2}}}{6b^{2}}-\frac{\left(-bx+2\right)^{\frac{7}{2}}\sqrt{x}}{8b^{3}}+\frac{\left(-bx+2\right)^{\frac{5}{2}}\sqrt{x}}{24b^{3}}+\frac{5\left(-bx+2\right)^{\frac{3}{2}}\sqrt{x}}{48b^{3}}+\frac{5\sqrt{-bx+2}\,\sqrt{x}}{16b^{3}}+\frac{5\sqrt{\left(-bx+2\right)x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{16\sqrt{-bx+2}\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(5/2),x)
```

```
[Out] -1/6/b*x^(5/2)*(-b*x+2)^(7/2)-1/6/b^2*x^(3/2)*(-b*x+2)^(7/2)-1/8/b^3*x^(1/2
)*(-b*x+2)^(7/2)+1/24*(-b*x+2)^(5/2)/b^3*x^(1/2)+5/48*(-b*x+2)^(3/2)/b^3*x^
(1/2)+5/16*(-b*x+2)^(1/2)/b^3*x^(1/2)+5/16*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2
)/b^(7/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.96, size = 209, normalized size = 1.39

$$\frac{\frac{15\,\sqrt{-bx+2}\,b^{5}}{\sqrt{x}}+\frac{85\,(-bx+2)^{\frac{3}{2}}\,b^{4}}{x^{\frac{3}{2}}}+\frac{198\,(-bx+2)^{\frac{5}{2}}\,b^{3}}{x^{\frac{5}{2}}}-\frac{198\,(-bx+2)^{\frac{7}{2}}\,b^{2}}{x^{\frac{7}{2}}}-\frac{85\,(-bx+2)^{\frac{9}{2}}\,b}{x^{\frac{9}{2}}}-\frac{15\,(-bx+2)^{\frac{11}{2}}}{x^{\frac{11}{2}}}}{24\left(b^{9}-\frac{6\,(bx-2)b^{8}}{x}+\frac{15\,(bx-2)^{2}b^{7}}{x^{2}}-\frac{20\,(bx-2)^{3}b^{6}}{x^{3}}+\frac{15\,(bx-2)^{4}b^{5}}{x^{4}}-\frac{6\,(bx-2)^{5}b^{4}}{x^{5}}+\frac{(bx-2)^{6}b^{3}}{x^{6}}\right)}-\frac{5\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(15*sqrt(-b*x + 2)*b^5/sqrt(x) + 85*(-b*x + 2)^(3/2)*b^4/x^(3/2) + 198
*(-b*x + 2)^(5/2)*b^3/x^(5/2) - 198*(-b*x + 2)^(7/2)*b^2/x^(7/2) - 85*(-b*x
 + 2)^(9/2)*b/x^(9/2) - 15*(-b*x + 2)^(11/2)/x^(11/2))/(b^9 - 6*(b*x - 2)*b
^8/x + 15*(b*x - 2)^2*b^7/x^2 - 20*(b*x - 2)^3*b^6/x^3 + 15*(b*x - 2)^4*b^5
/x^4 - 6*(b*x - 2)^5*b^4/x^5 + (b*x - 2)^6*b^3/x^6) - 5/8*arctan(sqrt(-b*x
+ 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(2-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)*(2 - b*x)^(5/2), x)
```

**sympy [A]** time = 22.84, size = 337, normalized size = 2.25

$$\begin{cases} \frac{ib^3x^{\frac{13}{2}}}{6\sqrt{bx-2}} - \frac{7ib^2x^{\frac{11}{2}}}{6\sqrt{bx-2}} + \frac{67ibx^{\frac{9}{2}}}{24\sqrt{bx-2}} - \frac{55ix^{\frac{7}{2}}}{24\sqrt{bx-2}} - \frac{ix^{\frac{5}{2}}}{48b\sqrt{bx-2}} - \frac{5ix^{\frac{3}{2}}}{48b^2\sqrt{bx-2}} + \frac{5i\sqrt{x}}{8b^3\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}} & \text{for}\ \frac{|bx|}{2} > 1 \\ -\frac{b^3x^{\frac{13}{2}}}{6\sqrt{-bx+2}} + \frac{7b^2x^{\frac{11}{2}}}{6\sqrt{-bx+2}} - \frac{67bx^{\frac{9}{2}}}{24\sqrt{-bx+2}} + \frac{55x^{\frac{7}{2}}}{24\sqrt{-bx+2}} + \frac{x^{\frac{5}{2}}}{48b\sqrt{-bx+2}} + \frac{5x^{\frac{3}{2}}}{48b^2\sqrt{-bx+2}} - \frac{5\sqrt{x}}{8b^3\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*b**3*x**(13/2)/(6*sqrt(b*x - 2)) - 7*I*b**2*x**(11/2)/(6*sqrt(
b*x - 2)) + 67*I*b*x**(9/2)/(24*sqrt(b*x - 2)) - 55*I*x**(7/2)/(24*sqrt(b*x
 - 2)) - I*x**(5/2)/(48*b*sqrt(b*x - 2)) - 5*I*x**(3/2)/(48*b**2*sqrt(b*x -
 2)) + 5*I*sqrt(x)/(8*b**3*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/(8*b**(7/2)), Abs(b*x)/2 > 1), (-b**3*x**(13/2)/(6*sqrt(-b*x + 2)) +
7*b**2*x**(11/2)/(6*sqrt(-b*x + 2)) - 67*b*x**(9/2)/(24*sqrt(-b*x + 2)) + 5
5*x**(7/2)/(24*sqrt(-b*x + 2)) + x**(5/2)/(48*b*sqrt(-b*x + 2)) + 5*x**(3/2
)/(48*b**2*sqrt(-b*x + 2)) - 5*sqrt(x)/(8*b**3*sqrt(-b*x + 2)) + 5*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/(8*b**(7/2)), True))
```

## 3.564 $\int x^{3/2}(2-bx)^{5/2}\,dx$

**Optimal**. Leaf size=128

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

**Rubi [A]** time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (x^(5/2)*(2 - b*x)^(5/2
))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}(2-bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \int x^{3/2}(2-bx)^{3/2}\,dx \\
&= \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{3}{4}\int x^{3/2}\sqrt{2-bx}\,dx \\
&= \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx \\
&= -\frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b} \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} +
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 79, normalized size = 0.62

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}} + \frac{\sqrt{x}\sqrt{2-bx}\left(8b^4x^4 - 42b^3x^3 + 62b^2x^2 - 5bx - 15\right)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x + 62*b^2*x^2 - 42*b^3*x^3 + 8*b^4*x^4))
/(40*b^2) + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.19, size = 104, normalized size = 0.81

$$\frac{3\sqrt{-b}\log\left(\sqrt{2-bx} - \sqrt{-b}\sqrt{x}\right)}{4b^3} + \frac{\sqrt{2-bx}\left(8b^4x^{9/2} - 42b^3x^{7/2} + 62b^2x^{5/2} - 5bx^{3/2} - 15\sqrt{x}\right)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] - 5*b*x^(3/2) + 62*b^2*x^(5/2) - 42*b^3*x^(7/2)
 + 8*b^4*x^(9/2)))/(40*b^2) + (3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2
- b*x]])/(4*b^3)
```

**fricas [A]** time = 1.32, size = 157, normalized size = 1.23

$$\left[\frac{\left(8\,b^{5}x^{4}-42\,b^{4}x^{3}+62\,b^{3}x^{2}-5\,b^{2}x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{40\,b^{3}},\ \frac{\left(8\,b^{5}x^{4}-42\,b^{4}x^{3}+62\,b^{3}x^{2}-5\,b^{2}x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{40\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 - 42*b^4*x^3 + 62*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2
)*sqrt(x) - 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^
3, 1/40*((8*b^5*x^4 - 42*b^4*x^3 + 62*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x +
 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-17.51342
60082,53.112478131]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,
0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+
%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[
0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%
}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20
,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]
%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%
%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[
4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}
+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46
```

,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]
%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%
%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at para
meters values [-62.3026123089,89.629912049]Warning, choosing root of [1,0,%
%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2
,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+
%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[
3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+
%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20
,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]
%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+
%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3
,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+
%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4
,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0
]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%
%%{16,[0,0]%%%}] at parameters values [-94.177692275,55.0343274642]Warning,
 choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%
{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2
]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%
%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3
,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+
%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3
,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%
}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-
48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]
%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-47.51193652
02,16.0204098616]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]
%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%
%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+
%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%
%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{
-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,
1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%
%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[
2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%
%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [-54.7543625063,66.0382199469]Warning, choosing root of [1,0,%%
%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,

```
2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%
%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3
,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%
%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,
[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%
%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%
%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,
3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%
%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,
[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]
%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%
%{16,[0,0]%%%}] at parameters values [-6.07356301835,51.8441526662]Warning,
 choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%
{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2
]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%
%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3
,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+
%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3
,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%
}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-
48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]
%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-2.287820476
57,4.66774101928]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]
%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%
%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+
%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%
%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{
-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,
1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%
%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[
2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%
%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [-10.7897139532,38.2197840363]1/b*(2*b^3*abs(b)/b^2*(2*((((5040
*b^19/100800/b^23*sqrt(-b*x+2)*sqrt(-b*x+2)-51660*b^19/100800/b^23)*sqrt(-b
*x+2)*sqrt(-b*x+2)+215460*b^19/100800/b^23)*sqrt(-b*x+2)*sqrt(-b*x+2)-46935
0*b^19/100800/b^23)*sqrt(-b*x+2)*sqrt(-b*x+2)+607950*b^19/100800/b^23)*sqrt
(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-63/8/b^3/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*
b)-sqrt(-b)*sqrt(-b*x+2))))-12*b^2*abs(b)/b^2*(2*(((-90*b^11/1440/b^14*sqrt
(-b*x+2)*sqrt(-b*x+2)+750*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)-2445*b^
```

```
11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)+4185*b^11/1440/b^14)*sqrt(-b*x+2)*s
qrt(-b*(-b*x+2)+2*b)-35/8/b^2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b
)*sqrt(-b*x+2))))+24*b*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqrt(-b*
x+2)-78*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b^5/144/b^7)*sqrt(-b*x+2
)*sqrt(-b*(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b
)*sqrt(-b*x+2))))+16*abs(b)/b^2/b*(2*(1/8*sqrt(-b*x+2)*sqrt(-b*x+2)-5/8)*sq
rt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b
)-sqrt(-b)*sqrt(-b*x+2)))))
```

**maple [A]** time = 0.01, size = 132, normalized size = 1.03

$$-\frac{(-bx+2)^{\frac{7}{2}} x^{\frac{3}{2}}}{5b}-\frac{3(-bx+2)^{\frac{7}{2}}\sqrt{x}}{20b^{2}}+\frac{(-bx+2)^{\frac{5}{2}}\sqrt{x}}{20b^{2}}+\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{8b^{2}}+\frac{3\sqrt{-bx+2}\,\sqrt{x}}{8b^{2}}+\frac{3\sqrt{(-bx+2)x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-bx^{2}+2x}}\right)}{8\sqrt{-bx+2}\,b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(5/2),x)
```

```
[Out] -1/5/b*x^(3/2)*(-b*x+2)^(7/2)-3/20/b^2*x^(1/2)*(-b*x+2)^(7/2)+1/20*(-b*x+2)
^(5/2)/b^2*x^(1/2)+1/8*(-b*x+2)^(3/2)/b^2*x^(1/2)+3/8*(-b*x+2)^(1/2)/b^2*x^
(1/2)+3/8*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(5/2)/x^(1/2)*arctan((x-1/b)/
(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima [B]** time = 3.01, size = 179, normalized size = 1.40

$$\frac{\frac{15\sqrt{-bx+2}\,b^{4}}{\sqrt{x}}+\frac{70(-bx+2)^{\frac{3}{2}}b^{3}}{x^{\frac{3}{2}}}+\frac{128(-bx+2)^{\frac{5}{2}}b^{2}}{x^{\frac{5}{2}}}-\frac{70(-bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15(-bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^{7}-\frac{5(bx-2)b^{6}}{x}+\frac{10(bx-2)^{2}b^{5}}{x^{2}}-\frac{10(bx-2)^{3}b^{4}}{x^{3}}+\frac{5(bx-2)^{4}b^{3}}{x^{4}}-\frac{(bx-2)^{5}b^{2}}{x^{5}}\right)}-\frac{3\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(-b*x + 2)*b^4/sqrt(x) + 70*(-b*x + 2)^(3/2)*b^3/x^(3/2) + 128
*(-b*x + 2)^(5/2)*b^2/x^(5/2) - 70*(-b*x + 2)^(7/2)*b/x^(7/2) - 15*(-b*x +
2)^(9/2)/x^(9/2))/(b^7 - 5*(b*x - 2)*b^6/x + 10*(b*x - 2)^2*b^5/x^2 - 10*(b
*x - 2)^3*b^4/x^3 + 5*(b*x - 2)^4*b^3/x^4 - (b*x - 2)^5*b^2/x^5) - 3/4*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(2-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(2 - b*x)^(5/2), x)
```

**sympy [A]** time = 14.30, size = 294, normalized size = 2.30

$$\begin{cases} \frac{ib^3x^{\frac{11}{2}}}{5\sqrt{bx-2}} - \frac{29ib^2x^{\frac{9}{2}}}{20\sqrt{bx-2}} + \frac{73ibx^{\frac{7}{2}}}{20\sqrt{bx-2}} - \frac{129ix^{\frac{5}{2}}}{40\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{8b\sqrt{bx-2}} + \frac{3i\sqrt{x}}{4b^2\sqrt{bx-2}} - \frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{for } \frac{|bx|}{2} > 1 \\ -\frac{b^3x^{\frac{11}{2}}}{5\sqrt{-bx+2}} + \frac{29b^2x^{\frac{9}{2}}}{20\sqrt{-bx+2}} - \frac{73bx^{\frac{7}{2}}}{20\sqrt{-bx+2}} + \frac{129x^{\frac{5}{2}}}{40\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{8b\sqrt{-bx+2}} - \frac{3\sqrt{x}}{4b^2\sqrt{-bx+2}} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*b**3*x**(11/2)/(5*sqrt(b*x - 2)) - 29*I*b**2*x**(9/2)/(20*sqrt
(b*x - 2)) + 73*I*b*x**(7/2)/(20*sqrt(b*x - 2)) - 129*I*x**(5/2)/(40*sqrt(b
*x - 2)) - I*x**(3/2)/(8*b*sqrt(b*x - 2)) + 3*I*sqrt(x)/(4*b**2*sqrt(b*x -
2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2)), Abs(b*x)/2 > 1), (
-b**3*x**(11/2)/(5*sqrt(-b*x + 2)) + 29*b**2*x**(9/2)/(20*sqrt(-b*x + 2)) -
 73*b*x**(7/2)/(20*sqrt(-b*x + 2)) + 129*x**(5/2)/(40*sqrt(-b*x + 2)) + x**
(3/2)/(8*b*sqrt(-b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(-b*x + 2)) + 3*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2)), True))
```

## 3.565 $\int \sqrt{x}(2-bx)^{5/2}\,dx$

**Optimal**. Leaf size=106

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{2-bx}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b}$$

**Rubi** [A] time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{2-bx}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 - b*x)^(5/2),x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 - b*x])/8 + (5*x^(3/2)
*(2 - b*x)^(3/2))/12 + (x^(3/2)*(2 - b*x)^(5/2))/4 + (5*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\,(2-bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{4}\int \sqrt{x}\,(2-bx)^{3/2}\,dx \\
&= \frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{4}\int \sqrt{x}\,\sqrt{2-bx}\,dx \\
&= \frac{5}{8}x^{3/2}\sqrt{2-bx}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx \\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2-bx}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{8b} \\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2-bx}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx}}\right.}{4b} \\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2-bx}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 71, normalized size = 0.67

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{\sqrt{x}\,\sqrt{2-bx}\,\left(6b^3x^3-34b^2x^2+59bx-15\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 + 59*b*x - 34*b^2*x^2 + 6*b^3*x^3))/(24*b) + (5
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.15, size = 94, normalized size = 0.89

$$\frac{5\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\,\sqrt{x}\right)}{4b^2}+\frac{\sqrt{2-bx}\,\left(6b^3x^{7/2}-34b^2x^{5/2}+59bx^{3/2}-15\sqrt{x}\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] + 59*b*x^(3/2) - 34*b^2*x^(5/2) + 6*b^3*x^(7/2)
))/(24*b) + (5*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*b^2)
```

**fricas [A]** time = 1.44, size = 141, normalized size = 1.33

$$\left[\frac{\left(6\,b^4x^3-34\,b^3x^2+59\,b^2x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{24\,b^2},\ \frac{\left(6\,b^4x^3-34\,b^3x^2+59\,b^2x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 - 34*b^3*x^2 + 59*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) -
15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^2, 1/24*((6*
b^4*x^3 - 34*b^3*x^2 + 59*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)
*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^2]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-17.51342
60082,53.112478131]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,
0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+
%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[
0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%
}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20
,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]
%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%
%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[
4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}
+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46
,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]
%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%
%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at para
meters values [-62.3026123089,89.629912049]Warning, choosing root of [1,0,%
%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2
```

,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+
%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[
3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+
%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20
,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]
%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+
%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3
,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+
%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4
,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0
]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%
%%{16,[0,0]%%%}] at parameters values [-94.177692275,55.0343274642]Warning,
 choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%
{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2
]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%
%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3
,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+
%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3
,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%
}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-
48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]
%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-47.51193652
02,16.0204098616]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]
%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%
%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+
%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%
%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{
-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,
1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%
%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[
2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%
%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [-54.7543625063,66.0382199469]Warning, choosing root of [1,0,%%
%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,
2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%
%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3
,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%
%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,
[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%

%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%
%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,
3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%
%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,
[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]
%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%
%{16,[0,0]%%%}] at parameters values [-6.07356301835,51.8441526662]Warning,
choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%
{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2
]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%
%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3
,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+
%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3
,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%
}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-
48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]
%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-2.287820476
57,4.66774101928]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]
%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%
%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+
%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%
%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{
-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,
1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%
%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[
2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%
%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [-10.7897139532,38.2197840363]1/b*(2*b^3*abs(b)/b^2*(2*(((-90*b
^11/1440/b^14*sqrt(-b*x+2)*sqrt(-b*x+2)+750*b^11/1440/b^14)*sqrt(-b*x+2)*sq
rt(-b*x+2)-2445*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)+4185*b^11/1440/b^
14)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-35/8/b^2/sqrt(-b)*ln(abs(sqrt(-b*(-b
*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))-12*b^2*abs(b)/b^2*(2*((12*b^5/144/b^7*s
qrt(-b*x+2)*sqrt(-b*x+2)-78*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b^5/
144/b^7)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-b*(
-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))-24*b*abs(b)/b^2/b*(2*(1/8*sqrt(-b*x+2
)*sqrt(-b*x+2)-5/8)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*ln(ab
s(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))-16*abs(b)/b^2*(1/2*sqrt(-b
*x+2)*sqrt(-b*(-b*x+2)+2*b)-2*b/2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqr
t(-b)*sqrt(-b*x+2)))))

**maple [A]** time = 0.00, size = 107, normalized size = 1.01

$$\frac{(-bx+2)^{\frac{5}{2}} x^{\frac{3}{2}}}{4}+\frac{5(-bx+2)^{\frac{3}{2}} x^{\frac{3}{2}}}{12}+\frac{5\sqrt{-bx+2}\, x^{\frac{3}{2}}}{8}-\frac{5\sqrt{-bx+2}\,\sqrt{x}}{8b}+\frac{5\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{8\sqrt{-bx+2}\, b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)*x^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(-b*x+2)^(5/2)+5/12*(-b*x+2)^(3/2)*x^(3/2)+5/8*(-b*x+2)^(1/2)*x
^(3/2)-5/8*(-b*x+2)^(1/2)/b*x^(1/2)+5/8*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b
^(3/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.89, size = 145, normalized size = 1.37

$$\frac{\frac{15\sqrt{-bx+2}\,b^{3}}{\sqrt{x}}+\frac{55(-bx+2)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}+\frac{73(-bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{15(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^{5}-\frac{4(bx-2)b^{4}}{x}+\frac{6(bx-2)^{2}b^{3}}{x^{2}}-\frac{4(bx-2)^{3}b^{2}}{x^{3}}+\frac{(bx-2)^{4}b}{x^{4}}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(-b*x + 2)*b^3/sqrt(x) + 55*(-b*x + 2)^(3/2)*b^2/x^(3/2) + 73*
(-b*x + 2)^(5/2)*b/x^(5/2) - 15*(-b*x + 2)^(7/2)/x^(7/2))/(b^5 - 4*(b*x - 2
)*b^4/x + 6*(b*x - 2)^2*b^3/x^2 - 4*(b*x - 2)^3*b^2/x^3 + (b*x - 2)^4*b/x^4
) - 5/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,(2-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(1/2)*(2 - b*x)^(5/2), x)
```

**sympy [A]** time = 8.59, size = 255, normalized size = 2.41

$$\begin{cases} \frac{ib^3x^{\frac{9}{2}}}{4\sqrt{bx-2}} - \frac{23ib^2x^{\frac{7}{2}}}{12\sqrt{bx-2}} + \frac{127ibx^{\frac{5}{2}}}{24\sqrt{bx-2}} - \frac{133ix^{\frac{3}{2}}}{24\sqrt{bx-2}} + \frac{5i\sqrt{x}}{4b\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}} & \text{for } \frac{|bx|}{2} > 1 \\ -\frac{b^3x^{\frac{9}{2}}}{4\sqrt{-bx+2}} + \frac{23b^2x^{\frac{7}{2}}}{12\sqrt{-bx+2}} - \frac{127bx^{\frac{5}{2}}}{24\sqrt{-bx+2}} + \frac{133x^{\frac{3}{2}}}{24\sqrt{-bx+2}} - \frac{5\sqrt{x}}{4b\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)*x**(1/2),x)
```

```
[Out] Piecewise((I*b**3*x**(9/2)/(4*sqrt(b*x - 2)) - 23*I*b**2*x**(7/2)/(12*sqrt(
b*x - 2)) + 127*I*b*x**(5/2)/(24*sqrt(b*x - 2)) - 133*I*x**(3/2)/(24*sqrt(b
*x - 2)) + 5*I*sqrt(x)/(4*b*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt
(x)/2)/(4*b**(3/2)), Abs(b*x)/2 > 1), (-b**3*x**(9/2)/(4*sqrt(-b*x + 2)) +
23*b**2*x**(7/2)/(12*sqrt(-b*x + 2)) - 127*b*x**(5/2)/(24*sqrt(-b*x + 2)) +
 133*x**(3/2)/(24*sqrt(-b*x + 2)) - 5*sqrt(x)/(4*b*sqrt(-b*x + 2)) + 5*asin
(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(3/2)), True))
```

## 3.566 $\int \frac{(2-bx)^{5/2}}{\sqrt{x}} dx$

**Optimal**. Leaf size=82

$$\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 - b*x])/2 + (5*Sqrt[x]*(2 - b*x)^(3/2))/6 + (Sqrt[x]*(2 -
 b*x)^(5/2))/3 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2-bx)^{5/2}}{\sqrt{x}}\,dx &= \frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{3}\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx \\
&= \frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{2}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx \\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx \\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+5\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right) \\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 58, normalized size = 0.71

$$
\frac{1}{6}\sqrt{x}\,\sqrt{2-bx}\,\left(2b^2x^2-13bx+33\right)+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(33 - 13*b*x + 2*b^2*x^2))/6 + (5*ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[2]])/Sqrt[b]
```

**IntegrateAlgebraic [A]** time = 0.14, size = 79, normalized size = 0.96

$$
\frac{1}{6}\sqrt{2-bx}\,\left(2b^2x^{5/2}-13bx^{3/2}+33\sqrt{x}\right)+\frac{5\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\,\sqrt{x}\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[2 - b*x]*(33*Sqrt[x] - 13*b*x^(3/2) + 2*b^2*x^(5/2)))/6 + (5*Sqrt[-b]
*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b
```

**fricas [A]** time = 1.26, size = 125, normalized size = 1.52

$$
\left[\frac{\left(2\,b^3x^2-13\,b^2x+33\,b\right)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6\,b},\ \frac{\left(2\,b^3x^2-13\,b^2x+33\,b\right)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - 13*b^2*x + 33*b)*sqrt(-b*x + 2)*sqrt(x) - 15*sqrt(-b)*lo
g(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b, 1/6*((2*b^3*x^2 - 13*b^2*
x + 33*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b
)*sqrt(x))))/b]
```

**giac** **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]1/abs(b)*b^2/b*(2*((1/6/b*sqrt(
-b*x+2)*sqrt(-b*x+2)+5/12/b)*sqrt(-b*x+2)*sqrt(-b*x+2)+5/4/b)*sqrt(-b*x+2)*
```

```
sqrt(-b*(-b*x+2)+2*b)+5/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt
(-b*x+2))))
```

**maple [A]** time = 0.00, size = 91, normalized size = 1.11

$$\frac{(-bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5\,(-bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{-bx+2}\,\sqrt{x}}{2}+\frac{5\sqrt{(-bx+2)\,x}\;\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(-b*x+2)^(5/2)*x^(1/2)+5/6*(-b*x+2)^(3/2)*x^(1/2)+5/2*(-b*x+2)^(1/2)*x^
(1/2)+5/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(1/2)/x^(1/2)*arctan((x-1/b)/
(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.98, size = 112, normalized size = 1.37

$$-\frac{5\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}+\frac{\frac{15\,\sqrt{-bx+2}\,b^{2}}{\sqrt{x}}+\frac{40\,(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{33\,(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{3}-\frac{3\,(bx-2)b^{2}}{x}+\frac{3\,(bx-2)^{2}b}{x^{2}}-\frac{(bx-2)^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/3*(15*sqrt(-b*x + 2
)*b^2/sqrt(x) + 40*(-b*x + 2)^(3/2)*b/x^(3/2) + 33*(-b*x + 2)^(5/2)/x^(5/2)
)/(b^3 - 3*(b*x - 2)*b^2/x + 3*(b*x - 2)^2*b/x^2 - (b*x - 2)^3/x^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2-b\,x)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(1/2),x)
```

```
[Out] int((2 - b*x)^(5/2)/x^(1/2), x)
```

**sympy [A]** time = 5.52, size = 209, normalized size = 2.55

$$\begin{cases} \frac{ib^3x^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{17ib^2x^{\frac{5}{2}}}{6\sqrt{bx-2}} + \frac{59ibx^{\frac{3}{2}}}{6\sqrt{bx-2}} - \frac{11i\sqrt{x}}{\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for } \frac{|bx|}{2} > 1 \\ -\frac{b^3x^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{17b^2x^{\frac{5}{2}}}{6\sqrt{-bx+2}} - \frac{59bx^{\frac{3}{2}}}{6\sqrt{-bx+2}} + \frac{11\sqrt{x}}{\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b**3*x**(7/2)/(3*sqrt(b*x - 2)) - 17*I*b**2*x**(5/2)/(6*sqrt(b
*x - 2)) + 59*I*b*x**(3/2)/(6*sqrt(b*x - 2)) - 11*I*sqrt(x)/sqrt(b*x - 2) -
 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x)/2 > 1), (-b**3*x**(
7/2)/(3*sqrt(-b*x + 2)) + 17*b**2*x**(5/2)/(6*sqrt(-b*x + 2)) - 59*b*x**(3/
2)/(6*sqrt(-b*x + 2)) + 11*sqrt(x)/sqrt(-b*x + 2) + 5*asin(sqrt(2)*sqrt(b)*
sqrt(x)/2)/sqrt(b), True))
```

## 3.567 $\int \frac{(2-bx)^{5/2}}{x^{3/2}} \, dx$

**Optimal**. Leaf size=82

$$-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}\,-15\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}\,-15\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-15*b*Sqrt[x]*Sqrt[2 - b*x])/2 - (5*b*Sqrt[x]*(2 - b*x)^(3/2))/2 - (2*(2 -
 b*x)^(5/2))/Sqrt[x] - 15*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(2-bx)^{5/2}}{\sqrt{x}} - (5b)\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx \\
&= -\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - \frac{1}{2}(15b)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx \\
&= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx} - \frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - \frac{1}{2}(15b)\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx \\
&= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx} - \frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - (15b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx} - \frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - 15\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

**Mathematica** [C] time = 0.01, size = 28, normalized size = 0.34

$$-\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{2}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/x^(3/2),x]
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -1/2, 1/2, (b*x)/2])/Sqrt[x]
```

**IntegrateAlgebraic** [A] time = 0.15, size = 68, normalized size = 0.83

$$\frac{\sqrt{2-bx}\,\left(b^2x^2-9bx-16\right)}{2\sqrt{x}} - 15\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - b*x)^(5/2)/x^(3/2),x]
[Out] (Sqrt[2 - b*x]*(-16 - 9*b*x + b^2*x^2))/(2*Sqrt[x]) - 15*Sqrt[-b]*Log[-(Sqr
t[-b]*Sqrt[x]) + Sqrt[2 - b*x]]
```

**fricas [A]** time = 1.32, size = 117, normalized size = 1.43

$$\left[\frac{15\sqrt{-b}\,x\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+\left(b^2x^2-9\,bx-16\right)\sqrt{-bx+2}\,\sqrt{x}}{2\,x},\ \frac{30\,\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\left(b^2x^2-9\,bx-16\right)\sqrt{-bx+2}\,\sqrt{x}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + (b^2*
x^2 - 9*b*x - 16)*sqrt(-b*x + 2)*sqrt(x))/x, 1/2*(30*sqrt(b)*x*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^2*x^2 - 9*b*x - 16)*sqrt(-b*x + 2)*sqrt(x
))/x]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
```

```
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]-b/abs(b)*b^2/b*(2*((-5/4-1/4*s
qrt(-b*x+2)*sqrt(-b*x+2))*sqrt(-b*x+2)*sqrt(-b*x+2)+15/2)*sqrt(-b*x+2)*sqrt
(-b*(-b*x+2)+2*b)/(-b*(-b*x+2)+2*b)+15/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b
)-sqrt(-b)*sqrt(-b*x+2))))
```

**maple [A]** time = 0.02, size = 106, normalized size = 1.29

$$-\frac{15\sqrt{(-bx+2)\,x}\;\sqrt{b}\;\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^2+2x}}\right)}{2\sqrt{-bx+2}\;\sqrt{x}}-\frac{\left(b^3x^3-11b^2x^2+2bx+32\right)\sqrt{(-bx+2)\,x}}{2\sqrt{-(bx-2)\,x}\;\sqrt{-bx+2}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(3/2),x)
```

```
[Out] -1/2*(b^3*x^3-11*b^2*x^2+2*b*x+32)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-
b*x+2)^(1/2)/x^(1/2)-15/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)*b^(1/2)/x^(1/2)
*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.92, size = 96, normalized size = 1.17

$$15\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-\frac{\frac{7\sqrt{-bx+2}\,b^2}{\sqrt{x}}+\frac{9(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}}{b^2-\frac{2(bx-2)b}{x}+\frac{(bx-2)^2}{x^2}}-\frac{8\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 15*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - (7*sqrt(-b*x + 2)*b^2
/sqrt(x) + 9*(-b*x + 2)^(3/2)*b/x^(3/2))/(b^2 - 2*(b*x - 2)*b/x + (b*x - 2)
^2/x^2) - 8*sqrt(-b*x + 2)/sqrt(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2-b\,x)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(3/2),x)
```

[Out] int((2 - b*x)^(5/2)/x^(3/2), x)

**sympy** [A]   time = 5.62, size = 202, normalized size = 2.46

$$\begin{cases} 15i\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right) + \frac{ib^3x^{\frac{5}{2}}}{2\sqrt{bx-2}} - \frac{11ib^2x^{\frac{3}{2}}}{2\sqrt{bx-2}} + \frac{ib\sqrt{x}}{\sqrt{bx-2}} + \frac{16i}{\sqrt{x}\,\sqrt{bx-2}} & \text{for } \frac{|bx|}{2} > 1 \\ -15\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right) - \frac{b^3x^{\frac{5}{2}}}{2\sqrt{-bx+2}} + \frac{11b^2x^{\frac{3}{2}}}{2\sqrt{-bx+2}} - \frac{b\sqrt{x}}{\sqrt{-bx+2}} - \frac{16}{\sqrt{x}\,\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(3/2),x)
```

```
[Out] Piecewise((15*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) + I*b**3*x**(5/2)/
(2*sqrt(b*x - 2)) - 11*I*b**2*x**(3/2)/(2*sqrt(b*x - 2)) + I*b*sqrt(x)/sqrt
(b*x - 2) + 16*I/(sqrt(x)*sqrt(b*x - 2)), Abs(b*x)/2 > 1), (-15*sqrt(b)*asi
n(sqrt(2)*sqrt(b)*sqrt(x)/2) - b**3*x**(5/2)/(2*sqrt(-b*x + 2)) + 11*b**2*x
**(3/2)/(2*sqrt(-b*x + 2)) - b*sqrt(x)/sqrt(-b*x + 2) - 16/(sqrt(x)*sqrt(-b
*x + 2)), True))
```

## 3.568 $\int \frac{(2-bx)^{5/2}}{x^{5/2}} \, dx$

**Optimal**. Leaf size=84

$$10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)+5b^2\sqrt{x}\sqrt{2-bx}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}$$

**Rubi** [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$5b^2\sqrt{x}\sqrt{2-bx}+10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 - b*x] + (10*b*(2 - b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 -
b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(2-bx)^{5/2}}{3x^{3/2}}-\frac{1}{3}(5b)\int \frac{(2-bx)^{3/2}}{x^{3/2}}\,dx\\
&= \frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(10b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= 5b^2\sqrt{x}\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.36

$$-\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{2}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/x^(5/2),x]
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -3/2, -1/2, (b*x)/2])/(3*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 70, normalized size = 0.83

$$\frac{\sqrt{2-bx}\left(3b^2x^2+28bx-8\right)}{3x^{3/2}}+10\sqrt{-b}\,b\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - b*x)^(5/2)/x^(5/2),x]
[Out] (Sqrt[2 - b*x]*(-8 + 28*b*x + 3*b^2*x^2))/(3*x^(3/2)) + 10*Sqrt[-b]*b*Log[-
(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]]
```

**fricas [A]** time = 0.86, size = 126, normalized size = 1.50

$$\left[\frac{15\,\sqrt{-b}\,bx^{2}\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+\left(3\,b^{2}x^{2}+28\,bx-8\right)\sqrt{-bx+2}\,\sqrt{x}}{3\,x^{2}},-\frac{30\,b^{\frac{3}{2}}x^{2}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\left(3\,b^{2}x^{2}+28\,bx-8\right)\sqrt{-bx+2}\,\sqrt{x}}{3\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*sqrt(-b)*b*x^2*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + (
3*b^2*x^2 + 28*b*x - 8)*sqrt(-b*x + 2)*sqrt(x))/x^2, -1/3*(30*b^(3/2)*x^2*a
rctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - (3*b^2*x^2 + 28*b*x - 8)*sqrt(-b*
x + 2)*sqrt(x))/x^2]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
```

```
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]1/abs(b)*b^2/b*(2*((9*b^4/18/b*
sqrt(-b*x+2)*sqrt(-b*x+2)-120*b^4/18/b)*sqrt(-b*x+2)*sqrt(-b*x+2)+180*b^4/1
8/b)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)/(-b*(-b*x+2)+2*b)^2+10*b^2/sqrt(-b)
*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))
```

**maple [A]** time = 0.02, size = 107, normalized size = 1.27

$$\frac{5\sqrt{(-bx+2)\,x}\;b^{\frac{3}{2}}\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{\sqrt{-bx+2}\;\sqrt{x}}-\frac{\left(3b^{3}x^{3}+22b^{2}x^{2}-64bx+16\right)\sqrt{(-bx+2)\,x}}{3\sqrt{-\left(bx-2\right)x}\;\sqrt{-bx+2}\;x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(5/2),x)
```

```
[Out] -1/3*(3*b^3*x^3+22*b^2*x^2-64*b*x+16)/x^(3/2)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*
x)^(1/2)/(-b*x+2)^(1/2)+5*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)*b^(3/2)/x^(1/2)
*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.89, size = 79, normalized size = 0.94

$$-10\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{8\,\sqrt{-bx+2}\,b}{\sqrt{x}}+\frac{2\,\sqrt{-bx+2}\,b^{2}}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}}-\frac{4\,(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -10*b^(3/2)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + 8*sqrt(-b*x + 2)*b/s
qrt(x) + 2*sqrt(-b*x + 2)*b^2/((b - (b*x - 2)/x)*sqrt(x)) - 4/3*(-b*x + 2)^
(3/2)/x^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2-b\,x)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(5/2),x)
```

```
[Out] int((2 - b*x)^(5/2)/x^(5/2), x)
```

**sympy [C]** time = 5.35, size = 221, normalized size = 2.63

$$\begin{cases} b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}+\frac{28b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3}+5ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-10ib^{\frac{3}{2}}\log\left(\frac{1}{\sqrt{b}\sqrt{x}}\right)+10b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)-\frac{8\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for } \frac{2}{|bx|}>1 \\ ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}+\frac{28ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}+5ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-10ib^{\frac{3}{2}}\log\left(\sqrt{1-\frac{2}{bx}}+1\right)-\frac{8i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(5/2),x)
```

```
[Out] Piecewise((b**(5/2)*x*sqrt(-1 + 2/(b*x)) + 28*b**(3/2)*sqrt(-1 + 2/(b*x))/3
 + 5*I*b**(3/2)*log(1/(b*x)) - 10*I*b**(3/2)*log(1/(sqrt(b)*sqrt(x))) + 10*
b**(3/2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2) - 8*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*
x), 2/Abs(b*x) > 1), (I*b**(5/2)*x*sqrt(1 - 2/(b*x)) + 28*I*b**(3/2)*sqrt(1
 - 2/(b*x))/3 + 5*I*b**(3/2)*log(1/(b*x)) - 10*I*b**(3/2)*log(sqrt(1 - 2/(b
*x)) + 1) - 8*I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

## 3.569 $\int \frac{x^{5/2}}{\sqrt{a+bx}} dx$

**Optimal**. Leaf size=101

$$-\frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}+\frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2}+\frac{x^{5/2}\sqrt{a+bx}}{3b}$$

**Rubi [A]** time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3}-\frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}-\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2}+\frac{x^{5/2}\sqrt{a+bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a + b*x])/(12*b^2
) + (x^(5/2)*Sqrt[a + b*x])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
 + b*x]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx &= \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{(5a)\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{6b} \\
&= -\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} + \frac{\left(5a^2\right)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{8b^2} \\
&= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{\left(5a^3\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{16b^3} \\
&= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{\left(5a^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{8b^3} \\
&= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{\left(5a^3\right)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^3} \\
&= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.16, size = 85, normalized size = 0.84

$$
\frac{\sqrt{a+bx}\left(\sqrt{b}\,\sqrt{x}\left(15a^2-10abx+8b^2x^2\right)-\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{24b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^2 - 10*a*b*x + 8*b^2*x^2) - (15*a^(5/
2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.10, size = 82, normalized size = 0.81

$$\frac{5a^3\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{8b^{7/2}}+\frac{\sqrt{a+bx}\left(15a^2\sqrt{x}-10abx^{3/2}+8b^2x^{5/2}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(15*a^2*Sqrt[x] - 10*a*b*x^(3/2) + 8*b^2*x^(5/2)))/(24*b^3)
+ (5*a^3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*b^(7/2))
```

**fricas [A]** time = 1.33, size = 140, normalized size = 1.39

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)+2\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{48\,b^4},\frac{15\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{24\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/24*(15*a^
3*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 - 10*a*b
^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 102, normalized size = 1.01

$$\frac{\sqrt{bx+a}\;x^{\frac{5}{2}}}{3b}-\frac{5\sqrt{bx+a}\;a\,x^{\frac{3}{2}}}{12b^2}-\frac{5\sqrt{(bx+a)\,x}\;a^3\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{16\sqrt{bx+a}\;b^{\frac{7}{2}}\sqrt{x}}+\frac{5\sqrt{bx+a}\;a^2\sqrt{x}}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/3*x^(5/2)*(b*x+a)^(1/2)/b-5/12*a*x^(3/2)*(b*x+a)^(1/2)/b^2+5/8*a^2*x^(1/2
)*(b*x+a)^(1/2)/b^3-5/16*a^3/b^(7/2)*((b*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/2
)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima [A]** time = 3.01, size = 146, normalized size = 1.45

$$\frac{5\,a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{7}{2}}}-\frac{\frac{33\,\sqrt{bx+a}\,a^3b^2}{\sqrt{x}}-\frac{40\,(bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}+\frac{15\,(bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^6-\frac{3\,(bx+a)b^5}{x}+\frac{3\,(bx+a)^2b^4}{x^2}-\frac{(bx+a)^3b^3}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(7/2) - 1/24*(33*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 40*(b*x + a)^(3/
2)*a^3*b/x^(3/2) + 15*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^6 - 3*(b*x + a)*b^5/x
 + 3*(b*x + a)^2*b^4/x^2 - (b*x + a)^3*b^3/x^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)/(a + b*x)^(1/2), x)
```

**sympy [A]** time = 8.52, size = 128, normalized size = 1.27

$$\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^3\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^2\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{1+\frac{bx}{a}}}-\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*sqrt(x)/(8*b**3*sqrt(1 + b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2*
sqrt(1 + b*x/a)) - sqrt(a)*x**(5/2)/(12*b*sqrt(1 + b*x/a)) - 5*a**3*asinh(s
qrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

## 3.570 $\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=77

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2}+\frac{x^{3/2}\sqrt{a+bx}}{2b}$$

**Rubi [A]** time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2}+\frac{x^{3/2}\sqrt{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (-3*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^2) + (x^(3/2)*Sqrt[a + b*x])/(2*b) + (3*a
^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx &= \frac{x^{3/2}\sqrt{a+bx}}{2b} - \frac{(3a)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{4b} \\
&= -\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{\left(3a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{8b^2} \\
&= -\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{4b^2} \\
&= -\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^2} \\
&= -\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 85, normalized size = 1.10

$$\frac{3a^{5/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)+\sqrt{b}\,\sqrt{x}\left(-3a^2-abx+2b^2x^2\right)}{4b^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(-3*a^2 - a*b*x + 2*b^2*x^2) + 3*a^(5/2)*Sqrt[1 + (b*x)/a]
*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 69, normalized size = 0.90

$$\frac{\sqrt{a+bx}\left(2bx^{3/2}-3a\sqrt{x}\right)}{4b^2}-\frac{3a^2\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(-3*a*Sqrt[x] + 2*b*x^(3/2)))/(4*b^2) - (3*a^2*Log[-(Sqrt[b]
*Sqrt[x]) + Sqrt[a + b*x]])/(4*b^(5/2))
```

**fricas [A]** time = 1.15, size = 119, normalized size = 1.55

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(2\,b^2x-3\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{8\,b^3},\,-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(2\,b^2x-3\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{4\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(-b)*arctan(sqr
t(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))
/b^3]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 84, normalized size = 1.09

$$\frac{\sqrt{bx+a}\;x^{\frac{3}{2}}}{2b}+\frac{3\sqrt{(bx+a)\,x}\;a^2\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8\sqrt{bx+a}\;b^{\frac{5}{2}}\sqrt{x}}-\frac{3\sqrt{bx+a}\;a\sqrt{x}}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+a)^(1/2)/b-3/4*a*x^(1/2)*(b*x+a)^(1/2)/b^2+3/8*a^2/b^(5/2)
*((b*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)
^(1/2))
```

**maxima [B]** time = 2.87, size = 112, normalized size = 1.45

$$-\frac{3\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{5}{2}}}+\frac{\frac{5\,\sqrt{bx+a}\,a^2b}{\sqrt{x}}-\frac{3\,(bx+a)^{\frac{3}{2}}\,a^2}{x^{\frac{3}{2}}}}{4\left(b^4-\frac{2\,(bx+a)b^3}{x}+\frac{(bx+a)^2b^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(5/2) + 1/4*(5*sqrt(b*x + a)*a^2*b/sqrt(x) - 3*(b*x + a)^(3/2)*a^
2/x^(3/2))/(b^4 - 2*(b*x + a)*b^3/x + (b*x + a)^2*b^2/x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a + b*x)^(1/2), x)
```

**sympy [A]** time = 4.30, size = 100, normalized size = 1.30

$$-\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,x^{\frac{3}{2}}}{4b\sqrt{1+\frac{bx}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*sqrt(x)/(4*b**2*sqrt(1 + b*x/a)) - sqrt(a)*x**(3/2)/(4*b*sqrt(1
 + b*x/a)) + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2)/
(2*sqrt(a)*sqrt(1 + b*x/a))
```

## 3.571 $\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=48

$$\frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^
(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx &= \frac{\sqrt{x}\,\sqrt{a+bx}}{b} - \frac{a\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{2b}\\
&= \frac{\sqrt{x}\,\sqrt{a+bx}}{b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{b}\\
&= \frac{\sqrt{x}\,\sqrt{a+bx}}{b} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b}\\
&= \frac{\sqrt{x}\,\sqrt{a+bx}}{b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 68, normalized size = 1.42

$$\frac{\sqrt{b}\,\sqrt{x}\,(a+bx) - a^{3/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(a + b*x) - a^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/(b^(3/2)*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.06, size = 49, normalized size = 1.02

$$\frac{a\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{b^{3/2}} + \frac{\sqrt{x}\,\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x])/b + (a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/b^(
3/2)
```

**fricas [A]** time = 1.47, size = 91, normalized size = 1.90

$$\left[\frac{a\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\sqrt{bx+a}\,b\sqrt{x}}{2\,b^2},\ \frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,b\sqrt{x}}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b^2, (a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x
))) + sqrt(b*x + a)*b*sqrt(x))/b^2]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.00, size = 65, normalized size = 1.35

$$-\frac{\sqrt{(bx+a)\,x}\;a\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{bx+a}\;b^{\frac{3}{2}}\sqrt{x}}+\frac{\sqrt{bx+a}\;\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*((b*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/
2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

**maxima [B]** time = 2.91, size = 73, normalized size = 1.52

$$\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}-\frac{\sqrt{bx+a}\,a}{\left(b^2-\frac{(bx+a)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(
x)))/b^(3/2) - sqrt(b*x + a)*a/((b^2 - (b*x + a)*b/x)*sqrt(x))
```

**mupad [B]** time = 0.55, size = 44, normalized size = 0.92

$$\frac{\sqrt{x}\,\sqrt{a+b\,x}}{b}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(1/2),x)
```

```
[Out] (x^(1/2)*(a + b*x)^(1/2))/b - (2*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2)
 - a^(1/2))))/b^(3/2)
```

**sympy [A]** time = 2.19, size = 44, normalized size = 0.92

$$\frac{\sqrt{a}\,\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}{b}-\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*sqrt(1 + b*x/a)/b - a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/
2)
```

## 3.572 $\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a + b*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 50, normalized size = 1.79

$$\frac{2\sqrt{a}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a + b*x]),x]
[Out] (2*Sqrt[a]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*S
qrt[a + b*x])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 30, normalized size = 1.07

$$-\frac{2\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[a + b*x]),x]
[Out] (-2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[b]
```

**fricas** [A] time = 1.04, size = 57, normalized size = 2.04

$$\left[\frac{\log\left(2\,bx+2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a)/sqrt(b), -2*sqrt(-b)*arct
an(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))/b]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.00, size = 48, normalized size = 1.71

$$\frac{\sqrt{(bx+a)\,x}\;\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{bx+a}\;\sqrt{b}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] ((b*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^
(1/2))/b^(1/2)
```

**maxima [B]** time = 2.95, size = 41, normalized size = 1.46

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/
sqrt(b)
```

**mupad [B]** time = 0.03, size = 30, normalized size = 1.07

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{-b}\;\sqrt{x}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(4*atan(((a + b*x)^(1/2) - a^(1/2))/((-b)^(1/2)*x^(1/2))))/(-b)^(1/2)
```

**sympy [A]** time = 1.10, size = 22, normalized size = 0.79

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

## 3.573 $\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=19

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

**Rubi [A]** time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx=-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

**Mathematica [A]** time = 0.00, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.02, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

**fricas [A]** time = 0.95, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

**giac [B]** time = 2.05, size = 33, normalized size = 1.74

$$-\frac{2\sqrt{bx+a}\,b^2}{\sqrt{(bx+a)b-ab}\,a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a*abs(b))
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.84

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2*(b*x+a)^(1/2)/a/x^(1/2)
```

**maxima [A]** time = 1.34, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

**mupad [B]** time = 0.35, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{a+b\,x}}{a\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(1/2)),x)
[Out] -(2*(a + b*x)^(1/2))/(a*x^(1/2))
```

**sympy [A]** time = 0.91, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(1/2),x)
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/a
```

## 3.574 $\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=44

$$\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{3ax^{3/2}} - \frac{(2b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a}$$
$$= -\frac{2\sqrt{a+bx}}{3ax^{3/2}} + \frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}$$

**Mathematica [A]** time = 0.01, size = 27, normalized size = 0.61

$$-\frac{2(a-2bx)\sqrt{a+bx}}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a + b*x]),x]
[Out] (-2*(a - 2*b*x)*Sqrt[a + b*x])/(3*a^2*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 29, normalized size = 0.66

$$\frac{2\sqrt{a+bx}\,(2bx-a)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*Sqrt[a + b*x]),x]
[Out] (2*Sqrt[a + b*x]*(-a + 2*b*x))/(3*a^2*x^(3/2))
```

**fricas [A]** time = 1.15, size = 23, normalized size = 0.52

$$\frac{2\,(2\,bx-a)\sqrt{bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*b*x - a)*sqrt(b*x + a)/(a^2*x^(3/2))
```

**giac [A]** time = 1.90, size = 50, normalized size = 1.14

$$\frac{2\left(\frac{2\,(bx+a)b^3}{a^2} - \frac{3\,b^3}{a}\right)\sqrt{bx+a}\,b}{3\,((bx+a)b-ab)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*(b*x + a)*b^3/a^2 - 3*b^3/a)*sqrt(b*x + a)*b/(((b*x + a)*b - a*b)^(3
/2)*abs(b))
```

**maple** [A] time = 0.00, size = 22, normalized size = 0.50

$$-\frac{2\sqrt{bx+a}\,\left(-2bx+a\right)}{3a^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(-2*b*x+a)/x^(3/2)/a^2
```

**maxima** [A] time = 1.30, size = 31, normalized size = 0.70

$$\frac{2\left(\frac{3\,\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^2
```

**mupad** [B] time = 0.34, size = 25, normalized size = 0.57

$$-\frac{\left(\frac{2}{3\,a}-\frac{4\,b\,x}{3\,a^2}\right)\,\sqrt{a+b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -((2/(3*a) - (4*b*x)/(3*a^2))*(a + b*x)^(1/2))/x^(3/2)
```

**sympy** [A] time = 1.92, size = 42, normalized size = 0.95

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3ax}+\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*a*x) + 4*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a**2
)
```

## 3.575 $\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=68

$$-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(5*a*x^(5/2)) + (8*b*Sqrt[a + b*x])/(15*a^2*x^(3/2)) - (
16*b^2*Sqrt[a + b*x])/(15*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{5ax^{5/2}} - \frac{(4b)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{5a}$$
$$= -\frac{2\sqrt{a+bx}}{5ax^{5/2}} + \frac{8b\sqrt{a+bx}}{15a^2x^{3/2}} + \frac{\left(8b^2\right)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{15a^2}$$
$$= -\frac{2\sqrt{a+bx}}{5ax^{5/2}} + \frac{8b\sqrt{a+bx}}{15a^2x^{3/2}} - \frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}$$

**Mathematica** [A]   time = 0.01, size = 40, normalized size = 0.59

$$-\frac{2\sqrt{a+bx}\left(3a^2-4abx+8b^2x^2\right)}{15a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a + b*x]),x]
[Out] (-2*Sqrt[a + b*x]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(5/2))
```

**IntegrateAlgebraic** [A]   time = 0.09, size = 40, normalized size = 0.59

$$-\frac{2\sqrt{a+bx}\left(3a^2-4abx+8b^2x^2\right)}{15a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*Sqrt[a + b*x]),x]
[Out] (-2*Sqrt[a + b*x]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(5/2))
```

**fricas** [A]   time = 1.21, size = 34, normalized size = 0.50

$$-\frac{2\left(8\,b^2x^2-4\,abx+3\,a^2\right)\sqrt{bx+a}}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x + a)/(a^3*x^(5/2))
```

**giac** [A] time = 1.67, size = 66, normalized size = 0.97

$$-\frac{2\left(\frac{15\,b^{5}}{a}+4\left(\frac{2\,(bx+a)b^{5}}{a^{3}}-\frac{5\,b^{5}}{a^{2}}\right)(bx+a)\right)\sqrt{bx+a}\,b}{15\,((bx+a)b-ab)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*b^5/a + 4*(2*(b*x + a)*b^5/a^3 - 5*b^5/a^2)*(b*x + a))*sqrt(b*x +
 a)*b/(((b*x + a)*b - a*b)^(5/2)*abs(b))
```

**maple** [A] time = 0.00, size = 35, normalized size = 0.51

$$-\frac{2\sqrt{bx+a}\,\left(8b^{2}x^{2}-4abx+3a^{2}\right)}{15a^{3}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)^(1/2)*(8*b^2*x^2-4*a*b*x+3*a^2)/x^(5/2)/a^3
```

**maxima** [A] time = 1.34, size = 46, normalized size = 0.68

$$-\frac{2\left(\frac{15\,\sqrt{bx+a}\,b^{2}}{\sqrt{x}}-\frac{10\,(bx+a)^{\frac{3}{2}}\,b}{x^{\frac{3}{2}}}+\frac{3\,(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(15*sqrt(b*x + a)*b^2/sqrt(x) - 10*(b*x + a)^(3/2)*b/x^(3/2) + 3*(b*x
 + a)^(5/2)/x^(5/2))/a^3
```

**mupad** [B] time = 0.35, size = 36, normalized size = 0.53

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{5\,a}+\frac{16\,b^{2}\,x^{2}}{15\,a^{3}}-\frac{8\,b\,x}{15\,a^{2}}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -((a + b*x)^(1/2)*(2/(5*a) + (16*b^2*x^2)/(15*a^3) - (8*b*x)/(15*a^2)))/x^(
5/2)
```

**sympy [B]** time = 6.25, size = 287, normalized size = 4.22

$$-\frac{6a^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{4a^3b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{6a^2b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{24ab^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{16b^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a)**(1/2),x)
```

```
[Out] -6*a**4*b**(9/2)*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 +
 15*a**3*b**6*x**4) - 4*a**3*b**(11/2)*x*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x*
*2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 6*a**2*b**(13/2)*x**2*sqrt(a/
(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 24
*a*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3
+ 15*a**3*b**6*x**4) - 16*b**(17/2)*x**4*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x*
*2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4)
```

## 3.576 $\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=92

$$\frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}+\frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(7*a*x^(7/2)) + (12*b*Sqrt[a + b*x])/(35*a^2*x^(5/2)) -
(16*b^2*Sqrt[a + b*x])/(35*a^3*x^(3/2)) + (32*b^3*Sqrt[a + b*x])/(35*a^4*Sq
rt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{7ax^{7/2}} - \frac{(6b)\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx}{7a}$$
$$= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} + \frac{\left(24b^2\right)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{35a^2}$$
$$= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} - \frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}} - \frac{\left(16b^3\right)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{35a^3}$$
$$= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} - \frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}} + \frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}$$

**Mathematica** [A] time = 0.01, size = 51, normalized size = 0.55

$$-\frac{2\sqrt{a+bx}\left(5a^3-6a^2bx+8ab^2x^2-16b^3x^3\right)}{35a^4x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(5*a^3 - 6*a^2*b*x + 8*a*b^2*x^2 - 16*b^3*x^3))/(35*a^4*x
^(7/2))
```

**IntegrateAlgebraic** [A] time = 0.10, size = 51, normalized size = 0.55

$$\frac{2\sqrt{a+bx}\left(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3\right)}{35a^4x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

```
[Out] (2*Sqrt[a + b*x]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35*a^4*x
^(7/2))
```

**fricas** [A] time = 1.21, size = 45, normalized size = 0.49

$$\frac{2\left(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3\right)\sqrt{bx+a}}{35\,a^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^4*x^(7
/2))
```

**giac [A]** time = 1.41, size = 82, normalized size = 0.89

$$-\frac{2\left(\frac{35\,b^7}{a}-2\left(\frac{35\,b^7}{a^2}+4\left(\frac{2\,(bx+a)b^7}{a^4}-\frac{7\,b^7}{a^3}\right)(bx+a)\right)(bx+a)\right)\sqrt{bx+a}\,b}{35\left((bx+a)b-ab\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*(35*b^7/a - 2*(35*b^7/a^2 + 4*(2*(b*x + a)*b^7/a^4 - 7*b^7/a^3)*(b*x
+ a))*(b*x + a))*sqrt(b*x + a)*b/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

**maple [A]** time = 0.00, size = 46, normalized size = 0.50

$$-\frac{2\sqrt{bx+a}\,\left(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3\right)}{35a^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)^(1/2)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^(7/2)/a^4
```

**maxima [A]** time = 1.30, size = 61, normalized size = 0.66

$$\frac{2\left(\frac{35\,\sqrt{bx+a}\,b^3}{\sqrt{x}}-\frac{35\,(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{21\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{5\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{35\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(b*x + a)*b^3/sqrt(x) - 35*(b*x + a)^(3/2)*b^2/x^(3/2) + 21*(b
*x + a)^(5/2)*b/x^(5/2) - 5*(b*x + a)^(7/2)/x^(7/2))/a^4
```

**mupad [B]** time = 0.38, size = 47, normalized size = 0.51

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{7\,a}+\frac{16\,b^2\,x^2}{35\,a^3}-\frac{32\,b^3\,x^3}{35\,a^4}-\frac{12\,b\,x}{35\,a^2}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -((a + b*x)^(1/2)*(2/(7*a) + (16*b^2*x^2)/(35*a^3) - (32*b^3*x^3)/(35*a^4)
- (12*b*x)/(35*a^2)))/x^(7/2)
```

**sympy [B]** time = 16.14, size = 488, normalized size = 5.30

$$-\frac{10a^6b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}-\frac{18a^5b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}-\frac{10a^4b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{10a^3b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{60a^2b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{80ab^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{32b^{\frac{31}{2}}x^6\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x+a)**(1/2),x)
```

```
[Out] -10*a**6*b**(19/2)*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x*
*4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) - 18*a**5*b**(21/2)*x*sqrt(a
/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5
+ 35*a**4*b**12*x**6) - 10*a**4*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(35*a**7*b
**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6)
+ 10*a**3*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b*
*10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 60*a**2*b**(27/2)*x*
*4*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b*
*11*x**5 + 35*a**4*b**12*x**6) + 80*a*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(35*
a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*
x**6) + 32*b**(31/2)*x**6*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b
**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6)
```

## 3.577 $\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=96

$$\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}-\frac{15a\sqrt{x}\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

**Rubi [A]** time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{15a\sqrt{x}\sqrt{a+bx}}{4b^3}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^(5/2))/(b*Sqrt[a + b*x]) - (15*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^3) + (5*
x^(3/2)*Sqrt[a + b*x])/(2*b^2) + (15*a^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
 b*x]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2x^{5/2}}{b\sqrt{a+bx}}+\frac{5\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{b}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{(15a)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{4b^2}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{\left(15a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{8b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 50, normalized size = 0.52

$$
\frac{2x^{7/2}\sqrt{\frac{bx}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a\sqrt{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 + (b*x)/a]*Hypergeometric2F1[3/2, 7/2, 9/2, -((b*x)/a)])/
(7*a*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.14, size = 82, normalized size = 0.85

$$\frac{-15a^2\sqrt{x}-5abx^{3/2}+2b^2x^{5/2}}{4b^3\sqrt{a+bx}}-\frac{15a^2\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-15*a^2*Sqrt[x] - 5*a*b*x^(3/2) + 2*b^2*x^(5/2))/(4*b^3*Sqrt[a + b*x]) - (
15*a^2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(4*b^(7/2))
```

**fricas [A]** time = 1.41, size = 175, normalized size = 1.82

$$\left[\frac{15\left(a^2bx+a^3\right)\sqrt{b}\log\left(2bx+2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2b^3x^2-5ab^2x-15a^2b\right)\sqrt{bx+a}\sqrt{x}}{8\left(b^5x+ab^4\right)},-\frac{15\left(a^2bx+a^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\left(2b^3x^2-5ab^2x-15a^2b\right)\sqrt{bx+a}\sqrt{x}}{4\left(b^5x+ab^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a^2*b*x + a^3)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x
) + a) + 2*(2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x
 + a*b^4), -1/4*(15*(a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/
(b*sqrt(x))) - (2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x + a*b^4)]
```

**giac [A]** time = 92.14, size = 131, normalized size = 1.36

$$\frac{\left(2\sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(\frac{2(bx+a)}{b^3}-\frac{9a}{b^3}\right)-\frac{32a^3}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)b^{\frac{3}{2}}}-\frac{15a^2\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{b^{\frac{5}{2}}}\right)|b|}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)/b^3 - 9*a/b^3) -
32*a^3/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*b^(3/2)
```

```
) - 15*a^2*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(5/2)
)*abs(b)/b^2
```

**maple [A]** time = 0.04, size = 119, normalized size = 1.24

$$\frac{\left(\frac{15a^2 \ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{7}{2}}}-\frac{2\sqrt{-\left(x+\frac{a}{b}\right)a+\left(x+\frac{a}{b}\right)^2 b}\;a^2}{\left(x+\frac{a}{b}\right)b^4}\right)\sqrt{(bx+a)\,x}}{\sqrt{bx+a}\;\sqrt{x}}-\frac{(-2bx+7a)\sqrt{bx+a}\;\sqrt{x}}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] -1/4*(-2*b*x+7*a)*(b*x+a)^(1/2)*x^(1/2)/b^3+(15/8/b^(7/2)*a^2*ln((b*x+1/2*a
)/b^(1/2)+(b*x^2+a*x)^(1/2))-2/b^4*a^2/(x+a/b)*(b*(x+a/b)^2-(x+a/b)*a)^(1/2
))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
```

**maxima [A]** time = 2.92, size = 131, normalized size = 1.36

$$-\frac{8\,a^2b^2-\frac{25\,(bx+a)a^2b}{x}+\frac{15\,(bx+a)^2a^2}{x^2}}{4\left(\frac{\sqrt{bx+a}\,b^5}{\sqrt{x}}-\frac{2\,(bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}-\frac{15\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a^2*b^2 - 25*(b*x + a)*a^2*b/x + 15*(b*x + a)^2*a^2/x^2)/(sqrt(b*x
+ a)*b^5/sqrt(x) - 2*(b*x + a)^(3/2)*b^4/x^(3/2) + (b*x + a)^(5/2)*b^3/x^(5
/2)) - 15/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x
+ a)/sqrt(x)))/b^(7/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)/(a + b*x)^(3/2), x)
```

**sympy [A]** time = 8.14, size = 105, normalized size = 1.09

$$-\frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1+\frac{bx}{a}}}-\frac{5\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{1+\frac{bx}{a}}}+\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] -15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 + b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b**2*
sqrt(1 + b*x/a)) + 15*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)) + x*
*(5/2)/(2*sqrt(a)*b*sqrt(1 + b*x/a))
```

# 3.578 $\int \frac{x^{3/2}}{(a+bx)^{3/2}} dx$

**Optimal**. Leaf size=68

$$-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

**Rubi** [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^(3/2))/(b*Sqrt[a + b*x]) + (3*Sqrt[x]*Sqrt[a + b*x])/b^2 - (3*a*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{b}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{(3a)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{2b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{(3a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 50, normalized size = 0.74

$$
\frac{2x^{5/2}\sqrt{\frac{bx}{a}+1}\;{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{bx}{a}\right)}{5a\sqrt{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 + (b*x)/a]*Hypergeometric2F1[3/2, 5/2, 7/2, -((b*x)/a)])/
(5*a*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.13, size = 61, normalized size = 0.90

$$\frac{3a\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{b^{5/2}}+\frac{3a\sqrt{x}+bx^{3/2}}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (3*a*Sqrt[x] + b*x^(3/2))/(b^2*Sqrt[a + b*x]) + (3*a*Log[-(Sqrt[b]*Sqrt[x])
 + Sqrt[a + b*x]])/b^(5/2)
```

**fricas [A]** time = 1.39, size = 145, normalized size = 2.13

$$\left[\frac{3\left(abx+a^2\right)\sqrt{b}\log\left(2bx-2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(b^2x+3ab\right)\sqrt{bx+a}\sqrt{x}}{2\left(b^4x+ab^3\right)}, \frac{3\left(abx+a^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(b^2x+3ab\right)\sqrt{bx+a}\sqrt{x}}{b^4x+ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + a^2)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) +
 a) + 2*(b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3), (3*(a*b*x +
 a^2)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 3*a*b)
*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3)]
```

**giac [B]** time = 93.12, size = 115, normalized size = 1.69

$$\frac{\left(\frac{8a^2\sqrt{b}}{\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab}+\frac{3a\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}}+\frac{2\sqrt{(bx+a)b-ab}\sqrt{bx+a}}{b}\right)|b|}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(8*a^2*sqrt(b)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a
*b) + 3*a*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b)
+ 2*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)/b)*abs(b)/b^3
```

**maple [B]** time = 0.03, size = 106, normalized size = 1.56

$$\frac{\left(-\frac{3a\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{5}{2}}}+\frac{2\sqrt{-\left(x+\frac{a}{b}\right)a+\left(x+\frac{a}{b}\right)^{2}b}\;a}{\left(x+\frac{a}{b}\right)b^{3}}\right)\sqrt{(bx+a)\,x}}{\sqrt{bx+a}\,\sqrt{x}}+\frac{\sqrt{bx+a}\,\sqrt{x}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b^2+(-3/2*a/b^(5/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x
)^(1/2))+2*a/b^3/(x+a/b)*(-(x+a/b)*a+(x+a/b)^2*b)^(1/2))*((b*x+a)*x)^(1/2)/
(b*x+a)^(1/2)/x^(1/2)
```

**maxima [A]** time = 3.02, size = 92, normalized size = 1.35

$$\frac{2\,ab-\frac{3\,(bx+a)a}{x}}{\frac{\sqrt{bx+a}\,b^{3}}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}}+\frac{3\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] (2*a*b - 3*(b*x + a)*a/x)/(sqrt(b*x + a)*b^3/sqrt(x) - (b*x + a)^(3/2)*b^2/
x^(3/2)) + 3/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x
 + a)/sqrt(x)))/b^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)/(a + b*x)^(3/2), x)
```

**sympy [A]** time = 3.68, size = 71, normalized size = 1.04

$$\frac{3\sqrt{a}\,\sqrt{x}}{b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] 3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 + b*x/a)) - 3*a*asinh(sqrt(b)*sqrt(x)/sqrt(a
))/b**(5/2) + x**(3/2)/(sqrt(a)*b*sqrt(1 + b*x/a))
```

## 3.579 $\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=48

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

**Rubi [A]** time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[a + b*x]) + (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx &= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{b}\\
&= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{b}\\
&= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b}\\
&= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.07, size = 64, normalized size = 1.33

$$
\frac{2\left(\sqrt{a}\,\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)-\sqrt{b}\,\sqrt{x}\right)}{b^{3/2}\sqrt{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-(Sqrt[b]*Sqrt[x]) + Sqrt[a]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]]))/(b^(3/2)*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 50, normalized size = 1.04

$$
-\frac{2\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[a + b*x]) - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
)/b^(3/2)
```

**fricas [A]** time = 1.33, size = 119, normalized size = 2.48

$$\left[\frac{(bx+a)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,\sqrt{bx+a}\,b\sqrt{x}}{b^3x+ab^2},-\frac{2\left((bx+a)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,b\sqrt{x}\right)}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sq
rt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2), -2*((b*x + a)*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2)]
```

**giac [B]** time = 94.86, size = 85, normalized size = 1.77

$$-\frac{\left(\frac{4\,a\sqrt{b}}{\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab}+\frac{\log\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(4*a*sqrt(b)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b) +
 log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b))*abs(b)/b
^2
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] int(x^(1/2)/(b*x+a)^(3/2),x)
```

**maxima [A]** time = 2.98, size = 57, normalized size = 1.19

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\,\sqrt{x}}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/
b^(3/2) - 2*sqrt(x)/(sqrt(b*x + a)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(a + b*x)^(3/2), x)
```

**sympy [A]** time = 1.78, size = 46, normalized size = 0.96

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{2\sqrt{x}}{\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*sqrt(x)/(sqrt(a)*b*sqrt(1 + b
*x/a))
```

## 3.580 $\int \frac{1}{\sqrt{x}\,(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=19

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

**Rubi** [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx)^{3/2}}\,dx=\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

**Mathematica** [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.02, size = 19, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])
```

**fricas [A]** time = 1.01, size = 22, normalized size = 1.16

$$\frac{2\sqrt{bx+a}\sqrt{x}}{abx+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)*sqrt(x)/(a*b*x + a^2)
```

**giac [B]** time = 1.11, size = 45, normalized size = 2.37

$$\frac{4b^{\frac{3}{2}}}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*abs(
b))
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.84

$$\frac{2\sqrt{x}}{\sqrt{bx+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 2*x^(1/2)/a/(b*x+a)^(1/2)
```

**maxima** [A] time = 1.32, size = 15, normalized size = 0.79

$$\frac{2\sqrt{x}}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)/(sqrt(b*x + a)*a)
```

**mupad** [B] time = 0.33, size = 22, normalized size = 1.16

$$\frac{2\sqrt{x}\,\sqrt{a+b\,x}}{a^2+b\,x\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^(3/2)),x)
```

```
[Out] (2*x^(1/2)*(a + b*x)^(1/2))/(a^2 + a*b*x)
```

**sympy** [A] time = 0.88, size = 17, normalized size = 0.89

$$\frac{2}{a\sqrt{b}\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] 2/(a*sqrt(b)*sqrt(a/(b*x) + 1))
```

## 3.581 $\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=39

$$\frac{2}{a\sqrt{x}\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

**Rubi [A]** time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{2}{a\sqrt{x}\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[x]*Sqrt[a + b*x]) - (4*Sqrt[a + b*x])/(a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx = \frac{2}{a\sqrt{x}\sqrt{a+bx}} + \frac{2\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{a\sqrt{x}\sqrt{a+bx}} - \frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 0.64

$$-\frac{2(a+2bx)}{a^2\sqrt{x}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^(3/2)),x]
[Out] (-2*(a + 2*b*x))/(a^2*Sqrt[x]*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 25, normalized size = 0.64

$$-\frac{2(a+2bx)}{a^2\sqrt{x}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x)^(3/2)),x]
[Out] (-2*(a + 2*b*x))/(a^2*Sqrt[x]*Sqrt[a + b*x])
```

**fricas [A]** time = 0.89, size = 34, normalized size = 0.87

$$-\frac{2\,(2\,bx+a)\sqrt{bx+a}\,\sqrt{x}}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] -2*(2*b*x + a)*sqrt(b*x + a)*sqrt(x)/(a^2*b*x^2 + a^3*x)
```

**giac [B]** time = 1.05, size = 82, normalized size = 2.10

$$-\frac{4\,b^{\frac{5}{2}}}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)a|b|}-\frac{2\,\sqrt{bx+a}\,b^2}{\sqrt{(bx+a)b-ab}\,a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*b^(5/2)/((((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*a*a
bs(b)) - 2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a^2*abs(b))
```

**maple** [A] time = 0.01, size = 22, normalized size = 0.56

$$-\frac{2\,(2bx+a)}{\sqrt{bx+a}\;a^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2*(2*b*x+a)/(b*x+a)^(1/2)/x^(1/2)/a^2
```

**maxima** [A] time = 1.32, size = 32, normalized size = 0.82

$$-\frac{2\,b\sqrt{x}}{\sqrt{bx+a}\,a^{2}}-\frac{2\,\sqrt{bx+a}}{a^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b*sqrt(x)/(sqrt(b*x + a)*a^2) - 2*sqrt(b*x + a)/(a^2*sqrt(x))
```

**mupad** [B] time = 0.39, size = 39, normalized size = 1.00

$$-\frac{2\,a\,\sqrt{a+b\,x}\,+4\,b\,x\,\sqrt{a+b\,x}}{\sqrt{x}\,\left(a^{3}+b\,x\,a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(3/2)),x)
```

```
[Out] -(2*a*(a + b*x)^(1/2) + 4*b*x*(a + b*x)^(1/2))/(x^(1/2)*(a^3 + a^2*b*x))
```

**sympy** [A] time = 1.60, size = 41, normalized size = 1.05

$$-\frac{2}{a\sqrt{b}\,x\sqrt{\frac{a}{bx}+1}}-\frac{4\sqrt{b}}{a^{2}\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(3/2),x)
[Out] -2/(a*sqrt(b)*x*sqrt(a/(b*x) + 1)) - 4*sqrt(b)/(a**2*sqrt(a/(b*x) + 1))
```

## 3.582 $\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=63

$$\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{2}{ax^{3/2}\sqrt{a+bx}}$$

**Rubi [A]** time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{2}{ax^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(3/2)*Sqrt[a + b*x]) - (8*Sqrt[a + b*x])/(3*a^2*x^(3/2)) + (16*b*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx = \frac{2}{ax^{3/2}\sqrt{a+bx}} + \frac{4\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{ax^{3/2}\sqrt{a+bx}} - \frac{8\sqrt{a+bx}}{3a^2x^{3/2}} - \frac{(8b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^2}$$
$$= \frac{2}{ax^{3/2}\sqrt{a+bx}} - \frac{8\sqrt{a+bx}}{3a^2x^{3/2}} + \frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}$$

**Mathematica** [A] time = 0.01, size = 38, normalized size = 0.60

$$-\frac{2\left(a^2-4abx-8b^2x^2\right)}{3a^3x^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^(3/2)),x]
[Out] (-2*(a^2 - 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a + b*x])
```

**IntegrateAlgebraic** [A] time = 0.11, size = 40, normalized size = 0.63

$$\frac{2\left(-a^2+4abx+8b^2x^2\right)}{3a^3x^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x)^(3/2)),x]
[Out] (2*(-a^2 + 4*a*b*x + 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a + b*x])
```

**fricas** [A] time = 1.35, size = 49, normalized size = 0.78

$$\frac{2\left(8\,b^2x^2+4\,abx-a^2\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^3bx^3+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] 2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b*x^3 + a^4*x^2)
```

**giac [B]** time = 1.20, size = 98, normalized size = 1.56

$$\frac{2\sqrt{bx+a}\left(\frac{5\,(bx+a)b^{2}|b|}{a^{3}}-\frac{6\,b^{2}|b|}{a^{2}}\right)}{3\left((bx+a)b-ab\right)^{\frac{3}{2}}}+\frac{4\,b^{\frac{7}{2}}}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)a^{2}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x + a)*(5*(b*x + a)*b^2*abs(b)/a^3 - 6*b^2*abs(b)/a^2)/((b*x + a
)*b - a*b)^(3/2) + 4*b^(7/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b -
a*b))^2 + a*b)*a^2*abs(b))
```

**maple [A]** time = 0.00, size = 33, normalized size = 0.52

$$-\frac{2\left(-8b^{2}x^{2}-4abx+a^{2}\right)}{3\sqrt{bx+a}\;a^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2/3*(-8*b^2*x^2-4*a*b*x+a^2)/(b*x+a)^(1/2)/x^(3/2)/a^3
```

**maxima [A]** time = 1.31, size = 50, normalized size = 0.79

$$\frac{2\,b^{2}\sqrt{x}}{\sqrt{bx+a}\,a^{3}}+\frac{2\left(\frac{6\,\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(b*x + a)*a^3) + 2/3*(6*sqrt(b*x + a)*b/sqrt(x) - (b*x +
 a)^(3/2)/x^(3/2))/a^3
```

**mupad [B]** time = 0.41, size = 46, normalized size = 0.73

$$\frac{\sqrt{a+b\,x}\,\left(\frac{8\,x}{3\,a^{2}}-\frac{2}{3\,a\,b}+\frac{16\,b\,x^{2}}{3\,a^{3}}\right)}{x^{5/2}+\frac{a\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(3/2)),x)
```

```
[Out] ((a + b*x)^(1/2)*((8*x)/(3*a^2) - 2/(3*a*b) + (16*b*x^2)/(3*a^3)))/(x^(5/2)
 + (a*x^(3/2))/b)
```

**sympy [B]** time = 3.98, size = 219, normalized size = 3.48

$$-\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{16b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2*a**3*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a*
*3*b**6*x**3) + 6*a**2*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**
4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*
a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 16*b**(15/2)*x**3*sqrt
(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3)
```

## 3.583 $\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=87

$$-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

**Rubi [A]** time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(5/2)*Sqrt[a + b*x]) - (12*Sqrt[a + b*x])/(5*a^2*x^(5/2)) + (16*b*Sq
rt[a + b*x])/(5*a^3*x^(3/2)) - (32*b^2*Sqrt[a + b*x])/(5*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx = \frac{2}{ax^{5/2}\sqrt{a+bx}} + \frac{6\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{ax^{5/2}\sqrt{a+bx}} - \frac{12\sqrt{a+bx}}{5a^2x^{5/2}} - \frac{(24b)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{5a^2}$$
$$= \frac{2}{ax^{5/2}\sqrt{a+bx}} - \frac{12\sqrt{a+bx}}{5a^2x^{5/2}} + \frac{16b\sqrt{a+bx}}{5a^3x^{3/2}} + \frac{\left(16b^2\right)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{5a^3}$$
$$= \frac{2}{ax^{5/2}\sqrt{a+bx}} - \frac{12\sqrt{a+bx}}{5a^2x^{5/2}} + \frac{16b\sqrt{a+bx}}{5a^3x^{3/2}} - \frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}$$

**Mathematica** [A] time = 0.01, size = 49, normalized size = 0.56

$$-\frac{2\left(a^3-2a^2bx+8ab^2x^2+16b^3x^3\right)}{5a^4x^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*(a^3 - 2*a^2*b*x + 8*a*b^2*x^2 + 16*b^3*x^3))/(5*a^4*x^(5/2)*Sqrt[a + b
*x])
```

**IntegrateAlgebraic** [A] time = 0.12, size = 49, normalized size = 0.56

$$-\frac{2\left(a^3-2a^2bx+8ab^2x^2+16b^3x^3\right)}{5a^4x^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*(a^3 - 2*a^2*b*x + 8*a*b^2*x^2 + 16*b^3*x^3))/(5*a^4*x^(5/2)*Sqrt[a + b
*x])
```

**fricas** [A] time = 1.18, size = 58, normalized size = 0.67

$$-\frac{2\left(16\,b^3x^3+8\,ab^2x^2-2\,a^2bx+a^3\right)\sqrt{bx+a}\,\sqrt{x}}{5\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b*x^4 + a^5*x^3)
```

**giac [A]** time = 1.21, size = 121, normalized size = 1.39

$$-\frac{4\,b^{\frac{9}{2}}}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)a^{3}|b|}-\frac{2\left(\frac{15\,b^{6}}{a^{2}|b|}+(bx+a)\left(\frac{11\,(bx+a)b^{6}}{a^{4}|b|}-\frac{25\,b^{6}}{a^{3}|b|}\right)\right)\sqrt{bx+a}}{5\,((bx+a)b-ab)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*b^(9/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*a^3
*abs(b)) - 2/5*(15*b^6/(a^2*abs(b)) + (b*x + a)*(11*(b*x + a)*b^6/(a^4*abs(
b)) - 25*b^6/(a^3*abs(b))))*sqrt(b*x + a)/((b*x + a)*b - a*b)^(5/2)
```

**maple [A]** time = 0.00, size = 44, normalized size = 0.51

$$-\frac{2\left(16b^{3}x^{3}+8a\,b^{2}x^{2}-2a^{2}bx+a^{3}\right)}{5\sqrt{bx+a}\;a^{4}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2/5*(16*b^3*x^3+8*a*b^2*x^2-2*a^2*b*x+a^3)/(b*x+a)^(1/2)/x^(5/2)/a^4
```

**maxima [A]** time = 1.31, size = 64, normalized size = 0.74

$$-\frac{2\,b^{3}\sqrt{x}}{\sqrt{bx+a}\,a^{4}}-\frac{2\left(\frac{15\,\sqrt{bx+a}\,b^{2}}{\sqrt{x}}-\frac{5\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{5\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b^3*sqrt(x)/(sqrt(b*x + a)*a^4) - 2/5*(15*sqrt(b*x + a)*b^2/sqrt(x) - 5*
(b*x + a)^(3/2)*b/x^(3/2) + (b*x + a)^(5/2)/x^(5/2))/a^4
```

**mupad [B]** time = 0.43, size = 58, normalized size = 0.67

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{5\,a\,b}-\frac{4\,x}{5\,a^{2}}+\frac{16\,b\,x^{2}}{5\,a^{3}}+\frac{32\,b^{2}\,x^{3}}{5\,a^{4}}\right)}{x^{7/2}+\frac{a\,x^{5/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(7/2)*(a + b*x)^(3/2)),x)

[Out] -((a + b*x)^(1/2)*(2/(5*a*b) - (4*x)/(5*a^2) + (16*b*x^2)/(5*a^3) + (32*b^2*x^3)/(5*a^4)))/(x^(7/2) + (a*x^(5/2))/b)

**sympy [B]** time = 11.15, size = 348, normalized size = 4.00

$$-\frac{2a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{10a^3b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{60a^2b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{80ab^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{32b^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(7/2)/(b*x+a)**(3/2),x)

[Out] -2*a**5*b**(19/2)*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 10*a**3*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 60*a**2*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 80*a*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 32*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5)

## 3.584 $\int \frac{x^{5/2}}{(a+bx)^{5/2}} \, dx$

**Optimal**. Leaf size=91

$$-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^(5/2))/(3*b*(a + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a + b*x]) + (
5*Sqrt[x]*Sqrt[a + b*x])/b^3 - (5*a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}+\frac{5\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx}{3b}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{b^2}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{2b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 50, normalized size = 0.55

$$\frac{2x^{7/2}\sqrt{\frac{bx}{a}+1}\;{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 + (b*x)/a]*Hypergeometric2F1[5/2, 7/2, 9/2, -((b*x)/a)])/
(7*a^2*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.15, size = 78, normalized size = 0.86

$$\frac{15a^2\sqrt{x}+20abx^{3/2}+3b^2x^{5/2}}{3b^3(a+bx)^{3/2}}+\frac{5a\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (15*a^2*Sqrt[x] + 20*a*b*x^(3/2) + 3*b^2*x^(5/2))/(3*b^3*(a + b*x)^(3/2)) +
 (5*a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/b^(7/2)
```

**fricas [A]** time = 1.42, size = 214, normalized size = 2.35

$$\left[\frac{15\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{b}\log\left(2bx-2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(3b^3x^2+20ab^2x+15a^2b\right)\sqrt{bx+a}\sqrt{x}}{6\left(b^6x^2+2ab^5x+a^2b^4\right)}, \frac{15\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(3b^3x^2+20ab^2x+15a^2b\right)\sqrt{bx+a}\sqrt{x}}{3\left(b^6x^2+2ab^5x+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*
sqrt(b)*sqrt(x) + a) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*
sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x +
a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 20*
a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

**giac [B]** time = 92.46, size = 197, normalized size = 2.16

$$\frac{\left(\frac{15a\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{b^{\frac{5}{2}}}+\frac{6\sqrt{(bx+a)b-ab}\sqrt{bx+a}}{b^3}+\frac{8\left(9a^2\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4\sqrt{b}+12a^3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{3}{2}}+7a^4b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3b^2}\right)|b|}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(15*a*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(5/2)
+ 6*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)/b^3 + 8*(9*a^2*(sqrt(b*x + a)*sqr
t(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + 12*a^3*(sqrt(b*x + a)*sqrt(b) -
```

```
sqrt((b*x + a)*b - a*b))^2*b^(3/2) + 7*a^4*b^(5/2))/(((sqrt(b*x + a)*sqrt(
b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*b^2))*abs(b)/b^2
```

**maple [B]** time = 0.05, size = 147, normalized size = 1.62

$$\frac{\left(-\frac{5a\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{7}{2}}}-\frac{2\sqrt{-\left(x+\frac{a}{b}\right)a+\left(x+\frac{a}{b}\right)^2 b}\;a^2}{3\left(x+\frac{a}{b}\right)^2 b^5}+\frac{14\sqrt{-\left(x+\frac{a}{b}\right)a+\left(x+\frac{a}{b}\right)^2 b}\;a}{3\left(x+\frac{a}{b}\right)b^4}\right)\sqrt{(bx+a)\,x}}{\sqrt{bx+a}\;\sqrt{x}}+\frac{\sqrt{bx+a}\;\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b^3+(-5/2/b^(7/2)*a*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x
)^(1/2))+14/3/b^4*a/(x+a/b)*(-(x+a/b)*a+(x+a/b)^2*b)^(1/2)-2/3/b^5*a^2/(x+a
/b)^2*(-(x+a/b)*a+(x+a/b)^2*b)^(1/2))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/
2)
```

**maxima [A]** time = 2.94, size = 109, normalized size = 1.20

$$\frac{2\,ab^2+\frac{10\,(bx+a)ab}{x}-\frac{15\,(bx+a)^2 a}{x^2}}{3\left(\frac{(bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}-\frac{(bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}+\frac{5\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a*b^2 + 10*(b*x + a)*a*b/x - 15*(b*x + a)^2*a/x^2)/((b*x + a)^(3/2)*
b^4/x^(3/2) - (b*x + a)^(5/2)*b^3/x^(5/2)) + 5/2*a*log(-(sqrt(b) - sqrt(b*x
 + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(7/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)/(a + b*x)^(5/2), x)
```

**sympy [B]** time = 7.61, size = 396, normalized size = 4.35

$$-\frac{15a^{\frac{81}{2}}b^{22}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x^{26}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{20a^{39}b^{\frac{47}{2}}x^{27}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{3a^{38}b^{\frac{49}{2}}x^{28}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] -15*a**(81/2)*b**22*x**(51/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a)
)/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*
x**(53/2)*sqrt(1 + b*x/a)) - 15*a**(79/2)*b**23*x**(53/2)*sqrt(1 + b*x/a)*a
sinh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x
/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a)) + 15*a**40*b**(45/2)
*x**26/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(5
3/2)*x**(53/2)*sqrt(1 + b*x/a)) + 20*a**39*b**(47/2)*x**27/(3*a**(79/2)*b**
(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 +
 b*x/a)) + 3*a**38*b**(49/2)*x**28/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1
+ b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a))
```

## 3.585 $\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=69

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*(a + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a + b*x]) + (2*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}+\frac{\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx}{b}\\ &= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{b^2}\\ &= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^2}\\ &= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 80, normalized size = 1.16

$$\frac{6\sqrt{a}\,(a+bx)\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-2\sqrt{b}\sqrt{x}\,(3a+4bx)}{3b^{5/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[b]*Sqrt[x]*(3*a + 4*b*x) + 6*Sqrt[a]*(a + b*x)*Sqrt[1 + (b*x)/a]*A
rcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(3*b^(5/2)*(a + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.14, size = 64, normalized size = 0.93

$$-\frac{2\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{b^{5/2}}-\frac{2\left(3a\sqrt{x}+4bx^{3/2}\right)}{3b^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(3*a*Sqrt[x] + 4*b*x^(3/2)))/(3*b^2*(a + b*x)^(3/2)) - (2*Log[-(Sqrt[b]
*Sqrt[x]) + Sqrt[a + b*x]])/b^(5/2)
```

**fricas [A]** time = 0.72, size = 186, normalized size = 2.70

$$\left[\frac{3\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\left(4\,b^{2}x+3\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(b^{5}x^{2}+2\,ab^{4}x+a^{2}b^{3}\right)},-\frac{2\left(3\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(4\,b^{2}x+3\,ab\right)\sqrt{bx+a}\,\sqrt{x}\right)}{3\left(b^{5}x^{2}+2\,ab^{4}x+a^{2}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(
b)*sqrt(x) + a) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x^2 + 2*a
*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqrt(b
*x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x^2 + 2*a*b^4*x + a^2*b^3)]
```

**giac [B]** time = 105.59, size = 165, normalized size = 2.39

$$-\frac{\left(\frac{3\,\log\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}\right)}{\sqrt{b}}+\frac{8\left(3\,a\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{4}\sqrt{b}+3\,a^{2}\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}b^{\frac{3}{2}}+2\,a^{3}b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)^{3}}\right)|b|}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b) +
8*(3*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + 3*a^2*
(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(3/2) + 2*a^3*b^(5/2)
)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3)*abs(b)/b^3
```

**maple [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] int(x^(3/2)/(b*x+a)^(5/2),x)
```

**maxima [A]** time = 2.83, size = 69, normalized size = 1.00

$$-\frac{2\left(b+\frac{3(bx+a)}{x}\right)x^{\frac{3}{2}}}{3(bx+a)^{\frac{3}{2}}b^{2}}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b + 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*b^2) - log(-(sqrt(b) - sq
rt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)/(a + b*x)^(5/2), x)
```

**sympy [B]** time = 4.03, size = 328, normalized size = 4.75

$$\frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{8a^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] 6*a**(39/2)*b**11*x**(27/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a))/
(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x*
*(29/2)*sqrt(1 + b*x/a)) + 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 + b*x/a)*asin
h(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a)
 + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/a)) - 6*a**19*b**(23/2)*x**
14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)
*x**(29/2)*sqrt(1 + b*x/a)) - 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(27/2
)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/
a))
```

## 3.586 $\int \frac{\sqrt{x}}{(a+bx)^{5/2}} \, dx$

**Optimal**. Leaf size=21

$$\frac{2x^{3/2}}{3a(a + bx)^{3/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{2x^{3/2}}{3a(a + bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^(5/2),x]
[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a + bx)^{5/2}} \, dx = \frac{2x^{3/2}}{3a(a + bx)^{3/2}}$$

**Mathematica [A]** time = 0.00, size = 21, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a + bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^(5/2),x]
[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 21, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x)^(5/2),x]
[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

**fricas [B]** time = 1.24, size = 33, normalized size = 1.57

$$\frac{2\sqrt{bx+a}\,x^{\frac{3}{2}}}{3\left(ab^2x^2+2\,a^2bx+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x + a)*x^(3/2)/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

**giac [B]** time = 1.63, size = 86, normalized size = 4.10

$$\frac{4\left(3\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4\sqrt{b}+a^2b^{\frac{5}{2}}\right)|b|}{3\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="giac")
[Out] 4/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + a^2*b^
(5/2))*abs(b)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^
3*b^2)
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.76

$$\frac{2x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(5/2),x)
```

```
[Out] 2/3*x^(3/2)/a/(b*x+a)^(3/2)
```

**maxima [A]** time = 1.33, size = 15, normalized size = 0.71

$$\frac{2\,x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] 2/3*x^(3/2)/((b*x + a)^(3/2)*a)
```

**mupad [B]** time = 0.24, size = 36, normalized size = 1.71

$$\frac{2\,x^{3/2}\,\sqrt{a+b\,x}}{3\,\left(a^3+2\,a^2\,b\,x+a\,b^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(5/2),x)
[Out] (2*x^(3/2)*(a + b*x)^(1/2))/(3*(a^3 + a*b^2*x^2 + 2*a^2*b*x))
```

**sympy [B]** time = 1.43, size = 42, normalized size = 2.00

$$\frac{2x^{\frac{3}{2}}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(5/2),x)
[Out] 2*x**(3/2)/(3*a**(5/2)*sqrt(1 + b*x/a) + 3*a**(3/2)*b*x*sqrt(1 + b*x/a))
```

## 3.587 $\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=43

$$\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x])/(3*a*(a + b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx &= \frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx}{3a}\\ &= \frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 0.67

$$\frac{2\sqrt{x}\,(3a+2bx)}{3a^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
[Out] (2*Sqrt[x]*(3*a + 2*b*x))/(3*a^2*(a + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 29, normalized size = 0.67

$$\frac{2\sqrt{x}\,(3a+2bx)}{3a^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
[Out] (2*Sqrt[x]*(3*a + 2*b*x))/(3*a^2*(a + b*x)^(3/2))
```

**fricas [A]** time = 1.49, size = 43, normalized size = 1.00

$$\frac{2\,(2\,bx+3\,a)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^2b^2x^2+2\,a^3bx+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*b*x + 3*a)*sqrt(b*x + a)*sqrt(x)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

**giac [B]** time = 1.50, size = 81, normalized size = 1.88

$$\frac{8\left(3\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)b^{\frac{5}{2}}}{3\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
[Out] 8/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*b^(5/2)/(
((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*abs(b))
```

**maple** [A] time = 0.00, size = 24, normalized size = 0.56

$$\frac{2\,(2bx+3a)\,\sqrt{x}}{3\,(bx+a)^{\frac{3}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 2/3*x^(1/2)*(2*b*x+3*a)/(b*x+a)^(3/2)/a^2
```

**maxima** [A] time = 1.34, size = 27, normalized size = 0.63

$$-\frac{2\left(b-\frac{3\,(bx+a)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b - 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*a^2)
```

**mupad** [B] time = 0.40, size = 54, normalized size = 1.26

$$\frac{6\,a\,\sqrt{x}\,\sqrt{a+b\,x}+4\,b\,x^{3/2}\,\sqrt{a+b\,x}}{3\,a^4+6\,a^3\,b\,x+3\,a^2\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^(5/2)),x)
```

```
[Out] (6*a*x^(1/2)*(a + b*x)^(1/2) + 4*b*x^(3/2)*(a + b*x)^(1/2))/(3*a^4 + 3*a^2*
b^2*x^2 + 6*a^3*b*x)
```

**sympy** [B] time = 1.90, size = 92, normalized size = 2.14

$$\frac{6a}{3a^3\sqrt{b}\,\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}+\frac{4bx}{3a^3\sqrt{b}\,\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] 6*a/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x) + 1)
) + 4*b*x/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x
) + 1))
```

## 3.588 $\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=64

$$-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}}+\frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}}+\frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*Sqrt[x]*(a + b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a + b*x]) - (16*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx = \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}} + \frac{4\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx}{3a}$$
$$= \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}} + \frac{8\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^2}$$
$$= \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}} - \frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 0.62

$$-\frac{2\left(3a^2+12abx+8b^2x^2\right)}{3a^3\sqrt{x}\,(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^(5/2)),x]
[Out] (-2*(3*a^2 + 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a + b*x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.10, size = 40, normalized size = 0.62

$$-\frac{2\left(3a^2+12abx+8b^2x^2\right)}{3a^3\sqrt{x}\,(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x)^(5/2)),x]
[Out] (-2*(3*a^2 + 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a + b*x)^(3/2))
```

**fricas** [A] time = 1.37, size = 58, normalized size = 0.91

$$-\frac{2\left(8\,b^2x^2+12\,abx+3\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] -2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b^2*x^3 + 2*
a^4*b*x^2 + a^5*x)
```

**giac [B]** time = 1.63, size = 159, normalized size = 2.48

$$-\frac{2\sqrt{bx+a}\,b^2}{\sqrt{(bx+a)b-ab}\,a^3|b|}-\frac{4\left(3\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4b^{\frac{5}{2}}+12\,a\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{7}{2}}+5\,a^2b^{\frac{9}{2}}\right)}{3\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a^3*abs(b)) - 4/3*(3*(sqrt(b*
x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b^(5/2) + 12*a*(sqrt(b*x + a)*s
qrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(7/2) + 5*a^2*b^(9/2))/(((sqrt(b*x +
a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*a^2*abs(b))
```

**maple [A]** time = 0.01, size = 35, normalized size = 0.55

$$-\frac{2\left(8b^2x^2+12abx+3a^2\right)}{3\left(bx+a\right)^{\frac{3}{2}}a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] -2/3*(8*b^2*x^2+12*a*b*x+3*a^2)/(b*x+a)^(3/2)/x^(1/2)/a^3
```

**maxima [A]** time = 1.34, size = 46, normalized size = 0.72

$$\frac{2\left(b^2-\frac{6\,(bx+a)b}{x}\right)x^{\frac{3}{2}}}{3\left(bx+a\right)^{\frac{3}{2}}a^3}-\frac{2\sqrt{bx+a}}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2 - 6*(b*x + a)*b/x)*x^(3/2)/((b*x + a)^(3/2)*a^3) - 2*sqrt(b*x + a)
/(a^3*sqrt(x))
```

**mupad [B]** time = 0.42, size = 71, normalized size = 1.11

$$-\frac{6\,a^2\,\sqrt{a+b\,x}+16\,b^2\,x^2\,\sqrt{a+b\,x}+24\,a\,b\,x\,\sqrt{a+b\,x}}{\sqrt{x}\,\left(x\,\left(6\,a^4\,b+3\,x\,a^3\,b^2\right)+3\,a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(5/2)),x)
```

```
[Out] -(6*a^2*(a + b*x)^(1/2) + 16*b^2*x^2*(a + b*x)^(1/2) + 24*a*b*x*(a + b*x)^(
1/2))/(x^(1/2)*(x*(6*a^4*b + 3*a^3*b^2*x) + 3*a^5))
```

**sympy [B]** time = 3.97, size = 153, normalized size = 2.39

$$-\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**2*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x + 3*a**3*b*
*6*x**2) - 24*a*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x
+ 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*
a**4*b**5*x + 3*a**3*b**6*x**2)
```

## 3.589 $\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=84

$$\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}+\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^(3/2)*(a + b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a + b*x]) - (16*Sqrt[
a + b*x])/(3*a^3*x^(3/2)) + (32*b*Sqrt[a + b*x])/(3*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx = \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{2\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx}{a}$$
$$= \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a+bx}} + \frac{8\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{a^2}$$
$$= \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a+bx}} - \frac{16\sqrt{a+bx}}{3a^3x^{3/2}} - \frac{(16b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^3}$$
$$= \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a+bx}} - \frac{16\sqrt{a+bx}}{3a^3x^{3/2}} + \frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}$$

**Mathematica** [A] time = 0.01, size = 49, normalized size = 0.58

$$-\frac{2\left(a^3-6a^2bx-24ab^2x^2-16b^3x^3\right)}{3a^4x^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^(5/2)),x]
[Out] (-2*(a^3 - 6*a^2*b*x - 24*a*b^2*x^2 - 16*b^3*x^3))/(3*a^4*x^(3/2)*(a + b*x)
^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.11, size = 51, normalized size = 0.61

$$\frac{2\left(-a^3+6a^2bx+24ab^2x^2+16b^3x^3\right)}{3a^4x^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x)^(5/2)),x]
[Out] (2*(-a^3 + 6*a^2*b*x + 24*a*b^2*x^2 + 16*b^3*x^3))/(3*a^4*x^(3/2)*(a + b*x)
^(3/2))
```

**fricas** [A] time = 1.04, size = 71, normalized size = 0.85

$$\frac{2\left(16\,b^3x^3+24\,ab^2x^2+6\,a^2bx-a^3\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

**giac [B]** time = 2.24, size = 175, normalized size = 2.08

$$\frac{2\sqrt{bx+a}\left(\frac{8(bx+a)b^2|b|}{a^4}-\frac{9b^2|b|}{a^3}\right)}{3((bx+a)b-ab)^{\frac{3}{2}}}+\frac{8\left(3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4b^{\frac{7}{2}}+9a\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{9}{2}}+4a^2b^{\frac{11}{2}}\right)}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3a^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x + a)*(8*(b*x + a)*b^2*abs(b)/a^4 - 9*b^2*abs(b)/a^3)/((b*x + a
)*b - a*b)^(3/2) + 8/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))
^4*b^(7/2) + 9*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(9/2
) + 4*a^2*b^(11/2))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 +
 a*b)^3*a^3*abs(b))
```

**maple [A]** time = 0.00, size = 44, normalized size = 0.52

$$-\frac{2\left(-16b^3x^3-24a\,b^2x^2-6a^2bx+a^3\right)}{3\,(bx+a)^{\frac{3}{2}}\,a^4x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] -2/3*(-16*b^3*x^3-24*a*b^2*x^2-6*a^2*b*x+a^3)/(b*x+a)^(3/2)/x^(3/2)/a^4
```

**maxima [A]** time = 1.27, size = 64, normalized size = 0.76

$$\frac{2\left(\frac{9\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^4}-\frac{2\left(b^3-\frac{9(bx+a)b^2}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(9*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^4 - 2/3*(b^3 -
9*(b*x + a)*b^2/x)*x^(3/2)/((b*x + a)^(3/2)*a^4)
```

**mupad [B]** time = 0.47, size = 88, normalized size = 1.05

$$\frac{32\,b^3\,x^3\,\sqrt{a+b\,x}\,-2\,a^3\,\sqrt{a+b\,x}\,+12\,a^2\,b\,x\,\sqrt{a+b\,x}\,+48\,a\,b^2\,x^2\,\sqrt{a+b\,x}}{x^{3/2}\,\left(x\,\left(6\,a^5\,b+3\,x\,a^4\,b^2\right)+3\,a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(5/2)),x)
```

```
[Out] (32*b^3*x^3*(a + b*x)^(1/2) - 2*a^3*(a + b*x)^(1/2) + 12*a^2*b*x*(a + b*x)^
(1/2) + 48*a*b^2*x^2*(a + b*x)^(1/2))/(x^(3/2)*(x*(6*a^5*b + 3*a^4*b^2*x) +
 3*a^6))
```

**sympy [B]** time = 7.06, size = 337, normalized size = 4.01

$$-\frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{80ab^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{32b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] -2*a**4*b**(19/2)*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*
a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(a/(b*x) + 1
)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**
4) + 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10
*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2)*x**3*sqrt(a
/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4
*b**12*x**4) + 32*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*
b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4)
```

## 3.590 $\int \frac{x^{5/2}}{\sqrt{a-bx}} dx$

**Optimal**. Leaf size=105

$$\frac{5a^3 \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}} - \frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a-bx}}{12b^2} - \frac{x^{5/2}\sqrt{a-bx}}{3b}$$

**Rubi [A]** time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3} + \frac{5a^3 \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}} - \frac{5ax^{3/2}\sqrt{a-bx}}{12b^2} - \frac{x^{5/2}\sqrt{a-bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a - b*x],x]
```

```
[Out] (-5*a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a - b*x])/(12*b^
2) - (x^(5/2)*Sqrt[a - b*x])/(3*b) + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a
 - b*x]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx &= -\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{(5a)\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{6b}\\
&= -\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^2\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{8b^2}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^3\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{16b^3}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{8b^3}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^3}\\
&= -\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 88, normalized size = 0.84

$$\frac{\sqrt{a-bx}\left(\frac{15a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}-\sqrt{b}\sqrt{x}\left(15a^2+10abx+8b^2x^2\right)\right)}{24b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-(Sqrt[b]*Sqrt[x]*(15*a^2 + 10*a*b*x + 8*b^2*x^2)) + (15*a^
(5/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(24*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 91, normalized size = 0.87

$$\frac{5a^3\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{8b^4}+\frac{\sqrt{a-bx}\,\left(-15a^2\sqrt{x}-10abx^{3/2}-8b^2x^{5/2}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/Sqrt[a - b*x],x]
[Out] (Sqrt[a - b*x]*(-15*a^2*Sqrt[x] - 10*a*b*x^(3/2) - 8*b^2*x^(5/2)))/(24*b^3)
 + (5*a^3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*b^4)
```

**fricas [A]** time = 1.35, size = 141, normalized size = 1.34

$$\left[-\frac{15\,a^3\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,\left(8\,b^3x^2+10\,ab^2x+15\,a^2b\right)\sqrt{-bx+a}\,\sqrt{x}}{48\,b^4},-\frac{15\,a^3\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(8\,b^3x^2+10\,ab^2x+15\,a^2b\right)\sqrt{-bx+a}\,\sqrt{x}}{24\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] [-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(8*b^3*x^2 + 10*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/24
*(15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (8*b^3*x^2 + 10
*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 0.01, size = 108, normalized size = 1.03

$$-\frac{\sqrt{-bx+a}\;x^{\frac{5}{2}}}{3b}-\frac{5\sqrt{-bx+a}\;a\,x^{\frac{3}{2}}}{12b^2}+\frac{5\sqrt{\left(-bx+a\right)x}\;a^3\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{16\sqrt{-bx+a}\;b^{\frac{7}{2}}\sqrt{x}}-\frac{5\sqrt{-bx+a}\;a^2\sqrt{x}}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -1/3*x^(5/2)*(-b*x+a)^(1/2)/b-5/12*a*x^(3/2)*(-b*x+a)^(1/2)/b^2-5/8*a^2*x^(
1/2)*(-b*x+a)^(1/2)/b^3+5/16*a^3/b^(7/2)*((-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a
)^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.96, size = 135, normalized size = 1.29

$$-\frac{5\,a^{3}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^{\frac{7}{2}}}-\frac{\frac{33\,\sqrt{-bx+a}\,a^{3}b^{2}}{\sqrt{x}}+\frac{40\,(-bx+a)^{\frac{3}{2}}a^{3}b}{x^{\frac{3}{2}}}+\frac{15\,(-bx+a)^{\frac{5}{2}}a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{6}-\frac{3\,(bx-a)b^{5}}{x}+\frac{3\,(bx-a)^{2}b^{4}}{x^{2}}-\frac{(bx-a)^{3}b^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) - 1/24*(33*sqrt(-
b*x + a)*a^3*b^2/sqrt(x) + 40*(-b*x + a)^(3/2)*a^3*b/x^(3/2) + 15*(-b*x + a
)^(5/2)*a^3/x^(5/2))/(b^6 - 3*(b*x - a)*b^5/x + 3*(b*x - a)^2*b^4/x^2 - (b*
x - a)^3*b^3/x^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{a-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)/(a - b*x)^(1/2), x)
```

**sympy [A]** time = 8.43, size = 270, normalized size = 2.57

$$\begin{cases} \frac{5ia^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{i\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{3}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}-\frac{ix^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{1-\frac{bx}{a}}}+\frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{1-\frac{bx}{a}}}+\frac{5a^{3}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(5/2)*sqrt(x)/(8*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(3/2)*x*
*(3/2)/(24*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(5/2)/(12*b*sqrt(-1 + b*x/
```

```
a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) - I*x**(7/2)/(3*
sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(5/2)*sqrt(x)/(8*b**3*sq
rt(1 - b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2*sqrt(1 - b*x/a)) + sqrt(a)*x*
*(5/2)/(12*b*sqrt(1 - b*x/a)) + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**
(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

## 3.591 $\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx$

**Optimal**. Leaf size=80

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$

**Rubi [A]** time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a - b*x],x]
```

```
[Out] (-3*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^2) - (x^(3/2)*Sqrt[a - b*x])/(2*b) + (3*a
^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx &= -\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{(3a)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{4b}\\ &= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{\left(3a^2\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{8b^2}\\ &= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{4b^2}\\ &= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^2}\\ &= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 86, normalized size = 1.08

$$\frac{3a^{5/2}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\sqrt{b}\sqrt{x}\left(-3a^2+abx+2b^2x^2\right)}{4b^{5/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(-3*a^2 + a*b*x + 2*b^2*x^2) + 3*a^(5/2)*Sqrt[1 - (b*x)/a]
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[a - b*x])
```

**IntegrateAlgebraic** [A] time = 0.10, size = 78, normalized size = 0.98

$$\frac{3a^2\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^3}+\frac{\sqrt{a-bx}\left(-3a\sqrt{x}-2bx^{3/2}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-3*a*Sqrt[x] - 2*b*x^(3/2)))/(4*b^2) + (3*a^2*Sqrt[-b]*Log[
-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(4*b^3)
```

**fricas [A]** time = 0.77, size = 119, normalized size = 1.49

$$\left[-\frac{3\,a^{2}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,\left(2\,b^{2}x+3\,ab\right)\sqrt{-bx+a}\,\sqrt{x}}{8\,b^{3}},\,-\frac{3\,a^{2}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(2\,b^{2}x+3\,ab\right)\sqrt{-bx+a}\,\sqrt{x}}{4\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*(2*b^2*x + 3*a*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(b)*arcta
n(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x + 3*a*b)*sqrt(-b*x + a)*sqrt
(x))/b^3]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [A]** time = 0.01, size = 89, normalized size = 1.11

$$-\frac{\sqrt{-bx+a}\;x^{\frac{3}{2}}}{2b}+\frac{3\sqrt{\left(-bx+a\right)x}\;a^{2}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{8\sqrt{-bx+a}\;b^{\frac{5}{2}}\sqrt{x}}-\frac{3\sqrt{-bx+a}\;a\sqrt{x}}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -1/2*x^(3/2)*(-b*x+a)^(1/2)/b-3/4*a*x^(1/2)*(-b*x+a)^(1/2)/b^2+3/8*a^2/b^(5
/2)*((-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*
x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.96, size = 98, normalized size = 1.22

$$-\frac{3\,a^{2}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{5}{2}}}-\frac{\frac{5\,\sqrt{-bx+a}\,a^{2}b}{\sqrt{x}}+\frac{3\,(-bx+a)^{\frac{3}{2}}\,a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{4}-\frac{2\,(bx-a)b^{3}}{x}+\frac{(bx-a)^{2}b^{2}}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) - 1/4*(5*sqrt(-b*
x + a)*a^2*b/sqrt(x) + 3*(-b*x + a)^(3/2)*a^2/x^(3/2))/(b^4 - 2*(b*x - a)*b
^3/x + (b*x - a)^2*b^2/x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{a-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a - b*x)^(1/2), x)
```

**sympy [A]** time = 4.30, size = 214, normalized size = 2.68

$$\begin{cases} \frac{3ia^{\frac{3}{2}}\sqrt{x}}{4b^{2}\sqrt{-1+\frac{bx}{a}}} - \frac{i\sqrt{a}\,x^{\frac{3}{2}}}{4b\sqrt{-1+\frac{bx}{a}}} - \frac{3ia^{2}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} - \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right| > 1 \\ -\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^{2}\sqrt{1-\frac{bx}{a}}} + \frac{\sqrt{a}\,x^{\frac{3}{2}}}{4b\sqrt{1-\frac{bx}{a}}} + \frac{3a^{2}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((3*I*a**(3/2)*sqrt(x)/(4*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(3
/2)/(4*b*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**
(5/2)) - I*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-3*a**(
3/2)*sqrt(x)/(4*b**2*sqrt(1 - b*x/a)) + sqrt(a)*x**(3/2)/(4*b*sqrt(1 - b*x/
a)) + 3*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2)/(2*sqrt(
a)*sqrt(1 - b*x/a)), True))
```

## 3.592 $\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx$

**Optimal**. Leaf size=50

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}$$

**Rubi [A]** time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/
b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx &= -\frac{\sqrt{x}\sqrt{a-bx}}{b}+\frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{2b}\\ &= -\frac{\sqrt{x}\sqrt{a-bx}}{b}+\frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= -\frac{\sqrt{x}\sqrt{a-bx}}{b}+\frac{a\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b}\\ &= -\frac{\sqrt{x}\sqrt{a-bx}}{b}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 71, normalized size = 1.42

$$\frac{a^{3/2}\sqrt{1-\frac{bx}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\sqrt{b}\sqrt{x}\,(bx-a)}{b^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a - b*x],x]
[Out] (Sqrt[b]*Sqrt[x]*(-a + b*x) + a^(3/2)*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/(b^(3/2)*Sqrt[a - b*x])
```

**IntegrateAlgebraic** [A] time = 0.07, size = 59, normalized size = 1.18

$$\frac{a\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b^2}-\frac{\sqrt{x}\sqrt{a-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[a - b*x],x]
[Out] -((Sqrt[x]*Sqrt[a - b*x])/b) + (a*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a
 - b*x]])/b^2
```

**fricas** [A] time = 1.21, size = 93, normalized size = 1.86

$$\left[-\frac{a\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}\,+a\right)+2\,\sqrt{-bx+a}\,b\sqrt{x}}{2\,b^2},-\frac{a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\sqrt{-bx+a}\,b\sqrt{x}}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) + 2*s
qrt(-b*x + a)*b*sqrt(x))/b^2, -(a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sq
rt(x))) + sqrt(-b*x + a)*b*sqrt(x))/b^2]
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [A] time = 0.01, size = 70, normalized size = 1.40

$$\frac{\sqrt{(-bx+a)\,x}\;a\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{2\sqrt{-bx+a}\;b^{\frac{3}{2}}\sqrt{x}}-\frac{\sqrt{-bx+a}\;\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*((-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a)
^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

**maxima** [A] time = 3.00, size = 56, normalized size = 1.12

$$-\frac{a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}-\frac{\sqrt{-bx+a}\,a}{\left(b^2-\frac{(bx-a)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) - sqrt(-b*x + a)*a/((b^
2 - (b*x - a)*b/x)*sqrt(x))
```

**mupad [B]** time = 0.52, size = 47, normalized size = 0.94

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-b\,x}-\sqrt{a}}\right)}{b^{3/2}}-\frac{\sqrt{x}\,\sqrt{a-b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(1/2),x)
```

```
[Out] (2*a*atan((b^(1/2)*x^(1/2))/((a - b*x)^(1/2) - a^(1/2))))/b^(3/2) - (x^(1/2
)*(a - b*x)^(1/2))/b
```

**sympy [A]** time = 2.28, size = 121, normalized size = 2.42

$$\begin{cases} -\frac{i\sqrt{a}\,\sqrt{x}\,\sqrt{-1+\frac{bx}{a}}}{b}-\frac{ia\,\mathrm{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{\sqrt{a}\,\sqrt{x}}{b\sqrt{1-\frac{bx}{a}}}+\frac{a\,\mathrm{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(a)*sqrt(x)*sqrt(-1 + b*x/a)/b - I*a*acosh(sqrt(b)*sqrt(x
)/sqrt(a))/b**(3/2), Abs(b*x/a) > 1), (-sqrt(a)*sqrt(x)/(b*sqrt(1 - b*x/a))
 + a*asin(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) + x**(3/2)/(sqrt(a)*sqrt(1 - b*
x/a)), True))
```

## 3.593 $\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx$

**Optimal**. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {63, 217, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a - b*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

**Mathematica** [A] time = 0.01, size = 52, normalized size = 1.79

$$\frac{2\sqrt{a}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a - b*x]),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sq
rt[a - b*x])
```

**IntegrateAlgebraic** [A] time = 0.06, size = 38, normalized size = 1.31

$$\frac{2\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[a - b*x]),x]
```

```
[Out] (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/b
```

**fricas** [A] time = 0.92, size = 57, normalized size = 1.97

$$\left[-\frac{\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)}{b}, -\frac{2\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)/b, -2*arctan
(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

**maple [B]** time = 0.01, size = 51, normalized size = 1.76

$$\frac{\sqrt{(-bx+a)\,x}\ \arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{\sqrt{-bx+a}\ \sqrt{b}\ \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+a)^(1/2),x)
[Out] ((-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a)^(1/2)/b^(1/2)*arctan((x-1/2*a/b)/(-b*x^
2+a*x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.88, size = 21, normalized size = 0.72

$$-\frac{2\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

**mupad [B]** time = 0.03, size = 27, normalized size = 0.93

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{a-b\,x}-\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(1/2)),x)
```

[Out] -(4*atan(((a - b*x)^(1/2) - a^(1/2))/(b^(1/2)*x^(1/2))))/b^(1/2)

**sympy** [A] time = 1.15, size = 54, normalized size = 1.86

$$\begin{cases} -\frac{2i \operatorname{acosh}{\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}} \right)}}{\sqrt{b}} & \text{for}\: \left|{\frac{bx}{a}}\right| > 1 \\ \frac{2 \operatorname{asin}{\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}} \right)}}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(1/2)/(-b*x+a)**(1/2),x)

[Out] Piecewise((-2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), Abs(b*x/a) > 1), (2*asin(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), True))

## 3.594 $\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx$

**Optimal**. Leaf size=20

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

**Rubi [A]** time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

**Mathematica [A]** time = 0.00, size = 20, normalized size = 1.00

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

**IntegrateAlgebraic** [A] time = 0.02, size = 20, normalized size = 1.00

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

**fricas** [A] time = 0.69, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-b*x + a)/(a*sqrt(x))
```

**giac** [B] time = 1.28, size = 35, normalized size = 1.75

$$-\frac{2\sqrt{-bx+a}\,b^2}{\sqrt{(bx-a)b+ab}\,a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a*abs(b))
```

**maple** [A] time = 0.00, size = 17, normalized size = 0.85

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(1/2),x)
[Out] -2*(-b*x+a)^(1/2)/a/x^(1/2)
```

**maxima** [A] time = 1.34, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-b*x + a)/(a*sqrt(x))
```

**mupad [B]** time = 0.40, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{a-b\,x}}{a\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(1/2)),x)
```

```
[Out] -(2*(a - b*x)^(1/2))/(a*x^(1/2))
```

**sympy [A]** time = 0.97, size = 46, normalized size = 2.30

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{2i\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/a, Abs(a/(b*x)) > 1), (-2*I*sqrt(b)
*sqrt(-a/(b*x) + 1)/a, True))
```

## 3.595 $\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx$

**Optimal**. Leaf size=46

$$-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(3*a*x^(3/2)) - (4*b*Sqrt[a - b*x])/(3*a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}}{3ax^{3/2}} + \frac{(2b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a}$$
$$= -\frac{2\sqrt{a-bx}}{3ax^{3/2}} - \frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 0.61

$$-\frac{2\sqrt{a-bx}\,(a+2bx)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[a - b*x]*(a + 2*b*x))/(3*a^2*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 28, normalized size = 0.61

$$-\frac{2\sqrt{a-bx}\,(a+2bx)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[a - b*x]*(a + 2*b*x))/(3*a^2*x^(3/2))
```

**fricas [A]** time = 1.77, size = 22, normalized size = 0.48

$$-\frac{2\,(2\,bx+a)\sqrt{-bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2/3*(2*b*x + a)*sqrt(-b*x + a)/(a^2*x^(3/2))
```

**giac [A]** time = 1.45, size = 54, normalized size = 1.17

$$-\frac{2\left(\frac{2\,(bx-a)b^3}{a^2}+\frac{3\,b^3}{a}\right)\sqrt{-bx+a}\,b}{3\,((bx-a)b+ab)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(2*(b*x - a)*b^3/a^2 + 3*b^3/a)*sqrt(-b*x + a)*b/(((b*x - a)*b + a*b)^(3/2)*abs(b))
```

**maple** [A] time = 0.00, size = 23, normalized size = 0.50

$$-\frac{2\sqrt{-bx+a}\,(2bx+a)}{3a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -2/3*(-b*x+a)^(1/2)*(2*b*x+a)/x^(3/2)/a^2
```

**maxima** [A] time = 1.30, size = 32, normalized size = 0.70

$$-\frac{2\left(\frac{3\sqrt{-bx+a}\,b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^2
```

**mupad** [B] time = 0.35, size = 26, normalized size = 0.57

$$-\frac{\left(\frac{2}{3\,a}+\frac{4\,b\,x}{3\,a^2}\right)\sqrt{a-b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^(1/2)),x)
```

```
[Out] -((2/(3*a) + (4*b*x)/(3*a^2))*(a - b*x)^(1/2))/x^(3/2)
```

**sympy** [A] time = 2.06, size = 177, normalized size = 3.85

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3ax}-\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3a^2} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{2ia^2b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2}+\frac{2iab^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2}-\frac{4ib^{\frac{7}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*a*x) - 4*b**(3/2)*sqrt(a/(b*x) -
 1)/(3*a**2), Abs(a/(b*x)) > 1), (2*I*a**2*b**(3/2)*sqrt(-a/(b*x) + 1)/(-3*
a**3*b*x + 3*a**2*b**2*x**2) + 2*I*a*b**(5/2)*x*sqrt(-a/(b*x) + 1)/(-3*a**3
*b*x + 3*a**2*b**2*x**2) - 4*I*b**(7/2)*x**2*sqrt(-a/(b*x) + 1)/(-3*a**3*b*
x + 3*a**2*b**2*x**2), True))
```

## 3.596 $\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx$

**Optimal**. Leaf size=100

$$-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}+\frac{2x^{5/2}}{b\sqrt{a-bx}}$$

**Rubi [A]** time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{2x^{5/2}}{b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2))/(b*Sqrt[a - b*x]) + (15*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^3) + (5*x
^(3/2)*Sqrt[a - b*x])/(2*b^2) - (15*a^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b
*x]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx &= \frac{2x^{5/2}}{b\sqrt{a-bx}}-\frac{5\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{b}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{(15a)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{4b^2}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{\left(15a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{8b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 51, normalized size = 0.51

$$
\frac{2x^{7/2}\sqrt{1-\frac{bx}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a\sqrt{a-bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 - (b*x)/a]*Hypergeometric2F1[3/2, 7/2, 9/2, (b*x)/a])/(7*
a*Sqrt[a - b*x])
```

**IntegrateAlgebraic [A]** time = 0.17, size = 100, normalized size = 1.00

$$-\frac{15a^2\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{4b^4}-\frac{\sqrt{a-bx}\,\left(15a^2\sqrt{x}-5abx^{3/2}-2b^2x^{5/2}\right)}{4b^3(bx-a)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] -1/4*(Sqrt[a - b*x]*(15*a^2*Sqrt[x] - 5*a*b*x^(3/2) - 2*b^2*x^(5/2)))/(b^3*
(-a + b*x)) - (15*a^2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(4
*b^4)
```

**fricas [A]** time = 1.37, size = 181, normalized size = 1.81

$$\left[-\frac{15\left(a^2bx-a^3\right)\sqrt{-b}\,\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\left(2\,b^3x^2+5\,ab^2x-15\,a^2b\right)\sqrt{-bx+a}\,\sqrt{x}}{8\left(b^5x-ab^4\right)},\ \frac{15\left(a^2bx-a^3\right)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(2\,b^3x^2+5\,ab^2x-15\,a^2b\right)\sqrt{-bx+a}\,\sqrt{x}}{4\left(b^5x-ab^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(a^2*b*x - a^3)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*s
qrt(x) + a) - 2*(2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/
(b^5*x - a*b^4), 1/4*(15*(a^2*b*x - a^3)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqr
t(b)*sqrt(x))) + (2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))
/(b^5*x - a*b^4)]
```

**giac [B]** time = 98.07, size = 154, normalized size = 1.54

$$\frac{\left(2\,\sqrt{(bx-a)b+ab}\,\sqrt{-bx+a}\left(\frac{2\,(bx-a)}{b^3}+\frac{9\,a}{b^3}\right)+\frac{32\,a^3}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)\sqrt{-b}\,b}-\frac{15\,a^2\log\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}\,b^2}\right)|b|}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)/b^3 + 9*a/b^3) +
 32*a^3/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)*sqrt
```

```
(-b)*b) - 15*a^2*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)
/(sqrt(-b)*b^2))*abs(b)/b^2
```

**maple [A]** time = 0.04, size = 127, normalized size = 1.27

$$\frac{\left(-\frac{15a^2\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{8b^{\frac{7}{2}}}-\frac{2\sqrt{-\left(x-\frac{a}{b}\right)a-\left(x-\frac{a}{b}\right)^2 b}\;a^2}{\left(x-\frac{a}{b}\right)b^4}\right)\sqrt{(-bx+a)\,x}}{\sqrt{-bx+a}\;\sqrt{x}}+\frac{(2bx+7a)\sqrt{-bx+a}\;\sqrt{x}}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(3/2),x)
```

```
[Out] 1/4*(2*b*x+7*a)/b^3*(-b*x+a)^(1/2)*x^(1/2)+(-15/8*a^2/b^(7/2)*arctan((x-1/2
*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))-2*a^2/b^4/(x-a/b)*(-b*(x-a/b)^2-(x-a/b)*a
)^(1/2))*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)
```

**maxima [A]** time = 2.92, size = 118, normalized size = 1.18

$$\frac{8\,a^2b^2-\frac{25\,(bx-a)a^2b}{x}+\frac{15\,(bx-a)^2a^2}{x^2}}{4\left(\frac{\sqrt{-bx+a}\,b^5}{\sqrt{x}}+\frac{2\,(-bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(-bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}+\frac{15\,a^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^2*b^2 - 25*(b*x - a)*a^2*b/x + 15*(b*x - a)^2*a^2/x^2)/(sqrt(-b*x
+ a)*b^5/sqrt(x) + 2*(-b*x + a)^(3/2)*b^4/x^(3/2) + (-b*x + a)^(5/2)*b^3/x^
(5/2)) + 15/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)/(a - b*x)^(3/2), x)
```

**sympy [A]** time = 8.03, size = 224, normalized size = 2.24

$$\begin{cases} -\frac{15ia^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{-1+\frac{bx}{a}}} + \frac{5i\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{-1+\frac{bx}{a}}} + \frac{15ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}} + \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1-\frac{bx}{a}}} - \frac{5\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{1-\frac{bx}{a}}} - \frac{15a^2\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}} - \frac{x^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-15*I*a**(3/2)*sqrt(x)/(4*b**3*sqrt(-1 + b*x/a)) + 5*I*sqrt(a)*x
**(3/2)/(4*b**2*sqrt(-1 + b*x/a)) + 15*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a)
)/(4*b**(7/2)) + I*x**(5/2)/(2*sqrt(a)*b*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1)
, (15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 - b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b**
2*sqrt(1 - b*x/a)) - 15*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)) - x
**(5/2)/(2*sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

## 3.597 $\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx$

**Optimal**. Leaf size=71

$$-\frac{3a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\sqrt{a-bx}}{b^2}+\frac{2x^{3/2}}{b\sqrt{a-bx}}$$

**Rubi [A]** time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$\frac{3\sqrt{x}\sqrt{a-bx}}{b^2}-\frac{3a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(3/2))/(b*Sqrt[a - b*x]) + (3*Sqrt[x]*Sqrt[a - b*x])/b^2 - (3*a*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx &= \frac{2x^{3/2}}{b\sqrt{a-bx}} - \frac{3\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{b}\\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2} - \frac{(3a)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{2b^2}\\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}\\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^2}\\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2} - \frac{3a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 51, normalized size = 0.72

$$\frac{2x^{5/2}\sqrt{1-\frac{bx}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{bx}{a}\right)}{5a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 - (b*x)/a]*Hypergeometric2F1[3/2, 5/2, 7/2, (b*x)/a])/(5*
a*Sqrt[a - b*x])
```

**IntegrateAlgebraic [A]** time = 0.14, size = 81, normalized size = 1.14

$$-\frac{3a\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b^3}-\frac{\sqrt{a-bx}\left(3a\sqrt{x}-bx^{3/2}\right)}{b^2(bx-a)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a - b*x)^(3/2),x]
```

```
[Out] -((Sqrt[a - b*x]*(3*a*Sqrt[x] - b*x^(3/2)))/(b^2*(-a + b*x))) - (3*a*Sqrt[-
b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/b^3
```

**fricas [A]** time = 1.30, size = 152, normalized size = 2.14

$$\left[-\frac{3\left(abx-a^2\right)\sqrt{-b}\log\left(-2bx-2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(b^2x-3ab\right)\sqrt{-bx+a}\sqrt{x}}{2\left(b^4x-ab^3\right)}, \frac{3\left(abx-a^2\right)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+\left(b^2x-3ab\right)\sqrt{-bx+a}\sqrt{x}}{b^4x-ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(a*b*x - a^2)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt
(x) + a) - 2*(b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3), (3*(a
*b*x - a^2)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (b^2*x - 3*a
*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3)]
```

**giac [B]** time = 111.13, size = 130, normalized size = 1.83

$$-\frac{\left(\frac{8a^2\sqrt{-b}}{\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab}+\frac{3a\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}}-\frac{2\sqrt{(bx-a)b+ab}\sqrt{-bx+a}}{b}\right)|b|}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(8*a^2*sqrt(-b)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2
 - a*b) + 3*a*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sq
rt(-b) - 2*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b)*abs(b)/b^3
```

**maple [B]** time = 0.03, size = 114, normalized size = 1.61

$$\frac{\left(-\frac{3a\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{2b^{\frac{5}{2}}}-\frac{2\sqrt{-\left(x-\frac{a}{b}\right)a-\left(x-\frac{a}{b}\right)^2 b}\;a}{\left(x-\frac{a}{b}\right)b^3}\right)\sqrt{(-bx+a)\,x}}{\sqrt{-bx+a}\;\sqrt{x}}+\frac{\sqrt{-bx+a}\;\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(3/2),x)
```

```
[Out] x^(1/2)*(-b*x+a)^(1/2)/b^2+(-3/2*a/b^(5/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^
(1/2)*b^(1/2))-2*a/b^3/(x-a/b)*(-(x-a/b)*a-(x-a/b)^2*b)^(1/2))*((-b*x+a)*x)
^(1/2)/(-b*x+a)^(1/2)/x^(1/2)
```

**maxima [A]** time = 2.95, size = 75, normalized size = 1.06

$$\frac{2\,ab-\frac{3\,(bx-a)a}{x}}{\frac{\sqrt{-bx+a}\,b^3}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}}+\frac{3\,a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] (2*a*b - 3*(b*x - a)*a/x)/(sqrt(-b*x + a)*b^3/sqrt(x) + (-b*x + a)^(3/2)*b^
2/x^(3/2)) + 3*a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(a-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)/(a - b*x)^(3/2), x)
```

**sympy [A]** time = 3.70, size = 155, normalized size = 2.18

$$\begin{cases} -\frac{3i\sqrt{a}\sqrt{x}}{b^2\sqrt{-1+\frac{bx}{a}}} + \frac{3ia\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}} + \frac{ix^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{3\sqrt{a}\sqrt{x}}{b^2\sqrt{1-\frac{bx}{a}}} - \frac{3a\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}} - \frac{x^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-3*I*sqrt(a)*sqrt(x)/(b**2*sqrt(-1 + b*x/a)) + 3*I*a*acosh(sqrt(
b)*sqrt(x)/sqrt(a))/b**(5/2) + I*x**(3/2)/(sqrt(a)*b*sqrt(-1 + b*x/a)), Abs
(b*x/a) > 1), (3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 - b*x/a)) - 3*a*asin(sqrt(b)*
sqrt(x)/sqrt(a))/b**(5/2) - x**(3/2)/(sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

## 3.598 $\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx$

**Optimal**. Leaf size=50

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 63, 217, 203}

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[a - b*x]) - (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

**Mathematica [A]** time = 0.08, size = 66, normalized size = 1.32

$$\frac{2\sqrt{b}\,\sqrt{x} - 2\sqrt{a}\,\sqrt{1-\frac{bx}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b]*Sqrt[x] - 2*Sqrt[a]*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/(b^(3/2)*Sqrt[a - b*x])
```

**IntegrateAlgebraic [A]** time = 0.11, size = 68, normalized size = 1.36

$$-\frac{2\sqrt{-b}\,\log\left(\sqrt{a-bx} - \sqrt{-b}\,\sqrt{x}\right)}{b^2} - \frac{2\sqrt{x}\,\sqrt{a-bx}}{b(bx-a)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a - b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a - b*x])/(b*(-a + b*x)) - (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt
[x]) + Sqrt[a - b*x]])/b^2
```

**fricas [A]** time = 1.21, size = 128, normalized size = 2.56

$$\left[-\frac{(bx-a)\sqrt{-b}\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,\sqrt{-bx+a}\,b\sqrt{x}}{b^3x-ab^2}, \frac{2\left((bx-a)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+a}\,b\sqrt{x}\right)}{b^3x-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - a)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*sqrt(-b*x + a)*b*sqrt(x))/(b^3*x - a*b^2), 2*((b*x - a)*sqrt(b)*arctan(s
qrt(-b*x + a)/(sqrt(b)*sqrt(x))) - sqrt(-b*x + a)*b*sqrt(x))/(b^3*x - a*b^2
)]
```

**giac [B]** time = 113.04, size = 98, normalized size = 1.96

$$-\frac{\left(\frac{4\,a\sqrt{-b}}{\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab}+\frac{\log\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(4*a*sqrt(-b)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b
) + log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sqrt(-b))*ab
s(b)/b^2
```

**maple [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(-bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(3/2),x)
```

```
[Out] int(x^(1/2)/(-b*x+a)^(3/2),x)
```

**maxima [A]** time = 3.01, size = 38, normalized size = 0.76

$$\frac{2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}+\frac{2\,\sqrt{x}}{\sqrt{-bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 2*sqrt(x)/(sqrt(-b*x +
 a)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(a-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(a - b*x)^(3/2), x)
```

**sympy [A]** time = 1.89, size = 102, normalized size = 2.04

$$\begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{2i\sqrt{x}}{\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right| > 1 \\ -\frac{2\,\mathrm{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} + \frac{2\sqrt{x}}{\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*I*sqrt(x)/(sqrt(
a)*b*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*asin(sqrt(b)*sqrt(x)/sqrt(a))/
b**(3/2) + 2*sqrt(x)/(sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

## 3.599 $\int \frac{1}{\sqrt{x}\,(a-bx)^{3/2}}\,dx$

**Optimal**. Leaf size=20

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

**Rubi [A]** time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a - b*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

**Mathematica [A]** time = 0.00, size = 20, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a - b*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])
```

**IntegrateAlgebraic [A]** time = 0.02, size = 20, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a - b*x)^(3/2)),x]
[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])
```

**fricas [A]** time = 1.03, size = 25, normalized size = 1.25

$$-\frac{2\sqrt{-bx+a}\sqrt{x}}{abx-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-b*x + a)*sqrt(x)/(a*b*x - a^2)
```

**giac [B]** time = 1.37, size = 53, normalized size = 2.65

$$-\frac{4\sqrt{-b}\,b}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
[Out] -4*sqrt(-b)*b/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b
)*abs(b))
```

**maple [A]** time = 0.00, size = 17, normalized size = 0.85

$$\frac{2\sqrt{x}}{\sqrt{-bx+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(3/2)/x^(1/2),x)
[Out] 2*x^(1/2)/a/(-b*x+a)^(1/2)
```

**maxima** [A] time = 1.30, size = 16, normalized size = 0.80

$$\frac{2\sqrt{x}}{\sqrt{-bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)/(sqrt(-b*x + a)*a)
```

**mupad** [B] time = 0.34, size = 24, normalized size = 1.20

$$\frac{2\sqrt{x}\,\sqrt{a-b\,x}}{a^2-a\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(3/2)),x)
```

```
[Out] (2*x^(1/2)*(a - b*x)^(1/2))/(a^2 - a*b*x)
```

**sympy** [A] time = 0.94, size = 44, normalized size = 2.20

$$\begin{cases} \frac{2}{a\sqrt{b}\sqrt{\frac{a}{bx}-1}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{2i}{a\sqrt{b}\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((2/(a*sqrt(b)*sqrt(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (-2*I/(a*sqr
t(b)*sqrt(-a/(b*x) + 1)), True))
```

## 3.600 $\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx$

**Optimal**. Leaf size=41

$$\frac{2}{a\sqrt{x}\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

**Rubi** [A] time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{2}{a\sqrt{x}\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[x]*Sqrt[a - b*x]) - (4*Sqrt[a - b*x])/(a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx = \frac{2}{a\sqrt{x}\sqrt{a-bx}} + \frac{2\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{a}$$
$$= \frac{2}{a\sqrt{x}\sqrt{a-bx}} - \frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

**Mathematica** [**A**] time = 0.01, size = 26, normalized size = 0.63

$$-\frac{2(a-2bx)}{a^2\sqrt{x}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a - b*x)^(3/2)),x]
[Out] (-2*(a - 2*b*x))/(a^2*Sqrt[x]*Sqrt[a - b*x])
```

**IntegrateAlgebraic** [**A**] time = 0.10, size = 37, normalized size = 0.90

$$-\frac{2\sqrt{a-bx}\,(2bx-a)}{a^2\sqrt{x}\,(bx-a)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a - b*x)^(3/2)),x]
[Out] (-2*Sqrt[a - b*x]*(-a + 2*b*x))/(a^2*Sqrt[x]*(-a + b*x))
```

**fricas** [**A**] time = 0.96, size = 38, normalized size = 0.93

$$-\frac{2\,(2\,bx-a)\sqrt{-bx+a}\,\sqrt{x}}{a^2bx^2-a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
[Out] -2*(2*b*x - a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b*x^2 - a^3*x)
```

**giac** [**B**] time = 1.44, size = 94, normalized size = 2.29

$$-\frac{4\sqrt{-b}\,b^2}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)a|b|}-\frac{2\sqrt{-bx+a}\,b^2}{\sqrt{(bx-a)b+ab}\,a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-b)*b^2/((((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a
*b)*a*abs(b)) - 2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a^2*abs(b))
```

**maple** [A] time = 0.00, size = 23, normalized size = 0.56

$$-\frac{2\left(-2bx+a\right)}{\sqrt{-bx+a}\;a^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(3/2),x)
```

```
[Out] -2*(-2*b*x+a)/(-b*x+a)^(1/2)/x^(1/2)/a^2
```

**maxima** [A] time = 1.30, size = 34, normalized size = 0.83

$$\frac{2\,b\sqrt{x}}{\sqrt{-bx+a}\,a^{2}}-\frac{2\,\sqrt{-bx+a}}{a^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(x)/(sqrt(-b*x + a)*a^2) - 2*sqrt(-b*x + a)/(a^2*sqrt(x))
```

**mupad** [B] time = 0.40, size = 42, normalized size = 1.02

$$-\frac{2\,a\,\sqrt{a-b\,x}\,-4\,b\,x\,\sqrt{a-b\,x}}{\sqrt{x}\,\left(a^{3}-a^{2}\,b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(3/2)),x)
```

```
[Out] -(2*a*(a - b*x)^(1/2) - 4*b*x*(a - b*x)^(1/2))/(x^(1/2)*(a^3 - a^2*b*x))
```

**sympy** [A] time = 1.68, size = 112, normalized size = 2.73

$$\begin{cases} -\frac{2}{a\sqrt{b}x\sqrt{\frac{a}{bx}-1}}+\frac{4\sqrt{b}}{a^{2}\sqrt{\frac{a}{bx}-1}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{2iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{a^{3}b-a^{2}b^{2}x}+\frac{4ib^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{a^{3}b-a^{2}b^{2}x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-2/(a*sqrt(b)*x*sqrt(a/(b*x) - 1)) + 4*sqrt(b)/(a**2*sqrt(a/(b*x
) - 1)), Abs(a/(b*x)) > 1), (-2*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/(a**3*b - a
**2*b**2*x) + 4*I*b**(5/2)*x*sqrt(-a/(b*x) + 1)/(a**3*b - a**2*b**2*x), Tru
e))
```

## 3.601 $\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx$

**Optimal**. Leaf size=66

$$-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}+\frac{2}{ax^{3/2}\sqrt{a-bx}}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{2}{ax^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(3/2)*Sqrt[a - b*x]) - (8*Sqrt[a - b*x])/(3*a^2*x^(3/2)) - (16*b*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx = \frac{2}{ax^{3/2}\sqrt{a-bx}} + \frac{4\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx}{a}$$
$$= \frac{2}{ax^{3/2}\sqrt{a-bx}} - \frac{8\sqrt{a-bx}}{3a^2x^{3/2}} + \frac{(8b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^2}$$
$$= \frac{2}{ax^{3/2}\sqrt{a-bx}} - \frac{8\sqrt{a-bx}}{3a^2x^{3/2}} - \frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}$$

**Mathematica** [A] time = 0.01, size = 39, normalized size = 0.59

$$-\frac{2\left(a^2+4abx-8b^2x^2\right)}{3a^3x^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a - b*x)^(3/2)),x]
[Out] (-2*(a^2 + 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a - b*x])
```

**IntegrateAlgebraic** [A] time = 0.15, size = 50, normalized size = 0.76

$$-\frac{2\sqrt{a-bx}\left(-a^2-4abx+8b^2x^2\right)}{3a^3x^{3/2}(bx-a)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a - b*x)^(3/2)),x]
[Out] (-2*Sqrt[a - b*x]*(-a^2 - 4*a*b*x + 8*b^2*x^2))/(3*a^3*x^(3/2)*(-a + b*x))
```

**fricas** [A] time = 1.20, size = 51, normalized size = 0.77

$$-\frac{2\left(8\,b^2x^2-4\,abx-a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{3\left(a^3bx^3-a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
[Out] -2/3*(8*b^2*x^2 - 4*a*b*x - a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b*x^3 - a^4*x^
2)
```

**giac [B]** time = 1.48, size = 112, normalized size = 1.70

$$-\frac{2\sqrt{-bx+a}\left(\frac{5(bx-a)b^2|b|}{a^3}+\frac{6b^2|b|}{a^2}\right)}{3\left((bx-a)b+ab\right)^{\frac{3}{2}}}-\frac{4\sqrt{-b}\,b^3}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(-b*x + a)*(5*(b*x - a)*b^2*abs(b)/a^3 + 6*b^2*abs(b)/a^2)/((b*x -
 a)*b + a*b)^(3/2) - 4*sqrt(-b)*b^3/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x
- a)*b + a*b))^2 - a*b)*a^2*abs(b))
```

**maple [A]** time = 0.00, size = 34, normalized size = 0.52

$$-\frac{2\left(-8b^2x^2+4abx+a^2\right)}{3\sqrt{-bx+a}\;a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(3/2),x)
```

```
[Out] -2/3*(-8*b^2*x^2+4*a*b*x+a^2)/(-b*x+a)^(1/2)/x^(3/2)/a^3
```

**maxima [A]** time = 1.35, size = 52, normalized size = 0.79

$$\frac{2\,b^2\sqrt{x}}{\sqrt{-bx+a}\,a^3}-\frac{2\left(\frac{6\sqrt{-bx+a}\,b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(-b*x + a)*a^3) - 2/3*(6*sqrt(-b*x + a)*b/sqrt(x) + (-b*
x + a)^(3/2)/x^(3/2))/a^3
```

**mupad [B]** time = 0.43, size = 48, normalized size = 0.73

$$\frac{\sqrt{a-b\,x}\,\left(\frac{8\,x}{3\,a^2}+\frac{2}{3\,a\,b}-\frac{16\,b\,x^2}{3\,a^3}\right)}{x^{5/2}-\frac{a\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^(3/2)),x)
```

```
[Out] ((a - b*x)^(1/2)*((8*x)/(3*a^2) + 2/(3*a*b) - (16*b*x^2)/(3*a^3)))/(x^(5/2)
 - (a*x^(3/2))/b)
```

**sympy [B]** time = 4.64, size = 452, normalized size = 6.85

$$\begin{cases} \frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3} + \frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3} - \frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3} + \frac{16b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}-1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{2ia^3b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3} + \frac{6ia^2b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3} - \frac{24iab^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3} + \frac{16ib^{\frac{15}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((2*a**3*b**(9/2)*sqrt(a/(b*x) - 1)/(-3*a**5*b**4*x + 6*a**4*b**5*
x**2 - 3*a**3*b**6*x**3) + 6*a**2*b**(11/2)*x*sqrt(a/(b*x) - 1)/(-3*a**5*b*
*4*x + 6*a**4*b**5*x**2 - 3*a**3*b**6*x**3) - 24*a*b**(13/2)*x**2*sqrt(a/(b
*x) - 1)/(-3*a**5*b**4*x + 6*a**4*b**5*x**2 - 3*a**3*b**6*x**3) + 16*b**(15
/2)*x**3*sqrt(a/(b*x) - 1)/(-3*a**5*b**4*x + 6*a**4*b**5*x**2 - 3*a**3*b**6
*x**3), Abs(a/(b*x)) > 1), (2*I*a**3*b**(9/2)*sqrt(-a/(b*x) + 1)/(-3*a**5*b
**4*x + 6*a**4*b**5*x**2 - 3*a**3*b**6*x**3) + 6*I*a**2*b**(11/2)*x*sqrt(-a
/(b*x) + 1)/(-3*a**5*b**4*x + 6*a**4*b**5*x**2 - 3*a**3*b**6*x**3) - 24*I*a
*b**(13/2)*x**2*sqrt(-a/(b*x) + 1)/(-3*a**5*b**4*x + 6*a**4*b**5*x**2 - 3*a
**3*b**6*x**3) + 16*I*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(-3*a**5*b**4*x + 6
*a**4*b**5*x**2 - 3*a**3*b**6*x**3), True))
```

## 3.602 $\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx$

**Optimal**. Leaf size=95

$$\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}+\frac{2x^{5/2}}{3b(a-bx)^{3/2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{3b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(5/2))/(3*b*(a - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a - b*x]) - (5
*Sqrt[x]*Sqrt[a - b*x])/b^3 + (5*a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx &= \frac{2x^{5/2}}{3b(a-bx)^{3/2}} - \frac{5\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx}{3b} \\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{a-bx}} + \frac{5\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{b^2} \\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{a-bx}} - \frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3} + \frac{(5a)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{2b^3} \\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{a-bx}} - \frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3} + \frac{(5a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b^3} \\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{a-bx}} - \frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3} + \frac{(5a)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^3} \\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{a-bx}} - \frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3} + \frac{5a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 51, normalized size = 0.54

$$\frac{2x^{7/2}\sqrt{1-\frac{bx}{a}}\;{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a^2\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 - (b*x)/a]*Hypergeometric2F1[5/2, 7/2, 9/2, (b*x)/a])/(7*
a^2*Sqrt[a - b*x])
```

**IntegrateAlgebraic [A]** time = 0.20, size = 96, normalized size = 1.01

$$\frac{\sqrt{a-bx}\left(-15a^2\sqrt{x}+20abx^{3/2}-3b^2x^{5/2}\right)}{3b^3(bx-a)^2}+\frac{5a\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^2*Sqrt[x] + 20*a*b*x^(3/2) - 3*b^2*x^(5/2)))/(3*b^3*(
-a + b*x)^2) + (5*a*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/b^4
```

**fricas [A]** time = 1.35, size = 215, normalized size = 2.26

$$\left[-\frac{15\left(ab^2x^2-2a^2bx+a^3\right)\sqrt{-b}\log\left(-2bx+2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(3b^3x^2-20ab^2x+15a^2b\right)\sqrt{-bx+a}\sqrt{x}}{6\left(b^6x^2-2ab^5x+a^2b^4\right)}, -\frac{15\left(ab^2x^2-2a^2bx+a^3\right)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+\left(3b^3x^2-20ab^2x+15a^2b\right)\sqrt{-bx+a}\sqrt{x}}{3\left(b^6x^2-2ab^5x+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x +
 a)*sqrt(-b)*sqrt(x) + a) + 2*(3*b^3*x^2 - 20*a*b^2*x + 15*a^2*b)*sqrt(-b*x
 + a)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), -1/3*(15*(a*b^2*x^2 - 2*a^2
*b*x + a^3)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (3*b^3*x^2 -
 20*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*
b^4)]
```

**giac [B]** time = 112.52, size = 221, normalized size = 2.33

$$\frac{\left(\frac{15a\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}b^2}-\frac{6\sqrt{(bx-a)b+ab}\sqrt{-bx+a}}{b^3}-\frac{8\left(9a^2\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4-12a^3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2b+7a^4b^2\right)}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3\sqrt{-b}b}\right)|b|}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(15*a*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/(sqrt(
-b)*b^2) - 6*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b^3 - 8*(9*a^2*(sqrt(-b
```

```
*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4 - 12*a^3*(sqrt(-b*x + a)*sqrt
(-b) - sqrt((b*x - a)*b + a*b))^2*b + 7*a^4*b^2)/(((sqrt(-b*x + a)*sqrt(-b)
 - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*sqrt(-b)*b))*abs(b)/b^2
```

**maple** [B] time = 0.04, size = 160, normalized size = 1.68

$$\frac{\left(\frac{5a\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{2b^{\frac{7}{2}}}+\frac{2\sqrt{-\left(x-\frac{a}{b}\right)a-\left(x-\frac{a}{b}\right)^2 b}\;a^2}{3\left(x-\frac{a}{b}\right)^2 b^5}+\frac{14\sqrt{-\left(x-\frac{a}{b}\right)a-\left(x-\frac{a}{b}\right)^2 b}\;a}{3\left(x-\frac{a}{b}\right)b^4}\right)\sqrt{(-bx+a)\,x}}{\sqrt{-bx+a}\,\sqrt{x}}-\frac{\sqrt{-bx+a}\,\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -x^(1/2)*(-b*x+a)^(1/2)/b^3+(5/2/b^(7/2)*a*arctan((x-1/2*a/b)/(-b*x^2+a*x)^
(1/2)*b^(1/2))+2/3/b^5*a^2/(x-a/b)^2*(-(x-a/b)*a-(x-a/b)^2*b)^(1/2)+14/3/b^
4*a/(x-a/b)*(-(x-a/b)*a-(x-a/b)^2*b)^(1/2))*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/
2)/x^(1/2)
```

**maxima** [A] time = 3.02, size = 94, normalized size = 0.99

$$\frac{2\,ab^2+\frac{10\,(bx-a)ab}{x}-\frac{15\,(bx-a)^2a}{x^2}}{3\left(\frac{(-bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(-bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}-\frac{5\,a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a*b^2 + 10*(b*x - a)*a*b/x - 15*(b*x - a)^2*a/x^2)/((-b*x + a)^(3/2)
*b^4/x^(3/2) + (-b*x + a)^(5/2)*b^3/x^(5/2)) - 5*a*arctan(sqrt(-b*x + a)/(s
qrt(b)*sqrt(x)))/b^(7/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)/(a - b*x)^(5/2), x)
```

**sympy [B]** time = 8.48, size = 971, normalized size = 10.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-30*I*a**(81/2)*b**22*x**(51/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*s
qrt(x)/sqrt(a))/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(7
7/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 15*pi*a**(81/2)*b**22*x**(51/2
)*sqrt(-1 + b*x/a)/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a*
*(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 30*I*a**(79/2)*b**23*x**(53
/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(6*a**(79/2)*b**(51/2)*
x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/
a)) - 15*pi*a**(79/2)*b**23*x**(53/2)*sqrt(-1 + b*x/a)/(6*a**(79/2)*b**(51/
2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b
*x/a)) + 30*I*a**40*b**(45/2)*x**26/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-
1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) - 40*I*a**39
*b**(47/2)*x**27/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(
77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 6*I*a**38*b**(49/2)*x**28/(6*
a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(
53/2)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (15*a**(81/2)*b**22*x**(51/2)*sqr
t(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)
*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) - 15*a*
*(79/2)*b**23*x**(53/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a*
*(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/
2)*sqrt(1 - b*x/a)) - 15*a**40*b**(45/2)*x**26/(3*a**(79/2)*b**(51/2)*x**(5
1/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) + 2
0*a**39*b**(47/2)*x**27/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) -
3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) - 3*a**38*b**(49/2)*x**28/
(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x*
*(53/2)*sqrt(1 - b*x/a)), True))
```

## 3.603 $\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx$

**Optimal**. Leaf size=72

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2x^{3/2}}{3b(a-bx)^{3/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 63, 217, 203}

$$-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{3b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*b*(a - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a - b*x]) + (2*Ar
cTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx &= \frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx}{b}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{b^2}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^2}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 82, normalized size = 1.14

$$\frac{2\left(\sqrt{b}\sqrt{x}(4bx-3a)+3\sqrt{a}(a-bx)\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\right)}{3b^{5/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*(Sqrt[b]*Sqrt[x]*(-3*a + 4*b*x) + 3*Sqrt[a]*(a - b*x)*Sqrt[1 - (b*x)/a]*
ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(3*b^(5/2)*(a - b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 82, normalized size = 1.14

$$\frac{2\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b^3}-\frac{2\sqrt{a-bx}\left(3a\sqrt{x}-4bx^{3/2}\right)}{3b^2(bx-a)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a - b*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a - b*x]*(3*a*Sqrt[x] - 4*b*x^(3/2)))/(3*b^2*(-a + b*x)^2) + (2*Sq
rt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/b^3
```

**fricas [A]** time = 1.56, size = 188, normalized size = 2.61

$$\left[-\frac{3\left(b^{2}x^{2}-2\,abx+a^{2}\right)\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\left(4\,b^{2}x-3\,ab\right)\sqrt{-bx+a}\,\sqrt{x}}{3\left(b^{5}x^{2}-2\,ab^{4}x+a^{2}b^{3}\right)},\,-\frac{2\left(3\left(b^{2}x^{2}-2\,abx+a^{2}\right)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(4\,b^{2}x-3\,ab\right)\sqrt{-bx+a}\,\sqrt{x}\right)}{3\left(b^{5}x^{2}-2\,ab^{4}x+a^{2}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*s
qrt(-b)*sqrt(x) + a) - 2*(4*b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^5*x^2
 - 2*a*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(b)*arctan(s
qrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (4*b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x)
)/(b^5*x^2 - 2*a*b^4*x + a^2*b^3)]
```

**giac [B]** time = 110.08, size = 194, normalized size = 2.69

$$\frac{\left(\frac{3\,\log\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}\right)}{\sqrt{-b}}+\frac{8\left(3\,a\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{4}\sqrt{-b}-3\,a^{2}\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}\sqrt{-b}\,b+2\,a^{3}\sqrt{-b}\,b^{2}\right)}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}-ab\right)^{3}}\right)|b|}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sqrt(-b)
+ 8*(3*a*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b) - 3
*a^2*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2*sqrt(-b)*b + 2*a
^3*sqrt(-b)*b^2)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a
*b)^3)*abs(b)/b^3
```

**maple [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(-bx+a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(5/2),x)
```

```
[Out] int(x^(3/2)/(-b*x+a)^(5/2),x)
```

**maxima [A]** time = 2.94, size = 52, normalized size = 0.72

$$\frac{2\left(b+\frac{3\,(bx-a)}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}b^{2}}-\frac{2\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b + 3*(b*x - a)/x)*x^(3/2)/((-b*x + a)^(3/2)*b^2) - 2*arctan(sqrt(-b*x
 + a)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{{\left(a-b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)/(a - b*x)^(5/2), x)
```

**sympy [B]** time = 4.50, size = 833, normalized size = 11.57

$$\begin{cases} -\frac{6ia^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{3\pi a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{6ia^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}-\frac{3\pi a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{6ia^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}-\frac{8ia^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ \frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}+\frac{8a^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-6*I*a**(39/2)*b**11*x**(27/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sq
rt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37
/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 3*pi*a**(39/2)*b**11*x**(27/2)*
sqrt(-1 + b*x/a)/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(
37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 6*I*a**(37/2)*b**12*x**(29/2)
*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**
(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a))
 - 3*pi*a**(37/2)*b**12*x**(29/2)*sqrt(-1 + b*x/a)/(3*a**(39/2)*b**(27/2)*x
**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a
)) + 6*I*a**19*b**(23/2)*x**14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b
*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) - 8*I*a**18*b**(2
```

```
5/2)*x**15/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*
b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (6*a**(39/2)*b**11*
x**(27/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27
/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 - b*
x/a)) - 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sq
rt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(2
9/2)*x**(29/2)*sqrt(1 - b*x/a)) - 6*a**19*b**(23/2)*x**14/(3*a**(39/2)*b**(
27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 -
b*x/a)) + 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 -
 b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 - b*x/a)), True))
```

## 3.604 $\int \frac{\sqrt{x}}{(a-bx)^{5/2}} dx$

**Optimal**. Leaf size=22

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x)^(5/2),x]
[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}} dx = \frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

**Mathematica [A]** time = 0.01, size = 22, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x)^(5/2),x]
[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.03, size = 22, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

**fricas [B]** time = 1.11, size = 34, normalized size = 1.55

$$\frac{2\sqrt{-bx+a}\,x^{\frac{3}{2}}}{3\left(ab^2x^2-2\,a^2bx+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-b*x + a)*x^(3/2)/(a*b^2*x^2 - 2*a^2*b*x + a^3)
```

**giac [B]** time = 1.63, size = 102, normalized size = 4.64

$$\frac{4\left(3\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}+a^2\sqrt{-b}\,b^2\right)|b|}{3\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b) + a^2
*sqrt(-b)*b^2)*abs(b)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))
^2 - a*b)^3*b^2)
```

**maple [A]** time = 0.00, size = 17, normalized size = 0.77

$$\frac{2x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(5/2),x)
```

```
[Out] 2/3*x^(3/2)/a/(-b*x+a)^(3/2)
```

**maxima [A]** time = 1.36, size = 16, normalized size = 0.73

$$\frac{2\,x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/((-b*x + a)^(3/2)*a)
```

**mupad [B]** time = 0.25, size = 37, normalized size = 1.68

$$\frac{2\,x^{3/2}\,\sqrt{a-b\,x}}{3\,\left(a^3-2\,a^2\,b\,x+a\,b^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(5/2),x)
```

```
[Out] (2*x^(3/2)*(a - b*x)^(1/2))/(3*(a^3 + a*b^2*x^2 - 2*a^2*b*x))
```

**sympy [B]** time = 1.51, size = 95, normalized size = 4.32

$$\begin{cases} \frac{2ix^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{2x^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((2*I*x**(3/2)/(-3*a**(5/2)*sqrt(-1 + b*x/a) + 3*a**(3/2)*b*x*sqrt
(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*x**(3/2)/(-3*a**(5/2)*sqrt(1 - b*x/a) +
 3*a**(3/2)*b*x*sqrt(1 - b*x/a)), True))
```

## 3.605 $\int \frac{1}{\sqrt{x}\,(a-bx)^{5/2}}\,dx$

**Optimal**. Leaf size=45

$$\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}+\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}+\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x])/(3*a*(a - b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a - b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a-bx)^{5/2}}\,dx &= \frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{x}\,(a-bx)^{3/2}}\,dx}{3a}\\ &= \frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 0.67

$$\frac{2\sqrt{x}\,(3a-2bx)}{3a^2(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
[Out] (2*Sqrt[x]*(3*a - 2*b*x))/(3*a^2*(a - b*x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.12, size = 30, normalized size = 0.67

$$-\frac{2\sqrt{x}\,(2bx-3a)}{3a^2(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
[Out] (-2*Sqrt[x]*(-3*a + 2*b*x))/(3*a^2*(a - b*x)^(3/2))
```

**fricas** [A] time = 1.25, size = 44, normalized size = 0.98

$$-\frac{2\,(2\,bx-3\,a)\sqrt{-bx+a}\,\sqrt{x}}{3\left(a^2b^2x^2-2\,a^3bx+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
[Out] -2/3*(2*b*x - 3*a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b^2*x^2 - 2*a^3*b*x + a^4)
```

**giac** [B] time = 1.43, size = 96, normalized size = 2.13

$$\frac{8\left(3\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)\sqrt{-b}\,b^2}{3\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
[Out] 8/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)*sqrt(-b
)*b^2/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*abs(
b))
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.56

$$\frac{2\left(-2bx+3a\right)\sqrt{x}}{3\left(-bx+a\right)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 2/3*x^(1/2)*(-2*b*x+3*a)/(-b*x+a)^(3/2)/a^2
```

**maxima [A]** time = 1.34, size = 30, normalized size = 0.67

$$\frac{2\left(b-\frac{3\left(bx-a\right)}{x}\right)x^{\frac{3}{2}}}{3\left(-bx+a\right)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b - 3*(b*x - a)/x)*x^(3/2)/((-b*x + a)^(3/2)*a^2)
```

**mupad [B]** time = 0.41, size = 56, normalized size = 1.24

$$\frac{6\,a\,\sqrt{x}\,\sqrt{a-b\,x}-4\,b\,x^{3/2}\,\sqrt{a-b\,x}}{3\,a^4-6\,a^3\,b\,x+3\,a^2\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (6*a*x^(1/2)*(a - b*x)^(1/2) - 4*b*x^(3/2)*(a - b*x)^(1/2))/(3*a^4 + 3*a^2*
b^2*x^2 - 6*a^3*b*x)
```

**sympy [C]** time = 2.00, size = 211, normalized size = 4.69

$$\begin{cases} \frac{6ia}{3ia^{3}\sqrt{b}\sqrt{\frac{a}{bx}-1}-3ia^{2}b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}}-\frac{4ibx}{3ia^{3}\sqrt{b}\sqrt{\frac{a}{bx}-1}-3ia^{2}b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ \frac{6ab}{3ia^{3}b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}-3ia^{2}b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}-\frac{4b^{2}x}{3ia^{3}b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}-3ia^{2}b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((6*I*a/(3*I*a**3*sqrt(b)*sqrt(a/(b*x) - 1) - 3*I*a**2*b**(3/2)*x*
sqrt(a/(b*x) - 1)) - 4*I*b*x/(3*I*a**3*sqrt(b)*sqrt(a/(b*x) - 1) - 3*I*a**2
*b**(3/2)*x*sqrt(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (6*a*b/(3*I*a**3*b**(3/2
)*sqrt(-a/(b*x) + 1) - 3*I*a**2*b**(5/2)*x*sqrt(-a/(b*x) + 1)) - 4*b**2*x/(
3*I*a**3*b**(3/2)*sqrt(-a/(b*x) + 1) - 3*I*a**2*b**(5/2)*x*sqrt(-a/(b*x) +
1)), True))
```

## 3.606 $\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx$

**Optimal**. Leaf size=67

$$-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}}+\frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}}+\frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a - b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*Sqrt[x]*(a - b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a - b*x]) - (16*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx = \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}} + \frac{4\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx}{3a}$$
$$= \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}} + \frac{8\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^2}$$
$$= \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}} - \frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}$$

**Mathematica** [A] time = 0.01, size = 41, normalized size = 0.61

$$-\frac{2\left(3a^2-12abx+8b^2x^2\right)}{3a^3\sqrt{x}\,(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a - b*x)^(5/2)),x]
[Out] (-2*(3*a^2 - 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a - b*x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.14, size = 41, normalized size = 0.61

$$-\frac{2\left(3a^2-12abx+8b^2x^2\right)}{3a^3\sqrt{x}\,(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a - b*x)^(5/2)),x]
[Out] (-2*(3*a^2 - 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a - b*x)^(3/2))
```

**fricas** [A] time = 1.19, size = 59, normalized size = 0.88

$$-\frac{2\left(8\,b^2x^2-12\,abx+3\,a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{3\left(a^3b^2x^3-2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
[Out] -2/3*(8*b^2*x^2 - 12*a*b*x + 3*a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b^2*x^3 - 2
*a^4*b*x^2 + a^5*x)
```

**giac [B]** time = 1.61, size = 189, normalized size = 2.82

$$-\frac{2\sqrt{-bx+a}\,b^2}{\sqrt{(bx-a)b+ab}\,a^3|b|}-\frac{4\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}\,b^2-12\,a\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\sqrt{-b}\,b^3+5\,a^2\sqrt{-b}\,b^4\right)}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a^3*abs(b)) - 4/3*(3*(sqrt(-
b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b)*b^2 - 12*a*(sqrt(-b
*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2*sqrt(-b)*b^3 + 5*a^2*sqrt(-b)
*b^4)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*a^2*
abs(b))
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.54

$$-\frac{2\left(8b^2x^2-12abx+3a^2\right)}{3\left(-bx+a\right)^{\frac{3}{2}}a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -2/3*(8*b^2*x^2-12*a*b*x+3*a^2)/(-b*x+a)^(3/2)/x^(1/2)/a^3
```

**maxima [A]** time = 1.29, size = 50, normalized size = 0.75

$$\frac{2\left(b^2-\frac{6\,(bx-a)b}{x}\right)x^{\frac{3}{2}}}{3\left(-bx+a\right)^{\frac{3}{2}}a^3}-\frac{2\sqrt{-bx+a}}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2 - 6*(b*x - a)*b/x)*x^(3/2)/((-b*x + a)^(3/2)*a^3) - 2*sqrt(-b*x +
a)/(a^3*sqrt(x))
```

**mupad [B]** time = 0.44, size = 73, normalized size = 1.09

$$\frac{6\,a^2\,\sqrt{a-b\,x}\,+16\,b^2\,x^2\,\sqrt{a-b\,x}\,-24\,a\,b\,x\,\sqrt{a-b\,x}}{\sqrt{x}\,\left(x\,\left(6\,a^4\,b-3\,a^3\,b^2\,x\right)-3\,a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (6*a^2*(a - b*x)^(1/2) + 16*b^2*x^2*(a - b*x)^(1/2) - 24*a*b*x*(a - b*x)^(1
/2))/(x^(1/2)*(x*(6*a^4*b - 3*a^3*b^2*x) - 3*a^5))
```

**sympy [B]** time = 4.27, size = 314, normalized size = 4.69

$$\begin{cases} -\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} + \frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} - \frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{6ia^2b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} + \frac{24iab^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} - \frac{16ib^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-6*a**2*b**(9/2)*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6*a**4*b**5*x
+ 3*a**3*b**6*x**2) + 24*a*b**(11/2)*x*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6*a
**4*b**5*x + 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) - 1)/(3*a**
5*b**4 - 6*a**4*b**5*x + 3*a**3*b**6*x**2), Abs(a/(b*x)) > 1), (-6*I*a**2*b
**(9/2)*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b**5*x + 3*a**3*b**6*x**2)
 + 24*I*a*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b**5*x + 3*a
**3*b**6*x**2) - 16*I*b**(13/2)*x**2*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a*
*4*b**5*x + 3*a**3*b**6*x**2), True))
```

## 3.607 $\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx$

**Optimal**. Leaf size=88

$$-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^(3/2)*(a - b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a - b*x]) - (16*Sqrt[
a - b*x])/(3*a^3*x^(3/2)) - (32*b*Sqrt[a - b*x])/(3*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx = \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{2\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx}{a}$$
$$= \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a-bx}} + \frac{8\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx}{a^2}$$
$$= \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a-bx}} - \frac{16\sqrt{a-bx}}{3a^3x^{3/2}} + \frac{(16b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^3}$$
$$= \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a-bx}} - \frac{16\sqrt{a-bx}}{3a^3x^{3/2}} - \frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}$$

**Mathematica** [A] time = 0.01, size = 50, normalized size = 0.57

$$-\frac{2\left(a^3+6a^2bx-24ab^2x^2+16b^3x^3\right)}{3a^4x^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*(a^3 + 6*a^2*b*x - 24*a*b^2*x^2 + 16*b^3*x^3))/(3*a^4*x^(3/2)*(a - b*x)
^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.15, size = 50, normalized size = 0.57

$$-\frac{2\left(a^3+6a^2bx-24ab^2x^2+16b^3x^3\right)}{3a^4x^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*(a^3 + 6*a^2*b*x - 24*a*b^2*x^2 + 16*b^3*x^3))/(3*a^4*x^(3/2)*(a - b*x)
^(3/2))
```

**fricas** [A] time = 1.23, size = 70, normalized size = 0.80

$$-\frac{2\left(16\,b^3x^3-24\,ab^2x^2+6\,a^2bx+a^3\right)\sqrt{-bx+a}\,\sqrt{x}}{3\left(a^4b^2x^4-2\,a^5bx^3+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*b^3*x^3 - 24*a*b^2*x^2 + 6*a^2*b*x + a^3)*sqrt(-b*x + a)*sqrt(x)/(
a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2)
```

**giac [B]** time = 1.70, size = 207, normalized size = 2.35

$$-\frac{2\sqrt{-bx+a}\left(\frac{8(bx-a)b^2|b|}{a^4}+\frac{9b^2|b|}{a^3}\right)}{3((bx-a)b+ab)^{\frac{3}{2}}}-\frac{8\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}b^3-9a\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\sqrt{-b}b^4+4a^2\sqrt{-b}b^5\right)}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3a^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(-b*x + a)*(8*(b*x - a)*b^2*abs(b)/a^4 + 9*b^2*abs(b)/a^3)/((b*x -
 a)*b + a*b)^(3/2) - 8/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a
*b))^4*sqrt(-b)*b^3 - 9*a*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b
))^2*sqrt(-b)*b^4 + 4*a^2*sqrt(-b)*b^5)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((
b*x - a)*b + a*b))^2 - a*b)^3*a^3*abs(b))
```

**maple [A]** time = 0.00, size = 45, normalized size = 0.51

$$-\frac{2\left(16b^3x^3-24a\,b^2x^2+6a^2bx+a^3\right)}{3\left(-bx+a\right)^{\frac{3}{2}}a^4x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -2/3*(16*b^3*x^3-24*a*b^2*x^2+6*a^2*b*x+a^3)/(-b*x+a)^(3/2)/x^(3/2)/a^4
```

**maxima [A]** time = 1.33, size = 68, normalized size = 0.77

$$-\frac{2\left(\frac{9\sqrt{-bx+a}\,b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^4}+\frac{2\left(b^3-\frac{9(bx-a)b^2}{x}\right)x^{\frac{3}{2}}}{3\left(-bx+a\right)^{\frac{3}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(9*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^4 + 2/3*(b^3
 - 9*(b*x - a)*b^2/x)*x^(3/2)/((-b*x + a)^(3/2)*a^4)
```

**mupad [B]** time = 0.47, size = 92, normalized size = 1.05

$$\frac{2\,a^3\,\sqrt{a-b\,x}\,+32\,b^3\,x^3\,\sqrt{a-b\,x}\,+12\,a^2\,b\,x\,\sqrt{a-b\,x}\,-48\,a\,b^2\,x^2\,\sqrt{a-b\,x}}{x^{3/2}\,\left(x\,\left(6\,a^5\,b-3\,a^4\,b^2\,x\right)-3\,a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (2*a^3*(a - b*x)^(1/2) + 32*b^3*x^3*(a - b*x)^(1/2) + 12*a^2*b*x*(a - b*x)^
(1/2) - 48*a*b^2*x^2*(a - b*x)^(1/2))/(x^(3/2)*(x*(6*a^5*b - 3*a^4*b^2*x) -
 3*a^6))
```

**sympy [B]** time = 13.45, size = 688, normalized size = 7.82

$$\begin{cases} \frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{80ab^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{32b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} & \text{for}\ \left|\frac{a}{bx}\right| > 1 \\ \frac{2ia^4b^{\frac{19}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{10ia^3b^{\frac{21}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{60ia^2b^{\frac{23}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{80iab^{\frac{25}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{32ib^{\frac{27}{2}}x^4\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((2*a**4*b**(19/2)*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**1
0*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(
a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a*
*4*b**12*x**4) - 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x +
 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2
)*x**3*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11
*x**3 + 3*a**4*b**12*x**4) - 32*b**(27/2)*x**4*sqrt(a/(b*x) - 1)/(-3*a**7*b
**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4), Abs(a/(
b*x)) > 1), (2*I*a**4*b**(19/2)*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6
*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*I*a**3*b**(21/2)*
x*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**
3 + 3*a**4*b**12*x**4) - 60*I*a**2*b**(23/2)*x**2*sqrt(-a/(b*x) + 1)/(-3*a*
*7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80
*I*a*b**(25/2)*x**3*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2
- 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) - 32*I*b**(27/2)*x**4*sqrt(-a/(b*x
) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**
12*x**4), True))
```

## 3.608 $\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=88

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{2b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{6b^2}+\frac{x^{5/2}\sqrt{bx+2}}{3b}$$

**Rubi [A]** time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{5x^{3/2}\sqrt{bx+2}}{6b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{2b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{x^{5/2}\sqrt{bx+2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[2 + b*x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(6*b^2) + (x^
(5/2)*Sqrt[2 + b*x])/(3*b) - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx = \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{3b}$$
$$= -\frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b^2}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b^3}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 0.68

$$\frac{\sqrt{x}\sqrt{bx+2}\left(2b^2x^2-5bx+15\right)}{6b^3} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2))/(6*b^3) - (5*ArcSinh[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

**IntegrateAlgebraic [A]** time = 0.09, size = 73, normalized size = 0.83

$$\frac{5\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{7/2}} + \frac{\sqrt{bx+2}\left(2b^2x^{5/2}-5bx^{3/2}+15\sqrt{x}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[2 + b*x]*(15*Sqrt[x] - 5*b*x^(3/2) + 2*b^2*x^(5/2)))/(6*b^3) + (5*Log
[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

**fricas [A]** time = 1.15, size = 124, normalized size = 1.41

$$\left[\frac{\left(2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b^4}, \frac{\left(2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(b
*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/6*((2*b^3*x^2 - 5*b^2*x + 1
5*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*s
qrt(x))))/b^4]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]2*abs(b)/b^2/b*(2*((12*b^5/144/b^7*
sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*x+2)+198*b^5/144
```

```
/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2
*b)-sqrt(b)*sqrt(b*x+2))))
```

**maple [A]** time = 0.00, size = 93, normalized size = 1.06

$$\frac{\sqrt{bx+2}\, x^{\frac{5}{2}}}{3b}-\frac{5\sqrt{bx+2}\, x^{\frac{3}{2}}}{6b^{2}}+\frac{5\sqrt{bx+2}\, \sqrt{x}}{2b^{3}}-\frac{5\sqrt{(bx+2)\, x}\, \ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\, x^{2}+2x}\right)}{2\sqrt{bx+2}\, b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/3*x^(5/2)*(b*x+2)^(1/2)/b-5/6*x^(3/2)*(b*x+2)^(1/2)/b^2+5/2*(b*x+2)^(1/2)
/b^3*x^(1/2)-5/2*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(7/2)/x^(1/2)*ln((b*x+1)
/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [B]** time = 2.97, size = 134, normalized size = 1.52

$$-\frac{\frac{33\sqrt{bx+2}\, b^{2}}{\sqrt{x}}-\frac{40\,(bx+2)^{\frac{3}{2}}\, b}{x^{\frac{3}{2}}}+\frac{15\,(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{6}-\frac{3\,(bx+2)b^{5}}{x}+\frac{3\,(bx+2)^{2}b^{4}}{x^{2}}-\frac{(bx+2)^{3}b^{3}}{x^{3}}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(33*sqrt(b*x + 2)*b^2/sqrt(x) - 40*(b*x + 2)^(3/2)*b/x^(3/2) + 15*(b*x
 + 2)^(5/2)/x^(5/2))/(b^6 - 3*(b*x + 2)*b^5/x + 3*(b*x + 2)^2*b^4/x^2 - (b*
x + 2)^3*b^3/x^3) + 5/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + s
qrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{b\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(5/2)/(b*x + 2)^(1/2), x)
```

**sympy [A]** time = 7.40, size = 95, normalized size = 1.08

$$\frac{x^{\frac{7}{2}}}{3\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{6b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{6b^2\sqrt{bx+2}}+\frac{5\sqrt{x}}{b^3\sqrt{bx+2}}-\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(7/2)/(3*sqrt(b*x + 2)) - x**(5/2)/(6*b*sqrt(b*x + 2)) + 5*x**(3/2)/(6*b
**2*sqrt(b*x + 2)) + 5*sqrt(x)/(b**3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(
b)*sqrt(x)/2)/b**(7/2)
```

## 3.609 $\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=67

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{3\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{x^{3/2}\sqrt{bx+2}}{2b}$$

**Rubi [A]** time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{x^{3/2}\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[2 + b*x],x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(2*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(2*b) + (3*Arc
Sinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx = \frac{x^{3/2}\sqrt{2+bx}}{2b} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b}$$
$$= -\frac{3\sqrt{x}\,\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b^2}$$
$$= -\frac{3\sqrt{x}\,\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{3\sqrt{x}\,\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

**Mathematica [A]** time = 0.03, size = 51, normalized size = 0.76

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}} + \frac{\sqrt{x}\,\sqrt{bx+2}\,(bx-3)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*(-3 + b*x)*Sqrt[2 + b*x])/(2*b^2) + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/b^(5/2)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 62, normalized size = 0.93

$$\frac{\sqrt{bx+2}\,\left(bx^{3/2}-3\sqrt{x}\right)}{2b^2} - \frac{3\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[2 + b*x]*(-3*Sqrt[x] + b*x^(3/2)))/(2*b^2) - (3*Log[-(Sqrt[b]*Sqrt[x]
) + Sqrt[2 + b*x]])/b^(5/2)
```

**fricas [A]** time = 1.00, size = 105, normalized size = 1.57

$$\left[\frac{\left(b^2x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{2\,b^3}, \frac{\left(b^2x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*
sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]2*abs(b)/b^2/b^2*(2*(1/8*sqrt(b*x+2
)*sqrt(b*x+2)-5/8)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqr
t(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
```

**maple** [A] time = 0.00, size = 78, normalized size = 1.16

$$\frac{\sqrt{bx+2}\,x^{\frac{3}{2}}}{2b}-\frac{3\sqrt{bx+2}\,\sqrt{x}}{2b^{2}}+\frac{3\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+2)^(1/2)/b-3/2*(b*x+2)^(1/2)/b^2*x^(1/2)+3/2*((b*x+2)*x)^(
1/2)/(b*x+2)^(1/2)/b^(5/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima** [B] time = 2.86, size = 102, normalized size = 1.52

$$\frac{\frac{5\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{3\,(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{4}-\frac{2\,(bx+2)b^{3}}{x}+\frac{(bx+2)^{2}b^{2}}{x^{2}}}-\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] (5*sqrt(b*x + 2)*b/sqrt(x) - 3*(b*x + 2)^(3/2)/x^(3/2))/(b^4 - 2*(b*x + 2)*
b^3/x + (b*x + 2)^2*b^2/x^2) - 3/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(
sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{b\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(1/2), x)
```

**sympy** [A] time = 3.67, size = 75, normalized size = 1.12

$$\frac{x^{\frac{5}{2}}}{2\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{2b\sqrt{bx+2}}-\frac{3\sqrt{x}}{b^{2}\sqrt{bx+2}}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(5/2)/(2*sqrt(b*x + 2)) - x**(3/2)/(2*b*sqrt(b*x + 2)) - 3*sqrt(x)/(b**2
*sqrt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

# 3.610 $\int \frac{\sqrt{x}}{\sqrt{2+bx}} dx$

**Optimal**. Leaf size=43

$$\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx &= \frac{\sqrt{x}\,\sqrt{2+bx}}{b} - \frac{\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{b} \\ &= \frac{\sqrt{x}\,\sqrt{2+bx}}{b} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b} \\ &= \frac{\sqrt{x}\,\sqrt{2+bx}}{b} - \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{\sqrt{x}\,\sqrt{bx+2}}{b} - \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[2 + b*x],x]
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.06, size = 49, normalized size = 1.14

$$\frac{2\log\left(\sqrt{bx+2} - \sqrt{b}\,\sqrt{x}\right)}{b^{3/2}} + \frac{\sqrt{x}\,\sqrt{bx+2}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[2 + b*x],x]
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b + (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(
3/2)
```

**fricas** [A] time = 1.02, size = 87, normalized size = 2.02

$$\left[\frac{\sqrt{bx+2}\,b\sqrt{x} + \sqrt{b}\,\log\left(bx - \sqrt{bx+2}\,\sqrt{b}\,\sqrt{x} + 1\right)}{b^2}, \frac{\sqrt{bx+2}\,b\sqrt{x} + 2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x)
 + 1))/b^2, (sqrt(b*x + 2)*b*sqrt(x) + 2*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt
(-b)/(b*sqrt(x))))/b^2]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]2*abs(b)/b^2/b*(1/2*sqrt(b*x+2)*sqr
t(b*(b*x+2)-2*b)+2*b/2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+
2))))
```

**maple [A]** time = 0.00, size = 62, normalized size = 1.44

$$\frac{\sqrt{bx+2}\,\sqrt{x}}{b}-\frac{\sqrt{(bx+2)\,x}\;\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\;b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] (b*x+2)^(1/2)/b*x^(1/2)-((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(3/2)/x^(1/2)*ln(
(b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

**maxima [B]** time = 2.92, size = 70, normalized size = 1.63

$$\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{bx+2}}{\left(b^{2}-\frac{(bx+2)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b
^(3/2) - 2*sqrt(b*x + 2)/((b^2 - (b*x + 2)*b/x)*sqrt(x))
```

**mupad [B]** time = 0.59, size = 43, normalized size = 1.00

$$\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{b\,x+2}}\right)}{b^{3/2}}+\frac{\sqrt{x}\,\sqrt{b\,x+2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(1/2),x)
```

```
[Out] (4*atanh((b^(1/2)*x^(1/2))/(2^(1/2) - (b*x + 2)^(1/2))))/b^(3/2) + (x^(1/2)
*(b*x + 2)^(1/2))/b
```

**sympy [A]** time = 1.93, size = 54, normalized size = 1.26

$$\frac{x^{\frac{3}{2}}}{\sqrt{bx+2}}+\frac{2\sqrt{x}}{b\sqrt{bx+2}}-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(3/2)/sqrt(b*x + 2) + 2*sqrt(x)/(b*sqrt(b*x + 2)) - 2*asinh(sqrt(2)*sqrt
(b)*sqrt(x)/2)/b**(3/2)
```

# 3.611 $\int \frac{1}{\sqrt{x}\sqrt{2+bx}} dx$

**Optimal**. Leaf size=24

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {54, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{2+bx}} dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}} dx, x, \sqrt{x}\right)\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

**IntegrateAlgebraic** [A] time = 0.04, size = 30, normalized size = 1.25

$$-\frac{2\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
[Out] (-2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

**fricas** [A] time = 1.28, size = 55, normalized size = 2.29

$$\left[\frac{\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] [log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1)/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x + 2)*sqrt(-b)/(b*sqrt(x)))/b]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
```

```
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]-2/abs(b)*b^2/b/sqrt(b)*ln(abs(sqrt
(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))
```

**maple [B]** time = 0.00, size = 46, normalized size = 1.92

$$\frac{\sqrt{(bx+2)\,x}\;\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{\sqrt{bx+2}\;\sqrt{b}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2
*x)^(1/2))
```

**maxima [B]** time = 2.96, size = 41, normalized size = 1.71

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
sqrt(b)
```

**mupad [B]** time = 0.04, size = 30, normalized size = 1.25

$$\frac{4\,\mathrm{atan}\left(\frac{\sqrt{2}-\sqrt{b\,x+2}}{\sqrt{-b}\,\sqrt{x}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((2^(1/2) - (b*x + 2)^(1/2))/((-b)^(1/2)*x^(1/2))))/(-b)^(1/2)
```

**sympy [A]** time = 1.02, size = 24, normalized size = 1.00

$$\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

## 3.612 $\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=16

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

**Rubi [A]** time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{2+bx}}{\sqrt{x}}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.02, size = 16, normalized size = 1.00

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*Sqrt[2 + b*x]),x]
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

**fricas [A]** time = 1.28, size = 12, normalized size = 0.75

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(b*x + 2)/sqrt(x)
```

**giac [B]** time = 1.11, size = 29, normalized size = 1.81

$$-\frac{\sqrt{bx+2}\,b^2}{\sqrt{(bx+2)b-2\,b}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] -sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b))
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(1/2),x)
[Out] -(b*x+2)^(1/2)/x^(1/2)
```

**maxima [A]** time = 1.35, size = 12, normalized size = 0.75

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(b*x + 2)/sqrt(x)
```

**mupad [B]** time = 0.33, size = 12, normalized size = 0.75

$$-\frac{\sqrt{b\,x+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(1/2)),x)
[Out] -(b*x + 2)^(1/2)/x^(1/2)
```

**sympy [A]** time = 0.88, size = 15, normalized size = 0.94

$$-\sqrt{b}\sqrt{1+\frac{2}{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(1/2),x)
[Out] -sqrt(b)*sqrt(1 + 2/(b*x))
```

## 3.613 $\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=38

$$\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -Sqrt[2 + b*x]/(3*x^(3/2)) + (b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{3x^{3/2}}-\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= -\frac{\sqrt{2+bx}}{3x^{3/2}}+\frac{b\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.61

$$\frac{(bx-1)\sqrt{bx+2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[2 + b*x]),x]
[Out] ((-1 + b*x)*Sqrt[2 + b*x])/(3*x^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.07, size = 23, normalized size = 0.61

$$\frac{(bx-1)\sqrt{bx+2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*Sqrt[2 + b*x]),x]
[Out] ((-1 + b*x)*Sqrt[2 + b*x])/(3*x^(3/2))
```

**fricas** [A] time = 1.11, size = 17, normalized size = 0.45

$$\frac{\sqrt{bx+2}\,(bx-1)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x + 2)*(b*x - 1)/x^(3/2)
```

**giac** [A] time = 1.09, size = 42, normalized size = 1.11

$$\frac{\left((bx+2)b^3-3\,b^3\right)\sqrt{bx+2}\,b}{3\,((bx+2)b-2\,b)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] 1/3*((b*x + 2)*b^3 - 3*b^3)*sqrt(b*x + 2)*b/(((b*x + 2)*b - 2*b)^(3/2)*abs(
b))
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.47

$$\frac{\sqrt{bx+2}\,(bx-1)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/3*(b*x+2)^(1/2)*(b*x-1)/x^(3/2)
```

**maxima** [A] time = 1.32, size = 26, normalized size = 0.68

$$\frac{\sqrt{bx+2}\,b}{2\sqrt{x}} - \frac{(bx+2)^{\frac{3}{2}}}{6\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/6*(b*x + 2)^(3/2)/x^(3/2)
```

**mupad** [B] time = 0.32, size = 17, normalized size = 0.45

$$\frac{\sqrt{b\,x+2}\,\left(\frac{b\,x}{3}-\frac{1}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] ((b*x + 2)^(1/2)*((b*x)/3 - 1/3))/x^(3/2)
```

**sympy** [A] time = 1.87, size = 34, normalized size = 0.89

$$\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3} - \frac{\sqrt{b}\,\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(1/2),x)
```

```
[Out] b**(3/2)*sqrt(1 + 2/(b*x))/3 - sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

## 3.614 $\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=59

$$-\frac{2b^2\sqrt{bx+2}}{15\sqrt{x}}+\frac{2b\sqrt{bx+2}}{15x^{3/2}}-\frac{\sqrt{bx+2}}{5x^{5/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2\sqrt{bx+2}}{15\sqrt{x}}+\frac{2b\sqrt{bx+2}}{15x^{3/2}}-\frac{\sqrt{bx+2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -Sqrt[2 + b*x]/(5*x^(5/2)) + (2*b*Sqrt[2 + b*x])/(15*x^(3/2)) - (2*b^2*Sqrt
[2 + b*x])/(15*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{5x^{5/2}} - \frac{1}{5}(2b)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx \\
&= -\frac{\sqrt{2+bx}}{5x^{5/2}} + \frac{2b\sqrt{2+bx}}{15x^{3/2}} + \frac{1}{15}\left(2b^2\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx \\
&= -\frac{\sqrt{2+bx}}{5x^{5/2}} + \frac{2b\sqrt{2+bx}}{15x^{3/2}} - \frac{2b^2\sqrt{2+bx}}{15\sqrt{x}}
\end{aligned}
$$

**Mathematica [A]** time = 0.01, size = 32, normalized size = 0.54

$$
-\frac{\sqrt{bx+2}\left(2b^2x^2-2bx+3\right)}{15x^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -1/15*(Sqrt[2 + b*x]*(3 - 2*b*x + 2*b^2*x^2))/x^(5/2)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 32, normalized size = 0.54

$$
\frac{\sqrt{bx+2}\left(-2b^2x^2+2bx-3\right)}{15x^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

```
[Out] (Sqrt[2 + b*x]*(-3 + 2*b*x - 2*b^2*x^2))/(15*x^(5/2))
```

**fricas [A]** time = 0.87, size = 26, normalized size = 0.44

$$
-\frac{\left(2\,b^2x^2-2\,bx+3\right)\sqrt{bx+2}}{15\,x^{\frac{5}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*b^2*x^2 - 2*b*x + 3)*sqrt(b*x + 2)/x^(5/2)
```

**giac [A]** time = 1.06, size = 55, normalized size = 0.93

$$-\frac{\left(15\,b^{5}+2\left((bx+2)b^{5}-5\,b^{5}\right)(bx+2)\right)\sqrt{bx+2}\,b}{15\left((bx+2)b-2\,b\right)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(15*b^5 + 2*((b*x + 2)*b^5 - 5*b^5)*(b*x + 2))*sqrt(b*x + 2)*b/(((b*x
 + 2)*b - 2*b)^(5/2)*abs(b))
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.46

$$-\frac{\sqrt{bx+2}\,\left(2b^{2}x^{2}-2bx+3\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+2)^(1/2),x)
```

```
[Out] -1/15*(b*x+2)^(1/2)*(2*b^2*x^2-2*b*x+3)/x^(5/2)
```

**maxima [A]** time = 1.31, size = 41, normalized size = 0.69

$$-\frac{\sqrt{bx+2}\,b^{2}}{4\,\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{20\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(b*x + 2)*b^2/sqrt(x) + 1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/20*(b*x
+ 2)^(5/2)/x^(5/2)
```

**mupad [B]** time = 0.32, size = 26, normalized size = 0.44

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^{2}\,x^{2}}{15}-\frac{2\,b\,x}{15}+\frac{1}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -((b*x + 2)^(1/2)*((2*b^2*x^2)/15 - (2*b*x)/15 + 1/5))/x^(5/2)
```

**sympy [B]** time = 6.10, size = 224, normalized size = 3.80

$$-\frac{2b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{6b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{3b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{4b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{12b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+2)**(1/2),x)
```

```
[Out] -2*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 + 60*b**4*
x**2) - 6*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 + 6
0*b**4*x**2) - 3*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x
**3 + 60*b**4*x**2) - 4*b**(11/2)*x*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b*
*5*x**3 + 60*b**4*x**2) - 12*b**(9/2)*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*
b**5*x**3 + 60*b**4*x**2)
```

## 3.615 $\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=80

$$\frac{2b^3\sqrt{bx+2}}{35\sqrt{x}}-\frac{2b^2\sqrt{bx+2}}{35x^{3/2}}+\frac{3b\sqrt{bx+2}}{35x^{5/2}}-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2\sqrt{bx+2}}{35x^{3/2}}+\frac{2b^3\sqrt{bx+2}}{35\sqrt{x}}+\frac{3b\sqrt{bx+2}}{35x^{5/2}}-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -Sqrt[2 + b*x]/(7*x^(7/2)) + (3*b*Sqrt[2 + b*x])/(35*x^(5/2)) - (2*b^2*Sqrt
[2 + b*x])/(35*x^(3/2)) + (2*b^3*Sqrt[2 + b*x])/(35*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{2+bx}}{7x^{7/2}} - \frac{1}{7}(3b)\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx$$
$$= -\frac{\sqrt{2+bx}}{7x^{7/2}} + \frac{3b\sqrt{2+bx}}{35x^{5/2}} + \frac{1}{35}\left(6b^2\right)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx$$
$$= -\frac{\sqrt{2+bx}}{7x^{7/2}} + \frac{3b\sqrt{2+bx}}{35x^{5/2}} - \frac{2b^2\sqrt{2+bx}}{35x^{3/2}} - \frac{1}{35}\left(2b^3\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx$$
$$= -\frac{\sqrt{2+bx}}{7x^{7/2}} + \frac{3b\sqrt{2+bx}}{35x^{5/2}} - \frac{2b^2\sqrt{2+bx}}{35x^{3/2}} + \frac{2b^3\sqrt{2+bx}}{35\sqrt{x}}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 0.50

$$\frac{\sqrt{bx+2}\left(2b^3x^3-2b^2x^2+3bx-5\right)}{35x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[2 + b*x]),x]
[Out] (Sqrt[2 + b*x]*(-5 + 3*b*x - 2*b^2*x^2 + 2*b^3*x^3))/(35*x^(7/2))
```

**IntegrateAlgebraic** [A] time = 0.08, size = 40, normalized size = 0.50

$$\frac{\sqrt{bx+2}\left(2b^3x^3-2b^2x^2+3bx-5\right)}{35x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(9/2)*Sqrt[2 + b*x]),x]
[Out] (Sqrt[2 + b*x]*(-5 + 3*b*x - 2*b^2*x^2 + 2*b^3*x^3))/(35*x^(7/2))
```

**fricas** [A] time = 0.81, size = 34, normalized size = 0.42

$$\frac{\left(2\,b^3x^3-2\,b^2x^2+3\,bx-5\right)\sqrt{bx+2}}{35\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] 1/35*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x - 5)*sqrt(b*x + 2)/x^(7/2)
```

**giac [A]** time = 1.03, size = 68, normalized size = 0.85

$$-\frac{\left(35\,b^{7}-\left(35\,b^{7}+2\left((bx+2)b^{7}-7\,b^{7}\right)(bx+2)\right)(bx+2)\right)\sqrt{bx+2}\,b}{35\left((bx+2)b-2\,b\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35*(35*b^7 - (35*b^7 + 2*((b*x + 2)*b^7 - 7*b^7)*(b*x + 2))*(b*x + 2))*s
qrt(b*x + 2)*b/(((b*x + 2)*b - 2*b)^(7/2)*abs(b))
```

**maple [A]** time = 0.01, size = 35, normalized size = 0.44

$$\frac{\sqrt{bx+2}\,\left(2b^{3}x^{3}-2b^{2}x^{2}+3bx-5\right)}{35x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/35*(b*x+2)^(1/2)*(2*b^3*x^3-2*b^2*x^2+3*b*x-5)/x^(7/2)
```

**maxima [A]** time = 1.29, size = 56, normalized size = 0.70

$$\frac{\sqrt{bx+2}\,b^{3}}{8\,\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}b^{2}}{8\,x^{\frac{3}{2}}}+\frac{3\,(bx+2)^{\frac{5}{2}}b}{40\,x^{\frac{5}{2}}}-\frac{(bx+2)^{\frac{7}{2}}}{56\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(b*x + 2)*b^3/sqrt(x) - 1/8*(b*x + 2)^(3/2)*b^2/x^(3/2) + 3/40*(b*x
 + 2)^(5/2)*b/x^(5/2) - 1/56*(b*x + 2)^(7/2)/x^(7/2)
```

**mupad [B]** time = 0.33, size = 33, normalized size = 0.41

$$\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^{3}\,x^{3}}{35}-\frac{2\,b^{2}\,x^{2}}{35}+\frac{3\,b\,x}{35}-\frac{1}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] ((b*x + 2)^(1/2)*((3*b*x)/35 - (2*b^2*x^2)/35 + (2*b^3*x^3)/35 - 1/7))/x^(7
/2)
```

**sympy [B]** time = 15.99, size = 374, normalized size = 4.68

$$\frac{2b^{\frac{31}{2}}x^6\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{10b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{15b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{5b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{10b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{36b^{\frac{21}{2}}x\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{40b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*b**(31/2)*x**6*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b*
*10*x**4 + 280*b**9*x**3) + 10*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(35*b**12*x
**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) + 15*b**(27/2)*x**4*
sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b*
*9*x**3) + 5*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x*
*5 + 420*b**10*x**4 + 280*b**9*x**3) - 10*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/
(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) - 36*b**(
21/2)*x*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4
+ 280*b**9*x**3) - 40*b**(19/2)*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**1
1*x**5 + 420*b**10*x**4 + 280*b**9*x**3)
```

# 3.616 $\int \frac{x^{5/2}}{(2+bx)^{3/2}} dx$

**Optimal**. Leaf size=86

$$\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{15\sqrt{x}\sqrt{bx+2}}{2b^3}+\frac{5x^{3/2}\sqrt{bx+2}}{2b^2}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$\frac{5x^{3/2}\sqrt{bx+2}}{2b^2}-\frac{15\sqrt{x}\sqrt{bx+2}}{2b^3}+\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*x^(5/2))/(b*Sqrt[2 + b*x]) - (15*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) + (5*x^
(3/2)*Sqrt[2 + b*x])/(2*b^2) + (15*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx &= -\frac{2x^{5/2}}{b\sqrt{2+bx}}+\frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{b}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}-\frac{15\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b^2}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.35

$$\frac{x^{7/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 + b*x)^(3/2),x]
[Out] (x^(7/2)*Hypergeometric2F1[3/2, 7/2, 9/2, -1/2*(b*x)])/(7*Sqrt[2])
```

**IntegrateAlgebraic [A]** time = 0.12, size = 72, normalized size = 0.84

$$\frac{b^2x^{5/2}-5bx^{3/2}-30\sqrt{x}}{2b^3\sqrt{bx+2}}-\frac{15\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-30*Sqrt[x] - 5*b*x^(3/2) + b^2*x^(5/2))/(2*b^3*Sqrt[2 + b*x]) - (15*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

**fricas [A]** time = 1.25, size = 152, normalized size = 1.77

$$\left[\frac{15\,(bx+2)\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+\left(b^3x^2-5\,b^2x-30\,b\right)\sqrt{bx+2}\,\sqrt{x}}{2\left(b^5x+2\,b^4\right)}, -\frac{30\,(bx+2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(b^3x^2-5\,b^2x-30\,b\right)\sqrt{bx+2}\,\sqrt{x}}{2\left(b^5x+2\,b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*(b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^3*x^2 - 5*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4), -1/2*(30*(b*x + 2)*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) - (b^3*x^2 - 5*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4)]
```

**giac [A]** time = 11.12, size = 119, normalized size = 1.38

$$\frac{\left(\sqrt{(bx+2)b-2\,b}\,\sqrt{bx+2}\left(\frac{bx+2}{b^3}-\frac{9}{b^3}\right)-\frac{15\,\log\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2\right)}{b^{\frac{5}{2}}}-\frac{64}{\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)b^{\frac{3}{2}}}\right)|b|}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*((b*x + 2)/b^3 - 9/b^3) - 15*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/b^(5/2) - 64/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*b^(3/2)))*abs(b)/b^2
```

**maple [A]** time = 0.03, size = 106, normalized size = 1.23

$$\frac{\left(\frac{15\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{2b^{\frac{7}{2}}}-\frac{8\sqrt{\left(x+\frac{2}{b}\right)^2 b-2x-\frac{4}{b}}}{\left(x+\frac{2}{b}\right)b^4}\right)\sqrt{(bx+2)\,x}}{\sqrt{bx+2}\,\sqrt{x}}+\frac{(bx-7)\sqrt{bx+2}\,\sqrt{x}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(3/2),x)
```

```
[Out] 1/2*(b*x-7)*(b*x+2)^(1/2)*x^(1/2)/b^3+(15/2/b^(7/2)*ln((b*x+1)/b^(1/2)+(b*x
^2+2*x)^(1/2))-8/b^4/(x+2/b)*(b*(x+2/b)^2-2*x-4/b)^(1/2))*((b*x+2)*x)^(1/2)
/(b*x+2)^(1/2)/x^(1/2)
```

**maxima [A]** time = 3.06, size = 119, normalized size = 1.38

$$-\frac{8\,b^2-\frac{25\,(bx+2)b}{x}+\frac{15\,(bx+2)^2}{x^2}}{\frac{\sqrt{bx+2}\,b^5}{\sqrt{x}}-\frac{2\,(bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}}-\frac{15\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(8*b^2 - 25*(b*x + 2)*b/x + 15*(b*x + 2)^2/x^2)/(sqrt(b*x + 2)*b^5/sqrt(x)
 - 2*(b*x + 2)^(3/2)*b^4/x^(3/2) + (b*x + 2)^(5/2)*b^3/x^(5/2)) - 15/2*log(
-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/
2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(b\,x+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(5/2)/(b*x + 2)^(3/2), x)
```

**sympy [A]** time = 7.10, size = 80, normalized size = 0.93

$$\frac{x^{\frac{5}{2}}}{2b\sqrt{bx+2}}-\frac{5x^{\frac{3}{2}}}{2b^2\sqrt{bx+2}}-\frac{15\sqrt{x}}{b^3\sqrt{bx+2}}+\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(3/2),x)
```

```
[Out] x**(5/2)/(2*b*sqrt(b*x + 2)) - 5*x**(3/2)/(2*b**2*sqrt(b*x + 2)) - 15*sqrt(
x)/(b**3*sqrt(b*x + 2)) + 15*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(7/2)
```

## 3.617 $\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=63

$$-\frac{6\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\sqrt{bx+2}}{b^2}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$\frac{3\sqrt{x}\sqrt{bx+2}}{b^2}-\frac{6\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*x^(3/2))/(b*Sqrt[2 + b*x]) + (3*Sqrt[x]*Sqrt[2 + b*x])/b^2 - (6*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx &= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{b}\\
&= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\sqrt{x}\,\sqrt{2+bx}}{b^2}-\frac{3\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\sqrt{x}\,\sqrt{2+bx}}{b^2}-\frac{6\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\sqrt{x}\,\sqrt{2+bx}}{b^2}-\frac{6\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.48

$$\frac{x^{5/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{bx}{2}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 + b*x)^(3/2),x]
[Out] (x^(5/2)*Hypergeometric2F1[3/2, 5/2, 7/2, -1/2*(b*x)])/(5*Sqrt[2])
```

**IntegrateAlgebraic [A]** time = 0.10, size = 59, normalized size = 0.94

$$\frac{6\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{b^{5/2}}+\frac{bx^{3/2}+6\sqrt{x}}{b^2\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (6*Sqrt[x] + b*x^(3/2))/(b^2*Sqrt[2 + b*x]) + (6*Log[-(Sqrt[b]*Sqrt[x]) + S
qrt[2 + b*x]])/b^(5/2)
```

**fricas [A]** time = 1.33, size = 134, normalized size = 2.13

$$\left[\frac{3\,(bx+2)\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+\left(b^2x+6\,b\right)\sqrt{bx+2}\,\sqrt{x}}{b^4x+2\,b^3},\frac{6\,(bx+2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(b^2x+6\,b\right)\sqrt{bx+2}\,\sqrt{x}}{b^4x+2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*(b*x + 2)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*x
 + 6*b)*sqrt(b*x + 2)*sqrt(x))/(b^4*x + 2*b^3), (6*(b*x + 2)*sqrt(-b)*arcta
n(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 6*b)*sqrt(b*x + 2)*sqrt(x)
)/(b^4*x + 2*b^3)]
```

**giac [B]** time = 10.16, size = 106, normalized size = 1.68

$$\frac{\left(\frac{3\,\log\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2\right)}{\sqrt{b}}+\frac{\sqrt{(bx+2)b-2\,b}\,\sqrt{bx+2}}{b}+\frac{16\,\sqrt{b}}{\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b}\right)|b|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] (3*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) + sqrt(
(b*x + 2)*b - 2*b)*sqrt(b*x + 2)/b + 16*sqrt(b)/((sqrt(b*x + 2)*sqrt(b) - s
qrt((b*x + 2)*b - 2*b))^2 + 2*b))*abs(b)/b^3
```

**maple [B]** time = 0.02, size = 100, normalized size = 1.59

$$\frac{\left(-\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{b^{\frac{5}{2}}}+\frac{4\sqrt{\left(x+\frac{2}{b}\right)^2b-2x-\frac{4}{b}}}{\left(x+\frac{2}{b}\right)b^3}\right)\sqrt{(bx+2)\,x}}{\sqrt{bx+2}\,\sqrt{x}}+\frac{\sqrt{bx+2}\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(3/2),x)
```

```
[Out] (b*x+2)^(1/2)/b^2*x^(1/2)+(-3/b^(5/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
+4/b^3/(x+2/b)*((x+2/b)^2*b-2*x-4/b)^(1/2))*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)
/x^(1/2)
```

**maxima** [A] time = 2.99, size = 90, normalized size = 1.43

$$\frac{2\left(2\,b-\frac{3\,(bx+2)}{x}\right)}{\frac{\sqrt{bx+2}\,b^{3}}{\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}}+\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(2*b - 3*(b*x + 2)/x)/(sqrt(b*x + 2)*b^3/sqrt(x) - (b*x + 2)^(3/2)*b^2/x^
(3/2)) + 3*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x)))/b^(5/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{(b\,x+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(3/2), x)
```

**sympy** [A] time = 3.06, size = 58, normalized size = 0.92

$$\frac{x^{\frac{3}{2}}}{b\sqrt{bx+2}}+\frac{6\sqrt{x}}{b^{2}\sqrt{bx+2}}-\frac{6\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(3/2),x)
```

```
[Out] x**(3/2)/(b*sqrt(b*x + 2)) + 6*sqrt(x)/(b**2*sqrt(b*x + 2)) - 6*asinh(sqrt(
2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

## 3.618 $\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=44

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 54, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(
3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx = -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

**Mathematica** [A] time = 0.03, size = 44, normalized size = 1.00

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}} - \frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 + b*x)^(3/2),x]
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(
3/2)
```

**IntegrateAlgebraic** [A] time = 0.08, size = 50, normalized size = 1.14

$$-\frac{2\log\left(\sqrt{bx+2} - \sqrt{b}\sqrt{x}\right)}{b^{3/2}} - \frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(2 + b*x)^(3/2),x]
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]]
)/b^(3/2)
```

**fricas** [A] time = 1.29, size = 117, normalized size = 2.66

$$\left[\frac{(bx+2)\sqrt{b}\,\log\left(bx + \sqrt{bx+2}\,\sqrt{b}\,\sqrt{x} + 1\right) - 2\,\sqrt{bx+2}\,b\sqrt{x}}{b^3x + 2\,b^2}, -\frac{2\left((bx+2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right) + \sqrt{bx+2}\,b\sqrt{x}\right)}{b^3x + 2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(b
*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2), -2*((b*x + 2)*sqrt(-b)*arctan(sqrt(b*x
+ 2)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2)]
```

**giac [B]** time = 10.61, size = 82, normalized size = 1.86

$$-\frac{\left(\frac{\log\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}\right)}{\sqrt{b}}+\frac{8\,\sqrt{b}}{\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}+2\,b}\right)|b|}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -(log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) + 8*sqrt
(b)/((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b))*abs(b)/b^2
```

**maple [A]** time = 0.11, size = 48, normalized size = 1.09

$$\frac{-\frac{\sqrt{\pi}\,\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{\sqrt{\frac{bx}{2}+1}}+2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{\pi}\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(3/2),x)
```

```
[Out] 2/b^(3/2)/Pi^(1/2)*(-1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)/(1/2*b*x+1)^(1/2)
+Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

**maxima [A]** time = 2.90, size = 57, normalized size = 1.30

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\,\sqrt{x}}{\sqrt{bx+2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
b^(3/2) - 2*sqrt(x)/(sqrt(b*x + 2)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(b\,x+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(1/2)/(b*x + 2)^(3/2), x)
```

**sympy [A]** time = 1.56, size = 41, normalized size = 0.93

$$-\frac{2\sqrt{x}}{b\sqrt{bx+2}}+\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(3/2),x)
```

```
[Out] -2*sqrt(x)/(b*sqrt(b*x + 2)) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

## 3.619 $\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=15

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]
[Out] Sqrt[x]/Sqrt[2 + b*x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{2+bx}}$$

**Mathematica [A]** time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]
[Out] Sqrt[x]/Sqrt[2 + b*x]
```

**IntegrateAlgebraic [A]** time = 0.02, size = 15, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 + b*x]
```

**fricas [A]** time = 1.32, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x)/sqrt(b*x + 2)
```

**giac [B]** time = 1.22, size = 44, normalized size = 2.93

$$\frac{4\,b^{\frac{3}{2}}}{\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}+2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(
b))
```

**maple [A]** time = 0.00, size = 12, normalized size = 0.80

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] x^(1/2)/(b*x+2)^(1/2)
```

**maxima** [A] time = 1.34, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/sqrt(b*x + 2)
```

**mupad** [B] time = 0.31, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{b\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] x^(1/2)/(b*x + 2)^(1/2)
```

**sympy** [A] time = 0.86, size = 15, normalized size = 1.00

$$\frac{1}{\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] 1/(sqrt(b)*sqrt(1 + 2/(b*x)))
```

## 3.620 $\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=32

$$\frac{1}{\sqrt{x}\sqrt{bx+2}}-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

**Rubi** [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{1}{\sqrt{x}\sqrt{bx+2}}-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(Sqrt[x]*Sqrt[2 + b*x]) - Sqrt[2 + b*x]/Sqrt[x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx &= \frac{1}{\sqrt{x}\sqrt{2+bx}}+\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= \frac{1}{\sqrt{x}\sqrt{2+bx}}-\frac{\sqrt{2+bx}}{\sqrt{x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 21, normalized size = 0.66

$$\frac{-bx-1}{\sqrt{x}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
[Out] (-1 - b*x)/(Sqrt[x]*Sqrt[2 + b*x])
```

**IntegrateAlgebraic** [A] time = 0.06, size = 21, normalized size = 0.66

$$\frac{-bx-1}{\sqrt{x}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
[Out] (-1 - b*x)/(Sqrt[x]*Sqrt[2 + b*x])
```

**fricas** [A] time = 1.00, size = 28, normalized size = 0.88

$$-\frac{\sqrt{bx+2}\,(bx+1)\sqrt{x}}{bx^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
[Out] -sqrt(b*x + 2)*(b*x + 1)*sqrt(x)/(b*x^2 + 2*x)
```

**giac** [B] time = 1.12, size = 74, normalized size = 2.31

$$-\frac{\sqrt{bx+2}\,b^2}{2\,\sqrt{(bx+2)b-2\,b}\,|b|}-\frac{2\,b^{\frac{5}{2}}}{\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="giac")
[Out] -1/2*sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b)) - 2*b^(5/2)/(((sqrt
(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(b))
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.56

$$-\frac{bx+1}{\sqrt{bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(3/2),x)
[Out] -(b*x+1)/(b*x+2)^(1/2)/x^(1/2)
```

**maxima** [A] time = 1.34, size = 26, normalized size = 0.81

$$-\frac{b\sqrt{x}}{2\sqrt{bx+2}}-\frac{\sqrt{bx+2}}{2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
[Out] -1/2*b*sqrt(x)/sqrt(b*x + 2) - 1/2*sqrt(b*x + 2)/sqrt(x)
```

**mupad** [B] time = 0.35, size = 17, normalized size = 0.53

$$-\frac{b\,x+1}{\sqrt{x}\,\sqrt{b\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(3/2)),x)
[Out] -(b*x + 1)/(x^(1/2)*(b*x + 2)^(1/2))
```

**sympy** [A] time = 1.54, size = 34, normalized size = 1.06

$$-\frac{\sqrt{b}}{\sqrt{1+\frac{2}{bx}}}-\frac{1}{\sqrt{b}\,x\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(3/2),x)
[Out] -sqrt(b)/sqrt(1 + 2/(b*x)) - 1/(sqrt(b)*x*sqrt(1 + 2/(b*x)))
```

## 3.621 $\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=53

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

**Rubi** [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x])/(3*x^(3/2)) + (2*b*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx = \frac{1}{x^{3/2}\sqrt{2+bx}} + 2\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx$$
$$= \frac{1}{x^{3/2}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3x^{3/2}} - \frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx$$
$$= \frac{1}{x^{3/2}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3x^{3/2}} + \frac{2b\sqrt{2+bx}}{3\sqrt{x}}$$

**Mathematica** [**A**] time = 0.01, size = 32, normalized size = 0.60

$$\frac{2b^2x^2+2bx-1}{3x^{3/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
[Out] (-1 + 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 + b*x])
```

**IntegrateAlgebraic** [**A**] time = 0.09, size = 32, normalized size = 0.60

$$\frac{2b^2x^2+2bx-1}{3x^{3/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
[Out] (-1 + 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 + b*x])
```

**fricas** [**A**] time = 0.64, size = 39, normalized size = 0.74

$$\frac{\left(2\,b^2x^2+2\,bx-1\right)\sqrt{bx+2}\,\sqrt{x}}{3\left(bx^3+2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(2*b^2*x^2 + 2*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^3 + 2*x^2)
```

**giac [B]** time = 1.23, size = 86, normalized size = 1.62

$$\frac{b^{\frac{7}{2}}}{\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}+2\,b\right)|b|}+\frac{\left(5\,(bx+2)b^{2}|b|-12\,b^{2}|b|\right)\sqrt{bx+2}}{12\,((bx+2)b-2\,b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] b^(7/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(b)
) + 1/12*(5*(b*x + 2)*b^2*abs(b) - 12*b^2*abs(b))*sqrt(b*x + 2)/((b*x + 2)*
b - 2*b)^(3/2)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.51

$$\frac{2b^{2}x^{2}+2bx-1}{3\sqrt{bx+2}\;x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(3/2),x)
```

```
[Out] 1/3*(2*b^2*x^2+2*b*x-1)/(b*x+2)^(1/2)/x^(3/2)
```

**maxima [A]** time = 1.37, size = 41, normalized size = 0.77

$$\frac{b^{2}\sqrt{x}}{4\,\sqrt{bx+2}}+\frac{\sqrt{bx+2}\,b}{2\,\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}}{12\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*sqrt(x)/sqrt(b*x + 2) + 1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/12*(b*x + 2
)^(3/2)/x^(3/2)
```

**mupad [B]** time = 0.38, size = 37, normalized size = 0.70

$$\frac{\sqrt{b\,x+2}\,\left(\frac{2\,x}{3}+\frac{2\,b\,x^2}{3}-\frac{1}{3\,b}\right)}{x^{5/2}+\frac{2\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] ((b*x + 2)^(1/2)*((2*x)/3 + (2*b*x^2)/3 - 1/(3*b)))/(x^(5/2) + (2*x^(3/2))/
b)
```

**sympy [B]** time = 3.86, size = 170, normalized size = 3.21

$$\frac{2b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}+\frac{6b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}+\frac{3b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}-\frac{2b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(3/2),x)
```

```
[Out] 2*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4*x)
 + 6*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4
*x) + 3*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4
*x) - 2*b**(9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4*x)
```

## 3.622 $\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=74

$$-\frac{2b^2\sqrt{bx+2}}{5\sqrt{x}}+\frac{2b\sqrt{bx+2}}{5x^{3/2}}-\frac{3\sqrt{bx+2}}{5x^{5/2}}+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2\sqrt{bx+2}}{5\sqrt{x}}+\frac{2b\sqrt{bx+2}}{5x^{3/2}}-\frac{3\sqrt{bx+2}}{5x^{5/2}}+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(x^(5/2)*Sqrt[2 + b*x]) - (3*Sqrt[2 + b*x])/(5*x^(5/2)) + (2*b*Sqrt[2 + b
*x])/(5*x^(3/2)) - (2*b^2*Sqrt[2 + b*x])/(5*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx &= \frac{1}{x^{5/2}\sqrt{2+bx}} + 3\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx \\
&= \frac{1}{x^{5/2}\sqrt{2+bx}} - \frac{3\sqrt{2+bx}}{5x^{5/2}} - \frac{1}{5}(6b)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx \\
&= \frac{1}{x^{5/2}\sqrt{2+bx}} - \frac{3\sqrt{2+bx}}{5x^{5/2}} + \frac{2b\sqrt{2+bx}}{5x^{3/2}} + \frac{1}{5}\left(2b^2\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx \\
&= \frac{1}{x^{5/2}\sqrt{2+bx}} - \frac{3\sqrt{2+bx}}{5x^{5/2}} + \frac{2b\sqrt{2+bx}}{5x^{3/2}} - \frac{2b^2\sqrt{2+bx}}{5\sqrt{x}}
\end{aligned}
$$

**Mathematica** [A] time = 0.01, size = 39, normalized size = 0.53

$$\frac{-2b^3x^3 - 2b^2x^2 + bx - 1}{5x^{5/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
[Out] (-1 + b*x - 2*b^2*x^2 - 2*b^3*x^3)/(5*x^(5/2)*Sqrt[2 + b*x])
```

**IntegrateAlgebraic** [A] time = 0.10, size = 39, normalized size = 0.53

$$\frac{-2b^3x^3 - 2b^2x^2 + bx - 1}{5x^{5/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
[Out] (-1 + b*x - 2*b^2*x^2 - 2*b^3*x^3)/(5*x^(5/2)*Sqrt[2 + b*x])
```

**fricas** [A] time = 1.22, size = 47, normalized size = 0.64

$$-\frac{\left(2\,b^3x^3 + 2\,b^2x^2 - bx + 1\right)\sqrt{bx+2}\,\sqrt{x}}{5\left(bx^4 + 2\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*b^3*x^3 + 2*b^2*x^2 - b*x + 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^4 + 2*x^3
)
```

**giac** [B] time = 1.11, size = 107, normalized size = 1.45

$$-\frac{b^{\frac{9}{2}}}{2\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}+2\,b\right)|b|}-\frac{\left(\frac{60\,b^{6}}{|b|}+\left(\frac{11\,(bx+2)b^{6}}{|b|}-\frac{50\,b^{6}}{|b|}\right)(bx+2)\right)\sqrt{bx+2}}{40\,((bx+2)b-2\,b)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*b^(9/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*a
bs(b)) - 1/40*(60*b^6/abs(b) + (11*(b*x + 2)*b^6/abs(b) - 50*b^6/abs(b))*(b
*x + 2))*sqrt(b*x + 2)/((b*x + 2)*b - 2*b)^(5/2)
```

**maple** [A] time = 0.00, size = 35, normalized size = 0.47

$$-\frac{2b^{3}x^{3}+2b^{2}x^{2}-bx+1}{5\sqrt{bx+2}\;x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+2)^(3/2),x)
```

```
[Out] -1/5*(2*b^3*x^3+2*b^2*x^2-b*x+1)/(b*x+2)^(1/2)/x^(5/2)
```

**maxima** [A] time = 1.30, size = 56, normalized size = 0.76

$$-\frac{b^{3}\sqrt{x}}{8\sqrt{bx+2}}-\frac{3\sqrt{bx+2}\,b^{2}}{8\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}b}{8\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{40\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*b^3*sqrt(x)/sqrt(b*x + 2) - 3/8*sqrt(b*x + 2)*b^2/sqrt(x) + 1/8*(b*x +
 2)^(3/2)*b/x^(3/2) - 1/40*(b*x + 2)^(5/2)/x^(5/2)
```

**mupad** [B] time = 0.43, size = 46, normalized size = 0.62

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b\,x^{2}}{5}-\frac{x}{5}+\frac{1}{5\,b}+\frac{2\,b^{2}\,x^{3}}{5}\right)}{x^{7/2}+\frac{2\,x^{5/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(7/2)*(b*x + 2)^(3/2)),x)

[Out] -((b*x + 2)^(1/2)*((2*b*x^2)/5 - x/5 + 1/(5*b) + (2*b^2*x^3)/5))/(x^(7/2) + (2*x^(5/2))/b)

**sympy [B]** time = 10.97, size = 269, normalized size = 3.64

$$-\frac{2b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{10b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{15b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{5b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{4b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(7/2)/(b*x+2)**(3/2),x)

[Out] -2*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) - 10*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) - 15*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) - 5*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) - 4*b**(19/2)*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2)

## 3.623 $\int \frac{x^{5/2}}{(2+bx)^{5/2}} dx$

**Optimal**. Leaf size=86

$$-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{bx+2}}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$-\frac{10x^{3/2}}{3b^2\sqrt{bx+2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{b^3}-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-2*x^(5/2))/(3*b*(2 + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 + b*x]) + (
5*Sqrt[x]*Sqrt[2 + b*x])/b^3 - (10*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx &= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}+\frac{5\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx}{3b}\\
&= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{b^2}\\
&= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\sqrt{x}\,\sqrt{2+bx}}{b^3}-\frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{b^3}\\
&= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\sqrt{x}\,\sqrt{2+bx}}{b^3}-\frac{10\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\sqrt{x}\,\sqrt{2+bx}}{b^3}-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.35

$$\frac{x^{7/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{2}\right)}{14\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[5/2, 7/2, 9/2, -1/2*(b*x)])/(14*Sqrt[2])
```

**IntegrateAlgebraic [A]** time = 0.13, size = 73, normalized size = 0.85

$$\frac{10\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{b^{7/2}}+\frac{3b^2x^{5/2}+40bx^{3/2}+60\sqrt{x}}{3b^3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (60*Sqrt[x] + 40*b*x^(3/2) + 3*b^2*x^(5/2))/(3*b^3*(2 + b*x)^(3/2)) + (10*L
og[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

**fricas [A]** time = 1.35, size = 186, normalized size = 2.16

$$\left[\frac{15\left(b^{2}x^{2}+4\,bx+4\right)\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+\left(3\,b^{3}x^{2}+40\,b^{2}x+60\,b\right)\sqrt{bx+2}\,\sqrt{x}}{3\left(b^{6}x^{2}+4\,b^{5}x+4\,b^{4}\right)},\ \frac{30\left(b^{2}x^{2}+4\,bx+4\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(3\,b^{3}x^{2}+40\,b^{2}x+60\,b\right)\sqrt{bx+2}\,\sqrt{x}}{3\left(b^{6}x^{2}+4\,b^{5}x+4\,b^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt
(x) + 1) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt(x))/(b^6*x^2 +
4*b^5*x + 4*b^4), 1/3*(30*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt
(x))/(b^6*x^2 + 4*b^5*x + 4*b^4)]
```

**giac [B]** time = 10.82, size = 182, normalized size = 2.12

$$\frac{\left(\frac{15\,\log\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}\right)}{b^{\frac{5}{2}}}+\frac{3\,\sqrt{(bx+2)b-2\,b}\,\sqrt{bx+2}}{b^{3}}+\frac{16\left(9\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{4}\sqrt{b}+24\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}b^{\frac{3}{2}}+28\,b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}+2\,b\right)^{3}b^{2}}\right)|b|}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(15*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/b^(5/2) +
3*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)/b^3 + 16*(9*(sqrt(b*x + 2)*sqrt(b)
- sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 24*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*
x + 2)*b - 2*b))^2*b^(3/2) + 28*b^(5/2))/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b
*x + 2)*b - 2*b))^2 + 2*b)^3*b^2))*abs(b)/b^2
```

**maple [B]** time = 0.04, size = 136, normalized size = 1.58

$$\frac{\left(-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{b^{\frac{7}{2}}}+\frac{28\sqrt{\left(x+\frac{2}{b}\right)^{2}b-2x-\frac{4}{b}}}{3\left(x+\frac{2}{b}\right)b^{4}}-\frac{8\sqrt{\left(x+\frac{2}{b}\right)^{2}b-2x-\frac{4}{b}}}{3\left(x+\frac{2}{b}\right)^{2}b^{5}}\right)\sqrt{(bx+2)\,x}}{\sqrt{bx+2}\,\sqrt{x}}+\frac{\sqrt{bx+2}\,\sqrt{x}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(5/2),x)
```

```
[Out] (b*x+2)^(1/2)/b^3*x^(1/2)+(-5/b^(7/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
+28/3/(x+2/b)*((x+2/b)^2*b-2*x-4/b)^(1/2)/b^4-8/3/b^5/(x+2/b)^2*((x+2/b)^2*
b-2*x-4/b)^(1/2))*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)
```

**maxima [A]** time = 3.00, size = 105, normalized size = 1.22

$$\frac{2\left(2\,b^{2}+\frac{10\,(bx+2)b}{x}-\frac{15\,(bx+2)^{2}}{x^{2}}\right)}{3\left(\frac{(bx+2)^{\frac{3}{2}}b^{4}}{x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}b^{3}}{x^{\frac{5}{2}}}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*b^2 + 10*(b*x + 2)*b/x - 15*(b*x + 2)^2/x^2)/((b*x + 2)^(3/2)*b^4/x^
(3/2) - (b*x + 2)^(5/2)*b^3/x^(5/2)) + 5*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt
(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(b\,x+2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(5/2)/(b*x + 2)^(5/2), x)
```

**sympy [B]** time = 6.62, size = 308, normalized size = 3.58

$$\frac{3b^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}+\frac{40b^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}+\frac{60b^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}-\frac{30b^{10}x^{\frac{27}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}-\frac{60b^{9}x^{\frac{25}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(5/2),x)
```

```
[Out] 3*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25
/2)*sqrt(b*x + 2)) + 40*b**(21/2)*x**14/(3*b**(27/2)*x**(27/2)*sqrt(b*x + 2
) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) + 60*b**(19/2)*x**13/(3*b**(27/2)*
x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 30*b**10*x
**(27/2)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27
```

```
/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 60*b**9*x**(25/2
)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27/2)*sqr
t(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2))
```

## 3.624 $\int \frac{x^{3/2}}{(2+bx)^{5/2}} \, dx$

**Optimal**. Leaf size=65

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{bx+2}}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 54, 215}

$$-\frac{2\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*(2 + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 + b*x]) + (2*A
rcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx = -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} + \frac{\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx}{b}$$
$$= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2+bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b^2}$$
$$= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2+bx}} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

**Mathematica [A]** time = 0.07, size = 52, normalized size = 0.80

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}} - \frac{4\sqrt{x}\,(2bx+3)}{3b^2(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-4*Sqrt[x]*(3 + 2*b*x))/(3*b^2*(2 + b*x)^(3/2)) + (2*ArcSinh[(Sqrt[b]*Sqrt
[x])/Sqrt[2]])/b^(5/2)
```

**IntegrateAlgebraic [A]** time = 0.12, size = 63, normalized size = 0.97

$$-\frac{2\log\left(\sqrt{bx+2} - \sqrt{b}\sqrt{x}\right)}{b^{5/2}} - \frac{4\left(2bx^{3/2}+3\sqrt{x}\right)}{3b^2(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-4*(3*Sqrt[x] + 2*b*x^(3/2)))/(3*b^2*(2 + b*x)^(3/2)) - (2*Log[-(Sqrt[b]*S
qrt[x]) + Sqrt[2 + b*x]])/b^(5/2)
```

**fricas [A]** time = 1.29, size = 171, normalized size = 2.63

$$\left[\frac{3\left(b^2x^2+4\,bx+4\right)\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}\,+1\right)-4\left(2\,b^2x+3\,b\right)\sqrt{bx+2}\,\sqrt{x}}{3\left(b^5x^2+4\,b^4x+4\,b^3\right)}, -\frac{2\left(3\left(b^2x^2+4\,bx+4\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+2\left(2\,b^2x+3\,b\right)\sqrt{bx+2}\,\sqrt{x}\right)}{3\left(b^5x^2+4\,b^4x+4\,b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(
x) + 1) - 4*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x + 4*b
^3), -2/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(
b*sqrt(x))) + 2*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x +
 4*b^3)]
```

**giac** [B] time = 10.77, size = 154, normalized size = 2.37

$$-\frac{\left(\frac{3\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^{2}\right)}{\sqrt{b}}+\frac{16\left(3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^{4}\sqrt{b}+6\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^{2}b^{\frac{3}{2}}+8b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^{2}+2b\right)^{3}}\right)|b|}{3b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) +
16*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 6*(sqrt
(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2*b^(3/2) + 8*b^(5/2))/((sqrt(
b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3)*abs(b)/b^3
```

**maple** [A] time = 0.04, size = 55, normalized size = 0.85

$$\frac{-\frac{\sqrt{\pi}\,\sqrt{2}\,(10bx+15)\sqrt{b}\,\sqrt{x}}{15\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}+2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{\pi}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(5/2),x)
```

```
[Out] 4/3/b^(5/2)/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(10*b*x+15)/(1
/2*b*x+1)^(3/2)+3/2*Pi^(1/2)*arcsinh(1/2*2^(1/2)*b^(1/2)*x^(1/2)))
```

**maxima** [A] time = 2.97, size = 69, normalized size = 1.06

$$-\frac{2\left(b+\frac{3(bx+2)}{x}\right)x^{\frac{3}{2}}}{3(bx+2)^{\frac{3}{2}}b^{2}}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b + 3*(b*x + 2)/x)*x^(3/2)/((b*x + 2)^(3/2)*b^2) - log(-(sqrt(b) - sq
rt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{(b\,x+2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(5/2), x)
```

**sympy [B]** time = 3.58, size = 257, normalized size = 3.95

$$-\frac{8b^{\frac{11}{2}}x^{8}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}-\frac{12b^{\frac{9}{2}}x^{7}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}+\frac{6b^{5}x^{\frac{15}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}+\frac{12b^{4}x^{\frac{13}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(5/2),x)
```

```
[Out] -8*b**(11/2)*x**8/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(13/2)*x**(13
/2)*sqrt(b*x + 2)) - 12*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2)
+ 6*b**(13/2)*x**(13/2)*sqrt(b*x + 2)) + 6*b**5*x**(15/2)*sqrt(b*x + 2)*asi
nh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(
13/2)*x**(13/2)*sqrt(b*x + 2)) + 12*b**4*x**(13/2)*sqrt(b*x + 2)*asinh(sqrt
(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(13/2)*x
**(13/2)*sqrt(b*x + 2))
```

## 3.625 $\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=18

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 + b*x)^(5/2),x]
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx=\frac{x^{3/2}}{3(2+bx)^{3/2}}$$

**Mathematica** [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 + b*x)^(5/2),x]
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 18, normalized size = 1.00

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(2 + b*x)^(5/2),x]
```

```
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

**fricas [B]** time = 1.19, size = 27, normalized size = 1.50

$$\frac{\sqrt{bx+2}\,x^{\frac{3}{2}}}{3\left(b^2x^2+4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x + 2)*x^(3/2)/(b^2*x^2 + 4*b*x + 4)
```

**giac [B]** time = 1.22, size = 82, normalized size = 4.56

$$\frac{4\left(3\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^4\sqrt{b}+4\,b^{\frac{5}{2}}\right)|b|}{3\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 4*b^(5
/2))*abs(b)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*
b^2)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.72

$$\frac{x^{\frac{3}{2}}}{3\left(bx+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(5/2),x)
```

```
[Out] 1/3*x^(3/2)/(b*x+2)^(3/2)
```

**maxima [A]** time = 1.33, size = 12, normalized size = 0.67

$$\frac{x^{\frac{3}{2}}}{3\,(bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="maxima")
[Out] 1/3*x^(3/2)/(b*x + 2)^(3/2)
```

**mupad [B]** time = 0.25, size = 12, normalized size = 0.67

$$\frac{x^{3/2}}{3\,(b\,x+2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(5/2),x)
[Out] x^(3/2)/(3*(b*x + 2)^(3/2))
```

**sympy [A]** time = 1.40, size = 27, normalized size = 1.50

$$\frac{x^{\frac{3}{2}}}{3bx\sqrt{bx+2}+6\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(5/2),x)
[Out] x**(3/2)/(3*b*x*sqrt(b*x + 2) + 6*sqrt(b*x + 2))
```

# 3.626 $\int \frac{1}{\sqrt{x}(2+bx)^{5/2}} dx$

**Optimal**. Leaf size=37

$$\frac{\sqrt{x}}{3\sqrt{bx+2}}+\frac{\sqrt{x}}{3(bx+2)^{3/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{\sqrt{x}}{3\sqrt{bx+2}}+\frac{\sqrt{x}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
```

```
[Out] Sqrt[x]/(3*(2 + b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(2+bx)^{5/2}} dx &= \frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{x}(2+bx)^{3/2}} dx\\ &= \frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2+bx}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 23, normalized size = 0.62

$$\frac{\sqrt{x}\,(bx+3)}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
[Out] (Sqrt[x]*(3 + b*x))/(3*(2 + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 23, normalized size = 0.62

$$\frac{\sqrt{x}\,(bx+3)}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
[Out] (Sqrt[x]*(3 + b*x))/(3*(2 + b*x)^(3/2))
```

**fricas [A]** time = 0.81, size = 32, normalized size = 0.86

$$\frac{(bx+3)\sqrt{bx+2}\,\sqrt{x}}{3\left(b^2x^2+4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
[Out] 1/3*(b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^2 + 4*b*x + 4)
```

**giac [B]** time = 1.22, size = 79, normalized size = 2.14

$$\frac{8\left(3\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)b^{\frac{5}{2}}}{3\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
[Out] 8/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*b^(5/2)/(
((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs(b))
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.49

$$\frac{(bx+3)\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*x^(1/2)*(b*x+3)/(b*x+2)^(3/2)
```

**maxima** [A] time = 1.32, size = 24, normalized size = 0.65

$$-\frac{\left(b-\frac{3(bx+2)}{x}\right)x^{\frac{3}{2}}}{6(bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(b - 3*(b*x + 2)/x)*x^(3/2)/(b*x + 2)^(3/2)
```

**mupad** [B] time = 0.36, size = 42, normalized size = 1.14

$$\frac{3\sqrt{x}\,\sqrt{b\,x+2}+b\,x^{3/2}\,\sqrt{b\,x+2}}{3\,b^2\,x^2+12\,b\,x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] (3*x^(1/2)*(b*x + 2)^(1/2) + b*x^(3/2)*(b*x + 2)^(1/2))/(12*b*x + 3*b^2*x^2
 + 12)
```

**sympy** [B] time = 1.84, size = 75, normalized size = 2.03

$$\frac{bx}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\sqrt{1+\frac{2}{bx}}}+\frac{3}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] b*x/(3*b**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x))) + 3/(3*b
**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x)))
```

## 3.627 $\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=55

$$-\frac{2\sqrt{bx+2}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{bx+2}}+\frac{1}{3\sqrt{x}\,(bx+2)^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2\sqrt{bx+2}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{bx+2}}+\frac{1}{3\sqrt{x}\,(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] 1/(3*Sqrt[x]*(2 + b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 + b*x]) - (2*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx &= \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}} + \frac{2}{3}\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx \\ &= \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}} + \frac{2}{3\sqrt{x}\,\sqrt{2+bx}} + \frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx \\ &= \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}} + \frac{2}{3\sqrt{x}\,\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 32, normalized size = 0.58

$$\frac{-2b^2x^2-6bx-3}{3\sqrt{x}\,(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
[Out] (-3 - 6*b*x - 2*b^2*x^2)/(3*Sqrt[x]*(2 + b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 32, normalized size = 0.58

$$\frac{-2b^2x^2-6bx-3}{3\sqrt{x}\,(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
[Out] (-3 - 6*b*x - 2*b^2*x^2)/(3*Sqrt[x]*(2 + b*x)^(3/2))
```

**fricas [A]** time = 1.47, size = 45, normalized size = 0.82

$$-\frac{\left(2\,b^2x^2+6\,bx+3\right)\sqrt{bx+2}\,\sqrt{x}}{3\left(b^2x^3+4\,bx^2+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
[Out] -1/3*(2*b^2*x^2 + 6*b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^3 + 4*b*x^2 + 4*x
)
```

**giac** [B] time = 1.36, size = 145, normalized size = 2.64

$$-\frac{\sqrt{bx+2}\,b^2}{4\,\sqrt{(bx+2)b-2\,b}\,|b|}-\frac{3\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^4 b^{\frac{5}{2}}+24\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2 b^{\frac{7}{2}}+20\,b^{\frac{9}{2}}}{3\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b)) - 1/3*(3*(sqrt(b*x
+ 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*b^(5/2) + 24*(sqrt(b*x + 2)*sqrt(
b) - sqrt((b*x + 2)*b - 2*b))^2*b^(7/2) + 20*b^(9/2))/(((sqrt(b*x + 2)*sqrt
(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs(b))
```

**maple** [A] time = 0.00, size = 27, normalized size = 0.49

$$-\frac{2b^2x^2+6bx+3}{3\,(bx+2)^{\frac{3}{2}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(5/2),x)
```

```
[Out] -1/3*(2*b^2*x^2+6*b*x+3)/(b*x+2)^(3/2)/x^(1/2)
```

**maxima** [A] time = 1.40, size = 40, normalized size = 0.73

$$\frac{\left(b^2-\frac{6\,(bx+2)b}{x}\right)x^{\frac{3}{2}}}{12\,(bx+2)^{\frac{3}{2}}}-\frac{\sqrt{bx+2}}{4\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2 - 6*(b*x + 2)*b/x)*x^(3/2)/(b*x + 2)^(3/2) - 1/4*sqrt(b*x + 2)/sq
rt(x)
```

**mupad** [B] time = 0.38, size = 57, normalized size = 1.04

$$-\frac{3\,\sqrt{b\,x+2}+6\,b\,x\,\sqrt{b\,x+2}+2\,b^2\,x^2\,\sqrt{b\,x+2}}{\sqrt{x}\,\left(x\,\left(3\,x\,b^2+12\,b\right)+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] -(3*(b*x + 2)^(1/2) + 6*b*x*(b*x + 2)^(1/2) + 2*b^2*x^2*(b*x + 2)^(1/2))/(x
^(1/2)*(x*(12*b + 3*b^2*x) + 12))
```

**sympy [B]** time = 3.89, size = 117, normalized size = 2.13

$$-\frac{2b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}-\frac{6b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}-\frac{3b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(5/2),x)
```

```
[Out] -2*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) - 6
*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) - 3*b**(
9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4)
```

## 3.628 $\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=71

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{1}{3x^{3/2}(bx+2)^{3/2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

**Rubi** [A] time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{1}{3x^{3/2}(bx+2)^{3/2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] 1/(3*x^(3/2)*(2 + b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x
])/(3*x^(3/2)) + (2*b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx &= \frac{1}{3x^{3/2}(2+bx)^{3/2}} + \int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx \\ &= \frac{1}{3x^{3/2}(2+bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2+bx}} + 2\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx \\ &= \frac{1}{3x^{3/2}(2+bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3x^{3/2}} - \frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx \\ &= \frac{1}{3x^{3/2}(2+bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3x^{3/2}} + \frac{2b\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 0.56

$$\frac{2b^3x^3+6b^2x^2+3bx-1}{3x^{3/2}(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] (-1 + 3*b*x + 6*b^2*x^2 + 2*b^3*x^3)/(3*x^(3/2)*(2 + b*x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.10, size = 40, normalized size = 0.56

$$\frac{2b^3x^3+6b^2x^2+3bx-1}{3x^{3/2}(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] (-1 + 3*b*x + 6*b^2*x^2 + 2*b^3*x^3)/(3*x^(3/2)*(2 + b*x)^(3/2))
```

**fricas** [A] time = 1.05, size = 55, normalized size = 0.77

$$\frac{\left(2\,b^3x^3+6\,b^2x^2+3\,bx-1\right)\sqrt{bx+2}\,\sqrt{x}}{3\left(b^2x^4+4\,bx^3+4\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^3*x^3 + 6*b^2*x^2 + 3*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^4 + 4*
b*x^3 + 4*x^2)
```

**giac [B]** time = 1.27, size = 158, normalized size = 2.23

$$\frac{\left(4\,(bx+2)b^2|b|-9\,b^2|b|\right)\sqrt{bx+2}}{12\,((bx+2)b-2\,b)^{\frac{3}{2}}}+\frac{3\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^4 b^{\frac{7}{2}}+18\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2 b^{\frac{9}{2}}+16\,b^{\frac{11}{2}}}{3\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(4*(b*x + 2)*b^2*abs(b) - 9*b^2*abs(b))*sqrt(b*x + 2)/((b*x + 2)*b - 2
*b)^(3/2) + 1/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*b^(7
/2) + 18*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2*b^(9/2) + 16*b
^(11/2))/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs
(b))
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.49

$$\frac{2b^3x^3+6b^2x^2+3bx-1}{3\,(bx+2)^{\frac{3}{2}}\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(5/2),x)
```

```
[Out] 1/3*(2*b^3*x^3+6*b^2*x^2+3*b*x-1)/(b*x+2)^(3/2)/x^(3/2)
```

**maxima [A]** time = 1.28, size = 55, normalized size = 0.77

$$\frac{3\,\sqrt{bx+2}\,b}{8\,\sqrt{x}}-\frac{\left(b^3-\frac{9\,(bx+2)b^2}{x}\right)x^{\frac{3}{2}}}{24\,(bx+2)^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{3}{2}}}{24\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(b*x + 2)*b/sqrt(x) - 1/24*(b^3 - 9*(b*x + 2)*b^2/x)*x^(3/2)/(b*x +
 2)^(3/2) - 1/24*(b*x + 2)^(3/2)/x^(3/2)
```

**mupad [B]** time = 0.42, size = 71, normalized size = 1.00

$$\frac{3\,b\,x\,\sqrt{b\,x+2}-\sqrt{b\,x+2}+6\,b^2\,x^2\,\sqrt{b\,x+2}+2\,b^3\,x^3\,\sqrt{b\,x+2}}{x^{3/2}\,\left(x\,\left(3\,x\,b^2+12\,b\right)+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] (3*b*x*(b*x + 2)^(1/2) - (b*x + 2)^(1/2) + 6*b^2*x^2*(b*x + 2)^(1/2) + 2*b^
3*x^3*(b*x + 2)^(1/2))/(x^(3/2)*(x*(12*b + 3*b^2*x) + 12))
```

**sympy [B]** time = 6.85, size = 257, normalized size = 3.62

$$\frac{2b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+\frac{10b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+\frac{15b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+\frac{5b^{\frac{21}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}-\frac{2b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(5/2),x)
```

```
[Out] 2*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10
*x**2 + 24*b**9*x) + 10*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18
*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 15*b**(23/2)*x**2*sqrt(1 + 2/(b*
x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 5*b**(21/2
)*x*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 + 24*b*
*9*x) - 2*b**(19/2)*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b*
*10*x**2 + 24*b**9*x)
```

## 3.629 $\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx$

**Optimal**. Leaf size=91

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}$$

**Rubi** [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{x^{5/2}\sqrt{2-bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(6*b^2) - (x
^(5/2)*Sqrt[2 - b*x])/(3*b) + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx = -\frac{x^{5/2}\sqrt{2-bx}}{3b} + \frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{3b}$$
$$= -\frac{5x^{3/2}\sqrt{2-bx}}{6b^2} - \frac{x^{5/2}\sqrt{2-bx}}{3b} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b^2}$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{6b^2} - \frac{x^{5/2}\sqrt{2-bx}}{3b} + \frac{5\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^3}$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{6b^2} - \frac{x^{5/2}\sqrt{2-bx}}{3b} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{6b^2} - \frac{x^{5/2}\sqrt{2-bx}}{3b} + \frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

**Mathematica** [A] time = 0.04, size = 61, normalized size = 0.67

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}} - \frac{\sqrt{x}\sqrt{2-bx}\left(2b^2x^2+5bx+15\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] -1/6*(Sqrt[x]*Sqrt[2 - b*x]*(15 + 5*b*x + 2*b^2*x^2))/b^3 + (5*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

**IntegrateAlgebraic** [A] time = 0.11, size = 82, normalized size = 0.90

$$\frac{5\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^4} + \frac{\sqrt{2-bx}\left(-2b^2x^{5/2}-5bx^{3/2}-15\sqrt{x}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] - 5*b*x^(3/2) - 2*b^2*x^(5/2)))/(6*b^3) + (5*Sq
rt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^4
```

**fricas** [A] time = 1.44, size = 125, normalized size = 1.37

$$\left[-\frac{\left(2\,b^3x^2+5\,b^2x+15\,b\right)\sqrt{-bx+2}\,\sqrt{x}+15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6\,b^4}, -\frac{\left(2\,b^3x^2+5\,b^2x+15\,b\right)\sqrt{-bx+2}\,\sqrt{x}+30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="fricas")

[Out] [-1/6*((2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^4, -1/6*((2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^4]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.6438432182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-29.292030761,78.6493344628]-2*abs(b)/b^2/b*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqrt(-b*x+2)-78*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+19

```
8*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqr
t(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))
```

**maple** [A] time = 0.00, size = 100, normalized size = 1.10

$$-\frac{\sqrt{-bx+2}\,x^{\frac{5}{2}}}{3b}-\frac{5\sqrt{-bx+2}\,x^{\frac{3}{2}}}{6b^{2}}-\frac{5\sqrt{-bx+2}\,\sqrt{x}}{2b^{3}}+\frac{5\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2\sqrt{-bx+2}\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -1/3*x^(5/2)*(-b*x+2)^(1/2)/b-5/6*x^(3/2)*(-b*x+2)^(1/2)/b^2-5/2*(-b*x+2)^(
1/2)/b^3*x^(1/2)+5/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(7/2)/x^(1/2)*arct
an((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima** [A] time = 3.03, size = 117, normalized size = 1.29

$$-\frac{\frac{33\sqrt{-bx+2}\,b^{2}}{\sqrt{x}}+\frac{40\,(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{15\,(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{6}-\frac{3\,(bx-2)b^{5}}{x}+\frac{3\,(bx-2)^{2}b^{4}}{x^{2}}-\frac{(bx-2)^{3}b^{3}}{x^{3}}\right)}-\frac{5\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(33*sqrt(-b*x + 2)*b^2/sqrt(x) + 40*(-b*x + 2)^(3/2)*b/x^(3/2) + 15*(-
b*x + 2)^(5/2)/x^(5/2))/(b^6 - 3*(b*x - 2)*b^5/x + 3*(b*x - 2)^2*b^4/x^2 -
(b*x - 2)^3*b^3/x^3) - 5*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{2-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)/(2 - b*x)^(1/2), x)
```

**sympy [A]** time = 7.51, size = 206, normalized size = 2.26

$$\begin{cases} -\frac{ix^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{ix^{\frac{5}{2}}}{6b\sqrt{bx-2}} - \frac{5ix^{\frac{3}{2}}}{6b^2\sqrt{bx-2}} + \frac{5i\sqrt{x}}{b^3\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{for } \frac{|bx|}{2} > 1 \\ \frac{x^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{x^{\frac{5}{2}}}{6b\sqrt{-bx+2}} + \frac{5x^{\frac{3}{2}}}{6b^2\sqrt{-bx+2}} - \frac{5\sqrt{x}}{b^3\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(7/2)/(3*sqrt(b*x - 2)) - I*x**(5/2)/(6*b*sqrt(b*x - 2)) -
 5*I*x**(3/2)/(6*b**2*sqrt(b*x - 2)) + 5*I*sqrt(x)/(b**3*sqrt(b*x - 2)) - 5
*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(7/2), Abs(b*x)/2 > 1), (x**(7/2)/(3
*sqrt(-b*x + 2)) + x**(5/2)/(6*b*sqrt(-b*x + 2)) + 5*x**(3/2)/(6*b**2*sqrt(
-b*x + 2)) - 5*sqrt(x)/(b**3*sqrt(-b*x + 2)) + 5*asin(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(7/2), True))
```

# 3.630 $\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx$

**Optimal**. Leaf size=69

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}$$

**Rubi [A]** time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{x^{3/2}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(2*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(2*b) + (3*Arc
Sin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx = -\frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

**Mathematica** [A] time = 0.03, size = 52, normalized size = 0.75

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}} - \frac{\sqrt{x}\sqrt{2-bx}\,(bx+3)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x]*(3 + b*x))/b^2 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sq
rt[2]])/b^(5/2)
```

**IntegrateAlgebraic** [A] time = 0.09, size = 72, normalized size = 1.04

$$\frac{3\sqrt{-b}\,\log\left(\sqrt{2-bx} - \sqrt{-b}\sqrt{x}\right)}{b^3} + \frac{\sqrt{2-bx}\,\left(-bx^{3/2} - 3\sqrt{x}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[2 - b*x]*(-3*Sqrt[x] - b*x^(3/2)))/(2*b^2) + (3*Sqrt[-b]*Log[-(Sqrt[-
b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^3
```

**fricas** [A] time = 1.34, size = 107, normalized size = 1.55

$$\left[-\frac{\left(b^2x + 3\,b\right)\sqrt{-bx+2}\,\sqrt{x} + 3\,\sqrt{-b}\,\log\left(-bx + \sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x} + 1\right)}{2\,b^3}, -\frac{\left(b^2x + 3\,b\right)\sqrt{-bx+2}\,\sqrt{x} + 6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(-b*x + sqrt(-b
*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, -1/2*((b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt
(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]2*abs(b)/b^2/b^2*(2*(1/8*sqrt(-
b*x+2)*sqrt(-b*x+2)-5/8)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*
ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))
```

**maple** [A] time = 0.00, size = 84, normalized size = 1.22

$$-\frac{\sqrt{-bx+2}\, x^{\frac{3}{2}}}{2b}-\frac{3\sqrt{-bx+2}\,\sqrt{x}}{2b^{2}}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2\sqrt{-bx+2}\, b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -1/2*x^(3/2)*(-b*x+2)^(1/2)/b-3/2*(-b*x+2)^(1/2)/b^2*x^(1/2)+3/2*((-b*x+2)*
x)^(1/2)/(-b*x+2)^(1/2)/b^(5/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b
^(1/2))
```

**maxima** [A] time = 2.93, size = 85, normalized size = 1.23

$$-\frac{\frac{5\sqrt{-bx+2}\,b}{\sqrt{x}}+\frac{3\,(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{4}-\frac{2\,(bx-2)b^{3}}{x}+\frac{(bx-2)^{2}b^{2}}{x^{2}}}-\frac{3\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(5*sqrt(-b*x + 2)*b/sqrt(x) + 3*(-b*x + 2)^(3/2)/x^(3/2))/(b^4 - 2*(b*x -
2)*b^3/x + (b*x - 2)^2*b^2/x^2) - 3*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))
)/b^(5/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{2-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)/(2 - b*x)^(1/2), x)
```

**sympy [A]** time = 3.59, size = 163, normalized size = 2.36

$$\begin{cases} -\frac{ix^{\frac{5}{2}}}{2\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{2b\sqrt{bx-2}} + \frac{3i\sqrt{x}}{b^2\sqrt{bx-2}} - \frac{3i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for } \frac{|bx|}{2} > 1 \\ \frac{x^{\frac{5}{2}}}{2\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{2b\sqrt{-bx+2}} - \frac{3\sqrt{x}}{b^2\sqrt{-bx+2}} + \frac{3\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(5/2)/(2*sqrt(b*x - 2)) - I*x**(3/2)/(2*b*sqrt(b*x - 2)) +
 3*I*sqrt(x)/(b**2*sqrt(b*x - 2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b*
*(5/2), Abs(b*x)/2 > 1), (x**(5/2)/(2*sqrt(-b*x + 2)) + x**(3/2)/(2*b*sqrt(
-b*x + 2)) - 3*sqrt(x)/(b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(5/2), True))
```

## 3.631 $\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx$

**Optimal**. Leaf size=45

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{2-bx}}{b}$$

**Rubi** [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{2-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[2 - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx = -\frac{\sqrt{x}\sqrt{2-bx}}{b} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{b} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}\sqrt{2-bx}}{b} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

**Mathematica [A]** time = 0.02, size = 45, normalized size = 1.00

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}} - \frac{\sqrt{x}\sqrt{2-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[2 - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2
)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 59, normalized size = 1.31

$$\frac{2\sqrt{-b}\log\left(\sqrt{2-bx} - \sqrt{-b}\sqrt{x}\right)}{b^2} - \frac{\sqrt{x}\sqrt{2-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[2 - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2
 - b*x]])/b^2
```

**fricas [A]** time = 1.28, size = 90, normalized size = 2.00

$$\left[-\frac{\sqrt{-bx+2}\,b\sqrt{x} + \sqrt{-b}\log\left(-bx + \sqrt{-bx+2}\sqrt{-b}\sqrt{x} + 1\right)}{b^2}, -\frac{\sqrt{-bx+2}\,b\sqrt{x} + 2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-b*x + 2)*b*sqrt(x) + sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*s
qrt(x) + 1))/b^2, -(sqrt(-b*x + 2)*b*sqrt(x) + 2*sqrt(b)*arctan(sqrt(-b*x +
 2)/(sqrt(b)*sqrt(x))))/b^2]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]-2*abs(b)/b^2/b*(1/2*sqrt(-b*x+
2)*sqrt(-b*(-b*x+2)+2*b)-2*b/2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-
b)*sqrt(-b*x+2))))
```

**maple** [A] time = 0.01, size = 67, normalized size = 1.49

$$-\frac{\sqrt{-bx+2}\,\sqrt{x}}{b}+\frac{\sqrt{(-bx+2)\,x}\ \arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{\sqrt{-bx+2}\ b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -(-b*x+2)^(1/2)/b*x^(1/2)+((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(3/2)/x^(1/2)
*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

**maxima** [A] time = 2.93, size = 52, normalized size = 1.16

$$-\frac{2\ \arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{-bx+2}}{\left(b^{2}-\frac{(bx-2)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2) - 2*sqrt(-b*x + 2)/((b^
2 - (b*x - 2)*b/x)*sqrt(x))
```

**mupad** [B] time = 0.52, size = 46, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{b}\ \sqrt{x}}{\sqrt{2}-\sqrt{2-b\,x}}\right)}{b^{3/2}}-\frac{\sqrt{x}\ \sqrt{2-b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(1/2),x)
```

```
[Out] - (4*atan((b^(1/2)*x^(1/2))/(2^(1/2) - (2 - b*x)^(1/2))))/b^(3/2) - (x^(1/2
)*(2 - b*x)^(1/2))/b
```

**sympy** [A] time = 1.97, size = 121, normalized size = 2.69

$$\begin{cases} -\frac{ix^{\frac{3}{2}}}{\sqrt{bx-2}}+\frac{2i\sqrt{x}}{b\sqrt{bx-2}}-\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2}>1\\ \frac{x^{\frac{3}{2}}}{\sqrt{-bx+2}}-\frac{2\sqrt{x}}{b\sqrt{-bx+2}}+\frac{2\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(3/2)/sqrt(b*x - 2) + 2*I*sqrt(x)/(b*sqrt(b*x - 2)) - 2*I*
acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), Abs(b*x)/2 > 1), (x**(3/2)/sqrt(
-b*x + 2) - 2*sqrt(x)/(b*sqrt(-b*x + 2)) + 2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2
)/b**(3/2), True))
```

## 3.632 $\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx$

**Optimal**. Leaf size=24

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {54, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[2 - b*x]),x]
```

```
[Out] (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[2 - b*x]),x]
```

```
[Out] (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

**IntegrateAlgebraic** [A] time = 0.06, size = 38, normalized size = 1.58

$$\frac{2\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[2 - b*x]),x]
```

```
[Out] (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b
```

**fricas** [A] time = 1.29, size = 56, normalized size = 2.33

$$\left[-\frac{\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{b}, -\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1)/b, -2*arctan(sqr
t(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
```

```
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]2/abs(b)*b^2/b/sqrt(-b)*ln(abs(
sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2)))
```

**maple [B]** time = 0.00, size = 50, normalized size = 2.08

$$\frac{\sqrt{(-bx+2)\,x}\;\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{\sqrt{-bx+2}\;\sqrt{b}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(1/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*
x)^(1/2)*b^(1/2))
```

**maxima [A]** time = 2.96, size = 21, normalized size = 0.88

$$-\frac{2\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

**mupad [B]** time = 0.03, size = 27, normalized size = 1.12

$$\frac{4\,\mathrm{atan}\left(\frac{\sqrt{2}-\sqrt{2-b\,x}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] (4*atan((2^(1/2) - (2 - b*x)^(1/2))/(b^(1/2)*x^(1/2))))/b^(1/2)
```

**sympy [A]** time = 1.08, size = 58, normalized size = 2.42

$$\begin{cases} -\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx|}{2} > 1 \\ \frac{2\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x)/2 > 1),
(2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), True))
```

## 3.633 $\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx$

**Optimal**. Leaf size=17

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx = -\frac{\sqrt{2-bx}}{\sqrt{x}}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.02, size = 17, normalized size = 1.00

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

**fricas [A]** time = 1.13, size = 13, normalized size = 0.76

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x + 2)/sqrt(x)
```

**giac [B]** time = 1.28, size = 30, normalized size = 1.76

$$-\frac{\sqrt{-bx+2}\,b^2}{\sqrt{(bx-2)b+2\,b}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b))
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.82

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -(-b*x+2)^(1/2)/x^(1/2)
```

**maxima [A]** time = 1.35, size = 13, normalized size = 0.76

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-b*x + 2)/sqrt(x)
```

**mupad** [B] time = 0.31, size = 13, normalized size = 0.76

$$-\frac{\sqrt{2-b\,x}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(2 - b*x)^(1/2)/x^(1/2)
```

**sympy** [A] time = 0.93, size = 39, normalized size = 2.29

$$\begin{cases} -\sqrt{b}\sqrt{-1+\frac{2}{bx}} & \text{for}\ \frac{2}{|bx|}>1 \\ -i\sqrt{b}\sqrt{1-\frac{2}{bx}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(b)*sqrt(-1 + 2/(b*x)), 2/Abs(b*x) > 1), (-I*sqrt(b)*sqrt(1
 - 2/(b*x)), True))
```

## 3.634 $\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx$

**Optimal**. Leaf size=40

$$-\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}$$

**Rubi** [A] time = 0.00, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -Sqrt[2 - b*x]/(3*x^(3/2)) - (b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx &= -\frac{\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\ &= -\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.60

$$-\frac{\sqrt{2-bx}\,(bx+1)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[2 - b*x]),x]
[Out] -1/3*(Sqrt[2 - b*x]*(1 + b*x))/x^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.09, size = 25, normalized size = 0.62

$$\frac{(-bx-1)\sqrt{2-bx}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*Sqrt[2 - b*x]),x]
[Out] ((-1 - b*x)*Sqrt[2 - b*x])/(3*x^(3/2))
```

**fricas** [A] time = 0.87, size = 18, normalized size = 0.45

$$-\frac{(bx+1)\sqrt{-bx+2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] -1/3*(b*x + 1)*sqrt(-b*x + 2)/x^(3/2)
```

**giac** [A] time = 1.12, size = 43, normalized size = 1.08

$$-\frac{\left((bx-2)b^3+3\,b^3\right)\sqrt{-bx+2}\,b}{3\,((bx-2)b+2\,b)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="giac")
[Out] -1/3*((b*x - 2)*b^3 + 3*b^3)*sqrt(-b*x + 2)*b/(((b*x - 2)*b + 2*b)^(3/2)*ab
s(b))
```

**maple** [A] time = 0.00, size = 19, normalized size = 0.48

$$-\frac{(bx+1)\sqrt{-bx+2}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -1/3*(b*x+1)/x^(3/2)*(-b*x+2)^(1/2)
```

**maxima** [A] time = 1.35, size = 28, normalized size = 0.70

$$-\frac{\sqrt{-bx+2}\,b}{2\sqrt{x}}-\frac{(-bx+2)^{\frac{3}{2}}}{6\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-b*x + 2)*b/sqrt(x) - 1/6*(-b*x + 2)^(3/2)/x^(3/2)
```

**mupad** [B] time = 0.29, size = 19, normalized size = 0.48

$$-\frac{\sqrt{2-b\,x}\,\left(\frac{b\,x}{3}+\frac{1}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -((2 - b*x)^(1/2)*((b*x)/3 + 1/3))/x^(3/2)
```

**sympy** [A] time = 1.96, size = 139, normalized size = 3.48

$$\begin{cases} -\frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3}-\frac{\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for}\ \frac{2}{|bx|}>1 \\ \frac{ib^{\frac{7}{2}}x^2\sqrt{1-\frac{2}{bx}}}{-3b^2x^2+6bx}-\frac{ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}}{-3b^2x^2+6bx}-\frac{2ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{-3b^2x^2+6bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-b**(3/2)*sqrt(-1 + 2/(b*x))/3 - sqrt(b)*sqrt(-1 + 2/(b*x))/(3*x
), 2/Abs(b*x) > 1), (I*b**(7/2)*x**2*sqrt(1 - 2/(b*x))/(-3*b**2*x**2 + 6*b*
x) - I*b**(5/2)*x*sqrt(1 - 2/(b*x))/(-3*b**2*x**2 + 6*b*x) - 2*I*b**(3/2)*s
qrt(1 - 2/(b*x))/(-3*b**2*x**2 + 6*b*x), True))
```

## 3.635 $\int \frac{x^{5/2}}{(2-bx)^{3/2}}\,dx$

**Optimal**. Leaf size=89

$$-\frac{15\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{15\sqrt{x}\sqrt{2-bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}+\frac{2x^{5/2}}{b\sqrt{2-bx}}$$

**Rubi [A]** time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}+\frac{15\sqrt{x}\sqrt{2-bx}}{2b^3}-\frac{15\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{b\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2))/(b*Sqrt[2 - b*x]) + (15*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) + (5*x^(
3/2)*Sqrt[2 - b*x])/(2*b^2) - (15*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(2-bx)^{3/2}}\,dx &= \frac{2x^{5/2}}{b\sqrt{2-bx}} - \frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{b}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b^2}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{2b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.34

$$\frac{x^{7/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[3/2, 7/2, 9/2, (b*x)/2])/(7*Sqrt[2])
```

**IntegrateAlgebraic [A]** time = 0.17, size = 88, normalized size = 0.99

$$\frac{\sqrt{2-bx}\,\left(b^2x^{5/2}+5bx^{3/2}-30\sqrt{x}\right)}{2b^3(bx-2)} - \frac{15\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\,\sqrt{x}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-30*Sqrt[x] + 5*b*x^(3/2) + b^2*x^(5/2)))/(2*b^3*(-2 + b*x)
) - (15*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^4
```

**fricas [A]** time = 1.23, size = 155, normalized size = 1.74

$$\left[-\frac{15\,(bx-2)\sqrt{-b}\,\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-\left(b^3x^2+5\,b^2x-30\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{2\left(b^5x-2\,b^4\right)},\ \frac{30\,(bx-2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\left(b^3x^2+5\,b^2x-30\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{2\left(b^5x-2\,b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(15*(b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1
) - (b^3*x^2 + 5*b^2*x - 30*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x - 2*b^4), 1/2
*(30*(b*x - 2)*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^3*x^2
+ 5*b^2*x - 30*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x - 2*b^4)]
```

**giac [B]** time = 10.85, size = 136, normalized size = 1.53

$$\frac{\left(\sqrt{(bx-2)b+2\,b}\,\sqrt{-bx+2}\left(\frac{bx-2}{b^3}+\frac{9}{b^3}\right)-\frac{15\,\log\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\right)}{\sqrt{-b}\,b^2}+\frac{64}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)\sqrt{-b}\,b}\right)|b|}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)*((b*x - 2)/b^3 + 9/b^3) - 15*lo
g((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/(sqrt(-b)*b^2) + 6
4/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*sqrt(-b)*b
))*abs(b)/b^2
```

**maple [B]** time = 0.03, size = 138, normalized size = 1.55

$$-\frac{\left(\frac{15\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^2+2x}}\right)}{2b^{\frac{7}{2}}}+\frac{8\sqrt{-\left(x-\frac{2}{b}\right)^2 b-2x+\frac{4}{b}}}{\left(x-\frac{2}{b}\right)b^4}\right)\sqrt{(-bx+2)\,x}}{\sqrt{-bx+2}\,\sqrt{x}}-\frac{(bx+7)\,(bx-2)\,\sqrt{(-bx+2)\,x}\,\sqrt{x}}{2\sqrt{-\,(bx-2)\,x}\,\sqrt{-bx+2}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(3/2),x)
```

```
[Out] -1/2*(b*x+7)*(b*x-2)*x^(1/2)/b^3/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-b*
x+2)^(1/2)-(15/2/b^(7/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))+8/b^4/(
x-2/b)*(-b*(x-2/b)^2-2*x+4/b)^(1/2))*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1
/2)
```

**maxima** [A] time = 2.99, size = 101, normalized size = 1.13

$$\frac{8\,b^2-\frac{25\,(bx-2)b}{x}+\frac{15\,(bx-2)^2}{x^2}}{\frac{\sqrt{-bx+2}\,b^5}{\sqrt{x}}+\frac{2\,(-bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(-bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}}+\frac{15\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] (8*b^2 - 25*(b*x - 2)*b/x + 15*(b*x - 2)^2/x^2)/(sqrt(-b*x + 2)*b^5/sqrt(x)
 + 2*(-b*x + 2)^(3/2)*b^4/x^(3/2) + (-b*x + 2)^(5/2)*b^3/x^(5/2)) + 15*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(2-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)/(2 - b*x)^(3/2), x)
```

**sympy** [A] time = 7.03, size = 173, normalized size = 1.94

$$\begin{cases} \frac{ix^{\frac{5}{2}}}{2b\sqrt{bx-2}}+\frac{5ix^{\frac{3}{2}}}{2b^2\sqrt{bx-2}}-\frac{15i\sqrt{x}}{b^3\sqrt{bx-2}}+\frac{15i\,\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{for } \frac{|bx|}{2}>1 \\ -\frac{x^{\frac{5}{2}}}{2b\sqrt{-bx+2}}-\frac{5x^{\frac{3}{2}}}{2b^2\sqrt{-bx+2}}+\frac{15\sqrt{x}}{b^3\sqrt{-bx+2}}-\frac{15\,\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((I*x**(5/2)/(2*b*sqrt(b*x - 2)) + 5*I*x**(3/2)/(2*b**2*sqrt(b*x -
 2)) - 15*I*sqrt(x)/(b**3*sqrt(b*x - 2)) + 15*I*acosh(sqrt(2)*sqrt(b)*sqrt(
```

```
x)/2)/b**(7/2), Abs(b*x)/2 > 1), (-x**(5/2)/(2*b*sqrt(-b*x + 2)) - 5*x**(3/
2)/(2*b**2*sqrt(-b*x + 2)) + 15*sqrt(x)/(b**3*sqrt(-b*x + 2)) - 15*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/b**(7/2), True))
```

## 3.636 $\int \frac{x^{3/2}}{(2-bx)^{3/2}} dx$

**Optimal**. Leaf size=65

$$-\frac{6\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\sqrt{2-bx}}{b^2}+\frac{2x^{3/2}}{b\sqrt{2-bx}}$$

**Rubi** [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$\frac{3\sqrt{x}\sqrt{2-bx}}{b^2}-\frac{6\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{b\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (2*x^(3/2))/(b*Sqrt[2 - b*x]) + (3*Sqrt[x]*Sqrt[2 - b*x])/b^2 - (6*ArcSin[(
Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx &= \frac{2x^{3/2}}{b\sqrt{2-bx}}-\frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{b}\\ &= \frac{2x^{3/2}}{b\sqrt{2-bx}}+\frac{3\sqrt{x}\sqrt{2-bx}}{b^2}-\frac{3\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b^2}\\ &= \frac{2x^{3/2}}{b\sqrt{2-bx}}+\frac{3\sqrt{x}\sqrt{2-bx}}{b^2}-\frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= \frac{2x^{3/2}}{b\sqrt{2-bx}}+\frac{3\sqrt{x}\sqrt{2-bx}}{b^2}-\frac{6\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.46

$$\frac{x^{5/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{bx}{2}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 - b*x)^(3/2),x]
[Out] (x^(5/2)*Hypergeometric2F1[3/2, 5/2, 7/2, (b*x)/2])/(5*Sqrt[2])
```

**IntegrateAlgebraic [A]** time = 0.15, size = 75, normalized size = 1.15

$$\frac{\sqrt{2-bx}\left(bx^{3/2}-6\sqrt{x}\right)}{b^2(bx-2)}-\frac{6\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(2 - b*x)^(3/2),x]
```

[Out] (Sqrt[2 - b*x]*(-6*Sqrt[x] + b*x^(3/2)))/(b^2*(-2 + b*x)) - (6*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^3

**fricas [A]** time = 1.30, size = 138, normalized size = 2.12

$$\left[-\frac{3\,(bx-2)\sqrt{-b}\,\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-\left(b^2x-6\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{b^4x-2\,b^3},\,\frac{6\,(bx-2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\left(b^2x-6\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{b^4x-2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="fricas")

[Out] [-(3*(b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - (b^2*x - 6*b)*sqrt(-b*x + 2)*sqrt(x))/(b^4*x - 2*b^3), (6*(b*x - 2)*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^2*x - 6*b)*sqrt(-b*x + 2)*sqrt(x))/(b^4*x - 2*b^3)]

**giac [B]** time = 10.41, size = 119, normalized size = 1.83

$$-\frac{\left(\frac{3\,\log\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\right)}{\sqrt{-b}}-\frac{\sqrt{(bx-2)b+2\,b}\,\sqrt{-bx+2}}{b}+\frac{16\,\sqrt{-b}}{\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b}\right)|b|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="giac")

[Out] -(3*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/sqrt(-b) - sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)/b + 16*sqrt(-b)/((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b))*abs(b)/b^3

**maple [B]** time = 0.03, size = 133, normalized size = 2.05

$$-\frac{\left(\frac{3\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^2+2x}}\right)}{b^{\frac{5}{2}}}+\frac{4\sqrt{-\left(x-\frac{2}{b}\right)^2 b-2x+\frac{4}{b}}}{\left(x-\frac{2}{b}\right)b^3}\right)\sqrt{(-bx+2)\,x}}{\sqrt{-bx+2}\,\sqrt{x}}-\frac{(bx-2)\sqrt{(-bx+2)\,x}\,\sqrt{x}}{\sqrt{-(bx-2)\,x}\,\sqrt{-bx+2}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(-b*x+2)^(3/2),x)

[Out] -1/b^2*(b*x-2)*x^(1/2)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)-(3/b^(5/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))+4/b^3/(x-2/b)*(-(x-2/b)^2*b-2*x+4/b)^(1/2))*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)

**maxima** [A] time = 3.00, size = 71, normalized size = 1.09

$$\frac{2\left(2\,b-\frac{3\,(bx-2)}{x}\right)}{\frac{\sqrt{-bx+2}\,b^3}{\sqrt{x}}+\frac{(-bx+2)^{\frac{3}{2}}\,b^2}{x^{\frac{3}{2}}}}+\frac{6\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(2*b - 3*(b*x - 2)/x)/(sqrt(-b*x + 2)*b^3/sqrt(x) + (-b*x + 2)^(3/2)*b^2/
x^(3/2)) + 6*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{(2-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)/(2 - b*x)^(3/2), x)
```

**sympy** [A] time = 3.20, size = 128, normalized size = 1.97

$$\begin{cases} \frac{ix^{\frac{3}{2}}}{b\sqrt{bx-2}}-\frac{6i\sqrt{x}}{b^2\sqrt{bx-2}}+\frac{6i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for } \frac{|bx|}{2}>1 \\ -\frac{x^{\frac{3}{2}}}{b\sqrt{-bx+2}}+\frac{6\sqrt{x}}{b^2\sqrt{-bx+2}}-\frac{6\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((I*x**(3/2)/(b*sqrt(b*x - 2)) - 6*I*sqrt(x)/(b**2*sqrt(b*x - 2))
+ 6*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2), Abs(b*x)/2 > 1), (-x**(3/2
)/(b*sqrt(-b*x + 2)) + 6*sqrt(x)/(b**2*sqrt(-b*x + 2)) - 6*asin(sqrt(2)*sqr
t(b)*sqrt(x)/2)/b**(5/2), True))
```

## 3.637 $\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx$

**Optimal**. Leaf size=45

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}}-\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}}-\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[2 - b*x]) - (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/
2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

**Mathematica** [A] time = 0.04, size = 45, normalized size = 1.00

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[2 - b*x]) - (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/
2)
```

**IntegrateAlgebraic** [A] time = 0.12, size = 66, normalized size = 1.47

$$-\frac{2\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^2} - \frac{2\sqrt{x}\sqrt{2-bx}}{b(bx-2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[2 - b*x])/(b*(-2 + b*x)) - (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt
[x]) + Sqrt[2 - b*x]])/b^2
```

**fricas** [A] time = 1.04, size = 122, normalized size = 2.71

$$\left[-\frac{(bx-2)\sqrt{-b}\log\left(-bx-\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)+2\sqrt{-bx+2}\,b\sqrt{x}}{b^3x-2b^2}, \frac{2\left((bx-2)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-\sqrt{-bx+2}\,b\sqrt{x}\right)}{b^3x-2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + 2*s
qrt(-b*x + 2)*b*sqrt(x))/(b^3*x - 2*b^2), 2*((b*x - 2)*sqrt(b)*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) - sqrt(-b*x + 2)*b*sqrt(x))/(b^3*x - 2*b^2)]
```

**giac [B]** time = 10.06, size = 92, normalized size = 2.04

$$-\frac{\left(\frac{\log\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2\right)}{\sqrt{-b}}+\frac{8\sqrt{-b}}{\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2-2b}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -(log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/sqrt(-b) + 8*s
qrt(-b)/((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b))*abs(
b)/b^2
```

**maple [A]** time = 0.05, size = 67, normalized size = 1.49

$$-\frac{2\left(\frac{\sqrt{\pi}\,\sqrt{2}\,(-b)^{\frac{3}{2}}\sqrt{x}}{2\sqrt{-\frac{bx}{2}+1}\,b}-\frac{\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}\right)}{\sqrt{-b}\,\sqrt{\pi}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(3/2),x)
```

```
[Out] -2/(-b)^(1/2)/Pi^(1/2)/b*(1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(3/2)/b/(-1/2*b
*x+1)^(1/2)-Pi^(1/2)*(-b)^(3/2)/b^(3/2)*arcsin(1/2*2^(1/2)*b^(1/2)*x^(1/2))
)
```

**maxima [A]** time = 2.98, size = 38, normalized size = 0.84

$$\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}+\frac{2\sqrt{x}}{\sqrt{-bx+2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2) + 2*sqrt(x)/(sqrt(-b*x +
 2)*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(2-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(2 - b*x)^(3/2), x)
```

**sympy [A]** time = 1.69, size = 92, normalized size = 2.04

$$\begin{cases} -\frac{2i\sqrt{x}}{b\sqrt{bx-2}}+\frac{2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ \frac{2\sqrt{x}}{b\sqrt{-bx+2}}-\frac{2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*I*sqrt(x)/(b*sqrt(b*x - 2)) + 2*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(3/2), Abs(b*x)/2 > 1), (2*sqrt(x)/(b*sqrt(-b*x + 2)) - 2*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), True))
```

## 3.638 $\int \frac{1}{\sqrt{x}(2-bx)^{3/2}}\,dx$

**Optimal**. Leaf size=16

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

**Rubi** [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 - b*x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(2-bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{2-bx}}$$

**Mathematica** [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 - b*x]
```

**IntegrateAlgebraic [A]** time = 0.03, size = 16, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 - b*x]
```

**fricas [A]** time = 1.16, size = 20, normalized size = 1.25

$$-\frac{\sqrt{-bx+2}\,\sqrt{x}}{bx-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x + 2)*sqrt(x)/(b*x - 2)
```

**giac [B]** time = 1.03, size = 50, normalized size = 3.12

$$-\frac{4\sqrt{-b}\,b}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-b)*b/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b
)*abs(b))
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$\frac{\sqrt{x}}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] x^(1/2)/(-b*x+2)^(1/2)
```

**maxima** [A] time = 1.27, size = 12, normalized size = 0.75

$$\frac{\sqrt{x}}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/sqrt(-b*x + 2)
```

**mupad** [B] time = 0.30, size = 12, normalized size = 0.75

$$\frac{\sqrt{x}}{\sqrt{2-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] x^(1/2)/(2 - b*x)^(1/2)
```

**sympy** [A] time = 0.93, size = 39, normalized size = 2.44

$$\begin{cases} \frac{1}{\sqrt{b}\sqrt{-1+\frac{2}{bx}}} & \text{for}\ \frac{2}{|bx|} > 1 \\ -\frac{i}{\sqrt{b}\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((1/(sqrt(b)*sqrt(-1 + 2/(b*x))), 2/Abs(b*x) > 1), (-I/(sqrt(b)*sq
rt(1 - 2/(b*x))), True))
```

# 3.639 $\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx$

**Optimal**. Leaf size=34

$$\frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

**Rubi [A]** time = 0.00, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] 1/(Sqrt[x]*Sqrt[2 - b*x]) - Sqrt[2 - b*x]/Sqrt[x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx &= \frac{1}{\sqrt{x}\sqrt{2-bx}}+\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\ &= \frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 21, normalized size = 0.62

$$\frac{bx-1}{\sqrt{x}\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 - b*x)^(3/2)),x]
[Out] (-1 + b*x)/(Sqrt[x]*Sqrt[2 - b*x])
```

**IntegrateAlgebraic** [A] time = 0.09, size = 29, normalized size = 0.85

$$\frac{(1-bx)\sqrt{2-bx}}{\sqrt{x}\,(bx-2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(2 - b*x)^(3/2)),x]
[Out] ((1 - b*x)*Sqrt[2 - b*x])/(Sqrt[x]*(-2 + b*x))
```

**fricas** [A] time = 1.13, size = 29, normalized size = 0.85

$$-\frac{(bx-1)\sqrt{-bx+2}\,\sqrt{x}}{bx^2-2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
[Out] -(b*x - 1)*sqrt(-b*x + 2)*sqrt(x)/(b*x^2 - 2*x)
```

**giac** [B] time = 1.09, size = 83, normalized size = 2.44

$$-\frac{\sqrt{-bx+2}\,b^2}{2\sqrt{(bx-2)b+2\,b}\,|b|}-\frac{2\sqrt{-b}\,b^2}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="giac")
[Out] -1/2*sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b)) - 2*sqrt(-b)*b^2/(
((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*abs(b))
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.53

$$\frac{bx-1}{\sqrt{-bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(3/2),x)
```

```
[Out] (b*x-1)/x^(1/2)/(-b*x+2)^(1/2)
```

**maxima** [A] time = 1.26, size = 28, normalized size = 0.82

$$\frac{b\sqrt{x}}{2\sqrt{-bx+2}}-\frac{\sqrt{-bx+2}}{2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*sqrt(x)/sqrt(-b*x + 2) - 1/2*sqrt(-b*x + 2)/sqrt(x)
```

**mupad** [B] time = 0.32, size = 27, normalized size = 0.79

$$\frac{b\sqrt{x}}{\sqrt{2-b\,x}}-\frac{1}{\sqrt{x}\,\sqrt{2-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] (b*x^(1/2))/(2 - b*x)^(1/2) - 1/(x^(1/2)*(2 - b*x)^(1/2))
```

**sympy** [A] time = 1.61, size = 90, normalized size = 2.65

$$\begin{cases} \frac{\sqrt{b}}{\sqrt{-1+\frac{2}{bx}}}-\frac{1}{\sqrt{b}\,x\sqrt{-1+\frac{2}{bx}}} & \text{for } \frac{2}{|bx|}>1 \\ \frac{ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}}{-b^2x+2b}-\frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{-b^2x+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((sqrt(b)/sqrt(-1 + 2/(b*x)) - 1/(sqrt(b)*x*sqrt(-1 + 2/(b*x))), 2
/Abs(b*x) > 1), (I*b**(5/2)*x*sqrt(1 - 2/(b*x))/(-b**2*x + 2*b) - I*b**(3/2
)*sqrt(1 - 2/(b*x))/(-b**2*x + 2*b), True))
```

## 3.640 $\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx$

**Optimal**. Leaf size=56

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

**Rubi** [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x])/(3*x^(3/2)) - (2*b*Sqrt[2 - b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx &= \frac{1}{x^{3/2}\sqrt{2-bx}} + 2\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx\\ &= \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} + \frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\ &= \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} - \frac{2b\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.59

$$\frac{2b^2x^2-2bx-1}{3x^{3/2}\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
[Out] (-1 - 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 - b*x])
```

**IntegrateAlgebraic** [A] time = 0.13, size = 40, normalized size = 0.71

$$\frac{\sqrt{2-bx}\left(-2b^2x^2+2bx+1\right)}{3x^{3/2}(bx-2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
[Out] (Sqrt[2 - b*x]*(1 + 2*b*x - 2*b^2*x^2))/(3*x^(3/2)*(-2 + b*x))
```

**fricas** [A] time = 1.26, size = 40, normalized size = 0.71

$$-\frac{\left(2\,b^2x^2-2\,bx-1\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(bx^3-2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
[Out] -1/3*(2*b^2*x^2 - 2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x)/(b*x^3 - 2*x^2)
```

**giac** [B] time = 1.17, size = 96, normalized size = 1.71

$$-\frac{\sqrt{-b}\,b^{3}}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^{2}-2\,b\right)|b|}-\frac{\left(5\,(bx-2)b^{2}|b|+12\,b^{2}|b|\right)\sqrt{-bx+2}}{12\,((bx-2)b+2\,b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(-b)*b^3/((((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b
)*abs(b)) - 1/12*(5*(b*x - 2)*b^2*abs(b) + 12*b^2*abs(b))*sqrt(-b*x + 2)/((
b*x - 2)*b + 2*b)^(3/2)
```

**maple** [A] time = 0.01, size = 28, normalized size = 0.50

$$\frac{2b^{2}x^{2}-2bx-1}{3\sqrt{-bx+2}\;x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(3/2),x)
```

```
[Out] 1/3*(2*b^2*x^2-2*b*x-1)/x^(3/2)/(-b*x+2)^(1/2)
```

**maxima** [A] time = 1.28, size = 44, normalized size = 0.79

$$\frac{b^{2}\sqrt{x}}{4\sqrt{-bx+2}}-\frac{\sqrt{-bx+2}\,b}{2\sqrt{x}}-\frac{(-bx+2)^{\frac{3}{2}}}{12\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*sqrt(x)/sqrt(-b*x + 2) - 1/2*sqrt(-b*x + 2)*b/sqrt(x) - 1/12*(-b*x
+ 2)^(3/2)/x^(3/2)
```

**mupad** [B] time = 0.36, size = 38, normalized size = 0.68

$$\frac{\sqrt{2-b\,x}\,\left(\frac{2\,x}{3}-\frac{2\,b\,x^2}{3}+\frac{1}{3\,b}\right)}{x^{5/2}-\frac{2\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] ((2 - b*x)^(1/2)*((2*x)/3 - (2*b*x^2)/3 + 1/(3*b)))/(x^(5/2) - (2*x^(3/2))/
b)
```

**sympy [B]** time = 4.29, size = 354, normalized size = 6.32

$$\begin{cases} \frac{2b^{\frac{15}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} - \frac{6b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} + \frac{3b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} + \frac{2b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} & \text{for } \frac{2}{|bx|} > 1 \\ \frac{2ib^{\frac{15}{2}}x^3\sqrt{1-\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} - \frac{6ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} + \frac{3ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} + \frac{2ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((2*b**(15/2)*x**3*sqrt(-1 + 2/(b*x))/(-3*b**6*x**3 + 12*b**5*x**2
 - 12*b**4*x) - 6*b**(13/2)*x**2*sqrt(-1 + 2/(b*x))/(-3*b**6*x**3 + 12*b**5
*x**2 - 12*b**4*x) + 3*b**(11/2)*x*sqrt(-1 + 2/(b*x))/(-3*b**6*x**3 + 12*b*
*5*x**2 - 12*b**4*x) + 2*b**(9/2)*sqrt(-1 + 2/(b*x))/(-3*b**6*x**3 + 12*b**
5*x**2 - 12*b**4*x), 2/Abs(b*x) > 1), (2*I*b**(15/2)*x**3*sqrt(1 - 2/(b*x))
/(-3*b**6*x**3 + 12*b**5*x**2 - 12*b**4*x) - 6*I*b**(13/2)*x**2*sqrt(1 - 2/
(b*x))/(-3*b**6*x**3 + 12*b**5*x**2 - 12*b**4*x) + 3*I*b**(11/2)*x*sqrt(1 -
 2/(b*x))/(-3*b**6*x**3 + 12*b**5*x**2 - 12*b**4*x) + 2*I*b**(9/2)*sqrt(1 -
 2/(b*x))/(-3*b**6*x**3 + 12*b**5*x**2 - 12*b**4*x), True))
```

## 3.641 $\int \frac{x^{5/2}}{(2-bx)^{5/2}}\,dx$

**Optimal**. Leaf size=89

$$\frac{10\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\sqrt{2-bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}+\frac{2x^{5/2}}{3b(2-bx)^{3/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}-\frac{5\sqrt{x}\sqrt{2-bx}}{b^3}+\frac{10\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{3b(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (2*x^(5/2))/(3*b*(2 - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 - b*x]) - (5
*Sqrt[x]*Sqrt[2 - b*x])/b^3 + (10*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(2-bx)^{5/2}}\,dx &= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{5\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx}{3b} \\
&= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{b^2} \\
&= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3} + \frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{b^3} \\
&= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3} + \frac{10\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^3} \\
&= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3} + \frac{10\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 30, normalized size = 0.34

$$\frac{x^{7/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{2}\right)}{14\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[5/2, 7/2, 9/2, (b*x)/2])/(14*Sqrt[2])
```

**IntegrateAlgebraic [A]** time = 0.19, size = 89, normalized size = 1.00

$$\frac{10\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\,\sqrt{x}\right)}{b^4} + \frac{\sqrt{2-bx}\,\left(-3b^2x^{5/2}+40bx^{3/2}-60\sqrt{x}\right)}{3b^3(bx-2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-60*Sqrt[x] + 40*b*x^(3/2) - 3*b^2*x^(5/2)))/(3*b^3*(-2 + b
*x)^2) + (10*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^4
```

**fricas [A]** time = 1.14, size = 187, normalized size = 2.10

$$\left[-\frac{15\left(b^{2}x^{2}-4\,bx+4\right)\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+\left(3\,b^{3}x^{2}-40\,b^{2}x+60\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^{6}x^{2}-4\,b^{5}x+4\,b^{4}\right)},\ -\frac{30\left(b^{2}x^{2}-4\,bx+4\right)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\left(3\,b^{3}x^{2}-40\,b^{2}x+60\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^{6}x^{2}-4\,b^{5}x+4\,b^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(15*(b^2*x^2 - 4*b*x + 4)*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)
*sqrt(x) + 1) + (3*b^3*x^2 - 40*b^2*x + 60*b)*sqrt(-b*x + 2)*sqrt(x))/(b^6*
x^2 - 4*b^5*x + 4*b^4), -1/3*(30*(b^2*x^2 - 4*b*x + 4)*sqrt(b)*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) + (3*b^3*x^2 - 40*b^2*x + 60*b)*sqrt(-b*x + 2)
*sqrt(x))/(b^6*x^2 - 4*b^5*x + 4*b^4)]
```

**giac [B]** time = 10.72, size = 200, normalized size = 2.25

$$\frac{\left(\frac{15\,\log\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^{2}\right)}{\sqrt{-b}\,b^{2}}-\frac{3\,\sqrt{(bx-2)b+2\,b}\,\sqrt{-bx+2}}{b^{3}}-\frac{16\left(9\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^{4}-24\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^{2}b+28\,b^{2}\right)}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^{2}-2\,b\right)^{3}\sqrt{-b}\,b}\right)|b|}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(15*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/(sqrt(-b
)*b^2) - 3*sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)/b^3 - 16*(9*(sqrt(-b*x +
2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4 - 24*(sqrt(-b*x + 2)*sqrt(-b) - sq
rt((b*x - 2)*b + 2*b))^2*b + 28*b^2)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x
 - 2)*b + 2*b))^2 - 2*b)^3*sqrt(-b)*b))*abs(b)/b^2
```

**maple [B]** time = 0.04, size = 168, normalized size = 1.89

$$\frac{\left(\frac{5\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{b^{\frac{7}{2}}}+\frac{28\sqrt{-\left(x-\frac{2}{b}\right)^{2}b-2x+\frac{4}{b}}}{3\left(x-\frac{2}{b}\right)b^{4}}+\frac{8\sqrt{-\left(x-\frac{2}{b}\right)^{2}b-2x+\frac{4}{b}}}{3\left(x-\frac{2}{b}\right)^{2}b^{5}}\right)\sqrt{(-bx+2)\,x}}{\sqrt{-bx+2}\,\sqrt{x}}+\frac{(bx-2)\,\sqrt{(-bx+2)\,x}\,\sqrt{x}}{\sqrt{-\,(bx-2)\,x}\,\sqrt{-bx+2}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(5/2),x)
```

```
[Out] 1/b^3*(b*x-2)*x^(1/2)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)+
(5/b^(7/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))+8/3/b^5/(x-2/b)^2*(-(
x-2/b)^2*b-2*x+4/b)^(1/2)+28/3/(x-2/b)*(-(x-2/b)^2*b-2*x+4/b)^(1/2)/b^4)*((
-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)
```

**maxima** [A]    time = 2.99, size = 86, normalized size = 0.97

$$\frac{2\left(2\,b^2+\frac{10\,(bx-2)b}{x}-\frac{15\,(bx-2)^2}{x^2}\right)}{3\left(\frac{(-bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(-bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}-\frac{10\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*b^2 + 10*(b*x - 2)*b/x - 15*(b*x - 2)^2/x^2)/((-b*x + 2)^(3/2)*b^4/x
^(3/2) + (-b*x + 2)^(5/2)*b^3/x^(5/2)) - 10*arctan(sqrt(-b*x + 2)/(sqrt(b)*
sqrt(x)))/b^(7/2)
```

**mupad** [F]    time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(2-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)/(2 - b*x)^(5/2), x)
```

**sympy** [B]    time = 6.80, size = 753, normalized size = 8.46

$$\begin{cases} -\frac{3ib^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}+\frac{40ib^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}-\frac{60ib^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}-\frac{30ib^{10}x^{\frac{27}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}+\frac{15\pi b^{10}x^{\frac{27}{2}}\sqrt{bx-2}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}+\frac{60ib^{9}x^{\frac{25}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}-\frac{30\pi b^{9}x^{\frac{25}{2}}\sqrt{bx-2}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}} & \text{for}\ \frac{|bx|}{2}>1 \\ \frac{3b^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}-\frac{40b^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}+\frac{60b^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}+\frac{30b^{10}x^{\frac{27}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}-\frac{60b^{9}x^{\frac{25}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-3*I*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b*
*(25/2)*x**(25/2)*sqrt(b*x - 2)) + 40*I*b**(21/2)*x**14/(3*b**(27/2)*x**(27
/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) - 60*I*b**(19/2)*x
**13/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x
```

```
- 2)) - 30*I*b**10*x**(27/2)*sqrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)
/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)
) + 15*pi*b**10*x**(27/2)*sqrt(b*x - 2)/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2
) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) + 60*I*b**9*x**(25/2)*sqrt(b*x - 2
)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6
*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) - 30*pi*b**9*x**(25/2)*sqrt(b*x - 2)/(3
*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)),
Abs(b*x)/2 > 1), (3*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) -
 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2)) - 40*b**(21/2)*x**14/(3*b**(27/2)*x*
*(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2)) + 60*b**(19/
2)*x**13/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt
(-b*x + 2)) + 30*b**10*x**(27/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)*sqrt(x
)/2)/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*
x + 2)) - 60*b**9*x**(25/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/
(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2
)), True))
```

## 3.642 $\int \frac{x^{3/2}}{(2-bx)^{5/2}}\,dx$

**Optimal**. Leaf size=67

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2x^{3/2}}{3b(2-bx)^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{3b(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*b*(2 - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 - b*x]) + (2*Ar
cSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{(2-bx)^{5/2}}\,dx = \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx}{b}$$
$$= \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2-bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b^2}$$
$$= \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2-bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= \frac{2x^{3/2}}{3b(2-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2-bx}} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

**Mathematica** [A] time = 0.06, size = 53, normalized size = 0.79

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}} + \frac{4\sqrt{x}\,(2bx-3)}{3b^2(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (4*Sqrt[x]*(-3 + 2*b*x))/(3*b^2*(2 - b*x)^(3/2)) + (2*ArcSin[(Sqrt[b]*Sqrt[
x])/Sqrt[2]])/b^(5/2)
```

**IntegrateAlgebraic** [A] time = 0.17, size = 79, normalized size = 1.18

$$\frac{2\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^3} + \frac{4\sqrt{2-bx}\left(2bx^{3/2}-3\sqrt{x}\right)}{3b^2(bx-2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (4*Sqrt[2 - b*x]*(-3*Sqrt[x] + 2*b*x^(3/2)))/(3*b^2*(-2 + b*x)^2) + (2*Sqrt
[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^3
```

**fricas** [A] time = 1.39, size = 173, normalized size = 2.58

$$\left[-\frac{3\left(b^2x^2-4\,bx+4\right)\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-4\left(2\,b^2x-3\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^5x^2-4\,b^4x+4\,b^3\right)}, -\frac{2\left(3\left(b^2x^2-4\,bx+4\right)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-2\left(2\,b^2x-3\,b\right)\sqrt{-bx+2}\,\sqrt{x}\right)}{3\left(b^5x^2-4\,b^4x+4\,b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 4*b*x + 4)*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*
sqrt(x) + 1) - 4*(2*b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x^2 - 4*b^4*x
 + 4*b^3), -2/3*(3*(b^2*x^2 - 4*b*x + 4)*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqr
t(b)*sqrt(x))) - 2*(2*b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x^2 - 4*b^4
*x + 4*b^3)]
```

**giac [B]** time = 10.63, size = 178, normalized size = 2.66

$$\frac{\left(\frac{3\log\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\right)}{\sqrt{-b}}+\frac{16\left(3\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^4\sqrt{-b}-6\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\sqrt{-b}\,b+8\,\sqrt{-b}\,b^2\right)}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3}\right)|b|}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/sqrt(-b)
+ 16*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b) - 6*
(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqrt(-b)*b + 8*sqrt(-
b)*b^2)/((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3)*ab
s(b)/b^3
```

**maple [A]** time = 0.04, size = 73, normalized size = 1.09

$$-\frac{4\left(-\frac{\sqrt{\pi}\,\sqrt{2}\,(-b)^{\frac{5}{2}}(-10bx+15)\sqrt{x}}{20\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}b^2}+\frac{3\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{2b^{\frac{5}{2}}}\right)}{3\,(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -4/3/(-b)^(3/2)/Pi^(1/2)/b*(-1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(5/2)*(-10*
b*x+15)/b^2/(-1/2*b*x+1)^(3/2)+3/2*Pi^(1/2)*(-b)^(5/2)/b^(5/2)*arcsin(1/2*2
^(1/2)*b^(1/2)*x^(1/2)))
```

**maxima [A]** time = 2.92, size = 50, normalized size = 0.75

$$\frac{2\left(b+\frac{3\,(bx-2)}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+2)^{\frac{3}{2}}b^2}-\frac{2\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b + 3*(b*x - 2)/x)*x^(3/2)/((-b*x + 2)^(3/2)*b^2) - 2*arctan(sqrt(-b*x
 + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(2-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)/(2 - b*x)^(5/2), x)
```

**sympy [B]** time = 3.70, size = 649, normalized size = 9.69

$$\begin{cases} \frac{8ib^{\frac{11}{2}}x^{8}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}} - \frac{12ib^{\frac{9}{2}}x^{7}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}} - \frac{6ib^{5}x^{\frac{15}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}} + \frac{3\pi b^{5}x^{\frac{15}{2}}\sqrt{bx-2}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}} + \frac{12ib^{4}x^{\frac{13}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}} - \frac{6\pi b^{4}x^{\frac{13}{2}}\sqrt{bx-2}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}} & \text{for}\: \frac{|bx|}{2} > 1 \\ -\frac{8b^{\frac{11}{2}}x^{8}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}} + \frac{12b^{\frac{9}{2}}x^{7}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}} + \frac{6b^{5}x^{\frac{15}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}} - \frac{12b^{4}x^{\frac{13}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((8*I*b**(11/2)*x**8/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(
13/2)*x**(13/2)*sqrt(b*x - 2)) - 12*I*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*
sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)) - 6*I*b**5*x**(15/2)*s
qrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b
*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)) + 3*pi*b**5*x**(15/2)*sqrt(b
*x - 2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b
*x - 2)) + 12*I*b**4*x**(13/2)*sqrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/
2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x -
2)) - 6*pi*b**4*x**(13/2)*sqrt(b*x - 2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2
) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)), Abs(b*x)/2 > 1), (-8*b**(11/2)*x*
*8/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13/2)*sqrt(-b*x
+ 2)) + 12*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2
)*x**(13/2)*sqrt(-b*x + 2)) + 6*b**5*x**(15/2)*sqrt(-b*x + 2)*asin(sqrt(2)*
sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(
13/2)*sqrt(-b*x + 2)) - 12*b**4*x**(13/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13/2)
*sqrt(-b*x + 2)), True))
```

## 3.643 $\int \frac{\sqrt{x}}{(2-bx)^{5/2}}\,dx$

**Optimal**. Leaf size=19

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 - b*x)^(5/2),x]
[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2-bx)^{5/2}}\,dx = \frac{x^{3/2}}{3(2-bx)^{3/2}}$$

**Mathematica [A]** time = 0.01, size = 19, normalized size = 1.00

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 - b*x)^(5/2),x]
[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 26, normalized size = 1.37

$$\frac{x^{3/2}\sqrt{2-bx}}{3(bx-2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(2 - b*x)^(5/2),x]
```

```
[Out] (x^(3/2)*Sqrt[2 - b*x])/(3*(-2 + b*x)^2)
```

**fricas [B]** time = 1.03, size = 28, normalized size = 1.47

$$\frac{\sqrt{-bx+2}\,x^{\frac{3}{2}}}{3\left(b^2x^2-4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-b*x + 2)*x^(3/2)/(b^2*x^2 - 4*b*x + 4)
```

**giac [B]** time = 1.25, size = 95, normalized size = 5.00

$$\frac{4\left(3\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^4\sqrt{-b}+4\,\sqrt{-b}\,b^2\right)|b|}{3\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b) + 4*s
qrt(-b)*b^2)*abs(b)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2
 - 2*b)^3*b^2)
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.74

$$\frac{x^{\frac{3}{2}}}{3\left(-bx+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(5/2),x)
```

```
[Out] 1/3*x^(3/2)/(-b*x+2)^(3/2)
```

**maxima [A]** time = 1.38, size = 13, normalized size = 0.68

$$\frac{x^{\frac{3}{2}}}{3\,(-bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
[Out] 1/3*x^(3/2)/(-b*x + 2)^(3/2)
```

**mupad [B]** time = 0.23, size = 13, normalized size = 0.68

$$\frac{x^{3/2}}{3\,(2-b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(5/2),x)
[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

**sympy [B]** time = 1.46, size = 65, normalized size = 3.42

$$\begin{cases} \frac{ix^{\frac{3}{2}}}{3bx\sqrt{bx-2}-6\sqrt{bx-2}} & \text{for}\ \frac{|bx|}{2}>1 \\ -\frac{x^{\frac{3}{2}}}{3bx\sqrt{-bx+2}-6\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(5/2),x)
[Out] Piecewise((I*x**(3/2)/(3*b*x*sqrt(b*x - 2) - 6*sqrt(b*x - 2)), Abs(b*x)/2 >
 1), (-x**(3/2)/(3*b*x*sqrt(-b*x + 2) - 6*sqrt(-b*x + 2)), True))
```

## 3.644 $\int \frac{1}{\sqrt{x}\,(2-bx)^{5/2}}\,dx$

**Optimal**. Leaf size=39

$$\frac{\sqrt{x}}{3\sqrt{2-bx}}+\frac{\sqrt{x}}{3(2-bx)^{3/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{\sqrt{x}}{3\sqrt{2-bx}}+\frac{\sqrt{x}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]
```

```
[Out] Sqrt[x]/(3*(2 - b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 - b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(2-bx)^{5/2}}\,dx &= \frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{x}\,(2-bx)^{3/2}}\,dx\\ &= \frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2-bx}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.62

$$-\frac{\sqrt{x}\,(bx-3)}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]
[Out] -1/3*(Sqrt[x]*(-3 + b*x))/(2 - b*x)^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.10, size = 31, normalized size = 0.79

$$-\frac{\sqrt{x}\,\sqrt{2-bx}\,(bx-3)}{3(bx-2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]
[Out] -1/3*(Sqrt[x]*Sqrt[2 - b*x]*(-3 + b*x))/(-2 + b*x)^2
```

**fricas** [A] time = 1.31, size = 33, normalized size = 0.85

$$-\frac{(bx-3)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^2x^2-4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
[Out] -1/3*(b*x - 3)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^2 - 4*b*x + 4)
```

**giac** [B] time = 1.10, size = 90, normalized size = 2.31

$$\frac{8\left(3\left(\sqrt{-bx+2}\,\sqrt{-b}\,-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)\sqrt{-b}\,b^2}{3\left(\left(\sqrt{-bx+2}\,\sqrt{-b}\,-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
[Out] 8/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*sqrt(-b
)*b^2/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3*abs(
b))
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.49

$$-\frac{(bx-3)\sqrt{x}}{3(-bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(5/2)/x^(1/2),x)
[Out] -1/3*x^(1/2)*(b*x-3)/(-b*x+2)^(3/2)
```

**maxima [A]** time = 1.30, size = 25, normalized size = 0.64

$$\frac{\left(b-\frac{3(bx-2)}{x}\right)x^{\frac{3}{2}}}{6(-bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
[Out] 1/6*(b - 3*(b*x - 2)/x)*x^(3/2)/(-b*x + 2)^(3/2)
```

**mupad [B]** time = 0.36, size = 45, normalized size = 1.15

$$\frac{3\sqrt{x}\sqrt{2-bx}-bx^{3/2}\sqrt{2-bx}}{3b^2x^2-12bx+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(5/2)),x)
[Out] (3*x^(1/2)*(2 - b*x)^(1/2) - b*x^(3/2)*(2 - b*x)^(1/2))/(3*b^2*x^2 - 12*b*x
 + 12)
```

**sympy [C]** time = 1.91, size = 177, normalized size = 4.54

$$\begin{cases} \frac{ibx}{3ib^{\frac{3}{2}}x\sqrt{-1+\frac{2}{bx}}-6i\sqrt{b}\sqrt{-1+\frac{2}{bx}}}-\frac{3i}{3ib^{\frac{3}{2}}x\sqrt{-1+\frac{2}{bx}}-6i\sqrt{b}\sqrt{-1+\frac{2}{bx}}} & \text{for}\ \frac{2}{|bx|}>1 \\ \frac{b^2x}{3ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}-6ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}-\frac{3b}{3ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}-6ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b*x/(3*I*b**(3/2)*x*sqrt(-1 + 2/(b*x)) - 6*I*sqrt(b)*sqrt(-1 +
 2/(b*x))) - 3*I/(3*I*b**(3/2)*x*sqrt(-1 + 2/(b*x)) - 6*I*sqrt(b)*sqrt(-1 +
 2/(b*x))), 2/Abs(b*x) > 1), (b**2*x/(3*I*b**(5/2)*x*sqrt(1 - 2/(b*x)) - 6*
I*b**(3/2)*sqrt(1 - 2/(b*x))) - 3*b/(3*I*b**(5/2)*x*sqrt(1 - 2/(b*x)) - 6*I
*b**(3/2)*sqrt(1 - 2/(b*x))), True))
```

## 3.645 $\int \frac{1}{x^{3/2}(2-bx)^{5/2}}\,dx$

**Optimal**. Leaf size=58

$$-\frac{2\sqrt{2-bx}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}+\frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2\sqrt{2-bx}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}+\frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] 1/(3*Sqrt[x]*(2 - b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 - b*x]) - (2*Sqrt[2 - b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{3/2}(2-bx)^{5/2}}\,dx = \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}} + \frac{2}{3}\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx$$
$$= \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}} + \frac{2}{3\sqrt{x}\,\sqrt{2-bx}} + \frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx$$
$$= \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}} + \frac{2}{3\sqrt{x}\,\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3\sqrt{x}}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.57

$$-\frac{2b^2x^2-6bx+3}{3\sqrt{x}\,(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] -1/3*(3 - 6*b*x + 2*b^2*x^2)/(Sqrt[x]*(2 - b*x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.12, size = 40, normalized size = 0.69

$$\frac{\sqrt{2-bx}\,\left(-2b^2x^2+6bx-3\right)}{3\sqrt{x}\,(bx-2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] (Sqrt[2 - b*x]*(-3 + 6*b*x - 2*b^2*x^2))/(3*Sqrt[x]*(-2 + b*x)^2)
```

**fricas** [A] time = 1.18, size = 46, normalized size = 0.79

$$-\frac{\left(2\,b^2x^2-6\,bx+3\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^2x^3-4\,bx^2+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 - 6*b*x + 3)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^3 - 4*b*x^2 + 4*
x)
```

**giac [B]** time = 1.12, size = 170, normalized size = 2.93

$$-\frac{\sqrt{-bx+2}\,b^2}{4\,\sqrt{(bx-2)b+2\,b}\,|b|}-\frac{3\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^4\sqrt{-b}\,b^2-24\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\sqrt{-b}\,b^3+20\,\sqrt{-b}\,b^4}{3\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b)) - 1/3*(3*(sqrt(-b*
x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b)*b^2 - 24*(sqrt(-b*x +
 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqrt(-b)*b^3 + 20*sqrt(-b)*b^4)/(
((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3*abs(b))
```

**maple [A]** time = 0.00, size = 28, normalized size = 0.48

$$-\frac{2b^2x^2-6bx+3}{3\,(-bx+2)^{\frac{3}{2}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -1/3*(2*b^2*x^2-6*b*x+3)/x^(1/2)/(-b*x+2)^(3/2)
```

**maxima [A]** time = 1.35, size = 42, normalized size = 0.72

$$\frac{\left(b^2-\frac{6\,(bx-2)b}{x}\right)x^{\frac{3}{2}}}{12\,(-bx+2)^{\frac{3}{2}}}-\frac{\sqrt{-bx+2}}{4\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2 - 6*(b*x - 2)*b/x)*x^(3/2)/(-b*x + 2)^(3/2) - 1/4*sqrt(-b*x + 2)/
sqrt(x)
```

**mupad [B]** time = 0.37, size = 59, normalized size = 1.02

$$\frac{3\,\sqrt{2-b\,x}\,-6\,b\,x\,\sqrt{2-b\,x}\,+2\,b^2\,x^2\,\sqrt{2-b\,x}}{\sqrt{x}\,\left(x\,\left(12\,b-3\,b^2\,x\right)-12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(5/2)),x)
```

```
[Out] (3*(2 - b*x)^(1/2) - 6*b*x*(2 - b*x)^(1/2) + 2*b^2*x^2*(2 - b*x)^(1/2))/(x^
(1/2)*(x*(12*b - 3*b^2*x) - 12))
```

**sympy [B]** time = 4.00, size = 243, normalized size = 4.19

$$\begin{cases} -\frac{2b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} + \frac{6b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} - \frac{3b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} & \text{for } \frac{2}{|bx|} > 1 \\ -\frac{2ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} + \frac{6ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} - \frac{3ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-2*b**(13/2)*x**2*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x +
12*b**4) + 6*b**(11/2)*x*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12*b
**4) - 3*b**(9/2)*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12*b**4), 2
/Abs(b*x) > 1), (-2*I*b**(13/2)*x**2*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b*
*5*x + 12*b**4) + 6*I*b**(11/2)*x*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b**5*
x + 12*b**4) - 3*I*b**(9/2)*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12
*b**4), True))
```

## 3.646 $\int \frac{1}{x^{5/2}(2-bx)^{5/2}} dx$

**Optimal**. Leaf size=75

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}+\frac{1}{3x^{3/2}(2-bx)^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

**Rubi** [A] time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}+\frac{1}{3x^{3/2}(2-bx)^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] 1/(3*x^(3/2)*(2 - b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x
])/(3*x^(3/2)) - (2*b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(2-bx)^{5/2}}\,dx &= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx \\ &= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2-bx}} + 2\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx \\ &= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} + \frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx \\ &= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} - \frac{2b\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

**Mathematica** [A]  time = 0.01, size = 41, normalized size = 0.55

$$-\frac{2b^3x^3 - 6b^2x^2 + 3bx + 1}{3x^{3/2}(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] -1/3*(1 + 3*b*x - 6*b^2*x^2 + 2*b^3*x^3)/(x^(3/2)*(2 - b*x)^(3/2))
```

**IntegrateAlgebraic** [A]  time = 0.12, size = 48, normalized size = 0.64

$$\frac{\sqrt{2-bx}\left(-2b^3x^3 + 6b^2x^2 - 3bx - 1\right)}{3x^{3/2}(bx-2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] (Sqrt[2 - b*x]*(-1 - 3*b*x + 6*b^2*x^2 - 2*b^3*x^3))/(3*x^(3/2)*(-2 + b*x)^
2)
```

**fricas** [A]  time = 1.10, size = 56, normalized size = 0.75

$$-\frac{\left(2\,b^3x^3 - 6\,b^2x^2 + 3\,bx + 1\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^2x^4 - 4\,bx^3 + 4\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^3*x^3 - 6*b^2*x^2 + 3*b*x + 1)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^4 -
4*b*x^3 + 4*x^2)
```

**giac [B]** time = 1.25, size = 183, normalized size = 2.44

$$-\frac{\left(4\,(bx-2)b^2|b|+9\,b^2|b|\right)\sqrt{-bx+2}}{12\,((bx-2)b+2\,b)^{\frac{3}{2}}}-\frac{3\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^4\sqrt{-b}\,b^3-18\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\sqrt{-b}\,b^4+16\,\sqrt{-b}\,b^5}{3\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(4*(b*x - 2)*b^2*abs(b) + 9*b^2*abs(b))*sqrt(-b*x + 2)/((b*x - 2)*b +
 2*b)^(3/2) - 1/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*
sqrt(-b)*b^3 - 18*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqr
t(-b)*b^4 + 16*sqrt(-b)*b^5)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b
+ 2*b))^2 - 2*b)^3*abs(b))
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.48

$$-\frac{2b^3x^3-6b^2x^2+3bx+1}{3\,(-bx+2)^{\frac{3}{2}}\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -1/3*(2*b^3*x^3-6*b^2*x^2+3*b*x+1)/x^(3/2)/(-b*x+2)^(3/2)
```

**maxima [A]** time = 1.35, size = 58, normalized size = 0.77

$$-\frac{3\,\sqrt{-bx+2}\,b}{8\,\sqrt{x}}+\frac{\left(b^3-\frac{9\,(bx-2)b^2}{x}\right)x^{\frac{3}{2}}}{24\,(-bx+2)^{\frac{3}{2}}}-\frac{(-bx+2)^{\frac{3}{2}}}{24\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/8*sqrt(-b*x + 2)*b/sqrt(x) + 1/24*(b^3 - 9*(b*x - 2)*b^2/x)*x^(3/2)/(-b*
x + 2)^(3/2) - 1/24*(-b*x + 2)^(3/2)/x^(3/2)
```

**mupad [B]** time = 0.44, size = 73, normalized size = 0.97

$$\frac{\sqrt{2-b\,x}+3\,b\,x\,\sqrt{2-b\,x}-6\,b^2\,x^2\,\sqrt{2-b\,x}+2\,b^3\,x^3\,\sqrt{2-b\,x}}{x^{3/2}\,\left(x\,\left(12\,b-3\,b^2\,x\right)-12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(5/2)),x)
```

```
[Out] ((2 - b*x)^(1/2) + 3*b*x*(2 - b*x)^(1/2) - 6*b^2*x^2*(2 - b*x)^(1/2) + 2*b^
3*x^3*(2 - b*x)^(1/2))/(x^(3/2)*(x*(12*b - 3*b^2*x) - 12))
```

**sympy [B]** time = 12.40, size = 529, normalized size = 7.05

$$\begin{cases} \frac{2b^{\frac{27}{2}}x^4\sqrt{-1+\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{10b^{\frac{25}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} + \frac{15b^{\frac{23}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{5b^{\frac{21}{2}}x\sqrt{-1+\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{2b^{\frac{19}{2}}\sqrt{-1+\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} & \text{for } \frac{2}{|bx|} > 1 \\ \frac{2ib^{\frac{27}{2}}x^4\sqrt{1-\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{10ib^{\frac{25}{2}}x^3\sqrt{1-\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} + \frac{15ib^{\frac{23}{2}}x^2\sqrt{1-\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{5ib^{\frac{21}{2}}x\sqrt{1-\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{2ib^{\frac{19}{2}}\sqrt{1-\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((2*b**(27/2)*x**4*sqrt(-1 + 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x*
*3 - 36*b**10*x**2 + 24*b**9*x) - 10*b**(25/2)*x**3*sqrt(-1 + 2/(b*x))/(-3*
b**12*x**4 + 18*b**11*x**3 - 36*b**10*x**2 + 24*b**9*x) + 15*b**(23/2)*x**2
*sqrt(-1 + 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x**3 - 36*b**10*x**2 + 24*b**
9*x) - 5*b**(21/2)*x*sqrt(-1 + 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x**3 - 36
*b**10*x**2 + 24*b**9*x) - 2*b**(19/2)*sqrt(-1 + 2/(b*x))/(-3*b**12*x**4 +
18*b**11*x**3 - 36*b**10*x**2 + 24*b**9*x), 2/Abs(b*x) > 1), (2*I*b**(27/2)
*x**4*sqrt(1 - 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x**3 - 36*b**10*x**2 + 24
*b**9*x) - 10*I*b**(25/2)*x**3*sqrt(1 - 2/(b*x))/(-3*b**12*x**4 + 18*b**11*
x**3 - 36*b**10*x**2 + 24*b**9*x) + 15*I*b**(23/2)*x**2*sqrt(1 - 2/(b*x))/(
-3*b**12*x**4 + 18*b**11*x**3 - 36*b**10*x**2 + 24*b**9*x) - 5*I*b**(21/2)*
x*sqrt(1 - 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x**3 - 36*b**10*x**2 + 24*b**
9*x) - 2*I*b**(19/2)*sqrt(1 - 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x**3 - 36*
b**10*x**2 + 24*b**9*x), True))
```

## 3.647 $\int \frac{\sqrt{x}}{\sqrt{1-x}} dx$

**Optimal**. Leaf size=27

$$-\sqrt{1-x}\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)$$

**Rubi [A]** time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 53, 619, 216}

$$-\sqrt{1-x}\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[1 - x],x]
```

```
[Out] -(Sqrt[1 - x]*Sqrt[x]) - ArcSin[1 - 2*x]/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx &= -\sqrt{1-x}\,\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx\\
&= -\sqrt{1-x}\,\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{x-x^2}}\,dx\\
&= -\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)\\
&= -\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)
\end{aligned}
$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 0.93

$$
-\sqrt{-((x-1)x)}-\sin^{-1}\left(\sqrt{1-x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[1 - x],x]
[Out] -Sqrt[-((-1 + x)*x)] - ArcSin[Sqrt[1 - x]]
```

**IntegrateAlgebraic [A]** time = 0.08, size = 39, normalized size = 1.44

$$
2\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)-\sqrt{1-x}\,\sqrt{x}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[1 - x],x]
[Out] -(Sqrt[1 - x]*Sqrt[x]) + 2*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - x])]
```

**fricas [A]** time = 1.52, size = 27, normalized size = 1.00

$$
-\sqrt{x}\,\sqrt{-x+1}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(x)*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))
```

**giac** [A] time = 1.17, size = 17, normalized size = 0.63

$$-\sqrt{x}\sqrt{-x+1} + \arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="giac")
[Out] -sqrt(x)*sqrt(-x + 1) + arcsin(sqrt(x))
```

**maple** [A] time = 0.01, size = 41, normalized size = 1.52

$$-\sqrt{-x+1}\,\sqrt{x} + \frac{\sqrt{(-x+1)\,x}\,\arcsin(2x-1)}{2\sqrt{-x+1}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1-x)^(1/2),x)
[Out] -(1-x)^(1/2)*x^(1/2)+1/2*(x*(1-x))^(1/2)/x^(1/2)/(1-x)^(1/2)*arcsin(-1+2*x)
```

**maxima** [A] time = 3.00, size = 37, normalized size = 1.37

$$\frac{\sqrt{-x+1}}{\sqrt{x}\left(\frac{x-1}{x}-1\right)} - \arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
[Out] sqrt(-x + 1)/(sqrt(x)*((x - 1)/x - 1)) - arctan(sqrt(-x + 1)/sqrt(x))
```

**mupad** [B] time = 0.57, size = 31, normalized size = 1.15

$$2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right) - \sqrt{x}\,\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1 - x)^(1/2),x)
[Out] 2*atan(x^(1/2)/((1 - x)^(1/2) - 1)) - x^(1/2)*(1 - x)^(1/2)
```

**sympy** [A] time = 1.65, size = 54, normalized size = 2.00

$$\begin{cases} -i\sqrt{x}\sqrt{x-1} - i\,\mathrm{acosh}\left(\sqrt{x}\right) & \text{for } |x| > 1 \\ \frac{x^{\frac{3}{2}}}{\sqrt{1-x}} - \frac{\sqrt{x}}{\sqrt{1-x}} + \mathrm{asin}\left(\sqrt{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(x)*sqrt(x - 1) - I*acosh(sqrt(x)), Abs(x) > 1), (x**(3/2
)/sqrt(1 - x) - sqrt(x)/sqrt(1 - x) + asin(sqrt(x)), True))
```

## 3.648 $\int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx$

**Optimal**. Leaf size=8

$$-\sin^{-1}(1-2x)$$

**Rubi [A]** time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {53, 619, 216}

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*Sqrt[x]),x]
```

```
[Out] -ArcSin[1 - 2*x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx &= \int \frac{1}{\sqrt{x-x^2}}\,dx \\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right) \\ &= -\sin^{-1}(1-2x) \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 12, normalized size = 1.50

$$-2\sin^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*Sqrt[x]),x]
[Out] -2*ArcSin[Sqrt[1 - x]]
```

**IntegrateAlgebraic** [A] time = 0.04, size = 8, normalized size = 1.00

$$2\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - x]*Sqrt[x]),x]
[Out] 2*ArcSin[Sqrt[x]]
```

**fricas** [B] time = 1.26, size = 14, normalized size = 1.75

$$-2\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-x + 1)/sqrt(x))
```

**giac** [A] time = 1.09, size = 6, normalized size = 0.75

$$2\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] 2*arcsin(sqrt(x))
```

**maple** [B] time = 0.00, size = 27, normalized size = 3.38

$$\frac{\sqrt{(-x+1)x}\ \arcsin(2x-1)}{\sqrt{-x+1}\ \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(1/2)/x^(1/2),x)
[Out] ((-x+1)*x)^(1/2)/(-x+1)^(1/2)/x^(1/2)*arcsin(2*x-1)
```

**maxima [B]** time = 2.95, size = 14, normalized size = 1.75

$$-2 \arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(-x + 1)/sqrt(x))
```

**mupad [B]** time = 0.05, size = 16, normalized size = 2.00

$$-4\,\mathrm{atan}\left(\frac{\sqrt{1-x}\,-1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(1 - x)^(1/2)),x)
[Out] -4*atan(((1 - x)^(1/2) - 1)/x^(1/2))
```

**sympy [A]** time = 0.97, size = 20, normalized size = 2.50

$$\begin{cases} -2i\,\mathrm{acosh}\left(\sqrt{x}\right) & \text{for}\ |x| > 1 \\ 2\,\mathrm{asin}\left(\sqrt{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/x**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(x)), Abs(x) > 1), (2*asin(sqrt(x)), True))
```

## 3.649 $\int \frac{1}{\sqrt{x}\sqrt{1-bx}}\,dx$

**Optimal**. Leaf size=19

$$\frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

**Rubi [A]** time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {54, 216}

$$\frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 - b*x]),x]
```

```
[Out] (2*ArcSin[Sqrt[b]*Sqrt[x]])/Sqrt[b]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1-bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 19, normalized size = 1.00

$$\frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[1 - b*x]),x]

[Out] (2*ArcSin[Sqrt[b]*Sqrt[x]])/Sqrt[b]

**IntegrateAlgebraic [A]** time = 0.05, size = 38, normalized size = 2.00

$$\frac{2\sqrt{-b}\,\log\left(\sqrt{1-bx}-\sqrt{-b}\,\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[1 - b*x]),x]

[Out] (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[1 - b*x]])/b

**fricas [A]** time = 1.16, size = 57, normalized size = 3.00

$$\left[-\frac{\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+1}\,\sqrt{-b}\,\sqrt{x}\,+1\right)}{b},-\frac{2\,\arctan\left(\frac{\sqrt{-bx+1}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="fricas")

[Out] [-sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + 1)*sqrt(-b)*sqrt(x) + 1)/b, -2*arctan(sqrt(-b*x + 1)/(sqrt(b)*sqrt(x)))/sqrt(b)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{

```
4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[1,1]%%%
}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[
0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-15.6438432182,61.793747834
9]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0
,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}
+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[
2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%
%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{
1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%
%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%
}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[
1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%
%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-29.292030761,78.649
3344628]2/abs(b)*b^2/b/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+1)+b)-sqrt(-b)*sqrt(-b
*x+1)))
```

**maple [B]** time = 0.01, size = 48, normalized size = 2.53

$$\frac{\sqrt{(-bx+1)\,x}\ \arctan\left(\frac{\left(x-\frac{1}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+x}}\right)}{\sqrt{-bx+1}\ \sqrt{b}\ \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+1)^(1/2),x)
```

```
[Out] (x*(-b*x+1))^(1/2)/x^(1/2)/(-b*x+1)^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2/b)/
(-b*x^2+x)^(1/2))
```

**maxima [A]** time = 2.89, size = 21, normalized size = 1.11

$$-\frac{2\ \arctan\left(\frac{\sqrt{-bx+1}}{\sqrt{b}\ \sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 1)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

**mupad [B]** time = 0.13, size = 23, normalized size = 1.21

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{1-b\,x}-1}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(1 - b*x)^(1/2)),x)
[Out] -(4*atan(((1 - b*x)^(1/2) - 1)/(b^(1/2)*x^(1/2))))/b^(1/2)
```

**sympy [A]** time = 1.06, size = 42, normalized size = 2.21

$$\begin{cases} -\frac{2i\,\mathrm{acosh}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}} & \text{for}\ |bx| > 1 \\ \frac{2\,\mathrm{asin}\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+1)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(b)*sqrt(x))/sqrt(b), Abs(b*x) > 1), (2*asin(sqrt
(b)*sqrt(x))/sqrt(b), True))
```

## 3.650 $\int x^{5/3}(a+bx)\,dx$

**Optimal**. Leaf size=21

$$\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

**Rubi [A]** time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(8/3))/8 + (3*b*x^(11/3))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)\,dx&=\int\left(ax^{5/3}+bx^{8/3}\right)\,dx\\&=\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{88}x^{8/3}(11a+8bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x),x]
```

```
[Out] (3*x^(8/3)*(11*a + 8*b*x))/88
```

**IntegrateAlgebraic** [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{3}{88}\left(11ax^{8/3}+8bx^{11/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)*(a + b*x),x]
```

```
[Out] (3*(11*a*x^(8/3) + 8*b*x^(11/3)))/88
```

**fricas** [A] time = 1.25, size = 18, normalized size = 0.86

$$\frac{3}{88}\left(8\,bx^3+11\,ax^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 3/88*(8*b*x^3 + 11*a*x^2)*x^(2/3)
```

**giac** [A] time = 0.98, size = 13, normalized size = 0.62

$$\frac{3}{11}\,bx^{\frac{11}{3}}+\frac{3}{8}\,ax^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="giac")
```

```
[Out] 3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

**maple** [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{3\left(8bx+11a\right)x^{\frac{8}{3}}}{88}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a),x)
```

```
[Out] 3/88*x^(8/3)*(8*b*x+11*a)
```

**maxima** [A] time = 1.30, size = 13, normalized size = 0.62

$$\frac{3}{11}\,bx^{\frac{11}{3}}+\frac{3}{8}\,ax^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

**mupad [B]** time = 0.03, size = 13, normalized size = 0.62

$$\frac{3\,x^{8/3}\,\left(11\,a+8\,b\,x\right)}{88}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x),x)
```

```
[Out] (3*x^(8/3)*(11*a + 8*b*x))/88
```

**sympy [A]** time = 2.01, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{8}{3}}}{8}+\frac{3bx^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(8/3)/8 + 3*b*x**(11/3)/11
```

## 3.651 $\int x^{4/3}(a + bx)\,dx$

**Optimal**. Leaf size=21

$$\frac{3}{7}ax^{7/3} + \frac{3}{10}bx^{10/3}$$

**Rubi** [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.091, Rules used = {43}

$$\frac{3}{7}ax^{7/3} + \frac{3}{10}bx^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(7/3))/7 + (3*b*x^(10/3))/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a + bx)\,dx &= \int \left(ax^{4/3} + bx^{7/3}\right)\,dx\\ &= \frac{3}{7}ax^{7/3} + \frac{3}{10}bx^{10/3}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{3}{70}x^{7/3}(10a + 7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x),x]
```

```
[Out] (3*x^(7/3)*(10*a + 7*b*x))/70
```

**IntegrateAlgebraic [A]** time = 0.01, size = 21, normalized size = 1.00

$$\frac{3}{70}\left(10ax^{7/3}+7bx^{10/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)*(a + b*x),x]
[Out] (3*(10*a*x^(7/3) + 7*b*x^(10/3)))/70
```

**fricas [A]** time = 1.24, size = 18, normalized size = 0.86

$$\frac{3}{70}\left(7\,bx^{3}+10\,ax^{2}\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="fricas")
[Out] 3/70*(7*b*x^3 + 10*a*x^2)*x^(1/3)
```

**giac [A]** time = 1.04, size = 13, normalized size = 0.62

$$\frac{3}{10}\,bx^{\frac{10}{3}}+\frac{3}{7}\,ax^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="giac")
[Out] 3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.67

$$\frac{3\,(7bx+10a)\,x^{\frac{7}{3}}}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a),x)
[Out] 3/70*x^(7/3)*(7*b*x+10*a)
```

**maxima [A]** time = 1.35, size = 13, normalized size = 0.62

$$\frac{3}{10}\,bx^{\frac{10}{3}}+\frac{3}{7}\,ax^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

**mupad [B]** time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{7/3}\,\left(10\,a+7\,b\,x\right)}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x),x)
```

```
[Out] (3*x^(7/3)*(10*a + 7*b*x))/70
```

**sympy [A]** time = 1.30, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{7}{3}}}{7} + \frac{3bx^{\frac{10}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(7/3)/7 + 3*b*x**(10/3)/10
```

## 3.652 $\int x^{2/3}(a + bx)\, dx$

**Optimal**. Leaf size=21

$$\frac{3}{5}ax^{5/3} + \frac{3}{8}bx^{8/3}$$

**Rubi** [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{5}ax^{5/3} + \frac{3}{8}bx^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(5/3))/5 + (3*b*x^(8/3))/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a + bx)\, dx &= \int \left(ax^{2/3} + bx^{5/3}\right) dx\\ &= \frac{3}{5}ax^{5/3} + \frac{3}{8}bx^{8/3}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{3}{40}x^{5/3}(8a + 5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x),x]
```

```
[Out] (3*x^(5/3)*(8*a + 5*b*x))/40
```

**IntegrateAlgebraic** [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{3}{40}\left(8ax^{5/3}+5bx^{8/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)*(a + b*x),x]
[Out] (3*(8*a*x^(5/3) + 5*b*x^(8/3)))/40
```

**fricas** [A] time = 1.24, size = 16, normalized size = 0.76

$$\frac{3}{40}\left(5\,bx^2+8\,ax\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="fricas")
[Out] 3/40*(5*b*x^2 + 8*a*x)*x^(2/3)
```

**giac** [A] time = 0.99, size = 13, normalized size = 0.62

$$\frac{3}{8}\,bx^{\frac{8}{3}}+\frac{3}{5}\,ax^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="giac")
[Out] 3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

**maple** [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{3\,(5bx+8a)\,x^{\frac{5}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a),x)
[Out] 3/40*x^(5/3)*(5*b*x+8*a)
```

**maxima** [A] time = 1.35, size = 13, normalized size = 0.62

$$\frac{3}{8}\,bx^{\frac{8}{3}}+\frac{3}{5}\,ax^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

**mupad [B]** time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{5/3}\,\left(8\,a+5\,b\,x\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x),x)
```

```
[Out] (3*x^(5/3)*(8*a + 5*b*x))/40
```

**sympy [A]** time = 0.45, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{5}{3}}}{5}+\frac{3bx^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(5/3)/5 + 3*b*x**(8/3)/8
```

## 3.653 $\int \sqrt[3]{x}(a+bx)\,dx$

**Optimal**. Leaf size=21

$$\frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}$$

**Rubi** [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(4/3))/4 + (3*b*x^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}(a+bx)\,dx &= \int \left(a\sqrt[3]{x}+bx^{4/3}\right)\,dx\\ &= \frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{3}{28}x^{4/3}(7a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x),x]
```

```
[Out] (3*x^(4/3)*(7*a + 4*b*x))/28
```

**IntegrateAlgebraic [A]** time = 0.01, size = 21, normalized size = 1.00

$$\frac{3}{28}\left(7ax^{4/3}+4bx^{7/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)*(a + b*x),x]
[Out] (3*(7*a*x^(4/3) + 4*b*x^(7/3)))/28
```

**fricas [A]** time = 1.17, size = 16, normalized size = 0.76

$$\frac{3}{28}\left(4\,bx^{2}+7\,ax\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="fricas")
[Out] 3/28*(4*b*x^2 + 7*a*x)*x^(1/3)
```

**giac [A]** time = 1.03, size = 13, normalized size = 0.62

$$\frac{3}{7}\,bx^{\frac{7}{3}}+\frac{3}{4}\,ax^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="giac")
[Out] 3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.67

$$\frac{3\left(4bx+7a\right)x^{\frac{4}{3}}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a),x)
[Out] 3/28*x^(4/3)*(4*b*x+7*a)
```

**maxima [A]** time = 1.37, size = 13, normalized size = 0.62

$$\frac{3}{7}\,bx^{\frac{7}{3}}+\frac{3}{4}\,ax^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

**mupad [B]** time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{4/3}\,\left(7\,a+4\,b\,x\right)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x),x)
```

```
[Out] (3*x^(4/3)*(7*a + 4*b*x))/28
```

**sympy [A]** time = 1.52, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{4}{3}}}{4} + \frac{3bx^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(4/3)/4 + 3*b*x**(7/3)/7
```

## 3.654 $\int \frac{a+bx}{\sqrt[3]{x}}\,dx$

**Optimal**. Leaf size=21

$$\frac{3}{2}ax^{2/3}+\frac{3}{5}bx^{5/3}$$

**Rubi [A]** time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{2}ax^{2/3}+\frac{3}{5}bx^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(1/3),x]
[Out] (3*a*x^(2/3))/2 + (3*b*x^(5/3))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt[3]{x}}\,dx&=\int\left(\frac{a}{\sqrt[3]{x}}+bx^{2/3}\right)dx\\&=\frac{3}{2}ax^{2/3}+\frac{3}{5}bx^{5/3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 0.81

$$\frac{3}{10}x^{2/3}(5a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(1/3),x]
[Out] (3*x^(2/3)*(5*a + 2*b*x))/10
```

**IntegrateAlgebraic** [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{3}{10}\left(5ax^{2/3}+2bx^{5/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(1/3),x]
[Out] (3*(5*a*x^(2/3) + 2*b*x^(5/3)))/10
```

**fricas** [A] time = 1.30, size = 13, normalized size = 0.62

$$\frac{3}{10}\,(2\,bx+5\,a)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="fricas")
[Out] 3/10*(2*b*x + 5*a)*x^(2/3)
```

**giac** [A] time = 1.12, size = 13, normalized size = 0.62

$$\frac{3}{5}\,bx^{\frac{5}{3}}+\frac{3}{2}\,ax^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="giac")
[Out] 3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

**maple** [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{3\,(2bx+5a)\,x^{\frac{2}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(1/3),x)
[Out] 3/10*x^(2/3)*(2*b*x+5*a)
```

**maxima** [A] time = 1.31, size = 13, normalized size = 0.62

$$\frac{3}{5}\,bx^{\frac{5}{3}}+\frac{3}{2}\,ax^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

**mupad [B]** time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{2/3}\,\left(5\,a+2\,b\,x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(1/3),x)
```

```
[Out] (3*x^(2/3)*(5*a + 2*b*x))/10
```

**sympy [A]** time = 1.66, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{2}{3}}}{2}+\frac{3bx^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(1/3),x)
```

```
[Out] 3*a*x**(2/3)/2 + 3*b*x**(5/3)/5
```

## 3.655 $\int \frac{a+bx}{x^{2/3}} \, dx$

**Optimal**. Leaf size=19

$$3a\sqrt[3]{x} + \frac{3}{4}bx^{4/3}$$

**Rubi [A]** time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$3a\sqrt[3]{x} + \frac{3}{4}bx^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(2/3),x]
```

```
[Out] 3*a*x^(1/3) + (3*b*x^(4/3))/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{2/3}} \, dx &= \int \left(\frac{a}{x^{2/3}} + b\sqrt[3]{x}\right) dx \\ &= 3a\sqrt[3]{x} + \frac{3}{4}bx^{4/3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 0.84

$$\frac{3}{4}\sqrt[3]{x}(4a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(2/3),x]
```

```
[Out] (3*x^(1/3)*(4*a + b*x))/4
```

**IntegrateAlgebraic** [A] time = 0.01, size = 20, normalized size = 1.05

$$\frac{3}{4}\left(4a\sqrt[3]{x}+bx^{4/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(2/3),x]
[Out] (3*(4*a*x^(1/3) + b*x^(4/3)))/4
```

**fricas** [A] time = 1.13, size = 12, normalized size = 0.63

$$\frac{3}{4}(bx+4a)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="fricas")
[Out] 3/4*(b*x + 4*a)*x^(1/3)
```

**giac** [A] time = 1.12, size = 13, normalized size = 0.68

$$\frac{3}{4}bx^{\frac{4}{3}}+3ax^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="giac")
[Out] 3/4*b*x^(4/3) + 3*a*x^(1/3)
```

**maple** [A] time = 0.00, size = 13, normalized size = 0.68

$$\frac{3(bx+4a)x^{\frac{1}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(2/3),x)
[Out] 3/4*x^(1/3)*(b*x+4*a)
```

**maxima** [A] time = 1.28, size = 13, normalized size = 0.68

$$\frac{3}{4}bx^{\frac{4}{3}}+3ax^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*b*x^(4/3) + 3*a*x^(1/3)
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.63

$$\frac{3\,x^{1/3}\,\left(4\,a+b\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(2/3),x)
```

```
[Out] (3*x^(1/3)*(4*a + b*x))/4
```

**sympy [A]** time = 1.49, size = 17, normalized size = 0.89

$$3a\sqrt[3]{x} + \frac{3bx^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(2/3),x)
```

```
[Out] 3*a*x**(1/3) + 3*b*x**(4/3)/4
```

## 3.656 $\int \frac{a+bx}{x^{4/3}}\,dx$

**Optimal**. Leaf size=19

$$\frac{3}{2}bx^{2/3}-\frac{3a}{\sqrt[3]{x}}$$

**Rubi [A]** time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{2}bx^{2/3}-\frac{3a}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(4/3),x]
```

```
[Out] (-3*a)/x^(1/3) + (3*b*x^(2/3))/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{4/3}}\,dx &= \int \left(\frac{a}{x^{4/3}}+\frac{b}{\sqrt[3]{x}}\right)dx\\ &= -\frac{3a}{\sqrt[3]{x}}+\frac{3}{2}bx^{2/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 16, normalized size = 0.84

$$\frac{3(bx-2a)}{2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(4/3),x]
```

```
[Out] (3*(-2*a + b*x))/(2*x^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.01, size = 16, normalized size = 0.84

$$\frac{3(bx-2a)}{2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(4/3),x]
[Out] (3*(-2*a + b*x))/(2*x^(1/3))
```

**fricas [A]** time = 1.00, size = 12, normalized size = 0.63

$$\frac{3\,(bx-2\,a)}{2\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="fricas")
[Out] 3/2*(b*x - 2*a)/x^(1/3)
```

**giac [A]** time = 1.13, size = 13, normalized size = 0.68

$$\frac{3}{2}\,bx^{\frac{2}{3}}-\frac{3\,a}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="giac")
[Out] 3/2*b*x^(2/3) - 3*a/x^(1/3)
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.74

$$-\frac{3\,(-bx+2a)}{2x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(4/3),x)
[Out] -3/2*(-b*x+2*a)/x^(1/3)
```

**maxima** [A] time = 1.34, size = 13, normalized size = 0.68

$$\frac{3}{2}bx^{\frac{2}{3}} - \frac{3\,a}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/2*b*x^(2/3) - 3*a/x^(1/3)
```

**mupad** [B] time = 0.03, size = 13, normalized size = 0.68

$$-\frac{6\,a - 3\,b\,x}{2\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(4/3),x)
```

```
[Out] -(6*a - 3*b*x)/(2*x^(1/3))
```

**sympy** [A] time = 0.39, size = 17, normalized size = 0.89

$$-\frac{3a}{\sqrt[3]{x}} + \frac{3bx^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(4/3),x)
```

```
[Out] -3*a/x**(1/3) + 3*b*x**(2/3)/2
```

## 3.657 $\int \frac{a+bx}{x^{5/3}} dx$

**Optimal**. Leaf size=19

$$3b\sqrt[3]{x} - \frac{3a}{2x^{2/3}}$$

**Rubi [A]** time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$3b\sqrt[3]{x} - \frac{3a}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(5/3),x]
```

```
[Out] (-3*a)/(2*x^(2/3)) + 3*b*x^(1/3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^{5/3}} dx &= \int \left(\frac{a}{x^{5/3}} + \frac{b}{x^{2/3}}\right) dx \\ &= -\frac{3a}{2x^{2/3}} + 3b\sqrt[3]{x} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 19, normalized size = 1.00

$$3b\sqrt[3]{x} - \frac{3a}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(5/3),x]
```

```
[Out] (-3*a)/(2*x^(2/3)) + 3*b*x^(1/3)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 17, normalized size = 0.89

$$\frac{3(2bx-a)}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(5/3),x]
[Out] (3*(-a + 2*b*x))/(2*x^(2/3))
```

**fricas** [A] time = 1.17, size = 13, normalized size = 0.68

$$\frac{3\,(2\,bx-a)}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="fricas")
[Out] 3/2*(2*b*x - a)/x^(2/3)
```

**giac** [A] time = 1.10, size = 13, normalized size = 0.68

$$3\,bx^{\frac{1}{3}}-\frac{3\,a}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="giac")
[Out] 3*b*x^(1/3) - 3/2*a/x^(2/3)
```

**maple** [A] time = 0.00, size = 12, normalized size = 0.63

$$-\frac{3\,(-2bx+a)}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(5/3),x)
[Out] -3/2*(-2*b*x+a)/x^(2/3)
```

**maxima** [A] time = 1.30, size = 13, normalized size = 0.68

$$3\,bx^{\frac{1}{3}}-\frac{3\,a}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="maxima")
```

```
[Out] 3*b*x^(1/3) - 3/2*a/x^(2/3)
```

**mupad [B]** time = 0.03, size = 13, normalized size = 0.68

$$-\frac{3\,a-6\,b\,x}{2\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(5/3),x)
```

```
[Out] -(3*a - 6*b*x)/(2*x^(2/3))
```

**sympy [A]** time = 0.45, size = 17, normalized size = 0.89

$$-\frac{3a}{2x^{\frac{2}{3}}}+3b\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(5/3),x)
```

```
[Out] -3*a/(2*x**(2/3)) + 3*b*x**(1/3)
```

## 3.658 $\int x^{5/3}(a+bx)^2\,dx$

**Optimal**. Leaf size=36

$$\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

**Rubi [A]** time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x)^2,x]
[Out] (3*a^2*x^(8/3))/8 + (6*a*b*x^(11/3))/11 + (3*b^2*x^(14/3))/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)^2\,dx &= \int \left(a^2x^{5/3}+2abx^{8/3}+b^2x^{11/3}\right)dx\\ &= \frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{616}x^{8/3}\left(77a^2+112abx+44b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x)^2,x]
[Out] (3*x^(8/3)*(77*a^2 + 112*a*b*x + 44*b^2*x^2))/616
```

**IntegrateAlgebraic** [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{616}x^{8/3}\left(77a^2+112abx+44b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(8/3)*(77*a^2 + 112*a*b*x + 44*b^2*x^2))/616
```

**fricas** [A] time = 1.30, size = 29, normalized size = 0.81

$$\frac{3}{616}\left(44\,b^2x^4+112\,abx^3+77\,a^2x^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/616*(44*b^2*x^4 + 112*a*b*x^3 + 77*a^2*x^2)*x^(2/3)
```

**giac** [A] time = 1.04, size = 24, normalized size = 0.67

$$\frac{3}{14}\,b^2x^{\frac{14}{3}}+\frac{6}{11}\,abx^{\frac{11}{3}}+\frac{3}{8}\,a^2x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{3\left(44b^2x^2+112abx+77a^2\right)x^{\frac{8}{3}}}{616}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a)^2,x)
```

```
[Out] 3/616*x^(8/3)*(44*b^2*x^2+112*a*b*x+77*a^2)
```

**maxima** [A] time = 1.31, size = 24, normalized size = 0.67

$$\frac{3}{14}\,b^2x^{\frac{14}{3}}+\frac{6}{11}\,abx^{\frac{11}{3}}+\frac{3}{8}\,a^2x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

**mupad [B]** time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{8/3}\,\left(77\,a^2+112\,a\,b\,x+44\,b^2\,x^2\right)}{616}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(8/3)*(77*a^2 + 44*b^2*x^2 + 112*a*b*x))/616
```

**sympy [A]** time = 3.75, size = 34, normalized size = 0.94

$$\frac{3a^2x^{\frac{8}{3}}}{8}+\frac{6abx^{\frac{11}{3}}}{11}+\frac{3b^2x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(8/3)/8 + 6*a*b*x**(11/3)/11 + 3*b**2*x**(14/3)/14
```

## 3.659 $\int x^{4/3}(a+bx)^2\,dx$

**Optimal**. Leaf size=36

$$\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

**Rubi [A]** time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(7/3))/7 + (3*a*b*x^(10/3))/5 + (3*b^2*x^(13/3))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)^2\,dx&=\int\left(a^2x^{4/3}+2abx^{7/3}+b^2x^{10/3}\right)dx\\&=\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{455}x^{7/3}\left(65a^2+91abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(7/3)*(65*a^2 + 91*a*b*x + 35*b^2*x^2))/455
```

**IntegrateAlgebraic** [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{455}x^{7/3}\left(65a^2+91abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(7/3)*(65*a^2 + 91*a*b*x + 35*b^2*x^2))/455
```

**fricas** [A] time = 1.02, size = 29, normalized size = 0.81

$$\frac{3}{455}\left(35\,b^2x^4+91\,abx^3+65\,a^2x^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/455*(35*b^2*x^4 + 91*a*b*x^3 + 65*a^2*x^2)*x^(1/3)
```

**giac** [A] time = 1.06, size = 24, normalized size = 0.67

$$\frac{3}{13}\,b^2x^{\frac{13}{3}}+\frac{3}{5}\,abx^{\frac{10}{3}}+\frac{3}{7}\,a^2x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{3\left(35b^2x^2+91abx+65a^2\right)x^{\frac{7}{3}}}{455}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a)^2,x)
```

```
[Out] 3/455*x^(7/3)*(35*b^2*x^2+91*a*b*x+65*a^2)
```

**maxima** [A] time = 1.32, size = 24, normalized size = 0.67

$$\frac{3}{13}\,b^2x^{\frac{13}{3}}+\frac{3}{5}\,abx^{\frac{10}{3}}+\frac{3}{7}\,a^2x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="maxima")
[Out] 3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

**mupad [B]** time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{7/3}\,\left(65\,a^2+91\,a\,b\,x+35\,b^2\,x^2\right)}{455}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x)^2,x)
[Out] (3*x^(7/3)*(65*a^2 + 35*b^2*x^2 + 91*a*b*x))/455
```

**sympy [A]** time = 2.65, size = 34, normalized size = 0.94

$$\frac{3a^2x^{\frac{7}{3}}}{7}+\frac{3abx^{\frac{10}{3}}}{5}+\frac{3b^2x^{\frac{13}{3}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a)**2,x)
[Out] 3*a**2*x**(7/3)/7 + 3*a*b*x**(10/3)/5 + 3*b**2*x**(13/3)/13
```

## 3.660 $\int x^{2/3}(a+bx)^2\,dx$

**Optimal**. Leaf size=36

$$\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

**Rubi [A]** time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(5/3))/5 + (3*a*b*x^(8/3))/4 + (3*b^2*x^(11/3))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)^2\,dx&=\int\left(a^2x^{2/3}+2abx^{5/3}+b^2x^{8/3}\right)dx\\&=\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{220}x^{5/3}\left(44a^2+55abx+20b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(5/3)*(44*a^2 + 55*a*b*x + 20*b^2*x^2))/220
```

**IntegrateAlgebraic [A]** time = 0.01, size = 34, normalized size = 0.94

$$\frac{3}{220}\left(44a^2x^{5/3}+55abx^{8/3}+20b^2x^{11/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)*(a + b*x)^2,x]
[Out] (3*(44*a^2*x^(5/3) + 55*a*b*x^(8/3) + 20*b^2*x^(11/3)))/220
```

**fricas [A]** time = 1.10, size = 27, normalized size = 0.75

$$\frac{3}{220}\left(20\,b^2x^3+55\,abx^2+44\,a^2x\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="fricas")
[Out] 3/220*(20*b^2*x^3 + 55*a*b*x^2 + 44*a^2*x)*x^(2/3)
```

**giac [A]** time = 1.05, size = 24, normalized size = 0.67

$$\frac{3}{11}\,b^2x^{\frac{11}{3}}+\frac{3}{4}\,abx^{\frac{8}{3}}+\frac{3}{5}\,a^2x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="giac")
[Out] 3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.69

$$\frac{3\left(20b^2x^2+55abx+44a^2\right)x^{\frac{5}{3}}}{220}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a)^2,x)
[Out] 3/220*x^(5/3)*(20*b^2*x^2+55*a*b*x+44*a^2)
```

**maxima [A]** time = 1.32, size = 24, normalized size = 0.67

$$\frac{3}{11}\,b^2x^{\frac{11}{3}}+\frac{3}{4}\,abx^{\frac{8}{3}}+\frac{3}{5}\,a^2x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

**mupad [B]** time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{5/3}\,\left(44\,a^2+55\,a\,b\,x+20\,b^2\,x^2\right)}{220}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(5/3)*(44*a^2 + 20*b^2*x^2 + 55*a*b*x))/220
```

**sympy [A]** time = 1.06, size = 34, normalized size = 0.94

$$\frac{3a^2x^{\frac{5}{3}}}{5}+\frac{3abx^{\frac{8}{3}}}{4}+\frac{3b^2x^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(5/3)/5 + 3*a*b*x**(8/3)/4 + 3*b**2*x**(11/3)/11
```

## 3.661 $\int \sqrt[3]{x}\,(a+bx)^2\,dx$

**Optimal**. Leaf size=36

$$\frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}$$

**Rubi [A]** time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(4/3))/4 + (6*a*b*x^(7/3))/7 + (3*b^2*x^(10/3))/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}\,(a+bx)^2\,dx&=\int \left(a^2\sqrt[3]{x}+2abx^{4/3}+b^2x^{7/3}\right)dx\\&=\frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{140}x^{4/3}\left(35a^2+40abx+14b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(4/3)*(35*a^2 + 40*a*b*x + 14*b^2*x^2))/140
```

**IntegrateAlgebraic** [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{3}{140}\left(35a^2x^{4/3}+40abx^{7/3}+14b^2x^{10/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)*(a + b*x)^2,x]
[Out] (3*(35*a^2*x^(4/3) + 40*a*b*x^(7/3) + 14*b^2*x^(10/3)))/140
```

**fricas** [A] time = 1.15, size = 27, normalized size = 0.75

$$\frac{3}{140}\left(14\,b^2x^3+40\,abx^2+35\,a^2x\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="fricas")
[Out] 3/140*(14*b^2*x^3 + 40*a*b*x^2 + 35*a^2*x)*x^(1/3)
```

**giac** [A] time = 1.04, size = 24, normalized size = 0.67

$$\frac{3}{10}\,b^2x^{\frac{10}{3}}+\frac{6}{7}\,abx^{\frac{7}{3}}+\frac{3}{4}\,a^2x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="giac")
[Out] 3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

**maple** [A] time = 0.01, size = 25, normalized size = 0.69

$$\frac{3\left(14b^2x^2+40abx+35a^2\right)x^{\frac{4}{3}}}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a)^2,x)
[Out] 3/140*x^(4/3)*(14*b^2*x^2+40*a*b*x+35*a^2)
```

**maxima** [A] time = 1.28, size = 24, normalized size = 0.67

$$\frac{3}{10}\,b^2x^{\frac{10}{3}}+\frac{6}{7}\,abx^{\frac{7}{3}}+\frac{3}{4}\,a^2x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

**mupad [B]** time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{4/3}\,\left(35\,a^2+40\,a\,b\,x+14\,b^2\,x^2\right)}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(4/3)*(35*a^2 + 14*b^2*x^2 + 40*a*b*x))/140
```

**sympy [C]** time = 2.22, size = 2633, normalized size = 73.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a)**2,x)
```

```
[Out] Piecewise((27*a**(34/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(-140*a**
8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*
a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3
*exp(2*I*pi/3)) + 27*a**(34/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7
*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I
*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 72*a**(31/3)*b*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2
*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(
a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3))
- 81*a**(31/3)*b*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**
(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/
3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 60*a**(28/3)*b**2*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp
(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)
*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)
) + 81*a**(28/3)*b**2*(a/b + x)**2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*
a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp
(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(25/3)*
b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*pi/3)/(-140*a**8*b**(
4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b
**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2
*I*pi/3)) - 27*a**(25/3)*b**3*(a/b + x)**3/(-140*a**8*b**(4/3)*exp(2*I*pi/3
) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x
```

```
)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 135*a
**(22/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(2*I*pi/3)/(-140*
a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 4
20*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)
**3*exp(2*I*pi/3)) - 132*a**(19/3)*b**5*(-1 + b*(a/b + x)/a)**(1/3)*(a/b +
x)**5*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(
a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 14
0*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 42*a**(16/3)*b**6*(-1 + b*(a
/b + x)/a)**(1/3)*(a/b + x)**6*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2*I*pi
/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b +
 x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)), Abs(
b*(a/b + x)/a) > 1), (-27*a**(34/3)*(1 - b*(a/b + x)/a)**(1/3)/(-140*a**8*b
**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**
6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*ex
p(2*I*pi/3)) + 27*a**(34/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b*
*(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi
/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 72*a**(31/3)*b*(1 -
b*(a/b + x)/a)**(1/3)*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**
7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*
I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 81*a**(31/3)*b*(
a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*ex
p(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(
13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(28/3)*b**2*(1 - b*(a/b + x)/a)**
(1/3)*(a/b + x)**2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a
/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140
*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 81*a**(28/3)*b**2*(a/b + x)**
2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*p
i/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(
a/b + x)**3*exp(2*I*pi/3)) + 60*a**(25/3)*b**3*(1 - b*(a/b + x)/a)**(1/3)*(
a/b + x)**3/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)
*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b
**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 27*a**(25/3)*b**3*(a/b + x)**3/(-140
*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) -
420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x
)**3*exp(2*I*pi/3)) - 135*a**(22/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b +
x)**4/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2
*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/
3)*(a/b + x)**3*exp(2*I*pi/3)) + 132*a**(19/3)*b**5*(1 - b*(a/b + x)/a)**(1
/3)*(a/b + x)**5/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b
 + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a
**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 42*a**(16/3)*b**6*(1 - b*(a/b +
 x)/a)**(1/3)*(a/b + x)**6/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**
(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/
3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)), True))
```

## 3.662 $\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx$

**Optimal**. Leaf size=36

$$\frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}$$

**Rubi** [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(1/3),x]
```

```
[Out] (3*a^2*x^(2/3))/2 + (6*a*b*x^(5/3))/5 + (3*b^2*x^(8/3))/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx &= \int \left(\frac{a^2}{\sqrt[3]{x}}+2abx^{2/3}+b^2x^{5/3}\right)dx\\ &= \frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{40}x^{2/3}\left(20a^2+16abx+5b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(20*a^2 + 16*a*b*x + 5*b^2*x^2))/40
```

**IntegrateAlgebraic** [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{3}{40}\left(20a^2x^{2/3}+16abx^{5/3}+5b^2x^{8/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^(1/3),x]
[Out] (3*(20*a^2*x^(2/3) + 16*a*b*x^(5/3) + 5*b^2*x^(8/3)))/40
```

**fricas** [A] time = 1.10, size = 24, normalized size = 0.67

$$\frac{3}{40}\left(5\,b^2x^2+16\,abx+20\,a^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="fricas")
[Out] 3/40*(5*b^2*x^2 + 16*a*b*x + 20*a^2)*x^(2/3)
```

**giac** [A] time = 1.11, size = 24, normalized size = 0.67

$$\frac{3}{8}\,b^2x^{\frac{8}{3}}+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{3}{2}\,a^2x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="giac")
[Out] 3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{3\left(5b^2x^2+16abx+20a^2\right)x^{\frac{2}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(1/3),x)
[Out] 3/40*x^(2/3)*(5*b^2*x^2+16*a*b*x+20*a^2)
```

**maxima** [A] time = 1.32, size = 24, normalized size = 0.67

$$\frac{3}{8}\,b^2x^{\frac{8}{3}}+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{3}{2}\,a^2x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

**mupad [B]** time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{2/3}\,\left(20\,a^2+16\,a\,b\,x+5\,b^2\,x^2\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(1/3),x)
```

```
[Out] (3*x^(2/3)*(20*a^2 + 5*b^2*x^2 + 16*a*b*x))/40
```

**sympy [C]** time = 2.05, size = 1765, normalized size = 49.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(1/3),x)
```

```
[Out] Piecewise((-27*a**(32/3)*(-1 + b*(a/b + x)/a)**(2/3)/(-40*a**8*b**(2/3) + 1
20*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(1
1/3)*(a/b + x)**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a*
*7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*
(a/b + x)**3) + 63*a**(29/3)*b*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)/(-40*a
**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2
 + 40*a**5*b**(11/3)*(a/b + x)**3) - 81*a**(29/3)*b*(a/b + x)*exp(2*I*pi/3)
/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b
+ x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 42*a**(26/3)*b**2*(-1 + b*(a/b
+ x)/a)**(2/3)*(a/b + x)**2/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x
) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 81*a
**(26/3)*b**2*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(
5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b +
x)**3) + 18*a**(23/3)*b**3*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(-40*a*
*8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2
+ 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(23/3)*b**3*(a/b + x)**3*exp(2*I*
pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*
(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(20/3)*b**4*(-1 + b*
(a/b + x)/a)**(2/3)*(a/b + x)**4/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/
b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) +
 15*a**(17/3)*b**5*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**5/(-40*a**8*b**(2
/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**
5*b**(11/3)*(a/b + x)**3), Abs(b*(a/b + x)/a) > 1), (-27*a**(32/3)*(1 - b*(
```

```
a/b + x)/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/
b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) +
 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x
) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 63*a
**(29/3)*b*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(2*I*pi/3)/(-40*a**8*b**
(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a
**5*b**(11/3)*(a/b + x)**3) - 81*a**(29/3)*b*(a/b + x)*exp(2*I*pi/3)/(-40*a
**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2
 + 40*a**5*b**(11/3)*(a/b + x)**3) - 42*a**(26/3)*b**2*(1 - b*(a/b + x)/a)*
*(2/3)*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a
/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3)
+ 81*a**(26/3)*b**2*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**
7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(
a/b + x)**3) + 18*a**(23/3)*b**3*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3*ex
p(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**
(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(23/3)*b**3*(a
/b + x)**3*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) -
 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(
20/3)*b**4*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(2*I*pi/3)/(-40*a**8*
b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 4
0*a**5*b**(11/3)*(a/b + x)**3) + 15*a**(17/3)*b**5*(1 - b*(a/b + x)/a)**(2/
3)*(a/b + x)**5*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b +
 x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3), Tru
e))
```

## 3.663 $\int \frac{(a+bx)^2}{x^{2/3}}\,dx$

**Optimal**. Leaf size=34

$$3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}$$

**Rubi** [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(2/3),x]
[Out] 3*a^2*x^(1/3) + (3*a*b*x^(4/3))/2 + (3*b^2*x^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{2/3}}\,dx &= \int \left(\frac{a^2}{x^{2/3}}+2ab\sqrt[3]{x}+b^2x^{4/3}\right)dx\\ &= 3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 28, normalized size = 0.82

$$\frac{3}{14}\sqrt[3]{x}\left(14a^2+7abx+2b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(2/3),x]
[Out] (3*x^(1/3)*(14*a^2 + 7*a*b*x + 2*b^2*x^2))/14
```

**IntegrateAlgebraic** [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{3}{14}\left(14a^2\sqrt[3]{x}+7abx^{4/3}+2b^2x^{7/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^(2/3),x]
[Out] (3*(14*a^2*x^(1/3) + 7*a*b*x^(4/3) + 2*b^2*x^(7/3)))/14
```

**fricas** [A] time = 1.33, size = 24, normalized size = 0.71

$$\frac{3}{14}\left(2\,b^2x^2+7\,abx+14\,a^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="fricas")
[Out] 3/14*(2*b^2*x^2 + 7*a*b*x + 14*a^2)*x^(1/3)
```

**giac** [A] time = 0.92, size = 24, normalized size = 0.71

$$\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{3}{2}\,abx^{\frac{4}{3}}+3\,a^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="giac")
[Out] 3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

**maple** [A] time = 0.01, size = 25, normalized size = 0.74

$$\frac{3\left(2b^2x^2+7abx+14a^2\right)x^{\frac{1}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(2/3),x)
[Out] 3/14*x^(1/3)*(2*b^2*x^2+7*a*b*x+14*a^2)
```

**maxima** [A] time = 1.38, size = 24, normalized size = 0.71

$$\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{3}{2}\,abx^{\frac{4}{3}}+3\,a^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

**mupad [B]** time = 0.03, size = 24, normalized size = 0.71

$$\frac{3\,x^{1/3}\,\left(14\,a^2+7\,a\,b\,x+2\,b^2\,x^2\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(2/3),x)
```

```
[Out] (3*x^(1/3)*(14*a^2 + 2*b^2*x^2 + 7*a*b*x))/14
```

**sympy [C]** time = 2.08, size = 1741, normalized size = 51.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(2/3),x)
```

```
[Out] Piecewise((-27*a**(31/3)*(-1 + b*(a/b + x)/a)**(1/3)/(-14*a**8*b**(1/3) + 4
2*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/
3)*(a/b + x)**3) + 27*a**(31/3)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b*
*(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b +
 x)**3) + 72*a**(28/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)/(-14*a**8*b*
*(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a*
*5*b**(10/3)*(a/b + x)**3) - 81*a**(28/3)*b*(a/b + x)*exp(I*pi/3)/(-14*a**8
*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14
*a**5*b**(10/3)*(a/b + x)**3) - 60*a**(25/3)*b**2*(-1 + b*(a/b + x)/a)**(1/
3)*(a/b + x)**2/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b
**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 81*a**(25/3)*b**2*
(a/b + x)**2*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) -
42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 18*a**(22
/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(-14*a**8*b**(1/3) + 42*a
**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*
(a/b + x)**3) - 27*a**(22/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/
3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b
**(10/3)*(a/b + x)**3) - 9*a**(19/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
+ x)**4/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*
(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 6*a**(16/3)*b**5*(-1 + b*(
a/b + x)/a)**(1/3)*(a/b + x)**5/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b
+ x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3), Abs
(b*(a/b + x)/a) > 1), (-27*a**(31/3)*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)
```

```
/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b +
x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 27*a**(31/3)*exp(I*pi/3)/(-14*a**
8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 1
4*a**5*b**(10/3)*(a/b + x)**3) + 72*a**(28/3)*b*(1 - b*(a/b + x)/a)**(1/3)*
(a/b + x)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*
a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 81*a**(28/3)
*b*(a/b + x)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) -
42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 60*a**(25
/3)*b**2*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I*pi/3)/(-14*a**8*b**(
1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5
*b**(10/3)*(a/b + x)**3) + 81*a**(25/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(-14*
a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2
+ 14*a**5*b**(10/3)*(a/b + x)**3) + 18*a**(22/3)*b**3*(1 - b*(a/b + x)/a)**
(1/3)*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b +
 x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 27*
a**(22/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/
3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)*
*3) - 9*a**(19/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(I*pi/3)/
(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x
)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 6*a**(16/3)*b**5*(1 - b*(a/b + x)/
a)**(1/3)*(a/b + x)**5*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a
/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3),
True))
```

## 3.664 $\int \frac{(a+bx)^2}{x^{4/3}}\,dx$

**Optimal**. Leaf size=32

$$-\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}$$

**Rubi** [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(4/3),x]
```

```
[Out] (-3*a^2)/x^(1/3) + 3*a*b*x^(2/3) + (3*b^2*x^(5/3))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{4/3}}\,dx &= \int \left(\frac{a^2}{x^{4/3}}+\frac{2ab}{\sqrt[3]{x}}+b^2x^{2/3}\right)dx\\ &= -\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{3\left(-5a^2+5abx+b^2x^2\right)}{5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(4/3),x]
```

[Out] (3*(-5*a^2 + 5*a*b*x + b^2*x^2))/(5*x^(1/3))

**IntegrateAlgebraic** [A] time = 0.02, size = 27, normalized size = 0.84

$$\frac{3\left(-5a^2+5abx+b^2x^2\right)}{5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x)^2/x^(4/3),x]

[Out] (3*(-5*a^2 + 5*a*b*x + b^2*x^2))/(5*x^(1/3))

**fricas** [A] time = 1.29, size = 23, normalized size = 0.72

$$\frac{3\left(b^2x^2+5\,abx-5\,a^2\right)}{5\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="fricas")

[Out] 3/5*(b^2*x^2 + 5*a*b*x - 5*a^2)/x^(1/3)

**giac** [A] time = 1.17, size = 24, normalized size = 0.75

$$\frac{3}{5}\,b^2x^{\frac{5}{3}}+3\,abx^{\frac{2}{3}}-\frac{3\,a^2}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="giac")

[Out] 3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)

**maple** [A] time = 0.00, size = 25, normalized size = 0.78

$$-\frac{3\left(-b^2x^2-5abx+5a^2\right)}{5x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^2/x^(4/3),x)

[Out] -3/5*(-b^2*x^2-5*a*b*x+5*a^2)/x^(1/3)

**maxima** [A] time = 1.36, size = 24, normalized size = 0.75

$$\frac{3}{5}\,b^2x^{\frac{5}{3}} + 3\,abx^{\frac{2}{3}} - \frac{3\,a^2}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)
```

**mupad** [B] time = 0.04, size = 24, normalized size = 0.75

$$\frac{-15\,a^2 + 15\,a\,b\,x + 3\,b^2\,x^2}{5\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(4/3),x)
```

```
[Out] (3*b^2*x^2 - 15*a^2 + 15*a*b*x)/(5*x^(1/3))
```

**sympy** [C] time = 2.09, size = 1826, normalized size = 57.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(4/3),x)
```

```
[Out] Piecewise((-27*a**(29/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/(
-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
 x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 27*a**(29/3)*b
**(1/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b*
*2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 63*a*
*(26/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(I*pi/3)/(-5*a**8
*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*
exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 81*a**(26/3)*b**(4/3)
*(a/b + x)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6
*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 42
*a**(23/3)*b**(7/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2*exp(I*pi/3)/(-
5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 81*a**(23/3)*b*
*(7/3)*(a/b + x)**2/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3)
- 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi
/3)) + 3*a**(20/3)*b**(10/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(I
*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**
2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 27*a**
```

```
(20/3)*b**(10/3)*(a/b + x)**3/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*ex
p(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**
3*exp(I*pi/3)) + 3*a**(17/3)*b**(13/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x
)**4*exp(I*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 1
5*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)
), Abs(b*(a/b + x)/a) > 1), (27*a**(29/3)*b**(1/3)*(1 - b*(a/b + x)/a)**(2/
3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a
/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 27*a**(29/
3)*b**(1/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**
6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 6
3*a**(26/3)*b**(4/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)/(-5*a**8*exp(I*pi
/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/
3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 81*a**(26/3)*b**(4/3)*(a/b + x
)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/
b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 42*a**(23/3
)*b**(7/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(-5*a**8*exp(I*pi/3) + 1
5*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*
a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 81*a**(23/3)*b**(7/3)*(a/b + x)**2/(-
5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 3*a**(20/3)*b**
(10/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(-5*a**8*exp(I*pi/3) + 15*a*
*7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5
*b**3*(a/b + x)**3*exp(I*pi/3)) + 27*a**(20/3)*b**(10/3)*(a/b + x)**3/(-5*a
**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)*
*2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 3*a**(17/3)*b**(13
/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(-5*a**8*exp(I*pi/3) + 15*a**7*
b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b*
*3*(a/b + x)**3*exp(I*pi/3)), True))
```

## 3.665 $\int \frac{(a+bx)^2}{x^{5/3}}\,dx$

**Optimal**. Leaf size=34

$$-\frac{3a^2}{2x^{2/3}}+6ab\sqrt[3]{x}+\frac{3}{4}b^2x^{4/3}$$

**Rubi** [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{3a^2}{2x^{2/3}}+6ab\sqrt[3]{x}+\frac{3}{4}b^2x^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(5/3),x]
[Out] (-3*a^2)/(2*x^(2/3)) + 6*a*b*x^(1/3) + (3*b^2*x^(4/3))/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{5/3}}\,dx &= \int \left(\frac{a^2}{x^{5/3}}+\frac{2ab}{x^{2/3}}+b^2\sqrt[3]{x}\right)dx\\ &= -\frac{3a^2}{2x^{2/3}}+6ab\sqrt[3]{x}+\frac{3}{4}b^2x^{4/3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 0.79

$$\frac{3\left(-2a^2+8abx+b^2x^2\right)}{4x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(5/3),x]
[Out] (3*(-2*a^2 + 8*a*b*x + b^2*x^2))/(4*x^(2/3))
```

**IntegrateAlgebraic** [A] time = 0.02, size = 27, normalized size = 0.79

$$\frac{3(-2a^2+8abx+b^2x^2)}{4x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^(5/3),x]
[Out] (3*(-2*a^2 + 8*a*b*x + b^2*x^2))/(4*x^(2/3))
```

**fricas** [A] time = 1.21, size = 23, normalized size = 0.68

$$\frac{3(b^2x^2+8\,abx-2\,a^2)}{4\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="fricas")
[Out] 3/4*(b^2*x^2 + 8*a*b*x - 2*a^2)/x^(2/3)
```

**giac** [A] time = 1.22, size = 24, normalized size = 0.71

$$\frac{3}{4}\,b^2x^{\frac{4}{3}}+6\,abx^{\frac{1}{3}}-\frac{3\,a^2}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="giac")
[Out] 3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.74

$$-\frac{3(-b^2x^2-8abx+2a^2)}{4x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(5/3),x)
[Out] -3/4*(-b^2*x^2-8*a*b*x+2*a^2)/x^(2/3)
```

**maxima** [A] time = 1.35, size = 24, normalized size = 0.71

$$\frac{3}{4}\,b^2x^{\frac{4}{3}}+6\,abx^{\frac{1}{3}}-\frac{3\,a^2}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

**mupad [B]** time = 0.04, size = 24, normalized size = 0.71

$$\frac{-6\,a^2+24\,a\,b\,x+3\,b^2\,x^2}{4\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(5/3),x)
```

```
[Out] (3*b^2*x^2 - 6*a^2 + 24*a*b*x)/(4*x^(2/3))
```

**sympy [C]** time = 2.06, size = 1957, normalized size = 57.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(5/3),x)
```

```
[Out] Piecewise((-27*a**(28/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)
/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*
(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 27*a
**(28/3)*b**(2/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3
) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(
2*I*pi/3)) + 72*a**(25/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*ex
p(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12
*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi
/3)) + 81*a**(25/3)*b**(5/3)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(
a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b
**3*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(22/3)*b**(8/3)*(-1 + b*(a/b + x)/a
)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b
 + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3
*(a/b + x)**3*exp(2*I*pi/3)) - 81*a**(22/3)*b**(8/3)*(a/b + x)**2/(-4*a**8*
exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)*
*2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) + 12*a**(19/3)*b
**(11/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*pi/3)/(-4*a**8*ex
p(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2
*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) + 27*a**(19/3)*b**
(11/3)*(a/b + x)**3/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi
/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*ex
p(2*I*pi/3)) + 3*a**(16/3)*b**(14/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*
*4*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3)
```

```
 - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2
*I*pi/3)), Abs(b*(a/b + x)/a) > 1), (27*a**(28/3)*b**(2/3)*(1 - b*(a/b + x)
/a)**(1/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*
a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/
3)) - 27*a**(28/3)*b**(2/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*ex
p(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b +
x)**3*exp(2*I*pi/3)) - 72*a**(25/3)*b**(5/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/
b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6
*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3))
+ 81*a**(25/3)*b**(5/3)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b +
 x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(
a/b + x)**3*exp(2*I*pi/3)) + 60*a**(22/3)*b**(8/3)*(1 - b*(a/b + x)/a)**(1/
3)*(a/b + x)**2/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3)
- 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*
I*pi/3)) - 81*a**(22/3)*b**(8/3)*(a/b + x)**2/(-4*a**8*exp(2*I*pi/3) + 12*a
**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4
*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 12*a**(19/3)*b**(11/3)*(1 - b*(a/b
 + x)/a)**(1/3)*(a/b + x)**3/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*e
xp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b +
 x)**3*exp(2*I*pi/3)) + 27*a**(19/3)*b**(11/3)*(a/b + x)**3/(-4*a**8*exp(2*
I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp
(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 3*a**(16/3)*b**(14/3
)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(-4*a**8*exp(2*I*pi/3) + 12*a**7*
b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**
5*b**3*(a/b + x)**3*exp(2*I*pi/3)), True))
```

## 3.666 $\int x^{5/3}(a+bx)^3\,dx$

**Optimal**. Leaf size=51

$$\frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

**Rubi** [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{11}a^2bx^{11/3}+\frac{3}{8}a^3x^{8/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(8/3))/8 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2*x^(14/3))/14 + (3*b^3*
x^(17/3))/17
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)^3\,dx&=\int\left(a^3x^{5/3}+3a^2bx^{8/3}+3ab^2x^{11/3}+b^3x^{14/3}\right)dx\\&=\frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{8/3}\left(1309a^3+2856a^2bx+2244ab^2x^2+616b^3x^3\right)}{10472}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(8/3)*(1309*a^3 + 2856*a^2*b*x + 2244*a*b^2*x^2 + 616*b^3*x^3))/10472
```

**IntegrateAlgebraic [A]** time = 0.01, size = 47, normalized size = 0.92

$$\frac{3\left(1309a^3x^{8/3}+2856a^2bx^{11/3}+2244ab^2x^{14/3}+616b^3x^{17/3}\right)}{10472}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)*(a + b*x)^3,x]
```

```
[Out] (3*(1309*a^3*x^(8/3) + 2856*a^2*b*x^(11/3) + 2244*a*b^2*x^(14/3) + 616*b^3*
x^(17/3)))/10472
```

**fricas [A]** time = 1.25, size = 40, normalized size = 0.78

$$\frac{3}{10472}\left(616\,b^3x^5+2244\,ab^2x^4+2856\,a^2bx^3+1309\,a^3x^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/10472*(616*b^3*x^5 + 2244*a*b^2*x^4 + 2856*a^2*b*x^3 + 1309*a^3*x^2)*x^(2
/3)
```

**giac [A]** time = 1.06, size = 35, normalized size = 0.69

$$\frac{3}{17}\,b^3x^{\frac{17}{3}}+\frac{9}{14}\,ab^2x^{\frac{14}{3}}+\frac{9}{11}\,a^2bx^{\frac{11}{3}}+\frac{3}{8}\,a^3x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^(
8/3)
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.71

$$\frac{3\left(616b^3x^3+2244a\,b^2x^2+2856a^2bx+1309a^3\right)x^{\frac{8}{3}}}{10472}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a)^3,x)
```

```
[Out] 3/10472*x^(8/3)*(616*b^3*x^3+2244*a*b^2*x^2+2856*a^2*b*x+1309*a^3)
```

**maxima** [A] time = 1.35, size = 35, normalized size = 0.69

$$\frac{3}{17}\,b^3x^{\frac{17}{3}}+\frac{9}{14}\,ab^2x^{\frac{14}{3}}+\frac{9}{11}\,a^2bx^{\frac{11}{3}}+\frac{3}{8}\,a^3x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^(
8/3)
```

**mupad** [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{8/3}}{8}+\frac{3\,b^3\,x^{17/3}}{17}+\frac{9\,a^2\,b\,x^{11/3}}{11}+\frac{9\,a\,b^2\,x^{14/3}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(8/3))/8 + (3*b^3*x^(17/3))/17 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2*
x^(14/3))/14
```

**sympy** [A] time = 6.39, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{8}{3}}}{8}+\frac{9a^2bx^{\frac{11}{3}}}{11}+\frac{9ab^2x^{\frac{14}{3}}}{14}+\frac{3b^3x^{\frac{17}{3}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(8/3)/8 + 9*a**2*b*x**(11/3)/11 + 9*a*b**2*x**(14/3)/14 + 3*b**3*
x**(17/3)/17
```

## 3.667 $\int x^{4/3}(a+bx)^3\,dx$

**Optimal**. Leaf size=51

$$\frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

**Rubi [A]** time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{10}a^2bx^{10/3}+\frac{3}{7}a^3x^{7/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(7/3))/7 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*x^(13/3))/13 + (3*b^3*
x^(16/3))/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)^3\,dx &= \int \left(a^3x^{4/3}+3a^2bx^{7/3}+3ab^2x^{10/3}+b^3x^{13/3}\right)dx\\ &= \frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{7/3}\left(1040a^3+2184a^2bx+1680ab^2x^2+455b^3x^3\right)}{7280}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(7/3)*(1040*a^3 + 2184*a^2*b*x + 1680*a*b^2*x^2 + 455*b^3*x^3))/7280
```

**IntegrateAlgebraic [A]** time = 0.01, size = 47, normalized size = 0.92

$$\frac{3\left(1040a^3x^{7/3}+2184a^2bx^{10/3}+1680ab^2x^{13/3}+455b^3x^{16/3}\right)}{7280}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)*(a + b*x)^3,x]
```

```
[Out] (3*(1040*a^3*x^(7/3) + 2184*a^2*b*x^(10/3) + 1680*a*b^2*x^(13/3) + 455*b^3*
x^(16/3)))/7280
```

**fricas [A]** time = 1.21, size = 40, normalized size = 0.78

$$\frac{3}{7280}\left(455\,b^3x^5+1680\,ab^2x^4+2184\,a^2bx^3+1040\,a^3x^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/7280*(455*b^3*x^5 + 1680*a*b^2*x^4 + 2184*a^2*b*x^3 + 1040*a^3*x^2)*x^(1/
3)
```

**giac [A]** time = 1.06, size = 35, normalized size = 0.69

$$\frac{3}{16}\,b^3x^{\frac{16}{3}}+\frac{9}{13}\,ab^2x^{\frac{13}{3}}+\frac{9}{10}\,a^2bx^{\frac{10}{3}}+\frac{3}{7}\,a^3x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^(
7/3)
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.71

$$\frac{3\left(455b^3x^3+1680a\,b^2x^2+2184a^2bx+1040a^3\right)x^{\frac{7}{3}}}{7280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a)^3,x)
```

```
[Out] 3/7280*x^(7/3)*(455*b^3*x^3+1680*a*b^2*x^2+2184*a^2*b*x+1040*a^3)
```

**maxima** [A] time = 1.29, size = 35, normalized size = 0.69

$$\frac{3}{16}\,b^3x^{\frac{16}{3}}+\frac{9}{13}\,ab^2x^{\frac{13}{3}}+\frac{9}{10}\,a^2bx^{\frac{10}{3}}+\frac{3}{7}\,a^3x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^(
7/3)
```

**mupad** [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{7/3}}{7}+\frac{3\,b^3\,x^{16/3}}{16}+\frac{9\,a^2\,b\,x^{10/3}}{10}+\frac{9\,a\,b^2\,x^{13/3}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(7/3))/7 + (3*b^3*x^(16/3))/16 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*
x^(13/3))/13
```

**sympy** [A] time = 4.55, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{7}{3}}}{7}+\frac{9a^2bx^{\frac{10}{3}}}{10}+\frac{9ab^2x^{\frac{13}{3}}}{13}+\frac{3b^3x^{\frac{16}{3}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(7/3)/7 + 9*a**2*b*x**(10/3)/10 + 9*a*b**2*x**(13/3)/13 + 3*b**3*
x**(16/3)/16
```

## 3.668 $\int x^{2/3}(a+bx)^3\,dx$

**Optimal**. Leaf size=51

$$\frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

**Rubi** [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{8}a^2bx^{8/3}+\frac{3}{5}a^3x^{5/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(5/3))/5 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x^(11/3))/11 + (3*b^3*x^
(14/3))/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)^3\,dx &= \int \left(a^3x^{2/3}+3a^2bx^{5/3}+3ab^2x^{8/3}+b^3x^{11/3}\right)dx\\ &= \frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{5/3}\left(616a^3+1155a^2bx+840ab^2x^2+220b^3x^3\right)}{3080}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(5/3)*(616*a^3 + 1155*a^2*b*x + 840*a*b^2*x^2 + 220*b^3*x^3))/3080
```

**IntegrateAlgebraic [A]** time = 0.01, size = 47, normalized size = 0.92

$$\frac{3\left(616a^3x^{5/3}+1155a^2bx^{8/3}+840ab^2x^{11/3}+220b^3x^{14/3}\right)}{3080}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)*(a + b*x)^3,x]
```

```
[Out] (3*(616*a^3*x^(5/3) + 1155*a^2*b*x^(8/3) + 840*a*b^2*x^(11/3) + 220*b^3*x^(
14/3)))/3080
```

**fricas [A]** time = 1.40, size = 38, normalized size = 0.75

$$\frac{3}{3080}\left(220\,b^3x^4+840\,ab^2x^3+1155\,a^2bx^2+616\,a^3x\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/3080*(220*b^3*x^4 + 840*a*b^2*x^3 + 1155*a^2*b*x^2 + 616*a^3*x)*x^(2/3)
```

**giac [A]** time = 1.07, size = 35, normalized size = 0.69

$$\frac{3}{14}\,b^3x^{\frac{14}{3}}+\frac{9}{11}\,ab^2x^{\frac{11}{3}}+\frac{9}{8}\,a^2bx^{\frac{8}{3}}+\frac{3}{5}\,a^3x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5/
3)
```

**maple [A]** time = 0.01, size = 36, normalized size = 0.71

$$\frac{3\left(220b^3x^3+840a\,b^2x^2+1155a^2bx+616a^3\right)x^{\frac{5}{3}}}{3080}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a)^3,x)
```

```
[Out] 3/3080*x^(5/3)*(220*b^3*x^3+840*a*b^2*x^2+1155*a^2*b*x+616*a^3)
```

**maxima** [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{3}{14}\,b^3x^{\frac{14}{3}}+\frac{9}{11}\,ab^2x^{\frac{11}{3}}+\frac{9}{8}\,a^2bx^{\frac{8}{3}}+\frac{3}{5}\,a^3x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5/
3)
```

**mupad** [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{5/3}}{5}+\frac{3\,b^3\,x^{14/3}}{14}+\frac{9\,a^2\,b\,x^{8/3}}{8}+\frac{9\,a\,b^2\,x^{11/3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(5/3))/5 + (3*b^3*x^(14/3))/14 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x^
(11/3))/11
```

**sympy** [A] time = 2.19, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{5}{3}}}{5}+\frac{9a^2bx^{\frac{8}{3}}}{8}+\frac{9ab^2x^{\frac{11}{3}}}{11}+\frac{3b^3x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(5/3)/5 + 9*a**2*b*x**(8/3)/8 + 9*a*b**2*x**(11/3)/11 + 3*b**3*x*
*(14/3)/14
```

## 3.669 $\int \sqrt[3]{x}\,(a+bx)^3\,dx$

**Optimal**. Leaf size=51

$$\frac{3}{4}a^3x^{4/3}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}$$

**Rubi [A]** time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{7}a^2bx^{7/3}+\frac{3}{4}a^3x^{4/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(4/3))/4 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^(10/3))/10 + (3*b^3*x^
(13/3))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}\,(a+bx)^3\,dx&=\int \left(a^3\sqrt[3]{x}+3a^2bx^{4/3}+3ab^2x^{7/3}+b^3x^{10/3}\right)dx\\&=\frac{3}{4}a^3x^{4/3}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{4/3}\left(455a^3+780a^2bx+546ab^2x^2+140b^3x^3\right)}{1820}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(4/3)*(455*a^3 + 780*a^2*b*x + 546*a*b^2*x^2 + 140*b^3*x^3))/1820
```

**IntegrateAlgebraic [A]** time = 0.01, size = 47, normalized size = 0.92

$$\frac{3\left(455a^3x^{4/3}+780a^2bx^{7/3}+546ab^2x^{10/3}+140b^3x^{13/3}\right)}{1820}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*(455*a^3*x^(4/3) + 780*a^2*b*x^(7/3) + 546*a*b^2*x^(10/3) + 140*b^3*x^(1
3/3)))/1820
```

**fricas [A]** time = 1.18, size = 38, normalized size = 0.75

$$\frac{3}{1820}\left(140\,b^3x^4+546\,ab^2x^3+780\,a^2bx^2+455\,a^3x\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/1820*(140*b^3*x^4 + 546*a*b^2*x^3 + 780*a^2*b*x^2 + 455*a^3*x)*x^(1/3)
```

**giac [A]** time = 1.16, size = 35, normalized size = 0.69

$$\frac{3}{13}\,b^3x^{\frac{13}{3}}+\frac{9}{10}\,ab^2x^{\frac{10}{3}}+\frac{9}{7}\,a^2bx^{\frac{7}{3}}+\frac{3}{4}\,a^3x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4/
3)
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.71

$$\frac{3\left(140b^3x^3+546a\,b^2x^2+780a^2bx+455a^3\right)x^{\frac{4}{3}}}{1820}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a)^3,x)
```

```
[Out] 3/1820*x^(4/3)*(140*b^3*x^3+546*a*b^2*x^2+780*a^2*b*x+455*a^3)
```

**maxima** [A] time = 1.35, size = 35, normalized size = 0.69

$$\frac{3}{13}\,b^3x^{\frac{13}{3}} + \frac{9}{10}\,ab^2x^{\frac{10}{3}} + \frac{9}{7}\,a^2bx^{\frac{7}{3}} + \frac{3}{4}\,a^3x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4/
3)
```

**mupad** [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{4/3}}{4} + \frac{3\,b^3\,x^{13/3}}{13} + \frac{9\,a^2\,b\,x^{7/3}}{7} + \frac{9\,a\,b^2\,x^{10/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(4/3))/4 + (3*b^3*x^(13/3))/13 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^
(10/3))/10
```

**sympy** [C] time = 3.22, size = 5012, normalized size = 98.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a)**3,x)
```

```
[Out] Piecewise((-243*a**(73/3)*(-1 + b*(a/b + x)/a)**(1/3)/(1820*a**20*b**(4/3)
- 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 364
00*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 1092
0*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a
**(73/3)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x)
+ 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 +
 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 +
1820*a**14*b**(22/3)*(a/b + x)**6) + 1377*a**(70/3)*b*(-1 + b*(a/b + x)/a)*
*(1/3)*(a/b + x)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27
300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 273
00*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820
*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(70/3)*b*(a/b + x)*exp(I*pi/3)/(18
20*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*
(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(
a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/
b + x)**6) - 3213*a**(67/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(
```

1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3
)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)
*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(
a/b + x)**6) + 3645*a**(67/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(1820*a**20*b**
(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2
 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4
- 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) +
 3927*a**(64/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(1820*a**20*b
**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)*
*2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**
4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6)
 - 4860*a**(64/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(1820*a**20*b**(4/3) - 1092
0*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**
17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**1
5*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 2163*a**(61
/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(1820*a**20*b**(4/3) - 10
920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a
**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a*
*15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 3645*a**(
61/3)*b**4*(a/b + x)**4*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(
7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)
*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*
(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1827*a**(58/3)*b**5*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5/(1820*a**20*b**(4/3) - 10920*a**19*b*
*(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/
3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3
)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(58/3)*b**5*(
a/b + x)**5*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b +
x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**
3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5
 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 6573*a**(55/3)*b**6*(-1 + b*(a/b +
x)/a)**(1/3)*(a/b + x)**6/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b
+ x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)
**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)*
*5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a**(55/3)*b**6*(a/b + x)**6*e
xp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a*
*18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**
16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14
*b**(22/3)*(a/b + x)**6) - 8787*a**(52/3)*b**7*(-1 + b*(a/b + x)/a)**(1/3)*
(a/b + x)**7/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*
a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a
**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**
14*b**(22/3)*(a/b + x)**6) + 6498*a**(49/3)*b**8*(-1 + b*(a/b + x)/a)**(1/3
)*(a/b + x)**8/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 2730
0*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300

*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a
**14*b**(22/3)*(a/b + x)**6) - 2562*a**(46/3)*b**9*(-1 + b*(a/b + x)/a)**(1
/3)*(a/b + x)**9/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27
300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 273
00*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820
*a**14*b**(22/3)*(a/b + x)**6) + 420*a**(43/3)*b**10*(-1 + b*(a/b + x)/a)**
(1/3)*(a/b + x)**10/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) +
 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 +
27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1
820*a**14*b**(22/3)*(a/b + x)**6), Abs(b*(a/b + x)/a) > 1), (-243*a**(73/3)
*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*
b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(1
3/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19
/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a**(73/3)*exp(I
*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*
b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b
**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**
(22/3)*(a/b + x)**6) + 1377*a**(70/3)*b*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x
)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300
*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*
a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a*
*14*b**(22/3)*(a/b + x)**6) - 1458*a**(70/3)*b*(a/b + x)*exp(I*pi/3)/(1820*
a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/
b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b
 + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b +
 x)**6) - 3213*a**(67/3)*b**2*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I
*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*
b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b
**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**
(22/3)*(a/b + x)**6) + 3645*a**(67/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(1820*a
**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b
 + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b
+ x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b +
x)**6) + 3927*a**(64/3)*b**3*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(I*
pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b
**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b*
*(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(
22/3)*(a/b + x)**6) - 4860*a**(64/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(1820*a*
*20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b
+ x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b +
 x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x
)**6) - 2163*a**(61/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(I*p
i/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b*
*(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**
(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(2

2/3)*(a/b + x)**6) + 3645*a**(61/3)*b**4*(a/b + x)**4*exp(I*pi/3)/(1820*a**
20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b +
x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b +
x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)
**6) - 1827*a**(58/3)*b**5*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5*exp(I*pi
/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**
(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(
16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22
/3)*(a/b + x)**6) - 1458*a**(58/3)*b**5*(a/b + x)**5*exp(I*pi/3)/(1820*a**2
0*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b +
x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x
)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)*
*6) + 6573*a**(55/3)*b**6*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**6*exp(I*pi/
3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(
10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(1
6/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/
3)*(a/b + x)**6) + 243*a**(55/3)*b**6*(a/b + x)**6*exp(I*pi/3)/(1820*a**20*
b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)
**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)*
*4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6
) - 8787*a**(52/3)*b**7*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**7*exp(I*pi/3)
/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10
/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/
3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)
*(a/b + x)**6) + 6498*a**(49/3)*b**8*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**
8*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300
*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*
a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a*
*14*b**(22/3)*(a/b + x)**6) - 2562*a**(46/3)*b**9*(1 - b*(a/b + x)/a)**(1/3
)*(a/b + x)**9*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b
+ x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x
)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)
**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 420*a**(43/3)*b**10*(1 - b*(a/b
+ x)/a)**(1/3)*(a/b + x)**10*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19
*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(
13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(1
9/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6), True))

## 3.670 $\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx$

**Optimal**. Leaf size=51

$$\frac{3}{2}a^3x^{2/3}+\frac{9}{5}a^2bx^{5/3}+\frac{9}{8}ab^2x^{8/3}+\frac{3}{11}b^3x^{11/3}$$

**Rubi** [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{5}a^2bx^{5/3}+\frac{3}{2}a^3x^{2/3}+\frac{9}{8}ab^2x^{8/3}+\frac{3}{11}b^3x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(1/3),x]
```

```
[Out] (3*a^3*x^(2/3))/2 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^(8/3))/8 + (3*b^3*x^(1
1/3))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx &= \int \left(\frac{a^3}{\sqrt[3]{x}}+3a^2bx^{2/3}+3ab^2x^{5/3}+b^3x^{8/3}\right)dx\\ &= \frac{3}{2}a^3x^{2/3}+\frac{9}{5}a^2bx^{5/3}+\frac{9}{8}ab^2x^{8/3}+\frac{3}{11}b^3x^{11/3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{3}{440}x^{2/3}\left(220a^3+264a^2bx+165ab^2x^2+40b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(220*a^3 + 264*a^2*b*x + 165*a*b^2*x^2 + 40*b^3*x^3))/440
```

**IntegrateAlgebraic [A]** time = 0.01, size = 47, normalized size = 0.92

$$\frac{3}{440}\left(220a^3x^{2/3}+264a^2bx^{5/3}+165ab^2x^{8/3}+40b^3x^{11/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(1/3),x]
[Out] (3*(220*a^3*x^(2/3) + 264*a^2*b*x^(5/3) + 165*a*b^2*x^(8/3) + 40*b^3*x^(11/
3)))/440
```

**fricas [A]** time = 0.78, size = 35, normalized size = 0.69

$$\frac{3}{440}\left(40\,b^3x^3+165\,ab^2x^2+264\,a^2bx+220\,a^3\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="fricas")
[Out] 3/440*(40*b^3*x^3 + 165*a*b^2*x^2 + 264*a^2*b*x + 220*a^3)*x^(2/3)
```

**giac [A]** time = 0.86, size = 35, normalized size = 0.69

$$\frac{3}{11}\,b^3x^{\frac{11}{3}}+\frac{9}{8}\,ab^2x^{\frac{8}{3}}+\frac{9}{5}\,a^2bx^{\frac{5}{3}}+\frac{3}{2}\,a^3x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="giac")
[Out] 3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3)
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.71

$$\frac{3\left(40b^3x^3+165a\,b^2x^2+264a^2bx+220a^3\right)x^{\frac{2}{3}}}{440}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(1/3),x)
[Out] 3/440*x^(2/3)*(40*b^3*x^3+165*a*b^2*x^2+264*a^2*b*x+220*a^3)
```

**maxima** [A] time = 1.33, size = 35, normalized size = 0.69

$$\frac{3}{11}\,b^3x^{\frac{11}{3}}+\frac{9}{8}\,ab^2x^{\frac{8}{3}}+\frac{9}{5}\,a^2bx^{\frac{5}{3}}+\frac{3}{2}\,a^3x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3)
```

**mupad** [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{2/3}}{2}+\frac{3\,b^3\,x^{11/3}}{11}+\frac{9\,a^2\,b\,x^{5/3}}{5}+\frac{9\,a\,b^2\,x^{8/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(1/3),x)
```

```
[Out] (3*a^3*x^(2/3))/2 + (3*b^3*x^(11/3))/11 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^
(8/3))/8
```

**sympy** [C] time = 3.19, size = 6246, normalized size = 122.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(1/3),x)
```

```
[Out] Piecewise((243*a**(71/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/(440*a**20
*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a*
*18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*e
xp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**
(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi
/3)) + 243*a**(71/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*
(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800
*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)*
*4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*
b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1296*a**(68/3)*b*(-1 + b*(a/b + x)/a)
**(2/3)*(a/b + x)*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*
b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/
3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(
a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 4
40*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1458*a**(68/3)*b*(a/b + x)/(
440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
+ 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
```

+ x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*
a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6
*exp(I*pi/3)) + 2808*a**(65/3)*b**2*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**
2*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b +
x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*
b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(
I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/
3)*(a/b + x)**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*(a/b + x)**2/(440*a**20*
b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**
18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*ex
p(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(
17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/
3)) - 3120*a**(62/3)*b**3*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(I*pi
/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*p
i/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*
(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) -
2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b +
x)**6*exp(I*pi/3)) - 4860*a**(62/3)*b**3*(a/b + x)**3/(440*a**20*b**(2/3)*e
xp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3
)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3)
+ 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b
 + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1710
*a**(59/3)*b**4*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(I*pi/3)/(440*a
**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 660
0*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)*
*3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15
*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(
I*pi/3)) + 3645*a**(59/3)*b**4*(a/b + x)**4/(440*a**20*b**(2/3)*exp(I*pi/3)
 - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x
)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**
16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*e
xp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 72*a**(56/3)*b
**5*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**5*exp(I*pi/3)/(440*a**20*b**(2/3
)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(
8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/
3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(
a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1
458*a**(56/3)*b**5*(a/b + x)**5/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**1
9*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*p
i/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)
*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) +
 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1104*a**(53/3)*b**6*(-1 +
b*(a/b + x)/a)**(2/3)*(a/b + x)**6*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi
/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b
+ x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*

a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**
5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 243*a**(53/
3)*b**6*(a/b + x)**6/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*
(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800
*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)*
*4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*
b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1152*a**(50/3)*b**7*(-1 + b*(a/b + x)
/a)**(2/3)*(a/b + x)**7*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*
a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp
(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(1
4/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/
3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 585*a**(47/3)*b**8*(-1
 + b*(a/b + x)/a)**(2/3)*(a/b + x)**8*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I
*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a
/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 66
00*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x
)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 120*a**(
44/3)*b**9*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**9*exp(I*pi/3)/(440*a**20*
b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**
18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*ex
p(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(
17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/
3)), Abs(b*(a/b + x)/a) > 1), (-243*a**(71/3)*(1 - b*(a/b + x)/a)**(2/3)/(4
40*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) +
 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b +
 x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a
**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*
exp(I*pi/3)) + 243*a**(71/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b
**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3
) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a
/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 44
0*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1296*a**(68/3)*b*(1 - b*(a/b
+ x)/a)**(2/3)*(a/b + x)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5
/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) -
8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b +
 x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a*
*14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1458*a**(68/3)*b*(a/b + x)/(440*a
**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 660
0*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)*
*3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15
*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(
I*pi/3)) - 2808*a**(65/3)*b**2*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(440
*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6
600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x
)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**

15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*ex
p(I*pi/3)) + 3645*a**(65/3)*b**2*(a/b + x)**2/(440*a**20*b**(2/3)*exp(I*pi/
3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b +
x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a
**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5
*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 3120*a**(62/
3)*b**3*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(440*a**20*b**(2/3)*exp(I*p
i/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b
+ x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600
*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)*
*5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 4860*a**(6
2/3)*b**3*(a/b + x)**3/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3
)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 88
00*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x
)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**1
4*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1710*a**(59/3)*b**4*(1 - b*(a/b + x
)/a)**(2/3)*(a/b + x)**4/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5
/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) -
8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b +
x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a*
*14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 3645*a**(59/3)*b**4*(a/b + x)**4/
(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
+ 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
+ x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640
*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**
6*exp(I*pi/3)) - 72*a**(56/3)*b**5*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**5/
(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
+ 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
+ x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640
*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**
6*exp(I*pi/3)) - 1458*a**(56/3)*b**5*(a/b + x)**5/(440*a**20*b**(2/3)*exp(I
*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a
/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 66
00*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x
)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1104*a**
(53/3)*b**6*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**6/(440*a**20*b**(2/3)*exp
(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*
(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) +
6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b +
x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 243*a*
*(53/3)*b**6*(a/b + x)**6/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(
5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) -
8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b
+ x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a
**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1152*a**(50/3)*b**7*(1 - b*(a/b

```
+ x)/a)**(2/3)*(a/b + x)**7/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b*
*(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3)
 - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/
b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440
*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 585*a**(47/3)*b**8*(1 - b*(a/b
 + x)/a)**(2/3)*(a/b + x)**8/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b
**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3
) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a
/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 44
0*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 120*a**(44/3)*b**9*(1 - b*(a/
b + x)/a)**(2/3)*(a/b + x)**9/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*
b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/
3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(
a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 4
40*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)), True))
```

## 3.671 $\int \frac{(a+bx)^3}{x^{2/3}}\,dx$

**Optimal**. Leaf size=49

$$3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}$$

**Rubi** [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{4}a^2bx^{4/3}+3a^3\sqrt[3]{x}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(2/3),x]
```

```
[Out] 3*a^3*x^(1/3) + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/3))/7 + (3*b^3*x^(10/3)
)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{2/3}}\,dx&=\int \left(\frac{a^3}{x^{2/3}}+3a^2b\sqrt[3]{x}+3ab^2x^{4/3}+b^3x^{7/3}\right)dx\\&=3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 39, normalized size = 0.80

$$\frac{3}{140}\sqrt[3]{x}\left(140a^3+105a^2bx+60ab^2x^2+14b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(2/3),x]
```

```
[Out] (3*x^(1/3)*(140*a^3 + 105*a^2*b*x + 60*a*b^2*x^2 + 14*b^3*x^3))/140
```

**IntegrateAlgebraic** [A] time = 0.02, size = 47, normalized size = 0.96

$$\frac{3}{140}\left(140a^3\sqrt[3]{x}+105a^2bx^{4/3}+60ab^2x^{7/3}+14b^3x^{10/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(2/3),x]
[Out] (3*(140*a^3*x^(1/3) + 105*a^2*b*x^(4/3) + 60*a*b^2*x^(7/3) + 14*b^3*x^(10/3
)))/140
```

**fricas** [A] time = 1.30, size = 35, normalized size = 0.71

$$\frac{3}{140}\left(14\,b^3x^3+60\,ab^2x^2+105\,a^2bx+140\,a^3\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="fricas")
[Out] 3/140*(14*b^3*x^3 + 60*a*b^2*x^2 + 105*a^2*b*x + 140*a^3)*x^(1/3)
```

**giac** [A] time = 0.94, size = 35, normalized size = 0.71

$$\frac{3}{10}\,b^3x^{\frac{10}{3}}+\frac{9}{7}\,ab^2x^{\frac{7}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+3\,a^3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="giac")
[Out] 3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

**maple** [A] time = 0.01, size = 36, normalized size = 0.73

$$\frac{3\left(14b^3x^3+60a\,b^2x^2+105a^2bx+140a^3\right)x^{\frac{1}{3}}}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(2/3),x)
[Out] 3/140*x^(1/3)*(14*b^3*x^3+60*a*b^2*x^2+105*a^2*b*x+140*a^3)
```

**maxima** [A] time = 1.29, size = 35, normalized size = 0.71

$$\frac{3}{10}\,b^3x^{\frac{10}{3}}+\frac{9}{7}\,ab^2x^{\frac{7}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+3\,a^3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

**mupad** [B] time = 0.04, size = 35, normalized size = 0.71

$$3\,a^3\,x^{1/3}+\frac{3\,b^3\,x^{10/3}}{10}+\frac{9\,a^2\,b\,x^{4/3}}{4}+\frac{9\,a\,b^2\,x^{7/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(2/3),x)
```

```
[Out] 3*a^3*x^(1/3) + (3*b^3*x^(10/3))/10 + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/3
))/7
```

**sympy** [C] time = 3.17, size = 6667, normalized size = 136.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(2/3),x)
```

```
[Out] Piecewise((243*a**(70/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(140*a**
20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 21
00*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840
*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)
**6*exp(2*I*pi/3)) + 243*a**(70/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*
a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*e
xp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16
*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*ex
p(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1377*a**(67
/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(140*a**20*b**(1/
3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*
b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp
(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b*
*(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2
*I*pi/3)) - 1458*a**(67/3)*b*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*pi/3) -
840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)*
```

```
*2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a
**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**
5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3213*a*
*(64/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(140*a*
*20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2
100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b +
 x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 84
0*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x
)**6*exp(2*I*pi/3)) + 3645*a**(64/3)*b**2*(a/b + x)**2/(140*a**20*b**(1/3)*
exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) - 3927*a**(61/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2
*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*e
xp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*
b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*ex
p(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b*
*(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 4860*a**(61/3)*b**3*(a/b + x)**3/(140
*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3)
 + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/
b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) -
 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b
 + x)**6*exp(2*I*pi/3)) + 2583*a**(58/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a
/b + x)**4*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(
4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/
3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)
*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3
) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3645*a**(58/3)*b**4*(
a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x
)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**
17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4
*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14
*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 693*a**(55/3)*b**5*(-1 + b*(a/b +
x)/a)**(1/3)*(a/b + x)**5*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) -
 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)
**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*
a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)*
*5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1458*a
**(55/3)*b**5*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b*
*(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*p
i/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/
3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi
/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 273*a**(52/3)*b**6*
(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**6*exp(2*I*pi/3)/(140*a**20*b**(1/3)*
```

exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) + 243*a**(52/3)*b**6*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I*pi/3)
- 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x
)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100
*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)
**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 387*a
**(49/3)*b**7*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**7*exp(2*I*pi/3)/(140*a
**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) +
2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b
+ x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 8
40*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b +
x)**6*exp(2*I*pi/3)) - 198*a**(46/3)*b**8*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
+ x)**8*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3
)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3)
- 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a
/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) +
 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 42*a**(43/3)*b**9*(-1 +
b*(a/b + x)/a)**(1/3)*(a/b + x)**9*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*
I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*
(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3
) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(
a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3))
, Abs(b*(a/b + x)/a) > 1), (-243*a**(70/3)*(1 - b*(a/b + x)/a)**(1/3)/(140*
a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) +
 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b
 + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) -
840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b +
 x)**6*exp(2*I*pi/3)) + 243*a**(70/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 8
40*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**
2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a*
*16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5
*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 1377*a**
(67/3)*b*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*p
i/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/
b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) +
 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b
 + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) -
1458*a**(67/3)*b*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b*
*(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*p
i/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/
3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi
/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 3213*a**(64/3)*b**2

```
*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(140*a**20*b**(1/3)*exp(2*I*pi/3)
- 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x
)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100
*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)
**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3645*
a**(64/3)*b**2*(a/b + x)**2/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b
**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*
pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13
/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*p
i/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3927*a**(61/3)*b**
3*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(140*a**20*b**(1/3)*exp(2*I*pi/3)
 - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 210
0*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x
)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 4860
*a**(61/3)*b**3*(a/b + x)**3/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*
b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I
*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(1
3/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*
pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 2583*a**(58/3)*b*
*4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3
) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
 x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 21
00*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b +
x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 364
5*a**(58/3)*b**4*(a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19
*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*
I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(
13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I
*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 693*a**(55/3)*b*
*5*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3
) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
 x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 21
00*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b +
x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 145
8*a**(55/3)*b**5*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19
*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*
I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(
13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I
*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 273*a**(52/3)*b*
*6*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I*pi/3
) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
 x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 21
00*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b +
x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 243
```

```
*a**(52/3)*b**6*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*
b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I
*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(1
3/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*
pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 387*a**(49/3)*b**
7*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**7/(140*a**20*b**(1/3)*exp(2*I*pi/3)
 - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 210
0*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x
)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 198*
a**(46/3)*b**8*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**8/(140*a**20*b**(1/3)*
exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) - 42*a**(43/3)*b**9*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**9/(140*a**
20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 21
00*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840
*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)
**6*exp(2*I*pi/3)), True))
```

## 3.672 $\int \frac{(a+bx)^3}{x^{4/3}}\,dx$

**Optimal**. Leaf size=49

$$-\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{2}a^2bx^{2/3}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}$$

**Rubi [A]** time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{2}a^2bx^{2/3}-\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(4/3),x]
```

```
[Out] (-3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/3))/5 + (3*b^3*x^(8/
3))/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{4/3}}\,dx &= \int \left(\frac{a^3}{x^{4/3}}+\frac{3a^2b}{\sqrt[3]{x}}+3ab^2x^{2/3}+b^3x^{5/3}\right)dx\\ &= -\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{2}a^2bx^{2/3}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 39, normalized size = 0.80

$$\frac{3\left(-40a^3+60a^2bx+24ab^2x^2+5b^3x^3\right)}{40\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(4/3),x]
[Out] (3*(-40*a^3 + 60*a^2*b*x + 24*a*b^2*x^2 + 5*b^3*x^3))/(40*x^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 39, normalized size = 0.80

$$\frac{3\left(-40a^3+60a^2bx+24ab^2x^2+5b^3x^3\right)}{40\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(4/3),x]
[Out] (3*(-40*a^3 + 60*a^2*b*x + 24*a*b^2*x^2 + 5*b^3*x^3))/(40*x^(1/3))
```

**fricas [A]** time = 0.98, size = 35, normalized size = 0.71

$$\frac{3\left(5\,b^3x^3+24\,ab^2x^2+60\,a^2bx-40\,a^3\right)}{40\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="fricas")
[Out] 3/40*(5*b^3*x^3 + 24*a*b^2*x^2 + 60*a^2*b*x - 40*a^3)/x^(1/3)
```

**giac [A]** time = 1.07, size = 35, normalized size = 0.71

$$\frac{3}{8}\,b^3x^{\frac{8}{3}}+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{2}\,a^2bx^{\frac{2}{3}}-\frac{3\,a^3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="giac")
[Out] 3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.73

$$-\frac{3\left(-5b^3x^3-24a\,b^2x^2-60a^2bx+40a^3\right)}{40x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(4/3),x)
[Out] -3/40*(-5*b^3*x^3-24*a*b^2*x^2-60*a^2*b*x+40*a^3)/x^(1/3)
```

**maxima [A]** time = 1.32, size = 35, normalized size = 0.71

$$\frac{3}{8}\,b^{3}x^{\frac{8}{3}}+\frac{9}{5}\,ab^{2}x^{\frac{5}{3}}+\frac{9}{2}\,a^{2}bx^{\frac{2}{3}}-\frac{3\,a^{3}}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

**mupad [B]** time = 0.04, size = 35, normalized size = 0.71

$$\frac{3\,b^{3}\,x^{8/3}}{8}-\frac{3\,a^{3}}{x^{1/3}}+\frac{9\,a^{2}\,b\,x^{2/3}}{2}+\frac{9\,a\,b^{2}\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(4/3),x)
```

```
[Out] (3*b^3*x^(8/3))/8 - (3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/3
))/5
```

**sympy [C]** time = 3.26, size = 4004, normalized size = 81.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(4/3),x)
```

```
[Out] Piecewise((243*a**(68/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)/(40*a**20 - 2
40*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b +
x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**1
4*b**6*(a/b + x)**6) - 243*a**(68/3)*b**(1/3)*exp(2*I*pi/3)/(40*a**20 - 240
*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)
**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*
b**6*(a/b + x)**6) - 1296*a**(65/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 8
00*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(
a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(65/3)*b**(4/3)*(a/b +
x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b +
x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**
15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 2808*a**(62/3)*b**(7/3
)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(40*a**20 - 240*a**19*b*(a/b + x
) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b
**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6
) - 3645*a**(62/3)*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3)/(40*a**20 - 240*a**1
```

```
9*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 +
 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*
(a/b + x)**6) - 3120*a**(59/3)*b**(10/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b +
 x)**3/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 80
0*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a
/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 4860*a**(59/3)*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b
 + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*
a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1830*a**(56/3)*b**(
13/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(40*a**20 - 240*a**19*b*(a/b
 + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**
16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x
)**6) - 3645*a**(56/3)*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 240
*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)
**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*
b**6*(a/b + x)**6) - 528*a**(53/3)*b**(16/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)**5/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(53/3)*b**(16/3)*(a/
b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*
(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 -
240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 96*a**(50/3)*b*
*(19/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**6/(40*a**20 - 240*a**19*b*(a
/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a
**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b +
 x)**6) - 243*a**(50/3)*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)/(40*a**20 - 24
0*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x
)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14
*b**6*(a/b + x)**6) - 48*a**(47/3)*b**(22/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)**7/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 15*a**(44/3)*b**(25/3)*(-1 +
 b*(a/b + x)/a)**(2/3)*(a/b + x)**8/(40*a**20 - 240*a**19*b*(a/b + x) + 600
*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/
b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6), Abs(
b*(a/b + x)/a) > 1), (243*a**(68/3)*b**(1/3)*(1 - b*(a/b + x)/a)**(2/3)*exp
(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 243*a**(68/3)*b**(1/3)*exp(2
*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 -
800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*
(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 1296*a**(65/3)*b**(4/3)*(1 - b
*(a/b + x)/a)**(2/3)*(a/b + x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b +
 x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16
*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)*
```

```
*6) + 1458*a**(65/3)*b**(4/3)*(a/b + x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19
*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 +
600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(
a/b + x)**6) + 2808*a**(62/3)*b**(7/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)
**2*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b +
 x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a*
*15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 3645*a**(62/3)*b**(7/
3)*(a/b + x)**2*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18
*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)
**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 3120*a**(
59/3)*b**(10/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(2*I*pi/3)/(40*a
**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3
*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 +
 40*a**14*b**6*(a/b + x)**6) + 4860*a**(59/3)*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 8
00*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(
a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1830*a**(56/3)*b**(13/3)*(1 - b
*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/
b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a*
*16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b +
x)**6) - 3645*a**(56/3)*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 24
0*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x
)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14
*b**6*(a/b + x)**6) - 528*a**(53/3)*b**(16/3)*(1 - b*(a/b + x)/a)**(2/3)*(a
/b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2
*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 -
 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(53/3)
*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 6
00*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(
a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 9
6*a**(50/3)*b**(19/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**6*exp(2*I*pi/3)
/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**1
7*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x
)**5 + 40*a**14*b**6*(a/b + x)**6) - 243*a**(50/3)*b**(19/3)*(a/b + x)**6*e
xp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**
2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b
**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 48*a**(47/3)*b**(22/3)*(1
- b*(a/b + x)/a)**(2/3)*(a/b + x)**7*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*
(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600
*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b
 + x)**6) + 15*a**(44/3)*b**(25/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**8*
exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)*
*2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*
b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6), True))
```

## 3.673 $\int \frac{(a+bx)^3}{x^{5/3}} \, dx$

**Optimal**. Leaf size=49

$$-\frac{3a^3}{2x^{2/3}}+9a^2b\sqrt[3]{x}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}$$

**Rubi [A]** time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$9a^2b\sqrt[3]{x}-\frac{3a^3}{2x^{2/3}}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(5/3),x]
```

```
[Out] (-3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/3))/4 + (3*b^3*x^(7/
3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{5/3}} \, dx &= \int \left(\frac{a^3}{x^{5/3}}+\frac{3a^2b}{x^{2/3}}+3ab^2\sqrt[3]{x}+b^3x^{4/3}\right) dx \\ &= -\frac{3a^3}{2x^{2/3}}+9a^2b\sqrt[3]{x}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 39, normalized size = 0.80

$$\frac{3\left(-14a^3+84a^2bx+21ab^2x^2+4b^3x^3\right)}{28x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(5/3),x]
[Out] (3*(-14*a^3 + 84*a^2*b*x + 21*a*b^2*x^2 + 4*b^3*x^3))/(28*x^(2/3))
```

**IntegrateAlgebraic** [A] time = 0.02, size = 39, normalized size = 0.80

$$\frac{3\left(-14a^3+84a^2bx+21ab^2x^2+4b^3x^3\right)}{28x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(5/3),x]
[Out] (3*(-14*a^3 + 84*a^2*b*x + 21*a*b^2*x^2 + 4*b^3*x^3))/(28*x^(2/3))
```

**fricas** [A] time = 1.31, size = 35, normalized size = 0.71

$$\frac{3\left(4\,b^3x^3+21\,ab^2x^2+84\,a^2bx-14\,a^3\right)}{28\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="fricas")
[Out] 3/28*(4*b^3*x^3 + 21*a*b^2*x^2 + 84*a^2*b*x - 14*a^3)/x^(2/3)
```

**giac** [A] time = 1.17, size = 35, normalized size = 0.71

$$\frac{3}{7}\,b^3x^{\frac{7}{3}}+\frac{9}{4}\,ab^2x^{\frac{4}{3}}+9\,a^2bx^{\frac{1}{3}}-\frac{3\,a^3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="giac")
[Out] 3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

**maple** [A] time = 0.00, size = 36, normalized size = 0.73

$$-\frac{3\left(-4b^3x^3-21a\,b^2x^2-84a^2bx+14a^3\right)}{28x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(5/3),x)
[Out] -3/28*(-4*b^3*x^3-21*a*b^2*x^2-84*a^2*b*x+14*a^3)/x^(2/3)
```

**maxima** [A] time = 1.36, size = 35, normalized size = 0.71

$$\frac{3}{7}\,b^3x^{\frac{7}{3}} + \frac{9}{4}\,ab^2x^{\frac{4}{3}} + 9\,a^2bx^{\frac{1}{3}} - \frac{3\,a^3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

**mupad** [B] time = 0.04, size = 35, normalized size = 0.71

$$\frac{3\,b^3\,x^{7/3}}{7} - \frac{3\,a^3}{2\,x^{2/3}} + 9\,a^2\,b\,x^{1/3} + \frac{9\,a\,b^2\,x^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(5/3),x)
```

```
[Out] (3*b^3*x^(7/3))/7 - (3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/3
))/4
```

**sympy** [C] time = 3.24, size = 3964, normalized size = 80.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(5/3),x)
```

```
[Out] Piecewise((243*a**(67/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)/(28*a**20 - 1
68*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b +
x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**1
4*b**6*(a/b + x)**6) - 243*a**(67/3)*b**(2/3)*exp(I*pi/3)/(28*a**20 - 168*a
**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**
3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b*
*6*(a/b + x)**6) - 1377*a**(64/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
 + x)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560
*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/
b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(64/3)*b**(5/3)*(a/b + x)
*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**
2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b
**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 3213*a**(61/3)*b**(8/3)*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(28*a**20 - 168*a**19*b*(a/b + x) +
420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*
(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) -
3645*a**(61/3)*b**(8/3)*(a/b + x)**2*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a
```

/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a
**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
 x)**6) - 3927*a**(58/3)*b**(11/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 4860*a**(58/3)*b**(11/3)*(a/b + x)**3*
exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2
 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b*
*5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 2625*a**(55/3)*b**(14/3)*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(28*a**20 - 168*a**19*b*(a/b + x) +
420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*
(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) -
3645*a**(55/3)*b**(14/3)*(a/b + x)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(
a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*
a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b
+ x)**6) - 903*a**(52/3)*b**(17/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(52/3)*b**(17/3)*(a/b + x)**5*
exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2
 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b*
*5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 147*a**(49/3)*b**(20/3)*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**6/(28*a**20 - 168*a**19*b*(a/b + x) + 4
20*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(
a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 2
43*a**(49/3)*b**(20/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/
b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a*
*16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
x)**6) - 33*a**(46/3)*b**(23/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**7/(2
8*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b
**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**
5 + 28*a**14*b**6*(a/b + x)**6) + 12*a**(43/3)*b**(26/3)*(-1 + b*(a/b + x)/
a)**(1/3)*(a/b + x)**8/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(
a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 1
68*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6), Abs(b*(a/b + x)/a
) > 1), (243*a**(67/3)*b**(2/3)*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)/(28*
a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**
3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5
+ 28*a**14*b**6*(a/b + x)**6) - 243*a**(67/3)*b**(2/3)*exp(I*pi/3)/(28*a**2
0 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a
/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28
*a**14*b**6*(a/b + x)**6) - 1377*a**(64/3)*b**(5/3)*(1 - b*(a/b + x)/a)**(1
/3)*(a/b + x)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**
2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4
- 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(64/3

```
)*b**(5/3)*(a/b + x)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a*
*18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b +
 x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 3213*a
**(61/3)*b**(8/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I*pi/3)/(28*a
**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3
*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 +
 28*a**14*b**6*(a/b + x)**6) - 3645*a**(61/3)*b**(8/3)*(a/b + x)**2*exp(I*p
i/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*
a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b
 + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 3927*a**(58/3)*b**(11/3)*(1 - b*(a
/b + x)/a)**(1/3)*(a/b + x)**3*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x
) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b
**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6
) + 4860*a**(58/3)*b**(11/3)*(a/b + x)**3*exp(I*pi/3)/(28*a**20 - 168*a**19
*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 +
420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(
a/b + x)**6) + 2625*a**(55/3)*b**(14/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x
)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b +
x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**
15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 3645*a**(55/3)*b**(14/
3)*(a/b + x)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b
**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**
4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 903*a**(52/
3)*b**(17/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5*exp(I*pi/3)/(28*a**20
- 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b
 + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a
**14*b**6*(a/b + x)**6) + 1458*a**(52/3)*b**(17/3)*(a/b + x)**5*exp(I*pi/3)
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 147*a**(49/3)*b**(20/3)*(1 - b*(a/b +
x)/a)**(1/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 4
20*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(
a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 2
43*a**(49/3)*b**(20/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/
b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a*
*16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
x)**6) - 33*a**(46/3)*b**(23/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**7*exp
(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 -
560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*
(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 12*a**(43/3)*b**(26/3)*(1 - b*
(a/b + x)/a)**(1/3)*(a/b + x)**8*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b +
 x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16
*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)*
*6), True))
```

## 3.674 $\int \frac{x^{5/3}}{a+bx} dx$

**Optimal**. Leaf size=125

$$-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}-\frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{8/3}}-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}$$

**Rubi [A]** time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 56, 617, 204, 31}

$$-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}-\frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{8/3}}-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x),x]
```

```
[Out] (-3*a*x^(2/3))/(2*b^2) + (3*x^(5/3))/(5*b) - (Sqrt[3]*a^(5/3)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(8/3) - (3*a^(5/3)*Log[a^(1/3
) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) + (a^(5/3)*Log[a + b*x])/(2*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{5/3}}{a+bx}\,dx = \frac{3x^{5/3}}{5b} - \frac{a\int \frac{x^{2/3}}{a+bx}\,dx}{b}$$

$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} + \frac{a^2\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{b^2}$$

$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} + \frac{a^{5/3}\log(a+bx)}{2b^{8/3}} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^3} - \frac{\left(3a^{5/3}\right)\text{Subst}\left(\int \right.}{2b^{8/}}$$

$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} - \frac{3a^{5/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}} + \frac{a^{5/3}\log(a+bx)}{2b^{8/3}} + \frac{\left(3a^{5/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{b^{8/3}}$$

$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} - \frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{8/3}} - \frac{3a^{5/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}} + \frac{a^{5/3}\log(a+bx)}{2b^{8/3}}$$

**Mathematica [C]** time = 0.01, size = 38, normalized size = 0.30

$$\frac{3x^{2/3}\left(5a\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx}{a}\right)-5a+2bx\right)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x),x]
```

```
[Out] (3*x^(2/3)*(-5*a + 2*b*x + 5*a*Hypergeometric2F1[2/3, 1, 5/3, -((b*x)/a)]))
/(10*b^2)
```

**IntegrateAlgebraic [A]** time = 0.10, size = 150, normalized size = 1.20

$$\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2b^{8/3}}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{8/3}}-\frac{\sqrt{3}a^{5/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{8/3}}+\frac{3\left(2bx^{5/3}-5ax^{2/3}\right)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)/(a + b*x),x]
```

```
[Out] (3*(-5*a*x^(2/3) + 2*b*x^(5/3)))/(10*b^2) - (Sqrt[3]*a^(5/3)*ArcTan[1/Sqrt[
3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]])/b^(8/3) - (a^(5/3)*Log[a^(1/3)
 + b^(1/3)*x^(1/3)])/b^(8/3) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/
3) + b^(2/3)*x^(2/3)])/(2*b^(8/3))
```

**fricas [A]** time = 0.85, size = 147, normalized size = 1.18

$$\frac{10\sqrt{3}a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)-5a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+10a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)+3(2bx-5a)x^{\frac{2}{3}}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/10*(10*sqrt(3)*a*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a^2/b
^2)^(1/3) + sqrt(3)*a)/a) - 5*a*(-a^2/b^2)^(1/3)*log(-b*x^(1/3)*(-a^2/b^2)^
(2/3) + a*x^(2/3) - a*(-a^2/b^2)^(1/3)) + 10*a*(-a^2/b^2)^(1/3)*log(b*(-a^2
/b^2)^(2/3) + a*x^(1/3)) + 3*(2*b*x - 5*a)*x^(2/3))/b^2
```

**giac [A]** time = 1.04, size = 138, normalized size = 1.10

$$-\frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b^2}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^4}+\frac{\left(-ab^2\right)^{\frac{2}{3}}a\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^4}+\frac{3\left(2b^4x^{\frac{5}{3}}-5ab^3x^{\frac{2}{3}}\right)}{10b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -a*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - sqrt(3)*(-a*b^2)^(2/
3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/2*
```

```
(-a*b^2)^(2/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 3
/10*(2*b^4*x^(5/3) - 5*a*b^3*x^(2/3))/b^5
```

**maple [A]** time = 0.01, size = 122, normalized size = 0.98

$$\frac{3x^{\frac{5}{3}}}{5b}+\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{a^{2}\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{a^{2}\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{3a\,x^{\frac{2}{3}}}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a),x)
```

```
[Out] 3/5*x^(5/3)/b-3/2*a*x^(2/3)/b^2-a^2/b^3/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))
+1/2*a^2/b^3/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+a^2/b^
3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima [A]** time = 3.00, size = 130, normalized size = 1.04

$$\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a^{2}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a^{2}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\left(2\,bx^{\frac{5}{3}}-5\,ax^{\frac{2}{3}}\right)}{10\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*
(a/b)^(1/3)) + 1/2*a^2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^
3*(a/b)^(1/3)) - a^2*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 3/10*(2
*b*x^(5/3) - 5*a*x^(2/3))/b^2
```

**mupad [B]** time = 0.24, size = 151, normalized size = 1.21

$$\frac{3\,x^{5/3}}{5\,b}+\frac{{\left(-a\right)}^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3}-\frac{9\,{\left(-a\right)}^{13/3}}{b^{10/3}}\right)}{b^{8/3}}-\frac{3\,a\,x^{2/3}}{2\,b^2}+\frac{{\left(-a\right)}^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3}-\frac{9\,{\left(-a\right)}^{13/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{8/3}}-\frac{{\left(-a\right)}^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3}-\frac{9\,{\left(-a\right)}^{13/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(a + b*x),x)
```

```
[Out] (3*x^(5/3))/(5*b) + ((-a)^(5/3)*log((9*a^4*x^(1/3))/b^3 - (9*(-a)^(13/3))/b
^(10/3)))/b^(8/3) - (3*a*x^(2/3))/(2*b^2) + ((-a)^(5/3)*log((9*a^4*x^(1/3))
/b^3 - (9*(-a)^(13/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(10/3))*((3^(1/2)*1i)/2 -
 1/2))/b^(8/3) - ((-a)^(5/3)*log((9*a^4*x^(1/3))/b^3 - (9*(-a)^(13/3)*((3^(
1/2)*1i)/2 + 1/2)^2)/b^(10/3))*((3^(1/2)*1i)/2 + 1/2))/b^(8/3)
```

**sympy [A]** time = 47.12, size = 241, normalized size = 1.93

$$\begin{cases} \tilde{\infty} x^{\frac{5}{3}} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{3x^{\frac{8}{3}}}{8a} & \text{for}\ b = 0 \\ \frac{3x^{\frac{5}{3}}}{5b} & \text{for}\ a = 0 \\ -\frac{(-1)^{\frac{2}{3}} a^{\frac{5}{3}} \log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}{b^4\left(\frac{1}{b}\right)^{\frac{4}{3}}} + \frac{(-1)^{\frac{2}{3}} a^{\frac{5}{3}} \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}{2b^4\left(\frac{1}{b}\right)^{\frac{4}{3}}} - \frac{(-1)^{\frac{2}{3}}\sqrt{3}a^{\frac{5}{3}}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{b^4\left(\frac{1}{b}\right)^{\frac{4}{3}}} - \frac{3ax^{\frac{2}{3}}}{2b^2} + \frac{3x^{\frac{5}{3}}}{5b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(8/3)/(8*a), Eq(b, 0))
, (3*x**(5/3)/(5*b), Eq(a, 0)), (-(-1)**(2/3)*a**(5/3)*log(-(-1)**(1/3)*a**
(1/3)*(1/b)**(1/3) + x**(1/3))/(b**4*(1/b)**(4/3)) + (-1)**(2/3)*a**(5/3)*l
og(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1
/b)**(1/3) + 4*x**(2/3))/(2*b**4*(1/b)**(4/3)) - (-1)**(2/3)*sqrt(3)*a**(5/
3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)
))/(b**4*(1/b)**(4/3)) - 3*a*x**(2/3)/(2*b**2) + 3*x**(5/3)/(5*b), True))
```

## 3.675 $\int \frac{x^{4/3}}{a+bx}\,dx$

**Optimal**. Leaf size=123

$$\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}-\frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{7/3}}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}$$

**Rubi [A]** time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 58, 617, 204, 31}

$$\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}-\frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{7/3}}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x),x]
```

```
[Out] (-3*a*x^(1/3))/b^2 + (3*x^(4/3))/(4*b) - (Sqrt[3]*a^(4/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(7/3) + (3*a^(4/3)*Log[a^(1/3) +
b^(1/3)*x^(1/3)])/(2*b^(7/3)) - (a^(4/3)*Log[a + b*x])/(2*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
```

```
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{4/3}}{a+bx}\,dx = \frac{3x^{4/3}}{4b} - \frac{a\int \frac{\sqrt[3]{x}}{a+bx}\,dx}{b}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} + \frac{a^2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{b^2}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}} + \frac{\left(3a^{5/3}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^{8/3}} + \frac{\left(3a^{4/3}\right)\text{Subst}\left(\int\right.}{2}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} + \frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}} + \frac{\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{b^{7/3}}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} - \frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{7/3}} + \frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}}$$

**Mathematica [A]** time = 0.06, size = 140, normalized size = 1.14

$$\frac{-2a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)+4a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)-4\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-12a\sqrt[3]{b}\sqrt[3]{x}+3b^{4/3}x^{4/3}}{4b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x),x]
```

```
[Out] (-12*a*b^(1/3)*x^(1/3) + 3*b^(4/3)*x^(4/3) - 4*Sqrt[3]*a^(4/3)*ArcTan[(1 -
(2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) + b^(1/3)*x^(
1/3)] - 2*a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])
/(4*b^(7/3))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 144, normalized size = 1.17

$$-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2b^{7/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}-\frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{7/3}}+\frac{3\sqrt[3]{x}\,(bx-4a)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)/(a + b*x),x]
```

```
[Out] (3*x^(1/3)*(-4*a + b*x))/(4*b^2) - (Sqrt[3]*a^(4/3)*ArcTan[1/Sqrt[3] - (2*b
^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(7/3) + (a^(4/3)*Log[a^(1/3) + b^(1/3
)*x^(1/3)])/b^(7/3) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2
/3)*x^(2/3)])/(2*b^(7/3))
```

**fricas [A]** time = 1.42, size = 116, normalized size = 0.94

$$\frac{4\sqrt{3}\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-2\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+4\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+3\,(bx-4\,a)x^{\frac{1}{3}}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(3)*a*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a/b)^(2/3) -
sqrt(3)*a)/a) - 2*a*(a/b)^(1/3)*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(
2/3)) + 4*a*(a/b)^(1/3)*log(x^(1/3) + (a/b)^(1/3)) + 3*(b*x - 4*a)*x^(1/3))
/b^2
```

**giac [A]** time = 1.21, size = 136, normalized size = 1.11

$$-\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b^2}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3}+\frac{\left(-ab^2\right)^{\frac{1}{3}}a\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^3}+\frac{3\left(b^3x^{\frac{4}{3}}-4\,ab^2x^{\frac{1}{3}}\right)}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -a*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + sqrt(3)*(-a*b^2)^(1/
3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/2*
(-a*b^2)^(1/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 3
/4*(b^3*x^(4/3) - 4*a*b^2*x^(1/3))/b^4
```

**maple [A]** time = 0.01, size = 121, normalized size = 0.98

$$\frac{3x^{\frac{4}{3}}}{4b}+\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{a^{2}\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{a^{2}\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{3a\,x^{\frac{1}{3}}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a),x)
```

```
[Out] 3/4*x^(4/3)/b-3*a*x^(1/3)/b^2+a^2/b^3/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1
/2*a^2/b^3/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+a^2/b^3/
(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima [A]** time = 3.05, size = 128, normalized size = 1.04

$$\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a^{2}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a^{2}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3\left(bx^{\frac{4}{3}}-4\,ax^{\frac{1}{3}}\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*
(a/b)^(2/3)) - 1/2*a^2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^
3*(a/b)^(2/3)) + a^2*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 3/4*(b*
x^(4/3) - 4*a*x^(1/3))/b^2
```

**mupad [B]** time = 0.07, size = 126, normalized size = 1.02

$$\frac{3\,x^{4/3}}{4\,b}-\frac{3\,a\,x^{1/3}}{b^{2}}+\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}}{b^{1/3}}+9\,a^{2}\,x^{1/3}\right)}{b^{7/3}}+\frac{a^{4/3}\,\ln\left(9\,a^{2}\,x^{1/3}+\frac{9\,a^{7/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{7/3}}-\frac{a^{4/3}\,\ln\left(9\,a^{2}\,x^{1/3}-\frac{9\,a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x),x)
```

```
[Out] (3*x^(4/3))/(4*b) - (3*a*x^(1/3))/b^2 + (a^(4/3)*log((9*a^(7/3))/b^(1/3) +
9*a^2*x^(1/3)))/b^(7/3) + (a^(4/3)*log(9*a^2*x^(1/3) + (9*a^(7/3)*((3^(1/2)
*1i)/2 - 1/2))/b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/b^(7/3) - (a^(4/3)*log(9*a^
2*x^(1/3) - (9*a^(7/3)*((3^(1/2)*1i)/2 + 1/2))/b^(1/3))*((3^(1/2)*1i)/2 + 1
/2))/b^(7/3)
```

**sympy [A]** time = 25.86, size = 240, normalized size = 1.95

$$\begin{cases} \tilde{\infty}x^{\frac{4}{3}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{7}{3}}}{7a} & \text{for } b = 0 \\ \frac{3x^{\frac{4}{3}}}{4b} & \text{for } a = 0 \\ -\frac{\sqrt[3]{-1}\,a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{b^2}+\frac{\sqrt[3]{-1}\,a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2b^2}+\frac{\sqrt[3]{-1}\sqrt{3}\,a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{b^2}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{\frac{4}{3}}}{4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(7/3)/(7*a), Eq(b, 0))
, (3*x**(4/3)/(4*b), Eq(a, 0)), (-(-1)**(1/3)*a**(4/3)*(1/b)**(1/3)*log(-(-
1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/b**2 + (-1)**(1/3)*a**(4/3)*(1/
b)**(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*
x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(2*b**2) + (-1)**(1/3)*sqrt(3)*a**(4/3)
*(1/b)**(1/3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(
1/b)**(1/3)))/b**2 - 3*a*x**(1/3)/b**2 + 3*x**(4/3)/(4*b), True))
```

## 3.676 $\int \frac{x^{2/3}}{a+bx} dx$

**Optimal**. Leaf size=111

$$\frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{5/3}}-\frac{a^{2/3}\log(a+bx)}{2b^{5/3}}+\frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{5/3}}+\frac{3x^{2/3}}{2b}$$

**Rubi** [A] time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 56, 617, 204, 31}

$$\frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{5/3}}-\frac{a^{2/3}\log(a+bx)}{2b^{5/3}}+\frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{5/3}}+\frac{3x^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x),x]
```

```
[Out] (3*x^(2/3))/(2*b) + (Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(
Sqrt[3]*a^(1/3))])/b^(5/3) + (3*a^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*
b^(5/3)) - (a^(2/3)*Log[a + b*x])/(2*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{2/3}}{a+bx}\,dx = \frac{3x^{2/3}}{2b} - \frac{a\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{b}$$

$$= \frac{3x^{2/3}}{2b} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^2} + \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{\vphantom{x}}\right.}{2b^{5/3}}$$

$$= \frac{3x^{2/3}}{2b} + \frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{5/3}} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}} - \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{b^{5/3}}$$

$$= \frac{3x^{2/3}}{2b} + \frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} + \frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{5/3}} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}}$$

**Mathematica [C]** time = 0.01, size = 29, normalized size = 0.26

$$-\frac{3x^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx}{a}\right)-1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x),x]
```

```
[Out] (-3*x^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -((b*x)/a)]))/(2*b)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 136, normalized size = 1.23

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2b^{5/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{5/3}}+\frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{5/3}}+\frac{3x^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)/(a + b*x),x]
```

```
[Out] (3*x^(2/3))/(2*b) + (Sqrt[3]*a^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))
/(Sqrt[3]*a^(1/3))])/b^(5/3) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/b^(
5/3) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(
2*b^(5/3))
```

**fricas [A]** time = 1.45, size = 128, normalized size = 1.15

$$-\frac{2\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)+\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)-3\,x^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a^2/b^2)^(
1/3) - sqrt(3)*a)/a) + (a^2/b^2)^(1/3)*log(-b*x^(1/3)*(a^2/b^2)^(2/3) + a*x
^(2/3) + a*(a^2/b^2)^(1/3)) - 2*(a^2/b^2)^(1/3)*log(b*(a^2/b^2)^(2/3) + a*x
^(1/3)) - 3*x^(2/3))/b
```

**giac [A]** time = 1.04, size = 118, normalized size = 1.06

$$\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b}+\frac{3\,x^{\frac{2}{3}}}{2\,b}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] (-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b + 3/2*x^(2/3)/b + sqrt(3)*(
-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b
^3 - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/
b^3
```

**maple [A]** time = 0.01, size = 107, normalized size = 0.96

$$-\frac{\sqrt{3}\, a \arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}+\frac{a \ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}-\frac{a \ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}}+\frac{3x^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a),x)
```

```
[Out] 3/2*x^(2/3)/b+a/b^2/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))-1/2*a/b^2/(a/b)^(1/
3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-a/b^2*3^(1/2)/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima [A]** time = 2.90, size = 114, normalized size = 1.03

$$-\frac{\sqrt{3}\, a \arctan\left(\frac{\sqrt{3}\left(2\, x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\, x^{\frac{2}{3}}}{2\, b}-\frac{a \log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\, b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a \log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(
a/b)^(1/3)) + 3/2*x^(2/3)/b - 1/2*a*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/
b)^(2/3))/(b^2*(a/b)^(1/3)) + a*log(x^(1/3) + (a/b)^(1/3))/(b^2*(a/b)^(1/3)
)
```

**mupad [B]** time = 0.15, size = 130, normalized size = 1.17

$$\frac{3\, x^{2/3}}{2\, b}+\frac{a^{2/3} \ln\left(\frac{9\, a^{7/3}}{b^{4/3}}+\frac{9\, a^{2}\, x^{1/3}}{b}\right)}{b^{5/3}}+\frac{a^{2/3} \ln\left(\frac{9\, a^{7/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)^{2}}{b^{4/3}}+\frac{9\, a^{2}\, x^{1/3}}{b}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)}{b^{5/3}}-\frac{a^{2/3} \ln\left(\frac{9\, a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)^{2}}{b^{4/3}}+\frac{9\, a^{2}\, x^{1/3}}{b}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)}{b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x),x)
```

```
[Out] (3*x^(2/3))/(2*b) + (a^(2/3)*log((9*a^(7/3))/b^(4/3) + (9*a^2*x^(1/3))/b))/
b^(5/3) + (a^(2/3)*log((9*a^(7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(4/3) + (9*a^
2*x^(1/3))/b)*((3^(1/2)*1i)/2 - 1/2))/b^(5/3) - (a^(2/3)*log((9*a^(7/3)*((3
^(1/2)*1i)/2 + 1/2)^2)/b^(4/3) + (9*a^2*x^(1/3))/b)*((3^(1/2)*1i)/2 + 1/2))
/b^(5/3)
```

**sympy [A]** time = 9.08, size = 228, normalized size = 2.05

$$\begin{cases} \tilde{\infty} x^{\frac{2}{3}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{5}{3}}}{5a} & \text{for}\: b = 0 \\ \frac{3x^{\frac{2}{3}}}{2b} & \text{for}\: a = 0 \\ \frac{(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}{b^2\sqrt[3]{\frac{1}{b}}} - \frac{(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}{2b^2\sqrt[3]{\frac{1}{b}}} + \frac{(-1)^{\frac{2}{3}}\sqrt{3}a^{\frac{2}{3}}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{b^2\sqrt[3]{\frac{1}{b}}} + \frac{3x^{\frac{2}{3}}}{2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a), Eq(b, 0))
, (3*x**(2/3)/(2*b), Eq(a, 0)), ((-1)**(2/3)*a**(2/3)*log(-(-1)**(1/3)*a**(
1/3)*(1/b)**(1/3) + x**(1/3))/(b**2*(1/b)**(1/3)) - (-1)**(2/3)*a**(2/3)*lo
g(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/
b)**(1/3) + 4*x**(2/3))/(2*b**2*(1/b)**(1/3)) + (-1)**(2/3)*sqrt(3)*a**(2/3
)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3))
)/(b**2*(1/b)**(1/3)) + 3*x**(2/3)/(2*b), True))
```

## 3.677 $\int \frac{\sqrt[3]{x}}{a+bx}\,dx$

**Optimal**. Leaf size=109

$$-\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}}+\frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}+\frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}}+\frac{3\sqrt[3]{x}}{b}$$

**Rubi [A]** time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 58, 617, 204, 31}

$$-\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}}+\frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}+\frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}}+\frac{3\sqrt[3]{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x),x]
```

```
[Out] (3*x^(1/3))/b + (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt
[3]*a^(1/3))])/b^(4/3) - (3*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(4
/3)) + (a^(1/3)*Log[a + b*x])/(2*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
```

```
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{a+bx}\,dx = \frac{3\sqrt[3]{x}}{b} - \frac{a\int \frac{1}{x^{2/3}(a+bx)}\,dx}{b}$$

$$= \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}} - \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^{5/3}} - \frac{\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx,\right.}{2b^{4/3}}$$

$$= \frac{3\sqrt[3]{x}}{b} - \frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}} - \frac{\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{b^{4/3}}$$

$$= \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}} - \frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}$$

**Mathematica [A]** time = 0.03, size = 126, normalized size = 1.16

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)+2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+6\sqrt[3]{b}\sqrt[3]{x}}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x),x]
```

```
[Out] (6*b^(1/3)*x^(1/3) + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1
/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + a^(1/3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*b^(4/3))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 135, normalized size = 1.24

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2b^{4/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{4/3}}+\frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}}+\frac{3\sqrt[3]{x}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)/(a + b*x),x]
```

```
[Out] (3*x^(1/3))/b + (Sqrt[3]*a^(1/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sq
rt[3]*a^(1/3))])/b^(4/3) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/b^(4/3)
 + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*b^
(4/3))
```

**fricas [A]** time = 1.28, size = 114, normalized size = 1.05

$$\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+6x^{\frac{1}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/b)^(2/3) -
sqrt(3)*a)/a) - (-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2
/3)) + 2*(-a/b)^(1/3)*log(x^(1/3) - (-a/b)^(1/3)) + 6*x^(1/3))/b
```

**giac [A]** time = 1.15, size = 119, normalized size = 1.09

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}+\frac{3x^{\frac{1}{3}}}{b}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] (-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b - sqrt(3)*(-a*b^2)^(1/3)*ar
ctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 + 3*x^(1/3)/b
```

```
 - 1/2*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^
2
```

**maple [A]** time = 0.01, size = 108, normalized size = 0.99

$$-\frac{\sqrt{3}\, a \arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}-\frac{a \ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}+\frac{a \ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}+\frac{3x^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a),x)
```

```
[Out] 3*x^(1/3)/b-a/b^2/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))+1/2*a/b^2/(a/b)^(2/3)
*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-a/b^2/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima [A]** time = 2.93, size = 115, normalized size = 1.06

$$-\frac{\sqrt{3}\, a \arctan\left(\frac{\sqrt{3}\left(2\, x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3\, x^{\frac{1}{3}}}{b}+\frac{a \log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\, b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a \log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(
a/b)^(2/3)) + 3*x^(1/3)/b + 1/2*a*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)
^(2/3))/(b^2*(a/b)^(2/3)) - a*log(x^(1/3) + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

**mupad [B]** time = 0.07, size = 126, normalized size = 1.16

$$\frac{3\,x^{1/3}}{b}+\frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}+9\,a\,b\,x^{1/3}\right)}{b^{4/3}}+\frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+9\,a\,b\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{4/3}}-\frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-9\,a\,b\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(a + b*x),x)
```

```
[Out] (3*x^(1/3))/b + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3) + 9*a*b*x^(1/3)))/b^(4
/3) + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/2 - 1/2) + 9*a*b*x
^(1/3))*((3^(1/2)*1i)/2 - 1/2))/b^(4/3) - ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2
/3)*((3^(1/2)*1i)/2 + 1/2) - 9*a*b*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/b^(4/3)
```

**sympy [A]** time = 6.10, size = 219, normalized size = 2.01

$$\begin{cases} \tilde{\infty}\sqrt[3]{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{4}{3}}}{4a} & \text{for } b = 0 \\ \frac{3\sqrt[3]{x}}{b} & \text{for } a = 0 \\ \frac{\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{b} - \frac{\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2b} - \frac{\sqrt[3]{-1}\sqrt{3}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{b} + \frac{3\sqrt[3]{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a), Eq(b, 0))
, (3*x**(1/3)/b, Eq(a, 0)), ((-1)**(1/3)*a**(1/3)*(1/b)**(1/3)*log(-(-1)**(
1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/b - (-1)**(1/3)*a**(1/3)*(1/b)**(1/3
)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)
*(1/b)**(1/3) + 4*x**(2/3))/(2*b) - (-1)**(1/3)*sqrt(3)*a**(1/3)*(1/b)**(1/
3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)
))/b + 3*x**(1/3)/b, True))
```

## 3.678 $\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx$

**Optimal**. Leaf size=100

$$-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}$$

**Rubi** [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(1/3
)*b^(2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[
a + b*x]/(2*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx = \frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}}$$

$$= -\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}\,b^{2/3}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}}$$

**Mathematica [C]** time = 0.01, size = 27, normalized size = 0.27

$$\frac{3x^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx}{a}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)),x]
[Out] (3*x^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, -((b*x)/a)])/(2*a)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 126, normalized size = 1.26

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2\sqrt[3]{a}\,b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(1/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(1
/3)*b^(2/3))) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(1/3)*b^(2/3)) + Log[a^(2
/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/(2*a^(1/3)*b^(2/3))
```

**fricas [A]** time = 1.30, size = 313, normalized size = 3.13

$$\left[\frac{\sqrt{3} a b \sqrt{\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{a}} \log\left(\frac{2 b^{2} x - a b + \sqrt{3} \left(a b x^{\frac{1}{3}} + \left(-a b^{2}\right)^{\frac{1}{3}} a + 2 \left(-a b^{2}\right)^{\frac{2}{3}} x^{\frac{2}{3}}\right) \sqrt{\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{a}} - 3 \left(-a b^{2}\right)^{\frac{2}{3}} x^{\frac{1}{3}}}{b x + a}\right) + \left(-a b^{2}\right)^{\frac{2}{3}} \log\left(b^{2} x^{\frac{2}{3}} + \left(-a b^{2}\right)^{\frac{1}{3}} b x^{\frac{1}{3}} + \left(-a b^{2}\right)^{\frac{2}{3}}\right) - 2 \left(-a b^{2}\right)^{\frac{2}{3}} \log\left(b x^{\frac{1}{3}} - \left(-a b^{2}\right)^{\frac{1}{3}}\right)}{2 a b^{2}}, \frac{2 \sqrt{3} a b \sqrt{-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{a}} \arctan\left(\frac{\sqrt{3} \left(2 b x^{\frac{1}{3}} + \left(-a b^{2}\right)^{\frac{1}{3}}\right) \sqrt{-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{a}}}{3 b}\right) + \left(-a b^{2}\right)^{\frac{2}{3}} \log\left(b^{2} x^{\frac{2}{3}} + \left(-a b^{2}\right)^{\frac{1}{3}} b x^{\frac{1}{3}} + \left(-a b^{2}\right)^{\frac{2}{3}}\right) - 2 \left(-a b^{2}\right)^{\frac{2}{3}} \log\left(b x^{\frac{1}{3}} - \left(-a b^{2}\right)^{\frac{1}{3}}\right)}{2 a b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x - a*b + sqrt(3)*(a*b*
x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/
a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x + a)) + (-a*b^2)^(2/3)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x^(1/
3) - (-a*b^2)^(1/3)))/(a*b^2), 1/2*(2*sqrt(3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*a
rctan(1/3*sqrt(3)*(2*b*x^(1/3) + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
 + (-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/
3)) - 2*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^2)]
```

**giac [A]** time = 1.17, size = 118, normalized size = 1.18

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} - \frac{\sqrt{3} \left(-a b^{2}\right)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3} \left(2 x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a b^{2}} + \frac{\left(-a b^{2}\right)^{\frac{2}{3}} \log\left(x^{\frac{2}{3}} + x^{\frac{1}{3}} \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2 a b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a - sqrt(3)*(-a*b^2)^(2/3)*a
rctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/2*(-
a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

**maple [A]** time = 0.00, size = 96, normalized size = 0.96

$$\frac{\sqrt{3}\, \arctan\left(\frac{\sqrt{3} \left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}} b} - \frac{\ln\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}} b} + \frac{\ln\left(x^{\frac{2}{3}} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a),x)
```

```
[Out] -1/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/2/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)
^(1/3)*x^(1/3)+(a/b)^(2/3))+3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x^(1/3)-1))
```

**maxima [A]** time = 2.83, size = 103, normalized size = 1.03

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^
(1/3)) + 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3
)) - log(x^(1/3) + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

**mupad [B]** time = 0.11, size = 120, normalized size = 1.20

$$\frac{\ln\left(9\,b\,x^{1/3}-9\,{(-a)}^{1/3}\,b^{2/3}\right)}{{(-a)}^{1/3}\,b^{2/3}}+\frac{\ln\left(9\,b\,x^{1/3}-\frac{9\,{(-a)}^{1/3}\,b^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,{(-a)}^{1/3}\,b^{2/3}}-\frac{\ln\left(9\,b\,x^{1/3}-\frac{9\,{(-a)}^{1/3}\,b^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,{(-a)}^{1/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)),x)
```

```
[Out] log(9*b*x^(1/3) - 9*(-a)^(1/3)*b^(2/3))/((-a)^(1/3)*b^(2/3)) + (log(9*b*x^(
1/3) - (9*(-a)^(1/3)*b^(2/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*(-
a)^(1/3)*b^(2/3)) - (log(9*b*x^(1/3) - (9*(-a)^(1/3)*b^(2/3)*(3^(1/2)*1i +
1)^2)/4)*(3^(1/2)*1i + 1))/(2*(-a)^(1/3)*b^(2/3))
```

**sympy [A]** time = 7.41, size = 212, normalized size = 2.12

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{2}{3}}}{2a} & \text{for } b = 0 \\ -\frac{3}{b\sqrt[3]{x}} & \text{for } a = 0 \\ -\frac{(-1)^{\frac{2}{3}} \log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}{\sqrt[3]{a}\, b\sqrt[3]{\frac{1}{b}}} + \frac{(-1)^{\frac{2}{3}} \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}{2\sqrt[3]{a}\, b\sqrt[3]{\frac{1}{b}}} - \frac{(-1)^{\frac{2}{3}}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{\sqrt[3]{a}\, b\sqrt[3]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a), Eq(b, 0))
, (-3/(b*x**(1/3)), Eq(a, 0)), (-(-1)**(2/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b
)**(1/3) + x**(1/3))/(a**(1/3)*b*(1/b)**(1/3)) + (-1)**(2/3)*log(4*(-1)**(2
/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) +
4*x**(2/3))/(2*a**(1/3)*b*(1/b)**(1/3)) - (-1)**(2/3)*sqrt(3)*atan(sqrt(3)/
3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(a**(1/3)*b*(
1/b)**(1/3)), True))
```

## 3.679 $\int \frac{1}{x^{2/3}(a+bx)}\,dx$

**Optimal**. Leaf size=100

$$\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

**Rubi [A]** time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {58, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(2/3
)*b^(1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[
a + b*x]/(2*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{2/3}(a+bx)}\,dx = -\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}$$

$$= \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}} + \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

**Mathematica [A]** time = 0.02, size = 103, normalized size = 1.03

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)),x]
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a
^(1/3) + b^(1/3)*x^(1/3)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)
*x^(2/3)])/(a^(2/3)*b^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 125, normalized size = 1.25

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2a^{2/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{2/3}\sqrt[3]{b}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(2/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(2
/3)*b^(1/3))) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(2/3)*b^(1/3)) - Log[a^(2
/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/(2*a^(2/3)*b^(1/3))
```

**fricas [A]** time = 1.49, size = 307, normalized size = 3.07

$$\left[\frac{\sqrt{3}\,ab\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^{2}+\sqrt{3}\left(2\,abx^{\frac{2}{3}}-\left(a^{2}b\right)^{\frac{1}{3}}a+\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}-3\left(a^{2}b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^{2}b\right)^{\frac{1}{3}}a-\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^{2}b\right)^{\frac{2}{3}}\right)}{2\,a^{2}b},\ \frac{2\sqrt{3}\,ab\sqrt{\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{3}\left(\left(a^{2}b\right)^{\frac{1}{3}}a-2\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}}{3\,a^{2}}\right)-\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^{2}b\right)^{\frac{1}{3}}a-\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^{2}b\right)^{\frac{2}{3}}\right)}{2\,a^{2}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x - a^2 + sqrt(3)*(2*a*
b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b)
 - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x + a)) - (a^2*b)^(2/3)*log(a*b*x^(2/3) +
(a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1/3)
+ (a^2*b)^(2/3)))/(a^2*b), 1/2*(2*sqrt(3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(
-1/3*sqrt(3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)
/b)/a^2) - (a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*
x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b)]
```

**giac [A]** time = 1.18, size = 117, normalized size = 1.17

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a + sqrt(3)*(-a*b^2)^(1/3)*a
rctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/2*(-a*
b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

**maple** [A] time = 0.01, size = 95, normalized size = 0.95

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a),x)
```

```
[Out] 1/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/2/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^
(1/3)*x^(1/3)+(a/b)^(2/3))+1/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x^(1/3)-1))
```

**maxima** [A] time = 2.96, size = 102, normalized size = 1.02

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^
(2/3)) - 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3
)) + log(x^(1/3) + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

**mupad** [B] time = 0.21, size = 110, normalized size = 1.10

$$\frac{\ln\left(9\,a^{1/3}\,b^{5/3}+9\,b^2\,x^{1/3}\right)}{a^{2/3}\,b^{1/3}}+\frac{\ln\left(9\,b^2\,x^{1/3}+\frac{9\,a^{1/3}\,b^{5/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{2/3}\,b^{1/3}}-\frac{\ln\left(9\,b^2\,x^{1/3}-\frac{9\,a^{1/3}\,b^{5/3}\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{2/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)),x)
```

```
[Out] log(9*a^(1/3)*b^(5/3) + 9*b^2*x^(1/3))/(a^(2/3)*b^(1/3)) + (log(9*b^2*x^(1/
3) + (9*a^(1/3)*b^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^(2/3)*b
```

```
^(1/3)) - (log(9*b^2*x^(1/3) - (9*a^(1/3)*b^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(
1/2)*1i + 1))/(2*a^(2/3)*b^(1/3))
```

**sympy** [A] time = 11.35, size = 212, normalized size = 2.12

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3\sqrt[3]{x}}{a} & \text{for } b = 0 \\ -\frac{3}{2bx^{\frac{2}{3}}} & \text{for } a = 0 \\ -\frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{a^{\frac{2}{3}}b\left(\frac{1}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt[3]{-1}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}b\left(\frac{1}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt[3]{-1}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{a^{\frac{2}{3}}b\left(\frac{1}{b}\right)^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a, Eq(b, 0)), (-
3/(2*b*x**(2/3)), Eq(a, 0)), (-(-1)**(1/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b)*
*(1/3) + x**(1/3))/(a**(2/3)*b*(1/b)**(2/3)) + (-1)**(1/3)*log(4*(-1)**(2/3
)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*
x**(2/3))/(2*a**(2/3)*b*(1/b)**(2/3)) + (-1)**(1/3)*sqrt(3)*atan(sqrt(3)/3
- 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(a**(2/3)*b*(1/
b)**(2/3)), True))
```

## 3.680 $\int \frac{1}{x^{4/3}(a+bx)}\,dx$

**Optimal**. Leaf size=109

$$\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}}-\frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}}+\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{3}{a\sqrt[3]{x}}$$

**Rubi** [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}}-\frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}}+\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{3}{a\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)),x]
```

```
[Out] -3/(a*x^(1/3)) + (Sqrt[3]*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqr
t[3]*a^(1/3))])/a^(4/3) + (3*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(
4/3)) - (b^(1/3)*Log[a + b*x])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = -\frac{3}{a\sqrt[3]{x}} - \frac{b\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{a}$$

$$= -\frac{3}{a\sqrt[3]{x}} - \frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2a} + \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx, \ldots\right.}{2a^{4/3}}$$

$$= -\frac{3}{a\sqrt[3]{x}} + \frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}} - \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\ldots}{\ldots}\right.}{a^{4/3}}$$

$$= -\frac{3}{a\sqrt[3]{x}} + \frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}}$$

**Mathematica [C]** time = 0.01, size = 25, normalized size = 0.23

$$-\frac{3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\frac{bx}{a}\right)}{a\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x)/a)])/(a*x^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 134, normalized size = 1.23

$$-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2a^{4/3}}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{4/3}}+\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{3}{a\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(4/3)*(a + b*x)),x]
```

```
[Out] -3/(a*x^(1/3)) + (Sqrt[3]*b^(1/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(S
qrt[3]*a^(1/3))])/a^(4/3) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/a^(4/3
) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*a
^(4/3))
```

**fricas [A]** time = 1.24, size = 113, normalized size = 1.04

$$-\frac{2\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+6x^{\frac{2}{3}}}{2ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)^(1/3) - 1/3*
sqrt(3)) + x*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)^(
1/3)) - 2*x*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) + 6*x^(2/3))/(a*x)
```

**giac [A]** time = 1.21, size = 125, normalized size = 1.15

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^{2}}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}b}-\frac{3}{ax^{\frac{1}{3}}}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + sqrt(3)*(-a*b^2)^(2/3
)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - 3/(
a*x^(1/3)) - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)
^(2/3))/(a^2*b)
```

**maple [A]** time = 0.01, size = 104, normalized size = 0.95

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}} a}+\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}} a}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{3}} a}-\frac{3}{a\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a),x)
```

```
[Out] -3/a/x^(1/3)+1/a/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))-1/2/a/(a/b)^(1/3)*ln(x
^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-1/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima [A]** time = 2.96, size = 111, normalized size = 1.02

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{3}{ax^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)
^(1/3)) - 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/
3)) + log(x^(1/3) + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 3/(a*x^(1/3))
```

**mupad [B]** time = 0.15, size = 124, normalized size = 1.14

$$\frac{b^{1/3}\,\ln\left(9\,a^{4/3}\,b^{8/3}+9\,a\,b^3\,x^{1/3}\right)}{a^{4/3}}-\frac{3}{a\,x^{1/3}}+\frac{b^{1/3}\,\ln\left(9\,a\,b^3\,x^{1/3}+9\,a^{4/3}\,b^{8/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}}-\frac{b^{1/3}\,\ln\left(9\,a\,b^3\,x^{1/3}+9\,a^{4/3}\,b^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)),x)
```

```
[Out] (b^(1/3)*log(9*a^(4/3)*b^(8/3) + 9*a*b^3*x^(1/3)))/a^(4/3) - 3/(a*x^(1/3))
+ (b^(1/3)*log(9*a*b^3*x^(1/3) + 9*a^(4/3)*b^(8/3)*((3^(1/2)*1i)/2 - 1/2)^2
```

```
)*((3^(1/2)*1i)/2 - 1/2))/a^(4/3) - (b^(1/3)*log(9*a*b^3*x^(1/3) + 9*a^(4/3
)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i)/2 + 1/2))/a^(4/3)
```

**sympy [A]** time = 25.29, size = 218, normalized size = 2.00

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{3}{4bx^{\frac{4}{3}}} & \text{for } a = 0 \\ -\frac{3}{a\sqrt[3]{x}} & \text{for } b = 0 \\ -\frac{3}{a\sqrt[3]{x}} + \frac{(-1)^{\frac{2}{3}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}{a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}} - \frac{(-1)^{\frac{2}{3}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}{2a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}} + \frac{(-1)^{\frac{2}{3}}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (-3/(4*b*x**(4/3)), Eq(a, 0)
), (-3/(a*x**(1/3)), Eq(b, 0)), (-3/(a*x**(1/3)) + (-1)**(2/3)*log(-(-1)**(
1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(a**(4/3)*(1/b)**(1/3)) - (-1)**(2/3
)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)
*(1/b)**(1/3) + 4*x**(2/3))/(2*a**(4/3)*(1/b)**(1/3)) + (-1)**(2/3)*sqrt(3)
*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))
/(a**(4/3)*(1/b)**(1/3)), True))
```

## 3.681 $\int \frac{1}{x^{5/3}(a+bx)}\,dx$

**Optimal**. Leaf size=111

$$-\frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}}+\frac{b^{2/3}\log(a+bx)}{2a^{5/3}}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}-\frac{3}{2ax^{2/3}}$$

**Rubi [A]** time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$-\frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}}+\frac{b^{2/3}\log(a+bx)}{2a^{5/3}}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}-\frac{3}{2ax^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)),x]
```

```
[Out] -3/(2*a*x^(2/3)) + (Sqrt[3]*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(S
qrt[3]*a^(1/3))])/a^(5/3) - (3*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a
^(5/3)) + (b^(2/3)*Log[a + b*x])/(2*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = -\frac{3}{2ax^{2/3}} - \frac{b\int \frac{1}{x^{2/3}(a+bx)}\,dx}{a}$$

$$= -\frac{3}{2ax^{2/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}} - \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\left(3b^{2/3}\right)\text{Subst}\left(\int \cdots\right)}{2a^{5/\cdots}}$$

$$= -\frac{3}{2ax^{2/3}} - \frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}} - \frac{\left(3b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\cdots\right)}{a^{5/3}}$$

$$= -\frac{3}{2ax^{2/3}} + \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} - \frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}}$$

**Mathematica [C]** time = 0.01, size = 27, normalized size = 0.24

$$-\frac{3\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{bx}{a}\right)}{2ax^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)),x]

[Out] (-3*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x)/a)])/(2*a*x^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 137, normalized size = 1.23

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}-\frac{3}{2ax^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/3)*(a + b*x)),x]
```

```
[Out] -3/(2*a*x^(2/3)) + (Sqrt[3]*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/
(Sqrt[3]*a^(1/3))])/a^(5/3) - (b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/a^(5
/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2
*a^(5/3))
```

**fricas [A]** time = 1.42, size = 147, normalized size = 1.32

$$\frac{2\sqrt{3}\,x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2\,x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3\,x^{\frac{1}{3}}}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(1/3)*(-b^2/a^2
)^(2/3) - sqrt(3)*b)/b) - x*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3)*
(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x*(-b^2/a^2)^(1/3)*log(b*x^(1/
3) - a*(-b^2/a^2)^(1/3)) - 3*x^(1/3))/(a*x)
```

**giac [A]** time = 1.14, size = 120, normalized size = 1.08

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,a^2}-\frac{3}{2\,ax^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 - sqrt(3)*(-a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/2*(-a
*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/2/(a
*x^(2/3))
```

**maple [A]** time = 0.01, size = 105, normalized size = 0.95

$$-\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}} a}-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}} a}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}} a}-\frac{3}{2a\, x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a),x)
```

```
[Out] -3/2/a/x^(2/3)-1/a/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))+1/2/a/(a/b)^(2/3)*ln
(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-1/a/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima [A]** time = 2.99, size = 112, normalized size = 1.01

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{3}{2\,ax^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)
^(2/3)) + 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/
3)) - log(x^(1/3) + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 3/2/(a*x^(2/3))
```

**mupad [B]** time = 0.07, size = 138, normalized size = 1.24

$$\frac{b^{2/3}\ln\left(9\,(-a)^{7/3}\,b^{8/3}-9\,a^2\,b^3\,x^{1/3}\right)}{(-a)^{5/3}}-\frac{3}{2\,a\,x^{2/3}}+\frac{b^{2/3}\ln\left(9\,(-a)^{7/3}\,b^{8/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-9\,a^2\,b^3\,x^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{5/3}}-\frac{b^{2/3}\ln\left(9\,(-a)^{7/3}\,b^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+9\,a^2\,b^3\,x^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)),x)
```

```
[Out] (b^(2/3)*log(9*(-a)^(7/3)*b^(8/3) - 9*a^2*b^3*x^(1/3)))/(-a)^(5/3) - 3/(2*a
*x^(2/3)) + (b^(2/3)*log(9*(-a)^(7/3)*b^(8/3)*((3^(1/2)*1i)/2 - 1/2) - 9*a^
```

```
2*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(-a)^(5/3) - (b^(2/3)*log(9*(-a)^(7/
3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2) + 9*a^2*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/
2))/(-a)^(5/3)
```

**sympy [A]** time = 34.96, size = 221, normalized size = 1.99

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{3}{2ax^{\frac{2}{3}}} & \text{for } b = 0 \\ -\frac{3}{5bx^{\frac{5}{3}}} & \text{for } a = 0 \\ -\frac{3}{2ax^{\frac{2}{3}}} + \frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}{a^{\frac{5}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt[3]{-1}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}{2a^{\frac{5}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt[3]{-1}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{a^{\frac{5}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (-3/(2*a*x**(2/3)), Eq(b, 0)
), (-3/(5*b*x**(5/3)), Eq(a, 0)), (-3/(2*a*x**(2/3)) + (-1)**(1/3)*log(-(-1
)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(a**(5/3)*(1/b)**(2/3)) - (-1)**
(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(
1/3)*(1/b)**(1/3) + 4*x**(2/3))/(2*a**(5/3)*(1/b)**(2/3)) - (-1)**(1/3)*sqr
t(3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/
3)))/(a**(5/3)*(1/b)**(2/3)), True))
```

## 3.682 $\int \frac{x^{5/3}}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=129

$$\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}+\frac{5a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}}-\frac{x^{5/3}}{b(a+bx)}+\frac{5x^{2/3}}{2b^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 50, 56, 617, 204, 31}

$$\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}+\frac{5a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}}-\frac{x^{5/3}}{b(a+bx)}+\frac{5x^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x)^2,x]
```

```
[Out] (5*x^(2/3))/(2*b^2) - x^(5/3)/(b*(a + b*x)) + (5*a^(2/3)*ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (5*a^(2/3)*Log[a
^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) - (5*a^(2/3)*Log[a + b*x])/(6*b^(8/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{5/3}}{(a+bx)^2}\,dx = -\frac{x^{5/3}}{b(a+bx)} + \frac{5\int \frac{x^{2/3}}{a+bx}\,dx}{3b}$$
$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} - \frac{(5a)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3b^2}$$
$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} - \frac{5a^{2/3}\log(a+bx)}{6b^{8/3}} - \frac{(5a)\,\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^3} + \frac{\left(5a^{2/3}\right)\mathrm{Subs}}{}$$
$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} + \frac{5a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}} - \frac{5a^{2/3}\log(a+bx)}{6b^{8/3}} - \frac{\left(5a^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{b^{8/3}}$$
$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} + \frac{5a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{8/3}} + \frac{5a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}} - \frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}$$

**Mathematica [C]** time = 0.00, size = 27, normalized size = 0.21

$$\frac{3x^{8/3}\,{}_2F_1\left(2,\frac{8}{3};\frac{11}{3};-\frac{bx}{a}\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x)^2,x]
[Out] (3*x^(8/3)*Hypergeometric2F1[2, 8/3, 11/3, -((b*x)/a)])/(8*a^2)
```

**IntegrateAlgebraic [A]** time = 0.18, size = 159, normalized size = 1.23

$$-\frac{5a^{2/3}\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{6b^{8/3}} + \frac{5a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{3b^{8/3}} + \frac{5a^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}} + \frac{5ax^{2/3}+3bx^{5/3}}{2b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)/(a + b*x)^2,x]
[Out] (5*a*x^(2/3) + 3*b*x^(5/3))/(2*b^2*(a + b*x)) + (5*a^(2/3)*ArcTan[1/Sqrt[3]
 - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (5*a^(2/3)*L
```

```
og[a^(1/3) + b^(1/3)*x^(1/3)])/(3*b^(8/3)) - (5*a^(2/3)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(6*b^(8/3))
```

**fricas [A]** time = 1.35, size = 162, normalized size = 1.26

$$-\frac{10\sqrt{3}\,(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)+5\,(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-10\,(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)-3\,(3\,bx+5\,a)x^{\frac{2}{3}}}{6\left(b^3x+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(10*sqrt(3)*(b*x + a)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*
(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(b*x + a)*(a^2/b^2)^(1/3)*log(-b*x^(1/3
)*(a^2/b^2)^(2/3) + a*x^(2/3) + a*(a^2/b^2)^(1/3)) - 10*(b*x + a)*(a^2/b^2)
^(1/3)*log(b*(a^2/b^2)^(2/3) + a*x^(1/3)) - 3*(3*b*x + 5*a)*x^(2/3))/(b^3*x
 + a*b^2)
```

**giac [A]** time = 1.05, size = 135, normalized size = 1.05

$$\frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b^2}+\frac{ax^{\frac{2}{3}}}{(bx+a)b^2}+\frac{3\,x^{\frac{2}{3}}}{2\,b^2}+\frac{5\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4}-\frac{5\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + a*x^(2/3)/((b*x + a
)*b^2) + 3/2*x^(2/3)/b^2 + 5/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2
*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 5/6*(-a*b^2)^(2/3)*log(x^(2/3)
 + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4
```

**maple [A]** time = 0.01, size = 123, normalized size = 0.95

$$\frac{a\,x^{\frac{2}{3}}}{(bx+a)\,b^2}-\frac{5\sqrt{3}\;a\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{5a\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{5a\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{3x^{\frac{2}{3}}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a)^2,x)
```

```
[Out] 3/2*x^(2/3)/b^2+a/b^2*x^(2/3)/(b*x+a)+5/3*a/b^3/(a/b)^(1/3)*ln(x^(1/3)+(a/b
)^(1/3))-5/6*a/b^3/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-
5/3*a/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima [A]** time = 2.96, size = 133, normalized size = 1.03

$$\frac{ax^{\frac{2}{3}}}{b^3x+ab^2}-\frac{5\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\,x^{\frac{2}{3}}}{2\,b^2}-\frac{5\,a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*x^(2/3)/(b^3*x + a*b^2) - 5/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (
a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 3/2*x^(2/3)/b^2 - 5/6*a*log(x^
(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) + 5/3*a*log(x^
(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

**mupad [B]** time = 0.26, size = 150, normalized size = 1.16

$$\frac{3\,x^{2/3}}{2\,b^2}+\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)}{3\,b^{8/3}}+\frac{a\,x^{2/3}}{x\,b^3+a\,b^2}+\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{8/3}}-\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(a + b*x)^2,x)
```

```
[Out] (3*x^(2/3))/(2*b^2) + (5*a^(2/3)*log((25*a^(7/3))/b^(10/3) + (25*a^2*x^(1/3
))/b^3))/(3*b^(8/3)) + (a*x^(2/3))/(a*b^2 + b^3*x) + (5*a^(2/3)*log((25*a^(
7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(10/3) + (25*a^2*x^(1/3))/b^3)*((3^(1/2)*1
i)/2 - 1/2))/(3*b^(8/3)) - (5*a^(2/3)*log((25*a^(7/3)*((3^(1/2)*1i)/2 + 1/2
)^2)/b^(10/3) + (25*a^2*x^(1/3))/b^3)*((3^(1/2)*1i)/2 + 1/2))/(3*b^(8/3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

## 3.683 $\int \frac{x^{4/3}}{(a+bx)^2} \, dx$

**Optimal**. Leaf size=125

$$-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}+\frac{4\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{x^{4/3}}{b(a+bx)}+\frac{4\sqrt[3]{x}}{b^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 50, 58, 617, 204, 31}

$$-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}+\frac{4\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{x^{4/3}}{b(a+bx)}+\frac{4\sqrt[3]{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x)^2,x]
```

```
[Out] (4*x^(1/3))/b^2 - x^(4/3)/(b*(a + b*x)) + (4*a^(1/3)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (2*a^(1/3)*Log[a^(1/
3) + b^(1/3)*x^(1/3)])/b^(7/3) + (2*a^(1/3)*Log[a + b*x])/(3*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{4/3}}{(a+bx)^2}\,dx = -\frac{x^{4/3}}{b(a+bx)} + \frac{4\int \frac{\sqrt[3]{x}}{a+bx}\,dx}{3b}$$
$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} - \frac{(4a)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3b^2}$$
$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} + \frac{2\sqrt[3]{a}\,\log(a+bx)}{3b^{7/3}} - \frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{b^{8/3}} - \frac{\left(2\sqrt[3]{a}\right)\text{Sub}}{}$$
$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} - \frac{2\sqrt[3]{a}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}} + \frac{2\sqrt[3]{a}\,\log(a+bx)}{3b^{7/3}} - \frac{\left(4\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{b^{7/3}}$$
$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} + \frac{4\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}} - \frac{2\sqrt[3]{a}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}} + \frac{2\sqrt[3]{a}\,\log(a+bx)}{3b^{7/3}}$$

**Mathematica** [C] time = 0.00, size = 27, normalized size = 0.22

$$\frac{3x^{7/3}\,{}_2F_1\left(2, \frac{7}{3}; \frac{10}{3}; -\frac{bx}{a}\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x)^2,x]
```

```
[Out] (3*x^(7/3)*Hypergeometric2F1[2, 7/3, 10/3, -((b*x)/a)])/(7*a^2)
```

**IntegrateAlgebraic** [A] time = 0.18, size = 156, normalized size = 1.25

$$\frac{2\sqrt[3]{a}\,\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{3b^{7/3}} - \frac{4\sqrt[3]{a}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{3b^{7/3}} + \frac{4\sqrt[3]{a}\,\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}} + \frac{4a\sqrt[3]{x} + 3bx^{4/3}}{b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)/(a + b*x)^2,x]
```

```
[Out] (4*a*x^(1/3) + 3*b*x^(4/3))/(b^2*(a + b*x)) + (4*a^(1/3)*ArcTan[1/Sqrt[3] -
 (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (4*a^(1/3)*Log
```

```
[a^(1/3) + b^(1/3)*x^(1/3)])/(3*b^(7/3)) + (2*a^(1/3)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(3*b^(7/3))
```

**fricas [A]** time = 1.40, size = 147, normalized size = 1.18

$$\frac{4\sqrt{3}\,(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-2\,(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+4\,(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+3\,(3\,bx+4\,a)x^{\frac{1}{3}}}{3\left(b^{3}x+ab^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(3)*(b*x + a)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/b
)^(2/3) - sqrt(3)*a)/a) - 2*(b*x + a)*(-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*(-
a/b)^(1/3) + (-a/b)^(2/3)) + 4*(b*x + a)*(-a/b)^(1/3)*log(x^(1/3) - (-a/b)^
(1/3)) + 3*(3*b*x + 4*a)*x^(1/3))/(b^3*x + a*b^2)
```

**giac [A]** time = 1.07, size = 135, normalized size = 1.08

$$\frac{4\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b^{2}}-\frac{4\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{3}}+\frac{ax^{\frac{1}{3}}}{(bx+a)b^{2}}+\frac{3\,x^{\frac{1}{3}}}{b^{2}}-\frac{2\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - 4/3*sqrt(3)*(-a*b^2
)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + a
*x^(1/3)/((b*x + a)*b^2) + 3*x^(1/3)/b^2 - 2/3*(-a*b^2)^(1/3)*log(x^(2/3) +
 x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3
```

**maple [A]** time = 0.01, size = 123, normalized size = 0.98

$$\frac{a\,x^{\frac{1}{3}}}{(bx+a)\,b^{2}}-\frac{4\sqrt{3}\;a\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{4a\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{2a\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{3x^{\frac{1}{3}}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a)^2,x)
```

```
[Out] 3*x^(1/3)/b^2+a/b^2*x^(1/3)/(b*x+a)-4/3*a/b^3/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^
(1/3))+2/3*a/b^3/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-4/
3*a/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima [A]** time = 2.93, size = 133, normalized size = 1.06

$$\frac{ax^{\frac{1}{3}}}{b^3x+ab^2} - \frac{4\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{3\,x^{\frac{1}{3}}}{b^2} + \frac{2\,a\log\left(x^{\frac{2}{3}} - x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{4\,a\log\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*x^(1/3)/(b^3*x + a*b^2) - 4/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (
a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 3*x^(1/3)/b^2 + 2/3*a*log(x^(2
/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 4/3*a*log(x^(1
/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

**mupad [B]** time = 0.15, size = 142, normalized size = 1.14

$$\frac{3\,x^{1/3}}{b^2} + \frac{a\,x^{1/3}}{x\,b^3 + a\,b^2} + \frac{4\,(-a)^{1/3}\,\ln\left(\frac{12\,(-a)^{4/3}}{b^{1/3}} + 12\,a\,x^{1/3}\right)}{3\,b^{7/3}} - \frac{4\,(-a)^{1/3}\,\ln\left(12\,a\,x^{1/3} - \frac{12\,(-a)^{4/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{1/3}}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{7/3}} + \frac{(-a)^{1/3}\,\ln\left(12\,a\,x^{1/3} + \frac{9\,(-a)^{4/3}\left(-\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)}{b^{1/3}}\right)\left(-\frac{2}{3} + \frac{\sqrt{3}\,2\mathrm{i}}{3}\right)}{b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x)^2,x)
```

```
[Out] (3*x^(1/3))/b^2 + (a*x^(1/3))/(a*b^2 + b^3*x) + (4*(-a)^(1/3)*log((12*(-a)^
(4/3))/b^(1/3) + 12*a*x^(1/3)))/(3*b^(7/3)) - (4*(-a)^(1/3)*log(12*a*x^(1/3
) - (12*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/b^(1/3))*((3^(1/2)*1i)/2 + 1/2))
/(3*b^(7/3)) + ((-a)^(1/3)*log(12*a*x^(1/3) + (9*(-a)^(4/3)*((3^(1/2)*2i)/3
 - 2/3))/b^(1/3))*((3^(1/2)*2i)/3 - 2/3))/b^(7/3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

## 3.684 $\int \frac{x^{2/3}}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=115

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{5/3}}-\frac{x^{2/3}}{b(a+bx)}$$

**Rubi [A]** time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{5/3}}-\frac{x^{2/3}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(2/3)/(b*(a + b*x))) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(
1/3)*b^(5/3)) + Log[a + b*x]/(3*a^(1/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{2/3}}{(a+bx)^2}\,dx &= -\frac{x^{2/3}}{b(a+bx)} + \frac{2\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3b} \\
&= -\frac{x^{2/3}}{b(a+bx)} + \frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{b^2} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}} \\
&= -\frac{x^{2/3}}{b(a+bx)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}} + \frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{5/3}} \\
&= -\frac{x^{2/3}}{b(a+bx)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}} + \frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.00, size = 27, normalized size = 0.23

$$
\frac{3x^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};-\frac{bx}{a}\right)}{5a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] (3*x^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, -((b*x)/a)])/(5*a^2)
```

**IntegrateAlgebraic [A]** time = 0.16, size = 145, normalized size = 1.26

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{3\sqrt[3]{a}\,b^{5/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3\sqrt[3]{a}\,b^{5/3}}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{5/3}}-\frac{x^{2/3}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(2/3)/(b*(a + b*x))) - (2*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[
3]*a^(1/3))]])/(Sqrt[3]*a^(1/3)*b^(5/3)) - (2*Log[a^(1/3) + b^(1/3)*x^(1/3)]
)/(3*a^(1/3)*b^(5/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(
2/3)]/(3*a^(1/3)*b^(5/3))
```

**fricas [B]** time = 1.32, size = 394, normalized size = 3.43

$$\left[-\frac{3\,ab^{2}x^{\frac{2}{3}}-3\sqrt{\frac{1}{3}}\left(ab^{2}x+a^{2}b\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^{2}x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}a+2\left(-ab^{2}\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}-3\left(-ab^{2}\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)-\left(-ab^{2}\right)^{\frac{2}{3}}(bx+a)\log\left(b^{2}x^{\frac{2}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{2}{3}}\right)+2\left(-ab^{2}\right)^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}}-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{3\left(ab^{4}x+a^{2}b^{3}\right)},\ -\frac{3\,ab^{2}x^{\frac{2}{3}}-6\sqrt{\frac{1}{3}}\left(ab^{2}x+a^{2}b\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}}{b}\right)-\left(-ab^{2}\right)^{\frac{2}{3}}(bx+a)\log\left(b^{2}x^{\frac{2}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{2}{3}}\right)+2\left(-ab^{2}\right)^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}}-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{3\left(ab^{4}x+a^{2}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/3*(3*a*b^2*x^(2/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((-a*b^2)^(1/3)/
a)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a
*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*
x + a)) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/
3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) - (-a*b^2)^
(1/3)))/(a*b^4*x + a^2*b^3), -1/3*(3*a*b^2*x^(2/3) - 6*sqrt(1/3)*(a*b^2*x +
 a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^2)^(1
/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b*x + a)*l
og(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^4*x + a^2*b^3)]
```

**giac [A]** time = 1.20, size = 136, normalized size = 1.18

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}-\frac{x^{\frac{2}{3}}}{(bx+a)b}-\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{3}}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) - x^(2/3)/((b*x +
a)*b) - 2/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a*b^3) + 1/3*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

**maple** [A] time = 0.01, size = 112, normalized size = 0.97

$$-\frac{x^{\frac{2}{3}}}{(bx+a)\,b}+\frac{2\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a)^2,x)
```

```
[Out] -x^(2/3)/b/(b*x+a)-2/3/b^2/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/3/b^2/(a/b
)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+2/3/b^2*3^(1/2)/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima** [A] time = 2.99, size = 120, normalized size = 1.04

$$-\frac{x^{\frac{2}{3}}}{b^{2}x+ab}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x^(2/3)/(b^2*x + a*b) + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^
(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) + 1/3*log(x^(2/3) - x^(1/3)*(a/b)^(1/
3) + (a/b)^(2/3))/(b^2*(a/b)^(1/3)) - 2/3*log(x^(1/3) + (a/b)^(1/3))/(b^2*(
a/b)^(1/3))
```

**mupad** [B] time = 0.24, size = 142, normalized size = 1.23

$$\frac{2\ln\left(\frac{4\,x^{1/3}}{b}-\frac{4\,{(-a)}^{1/3}}{b^{4/3}}\right)}{3\,{(-a)}^{1/3}\,b^{5/3}}-\frac{x^{2/3}}{b\,\left(a+b\,x\right)}+\frac{\ln\left(\frac{4\,x^{1/3}}{b}-\frac{{(-a)}^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{b^{4/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{3\,{(-a)}^{1/3}\,b^{5/3}}-\frac{\ln\left(\frac{4\,x^{1/3}}{b}-\frac{{(-a)}^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{b^{4/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{3\,{(-a)}^{1/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x)^2,x)
```

```
[Out] (2*log((4*x^(1/3))/b - (4*(-a)^(1/3))/b^(4/3)))/(3*(-a)^(1/3)*b^(5/3)) - x^
(2/3)/(b*(a + b*x)) + (log((4*x^(1/3))/b - ((-a)^(1/3)*(3^(1/2)*1i - 1)^2)/
b^(4/3))*(3^(1/2)*1i - 1))/(3*(-a)^(1/3)*b^(5/3)) - (log((4*x^(1/3))/b - ((
-a)^(1/3)*(3^(1/2)*1i + 1)^2)/b^(4/3))*(3^(1/2)*1i + 1))/(3*(-a)^(1/3)*b^(5
/3))
```

**sympy [A]** time = 106.98, size = 787, normalized size = 6.84

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{5}{3}}}{5a^2} & \text{for } b = 0 \\ -\frac{3}{b^2\sqrt[3]{x}} & \text{for } a = 0 \\ -\frac{3\sqrt[3]{-1}\sqrt[3]{a}bx^{\frac{2}{3}}\sqrt[3]{\frac{1}{b}}}{3\sqrt[3]{-1}a^{\frac{4}{3}}b^2\sqrt[3]{\frac{1}{b}}+3\sqrt[3]{-1}\sqrt[3]{a}b^3x\sqrt[3]{\frac{1}{b}}} + \frac{2a\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{3\sqrt[3]{-1}a^{\frac{4}{3}}b^2\sqrt[3]{\frac{1}{b}}+3\sqrt[3]{-1}\sqrt[3]{a}b^3x\sqrt[3]{\frac{1}{b}}} - \frac{a\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{3\sqrt[3]{-1}a^{\frac{4}{3}}b^2\sqrt[3]{\frac{1}{b}}+3\sqrt[3]{-1}\sqrt[3]{a}b^3x\sqrt[3]{\frac{1}{b}}} + \frac{2\sqrt{3}a\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{3\sqrt[3]{-1}a^{\frac{4}{3}}b^2\sqrt[3]{\frac{1}{b}}+3\sqrt[3]{-1}\sqrt[3]{a}b^3x\sqrt[3]{\frac{1}{b}}} + \frac{2a\log(2)}{3\sqrt[3]{-1}a^{\frac{4}{3}}b^2\sqrt[3]{\frac{1}{b}}+3\sqrt[3]{-1}\sqrt[3]{a}b^3x\sqrt[3]{\frac{1}{b}}} + \frac{2bx\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{3\sqrt[3]{-1}a^{\frac{4}{3}}b^2\sqrt[3]{\frac{1}{b}}+3\sqrt[3]{-1}\sqrt[3]{a}b^3x\sqrt[3]{\frac{1}{b}}} - \frac{bx\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{3\sqrt[3]{-1}a^{\frac{4}{3}}b^2\sqrt[3]{\frac{1}{b}}+3\sqrt[3]{-1}\sqrt[3]{a}b^3x\sqrt[3]{\frac{1}{b}}} + \frac{2\sqrt{3}bx\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{3\sqrt[3]{-1}a^{\frac{4}{3}}b^2\sqrt[3]{\frac{1}{b}}+3\sqrt[3]{-1}\sqrt[3]{a}b^3x\sqrt[3]{\frac{1}{b}}} + \frac{2bx\log(2)}{3\sqrt[3]{-1}a^{\frac{4}{3}}b^2\sqrt[3]{\frac{1}{b}}+3\sqrt[3]{-1}\sqrt[3]{a}b^3x\sqrt[3]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a**2), Eq(b,
0)), (-3/(b**2*x**(1/3)), Eq(a, 0)), (-3*(-1)**(1/3)*a**(1/3)*b*x**(2/3)*(1
/b)**(1/3)/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/
3)*b**3*x*(1/b)**(1/3)) + 2*a*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(
1/3))/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/3)*b*
*3*x*(1/b)**(1/3)) - a*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1
/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(3*(-1)**(1/3)*a**(4/3)*b*
*2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/3)*b**3*x*(1/b)**(1/3)) + 2*sqrt(3)*a
*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))
/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/3)*b**3*x*
(1/b)**(1/3)) + 2*a*log(2)/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-
1)**(1/3)*a**(1/3)*b**3*x*(1/b)**(1/3)) + 2*b*x*log(-(-1)**(1/3)*a**(1/3)*(
1/b)**(1/3) + x**(1/3))/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)*
*(1/3)*a**(1/3)*b**3*x*(1/b)**(1/3)) - b*x*log(4*(-1)**(2/3)*a**(2/3)*(1/b)
**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(3*(-1
)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/3)*b**3*x*(1/b)**
(1/3)) + 2*sqrt(3)*b*x*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a
**(1/3)*(1/b)**(1/3)))/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)**
(1/3)*a**(1/3)*b**3*x*(1/b)**(1/3)) + 2*b*x*log(2)/(3*(-1)**(1/3)*a**(4/3)*
b**2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/3)*b**3*x*(1/b)**(1/3)), True))
```

# 3.685 $\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=117

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

**Rubi** [A] time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(1/3)/(b*(a + b*x))) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))]/(Sqrt[3]*a^(2/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(2/
3)*b^(4/3)) - Log[a + b*x]/(6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
```

```
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx &= -\frac{\sqrt[3]{x}}{b(a+bx)}+\frac{\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3b}\\
&= -\frac{\sqrt[3]{x}}{b(a+bx)}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{5/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx,x,\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}\\
&= -\frac{\sqrt[3]{x}}{b(a+bx)}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}b^{4/3}}\\
&= -\frac{\sqrt[3]{x}}{b(a+bx)}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.00, size = 27, normalized size = 0.23

$$\frac{3x^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};-\frac{bx}{a}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x)^2,x]
```

[Out] (3*x^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, -((b*x)/a)])/(4*a^2)

**IntegrateAlgebraic [A]** time = 0.17, size = 145, normalized size = 1.24

$$-\frac{\log \left(a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} \sqrt[3]{x}+b^{2/3} x^{2/3}\right)}{6 a^{2/3} b^{4/3}}+\frac{\log \left(\sqrt[3]{a}+\sqrt[3]{b} \sqrt[3]{x}\right)}{3 a^{2/3} b^{4/3}}-\frac{\tan ^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2 \sqrt[3]{b} \sqrt[3]{x}}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} b^{4/3}}-\frac{\sqrt[3]{x}}{b(a+b x)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(1/3)/(a + b*x)^2,x]

[Out] -(x^(1/3)/(b*(a + b*x))) - ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(2/3)*b^(4/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/(6*a^(2/3)*b^(4/3))

**fricas [B]** time = 1.25, size = 389, normalized size = 3.32

$$\left[-\frac{6 a^{2} b x^{\frac{1}{3}}-3 \sqrt{\frac{1}{3}}\left(a b^{2} x+a^{2} b\right) \sqrt{-\frac{\left(a^{2} b\right)^{\frac{1}{3}}}{b}} \log \left(\frac{2 a b x-a^{2}+3 \sqrt{\frac{1}{3}}\left(2 a b x^{\frac{2}{3}}-\left(a^{2} b\right)^{\frac{1}{3}} a+\left(a^{2} b\right)^{\frac{2}{3}} x^{\frac{1}{3}}\right) \sqrt{-\frac{\left(a^{2} b\right)^{\frac{1}{3}}}{b}}-3\left(a^{2} b\right)^{\frac{1}{3}} a x^{\frac{1}{3}}}{b x+a}\right)+\left(a^{2} b\right)^{\frac{2}{3}}(b x+a) \log \left(a b x^{\frac{2}{3}}+\left(a^{2} b\right)^{\frac{1}{3}} a-\left(a^{2} b\right)^{\frac{2}{3}} x^{\frac{1}{3}}\right)-2\left(a^{2} b\right)^{\frac{2}{3}}(b x+a) \log \left(a b x^{\frac{1}{3}}+\left(a^{2} b\right)^{\frac{2}{3}}\right)}{6\left(a^{2} b^{3} x+a^{3} b^{2}\right)},-\frac{6 a^{2} b x^{\frac{1}{3}}-6 \sqrt{\frac{1}{3}}\left(a b^{2} x+a^{2} b\right) \sqrt{\frac{\left(a^{2} b\right)^{\frac{1}{3}}}{b}} \arctan \left(-\frac{\sqrt{\frac{1}{3}}\left(\left(a^{2} b\right)^{\frac{1}{3}} a-2\left(a^{2} b\right)^{\frac{2}{3}} x^{\frac{1}{3}}\right) \sqrt{\frac{\left(a^{2} b\right)^{\frac{1}{3}}}{b}}}{a^{2}}\right)+\left(a^{2} b\right)^{\frac{2}{3}}(b x+a) \log \left(a b x^{\frac{2}{3}}+\left(a^{2} b\right)^{\frac{1}{3}} a-\left(a^{2} b\right)^{\frac{2}{3}} x^{\frac{1}{3}}\right)-2\left(a^{2} b\right)^{\frac{2}{3}}(b x+a) \log \left(a b x^{\frac{1}{3}}+\left(a^{2} b\right)^{\frac{2}{3}}\right)}{6\left(a^{2} b^{3} x+a^{3} b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="fricas")

[Out] [-1/6*(6*a^2*b*x^(1/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x + a)) + (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b^3*x + a^3*b^2), -1/6*(6*a^2*b*x^(1/3) - 6*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b^3*x + a^3*b^2)]

**giac [A]** time = 1.13, size = 136, normalized size = 1.16

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log \left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 a b}+\frac{\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}} \arctan \left(\frac{\sqrt{3}\left(2 x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 a b^{2}}-\frac{x^{\frac{1}{3}}}{(b x+a) b}+\frac{\left(-a b^{2}\right)^{\frac{1}{3}} \log \left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 a b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(-a*
b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b
^2) - x^(1/3)/((b*x + a)*b) + 1/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

**maple** [A] time = 0.01, size = 112, normalized size = 0.96

$$-\frac{x^{\frac{1}{3}}}{(bx+a)\,b}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}+\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a)^2,x)
```

```
[Out] -x^(1/3)/b/(b*x+a)+1/3/b^2/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b^2/(a/b
)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b^2/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima** [A] time = 3.03, size = 120, normalized size = 1.03

$$-\frac{x^{\frac{1}{3}}}{b^{2}x+ab}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x^(1/3)/(b^2*x + a*b) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^
(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - 1/6*log(x^(2/3) - x^(1/3)*(a/b)^(1/
3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*log(x^(1/3) + (a/b)^(1/3))/(b^2*(
a/b)^(2/3))
```

**mupad** [B] time = 0.06, size = 120, normalized size = 1.03

$$\frac{\ln\left(3\,b\,x^{1/3}+3\,a^{1/3}\,b^{2/3}\right)}{3\,a^{2/3}\,b^{4/3}}-\frac{x^{1/3}}{b\,\left(a+b\,x\right)}+\frac{\ln\left(3\,b\,x^{1/3}+\frac{3\,a^{1/3}\,b^{2/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,a^{2/3}\,b^{4/3}}-\frac{\ln\left(3\,b\,x^{1/3}-\frac{3\,a^{1/3}\,b^{2/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,a^{2/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(a + b*x)^2,x)
```

```
[Out] log(3*b*x^(1/3) + 3*a^(1/3)*b^(2/3))/(3*a^(2/3)*b^(4/3)) - x^(1/3)/(b*(a +
b*x)) + (log(3*b*x^(1/3) + (3*a^(1/3)*b^(2/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)
*1i - 1))/(6*a^(2/3)*b^(4/3)) - (log(3*b*x^(1/3) - (3*a^(1/3)*b^(2/3)*(3^(1
/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(4/3))
```

**sympy [A]** time = 71.72, size = 607, normalized size = 5.19

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{4}{3}}}{4a^{2}} & \text{for}\: b = 0 \\ -\frac{3}{2b^{2}x^{\frac{2}{3}}} & \text{for}\: a = 0 \\ -\frac{2\sqrt[3]{-1}a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{6a^{2}b+6ab^{2}x} + \frac{\sqrt[3]{-1}a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{6a^{2}b+6ab^{2}x} + \frac{2\sqrt[3]{-1}\sqrt{3}a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{6a^{2}b+6ab^{2}x} - \frac{2\sqrt[3]{-1}a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log(2)}{6a^{2}b+6ab^{2}x} - \frac{2\sqrt[3]{-1}\sqrt[3]{a}bx\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{6a^{2}b+6ab^{2}x} + \frac{\sqrt[3]{-1}\sqrt[3]{a}bx\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{6a^{2}b+6ab^{2}x} + \frac{2\sqrt[3]{-1}\sqrt{3}\sqrt[3]{a}bx\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{6a^{2}b+6ab^{2}x} - \frac{2\sqrt[3]{-1}\sqrt[3]{a}bx\sqrt[3]{\frac{1}{b}}\log(2)}{6a^{2}b+6ab^{2}x} - \frac{6a\sqrt[3]{x}}{6a^{2}b+6ab^{2}x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a**2), Eq(b,
0)), (-3/(2*b**2*x**(2/3)), Eq(a, 0)), (-2*(-1)**(1/3)*a**(4/3)*(1/b)**(1/3
)*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(6*a**2*b + 6*a*b**2*x
) + (-1)**(1/3)*a**(4/3)*(1/b)**(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/
3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(6*a**2*b +
 6*a*b**2*x) + 2*(-1)**(1/3)*sqrt(3)*a**(4/3)*(1/b)**(1/3)*atan(sqrt(3)/3 -
 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(6*a**2*b + 6*a*
b**2*x) - 2*(-1)**(1/3)*a**(4/3)*(1/b)**(1/3)*log(2)/(6*a**2*b + 6*a*b**2*x
) - 2*(-1)**(1/3)*a**(1/3)*b*x*(1/b)**(1/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b)
**(1/3) + x**(1/3))/(6*a**2*b + 6*a*b**2*x) + (-1)**(1/3)*a**(1/3)*b*x*(1/b
)**(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x
**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(6*a**2*b + 6*a*b**2*x) + 2*(-1)**(1/3)*
sqrt(3)*a**(1/3)*b*x*(1/b)**(1/3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x*
*(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(6*a**2*b + 6*a*b**2*x) - 2*(-1)**(1/3)*a
**(1/3)*b*x*(1/b)**(1/3)*log(2)/(6*a**2*b + 6*a*b**2*x) - 6*a*x**(1/3)/(6*a
**2*b + 6*a*b**2*x), True))
```

## 3.686 $\int \frac{1}{\sqrt[3]{x}\,(a+bx)^2}\,dx$

**Optimal**. Leaf size=116

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}+\frac{x^{2/3}}{a(a+bx)}$$

**Rubi [A]** time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}+\frac{x^{2/3}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] x^(2/3)/(a*(a + b*x)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/
3))]/(Sqrt[3]*a^(4/3)*b^(2/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(4/3)*
b^(2/3)) + Log[a + b*x]/(6*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx &= \frac{x^{2/3}}{a(a+bx)}+\frac{\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{3a}\\
&= \frac{x^{2/3}}{a(a+bx)}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{2ab}-\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx,x,\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}\\
&= \frac{x^{2/3}}{a(a+bx)}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{4/3}b^{2/3}}\\
&= \frac{x^{2/3}}{a(a+bx)}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.00, size = 27, normalized size = 0.23

$$\frac{3x^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};-\frac{bx}{a}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] (3*x^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, -((b*x)/a)])/(2*a^2)
```

**IntegrateAlgebraic [A]** time = 0.16, size = 144, normalized size = 1.24

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{6a^{4/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}+\frac{x^{2/3}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] x^(2/3)/(a*(a + b*x)) - ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(
1/3))]/(Sqrt[3]*a^(4/3)*b^(2/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(4/3
)*b^(2/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/(6*a^
(4/3)*b^(2/3))
```

**fricas [B]** time = 1.44, size = 396, normalized size = 3.41

$$\left[\frac{6ab^2x^{\frac{2}{3}}+3\sqrt{\frac{1}{3}}\left(ab^2x+a^2b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+\left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-2\left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)}{6\left(a^2b^3x+a^3b^2\right)},\ \frac{6ab^2x^{\frac{2}{3}}+6\sqrt{\frac{1}{3}}\left(ab^2x+a^2b\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)+\left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-2\left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)}{6\left(a^2b^3x+a^3b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(6*a*b^2*x^(2/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*
b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
 + a)) + (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3
) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) - (-a*b^2)^(
1/3)))/(a^2*b^3*x + a^3*b^2), 1/6*(6*a*b^2*x^(2/3) + 6*sqrt(1/3)*(a*b^2*x +
 a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^2)^(1
/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*(b*x + a)*l
og(b*x^(1/3) - (-a*b^2)^(1/3)))/(a^2*b^3*x + a^3*b^2)]
```

**giac [A]** time = 1.11, size = 132, normalized size = 1.14

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2}+\frac{x^{\frac{2}{3}}}{(bx+a)a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + x^(2/3)/((b*x + a)
*a) - 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^2*b^2) + 1/6*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)
```

**maple** [A] time = 0.01, size = 120, normalized size = 1.03

$$\frac{x^{\frac{2}{3}}}{(bx+a)\,a}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a)^2,x)
```

```
[Out] x^(2/3)/a/(b*x+a)-1/3/a/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/a/b/(a/b)
^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/a*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima** [A] time = 2.96, size = 127, normalized size = 1.09

$$\frac{x^{\frac{2}{3}}}{abx+a^{2}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x^(2/3)/(a*b*x + a^2) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(
1/3))/(a/b)^(1/3))/(a*b*(a/b)^(1/3)) + 1/6*log(x^(2/3) - x^(1/3)*(a/b)^(1/3
) + (a/b)^(2/3))/(a*b*(a/b)^(1/3)) - 1/3*log(x^(1/3) + (a/b)^(1/3))/(a*b*(a
/b)^(1/3))
```

**mupad** [B] time = 0.36, size = 144, normalized size = 1.24

$$\frac{x^{2/3}}{a\,\left(a+b\,x\right)}+\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{{\left(-1\right)}^{2/3}\,b^{2/3}}{a^{5/3}}+\frac{b\,x^{1/3}}{a^2}\right)}{3\,a^{4/3}\,b^{2/3}}-\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{b\,x^{1/3}}{a^2}+\frac{{\left(-1\right)}^{2/3}\,b^{2/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{4/3}\,b^{2/3}}+\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{b\,x^{1/3}}{a^2}+\frac{9\,{\left(-1\right)}^{2/3}\,b^{2/3}\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{a^{5/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{a^{4/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)^2),x)
```

```
[Out] x^(2/3)/(a*(a + b*x)) + ((-1)^(1/3)*log(((-1)^(2/3)*b^(2/3))/a^(5/3) + (b*x
^(1/3))/a^2))/(3*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*log((b*x^(1/3))/a^2 + ((-1)
^(2/3)*b^(2/3)*((3^(1/2)*1i)/2 + 1/2)^2)/a^(5/3))*((3^(1/2)*1i)/2 + 1/2))/(
3*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*log((b*x^(1/3))/a^2 + (9*(-1)^(2/3)*b^(2/3
)*((3^(1/2)*1i)/6 - 1/6)^2)/a^(5/3))*((3^(1/2)*1i)/6 - 1/6))/(a^(4/3)*b^(2/
3))
```

**sympy [A]** time = 79.66, size = 774, normalized size = 6.67

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{2}{3}}}{2a^{2}} & \text{for } b = 0 \\ -\frac{3}{4b^{2}x^{\frac{4}{3}}} & \text{for } a = 0 \\ \frac{6\sqrt[3]{-1}\sqrt[3]{a}bx^{\frac{2}{3}}\sqrt[3]{\frac{1}{b}}}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}} + 6\sqrt[3]{-1}a^{\frac{4}{3}}b^{2}x\sqrt[3]{\frac{1}{b}}} + \frac{2a\log{\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}} + 6\sqrt[3]{-1}a^{\frac{4}{3}}b^{2}x\sqrt[3]{\frac{1}{b}}} - \frac{a\log{\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}} + 6\sqrt[3]{-1}a^{\frac{4}{3}}b^{2}x\sqrt[3]{\frac{1}{b}}} + \frac{2\sqrt{3}a\operatorname{atan}{\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}} + 6\sqrt[3]{-1}a^{\frac{4}{3}}b^{2}x\sqrt[3]{\frac{1}{b}}} + \frac{2a\log{(2)}}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}} + 6\sqrt[3]{-1}a^{\frac{4}{3}}b^{2}x\sqrt[3]{\frac{1}{b}}} + \frac{2bx\log{\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}} + 6\sqrt[3]{-1}a^{\frac{4}{3}}b^{2}x\sqrt[3]{\frac{1}{b}}} - \frac{bx\log{\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}} + 6\sqrt[3]{-1}a^{\frac{4}{3}}b^{2}x\sqrt[3]{\frac{1}{b}}} + \frac{2\sqrt{3}bx\operatorname{atan}{\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}} + 6\sqrt[3]{-1}a^{\frac{4}{3}}b^{2}x\sqrt[3]{\frac{1}{b}}} + \frac{2bx\log{(2)}}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}} + 6\sqrt[3]{-1}a^{\frac{4}{3}}b^{2}x\sqrt[3]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a**2), Eq(b,
0)), (-3/(4*b**2*x**(4/3)), Eq(a, 0)), (6*(-1)**(1/3)*a**(1/3)*b*x**(2/3)*(
1/b)**(1/3)/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)
*b**2*x*(1/b)**(1/3)) + 2*a*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/
3))/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b**2*x*
(1/b)**(1/3)) - a*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a
**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)
**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b**2*x*(1/b)**(1/3)) + 2*sqrt(3)*a*atan(sq
rt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(6*(-1)
**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b**2*x*(1/b)**(1/3
)) + 2*a*log(2)/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(
4/3)*b**2*x*(1/b)**(1/3)) + 2*b*x*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) +
x**(1/3))/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b
**2*x*(1/b)**(1/3)) - b*x*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)*
*(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(6*(-1)**(1/3)*a**(7/3)
*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b**2*x*(1/b)**(1/3)) + 2*sqrt(3)*b
*x*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)
))/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b**2*x*(
1/b)**(1/3)) + 2*b*x*log(2)/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)
**(1/3)*a**(4/3)*b**2*x*(1/b)**(1/3)), True))
```

## 3.687 $\int \frac{1}{x^{2/3}(a+bx)^2}\,dx$

**Optimal**. Leaf size=113

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

**Rubi [A]** time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] x^(1/3)/(a*(a + b*x)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(5/3)*b^(1/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(5/3
)*b^(1/3)) - Log[a + b*x]/(3*a^(5/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
```

```
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{2/3}(a+bx)^2}\,dx &= \frac{\sqrt[3]{x}}{a(a+bx)} + \frac{2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3a} \\
&= \frac{\sqrt[3]{x}}{a(a+bx)} - \frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx, x, \sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}} \\
&= \frac{\sqrt[3]{x}}{a(a+bx)} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}} \\
&= \frac{\sqrt[3]{x}}{a(a+bx)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}
\end{aligned}$$

**Mathematica [C]** time = 0.00, size = 25, normalized size = 0.22

$$\frac{3\sqrt[3]{x}\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};-\frac{bx}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] (3*x^(1/3)*Hypergeometric2F1[1/3, 2, 4/3, -((b*x)/a)])/a^2
```

**IntegrateAlgebraic [A]** time = 0.16, size = 144, normalized size = 1.27

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] x^(1/3)/(a*(a + b*x)) - (2*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(1/3)) + (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(
3*a^(5/3)*b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3
)]/(3*a^(5/3)*b^(1/3))
```

**fricas [B]** time = 1.19, size = 387, normalized size = 3.42

$$\left[\frac{3\,a^{2}bx^{\frac{1}{3}}+3\sqrt{\frac{1}{3}}\left(ab^{2}x+a^{2}b\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^{2}+3\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}}-\left(a^{2}b\right)^{\frac{1}{3}}a+\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}-3\left(a^{2}b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-\left(a^{2}b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}}+\left(a^{2}b\right)^{\frac{1}{3}}a-\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^{2}b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}}+\left(a^{2}b\right)^{\frac{2}{3}}\right)}{3\left(a^{3}b^{2}x+a^{4}b\right)},\ \frac{3\,a^{2}bx^{\frac{1}{3}}+6\sqrt{\frac{1}{3}}\left(ab^{2}x+a^{2}b\right)\sqrt{\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(a^{2}b\right)^{\frac{1}{3}}a-2\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-\left(a^{2}b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}}+\left(a^{2}b\right)^{\frac{1}{3}}a-\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^{2}b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}}+\left(a^{2}b\right)^{\frac{2}{3}}\right)}{3\left(a^{3}b^{2}x+a^{4}b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^2*b*x^(1/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*
b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
+ a)) - (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)
^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)
))/(a^3*b^2*x + a^4*b), 1/3*(3*a^2*b*x^(1/3) + 6*sqrt(1/3)*(a*b^2*x + a^2*b
)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3
)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(
2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*(b*x + a)
*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^3*b^2*x + a^4*b)]
```

**giac [A]** time = 1.04, size = 132, normalized size = 1.17

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{2}}+\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}b}+\frac{x^{\frac{1}{3}}}{(bx+a)a}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + 2/3*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b
) + x^(1/3)/((b*x + a)*a) + 1/3*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^2*b)
```

**maple** [A] time = 0.01, size = 120, normalized size = 1.06

$$\frac{x^{\frac{1}{3}}}{(bx+a)\,a}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a)^2,x)
```

```
[Out] x^(1/3)/a/(b*x+a)+2/3/a/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/3/a/b/(a/b)
^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+2/3/a/b/(a/b)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima** [A] time = 3.01, size = 127, normalized size = 1.12

$$\frac{x^{\frac{1}{3}}}{abx+a^{2}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x^(1/3)/(a*b*x + a^2) + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(
1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/3*log(x^(2/3) - x^(1/3)*(a/b)^(1/3
) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) + 2/3*log(x^(1/3) + (a/b)^(1/3))/(a*b*(a
/b)^(2/3))
```

**mupad** [B] time = 0.22, size = 134, normalized size = 1.19

$$\frac{2\,\ln\left(\frac{6\,b^{5/3}}{a^{2/3}}+\frac{6\,b^2\,x^{1/3}}{a}\right)}{3\,a^{5/3}\,b^{1/3}}+\frac{x^{1/3}}{a\,\left(a+b\,x\right)}+\frac{\ln\left(\frac{6\,b^2\,x^{1/3}}{a}+\frac{3\,b^{5/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{3\,a^{5/3}\,b^{1/3}}-\frac{\ln\left(\frac{6\,b^2\,x^{1/3}}{a}-\frac{3\,b^{5/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{a^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{3\,a^{5/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)^2),x)
```

```
[Out] (2*log((6*b^(5/3))/a^(2/3) + (6*b^2*x^(1/3))/a))/(3*a^(5/3)*b^(1/3)) + x^(1
/3)/(a*(a + b*x)) + (log((6*b^2*x^(1/3))/a + (3*b^(5/3)*(3^(1/2)*1i - 1))/a
^(2/3))*(3^(1/2)*1i - 1))/(3*a^(5/3)*b^(1/3)) - (log((6*b^2*x^(1/3))/a - (3
*b^(5/3)*(3^(1/2)*1i + 1))/a^(2/3))*(3^(1/2)*1i + 1))/(3*a^(5/3)*b^(1/3))
```

**sympy [A]** time = 115.98, size = 590, normalized size = 5.22

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3\sqrt[3]{x}}{a^{2}} & \text{for } b = 0 \\ -\frac{3}{5b^{2}x^{\frac{5}{3}}} & \text{for } a = 0 \\ -\frac{2\sqrt[3]{-1}\,a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{3a^{3}+3a^{2}bx} + \frac{\sqrt[3]{-1}\,a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{3a^{3}+3a^{2}bx} + \frac{2\sqrt[3]{-1}\sqrt{3}\,a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{3a^{3}+3a^{2}bx} - \frac{2\sqrt[3]{-1}\,a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log(2)}{3a^{3}+3a^{2}bx} - \frac{2\sqrt[3]{-1}\sqrt[3]{a}\,bx\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{3a^{3}+3a^{2}bx} + \frac{\sqrt[3]{-1}\sqrt[3]{a}\,bx\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{3a^{3}+3a^{2}bx} + \frac{2\sqrt[3]{-1}\sqrt{3}\sqrt[3]{a}\,bx\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{3a^{3}+3a^{2}bx} - \frac{2\sqrt[3]{-1}\sqrt[3]{a}\,bx\sqrt[3]{\frac{1}{b}}\log(2)}{3a^{3}+3a^{2}bx} + \frac{3a\sqrt[3]{x}}{3a^{3}+3a^{2}bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a**2, Eq(b, 0)),
 (-3/(5*b**2*x**(5/3)), Eq(a, 0)), (-2*(-1)**(1/3)*a**(4/3)*(1/b)**(1/3)*lo
g(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(3*a**3 + 3*a**2*b*x) + (-
1)**(1/3)*a**(4/3)*(1/b)**(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4
*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(3*a**3 + 3*a**2*
b*x) + 2*(-1)**(1/3)*sqrt(3)*a**(4/3)*(1/b)**(1/3)*atan(sqrt(3)/3 - 2*(-1)*
*(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(3*a**3 + 3*a**2*b*x) -
2*(-1)**(1/3)*a**(4/3)*(1/b)**(1/3)*log(2)/(3*a**3 + 3*a**2*b*x) - 2*(-1)**
(1/3)*a**(1/3)*b*x*(1/b)**(1/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x*
*(1/3))/(3*a**3 + 3*a**2*b*x) + (-1)**(1/3)*a**(1/3)*b*x*(1/b)**(1/3)*log(4
*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)*
*(1/3) + 4*x**(2/3))/(3*a**3 + 3*a**2*b*x) + 2*(-1)**(1/3)*sqrt(3)*a**(1/3)
*b*x*(1/b)**(1/3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/
3)*(1/b)**(1/3)))/(3*a**3 + 3*a**2*b*x) - 2*(-1)**(1/3)*a**(1/3)*b*x*(1/b)*
*(1/3)*log(2)/(3*a**3 + 3*a**2*b*x) + 3*a*x**(1/3)/(3*a**3 + 3*a**2*b*x), T
rue))
```

## 3.688 $\int \frac{1}{x^{4/3}(a+bx)^2} dx$

**Optimal**. Leaf size=124

$$\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{7/3}}-\frac{2\sqrt[3]{b}\log(a+bx)}{3a^{7/3}}+\frac{4\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}-\frac{4}{a^2\sqrt[3]{x}}+\frac{1}{a\sqrt[3]{x}(a+bx)}$$

**Rubi [A]** time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{7/3}}-\frac{2\sqrt[3]{b}\log(a+bx)}{3a^{7/3}}+\frac{4\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}-\frac{4}{a^2\sqrt[3]{x}}+\frac{1}{a\sqrt[3]{x}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)^2),x]
```

```
[Out] -4/(a^2*x^(1/3)) + 1/(a*x^(1/3)*(a + b*x)) + (4*b^(1/3)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b^(1/3)*Log[a^
(1/3) + b^(1/3)*x^(1/3)])/a^(7/3) - (2*b^(1/3)*Log[a + b*x])/(3*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{4\int \frac{1}{x^{4/3}(a+bx)}\,dx}{3a}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} - \frac{(4b)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3a^2}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} - \frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{a^2} + \frac{\left(2\sqrt[3]{b}\right)}{}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{7/3}} - \frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}} - \frac{\left(4\sqrt[3]{b}\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{7/3}} - \frac{2\sqrt[3]{b}\,\log(a}{3a^{7/3}}$$

**Mathematica [C]** time = 0.01, size = 25, normalized size = 0.20

$$-\frac{3\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};-\frac{bx}{a}\right)}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)^2),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 2, 2/3, -((b*x)/a)])/(a^2*x^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 152, normalized size = 1.23

$$-\frac{2\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{3a^{7/3}}+\frac{4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}}+\frac{4\sqrt[3]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{-3a-4bx}{a^2\sqrt[3]{x}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(4/3)*(a + b*x)^2),x]
```

```
[Out] (-3*a - 4*b*x)/(a^2*x^(1/3)*(a + b*x)) + (4*b^(1/3)*ArcTan[1/Sqrt[3] - (2*b
^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (4*b^(1/3)*Log[a^(1
/3) + b^(1/3)*x^(1/3)])/(3*a^(7/3)) - (2*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(3*a^(7/3))
```

**fricas [A]** time = 1.01, size = 156, normalized size = 1.26

$$-\frac{4\sqrt{3}\left(bx^2+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2\left(bx^2+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-4\left(bx^2+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+3\,(4\,bx+3\,a)x^{\frac{2}{3}}}{3\left(a^2bx^2+a^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(3)*(b*x^2 + a*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)^
(1/3) - 1/3*sqrt(3)) + 2*(b*x^2 + a*x)*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/
3) + b*x^(2/3) + a*(b/a)^(1/3)) - 4*(b*x^2 + a*x)*(b/a)^(1/3)*log(a*(b/a)^(
2/3) + b*x^(1/3)) + 3*(4*b*x + 3*a)*x^(2/3))/(a^2*b*x^2 + a^3*x)
```

**giac [A]** time = 0.97, size = 145, normalized size = 1.17

$$\frac{4\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3}+\frac{4\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b}-\frac{4\,bx+3\,a}{\left(bx^{\frac{4}{3}}+ax^{\frac{1}{3}}\right)a^2}-\frac{2\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4/3*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 4/3*sqrt(3)*(-a*b
^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*
```

```
b) - (4*b*x + 3*a)/((b*x^(4/3) + a*x^(1/3))*a^2) - 2/3*(-a*b^2)^(2/3)*log(x
^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b)
```

**maple** [A] time = 0.01, size = 121, normalized size = 0.98

$$-\frac{b\,x^{\frac{2}{3}}}{(bx+a)\,a^{2}}-\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}+\frac{4\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}-\frac{2\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}-\frac{3}{a^{2}x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a)^2,x)
```

```
[Out] -3/a^2/x^(1/3)-1/a^2*b*x^(2/3)/(b*x+a)+4/3/a^2/(a/b)^(1/3)*ln(x^(1/3)+(a/b)
^(1/3))-2/3/a^2/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-4/3
/a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima** [A] time = 3.04, size = 132, normalized size = 1.06

$$-\frac{4\,bx+3\,a}{a^{2}bx^{\frac{4}{3}}+a^{3}x^{\frac{1}{3}}}-\frac{4\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(4*b*x + 3*a)/(a^2*b*x^(4/3) + a^3*x^(1/3)) - 4/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(1/3)) - 2/3*log(x^(2/
3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(1/3)) + 4/3*log(x^(1/3)
 + (a/b)^(1/3))/(a^2*(a/b)^(1/3))
```

**mupad** [B] time = 0.15, size = 151, normalized size = 1.22

$$\frac{4\,b^{1/3}\,\ln\left(16\,a^{7/3}\,b^{8/3}+16\,a^2\,b^3\,x^{1/3}\right)}{3\,a^{7/3}}-\frac{\frac{3}{a}+\frac{4\,b\,x}{a^2}}{a\,x^{1/3}+b\,x^{4/3}}-\frac{4\,b^{1/3}\,\ln\left(16\,a^{7/3}\,b^{8/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2+16\,a^2\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{7/3}}+\frac{b^{1/3}\,\ln\left(9\,a^{7/3}\,b^{8/3}\,{\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}^2+16\,a^2\,b^3\,x^{1/3}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)^2),x)
```

```
[Out] (4*b^(1/3)*log(16*a^(7/3)*b^(8/3) + 16*a^2*b^3*x^(1/3)))/(3*a^(7/3)) - (3/a
 + (4*b*x)/a^2)/(a*x^(1/3) + b*x^(4/3)) - (4*b^(1/3)*log(16*a^(7/3)*b^(8/3)
*((3^(1/2)*1i)/2 + 1/2)^2 + 16*a^2*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(3*
a^(7/3)) + (b^(1/3)*log(9*a^(7/3)*b^(8/3)*((3^(1/2)*2i)/3 - 2/3)^2 + 16*a^2
*b^3*x^(1/3))*((3^(1/2)*2i)/3 - 2/3))/a^(7/3)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

## 3.689 $\int \frac{1}{x^{5/3}(a+bx)^2} dx$

**Optimal**. Leaf size=128

$$-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}}+\frac{5b^{2/3}\log(a+bx)}{6a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}}-\frac{5}{2a^2x^{2/3}}+\frac{1}{ax^{2/3}(a+bx)}$$

**Rubi [A]** time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}}+\frac{5b^{2/3}\log(a+bx)}{6a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}}-\frac{5}{2a^2x^{2/3}}+\frac{1}{ax^{2/3}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] -5/(2*a^2*x^(2/3)) + 1/(a*x^(2/3)*(a + b*x)) + (5*b^(2/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) - (5*b^(2/3)*Log[
a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(8/3)) + (5*b^(2/3)*Log[a + b*x])/(6*a^(8/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
```

```
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx = \frac{1}{ax^{2/3}(a+bx)} + \frac{5\int \frac{1}{x^{5/3}(a+bx)}\,dx}{3a}$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} - \frac{(5b)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3a^2}$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} + \frac{5b^{2/3}\log(a+bx)}{6a^{8/3}} - \frac{\left(5\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2a^{7/3}} -$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} - \frac{5b^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{8/3}} + \frac{5b^{2/3}\log(a+bx)}{6a^{8/3}} - \frac{\left(5b^{2/3}\right)\text{Subst}}{}$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} + \frac{5b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{8/3}} - \frac{5b^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{8/3}} + \frac{5b^{2/3}\log}{6a}$$

**Mathematica [C]** time = 0.01, size = 27, normalized size = 0.21

$$-\frac{3\,{}_2F_1\left(-\frac{2}{3},2;\frac{1}{3};-\frac{bx}{a}\right)}{2a^2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] (-3*Hypergeometric2F1[-2/3, 2, 1/3, -((b*x)/a)])/(2*a^2*x^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.18, size = 155, normalized size = 1.21

$$\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{6a^{8/3}}-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}+\frac{-3a-5bx}{2a^2x^{2/3}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] (-3*a - 5*b*x)/(2*a^2*x^(2/3)*(a + b*x)) + (5*b^(2/3)*ArcTan[1/Sqrt[3] - (2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) - (5*b^(2/3)*Log[a^
(1/3) + b^(1/3)*x^(1/3)])/(3*a^(8/3)) + (5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(6*a^(8/3))
```

**fricas [B]** time = 0.65, size = 189, normalized size = 1.48

$$\frac{10\sqrt{3}\left(bx^2+ax\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-5\left(bx^2+ax\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+10\left(bx^2+ax\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3\,(5\,bx+3\,a)x^{\frac{1}{3}}}{6\left(a^2bx^2+a^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(10*sqrt(3)*(b*x^2 + a*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(1
/3)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(b*x^2 + a*x)*(-b^2/a^2)^(1/3)*log
(b^2*x^(2/3) + a*b*x^(1/3)*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 10*(b
*x^2 + a*x)*(-b^2/a^2)^(1/3)*log(b*x^(1/3) - a*(-b^2/a^2)^(1/3)) - 3*(5*b*x
 + 3*a)*x^(1/3))/(a^2*b*x^2 + a^3*x)
```

**giac [A]** time = 1.01, size = 137, normalized size = 1.07

$$\frac{5\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3}-\frac{5\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3}-\frac{bx^{\frac{1}{3}}}{(bx+a)a^2}-\frac{5\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3}-\frac{3}{2\,a^2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5/3*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 5/3*sqrt(3)*(-a*b
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 -
 b*x^(1/3)/((b*x + a)*a^2) - 5/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b
)^(1/3) + (-a/b)^(2/3))/a^3 - 3/2/(a^2*x^(2/3))
```

**maple [A]** time = 0.01, size = 121, normalized size = 0.95

$$-\frac{b\,x^{\frac{1}{3}}}{(bx+a)\,a^{2}}-\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{3}{2a^{2}x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a)^2,x)
```

```
[Out] -3/2/a^2/x^(2/3)-1/a^2*b*x^(1/3)/(b*x+a)-5/3/a^2/(a/b)^(2/3)*ln(x^(1/3)+(a/
b)^(1/3))+5/6/a^2/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-5
/3/a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima [A]** time = 2.94, size = 132, normalized size = 1.03

$$-\frac{5\,bx+3\,a}{2\left(a^{2}bx^{\frac{5}{3}}+a^{3}x^{\frac{2}{3}}\right)}-\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(5*b*x + 3*a)/(a^2*b*x^(5/3) + a^3*x^(2/3)) - 5/3*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(2/3)) + 5/6*log(x
^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(2/3)) - 5/3*log(x^(
1/3) + (a/b)^(1/3))/(a^2*(a/b)^(2/3))
```

**mupad [B]** time = 0.17, size = 166, normalized size = 1.30

$$\frac{5\,{(-1)}^{1/3}\,b^{2/3}\,\ln\left(15\,{(-1)}^{1/3}\,a^{13/3}\,b^{8/3}-15\,a^4\,b^3\,x^{1/3}\right)}{3\,a^{8/3}}-\frac{\frac{3}{2\,a}+\frac{5\,b\,x}{2\,a^2}}{a\,x^{2/3}+b\,x^{5/3}}+\frac{5\,{(-1)}^{1/3}\,b^{2/3}\,\ln\left(15\,a^4\,b^3\,x^{1/3}-15\,{(-1)}^{1/3}\,a^{13/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{8/3}}-\frac{5\,{(-1)}^{1/3}\,b^{2/3}\,\ln\left(15\,a^4\,b^3\,x^{1/3}+15\,{(-1)}^{1/3}\,a^{13/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)^2),x)
```

```
[Out] (5*(-1)^(1/3)*b^(2/3)*log(15*(-1)^(1/3)*a^(13/3)*b^(8/3) - 15*a^4*b^3*x^(1/
3)))/(3*a^(8/3)) - (3/(2*a) + (5*b*x)/(2*a^2))/(a*x^(2/3) + b*x^(5/3)) + (5
*(-1)^(1/3)*b^(2/3)*log(15*a^4*b^3*x^(1/3) - 15*(-1)^(1/3)*a^(13/3)*b^(8/3)
*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(3*a^(8/3)) - (5*(-1)^(1/3
)*b^(2/3)*log(15*a^4*b^3*x^(1/3) + 15*(-1)^(1/3)*a^(13/3)*b^(8/3)*((3^(1/2)
*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(8/3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

## 3.690 $\int \frac{x^{5/3}}{(a+bx)^3} dx$

**Optimal**. Leaf size=140

$$-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{8/3}}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{x^{5/3}}{2b(a+bx)^2}$$

**Rubi** [A] time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 56, 617, 204, 31}

$$-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{8/3}}-\frac{x^{5/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] -x^(5/3)/(2*b*(a + b*x)^2) - (5*x^(2/3))/(6*b^2*(a + b*x)) - (5*ArcTan[(a^(
1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(8/3)) -
 (5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(1/3)*b^(8/3)) + (5*Log[a + b*x])/
(18*a^(1/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
```

```
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{5/3}}{(a+bx)^3}\,dx = -\frac{x^{5/3}}{2b(a+bx)^2}+\frac{5\int \frac{x^{2/3}}{(a+bx)^2}\,dx}{6b}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}+\frac{5\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9b^2}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}+\frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{6b^3}-\frac{5\,\mathrm{Subst}\left(\int\right.}{6}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}+\frac{5\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,\right.}{3\sqrt[3]{a}\,b^{8/3}}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}$$

**Mathematica [C]** time = 0.00, size = 27, normalized size = 0.19

$$\frac{3x^{8/3}\,{}_2F_1\left(\frac{8}{3},3;\frac{11}{3};-\frac{bx}{a}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(8/3)*Hypergeometric2F1[8/3, 3, 11/3, -((b*x)/a)])/(8*a^3)
```

**IntegrateAlgebraic [A]** time = 0.26, size = 161, normalized size = 1.15

$$\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{18\sqrt[3]{a}\,b^{8/3}}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9\sqrt[3]{a}\,b^{8/3}}-\frac{5\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{8/3}}+\frac{-5ax^{2/3}-8bx^{5/3}}{6b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] (-5*a*x^(2/3) - 8*b*x^(5/3))/(6*b^2*(a + b*x)^2) - (5*ArcTan[1/Sqrt[3] - (2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(8/3)) - (5*Log[
a^(1/3) + b^(1/3)*x^(1/3)])/(9*a^(1/3)*b^(8/3)) + (5*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(1/3)*b^(8/3))
```

**fricas [B]** time = 1.03, size = 506, normalized size = 3.61

$$\left[\frac{15\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+5\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-10\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)-3\left(8ab^3x+5a^2b^2\right)x^{\frac{2}{3}}}{18\left(ab^6x^2+2a^2b^5x+a^3b^4\right)},\ \frac{30\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)+5\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-10\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)-3\left(8ab^3x+5a^2b^2\right)x^{\frac{2}{3}}}{18\left(ab^6x^2+2a^2b^5x+a^3b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*
b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
 + a)) + 5*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b
^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 10*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^
2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) - 3*(8*a*b^3*x + 5*a^2*b^2)*x^(2/3
))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4), 1/18*(30*sqrt(1/3)*(a*b^3*x^2 + 2*a
^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(b^2*x^2 + 2*a*b*x + a^2)*(-a
*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) -
10*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3))
 - 3*(8*a*b^3*x + 5*a^2*b^2)*x^(2/3))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4)]
```

**giac [A]** time = 1.06, size = 146, normalized size = 1.04

$$-\frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2}-\frac{5\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4}-\frac{8\,bx^{\frac{5}{3}}+5\,ax^{\frac{2}{3}}}{6\,(bx+a)^2b^2}+\frac{5\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -5/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) - 5/9*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^4) - 1/6*(8*b*x^(5/3) + 5*a*x^(2/3))/((b*x + a)^2*b^2) + 5/18*(-a*b^2)^(
2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)
```

**maple** [A] time = 0.01, size = 124, normalized size = 0.89

$$\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{-\frac{4x^{\frac{5}{3}}}{3b}-\frac{5a\,x^{\frac{2}{3}}}{6b^{2}}}{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a)^3,x)
```

```
[Out] 3*(-4/9/b*x^(5/3)-5/18*a/b^2*x^(2/3))/(b*x+a)^2-5/9/b^3/(a/b)^(1/3)*ln(x^(1
/3)+(a/b)^(1/3))+5/18/b^3/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^
(2/3))+5/9/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3
)-1))
```

**maxima** [A] time = 2.95, size = 143, normalized size = 1.02

$$-\frac{8\,bx^{\frac{5}{3}}+5\,ax^{\frac{2}{3}}}{6\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)}+\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(8*b*x^(5/3) + 5*a*x^(2/3))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 5/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1
/3)) + 5/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/
3)) - 5/9*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

**mupad [B]** time = 0.17, size = 165, normalized size = 1.18

$$\frac{5\ln\left(\frac{25x^{1/3}}{9b^3}-\frac{25(-a)^{1/3}}{9b^{10/3}}\right)}{9(-a)^{1/3}b^{8/3}}-\frac{\frac{4x^{5/3}}{3b}+\frac{5ax^{2/3}}{6b^2}}{a^2+2abx+b^2x^2}+\frac{\ln\left(\frac{25x^{1/3}}{9b^3}-\frac{(-a)^{1/3}\left(-5+\sqrt{3}\,5i\right)^2}{36b^{10/3}}\right)\left(-5+\sqrt{3}\,5i\right)}{18(-a)^{1/3}b^{8/3}}-\frac{\ln\left(\frac{25x^{1/3}}{9b^3}-\frac{(-a)^{1/3}\left(5+\sqrt{3}\,5i\right)^2}{36b^{10/3}}\right)\left(5+\sqrt{3}\,5i\right)}{18(-a)^{1/3}b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(a + b*x)^3,x)
```

```
[Out] (5*log((25*x^(1/3))/(9*b^3) - (25*(-a)^(1/3))/(9*b^(10/3))))/(9*(-a)^(1/3)*
b^(8/3)) - ((4*x^(5/3))/(3*b) + (5*a*x^(2/3))/(6*b^2))/(a^2 + b^2*x^2 + 2*a
*b*x) + (log((25*x^(1/3))/(9*b^3) - ((-a)^(1/3)*(3^(1/2)*5i - 5)^2)/(36*b^(
10/3)))*(3^(1/2)*5i - 5))/(18*(-a)^(1/3)*b^(8/3)) - (log((25*x^(1/3))/(9*b^
3) - ((-a)^(1/3)*(3^(1/2)*5i + 5)^2)/(36*b^(10/3)))*(3^(1/2)*5i + 5))/(18*(
-a)^(1/3)*b^(8/3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.691 $\int \frac{x^{4/3}}{(a+bx)^3} dx$

**Optimal**. Leaf size=140

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{x^{4/3}}{2b(a+bx)^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{x^{4/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] -x^(4/3)/(2*b*(a + b*x)^2) - (2*x^(1/3))/(3*b^2*(a + b*x)) - (2*ArcTan[(a^(
1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(7/3)) +
 Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(2/3)*b^(7/3)) - Log[a + b*x]/(9*a^(2/
3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
```

```
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{4/3}}{(a+bx)^3}\,dx = -\frac{x^{4/3}}{2b(a+bx)^2}+\frac{2\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx}{3b}$$

$$= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}+\frac{2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9b^2}$$

$$= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{3\sqrt[3]{a}\,b^{8/3}}+\frac{\text{Subst}\left(\int \frac{}{}\right.}{3a^2}$$

$$= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}+\frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1\right.}{3a^{2/3}b^{7/3}}$$

$$= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}$$

**Mathematica [C]** time = 0.00, size = 27, normalized size = 0.19

$$\frac{3x^{7/3}\,{}_2F_1\left(\frac{7}{3},3;\frac{10}{3};-\frac{bx}{a}\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(7/3)*Hypergeometric2F1[7/3, 3, 10/3, -((b*x)/a)])/(7*a^3)
```

**IntegrateAlgebraic [A]** time = 0.25, size = 161, normalized size = 1.15

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{9a^{2/3}b^{7/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{2/3}b^{7/3}}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}+\frac{-4a\sqrt[3]{x}-7bx^{4/3}}{6b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] (-4*a*x^(1/3) - 7*b*x^(4/3))/(6*b^2*(a + b*x)^2) - (2*ArcTan[1/Sqrt[3] - (2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(7/3)) + (2*Log[
a^(1/3) + b^(1/3)*x^(1/3)])/(9*a^(2/3)*b^(7/3)) - Log[a^(2/3) - a^(1/3)*b^(
1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/(9*a^(2/3)*b^(7/3))
```

**fricas [B]** time = 1.11, size = 503, normalized size = 3.59

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^2+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-2\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+4\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)-3\left(7a^2b^2x+4a^3b\right)x^{\frac{1}{3}}}{18\left(a^2b^5x^2+2a^3b^4x+a^4b^3\right)}, \frac{12\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(a^2b\right)^{\frac{1}{3}}a-2\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-2\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+4\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)-3\left(7a^2b^2x+4a^3b\right)x^{\frac{1}{3}}}{18\left(a^2b^5x^2+2a^3b^4x+a^4b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b
)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x +
 a)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^
(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3
)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3))/(a^
2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(12*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b
^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*
(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*x^2 + 2*a*b*x +
a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3
)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2
/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*
b^3)]
```

**giac [A]** time = 1.13, size = 146, normalized size = 1.04

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^{2}}+\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{3}}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^{3}}-\frac{7\,bx^{\frac{4}{3}}+4\,ax^{\frac{1}{3}}}{6\,(bx+a)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) + 2/9*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^3) + 1/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a*b^3) - 1/6*(7*b*x^(4/3) + 4*a*x^(1/3))/((b*x + a)^2*b^2)
```

**maple [A]** time = 0.01, size = 124, normalized size = 0.89

$$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{-\frac{7x^{\frac{4}{3}}}{6b}-\frac{2a\,x^{\frac{1}{3}}}{3b^{2}}}{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a)^3,x)
```

```
[Out] 3*(-7/18/b*x^(4/3)-2/9*a/b^2*x^(1/3))/(b*x+a)^2+2/9/b^3/(a/b)^(2/3)*ln(x^(1
/3)+(a/b)^(1/3))-1/9/b^3/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(
2/3))+2/9/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)
-1))
```

**maxima [A]** time = 2.88, size = 143, normalized size = 1.02

$$-\frac{7\,bx^{\frac{4}{3}}+4\,ax^{\frac{1}{3}}}{6\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(7*b*x^(4/3) + 4*a*x^(1/3))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 2/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2
/3)) - 1/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3
)) + 2/9*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

**mupad [B]** time = 0.07, size = 139, normalized size = 0.99

$$\frac{2\,\ln\left(2\,x^{1/3}+\frac{2\,a^{1/3}}{b^{1/3}}\right)}{9\,a^{2/3}\,b^{7/3}}-\frac{\frac{7\,x^{4/3}}{6\,b}+\frac{2\,a\,x^{1/3}}{3\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\ln\left(2\,x^{1/3}+\frac{a^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{b^{1/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,a^{2/3}\,b^{7/3}}-\frac{\ln\left(2\,x^{1/3}-\frac{a^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{b^{1/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,a^{2/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x)^3,x)
```

```
[Out] (2*log(2*x^(1/3) + (2*a^(1/3))/b^(1/3)))/(9*a^(2/3)*b^(7/3)) - ((7*x^(4/3))
/(6*b) + (2*a*x^(1/3))/(3*b^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (log(2*x^(1/3)
+ (a^(1/3)*(3^(1/2)*1i - 1))/b^(1/3))*(3^(1/2)*1i - 1))/(9*a^(2/3)*b^(7/3))
 - (log(2*x^(1/3) - (a^(1/3)*(3^(1/2)*1i + 1))/b^(1/3))*(3^(1/2)*1i + 1))/(
9*a^(2/3)*b^(7/3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.692 $\int \frac{x^{2/3}}{(a+bx)^3} \, dx$

**Optimal**. Leaf size=143

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{x^{2/3}}{2b(a+bx)^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{x^{2/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] -x^(2/3)/(2*b*(a + b*x)^2) + x^(2/3)/(3*a*b*(a + b*x)) - ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - Log[a^(
1/3) + b^(1/3)*x^(1/3)]/(6*a^(4/3)*b^(5/3)) + Log[a + b*x]/(18*a^(4/3)*b^(5
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{2/3}}{(a+bx)^3}\,dx = -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{\int \frac{1}{\sqrt[3]{x}\,(a+bx)^2}\,dx}{3b}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} + \frac{\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9ab}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} + \frac{\log(a+bx)}{18a^{4/3}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6ab^2} - \frac{\text{Subst}\left(\int \frac{}{\frac{\sqrt[3]{}}{\sqrt[3]{}}}\right.}{6a^{}}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \frac{\log(a+bx)}{18a^{4/3}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \right.}{3a^{4/3}b^{5/3}}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \frac{\log(a+bx)}{18a^{4/3}b^{5/3}}$$

**Mathematica [C]** time = 0.01, size = 27, normalized size = 0.19

$$\frac{3x^{5/3}\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};-\frac{bx}{a}\right)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(5/3)*Hypergeometric2F1[5/3, 3, 8/3, -((b*x)/a)])/(5*a^3)
```

**IntegrateAlgebraic [A]** time = 0.24, size = 164, normalized size = 1.15

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{18a^{4/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{4/3}b^{5/3}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}} + \frac{2bx^{5/3} - ax^{2/3}}{6ab(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] (-(a*x^(2/3)) + 2*b*x^(5/3))/(6*a*b*(a + b*x)^2) - ArcTan[1/Sqrt[3] - (2*b^
(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - Log[a^(1/3)
```

```
 + b^(1/3)*x^(1/3)]/(9*a^(4/3)*b^(5/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(
1/3) + b^(2/3)*x^(2/3)]/(18*a^(4/3)*b^(5/3))
```

**fricas [B]** time = 0.99, size = 508, normalized size = 3.55

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^{3}x^{2}+2a^{2}b^{2}x+a^{3}b\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^{2}x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}a+2\left(-ab^{2}\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}-3\left(-ab^{2}\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+\left(b^{2}x^{2}+2abx+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{\frac{2}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-2\left(b^{2}x^{2}+2abx+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^{2}\right)^{\frac{1}{3}}\right)+3\left(2ab^{3}x-a^{2}b^{2}\right)x^{\frac{2}{3}}}{18\left(a^{2}b^{5}x^{2}+2a^{3}b^{4}x+a^{4}b^{3}\right)}, \frac{6\sqrt{\frac{1}{3}}\left(ab^{3}x^{2}+2a^{2}b^{2}x+a^{3}b\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}}{b}\right)+\left(b^{2}x^{2}+2abx+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{\frac{2}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-2\left(b^{2}x^{2}+2abx+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^{2}\right)^{\frac{1}{3}}\right)+3\left(2ab^{3}x-a^{2}b^{2}\right)x^{\frac{2}{3}}}{18\left(a^{2}b^{5}x^{2}+2a^{3}b^{4}x+a^{4}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b
^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
+ a)) + (b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)
^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(
2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(2*a*b^3*x - a^2*b^2)*x^(2/3))/(a^
2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^
2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^
2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(
2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(b^2*
x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(2*
a*b^3*x - a^2*b^2)*x^(2/3))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3)]
```

**giac [A]** time = 1.06, size = 149, normalized size = 1.04

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^{2}b}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^{2}b^{3}}+\frac{2bx^{\frac{5}{3}}-ax^{\frac{2}{3}}}{6(bx+a)^{2}ab}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) - 1/9*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^2*b^3) + 1/6*(2*b*x^(5/3) - a*x^(2/3))/((b*x + a)^2*a*b) + 1/18*(-a*b^2)^(
2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

**maple [A]** time = 0.01, size = 132, normalized size = 0.92

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a\,b^{2}}-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}} a\,b^{2}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}} a\,b^{2}}+\frac{\frac{x^{\frac{5}{3}}}{3a}-\frac{x^{\frac{2}{3}}}{6b}}{\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a)^3,x)
```

```
[Out] 3*(1/9/a*x^(5/3)-1/18/b*x^(2/3))/(b*x+a)^2-1/9/b^2/a/(a/b)^(1/3)*ln(x^(1/3)
+(a/b)^(1/3))+1/18/b^2/a/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(
2/3))+1/9/b^2/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/
3)-1))
```

**maxima [A]** time = 2.92, size = 153, normalized size = 1.07

$$\frac{2\,bx^{\frac{5}{3}}-ax^{\frac{2}{3}}}{6\left(ab^{3}x^{2}+2\,a^{2}b^{2}x+a^{3}b\right)}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*x^(5/3) - a*x^(2/3))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(
1/3)) + 1/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^
(1/3)) - 1/9*log(x^(1/3) + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

**mupad [B]** time = 0.26, size = 172, normalized size = 1.20

$$\frac{\frac{x^{5/3}}{3\,a}-\frac{x^{2/3}}{6\,b}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\ln\left(\frac{1}{9\,a^{5/3}\,{\left(-b\right)}^{4/3}}+\frac{x^{1/3}}{9\,a^2\,b}\right)}{9\,a^{4/3}\,{\left(-b\right)}^{5/3}}+\frac{\ln\left(\frac{x^{1/3}}{9\,a^2\,b}+\frac{{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{36\,a^{5/3}\,{\left(-b\right)}^{4/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{18\,a^{4/3}\,{\left(-b\right)}^{5/3}}-\frac{\ln\left(\frac{x^{1/3}}{9\,a^2\,b}+\frac{{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{36\,a^{5/3}\,{\left(-b\right)}^{4/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{18\,a^{4/3}\,{\left(-b\right)}^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x)^3,x)
```

```
[Out] (x^(5/3)/(3*a) - x^(2/3)/(6*b))/(a^2 + b^2*x^2 + 2*a*b*x) + log(1/(9*a^(5/3
)*(-b)^(4/3)) + x^(1/3)/(9*a^2*b))/(9*a^(4/3)*(-b)^(5/3)) + (log(x^(1/3)/(9
*a^2*b) + (3^(1/2)*1i - 1)^2/(36*a^(5/3)*(-b)^(4/3)))*(3^(1/2)*1i - 1))/(18
*a^(4/3)*(-b)^(5/3)) - (log(x^(1/3)/(9*a^2*b) + (3^(1/2)*1i + 1)^2/(36*a^(5
/3)*(-b)^(4/3)))*(3^(1/2)*1i + 1))/(18*a^(4/3)*(-b)^(5/3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.693 $\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx$

**Optimal**. Leaf size=143

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

**Rubi** [A] time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] -x^(1/3)/(2*b*(a + b*x)^2) + x^(1/3)/(6*a*b*(a + b*x)) - ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + Log[a^(
1/3) + b^(1/3)*x^(1/3)]/(6*a^(5/3)*b^(4/3)) - Log[a + b*x]/(18*a^(5/3)*b^(4
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx = -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\int \frac{1}{x^{2/3}(a+bx)^2}\,dx}{6b}$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} + \frac{\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9ab}$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} - \frac{\log(a+bx)}{18a^{5/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{}{}\right.}{6a^{5}}$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}} - \frac{\log(a+bx)}{18a^{5/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{3a^{5/3}b^{4/3}}$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}} - \frac{\log(a+bx)}{18a^{5/3}b^{4/3}}$$

**Mathematica [C]** time = 0.00, size = 27, normalized size = 0.19

$$\frac{3x^{4/3}\,{}_2F_1\left(\frac{4}{3}, 3; \frac{7}{3}; -\frac{bx}{a}\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(4/3)*Hypergeometric2F1[4/3, 3, 7/3, -((b*x)/a)])/(4*a^3)
```

**IntegrateAlgebraic [A]** time = 0.24, size = 163, normalized size = 1.14

$$-\frac{\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{18a^{5/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{5/3}b^{4/3}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}} + \frac{bx^{4/3} - 2a\sqrt[3]{x}}{6ab(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] (-2*a*x^(1/3) + b*x^(4/3))/(6*a*b*(a + b*x)^2) - ArcTan[1/Sqrt[3] - (2*b^(1
/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + Log[a^(1/3) +
```

```
b^(1/3)*x^(1/3)]/(9*a^(5/3)*b^(4/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/
3) + b^(2/3)*x^(2/3)]/(18*a^(5/3)*b^(4/3))
```

**fricas [B]** time = 1.21, size = 501, normalized size = 3.50

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^{3}x^{2}+2a^{2}b^{2}x+a^{3}b\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^{2}+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-\left(a^{2}b\right)^{\frac{1}{3}}a+\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}-3\left(a^{2}b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-\left(b^{2}x^{2}+2abx+a^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^{2}b\right)^{\frac{1}{3}}a-\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(b^{2}x^{2}+2abx+a^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^{2}b\right)^{\frac{2}{3}}\right)+3\left(a^{2}b^{2}x-2a^{3}b\right)x^{\frac{1}{3}}}{18\left(a^{3}b^{4}x^{2}+2a^{4}b^{3}x+a^{5}b^{2}\right)}, \frac{6\sqrt{\frac{1}{3}}\left(ab^{3}x^{2}+2a^{2}b^{2}x+a^{3}b\right)\sqrt{\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(a^{2}b\right)^{\frac{1}{3}}a-2\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-\left(b^{2}x^{2}+2abx+a^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^{2}b\right)^{\frac{1}{3}}a-\left(a^{2}b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(b^{2}x^{2}+2abx+a^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^{2}b\right)^{\frac{2}{3}}\right)+3\left(a^{2}b^{2}x-2a^{3}b\right)x^{\frac{1}{3}}}{18\left(a^{3}b^{4}x^{2}+2a^{4}b^{3}x+a^{5}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b
)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x +
 a)) - (b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1
/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*
log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(a^3*b^
4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x
+ a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*
b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^2 + 2*a*b*x + a^2)*(a
^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*
(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) +
3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)]
```

**giac [A]** time = 1.11, size = 148, normalized size = 1.03

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}b}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b^{2}}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b^{2}}+\frac{bx^{\frac{4}{3}}-2\,ax^{\frac{1}{3}}}{6\left(bx+a\right)^{2}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) + 1/9*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^2*b^2) + 1/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^2*b^2) + 1/6*(b*x^(4/3) - 2*a*x^(1/3))/((b*x + a)^2*a*b)
```

**maple** [A] time = 0.01, size = 132, normalized size = 0.92

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{\frac{x^{\frac{4}{3}}}{6a}-\frac{x^{\frac{1}{3}}}{3b}}{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a)^3,x)
```

```
[Out] 3*(1/18/a*x^(4/3)-1/9/b*x^(1/3))/(b*x+a)^2+1/9/b^2/a/(a/b)^(2/3)*ln(x^(1/3)
+(a/b)^(1/3))-1/18/b^2/a/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(
2/3))+1/9/b^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/
3)-1))
```

**maxima** [A] time = 2.96, size = 152, normalized size = 1.06

$$\frac{bx^{\frac{4}{3}}-2\,ax^{\frac{1}{3}}}{6\left(ab^{3}x^{2}+2\,a^{2}b^{2}x+a^{3}b\right)}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^(4/3) - 2*a*x^(1/3))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(
2/3)) - 1/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^
(2/3)) + 1/9*log(x^(1/3) + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

**mupad** [B] time = 0.24, size = 146, normalized size = 1.02

$$\frac{\frac{x^{4/3}}{6\,a}-\frac{x^{1/3}}{3\,b}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\ln\left(\frac{b^{2/3}}{a^{2/3}}+\frac{b\,x^{1/3}}{a}\right)}{9\,a^{5/3}\,b^{4/3}}+\frac{\ln\left(\frac{b\,x^{1/3}}{a}+\frac{b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{5/3}\,b^{4/3}}-\frac{\ln\left(\frac{b\,x^{1/3}}{a}-\frac{b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{5/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(a + b*x)^3,x)
```

```
[Out] (x^(4/3)/(6*a) - x^(1/3)/(3*b))/(a^2 + b^2*x^2 + 2*a*b*x) + log(b^(2/3)/a^(
2/3) + (b*x^(1/3))/a)/(9*a^(5/3)*b^(4/3)) + (log((b*x^(1/3))/a + (b^(2/3)*(
3^(1/2)*1i - 1))/(2*a^(2/3)))*(3^(1/2)*1i - 1))/(18*a^(5/3)*b^(4/3)) - (log
((b*x^(1/3))/a - (b^(2/3)*(3^(1/2)*1i + 1))/(2*a^(2/3)))*(3^(1/2)*1i + 1))/
(18*a^(5/3)*b^(4/3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.694 $\int \frac{1}{\sqrt[3]{x}\,(a+bx)^3}\,dx$

**Optimal**. Leaf size=140

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}}+\frac{\log(a+bx)}{9a^{7/3}b^{2/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}+\frac{2x^{2/3}}{3a^2(a+bx)}+\frac{x^{2/3}}{2a(a+bx)^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}}+\frac{\log(a+bx)}{9a^{7/3}b^{2/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}+\frac{2x^{2/3}}{3a^2(a+bx)}+\frac{x^{2/3}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)^3),x]
```

```
[Out] x^(2/3)/(2*a*(a + b*x)^2) + (2*x^(2/3))/(3*a^2*(a + b*x)) - (2*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(2/3)) -
Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(7/3)*b^(2/3)) + Log[a + b*x]/(9*a^(7/3
)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
```

```
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx = \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx}{3a}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} + \frac{2\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{9a^2}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{3a^2b} - \frac{\text{Subst}\left(\int \frac{}{}\right.}{3a}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{3a^{7/3}b^{2/3}}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}}$$

**Mathematica [C]** time = 0.00, size = 27, normalized size = 0.19

$$\frac{3x^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};-\frac{bx}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)^3),x]
```

```
[Out] (3*x^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, -((b*x)/a)])/(2*a^3)
```

**IntegrateAlgebraic [A]** time = 0.16, size = 157, normalized size = 1.12

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{9a^{7/3}b^{2/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{7/3}b^{2/3}}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}+\frac{x^{2/3}(7a+4bx)}{6a^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(1/3)*(a + b*x)^3),x]
```

```
[Out] (x^(2/3)*(7*a + 4*b*x))/(6*a^2*(a + b*x)^2) - (2*ArcTan[1/Sqrt[3] - (2*b^(1
/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]])/(3*Sqrt[3]*a^(7/3)*b^(2/3)) - (2*Log[a^(1/
3) + b^(1/3)*x^(1/3)]])/(9*a^(7/3)*b^(2/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*
x^(1/3) + b^(2/3)*x^(2/3)]/(9*a^(7/3)*b^(2/3))
```

**fricas [B]** time = 0.89, size = 510, normalized size = 3.64

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+2\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-4\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)+3\left(4ab^3x+7a^2b^2\right)x^{\frac{2}{3}}}{18\left(a^3b^4x^2+2a^4b^3x+a^5b^2\right)}, \frac{12\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)+2\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-4\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)+3\left(4ab^3x+7a^2b^2\right)x^{\frac{2}{3}}}{18\left(a^3b^4x^2+2a^4b^3x+a^5b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b
^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
+ a)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^
2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 4*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)
^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(4*a*b^3*x + 7*a^2*b^2)*x^(2/3))
/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(12*sqrt(1/3)*(a*b^3*x^2 + 2*a
^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a
*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) -
4*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3))
+ 3*(4*a*b^3*x + 7*a^2*b^2)*x^(2/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)]
```

**giac [A]** time = 1.06, size = 143, normalized size = 1.02

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^3}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^3b^2}+\frac{4bx^{\frac{5}{3}}+7ax^{\frac{2}{3}}}{6(bx+a)^2a^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 2/9*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
^2) + 1/6*(4*b*x^(5/3) + 7*a*x^(2/3))/((b*x + a)^2*a^2) + 1/9*(-a*b^2)^(2/3
)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2)
```

**maple [A]** time = 0.01, size = 136, normalized size = 0.97

$$\frac{x^{\frac{2}{3}}}{2\left(bx+a\right)^{2}a}+\frac{2x^{\frac{2}{3}}}{3\left(bx+a\right)a^{2}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a)^3,x)
```

```
[Out] 1/2*x^(2/3)/a/(b*x+a)^2+2/3*x^(2/3)/a^2/(b*x+a)-2/9/a^2/b/(a/b)^(1/3)*ln(x^
(1/3)+(a/b)^(1/3))+1/9/a^2/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/
b)^(2/3))+2/9/a^2*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
^(1/3)-1))
```

**maxima [A]** time = 2.96, size = 151, normalized size = 1.08

$$\frac{4\,bx^{\frac{5}{3}}+7\,ax^{\frac{2}{3}}}{6\left(a^{2}b^{2}x^{2}+2\,a^{3}bx+a^{4}\right)}+\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^{2}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{2}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(4*b*x^(5/3) + 7*a*x^(2/3))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 2/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(
1/3)) + 1/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(
1/3)) - 2/9*log(x^(1/3) + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

**mupad** [B] time = 0.19, size = 167, normalized size = 1.19

$$\frac{\frac{7x^{2/3}}{6a}+\frac{2bx^{5/3}}{3a^2}}{a^2+2abx+b^2x^2}+\frac{2\ln\left(\frac{4bx^{1/3}}{9a^4}-\frac{4b^{2/3}}{9(-a)^{11/3}}\right)}{9(-a)^{7/3}b^{2/3}}+\frac{\ln\left(\frac{4bx^{1/3}}{9a^4}-\frac{b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{9(-a)^{11/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9(-a)^{7/3}b^{2/3}}-\frac{\ln\left(\frac{4bx^{1/3}}{9a^4}-\frac{b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{9(-a)^{11/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9(-a)^{7/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)^3),x)
```

```
[Out] ((7*x^(2/3))/(6*a) + (2*b*x^(5/3))/(3*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (2*
log((4*b*x^(1/3))/(9*a^4) - (4*b^(2/3))/(9*(-a)^(11/3))))/(9*(-a)^(7/3)*b^(
2/3)) + (log((4*b*x^(1/3))/(9*a^4) - (b^(2/3)*(3^(1/2)*1i - 1)^2)/(9*(-a)^(
11/3)))*(3^(1/2)*1i - 1))/(9*(-a)^(7/3)*b^(2/3)) - (log((4*b*x^(1/3))/(9*a^
4) - (b^(2/3)*(3^(1/2)*1i + 1)^2)/(9*(-a)^(11/3)))*(3^(1/2)*1i + 1))/(9*(-a
)^(7/3)*b^(2/3))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.695 $\int \frac{1}{x^{2/3}(a+bx)^3}\,dx$

**Optimal**. Leaf size=140

$$\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] x^(1/3)/(2*a*(a + b*x)^2) + (5*x^(1/3))/(6*a^2*(a + b*x)) - (5*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(1/3)) +
(5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(8/3)*b^(1/3)) - (5*Log[a + b*x])/(
18*a^(8/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
```

```
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{2/3}(a+bx)^3}\,dx = \frac{\sqrt[3]{x}}{2a(a+bx)^2} + \frac{5\int \frac{1}{x^{2/3}(a+bx)^2}\,dx}{6a}$$

$$= \frac{\sqrt[3]{x}}{2a(a+bx)^2} + \frac{5\sqrt[3]{x}}{6a^2(a+bx)} + \frac{5\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9a^2}$$

$$= \frac{\sqrt[3]{x}}{2a(a+bx)^2} + \frac{5\sqrt[3]{x}}{6a^2(a+bx)} - \frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6a^{7/3}b^{2/3}} + \frac{5\,\mathrm{Subs}}{}$$

$$= \frac{\sqrt[3]{x}}{2a(a+bx)^2} + \frac{5\sqrt[3]{x}}{6a^2(a+bx)} + \frac{5\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}} - \frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{3a^{8/3}}$$

$$= \frac{\sqrt[3]{x}}{2a(a+bx)^2} + \frac{5\sqrt[3]{x}}{6a^2(a+bx)} - \frac{5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}} + \frac{5\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}} - \frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}$$

**Mathematica** [C] time = 0.00, size = 25, normalized size = 0.18

$$\frac{3\sqrt[3]{x}\,{}_2F_1\left(\frac{1}{3},3;\frac{4}{3};-\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] (3*x^(1/3)*Hypergeometric2F1[1/3, 3, 4/3, -((b*x)/a)])/a^3
```

**IntegrateAlgebraic [A]**  time = 0.15, size = 157, normalized size = 1.12

$$-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{18a^{8/3}\sqrt[3]{b}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}(8a+5bx)}{6a^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] (x^(1/3)*(8*a + 5*b*x))/(6*a^2*(a + b*x)^2) - (5*ArcTan[1/Sqrt[3] - (2*b^(1
/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(1/3)) + (5*Log[a^(1/
3) + b^(1/3)*x^(1/3)])/(9*a^(8/3)*b^(1/3)) - (5*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(8/3)*b^(1/3))
```

**fricas [B]**  time = 1.37, size = 499, normalized size = 3.56

$$\left[\frac{15\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^2+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-5\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+10\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)+3\left(5a^2b^2x+8a^3b\right)x^{\frac{1}{3}}}{18\left(a^4b^3x^2+2a^5b^2x+a^6b\right)}, \frac{30\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(a^2b\right)^{\frac{1}{3}}a-2\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-5\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+10\left(b^2x^2+2abx+a^2\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)+3\left(5a^2b^2x+8a^3b\right)x^{\frac{1}{3}}}{18\left(a^4b^3x^2+2a^5b^2x+a^6b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*
b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
+ a)) - 5*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)
^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2
/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1/3))/(
a^4*b^3*x^2 + 2*a^5*b^2*x + a^6*b), 1/18*(30*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b
^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*
(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^2 + 2*a*b*x +
a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3
)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(
2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1/3))/(a^4*b^3*x^2 + 2*a^5*b^2*x + a^6
*b)]
```

**giac [A]** time = 1.00, size = 143, normalized size = 1.02

$$-\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{3}}+\frac{5\,\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}b}+\frac{5\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{3}b}+\frac{5\,bx^{\frac{4}{3}}+8\,ax^{\frac{1}{3}}}{6\,(bx+a)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -5/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 5/9*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
) + 5/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/
(a^3*b) + 1/6*(5*b*x^(4/3) + 8*a*x^(1/3))/((b*x + a)^2*a^2)
```

**maple [A]** time = 0.01, size = 136, normalized size = 0.97

$$\frac{x^{\frac{1}{3}}}{2\,(bx+a)^{2}\,a}+\frac{5x^{\frac{1}{3}}}{6\,(bx+a)\,a^{2}}+\frac{5\sqrt{3}\;\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a)^3,x)
```

```
[Out] 1/2*x^(1/3)/a/(b*x+a)^2+5/6*x^(1/3)/a^2/(b*x+a)+5/9/a^2/b/(a/b)^(2/3)*ln(x^
(1/3)+(a/b)^(1/3))-5/18/a^2/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a
/b)^(2/3))+5/9/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x^(1/3)-1))
```

**maxima [A]** time = 3.00, size = 151, normalized size = 1.08

$$\frac{5\,bx^{\frac{4}{3}}+8\,ax^{\frac{1}{3}}}{6\left(a^{2}b^{2}x^{2}+2\,a^{3}bx+a^{4}\right)}+\frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(5*b*x^(4/3) + 8*a*x^(1/3))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 5/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(
2/3)) - 5/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^
(2/3)) + 5/9*log(x^(1/3) + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

**mupad [B]** time = 0.24, size = 157, normalized size = 1.12

$$\frac{\frac{4\,x^{1/3}}{3\,a}+\frac{5\,b\,x^{4/3}}{6\,a^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{5\,\ln\left(\frac{5\,b^{5/3}}{a^{5/3}}+\frac{5\,b^2\,x^{1/3}}{a^2}\right)}{9\,a^{8/3}\,b^{1/3}}+\frac{\ln\left(\frac{5\,b^2\,x^{1/3}}{a^2}+\frac{b^{5/3}\,\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{2\,a^{5/3}}\right)\,\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{18\,a^{8/3}\,b^{1/3}}-\frac{\ln\left(\frac{5\,b^2\,x^{1/3}}{a^2}-\frac{b^{5/3}\,\left(5+\sqrt{3}\,5\mathrm{i}\right)}{2\,a^{5/3}}\right)\,\left(5+\sqrt{3}\,5\mathrm{i}\right)}{18\,a^{8/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)^3),x)
```

```
[Out] ((4*x^(1/3))/(3*a) + (5*b*x^(4/3))/(6*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (5*
log((5*b^(5/3))/a^(5/3) + (5*b^2*x^(1/3))/a^2))/(9*a^(8/3)*b^(1/3)) + (log(
(5*b^2*x^(1/3))/a^2 + (b^(5/3)*(3^(1/2)*5i - 5))/(2*a^(5/3)))*(3^(1/2)*5i -
 5))/(18*a^(8/3)*b^(1/3)) - (log((5*b^2*x^(1/3))/a^2 - (b^(5/3)*(3^(1/2)*5i
 + 5))/(2*a^(5/3)))*(3^(1/2)*5i + 5))/(18*a^(8/3)*b^(1/3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.696 $\int \frac{1}{x^{4/3}(a+bx)^3}\,dx$

**Optimal**. Leaf size=152

$$\frac{7\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{10/3}}-\frac{7\sqrt[3]{b}\log(a+bx)}{9a^{10/3}}+\frac{14\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{14}{3a^3\sqrt[3]{x}}+\frac{7}{6a^2\sqrt[3]{x}(a+bx)}+\frac{1}{2a\sqrt[3]{x}(a+b$$

**Rubi** [A] time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$\frac{7}{6a^2\sqrt[3]{x}(a+bx)}+\frac{7\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{10/3}}-\frac{7\sqrt[3]{b}\log(a+bx)}{9a^{10/3}}+\frac{14\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{14}{3a^3\sqrt[3]{x}}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)^3),x]
```

```
[Out] -14/(3*a^3*x^(1/3)) + 1/(2*a*x^(1/3)*(a + b*x)^2) + 7/(6*a^2*x^(1/3)*(a + b
*x)) + (14*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(10/3)) + (7*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(10
/3)) - (7*b^(1/3)*Log[a + b*x])/(9*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx = \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7\int \frac{1}{x^{4/3}(a+bx)^2}\,dx}{6a}$$

$$= \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{14\int \frac{1}{x^{4/3}(a+bx)}\,dx}{9a^2}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} - \frac{(14b)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9a^3}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} - \frac{7\sqrt[3]{b}\,\log(a+bx)}{9a^{10/3}} - \frac{7\,\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2 \cdots}\right.}{3a^3}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{7\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{10/3}} - \frac{7\sqrt[3]{b}\,\log(a+b \cdots}{9a^{10/3}}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{14\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} + \frac{7\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \cdots\right.}{3a^{10/3}}$$

**Mathematica [C]** time = 0.01, size = 25, normalized size = 0.16

$$-\frac{3\,{}_2F_1\left(-\frac{1}{3},3;\frac{2}{3};-\frac{bx}{a}\right)}{a^3\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)^3),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 3, 2/3, -((b*x)/a)])/(a^3*x^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.27, size = 168, normalized size = 1.11

$$-\frac{7\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{9a^{10/3}}+\frac{14\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{10/3}}+\frac{14\sqrt[3]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}+\frac{-18a^2-49abx-28b^2x^2}{6a^3\sqrt[3]{x}\,(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(4/3)*(a + b*x)^3),x]
```

```
[Out] (-18*a^2 - 49*a*b*x - 28*b^2*x^2)/(6*a^3*x^(1/3)*(a + b*x)^2) + (14*b^(1/3)
*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1
0/3)) + (14*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(9*a^(10/3)) - (7*b^(1/
3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(9*a^(10/3))
```

**fricas [A]** time = 1.29, size = 211, normalized size = 1.39

$$-\frac{28\sqrt{3}\left(b^2x^3+2\,abx^2+a^2x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14\left(b^2x^3+2\,abx^2+a^2x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-28\left(b^2x^3+2\,abx^2+a^2x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+3\left(28\,b^2x^2+49\,abx+18\,a^2\right)x^{\frac{2}{3}}}{18\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/18*(28*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*arctan(2/3*sqrt
(3)*x^(1/3)*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(
b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)^(1/3)) - 28*(b^
2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) + 3*(
28*b^2*x^2 + 49*a*b*x + 18*a^2)*x^(2/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x
)
```

**giac [A]** time = 1.16, size = 155, normalized size = 1.02

$$\frac{14\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4}+\frac{14\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b}-\frac{3}{a^3x^{\frac{1}{3}}}-\frac{7\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^4b}-\frac{10\,b^2x^{\frac{5}{3}}+13\,abx^{\frac{2}{3}}}{6\,(bx+a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 14/9*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 + 14/9*sqrt(3)*(-a
*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^
4*b) - 3/(a^3*x^(1/3)) - 7/9*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^4*b) - 1/6*(10*b^2*x^(5/3) + 13*a*b*x^(2/3))/((b*x +
 a)^2*a^3)
```

**maple** [A] time = 0.02, size = 139, normalized size = 0.91

$$-\frac{5b^2x^{\frac{5}{3}}}{3\left(bx+a\right)^2a^3}-\frac{13b\,x^{\frac{2}{3}}}{6\left(bx+a\right)^2a^2}-\frac{14\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{14\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{7\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{3}{a^3x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a)^3,x)
```

```
[Out] -3/a^3/x^(1/3)-5/3*b^2/a^3/(b*x+a)^2*x^(5/3)-13/6*b/a^2/(b*x+a)^2*x^(2/3)+1
4/9/a^3/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))-7/9/a^3/(a/b)^(1/3)*ln(x^(2/3)-
(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-14/9/a^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima** [A] time = 2.99, size = 154, normalized size = 1.01

$$-\frac{28\,b^2x^2+49\,abx+18\,a^2}{6\left(a^3b^2x^{\frac{7}{3}}+2\,a^4bx^{\frac{4}{3}}+a^5x^{\frac{1}{3}}\right)}-\frac{14\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(28*b^2*x^2 + 49*a*b*x + 18*a^2)/(a^3*b^2*x^(7/3) + 2*a^4*b*x^(4/3) +
a^5*x^(1/3)) - 14/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a
/b)^(1/3))/(a^3*(a/b)^(1/3)) - 7/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(1/3)) + 14/9*log(x^(1/3) + (a/b)^(1/3))/(a^3*(a/b)^(1/
3))
```

**mupad** [**B**] time = 0.09, size = 174, normalized size = 1.14

$$\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}+588\,a^3\,b^3\,x^{1/3}\right)}{9\,a^{10/3}}-\frac{\frac{3}{a}+\frac{14\,b^2\,x^2}{3\,a^3}+\frac{49\,b\,x}{6\,a^2}}{a^2\,x^{1/3}+b^2\,x^{7/3}+2\,a\,b\,x^{4/3}}+\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}^2+588\,a^3\,b^3\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}}-\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}^2+588\,a^3\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)^3),x)
```

```
[Out] (14*b^(1/3)*log(588*a^(10/3)*b^(8/3) + 588*a^3*b^3*x^(1/3)))/(9*a^(10/3)) -
 (3/a + (14*b^2*x^2)/(3*a^3) + (49*b*x)/(6*a^2))/(a^2*x^(1/3) + b^2*x^(7/3)
 + 2*a*b*x^(4/3)) + (14*b^(1/3)*log(588*a^(10/3)*b^(8/3)*((3^(1/2)*1i)/2 -
1/2)^2 + 588*a^3*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(10/3)) - (14*b^
(1/3)*log(588*a^(10/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2 + 588*a^3*b^3*x^(1/
3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(10/3))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.697 $\int \frac{1}{x^{5/3}(a+bx)^3} \, dx$

**Optimal**. Leaf size=152

$$-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}-\frac{10}{3a^3x^{2/3}}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{1}{2ax^{2/3}(a$$

**Rubi [A]** time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}+\frac{4}{3a^2x^{2/3}(a+bx)}-\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] -10/(3*a^3*x^(2/3)) + 1/(2*a*x^(2/3)*(a + b*x)^2) + 4/(3*a^2*x^(2/3)*(a + b
*x)) + (20*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(11/3)) - (10*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(1
1/3)) + (10*b^(2/3)*Log[a + b*x])/(9*a^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
```

```
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/3}(a+bx)^3}\,dx &= \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4\int \frac{1}{x^{5/3}(a+bx)^2}\,dx}{3a} \\
&= \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} + \frac{20\int \frac{1}{x^{5/3}(a+bx)}\,dx}{9a^2} \\
&= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} - \frac{(20b)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9a^3} \\
&= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} + \frac{10b^{2/3}\log(a+bx)}{9a^{11/3}} - \frac{\left(10\sqrt[3]{b}\right)\text{Subst}\left(\int \cdots\right)}{3a\cdots} \\
&= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} - \frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}} + \frac{10b^{2/3}\log(\cdots}{9a^{11/3}} \\
&= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} + \frac{20b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{11/3}} - \frac{10b^{2/3}\log\left(\sqrt[3]{\cdots}\right.}{3a^{11\cdots}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 27, normalized size = 0.18

$$-\frac{3\,{}_2F_1\left(-\frac{2}{3},3;\frac{1}{3};-\frac{bx}{a}\right)}{2a^3x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] (-3*Hypergeometric2F1[-2/3, 3, 1/3, -((b*x)/a)])/(2*a^3*x^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.27, size = 168, normalized size = 1.11

$$\frac{10b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{9a^{11/3}}-\frac{20b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{11/3}}+\frac{20b^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}+\frac{-9a^2-32abx-20b^2x^2}{6a^3x^{2/3}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] (-9*a^2 - 32*a*b*x - 20*b^2*x^2)/(6*a^3*x^(2/3)*(a + b*x)^2) + (20*b^(2/3)*
ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11
/3)) - (20*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(9*a^(11/3)) + (10*b^(2/
3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(9*a^(11/3))
```

**fricas [B]** time = 0.67, size = 244, normalized size = 1.61

$$\frac{40\sqrt{3}\left(b^2x^3+2\,abx^2+a^2x\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-20\left(b^2x^3+2\,abx^2+a^2x\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+40\left(b^2x^3+2\,abx^2+a^2x\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3\left(20\,b^2x^2+32\,abx+9\,a^2\right)x^{\frac{1}{3}}}{18\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(40*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*arctan(1/3*
(2*sqrt(3)*a*x^(1/3)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 20*(b^2*x^3 + 2*a*b
*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3)*(-b^2/a^2)^(1/
3) + a^2*(-b^2/a^2)^(2/3)) + 40*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2)^(1
/3)*log(b*x^(1/3) - a*(-b^2/a^2)^(1/3)) - 3*(20*b^2*x^2 + 32*a*b*x + 9*a^2)
*x^(1/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)
```

**giac [A]** time = 1.08, size = 150, normalized size = 0.99

$$\frac{20\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4}-\frac{20\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4}-\frac{10\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^4}-\frac{20\,b^2x^2+32\,abx+9\,a^2}{6\left(bx^{\frac{4}{3}}+ax^{\frac{1}{3}}\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 20/9*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 - 20/9*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4
 - 10/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/a
^4 - 1/6*(20*b^2*x^2 + 32*a*b*x + 9*a^2)/((b*x^(4/3) + a*x^(1/3))^2*a^3)
```

**maple** [A] time = 0.02, size = 139, normalized size = 0.91

$$-\frac{11b^2x^{\frac{4}{3}}}{6\,(bx+a)^2\,a^3}-\frac{7b\,x^{\frac{1}{3}}}{3\,(bx+a)^2\,a^2}-\frac{20\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{20\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}+\frac{10\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{3}{2a^3x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a)^3,x)
```

```
[Out] -3/2/a^3/x^(2/3)-11/6/a^3*b^2/(b*x+a)^2*x^(4/3)-7/3/a^2*b/(b*x+a)^2*x^(1/3)
-20/9/a^3/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))+10/9/a^3/(a/b)^(2/3)*ln(x^(2/
3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-20/9/a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

**maxima** [A] time = 2.99, size = 154, normalized size = 1.01

$$-\frac{20\,b^2x^2+32\,abx+9\,a^2}{6\left(a^3b^2x^{\frac{8}{3}}+2\,a^4bx^{\frac{5}{3}}+a^5x^{\frac{2}{3}}\right)}-\frac{20\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{20\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(20*b^2*x^2 + 32*a*b*x + 9*a^2)/(a^3*b^2*x^(8/3) + 2*a^4*b*x^(5/3) + a
^5*x^(2/3)) - 20/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/
b)^(1/3))/(a^3*(a/b)^(2/3)) + 10/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(2/3)) - 20/9*log(x^(1/3) + (a/b)^(1/3))/(a^3*(a/b)^(2/
3))
```

**mupad** [B] time = 0.17, size = 182, normalized size = 1.20

$$\frac{20\,b^{2/3}\,\ln\left(540\,{(-a)}^{19/3}\,b^{8/3}-540\,a^6\,b^3\,x^{1/3}\right)}{9\,{(-a)}^{11/3}}-\frac{\frac{3}{2\,a}+\frac{10\,b^2\,x^2}{3\,a^3}+\frac{16\,b\,x}{3\,a^2}}{a^2\,x^{2/3}+b^2\,x^{8/3}+2\,a\,b\,x^{5/3}}+\frac{20\,b^{2/3}\,\ln\left(540\,{(-a)}^{19/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-540\,a^6\,b^3\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,{(-a)}^{11/3}}-\frac{20\,b^{2/3}\,\ln\left(540\,{(-a)}^{19/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)+540\,a^6\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,{(-a)}^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)^3),x)
```

```
[Out] (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3) - 540*a^6*b^3*x^(1/3)))/(9*(-a)^(11
/3)) - (3/(2*a) + (10*b^2*x^2)/(3*a^3) + (16*b*x)/(3*a^2))/(a^2*x^(2/3) + b
^2*x^(8/3) + 2*a*b*x^(5/3)) + (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3)*((3^(
1/2)*1i)/2 - 1/2) - 540*a^6*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*(-a)^(1
1/3)) - (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2) + 54
0*a^6*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*(-a)^(11/3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.698 $\int \frac{\sqrt[4]{1-x}}{1+x} \, dx$

**Optimal**. Leaf size=58

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {50, 63, 212, 206, 203}

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{1-x}}{1+x}\,dx &= 4\sqrt[4]{1-x}+2\int \frac{1}{(1-x)^{3/4}(1+x)}\,dx\\
&= 4\sqrt[4]{1-x}-8\,\text{Subst}\left(\int \frac{1}{2-x^4}\,dx,x,\sqrt[4]{1-x}\right)\\
&= 4\sqrt[4]{1-x}-\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx,x,\sqrt[4]{1-x}\right)-\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx,x,\sqrt[4]{1-x}\right)\\
&= 4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)
\end{aligned}
$$

**Mathematica** [A] time = 0.02, size = 58, normalized size = 1.00

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

**IntegrateAlgebraic** [A] time = 0.08, size = 58, normalized size = 1.00

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

**fricas [A]** time = 0.96, size = 82, normalized size = 1.41

$$4 \cdot 2^{\frac{1}{4}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{3}{4}} \sqrt{\sqrt{2} + \sqrt{-x+1}} - \frac{1}{2} \cdot 2^{\frac{3}{4}} (-x+1)^{\frac{1}{4}}\right) - 2^{\frac{1}{4}} \log\left(2^{\frac{1}{4}} + (-x+1)^{\frac{1}{4}}\right) + 2^{\frac{1}{4}} \log\left(-2^{\frac{1}{4}} + (-x+1)^{\frac{1}{4}}\right) + 4(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="fricas")
```

```
[Out] 4*2^(1/4)*arctan(1/2*2^(3/4)*sqrt(sqrt(2) + sqrt(-x + 1)) - 1/2*2^(3/4)*(-x
 + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (-x + 1)^(1/4)) + 2^(1/4)*log(-2^(1/4)
 + (-x + 1)^(1/4)) + 4*(-x + 1)^(1/4)
```

**giac [A]** time = 1.17, size = 64, normalized size = 1.10

$$-2 \cdot 2^{\frac{1}{4}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{3}{4}} (-x+1)^{\frac{1}{4}}\right) - 2^{\frac{1}{4}} \log\left(2^{\frac{1}{4}} + (-x+1)^{\frac{1}{4}}\right) + 2^{\frac{1}{4}} \log\left(\left|-2^{\frac{1}{4}} + (-x+1)^{\frac{1}{4}}\right|\right) + 4(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="giac")
```

```
[Out] -2*2^(1/4)*arctan(1/2*2^(3/4)*(-x + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (-x +
 1)^(1/4)) + 2^(1/4)*log(abs(-2^(1/4) + (-x + 1)^(1/4))) + 4*(-x + 1)^(1/4)
```

**maple [A]** time = 0.01, size = 62, normalized size = 1.07

$$-2\, 2^{\frac{1}{4}} \arctan\left(\frac{(-x+1)^{\frac{1}{4}}\, 2^{\frac{3}{4}}}{2}\right) - 2^{\frac{1}{4}} \ln\left(\frac{(-x+1)^{\frac{1}{4}} + 2^{\frac{1}{4}}}{(-x+1)^{\frac{1}{4}} - 2^{\frac{1}{4}}}\right) + 4(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/4)/(1+x),x)
```

```
[Out] 4*(-x+1)^(1/4)-2*2^(1/4)*arctan(1/2*(-x+1)^(1/4)*2^(3/4))-2^(1/4)*ln(((-x+1
)^(1/4)+2^(1/4))/((-x+1)^(1/4)-2^(1/4)))
```

**maxima [A]** time = 2.91, size = 61, normalized size = 1.05

$$-2 \cdot 2^{\frac{1}{4}} \arctan\left(\frac{1}{2} \cdot 2^{\frac{3}{4}} (-x+1)^{\frac{1}{4}}\right) + 2^{\frac{1}{4}} \log\left(-\frac{2^{\frac{1}{4}} - (-x+1)^{\frac{1}{4}}}{2^{\frac{1}{4}} + (-x+1)^{\frac{1}{4}}}\right) + 4(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="maxima")
```

```
[Out] -2*2^(1/4)*arctan(1/2*2^(3/4)*(-x + 1)^(1/4)) + 2^(1/4)*log(-(2^(1/4) - (-x
 + 1)^(1/4))/(2^(1/4) + (-x + 1)^(1/4))) + 4*(-x + 1)^(1/4)
```

**mupad [B]** time = 0.07, size = 46, normalized size = 0.79

$$4\,(1-x)^{1/4}-2\,2^{1/4}\,\mathrm{atanh}\left(\frac{2^{3/4}\,(1-x)^{1/4}}{2}\right)-2\,2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,(1-x)^{1/4}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/4)/(x + 1),x)
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*atanh((2^(3/4)*(1 - x)^(1/4))/2) - 2*2^(1/4)*at
an((2^(3/4)*(1 - x)^(1/4))/2)
```

**sympy [C]** time = 2.35, size = 243, normalized size = 4.19

$$\frac{5\sqrt[4]{-1}\,\sqrt[4]{x-1}\,\Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{9}{4}\right)}+\frac{5\sqrt[4]{-2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5\,(-1)^{\frac{3}{4}}\sqrt[4]{2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{3i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt[4]{-2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{5i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{5\,(-1)^{\frac{3}{4}}\sqrt[4]{2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{7i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/4)/(1+x),x)
```

```
[Out] 5*(-1)**(1/4)*(x - 1)**(1/4)*gamma(5/4)/gamma(9/4) + 5*(-2)**(1/4)*exp(-I*p
i/4)*log(-2**(3/4)*(x - 1)**(1/4)*exp_polar(I*pi/4)/2 + 1)*gamma(5/4)/(4*ga
mma(9/4)) - 5*(-1)**(3/4)*2**(1/4)*exp(-I*pi/4)*log(-2**(3/4)*(x - 1)**(1/4
)*exp_polar(3*I*pi/4)/2 + 1)*gamma(5/4)/(4*gamma(9/4)) - 5*(-2)**(1/4)*exp(
-I*pi/4)*log(-2**(3/4)*(x - 1)**(1/4)*exp_polar(5*I*pi/4)/2 + 1)*gamma(5/4)
/(4*gamma(9/4)) + 5*(-1)**(3/4)*2**(1/4)*exp(-I*pi/4)*log(-2**(3/4)*(x - 1)
**(1/4)*exp_polar(7*I*pi/4)/2 + 1)*gamma(5/4)/(4*gamma(9/4))
```

## 3.699 $\int x^m(a+bx)^{10}\,dx$

**Optimal**. Leaf size=187

$$\frac{a^{10}x^{m+1}}{m+1}+\frac{10a^9bx^{m+2}}{m+2}+\frac{45a^8b^2x^{m+3}}{m+3}+\frac{120a^7b^3x^{m+4}}{m+4}+\frac{210a^6b^4x^{m+5}}{m+5}+\frac{252a^5b^5x^{m+6}}{m+6}+\frac{210a^4b^6x^{m+7}}{m+7}+\frac{120a^3b^7x^{m+8}}{m+8}+$$

**Rubi** [A] time = 0.08, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{45a^8b^2x^{m+3}}{m+3}+\frac{120a^7b^3x^{m+4}}{m+4}+\frac{210a^6b^4x^{m+5}}{m+5}+\frac{252a^5b^5x^{m+6}}{m+6}+\frac{210a^4b^6x^{m+7}}{m+7}+\frac{120a^3b^7x^{m+8}}{m+8}+\frac{45a^2b^8x^{m+9}}{m+9}+\frac{10a^9bx^{m+2}}{m+2}+\frac{a^{10}x^{m+1}}{m+1}+\frac{10ab^9x^{m+10}}{m+10}+\frac{b^{10}x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^10,x]
```

```
[Out] (a^10*x^(1 + m))/(1 + m) + (10*a^9*b*x^(2 + m))/(2 + m) + (45*a^8*b^2*x^(3
+ m))/(3 + m) + (120*a^7*b^3*x^(4 + m))/(4 + m) + (210*a^6*b^4*x^(5 + m))/(
5 + m) + (252*a^5*b^5*x^(6 + m))/(6 + m) + (210*a^4*b^6*x^(7 + m))/(7 + m)
+ (120*a^3*b^7*x^(8 + m))/(8 + m) + (45*a^2*b^8*x^(9 + m))/(9 + m) + (10*a*
b^9*x^(10 + m))/(10 + m) + (b^10*x^(11 + m))/(11 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^m(a+bx)^{10}\,dx=\int\left(a^{10}x^m+10a^9bx^{1+m}+45a^8b^2x^{2+m}+120a^7b^3x^{3+m}+210a^6b^4x^{4+m}+252a^5b^5x^{5+m}+21\right.$$

$$=\frac{a^{10}x^{1+m}}{1+m}+\frac{10a^9bx^{2+m}}{2+m}+\frac{45a^8b^2x^{3+m}}{3+m}+\frac{120a^7b^3x^{4+m}}{4+m}+\frac{210a^6b^4x^{5+m}}{5+m}+\frac{252a^5b^5x^{6+m}}{6+m}+$$

**Mathematica** [A] time = 0.11, size = 166, normalized size = 0.89

$$x^{m+1}\left(\frac{a^{10}}{m+1}+\frac{10a^9bx}{m+2}+\frac{45a^8b^2x^2}{m+3}+\frac{120a^7b^3x^3}{m+4}+\frac{210a^6b^4x^4}{m+5}+\frac{252a^5b^5x^5}{m+6}+\frac{210a^4b^6x^6}{m+7}+\frac{120a^3b^7x^7}{m+8}+\frac{45a^2b^8x^8}{m+9}+\frac{10ab^9x^9}{m+10}+\frac{b^{10}x^{10}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^10,x]
```

```
[Out] x^(1 + m)*(a^10/(1 + m) + (10*a^9*b*x)/(2 + m) + (45*a^8*b^2*x^2)/(3 + m) +
 (120*a^7*b^3*x^3)/(4 + m) + (210*a^6*b^4*x^4)/(5 + m) + (252*a^5*b^5*x^5)/
(6 + m) + (210*a^4*b^6*x^6)/(7 + m) + (120*a^3*b^7*x^7)/(8 + m) + (45*a^2*b
^8*x^8)/(9 + m) + (10*a*b^9*x^9)/(10 + m) + (b^10*x^10)/(11 + m))
```

**IntegrateAlgebraic [F]** time = 0.18, size = 0, normalized size = 0.00

$$\int x^m(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^10,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^10, x]
```

**fricas [B]** time = 1.17, size = 1277, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] ((b^10*m^10 + 55*b^10*m^9 + 1320*b^10*m^8 + 18150*b^10*m^7 + 157773*b^10*m^
6 + 902055*b^10*m^5 + 3416930*b^10*m^4 + 8409500*b^10*m^3 + 12753576*b^10*m
^2 + 10628640*b^10*m + 3628800*b^10)*x^11 + 10*(a*b^9*m^10 + 56*a*b^9*m^9 +
 1365*a*b^9*m^8 + 19020*a*b^9*m^7 + 167223*a*b^9*m^6 + 965328*a*b^9*m^5 + 3
686255*a*b^9*m^4 + 9133180*a*b^9*m^3 + 13926276*a*b^9*m^2 + 11655216*a*b^9*
m + 3991680*a*b^9)*x^10 + 45*(a^2*b^8*m^10 + 57*a^2*b^8*m^9 + 1412*a^2*b^8*
m^8 + 19962*a^2*b^8*m^7 + 177765*a^2*b^8*m^6 + 1037673*a^2*b^8*m^5 + 400047
8*a^2*b^8*m^4 + 9991428*a^2*b^8*m^3 + 15335224*a^2*b^8*m^2 + 12900960*a^2*b
^8*m + 4435200*a^2*b^8)*x^9 + 120*(a^3*b^7*m^10 + 58*a^3*b^7*m^9 + 1461*a^3
*b^7*m^8 + 20982*a^3*b^7*m^7 + 189567*a^3*b^7*m^6 + 1121022*a^3*b^7*m^5 + 4
371359*a^3*b^7*m^4 + 11024858*a^3*b^7*m^3 + 17059212*a^3*b^7*m^2 + 14444280
*a^3*b^7*m + 4989600*a^3*b^7)*x^8 + 210*(a^4*b^6*m^10 + 59*a^4*b^6*m^9 + 15
12*a^4*b^6*m^8 + 22086*a^4*b^6*m^7 + 202821*a^4*b^6*m^6 + 1217811*a^4*b^6*m
^5 + 4814858*a^4*b^6*m^4 + 12291724*a^4*b^6*m^3 + 19216008*a^4*b^6*m^2 + 16
405920*a^4*b^6*m + 5702400*a^4*b^6)*x^7 + 252*(a^5*b^5*m^10 + 60*a^5*b^5*m^
9 + 1565*a^5*b^5*m^8 + 23280*a^5*b^5*m^7 + 217743*a^5*b^5*m^6 + 1331100*a^5
*b^5*m^5 + 5352935*a^5*b^5*m^4 + 13878120*a^5*b^5*m^3 + 21989356*a^5*b^5*m^
2 + 18981840*a^5*b^5*m + 6652800*a^5*b^5)*x^6 + 210*(a^6*b^4*m^10 + 61*a^6*
b^4*m^9 + 1620*a^6*b^4*m^8 + 24570*a^6*b^4*m^7 + 234573*a^6*b^4*m^6 + 14646
93*a^6*b^4*m^5 + 6016070*a^6*b^4*m^4 + 15915380*a^6*b^4*m^3 + 25681176*a^6*
b^4*m^2 + 22512096*a^6*b^4*m + 7983360*a^6*b^4)*x^5 + 120*(a^7*b^3*m^10 + 6
2*a^7*b^3*m^9 + 1677*a^7*b^3*m^8 + 25962*a^7*b^3*m^7 + 253575*a^7*b^3*m^6 +
```

```
 1623258*a^7*b^3*m^5 + 6846503*a^7*b^3*m^4 + 18609718*a^7*b^3*m^3 + 3081920
4*a^7*b^3*m^2 + 27641160*a^7*b^3*m + 9979200*a^7*b^3)*x^4 + 45*(a^8*b^2*m^1
0 + 63*a^8*b^2*m^9 + 1736*a^8*b^2*m^8 + 27462*a^8*b^2*m^7 + 275037*a^8*b^2*
m^6 + 1812447*a^8*b^2*m^5 + 7902194*a^8*b^2*m^4 + 22289148*a^8*b^2*m^3 + 38
390632*a^8*b^2*m^2 + 35746080*a^8*b^2*m + 13305600*a^8*b^2)*x^3 + 10*(a^9*b
*m^10 + 64*a^9*b*m^9 + 1797*a^9*b*m^8 + 29076*a^9*b*m^7 + 299271*a^9*b*m^6
+ 2039016*a^9*b*m^5 + 9261503*a^9*b*m^4 + 27472724*a^9*b*m^3 + 50312628*a^9
*b*m^2 + 50292720*a^9*b*m + 19958400*a^9*b)*x^2 + (a^10*m^10 + 65*a^10*m^9
+ 1860*a^10*m^8 + 30810*a^10*m^7 + 326613*a^10*m^6 + 2310945*a^10*m^5 + 110
28590*a^10*m^4 + 34967140*a^10*m^3 + 70290936*a^10*m^2 + 80627040*a^10*m +
39916800*a^10)*x)*x^m/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 +
 2637558*m^6 + 13339535*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2
+ 120543840*m + 39916800)
```

**giac [B]** time = 1.20, size = 1925, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] (b^10*m^10*x^11*x^m + 10*a*b^9*m^10*x^10*x^m + 55*b^10*m^9*x^11*x^m + 45*a^
2*b^8*m^10*x^9*x^m + 560*a*b^9*m^9*x^10*x^m + 1320*b^10*m^8*x^11*x^m + 120*
a^3*b^7*m^10*x^8*x^m + 2565*a^2*b^8*m^9*x^9*x^m + 13650*a*b^9*m^8*x^10*x^m
+ 18150*b^10*m^7*x^11*x^m + 210*a^4*b^6*m^10*x^7*x^m + 6960*a^3*b^7*m^9*x^8
*x^m + 63540*a^2*b^8*m^8*x^9*x^m + 190200*a*b^9*m^7*x^10*x^m + 157773*b^10*
m^6*x^11*x^m + 252*a^5*b^5*m^10*x^6*x^m + 12390*a^4*b^6*m^9*x^7*x^m + 17532
0*a^3*b^7*m^8*x^8*x^m + 898290*a^2*b^8*m^7*x^9*x^m + 1672230*a*b^9*m^6*x^10
*x^m + 902055*b^10*m^5*x^11*x^m + 210*a^6*b^4*m^10*x^5*x^m + 15120*a^5*b^5*
m^9*x^6*x^m + 317520*a^4*b^6*m^8*x^7*x^m + 2517840*a^3*b^7*m^7*x^8*x^m + 79
99425*a^2*b^8*m^6*x^9*x^m + 9653280*a*b^9*m^5*x^10*x^m + 3416930*b^10*m^4*x
^11*x^m + 120*a^7*b^3*m^10*x^4*x^m + 12810*a^6*b^4*m^9*x^5*x^m + 394380*a^5
*b^5*m^8*x^6*x^m + 4638060*a^4*b^6*m^7*x^7*x^m + 22748040*a^3*b^7*m^6*x^8*x
^m + 46695285*a^2*b^8*m^5*x^9*x^m + 36862550*a*b^9*m^4*x^10*x^m + 8409500*b
^10*m^3*x^11*x^m + 45*a^8*b^2*m^10*x^3*x^m + 7440*a^7*b^3*m^9*x^4*x^m + 340
200*a^6*b^4*m^8*x^5*x^m + 5866560*a^5*b^5*m^7*x^6*x^m + 42592410*a^4*b^6*m^
6*x^7*x^m + 134522640*a^3*b^7*m^5*x^8*x^m + 180021510*a^2*b^8*m^4*x^9*x^m +
 91331800*a*b^9*m^3*x^10*x^m + 12753576*b^10*m^2*x^11*x^m + 10*a^9*b*m^10*x
^2*x^m + 2835*a^8*b^2*m^9*x^3*x^m + 201240*a^7*b^3*m^8*x^4*x^m + 5159700*a^
6*b^4*m^7*x^5*x^m + 54871236*a^5*b^5*m^6*x^6*x^m + 255740310*a^4*b^6*m^5*x^
7*x^m + 524563080*a^3*b^7*m^4*x^8*x^m + 449614260*a^2*b^8*m^3*x^9*x^m + 139
262760*a*b^9*m^2*x^10*x^m + 10628640*b^10*m*x^11*x^m + a^10*m^10*x*x^m + 64
0*a^9*b*m^9*x^2*x^m + 78120*a^8*b^2*m^8*x^3*x^m + 3115440*a^7*b^3*m^7*x^4*x
^m + 49260330*a^6*b^4*m^6*x^5*x^m + 335437200*a^5*b^5*m^5*x^6*x^m + 1011120
180*a^4*b^6*m^4*x^7*x^m + 1322982960*a^3*b^7*m^3*x^8*x^m + 690085080*a^2*b^
```

```
8*m^2*x^9*x^m + 116552160*a*b^9*m*x^10*x^m + 3628800*b^10*x^11*x^m + 65*a^1
0*m^9*x*x^m + 17970*a^9*b*m^8*x^2*x^m + 1235790*a^8*b^2*m^7*x^3*x^m + 30429
000*a^7*b^3*m^6*x^4*x^m + 307585530*a^6*b^4*m^5*x^5*x^m + 1348939620*a^5*b^
5*m^4*x^6*x^m + 2581262040*a^4*b^6*m^3*x^7*x^m + 2047105440*a^3*b^7*m^2*x^8
*x^m + 580543200*a^2*b^8*m*x^9*x^m + 39916800*a*b^9*x^10*x^m + 1860*a^10*m^
8*x*x^m + 290760*a^9*b*m^7*x^2*x^m + 12376665*a^8*b^2*m^6*x^3*x^m + 1947909
60*a^7*b^3*m^5*x^4*x^m + 1263374700*a^6*b^4*m^4*x^5*x^m + 3497286240*a^5*b^
5*m^3*x^6*x^m + 4035361680*a^4*b^6*m^2*x^7*x^m + 1733313600*a^3*b^7*m*x^8*x
^m + 199584000*a^2*b^8*x^9*x^m + 30810*a^10*m^7*x*x^m + 2992710*a^9*b*m^6*x
^2*x^m + 81560115*a^8*b^2*m^5*x^3*x^m + 821580360*a^7*b^3*m^4*x^4*x^m + 334
2229800*a^6*b^4*m^3*x^5*x^m + 5541317712*a^5*b^5*m^2*x^6*x^m + 3445243200*a
^4*b^6*m*x^7*x^m + 598752000*a^3*b^7*x^8*x^m + 326613*a^10*m^6*x*x^m + 2039
0160*a^9*b*m^5*x^2*x^m + 355598730*a^8*b^2*m^4*x^3*x^m + 2233166160*a^7*b^3
*m^3*x^4*x^m + 5393046960*a^6*b^4*m^2*x^5*x^m + 4783423680*a^5*b^5*m*x^6*x^
m + 1197504000*a^4*b^6*x^7*x^m + 2310945*a^10*m^5*x*x^m + 92615030*a^9*b*m^
4*x^2*x^m + 1003011660*a^8*b^2*m^3*x^3*x^m + 3698304480*a^7*b^3*m^2*x^4*x^m
 + 4727540160*a^6*b^4*m*x^5*x^m + 1676505600*a^5*b^5*x^6*x^m + 11028590*a^1
0*m^4*x*x^m + 274727240*a^9*b*m^3*x^2*x^m + 1727578440*a^8*b^2*m^2*x^3*x^m
+ 3316939200*a^7*b^3*m*x^4*x^m + 1676505600*a^6*b^4*x^5*x^m + 34967140*a^10
*m^3*x*x^m + 503126280*a^9*b*m^2*x^2*x^m + 1608573600*a^8*b^2*m*x^3*x^m + 1
197504000*a^7*b^3*x^4*x^m + 70290936*a^10*m^2*x*x^m + 502927200*a^9*b*m*x^2
*x^m + 598752000*a^8*b^2*x^3*x^m + 80627040*a^10*m*x*x^m + 199584000*a^9*b*
x^2*x^m + 39916800*a^10*x*x^m)/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357
423*m^7 + 2637558*m^6 + 13339535*m^5 + 45995730*m^4 + 105258076*m^3 + 15091
7976*m^2 + 120543840*m + 39916800)
```

**maple [B]** time = 0.01, size = 1535, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^10,x)
```

```
[Out] x^(m+1)*(b^10*m^10*x^10+10*a*b^9*m^10*x^9+55*b^10*m^9*x^10+45*a^2*b^8*m^10*
x^8+560*a*b^9*m^9*x^9+1320*b^10*m^8*x^10+120*a^3*b^7*m^10*x^7+2565*a^2*b^8*
m^9*x^8+13650*a*b^9*m^8*x^9+18150*b^10*m^7*x^10+210*a^4*b^6*m^10*x^6+6960*a
^3*b^7*m^9*x^7+63540*a^2*b^8*m^8*x^8+190200*a*b^9*m^7*x^9+157773*b^10*m^6*x
^10+252*a^5*b^5*m^10*x^5+12390*a^4*b^6*m^9*x^6+175320*a^3*b^7*m^8*x^7+89829
0*a^2*b^8*m^7*x^8+1672230*a*b^9*m^6*x^9+902055*b^10*m^5*x^10+210*a^6*b^4*m^
10*x^4+15120*a^5*b^5*m^9*x^5+317520*a^4*b^6*m^8*x^6+2517840*a^3*b^7*m^7*x^7
+7999425*a^2*b^8*m^6*x^8+9653280*a*b^9*m^5*x^9+3416930*b^10*m^4*x^10+120*a^
7*b^3*m^10*x^3+12810*a^6*b^4*m^9*x^4+394380*a^5*b^5*m^8*x^5+4638060*a^4*b^6
*m^7*x^6+22748040*a^3*b^7*m^6*x^7+46695285*a^2*b^8*m^5*x^8+36862550*a*b^9*m
^4*x^9+8409500*b^10*m^3*x^10+45*a^8*b^2*m^10*x^2+7440*a^7*b^3*m^9*x^3+34020
0*a^6*b^4*m^8*x^4+5866560*a^5*b^5*m^7*x^5+42592410*a^4*b^6*m^6*x^6+13452264
```

```
0*a^3*b^7*m^5*x^7+180021510*a^2*b^8*m^4*x^8+91331800*a*b^9*m^3*x^9+12753576
*b^10*m^2*x^10+10*a^9*b*m^10*x+2835*a^8*b^2*m^9*x^2+201240*a^7*b^3*m^8*x^3+
5159700*a^6*b^4*m^7*x^4+54871236*a^5*b^5*m^6*x^5+255740310*a^4*b^6*m^5*x^6+
524563080*a^3*b^7*m^4*x^7+449614260*a^2*b^8*m^3*x^8+139262760*a*b^9*m^2*x^9
+10628640*b^10*m*x^10+a^10*m^10+640*a^9*b*m^9*x+78120*a^8*b^2*m^8*x^2+31154
40*a^7*b^3*m^7*x^3+49260330*a^6*b^4*m^6*x^4+335437200*a^5*b^5*m^5*x^5+10111
20180*a^4*b^6*m^4*x^6+1322982960*a^3*b^7*m^3*x^7+690085080*a^2*b^8*m^2*x^8+
116552160*a*b^9*m*x^9+3628800*b^10*x^10+65*a^10*m^9+17970*a^9*b*m^8*x+12357
90*a^8*b^2*m^7*x^2+30429000*a^7*b^3*m^6*x^3+307585530*a^6*b^4*m^5*x^4+13489
39620*a^5*b^5*m^4*x^5+2581262040*a^4*b^6*m^3*x^6+2047105440*a^3*b^7*m^2*x^7
+580543200*a^2*b^8*m*x^8+39916800*a*b^9*x^9+1860*a^10*m^8+290760*a^9*b*m^7*
x+12376665*a^8*b^2*m^6*x^2+194790960*a^7*b^3*m^5*x^3+1263374700*a^6*b^4*m^4
*x^4+3497286240*a^5*b^5*m^3*x^5+4035361680*a^4*b^6*m^2*x^6+1733313600*a^3*b
^7*m*x^7+199584000*a^2*b^8*x^8+30810*a^10*m^7+2992710*a^9*b*m^6*x+81560115*
a^8*b^2*m^5*x^2+821580360*a^7*b^3*m^4*x^3+3342229800*a^6*b^4*m^3*x^4+554131
7712*a^5*b^5*m^2*x^5+3445243200*a^4*b^6*m*x^6+598752000*a^3*b^7*x^7+326613*
a^10*m^6+20390160*a^9*b*m^5*x+355598730*a^8*b^2*m^4*x^2+2233166160*a^7*b^3*
m^3*x^3+5393046960*a^6*b^4*m^2*x^4+4783423680*a^5*b^5*m*x^5+1197504000*a^4*
b^6*x^6+2310945*a^10*m^5+92615030*a^9*b*m^4*x+1003011660*a^8*b^2*m^3*x^2+36
98304480*a^7*b^3*m^2*x^3+4727540160*a^6*b^4*m*x^4+1676505600*a^5*b^5*x^5+11
028590*a^10*m^4+274727240*a^9*b*m^3*x+1727578440*a^8*b^2*m^2*x^2+3316939200
*a^7*b^3*m*x^3+1676505600*a^6*b^4*x^4+34967140*a^10*m^3+503126280*a^9*b*m^2
*x+1608573600*a^8*b^2*m*x^2+1197504000*a^7*b^3*x^3+70290936*a^10*m^2+502927
200*a^9*b*m*x+598752000*a^8*b^2*x^2+80627040*a^10*m+199584000*a^9*b*x+39916
800*a^10)/(11+m)/(10+m)/(9+m)/(8+m)/(7+m)/(6+m)/(5+m)/(m+4)/(m+3)/(m+2)/(m+
1)
```

**maxima** [A] time = 1.39, size = 187, normalized size = 1.00

$$\frac{b^{10}x^{m+11}}{m+11}+\frac{10\,ab^{9}x^{m+10}}{m+10}+\frac{45\,a^{2}b^{8}x^{m+9}}{m+9}+\frac{120\,a^{3}b^{7}x^{m+8}}{m+8}+\frac{210\,a^{4}b^{6}x^{m+7}}{m+7}+\frac{252\,a^{5}b^{5}x^{m+6}}{m+6}+\frac{210\,a^{6}b^{4}x^{m+5}}{m+5}+\frac{120\,a^{7}b^{3}x^{m+4}}{m+4}+\frac{45\,a^{8}b^{2}x^{m+3}}{m+3}+\frac{10\,a^{9}bx^{m+2}}{m+2}+\frac{a^{10}x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] b^10*x^(m + 11)/(m + 11) + 10*a*b^9*x^(m + 10)/(m + 10) + 45*a^2*b^8*x^(m +
 9)/(m + 9) + 120*a^3*b^7*x^(m + 8)/(m + 8) + 210*a^4*b^6*x^(m + 7)/(m + 7)
 + 252*a^5*b^5*x^(m + 6)/(m + 6) + 210*a^6*b^4*x^(m + 5)/(m + 5) + 120*a^7*
b^3*x^(m + 4)/(m + 4) + 45*a^8*b^2*x^(m + 3)/(m + 3) + 10*a^9*b*x^(m + 2)/(
m + 2) + a^10*x^(m + 1)/(m + 1)
```

**mupad** [B] time = 1.37, size = 1274, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^10,x)

[Out] (a^10*x*x^m*(80627040*m + 70290936*m^2 + 34967140*m^3 + 11028590*m^4 + 2310
945*m^5 + 326613*m^6 + 30810*m^7 + 1860*m^8 + 65*m^9 + m^10 + 39916800))/(1
20543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 +
2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800
) + (b^10*x^m*x^11*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^4 +
 902055*m^5 + 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 3628800))
/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5
 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916
800) + (45*a^2*b^8*x^m*x^9*(12900960*m + 15335224*m^2 + 9991428*m^3 + 40004
78*m^4 + 1037673*m^5 + 177765*m^6 + 19962*m^7 + 1412*m^8 + 57*m^9 + m^10 +
4435200))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 133
39535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^1
1 + 39916800) + (120*a^3*b^7*x^m*x^8*(14444280*m + 17059212*m^2 + 11024858*
m^3 + 4371359*m^4 + 1121022*m^5 + 189567*m^6 + 20982*m^7 + 1461*m^8 + 58*m^
9 + m^10 + 4989600))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 4599573
0*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66
*m^10 + m^11 + 39916800) + (210*a^4*b^6*x^m*x^7*(16405920*m + 19216008*m^2
+ 12291724*m^3 + 4814858*m^4 + 1217811*m^5 + 202821*m^6 + 22086*m^7 + 1512*
m^8 + 59*m^9 + m^10 + 5702400))/(120543840*m + 150917976*m^2 + 105258076*m^
3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 19
25*m^9 + 66*m^10 + m^11 + 39916800) + (252*a^5*b^5*x^m*x^6*(18981840*m + 21
989356*m^2 + 13878120*m^3 + 5352935*m^4 + 1331100*m^5 + 217743*m^6 + 23280*
m^7 + 1565*m^8 + 60*m^9 + m^10 + 6652800))/(120543840*m + 150917976*m^2 + 1
05258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 326
70*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (210*a^6*b^4*x^m*x^5*(2251
2096*m + 25681176*m^2 + 15915380*m^3 + 6016070*m^4 + 1464693*m^5 + 234573*m
^6 + 24570*m^7 + 1620*m^8 + 61*m^9 + m^10 + 7983360))/(120543840*m + 150917
976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 35742
3*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (120*a^7*b^3*x^
m*x^4*(27641160*m + 30819204*m^2 + 18609718*m^3 + 6846503*m^4 + 1623258*m^5
 + 253575*m^6 + 25962*m^7 + 1677*m^8 + 62*m^9 + m^10 + 9979200))/(120543840
*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*
m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (45*
a^8*b^2*x^m*x^3*(35746080*m + 38390632*m^2 + 22289148*m^3 + 7902194*m^4 + 1
812447*m^5 + 275037*m^6 + 27462*m^7 + 1736*m^8 + 63*m^9 + m^10 + 13305600))
/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5
 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916
800) + (10*a*b^9*x^m*x^10*(11655216*m + 13926276*m^2 + 9133180*m^3 + 368625
5*m^4 + 965328*m^5 + 167223*m^6 + 19020*m^7 + 1365*m^8 + 56*m^9 + m^10 + 39
91680))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339
535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11
+ 39916800) + (10*a^9*b*x^m*x^2*(50292720*m + 50312628*m^2 + 27472724*m^3 +
 9261503*m^4 + 2039016*m^5 + 299271*m^6 + 29076*m^7 + 1797*m^8 + 64*m^9 + m
```

```
^10 + 19958400))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^
4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^1
0 + m^11 + 39916800)
```

**sympy [A]** time = 6.93, size = 9996, normalized size = 53.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**10,x)
```

```
[Out] Piecewise((-a**10/(10*x**10) - 10*a**9*b/(9*x**9) - 45*a**8*b**2/(8*x**8) -
 120*a**7*b**3/(7*x**7) - 35*a**6*b**4/x**6 - 252*a**5*b**5/(5*x**5) - 105*
a**4*b**6/(2*x**4) - 40*a**3*b**7/x**3 - 45*a**2*b**8/(2*x**2) - 10*a*b**9/
x + b**10*log(x), Eq(m, -11)), (-a**10/(9*x**9) - 5*a**9*b/(4*x**8) - 45*a*
*8*b**2/(7*x**7) - 20*a**7*b**3/x**6 - 42*a**6*b**4/x**5 - 63*a**5*b**5/x**
4 - 70*a**4*b**6/x**3 - 60*a**3*b**7/x**2 - 45*a**2*b**8/x + 10*a*b**9*log(
x) + b**10*x, Eq(m, -10)), (-a**10/(8*x**8) - 10*a**9*b/(7*x**7) - 15*a**8*
b**2/(2*x**6) - 24*a**7*b**3/x**5 - 105*a**6*b**4/(2*x**4) - 84*a**5*b**5/x
**3 - 105*a**4*b**6/x**2 - 120*a**3*b**7/x + 45*a**2*b**8*log(x) + 10*a*b**
9*x + b**10*x**2/2, Eq(m, -9)), (-a**10/(7*x**7) - 5*a**9*b/(3*x**6) - 9*a*
*8*b**2/x**5 - 30*a**7*b**3/x**4 - 70*a**6*b**4/x**3 - 126*a**5*b**5/x**2 -
 210*a**4*b**6/x + 120*a**3*b**7*log(x) + 45*a**2*b**8*x + 5*a*b**9*x**2 +
b**10*x**3/3, Eq(m, -8)), (-a**10/(6*x**6) - 2*a**9*b/x**5 - 45*a**8*b**2/(
4*x**4) - 40*a**7*b**3/x**3 - 105*a**6*b**4/x**2 - 252*a**5*b**5/x + 210*a*
*4*b**6*log(x) + 120*a**3*b**7*x + 45*a**2*b**8*x**2/2 + 10*a*b**9*x**3/3 +
 b**10*x**4/4, Eq(m, -7)), (-a**10/(5*x**5) - 5*a**9*b/(2*x**4) - 15*a**8*b
**2/x**3 - 60*a**7*b**3/x**2 - 210*a**6*b**4/x + 252*a**5*b**5*log(x) + 210
*a**4*b**6*x + 60*a**3*b**7*x**2 + 15*a**2*b**8*x**3 + 5*a*b**9*x**4/2 + b*
*10*x**5/5, Eq(m, -6)), (-a**10/(4*x**4) - 10*a**9*b/(3*x**3) - 45*a**8*b**
2/(2*x**2) - 120*a**7*b**3/x + 210*a**6*b**4*log(x) + 252*a**5*b**5*x + 105
*a**4*b**6*x**2 + 40*a**3*b**7*x**3 + 45*a**2*b**8*x**4/4 + 2*a*b**9*x**5 +
 b**10*x**6/6, Eq(m, -5)), (-a**10/(3*x**3) - 5*a**9*b/x**2 - 45*a**8*b**2/
x + 120*a**7*b**3*log(x) + 210*a**6*b**4*x + 126*a**5*b**5*x**2 + 70*a**4*b
**6*x**3 + 30*a**3*b**7*x**4 + 9*a**2*b**8*x**5 + 5*a*b**9*x**6/3 + b**10*x
**7/7, Eq(m, -4)), (-a**10/(2*x**2) - 10*a**9*b/x + 45*a**8*b**2*log(x) + 1
20*a**7*b**3*x + 105*a**6*b**4*x**2 + 84*a**5*b**5*x**3 + 105*a**4*b**6*x**
4/2 + 24*a**3*b**7*x**5 + 15*a**2*b**8*x**6/2 + 10*a*b**9*x**7/7 + b**10*x*
*8/8, Eq(m, -3)), (-a**10/x + 10*a**9*b*log(x) + 45*a**8*b**2*x + 60*a**7*b
**3*x**2 + 70*a**6*b**4*x**3 + 63*a**5*b**5*x**4 + 42*a**4*b**6*x**5 + 20*a
**3*b**7*x**6 + 45*a**2*b**8*x**7/7 + 5*a*b**9*x**8/4 + b**10*x**9/9, Eq(m,
 -2)), (a**10*log(x) + 10*a**9*b*x + 45*a**8*b**2*x**2/2 + 40*a**7*b**3*x**
3 + 105*a**6*b**4*x**4/2 + 252*a**5*b**5*x**5/5 + 35*a**4*b**6*x**6 + 120*a
**3*b**7*x**7/7 + 45*a**2*b**8*x**8/8 + 10*a*b**9*x**9/9 + b**10*x**10/10,
Eq(m, -1)), (a**10*m**10*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
```

+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 65*a**10*m**9*x*x**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 1860*a**10*m**8*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m
**3 + 150917976*m**2 + 120543840*m + 39916800) + 30810*a**10*m**7*x*x**m/(m
**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 326613*a**10*m**6*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 2310945*a**10*m**5*x
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 11028590*a**10*m**4*x*x**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 34967140*a
**10*m**3*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 70290936*a**10*m**2*x*x**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
 80627040*a**10*m*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 35742
3*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15
0917976*m**2 + 120543840*m + 39916800) + 39916800*a**10*x*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 10*a**9*b*m**10*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
 + 150917976*m**2 + 120543840*m + 39916800) + 640*a**9*b*m**9*x**2*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 17970*a**9*b*m**8*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32
670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105
258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 290760*a**9*b*m**7
*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 2992710*a**9*b*m**6*x**2*x**m/(m**11 + 66*m**10
+ 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459
95730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 20
390160*a**9*b*m**5*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 3
57423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
+ 150917976*m**2 + 120543840*m + 39916800) + 92615030*a**9*b*m**4*x**2*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840

*m + 39916800) + 274727240*a**9*b*m**3*x**2*x**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 503126280*
a**9*b*m**2*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m
**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091
7976*m**2 + 120543840*m + 39916800) + 502927200*a**9*b*m*x**2*x**m/(m**11 +
 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 199584000*a**9*b*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m*
*8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076
*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 45*a**8*b**2*m**10*x**3*
x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**
6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12054
3840*m + 39916800) + 2835*a**8*b**2*m**9*x**3*x**m/(m**11 + 66*m**10 + 1925
*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*
m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 78120*a*
*8*b**2*m**8*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*
m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1509
17976*m**2 + 120543840*m + 39916800) + 1235790*a**8*b**2*m**7*x**3*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 12376665*a**8*b**2*m**6*x**3*x**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 81560115*a**
8*b**2*m**5*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m
**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091
7976*m**2 + 120543840*m + 39916800) + 355598730*a**8*b**2*m**4*x**3*x**m/(m
**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 1003011660*a**8*b**2*m**3*x**3*x**m/(m**11 + 66*m**10 + 1925*
m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m
**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 172757844
0*a**8*b**2*m**2*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 1608573600*a**8*b**2*m*x**3*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 598752000*a**8*b**2*x**3*x**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 120*a**7*b**
3*m**10*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 7440*a**7*b**3*m**9*x**4*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680

0) + 201240*a**7*b**3*m**8*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3115440*a**7*b**3*m**7
*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 30429000*a**7*b**3*m**6*x**4*x**m/(m**11 + 66*m*
*10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
 + 194790960*a**7*b**3*m**5*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 821580360*a**7*b**3*m*
*4*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
 + 120543840*m + 39916800) + 2233166160*a**7*b**3*m**3*x**4*x**m/(m**11 + 6
6*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m*
*5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399168
00) + 3698304480*a**7*b**3*m**2*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3316939200*a**7*b
**3*m*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
**2 + 120543840*m + 39916800) + 1197504000*a**7*b**3*x**4*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 210*a**6*b**4*m**10*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12810*a**6*b**4*m**9*x**5
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 340200*a**6*b**4*m**8*x**5*x**m/(m**11 + 66*m**10 + 1
925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459957
30*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 51597
00*a**6*b**4*m**7*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 35
7423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
 150917976*m**2 + 120543840*m + 39916800) + 49260330*a**6*b**4*m**6*x**5*x*
*m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6
 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205438
40*m + 39916800) + 307585530*a**6*b**4*m**5*x**5*x**m/(m**11 + 66*m**10 + 1
925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459957
30*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12633
74700*a**6*b**4*m**4*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**
3 + 150917976*m**2 + 120543840*m + 39916800) + 3342229800*a**6*b**4*m**3*x*
*5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12

0543840*m + 39916800) + 5393046960*a**6*b**4*m**2*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 4727540160*a**6*b**4*m*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1676505600*a**6*b**4*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 252*a**5*b**5*m**10*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 15120*a**5*b**5*m**9*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 394380*a**5*b**5*m**8*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 5866560*a**5*b**5*m**7*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 54871236*a**5*b**5*m**6*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 335437200*a**5*b**5*m**5*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1348939620*a**5*b**5*m**4*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3497286240*a**5*b**5*m**3*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 5541317712*a**5*b**5*m**2*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 4783423680*a**5*b**5*m*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1676505600*a**5*b**5*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 210*a**4*b**6*m**10*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12390*a**4*b**6*m**9*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 317520*a**4*b**6*m**8*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m*

*2 + 120543840*m + 39916800) + 4638060*a**4*b**6*m**7*x**7*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 42592410*a**4*b**6*m**6*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 255740310*a**4*b**6*
m**5*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2
637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m*
*2 + 120543840*m + 39916800) + 1011120180*a**4*b**6*m**4*x**7*x**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 2581262040*a**4*b**6*m**3*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 +
32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 4035361680*a**4
*b**6*m**2*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m*
*7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917
976*m**2 + 120543840*m + 39916800) + 3445243200*a**4*b**6*m*x**7*x**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 1197504000*a**4*b**6*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 120*a**3*b**7*m**
10*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
+ 120543840*m + 39916800) + 6960*a**3*b**7*m**9*x**8*x**m/(m**11 + 66*m**1
0 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4
5995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
175320*a**3*b**7*m**8*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 2517840*a**3*b**7*m**7*x**8
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 22748040*a**3*b**7*m**6*x**8*x**m/(m**11 + 66*m**10 +
1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4599
5730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 134
522640*a**3*b**7*m**5*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 524563080*a**3*b**7*m**4*x*
*8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 1322982960*a**3*b**7*m**3*x**8*x**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
2047105440*a**3*b**7*m**2*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580

76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1733313600*a**3*b**7*m
*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 598752000*a**3*b**7*x**8*x**m/(m**11 + 66*m**10
+ 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459
95730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 45
*a**2*b**8*m**10*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 2565*a**2*b**8*m**9*x**9*x**m/(m
**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 63540*a**2*b**8*m**8*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 898290*a**2*b*
*8*m**7*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 7999425*a**2*b**8*m**6*x**9*x**m/(m**11 +
 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 46695285*a**2*b**8*m**5*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 180021510*a**2*b*
*8*m**4*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 449614260*a**2*b**8*m**3*x**9*x**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 690085080*a**2*b**8*m**2*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 580543200*a**2
*b**8*m*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 199584000*a**2*b**8*x**9*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 10*a*b**9*m**10*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 560*a*b**9*m**9*x**10*x**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800) + 13650*a*b**9*m**8*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 190200*a*b**9*
m**7*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m

```
**2 + 120543840*m + 39916800) + 1672230*a*b**9*m**6*x**10*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 9653280*a*b**9*m**5*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m*
*8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076
*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 36862550*a*b**9*m**4*x**
10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 91331800*a*b**9*m**3*x**10*x**m/(m**11 + 66*m**10 +
 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4599
5730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 139
262760*a*b**9*m**2*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
 + 150917976*m**2 + 120543840*m + 39916800) + 116552160*a*b**9*m*x**10*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 39916800*a*b**9*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 +
 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + b**10*m**10*x**
11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 55*b**10*m**9*x**11*x**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1320*b**10
*m**8*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 18150*b**10*m**7*x**11*x**m/(m**11 + 66*m*
*10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
+ 157773*b**10*m**6*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**
3 + 150917976*m**2 + 120543840*m + 39916800) + 902055*b**10*m**5*x**11*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 3416930*b**10*m**4*x**11*x**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 8409500*b**1
0*m**3*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 12753576*b**10*m**2*x**11*x**m/(m**11 + 6
6*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m*
*5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399168
00) + 10628640*b**10*m*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3628800*b**10*x**11*x**m/
```

```
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800), True))
```

## 3.700 $\int x^m (a+bx)^7 \, dx$

**Optimal**. Leaf size=133

$$\frac{a^7 x^{m+1}}{m+1}+\frac{7 a^6 b x^{m+2}}{m+2}+\frac{21 a^5 b^2 x^{m+3}}{m+3}+\frac{35 a^4 b^3 x^{m+4}}{m+4}+\frac{35 a^3 b^4 x^{m+5}}{m+5}+\frac{21 a^2 b^5 x^{m+6}}{m+6}+\frac{7 a b^6 x^{m+7}}{m+7}+\frac{b^7 x^{m+8}}{m+8}$$

**Rubi [A]** time = 0.05, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21 a^5 b^2 x^{m+3}}{m+3}+\frac{35 a^4 b^3 x^{m+4}}{m+4}+\frac{35 a^3 b^4 x^{m+5}}{m+5}+\frac{21 a^2 b^5 x^{m+6}}{m+6}+\frac{7 a^6 b x^{m+2}}{m+2}+\frac{a^7 x^{m+1}}{m+1}+\frac{7 a b^6 x^{m+7}}{m+7}+\frac{b^7 x^{m+8}}{m+8}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^7,x]
```

```
[Out] (a^7*x^(1 + m))/(1 + m) + (7*a^6*b*x^(2 + m))/(2 + m) + (21*a^5*b^2*x^(3 +
m))/(3 + m) + (35*a^4*b^3*x^(4 + m))/(4 + m) + (35*a^3*b^4*x^(5 + m))/(5 +
m) + (21*a^2*b^5*x^(6 + m))/(6 + m) + (7*a*b^6*x^(7 + m))/(7 + m) + (b^7*x^
(8 + m))/(8 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m (a+bx)^7 \, dx &= \int \left(a^7 x^m + 7a^6 b x^{1+m} + 21 a^5 b^2 x^{2+m} + 35 a^4 b^3 x^{3+m} + 35 a^3 b^4 x^{4+m} + 21 a^2 b^5 x^{5+m} + 7 a b^6 x^{6+m}\right.\\ &= \frac{a^7 x^{1+m}}{1+m}+\frac{7 a^6 b x^{2+m}}{2+m}+\frac{21 a^5 b^2 x^{3+m}}{3+m}+\frac{35 a^4 b^3 x^{4+m}}{4+m}+\frac{35 a^3 b^4 x^{5+m}}{5+m}+\frac{21 a^2 b^5 x^{6+m}}{6+m}+\frac{7 a b^6 x^{7+m}}{7+m}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 118, normalized size = 0.89

$$x^{m+1} \left(\frac{a^7}{m+1}+\frac{7 a^6 b x}{m+2}+\frac{21 a^5 b^2 x^2}{m+3}+\frac{35 a^4 b^3 x^3}{m+4}+\frac{35 a^3 b^4 x^4}{m+5}+\frac{21 a^2 b^5 x^5}{m+6}+\frac{7 a b^6 x^6}{m+7}+\frac{b^7 x^7}{m+8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^7,x]
```

```
[Out] x^(1 + m)*(a^7/(1 + m) + (7*a^6*b*x)/(2 + m) + (21*a^5*b^2*x^2)/(3 + m) + (
35*a^4*b^3*x^3)/(4 + m) + (35*a^3*b^4*x^4)/(5 + m) + (21*a^2*b^5*x^5)/(6 +
m) + (7*a*b^6*x^6)/(7 + m) + (b^7*x^7)/(8 + m))
```

**IntegrateAlgebraic [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int x^m(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^7,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^7, x]
```

**fricas [B]** time = 1.35, size = 665, normalized size = 5.00

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] ((b^7*m^7 + 28*b^7*m^6 + 322*b^7*m^5 + 1960*b^7*m^4 + 6769*b^7*m^3 + 13132*
b^7*m^2 + 13068*b^7*m + 5040*b^7)*x^8 + 7*(a*b^6*m^7 + 29*a*b^6*m^6 + 343*a
*b^6*m^5 + 2135*a*b^6*m^4 + 7504*a*b^6*m^3 + 14756*a*b^6*m^2 + 14832*a*b^6*
m + 5760*a*b^6)*x^7 + 21*(a^2*b^5*m^7 + 30*a^2*b^5*m^6 + 366*a^2*b^5*m^5 +
2340*a^2*b^5*m^4 + 8409*a^2*b^5*m^3 + 16830*a^2*b^5*m^2 + 17144*a^2*b^5*m +
 6720*a^2*b^5)*x^6 + 35*(a^3*b^4*m^7 + 31*a^3*b^4*m^6 + 391*a^3*b^4*m^5 + 2
581*a^3*b^4*m^4 + 9544*a^3*b^4*m^3 + 19564*a^3*b^4*m^2 + 20304*a^3*b^4*m +
8064*a^3*b^4)*x^5 + 35*(a^4*b^3*m^7 + 32*a^4*b^3*m^6 + 418*a^4*b^3*m^5 + 28
64*a^4*b^3*m^4 + 10993*a^4*b^3*m^3 + 23312*a^4*b^3*m^2 + 24876*a^4*b^3*m +
10080*a^4*b^3)*x^4 + 21*(a^5*b^2*m^7 + 33*a^5*b^2*m^6 + 447*a^5*b^2*m^5 + 3
195*a^5*b^2*m^4 + 12864*a^5*b^2*m^3 + 28692*a^5*b^2*m^2 + 32048*a^5*b^2*m +
 13440*a^5*b^2)*x^3 + 7*(a^6*b*m^7 + 34*a^6*b*m^6 + 478*a^6*b*m^5 + 3580*a^
6*b*m^4 + 15289*a^6*b*m^3 + 36706*a^6*b*m^2 + 44712*a^6*b*m + 20160*a^6*b)*
x^2 + (a^7*m^7 + 35*a^7*m^6 + 511*a^7*m^5 + 4025*a^7*m^4 + 18424*a^7*m^3 +
48860*a^7*m^2 + 69264*a^7*m + 40320*a^7)*x)*x^m/(m^8 + 36*m^7 + 546*m^6 + 4
536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)
```

**giac [B]** time = 1.40, size = 992, normalized size = 7.46

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] (b^7*m^7*x^8*x^m + 7*a*b^6*m^7*x^7*x^m + 28*b^7*m^6*x^8*x^m + 21*a^2*b^5*m^
7*x^6*x^m + 203*a*b^6*m^6*x^7*x^m + 322*b^7*m^5*x^8*x^m + 35*a^3*b^4*m^7*x^
5*x^m + 630*a^2*b^5*m^6*x^6*x^m + 2401*a*b^6*m^5*x^7*x^m + 1960*b^7*m^4*x^8
*x^m + 35*a^4*b^3*m^7*x^4*x^m + 1085*a^3*b^4*m^6*x^5*x^m + 7686*a^2*b^5*m^5
*x^6*x^m + 14945*a*b^6*m^4*x^7*x^m + 6769*b^7*m^3*x^8*x^m + 21*a^5*b^2*m^7*
x^3*x^m + 1120*a^4*b^3*m^6*x^4*x^m + 13685*a^3*b^4*m^5*x^5*x^m + 49140*a^2*
b^5*m^4*x^6*x^m + 52528*a*b^6*m^3*x^7*x^m + 13132*b^7*m^2*x^8*x^m + 7*a^6*b
*m^7*x^2*x^m + 693*a^5*b^2*m^6*x^3*x^m + 14630*a^4*b^3*m^5*x^4*x^m + 90335*
a^3*b^4*m^4*x^5*x^m + 176589*a^2*b^5*m^3*x^6*x^m + 103292*a*b^6*m^2*x^7*x^m
 + 13068*b^7*m*x^8*x^m + a^7*m^7*x*x^m + 238*a^6*b*m^6*x^2*x^m + 9387*a^5*b
^2*m^5*x^3*x^m + 100240*a^4*b^3*m^4*x^4*x^m + 334040*a^3*b^4*m^3*x^5*x^m +
353430*a^2*b^5*m^2*x^6*x^m + 103824*a*b^6*m*x^7*x^m + 5040*b^7*x^8*x^m + 35
*a^7*m^6*x*x^m + 3346*a^6*b*m^5*x^2*x^m + 67095*a^5*b^2*m^4*x^3*x^m + 38475
5*a^4*b^3*m^3*x^4*x^m + 684740*a^3*b^4*m^2*x^5*x^m + 360024*a^2*b^5*m*x^6*x
^m + 40320*a*b^6*x^7*x^m + 511*a^7*m^5*x*x^m + 25060*a^6*b*m^4*x^2*x^m + 27
0144*a^5*b^2*m^3*x^3*x^m + 815920*a^4*b^3*m^2*x^4*x^m + 710640*a^3*b^4*m*x^
5*x^m + 141120*a^2*b^5*x^6*x^m + 4025*a^7*m^4*x*x^m + 107023*a^6*b*m^3*x^2*
x^m + 602532*a^5*b^2*m^2*x^3*x^m + 870660*a^4*b^3*m*x^4*x^m + 282240*a^3*b^
4*x^5*x^m + 18424*a^7*m^3*x*x^m + 256942*a^6*b*m^2*x^2*x^m + 673008*a^5*b^2
*m*x^3*x^m + 352800*a^4*b^3*x^4*x^m + 48860*a^7*m^2*x*x^m + 312984*a^6*b*m*
x^2*x^m + 282240*a^5*b^2*x^3*x^m + 69264*a^7*m*x*x^m + 141120*a^6*b*x^2*x^m
 + 40320*a^7*x*x^m)/(m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*
m^3 + 118124*m^2 + 109584*m + 40320)
```

**maple [B]** time = 0.01, size = 782, normalized size = 5.88

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^7,x)
```

```
[Out] x^(m+1)*(b^7*m^7*x^7+7*a*b^6*m^7*x^6+28*b^7*m^6*x^7+21*a^2*b^5*m^7*x^5+203*
a*b^6*m^6*x^6+322*b^7*m^5*x^7+35*a^3*b^4*m^7*x^4+630*a^2*b^5*m^6*x^5+2401*a
*b^6*m^5*x^6+1960*b^7*m^4*x^7+35*a^4*b^3*m^7*x^3+1085*a^3*b^4*m^6*x^4+7686*
a^2*b^5*m^5*x^5+14945*a*b^6*m^4*x^6+6769*b^7*m^3*x^7+21*a^5*b^2*m^7*x^2+112
0*a^4*b^3*m^6*x^3+13685*a^3*b^4*m^5*x^4+49140*a^2*b^5*m^4*x^5+52528*a*b^6*m
^3*x^6+13132*b^7*m^2*x^7+7*a^6*b*m^7*x+693*a^5*b^2*m^6*x^2+14630*a^4*b^3*m^
5*x^3+90335*a^3*b^4*m^4*x^4+176589*a^2*b^5*m^3*x^5+103292*a*b^6*m^2*x^6+130
68*b^7*m*x^7+a^7*m^7+238*a^6*b*m^6*x+9387*a^5*b^2*m^5*x^2+100240*a^4*b^3*m^
4*x^3+334040*a^3*b^4*m^3*x^4+353430*a^2*b^5*m^2*x^5+103824*a*b^6*m*x^6+5040
*b^7*x^7+35*a^7*m^6+3346*a^6*b*m^5*x+67095*a^5*b^2*m^4*x^2+384755*a^4*b^3*m
^3*x^3+684740*a^3*b^4*m^2*x^4+360024*a^2*b^5*m*x^5+40320*a*b^6*x^6+511*a^7*
m^5+25060*a^6*b*m^4*x+270144*a^5*b^2*m^3*x^2+815920*a^4*b^3*m^2*x^3+710640*
```

```
a^3*b^4*m*x^4+141120*a^2*b^5*x^5+4025*a^7*m^4+107023*a^6*b*m^3*x+602532*a^5
*b^2*m^2*x^2+870660*a^4*b^3*m*x^3+282240*a^3*b^4*x^4+18424*a^7*m^3+256942*a
^6*b*m^2*x+673008*a^5*b^2*m*x^2+352800*a^4*b^3*x^3+48860*a^7*m^2+312984*a^6
*b*m*x+282240*a^5*b^2*x^2+69264*a^7*m+141120*a^6*b*x+40320*a^7)/(m+8)/(m+7)
/(m+6)/(m+5)/(m+4)/(m+3)/(m+2)/(m+1)
```

**maxima [A]** time = 1.37, size = 133, normalized size = 1.00

$$\frac{b^7x^{m+8}}{m+8}+\frac{7\,ab^6x^{m+7}}{m+7}+\frac{21\,a^2b^5x^{m+6}}{m+6}+\frac{35\,a^3b^4x^{m+5}}{m+5}+\frac{35\,a^4b^3x^{m+4}}{m+4}+\frac{21\,a^5b^2x^{m+3}}{m+3}+\frac{7\,a^6bx^{m+2}}{m+2}+\frac{a^7x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] b^7*x^(m + 8)/(m + 8) + 7*a*b^6*x^(m + 7)/(m + 7) + 21*a^2*b^5*x^(m + 6)/(m
 + 6) + 35*a^3*b^4*x^(m + 5)/(m + 5) + 35*a^4*b^3*x^(m + 4)/(m + 4) + 21*a^
5*b^2*x^(m + 3)/(m + 3) + 7*a^6*b*x^(m + 2)/(m + 2) + a^7*x^(m + 1)/(m + 1)
```

**mupad [B]** time = 0.78, size = 683, normalized size = 5.14

$$\frac{a^7\,x\,x^m\,\left(m^7+35\,m^6+511\,m^5+4025\,m^4+18424\,m^3+48860\,m^2+69264\,m+40320\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{b^7\,x^m\,x^8\,\left(m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{21\,a^2\,b^5\,x^m\,x^6\,\left(m^7+30\,m^6+366\,m^5+2340\,m^4+8409\,m^3+16830\,m^2+17144\,m+6720\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{35\,a^3\,b^4\,x^m\,x^5\,\left(m^7+31\,m^6+391\,m^5+2581\,m^4+9544\,m^3+19564\,m^2+20304\,m+8064\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{35\,a^4\,b^3\,x^m\,x^4\,\left(m^7+32\,m^6+418\,m^5+2864\,m^4+10993\,m^3+23312\,m^2+24876\,m+10080\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{21\,a^5\,b^2\,x^m\,x^3\,\left(m^7+33\,m^6+447\,m^5+3195\,m^4+12864\,m^3+28692\,m^2+32048\,m+13440\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{7\,a\,b^6\,x^m\,x^7\,\left(m^7+29\,m^6+343\,m^5+2135\,m^4+7504\,m^3+14756\,m^2+14832\,m+5760\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{7\,a^6\,b\,x^m\,x^2\,\left(m^7+34\,m^6+478\,m^5+3580\,m^4+15289\,m^3+36706\,m^2+44712\,m+20160\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^7,x)
```

```
[Out] (a^7*x*x^m*(69264*m + 48860*m^2 + 18424*m^3 + 4025*m^4 + 511*m^5 + 35*m^6 +
 m^7 + 40320))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
546*m^6 + 36*m^7 + m^8 + 40320) + (b^7*x^m*x^8*(13068*m + 13132*m^2 + 6769*
m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))/(109584*m + 118124*m^2 + 6
7284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (21*a^2
*b^5*x^m*x^6*(17144*m + 16830*m^2 + 8409*m^3 + 2340*m^4 + 366*m^5 + 30*m^6
+ m^7 + 6720))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
546*m^6 + 36*m^7 + m^8 + 40320) + (35*a^3*b^4*x^m*x^5*(20304*m + 19564*m^2
+ 9544*m^3 + 2581*m^4 + 391*m^5 + 31*m^6 + m^7 + 8064))/(109584*m + 118124*
m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) +
(35*a^4*b^3*x^m*x^4*(24876*m + 23312*m^2 + 10993*m^3 + 2864*m^4 + 418*m^5 +
 32*m^6 + m^7 + 10080))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 45
36*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (21*a^5*b^2*x^m*x^3*(32048*m + 2
8692*m^2 + 12864*m^3 + 3195*m^4 + 447*m^5 + 33*m^6 + m^7 + 13440))/(109584*
m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8
+ 40320) + (7*a*b^6*x^m*x^7*(14832*m + 14756*m^2 + 7504*m^3 + 2135*m^4 + 34
3*m^5 + 29*m^6 + m^7 + 5760))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^
4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (7*a^6*b*x^m*x^2*(44712*m
+ 36706*m^2 + 15289*m^3 + 3580*m^4 + 478*m^5 + 34*m^6 + m^7 + 20160))/(1095
```

```
84*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m
^8 + 40320)
```

**sympy [A]** time = 3.34, size = 4257, normalized size = 32.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**7,x)
```

```
[Out] Piecewise((-a**7/(7*x**7) - 7*a**6*b/(6*x**6) - 21*a**5*b**2/(5*x**5) - 35*
a**4*b**3/(4*x**4) - 35*a**3*b**4/(3*x**3) - 21*a**2*b**5/(2*x**2) - 7*a*b*
*6/x + b**7*log(x), Eq(m, -8)), (-a**7/(6*x**6) - 7*a**6*b/(5*x**5) - 21*a*
*5*b**2/(4*x**4) - 35*a**4*b**3/(3*x**3) - 35*a**3*b**4/(2*x**2) - 21*a**2*
b**5/x + 7*a*b**6*log(x) + b**7*x, Eq(m, -7)), (-a**7/(5*x**5) - 7*a**6*b/(
4*x**4) - 7*a**5*b**2/x**3 - 35*a**4*b**3/(2*x**2) - 35*a**3*b**4/x + 21*a*
*2*b**5*log(x) + 7*a*b**6*x + b**7*x**2/2, Eq(m, -6)), (-a**7/(4*x**4) - 7*
a**6*b/(3*x**3) - 21*a**5*b**2/(2*x**2) - 35*a**4*b**3/x + 35*a**3*b**4*log
(x) + 21*a**2*b**5*x + 7*a*b**6*x**2/2 + b**7*x**3/3, Eq(m, -5)), (-a**7/(3
*x**3) - 7*a**6*b/(2*x**2) - 21*a**5*b**2/x + 35*a**4*b**3*log(x) + 35*a**3
*b**4*x + 21*a**2*b**5*x**2/2 + 7*a*b**6*x**3/3 + b**7*x**4/4, Eq(m, -4)),
(-a**7/(2*x**2) - 7*a**6*b/x + 21*a**5*b**2*log(x) + 35*a**4*b**3*x + 35*a*
*3*b**4*x**2/2 + 7*a**2*b**5*x**3 + 7*a*b**6*x**4/4 + b**7*x**5/5, Eq(m, -3
)), (-a**7/x + 7*a**6*b*log(x) + 21*a**5*b**2*x + 35*a**4*b**3*x**2/2 + 35*
a**3*b**4*x**3/3 + 21*a**2*b**5*x**4/4 + 7*a*b**6*x**5/5 + b**7*x**6/6, Eq(
m, -2)), (a**7*log(x) + 7*a**6*b*x + 21*a**5*b**2*x**2/2 + 35*a**4*b**3*x**
3/3 + 35*a**3*b**4*x**4/4 + 21*a**2*b**5*x**5/5 + 7*a*b**6*x**6/6 + b**7*x*
*7/7, Eq(m, -1)), (a**7*m**7*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5
+ 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**7*m**6*
x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 1
18124*m**2 + 109584*m + 40320) + 511*a**7*m**5*x*x**m/(m**8 + 36*m**7 + 546
*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 4032
0) + 4025*a**7*m**4*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m
**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 18424*a**7*m**3*x*x**m
/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*
m**2 + 109584*m + 40320) + 48860*a**7*m**2*x*x**m/(m**8 + 36*m**7 + 546*m**
6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
 69264*a**7*m*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 +
67284*m**3 + 118124*m**2 + 109584*m + 40320) + 40320*a**7*x*x**m/(m**8 + 36
*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 1095
84*m + 40320) + 7*a**6*b*m**7*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 238*a**6*
b*m**6*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 6728
4*m**3 + 118124*m**2 + 109584*m + 40320) + 3346*a**6*b*m**5*x**2*x**m/(m**8
 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
```

109584*m + 40320) + 25060*a**6*b*m**4*x**2*x**m/(m**8 + 36*m**7 + 546*m**6
+ 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
107023*a**6*b*m**3*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449
*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 256942*a**6*b*m**2*x
**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
118124*m**2 + 109584*m + 40320) + 312984*a**6*b*m*x**2*x**m/(m**8 + 36*m**
7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m
+ 40320) + 141120*a**6*b*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5
+ 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 21*a**5*b**2*
m**7*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*
m**3 + 118124*m**2 + 109584*m + 40320) + 693*a**5*b**2*m**6*x**3*x**m/(m**8
+ 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
109584*m + 40320) + 9387*a**5*b**2*m**5*x**3*x**m/(m**8 + 36*m**7 + 546*m*
*6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
+ 67095*a**5*b**2*m**4*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2
2449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 270144*a**5*b**2
*m**3*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284
*m**3 + 118124*m**2 + 109584*m + 40320) + 602532*a**5*b**2*m**2*x**3*x**m/(
m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m*
*2 + 109584*m + 40320) + 673008*a**5*b**2*m*x**3*x**m/(m**8 + 36*m**7 + 546
*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 4032
0) + 282240*a**5*b**2*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22
449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**4*b**3*m**7
*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3
+ 118124*m**2 + 109584*m + 40320) + 1120*a**4*b**3*m**6*x**4*x**m/(m**8 +
36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10
9584*m + 40320) + 14630*a**4*b**3*m**5*x**4*x**m/(m**8 + 36*m**7 + 546*m**6
+ 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
100240*a**4*b**3*m**4*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22
449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 384755*a**4*b**3*
m**3*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*
m**3 + 118124*m**2 + 109584*m + 40320) + 815920*a**4*b**3*m**2*x**4*x**m/(m
**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**
2 + 109584*m + 40320) + 870660*a**4*b**3*m*x**4*x**m/(m**8 + 36*m**7 + 546*
m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320
) + 352800*a**4*b**3*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 224
49*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**3*b**4*m**7*
x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3
+ 118124*m**2 + 109584*m + 40320) + 1085*a**3*b**4*m**6*x**5*x**m/(m**8 + 3
6*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109
584*m + 40320) + 13685*a**3*b**4*m**5*x**5*x**m/(m**8 + 36*m**7 + 546*m**6
+ 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 9
0335*a**3*b**4*m**4*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2244
9*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 334040*a**3*b**4*m*
*3*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m*

*3 + 118124*m**2 + 109584*m + 40320) + 684740*a**3*b**4*m**2*x**5*x**m/(m**
8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2
+ 109584*m + 40320) + 710640*a**3*b**4*m*x**5*x**m/(m**8 + 36*m**7 + 546*m*
*6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
+ 282240*a**3*b**4*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449
*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 21*a**2*b**5*m**7*x*
*6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
118124*m**2 + 109584*m + 40320) + 630*a**2*b**5*m**6*x**6*x**m/(m**8 + 36*m
**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584
*m + 40320) + 7686*a**2*b**5*m**5*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 45
36*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 49140
*a**2*b**5*m**4*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m*
*4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 176589*a**2*b**5*m**3*x
**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
118124*m**2 + 109584*m + 40320) + 353430*a**2*b**5*m**2*x**6*x**m/(m**8 +
36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10
9584*m + 40320) + 360024*a**2*b**5*m*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 +
4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 14
1120*a**2*b**5*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**
4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 7*a*b**6*m**7*x**7*x**m/
(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m
**2 + 109584*m + 40320) + 203*a*b**6*m**6*x**7*x**m/(m**8 + 36*m**7 + 546*m
**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
+ 2401*a*b**6*m**5*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2244
9*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 14945*a*b**6*m**4*x
**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
118124*m**2 + 109584*m + 40320) + 52528*a*b**6*m**3*x**7*x**m/(m**8 + 36*m
**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584
*m + 40320) + 103292*a*b**6*m**2*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 453
6*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 103824
*a*b**6*m*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 6
7284*m**3 + 118124*m**2 + 109584*m + 40320) + 40320*a*b**6*x**7*x**m/(m**8
+ 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
109584*m + 40320) + b**7*m**7*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 28*b**7*m
**6*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m
**3 + 118124*m**2 + 109584*m + 40320) + 322*b**7*m**5*x**8*x**m/(m**8 + 36*
m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10958
4*m + 40320) + 1960*b**7*m**4*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 6769*b**7
*m**3*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284
*m**3 + 118124*m**2 + 109584*m + 40320) + 13132*b**7*m**2*x**8*x**m/(m**8 +
36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 1
09584*m + 40320) + 13068*b**7*m*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536
*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 5040*b*

```
*7*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m*
*3 + 118124*m**2 + 109584*m + 40320), True))
```

## 3.701 $\int x^m(a+bx)^3\,dx$

**Optimal**. Leaf size=61

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+2}}{m+2}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

**Rubi** [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^3\,dx&=\int\left(a^3x^m+3a^2bx^{1+m}+3ab^2x^{2+m}+b^3x^{3+m}\right)dx\\&=\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 54, normalized size = 0.89

$$x^{m+1}\left(\frac{a^3}{m+1}+\frac{3a^2bx}{m+2}+\frac{3ab^2x^2}{m+3}+\frac{b^3x^3}{m+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^3
*x^3)/(4 + m))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^m(a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^3, x]
```

**fricas [B]** time = 1.03, size = 157, normalized size = 2.57

$$\frac{\left(\left(b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3\right)x^4+3\left(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2\right)x^3+3\left(a^2bm^3+8\,a^2bm^2+19\,a^2bm+12\,a^2b\right)x^2+\left(a^3m^3+9\,a^3m^2+26\,a^3m+24\,a^3\right)x\right)x^m}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
+ 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*
a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*x^m/(m^4 + 10*m^3
 + 35*m^2 + 50*m + 24)
```

**giac [B]** time = 1.07, size = 224, normalized size = 3.67

$$\frac{b^3m^3x^4x^m+3\,ab^2m^3x^3x^m+6\,b^3m^2x^4x^m+3\,a^2bm^3x^2x^m+21\,ab^2m^2x^3x^m+11\,b^3mx^4x^m+a^3m^3xx^m+24\,a^2bm^2x^2x^m+42\,ab^2mx^3x^m+6\,b^3x^4x^m+9\,a^3m^2xx^m+57\,a^2bmx^2x^m+24\,ab^2x^3x^m+26\,a^3mxx^m+36\,a^2bx^2x^m+24\,a^3xx^m}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^4*x^m + 3*a*b^2*m^3*x^3*x^m + 6*b^3*m^2*x^4*x^m + 3*a^2*b*m^3*x^
2*x^m + 21*a*b^2*m^2*x^3*x^m + 11*b^3*m*x^4*x^m + a^3*m^3*x*x^m + 24*a^2*b*
m^2*x^2*x^m + 42*a*b^2*m*x^3*x^m + 6*b^3*x^4*x^m + 9*a^3*m^2*x*x^m + 57*a^2
*b*m*x^2*x^m + 24*a*b^2*x^3*x^m + 26*a^3*m*x*x^m + 36*a^2*b*x^2*x^m + 24*a^
3*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

**maple [B]** time = 0.00, size = 170, normalized size = 2.79

$$\frac{\left(b^3m^3x^3+3a\,b^2m^3x^2+6b^3m^2x^3+3a^2b\,m^3x+21a\,b^2m^2x^2+11b^3m\,x^3+a^3m^3+24a^2b\,m^2x+42a\,b^2m\,x^2+6b^3x^3+9a^3m^2+57a^2bmx+24a\,b^2x^2+26a^3m+36a^2bx+24a^3\right)x^{m+1}}{(m+4)\,(m+3)\,(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^3,x)
```

```
[Out] (b^3*m^3*x^3+3*a*b^2*m^3*x^2+6*b^3*m^2*x^3+3*a^2*b*m^3*x+21*a*b^2*m^2*x^2+1
1*b^3*m*x^3+a^3*m^3+24*a^2*b*m^2*x+42*a*b^2*m*x^2+6*b^3*x^3+9*a^3*m^2+57*a^
2*b*m*x+24*a*b^2*x^2+26*a^3*m+36*a^2*b*x+24*a^3)/(m+4)/(m+3)/(m+2)/(m+1)*x^
(m+1)
```

**maxima** [A] time = 1.30, size = 61, normalized size = 1.00

$$\frac{b^3 x^{m+4}}{m+4} + \frac{3\,ab^2 x^{m+3}}{m+3} + \frac{3\,a^2 b x^{m+2}}{m+2} + \frac{a^3 x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 4)/(m + 4) + 3*a*b^2*x^(m + 3)/(m + 3) + 3*a^2*b*x^(m + 2)/(m +
2) + a^3*x^(m + 1)/(m + 1)
```

**mupad** [B] time = 0.44, size = 167, normalized size = 2.74

$$x^m \left(\frac{a^3\,x\,\left(m^3+9\,m^2+26\,m+24\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{b^3\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a\,b^2\,x^3\,\left(m^3+7\,m^2+14\,m+8\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a^2\,b\,x^2\,\left(m^3+8\,m^2+19\,m+12\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^3,x)
```

```
[Out] x^m*((a^3*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (b^3*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) +
(3*a*b^2*x^3*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (3*a^2*b*x^2*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4))
```

**sympy** [A] time = 0.92, size = 663, normalized size = 10.87

$$\begin{cases} -\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - \frac{3ab^2}{x} + b^3\log(x) & \text{for } m = -4 \\ -\frac{a^3}{2x^2} - \frac{3a^2b}{x} + 3ab^2\log(x) + b^3x & \text{for } m = -3 \\ -\frac{a^3}{x} + 3a^2b\log(x) + 3ab^2x + \frac{b^3x^2}{2} & \text{for } m = -2 \\ a^3\log(x) + 3a^2bx + \frac{3ab^2x^2}{2} + \frac{b^3x^3}{3} & \text{for } m = -1 \\ \frac{a^3m^3xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{9a^3m^2xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{26a^3mxx^m}{m^4+10m^3+35m^2+50m+24} + \frac{24a^3xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{3a^2bm^3x^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{24a^2bm^2x^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{57a^2bmx^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{36a^2bx^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{3ab^2m^3x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{21ab^2m^2x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{42ab^2mx^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{24ab^2x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{b^3m^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^3m^2x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{11b^3mx^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x), E
q(m, -4)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*x, Eq(m, -
3)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq(m, -2)), (a*
*3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3, Eq(m, -1)), (a**3*m
**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*a**3*m**2*x*x**m/(m**
4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*a**3*m*x*x**m/(m**4 + 10*m**3 + 35*
```

```
m**2 + 50*m + 24) + 24*a**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
 3*a**2*b*m**3*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**2*b
*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 57*a**2*b*m*x**2*x
**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 36*a**2*b*x**2*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) + 3*a*b**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 21*a*b**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 5
0*m + 24) + 42*a*b**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*a*b**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + b**3*m**3*x**4
*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*m**2*x**4*x**m/(m**4
+ 10*m**3 + 35*m**2 + 50*m + 24) + 11*b**3*m*x**4*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24
), True))
```

## 3.702 $\int x^m (a+bx)^2\,dx$

**Optimal**. Leaf size=43

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

**Rubi** [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 +
m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^2\,dx&=\int\left(a^2x^m+2abx^{1+m}+b^2x^{2+m}\right)dx\\&=\frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 38, normalized size = 0.88

$$x^{m+1}\left(\frac{a^2}{m+1}+\frac{2abx}{m+2}+\frac{b^2x^2}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^m(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^2, x]
```

**fricas [A]** time = 1.17, size = 85, normalized size = 1.98

$$\frac{\left(\left(b^2m^2+3\,b^2m+2\,b^2\right)x^3+2\left(abm^2+4\,abm+3\,ab\right)x^2+\left(a^2m^2+5\,a^2m+6\,a^2\right)x\right)x^m}{m^3+6\,m^2+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2
*m^2 + 5*a^2*m + 6*a^2)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

**giac [B]** time = 1.04, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^3x^m+2\,abm^2x^2x^m+3\,b^2mx^3x^m+a^2m^2xx^m+8\,abmx^2x^m+2\,b^2x^3x^m+5\,a^2mxx^m+6\,abx^2x^m+6\,a^2xx^m}{m^3+6\,m^2+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^3*x^m + 2*a*b*m^2*x^2*x^m + 3*b^2*m*x^3*x^m + a^2*m^2*x*x^m + 8*
a*b*m*x^2*x^m + 2*b^2*x^3*x^m + 5*a^2*m*x*x^m + 6*a*b*x^2*x^m + 6*a^2*x*x^m
)/(m^3 + 6*m^2 + 11*m + 6)
```

**maple [A]** time = 0.00, size = 87, normalized size = 2.02

$$\frac{\left(b^2m^2x^2+2ab\,m^2x+3b^2m\,x^2+a^2m^2+8abmx+2b^2x^2+5a^2m+6abx+6a^2\right)x^{m+1}}{(m+3)\,(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^2,x)
```

```
[Out] (b^2*m^2*x^2+2*a*b*m^2*x+3*b^2*m*x^2+a^2*m^2+8*a*b*m*x+2*b^2*x^2+5*a^2*m+6*
a*b*x+6*a^2)/(m+3)/(m+2)/(m+1)*x^(m+1)
```

**maxima** [A] time = 1.32, size = 43, normalized size = 1.00

$$\frac{b^2x^{m+3}}{m+3} + \frac{2\,abx^{m+2}}{m+2} + \frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 3)/(m + 3) + 2*a*b*x^(m + 2)/(m + 2) + a^2*x^(m + 1)/(m + 1)
```

**mupad** [B] time = 0.37, size = 93, normalized size = 2.16

$$x^m\left(\frac{a^2\,x\,\left(m^2+5\,m+6\right)}{m^3+6\,m^2+11\,m+6}+\frac{b^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6}+\frac{2\,a\,b\,x^2\,\left(m^2+4\,m+3\right)}{m^3+6\,m^2+11\,m+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^2,x)
```

```
[Out] x^m*((a^2*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (b^2*x^3*(3*m + m^2
 + 2))/(11*m + 6*m^2 + m^3 + 6) + (2*a*b*x^2*(4*m + m^2 + 3))/(11*m + 6*m^2
 + m^3 + 6))
```

**sympy** [A] time = 0.55, size = 299, normalized size = 6.95

$$\begin{cases} -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2\log(x) & \text{for } m = -3 \\ -\frac{a^2}{x} + 2ab\log(x) + b^2x & \text{for } m = -2 \\ a^2\log(x) + 2abx + \frac{b^2x^2}{2} & \text{for } m = -1 \\ \frac{a^2m^2xx^m}{m^3+6m^2+11m+6} + \frac{5a^2mxx^m}{m^3+6m^2+11m+6} + \frac{6a^2xx^m}{m^3+6m^2+11m+6} + \frac{2abm^2x^2x^m}{m^3+6m^2+11m+6} + \frac{8abmx^2x^m}{m^3+6m^2+11m+6} + \frac{6abx^2x^m}{m^3+6m^2+11m+6} + \frac{b^2m^2x^3x^m}{m^3+6m^2+11m+6} + \frac{3b^2mx^3x^m}{m^3+6m^2+11m+6} + \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(m, -3)), (-a**2/x + 2
*a*b*log(x) + b**2*x, Eq(m, -2)), (a**2*log(x) + 2*a*b*x + b**2*x**2/2, Eq(
m, -1)), (a**2*m**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*a**2*m*x*x**m/(m*
*3 + 6*m**2 + 11*m + 6) + 6*a**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*a*b*
m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*b*m*x**2*x**m/(m**3 + 6*m**
2 + 11*m + 6) + 6*a*b*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3
*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*b**2*m*x**3*x**m/(m**3 + 6*m**2 + 11*m
 + 6) + 2*b**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

## 3.703 $\int x^m(a+bx)\,dx$

**Optimal**. Leaf size=25

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

**Rubi [A]** time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x),x]
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)\,dx&=\int\left(ax^m+bx^{1+m}\right)\,dx\\&=\frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 22, normalized size = 0.88

$$x^{m+1}\left(\frac{a}{m+1}+\frac{bx}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x),x]
[Out] x^(1 + m)*(a/(1 + m) + (b*x)/(2 + m))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^m(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x),x]
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x), x]
```

**fricas [A]** time = 0.79, size = 33, normalized size = 1.32

$$\frac{\left((bm+b)x^2+(am+2a)x\right)x^m}{m^2+3m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="fricas")
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

**giac [A]** time = 1.02, size = 43, normalized size = 1.72

$$\frac{bmx^2x^m+amxx^m+bx^2x^m+2\,axx^m}{m^2+3m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="giac")
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

**maple [A]** time = 0.00, size = 31, normalized size = 1.24

$$\frac{(bmx+am+bx+2a)\,x^{m+1}}{(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a),x)
[Out] (b*m*x+a*m+b*x+2*a)/(m+2)/(m+1)*x^(m+1)
```

**maxima [A]** time = 1.35, size = 25, normalized size = 1.00

$$\frac{bx^{m+2}}{m+2}+\frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*x^(m + 2)/(m + 2) + a*x^(m + 1)/(m + 1)
```

**mupad [B]** time = 0.30, size = 30, normalized size = 1.20

$$\frac{x^{m+1}\,\left(2\,a+a\,m+b\,x+b\,m\,x\right)}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x),x)
```

```
[Out] (x^(m + 1)*(2*a + a*m + b*x + b*m*x))/(3*m + m^2 + 2)
```

**sympy [A]** time = 0.31, size = 87, normalized size = 3.48

$$\begin{cases} -\frac{a}{x} + b\log{\left(x \right)} & \text{for}\: m = -2 \\ a\log{\left(x \right)} + bx & \text{for}\: m = -1 \\ \frac{amxx^{m}}{m^{2}+3m+2} + \frac{2axx^{m}}{m^{2}+3m+2} + \frac{bmx^{2}x^{m}}{m^{2}+3m+2} + \frac{bx^{2}x^{m}}{m^{2}+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a),x)
```

```
[Out] Piecewise((-a/x + b*log(x), Eq(m, -2)), (a*log(x) + b*x, Eq(m, -1)), (a*m*x
*x**m/(m**2 + 3*m + 2) + 2*a*x*x**m/(m**2 + 3*m + 2) + b*m*x**2*x**m/(m**2
+ 3*m + 2) + b*x**2*x**m/(m**2 + 3*m + 2), True))
```

## 3.704 $\int x^3(a+bx)^n\,dx$

**Optimal**. Leaf size=83

$$-\frac{a^3(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2(a+bx)^{n+2}}{b^4(n+2)}-\frac{3a(a+bx)^{n+3}}{b^4(n+3)}+\frac{(a+bx)^{n+4}}{b^4(n+4)}$$

**Rubi [A]** time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^3(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2(a+bx)^{n+2}}{b^4(n+2)}-\frac{3a(a+bx)^{n+3}}{b^4(n+3)}+\frac{(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^n,x]
```

```
[Out] -((a^3*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (3*a^2*(a + b*x)^(2 + n))/(b^4*(
2 + n)) - (3*a*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (a + b*x)^(4 + n)/(b^4*(4
 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^n\,dx&=\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx\\&=-\frac{a^3(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2(a+bx)^{2+n}}{b^4(2+n)}-\frac{3a(a+bx)^{3+n}}{b^4(3+n)}+\frac{(a+bx)^{4+n}}{b^4(4+n)}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 67, normalized size = 0.81

$$\frac{(a+bx)^{n+1}\left(-\frac{a^3}{n+1}+\frac{3a^2(a+bx)}{n+2}-\frac{3a(a+bx)^2}{n+3}+\frac{(a+bx)^3}{n+4}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-(a^3/(1 + n)) + (3*a^2*(a + b*x))/(2 + n) - (3*a*(a + b*x)^2)/(3 + n) + (a + b*x)^3/(4 + n)))/b^4
```

**IntegrateAlgebraic [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^3*(a + b*x)^n, x]
```

**fricas [A]** time = 1.29, size = 143, normalized size = 1.72

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\,\left(a^2b^2n^2+a^2b^2n\right)x^2\right)(bx+a)^n}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*(b*x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

**giac [B]** time = 1.21, size = 226, normalized size = 2.72

$$\frac{(bx+a)^nb^4n^3x^4+(bx+a)^nab^3n^3x^3+6\,(bx+a)^nb^4n^2x^4+3\,(bx+a)^nab^3n^2x^3+11\,(bx+a)^nb^4nx^4-3\,(bx+a)^na^2b^2n^2x^2+2\,(bx+a)^nab^3nx^3+6\,(bx+a)^nb^4x^4-3\,(bx+a)^na^2b^2nx^2+6\,(bx+a)^na^3bnx-6\,(bx+a)^na^4}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*n^3*x^4 + (b*x + a)^n*a*b^3*n^3*x^3 + 6*(b*x + a)^n*b^4*n^2*x^4 + 3*(b*x + a)^n*a*b^3*n^2*x^3 + 11*(b*x + a)^n*b^4*n*x^4 - 3*(b*x + a)^n*a^2*b^2*n^2*x^2 + 2*(b*x + a)^n*a*b^3*n*x^3 + 6*(b*x + a)^n*b^4*x^4 - 3*(b*x + a)^n*a^2*b^2*n*x^2 + 6*(b*x + a)^n*a^3*b*n*x - 6*(b*x + a)^n*a^4)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

**maple [A]** time = 0.01, size = 126, normalized size = 1.52

$$-\frac{\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)(bx+a)^{n+1}}{\left(n^4+10n^3+35n^2+50n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3+9*a
*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/b^4/(n^4+10*n
^3+35*n^2+50*n+24)
```

**maxima** [A] time = 1.36, size = 101, normalized size = 1.22

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4)
```

**mupad** [B] time = 0.53, size = 176, normalized size = 2.12

$$(a+b\,x)^n\left(\frac{x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^3\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^2\,\left(n+1\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 +
 24) - (6*a^4)/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*n*x)/(b^3
*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^3*(3*n + n^2 + 2))/(b*(50*n
+ 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^2*(n + 1))/(b^2*(50*n + 35*n^2
+ 10*n^3 + n^4 + 24)))
```

**sympy** [A] time = 2.33, size = 1318, normalized size = 15.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a
**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2
*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*
b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3
*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log
(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 1
8*a*b**2*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
```

```
 + 6*b**3*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2
+ 6*b**7*x**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x
 + 2*b**6*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2
*b*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2
*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a
**3/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) -
 3*a*b**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(
n, -2)), (-a**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3
*b), Eq(n, -1)), (-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**
4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b
*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*
a**2*b**2*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**
3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(
b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n
*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4) + b**4*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n
**2 + 50*b**4*n + 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)*
*n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4
*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4), True))
```

## 3.705 $\int x^2(a+bx)^n\,dx$

**Optimal**. Leaf size=60

$$\frac{a^2(a+bx)^{n+1}}{b^3(n+1)}-\frac{2a(a+bx)^{n+2}}{b^3(n+2)}+\frac{(a+bx)^{n+3}}{b^3(n+3)}$$

**Rubi** [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{n+1}}{b^3(n+1)}-\frac{2a(a+bx)^{n+2}}{b^3(n+2)}+\frac{(a+bx)^{n+3}}{b^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n,x]
```

```
[Out] (a^2*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*(a + b*x)^(2 + n))/(b^3*(2 + n
)) + (a + b*x)^(3 + n)/(b^3*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\,dx&=\int\left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx\\&=\frac{a^2(a+bx)^{1+n}}{b^3(1+n)}-\frac{2a(a+bx)^{2+n}}{b^3(2+n)}+\frac{(a+bx)^{3+n}}{b^3(3+n)}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 57, normalized size = 0.95

$$\frac{(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

**IntegrateAlgebraic [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x)^n, x]
```

**fricas [A]** time = 1.08, size = 96, normalized size = 1.60

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)(bx+a)^n}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

**giac [B]** time = 1.07, size = 140, normalized size = 2.33

$$\frac{(bx+a)^nb^3n^2x^3+(bx+a)^nab^2n^2x^2+3\,(bx+a)^nb^3nx^3+(bx+a)^nab^2nx^2+2\,(bx+a)^nb^3x^3-2\,(bx+a)^na^2bnx+2\,(bx+a)^na^3}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*n^2*x^3 + (b*x + a)^n*a*b^2*n^2*x^2 + 3*(b*x + a)^n*b^3*n*
x^3 + (b*x + a)^n*a*b^2*n*x^2 + 2*(b*x + a)^n*b^3*x^3 - 2*(b*x + a)^n*a^2*b
*n*x + 2*(b*x + a)^n*a^3)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

**maple [A]** time = 0.01, size = 73, normalized size = 1.22

$$\frac{\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2\right)(bx+a)^{n+1}}{\left(n^3+6n^2+11n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n,x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)/b
^3/(n^3+6*n^2+11*n+6)
```

**maxima [A]** time = 1.37, size = 68, normalized size = 1.13

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

**mupad [B]** time = 0.56, size = 192, normalized size = 3.20

$$\begin{cases} \frac{2\,a^2\,\ln(a+b\,x)+b^2\,x^2-2\,a\,b\,x}{2\,b^3} & \text{if } n=-1\\ \frac{x}{b^2}-\frac{a^2}{b^3\,(a+b\,x)}-\frac{2\,a\,\ln(a+b\,x)}{b^3} & \text{if } n=-2\\ \frac{\ln(a+b\,x)+\frac{2\,a}{a+b\,x}-\frac{a^2}{2\,(a+b\,x)^2}}{b^3} & \text{if } n=-3\\ \frac{2\,(a+b\,x)^{n+1}\,\left(8\,a^2-8\,a\,b\,n\,x-8\,a\,b\,x+4\,b^2\,n^2\,x^2+12\,b^2\,n\,x^2+8\,b^2\,x^2\right)}{b^3\,\left(8\,n^3+48\,n^2+88\,n+48\right)} & \text{if } n\neq -1\wedge n\neq -2\wedge n\neq -3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^n,x)
```

```
[Out] piecewise(n == -1, (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3), n == -
2, x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*log(a + b*x))/b^3, n == -3, (log(a +
b*x) + (2*a)/(a + b*x) - a^2/(2*(a + b*x)^2))/b^3, n ~= -1 & n ~= -2 & n ~=
 -3, (2*(a + b*x)^(n + 1)*(8*a^2 + 8*b^2*x^2 + 12*b^2*n*x^2 - 8*a*b*x + 4*b
^2*n^2*x^2 - 8*a*b*n*x))/(b^3*(88*n + 48*n^2 + 8*n^3 + 48)))
```

**sympy [A]** time = 1.32, size = 597, normalized size = 9.95

$$\begin{cases} \frac{a^n x^3}{3} & \text{for } b=0\\ \frac{2a^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{2b^2x^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} & \text{for } n=-3\\ -\frac{2a^2\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{2a^2}{ab^3+b^4x}-\frac{2abx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}+\frac{b^2x^2}{ab^3+b^4x} & \text{for } n=-2\\ \frac{a^2\log\left(\frac{a}{b}+x\right)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b} & \text{for } n=-1\\ \frac{2a^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}-\frac{2a^2bnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2n^2x^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{ab^2nx^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{b^3n^2x^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{3b^3nx^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{2b^3x^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*
b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4
*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(2*a
**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)/(a*b**3 +
b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*x)
 + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b**3 - a*x/b
**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*
n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3
*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b**3*n**3 + 6*b*
*3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(b**3*n**3 + 6*b
**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*n**3 + 6*b*
*3*n**2 + 11*b**3*n + 6*b**3), True))
```

## 3.706 $\int x(a+bx)^n\,dx$

**Optimal**. Leaf size=39

$$\frac{(a+bx)^{n+2}}{b^2(n+2)}-\frac{a(a+bx)^{n+1}}{b^2(n+1)}$$

**Rubi** [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^{n+2}}{b^2(n+2)}-\frac{a(a+bx)^{n+1}}{b^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n,x]
```

```
[Out] -((a*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (a + b*x)^(2 + n)/(b^2*(2 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n\,dx&=\int\left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx\\&=-\frac{a(a+bx)^{1+n}}{b^2(1+n)}+\frac{(a+bx)^{2+n}}{b^2(2+n)}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n))
```

**IntegrateAlgebraic [F]** time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^n,x]
[Out] Defer[IntegrateAlgebraic][x*(a + b*x)^n, x]
```

**fricas [A]** time = 1.14, size = 53, normalized size = 1.36

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)(bx+a)^n}{b^2n^2+3\,b^2n+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="fricas")
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*(b*x + a)^n/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

**giac [A]** time = 1.05, size = 76, normalized size = 1.95

$$\frac{(bx+a)^n b^2nx^2+(bx+a)^n abnx+(bx+a)^n b^2x^2-(bx+a)^n a^2}{b^2n^2+3\,b^2n+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="giac")
[Out] ((b*x + a)^n*b^2*n*x^2 + (b*x + a)^n*a*b*n*x + (b*x + a)^n*b^2*x^2 - (b*x +
 a)^n*a^2)/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.92

$$-\frac{(-xnb-bx+a)\,(bx+a)^{n+1}}{\left(n^2+3n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n,x)
[Out] -(b*x+a)^(n+1)*(-b*n*x-b*x+a)/b^2/(n^2+3*n+2)
```

**maxima [A]** time = 1.31, size = 42, normalized size = 1.08

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2)
```

**mupad [B]** time = 0.38, size = 94, normalized size = 2.41

$$\begin{cases} -\frac{a\,\ln(a+b\,x)-b\,x}{b^2} & \text{if}\ \ n=-1\\ \frac{\ln(a+b\,x)+\frac{a}{a+b\,x}}{b^2} & \text{if}\ \ n=-2\\ \frac{2\left(\frac{(a+b\,x)^{n+2}}{2\,n+4}-\frac{a\,(a+b\,x)^{n+1}}{2\,n+2}\right)}{b^2} & \text{if}\ \ n\neq -1\wedge n\neq -2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^n,x)
```

```
[Out] piecewise(n == -1, -(a*log(a + b*x) - b*x)/b^2, n == -2, (log(a + b*x) + a/
(a + b*x))/b^2, n ~= -1 & n ~= -2, (2*((a + b*x)^(n + 2)/(2*n + 4) - (a*(a
+ b*x)^(n + 1))/(2*n + 2)))/b^2)
```

**sympy [A]** time = 0.70, size = 201, normalized size = 5.15

$$\begin{cases} \frac{a^n x^2}{2} & \text{for}\ b=0\\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x}+\frac{a}{ab^2+b^3x}+\frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for}\ n=-2\\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2}+\frac{x}{b} & \text{for}\ n=-1\\ -\frac{a^2(a+bx)^n}{b^2n^2+3b^2n+2b^2}+\frac{abnx(a+bx)^n}{b^2n^2+3b^2n+2b^2}+\frac{b^2nx^2(a+bx)^n}{b^2n^2+3b^2n+2b^2}+\frac{b^2x^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a
*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2)), (-a*log(a
/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n +
 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*n*x*
*2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a + b*x)**n/(b
**2*n**2 + 3*b**2*n + 2*b**2), True))
```

## 3.707 $\int (a+bx)^n \, dx$

**Optimal**. Leaf size=18

$$\frac{(a+bx)^{n+1}}{b(n+1)}$$

**Rubi [A]** time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n,x]
```

```
[Out] (a + b*x)^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^n \, dx = \frac{(a+bx)^{1+n}}{b(1+n)}$$

**Mathematica [A]** time = 0.01, size = 17, normalized size = 0.94

$$\frac{(a+bx)^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n,x]
```

```
[Out] (a + b*x)^(1 + n)/(b + b*n)
```

**IntegrateAlgebraic [F]** time = 0.01, size = 0, normalized size = 0.00

$$\int (a+bx)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n, x]
```

**fricas [A]** time = 1.23, size = 20, normalized size = 1.11

$$\frac{(bx+a)(bx+a)^n}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b*x + a)^n/(b*n + b)
```

**giac [A]** time = 1.14, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="giac")
```

```
[Out] (b*x + a)^(n + 1)/(b*(n + 1))
```

**maple [A]** time = 0.00, size = 19, normalized size = 1.06

$$\frac{(bx+a)^{n+1}}{(n+1)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n,x)
```

```
[Out] (b*x+a)^(n+1)/b/(n+1)
```

**maxima [A]** time = 1.30, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)/(b*(n + 1))
```

**mupad [B]** time = 0.20, size = 18, normalized size = 1.00

$$\frac{(a+b\,x)^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n,x)
```

```
[Out] (a + b*x)^(n + 1)/(b*(n + 1))
```

**sympy [A]** time = 0.07, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(a+bx)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n,x)
```

```
[Out] Piecewise(((a + b*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(a + b*x), True))/b
```

## 3.708 $\int x^{-4+n}(a+bx)^{-n}\,dx$

**Optimal**. Leaf size=110

$$-\frac{2b^2x^{n-1}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}+\frac{2bx^{n-2}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{x^{n-3}(a+bx)^{1-n}}{a(3-n)}$$

**Rubi [A]** time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2x^{n-1}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}+\frac{2bx^{n-2}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{x^{n-3}(a+bx)^{1-n}}{a(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-3 + n)*(a + b*x)^(1 - n))/(a*(3 - n))) + (2*b*x^(-2 + n)*(a + b*x)^(
1 - n))/(a^2*(2 - n)*(3 - n)) - (2*b^2*x^(-1 + n)*(a + b*x)^(1 - n))/(a^3*(
1 - n)*(2 - n)*(3 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int x^{-4+n}(a+bx)^{-n}\,dx = -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)} - \frac{(2b)\int x^{-3+n}(a+bx)^{-n}\,dx}{a(3-n)}$$
$$= -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)} + \frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)} + \frac{\left(2b^2\right)\int x^{-2+n}(a+bx)^{-n}\,dx}{a^2(2-n)(3-n)}$$
$$= -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)} + \frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)} - \frac{2b^2x^{-1+n}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}$$

**Mathematica [A]** time = 0.03, size = 64, normalized size = 0.58

$$\frac{x^{n-3}(a+bx)^{1-n}\left(a^2\left(n^2-3n+2\right)+2ab(n-1)x+2b^2x^2\right)}{a^3(n-3)(n-2)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(-3 + n)*(a + b*x)^(1 - n)*(a^2*(2 - 3*n + n^2) + 2*a*b*(-1 + n)*x + 2*b
^2*x^2))/(a^3*(-3 + n)*(-2 + n)*(-1 + n))
```

**IntegrateAlgebraic [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int x^{-4+n}(a+bx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-4 + n)/(a + b*x)^n, x]
```

**fricas [A]** time = 0.99, size = 104, normalized size = 0.95

$$\frac{\left(2\,ab^2nx^3+2\,b^3x^4+\left(a^2bn^2-a^2bn\right)x^2+\left(a^3n^2-3\,a^3n+2\,a^3\right)x\right)x^{n-4}}{\left(a^3n^3-6\,a^3n^2+11\,a^3n-6\,a^3\right)(bx+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] (2*a*b^2*n*x^3 + 2*b^3*x^4 + (a^2*b*n^2 - a^2*b*n)*x^2 + (a^3*n^2 - 3*a^3*n
 + 2*a^3)*x)*x^(n - 4)/((a^3*n^3 - 6*a^3*n^2 + 11*a^3*n - 6*a^3)*(b*x + a)^
n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-4}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 4)/(b*x + a)^n, x)
```

**maple [A]** time = 0.01, size = 77, normalized size = 0.70

$$\frac{(bx+a)\left(a^2n^2+2abnx+2b^2x^2-3a^2n-2abx+2a^2\right)x^{n-3}\,(bx+a)^{-n}}{(n-3)\,(n-2)\,(n-1)\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4+n)/((b*x+a)^n),x)
```

```
[Out] x^(-3+n)*(b*x+a)*(a^2*n^2+2*a*b*n*x+2*b^2*x^2-3*a^2*n-2*a*b*x+2*a^2)/((b*x+
a)^n)/(-3+n)/(-2+n)/(-1+n)/a^3
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-4}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 4)/(b*x + a)^n, x)
```

**mupad [B]** time = 0.52, size = 136, normalized size = 1.24

$$\frac{\frac{x\,x^{n-4}\,\left(n^2-3\,n+2\right)}{n^3-6\,n^2+11\,n-6}+\frac{2\,b^3\,x^{n-4}\,x^4}{a^3\,\left(n^3-6\,n^2+11\,n-6\right)}+\frac{2\,b^2\,n\,x^{n-4}\,x^3}{a^2\,\left(n^3-6\,n^2+11\,n-6\right)}+\frac{b\,n\,x^{n-4}\,x^2\,(n-1)}{a\,\left(n^3-6\,n^2+11\,n-6\right)}}{(a+b\,x)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 4)/(a + b*x)^n,x)
```

```
[Out] ((x*x^(n - 4)*(n^2 - 3*n + 2))/(11*n - 6*n^2 + n^3 - 6) + (2*b^3*x^(n - 4)*
x^4)/(a^3*(11*n - 6*n^2 + n^3 - 6)) + (2*b^2*n*x^(n - 4)*x^3)/(a^2*(11*n -
```

```
6*n^2 + n^3 - 6)) + (b*n*x^(n - 4)*x^2*(n - 1))/(a*(11*n - 6*n^2 + n^3 - 6)
))/(a + b*x)^n
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4+n)/((b*x+a)**n),x)
```

```
[Out] Timed out
```

## 3.709 $\int x^{-3+n}(a+bx)^{-n}\,dx$

**Optimal**. Leaf size=64

$$\frac{bx^{n-1}(a+bx)^{1-n}}{a^2(1-n)(2-n)}-\frac{x^{n-2}(a+bx)^{1-n}}{a(2-n)}$$

**Rubi [A]** time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{bx^{n-1}(a+bx)^{1-n}}{a^2(1-n)(2-n)}-\frac{x^{n-2}(a+bx)^{1-n}}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-2 + n)*(a + b*x)^(1 - n))/(a*(2 - n))) + (b*x^(-1 + n)*(a + b*x)^(1
- n))/(a^2*(1 - n)*(2 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{-3+n}(a+bx)^{-n}\,dx&=-\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}-\frac{b\int x^{-2+n}(a+bx)^{-n}\,dx}{a(2-n)}\\&=-\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}+\frac{bx^{-1+n}(a+bx)^{1-n}}{a^2(1-n)(2-n)}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 39, normalized size = 0.61

$$\frac{x^{n-2}(a+bx)^{1-n}(a(n-1)+bx)}{a^2(n-2)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + n)/(a + b*x)^n,x]
[Out] (x^(-2 + n)*(a + b*x)^(1 - n)*(a*(-1 + n) + b*x))/(a^2*(-2 + n)*(-1 + n))
```

**IntegrateAlgebraic** [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^{-3+n}(a+bx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-3 + n)/(a + b*x)^n,x]
[Out] Defer[IntegrateAlgebraic][x^(-3 + n)/(a + b*x)^n, x]
```

**fricas** [A] time = 1.23, size = 64, normalized size = 1.00

$$\frac{\left(abnx^2+b^2x^3+\left(a^2n-a^2\right)x\right)x^{n-3}}{\left(a^2n^2-3\,a^2n+2\,a^2\right)(bx+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="fricas")
[Out] (a*b*n*x^2 + b^2*x^3 + (a^2*n - a^2)*x)*x^(n - 3)/((a^2*n^2 - 3*a^2*n + 2*a
^2)*(b*x + a)^n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{n-3}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="giac")
[Out] integrate(x^(n - 3)/(b*x + a)^n, x)
```

**maple** [A] time = 0.00, size = 44, normalized size = 0.69

$$\frac{(an+bx-a)\,(bx+a)\,x^{n-2}\,(bx+a)^{-n}}{(n-2)\,(n-1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-3)/((b*x+a)^n),x)
```

```
[Out] x^(n-2)*(a*n+b*x-a)*(b*x+a)/((b*x+a)^n)/(n-2)/(n-1)/a^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-3}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 3)/(b*x + a)^n, x)
```

**mupad [B]** time = 0.45, size = 80, normalized size = 1.25

$$\frac{\frac{x\,x^{n-3}\,(n-1)}{n^2-3\,n+2}+\frac{b^2\,x^{n-3}\,x^3}{a^2\,\left(n^2-3\,n+2\right)}+\frac{b\,n\,x^{n-3}\,x^2}{a\,\left(n^2-3\,n+2\right)}}{{(a+b\,x)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 3)/(a + b*x)^n,x)
```

```
[Out] ((x*x^(n - 3)*(n - 1))/(n^2 - 3*n + 2) + (b^2*x^(n - 3)*x^3)/(a^2*(n^2 - 3*
n + 2)) + (b*n*x^(n - 3)*x^2)/(a*(n^2 - 3*n + 2)))/(a + b*x)^n
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+n)/((b*x+a)**n),x)
```

```
[Out] Timed out
```

## 3.710 $\int x^{-2+n}(a+bx)^{-n}\,dx$

**Optimal**. Leaf size=28

$$-\frac{x^{n-1}(a+bx)^{1-n}}{a(1-n)}$$

**Rubi [A]** time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{x^{n-1}(a+bx)^{1-n}}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + n)/(a + b*x)^n,x]
[Out] -((x^(-1 + n)*(a + b*x)^(1 - n))/(a*(1 - n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int x^{-2+n}(a+bx)^{-n}\,dx=-\frac{x^{-1+n}(a+bx)^{1-n}}{a(1-n)}$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 0.89

$$\frac{x^{n-1}(a+bx)^{1-n}}{a(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + n)/(a + b*x)^n,x]
[Out] (x^(-1 + n)*(a + b*x)^(1 - n))/(a*(-1 + n))
```

**IntegrateAlgebraic [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int x^{-2+n}(a+bx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-2 + n)/(a + b*x)^n,x]
[Out] Defer[IntegrateAlgebraic][x^(-2 + n)/(a + b*x)^n, x]
```

**fricas [A]** time = 0.97, size = 33, normalized size = 1.18

$$\frac{\left(bx^2+ax\right)x^{n-2}}{(an-a)(bx+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="fricas")
[Out] (b*x^2 + a*x)*x^(n - 2)/((a*n - a)*(b*x + a)^n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-2}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="giac")
[Out] integrate(x^(n - 2)/(b*x + a)^n, x)
```

**maple [A]** time = 0.00, size = 29, normalized size = 1.04

$$\frac{(bx+a)\,x^{n-1}\,(bx+a)^{-n}}{(n-1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-2)/((b*x+a)^n),x)
[Out] (b*x+a)*x^(n-1)/a/(n-1)/((b*x+a)^n)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-2}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 2)/(b*x + a)^n, x)
```

**mupad [B]** time = 0.35, size = 29, normalized size = 1.04

$$\frac{x^n\,\left(a+b\,x\right)}{a\,x\,\left(n-1\right)\,{\left(a+b\,x\right)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 2)/(a + b*x)^n,x)
```

```
[Out] (x^n*(a + b*x))/(a*x*(n - 1)*(a + b*x)^n)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+n)/((b*x+a)**n),x)
```

```
[Out] Timed out
```

## 3.711 $\int x^{-1+n}(a+bx)^{-1-n}\,dx$

**Optimal**. Leaf size=19

$$\frac{x^n(a+bx)^{-n}}{an}$$

**Rubi** [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{x^n(a+bx)^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] x^n/(a*n*(a + b*x)^n)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int x^{-1+n}(a+bx)^{-1-n}\,dx=\frac{x^n(a+bx)^{-n}}{an}$$

**Mathematica** [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^n(a+bx)^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] x^n/(a*n*(a + b*x)^n)
```

**IntegrateAlgebraic** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^{-1+n}(a+bx)^{-1-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n)*(a + b*x)^(-1 - n), x]
```

**fricas [A]** time = 0.95, size = 32, normalized size = 1.68

$$\frac{\left(bx^2+ax\right)(bx+a)^{-n-1}x^{n-1}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^(-n - 1)*x^(n - 1)/(a*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-1}x^{n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 1)*x^(n - 1), x)
```

**maple [A]** time = 0.00, size = 20, normalized size = 1.05

$$\frac{x^n\,(bx+a)^{-n}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b*x+a)^(-1-n),x)
```

```
[Out] (b*x+a)^(-n)*x^n/a/n
```

**maxima [A]** time = 1.30, size = 22, normalized size = 1.16

$$\frac{e^{\left(-n\log(bx+a)+n\log(x)\right)}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="maxima")
```

```
[Out] e^(-n*log(b*x + a) + n*log(x))/(a*n)
```

**mupad [B]** time = 0.50, size = 19, normalized size = 1.00

$$\frac{x^n}{a\,n\,(a+b\,x)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x)^(n + 1),x)
[Out] x^n/(a*n*(a + b*x)^n)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b*x+a)**(-1-n),x)
[Out] Timed out
```

## 3.712 $\int x^{-3-n}(a+bx)^n\,dx$

**Optimal**. Leaf size=58

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

**Rubi [A]** time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - n)*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{-3-n}(a+bx)^n\,dx&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-n}(a+bx)^n\,dx}{a(2+n)}\\&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 0.69

$$-\frac{x^{-n-2}(an+a-bx)(a+bx)^{n+1}}{a^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - n)*(a + b*x)^n,x]
[Out] -((x^(-2 - n)*(a + a*n - b*x)*(a + b*x)^(1 + n))/(a^2*(1 + n)*(2 + n)))
```

**IntegrateAlgebraic** [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^{-3-n}(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-3 - n)*(a + b*x)^n,x]
[Out] Defer[IntegrateAlgebraic][x^(-3 - n)*(a + b*x)^n, x]
```

**fricas** [A] time = 1.15, size = 64, normalized size = 1.10

$$-\frac{\left(abnx^2-b^2x^3+\left(a^2n+a^2\right)x\right)(bx+a)^nx^{-n-3}}{a^2n^2+3\,a^2n+2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="fricas")
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x + a)^n*x^(-n - 3)/(a^2*n^2 +
3*a^2*n + 2*a^2)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^nx^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="giac")
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

**maple** [A] time = 0.00, size = 41, normalized size = 0.71

$$-\frac{(an-bx+a)\,x^{-n-2}\,(bx+a)^{n+1}}{(n+2)\,(n+1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-n)*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(n+1)*x^(-2-n)*(a*n-b*x+a)/(n+2)/(n+1)/a^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n x^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

**mupad [B]** time = 0.50, size = 86, normalized size = 1.48

$$-(a+b\,x)^n\left(\frac{x\,(n+1)}{x^{n+3}\,\left(n^2+3\,n+2\right)}-\frac{b^2\,x^3}{a^2\,x^{n+3}\,\left(n^2+3\,n+2\right)}+\frac{b\,n\,x^2}{a\,x^{n+3}\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(n + 3),x)
```

```
[Out] -(a + b*x)^n*((x*(n + 1))/(x^(n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(a^2*x^(n
 + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(n + 3)*(3*n + n^2 + 2)))
```

**sympy [A]** time = 94.82, size = 323, normalized size = 5.57

$$\begin{cases} -\frac{b^n}{2x^2} & \text{for}\ a=0\\ \frac{a\log(x)}{a^3+a^2bx}-\frac{a\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx}+\frac{a}{a^3+a^2bx}+\frac{bx\log(x)}{a^3+a^2bx}-\frac{bx\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} & \text{for}\ n=-2\\ -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log\left(\frac{a}{b}+x\right)}{a^2} & \text{for}\ n=-1\\ -\frac{a^2n(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n}-\frac{a^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n}-\frac{abnx(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n}+\frac{b^2x^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-n)*(b*x+a)**n,x)
```

```
[Out] Piecewise((-b**n/(2*x**2), Eq(a, 0)), (a*log(x)/(a**3 + a**2*b*x) - a*log(a
/b + x)/(a**3 + a**2*b*x) + a/(a**3 + a**2*b*x) + b*x*log(x)/(a**3 + a**2*b
*x) - b*x*log(a/b + x)/(a**3 + a**2*b*x), Eq(n, -2)), (-1/(a*x) - b*log(x)/
a**2 + b*log(a/b + x)/a**2, Eq(n, -1)), (-a**2*n*(a + b*x)**n/(a**2*n**2*x*
*2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a**2*(a + b*x)**n/(a**2*
```

```
n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a*b*n*x*(a + b*x)
**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) + b**2*x*
*2*(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**
n), True))
```

## 3.713 $\int x^{2n-3(1+n)}(a+bx)^n\,dx$

**Optimal**. Leaf size=58

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

**Rubi [A]** time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{2n-3(1+n)}(a+bx)^n\,dx&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-n}(a+bx)^n\,dx}{a(2+n)}\\&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 40, normalized size = 0.69

$$-\frac{x^{-n-2}(an+a-bx)(a+bx)^{n+1}}{a^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + a*n - b*x)*(a + b*x)^(1 + n))/(a^2*(1 + n)*(2 + n)))
```

**IntegrateAlgebraic** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^{2n-3(1+n)}(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(2*n - 3*(1 + n))*(a + b*x)^n, x]
```

**fricas** [A] time = 1.02, size = 64, normalized size = 1.10

$$-\frac{\left(abnx^2-b^2x^3+\left(a^2n+a^2\right)x\right)(bx+a)^nx^{-n-3}}{a^2n^2+3\,a^2n+2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x + a)^n*x^(-n - 3)/(a^2*n^2 +
3*a^2*n + 2*a^2)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^nx^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

**maple** [A] time = 0.00, size = 41, normalized size = 0.71

$$-\frac{(an-bx+a)\,x^{-n-2}\,(bx+a)^{n+1}}{(n+2)\,(n+1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-n)*(b*x+a)^n,x)
```

```
[Out] -(a*n-b*x+a)/(n+2)/(n+1)/a^2*x^(-n-2)*(b*x+a)^(n+1)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n x^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

**mupad [B]** time = 0.00, size = 86, normalized size = 1.48

$$-(a+b\,x)^n\left(\frac{x\,(n+1)}{x^{n+3}\,\left(n^2+3\,n+2\right)}-\frac{b^2\,x^3}{a^2\,x^{n+3}\,\left(n^2+3\,n+2\right)}+\frac{b\,n\,x^2}{a\,x^{n+3}\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(n + 3),x)
```

```
[Out] -(a + b*x)^n*((x*(n + 1))/(x^(n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(a^2*x^(n
 + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(n + 3)*(3*n + n^2 + 2)))
```

**sympy [A]** time = 94.49, size = 323, normalized size = 5.57

$$\begin{cases} -\frac{b^n}{2x^2} & \text{for}\ a=0 \\ \frac{a\log(x)}{a^3+a^2bx}-\frac{a\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx}+\frac{a}{a^3+a^2bx}+\frac{bx\log(x)}{a^3+a^2bx}-\frac{bx\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} & \text{for}\ n=-2 \\ -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log\left(\frac{a}{b}+x\right)}{a^2} & \text{for}\ n=-1 \\ -\frac{a^2n(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n}-\frac{a^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n}-\frac{abnx(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n}+\frac{b^2x^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-n)*(b*x+a)**n,x)
```

```
[Out] Piecewise((-b**n/(2*x**2), Eq(a, 0)), (a*log(x)/(a**3 + a**2*b*x) - a*log(a
/b + x)/(a**3 + a**2*b*x) + a/(a**3 + a**2*b*x) + b*x*log(x)/(a**3 + a**2*b
*x) - b*x*log(a/b + x)/(a**3 + a**2*b*x), Eq(n, -2)), (-1/(a*x) - b*log(x)/
a**2 + b*log(a/b + x)/a**2, Eq(n, -1)), (-a**2*n*(a + b*x)**n/(a**2*n**2*x*
*2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a**2*(a + b*x)**n/(a**2*
```

```
n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a*b*n*x*(a + b*x)
**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) + b**2*x*
*2*(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**
n), True))
```

# 3.714 $\int x^3\sqrt{cx^2}\,(a+bx)\,dx$

**Optimal**. Leaf size=35

$$\frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^4*Sqrt[c*x^2])/5 + (b*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x^4(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(ax^4+bx^5\right)\,dx}{x}\\ &= \frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{30}x^4\sqrt{cx^2}\,(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^4*Sqrt[c*x^2]*(6*a + 5*b*x))/30
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.69

$$\frac{1}{30}x^4\sqrt{cx^2}\,(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^4*Sqrt[c*x^2]*(6*a + 5*b*x))/30
```

**fricas** [A] time = 0.72, size = 22, normalized size = 0.63

$$\frac{1}{30}\left(5\,bx^5+6\,ax^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/30*(5*b*x^5 + 6*a*x^4)*sqrt(c*x^2)
```

**giac** [A] time = 0.89, size = 22, normalized size = 0.63

$$\frac{1}{30}\left(5\,bx^6\mathrm{sgn}(x)+6\,ax^5\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.01, size = 21, normalized size = 0.60

$$\frac{(5bx+6a)\sqrt{c\,x^2}\;x^4}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)*(c*x^2)^(1/2),x)
```

```
[Out] 1/30*x^4*(5*b*x+6*a)*(c*x^2)^(1/2)
```

**maxima [A]** time = 1.33, size = 33, normalized size = 0.94

$$\frac{\left(cx^2\right)^{\frac{3}{2}}bx^3}{6\,c}+\frac{\left(cx^2\right)^{\frac{3}{2}}ax^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(3/2)*b*x^3/c + 1/5*(c*x^2)^(3/2)*a*x^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int x^3\,\sqrt{c\,x^2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] int(x^3*(c*x^2)^(1/2)*(a + b*x), x)
```

**sympy [A]** time = 0.43, size = 36, normalized size = 1.03

$$\frac{a\sqrt{c}\,x^4\sqrt{x^2}}{5}+\frac{b\sqrt{c}\,x^5\sqrt{x^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x**4*sqrt(x**2)/5 + b*sqrt(c)*x**5*sqrt(x**2)/6
```

## 3.715 $\int x^2\sqrt{cx^2}\,(a+bx)\,dx$

**Optimal**. Leaf size=35

$$\frac{1}{4}ax^3\sqrt{cx^2}\,+\frac{1}{5}bx^4\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{4}ax^3\sqrt{cx^2}\,+\frac{1}{5}bx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^3*Sqrt[c*x^2])/4 + (b*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\,\int x^3(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,\int \left(ax^3+bx^4\right)\,dx}{x}\\ &= \frac{1}{4}ax^3\sqrt{cx^2}\,+\frac{1}{5}bx^4\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{20}x^3\sqrt{cx^2}\,(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^3*Sqrt[c*x^2]*(5*a + 4*b*x))/20
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.69

$$\frac{1}{20}x^3\sqrt{cx^2}\,(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^3*Sqrt[c*x^2]*(5*a + 4*b*x))/20
```

**fricas** [A] time = 0.97, size = 22, normalized size = 0.63

$$\frac{1}{20}\left(4\,bx^4+5\,ax^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/20*(4*b*x^4 + 5*a*x^3)*sqrt(c*x^2)
```

**giac** [A] time = 1.03, size = 22, normalized size = 0.63

$$\frac{1}{20}\left(4\,bx^5\mathrm{sgn}(x)+5\,ax^4\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{(4bx+5a)\sqrt{c\,x^2}\;x^3}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)*(c*x^2)^(1/2),x)
```

```
[Out] 1/20*x^3*(4*b*x+5*a)*(c*x^2)^(1/2)
```

**maxima [A]** time = 1.37, size = 31, normalized size = 0.89

$$\frac{\left(cx^2\right)^{\frac{3}{2}}bx^2}{5\,c}+\frac{\left(cx^2\right)^{\frac{3}{2}}ax}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(c*x^2)^(3/2)*b*x^2/c + 1/4*(c*x^2)^(3/2)*a*x/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\,\sqrt{c\,x^2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] int(x^2*(c*x^2)^(1/2)*(a + b*x), x)
```

**sympy [A]** time = 0.34, size = 36, normalized size = 1.03

$$\frac{a\sqrt{c}\,x^3\sqrt{x^2}}{4}+\frac{b\sqrt{c}\,x^4\sqrt{x^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x**3*sqrt(x**2)/4 + b*sqrt(c)*x**4*sqrt(x**2)/5
```

## 3.716 $\int x\sqrt{cx^2}\,(a+bx)\,dx$

**Optimal**. Leaf size=35

$$\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^2*Sqrt[c*x^2])/3 + (b*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}\,(a+bx)\,dx&=\frac{\sqrt{cx^2}\int x^2(a+bx)\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(ax^2+bx^3\right)\,dx}{x}\\&=\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{12}x^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.69

$$\frac{1}{12}x^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

**fricas** [A] time = 0.97, size = 22, normalized size = 0.63

$$\frac{1}{12}\left(3\,bx^3+4\,ax^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)
```

**giac** [A] time = 0.85, size = 22, normalized size = 0.63

$$\frac{1}{12}\left(3\,bx^4\mathrm{sgn}(x)+4\,ax^3\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{(3bx+4a)\sqrt{c\,x^2}\;x^2}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*(c*x^2)^(1/2),x)
```

```
[Out] 1/12*x^2*(3*b*x+4*a)*(c*x^2)^(1/2)
```

**maxima [A]** time = 1.35, size = 28, normalized size = 0.80

$$\frac{\left(cx^2\right)^{\frac{3}{2}}bx}{4\,c}+\frac{\left(cx^2\right)^{\frac{3}{2}}a}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2)^(3/2)*b*x/c + 1/3*(c*x^2)^(3/2)*a/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int x\,\sqrt{c\,x^2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] int(x*(c*x^2)^(1/2)*(a + b*x), x)
```

**sympy [A]** time = 0.27, size = 36, normalized size = 1.03

$$\frac{a\sqrt{c}\,x^2\sqrt{x^2}}{3}+\frac{b\sqrt{c}\,x^3\sqrt{x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x**2*sqrt(x**2)/3 + b*sqrt(c)*x**3*sqrt(x**2)/4
```

# 3.717 $\int \sqrt{cx^2}\,(a+bx)\,dx$

**Optimal**. Leaf size=33

$$\frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x*Sqrt[c*x^2])/2 + (b*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(ax+bx^2\right)\,dx}{x}\\ &= \frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 22, normalized size = 0.67

$$\frac{1}{6}x\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x),x]
[Out] (x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

**IntegrateAlgebraic** [A] time = 0.02, size = 22, normalized size = 0.67

$$\frac{1}{6}x\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]*(a + b*x),x]
[Out] (x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

**fricas** [A] time = 0.95, size = 20, normalized size = 0.61

$$\frac{1}{6}\left(2\,bx^2+3\,ax\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)
```

**giac** [A] time = 1.05, size = 22, normalized size = 0.67

$$\frac{1}{6}\left(2\,bx^3\mathrm{sgn}(x)+3\,ax^2\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.01, size = 19, normalized size = 0.58

$$\frac{(2bx+3a)\sqrt{c\,x^2}\;x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2),x)
```

```
[Out] 1/6*x*(2*b*x+3*a)*(c*x^2)^(1/2)
```

**maxima** [A] time = 1.32, size = 25, normalized size = 0.76

$$\frac{1}{2}\sqrt{cx^2}\,ax + \frac{\left(cx^2\right)^{\frac{3}{2}}b}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2)*a*x + 1/3*(c*x^2)^(3/2)*b/c
```

**mupad** [B] time = 0.54, size = 20, normalized size = 0.61

$$\frac{\sqrt{c}\,\left(2\,b\,\sqrt{x^6}+3\,a\,x\,|x|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] (c^(1/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

**sympy** [A] time = 0.22, size = 34, normalized size = 1.03

$$\frac{a\sqrt{c}\,x\sqrt{x^2}}{2} + \frac{b\sqrt{c}\,x^2\sqrt{x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x*sqrt(x**2)/2 + b*sqrt(c)*x**2*sqrt(x**2)/3
```

# 3.718 $\int \frac{\sqrt{cx^2}(a+bx)}{x} dx$

**Optimal**. Leaf size=27

$$a\sqrt{cx^2} + \frac{1}{2}bx\sqrt{cx^2}$$

**Rubi [A]** time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.056, Rules used = {15}

$$a\sqrt{cx^2} + \frac{1}{2}bx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

```
[Out] a*Sqrt[c*x^2] + (b*x*Sqrt[c*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)}{x} dx &= \frac{\sqrt{cx^2} \int (a+bx) dx}{x} \\ &= a\sqrt{cx^2} + \frac{1}{2}bx\sqrt{cx^2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 24, normalized size = 0.89

$$\frac{cx^2(2a+bx)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

```
[Out] (c*x^2*(2*a + b*x))/(2*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.02, size = 20, normalized size = 0.74

$$\frac{1}{2}\sqrt{cx^2}\,(2a+bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

```
[Out] (Sqrt[c*x^2]*(2*a + b*x))/2
```

**fricas** [A] time = 0.92, size = 16, normalized size = 0.59

$$\frac{1}{2}\,\sqrt{cx^2}\,(bx+2\,a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)
```

**giac** [A] time = 1.09, size = 17, normalized size = 0.63

$$\frac{1}{2}\left(bx^2+2\,ax\right)\sqrt{c}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*sqrt(c)*sgn(x)
```

**maple** [A] time = 0.00, size = 17, normalized size = 0.63

$$\frac{(bx+2a)\,\sqrt{c\,x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x,x)
```

```
[Out] 1/2*(b*x+2*a)*(c*x^2)^(1/2)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad** [**B**] time = 0.19, size = 14, normalized size = 0.52

$$\frac{\sqrt{c}\ |x|\ (2\,a + b\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x,x)
```

```
[Out] (c^(1/2)*abs(x)*(2*a + b*x))/2
```

**sympy** [**A**] time = 0.23, size = 29, normalized size = 1.07

$$a\sqrt{c}\,\sqrt{x^2} + \frac{b\sqrt{c}\,x\sqrt{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x,x)
```

```
[Out] a*sqrt(c)*sqrt(x**2) + b*sqrt(c)*x*sqrt(x**2)/2
```

## 3.719 $\int \frac{\sqrt{cx^2}(a+bx)}{x^2} dx$

**Optimal**. Leaf size=28

$$\frac{a\sqrt{cx^2}\log(x)}{x}+b\sqrt{cx^2}$$

**Rubi** [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{a\sqrt{cx^2}\log(x)}{x}+b\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
```

```
[Out] b*Sqrt[c*x^2] + (a*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)}{x^2}dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x}dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(b+\frac{a}{x}\right)dx}{x}\\
&= b\sqrt{cx^2}+\frac{a\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 20, normalized size = 0.71

$$\frac{cx(a\log(x)+bx)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
[Out] (c*x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

**IntegrateAlgebraic** [A] time = 0.02, size = 19, normalized size = 0.68

$$\sqrt{cx^2}\left(\frac{a\log(x)}{x}+b\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
[Out] Sqrt[c*x^2]*(b + (a*Log[x])/x)
```

**fricas** [A] time = 1.27, size = 19, normalized size = 0.68

$$\frac{\sqrt{cx^2}\left(bx+a\log(x)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x + a*log(x))/x
```

**giac** [A] time = 0.92, size = 17, normalized size = 0.61

$$\left(bx\operatorname{sgn}(x)+a\log\left(|x|\right)\operatorname{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="giac")
[Out] (b*x*sgn(x) + a*log(abs(x))*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.02, size = 20, normalized size = 0.71

$$\frac{\sqrt{c\,x^2}\;\left(a\ln(x)+bx\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^2,x)
```

```
[Out] (c*x^2)^(1/2)/x*(a*ln(x)+b*x)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^2,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x))/x^2, x)
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)/x**2, x)
```

## 3.720 $\int \frac{\sqrt{cx^2}(a+bx)}{x^3} dx$

**Optimal**. Leaf size=32

$$\frac{b\sqrt{cx^2}\log(x)}{x} - \frac{a\sqrt{cx^2}}{x^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{b\sqrt{cx^2}\log(x)}{x} - \frac{a\sqrt{cx^2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
```

```
[Out] -((a*Sqrt[c*x^2])/x^2) + (b*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)}{x^3} dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x^2} dx}{x} \\
&= \frac{\sqrt{cx^2}\int \left(\frac{a}{x^2}+\frac{b}{x}\right) dx}{x} \\
&= -\frac{a\sqrt{cx^2}}{x^2} + \frac{b\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 20, normalized size = 0.62

$$\frac{c(bx\log(x)-a)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
[Out] (c*(-a + b*x*Log[x]))/Sqrt[c*x^2]
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.75

$$\sqrt{cx^2}\left(\frac{b\log(x)}{x}-\frac{a}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
[Out] Sqrt[c*x^2]*(-(a/x^2) + (b*Log[x])/x)
```

**fricas** [A] time = 1.23, size = 20, normalized size = 0.62

$$\frac{\sqrt{cx^2}\left(bx\log(x)-a\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/x^2
```

**giac** [A] time = 1.10, size = 20, normalized size = 0.62

$$\left(b\log\left(|x|\right)\operatorname{sgn}(x)-\frac{a\operatorname{sgn}(x)}{x}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="giac")
[Out] (b*log(abs(x))*sgn(x) - a*sgn(x)/x)*sqrt(c)
```

**maple** [A] time = 0.01, size = 21, normalized size = 0.66

$$\frac{\sqrt{c\,x^2}\,\left(bx\ln(x)-a\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^3,x)
```

```
[Out] (c*x^2)^(1/2)*(b*ln(x)*x-a)/x^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^3,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x))/x^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2}} \left(a + bx\right)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)/x**3, x)
```

## 3.721 $\int \frac{\sqrt{cx^2}(a+bx)}{x^4}\,dx$

**Optimal**. Leaf size=26

$$-\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}$$

**Rubi** [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
```

```
[Out] -(Sqrt[c*x^2]*(a + b*x)^2)/(2*a*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)}{x^4}\,dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x^3}\,dx}{x}\\ &= -\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 22, normalized size = 0.85

$$-\frac{\sqrt{cx^2}(a+2bx)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
[Out] -1/2*(Sqrt[c*x^2]*(a + 2*b*x))/x^3
```

**IntegrateAlgebraic [A]** time = 0.02, size = 24, normalized size = 0.92

$$\frac{\sqrt{cx^2}\,(-a-2bx)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
[Out] (Sqrt[c*x^2]*(-a - 2*b*x))/(2*x^3)
```

**fricas [A]** time = 1.06, size = 18, normalized size = 0.69

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/x^3
```

**giac [A]** time = 0.97, size = 19, normalized size = 0.73

$$-\frac{\left(2\,bx\mathrm{sgn}(x)+a\mathrm{sgn}(x)\right)\sqrt{c}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="giac")
[Out] -1/2*(2*b*x*sgn(x) + a*sgn(x))*sqrt(c)/x^2
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.73

$$-\frac{(2bx+a)\,\sqrt{c\,x^2}}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(2*b*x+a)*(c*x^2)^(1/2)/x^3
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [B]** time = 0.14, size = 28, normalized size = 1.08

$$-\frac{a\,\sqrt{c}\,x^2+2\,b\,\sqrt{c}\,x^3}{2\,x\,{\left(x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^4,x)
```

```
[Out] -(a*c^(1/2)*x^2 + 2*b*c^(1/2)*x^3)/(2*x*(x^2)^(3/2))
```

**sympy [A]** time = 0.51, size = 36, normalized size = 1.38

$$-\frac{a\sqrt{c}\,\sqrt{x^2}}{2x^3}-\frac{b\sqrt{c}\,\sqrt{x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**4,x)
```

```
[Out] -a*sqrt(c)*sqrt(x**2)/(2*x**3) - b*sqrt(c)*sqrt(x**2)/x**2
```

# 3.722 $\int x^3 \left(cx^2\right)^{3/2} (a+bx)\,dx$

**Optimal**. Leaf size=37

$$\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^6*Sqrt[c*x^2])/7 + (b*c*x^7*Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(cx^2\right)^{3/2}(a+bx)\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^6(a+bx)\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(ax^6+bx^7\right)\,dx}{x}\\&=\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.65

$$\frac{1}{56}x^4\left(cx^2\right)^{3/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^4*(c*x^2)^(3/2)*(8*a + 7*b*x))/56
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.65

$$\frac{1}{56}x^4\left(cx^2\right)^{3/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^4*(c*x^2)^(3/2)*(8*a + 7*b*x))/56
```

**fricas** [A] time = 0.91, size = 24, normalized size = 0.65

$$\frac{1}{56}\left(7\,bcx^7+8\,acx^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/56*(7*b*c*x^7 + 8*a*c*x^6)*sqrt(c*x^2)
```

**giac** [A] time = 1.17, size = 22, normalized size = 0.59

$$\frac{1}{56}\left(7\,bx^8\mathrm{sgn}(x)+8\,ax^7\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
[Out] 1/56*(7*b*x^8*sgn(x) + 8*a*x^7*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.57

$$\frac{(7bx+8a)\left(c\,x^2\right)^{\frac{3}{2}}x^4}{56}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/56*x^4*(7*b*x+8*a)*(c*x^2)^(3/2)
```

**maxima [A]** time = 1.33, size = 33, normalized size = 0.89

$$\frac{\left(cx^2\right)^{\frac{5}{2}} bx^3}{8\,c} + \frac{\left(cx^2\right)^{\frac{5}{2}} ax^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2)^(5/2)*b*x^3/c + 1/7*(c*x^2)^(5/2)*a*x^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int x^3 \left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] int(x^3*(c*x^2)^(3/2)*(a + b*x), x)
```

**sympy [A]** time = 1.16, size = 36, normalized size = 0.97

$$\frac{ac^{\frac{3}{2}}x^4\left(x^2\right)^{\frac{3}{2}}}{7} + \frac{bc^{\frac{3}{2}}x^5\left(x^2\right)^{\frac{3}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x**4*(x**2)**(3/2)/7 + b*c**(3/2)*x**5*(x**2)**(3/2)/8
```

## 3.723 $\int x^2 \left(cx^2\right)^{3/2} (a+bx)\,dx$

**Optimal**. Leaf size=37

$$\frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^5*Sqrt[c*x^2])/6 + (b*c*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2 \left(cx^2\right)^{3/2} (a+bx)\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^5(a+bx)\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^5+bx^6\right)\,dx}{x}\\ &= \frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.65

$$\frac{1}{42}x^3\left(cx^2\right)^{3/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^3*(c*x^2)^(3/2)*(7*a + 6*b*x))/42
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.65

$$\frac{1}{42}x^3\left(cx^2\right)^{3/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^3*(c*x^2)^(3/2)*(7*a + 6*b*x))/42
```

**fricas** [A] time = 1.08, size = 24, normalized size = 0.65

$$\frac{1}{42}\left(6\,bcx^6+7\,acx^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/42*(6*b*c*x^6 + 7*a*c*x^5)*sqrt(c*x^2)
```

**giac** [A] time = 0.93, size = 22, normalized size = 0.59

$$\frac{1}{42}\left(6\,bx^7\mathrm{sgn}(x)+7\,ax^6\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
[Out] 1/42*(6*b*x^7*sgn(x) + 7*a*x^6*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.57

$$\frac{(6bx+7a)\left(c\,x^2\right)^{\frac{3}{2}}x^3}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/42*x^3*(6*b*x+7*a)*(c*x^2)^(3/2)
```

**maxima [A]** time = 1.34, size = 31, normalized size = 0.84

$$\frac{\left(cx^2\right)^{\frac{5}{2}}bx^2}{7c}+\frac{\left(cx^2\right)^{\frac{5}{2}}ax}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2)^(5/2)*b*x^2/c + 1/6*(c*x^2)^(5/2)*a*x/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] int(x^2*(c*x^2)^(3/2)*(a + b*x), x)
```

**sympy [A]** time = 0.91, size = 36, normalized size = 0.97

$$\frac{ac^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}{6}+\frac{bc^{\frac{3}{2}}x^4\left(x^2\right)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x**3*(x**2)**(3/2)/6 + b*c**(3/2)*x**4*(x**2)**(3/2)/7
```

## 3.724 $\int x\left(cx^2\right)^{3/2}(a+bx)\,dx$

**Optimal**. Leaf size=37

$$\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^4*Sqrt[c*x^2])/5 + (b*c*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{3/2}(a+bx)\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(ax^4+bx^5\right)\,dx}{x}\\&=\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.65

$$\frac{1}{30}x^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.65

$$\frac{1}{30}x^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

**fricas** [A] time = 0.86, size = 24, normalized size = 0.65

$$\frac{1}{30}\left(5\,bcx^5+6\,acx^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/30*(5*b*c*x^5 + 6*a*c*x^4)*sqrt(c*x^2)
```

**giac** [A] time = 0.99, size = 22, normalized size = 0.59

$$\frac{1}{30}\left(5\,bx^6\mathrm{sgn}(x)+6\,ax^5\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
[Out] 1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.57

$$\frac{(5bx+6a)\left(c\,x^2\right)^{\frac{3}{2}}x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/30*x^2*(5*b*x+6*a)*(c*x^2)^(3/2)
```

**maxima [A]** time = 1.35, size = 28, normalized size = 0.76

$$\frac{\left(cx^2\right)^{\frac{5}{2}} bx}{6\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}} a}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(5/2)*b*x/c + 1/5*(c*x^2)^(5/2)*a/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int x\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] int(x*(c*x^2)^(3/2)*(a + b*x), x)
```

**sympy [A]** time = 0.73, size = 36, normalized size = 0.97

$$\frac{ac^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}{5}+\frac{bc^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x**2*(x**2)**(3/2)/5 + b*c**(3/2)*x**3*(x**2)**(3/2)/6
```

## 3.725 $\int \left(cx^2\right)^{3/2} (a + bx)\, dx$

**Optimal**. Leaf size=37

$$\frac{1}{4}acx^3\sqrt{cx^2} + \frac{1}{5}bcx^4\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{1}{4}acx^3\sqrt{cx^2} + \frac{1}{5}bcx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^3*Sqrt[c*x^2])/4 + (b*c*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(cx^2\right)^{3/2} (a + bx)\, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(a + bx)\, dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^3 + bx^4\right)\, dx}{x} \\
&= \frac{1}{4}acx^3\sqrt{cx^2} + \frac{1}{5}bcx^4\sqrt{cx^2}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 22, normalized size = 0.59

$$\frac{1}{20}x\left(cx^2\right)^{3/2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

**IntegrateAlgebraic** [A] time = 0.02, size = 22, normalized size = 0.59

$$\frac{1}{20}x\left(cx^2\right)^{3/2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

**fricas** [A] time = 0.83, size = 24, normalized size = 0.65

$$\frac{1}{20}\left(4\,bcx^4+5\,acx^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/20*(4*b*c*x^4 + 5*a*c*x^3)*sqrt(c*x^2)
```

**giac** [A] time = 0.88, size = 22, normalized size = 0.59

$$\frac{1}{20}\left(4\,bx^5\mathrm{sgn}(x)+5\,ax^4\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
[Out] 1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 19, normalized size = 0.51

$$\frac{(4bx+5a)\left(c\,x^2\right)^{\frac{3}{2}}x}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/20*x*(4*b*x+5*a)*(c*x^2)^(3/2)
```

**maxima [A]** time = 1.30, size = 25, normalized size = 0.68

$$\frac{1}{4}\left(cx^{2}\right)^{\frac{3}{2}}ax+\frac{\left(cx^{2}\right)^{\frac{5}{2}}b}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2)^(3/2)*a*x + 1/5*(c*x^2)^(5/2)*b/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] int((c*x^2)^(3/2)*(a + b*x), x)
```

**sympy [A]** time = 0.56, size = 34, normalized size = 0.92

$$\frac{ac^{\frac{3}{2}}x\left(x^{2}\right)^{\frac{3}{2}}}{4}+\frac{bc^{\frac{3}{2}}x^{2}\left(x^{2}\right)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x*(x**2)**(3/2)/4 + b*c**(3/2)*x**2*(x**2)**(3/2)/5
```

## 3.726 $\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx$

**Optimal**. Leaf size=37

$$\frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x,x]
```

```
[Out] (a*c*x^2*Sqrt[c*x^2])/3 + (b*c*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^2+bx^3\right)\,dx}{x}\\
&= \frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 25, normalized size = 0.68

$$\frac{1}{12}cx^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x,x]
```

```
[Out] (c*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

**IntegrateAlgebraic** [A] time = 0.02, size = 21, normalized size = 0.57

$$\frac{1}{12}\left(cx^2\right)^{3/2}(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x))/x,x]
```

```
[Out] ((c*x^2)^(3/2)*(4*a + 3*b*x))/12
```

**fricas** [A] time = 1.09, size = 24, normalized size = 0.65

$$\frac{1}{12}\left(3\,bcx^3+4\,acx^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c*x^3 + 4*a*c*x^2)*sqrt(c*x^2)
```

**giac** [A] time = 1.09, size = 22, normalized size = 0.59

$$\frac{1}{12}\left(3\,bx^4\mathrm{sgn}(x)+4\,ax^3\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="giac")
```

```
[Out] 1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.49

$$\frac{(3bx+4a)\left(c\,x^2\right)^{\frac{3}{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x,x)
```

```
[Out] 1/12*(3*b*x+4*a)*(c*x^2)^(3/2)
```

**maxima [A]** time = 1.34, size = 22, normalized size = 0.59

$$\frac{1}{4}\left(cx^2\right)^{\frac{3}{2}}bx+\frac{1}{3}\left(cx^2\right)^{\frac{3}{2}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2)^(3/2)*b*x + 1/3*(c*x^2)^(3/2)*a
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x))/x, x)
```

**sympy [A]** time = 0.58, size = 31, normalized size = 0.84

$$\frac{ac^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{3}+\frac{bc^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x,x)
```

```
[Out] a*c**(3/2)*(x**2)**(3/2)/3 + b*c**(3/2)*x*(x**2)**(3/2)/4
```

## 3.727 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^2} dx$

**Optimal**. Leaf size=35

$$\frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
```

```
[Out] (a*c*x*Sqrt[c*x^2])/2 + (b*c*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)}{x^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)\,dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax+bx^2\right)\,dx}{x} \\
&= \frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 23, normalized size = 0.66

$$\frac{1}{6}cx\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
[Out] (c*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.69

$$\frac{\left(cx^2\right)^{3/2}(3a+2bx)}{6x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
[Out] ((c*x^2)^(3/2)*(3*a + 2*b*x))/(6*x)
```

**fricas** [A] time = 1.01, size = 22, normalized size = 0.63

$$\frac{1}{6}\left(2\,bcx^2+3\,acx\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="fricas")
[Out] 1/6*(2*b*c*x^2 + 3*a*c*x)*sqrt(c*x^2)
```

**giac** [A] time = 1.08, size = 22, normalized size = 0.63

$$\frac{1}{6}\left(2\,bx^3\mathrm{sgn}(x)+3\,ax^2\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="giac")
[Out] 1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{3}{2}}}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^2,x)
```

```
[Out] 1/6/x*(2*b*x+3*a)*(c*x^2)^(3/2)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad** [B] time = 0.27, size = 20, normalized size = 0.57

$$\frac{c^{3/2}\left(2\,b\,\sqrt{x^6}\;+3\,a\,x\,|x|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^2,x)
```

```
[Out] (c^(3/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

**sympy** [A] time = 0.57, size = 31, normalized size = 0.89

$$\frac{ac^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{2x}+\frac{bc^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**2,x)
```

```
[Out] a*c**(3/2)*(x**2)**(3/2)/(2*x) + b*c**(3/2)*(x**2)**(3/2)/3
```

## 3.728 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^3}\,dx$

**Optimal**. Leaf size=29

$$ac\sqrt{cx^2}+\frac{1}{2}bcx\sqrt{cx^2}$$

**Rubi [A]** time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15}

$$ac\sqrt{cx^2}+\frac{1}{2}bcx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

```
[Out] a*c*Sqrt[c*x^2] + (b*c*x*Sqrt[c*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)}{x^3}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)\,dx}{x}\\ &= ac\sqrt{cx^2}+\frac{1}{2}bcx\sqrt{cx^2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 21, normalized size = 0.72

$$\frac{1}{2}c\sqrt{cx^2}\,(2a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(2*a + b*x))/2
```

**IntegrateAlgebraic** [A] time = 0.02, size = 23, normalized size = 0.79

$$\frac{(cx^2)^{3/2}(2a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

```
[Out] ((c*x^2)^(3/2)*(2*a + b*x))/(2*x^2)
```

**fricas** [A] time = 1.04, size = 18, normalized size = 0.62

$$\frac{1}{2}(bcx+2\,ac)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x + 2*a*c)*sqrt(c*x^2)
```

**giac** [A] time = 0.99, size = 17, normalized size = 0.59

$$\frac{1}{2}\left(bx^2+2\,ax\right)c^{\frac{3}{2}}\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c^(3/2)*sgn(x)
```

**maple** [A] time = 0.00, size = 20, normalized size = 0.69

$$\frac{(bx+2a)\left(c\,x^2\right)^{\frac{3}{2}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^3,x)
```

```
[Out] 1/2/x^2*(b*x+2*a)*(c*x^2)^(3/2)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad** [**B**] time = 0.22, size = 14, normalized size = 0.48

$$\frac{c^{3/2}\,|x|\,\left(2\,a+b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^3,x)
```

```
[Out] (c^(3/2)*abs(x)*(2*a + b*x))/2
```

**sympy** [**A**] time = 0.74, size = 32, normalized size = 1.10

$$\frac{ac^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{x^{2}}+\frac{bc^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**3,x)
```

```
[Out] a*c**(3/2)*(x**2)**(3/2)/x**2 + b*c**(3/2)*(x**2)**(3/2)/(2*x)
```

## 3.729 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}\,dx$

**Optimal**. Leaf size=30

$$\frac{ac\sqrt{cx^2}\log(x)}{x}+bc\sqrt{cx^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ac\sqrt{cx^2}\log(x)}{x}+bc\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
```

```
[Out] b*c*Sqrt[c*x^2] + (a*c*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{a+bx}{x}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(b+\frac{a}{x}\right)dx}{x}\\
&= bc\sqrt{cx^2}+\frac{ac\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 21, normalized size = 0.70

$$\frac{\left(cx^2\right)^{3/2}(a\log(x)+bx)}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
[Out] ((c*x^2)^(3/2)*(b*x + a*Log[x]))/x^3
```

**IntegrateAlgebraic** [A] time = 0.02, size = 23, normalized size = 0.77

$$\left(cx^2\right)^{3/2}\left(\frac{a\log(x)}{x^3}+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
[Out] (c*x^2)^(3/2)*(b/x^2 + (a*Log[x])/x^3)
```

**fricas** [A] time = 0.88, size = 21, normalized size = 0.70

$$\frac{\left(bcx+ac\log(x)\right)\sqrt{cx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="fricas")
[Out] (b*c*x + a*c*log(x))*sqrt(c*x^2)/x
```

**giac** [A] time = 0.96, size = 17, normalized size = 0.57

$$\left(bx\mathrm{sgn}(x)+a\log\left(|x|\right)\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="giac")
[Out] (b*x*sgn(x) + a*log(abs(x))*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 20, normalized size = 0.67

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}(a\ln(x)+bx)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^4,x)
```

```
[Out] (c*x^2)^(3/2)/x^3*(a*ln(x)+b*x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^4,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x))/x^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{3}{2}}\left(a+bx\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**4,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)/x**4, x)
```

## 3.730 $\int x^3 \left(cx^2\right)^{5/2} (a+bx)\,dx$

**Optimal**. Leaf size=41

$$\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^8*Sqrt[c*x^2])/9 + (b*c^2*x^9*Sqrt[c*x^2])/10
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(cx^2\right)^{5/2}(a+bx)\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^8(a+bx)\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int\left(ax^8+bx^9\right)\,dx}{x}\\&=\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.59

$$\frac{1}{90}x^4\left(cx^2\right)^{5/2}(10a+9bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^4*(c*x^2)^(5/2)*(10*a + 9*b*x))/90
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{1}{90}x^4\left(cx^2\right)^{5/2}(10a+9bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^4*(c*x^2)^(5/2)*(10*a + 9*b*x))/90
```

**fricas** [A] time = 1.22, size = 28, normalized size = 0.68

$$\frac{1}{90}\left(9\,bc^2x^9+10\,ac^2x^8\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/90*(9*b*c^2*x^9 + 10*a*c^2*x^8)*sqrt(c*x^2)
```

**giac** [A] time = 0.97, size = 28, normalized size = 0.68

$$\frac{1}{90}\left(9\,bc^2x^{10}\mathrm{sgn}(x)+10\,ac^2x^9\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
[Out] 1/90*(9*b*c^2*x^10*sgn(x) + 10*a*c^2*x^9*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.51

$$\frac{(9bx+10a)\left(c\,x^2\right)^{\frac{5}{2}}x^4}{90}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(5/2)*(b*x+a),x)
[Out] 1/90*x^4*(9*b*x+10*a)*(c*x^2)^(5/2)
```

**maxima [A]** time = 1.24, size = 33, normalized size = 0.80

$$\frac{\left(cx^2\right)^{\frac{7}{2}} bx^3}{10\,c} + \frac{\left(cx^2\right)^{\frac{7}{2}} ax^2}{9\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/10*(c*x^2)^(7/2)*b*x^3/c + 1/9*(c*x^2)^(7/2)*a*x^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \left(c\,x^2\right)^{5/2}\,(a + b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(5/2)*(a + b*x),x)
[Out] int(x^3*(c*x^2)^(5/2)*(a + b*x), x)
```

**sympy [A]** time = 2.54, size = 36, normalized size = 0.88

$$\frac{ac^{\frac{5}{2}}x^4\left(x^2\right)^{\frac{5}{2}}}{9} + \frac{bc^{\frac{5}{2}}x^5\left(x^2\right)^{\frac{5}{2}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(5/2)*(b*x+a),x)
[Out] a*c**(5/2)*x**4*(x**2)**(5/2)/9 + b*c**(5/2)*x**5*(x**2)**(5/2)/10
```

## 3.731 $\int x^2 \left(cx^2\right)^{5/2} (a + bx) \, dx$

**Optimal**. Leaf size=41

$$\frac{1}{8}ac^2x^7\sqrt{cx^2} + \frac{1}{9}bc^2x^8\sqrt{cx^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{8}ac^2x^7\sqrt{cx^2} + \frac{1}{9}bc^2x^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^7*Sqrt[c*x^2])/8 + (b*c^2*x^8*Sqrt[c*x^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(cx^2\right)^{5/2} (a + bx) \, dx &= \frac{\left(c^2\sqrt{cx^2}\right) \int x^7(a + bx) \, dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right) \int \left(ax^7 + bx^8\right) \, dx}{x} \\
&= \frac{1}{8}ac^2x^7\sqrt{cx^2} + \frac{1}{9}bc^2x^8\sqrt{cx^2}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.59

$$\frac{1}{72}x^3\left(cx^2\right)^{5/2}(9a+8bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^3*(c*x^2)^(5/2)*(9*a + 8*b*x))/72
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{1}{72}x^3\left(cx^2\right)^{5/2}(9a+8bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^3*(c*x^2)^(5/2)*(9*a + 8*b*x))/72
```

**fricas** [A] time = 0.59, size = 28, normalized size = 0.68

$$\frac{1}{72}\left(8\,bc^2x^8+9\,ac^2x^7\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/72*(8*b*c^2*x^8 + 9*a*c^2*x^7)*sqrt(c*x^2)
```

**giac** [A] time = 0.80, size = 28, normalized size = 0.68

$$\frac{1}{72}\left(8\,bc^2x^9\mathrm{sgn}(x)+9\,ac^2x^8\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
[Out] 1/72*(8*b*c^2*x^9*sgn(x) + 9*a*c^2*x^8*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.51

$$\frac{(8bx+9a)\left(c\,x^2\right)^{\frac{5}{2}}x^3}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] 1/72*x^3*(8*b*x+9*a)*(c*x^2)^(5/2)
```

**maxima [A]** time = 1.30, size = 31, normalized size = 0.76

$$\frac{\left(cx^2\right)^{\frac{7}{2}}bx^2}{9\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}ax}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/9*(c*x^2)^(7/2)*b*x^2/c + 1/8*(c*x^2)^(7/2)*a*x/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(5/2)*(a + b*x),x)
```

```
[Out] int(x^2*(c*x^2)^(5/2)*(a + b*x), x)
```

**sympy [A]** time = 2.10, size = 36, normalized size = 0.88

$$\frac{ac^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}{8}+\frac{bc^{\frac{5}{2}}x^4\left(x^2\right)^{\frac{5}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*c**(5/2)*x**3*(x**2)**(5/2)/8 + b*c**(5/2)*x**4*(x**2)**(5/2)/9
```

## 3.732 $\int x\left(cx^2\right)^{5/2}(a+bx)\,dx$

**Optimal**. Leaf size=41

$$\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^6*Sqrt[c*x^2])/7 + (b*c^2*x^7*Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{5/2}(a+bx)\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^6(a+bx)\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int\left(ax^6+bx^7\right)\,dx}{x}\\&=\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.59

$$\frac{1}{56}x^2\left(cx^2\right)^{5/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^2*(c*x^2)^(5/2)*(8*a + 7*b*x))/56
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{1}{56}x^2\left(cx^2\right)^{5/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^2*(c*x^2)^(5/2)*(8*a + 7*b*x))/56
```

**fricas** [A] time = 0.98, size = 28, normalized size = 0.68

$$\frac{1}{56}\left(7\,bc^2x^7+8\,ac^2x^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/56*(7*b*c^2*x^7 + 8*a*c^2*x^6)*sqrt(c*x^2)
```

**giac** [A] time = 1.03, size = 28, normalized size = 0.68

$$\frac{1}{56}\left(7\,bc^2x^8\mathrm{sgn}(x)+8\,ac^2x^7\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
[Out] 1/56*(7*b*c^2*x^8*sgn(x) + 8*a*c^2*x^7*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.51

$$\frac{(7bx+8a)\left(c\,x^2\right)^{\frac{5}{2}}x^2}{56}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] 1/56*x^2*(7*b*x+8*a)*(c*x^2)^(5/2)
```

**maxima [A]** time = 1.35, size = 28, normalized size = 0.68

$$\frac{\left(cx^2\right)^{\frac{7}{2}} bx}{8\,c} + \frac{\left(cx^2\right)^{\frac{7}{2}} a}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2)^(7/2)*b*x/c + 1/7*(c*x^2)^(7/2)*a/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(c\,x^2\right)^{5/2}\,(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(a + b*x),x)
```

```
[Out] int(x*(c*x^2)^(5/2)*(a + b*x), x)
```

**sympy [A]** time = 1.74, size = 36, normalized size = 0.88

$$\frac{ac^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}{7} + \frac{bc^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*c**(5/2)*x**2*(x**2)**(5/2)/7 + b*c**(5/2)*x**3*(x**2)**(5/2)/8
```

## 3.733 $\int \left(cx^2\right)^{5/2} (a+bx)\, dx$

**Optimal**. Leaf size=41

$$\frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^5*Sqrt[c*x^2])/6 + (b*c^2*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^{5/2} (a+bx)\, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)\, dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^5+bx^6\right)\, dx}{x}\\ &= \frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 22, normalized size = 0.54

$$\frac{1}{42}x\left(cx^2\right)^{5/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x*(c*x^2)^(5/2)*(7*a + 6*b*x))/42
```

**IntegrateAlgebraic** [A] time = 0.02, size = 22, normalized size = 0.54

$$\frac{1}{42}x\left(cx^2\right)^{5/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x*(c*x^2)^(5/2)*(7*a + 6*b*x))/42
```

**fricas** [A] time = 0.99, size = 28, normalized size = 0.68

$$\frac{1}{42}\left(6\,bc^2x^6+7\,ac^2x^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/42*(6*b*c^2*x^6 + 7*a*c^2*x^5)*sqrt(c*x^2)
```

**giac** [A] time = 0.95, size = 28, normalized size = 0.68

$$\frac{1}{42}\left(6\,bc^2x^7\mathrm{sgn}(x)+7\,ac^2x^6\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/42*(6*b*c^2*x^7*sgn(x) + 7*a*c^2*x^6*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 19, normalized size = 0.46

$$\frac{(6bx+7a)\left(c\,x^2\right)^{\frac{5}{2}}x}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] 1/42*x*(6*b*x+7*a)*(c*x^2)^(5/2)
```

**maxima** [A] time = 1.34, size = 25, normalized size = 0.61

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}ax+\frac{\left(cx^2\right)^{\frac{7}{2}}b}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(5/2)*a*x + 1/7*(c*x^2)^(7/2)*b/c
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x),x)
```

```
[Out] int((c*x^2)^(5/2)*(a + b*x), x)
```

**sympy** [A] time = 1.41, size = 34, normalized size = 0.83

$$\frac{ac^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}{6}+\frac{bc^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*c**(5/2)*x*(x**2)**(5/2)/6 + b*c**(5/2)*x**2*(x**2)**(5/2)/7
```

## 3.734 $\int \frac{(cx^2)^{5/2}(a+bx)}{x}\,dx$

**Optimal**. Leaf size=41

$$\frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x,x]
```

```
[Out] (a*c^2*x^4*Sqrt[c*x^2])/5 + (b*c^2*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^4+bx^5\right)\,dx}{x}\\ &= \frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 25, normalized size = 0.61

$$\frac{1}{30}cx^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x,x]
[Out] (c*x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

**IntegrateAlgebraic** [A] time = 0.02, size = 21, normalized size = 0.51

$$\frac{1}{30}\left(cx^2\right)^{5/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x))/x,x]
[Out] ((c*x^2)^(5/2)*(6*a + 5*b*x))/30
```

**fricas** [A] time = 1.21, size = 28, normalized size = 0.68

$$\frac{1}{30}\left(5\,bc^2x^5+6\,ac^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="fricas")
[Out] 1/30*(5*b*c^2*x^5 + 6*a*c^2*x^4)*sqrt(c*x^2)
```

**giac** [A] time = 1.07, size = 28, normalized size = 0.68

$$\frac{1}{30}\left(5\,bc^2x^6\mathrm{sgn}(x)+6\,ac^2x^5\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="giac")
[Out] 1/30*(5*b*c^2*x^6*sgn(x) + 6*a*c^2*x^5*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.44

$$\frac{(5bx+6a)\left(c\,x^2\right)^{\frac{5}{2}}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x,x)
[Out] 1/30*(5*b*x+6*a)*(c*x^2)^(5/2)
```

**maxima [A]** time = 1.40, size = 22, normalized size = 0.54

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}bx+\frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(5/2)*b*x + 1/5*(c*x^2)^(5/2)*a
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x,x)
[Out] int(((c*x^2)^(5/2)*(a + b*x))/x, x)
```

**sympy [A]** time = 1.43, size = 31, normalized size = 0.76

$$\frac{ac^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5}+\frac{bc^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x,x)
[Out] a*c**(5/2)*(x**2)**(5/2)/5 + b*c**(5/2)*x*(x**2)**(5/2)/6
```

## 3.735 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^2} dx$

**Optimal**. Leaf size=41

$$\frac{1}{4}ac^2x^3\sqrt{cx^2}+\frac{1}{5}bc^2x^4\sqrt{cx^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{4}ac^2x^3\sqrt{cx^2}+\frac{1}{5}bc^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

```
[Out] (a*c^2*x^3*Sqrt[c*x^2])/4 + (b*c^2*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^3+bx^4\right)\,dx}{x}\\ &= \frac{1}{4}ac^2x^3\sqrt{cx^2}+\frac{1}{5}bc^2x^4\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 23, normalized size = 0.56

$$\frac{1}{20}cx\left(cx^2\right)^{3/2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

```
[Out] (c*x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{\left(cx^2\right)^{5/2}(5a+4bx)}{20x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

```
[Out] ((c*x^2)^(5/2)*(5*a + 4*b*x))/(20*x)
```

**fricas** [A] time = 0.79, size = 28, normalized size = 0.68

$$\frac{1}{20}\left(4\,bc^2x^4+5\,ac^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c^2*x^4 + 5*a*c^2*x^3)*sqrt(c*x^2)
```

**giac** [A] time = 1.10, size = 28, normalized size = 0.68

$$\frac{1}{20}\left(4\,bc^2x^5\mathrm{sgn}(x)+5\,ac^2x^4\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/20*(4*b*c^2*x^5*sgn(x) + 5*a*c^2*x^4*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.51

$$\frac{(4bx+5a)\left(c\,x^2\right)^{\frac{5}{2}}}{20x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^2,x)
```

```
[Out] 1/20/x*(4*b*x+5*a)*(c*x^2)^(5/2)
```

**maxima** [A] time = 1.28, size = 24, normalized size = 0.59

$$\frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}b+\frac{\left(cx^2\right)^{\frac{5}{2}}a}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(c*x^2)^(5/2)*b + 1/4*(c*x^2)^(5/2)*a/x
```

**mupad** [B] time = 0.28, size = 25, normalized size = 0.61

$$\frac{c^{5/2}\left(4\,b\,\sqrt{x^{10}}+5\,a\,x^3\,\sqrt{x^2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^2,x)
```

```
[Out] (c^(5/2)*(4*b*(x^10)^(1/2) + 5*a*x^3*(x^2)^(1/2)))/20
```

**sympy** [A] time = 1.53, size = 31, normalized size = 0.76

$$\frac{ac^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{4x}+\frac{bc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**2,x)
```

```
[Out] a*c**(5/2)*(x**2)**(5/2)/(4*x) + b*c**(5/2)*(x**2)**(5/2)/5
```

## 3.736 $\int \frac{\left(cx^2\right)^{5/2}(a+bx)}{x^3}\,dx$

**Optimal**. Leaf size=41

$$\frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
```

```
[Out] (a*c^2*x^2*Sqrt[c*x^2])/3 + (b*c^2*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cx^2\right)^{5/2}(a+bx)}{x^3}\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^2+bx^3\right)\,dx}{x}\\&=\frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 27, normalized size = 0.66

$$\frac{1}{12}c^2x^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
[Out] (c^2*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{\left(cx^2\right)^{5/2}(4a+3bx)}{12x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
[Out] ((c*x^2)^(5/2)*(4*a + 3*b*x))/(12*x^2)
```

**fricas** [A] time = 1.08, size = 28, normalized size = 0.68

$$\frac{1}{12}\left(3\,bc^2x^3+4\,ac^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="fricas")
[Out] 1/12*(3*b*c^2*x^3 + 4*a*c^2*x^2)*sqrt(c*x^2)
```

**giac** [A] time = 0.99, size = 28, normalized size = 0.68

$$\frac{1}{12}\left(3\,bc^2x^4\mathrm{sgn}(x)+4\,ac^2x^3\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="giac")
[Out] 1/12*(3*b*c^2*x^4*sgn(x) + 4*a*c^2*x^3*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.51

$$\frac{(3bx+4a)\left(c\,x^2\right)^{\frac{5}{2}}}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^3,x)
```

```
[Out] 1/12/x^2*(3*b*x+4*a)*(c*x^2)^(5/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [B]** time = 0.27, size = 25, normalized size = 0.61

$$\frac{c^{5/2}\left(4\,a\,\sqrt{x^6}+3\,b\,x^3\,\sqrt{x^2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^3,x)
```

```
[Out] (c^(5/2)*(4*a*(x^6)^(1/2) + 3*b*x^3*(x^2)^(1/2)))/12
```

**sympy [A]** time = 1.58, size = 34, normalized size = 0.83

$$\frac{ac^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^2}+\frac{bc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**3,x)
```

```
[Out] a*c**(5/2)*(x**2)**(5/2)/(3*x**2) + b*c**(5/2)*(x**2)**(5/2)/(4*x)
```

## 3.737 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^4} dx$

**Optimal**. Leaf size=39

$$\frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

```
[Out] (a*c^2*x*Sqrt[c*x^2])/2 + (b*c^2*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x^4}dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax+bx^2\right)\,dx}{x}\\ &= \frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 25, normalized size = 0.64

$$\frac{1}{6}c^2x\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

```
[Out] (c^2*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.62

$$\frac{\left(cx^2\right)^{5/2}(3a+2bx)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

```
[Out] ((c*x^2)^(5/2)*(3*a + 2*b*x))/(6*x^3)
```

**fricas** [A] time = 0.77, size = 26, normalized size = 0.67

$$\frac{1}{6}\left(2\,bc^2x^2+3\,ac^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c^2*x^2 + 3*a*c^2*x)*sqrt(c*x^2)
```

**giac** [A] time = 1.14, size = 28, normalized size = 0.72

$$\frac{1}{6}\left(2\,bc^2x^3\mathrm{sgn}(x)+3\,ac^2x^2\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c^2*x^3*sgn(x) + 3*a*c^2*x^2*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.54

$$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{5}{2}}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^4,x)
```

```
[Out] 1/6/x^3*(2*b*x+3*a)*(c*x^2)^(5/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [B]** time = 0.26, size = 20, normalized size = 0.51

$$\frac{c^{5/2}\,\left(2\,b\,\sqrt{x^6}+3\,a\,x\,|x|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^4,x)
```

```
[Out] (c^(5/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

**sympy [A]** time = 1.62, size = 36, normalized size = 0.92

$$\frac{ac^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{2x^{3}}+\frac{bc^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{3x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**4,x)
```

```
[Out] a*c**(5/2)*(x**2)**(5/2)/(2*x**3) + b*c**(5/2)*(x**2)**(5/2)/(3*x**2)
```

## 3.738 $\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=35

$$\frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^4)/(3*Sqrt[c*x^2]) + (b*x^5)/(4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int x^2(a+bx)\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(ax^2+bx^3\right)dx}{\sqrt{cx^2}}\\
&= \frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 24, normalized size = 0.69

$$\frac{x^4(4a+3bx)}{12\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/Sqrt[c*x^2],x]
[Out] (x^4*(4*a + 3*b*x))/(12*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.02, size = 27, normalized size = 0.77

$$\frac{x^2\sqrt{cx^2}\,(4a+3bx)}{12c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x))/Sqrt[c*x^2],x]
[Out] (x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/(12*c)
```

**fricas** [A] time = 1.26, size = 25, normalized size = 0.71

$$\frac{\left(3\,bx^3+4\,ax^2\right)\sqrt{cx^2}}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)/c
```

**giac** [A] time = 1.30, size = 26, normalized size = 0.74

$$\frac{1}{12}\,\sqrt{cx^2}\left(\frac{3\,bx}{c}+\frac{4\,a}{c}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/12*sqrt(c*x^2)*(3*b*x/c + 4*a/c)*x^2
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{(3bx+4a)\,x^4}{12\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/12*x^4*(3*b*x+4*a)/(c*x^2)^(1/2)
```

**maxima [A]** time = 1.40, size = 33, normalized size = 0.94

$$\frac{\sqrt{cx^2}\,bx^3}{4\,c}+\frac{\sqrt{cx^2}\,ax^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^2)*b*x^3/c + 1/3*sqrt(c*x^2)*a*x^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\,\left(a+b\,x\right)}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*x))/(c*x^2)^(1/2), x)
```

**sympy [A]** time = 0.61, size = 36, normalized size = 1.03

$$\frac{ax^4}{3\sqrt{c}\,\sqrt{x^2}}+\frac{bx^5}{4\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**4/(3*sqrt(c)*sqrt(x**2)) + b*x**5/(4*sqrt(c)*sqrt(x**2))
```

## 3.739 $\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=35

$$\frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^3)/(2*Sqrt[c*x^2]) + (b*x^4)/(3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int x(a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(ax+bx^2\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 24, normalized size = 0.69

$$\frac{x^3(3a+2bx)}{6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/Sqrt[c*x^2],x]
[Out] (x^3*(3*a + 2*b*x))/(6*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.02, size = 25, normalized size = 0.71

$$\frac{x\sqrt{cx^2}\,(3a+2bx)}{6c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x))/Sqrt[c*x^2],x]
[Out] (x*Sqrt[c*x^2]*(3*a + 2*b*x))/(6*c)
```

**fricas** [A] time = 1.02, size = 23, normalized size = 0.66

$$\frac{\left(2\,bx^2+3\,ax\right)\sqrt{cx^2}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)/c
```

**giac** [A] time = 1.16, size = 24, normalized size = 0.69

$$\frac{1}{6}\,\sqrt{cx^2}\left(\frac{2\,bx}{c}+\frac{3\,a}{c}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(c*x^2)*(2*b*x/c + 3*a/c)*x
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{(2bx+3a)\,x^3}{6\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/6*x^3*(2*b*x+3*a)/(c*x^2)^(1/2)
```

**maxima [A]** time = 1.30, size = 26, normalized size = 0.74

$$\frac{\sqrt{cx^2}\,bx^2}{3\,c}+\frac{ax^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2)*b*x^2/c + 1/2*a*x^2/sqrt(c)
```

**mupad [B]** time = 0.25, size = 23, normalized size = 0.66

$$\frac{2\,b\,\sqrt{x^6}\,+3\,a\,x\,\sqrt{x^2}}{6\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] (2*b*(x^6)^(1/2) + 3*a*x*(x^2)^(1/2))/(6*c^(1/2))
```

**sympy [A]** time = 0.52, size = 36, normalized size = 1.03

$$\frac{ax^3}{2\sqrt{c}\,\sqrt{x^2}}+\frac{bx^4}{3\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**3/(2*sqrt(c)*sqrt(x**2)) + b*x**4/(3*sqrt(c)*sqrt(x**2))
```

# 3.740 $\int \frac{x(a+bx)}{\sqrt{cx^2}} \, dx$

**Optimal**. Leaf size=32

$$\frac{ax^2}{\sqrt{cx^2}} + \frac{bx^3}{2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {15}

$$\frac{ax^2}{\sqrt{cx^2}} + \frac{bx^3}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^2)/Sqrt[c*x^2] + (b*x^3)/(2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x(a+bx)}{\sqrt{cx^2}} \, dx &= \frac{x \int (a+bx) \, dx}{\sqrt{cx^2}} \\ &= \frac{ax^2}{\sqrt{cx^2}} + \frac{bx^3}{2\sqrt{cx^2}} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 23, normalized size = 0.72

$$\frac{x^2(2a+bx)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x^2*(2*a + b*x))/(2*Sqrt[c*x^2])
```

**IntegrateAlgebraic [A]** time = 0.02, size = 23, normalized size = 0.72

$$\frac{\sqrt{cx^2}\,(2a+bx)}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x))/Sqrt[c*x^2],x]
[Out] (Sqrt[c*x^2]*(2*a + b*x))/(2*c)
```

**fricas [A]** time = 1.18, size = 19, normalized size = 0.59

$$\frac{\sqrt{cx^2}\,(bx+2\,a)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)/c
```

**giac [A]** time = 1.11, size = 22, normalized size = 0.69

$$\frac{1}{2}\sqrt{cx^2}\left(\frac{bx}{c}+\frac{2\,a}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(c*x^2)*(b*x/c + 2*a/c)
```

**maple [A]** time = 0.00, size = 20, normalized size = 0.62

$$\frac{(bx+2a)\,x^2}{2\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(1/2),x)
[Out] 1/2*x^2*(b*x+2*a)/(c*x^2)^(1/2)
```

**maxima** [A] time = 1.30, size = 22, normalized size = 0.69

$$\frac{bx^2}{2\sqrt{c}} + \frac{\sqrt{cx^2}\,a}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2/sqrt(c) + sqrt(c*x^2)*a/c
```

**mupad** [B] time = 0.22, size = 19, normalized size = 0.59

$$\frac{2\,a\,|x| + b\,x\,\sqrt{x^2}}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] (2*a*abs(x) + b*x*(x^2)^(1/2))/(2*c^(1/2))
```

**sympy** [A] time = 0.46, size = 34, normalized size = 1.06

$$\frac{ax^2}{\sqrt{c}\,\sqrt{x^2}} + \frac{bx^3}{2\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**2/(sqrt(c)*sqrt(x**2)) + b*x**3/(2*sqrt(c)*sqrt(x**2))
```

## 3.741 $\int \frac{a+bx}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=29

$$\frac{ax\log(x)}{\sqrt{cx^2}}+\frac{bx^2}{\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{ax\log(x)}{\sqrt{cx^2}}+\frac{bx^2}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[c*x^2],x]
```

```
[Out] (b*x^2)/Sqrt[c*x^2] + (a*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(b+\frac{a}{x}\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{bx^2}{\sqrt{cx^2}}+\frac{ax\log(x)}{\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 19, normalized size = 0.66

$$\frac{x(a\log(x)+bx)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[c*x^2],x]
```

```
[Out] (x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

**IntegrateAlgebraic** [A] time = 0.02, size = 26, normalized size = 0.90

$$\sqrt{cx^2}\left(\frac{a\log(x)}{cx}+\frac{b}{c}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/Sqrt[c*x^2],x]
```

```
[Out] Sqrt[c*x^2]*(b/c + (a*Log[x])/(c*x))
```

**fricas** [A] time = 1.02, size = 22, normalized size = 0.76

$$\frac{\sqrt{cx^2}\left(bx+a\log(x)\right)}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a*log(x))/(c*x)
```

**giac** [A] time = 1.32, size = 35, normalized size = 1.21

$$-\frac{a\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)}{\sqrt{c}}+\frac{\sqrt{cx^2}\,b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) + sqrt(c*x^2)*b/c
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.62

$$\frac{(a\ln(x)+bx)\,x}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(1/2),x)
[Out] 1/(c*x^2)^(1/2)*x*(a*ln(x)+b*x)
```

**maxima [A]** time = 1.32, size = 20, normalized size = 0.69

$$\frac{a\log(x)}{\sqrt{c}}+\frac{\sqrt{cx^2}\,b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] a*log(x)/sqrt(c) + sqrt(c*x^2)*b/c
```

**mupad [B]** time = 0.51, size = 17, normalized size = 0.59

$$\frac{b\,|x|+a\,\ln\left(c\,x\right)\,\mathrm{sign}(x)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(1/2),x)
[Out] (b*abs(x) + a*log(c*x)*sign(x))/c^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+bx}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(1/2),x)
[Out] Integral((a + b*x)/sqrt(c*x**2), x)
```

## 3.742 $\int \frac{a+bx}{x\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=27

$$\frac{bx\log(x)}{\sqrt{cx^2}}-\frac{a}{\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{bx\log(x)}{\sqrt{cx^2}}-\frac{a}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*Sqrt[c*x^2]),x]
```

```
[Out] -(a/Sqrt[c*x^2]) + (b*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a}{\sqrt{cx^2}}+\frac{bx\log(x)}{\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.85

$$\frac{cx^2(bx\log(x)-a)}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*Sqrt[c*x^2]),x]
[Out] (c*x^2*(-a + b*x*Log[x]))/(c*x^2)^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.02, size = 30, normalized size = 1.11

$$\sqrt{cx^2}\left(\frac{b\log(x)}{cx}-\frac{a}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x*Sqrt[c*x^2]),x]
[Out] Sqrt[c*x^2]*(-(a/(c*x^2)) + (b*Log[x])/(c*x))
```

**fricas** [A] time = 1.01, size = 23, normalized size = 0.85

$$\frac{\sqrt{cx^2}\left(bx\log(x)-a\right)}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c*x^2)
```

**giac** [B] time = 0.98, size = 47, normalized size = 1.74

$$-\frac{b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)-\frac{2\,a\sqrt{c}}{\sqrt{c}\,x-\sqrt{cx^2}}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="giac")
[Out] -(b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - 2*a*sqrt(c)/(sqrt(c)*x - sqrt(c*x^
2)))/sqrt(c)
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.67

$$\frac{bx\ln(x)-a}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(1/2),x)
[Out] (b*x*ln(x)-a)/(c*x^2)^(1/2)
```

**maxima** [A] time = 1.33, size = 17, normalized size = 0.63

$$\frac{b\log(x)}{\sqrt{c}}-\frac{a}{\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] b*log(x)/sqrt(c) - a/(sqrt(c)*x)
```

**mupad** [B] time = 1.22, size = 22, normalized size = 0.81

$$-\frac{\frac{a}{\sqrt{x^2}}-b\,\ln\left(c\,x\right)\,\mathrm{sign}(x)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(1/2)),x)
[Out] -(a/(x^2)^(1/2) - b*log(c*x)*sign(x))/c^(1/2)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+bx}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(1/2),x)
[Out] Integral((a + b*x)/(x*sqrt(c*x**2)), x)
```

## 3.743 $\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=26

$$-\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] -(a + b*x)^2/(2*a*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2ax\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{cx(-a-2bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
[Out] (c*x*(-a - 2*b*x))/(2*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.02, size = 27, normalized size = 1.04

$$\frac{\sqrt{cx^2}\,(-a-2bx)}{2cx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
[Out] (Sqrt[c*x^2]*(-a - 2*b*x))/(2*c*x^3)
```

**fricas** [A] time = 1.03, size = 21, normalized size = 0.81

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c*x^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

**maple** [A] time = 0.00, size = 19, normalized size = 0.73

$$-\frac{2bx+a}{2\sqrt{c\,x^2}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(1/2),x)
```

```
[Out] -1/2*(2*b*x+a)/x/(c*x^2)^(1/2)
```

**maxima** [A] time = 1.25, size = 19, normalized size = 0.73

$$-\frac{b}{\sqrt{c}\,x}-\frac{a}{2\,\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b/(sqrt(c)*x) - 1/2*a/(sqrt(c)*x^2)
```

**mupad** [B] time = 0.16, size = 25, normalized size = 0.96

$$-\frac{2\,b\,x^3+a\,x^2}{2\,\sqrt{c}\,x\,{\left(x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(1/2)),x)
```

```
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(1/2)*x*(x^2)^(3/2))
```

**sympy** [A] time = 0.54, size = 31, normalized size = 1.19

$$-\frac{a}{2\sqrt{c}\,x\sqrt{x^2}}-\frac{b}{\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(1/2),x)
```

```
[Out] -a/(2*sqrt(c)*x*sqrt(x**2)) - b/(sqrt(c)*sqrt(x**2))
```

## 3.744 $\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=35

$$-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] -a/(3*x^2*Sqrt[c*x^2]) - b/(2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^4}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 22, normalized size = 0.63

$$\frac{c(-2a-3bx)}{6\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
[Out] (c*(-2*a - 3*b*x))/(6*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.02, size = 27, normalized size = 0.77

$$\frac{\sqrt{cx^2}\,(-2a-3bx)}{6cx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
[Out] (Sqrt[c*x^2]*(-2*a - 3*b*x))/(6*c*x^4)
```

**fricas** [A] time = 0.90, size = 23, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c*x^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\sqrt{cx^2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a)/(sqrt(c*x^2)*x^3), x)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.60

$$-\frac{3bx+2a}{6\sqrt{c\,x^2}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(1/2),x)
```

```
[Out] -1/6*(3*b*x+2*a)/x^2/(c*x^2)^(1/2)
```

**maxima** [A] time = 1.28, size = 19, normalized size = 0.54

$$-\frac{b}{2\sqrt{c}\,x^2}-\frac{a}{3\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b/(sqrt(c)*x^2) - 1/3*a/(sqrt(c)*x^3)
```

**mupad** [B] time = 0.15, size = 26, normalized size = 0.74

$$-\frac{2\,a\,\sqrt{x^2}\,+3\,b\,x\,\sqrt{x^2}}{6\,\sqrt{c}\,\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(1/2)),x)
```

```
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(1/2)*x^4)
```

**sympy** [A] time = 0.64, size = 36, normalized size = 1.03

$$-\frac{a}{3\sqrt{c}\,x^2\sqrt{x^2}}-\frac{b}{2\sqrt{c}\,x\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(1/2),x)
```

```
[Out] -a/(3*sqrt(c)*x**2*sqrt(x**2)) - b/(2*sqrt(c)*x*sqrt(x**2))
```

## 3.745 $\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=35

$$-\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] -a/(4*x^3*Sqrt[c*x^2]) - b/(3*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{\sqrt{cx^2}}\\
&= -\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.69

$$\frac{-3a-4bx}{12x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
[Out] (-3*a - 4*b*x)/(12*x^3*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.02, size = 27, normalized size = 0.77

$$\frac{\sqrt{cx^2}\,(-3a-4bx)}{12cx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
[Out] (Sqrt[c*x^2]*(-3*a - 4*b*x))/(12*c*x^5)
```

**fricas** [A] time = 1.10, size = 23, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(4\,bx+3\,a)}{12\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c*x^5)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.60

$$-\frac{4bx+3a}{12\sqrt{c\,x^2}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(1/2),x)
```

```
[Out] -1/12*(4*b*x+3*a)/x^3/(c*x^2)^(1/2)
```

**maxima** [A] time = 1.30, size = 19, normalized size = 0.54

$$-\frac{b}{3\sqrt{c}x^3}-\frac{a}{4\sqrt{c}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*b/(sqrt(c)*x^3) - 1/4*a/(sqrt(c)*x^4)
```

**mupad** [B] time = 0.15, size = 26, normalized size = 0.74

$$-\frac{3a\sqrt{x^2}+4bx\sqrt{x^2}}{12\sqrt{c}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(1/2)),x)
```

```
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(1/2)*x^5)
```

**sympy** [A] time = 0.81, size = 37, normalized size = 1.06

$$-\frac{a}{4\sqrt{c}x^3\sqrt{x^2}}-\frac{b}{3\sqrt{c}x^2\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(1/2),x)
```

```
[Out] -a/(4*sqrt(c)*x**3*sqrt(x**2)) - b/(3*sqrt(c)*x**2*sqrt(x**2))
```

# 3.746 $\int \frac{x^3(a+bx)}{\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=38

$$\frac{ax^2}{c\sqrt{cx^2}}+\frac{bx^3}{2c\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15}

$$\frac{ax^2}{c\sqrt{cx^2}}+\frac{bx^3}{2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (a*x^2)/(c*Sqrt[c*x^2]) + (b*x^3)/(2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int (a+bx)\,dx}{c\sqrt{cx^2}}\\ &= \frac{ax^2}{c\sqrt{cx^2}}+\frac{bx^3}{2c\sqrt{cx^2}}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 23, normalized size = 0.61

$$\frac{x^4(2a+bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (x^4*(2*a + b*x))/(2*(c*x^2)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 23, normalized size = 0.61

$$\frac{x^4(2a+bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (x^4*(2*a + b*x))/(2*(c*x^2)^(3/2))
```

**fricas [A]** time = 1.05, size = 19, normalized size = 0.50

$$\frac{\sqrt{cx^2}\,(bx+2\,a)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)/c^2
```

**giac [A]** time = 1.01, size = 25, normalized size = 0.66

$$\frac{\sqrt{cx^2}\left(\frac{bx}{c}+\frac{2\,a}{c}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
[Out] 1/2*sqrt(c*x^2)*(b*x/c + 2*a/c)/c
```

**maple [A]** time = 0.00, size = 20, normalized size = 0.53

$$\frac{(bx+2a)\,x^4}{2\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] 1/2*x^4*(b*x+2*a)/(c*x^2)^(3/2)
```

**maxima** [A] time = 1.31, size = 32, normalized size = 0.84

$$\frac{bx^3}{2\sqrt{cx^2}\,c}+\frac{ax^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/2*b*x^3/(sqrt(c*x^2)*c) + a*x^2/(sqrt(c*x^2)*c)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\,\left(a+b\,x\right)}{{\left(c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(3/2),x)
[Out] int((x^3*(a + b*x))/(c*x^2)^(3/2), x)
```

**sympy** [A] time = 0.64, size = 34, normalized size = 0.89

$$\frac{ax^4}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{bx^5}{2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(3/2),x)
[Out] a*x**4/(c**(3/2)*(x**2)**(3/2)) + b*x**5/(2*c**(3/2)*(x**2)**(3/2))
```

## 3.747 $\int \frac{x^2(a+bx)}{\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=35

$$\frac{ax\log(x)}{c\sqrt{cx^2}}+\frac{bx^2}{c\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ax\log(x)}{c\sqrt{cx^2}}+\frac{bx^2}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (b*x^2)/(c*Sqrt[c*x^2]) + (a*x*Log[x])/(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(b+\frac{a}{x}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{bx^2}{c\sqrt{cx^2}}+\frac{ax\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{x^3(a\log(x)+bx)}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (x^3*(b*x + a*Log[x]))/(c*x^2)^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.02, size = 23, normalized size = 0.66

$$\frac{ax^3\log(x)+bx^4}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (b*x^4 + a*x^3*Log[x])/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.11, size = 22, normalized size = 0.63

$$\frac{\sqrt{cx^2}\left(bx+a\log(x)\right)}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x + a*log(x))/(c^2*x)
```

**giac** [A] time = 0.99, size = 40, normalized size = 1.14

$$-\frac{\frac{a\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{\sqrt{c}}-\frac{\sqrt{cx^2}b}{c}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
[Out] -(a*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) - sqrt(c*x^2)*b/c)/c
```

**maple** [A] time = 0.00, size = 20, normalized size = 0.57

$$\frac{(a\ln(x)+bx)\,x^3}{(c\,x^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] 1/(c*x^2)^(3/2)*x^3*(a*ln(x)+b*x)
```

**maxima** [A] time = 1.35, size = 23, normalized size = 0.66

$$\frac{bx^2}{\sqrt{cx^2}\,c}+\frac{a\log(x)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x^2/(sqrt(c*x^2)*c) + a*log(x)/c^(3/2)
```

**mupad** [B] time = 0.32, size = 30, normalized size = 0.86

$$\frac{b\,|x|}{c^{3/2}}+\frac{a\,\ln\left(x+|x|\right)}{c^{3/2}}-\frac{a\,x}{c^{3/2}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] (b*abs(x))/c^(3/2) + (a*log(x + abs(x)))/c^(3/2) - (a*x)/(c^(3/2)*(x^2)^(1/
2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\,(a+bx)}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x)/(c*x**2)**(3/2), x)
```

## 3.748 $\int \frac{x(a+bx)}{\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=33

$$\frac{bx\log(x)}{c\sqrt{cx^2}}-\frac{a}{c\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{bx\log(x)}{c\sqrt{cx^2}}-\frac{a}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] -(a/(c*Sqrt[c*x^2])) + (b*x*Log[x])/(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{c\sqrt{cx^2}}+\frac{bx\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 22, normalized size = 0.67

$$\frac{x^2(bx\log(x)-a)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (x^2*(-a + b*x*Log[x]))/(c*x^2)^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.73

$$\frac{bx^3\log(x)-ax^2}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (-(a*x^2) + b*x^3*Log[x])/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.07, size = 23, normalized size = 0.70

$$\frac{\sqrt{cx^2}\left(bx\log(x)-a\right)}{c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c^2*x^2)
```

**giac** [A] time = 1.22, size = 47, normalized size = 1.42

$$-\frac{b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)-\frac{2\,a\sqrt{c}}{\sqrt{c}\,x-\sqrt{cx^2}}}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
[Out] -(b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - 2*a*sqrt(c)/(sqrt(c)*x - sqrt(c*x^
2)))/c^(3/2)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.64

$$\frac{(bx\ln(x)-a)\,x^2}{\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(3/2),x)
[Out] x^2*(b*x*ln(x)-a)/(c*x^2)^(3/2)
```

**maxima** [A] time = 1.31, size = 21, normalized size = 0.64

$$\frac{b\log(x)}{c^{\frac{3}{2}}}-\frac{a}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] b*log(x)/c^(3/2) - a/(sqrt(c*x^2)*c)
```

**mupad** [B] time = 0.25, size = 28, normalized size = 0.85

$$-\frac{a+b\,x-b\,\ln\left(x+|x|\right)\,\sqrt{x^2}}{c^{3/2}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(3/2),x)
[Out] -(a + b*x - b*log(x + abs(x))*(x^2)^(1/2))/(c^(3/2)*(x^2)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\,(a+bx)}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(3/2),x)
[Out] Integral(x*(a + b*x)/(c*x**2)**(3/2), x)
```

## 3.749 $\int \frac{a+bx}{\left(cx^2\right)^{3/2}} \, dx$

**Optimal**. Leaf size=29

$$-\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c*x^2)^(3/2),x]
```

```
[Out] -(a + b*x)^2/(2*a*c*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{\left(cx^2\right)^{3/2}} \, dx &= \frac{x \int \frac{a+bx}{x^3} \, dx}{c\sqrt{cx^2}} \\ &= -\frac{(a+bx)^2}{2acx\sqrt{cx^2}} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 22, normalized size = 0.76

$$\frac{x(-a-2bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c*x^2)^(3/2),x]
[Out] (x*(-a - 2*b*x))/(2*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.02, size = 20, normalized size = 0.69

$$-\frac{x(a+2bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(c*x^2)^(3/2),x]
[Out] -1/2*(x*(a + 2*b*x))/(c*x^2)^(3/2)
```

**fricas** [A] time = 0.71, size = 21, normalized size = 0.72

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c^2*x^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

**maple** [A] time = 0.00, size = 17, normalized size = 0.59

$$-\frac{(2bx+a)\,x}{2\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] -1/2*x*(2*b*x+a)/(c*x^2)^(3/2)
```

**maxima** [A] time = 1.35, size = 23, normalized size = 0.79

$$-\frac{b}{\sqrt{cx^2}\,c}-\frac{a}{2\,c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -b/(sqrt(c*x^2)*c) - 1/2*a/(c^(3/2)*x^2)
```

**mupad** [B] time = 0.15, size = 25, normalized size = 0.86

$$-\frac{2\,b\,x^3+a\,x^2}{2\,c^{3/2}\,x\,\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(3/2),x)
```

```
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(3/2)*x*(x^2)^(3/2))
```

**sympy** [A] time = 0.54, size = 34, normalized size = 1.17

$$-\frac{ax}{2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}-\frac{bx^2}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] -a*x/(2*c**(3/2)*(x**2)**(3/2)) - b*x**2/(c**(3/2)*(x**2)**(3/2))
```

# 3.750 $\int \frac{a+bx}{x(cx^2)^{3/2}} \, dx$

**Optimal**. Leaf size=41

$$-\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*(c*x^2)^(3/2)),x]
```

```
[Out] -a/(3*c*x^2*Sqrt[c*x^2]) - b/(2*c*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x\left(cx^2\right)^{3/2}} \, dx &= \frac{x\int \frac{a+bx}{x^4} \, dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right) \, dx}{c\sqrt{cx^2}}\\
&= -\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 25, normalized size = 0.61

$$\frac{cx^2(-2a-3bx)}{6\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (c*x^2*(-2*a - 3*b*x))/(6*(c*x^2)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.02, size = 21, normalized size = 0.51

$$\frac{-2a-3bx}{6\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (-2*a - 3*b*x)/(6*(c*x^2)^(3/2))
```

**fricas** [A] time = 1.76, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^2*x^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(cx^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)/((c*x^2)^(3/2)*x), x)
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.44

$$-\frac{3bx + 2a}{6\left(c\,x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(3/2),x)
[Out] -1/6*(3*b*x+2*a)/(c*x^2)^(3/2)
```

**maxima** [A] time = 1.32, size = 19, normalized size = 0.46

$$-\frac{b}{2\,c^{\frac{3}{2}}x^{2}} - \frac{a}{3\,c^{\frac{3}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*b/(c^(3/2)*x^2) - 1/3*a/(c^(3/2)*x^3)
```

**mupad** [B] time = 0.16, size = 26, normalized size = 0.63

$$-\frac{2\,a\,\sqrt{x^{2}}\,+3\,b\,x\,\sqrt{x^{2}}}{6\,c^{3/2}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(3/2)),x)
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(3/2)*x^4)
```

**sympy** [A] time = 0.63, size = 32, normalized size = 0.78

$$-\frac{a}{3c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}} - \frac{bx}{2c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(3/2),x)
[Out] -a/(3*c**(3/2)*(x**2)**(3/2)) - b*x/(2*c**(3/2)*(x**2)**(3/2))
```

## 3.751 $\int \frac{a+bx}{x^2(cx^2)^{3/2}}\,dx$

**Optimal**. Leaf size=41

$$-\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -a/(4*c*x^3*Sqrt[c*x^2]) - b/(3*c*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}(3a+4bx)}{12c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
[Out] -1/12*(Sqrt[c*x^2]*(3*a + 4*b*x))/(c^2*x^5)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 24, normalized size = 0.59

$$\frac{-3a-4bx}{12x\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
[Out] (-3*a - 4*b*x)/(12*x*(c*x^2)^(3/2))
```

**fricas [A]** time = 1.07, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}(4\,bx+3\,a)}{12\,c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^2*x^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.51

$$-\frac{4bx+3a}{12\left(c\,x^2\right)^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(3/2),x)
```

```
[Out] -1/12*(4*b*x+3*a)/x/(c*x^2)^(3/2)
```

**maxima [A]** time = 1.34, size = 19, normalized size = 0.46

$$-\frac{b}{3\,c^{\frac{3}{2}}x^3}-\frac{a}{4\,c^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*b/(c^(3/2)*x^3) - 1/4*a/(c^(3/2)*x^4)
```

**mupad [B]** time = 0.15, size = 26, normalized size = 0.63

$$-\frac{3\,a\,\sqrt{x^2}\,+4\,b\,x\,\sqrt{x^2}}{12\,c^{3/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(3/2)),x)
```

```
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(3/2)*x^5)
```

**sympy [A]** time = 0.77, size = 32, normalized size = 0.78

$$-\frac{a}{4c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}-\frac{b}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(3/2),x)
```

```
[Out] -a/(4*c**(3/2)*x*(x**2)**(3/2)) - b/(3*c**(3/2)*(x**2)**(3/2))
```

## 3.752 $\int \frac{a+bx}{x^3\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=41

$$-\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] -a/(5*c*x^4*Sqrt[c*x^2]) - b/(4*c*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^3\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^6}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^6}+\frac{b}{x^5}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 22, normalized size = 0.54

$$\frac{c(-4a-5bx)}{20\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
[Out] (c*(-4*a - 5*b*x))/(20*(c*x^2)^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 24, normalized size = 0.59

$$\frac{-4a-5bx}{20x^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
[Out] (-4*a - 5*b*x)/(20*x^2*(c*x^2)^(3/2))
```

**fricas [A]** time = 0.96, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(5\,bx+4\,a)}{20\,c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^2*x^6)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(cx^2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*x + a)/((c*x^2)^(3/2)*x^3), x)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{5bx+4a}{20\left(c\,x^{2}\right)^{\frac{3}{2}}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(3/2),x)
[Out] -1/20*(5*b*x+4*a)/x^2/(c*x^2)^(3/2)
```

**maxima** [A] time = 1.28, size = 19, normalized size = 0.46

$$-\frac{b}{4\,c^{\frac{3}{2}}x^{4}}-\frac{a}{5\,c^{\frac{3}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/4*b/(c^(3/2)*x^4) - 1/5*a/(c^(3/2)*x^5)
```

**mupad** [B] time = 0.15, size = 26, normalized size = 0.63

$$-\frac{4\,a\,\sqrt{x^{2}}\,+5\,b\,x\,\sqrt{x^{2}}}{20\,c^{3/2}\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(3/2)),x)
[Out] -(4*a*(x^2)^(1/2) + 5*b*x*(x^2)^(1/2))/(20*c^(3/2)*x^6)
```

**sympy** [A] time = 0.93, size = 36, normalized size = 0.88

$$-\frac{a}{5c^{\frac{3}{2}}x^{2}\left(x^{2}\right)^{\frac{3}{2}}}-\frac{b}{4c^{\frac{3}{2}}x\left(x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(3/2),x)
[Out] -a/(5*c**(3/2)*x**2*(x**2)**(3/2)) - b/(4*c**(3/2)*x*(x**2)**(3/2))
```

## 3.753 $\int \frac{a+bx}{x^4(cx^2)^{3/2}}\,dx$

**Optimal**. Leaf size=41

$$-\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] -a/(6*c*x^5*Sqrt[c*x^2]) - b/(5*c*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^4\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^7}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^7}+\frac{b}{x^6}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 0.59

$$\frac{-5a-6bx}{30x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
[Out] (-5*a - 6*b*x)/(30*x^3*(c*x^2)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 24, normalized size = 0.59

$$\frac{-5a-6bx}{30x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
[Out] (-5*a - 6*b*x)/(30*x^3*(c*x^2)^(3/2))
```

**fricas [A]** time = 1.28, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(6\,bx+5\,a)}{30\,c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^2*x^7)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.51

$$-\frac{6bx+5a}{30\left(c\,x^2\right)^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(3/2),x)
```

```
[Out] -1/30*(6*b*x+5*a)/x^3/(c*x^2)^(3/2)
```

**maxima [A]** time = 1.35, size = 19, normalized size = 0.46

$$-\frac{b}{5\,c^{\frac{3}{2}}x^5}-\frac{a}{6\,c^{\frac{3}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*b/(c^(3/2)*x^5) - 1/6*a/(c^(3/2)*x^6)
```

**mupad [B]** time = 0.15, size = 26, normalized size = 0.63

$$-\frac{5\,a\,\sqrt{x^2}\,+6\,b\,x\,\sqrt{x^2}}{30\,c^{3/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(3/2)),x)
```

```
[Out] -(5*a*(x^2)^(1/2) + 6*b*x*(x^2)^(1/2))/(30*c^(3/2)*x^7)
```

**sympy [A]** time = 1.16, size = 37, normalized size = 0.90

$$-\frac{a}{6c^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}-\frac{b}{5c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(3/2),x)
```

```
[Out] -a/(6*c**(3/2)*x**3*(x**2)**(3/2)) - b/(5*c**(3/2)*x**2*(x**2)**(3/2))
```

## 3.754 $\int \frac{x^3(a+bx)}{\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=33

$$\frac{bx\log(x)}{c^2\sqrt{cx^2}}-\frac{a}{c^2\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{bx\log(x)}{c^2\sqrt{cx^2}}-\frac{a}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -(a/(c^2*Sqrt[c*x^2])) + (b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{c^2\sqrt{cx^2}}+\frac{bx\log(x)}{c^2\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 22, normalized size = 0.67

$$\frac{bx\log(x)-a}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
[Out] (-a + b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 24, normalized size = 0.73

$$\frac{bx^5\log(x)-ax^4}{\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
[Out] (-(a*x^4) + b*x^5*Log[x])/(c*x^2)^(5/2)
```

**fricas** [A] time = 0.79, size = 23, normalized size = 0.70

$$\frac{\sqrt{cx^2}\left(bx\log(x)-a\right)}{c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c^3*x^2)
```

**giac** [A] time = 1.02, size = 47, normalized size = 1.42

$$-\frac{b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)-\frac{2\,a\sqrt{c}}{\sqrt{c}\,x-\sqrt{cx^2}}}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -(b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - 2*a*sqrt(c)/(sqrt(c)*x - sqrt(c*x^
2)))/c^(5/2)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.64

$$\frac{(bx\ln(x)-a)\,x^4}{\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(5/2),x)
[Out] x^4*(b*x*ln(x)-a)/(c*x^2)^(5/2)
```

**maxima** [A] time = 1.43, size = 24, normalized size = 0.73

$$-\frac{ax^2}{\left(cx^2\right)^{\frac{3}{2}}c}+\frac{b\log(x)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -a*x^2/((c*x^2)^(3/2)*c) + b*log(x)/c^(5/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{x^3\,(a+b\,x)}{\left(c\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(5/2),x)
[Out] int((x^3*(a + b*x))/(c*x^2)^(5/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\,(a+bx)}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(5/2),x)
[Out] Integral(x**3*(a + b*x)/(c*x**2)**(5/2), x)
```

## 3.755 $\int \frac{x^2(a+bx)}{\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=29

$$-\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -(a + b*x)^2/(2*a*c^2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.83

$$\frac{x^3(-a-2bx)}{2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
[Out] (x^3*(-a - 2*b*x))/(2*(c*x^2)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.02, size = 22, normalized size = 0.76

$$-\frac{x^3(a+2bx)}{2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
[Out] -1/2*(x^3*(a + 2*b*x))/(c*x^2)^(5/2)
```

**fricas** [A] time = 1.05, size = 21, normalized size = 0.72

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c^3*x^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

**maple** [A] time = 0.00, size = 19, normalized size = 0.66

$$-\frac{(2bx+a)\,x^3}{2\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] -1/2*x^3*(2*b*x+a)/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.34, size = 26, normalized size = 0.90

$$-\frac{bx^2}{\left(cx^2\right)^{\frac{3}{2}}c}-\frac{a}{2\,c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -b*x^2/((c*x^2)^(3/2)*c) - 1/2*a/(c^(5/2)*x^2)
```

**mupad [B]** time = 0.15, size = 25, normalized size = 0.86

$$-\frac{2\,b\,x^3+a\,x^2}{2\,c^{5/2}\,x\,\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(5/2),x)
```

```
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(5/2)*x*(x^2)^(3/2))
```

**sympy [A]** time = 0.93, size = 36, normalized size = 1.24

$$-\frac{ax^3}{2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{bx^4}{c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x**3/(2*c**(5/2)*(x**2)**(5/2)) - b*x**4/(c**(5/2)*(x**2)**(5/2))
```

## 3.756 $\int \frac{x(a+bx)}{\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=41

$$-\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$-\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -a/(3*c^2*x^2*Sqrt[c*x^2]) - b/(2*c^2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx)}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^4}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 24, normalized size = 0.59

$$\frac{x^2(-2a-3bx)}{6\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/(c*x^2)^(5/2),x]
[Out] (x^2*(-2*a - 3*b*x))/(6*(c*x^2)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.59

$$-\frac{x^2(2a+3bx)}{6\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x))/(c*x^2)^(5/2),x]
[Out] -1/6*(x^2*(2*a + 3*b*x))/(c*x^2)^(5/2)
```

**fricas** [A] time = 1.09, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^3*x^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx+a)x}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)*x/(c*x^2)^(5/2), x)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{(3bx+2a)\,x^{2}}{6\left(c\,x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(5/2),x)
[Out] -1/6*x^2*(3*b*x+2*a)/(c*x^2)^(5/2)
```

**maxima** [A] time = 1.39, size = 23, normalized size = 0.56

$$-\frac{a}{3\left(cx^{2}\right)^{\frac{3}{2}}c}-\frac{b}{2\,c^{\frac{5}{2}}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/3*a/((c*x^2)^(3/2)*c) - 1/2*b/(c^(5/2)*x^2)
```

**mupad** [B] time = 0.15, size = 26, normalized size = 0.63

$$-\frac{2\,a\,\sqrt{x^{2}}+3\,b\,x\,\sqrt{x^{2}}}{6\,c^{5/2}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(5/2),x)
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(5/2)*x^4)
```

**sympy** [A] time = 0.92, size = 37, normalized size = 0.90

$$-\frac{ax^{2}}{3c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{bx^{3}}{2c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(5/2),x)
[Out] -a*x**2/(3*c**(5/2)*(x**2)**(5/2)) - b*x**3/(2*c**(5/2)*(x**2)**(5/2))
```

## 3.757 $\int \frac{a+bx}{(cx^2)^{5/2}}\,dx$

**Optimal**. Leaf size=41

$$-\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$-\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c*x^2)^(5/2),x]
```

```
[Out] -a/(4*c^2*x^3*Sqrt[c*x^2]) - b/(3*c^2*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(3a+4bx)}{12c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c*x^2)^(5/2),x]
[Out] -1/12*(Sqrt[c*x^2]*(3*a + 4*b*x))/(c^3*x^5)
```

**IntegrateAlgebraic** [A] time = 0.02, size = 22, normalized size = 0.54

$$-\frac{x(3a+4bx)}{12\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(c*x^2)^(5/2),x]
[Out] -1/12*(x*(3*a + 4*b*x))/(c*x^2)^(5/2)
```

**fricas** [A] time = 1.00, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(4\,bx+3\,a)}{12\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^3*x^5)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

**maple** [A] time = 0.00, size = 19, normalized size = 0.46

$$-\frac{(4bx+3a)\,x}{12\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] -1/12*x*(4*b*x+3*a)/(c*x^2)^(5/2)
```

**maxima** [A] time = 1.37, size = 23, normalized size = 0.56

$$-\frac{b}{3\left(cx^{2}\right)^{\frac{3}{2}}c}-\frac{a}{4\,c^{\frac{5}{2}}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*b/((c*x^2)^(3/2)*c) - 1/4*a/(c^(5/2)*x^4)
```

**mupad** [B] time = 0.16, size = 26, normalized size = 0.63

$$-\frac{3\,a\,\sqrt{x^2}\,+4\,b\,x\,\sqrt{x^2}}{12\,c^{5/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(5/2),x)
```

```
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(5/2)*x^5)
```

**sympy** [A] time = 0.92, size = 36, normalized size = 0.88

$$-\frac{ax}{4c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{bx^{2}}{3c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x/(4*c**(5/2)*(x**2)**(5/2)) - b*x**2/(3*c**(5/2)*(x**2)**(5/2))
```

## 3.758 $\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=41

$$-\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(5*c^2*x^4*Sqrt[c*x^2]) - b/(4*c^2*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^6}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^6}+\frac{b}{x^5}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(4a+5bx)}{20c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*(c*x^2)^(5/2)),x]
[Out] -1/20*(Sqrt[c*x^2]*(4*a + 5*b*x))/(c^3*x^6)
```

**IntegrateAlgebraic** [A] time = 0.02, size = 21, normalized size = 0.51

$$\frac{-4a-5bx}{20\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x*(c*x^2)^(5/2)),x]
[Out] (-4*a - 5*b*x)/(20*(c*x^2)^(5/2))
```

**fricas** [A] time = 0.74, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(5\,bx+4\,a)}{20\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^3*x^6)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(cx^2\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)/((c*x^2)^(5/2)*x), x)
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.44

$$-\frac{5bx+4a}{20\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(5/2),x)
```

```
[Out] -1/20*(5*b*x+4*a)/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.33, size = 19, normalized size = 0.46

$$-\frac{b}{4\,c^{\frac{5}{2}}x^{4}}-\frac{a}{5\,c^{\frac{5}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*b/(c^(5/2)*x^4) - 1/5*a/(c^(5/2)*x^5)
```

**mupad [B]** time = 0.16, size = 26, normalized size = 0.63

$$-\frac{4\,a\,\sqrt{x^2}\,+5\,b\,x\,\sqrt{x^2}}{20\,c^{5/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(5/2)),x)
```

```
[Out] -(4*a*(x^2)^(1/2) + 5*b*x*(x^2)^(1/2))/(20*c^(5/2)*x^6)
```

**sympy [A]** time = 1.12, size = 32, normalized size = 0.78

$$-\frac{a}{5c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{bx}{4c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(5/2),x)
```

```
[Out] -a/(5*c**(5/2)*(x**2)**(5/2)) - b*x/(4*c**(5/2)*(x**2)**(5/2))
```

## 3.759 $\int \frac{a+bx}{x^2(cx^2)^{5/2}}\,dx$

**Optimal**. Leaf size=41

$$-\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(6*c^2*x^5*Sqrt[c*x^2]) - b/(5*c^2*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^7}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^7}+\frac{b}{x^6}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(5a+6bx)}{30c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -1/30*(Sqrt[c*x^2]*(5*a + 6*b*x))/(c^3*x^7)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 24, normalized size = 0.59

$$\frac{-5a-6bx}{30x\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] (-5*a - 6*b*x)/(30*x*(c*x^2)^(5/2))
```

**fricas** [A] time = 1.03, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(6\,bx+5\,a)}{30\,c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^3*x^7)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{6bx+5a}{30\left(c\,x^2\right)^{\frac{5}{2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(5/2),x)
```

```
[Out] -1/30*(6*b*x+5*a)/x/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.32, size = 19, normalized size = 0.46

$$-\frac{b}{5\,c^{\frac{5}{2}}x^5}-\frac{a}{6\,c^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*b/(c^(5/2)*x^5) - 1/6*a/(c^(5/2)*x^6)
```

**mupad [B]** time = 0.16, size = 26, normalized size = 0.63

$$-\frac{5\,a\,\sqrt{x^2}+6\,b\,x\,\sqrt{x^2}}{30\,c^{5/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(5/2)),x)
```

```
[Out] -(5*a*(x^2)^(1/2) + 6*b*x*(x^2)^(1/2))/(30*c^(5/2)*x^7)
```

**sympy [A]** time = 1.34, size = 32, normalized size = 0.78

$$-\frac{a}{6c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}-\frac{b}{5c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(5/2),x)
```

```
[Out] -a/(6*c**(5/2)*x*(x**2)**(5/2)) - b/(5*c**(5/2)*(x**2)**(5/2))
```

## 3.760 $\int \frac{a+bx}{x^3\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=41

$$-\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(7*c^2*x^6*Sqrt[c*x^2]) - b/(6*c^2*x^5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^3\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^8}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^8}+\frac{b}{x^7}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 22, normalized size = 0.54

$$\frac{c(-6a-7bx)}{42\left(cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
[Out] (c*(-6*a - 7*b*x))/(42*(c*x^2)^(7/2))
```

**IntegrateAlgebraic** [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{-6a-7bx}{42x^2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
[Out] (-6*a - 7*b*x)/(42*x^2*(c*x^2)^(5/2))
```

**fricas** [A] time = 1.10, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(7\,bx+6\,a)}{42\,c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/42*sqrt(c*x^2)*(7*b*x + 6*a)/(c^3*x^8)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(cx^2\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)/((c*x^2)^(5/2)*x^3), x)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{7bx+6a}{42\left(c\,x^{2}\right)^{\frac{5}{2}}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(5/2),x)
[Out] -1/42*(7*b*x+6*a)/x^2/(c*x^2)^(5/2)
```

**maxima** [A] time = 1.37, size = 19, normalized size = 0.46

$$-\frac{b}{6\,c^{\frac{5}{2}}x^{6}}-\frac{a}{7\,c^{\frac{5}{2}}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/6*b/(c^(5/2)*x^6) - 1/7*a/(c^(5/2)*x^7)
```

**mupad** [B] time = 0.16, size = 26, normalized size = 0.63

$$-\frac{6\,a\,\sqrt{x^{2}}\,+7\,b\,x\,\sqrt{x^{2}}}{42\,c^{5/2}\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(5/2)),x)
[Out] -(6*a*(x^2)^(1/2) + 7*b*x*(x^2)^(1/2))/(42*c^(5/2)*x^8)
```

**sympy** [A] time = 1.64, size = 36, normalized size = 0.88

$$-\frac{a}{7c^{\frac{5}{2}}x^{2}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{b}{6c^{\frac{5}{2}}x\left(x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(5/2),x)
[Out] -a/(7*c**(5/2)*x**2*(x**2)**(5/2)) - b/(6*c**(5/2)*x*(x**2)**(5/2))
```

## 3.761 $\int \frac{a+bx}{x^4(cx^2)^{5/2}}\,dx$

**Optimal**. Leaf size=41

$$-\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(8*c^2*x^7*Sqrt[c*x^2]) - b/(7*c^2*x^6*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^4\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^9}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^9}+\frac{b}{x^8}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.59

$$\frac{-7a-8bx}{56x^3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
[Out] (-7*a - 8*b*x)/(56*x^3*(c*x^2)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 24, normalized size = 0.59

$$\frac{-7a-8bx}{56x^3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
[Out] (-7*a - 8*b*x)/(56*x^3*(c*x^2)^(5/2))
```

**fricas** [A] time = 0.83, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(8\,bx+7\,a)}{56\,c^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/56*sqrt(c*x^2)*(8*b*x + 7*a)/(c^3*x^9)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{8bx+7a}{56\left(c\,x^2\right)^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(5/2),x)
```

```
[Out] -1/56*(8*b*x+7*a)/x^3/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.34, size = 19, normalized size = 0.46

$$-\frac{b}{7\,c^{\frac{5}{2}}x^7}-\frac{a}{8\,c^{\frac{5}{2}}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/7*b/(c^(5/2)*x^7) - 1/8*a/(c^(5/2)*x^8)
```

**mupad [B]** time = 0.16, size = 26, normalized size = 0.63

$$-\frac{7\,a\,\sqrt{x^2}\,+8\,b\,x\,\sqrt{x^2}}{56\,c^{5/2}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(5/2)),x)
```

```
[Out] -(7*a*(x^2)^(1/2) + 8*b*x*(x^2)^(1/2))/(56*c^(5/2)*x^9)
```

**sympy [A]** time = 1.97, size = 37, normalized size = 0.90

$$-\frac{a}{8c^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}-\frac{b}{7c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(5/2),x)
```

```
[Out] -a/(8*c**(5/2)*x**3*(x**2)**(5/2)) - b/(7*c**(5/2)*x**2*(x**2)**(5/2))
```

## 3.762 $\int x^3\sqrt{cx^2}\,(a+bx)^2\,dx$

**Optimal**. Leaf size=57

$$\frac{1}{5}a^2x^4\sqrt{cx^2}+\frac{1}{3}abx^5\sqrt{cx^2}+\frac{1}{7}b^2x^6\sqrt{cx^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{5}a^2x^4\sqrt{cx^2}+\frac{1}{3}abx^5\sqrt{cx^2}+\frac{1}{7}b^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4*Sqrt[c*x^2])/5 + (a*b*x^5*Sqrt[c*x^2])/3 + (b^2*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{cx^2}\,(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\,\int x^4(a+bx)^2\,dx}{x}\\
&= \frac{\sqrt{cx^2}\,\int \left(a^2x^4+2abx^5+b^2x^6\right)\,dx}{x}\\
&= \frac{1}{5}a^2x^4\sqrt{cx^2}+\frac{1}{3}abx^5\sqrt{cx^2}+\frac{1}{7}b^2x^6\sqrt{cx^2}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{1}{105}x^4\sqrt{cx^2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^4*Sqrt[c*x^2]*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.61

$$\frac{1}{105}x^4\sqrt{cx^2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^4*Sqrt[c*x^2]*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

**fricas** [A] time = 0.86, size = 33, normalized size = 0.58

$$\frac{1}{105}\left(15\,b^2x^6+35\,abx^5+21\,a^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/105*(15*b^2*x^6 + 35*a*b*x^5 + 21*a^2*x^4)*sqrt(c*x^2)
```

**giac** [A] time = 1.08, size = 35, normalized size = 0.61

$$\frac{1}{105}\left(15\,b^2x^7\mathrm{sgn}(x)+35\,abx^6\mathrm{sgn}(x)+21\,a^2x^5\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.56

$$\frac{\left(15b^2x^2+35abx+21a^2\right)\sqrt{c\,x^2}\;x^4}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/105*x^4*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(1/2)
```

**maxima [A]** time = 1.39, size = 54, normalized size = 0.95

$$\frac{(cx^2)^{\frac{3}{2}}\,b^2x^4}{7\,c}+\frac{(cx^2)^{\frac{3}{2}}\,abx^3}{3\,c}+\frac{(cx^2)^{\frac{3}{2}}\,a^2x^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2)^(3/2)*b^2*x^4/c + 1/3*(c*x^2)^(3/2)*a*b*x^3/c + 1/5*(c*x^2)^(3/
2)*a^2*x^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\,\sqrt{c\,x^2}\,\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(x^3*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

**sympy [A]** time = 0.59, size = 60, normalized size = 1.05

$$\frac{a^2\sqrt{c}\,x^4\sqrt{x^2}}{5}+\frac{ab\sqrt{c}\,x^5\sqrt{x^2}}{3}+\frac{b^2\sqrt{c}\,x^6\sqrt{x^2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x**4*sqrt(x**2)/5 + a*b*sqrt(c)*x**5*sqrt(x**2)/3 + b**2*sqrt(
c)*x**6*sqrt(x**2)/7
```

## 3.763 $\int x^2\sqrt{cx^2}\,(a+bx)^2\,dx$

**Optimal**. Leaf size=57

$$\frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*x^4*Sqrt[c*x^2])/5 + (b^2*x^5*Sqrt[c*x^2])
/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{cx^2}\,(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x^3(a+bx)^2\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int\left(a^2x^3+2abx^4+b^2x^5\right)dx}{x}\\
&= \frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{1}{60}x^3\sqrt{cx^2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^3*Sqrt[c*x^2]*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

**IntegrateAlgebraic** [A] time = 0.02, size = 35, normalized size = 0.61

$$\frac{1}{60}x^3\sqrt{cx^2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^3*Sqrt[c*x^2]*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

**fricas** [A] time = 0.75, size = 33, normalized size = 0.58

$$\frac{1}{60}\left(10\,b^2x^5+24\,abx^4+15\,a^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/60*(10*b^2*x^5 + 24*a*b*x^4 + 15*a^2*x^3)*sqrt(c*x^2)
```

**giac** [A] time = 1.04, size = 35, normalized size = 0.61

$$\frac{1}{60}\left(10\,b^2x^6\mathrm{sgn}(x)+24\,abx^5\mathrm{sgn}(x)+15\,a^2x^4\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.56

$$\frac{\left(10b^2x^2+24abx+15a^2\right)\sqrt{c\,x^2}\;x^3}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/60*x^3*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(1/2)
```

**maxima** [A] time = 1.35, size = 52, normalized size = 0.91

$$\frac{\left(cx^{2}\right)^{\frac{3}{2}} b^{2}x^{3}}{6\,c}+\frac{2\,\left(cx^{2}\right)^{\frac{3}{2}} abx^{2}}{5\,c}+\frac{\left(cx^{2}\right)^{\frac{3}{2}} a^{2}x}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(3/2)*b^2*x^3/c + 2/5*(c*x^2)^(3/2)*a*b*x^2/c + 1/4*(c*x^2)^(3/
2)*a^2*x/c
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\sqrt{c\,x^2}\,\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(x^2*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

**sympy** [A] time = 0.46, size = 61, normalized size = 1.07

$$\frac{a^{2}\sqrt{c}\,x^{3}\sqrt{x^{2}}}{4}+\frac{2ab\sqrt{c}\,x^{4}\sqrt{x^{2}}}{5}+\frac{b^{2}\sqrt{c}\,x^{5}\sqrt{x^{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x**3*sqrt(x**2)/4 + 2*a*b*sqrt(c)*x**4*sqrt(x**2)/5 + b**2*sqr
t(c)*x**5*sqrt(x**2)/6
```

## 3.764 $\int x\sqrt{cx^2}\,(a+bx)^2\,dx$

**Optimal**. Leaf size=57

$$\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2*Sqrt[c*x^2])/3 + (a*b*x^3*Sqrt[c*x^2])/2 + (b^2*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}\,(a+bx)^2\,dx&=\frac{\sqrt{cx^2}\int x^2(a+bx)^2\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(a^2x^2+2abx^3+b^2x^4\right)dx}{x}\\&=\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{1}{30}x^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

**IntegrateAlgebraic** [A] time = 0.02, size = 35, normalized size = 0.61

$$\frac{1}{30}x^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

**fricas** [A] time = 1.11, size = 33, normalized size = 0.58

$$\frac{1}{30}\left(6\,b^2x^4+15\,abx^3+10\,a^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)
```

**giac** [A] time = 1.11, size = 35, normalized size = 0.61

$$\frac{1}{30}\left(6\,b^2x^5\mathrm{sgn}(x)+15\,abx^4\mathrm{sgn}(x)+10\,a^2x^3\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.56

$$\frac{\left(6b^2x^2+15abx+10a^2\right)\sqrt{c\,x^2}\;x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/30*x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(1/2)
```

**maxima [A]** time = 1.31, size = 49, normalized size = 0.86

$$\frac{\left(cx^2\right)^{\frac{3}{2}} b^2x^2}{5\,c} + \frac{\left(cx^2\right)^{\frac{3}{2}} abx}{2\,c} + \frac{\left(cx^2\right)^{\frac{3}{2}} a^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(c*x^2)^(3/2)*b^2*x^2/c + 1/2*(c*x^2)^(3/2)*a*b*x/c + 1/3*(c*x^2)^(3/2)
*a^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{c\,x^2}\,\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(x*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

**sympy [A]** time = 0.37, size = 60, normalized size = 1.05

$$\frac{a^2\sqrt{c}\,x^2\sqrt{x^2}}{3} + \frac{ab\sqrt{c}\,x^3\sqrt{x^2}}{2} + \frac{b^2\sqrt{c}\,x^4\sqrt{x^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x**2*sqrt(x**2)/3 + a*b*sqrt(c)*x**3*sqrt(x**2)/2 + b**2*sqrt(
c)*x**4*sqrt(x**2)/5
```

## 3.765 $\int \sqrt{cx^2} (a+bx)^2 \, dx$

**Optimal**. Leaf size=55

$$\frac{1}{2}a^2x\sqrt{cx^2} + \frac{2}{3}abx^2\sqrt{cx^2} + \frac{1}{4}b^2x^3\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{1}{2}a^2x\sqrt{cx^2} + \frac{2}{3}abx^2\sqrt{cx^2} + \frac{1}{4}b^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x*Sqrt[c*x^2])/2 + (2*a*b*x^2*Sqrt[c*x^2])/3 + (b^2*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{cx^2} (a+bx)^2 \, dx &= \frac{\sqrt{cx^2} \int x(a+bx)^2 \, dx}{x} \\
&= \frac{\sqrt{cx^2} \int \left(a^2x + 2abx^2 + b^2x^3\right) dx}{x} \\
&= \frac{1}{2}a^2x\sqrt{cx^2} + \frac{2}{3}abx^2\sqrt{cx^2} + \frac{1}{4}b^2x^3\sqrt{cx^2}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.60

$$\frac{1}{12}x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

**IntegrateAlgebraic** [A] time = 0.03, size = 33, normalized size = 0.60

$$\frac{1}{12}x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

**fricas** [A] time = 1.12, size = 31, normalized size = 0.56

$$\frac{1}{12}\left(3\,b^2x^3+8\,abx^2+6\,a^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)
```

**giac** [A] time = 0.95, size = 35, normalized size = 0.64

$$\frac{1}{12}\left(3\,b^2x^4\mathrm{sgn}(x)+8\,abx^3\mathrm{sgn}(x)+6\,a^2x^2\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.00, size = 30, normalized size = 0.55

$$\frac{\left(3b^2x^2+8abx+6a^2\right)\sqrt{c\,x^2}\;x}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/12*x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(1/2)
```

**maxima [A]** time = 1.35, size = 44, normalized size = 0.80

$$\frac{1}{2}\sqrt{cx^2}\,a^2x+\frac{\left(cx^2\right)^{\frac{3}{2}}b^2x}{4\,c}+\frac{2\left(cx^2\right)^{\frac{3}{2}}ab}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2)*a^2*x + 1/4*(c*x^2)^(3/2)*b^2*x/c + 2/3*(c*x^2)^(3/2)*a*b/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{c\,x^2}\,\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(1/2)*(a + b*x)^2, x)
```

**sympy [A]** time = 0.30, size = 60, normalized size = 1.09

$$\frac{a^2\sqrt{c}\,x\sqrt{x^2}}{2}+\frac{2ab\sqrt{c}\,x^2\sqrt{x^2}}{3}+\frac{b^2\sqrt{c}\,x^3\sqrt{x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x*sqrt(x**2)/2 + 2*a*b*sqrt(c)*x**2*sqrt(x**2)/3 + b**2*sqrt(c
)*x**3*sqrt(x**2)/4
```

## 3.766 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x}\,dx$

**Optimal**. Leaf size=26

$$\frac{\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

**Rubi [A]** time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]

[Out] (Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x}\,dx &= \frac{\sqrt{cx^2}\int (a+bx)^2\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,(a+bx)^3}{3bx}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 0.96

$$\frac{cx(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]
```

```
[Out] (c*x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

**IntegrateAlgebraic [A]** time = 0.03, size = 31, normalized size = 1.19

$$\frac{1}{3}\sqrt{cx^2}\left(3a^2+3abx+b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]
```

```
[Out] (Sqrt[c*x^2]*(3*a^2 + 3*a*b*x + b^2*x^2))/3
```

**fricas [A]** time = 1.08, size = 27, normalized size = 1.04

$$\frac{1}{3}\left(b^2x^2+3\,abx+3\,a^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)
```

**giac [A]** time = 0.97, size = 29, normalized size = 1.12

$$\frac{1}{3}\left(\frac{(bx+a)^3\mathrm{sgn}(x)}{b}-\frac{a^3\mathrm{sgn}(x)}{b}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*sqrt(c)
```

**maple [A]** time = 0.00, size = 28, normalized size = 1.08

$$\frac{\left(b^2x^2+3abx+3a^2\right)\sqrt{c\,x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x,x)
```

```
[Out] 1/3*(b^2*x^2+3*a*b*x+3*a^2)*(c*x^2)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x, x)
```

**sympy [B]** time = 0.30, size = 51, normalized size = 1.96

$$a^2\sqrt{c}\,\sqrt{x^2}+ab\sqrt{c}\,x\sqrt{x^2}+\frac{b^2\sqrt{c}\,x^2\sqrt{x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x,x)
```

```
[Out] a**2*sqrt(c)*sqrt(x**2) + a*b*sqrt(c)*x*sqrt(x**2) + b**2*sqrt(c)*x**2*sqrt
(x**2)/3
```

## 3.767 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2} dx$

**Optimal**. Leaf size=49

$$\frac{a^2\sqrt{cx^2}\log(x)}{x}+2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}\log(x)}{x}+2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
```

```
[Out] 2*a*b*Sqrt[c*x^2] + (b^2*x*Sqrt[c*x^2])/2 + (a^2*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2} dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x} dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(2ab+\frac{a^2}{x}+b^2x\right) dx}{x}\\
&= 2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}+\frac{a^2\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.67

$$\frac{cx\left(2a^2\log(x)+bx(4a+bx)\right)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
[Out] (c*x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 34, normalized size = 0.69

$$\sqrt{cx^2}\left(\frac{a^2\log(x)}{x}+\frac{1}{2}\left(4ab+b^2x\right)\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
[Out] Sqrt[c*x^2]*((4*a*b + b^2*x)/2 + (a^2*Log[x])/x)
```

**fricas** [A] time = 0.76, size = 32, normalized size = 0.65

$$\frac{\left(b^2x^2+4\,abx+2\,a^2\log(x)\right)\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/x
```

**giac** [A] time = 1.08, size = 32, normalized size = 0.65

$$\frac{1}{2}\left(b^2x^2\operatorname{sgn}(x)+4\,abx\operatorname{sgn}(x)+2\,a^2\log\left(|x|\right)\operatorname{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="giac")
[Out] 1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*sqrt(c)
```

**maple** [A] time = 0.01, size = 33, normalized size = 0.67

$$\frac{\sqrt{c\,x^2}\,\left(b^2x^2+2a^2\ln(x)+4abx\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^2,x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^2,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2}} \left(a + bx\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**2, x)
```

## 3.768 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx$

**Optimal**. Leaf size=49

$$-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\,\log(x)}{x}+b^2\sqrt{cx^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\,\log(x)}{x}+b^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
```

```
[Out] b^2*Sqrt[c*x^2] - (a^2*Sqrt[c*x^2])/x^2 + (2*a*b*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x^2}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{x}\\ &= b^2\sqrt{cx^2}-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\,\log(x)}{x}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 31, normalized size = 0.63

$$\frac{c\left(-a^2+2abx\log(x)+b^2x^2\right)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
[Out] (c*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/Sqrt[c*x^2]
```

**IntegrateAlgebraic** [A] time = 0.03, size = 37, normalized size = 0.76

$$\sqrt{cx^2}\left(\frac{b^2x^2-a^2}{x^2}+\frac{2ab\log(x)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
[Out] Sqrt[c*x^2]*((-a^2 + b^2*x^2)/x^2 + (2*a*b*Log[x])/x)
```

**fricas** [A] time = 1.21, size = 31, normalized size = 0.63

$$\frac{\left(b^2x^2+2\,abx\log(x)-a^2\right)\sqrt{cx^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/x^2
```

**giac** [A] time = 1.02, size = 31, normalized size = 0.63

$$\left(b^2x\mathrm{sgn}(x)+2\,ab\log\left(|x|\right)\mathrm{sgn}(x)-\frac{a^2\mathrm{sgn}(x)}{x}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="giac")
[Out] (b^2*x*sgn(x) + 2*a*b*log(abs(x))*sgn(x) - a^2*sgn(x)/x)*sqrt(c)
```

**maple** [A] time = 0.01, size = 32, normalized size = 0.65

$$\frac{\sqrt{c\,x^2}\,\left(2abx\ln(x)+b^2x^2-a^2\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^3,x)
```

```
[Out] (c*x^2)^(1/2)*(2*a*b*ln(x)*x+b^2*x^2-a^2)/x^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^3,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,\left(a+bx\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**3, x)
```

## 3.769 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4} dx$

**Optimal**. Leaf size=54

$$-\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}$$

**Rubi [A]** time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
```

```
[Out] -(a^2*Sqrt[c*x^2])/(2*x^3) - (2*a*b*Sqrt[c*x^2])/x^2 + (b^2*Sqrt[c*x^2]*Log
[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4} dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x^3} dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right) dx}{x}\\ &= -\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 36, normalized size = 0.67

$$\frac{\sqrt{cx^2}\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
[Out] (Sqrt[c*x^2]*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*x^3)
```

**IntegrateAlgebraic** [A] time = 0.04, size = 38, normalized size = 0.70

$$\sqrt{cx^2}\left(\frac{-a^2-4abx}{2x^3}+\frac{b^2\log(x)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
[Out] Sqrt[c*x^2]*((-a^2 - 4*a*b*x)/(2*x^3) + (b^2*Log[x])/x)
```

**fricas** [A] time = 1.36, size = 33, normalized size = 0.61

$$\frac{\left(2\,b^2x^2\log(x)-4\,abx-a^2\right)\sqrt{cx^2}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/x^3
```

**giac** [A] time = 0.99, size = 35, normalized size = 0.65

$$\frac{1}{2}\left(2\,b^2\log\left(|x|\right)\operatorname{sgn}(x)-\frac{4\,abx\operatorname{sgn}(x)+a^2\operatorname{sgn}(x)}{x^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="giac")
[Out] 1/2*(2*b^2*log(abs(x))*sgn(x) - (4*a*b*x*sgn(x) + a^2*sgn(x))/x^2)*sqrt(c)
```

**maple** [A] time = 0.01, size = 34, normalized size = 0.63

$$\frac{\sqrt{c\,x^2}\,\left(2b^2x^2\ln(x)-4abx-a^2\right)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^4,x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/x^3
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^4,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**4, x)
```

## 3.770 $\int x^3 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

**Optimal**. Leaf size=60

$$\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^6*Sqrt[c*x^2])/7 + (a*b*c*x^7*Sqrt[c*x^2])/4 + (b^2*c*x^8*Sqrt[c*x
^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^6(a+bx)^2 \, dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^6+2abx^7+b^2x^8\right) dx}{x}\\ &= \frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{252}x^4\left(cx^2\right)^{3/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x^4*(c*x^2)^(3/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.58

$$\frac{1}{252}x^4\left(cx^2\right)^{3/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x^4*(c*x^2)^(3/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

**fricas** [A] time = 0.84, size = 36, normalized size = 0.60

$$\frac{1}{252}\left(28\,b^2cx^8+63\,abcx^7+36\,a^2cx^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^2*c*x^8 + 63*a*b*c*x^7 + 36*a^2*c*x^6)*sqrt(c*x^2)
```

**giac** [A] time = 1.09, size = 35, normalized size = 0.58

$$\frac{1}{252}\left(28\,b^2x^9\mathrm{sgn}(x)+63\,abx^8\mathrm{sgn}(x)+36\,a^2x^7\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/252*(28*b^2*x^9*sgn(x) + 63*a*b*x^8*sgn(x) + 36*a^2*x^7*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.01, size = 32, normalized size = 0.53

$$\frac{\left(28b^2x^2+63abx+36a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}x^4}{252}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/252*x^4*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(3/2)
```

**maxima [A]** time = 1.35, size = 54, normalized size = 0.90

$$\frac{\left(cx^{2}\right)^{\frac{5}{2}} b^{2} x^{4}}{9 \, c} + \frac{\left(cx^{2}\right)^{\frac{5}{2}} a b x^{3}}{4 \, c} + \frac{\left(cx^{2}\right)^{\frac{5}{2}} a^{2} x^{2}}{7 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(c*x^2)^(5/2)*b^2*x^4/c + 1/4*(c*x^2)^(5/2)*a*b*x^3/c + 1/7*(c*x^2)^(5/
2)*a^2*x^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \left(c\,x^2\right)^{3/2} \left(a+b\,x\right)^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int(x^3*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

**sympy [A]** time = 1.50, size = 60, normalized size = 1.00

$$\frac{a^{2} c^{\frac{3}{2}} x^{4} \left(x^{2}\right)^{\frac{3}{2}}}{7} + \frac{a b c^{\frac{3}{2}} x^{5} \left(x^{2}\right)^{\frac{3}{2}}}{4} + \frac{b^{2} c^{\frac{3}{2}} x^{6} \left(x^{2}\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x**4*(x**2)**(3/2)/7 + a*b*c**(3/2)*x**5*(x**2)**(3/2)/4 + b*
*2*c**(3/2)*x**6*(x**2)**(3/2)/9
```

## 3.771 $\int x^2 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

**Optimal**. Leaf size=60

$$\frac{1}{6}a^2cx^5\sqrt{cx^2} + \frac{2}{7}abcx^6\sqrt{cx^2} + \frac{1}{8}b^2cx^7\sqrt{cx^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{6}a^2cx^5\sqrt{cx^2} + \frac{2}{7}abcx^6\sqrt{cx^2} + \frac{1}{8}b^2cx^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^5*Sqrt[c*x^2])/6 + (2*a*b*c*x^6*Sqrt[c*x^2])/7 + (b^2*c*x^7*Sqrt[c
*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^5(a+bx)^2 \, dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^5+2abx^6+b^2x^7\right) dx}{x}\\
&= \frac{1}{6}a^2cx^5\sqrt{cx^2} + \frac{2}{7}abcx^6\sqrt{cx^2} + \frac{1}{8}b^2cx^7\sqrt{cx^2}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{168}x^3\left(cx^2\right)^{3/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x^3*(c*x^2)^(3/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.58

$$\frac{1}{168}x^3\left(cx^2\right)^{3/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x^3*(c*x^2)^(3/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

**fricas** [A] time = 1.14, size = 36, normalized size = 0.60

$$\frac{1}{168}\left(21\,b^2cx^7+48\,abcx^6+28\,a^2cx^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
[Out] 1/168*(21*b^2*c*x^7 + 48*a*b*c*x^6 + 28*a^2*c*x^5)*sqrt(c*x^2)
```

**giac** [A] time = 1.08, size = 35, normalized size = 0.58

$$\frac{1}{168}\left(21\,b^2x^8\mathrm{sgn}(x)+48\,abx^7\mathrm{sgn}(x)+28\,a^2x^6\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
[Out] 1/168*(21*b^2*x^8*sgn(x) + 48*a*b*x^7*sgn(x) + 28*a^2*x^6*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.53

$$\frac{\left(21b^2x^2+48abx+28a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}x^3}{168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/168*x^3*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(3/2)
```

**maxima** [A] time = 1.33, size = 52, normalized size = 0.87

$$\frac{\left(cx^2\right)^{\frac{5}{2}}b^2x^3}{8\,c}+\frac{2\,\left(cx^2\right)^{\frac{5}{2}}abx^2}{7\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}a^2x}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2)^(5/2)*b^2*x^3/c + 2/7*(c*x^2)^(5/2)*a*b*x^2/c + 1/6*(c*x^2)^(5/
2)*a^2*x/c
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\left(c\,x^2\right)^{3/2}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int(x^2*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

**sympy** [A] time = 1.22, size = 61, normalized size = 1.02

$$\frac{a^2c^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}{6}+\frac{2abc^{\frac{3}{2}}x^4\left(x^2\right)^{\frac{3}{2}}}{7}+\frac{b^2c^{\frac{3}{2}}x^5\left(x^2\right)^{\frac{3}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x**3*(x**2)**(3/2)/6 + 2*a*b*c**(3/2)*x**4*(x**2)**(3/2)/7 +
b**2*c**(3/2)*x**5*(x**2)**(3/2)/8
```

# 3.772 $\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx$

**Optimal**. Leaf size=60

$$\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^4*Sqrt[c*x^2])/5 + (a*b*c*x^5*Sqrt[c*x^2])/3 + (b^2*c*x^6*Sqrt[c*x
^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)^2\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(a^2x^4+2abx^5+b^2x^6\right)dx}{x}\\&=\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{105}x^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.58

$$\frac{1}{105}x^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

**fricas** [A] time = 1.21, size = 36, normalized size = 0.60

$$\frac{1}{105}\left(15\,b^2cx^6+35\,abcx^5+21\,a^2cx^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
[Out] 1/105*(15*b^2*c*x^6 + 35*a*b*c*x^5 + 21*a^2*c*x^4)*sqrt(c*x^2)
```

**giac** [A] time = 1.18, size = 35, normalized size = 0.58

$$\frac{1}{105}\left(15\,b^2x^7\mathrm{sgn}(x)+35\,abx^6\mathrm{sgn}(x)+21\,a^2x^5\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
[Out] 1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.53

$$\frac{\left(15b^2x^2+35abx+21a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}x^2}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/105*x^2*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(3/2)
```

**maxima [A]** time = 1.31, size = 49, normalized size = 0.82

$$\frac{\left(cx^2\right)^{\frac{5}{2}} b^2 x^2}{7\,c} + \frac{\left(cx^2\right)^{\frac{5}{2}} abx}{3\,c} + \frac{\left(cx^2\right)^{\frac{5}{2}} a^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2)^(5/2)*b^2*x^2/c + 1/3*(c*x^2)^(5/2)*a*b*x/c + 1/5*(c*x^2)^(5/2)
*a^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(c\,x^2\right)^{3/2}\,(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int(x*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

**sympy [A]** time = 0.97, size = 60, normalized size = 1.00

$$\frac{a^2 c^{\frac{3}{2}} x^2 \left(x^2\right)^{\frac{3}{2}}}{5} + \frac{abc^{\frac{3}{2}} x^3 \left(x^2\right)^{\frac{3}{2}}}{3} + \frac{b^2 c^{\frac{3}{2}} x^4 \left(x^2\right)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x**2*(x**2)**(3/2)/5 + a*b*c**(3/2)*x**3*(x**2)**(3/2)/3 + b*
*2*c**(3/2)*x**4*(x**2)**(3/2)/7
```

## 3.773 $\int \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

**Optimal**. Leaf size=60

$$\frac{1}{4}a^2cx^3\sqrt{cx^2}+\frac{2}{5}abcx^4\sqrt{cx^2}+\frac{1}{6}b^2cx^5\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{1}{4}a^2cx^3\sqrt{cx^2}+\frac{2}{5}abcx^4\sqrt{cx^2}+\frac{1}{6}b^2cx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^3*Sqrt[c*x^2])/4 + (2*a*b*c*x^4*Sqrt[c*x^2])/5 + (b^2*c*x^5*Sqrt[c
*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^{3/2} (a+bx)^2 \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx)^2 \, dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^3+2abx^4+b^2x^5\right) dx}{x}\\ &= \frac{1}{4}a^2cx^3\sqrt{cx^2}+\frac{2}{5}abcx^4\sqrt{cx^2}+\frac{1}{6}b^2cx^5\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.55

$$\frac{1}{60}x\left(cx^2\right)^{3/2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

**IntegrateAlgebraic** [A] time = 0.03, size = 33, normalized size = 0.55

$$\frac{1}{60}x\left(cx^2\right)^{3/2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

**fricas** [A] time = 0.93, size = 36, normalized size = 0.60

$$\frac{1}{60}\left(10\,b^2cx^5+24\,abcx^4+15\,a^2cx^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*c*x^5 + 24*a*b*c*x^4 + 15*a^2*c*x^3)*sqrt(c*x^2)
```

**giac** [A] time = 1.13, size = 35, normalized size = 0.58

$$\frac{1}{60}\left(10\,b^2x^6\mathrm{sgn}(x)+24\,abx^5\mathrm{sgn}(x)+15\,a^2x^4\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.01, size = 30, normalized size = 0.50

$$\frac{\left(10b^2x^2+24abx+15a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}x}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/60*x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(3/2)
```

**maxima [A]** time = 1.36, size = 44, normalized size = 0.73

$$\frac{1}{4}\left(cx^2\right)^{\frac{3}{2}}a^2x+\frac{\left(cx^2\right)^{\frac{5}{2}}b^2x}{6\,c}+\frac{2\left(cx^2\right)^{\frac{5}{2}}ab}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2)^(3/2)*a^2*x + 1/6*(c*x^2)^(5/2)*b^2*x/c + 2/5*(c*x^2)^(5/2)*a*b
/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \left(c\,x^2\right)^{3/2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(3/2)*(a + b*x)^2, x)
```

**sympy [A]** time = 0.77, size = 60, normalized size = 1.00

$$\frac{a^2c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}{4}+\frac{2abc^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}{5}+\frac{b^2c^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x*(x**2)**(3/2)/4 + 2*a*b*c**(3/2)*x**2*(x**2)**(3/2)/5 + b**
2*c**(3/2)*x**3*(x**2)**(3/2)/6
```

## 3.774 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx$

**Optimal**. Leaf size=60

$$\frac{1}{3}a^2cx^2\sqrt{cx^2}+\frac{1}{2}abcx^3\sqrt{cx^2}+\frac{1}{5}b^2cx^4\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{3}a^2cx^2\sqrt{cx^2}+\frac{1}{2}abcx^3\sqrt{cx^2}+\frac{1}{5}b^2cx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
```

```
[Out] (a^2*c*x^2*Sqrt[c*x^2])/3 + (a*b*c*x^3*Sqrt[c*x^2])/2 + (b^2*c*x^4*Sqrt[c*x
^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)^2\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^2+2abx^3+b^2x^4\right)\,dx}{x}\\ &= \frac{1}{3}a^2cx^2\sqrt{cx^2}+\frac{1}{2}abcx^3\sqrt{cx^2}+\frac{1}{5}b^2cx^4\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 36, normalized size = 0.60

$$\frac{1}{30}cx^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
[Out] (c*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

**IntegrateAlgebraic** [A] time = 0.03, size = 32, normalized size = 0.53

$$\frac{1}{30}\left(cx^2\right)^{3/2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
[Out] ((c*x^2)^(3/2)*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

**fricas** [A] time = 1.07, size = 36, normalized size = 0.60

$$\frac{1}{30}\left(6\,b^2cx^4+15\,abcx^3+10\,a^2cx^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="fricas")
[Out] 1/30*(6*b^2*c*x^4 + 15*a*b*c*x^3 + 10*a^2*c*x^2)*sqrt(c*x^2)
```

**giac** [A] time = 0.95, size = 35, normalized size = 0.58

$$\frac{1}{30}\left(6\,b^2x^5\mathrm{sgn}(x)+15\,abx^4\mathrm{sgn}(x)+10\,a^2x^3\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="giac")
[Out] 1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 29, normalized size = 0.48

$$\frac{\left(6b^2x^2+15abx+10a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x,x)
```

```
[Out] 1/30*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(3/2)
```

**maxima [A]** time = 1.30, size = 40, normalized size = 0.67

$$\frac{1}{2}\left(cx^2\right)^{\frac{3}{2}}abx + \frac{1}{3}\left(cx^2\right)^{\frac{3}{2}}a^2 + \frac{\left(cx^2\right)^{\frac{5}{2}}b^2}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^2)^(3/2)*a*b*x + 1/3*(c*x^2)^(3/2)*a^2 + 1/5*(c*x^2)^(5/2)*b^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x, x)
```

**sympy [A]** time = 0.79, size = 54, normalized size = 0.90

$$\frac{a^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{3} + \frac{abc^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}{2} + \frac{b^2c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x,x)
```

```
[Out] a**2*c**(3/2)*(x**2)**(3/2)/3 + a*b*c**(3/2)*x*(x**2)**(3/2)/2 + b**2*c**(3
/2)*x**2*(x**2)**(3/2)/5
```

## 3.775 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2} dx$

**Optimal**. Leaf size=58

$$\frac{1}{2}a^2cx\sqrt{cx^2} + \frac{2}{3}abcx^2\sqrt{cx^2} + \frac{1}{4}b^2cx^3\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{2}a^2cx\sqrt{cx^2} + \frac{2}{3}abcx^2\sqrt{cx^2} + \frac{1}{4}b^2cx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (a^2*c*x*Sqrt[c*x^2])/2 + (2*a*b*c*x^2*Sqrt[c*x^2])/3 + (b^2*c*x^3*Sqrt[c*x
^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)^2\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x+2abx^2+b^2x^3\right)dx}{x}\\
&= \frac{1}{2}a^2cx\sqrt{cx^2} + \frac{2}{3}abcx^2\sqrt{cx^2} + \frac{1}{4}b^2cx^3\sqrt{cx^2}
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 34, normalized size = 0.59

$$\frac{1}{12}cx\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (c*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.60

$$\frac{\left(cx^2\right)^{3/2}\left(6a^2+8abx+3b^2x^2\right)}{12x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] ((c*x^2)^(3/2)*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*x)
```

**fricas** [A] time = 1.06, size = 34, normalized size = 0.59

$$\frac{1}{12}\left(3\,b^2cx^3+8\,abcx^2+6\,a^2cx\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c*x^3 + 8*a*b*c*x^2 + 6*a^2*c*x)*sqrt(c*x^2)
```

**giac** [A] time = 1.03, size = 35, normalized size = 0.60

$$\frac{1}{12}\left(3\,b^2x^4\mathrm{sgn}(x)+8\,abx^3\mathrm{sgn}(x)+6\,a^2x^2\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.55

$$\frac{\left(3b^2x^2+8abx+6a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^2,x)
```

```
[Out] 1/12/x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(3/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}\,{\left(a+b\,x\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^2, x)
```

**sympy [A]** time = 0.80, size = 54, normalized size = 0.93

$$\frac{a^{2} c^{\frac{3}{2}} \left(x^{2}\right)^{\frac{3}{2}}}{2 x} + \frac{2 a b c^{\frac{3}{2}} \left(x^{2}\right)^{\frac{3}{2}}}{3} + \frac{b^{2} c^{\frac{3}{2}} x \left(x^{2}\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**2,x)
```

```
[Out] a**2*c**(3/2)*(x**2)**(3/2)/(2*x) + 2*a*b*c**(3/2)*(x**2)**(3/2)/3 + b**2*c
**(3/2)*x*(x**2)**(3/2)/4
```

## 3.776 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3} \, dx$

**Optimal**. Leaf size=27

$$\frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

**Rubi [A]** time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)^2\,dx}{x}\\ &= \frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 0.96

$$\frac{(cx^2)^{3/2}(a+bx)^3}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^3)/(3*b*x^3)
```

**IntegrateAlgebraic** [A]   time = 0.03, size = 34, normalized size = 1.26

$$\frac{\left(cx^2\right)^{3/2}\left(3a^2+3abx+b^2x^2\right)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] ((c*x^2)^(3/2)*(3*a^2 + 3*a*b*x + b^2*x^2))/(3*x^2)
```

**fricas** [A]   time = 1.06, size = 30, normalized size = 1.11

$$\frac{1}{3}\left(b^2cx^2+3\,abcx+3\,a^2c\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c*x^2 + 3*a*b*c*x + 3*a^2*c)*sqrt(c*x^2)
```

**giac** [A]   time = 1.22, size = 29, normalized size = 1.07

$$\frac{1}{3}\left(\frac{(bx+a)^3\mathrm{sgn}(x)}{b}-\frac{a^3\mathrm{sgn}(x)}{b}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*c^(3/2)
```

**maple** [A]   time = 0.00, size = 31, normalized size = 1.15

$$\frac{\left(b^2x^2+3abx+3a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^3,x)
```

```
[Out] 1/3/x^2*(b^2*x^2+3*a*b*x+3*a^2)*(c*x^2)^(3/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^3, x)
```

**sympy [B]** time = 0.94, size = 51, normalized size = 1.89

$$\frac{a^{2} c^{\frac{3}{2}} \left(x^{2}\right)^{\frac{3}{2}}}{x^{2}} + \frac{a b c^{\frac{3}{2}} \left(x^{2}\right)^{\frac{3}{2}}}{x} + \frac{b^{2} c^{\frac{3}{2}} \left(x^{2}\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**3,x)
```

```
[Out] a**2*c**(3/2)*(x**2)**(3/2)/x**2 + a*b*c**(3/2)*(x**2)**(3/2)/x + b**2*c**(
3/2)*(x**2)**(3/2)/3
```

## 3.777 $\int \frac{\left(cx^2\right)^{3/2}(a+bx)^2}{x^4}\,dx$

**Optimal**. Leaf size=52

$$\frac{a^2c\sqrt{cx^2}\,\log(x)}{x}+2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}\,\log(x)}{x}+2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] 2*a*b*c*Sqrt[c*x^2] + (b^2*c*x*Sqrt[c*x^2])/2 + (a^2*c*Sqrt[c*x^2]*Log[x])/
x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^2}{x}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{x}$$
$$= 2abc\sqrt{cx^2} + \frac{1}{2}b^2cx\sqrt{cx^2} + \frac{a^2c\sqrt{cx^2}\,\log(x)}{x}$$

**Mathematica** [A] time = 0.01, size = 34, normalized size = 0.65

$$\frac{(cx^2)^{3/2}\left(2a^2\log(x)+bx(4a+bx)\right)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
[Out] ((c*x^2)^(3/2)*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*x^3)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 37, normalized size = 0.71

$$(cx^2)^{3/2}\left(\frac{a^2\log(x)}{x^3}+\frac{4ab+b^2x}{2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
[Out] (c*x^2)^(3/2)*((4*a*b + b^2*x)/(2*x^2) + (a^2*Log[x])/x^3)
```

**fricas** [A] time = 1.13, size = 35, normalized size = 0.67

$$\frac{\left(b^2cx^2+4\,abcx+2\,a^2c\log(x)\right)\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
[Out] 1/2*(b^2*c*x^2 + 4*a*b*c*x + 2*a^2*c*log(x))*sqrt(c*x^2)/x
```

**giac** [A] time = 1.20, size = 32, normalized size = 0.62

$$\frac{1}{2}\left(b^2x^2\mathrm{sgn}(x) + 4\,abx\mathrm{sgn}(x) + 2\,a^2\log\left(|x|\right)\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="giac")
[Out] 1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*c^(3/2)
```

**maple** [A] time = 0.00, size = 33, normalized size = 0.63

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(b^2x^2 + 2a^2\ln(x) + 4abx\right)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^4,x)
[Out] 1/2*(c*x^2)^(3/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x^3
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x)
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^4, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**4,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**2/x**4, x)
```

## 3.778 $\int x \left(cx^2\right)^{5/2} (a+bx)^2 \, dx$

**Optimal**. Leaf size=66

$$\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*x^6*Sqrt[c*x^2])/7 + (a*b*c^2*x^7*Sqrt[c*x^2])/4 + (b^2*c^2*x^8*Sq
rt[c*x^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x \left(cx^2\right)^{5/2} (a+bx)^2 \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^6(a+bx)^2 \, dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^6+2abx^7+b^2x^8\right) dx}{x}\\ &= \frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.53

$$\frac{1}{252}x^2\left(cx^2\right)^{5/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
[Out] (x^2*(c*x^2)^(5/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{1}{252}x^2\left(cx^2\right)^{5/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
[Out] (x^2*(c*x^2)^(5/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

**fricas** [A] time = 0.94, size = 42, normalized size = 0.64

$$\frac{1}{252}\left(28\,b^2c^2x^8+63\,abc^2x^7+36\,a^2c^2x^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
[Out] 1/252*(28*b^2*c^2*x^8 + 63*a*b*c^2*x^7 + 36*a^2*c^2*x^6)*sqrt(c*x^2)
```

**giac** [A] time = 1.22, size = 44, normalized size = 0.67

$$\frac{1}{252}\left(28\,b^2c^2x^9\mathrm{sgn}(x)+63\,abc^2x^8\mathrm{sgn}(x)+36\,a^2c^2x^7\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
[Out] 1/252*(28*b^2*c^2*x^9*sgn(x) + 63*a*b*c^2*x^8*sgn(x) + 36*a^2*c^2*x^7*sgn(x
))*sqrt(c)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.48

$$\frac{\left(28b^2x^2+63abx+36a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}x^2}{252}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] 1/252*x^2*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(5/2)
```

**maxima [A]** time = 1.34, size = 49, normalized size = 0.74

$$\frac{\left(cx^2\right)^{\frac{7}{2}}b^2x^2}{9\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}abx}{4\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}a^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(c*x^2)^(7/2)*b^2*x^2/c + 1/4*(c*x^2)^(7/2)*a*b*x/c + 1/7*(c*x^2)^(7/2)
*a^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(c\,x^2\right)^{5/2}\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] int(x*(c*x^2)^(5/2)*(a + b*x)^2, x)
```

**sympy [A]** time = 2.19, size = 60, normalized size = 0.91

$$\frac{a^2c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}{7}+\frac{abc^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}{4}+\frac{b^2c^{\frac{5}{2}}x^4\left(x^2\right)^{\frac{5}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(5/2)*x**2*(x**2)**(5/2)/7 + a*b*c**(5/2)*x**3*(x**2)**(5/2)/4 + b*
*2*c**(5/2)*x**4*(x**2)**(5/2)/9
```

## 3.779 $\int \left(cx^2\right)^{5/2} (a+bx)^2 \, dx$

**Optimal**. Leaf size=66

$$\frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*x^5*Sqrt[c*x^2])/6 + (2*a*b*c^2*x^6*Sqrt[c*x^2])/7 + (b^2*c^2*x^7*
Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(cx^2\right)^{5/2} (a+bx)^2 \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)^2 \, dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^5+2abx^6+b^2x^7\right) dx}{x} \\
&= \frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.50

$$\frac{1}{168}x\left(cx^2\right)^{5/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (x*(c*x^2)^(5/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

**IntegrateAlgebraic** [A] time = 0.03, size = 33, normalized size = 0.50

$$\frac{1}{168}x\left(cx^2\right)^{5/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (x*(c*x^2)^(5/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

**fricas** [A] time = 1.02, size = 42, normalized size = 0.64

$$\frac{1}{168}\left(21\,b^2c^2x^7+48\,abc^2x^6+28\,a^2c^2x^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^2*c^2*x^7 + 48*a*b*c^2*x^6 + 28*a^2*c^2*x^5)*sqrt(c*x^2)
```

**giac** [A] time = 1.05, size = 44, normalized size = 0.67

$$\frac{1}{168}\left(21\,b^2c^2x^8\mathrm{sgn}(x)+48\,abc^2x^7\mathrm{sgn}(x)+28\,a^2c^2x^6\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/168*(21*b^2*c^2*x^8*sgn(x) + 48*a*b*c^2*x^7*sgn(x) + 28*a^2*c^2*x^6*sgn(x
))*sqrt(c)
```

**maple** [A] time = 0.00, size = 30, normalized size = 0.45

$$\frac{\left(21b^2x^2+48abx+28a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}x}{168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] 1/168*x*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(5/2)
```

**maxima [A]** time = 1.32, size = 44, normalized size = 0.67

$$\frac{1}{6}\left(cx^{2}\right)^{\frac{5}{2}}a^{2}x+\frac{\left(cx^{2}\right)^{\frac{7}{2}}b^{2}x}{8\,c}+\frac{2\left(cx^{2}\right)^{\frac{7}{2}}ab}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(5/2)*a^2*x + 1/8*(c*x^2)^(7/2)*b^2*x/c + 2/7*(c*x^2)^(7/2)*a*b
/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \left(c\,x^{2}\right)^{5/2}\,\left(a+b\,x\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(5/2)*(a + b*x)^2, x)
```

**sympy [A]** time = 1.80, size = 60, normalized size = 0.91

$$\frac{a^{2}c^{\frac{5}{2}}x\left(x^{2}\right)^{\frac{5}{2}}}{6}+\frac{2abc^{\frac{5}{2}}x^{2}\left(x^{2}\right)^{\frac{5}{2}}}{7}+\frac{b^{2}c^{\frac{5}{2}}x^{3}\left(x^{2}\right)^{\frac{5}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(5/2)*x*(x**2)**(5/2)/6 + 2*a*b*c**(5/2)*x**2*(x**2)**(5/2)/7 + b**
2*c**(5/2)*x**3*(x**2)**(5/2)/8
```

## 3.780 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx$

**Optimal**. Leaf size=66

$$\frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
```

```
[Out] (a^2*c^2*x^4*Sqrt[c*x^2])/5 + (a*b*c^2*x^5*Sqrt[c*x^2])/3 + (b^2*c^2*x^6*Sq
rt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^4+2abx^5+b^2x^6\right)dx}{x}\\ &= \frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 36, normalized size = 0.55

$$\frac{1}{105}cx^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
[Out] (c*x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

**IntegrateAlgebraic** [A] time = 0.03, size = 32, normalized size = 0.48

$$\frac{1}{105}\left(cx^2\right)^{5/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
[Out] ((c*x^2)^(5/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

**fricas** [A] time = 0.82, size = 42, normalized size = 0.64

$$\frac{1}{105}\left(15\,b^2c^2x^6+35\,abc^2x^5+21\,a^2c^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="fricas")
[Out] 1/105*(15*b^2*c^2*x^6 + 35*a*b*c^2*x^5 + 21*a^2*c^2*x^4)*sqrt(c*x^2)
```

**giac** [A] time = 0.96, size = 44, normalized size = 0.67

$$\frac{1}{105}\left(15\,b^2c^2x^7\mathrm{sgn}(x)+35\,abc^2x^6\mathrm{sgn}(x)+21\,a^2c^2x^5\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="giac")
[Out] 1/105*(15*b^2*c^2*x^7*sgn(x) + 35*a*b*c^2*x^6*sgn(x) + 21*a^2*c^2*x^5*sgn(x
))*sqrt(c)
```

**maple** [A] time = 0.00, size = 29, normalized size = 0.44

$$\frac{\left(15b^2x^2+35abx+21a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x,x)
```

```
[Out] 1/105*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(5/2)
```

**maxima** [A] time = 1.28, size = 40, normalized size = 0.61

$$\frac{1}{3}\left(cx^2\right)^{\frac{5}{2}}abx+\frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}a^2+\frac{\left(cx^2\right)^{\frac{7}{2}}b^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^2)^(5/2)*a*b*x + 1/5*(c*x^2)^(5/2)*a^2 + 1/7*(c*x^2)^(7/2)*b^2/c
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x, x)
```

**sympy** [A] time = 1.82, size = 54, normalized size = 0.82

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5}+\frac{abc^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}{3}+\frac{b^2c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/5 + a*b*c**(5/2)*x*(x**2)**(5/2)/3 + b**2*c**(5
/2)*x**2*(x**2)**(5/2)/7
```

# 3.781 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2}\,dx$

**Optimal**. Leaf size=66

$$\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (a^2*c^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*c^2*x^4*Sqrt[c*x^2])/5 + (b^2*c^2*x^5*
Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2}\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)^2\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^3+2abx^4+b^2x^5\right)dx}{x}\\&=\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 34, normalized size = 0.52

$$\frac{1}{60}cx\left(cx^2\right)^{3/2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
[Out] (c*x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{\left(cx^2\right)^{5/2}\left(15a^2+24abx+10b^2x^2\right)}{60x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
[Out] ((c*x^2)^(5/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/(60*x)
```

**fricas** [A] time = 1.11, size = 42, normalized size = 0.64

$$\frac{1}{60}\left(10\,b^2c^2x^5+24\,abc^2x^4+15\,a^2c^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] 1/60*(10*b^2*c^2*x^5 + 24*a*b*c^2*x^4 + 15*a^2*c^2*x^3)*sqrt(c*x^2)
```

**giac** [A] time = 0.99, size = 44, normalized size = 0.67

$$\frac{1}{60}\left(10\,b^2c^2x^6\mathrm{sgn}(x)+24\,abc^2x^5\mathrm{sgn}(x)+15\,a^2c^2x^4\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="giac")
[Out] 1/60*(10*b^2*c^2*x^6*sgn(x) + 24*a*b*c^2*x^5*sgn(x) + 15*a^2*c^2*x^4*sgn(x)
)*sqrt(c)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.48

$$\frac{\left(10b^2x^2+24abx+15a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{60x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^2,x)
```

```
[Out] 1/60/x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(5/2)
```

**maxima [A]** time = 1.37, size = 40, normalized size = 0.61

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}b^2x + \frac{2}{5}\left(cx^2\right)^{\frac{5}{2}}ab + \frac{\left(cx^2\right)^{\frac{5}{2}}a^2}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(5/2)*b^2*x + 2/5*(c*x^2)^(5/2)*a*b + 1/4*(c*x^2)^(5/2)*a^2/x
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^2, x)
```

**sympy [A]** time = 1.84, size = 54, normalized size = 0.82

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{4x} + \frac{2abc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5} + \frac{b^2c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**2,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/(4*x) + 2*a*b*c**(5/2)*(x**2)**(5/2)/5 + b**2*c
**(5/2)*x*(x**2)**(5/2)/6
```

## 3.782 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx$

**Optimal**. Leaf size=66

$$\frac{1}{3}a^2c^2x^2\sqrt{cx^2}+\frac{1}{2}abc^2x^3\sqrt{cx^2}+\frac{1}{5}b^2c^2x^4\sqrt{cx^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{3}a^2c^2x^2\sqrt{cx^2}+\frac{1}{2}abc^2x^3\sqrt{cx^2}+\frac{1}{5}b^2c^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (a^2*c^2*x^2*Sqrt[c*x^2])/3 + (a*b*c^2*x^3*Sqrt[c*x^2])/2 + (b^2*c^2*x^4*Sq
rt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^2+2abx^3+b^2x^4\right)dx}{x}\\ &= \frac{1}{3}a^2c^2x^2\sqrt{cx^2}+\frac{1}{2}abc^2x^3\sqrt{cx^2}+\frac{1}{5}b^2c^2x^4\sqrt{cx^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 38, normalized size = 0.58

$$\frac{1}{30}c^2x^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (c^2*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{\left(cx^2\right)^{5/2}\left(10a^2+15abx+6b^2x^2\right)}{30x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] ((c*x^2)^(5/2)*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/(30*x^2)
```

**fricas** [A] time = 1.19, size = 42, normalized size = 0.64

$$\frac{1}{30}\left(6\,b^2c^2x^4+15\,abc^2x^3+10\,a^2c^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*c^2*x^4 + 15*a*b*c^2*x^3 + 10*a^2*c^2*x^2)*sqrt(c*x^2)
```

**giac** [A] time = 0.94, size = 44, normalized size = 0.67

$$\frac{1}{30}\left(6\,b^2c^2x^5\mathrm{sgn}(x)+15\,abc^2x^4\mathrm{sgn}(x)+10\,a^2c^2x^3\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*c^2*x^5*sgn(x) + 15*a*b*c^2*x^4*sgn(x) + 10*a^2*c^2*x^3*sgn(x))
*sqrt(c)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.48

$$\frac{\left(6b^2x^2+15abx+10a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{30x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^3,x)
```

```
[Out] 1/30/x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(5/2)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^3, x)
```

**sympy** [**A**] time = 1.95, size = 54, normalized size = 0.82

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^2}+\frac{abc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{2x}+\frac{b^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**3,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/(3*x**2) + a*b*c**(5/2)*(x**2)**(5/2)/(2*x) + b
**2*c**(5/2)*(x**2)**(5/2)/5
```

## 3.783 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4} dx$

**Optimal**. Leaf size=64

$$\frac{1}{2}a^2c^2x\sqrt{cx^2} + \frac{2}{3}abc^2x^2\sqrt{cx^2} + \frac{1}{4}b^2c^2x^3\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{2}a^2c^2x\sqrt{cx^2} + \frac{2}{3}abc^2x^2\sqrt{cx^2} + \frac{1}{4}b^2c^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] (a^2*c^2*x*Sqrt[c*x^2])/2 + (2*a*b*c^2*x^2*Sqrt[c*x^2])/3 + (b^2*c^2*x^3*Sq
rt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)^2 dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x + 2abx^2 + b^2x^3\right) dx}{x} \\
&= \frac{1}{2}a^2c^2x\sqrt{cx^2} + \frac{2}{3}abc^2x^2\sqrt{cx^2} + \frac{1}{4}b^2c^2x^3\sqrt{cx^2}
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 36, normalized size = 0.56

$$\frac{1}{12}c^2x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
[Out] (c^2*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.55

$$\frac{\left(cx^2\right)^{5/2}\left(6a^2+8abx+3b^2x^2\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
[Out] ((c*x^2)^(5/2)*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*x^3)
```

**fricas** [A] time = 1.18, size = 40, normalized size = 0.62

$$\frac{1}{12}\left(3\,b^2c^2x^3+8\,abc^2x^2+6\,a^2c^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
[Out] 1/12*(3*b^2*c^2*x^3 + 8*a*b*c^2*x^2 + 6*a^2*c^2*x)*sqrt(c*x^2)
```

**giac** [A] time = 1.05, size = 44, normalized size = 0.69

$$\frac{1}{12}\left(3\,b^2c^2x^4\mathrm{sgn}(x)+8\,abc^2x^3\mathrm{sgn}(x)+6\,a^2c^2x^2\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="giac")
[Out] 1/12*(3*b^2*c^2*x^4*sgn(x) + 8*a*b*c^2*x^3*sgn(x) + 6*a^2*c^2*x^2*sgn(x))*s
qrt(c)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.50

$$\frac{\left(3b^2x^2+8abx+6a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^4,x)
```

```
[Out] 1/12/x^3*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(5/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^4, x)
```

**sympy [A]** time = 2.01, size = 60, normalized size = 0.94

$$\frac{a^{2} c^{\frac{5}{2}} \left(x^{2}\right)^{\frac{5}{2}}}{2 x^{3}} + \frac{2 a b c^{\frac{5}{2}} \left(x^{2}\right)^{\frac{5}{2}}}{3 x^{2}} + \frac{b^{2} c^{\frac{5}{2}} \left(x^{2}\right)^{\frac{5}{2}}}{4 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**4,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/(2*x**3) + 2*a*b*c**(5/2)*(x**2)**(5/2)/(3*x**2
) + b**2*c**(5/2)*(x**2)**(5/2)/(4*x)
```

## 3.784 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5} dx$

**Optimal**. Leaf size=29

$$\frac{c^2\sqrt{cx^2}(a+bx)^3}{3bx}$$

**Rubi [A]** time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{c^2\sqrt{cx^2}(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int (a+bx)^2 dx}{x} \\ &= \frac{c^2\sqrt{cx^2}(a+bx)^3}{3bx}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 0.90

$$\frac{(cx^2)^{5/2}(a+bx)^3}{3bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] ((c*x^2)^(5/2)*(a + b*x)^3)/(3*b*x^5)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 34, normalized size = 1.17

$$\frac{\left(cx^2\right)^{5/2}\left(3a^2+3abx+b^2x^2\right)}{3x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] ((c*x^2)^(5/2)*(3*a^2 + 3*a*b*x + b^2*x^2))/(3*x^4)
```

**fricas [A]** time = 0.97, size = 36, normalized size = 1.24

$$\frac{1}{3}\left(b^2c^2x^2+3\,abc^2x+3\,a^2c^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c^2*x^2 + 3*a*b*c^2*x + 3*a^2*c^2)*sqrt(c*x^2)
```

**giac [A]** time = 1.12, size = 41, normalized size = 1.41

$$\frac{1}{3}\left(b^2c^2x^3\mathrm{sgn}(x)+3\,abc^2x^2\mathrm{sgn}(x)+3\,a^2c^2x\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*c^2*x^3*sgn(x) + 3*a*b*c^2*x^2*sgn(x) + 3*a^2*c^2*x*sgn(x))*sqrt(c
)
```

**maple [A]** time = 0.00, size = 31, normalized size = 1.07

$$\frac{\left(b^2x^2+3abx+3a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^5,x)
```

[Out] 1/3/x^4*(b^2*x^2+3*a*b*x+3*a^2)*(c*x^2)^(5/2)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x)

[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^5, x)

**sympy** [B] time = 2.03, size = 56, normalized size = 1.93

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{x^4}+\frac{abc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{x^3}+\frac{b^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**5,x)

[Out] a**2*c**(5/2)*(x**2)**(5/2)/x**4 + a*b*c**(5/2)*(x**2)**(5/2)/x**3 + b**2*c**(5/2)*(x**2)**(5/2)/(3*x**2)

# 3.785 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6}\,dx$

**Optimal**. Leaf size=58

$$\frac{a^2c^2\sqrt{cx^2}\,\log(x)}{x}+2abc^2\sqrt{cx^2}+\frac{1}{2}b^2c^2x\sqrt{cx^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}\,\log(x)}{x}+2abc^2\sqrt{cx^2}+\frac{1}{2}b^2c^2x\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
```

```
[Out] 2*a*b*c^2*Sqrt[c*x^2] + (b^2*c^2*x*Sqrt[c*x^2])/2 + (a^2*c^2*Sqrt[c*x^2]*Lo
g[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(cx^2\right)^{5/2}(a+bx)^2}{x^6}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{(a+bx)^2}{x}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{x}$$
$$= 2abc^2\sqrt{cx^2} + \frac{1}{2}b^2c^2x\sqrt{cx^2} + \frac{a^2c^2\sqrt{cx^2}\,\log(x)}{x}$$

**Mathematica** [**A**] time = 0.01, size = 35, normalized size = 0.60

$$\frac{c^3x\left(2a^2\log(x)+bx(4a+bx)\right)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
[Out] (c^3*x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [**A**] time = 0.03, size = 37, normalized size = 0.64

$$\left(cx^2\right)^{5/2}\left(\frac{a^2\log(x)}{x^5}+\frac{4ab+b^2x}{2x^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
[Out] (c*x^2)^(5/2)*((4*a*b + b^2*x)/(2*x^4) + (a^2*Log[x])/x^5)
```

**fricas** [**A**] time = 1.09, size = 41, normalized size = 0.71

$$\frac{\left(b^2c^2x^2+4\,abc^2x+2\,a^2c^2\log(x)\right)\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="fricas")
[Out] 1/2*(b^2*c^2*x^2 + 4*a*b*c^2*x + 2*a^2*c^2*log(x))*sqrt(c*x^2)/x
```

**giac** [A] time = 1.08, size = 41, normalized size = 0.71

$$\frac{1}{2}\left(b^2c^2x^2\mathrm{sgn}(x) + 4\,abc^2x\mathrm{sgn}(x) + 2\,a^2c^2\log\left(|x|\right)\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="giac")
[Out] 1/2*(b^2*c^2*x^2*sgn(x) + 4*a*b*c^2*x*sgn(x) + 2*a^2*c^2*log(abs(x))*sgn(x)
)*sqrt(c)
```

**maple** [A] time = 0.01, size = 33, normalized size = 0.57

$$\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(b^2x^2 + 2a^2\ln(x) + 4abx\right)}{2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^6,x)
[Out] 1/2*(c*x^2)^(5/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x^5
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x)
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^6, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**6,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**2/x**6, x)
```

## 3.786 $\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=57

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x^4)/(3*Sqrt[c*x^2]) + (a*b*x^5)/(2*Sqrt[c*x^2]) + (b^2*x^6)/(5*Sqrt[c
*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int x^2(a+bx)^2\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(a^2x^2+2abx^3+b^2x^4\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{x^4 (10a^2 + 15abx + 6b^2x^2)}{30\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x^4*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/(30*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 38, normalized size = 0.67

$$\frac{x^2\sqrt{cx^2}\,(10a^2 + 15abx + 6b^2x^2)}{30c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/(30*c)
```

**fricas** [A] time = 1.24, size = 36, normalized size = 0.63

$$\frac{(6\,b^2x^4 + 15\,abx^3 + 10\,a^2x^2)\sqrt{cx^2}}{30\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)/c
```

**giac** [A] time = 0.94, size = 41, normalized size = 0.72

$$\frac{1}{30}\sqrt{cx^2}\left(3\left(\frac{2\,b^2x}{c} + \frac{5\,ab}{c}\right)x + \frac{10\,a^2}{c}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/30*sqrt(c*x^2)*(3*(2*b^2*x/c + 5*a*b/c)*x + 10*a^2/c)*x^2
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.56

$$\frac{(6b^2x^2 + 15abx + 10a^2)\,x^4}{30\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/30*x^4*(6*b^2*x^2+15*a*b*x+10*a^2)/(c*x^2)^(1/2)
```

**maxima [A]** time = 1.34, size = 54, normalized size = 0.95

$$\frac{\sqrt{cx^2}\,b^2x^4}{5\,c} + \frac{\sqrt{cx^2}\,abx^3}{2\,c} + \frac{\sqrt{cx^2}\,a^2x^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c*x^2)*b^2*x^4/c + 1/2*sqrt(c*x^2)*a*b*x^3/c + 1/3*sqrt(c*x^2)*a^2
*x^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,(a+b\,x)^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

**sympy [A]** time = 0.79, size = 60, normalized size = 1.05

$$\frac{a^2x^4}{3\sqrt{c}\,\sqrt{x^2}} + \frac{abx^5}{2\sqrt{c}\,\sqrt{x^2}} + \frac{b^2x^6}{5\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**4/(3*sqrt(c)*sqrt(x**2)) + a*b*x**5/(2*sqrt(c)*sqrt(x**2)) + b**2*x
**6/(5*sqrt(c)*sqrt(x**2))
```

## 3.787 $\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}} \, dx$

**Optimal**. Leaf size=57

$$\frac{a^2x^3}{2\sqrt{cx^2}} + \frac{2abx^4}{3\sqrt{cx^2}} + \frac{b^2x^5}{4\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x^3}{2\sqrt{cx^2}} + \frac{2abx^4}{3\sqrt{cx^2}} + \frac{b^2x^5}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x^3)/(2*Sqrt[c*x^2]) + (2*a*b*x^4)/(3*Sqrt[c*x^2]) + (b^2*x^5)/(4*Sqrt
[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}} \, dx &= \frac{x \int x(a+bx)^2 \, dx}{\sqrt{cx^2}} \\
&= \frac{x \int \left(a^2x + 2abx^2 + b^2x^3\right) \, dx}{\sqrt{cx^2}} \\
&= \frac{a^2x^3}{2\sqrt{cx^2}} + \frac{2abx^4}{3\sqrt{cx^2}} + \frac{b^2x^5}{4\sqrt{cx^2}}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{x^3\left(6a^2+8abx+3b^2x^2\right)}{12\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x^3*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 36, normalized size = 0.63

$$\frac{x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)}{12c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*c)
```

**fricas** [A] time = 1.07, size = 34, normalized size = 0.60

$$\frac{\left(3\,b^2x^3+8\,abx^2+6\,a^2x\right)\sqrt{cx^2}}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)/c
```

**giac** [A] time = 1.25, size = 38, normalized size = 0.67

$$\frac{1}{12}\sqrt{cx^2}\left(\left(\frac{3\,b^2x}{c}+\frac{8\,ab}{c}\right)x+\frac{6\,a^2}{c}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/12*sqrt(c*x^2)*((3*b^2*x/c + 8*a*b/c)*x + 6*a^2/c)*x
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.56

$$\frac{\left(3b^2x^2+8abx+6a^2\right)x^3}{12\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/12*x^3*(3*b^2*x^2+8*a*b*x+6*a^2)/(c*x^2)^(1/2)
```

**maxima [A]** time = 1.32, size = 47, normalized size = 0.82

$$\frac{\sqrt{cx^2}\, b^2 x^3}{4\,c} + \frac{2\,\sqrt{cx^2}\, abx^2}{3\,c} + \frac{a^2x^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^2)*b^2*x^3/c + 2/3*sqrt(c*x^2)*a*b*x^2/c + 1/2*a^2*x^2/sqrt(c)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,{\left(a+b\,x\right)}^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

**sympy [A]** time = 0.65, size = 61, normalized size = 1.07

$$\frac{a^2x^3}{2\sqrt{c}\,\sqrt{x^2}} + \frac{2abx^4}{3\sqrt{c}\,\sqrt{x^2}} + \frac{b^2x^5}{4\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**3/(2*sqrt(c)*sqrt(x**2)) + 2*a*b*x**4/(3*sqrt(c)*sqrt(x**2)) + b**2
*x**5/(4*sqrt(c)*sqrt(x**2))
```

## 3.788 $\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=24

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 32}

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int (a+bx)^2\,dx}{\sqrt{cx^2}}\\ &= \frac{x(a+bx)^3}{3b\sqrt{cx^2}}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 24, normalized size = 1.00

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

**IntegrateAlgebraic [A]** time = 0.03, size = 34, normalized size = 1.42

$$\frac{\sqrt{cx^2}\left(3a^2+3abx+b^2x^2\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (Sqrt[c*x^2]*(3*a^2 + 3*a*b*x + b^2*x^2))/(3*c)
```

**fricas [A]** time = 1.16, size = 30, normalized size = 1.25

$$\frac{\left(b^2x^2+3\,abx+3\,a^2\right)\sqrt{cx^2}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c
```

**giac [A]** time = 1.15, size = 36, normalized size = 1.50

$$\frac{1}{3}\sqrt{cx^2}\left(\left(\frac{b^2x}{c}+\frac{3\,ab}{c}\right)x+\frac{3\,a^2}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(c*x^2)*((b^2*x/c + 3*a*b/c)*x + 3*a^2/c)
```

**maple [A]** time = 0.00, size = 31, normalized size = 1.29

$$\frac{\left(b^2x^2+3abx+3a^2\right)x^2}{3\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/3*x^2*(b^2*x^2+3*a*b*x+3*a^2)/(c*x^2)^(1/2)
```

**maxima [B]** time = 1.37, size = 42, normalized size = 1.75

$$\frac{\sqrt{cx^2}\,b^2x^2}{3\,c} + \frac{abx^2}{\sqrt{c}} + \frac{\sqrt{cx^2}\,a^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2)*b^2*x^2/c + a*b*x^2/sqrt(c) + sqrt(c*x^2)*a^2/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,(a+b\,x)^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

**sympy [B]** time = 0.54, size = 56, normalized size = 2.33

$$\frac{a^2x^2}{\sqrt{c}\,\sqrt{x^2}} + \frac{abx^3}{\sqrt{c}\,\sqrt{x^2}} + \frac{b^2x^4}{3\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**2/(sqrt(c)*sqrt(x**2)) + a*b*x**3/(sqrt(c)*sqrt(x**2)) + b**2*x**4/
(3*sqrt(c)*sqrt(x**2))
```

## 3.789 $\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=52

$$\frac{a^2x\log(x)}{\sqrt{cx^2}}+\frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^2x\log(x)}{\sqrt{cx^2}}+\frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[c*x^2],x]
```

```
[Out] (2*a*b*x^2)/Sqrt[c*x^2] + (b^2*x^3)/(2*Sqrt[c*x^2]) + (a^2*x*Log[x])/Sqrt[c
*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{\sqrt{cx^2}}$$
$$= \frac{2abx^2}{\sqrt{cx^2}} + \frac{b^2x^3}{2\sqrt{cx^2}} + \frac{a^2x\log(x)}{\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.00, size = 32, normalized size = 0.62

$$\frac{x\left(2a^2\log(x)+bx(4a+bx)\right)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[c*x^2],x]
[Out] (x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

**IntegrateAlgebraic [A]** time = 0.03, size = 40, normalized size = 0.77

$$\sqrt{cx^2}\left(\frac{a^2\log(x)}{cx}+\frac{4ab+b^2x}{2c}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/Sqrt[c*x^2],x]
[Out] Sqrt[c*x^2]*((4*a*b + b^2*x)/(2*c) + (a^2*Log[x])/(c*x))
```

**fricas [A]** time = 0.99, size = 35, normalized size = 0.67

$$\frac{\left(b^2x^2+4\,abx+2\,a^2\log(x)\right)\sqrt{cx^2}}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c*x)
```

**giac [A]** time = 1.12, size = 50, normalized size = 0.96

$$-\frac{a^2 \log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2}\right|\right)}{\sqrt{c}} + \frac{1}{2}\,\sqrt{cx^2}\left(\frac{b^2 x}{c} + \frac{4\,ab}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) + 1/2*sqrt(c*x^2)*(b^2*x/c
+ 4*a*b/c)
```

**maple [A]** time = 0.00, size = 31, normalized size = 0.60

$$\frac{\left(b^2 x^2 + 2 a^2 \ln(x) + 4 abx\right) x}{2\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(1/2)
```

**maxima [A]** time = 1.35, size = 35, normalized size = 0.67

$$\frac{b^2 x^2}{2\,\sqrt{c}} + \frac{a^2 \log(x)}{\sqrt{c}} + \frac{2\,\sqrt{cx^2}\,ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/sqrt(c) + a^2*log(x)/sqrt(c) + 2*sqrt(c*x^2)*a*b/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + b\,x)^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c*x^2)^(1/2),x)
```

```
[Out] int((a + b*x)^2/(c*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)^2}{\sqrt{cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2/sqrt(c*x**2), x)
```

## 3.790 $\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=47

$$-\frac{a^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
```

```
[Out] -(a^2/Sqrt[c*x^2]) + (b^2*x^2)/Sqrt[c*x^2] + (2*a*b*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a^2}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 34, normalized size = 0.72

$$\frac{cx^2\left(-a^2+2abx\log(x)+b^2x^2\right)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
```

```
[Out] (c*x^2*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/(c*x^2)^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 43, normalized size = 0.91

$$\sqrt{cx^2}\left(\frac{b^2x^2-a^2}{cx^2}+\frac{2ab\log(x)}{cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
```

```
[Out] Sqrt[c*x^2]*((-a^2 + b^2*x^2)/(c*x^2) + (2*a*b*Log[x])/(c*x))
```

**fricas** [A] time = 1.03, size = 34, normalized size = 0.72

$$\frac{\left(b^2x^2+2\,abx\log(x)-a^2\right)\sqrt{cx^2}}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c*x^2)
```

**giac** [A] time = 0.94, size = 65, normalized size = 1.38

$$\frac{\sqrt{cx^2}\,b^2}{c}-\frac{2\left(ab\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)-\frac{a^2\sqrt{c}}{\sqrt{c}\,x-\sqrt{cx^2}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2)*b^2/c - 2*(a*b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - a^2*sqrt(c)
/(sqrt(c)*x - sqrt(c*x^2)))/sqrt(c)
```

**maple [A]** time = 0.00, size = 29, normalized size = 0.62

$$\frac{2abx\ln(x)+b^2x^2-a^2}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(1/2),x)
[Out] (2*a*b*x*ln(x)+b^2*x^2-a^2)/(c*x^2)^(1/2)
```

**maxima [A]** time = 1.36, size = 35, normalized size = 0.74

$$\frac{2\,ab\log(x)}{\sqrt{c}}+\frac{\sqrt{cx^2}\,b^2}{c}-\frac{a^2}{\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 2*a*b*log(x)/sqrt(c) + sqrt(c*x^2)*b^2/c - a^2/(sqrt(c)*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^2}{x\,\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(1/2)),x)
[Out] int((a + b*x)^2/(x*(c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(1/2),x)
[Out] Integral((a + b*x)**2/(x*sqrt(c*x**2)), x)
```

## 3.791 $\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=49

$$-\frac{a^2}{2x\sqrt{cx^2}}-\frac{2ab}{\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{2x\sqrt{cx^2}}-\frac{2ab}{\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] (-2*a*b)/Sqrt[c*x^2] - a^2/(2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{\sqrt{cx^2}}\\
&= -\frac{2ab}{\sqrt{cx^2}}-\frac{a^2}{2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.71

$$\frac{cx\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
[Out] (c*x*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 44, normalized size = 0.90

$$\sqrt{cx^2}\left(\frac{-a^2-4abx}{2cx^3}+\frac{b^2\log(x)}{cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
[Out] Sqrt[c*x^2]*((-a^2 - 4*a*b*x)/(2*c*x^3) + (b^2*Log[x])/(c*x))
```

**fricas** [A] time = 1.09, size = 36, normalized size = 0.73

$$\frac{\left(2\,b^2x^2\log(x)-4\,abx-a^2\right)\sqrt{cx^2}}{2\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c*x^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

**maple** [A] time = 0.01, size = 34, normalized size = 0.69

$$\frac{2b^2x^2\ln(x)-4abx-a^2}{2\sqrt{c\,x^2}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2/x*(2*b^2*x^2*ln(x)-4*a*b*x-a^2)/(c*x^2)^(1/2)
```

**maxima [A]** time = 1.29, size = 31, normalized size = 0.63

$$\frac{b^2\log(x)}{\sqrt{c}} - \frac{2\,ab}{\sqrt{c}\,x} - \frac{a^2}{2\,\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*log(x)/sqrt(c) - 2*a*b/(sqrt(c)*x) - 1/2*a^2/(sqrt(c)*x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^2}{x^2\,\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x)^2/(x^2*(c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2/(x**2*sqrt(c*x**2)), x)
```

## 3.792 $\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=26

$$-\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] -(a + b*x)^3/(3*a*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^4}\,dx}{\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 1.27

$$\frac{c\left(-a^2-3abx-3b^2x^2\right)}{3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
[Out] (c*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 38, normalized size = 1.46

$$\frac{\sqrt{cx^2}\left(-a^2-3abx-3b^2x^2\right)}{3cx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
[Out] (Sqrt[c*x^2]*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*c*x^4)
```

**fricas** [A] time = 1.07, size = 32, normalized size = 1.23

$$-\frac{\left(3\,b^2x^2+3\,abx+a^2\right)\sqrt{cx^2}}{3\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c*x^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx+a)^2}{\sqrt{cx^2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2/(sqrt(c*x^2)*x^3), x)
```

**maple** [A] time = 0.00, size = 30, normalized size = 1.15

$$-\frac{3b^2x^2+3abx+a^2}{3\sqrt{c\,x^2}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(1/2),x)
```

```
[Out] -1/3*(3*b^2*x^2+3*a*b*x+a^2)/x^2/(c*x^2)^(1/2)
```

**maxima** [A] time = 1.33, size = 33, normalized size = 1.27

$$-\frac{b^2}{\sqrt{c}\,x}-\frac{ab}{\sqrt{c}\,x^2}-\frac{a^2}{3\,\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b^2/(sqrt(c)*x) - a*b/(sqrt(c)*x^2) - 1/3*a^2/(sqrt(c)*x^3)
```

**mupad** [B] time = 0.18, size = 33, normalized size = 1.27

$$-\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,\sqrt{c}\,{\left(x^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(1/2)),x)
```

```
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(1/2)*(x^2)^(5/2))
```

**sympy** [B] time = 0.66, size = 53, normalized size = 2.04

$$-\frac{a^2}{3\sqrt{c}\,x^2\sqrt{x^2}}-\frac{ab}{\sqrt{c}\,x\sqrt{x^2}}-\frac{b^2}{\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(1/2),x)
```

```
[Out] -a**2/(3*sqrt(c)*x**2*sqrt(x**2)) - a*b/(sqrt(c)*x*sqrt(x**2)) - b**2/(sqrt
(c)*sqrt(x**2))
```

## 3.793 $\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=57

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] -a^2/(4*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*x^2*Sqrt[c*x^2]) - b^2/(2*x*Sqrt[c*x^
2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{a^2}{4x^3\sqrt{cx^2}} - \frac{2ab}{3x^2\sqrt{cx^2}} - \frac{b^2}{2x\sqrt{cx^2}}$$

**Mathematica** [**A**] time = 0.01, size = 35, normalized size = 0.61

$$\frac{-3a^2-8abx-6b^2x^2}{12x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
[Out] (-3*a^2 - 8*a*b*x - 6*b^2*x^2)/(12*x^3*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [**A**] time = 0.03, size = 38, normalized size = 0.67

$$\frac{\sqrt{cx^2}\left(-3a^2-8abx-6b^2x^2\right)}{12cx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
[Out] (Sqrt[c*x^2]*(-3*a^2 - 8*a*b*x - 6*b^2*x^2))/(12*c*x^5)
```

**fricas** [**A**] time = 1.25, size = 34, normalized size = 0.60

$$-\frac{\left(6\,b^2x^2+8\,abx+3\,a^2\right)\sqrt{cx^2}}{12\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c*x^5)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

**maple** [A] time = 0.01, size = 32, normalized size = 0.56

$$-\frac{6b^2x^2 + 8abx + 3a^2}{12\sqrt{c\,x^2}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(1/2),x)
```

```
[Out] -1/12*(6*b^2*x^2+8*a*b*x+3*a^2)/x^3/(c*x^2)^(1/2)
```

**maxima** [A] time = 1.33, size = 33, normalized size = 0.58

$$-\frac{b^2}{2\,\sqrt{c}\,x^2} - \frac{2\,ab}{3\,\sqrt{c}\,x^3} - \frac{a^2}{4\,\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b^2/(sqrt(c)*x^2) - 2/3*a*b/(sqrt(c)*x^3) - 1/4*a^2/(sqrt(c)*x^4)
```

**mupad** [B] time = 0.19, size = 42, normalized size = 0.74

$$-\frac{3\,a^2\,\sqrt{x^2}\, + 6\,b^2\,x^2\,\sqrt{x^2}\, + 8\,a\,b\,x\,\sqrt{x^2}}{12\,\sqrt{c}\;x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(1/2)),x)
```

```
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(1
/2)*x^5)
```

**sympy** [A] time = 0.82, size = 61, normalized size = 1.07

$$-\frac{a^2}{4\sqrt{c}\,x^3\sqrt{x^2}} - \frac{2ab}{3\sqrt{c}\,x^2\sqrt{x^2}} - \frac{b^2}{2\sqrt{c}\,x\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(1/2),x)
```

```
[Out] -a**2/(4*sqrt(c)*x**3*sqrt(x**2)) - 2*a*b/(3*sqrt(c)*x**2*sqrt(x**2)) - b**
2/(2*sqrt(c)*x*sqrt(x**2))
```

# 3.794 $\int \frac{x^3(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=27

$$\frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*x)^3)/(3*b*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int (a+bx)^2\,dx}{c\sqrt{cx^2}}\\ &= \frac{x(a+bx)^3}{3bc\sqrt{cx^2}}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 26, normalized size = 0.96

$$\frac{x^3(a+bx)^3}{3b\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^3)/(3*b*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 34, normalized size = 1.26

$$\frac{x^4\left(3a^2+3abx+b^2x^2\right)}{3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^4*(3*a^2 + 3*a*b*x + b^2*x^2))/(3*(c*x^2)^(3/2))
```

**fricas** [A] time = 1.06, size = 30, normalized size = 1.11

$$\frac{\left(b^2x^2+3\,abx+3\,a^2\right)\sqrt{cx^2}}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c^2
```

**giac** [A] time = 1.05, size = 39, normalized size = 1.44

$$\frac{\sqrt{cx^2}\left(\left(\frac{b^2x}{c}+\frac{3\,ab}{c}\right)x+\frac{3\,a^2}{c}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(c*x^2)*((b^2*x/c + 3*a*b/c)*x + 3*a^2/c)/c
```

**maple** [A] time = 0.00, size = 31, normalized size = 1.15

$$\frac{\left(b^2x^2+3abx+3a^2\right)x^4}{3\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] 1/3*x^4*(b^2*x^2+3*a*b*x+3*a^2)/(c*x^2)^(3/2)
```

**maxima [B]** time = 1.36, size = 52, normalized size = 1.93

$$\frac{b^2x^4}{3\sqrt{cx^2}\,c}+\frac{abx^3}{\sqrt{cx^2}\,c}+\frac{a^2x^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^4/(sqrt(c*x^2)*c) + a*b*x^3/(sqrt(c*x^2)*c) + a^2*x^2/(sqrt(c*x^2
)*c)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3\,(a+b\,x)^2}{\left(c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

**sympy [B]** time = 0.80, size = 56, normalized size = 2.07

$$\frac{a^2x^4}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{abx^5}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{b^2x^6}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] a**2*x**4/(c**(3/2)*(x**2)**(3/2)) + a*b*x**5/(c**(3/2)*(x**2)**(3/2)) + b*
*2*x**6/(3*c**(3/2)*(x**2)**(3/2))
```

## 3.795 $\int \frac{x^2(a+bx)^2}{(cx^2)^{3/2}}\,dx$

**Optimal**. Leaf size=61

$$\frac{a^2x\log(x)}{c\sqrt{cx^2}}+\frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x\log(x)}{c\sqrt{cx^2}}+\frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (2*a*b*x^2)/(c*Sqrt[c*x^2]) + (b^2*x^3)/(2*c*Sqrt[c*x^2]) + (a^2*x*Log[x])/
(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{2abx^2}{c\sqrt{cx^2}} + \frac{b^2x^3}{2c\sqrt{cx^2}} + \frac{a^2x\log(x)}{c\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 34, normalized size = 0.56

$$\frac{x^3\left(2a^2\log(x)+bx(4a+bx)\right)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] (x^3*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 39, normalized size = 0.64

$$\frac{a^2x^3\log(x)+\frac{1}{2}\left(4abx^4+b^2x^5\right)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] ((4*a*b*x^4 + b^2*x^5)/2 + a^2*x^3*Log[x])/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.18, size = 35, normalized size = 0.57

$$\frac{\left(b^2x^2+4\,abx+2\,a^2\log(x)\right)\sqrt{cx^2}}{2\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c^2*x)
```

**giac [A]** time = 1.10, size = 55, normalized size = 0.90

$$-\frac{\frac{2\,a^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)}{\sqrt{c}}-\sqrt{cx^2}\left(\frac{b^2x}{c}+\frac{4\,ab}{c}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) - sqrt(c*x^2)*(b^2*x
/c + 4*a*b/c))/c
```

**maple [A]** time = 0.00, size = 33, normalized size = 0.54

$$\frac{\left(b^2x^2+2a^2\ln(x)+4abx\right)x^3}{2\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] 1/2*x^3*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(3/2)
```

**maxima [A]** time = 1.33, size = 45, normalized size = 0.74

$$\frac{b^2x^3}{2\sqrt{cx^2}\,c}+\frac{2\,abx^2}{\sqrt{cx^2}\,c}+\frac{a^2\log(x)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^3/(sqrt(c*x^2)*c) + 2*a*b*x^2/(sqrt(c*x^2)*c) + a^2*log(x)/c^(3/2
)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^2\,(a+b\,x)^2}{\left(c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + bx)^2}{(cx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(3/2),x)
[Out] Integral(x**2*(a + b*x)**2/(c*x**2)**(3/2), x)
```

## 3.796 $\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx$

**Optimal**. Leaf size=56

$$-\frac{a^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] -(a^2/(c*Sqrt[c*x^2])) + (b^2*x^2)/(c*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(c*Sq
rt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2}{c\sqrt{cx^2}} + \frac{b^2x^2}{c\sqrt{cx^2}} + \frac{2abx\log(x)}{c\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.59

$$\frac{x^2\left(-a^2+2abx\log(x)+b^2x^2\right)}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] (x^2*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/(c*x^2)^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.62

$$\frac{-a^2x^2+2abx^3\log(x)+b^2x^4}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] (-(a^2*x^2) + b^2*x^4 + 2*a*b*x^3*Log[x])/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.20, size = 34, normalized size = 0.61

$$\frac{\left(b^2x^2+2\,abx\log(x)-a^2\right)\sqrt{cx^2}}{c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^2*x^2)
```

**giac** [A] time = 1.08, size = 69, normalized size = 1.23

$$\frac{\frac{\sqrt{cx^2}\,b^2}{c}-\frac{2\left(ab\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)-\frac{a^2\sqrt{c}}{\sqrt{c}\,x-\sqrt{cx^2}}\right)}{\sqrt{c}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(c*x^2)*b^2/c - 2*(a*b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - a^2*sqrt(c
)/(sqrt(c)*x - sqrt(c*x^2)))/sqrt(c))/c
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.57

$$\frac{\left(2abx\ln(x)+b^2x^2-a^2\right)x^2}{\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] x^2*(2*a*b*x*ln(x)+b^2*x^2-a^2)/(c*x^2)^(3/2)
```

**maxima** [A] time = 1.38, size = 42, normalized size = 0.75

$$\frac{b^2x^2}{\sqrt{cx^2}\,c}+\frac{2\,ab\log(x)}{c^{\frac{3}{2}}}-\frac{a^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*x^2/(sqrt(c*x^2)*c) + 2*a*b*log(x)/c^(3/2) - a^2/(sqrt(c*x^2)*c)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,(a+b\,x)^2}{\left(c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x (a + bx)^2}{\left(cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x)**2/(c*x**2)**(3/2), x)
```

## 3.797 $\int \frac{(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=58

$$-\frac{a^2}{2cx\sqrt{cx^2}}-\frac{2ab}{c\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$-\frac{a^2}{2cx\sqrt{cx^2}}-\frac{2ab}{c\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c*x^2)^(3/2),x]
```

```
[Out] (-2*a*b)/(c*Sqrt[c*x^2]) - a^2/(2*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c*Sqrt
[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{c\sqrt{cx^2}} \\
&= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{c\sqrt{cx^2}} \\
&= -\frac{2ab}{c\sqrt{cx^2}}-\frac{a^2}{2cx\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}
\end{aligned}
$$

**Mathematica** [A] time = 0.00, size = 34, normalized size = 0.59

$$
\frac{x\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2\left(cx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c*x^2)^(3/2),x]
[Out] (x*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 38, normalized size = 0.66

$$
\frac{\frac{1}{2}\left(a^2(-x)-4abx^2\right)+b^2x^3\log(x)}{\left(cx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c*x^2)^(3/2),x]
[Out] ((-(a^2*x) - 4*a*b*x^2)/2 + b^2*x^3*Log[x])/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.04, size = 36, normalized size = 0.62

$$
\frac{\left(2\,b^2x^2\log(x)-4\,abx-a^2\right)\sqrt{cx^2}}{2\,c^2x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^2*x^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.55

$$\frac{\left(2b^2x^2\ln(x)-4abx-a^2\right)x}{2\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(3/2),x)
[Out] 1/2*x*(2*b^2*x^2*ln(x)-4*a*b*x-a^2)/(c*x^2)^(3/2)
```

**maxima [A]** time = 1.33, size = 35, normalized size = 0.60

$$\frac{b^2\log(x)}{c^{\frac{3}{2}}}-\frac{2\,ab}{\sqrt{cx^2}\,c}-\frac{a^2}{2\,c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] b^2*log(x)/c^(3/2) - 2*a*b/(sqrt(c*x^2)*c) - 1/2*a^2/(c^(3/2)*x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{(a+b\,x)}^2}{{\left(c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c*x^2)^(3/2),x)
[Out] int((a + b*x)^2/(c*x^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx)^2}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Integral((a + b*x)**2/(c*x**2)**(3/2), x)
```

## 3.798 $\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=29

$$-\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
```

```
[Out] -(a + b*x)^3/(3*a*c*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^4}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 36, normalized size = 1.24

$$\frac{cx^2\left(-a^2-3abx-3b^2x^2\right)}{3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
[Out] (c*x^2*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 32, normalized size = 1.10

$$\frac{-a^2-3abx-3b^2x^2}{3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
[Out] (-a^2 - 3*a*b*x - 3*b^2*x^2)/(3*(c*x^2)^(3/2))
```

**fricas** [A] time = 0.80, size = 32, normalized size = 1.10

$$-\frac{\left(3\,b^2x^2+3\,abx+a^2\right)\sqrt{cx^2}}{3\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^2*x^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2}{\left(cx^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2/((c*x^2)^(3/2)*x), x)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.93

$$-\frac{3b^{2}x^{2}+3abx+a^{2}}{3\left(c\,x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(3/2),x)
```

```
[Out] -1/3*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(3/2)
```

**maxima [A]** time = 1.36, size = 37, normalized size = 1.28

$$-\frac{b^{2}}{\sqrt{cx^{2}}\,c}-\frac{ab}{c^{\frac{3}{2}}x^{2}}-\frac{a^{2}}{3\,c^{\frac{3}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -b^2/(sqrt(c*x^2)*c) - a*b/(c^(3/2)*x^2) - 1/3*a^2/(c^(3/2)*x^3)
```

**mupad [B]** time = 0.19, size = 33, normalized size = 1.14

$$-\frac{a^{2}\,x^{2}+3\,a\,b\,x^{3}+3\,b^{2}\,x^{4}}{3\,c^{3/2}\,\left(x^{2}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(3/2)),x)
```

```
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(3/2)*(x^2)^(5/2))
```

**sympy [B]** time = 0.67, size = 53, normalized size = 1.83

$$-\frac{a^{2}}{3c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}-\frac{abx}{c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}-\frac{b^{2}x^{2}}{c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(3*c**(3/2)*(x**2)**(3/2)) - a*b*x/(c**(3/2)*(x**2)**(3/2)) - b**2*x*
*2/(c**(3/2)*(x**2)**(3/2))
```

## 3.799 $\int \frac{(a+bx)^2}{x^2(cx^2)^{3/2}}\,dx$

**Optimal**. Leaf size=66

$$-\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -a^2/(4*c*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c*x^2*Sqrt[c*x^2]) - b^2/(2*c*x*Sqr
t[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{3/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\left(3a^2+8abx+6b^2x^2\right)}{12c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
[Out] -1/12*(Sqrt[c*x^2]*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c^2*x^5)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 35, normalized size = 0.53

$$\frac{-3a^2-8abx-6b^2x^2}{12x\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
[Out] (-3*a^2 - 8*a*b*x - 6*b^2*x^2)/(12*x*(c*x^2)^(3/2))
```

**fricas [A]** time = 0.99, size = 34, normalized size = 0.52

$$-\frac{\left(6\,b^2x^2+8\,abx+3\,a^2\right)\sqrt{cx^2}}{12\,c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^2*x^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

**maple [A]** time = 0.01, size = 32, normalized size = 0.48

$$-\frac{6b^2x^2 + 8abx + 3a^2}{12\left(c\,x^2\right)^{\frac{3}{2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(3/2),x)
[Out] -1/12*(6*b^2*x^2+8*a*b*x+3*a^2)/x/(c*x^2)^(3/2)
```

**maxima [A]** time = 1.31, size = 33, normalized size = 0.50

$$-\frac{b^2}{2\,c^{\frac{3}{2}}x^2} - \frac{2\,ab}{3\,c^{\frac{3}{2}}x^3} - \frac{a^2}{4\,c^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*b^2/(c^(3/2)*x^2) - 2/3*a*b/(c^(3/2)*x^3) - 1/4*a^2/(c^(3/2)*x^4)
```

**mupad [B]** time = 0.19, size = 42, normalized size = 0.64

$$-\frac{3\,a^2\,\sqrt{x^2}\, + 6\,b^2\,x^2\,\sqrt{x^2}\, + 8\,a\,b\,x\,\sqrt{x^2}}{12\,c^{3/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(3/2)),x)
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(3
/2)*x^5)
```

**sympy [A]** time = 0.81, size = 56, normalized size = 0.85

$$-\frac{a^2}{4c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}} - \frac{2ab}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} - \frac{b^2x}{2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(4*c**(3/2)*x*(x**2)**(3/2)) - 2*a*b/(3*c**(3/2)*(x**2)**(3/2)) - b**
2*x/(2*c**(3/2)*(x**2)**(3/2))
```

## 3.800 $\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=66

$$-\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] -a^2/(5*c*x^4*Sqrt[c*x^2]) - (a*b)/(2*c*x^3*Sqrt[c*x^2]) - b^2/(3*c*x^2*Sqr
t[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^6}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2}{5cx^4\sqrt{cx^2}} - \frac{ab}{2cx^3\sqrt{cx^2}} - \frac{b^2}{3cx^2\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.50

$$\frac{c\left(-6a^2-15abx-10b^2x^2\right)}{30\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
[Out] (c*(-6*a^2 - 15*a*b*x - 10*b^2*x^2))/(30*(c*x^2)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{-6a^2-15abx-10b^2x^2}{30x^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
[Out] (-6*a^2 - 15*a*b*x - 10*b^2*x^2)/(30*x^2*(c*x^2)^(3/2))
```

**fricas** [A] time = 1.28, size = 34, normalized size = 0.52

$$-\frac{\left(10\,b^2x^2+15\,abx+6\,a^2\right)\sqrt{cx^2}}{30\,c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^2*x^6)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2}{(cx^2)^{\frac{3}{2}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2/((c*x^2)^(3/2)*x^3), x)
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.48

$$-\frac{10b^2x^2+15abx+6a^2}{30\left(c\,x^2\right)^{\frac{3}{2}}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(3/2),x)
[Out] -1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/x^2/(c*x^2)^(3/2)
```

**maxima [A]** time = 1.36, size = 33, normalized size = 0.50

$$-\frac{b^2}{3\,c^{\frac{3}{2}}x^3}-\frac{ab}{2\,c^{\frac{3}{2}}x^4}-\frac{a^2}{5\,c^{\frac{3}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/3*b^2/(c^(3/2)*x^3) - 1/2*a*b/(c^(3/2)*x^4) - 1/5*a^2/(c^(3/2)*x^5)
```

**mupad [B]** time = 0.20, size = 42, normalized size = 0.64

$$-\frac{6\,a^2\,\sqrt{x^2}\;+10\,b^2\,x^2\,\sqrt{x^2}\;+15\,a\,b\,x\,\sqrt{x^2}}{30\,c^{3/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(3/2)),x)
[Out] -(6*a^2*(x^2)^(1/2) + 10*b^2*x^2*(x^2)^(1/2) + 15*a*b*x*(x^2)^(1/2))/(30*c^
(3/2)*x^6)
```

**sympy [A]** time = 0.98, size = 56, normalized size = 0.85

$$-\frac{a^2}{5c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}-\frac{ab}{2c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}-\frac{b^2}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(5*c**(3/2)*x**2*(x**2)**(3/2)) - a*b/(2*c**(3/2)*x*(x**2)**(3/2)) -
b**2/(3*c**(3/2)*(x**2)**(3/2))
```

## 3.801 $\int \frac{(a+bx)^2}{x^4 (cx^2)^{3/2}} \, dx$

**Optimal**. Leaf size=66

$$-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] -a^2/(6*c*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c*x^4*Sqrt[c*x^2]) - b^2/(4*c*x^3*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{3/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^7}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2}{6cx^5\sqrt{cx^2}} - \frac{2ab}{5cx^4\sqrt{cx^2}} - \frac{b^2}{4cx^3\sqrt{cx^2}}$$

**Mathematica** [**A**] time = 0.01, size = 35, normalized size = 0.53

$$\frac{-10a^2-24abx-15b^2x^2}{60x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
[Out] (-10*a^2 - 24*a*b*x - 15*b^2*x^2)/(60*x^3*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [**A**] time = 0.03, size = 35, normalized size = 0.53

$$\frac{-10a^2-24abx-15b^2x^2}{60x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
[Out] (-10*a^2 - 24*a*b*x - 15*b^2*x^2)/(60*x^3*(c*x^2)^(3/2))
```

**fricas** [**A**] time = 1.15, size = 34, normalized size = 0.52

$$-\frac{\left(15\,b^2x^2+24\,abx+10\,a^2\right)\sqrt{cx^2}}{60\,c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^2*x^7)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.48

$$-\frac{15b^2x^2+24abx+10a^2}{60\left(c\,x^2\right)^{\frac{3}{2}}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(3/2),x)
[Out] -1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x^3/(c*x^2)^(3/2)
```

**maxima [A]** time = 1.29, size = 33, normalized size = 0.50

$$-\frac{b^2}{4\,c^{\frac{3}{2}}x^4}-\frac{2\,ab}{5\,c^{\frac{3}{2}}x^5}-\frac{a^2}{6\,c^{\frac{3}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/4*b^2/(c^(3/2)*x^4) - 2/5*a*b/(c^(3/2)*x^5) - 1/6*a^2/(c^(3/2)*x^6)
```

**mupad [B]** time = 0.18, size = 42, normalized size = 0.64

$$-\frac{10\,a^2\,\sqrt{x^2}\,+15\,b^2\,x^2\,\sqrt{x^2}\,+24\,a\,b\,x\,\sqrt{x^2}}{60\,c^{3/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(3/2)),x)
[Out] -(10*a^2*(x^2)^(1/2) + 15*b^2*x^2*(x^2)^(1/2) + 24*a*b*x*(x^2)^(1/2))/(60*c
^(3/2)*x^7)
```

**sympy [A]** time = 1.18, size = 61, normalized size = 0.92

$$-\frac{a^2}{6c^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}-\frac{2ab}{5c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}-\frac{b^2}{4c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(6*c**(3/2)*x**3*(x**2)**(3/2)) - 2*a*b/(5*c**(3/2)*x**2*(x**2)**(3/2
)) - b**2/(4*c**(3/2)*x*(x**2)**(3/2))
```

## 3.802 $\int \frac{x^3(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=56

$$-\frac{a^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -(a^2/(c^2*Sqrt[c*x^2])) + (b^2*x^2)/(c^2*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(
c^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 0.59

$$\frac{-a^2+2abx\log(x)+b^2x^2}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] (-a^2 + b^2*x^2 + 2*a*b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

**IntegrateAlgebraic [A]** time = 0.04, size = 35, normalized size = 0.62

$$\frac{-a^2x^4+2abx^5\log(x)+b^2x^6}{\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] (-(a^2*x^4) + b^2*x^6 + 2*a*b*x^5*Log[x])/(c*x^2)^(5/2)
```

**fricas [A]** time = 0.97, size = 34, normalized size = 0.61

$$\frac{\left(b^2x^2+2\,abx\log(x)-a^2\right)\sqrt{cx^2}}{c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^3*x^2)
```

**giac [A]** time = 1.07, size = 65, normalized size = 1.16

$$\frac{\sqrt{cx^2}\,b^2}{c^3} - \frac{2\left(ab\log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2}\right|\right) - \frac{a^2\sqrt{c}}{\sqrt{c}\,x - \sqrt{cx^2}}\right)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2)*b^2/c^3 - 2*(a*b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - a^2*sqrt(
c)/(sqrt(c)*x - sqrt(c*x^2)))/c^(5/2)
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.57

$$\frac{\left(2abx\ln(x) + b^2x^2 - a^2\right)x^4}{\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] x^4*(2*a*b*x*ln(x)+b^2*x^2-a^2)/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.45, size = 45, normalized size = 0.80

$$\frac{b^2x^4}{\left(cx^2\right)^{\frac{3}{2}}c} - \frac{a^2x^2}{\left(cx^2\right)^{\frac{3}{2}}c} + \frac{2\,ab\log(x)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b^2*x^4/((c*x^2)^(3/2)*c) - a^2*x^2/((c*x^2)^(3/2)*c) + 2*a*b*log(x)/c^(5/2
)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,(a+b\,x)^2}{\left(c\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + bx)^2}{(cx^2)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*x)**2/(c*x**2)**(5/2), x)
```

## 3.803 $\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=58

$$-\frac{a^2}{2c^2x\sqrt{cx^2}}-\frac{2ab}{c^2\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{2c^2x\sqrt{cx^2}}-\frac{2ab}{c^2\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (-2*a*b)/(c^2*Sqrt[c*x^2]) - a^2/(2*c^2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c^
2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{2ab}{c^2\sqrt{cx^2}} - \frac{a^2}{2c^2x\sqrt{cx^2}} + \frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 36, normalized size = 0.62

$$\frac{x^3\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] (x^3*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 40, normalized size = 0.69

$$\frac{\frac{1}{2}\left(-a^2x^3-4abx^4\right)+b^2x^5\log(x)}{\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] ((-(a^2*x^3) - 4*a*b*x^4)/2 + b^2*x^5*Log[x])/(c*x^2)^(5/2)
```

**fricas** [A] time = 0.84, size = 36, normalized size = 0.62

$$\frac{\left(2\,b^2x^2\log(x)-4\,abx-a^2\right)\sqrt{cx^2}}{2\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^3*x^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

**maple [A]** time = 0.01, size = 34, normalized size = 0.59

$$\frac{\left(2b^2x^2\ln(x)-4abx-a^2\right)x^3}{2\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(5/2),x)
[Out] 1/2*x^3*(2*b^2*x^2*ln(x)-4*a*b*x-a^2)/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.40, size = 38, normalized size = 0.66

$$-\frac{2\,abx^2}{\left(cx^2\right)^{\frac{3}{2}}c}+\frac{b^2\log(x)}{c^{\frac{5}{2}}}-\frac{a^2}{2\,c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -2*a*b*x^2/((c*x^2)^(3/2)*c) + b^2*log(x)/c^(5/2) - 1/2*a^2/(c^(5/2)*x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,(a+b\,x)^2}{\left(c\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^2)/(c*x^2)^(5/2),x)
[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + bx)^2}{(cx^2)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**2/(c*x**2)**(5/2), x)
```

## 3.804 $\int \frac{x(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=29

$$-\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -(a + b*x)^3/(3*a*c^2*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^4}\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 1.21

$$\frac{x^2\left(-a^2-3abx-3b^2x^2\right)}{3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] (x^2*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 33, normalized size = 1.14

$$-\frac{x^2\left(a^2+3abx+3b^2x^2\right)}{3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] -1/3*(x^2*(a^2 + 3*a*b*x + 3*b^2*x^2))/(c*x^2)^(5/2)
```

**fricas** [A] time = 0.86, size = 32, normalized size = 1.10

$$-\frac{\left(3\,b^2x^2+3\,abx+a^2\right)\sqrt{cx^2}}{3\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^3*x^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2x}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2*x/(c*x^2)^(5/2), x)
```

**maple** [A] time = 0.00, size = 30, normalized size = 1.03

$$-\frac{\left(3b^{2}x^{2}+3abx+a^{2}\right)x^{2}}{3\left(c\,x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] -1/3*x^2*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(5/2)
```

**maxima** [A] time = 1.38, size = 44, normalized size = 1.52

$$-\frac{b^{2}x^{2}}{\left(cx^{2}\right)^{\frac{3}{2}}c}-\frac{a^{2}}{3\left(cx^{2}\right)^{\frac{3}{2}}c}-\frac{ab}{c^{\frac{5}{2}}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -b^2*x^2/((c*x^2)^(3/2)*c) - 1/3*a^2/((c*x^2)^(3/2)*c) - a*b/(c^(5/2)*x^2)
```

**mupad** [B] time = 0.18, size = 33, normalized size = 1.14

$$-\frac{a^{2}\,x^{2}+3\,a\,b\,x^{3}+3\,b^{2}\,x^{4}}{3\,c^{5/2}\,\left(x^{2}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

```
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(5/2)*(x^2)^(5/2))
```

**sympy** [B] time = 0.96, size = 58, normalized size = 2.00

$$-\frac{a^{2}x^{2}}{3c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{abx^{3}}{c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{b^{2}x^{4}}{c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2*x**2/(3*c**(5/2)*(x**2)**(5/2)) - a*b*x**3/(c**(5/2)*(x**2)**(5/2)) -
 b**2*x**4/(c**(5/2)*(x**2)**(5/2))
```

## 3.805 $\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx$

**Optimal**. Leaf size=66

$$-\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$-\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c*x^2)^(5/2),x]
```

```
[Out] -a^2/(4*c^2*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c^2*x^2*Sqrt[c*x^2]) - b^2/(2*c^2
*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.00, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\left(3a^2+8abx+6b^2x^2\right)}{12c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c*x^2)^(5/2),x]
[Out] -1/12*(Sqrt[c*x^2]*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c^3*x^5)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 33, normalized size = 0.50

$$-\frac{x\left(3a^2+8abx+6b^2x^2\right)}{12\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c*x^2)^(5/2),x]
[Out] -1/12*(x*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c*x^2)^(5/2)
```

**fricas [A]** time = 0.83, size = 34, normalized size = 0.52

$$-\frac{\left(6\,b^2x^2+8\,abx+3\,a^2\right)\sqrt{cx^2}}{12\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^3*x^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

**maple [A]** time = 0.00, size = 30, normalized size = 0.45

$$-\frac{\left(6b^2x^2+8abx+3a^2\right)x}{12\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(5/2),x)
[Out] -1/12*x*(6*b^2*x^2+8*a*b*x+3*a^2)/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.36, size = 37, normalized size = 0.56

$$-\frac{2\,ab}{3\,\left(cx^2\right)^{\frac{3}{2}}c}-\frac{b^2}{2\,c^{\frac{5}{2}}x^2}-\frac{a^2}{4\,c^{\frac{5}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -2/3*a*b/((c*x^2)^(3/2)*c) - 1/2*b^2/(c^(5/2)*x^2) - 1/4*a^2/(c^(5/2)*x^4)
```

**mupad [B]** time = 0.17, size = 42, normalized size = 0.64

$$-\frac{3\,a^2\,\sqrt{x^2}+6\,b^2\,x^2\,\sqrt{x^2}+8\,a\,b\,x\,\sqrt{x^2}}{12\,c^{5/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c*x^2)^(5/2),x)
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(5
/2)*x^5)
```

**sympy [A]** time = 0.96, size = 61, normalized size = 0.92

$$-\frac{a^2x}{4c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{2abx^2}{3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{b^2x^3}{2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2*x/(4*c**(5/2)*(x**2)**(5/2)) - 2*a*b*x**2/(3*c**(5/2)*(x**2)**(5/2))
- b**2*x**3/(2*c**(5/2)*(x**2)**(5/2))
```

## 3.806 $\int \frac{(a+bx)^2}{x\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=66

$$-\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -a^2/(5*c^2*x^4*Sqrt[c*x^2]) - (a*b)/(2*c^2*x^3*Sqrt[c*x^2]) - b^2/(3*c^2*x
^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^6}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\left(6a^2+15abx+10b^2x^2\right)}{30c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
[Out] -1/30*(Sqrt[c*x^2]*(6*a^2 + 15*a*b*x + 10*b^2*x^2))/(c^3*x^6)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 32, normalized size = 0.48

$$\frac{-6a^2-15abx-10b^2x^2}{30\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
[Out] (-6*a^2 - 15*a*b*x - 10*b^2*x^2)/(30*(c*x^2)^(5/2))
```

**fricas** [A] time = 1.49, size = 34, normalized size = 0.52

$$-\frac{\left(10\,b^2x^2+15\,abx+6\,a^2\right)\sqrt{cx^2}}{30\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^3*x^6)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2}{(cx^2)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2/((c*x^2)^(5/2)*x), x)
```

**maple [A]** time = 0.01, size = 29, normalized size = 0.44

$$-\frac{10b^2x^2+15abx+6a^2}{30\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(5/2),x)
```

```
[Out] -1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.31, size = 37, normalized size = 0.56

$$-\frac{b^2}{3\left(cx^2\right)^{\frac{3}{2}}c}-\frac{ab}{2\,c^{\frac{5}{2}}x^4}-\frac{a^2}{5\,c^{\frac{5}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*b^2/((c*x^2)^(3/2)*c) - 1/2*a*b/(c^(5/2)*x^4) - 1/5*a^2/(c^(5/2)*x^5)
```

**mupad [B]** time = 0.18, size = 42, normalized size = 0.64

$$-\frac{6\,a^2\,\sqrt{x^2}+10\,b^2\,x^2\,\sqrt{x^2}+15\,a\,b\,x\,\sqrt{x^2}}{30\,c^{5/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(5/2)),x)
```

```
[Out] -(6*a^2*(x^2)^(1/2) + 10*b^2*x^2*(x^2)^(1/2) + 15*a*b*x*(x^2)^(1/2))/(30*c^
(5/2)*x^6)
```

**sympy [A]** time = 1.15, size = 56, normalized size = 0.85

$$-\frac{a^2}{5c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{abx}{2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{b^2x^2}{3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(5*c**(5/2)*(x**2)**(5/2)) - a*b*x/(2*c**(5/2)*(x**2)**(5/2)) - b**2*
x**2/(3*c**(5/2)*(x**2)**(5/2))
```

## 3.807 $\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=66

$$-\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -a^2/(6*c^2*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c^2*x^4*Sqrt[c*x^2]) - b^2/(4*c^2
*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^2 \left(cx^2\right)^{5/2}} \, dx = \frac{x \int \frac{(a+bx)^2}{x^7} \, dx}{c^2 \sqrt{cx^2}}$$
$$= \frac{x \int \left(\frac{a^2}{x^7} + \frac{2ab}{x^6} + \frac{b^2}{x^5}\right) dx}{c^2 \sqrt{cx^2}}$$
$$= -\frac{a^2}{6c^2x^5\sqrt{cx^2}} - \frac{2ab}{5c^2x^4\sqrt{cx^2}} - \frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2} \left(10a^2 + 24abx + 15b^2x^2\right)}{60c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
[Out] -1/60*(Sqrt[c*x^2]*(10*a^2 + 24*a*b*x + 15*b^2*x^2))/(c^3*x^7)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{-10a^2 - 24abx - 15b^2x^2}{60x \left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
[Out] (-10*a^2 - 24*a*b*x - 15*b^2*x^2)/(60*x*(c*x^2)^(5/2))
```

**fricas** [A] time = 0.77, size = 34, normalized size = 0.52

$$-\frac{\left(15\,b^2x^2 + 24\,abx + 10\,a^2\right)\sqrt{cx^2}}{60\,c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^3*x^7)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.48

$$-\frac{15b^2x^2 + 24abx + 10a^2}{60\left(c\,x^2\right)^{\frac{5}{2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(5/2),x)
[Out] -1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.34, size = 33, normalized size = 0.50

$$-\frac{b^2}{4\,c^{\frac{5}{2}}x^4} - \frac{2\,ab}{5\,c^{\frac{5}{2}}x^5} - \frac{a^2}{6\,c^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/4*b^2/(c^(5/2)*x^4) - 2/5*a*b/(c^(5/2)*x^5) - 1/6*a^2/(c^(5/2)*x^6)
```

**mupad [B]** time = 0.18, size = 42, normalized size = 0.64

$$-\frac{10\,a^2\,\sqrt{x^2}\,+15\,b^2\,x^2\,\sqrt{x^2}\,+24\,a\,b\,x\,\sqrt{x^2}}{60\,c^{5/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(5/2)),x)
[Out] -(10*a^2*(x^2)^(1/2) + 15*b^2*x^2*(x^2)^(1/2) + 24*a*b*x*(x^2)^(1/2))/(60*c
^(5/2)*x^7)
```

**sympy [A]** time = 1.40, size = 56, normalized size = 0.85

$$-\frac{a^2}{6c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}} - \frac{2ab}{5c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} - \frac{b^2x}{4c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(6*c**(5/2)*x*(x**2)**(5/2)) - 2*a*b/(5*c**(5/2)*(x**2)**(5/2)) - b**
2*x/(4*c**(5/2)*(x**2)**(5/2))
```

## 3.808 $\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=66

$$-\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] -a^2/(7*c^2*x^6*Sqrt[c*x^2]) - (a*b)/(3*c^2*x^5*Sqrt[c*x^2]) - b^2/(5*c^2*x
^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^3 \left(cx^2\right)^{5/2}} \, dx = \frac{x \int \frac{(a+bx)^2}{x^8} \, dx}{c^2 \sqrt{cx^2}}$$
$$= \frac{x \int \left(\frac{a^2}{x^8} + \frac{2ab}{x^7} + \frac{b^2}{x^6}\right) dx}{c^2 \sqrt{cx^2}}$$
$$= -\frac{a^2}{7c^2x^6\sqrt{cx^2}} - \frac{ab}{3c^2x^5\sqrt{cx^2}} - \frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.50

$$\frac{c \left(-15a^2 - 35abx - 21b^2x^2\right)}{105 \left(cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
[Out] (c*(-15*a^2 - 35*a*b*x - 21*b^2*x^2))/(105*(c*x^2)^(7/2))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{-15a^2 - 35abx - 21b^2x^2}{105x^2 \left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
[Out] (-15*a^2 - 35*a*b*x - 21*b^2*x^2)/(105*x^2*(c*x^2)^(5/2))
```

**fricas** [A] time = 0.74, size = 34, normalized size = 0.52

$$-\frac{\left(21\,b^2x^2 + 35\,abx + 15\,a^2\right)\sqrt{cx^2}}{105\,c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)*sqrt(c*x^2)/(c^3*x^8)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2}{\left(cx^2\right)^{\frac{5}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2/((c*x^2)^(5/2)*x^3), x)
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.48

$$-\frac{21b^2x^2+35abx+15a^2}{105\left(c\,x^2\right)^{\frac{5}{2}} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(5/2),x)
[Out] -1/105*(21*b^2*x^2+35*a*b*x+15*a^2)/x^2/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.37, size = 33, normalized size = 0.50

$$-\frac{b^2}{5\,c^{\frac{5}{2}}x^5}-\frac{ab}{3\,c^{\frac{5}{2}}x^6}-\frac{a^2}{7\,c^{\frac{5}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/5*b^2/(c^(5/2)*x^5) - 1/3*a*b/(c^(5/2)*x^6) - 1/7*a^2/(c^(5/2)*x^7)
```

**mupad [B]** time = 0.18, size = 42, normalized size = 0.64

$$-\frac{15\,a^2\,\sqrt{x^2}+21\,b^2\,x^2\,\sqrt{x^2}+35\,a\,b\,x\,\sqrt{x^2}}{105\,c^{5/2}\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(5/2)),x)
[Out] -(15*a^2*(x^2)^(1/2) + 21*b^2*x^2*(x^2)^(1/2) + 35*a*b*x*(x^2)^(1/2))/(105*
c^(5/2)*x^8)
```

**sympy [A]** time = 1.70, size = 56, normalized size = 0.85

$$-\frac{a^2}{7c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}-\frac{ab}{3c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}-\frac{b^2}{5c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(7*c**(5/2)*x**2*(x**2)**(5/2)) - a*b/(3*c**(5/2)*x*(x**2)**(5/2)) -
b**2/(5*c**(5/2)*(x**2)**(5/2))
```

## 3.809 $\int \frac{(a+bx)^2}{x^4 \left(cx^2\right)^{5/2}} \, dx$

**Optimal**. Leaf size=66

$$-\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] -a^2/(8*c^2*x^7*Sqrt[c*x^2]) - (2*a*b)/(7*c^2*x^6*Sqrt[c*x^2]) - b^2/(6*c^2
*x^5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^9}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^9}+\frac{2ab}{x^8}+\frac{b^2}{x^7}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.53

$$\frac{-21a^2-48abx-28b^2x^2}{168x^3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
[Out] (-21*a^2 - 48*a*b*x - 28*b^2*x^2)/(168*x^3*(c*x^2)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{-21a^2-48abx-28b^2x^2}{168x^3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
[Out] (-21*a^2 - 48*a*b*x - 28*b^2*x^2)/(168*x^3*(c*x^2)^(5/2))
```

**fricas** [A] time = 1.20, size = 34, normalized size = 0.52

$$-\frac{\left(28\,b^2x^2+48\,abx+21\,a^2\right)\sqrt{cx^2}}{168\,c^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/168*(28*b^2*x^2 + 48*a*b*x + 21*a^2)*sqrt(c*x^2)/(c^3*x^9)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

**maple [A]** time = 0.01, size = 32, normalized size = 0.48

$$-\frac{28b^2x^2 + 48abx + 21a^2}{168\left(c\,x^2\right)^{\frac{5}{2}}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(5/2),x)
```

```
[Out] -1/168*(28*b^2*x^2+48*a*b*x+21*a^2)/x^3/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.35, size = 33, normalized size = 0.50

$$-\frac{b^2}{6\,c^{\frac{5}{2}}x^6} - \frac{2\,ab}{7\,c^{\frac{5}{2}}x^7} - \frac{a^2}{8\,c^{\frac{5}{2}}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b^2/(c^(5/2)*x^6) - 2/7*a*b/(c^(5/2)*x^7) - 1/8*a^2/(c^(5/2)*x^8)
```

**mupad [B]** time = 0.18, size = 42, normalized size = 0.64

$$-\frac{21\,a^2\,\sqrt{x^2}\,+28\,b^2\,x^2\,\sqrt{x^2}\,+48\,a\,b\,x\,\sqrt{x^2}}{168\,c^{5/2}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(5/2)),x)
```

```
[Out] -(21*a^2*(x^2)^(1/2) + 28*b^2*x^2*(x^2)^(1/2) + 48*a*b*x*(x^2)^(1/2))/(168*
c^(5/2)*x^9)
```

**sympy [A]** time = 2.03, size = 61, normalized size = 0.92

$$-\frac{a^2}{8c^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}} - \frac{2ab}{7c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}} - \frac{b^2}{6c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(8*c**(5/2)*x**3*(x**2)**(5/2)) - 2*a*b/(7*c**(5/2)*x**2*(x**2)**(5/2
)) - b**2/(6*c**(5/2)*x*(x**2)**(5/2))
```

## 3.810 $\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx$

**Optimal**. Leaf size=102

$$\frac{a^4\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}$$

**Rubi [A]** time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}+\frac{a^4\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] -((a^3*Sqrt[c*x^2])/b^4) + (a^2*x*Sqrt[c*x^2])/(2*b^3) - (a*x^2*Sqrt[c*x^2]
)/(3*b^2) + (x^3*Sqrt[c*x^2])/(4*b) + (a^4*Sqrt[c*x^2]*Log[a + b*x])/(b^5*x
)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx = \frac{\sqrt{cx^2}\int \frac{x^4}{a+bx}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}$$
$$= -\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}+\frac{a^4\sqrt{cx^2}\log(a+bx)}{b^5x}$$

**Mathematica [A]** time = 0.02, size = 63, normalized size = 0.62

$$\frac{cx\left(12a^4\log(a+bx)+bx\left(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3\right)\right)}{12b^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a^4*Log[a +
b*x]))/(12*b^5*Sqrt[c*x^2])
```

**IntegrateAlgebraic [A]** time = 0.06, size = 64, normalized size = 0.63

$$\sqrt{cx^2}\left(\frac{a^4\log(a+bx)}{b^5x}+\frac{-12a^3+6a^2bx-4ab^2x^2+3b^3x^3}{12b^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] Sqrt[c*x^2]*((-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3)/(12*b^4) + (a^
4*Log[a + b*x])/(b^5*x))
```

**fricas [A]** time = 0.97, size = 62, normalized size = 0.61

$$\frac{\left(3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log\left(bx+a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))*sqrt(c*x^2)/(b^5*x)
```

**giac [A]** time = 1.11, size = 81, normalized size = 0.79

$$\frac{1}{12}\sqrt{c}\left(\frac{12\,a^4\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^5}-\frac{12\,a^4\log\left(|a|\right)\mathrm{sgn}(x)}{b^5}+\frac{3\,b^3x^4\mathrm{sgn}(x)-4\,ab^2x^3\mathrm{sgn}(x)+6\,a^2bx^2\mathrm{sgn}(x)-12\,a^3x\mathrm{sgn}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn(
x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12*a
^3*x*sgn(x))/b^4)
```

**maple [A]** time = 0.01, size = 63, normalized size = 0.62

$$\frac{\sqrt{c\,x^2}\,\left(3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln\left(bx+a\right)-12a^3bx\right)}{12b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] 1/12*(c*x^2)^(1/2)*(3*b^4*x^4-4*x^3*a*b^3+6*x^2*a^2*b^2+12*a^4*ln(b*x+a)-12
*b*x*a^3)/x/b^5
```

**maxima [A]** time = 1.57, size = 128, normalized size = 1.25

$$\frac{(-1)^{\frac{2\,cx}{b}}\,a^4\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^4\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5}+\frac{\sqrt{cx^2}\,a^2x}{2\,b^3}+\frac{\left(cx^2\right)^{\frac{3}{2}}\,x}{4\,bc}-\frac{\sqrt{cx^2}\,a^3}{b^4}-\frac{\left(cx^2\right)^{\frac{3}{2}}\,a}{3\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*sqrt(c)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*sqrt(c)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/2*sqrt(c*x^2)*a^2*x/b^3 + 1/4*(c*x^2
)^(3/2)*x/(b*c) - sqrt(c*x^2)*a^3/b^4 - 1/3*(c*x^2)^(3/2)*a/(b^2*c)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c*x^2)^(1/2))/(a + b*x),x)
```

```
[Out] int((x^3*(c*x^2)^(1/2))/(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x**3*sqrt(c*x**2)/(a + b*x), x)
```

## 3.811 $\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx$

**Optimal**. Leaf size=80

$$-\frac{a^3\sqrt{cx^2}\,\log(a+bx)}{b^4x}+\frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}$$

**Rubi** [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}}{b^3}-\frac{a^3\sqrt{cx^2}\,\log(a+bx)}{b^4x}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (a^2*Sqrt[c*x^2])/b^3 - (a*x*Sqrt[c*x^2])/(2*b^2) + (x^2*Sqrt[c*x^2])/(3*b)
 - (a^3*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^3}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}\\ &= \frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}-\frac{a^3\sqrt{cx^2}\,\log(a+bx)}{b^4x}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 52, normalized size = 0.65

$$\frac{cx\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*Sqrt[
c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 54, normalized size = 0.68

$$\sqrt{cx^2}\left(\frac{6a^2-3abx+2b^2x^2}{6b^3}-\frac{a^3\log(a+bx)}{b^4x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] Sqrt[c*x^2]*((6*a^2 - 3*a*b*x + 2*b^2*x^2)/(6*b^3) - (a^3*Log[a + b*x])/(b^
4*x))
```

**fricas** [A] time = 1.07, size = 51, normalized size = 0.64

$$\frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*x)
```

**giac** [A] time = 0.94, size = 69, normalized size = 0.86

$$-\frac{1}{6}\sqrt{c}\left(\frac{6\,a^3\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^4}-\frac{6\,a^3\log\left(|a|\right)\mathrm{sgn}(x)}{b^4}-\frac{2\,b^2x^3\mathrm{sgn}(x)-3\,abx^2\mathrm{sgn}(x)+6\,a^2x\mathrm{sgn}(x)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(c)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x)
/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

**maple** [A] time = 0.01, size = 52, normalized size = 0.65

$$-\frac{\sqrt{c\,x^2}\,\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln\left(bx+a\right)-6a^2bx\right)}{6b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(1/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x/b^4
```

**maxima** [A] time = 1.56, size = 110, normalized size = 1.38

$$-\frac{(-1)^{\frac{2cx}{b}}a^3\sqrt{c}\log\left(\frac{2cx}{b}\right)}{b^4}-\frac{(-1)^{\frac{2acx}{b}}a^3\sqrt{c}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^4}-\frac{\sqrt{cx^2}\,ax}{2\,b^2}+\frac{\sqrt{cx^2}\,a^2}{b^3}+\frac{\left(cx^2\right)^{\frac{3}{2}}}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*sqrt(c)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*sqrt(c)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*x/b^2 + sqrt(c*x^2)
*a^2/b^3 + 1/3*(c*x^2)^(3/2)/(b*c)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^(1/2))/(a + b*x),x)
```

```
[Out] int((x^2*(c*x^2)^(1/2))/(a + b*x), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x**2*sqrt(c*x**2)/(a + b*x), x)
```

## 3.812 $\int \frac{x\sqrt{cx^2}}{a+bx}\,dx$

**Optimal**. Leaf size=58

$$\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}$$

**Rubi [A]** time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] -((a*Sqrt[c*x^2])/b^2) + (x*Sqrt[c*x^2])/(2*b) + (a^2*Sqrt[c*x^2]*Log[a + b
*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^2}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}\\ &= -\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}+\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 0.69

$$\frac{cx\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c*x^2])/(a + b*x),x]
[Out] (c*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 41, normalized size = 0.71

$$\sqrt{cx^2}\left(\frac{a^2\log(a+bx)}{b^3x}+\frac{bx-2a}{2b^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[c*x^2])/(a + b*x),x]
[Out] Sqrt[c*x^2]*((-2*a + b*x)/(2*b^2) + (a^2*Log[a + b*x])/(b^3*x))
```

**fricas** [A] time = 1.19, size = 39, normalized size = 0.67

$$\frac{\left(b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

**giac** [A] time = 1.02, size = 54, normalized size = 0.93

$$\frac{1}{2}\sqrt{c}\left(\frac{2\,a^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3}-\frac{2\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^3}+\frac{bx^2\mathrm{sgn}(x)-2\,ax\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
[Out] 1/2*sqrt(c)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)/
b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

**maple** [A] time = 0.01, size = 40, normalized size = 0.69

$$\frac{\sqrt{c\,x^2}\,\left(b^2x^2+2a^2\ln\left(bx+a\right)-2abx\right)}{2b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/x/b^3
```

**maxima [A]** time = 1.49, size = 91, normalized size = 1.57

$$\frac{(-1)^{\frac{2\,cx}{b}}\,a^2\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,x}{2\,b}-\frac{\sqrt{cx^2}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^2*sqrt(c)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*sqrt(c)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*x/b - sqrt(c*x^2)*a/b^
2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(1/2))/(a + b*x),x)
```

```
[Out] int((x*(c*x^2)^(1/2))/(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x*sqrt(c*x**2)/(a + b*x), x)
```

## 3.813 $\int \frac{\sqrt{cx^2}}{a+bx}\,dx$

**Optimal**. Leaf size=38

$$\frac{\sqrt{cx^2}}{b}-\frac{a\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

**Rubi** [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{\sqrt{cx^2}}{b}-\frac{a\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(a + b*x),x]
```

```
[Out] Sqrt[c*x^2]/b - (a*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x}{a+bx}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}\\
&= \frac{\sqrt{cx^2}}{b}-\frac{a\sqrt{cx^2}\,\log(a+bx)}{b^2x}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{cx(bx - a\log(a + bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(a + b*x),x]
[Out] (c*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 29, normalized size = 0.76

$$\sqrt{cx^2}\left(\frac{1}{b} - \frac{a\log(a + bx)}{b^2x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(a + b*x),x]
[Out] Sqrt[c*x^2]*(b^(-1) - (a*Log[a + b*x])/(b^2*x))
```

**fricas** [A] time = 1.04, size = 27, normalized size = 0.71

$$\frac{\sqrt{cx^2}\left(bx - a\log\left(bx + a\right)\right)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*x)
```

**giac** [A] time = 0.96, size = 37, normalized size = 0.97

$$\sqrt{c}\left(\frac{x\mathrm{sgn}(x)}{b} - \frac{a\log\left(|bx + a|\right)\mathrm{sgn}(x)}{b^2} + \frac{a\log\left(|a|\right)\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
[Out] sqrt(c)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x)
/b^2)
```

**maple** [A] time = 0.00, size = 29, normalized size = 0.76

$$-\frac{\sqrt{c\,x^2}\,\left(a\ln\left(bx + a\right) - bx\right)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] -(c*x^2)^(1/2)*(a*ln(b*x+a)-b*x)/b^2/x
```

**maxima [B]** time = 1.45, size = 74, normalized size = 1.95

$$-\frac{(-1)^{\frac{2cx}{b}}a\sqrt{c}\log\left(\frac{2cx}{b}\right)}{b^2}-\frac{(-1)^{\frac{2acx}{b}}a\sqrt{c}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2}+\frac{\sqrt{cx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a*sqrt(c)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*sqrt(c)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(a + b*x),x)
```

```
[Out] int((c*x^2)^(1/2)/(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(a + b*x), x)
```

## 3.814 $\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx$

**Optimal**. Leaf size=22

$$\frac{\sqrt{cx^2}\,\log(a+bx)}{bx}$$

**Rubi** [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 31}

$$\frac{\sqrt{cx^2}\,\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,\log(a+bx)}{bx}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{cx\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x*(a + b*x)),x]
```

```
[Out] (c*x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.02, size = 22, normalized size = 1.00

$$\frac{\sqrt{cx^2}\,\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

**fricas** [A] time = 1.17, size = 20, normalized size = 0.91

$$\frac{\sqrt{cx^2}\,\log(bx+a)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*log(b*x + a)/(b*x)
```

**giac** [A] time = 0.95, size = 28, normalized size = 1.27

$$\sqrt{c}\left(\frac{\log(|bx+a|)\,\mathrm{sgn}(x)}{b}-\frac{\log(|a|)\,\mathrm{sgn}(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] sqrt(c)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\sqrt{c\,x^2}\,\ln(bx+a)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x/(b*x+a),x)
```

```
[Out] ln(b*x+a)*(c*x^2)^(1/2)/b/x
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{c\,x^2}}{x\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x*(a + b*x)), x)
```

## 3.815 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx$

**Optimal**. Leaf size=42

$$\frac{\sqrt{cx^2}\,\log(x)}{ax}-\frac{\sqrt{cx^2}\,\log(a+bx)}{ax}$$

**Rubi** [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{\sqrt{cx^2}\,\log(x)}{ax}-\frac{\sqrt{cx^2}\,\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*Log[x])/(a*x) - (Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \frac{1}{x}\,dx}{ax} - \frac{\left(b\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}\\ &= \frac{\sqrt{cx^2}\log(x)}{ax} - \frac{\sqrt{cx^2}\log(a+bx)}{ax}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 26, normalized size = 0.62

$$\frac{cx(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
[Out] (c*x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 37, normalized size = 0.88

$$\sqrt{cx^2}\left(\frac{\log(x)}{ax} - \frac{\log\left(a^2+abx\right)}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
[Out] Sqrt[c*x^2]*(Log[x]/(a*x) - Log[a^2 + a*b*x]/(a*x))
```

**fricas** [A] time = 1.42, size = 64, normalized size = 1.52

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="fricas")
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*x +
 a)*sqrt(-c)/(a*c*x))/a]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple [A]** time = 0.01, size = 26, normalized size = 0.62

$$\frac{\sqrt{c\,x^2}\,\left(\ln(x)-\ln\left(bx+a\right)\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^2/(b*x+a),x)
```

```
[Out] (c*x^2)^(1/2)*(ln(x)-ln(b*x+a))/x/a
```

**maxima [A]** time = 1.38, size = 24, normalized size = 0.57

$$-\frac{\sqrt{c}\,\log\left(bx+a\right)}{a}+\frac{\sqrt{c}\,\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(c)*log(b*x + a)/a + sqrt(c)*log(x)/a
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{x^2\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^2*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^2*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c x^2}}{x^2 (a + b x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**2/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**2*(a + b*x)), x)
```

## 3.816 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx$

**Optimal**. Leaf size=61

$$-\frac{b\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\,\log(a+bx)}{a^2x}-\frac{\sqrt{cx^2}}{ax^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{b\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\,\log(a+bx)}{a^2x}-\frac{\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

```
[Out] -(Sqrt[c*x^2]/(a*x^2)) - (b*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*Sqrt[c*x^2]*Lo
g[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx&=\frac{\sqrt{cx^2}\int \frac{1}{x^2(a+bx)}\,dx}{x}\\&=\frac{\sqrt{cx^2}\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{x}\\&=-\frac{\sqrt{cx^2}}{ax^2}-\frac{b\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\,\log(a+bx)}{a^2x}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 32, normalized size = 0.52

$$-\frac{c(-bx\log(a+bx)+a+bx\log(x))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

```
[Out] -((c*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 44, normalized size = 0.72

$$\sqrt{cx^2}\left(-\frac{b\log(x)}{a^2x}+\frac{b\log(a+bx)}{a^2x}-\frac{1}{ax^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*(-(1/(a*x^2)) - (b*Log[x])/(a^2*x) + (b*Log[a + b*x])/(a^2*x))
```

**fricas** [A] time = 0.77, size = 31, normalized size = 0.51

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*x^2)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple** [A] time = 0.01, size = 33, normalized size = 0.54

$$-\frac{\sqrt{c\,x^2}\;(bx\ln(x)-bx\ln(bx+a)+a)}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^3/(b*x+a),x)
[Out] -(c*x^2)^(1/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a)/x^2/a^2
```

**maxima** [A] time = 1.38, size = 37, normalized size = 0.61

$$\frac{b\sqrt{c}\,\log(bx+a)}{a^2}-\frac{b\sqrt{c}\,\log(x)}{a^2}-\frac{\sqrt{c}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="maxima")
[Out] b*sqrt(c)*log(b*x + a)/a^2 - b*sqrt(c)*log(x)/a^2 - sqrt(c)/(a*x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{x^3\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^3*(a + b*x)),x)
[Out] int((c*x^2)^(1/2)/(x^3*(a + b*x)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^3\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**3/(b*x+a),x)
[Out] Integral(sqrt(c*x**2)/(x**3*(a + b*x)), x)
```

## 3.817 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx$

**Optimal**. Leaf size=84

$$\frac{b^2\sqrt{cx^2}\,\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\,\log(a+bx)}{a^3x}+\frac{b\sqrt{cx^2}}{a^2x^2}-\frac{\sqrt{cx^2}}{2ax^3}$$

**Rubi** [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2\sqrt{cx^2}\,\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\,\log(a+bx)}{a^3x}+\frac{b\sqrt{cx^2}}{a^2x^2}-\frac{\sqrt{cx^2}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

```
[Out] -Sqrt[c*x^2]/(2*a*x^3) + (b*Sqrt[c*x^2])/(a^2*x^2) + (b^2*Sqrt[c*x^2]*Log[x
])/(a^3*x) - (b^2*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x^3(a+bx)}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{x}\\
&= -\frac{\sqrt{cx^2}}{2ax^3}+\frac{b\sqrt{cx^2}}{a^2x^2}+\frac{b^2\sqrt{cx^2}\,\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\,\log(a+bx)}{a^3x}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 53, normalized size = 0.63

$$\frac{\sqrt{cx^2}\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x])
)/(2*a^3*x^3)
```

**IntegrateAlgebraic** [A] time = 0.05, size = 58, normalized size = 0.69

$$\sqrt{cx^2}\left(\frac{b^2\log(x)}{a^3x}-\frac{b^2\log(a+bx)}{a^3x}+\frac{2bx-a}{2a^2x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*((-a + 2*b*x)/(2*a^2*x^3) + (b^2*Log[x])/(a^3*x) - (b^2*Log[a +
 b*x])/(a^3*x))
```

**fricas** [A] time = 1.15, size = 44, normalized size = 0.52

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*x^3)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple** [A] time = 0.01, size = 51, normalized size = 0.61

$$\frac{\sqrt{c\,x^{2}}\,\left(2b^{2}x^{2}\ln(x)-2b^{2}x^{2}\ln\left(bx+a\right)+2abx-a^{2}\right)}{2a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^4/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(2*b^2*x^2*ln(x)-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/a^3/x^3
```

**maxima** [A] time = 1.36, size = 52, normalized size = 0.62

$$-\frac{b^{2}\sqrt{c}\,\log\left(bx+a\right)}{a^{3}}+\frac{b^{2}\sqrt{c}\,\log(x)}{a^{3}}+\frac{2\,b\sqrt{c}\,x-a\sqrt{c}}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^2*sqrt(c)*log(b*x + a)/a^3 + b^2*sqrt(c)*log(x)/a^3 + 1/2*(2*b*sqrt(c)*x
 - a*sqrt(c))/(a^2*x^2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^{2}}}{x^{4}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^4*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^4*(a + b*x)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2}}}{x^{4}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**4/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**4*(a + b*x)), x)
```

## 3.818 $\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx$

**Optimal**. Leaf size=107

$$\frac{a^4c\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}$$

**Rubi [A]** time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}+\frac{a^4c\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] -((a^3*c*Sqrt[c*x^2])/b^4) + (a^2*c*x*Sqrt[c*x^2])/(2*b^3) - (a*c*x^2*Sqrt[
c*x^2])/(3*b^2) + (c*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c*Sqrt[c*x^2]*Log[a + b*
x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x\left(cx^2\right)^{3/2}}{a+bx}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^4}{a+bx}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}$$
$$= -\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}+\frac{a^4c\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

**Mathematica** [A] time = 0.01, size = 64, normalized size = 0.60

$$\frac{\left(cx^2\right)^{3/2}\left(12a^4\log(a+bx)+bx\left(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3\right)\right)}{12b^5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a^
4*Log[a + b*x]))/(12*b^5*x^3)
```

**IntegrateAlgebraic** [A] time = 0.05, size = 67, normalized size = 0.63

$$\left(cx^2\right)^{3/2}\left(\frac{a^4\log(a+bx)}{b^5x^3}+\frac{-12a^3+6a^2bx-4ab^2x^2+3b^3x^3}{12b^4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] (c*x^2)^(3/2)*((-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3)/(12*b^4*x^2)
 + (a^4*Log[a + b*x])/(b^5*x^3))
```

**fricas** [A] time = 1.04, size = 67, normalized size = 0.63

$$\frac{\left(3\,b^4cx^4-4\,ab^3cx^3+6\,a^2b^2cx^2-12\,a^3bcx+12\,a^4c\log\left(bx+a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*c*x^4 - 4*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 - 12*a^3*b*c*x + 12*a^4
*c*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

**giac [A]** time = 1.00, size = 81, normalized size = 0.76

$$\frac{1}{12}\,c^{\frac{3}{2}}\left(\frac{12\,a^4\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^5}-\frac{12\,a^4\log\left(|a|\right)\mathrm{sgn}(x)}{b^5}+\frac{3\,b^3x^4\mathrm{sgn}(x)-4\,ab^2x^3\mathrm{sgn}(x)+6\,a^2bx^2\mathrm{sgn}(x)-12\,a^3x\mathrm{sgn}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*c^(3/2)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn(
x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12*a
^3*x*sgn(x))/b^4)
```

**maple [A]** time = 0.01, size = 63, normalized size = 0.59

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln\left(bx+a\right)-12a^3bx\right)}{12b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/12*(c*x^2)^(3/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-12
*a^3*b*x)/x^3/b^5
```

**maxima [A]** time = 1.62, size = 124, normalized size = 1.16

$$\frac{(-1)^{\frac{2\,cx}{b}}\,a^4c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^4c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5}+\frac{\left(cx^2\right)^{\frac{3}{2}}x}{4\,b}+\frac{\sqrt{cx^2}\,a^2cx}{2\,b^3}-\frac{\left(cx^2\right)^{\frac{3}{2}}a}{3\,b^2}-\frac{\sqrt{cx^2}\,a^3c}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*c^(3/2)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*c^(3/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/4*(c*x^2)^(3/2)*x/b + 1/2*sqrt(c*x^2
)*a^2*c*x/b^3 - 1/3*(c*x^2)^(3/2)*a/b^2 - sqrt(c*x^2)*a^3*c/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(c\,x^2\right)^{3/2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(3/2))/(a + b*x),x)
```

```
[Out] int((x*(c*x^2)^(3/2))/(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(cx^2\right)^{\frac{3}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)/(a + b*x), x)
```

## 3.819 $\int \frac{(cx^2)^{3/2}}{a+bx}\,dx$

**Optimal**. Leaf size=84

$$-\frac{a^3c\sqrt{cx^2}\,\log(a+bx)}{b^4x}+\frac{a^2c\sqrt{cx^2}}{b^3}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}$$

**Rubi** [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}}{b^3}-\frac{a^3c\sqrt{cx^2}\,\log(a+bx)}{b^4x}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(a + b*x),x]

[Out] (a^2*c*Sqrt[c*x^2])/b^3 - (a*c*x*Sqrt[c*x^2])/(2*b^2) + (c*x^2*Sqrt[c*x^2])
/(3*b) - (a^3*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{a+bx}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^3}{a+bx}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}$$
$$= \frac{a^2c\sqrt{cx^2}}{b^3}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}-\frac{a^3c\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

**Mathematica** [A] time = 0.01, size = 53, normalized size = 0.63

$$\frac{(cx^2)^{3/2}\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6
*b^4*x^3)
```

**IntegrateAlgebraic** [A] time = 0.04, size = 57, normalized size = 0.68

$$(cx^2)^{3/2}\left(\frac{6a^2-3abx+2b^2x^2}{6b^3x^2}-\frac{a^3\log(a+bx)}{b^4x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] (c*x^2)^(3/2)*((6*a^2 - 3*a*b*x + 2*b^2*x^2)/(6*b^3*x^2) - (a^3*Log[a + b*x
])/(b^4*x^3))
```

**fricas** [A] time = 1.31, size = 55, normalized size = 0.65

$$\frac{\left(2\,b^3cx^3-3\,ab^2cx^2+6\,a^2bcx-6\,a^3c\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*c*x^3 - 3*a*b^2*c*x^2 + 6*a^2*b*c*x - 6*a^3*c*log(b*x + a))*sqrt
(c*x^2)/(b^4*x)
```

**giac [A]** time = 1.15, size = 69, normalized size = 0.82

$$-\frac{1}{6}\,c^{\frac{3}{2}}\left(\frac{6\,a^{3}\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^{4}}-\frac{6\,a^{3}\log\left(|a|\right)\mathrm{sgn}(x)}{b^{4}}-\frac{2\,b^{2}x^{3}\mathrm{sgn}(x)-3\,abx^{2}\mathrm{sgn}(x)+6\,a^{2}x\mathrm{sgn}(x)}{b^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*c^(3/2)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x)
/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

**maple [A]** time = 0.00, size = 52, normalized size = 0.62

$$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(-2b^{3}x^{3}+3a\,b^{2}x^{2}+6a^{3}\ln\left(bx+a\right)-6a^{2}bx\right)}{6b^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(3/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x^3/b
^4
```

**maxima [A]** time = 1.54, size = 109, normalized size = 1.30

$$-\frac{(-1)^{\frac{2\,cx}{b}}\,a^{3}c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^{4}}-\frac{(-1)^{\frac{2\,acx}{b}}\,a^{3}c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^{4}}-\frac{\sqrt{cx^{2}}\,acx}{2\,b^{2}}+\frac{\left(cx^{2}\right)^{\frac{3}{2}}}{3\,b}+\frac{\sqrt{cx^{2}}\,a^{2}c}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*c^(3/2)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*c^(3/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*c*x/b^2 + 1/3*(c*x^
2)^(3/2)/b + sqrt(c*x^2)*a^2*c/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^{2}\right)^{3/2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(a + b*x),x)
```

```
[Out] int((c*x^2)^(3/2)/(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(a + b*x), x)
```

## 3.820 $\int \frac{(cx^2)^{3/2}}{x(a+bx)} \, dx$

**Optimal**. Leaf size=61

$$\frac{a^2c\sqrt{cx^2}\,\log(a+bx)}{b^3x} - \frac{ac\sqrt{cx^2}}{b^2} + \frac{cx\sqrt{cx^2}}{2b}$$

**Rubi** [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}\,\log(a+bx)}{b^3x} - \frac{ac\sqrt{cx^2}}{b^2} + \frac{cx\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x*(a + b*x)),x]
```

```
[Out] -((a*c*Sqrt[c*x^2])/b^2) + (c*x*Sqrt[c*x^2])/(2*b) + (a^2*c*Sqrt[c*x^2]*Log
[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^2}{a+bx}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}$$
$$= -\frac{ac\sqrt{cx^2}}{b^2}+\frac{cx\sqrt{cx^2}}{2b}+\frac{a^2c\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

**Mathematica** [A] time = 0.00, size = 42, normalized size = 0.69

$$\frac{c^2x\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x*(a + b*x)),x]
[Out] (c^2*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 44, normalized size = 0.72

$$(cx^2)^{3/2}\left(\frac{a^2\log(a+bx)}{b^3x^3}+\frac{bx-2a}{2b^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x*(a + b*x)),x]
[Out] (c*x^2)^(3/2)*((-2*a + b*x)/(2*b^2*x^2) + (a^2*Log[a + b*x])/(b^3*x^3))
```

**fricas** [A] time = 1.42, size = 42, normalized size = 0.69

$$\frac{\left(b^2cx^2-2\,abcx+2\,a^2c\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*c*x^2 - 2*a*b*c*x + 2*a^2*c*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

**giac [A]** time = 1.12, size = 54, normalized size = 0.89

$$\frac{1}{2}\,c^{\frac{3}{2}}\left(\frac{2\,a^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3}-\frac{2\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^3}+\frac{bx^2\mathrm{sgn}(x)-2\,ax\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*c^(3/2)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)/
b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

**maple [A]** time = 0.00, size = 40, normalized size = 0.66

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(b^2x^2+2a^2\ln\left(bx+a\right)-2abx\right)}{2b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x^3
```

**maxima [A]** time = 1.48, size = 93, normalized size = 1.52

$$\frac{(-1)^{\frac{2\,cx}{b}}\,a^2c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,cx}{2\,b}-\frac{\sqrt{cx^2}\,ac}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^2*c^(3/2)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*c^(3/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*c*x/b - sqrt(c*x^2)*a*
c/b^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x*(a + b*x)), x)
```

## 3.821 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)} \, dx$

**Optimal**. Leaf size=40

$$\frac{c\sqrt{cx^2}}{b} - \frac{ac\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

**Rubi [A]** time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{c\sqrt{cx^2}}{b} - \frac{ac\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2])/b - (a*c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^2(a+bx)} \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x}{a+bx} \, dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx}{x} \\
&= \frac{c\sqrt{cx^2}}{b} - \frac{ac\sqrt{cx^2}\,\log(a+bx)}{b^2x}
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 30, normalized size = 0.75

$$\frac{c^2x(bx - a\log(a + bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c^2*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 33, normalized size = 0.82

$$\left(cx^2\right)^{3/2}\left(\frac{1}{bx^2} - \frac{a\log(a + bx)}{b^2x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c*x^2)^(3/2)*(1/(b*x^2) - (a*Log[a + b*x])/(b^2*x^3))
```

**fricas** [A] time = 0.84, size = 29, normalized size = 0.72

$$\frac{\left(bcx - ac\log\left(bx + a\right)\right)\sqrt{cx^2}}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c*x - a*c*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

**giac** [A] time = 1.00, size = 37, normalized size = 0.92

$$c^{\frac{3}{2}}\left(\frac{x\mathrm{sgn}(x)}{b} - \frac{a\log\left(|bx + a|\right)\mathrm{sgn}(x)}{b^2} + \frac{a\log\left(|a|\right)\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] c^(3/2)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x)
/b^2)
```

**maple** [A] time = 0.00, size = 29, normalized size = 0.72

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(a\ln\left(bx + a\right) - bx\right)}{b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^2/(b*x+a),x)
```

```
[Out] -(c*x^2)^(3/2)*(a*ln(b*x+a)-b*x)/b^2/x^3
```

**maxima [B]** time = 1.48, size = 75, normalized size = 1.88

$$-\frac{(-1)^{\frac{2\,cx}{b}}ac^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^2}-\frac{(-1)^{\frac{2\,acx}{b}}ac^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2}+\frac{\sqrt{cx^2}\,c}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a*c^(3/2)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*c^(3/2)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)*c/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{x^2\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^2*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^2*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{3}{2}}}{x^{2}\left(a+bx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**2/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**2*(a + b*x)), x)
```

## 3.822 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)} \, dx$

**Optimal**. Leaf size=23

$$\frac{c\sqrt{cx^2}\log(a+bx)}{bx}$$

**Rubi** [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 31}

$$\frac{c\sqrt{cx^2}\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(cx^2)^{3/2}}{x^3(a+bx)} \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{a+bx} \, dx}{x} \\ &= \frac{c\sqrt{cx^2}\log(a+bx)}{bx} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{(cx^2)^{3/2}\log(a+bx)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*Log[a + b*x])/(b*x^3)
```

**IntegrateAlgebraic** [A] time = 0.02, size = 22, normalized size = 0.96

$$\frac{(cx^2)^{3/2}\log(a+bx)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*Log[a + b*x])/(b*x^3)
```

**fricas** [A] time = 1.05, size = 21, normalized size = 0.91

$$\frac{\sqrt{cx^2}\,c\log(bx+a)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*c*log(b*x + a)/(b*x)
```

**giac** [A] time = 1.14, size = 28, normalized size = 1.22

$$c^{\frac{3}{2}}\left(\frac{\log(|bx+a|)\operatorname{sgn}(x)}{b}-\frac{\log(|a|)\operatorname{sgn}(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] c^(3/2)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.91

$$\frac{(c\,x^2)^{\frac{3}{2}}\ln(bx+a)}{b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^3/(b*x+a),x)
```

```
[Out] (c*x^2)^(3/2)/x^3*ln(b*x+a)/b
```

**maxima [A]** time = 1.36, size = 13, normalized size = 0.57

$$\frac{c^{\frac{3}{2}} \log (bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="maxima")
[Out] c^(3/2)*log(b*x + a)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^3\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^3*(a + b*x)),x)
[Out] int((c*x^2)^(3/2)/(x^3*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{3}{2}}}{x^{3} \left(a + bx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**3/(b*x+a),x)
[Out] Integral((c*x**2)**(3/2)/(x**3*(a + b*x)), x)
```

## 3.823 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)} dx$

**Optimal**. Leaf size=44

$$\frac{c\sqrt{cx^2}\,\log(x)}{ax}-\frac{c\sqrt{cx^2}\,\log(a+bx)}{ax}$$

**Rubi [A]** time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{c\sqrt{cx^2}\,\log(x)}{ax}-\frac{c\sqrt{cx^2}\,\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2]*Log[x])/(a*x) - (c*Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x}\,dx}{ax} - \frac{\left(bc\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}$$
$$= \frac{c\sqrt{cx^2}\,\log(x)}{ax} - \frac{c\sqrt{cx^2}\,\log(a+bx)}{ax}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 0.61

$$\frac{(cx^2)^{3/2}(\log(x)-\log(a+bx))}{ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
[Out] ((c*x^2)^(3/2)*(Log[x] - Log[a + b*x]))/(a*x^3)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 37, normalized size = 0.84

$$(cx^2)^{3/2}\left(\frac{\log(x)}{ax^3} - \frac{\log\left(a^2+abx\right)}{ax^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
[Out] (c*x^2)^(3/2)*(Log[x]/(a*x^3) - Log[a^2 + a*b*x]/(a*x^3))
```

**fricas** [A] time = 0.79, size = 66, normalized size = 1.50

$$\left[\frac{\sqrt{cx^2}\,c\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}\,c\arctan\left(\frac{\sqrt{cx^2}\,(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*c*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c*arctan(sqrt(c*x^2)*(2*b
*x + a)*sqrt(-c)/(a*c*x))/a]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple [A]** time = 0.00, size = 26, normalized size = 0.59

$$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(\ln(x)-\ln\left(bx+a\right)\right)}{a\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^4/(b*x+a),x)
```

```
[Out] (c*x^2)^(3/2)*(ln(x)-ln(b*x+a))/a/x^3
```

**maxima [A]** time = 1.38, size = 24, normalized size = 0.55

$$-\frac{c^{\frac{3}{2}}\log\left(bx+a\right)}{a}+\frac{c^{\frac{3}{2}}\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^(3/2)*log(b*x + a)/a + c^(3/2)*log(x)/a
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^4\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^4*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^4*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^4\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**4/(b*x+a),x)
[Out] Integral((c*x**2)**(3/2)/(x**4*(a + b*x)), x)
```

## 3.824 $\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx$

**Optimal**. Leaf size=64

$$-\frac{bc\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c\sqrt{cx^2}}{ax^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{bc\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(a*x^2)) - (b*c*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c*Sqrt[c
*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{ax^2} - \frac{bc\sqrt{cx^2}\,\log(x)}{a^2x} + \frac{bc\sqrt{cx^2}\,\log(a+bx)}{a^2x}$$

**Mathematica [A]** time = 0.01, size = 34, normalized size = 0.53

$$-\frac{c^2(-bx\log(a+bx)+a+bx\log(x))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
[Out] -((c^2*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

**IntegrateAlgebraic [A]** time = 0.04, size = 44, normalized size = 0.69

$$(cx^2)^{3/2}\left(-\frac{b\log(x)}{a^2x^3} + \frac{b\log(a+bx)}{a^2x^3} - \frac{1}{ax^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
[Out] (c*x^2)^(3/2)*(-(1/(a*x^4)) - (b*Log[x])/(a^2*x^3) + (b*Log[a + b*x])/(a^2*
x^3))
```

**fricas [A]** time = 1.10, size = 33, normalized size = 0.52

$$\frac{\left(bcx\log\left(\frac{bx+a}{x}\right) - ac\right)\sqrt{cx^2}}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="fricas")
[Out] (b*c*x*log((b*x + a)/x) - a*c)*sqrt(c*x^2)/(a^2*x^2)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple** [A] time = 0.01, size = 33, normalized size = 0.52

$$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(bx\ln(x)-bx\ln\left(bx+a\right)+a\right)}{a^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^5/(b*x+a),x)
```

```
[Out] -(c*x^2)^(3/2)*(b*x*ln(x)-b*x*ln(b*x+a)+a)/x^4/a^2
```

**maxima** [A] time = 1.36, size = 37, normalized size = 0.58

$$\frac{bc^{\frac{3}{2}}\log\left(bx+a\right)}{a^{2}}-\frac{bc^{\frac{3}{2}}\log(x)}{a^{2}}-\frac{c^{\frac{3}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(3/2)*log(b*x + a)/a^2 - b*c^(3/2)*log(x)/a^2 - c^(3/2)/(a*x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(c\,x^{2}\right)^{3/2}}{x^{5}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^5*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^5*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^5\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**5/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**5*(a + b*x)), x)
```

## 3.825 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx$

**Optimal**. Leaf size=88

$$\frac{b^2c\sqrt{cx^2}\,\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\,\log(a+bx)}{a^3x}+\frac{bc\sqrt{cx^2}}{a^2x^2}-\frac{c\sqrt{cx^2}}{2ax^3}$$

**Rubi** [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2c\sqrt{cx^2}\,\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\,\log(a+bx)}{a^3x}+\frac{bc\sqrt{cx^2}}{a^2x^2}-\frac{c\sqrt{cx^2}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

```
[Out] -(c*Sqrt[c*x^2])/(2*a*x^3) + (b*c*Sqrt[c*x^2])/(a^2*x^2) + (b^2*c*Sqrt[c*x^
2]*Log[x])/(a^3*x) - (b^2*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\left(cx^2\right)^{3/2}}{x^6(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^3(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{2ax^3}+\frac{bc\sqrt{cx^2}}{a^2x^2}+\frac{b^2c\sqrt{cx^2}\,\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\,\log(a+bx)}{a^3x}$$

**Mathematica [A]** time = 0.01, size = 53, normalized size = 0.60

$$\frac{\left(cx^2\right)^{3/2}\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x
]))/(2*a^3*x^5)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 58, normalized size = 0.66

$$\left(cx^2\right)^{3/2}\left(\frac{b^2\log(x)}{a^3x^3}-\frac{b^2\log(a+bx)}{a^3x^3}+\frac{2bx-a}{2a^2x^5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

```
[Out] (c*x^2)^(3/2)*((-a + 2*b*x)/(2*a^2*x^5) + (b^2*Log[x])/(a^3*x^3) - (b^2*Log
[a + b*x])/(a^3*x^3))
```

**fricas [A]** time = 0.91, size = 47, normalized size = 0.53

$$\frac{\left(2\,b^2cx^2\log\left(\frac{x}{bx+a}\right)+2\,abcx-a^2c\right)\sqrt{cx^2}}{2\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*c*x^2*log(x/(b*x + a)) + 2*a*b*c*x - a^2*c)*sqrt(c*x^2)/(a^3*x^3
)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple [A]** time = 0.00, size = 51, normalized size = 0.58

$$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(2 b^{2} x^{2} \ln \left(x \right)-2 b^{2} x^{2} \ln \left(b x +a \right)+2 a b x -a^{2}\right)}{2 a^{3} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^6/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x-a^2)/x^5/a^3
```

**maxima [A]** time = 1.45, size = 52, normalized size = 0.59

$$-\frac{b^{2} c^{\frac{3}{2}} \log \left(b x + a\right)}{a^{3}} + \frac{b^{2} c^{\frac{3}{2}} \log\left(x\right)}{a^{3}} + \frac{2 \, b c^{\frac{3}{2}} x - a c^{\frac{3}{2}}}{2 \, a^{2} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^2*c^(3/2)*log(b*x + a)/a^3 + b^2*c^(3/2)*log(x)/a^3 + 1/2*(2*b*c^(3/2)*x
 - a*c^(3/2))/(a^2*x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{x^6\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^6*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^6*(a + b*x)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^6\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**6/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**6*(a + b*x)), x)
```

## 3.826 $\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx$

**Optimal**. Leaf size=112

$$-\frac{b^3c\sqrt{cx^2}\,\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\,\log(a+bx)}{a^4x}-\frac{b^2c\sqrt{cx^2}}{a^3x^2}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{c\sqrt{cx^2}}{3ax^4}$$

**Rubi [A]** time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{b^2c\sqrt{cx^2}}{a^3x^2}-\frac{b^3c\sqrt{cx^2}\,\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\,\log(a+bx)}{a^4x}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{c\sqrt{cx^2}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

```
[Out] -(c*Sqrt[c*x^2])/(3*a*x^4) + (b*c*Sqrt[c*x^2])/(2*a^2*x^3) - (b^2*c*Sqrt[c*
x^2])/(a^3*x^2) - (b^3*c*Sqrt[c*x^2]*Log[x])/(a^4*x) + (b^3*c*Sqrt[c*x^2]*L
og[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\left(cx^2\right)^{3/2}}{x^7(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^4(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{3ax^4}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{b^2c\sqrt{cx^2}}{a^3x^2}-\frac{b^3c\sqrt{cx^2}\,\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\,\log(a+bx)}{a^4x}$$

**Mathematica** [A] time = 0.03, size = 65, normalized size = 0.58

$$-\frac{\left(cx^2\right)^{3/2}\left(a\left(2a^2-3abx+6b^2x^2\right)-6b^3x^3\log(a+bx)+6b^3x^3\log(x)\right)}{6a^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

```
[Out] -1/6*((c*x^2)^(3/2)*(a*(2*a^2 - 3*a*b*x + 6*b^2*x^2) + 6*b^3*x^3*Log[x] - 6
*b^3*x^3*Log[a + b*x]))/(a^4*x^6)
```

**IntegrateAlgebraic** [A] time = 0.07, size = 69, normalized size = 0.62

$$\left(cx^2\right)^{3/2}\left(-\frac{b^3\log(x)}{a^4x^3}+\frac{b^3\log(a+bx)}{a^4x^3}+\frac{-2a^2+3abx-6b^2x^2}{6a^3x^6}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

```
[Out] (c*x^2)^(3/2)*((-2*a^2 + 3*a*b*x - 6*b^2*x^2)/(6*a^3*x^6) - (b^3*Log[x])/(a
^4*x^3) + (b^3*Log[a + b*x])/(a^4*x^3))
```

**fricas** [A] time = 1.01, size = 59, normalized size = 0.53

$$\frac{\left(6\,b^3cx^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2cx^2+3\,a^2bcx-2\,a^3c\right)\sqrt{cx^2}}{6\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*c*x^3*log((b*x + a)/x) - 6*a*b^2*c*x^2 + 3*a^2*b*c*x - 2*a^3*c)*
sqrt(c*x^2)/(a^4*x^4)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple [A]** time = 0.01, size = 62, normalized size = 0.55

$$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(6 b^{3} x^{3} \ln \left(x \right)-6 b^{3} x^{3} \ln \left(b x +a \right)+6 a \,b^{2} x^{2}-3 a^{2} b x +2 a^{3}\right)}{6 a^{4} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^7/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(3/2)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*a^2*b
*x+2*a^3)/x^6/a^4
```

**maxima [A]** time = 1.45, size = 66, normalized size = 0.59

$$\frac{b^{3} c^{\frac{3}{2}} \log \left(b x + a\right)}{a^{4}} - \frac{b^{3} c^{\frac{3}{2}} \log\left(x\right)}{a^{4}} - \frac{6 \, b^{2} c^{\frac{3}{2}} x^{2} - 3 \, a b c^{\frac{3}{2}} x + 2 \, a^{2} c^{\frac{3}{2}}}{6 \, a^{3} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*c^(3/2)*log(b*x + a)/a^4 - b^3*c^(3/2)*log(x)/a^4 - 1/6*(6*b^2*c^(3/2)*
x^2 - 3*a*b*c^(3/2)*x + 2*a^2*c^(3/2))/(a^3*x^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{x^7\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^7*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^7*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x^7 (a + bx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**7/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**7*(a + b*x)), x)
```

## 3.827 $\int \frac{(cx^2)^{5/2}}{a+bx} dx$

**Optimal**. Leaf size=142

$$-\frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}+\frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}$$

**Rubi** [A] time = 0.04, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(a + b*x),x]
```

```
[Out] (a^4*c^2*Sqrt[c*x^2])/b^5 - (a^3*c^2*x*Sqrt[c*x^2])/(2*b^4) + (a^2*c^2*x^2*
Sqrt[c*x^2])/(3*b^3) - (a*c^2*x^3*Sqrt[c*x^2])/(4*b^2) + (c^2*x^4*Sqrt[c*x^
2])/(5*b) - (a^5*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^6*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{a+bx}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^5}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right)dx}{x}$$
$$= \frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}-\frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}$$

**Mathematica** [A] time = 0.02, size = 76, normalized size = 0.54

$$\frac{c^3x\left(bx\left(60a^4-30a^3bx+20a^2b^2x^2-15ab^3x^3+12b^4x^4\right)-60a^5\log(a+bx)\right)}{60b^6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(a + b*x),x]
```

```
[Out] (c^3*x*(b*x*(60*a^4 - 30*a^3*b*x + 20*a^2*b^2*x^2 - 15*a*b^3*x^3 + 12*b^4*x
^4) - 60*a^5*Log[a + b*x]))/(60*b^6*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.06, size = 79, normalized size = 0.56

$$\left(cx^2\right)^{5/2}\left(\frac{60a^4-30a^3bx+20a^2b^2x^2-15ab^3x^3+12b^4x^4}{60b^5x^4}-\frac{a^5\log(a+bx)}{b^6x^5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(a + b*x),x]
```

```
[Out] (c*x^2)^(5/2)*((60*a^4 - 30*a^3*b*x + 20*a^2*b^2*x^2 - 15*a*b^3*x^3 + 12*b^
4*x^4)/(60*b^5*x^4) - (a^5*Log[a + b*x])/(b^6*x^5))
```

**fricas** [A] time = 1.34, size = 91, normalized size = 0.64

$$\frac{\left(12\,b^5c^2x^5-15\,ab^4c^2x^4+20\,a^2b^3c^2x^3-30\,a^3b^2c^2x^2+60\,a^4bc^2x-60\,a^5c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{60\,b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*c^2*x^5 - 15*a*b^4*c^2*x^4 + 20*a^2*b^3*c^2*x^3 - 30*a^3*b^2*c
^2*x^2 + 60*a^4*b*c^2*x - 60*a^5*c^2*log(b*x + a))*sqrt(c*x^2)/(b^6*x)
```

**giac** [A] time = 1.18, size = 116, normalized size = 0.82

$$-\frac{1}{60}\left(\frac{60\,a^5c^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^6}-\frac{60\,a^5c^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^6}-\frac{12\,b^4c^2x^5\mathrm{sgn}(x)-15\,ab^3c^2x^4\mathrm{sgn}(x)+20\,a^2b^2c^2x^3\mathrm{sgn}(x)-30\,a^3bc^2x^2\mathrm{sgn}(x)+60\,a^4c^2x\mathrm{sgn}(x)}{b^5}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/60*(60*a^5*c^2*log(abs(b*x + a))*sgn(x)/b^6 - 60*a^5*c^2*log(abs(a))*sgn
(x)/b^6 - (12*b^4*c^2*x^5*sgn(x) - 15*a*b^3*c^2*x^4*sgn(x) + 20*a^2*b^2*c^2
*x^3*sgn(x) - 30*a^3*b*c^2*x^2*sgn(x) + 60*a^4*c^2*x*sgn(x))/b^5)*sqrt(c)
```

**maple** [A] time = 0.01, size = 74, normalized size = 0.52

$$-\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(-12b^5x^5+15a\,b^4x^4-20a^2b^3x^3+30a^3b^2x^2+60a^5\ln\left(bx+a\right)-60a^4bx\right)}{60b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(b*x+a),x)
```

```
[Out] -1/60*(c*x^2)^(5/2)*(-12*b^5*x^5+15*a*b^4*x^4-20*a^2*b^3*x^3+30*a^3*b^2*x^2
+60*a^5*ln(b*x+a)-60*a^4*b*x)/x^5/b^6
```

**maxima** [A] time = 1.58, size = 146, normalized size = 1.03

$$-\frac{(-1)^{\frac{2cx}{b}}\,a^5c^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^6}-\frac{(-1)^{\frac{2acx}{b}}\,a^5c^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^6}-\frac{\left(cx^2\right)^{\frac{3}{2}}acx}{4\,b^2}-\frac{\sqrt{cx^2}\,a^3c^2x}{2\,b^4}+\frac{\left(cx^2\right)^{\frac{5}{2}}}{5\,b}+\frac{\left(cx^2\right)^{\frac{3}{2}}a^2c}{3\,b^3}+\frac{\sqrt{cx^2}\,a^4c^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^5*c^(5/2)*log(2*c*x/b)/b^6 - (-1)^(2*a*c*x/b)*a^5*c^(5/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^6 - 1/4*(c*x^2)^(3/2)*a*c*x/b^2 - 1/2*sqr
t(c*x^2)*a^3*c^2*x/b^4 + 1/5*(c*x^2)^(5/2)/b + 1/3*(c*x^2)^(3/2)*a^2*c/b^3
+ sqrt(c*x^2)*a^4*c^2/b^5
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c\,x^2\right)^{5/2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(a + b*x),x)
```

```
[Out] int((c*x^2)^(5/2)/(a + b*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}}}{a + bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(a + b*x), x)
```

## 3.828 $\int \frac{(cx^2)^{5/2}}{x(a+bx)} dx$

**Optimal**. Leaf size=117

$$\frac{a^4c^2\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}$$

**Rubi [A]** time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}+\frac{a^4c^2\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```

```
[Out] -((a^3*c^2*Sqrt[c*x^2])/b^4) + (a^2*c^2*x*Sqrt[c*x^2])/(2*b^3) - (a*c^2*x^2
*Sqrt[c*x^2])/(3*b^2) + (c^2*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c^2*Sqrt[c*x^2]*
Log[a + b*x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^4}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}$$
$$= -\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}+\frac{a^4c^2\sqrt{cx^2}\log(a+bx)}{b^5x}$$

**Mathematica [A]** time = 0.01, size = 65, normalized size = 0.56

$$\frac{c\left(cx^2\right)^{3/2}\left(12a^4\log(a+bx)+bx\left(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3\right)\right)}{12b^5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```

```
[Out] (c*(c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*
a^4*Log[a + b*x]))/(12*b^5*x^3)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 67, normalized size = 0.57

$$\left(cx^2\right)^{5/2}\left(\frac{a^4\log(a+bx)}{b^5x^5}+\frac{-12a^3+6a^2bx-4ab^2x^2+3b^3x^3}{12b^4x^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```

```
[Out] (c*x^2)^(5/2)*((-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3)/(12*b^4*x^4)
 + (a^4*Log[a + b*x])/(b^5*x^5))
```

**fricas [A]** time = 1.19, size = 77, normalized size = 0.66

$$\frac{\left(3\,b^4c^2x^4-4\,ab^3c^2x^3+6\,a^2b^2c^2x^2-12\,a^3bc^2x+12\,a^4c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*c^2*x^4 - 4*a*b^3*c^2*x^3 + 6*a^2*b^2*c^2*x^2 - 12*a^3*b*c^2*x
+ 12*a^4*c^2*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

**giac [A]** time = 1.01, size = 99, normalized size = 0.85

$$\frac{1}{12}\left(\frac{12\,a^4c^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^5}-\frac{12\,a^4c^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^5}+\frac{3\,b^3c^2x^4\mathrm{sgn}(x)-4\,ab^2c^2x^3\mathrm{sgn}(x)+6\,a^2bc^2x^2\mathrm{sgn}(x)-12\,a^3c^2x\mathrm{sgn}(x)}{b^4}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(12*a^4*c^2*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*c^2*log(abs(a))*sgn(
x)/b^5 + (3*b^3*c^2*x^4*sgn(x) - 4*a*b^2*c^2*x^3*sgn(x) + 6*a^2*b*c^2*x^2*s
gn(x) - 12*a^3*c^2*x*sgn(x))/b^4)*sqrt(c)
```

**maple [A]** time = 0.01, size = 63, normalized size = 0.54

$$\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln\left(bx+a\right)-12a^3bx\right)}{12b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x/(b*x+a),x)
```

```
[Out] 1/12*(c*x^2)^(5/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-12
*a^3*b*x)/b^5/x^5
```

**maxima [A]** time = 1.60, size = 130, normalized size = 1.11

$$\frac{(-1)^{\frac{2cx}{b}}\,a^4c^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2acx}{b}}\,a^4c^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^5}+\frac{\left(cx^2\right)^{\frac{3}{2}}cx}{4b}+\frac{\sqrt{cx^2}\,a^2c^2x}{2b^3}-\frac{\left(cx^2\right)^{\frac{3}{2}}ac}{3b^2}-\frac{\sqrt{cx^2}\,a^3c^2}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*c^(5/2)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*c^(5/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/4*(c*x^2)^(3/2)*c*x/b + 1/2*sqrt(c*x
^2)*a^2*c^2*x/b^3 - 1/3*(c*x^2)^(3/2)*a*c/b^2 - sqrt(c*x^2)*a^3*c^2/b^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x*(a + b*x)), x)
```

## 3.829 $\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx$

**Optimal**. Leaf size=92

$$-\frac{a^3c^2\sqrt{cx^2}\,\log(a+bx)}{b^4x}+\frac{a^2c^2\sqrt{cx^2}}{b^3}-\frac{ac^2x\sqrt{cx^2}}{2b^2}+\frac{c^2x^2\sqrt{cx^2}}{3b}$$

**Rubi** [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}}{b^3}-\frac{a^3c^2\sqrt{cx^2}\,\log(a+bx)}{b^4x}-\frac{ac^2x\sqrt{cx^2}}{2b^2}+\frac{c^2x^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

```
[Out] (a^2*c^2*Sqrt[c*x^2])/b^3 - (a*c^2*x*Sqrt[c*x^2])/(2*b^2) + (c^2*x^2*Sqrt[c
*x^2])/(3*b) - (a^3*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^3}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^2}{b^3} - \frac{ax}{b^2} + \frac{x^2}{b} - \frac{a^3}{b^3(a+bx)}\right)dx}{x}$$
$$= \frac{a^2c^2\sqrt{cx^2}}{b^3} - \frac{ac^2x\sqrt{cx^2}}{2b^2} + \frac{c^2x^2\sqrt{cx^2}}{3b} - \frac{a^3c^2\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

**Mathematica** [A] time = 0.01, size = 54, normalized size = 0.59

$$\frac{c\,(cx^2)^{3/2}\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c*(c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/
(6*b^4*x^3)
```

**IntegrateAlgebraic** [A] time = 0.05, size = 57, normalized size = 0.62

$$(cx^2)^{5/2}\left(\frac{6a^2-3abx+2b^2x^2}{6b^3x^4} - \frac{a^3\log(a+bx)}{b^4x^5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c*x^2)^(5/2)*((6*a^2 - 3*a*b*x + 2*b^2*x^2)/(6*b^3*x^4) - (a^3*Log[a + b*x
])/(b^4*x^5))
```

**fricas** [A] time = 0.87, size = 63, normalized size = 0.68

$$\frac{\left(2\,b^3c^2x^3 - 3\,ab^2c^2x^2 + 6\,a^2bc^2x - 6\,a^3c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*c^2*x^3 - 3*a*b^2*c^2*x^2 + 6*a^2*b*c^2*x - 6*a^3*c^2*log(b*x +
a))*sqrt(c*x^2)/(b^4*x)
```

**giac [A]** time = 0.96, size = 84, normalized size = 0.91

$$-\frac{1}{6}\left(\frac{6\,a^3c^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^4}-\frac{6\,a^3c^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^4}-\frac{2\,b^2c^2x^3\mathrm{sgn}(x)-3\,abc^2x^2\mathrm{sgn}(x)+6\,a^2c^2x\mathrm{sgn}(x)}{b^3}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*(6*a^3*c^2*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*c^2*log(abs(a))*sgn(x)
/b^4 - (2*b^2*c^2*x^3*sgn(x) - 3*a*b*c^2*x^2*sgn(x) + 6*a^2*c^2*x*sgn(x))/b
^3)*sqrt(c)
```

**maple [A]** time = 0.00, size = 52, normalized size = 0.57

$$-\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln\left(bx+a\right)-6a^2bx\right)}{6b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^2/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(5/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x^5/b
^4
```

**maxima [A]** time = 1.56, size = 114, normalized size = 1.24

$$-\frac{\left(-1\right)^{\frac{2\,cx}{b}}a^3c^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^4}-\frac{\left(-1\right)^{\frac{2\,acx}{b}}a^3c^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4}-\frac{\sqrt{cx^2}\,ac^2x}{2\,b^2}+\frac{\left(cx^2\right)^{\frac{3}{2}}c}{3\,b}+\frac{\sqrt{cx^2}\,a^2c^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*c^(5/2)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*c^(5/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*c^2*x/b^2 + 1/3*(c*
x^2)^(3/2)*c/b + sqrt(c*x^2)*a^2*c^2/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x^2\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^2*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^2*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^2\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**2/(b*x+a),x)
[Out] Integral((c*x**2)**(5/2)/(x**2*(a + b*x)), x)
```

## 3.830 $\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx$

**Optimal**. Leaf size=67

$$\frac{a^2c^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}-\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}$$

**Rubi [A]** time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}-\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
```

```
[Out] -((a*c^2*Sqrt[c*x^2])/b^2) + (c^2*x*Sqrt[c*x^2])/(2*b) + (a^2*c^2*Sqrt[c*x^
2]*Log[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^2}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}$$
$$= -\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}+\frac{a^2c^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

**Mathematica** [A] time = 0.01, size = 42, normalized size = 0.63

$$\frac{c^3x\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
[Out] (c^3*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 44, normalized size = 0.66

$$(cx^2)^{5/2}\left(\frac{a^2\log(a+bx)}{b^3x^5}+\frac{bx-2a}{2b^2x^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
[Out] (c*x^2)^(5/2)*((-2*a + b*x)/(2*b^2*x^4) + (a^2*Log[a + b*x])/(b^3*x^5))
```

**fricas** [A] time = 1.23, size = 48, normalized size = 0.72

$$\frac{\left(b^2c^2x^2-2\,abc^2x+2\,a^2c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*c^2*x^2 - 2*a*b*c^2*x + 2*a^2*c^2*log(b*x + a))*sqrt(c*x^2)/(b^3*x
)
```

**giac [A]** time = 1.13, size = 66, normalized size = 0.99

$$\frac{1}{2}\left(\frac{2\,a^2c^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3}-\frac{2\,a^2c^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^3}+\frac{bc^2x^2\mathrm{sgn}(x)-2\,ac^2x\mathrm{sgn}(x)}{b^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*c^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*c^2*log(abs(a))*sgn(x)/
b^3 + (b*c^2*x^2*sgn(x) - 2*a*c^2*x*sgn(x))/b^2)*sqrt(c)
```

**maple [A]** time = 0.01, size = 40, normalized size = 0.60

$$\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(b^2x^2+2a^2\ln\left(bx+a\right)-2abx\right)}{2b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^3/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(5/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/x^5/b^3
```

**maxima [A]** time = 1.50, size = 97, normalized size = 1.45

$$\frac{(-1)^{\frac{2\,cx}{b}}\,a^2c^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2c^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,c^2x}{2\,b}-\frac{\sqrt{cx^2}\,ac^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^2*c^(5/2)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*c^(5/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*c^2*x/b - sqrt(c*x^2)*
a*c^2/b^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c\,x^2\right)^{5/2}}{x^3\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^3*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^3*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^3\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**3/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**3*(a + b*x)), x)
```

## 3.831 $\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx$

**Optimal**. Leaf size=44

$$\frac{c^2\sqrt{cx^2}}{b}-\frac{ac^2\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

**Rubi [A]** time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{c^2\sqrt{cx^2}}{b}-\frac{ac^2\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2])/b - (a*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}}{x^4(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x}{a+bx}\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}\\ &= \frac{c^2\sqrt{cx^2}}{b}-\frac{ac^2\sqrt{cx^2}\,\log(a+bx)}{b^2x}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 30, normalized size = 0.68

$$\frac{c^3x(bx - a\log(a + bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
[Out] (c^3*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 33, normalized size = 0.75

$$\left(cx^2\right)^{5/2}\left(\frac{1}{bx^4} - \frac{a\log(a + bx)}{b^2x^5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
[Out] (c*x^2)^(5/2)*(1/(b*x^4) - (a*Log[a + b*x])/(b^2*x^5))
```

**fricas** [A] time = 0.68, size = 33, normalized size = 0.75

$$\frac{\left(bc^2x - ac^2\log\left(bx + a\right)\right)\sqrt{cx^2}}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="fricas")
[Out] (b*c^2*x - a*c^2*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

**giac** [A] time = 1.04, size = 46, normalized size = 1.05

$$\left(\frac{c^2x\mathrm{sgn}(x)}{b} - \frac{ac^2\log\left(|bx + a|\right)\mathrm{sgn}(x)}{b^2} + \frac{ac^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="giac")
[Out] (c^2*x*sgn(x)/b - a*c^2*log(abs(b*x + a))*sgn(x)/b^2 + a*c^2*log(abs(a))*sg
n(x)/b^2)*sqrt(c)
```

**maple** [A] time = 0.00, size = 29, normalized size = 0.66

$$-\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(a\ln\left(bx + a\right) - bx\right)}{b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^4/(b*x+a),x)
```

```
[Out] -(c*x^2)^(5/2)*(a*ln(b*x+a)-b*x)/x^5/b^2
```

**maxima [A]** time = 1.51, size = 77, normalized size = 1.75

$$-\frac{(-1)^{\frac{2cx}{b}}ac^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^2}-\frac{(-1)^{\frac{2acx}{b}}ac^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2}+\frac{\sqrt{cx^2}\,c^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a*c^(5/2)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*c^(5/2)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)*c^2/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x^4\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^4*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^4*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^4\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**4/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**4*(a + b*x)), x)
```

## 3.832 $\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx$

**Optimal**. Leaf size=25

$$\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{bx}$$

**Rubi** [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 31}

$$\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{x}\\ &= \frac{c^2\sqrt{cx^2}\,\log(a+bx)}{bx}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{(cx^2)^{5/2}\log(a+bx)}{bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(5/2)*Log[a + b*x])/(b*x^5)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 22, normalized size = 0.88

$$\frac{(cx^2)^{5/2}\log(a+bx)}{bx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(5/2)*Log[a + b*x])/(b*x^5)
```

**fricas [A]** time = 1.36, size = 23, normalized size = 0.92

$$\frac{\sqrt{cx^2}\,c^2\log(bx+a)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*c^2*log(b*x + a)/(b*x)
```

**giac [A]** time = 1.10, size = 34, normalized size = 1.36

$$\left(\frac{c^2\log(|bx+a|)\operatorname{sgn}(x)}{b}-\frac{c^2\log(|a|)\operatorname{sgn}(x)}{b}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] (c^2*log(abs(b*x + a))*sgn(x)/b - c^2*log(abs(a))*sgn(x)/b)*sqrt(c)
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.84

$$\frac{(c\,x^2)^{\frac{5}{2}}\ln(bx+a)}{b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^5/(b*x+a),x)
```

```
[Out] (c*x^2)^(5/2)/x^5*ln(b*x+a)/b
```

**maxima [A]** time = 1.36, size = 13, normalized size = 0.52

$$\frac{c^{\frac{5}{2}}\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] c^(5/2)*log(b*x + a)/b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x^5\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^5*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^5*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{5}{2}}}{x^{5}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**5/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**5*(a + b*x)), x)
```

## 3.833 $\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx$

**Optimal**. Leaf size=48

$$\frac{c^2\sqrt{cx^2}\,\log(x)}{ax}-\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{ax}$$

**Rubi [A]** time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{c^2\sqrt{cx^2}\,\log(x)}{ax}-\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2]*Log[x])/(a*x) - (c^2*Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x}\,dx}{ax} - \frac{\left(bc^2\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}$$
$$= \frac{c^2\sqrt{cx^2}\,\log(x)}{ax} - \frac{c^2\sqrt{cx^2}\,\log(a+bx)}{ax}$$

**Mathematica** [A] time = 0.01, size = 28, normalized size = 0.58

$$\frac{c^3x(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
[Out] (c^3*x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 37, normalized size = 0.77

$$\left(cx^2\right)^{5/2}\left(\frac{\log(x)}{ax^5} - \frac{\log\left(a^2+abx\right)}{ax^5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
[Out] (c*x^2)^(5/2)*(Log[x]/(a*x^5) - Log[a^2 + a*b*x]/(a*x^5))
```

**fricas** [A] time = 1.54, size = 70, normalized size = 1.46

$$\left[\frac{\sqrt{cx^2}\,c^2\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{cx^2}\,(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*c^2*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c^2*arctan(sqrt(c*x^2)*
(2*b*x + a)*sqrt(-c)/(a*c*x))/a]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.56

$$-\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \left(-\ln(x)+\ln\left(bx+a\right)\right)}{a\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^6/(b*x+a),x)
```

```
[Out] -(c*x^2)^(5/2)*(-ln(x)+ln(b*x+a))/x^5/a
```

**maxima [A]** time = 1.35, size = 24, normalized size = 0.50

$$-\frac{c^{\frac{5}{2}} \log\left(bx+a\right)}{a}+\frac{c^{\frac{5}{2}} \log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^(5/2)*log(b*x + a)/a + c^(5/2)*log(x)/a
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x^6\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^6*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^6*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^6\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**6/(b*x+a),x)
[Out] Integral((c*x**2)**(5/2)/(x**6*(a + b*x)), x)
```

## 3.834 $\int \frac{(cx^2)^{5/2}}{x^7(a+bx)} dx$

**Optimal**. Leaf size=70

$$-\frac{bc^2\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c^2\sqrt{cx^2}}{ax^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{bc^2\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c^2\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
```

```
[Out] -((c^2*Sqrt[c*x^2])/(a*x^2)) - (b*c^2*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c^2*
Sqrt[c*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\left(cx^2\right)^{5/2}}{x^7(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{x}$$
$$= -\frac{c^2\sqrt{cx^2}}{ax^2}-\frac{bc^2\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\,\log(a+bx)}{a^2x}$$

**Mathematica** [A] time = 0.01, size = 34, normalized size = 0.49

$$-\frac{c^3(-bx\log(a+bx)+a+bx\log(x))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
[Out] -((c^3*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 44, normalized size = 0.63

$$\left(cx^2\right)^{5/2}\left(-\frac{b\log(x)}{a^2x^5}+\frac{b\log(a+bx)}{a^2x^5}-\frac{1}{ax^6}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
[Out] (c*x^2)^(5/2)*(-(1/(a*x^6)) - (b*Log[x])/(a^2*x^5) + (b*Log[a + b*x])/(a^2*
x^5))
```

**fricas** [A] time = 0.94, size = 37, normalized size = 0.53

$$\frac{\left(bc^2x\log\left(\frac{bx+a}{x}\right)-ac^2\right)\sqrt{cx^2}}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="fricas")
[Out] (b*c^2*x*log((b*x + a)/x) - a*c^2)*sqrt(c*x^2)/(a^2*x^2)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple [A]** time = 0.01, size = 33, normalized size = 0.47

$$-\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \left(bx\ln(x) - bx\ln\left(bx+a\right) + a\right)}{a^{2} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^7/(b*x+a),x)
```

```
[Out] -(c*x^2)^(5/2)*(b*x*ln(x)-b*x*ln(b*x+a)+a)/x^6/a^2
```

**maxima [A]** time = 1.39, size = 37, normalized size = 0.53

$$\frac{bc^{\frac{5}{2}}\log\left(bx+a\right)}{a^{2}} - \frac{bc^{\frac{5}{2}}\log(x)}{a^{2}} - \frac{c^{\frac{5}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(5/2)*log(b*x + a)/a^2 - b*c^(5/2)*log(x)/a^2 - c^(5/2)/(a*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^{2}\right)^{5/2}}{x^{7}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^7*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^7*(a + b*x)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^7\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**7/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**7*(a + b*x)), x)
```

## 3.835 $\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx$

**Optimal**. Leaf size=83

$$-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}+\frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (a^2*x^2)/(b^3*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*Sqrt[c*x^2]) + x^4/(3*b*Sqrt[c
*x^2]) - (a^3*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx = \frac{x\int \frac{x^3}{a+bx}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 51, normalized size = 0.61

$$\frac{x\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*Sqrt[c*
x^2])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 60, normalized size = 0.72

$$\sqrt{cx^2}\left(\frac{6a^2-3abx+2b^2x^2}{6b^3c}-\frac{a^3\log(a+bx)}{b^4cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*((6*a^2 - 3*a*b*x + 2*b^2*x^2)/(6*b^3*c) - (a^3*Log[a + b*x])/(
b^4*c*x))
```

**fricas** [A] time = 1.39, size = 54, normalized size = 0.65

$$\frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*c*x)
```

**giac [A]** time = 1.15, size = 81, normalized size = 0.98

$$\frac{1}{6}\sqrt{cx^2}\left(x\left(\frac{2x}{bc}-\frac{3a}{b^2c}\right)+\frac{6a^2}{b^3c}\right)+\frac{a^3\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^2}\right)b\sqrt{c}\,-2\,ac\right|\right)}{b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2)*(x*(2*x/(b*c) - 3*a/(b^2*c)) + 6*a^2/(b^3*c)) + a^3*log(abs
(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b^4*sqrt(c))
```

**maple [A]** time = 0.01, size = 50, normalized size = 0.60

$$-\frac{\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln\left(bx+a\right)-6a^2bx\right)x}{6\sqrt{c\,x^2}\;b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -1/6*x*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/(c*x^2)^(1/2)/b^4
```

**maxima [A]** time = 1.52, size = 142, normalized size = 1.71

$$\frac{\sqrt{cx^2}\,x^2}{3\,bc}-\frac{7\,ax^2}{6\,b^2\sqrt{c}}-\frac{(-1)^{\frac{2\,acx}{b}}\,a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4\sqrt{c}}+\frac{2\sqrt{cx^2}\,ax}{3\,b^2c}-\frac{14\,a^2x}{3\,b^3\sqrt{c}}-\frac{a^3\log\left(bx\right)}{b^4\sqrt{c}}+\frac{17\sqrt{cx^2}\,a^2}{3\,b^3c}-\frac{7\,a^3}{2\,b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2)*x^2/(b*c) - 7/6*a*x^2/(b^2*sqrt(c)) - (-1)^(2*a*c*x/b)*a^3*
log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*sqrt(c)) + 2/3*sqrt(c*x^2)*a*x/(b^2*c)
- 14/3*a^2*x/(b^3*sqrt(c)) - a^3*log(b*x)/(b^4*sqrt(c)) + 17/3*sqrt(c*x^2)*
a^2/(b^3*c) - 7/2*a^3/(b^4*sqrt(c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4}{\sqrt{c\,x^2}\;\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(x^4/((c*x^2)^(1/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(c*x**2)*(a + b*x)), x)
```

## 3.836 $\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx$

**Optimal**. Leaf size=61

$$\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}-\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}-\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -((a*x^2)/(b^2*Sqrt[c*x^2])) + x^3/(2*b*Sqrt[c*x^2]) + (a^2*x*Log[a + b*x])
/(b^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{x^2}{a+bx}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 39, normalized size = 0.64

$$\frac{x\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] (x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 47, normalized size = 0.77

$$\sqrt{cx^2}\left(\frac{a^2\log(a+bx)}{b^3cx}+\frac{bx-2a}{2b^2c}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] Sqrt[c*x^2]*((-2*a + b*x)/(2*b^2*c) + (a^2*Log[a + b*x])/(b^3*c*x))
```

**fricas** [A] time = 1.35, size = 42, normalized size = 0.69

$$\frac{\left(b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c*x)
```

**giac [A]** time = 1.09, size = 67, normalized size = 1.10

$$\frac{1}{2}\sqrt{cx^2}\left(\frac{x}{bc}-\frac{2a}{b^2c}\right)-\frac{a^2\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b\sqrt{c}-2ac\right|\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2)*(x/(b*c) - 2*a/(b^2*c)) - a^2*log(abs(-(sqrt(c)*x - sqrt(c*
x^2))*b*sqrt(c) - 2*a*c))/(b^3*sqrt(c))
```

**maple [A]** time = 0.00, size = 38, normalized size = 0.62

$$\frac{\left(b^2x^2+2a^2\ln\left(bx+a\right)-2abx\right)x}{2\sqrt{c\,x^2}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(1/2)/b^3
```

**maxima [A]** time = 1.49, size = 100, normalized size = 1.64

$$\frac{x^2}{2b\sqrt{c}}+\frac{(-1)^{\frac{2acx}{b}}a^2\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^3\sqrt{c}}+\frac{2ax}{b^2\sqrt{c}}+\frac{a^2\log\left(bx\right)}{b^3\sqrt{c}}-\frac{3\sqrt{cx^2}a}{b^2c}+\frac{3a^2}{2b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/(b*sqrt(c)) + (-1)^(2*a*c*x/b)*a^2*log(-2*a*c*x/(b*abs(b*x + a)))/(
b^3*sqrt(c)) + 2*a*x/(b^2*sqrt(c)) + a^2*log(b*x)/(b^3*sqrt(c)) - 3*sqrt(c*
x^2)*a/(b^2*c) + 3/2*a^2/(b^3*sqrt(c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^3}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(x^3/((c*x^2)^(1/2)*(a + b*x)), x)
```

**sympy [F]**     time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)/(c*x**2)**(1/2),x)
[Out] Integral(x**3/(sqrt(c*x**2)*(a + b*x)), x)
```

## 3.837 $\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx$

**Optimal**. Leaf size=39

$$\frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] x^2/(b*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx &= \frac{x\int \frac{x}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= \frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 0.69

$$\frac{x(bx - a\log(a+bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] (x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 36, normalized size = 0.92

$$\sqrt{cx^2}\left(\frac{1}{bc} - \frac{a\log(a+bx)}{b^2cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] Sqrt[c*x^2]*(1/(b*c) - (a*Log[a + b*x])/(b^2*c*x))
```

**fricas** [A] time = 0.95, size = 30, normalized size = 0.77

$$\frac{\sqrt{cx^2}\left(bx - a\log\left(bx+a\right)\right)}{b^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c*x)
```

**giac** [A] time = 1.19, size = 51, normalized size = 1.31

$$\frac{a\log\left(\left|-\left(\sqrt{c}\,x - \sqrt{cx^2}\right)b\sqrt{c} - 2\,ac\right|\right)}{b^2\sqrt{c}} + \frac{\sqrt{cx^2}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] a*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b^2*sqrt(c)) + sq
rt(c*x^2)/(b*c)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.69

$$-\frac{(a\ln(bx+a)-bx)\,x}{\sqrt{c\,x^2}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -x*(a*ln(b*x+a)-b*x)/(c*x^2)^(1/2)/b^2
```

**maxima [A]** time = 1.47, size = 64, normalized size = 1.64

$$-\frac{(-1)^{\frac{2acx}{b}}\,a\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2\sqrt{c}}-\frac{a\log(bx)}{b^2\sqrt{c}}+\frac{\sqrt{cx^2}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(b^2*sqrt(c)) - a*log(b*
x)/(b^2*sqrt(c)) + sqrt(c*x^2)/(b*c)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\sqrt{c\,x^2}\;(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(x^2/((c*x^2)^(1/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^2}\;(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(c*x**2)*(a + b*x)), x)
```

## 3.838 $\int \frac{x}{\sqrt{cx^2}(a+bx)} \, dx$

**Optimal**. Leaf size=20

$$\frac{x \log(a+bx)}{b\sqrt{cx^2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 31}

$$\frac{x \log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{cx^2}(a+bx)} \, dx &= \frac{x \int \frac{1}{a+bx} \, dx}{\sqrt{cx^2}} \\ &= \frac{x \log(a+bx)}{b\sqrt{cx^2}} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{x \log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] (x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

**IntegrateAlgebraic [A]** time = 0.02, size = 25, normalized size = 1.25

$$\frac{\sqrt{cx^2}\log(a+bx)}{bcx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] (Sqrt[c*x^2]*Log[a + b*x])/(b*c*x)
```

**fricas [A]** time = 0.63, size = 23, normalized size = 1.15

$$\frac{\sqrt{cx^2}\log(bx+a)}{bcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*log(b*x + a)/(b*c*x)
```

**giac [A]** time = 0.90, size = 36, normalized size = 1.80

$$-\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b\sqrt{c}-2ac\right|\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] -log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b*sqrt(c))
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.95

$$\frac{x\ln(bx+a)}{\sqrt{cx^2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] x*ln(b*x+a)/b/(c*x^2)^(1/2)
```

**maxima [B]** time = 1.46, size = 46, normalized size = 2.30

$$\frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b\sqrt{c}}+\frac{\log(bx)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(b*sqrt(c)) + log(b*x)/(b*s
qrt(c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.05

$$\int\frac{x}{\sqrt{c\,x^2}\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(x/((c*x^2)^(1/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*x**2)*(a + b*x)), x)
```

## 3.839 $\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx$

**Optimal**. Leaf size=38

$$\frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {15, 36, 29, 31}

$$\frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*Log[x])/(a*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx = \frac{x\int \frac{1}{x(a+bx)}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \frac{1}{x}\,dx}{a\sqrt{cx^2}} - \frac{(bx)\int \frac{1}{a+bx}\,dx}{a\sqrt{cx^2}}$$
$$= \frac{x\log(x)}{a\sqrt{cx^2}} - \frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.00, size = 25, normalized size = 0.66

$$\frac{x(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] (x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [A] time = 0.03, size = 43, normalized size = 1.13

$$\sqrt{cx^2}\left(\frac{\log(x)}{acx} - \frac{\log\left(a^2+abx\right)}{acx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] Sqrt[c*x^2]*(Log[x]/(a*c*x) - Log[a^2 + a*b*x]/(a*c*x))
```

**fricas** [A] time = 0.94, size = 70, normalized size = 1.84

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{acx}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*c*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*x
 + a)*sqrt(-c)/(a*c*x))/(a*c)]
```

**giac** [A] time = 0.97, size = 59, normalized size = 1.55

$$\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2a\sqrt{c}\right|\right)}{a\sqrt{c}}-\frac{\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a*sqrt(c)) - log(abs(
-sqrt(c)*x + sqrt(c*x^2)))/(a*sqrt(c))
```

**maple** [A] time = 0.00, size = 24, normalized size = 0.63

$$\frac{(\ln(x)-\ln(bx+a))\,x}{\sqrt{c\,x^2}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] x*(ln(x)-ln(b*x+a))/(c*x^2)^(1/2)/a
```

**maxima** [A] time = 1.46, size = 35, normalized size = 0.92

$$-\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(a*sqrt(c))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(1/((c*x^2)^(1/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*x**2)*(a + b*x)), x)
```

## 3.840 $\int \frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx$

**Optimal**. Leaf size=54

$$-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}-\frac{1}{a\sqrt{cx^2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}-\frac{1}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -(1/(a*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*Sqrt[c*x^2]) + (b*x*Log[a + b*x])/
(a^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{1}{x^2(a+bx)}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{1}{a\sqrt{cx^2}}-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 36, normalized size = 0.67

$$\frac{cx^2(bx\log(a+bx)-a-bx\log(x))}{a^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
[Out] (c*x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 53, normalized size = 0.98

$$\sqrt{cx^2}\left(-\frac{b\log(x)}{a^2cx}+\frac{b\log(a+bx)}{a^2cx}-\frac{1}{acx^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
[Out] Sqrt[c*x^2]*(-(1/(a*c*x^2)) - (b*Log[x])/(a^2*c*x) + (b*Log[a + b*x])/(a^2*
c*x))
```

**fricas** [A] time = 0.77, size = 34, normalized size = 0.63

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c*x^2)
```

**giac [A]** time = 1.19, size = 91, normalized size = 1.69

$$-\sqrt{c}\left(\frac{b\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^2}\right)b-2\,a\sqrt{c}\right|\right)}{a^2c}-\frac{b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)}{a^2c}-\frac{2}{\left(\sqrt{c}\,x-\sqrt{cx^2}\right)a\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(c)*(b*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a^2*c) -
b*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/(a^2*c) - 2/((sqrt(c)*x - sqrt(c*x^2))
*a*sqrt(c)))
```

**maple [A]** time = 0.01, size = 30, normalized size = 0.56

$$-\frac{bx\ln(x)-bx\ln(bx+a)+a}{\sqrt{c\,x^2}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -(b*x*ln(x)-b*x*ln(b*x+a)+a)/(c*x^2)^(1/2)/a^2
```

**maxima [A]** time = 1.37, size = 37, normalized size = 0.69

$$\frac{b\log(bx+a)}{a^2\sqrt{c}}-\frac{b\log(x)}{a^2\sqrt{c}}-\frac{1}{a\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/(a^2*sqrt(c)) - b*log(x)/(a^2*sqrt(c)) - 1/(a*sqrt(c)*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{x\sqrt{c\,x^2}\;(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(1/(x*(c*x^2)^(1/2)*(a + b*x)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*x**2)*(a + b*x)), x)
```

## 3.841 $\int \frac{1}{x^2 \sqrt{cx^2} (a+bx)} \, dx$

**Optimal**. Leaf size=77

$$\frac{b^2 x \log(x)}{a^3 \sqrt{cx^2}} - \frac{b^2 x \log(a+bx)}{a^3 \sqrt{cx^2}} + \frac{b}{a^2 \sqrt{cx^2}} - \frac{1}{2ax\sqrt{cx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2 x \log(x)}{a^3 \sqrt{cx^2}} - \frac{b^2 x \log(a+bx)}{a^3 \sqrt{cx^2}} + \frac{b}{a^2 \sqrt{cx^2}} - \frac{1}{2ax\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] b/(a^2*Sqrt[c*x^2]) - 1/(2*a*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*Sqrt[c*x^
2]) - (b^2*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{1}{x^3(a+bx)}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 52, normalized size = 0.68

$$\frac{cx\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (c*x*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x]))/(2*a^3
*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 67, normalized size = 0.87

$$\sqrt{cx^2}\left(\frac{b^2\log(x)}{a^3cx}-\frac{b^2\log(a+bx)}{a^3cx}+\frac{2bx-a}{2a^2cx^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*((-a + 2*b*x)/(2*a^2*c*x^3) + (b^2*Log[x])/(a^3*c*x) - (b^2*Log
[a + b*x])/(a^3*c*x))
```

**fricas** [A] time = 0.95, size = 47, normalized size = 0.61

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c*x^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

**maple [A]** time = 0.00, size = 51, normalized size = 0.66

$$\frac{2b^2x^2\ln(x) - 2b^2x^2\ln(bx+a) + 2abx - a^2}{2\sqrt{c\,x^2}\;a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/2/x*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x-a^2)/(c*x^2)^(1/2)/a^3
```

**maxima [A]** time = 1.38, size = 55, normalized size = 0.71

$$-\frac{b^2\log(bx+a)}{a^3\sqrt{c}} + \frac{b^2\log(x)}{a^3\sqrt{c}} + \frac{2\,b\sqrt{c}\,x - a\sqrt{c}}{2\,a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/(a^3*sqrt(c)) + b^2*log(x)/(a^3*sqrt(c)) + 1/2*(2*b*sqrt(
c)*x - a*sqrt(c))/(a^2*c*x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{c\,x^2}\;(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)), x)
```

## 3.842 $\int \frac{1}{x^3\sqrt{cx^2}\,(a+bx)}\,dx$

**Optimal**. Leaf size=100

$$-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}-\frac{b^2}{a^3\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{b^2}{a^3\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -(b^2/(a^3*Sqrt[c*x^2])) - 1/(3*a*x^2*Sqrt[c*x^2]) + b/(2*a^2*x*Sqrt[c*x^2]
) - (b^3*x*Log[x])/(a^4*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*Sqrt[c*x^2
])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{1}{x^4(a+bx)}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{b^2}{a^3\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.01, size = 63, normalized size = 0.63

$$\frac{c\left(a\left(-2a^2+3abx-6b^2x^2\right)+6b^3x^3\log(a+bx)-6b^3x^3\log(x)\right)}{6a^4\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (c*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Log[a +
 b*x]))/(6*a^4*(c*x^2)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 78, normalized size = 0.78

$$\sqrt{cx^2}\left(-\frac{b^3\log(x)}{a^4cx}+\frac{b^3\log(a+bx)}{a^4cx}+\frac{-2a^2+3abx-6b^2x^2}{6a^3cx^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*((-2*a^2 + 3*a*b*x - 6*b^2*x^2)/(6*a^3*c*x^4) - (b^3*Log[x])/(a
^4*c*x) + (b^3*Log[a + b*x])/(a^4*c*x))
```

**fricas [A]** time = 0.84, size = 58, normalized size = 0.58

$$\frac{\left(6\,b^3x^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3\right)\sqrt{cx^2}}{6\,a^4cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*x
^2)/(a^4*c*x^4)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2}\,(bx+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2)*(b*x + a)*x^3), x)
```

**maple [A]** time = 0.00, size = 62, normalized size = 0.62

$$-\frac{6b^3x^3\ln(x)-6b^3x^3\ln(bx+a)+6a\,b^2x^2-3a^2bx+2a^3}{6\sqrt{c\,x^2}\;a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -1/6/x^2*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*a*b^2*x^2-3*a^2*b*x+2*a^3)/
(c*x^2)^(1/2)/a^4
```

**maxima [A]** time = 1.40, size = 69, normalized size = 0.69

$$\frac{b^3\log(bx+a)}{a^4\sqrt{c}}-\frac{b^3\log(x)}{a^4\sqrt{c}}-\frac{6\,b^2\sqrt{c}\,x^2-3\,ab\sqrt{c}\,x+2\,a^2\sqrt{c}}{6\,a^3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/(a^4*sqrt(c)) - b^3*log(x)/(a^4*sqrt(c)) - 1/6*(6*b^2*sqrt
(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c*x^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(1/(x^3*(c*x^2)^(1/2)*(a + b*x)), x)
```

**sympy [F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)/(c*x**2)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(c*x**2)*(a + b*x)), x)
```

## 3.843 $\int \frac{x^6}{\left(cx^2\right)^{3/2}(a+bx)}\,dx$

**Optimal**. Leaf size=95

$$-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}+\frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (a^2*x^2)/(b^3*c*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*c*Sqrt[c*x^2]) + x^4/(3*b*c*
Sqrt[c*x^2]) - (a^3*x*Log[a + b*x])/(b^4*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{x^3}{a+bx}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.01, size = 53, normalized size = 0.56

$$\frac{x^3\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*(c*x^
2)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 59, normalized size = 0.62

$$\frac{\frac{6a^2x^4-3abx^5+2b^2x^6}{6b^3}-\frac{a^3x^3\log(a+bx)}{b^4}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] ((6*a^2*x^4 - 3*a*b*x^5 + 2*b^2*x^6)/(6*b^3) - (a^3*x^3*Log[a + b*x])/b^4)/
(c*x^2)^(3/2)
```

**fricas [A]** time = 1.15, size = 54, normalized size = 0.57

$$\frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*c^2*x)
```

**giac** [A] time = 1.18, size = 86, normalized size = 0.91

$$\frac{\sqrt{cx^2}\left(x\left(\frac{2x}{bc}-\frac{3a}{b^2c}\right)+\frac{6a^2}{b^3c}\right)+\frac{6a^3\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b\sqrt{c}-2ac\right|\right)}{b^4\sqrt{c}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(c*x^2)*(x*(2*x/(b*c) - 3*a/(b^2*c)) + 6*a^2/(b^3*c)) + 6*a^3*log(
abs(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b^4*sqrt(c)))/c
```

**maple** [A] time = 0.01, size = 52, normalized size = 0.55

$$-\frac{\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln\left(bx+a\right)-6a^2bx\right)x^3}{6\left(c\,x^2\right)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -1/6*x^3*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/(c*x^2)^(3/2)/b
^4
```

**maxima** [A] time = 1.79, size = 162, normalized size = 1.71

$$\frac{x^4}{3\sqrt{cx^2}\,bc}-\frac{ax^3}{2\sqrt{cx^2}\,b^2c}+\frac{a^2x^2}{\sqrt{cx^2}\,b^3c}-\frac{(-1)^{\frac{2acx}{b}}a^3\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^4c^{\frac{3}{2}}}+\frac{29a^3x}{6\sqrt{cx^2}\,b^4c}-\frac{a^3\log\left(bx\right)}{b^4c^{\frac{3}{2}}}-\frac{2a^4}{\sqrt{cx^2}\,b^5c}+\frac{2a^4}{b^5c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^4/(sqrt(c*x^2)*b*c) - 1/2*a*x^3/(sqrt(c*x^2)*b^2*c) + a^2*x^2/(sqrt(c
*x^2)*b^3*c) - (-1)^(2*a*c*x/b)*a^3*log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*c^(
3/2)) + 29/6*a^3*x/(sqrt(c*x^2)*b^4*c) - a^3*log(b*x)/(b^4*c^(3/2)) - 2*a^4
/(sqrt(c*x^2)*b^5*c) + 2*a^4/(b^5*c^(3/2)*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^6/((c*x^2)^(3/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(cx^2\right)^{\frac{3}{2}}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**6/((c*x**2)**(3/2)*(a + b*x)), x)
```

## 3.844 $\int \frac{x^5}{(cx^2)^{3/2}(a+bx)} dx$

**Optimal**. Leaf size=70

$$\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}} - \frac{ax^2}{b^2c\sqrt{cx^2}} + \frac{x^3}{2bc\sqrt{cx^2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}} - \frac{ax^2}{b^2c\sqrt{cx^2}} + \frac{x^3}{2bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -((a*x^2)/(b^2*c*Sqrt[c*x^2])) + x^3/(2*b*c*Sqrt[c*x^2]) + (a^2*x*Log[a + b
*x])/(b^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{x^2}{a+bx}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{ax^2}{b^2c\sqrt{cx^2}} + \frac{x^3}{2bc\sqrt{cx^2}} + \frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 41, normalized size = 0.59

$$\frac{x^3\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^3*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 46, normalized size = 0.66

$$\frac{\frac{a^2x^3\log(a+bx)}{b^3}+\frac{bx^5-2ax^4}{2b^2}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] ((-2*a*x^4 + b*x^5)/(2*b^2) + (a^2*x^3*Log[a + b*x])/b^3)/(c*x^2)^(3/2)
```

**fricas** [A] time = 0.83, size = 42, normalized size = 0.60

$$\frac{\left(b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c^2*x)
```

**giac [A]** time = 0.96, size = 71, normalized size = 1.01

$$\frac{\sqrt{cx^2}\left(\frac{x}{bc}-\frac{2a}{b^2c}\right)-\frac{2a^2\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b\sqrt{c}-2ac\right|\right)}{b^3\sqrt{c}}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(c*x^2)*(x/(b*c) - 2*a/(b^2*c)) - 2*a^2*log(abs(-(sqrt(c)*x - sqrt
(c*x^2))*b*sqrt(c) - 2*a*c))/(b^3*sqrt(c)))/c
```

**maple [A]** time = 0.00, size = 40, normalized size = 0.57

$$\frac{\left(b^2x^2+2a^2\ln\left(bx+a\right)-2abx\right)x^3}{2\left(c\,x^2\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/2*x^3*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(3/2)/b^3
```

**maxima [B]** time = 1.66, size = 140, normalized size = 2.00

$$\frac{x^3}{2\sqrt{cx^2}\,bc}-\frac{ax^2}{\sqrt{cx^2}\,b^2c}+\frac{(-1)^{\frac{2acx}{b}}a^2\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^3c^{\frac{3}{2}}}-\frac{7a^2x}{2\sqrt{cx^2}\,b^3c}+\frac{a^2\log\left(bx\right)}{b^3c^{\frac{3}{2}}}+\frac{2a^3}{\sqrt{cx^2}\,b^4c}-\frac{2a^3}{b^4c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/(sqrt(c*x^2)*b*c) - a*x^2/(sqrt(c*x^2)*b^2*c) + (-1)^(2*a*c*x/b)*a^
2*log(-2*a*c*x/(b*abs(b*x + a)))/(b^3*c^(3/2)) - 7/2*a^2*x/(sqrt(c*x^2)*b^3
*c) + a^2*log(b*x)/(b^3*c^(3/2)) + 2*a^3/(sqrt(c*x^2)*b^4*c) - 2*a^3/(b^4*c
^(3/2)*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^5/((c*x^2)^(3/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(cx^2\right)^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**5/((c*x**2)**(3/2)*(a + b*x)), x)
```

## 3.845 $\int \frac{x^4}{\left(cx^2\right)^{3/2}(a+bx)}\,dx$

**Optimal**. Leaf size=45

$$\frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] x^2/(b*c*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(cx^2\right)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{x}{a+bx}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{c\sqrt{cx^2}}\\
&= \frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 29, normalized size = 0.64

$$\frac{x^3(bx - a\log(a+bx))}{b^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^3*(b*x - a*Log[a + b*x]))/(b^2*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 33, normalized size = 0.73

$$\frac{\frac{x^4}{b} - \frac{ax^3\log(a+bx)}{b^2}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^4/b - (a*x^3*Log[a + b*x])/b^2)/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.11, size = 30, normalized size = 0.67

$$\frac{\sqrt{cx^2}\left(bx - a\log\left(bx+a\right)\right)}{b^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c^2*x)
```

**giac** [A] time = 1.09, size = 55, normalized size = 1.22

$$\frac{\frac{a\log\left(\left|-\left(\sqrt{c}\,x - \sqrt{cx^2}\right)b\sqrt{c} - 2\,ac\right|\right)}{b^2\sqrt{c}} + \frac{\sqrt{cx^2}}{bc}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
[Out] (a*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b^2*sqrt(c)) + s
qrt(c*x^2)/(b*c))/c
```

**maple** [A] time = 0.00, size = 29, normalized size = 0.64

$$-\frac{\left(a\ln\left(bx+a\right)-bx\right)x^{3}}{\left(c\,x^{2}\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -x^3*(a*ln(b*x+a)-b*x)/(c*x^2)^(3/2)/b^2
```

**maxima** [B] time = 1.60, size = 116, normalized size = 2.58

$$\frac{x^{2}}{\sqrt{cx^{2}}\,bc}-\frac{\left(-1\right)^{\frac{2\,acx}{b}}a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^{2}c^{\frac{3}{2}}}+\frac{2\,ax}{\sqrt{cx^{2}}\,b^{2}c}-\frac{a\log\left(bx\right)}{b^{2}c^{\frac{3}{2}}}-\frac{2\,a^{2}}{\sqrt{cx^{2}}\,b^{3}c}+\frac{2\,a^{2}}{b^{3}c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(c*x^2)*b*c) - (-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(
b^2*c^(3/2)) + 2*a*x/(sqrt(c*x^2)*b^2*c) - a*log(b*x)/(b^2*c^(3/2)) - 2*a^2
/(sqrt(c*x^2)*b^3*c) + 2*a^2/(b^3*c^(3/2)*x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^4/((c*x^2)^(3/2)*(a + b*x)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(cx^{2}\right)^{\frac{3}{2}}\left(a+bx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**4/((c*x**2)**(3/2)*(a + b*x)), x)
```

# 3.846 $\int \frac{x^3}{(cx^2)^{3/2}(a+bx)} dx$

**Optimal**. Leaf size=23

$$\frac{x\log(a+bx)}{bc\sqrt{cx^2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 31}

$$\frac{x\log(a+bx)}{bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*Log[a + b*x])/(b*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(cx^2)^{3/2}(a+bx)} dx &= \frac{x\int \frac{1}{a+bx} dx}{c\sqrt{cx^2}} \\ &= \frac{x\log(a+bx)}{bc\sqrt{cx^2}}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 22, normalized size = 0.96

$$\frac{x^3\log(a+bx)}{b(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*Log[a + b*x])/(b*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 22, normalized size = 0.96

$$\frac{x^3 \log(a+bx)}{b\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*Log[a + b*x])/(b*(c*x^2)^(3/2))
```

**fricas** [A] time = 1.09, size = 23, normalized size = 1.00

$$\frac{\sqrt{cx^2}\,\log\left(bx+a\right)}{bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*log(b*x + a)/(b*c^2*x)
```

**giac** [A] time = 0.97, size = 36, normalized size = 1.57

$$-\frac{\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^2}\right)b\sqrt{c}\,-2\,ac\right|\right)}{bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b*c^(3/2))
```

**maple** [A] time = 0.00, size = 21, normalized size = 0.91

$$\frac{x^3\ln\left(bx+a\right)}{\left(c\,x^2\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/(c*x^2)^(3/2)*x^3*ln(b*x+a)/b
```

**maxima [B]** time = 1.60, size = 74, normalized size = 3.22

$$\frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{bc^{\frac{3}{2}}}+\frac{\log(bx)}{bc^{\frac{3}{2}}}+\frac{2a}{\sqrt{cx^2}\,b^2c}-\frac{2a}{b^2c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(b*c^(3/2)) + log(b*x)/(b*c
^(3/2)) + 2*a/(sqrt(c*x^2)*b^2*c) - 2*a/(b^2*c^(3/2)*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^3/((c*x^2)^(3/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**3/((c*x**2)**(3/2)*(a + b*x)), x)
```

## 3.847 $\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx$

**Optimal**. Leaf size=44

$$\frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*Log[x])/(a*c*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(cx^2\right)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{1}{x(a+bx)}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \frac{1}{x}\,dx}{ac\sqrt{cx^2}} - \frac{(bx)\int \frac{1}{a+bx}\,dx}{ac\sqrt{cx^2}}$$
$$= \frac{x\log(x)}{ac\sqrt{cx^2}} - \frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 0.61

$$\frac{x^3(\log(x)-\log(a+bx))}{a\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^3*(Log[x] - Log[a + b*x]))/(a*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 37, normalized size = 0.84

$$\frac{\frac{x^3\log(x)}{a}-\frac{x^3\log\left(a^2+abx\right)}{a}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] ((x^3*Log[x])/a - (x^3*Log[a^2 + a*b*x])/a)/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.29, size = 70, normalized size = 1.59

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{ac^2x}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*c^2*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b
*x + a)*sqrt(-c)/(a*c*x))/(a*c^2)]
```

**giac [A]** time = 1.05, size = 63, normalized size = 1.43

$$\frac{\frac{\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^2}\right)b-2\,a\sqrt{c}\right|\right)}{a\sqrt{c}}-\frac{\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)}{a\sqrt{c}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] (log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a*sqrt(c)) - log(abs
(-sqrt(c)*x + sqrt(c*x^2)))/(a*sqrt(c)))/c
```

**maple [A]** time = 0.00, size = 26, normalized size = 0.59

$$\frac{(\ln(x)-\ln(bx+a))\,x^3}{\left(c\,x^2\right)^{\frac{3}{2}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] x^3*(ln(x)-ln(b*x+a))/(c*x^2)^(3/2)/a
```

**maxima [A]** time = 1.43, size = 35, normalized size = 0.80

$$-\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(a*c^(3/2))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^2/((c*x^2)^(3/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^2\right)^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**2/((c*x**2)**(3/2)*(a + b*x)), x)
```

## 3.848 $\int \frac{x}{(cx^2)^{3/2}(a+bx)}\,dx$

**Optimal**. Leaf size=63

$$-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}-\frac{1}{ac\sqrt{cx^2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 44}

$$-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}-\frac{1}{ac\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -(1/(a*c*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*c*Sqrt[c*x^2]) + (b*x*Log[a + b*
x])/(a^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{x}{\left(cx^2\right)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{1}{x^2(a+bx)}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{1}{ac\sqrt{cx^2}} - \frac{bx\log(x)}{a^2c\sqrt{cx^2}} + \frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.56

$$\frac{x^2(bx\log(a+bx) - a - bx\log(x))}{a^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 44, normalized size = 0.70

$$\frac{-\frac{bx^3\log(x)}{a^2} + \frac{bx^3\log(a+bx)}{a^2} - \frac{x^2}{a}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (-(x^2/a) - (b*x^3*Log[x])/a^2 + (b*x^3*Log[a + b*x])/a^2)/(c*x^2)^(3/2)
```

**fricas** [A] time = 0.95, size = 34, normalized size = 0.54

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right) - a\right)}{a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c^2*x^2)
```

**giac** [A] time = 1.08, size = 91, normalized size = 1.44

$$-\frac{\frac{b\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2a\sqrt{c}\right|\right)}{a^2c}-\frac{b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{a^2c}-\frac{2}{\left(\sqrt{c}x-\sqrt{cx^2}\right)a\sqrt{c}}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a^2*c) - b*log(ab
s(-sqrt(c)*x + sqrt(c*x^2)))/(a^2*c) - 2/((sqrt(c)*x - sqrt(c*x^2))*a*sqrt(
c)))/sqrt(c)
```

**maple** [A] time = 0.01, size = 33, normalized size = 0.52

$$-\frac{\left(bx\ln(x)-bx\ln\left(bx+a\right)+a\right)x^2}{\left(c\,x^2\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -x^2*(b*x*ln(x)-b*x*ln(b*x+a)+a)/(c*x^2)^(3/2)/a^2
```

**maxima** [A] time = 1.48, size = 51, normalized size = 0.81

$$\frac{(-1)^{\frac{2\,acx}{b}}\,b\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^2c^{\frac{3}{2}}}-\frac{1}{\sqrt{cx^2}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*a*c*x/b)*b*log(-2*a*c*x/(b*abs(b*x + a)))/(a^2*c^(3/2)) - 1/(sqrt(c
*x^2)*a*c)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x/((c*x^2)^(3/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^2\right)^{\frac{3}{2}} (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x/((c*x**2)**(3/2)*(a + b*x)), x)
```

## 3.849 $\int \frac{1}{\left(cx^2\right)^{3/2}(a+bx)}\,dx$

**Optimal**. Leaf size=89

$$\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}+\frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 44}

$$\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}+\frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] b/(a^2*c*Sqrt[c*x^2]) - 1/(2*a*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*c*Sqr
t[c*x^2]) - (b^2*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{1}{x^3(a+bx)}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 51, normalized size = 0.57

$$\frac{x\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x]))/(2*a^3*(
c*x^2)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 58, normalized size = 0.65

$$\frac{\frac{b^2x^3\log(x)}{a^3}-\frac{b^2x^3\log(a+bx)}{a^3}+\frac{2bx^2-ax}{2a^2}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] ((-(a*x) + 2*b*x^2)/(2*a^2) + (b^2*x^3*Log[x])/a^3 - (b^2*x^3*Log[a + b*x])
/a^3)/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.16, size = 47, normalized size = 0.53

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c^2*x^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

**maple [A]** time = 0.01, size = 49, normalized size = 0.55

$$\frac{\left(2b^2x^2\ln(x)-2b^2x^2\ln\left(bx+a\right)+2abx-a^2\right)x}{2\left(c\,x^2\right)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/2*x*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x-a^2)/(c*x^2)^(3/2)/a^3
```

**maxima [A]** time = 1.54, size = 65, normalized size = 0.73

$$-\frac{(-1)^{\frac{2\,acx}{b}}b^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^3c^{\frac{3}{2}}}+\frac{b}{\sqrt{cx^2}\,a^2c}-\frac{1}{2\,ac^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*b^2*log(-2*a*c*x/(b*abs(b*x + a)))/(a^3*c^(3/2)) + b/(sqr
t(c*x^2)*a^2*c) - 1/2/(a*c^(3/2)*x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(1/((c*x^2)^(3/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2\right)^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2)**(3/2)/(b*x+a),x)
[Out] Integral(1/((c*x**2)**(3/2)*(a + b*x)), x)
```

## 3.850 $\int \frac{1}{x\left(cx^2\right)^{3/2}(a+bx)}\,dx$

**Optimal**. Leaf size=115

$$-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}-\frac{b^2}{a^3c\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{b^2}{a^3c\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -(b^2/(a^3*c*Sqrt[c*x^2])) - 1/(3*a*c*x^2*Sqrt[c*x^2]) + b/(2*a^2*c*x*Sqrt[
c*x^2]) - (b^3*x*Log[x])/(a^4*c*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*c*
Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\left(cx^2\right)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{1}{x^4(a+bx)}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{b^2}{a^3c\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 66, normalized size = 0.57

$$\frac{cx^2\left(a\left(-2a^2+3abx-6b^2x^2\right)+6b^3x^3\log(a+bx)-6b^3x^3\log(x)\right)}{6a^4\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (c*x^2*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Log
[a + b*x]))/(6*a^4*(c*x^2)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.06, size = 66, normalized size = 0.57

$$\frac{-\frac{b^3x^3\log(x)}{a^4}+\frac{b^3x^3\log(a+bx)}{a^4}+\frac{-2a^2+3abx-6b^2x^2}{6a^3}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] ((-2*a^2 + 3*a*b*x - 6*b^2*x^2)/(6*a^3) - (b^3*x^3*Log[x])/a^4 + (b^3*x^3*L
og[a + b*x])/a^4)/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.05, size = 58, normalized size = 0.50

$$\frac{\left(6\,b^3x^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3\right)\sqrt{cx^2}}{6\,a^4c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*x
^2)/(a^4*c^2*x^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2\right)^{\frac{3}{2}}(bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^2)^(3/2)*(b*x + a)*x), x)
```

**maple** [A] time = 0.01, size = 59, normalized size = 0.51

$$-\frac{6b^3x^3\ln(x) - 6b^3x^3\ln(bx+a) + 6a\,b^2x^2 - 3a^2bx + 2a^3}{6\left(c\,x^2\right)^{\frac{3}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -1/6*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*a*b^2*x^2-3*a^2*b*x+2*a^3)/(c*x
^2)^(3/2)/a^4
```

**maxima** [A] time = 1.36, size = 69, normalized size = 0.60

$$\frac{b^3\log(bx+a)}{a^4c^{\frac{3}{2}}} - \frac{b^3\log(x)}{a^4c^{\frac{3}{2}}} - \frac{6\,b^2\sqrt{c}\,x^2 - 3\,ab\sqrt{c}\,x + 2\,a^2\sqrt{c}}{6\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/(a^4*c^(3/2)) - b^3*log(x)/(a^4*c^(3/2)) - 1/6*(6*b^2*sqrt
(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c^2*x^3)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(c\,x^2\right)^{3/2}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(1/(x*(c*x^2)^(3/2)*(a + b*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(cx^2\right)^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(1/(x*(c*x**2)**(3/2)*(a + b*x)), x)
```

## 3.851 $\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=106

$$-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3\sqrt{cx^2}\,\log(a+bx)}{b^5x}+\frac{3a^2\sqrt{cx^2}}{b^4}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}+\frac{3a^2\sqrt{cx^2}}{b^4}-\frac{4a^3\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (3*a^2*Sqrt[c*x^2])/b^4 - (a*x*Sqrt[c*x^2])/b^3 + (x^2*Sqrt[c*x^2])/(3*b^2)
 - (a^4*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*Sqrt[c*x^2]*Log[a + b*x])/(
b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{x^4}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{3a^2}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{b^2} + \frac{a^4}{b^4(a+bx)^2} - \frac{4a^3}{b^4(a+bx)}\right)dx}{x}$$
$$= \frac{3a^2\sqrt{cx^2}}{b^4} - \frac{ax\sqrt{cx^2}}{b^3} + \frac{x^2\sqrt{cx^2}}{3b^2} - \frac{a^4\sqrt{cx^2}}{b^5x(a+bx)} - \frac{4a^3\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

**Mathematica** [A] time = 0.03, size = 81, normalized size = 0.76

$$\frac{cx\left(-3a^4+9a^3bx-12a^3(a+bx)\log(a+bx)+6a^2b^2x^2-2ab^3x^3+b^4x^4\right)}{3b^5\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*(
a + b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.08, size = 85, normalized size = 0.80

$$\sqrt{cx^2}\left(\frac{-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4}{3b^5x(a+bx)} - \frac{4a^3\log(a+bx)}{b^5x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] Sqrt[c*x^2]*((-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4)/(
3*b^5*x*(a + b*x)) - (4*a^3*Log[a + b*x])/(b^5*x))
```

**fricas** [A] time = 1.14, size = 83, normalized size = 0.78

$$\frac{\left(b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4 - 12\left(a^3bx + a^4\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{3\left(b^6x^2 + ab^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

**giac [A]** time = 1.00, size = 96, normalized size = 0.91

$$-\frac{1}{3}\sqrt{c}\left(\frac{12\,a^3\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^5}+\frac{3\,a^4\operatorname{sgn}(x)}{(bx+a)b^5}-\frac{3\left(4\,a^3\log\left(|a|\right)+a^3\right)\operatorname{sgn}(x)}{b^5}-\frac{b^4x^3\operatorname{sgn}(x)-3\,ab^3x^2\operatorname{sgn}(x)+9\,a^2b^2x\operatorname{sgn}(x)}{b^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(c)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a)
*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^3*
x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

**maple [A]** time = 0.01, size = 88, normalized size = 0.83

$$-\frac{\sqrt{c\,x^2}\,\left(-b^4x^4+2a\,b^3x^3+12a^3bx\ln\left(bx+a\right)-6a^2b^2x^2+12a^4\ln\left(bx+a\right)-9a^3bx+3a^4\right)}{3\left(bx+a\right)b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] -1/3*(c*x^2)^(1/2)*(-b^4*x^4+2*a*b^3*x^3+12*ln(b*x+a)*x*a^3*b-6*a^2*b^2*x^2
+12*a^4*ln(b*x+a)-9*a^3*b*x+3*a^4)/x/b^5/(b*x+a)
```

**maxima [A]** time = 1.52, size = 135, normalized size = 1.27

$$\frac{\sqrt{cx^2}\,a^3}{b^5x+ab^4}-\frac{4\,(-1)^{\frac{2\,cx}{b}}\,a^3\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^5}-\frac{4\,(-1)^{\frac{2\,acx}{b}}\,a^3\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5}-\frac{\sqrt{cx^2}\,ax}{b^3}+\frac{3\,\sqrt{cx^2}\,a^2}{b^4}+\frac{\left(cx^2\right)^{\frac{3}{2}}}{3\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a^3/(b^5*x + a*b^4) - 4*(-1)^(2*c*x/b)*a^3*sqrt(c)*log(2*c*x/b)
/b^5 - 4*(-1)^(2*a*c*x/b)*a^3*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^5 -
sqrt(c*x^2)*a*x/b^3 + 3*sqrt(c*x^2)*a^2/b^4 + 1/3*(c*x^2)^(3/2)/(b^2*c)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c\,x^2}}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x^3*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*sqrt(c*x**2)/(a + b*x)**2, x)
```

## 3.852 $\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=85

$$\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\,\log(a+bx)}{b^4x}-\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\,\log(a+bx)}{b^4x}-\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (-2*a*Sqrt[c*x^2])/b^3 + (x*Sqrt[c*x^2])/(2*b^2) + (a^3*Sqrt[c*x^2])/(b^4*x
*(a + b*x)) + (3*a^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{x^3}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{x}$$
$$= -\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}+\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

**Mathematica [A]** time = 0.02, size = 70, normalized size = 0.82

$$\frac{cx\left(2a^3-4a^2bx+6a^2(a+bx)\log(a+bx)-3ab^2x^2+b^3x^3\right)}{2b^4\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a + b
*x]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 74, normalized size = 0.87

$$\sqrt{cx^2}\left(\frac{3a^2\log(a+bx)}{b^4x}+\frac{2a^3-4a^2bx-3ab^2x^2+b^3x^3}{2b^4x(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] Sqrt[c*x^2]*((2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3)/(2*b^4*x*(a + b*x)
) + (3*a^2*Log[a + b*x])/(b^4*x))
```

**fricas [A]** time = 0.85, size = 72, normalized size = 0.85

$$\frac{\left(b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\left(a^2bx+a^3\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\left(b^5x^2+ab^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)
```

**giac [A]** time = 1.06, size = 80, normalized size = 0.94

$$\frac{1}{2}\sqrt{c}\left(\frac{6\,a^2\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^4}+\frac{2\,a^3\operatorname{sgn}(x)}{(bx+a)b^4}-\frac{2\left(3\,a^2\log\left(|a|\right)+a^2\right)\operatorname{sgn}(x)}{b^4}+\frac{b^2x^2\operatorname{sgn}(x)-4\,abx\operatorname{sgn}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*b
^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*sg
n(x))/b^4)
```

**maple [A]** time = 0.01, size = 76, normalized size = 0.89

$$\frac{\sqrt{c\,x^2}\,\left(b^3x^3+6a^2bx\ln\left(bx+a\right)-3a\,b^2x^2+6a^3\ln\left(bx+a\right)-4a^2bx+2a^3\right)}{2\left(bx+a\right)b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(b^3*x^3+6*ln(b*x+a)*x*a^2*b-3*a*b^2*x^2+6*a^3*ln(b*x+a)-
4*a^2*b*x+2*a^3)/x/b^4/(b*x+a)
```

**maxima [A]** time = 1.55, size = 118, normalized size = 1.39

$$-\frac{\sqrt{cx^2}\,a^2}{b^4x+ab^3}+\frac{3\,(-1)^{\frac{2\,cx}{b}}\,a^2\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^4}+\frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4}+\frac{\sqrt{cx^2}\,x}{2\,b^2}-\frac{2\,\sqrt{cx^2}\,a}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)*a^2/(b^4*x + a*b^3) + 3*(-1)^(2*c*x/b)*a^2*sqrt(c)*log(2*c*x/b
)/b^4 + 3*(-1)^(2*a*c*x/b)*a^2*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 +
 1/2*sqrt(c*x^2)*x/b^2 - 2*sqrt(c*x^2)*a/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c\,x^2}}{{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x^2*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c*x**2)/(a + b*x)**2, x)
```

## 3.853 $\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=65

$$-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\,\log(a+bx)}{b^3x}+\frac{\sqrt{cx^2}}{b^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\,\log(a+bx)}{b^3x}+\frac{\sqrt{cx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] Sqrt[c*x^2]/b^2 - (a^2*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*Sqrt[c*x^2]*Lo
g[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{x^2}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{1}{b^2} + \frac{a^2}{b^2(a+bx)^2} - \frac{2a}{b^2(a+bx)}\right)dx}{x}$$
$$= \frac{\sqrt{cx^2}}{b^2} - \frac{a^2\sqrt{cx^2}}{b^3x(a+bx)} - \frac{2a\sqrt{cx^2}\log(a+bx)}{b^3x}$$

**Mathematica** [A] time = 0.02, size = 53, normalized size = 0.82

$$\frac{cx\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
[Out] (c*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^2
]*(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.06, size = 57, normalized size = 0.88

$$\sqrt{cx^2}\left(\frac{-a^2+abx+b^2x^2}{b^3x(a+bx)} - \frac{2a\log(a+bx)}{b^3x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
[Out] Sqrt[c*x^2]*((-a^2 + a*b*x + b^2*x^2)/(b^3*x*(a + b*x)) - (2*a*Log[a + b*x]
)/(b^3*x))
```

**fricas** [A] time = 1.02, size = 57, normalized size = 0.88

$$\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4x^2+ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*x^2
 + a*b^3*x)
```

**giac** [A] time = 0.92, size = 58, normalized size = 0.89

$$\sqrt{c}\left(\frac{x\mathrm{sgn}(x)}{b^2}-\frac{2\,a\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3}+\frac{\left(2\,a\log\left(|a|\right)+a\right)\mathrm{sgn}(x)}{b^3}-\frac{a^2\mathrm{sgn}(x)}{(bx+a)b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] sqrt(c)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a))
 + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

**maple** [A] time = 0.01, size = 62, normalized size = 0.95

$$-\frac{\sqrt{c\,x^2}\,\left(2abx\ln\left(bx+a\right)-b^2x^2+2a^2\ln\left(bx+a\right)-abx+a^2\right)}{\left(bx+a\right)b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(1/2)*(2*ln(b*x+a)*x*a*b-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/x/b^3/
(b*x+a)
```

**maxima** [A] time = 1.51, size = 96, normalized size = 1.48

$$\frac{\sqrt{cx^2}\,a}{b^3x+ab^2}-\frac{2\,(-1)^{\frac{2\,cx}{b}}\,a\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^3}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a/(b^3*x + a*b^2) - 2*(-1)^(2*c*x/b)*a*sqrt(c)*log(2*c*x/b)/b^3
 - 2*(-1)^(2*a*c*x/b)*a*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + sqrt(c
*x^2)/b^2
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{c\,x^2}}{{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x*sqrt(c*x**2)/(a + b*x)**2, x)
```

## 3.854 $\int \frac{\sqrt{cx^2}}{(a+bx)^2} \, dx$

**Optimal**. Leaf size=47

$$\frac{a\sqrt{cx^2}}{b^2x(a+bx)}+\frac{\sqrt{cx^2}\log(a+bx)}{b^2x}$$

**Rubi [A]** time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a\sqrt{cx^2}}{b^2x(a+bx)}+\frac{\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(a + b*x)^2,x]
```

```
[Out] (a*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}}{(a+bx)^2} \, dx &= \frac{\sqrt{cx^2} \int \frac{x}{(a+bx)^2} \, dx}{x}\\
&= \frac{\sqrt{cx^2} \int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right) \, dx}{x}\\
&= \frac{a\sqrt{cx^2}}{b^2x(a+bx)}+\frac{\sqrt{cx^2}\log(a+bx)}{b^2x}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 36, normalized size = 0.77

$$\frac{cx((a+bx)\log(a+bx)+a)}{b^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(a + b*x)^2,x]
[Out] (c*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 39, normalized size = 0.83

$$\sqrt{cx^2}\left(\frac{a}{b^2x(a+bx)}+\frac{\log(a+bx)}{b^2x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(a + b*x)^2,x]
[Out] Sqrt[c*x^2]*(a/(b^2*x*(a + b*x)) + Log[a + b*x]/(b^2*x))
```

**fricas** [A] time = 0.67, size = 38, normalized size = 0.81

$$\frac{\sqrt{cx^2}\left((bx+a)\log(bx+a)+a\right)}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*x^2 + a*b^2*x)
```

**giac** [A] time = 1.06, size = 46, normalized size = 0.98

$$-\sqrt{c}\left(\frac{\left(\log(|a|)+1\right)\mathrm{sgn}(x)}{b^2}-\frac{\log(|bx+a|)\,\mathrm{sgn}(x)}{b^2}-\frac{a\mathrm{sgn}(x)}{(bx+a)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
[Out] -sqrt(c)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*s
gn(x)/((b*x + a)*b^2))
```

**maple [A]** time = 0.01, size = 41, normalized size = 0.87

$$\frac{\sqrt{c\,x^{2}}\,\left(bx\ln\left(bx+a\right)+a\ln\left(bx+a\right)+a\right)}{\left(bx+a\right)b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] (c*x^2)^(1/2)*(b*x*ln(b*x+a)+a*ln(b*x+a)+a)/x/b^2/(b*x+a)
```

**maxima [A]** time = 1.47, size = 79, normalized size = 1.68

$$\frac{\left(-1\right)^{\frac{2\,cx}{b}}\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^{2}}+\frac{\left(-1\right)^{\frac{2\,acx}{b}}\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^{2}}-\frac{\sqrt{cx^{2}}}{b^{2}x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*sqrt(c)*log(2*c*x/b)/b^2 + (-1)^(2*a*c*x/b)*sqrt(c)*log(-2*a
*c*x/(b*abs(b*x + a)))/b^2 - sqrt(c*x^2)/(b^2*x + a*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(1/2)/(a + b*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^{2}}}{\left(a+bx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(a + b*x)**2, x)
```

## 3.855 $\int \frac{\sqrt{cx^2}}{x(a+bx)^2} dx$

**Optimal**. Leaf size=24

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

**Rubi** [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(b*x*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{cx^2}}{x(a+bx)^2} dx &= \frac{\sqrt{cx^2} \int \frac{1}{(a+bx)^2} dx}{x} \\ &= -\frac{\sqrt{cx^2}}{bx(a+bx)} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.96

$$-\frac{cx}{b\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
[Out] -((c*x)/(b*Sqrt[c*x^2]*(a + b*x)))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 24, normalized size = 1.00

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
[Out] -(Sqrt[c*x^2]/(b*x*(a + b*x)))
```

**fricas [A]** time = 1.25, size = 23, normalized size = 0.96

$$-\frac{\sqrt{cx^2}}{b^2x^2+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="fricas")
[Out] -sqrt(c*x^2)/(b^2*x^2 + a*b*x)
```

**giac [A]** time = 1.02, size = 29, normalized size = 1.21

$$-\sqrt{c}\left(\frac{\text{sgn}(x)}{(bx+a)b}-\frac{\text{sgn}(x)}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="giac")
[Out] -sqrt(c)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

**maple [A]** time = 0.00, size = 23, normalized size = 0.96

$$-\frac{\sqrt{c\,x^2}}{(bx+a)\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(1/2)/b/x/(b*x+a)
```

**maxima [A]** time = 1.36, size = 16, normalized size = 0.67

$$-\frac{\sqrt{c}}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c)/(b^2*x + a*b)
```

**mupad [B]** time = 0.16, size = 22, normalized size = 0.92

$$-\frac{\sqrt{c\,x^2}}{b\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x*(a + b*x)^2),x)
```

```
[Out] -(c*x^2)^(1/2)/(b*x*(a + b*x))
```

**sympy [A]** time = 0.82, size = 39, normalized size = 1.62

$$\begin{cases} -\frac{\sqrt{c}\,\sqrt{x^2}}{abx+b^2x^2} & \text{for}\ b \neq 0 \\ \frac{\sqrt{c}\,\sqrt{x^2}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x/(b*x+a)**2,x)
```

```
[Out] Piecewise((-sqrt(c)*sqrt(x**2)/(a*b*x + b**2*x**2), Ne(b, 0)), (sqrt(c)*sqr
t(x**2)/a**2, True))
```

## 3.856 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2} dx$

**Optimal**. Leaf size=65

$$-\frac{\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{\sqrt{cx^2}\log(x)}{a^2x}+\frac{\sqrt{cx^2}}{ax(a+bx)}$$

**Rubi [A]** time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{\sqrt{cx^2}\log(x)}{a^2x}+\frac{\sqrt{cx^2}}{ax(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]/(a*x*(a + b*x)) + (Sqrt[c*x^2]*Log[x])/(a^2*x) - (Sqrt[c*x^2]*L
og[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx}{x}$$
$$= \frac{\sqrt{cx^2}}{ax(a+bx)} + \frac{\sqrt{cx^2}\log(x)}{a^2x} - \frac{\sqrt{cx^2}\log(a+bx)}{a^2x}$$

**Mathematica [A]** time = 0.02, size = 45, normalized size = 0.69

$$\frac{cx(\log(x)(a+bx)-(a+bx)\log(a+bx)+a)}{a^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

```
[Out] (c*x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a +
 b*x))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 48, normalized size = 0.74

$$\sqrt{cx^2}\left(-\frac{\log(a+bx)}{a^2x} + \frac{\log(x)}{a^2x} + \frac{1}{ax(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*(1/(a*x*(a + b*x)) + Log[x]/(a^2*x) - Log[a + b*x]/(a^2*x))
```

**fricas [A]** time = 1.05, size = 42, normalized size = 0.65

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*x^2 + a^3*x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple [A]** time = 0.01, size = 52, normalized size = 0.80

$$\frac{\sqrt{c\,x^{2}}\,\left(bx\ln(x)-bx\ln\left(bx+a\right)+a\ln(x)-a\ln\left(bx+a\right)+a\right)}{\left(bx+a\right)a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^2/(b*x+a)^2,x)
```

```
[Out] (c*x^2)^(1/2)*(b*x*ln(x)-b*x*ln(b*x+a)+a*ln(x)-a*ln(b*x+a)+a)/x/a^2/(b*x+a)
```

**maxima [A]** time = 1.32, size = 38, normalized size = 0.58

$$\frac{\sqrt{c}}{abx+a^{2}}-\frac{\sqrt{c}\,\log\left(bx+a\right)}{a^{2}}+\frac{\sqrt{c}\,\log(x)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c)/(a*b*x + a^2) - sqrt(c)*log(b*x + a)/a^2 + sqrt(c)*log(x)/a^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{c\,x^{2}}}{x^{2}\,{\left(a+b\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^2*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^2*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**2/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**2*(a + b*x)**2), x)
```

## 3.857 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx$

**Optimal**. Leaf size=87

$$-\frac{2b\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\,\log(a+bx)}{a^3x}-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{\sqrt{cx^2}}{a^2x^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2b\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\,\log(a+bx)}{a^3x}-\frac{\sqrt{cx^2}}{a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(a^2*x^2)) - (b*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*Sqrt[c*
x^2]*Log[x])/(a^3*x) + (2*b*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{1}{x^2(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)dx}{x}$$
$$= -\frac{\sqrt{cx^2}}{a^2x^2} - \frac{b\sqrt{cx^2}}{a^2x(a+bx)} - \frac{2b\sqrt{cx^2}\log(x)}{a^3x} + \frac{2b\sqrt{cx^2}\log(a+bx)}{a^3x}$$

**Mathematica [A]** time = 0.03, size = 57, normalized size = 0.66

$$-\frac{c(a(a+2bx)+2bx\log(x)(a+bx)-2bx(a+bx)\log(a+bx))}{a^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*(a*(a + 2*b*x) + 2*b*x*(a + b*x)*Log[x] - 2*b*x*(a + b*x)*Log[a + b*x]
))/(a^3*Sqrt[c*x^2]*(a + b*x)))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 59, normalized size = 0.68

$$\sqrt{cx^2}\left(-\frac{2b\log(x)}{a^3x} + \frac{2b\log(a+bx)}{a^3x} + \frac{-a-2bx}{a^2x^2(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-a - 2*b*x)/(a^2*x^2*(a + b*x)) - (2*b*Log[x])/(a^3*x) + (2*b
*Log[a + b*x])/(a^3*x))
```

**fricas [A]** time = 1.04, size = 60, normalized size = 0.69

$$-\frac{\left(2\,abx + a^2 - 2\left(b^2x^2 + abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bx^3 + a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
x^3 + a^4*x^2)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple [A]** time = 0.01, size = 74, normalized size = 0.85

$$-\frac{\sqrt{c\,x^2}\,\left(2b^2x^2\ln(x)-2b^2x^2\ln\left(bx+a\right)+2abx\ln(x)-2abx\ln\left(bx+a\right)+2abx+a^2\right)}{\left(bx+a\right)a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^3/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(1/2)*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x*ln(x)-2*a*b*x*l
n(b*x+a)+2*a*b*x+a^2)/x^2/a^3/(b*x+a)
```

**maxima [A]** time = 1.40, size = 58, normalized size = 0.67

$$-\frac{2\,b\sqrt{c}\,x+a\sqrt{c}}{a^2bx^2+a^3x}+\frac{2\,b\sqrt{c}\,\log\left(bx+a\right)}{a^3}-\frac{2\,b\sqrt{c}\,\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*sqrt(c)*x + a*sqrt(c))/(a^2*b*x^2 + a^3*x) + 2*b*sqrt(c)*log(b*x + a)
/a^3 - 2*b*sqrt(c)*log(x)/a^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2}}{x^3\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^3*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^3*(a + b*x)^2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**3/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**3*(a + b*x)**2), x)
```

## 3.858 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx$

**Optimal**. Leaf size=112

$$\frac{3b^2\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\,\log(a+bx)}{a^4x}+\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{2b\sqrt{cx^2}}{a^3x^2}-\frac{\sqrt{cx^2}}{2a^2x^3}$$

**Rubi** [A] time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\,\log(a+bx)}{a^4x}+\frac{2b\sqrt{cx^2}}{a^3x^2}-\frac{\sqrt{cx^2}}{2a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] -Sqrt[c*x^2]/(2*a^2*x^3) + (2*b*Sqrt[c*x^2])/(a^3*x^2) + (b^2*Sqrt[c*x^2])/
(a^3*x*(a + b*x)) + (3*b^2*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2*Sqrt[c*x^2]
*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{1}{x^3(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x^3} - \frac{2b}{a^3x^2} + \frac{3b^2}{a^4x} - \frac{b^3}{a^3(a+bx)^2} - \frac{3b^3}{a^4(a+bx)}\right)dx}{x}$$
$$= -\frac{\sqrt{cx^2}}{2a^2x^3} + \frac{2b\sqrt{cx^2}}{a^3x^2} + \frac{b^2\sqrt{cx^2}}{a^3x(a+bx)} + \frac{3b^2\sqrt{cx^2}\log(x)}{a^4x} - \frac{3b^2\sqrt{cx^2}\log(a+bx)}{a^4x}$$

**Mathematica [A]** time = 0.02, size = 82, normalized size = 0.73

$$\frac{\sqrt{cx^2}\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] (Sqrt[c*x^2]*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] -
 6*b^2*x^2*(a + b*x)*Log[a + b*x]))/(2*a^4*x^3*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 77, normalized size = 0.69

$$\sqrt{cx^2}\left(\frac{3b^2\log(x)}{a^4x} - \frac{3b^2\log(a+bx)}{a^4x} + \frac{-a^2+3abx+6b^2x^2}{2a^3x^3(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-a^2 + 3*a*b*x + 6*b^2*x^2)/(2*a^3*x^3*(a + b*x)) + (3*b^2*Lo
g[x])/(a^4*x) - (3*b^2*Log[a + b*x])/(a^4*x))
```

**fricas [A]** time = 1.16, size = 77, normalized size = 0.69

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\left(b^3x^3+ab^2x^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple [A]** time = 0.02, size = 95, normalized size = 0.85

$$\frac{\sqrt{c\,x^{2}}\,\left(6b^{3}x^{3}\ln(x)-6b^{3}x^{3}\ln\left(bx+a\right)+6a\,b^{2}x^{2}\ln(x)-6a\,b^{2}x^{2}\ln\left(bx+a\right)+6a\,b^{2}x^{2}+3a^{2}bx-a^{3}\right)}{2\left(bx+a\right)a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^4/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*ln(x)*x^2*a*b^2-6*
ln(b*x+a)*x^2*a*b^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/x^3/a^4/(b*x+a)
```

**maxima [A]** time = 1.42, size = 79, normalized size = 0.71

$$\frac{6\,b^{2}\sqrt{c}\,x^{2}+3\,ab\sqrt{c}\,x-a^{2}\sqrt{c}}{2\left(a^{3}bx^{3}+a^{4}x^{2}\right)}-\frac{3\,b^{2}\sqrt{c}\,\log\left(bx+a\right)}{a^{4}}+\frac{3\,b^{2}\sqrt{c}\,\log(x)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*sqrt(c)*x^2 + 3*a*b*sqrt(c)*x - a^2*sqrt(c))/(a^3*b*x^3 + a^4*x^
2) - 3*b^2*sqrt(c)*log(b*x + a)/a^4 + 3*b^2*sqrt(c)*log(x)/a^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^{2}}}{x^{4}\,{\left(a+b\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^4*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^4*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^4 (a + bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**4/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**4*(a + b*x)**2), x)
```

## 3.859 $\int \frac{x(cx^2)^{3/2}}{(a+bx)^2} dx$

**Optimal**. Leaf size=111

$$-\frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}+\frac{3a^2c\sqrt{cx^2}}{b^4}-\frac{acx\sqrt{cx^2}}{b^3}+\frac{cx^2\sqrt{cx^2}}{3b^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)}+\frac{3a^2c\sqrt{cx^2}}{b^4}-\frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{acx\sqrt{cx^2}}{b^3}+\frac{cx^2\sqrt{cx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

```
[Out] (3*a^2*c*Sqrt[c*x^2])/b^4 - (a*c*x*Sqrt[c*x^2])/b^3 + (c*x^2*Sqrt[c*x^2])/(
3*b^2) - (a^4*c*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*c*Sqrt[c*x^2]*Log[a
 + b*x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x\left(cx^2\right)^{3/2}}{(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^4}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{3a^2}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{b^2} + \frac{a^4}{b^4(a+bx)^2} - \frac{4a^3}{b^4(a+bx)}\right)dx}{x}$$
$$= \frac{3a^2c\sqrt{cx^2}}{b^4} - \frac{acx\sqrt{cx^2}}{b^3} + \frac{cx^2\sqrt{cx^2}}{3b^2} - \frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)} - \frac{4a^3c\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

**Mathematica [A]** time = 0.02, size = 82, normalized size = 0.74

$$\frac{\left(cx^2\right)^{3/2}\left(-3a^4+9a^3bx-12a^3(a+bx)\log(a+bx)+6a^2b^2x^2-2ab^3x^3+b^4x^4\right)}{3b^5x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

```
[Out] ((c*x^2)^(3/2)*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4
- 12*a^3*(a + b*x)*Log[a + b*x]))/(3*b^5*x^3*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 85, normalized size = 0.77

$$\left(cx^2\right)^{3/2}\left(\frac{-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4}{3b^5x^3(a+bx)} - \frac{4a^3\log(a+bx)}{b^5x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

```
[Out] (c*x^2)^(3/2)*((-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4)
/(3*b^5*x^3*(a + b*x)) - (4*a^3*Log[a + b*x])/(b^5*x^3))
```

**fricas [A]** time = 1.10, size = 91, normalized size = 0.82

$$\frac{\left(b^4cx^4 - 2\,ab^3cx^3 + 6\,a^2b^2cx^2 + 9\,a^3bcx - 3\,a^4c - 12\left(a^3bcx + a^4c\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{3\left(b^6x^2 + ab^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*c*x^4 - 2*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 + 9*a^3*b*c*x - 3*a^4*c -
12*(a^3*b*c*x + a^4*c)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

**giac** [A] time = 1.17, size = 96, normalized size = 0.86

$$-\frac{1}{3}\,c^{\frac{3}{2}}\left(\frac{12\,a^{3}\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^{5}}+\frac{3\,a^{4}\mathrm{sgn}(x)}{(bx+a)b^{5}}-\frac{3\left(4\,a^{3}\log\left(|a|\right)+a^{3}\right)\mathrm{sgn}(x)}{b^{5}}-\frac{b^{4}x^{3}\mathrm{sgn}(x)-3\,ab^{3}x^{2}\mathrm{sgn}(x)+9\,a^{2}b^{2}x\mathrm{sgn}(x)}{b^{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*c^(3/2)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a)
*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^3*
x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

**maple** [A] time = 0.01, size = 88, normalized size = 0.79

$$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(-b^{4}x^{4}+2a\,b^{3}x^{3}+12a^{3}bx\ln\left(bx+a\right)-6a^{2}b^{2}x^{2}+12a^{4}\ln\left(bx+a\right)-9a^{3}bx+3a^{4}\right)}{3\left(bx+a\right)b^{5}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -1/3*(c*x^2)^(3/2)*(-b^4*x^4+2*a*b^3*x^3+12*a^3*b*x*ln(b*x+a)-6*a^2*b^2*x^2
+12*a^4*ln(b*x+a)-9*a^3*b*x+3*a^4)/x^3/b^5/(b*x+a)
```

**maxima** [A] time = 1.62, size = 132, normalized size = 1.19

$$\frac{\left(cx^{2}\right)^{\frac{3}{2}}a}{b^{3}x+ab^{2}}-\frac{4\left(-1\right)^{\frac{2cx}{b}}a^{3}c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^{5}}-\frac{4\left(-1\right)^{\frac{2acx}{b}}a^{3}c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^{5}}-\frac{2\sqrt{cx^{2}}\,acx}{b^{3}}+\frac{\left(cx^{2}\right)^{\frac{3}{2}}}{3\,b^{2}}+\frac{4\sqrt{cx^{2}}\,a^{2}c}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (c*x^2)^(3/2)*a/(b^3*x + a*b^2) - 4*(-1)^(2*c*x/b)*a^3*c^(3/2)*log(2*c*x/b)
/b^5 - 4*(-1)^(2*a*c*x/b)*a^3*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^5 -
2*sqrt(c*x^2)*a*c*x/b^3 + 1/3*(c*x^2)^(3/2)/b^2 + 4*sqrt(c*x^2)*a^2*c/b^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(c\,x^{2}\right)^{3/2}}{\left(a+b\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(3/2))/(a + b*x)^2,x)
```

```
[Out] int((x*(c*x^2)^(3/2))/(a + b*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(cx^2\right)^{\frac{3}{2}}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)/(a + b*x)**2, x)
```

## 3.860 $\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=89

$$\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\,\log(a+bx)}{b^4x}-\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}$$

**Rubi [A]** time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\,\log(a+bx)}{b^4x}-\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (-2*a*c*Sqrt[c*x^2])/b^3 + (c*x*Sqrt[c*x^2])/(2*b^2) + (a^3*c*Sqrt[c*x^2])/
(b^4*x*(a + b*x)) + (3*a^2*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^3}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{x}$$
$$= -\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}+\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\log(a+bx)}{b^4x}$$

**Mathematica [A]** time = 0.02, size = 71, normalized size = 0.80

$$\frac{(cx^2)^{3/2}\left(2a^3-4a^2bx+6a^2(a+bx)\log(a+bx)-3ab^2x^2+b^3x^3\right)}{2b^4x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] ((c*x^2)^(3/2)*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)
*Log[a + b*x]))/(2*b^4*x^3*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 74, normalized size = 0.83

$$(cx^2)^{3/2}\left(\frac{3a^2\log(a+bx)}{b^4x^3}+\frac{2a^3-4a^2bx-3ab^2x^2+b^3x^3}{2b^4x^3(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (c*x^2)^(3/2)*((2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3)/(2*b^4*x^3*(a +
b*x)) + (3*a^2*Log[a + b*x])/(b^4*x^3))
```

**fricas [A]** time = 1.10, size = 79, normalized size = 0.89

$$\frac{\left(b^3cx^3-3\,ab^2cx^2-4\,a^2bcx+2\,a^3c+6\left(a^2bcx+a^3c\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\left(b^5x^2+ab^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*c*x^3 - 3*a*b^2*c*x^2 - 4*a^2*b*c*x + 2*a^3*c + 6*(a^2*b*c*x + a^3
*c)*log(b*x + a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)
```

**giac [A]** time = 0.95, size = 80, normalized size = 0.90

$$\frac{1}{2}c^{\frac{3}{2}}\left(\frac{6a^2\log(|bx+a|)\operatorname{sgn}(x)}{b^4}+\frac{2a^3\operatorname{sgn}(x)}{(bx+a)b^4}-\frac{2\left(3a^2\log(|a|)+a^2\right)\operatorname{sgn}(x)}{b^4}+\frac{b^2x^2\operatorname{sgn}(x)-4abx\operatorname{sgn}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^(3/2)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*b
^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*sg
n(x))/b^4)
```

**maple [A]** time = 0.00, size = 76, normalized size = 0.85

$$\frac{\left(cx^2\right)^{\frac{3}{2}}\left(b^3x^3+6a^2bx\ln(bx+a)-3ab^2x^2+6a^3\ln(bx+a)-4a^2bx+2a^3\right)}{2(bx+a)b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(b^3*x^3+6*a^2*b*x*ln(b*x+a)-3*a*b^2*x^2+6*a^3*ln(b*x+a)-
4*a^2*b*x+2*a^3)/x^3/b^4/(b*x+a)
```

**maxima [A]** time = 1.55, size = 115, normalized size = 1.29

$$\frac{3(-1)^{\frac{2cx}{b}}a^2c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^4}+\frac{3(-1)^{\frac{2acx}{b}}a^2c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^4}-\frac{\left(cx^2\right)^{\frac{3}{2}}}{b^2x+ab}+\frac{3\sqrt{cx^2}\,cx}{2b^2}-\frac{3\sqrt{cx^2}\,ac}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3*(-1)^(2*c*x/b)*a^2*c^(3/2)*log(2*c*x/b)/b^4 + 3*(-1)^(2*a*c*x/b)*a^2*c^(3
/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - (c*x^2)^(3/2)/(b^2*x + a*b) + 3/2*
sqrt(c*x^2)*c*x/b^2 - 3*sqrt(c*x^2)*a*c/b^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(cx^2\right)^{3/2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(a + b*x)^2,x)
[Out] int((c*x^2)^(3/2)/(a + b*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/(b*x+a)**2,x)
[Out] Integral((c*x**2)**(3/2)/(a + b*x)**2, x)
```

## 3.861 $\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx$

**Optimal**. Leaf size=68

$$-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\,\log(a+bx)}{b^3x}+\frac{c\sqrt{cx^2}}{b^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\,\log(a+bx)}{b^3x}+\frac{c\sqrt{cx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

```
[Out] (c*Sqrt[c*x^2])/b^2 - (a^2*c*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*c*Sqrt[c
*x^2]*Log[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^2}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{x}$$
$$= \frac{c\sqrt{cx^2}}{b^2}-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

**Mathematica** [A] time = 0.01, size = 55, normalized size = 0.81

$$\frac{c^2x\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

```
[Out] (c^2*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x
^2]*(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 57, normalized size = 0.84

$$(cx^2)^{3/2}\left(\frac{-a^2+abx+b^2x^2}{b^3x^3(a+bx)}-\frac{2a\log(a+bx)}{b^3x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

```
[Out] (c*x^2)^(3/2)*((-a^2 + a*b*x + b^2*x^2)/(b^3*x^3*(a + b*x)) - (2*a*Log[a +
b*x])/(b^3*x^3))
```

**fricas** [A] time = 1.26, size = 63, normalized size = 0.93

$$\frac{\left(b^2cx^2+abcx-a^2c-2\left(abcx+a^2c\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4x^2+ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x^2 + a*b*c*x - a^2*c - 2*(a*b*c*x + a^2*c)*log(b*x + a))*sqrt(c*x^2
)/(b^4*x^2 + a*b^3*x)
```

**giac** [A] time = 0.96, size = 58, normalized size = 0.85

$$c^{\frac{3}{2}}\left(\frac{x\mathrm{sgn}(x)}{b^2} - \frac{2\,a\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3} + \frac{\left(2\,a\log\left(|a|\right)+a\right)\mathrm{sgn}(x)}{b^3} - \frac{a^2\mathrm{sgn}(x)}{(bx+a)b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] c^(3/2)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a))
 + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

**maple** [A] time = 0.01, size = 62, normalized size = 0.91

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(2abx\ln\left(bx+a\right)-b^2x^2+2a^2\ln\left(bx+a\right)-abx+a^2\right)}{\left(bx+a\right)b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(3/2)*(2*a*b*x*ln(b*x+a)-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/x^3/b^
3/(b*x+a)
```

**maxima** [A] time = 1.42, size = 98, normalized size = 1.44

$$\frac{\sqrt{cx^2}\,ac}{b^3x+ab^2} - \frac{2\,(-1)^{\frac{2\,cx}{b}}\,ac^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3} - \frac{2\,(-1)^{\frac{2\,acx}{b}}\,ac^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3} + \frac{\sqrt{cx^2}\,c}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a*c/(b^3*x + a*b^2) - 2*(-1)^(2*c*x/b)*a*c^(3/2)*log(2*c*x/b)/b
^3 - 2*(-1)^(2*a*c*x/b)*a*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + sqrt
(c*x^2)*c/b^2
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x*(a + b*x)^2),x)
[Out] int((c*x^2)^(3/2)/(x*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x/(b*x+a)**2,x)
[Out] Integral((c*x**2)**(3/2)/(x*(a + b*x)**2), x)
```

## 3.862 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2} \, dx$

**Optimal**. Leaf size=49

$$\frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}$$

**Rubi [A]** time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
```

```
[Out] (a*c*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2} \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x}{(a+bx)^2} \, dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right) dx}{x}\\
&= \frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 38, normalized size = 0.78

$$\frac{c^2x((a+bx)\log(a+bx)+a)}{b^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
[Out] (c^2*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 39, normalized size = 0.80

$$\left(cx^2\right)^{3/2}\left(\frac{a}{b^2x^3(a+bx)}+\frac{\log(a+bx)}{b^2x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
[Out] (c*x^2)^(3/2)*(a/(b^2*x^3*(a + b*x)) + Log[a + b*x]/(b^2*x^3))
```

**fricas** [A] time = 1.41, size = 43, normalized size = 0.88

$$\frac{\sqrt{cx^2}\left(ac+(bcx+ac)\log\left(bx+a\right)\right)}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="fricas")
[Out] sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(b*x + a))/(b^3*x^2 + a*b^2*x)
```

**giac** [A] time = 0.96, size = 46, normalized size = 0.94

$$-c^{\frac{3}{2}}\left(\frac{\left(\log\left(|a|\right)+1\right)\mathrm{sgn}(x)}{b^2}-\frac{\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^2}-\frac{a\mathrm{sgn}(x)}{(bx+a)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="giac")
[Out] -c^(3/2)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*s
gn(x)/((b*x + a)*b^2))
```

**maple [A]** time = 0.00, size = 41, normalized size = 0.84

$$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}} \left(bx\ln\left(bx+a\right)+a\ln\left(bx+a\right)+a\right)}{\left(bx+a\right)b^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^2/(b*x+a)^2,x)
```

```
[Out] (c*x^2)^(3/2)*(b*x*ln(b*x+a)+a*ln(b*x+a)+a)/x^3/b^2/(b*x+a)
```

**maxima [A]** time = 1.48, size = 80, normalized size = 1.63

$$\frac{\left(-1\right)^{\frac{2cx}{b}}c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^{2}}+\frac{\left(-1\right)^{\frac{2acx}{b}}c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^{2}}-\frac{\sqrt{cx^{2}}\,c}{b^{2}x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*c^(3/2)*log(2*c*x/b)/b^2 + (-1)^(2*a*c*x/b)*c^(3/2)*log(-2*a
*c*x/(b*abs(b*x + a)))/b^2 - sqrt(c*x^2)*c/(b^2*x + a*b)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^2\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^2*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^2*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{3}{2}}}{x^{2}\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**2/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**2*(a + b*x)**2), x)
```

## 3.863 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2} \, dx$

**Optimal**. Leaf size=25

$$-\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

**Rubi [A]** time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$-\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(b*x*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2} \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{(a+bx)^2} \, dx}{x}\\ &= -\frac{c\sqrt{cx^2}}{bx(a+bx)}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 0.96

$$-\frac{(cx^2)^{3/2}}{bx^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*x^2)^(3/2)/(b*x^3*(a + b*x)))
```

**IntegrateAlgebraic [A]** time = 0.03, size = 24, normalized size = 0.96

$$-\frac{(cx^2)^{3/2}}{bx^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*x^2)^(3/2)/(b*x^3*(a + b*x)))
```

**fricas [A]** time = 1.09, size = 24, normalized size = 0.96

$$-\frac{\sqrt{cx^2}\,c}{b^2x^2+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)*c/(b^2*x^2 + a*b*x)
```

**giac [A]** time = 1.14, size = 29, normalized size = 1.16

$$-c^{\frac{3}{2}}\left(\frac{\operatorname{sgn}(x)}{(bx+a)b}-\frac{\operatorname{sgn}(x)}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -c^(3/2)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

**maple [A]** time = 0.00, size = 23, normalized size = 0.92

$$-\frac{(c\,x^2)^{\frac{3}{2}}}{(bx+a)\,b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^3/(b*x+a)^2,x)
```

[Out] -1/(b*x+a)/b*(c*x^2)^(3/2)/x^3

**maxima [A]** time = 1.33, size = 16, normalized size = 0.64

$$-\frac{c^{\frac{3}{2}}}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="maxima")

[Out] -c^(3/2)/(b^2*x + a*b)

**mupad [B]** time = 0.15, size = 24, normalized size = 0.96

$$-\frac{c^{3/2}\sqrt{x^2}}{b^2x^2 + abx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2)^(3/2)/(x^3*(a + b*x)^2),x)

[Out] -(c^(3/2)*(x^2)^(1/2))/(b^2*x^2 + a*b*x)

**sympy [A]** time = 2.23, size = 44, normalized size = 1.76

$$\begin{cases} -\frac{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{abx^3+b^2x^4} & \text{for } b \neq 0 \\ \frac{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{a^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(3/2)/x**3/(b*x+a)**2,x)

[Out] Piecewise((-c**(3/2)*(x**2)**(3/2)/(a*b*x**3 + b**2*x**4), Ne(b, 0)), (c**(3/2)*(x**2)**(3/2)/(a**2*x**2), True))

## 3.864 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx$

**Optimal**. Leaf size=68

$$-\frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{c\sqrt{cx^2}\log(x)}{a^2x}+\frac{c\sqrt{cx^2}}{ax(a+bx)}$$

**Rubi [A]** time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{c\sqrt{cx^2}\log(x)}{a^2x}+\frac{c\sqrt{cx^2}}{ax(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

```
[Out] (c*Sqrt[c*x^2])/(a*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[x])/(a^2*x) - (c*Sqrt[
c*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx}{x}$$
$$= \frac{c\sqrt{cx^2}}{ax(a+bx)} + \frac{c\sqrt{cx^2}\log(x)}{a^2x} - \frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 0.68

$$\frac{(cx^2)^{3/2}(\log(x)(a+bx)-(a+bx)\log(a+bx)+a)}{a^2x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

```
[Out] ((c*x^2)^(3/2)*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*x^3*(a
 + b*x))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 48, normalized size = 0.71

$$(cx^2)^{3/2}\left(-\frac{\log(a+bx)}{a^2x^3} + \frac{\log(x)}{a^2x^3} + \frac{1}{ax^3(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

```
[Out] (c*x^2)^(3/2)*(1/(a*x^3*(a + b*x)) + Log[x]/(a^2*x^3) - Log[a + b*x]/(a^2*x
^3))
```

**fricas** [A] time = 1.12, size = 47, normalized size = 0.69

$$\frac{\sqrt{cx^2}\left(ac+(bcx+ac)\log\left(\frac{x}{bx+a}\right)\right)}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(x/(b*x + a)))/(a^2*b*x^2 + a^3*x)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple** [A] time = 0.01, size = 52, normalized size = 0.76

$$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(bx\ln(x)-bx\ln\left(bx+a\right)+a\ln(x)-a\ln\left(bx+a\right)+a\right)}{\left(bx+a\right)a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^4/(b*x+a)^2,x)
```

```
[Out] (c*x^2)^(3/2)*(b*x*ln(x)-b*x*ln(b*x+a)+a*ln(x)-a*ln(b*x+a)+a)/x^3/a^2/(b*x+
a)
```

**maxima** [A] time = 1.40, size = 38, normalized size = 0.56

$$\frac{c^{\frac{3}{2}}}{abx+a^{2}}-\frac{c^{\frac{3}{2}}\log\left(bx+a\right)}{a^{2}}+\frac{c^{\frac{3}{2}}\log(x)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] c^(3/2)/(a*b*x + a^2) - c^(3/2)*log(b*x + a)/a^2 + c^(3/2)*log(x)/a^2
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c\,x^{2}\right)^{3/2}}{x^{4}\,\left(a+b\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^4*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^4*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^4\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**4/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**4*(a + b*x)**2), x)
```

## 3.865 $\int \frac{\left(cx^2\right)^{3/2}}{x^5(a+bx)^2}\,dx$

**Optimal**. Leaf size=91

$$-\frac{2bc\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\,\log(a+bx)}{a^3x}-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{c\sqrt{cx^2}}{a^2x^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2bc\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\,\log(a+bx)}{a^3x}-\frac{c\sqrt{cx^2}}{a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(a^2*x^2)) - (b*c*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*c
*Sqrt[c*x^2]*Log[x])/(a^3*x) + (2*b*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{a^2x^2}-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2bc\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\,\log(a+bx)}{a^3x}$$

**Mathematica [A]** time = 0.02, size = 59, normalized size = 0.65

$$-\frac{c^2(a(a+2bx)+2bx\log(x)(a+bx)-2bx(a+bx)\log(a+bx))}{a^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

```
[Out] -((c^2*(a*(a + 2*b*x) + 2*b*x*(a + b*x)*Log[x] - 2*b*x*(a + b*x)*Log[a + b*
x]))/(a^3*Sqrt[c*x^2]*(a + b*x)))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 59, normalized size = 0.65

$$(cx^2)^{3/2}\left(-\frac{2b\log(x)}{a^3x^3}+\frac{2b\log(a+bx)}{a^3x^3}+\frac{-a-2bx}{a^2x^4(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

```
[Out] (c*x^2)^(3/2)*((-a - 2*b*x)/(a^2*x^4*(a + b*x)) - (2*b*Log[x])/(a^3*x^3) +
(2*b*Log[a + b*x])/(a^3*x^3))
```

**fricas [A]** time = 1.16, size = 65, normalized size = 0.71

$$-\frac{\left(2\,abcx+a^2c-2\left(b^2cx^2+abcx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bx^3+a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*c*x + a^2*c - 2*(b^2*c*x^2 + a*b*c*x)*log((b*x + a)/x))*sqrt(c*x^2)
/(a^3*b*x^3 + a^4*x^2)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple [A]** time = 0.01, size = 74, normalized size = 0.81

$$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(2b^{2}x^{2}\ln(x)-2b^{2}x^{2}\ln\left(bx+a\right)+2abx\ln(x)-2abx\ln\left(bx+a\right)+2abx+a^{2}\right)}{\left(bx+a\right)a^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^5/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(3/2)*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x*ln(x)-2*a*b*x*l
n(b*x+a)+2*a*b*x+a^2)/x^4/a^3/(b*x+a)
```

**maxima [A]** time = 1.43, size = 58, normalized size = 0.64

$$\frac{2\,bc^{\frac{3}{2}}\log\left(bx+a\right)}{a^{3}}-\frac{2\,bc^{\frac{3}{2}}\log(x)}{a^{3}}-\frac{2\,bc^{\frac{3}{2}}x+ac^{\frac{3}{2}}}{a^{2}bx^{2}+a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*b*c^(3/2)*log(b*x + a)/a^3 - 2*b*c^(3/2)*log(x)/a^3 - (2*b*c^(3/2)*x + a*
c^(3/2))/(a^2*b*x^2 + a^3*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{x^5\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^5*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^5*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^5 \left(a + bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**5/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**5*(a + b*x)**2), x)
```

## 3.866 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx$

**Optimal**. Leaf size=117

$$\frac{3b^2c\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\,\log(a+bx)}{a^4x}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{2bc\sqrt{cx^2}}{a^3x^2}-\frac{c\sqrt{cx^2}}{2a^2x^3}$$

**Rubi [A]** time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\,\log(a+bx)}{a^4x}+\frac{2bc\sqrt{cx^2}}{a^3x^2}-\frac{c\sqrt{cx^2}}{2a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

```
[Out] -(c*Sqrt[c*x^2])/(2*a^2*x^3) + (2*b*c*Sqrt[c*x^2])/(a^3*x^2) + (b^2*c*Sqrt[
c*x^2])/(a^3*x*(a + b*x)) + (3*b^2*c*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2*c
*Sqrt[c*x^2]*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^3(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{2a^2x^3}+\frac{2bc\sqrt{cx^2}}{a^3x^2}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\,\log(a+bx)}{a^4x}$$

**Mathematica [A]** time = 0.02, size = 82, normalized size = 0.70

$$\frac{(cx^2)^{3/2}\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4x^5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

```
[Out] ((c*x^2)^(3/2)*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x]
 - 6*b^2*x^2*(a + b*x)*Log[a + b*x]))/(2*a^4*x^5*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 77, normalized size = 0.66

$$(cx^2)^{3/2}\left(\frac{3b^2\log(x)}{a^4x^3}-\frac{3b^2\log(a+bx)}{a^4x^3}+\frac{-a^2+3abx+6b^2x^2}{2a^3x^5(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

```
[Out] (c*x^2)^(3/2)*((-a^2 + 3*a*b*x + 6*b^2*x^2)/(2*a^3*x^5*(a + b*x)) + (3*b^2*
Log[x])/(a^4*x^3) - (3*b^2*Log[a + b*x])/(a^4*x^3))
```

**fricas [A]** time = 0.93, size = 82, normalized size = 0.70

$$\frac{\left(6\,ab^2cx^2+3\,a^2bcx-a^3c+6\left(b^3cx^3+ab^2cx^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*c*x^2 + 3*a^2*b*c*x - a^3*c + 6*(b^3*c*x^3 + a*b^2*c*x^2)*log(
x/(b*x + a)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

**maple [A]** time = 0.01, size = 95, normalized size = 0.81

$$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(6b^{3}x^{3}\ln(x)-6b^{3}x^{3}\ln\left(bx+a\right)+6a\,b^{2}x^{2}\ln(x)-6a\,b^{2}x^{2}\ln\left(bx+a\right)+6a\,b^{2}x^{2}+3a^{2}bx-a^{3}\right)}{2\left(bx+a\right)a^{4}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^6/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*a*b^2*x^2*ln(x)-6*
a*b^2*x^2*ln(b*x+a)+6*a*b^2*x^2+3*a^2*b*x-a^3)/x^5/a^4/(b*x+a)
```

**maxima [A]** time = 1.33, size = 79, normalized size = 0.68

$$-\frac{3\,b^{2}c^{\frac{3}{2}}\log\left(bx+a\right)}{a^{4}}+\frac{3\,b^{2}c^{\frac{3}{2}}\log(x)}{a^{4}}+\frac{6\,b^{2}c^{\frac{3}{2}}x^{2}+3\,abc^{\frac{3}{2}}x-a^{2}c^{\frac{3}{2}}}{2\left(a^{3}bx^{3}+a^{4}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3*b^2*c^(3/2)*log(b*x + a)/a^4 + 3*b^2*c^(3/2)*log(x)/a^4 + 1/2*(6*b^2*c^(
3/2)*x^2 + 3*a*b*c^(3/2)*x - a^2*c^(3/2))/(a^3*b*x^3 + a^4*x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{x^6\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^6*(a + b*x)^2),x)
[Out] int((c*x^2)^(3/2)/(x^6*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^6\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**6/(b*x+a)**2,x)
[Out] Integral((c*x**2)**(3/2)/(x**6*(a + b*x)**2), x)
```

## 3.867 $\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2} dx$

**Optimal**. Leaf size=107

$$-\frac{a^4x}{b^5\sqrt{cx^2}(a+bx)}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}+\frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^4x}{b^5\sqrt{cx^2}(a+bx)}+\frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (3*a^2*x^2)/(b^4*Sqrt[c*x^2]) - (a*x^3)/(b^3*Sqrt[c*x^2]) + x^4/(3*b^2*Sqrt
[c*x^2]) - (a^4*x)/(b^5*Sqrt[c*x^2]*(a + b*x)) - (4*a^3*x*Log[a + b*x])/(b^
5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{x^4}{(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}-\frac{a^4x}{b^5\sqrt{cx^2}\,(a+bx)}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.02, size = 80, normalized size = 0.75

$$\frac{x\left(-3a^4+9a^3bx-12a^3(a+bx)\log(a+bx)+6a^2b^2x^2-2ab^3x^3+b^4x^4\right)}{3b^5\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*(a
+ b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 91, normalized size = 0.85

$$\sqrt{cx^2}\left(\frac{-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4}{3b^5cx(a+bx)}-\frac{4a^3\log(a+bx)}{b^5cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4)/(
3*b^5*c*x*(a + b*x)) - (4*a^3*Log[a + b*x])/(b^5*c*x))
```

**fricas [A]** time = 0.97, size = 85, normalized size = 0.79

$$\frac{\left(b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4-12\left(a^3bx+a^4\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{3\left(b^6cx^2+ab^5cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*c*x^2 + a*b^5*c*x)
```

**giac [A]** time = 1.13, size = 155, normalized size = 1.45

$$\frac{\frac{(bx+a)^3\left(\frac{6a}{bx+a}-\frac{18a^2}{(bx+a)^2}-1\right)}{b^5\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{12a^3\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^5\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{3a^4}{(bx+a)b^5\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x + a)^3*(6*a/(b*x + a) - 18*a^2/(b*x + a)^2 - 1)/(b^5*sgn(-b/(b*x
+ a) + a*b/(b*x + a)^2)) - 12*a^3*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b
^5*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) + 3*a^4/((b*x + a)*b^5*sgn(-b/(b*x
+ a) + a*b/(b*x + a)^2)))/sqrt(c)
```

**maple [A]** time = 0.01, size = 86, normalized size = 0.80

$$-\frac{\left(-b^4x^4+2a\,b^3x^3+12a^3bx\ln(bx+a)-6a^2b^2x^2+12a^4\ln(bx+a)-9a^3bx+3a^4\right)x}{3\sqrt{c\,x^2}\,\left(bx+a\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -1/3*x*(-b^4*x^4+2*a*b^3*x^3+12*a^3*b*x*ln(b*x+a)-6*a^2*b^2*x^2+12*a^4*ln(b
*x+a)-9*a^3*b*x+3*a^4)/(c*x^2)^(1/2)/b^5/(b*x+a)
```

**maxima [A]** time = 1.56, size = 168, normalized size = 1.57

$$\frac{\sqrt{cx^2}\,a^3}{b^5cx+ab^4c}+\frac{\sqrt{cx^2}\,x^2}{3\,b^2c}-\frac{5\,ax^2}{3\,b^3\sqrt{c}}-\frac{4\,(-1)^{\frac{2\,acx}{b}}\,a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5\sqrt{c}}+\frac{2\,\sqrt{cx^2}\,ax}{3\,b^3c}-\frac{20\,a^2x}{3\,b^4\sqrt{c}}-\frac{4\,a^3\log(bx)}{b^5\sqrt{c}}+\frac{29\,\sqrt{cx^2}\,a^2}{3\,b^4c}-\frac{5\,a^3}{b^5\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a^3/(b^5*c*x + a*b^4*c) + 1/3*sqrt(c*x^2)*x^2/(b^2*c) - 5/3*a*x
^2/(b^3*sqrt(c)) - 4*(-1)^(2*a*c*x/b)*a^3*log(-2*a*c*x/(b*abs(b*x + a)))/(b
^5*sqrt(c)) + 2/3*sqrt(c*x^2)*a*x/(b^3*c) - 20/3*a^2*x/(b^4*sqrt(c)) - 4*a^
3*log(b*x)/(b^5*sqrt(c)) + 29/3*sqrt(c*x^2)*a^2/(b^4*c) - 5*a^3/(b^5*sqrt(c
))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(1/2)*(a + b*x)^2),x)
[Out] int(x^5/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{cx^2}\,(a+bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**2/(c*x**2)**(1/2),x)
[Out] Integral(x**5/(sqrt(c*x**2)*(a + b*x)**2), x)
```

## 3.868 $\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2} dx$

**Optimal**. Leaf size=86

$$\frac{a^3x}{b^4\sqrt{cx^2}(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^3x}{b^4\sqrt{cx^2}(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (-2*a*x^2)/(b^3*Sqrt[c*x^2]) + x^3/(2*b^2*Sqrt[c*x^2]) + (a^3*x)/(b^4*Sqrt[
c*x^2]*(a + b*x)) + (3*a^2*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{x^3}{(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}+\frac{a^3x}{b^4\sqrt{cx^2}\,(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.01, size = 69, normalized size = 0.80

$$\frac{x\left(2a^3-4a^2bx+6a^2(a+bx)\log(a+bx)-3ab^2x^2+b^3x^3\right)}{2b^4\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a + b*x
]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 80, normalized size = 0.93

$$\sqrt{cx^2}\left(\frac{3a^2\log(a+bx)}{b^4cx}+\frac{2a^3-4a^2bx-3ab^2x^2+b^3x^3}{2b^4cx(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3)/(2*b^4*c*x*(a + b*
x)) + (3*a^2*Log[a + b*x])/(b^4*c*x))
```

**fricas [A]** time = 0.99, size = 74, normalized size = 0.86

$$\frac{\left(b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\left(a^2bx+a^3\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\left(b^5cx^2+ab^4cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))*sqrt(c*x^2)/(b^5*c*x^2 + a*b^4*c*x)
```

**giac [A]** time = 1.04, size = 143, normalized size = 1.66

$$\frac{\frac{(bx+a)^2\left(\frac{6a}{bx+a}-1\right)}{b^4\text{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{6a^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4\text{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{2a^3}{(bx+a)b^4\text{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((b*x + a)^2*(6*a/(b*x + a) - 1)/(b^4*sgn(-b/(b*x + a) + a*b/(b*x + a)^
2)) + 6*a^2*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b^4*sgn(-b/(b*x + a) +
a*b/(b*x + a)^2)) - 2*a^3/((b*x + a)*b^4*sgn(-b/(b*x + a) + a*b/(b*x + a)^2
)))/sqrt(c)
```

**maple [A]** time = 0.00, size = 74, normalized size = 0.86

$$\frac{\left(b^3x^3+6a^2bx\ln(bx+a)-3a\,b^2x^2+6a^3\ln(bx+a)-4a^2bx+2a^3\right)x}{2\sqrt{c\,x^2}\,(bx+a)\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(b^3*x^3+6*a^2*b*x*ln(b*x+a)-3*a*b^2*x^2+6*a^3*ln(b*x+a)-4*a^2*b*x+2*
a^3)/(c*x^2)^(1/2)/b^4/(b*x+a)
```

**maxima [A]** time = 1.55, size = 129, normalized size = 1.50

$$-\frac{\sqrt{cx^2}\,a^2}{b^4cx+ab^3c}+\frac{x^2}{2\,b^2\sqrt{c}}+\frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4\sqrt{c}}+\frac{2\,ax}{b^3\sqrt{c}}+\frac{3\,a^2\log(bx)}{b^4\sqrt{c}}-\frac{4\sqrt{cx^2}\,a}{b^3c}+\frac{3\,a^2}{2\,b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)*a^2/(b^4*c*x + a*b^3*c) + 1/2*x^2/(b^2*sqrt(c)) + 3*(-1)^(2*a*
c*x/b)*a^2*log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*sqrt(c)) + 2*a*x/(b^3*sqrt(c
)) + 3*a^2*log(b*x)/(b^4*sqrt(c)) - 4*sqrt(c*x^2)*a/(b^3*c) + 3/2*a^2/(b^4*
sqrt(c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(1/2)*(a + b*x)^2),x)
[Out] int(x^4/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^2}\,\left(a+bx\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**2/(c*x**2)**(1/2),x)
[Out] Integral(x**4/(sqrt(c*x**2)*(a + b*x)**2), x)
```

## 3.869 $\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx$

**Optimal**. Leaf size=64

$$-\frac{a^2x}{b^3\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}+\frac{x^2}{b^2\sqrt{cx^2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2x}{b^3\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}+\frac{x^2}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] x^2/(b^2*Sqrt[c*x^2]) - (a^2*x)/(b^3*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Log[a
+ b*x])/(b^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{x\int \frac{x^2}{(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{x^2}{b^2\sqrt{cx^2}}-\frac{a^2x}{b^3\sqrt{cx^2}(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 52, normalized size = 0.81

$$\frac{x\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^2]*
(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.06, size = 63, normalized size = 0.98

$$\sqrt{cx^2}\left(\frac{-a^2+abx+b^2x^2}{b^3cx(a+bx)}-\frac{2a\log(a+bx)}{b^3cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-a^2 + a*b*x + b^2*x^2)/(b^3*c*x*(a + b*x)) - (2*a*Log[a + b*
x])/(b^3*c*x))
```

**fricas** [A] time = 1.05, size = 59, normalized size = 0.92

$$\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4cx^2+ab^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c*x
^2 + a*b^3*c*x)
```

**giac [B]** time = 1.13, size = 127, normalized size = 1.98

$$-\frac{\frac{2\,a\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{bx+a}{b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{a^2}{(bx+a)b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b^3*sgn(-b/(b*x + a) + a*b/(b
*x + a)^2)) + (b*x + a)/(b^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - a^2/((b
*x + a)*b^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/sqrt(c)
```

**maple [A]** time = 0.01, size = 60, normalized size = 0.94

$$-\frac{\left(2abx\ln(bx+a)-b^2x^2+2a^2\ln(bx+a)-abx+a^2\right)x}{\sqrt{c\,x^2}\;(bx+a)\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -x*(2*a*b*x*ln(b*x+a)-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/(c*x^2)^(1/2)/b^3/
(b*x+a)
```

**maxima [A]** time = 1.45, size = 88, normalized size = 1.38

$$\frac{\sqrt{cx^2}\,a}{b^3cx+ab^2c}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3\sqrt{c}}-\frac{2\,a\log(bx)}{b^3\sqrt{c}}+\frac{\sqrt{cx^2}}{b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a/(b^3*c*x + a*b^2*c) - 2*(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*ab
s(b*x + a)))/(b^3*sqrt(c)) - 2*a*log(b*x)/(b^3*sqrt(c)) + sqrt(c*x^2)/(b^2*
c)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{c\,x^2}\,{(a+b\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(x^3/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(c*x**2)*(a + b*x)**2), x)
```

## 3.870 $\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2} dx$

**Optimal**. Leaf size=43

$$\frac{ax}{b^2\sqrt{cx^2}(a+bx)} + \frac{x\log(a+bx)}{b^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{ax}{b^2\sqrt{cx^2}(a+bx)} + \frac{x\log(a+bx)}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (a*x)/(b^2*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2} dx &= \frac{x\int \frac{x}{(a+bx)^2} dx}{\sqrt{cx^2}} \\
&= \frac{x\int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right) dx}{\sqrt{cx^2}} \\
&= \frac{ax}{b^2\sqrt{cx^2}(a+bx)} + \frac{x\log(a+bx)}{b^2\sqrt{cx^2}}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.81

$$\frac{x((a+bx)\log(a+bx)+a)}{b^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] (x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 45, normalized size = 1.05

$$\sqrt{cx^2}\left(\frac{a}{b^2cx(a+bx)}+\frac{\log(a+bx)}{b^2cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] Sqrt[c*x^2]*(a/(b^2*c*x*(a + b*x)) + Log[a + b*x]/(b^2*c*x))
```

**fricas** [A] time = 1.20, size = 40, normalized size = 0.93

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(bx+a\right)+a\right)}{b^3cx^2+ab^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c*x^2 + a*b^2*c*x)
```

**giac** [B] time = 1.15, size = 89, normalized size = 2.07

$$\frac{\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{a}{(bx+a)b^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] (log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b^2*sgn(-b/(b*x + a) + a*b/(b*x +
a)^2)) - a/((b*x + a)*b^2*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/sqrt(c)
```

**maple** [A] time = 0.00, size = 39, normalized size = 0.91

$$\frac{(bx\ln(bx+a)+a\ln(bx+a)+a)\,x}{\sqrt{c\,x^2}\,(bx+a)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] x*(b*x*ln(b*x+a)+a*ln(b*x+a)+a)/(c*x^2)^(1/2)/b^2/(b*x+a)
```

**maxima** [A] time = 1.41, size = 68, normalized size = 1.58

$$-\frac{\sqrt{cx^2}}{b^2cx+abc}+\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2\sqrt{c}}+\frac{\log(bx)}{b^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)/(b^2*c*x + a*b*c) + (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x +
 a)))/(b^2*sqrt(c)) + log(b*x)/(b^2*sqrt(c))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^2}{\sqrt{c\,x^2}\,(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(x^2/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(c*x**2)*(a + b*x)**2), x)
```

## 3.871 $\int \frac{x}{\sqrt{cx^2}(a+bx)^2} dx$

**Optimal**. Leaf size=22

$$-\frac{x}{b\sqrt{cx^2}(a+bx)}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 32}

$$-\frac{x}{b\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{cx^2}(a+bx)^2} dx &= \frac{x \int \frac{1}{(a+bx)^2} dx}{\sqrt{cx^2}} \\ &= -\frac{x}{b\sqrt{cx^2}(a+bx)} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 22, normalized size = 1.00

$$-\frac{x}{b\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] -(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

**IntegrateAlgebraic [A]** time = 0.03, size = 27, normalized size = 1.23

$$-\frac{\sqrt{cx^2}}{bcx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] -(Sqrt[c*x^2]/(b*c*x*(a + b*x)))
```

**fricas [A]** time = 1.08, size = 25, normalized size = 1.14

$$-\frac{\sqrt{cx^2}}{b^2cx^2+abcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(c*x^2)/(b^2*c*x^2 + a*b*c*x)
```

**giac [A]** time = 1.16, size = 38, normalized size = 1.73

$$\frac{1}{(bx+a)b\sqrt{c}\,\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/((b*x + a)*b*sqrt(c)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2))
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.95

$$-\frac{x}{(bx+a)\sqrt{c\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -x/b/(b*x+a)/(c*x^2)^(1/2)
```

**maxima [A]** time = 1.45, size = 21, normalized size = 0.95

$$\frac{\sqrt{cx^2}}{abcx + a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(c*x^2)/(a*b*c*x + a^2*c)
```

**mupad [B]** time = 0.16, size = 25, normalized size = 1.14

$$-\frac{\sqrt{c\,x^2}}{b\,c\,x\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(1/2)*(a + b*x)^2),x)
[Out] -(c*x^2)^(1/2)/(b*c*x*(a + b*x))
```

**sympy [A]** time = 1.17, size = 85, normalized size = 3.86

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{c}\sqrt{x^2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{\tilde{\infty}x^2}{\sqrt{c}\sqrt{x^2}} & \text{for } a = -bx \\ \frac{x^2}{a^2\sqrt{c}\sqrt{x^2}} & \text{for } b = 0 \\ -\frac{x}{ab\sqrt{c}\sqrt{x^2} + b^2\sqrt{c}\,x\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**2/(c*x**2)**(1/2),x)
[Out] Piecewise((zoo/(sqrt(c)*sqrt(x**2)), Eq(a, 0) & Eq(b, 0)), (zoo*x**2/(sqrt(
c)*sqrt(x**2)), Eq(a, -b*x)), (x**2/(a**2*sqrt(c)*sqrt(x**2)), Eq(b, 0)), (
-x/(a*b*sqrt(c)*sqrt(x**2) + b**2*sqrt(c)*x*sqrt(x**2)), True))
```

## 3.872 $\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx$

**Optimal**. Leaf size=59

$$-\frac{x\log(a+bx)}{a^2\sqrt{cx^2}}+\frac{x\log(x)}{a^2\sqrt{cx^2}}+\frac{x}{a\sqrt{cx^2}\,(a+bx)}$$

**Rubi** [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 44}

$$-\frac{x\log(a+bx)}{a^2\sqrt{cx^2}}+\frac{x\log(x)}{a^2\sqrt{cx^2}}+\frac{x}{a\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] x/(a*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*Sqrt[c*x^2]) - (x*Log[a + b*x
])/(a^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{x}{a\sqrt{cx^2}(a+bx)} + \frac{x\log(x)}{a^2\sqrt{cx^2}} - \frac{x\log(a+bx)}{a^2\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 44, normalized size = 0.75

$$\frac{x(\log(x)(a+bx) - (a+bx)\log(a+bx) + a)}{a^2\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] (x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a + b
*x))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 57, normalized size = 0.97

$$\sqrt{cx^2}\left(-\frac{\log(a+bx)}{a^2cx} + \frac{\log(x)}{a^2cx} + \frac{1}{acx(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] Sqrt[c*x^2]*(1/(a*c*x*(a + b*x)) + Log[x]/(a^2*c*x) - Log[a + b*x]/(a^2*c*x
))
```

**fricas** [A] time = 0.99, size = 44, normalized size = 0.75

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right) + a\right)}{a^2bcx^2 + a^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c*x^2 + a^3*c*x)
```

**giac** [A] time = 1.08, size = 86, normalized size = 1.46

$$-\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^2\sqrt{c}\,\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{1}{(bx+a)a\sqrt{c}\,\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-a/(b*x + a) + 1))/(a^2*sqrt(c)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2
)) - 1/((b*x + a)*a*sqrt(c)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2))
```

**maple** [A] time = 0.01, size = 50, normalized size = 0.85

$$\frac{(bx\ln(x)-bx\ln(bx+a)+a\ln(x)-a\ln(bx+a)+a)\,x}{\sqrt{c\,x^2}\;(bx+a)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] x*(b*x*ln(x)-b*x*ln(b*x+a)+a*ln(x)-a*ln(b*x+a)+a)/(c*x^2)^(1/2)/a^2/(b*x+a)
```

**maxima** [A] time = 1.46, size = 61, normalized size = 1.03

$$-\frac{\sqrt{cx^2}\,b}{a^2bcx+a^3c}-\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)*b/(a^2*b*c*x + a^3*c) - (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b
*x + a)))/(a^2*sqrt(c))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{c\,x^2}\;(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*x**2)*(a + b*x)**2), x)
```

## 3.873 $\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx$

**Optimal**. Leaf size=78

$$-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}-\frac{bx}{a^2\sqrt{cx^2}\,(a+bx)}-\frac{1}{a^2\sqrt{cx^2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{bx}{a^2\sqrt{cx^2}\,(a+bx)}-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}-\frac{1}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*Sqrt[c*x^2])) - (b*x)/(a^2*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log[x])
/(a^3*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x^2(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{1}{a^2\sqrt{cx^2}} - \frac{bx}{a^2\sqrt{cx^2}(a+bx)} - \frac{2bx\log(x)}{a^3\sqrt{cx^2}} + \frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.03, size = 60, normalized size = 0.77

$$\frac{cx^2(-a(a+2bx) - 2bx\log(x)(a+bx) + 2bx(a+bx)\log(a+bx))}{a^3\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (c*x^2*(-(a*(a + 2*b*x)) - 2*b*x*(a + b*x)*Log[x] + 2*b*x*(a + b*x)*Log[a +
 b*x]))/(a^3*(c*x^2)^(3/2)*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 68, normalized size = 0.87

$$\sqrt{cx^2}\left(-\frac{2b\log(x)}{a^3cx} + \frac{2b\log(a+bx)}{a^3cx} + \frac{-a-2bx}{a^2cx^2(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-a - 2*b*x)/(a^2*c*x^2*(a + b*x)) - (2*b*Log[x])/(a^3*c*x) +
(2*b*Log[a + b*x])/(a^3*c*x))
```

**fricas [A]** time = 0.95, size = 62, normalized size = 0.79

$$-\frac{\left(2\,abx + a^2 - 2\left(b^2x^2 + abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bcx^3 + a^4cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
c*x^3 + a^4*c*x^2)
```

**giac [A]** time = 1.14, size = 126, normalized size = 1.62

$$\frac{b\left(\frac{2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3\operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{1}{(bx+a)a^2\operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{1}{a^3\left(\frac{a}{bx+a}-1\right)\operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b*(2*log(abs(-a/(b*x + a) + 1))/(a^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) +
 1/((b*x + a)*a^2*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - 1/(a^3*(a/(b*x + a
) - 1)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/sqrt(c)
```

**maple [A]** time = 0.01, size = 71, normalized size = 0.91

$$-\frac{2b^2x^2\ln(x)-2b^2x^2\ln(bx+a)+2abx\ln(x)-2abx\ln(bx+a)+2abx+a^2}{\sqrt{c\,x^2}\,\,(bx+a)\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x*ln(x)-2*a*b*x*ln(b*x+a)+2*a*b
*x+a^2)/(c*x^2)^(1/2)/a^3/(b*x+a)
```

**maxima [A]** time = 1.43, size = 57, normalized size = 0.73

$$-\frac{2\,bx+a}{a^2b\sqrt{c}\,x^2+a^3\sqrt{c}\,x}+\frac{2\,b\log\left(bx+a\right)}{a^3\sqrt{c}}-\frac{2\,b\log(x)}{a^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*sqrt(c)*x^2 + a^3*sqrt(c)*x) + 2*b*log(b*x + a)/(a^3*sq
rt(c)) - 2*b*log(x)/(a^3*sqrt(c))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/(x*(c*x^2)^(1/2)*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*x**2)*(a + b*x)**2), x)
```

## 3.874 $\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)^2}\,dx$

**Optimal**. Leaf size=103

$$\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{b^2x}{a^3\sqrt{cx^2}\,(a+bx)}+\frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2x}{a^3\sqrt{cx^2}\,(a+bx)}+\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (2*b)/(a^3*Sqrt[c*x^2]) - 1/(2*a^2*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*Sqrt[c*x^2
]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*Sqrt[c*x^2]) - (3*b^2*x*Log[a + b*x])/
(a^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{1}{x^3(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}+\frac{b^2x}{a^3\sqrt{cx^2}\,(a+bx)}+\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.02, size = 81, normalized size = 0.79

$$\frac{cx\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (c*x*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] - 6*b^2*x
^2*(a + b*x)*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2)*(a + b*x))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 86, normalized size = 0.83

$$\sqrt{cx^2}\left(\frac{3b^2\log(x)}{a^4cx}-\frac{3b^2\log(a+bx)}{a^4cx}+\frac{-a^2+3abx+6b^2x^2}{2a^3cx^3(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-a^2 + 3*a*b*x + 6*b^2*x^2)/(2*a^3*c*x^3*(a + b*x)) + (3*b^2*
Log[x])/(a^4*c*x) - (3*b^2*Log[a + b*x])/(a^4*c*x))
```

**fricas [A]** time = 1.28, size = 79, normalized size = 0.77

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\left(b^3x^3+ab^2x^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bcx^4+a^5cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*c*x^4 + a^5*c*x^3)
```

**giac** [A] time = 1.22, size = 152, normalized size = 1.48

$$-\frac{\frac{6\,b^2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^4\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{2\,b^2}{(bx+a)a^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{\frac{6\,ab^2}{bx+a}-5\,b^2}{a^4\left(\frac{a}{bx+a}-1\right)^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(6*b^2*log(abs(-a/(b*x + a) + 1))/(a^4*sgn(-b/(b*x + a) + a*b/(b*x + a
)^2)) + 2*b^2/((b*x + a)*a^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - (6*a*b^
2/(b*x + a) - 5*b^2)/(a^4*(a/(b*x + a) - 1)^2*sgn(-b/(b*x + a) + a*b/(b*x +
 a)^2)))/sqrt(c)
```

**maple** [A] time = 0.01, size = 95, normalized size = 0.92

$$\frac{6b^3x^3\ln(x)-6b^3x^3\ln(bx+a)+6a\,b^2x^2\ln(x)-6a\,b^2x^2\ln(bx+a)+6a\,b^2x^2+3a^2bx-a^3}{2\sqrt{c\,x^2}\,(bx+a)\,a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2/x*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*a*b^2*x^2*ln(x)-6*a*b^2*x^2*ln
(b*x+a)+6*a*b^2*x^2+3*a^2*b*x-a^3)/(c*x^2)^(1/2)/a^4/(b*x+a)
```

**maxima** [A] time = 1.41, size = 76, normalized size = 0.74

$$\frac{6\,b^2x^2+3\,abx-a^2}{2\left(a^3b\sqrt{c}\,x^3+a^4\sqrt{c}\,x^2\right)}-\frac{3\,b^2\log(bx+a)}{a^4\sqrt{c}}+\frac{3\,b^2\log(x)}{a^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*sqrt(c)*x^3 + a^4*sqrt(c)*x^2) - 3*b
^2*log(b*x + a)/(a^4*sqrt(c)) + 3*b^2*log(x)/(a^4*sqrt(c))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{cx^2} (a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)**2), x)
```

## 3.875 $\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx$

**Optimal**. Leaf size=73

$$-\frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}+\frac{x^2}{b^2c\sqrt{cx^2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}+\frac{x^2}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] x^2/(b^2*c*Sqrt[c*x^2]) - (a^2*x)/(b^3*c*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Lo
g[a + b*x])/(b^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx = \frac{x\int \frac{x^2}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{x^2}{b^2c\sqrt{cx^2}}-\frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 54, normalized size = 0.74

$$\frac{x^3\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*(c*x^2)^(3
/2)*(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.06, size = 59, normalized size = 0.81

$$\frac{\frac{-a^2x^3+abx^4+b^2x^5}{b^3(a+bx)}-\frac{2ax^3\log(a+bx)}{b^3}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] ((-(a^2*x^3) + a*b*x^4 + b^2*x^5)/(b^3*(a + b*x)) - (2*a*x^3*Log[a + b*x])/
b^3)/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.00, size = 63, normalized size = 0.86

$$\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4c^2x^2+ab^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c^2
*x^2 + a*b^3*c^2*x)
```

**giac [A]** time = 1.26, size = 127, normalized size = 1.74

$$-\frac{\frac{2\,a\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{bx+a}{b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{a^2}{(bx+a)b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b^3*sgn(-b/(b*x + a) + a*b/(b
*x + a)^2)) + (b*x + a)/(b^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - a^2/((b
*x + a)*b^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/c^(3/2)
```

**maple [A]** time = 0.00, size = 62, normalized size = 0.85

$$-\frac{\left(2abx\ln\left(bx+a\right)-b^2x^2+2a^2\ln\left(bx+a\right)-abx+a^2\right)x^3}{\left(c\,x^2\right)^{\frac{3}{2}}\left(bx+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -x^3*(2*a*b*x*ln(b*x+a)-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/(c*x^2)^(3/2)/b^
3/(b*x+a)
```

**maxima [B]** time = 1.61, size = 149, normalized size = 2.04

$$\frac{a^3}{\sqrt{cx^2}\,b^5cx+\sqrt{cx^2}\,ab^4c}+\frac{x^2}{\sqrt{cx^2}\,b^2c}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3c^{\frac{3}{2}}}+\frac{2\,ax}{\sqrt{cx^2}\,b^3c}-\frac{2\,a\log\left(bx\right)}{b^3c^{\frac{3}{2}}}-\frac{5\,a^2}{\sqrt{cx^2}\,b^4c}+\frac{4\,a^2}{b^4c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^3/(sqrt(c*x^2)*b^5*c*x + sqrt(c*x^2)*a*b^4*c) + x^2/(sqrt(c*x^2)*b^2*c) -
 2*(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(b^3*c^(3/2)) + 2*a*x/
(sqrt(c*x^2)*b^3*c) - 2*a*log(b*x)/(b^3*c^(3/2)) - 5*a^2/(sqrt(c*x^2)*b^4*c
) + 4*a^2/(b^4*c^(3/2)*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(3/2)*(a + b*x)^2),x)
[Out] int(x^5/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2)**(3/2)/(b*x+a)**2,x)
[Out] Integral(x**5/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

## 3.876 $\int \frac{x^4}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx$

**Optimal**. Leaf size=49

$$\frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (a*x)/(b^2*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{x}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 37, normalized size = 0.76

$$\frac{x^3((a+bx)\log(a+bx)+a)}{b^2\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
[Out] (x^3*(a + (a + b*x)*Log[a + b*x]))/(b^2*(c*x^2)^(3/2)*(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 39, normalized size = 0.80

$$\frac{\frac{ax^3}{b^2(a+bx)}+\frac{x^3\log(a+bx)}{b^2}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
[Out] ((a*x^3)/(b^2*(a + b*x)) + (x^3*Log[a + b*x])/b^2)/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.12, size = 44, normalized size = 0.90

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(bx+a\right)+a\right)}{b^3c^2x^2+ab^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c^2*x^2 + a*b^2*c^2*x)
```

**giac** [A] time = 1.03, size = 89, normalized size = 1.82

$$\frac{\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{a}{(bx+a)b^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b^2*sgn(-b/(b*x + a) + a*b/(b*x +
a)^2)) - a/((b*x + a)*b^2*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/c^(3/2)
```

**maple [A]** time = 0.00, size = 41, normalized size = 0.84

$$\frac{(bx\ln(bx+a)+a\ln(bx+a)+a)\,x^3}{\left(c\,x^2\right)^{\frac{3}{2}}(bx+a)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] x^3*(b*x*ln(b*x+a)+a*ln(b*x+a)+a)/(c*x^2)^(3/2)/b^2/(b*x+a)
```

**maxima [B]** time = 1.56, size = 108, normalized size = 2.20

$$-\frac{a^2}{\sqrt{cx^2}\,b^4cx+\sqrt{cx^2}\,ab^3c}+\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2c^{\frac{3}{2}}}+\frac{\log(bx)}{b^2c^{\frac{3}{2}}}+\frac{3\,a}{\sqrt{cx^2}\,b^3c}-\frac{2\,a}{b^3c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^2/(sqrt(c*x^2)*b^4*c*x + sqrt(c*x^2)*a*b^3*c) + (-1)^(2*a*c*x/b)*log(-2*
a*c*x/(b*abs(b*x + a)))/(b^2*c^(3/2)) + log(b*x)/(b^2*c^(3/2)) + 3*a/(sqrt(
c*x^2)*b^3*c) - 2*a/(b^3*c^(3/2)*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\left(c\,x^2\right)^{3/2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(x^4/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(cx^2\right)^{\frac{3}{2}}(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2)**(3/2)/(b*x+a)**2,x)
[Out] Integral(x**4/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

## 3.877 $\int \frac{x^3}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx$

**Optimal**. Leaf size=25

$$-\frac{x}{bc\sqrt{cx^2}\,(a+bx)}$$

**Rubi** [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$-\frac{x}{bc\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(x/(b*c*Sqrt[c*x^2]*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{x}{bc\sqrt{cx^2}\,(a+bx)}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 0.96

$$-\frac{x^3}{b\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(x^3/(b*(c*x^2)^(3/2)*(a + b*x)))
```

**IntegrateAlgebraic** [A] time = 0.03, size = 24, normalized size = 0.96

$$-\frac{x^3}{b\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(x^3/(b*(c*x^2)^(3/2)*(a + b*x)))
```

**fricas** [A] time = 0.96, size = 29, normalized size = 1.16

$$-\frac{\sqrt{cx^2}}{b^2c^2x^2+abc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)/(b^2*c^2*x^2 + a*b*c^2*x)
```

**giac** [A] time = 1.16, size = 38, normalized size = 1.52

$$\frac{1}{(bx+a)bc^{\frac{3}{2}}\operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/((b*x + a)*b*c^(3/2)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2))
```

**maple** [A] time = 0.00, size = 23, normalized size = 0.92

$$-\frac{x^3}{(bx+a)\left(c\,x^2\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -1/(b*x+a)/b*x^3/(c*x^2)^(3/2)
```

**maxima [B]** time = 1.48, size = 47, normalized size = 1.88

$$\frac{a}{\sqrt{cx^2}\,b^3cx + \sqrt{cx^2}\,ab^2c} - \frac{1}{\sqrt{cx^2}\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a/(sqrt(c*x^2)*b^3*c*x + sqrt(c*x^2)*a*b^2*c) - 1/(sqrt(c*x^2)*b^2*c)
```

**mupad [B]** time = 0.17, size = 25, normalized size = 1.00

$$-\frac{\sqrt{c\,x^2}}{b\,c^2\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] -(c*x^2)^(1/2)/(b*c^2*x*(a + b*x))
```

**sympy [A]** time = 1.94, size = 90, normalized size = 3.60

$$\begin{cases} \frac{\tilde{\infty} x^2}{c^{\frac{3}{2}} \left(x^2\right)^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{\tilde{\infty} x^4}{c^{\frac{3}{2}} \left(x^2\right)^{\frac{3}{2}}} & \text{for}\: a = -bx \\ \frac{x^4}{a^2 c^{\frac{3}{2}} \left(x^2\right)^{\frac{3}{2}}} & \text{for}\: b = 0 \\ -\frac{x^3}{abc^{\frac{3}{2}} \left(x^2\right)^{\frac{3}{2}} + b^2 c^{\frac{3}{2}} x \left(x^2\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**2/(c**(3/2)*(x**2)**(3/2)), Eq(a, 0) & Eq(b, 0)), (zoo*x*
*4/(c**(3/2)*(x**2)**(3/2)), Eq(a, -b*x)), (x**4/(a**2*c**(3/2)*(x**2)**(3/
2)), Eq(b, 0)), (-x**3/(a*b*c**(3/2)*(x**2)**(3/2) + b**2*c**(3/2)*x*(x**2)
**(3/2)), True))
```

## 3.878 $\int \frac{x^2}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx$

**Optimal**. Leaf size=68

$$-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}+\frac{x}{ac\sqrt{cx^2}\,(a+bx)}$$

**Rubi [A]** time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}+\frac{x}{ac\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] x/(a*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*c*Sqrt[c*x^2]) - (x*Log[a +
 b*x])/(a^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x(a+bx)^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{x}{ac\sqrt{cx^2}\,(a+bx)} + \frac{x\log(x)}{a^2c\sqrt{cx^2}} - \frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 46, normalized size = 0.68

$$\frac{x^3(\log(x)(a+bx)-(a+bx)\log(a+bx)+a)}{a^2\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
[Out] (x^3*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*(c*x^2)^(3/2)*(a
 + b*x))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 48, normalized size = 0.71

$$\frac{-\frac{x^3\log(a+bx)}{a^2}+\frac{x^3\log(x)}{a^2}+\frac{x^3}{a(a+bx)}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
[Out] (x^3/(a*(a + b*x)) + (x^3*Log[x])/a^2 - (x^3*Log[a + b*x])/a^2)/(c*x^2)^(3/
2)
```

**fricas** [A] time = 1.02, size = 48, normalized size = 0.71

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bc^2x^2+a^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c^2*x^2 + a^3*c^2*x)
```

**giac** [A] time = 1.06, size = 83, normalized size = 1.22

$$-\frac{\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{1}{(bx+a)a\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(log(abs(-a/(b*x + a) + 1))/(a^2*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) + 1/
((b*x + a)*a*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/c^(3/2)
```

**maple** [A] time = 0.01, size = 52, normalized size = 0.76

$$\frac{(bx\ln(x)-bx\ln(bx+a)+a\ln(x)-a\ln(bx+a)+a)\,x^3}{\left(c\,x^2\right)^{\frac{3}{2}}(bx+a)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] x^3*(b*x*ln(x)-b*x*ln(b*x+a)+a*ln(x)-a*ln(b*x+a)+a)/(c*x^2)^(3/2)/a^2/(b*x+
a)
```

**maxima** [A] time = 1.52, size = 82, normalized size = 1.21

$$-\frac{1}{\sqrt{cx^2}\,b^2cx+\sqrt{cx^2}\,abc}-\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^2c^{\frac{3}{2}}}+\frac{1}{\sqrt{cx^2}\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/(sqrt(c*x^2)*b^2*c*x + sqrt(c*x^2)*a*b*c) - (-1)^(2*a*c*x/b)*log(-2*a*c*
x/(b*abs(b*x + a)))/(a^2*c^(3/2)) + 1/(sqrt(c*x^2)*a*b*c)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(x^2/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^2\right)^{\frac{3}{2}} (a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

## 3.879 $\int \frac{x}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx$

**Optimal**. Leaf size=90

$$-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}-\frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)}-\frac{1}{a^2c\sqrt{cx^2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 44}

$$-\frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)}-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}-\frac{1}{a^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*c*Sqrt[c*x^2])) - (b*x)/(a^2*c*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log
[x])/(a^3*c*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{x}{\left(cx^2\right)^{3/2} (a+bx)^2} \, dx = \frac{x \int \frac{1}{x^2(a+bx)^2} \, dx}{c\sqrt{cx^2}}$$
$$= \frac{x \int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right) dx}{c\sqrt{cx^2}}$$
$$= -\frac{1}{a^2c\sqrt{cx^2}} - \frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)} - \frac{2bx\log(x)}{a^3c\sqrt{cx^2}} + \frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 59, normalized size = 0.66

$$\frac{x^2(-a(a+2bx) - 2bx\log(x)(a+bx) + 2bx(a+bx)\log(a+bx))}{a^3\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^2*(-(a*(a + 2*b*x)) - 2*b*x*(a + b*x)*Log[x] + 2*b*x*(a + b*x)*Log[a + b
*x]))/(a^3*(c*x^2)^(3/2)*(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.06, size = 61, normalized size = 0.68

$$\frac{-\frac{2bx^3\log(x)}{a^3} + \frac{2bx^3\log(a+bx)}{a^3} + \frac{-ax^2-2bx^3}{a^2(a+bx)}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] ((-(a*x^2) - 2*b*x^3)/(a^2*(a + b*x)) - (2*b*x^3*Log[x])/a^3 + (2*b*x^3*Log
[a + b*x])/a^3)/(c*x^2)^(3/2)
```

**fricas** [A] time = 1.21, size = 66, normalized size = 0.73

$$-\frac{\left(2\,abx + a^2 - 2\left(b^2x^2 + abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bc^2x^3 + a^4c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
c^2*x^3 + a^4*c^2*x^2)
```

**giac** [A] time = 1.11, size = 137, normalized size = 1.52

$$\frac{\frac{2\,b^2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{b^2}{(bx+a)a^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{b^2}{a^3\left(\frac{a}{bx+a}-1\right)\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (2*b^2*log(abs(-a/(b*x + a) + 1))/(a^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2))
 + b^2/((b*x + a)*a^2*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - b^2/(a^3*(a/(b
*x + a) - 1)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/(b*c^(3/2))
```

**maple** [A] time = 0.00, size = 74, normalized size = 0.82

$$-\frac{\left(2b^2x^2\ln(x)-2b^2x^2\ln(bx+a)+2abx\ln(x)-2abx\ln(bx+a)+2abx+a^2\right)x^2}{\left(c\,x^2\right)^{\frac{3}{2}}(bx+a)\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -x^2*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x*ln(x)-2*a*b*x*ln(b*x+a)+2
*a*b*x+a^2)/(c*x^2)^(3/2)/a^3/(b*x+a)
```

**maxima** [A] time = 1.41, size = 79, normalized size = 0.88

$$\frac{1}{\sqrt{cx^2}\,abcx+\sqrt{cx^2}\,a^2c}+\frac{2\,(-1)^{\frac{2\,acx}{b}}\,b\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^3c^{\frac{3}{2}}}-\frac{2}{\sqrt{cx^2}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/(sqrt(c*x^2)*a*b*c*x + sqrt(c*x^2)*a^2*c) + 2*(-1)^(2*a*c*x/b)*b*log(-2*a
*c*x/(b*abs(b*x + a)))/(a^3*c^(3/2)) - 2/(sqrt(c*x^2)*a^2*c)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(x/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

## 3.880 $\int \frac{1}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx$

**Optimal**. Leaf size=118

$$\frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}+\frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)}+\frac{2b}{a^3c\sqrt{cx^2}}-\frac{1}{2a^2cx\sqrt{cx^2}}$$

**Rubi** [A] time = 0.03, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 44}

$$\frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)}+\frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}+\frac{2b}{a^3c\sqrt{cx^2}}-\frac{1}{2a^2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (2*b)/(a^3*c*Sqrt[c*x^2]) - 1/(2*a^2*c*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*c*Sqrt
[c*x^2]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*c*Sqrt[c*x^2]) - (3*b^2*x*Log[a
+ b*x])/(a^4*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x^3(a+bx)^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x^3} - \frac{2b}{a^3x^2} + \frac{3b^2}{a^4x} - \frac{b^3}{a^3(a+bx)^2} - \frac{3b^3}{a^4(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{2b}{a^3c\sqrt{cx^2}} - \frac{1}{2a^2cx\sqrt{cx^2}} + \frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)} + \frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}} - \frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.01, size = 80, normalized size = 0.68

$$\frac{x\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] - 6*b^2*x^2
*(a + b*x)*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2)*(a + b*x))
```

**IntegrateAlgebraic** [A] time = 0.07, size = 77, normalized size = 0.65

$$\frac{\frac{3b^2x^3\log(x)}{a^4} - \frac{3b^2x^3\log(a+bx)}{a^4} + \frac{-a^2x+3abx^2+6b^2x^3}{2a^3(a+bx)}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] ((-(a^2*x) + 3*a*b*x^2 + 6*b^2*x^3)/(2*a^3*(a + b*x)) + (3*b^2*x^3*Log[x])/
a^4 - (3*b^2*x^3*Log[a + b*x])/a^4)/(c*x^2)^(3/2)
```

**fricas** [A] time = 0.98, size = 83, normalized size = 0.70

$$\frac{\left(6\,ab^2x^2 + 3\,a^2bx - a^3 + 6\left(b^3x^3 + ab^2x^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bc^2x^4 + a^5c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*c^2*x^4 + a^5*c^2*x^3)
```

**giac [A]** time = 1.03, size = 152, normalized size = 1.29

$$-\frac{\frac{6\,b^2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^4\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{2\,b^2}{(bx+a)a^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{\frac{6\,ab^2}{bx+a}-5\,b^2}{a^4\left(\frac{a}{bx+a}-1\right)^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(6*b^2*log(abs(-a/(b*x + a) + 1))/(a^4*sgn(-b/(b*x + a) + a*b/(b*x + a
)^2)) + 2*b^2/((b*x + a)*a^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - (6*a*b^
2/(b*x + a) - 5*b^2)/(a^4*(a/(b*x + a) - 1)^2*sgn(-b/(b*x + a) + a*b/(b*x +
 a)^2)))/c^(3/2)
```

**maple [A]** time = 0.01, size = 93, normalized size = 0.79

$$\frac{\left(6b^3x^3\ln(x)-6b^3x^3\ln(bx+a)+6a\,b^2x^2\ln(x)-6a\,b^2x^2\ln(bx+a)+6a\,b^2x^2+3a^2bx-a^3\right)x}{2\left(c\,x^2\right)^{\frac{3}{2}}(bx+a)\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 1/2*x*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*a*b^2*x^2*ln(x)-6*a*b^2*x^2*ln
(b*x+a)+6*a*b^2*x^2+3*a^2*b*x-a^3)/(c*x^2)^(3/2)/a^4/(b*x+a)
```

**maxima [A]** time = 1.46, size = 98, normalized size = 0.83

$$-\frac{b}{\sqrt{cx^2}\,a^2bcx+\sqrt{cx^2}\,a^3c}-\frac{3\,(-1)^{\frac{2\,acx}{b}}\,b^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^4c^{\frac{3}{2}}}+\frac{3\,b}{\sqrt{cx^2}\,a^3c}-\frac{1}{2\,a^2c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -b/(sqrt(c*x^2)*a^2*b*c*x + sqrt(c*x^2)*a^3*c) - 3*(-1)^(2*a*c*x/b)*b^2*log
(-2*a*c*x/(b*abs(b*x + a)))/(a^4*c^(3/2)) + 3*b/(sqrt(c*x^2)*a^3*c) - 1/2/(
a^2*c^(3/2)*x^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(1/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(1/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

## 3.881 $\int x^2\sqrt{cx^2}\,(a+bx)^n\,dx$

**Optimal**. Leaf size=131

$$-\frac{a^3\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

**Rubi** [A] time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] -((a^3*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(
b^4*(3 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\sqrt{cx^2}\,(a+bx)^n\,dx = \frac{\sqrt{cx^2}\,\int x^3(a+bx)^n\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\,\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}$$
$$= -\frac{a^3\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{\sqrt{cx^2}\,(a+bx)^{4+n}}{b^4(4+n)x}$$

**Mathematica [A]** time = 0.07, size = 97, normalized size = 0.74

$$\frac{cx(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2
)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic [F]** time = 0.22, size = 0, normalized size = 0.00

$$\int x^2\sqrt{cx^2}\,(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*Sqrt[c*x^2]*(a + b*x)^n, x]
```

**fricas [A]** time = 1.24, size = 153, normalized size = 1.17

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
```

```
t(c*x^2)*(b*x + a)^n/((b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^
4)*x)
```

**giac [B]** time = 1.11, size = 300, normalized size = 2.29

$$\left(\frac{6\,a^4 a^n \operatorname{sgn}(x)}{b^4 n^4 + 10\,b^4 n^3 + 35\,b^4 n^2 + 50\,b^4 n + 24\,b^4} + \frac{(bx+a)^n b^4 n^3 x^4 \operatorname{sgn}(x) + (bx+a)^n a b^3 n^3 x^3 \operatorname{sgn}(x) + 6\,(bx+a)^n b^4 n^2 x^4 \operatorname{sgn}(x) + 3\,(bx+a)^n a b^3 n^2 x^3 \operatorname{sgn}(x) + 11\,(bx+a)^n b^4 n x^4 \operatorname{sgn}(x) - 3\,(bx+a)^n a^2 b^2 n^2 x^2 \operatorname{sgn}(x) + 2\,(bx+a)^n a b^3 n x^3 \operatorname{sgn}(x) + 6\,(bx+a)^n b^4 x^4 \operatorname{sgn}(x) - 3\,(bx+a)^n a^2 b^2 n x^2 \operatorname{sgn}(x) + 6\,(bx+a)^n a^3 b n x \operatorname{sgn}(x) - 6\,(bx+a)^n a^4 \operatorname{sgn}(x)}{b^4 n^4 + 10\,b^4 n^3 + 35\,b^4 n^2 + 50\,b^4 n + 24\,b^4}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (6*a^4*a^n*sgn(x)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4) +
 ((b*x + a)^n*b^4*n^3*x^4*sgn(x) + (b*x + a)^n*a*b^3*n^3*x^3*sgn(x) + 6*(b*
x + a)^n*b^4*n^2*x^4*sgn(x) + 3*(b*x + a)^n*a*b^3*n^2*x^3*sgn(x) + 11*(b*x
+ a)^n*b^4*n*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b^2*n^2*x^2*sgn(x) + 2*(b*x + a
)^n*a*b^3*n*x^3*sgn(x) + 6*(b*x + a)^n*b^4*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b
^2*n*x^2*sgn(x) + 6*(b*x + a)^n*a^3*b*n*x*sgn(x) - 6*(b*x + a)^n*a^4*sgn(x)
)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4))*sqrt(c)
```

**maple [A]** time = 0.01, size = 136, normalized size = 1.04

$$-\frac{\sqrt{c\,x^2}\,\left(-b^3 n^3 x^3 - 6b^3 n^2 x^3 + 3a\,b^2 n^2 x^2 - 11b^3 n\,x^3 + 9a\,b^2 n\,x^2 - 6b^3 x^3 - 6a^2 bnx + 6a\,b^2 x^2 - 6a^2 bx + 6a^3\right)(bx+a)^{n+1}}{\left(n^4 + 10n^3 + 35n^2 + 50n + 24\right) b^4 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(c*x^2)^(1/2),x)
```

```
[Out] -(c*x^2)^(1/2)*(b*x+a)^(n+1)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

**maxima [A]** time = 1.46, size = 116, normalized size = 0.89

$$\frac{\left(\left(n^3 + 6\,n^2 + 11\,n + 6\right)b^4\sqrt{c}\,x^4 + \left(n^3 + 3\,n^2 + 2\,n\right)ab^3\sqrt{c}\,x^3 - 3\left(n^2 + n\right)a^2b^2\sqrt{c}\,x^2 + 6\,a^3b\sqrt{c}\,nx - 6\,a^4\sqrt{c}\right)(bx+a)^n}{\left(n^4 + 10\,n^3 + 35\,n^2 + 50\,n + 24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*sqrt(c)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*sqrt(
c)*x^3 - 3*(n^2 + n)*a^2*b^2*sqrt(c)*x^2 + 6*a^3*b*sqrt(c)*n*x - 6*a^4*sqrt
(c))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

**mupad [B]** time = 0.35, size = 214, normalized size = 1.63

$$\frac{(a+b\,x)^n\left(\frac{x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24} - \frac{6\,a^4\,\sqrt{c\,x^2}}{b^4\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} + \frac{6\,a^3\,n\,x\,\sqrt{c\,x^2}}{b^3\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} + \frac{a\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} - \frac{3\,a^2\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b^2\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 +
 10*n^3 + n^4 + 24) - (6*a^4*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 10*n^3 +
n^4 + 24)) + (6*a^3*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 +
 24)) + (a*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3
+ n^4 + 24)) - (3*a^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2*(50*n + 35*n^2 + 10
*n^3 + n^4 + 24))))/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n \sqrt{c} x^3 \sqrt{x^2}}{4} & \text{for } b = 0 \\ \int \frac{x^2 \sqrt{cx^2}}{(a+bx)^4}\, dx & \text{for } n = -4 \\ \int \frac{x^2 \sqrt{cx^2}}{(a+bx)^3}\, dx & \text{for } n = -3 \\ \int \frac{x^2 \sqrt{cx^2}}{(a+bx)^2}\, dx & \text{for } n = -2 \\ \int \frac{x^2 \sqrt{cx^2}}{a+bx}\, dx & \text{for } n = -1 \\ -\frac{6a^4\sqrt{c}(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{6a^3b\sqrt{c}nx(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} - \frac{3a^2b^2\sqrt{c}n^2x^2(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} - \frac{3a^2b^2\sqrt{c}nx^2(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{ab^3\sqrt{c}n^3x^3(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{3ab^3\sqrt{c}n^2x^3(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{2ab^3\sqrt{c}nx^3(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{b^4\sqrt{c}n^3x^4(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{6b^4\sqrt{c}n^2x^4(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{11b^4\sqrt{c}nx^4(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} + \frac{6b^4\sqrt{c}x^4(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*sqrt(c)*x**3*sqrt(x**2)/4, Eq(b, 0)), (Integral(x**2*sqrt(c
*x**2)/(a + b*x)**4, x), Eq(n, -4)), (Integral(x**2*sqrt(c*x**2)/(a + b*x)*
*3, x), Eq(n, -3)), (Integral(x**2*sqrt(c*x**2)/(a + b*x)**2, x), Eq(n, -2)
), (Integral(x**2*sqrt(c*x**2)/(a + b*x), x), Eq(n, -1)), (-6*a**4*sqrt(c)*
(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50
*b**4*n*x + 24*b**4*x) + 6*a**3*b*sqrt(c)*n*x*(a + b*x)**n*sqrt(x**2)/(b**4
*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) - 3*a*
*2*b**2*sqrt(c)*n**2*x**2*(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n*
*3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) - 3*a**2*b**2*sqrt(c)*n*x*
*2*(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x +
 50*b**4*n*x + 24*b**4*x) + a*b**3*sqrt(c)*n**3*x**3*(a + b*x)**n*sqrt(x**2
)/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x)
 + 3*a*b**3*sqrt(c)*n**2*x**3*(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**
4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + 2*a*b**3*sqrt(c)*n*x
**3*(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x
+ 50*b**4*n*x + 24*b**4*x) + b**4*sqrt(c)*n**3*x**4*(a + b*x)**n*sqrt(x**2)
/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x)
+ 6*b**4*sqrt(c)*n**2*x**4*(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n
**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + 11*b**4*sqrt(c)*n*x**4*
(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50
*b**4*n*x + 24*b**4*x) + 6*b**4*sqrt(c)*x**4*(a + b*x)**n*sqrt(x**2)/(b**4*
n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x), True))
```

## 3.882 $\int x\sqrt{cx^2}\,(a+bx)^n\,dx$

**Optimal**. Leaf size=96

$$\frac{a^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2a\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

**Rubi [A]** time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2a\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (a^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*Sqrt[c*x^2]*(a +
 b*x)^(2 + n))/(b^3*(2 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3*(3 +
n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\sqrt{cx^2}(a+bx)^n\,dx = \frac{\sqrt{cx^2}\int x^2(a+bx)^n\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int\left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}$$
$$= \frac{a^2\sqrt{cx^2}(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2a\sqrt{cx^2}(a+bx)^{2+n}}{b^3(2+n)x}+\frac{\sqrt{cx^2}(a+bx)^{3+n}}{b^3(3+n)x}$$

**Mathematica** [A] time = 0.05, size = 68, normalized size = 0.71

$$\frac{cx(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))
/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [F] time = 0.20, size = 0, normalized size = 0.00

$$\int x\sqrt{cx^2}(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*Sqrt[c*x^2]*(a + b*x)^n, x]
```

**fricas** [A] time = 1.56, size = 106, normalized size = 1.10

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
*x)
```

**giac [B]** time = 0.96, size = 200, normalized size = 2.08

$$-\left(\frac{2\,a^{3}a^{n}\operatorname{sgn}(x)}{b^{3}n^{3}+6\,b^{3}n^{2}+11\,b^{3}n+6\,b^{3}}-\frac{(bx+a)^{n}b^{3}n^{2}x^{3}\operatorname{sgn}(x)+(bx+a)^{n}ab^{2}n^{2}x^{2}\operatorname{sgn}(x)+3\,(bx+a)^{n}b^{3}nx^{3}\operatorname{sgn}(x)+(bx+a)^{n}ab^{2}nx^{2}\operatorname{sgn}(x)+2\,(bx+a)^{n}b^{3}x^{3}\operatorname{sgn}(x)-2\,(bx+a)^{n}a^{2}bnx\operatorname{sgn}(x)+2\,(bx+a)^{n}a^{3}\operatorname{sgn}(x)}{b^{3}n^{3}+6\,b^{3}n^{2}+11\,b^{3}n+6\,b^{3}}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a^3*a^n*sgn(x)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3) - ((b*x + a)^n*
b^3*n^2*x^3*sgn(x) + (b*x + a)^n*a*b^2*n^2*x^2*sgn(x) + 3*(b*x + a)^n*b^3*n
*x^3*sgn(x) + (b*x + a)^n*a*b^2*n*x^2*sgn(x) + 2*(b*x + a)^n*b^3*x^3*sgn(x)
 - 2*(b*x + a)^n*a^2*b*n*x*sgn(x) + 2*(b*x + a)^n*a^3*sgn(x))/(b^3*n^3 + 6*
b^3*n^2 + 11*b^3*n + 6*b^3))*sqrt(c)
```

**maple [A]** time = 0.01, size = 83, normalized size = 0.86

$$\frac{\left(b^{2}n^{2}x^{2}+3b^{2}n\,x^{2}-2abnx+2b^{2}x^{2}-2abx+2a^{2}\right)\sqrt{c\,x^{2}}\,\left(bx+a\right)^{n+1}}{\left(n^{3}+6n^{2}+11n+6\right)b^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(c*x^2)^(1/2),x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(1/2)/x/b^3/(n^3+6*n^2+11*n+6)
```

**maxima [A]** time = 1.41, size = 80, normalized size = 0.83

$$\frac{\left(\left(n^{2}+3\,n+2\right)b^{3}\sqrt{c}\,x^{3}+\left(n^{2}+n\right)ab^{2}\sqrt{c}\,x^{2}-2\,a^{2}b\sqrt{c}\,nx+2\,a^{3}\sqrt{c}\right)(bx+a)^{n}}{\left(n^{3}+6\,n^{2}+11\,n+6\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

**mupad [B]** time = 0.25, size = 142, normalized size = 1.48

$$\frac{(a+b\,x)^{n}\,\left(\frac{2\,a^{3}\,\sqrt{c\,x^{2}}}{b^{3}\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}+\frac{x^{3}\,\sqrt{c\,x^{2}}\,\left(n^{2}+3\,n+2\right)}{n^{3}+6\,n^{2}+11\,n+6}-\frac{2\,a^{2}\,n\,x\,\sqrt{c\,x^{2}}}{b^{2}\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}+\frac{a\,n\,x^{2}\,\sqrt{c\,x^{2}}\,(n+1)}{b\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((2*a^3*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (x^3*(
c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) - (2*a^2*n*x*(c*x^2)
^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b
*(11*n + 6*n^2 + n^3 + 6))))/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^{n}\sqrt{c}x^{2}\sqrt{x^{2}}}{3} & \text{for}\: b = 0 \\ \int \frac{x\sqrt{cx^{2}}}{(a+bx)^{3}}\, dx & \text{for}\: n = -3 \\ \int \frac{x\sqrt{cx^{2}}}{(a+bx)^{2}}\, dx & \text{for}\: n = -2 \\ \int \frac{x\sqrt{cx^{2}}}{a+bx}\, dx & \text{for}\: n = -1 \\ \frac{2a^{3}\sqrt{c}(a+bx)^{n}\sqrt{x^{2}}}{b^{3}n^{3}x+6b^{3}n^{2}x+11b^{3}nx+6b^{3}x} - \frac{2a^{2}b\sqrt{c}nx(a+bx)^{n}\sqrt{x^{2}}}{b^{3}n^{3}x+6b^{3}n^{2}x+11b^{3}nx+6b^{3}x} + \frac{ab^{2}\sqrt{c}n^{2}x^{2}(a+bx)^{n}\sqrt{x^{2}}}{b^{3}n^{3}x+6b^{3}n^{2}x+11b^{3}nx+6b^{3}x} + \frac{ab^{2}\sqrt{c}nx^{2}(a+bx)^{n}\sqrt{x^{2}}}{b^{3}n^{3}x+6b^{3}n^{2}x+11b^{3}nx+6b^{3}x} + \frac{b^{3}\sqrt{c}n^{2}x^{3}(a+bx)^{n}\sqrt{x^{2}}}{b^{3}n^{3}x+6b^{3}n^{2}x+11b^{3}nx+6b^{3}x} + \frac{3b^{3}\sqrt{c}nx^{3}(a+bx)^{n}\sqrt{x^{2}}}{b^{3}n^{3}x+6b^{3}n^{2}x+11b^{3}nx+6b^{3}x} + \frac{2b^{3}\sqrt{c}x^{3}(a+bx)^{n}\sqrt{x^{2}}}{b^{3}n^{3}x+6b^{3}n^{2}x+11b^{3}nx+6b^{3}x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*sqrt(c)*x**2*sqrt(x**2)/3, Eq(b, 0)), (Integral(x*sqrt(c*x*
*2)/(a + b*x)**3, x), Eq(n, -3)), (Integral(x*sqrt(c*x**2)/(a + b*x)**2, x)
, Eq(n, -2)), (Integral(x*sqrt(c*x**2)/(a + b*x), x), Eq(n, -1)), (2*a**3*s
qrt(c)*(a + b*x)**n*sqrt(x**2)/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x +
 6*b**3*x) - 2*a**2*b*sqrt(c)*n*x*(a + b*x)**n*sqrt(x**2)/(b**3*n**3*x + 6*
b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + a*b**2*sqrt(c)*n**2*x**2*(a + b*x)*
*n*sqrt(x**2)/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + a*b*
*2*sqrt(c)*n*x**2*(a + b*x)**n*sqrt(x**2)/(b**3*n**3*x + 6*b**3*n**2*x + 11
*b**3*n*x + 6*b**3*x) + b**3*sqrt(c)*n**2*x**3*(a + b*x)**n*sqrt(x**2)/(b**
3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + 3*b**3*sqrt(c)*n*x**3*
(a + b*x)**n*sqrt(x**2)/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3
*x) + 2*b**3*sqrt(c)*x**3*(a + b*x)**n*sqrt(x**2)/(b**3*n**3*x + 6*b**3*n**
2*x + 11*b**3*n*x + 6*b**3*x), True))
```

# 3.883 $\int \sqrt{cx^2}\,(a+bx)^n\,dx$

**Optimal**. Leaf size=63

$$\frac{\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{a\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

**Rubi [A]** time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{a\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] -((a*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (Sqrt[c*x^2]*(a + b*
x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx^2}\,(a+bx)^n\,dx &= \frac{\sqrt{cx^2}\int x(a+bx)^n\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}\\ &= -\frac{a\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 44, normalized size = 0.70

$$\frac{cx(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^2]
)
```

**IntegrateAlgebraic** [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2}\,(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[c*x^2]*(a + b*x)^n, x]
```

**fricas** [A] time = 0.79, size = 63, normalized size = 1.00

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2n^2+3\,b^2n+2\,b^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*n^2 + 3*b
^2*n + 2*b^2)*x)
```

**giac** [B] time = 1.14, size = 119, normalized size = 1.89

$$\left(\frac{a^2a^n\mathrm{sgn}(x)}{b^2n^2+3\,b^2n+2\,b^2}+\frac{(bx+a)^nb^2nx^2\mathrm{sgn}(x)+(bx+a)^nabnx\mathrm{sgn}(x)+(bx+a)^nb^2x^2\mathrm{sgn}(x)-(bx+a)^na^2\mathrm{sgn}(x)}{b^2n^2+3\,b^2n+2\,b^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (a^2*a^n*sgn(x)/(b^2*n^2 + 3*b^2*n + 2*b^2) + ((b*x + a)^n*b^2*n*x^2*sgn(x)
 + (b*x + a)^n*a*b*n*x*sgn(x) + (b*x + a)^n*b^2*x^2*sgn(x) - (b*x + a)^n*a^
2*sgn(x))/(b^2*n^2 + 3*b^2*n + 2*b^2))*sqrt(c)
```

**maple** [A] time = 0.00, size = 46, normalized size = 0.73

$$-\frac{\sqrt{c\,x^2}\,\left(-xnb-bx+a\right)\left(bx+a\right)^{n+1}}{\left(n^2+3n+2\right)b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2),x)
```

```
[Out] -(c*x^2)^(1/2)*(b*x+a)^(n+1)*(-b*n*x-b*x+a)/x/b^2/(n^2+3*n+2)
```

**maxima** [A] time = 1.45, size = 51, normalized size = 0.81

$$\frac{\left(b^2\sqrt{c}\,(n+1)x^2+ab\sqrt{c}\,nx-a^2\sqrt{c}\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*sqrt(c)*(n + 1)*x^2 + a*b*sqrt(c)*n*x - a^2*sqrt(c))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

**mupad** [B] time = 0.22, size = 85, normalized size = 1.35

$$\frac{(a+b\,x)^n\left(\frac{x^2\,\sqrt{c\,x^2}\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,\sqrt{c\,x^2}}{b^2\left(n^2+3\,n+2\right)}+\frac{a\,n\,x\,\sqrt{c\,x^2}}{b\left(n^2+3\,n+2\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*(c*x^2)^(1
/2))/(b^2*(3*n + n^2 + 2)) + (a*n*x*(c*x^2)^(1/2))/(b*(3*n + n^2 + 2))))/x
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n\sqrt{c}\,x\sqrt{x^2}}{2} & \text{for } b = 0 \\ \int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx & \text{for } n = -2 \\ \int \frac{\sqrt{cx^2}}{a+bx}\,dx & \text{for } n = -1 \\ -\frac{a^2\sqrt{c}\,(a+bx)^n\sqrt{x^2}}{b^2n^2x+3b^2nx+2b^2x}+\frac{ab\sqrt{c}\,nx(a+bx)^n\sqrt{x^2}}{b^2n^2x+3b^2nx+2b^2x}+\frac{b^2\sqrt{c}\,nx^2(a+bx)^n\sqrt{x^2}}{b^2n^2x+3b^2nx+2b^2x}+\frac{b^2\sqrt{c}\,x^2(a+bx)^n\sqrt{x^2}}{b^2n^2x+3b^2nx+2b^2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*sqrt(c)*x*sqrt(x**2)/2, Eq(b, 0)), (Integral(sqrt(c*x**2)/(
a + b*x)**2, x), Eq(n, -2)), (Integral(sqrt(c*x**2)/(a + b*x), x), Eq(n, -1
)), (-a**2*sqrt(c)*(a + b*x)**n*sqrt(x**2)/(b**2*n**2*x + 3*b**2*n*x + 2*b*
*2*x) + a*b*sqrt(c)*n*x*(a + b*x)**n*sqrt(x**2)/(b**2*n**2*x + 3*b**2*n*x +
 2*b**2*x) + b**2*sqrt(c)*n*x**2*(a + b*x)**n*sqrt(x**2)/(b**2*n**2*x + 3*b
**2*n*x + 2*b**2*x) + b**2*sqrt(c)*x**2*(a + b*x)**n*sqrt(x**2)/(b**2*n**2*
x + 3*b**2*n*x + 2*b**2*x), True))
```

## 3.884 $\int \frac{\sqrt{cx^2}(a+bx)^n}{x} dx$

**Optimal**. Leaf size=30

$$\frac{\sqrt{cx^2}(a+bx)^{n+1}}{b(n+1)x}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{\sqrt{cx^2}(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]
```

```
[Out] (Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)^n}{x} dx &= \frac{\sqrt{cx^2} \int (a+bx)^n dx}{x} \\ &= \frac{\sqrt{cx^2}(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 0.97

$$\frac{cx(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]
[Out] (c*x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic [F]** time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]
[Out] Defer[IntegrateAlgebraic][(Sqrt[c*x^2]*(a + b*x)^n)/x, x]
```

**fricas [A]** time = 0.99, size = 30, normalized size = 1.00

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*n + b)*x)
```

**giac [A]** time = 1.01, size = 42, normalized size = 1.40

$$-\sqrt{c}\left(\frac{a^{n+1}\mathrm{sgn}(x)}{bn+b}-\frac{(bx+a)^{n+1}\mathrm{sgn}(x)}{b(n+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="giac")
[Out] -sqrt(c)*(a^(n + 1)*sgn(x)/(b*n + b) - (b*x + a)^(n + 1)*sgn(x)/(b*(n + 1))
)
```

**maple [A]** time = 0.00, size = 29, normalized size = 0.97

$$\frac{\sqrt{c\,x^2}\;(bx+a)^{n+1}}{(n+1)\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x,x)
```

```
[Out] (b*x+a)^(n+1)*(c*x^2)^(1/2)/b/(n+1)/x
```

**maxima [A]** time = 1.40, size = 28, normalized size = 0.93

$$\frac{\left(b\sqrt{c}\,x + a\sqrt{c}\right)(bx + a)^n}{b(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*(n + 1))
```

**mupad [B]** time = 0.23, size = 31, normalized size = 1.03

$$\frac{\sqrt{c\,x^2}\,\left(a + b\,x\right)^n\,\left(a + b\,x\right)}{b\,x\,\left(n + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((c*x^2)^(1/2)*(a + b*x)^n*(a + b*x))/(b*x*(n + 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\sqrt{c}\sqrt{x^2}}{a} & \text{for}\: b = 0 \wedge n = -1 \\ a^n\sqrt{c}\sqrt{x^2} & \text{for}\: b = 0 \\ \int \frac{\sqrt{cx^2}}{x(a+bx)}\, dx & \text{for}\: n = -1 \\ \frac{a\sqrt{c}\,(a+bx)^n\sqrt{x^2}}{bnx+bx} + \frac{b\sqrt{c}\,x(a+bx)^n\sqrt{x^2}}{bnx+bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x,x)
```

```
[Out] Piecewise((sqrt(c)*sqrt(x**2)/a, Eq(b, 0) & Eq(n, -1)), (a**n*sqrt(c)*sqrt(
x**2), Eq(b, 0)), (Integral(sqrt(c*x**2)/(x*(a + b*x)), x), Eq(n, -1)), (a*
sqrt(c)*(a + b*x)**n*sqrt(x**2)/(b*n*x + b*x) + b*sqrt(c)*x*(a + b*x)**n*sq
rt(x**2)/(b*n*x + b*x), True))
```

## 3.885 $\int x\left(cx^2\right)^{3/2}(a+bx)^n\,dx$

**Optimal**. Leaf size=169

$$\frac{a^4c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c\sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac\sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

**Rubi** [A] time = 0.06, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{a^4c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c\sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac\sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] (a^4*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c*Sqrt[c*x^2
]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c*Sqrt[c*x^2]*(a + b*x)^(3 +
n))/(b^5*(3 + n)*x) - (4*a*c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4 + n)*x)
 + (c*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\left(cx^2\right)^{3/2}(a+bx)^n\,dx=\frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)^n\,dx}{x}$$
$$=\frac{\left(c\sqrt{cx^2}\right)\int\left(\frac{a^4(a+bx)^n}{b^4}-\frac{4a^3(a+bx)^{1+n}}{b^4}+\frac{6a^2(a+bx)^{2+n}}{b^4}-\frac{4a(a+bx)^{3+n}}{b^4}+\frac{(a+bx)^{4+n}}{b^4}\right)dx}{x}$$
$$=\frac{a^4c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^5(1+n)x}-\frac{4a^3c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^5(2+n)x}+\frac{6a^2c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^5(3+n)x}-\frac{4ac\sqrt{cx^2}\,(a}{b^5(4+}$$

**Mathematica** [A] time = 0.08, size = 132, normalized size = 0.78

$$\frac{\left(cx^2\right)^{3/2}(a+bx)^{n+1}\left(24a^4-24a^3b(n+1)x+12a^2b^2\left(n^2+3n+2\right)x^2-4ab^3\left(n^3+6n^2+11n+6\right)x^3+b^4\left(n^4+10n^3+35n^2+50n+24\right)x^4\right)}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n)*(24*a^4 - 24*a^3*b*(1 + n)*x + 12*a^2*b^2*
(2 + 3*n + n^2)*x^2 - 4*a*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + b^4*(24 + 50*n
 + 35*n^2 + 10*n^3 + n^4)*x^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n
)*x^3)
```

**IntegrateAlgebraic** [F] time = 0.22, size = 0, normalized size = 0.00

$$\int x\left(cx^2\right)^{3/2}(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(c*x^2)^(3/2)*(a + b*x)^n, x]
```

**fricas** [A] time = 1.34, size = 233, normalized size = 1.38

$$-\frac{\left(24\,a^4bcnx-24\,a^5c-\left(b^5cn^4+10\,b^5cn^3+35\,b^5cn^2+50\,b^5cn+24\,b^5c\right)x^5-\left(ab^4cn^4+6\,ab^4cn^3+11\,ab^4cn^2+6\,ab^4cn\right)x^4+4\left(a^2b^3cn^3+3\,a^2b^3cn^2+2\,a^2b^3cn\right)x^3-12\left(a^3b^2cn^2+a^3b^2cn\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\,b^5n+120\,b^5\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(24*a^4*b*c*n*x - 24*a^5*c - (b^5*c*n^4 + 10*b^5*c*n^3 + 35*b^5*c*n^2 + 50
*b^5*c*n + 24*b^5*c)*x^5 - (a*b^4*c*n^4 + 6*a*b^4*c*n^3 + 11*a*b^4*c*n^2 +
```

```
6*a*b^4*c*n)*x^4 + 4*(a^2*b^3*c*n^3 + 3*a^2*b^3*c*n^2 + 2*a^2*b^3*c*n)*x^3
- 12*(a^3*b^2*c*n^2 + a^3*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^5*n^5 +
 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)*x)
```

**giac [B]** time = 1.20, size = 426, normalized size = 2.52

$$-\left(\frac{24\,a^{5}a^{n}\operatorname{sgn}(x)}{b^{5}n^{5}+15\,b^{5}n^{4}+85\,b^{5}n^{3}+225\,b^{5}n^{2}+274\,b^{5}n+120\,b^{5}}-\frac{(bx+a)^{n}b^{5}n^{4}x^{5}\operatorname{sgn}(x)+(bx+a)^{n}ab^{4}n^{4}x^{4}\operatorname{sgn}(x)+10\,(bx+a)^{n}b^{5}n^{3}x^{5}\operatorname{sgn}(x)+6\,(bx+a)^{n}ab^{4}n^{3}x^{4}\operatorname{sgn}(x)+35\,(bx+a)^{n}b^{5}n^{2}x^{5}\operatorname{sgn}(x)-4\,(bx+a)^{n}a^{2}b^{3}n^{3}x^{3}\operatorname{sgn}(x)+11\,(bx+a)^{n}ab^{4}n^{2}x^{4}\operatorname{sgn}(x)+50\,(bx+a)^{n}b^{5}nx^{5}\operatorname{sgn}(x)-12\,(bx+a)^{n}a^{2}b^{3}n^{2}x^{3}\operatorname{sgn}(x)+6\,(bx+a)^{n}ab^{4}nx^{4}\operatorname{sgn}(x)+24\,(bx+a)^{n}b^{5}x^{5}\operatorname{sgn}(x)+12\,(bx+a)^{n}a^{3}b^{2}n^{2}x^{2}\operatorname{sgn}(x)-8\,(bx+a)^{n}a^{2}b^{3}nx^{3}\operatorname{sgn}(x)+12\,(bx+a)^{n}a^{3}b^{2}nx^{2}\operatorname{sgn}(x)-24\,(bx+a)^{n}a^{4}bnx\operatorname{sgn}(x)+24\,(bx+a)^{n}a^{5}\operatorname{sgn}(x)}{b^{5}n^{5}+15\,b^{5}n^{4}+85\,b^{5}n^{3}+225\,b^{5}n^{2}+274\,b^{5}n+120\,b^{5}}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] -(24*a^5*a^n*sgn(x)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*
b^5*n + 120*b^5) - ((b*x + a)^n*b^5*n^4*x^5*sgn(x) + (b*x + a)^n*a*b^4*n^4*
x^4*sgn(x) + 10*(b*x + a)^n*b^5*n^3*x^5*sgn(x) + 6*(b*x + a)^n*a*b^4*n^3*x^
4*sgn(x) + 35*(b*x + a)^n*b^5*n^2*x^5*sgn(x) - 4*(b*x + a)^n*a^2*b^3*n^3*x^
3*sgn(x) + 11*(b*x + a)^n*a*b^4*n^2*x^4*sgn(x) + 50*(b*x + a)^n*b^5*n*x^5*s
gn(x) - 12*(b*x + a)^n*a^2*b^3*n^2*x^3*sgn(x) + 6*(b*x + a)^n*a*b^4*n*x^4*s
gn(x) + 24*(b*x + a)^n*b^5*x^5*sgn(x) + 12*(b*x + a)^n*a^3*b^2*n^2*x^2*sgn(
x) - 8*(b*x + a)^n*a^2*b^3*n*x^3*sgn(x) + 12*(b*x + a)^n*a^3*b^2*n*x^2*sgn(
x) - 24*(b*x + a)^n*a^4*b*n*x*sgn(x) + 24*(b*x + a)^n*a^5*sgn(x))/(b^5*n^5
+ 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5))*c^(3/2)
```

**maple [A]** time = 0.01, size = 199, normalized size = 1.18

$$\frac{\left(b^{4}n^{4}x^{4}+10b^{4}n^{3}x^{4}-4a\,b^{3}n^{3}x^{3}+35b^{4}n^{2}x^{4}-24a\,b^{3}n^{2}x^{3}+50b^{4}n\,x^{4}+12a^{2}b^{2}n^{2}x^{2}-44a\,b^{3}n\,x^{3}+24b^{4}x^{4}+36a^{2}b^{2}n\,x^{2}-24a\,b^{3}x^{3}-24a^{3}bnx+24a^{2}b^{2}x^{2}-24a^{3}bx+24a^{4}\right)\left(c\,x^{2}\right)^{\frac{3}{2}}\left(bx+a\right)^{n+1}}{\left(n^{5}+15n^{4}+85n^{3}+225n^{2}+274n+120\right)b^{5}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] (b*x+a)^(n+1)*(b^4*n^4*x^4+10*b^4*n^3*x^4-4*a*b^3*n^3*x^3+35*b^4*n^2*x^4-24
*a*b^3*n^2*x^3+50*b^4*n*x^4+12*a^2*b^2*n^2*x^2-44*a*b^3*n*x^3+24*b^4*x^4+36
*a^2*b^2*n*x^2-24*a*b^3*x^3-24*a^3*b*n*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)*
(c*x^2)^(3/2)/x^3/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

**maxima [A]** time = 1.44, size = 157, normalized size = 0.93

$$\frac{\left(\left(n^{4}+10\,n^{3}+35\,n^{2}+50\,n+24\right)b^{5}c^{\frac{3}{2}}x^{5}+\left(n^{4}+6\,n^{3}+11\,n^{2}+6\,n\right)ab^{4}c^{\frac{3}{2}}x^{4}-4\left(n^{3}+3\,n^{2}+2\,n\right)a^{2}b^{3}c^{\frac{3}{2}}x^{3}+12\left(n^{2}+n\right)a^{3}b^{2}c^{\frac{3}{2}}x^{2}-24\,a^{4}bc^{\frac{3}{2}}nx+24\,a^{5}c^{\frac{3}{2}}\right)(bx+a)^{n}}{\left(n^{5}+15\,n^{4}+85\,n^{3}+225\,n^{2}+274\,n+120\right)b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^(3/2)*x^5 + (n^4 + 6*n^3 + 11*n^
2 + 6*n)*a*b^4*c^(3/2)*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^(3/2)*x^3 + 12
```

```
*(n^2 + n)*a^3*b^2*c^(3/2)*x^2 - 24*a^4*b*c^(3/2)*n*x + 24*a^5*c^(3/2))*(b*
x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

**mupad [B]** time = 0.41, size = 307, normalized size = 1.82

$$\frac{(a+b\,x)^n\,\left(\frac{24\,a^5\,c\,\sqrt{c\,x^2}}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{c\,x^5\,\sqrt{c\,x^2}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}-\frac{24\,a^4\,c\,n\,x\,\sqrt{c\,x^2}}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,c\,n\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{12\,a^3\,c\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{4\,a^2\,c\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((24*a^5*c*(c*x^2)^(1/2))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5 + 120)) + (c*x^5*(c*x^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24
))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) - (24*a^4*c*n*x*(c*x^2)^
(1/2))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*c*n*x^4*(
c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^
4 + n^5 + 120)) + (12*a^3*c*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^3*(274*n + 225*
n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*c*n*x^3*(c*x^2)^(1/2)*(3*n + n
^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))))/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)*(a + b*x)**n, x)
```

## 3.886 $\int \left(cx^2\right)^{3/2} (a+bx)^n \, dx$

**Optimal**. Leaf size=135

$$-\frac{a^3c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

**Rubi [A]** time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$-\frac{a^3c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] -((a^3*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c*Sqrt[c*
x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c*Sqrt[c*x^2]*(a + b*x)^(3 +
 n))/(b^4*(3 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(cx^2\right)^{3/2} (a+bx)^n\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}$$
$$= -\frac{a^3c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3ac\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{c\sqrt{cx^2}\,(a+b}{b^4(4+n}$$

**Mathematica [A]** time = 0.05, size = 98, normalized size = 0.73

$$\frac{\left(cx^2\right)^{3/2}(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x)^n,x]
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 +
 3*n + n^2)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(
3 + n)*(4 + n)*x^3)
```

**IntegrateAlgebraic [F]** time = 0.21, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{3/2}(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)*(a + b*x)^n,x]
[Out] Defer[IntegrateAlgebraic][(c*x^2)^(3/2)*(a + b*x)^n, x]
```

**fricas [A]** time = 1.06, size = 164, normalized size = 1.21

$$\frac{\left(6\,a^3bcnx-6\,a^4c+\left(b^4cn^3+6\,b^4cn^2+11\,b^4cn+6\,b^4c\right)x^4+\left(ab^3cn^3+3\,ab^3cn^2+2\,ab^3cn\right)x^3-3\left(a^2b^2cn^2+a^2b^2cn\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
[Out] (6*a^3*b*c*n*x - 6*a^4*c + (b^4*c*n^3 + 6*b^4*c*n^2 + 11*b^4*c*n + 6*b^4*c)
*x^4 + (a*b^3*c*n^3 + 3*a*b^3*c*n^2 + 2*a*b^3*c*n)*x^3 - 3*(a^2*b^2*c*n^2 +
```

```
a^2*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^4*n^4 + 10*b^4*n^3 + 35*b^4*
n^2 + 50*b^4*n + 24*b^4)*x)
```

**giac [B]** time = 1.14, size = 300, normalized size = 2.22

$$\left(\frac{6\,a^4a^n\mathrm{sgn}(x)}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}+\frac{(bx+a)^n b^4n^3x^4\mathrm{sgn}(x)+(bx+a)^n ab^3n^3x^3\mathrm{sgn}(x)+6\,(bx+a)^n b^4n^2x^4\mathrm{sgn}(x)+3\,(bx+a)^n ab^3n^2x^3\mathrm{sgn}(x)+11\,(bx+a)^n b^4nx^4\mathrm{sgn}(x)-3\,(bx+a)^n a^2b^2n^2x^2\mathrm{sgn}(x)+2\,(bx+a)^n ab^3nx^3\mathrm{sgn}(x)+6\,(bx+a)^n b^4x^4\mathrm{sgn}(x)-3\,(bx+a)^n a^2b^2nx^2\mathrm{sgn}(x)+6\,(bx+a)^n a^3bnx\mathrm{sgn}(x)-6\,(bx+a)^n a^4\mathrm{sgn}(x)}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] (6*a^4*a^n*sgn(x)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4) +
 ((b*x + a)^n*b^4*n^3*x^4*sgn(x) + (b*x + a)^n*a*b^3*n^3*x^3*sgn(x) + 6*(b*
x + a)^n*b^4*n^2*x^4*sgn(x) + 3*(b*x + a)^n*a*b^3*n^2*x^3*sgn(x) + 11*(b*x
+ a)^n*b^4*n*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b^2*n^2*x^2*sgn(x) + 2*(b*x + a
)^n*a*b^3*n*x^3*sgn(x) + 6*(b*x + a)^n*b^4*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b
^2*n*x^2*sgn(x) + 6*(b*x + a)^n*a^3*b*n*x*sgn(x) - 6*(b*x + a)^n*a^4*sgn(x)
)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4))*c^(3/2)
```

**maple [A]** time = 0.01, size = 136, normalized size = 1.01

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)(bx+a)^{n+1}}{\left(n^4+10n^3+35n^2+50n+24\right)b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(n+1)*(c*x^2)^(3/2)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x^3/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

**maxima [A]** time = 1.44, size = 116, normalized size = 0.86

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4c^{\frac{3}{2}}x^4+\left(n^3+3\,n^2+2\,n\right)ab^3c^{\frac{3}{2}}x^3-3\left(n^2+n\right)a^2b^2c^{\frac{3}{2}}x^2+6\,a^3bc^{\frac{3}{2}}nx-6\,a^4c^{\frac{3}{2}}\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*c^(3/2)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^(3/
2)*x^3 - 3*(n^2 + n)*a^2*b^2*c^(3/2)*x^2 + 6*a^3*b*c^(3/2)*n*x - 6*a^4*c^(3
/2))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

**mupad [B]** time = 0.32, size = 219, normalized size = 1.62

$$\frac{(a+bx)^n\left(\frac{c x^4\sqrt{c x^2}\left(n^3+6n^2+11n+6\right)}{n^4+10n^3+35n^2+50n+24}-\frac{6a^4c\sqrt{c x^2}}{b^4\left(n^4+10n^3+35n^2+50n+24\right)}+\frac{6a^3cnx\sqrt{c x^2}}{b^3\left(n^4+10n^3+35n^2+50n+24\right)}-\frac{3a^2cnx^2\sqrt{c x^2}\,(n+1)}{b^2\left(n^4+10n^3+35n^2+50n+24\right)}+\frac{acnx^3\sqrt{c x^2}\left(n^2+3n+2\right)}{b\left(n^4+10n^3+35n^2+50n+24\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((c*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2
 + 10*n^3 + n^4 + 24) - (6*a^4*c*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 10*n^
3 + n^4 + 24)) + (6*a^3*c*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10*n^3 +
 n^4 + 24)) - (3*a^2*c*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2*(50*n + 35*n^2 + 1
0*n^3 + n^4 + 24)) + (a*c*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n + 3
5*n^2 + 10*n^3 + n^4 + 24))))/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{\frac{3}{2}} (a+bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n, x)
```

## 3.887 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x}\,dx$

**Optimal**. Leaf size=99

$$\frac{a^2c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

**Rubi [A]** time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
```

```
[Out] (a^2*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c*Sqrt[c*x^2]*
(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3
*(3 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}(a+bx)^n}{x}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}$$
$$= \frac{a^2c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x} - \frac{2ac\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x} + \frac{c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

**Mathematica [A]** time = 0.03, size = 70, normalized size = 0.71

$$\frac{c^2x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
```

```
[Out] (c^2*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
))/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic [F]** time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{3/2}(a+bx)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(3/2)*(a + b*x)^n)/x, x]
```

**fricas [A]** time = 1.02, size = 113, normalized size = 1.14

$$-\frac{\left(2\,a^2bcnx-2\,a^3c-\left(b^3cn^2+3\,b^3cn+2\,b^3c\right)x^3-\left(ab^2cn^2+ab^2cn\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*c*n*x - 2*a^3*c - (b^3*c*n^2 + 3*b^3*c*n + 2*b^3*c)*x^3 - (a*b^2*
c*n^2 + a*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6*b^3*n^2 + 11*
b^3*n + 6*b^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x, x)
```

**maple [A]** time = 0.01, size = 83, normalized size = 0.84

$$\frac{\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}(bx+a)^{n+1}}{\left(n^3+6n^2+11n+6\right)b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x,x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(3/2)/x^3/b^3/(n^3+6*n^2+11*n+6)
```

**maxima [A]** time = 1.43, size = 80, normalized size = 0.81

$$\frac{\left(\left(n^2+3\,n+2\right)b^3c^{\frac{3}{2}}x^3+\left(n^2+n\right)ab^2c^{\frac{3}{2}}x^2-2\,a^2bc^{\frac{3}{2}}nx+2\,a^3c^{\frac{3}{2}}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*c^(3/2)*x^3 + (n^2 + n)*a*b^2*c^(3/2)*x^2 - 2*a^2*b*c^
(3/2)*n*x + 2*a^3*c^(3/2))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

**mupad [B]** time = 0.26, size = 146, normalized size = 1.47

$$\frac{(a+b\,x)^n\left(\frac{c\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{2\,a^3\,c\,\sqrt{c\,x^2}}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,c\,n\,x\,\sqrt{c\,x^2}}{b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,c\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((a + b*x)^n*((c*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6
) + (2*a^3*c*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*c*n*x*(
c*x^2)^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*c*n*x^2*(c*x^2)^(1/2)*(n
+ 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}(a+bx)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x, x)
```

## 3.888 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^2}\,dx$

**Optimal**. Leaf size=65

$$\frac{c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

**Rubi** [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] -((a*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c*Sqrt[c*x^2]*(a
+ b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}$$
$$= -\frac{ac\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}$$

**Mathematica [A]** time = 0.01, size = 46, normalized size = 0.71

$$\frac{c^2x(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] (c^2*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

**IntegrateAlgebraic [F]** time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(3/2)*(a + b*x)^n)/x^2, x]
```

**fricas [A]** time = 0.84, size = 68, normalized size = 1.05

$$\frac{\left(abcnx-a^2c+\left(b^2cn+b^2c\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2n^2+3\,b^2n+2\,b^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] (a*b*c*n*x - a^2*c + (b^2*c*n + b^2*c)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*n
^2 + 3*b^2*n + 2*b^2)*x)
```

**giac [A]** time = 0.91, size = 119, normalized size = 1.83

$$\left(\frac{a^2a^n\mathrm{sgn}(x)}{b^2n^2+3\,b^2n+2\,b^2}+\frac{(bx+a)^n b^2nx^2\mathrm{sgn}(x)+(bx+a)^n abnx\mathrm{sgn}(x)+(bx+a)^n b^2x^2\mathrm{sgn}(x)-(bx+a)^n a^2\mathrm{sgn}(x)}{b^2n^2+3\,b^2n+2\,b^2}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="giac")
```

```
[Out] (a^2*a^n*sgn(x)/(b^2*n^2 + 3*b^2*n + 2*b^2) + ((b*x + a)^n*b^2*n*x^2*sgn(x)
 + (b*x + a)^n*a*b*n*x*sgn(x) + (b*x + a)^n*b^2*x^2*sgn(x) - (b*x + a)^n*a^
2*sgn(x))/(b^2*n^2 + 3*b^2*n + 2*b^2))*c^(3/2)
```

**maple [A]** time = 0.00, size = 46, normalized size = 0.71

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(-xnb-bx+a\right)\left(bx+a\right)^{n+1}}{\left(n^2+3n+2\right)b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^2,x)
```

```
[Out] -(b*x+a)^(n+1)*(c*x^2)^(3/2)*(-b*n*x-b*x+a)/x^3/b^2/(n^2+3*n+2)
```

**maxima [A]** time = 1.43, size = 51, normalized size = 0.78

$$\frac{\left(b^2c^{\frac{3}{2}}(n+1)x^2+abc^{\frac{3}{2}}nx-a^2c^{\frac{3}{2}}\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] (b^2*c^(3/2)*(n + 1)*x^2 + a*b*c^(3/2)*n*x - a^2*c^(3/2))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

**mupad [B]** time = 0.23, size = 88, normalized size = 1.35

$$\frac{(a+b\,x)^n\left(\frac{c\,x^2\,\sqrt{c\,x^2}\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,c\,\sqrt{c\,x^2}}{b^2\left(n^2+3\,n+2\right)}+\frac{a\,c\,n\,x\,\sqrt{c\,x^2}}{b\left(n^2+3\,n+2\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x)
```

```
[Out] ((a + b*x)^n*((c*x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*c*(c*x^2
)^(1/2))/(b^2*(3*n + n^2 + 2)) + (a*c*n*x*(c*x^2)^(1/2))/(b*(3*n + n^2 + 2)
)))/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n c^{\frac{3}{2}} \left(x^2\right)^{\frac{3}{2}}}{2x} & \text{for } b = 0 \\ \int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^2 (a+bx)^2}\, dx & \text{for } n = -2 \\ \int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^2 (a+bx)}\, dx & \text{for } n = -1 \\ -\frac{a^2 c^{\frac{3}{2}} (a+bx)^n \left(x^2\right)^{\frac{3}{2}}}{b^2 n^2 x^3 + 3b^2 n x^3 + 2b^2 x^3} + \frac{abc^{\frac{3}{2}} n x (a+bx)^n \left(x^2\right)^{\frac{3}{2}}}{b^2 n^2 x^3 + 3b^2 n x^3 + 2b^2 x^3} + \frac{b^2 c^{\frac{3}{2}} n x^2 (a+bx)^n \left(x^2\right)^{\frac{3}{2}}}{b^2 n^2 x^3 + 3b^2 n x^3 + 2b^2 x^3} + \frac{b^2 c^{\frac{3}{2}} x^2 (a+bx)^n \left(x^2\right)^{\frac{3}{2}}}{b^2 n^2 x^3 + 3b^2 n x^3 + 2b^2 x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**2,x)
```

```
[Out] Piecewise((a**n*c**(3/2)*(x**2)**(3/2)/(2*x), Eq(b, 0)), (Integral((c*x**2)
**(3/2)/(x**2*(a + b*x)**2), x), Eq(n, -2)), (Integral((c*x**2)**(3/2)/(x**
2*(a + b*x)), x), Eq(n, -1)), (-a**2*c**(3/2)*(a + b*x)**n*(x**2)**(3/2)/(b
**2*n**2*x**3 + 3*b**2*n*x**3 + 2*b**2*x**3) + a*b*c**(3/2)*n*x*(a + b*x)**
n*(x**2)**(3/2)/(b**2*n**2*x**3 + 3*b**2*n*x**3 + 2*b**2*x**3) + b**2*c**(3
/2)*n*x**2*(a + b*x)**n*(x**2)**(3/2)/(b**2*n**2*x**3 + 3*b**2*n*x**3 + 2*b
**2*x**3) + b**2*c**(3/2)*x**2*(a + b*x)**n*(x**2)**(3/2)/(b**2*n**2*x**3 +
 3*b**2*n*x**3 + 2*b**2*x**3), True))
```

## 3.889 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^3} dx$

**Optimal**. Leaf size=31

$$\frac{c\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

**Rubi [A]** time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{c\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}\,(a+bx)^n}{x^3}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)^n\,dx}{x}\\ &= \frac{c\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 0.97

$$\frac{(cx^2)^{3/2}\,(a+bx)^{n+1}}{b(n+1)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n))/(b*(1 + n)*x^3)
```

**IntegrateAlgebraic [F]** time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{3/2} (a+bx)^n}{x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(3/2)*(a + b*x)^n)/x^3, x]
```

**fricas [A]** time = 1.01, size = 33, normalized size = 1.06

$$\frac{(bcx+ac)\sqrt{cx^2}\,(bx+a)^n}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="fricas")
[Out] (b*c*x + a*c)*sqrt(c*x^2)*(b*x + a)^n/((b*n + b)*x)
```

**giac [A]** time = 1.09, size = 42, normalized size = 1.35

$$-c^{\frac{3}{2}}\left(\frac{a^{n+1}\operatorname{sgn}(x)}{bn+b} - \frac{(bx+a)^{n+1}\operatorname{sgn}(x)}{b(n+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="giac")
[Out] -c^(3/2)*(a^(n + 1)*sgn(x)/(b*n + b) - (b*x + a)^(n + 1)*sgn(x)/(b*(n + 1))
)
```

**maple [A]** time = 0.00, size = 29, normalized size = 0.94

$$\frac{(c\,x^2)^{\frac{3}{2}}\,(bx+a)^{n+1}}{(n+1)\,b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^3,x)
```

```
[Out] (b*x+a)^(n+1)/b/(n+1)*(c*x^2)^(3/2)/x^3
```

**maxima [A]** time = 1.42, size = 28, normalized size = 0.90

$$\frac{\left(bc^{\frac{3}{2}}x + ac^{\frac{3}{2}}\right)(bx+a)^n}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] (b*c^(3/2)*x + a*c^(3/2))*(b*x + a)^n/(b*(n + 1))
```

**mupad [B]** time = 0.23, size = 45, normalized size = 1.45

$$\frac{\left(\frac{c\,x\,\sqrt{c\,x^2}}{n+1} + \frac{a\,c\,\sqrt{c\,x^2}}{b\,(n+1)}\right)\,(a+b\,x)^n}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x)
```

```
[Out] (((c*x*(c*x^2)^(1/2))/(n + 1) + (a*c*(c*x^2)^(1/2))/(b*(n + 1)))*(a + b*x)^
n)/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{ax^2} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{x^2} & \text{for } b = 0 \\ \int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^3(a+bx)}\,dx & \text{for } n = -1 \\ \frac{ac^{\frac{3}{2}}(a+bx)^n\left(x^2\right)^{\frac{3}{2}}}{bnx^3+bx^3} + \frac{bc^{\frac{3}{2}}x(a+bx)^n\left(x^2\right)^{\frac{3}{2}}}{bnx^3+bx^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**3,x)
```

```
[Out] Piecewise(((c**(3/2)*(x**2)**(3/2)/(a*x**2), Eq(b, 0) & Eq(n, -1)), (a**n*c*
*(3/2)*(x**2)**(3/2)/x**2, Eq(b, 0)), (Integral((c*x**2)**(3/2)/(x**3*(a +
```

```
b*x)), x), Eq(n, -1)), (a*c**(3/2)*(a + b*x)**n*(x**2)**(3/2)/(b*n*x**3 + b
*x**3) + b*c**(3/2)*x*(a + b*x)**n*(x**2)**(3/2)/(b*n*x**3 + b*x**3), True)
)
```

## 3.890 $\int \left(cx^2\right)^{5/2} (a+bx)^n \, dx$

**Optimal**. Leaf size=217

$$-\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^6(n+1)x}+\frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^6(n+2)x}-\frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^6(n+3)x}+\frac{10a^2c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^6(n+4)x}-\frac{5ac^2\sqrt{cx^2}\,(a}{b^6(n+}$$

**Rubi [A]** time = 0.07, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$-\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^6(n+1)x}+\frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^6(n+2)x}-\frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^6(n+3)x}+\frac{10a^2c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^6(n+4)x}-\frac{5ac^2\sqrt{cx^2}\,(a+bx)^{n+5}}{b^6(n+5)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+6}}{b^6(n+6)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] -((a^5*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^6*(1 + n)*x)) + (5*a^4*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^6*(2 + n)*x) - (10*a^3*c^2*Sqrt[c*x^2]*(a +
b*x)^(3 + n))/(b^6*(3 + n)*x) + (10*a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/
(b^6*(4 + n)*x) - (5*a*c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^6*(5 + n)*x) +
 (c^2*Sqrt[c*x^2]*(a + b*x)^(6 + n))/(b^6*(6 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(cx^2\right)^{5/2} (a+bx)^n \, dx = \frac{\left(c^2\sqrt{cx^2}\right) \int x^5 (a+bx)^n \, dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right) \int \left(-\frac{a^5(a+bx)^n}{b^5} + \frac{5a^4(a+bx)^{1+n}}{b^5} - \frac{10a^3(a+bx)^{2+n}}{b^5} + \frac{10a^2(a+bx)^{3+n}}{b^5} - \frac{5a(a+bx)^{4+n}}{b^5} + \frac{(a+b}{b}\right.}{x}$$
$$= -\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^6(1+n)x} + \frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^6(2+n)x} - \frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^6(3+n)x} + \frac{10a^2c^2}{b}$$

**Mathematica [A]** time = 0.11, size = 172, normalized size = 0.79

$$\frac{c^3x(a+bx)^{n+1}\left(-120a^5+120a^4b(n+1)x-60a^3b^2\left(n^2+3n+2\right)x^2+20a^2b^3\left(n^3+6n^2+11n+6\right)x^3-5ab^4\left(n^4+10n^3+35n^2+50n+24\right)x^4+b^5\left(n^5+15n^4+85n^3+225n^2+274n+120\right)x^5\right)}{b^6(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(-120*a^5 + 120*a^4*b*(1 + n)*x - 60*a^3*b^2*(2 +
3*n + n^2)*x^2 + 20*a^2*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 - 5*a*b^4*(24 + 50
*n + 35*n^2 + 10*n^3 + n^4)*x^4 + b^5*(120 + 274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5)*x^5))/(b^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*Sqrt[
c*x^2])
```

**IntegrateAlgebraic [F]** time = 0.23, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{5/2} (a+bx)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(c*x^2)^(5/2)*(a + b*x)^n, x]
```

**fricas [A]** time = 0.89, size = 352, normalized size = 1.62

$$\frac{\left(120\,a^5bc^2nx - 120\,a^6c^2 + \left(b^6c^2n^5 + 15\,b^6c^2n^4 + 85\,b^6c^2n^3 + 225\,b^6c^2n^2 + 274\,b^6c^2n + 120\,b^6c^2\right)x^6 + \left(ab^5c^2n^5 + 10\,ab^5c^2n^4 + 35\,ab^5c^2n^3 + 50\,ab^5c^2n^2 + 24\,ab^5c^2n\right)x^5 - 5\left(a^2b^4c^2n^4 + 6\,a^2b^4c^2n^3 + 11\,a^2b^4c^2n^2 + 6\,a^2b^4c^2n\right)x^4 + 20\left(a^3b^3c^2n^3 + 3\,a^3b^3c^2n^2 + 2\,a^3b^3c^2n\right)x^3 - 60\left(a^4b^2c^2n^2 + a^4b^2c^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^6n^6 + 21\,b^6n^5 + 175\,b^6n^4 + 735\,b^6n^3 + 1624\,b^6n^2 + 1764\,b^6n + 720\,b^6\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (120*a^5*b*c^2*n*x - 120*a^6*c^2 + (b^6*c^2*n^5 + 15*b^6*c^2*n^4 + 85*b^6*c
^2*n^3 + 225*b^6*c^2*n^2 + 274*b^6*c^2*n + 120*b^6*c^2)*x^6 + (a*b^5*c^2*n^
```

```
5 + 10*a*b^5*c^2*n^4 + 35*a*b^5*c^2*n^3 + 50*a*b^5*c^2*n^2 + 24*a*b^5*c^2*n
)*x^5 - 5*(a^2*b^4*c^2*n^4 + 6*a^2*b^4*c^2*n^3 + 11*a^2*b^4*c^2*n^2 + 6*a^2
*b^4*c^2*n)*x^4 + 20*(a^3*b^3*c^2*n^3 + 3*a^3*b^3*c^2*n^2 + 2*a^3*b^3*c^2*n
)*x^3 - 60*(a^4*b^2*c^2*n^2 + a^4*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/(
(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6
*n + 720*b^6)*x)
```

**giac [B]** time = 1.06, size = 640, normalized size = 2.95

$$\left(\frac{120\,a^{6}a^{n}c^{2}\mathrm{sgn}(x)}{b^{6}n^{6}+21\,b^{6}n^{5}+175\,b^{6}n^{4}+735\,b^{6}n^{3}+1624\,b^{6}n^{2}+1764\,b^{6}n+720\,b^{6}}+\frac{(bx+a)^{n}b^{6}c^{2}n^{5}x^{6}\mathrm{sgn}(x)+(bx+a)^{n}ab^{5}c^{2}n^{5}x^{5}\mathrm{sgn}(x)+15\,(bx+a)^{n}b^{6}c^{2}n^{4}x^{6}\mathrm{sgn}(x)+10\,(bx+a)^{n}ab^{5}c^{2}n^{4}x^{5}\mathrm{sgn}(x)+85\,(bx+a)^{n}b^{6}c^{2}n^{3}x^{6}\mathrm{sgn}(x)-5\,(bx+a)^{n}a^{2}b^{4}c^{2}n^{4}x^{4}\mathrm{sgn}(x)+35\,(bx+a)^{n}ab^{5}c^{2}n^{3}x^{5}\mathrm{sgn}(x)+225\,(bx+a)^{n}b^{6}c^{2}n^{2}x^{6}\mathrm{sgn}(x)-30\,(bx+a)^{n}a^{2}b^{4}c^{2}n^{3}x^{4}\mathrm{sgn}(x)+50\,(bx+a)^{n}ab^{5}c^{2}n^{2}x^{5}\mathrm{sgn}(x)+274\,(bx+a)^{n}b^{6}c^{2}nx^{6}\mathrm{sgn}(x)+20\,(bx+a)^{n}a^{3}b^{3}c^{2}n^{3}x^{3}\mathrm{sgn}(x)-55\,(bx+a)^{n}a^{2}b^{4}c^{2}n^{2}x^{4}\mathrm{sgn}(x)+24\,(bx+a)^{n}ab^{5}c^{2}nx^{5}\mathrm{sgn}(x)+120\,(bx+a)^{n}b^{6}c^{2}x^{6}\mathrm{sgn}(x)+60\,(bx+a)^{n}a^{3}b^{3}c^{2}n^{2}x^{3}\mathrm{sgn}(x)-30\,(bx+a)^{n}a^{2}b^{4}c^{2}nx^{4}\mathrm{sgn}(x)-60\,(bx+a)^{n}a^{4}b^{2}c^{2}n^{2}x^{2}\mathrm{sgn}(x)+40\,(bx+a)^{n}a^{3}b^{3}c^{2}nx^{3}\mathrm{sgn}(x)-60\,(bx+a)^{n}a^{4}b^{2}c^{2}nx^{2}\mathrm{sgn}(x)+120\,(bx+a)^{n}a^{5}bc^{2}nx\,\mathrm{sgn}(x)-120\,(bx+a)^{n}a^{6}c^{2}\mathrm{sgn}(x)}{b^{6}n^{6}+21\,b^{6}n^{5}+175\,b^{6}n^{4}+735\,b^{6}n^{3}+1624\,b^{6}n^{2}+1764\,b^{6}n+720\,b^{6}}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] (120*a^6*a^n*c^2*sgn(x)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 +
 1624*b^6*n^2 + 1764*b^6*n + 720*b^6) + ((b*x + a)^n*b^6*c^2*n^5*x^6*sgn(x)
 + (b*x + a)^n*a*b^5*c^2*n^5*x^5*sgn(x) + 15*(b*x + a)^n*b^6*c^2*n^4*x^6*sg
n(x) + 10*(b*x + a)^n*a*b^5*c^2*n^4*x^5*sgn(x) + 85*(b*x + a)^n*b^6*c^2*n^3
*x^6*sgn(x) - 5*(b*x + a)^n*a^2*b^4*c^2*n^4*x^4*sgn(x) + 35*(b*x + a)^n*a*b
^5*c^2*n^3*x^5*sgn(x) + 225*(b*x + a)^n*b^6*c^2*n^2*x^6*sgn(x) - 30*(b*x +
a)^n*a^2*b^4*c^2*n^3*x^4*sgn(x) + 50*(b*x + a)^n*a*b^5*c^2*n^2*x^5*sgn(x) +
 274*(b*x + a)^n*b^6*c^2*n*x^6*sgn(x) + 20*(b*x + a)^n*a^3*b^3*c^2*n^3*x^3*
sgn(x) - 55*(b*x + a)^n*a^2*b^4*c^2*n^2*x^4*sgn(x) + 24*(b*x + a)^n*a*b^5*c
^2*n*x^5*sgn(x) + 120*(b*x + a)^n*b^6*c^2*x^6*sgn(x) + 60*(b*x + a)^n*a^3*b
^3*c^2*n^2*x^3*sgn(x) - 30*(b*x + a)^n*a^2*b^4*c^2*n*x^4*sgn(x) - 60*(b*x +
 a)^n*a^4*b^2*c^2*n^2*x^2*sgn(x) + 40*(b*x + a)^n*a^3*b^3*c^2*n*x^3*sgn(x)
- 60*(b*x + a)^n*a^4*b^2*c^2*n*x^2*sgn(x) + 120*(b*x + a)^n*a^5*b*c^2*n*x*s
gn(x) - 120*(b*x + a)^n*a^6*c^2*sgn(x))/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4
 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6))*sqrt(c)
```

**maple [A]** time = 0.01, size = 280, normalized size = 1.29

$$-\frac{\left(c\,x^{2}\right)^{\frac{5}{2}}\left(-b^{5}n^{5}x^{5}-15b^{5}n^{4}x^{5}+5a\,b^{4}n^{4}x^{4}-85b^{5}n^{3}x^{5}+50a\,b^{4}n^{3}x^{4}-225b^{5}n^{2}x^{5}-20a^{2}b^{3}n^{3}x^{3}+175a\,b^{4}n^{2}x^{4}-274b^{5}n\,x^{5}-120a^{2}b^{3}n^{2}x^{3}+250a\,b^{4}n\,x^{4}-120b^{5}x^{5}+60a^{3}b^{2}n^{2}x^{2}-220a^{2}b^{3}n\,x^{3}+120a\,b^{4}x^{4}+180a^{3}b^{2}n\,x^{2}-120a^{2}b^{3}x^{3}-120a^{4}bnx+120a^{3}b^{2}x^{2}-120a^{4}bx+120a^{5}\right)\left(bx+a\right)^{n+1}}{\left(n^{6}+21n^{5}+175n^{4}+735n^{3}+1624n^{2}+1764n+720\right)b^{6}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(n+1)*(c*x^2)^(5/2)*(-b^5*n^5*x^5-15*b^5*n^4*x^5+5*a*b^4*n^4*x^4-8
5*b^5*n^3*x^5+50*a*b^4*n^3*x^4-225*b^5*n^2*x^5-20*a^2*b^3*n^3*x^3+175*a*b^4
*n^2*x^4-274*b^5*n*x^5-120*a^2*b^3*n^2*x^3+250*a*b^4*n*x^4-120*b^5*x^5+60*a
^3*b^2*n^2*x^2-220*a^2*b^3*n*x^3+120*a*b^4*x^4+180*a^3*b^2*n*x^2-120*a^2*b^
3*x^3-120*a^4*b*n*x+120*a^3*b^2*x^2-120*a^4*b*x+120*a^5)/x^5/b^6/(n^6+21*n^
5+175*n^4+735*n^3+1624*n^2+1764*n+720)
```

**maxima** [A] time = 1.53, size = 203, normalized size = 0.94

$$\frac{\left(\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^6c^{\frac{5}{2}}x^6+\left(n^5+10\,n^4+35\,n^3+50\,n^2+24\,n\right)ab^5c^{\frac{5}{2}}x^5-5\left(n^4+6\,n^3+11\,n^2+6\,n\right)a^2b^4c^{\frac{5}{2}}x^4+20\left(n^3+3\,n^2+2\,n\right)a^3b^3c^{\frac{5}{2}}x^3-60\left(n^2+n\right)a^4b^2c^{\frac{5}{2}}x^2+120\,a^5bc^{\frac{5}{2}}nx-120\,a^6c^{\frac{5}{2}}\right)(bx+a)^n}{\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^6*c^(5/2)*x^6 + (n^5 + 1
0*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*c^(5/2)*x^5 - 5*(n^4 + 6*n^3 + 11*n^2
 + 6*n)*a^2*b^4*c^(5/2)*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*c^(5/2)*x^3 -
60*(n^2 + n)*a^4*b^2*c^(5/2)*x^2 + 120*a^5*b*c^(5/2)*n*x - 120*a^6*c^(5/2))
*(b*x + a)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*
b^6)
```

**mupad** [B] time = 0.50, size = 424, normalized size = 1.95

$$\frac{(a+b\,x)^n\left(\frac{c^2\,x^6\,\sqrt{c\,x^2}\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720}-\frac{120\,a^6\,c^2\,\sqrt{c\,x^2}}{b^6\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{120\,a^5\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^5\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{5\,a^2\,c^2\,n\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{b^2\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{60\,a^4\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^4\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{a\,c^2\,n\,x^5\,\sqrt{c\,x^2}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{b\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{20\,a^3\,c^2\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b^3\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^6*(c*x^2)^(1/2)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n
^5 + 120))/(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) - (
120*a^6*c^2*(c*x^2)^(1/2))/(b^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21
*n^5 + n^6 + 720)) + (120*a^5*c^2*n*x*(c*x^2)^(1/2))/(b^5*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (5*a^2*c^2*n*x^4*(c*x^2)^(1/
2)*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 +
21*n^5 + n^6 + 720)) - (60*a^4*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^4*(1764*
n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*c^2*n*x^5*(c*x
^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*
n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (20*a^3*c^2*n*x^3*(c*x^2)^(1/2)*(3*n
 + n^2 + 2))/(b^3*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 7
20))))/x
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{\frac{5}{2}}\,(a+bx)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n, x)
```

## 3.891 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x} dx$

**Optimal**. Leaf size=179

$$\frac{a^4c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

**Rubi [A]** time = 0.05, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^4c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
```

```
[Out] (a^4*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c^2*Sqrt[c
*x^2]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c^2*Sqrt[c*x^2]*(a + b*x)
^(3 + n))/(b^5*(3 + n)*x) - (4*a*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4
 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^4(a+bx)^n}{b^4} - \frac{4a^3(a+bx)^{1+n}}{b^4} + \frac{6a^2(a+bx)^{2+n}}{b^4} - \frac{4a(a+bx)^{3+n}}{b^4} + \frac{(a+bx)^{4+n}}{b^4}\right)dx}{x}$$
$$= \frac{a^4c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^5(1+n)x} - \frac{4a^3c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^5(2+n)x} + \frac{6a^2c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^5(3+n)x} - \frac{4ac^2\sqrt{cx^2}}{b^5(4}$$

**Mathematica [A]** time = 0.02, size = 133, normalized size = 0.74

$$\frac{c\left(cx^2\right)^{3/2}(a+bx)^{n+1}\left(24a^4-24a^3b(n+1)x+12a^2b^2\left(n^2+3n+2\right)x^2-4ab^3\left(n^3+6n^2+11n+6\right)x^3+b^4\left(n^4+10n^3+35n^2+50n+24\right)x^4\right)}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
```

```
[Out] (c*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*(24*a^4 - 24*a^3*b*(1 + n)*x + 12*a^2*b^
2*(2 + 3*n + n^2)*x^2 - 4*a*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + b^4*(24 + 50
*n + 35*n^2 + 10*n^3 + n^4)*x^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 +
 n)*x^3)
```

**IntegrateAlgebraic [F]** time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(5/2)*(a + b*x)^n)/x, x]
```

**fricas [A]** time = 1.15, size = 265, normalized size = 1.48

$$-\frac{\left(24\,a^4bc^2nx-24\,a^5c^2-\left(b^5c^2n^4+10\,b^5c^2n^3+35\,b^5c^2n^2+50\,b^5c^2n+24\,b^5c^2\right)x^5-\left(ab^4c^2n^4+6\,ab^4c^2n^3+11\,ab^4c^2n^2+6\,ab^4c^2n\right)x^4+4\left(a^2b^3c^2n^3+3\,a^2b^3c^2n^2+2\,a^2b^3c^2n\right)x^3-12\left(a^3b^2c^2n^2+a^3b^2c^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\,b^5n+120\,b^5\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] -(24*a^4*b*c^2*n*x - 24*a^5*c^2 - (b^5*c^2*n^4 + 10*b^5*c^2*n^3 + 35*b^5*c^
2*n^2 + 50*b^5*c^2*n + 24*b^5*c^2)*x^5 - (a*b^4*c^2*n^4 + 6*a*b^4*c^2*n^3 +
```

```
11*a*b^4*c^2*n^2 + 6*a*b^4*c^2*n)*x^4 + 4*(a^2*b^3*c^2*n^3 + 3*a^2*b^3*c^2
*n^2 + 2*a^2*b^3*c^2*n)*x^3 - 12*(a^3*b^2*c^2*n^2 + a^3*b^2*c^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 27
4*b^5*n + 120*b^5)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}} (bx+a)^n}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x, x)
```

**maple [A]** time = 0.01, size = 199, normalized size = 1.11

$$\frac{\left(b^4n^4x^4+10b^4n^3x^4-4a\,b^3n^3x^3+35b^4n^2x^4-24a\,b^3n^2x^3+50b^4n\,x^4+12a^2b^2n^2x^2-44a\,b^3n\,x^3+24b^4x^4+36a^2b^2n\,x^2-24a\,b^3x^3-24a^3bnx+24a^2b^2x^2-24a^3bx+24a^4\right)\left(c\,x^2\right)^{\frac{5}{2}}(bx+a)^{n+1}}{\left(n^5+15n^4+85n^3+225n^2+274n+120\right)b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x,x)
```

```
[Out] (b*x+a)^(n+1)*(b^4*n^4*x^4+10*b^4*n^3*x^4-4*a*b^3*n^3*x^3+35*b^4*n^2*x^4-24
*a*b^3*n^2*x^3+50*b^4*n*x^4+12*a^2*b^2*n^2*x^2-44*a*b^3*n*x^3+24*b^4*x^4+36
*a^2*b^2*n*x^2-24*a*b^3*x^3-24*a^3*b*n*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)*
(c*x^2)^(5/2)/x^5/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

**maxima [A]** time = 1.45, size = 157, normalized size = 0.88

$$\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5c^{\frac{5}{2}}x^5+\left(n^4+6\,n^3+11\,n^2+6\,n\right)ab^4c^{\frac{5}{2}}x^4-4\left(n^3+3\,n^2+2\,n\right)a^2b^3c^{\frac{5}{2}}x^3+12\left(n^2+n\right)a^3b^2c^{\frac{5}{2}}x^2-24\,a^4bc^{\frac{5}{2}}nx+24\,a^5c^{\frac{5}{2}}\right)(bx+a)^n}{\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^(5/2)*x^5 + (n^4 + 6*n^3 + 11*n^
2 + 6*n)*a*b^4*c^(5/2)*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^(5/2)*x^3 + 12
*(n^2 + n)*a^3*b^2*c^(5/2)*x^2 - 24*a^4*b*c^(5/2)*n*x + 24*a^5*c^(5/2))*(b*
x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

**mupad [B]** time = 0.38, size = 319, normalized size = 1.78

$$\frac{(a+b\,x)^n\left(\frac{c^2\,x^5\,\sqrt{c\,x^2}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}+\frac{24\,a^5\,c^2\,\sqrt{c\,x^2}}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{24\,a^4\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,c^2\,n\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{12\,a^3\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{4\,a^2\,c^2\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^5*(c*x^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(
274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (24*a^5*c^2*(c*x^2)^(1/2))
/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (24*a^4*c^2*n*x*(c
*x^2)^(1/2))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*c^2
*n*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3
 + 15*n^4 + n^5 + 120)) + (12*a^3*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^3*(27
4*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*c^2*n*x^3*(c*x^2)^(1
/2)*(3*n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
)/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}} (a+bx)^n}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x, x)
```

## 3.892 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^2}\,dx$

**Optimal**. Leaf size=143

$$-\frac{a^3c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

**Rubi [A]** time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] -((a^3*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c^2*Sqrt[c*x^2]*(a + b*x
)^(3 + n))/(b^4*(3 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 +
n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(cx^2\right)^{5/2} (a+bx)^n}{x^2} dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)^n dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^3(a+bx)^n}{b^3} + \frac{3a^2(a+bx)^{1+n}}{b^3} - \frac{3a(a+bx)^{2+n}}{b^3} + \frac{(a+bx)^{3+n}}{b^3}\right) dx}{x}$$
$$= -\frac{a^3c^2\sqrt{cx^2}(a+bx)^{1+n}}{b^4(1+n)x} + \frac{3a^2c^2\sqrt{cx^2}(a+bx)^{2+n}}{b^4(2+n)x} - \frac{3ac^2\sqrt{cx^2}(a+bx)^{3+n}}{b^4(3+n)x} + \frac{c^2\sqrt{cx^2}(}{b^4(4}$$

**Mathematica** [A] time = 0.02, size = 99, normalized size = 0.69

$$\frac{c\left(cx^2\right)^{3/2}(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] (c*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2
 + 3*n + n^2)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)
*(3 + n)*(4 + n)*x^3)
```

**IntegrateAlgebraic** [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(5/2)*(a + b*x)^n)/x^2, x]
```

**fricas** [A] time = 1.15, size = 186, normalized size = 1.30

$$\frac{\left(6\,a^3bc^2nx-6\,a^4c^2+\left(b^4c^2n^3+6\,b^4c^2n^2+11\,b^4c^2n+6\,b^4c^2\right)x^4+\left(ab^3c^2n^3+3\,ab^3c^2n^2+2\,ab^3c^2n\right)x^3-3\left(a^2b^2c^2n^2+a^2b^2c^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*c^2*n*x - 6*a^4*c^2 + (b^4*c^2*n^3 + 6*b^4*c^2*n^2 + 11*b^4*c^2*n
+ 6*b^4*c^2)*x^4 + (a*b^3*c^2*n^3 + 3*a*b^3*c^2*n^2 + 2*a*b^3*c^2*n)*x^3 -
```

```
3*(a^2*b^2*c^2*n^2 + a^2*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^4*n^4
+ 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}} (bx+a)^n}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^2, x)
```

**maple [A]** time = 0.01, size = 136, normalized size = 0.95

$$-\frac{\left(c\,x^2\right)^{\frac{5}{2}} \left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)(bx+a)^{n+1}}{\left(n^4+10n^3+35n^2+50n+24\right)b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^2,x)
```

```
[Out] -(b*x+a)^(n+1)*(c*x^2)^(5/2)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x^5/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

**maxima [A]** time = 1.45, size = 116, normalized size = 0.81

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4c^{\frac{5}{2}}x^4+\left(n^3+3\,n^2+2\,n\right)ab^3c^{\frac{5}{2}}x^3-3\left(n^2+n\right)a^2b^2c^{\frac{5}{2}}x^2+6\,a^3bc^{\frac{5}{2}}nx-6\,a^4c^{\frac{5}{2}}\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*c^(5/2)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^(5/
2)*x^3 - 3*(n^2 + n)*a^2*b^2*c^(5/2)*x^2 + 6*a^3*b*c^(5/2)*n*x - 6*a^4*c^(5
/2))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

**mupad [B]** time = 0.32, size = 229, normalized size = 1.60

$$\frac{(a+b\,x)^n \left(\frac{c^2\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,c^2\,\sqrt{c\,x^2}}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,c^2\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n
^2 + 10*n^3 + n^4 + 24) - (6*a^4*c^2*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 1
0*n^3 + n^4 + 24)) + (6*a^3*c^2*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10
*n^3 + n^4 + 24)) + (a*c^2*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n +
35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2
*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}} (a + bx)^n}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**2,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**2, x)
```

## 3.893 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^3} dx$

**Optimal**. Leaf size=105

$$\frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

**Rubi [A]** time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c^2*Sqrt[c*x
^2]*(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(3 + n)
)/(b^3*(3 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^3}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}$$
$$= \frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x} - \frac{2ac^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x} + \frac{c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

**Mathematica** [A] time = 0.05, size = 70, normalized size = 0.67

$$\frac{c^3x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
))/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(5/2)*(a + b*x)^n)/x^3, x]
```

**fricas** [A] time = 1.05, size = 127, normalized size = 1.21

$$-\frac{\left(2\,a^2bc^2nx-2\,a^3c^2-\left(b^3c^2n^2+3\,b^3c^2n+2\,b^3c^2\right)x^3-\left(ab^2c^2n^2+ab^2c^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*c^2*n*x - 2*a^3*c^2 - (b^3*c^2*n^2 + 3*b^3*c^2*n + 2*b^3*c^2)*x^3
 - (a*b^2*c^2*n^2 + a*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6
*b^3*n^2 + 11*b^3*n + 6*b^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}}(bx+a)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^3, x)
```

**maple [A]** time = 0.01, size = 83, normalized size = 0.79

$$\frac{(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2)\,(c\,x^2)^{\frac{5}{2}}\,(bx+a)^{n+1}}{(n^3+6n^2+11n+6)\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^3,x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(5/2)/x^5/b^3/(n^3+6*n^2+11*n+6)
```

**maxima [A]** time = 1.36, size = 80, normalized size = 0.76

$$\frac{\left((n^2+3\,n+2)b^3c^{\frac{5}{2}}x^3+(n^2+n)ab^2c^{\frac{5}{2}}x^2-2\,a^2bc^{\frac{5}{2}}nx+2\,a^3c^{\frac{5}{2}}\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*c^(5/2)*x^3 + (n^2 + n)*a*b^2*c^(5/2)*x^2 - 2*a^2*b*c^
(5/2)*n*x + 2*a^3*c^(5/2))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

**mupad [B]** time = 0.27, size = 154, normalized size = 1.47

$$\frac{(a+b\,x)^n\left(\frac{2\,a^3\,c^2\,\sqrt{c\,x^2}}{b^3\,(n^3+6\,n^2+11\,n+6)}+\frac{c^2\,x^3\,\sqrt{c\,x^2}\,(n^2+3\,n+2)}{n^3+6\,n^2+11\,n+6}-\frac{2\,a^2\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^2\,(n^3+6\,n^2+11\,n+6)}+\frac{a\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b\,(n^3+6\,n^2+11\,n+6)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x)
```

```
[Out] ((a + b*x)^n*((2*a^3*c^2*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (c
^2*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) - (2*a^2*c^2
*n*x*(c*x^2)^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*c^2*n*x^2*(c*x^2)^(
1/2)*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**3,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**3, x)
```

## 3.894 $\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^4}\,dx$

**Optimal**. Leaf size=69

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

**Rubi** [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] -((a*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^4}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}$$
$$= -\frac{ac^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}$$

**Mathematica [A]** time = 0.01, size = 46, normalized size = 0.67

$$\frac{c^3x(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

**IntegrateAlgebraic [F]** time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(5/2)*(a + b*x)^n)/x^4, x]
```

**fricas [A]** time = 1.05, size = 76, normalized size = 1.10

$$\frac{\left(abc^2nx-a^2c^2+\left(b^2c^2n+b^2c^2\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2n^2+3\,b^2n+2\,b^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="fricas")
```

```
[Out] (a*b*c^2*n*x - a^2*c^2 + (b^2*c^2*n + b^2*c^2)*x^2)*sqrt(c*x^2)*(b*x + a)^n
/((b^2*n^2 + 3*b^2*n + 2*b^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(bx+a)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="giac")
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^4, x)
```

**maple [A]** time = 0.00, size = 46, normalized size = 0.67

$$-\frac{\left(c\,x^2\right)^{\frac{5}{2}}(-xnb-bx+a)\,(bx+a)^{n+1}}{\left(n^2+3n+2\right)b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^4,x)
[Out] -(b*x+a)^(n+1)*(c*x^2)^(5/2)*(-b*n*x-b*x+a)/x^5/b^2/(n^2+3*n+2)
```

**maxima [A]** time = 1.45, size = 51, normalized size = 0.74

$$\frac{\left(b^2c^{\frac{5}{2}}(n+1)x^2+abc^{\frac{5}{2}}nx-a^2c^{\frac{5}{2}}\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="maxima")
[Out] (b^2*c^(5/2)*(n + 1)*x^2 + a*b*c^(5/2)*n*x - a^2*c^(5/2))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

**mupad [B]** time = 0.24, size = 94, normalized size = 1.36

$$\frac{(a+b\,x)^n\left(\frac{c^2\,x^2\,\sqrt{c\,x^2}\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,c^2\,\sqrt{c\,x^2}}{b^2\left(n^2+3\,n+2\right)}+\frac{a\,c^2\,n\,x\,\sqrt{c\,x^2}}{b\left(n^2+3\,n+2\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*c^2*(c
*x^2)^(1/2))/(b^2*(3*n + n^2 + 2)) + (a*c^2*n*x*(c*x^2)^(1/2))/(b*(3*n + n^
2 + 2))))/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n c^{\frac{5}{2}} (x^2)^{\frac{5}{2}}}{2x^3} & \text{for } b = 0 \\ \int \frac{(cx^2)^{\frac{5}{2}}}{x^4(a+bx)^2}\, dx & \text{for } n = -2 \\ \int \frac{(cx^2)^{\frac{5}{2}}}{x^4(a+bx)}\, dx & \text{for } n = -1 \\ -\frac{a^2 c^{\frac{5}{2}} (a+bx)^n (x^2)^{\frac{5}{2}}}{b^2n^2x^5+3b^2nx^5+2b^2x^5} + \frac{abc^{\frac{5}{2}} nx(a+bx)^n (x^2)^{\frac{5}{2}}}{b^2n^2x^5+3b^2nx^5+2b^2x^5} + \frac{b^2c^{\frac{5}{2}} nx^2(a+bx)^n (x^2)^{\frac{5}{2}}}{b^2n^2x^5+3b^2nx^5+2b^2x^5} + \frac{b^2c^{\frac{5}{2}} x^2(a+bx)^n (x^2)^{\frac{5}{2}}}{b^2n^2x^5+3b^2nx^5+2b^2x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**4,x)
```

```
[Out] Piecewise((a**n*c**(5/2)*(x**2)**(5/2)/(2*x**3), Eq(b, 0)), (Integral((c*x*
*2)**(5/2)/(x**4*(a + b*x)**2), x), Eq(n, -2)), (Integral((c*x**2)**(5/2)/(
x**4*(a + b*x)), x), Eq(n, -1)), (-a**2*c**(5/2)*(a + b*x)**n*(x**2)**(5/2)
/(b**2*n**2*x**5 + 3*b**2*n*x**5 + 2*b**2*x**5) + a*b*c**(5/2)*n*x*(a + b*x
)**n*(x**2)**(5/2)/(b**2*n**2*x**5 + 3*b**2*n*x**5 + 2*b**2*x**5) + b**2*c*
*(5/2)*n*x**2*(a + b*x)**n*(x**2)**(5/2)/(b**2*n**2*x**5 + 3*b**2*n*x**5 +
2*b**2*x**5) + b**2*c**(5/2)*x**2*(a + b*x)**n*(x**2)**(5/2)/(b**2*n**2*x**
5 + 3*b**2*n*x**5 + 2*b**2*x**5), True))
```

## 3.895 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^5}\,dx$

**Optimal**. Leaf size=33

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

**Rubi** [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^5}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int (a+bx)^n\,dx}{x}\\ &= \frac{c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 31, normalized size = 0.94

$$\frac{c^3x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic [F]** time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(5/2)*(a + b*x)^n)/x^5, x]
```

**fricas [A]** time = 0.84, size = 37, normalized size = 1.12

$$\frac{\left(bc^2x+ac^2\right)\sqrt{cx^2}\,(bx+a)^n}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="fricas")
```

```
[Out] (b*c^2*x + a*c^2)*sqrt(c*x^2)*(b*x + a)^n/((b*n + b)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(bx+a)^n}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^5, x)
```

**maple [A]** time = 0.00, size = 29, normalized size = 0.88

$$\frac{\left(c\,x^2\right)^{\frac{5}{2}}(bx+a)^{n+1}}{(n+1)\,b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^5,x)
```

```
[Out] (b*x+a)^(n+1)/b/(n+1)*(c*x^2)^(5/2)/x^5
```

**maxima [A]** time = 1.39, size = 28, normalized size = 0.85

$$\frac{\left(bc^{\frac{5}{2}}x + ac^{\frac{5}{2}}\right)(bx + a)^n}{b(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="maxima")
```

```
[Out] (b*c^(5/2)*x + a*c^(5/2))*(b*x + a)^n/(b*(n + 1))
```

**mupad [B]** time = 0.23, size = 49, normalized size = 1.48

$$\frac{\left(\frac{c^2\,x\,\sqrt{c\,x^2}}{n+1} + \frac{a\,c^2\,\sqrt{c\,x^2}}{b\,(n+1)}\right)\,(a + b\,x)^n}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x)
```

```
[Out] (((c^2*x*(c*x^2)^(1/2))/(n + 1) + (a*c^2*(c*x^2)^(1/2))/(b*(n + 1)))*(a + b
*x)^n)/x
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{ax^4} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{x^4} & \text{for } b = 0 \\ \int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^5(a+bx)}\, dx & \text{for } n = -1 \\ \frac{ac^{\frac{5}{2}}(a+bx)^n\left(x^2\right)^{\frac{5}{2}}}{bnx^5+bx^5} + \frac{bc^{\frac{5}{2}}x(a+bx)^n\left(x^2\right)^{\frac{5}{2}}}{bnx^5+bx^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**5,x)
```

```
[Out] Piecewise((c**(5/2)*(x**2)**(5/2)/(a*x**4), Eq(b, 0) & Eq(n, -1)), (a**n*c*
*(5/2)*(x**2)**(5/2)/x**4, Eq(b, 0)), (Integral((c*x**2)**(5/2)/(x**5*(a +
```

```
b*x)), x), Eq(n, -1)), (a*c**(5/2)*(a + b*x)**n*(x**2)**(5/2)/(b*n*x**5 + b
*x**5) + b*c**(5/2)*x*(a + b*x)**n*(x**2)**(5/2)/(b*n*x**5 + b*x**5), True)
)
```

## 3.896 $\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=123

$$-\frac{a^3x(a+bx)^{n+1}}{b^4(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4(n+4)\sqrt{cx^2}}$$

**Rubi [A]** time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*x)
^(2 + n))/(b^4*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*(3 + n
)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*(4 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}}\,dx = \frac{x\int x^3(a+bx)^n\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{a^3x(a+bx)^{1+n}}{b^4(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4(4+n)\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.04, size = 96, normalized size = 0.78

$$\frac{x(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2)*
x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 +
n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^4*(a + b*x)^n)/Sqrt[c*x^2], x]
```

**fricas** [A] time = 1.06, size = 158, normalized size = 1.28

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4cn^4+10\,b^4cn^3+35\,b^4cn^2+50\,b^4cn+24\,b^4c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c*n^4 + 10*b^4*c*n^3 + 35*b^4*c*n^2 + 50*b^4*c*n
 + 24*b^4*c)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^4}{\sqrt{cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^4/sqrt(c*x^2), x)
```

**maple [A]** time = 0.01, size = 134, normalized size = 1.09

$$-\frac{\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)x\,(bx+a)^{n+1}}{\sqrt{c\,x^2}\,\left(n^4+10n^3+35n^2+50n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] -(b*x+a)^(n+1)*x*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3+9
*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(1/
2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

**maxima [A]** time = 1.45, size = 104, normalized size = 0.85

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\,\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*sqrt(c))
```

**mupad [B]** time = 0.37, size = 186, normalized size = 1.51

$$\frac{(a+b\,x)^n\,\left(\frac{x^5\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,x}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,(n+1)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4
+ 24) - (6*a^4*x)/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*n*x^2)
/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n + n^2 + 2))/(b*(
50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1))/(b^2*(50*n + 35
*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**5/(4*sqrt(c)*sqrt(x**2)), Eq(b, 0)), (Integral(x**4/(sqr
t(c*x**2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**4/(sqrt(c*x**2)*(a +
b*x)**3), x), Eq(n, -3)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)**2), x), E
q(n, -2)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)), x), Eq(n, -1)), (-6*a**
4*x*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(
x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24
*b**4*sqrt(c)*sqrt(x**2)) + 6*a**3*b*n*x**2*(a + b*x)**n/(b**4*sqrt(c)*n**4
*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x
**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)) - 3*a**2*
b**2*n**2*x**3*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)
*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt
(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)) - 3*a**2*b**2*n*x**3*(a + b*x)**n/(b**
4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(
c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x*
*2)) + a*b**3*n**3*x**4*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**
4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(
c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)) + 3*a*b**3*n**2*x**4*(a + b*x
)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b
**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c
)*sqrt(x**2)) + 2*a*b**3*n*x**4*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2)
+ 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b*
*4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)) + b**4*n**3*x**5*(a +
 b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) +
35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sq
rt(c)*sqrt(x**2)) + 6*b**4*n**2*x**5*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x
**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) +
50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)) + 11*b**4*n*x**5
*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**
2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b*
```

```
*4*sqrt(c)*sqrt(x**2)) + 6*b**4*x**5*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x
**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) +
50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)), True))
```

## 3.897 $\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=90

$$\frac{a^2x(a+bx)^{n+1}}{b^3(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3(n+3)\sqrt{cx^2}}$$

**Rubi** [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x(a+bx)^{n+1}}{b^3(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2 +
 n))/(b^3*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*(3 + n)*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}}\,dx = \frac{x\int x^2(a+bx)^n\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{a^2x(a+bx)^{1+n}}{b^3(1+n)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+n}}{b^3(2+n)\sqrt{cx^2}} + \frac{x(a+bx)^{3+n}}{b^3(3+n)\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.03, size = 67, normalized size = 0.74

$$\frac{x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(
b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic [F]** time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^3*(a + b*x)^n)/Sqrt[c*x^2], x]
```

**fricas [A]** time = 1.33, size = 110, normalized size = 1.22

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3cn^3+6\,b^3cn^2+11\,b^3cn+6\,b^3c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c*n^3 + 6*b^3*c*n^2 + 11*b^3*c*n +
6*b^3*c)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^3}{\sqrt{cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^3/sqrt(c*x^2), x)
```

**maple [A]** time = 0.00, size = 81, normalized size = 0.90

$$\frac{\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2\right)x\,(bx+a)^{n+1}}{\sqrt{c\,x^2}\,\left(n^3+6n^2+11n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
/(c*x^2)^(1/2)/b^3/(n^3+6*n^2+11*n+6)
```

**maxima [A]** time = 1.46, size = 83, normalized size = 0.92

$$\frac{\left(\left(n^2+3\,n+2\right)b^3\sqrt{c}\,x^3+\left(n^2+n\right)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c)
```

**mupad [B]** time = 0.29, size = 121, normalized size = 1.34

$$\frac{(a+b\,x)^n\,\left(\frac{x^4\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{2\,a^3\,x}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,n\,x^2}{b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^3\,(n+1)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (2*a^3*x)/(b
^3*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*(11*n + 6*n^2 + n^3 + 6))
 + (a*n*x^3*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/(c*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^4}{3\sqrt{c}\sqrt{x^2}} & \text{for } b = 0 \\ \int \frac{x^3}{\sqrt{cx^2}(a+bx)^3}\,dx & \text{for } n = -3 \\ \int \frac{x^3}{\sqrt{cx^2}(a+bx)^2}\,dx & \text{for } n = -2 \\ \int \frac{x^3}{\sqrt{cx^2}(a+bx)}\,dx & \text{for } n = -1 \\ \frac{2a^3x(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} - \frac{2a^2bnx^2(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} + \frac{ab^2n^2x^3(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} + \frac{ab^2nx^3(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} + \frac{b^3n^2x^4(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} + \frac{3b^3nx^4(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} + \frac{2b^3x^4(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**4/(3*sqrt(c)*sqrt(x**2)), Eq(b, 0)), (Integral(x**3/(sqr
t(c*x**2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**3/(sqrt(c*x**2)*(a +
b*x)**2), x), Eq(n, -2)), (Integral(x**3/(sqrt(c*x**2)*(a + b*x)), x), Eq(n
, -1)), (2*a**3*x*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2) + 6*b**3*sqrt(
c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*sqrt(c)*sqrt(x**
2)) - 2*a**2*b*n*x**2*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2) + 6*b**3*s
qrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*sqrt(c)*sqrt
(x**2)) + a*b**2*n**2*x**3*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2) + 6*b
**3*sqrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*sqrt(c)
*sqrt(x**2)) + a*b**2*n*x**3*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2) + 6
*b**3*sqrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*sqrt(
c)*sqrt(x**2)) + b**3*n**2*x**4*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2)
+ 6*b**3*sqrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*sq
rt(c)*sqrt(x**2)) + 3*b**3*n*x**4*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2
) + 6*b**3*sqrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*
sqrt(c)*sqrt(x**2)) + 2*b**3*x**4*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2
) + 6*b**3*sqrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*
sqrt(c)*sqrt(x**2)), True))
```

## 3.898 $\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=59

$$\frac{x(a+bx)^{n+2}}{b^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2(n+1)\sqrt{cx^2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{x(a+bx)^{n+2}}{b^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 + n)
)/(b^2*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}}\,dx = \frac{x\int x(a+bx)^n\,dx}{\sqrt{cx^2}}$$

$$= \frac{x\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{\sqrt{cx^2}}$$

$$= -\frac{ax(a+bx)^{1+n}}{b^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2(2+n)\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.02, size = 43, normalized size = 0.73

$$\frac{x(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
[Out] (x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
[Out] Defer[IntegrateAlgebraic][(x^2*(a + b*x)^n)/Sqrt[c*x^2], x]
```

**fricas** [A] time = 0.96, size = 66, normalized size = 1.12

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2cn^2+3\,b^2cn+2\,b^2c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c*n^2 + 3
*b^2*c*n + 2*b^2*c)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^2}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a)^n*x^2/sqrt(c*x^2), x)
```

**maple [A]** time = 0.00, size = 44, normalized size = 0.75

$$-\frac{(-xnb-bx+a)\,x\,(bx+a)^{n+1}}{\sqrt{c\,x^2}\,\left(n^2+3n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n/(c*x^2)^(1/2),x)
[Out] -(b*x+a)^(n+1)*x*(-b*n*x-b*x+a)/(c*x^2)^(1/2)/b^2/(n^2+3*n+2)
```

**maxima [A]** time = 1.49, size = 45, normalized size = 0.76

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*sqrt(c))
```

**mupad [B]** time = 0.28, size = 71, normalized size = 1.20

$$\frac{(a+b\,x)^n\,\left(\frac{x^3\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,x}{b^2\,\left(n^2+3\,n+2\right)}+\frac{a\,n\,x^2}{b\,\left(n^2+3\,n+2\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^n)/(c*x^2)^(1/2),x)
[Out] ((a + b*x)^n*((x^3*(n + 1))/(3*n + n^2 + 2) - (a^2*x)/(b^2*(3*n + n^2 + 2))
 + (a*n*x^2)/(b*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^3}{2\sqrt{c}\sqrt{x^2}} & \text{for } b = 0 \\ \int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx & \text{for } n = -2 \\ \int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx & \text{for } n = -1 \\ -\frac{a^2x(a+bx)^n}{b^2\sqrt{c}n^2\sqrt{x^2}+3b^2\sqrt{c}n\sqrt{x^2}+2b^2\sqrt{c}\sqrt{x^2}} + \frac{abnx^2(a+bx)^n}{b^2\sqrt{c}n^2\sqrt{x^2}+3b^2\sqrt{c}n\sqrt{x^2}+2b^2\sqrt{c}\sqrt{x^2}} + \frac{b^2nx^3(a+bx)^n}{b^2\sqrt{c}n^2\sqrt{x^2}+3b^2\sqrt{c}n\sqrt{x^2}+2b^2\sqrt{c}\sqrt{x^2}} + \frac{b^2x^3(a+bx)^n}{b^2\sqrt{c}n^2\sqrt{x^2}+3b^2\sqrt{c}n\sqrt{x^2}+2b^2\sqrt{c}\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**3/(2*sqrt(c)*sqrt(x**2)), Eq(b, 0)), (Integral(x**2/(sqr
t(c*x**2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**2/(sqrt(c*x**2)*(a +
b*x)), x), Eq(n, -1)), (-a**2*x*(a + b*x)**n/(b**2*sqrt(c)*n**2*sqrt(x**2)
+ 3*b**2*sqrt(c)*n*sqrt(x**2) + 2*b**2*sqrt(c)*sqrt(x**2)) + a*b*n*x**2*(a
+ b*x)**n/(b**2*sqrt(c)*n**2*sqrt(x**2) + 3*b**2*sqrt(c)*n*sqrt(x**2) + 2*b
**2*sqrt(c)*sqrt(x**2)) + b**2*n*x**3*(a + b*x)**n/(b**2*sqrt(c)*n**2*sqrt(
x**2) + 3*b**2*sqrt(c)*n*sqrt(x**2) + 2*b**2*sqrt(c)*sqrt(x**2)) + b**2*x**
3*(a + b*x)**n/(b**2*sqrt(c)*n**2*sqrt(x**2) + 3*b**2*sqrt(c)*n*sqrt(x**2)
+ 2*b**2*sqrt(c)*sqrt(x**2)), True))
```

## 3.899 $\int \frac{x(a+bx)^n}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=28

$$\frac{x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic [F]** time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx)^n}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x*(a + b*x)^n)/Sqrt[c*x^2], x]
```

**fricas [A]** time = 1.69, size = 33, normalized size = 1.18

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{(bcn+bc)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c*n + b*c)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x/sqrt(c*x^2), x)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.96

$$\frac{x\,(bx+a)^{n+1}}{(n+1)\sqrt{c\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n/(c*x^2)^(1/2),x)
```

[Out] x*(b*x+a)^(n+1)/b/(n+1)/(c*x^2)^(1/2)

**maxima** [A] time = 1.44, size = 31, normalized size = 1.11

$$\frac{\left(b\sqrt{c}\,x + a\sqrt{c}\right)(bx+a)^n}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")

[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c*(n + 1))

**mupad** [B] time = 0.22, size = 36, normalized size = 1.29

$$\frac{\left(\frac{x^2}{n+1} + \frac{a\,x}{b\,(n+1)}\right)(a+b\,x)^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x)^n)/(c*x^2)^(1/2),x)

[Out] ((x^2/(n + 1) + (a*x)/(b*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^2}{a\sqrt{c}\sqrt{x^2}} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n x^2}{\sqrt{c}\sqrt{x^2}} & \text{for } b = 0 \\ \int \frac{x}{\sqrt{cx^2}\,(a+bx)}\,dx & \text{for } n = -1 \\ \frac{ax(a+bx)^n}{b\sqrt{c}\,n\sqrt{x^2} + b\sqrt{c}\sqrt{x^2}} + \frac{bx^2(a+bx)^n}{b\sqrt{c}\,n\sqrt{x^2} + b\sqrt{c}\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)**n/(c*x**2)**(1/2),x)

[Out] Piecewise((x**2/(a*sqrt(c)*sqrt(x**2)), Eq(b, 0) & Eq(n, -1)), (a**n*x**2/(
sqrt(c)*sqrt(x**2)), Eq(b, 0)), (Integral(x/(sqrt(c*x**2)*(a + b*x)), x), E
q(n, -1)), (a*x*(a + b*x)**n/(b*sqrt(c)*n*sqrt(x**2) + b*sqrt(c)*sqrt(x**2)
) + b*x**2*(a + b*x)**n/(b*sqrt(c)*n*sqrt(x**2) + b*sqrt(c)*sqrt(x**2)), Tr
ue))

## 3.900 $\int \frac{x^6(a+bx)^n}{(cx^2)^{3/2}}\,dx$

**Optimal**. Leaf size=135

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c(n+4)\sqrt{cx^2}}$$

**Rubi [A]** time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*
x)^(2 + n))/(b^4*c*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*c*
(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c*(4 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx = \frac{x\int x^3(a+bx)^n\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^3x(a+bx)^{1+n}}{b^4c(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c(4+n)\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.04, size = 98, normalized size = 0.73

$$\frac{x^3(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
[Out] (x^3*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2
)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*(c*x^2)^(3/2))
```

**IntegrateAlgebraic [F]** time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
[Out] Defer[IntegrateAlgebraic][(x^6*(a + b*x)^n)/(c*x^2)^(3/2), x]
```

**fricas [A]** time = 0.99, size = 168, normalized size = 1.24

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4c^2n^4+10\,b^4c^2n^3+35\,b^4c^2n^2+50\,b^4c^2n+24\,b^4c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c^2*n^4 + 10*b^4*c^2*n^3 + 35*b^4*c^2*n^2 + 50*b
^4*c^2*n + 24*b^4*c^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^6}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^6/(c*x^2)^(3/2), x)
```

**maple [A]** time = 0.01, size = 136, normalized size = 1.01

$$-\frac{\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)x^3\,(bx+a)^{n+1}}{\left(c\,x^2\right)^{\frac{3}{2}}\left(n^4+10n^3+35n^2+50n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] -(b*x+a)^(n+1)*x^3*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3
+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(
3/2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

**maxima [A]** time = 1.46, size = 104, normalized size = 0.77

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*c^(3/2))
```

**mupad [B]** time = 0.40, size = 201, normalized size = 1.49

$$\frac{(a+b\,x)^n\left(\frac{x^5\left(n^3+6\,n^2+11\,n+6\right)}{c\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{6\,a^4\,x}{b^4\,c\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,c\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\left(n^2+3\,n+2\right)}{b\,c\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,(n+1)}{b^2\,c\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(a + b*x)^n)/(c*x^2)^(3/2),x)

[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (6*a^4*x)/(b^4*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*n*x^2)/(b^3*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n + n^2 + 2))/(b*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1))/(b^2*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(b*x+a)**n/(c*x**2)**(3/2),x)

[Out] Piecewise((a**n*x**7/(4*c**(3/2)*(x**2)**(3/2)), Eq(b, 0)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*x)), x), Eq(n, -1)), (-6*a**4*x**3*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) + 6*a**3*b*n*x**4*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) - 3*a**2*b**2*n**2*x**5*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) - 3*a**2*b**2*n*x**5*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) + a*b**3*n**3*x**6*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) + 3*a*b**3*n**2*x**6*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) + 2*a*b**3*n*x**6*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) + b**4*n**3*x**7*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) + 6*b**4*n**2*x**7*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) +

```
 24*b**4*c**(3/2)*(x**2)**(3/2)) + 11*b**4*n*x**7*(a + b*x)**n/(b**4*c**(3/
2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3
/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2
)*(x**2)**(3/2)) + 6*b**4*x**7*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/
2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3
/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)), T
rue))
```

## 3.901 $\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=99

$$\frac{a^2x(a+bx)^{n+1}}{b^3c(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c(n+3)\sqrt{cx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x(a+bx)^{n+1}}{b^3c(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2
 + n))/(b^3*c*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c*(3 + n)*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx = \frac{x\int x^2(a+bx)^n\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{a^2x(a+bx)^{1+n}}{b^3c(1+n)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+n}}{b^3c(2+n)\sqrt{cx^2}} + \frac{x(a+bx)^{3+n}}{b^3c(3+n)\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.03, size = 69, normalized size = 0.70

$$\frac{x^3(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))
/(b^3*(1 + n)*(2 + n)*(3 + n)*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^5*(a + b*x)^n)/(c*x^2)^(3/2), x]
```

**fricas** [A] time = 1.17, size = 118, normalized size = 1.19

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3c^2n^3+6\,b^3c^2n^2+11\,b^3c^2n+6\,b^3c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c^2*n^3 + 6*b^3*c^2*n^2 + 11*b^3*c^
2*n + 6*b^3*c^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^5}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^5/(c*x^2)^(3/2), x)
```

**maple [A]** time = 0.00, size = 83, normalized size = 0.84

$$\frac{\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2\right)x^3\,(bx+a)^{n+1}}{\left(c\,x^2\right)^{\frac{3}{2}}\left(n^3+6n^2+11n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
^3/(c*x^2)^(3/2)/b^3/(n^3+6*n^2+11*n+6)
```

**maxima [A]** time = 1.46, size = 83, normalized size = 0.84

$$\frac{\left(\left(n^2+3\,n+2\right)b^3\sqrt{c}\,x^3+\left(n^2+n\right)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c^2)
```

**mupad [B]** time = 0.31, size = 133, normalized size = 1.34

$$\frac{(a+b\,x)^n\left(\frac{x^4\left(n^2+3\,n+2\right)}{c\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,a^3\,x}{b^3\,c\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,n\,x^2}{b^2\,c\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^3\,(n+1)}{b\,c\left(n^3+6\,n^2+11\,n+6\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(c*(11*n + 6*n^2 + n^3 + 6)) + (2*a^3*x
)/(b^3*c*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*c*(11*n + 6*n^2 + n
^3 + 6)) + (a*n*x^3*(n + 1))/(b*c*(11*n + 6*n^2 + n^3 + 6))))/(c*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^6}{3c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} & \text{for } b = 0 \\ \int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)^3}\,dx & \text{for } n = -3 \\ \int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)^2}\,dx & \text{for } n = -2 \\ \int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx & \text{for } n = -1 \\ \frac{2a^3x^3(a+bx)^n}{D} - \frac{2a^2bnx^4(a+bx)^n}{D} + \frac{ab^2n^2x^5(a+bx)^n}{D} + \frac{ab^2nx^5(a+bx)^n}{D} + \frac{b^3n^2x^6(a+bx)^n}{D} + \frac{3b^3nx^6(a+bx)^n}{D} + \frac{2b^3x^6(a+bx)^n}{D} & \text{otherwise} \end{cases}$$

$$D = b^3c^{\frac{3}{2}}n^3(x^2)^{\frac{3}{2}} + 6b^3c^{\frac{3}{2}}n^2(x^2)^{\frac{3}{2}} + 11b^3c^{\frac{3}{2}}n(x^2)^{\frac{3}{2}} + 6b^3c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((a**n*x**6/(3*c**(3/2)*(x**2)**(3/2)), Eq(b, 0)), (Integral(x**5/
((c*x**2)**(3/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**5/((c*x**2)**(
3/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**5/((c*x**2)**(3/2)*(a + b*
x)), x), Eq(n, -1)), (2*a**3*x**3*(a + b*x)**n/(b**3*c**(3/2)*n**3*(x**2)**
(3/2) + 6*b**3*c**(3/2)*n**2*(x**2)**(3/2) + 11*b**3*c**(3/2)*n*(x**2)**(3/
2) + 6*b**3*c**(3/2)*(x**2)**(3/2)) - 2*a**2*b*n*x**4*(a + b*x)**n/(b**3*c*
*(3/2)*n**3*(x**2)**(3/2) + 6*b**3*c**(3/2)*n**2*(x**2)**(3/2) + 11*b**3*c*
*(3/2)*n*(x**2)**(3/2) + 6*b**3*c**(3/2)*(x**2)**(3/2)) + a*b**2*n**2*x**5*
(a + b*x)**n/(b**3*c**(3/2)*n**3*(x**2)**(3/2) + 6*b**3*c**(3/2)*n**2*(x**2
)**(3/2) + 11*b**3*c**(3/2)*n*(x**2)**(3/2) + 6*b**3*c**(3/2)*(x**2)**(3/2)
) + a*b**2*n*x**5*(a + b*x)**n/(b**3*c**(3/2)*n**3*(x**2)**(3/2) + 6*b**3*c
**(3/2)*n**2*(x**2)**(3/2) + 11*b**3*c**(3/2)*n*(x**2)**(3/2) + 6*b**3*c**(
3/2)*(x**2)**(3/2)) + b**3*n**2*x**6*(a + b*x)**n/(b**3*c**(3/2)*n**3*(x**2
)**(3/2) + 6*b**3*c**(3/2)*n**2*(x**2)**(3/2) + 11*b**3*c**(3/2)*n*(x**2)**
(3/2) + 6*b**3*c**(3/2)*(x**2)**(3/2)) + 3*b**3*n*x**6*(a + b*x)**n/(b**3*c
**(3/2)*n**3*(x**2)**(3/2) + 6*b**3*c**(3/2)*n**2*(x**2)**(3/2) + 11*b**3*c
**(3/2)*n*(x**2)**(3/2) + 6*b**3*c**(3/2)*(x**2)**(3/2)) + 2*b**3*x**6*(a +
 b*x)**n/(b**3*c**(3/2)*n**3*(x**2)**(3/2) + 6*b**3*c**(3/2)*n**2*(x**2)**(
3/2) + 11*b**3*c**(3/2)*n*(x**2)**(3/2) + 6*b**3*c**(3/2)*(x**2)**(3/2)), T
rue))
```

## 3.902 $\int \frac{x^4(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=65

$$\frac{x(a+bx)^{n+2}}{b^2c(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c(n+1)\sqrt{cx^2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{x(a+bx)^{n+2}}{b^2c(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 +
n))/(b^2*c*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx = \frac{x\int x(a+bx)^n\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{ax(a+bx)^{1+n}}{b^2c(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c(2+n)\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.02, size = 45, normalized size = 0.69

$$\frac{x^3(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*(c*x^2)^(3/
2))
```

**IntegrateAlgebraic [F]** time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^4*(a + b*x)^n)/(c*x^2)^(3/2), x]
```

**fricas [A]** time = 1.59, size = 72, normalized size = 1.11

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2c^2n^2+3\,b^2c^2n+2\,b^2c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c^2*n^2 +
 3*b^2*c^2*n + 2*b^2*c^2)*x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^4}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^4/(c*x^2)^(3/2), x)
```

**maple** [A] time = 0.00, size = 46, normalized size = 0.71

$$-\frac{(-xnb-bx+a)\,x^3\,(bx+a)^{n+1}}{\left(c\,x^2\right)^{\frac{3}{2}}\left(n^2+3n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] -(b*x+a)^(n+1)*x^3*(-b*n*x-b*x+a)/(c*x^2)^(3/2)/b^2/(n^2+3*n+2)
```

**maxima** [A] time = 1.47, size = 45, normalized size = 0.69

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*c^(3/2))
```

**mupad** [B] time = 0.29, size = 80, normalized size = 1.23

$$\frac{(a+b\,x)^n\left(\frac{x^3\,(n+1)}{c\left(n^2+3\,n+2\right)}-\frac{a^2\,x}{b^2\,c\left(n^2+3\,n+2\right)}+\frac{a\,n\,x^2}{b\,c\left(n^2+3\,n+2\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((a + b*x)^n*((x^3*(n + 1))/(c*(3*n + n^2 + 2)) - (a^2*x)/(b^2*c*(3*n + n^2
 + 2)) + (a*n*x^2)/(b*c*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^5}{2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} & \text{for } b = 0 \\ \int \frac{x^4}{\left(cx^2\right)^{\frac{3}{2}}(a+bx)^2}\,dx & \text{for } n = -2 \\ \int \frac{x^4}{\left(cx^2\right)^{\frac{3}{2}}(a+bx)}\,dx & \text{for } n = -1 \\ -\frac{a^2x^3(a+bx)^n}{b^2c^{\frac{3}{2}}n^2\left(x^2\right)^{\frac{3}{2}}+3b^2c^{\frac{3}{2}}n\left(x^2\right)^{\frac{3}{2}}+2b^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} + \frac{abnx^4(a+bx)^n}{b^2c^{\frac{3}{2}}n^2\left(x^2\right)^{\frac{3}{2}}+3b^2c^{\frac{3}{2}}n\left(x^2\right)^{\frac{3}{2}}+2b^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} + \frac{b^2nx^5(a+bx)^n}{b^2c^{\frac{3}{2}}n^2\left(x^2\right)^{\frac{3}{2}}+3b^2c^{\frac{3}{2}}n\left(x^2\right)^{\frac{3}{2}}+2b^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} + \frac{b^2x^5(a+bx)^n}{b^2c^{\frac{3}{2}}n^2\left(x^2\right)^{\frac{3}{2}}+3b^2c^{\frac{3}{2}}n\left(x^2\right)^{\frac{3}{2}}+2b^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((a**n*x**5/(2*c**(3/2)*(x**2)**(3/2)), Eq(b, 0)), (Integral(x**4/
((c*x**2)**(3/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**4/((c*x**2)**(
3/2)*(a + b*x)), x), Eq(n, -1)), (-a**2*x**3*(a + b*x)**n/(b**2*c**(3/2)*n*
*2*(x**2)**(3/2) + 3*b**2*c**(3/2)*n*(x**2)**(3/2) + 2*b**2*c**(3/2)*(x**2)
**(3/2)) + a*b*n*x**4*(a + b*x)**n/(b**2*c**(3/2)*n**2*(x**2)**(3/2) + 3*b*
*2*c**(3/2)*n*(x**2)**(3/2) + 2*b**2*c**(3/2)*(x**2)**(3/2)) + b**2*n*x**5*
(a + b*x)**n/(b**2*c**(3/2)*n**2*(x**2)**(3/2) + 3*b**2*c**(3/2)*n*(x**2)**
(3/2) + 2*b**2*c**(3/2)*(x**2)**(3/2)) + b**2*x**5*(a + b*x)**n/(b**2*c**(3
/2)*n**2*(x**2)**(3/2) + 3*b**2*c**(3/2)*n*(x**2)**(3/2) + 2*b**2*c**(3/2)*
(x**2)**(3/2)), True))
```

## 3.903 $\int \frac{x^3(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=31

$$\frac{x(a+bx)^{n+1}}{bc(n+1)\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{bc(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*c*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{c\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{bc(1+n)\sqrt{cx^2}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 0.97

$$\frac{x^3(a+bx)^{n+1}}{b(n+1)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n))/(b*(1 + n)*(c*x^2)^(3/2))
```

**IntegrateAlgebraic [F]** time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx)^n}{(cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^3*(a + b*x)^n)/(c*x^2)^(3/2), x]
```

**fricas [A]** time = 0.91, size = 37, normalized size = 1.19

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{(bc^2n+bc^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c^2*n + b*c^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^3}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^3/(c*x^2)^(3/2), x)
```

**maple [A]** time = 0.00, size = 29, normalized size = 0.94

$$\frac{x^3\,(bx+a)^{n+1}}{(n+1)\,(c\,x^2)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(3/2),x)
[Out] (b*x+a)^(n+1)/b/(n+1)*x^3/(c*x^2)^(3/2)
```

**maxima [A]** time = 1.44, size = 31, normalized size = 1.00

$$\frac{\left(b\sqrt{c}\,x+a\sqrt{c}\right)(bx+a)^n}{bc^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c^2*(n + 1))
```

**mupad [B]** time = 0.23, size = 42, normalized size = 1.35

$$\frac{\left(\frac{x^2}{c\,(n+1)}+\frac{a\,x}{b\,c\,(n+1)}\right)\,(a+b\,x)^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^n)/(c*x^2)^(3/2),x)
[Out] ((x^2/(c*(n + 1)) + (a*x)/(b*c*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^4}{ac^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} & \text{for } b=0 \wedge n=-1 \\ \frac{a^n x^4}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} & \text{for } b=0 \\ \int \frac{x^3}{\left(cx^2\right)^{\frac{3}{2}}(a+bx)}\,dx & \text{for } n=-1 \\ \frac{ax^3(a+bx)^n}{bc^{\frac{3}{2}}n\left(x^2\right)^{\frac{3}{2}}+bc^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{bx^4(a+bx)^n}{bc^{\frac{3}{2}}n\left(x^2\right)^{\frac{3}{2}}+bc^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(3/2),x)
[Out] Piecewise((x**4/(a*c**(3/2)*(x**2)**(3/2)), Eq(b, 0) & Eq(n, -1)), (a**n*x*
*4/(c**(3/2)*(x**2)**(3/2)), Eq(b, 0)), (Integral(x**3/((c*x**2)**(3/2)*(a
+ b*x)), x), Eq(n, -1)), (a*x**3*(a + b*x)**n/(b*c**(3/2)*n*(x**2)**(3/2) +
 b*c**(3/2)*(x**2)**(3/2)) + b*x**4*(a + b*x)**n/(b*c**(3/2)*n*(x**2)**(3/2
) + b*c**(3/2)*(x**2)**(3/2)), True))
```

## 3.904 $\int \frac{x^8(a+bx)^n}{\left(cx^2\right)^{5/2}} \, dx$

**Optimal**. Leaf size=135

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c^2(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c^2(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c^2(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c^2(n+4)\sqrt{cx^2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c^2(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c^2(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c^2(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c^2(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c^2*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a +
b*x)^(2 + n))/(b^4*c^2*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^
4*c^2*(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c^2*(4 + n)*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^8(a+bx)^n}{(cx^2)^{5/2}}\,dx = \frac{x\int x^3(a+bx)^n\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^3x(a+bx)^{1+n}}{b^4c^2(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c^2(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c^2(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c^2(4+n)\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.04, size = 99, normalized size = 0.73

$$\frac{x(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4c^2(n+1)(n+2)(n+3)(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2)*
x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*c^2*(1 + n)*(2 + n)*(3 + n)*(
4 + n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic [F]** time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{x^8(a+bx)^n}{(cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^8*(a + b*x)^n)/(c*x^2)^(5/2), x]
```

**fricas [A]** time = 0.95, size = 168, normalized size = 1.24

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4c^3n^4+10\,b^4c^3n^3+35\,b^4c^3n^2+50\,b^4c^3n+24\,b^4c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c^3*n^4 + 10*b^4*c^3*n^3 + 35*b^4*c^3*n^2 + 50*b
^4*c^3*n + 24*b^4*c^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^8}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^8/(c*x^2)^(5/2), x)
```

**maple [A]** time = 0.01, size = 136, normalized size = 1.01

$$-\frac{\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)x^5\,(bx+a)^{n+1}}{\left(c\,x^2\right)^{\frac{5}{2}}\left(n^4+10n^3+35n^2+50n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] -(b*x+a)^(n+1)*x^5*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3
+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(
5/2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

**maxima [A]** time = 1.47, size = 104, normalized size = 0.77

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*c^(5/2))
```

**mupad [B]** time = 0.41, size = 201, normalized size = 1.49

$$\frac{(a+b\,x)^n\left(\frac{x^5\left(n^3+6\,n^2+11\,n+6\right)}{c^2\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{6\,a^4\,x}{b^4\,c^2\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,c^2\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\left(n^2+3\,n+2\right)}{b\,c^2\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,(n+1)}{b^2\,c^2\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^8*(a + b*x)^n)/(c*x^2)^(5/2),x)

[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (6*a^4*x)/(b^4*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*n*x^2)/(b^3*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n + n^2 + 2))/(b*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1))/(b^2*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(b*x+a)**n/(c*x**2)**(5/2),x)

[Out] Piecewise((a**n*x**9/(4*c**(5/2)*(x**2)**(5/2)), Eq(b, 0)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*x)), x), Eq(n, -1)), (-6*a**4*x**5*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) + 6*a**3*b*n*x**6*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) - 3*a**2*b**2*n**2*x**7*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) - 3*a**2*b**2*n*x**7*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) + a*b**3*n**3*x**8*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) + 3*a*b**3*n**2*x**8*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) + 2*a*b**3*n*x**8*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) + b**4*n**3*x**9*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) + 6*b**4*n**2*x**9*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) +

```
 24*b**4*c**(5/2)*(x**2)**(5/2)) + 11*b**4*n*x**9*(a + b*x)**n/(b**4*c**(5/
2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5
/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2
)*(x**2)**(5/2)) + 6*b**4*x**9*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/
2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5
/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)), T
rue))
```

## 3.905 $\int \frac{x^7(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=99

$$\frac{a^2x(a+bx)^{n+1}}{b^3c^2(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c^2(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c^2(n+3)\sqrt{cx^2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x(a+bx)^{n+1}}{b^3c^2(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c^2(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c^2(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c^2*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^
(2 + n))/(b^3*c^2*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c^2*(3
+ n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^7(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx = \frac{x\int x^2(a+bx)^n\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{c^2\sqrt{cx^2}}$$
$$= \frac{a^2x(a+bx)^{1+n}}{b^3c^2(1+n)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+n}}{b^3c^2(2+n)\sqrt{cx^2}} + \frac{x(a+bx)^{3+n}}{b^3c^2(3+n)\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.03, size = 70, normalized size = 0.71

$$\frac{x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3c^2(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] (x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(
b^3*c^2*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^7(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] Defer[IntegrateAlgebraic][(x^7*(a + b*x)^n)/(c*x^2)^(5/2), x]
```

**fricas** [A] time = 1.87, size = 118, normalized size = 1.19

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3c^3n^3+6\,b^3c^3n^2+11\,b^3c^3n+6\,b^3c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c^3*n^3 + 6*b^3*c^3*n^2 + 11*b^3*c^
3*n + 6*b^3*c^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^7}{(cx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^7/(c*x^2)^(5/2), x)
```

**maple [A]** time = 0.01, size = 83, normalized size = 0.84

$$\frac{\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2\right)x^5\,(bx+a)^{n+1}}{(c\,x^2)^{\frac{5}{2}}\left(n^3+6n^2+11n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
^5/(c*x^2)^(5/2)/b^3/(n^3+6*n^2+11*n+6)
```

**maxima [A]** time = 1.45, size = 83, normalized size = 0.84

$$\frac{\left(\left(n^2+3\,n+2\right)b^3\sqrt{c}\,x^3+\left(n^2+n\right)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c^3)
```

**mupad [B]** time = 0.35, size = 133, normalized size = 1.34

$$\frac{(a+b\,x)^n\left(\frac{x^4\,\left(n^2+3\,n+2\right)}{c^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,a^3\,x}{b^3\,c^2\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,n\,x^2}{b^2\,c^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^3\,(n+1)}{b\,c^2\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(c^2*(11*n + 6*n^2 + n^3 + 6)) + (2*a^3
*x)/(b^3*c^2*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*c^2*(11*n + 6*n
^2 + n^3 + 6)) + (a*n*x^3*(n + 1))/(b*c^2*(11*n + 6*n^2 + n^3 + 6))))/(c*x^
2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^8}{3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} & \text{for } b = 0 \\ \int \frac{x^7}{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^3}\,dx & \text{for } n = -3 \\ \int \frac{x^7}{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^2}\,dx & \text{for } n = -2 \\ \int \frac{x^7}{\left(cx^2\right)^{\frac{5}{2}}(a+bx)}\,dx & \text{for } n = -1 \\ \frac{2a^3x^5(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} - \frac{2a^2bnx^6(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{ab^2n^2x^7(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{ab^2nx^7(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{b^3n^2x^8(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{3b^3nx^8(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{2b^3x^8(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((a**n*x**8/(3*c**(5/2)*(x**2)**(5/2)), Eq(b, 0)), (Integral(x**7/
((c*x**2)**(5/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**7/((c*x**2)**(
5/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**7/((c*x**2)**(5/2)*(a + b*
x)), x), Eq(n, -1)), (2*a**3*x**5*(a + b*x)**n/(b**3*c**(5/2)*n**3*(x**2)**
(5/2) + 6*b**3*c**(5/2)*n**2*(x**2)**(5/2) + 11*b**3*c**(5/2)*n*(x**2)**(5/
2) + 6*b**3*c**(5/2)*(x**2)**(5/2)) - 2*a**2*b*n*x**6*(a + b*x)**n/(b**3*c*
*(5/2)*n**3*(x**2)**(5/2) + 6*b**3*c**(5/2)*n**2*(x**2)**(5/2) + 11*b**3*c*
*(5/2)*n*(x**2)**(5/2) + 6*b**3*c**(5/2)*(x**2)**(5/2)) + a*b**2*n**2*x**7*
(a + b*x)**n/(b**3*c**(5/2)*n**3*(x**2)**(5/2) + 6*b**3*c**(5/2)*n**2*(x**2
)**(5/2) + 11*b**3*c**(5/2)*n*(x**2)**(5/2) + 6*b**3*c**(5/2)*(x**2)**(5/2)
) + a*b**2*n*x**7*(a + b*x)**n/(b**3*c**(5/2)*n**3*(x**2)**(5/2) + 6*b**3*c
**(5/2)*n**2*(x**2)**(5/2) + 11*b**3*c**(5/2)*n*(x**2)**(5/2) + 6*b**3*c**(
5/2)*(x**2)**(5/2)) + b**3*n**2*x**8*(a + b*x)**n/(b**3*c**(5/2)*n**3*(x**2
)**(5/2) + 6*b**3*c**(5/2)*n**2*(x**2)**(5/2) + 11*b**3*c**(5/2)*n*(x**2)**
(5/2) + 6*b**3*c**(5/2)*(x**2)**(5/2)) + 3*b**3*n*x**8*(a + b*x)**n/(b**3*c
**(5/2)*n**3*(x**2)**(5/2) + 6*b**3*c**(5/2)*n**2*(x**2)**(5/2) + 11*b**3*c
**(5/2)*n*(x**2)**(5/2) + 6*b**3*c**(5/2)*(x**2)**(5/2)) + 2*b**3*x**8*(a +
 b*x)**n/(b**3*c**(5/2)*n**3*(x**2)**(5/2) + 6*b**3*c**(5/2)*n**2*(x**2)**(
5/2) + 11*b**3*c**(5/2)*n*(x**2)**(5/2) + 6*b**3*c**(5/2)*(x**2)**(5/2)), T
rue))
```

## 3.906 $\int \frac{x^6(a+bx)^n}{(cx^2)^{5/2}}\,dx$

**Optimal**. Leaf size=65

$$\frac{x(a+bx)^{n+2}}{b^2c^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c^2(n+1)\sqrt{cx^2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{x(a+bx)^{n+2}}{b^2c^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 + n))/(b^2*c^2*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx = \frac{x\int x(a+bx)^n\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{ax(a+bx)^{1+n}}{b^2c^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c^2(2+n)\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 0.71

$$\frac{x(a+bx)^{n+1}(b(n+1)x-a)}{b^2c^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*c^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

**IntegrateAlgebraic** [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^6*(a + b*x)^n)/(c*x^2)^(5/2), x]
```

**fricas** [A] time = 0.92, size = 72, normalized size = 1.11

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2c^3n^2+3\,b^2c^3n+2\,b^2c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c^3*n^2 +
 3*b^2*c^3*n + 2*b^2*c^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^6}{\left(cx^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^6/(c*x^2)^(5/2), x)
```

**maple [A]** time = 0.00, size = 46, normalized size = 0.71

$$-\frac{(-xnb-bx+a)\, x^5\, (bx+a)^{n+1}}{\left(c\, x^2\right)^{\frac{5}{2}} \left(n^2+3n+2\right) b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] -(b*x+a)^(n+1)*x^5*(-b*n*x-b*x+a)/(c*x^2)^(5/2)/b^2/(n^2+3*n+2)
```

**maxima [A]** time = 1.41, size = 45, normalized size = 0.69

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*c^(5/2))
```

**mupad [B]** time = 0.29, size = 80, normalized size = 1.23

$$\frac{(a+b\,x)^n \left(\frac{x^3\,(n+1)}{c^2\,\left(n^2+3\,n+2\right)}-\frac{a^2\,x}{b^2\,c^2\,\left(n^2+3\,n+2\right)}+\frac{a\,n\,x^2}{b\,c^2\,\left(n^2+3\,n+2\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((a + b*x)^n*((x^3*(n + 1))/(c^2*(3*n + n^2 + 2)) - (a^2*x)/(b^2*c^2*(3*n +
 n^2 + 2)) + (a*n*x^2)/(b*c^2*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^7}{2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} & \text{for } b = 0 \\ \int \frac{x^6}{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^2}\,dx & \text{for } n = -2 \\ \int \frac{x^6}{\left(cx^2\right)^{\frac{5}{2}}(a+bx)}\,dx & \text{for } n = -1 \\ -\frac{a^2x^5(a+bx)^n}{b^2c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+3b^2c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+2b^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{abnx^6(a+bx)^n}{b^2c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+3b^2c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+2b^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{b^2nx^7(a+bx)^n}{b^2c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+3b^2c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+2b^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{b^2x^7(a+bx)^n}{b^2c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+3b^2c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+2b^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((a**n*x**7/(2*c**(5/2)*(x**2)**(5/2)), Eq(b, 0)), (Integral(x**6/
((c*x**2)**(5/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**6/((c*x**2)**(
5/2)*(a + b*x)), x), Eq(n, -1)), (-a**2*x**5*(a + b*x)**n/(b**2*c**(5/2)*n*
*2*(x**2)**(5/2) + 3*b**2*c**(5/2)*n*(x**2)**(5/2) + 2*b**2*c**(5/2)*(x**2)
**(5/2)) + a*b*n*x**6*(a + b*x)**n/(b**2*c**(5/2)*n**2*(x**2)**(5/2) + 3*b*
*2*c**(5/2)*n*(x**2)**(5/2) + 2*b**2*c**(5/2)*(x**2)**(5/2)) + b**2*n*x**7*
(a + b*x)**n/(b**2*c**(5/2)*n**2*(x**2)**(5/2) + 3*b**2*c**(5/2)*n*(x**2)**
(5/2) + 2*b**2*c**(5/2)*(x**2)**(5/2)) + b**2*x**7*(a + b*x)**n/(b**2*c**(5
/2)*n**2*(x**2)**(5/2) + 3*b**2*c**(5/2)*n*(x**2)**(5/2) + 2*b**2*c**(5/2)*
(x**2)**(5/2)), True))
```

## 3.907 $\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=31

$$\frac{x(a+bx)^{n+1}}{bc^2(n+1)\sqrt{cx^2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{bc^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] (x*(a + b*x)^(1 + n))/(b*c^2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{bc^2(1+n)\sqrt{cx^2}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 1.00

$$\frac{x(a+bx)^{n+1}}{bc^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*c^2*(1 + n)*Sqrt[c*x^2])
```

**IntegrateAlgebraic [F]** time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^5*(a + b*x)^n)/(c*x^2)^(5/2), x]
```

**fricas [A]** time = 1.49, size = 37, normalized size = 1.19

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{\left(bc^3n+bc^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c^3*n + b*c^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^5}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^5/(c*x^2)^(5/2), x)
```

**maple [A]** time = 0.00, size = 29, normalized size = 0.94

$$\frac{x^5\,(bx+a)^{n+1}}{(n+1)\left(c\,x^2\right)^{\frac{5}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^n/(c*x^2)^(5/2),x)
[Out] (b*x+a)^(n+1)/b/(n+1)*x^5/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.45, size = 31, normalized size = 1.00

$$\frac{\left(b\sqrt{c}\,x+a\sqrt{c}\right)(bx+a)^n}{bc^3(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c^3*(n + 1))
```

**mupad [B]** time = 0.23, size = 42, normalized size = 1.35

$$\frac{\left(\frac{x^2}{c^2\,(n+1)}+\frac{a\,x}{b\,c^2\,(n+1)}\right)\,(a+b\,x)^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x)^n)/(c*x^2)^(5/2),x)
[Out] ((x^2/(c^2*(n + 1)) + (a*x)/(b*c^2*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^6}{ac^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} & \text{for } b=0 \wedge n=-1 \\ \frac{a^n x^6}{c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} & \text{for } b=0 \\ \int \frac{x^5}{\left(cx^2\right)^{\frac{5}{2}}(a+bx)}\,dx & \text{for } n=-1 \\ \frac{ax^5(a+bx)^n}{bc^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+bc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}+\frac{bx^6(a+bx)^n}{bc^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+bc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**n/(c*x**2)**(5/2),x)
[Out] Piecewise((x**6/(a*c**(5/2)*(x**2)**(5/2)), Eq(b, 0) & Eq(n, -1)), (a**n*x*
*6/(c**(5/2)*(x**2)**(5/2)), Eq(b, 0)), (Integral(x**5/((c*x**2)**(5/2)*(a
+ b*x)), x), Eq(n, -1)), (a*x**5*(a + b*x)**n/(b*c**(5/2)*n*(x**2)**(5/2) +
 b*c**(5/2)*(x**2)**(5/2)) + b*x**6*(a + b*x)**n/(b*c**(5/2)*n*(x**2)**(5/2
) + b*c**(5/2)*(x**2)**(5/2)), True))
```

# 3.908 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx$

**Optimal**. Leaf size=65

$$\frac{ac^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{bc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}$$

**Rubi [A]** time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$\frac{ac^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{bc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (b*c^2*(d*x)^(7 + m)*Sq
rt[c*x^2])/(d^7*(7 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(dx)^m(a+bx)\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int (dx)^{5+m}(a+bx)\,dx}{d^5x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a(dx)^{5+m}+\frac{b(dx)^{6+m}}{d}\right)dx}{d^5x}$$
$$= \frac{ac^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}+\frac{bc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}$$

**Mathematica** [A] time = 0.03, size = 38, normalized size = 0.58

$$\frac{x\left(cx^2\right)^{5/2}(dx)^m(a(m+7)+b(m+6)x)}{(m+6)(m+7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x*(d*x)^m*(c*x^2)^(5/2)*(a*(7 + m) + b*(6 + m)*x))/((6 + m)*(7 + m))
```

**IntegrateAlgebraic** [F] time = 0.46, size = 0, normalized size = 0.00

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(c*x^2)^(5/2)*(a + b*x), x]
```

**fricas** [A] time = 1.41, size = 58, normalized size = 0.89

$$\frac{\left(\left(bc^2m+6\,bc^2\right)x^6+\left(ac^2m+7\,ac^2\right)x^5\right)\sqrt{cx^2}\,\left(dx\right)^m}{m^2+13\,m+42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*c^2*m + 6*b*c^2)*x^6 + (a*c^2*m + 7*a*c^2)*x^5)*sqrt(c*x^2)*(d*x)^m/(m^
2 + 13*m + 42)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limit
```

**maple [A]** time = 0.00, size = 40, normalized size = 0.62

$$\frac{(bmx + am + 6bx + 7a)\left(c\,x^{2}\right)^{\frac{5}{2}} x\,(dx)^{m}}{(m+7)\,(m+6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x)
```

```
[Out] x*(b*m*x+a*m+6*b*x+7*a)*(d*x)^m*(c*x^2)^(5/2)/(m+7)/(m+6)
```

**maxima [A]** time = 1.52, size = 39, normalized size = 0.60

$$\frac{bc^{\frac{5}{2}}d^{m}x^{7}x^{m}}{m+7} + \frac{ac^{\frac{5}{2}}d^{m}x^{6}x^{m}}{m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a*c^(5/2)*d^m*x^6*x^m/(m + 6)
```

**mupad [B]** time = 0.27, size = 44, normalized size = 0.68

$$\frac{c^{2}\,x^{5}\,(d\,x)^{m}\,\sqrt{c\,x^{2}}\;(7\,a + a\,m + 6\,b\,x + b\,m\,x)}{m^{2} + 13\,m + 42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x),x)
```

[Out] (c^2*x^5*(d*x)^m*(c*x^2)^(1/2)*(7*a + a*m + 6*b*x + b*m*x))/(13*m + m^2 + 4
2)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a(cx^2)^{\frac{5}{2}}}{x^7}\,dx + \int \frac{b(cx^2)^{\frac{5}{2}}}{x^6}\,dx}{d^7} & \text{for } m = -7 \\ \frac{\int \frac{a(cx^2)^{\frac{5}{2}}}{x^6}\,dx + \int \frac{b(cx^2)^{\frac{5}{2}}}{x^5}\,dx}{d^6} & \text{for } m = -6 \\ \frac{ac^{\frac{5}{2}}d^m m x x^m (x^2)^{\frac{5}{2}}}{m^2+13m+42} + \frac{7ac^{\frac{5}{2}}d^m x x^m (x^2)^{\frac{5}{2}}}{m^2+13m+42} + \frac{bc^{\frac{5}{2}}d^m m x^2 x^m (x^2)^{\frac{5}{2}}}{m^2+13m+42} + \frac{6bc^{\frac{5}{2}}d^m x^2 x^m (x^2)^{\frac{5}{2}}}{m^2+13m+42} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*(c*x**2)**(5/2)/x**7, x) + Integral(b*(c*x**2)**(5/2
)/x**6, x))/d**7, Eq(m, -7)), ((Integral(a*(c*x**2)**(5/2)/x**6, x) + Integ
ral(b*(c*x**2)**(5/2)/x**5, x))/d**6, Eq(m, -6)), (a*c**(5/2)*d**m*m*x*x**m
*(x**2)**(5/2)/(m**2 + 13*m + 42) + 7*a*c**(5/2)*d**m*x*x**m*(x**2)**(5/2)/
(m**2 + 13*m + 42) + b*c**(5/2)*d**m*m*x**2*x**m*(x**2)**(5/2)/(m**2 + 13*m
 + 42) + 6*b*c**(5/2)*d**m*x**2*x**m*(x**2)**(5/2)/(m**2 + 13*m + 42), True
))
```

## 3.909 $\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx$

**Optimal**. Leaf size=61

$$\frac{ac\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{bc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}$$

**Rubi** [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$\frac{ac\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{bc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (b*c*(d*x)^(5 + m)*Sqrt[c
*x^2])/(d^5*(5 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^3(dx)^m(a+bx)\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int (dx)^{3+m}(a+bx)\,dx}{d^3x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(a(dx)^{3+m}+\frac{b(dx)^{4+m}}{d}\right)dx}{d^3x}$$
$$= \frac{ac(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x} + \frac{bc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}$$

**Mathematica** [A] time = 0.02, size = 38, normalized size = 0.62

$$\frac{x\left(cx^2\right)^{3/2}(dx)^m(a(m+5)+b(m+4)x)}{(m+4)(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x*(d*x)^m*(c*x^2)^(3/2)*(a*(5 + m) + b*(4 + m)*x))/((4 + m)*(5 + m))
```

**IntegrateAlgebraic** [F] time = 0.39, size = 0, normalized size = 0.00

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(c*x^2)^(3/2)*(a + b*x), x]
```

**fricas** [A] time = 1.49, size = 50, normalized size = 0.82

$$\frac{\left((bcm+4\,bc)x^4+(acm+5\,ac)x^3\right)\sqrt{cx^2}\,(dx)^m}{m^2+9\,m+20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

[Out] ((b*c*m + 4*b*c)*x^4 + (a*c*m + 5*a*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(m^2 + 9*m + 20)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)] Undef/Unsigned Inf encountered in limit

**maple [A]** time = 0.00, size = 40, normalized size = 0.66

$$\frac{(bmx + am + 4bx + 5a)\left(c\,x^2\right)^{\frac{3}{2}} x\,(dx)^m}{(m+5)\,(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x)

[Out] x*(b*m*x+a*m+4*b*x+5*a)*(d*x)^m*(c*x^2)^(3/2)/(m+5)/(m+4)

**maxima [A]** time = 1.55, size = 39, normalized size = 0.64

$$\frac{bc^{\frac{3}{2}}d^m x^5 x^m}{m+5} + \frac{ac^{\frac{3}{2}}d^m x^4 x^m}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")

[Out] b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a*c^(3/2)*d^m*x^4*x^m/(m + 4)

**mupad [B]** time = 0.24, size = 42, normalized size = 0.69

$$\frac{c\,x^3\,(d\,x)^m\,\sqrt{c\,x^2}\,\,(5\,a + a\,m + 4\,b\,x + b\,m\,x)}{m^2 + 9\,m + 20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x),x)

```
[Out] (c*x^3*(d*x)^m*(c*x^2)^(1/2)*(5*a + a*m + 4*b*x + b*m*x))/(9*m + m^2 + 20)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a(cx^2)^{\frac{3}{2}}}{x^5}\,dx+\int \frac{b(cx^2)^{\frac{3}{2}}}{x^4}\,dx}{d^5} & \text{for } m=-5 \\ \frac{\int \frac{a(cx^2)^{\frac{3}{2}}}{x^4}\,dx+\int \frac{b(cx^2)^{\frac{3}{2}}}{x^3}\,dx}{d^4} & \text{for } m=-4 \\ \frac{ac^{\frac{3}{2}}d^m m x x^m (x^2)^{\frac{3}{2}}}{m^2+9m+20} + \frac{5ac^{\frac{3}{2}}d^m x x^m (x^2)^{\frac{3}{2}}}{m^2+9m+20} + \frac{bc^{\frac{3}{2}}d^m m x^2 x^m (x^2)^{\frac{3}{2}}}{m^2+9m+20} + \frac{4bc^{\frac{3}{2}}d^m x^2 x^m (x^2)^{\frac{3}{2}}}{m^2+9m+20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*(c*x**2)**(3/2)/x**5, x) + Integral(b*(c*x**2)**(3/2
)/x**4, x))/d**5, Eq(m, -5)), ((Integral(a*(c*x**2)**(3/2)/x**4, x) + Integ
ral(b*(c*x**2)**(3/2)/x**3, x))/d**4, Eq(m, -4)), (a*c**(3/2)*d**m*m*x*x**m
*(x**2)**(3/2)/(m**2 + 9*m + 20) + 5*a*c**(3/2)*d**m*x*x**m*(x**2)**(3/2)/(
m**2 + 9*m + 20) + b*c**(3/2)*d**m*m*x**2*x**m*(x**2)**(3/2)/(m**2 + 9*m +
20) + 4*b*c**(3/2)*d**m*x**2*x**m*(x**2)**(3/2)/(m**2 + 9*m + 20), True))
```

## 3.910 $\int (dx)^m \sqrt{cx^2} (a + bx) \, dx$

**Optimal**. Leaf size=59

$$\frac{a\sqrt{cx^2}\,(dx)^{m+2}}{d^2(m+2)x} + \frac{b\sqrt{cx^2}\,(dx)^{m+3}}{d^3(m+3)x}$$

**Rubi** [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$\frac{a\sqrt{cx^2}\,(dx)^{m+2}}{d^2(m+2)x} + \frac{b\sqrt{cx^2}\,(dx)^{m+3}}{d^3(m+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (b*(d*x)^(3 + m)*Sqrt[c*x^2
])/(d^3*(3 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \sqrt{cx^2}\,(a+bx)\,dx = \frac{\sqrt{cx^2}\,\int x(dx)^m(a+bx)\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\,\int (dx)^{1+m}(a+bx)\,dx}{dx}$$
$$= \frac{\sqrt{cx^2}\,\int \left(a(dx)^{1+m} + \frac{b(dx)^{2+m}}{d}\right)dx}{dx}$$
$$= \frac{a(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x} + \frac{b(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}$$

**Mathematica** [A] time = 0.02, size = 38, normalized size = 0.64

$$\frac{x\sqrt{cx^2}\,(dx)^m(a(m+3)+b(m+2)x)}{(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a*(3 + m) + b*(2 + m)*x))/((2 + m)*(3 + m))
```

**IntegrateAlgebraic** [F] time = 0.35, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{cx^2}\,(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
[Out] Defer[IntegrateAlgebraic][(d*x)^m*Sqrt[c*x^2]*(a + b*x), x]
```

**fricas** [A] time = 1.52, size = 44, normalized size = 0.75

$$\frac{\left((bm+2\,b)x^2+(am+3\,a)x\right)\sqrt{cx^2}\,\,(dx)^m}{m^2+5\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="fricas")
[Out] ((b*m + 2*b)*x^2 + (a*m + 3*a)*x)*sqrt(c*x^2)*(d*x)^m/(m^2 + 5*m + 6)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limit
```

**maple [A]** time = 0.00, size = 40, normalized size = 0.68

$$\frac{(bmx + am + 2bx + 3a)\sqrt{c\,x^2}\;x\,(dx)^m}{(m+3)\,(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x)
```

```
[Out] x*(b*m*x+a*m+2*b*x+3*a)*(d*x)^m*(c*x^2)^(1/2)/(m+3)/(m+2)
```

**maxima [A]** time = 1.52, size = 39, normalized size = 0.66

$$\frac{b\sqrt{c}\,d^m x^3 x^m}{m+3} + \frac{a\sqrt{c}\,d^m x^2 x^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a*sqrt(c)*d^m*x^2*x^m/(m + 2)
```

**mupad [B]** time = 0.21, size = 39, normalized size = 0.66

$$\frac{x\,(d\,x)^m\,\sqrt{c\,x^2}\;(3\,a + a\,m + 2\,b\,x + b\,m\,x)}{m^2 + 5\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] (x*(d*x)^m*(c*x^2)^(1/2)*(3*a + a*m + 2*b*x + b*m*x))/(5*m + m^2 + 6)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a\sqrt{cx^2}}{x^3}\,dx+\int \frac{b\sqrt{cx^2}}{x^2}\,dx}{d^3} & \text{for } m=-3 \\ \frac{\int \frac{a\sqrt{cx^2}}{x^2}\,dx+\int \frac{b\sqrt{cx^2}}{x}\,dx}{d^2} & \text{for } m=-2 \\ \frac{a\sqrt{c}\,d^m m x x^m \sqrt{x^2}}{m^2+5m+6}+\frac{3a\sqrt{c}\,d^m x x^m \sqrt{x^2}}{m^2+5m+6}+\frac{b\sqrt{c}\,d^m m x^2 x^m \sqrt{x^2}}{m^2+5m+6}+\frac{2b\sqrt{c}\,d^m x^2 x^m \sqrt{x^2}}{m^2+5m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*sqrt(c*x**2)/x**3, x) + Integral(b*sqrt(c*x**2)/x**2
, x))/d**3, Eq(m, -3)), ((Integral(a*sqrt(c*x**2)/x**2, x) + Integral(b*sqr
t(c*x**2)/x, x))/d**2, Eq(m, -2)), (a*sqrt(c)*d**m*m*x*x**m*sqrt(x**2)/(m**
2 + 5*m + 6) + 3*a*sqrt(c)*d**m*x*x**m*sqrt(x**2)/(m**2 + 5*m + 6) + b*sqrt
(c)*d**m*m*x**2*x**m*sqrt(x**2)/(m**2 + 5*m + 6) + 2*b*sqrt(c)*d**m*x**2*x*
*m*sqrt(x**2)/(m**2 + 5*m + 6), True))
```

## 3.911 $\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=48

$$\frac{ax(dx)^m}{m\sqrt{cx^2}}+\frac{bx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$\frac{ax(dx)^m}{m\sqrt{cx^2}}+\frac{bx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)}{x}\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int (dx)^{-1+m}(a+bx)\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int \left(a(dx)^{-1+m} + \frac{b(dx)^m}{d}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{ax(dx)^m}{m\sqrt{cx^2}} + \frac{bx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.02, size = 33, normalized size = 0.69

$$\frac{x(dx)^m(am+a+bmx)}{m(m+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
[Out] (x*(d*x)^m*(a + a*m + b*m*x))/(m*(1 + m)*Sqrt[c*x^2])
```

**IntegrateAlgebraic** [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x))/Sqrt[c*x^2], x]
```

**fricas** [A] time = 0.75, size = 36, normalized size = 0.75

$$\frac{(bmx+am+a)\sqrt{cx^2}\,(dx)^m}{\left(cm^2+cm\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*m*x + a*m + a)*sqrt(c*x^2)*(d*x)^m/((c*m^2 + c*m)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)\,(dx)^m}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(d*x)^m/sqrt(c*x^2), x)
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.67

$$\frac{(bmx+am+a)\,x\,(dx)^m}{(m+1)\,\sqrt{c\,x^2}\;m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] x*(b*m*x+a*m+a)*(d*x)^m/(m+1)/m/(c*x^2)^(1/2)
```

**maxima [A]** time = 1.48, size = 32, normalized size = 0.67

$$\frac{bd^m x x^m}{\sqrt{c}\,(m+1)} + \frac{ad^m x^m}{\sqrt{c}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a*d^m*x^m/(sqrt(c)*m)
```

**mupad [B]** time = 0.21, size = 30, normalized size = 0.62

$$\frac{\left(\frac{a\,x}{m}+\frac{b\,x^2}{m+1}\right)\,{\left(d\,x\right)}^m}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] (((a*x)/m + (b*x^2)/(m + 1))*(d*x)^m)/(c*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{b}{\sqrt{cx^2}}\,dx + \int \frac{a}{x\sqrt{cx^2}}\,dx}{d} & \text{for } m = -1 \\ \int \frac{a+bx}{\sqrt{cx^2}}\,dx & \text{for } m = 0 \\ \frac{ad^m m x x^m}{\sqrt{c}\,m^2\sqrt{x^2} + \sqrt{c}\,m\sqrt{x^2}} + \frac{ad^m x x^m}{\sqrt{c}\,m^2\sqrt{x^2} + \sqrt{c}\,m\sqrt{x^2}} + \frac{bd^m m x^2 x^m}{\sqrt{c}\,m^2\sqrt{x^2} + \sqrt{c}\,m\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise(((Integral(b/sqrt(c*x**2), x) + Integral(a/(x*sqrt(c*x**2)), x))/
d, Eq(m, -1)), (Integral((a + b*x)/sqrt(c*x**2), x), Eq(m, 0)), (a*d**m*m*x
*x**m/(sqrt(c)*m**2*sqrt(x**2) + sqrt(c)*m*sqrt(x**2)) + a*d**m*x*x**m/(sqr
t(c)*m**2*sqrt(x**2) + sqrt(c)*m*sqrt(x**2)) + b*d**m*m*x**2*x**m/(sqrt(c)*
m**2*sqrt(x**2) + sqrt(c)*m*sqrt(x**2)), True))
```

## 3.912 $\int \frac{(dx)^m(a+bx)}{\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=65

$$-\frac{ad^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}$$

**Rubi** [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$-\frac{ad^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] -((a*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (b*d*x*(d*x)^(-1 + m)
)/(c*(1 - m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)}{x^3}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int (dx)^{-3+m}(a+bx)\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int \left(a(dx)^{-3+m}+\frac{b(dx)^{-2+m}}{d}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{ad^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.03, size = 38, normalized size = 0.58

$$\frac{x(dx)^m(a(m-1)+b(m-2)x)}{(m-2)(m-1)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (x*(d*x)^m*(a*(-1 + m) + b*(-2 + m)*x))/((-2 + m)*(-1 + m)*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x))/(c*x^2)^(3/2), x]
```

**fricas** [A] time = 1.06, size = 53, normalized size = 0.82

$$\frac{\sqrt{cx^2}\,(am+(bm-2\,b)x-a)\,(dx)^m}{\left(c^2m^2-3\,c^2m+2\,c^2\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*m + (b*m - 2*b)*x - a)*(d*x)^m/((c^2*m^2 - 3*c^2*m + 2*c^2)*
x^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)(dx)^m}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(d*x)^m/(c*x^2)^(3/2), x)
```

**maple [A]** time = 0.00, size = 40, normalized size = 0.62

$$\frac{(bmx+am-2bx-a)\,x\,(dx)^m}{(m-1)\,(m-2)\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] x*(b*m*x+a*m-2*b*x-a)*(d*x)^m/(m-1)/(m-2)/(c*x^2)^(3/2)
```

**maxima [A]** time = 1.51, size = 39, normalized size = 0.60

$$\frac{bd^m x^m}{c^{\frac{3}{2}}(m-1)x} + \frac{ad^m x^m}{c^{\frac{3}{2}}(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a*d^m*x^m/(c^(3/2)*(m - 2)*x^2)
```

**mupad [B]** time = 0.26, size = 48, normalized size = 0.74

$$\frac{b\,(d\,x)^m}{c\,\sqrt{c\,x^2}\,\,(m-1)} + \frac{a\,(d\,x)^m}{c\,x\,\sqrt{c\,x^2}\,\,(m-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] (b*(d*x)^m)/(c*(c*x^2)^(1/2)*(m - 1)) + (a*(d*x)^m)/(c*x*(c*x^2)^(1/2)*(m -
 2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} d\left(\int \frac{ax}{(cx^2)^{\frac{3}{2}}}\,dx + \int \frac{bx^2}{(cx^2)^{\frac{3}{2}}}\,dx\right) & \text{for } m = 1 \\ d^2\left(\int \frac{ax^2}{(cx^2)^{\frac{3}{2}}}\,dx + \int \frac{bx^3}{(cx^2)^{\frac{3}{2}}}\,dx\right) & \text{for } m = 2 \\ \frac{ad^m mxx^m}{c^{\frac{3}{2}}m^2(x^2)^{\frac{3}{2}} - 3c^{\frac{3}{2}}m(x^2)^{\frac{3}{2}} + 2c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} - \frac{ad^m xx^m}{c^{\frac{3}{2}}m^2(x^2)^{\frac{3}{2}} - 3c^{\frac{3}{2}}m(x^2)^{\frac{3}{2}} + 2c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} + \frac{bd^m mx^2x^m}{c^{\frac{3}{2}}m^2(x^2)^{\frac{3}{2}} - 3c^{\frac{3}{2}}m(x^2)^{\frac{3}{2}} + 2c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} - \frac{2bd^m x^2x^m}{c^{\frac{3}{2}}m^2(x^2)^{\frac{3}{2}} - 3c^{\frac{3}{2}}m(x^2)^{\frac{3}{2}} + 2c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((d*(Integral(a*x/(c*x**2)**(3/2), x) + Integral(b*x**2/(c*x**2)**
(3/2), x)), Eq(m, 1)), (d**2*(Integral(a*x**2/(c*x**2)**(3/2), x) + Integra
l(b*x**3/(c*x**2)**(3/2), x)), Eq(m, 2)), (a*d**m*m*x*x**m/(c**(3/2)*m**2*(
x**2)**(3/2) - 3*c**(3/2)*m*(x**2)**(3/2) + 2*c**(3/2)*(x**2)**(3/2)) - a*d
**m*x*x**m/(c**(3/2)*m**2*(x**2)**(3/2) - 3*c**(3/2)*m*(x**2)**(3/2) + 2*c*
*(3/2)*(x**2)**(3/2)) + b*d**m*m*x**2*x**m/(c**(3/2)*m**2*(x**2)**(3/2) - 3
*c**(3/2)*m*(x**2)**(3/2) + 2*c**(3/2)*(x**2)**(3/2)) - 2*b*d**m*x**2*x**m/
(c**(3/2)*m**2*(x**2)**(3/2) - 3*c**(3/2)*m*(x**2)**(3/2) + 2*c**(3/2)*(x**
2)**(3/2)), True))
```

## 3.913 $\int \frac{(dx)^m(a+bx)}{\left(cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=67

$$-\frac{ad^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{bd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}$$

**Rubi** [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$-\frac{ad^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{bd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -((a*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (b*d^3*x*(d*x)^(-3
+ m))/(c^2*(3 - m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int (dx)^{-5+m}(a+bx)\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int \left(a(dx)^{-5+m}+\frac{b(dx)^{-4+m}}{d}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{ad^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}} - \frac{bd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.03, size = 38, normalized size = 0.57

$$\frac{x(dx)^m(a(m-3)+b(m-4)x)}{(m-4)(m-3)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
[Out] (x*(d*x)^m*(a*(-3 + m) + b*(-4 + m)*x))/((-4 + m)*(-3 + m)*(c*x^2)^(5/2))
```

**IntegrateAlgebraic [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x))/(c*x^2)^(5/2), x]
```

**fricas [A]** time = 1.21, size = 53, normalized size = 0.79

$$\frac{\sqrt{cx^2}\,(am+(bm-4\,b)x-3\,a)\,(dx)^m}{\left(c^3m^2-7\,c^3m+12\,c^3\right)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*m + (b*m - 4*b)*x - 3*a)*(d*x)^m/((c^3*m^2 - 7*c^3*m + 12*c^
3)*x^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)(dx)^m}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(d*x)^m/(c*x^2)^(5/2), x)
```

**maple [A]** time = 0.00, size = 40, normalized size = 0.60

$$\frac{(bmx+am-4bx-3a)\,x\,(dx)^m}{(m-3)(m-4)\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] x*(b*m*x+a*m-4*b*x-3*a)*(d*x)^m/(m-3)/(-4+m)/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.52, size = 39, normalized size = 0.58

$$\frac{bd^m x^m}{c^{\frac{5}{2}}(m-3)x^3}+\frac{ad^m x^m}{c^{\frac{5}{2}}(m-4)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) + a*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

**mupad [B]** time = 0.28, size = 47, normalized size = 0.70

$$-\frac{(d\,x)^m\,\left(3\,a-a\,m+4\,b\,x-b\,m\,x\right)}{c^2\,x^3\,\sqrt{c\,x^2}\,\left(m^2-7\,m+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x)
```

```
[Out] -((d*x)^m*(3*a - a*m + 4*b*x - b*m*x))/(c^2*x^3*(c*x^2)^(1/2)*(m^2 - 7*m +
12))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} d^3\left(\int \frac{ax^3}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{bx^4}{(cx^2)^{\frac{5}{2}}}\,dx\right) & \text{for } m = 3 \\ d^4\left(\int \frac{ax^4}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{bx^5}{(cx^2)^{\frac{5}{2}}}\,dx\right) & \text{for } m = 4 \\ \frac{ad^m mxx^m}{c^{\frac{5}{2}}m^2(x^2)^{\frac{5}{2}} - 7c^{\frac{5}{2}}m(x^2)^{\frac{5}{2}} + 12c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} - \frac{3ad^m xx^m}{c^{\frac{5}{2}}m^2(x^2)^{\frac{5}{2}} - 7c^{\frac{5}{2}}m(x^2)^{\frac{5}{2}} + 12c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} + \frac{bd^m mx^2x^m}{c^{\frac{5}{2}}m^2(x^2)^{\frac{5}{2}} - 7c^{\frac{5}{2}}m(x^2)^{\frac{5}{2}} + 12c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} - \frac{4bd^m x^2x^m}{c^{\frac{5}{2}}m^2(x^2)^{\frac{5}{2}} - 7c^{\frac{5}{2}}m(x^2)^{\frac{5}{2}} + 12c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((d**3*(Integral(a*x**3/(c*x**2)**(5/2), x) + Integral(b*x**4/(c*x
**2)**(5/2), x)), Eq(m, 3)), (d**4*(Integral(a*x**4/(c*x**2)**(5/2), x) + I
ntegral(b*x**5/(c*x**2)**(5/2), x)), Eq(m, 4)), (a*d**m*m*x*x**m/(c**(5/2)*
m**2*(x**2)**(5/2) - 7*c**(5/2)*m*(x**2)**(5/2) + 12*c**(5/2)*(x**2)**(5/2)
) - 3*a*d**m*x*x**m/(c**(5/2)*m**2*(x**2)**(5/2) - 7*c**(5/2)*m*(x**2)**(5/
2) + 12*c**(5/2)*(x**2)**(5/2)) + b*d**m*m*x**2*x**m/(c**(5/2)*m**2*(x**2)*
*(5/2) - 7*c**(5/2)*m*(x**2)**(5/2) + 12*c**(5/2)*(x**2)**(5/2)) - 4*b*d**m
*x**2*x**m/(c**(5/2)*m**2*(x**2)**(5/2) - 7*c**(5/2)*m*(x**2)**(5/2) + 12*c
**(5/2)*(x**2)**(5/2)), True))
```

## 3.914 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2 \, dx$

**Optimal**. Leaf size=103

$$\frac{a^2c^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{2abc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}+\frac{b^2c^2\sqrt{cx^2}\,(dx)^{m+8}}{d^8(m+8)x}$$

**Rubi [A]** time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2c^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{2abc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}+\frac{b^2c^2\sqrt{cx^2}\,(dx)^{m+8}}{d^8(m+8)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (2*a*b*c^2*(d*x)^(7 +
 m)*Sqrt[c*x^2])/(d^7*(7 + m)*x) + (b^2*c^2*(d*x)^(8 + m)*Sqrt[c*x^2])/(d^8
*(8 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(dx)^m(a+bx)^2\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int (dx)^{5+m}(a+bx)^2\,dx}{d^5x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2(dx)^{5+m}+\frac{2ab(dx)^{6+m}}{d}+\frac{b^2(dx)^{7+m}}{d^2}\right)dx}{d^5x}$$
$$= \frac{a^2c^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}+\frac{2abc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}+\frac{b^2c^2(dx)^{8+m}\sqrt{cx^2}}{d^8(8+m)x}$$

**Mathematica** [A] time = 0.05, size = 48, normalized size = 0.47

$$x\left(cx^2\right)^{5/2}(dx)^m\left(\frac{a^2}{m+6}+\frac{2abx}{m+7}+\frac{b^2x^2}{m+8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] x*(d*x)^m*(c*x^2)^(5/2)*(a^2/(6 + m) + (2*a*b*x)/(7 + m) + (b^2*x^2)/(8 + m
))
```

**IntegrateAlgebraic** [F] time = 0.85, size = 0, normalized size = 0.00

$$\int (dx)^m \left(cx^2\right)^{5/2}(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2, x]
```

**fricas** [A] time = 1.28, size = 123, normalized size = 1.19

$$\frac{\left(\left(b^2c^2m^2+13\,b^2c^2m+42\,b^2c^2\right)x^7+2\left(abc^2m^2+14\,abc^2m+48\,abc^2\right)x^6+\left(a^2c^2m^2+15\,a^2c^2m+56\,a^2c^2\right)x^5\right)\sqrt{cx^2}\,(dx)^m}{m^3+21\,m^2+146\,m+336}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*c^2*m^2 + 13*b^2*c^2*m + 42*b^2*c^2)*x^7 + 2*(a*b*c^2*m^2 + 14*a*b*c^
2*m + 48*a*b*c^2)*x^6 + (a^2*c^2*m^2 + 15*a^2*c^2*m + 56*a^2*c^2)*x^5)*sqrt
(c*x^2)*(d*x)^m/(m^3 + 21*m^2 + 146*m + 336)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limit
```

**maple [A]** time = 0.00, size = 95, normalized size = 0.92

$$\frac{\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+13 b^{2} m\, x^{2}+a^{2} m^{2}+28 a b m x+42 b^{2} x^{2}+15 a^{2} m+96 a b x+56 a^{2}\right) \left(c\, x^{2}\right)^{\frac{5}{2}} x\left(d x\right)^{m}}{\left(m+8\right) \left(m+7\right) \left(m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+13*b^2*m*x^2+a^2*m^2+28*a*b*m*x+42*b^2*x^2+15*a^
2*m+96*a*b*x+56*a^2)*(d*x)^m*(c*x^2)^(5/2)/(m+8)/(m+7)/(m+6)
```

**maxima [A]** time = 1.60, size = 64, normalized size = 0.62

$$\frac{b^{2} c^{\frac{5}{2}} d^{m} x^{8} x^{m}}{m+8}+\frac{2 \, a b c^{\frac{5}{2}} d^{m} x^{7} x^{m}}{m+7}+\frac{a^{2} c^{\frac{5}{2}} d^{m} x^{6} x^{m}}{m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(5/2)*d^m*x^8*x^m/(m + 8) + 2*a*b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a^2*c
^(5/2)*d^m*x^6*x^m/(m + 6)
```

**mupad [B]** time = 0.31, size = 127, normalized size = 1.23

$$\left(d\,x\right)^m\,\left(\frac{a^2\,c^2\,x^5\,\sqrt{c\,x^2}\,\left(m^2+15\,m+56\right)}{m^3+21\,m^2+146\,m+336}+\frac{b^2\,c^2\,x^7\,\sqrt{c\,x^2}\,\left(m^2+13\,m+42\right)}{m^3+21\,m^2+146\,m+336}+\frac{2\,a\,b\,c^2\,x^6\,\sqrt{c\,x^2}\,\left(m^2+14\,m+48\right)}{m^3+21\,m^2+146\,m+336}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*c^2*x^5*(c*x^2)^(1/2)*(15*m + m^2 + 56))/(146*m + 21*m^2 + m^
3 + 336) + (b^2*c^2*x^7*(c*x^2)^(1/2)*(13*m + m^2 + 42))/(146*m + 21*m^2 +
m^3 + 336) + (2*a*b*c^2*x^6*(c*x^2)^(1/2)*(14*m + m^2 + 48))/(146*m + 21*m^
2 + m^3 + 336))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a^2(cx^2)^{\frac{5}{2}}}{x^8}\,dx + \int \frac{b^2(cx^2)^{\frac{5}{2}}}{x^6}\,dx + \int \frac{2ab(cx^2)^{\frac{5}{2}}}{x^7}\,dx}{d^8} & \text{for } m = -8 \\ \frac{\int \frac{a^2(cx^2)^{\frac{5}{2}}}{x^7}\,dx + \int \frac{b^2(cx^2)^{\frac{5}{2}}}{x^5}\,dx + \int \frac{2ab(cx^2)^{\frac{5}{2}}}{x^6}\,dx}{d^7} & \text{for } m = -7 \\ \frac{\int \frac{a^2(cx^2)^{\frac{5}{2}}}{x^6}\,dx + \int \frac{b^2(cx^2)^{\frac{5}{2}}}{x^4}\,dx + \int \frac{2ab(cx^2)^{\frac{5}{2}}}{x^5}\,dx}{d^6} & \text{for } m = -6 \\ \frac{a^2c^{\frac{5}{2}}d^m m^2 x x^m (x^2)^{\frac{5}{2}}}{m^3+21m^2+146m+336} + \frac{15a^2c^{\frac{5}{2}}d^m m x x^m (x^2)^{\frac{5}{2}}}{m^3+21m^2+146m+336} + \frac{56a^2c^{\frac{5}{2}}d^m x x^m (x^2)^{\frac{5}{2}}}{m^3+21m^2+146m+336} + \frac{2abc^{\frac{5}{2}}d^m m^2 x^2 x^m (x^2)^{\frac{5}{2}}}{m^3+21m^2+146m+336} + \frac{28abc^{\frac{5}{2}}d^m m x^2 x^m (x^2)^{\frac{5}{2}}}{m^3+21m^2+146m+336} + \frac{96abc^{\frac{5}{2}}d^m x^2 x^m (x^2)^{\frac{5}{2}}}{m^3+21m^2+146m+336} + \frac{b^2c^{\frac{5}{2}}d^m m^2 x^3 x^m (x^2)^{\frac{5}{2}}}{m^3+21m^2+146m+336} + \frac{13b^2c^{\frac{5}{2}}d^m m x^3 x^m (x^2)^{\frac{5}{2}}}{m^3+21m^2+146m+336} + \frac{42b^2c^{\frac{5}{2}}d^m x^3 x^m (x^2)^{\frac{5}{2}}}{m^3+21m^2+146m+336} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise(((Integral(a**2*(c*x**2)**(5/2)/x**8, x) + Integral(b**2*(c*x**2)
**(5/2)/x**6, x) + Integral(2*a*b*(c*x**2)**(5/2)/x**7, x))/d**8, Eq(m, -8)
), ((Integral(a**2*(c*x**2)**(5/2)/x**7, x) + Integral(b**2*(c*x**2)**(5/2)
/x**5, x) + Integral(2*a*b*(c*x**2)**(5/2)/x**6, x))/d**7, Eq(m, -7)), ((In
tegral(a**2*(c*x**2)**(5/2)/x**6, x) + Integral(b**2*(c*x**2)**(5/2)/x**4,
x) + Integral(2*a*b*(c*x**2)**(5/2)/x**5, x))/d**6, Eq(m, -6)), (a**2*c**(5
/2)*d**m*m**2*x*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146*m + 336) + 15*a**2
*c**(5/2)*d**m*m*x*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146*m + 336) + 56*a
**2*c**(5/2)*d**m*x*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146*m + 336) + 2*a
*b*c**(5/2)*d**m*m**2*x**2*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146*m + 336
) + 28*a*b*c**(5/2)*d**m*m*x**2*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146*m
+ 336) + 96*a*b*c**(5/2)*d**m*x**2*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146
*m + 336) + b**2*c**(5/2)*d**m*m**2*x**3*x**m*(x**2)**(5/2)/(m**3 + 21*m**2
 + 146*m + 336) + 13*b**2*c**(5/2)*d**m*m*x**3*x**m*(x**2)**(5/2)/(m**3 + 2
1*m**2 + 146*m + 336) + 42*b**2*c**(5/2)*d**m*x**3*x**m*(x**2)**(5/2)/(m**3
 + 21*m**2 + 146*m + 336), True))
```

## 3.915 $\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

**Optimal**. Leaf size=97

$$\frac{a^2c\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{2abc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}+\frac{b^2c\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}$$

**Rubi** [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2c\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{2abc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}+\frac{b^2c\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (2*a*b*c*(d*x)^(5 + m)*
Sqrt[c*x^2])/(d^5*(5 + m)*x) + (b^2*c*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 +
m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(dx)^m(a+bx)^2\,dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int (dx)^{3+m}(a+bx)^2\,dx}{d^3x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2(dx)^{3+m}+\frac{2ab(dx)^{4+m}}{d}+\frac{b^2(dx)^{5+m}}{d^2}\right)dx}{d^3x} \\
&= \frac{a^2c(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}+\frac{2abc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}+\frac{b^2c(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 48, normalized size = 0.49

$$
x\left(cx^2\right)^{3/2}(dx)^m\left(\frac{a^2}{m+4}+\frac{2abx}{m+5}+\frac{b^2x^2}{m+6}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] x*(d*x)^m*(c*x^2)^(3/2)*(a^2/(4 + m) + (2*a*b*x)/(5 + m) + (b^2*x^2)/(6 + m
))
```

**IntegrateAlgebraic [F]** time = 0.62, size = 0, normalized size = 0.00

$$
\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2, x]
```

**fricas [A]** time = 1.42, size = 105, normalized size = 1.08

$$
\frac{\left(\left(b^2cm^2+9\,b^2cm+20\,b^2c\right)x^5+2\left(abcm^2+10\,abcm+24\,abc\right)x^4+\left(a^2cm^2+11\,a^2cm+30\,a^2c\right)x^3\right)\sqrt{cx^2}\,(dx)^m}{m^3+15\,m^2+74\,m+120}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*c*m^2 + 9*b^2*c*m + 20*b^2*c)*x^5 + 2*(a*b*c*m^2 + 10*a*b*c*m + 24*a*
b*c)*x^4 + (a^2*c*m^2 + 11*a^2*c*m + 30*a^2*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(m^
3 + 15*m^2 + 74*m + 120)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limit
```

**maple [A]** time = 0.00, size = 95, normalized size = 0.98

$$\frac{\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+9 b^{2} m\, x^{2}+a^{2} m^{2}+20 a b m x+20 b^{2} x^{2}+11 a^{2} m+48 a b x+30 a^{2}\right) \left(c\, x^{2}\right)^{\frac{3}{2}} x \left(d x\right)^{m}}{\left(m+6\right) \left(m+5\right) \left(m+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+9*b^2*m*x^2+a^2*m^2+20*a*b*m*x+20*b^2*x^2+11*a^2
*m+48*a*b*x+30*a^2)*(d*x)^m*(c*x^2)^(3/2)/(m+6)/(m+5)/(m+4)
```

**maxima [A]** time = 1.55, size = 64, normalized size = 0.66

$$\frac{b^{2} c^{\frac{3}{2}} d^{m} x^{6} x^{m}}{m+6}+\frac{2 \, a b c^{\frac{3}{2}} d^{m} x^{5} x^{m}}{m+5}+\frac{a^{2} c^{\frac{3}{2}} d^{m} x^{4} x^{m}}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(3/2)*d^m*x^6*x^m/(m + 6) + 2*a*b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a^2*c
^(3/2)*d^m*x^4*x^m/(m + 4)
```

**mupad [B]** time = 0.28, size = 121, normalized size = 1.25

$$\left(d\,x\right)^m\,\left(\frac{a^2\,c\,x^3\,\sqrt{c\,x^2}\,\left(m^2+11\,m+30\right)}{m^3+15\,m^2+74\,m+120}+\frac{b^2\,c\,x^5\,\sqrt{c\,x^2}\,\left(m^2+9\,m+20\right)}{m^3+15\,m^2+74\,m+120}+\frac{2\,a\,b\,c\,x^4\,\sqrt{c\,x^2}\,\left(m^2+10\,m+24\right)}{m^3+15\,m^2+74\,m+120}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*c*x^3*(c*x^2)^(1/2)*(11*m + m^2 + 30))/(74*m + 15*m^2 + m^3 +
 120) + (b^2*c*x^5*(c*x^2)^(1/2)*(9*m + m^2 + 20))/(74*m + 15*m^2 + m^3 + 1
20) + (2*a*b*c*x^4*(c*x^2)^(1/2)*(10*m + m^2 + 24))/(74*m + 15*m^2 + m^3 +
120))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a^2(cx^2)^{\frac{3}{2}}}{x^6}\,dx+\int \frac{b^2(cx^2)^{\frac{3}{2}}}{x^4}\,dx+\int \frac{2ab(cx^2)^{\frac{3}{2}}}{x^5}\,dx}{d^6} & \text{for } m=-6 \\ \frac{\int \frac{a^2(cx^2)^{\frac{3}{2}}}{x^5}\,dx+\int \frac{b^2(cx^2)^{\frac{3}{2}}}{x^3}\,dx+\int \frac{2ab(cx^2)^{\frac{3}{2}}}{x^4}\,dx}{d^5} & \text{for } m=-5 \\ \frac{\int \frac{a^2(cx^2)^{\frac{3}{2}}}{x^4}\,dx+\int \frac{b^2(cx^2)^{\frac{3}{2}}}{x^2}\,dx+\int \frac{2ab(cx^2)^{\frac{3}{2}}}{x^3}\,dx}{d^4} & \text{for } m=-4 \\ \frac{a^2c^{\frac{3}{2}}d^m m^2 x x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120}+\frac{11a^2c^{\frac{3}{2}}d^m m x x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120}+\frac{30a^2c^{\frac{3}{2}}d^m x x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120}+\frac{2abc^{\frac{3}{2}}d^m m^2 x^2 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120}+\frac{20abc^{\frac{3}{2}}d^m m x^2 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120}+\frac{48abc^{\frac{3}{2}}d^m x^2 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120}+\frac{b^2c^{\frac{3}{2}}d^m m^2 x^3 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120}+\frac{9b^2c^{\frac{3}{2}}d^m m x^3 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120}+\frac{20b^2c^{\frac{3}{2}}d^m x^3 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise(((Integral(a**2*(c*x**2)**(3/2)/x**6, x) + Integral(b**2*(c*x**2)
**(3/2)/x**4, x) + Integral(2*a*b*(c*x**2)**(3/2)/x**5, x))/d**6, Eq(m, -6)
), ((Integral(a**2*(c*x**2)**(3/2)/x**5, x) + Integral(b**2*(c*x**2)**(3/2)
/x**3, x) + Integral(2*a*b*(c*x**2)**(3/2)/x**4, x))/d**5, Eq(m, -5)), ((In
tegral(a**2*(c*x**2)**(3/2)/x**4, x) + Integral(b**2*(c*x**2)**(3/2)/x**2,
x) + Integral(2*a*b*(c*x**2)**(3/2)/x**3, x))/d**4, Eq(m, -4)), (a**2*c**(3
/2)*d**m*m**2*x*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 120) + 11*a**2*
c**(3/2)*d**m*m*x*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 120) + 30*a**
2*c**(3/2)*d**m*x*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 120) + 2*a*b*
c**(3/2)*d**m*m**2*x**2*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 120) +
20*a*b*c**(3/2)*d**m*m*x**2*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 120
) + 48*a*b*c**(3/2)*d**m*x**2*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 1
20) + b**2*c**(3/2)*d**m*m**2*x**3*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*
m + 120) + 9*b**2*c**(3/2)*d**m*m*x**3*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 +
 74*m + 120) + 20*b**2*c**(3/2)*d**m*x**3*x**m*(x**2)**(3/2)/(m**3 + 15*m**
2 + 74*m + 120), True))
```

## 3.916 $\int (dx)^m \sqrt{cx^2} (a + bx)^2 \, dx$

**Optimal**. Leaf size=94

$$\frac{a^2\sqrt{cx^2}\,(dx)^{m+2}}{d^2(m+2)x} + \frac{2ab\sqrt{cx^2}\,(dx)^{m+3}}{d^3(m+3)x} + \frac{b^2\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}$$

**Rubi** [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2\sqrt{cx^2}\,(dx)^{m+2}}{d^2(m+2)x} + \frac{2ab\sqrt{cx^2}\,(dx)^{m+3}}{d^3(m+3)x} + \frac{b^2\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (2*a*b*(d*x)^(3 + m)*Sqrt
[c*x^2])/(d^3*(3 + m)*x) + (b^2*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m\sqrt{cx^2}\,(a+bx)^2\,dx = \frac{\sqrt{cx^2}\int x(dx)^m(a+bx)^2\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int (dx)^{1+m}(a+bx)^2\,dx}{dx}$$
$$= \frac{\sqrt{cx^2}\int \left(a^2(dx)^{1+m}+\frac{2ab(dx)^{2+m}}{d}+\frac{b^2(dx)^{3+m}}{d^2}\right)dx}{dx}$$
$$= \frac{a^2(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x}+\frac{2ab(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}+\frac{b^2(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}$$

**Mathematica [A]** time = 0.04, size = 72, normalized size = 0.77

$$\frac{x\sqrt{cx^2}\,(dx)^m\left(a^2\left(m^2+7m+12\right)+2ab\left(m^2+6m+8\right)x+b^2\left(m^2+5m+6\right)x^2\right)}{(m+2)(m+3)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a^2*(12 + 7*m + m^2) + 2*a*b*(8 + 6*m + m^2)*x + b^
2*(6 + 5*m + m^2)*x^2))/((2 + m)*(3 + m)*(4 + m))
```

**IntegrateAlgebraic [F]** time = 0.55, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{cx^2}\,(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2, x]
```

**fricas [A]** time = 1.55, size = 94, normalized size = 1.00

$$\frac{\left(\left(b^2m^2+5\,b^2m+6\,b^2\right)x^3+2\left(abm^2+6\,abm+8\,ab\right)x^2+\left(a^2m^2+7\,a^2m+12\,a^2\right)x\right)\sqrt{cx^2}\;(dx)^m}{m^3+9\,m^2+26\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 5*b^2*m + 6*b^2)*x^3 + 2*(a*b*m^2 + 6*a*b*m + 8*a*b)*x^2 + (a^2
*m^2 + 7*a^2*m + 12*a^2)*x)*sqrt(c*x^2)*(d*x)^m/(m^3 + 9*m^2 + 26*m + 24)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limit
```

**maple [A]** time = 0.00, size = 95, normalized size = 1.01

$$\frac{\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+5 b^{2} m\, x^{2}+a^{2} m^{2}+12 a b m x+6 b^{2} x^{2}+7 a^{2} m+16 a b x+12 a^{2}\right) \sqrt{c\, x^{2}}\, x \left(d x\right)^{m}}{\left(m+4\right) \left(m+3\right) \left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+5*b^2*m*x^2+a^2*m^2+12*a*b*m*x+6*b^2*x^2+7*a^2*m
+16*a*b*x+12*a^2)*(d*x)^m*(c*x^2)^(1/2)/(m+4)/(m+3)/(m+2)
```

**maxima [A]** time = 1.52, size = 64, normalized size = 0.68

$$\frac{b^{2} \sqrt{c} d^{m} x^{4} x^{m}}{m+4}+\frac{2 \, a b \sqrt{c} d^{m} x^{3} x^{m}}{m+3}+\frac{a^{2} \sqrt{c} d^{m} x^{2} x^{m}}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*sqrt(c)*d^m*x^4*x^m/(m + 4) + 2*a*b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a^2*s
qrt(c)*d^m*x^2*x^m/(m + 2)
```

**mupad [B]** time = 0.26, size = 116, normalized size = 1.23

$$(d\,x)^m\,\left(\frac{a^2\,x\,\sqrt{c\,x^2}\,\left(m^2+7\,m+12\right)}{m^3+9\,m^2+26\,m+24}+\frac{b^2\,x^3\,\sqrt{c\,x^2}\,\left(m^2+5\,m+6\right)}{m^3+9\,m^2+26\,m+24}+\frac{2\,a\,b\,x^2\,\sqrt{c\,x^2}\,\left(m^2+6\,m+8\right)}{m^3+9\,m^2+26\,m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*x*(c*x^2)^(1/2)*(7*m + m^2 + 12))/(26*m + 9*m^2 + m^3 + 24) +
 (b^2*x^3*(c*x^2)^(1/2)*(5*m + m^2 + 6))/(26*m + 9*m^2 + m^3 + 24) + (2*a*b
*x^2*(c*x^2)^(1/2)*(6*m + m^2 + 8))/(26*m + 9*m^2 + m^3 + 24))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a^{2}\sqrt{cx^{2}}}{x^{4}}\,dx + \int \frac{b^{2}\sqrt{cx^{2}}}{x^{2}}\,dx + \int \frac{2ab\sqrt{cx^{2}}}{x^{3}}\,dx}{d^{4}} & \text{for}\: m = -4 \\ \frac{\int \frac{a^{2}\sqrt{cx^{2}}}{x^{3}}\,dx + \int \frac{b^{2}\sqrt{cx^{2}}}{x}\,dx + \int \frac{2ab\sqrt{cx^{2}}}{x^{2}}\,dx}{d^{3}} & \text{for}\: m = -3 \\ \frac{\int b^{2}\sqrt{cx^{2}}\,dx + \int \frac{a^{2}\sqrt{cx^{2}}}{x^{2}}\,dx + \int \frac{2ab\sqrt{cx^{2}}}{x}\,dx}{d^{2}} & \text{for}\: m = -2 \\ \frac{a^{2}\sqrt{c}d^{m}m^{2}xx^{m}\sqrt{x^{2}}}{m^{3}+9m^{2}+26m+24} + \frac{7a^{2}\sqrt{c}d^{m}mxx^{m}\sqrt{x^{2}}}{m^{3}+9m^{2}+26m+24} + \frac{12a^{2}\sqrt{c}d^{m}xx^{m}\sqrt{x^{2}}}{m^{3}+9m^{2}+26m+24} + \frac{2ab\sqrt{c}d^{m}m^{2}x^{2}x^{m}\sqrt{x^{2}}}{m^{3}+9m^{2}+26m+24} + \frac{12ab\sqrt{c}d^{m}mx^{2}x^{m}\sqrt{x^{2}}}{m^{3}+9m^{2}+26m+24} + \frac{16ab\sqrt{c}d^{m}x^{2}x^{m}\sqrt{x^{2}}}{m^{3}+9m^{2}+26m+24} + \frac{b^{2}\sqrt{c}d^{m}m^{2}x^{3}x^{m}\sqrt{x^{2}}}{m^{3}+9m^{2}+26m+24} + \frac{5b^{2}\sqrt{c}d^{m}mx^{3}x^{m}\sqrt{x^{2}}}{m^{3}+9m^{2}+26m+24} + \frac{6b^{2}\sqrt{c}d^{m}x^{3}x^{m}\sqrt{x^{2}}}{m^{3}+9m^{2}+26m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise(((Integral(a**2*sqrt(c*x**2)/x**4, x) + Integral(b**2*sqrt(c*x**2
)/x**2, x) + Integral(2*a*b*sqrt(c*x**2)/x**3, x))/d**4, Eq(m, -4)), ((Inte
gral(a**2*sqrt(c*x**2)/x**3, x) + Integral(b**2*sqrt(c*x**2)/x, x) + Integr
al(2*a*b*sqrt(c*x**2)/x**2, x))/d**3, Eq(m, -3)), ((Integral(b**2*sqrt(c*x*
*2), x) + Integral(a**2*sqrt(c*x**2)/x**2, x) + Integral(2*a*b*sqrt(c*x**2)
/x, x))/d**2, Eq(m, -2)), (a**2*sqrt(c)*d**m*m**2*x*x**m*sqrt(x**2)/(m**3 +
 9*m**2 + 26*m + 24) + 7*a**2*sqrt(c)*d**m*m*x*x**m*sqrt(x**2)/(m**3 + 9*m*
*2 + 26*m + 24) + 12*a**2*sqrt(c)*d**m*x*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 2
6*m + 24) + 2*a*b*sqrt(c)*d**m*m**2*x**2*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 2
6*m + 24) + 12*a*b*sqrt(c)*d**m*m*x**2*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 26*
m + 24) + 16*a*b*sqrt(c)*d**m*x**2*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 26*m +
24) + b**2*sqrt(c)*d**m*m**2*x**3*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 26*m + 2
4) + 5*b**2*sqrt(c)*d**m*m*x**3*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 26*m + 24)
 + 6*b**2*sqrt(c)*d**m*x**3*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 26*m + 24), Tr
ue))
```

## 3.917 $\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx$

**Optimal**. Leaf size=81

$$\frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}+\frac{b^2x(dx)^{m+2}}{d^2(m+2)\sqrt{cx^2}}$$

**Rubi** [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}+\frac{b^2x(dx)^{m+2}}{d^2(m+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (2*a*b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c
*x^2]) + (b^2*x*(d*x)^(2 + m))/(d^2*(2 + m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^2}{x}\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int (dx)^{-1+m}(a+bx)^2\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int \left(a^2(dx)^{-1+m} + \frac{2ab(dx)^m}{d} + \frac{b^2(dx)^{1+m}}{d^2}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{a^2x(dx)^m}{m\sqrt{cx^2}} + \frac{2abx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}} + \frac{b^2x(dx)^{2+m}}{d^2(2+m)\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.03, size = 62, normalized size = 0.77

$$\frac{x(dx)^m\left(a^2\left(m^2+3m+2\right)+2abm(m+2)x+b^2m(m+1)x^2\right)}{m(m+1)(m+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x*(d*x)^m*(a^2*(2 + 3*m + m^2) + 2*a*b*m*(2 + m)*x + b^2*m*(1 + m)*x^2))/(
m*(1 + m)*(2 + m)*Sqrt[c*x^2])
```

**IntegrateAlgebraic [F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2], x]
```

**fricas [A]** time = 1.18, size = 85, normalized size = 1.05

$$\frac{\left(a^2m^2+3\,a^2m+\left(b^2m^2+b^2m\right)x^2+2\,a^2+2\left(abm^2+2\,abm\right)x\right)\sqrt{cx^2}\,(dx)^m}{\left(cm^3+3\,cm^2+2\,cm\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 + 3*a^2*m + (b^2*m^2 + b^2*m)*x^2 + 2*a^2 + 2*(a*b*m^2 + 2*a*b*m)*
x)*sqrt(c*x^2)*(d*x)^m/((c*m^3 + 3*c*m^2 + 2*c*m)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2 (dx)^m}{\sqrt{cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(d*x)^m/sqrt(c*x^2), x)
```

**maple [A]** time = 0.00, size = 79, normalized size = 0.98

$$\frac{\left(b^2m^2x^2+2ab\,m^2x+b^2m\,x^2+a^2m^2+4abmx+3a^2m+2a^2\right)x\,(dx)^m}{(m+2)\,(m+1)\,\sqrt{c\,x^2}\;m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+b^2*m*x^2+a^2*m^2+4*a*b*m*x+3*a^2*m+2*a^2)*(d*x)
^m/(m+2)/(m+1)/m/(c*x^2)^(1/2)
```

**maxima [A]** time = 1.64, size = 57, normalized size = 0.70

$$\frac{b^2d^mx^2x^m}{\sqrt{c}\,(m+2)}+\frac{2\,abd^mxx^m}{\sqrt{c}\,(m+1)}+\frac{a^2d^mx^m}{\sqrt{c}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^2*x^m/(sqrt(c)*(m + 2)) + 2*a*b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a^2
*d^m*x^m/(sqrt(c)*m)
```

**mupad [B]** time = 0.26, size = 62, normalized size = 0.77

$$\frac{(d\,x)^m\,\left(\frac{a^2\,x}{m}+\frac{b^2\,x^3\,(m+1)}{m^2+3\,m+2}+\frac{2\,a\,b\,x^2\,(m+2)}{m^2+3\,m+2}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] ((d*x)^m*((a^2*x)/m + (b^2*x^3*(m + 1))/(3*m + m^2 + 2) + (2*a*b*x^2*(m + 2
))/(3*m + m^2 + 2)))/(c*x^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{b^2}{\sqrt{cx^2}}\,dx+\int \frac{a^2}{x^2\sqrt{cx^2}}\,dx+\int \frac{2ab}{x\sqrt{cx^2}}\,dx}{d^2} & \text{for } m=-2 \\ \frac{\int \frac{2ab}{\sqrt{cx^2}}\,dx+\int \frac{a^2}{x\sqrt{cx^2}}\,dx+\int \frac{b^2x}{\sqrt{cx^2}}\,dx}{d} & \text{for } m=-1 \\ \int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx & \text{for } m=0 \\ \frac{a^2d^mm^2xx^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{3a^2d^mmxx^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{2a^2d^mxx^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{2abd^mm^2x^2x^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{4abd^mmx^2x^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{b^2d^mm^2x^3x^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{b^2d^mmx^3x^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise(((Integral(b**2/sqrt(c*x**2), x) + Integral(a**2/(x**2*sqrt(c*x**
2)), x) + Integral(2*a*b/(x*sqrt(c*x**2)), x))/d**2, Eq(m, -2)), ((Integral
(2*a*b/sqrt(c*x**2), x) + Integral(a**2/(x*sqrt(c*x**2)), x) + Integral(b**
2*x/sqrt(c*x**2), x))/d, Eq(m, -1)), (Integral((a + b*x)**2/sqrt(c*x**2), x
), Eq(m, 0)), (a**2*d**m*m**2*x*x**m/(sqrt(c)*m**3*sqrt(x**2) + 3*sqrt(c)*m
**2*sqrt(x**2) + 2*sqrt(c)*m*sqrt(x**2)) + 3*a**2*d**m*m*x*x**m/(sqrt(c)*m*
*3*sqrt(x**2) + 3*sqrt(c)*m**2*sqrt(x**2) + 2*sqrt(c)*m*sqrt(x**2)) + 2*a**
2*d**m*x*x**m/(sqrt(c)*m**3*sqrt(x**2) + 3*sqrt(c)*m**2*sqrt(x**2) + 2*sqrt
(c)*m*sqrt(x**2)) + 2*a*b*d**m*m**2*x**2*x**m/(sqrt(c)*m**3*sqrt(x**2) + 3*
sqrt(c)*m**2*sqrt(x**2) + 2*sqrt(c)*m*sqrt(x**2)) + 4*a*b*d**m*m*x**2*x**m/
(sqrt(c)*m**3*sqrt(x**2) + 3*sqrt(c)*m**2*sqrt(x**2) + 2*sqrt(c)*m*sqrt(x**
2)) + b**2*d**m*m**2*x**3*x**m/(sqrt(c)*m**3*sqrt(x**2) + 3*sqrt(c)*m**2*sq
rt(x**2) + 2*sqrt(c)*m*sqrt(x**2)) + b**2*d**m*m*x**3*x**m/(sqrt(c)*m**3*sq
rt(x**2) + 3*sqrt(c)*m**2*sqrt(x**2) + 2*sqrt(c)*m*sqrt(x**2)), True))
```

## 3.918 $\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=93

$$-\frac{a^2d^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

**Rubi** [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$-\frac{a^2d^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] -((a^2*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (2*a*b*d*x*(d*x)^(-
1 + m))/(c*(1 - m)*Sqrt[c*x^2]) + (b^2*x*(d*x)^m)/(c*m*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^2}{x^3}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{\left(d^3x\right)\int (dx)^{-3+m}(a+bx)^2\,dx}{c\sqrt{cx^2}}$$
$$= \frac{\left(d^3x\right)\int \left(a^2(dx)^{-3+m}+\frac{2ab(dx)^{-2+m}}{d}+\frac{b^2(dx)^{-1+m}}{d^2}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2d^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

**Mathematica** [A] time = 0.04, size = 62, normalized size = 0.67

$$\frac{x(dx)^m\left(a^2(m-1)m+2ab(m-2)mx+b^2\left(m^2-3m+2\right)x^2\right)}{(m-2)(m-1)m\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a^2*(-1 + m)*m + 2*a*b*(-2 + m)*m*x + b^2*(2 - 3*m + m^2)*x^2))
/((-2 + m)*(-1 + m)*m*(c*x^2)^(3/2))
```

**IntegrateAlgebraic** [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2), x]
```

**fricas** [A] time = 1.31, size = 92, normalized size = 0.99

$$\frac{\left(a^2m^2-a^2m+\left(b^2m^2-3\,b^2m+2\,b^2\right)x^2+2\left(abm^2-2\,abm\right)x\right)\sqrt{cx^2}\;(dx)^m}{\left(c^2m^3-3\,c^2m^2+2\,c^2m\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 - a^2*m + (b^2*m^2 - 3*b^2*m + 2*b^2)*x^2 + 2*(a*b*m^2 - 2*a*b*m)*
x)*sqrt(c*x^2)*(d*x)^m/((c^2*m^3 - 3*c^2*m^2 + 2*c^2*m)*x^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2 (dx)^m}{(cx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(d*x)^m/(c*x^2)^(3/2), x)
```

**maple [A]** time = 0.01, size = 83, normalized size = 0.89

$$\frac{\left(b^2m^2x^2 + 2ab\,m^2x - 3b^2m\,x^2 + a^2m^2 - 4abmx + 2b^2x^2 - a^2m\right) x\,(dx)^m}{(m-1)\,(m-2)\left(c\,x^2\right)^{\frac{3}{2}}\, m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x-3*b^2*m*x^2+a^2*m^2-4*a*b*m*x+2*b^2*x^2-a^2*m)*(
d*x)^m/m/(m-1)/(m-2)/(c*x^2)^(3/2)
```

**maxima [A]** time = 1.58, size = 59, normalized size = 0.63

$$\frac{b^2d^mx^m}{c^{\frac{3}{2}}m} + \frac{2\,abd^mx^m}{c^{\frac{3}{2}}(m-1)x} + \frac{a^2d^mx^m}{c^{\frac{3}{2}}(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^m/(c^(3/2)*m) + 2*a*b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a^2*d^m*x^m/(
c^(3/2)*(m - 2)*x^2)
```

**mupad [B]** time = 0.32, size = 66, normalized size = 0.71

$$\frac{a^2\,(d\,x)^m}{c\,x\,\sqrt{c\,x^2}\,\,(m-2)} + \frac{b\,(d\,x)^m\,\,(2\,a\,m - b\,x + b\,m\,x)}{c\,m\,\sqrt{c\,x^2}\,\,(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] (a^2*(d*x)^m)/(c*x*(c*x^2)^(1/2)*(m - 2)) + (b*(d*x)^m*(2*a*m - b*x + b*m*x
))/(c*m*(c*x^2)^(1/2)*(m - 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \frac{(a+bx)^2}{(cx^2)^{\frac{3}{2}}}\, dx & \text{for}\ m = 0 \\ d\left(\int \frac{a^2x}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{b^2x^3}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{2abx^2}{(cx^2)^{\frac{3}{2}}}\, dx\right) & \text{for}\ m = 1 \\ d^2\left(\int \frac{a^2x^2}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{b^2x^4}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{2abx^3}{(cx^2)^{\frac{3}{2}}}\, dx\right) & \text{for}\ m = 2 \\ \frac{a^2d^mm^2xx^m}{c^{\frac{3}{2}}m^3(x^2)^{\frac{3}{2}}-3c^{\frac{3}{2}}m^2(x^2)^{\frac{3}{2}}+2c^{\frac{3}{2}}m(x^2)^{\frac{3}{2}}} - \frac{a^2d^mmxx^m}{c^{\frac{3}{2}}m^3(x^2)^{\frac{3}{2}}-3c^{\frac{3}{2}}m^2(x^2)^{\frac{3}{2}}+2c^{\frac{3}{2}}m(x^2)^{\frac{3}{2}}} + \frac{2abd^mm^2x^2x^m}{c^{\frac{3}{2}}m^3(x^2)^{\frac{3}{2}}-3c^{\frac{3}{2}}m^2(x^2)^{\frac{3}{2}}+2c^{\frac{3}{2}}m(x^2)^{\frac{3}{2}}} - \frac{4abd^mmx^2x^m}{c^{\frac{3}{2}}m^3(x^2)^{\frac{3}{2}}-3c^{\frac{3}{2}}m^2(x^2)^{\frac{3}{2}}+2c^{\frac{3}{2}}m(x^2)^{\frac{3}{2}}} + \frac{b^2d^mm^2x^3x^m}{c^{\frac{3}{2}}m^3(x^2)^{\frac{3}{2}}-3c^{\frac{3}{2}}m^2(x^2)^{\frac{3}{2}}+2c^{\frac{3}{2}}m(x^2)^{\frac{3}{2}}} - \frac{3b^2d^mmx^3x^m}{c^{\frac{3}{2}}m^3(x^2)^{\frac{3}{2}}-3c^{\frac{3}{2}}m^2(x^2)^{\frac{3}{2}}+2c^{\frac{3}{2}}m(x^2)^{\frac{3}{2}}} + \frac{2b^2d^mx^3x^m}{c^{\frac{3}{2}}m^3(x^2)^{\frac{3}{2}}-3c^{\frac{3}{2}}m^2(x^2)^{\frac{3}{2}}+2c^{\frac{3}{2}}m(x^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((Integral((a + b*x)**2/(c*x**2)**(3/2), x), Eq(m, 0)), (d*(Integr
al(a**2*x/(c*x**2)**(3/2), x) + Integral(b**2*x**3/(c*x**2)**(3/2), x) + In
tegral(2*a*b*x**2/(c*x**2)**(3/2), x)), Eq(m, 1)), (d**2*(Integral(a**2*x**
2/(c*x**2)**(3/2), x) + Integral(b**2*x**4/(c*x**2)**(3/2), x) + Integral(2
*a*b*x**3/(c*x**2)**(3/2), x)), Eq(m, 2)), (a**2*d**m*m**2*x*x**m/(c**(3/2)
*m**3*(x**2)**(3/2) - 3*c**(3/2)*m**2*(x**2)**(3/2) + 2*c**(3/2)*m*(x**2)**
(3/2)) - a**2*d**m*m*x*x**m/(c**(3/2)*m**3*(x**2)**(3/2) - 3*c**(3/2)*m**2*
(x**2)**(3/2) + 2*c**(3/2)*m*(x**2)**(3/2)) + 2*a*b*d**m*m**2*x**2*x**m/(c*
*(3/2)*m**3*(x**2)**(3/2) - 3*c**(3/2)*m**2*(x**2)**(3/2) + 2*c**(3/2)*m*(x
**2)**(3/2)) - 4*a*b*d**m*m*x**2*x**m/(c**(3/2)*m**3*(x**2)**(3/2) - 3*c**(
3/2)*m**2*(x**2)**(3/2) + 2*c**(3/2)*m*(x**2)**(3/2)) + b**2*d**m*m**2*x**3
*x**m/(c**(3/2)*m**3*(x**2)**(3/2) - 3*c**(3/2)*m**2*(x**2)**(3/2) + 2*c**(
3/2)*m*(x**2)**(3/2)) - 3*b**2*d**m*m*x**3*x**m/(c**(3/2)*m**3*(x**2)**(3/2
) - 3*c**(3/2)*m**2*(x**2)**(3/2) + 2*c**(3/2)*m*(x**2)**(3/2)) + 2*b**2*d*
*m*x**3*x**m/(c**(3/2)*m**3*(x**2)**(3/2) - 3*c**(3/2)*m**2*(x**2)**(3/2) +
 2*c**(3/2)*m*(x**2)**(3/2)), True))
```

## 3.919 $\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{5/2}} \, dx$

**Optimal**. Leaf size=105

$$-\frac{a^2d^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{m-2}}{c^2(2-m)\sqrt{cx^2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$-\frac{a^2d^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{m-2}}{c^2(2-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -((a^2*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (2*a*b*d^3*x*(d*x
)^(-3 + m))/(c^2*(3 - m)*Sqrt[c*x^2]) - (b^2*d^2*x*(d*x)^(-2 + m))/(c^2*(2
- m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^2}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{\left(d^5x\right)\int (dx)^{-5+m}(a+bx)^2\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{\left(d^5x\right)\int \left(a^2(dx)^{-5+m}+\frac{2ab(dx)^{-4+m}}{d}+\frac{b^2(dx)^{-3+m}}{d^2}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2d^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{-2+m}}{c^2(2-m)\sqrt{cx^2}}$$

**Mathematica [A]** time = 0.05, size = 72, normalized size = 0.69

$$\frac{x(dx)^m\left(a^2\left(m^2-5m+6\right)+2ab\left(m^2-6m+8\right)x+b^2\left(m^2-7m+12\right)x^2\right)}{(m-4)(m-3)(m-2)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] (x*(d*x)^m*(a^2*(6 - 5*m + m^2) + 2*a*b*(8 - 6*m + m^2)*x + b^2*(12 - 7*m +
 m^2)*x^2))/((-4 + m)*(-3 + m)*(-2 + m)*(c*x^2)^(5/2))
```

**IntegrateAlgebraic [F]** time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2), x]
```

**fricas [A]** time = 1.41, size = 106, normalized size = 1.01

$$\frac{\left(a^2m^2-5\,a^2m+\left(b^2m^2-7\,b^2m+12\,b^2\right)x^2+6\,a^2+2\left(abm^2-6\,abm+8\,ab\right)x\right)\sqrt{cx^2}\,\left(dx\right)^m}{\left(c^3m^3-9\,c^3m^2+26\,c^3m-24\,c^3\right)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 - 5*a^2*m + (b^2*m^2 - 7*b^2*m + 12*b^2)*x^2 + 6*a^2 + 2*(a*b*m^2
- 6*a*b*m + 8*a*b)*x)*sqrt(c*x^2)*(d*x)^m/((c^3*m^3 - 9*c^3*m^2 + 26*c^3*m
- 24*c^3)*x^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2 (dx)^m}{(cx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(d*x)^m/(c*x^2)^(5/2), x)
```

**maple [A]** time = 0.01, size = 95, normalized size = 0.90

$$\frac{\left(b^2m^2x^2+2ab\,m^2x-7b^2m\,x^2+a^2m^2-12abmx+12b^2x^2-5a^2m+16abx+6a^2\right)x\,(dx)^m}{(m-2)\,(m-3)\,(m-4)\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x-7*b^2*m*x^2+a^2*m^2-12*a*b*m*x+12*b^2*x^2-5*a^2*
m+16*a*b*x+6*a^2)*(d*x)^m/(m-2)/(m-3)/(m-4)/(c*x^2)^(5/2)
```

**maxima [A]** time = 1.57, size = 64, normalized size = 0.61

$$\frac{b^2d^mx^m}{c^{\frac{5}{2}}(m-2)x^2}+\frac{2\,abd^mx^m}{c^{\frac{5}{2}}(m-3)x^3}+\frac{a^2d^mx^m}{c^{\frac{5}{2}}(m-4)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^m/(c^(5/2)*(m - 2)*x^2) + 2*a*b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) + a
^2*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

**mupad [B]** time = 0.34, size = 82, normalized size = 0.78

$$\frac{a^2\,(d\,x)^m}{c^2\,x^3\,\sqrt{c\,x^2}\,\,(m-4)}+\frac{b^2\,(d\,x)^m}{c^2\,x\,\sqrt{c\,x^2}\,\,(m-2)}+\frac{2\,a\,b\,(d\,x)^m}{c^2\,x^2\,\sqrt{c\,x^2}\,\,(m-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

```
[Out] (a^2*(d*x)^m)/(c^2*x^3*(c*x^2)^(1/2)*(m - 4)) + (b^2*(d*x)^m)/(c^2*x*(c*x^2
)^(1/2)*(m - 2)) + (2*a*b*(d*x)^m)/(c^2*x^2*(c*x^2)^(1/2)*(m - 3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((d**2*(Integral(a**2*x**2/(c*x**2)**(5/2), x) + Integral(b**2*x**
4/(c*x**2)**(5/2), x) + Integral(2*a*b*x**3/(c*x**2)**(5/2), x)), Eq(m, 2))
, (d**3*(Integral(a**2*x**3/(c*x**2)**(5/2), x) + Integral(b**2*x**5/(c*x**
2)**(5/2), x) + Integral(2*a*b*x**4/(c*x**2)**(5/2), x)), Eq(m, 3)), (d**4*
(Integral(a**2*x**4/(c*x**2)**(5/2), x) + Integral(b**2*x**6/(c*x**2)**(5/2
), x) + Integral(2*a*b*x**5/(c*x**2)**(5/2), x)), Eq(m, 4)), (a**2*d**m*m**
2*x*x**m/(c**(5/2)*m**3*(x**2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*
c**(5/2)*m*(x**2)**(5/2) - 24*c**(5/2)*(x**2)**(5/2)) - 5*a**2*d**m*m*x*x**
m/(c**(5/2)*m**3*(x**2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2
)*m*(x**2)**(5/2) - 24*c**(5/2)*(x**2)**(5/2)) + 6*a**2*d**m*x*x**m/(c**(5/
2)*m**3*(x**2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2
)**(5/2) - 24*c**(5/2)*(x**2)**(5/2)) + 2*a*b*d**m*m**2*x**2*x**m/(c**(5/2)
*m**3*(x**2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)*
*(5/2) - 24*c**(5/2)*(x**2)**(5/2)) - 12*a*b*d**m*m*x**2*x**m/(c**(5/2)*m**
3*(x**2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)**(5/
2) - 24*c**(5/2)*(x**2)**(5/2)) + 16*a*b*d**m*x**2*x**m/(c**(5/2)*m**3*(x**
2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)**(5/2) - 2
4*c**(5/2)*(x**2)**(5/2)) + b**2*d**m*m**2*x**3*x**m/(c**(5/2)*m**3*(x**2)*
*(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)**(5/2) - 24*c
**(5/2)*(x**2)**(5/2)) - 7*b**2*d**m*m*x**3*x**m/(c**(5/2)*m**3*(x**2)**(5/
2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)**(5/2) - 24*c**(5
/2)*(x**2)**(5/2)) + 12*b**2*d**m*x**3*x**m/(c**(5/2)*m**3*(x**2)**(5/2) -
9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)**(5/2) - 24*c**(5/2)*(
x**2)**(5/2)), True))
```

## 3.920 $\int x^3 \left(cx^2\right)^p (a+bx)^{-5-2p}\,dx$

**Optimal**. Leaf size=33

$$\frac{x^4 \left(cx^2\right)^p (a+bx)^{-2(p+2)}}{2a(p+2)}$$

**Rubi** [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$\frac{x^4 \left(cx^2\right)^p (a+bx)^{-2(p+2)}}{2a(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
```

```
[Out] (x^4*(c*x^2)^p)/(2*a*(2 + p)*(a + b*x)^(2*(2 + p)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(cx^2\right)^p (a+bx)^{-5-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{3+2p}(a+bx)^{-5-2p}\,dx\\ &= \frac{x^4 \left(cx^2\right)^p (a+bx)^{-2(2+p)}}{2a(2+p)}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 32, normalized size = 0.97

$$\frac{x^4 \left(cx^2\right)^p (a+bx)^{-2p-4}}{a(2p+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
[Out] (x^4*(c*x^2)^p*(a + b*x)^(-4 - 2*p))/(a*(4 + 2*p))
```

**IntegrateAlgebraic** [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x^3 \left(cx^2\right)^p (a+bx)^{-5-2p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
[Out] Defer[IntegrateAlgebraic][x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p), x]
```

**fricas** [A] time = 1.22, size = 40, normalized size = 1.21

$$\frac{\left(bx^5+ax^4\right)\left(cx^2\right)^p (bx+a)^{-2\,p-5}}{2\left(ap+2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="fricas")
[Out] 1/2*(b*x^5 + a*x^4)*(c*x^2)^p*(b*x + a)^(-2*p - 5)/(a*p + 2*a)
```

**giac** [B] time = 1.26, size = 74, normalized size = 2.24

$$\frac{\left(cx^2\right)^p bx^5 e^{\left(-2\,p\log(bx+a)-5\,\log(bx+a)\right)}+\left(cx^2\right)^p ax^4 e^{\left(-2\,p\log(bx+a)-5\,\log(bx+a)\right)}}{2\left(ap+2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="giac")
[Out] 1/2*((c*x^2)^p*b*x^5*e^(-2*p*log(b*x + a) - 5*log(b*x + a)) + (c*x^2)^p*a*x
^4*e^(-2*p*log(b*x + a) - 5*log(b*x + a)))/(a*p + 2*a)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.97

$$\frac{x^4 \left(c\,x^2\right)^p (bx+a)^{-2p-4}}{2\left(p+2\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x)
[Out] 1/2*(b*x+a)^(-4-2*p)*x^4/a/(2+p)*(c*x^2)^p
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (bx+a)^{-2\,p-5} x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 5)*x^3, x)
```

**mupad** [B] time = 0.27, size = 33, normalized size = 1.00

$$\frac{x^4 \left(c\,x^2\right)^p}{2\,a\,\left(p+2\right)\,(a+b\,x)^{2\,p+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c*x^2)^p)/(a + b*x)^(2*p + 5),x)
[Out] (x^4*(c*x^2)^p)/(2*a*(p + 2)*(a + b*x)^(2*p + 4))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**p*(b*x+a)**(-5-2*p),x)
[Out] Timed out
```

## 3.921 $\int x^2 \left(cx^2\right)^p (a+bx)^{-4-2p}\,dx$

**Optimal**. Leaf size=32

$$\frac{x^3 \left(cx^2\right)^p (a+bx)^{-2p-3}}{a(2p+3)}$$

**Rubi [A]** time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$\frac{x^3 \left(cx^2\right)^p (a+bx)^{-2p-3}}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]
```

```
[Out] (x^3*(c*x^2)^p*(a + b*x)^(-3 - 2*p))/(a*(3 + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(cx^2\right)^p (a+bx)^{-4-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2+2p}(a+bx)^{-4-2p}\,dx\\ &= \frac{x^3\left(cx^2\right)^p (a+bx)^{-3-2p}}{a(3+2p)}\end{aligned}$$

**Mathematica** [A]   time = 0.02, size = 34, normalized size = 1.06

$$\frac{x^3\left(cx^2\right)^p(a+bx)^{1-2(p+2)}}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]
[Out] (x^3*(c*x^2)^p*(a + b*x)^(1 - 2*(2 + p)))/(a*(3 + 2*p))
```

**IntegrateAlgebraic** [F]   time = 0.08, size = 0, normalized size = 0.00

$$\int x^2\left(cx^2\right)^p(a+bx)^{-4-2p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]
[Out] Defer[IntegrateAlgebraic][x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p), x]
```

**fricas** [A]   time = 1.23, size = 40, normalized size = 1.25

$$\frac{\left(bx^4+ax^3\right)\left(cx^2\right)^p(bx+a)^{-2p-4}}{2ap+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="fricas")
[Out] (b*x^4 + a*x^3)*(c*x^2)^p*(b*x + a)^(-2*p - 4)/(2*a*p + 3*a)
```

**giac** [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^2\right)^p(bx+a)^{-2p-4}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 4)*x^2, x)
```

**maple** [A]   time = 0.00, size = 33, normalized size = 1.03

$$\frac{x^3\left(c\,x^2\right)^p(bx+a)^{-2p-3}}{\left(2p+3\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^p*(b*x+a)^(-2*p-4),x)
[Out] x^3*(c*x^2)^p*(b*x+a)^(-3-2*p)/a/(3+2*p)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (bx+a)^{-2p-4} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 4)*x^2, x)
```

**mupad [B]** time = 0.24, size = 34, normalized size = 1.06

$$\frac{x^3 \left(c\,x^2\right)^p}{a\,\left(2\,p+3\right)\,{\left(a+b\,x\right)}^{2\,p+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^p)/(a + b*x)^(2*p + 4),x)
[Out] (x^3*(c*x^2)^p)/(a*(2*p + 3)*(a + b*x)^(2*p + 3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**p*(b*x+a)**(-4-2*p),x)
[Out] Timed out
```

## 3.922 $\int x \left(cx^2\right)^p (a+bx)^{-3-2p} \, dx$

**Optimal**. Leaf size=33

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2(p+1)}}{2a(p+1)}$$

**Rubi** [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 37}

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2(p+1)}}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]
```

```
[Out] (x^2*(c*x^2)^p)/(2*a*(1 + p)*(a + b*x)^(2*(1 + p)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x \left(cx^2\right)^p (a+bx)^{-3-2p} \, dx &= \left(x^{-2p} \left(cx^2\right)^p\right) \int x^{1+2p}(a+bx)^{-3-2p} \, dx\\ &= \frac{x^2 \left(cx^2\right)^p (a+bx)^{-2(1+p)}}{2a(1+p)}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 32, normalized size = 0.97

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2p-2}}{a(2p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]
[Out] (x^2*(c*x^2)^p*(a + b*x)^(-2 - 2*p))/(a*(2 + 2*p))
```

**IntegrateAlgebraic** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x \left(cx^2\right)^p (a+bx)^{-3-2p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]
[Out] Defer[IntegrateAlgebraic][x*(c*x^2)^p*(a + b*x)^(-3 - 2*p), x]
```

**fricas** [A] time = 1.51, size = 38, normalized size = 1.15

$$\frac{\left(bx^3+ax^2\right)\left(cx^2\right)^p (bx+a)^{-2p-3}}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="fricas")
[Out] 1/2*(b*x^3 + a*x^2)*(c*x^2)^p*(b*x + a)^(-2*p - 3)/(a*p + a)
```

**giac** [B] time = 1.07, size = 72, normalized size = 2.18

$$\frac{\left(cx^2\right)^p bx^3 e^{\left(-2\,p\log(bx+a)-3\,\log(bx+a)\right)}+\left(cx^2\right)^p ax^2 e^{\left(-2\,p\log(bx+a)-3\,\log(bx+a)\right)}}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="giac")
[Out] 1/2*((c*x^2)^p*b*x^3*e^(-2*p*log(b*x + a) - 3*log(b*x + a)) + (c*x^2)^p*a*x
^2*e^(-2*p*log(b*x + a) - 3*log(b*x + a)))/(a*p + a)
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.97

$$\frac{x^{2} \left(c\,x^{2}\right)^{p} \left(bx+a\right)^{-2p-2}}{2 \left(p+1\right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^p*(b*x+a)^(-2*p-3),x)
[Out] 1/2*(b*x+a)^(-2-2*p)*x^2/a/(1+p)*(c*x^2)^p
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2}\right)^{p} \left(bx+a\right)^{-2\,p-3} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 3)*x, x)
```

**mupad [B]** time = 0.22, size = 33, normalized size = 1.00

$$\frac{x^{2} \left(c\,x^{2}\right)^{p}}{2\,a\,\left(p+1\right)\,\left(a+b\,x\right)^{2\,p+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^p)/(a + b*x)^(2*p + 3),x)
[Out] (x^2*(c*x^2)^p)/(2*a*(p + 1)*(a + b*x)^(2*p + 2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**p*(b*x+a)**(-3-2*p),x)
[Out] Timed out
```

## 3.923 $\int \left(cx^2\right)^p (a+bx)^{-2-2p}\,dx$

**Optimal**. Leaf size=30

$$\frac{x\left(cx^2\right)^p (a+bx)^{-2p-1}}{a(2p+1)}$$

**Rubi [A]** time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {15, 37}

$$\frac{x\left(cx^2\right)^p (a+bx)^{-2p-1}}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]
```

```
[Out] (x*(c*x^2)^p*(a + b*x)^(-1 - 2*p))/(a*(1 + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^p (a+bx)^{-2-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(a+bx)^{-2-2p}\,dx\\ &= \frac{x\left(cx^2\right)^p (a+bx)^{-1-2p}}{a(1+2p)}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 28, normalized size = 0.93

$$\frac{x\left(cx^2\right)^p(a+bx)^{-2p-1}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]
[Out] (x*(c*x^2)^p*(a + b*x)^(-1 - 2*p))/(a + 2*a*p)
```

**IntegrateAlgebraic** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p(a+bx)^{-2-2p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]
[Out] Defer[IntegrateAlgebraic][(c*x^2)^p*(a + b*x)^(-2 - 2*p), x]
```

**fricas** [A] time = 1.37, size = 36, normalized size = 1.20

$$\frac{\left(bx^2+ax\right)\left(cx^2\right)^p(bx+a)^{-2\,p-2}}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="fricas")
[Out] (b*x^2 + a*x)*(c*x^2)^p*(b*x + a)^(-2*p - 2)/(2*a*p + a)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p(bx+a)^{-2\,p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 2), x)
```

**maple** [A] time = 0.00, size = 31, normalized size = 1.03

$$\frac{x\left(c\,x^2\right)^p(bx+a)^{-2p-1}}{\left(2p+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-2*p-2),x)
```

```
[Out] x*(c*x^2)^p*(b*x+a)^(-1-2*p)/a/(1+2*p)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (bx+a)^{-2p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 2), x)
```

**mupad [B]** time = 0.20, size = 32, normalized size = 1.07

$$\frac{x\left(c\,x^2\right)^p}{a\,\left(2\,p+1\right)\,\left(a+b\,x\right)^{2\,p+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/(a + b*x)^(2*p + 2),x)
```

```
[Out] (x*(c*x^2)^p)/(a*(2*p + 1)*(a + b*x)^(2*p + 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{b^{-2p}c^p x^{-2p}\left(x^2\right)^p}{b^2x} & \text{for } a=0 \\ \frac{0^{-2p-2}c^p x\left(x^2\right)^p}{2p+1} & \text{for } a=-bx \\ \frac{c^p x\left(0^{\frac{1}{p}}\right)^{-2p-2}\left(x^2\right)^p}{2p+1} & \text{for } a=0^{\frac{1}{p}}-bx \\ \int \frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx & \text{for } p=-\frac{1}{2} \\ \frac{a^3c^px\left(x^2\right)^p}{2a^5p(a+bx)^{2p}+a^5(a+bx)^{2p}+8a^4bpx(a+bx)^{2p}+4a^4bx(a+bx)^{2p}+12a^3b^2px^2(a+bx)^{2p}+6a^3b^2x^2(a+bx)^{2p}+8a^2b^3px^3(a+bx)^{2p}+4a^2b^3x^3(a+bx)^{2p}+2ab^4px^4(a+bx)^{2p}+ab^4x^4(a+bx)^{2p}} + \frac{2a^2bc^px^2\left(x^2\right)^p}{2a^5p(a+bx)^{2p}+a^5(a+bx)^{2p}+8a^4bpx(a+bx)^{2p}+4a^4bx(a+bx)^{2p}+12a^3b^2px^2(a+bx)^{2p}+6a^3b^2x^2(a+bx)^{2p}+8a^2b^3px^3(a+bx)^{2p}+4a^2b^3x^3(a+bx)^{2p}+2ab^4px^4(a+bx)^{2p}+ab^4x^4(a+bx)^{2p}} + \frac{ab^2c^px^3\left(x^2\right)^p}{2a^5p(a+bx)^{2p}+a^5(a+bx)^{2p}+8a^4bpx(a+bx)^{2p}+4a^4bx(a+bx)^{2p}+12a^3b^2px^2(a+bx)^{2p}+6a^3b^2x^2(a+bx)^{2p}+8a^2b^3px^3(a+bx)^{2p}+4a^2b^3x^3(a+bx)^{2p}+2ab^4px^4(a+bx)^{2p}+ab^4x^4(a+bx)^{2p}} + \frac{bc^px^2\left(x^2\right)^p}{2a^3p(a+bx)^{2p}+a^3(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2a^2bx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}+ab^2x^2(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(-2-2*p),x)
```

```
[Out] Piecewise((-b**(-2*p)*c**p*x**(-2*p)*(x**2)**p/(b**2*x), Eq(a, 0)), (0**(-2
*p - 2)*c**p*x*(x**2)**p/(2*p + 1), Eq(a, -b*x)), (c**p*x*(0**(1/p))**(-2*p
 - 2)*(x**2)**p/(2*p + 1), Eq(a, 0**(1/p) - b*x)), (Integral(1/(sqrt(c*x**2
)*(a + b*x)), x), Eq(p, -1/2)), (a**3*c**p*x*(x**2)**p/(2*a**5*p*(a + b*x)*
*(2*p) + a**5*(a + b*x)**(2*p) + 8*a**4*b*p*x*(a + b*x)**(2*p) + 4*a**4*b*x
*(a + b*x)**(2*p) + 12*a**3*b**2*p*x**2*(a + b*x)**(2*p) + 6*a**3*b**2*x**2
*(a + b*x)**(2*p) + 8*a**2*b**3*p*x**3*(a + b*x)**(2*p) + 4*a**2*b**3*x**3*
(a + b*x)**(2*p) + 2*a*b**4*p*x**4*(a + b*x)**(2*p) + a*b**4*x**4*(a + b*x)
**(2*p)) + 2*a**2*b*c**p*x**2*(x**2)**p/(2*a**5*p*(a + b*x)**(2*p) + a**5*(
a + b*x)**(2*p) + 8*a**4*b*p*x*(a + b*x)**(2*p) + 4*a**4*b*x*(a + b*x)**(2*
```

```
p) + 12*a**3*b**2*p*x**2*(a + b*x)**(2*p) + 6*a**3*b**2*x**2*(a + b*x)**(2*
p) + 8*a**2*b**3*p*x**3*(a + b*x)**(2*p) + 4*a**2*b**3*x**3*(a + b*x)**(2*p
) + 2*a*b**4*p*x**4*(a + b*x)**(2*p) + a*b**4*x**4*(a + b*x)**(2*p)) + a*b*
*2*c**p*x**3*(x**2)**p/(2*a**5*p*(a + b*x)**(2*p) + a**5*(a + b*x)**(2*p) +
 8*a**4*b*p*x*(a + b*x)**(2*p) + 4*a**4*b*x*(a + b*x)**(2*p) + 12*a**3*b**2
*p*x**2*(a + b*x)**(2*p) + 6*a**3*b**2*x**2*(a + b*x)**(2*p) + 8*a**2*b**3*
p*x**3*(a + b*x)**(2*p) + 4*a**2*b**3*x**3*(a + b*x)**(2*p) + 2*a*b**4*p*x*
*4*(a + b*x)**(2*p) + a*b**4*x**4*(a + b*x)**(2*p)) + b*c**p*x**2*(x**2)**p
/(2*a**3*p*(a + b*x)**(2*p) + a**3*(a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x
)**(2*p) + 2*a**2*b*x*(a + b*x)**(2*p) + 2*a*b**2*p*x**2*(a + b*x)**(2*p) +
 a*b**2*x**2*(a + b*x)**(2*p)), True))
```

## 3.924 $\int \frac{(cx^2)^p (a+bx)^{-1-2p}}{x}\,dx$

**Optimal**. Leaf size=26

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

**Rubi [A]** time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
```

```
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{(cx^2)^p (a+bx)^{-1-2p}}{x}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{-1+2p}(a+bx)^{-1-2p}\,dx \\ &= \frac{(cx^2)^p (a+bx)^{-2p}}{2ap} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

**IntegrateAlgebraic** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p (a+bx)^{-1-2p}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
[Out] Defer[IntegrateAlgebraic][((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x, x]
```

**fricas** [A] time = 1.18, size = 31, normalized size = 1.19

$$\frac{(bx+a)(cx^2)^p (bx+a)^{-2p-1}}{2ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="fricas")
[Out] 1/2*(b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p - 1)/(a*p)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p (bx+a)^{-2p-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 1)/x, x)
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.96

$$\frac{(c\,x^2)^p (bx+a)^{-2p}}{2ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-2*p-1)/x,x)
```

```
[Out] 1/2*(b*x+a)^(-2*p)/a/p*(c*x^2)^p
```

**maxima [A]** time = 1.45, size = 27, normalized size = 1.04

$$\frac{c^{p}e^{\left(-2\,p\log\left(bx+a\right)+2\,p\log\left(x\right)\right)}}{2\,ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*c^p*e^(-2*p*log(b*x + a) + 2*p*log(x))/(a*p)
```

**mupad [B]** time = 0.26, size = 26, normalized size = 1.00

$$\frac{{\left(c\,x^2\right)}^p}{2\,a\,p\,{\left(a+b\,x\right)}^{2\,p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/(x*(a + b*x)^(2*p + 1)),x)
```

```
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

**sympy [A]** time = 59.69, size = 264, normalized size = 10.15

$$\begin{cases} -\frac{b^{-2p}c^{p}x^{-2p}\left(x^{2}\right)^{p}}{bx} & \text{for}\: a = 0 \\ \frac{0^{-2p-1}c^{p}\left(x^{2}\right)^{p}}{2p} & \text{for}\: a = -bx \\ \frac{c^{p}\left(0^{\frac{1}{p}}\right)^{-2p-1}\left(x^{2}\right)^{p}}{2p} & \text{for}\: a = 0^{\frac{1}{p}} - bx \\ \frac{\log{\left(x \right)}}{a} - \frac{\log{\left(\frac{a}{b} + x \right)}}{a} & \text{for}\: p = 0 \\ \frac{a^{2}c^{p}\left(x^{2}\right)^{p}}{2a^{3}p(a+bx)^{2p}+4a^{2}bpx(a+bx)^{2p}+2ab^{2}px^{2}(a+bx)^{2p}} + \frac{abc^{p}x\left(x^{2}\right)^{p}}{2a^{3}p(a+bx)^{2p}+4a^{2}bpx(a+bx)^{2p}+2ab^{2}px^{2}(a+bx)^{2p}} + \frac{bc^{p}x\left(x^{2}\right)^{p}}{2a^{2}p(a+bx)^{2p}+2abpx(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(-1-2*p)/x,x)
```

```
[Out] Piecewise((-b**(-2*p)*c**p*x**(-2*p)*(x**2)**p/(b*x), Eq(a, 0)), (0**(-2*p
- 1)*c**p*(x**2)**p/(2*p), Eq(a, -b*x)), (c**p*(0**(1/p))**(-2*p - 1)*(x**2
```

```
)**p/(2*p), Eq(a, 0**(1/p) - b*x)), (log(x)/a - log(a/b + x)/a, Eq(p, 0)),
(a**2*c**p*(x**2)**p/(2*a**3*p*(a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x)**(
2*p) + 2*a*b**2*p*x**2*(a + b*x)**(2*p)) + a*b*c**p*x*(x**2)**p/(2*a**3*p*(
a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x)**(2*p) + 2*a*b**2*p*x**2*(a + b*x)
**(2*p)) + b*c**p*x*(x**2)**p/(2*a**2*p*(a + b*x)**(2*p) + 2*a*b*p*x*(a + b
*x)**(2*p)), True))
```

## 3.925 $\int \frac{(cx^2)^p (a+bx)^{-2p}}{x^2} \, dx$

**Optimal**. Leaf size=33

$$-\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}$$

**Rubi [A]** time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
```

```
[Out] -(((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*(1 - 2*p)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{(cx^2)^p (a+bx)^{-2p}}{x^2} \, dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-2+2p} (a+bx)^{-2p} \, dx \\ &= -\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 32, normalized size = 0.97

$$\frac{(cx^2)^p (a+bx)^{1-2p}}{a(2p-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
[Out] ((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*(-1 + 2*p)*x)
```

**IntegrateAlgebraic** [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p (a+bx)^{-2p}}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
[Out] Defer[IntegrateAlgebraic][(c*x^2)^p/(x^2*(a + b*x)^(2*p)), x]
```

**fricas** [A] time = 1.17, size = 37, normalized size = 1.12

$$\frac{(bx+a)(cx^2)^p}{(2\,ap-a)(bx+a)^{2\,p}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="fricas")
[Out] (b*x + a)*(c*x^2)^p/((2*a*p - a)*(b*x + a)^(2*p)*x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p}{(bx+a)^{2\,p}x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="giac")
[Out] integrate((c*x^2)^p/((b*x + a)^(2*p)*x^2), x)
```

**maple** [A] time = 0.00, size = 38, normalized size = 1.15

$$\frac{(bx+a)\left(c\,x^{2}\right)^{p}(bx+a)^{-2p}}{\left(2p-1\right)ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/x^2/((b*x+a)^(2*p)),x)
[Out] (b*x+a)/x/a/(2*p-1)*(c*x^2)^p/((b*x+a)^(2*p))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^{2}\right)^{p}}{(bx+a)^{2\,p}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="maxima")
[Out] integrate((c*x^2)^p/((b*x + a)^(2*p)*x^2), x)
```

**mupad** [B] time = 0.24, size = 32, normalized size = 0.97

$$\frac{\left(c\,x^{2}\right)^{p}(a+b\,x)^{1-2\,p}}{a\,x\,\left(2\,p-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/(x^2*(a + b*x)^(2*p)),x)
[Out] ((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*x*(2*p - 1))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{\sqrt{c}\,\sqrt{x^{2}}}{bx^{2}} & \text{for } a=0\wedge p=\frac{1}{2} \\ -\frac{b^{-2p}c^{p}x^{-2p}\left(x^{2}\right)^{p}}{x} & \text{for } a=0 \\ \int\frac{\sqrt{cx^{2}}}{x^{2}(a+bx)}\,dx & \text{for } p=\frac{1}{2} \\ \frac{ac^{p}\left(x^{2}\right)^{p}}{2apx(a+bx)^{2p}-ax(a+bx)^{2p}}+\frac{bc^{p}x\left(x^{2}\right)^{p}}{2apx(a+bx)^{2p}-ax(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p/x**2/((b*x+a)**(2*p)),x)
```

```
[Out] Piecewise((-sqrt(c)*sqrt(x**2)/(b*x**2), Eq(a, 0) & Eq(p, 1/2)), (-b**(-2*p
)*c**p*x**(-2*p)*(x**2)**p/x, Eq(a, 0)), (Integral(sqrt(c*x**2)/(x**2*(a +
b*x)), x), Eq(p, 1/2)), (a*c**p*(x**2)**p/(2*a*p*x*(a + b*x)**(2*p) - a*x*(
a + b*x)**(2*p)) + b*c**p*x*(x**2)**p/(2*a*p*x*(a + b*x)**(2*p) - a*x*(a +
b*x)**(2*p)), True))
```

## 3.926 $\int \frac{(cx^2)^p (a+bx)^{1-2p}}{x^3} dx$

**Optimal**. Leaf size=35

$$-\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
```

```
[Out] -((c*x^2)^p*(a + b*x)^(2 - 2*p))/(2*a*(1 - p)*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^p (a+bx)^{1-2p}}{x^3} dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-3+2p}(a+bx)^{1-2p} dx \\
&= -\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 32, normalized size = 0.91

$$\frac{(cx^2)^p (a+bx)^{2-2p}}{a(2p-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
[Out] ((c*x^2)^p*(a + b*x)^(2 - 2*p))/(a*(-2 + 2*p)*x^2)
```

**IntegrateAlgebraic** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p (a+bx)^{1-2p}}{x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
[Out] Defer[IntegrateAlgebraic][((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3, x]
```

**fricas** [A] time = 0.88, size = 37, normalized size = 1.06

$$\frac{(bx+a)(cx^2)^p (bx+a)^{-2p+1}}{2(ap-a)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="fricas")
[Out] 1/2*(b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p + 1)/((a*p - a)*x^2)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p (bx+a)^{-2p+1}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 1)/x^3, x)
```

**maple** [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{(c\,x^2)^p (bx+a)^{-2p+2}}{2(p-1)\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x)
```

```
[Out] 1/2*(b*x+a)^(2-2*p)/x^2/a/(p-1)*(c*x^2)^p
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p (bx+a)^{-2\,p+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 1)/x^3, x)
```

**mupad [B]** time = 0.25, size = 50, normalized size = 1.43

$$\frac{\left(\frac{\left(c\,x^2\right)^p}{2\,\left(p-1\right)}+\frac{b\,x\,\left(c\,x^2\right)^p}{2\,a\,\left(p-1\right)}\right)\,{\left(a+b\,x\right)}^{1-2\,p}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x)
```

```
[Out] (((c*x^2)^p/(2*(p - 1)) + (b*x*(c*x^2)^p)/(2*a*(p - 1)))*(a + b*x)^(1 - 2*p
))/x^2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p (a+bx)^{1-2p}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(1-2*p)/x**3,x)
```

```
[Out] Integral((c*x**2)**p*(a + b*x)**(1 - 2*p)/x**3, x)
```

## 3.927 $\int \frac{(cx^2)^p (a+bx)^{2-2p}}{x^4} dx$

**Optimal**. Leaf size=33

$$-\frac{(cx^2)^p (a+bx)^{3-2p}}{a(3-2p)x^3}$$

**Rubi** [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{3-2p}}{a(3-2p)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]
```

```
[Out] -(((c*x^2)^p*(a + b*x)^(3 - 2*p))/(a*(3 - 2*p)*x^3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p (a+bx)^{2-2p}}{x^4} dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-4+2p}(a+bx)^{2-2p} dx \\ &= -\frac{(cx^2)^p (a+bx)^{3-2p}}{a(3-2p)x^3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 32, normalized size = 0.97

$$\frac{(cx^2)^p (a+bx)^{3-2p}}{a(2p-3)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]
[Out] ((c*x^2)^p*(a + b*x)^(3 - 2*p))/(a*(-3 + 2*p)*x^3)
```

**IntegrateAlgebraic** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p (a+bx)^{2-2p}}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]
[Out] Defer[IntegrateAlgebraic][((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4, x]
```

**fricas** [A] time = 1.32, size = 37, normalized size = 1.12

$$\frac{(bx+a)\left(cx^2\right)^p (bx+a)^{-2\,p+2}}{\left(2\,ap-3\,a\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="fricas")
[Out] (b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p + 2)/((2*a*p - 3*a)*x^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p (bx+a)^{-2\,p+2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 2)/x^4, x)
```

**maple** [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{\left(c\,x^2\right)^p (bx+a)^{-2p+3}}{\left(2p-3\right)a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-2*p+2)/x^4,x)
```

```
[Out] (b*x+a)^(3-2*p)/x^3/a/(2*p-3)*(c*x^2)^p
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p (bx+a)^{-2\,p+2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 2)/x^4, x)
```

**mupad [B]** time = 0.25, size = 51, normalized size = 1.55

$$\frac{\left(\frac{\left(c\,x^2\right)^p}{2\,p-3}+\frac{b\,x\,\left(c\,x^2\right)^p}{a\,\left(2\,p-3\right)}\right)\,{\left(a+b\,x\right)}^{2-2\,p}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x)
```

```
[Out] (((c*x^2)^p/(2*p - 3) + (b*x*(c*x^2)^p)/(a*(2*p - 3)))*(a + b*x)^(2 - 2*p))
/x^3
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(2-2*p)/x**4,x)
```

```
[Out] Timed out
```

## 3.928 $\int x^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx$

**Optimal**. Leaf size=38

$$\frac{x^{m+1}\left(cx^2\right)^p (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

**Rubi [A]** time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {15, 37}

$$\frac{x^{m+1}\left(cx^2\right)^p (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^{-2-m-2p}\,dx\\ &= \frac{x^{1+m}\left(cx^2\right)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 38, normalized size = 1.00

$$\frac{x^{m+1}\left(cx^2\right)^p(a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

**IntegrateAlgebraic** [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x^m\left(cx^2\right)^p(a+bx)^{-2-m-2p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
[Out] Defer[IntegrateAlgebraic][x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p), x]
```

**fricas** [A] time = 1.42, size = 49, normalized size = 1.29

$$\frac{\left(bx^2+ax\right)(bx+a)^{-m-2\,p-2}x^me^{\left(p\log(c)+2\,p\log(x)\right)}}{am+2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="fricas")
[Out] (b*x^2 + a*x)*(b*x + a)^(-m - 2*p - 2)*x^m*e^(p*log(c) + 2*p*log(x))/(a*m +
 2*a*p + a)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^2\right)^p(bx+a)^{-m-2\,p-2}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*x^m, x)
```

**maple** [A] time = 0.00, size = 39, normalized size = 1.03

$$\frac{x^{m+1}\left(c\,x^2\right)^p(bx+a)^{-m-2p-1}}{\left(m+2p+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x)
```

```
[Out] x^(m+1)*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (bx+a)^{-m-2p-2} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*x^m, x)
```

**mupad [B]** time = 0.34, size = 50, normalized size = 1.32

$$\frac{x\,x^m\,\left(c\,x^2\right)^p}{a\,{\left(a+b\,x\right)}^m\,{\left(a+b\,x\right)}^{2\,p}\,\left(a+b\,x\right)\,\left(m+2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c*x^2)^p)/(a + b*x)^(m + 2*p + 2),x)
```

```
[Out] (x*x^m*(c*x^2)^p)/(a*(a + b*x)^m*(a + b*x)^(2*p)*(a + b*x)*(m + 2*p + 1))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**2)**p*(b*x+a)**(-2-m-2*p),x)
```

```
[Out] Timed out
```

## 3.929 $\int (dx)^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx$

**Optimal**. Leaf size=39

$$\frac{x\left(cx^2\right)^p (dx)^m (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

**Rubi [A]** time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 20, 37}

$$\frac{x\left(cx^2\right)^p (dx)^m (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx = \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(dx)^m(a+bx)^{-2-m-2p}\,dx$$
$$= \left(x^{-m-2p}(dx)^m\left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^{-2-m-2p}\,dx$$
$$= \frac{x(dx)^m\left(cx^2\right)^p(a+bx)^{-1-m-2p}}{a(1+m+2p)}$$

**Mathematica [A]** time = 0.02, size = 39, normalized size = 1.00

$$\frac{x\left(cx^2\right)^p(dx)^m(a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

**IntegrateAlgebraic [F]** time = 0.12, size = 0, normalized size = 0.00

$$\int (dx)^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p), x]
```

**fricas [A]** time = 1.36, size = 57, normalized size = 1.46

$$\frac{\left(bx^2+ax\right)(bx+a)^{-m-2p-2}(dx)^m e^{\left(2p\log(dx)+p\log\left(\frac{c}{d^2}\right)\right)}}{am+2ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="fricas")
[Out] (b*x^2 + a*x)*(b*x + a)^(-m - 2*p - 2)*(d*x)^m*e^(2*p*log(d*x) + p*log(c/d^
2))/(a*m + 2*a*p + a)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (bx+a)^{-m-2p-2}(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*(d*x)^m, x)
```

**maple** [A] time = 0.00, size = 40, normalized size = 1.03

$$\frac{x\left(c\,x^{2}\right)^{p}\left(dx\right)^{m}\left(bx+a\right)^{-m-2p-1}}{\left(m+2p+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x)
[Out] x*(d*x)^m*(c*x^2)^p*(b*x+a)^(-m-2*p-1)/a/(m+2*p+1)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^{2}\right)^{p}\left(bx+a\right)^{-m-2\,p-2}\left(dx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*(d*x)^m, x)
```

**mupad** [B] time = 0.26, size = 39, normalized size = 1.00

$$\frac{x\,\left(d\,x\right)^{m}\,\left(c\,x^{2}\right)^{p}}{a\,\left(a+b\,x\right)^{m+2\,p+1}\,\left(m+2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(c*x^2)^p)/(a + b*x)^(m + 2*p + 2),x)
[Out] (x*(d*x)^m*(c*x^2)^p)/(a*(a + b*x)^(m + 2*p + 1)*(m + 2*p + 1))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**p*(b*x+a)**(-2-m-2*p),x)
[Out] Timed out
```

## 3.930 $\int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3} dx$

**Optimal**. Leaf size=17

$$\frac{b^2(a+bx)^3}{3d^3}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{b^2(a+bx)^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^3)/(3*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int (a+bx)^2 dx}{d^3}\\ &= \frac{b^2(a+bx)^3}{3d^3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{b^2(a+bx)^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^3)/(3*d^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/((a*d)/b + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/((a*d)/b + d*x)^3, x]
```

**fricas [B]** time = 1.19, size = 31, normalized size = 1.82

$$\frac{b^5x^3+3\,ab^4x^2+3\,a^2b^3x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

**giac [B]** time = 1.09, size = 31, normalized size = 1.82

$$\frac{b^5x^3+3\,ab^4x^2+3\,a^2b^3x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.94

$$\frac{(bx+a)^3\,b^2}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(a*d/b+d*x)^3,x)
```

```
[Out] 1/3*b^2*(b*x+a)^3/d^3
```

**maxima [B]** time = 1.37, size = 31, normalized size = 1.82

$$\frac{b^5x^3 + 3\,ab^4x^2 + 3\,a^2b^3x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

**mupad [B]** time = 0.05, size = 27, normalized size = 1.59

$$\frac{b^3\,x\,\left(3\,a^2+3\,a\,b\,x+b^2\,x^2\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^3*x*(3*a^2 + b^2*x^2 + 3*a*b*x))/(3*d^3)
```

**sympy [B]** time = 0.12, size = 34, normalized size = 2.00

$$\frac{a^2b^3x}{d^3} + \frac{ab^4x^2}{d^3} + \frac{b^5x^3}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(a*d/b+d*x)**3,x)
```

```
[Out] a**2*b**3*x/d**3 + a*b**4*x**2/d**3 + b**5*x**3/(3*d**3)
```

## 3.931 $\int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3} dx$

**Optimal**. Leaf size=23

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

**Rubi** [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {21}

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/((a*d)/b + d*x)^3,x]
```

```
[Out] (a*b^3*x)/d^3 + (b^4*x^2)/(2*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int (a+bx) dx}{d^3} \\ &= \frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{b^3\left(ax+\frac{bx^2}{2}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/((a*d)/b + d*x)^3,x]
[Out] (b^3*(a*x + (b*x^2)/2))/d^3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/((a*d)/b + d*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)^4/((a*d)/b + d*x)^3, x]
```

**fricas [A]** time = 1.25, size = 20, normalized size = 0.87

$$\frac{b^4x^2+2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

**giac [A]** time = 1.19, size = 20, normalized size = 0.87

$$\frac{b^4x^2+2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="giac")
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

**maple [A]** time = 0.00, size = 18, normalized size = 0.78

$$\frac{\left(\frac{1}{2}b\,x^2+ax\right)b^3}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(a*d/b+d*x)^3,x)
[Out] b^3/d^3*(1/2*b*x^2+a*x)
```

**maxima** [A] time = 1.24, size = 20, normalized size = 0.87

$$\frac{b^4x^2 + 2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

**mupad** [B] time = 0.03, size = 16, normalized size = 0.70

$$\frac{b^3\,x\,\left(2\,a + b\,x\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^3*x*(2*a + b*x))/(2*d^3)
```

**sympy** [A] time = 0.12, size = 20, normalized size = 0.87

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(a*d/b+d*x)**3,x)
```

```
[Out] a*b**3*x/d**3 + b**4*x**2/(2*d**3)
```

## 3.932 $\int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

**Optimal**. Leaf size=8

$$\frac{b^3x}{d^3}$$

**Rubi [A]** time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 8}

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int 1\,dx}{d^3}\\ &= \frac{b^3x}{d^3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/((a*d)/b + d*x)^3,x]
[Out] (b^3*x)/d^3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/((a*d)/b + d*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)^3/((a*d)/b + d*x)^3, x]
```

**fricas [A]** time = 1.35, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] b^3*x/d^3
```

**giac [A]** time = 1.00, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="giac")
[Out] b^3*x/d^3
```

**maple [A]** time = 0.00, size = 9, normalized size = 1.12

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(a*d/b+d*x)^3,x)
```

```
[Out] b^3*x/d^3
```

**maxima** [A] time = 1.27, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3
```

**mupad** [B] time = 0.01, size = 8, normalized size = 1.00

$$\frac{b^3\,x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^3*x)/d^3
```

**sympy** [A] time = 0.11, size = 7, normalized size = 0.88

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(a*d/b+d*x)**3,x)
```

```
[Out] b**3*x/d**3
```

## 3.933 $\int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

**Optimal**. Leaf size=13

$$\frac{b^2\log(a+bx)}{d^3}$$

**Rubi [A]** time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 31}

$$\frac{b^2\log(a+bx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*Log[a + b*x])/d^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{a+bx}\,dx}{d^3}\\ &= \frac{b^2\log(a+bx)}{d^3}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{b^2 \log(a+bx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/((a*d)/b + d*x)^3,x]
[Out] (b^2*Log[a + b*x])/d^3
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/((a*d)/b + d*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/((a*d)/b + d*x)^3, x]
```

**fricas** [A] time = 1.22, size = 13, normalized size = 1.00

$$\frac{b^2 \log(bx+a)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] b^2*log(b*x + a)/d^3
```

**giac** [A] time = 1.07, size = 14, normalized size = 1.08

$$\frac{b^2 \log(|bx+a|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="giac")
[Out] b^2*log(abs(b*x + a))/d^3
```

**maple** [A] time = 0.00, size = 14, normalized size = 1.08

$$\frac{b^2 \ln(bx+a)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(a*d/b+d*x)^3,x)
```

```
[Out] b^2*ln(b*x+a)/d^3
```

**maxima [A]** time = 1.31, size = 13, normalized size = 1.00

$$\frac{b^2 \log(bx + a)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/d^3
```

**mupad [B]** time = 0.05, size = 13, normalized size = 1.00

$$\frac{b^2\ \ln(a + b\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^2*log(a + b*x))/d^3
```

**sympy [A]** time = 0.11, size = 19, normalized size = 1.46

$$\frac{b^2 \log\left(ad^3 + bd^3x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(a*d/b+d*x)**3,x)
```

```
[Out] b**2*log(a*d**3 + b*d**3*x)/d**3
```

## 3.934 $\int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

**Optimal**. Leaf size=15

$$-\frac{b^2}{d^3(a+bx)}$$

**Rubi [A]** time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$-\frac{b^2}{d^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((a*d)/b + d*x)^3,x]
```

```
[Out] -(b^2/(d^3*(a + b*x)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^2}\,dx}{d^3}\\ &= -\frac{b^2}{d^3(a+bx)}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{b^2}{d^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/((a*d)/b + d*x)^3,x]
[Out] -(b^2/(d^3*(a + b*x)))
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/((a*d)/b + d*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)/((a*d)/b + d*x)^3, x]
```

**fricas** [A] time = 1.34, size = 19, normalized size = 1.27

$$-\frac{b^2}{bd^3x+ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] -b^2/(b*d^3*x + a*d^3)
```

**giac** [A] time = 0.99, size = 15, normalized size = 1.00

$$-\frac{b^2}{(bx+a)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="giac")
[Out] -b^2/((b*x + a)*d^3)
```

**maple** [A] time = 0.00, size = 16, normalized size = 1.07

$$-\frac{b^2}{(bx+a)\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(a*d/b+d*x)^3,x)
```

```
[Out] -b^2/d^3/(b*x+a)
```

**maxima** [A] time = 1.34, size = 19, normalized size = 1.27

$$-\frac{b^2}{bd^3x + ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -b^2/(b*d^3*x + a*d^3)
```

**mupad** [B] time = 0.04, size = 15, normalized size = 1.00

$$-\frac{b^2}{d^3\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(d*x + (a*d)/b)^3,x)
```

```
[Out] -b^2/(d^3*(a + b*x))
```

**sympy** [A] time = 0.18, size = 19, normalized size = 1.27

$$-\frac{b^3}{abd^3 + b^2d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(a*b*d**3 + b**2*d**3*x)
```

## 3.935 $\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3}\,dx$

**Optimal**. Leaf size=17

$$-\frac{b^2}{3d^3(a+bx)^3}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{b^2}{3d^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*((a*d)/b + d*x)^3),x]
```

```
[Out] -b^2/(3*d^3*(a + b*x)^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^4}\,dx}{d^3}\\ &= -\frac{b^2}{3d^3(a+bx)^3}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{b^2}{3d^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*((a*d)/b + d*x)^3),x]
[Out] -1/3*b^2/(d^3*(a + b*x)^3)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*((a*d)/b + d*x)^3),x]
[Out] IntegrateAlgebraic[1/((a + b*x)*((a*d)/b + d*x)^3), x]
```

**fricas** [B] time = 1.33, size = 47, normalized size = 2.76

$$-\frac{b^2}{3\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] -1/3*b^2/(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
```

**giac** [A] time = 1.19, size = 15, normalized size = 0.88

$$-\frac{b^2}{3\,(bx+a)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="giac")
[Out] -1/3*b^2/((b*x + a)^3*d^3)
```

**maple** [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{b^2}{3\,(bx+a)^3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(a*d/b+d*x)^3,x)
```

```
[Out] -1/3*b^2/d^3/(b*x+a)^3
```

**maxima [B]** time = 1.31, size = 47, normalized size = 2.76

$$-\frac{b^2}{3\left(b^3d^3x^3 + 3\,ab^2d^3x^2 + 3\,a^2bd^3x + a^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*b^2/(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
```

**mupad [B]** time = 0.15, size = 49, normalized size = 2.88

$$-\frac{b^2}{3\,\left(a^3\,d^3 + 3\,a^2\,b\,d^3\,x + 3\,a\,b^2\,d^3\,x^2 + b^3\,d^3\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)),x)
```

```
[Out] -b^2/(3*(a^3*d^3 + b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x))
```

**sympy [B]** time = 0.29, size = 53, normalized size = 3.12

$$-\frac{b^3}{3a^3bd^3 + 9a^2b^2d^3x + 9ab^3d^3x^2 + 3b^4d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(3*a**3*b*d**3 + 9*a**2*b**2*d**3*x + 9*a*b**3*d**3*x**2 + 3*b**4*d**
3*x**3)
```

## 3.936 $\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3}\,dx$

**Optimal**. Leaf size=17

$$-\frac{b^2}{4d^3(a+bx)^4}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{b^2}{4d^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]
```

```
[Out] -b^2/(4*d^3*(a + b*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3}\,dx = \frac{b^3\int \frac{1}{(a+bx)^5}\,dx}{d^3}$$
$$= -\frac{b^2}{4d^3(a+bx)^4}$$

**Mathematica** [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{b^2}{4d^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/4*b^2/(d^3*(a + b*x)^4)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*((a*d)/b + d*x)^3), x]
```

**fricas** [B] time = 1.36, size = 61, normalized size = 3.59

$$-\frac{b^2}{4\left(b^4d^3x^4+4\,ab^3d^3x^3+6\,a^2b^2d^3x^2+4\,a^3bd^3x+a^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*b^2/(b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^3*b*d^3*x
 + a^4*d^3)
```

**giac** [A] time = 0.91, size = 15, normalized size = 0.88

$$-\frac{b^2}{4\,(bx+a)^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*b^2/((b*x + a)^4*d^3)
```

**maple** [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{b^2}{4\,(bx+a)^4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(a*d/b+d*x)^3,x)
```

```
[Out] -1/4*b^2/d^3/(b*x+a)^4
```

**maxima** [B] time = 1.35, size = 61, normalized size = 3.59

$$-\frac{b^2}{4\left(b^4d^3x^4+4\,ab^3d^3x^3+6\,a^2b^2d^3x^2+4\,a^3bd^3x+a^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2/(b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^3*b*d^3*x
 + a^4*d^3)
```

**mupad** [B] time = 0.06, size = 63, normalized size = 3.71

$$-\frac{b^2}{4\,\left(a^4\,d^3+4\,a^3\,b\,d^3\,x+6\,a^2\,b^2\,d^3\,x^2+4\,a\,b^3\,d^3\,x^3+b^4\,d^3\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)^2),x)
```

```
[Out] -b^2/(4*(a^4*d^3 + b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^
3*b*d^3*x))
```

**sympy** [B] time = 0.36, size = 68, normalized size = 4.00

$$-\frac{b^3}{4a^4bd^3+16a^3b^2d^3x+24a^2b^3d^3x^2+16ab^4d^3x^3+4b^5d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(4*a**4*b*d**3 + 16*a**3*b**2*d**3*x + 24*a**2*b**3*d**3*x**2 + 16*a*
b**4*d**3*x**3 + 4*b**5*d**3*x**4)
```

## 3.937 $\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx$

**Optimal**. Leaf size=17

$$-\frac{b^2}{5d^3(a+bx)^5}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{b^2}{5d^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
```

```
[Out] -b^2/(5*d^3*(a + b*x)^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^6}\,dx}{d^3}\\ &= -\frac{b^2}{5d^3(a+bx)^5}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{b^2}{5d^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
[Out] -1/5*b^2/(d^3*(a + b*x)^5)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
[Out] IntegrateAlgebraic[1/((a + b*x)^3*((a*d)/b + d*x)^3), x]
```

**fricas** [B] time = 0.88, size = 75, normalized size = 4.41

$$-\frac{b^2}{5\left(b^5d^3x^5+5\,ab^4d^3x^4+10\,a^2b^3d^3x^3+10\,a^3b^2d^3x^2+5\,a^4bd^3x+a^5d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] -1/5*b^2/(b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^2*b^3*d^3*x^3 + 10*a^3*b^2*d
^3*x^2 + 5*a^4*b*d^3*x + a^5*d^3)
```

**giac** [A] time = 0.96, size = 15, normalized size = 0.88

$$-\frac{b^2}{5\,(bx+a)^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="giac")
[Out] -1/5*b^2/((b*x + a)^5*d^3)
```

**maple** [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{b^{2}}{5\left(bx+a\right)^{5}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(a*d/b+d*x)^3,x)
```

```
[Out] -1/5*b^2/d^3/(b*x+a)^5
```

**maxima** [B] time = 1.33, size = 75, normalized size = 4.41

$$-\frac{b^{2}}{5\left(b^{5}d^{3}x^{5}+5\,ab^{4}d^{3}x^{4}+10\,a^{2}b^{3}d^{3}x^{3}+10\,a^{3}b^{2}d^{3}x^{2}+5\,a^{4}bd^{3}x+a^{5}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/5*b^2/(b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^2*b^3*d^3*x^3 + 10*a^3*b^2*d
^3*x^2 + 5*a^4*b*d^3*x + a^5*d^3)
```

**mupad** [B] time = 0.05, size = 77, normalized size = 4.53

$$-\frac{b^{2}}{5\,\left(a^{5}\,d^{3}+5\,a^{4}\,b\,d^{3}\,x+10\,a^{3}\,b^{2}\,d^{3}\,x^{2}+10\,a^{2}\,b^{3}\,d^{3}\,x^{3}+5\,a\,b^{4}\,d^{3}\,x^{4}+b^{5}\,d^{3}\,x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)^3),x)
```

```
[Out] -b^2/(5*(a^5*d^3 + b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^3*b^2*d^3*x^2 + 10*
a^2*b^3*d^3*x^3 + 5*a^4*b*d^3*x))
```

**sympy** [B] time = 0.42, size = 83, normalized size = 4.88

$$-\frac{b^{3}}{5a^{5}bd^{3}+25a^{4}b^{2}d^{3}x+50a^{3}b^{3}d^{3}x^{2}+50a^{2}b^{4}d^{3}x^{3}+25ab^{5}d^{3}x^{4}+5b^{6}d^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(5*a**5*b*d**3 + 25*a**4*b**2*d**3*x + 50*a**3*b**3*d**3*x**2 + 50*a*
*2*b**4*d**3*x**3 + 25*a*b**5*d**3*x**4 + 5*b**6*d**3*x**5)
```

## 3.938 $\int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=17

$$\frac{b^5(c+dx)^3}{3d^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (b^5*(c + d*x)^3)/(3*d^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3}\,dx &= \frac{b^5\int (c+dx)^2\,dx}{d^5}\\ &= \frac{b^5(c+dx)^3}{3d^6}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^5/(c + d*x)^3,x]
[Out] (b^5*(c + d*x)^3)/(3*d^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((b*c)/d + b*x)^5/(c + d*x)^3,x]
[Out] IntegrateAlgebraic[((b*c)/d + b*x)^5/(c + d*x)^3, x]
```

**fricas [B]** time = 1.25, size = 35, normalized size = 2.06

$$\frac{b^5d^2x^3+3\,b^5cdx^2+3\,b^5c^2x}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="fricas")
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

**giac [B]** time = 1.01, size = 35, normalized size = 2.06

$$\frac{b^5d^2x^3+3\,b^5cdx^2+3\,b^5c^2x}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="giac")
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.94

$$\frac{(dx+c)^3\,b^5}{3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^5/(d*x+c)^3,x)
```

[Out] 1/3*b^5*(d*x+c)^3/d^6

**maxima [B]** time = 1.37, size = 35, normalized size = 2.06

$$\frac{b^5d^2x^3 + 3\,b^5cdx^2 + 3\,b^5c^2x}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="maxima")

[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5

**mupad [B]** time = 0.16, size = 27, normalized size = 1.59

$$\frac{b^5\,x\,\left(3\,c^2+3\,c\,d\,x+d^2\,x^2\right)}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + (b*c)/d)^5/(c + d*x)^3,x)

[Out] (b^5*x*(3*c^2 + d^2*x^2 + 3*c*d*x))/(3*d^5)

**sympy [B]** time = 0.13, size = 34, normalized size = 2.00

$$\frac{b^5c^2x}{d^5} + \frac{b^5cx^2}{d^4} + \frac{b^5x^3}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*c/d+b*x)**5/(d*x+c)**3,x)

[Out] b**5*c**2*x/d**5 + b**5*c*x**2/d**4 + b**5*x**3/(3*d**3)

## 3.939 $\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3} dx$

**Optimal**. Leaf size=23

$$\frac{b^4cx}{d^4} + \frac{b^4x^2}{2d^3}$$

**Rubi [A]** time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {21}

$$\frac{b^4cx}{d^4} + \frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (b^4*c*x)/d^4 + (b^4*x^2)/(2*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3} dx &= \frac{b^4 \int (c+dx)\, dx}{d^4} \\ &= \frac{b^4cx}{d^4} + \frac{b^4x^2}{2d^3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 19, normalized size = 0.83

$$\frac{b^4\left(cx + \frac{dx^2}{2}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (b^4*(c*x + (d*x^2)/2))/d^4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[((b*c)/d + b*x)^4/(c + d*x)^3, x]
```

**fricas [A]** time = 1.26, size = 21, normalized size = 0.91

$$\frac{b^4dx^2+2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

**giac [A]** time = 1.04, size = 21, normalized size = 0.91

$$\frac{b^4dx^2+2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

**maple [A]** time = 0.00, size = 18, normalized size = 0.78

$$\frac{\left(\frac{1}{2}d\,x^2+cx\right)b^4}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^4/(d*x+c)^3,x)
```

```
[Out] b^4/d^4*(c*x+1/2*d*x^2)
```

**maxima** [A] time = 1.32, size = 21, normalized size = 0.91

$$\frac{b^4dx^2 + 2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

**mupad** [B] time = 0.03, size = 16, normalized size = 0.70

$$\frac{b^4\,x\,\left(2\,c + d\,x\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^4/(c + d*x)^3,x)
```

```
[Out] (b^4*x*(2*c + d*x))/(2*d^4)
```

**sympy** [A] time = 0.12, size = 20, normalized size = 0.87

$$\frac{b^4cx}{d^4} + \frac{b^4x^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**4/(d*x+c)**3,x)
```

```
[Out] b**4*c*x/d**4 + b**4*x**2/(2*d**3)
```

## 3.940 $\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=8

$$\frac{b^3x}{d^3}$$

**Rubi [A]** time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 8}

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3}\,dx &= \frac{b^3\int 1\,dx}{d^3}\\ &= \frac{b^3x}{d^3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((b*c)/d + b*x)^3/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[((b*c)/d + b*x)^3/(c + d*x)^3, x]
```

**fricas [A]** time = 1.19, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] b^3*x/d^3
```

**giac [A]** time = 1.05, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3
```

**maple [A]** time = 0.00, size = 9, normalized size = 1.12

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^3/(d*x+c)^3,x)
```

```
[Out] b^3/d^3*x
```

**maxima** [A] time = 1.33, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="maxima")
[Out] b^3*x/d^3
```

**mupad** [B] time = 0.01, size = 8, normalized size = 1.00

$$\frac{b^3\,x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^3/(c + d*x)^3,x)
[Out] (b^3*x)/d^3
```

**sympy** [A] time = 0.10, size = 7, normalized size = 0.88

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**3/(d*x+c)**3,x)
[Out] b**3*x/d**3
```

## 3.941 $\int \frac{\left(\frac{bc}{d}+bx\right)^2}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=13

$$\frac{b^2 \log(c+dx)}{d^3}$$

**Rubi** [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 31}

$$\frac{b^2 \log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Log[c + d*x])/d^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^2}{(c+dx)^3}\,dx &= \frac{b^2 \int \frac{1}{c+dx}\,dx}{d^2}\\ &= \frac{b^2 \log(c+dx)}{d^3}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{b^2 \log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Log[c + d*x])/d^3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bc}{d} + bx\right)^2}{(c + dx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((b*c)/d + b*x)^2/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[((b*c)/d + b*x)^2/(c + d*x)^3, x]
```

**fricas [A]** time = 1.27, size = 13, normalized size = 1.00

$$\frac{b^2 \log(dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] b^2*log(d*x + c)/d^3
```

**giac [A]** time = 0.89, size = 14, normalized size = 1.08

$$\frac{b^2 \log(|dx + c|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(d*x + c))/d^3
```

**maple [A]** time = 0.00, size = 14, normalized size = 1.08

$$\frac{b^2 \ln(dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^2/(d*x+c)^3,x)
```

```
[Out] b^2*ln(d*x+c)/d^3
```

**maxima [A]** time = 1.37, size = 13, normalized size = 1.00

$$\frac{b^2 \log(dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(d*x + c)/d^3
```

**mupad [B]** time = 0.14, size = 13, normalized size = 1.00

$$\frac{b^2 \ln(c + d\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^2/(c + d*x)^3,x)
```

```
[Out] (b^2*log(c + d*x))/d^3
```

**sympy [A]** time = 0.10, size = 17, normalized size = 1.31

$$\frac{b^2 \log\left(cd^2 + d^3x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**2/(d*x+c)**3,x)
```

```
[Out] b**2*log(c*d**2 + d**3*x)/d**3
```

## 3.942 $\int \frac{\frac{bc}{d}+bx}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=13

$$-\frac{b}{d^2(c+dx)}$$

**Rubi [A]** time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$-\frac{b}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)/(c + d*x)^3,x]
```

```
[Out] -(b/(d^2*(c + d*x)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{bc}{d}+bx}{(c+dx)^3}\,dx &= \frac{b\int \frac{1}{(c+dx)^2}\,dx}{d}\\ &= -\frac{b}{d^2(c+dx)}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 13, normalized size = 1.00

$$-\frac{b}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)/(c + d*x)^3,x]
```

```
[Out] -(b/(d^2*(c + d*x)))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\frac{bc}{d} + bx}{(c + dx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((b*c)/d + b*x)/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[((b*c)/d + b*x)/(c + d*x)^3, x]
```

**fricas [A]** time = 1.20, size = 16, normalized size = 1.23

$$-\frac{b}{d^3x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -b/(d^3*x + c*d^2)
```

**giac [A]** time = 1.00, size = 13, normalized size = 1.00

$$-\frac{b}{(dx + c)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -b/((d*x + c)*d^2)
```

**maple [A]** time = 0.00, size = 14, normalized size = 1.08

$$-\frac{b}{(dx + c)\, d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)/(d*x+c)^3,x)
```

```
[Out] -b/d^2/(d*x+c)
```

**maxima [A]** time = 1.38, size = 16, normalized size = 1.23

$$-\frac{b}{d^3x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -b/(d^3*x + c*d^2)
```

**mupad [B]** time = 0.04, size = 13, normalized size = 1.00

$$-\frac{b}{d^2\,\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)/(c + d*x)^3,x)
```

```
[Out] -b/(d^2*(c + d*x))
```

**sympy [A]** time = 0.15, size = 12, normalized size = 0.92

$$-\frac{b}{cd^2 + d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)**3,x)
```

```
[Out] -b/(c*d**2 + d**3*x)
```

## 3.943 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx$

**Optimal**. Leaf size=14

$$-\frac{1}{3b(c+dx)^3}$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{1}{3b(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x)*(c + d*x)^3),x]
```

```
[Out] -1/(3*b*(c + d*x)^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx &= \frac{d\int \frac{1}{(c+dx)^4}\,dx}{b}\\ &= -\frac{1}{3b(c+dx)^3}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{3b(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)*(c + d*x)^3),x]
```

```
[Out] -1/3*1/(b*(c + d*x)^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(((b*c)/d + b*x)*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(((b*c)/d + b*x)*(c + d*x)^3), x]
```

**fricas [B]** time = 1.29, size = 36, normalized size = 2.57

$$-\frac{1}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
```

**giac [A]** time = 1.14, size = 12, normalized size = 0.86

$$-\frac{1}{3\,(dx+c)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/3/((d*x + c)^3*b)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{3\,(dx+c)^3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)/(d*x+c)^3,x)
```

```
[Out] -1/3/b/(d*x+c)^3
```

**maxima [B]** time = 1.34, size = 36, normalized size = 2.57

$$-\frac{1}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/3/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
```

**mupad [B]** time = 0.05, size = 38, normalized size = 2.71

$$-\frac{1}{3\,b\,c^3+9\,b\,c^2\,d\,x+9\,b\,c\,d^2\,x^2+3\,b\,d^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)*(c + d*x)^3),x)
```

```
[Out] -1/(3*b*c^3 + 3*b*d^3*x^3 + 9*b*c^2*d*x + 9*b*c*d^2*x^2)
```

**sympy [B]** time = 0.28, size = 44, normalized size = 3.14

$$-\frac{d}{3bc^3d+9bc^2d^2x+9bcd^3x^2+3bd^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)**3,x)
```

```
[Out] -d/(3*b*c**3*d + 9*b*c**2*d**2*x + 9*b*c*d**3*x**2 + 3*b*d**4*x**3)
```

## 3.944 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2 (c+dx)^3} \, dx$

**Optimal**. Leaf size=15

$$-\frac{d}{4b^2(c+dx)^4}$$

**Rubi [A]** time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{d}{4b^2(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -d/(4*b^2*(c + d*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2 (c+dx)^3} \, dx &= \frac{d^2 \int \frac{1}{(c+dx)^5} \, dx}{b^2}\\ &= -\frac{d}{4b^2(c+dx)^4}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{d}{4b^2(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]
[Out] -1/4*d/(b^2*(c + d*x)^4)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]
[Out] IntegrateAlgebraic[1/(((b*c)/d + b*x)^2*(c + d*x)^3), x]
```

**fricas** [B] time = 1.31, size = 59, normalized size = 3.93

$$-\frac{d}{4\left(b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="fricas")
[Out] -1/4*d/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4)
```

**giac** [A] time = 0.98, size = 20, normalized size = 1.33

$$-\frac{b^2}{4\left(bx+\frac{bc}{d}\right)^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="giac")
[Out] -1/4*b^2/((b*x + b*c/d)^4*d^3)
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.93

$$-\frac{d}{4\left(dx+c\right)^{4}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)^2/(d*x+c)^3,x)
```

```
[Out] -1/4*d/b^2/(d*x+c)^4
```

**maxima [B]** time = 1.35, size = 59, normalized size = 3.93

$$-\frac{d}{4\left(b^{2}d^{4}x^{4}+4\,b^{2}cd^{3}x^{3}+6\,b^{2}c^{2}d^{2}x^{2}+4\,b^{2}c^{3}dx+b^{2}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*d/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4)
```

**mupad [B]** time = 0.05, size = 61, normalized size = 4.07

$$-\frac{d}{4\,\left(b^2\,c^4+4\,b^2\,c^3\,d\,x+6\,b^2\,c^2\,d^2\,x^2+4\,b^2\,c\,d^3\,x^3+b^2\,d^4\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)^2*(c + d*x)^3),x)
```

```
[Out] -d/(4*(b^2*c^4 + b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*
c^3*d*x))
```

**sympy [B]** time = 0.36, size = 68, normalized size = 4.53

$$-\frac{d^{2}}{4b^{2}c^{4}d+16b^{2}c^{3}d^{2}x+24b^{2}c^{2}d^{3}x^{2}+16b^{2}cd^{4}x^{3}+4b^{2}d^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)**2/(d*x+c)**3,x)
```

```
[Out] -d**2/(4*b**2*c**4*d + 16*b**2*c**3*d**2*x + 24*b**2*c**2*d**3*x**2 + 16*b*
*2*c*d**4*x**3 + 4*b**2*d**5*x**4)
```

## 3.945 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)^3 (c+dx)^3}\,dx$

**Optimal**. Leaf size=17

$$-\frac{d^2}{5b^3(c+dx)^5}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{d^2}{5b^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -d^2/(5*b^3*(c + d*x)^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)^3 (c+dx)^3}\,dx &= \frac{d^3 \int \frac{1}{(c+dx)^6}\,dx}{b^3}\\ &= -\frac{d^2}{5b^3(c+dx)^5}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{d^2}{5b^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -1/5*d^2/(b^3*(c + d*x)^5)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{bc}{d}+bx\right)^3 (c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(((b*c)/d + b*x)^3*(c + d*x)^3), x]
```

**fricas** [B] time = 1.36, size = 75, normalized size = 4.41

$$-\frac{d^2}{5\left(b^3d^5x^5+5\,b^3cd^4x^4+10\,b^3c^2d^3x^3+10\,b^3c^3d^2x^2+5\,b^3c^4dx+b^3c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/5*d^2/(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d
^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)
```

**giac** [A] time = 0.92, size = 15, normalized size = 0.88

$$-\frac{d^2}{5\,(dx+c)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/5*d^2/((d*x + c)^5*b^3)
```

**maple** [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{d^2}{5\,(dx+c)^5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)^3/(d*x+c)^3,x)
```

```
[Out] -1/5*d^2/b^3/(d*x+c)^5
```

**maxima** [B] time = 1.36, size = 75, normalized size = 4.41

$$-\frac{d^2}{5\left(b^3d^5x^5+5\,b^3cd^4x^4+10\,b^3c^2d^3x^3+10\,b^3c^3d^2x^2+5\,b^3c^4dx+b^3c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/5*d^2/(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d
^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)
```

**mupad** [B] time = 0.17, size = 77, normalized size = 4.53

$$-\frac{d^2}{5\,\left(b^3\,c^5+5\,b^3\,c^4\,d\,x+10\,b^3\,c^3\,d^2\,x^2+10\,b^3\,c^2\,d^3\,x^3+5\,b^3\,c\,d^4\,x^4+b^3\,d^5\,x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)^3*(c + d*x)^3),x)
```

```
[Out] -d^2/(5*(b^3*c^5 + b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^3*d^2*x^2 + 10*
b^3*c^2*d^3*x^3 + 5*b^3*c^4*d*x))
```

**sympy** [B] time = 0.42, size = 83, normalized size = 4.88

$$-\frac{d^3}{5b^3c^5d+25b^3c^4d^2x+50b^3c^3d^3x^2+50b^3c^2d^4x^3+25b^3cd^5x^4+5b^3d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)**3/(d*x+c)**3,x)
```

```
[Out] -d**3/(5*b**3*c**5*d + 25*b**3*c**4*d**2*x + 50*b**3*c**3*d**3*x**2 + 50*b*
*3*c**2*d**4*x**3 + 25*b**3*c*d**5*x**4 + 5*b**3*d**6*x**5)
```

## 3.946 $\int (a+bx)^5(ac+bcx)^n\,dx$

**Optimal**. Leaf size=24

$$\frac{(ac+bcx)^{n+6}}{bc^6(n+6)}$$

**Rubi [A]** time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{(ac+bcx)^{n+6}}{bc^6(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^n,x]
```

```
[Out] (a*c + b*c*x)^(6 + n)/(b*c^6*(6 + n))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^n\,dx &= \frac{\int (ac+bcx)^{5+n}\,dx}{c^5}\\ &= \frac{(ac+bcx)^{6+n}}{bc^6(6+n)}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 25, normalized size = 1.04

$$\frac{(a+bx)^6(c(a+bx))^n}{b(n+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^n,x]
```

```
[Out] ((a + b*x)^6*(c*(a + b*x))^n)/(b*(6 + n))
```

**IntegrateAlgebraic [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int (a+bx)^5(ac+bcx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^5*(a*c + b*c*x)^n, x]
```

**fricas [B]** time = 1.00, size = 80, normalized size = 3.33

$$\frac{\left(b^{6}x^{6}+6\,ab^{5}x^{5}+15\,a^{2}b^{4}x^{4}+20\,a^{3}b^{3}x^{3}+15\,a^{4}b^{2}x^{2}+6\,a^{5}bx+a^{6}\right)(bcx+ac)^{n}}{bn+6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] (b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 +
 6*a^5*b*x + a^6)*(b*c*x + a*c)^n/(b*n + 6*b)
```

**giac [B]** time = 1.13, size = 141, normalized size = 5.88

$$\frac{(bcx+ac)^{n}b^{6}x^{6}+6\,(bcx+ac)^{n}ab^{5}x^{5}+15\,(bcx+ac)^{n}a^{2}b^{4}x^{4}+20\,(bcx+ac)^{n}a^{3}b^{3}x^{3}+15\,(bcx+ac)^{n}a^{4}b^{2}x^{2}+6\,(bcx+ac)^{n}a^{5}bx+(bcx+ac)^{n}a^{6}}{bn+6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((b*c*x + a*c)^n*b^6*x^6 + 6*(b*c*x + a*c)^n*a*b^5*x^5 + 15*(b*c*x + a*c)^n
*a^2*b^4*x^4 + 20*(b*c*x + a*c)^n*a^3*b^3*x^3 + 15*(b*c*x + a*c)^n*a^4*b^2*
x^2 + 6*(b*c*x + a*c)^n*a^5*b*x + (b*c*x + a*c)^n*a^6)/(b*n + 6*b)
```

**maple [A]** time = 0.00, size = 27, normalized size = 1.12

$$\frac{(bx+a)^{6}\,(bcx+ac)^{n}}{(n+6)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^n,x)
```

```
[Out] (b*x+a)^6/b/(6+n)*(b*c*x+a*c)^n
```

**maxima [B]** time = 1.81, size = 649, normalized size = 27.04

$$\frac{5\left(b^{2}c^{n}(n+1)x^{2}+abc^{n}nx-a^{2}c^{n}\right)(bx+a)^{n}a^{4}}{\left(n^{2}+3n+2\right)b}+\frac{10\left(\left(n^{2}+3n+2\right)b^{3}c^{n}x^{3}+\left(n^{2}+n\right)ab^{2}c^{n}x^{2}-2a^{2}bc^{n}nx+2a^{3}c^{n}\right)(bx+a)^{n}a^{3}}{\left(n^{3}+6n^{2}+11n+6\right)b}+\frac{(bcx+ac)^{n+1}a^{5}}{bc(n+1)}+\frac{10\left(\left(n^{3}+6n^{2}+11n+6\right)b^{4}c^{n}x^{4}+\left(n^{3}+3n^{2}+2n\right)ab^{3}c^{n}x^{3}-3\left(n^{2}+n\right)a^{2}b^{2}c^{n}x^{2}+6a^{3}bc^{n}nx-6a^{4}c^{n}\right)(bx+a)^{n}a^{2}}{\left(n^{4}+10n^{3}+35n^{2}+50n+24\right)b}+\frac{5\left(\left(n^{4}+10n^{3}+35n^{2}+50n+24\right)b^{5}c^{n}x^{5}+\left(n^{4}+6n^{3}+11n^{2}+6n\right)ab^{4}c^{n}x^{4}-4\left(n^{3}+3n^{2}+2n\right)a^{2}b^{3}c^{n}x^{3}+12\left(n^{2}+n\right)a^{3}b^{2}c^{n}x^{2}-24a^{4}bc^{n}nx+24a^{5}c^{n}\right)(bx+a)^{n}a}{\left(n^{5}+15n^{4}+85n^{3}+225n^{2}+274n+120\right)b}+\frac{\left(\left(n^{5}+15n^{4}+85n^{3}+225n^{2}+274n+120\right)b^{6}c^{n}x^{6}+\left(n^{5}+10n^{4}+35n^{3}+50n^{2}+24n\right)ab^{5}c^{n}x^{5}-5\left(n^{4}+6n^{3}+11n^{2}+6n\right)a^{2}b^{4}c^{n}x^{4}+20\left(n^{3}+3n^{2}+2n\right)a^{3}b^{3}c^{n}x^{3}-60\left(n^{2}+n\right)a^{4}b^{2}c^{n}x^{2}+120a^{5}bc^{n}nx-120a^{6}c^{n}\right)(bx+a)^{n}}{\left(n^{6}+21n^{5}+175n^{4}+735n^{3}+1624n^{2}+1764n+720\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] 5*(b^2*c^n*(n + 1)*x^2 + a*b*c^n*n*x - a^2*c^n)*(b*x + a)^n*a^4/((n^2 + 3*n
 + 2)*b) + 10*((n^2 + 3*n + 2)*b^3*c^n*x^3 + (n^2 + n)*a*b^2*c^n*x^2 - 2*a^
2*b*c^n*n*x + 2*a^3*c^n)*(b*x + a)^n*a^3/((n^3 + 6*n^2 + 11*n + 6)*b) + (b*
c*x + a*c)^(n + 1)*a^5/(b*c*(n + 1)) + 10*((n^3 + 6*n^2 + 11*n + 6)*b^4*c^n
*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^n*x^3 - 3*(n^2 + n)*a^2*b^2*c^n*x^2 + 6*
a^3*b*c^n*n*x - 6*a^4*c^n)*(b*x + a)^n*a^2/((n^4 + 10*n^3 + 35*n^2 + 50*n +
 24)*b) + 5*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^n*x^5 + (n^4 + 6*n^3
 + 11*n^2 + 6*n)*a*b^4*c^n*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^n*x^3 + 12
*(n^2 + n)*a^3*b^2*c^n*x^2 - 24*a^4*b*c^n*n*x + 24*a^5*c^n)*(b*x + a)^n*a/(
(n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b) + ((n^5 + 15*n^4 + 85*n^
3 + 225*n^2 + 274*n + 120)*b^6*c^n*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 +
24*n)*a*b^5*c^n*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*c^n*x^4 + 20*(
n^3 + 3*n^2 + 2*n)*a^3*b^3*c^n*x^3 - 60*(n^2 + n)*a^4*b^2*c^n*x^2 + 120*a^5
*b*c^n*n*x - 120*a^6*c^n)*(b*x + a)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 +
1624*n^2 + 1764*n + 720)*b)
```

**mupad [B]** time = 0.33, size = 107, normalized size = 4.46

$$(a\,c+b\,c\,x)^n\,\left(\frac{a^6}{b\,\left(n+6\right)}+\frac{b^5\,x^6}{n+6}+\frac{6\,a^5\,x}{n+6}+\frac{15\,a^4\,b\,x^2}{n+6}+\frac{6\,a\,b^4\,x^5}{n+6}+\frac{20\,a^3\,b^2\,x^3}{n+6}+\frac{15\,a^2\,b^3\,x^4}{n+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^n*(a + b*x)^5,x)
```

```
[Out] (a*c + b*c*x)^n*(a^6/(b*(n + 6)) + (b^5*x^6)/(n + 6) + (6*a^5*x)/(n + 6) +
(15*a^4*b*x^2)/(n + 6) + (6*a*b^4*x^5)/(n + 6) + (20*a^3*b^2*x^3)/(n + 6) +
 (15*a^2*b^3*x^4)/(n + 6))
```

**sympy [A]** time = 2.29, size = 212, normalized size = 8.83

$$\begin{cases} \frac{x}{ac^{6}} & \text{for}\: b = 0 \wedge n = -6 \\ a^{5} x \left(ac\right)^{n} & \text{for}\: b = 0 \\ \frac{\log{\left(\frac{a}{b} + x \right)}}{bc^{6}} & \text{for}\: n = -6 \\ \frac{a^{6} \left(ac + bcx\right)^{n}}{bn + 6b} + \frac{6 a^{5} b x \left(ac + bcx\right)^{n}}{bn + 6b} + \frac{15 a^{4} b^{2} x^{2} \left(ac + bcx\right)^{n}}{bn + 6b} + \frac{20 a^{3} b^{3} x^{3} \left(ac + bcx\right)^{n}}{bn + 6b} + \frac{15 a^{2} b^{4} x^{4} \left(ac + bcx\right)^{n}}{bn + 6b} + \frac{6 a b^{5} x^{5} \left(ac + bcx\right)^{n}}{bn + 6b} + \frac{b^{6} x^{6} \left(ac + bcx\right)^{n}}{bn + 6b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((x/(a*c**6), Eq(b, 0) & Eq(n, -6)), (a**5*x*(a*c)**n, Eq(b, 0)),
(log(a/b + x)/(b*c**6), Eq(n, -6)), (a**6*(a*c + b*c*x)**n/(b*n + 6*b) + 6*
a**5*b*x*(a*c + b*c*x)**n/(b*n + 6*b) + 15*a**4*b**2*x**2*(a*c + b*c*x)**n/
(b*n + 6*b) + 20*a**3*b**3*x**3*(a*c + b*c*x)**n/(b*n + 6*b) + 15*a**2*b**4
*x**4*(a*c + b*c*x)**n/(b*n + 6*b) + 6*a*b**5*x**5*(a*c + b*c*x)**n/(b*n +
6*b) + b**6*x**6*(a*c + b*c*x)**n/(b*n + 6*b), True))
```

## 3.947 $\int (a+bx)^5 (ac+bcx)^3 \, dx$

**Optimal**. Leaf size=17

$$\frac{c^3(a+bx)^9}{9b}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{c^3(a+bx)^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^3,x]
```

```
[Out] (c^3*(a + b*x)^9)/(9*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (a+bx)^5 (ac+bcx)^3 \, dx &= c^3 \int (a+bx)^8 \, dx \\ &= \frac{c^3(a+bx)^9}{9b} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{c^3(a+bx)^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^3,x]
```

```
[Out] (c^3*(a + b*x)^9)/(9*b)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5(ac+bcx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^3, x]
```

**fricas [B]** time = 1.25, size = 113, normalized size = 6.65

$$\frac{1}{9}x^9c^3b^8 + x^8c^3b^7a + 4x^7c^3b^6a^2 + \frac{28}{3}x^6c^3b^5a^3 + 14x^5c^3b^4a^4 + 14x^4c^3b^3a^5 + \frac{28}{3}x^3c^3b^2a^6 + 4x^2c^3ba^7 + xc^3a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^3*b^8 + x^8*c^3*b^7*a + 4*x^7*c^3*b^6*a^2 + 28/3*x^6*c^3*b^5*a^3
+ 14*x^5*c^3*b^4*a^4 + 14*x^4*c^3*b^3*a^5 + 28/3*x^3*c^3*b^2*a^6 + 4*x^2*c^
3*b*a^7 + x*c^3*a^8
```

**giac [B]** time = 0.96, size = 113, normalized size = 6.65

$$\frac{1}{9}\,b^8c^3x^9 + ab^7c^3x^8 + 4\,a^2b^6c^3x^7 + \frac{28}{3}\,a^3b^5c^3x^6 + 14\,a^4b^4c^3x^5 + 14\,a^5b^3c^3x^4 + \frac{28}{3}\,a^6b^2c^3x^3 + 4\,a^7bc^3x^2 + a^8c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

**maple [B]** time = 0.00, size = 114, normalized size = 6.71

$$\frac{1}{9}b^8c^3x^9 + a\,b^7c^3x^8 + 4a^2b^6c^3x^7 + \frac{28}{3}a^3b^5c^3x^6 + 14a^4b^4c^3x^5 + 14a^5b^3c^3x^4 + \frac{28}{3}a^6b^2c^3x^3 + 4a^7b\,c^3x^2 + a^8c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^3,x)
```

```
[Out] 1/9*b^8*c^3*x^9+a*b^7*c^3*x^8+4*a^2*b^6*c^3*x^7+28/3*a^3*b^5*c^3*x^6+14*a^4
*b^4*c^3*x^5+14*a^5*b^3*c^3*x^4+28/3*a^6*c^3*b^2*x^3+4*a^7*c^3*b*x^2+a^8*c^
3*x
```

**maxima [B]** time = 1.31, size = 113, normalized size = 6.65

$$\frac{1}{9}\,b^8c^3x^9 + ab^7c^3x^8 + 4\,a^2b^6c^3x^7 + \frac{28}{3}\,a^3b^5c^3x^6 + 14\,a^4b^4c^3x^5 + 14\,a^5b^3c^3x^4 + \frac{28}{3}\,a^6b^2c^3x^3 + 4\,a^7bc^3x^2 + a^8c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

**mupad [B]** time = 0.05, size = 113, normalized size = 6.65

$$a^8\,c^3\,x + 4\,a^7\,b\,c^3\,x^2 + \frac{28\,a^6\,b^2\,c^3\,x^3}{3} + 14\,a^5\,b^3\,c^3\,x^4 + 14\,a^4\,b^4\,c^3\,x^5 + \frac{28\,a^3\,b^5\,c^3\,x^6}{3} + 4\,a^2\,b^6\,c^3\,x^7 + a\,b^7\,c^3\,x^8 + \frac{b^8\,c^3\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^3*(a + b*x)^5,x)
```

```
[Out] a^8*c^3*x + (b^8*c^3*x^9)/9 + 4*a^7*b*c^3*x^2 + a*b^7*c^3*x^8 + (28*a^6*b^2
*c^3*x^3)/3 + 14*a^5*b^3*c^3*x^4 + 14*a^4*b^4*c^3*x^5 + (28*a^3*b^5*c^3*x^6
)/3 + 4*a^2*b^6*c^3*x^7
```

**sympy [B]** time = 0.10, size = 124, normalized size = 7.29

$$a^8c^3x + 4a^7bc^3x^2 + \frac{28a^6b^2c^3x^3}{3} + 14a^5b^3c^3x^4 + 14a^4b^4c^3x^5 + \frac{28a^3b^5c^3x^6}{3} + 4a^2b^6c^3x^7 + ab^7c^3x^8 + \frac{b^8c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**3,x)
```

```
[Out] a**8*c**3*x + 4*a**7*b*c**3*x**2 + 28*a**6*b**2*c**3*x**3/3 + 14*a**5*b**3*
c**3*x**4 + 14*a**4*b**4*c**3*x**5 + 28*a**3*b**5*c**3*x**6/3 + 4*a**2*b**6
*c**3*x**7 + a*b**7*c**3*x**8 + b**8*c**3*x**9/9
```

## 3.948 $\int (a+bx)^5 (ac+bcx)^2 \, dx$

**Optimal**. Leaf size=17

$$\frac{c^2 (a+bx)^8}{8b}$$

**Rubi** [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{c^2 (a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] (c^2*(a + b*x)^8)/(8*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (a+bx)^5 (ac+bcx)^2 \, dx &= c^2 \int (a+bx)^7 \, dx \\ &= \frac{c^2 (a+bx)^8}{8b} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{c^2 (a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] (c^2*(a + b*x)^8)/(8*b)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5(ac+bcx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^2, x]
```

**fricas [B]** time = 0.98, size = 99, normalized size = 5.82

$$\frac{1}{8}x^8c^2b^7 + x^7c^2b^6a + \frac{7}{2}x^6c^2b^5a^2 + 7x^5c^2b^4a^3 + \frac{35}{4}x^4c^2b^3a^4 + 7x^3c^2b^2a^5 + \frac{7}{2}x^2c^2ba^6 + xc^2a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c^2*b^7 + x^7*c^2*b^6*a + 7/2*x^6*c^2*b^5*a^2 + 7*x^5*c^2*b^4*a^3 +
 35/4*x^4*c^2*b^3*a^4 + 7*x^3*c^2*b^2*a^5 + 7/2*x^2*c^2*b*a^6 + x*c^2*a^7
```

**giac [B]** time = 0.99, size = 99, normalized size = 5.82

$$\frac{1}{8}\,b^7c^2x^8 + ab^6c^2x^7 + \frac{7}{2}\,a^2b^5c^2x^6 + 7\,a^3b^4c^2x^5 + \frac{35}{4}\,a^4b^3c^2x^4 + 7\,a^5b^2c^2x^3 + \frac{7}{2}\,a^6bc^2x^2 + a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

**maple [B]** time = 0.00, size = 100, normalized size = 5.88

$$\frac{1}{8}b^7c^2x^8 + a\,b^6c^2x^7 + \frac{7}{2}a^2b^5c^2x^6 + 7a^3b^4c^2x^5 + \frac{35}{4}a^4b^3c^2x^4 + 7a^5b^2c^2x^3 + \frac{7}{2}a^6b\,c^2x^2 + a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^2,x)
```

```
[Out] 1/8*b^7*c^2*x^8+a*b^6*c^2*x^7+7/2*a^2*b^5*c^2*x^6+7*a^3*b^4*c^2*x^5+35/4*a^
4*b^3*c^2*x^4+7*a^5*b^2*c^2*x^3+7/2*a^6*c^2*b*x^2+a^7*c^2*x
```

**maxima [B]** time = 1.36, size = 99, normalized size = 5.82

$$\frac{1}{8}\,b^7c^2x^8 + ab^6c^2x^7 + \frac{7}{2}\,a^2b^5c^2x^6 + 7\,a^3b^4c^2x^5 + \frac{35}{4}\,a^4b^3c^2x^4 + 7\,a^5b^2c^2x^3 + \frac{7}{2}\,a^6bc^2x^2 + a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

**mupad [B]** time = 0.04, size = 99, normalized size = 5.82

$$a^7\,c^2\,x + \frac{7\,a^6\,b\,c^2\,x^2}{2} + 7\,a^5\,b^2\,c^2\,x^3 + \frac{35\,a^4\,b^3\,c^2\,x^4}{4} + 7\,a^3\,b^4\,c^2\,x^5 + \frac{7\,a^2\,b^5\,c^2\,x^6}{2} + a\,b^6\,c^2\,x^7 + \frac{b^7\,c^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^2*(a + b*x)^5,x)
```

```
[Out] a^7*c^2*x + (b^7*c^2*x^8)/8 + (7*a^6*b*c^2*x^2)/2 + a*b^6*c^2*x^7 + 7*a^5*b
^2*c^2*x^3 + (35*a^4*b^3*c^2*x^4)/4 + 7*a^3*b^4*c^2*x^5 + (7*a^2*b^5*c^2*x^
6)/2
```

**sympy [B]** time = 0.10, size = 110, normalized size = 6.47

$$a^7c^2x + \frac{7a^6bc^2x^2}{2} + 7a^5b^2c^2x^3 + \frac{35a^4b^3c^2x^4}{4} + 7a^3b^4c^2x^5 + \frac{7a^2b^5c^2x^6}{2} + ab^6c^2x^7 + \frac{b^7c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**2,x)
```

```
[Out] a**7*c**2*x + 7*a**6*b*c**2*x**2/2 + 7*a**5*b**2*c**2*x**3 + 35*a**4*b**3*c
**2*x**4/4 + 7*a**3*b**4*c**2*x**5 + 7*a**2*b**5*c**2*x**6/2 + a*b**6*c**2*
x**7 + b**7*c**2*x**8/8
```

## 3.949 $\int (a+bx)^5(ac+bcx)\,dx$

**Optimal**. Leaf size=15

$$\frac{c(a+bx)^7}{7b}$$

**Rubi [A]** time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {21, 32}

$$\frac{c(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] (c*(a + b*x)^7)/(7*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)\,dx &= c\int (a+bx)^6\,dx\\ &= \frac{c(a+bx)^7}{7b}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 15, normalized size = 1.00

$$\frac{c(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] (c*(a + b*x)^7)/(7*b)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5(ac+bcx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x), x]
```

**fricas [B]** time = 1.27, size = 71, normalized size = 4.73

$$\frac{1}{7}x^7cb^6 + x^6cb^5a + 3x^5cb^4a^2 + 5x^4cb^3a^3 + 5x^3cb^2a^4 + 3x^2cba^5 + xca^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c*b^6 + x^6*c*b^5*a + 3*x^5*c*b^4*a^2 + 5*x^4*c*b^3*a^3 + 5*x^3*c*b
^2*a^4 + 3*x^2*c*b*a^5 + x*c*a^6
```

**giac [B]** time = 1.09, size = 71, normalized size = 4.73

$$\frac{1}{7}\,b^6cx^7 + ab^5cx^6 + 3\,a^2b^4cx^5 + 5\,a^3b^3cx^4 + 5\,a^4b^2cx^3 + 3\,a^5bcx^2 + a^6cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

**maple [B]** time = 0.00, size = 72, normalized size = 4.80

$$\frac{1}{7}b^6c\,x^7 + a\,b^5c\,x^6 + 3a^2b^4c\,x^5 + 5a^3b^3c\,x^4 + 5a^4b^2c\,x^3 + 3a^5bc\,x^2 + a^6cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c),x)
```

```
[Out] 1/7*b^6*c*x^7+a*b^5*c*x^6+3*a^2*b^4*c*x^5+5*a^3*b^3*c*x^4+5*a^4*b^2*c*x^3+3
*a^5*b*c*x^2+a^6*c*x
```

**maxima** [B] time = 1.35, size = 71, normalized size = 4.73

$$\frac{1}{7}\,b^6cx^7 + ab^5cx^6 + 3\,a^2b^4cx^5 + 5\,a^3b^3cx^4 + 5\,a^4b^2cx^3 + 3\,a^5bcx^2 + a^6cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

**mupad** [B] time = 0.03, size = 71, normalized size = 4.73

$$c\,a^6\,x + 3\,c\,a^5\,b\,x^2 + 5\,c\,a^4\,b^2\,x^3 + 5\,c\,a^3\,b^3\,x^4 + 3\,c\,a^2\,b^4\,x^5 + c\,a\,b^5\,x^6 + \frac{c\,b^6\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)*(a + b*x)^5,x)
```

```
[Out] (b^6*c*x^7)/7 + a^6*c*x + 5*a^4*b^2*c*x^3 + 5*a^3*b^3*c*x^4 + 3*a^2*b^4*c*x
^5 + 3*a^5*b*c*x^2 + a*b^5*c*x^6
```

**sympy** [B] time = 0.08, size = 78, normalized size = 5.20

$$a^6cx + 3a^5bcx^2 + 5a^4b^2cx^3 + 5a^3b^3cx^4 + 3a^2b^4cx^5 + ab^5cx^6 + \frac{b^6cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c),x)
```

```
[Out] a**6*c*x + 3*a**5*b*c*x**2 + 5*a**4*b**2*c*x**3 + 5*a**3*b**3*c*x**4 + 3*a*
*2*b**4*c*x**5 + a*b**5*c*x**6 + b**6*c*x**7/7
```

## 3.950 $\int \frac{(a+bx)^5}{ac+bcx}\,dx$

**Optimal**. Leaf size=17

$$\frac{(a+bx)^5}{5bc}$$

**Rubi** [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{(a+bx)^5}{5bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x),x]
```

```
[Out] (a + b*x)^5/(5*b*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{ac+bcx}\,dx &= \frac{\int (a+bx)^4\,dx}{c}\\ &= \frac{(a+bx)^5}{5bc}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^5}{5bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x),x]
```

```
[Out] (a + b*x)^5/(5*b*c)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{ac+bcx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x), x]
```

**fricas [B]** time = 1.10, size = 48, normalized size = 2.82

$$\frac{b^4x^5 + 5\,ab^3x^4 + 10\,a^2b^2x^3 + 10\,a^3bx^2 + 5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

**giac [B]** time = 0.89, size = 48, normalized size = 2.82

$$\frac{b^4x^5 + 5\,ab^3x^4 + 10\,a^2b^2x^3 + 10\,a^3bx^2 + 5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.94

$$\frac{(bx+a)^5}{5bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c),x)
```

```
[Out] 1/5*(b*x+a)^5/b/c
```

**maxima [B]** time = 1.35, size = 48, normalized size = 2.82

$$\frac{b^4x^5 + 5\,ab^3x^4 + 10\,a^2b^2x^3 + 10\,a^3bx^2 + 5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

**mupad [B]** time = 0.03, size = 57, normalized size = 3.35

$$\frac{a^4\,x}{c} + \frac{b^4\,x^5}{5\,c} + \frac{2\,a^3\,b\,x^2}{c} + \frac{a\,b^3\,x^4}{c} + \frac{2\,a^2\,b^2\,x^3}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x),x)
```

```
[Out] (a^4*x)/c + (b^4*x^5)/(5*c) + (2*a^3*b*x^2)/c + (a*b^3*x^4)/c + (2*a^2*b^2*
x^3)/c
```

**sympy [B]** time = 0.10, size = 51, normalized size = 3.00

$$\frac{a^4x}{c} + \frac{2a^3bx^2}{c} + \frac{2a^2b^2x^3}{c} + \frac{ab^3x^4}{c} + \frac{b^4x^5}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c),x)
```

```
[Out] a**4*x/c + 2*a**3*b*x**2/c + 2*a**2*b**2*x**3/c + a*b**3*x**4/c + b**4*x**5
/(5*c)
```

## 3.951 $\int \frac{(a+bx)^5}{(ac+bcx)^2} \, dx$

**Optimal**. Leaf size=17

$$\frac{(a+bx)^4}{4bc^2}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{(a+bx)^4}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^2,x]
```

```
[Out] (a + b*x)^4/(4*b*c^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^2} \, dx &= \frac{\int (a+bx)^3 \, dx}{c^2}\\ &= \frac{(a+bx)^4}{4bc^2}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^4}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^2,x]
```

```
[Out] (a + b*x)^4/(4*b*c^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^2, x]
```

**fricas [B]** time = 1.18, size = 37, normalized size = 2.18

$$\frac{b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^2
```

**giac [A]** time = 0.98, size = 18, normalized size = 1.06

$$\frac{(bcx+ac)^4}{4\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*c*x + a*c)^4/(b*c^6)
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.94

$$\frac{(bx+a)^4}{4b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^2,x)
```

```
[Out] 1/4*(b*x+a)^4/b/c^2
```

**maxima [B]** time = 1.31, size = 37, normalized size = 2.18

$$\frac{b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="maxima")
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^2
```

**mupad [B]** time = 0.05, size = 43, normalized size = 2.53

$$\frac{a^3\,x}{c^2} + \frac{b^3\,x^4}{4\,c^2} + \frac{3\,a^2\,b\,x^2}{2\,c^2} + \frac{a\,b^2\,x^3}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^2,x)
[Out] (a^3*x)/c^2 + (b^3*x^4)/(4*c^2) + (3*a^2*b*x^2)/(2*c^2) + (a*b^2*x^3)/c^2
```

**sympy [B]** time = 0.11, size = 46, normalized size = 2.71

$$\frac{a^3x}{c^2} + \frac{3a^2bx^2}{2c^2} + \frac{ab^2x^3}{c^2} + \frac{b^3x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**2,x)
[Out] a**3*x/c**2 + 3*a**2*b*x**2/(2*c**2) + a*b**2*x**3/c**2 + b**3*x**4/(4*c**2
)
```

## 3.952 $\int \frac{(a+bx)^5}{(ac+bcx)^3}\,dx$

**Optimal**. Leaf size=17

$$\frac{(a+bx)^3}{3bc^3}$$

**Rubi** [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{(a+bx)^3}{3bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^3,x]
```

```
[Out] (a + b*x)^3/(3*b*c^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^3}\,dx &= \frac{\int (a+bx)^2\,dx}{c^3}\\ &= \frac{(a+bx)^3}{3bc^3} \end{aligned}$$

**Mathematica** [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^3}{3bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^3,x]
```

```
[Out] (a + b*x)^3/(3*b*c^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^3, x]
```

**fricas [A]** time = 0.71, size = 26, normalized size = 1.53

$$\frac{b^2x^3 + 3\,abx^2 + 3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

**giac [A]** time = 1.00, size = 26, normalized size = 1.53

$$\frac{b^2x^3 + 3\,abx^2 + 3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.94

$$\frac{(bx+a)^3}{3b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^3,x)
```

```
[Out] 1/3*(b*x+a)^3/b/c^3
```

**maxima** [A] time = 1.31, size = 26, normalized size = 1.53

$$\frac{b^2x^3 + 3\,abx^2 + 3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

**mupad** [B] time = 0.04, size = 24, normalized size = 1.41

$$\frac{x\,\left(3\,a^2+3\,a\,b\,x+b^2\,x^2\right)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^3,x)
```

```
[Out] (x*(3*a^2 + b^2*x^2 + 3*a*b*x))/(3*c^3)
```

**sympy** [B] time = 0.11, size = 29, normalized size = 1.71

$$\frac{a^2x}{c^3} + \frac{abx^2}{c^3} + \frac{b^2x^3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**3,x)
```

```
[Out] a**2*x/c**3 + a*b*x**2/c**3 + b**2*x**3/(3*c**3)
```

## 3.953 $\int \frac{(a+bx)^5}{(ac+bcx)^4}\,dx$

**Optimal**. Leaf size=18

$$\frac{ax}{c^4}+\frac{bx^2}{2c^4}$$

**Rubi** [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {21}

$$\frac{ax}{c^4}+\frac{bx^2}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^4,x]
```

```
[Out] (a*x)/c^4 + (b*x^2)/(2*c^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^4}\,dx &= \frac{\int (a+bx)\,dx}{c^4}\\ &= \frac{ax}{c^4}+\frac{bx^2}{2c^4}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 16, normalized size = 0.89

$$\frac{ax+\frac{bx^2}{2}}{c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^4,x]
```

```
[Out] (a*x + (b*x^2)/2)/c^4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^4,x]
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^4, x]
```

**fricas [A]** time = 1.36, size = 15, normalized size = 0.83

$$\frac{bx^2+2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="fricas")
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

**giac [A]** time = 1.21, size = 15, normalized size = 0.83

$$\frac{bx^2+2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="giac")
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

**maple [A]** time = 0.00, size = 15, normalized size = 0.83

$$\frac{\frac{1}{2}b\,x^2+ax}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^4,x)
[Out] 1/c^4*(1/2*b*x^2+a*x)
```

**maxima [A]** time = 1.38, size = 15, normalized size = 0.83

$$\frac{bx^2+2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

**mupad [B]** time = 0.02, size = 13, normalized size = 0.72

$$\frac{x\,\left(2\,a+b\,x\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^4,x)
```

```
[Out] (x*(2*a + b*x))/(2*c^4)
```

**sympy [A]** time = 0.11, size = 15, normalized size = 0.83

$$\frac{ax}{c^4} + \frac{bx^2}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**4,x)
```

```
[Out] a*x/c**4 + b*x**2/(2*c**4)
```

## 3.954 $\int \frac{(a+bx)^5}{(ac+bcx)^5} dx$

**Optimal**. Leaf size=5

$$\frac{x}{c^5}$$

**Rubi [A]** time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 8}

$$\frac{x}{c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^5,x]
```

```
[Out] x/c^5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^5} dx &= \frac{\int 1 dx}{c^5} \\ &= \frac{x}{c^5} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^5,x]
[Out] x/c^5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^5} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^5,x]
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^5, x]
```

**fricas [A]** time = 1.22, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="fricas")
[Out] x/c^5
```

**giac [B]** time = 1.11, size = 15, normalized size = 3.00

$$\frac{bcx+ac}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="giac")
[Out] (b*c*x + a*c)/(b*c^6)
```

**maple [A]** time = 0.00, size = 6, normalized size = 1.20

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^5,x)
[Out] x/c^5
```

**maxima [A]** time = 1.40, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] x/c^5
```

**mupad [B]** time = 0.01, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^5,x)
```

```
[Out] x/c^5
```

**sympy [A]** time = 0.11, size = 3, normalized size = 0.60

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**5,x)
```

```
[Out] x/c**5
```

## 3.955 $\int \frac{(a+bx)^5}{(ac+bcx)^6} dx$

**Optimal**. Leaf size=13

$$\frac{\log(a+bx)}{bc^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 31}

$$\frac{\log(a+bx)}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^6,x]
```

```
[Out] Log[a + b*x]/(b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^6}\,dx &= \frac{\int \frac{1}{a+bx}\,dx}{c^6}\\ &= \frac{\log(a+bx)}{bc^6}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 13, normalized size = 1.00

$$\frac{\log(a+bx)}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^6,x]
```

```
[Out] Log[a + b*x]/(b*c^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^6, x]
```

**fricas [A]** time = 1.33, size = 13, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] log(b*x + a)/(b*c^6)
```

**giac [A]** time = 0.95, size = 14, normalized size = 1.08

$$\frac{\log\left(|bx+a|\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/(b*c^6)
```

**maple [A]** time = 0.00, size = 14, normalized size = 1.08

$$\frac{\ln\left(bx+a\right)}{b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^6,x)
```

```
[Out] ln(b*x+a)/b/c^6
```

**maxima** [A] time = 1.31, size = 13, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] log(b*x + a)/(b*c^6)
```

**mupad** [B] time = 0.04, size = 13, normalized size = 1.00

$$\frac{\ln\left(a+b\,x\right)}{b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^6,x)
```

```
[Out] log(a + b*x)/(b*c^6)
```

**sympy** [A] time = 0.12, size = 17, normalized size = 1.31

$$\frac{\log\left(ac^6+bc^6x\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**6,x)
```

```
[Out] log(a*c**6 + b*c**6*x)/(b*c**6)
```

## 3.956 $\int \frac{(a+bx)^5}{(ac+bcx)^7} dx$

**Optimal**. Leaf size=15

$$-\frac{1}{bc^7(a+bx)}$$

**Rubi** [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$-\frac{1}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^7,x]
```

```
[Out] -(1/(b*c^7*(a + b*x)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^7} dx &= \frac{\int \frac{1}{(a+bx)^2} dx}{c^7}\\ &= -\frac{1}{bc^7(a+bx)}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^7,x]
```

```
[Out] -(1/(b*c^7*(a + b*x)))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^7, x]
```

**fricas [A]** time = 1.22, size = 19, normalized size = 1.27

$$-\frac{1}{b^2c^7x+abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="fricas")
```

```
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

**giac [A]** time = 1.09, size = 15, normalized size = 1.00

$$-\frac{1}{(bx+a)bc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b*c^7)
```

**maple [A]** time = 0.00, size = 16, normalized size = 1.07

$$-\frac{1}{(bx+a)\,b\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^7,x)
```

```
[Out] -1/b/c^7/(b*x+a)
```

**maxima** [A] time = 1.33, size = 19, normalized size = 1.27

$$-\frac{1}{b^2c^7x + abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="maxima")
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

**mupad** [B] time = 0.05, size = 19, normalized size = 1.27

$$-\frac{1}{x\,b^2\,c^7 + a\,b\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^7,x)
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

**sympy** [A] time = 0.20, size = 17, normalized size = 1.13

$$-\frac{1}{abc^7 + b^2c^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**7,x)
[Out] -1/(a*b*c**7 + b**2*c**7*x)
```

## 3.957 $\int \frac{(a+bx)^5}{(ac+bcx)^8}\,dx$

**Optimal**. Leaf size=17

$$-\frac{1}{2bc^8(a+bx)^2}$$

**Rubi** [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$-\frac{1}{2bc^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^8,x]
```

```
[Out] -1/(2*b*c^8*(a + b*x)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^8}\,dx &= \frac{\int \frac{1}{(a+bx)^3}\,dx}{c^8}\\ &= -\frac{1}{2bc^8(a+bx)^2}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2bc^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^8,x]
```

```
[Out] -1/2*1/(b*c^8*(a + b*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^8, x]
```

**fricas [B]** time = 1.36, size = 33, normalized size = 1.94

$$-\frac{1}{2\left(b^3c^8x^2+2\,ab^2c^8x+a^2bc^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*c^8*x^2 + 2*a*b^2*c^8*x + a^2*b*c^8)
```

**giac [A]** time = 1.05, size = 15, normalized size = 0.88

$$-\frac{1}{2\,(bx+a)^2bc^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="giac")
```

```
[Out] -1/2/((b*x + a)^2*b*c^8)
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.94

$$-\frac{1}{2\,(bx+a)^2\,b\,c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^8,x)
```

```
[Out] -1/2/b/c^8/(b*x+a)^2
```

**maxima [B]** time = 1.33, size = 33, normalized size = 1.94

$$-\frac{1}{2\left(b^{3}c^{8}x^{2}+2\,ab^{2}c^{8}x+a^{2}bc^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="maxima")
[Out] -1/2/(b^3*c^8*x^2 + 2*a*b^2*c^8*x + a^2*b*c^8)
```

**mupad [B]** time = 0.15, size = 35, normalized size = 2.06

$$-\frac{1}{2\,a^{2}\,b\,c^{8}+4\,a\,b^{2}\,c^{8}\,x+2\,b^{3}\,c^{8}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^8,x)
[Out] -1/(2*a^2*b*c^8 + 2*b^3*c^8*x^2 + 4*a*b^2*c^8*x)
```

**sympy [B]** time = 0.26, size = 36, normalized size = 2.12

$$-\frac{1}{2a^{2}bc^{8}+4ab^{2}c^{8}x+2b^{3}c^{8}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**8,x)
[Out] -1/(2*a**2*b*c**8 + 4*a*b**2*c**8*x + 2*b**3*c**8*x**2)
```

## 3.958 $\int \frac{1}{\sqrt{-2-3x}\,\sqrt{2+3x}}\,dx$

**Optimal**. Leaf size=28

$$\frac{\sqrt{3x+2}\,\log(3x+2)}{3\sqrt{-3x-2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {23, 31}

$$\frac{\sqrt{3x+2}\,\log(3x+2)}{3\sqrt{-3x-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] (Sqrt[2 + 3*x]*Log[2 + 3*x])/(3*Sqrt[-2 - 3*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-3x}\,\sqrt{2+3x}}\,dx &= \frac{\sqrt{2+3x}\int \frac{1}{2+3x}\,dx}{\sqrt{-2-3x}}\\ &= \frac{\sqrt{2+3x}\,\log(2+3x)}{3\sqrt{-2-3x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

**IntegrateAlgebraic [F]** time = 1.78, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2-3x}\,\sqrt{2+3x}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]
[Out] Defer[IntegrateAlgebraic][1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]), x]
```

**fricas [A]** time = 1.27, size = 1, normalized size = 0.04

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
[Out] 0
```

**giac [C]** time = 1.01, size = 11, normalized size = 0.39

$$-\frac{1}{3}i\,\log\left(|3\,x+2|\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
[Out] -1/3*I*log(abs(3*x + 2))*sgn(x)
```

**maple [A]** time = 0.00, size = 23, normalized size = 0.82

$$\frac{\sqrt{3x+2}\,\ln\left(3x+2\right)}{3\sqrt{-3x-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*x)^(1/2)/(3*x+2)^(1/2),x)
[Out] 1/3*ln(3*x+2)*(3*x+2)^(1/2)/(-2-3*x)^(1/2)
```

**maxima** [C] time = 2.99, size = 6, normalized size = 0.21

$$\frac{1}{3}i \log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*I*log(x + 2/3)
```

**mupad** [B] time = 0.22, size = 35, normalized size = 1.25

$$-\frac{4\,\mathrm{atan}\left(\frac{-\sqrt{-3\,x-2}+\sqrt{2}\,1\mathrm{i}}{\sqrt{2}-\sqrt{3\,x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- 3*x - 2)^(1/2)*(3*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((2^(1/2)*1i - (- 3*x - 2)^(1/2))/(2^(1/2) - (3*x + 2)^(1/2))))/3
```

**sympy** [C] time = 1.46, size = 53, normalized size = 1.89

$$\begin{cases} -\frac{i\log\left(x+\frac{2}{3}\right)}{3} & \text{for}\ \left|x+\frac{2}{3}\right|<1 \\ \frac{i\log\left(\frac{1}{x+\frac{2}{3}}\right)}{3} & \text{for}\ \frac{1}{\left|x+\frac{2}{3}\right|}<1 \\ \frac{iG_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle|x+\frac{2}{3}\right)}{3}-\frac{iG_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle|x+\frac{2}{3}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((-I*log(x + 2/3)/3, Abs(x + 2/3) < 1), (I*log(1/(x + 2/3))/3, 1/A
bs(x + 2/3) < 1), (I*meijerg(((), (1, 1)), ((0, 0), ()), x + 2/3)/3 - I*mei
jerg(((1, 1), ()), ((), (0, 0)), x + 2/3)/3, True))
```

## 3.959 $\int (a+bx)(ac-bcx)^3\,dx$

**Optimal**. Leaf size=38

$$\frac{c^3(a-bx)^5}{5b}-\frac{ac^3(a-bx)^4}{2b}$$

**Rubi [A]** time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{c^3(a-bx)^5}{5b}-\frac{ac^3(a-bx)^4}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^3,x]
```

```
[Out] -(a*c^3*(a - b*x)^4)/(2*b) + (c^3*(a - b*x)^5)/(5*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^3\,dx&=\int \left(2a(ac-bcx)^3-\frac{(ac-bcx)^4}{c}\right)dx\\&=-\frac{ac^3(a-bx)^4}{2b}+\frac{c^3(a-bx)^5}{5b}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 40, normalized size = 1.05

$$c^3\left(a^4x-a^3bx^2+\frac{1}{2}ab^3x^4-\frac{1}{5}b^4x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^3,x]
```

```
[Out] c^3*(a^4*x - a^3*b*x^2 + (a*b^3*x^4)/2 - (b^4*x^5)/5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(ac-bcx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x)^3, x]
```

**fricas [A]** time = 1.29, size = 44, normalized size = 1.16

$$-\frac{1}{5}x^5c^3b^4+\frac{1}{2}x^4c^3b^3a-x^2c^3ba^3+xc^3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="fricas")
[Out] -1/5*x^5*c^3*b^4 + 1/2*x^4*c^3*b^3*a - x^2*c^3*b*a^3 + x*c^3*a^4
```

**giac [A]** time = 1.00, size = 44, normalized size = 1.16

$$-\frac{1}{5}\,b^4c^3x^5+\frac{1}{2}\,ab^3c^3x^4-a^3bc^3x^2+a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="giac")
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

**maple [A]** time = 0.00, size = 45, normalized size = 1.18

$$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3b\,c^3x^2+a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^3,x)
[Out] -1/5*b^4*c^3*x^5+1/2*a*b^3*c^3*x^4-a^3*c^3*b*x^2+a^4*c^3*x
```

**maxima [A]** time = 1.23, size = 44, normalized size = 1.16

$$-\frac{1}{5}\,b^4c^3x^5+\frac{1}{2}\,ab^3c^3x^4-a^3bc^3x^2+a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

**mupad [B]** time = 0.16, size = 44, normalized size = 1.16

$$a^4\,c^3\,x-a^3\,b\,c^3\,x^2+\frac{a\,b^3\,c^3\,x^4}{2}-\frac{b^4\,c^3\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3*(a + b*x),x)
```

```
[Out] a^4*c^3*x - (b^4*c^3*x^5)/5 - a^3*b*c^3*x^2 + (a*b^3*c^3*x^4)/2
```

**sympy [A]** time = 0.08, size = 44, normalized size = 1.16

$$a^{4}c^{3}x-a^{3}bc^{3}x^{2}+\frac{ab^{3}c^{3}x^{4}}{2}-\frac{b^{4}c^{3}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**3,x)
```

```
[Out] a**4*c**3*x - a**3*b*c**3*x**2 + a*b**3*c**3*x**4/2 - b**4*c**3*x**5/5
```

## 3.960 $\int (a+bx)(ac-bcx)^2\,dx$

**Optimal**. Leaf size=38

$$\frac{c^2(a-bx)^4}{4b}-\frac{2ac^2(a-bx)^3}{3b}$$

**Rubi** [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{c^2(a-bx)^4}{4b}-\frac{2ac^2(a-bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^2,x]
```

```
[Out] (-2*a*c^2*(a - b*x)^3)/(3*b) + (c^2*(a - b*x)^4)/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^2\,dx &= \int \left(2a(ac-bcx)^2-\frac{(ac-bcx)^3}{c}\right)dx\\ &= -\frac{2ac^2(a-bx)^3}{3b}+\frac{c^2(a-bx)^4}{4b}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 42, normalized size = 1.11

$$c^2\left(a^3x-\frac{1}{2}a^2bx^2-\frac{1}{3}ab^2x^3+\frac{b^3x^4}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^2,x]
```

```
[Out] c^2*(a^3*x - (a^2*b*x^2)/2 - (a*b^2*x^3)/3 + (b^3*x^4)/4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)(ac - bcx)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x)^2,x]
[Out] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x)^2, x]
```

**fricas [A]** time = 0.73, size = 44, normalized size = 1.16

$$\frac{1}{4}x^4c^2b^3 - \frac{1}{3}x^3c^2b^2a - \frac{1}{2}x^2c^2ba^2 + xc^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="fricas")
[Out] 1/4*x^4*c^2*b^3 - 1/3*x^3*c^2*b^2*a - 1/2*x^2*c^2*b*a^2 + x*c^2*a^3
```

**giac [A]** time = 1.04, size = 44, normalized size = 1.16

$$\frac{1}{4} \, b^3c^2x^4 - \frac{1}{3} \, ab^2c^2x^3 - \frac{1}{2} \, a^2bc^2x^2 + a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="giac")
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

**maple [A]** time = 0.00, size = 45, normalized size = 1.18

$$\frac{1}{4}b^3c^2x^4 - \frac{1}{3}a\,b^2c^2x^3 - \frac{1}{2}a^2b\,c^2x^2 + a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^2,x)
[Out] 1/4*b^3*c^2*x^4-1/3*a*b^2*c^2*x^3-1/2*a^2*c^2*b*x^2+a^3*c^2*x
```

**maxima [A]** time = 1.31, size = 44, normalized size = 1.16

$$\frac{1}{4} \, b^3c^2x^4 - \frac{1}{3} \, ab^2c^2x^3 - \frac{1}{2} \, a^2bc^2x^2 + a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

**mupad [B]** time = 0.05, size = 44, normalized size = 1.16

$$a^3\,c^2\,x-\frac{a^2\,b\,c^2\,x^2}{2}-\frac{a\,b^2\,c^2\,x^3}{3}+\frac{b^3\,c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2*(a + b*x),x)
```

```
[Out] a^3*c^2*x + (b^3*c^2*x^4)/4 - (a^2*b*c^2*x^2)/2 - (a*b^2*c^2*x^3)/3
```

**sympy [A]** time = 0.07, size = 46, normalized size = 1.21

$$a^{3}c^{2}x-\frac{a^{2}bc^{2}x^{2}}{2}-\frac{ab^{2}c^{2}x^{3}}{3}+\frac{b^{3}c^{2}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**2,x)
```

```
[Out] a**3*c**2*x - a**2*b*c**2*x**2/2 - a*b**2*c**2*x**3/3 + b**3*c**2*x**4/4
```

## 3.961 $\int (a+bx)(ac-bcx)\,dx$

**Optimal**. Leaf size=18

$$a^2cx-\frac{1}{3}b^2cx^3$$

**Rubi [A]** time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {41}

$$a^2cx-\frac{1}{3}b^2cx^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x),x]
```

```
[Out] a^2*c*x - (b^2*c*x^3)/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)\,dx &= \int \left(a^2c-b^2cx^2\right)\,dx\\ &= a^2cx-\frac{1}{3}b^2cx^3\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 18, normalized size = 1.00

$$c\left(a^2x-\frac{b^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x),x]
```

```
[Out] c*(a^2*x - (b^2*x^3)/3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)(ac - bcx)\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x),x]
[Out] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x), x]
```

**fricas [A]** time = 1.03, size = 16, normalized size = 0.89

$$-\frac{1}{3} x^3 c b^2 + x c a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="fricas")
[Out] -1/3*x^3*c*b^2 + x*c*a^2
```

**giac [A]** time = 1.01, size = 16, normalized size = 0.89

$$-\frac{1}{3} b^2 c x^3 + a^2 c x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="giac")
[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

**maple [A]** time = 0.00, size = 17, normalized size = 0.94

$$-\frac{1}{3} b^2 c\, x^3 + a^2 c x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c),x)
[Out] a^2*c*x-1/3*b^2*c*x^3
```

**maxima [A]** time = 1.35, size = 16, normalized size = 0.89

$$-\frac{1}{3} b^2 c x^3 + a^2 c x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="maxima")
[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

**mupad [B]** time = 0.02, size = 18, normalized size = 1.00

$$\frac{c\,x\,\left(3\,a^2-b^2\,x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)*(a + b*x),x)
[Out] (c*x*(3*a^2 - b^2*x^2))/3
```

**sympy [A]** time = 0.06, size = 15, normalized size = 0.83

$$a^2cx - \frac{b^2cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x)
[Out] a**2*c*x - b**2*c*x**3/3
```

## 3.962 $\int (a+bx)\,dx$

**Optimal**. Leaf size=12

$$ax+\frac{bx^2}{2}$$

**Rubi [A]** time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x,x]
[Out] a*x + (b*x^2)/2
```

Rubi steps

$$\int (a+bx)\,dx=ax+\frac{bx^2}{2}$$

**Mathematica [A]** time = 0.00, size = 12, normalized size = 1.00

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x,x]
[Out] a*x + (b*x^2)/2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + b*x,x]
[Out] IntegrateAlgebraic[a + b*x, x]
```

**fricas** [**A**] time = 1.07, size = 10, normalized size = 0.83

$$\frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*b + x*a
```

**giac** [**A**] time = 0.86, size = 10, normalized size = 0.83

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

**maple** [**A**] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2}b\,x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a,x)
```

```
[Out] 1/2*b*x^2+a*x
```

**maxima** [**A**] time = 1.34, size = 10, normalized size = 0.83

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

**mupad** [**B**] time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^2}{2} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x,x)
```

```
[Out] a*x + (b*x^2)/2
```

**sympy [A]** time = 0.06, size = 8, normalized size = 0.67

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] a*x + b*x**2/2
```

## 3.963 $\int \frac{a+bx}{ac-bcx} \, dx$

**Optimal**. Leaf size=23

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x),x]
```

```
[Out] -(x/c) - (2*a*Log[a - b*x])/(b*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{ac-bcx} \, dx &= \int \left(-\frac{1}{c}+\frac{2a}{c(a-bx)}\right) dx\\ &= -\frac{x}{c}-\frac{2a\log(a-bx)}{bc}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x),x]
```

```
[Out] -(x/c) - (2*a*Log[a - b*x])/(b*c)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{ac-bcx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x),x]
[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x), x]
```

**fricas [A]** time = 1.31, size = 23, normalized size = 1.00

$$-\frac{bx+2\,a\log\left(bx-a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
[Out] -(b*x + 2*a*log(b*x - a))/(b*c)
```

**giac [A]** time = 0.92, size = 25, normalized size = 1.09

$$-\frac{x}{c}-\frac{2\,a\log\left(|bx-a|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="giac")
[Out] -x/c - 2*a*log(abs(b*x - a))/(b*c)
```

**maple [A]** time = 0.00, size = 25, normalized size = 1.09

$$-\frac{2a\ln\left(bx-a\right)}{bc}-\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c),x)
[Out] -x/c-2/c*a/b*ln(b*x-a)
```

**maxima [A]** time = 1.31, size = 24, normalized size = 1.04

$$-\frac{x}{c}-\frac{2\,a\log\left(bx-a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -x/c - 2*a*log(b*x - a)/(b*c)
```

**mupad [B]** time = 0.05, size = 23, normalized size = 1.00

$$-\frac{b\,x+2\,a\,\ln\left(b\,x-a\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x),x)
```

```
[Out] -(b*x + 2*a*log(b*x - a))/(b*c)
```

**sympy [A]** time = 0.14, size = 17, normalized size = 0.74

$$-\frac{2a\log\left(-a+bx\right)}{bc}-\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x)
```

```
[Out] -2*a*log(-a + b*x)/(b*c) - x/c
```

## 3.964 $\int \frac{a+bx}{(ac-bcx)^2} \, dx$

**Optimal**. Leaf size=32

$$\frac{2a}{bc^2(a-bx)}+\frac{\log(a-bx)}{bc^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2a}{bc^2(a-bx)}+\frac{\log(a-bx)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^2,x]
```

```
[Out] (2*a)/(b*c^2*(a - b*x)) + Log[a - b*x]/(b*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^2}\,dx &= \int \left(\frac{2a}{c^2(a-bx)^2}-\frac{1}{c^2(a-bx)}\right)dx\\ &= \frac{2a}{bc^2(a-bx)}+\frac{\log(a-bx)}{bc^2}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 28, normalized size = 0.88

$$\frac{\log(c(a-bx))+\frac{2a}{a-bx}}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^2,x]
```

[Out] ((2*a)/(a - b*x) + Log[c*(a - b*x)])/(b*c^2)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(ac-bcx)^2}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^2,x]

[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^2, x]

**fricas [A]** time = 1.25, size = 39, normalized size = 1.22

$$\frac{(bx-a)\log(bx-a)-2a}{b^2c^2x-abc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")

[Out] ((b*x - a)*log(b*x - a) - 2*a)/(b^2*c^2*x - a*b*c^2)

**giac [B]** time = 1.11, size = 81, normalized size = 2.53

$$-\frac{\frac{a}{(bcx-ac)b}+\frac{\log\left(\frac{|bcx-ac|}{(bcx-ac)^2|b||c|}\right)}{bc}}{c}-\frac{a}{(bcx-ac)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="giac")

[Out] -(a/((b*c*x - a*c)*b) + log(abs(b*c*x - a*c)/((b*c*x - a*c)^2*abs(b)*abs(c)
))/(b*c))/c - a/((b*c*x - a*c)*b*c)

**maple [A]** time = 0.01, size = 35, normalized size = 1.09

$$-\frac{2a}{(bx-a)\,b\,c^2}+\frac{\ln(bx-a)}{b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)/(-b*c*x+a*c)^2,x)

[Out] -2/c^2*a/b/(b*x-a)+1/c^2/b*ln(b*x-a)

**maxima** [A] time = 1.31, size = 37, normalized size = 1.16

$$-\frac{2a}{b^2c^2x - abc^2} + \frac{\log(bx - a)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")
[Out] -2*a/(b^2*c^2*x - a*b*c^2) + log(b*x - a)/(b*c^2)
```

**mupad** [B] time = 0.05, size = 37, normalized size = 1.16

$$\frac{\ln(b\,x - a)}{b\,c^2} + \frac{2\,a}{b\,\left(a\,c^2 - b\,c^2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^2,x)
[Out] log(b*x - a)/(b*c^2) + (2*a)/(b*(a*c^2 - b*c^2*x))
```

**sympy** [A] time = 0.19, size = 29, normalized size = 0.91

$$-\frac{2a}{-abc^2 + b^2c^2x} + \frac{\log(-a + bx)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**2,x)
[Out] -2*a/(-a*b*c**2 + b**2*c**2*x) + log(-a + b*x)/(b*c**2)
```

## 3.965 $\int \frac{a+bx}{(ac-bcx)^3} dx$

**Optimal**. Leaf size=13

$$\frac{x}{c^3(a-bx)^2}$$

**Rubi** [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {34}

$$\frac{x}{c^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^3,x]
```

```
[Out] x/(c^3*(a - b*x)^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int \frac{a+bx}{(ac-bcx)^3} dx = \frac{x}{c^3(a-bx)^2}$$

**Mathematica** [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{x}{c^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^3,x]
```

```
[Out] x/(c^3*(a - b*x)^2)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(ac-bcx)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^3, x]
```

**fricas [B]** time = 0.68, size = 30, normalized size = 2.31

$$\frac{x}{b^2c^3x^2 - 2\,abc^3x + a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

**giac [A]** time = 0.89, size = 14, normalized size = 1.08

$$\frac{x}{(bx-a)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] x/((b*x - a)^2*c^3)
```

**maple [B]** time = 0.00, size = 33, normalized size = 2.54

$$\frac{\frac{a}{(bx-a)^2b} + \frac{1}{(bx-a)b}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^3,x)
```

```
[Out] 1/c^3*(a/b/(b*x-a)^2+1/b/(b*x-a))
```

**maxima [B]** time = 1.31, size = 30, normalized size = 2.31

$$\frac{x}{b^2c^3x^2 - 2\,abc^3x + a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

**mupad [B]** time = 0.15, size = 13, normalized size = 1.00

$$\frac{x}{c^3\,{\left(a-b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^3,x)
[Out] x/(c^3*(a - b*x)^2)
```

**sympy [B]** time = 0.24, size = 27, normalized size = 2.08

$$\frac{x}{a^2c^3 - 2abc^3x + b^2c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**3,x)
[Out] x/(a**2*c**3 - 2*a*b*c**3*x + b**2*c**3*x**2)
```

## 3.966 $\int \frac{a+bx}{(ac-bcx)^4} \, dx$

**Optimal**. Leaf size=38

$$\frac{2a}{3bc^4(a-bx)^3} - \frac{1}{2bc^4(a-bx)^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2a}{3bc^4(a-bx)^3} - \frac{1}{2bc^4(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^4,x]
```

```
[Out] (2*a)/(3*b*c^4*(a - b*x)^3) - 1/(2*b*c^4*(a - b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^4} \, dx &= \int \left(\frac{2a}{c^4(a-bx)^4} - \frac{1}{c^4(a-bx)^3}\right) dx \\ &= \frac{2a}{3bc^4(a-bx)^3} - \frac{1}{2bc^4(a-bx)^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{a+3bx}{6bc^4(bx-a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^4,x]
```

```
[Out] -1/6*(a + 3*b*x)/(b*c^4*(-a + b*x)^3)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(ac-bcx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^4, x]
```

**fricas** [A] time = 1.24, size = 54, normalized size = 1.42

$$-\frac{3\,bx+a}{6\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*c
^4)
```

**giac** [A] time = 0.98, size = 23, normalized size = 0.61

$$-\frac{3\,bx+a}{6\,(bx-a)^3bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x + a)/((b*x - a)^3*b*c^4)
```

**maple** [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{-\frac{2a}{3(bx-a)^3b}-\frac{1}{2(bx-a)^2b}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^4,x)
```

```
[Out] 1/c^4*(-1/2/b/(b*x-a)^2-2/3*a/b/(b*x-a)^3)
```

**maxima** [A] time = 1.34, size = 54, normalized size = 1.42

$$-\frac{3\,bx+a}{6\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*c
^4)
```

**mupad [B]** time = 0.05, size = 54, normalized size = 1.42

$$\frac{\frac{x}{2}+\frac{a}{6\,b}}{a^3\,c^4-3\,a^2\,b\,c^4\,x+3\,a\,b^2\,c^4\,x^2-b^3\,c^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^4,x)
```

```
[Out] (x/2 + a/(6*b))/(a^3*c^4 - b^3*c^4*x^3 + 3*a*b^2*c^4*x^2 - 3*a^2*b*c^4*x)
```

**sympy [A]** time = 0.31, size = 56, normalized size = 1.47

$$\frac{-a-3bx}{-6a^3bc^4+18a^2b^2c^4x-18ab^3c^4x^2+6b^4c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**4,x)
```

```
[Out] (-a - 3*b*x)/(-6*a**3*b*c**4 + 18*a**2*b**2*c**4*x - 18*a*b**3*c**4*x**2 +
6*b**4*c**4*x**3)
```

## 3.967 $\int \frac{a+bx}{(ac-bcx)^5} dx$

**Optimal**. Leaf size=38

$$\frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}$$

**Rubi [A]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^5,x]
[Out] a/(2*b*c^5*(a - b*x)^4) - 1/(3*b*c^5*(a - b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^5} dx &= \int \left(\frac{2a}{c^5(a-bx)^5}-\frac{1}{c^5(a-bx)^4}\right) dx\\ &= \frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 0.63

$$\frac{a+2bx}{6bc^5(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^5,x]
[Out] (a + 2*b*x)/(6*b*c^5*(a - b*x)^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(ac-bcx)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^5, x]
```

**fricas [A]** time = 1.41, size = 67, normalized size = 1.76

$$\frac{2\,bx+a}{6\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3*
b^2*c^5*x + a^4*b*c^5)
```

**giac [A]** time = 0.95, size = 40, normalized size = 1.05

$$\frac{a}{2\,(bcx-ac)^4bc}+\frac{1}{3\,(bcx-ac)^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="giac")
```

```
[Out] 1/2*a/((b*c*x - a*c)^4*b*c) + 1/3/((b*c*x - a*c)^3*b*c^2)
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.92

$$\frac{\frac{a}{2(bx-a)^4b}+\frac{1}{3(bx-a)^3b}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^5,x)
```

```
[Out] 1/c^5*(1/2*a/b/(b*x-a)^4+1/3/b/(b*x-a)^3)
```

**maxima [A]** time = 1.35, size = 67, normalized size = 1.76

$$\frac{2\,bx+a}{6\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3*
b^2*c^5*x + a^4*b*c^5)
```

**mupad [B]** time = 0.17, size = 67, normalized size = 1.76

$$\frac{\frac{x}{3}+\frac{a}{6\,b}}{a^4\,c^5-4\,a^3\,b\,c^5\,x+6\,a^2\,b^2\,c^5\,x^2-4\,a\,b^3\,c^5\,x^3+b^4\,c^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^5,x)
```

```
[Out] (x/3 + a/(6*b))/(a^4*c^5 + b^4*c^5*x^4 - 4*a*b^3*c^5*x^3 + 6*a^2*b^2*c^5*x^
2 - 4*a^3*b*c^5*x)
```

**sympy [B]** time = 0.39, size = 73, normalized size = 1.92

$$-\frac{-a-2bx}{6a^4bc^5-24a^3b^2c^5x+36a^2b^3c^5x^2-24ab^4c^5x^3+6b^5c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**5,x)
```

```
[Out] -(-a - 2*b*x)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36*a**2*b**3*c**5*x**2
 - 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

## 3.968 $\int \frac{a+bx}{(ac-bcx)^6} \, dx$

**Optimal**. Leaf size=38

$$\frac{2a}{5bc^6(a-bx)^5} - \frac{1}{4bc^6(a-bx)^4}$$

**Rubi [A]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2a}{5bc^6(a-bx)^5} - \frac{1}{4bc^6(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^6,x]
```

```
[Out] (2*a)/(5*b*c^6*(a - b*x)^5) - 1/(4*b*c^6*(a - b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^6} \, dx &= \int \left(\frac{2a}{c^6(a-bx)^6} - \frac{1}{c^6(a-bx)^5}\right) dx\\ &= \frac{2a}{5bc^6(a-bx)^5} - \frac{1}{4bc^6(a-bx)^4}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 27, normalized size = 0.71

$$-\frac{3a+5bx}{20bc^6(bx-a)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^6,x]
```

```
[Out] -1/20*(3*a + 5*b*x)/(b*c^6*(-a + b*x)^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx}{(ac - bcx)^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^6, x]
```

**fricas [B]** time = 1.21, size = 84, normalized size = 2.21

$$-\frac{5\,bx + 3\,a}{20\left(b^6c^6x^5 - 5\,ab^5c^6x^4 + 10\,a^2b^4c^6x^3 - 10\,a^3b^3c^6x^2 + 5\,a^4b^2c^6x - a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 - 1
0*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

**giac [A]** time = 1.04, size = 25, normalized size = 0.66

$$-\frac{5\,bx + 3\,a}{20\,(bx - a)^5bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="giac")
```

```
[Out] -1/20*(5*b*x + 3*a)/((b*x - a)^5*b*c^6)
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.92

$$\frac{-\frac{2a}{5(bx-a)^5b} - \frac{1}{4(bx-a)^4b}}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^6,x)
```

```
[Out] 1/c^6*(-1/4/b/(b*x-a)^4-2/5*a/b/(b*x-a)^5)
```

**maxima [B]** time = 1.31, size = 84, normalized size = 2.21

$$-\frac{5\,bx + 3\,a}{20\left(b^6c^6x^5 - 5\,ab^5c^6x^4 + 10\,a^2b^4c^6x^3 - 10\,a^3b^3c^6x^2 + 5\,a^4b^2c^6x - a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 - 1
0*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

**mupad [B]** time = 0.08, size = 82, normalized size = 2.16

$$\frac{\frac{x}{4}+\frac{3\,a}{20\,b}}{a^5\,c^6-5\,a^4\,b\,c^6\,x+10\,a^3\,b^2\,c^6\,x^2-10\,a^2\,b^3\,c^6\,x^3+5\,a\,b^4\,c^6\,x^4-b^5\,c^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^6,x)
```

```
[Out] (x/4 + (3*a)/(20*b))/(a^5*c^6 - b^5*c^6*x^5 + 5*a*b^4*c^6*x^4 + 10*a^3*b^2*
c^6*x^2 - 10*a^2*b^3*c^6*x^3 - 5*a^4*b*c^6*x)
```

**sympy [B]** time = 0.46, size = 88, normalized size = 2.32

$$\frac{-3a-5bx}{-20a^5bc^6+100a^4b^2c^6x-200a^3b^3c^6x^2+200a^2b^4c^6x^3-100ab^5c^6x^4+20b^6c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**6,x)
```

```
[Out] (-3*a - 5*b*x)/(-20*a**5*b*c**6 + 100*a**4*b**2*c**6*x - 200*a**3*b**3*c**6
*x**2 + 200*a**2*b**4*c**6*x**3 - 100*a*b**5*c**6*x**4 + 20*b**6*c**6*x**5)
```

## 3.969 $\int (a+bx)^2(ac-bcx)^3\,dx$

**Optimal**. Leaf size=57

$$-\frac{a^2c^3(a-bx)^4}{b}-\frac{c^3(a-bx)^6}{6b}+\frac{4ac^3(a-bx)^5}{5b}$$

**Rubi** [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{a^2c^3(a-bx)^4}{b}-\frac{c^3(a-bx)^6}{6b}+\frac{4ac^3(a-bx)^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] -((a^2*c^3*(a - b*x)^4)/b) + (4*a*c^3*(a - b*x)^5)/(5*b) - (c^3*(a - b*x)^6
)/(6*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^3\,dx &= \int \left(4a^2(ac-bcx)^3-\frac{4a(ac-bcx)^4}{c}+\frac{(ac-bcx)^5}{c^2}\right)dx\\ &= -\frac{a^2c^3(a-bx)^4}{b}+\frac{4ac^3(a-bx)^5}{5b}-\frac{c^3(a-bx)^6}{6b}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 68, normalized size = 1.19

$$c^3\left(a^5x-\frac{1}{2}a^4bx^2-\frac{2}{3}a^3b^2x^3+\frac{1}{2}a^2b^3x^4+\frac{1}{5}ab^4x^5-\frac{1}{6}b^5x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] c^3*(a^5*x - (a^4*b*x^2)/2 - (2*a^3*b^2*x^3)/3 + (a^2*b^3*x^4)/2 + (a*b^4*x
^5)/5 - (b^5*x^6)/6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(ac-bcx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x)^3, x]
```

**fricas [A]** time = 1.14, size = 72, normalized size = 1.26

$$-\frac{1}{6}x^6c^3b^5+\frac{1}{5}x^5c^3b^4a+\frac{1}{2}x^4c^3b^3a^2-\frac{2}{3}x^3c^3b^2a^3-\frac{1}{2}x^2c^3ba^4+xc^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/6*x^6*c^3*b^5 + 1/5*x^5*c^3*b^4*a + 1/2*x^4*c^3*b^3*a^2 - 2/3*x^3*c^3*b^
2*a^3 - 1/2*x^2*c^3*b*a^4 + x*c^3*a^5
```

**giac [A]** time = 1.04, size = 72, normalized size = 1.26

$$-\frac{1}{6}\,b^5c^3x^6+\frac{1}{5}\,ab^4c^3x^5+\frac{1}{2}\,a^2b^3c^3x^4-\frac{2}{3}\,a^3b^2c^3x^3-\frac{1}{2}\,a^4bc^3x^2+a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

**maple [A]** time = 0.00, size = 73, normalized size = 1.28

$$-\frac{1}{6}b^5c^3x^6+\frac{1}{5}a\,b^4c^3x^5+\frac{1}{2}a^2b^3c^3x^4-\frac{2}{3}a^3b^2c^3x^3-\frac{1}{2}a^4b\,c^3x^2+a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^3,x)
```

```
[Out] -1/6*b^5*c^3*x^6+1/5*a*b^4*c^3*x^5+1/2*a^2*b^3*c^3*x^4-2/3*a^3*c^3*b^2*x^3-
1/2*a^4*c^3*b*x^2+a^5*c^3*x
```

**maxima** [A] time = 1.29, size = 72, normalized size = 1.26

$$-\frac{1}{6}\,b^{5}c^{3}x^{6}+\frac{1}{5}\,ab^{4}c^{3}x^{5}+\frac{1}{2}\,a^{2}b^{3}c^{3}x^{4}-\frac{2}{3}\,a^{3}b^{2}c^{3}x^{3}-\frac{1}{2}\,a^{4}bc^{3}x^{2}+a^{5}c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

**mupad** [B] time = 0.03, size = 72, normalized size = 1.26

$$a^5\,c^3\,x-\frac{a^4\,b\,c^3\,x^2}{2}-\frac{2\,a^3\,b^2\,c^3\,x^3}{3}+\frac{a^2\,b^3\,c^3\,x^4}{2}+\frac{a\,b^4\,c^3\,x^5}{5}-\frac{b^5\,c^3\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3*(a + b*x)^2,x)
```

```
[Out] a^5*c^3*x - (b^5*c^3*x^6)/6 - (a^4*b*c^3*x^2)/2 + (a*b^4*c^3*x^5)/5 - (2*a^
3*b^2*c^3*x^3)/3 + (a^2*b^3*c^3*x^4)/2
```

**sympy** [A] time = 0.09, size = 78, normalized size = 1.37

$$a^{5}c^{3}x-\frac{a^{4}bc^{3}x^{2}}{2}-\frac{2a^{3}b^{2}c^{3}x^{3}}{3}+\frac{a^{2}b^{3}c^{3}x^{4}}{2}+\frac{ab^{4}c^{3}x^{5}}{5}-\frac{b^{5}c^{3}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**3,x)
```

```
[Out] a**5*c**3*x - a**4*b*c**3*x**2/2 - 2*a**3*b**2*c**3*x**3/3 + a**2*b**3*c**3
*x**4/2 + a*b**4*c**3*x**5/5 - b**5*c**3*x**6/6
```

## 3.970 $\int (a+bx)^2(ac-bcx)^2\,dx$

**Optimal**. Leaf size=38

$$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$$

**Rubi [A]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {41, 194}

$$-\frac{2}{3}a^2b^2c^2x^3+a^4c^2x+\frac{1}{5}b^4c^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

```
[Out] a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^2\,dx &= \int \left(a^2c-b^2cx^2\right)^2\,dx\\ &= \int \left(a^4c^2-2a^2b^2c^2x^2+b^4c^2x^4\right)dx\\ &= a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 38, normalized size = 1.00

$$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^2,x]
[Out] a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(ac-bcx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x)^2,x]
[Out] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x)^2, x]
```

**fricas [A]** time = 0.49, size = 34, normalized size = 0.89

$$\frac{1}{5}x^5c^2b^4 - \frac{2}{3}x^3c^2b^2a^2 + xc^2a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="fricas")
[Out] 1/5*x^5*c^2*b^4 - 2/3*x^3*c^2*b^2*a^2 + x*c^2*a^4
```

**giac [A]** time = 1.10, size = 34, normalized size = 0.89

$$\frac{1}{5}\,b^4c^2x^5 - \frac{2}{3}\,a^2b^2c^2x^3 + a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="giac")
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.92

$$\frac{1}{5}b^4c^2x^5 - \frac{2}{3}a^2b^2c^2x^3 + a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^2,x)
[Out] a^4*c^2*x-2/3*a^2*b^2*c^2*x^3+1/5*b^4*c^2*x^5
```

**maxima** [A] time = 1.34, size = 34, normalized size = 0.89

$$\frac{1}{5}\,b^4c^2x^5 - \frac{2}{3}\,a^2b^2c^2x^3 + a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

**mupad** [B] time = 0.04, size = 31, normalized size = 0.82

$$\frac{c^2\,x\,\left(15\,a^4-10\,a^2\,b^2\,x^2+3\,b^4\,x^4\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2*(a + b*x)^2,x)
```

```
[Out] (c^2*x*(15*a^4 + 3*b^4*x^4 - 10*a^2*b^2*x^2))/15
```

**sympy** [A] time = 0.08, size = 36, normalized size = 0.95

$$a^4c^2x - \frac{2a^2b^2c^2x^3}{3} + \frac{b^4c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**2,x)
```

```
[Out] a**4*c**2*x - 2*a**2*b**2*c**2*x**3/3 + b**4*c**2*x**5/5
```

## 3.971 $\int (a+bx)^2(ac-bcx)\,dx$

**Optimal**. Leaf size=32

$$\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

**Rubi [A]** time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x),x]
```

```
[Out] (2*a*c*(a + b*x)^3)/(3*b) - (c*(a + b*x)^4)/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)\,dx &= \int \left(2ac(a+bx)^2-c(a+bx)^3\right)\,dx\\ &= \frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 40, normalized size = 1.25

$$c\left(a^3x+\frac{1}{2}a^2bx^2-\frac{1}{3}ab^2x^3-\frac{1}{4}b^3x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x),x]
```

```
[Out] c*(a^3*x + (a^2*b*x^2)/2 - (a*b^2*x^3)/3 - (b^3*x^4)/4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(ac-bcx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x),x]
[Out] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x), x]
```

**fricas [A]** time = 1.13, size = 36, normalized size = 1.12

$$-\frac{1}{4}x^4cb^3-\frac{1}{3}x^3cb^2a+\frac{1}{2}x^2cba^2+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="fricas")
[Out] -1/4*x^4*c*b^3 - 1/3*x^3*c*b^2*a + 1/2*x^2*c*b*a^2 + x*c*a^3
```

**giac [A]** time = 1.02, size = 36, normalized size = 1.12

$$-\frac{1}{4}\,b^3cx^4-\frac{1}{3}\,ab^2cx^3+\frac{1}{2}\,a^2bcx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="giac")
[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

**maple [A]** time = 0.00, size = 37, normalized size = 1.16

$$-\frac{1}{4}b^3c\,x^4-\frac{1}{3}a\,b^2c\,x^3+\frac{1}{2}a^2bc\,x^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c),x)
[Out] -1/4*b^3*c*x^4-1/3*a*b^2*c*x^3+1/2*a^2*b*c*x^2+a^3*c*x
```

**maxima [A]** time = 1.32, size = 36, normalized size = 1.12

$$-\frac{1}{4}\,b^3cx^4-\frac{1}{3}\,ab^2cx^3+\frac{1}{2}\,a^2bcx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="maxima")
[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

**mupad** [**B**] time = 0.05, size = 36, normalized size = 1.12

$$c\,a^3\,x+\frac{c\,a^2\,b\,x^2}{2}-\frac{c\,a\,b^2\,x^3}{3}-\frac{c\,b^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)*(a + b*x)^2,x)
[Out] a^3*c*x - (b^3*c*x^4)/4 + (a^2*b*c*x^2)/2 - (a*b^2*c*x^3)/3
```

**sympy** [**A**] time = 0.07, size = 39, normalized size = 1.22

$$a^{3}cx+\frac{a^{2}bcx^{2}}{2}-\frac{ab^{2}cx^{3}}{3}-\frac{b^{3}cx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c),x)
[Out] a**3*c*x + a**2*b*c*x**2/2 - a*b**2*c*x**3/3 - b**3*c*x**4/4
```

## 3.972 $\int (a+bx)^2 \, dx$

**Optimal**. Leaf size=14

$$\frac{(a+bx)^3}{3b}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2,x]
```

```
[Out] (a + b*x)^3/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^2 \, dx = \frac{(a+bx)^3}{3b}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2,x]
```

```
[Out] (a + b*x)^3/(3*b)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2,x]
[Out] IntegrateAlgebraic[(a + b*x)^2, x]
```

**fricas [A]** time = 1.13, size = 20, normalized size = 1.43

$$\frac{1}{3}x^3b^2 + x^2ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="fricas")
[Out] 1/3*x^3*b^2 + x^2*b*a + x*a^2
```

**giac [A]** time = 0.97, size = 12, normalized size = 0.86

$$\frac{(bx + a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="giac")
[Out] 1/3*(b*x + a)^3/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx + a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2,x)
[Out] 1/3*(b*x+a)^3/b
```

**maxima [A]** time = 1.27, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

**mupad [B]** time = 0.03, size = 20, normalized size = 1.43

$$a^2\,x+a\,b\,x^2+\frac{b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2,x)
[Out] a^2*x + (b^2*x^3)/3 + a*b*x^2
```

**sympy [B]** time = 0.06, size = 19, normalized size = 1.36

$$a^2x+abx^2+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2,x)
[Out] a**2*x + a*b*x**2 + b**2*x**3/3
```

## 3.973 $\int \frac{(a+bx)^2}{ac-bcx}\,dx$

**Optimal**. Leaf size=43

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{(a+bx)^2}{2bc}-\frac{2ax}{c}$$

**Rubi [A]** time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{(a+bx)^2}{2bc}-\frac{2ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x),x]
```

```
[Out] (-2*a*x)/c - (a + b*x)^2/(2*b*c) - (4*a^2*Log[a - b*x])/(b*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{ac-bcx}\,dx &= \int \left(-\frac{2a}{c}-\frac{a+bx}{c}+\frac{4a^2}{ac-bcx}\right)dx\\ &= -\frac{2ax}{c}-\frac{(a+bx)^2}{2bc}-\frac{4a^2\log(a-bx)}{bc}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 37, normalized size = 0.86

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{3ax}{c}-\frac{bx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x),x]
```

```
[Out] (-3*a*x)/c - (b*x^2)/(2*c) - (4*a^2*Log[a - b*x])/(b*c)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{ac-bcx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x),x]
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x), x]
```

**fricas [A]** time = 1.23, size = 34, normalized size = 0.79

$$-\frac{b^2x^2+6\,abx+8\,a^2\log\left(bx-a\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
[Out] -1/2*(b^2*x^2 + 6*a*b*x + 8*a^2*log(b*x - a))/(b*c)
```

**giac [A]** time = 0.87, size = 46, normalized size = 1.07

$$-\frac{4\,a^2\log\left(|bx-a|\right)}{bc}-\frac{b^3cx^2+6\,ab^2cx}{2\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="giac")
[Out] -4*a^2*log(abs(b*x - a))/(b*c) - 1/2*(b^3*c*x^2 + 6*a*b^2*c*x)/(b^2*c^2)
```

**maple [A]** time = 0.00, size = 37, normalized size = 0.86

$$-\frac{b\,x^2}{2c}-\frac{4a^2\ln\left(bx-a\right)}{bc}-\frac{3ax}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c),x)
[Out] -1/2/c*x^2*b-3*a*x/c-4/c*a^2/b*ln(b*x-a)
```

**maxima [A]** time = 1.29, size = 35, normalized size = 0.81

$$-\frac{4\,a^2\log\left(bx-a\right)}{bc}-\frac{bx^2+6\,ax}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -4*a^2*log(b*x - a)/(b*c) - 1/2*(b*x^2 + 6*a*x)/c
```

**mupad [B]** time = 0.05, size = 34, normalized size = 0.79

$$-\frac{8\,a^2\,\ln\left(b\,x-a\right)+b^2\,x^2+6\,a\,b\,x}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x),x)
```

```
[Out] -(8*a^2*log(b*x - a) + b^2*x^2 + 6*a*b*x)/(2*b*c)
```

**sympy [A]** time = 0.17, size = 31, normalized size = 0.72

$$-\frac{4a^2\log\left(-a+bx\right)}{bc}-\frac{3ax}{c}-\frac{bx^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c),x)
```

```
[Out] -4*a**2*log(-a + b*x)/(b*c) - 3*a*x/c - b*x**2/(2*c)
```

## 3.974 $\int \frac{(a+bx)^2}{(ac-bcx)^2}\,dx$

**Optimal**. Leaf size=41

$$\frac{4a^2}{bc^2(a-bx)}+\frac{4a\log(a-bx)}{bc^2}+\frac{x}{c^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{4a^2}{bc^2(a-bx)}+\frac{4a\log(a-bx)}{bc^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

```
[Out] x/c^2 + (4*a^2)/(b*c^2*(a - b*x)) + (4*a*Log[a - b*x])/(b*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^2}\,dx &= \int \left(\frac{1}{c^2}+\frac{4a^2}{c^2(a-bx)^2}-\frac{4a}{c^2(a-bx)}\right)dx\\ &= \frac{x}{c^2}+\frac{4a^2}{bc^2(a-bx)}+\frac{4a\log(a-bx)}{bc^2}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 35, normalized size = 0.85

$$\frac{\frac{4a^2}{b(a-bx)}+\frac{4a\log(a-bx)}{b}+x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

```
[Out] (x + (4*a^2)/(b*(a - b*x)) + (4*a*Log[a - b*x])/b)/c^2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^2, x]
```

**fricas [A]** time = 0.60, size = 57, normalized size = 1.39

$$\frac{b^2x^2 - abx - 4\,a^2 + 4\left(abx - a^2\right)\log\left(bx - a\right)}{b^2c^2x - abc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 - a*b*x - 4*a^2 + 4*(a*b*x - a^2)*log(b*x - a))/(b^2*c^2*x - a*b*c
^2)
```

**giac [A]** time = 1.08, size = 79, normalized size = 1.93

$$-\frac{4\,a^2}{(bcx-ac)bc} - \frac{4\,a\log\left(\frac{|bcx-ac|}{(bcx-ac)^2|b||c|}\right)}{bc^2} + \frac{bcx-ac}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -4*a^2/((b*c*x - a*c)*b*c) - 4*a*log(abs(b*c*x - a*c)/((b*c*x - a*c)^2*abs(
b)*abs(c)))/(b*c^2) + (b*c*x - a*c)/(b*c^3)
```

**maple [A]** time = 0.01, size = 44, normalized size = 1.07

$$-\frac{4a^2}{(bx-a)\,b\,c^2} + \frac{4a\ln\left(bx-a\right)}{b\,c^2} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^2,x)
```

```
[Out] x/c^2-4/c^2*a^2/b/(b*x-a)+4/c^2*a/b*ln(b*x-a)
```

**maxima** [A] time = 1.38, size = 46, normalized size = 1.12

$$-\frac{4\,a^2}{b^2c^2x - abc^2} + \frac{x}{c^2} + \frac{4\,a\log\left(bx - a\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -4*a^2/(b^2*c^2*x - a*b*c^2) + x/c^2 + 4*a*log(b*x - a)/(b*c^2)
```

**mupad** [B] time = 0.15, size = 46, normalized size = 1.12

$$\frac{x}{c^2} + \frac{4\,a^2}{b\,\left(a\,c^2 - b\,c^2\,x\right)} + \frac{4\,a\,\ln\left(b\,x - a\right)}{b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^2,x)
```

```
[Out] x/c^2 + (4*a^2)/(b*(a*c^2 - b*c^2*x)) + (4*a*log(b*x - a))/(b*c^2)
```

**sympy** [A] time = 0.20, size = 39, normalized size = 0.95

$$-\frac{4a^2}{-abc^2 + b^2c^2x} + \frac{4a\log\left(-a + bx\right)}{bc^2} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**2,x)
```

```
[Out] -4*a**2/(-a*b*c**2 + b**2*c**2*x) + 4*a*log(-a + b*x)/(b*c**2) + x/c**2
```

## 3.975 $\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx$

**Optimal**. Leaf size=52

$$\frac{2a^2}{bc^3(a-bx)^2}-\frac{4a}{bc^3(a-bx)}-\frac{\log(a-bx)}{bc^3}$$

**Rubi [A]** time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{2a^2}{bc^3(a-bx)^2}-\frac{4a}{bc^3(a-bx)}-\frac{\log(a-bx)}{bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^3,x]
```

```
[Out] (2*a^2)/(b*c^3*(a - b*x)^2) - (4*a)/(b*c^3*(a - b*x)) - Log[a - b*x]/(b*c^3
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx &= \int \left(\frac{4a^2}{c^3(a-bx)^3}-\frac{4a}{c^3(a-bx)^2}+\frac{1}{c^3(a-bx)}\right)dx\\ &= \frac{2a^2}{bc^3(a-bx)^2}-\frac{4a}{bc^3(a-bx)}-\frac{\log(a-bx)}{bc^3}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 33, normalized size = 0.63

$$-\frac{\frac{2a(a-2bx)}{(a-bx)^2}+\log(a-bx)}{bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^3,x]
[Out] -(((2*a*(a - 2*b*x))/(a - b*x)^2 + Log[a - b*x])/(b*c^3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^3, x]
```

**fricas [A]** time = 1.03, size = 69, normalized size = 1.33

$$\frac{4\,abx-2\,a^2-\left(b^2x^2-2\,abx+a^2\right)\log\left(bx-a\right)}{b^3c^3x^2-2\,ab^2c^3x+a^2bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
[Out] (4*a*b*x - 2*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x - a))/(b^3*c^3*x^2 - 2
*a*b^2*c^3*x + a^2*b*c^3)
```

**giac [A]** time = 1.07, size = 46, normalized size = 0.88

$$-\frac{\log\left(|bx-a|\right)}{bc^3}+\frac{2\left(2\,abx-a^2\right)}{(bx-a)^2bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="giac")
[Out] -log(abs(b*x - a))/(b*c^3) + 2*(2*a*b*x - a^2)/((b*x - a)^2*b*c^3)
```

**maple [A]** time = 0.01, size = 56, normalized size = 1.08

$$\frac{2a^2}{(bx-a)^2\,b\,c^3}+\frac{4a}{(bx-a)\,b\,c^3}-\frac{\ln\left(bx-a\right)}{b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^3,x)
[Out] 2/c^3*a^2/b/(b*x-a)^2+4/c^3*a/b/(b*x-a)-1/c^3/b*ln(b*x-a)
```

**maxima** [A] time = 1.33, size = 61, normalized size = 1.17

$$\frac{2\left(2\,abx - a^2\right)}{b^3c^3x^2 - 2\,ab^2c^3x + a^2bc^3} - \frac{\log\left(bx - a\right)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 2*(2*a*b*x - a^2)/(b^3*c^3*x^2 - 2*a*b^2*c^3*x + a^2*b*c^3) - log(b*x - a)/
(b*c^3)
```

**mupad** [B] time = 0.17, size = 59, normalized size = 1.13

$$\frac{4\,a\,x - \frac{2\,a^2}{b}}{a^2\,c^3 - 2\,a\,b\,c^3\,x + b^2\,c^3\,x^2} - \frac{\ln\left(b\,x - a\right)}{b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^3,x)
```

```
[Out] (4*a*x - (2*a^2)/b)/(a^2*c^3 + b^2*c^3*x^2 - 2*a*b*c^3*x) - log(b*x - a)/(b
*c^3)
```

**sympy** [A] time = 0.31, size = 54, normalized size = 1.04

$$-\frac{2a^2 - 4abx}{a^2bc^3 - 2ab^2c^3x + b^3c^3x^2} - \frac{\log\left(-a + bx\right)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**3,x)
```

```
[Out] -(2*a**2 - 4*a*b*x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2) - log(
-a + b*x)/(b*c**3)
```

## 3.976 $\int \frac{(a+bx)^2}{(ac-bcx)^4} dx$

**Optimal**. Leaf size=28

$$\frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

**Rubi** [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] (a + b*x)^3/(6*a*b*c^4*(a - b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^2}{(ac-bcx)^4} dx = \frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

**Mathematica** [A] time = 0.02, size = 31, normalized size = 1.11

$$-\frac{a^2+3b^2x^2}{3bc^4(bx-a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] -1/3*(a^2 + 3*b^2*x^2)/(b*c^4*(-a + b*x)^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^4, x]
```

**fricas [B]** time = 1.30, size = 60, normalized size = 2.14

$$-\frac{3\,b^2x^2+a^2}{3\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a
^3*b*c^4)
```

**giac [A]** time = 0.98, size = 29, normalized size = 1.04

$$-\frac{3\,b^2x^2+a^2}{3\,(bx-a)^3bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/((b*x - a)^3*b*c^4)
```

**maple [A]** time = 0.00, size = 52, normalized size = 1.86

$$\frac{-\frac{4a^2}{3(bx-a)^3b}-\frac{2a}{(bx-a)^2b}-\frac{1}{(bx-a)b}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^4,x)
```

```
[Out] 1/c^4*(-2/(b*x-a)^2*a/b-4/3*a^2/b/(b*x-a)^3-1/(b*x-a)/b)
```

**maxima [B]** time = 1.33, size = 60, normalized size = 2.14

$$-\frac{3\,b^2x^2+a^2}{3\,\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a
^3*b*c^4)
```

**mupad [B]** time = 0.05, size = 58, normalized size = 2.07

$$\frac{b\,x^2+\frac{a^2}{3\,b}}{a^3\,c^4-3\,a^2\,b\,c^4\,x+3\,a\,b^2\,c^4\,x^2-b^3\,c^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^4,x)
```

```
[Out] (b*x^2 + a^2/(3*b))/(a^3*c^4 - b^3*c^4*x^3 + 3*a*b^2*c^4*x^2 - 3*a^2*b*c^4*
x)
```

**sympy [B]** time = 0.35, size = 61, normalized size = 2.18

$$\frac{-a^2-3b^2x^2}{-3a^3bc^4+9a^2b^2c^4x-9ab^3c^4x^2+3b^4c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**4,x)
```

```
[Out] (-a**2 - 3*b**2*x**2)/(-3*a**3*b*c**4 + 9*a**2*b**2*c**4*x - 9*a*b**3*c**4*
x**2 + 3*b**4*c**4*x**3)
```

## 3.977 $\int \frac{(a+bx)^2}{(ac-bcx)^5} dx$

**Optimal**. Leaf size=56

$$\frac{a^2}{bc^5(a-bx)^4}-\frac{4a}{3bc^5(a-bx)^3}+\frac{1}{2bc^5(a-bx)^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{a^2}{bc^5(a-bx)^4}-\frac{4a}{3bc^5(a-bx)^3}+\frac{1}{2bc^5(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^5,x]
```

```
[Out] a^2/(b*c^5*(a - b*x)^4) - (4*a)/(3*b*c^5*(a - b*x)^3) + 1/(2*b*c^5*(a - b*x
)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^5} dx &= \int \left(\frac{4a^2}{c^5(a-bx)^5}-\frac{4a}{c^5(a-bx)^4}+\frac{1}{c^5(a-bx)^3}\right) dx\\ &= \frac{a^2}{bc^5(a-bx)^4}-\frac{4a}{3bc^5(a-bx)^3}+\frac{1}{2bc^5(a-bx)^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 35, normalized size = 0.62

$$\frac{a^2+2abx+3b^2x^2}{6bc^5(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^5,x]
```

```
[Out] (a^2 + 2*a*b*x + 3*b^2*x^2)/(6*b*c^5*(a - b*x)^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^5, x]
```

**fricas [A]** time = 1.16, size = 78, normalized size = 1.39

$$\frac{3\,b^2x^2+2\,abx+a^2}{6\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*
c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

**giac [A]** time = 1.07, size = 64, normalized size = 1.14

$$\frac{\frac{6\,a^2}{(bcx-ac)^4b}+\frac{8\,a}{(bcx-ac)^3bc}+\frac{3}{(bcx-ac)^2bc^2}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="giac")
```

```
[Out] 1/6*(6*a^2/((b*c*x - a*c)^4*b) + 8*a/((b*c*x - a*c)^3*b*c) + 3/((b*c*x - a*
c)^2*b*c^2))/c
```

**maple [A]** time = 0.00, size = 51, normalized size = 0.91

$$\frac{\frac{a^2}{(bx-a)^4b}+\frac{4a}{3(bx-a)^3b}+\frac{1}{2(bx-a)^2b}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^5,x)
```

```
[Out] 1/c^5*(a^2/b/(b*x-a)^4+1/2/(b*x-a)^2/b+4/3/(b*x-a)^3*a/b)
```

**maxima** [A] time = 1.38, size = 78, normalized size = 1.39

$$\frac{3\,b^2x^2 + 2\,abx + a^2}{6\left(b^5c^5x^4 - 4\,ab^4c^5x^3 + 6\,a^2b^3c^5x^2 - 4\,a^3b^2c^5x + a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*
c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

**mupad** [B] time = 0.05, size = 76, normalized size = 1.36

$$\frac{\frac{a\,x}{3} + \frac{b\,x^2}{2} + \frac{a^2}{6\,b}}{a^4\,c^5 - 4\,a^3\,b\,c^5\,x + 6\,a^2\,b^2\,c^5\,x^2 - 4\,a\,b^3\,c^5\,x^3 + b^4\,c^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^5,x)
```

```
[Out] ((a*x)/3 + (b*x^2)/2 + a^2/(6*b))/(a^4*c^5 + b^4*c^5*x^4 - 4*a*b^3*c^5*x^3
+ 6*a^2*b^2*c^5*x^2 - 4*a^3*b*c^5*x)
```

**sympy** [A] time = 0.44, size = 85, normalized size = 1.52

$$-\frac{-a^2 - 2abx - 3b^2x^2}{6a^4bc^5 - 24a^3b^2c^5x + 36a^2b^3c^5x^2 - 24ab^4c^5x^3 + 6b^5c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**5,x)
```

```
[Out] -(-a**2 - 2*a*b*x - 3*b**2*x**2)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36*
a**2*b**3*c**5*x**2 - 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

## 3.978 $\int \frac{(a+bx)^2}{(ac-bcx)^6} dx$

**Optimal**. Leaf size=57

$$\frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}$$

**Rubi** [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] (4*a^2)/(5*b*c^6*(a - b*x)^5) - a/(b*c^6*(a - b*x)^4) + 1/(3*b*c^6*(a - b*x
)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx &= \int \left(\frac{4a^2}{c^6(a-bx)^6}-\frac{4a}{c^6(a-bx)^5}+\frac{1}{c^6(a-bx)^4}\right)dx\\ &= \frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 38, normalized size = 0.67

$$-\frac{2a^2+5abx+5b^2x^2}{15bc^6(bx-a)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] -1/15*(2*a^2 + 5*a*b*x + 5*b^2*x^2)/(b*c^6*(-a + b*x)^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^6, x]
```

**fricas [A]** time = 1.19, size = 95, normalized size = 1.67

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2
*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

**giac [A]** time = 0.85, size = 36, normalized size = 0.63

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\,(bx-a)^5bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="giac")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/((b*x - a)^5*b*c^6)
```

**maple [A]** time = 0.00, size = 52, normalized size = 0.91

$$\frac{-\frac{4a^2}{5(bx-a)^5b}-\frac{a}{(bx-a)^4b}-\frac{1}{3(bx-a)^3b}}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^6,x)
```

```
[Out] 1/c^6*(-1/(b*x-a)^4*a/b-1/3/(b*x-a)^3/b-4/5*a^2/b/(b*x-a)^5)
```

**maxima [A]** time = 1.39, size = 95, normalized size = 1.67

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2
*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

**mupad [B]** time = 0.19, size = 91, normalized size = 1.60

$$\frac{\frac{a\,x}{3}+\frac{b\,x^2}{3}+\frac{2\,a^2}{15\,b}}{a^5\,c^6-5\,a^4\,b\,c^6\,x+10\,a^3\,b^2\,c^6\,x^2-10\,a^2\,b^3\,c^6\,x^3+5\,a\,b^4\,c^6\,x^4-b^5\,c^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^6,x)
```

```
[Out] ((a*x)/3 + (b*x^2)/3 + (2*a^2)/(15*b))/(a^5*c^6 - b^5*c^6*x^5 + 5*a*b^4*c^6
*x^4 + 10*a^3*b^2*c^6*x^2 - 10*a^2*b^3*c^6*x^3 - 5*a^4*b*c^6*x)
```

**sympy [B]** time = 0.51, size = 100, normalized size = 1.75

$$\frac{-2a^2-5abx-5b^2x^2}{-15a^5bc^6+75a^4b^2c^6x-150a^3b^3c^6x^2+150a^2b^4c^6x^3-75ab^5c^6x^4+15b^6c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**6,x)
```

```
[Out] (-2*a**2 - 5*a*b*x - 5*b**2*x**2)/(-15*a**5*b*c**6 + 75*a**4*b**2*c**6*x -
150*a**3*b**3*c**6*x**2 + 150*a**2*b**4*c**6*x**3 - 75*a*b**5*c**6*x**4 + 1
5*b**6*c**6*x**5)
```

## 3.979 $\int \frac{(a+bx)^2}{(ac-bcx)^7} dx$

**Optimal**. Leaf size=59

$$\frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}$$

**Rubi** [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] (2*a^2)/(3*b*c^7*(a - b*x)^6) - (4*a)/(5*b*c^7*(a - b*x)^5) + 1/(4*b*c^7*(a
 - b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx &= \int \left(\frac{4a^2}{c^7(a-bx)^7}-\frac{4a}{c^7(a-bx)^6}+\frac{1}{c^7(a-bx)^5}\right)dx\\ &= \frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 37, normalized size = 0.63

$$\frac{7a^2+18abx+15b^2x^2}{60bc^7(a-bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] (7*a^2 + 18*a*b*x + 15*b^2*x^2)/(60*b*c^7*(a - b*x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^7, x]
```

**fricas [A]** time = 0.99, size = 108, normalized size = 1.83

$$\frac{15\,b^2x^2+18\,abx+7\,a^2}{60\left(b^7c^7x^6-6\,ab^6c^7x^5+15\,a^2b^5c^7x^4-20\,a^3b^4c^7x^3+15\,a^4b^3c^7x^2-6\,a^5b^2c^7x+a^6bc^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a^
2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x +
 a^6*b*c^7)
```

**giac [A]** time = 0.93, size = 36, normalized size = 0.61

$$\frac{15\,b^2x^2+18\,abx+7\,a^2}{60\,(bx-a)^6bc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="giac")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/((b*x - a)^6*b*c^7)
```

**maple [A]** time = 0.01, size = 52, normalized size = 0.88

$$\frac{\frac{2a^2}{3(bx-a)^6b}+\frac{4a}{5(bx-a)^5b}+\frac{1}{4(bx-a)^4b}}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^7,x)
```

```
[Out] 1/c^7*(1/4/(b*x-a)^4/b+2/3*a^2/b/(b*x-a)^6+4/5/(b*x-a)^5*a/b)
```

**maxima [A]** time = 1.42, size = 108, normalized size = 1.83

$$\frac{15\,b^2x^2+18\,abx+7\,a^2}{60\left(b^7c^7x^6-6\,ab^6c^7x^5+15\,a^2b^5c^7x^4-20\,a^3b^4c^7x^3+15\,a^4b^3c^7x^2-6\,a^5b^2c^7x+a^6bc^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a^
2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x +
 a^6*b*c^7)
```

**mupad [B]** time = 0.11, size = 104, normalized size = 1.76

$$\frac{\frac{3\,a\,x}{10}+\frac{b\,x^2}{4}+\frac{7\,a^2}{60\,b}}{a^6\,c^7-6\,a^5\,b\,c^7\,x+15\,a^4\,b^2\,c^7\,x^2-20\,a^3\,b^3\,c^7\,x^3+15\,a^2\,b^4\,c^7\,x^4-6\,a\,b^5\,c^7\,x^5+b^6\,c^7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^7,x)
```

```
[Out] ((3*a*x)/10 + (b*x^2)/4 + (7*a^2)/(60*b))/(a^6*c^7 + b^6*c^7*x^6 - 6*a*b^5*
c^7*x^5 + 15*a^4*b^2*c^7*x^2 - 20*a^3*b^3*c^7*x^3 + 15*a^2*b^4*c^7*x^4 - 6*
a^5*b*c^7*x)
```

**sympy [B]** time = 0.60, size = 117, normalized size = 1.98

$$-\frac{-7a^2-18abx-15b^2x^2}{60a^6bc^7-360a^5b^2c^7x+900a^4b^3c^7x^2-1200a^3b^4c^7x^3+900a^2b^5c^7x^4-360ab^6c^7x^5+60b^7c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**7,x)
```

```
[Out] -(-7*a**2 - 18*a*b*x - 15*b**2*x**2)/(60*a**6*b*c**7 - 360*a**5*b**2*c**7*x
 + 900*a**4*b**3*c**7*x**2 - 1200*a**3*b**4*c**7*x**3 + 900*a**2*b**5*c**7*
x**4 - 360*a*b**6*c**7*x**5 + 60*b**7*c**7*x**6)
```

## 3.980 $\int \frac{(ac-bcx)^3}{a+bx} \, dx$

**Optimal**. Leaf size=61

$$\frac{8a^3c^3\log(a+bx)}{b}-4a^2c^3x+\frac{c^3(a-bx)^3}{3b}+\frac{ac^3(a-bx)^2}{b}$$

**Rubi** [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{8a^3c^3\log(a+bx)}{b}-4a^2c^3x+\frac{c^3(a-bx)^3}{3b}+\frac{ac^3(a-bx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] -4*a^2*c^3*x + (a*c^3*(a - b*x)^2)/b + (c^3*(a - b*x)^3)/(3*b) + (8*a^3*c^3
*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^3}{a+bx}\,dx &= \int \left(-4a^2c^3+\frac{8a^3c^3}{a+bx}-2ac^2(ac-bcx)-c(ac-bcx)^2\right)dx\\ &= -4a^2c^3x+\frac{ac^3(a-bx)^2}{b}+\frac{c^3(a-bx)^3}{3b}+\frac{8a^3c^3\log(a+bx)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 42, normalized size = 0.69

$$c^3\left(\frac{8a^3\log(a+bx)}{b}-7a^2x+2abx^2-\frac{b^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] c^3*(-7*a^2*x + 2*a*b*x^2 - (b^2*x^3)/3 + (8*a^3*Log[a + b*x])/b)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ac-bcx)^3}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(a*c - b*c*x)^3/(a + b*x), x]
```

**fricas [A]** time = 1.37, size = 52, normalized size = 0.85

$$-\frac{b^3c^3x^3 - 6\,ab^2c^3x^2 + 21\,a^2bc^3x - 24\,a^3c^3\log\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*c^3*x^3 - 6*a*b^2*c^3*x^2 + 21*a^2*b*c^3*x - 24*a^3*c^3*log(b*x +
 a))/b
```

**giac [A]** time = 1.15, size = 59, normalized size = 0.97

$$\frac{8\,a^3c^3\log\left(|bx+a|\right)}{b} - \frac{b^5c^3x^3 - 6\,ab^4c^3x^2 + 21\,a^2b^3c^3x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 8*a^3*c^3*log(abs(b*x + a))/b - 1/3*(b^5*c^3*x^3 - 6*a*b^4*c^3*x^2 + 21*a^2
*b^3*c^3*x)/b^3
```

**maple [A]** time = 0.00, size = 49, normalized size = 0.80

$$-\frac{b^2c^3x^3}{3} + 2ab\,c^3x^2 + \frac{8a^3c^3\ln\left(bx+a\right)}{b} - 7a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^3/(b*x+a),x)
```

```
[Out] -1/3*c^3*b^2*x^3+2*c^3*b*x^2*a-7*a^2*c^3*x+8*a^3*c^3*ln(b*x+a)/b
```

**maxima** [A] time = 1.34, size = 48, normalized size = 0.79

$$-\frac{1}{3}\,b^2c^3x^3+2\,abc^3x^2-7\,a^2c^3x+\frac{8\,a^3c^3\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*b^2*c^3*x^3 + 2*a*b*c^3*x^2 - 7*a^2*c^3*x + 8*a^3*c^3*log(b*x + a)/b
```

**mupad** [B] time = 0.05, size = 48, normalized size = 0.79

$$\frac{8\,a^3\,c^3\,\ln\left(a+b\,x\right)}{b}-\frac{b^2\,c^3\,x^3}{3}-7\,a^2\,c^3\,x+2\,a\,b\,c^3\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3/(a + b*x),x)
```

```
[Out] (8*a^3*c^3*log(a + b*x))/b - (b^2*c^3*x^3)/3 - 7*a^2*c^3*x + 2*a*b*c^3*x^2
```

**sympy** [A] time = 0.18, size = 49, normalized size = 0.80

$$\frac{8a^3c^3\log\left(a+bx\right)}{b}-7a^2c^3x+2abc^3x^2-\frac{b^2c^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**3/(b*x+a),x)
```

```
[Out] 8*a**3*c**3*log(a + b*x)/b - 7*a**2*c**3*x + 2*a*b*c**3*x**2 - b**2*c**3*x*
*3/3
```

## 3.981 $\int \frac{(ac-bcx)^2}{a+bx}\,dx$

**Optimal**. Leaf size=43

$$\frac{4a^2c^2\log(a+bx)}{b}+\frac{c^2(a-bx)^2}{2b}-2ac^2x$$

**Rubi [A]** time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{4a^2c^2\log(a+bx)}{b}+\frac{c^2(a-bx)^2}{2b}-2ac^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^2/(a + b*x),x]
```

```
[Out] -2*a*c^2*x + (c^2*(a - b*x)^2)/(2*b) + (4*a^2*c^2*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^2}{a+bx}\,dx&=\int \left(-2ac^2+\frac{4a^2c^2}{a+bx}-c(ac-bcx)\right)dx\\&=-2ac^2x+\frac{c^2(a-bx)^2}{2b}+\frac{4a^2c^2\log(a+bx)}{b}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 0.72

$$c^2\left(\frac{4a^2\log(a+bx)}{b}-3ax+\frac{bx^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^2/(a + b*x),x]
```

```
[Out] c^2*(-3*a*x + (b*x^2)/2 + (4*a^2*Log[a + b*x])/b)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ac-bcx)^2}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)^2/(a + b*x),x]
[Out] IntegrateAlgebraic[(a*c - b*c*x)^2/(a + b*x), x]
```

**fricas [A]** time = 1.23, size = 38, normalized size = 0.88

$$\frac{b^2c^2x^2 - 6\,abc^2x + 8\,a^2c^2\log\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*c^2*x^2 - 6*a*b*c^2*x + 8*a^2*c^2*log(b*x + a))/b
```

**giac [A]** time = 0.87, size = 45, normalized size = 1.05

$$\frac{4\,a^2c^2\log\left(|bx+a|\right)}{b} + \frac{b^3c^2x^2 - 6\,ab^2c^2x}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="giac")
[Out] 4*a^2*c^2*log(abs(b*x + a))/b + 1/2*(b^3*c^2*x^2 - 6*a*b^2*c^2*x)/b^2
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.81

$$\frac{b\,c^2x^2}{2} + \frac{4a^2c^2\ln\left(bx+a\right)}{b} - 3a\,c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^2/(b*x+a),x)
[Out] 1/2*c^2*x^2*b-3*a*c^2*x+4*a^2*c^2*ln(b*x+a)/b
```

**maxima [A]** time = 1.34, size = 34, normalized size = 0.79

$$\frac{1}{2}\,bc^2x^2 - 3\,ac^2x + \frac{4\,a^2c^2\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*b*c^2*x^2 - 3*a*c^2*x + 4*a^2*c^2*log(b*x + a)/b
```

**mupad [B]** time = 0.15, size = 32, normalized size = 0.74

$$\frac{c^2\,\left(8\,a^2\,\ln\left(a+b\,x\right)+b^2\,x^2-6\,a\,b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2/(a + b*x),x)
```

```
[Out] (c^2*(8*a^2*log(a + b*x) + b^2*x^2 - 6*a*b*x))/(2*b)
```

**sympy [A]** time = 0.15, size = 34, normalized size = 0.79

$$\frac{4a^2c^2\log\left(a+bx\right)}{b}-3ac^2x+\frac{bc^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**2/(b*x+a),x)
```

```
[Out] 4*a**2*c**2*log(a + b*x)/b - 3*a*c**2*x + b*c**2*x**2/2
```

## 3.982 $\int \frac{ac-bcx}{a+bx}\,dx$

**Optimal**. Leaf size=18

$$\frac{2ac\log(a+bx)}{b}-cx$$

**Rubi** [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2ac\log(a+bx)}{b}-cx$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)/(a + b*x),x]
```

```
[Out] -(c*x) + (2*a*c*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac-bcx}{a+bx}\,dx &= \int \left(-c+\frac{2ac}{a+bx}\right)dx\\ &= -cx+\frac{2ac\log(a+bx)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 18, normalized size = 1.00

$$c\left(\frac{2a\log(a+bx)}{b}-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)/(a + b*x),x]
```

```
[Out] c*(-x + (2*a*Log[a + b*x])/b)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac - bcx}{a + bx}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)/(a + b*x),x]
[Out] IntegrateAlgebraic[(a*c - b*c*x)/(a + b*x), x]
```

**fricas [A]** time = 1.35, size = 20, normalized size = 1.11

$$-\frac{bcx - 2\, ac \log\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="fricas")
[Out] -(b*c*x - 2*a*c*log(b*x + a))/b
```

**giac [A]** time = 1.01, size = 19, normalized size = 1.06

$$-cx + \frac{2\, ac \log\left(|bx + a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="giac")
[Out] -c*x + 2*a*c*log(abs(b*x + a))/b
```

**maple [A]** time = 0.00, size = 19, normalized size = 1.06

$$\frac{2ac \ln\left(bx + a\right)}{b} - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)/(b*x+a),x)
[Out] -c*x+2*a*c*ln(b*x+a)/b
```

**maxima [A]** time = 1.34, size = 18, normalized size = 1.00

$$-cx + \frac{2\, ac \log\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -c*x + 2*a*c*log(b*x + a)/b
```

**mupad [B]** time = 0.04, size = 18, normalized size = 1.00

$$\frac{2\,a\,c\,\ln\left(a+b\,x\right)}{b}-c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)/(a + b*x),x)
```

```
[Out] (2*a*c*log(a + b*x))/b - c*x
```

**sympy [A]** time = 0.12, size = 15, normalized size = 0.83

$$\frac{2ac\log\left(a+bx\right)}{b}-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x)
```

```
[Out] 2*a*c*log(a + b*x)/b - c*x
```

## 3.983 $\int \frac{1}{a+bx} dx$

**Optimal**. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

**Mathematica [A]** time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+bx} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-1), x]
```

**fricas [A]** time = 1.16, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

**giac [A]** time = 1.06, size = 11, normalized size = 1.10

$$\frac{\log(|bx+a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

**maple [A]** time = 0.00, size = 11, normalized size = 1.10

$$\frac{\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x)
```

```
[Out] 1/b*ln(b*x+a)
```

**maxima [A]** time = 1.33, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

**mupad** [B] time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x),x)
```

```
[Out] log(a + b*x)/b
```

**sympy** [A] time = 0.07, size = 7, normalized size = 0.70

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(a + b*x)/b
```

## 3.984 $\int \frac{1}{(a+bx)(ac-bcx)}\,dx$

**Optimal**. Leaf size=17

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

**Rubi** [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {35, 208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b*c)
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(ac-bcx)}\,dx &= \int \frac{1}{a^2c-b^2cx^2}\,dx\\ &= \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b*c)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(ac-bcx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)), x]
```

**fricas [A]** time = 1.23, size = 28, normalized size = 1.65

$$\frac{\log(bx+a) - \log(bx-a)}{2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/2*(log(b*x + a) - log(b*x - a))/(a*b*c)
```

**giac [B]** time = 1.20, size = 39, normalized size = 2.29

$$\frac{\log(|bx+a|)}{2\,abc} - \frac{\log(|bx-a|)}{2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x + a))/(a*b*c) - 1/2*log(abs(b*x - a))/(a*b*c)
```

**maple [B]** time = 0.01, size = 38, normalized size = 2.24

$$-\frac{\ln(bx-a)}{2abc} + \frac{\ln(bx+a)}{2abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c),x)
```

```
[Out] 1/2/c/b/a*ln(b*x+a)-1/2/c/b/a*ln(b*x-a)
```

**maxima [B]** time = 1.40, size = 37, normalized size = 2.18

$$\frac{\log(bx+a)}{2\,abc} - \frac{\log(bx-a)}{2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
[Out] 1/2*log(b*x + a)/(a*b*c) - 1/2*log(b*x - a)/(a*b*c)
```

**mupad [B]** time = 0.17, size = 17, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{a\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)*(a + b*x)),x)
[Out] atanh((b*x)/a)/(a*b*c)
```

**sympy [B]** time = 0.17, size = 22, normalized size = 1.29

$$-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2} - \frac{\log\left(\frac{a}{b}+x\right)}{2}}{abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x)
[Out] -(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b*c)
```

## 3.985 $\int \frac{1}{(a+bx)(ac-bcx)^2} dx$

**Optimal**. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2} + \frac{1}{2abc^2(a-bx)}$$

**Rubi [A]** time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2} + \frac{1}{2abc^2(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

```
[Out] 1/(2*a*b*c^2*(a - b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)(ac-bcx)^2} dx &= \int \left(\frac{1}{2ac^2(a-bx)^2} + \frac{1}{2ac^2\left(a^2-b^2x^2\right)}\right) dx \\
&= \frac{1}{2abc^2(a-bx)} + \frac{\int \frac{1}{a^2-b^2x^2} dx}{2ac^2} \\
&= \frac{1}{2abc^2(a-bx)} + \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 53, normalized size = 1.26

$$\frac{(bx-a)\log(a-bx)+(a-bx)\log(a+bx)+2a}{4a^2bc^2(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

```
[Out] (2*a + (-a + b*x)*Log[a - b*x] + (a - b*x)*Log[a + b*x])/(4*a^2*b*c^2*(a - b*x))
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)^2), x]
```

**fricas** [A] time = 1.41, size = 60, normalized size = 1.43

$$\frac{(bx-a)\log(bx+a)-(bx-a)\log(bx-a)-2\,a}{4\left(a^2b^2c^2x-a^3bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b*x - a)*log(b*x + a) - (b*x - a)*log(b*x - a) - 2*a)/(a^2*b^2*c^2*x - a^3*b*c^2)
```

**giac** [A] time = 1.03, size = 53, normalized size = 1.26

$$-\frac{1}{2\,(bcx-ac)abc}+\frac{\log\left(\left|-\frac{2\,ac}{bcx-ac}-1\right|\right)}{4\,a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -1/2/((b*c*x - a*c)*a*b*c) + 1/4*log(abs(-2*a*c/(b*c*x - a*c) - 1))/(a^2*b*c^2)
```

**maple** [A] time = 0.01, size = 58, normalized size = 1.38

$$-\frac{1}{2\left(bx-a\right)ab\,c^{2}}-\frac{\ln\left(bx-a\right)}{4a^{2}b\,c^{2}}+\frac{\ln\left(bx+a\right)}{4a^{2}b\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c)^2,x)
```

```
[Out] 1/4/c^2/a^2/b*ln(b*x+a)-1/4/c^2/a^2/b*ln(b*x-a)-1/2/c^2/b/a/(b*x-a)
```

**maxima** [A] time = 1.26, size = 60, normalized size = 1.43

$$-\frac{1}{2\left(ab^{2}c^{2}x-a^{2}bc^{2}\right)}+\frac{\log\left(bx+a\right)}{4\,a^{2}bc^{2}}-\frac{\log\left(bx-a\right)}{4\,a^{2}bc^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(a*b^2*c^2*x - a^2*b*c^2) + 1/4*log(b*x + a)/(a^2*b*c^2) - 1/4*log(b*x
 - a)/(a^2*b*c^2)
```

**mupad** [B] time = 0.07, size = 42, normalized size = 1.00

$$\frac{1}{2\,a\,b\,\left(a\,c^{2}-b\,c^{2}\,x\right)}+\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^{2}\,b\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^2*(a + b*x)),x)
```

```
[Out] 1/(2*a*b*(a*c^2 - b*c^2*x)) + atanh((b*x)/a)/(2*a^2*b*c^2)
```

**sympy** [A] time = 0.28, size = 48, normalized size = 1.14

$$-\frac{1}{-2a^{2}bc^{2}+2ab^{2}c^{2}x}+\frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^{2}bc^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)**2,x)
```

```
[Out] -1/(-2*a**2*b*c**2 + 2*a*b**2*c**2*x) + (-log(-a/b + x)/4 + log(a/b + x)/4)
/(a**2*b*c**2)
```

## 3.986 $\int \frac{1}{(a+bx)(ac-bcx)^3} \, dx$

**Optimal**. Leaf size=63

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}+\frac{1}{4a^2bc^3(a-bx)}+\frac{1}{4abc^3(a-bx)^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{1}{4a^2bc^3(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}+\frac{1}{4abc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] 1/(4*a*b*c^3*(a - b*x)^2) + 1/(4*a^2*b*c^3*(a - b*x)) + ArcTanh[(b*x)/a]/(4
*a^3*b*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx = \int \left(\frac{1}{2ac^3(a-bx)^3} + \frac{1}{4a^2c^3(a-bx)^2} + \frac{1}{4a^2c^3\left(a^2-b^2x^2\right)}\right)dx$$
$$= \frac{1}{4abc^3(a-bx)^2} + \frac{1}{4a^2bc^3(a-bx)} + \frac{\int \frac{1}{a^2-b^2x^2}\,dx}{4a^2c^3}$$
$$= \frac{1}{4abc^3(a-bx)^2} + \frac{1}{4a^2bc^3(a-bx)} + \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}$$

**Mathematica [A]** time = 0.02, size = 65, normalized size = 1.03

$$\frac{2a(2a-bx)+(a-bx)^2(-\log(a-bx))+(a-bx)^2\log(a+bx)}{8a^3bc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] (2*a*(2*a - b*x) - (a - b*x)^2*Log[a - b*x] + (a - b*x)^2*Log[a + b*x])/(8*
a^3*b*c^3*(a - b*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)^3), x]
```

**fricas [A]** time = 1.49, size = 98, normalized size = 1.56

$$-\frac{2\,abx-4\,a^2-\left(b^2x^2-2\,abx+a^2\right)\log\left(bx+a\right)+\left(b^2x^2-2\,abx+a^2\right)\log\left(bx-a\right)}{8\left(a^3b^3c^3x^2-2\,a^4b^2c^3x+a^5bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*b*x - 4*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x + a) + (b^2*x^2 -
 2*a*b*x + a^2)*log(b*x - a))/(a^3*b^3*c^3*x^2 - 2*a^4*b^2*c^3*x + a^5*b*c^
3)
```

**giac** [A] time = 0.87, size = 69, normalized size = 1.10

$$\frac{\log\left(|bx+a|\right)}{8\,a^3bc^3}-\frac{\log\left(|bx-a|\right)}{8\,a^3bc^3}-\frac{abx-2\,a^2}{4\,(bx-a)^2a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(b*x + a))/(a^3*b*c^3) - 1/8*log(abs(b*x - a))/(a^3*b*c^3) - 1/4
*(a*b*x - 2*a^2)/((b*x - a)^2*a^3*b*c^3)
```

**maple** [A] time = 0.01, size = 78, normalized size = 1.24

$$\frac{1}{4\,(bx-a)^2\,ab\,c^3}-\frac{1}{4\,(bx-a)\,a^2b\,c^3}-\frac{\ln\left(bx-a\right)}{8a^3b\,c^3}+\frac{\ln\left(bx+a\right)}{8a^3b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c)^3,x)
```

```
[Out] 1/8/c^3/a^3/b*ln(b*x+a)-1/8/c^3/a^3/b*ln(b*x-a)-1/4/c^3/a^2/b/(b*x-a)+1/4/c
^3/b/a/(b*x-a)^2
```

**maxima** [A] time = 1.35, size = 82, normalized size = 1.30

$$-\frac{bx-2\,a}{4\left(a^2b^3c^3x^2-2\,a^3b^2c^3x+a^4bc^3\right)}+\frac{\log\left(bx+a\right)}{8\,a^3bc^3}-\frac{\log\left(bx-a\right)}{8\,a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x - 2*a)/(a^2*b^3*c^3*x^2 - 2*a^3*b^2*c^3*x + a^4*b*c^3) + 1/8*log(
b*x + a)/(a^3*b*c^3) - 1/8*log(b*x - a)/(a^3*b*c^3)
```

**mupad** [B] time = 0.08, size = 64, normalized size = 1.02

$$\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{4\,a^3\,b\,c^3}-\frac{\frac{x}{4\,a^2}-\frac{1}{2\,a\,b}}{a^2\,c^3-2\,a\,b\,c^3\,x+b^2\,c^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^3*(a + b*x)),x)
```

```
[Out] atanh((b*x)/a)/(4*a^3*b*c^3) - (x/(4*a^2) - 1/(2*a*b))/(a^2*c^3 + b^2*c^3*x
^2 - 2*a*b*c^3*x)
```

**sympy [A]** time = 0.38, size = 71, normalized size = 1.13

$$-\frac{-2a+bx}{4a^4bc^3-8a^3b^2c^3x+4a^2b^3c^3x^2}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{8}-\frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)**3,x)
```

```
[Out] -(-2*a + b*x)/(4*a**4*b*c**3 - 8*a**3*b**2*c**3*x + 4*a**2*b**3*c**3*x**2)
- (log(-a/b + x)/8 - log(a/b + x)/8)/(a**3*b*c**3)
```

## 3.987 $\int \frac{(ac-bcx)^3}{(a+bx)^2} \, dx$

**Optimal**. Leaf size=54

$$-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}+5ac^3x-\frac{1}{2}bc^3x^2$$

**Rubi** [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}+5ac^3x-\frac{1}{2}bc^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

```
[Out] 5*a*c^3*x - (b*c^3*x^2)/2 - (8*a^3*c^3)/(b*(a + b*x)) - (12*a^2*c^3*Log[a +
 b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx &= \int \left(5ac^3-bc^3x+\frac{8a^3c^3}{(a+bx)^2}-\frac{12a^2c^3}{a+bx}\right)dx\\ &= 5ac^3x-\frac{1}{2}bc^3x^2-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 0.85

$$c^3\left(-\frac{8a^3}{b(a+bx)}-\frac{12a^2\log(a+bx)}{b}+5ax-\frac{bx^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

```
[Out] c^3*(5*a*x - (b*x^2)/2 - (8*a^3)/(b*(a + b*x)) - (12*a^2*Log[a + b*x])/b)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ac - bcx)^3}{(a + bx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a*c - b*c*x)^3/(a + b*x)^2, x]
```

**fricas [A]** time = 0.77, size = 79, normalized size = 1.46

$$-\frac{b^3c^3x^3 - 9\,ab^2c^3x^2 - 10\,a^2bc^3x + 16\,a^3c^3 + 24\left(a^2bc^3x + a^3c^3\right)\log\left(bx + a\right)}{2\left(b^2x + ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*c^3*x^3 - 9*a*b^2*c^3*x^2 - 10*a^2*b*c^3*x + 16*a^3*c^3 + 24*(a^2
*b*c^3*x + a^3*c^3)*log(b*x + a))/(b^2*x + a*b)
```

**giac [A]** time = 1.12, size = 80, normalized size = 1.48

$$\frac{12\,a^2c^3\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} - \frac{8\,a^3c^3}{(bx + a)b} + \frac{\left(\frac{12\,ac^3}{bx+a} - c^3\right)(bx + a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 12*a^2*c^3*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - 8*a^3*c^3/((b*x + a)*
b) + 1/2*(12*a*c^3/(b*x + a) - c^3)*(b*x + a)^2/b
```

**maple [A]** time = 0.01, size = 53, normalized size = 0.98

$$-\frac{b\,c^3x^2}{2} - \frac{8a^3c^3}{(bx + a)\,b} - \frac{12a^2c^3\ln\left(bx + a\right)}{b} + 5a\,c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^3/(b*x+a)^2,x)
```

```
[Out] 5*a*c^3*x-1/2*b*c^3*x^2-8*a^3*c^3/b/(b*x+a)-12*a^2*c^3*ln(b*x+a)/b
```

**maxima** [A] time = 1.29, size = 53, normalized size = 0.98

$$-\frac{1}{2}\,bc^3x^2 - \frac{8\,a^3c^3}{b^2x + ab} + 5\,ac^3x - \frac{12\,a^2c^3\log\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*c^3*x^2 - 8*a^3*c^3/(b^2*x + a*b) + 5*a*c^3*x - 12*a^2*c^3*log(b*x +
 a)/b
```

**mupad** [B] time = 0.05, size = 52, normalized size = 0.96

$$5\,a\,c^3\,x - \frac{b\,c^3\,x^2}{2} - \frac{12\,a^2\,c^3\,\ln\left(a+b\,x\right)}{b} - \frac{8\,a^3\,c^3}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3/(a + b*x)^2,x)
```

```
[Out] 5*a*c^3*x - (b*c^3*x^2)/2 - (12*a^2*c^3*log(a + b*x))/b - (8*a^3*c^3)/(b*(a
 + b*x))
```

**sympy** [A] time = 0.25, size = 51, normalized size = 0.94

$$-\frac{8a^3c^3}{ab + b^2x} - \frac{12a^2c^3\log\left(a + bx\right)}{b} + 5ac^3x - \frac{bc^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**3/(b*x+a)**2,x)
```

```
[Out] -8*a**3*c**3/(a*b + b**2*x) - 12*a**2*c**3*log(a + b*x)/b + 5*a*c**3*x - b*
c**3*x**2/2
```

## 3.988 $\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=39

$$-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}+c^2x$$

**Rubi** [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^2/(a + b*x)^2,x]
```

```
[Out] c^2*x - (4*a^2*c^2)/(b*(a + b*x)) - (4*a*c^2*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx &= \int \left(c^2+\frac{4a^2c^2}{(a+bx)^2}-\frac{4ac^2}{a+bx}\right)dx\\ &= c^2x-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 33, normalized size = 0.85

$$c^2\left(-\frac{4a^2}{b(a+bx)}-\frac{4a\log(a+bx)}{b}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^2/(a + b*x)^2,x]
```

```
[Out] c^2*(x - (4*a^2)/(b*(a + b*x)) - (4*a*Log[a + b*x])/b)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)^2/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a*c - b*c*x)^2/(a + b*x)^2, x]
```

**fricas [A]** time = 1.25, size = 61, normalized size = 1.56

$$\frac{b^2c^2x^2 + abc^2x - 4\,a^2c^2 - 4\left(abc^2x + a^2c^2\right)\log\left(bx+a\right)}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c^2*x^2 + a*b*c^2*x - 4*a^2*c^2 - 4*(a*b*c^2*x + a^2*c^2)*log(b*x + a)
)/(b^2*x + a*b)
```

**giac [A]** time = 1.18, size = 59, normalized size = 1.51

$$\frac{4\,ac^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} + \frac{(bx+a)c^2}{b} - \frac{4\,a^2c^2}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4*a*c^2*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b + (b*x + a)*c^2/b - 4*a^2*
c^2/((b*x + a)*b)
```

**maple [A]** time = 0.01, size = 40, normalized size = 1.03

$$-\frac{4a^2c^2}{(bx+a)\,b} - \frac{4a\,c^2\ln\left(bx+a\right)}{b} + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^2/(b*x+a)^2,x)
```

```
[Out] c^2*x-4*a^2*c^2/b/(b*x+a)-4*a*c^2*ln(b*x+a)/b
```

**maxima [A]** time = 1.36, size = 40, normalized size = 1.03

$$-\frac{4\,a^2c^2}{b^2x+ab}+c^2x-\frac{4\,ac^2\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="maxima")
[Out] -4*a^2*c^2/(b^2*x + a*b) + c^2*x - 4*a*c^2*log(b*x + a)/b
```

**mupad [B]** time = 0.17, size = 39, normalized size = 1.00

$$c^2\,x-\frac{4\,a\,c^2\,\ln\left(a+b\,x\right)}{b}-\frac{4\,a^2\,c^2}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2/(a + b*x)^2,x)
[Out] c^2*x - (4*a*c^2*log(a + b*x))/b - (4*a^2*c^2)/(b*(a + b*x))
```

**sympy [A]** time = 0.19, size = 36, normalized size = 0.92

$$-\frac{4a^2c^2}{ab+b^2x}-\frac{4ac^2\log\left(a+bx\right)}{b}+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**2/(b*x+a)**2,x)
[Out] -4*a**2*c**2/(a*b + b**2*x) - 4*a*c**2*log(a + b*x)/b + c**2*x
```

## 3.989 $\int \frac{ac-bcx}{(a+bx)^2} dx$

**Optimal**. Leaf size=27

$$-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

**Rubi [A]** time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)/(a + b*x)^2,x]
```

```
[Out] (-2*a*c)/(b*(a + b*x)) - (c*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac-bcx}{(a+bx)^2} dx &= \int \left(\frac{2ac}{(a+bx)^2}-\frac{c}{a+bx}\right) dx\\ &= -\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 23, normalized size = 0.85

$$-\frac{c\left(\frac{2a}{a+bx}+\log(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)/(a + b*x)^2,x]
```

```
[Out] -((c*((2*a)/(a + b*x) + Log[a + b*x]))/b)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac - bcx}{(a + bx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a*c - b*c*x)/(a + b*x)^2, x]
```

**fricas [A]** time = 1.19, size = 33, normalized size = 1.22

$$-\frac{2\,ac + (bcx + ac)\log{(bx + a)}}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c + (b*c*x + a*c)*log(b*x + a))/(b^2*x + a*b)
```

**giac [A]** time = 1.02, size = 54, normalized size = 2.00

$$c\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} - \frac{a}{(bx + a)b}\right) - \frac{ac}{(bx + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] c*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b)) - a*c/((b*x
+ a)*b)
```

**maple [A]** time = 0.00, size = 28, normalized size = 1.04

$$-\frac{2ac}{(bx + a)\,b} - \frac{c\ln{(bx + a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)/(b*x+a)^2,x)
```

```
[Out] -2*a*c/b/(b*x+a)-c*ln(b*x+a)/b
```

**maxima** [A] time = 1.34, size = 28, normalized size = 1.04

$$-\frac{2\,ac}{b^2x+ab}-\frac{c\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*a*c/(b^2*x + a*b) - c*log(b*x + a)/b
```

**mupad** [B] time = 0.04, size = 27, normalized size = 1.00

$$-\frac{c\;\ln\left(a+b\,x\right)}{b}-\frac{2\,a\,c}{b\;\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)/(a + b*x)^2,x)
```

```
[Out] - (c*log(a + b*x))/b - (2*a*c)/(b*(a + b*x))
```

**sympy** [A] time = 0.17, size = 24, normalized size = 0.89

$$-\frac{2ac}{ab+b^2x}-\frac{c\log\left(a+bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)**2,x)
```

```
[Out] -2*a*c/(a*b + b**2*x) - c*log(a + b*x)/b
```

# 3.990 $\int \frac{1}{(a+bx)^2} dx$

**Optimal**. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

**Rubi** [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

**Mathematica** [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-2), x]
```

**fricas [A]** time = 1.28, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

**giac [A]** time = 1.11, size = 12, normalized size = 1.00

$$-\frac{1}{(bx + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

**maple [A]** time = 0.00, size = 13, normalized size = 1.08

$$-\frac{1}{(bx + a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x)
```

```
[Out] -1/(b*x+a)/b
```

**maxima [A]** time = 1.37, size = 12, normalized size = 1.00

$$-\frac{1}{(bx + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

**mupad [B]** time = 0.02, size = 12, normalized size = 1.00

$$-\frac{1}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^2,x)
[Out] -1/(b*(a + b*x))
```

**sympy [A]** time = 0.14, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
[Out] -1/(a*b + b**2*x)
```

## 3.991 $\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx$

**Optimal**. Leaf size=41

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}-\frac{1}{2abc(a+bx)}$$

**Rubi [A]** time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}-\frac{1}{2abc(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] -1/(2*a*b*c*(a + b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx &= \int \left(\frac{1}{2ac(a+bx)^2}+\frac{1}{2ac\left(a^2-b^2x^2\right)}\right)dx\\
&= -\frac{1}{2abc(a+bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{2ac}\\
&= -\frac{1}{2abc(a+bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 50, normalized size = 1.22

$$\frac{-(a+bx)\log(a-bx)+(a+bx)\log(a+bx)-2a}{4a^2bc(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] (-2*a - (a + b*x)*Log[a - b*x] + (a + b*x)*Log[a + b*x])/(4*a^2*b*c*(a + b*
x))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)), x]
```

**fricas [A]** time = 1.01, size = 51, normalized size = 1.24

$$\frac{(bx+a)\log(bx+a)-(bx+a)\log(bx-a)-2\,a}{4\,\left(a^2b^2cx+a^3bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x + a)*log(b*x + a) - (b*x + a)*log(b*x - a) - 2*a)/(a^2*b^2*c*x +
a^3*b*c)
```

**giac [A]** time = 0.95, size = 44, normalized size = 1.07

$$-\frac{\log\left(\left|-\frac{2\,a}{bx+a}+1\right|\right)}{4\,a^2bc}-\frac{1}{2\,(bx+a)abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(-2*a/(b*x + a) + 1))/(a^2*b*c) - 1/2/((b*x + a)*a*b*c)
```

**maple [A]** time = 0.01, size = 56, normalized size = 1.37

$$-\frac{1}{2\left(bx+a\right)abc}-\frac{\ln\left(bx-a\right)}{4a^{2}bc}+\frac{\ln\left(bx+a\right)}{4a^{2}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c),x)
```

```
[Out] 1/4/c/a^2/b*ln(b*x+a)-1/2/a/b/c/(b*x+a)-1/4/c/a^2/b*ln(b*x-a)
```

**maxima [A]** time = 1.38, size = 55, normalized size = 1.34

$$-\frac{1}{2\left(ab^{2}cx+a^{2}bc\right)}+\frac{\log\left(bx+a\right)}{4\,a^{2}bc}-\frac{\log\left(bx-a\right)}{4\,a^{2}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -1/2/(a*b^2*c*x + a^2*b*c) + 1/4*log(b*x + a)/(a^2*b*c) - 1/4*log(b*x - a)/
(a^2*b*c)
```

**mupad [B]** time = 0.18, size = 37, normalized size = 0.90

$$\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^2\,b\,c}-\frac{1}{2\,a\,b\,\left(a\,c+b\,c\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)*(a + b*x)^2),x)
```

```
[Out] atanh((b*x)/a)/(2*a^2*b*c) - 1/(2*a*b*(a*c + b*c*x))
```

**sympy [A]** time = 0.28, size = 44, normalized size = 1.07

$$-\frac{1}{2a^{2}bc+2ab^{2}cx}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{4}-\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^{2}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c),x)
```

```
[Out] -1/(2*a**2*b*c + 2*a*b**2*c*x) - (log(-a/b + x)/4 - log(a/b + x)/4)/(a**2*b
*c)
```

## 3.992 $\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx$

**Optimal**. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}+\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}$$

**Rubi [A]** time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {41, 199, 208}

$$\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] x/(2*a^2*c^2*(a^2 - b^2*x^2)) + ArcTanh[(b*x)/a]/(2*a^3*b*c^2)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx = \int \frac{1}{\left(a^2c-b^2cx^2\right)^2}\,dx$$
$$= \frac{x}{2a^2c^2\left(a^2-b^2x^2\right)} + \frac{\int \frac{1}{a^2c-b^2cx^2}\,dx}{2a^2c}$$
$$= \frac{x}{2a^2c^2\left(a^2-b^2x^2\right)} + \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}$$

**Mathematica [A]** time = 0.02, size = 74, normalized size = 1.61

$$\frac{\left(b^2x^2-a^2\right)\log(a-bx)+\left(a^2-b^2x^2\right)\log(a+bx)+2abx}{4a^3bc^2(a-bx)(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] (2*a*b*x + (-a^2 + b^2*x^2)*Log[a - b*x] + (a^2 - b^2*x^2)*Log[a + b*x])/(4
*a^3*b*c^2*(a - b*x)*(a + b*x))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)^2), x]
```

**fricas [A]** time = 1.15, size = 76, normalized size = 1.65

$$-\frac{2\,abx-\left(b^2x^2-a^2\right)\log\left(bx+a\right)+\left(b^2x^2-a^2\right)\log\left(bx-a\right)}{4\left(a^3b^3c^2x^2-a^5bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*x - (b^2*x^2 - a^2)*log(b*x + a) + (b^2*x^2 - a^2)*log(b*x - a)
)/(a^3*b^3*c^2*x^2 - a^5*b*c^2)
```

**giac** [A] time = 1.07, size = 83, normalized size = 1.80

$$-\frac{1}{4\,(bcx-ac)a^2bc}+\frac{\log\left(\left|-\frac{2\,ac}{bcx-ac}-1\right|\right)}{4\,a^3bc^2}+\frac{1}{8\,a^3b\left(\frac{2\,ac}{bcx-ac}+1\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -1/4/((b*c*x - a*c)*a^2*b*c) + 1/4*log(abs(-2*a*c/(b*c*x - a*c) - 1))/(a^3*
b*c^2) + 1/8/(a^3*b*(2*a*c/(b*c*x - a*c) + 1)*c^2)
```

**maple** [A] time = 0.01, size = 76, normalized size = 1.65

$$-\frac{1}{4\,(bx+a)\,a^2b\,c^2}-\frac{1}{4\,(bx-a)\,a^2b\,c^2}-\frac{\ln(bx-a)}{4a^3b\,c^2}+\frac{\ln(bx+a)}{4a^3b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c)^2,x)
```

```
[Out] 1/4/c^2/a^3/b*ln(b*x+a)-1/4/c^2/a^2/b/(b*x+a)-1/4/c^2/a^3/b*ln(b*x-a)-1/4/c
^2/a^2/b/(b*x-a)
```

**maxima** [A] time = 1.32, size = 64, normalized size = 1.39

$$-\frac{x}{2\left(a^2b^2c^2x^2-a^4c^2\right)}+\frac{\log(bx+a)}{4\,a^3bc^2}-\frac{\log(bx-a)}{4\,a^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(a^2*b^2*c^2*x^2 - a^4*c^2) + 1/4*log(b*x + a)/(a^3*b*c^2) - 1/4*log
(b*x - a)/(a^3*b*c^2)
```

**mupad** [B] time = 0.18, size = 46, normalized size = 1.00

$$\frac{x}{2\,a^2\,\left(a^2\,c^2-b^2\,c^2\,x^2\right)}+\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^3\,b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^2*(a + b*x)^2),x)
```

```
[Out] x/(2*a^2*(a^2*c^2 - b^2*c^2*x^2)) + atanh((b*x)/a)/(2*a^3*b*c^2)
```

**sympy [A]** time = 0.27, size = 49, normalized size = 1.07

$$-\frac{x}{-2a^4c^2+2a^2b^2c^2x^2}+\frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c)**2,x)
```

```
[Out] -x/(-2*a**4*c**2 + 2*a**2*b**2*c**2*x**2) + (-log(-a/b + x)/4 + log(a/b + x
)/4)/(a**3*b*c**2)
```

## 3.993 $\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx$

**Optimal**. Leaf size=83

$$\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{1}{8a^2bc^3(a-bx)^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{1}{8a^2bc^3(a-bx)^2}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] 1/(8*a^2*b*c^3*(a - b*x)^2) + 1/(4*a^3*b*c^3*(a - b*x)) - 1/(8*a^3*b*c^3*(a
 + b*x)) + (3*ArcTanh[(b*x)/a])/(8*a^4*b*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx = \int \left(\frac{1}{4a^2c^3(a-bx)^3} + \frac{1}{4a^3c^3(a-bx)^2} + \frac{1}{8a^3c^3(a+bx)^2} + \frac{3}{8a^3c^3\left(a^2-b^2x^2\right)}\right)dx$$
$$= \frac{1}{8a^2bc^3(a-bx)^2} + \frac{1}{4a^3bc^3(a-bx)} - \frac{1}{8a^3bc^3(a+bx)} + \frac{3\int \frac{1}{a^2-b^2x^2}\,dx}{8a^3c^3}$$
$$= \frac{1}{8a^2bc^3(a-bx)^2} + \frac{1}{4a^3bc^3(a-bx)} - \frac{1}{8a^3bc^3(a+bx)} + \frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

**Mathematica** [A] time = 0.04, size = 68, normalized size = 0.82

$$\frac{\frac{2a\left(2a^2+3abx-3b^2x^2\right)}{(a-bx)^2(a+bx)} - 3\log(a-bx) + 3\log(a+bx)}{16a^4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] ((2*a*(2*a^2 + 3*a*b*x - 3*b^2*x^2))/((a - b*x)^2*(a + b*x)) - 3*Log[a - b*
x] + 3*Log[a + b*x])/(16*a^4*b*c^3)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)^3), x]
```

**fricas** [A] time = 1.17, size = 146, normalized size = 1.76

$$-\frac{6\,ab^2x^2 - 6\,a^2bx - 4\,a^3 - 3\left(b^3x^3 - ab^2x^2 - a^2bx + a^3\right)\log\left(bx+a\right) + 3\left(b^3x^3 - ab^2x^2 - a^2bx + a^3\right)\log\left(bx-a\right)}{16\left(a^4b^4c^3x^3 - a^5b^3c^3x^2 - a^6b^2c^3x + a^7bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*b^2*x^2 - 6*a^2*b*x - 4*a^3 - 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x +
 a^3)*log(b*x + a) + 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x + a^3)*log(b*x - a))/
(a^4*b^4*c^3*x^3 - a^5*b^3*c^3*x^2 - a^6*b^2*c^3*x + a^7*b*c^3)
```

**giac [A]** time = 0.96, size = 81, normalized size = 0.98

$$-\frac{3 \log\left(\left|-\frac{2a}{bx+a}+1\right|\right)}{16 a^4bc^3}-\frac{1}{8(bx+a)a^3bc^3}+\frac{\frac{12a}{bx+a}-5}{32 a^4bc^3\left(\frac{2a}{bx+a}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -3/16*log(abs(-2*a/(b*x + a) + 1))/(a^4*b*c^3) - 1/8/((b*x + a)*a^3*b*c^3)
+ 1/32*(12*a/(b*x + a) - 5)/(a^4*b*c^3*(2*a/(b*x + a) - 1)^2)
```

**maple [A]** time = 0.01, size = 96, normalized size = 1.16

$$\frac{1}{8(bx-a)^2a^2bc^3}-\frac{1}{8(bx+a)a^3bc^3}-\frac{1}{4(bx-a)a^3bc^3}-\frac{3\ln(bx-a)}{16a^4bc^3}+\frac{3\ln(bx+a)}{16a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c)^3,x)
```

```
[Out] 3/16/c^3/a^4/b*ln(b*x+a)-1/8/a^3/b/c^3/(b*x+a)-3/16/c^3/a^4/b*ln(b*x-a)-1/4
/c^3/a^3/b/(b*x-a)+1/8/c^3/a^2/b/(b*x-a)^2
```

**maxima [A]** time = 1.35, size = 108, normalized size = 1.30

$$-\frac{3b^2x^2-3abx-2a^2}{8\left(a^3b^4c^3x^3-a^4b^3c^3x^2-a^5b^2c^3x+a^6bc^3\right)}+\frac{3\log(bx+a)}{16a^4bc^3}-\frac{3\log(bx-a)}{16a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^2*x^2 - 3*a*b*x - 2*a^2)/(a^3*b^4*c^3*x^3 - a^4*b^3*c^3*x^2 - a^5
*b^2*c^3*x + a^6*b*c^3) + 3/16*log(b*x + a)/(a^4*b*c^3) - 3/16*log(b*x - a)
/(a^4*b*c^3)
```

**mupad [B]** time = 0.10, size = 86, normalized size = 1.04

$$\frac{\frac{3x}{8a^2}+\frac{1}{4ab}-\frac{3bx^2}{8a^3}}{a^3c^3-a^2bc^3x-ab^2c^3x^2+b^3c^3x^3}+\frac{3\,\mathrm{atanh}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^3*(a + b*x)^2),x)
```

```
[Out] ((3*x)/(8*a^2) + 1/(4*a*b) - (3*b*x^2)/(8*a^3))/(a^3*c^3 + b^3*c^3*x^3 - a*
b^2*c^3*x^2 - a^2*b*c^3*x) + (3*atanh((b*x)/a))/(8*a^4*b*c^3)
```

**sympy [A]** time = 0.51, size = 104, normalized size = 1.25

$$-\frac{-2a^2-3abx+3b^2x^2}{8a^6bc^3-8a^5b^2c^3x-8a^4b^3c^3x^2+8a^3b^4c^3x^3}-\frac{\frac{3\log\left(-\frac{a}{b}+x\right)}{16}-\frac{3\log\left(\frac{a}{b}+x\right)}{16}}{a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c)**3,x)
```

```
[Out] -(-2*a**2 - 3*a*b*x + 3*b**2*x**2)/(8*a**6*b*c**3 - 8*a**5*b**2*c**3*x - 8*
a**4*b**3*c**3*x**2 + 8*a**3*b**4*c**3*x**3) - (3*log(-a/b + x)/16 - 3*log(
a/b + x)/16)/(a**4*b*c**3)
```

## 3.994 $\int (1-x)^{9/2}\sqrt{1+x}\,dx$

**Optimal**. Leaf size=108

$$\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}+\frac{3}{10}(x+1)^{3/2}(1-x)^{7/2}+\frac{21}{40}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{8}(x+1)^{3/2}(1-x)^{3/2}+\frac{21}{16}x\sqrt{x+1}\,\sqrt{1-x}+\frac{21}{16}\sin^{-1}(x$$

**Rubi** [A] time = 0.02, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}+\frac{3}{10}(x+1)^{3/2}(1-x)^{7/2}+\frac{21}{40}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{8}(x+1)^{3/2}(1-x)^{3/2}+\frac{21}{16}x\sqrt{x+1}\,\sqrt{1-x}+\frac{21}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

```
[Out] (21*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/8 + (21
*(1 - x)^(5/2)*(1 + x)^(3/2))/40 + (3*(1 - x)^(7/2)*(1 + x)^(3/2))/10 + ((1
 - x)^(9/2)*(1 + x)^(3/2))/6 + (21*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{9/2}\sqrt{1+x}\,dx = \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2} + \frac{3}{2}\int (1-x)^{7/2}\sqrt{1+x}\,dx$$
$$= \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2} + \frac{21}{10}\int (1-x)^{5/2}\sqrt{1+x}\,dx$$
$$= \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2} + \frac{21}{8}\int (1-x)^{3/2}\sqrt{}$$
$$= \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1-x)^{9/2}(1+x$$
$$= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3}$$
$$= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3}$$
$$= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3}$$

**Mathematica [A]** time = 0.05, size = 60, normalized size = 0.56

$$\frac{1}{240}\left(\sqrt{1-x^2}\left(40x^5-192x^4+350x^3-256x^2-75x+448\right)-630\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x^2]*(448 - 75*x - 256*x^2 + 350*x^3 - 192*x^4 + 40*x^5) - 630*Ar
cSin[Sqrt[1 - x]/Sqrt[2]])/240
```

**IntegrateAlgebraic [A]** time = 0.09, size = 128, normalized size = 1.19

$$\frac{\sqrt{x+1}\left(\frac{315(x+1)^5}{(1-x)^5}+\frac{1785(x+1)^4}{(1-x)^4}+\frac{4158(x+1)^3}{(1-x)^3}+\frac{5058(x+1)^2}{(1-x)^2}+\frac{3335(x+1)}{1-x}-315\right)}{120\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^6}+\frac{21}{8}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(-315 + (3335*(1 + x))/(1 - x) + (5058*(1 + x)^2)/(1 - x)^2 +
(4158*(1 + x)^3)/(1 - x)^3 + (1785*(1 + x)^4)/(1 - x)^4 + (315*(1 + x)^5)/(
```

```
1 - x)^5))/(120*Sqrt[1 - x]*(1 + (1 + x)/(1 - x))^6) + (21*ArcTan[Sqrt[1 +
x]/Sqrt[1 - x]])/8
```

**fricas [A]** time = 0.76, size = 62, normalized size = 0.57

$$\frac{1}{240}\left(40\,x^{5}-192\,x^{4}+350\,x^{3}-256\,x^{2}-75\,x+448\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{21}{8}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}\,-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/240*(40*x^5 - 192*x^4 + 350*x^3 - 256*x^2 - 75*x + 448)*sqrt(x + 1)*sqrt(
-x + 1) - 21/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac [B]** time = 1.27, size = 185, normalized size = 1.71

$$\frac{1}{240}\left(\left(2\left(\left(4\left(5\,x-26\right)\left(x+1\right)+321\right)\left(x+1\right)-451\right)\left(x+1\right)+745\right)\left(x+1\right)-405\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{40}\left(\left(2\left(3\left(4\,x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{12}\left(\left(2\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{3}\left(\left(2\,x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{2}\,\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{21}{8}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
1) - 405)*sqrt(x + 1)*sqrt(-x + 1) - 1/40*((2*(3*(4*x - 17)*(x + 1) + 133)*
(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10
)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) + 1/3*((2*x - 5)*(x
+ 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 3/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) +
 sqrt(x + 1)*sqrt(-x + 1) + 21/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [A]** time = 0.01, size = 113, normalized size = 1.05

$$\frac{21\sqrt{\left(x+1\right)\left(-x+1\right)}\;\arcsin\left(x\right)}{16\sqrt{x+1}\;\sqrt{-x+1}}+\frac{\left(-x+1\right)^{\frac{9}{2}}\left(x+1\right)^{\frac{3}{2}}}{6}+\frac{3\left(-x+1\right)^{\frac{7}{2}}\left(x+1\right)^{\frac{3}{2}}}{10}+\frac{21\left(-x+1\right)^{\frac{5}{2}}\left(x+1\right)^{\frac{3}{2}}}{40}+\frac{7\left(-x+1\right)^{\frac{3}{2}}\left(x+1\right)^{\frac{3}{2}}}{8}+\frac{21\sqrt{-x+1}\;\left(x+1\right)^{\frac{3}{2}}}{16}-\frac{21\sqrt{-x+1}\;\sqrt{x+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(9/2)*(1+x)^(1/2),x)
```

```
[Out] 1/6*(-x+1)^(9/2)*(1+x)^(3/2)+3/10*(-x+1)^(7/2)*(1+x)^(3/2)+21/40*(-x+1)^(5/
2)*(1+x)^(3/2)+7/8*(-x+1)^(3/2)*(1+x)^(3/2)+21/16*(-x+1)^(1/2)*(1+x)^(3/2)-
21/16*(-x+1)^(1/2)*(1+x)^(1/2)+21/16*((1+x)*(-x+1))^(1/2)/(1+x)^(1/2)/(-x+1
)^(1/2)*arcsin(x)
```

**maxima [A]** time = 3.00, size = 68, normalized size = 0.63

$$-\frac{1}{6}\left(-x^{2}+1\right)^{\frac{3}{2}}x^{3}+\frac{4}{5}\left(-x^{2}+1\right)^{\frac{3}{2}}x^{2}-\frac{13}{8}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{28}{15}\left(-x^{2}+1\right)^{\frac{3}{2}}+\frac{21}{16}\sqrt{-x^{2}+1}\,x+\frac{21}{16}\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(3/2)*x^3 + 4/5*(-x^2 + 1)^(3/2)*x^2 - 13/8*(-x^2 + 1)^(3/2
)*x + 28/15*(-x^2 + 1)^(3/2) + 21/16*sqrt(-x^2 + 1)*x + 21/16*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(9/2)*(x + 1)^(1/2), x)
```

**sympy [A]** time = 48.59, size = 289, normalized size = 2.68

$$\begin{cases} -\frac{21i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}} - \frac{59i(x+1)^{\frac{11}{2}}}{30\sqrt{x-1}} + \frac{1151i(x+1)^{\frac{9}{2}}}{120\sqrt{x-1}} - \frac{2947i(x+1)^{\frac{7}{2}}}{120\sqrt{x-1}} + \frac{8171i(x+1)^{\frac{5}{2}}}{240\sqrt{x-1}} - \frac{1045i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}} + \frac{21i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{21\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}} + \frac{59(x+1)^{\frac{11}{2}}}{30\sqrt{1-x}} - \frac{1151(x+1)^{\frac{9}{2}}}{120\sqrt{1-x}} + \frac{2947(x+1)^{\frac{7}{2}}}{120\sqrt{1-x}} - \frac{8171(x+1)^{\frac{5}{2}}}{240\sqrt{1-x}} + \frac{1045(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}} - \frac{21\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-21*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(13/2)/(6*sqrt
(x - 1)) - 59*I*(x + 1)**(11/2)/(30*sqrt(x - 1)) + 1151*I*(x + 1)**(9/2)/(1
20*sqrt(x - 1)) - 2947*I*(x + 1)**(7/2)/(120*sqrt(x - 1)) + 8171*I*(x + 1)*
*(5/2)/(240*sqrt(x - 1)) - 1045*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 21*I*sq
rt(x + 1)/(8*sqrt(x - 1)), Abs(x + 1)/2 > 1), (21*asin(sqrt(2)*sqrt(x + 1)/
2)/8 - (x + 1)**(13/2)/(6*sqrt(1 - x)) + 59*(x + 1)**(11/2)/(30*sqrt(1 - x)
) - 1151*(x + 1)**(9/2)/(120*sqrt(1 - x)) + 2947*(x + 1)**(7/2)/(120*sqrt(1
 - x)) - 8171*(x + 1)**(5/2)/(240*sqrt(1 - x)) + 1045*(x + 1)**(3/2)/(48*sq
rt(1 - x)) - 21*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

## 3.995 $\int (1-x)^{7/2}\sqrt{1+x}\,dx$

**Optimal**. Leaf size=88

$$\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}+\frac{7}{20}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{7}{8}\sin^{-1}(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}+\frac{7}{20}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{7}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + (7*(
1 - x)^(5/2)*(1 + x)^(3/2))/20 + ((1 - x)^(7/2)*(1 + x)^(3/2))/5 + (7*ArcSi
n[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{7/2}\sqrt{1+x}\,dx &= \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}+\frac{7}{5}\int (1-x)^{5/2}\sqrt{1+x}\,dx\\
&= \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}+\frac{7}{4}\int (1-x)^{3/2}\sqrt{1+x}\,dx\\
&= \frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}+\frac{7}{4}\int \sqrt{1-x}\,\sqrt{1+\\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x}\,+\frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}\\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x}\,+\frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}\\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x}\,+\frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 56, normalized size = 0.64

$$\frac{1}{120}\sqrt{1-x^2}\left(-24x^4+90x^3-112x^2+15x+136\right)-\frac{7}{4}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x^2]*(136 + 15*x - 112*x^2 + 90*x^3 - 24*x^4))/120 - (7*ArcSin[Sq
rt[1 - x]/Sqrt[2]])/4
```

**IntegrateAlgebraic [A]** time = 0.08, size = 114, normalized size = 1.30

$$\frac{\sqrt{x+1}\left(\frac{105(x+1)^4}{(1-x)^4}+\frac{490(x+1)^3}{(1-x)^3}+\frac{896(x+1)^2}{(1-x)^2}+\frac{790(x+1)}{1-x}-105\right)}{60\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^5}+\frac{7}{4}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(-105 + (790*(1 + x))/(1 - x) + (896*(1 + x)^2)/(1 - x)^2 + (4
90*(1 + x)^3)/(1 - x)^3 + (105*(1 + x)^4)/(1 - x)^4))/(60*Sqrt[1 - x]*(1 +
(1 + x)/(1 - x))^5) + (7*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]])/4
```

**fricas** [A] time = 1.32, size = 57, normalized size = 0.65

$$-\frac{1}{120}\left(24\,x^{4}-90\,x^{3}+112\,x^{2}-15\,x-136\right)\sqrt{x+1}\,\sqrt{-x+1}\,-\frac{7}{4}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}\,-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(24*x^4 - 90*x^3 + 112*x^2 - 15*x - 136)*sqrt(x + 1)*sqrt(-x + 1) -
7/4*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac** [A] time = 1.06, size = 115, normalized size = 1.31

$$-\frac{1}{120}\left(\left(2\left(3\left(4\,x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\,\sqrt{-x+1}\,+\frac{1}{12}\left(\left(2\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}\,-\sqrt{x+1}\,\left(x-2\right)\sqrt{-x+1}\,+\sqrt{x+1}\,\sqrt{-x+1}\,+\frac{7}{4}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(
x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(
x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-
x + 1) + 7/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple** [A] time = 0.00, size = 99, normalized size = 1.12

$$\frac{7\sqrt{\left(x+1\right)\left(-x+1\right)}\,\arcsin\left(x\right)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{\left(-x+1\right)^{\frac{7}{2}}\left(x+1\right)^{\frac{3}{2}}}{5}+\frac{7\left(-x+1\right)^{\frac{5}{2}}\left(x+1\right)^{\frac{3}{2}}}{20}+\frac{7\left(-x+1\right)^{\frac{3}{2}}\left(x+1\right)^{\frac{3}{2}}}{12}+\frac{7\sqrt{-x+1}\,\left(x+1\right)^{\frac{3}{2}}}{8}-\frac{7\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)*(x+1)^(1/2),x)
```

```
[Out] 1/5*(-x+1)^(7/2)*(x+1)^(3/2)+7/20*(-x+1)^(5/2)*(x+1)^(3/2)+7/12*(-x+1)^(3/2
)*(x+1)^(3/2)+7/8*(-x+1)^(1/2)*(x+1)^(3/2)-7/8*(-x+1)^(1/2)*(x+1)^(1/2)+7/8
*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima** [A] time = 2.97, size = 54, normalized size = 0.61

$$\frac{1}{5}\left(-x^{2}+1\right)^{\frac{3}{2}}x^{2}-\frac{3}{4}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{17}{15}\left(-x^{2}+1\right)^{\frac{3}{2}}+\frac{7}{8}\sqrt{-x^{2}+1}\,x+\frac{7}{8}\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="maxima")
```

[Out] 1/5*(-x^2 + 1)^(3/2)*x^2 - 3/4*(-x^2 + 1)^(3/2)*x + 17/15*(-x^2 + 1)^(3/2) + 7/8*sqrt(-x^2 + 1)*x + 7/8*arcsin(x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x)^(7/2)*(x + 1)^(1/2),x)

[Out] int((1 - x)^(7/2)*(x + 1)^(1/2), x)

**sympy** [A] time = 21.07, size = 253, normalized size = 2.88

$$\begin{cases} -\frac{7i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}} + \frac{39i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}} - \frac{449i(x+1)^{\frac{7}{2}}}{60\sqrt{x-1}} + \frac{1657i(x+1)^{\frac{5}{2}}}{120\sqrt{x-1}} - \frac{263i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}} + \frac{7i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{7\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}} - \frac{39(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}} + \frac{449(x+1)^{\frac{7}{2}}}{60\sqrt{1-x}} - \frac{1657(x+1)^{\frac{5}{2}}}{120\sqrt{1-x}} + \frac{263(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}} - \frac{7\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)**(7/2)*(1+x)**(1/2),x)

[Out] Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt(x - 1)) + 39*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 449*I*(x + 1)**(7/2)/(60*sqrt(x - 1)) + 1657*I*(x + 1)**(5/2)/(120*sqrt(x - 1)) - 263*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 7*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), (7*asin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 39*(x + 1)**(9/2)/(20*sqrt(1 - x)) + 449*(x + 1)**(7/2)/(60*sqrt(1 - x)) - 1657*(x + 1)**(5/2)/(120*sqrt(1 - x)) + 263*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 7*sqrt(x + 1)/(4*sqrt(1 - x)), True))

## 3.996 $\int (1-x)^{5/2}\sqrt{1+x}\,dx$

**Optimal**. Leaf size=68

$$\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}+\frac{5}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{5}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{5}{8}\sin^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}+\frac{5}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{5}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{5}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + ((1
- x)^(5/2)*(1 + x)^(3/2))/4 + (5*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{5/2}\sqrt{1+x}\,dx &= \frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{4}\int (1-x)^{3/2}\sqrt{1+x}\,dx\\
&= \frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{4}\int \sqrt{1-x}\sqrt{1+x}\,dx\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\sin^{-1}(x)
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 50, normalized size = 0.74

$$\frac{1}{24}\left(\sqrt{1-x^2}\left(6x^3-16x^2+9x+16\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x^2]*(16 + 9*x - 16*x^2 + 6*x^3) - 30*ArcSin[Sqrt[1 - x]/Sqrt[2]]
)/24
```

**IntegrateAlgebraic [A]** time = 0.08, size = 100, normalized size = 1.47

$$\frac{\sqrt{x+1}\left(\frac{15(x+1)^3}{(1-x)^3}+\frac{55(x+1)^2}{(1-x)^2}+\frac{73(x+1)}{1-x}-15\right)}{12\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^4}+\frac{5}{4}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(-15 + (73*(1 + x))/(1 - x) + (55*(1 + x)^2)/(1 - x)^2 + (15*(
1 + x)^3)/(1 - x)^3))/(12*Sqrt[1 - x]*(1 + (1 + x)/(1 - x))^4) + (5*ArcTan[
Sqrt[1 + x]/Sqrt[1 - x]])/4
```

**fricas [A]** time = 1.36, size = 52, normalized size = 0.76

$$\frac{1}{24}\left(6\,x^3-16\,x^2+9\,x+16\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{4}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 - 16*x^2 + 9*x + 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sqr
t(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac [B]** time = 1.29, size = 101, normalized size = 1.49

$$\frac{1}{24}\left(\left(2\left(3x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{6}\left(\left(2x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{5}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) -
1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x -
 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5/4*arcsin(1/2*sqrt(2)*sqrt(x
 + 1))
```

**maple [A]** time = 0.01, size = 85, normalized size = 1.25

$$\frac{5\sqrt{\left(x+1\right)\left(-x+1\right)}\,\arcsin\left(x\right)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{\left(-x+1\right)^{\frac{5}{2}}\left(x+1\right)^{\frac{3}{2}}}{4}+\frac{5\left(-x+1\right)^{\frac{3}{2}}\left(x+1\right)^{\frac{3}{2}}}{12}+\frac{5\sqrt{-x+1}\,\left(x+1\right)^{\frac{3}{2}}}{8}-\frac{5\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)*(x+1)^(1/2),x)
```

```
[Out] 1/4*(-x+1)^(5/2)*(x+1)^(3/2)+5/12*(-x+1)^(3/2)*(x+1)^(3/2)+5/8*(-x+1)^(1/2)
*(x+1)^(3/2)-5/8*(-x+1)^(1/2)*(x+1)^(1/2)+5/8*((x+1)*(-x+1))^(1/2)/(x+1)^(1
/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 2.95, size = 40, normalized size = 0.59

$$-\frac{1}{4}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{2}{3}\left(-x^{2}+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^{2}+1}\,x+\frac{5}{8}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)*x + 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/8
*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(1/2), x)
```

**sympy [A]** time = 9.03, size = 218, normalized size = 3.21

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}} - \frac{23i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}} + \frac{127i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}} - \frac{133i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}} + \frac{23(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}} - \frac{127(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}} + \frac{133(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}} - \frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) - 23*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) + 127*I*(x + 1)**(5/2)/(24*sq
rt(x - 1)) - 133*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*sqr
t(x - 1)), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(
9/2)/(4*sqrt(1 - x)) + 23*(x + 1)**(7/2)/(12*sqrt(1 - x)) - 127*(x + 1)**(5
/2)/(24*sqrt(1 - x)) + 133*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x + 1)/
(4*sqrt(1 - x)), True))
```

## 3.997 $\int (1-x)^{3/2}\sqrt{1+x}\,dx$

**Optimal**. Leaf size=48

$$\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{3/2}\sqrt{1+x}\,dx &= \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 44, normalized size = 0.92

$$\frac{1}{6}\left(-2x^2+3x+2\right)\sqrt{1-x^2}-\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*Sqrt[1 + x],x]
[Out] ((2 + 3*x - 2*x^2)*Sqrt[1 - x^2])/6 - ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

**IntegrateAlgebraic [A]** time = 0.07, size = 82, normalized size = 1.71

$$\frac{\sqrt{x+1}\left(\frac{3(x+1)^2}{(1-x)^2}+\frac{8(x+1)}{1-x}-3\right)}{3\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^3}+\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)*Sqrt[1 + x],x]
[Out] (Sqrt[1 + x]*(-3 + (8*(1 + x))/(1 - x) + (3*(1 + x)^2)/(1 - x)^2))/(3*Sqrt[
1 - x]*(1 + (1 + x)/(1 - x))^3) + ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

**fricas [A]** time = 1.36, size = 47, normalized size = 0.98

$$-\frac{1}{6}\left(2\,x^2-3\,x-2\right)\sqrt{x+1}\,\sqrt{-x+1}\,-\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}\,-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^2 - 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(
-x + 1) - 1)/x)
```

**giac** [A] time = 1.02, size = 50, normalized size = 1.04

$$-\frac{1}{6}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x
 + 1) + arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple** [B] time = 0.00, size = 71, normalized size = 1.48

$$\frac{\sqrt{(x+1)(-x+1)}\arcsin(x)}{2\sqrt{x+1}\sqrt{-x+1}}+\frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{3}+\frac{\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{2}-\frac{\sqrt{-x+1}\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)*(x+1)^(1/2),x)
```

```
[Out] 1/3*(-x+1)^(3/2)*(x+1)^(3/2)+1/2*(-x+1)^(1/2)*(x+1)^(3/2)-1/2*(-x+1)^(1/2)*
(x+1)^(1/2)+1/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima** [A] time = 2.90, size = 28, normalized size = 0.58

$$\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (1-x)^{3/2}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(3/2)*(x + 1)^(1/2), x)
```

**sympy [B]** time = 4.49, size = 168, normalized size = 3.50

$$\begin{cases} -i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} + \frac{11i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{17i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} - \frac{11(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{17(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x - 1
)) + 11*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 17*I*(x + 1)**(3/2)/(6*sqrt(x -
1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (asin(sqrt(2)*sqrt(x +
1)/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) - 11*(x + 1)**(5/2)/(6*sqrt(1 - x))
+ 17*(x + 1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

## 3.998 $\int \sqrt{1-x}\sqrt{1+x}\,dx$

**Optimal**. Leaf size=28

$$\frac{1}{2}\sqrt{1-x}\,\sqrt{x+1}\,x+\frac{1}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {38, 41, 216}

$$\frac{1}{2}\sqrt{1-x}\,\sqrt{x+1}\,x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ArcSin[x]/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{1-x}\sqrt{1+x}\,dx = \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$$
$$= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{2}\sin^{-1}(x)$$

**Mathematica** [A] time = 0.01, size = 20, normalized size = 0.71

$$\frac{1}{2}\left(\sqrt{1-x^2}\,x+\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*Sqrt[1 + x],x]
[Out] (x*Sqrt[1 - x^2] + ArcSin[x])/2
```

**IntegrateAlgebraic** [B] time = 0.06, size = 73, normalized size = 2.61

$$\frac{\frac{\sqrt{1-x}}{\sqrt{x+1}}-\frac{(1-x)^{3/2}}{(x+1)^{3/2}}}{\left(\frac{1-x}{x+1}+1\right)^2}-\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]*Sqrt[1 + x],x]
[Out] (-((1 - x)^(3/2)/(1 + x)^(3/2)) + Sqrt[1 - x]/Sqrt[1 + x])/(1 + (1 - x)/(1
+ x))^2 - ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

**fricas** [A] time = 0.88, size = 38, normalized size = 1.36

$$\frac{1}{2}\sqrt{x+1}\,x\sqrt{-x+1} - \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x + 1)*x*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac [B]** time = 1.04, size = 42, normalized size = 1.50

$$\frac{1}{2}\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + arcsin(1/
2*sqrt(2)*sqrt(x + 1))
```

**maple [B]** time = 0.00, size = 57, normalized size = 2.04

$$\frac{\sqrt{(x+1)\,(-x+1)}\;\arcsin(x)}{2\sqrt{x+1}\,\sqrt{-x+1}}-\frac{(-x+1)^{\frac{3}{2}}\,\sqrt{x+1}}{2}+\frac{\sqrt{-x+1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)*(x+1)^(1/2),x)
```

```
[Out] -1/2*(-x+1)^(3/2)*(x+1)^(1/2)+1/2*(-x+1)^(1/2)*(x+1)^(1/2)+1/2*((x+1)*(-x+1
))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 2.98, size = 17, normalized size = 0.61

$$\frac{1}{2}\,\sqrt{-x^2+1}\,x+\frac{1}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

**mupad [B]** time = 0.20, size = 37, normalized size = 1.32

$$\frac{x\,\sqrt{1-x}\,\sqrt{x+1}}{2}-\frac{\ln\left(x-\sqrt{1-x}\,\sqrt{x+1}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(1/2),x)
```

```
[Out] (x*(1 - x)^(1/2)*(x + 1)^(1/2))/2 - (log(x - (1 - x)^(1/2)*(x + 1)^(1/2)*1i
)*1i)/2
```

**sympy [B]** time = 2.73, size = 133, normalized size = 4.75

$$\begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} - \frac{3i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} + \frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} + \frac{3(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} - \frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x - 1
)) - 3*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x
+ 1)/2 > 1), (asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x))
+ 3*(x + 1)**(3/2)/(2*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

## 3.999 $\int \frac{\sqrt{1+x}}{\sqrt{1-x}} dx$

**Optimal**. Leaf size=21

$$\sin^{-1}(x) - \sqrt{1-x}\sqrt{x+1}$$

**Rubi [A]** time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\sin^{-1}(x) - \sqrt{1-x}\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/Sqrt[1 - x],x]
```

```
[Out] -(Sqrt[1 - x]*Sqrt[1 + x]) + ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = -\sqrt{1-x}\,\sqrt{1+x} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= -\sqrt{1-x}\,\sqrt{1+x} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\sqrt{1-x}\,\sqrt{1+x} + \sin^{-1}(x)$$

**Mathematica [A]** time = 0.01, size = 32, normalized size = 1.52

$$-\sqrt{1-x^2} - 2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x],x]
[Out] -Sqrt[1 - x^2] - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

**IntegrateAlgebraic [C]** time = 0.09, size = 45, normalized size = 2.14

$$-\sqrt{1-x}\,\sqrt{x+1} + 2i\log\left(\sqrt{1-x} - i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/Sqrt[1 - x],x]
[Out] -(Sqrt[1 - x]*Sqrt[1 + x]) + (2*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

**fricas [B]** time = 1.11, size = 37, normalized size = 1.76

$$-\sqrt{x+1}\,\sqrt{-x+1} - 2\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac [A]** time = 1.01, size = 28, normalized size = 1.33

$$-\sqrt{x+1}\,\sqrt{-x+1} + 2\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [B]** time = 0.01, size = 42, normalized size = 2.00

$$\frac{\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\ \sqrt{-x+1}}-\sqrt{-x+1}\ \sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(1/2),x)
```

```
[Out] -(-x+1)^(1/2)*(x+1)^(1/2)+((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arc
sin(x)
```

**maxima [A]** time = 2.90, size = 14, normalized size = 0.67

$$-\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) + arcsin(x)
```

**mupad [B]** time = 0.14, size = 14, normalized size = 0.67

$$\operatorname{asin}(x)-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(1/2),x)
```

```
[Out] asin(x) - (1 - x^2)^(1/2)
```

**sympy [B]** time = 1.84, size = 100, normalized size = 4.76

$$\begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}}+\frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1)
 + 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (2*asin(sqrt(2)*sqrt(x +
 1)/2) + (x + 1)**(3/2)/sqrt(1 - x) - 2*sqrt(x + 1)/sqrt(1 - x), True))
```

## 3.1000 $\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx$

**Optimal**. Leaf size=23

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(3/2),x]
```

```
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx = \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \sin^{-1}(x)$$

**Mathematica** [A] time = 0.01, size = 36, normalized size = 1.57

$$2\left(\frac{\sqrt{x+1}}{\sqrt{1-x}} + \sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(3/2),x]
```

```
[Out] 2*(Sqrt[1 + x]/Sqrt[1 - x] + ArcSin[Sqrt[1 - x]/Sqrt[2]])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 39, normalized size = 1.70

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}} - 2\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(3/2),x]
```

```
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - 2*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

**fricas** [B] time = 1.27, size = 48, normalized size = 2.09

$$\frac{2\left((x-1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right) + x - \sqrt{x+1}\,\sqrt{-x+1} - 1\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + x - sqrt(x + 1)*sqrt(
-x + 1) - 1)/(x - 1)
```

**giac [A]** time = 1.06, size = 33, normalized size = 1.43

$$-\frac{2\sqrt{x+1}\sqrt{-x+1}}{x-1}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="giac")
[Out] -2*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [B]** time = 0.03, size = 64, normalized size = 2.78

$$-\frac{\sqrt{(x+1)(-x+1)}\arcsin(x)}{\sqrt{x+1}\sqrt{-x+1}}+\frac{2\sqrt{x+1}\sqrt{(x+1)(-x+1)}}{\sqrt{-(x+1)(x-1)}\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(3/2),x)
[Out] 2*(x+1)^(1/2)/(-(x+1)*(-1+x))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)-((x+1
)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 2.95, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{-x^2+1}}{x-1}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
[Out] -2*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x+1}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(3/2),x)
[Out] int((x + 1)^(1/2)/(1 - x)^(3/2), x)
```

**sympy [A]** time = 1.62, size = 71, normalized size = 3.09

$$\begin{cases} 2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1\\ -2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1),
Abs(x + 1)/2 > 1), (-2*asin(sqrt(2)*sqrt(x + 1)/2) + 2*sqrt(x + 1)/sqrt(1 -
 x), True))
```

## 3.1001 $\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} dx$

**Optimal**. Leaf size=20

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(5/2),x]
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} dx = \frac{(1+x)^{3/2}}{3(1-x)^{3/2}}$$

**Mathematica [A]** time = 0.01, size = 20, normalized size = 1.00

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(5/2),x]
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 20, normalized size = 1.00

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(5/2),x]
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

**fricas [B]** time = 1.25, size = 33, normalized size = 1.65

$$\frac{x^2 + (x+1)^{\frac{3}{2}}\sqrt{-x+1} - 2x + 1}{3\left(x^2 - 2x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="fricas")
[Out] 1/3*(x^2 + (x + 1)^(3/2)*sqrt(-x + 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

**giac [A]** time = 0.94, size = 19, normalized size = 0.95

$$\frac{(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{3(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="giac")
[Out] 1/3*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^2
```

**maple [A]** time = 0.00, size = 15, normalized size = 0.75

$$\frac{(x+1)^{\frac{3}{2}}}{3(-x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(5/2),x)
[Out] 1/3*(x+1)^(3/2)/(-x+1)^(3/2)
```

**maxima** [B] time = 1.26, size = 38, normalized size = 1.90

$$\frac{2\sqrt{-x^2+1}}{3\left(x^2-2x+1\right)}+\frac{\sqrt{-x^2+1}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x - 1)
```

**mupad** [B] time = 0.27, size = 34, normalized size = 1.70

$$\frac{\left(\frac{x\sqrt{x+1}}{3}+\frac{\sqrt{x+1}}{3}\right)\sqrt{1-x}}{x^2-2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(5/2),x)
```

```
[Out] (((x*(x + 1)^(1/2))/3 + (x + 1)^(1/2)/3)*(1 - x)^(1/2))/(x^2 - 2*x + 1)
```

**sympy** [A] time = 1.67, size = 61, normalized size = 3.05

$$\begin{cases} \frac{i(x+1)^{\frac{3}{2}}}{3\sqrt{x-1}\,(x+1)-6\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -\frac{(x+1)^{\frac{3}{2}}}{3\sqrt{1-x}\,(x+1)-6\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(3/2)/(3*sqrt(x - 1)*(x + 1) - 6*sqrt(x - 1)), Abs(x
+ 1)/2 > 1), (-(x + 1)**(3/2)/(3*sqrt(1 - x)*(x + 1) - 6*sqrt(1 - x)), True
))
```

## 3.1002 $\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} \, dx$

**Optimal**. Leaf size=41

$$\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(7/2),x]
```

```
[Out] (1 + x)^(3/2)/(5*(1 - x)^(5/2)) + (1 + x)^(3/2)/(15*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} \, dx &= \frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{1}{5}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} \, dx\\ &= \frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{(1+x)^{3/2}}{15(1-x)^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 23, normalized size = 0.56

$$-\frac{(x-4)(x+1)^{3/2}}{15(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(7/2),x]
[Out] -1/15*((-4 + x)*(1 + x)^(3/2))/(1 - x)^(5/2)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 34, normalized size = 0.83

$$\frac{(x+1)^{3/2}\left(\frac{3(x+1)}{1-x}+5\right)}{30(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(7/2),x]
[Out] ((1 + x)^(3/2)*(5 + (3*(1 + x))/(1 - x)))/(30*(1 - x)^(3/2))
```

**fricas [A]** time = 0.92, size = 53, normalized size = 1.29

$$\frac{4\,x^3-12\,x^2+\left(x^2-3\,x-4\right)\sqrt{x+1}\,\sqrt{-x+1}\,+12\,x-4}{15\,\left(x^3-3\,x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="fricas")
[Out] 1/15*(4*x^3 - 12*x^2 + (x^2 - 3*x - 4)*sqrt(x + 1)*sqrt(-x + 1) + 12*x - 4)
/(x^3 - 3*x^2 + 3*x - 1)
```

**giac [A]** time = 1.05, size = 22, normalized size = 0.54

$$\frac{(x+1)^{\frac{3}{2}}(x-4)\sqrt{-x+1}}{15\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="giac")
[Out] 1/15*(x + 1)^(3/2)*(x - 4)*sqrt(-x + 1)/(x - 1)^3
```

**maple** [A] time = 0.00, size = 18, normalized size = 0.44

$$-\frac{(x+1)^{\frac{3}{2}}(x-4)}{15(-x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(7/2),x)
```

```
[Out] -1/15*(x+1)^(3/2)*(x-4)/(-x+1)^(5/2)
```

**maxima** [B] time = 1.40, size = 64, normalized size = 1.56

$$-\frac{2\sqrt{-x^2+1}}{5\left(x^3-3x^2+3x-1\right)}-\frac{\sqrt{-x^2+1}}{15\left(x^2-2x+1\right)}+\frac{\sqrt{-x^2+1}}{15(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 1/15*sqrt(-x^2 + 1)/(x^2 - 2*
x + 1) + 1/15*sqrt(-x^2 + 1)/(x - 1)
```

**mupad** [B] time = 0.24, size = 50, normalized size = 1.22

$$-\frac{\sqrt{1-x}\left(\frac{x\sqrt{x+1}}{5}+\frac{4\sqrt{x+1}}{15}-\frac{x^2\sqrt{x+1}}{15}\right)}{x^3-3x^2+3x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(7/2),x)
```

```
[Out] -((1 - x)^(1/2)*((x*(x + 1)^(1/2))/5 + (4*(x + 1)^(1/2))/15 - (x^2*(x + 1)^
(1/2))/15))/(3*x - 3*x^2 + x^3 - 1)
```

**sympy** [B] time = 6.55, size = 173, normalized size = 4.22

$$\begin{cases} \frac{i(x+1)^{\frac{5}{2}}}{15\sqrt{x-1}(x+1)^2-60\sqrt{x-1}(x+1)+60\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{15\sqrt{x-1}(x+1)^2-60\sqrt{x-1}(x+1)+60\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1 \\ -\frac{(x+1)^{\frac{5}{2}}}{15\sqrt{1-x}(x+1)^2-60\sqrt{1-x}(x+1)+60\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{15\sqrt{1-x}(x+1)^2-60\sqrt{1-x}(x+1)+60\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(5/2)/(15*sqrt(x - 1)*(x + 1)**2 - 60*sqrt(x - 1)*(x
+ 1) + 60*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(15*sqrt(x - 1)*(x + 1)**2 - 60
*sqrt(x - 1)*(x + 1) + 60*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-(x + 1)**(5/2)
/(15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*sqrt(1 - x)) + 5*
(x + 1)**(3/2)/(15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*sqr
t(1 - x)), True))
```

## 3.1003 $\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx$

**Optimal**. Leaf size=61

$$\frac{2(x+1)^{3/2}}{105(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{3/2}}{105(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(3/2)/(7*(1 - x)^(7/2)) + (2*(1 + x)^(3/2))/(35*(1 - x)^(5/2)) + (2
*(1 + x)^(3/2))/(105*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx = \frac{(1+x)^{3/2}}{7(1-x)^{7/2}} + \frac{2}{7}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx$$
$$= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}} + \frac{2(1+x)^{3/2}}{35(1-x)^{5/2}} + \frac{2}{35}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx$$
$$= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}} + \frac{2(1+x)^{3/2}}{35(1-x)^{5/2}} + \frac{2(1+x)^{3/2}}{105(1-x)^{3/2}}$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 0.49

$$\frac{(x+1)^{3/2}\left(2x^2-10x+23\right)}{105(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(9/2),x]
[Out] ((1 + x)^(3/2)*(23 - 10*x + 2*x^2))/(105*(1 - x)^(7/2))
```

**IntegrateAlgebraic** [A] time = 0.07, size = 48, normalized size = 0.79

$$\frac{(x+1)^{3/2}\left(\frac{15(x+1)^2}{(1-x)^2}+\frac{42(x+1)}{1-x}+35\right)}{420(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(9/2),x]
[Out] ((1 + x)^(3/2)*(35 + (42*(1 + x))/(1 - x) + (15*(1 + x)^2)/(1 - x)^2))/(420
*(1 - x)^(3/2))
```

**fricas** [A] time = 1.17, size = 70, normalized size = 1.15

$$\frac{23\,x^4-92\,x^3+138\,x^2+\left(2\,x^3-8\,x^2+13\,x+23\right)\sqrt{x+1}\,\sqrt{-x+1}\,-92\,x+23}{105\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="fricas")
[Out] 1/105*(23*x^4 - 92*x^3 + 138*x^2 + (2*x^3 - 8*x^2 + 13*x + 23)*sqrt(x + 1)*
sqrt(-x + 1) - 92*x + 23)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

**giac** [A] time = 1.22, size = 29, normalized size = 0.48

$$\frac{(2\,(x+1)(x-6)+35)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{105\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(2*(x + 1)*(x - 6) + 35)*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^4
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.41

$$\frac{(x+1)^{\frac{3}{2}}\left(2x^2-10x+23\right)}{105\,(-x+1)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(9/2),x)
```

```
[Out] 1/105*(x+1)^(3/2)*(2*x^2-10*x+23)/(-x+1)^(7/2)
```

**maxima** [B] time = 1.27, size = 95, normalized size = 1.56

$$\frac{2\sqrt{-x^2+1}}{7\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}+\frac{\sqrt{-x^2+1}}{35\left(x^3-3\,x^2+3\,x-1\right)}-\frac{2\sqrt{-x^2+1}}{105\left(x^2-2\,x+1\right)}+\frac{2\sqrt{-x^2+1}}{105\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/35*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) - 2/105*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/105*sqrt(-
x^2 + 1)/(x - 1)
```

**mupad** [B] time = 0.27, size = 64, normalized size = 1.05

$$\frac{\sqrt{1-x}\,\left(\frac{13\,x\,\sqrt{x+1}}{105}+\frac{23\,\sqrt{x+1}}{105}-\frac{8\,x^2\,\sqrt{x+1}}{105}+\frac{2\,x^3\,\sqrt{x+1}}{105}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((13*x*(x + 1)^(1/2))/105 + (23*(x + 1)^(1/2))/105 - (8*x^2*
(x + 1)^(1/2))/105 + (2*x^3*(x + 1)^(1/2))/105))/(6*x^2 - 4*x - 4*x^3 + x^4
 + 1)
```

**sympy [B]** time = 19.92, size = 568, normalized size = 9.31

$$\begin{cases} \frac{2i(x+1)^{\frac{9}{2}}}{105\sqrt{x-1}\,(x+1)^4-840\sqrt{x-1}\,(x+1)^3+2520\sqrt{x-1}\,(x+1)^2-3360\sqrt{x-1}\,(x+1)+1680\sqrt{x-1}} - \frac{18i(x+1)^{\frac{7}{2}}}{105\sqrt{x-1}\,(x+1)^4-840\sqrt{x-1}\,(x+1)^3+2520\sqrt{x-1}\,(x+1)^2-3360\sqrt{x-1}\,(x+1)+1680\sqrt{x-1}} + \frac{63i(x+1)^{\frac{5}{2}}}{105\sqrt{x-1}\,(x+1)^4-840\sqrt{x-1}\,(x+1)^3+2520\sqrt{x-1}\,(x+1)^2-3360\sqrt{x-1}\,(x+1)+1680\sqrt{x-1}} - \frac{70i(x+1)^{\frac{3}{2}}}{105\sqrt{x-1}\,(x+1)^4-840\sqrt{x-1}\,(x+1)^3+2520\sqrt{x-1}\,(x+1)^2-3360\sqrt{x-1}\,(x+1)+1680\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -\frac{2(x+1)^{\frac{9}{2}}}{105\sqrt{1-x}\,(x+1)^4-840\sqrt{1-x}\,(x+1)^3+2520\sqrt{1-x}\,(x+1)^2-3360\sqrt{1-x}\,(x+1)+1680\sqrt{1-x}} + \frac{18(x+1)^{\frac{7}{2}}}{105\sqrt{1-x}\,(x+1)^4-840\sqrt{1-x}\,(x+1)^3+2520\sqrt{1-x}\,(x+1)^2-3360\sqrt{1-x}\,(x+1)+1680\sqrt{1-x}} - \frac{63(x+1)^{\frac{5}{2}}}{105\sqrt{1-x}\,(x+1)^4-840\sqrt{1-x}\,(x+1)^3+2520\sqrt{1-x}\,(x+1)^2-3360\sqrt{1-x}\,(x+1)+1680\sqrt{1-x}} + \frac{70(x+1)^{\frac{3}{2}}}{105\sqrt{1-x}\,(x+1)^4-840\sqrt{1-x}\,(x+1)^3+2520\sqrt{1-x}\,(x+1)^2-3360\sqrt{1-x}\,(x+1)+1680\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**(9/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sqrt(x - 1)
*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x + 1) + 1680
*sqrt(x - 1)) - 18*I*(x + 1)**(7/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sqrt(
x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x + 1)
+ 1680*sqrt(x - 1)) + 63*I*(x + 1)**(5/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840
*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x
 + 1) + 1680*sqrt(x - 1)) - 70*I*(x + 1)**(3/2)/(105*sqrt(x - 1)*(x + 1)**4
 - 840*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x -
 1)*(x + 1) + 1680*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-2*(x + 1)**(9/2)/(105
*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 - x)*(x
+ 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) + 18*(x + 1)**(7/2)/
(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 - x)
*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) - 63*(x + 1)**(5
/2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1
- x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) + 70*(x + 1)
**(3/2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqr
t(1 - x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)), True))
```

## 3.1004 $\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx$

**Optimal**. Leaf size=81

$$\frac{2(x+1)^{3/2}}{315(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{105(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{21(1-x)^{7/2}}+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{3/2}}{315(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{105(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{21(1-x)^{7/2}}+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(11/2),x]
```

```
[Out] (1 + x)^(3/2)/(9*(1 - x)^(9/2)) + (1 + x)^(3/2)/(21*(1 - x)^(7/2)) + (2*(1
+ x)^(3/2))/(105*(1 - x)^(5/2)) + (2*(1 + x)^(3/2))/(315*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{1}{3}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2}{21}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{5/2}}+\frac{2}{105}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{5/2}}+\frac{2(1+x)^{3/2}}{315(1-x)^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 35, normalized size = 0.43

$$\frac{(x+1)^{3/2}\left(-2x^3+12x^2-33x+58\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(11/2),x]
[Out] ((1 + x)^(3/2)*(58 - 33*x + 12*x^2 - 2*x^3))/(315*(1 - x)^(9/2))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 77, normalized size = 0.95

$$\frac{\frac{35(x+1)^{9/2}}{(1-x)^{9/2}}+\frac{135(x+1)^{7/2}}{(1-x)^{7/2}}+\frac{189(x+1)^{5/2}}{(1-x)^{5/2}}+\frac{105(x+1)^{3/2}}{(1-x)^{3/2}}}{2520}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(11/2),x]
[Out] ((105*(1 + x)^(3/2))/(1 - x)^(3/2) + (189*(1 + x)^(5/2))/(1 - x)^(5/2) + (1
35*(1 + x)^(7/2))/(1 - x)^(7/2) + (35*(1 + x)^(9/2))/(1 - x)^(9/2))/2520
```

**fricas [A]** time = 1.31, size = 85, normalized size = 1.05

$$\frac{58\,x^5-290\,x^4+580\,x^3-580\,x^2+\left(2\,x^4-10\,x^3+21\,x^2-25\,x-58\right)\sqrt{x+1}\,\sqrt{-x+1}\,+290\,x-58}{315\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(58*x^5 - 290*x^4 + 580*x^3 - 580*x^2 + (2*x^4 - 10*x^3 + 21*x^2 - 25
*x - 58)*sqrt(x + 1)*sqrt(-x + 1) + 290*x - 58)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1)
```

**giac** [A] time = 1.12, size = 35, normalized size = 0.43

$$\frac{((2(x+1)(x-8)+63)(x+1)-105)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{315(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*((2*(x + 1)*(x - 8) + 63)*(x + 1) - 105)*(x + 1)^(3/2)*sqrt(-x + 1)/(
x - 1)^5
```

**maple** [A] time = 0.00, size = 30, normalized size = 0.37

$$-\frac{(x+1)^{\frac{3}{2}}\left(2x^3-12x^2+33x-58\right)}{315(-x+1)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(11/2),x)
```

```
[Out] -1/315*(x+1)^(3/2)*(2*x^3-12*x^2+33*x-58)/(-x+1)^(9/2)
```

**maxima** [B] time = 1.35, size = 131, normalized size = 1.62

$$-\frac{2\sqrt{-x^2+1}}{9\left(x^5-5x^4+10x^3-10x^2+5x-1\right)}-\frac{\sqrt{-x^2+1}}{63\left(x^4-4x^3+6x^2-4x+1\right)}+\frac{\sqrt{-x^2+1}}{105\left(x^3-3x^2+3x-1\right)}-\frac{2\sqrt{-x^2+1}}{315\left(x^2-2x+1\right)}+\frac{2\sqrt{-x^2+1}}{315(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 1/63*sqrt(-
x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/105*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) - 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/315*sqrt(-x^2 + 1)/
(x - 1)
```

**mupad** [B] time = 0.28, size = 80, normalized size = 0.99

$$-\frac{\sqrt{1-x}\left(\frac{5x\sqrt{x+1}}{63}+\frac{58\sqrt{x+1}}{315}-\frac{x^2\sqrt{x+1}}{15}+\frac{2x^3\sqrt{x+1}}{63}-\frac{2x^4\sqrt{x+1}}{315}\right)}{x^5-5x^4+10x^3-10x^2+5x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(11/2),x)
```

```
[Out] -((1 - x)^(1/2)*((5*x*(x + 1)^(1/2))/63 + (58*(x + 1)^(1/2))/315 - (x^2*(x
+ 1)^(1/2))/15 + (2*x^3*(x + 1)^(1/2))/63 - (2*x^4*(x + 1)^(1/2))/315))/(5*
x - 10*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

**sympy [B]** time = 53.78, size = 1562, normalized size = 19.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(11/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**(15/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x -
1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4
 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*s
qrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) - 30*I*(x + 1)**(13/2)/(315*sqrt(x
- 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**
5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*s
qrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 1
95*I*(x + 1)**(11/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)
**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*
sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)
*(x + 1) - 40320*sqrt(x - 1)) - 715*I*(x + 1)**(9/2)/(315*sqrt(x - 1)*(x +
1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*
sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)
*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 1530*I*(x +
 1)**(7/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 2646
0*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1
)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) -
 40320*sqrt(x - 1)) - 1764*I*(x + 1)**(5/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4
410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x -
1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)*
*2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 840*I*(x + 1)**(3/2)
/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x -
 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)*
*3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqr
t(x - 1)), Abs(x + 1)/2 > 1), (-2*(x + 1)**(15/2)/(315*sqrt(1 - x)*(x + 1)*
*7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqr
t(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x
 + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) + 30*(x + 1)**(1
3/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt
(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x +
 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320
```

```
*sqrt(1 - x)) - 195*(x + 1)**(11/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt
(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x +
1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141
120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) + 715*(x + 1)**(9/2)/(315*sqrt
(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1
)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 21168
0*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x))
- 1530*(x + 1)**(7/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1
)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400
*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x
)*(x + 1) - 40320*sqrt(1 - x)) + 1764*(x + 1)**(5/2)/(315*sqrt(1 - x)*(x +
1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*
sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)
*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) - 840*(x + 1)
**(3/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*s
qrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(
x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40
320*sqrt(1 - x)), True))
```

## 3.1005 $\int \frac{\sqrt{1+x}}{(1-x)^{13/2}} \, dx$

**Optimal**. Leaf size=101

$$\frac{8(x+1)^{3/2}}{3465(1-x)^{3/2}}+\frac{8(x+1)^{3/2}}{1155(1-x)^{5/2}}+\frac{4(x+1)^{3/2}}{231(1-x)^{7/2}}+\frac{4(x+1)^{3/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8(x+1)^{3/2}}{3465(1-x)^{3/2}}+\frac{8(x+1)^{3/2}}{1155(1-x)^{5/2}}+\frac{4(x+1)^{3/2}}{231(1-x)^{7/2}}+\frac{4(x+1)^{3/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(3/2)/(11*(1 - x)^(11/2)) + (4*(1 + x)^(3/2))/(99*(1 - x)^(9/2)) +
(4*(1 + x)^(3/2))/(231*(1 - x)^(7/2)) + (8*(1 + x)^(3/2))/(1155*(1 - x)^(5/
2)) + (8*(1 + x)^(3/2))/(3465*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx &= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4}{11}\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4(1+x)^{3/2}}{99(1-x)^{9/2}}+\frac{4}{33}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4(1+x)^{3/2}}{99(1-x)^{9/2}}+\frac{4(1+x)^{3/2}}{231(1-x)^{7/2}}+\frac{8}{231}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4(1+x)^{3/2}}{99(1-x)^{9/2}}+\frac{4(1+x)^{3/2}}{231(1-x)^{7/2}}+\frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}}+\frac{8\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx}{1155}\\ &= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4(1+x)^{3/2}}{99(1-x)^{9/2}}+\frac{4(1+x)^{3/2}}{231(1-x)^{7/2}}+\frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}}+\frac{8(1+x)^{3/2}}{3465(1-x)^{3/2}}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 40, normalized size = 0.40

$$\frac{(x+1)^{3/2}\left(8x^4-56x^3+180x^2-364x+547\right)}{3465(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(3/2)*(547 - 364*x + 180*x^2 - 56*x^3 + 8*x^4))/(3465*(1 - x)^(11/
2))
```

**IntegrateAlgebraic** [A] time = 0.07, size = 95, normalized size = 0.94

$$\frac{\frac{315(x+1)^{11/2}}{(1-x)^{11/2}}+\frac{1540(x+1)^{9/2}}{(1-x)^{9/2}}+\frac{2970(x+1)^{7/2}}{(1-x)^{7/2}}+\frac{2772(x+1)^{5/2}}{(1-x)^{5/2}}+\frac{1155(x+1)^{3/2}}{(1-x)^{3/2}}}{55440}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] ((1155*(1 + x)^(3/2))/(1 - x)^(3/2) + (2772*(1 + x)^(5/2))/(1 - x)^(5/2) +
(2970*(1 + x)^(7/2))/(1 - x)^(7/2) + (1540*(1 + x)^(9/2))/(1 - x)^(9/2) + (
315*(1 + x)^(11/2))/(1 - x)^(11/2))/55440
```

**fricas** [A] time = 0.99, size = 100, normalized size = 0.99

$$\frac{547\,x^6-3282\,x^5+8205\,x^4-10940\,x^3+8205\,x^2+\left(8\,x^5-48\,x^4+124\,x^3-184\,x^2+183\,x+547\right)\sqrt{x+1}\,\sqrt{-x+1}-3282\,x+547}{3465\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(547*x^6 - 3282*x^5 + 8205*x^4 - 10940*x^3 + 8205*x^2 + (8*x^5 - 48*
x^4 + 124*x^3 - 184*x^2 + 183*x + 547)*sqrt(x + 1)*sqrt(-x + 1) - 3282*x +
547)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

**giac [A]** time = 1.36, size = 42, normalized size = 0.42

$$\frac{(4\,((2\,(x+1)(x-10)+99)(x+1)-231)(x+1)+1155)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{3465\,(x-1)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] 1/3465*(4*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1) + 1155)*(x + 1)
^(3/2)*sqrt(-x + 1)/(x - 1)^6
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.35

$$\frac{(x+1)^{\frac{3}{2}}\left(8x^{4}-56x^{3}+180x^{2}-364x+547\right)}{3465\,(-x+1)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(13/2),x)
```

```
[Out] 1/3465*(x+1)^(3/2)*(8*x^4-56*x^3+180*x^2-364*x+547)/(-x+1)^(11/2)
```

**maxima [B]** time = 1.32, size = 172, normalized size = 1.70

$$\frac{2\sqrt{-x^{2}+1}}{11\left(x^{6}-6x^{5}+15x^{4}-20x^{3}+15x^{2}-6x+1\right)}+\frac{\sqrt{-x^{2}+1}}{99\left(x^{5}-5x^{4}+10x^{3}-10x^{2}+5x-1\right)}-\frac{4\sqrt{-x^{2}+1}}{693\left(x^{4}-4x^{3}+6x^{2}-4x+1\right)}+\frac{4\sqrt{-x^{2}+1}}{1155\left(x^{3}-3x^{2}+3x-1\right)}-\frac{8\sqrt{-x^{2}+1}}{3465\left(x^{2}-2x+1\right)}+\frac{8\sqrt{-x^{2}+1}}{3465\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] 2/11*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 1/
99*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 4/693*sqrt(-x
^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/1155*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) - 8/3465*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 8/3465*sqrt(-x^2 + 1
)/(x - 1)
```

**mupad [B]** time = 0.29, size = 94, normalized size = 0.93

$$\frac{\sqrt{1-x}\,\left(\frac{61\,x\,\sqrt{x+1}}{1155}+\frac{547\,\sqrt{x+1}}{3465}-\frac{184\,x^2\,\sqrt{x+1}}{3465}+\frac{124\,x^3\,\sqrt{x+1}}{3465}-\frac{16\,x^4\,\sqrt{x+1}}{1155}+\frac{8\,x^5\,\sqrt{x+1}}{3465}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((61*x*(x + 1)^(1/2))/1155 + (547*(x + 1)^(1/2))/3465 - (184
*x^2*(x + 1)^(1/2))/3465 + (124*x^3*(x + 1)^(1/2))/3465 - (16*x^4*(x + 1)^(
1/2))/1155 + (8*x^5*(x + 1)^(1/2))/3465))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 -
 6*x^5 + x^6 + 1)
```

**sympy [B]** time = 135.09, size = 3650, normalized size = 36.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(13/2),x)
```

```
[Out] Piecewise((8*I*(x + 1)**(23/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x
 - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x
+ 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6
 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 14
6361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760
*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 184*I*(x + 1)**(21/2)/(3465*s
qrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)
*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)
**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 -
146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574
400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x
- 1)) + 1932*I*(x + 1)**(19/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x
 - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x
+ 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6
 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 14
6361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760
*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 12236*I*(x + 1)**(17/2)/(3465
*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x -
1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x +
1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5
- 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 975
74400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(
x - 1)) + 52003*I*(x + 1)**(15/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqr
t(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*
```

(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)
**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 +
146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029
760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 155316*I*(x + 1)**(13/2)/(
3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(
x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(
x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)
**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 -
97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*s
qrt(x - 1)) + 329588*I*(x + 1)**(11/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 7623
0*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x
- 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x
+ 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)
**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 +
39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 488224*I*(x + 1)**(9/
2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*s
qrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x -
1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x
+ 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)*
*3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 70963
20*sqrt(x - 1)) + 479952*I*(x + 1)**(7/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 7
6230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt
(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)
*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x +
1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2
+ 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 280896*I*(x + 1)**
(5/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 76230
0*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x
- 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*
(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x +
1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 70
96320*sqrt(x - 1)) + 73920*I*(x + 1)**(3/2)/(3465*sqrt(x - 1)*(x + 1)**11 -
76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sq
rt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x -
1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x
+ 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)*
*2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)), Abs(x + 1)/2 > 1)
, (-8*(x + 1)**(23/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x
+ 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 +
18295200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453
120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*s
qrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 -
x)*(x + 1) - 7096320*sqrt(1 - x)) + 184*(x + 1)**(21/2)/(3465*sqrt(1 - x)*
(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9
- 4573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 51226

560*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*s
qrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1
 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 193
2*(x + 1)**(19/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)
**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 182
95200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*
sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(
1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*
(x + 1) - 7096320*sqrt(1 - x)) + 12236*(x + 1)**(17/2)/(3465*sqrt(1 - x)*(x
 + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 -
 4573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 5122656
0*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqr
t(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 -
x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 52003
*(x + 1)**(15/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)*
*10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 1829
5200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*s
qrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1
 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(
x + 1) - 7096320*sqrt(1 - x)) + 155316*(x + 1)**(13/2)/(3465*sqrt(1 - x)*(x
 + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 -
 4573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 5122656
0*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqr
t(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 -
x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 32958
8*(x + 1)**(11/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)
**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 182
95200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*
sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(
1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*
(x + 1) - 7096320*sqrt(1 - x)) + 488224*(x + 1)**(9/2)/(3465*sqrt(1 - x)*(x
 + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 -
 4573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 5122656
0*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqr
t(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 -
x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 47995
2*(x + 1)**(7/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)*
*10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 1829
5200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*s
qrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1
 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(
x + 1) - 7096320*sqrt(1 - x)) + 280896*(x + 1)**(5/2)/(3465*sqrt(1 - x)*(x
+ 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 -
4573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 51226560
*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt

```
(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x
)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 73920*
(x + 1)**(3/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**1
0 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 182952
00*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*sqr
t(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1 -
 x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x
+ 1) - 7096320*sqrt(1 - x)), True))
```

## 3.1006 $\int (1-x)^{9/2}(1+x)^{3/2}\,dx$

**Optimal**. Leaf size=109

$$\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}+\frac{3}{14}(x+1)^{5/2}(1-x)^{7/2}+\frac{3}{10}(x+1)^{5/2}(1-x)^{5/2}+\frac{3}{8}x(x+1)^{3/2}(1-x)^{3/2}+\frac{9}{16}x\sqrt{x+1}\,\sqrt{1-x}+\frac{9}{16}\sin^{-1}(x)$$

**Rubi** [A] time = 0.02, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}+\frac{3}{14}(x+1)^{5/2}(1-x)^{7/2}+\frac{3}{10}(x+1)^{5/2}(1-x)^{5/2}+\frac{3}{8}x(x+1)^{3/2}(1-x)^{3/2}+\frac{9}{16}x\sqrt{x+1}\,\sqrt{1-x}+\frac{9}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

```
[Out] (9*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (3*(1 - x)^(3/2)*x*(1 + x)^(3/2))/8 + (3
*(1 - x)^(5/2)*(1 + x)^(5/2))/10 + (3*(1 - x)^(7/2)*(1 + x)^(5/2))/14 + ((1
 - x)^(9/2)*(1 + x)^(5/2))/7 + (9*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{9/2}(1+x)^{3/2}\,dx = \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{9}{7}\int (1-x)^{7/2}(1+x)^{3/2}\,dx$$
$$= \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{3}{2}\int (1-x)^{5/2}(1+x)^{3/2}\,dx$$
$$= \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{3}{2}\int (1-x)^{3/2}$$
$$= \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1-x)^{9/2}(1$$
$$= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1$$
$$= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1$$
$$= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1$$

**Mathematica** [A] time = 0.06, size = 66, normalized size = 0.61

$$\frac{1}{560}\sqrt{1-x^2}\left(80x^6-280x^5+208x^4+350x^3-656x^2+245x+368\right)-\frac{9}{8}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(368 + 245*x - 656*x^2 + 350*x^3 + 208*x^4 - 280*x^5 + 80*x^
6))/560 - (9*ArcSin[Sqrt[1 - x]/Sqrt[2]])/8
```

**IntegrateAlgebraic** [A] time = 0.16, size = 169, normalized size = 1.55

$$\frac{-\frac{315(1-x)^{13/2}}{(x+1)^{13/2}}-\frac{2100(1-x)^{11/2}}{(x+1)^{11/2}}+\frac{8393(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{9216(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{5943(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{2100(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{315\sqrt{1-x}}{\sqrt{x+1}}}{280\left(\frac{1-x}{x+1}+1\right)^7}-\frac{9}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

```
[Out] ((-315*(1 - x)^(13/2))/(1 + x)^(13/2) - (2100*(1 - x)^(11/2))/(1 + x)^(11/2
) + (8393*(1 - x)^(9/2))/(1 + x)^(9/2) + (9216*(1 - x)^(7/2))/(1 + x)^(7/2)
 + (5943*(1 - x)^(5/2))/(1 + x)^(5/2) + (2100*(1 - x)^(3/2))/(1 + x)^(3/2)
```

```
+ (315*Sqrt[1 - x])/Sqrt[1 + x])/(280*(1 + (1 - x)/(1 + x))^7) - (9*ArcTan[
Sqrt[1 - x]/Sqrt[1 + x]])/8
```

**fricas [A]** time = 1.16, size = 67, normalized size = 0.61

$$\frac{1}{560}\left(80\,x^{6}-280\,x^{5}+208\,x^{4}+350\,x^{3}-656\,x^{2}+245\,x+368\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{9}{8}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/560*(80*x^6 - 280*x^5 + 208*x^4 + 350*x^3 - 656*x^2 + 245*x + 368)*sqrt(x
 + 1)*sqrt(-x + 1) - 9/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac [B]** time = 1.26, size = 237, normalized size = 2.17

$$\frac{1}{1680}\left(\left(2\left(\left(4\left(5\left(6x-37\right)(x+1)+661\right)(x+1)-4551\right)(x+1)+4781\right)(x+1)-6335\right)(x+1)+2835\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{120}\left(\left(2\left(\left(4\left(5x-26\right)(x+1)+321\right)(x+1)-451\right)(x+1)+745\right)(x+1)-405\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{120}\left(\left(2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{6}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{6}\left(\left(2x-5\right)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{9}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(x + 1) + 4781)
*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) - 1/120*((2*((4*(
5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x + 1) - 405)*sqrt(
x + 1)*sqrt(-x + 1) - 1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)
*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/6*((2*(3*x - 10)*(x + 1) + 43)
*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(
x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-
x + 1) + 9/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [A]** time = 0.00, size = 127, normalized size = 1.17

$$\frac{9\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{16\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{9}{2}}(x+1)^{\frac{5}{2}}}{7}+\frac{3(-x+1)^{\frac{7}{2}}(x+1)^{\frac{5}{2}}}{14}+\frac{3(-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}{10}+\frac{3(-x+1)^{\frac{3}{2}}(x+1)^{\frac{5}{2}}}{8}+\frac{3\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{8}-\frac{3\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{16}-\frac{9\sqrt{-x+1}\,\sqrt{x+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(9/2)*(x+1)^(3/2),x)
```

```
[Out] 1/7*(-x+1)^(9/2)*(x+1)^(5/2)+3/14*(-x+1)^(7/2)*(x+1)^(5/2)+3/10*(-x+1)^(5/2
)*(x+1)^(5/2)+3/8*(-x+1)^(3/2)*(x+1)^(5/2)+3/8*(-x+1)^(1/2)*(x+1)^(5/2)-3/1
6*(-x+1)^(1/2)*(x+1)^(3/2)-9/16*(-x+1)^(1/2)*(x+1)^(1/2)+9/16*((x+1)*(-x+1)
)^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 3.03, size = 66, normalized size = 0.61

$$\frac{1}{7}\left(-x^{2}+1\right)^{\frac{5}{2}}x^{2}-\frac{1}{2}\left(-x^{2}+1\right)^{\frac{5}{2}}x+\frac{23}{35}\left(-x^{2}+1\right)^{\frac{5}{2}}+\frac{3}{8}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{9}{16}\sqrt{-x^{2}+1}\,x+\frac{9}{16}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*(-x^2 + 1)^(5/2)*x^2 - 1/2*(-x^2 + 1)^(5/2)*x + 23/35*(-x^2 + 1)^(5/2)
+ 3/8*(-x^2 + 1)^(3/2)*x + 9/16*sqrt(-x^2 + 1)*x + 9/16*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2}(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(9/2)*(x + 1)^(3/2), x)
```

**sympy [A]** time = 75.20, size = 325, normalized size = 2.98

$$\begin{cases} -\frac{9i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{i(x+1)^{\frac{15}{2}}}{7\sqrt{x-1}} - \frac{23i(x+1)^{\frac{13}{2}}}{14\sqrt{x-1}} + \frac{541i(x+1)^{\frac{11}{2}}}{70\sqrt{x-1}} - \frac{5249i(x+1)^{\frac{9}{2}}}{280\sqrt{x-1}} + \frac{6653i(x+1)^{\frac{7}{2}}}{280\sqrt{x-1}} - \frac{1027i(x+1)^{\frac{5}{2}}}{80\sqrt{x-1}} - \frac{3i(x+1)^{\frac{3}{2}}}{16\sqrt{x-1}} + \frac{9i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{9\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{(x+1)^{\frac{15}{2}}}{7\sqrt{1-x}} + \frac{23(x+1)^{\frac{13}{2}}}{14\sqrt{1-x}} - \frac{541(x+1)^{\frac{11}{2}}}{70\sqrt{1-x}} + \frac{5249(x+1)^{\frac{9}{2}}}{280\sqrt{1-x}} - \frac{6653(x+1)^{\frac{7}{2}}}{280\sqrt{1-x}} + \frac{1027(x+1)^{\frac{5}{2}}}{80\sqrt{1-x}} + \frac{3(x+1)^{\frac{3}{2}}}{16\sqrt{1-x}} - \frac{9\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-9*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(15/2)/(7*sqrt(
x - 1)) - 23*I*(x + 1)**(13/2)/(14*sqrt(x - 1)) + 541*I*(x + 1)**(11/2)/(70
*sqrt(x - 1)) - 5249*I*(x + 1)**(9/2)/(280*sqrt(x - 1)) + 6653*I*(x + 1)**(
7/2)/(280*sqrt(x - 1)) - 1027*I*(x + 1)**(5/2)/(80*sqrt(x - 1)) - 3*I*(x +
1)**(3/2)/(16*sqrt(x - 1)) + 9*I*sqrt(x + 1)/(8*sqrt(x - 1)), Abs(x + 1)/2
> 1), (9*asin(sqrt(2)*sqrt(x + 1)/2)/8 - (x + 1)**(15/2)/(7*sqrt(1 - x)) +
23*(x + 1)**(13/2)/(14*sqrt(1 - x)) - 541*(x + 1)**(11/2)/(70*sqrt(1 - x))
+ 5249*(x + 1)**(9/2)/(280*sqrt(1 - x)) - 6653*(x + 1)**(7/2)/(280*sqrt(1 -
 x)) + 1027*(x + 1)**(5/2)/(80*sqrt(1 - x)) + 3*(x + 1)**(3/2)/(16*sqrt(1 -
 x)) - 9*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

## 3.1007 $\int (1-x)^{7/2}(1+x)^{3/2}\,dx$

**Optimal**. Leaf size=89

$$\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}+\frac{7}{30}(x+1)^{5/2}(1-x)^{5/2}+\frac{7}{24}x(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{16}x\sqrt{x+1}\,\sqrt{1-x}+\frac{7}{16}\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}+\frac{7}{30}(x+1)^{5/2}(1-x)^{5/2}+\frac{7}{24}x(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{16}x\sqrt{x+1}\,\sqrt{1-x}+\frac{7}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
7*(1 - x)^(5/2)*(1 + x)^(5/2))/30 + ((1 - x)^(7/2)*(1 + x)^(5/2))/6 + (7*Ar
cSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{7/2}(1+x)^{3/2}\,dx &= \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2} + \frac{7}{6}\int (1-x)^{5/2}(1+x)^{3/2}\,dx \\
&= \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2} + \frac{7}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx \\
&= \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2} + \frac{7}{8}\int \sqrt{1-x} \\
&= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1 \\
&= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1 \\
&= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 61, normalized size = 0.69

$$\frac{1}{240}\sqrt{1-x^2}\left(-40x^5+96x^4+10x^3-192x^2+135x+96\right)-\frac{7}{8}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(96 + 135*x - 192*x^2 + 10*x^3 + 96*x^4 - 40*x^5))/240 - (7*
ArcSin[Sqrt[1 - x]/Sqrt[2]])/8
```

**IntegrateAlgebraic [A]** time = 0.13, size = 151, normalized size = 1.70

$$\frac{-\frac{105(1-x)^{11/2}}{(x+1)^{11/2}}-\frac{595(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{1686(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{1386(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{595(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{105\sqrt{1-x}}{\sqrt{x+1}}}{120\left(\frac{1-x}{x+1}+1\right)^6}-\frac{7}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

```
[Out] ((-105*(1 - x)^(11/2))/(1 + x)^(11/2) - (595*(1 - x)^(9/2))/(1 + x)^(9/2) +
 (1686*(1 - x)^(7/2))/(1 + x)^(7/2) + (1386*(1 - x)^(5/2))/(1 + x)^(5/2) +
(595*(1 - x)^(3/2))/(1 + x)^(3/2) + (105*Sqrt[1 - x])/Sqrt[1 + x])/(120*(1
+ (1 - x)/(1 + x))^6) - (7*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/8
```

**fricas [A]** time = 1.15, size = 62, normalized size = 0.70

$$-\frac{1}{240}\left(40\,x^{5}-96\,x^{4}-10\,x^{3}+192\,x^{2}-135\,x-96\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{7}{8}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(40*x^5 - 96*x^4 - 10*x^3 + 192*x^2 - 135*x - 96)*sqrt(x + 1)*sqrt(-
x + 1) - 7/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac [B]** time = 1.33, size = 185, normalized size = 2.08

$$-\frac{1}{240}\left((2\left((4\left(5x-26\right)(x+1)+321)(x+1)-451\right)(x+1)+745)(x+1)-405\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{120}\left((2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{12}\left((2\left(3x-10\right)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\,\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{7}{8}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
 1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/120*((2*(3*(4*x - 17)*(x + 1) + 133
)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x -
10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x - 5)*(
x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1)
 + sqrt(x + 1)*sqrt(-x + 1) + 7/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [A]** time = 0.00, size = 113, normalized size = 1.27

$$\frac{7\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{16\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{7}{2}}\,(x+1)^{\frac{5}{2}}}{6}+\frac{7\,(-x+1)^{\frac{5}{2}}\,(x+1)^{\frac{5}{2}}}{30}+\frac{7\,(-x+1)^{\frac{3}{2}}\,(x+1)^{\frac{5}{2}}}{24}+\frac{7\sqrt{-x+1}\,\left(x+1\right)^{\frac{5}{2}}}{24}-\frac{7\sqrt{-x+1}\,\left(x+1\right)^{\frac{3}{2}}}{48}-\frac{7\sqrt{-x+1}\,\sqrt{x+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)*(x+1)^(3/2),x)
```

```
[Out] 1/6*(-x+1)^(7/2)*(x+1)^(5/2)+7/30*(-x+1)^(5/2)*(x+1)^(5/2)+7/24*(-x+1)^(3/2
)*(x+1)^(5/2)+7/24*(-x+1)^(1/2)*(x+1)^(5/2)-7/48*(-x+1)^(1/2)*(x+1)^(3/2)-7
/16*(-x+1)^(1/2)*(x+1)^(1/2)+7/16*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(
1/2)*arcsin(x)
```

**maxima [A]** time = 2.90, size = 52, normalized size = 0.58

$$-\frac{1}{6}\left(-x^{2}+1\right)^{\frac{5}{2}}x+\frac{2}{5}\left(-x^{2}+1\right)^{\frac{5}{2}}+\frac{7}{24}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{7}{16}\,\sqrt{-x^{2}+1}\,x+\frac{7}{16}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(5/2)*x + 2/5*(-x^2 + 1)^(5/2) + 7/24*(-x^2 + 1)^(3/2)*x +
7/16*sqrt(-x^2 + 1)*x + 7/16*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2} (x+1)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(7/2)*(x + 1)^(3/2), x)
```

**sympy [A]** time = 32.99, size = 289, normalized size = 3.25

$$\begin{cases} -\frac{7i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}} + \frac{47i(x+1)^{\frac{11}{2}}}{30\sqrt{x-1}} - \frac{683i(x+1)^{\frac{9}{2}}}{120\sqrt{x-1}} + \frac{1151i(x+1)^{\frac{7}{2}}}{120\sqrt{x-1}} - \frac{1543i(x+1)^{\frac{5}{2}}}{240\sqrt{x-1}} - \frac{7i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}} + \frac{7i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{7\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}} - \frac{47(x+1)^{\frac{11}{2}}}{30\sqrt{1-x}} + \frac{683(x+1)^{\frac{9}{2}}}{120\sqrt{1-x}} - \frac{1151(x+1)^{\frac{7}{2}}}{120\sqrt{1-x}} + \frac{1543(x+1)^{\frac{5}{2}}}{240\sqrt{1-x}} + \frac{7(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}} - \frac{7\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 - I*(x + 1)**(13/2)/(6*sqrt(
x - 1)) + 47*I*(x + 1)**(11/2)/(30*sqrt(x - 1)) - 683*I*(x + 1)**(9/2)/(120
*sqrt(x - 1)) + 1151*I*(x + 1)**(7/2)/(120*sqrt(x - 1)) - 1543*I*(x + 1)**(
5/2)/(240*sqrt(x - 1)) - 7*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 7*I*sqrt(x +
 1)/(8*sqrt(x - 1)), Abs(x + 1)/2 > 1), (7*asin(sqrt(2)*sqrt(x + 1)/2)/8 +
(x + 1)**(13/2)/(6*sqrt(1 - x)) - 47*(x + 1)**(11/2)/(30*sqrt(1 - x)) + 683
*(x + 1)**(9/2)/(120*sqrt(1 - x)) - 1151*(x + 1)**(7/2)/(120*sqrt(1 - x)) +
 1543*(x + 1)**(5/2)/(240*sqrt(1 - x)) + 7*(x + 1)**(3/2)/(48*sqrt(1 - x))
- 7*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

# 3.1008 $\int (1-x)^{5/2}(1+x)^{3/2}\,dx$

**Optimal**. Leaf size=69

$$\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + ((1 -
 x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{5/2}(1+x)^{3/2}\,dx &= \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\
&= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+
\end{aligned}$$

$$\begin{aligned}
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\sin^{-1}(x)
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 55, normalized size = 0.80

$$\frac{1}{40}\left(\sqrt{1-x^2}\left(8x^4-10x^3-16x^2+25x+8\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(8 + 25*x - 16*x^2 - 10*x^3 + 8*x^4) - 30*ArcSin[Sqrt[1 - x]
/Sqrt[2]])/40
```

**IntegrateAlgebraic [A]** time = 0.11, size = 133, normalized size = 1.93

$$\frac{-\frac{15(1-x)^{9/2}}{(x+1)^{9/2}}-\frac{70(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{128(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{70(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{15\sqrt{1-x}}{\sqrt{x+1}}}{20\left(\frac{1-x}{x+1}+1\right)^5}-\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] ((-15*(1 - x)^(9/2))/(1 + x)^(9/2) - (70*(1 - x)^(7/2))/(1 + x)^(7/2) + (12
8*(1 - x)^(5/2))/(1 + x)^(5/2) + (70*(1 - x)^(3/2))/(1 + x)^(3/2) + (15*Sqr
t[1 - x])/Sqrt[1 + x])/(20*(1 + (1 - x)/(1 + x))^5) - (3*ArcTan[Sqrt[1 - x]
/Sqrt[1 + x]])/4
```

**fricas [A]** time = 1.22, size = 57, normalized size = 0.83

$$\frac{1}{40}\left(8\,x^4-10\,x^3-16\,x^2+25\,x+8\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{4}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 10*x^3 - 16*x^2 + 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*ar
ctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac** [A] time = 1.16, size = 91, normalized size = 1.32

$$\frac{1}{120}\left((2(3(4x-17)(x+1)+133)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{3}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x
 + 1)*sqrt(-x + 1) - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) +
 sqrt(x + 1)*sqrt(-x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple** [A] time = 0.00, size = 99, normalized size = 1.43

$$\frac{3\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}{5}+\frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{5}{2}}}{4}+\frac{\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{4}-\frac{\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{8}-\frac{3\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)*(x+1)^(3/2),x)
```

```
[Out] 1/5*(-x+1)^(5/2)*(x+1)^(5/2)+1/4*(-x+1)^(3/2)*(x+1)^(5/2)+1/4*(-x+1)^(1/2)*
(x+1)^(5/2)-1/8*(-x+1)^(1/2)*(x+1)^(3/2)-3/8*(-x+1)^(1/2)*(x+1)^(1/2)+3/8*(
(x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima** [A] time = 2.98, size = 40, normalized size = 0.58

$$\frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}\,x+\frac{3}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8*
arcsin(x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(3/2), x)
```

**sympy [B]** time = 15.25, size = 250, normalized size = 3.62

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}} - \frac{29i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}} + \frac{73i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}} - \frac{129i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}} + \frac{29(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}} - \frac{73(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}} + \frac{129(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}} - \frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) - 29*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) + 73*I*(x + 1)**(7/2)/(20*sq
rt(x - 1)) - 129*I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8*sq
rt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), (3*asin(sq
rt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(11/2)/(5*sqrt(1 - x)) + 29*(x + 1)**(9/2
)/(20*sqrt(1 - x)) - 73*(x + 1)**(7/2)/(20*sqrt(1 - x)) + 129*(x + 1)**(5/2
)/(40*sqrt(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt
(1 - x)), True))
```

## 3.1009 $\int (1-x)^{3/2}(1+x)^{3/2}\,dx$

**Optimal**. Leaf size=49

$$\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {38, 41, 216}

$$\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + (3*Ar
cSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{3/2}(1+x)^{3/2}\,dx = \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx$$
$$= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{8}\sin^{-1}(x)$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 0.59

$$\frac{1}{8}\left(x\sqrt{1-x^2}\left(5-2x^2\right)+3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] (x*(5 - 2*x^2)*Sqrt[1 - x^2] + 3*ArcSin[x])/8
```

**IntegrateAlgebraic [B]** time = 0.10, size = 115, normalized size = 2.35

$$\frac{-\frac{3(1-x)^{7/2}}{(x+1)^{7/2}}-\frac{11(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{11(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{3\sqrt{1-x}}{\sqrt{x+1}}}{4\left(\frac{1-x}{x+1}+1\right)^4}-\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] ((-3*(1 - x)^(7/2))/(1 + x)^(7/2) - (11*(1 - x)^(5/2))/(1 + x)^(5/2) + (11*
(1 - x)^(3/2))/(1 + x)^(3/2) + (3*Sqrt[1 - x])/Sqrt[1 + x])/(4*(1 + (1 - x)
/(1 + x))^4) - (3*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/4
```

**fricas [A]** time = 1.19, size = 46, normalized size = 0.94

$$-\frac{1}{8}\left(2x^3-5x\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^3 - 5*x)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*arctan((sqrt(x + 1)*sqrt(
-x + 1) - 1)/x)
```

**giac [B]** time = 1.12, size = 101, normalized size = 2.06

$$-\frac{1}{24}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{6}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{3}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) -
 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*(x
- 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(
x + 1))
```

**maple [B]** time = 0.00, size = 85, normalized size = 1.73

$$\frac{3\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{5}{2}}}{4}+\frac{\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{4}-\frac{\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{8}-\frac{3\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)*(x+1)^(3/2),x)
```

```
[Out] 1/4*(-x+1)^(3/2)*(x+1)^(5/2)+1/4*(-x+1)^(1/2)*(x+1)^(5/2)-1/8*(-x+1)^(1/2)*
(x+1)^(3/2)-3/8*(-x+1)^(1/2)*(x+1)^(1/2)+3/8*((x+1)*(-x+1))^(1/2)/(x+1)^(1/
2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 2.91, size = 29, normalized size = 0.59

$$\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}\,x+\frac{3}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int (1-x)^{3/2}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(3/2)*(x + 1)^(3/2), x)
```

**sympy [B]** time = 7.46, size = 214, normalized size = 4.37

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}} + \frac{5i(x+1)^{\frac{7}{2}}}{4\sqrt{x-1}} - \frac{13i(x+1)^{\frac{5}{2}}}{8\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}} - \frac{5(x+1)^{\frac{7}{2}}}{4\sqrt{1-x}} + \frac{13(x+1)^{\frac{5}{2}}}{8\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}} - \frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) + 5*I*(x + 1)**(7/2)/(4*sqrt(x - 1)) - 13*I*(x + 1)**(5/2)/(8*sqrt(x
 - 1)) - I*(x + 1)**(3/2)/(8*sqrt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1))
, Abs(x + 1)/2 > 1), (3*asin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(9/2)/(4*s
qrt(1 - x)) - 5*(x + 1)**(7/2)/(4*sqrt(1 - x)) + 13*(x + 1)**(5/2)/(8*sqrt(
1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1 - x)), T
rue))
```

## 3.1010 $\int \sqrt{1-x}\,(1+x)^{3/2}\,dx$

**Optimal**. Leaf size=48

$$-\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$-\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 - ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x}(1+x)^{3/2}\,dx &= -\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\int \sqrt{1-x}\sqrt{1+x}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 44, normalized size = 0.92

$$\frac{1}{6}\sqrt{1-x^2}\left(2x^2+3x-2\right)-\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(1 + x)^(3/2),x]
[Out] (Sqrt[1 - x^2]*(-2 + 3*x + 2*x^2))/6 - ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

**IntegrateAlgebraic [A]** time = 0.07, size = 95, normalized size = 1.98

$$\frac{-\frac{3(1-x)^{5/2}}{(x+1)^{5/2}}-\frac{8(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{3\sqrt{1-x}}{\sqrt{x+1}}}{3\left(\frac{1-x}{x+1}+1\right)^3}-\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]*(1 + x)^(3/2),x]
[Out] ((-3*(1 - x)^(5/2))/(1 + x)^(5/2) - (8*(1 - x)^(3/2))/(1 + x)^(3/2) + (3*Sq
rt[1 - x])/Sqrt[1 + x])/(3*(1 + (1 - x)/(1 + x))^3) - ArcTan[Sqrt[1 - x]/Sq
rt[1 + x]]
```

**fricas [A]** time = 1.28, size = 47, normalized size = 0.98

$$\frac{1}{6}\left(2\,x^2+3\,x-2\right)\sqrt{x+1}\,\sqrt{-x+1}-\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 + 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(-
x + 1) - 1)/x)
```

**giac [A]** time = 0.92, size = 66, normalized size = 1.38

$$\frac{1}{6}((2x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1}+\sqrt{x+1}(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*(x - 2)*
sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [B]** time = 0.01, size = 71, normalized size = 1.48

$$\frac{\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{2\sqrt{x+1}\,\sqrt{-x+1}}+\frac{\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{3}-\frac{\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{6}-\frac{\sqrt{-x+1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)*(x+1)^(3/2),x)
```

```
[Out] 1/3*(-x+1)^(1/2)*(x+1)^(5/2)-1/6*(-x+1)^(1/2)*(x+1)^(3/2)-1/2*(-x+1)^(1/2)*
(x+1)^(1/2)+1/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 2.97, size = 28, normalized size = 0.58

$$-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{1-x}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(1/2)*(x + 1)^(3/2), x)
```

**sympy [B]** time = 4.82, size = 165, normalized size = 3.44

$$\begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} - \frac{5i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} + \frac{5(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x - 1
)) - 5*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - I*(x + 1)**(3/2)/(6*sqrt(x - 1))
+ I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (asin(sqrt(2)*sqrt(x + 1)/2
) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 5*(x + 1)**(5/2)/(6*sqrt(1 - x)) + (x
+ 1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

## 3.1011 $\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx$

**Optimal**. Leaf size=47

$$-\frac{1}{2}\sqrt{1-x}\,(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\,\sqrt{x+1}\,+\frac{3}{2}\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$-\frac{1}{2}\sqrt{1-x}\,(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\,\sqrt{x+1}\,+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

```
[Out] (-3*Sqrt[1 - x]*Sqrt[1 + x])/2 - (Sqrt[1 - x]*(1 + x)^(3/2))/2 + (3*ArcSin[
x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx = -\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{3}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$$
$$= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{3}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{3}{2}\sin^{-1}(x)$$

**Mathematica** [A] time = 0.01, size = 37, normalized size = 0.79

$$-\frac{1}{2}\sqrt{1-x^2}\,(x+4) - 3\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

```
[Out] -1/2*((4 + x)*Sqrt[1 - x^2]) - 3*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

**IntegrateAlgebraic** [A] time = 0.06, size = 68, normalized size = 1.45

$$-\frac{\sqrt{1-x}\left(\frac{3(1-x)}{x+1}+5\right)}{\sqrt{x+1}\left(\frac{1-x}{x+1}+1\right)^2} - 3\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

```
[Out] -((Sqrt[1 - x]*(5 + (3*(1 - x))/(1 + x)))/(Sqrt[1 + x]*(1 + (1 - x)/(1 + x)
)^2)) - 3*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

**fricas** [A] time = 1.24, size = 40, normalized size = 0.85

$$-\frac{1}{2}\,(x+4)\sqrt{x+1}\,\sqrt{-x+1} - 3\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
- 1)/x)
```

**giac** [A] time = 1.17, size = 31, normalized size = 0.66

$$-\frac{1}{2}(x+4)\sqrt{x+1}\sqrt{-x+1}+3\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) + 3*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple** [A] time = 0.00, size = 57, normalized size = 1.21

$$\frac{3\sqrt{(x+1)(-x+1)}\arcsin(x)}{2\sqrt{x+1}\sqrt{-x+1}}-\frac{\sqrt{-x+1}(x+1)^{\frac{3}{2}}}{2}-\frac{3\sqrt{-x+1}\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(1/2),x)
```

```
[Out] -1/2*(-x+1)^(1/2)*(x+1)^(3/2)-3/2*(-x+1)^(1/2)*(x+1)^(1/2)+3/2*((x+1)*(-x+1
))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima** [A] time = 3.09, size = 28, normalized size = 0.60

$$-\frac{1}{2}\sqrt{-x^2+1}x-2\sqrt{-x^2+1}+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(x+1)^{3/2}}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(1/2),x)
```

```
[Out] int((x + 1)^(3/2)/(1 - x)^(1/2), x)
```

**sympy** [A] time = 3.28, size = 136, normalized size = 2.89

$$\begin{cases} -3i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ 3\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} - \frac{3\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) - I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + 3*I*sqrt(x + 1)/sqrt(x - 1), Abs(
x + 1)/2 > 1), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1 -
x)) + (x + 1)**(3/2)/(2*sqrt(1 - x)) - 3*sqrt(x + 1)/sqrt(1 - x), True))
```

## 3.1012 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}} dx$

**Optimal**. Leaf size=41

$$\frac{2(x+1)^{3/2}}{\sqrt{1-x}}+3\sqrt{1-x}\,\sqrt{x+1}\,-3\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$\frac{2(x+1)^{3/2}}{\sqrt{1-x}}+3\sqrt{1-x}\,\sqrt{x+1}\,-3\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

```
[Out] 3*Sqrt[1 - x]*Sqrt[1 + x] + (2*(1 + x)^(3/2))/Sqrt[1 - x] - 3*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx &= \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\sin^{-1}(x)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.85

$$\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{1-x}{2}\right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*Hypergeometric2F1[-3/2, -1/2, 1/2, (1 - x)/2])/Sqrt[1 - x]
```

**IntegrateAlgebraic [C]** time = 0.15, size = 59, normalized size = 1.44

$$\frac{\sqrt{1-x}\left((x+1)^{3/2}-6\sqrt{x+1}\right)}{x-1} - 6i\log\left(\sqrt{1-x} - i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*(-6*Sqrt[1 + x] + (1 + x)^(3/2)))/(-1 + x) - (6*I)*Log[Sqrt[1
- x] - I*Sqrt[1 + x]]
```

**fricas** [A] time = 0.73, size = 52, normalized size = 1.27

$$\frac{\sqrt{x+1}\,(x-5)\sqrt{-x+1}+6\,(x-1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+5\,x-5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(x + 1)*(x - 5)*sqrt(-x + 1) + 6*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x +
 1) - 1)/x) + 5*x - 5)/(x - 1)
```

**giac** [A] time = 1.19, size = 35, normalized size = 0.85

$$\frac{\sqrt{x+1}\,(x-5)\sqrt{-x+1}}{x-1}-6\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*(x - 5)*sqrt(-x + 1)/(x - 1) - 6*arcsin(1/2*sqrt(2)*sqrt(x + 1)
)
```

**maple** [B] time = 0.02, size = 72, normalized size = 1.76

$$-\frac{3\sqrt{(x+1)\,(-x+1)}\;\arcsin(x)}{\sqrt{x+1}\;\sqrt{-x+1}}-\frac{\left(x^2-4x-5\right)\sqrt{(x+1)\,(-x+1)}}{\sqrt{-\,(x+1)\,(x-1)}\;\sqrt{-x+1}\;\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(3/2),x)
```

```
[Out] -(x^2-4*x-5)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)/(x+1)^(
1/2)-3*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima** [A] time = 2.99, size = 42, normalized size = 1.02

$$-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2-2\,x+1}-\frac{6\,\sqrt{-x^2+1}}{x-1}-3\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-x^2 + 1)^(3/2)/(x^2 - 2*x + 1) - 6*sqrt(-x^2 + 1)/(x - 1) - 3*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(3/2),x)
```

```
[Out] int((x + 1)^(3/2)/(1 - x)^(3/2), x)
```

**sympy [A]** time = 2.92, size = 100, normalized size = 2.44

$$\begin{cases} 6i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} - \frac{6i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -6\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} + \frac{6\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1)
- 6*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (-6*asin(sqrt(2)*sqrt(x +
 1)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 6*sqrt(x + 1)/sqrt(1 - x), True))
```

## 3.1013 $\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx$

**Optimal**. Leaf size=41

$$\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}}+\sin^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + x])/Sqrt[1 - x] + (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + ArcSin
[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx = \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} - \int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx$$
$$= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \sin^{-1}(x)$$

**Mathematica** [C] time = 0.01, size = 37, normalized size = 0.90

$$\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1-x}{2}\right)}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(5/2),x]
[Out] (4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - x)/2])/(3*(1 - x)^(3/2)
)
```

**IntegrateAlgebraic** [A] time = 0.06, size = 55, normalized size = 1.34

$$-\frac{2\left(\frac{3(1-x)}{x+1}-1\right)(x+1)^{3/2}}{3(1-x)^{3/2}} - 2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(5/2),x]
[Out] (-2*(1 + x)^(3/2)*(-1 + (3*(1 - x))/(1 + x)))/(3*(1 - x)^(3/2)) - 2*ArcTan[
Sqrt[1 - x]/Sqrt[1 + x]]
```

**fricas** [B] time = 1.31, size = 71, normalized size = 1.73

$$-\frac{2\left(2\,x^{2} - 2\,(2\,x-1)\sqrt{x+1}\,\sqrt{-x+1} + 3\left(x^{2} - 2\,x + 1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right) - 4\,x + 2\right)}{3\left(x^{2} - 2\,x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*x^2 - 2*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*(x^2 - 2*x + 1)*arct
an((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 4*x + 2)/(x^2 - 2*x + 1)
```

**giac [A]** time = 1.02, size = 38, normalized size = 0.93

$$\frac{4\,(2\,x-1)\sqrt{x+1}\,\sqrt{-x+1}}{3\,(x-1)^{2}}+2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 2*arcsin(1/2*sqrt(2)*sqr
t(x + 1))
```

**maple [B]** time = 0.02, size = 76, normalized size = 1.85

$$\frac{\sqrt{(x+1)\,(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\,\sqrt{-x+1}}-\frac{4\left(2x^{2}+x-1\right)\sqrt{(x+1)\,(-x+1)}}{3\,(x-1)\,\sqrt{-\,(x+1)\,(x-1)}\,\sqrt{-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(5/2),x)
```

```
[Out] -4/3*(2*x^2+x-1)/(x-1)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/
2)/(x+1)^(1/2)+((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [B]** time = 2.97, size = 66, normalized size = 1.61

$$-\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{3\left(x^{3}-3\,x^{2}+3\,x-1\right)}+\frac{2\,\sqrt{-x^{2}+1}}{3\left(x^{2}-2\,x+1\right)}+\frac{7\,\sqrt{-x^{2}+1}}{3\,(x-1)}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x^2 + 3*x - 1) + 2/3*sqrt(-x^2 + 1)/(x^2 - 2
*x + 1) + 7/3*sqrt(-x^2 + 1)/(x - 1) + arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{(1-x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(5/2),x)
```

```
[Out] int((x + 1)^(3/2)/(1 - x)^(5/2), x)
```

**sympy [B]** time = 3.70, size = 500, normalized size = 12.20

$$\begin{cases} \frac{6i\sqrt{x-1}\,(x+1)^{\frac{15}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} - \frac{3\pi\sqrt{x-1}\,(x+1)^{\frac{15}{2}}}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} - \frac{12i\sqrt{x-1}\,(x+1)^{\frac{13}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} + \frac{6\pi\sqrt{x-1}\,(x+1)^{\frac{13}{2}}}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} - \frac{8i(x+1)^{8}}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} + \frac{12i(x+1)^{7}}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{6\sqrt{1-x}\,(x+1)^{\frac{15}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}} - \frac{12\sqrt{1-x}\,(x+1)^{\frac{13}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}} - \frac{8(x+1)^{8}}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}} + \frac{12(x+1)^{7}}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((6*I*sqrt(x - 1)*(x + 1)**(15/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(-3
*sqrt(x - 1)*(x + 1)**(15/2) + 6*sqrt(x - 1)*(x + 1)**(13/2)) - 3*pi*sqrt(x
 - 1)*(x + 1)**(15/2)/(-3*sqrt(x - 1)*(x + 1)**(15/2) + 6*sqrt(x - 1)*(x +
1)**(13/2)) - 12*I*sqrt(x - 1)*(x + 1)**(13/2)*acosh(sqrt(2)*sqrt(x + 1)/2)
/(-3*sqrt(x - 1)*(x + 1)**(15/2) + 6*sqrt(x - 1)*(x + 1)**(13/2)) + 6*pi*sq
rt(x - 1)*(x + 1)**(13/2)/(-3*sqrt(x - 1)*(x + 1)**(15/2) + 6*sqrt(x - 1)*(
x + 1)**(13/2)) - 8*I*(x + 1)**8/(-3*sqrt(x - 1)*(x + 1)**(15/2) + 6*sqrt(x
 - 1)*(x + 1)**(13/2)) + 12*I*(x + 1)**7/(-3*sqrt(x - 1)*(x + 1)**(15/2) +
6*sqrt(x - 1)*(x + 1)**(13/2)), Abs(x + 1)/2 > 1), (6*sqrt(1 - x)*(x + 1)**
(15/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(
1 - x)*(x + 1)**(13/2)) - 12*sqrt(1 - x)*(x + 1)**(13/2)*asin(sqrt(2)*sqrt(
x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2)) -
 8*(x + 1)**8/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2
)) + 12*(x + 1)**7/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**
(13/2)), True))
```

## 3.1014 $\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx$

**Optimal**. Leaf size=20

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

```
[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx = \frac{(1+x)^{5/2}}{5(1-x)^{5/2}}$$

**Mathematica** [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

```
[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.07, size = 20, normalized size = 1.00

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

```
[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

**fricas** [B] time = 1.00, size = 52, normalized size = 2.60

$$\frac{x^3-3x^2-\left(x^2+2x+1\right)\sqrt{x+1}\sqrt{-x+1}+3x-1}{5\left(x^3-3x^2+3x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(x^3 - 3*x^2 - (x^2 + 2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*x - 1)/(x^3
 - 3*x^2 + 3*x - 1)
```

**giac** [A] time = 1.06, size = 19, normalized size = 0.95

$$-\frac{(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{5(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^3
```

**maple** [A] time = 0.00, size = 15, normalized size = 0.75

$$\frac{(x+1)^{\frac{5}{2}}}{5(-x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(7/2),x)
```

```
[Out] 1/5*(x+1)^(5/2)/(-x+1)^(5/2)
```

**maxima [B]** time = 1.28, size = 94, normalized size = 4.70

$$\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^4-4\,x^3+6\,x^2-4\,x+1}+\frac{6\,\sqrt{-x^2+1}}{5\left(x^3-3\,x^2+3\,x-1\right)}+\frac{\sqrt{-x^2+1}}{5\left(x^2-2\,x+1\right)}-\frac{\sqrt{-x^2+1}}{5\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 6/5*sqrt(-x^2 + 1)/(x^3
- 3*x^2 + 3*x - 1) + 1/5*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/5*sqrt(-x^2 + 1
)/(x - 1)
```

**mupad [B]** time = 0.25, size = 50, normalized size = 2.50

$$-\frac{\sqrt{1-x}\,\left(\frac{2\,x\,\sqrt{x+1}}{5}+\frac{\sqrt{x+1}}{5}+\frac{x^2\,\sqrt{x+1}}{5}\right)}{x^3-3\,x^2+3\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(7/2),x)
```

```
[Out] -((1 - x)^(1/2)*((2*x*(x + 1)^(1/2))/5 + (x + 1)^(1/2)/5 + (x^2*(x + 1)^(1/
2))/5))/(3*x - 3*x^2 + x^3 - 1)
```

**sympy [B]** time = 6.26, size = 88, normalized size = 4.40

$$\begin{cases} -\frac{i(x+1)^{\frac{5}{2}}}{5\sqrt{x-1}\,(x+1)^2-20\sqrt{x-1}\,(x+1)+20\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{(x+1)^{\frac{5}{2}}}{5\sqrt{1-x}\,(x+1)^2-20\sqrt{1-x}\,(x+1)+20\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((-I*(x + 1)**(5/2)/(5*sqrt(x - 1)*(x + 1)**2 - 20*sqrt(x - 1)*(x
+ 1) + 20*sqrt(x - 1)), Abs(x + 1)/2 > 1), ((x + 1)**(5/2)/(5*sqrt(1 - x)*(
x + 1)**2 - 20*sqrt(1 - x)*(x + 1) + 20*sqrt(1 - x)), True))
```

## 3.1015 $\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}} dx$

**Optimal**. Leaf size=41

$$\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(5/2)/(7*(1 - x)^(7/2)) + (1 + x)^(5/2)/(35*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx &= \frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{1}{7}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{(1+x)^{5/2}}{35(1-x)^{5/2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.56

$$-\frac{(x-6)(x+1)^{5/2}}{35(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(9/2),x]
```

```
[Out] -1/35*((-6 + x)*(1 + x)^(5/2))/(1 - x)^(7/2)
```

**IntegrateAlgebraic** [A] time = 0.07, size = 34, normalized size = 0.83

$$\frac{(x+1)^{7/2}\left(\frac{7(1-x)}{x+1}+5\right)}{70(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(9/2),x]
```

```
[Out] ((1 + x)^(7/2)*(5 + (7*(1 - x))/(1 + x)))/(70*(1 - x)^(7/2))
```

**fricas** [B] time = 1.15, size = 69, normalized size = 1.68

$$\frac{6x^4-24x^3+36x^2-\left(x^3-4x^2-11x-6\right)\sqrt{x+1}\sqrt{-x+1}-24x+6}{35\left(x^4-4x^3+6x^2-4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(6*x^4 - 24*x^3 + 36*x^2 - (x^3 - 4*x^2 - 11*x - 6)*sqrt(x + 1)*sqrt(-
x + 1) - 24*x + 6)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

**giac** [A] time = 1.13, size = 22, normalized size = 0.54

$$-\frac{(x+1)^{\frac{5}{2}}(x-6)\sqrt{-x+1}}{35\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(x + 1)^(5/2)*(x - 6)*sqrt(-x + 1)/(x - 1)^4
```

**maple [A]** time = 0.00, size = 18, normalized size = 0.44

$$-\frac{(x+1)^{\frac{5}{2}}(x-6)}{35(-x+1)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(9/2),x)
```

```
[Out] -1/35*(x+1)^(5/2)*(x-6)/(-x+1)^(7/2)
```

**maxima [B]** time = 1.38, size = 131, normalized size = 3.20

$$-\frac{(-x^2+1)^{\frac{3}{2}}}{2(x^5-5x^4+10x^3-10x^2+5x-1)}-\frac{3\sqrt{-x^2+1}}{7(x^4-4x^3+6x^2-4x+1)}-\frac{3\sqrt{-x^2+1}}{70(x^3-3x^2+3x-1)}+\frac{\sqrt{-x^2+1}}{35(x^2-2x+1)}-\frac{\sqrt{-x^2+1}}{35(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*(-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 3/7*sqrt(
-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/70*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) + 1/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/35*sqrt(-x^2 + 1)/(x
 - 1)
```

**mupad [B]** time = 0.27, size = 64, normalized size = 1.56

$$\frac{\sqrt{1-x}\left(\frac{11x\sqrt{x+1}}{35}+\frac{6\sqrt{x+1}}{35}+\frac{4x^2\sqrt{x+1}}{35}-\frac{x^3\sqrt{x+1}}{35}\right)}{x^4-4x^3+6x^2-4x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((11*x*(x + 1)^(1/2))/35 + (6*(x + 1)^(1/2))/35 + (4*x^2*(x
+ 1)^(1/2))/35 - (x^3*(x + 1)^(1/2))/35))/(6*x^2 - 4*x - 4*x^3 + x^4 + 1)
```

**sympy [B]** time = 19.08, size = 228, normalized size = 5.56

$$\begin{cases} -\frac{i(x+1)^{\frac{7}{2}}}{35\sqrt{x-1}(x+1)^3-210\sqrt{x-1}(x+1)^2+420\sqrt{x-1}(x+1)-280\sqrt{x-1}}+\frac{7i(x+1)^{\frac{5}{2}}}{35\sqrt{x-1}(x+1)^3-210\sqrt{x-1}(x+1)^2+420\sqrt{x-1}(x+1)-280\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1 \\ \frac{(x+1)^{\frac{7}{2}}}{35\sqrt{1-x}(x+1)^3-210\sqrt{1-x}(x+1)^2+420\sqrt{1-x}(x+1)-280\sqrt{1-x}}-\frac{7(x+1)^{\frac{5}{2}}}{35\sqrt{1-x}(x+1)^3-210\sqrt{1-x}(x+1)^2+420\sqrt{1-x}(x+1)-280\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((-I*(x + 1)**(7/2)/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*(
x + 1)**2 + 420*sqrt(x - 1)*(x + 1) - 280*sqrt(x - 1)) + 7*I*(x + 1)**(5/2)
/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*(x + 1)**2 + 420*sqrt(x - 1)*
(x + 1) - 280*sqrt(x - 1)), Abs(x + 1)/2 > 1), ((x + 1)**(7/2)/(35*sqrt(1 -
 x)*(x + 1)**3 - 210*sqrt(1 - x)*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) - 280
*sqrt(1 - x)) - 7*(x + 1)**(5/2)/(35*sqrt(1 - x)*(x + 1)**3 - 210*sqrt(1 -
x)*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) - 280*sqrt(1 - x)), True))
```

## 3.1016 $\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx$

**Optimal**. Leaf size=61

$$\frac{2(x+1)^{5/2}}{315(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{5/2}}{315(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] (1 + x)^(5/2)/(9*(1 - x)^(9/2)) + (2*(1 + x)^(5/2))/(63*(1 - x)^(7/2)) + (2
*(1 + x)^(5/2))/(315*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx = \frac{(1+x)^{5/2}}{9(1-x)^{9/2}} + \frac{2}{9}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx$$
$$= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}} + \frac{2(1+x)^{5/2}}{63(1-x)^{7/2}} + \frac{2}{63}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx$$
$$= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}} + \frac{2(1+x)^{5/2}}{63(1-x)^{7/2}} + \frac{2(1+x)^{5/2}}{315(1-x)^{5/2}}$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 0.49

$$\frac{(x+1)^{5/2}\left(2x^2-14x+47\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] ((1 + x)^(5/2)*(47 - 14*x + 2*x^2))/(315*(1 - x)^(9/2))
```

**IntegrateAlgebraic** [A] time = 0.07, size = 48, normalized size = 0.79

$$\frac{(x+1)^{9/2}\left(\frac{63(1-x)^2}{(x+1)^2}+\frac{90(1-x)}{x+1}+35\right)}{1260(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] ((1 + x)^(9/2)*(35 + (63*(1 - x)^2)/(1 + x)^2 + (90*(1 - x))/(1 + x)))/(126
0*(1 - x)^(9/2))
```

**fricas** [A] time = 0.67, size = 86, normalized size = 1.41

$$\frac{47\,x^5-235\,x^4+470\,x^3-470\,x^2-\left(2\,x^4-10\,x^3+21\,x^2+80\,x+47\right)\sqrt{x+1}\,\sqrt{-x+1}+235\,x-47}{315\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(47*x^5 - 235*x^4 + 470*x^3 - 470*x^2 - (2*x^4 - 10*x^3 + 21*x^2 + 80
*x + 47)*sqrt(x + 1)*sqrt(-x + 1) + 235*x - 47)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1)
```

**giac [A]** time = 1.11, size = 29, normalized size = 0.48

$$-\frac{(2(x+1)(x-8)+63)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{315(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="giac")
[Out] -1/315*(2*(x + 1)*(x - 8) + 63)*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^5
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.41

$$\frac{(x+1)^{\frac{5}{2}}\left(2x^2-14x+47\right)}{315(-x+1)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(11/2),x)
[Out] 1/315*(x+1)^(5/2)*(2*x^2-14*x+47)/(-x+1)^(9/2)
```

**maxima [B]** time = 1.37, size = 172, normalized size = 2.82

$$\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^6-6x^5+15x^4-20x^3+15x^2-6x+1\right)}+\frac{2\sqrt{-x^2+1}}{9\left(x^5-5x^4+10x^3-10x^2+5x-1\right)}+\frac{\sqrt{-x^2+1}}{63\left(x^4-4x^3+6x^2-4x+1\right)}-\frac{\sqrt{-x^2+1}}{105\left(x^3-3x^2+3x-1\right)}+\frac{2\sqrt{-x^2+1}}{315\left(x^2-2x+1\right)}-\frac{2\sqrt{-x^2+1}}{315(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="maxima")
[Out] 1/3*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 2
/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 1/63*sqrt(-x^
2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/105*sqrt(-x^2 + 1)/(x^3 - 3*x^2
+ 3*x - 1) + 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/315*sqrt(-x^2 + 1)/(x
 - 1)
```

**mupad [B]** time = 0.32, size = 80, normalized size = 1.31

$$-\frac{\sqrt{1-x}\,\left(\frac{16\,x\,\sqrt{x+1}}{63}+\frac{47\,\sqrt{x+1}}{315}+\frac{x^2\,\sqrt{x+1}}{15}-\frac{2\,x^3\,\sqrt{x+1}}{63}+\frac{2\,x^4\,\sqrt{x+1}}{315}\right)}{x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(11/2),x)
```

```
[Out] -((1 - x)^(1/2)*((16*x*(x + 1)^(1/2))/63 + (47*(x + 1)^(1/2))/315 + (x^2*(x
 + 1)^(1/2))/15 - (2*x^3*(x + 1)^(1/2))/63 + (2*x^4*(x + 1)^(1/2))/315))/(5
*x - 10*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

**sympy [B]** time = 51.65, size = 677, normalized size = 11.10

$$\begin{cases} -\frac{2i(x+1)^{\frac{11}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}}+\frac{22i(x+1)^{\frac{9}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}}-\frac{99i(x+1)^{\frac{7}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}}+\frac{126i(x+1)^{\frac{5}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{2(x+1)^{\frac{11}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}}-\frac{22(x+1)^{\frac{9}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}}+\frac{99(x+1)^{\frac{7}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}}-\frac{126(x+1)^{\frac{5}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(11/2),x)
```

```
[Out] Piecewise((-2*I*(x + 1)**(11/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x -
 1)*(x + 1)**4 + 12600*sqrt(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**
2 + 25200*sqrt(x - 1)*(x + 1) - 10080*sqrt(x - 1)) + 22*I*(x + 1)**(9/2)/(3
15*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x - 1)*(x + 1)**4 + 12600*sqrt(x - 1)
*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**2 + 25200*sqrt(x - 1)*(x + 1) - 10
080*sqrt(x - 1)) - 99*I*(x + 1)**(7/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*s
qrt(x - 1)*(x + 1)**4 + 12600*sqrt(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x
 + 1)**2 + 25200*sqrt(x - 1)*(x + 1) - 10080*sqrt(x - 1)) + 126*I*(x + 1)**
(5/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x - 1)*(x + 1)**4 + 12600*sqr
t(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**2 + 25200*sqrt(x - 1)*(x +
 1) - 10080*sqrt(x - 1)), Abs(x + 1)/2 > 1), (2*(x + 1)**(11/2)/(315*sqrt(1
 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)*
*3 - 25200*sqrt(1 - x)*(x + 1)**2 + 25200*sqrt(1 - x)*(x + 1) - 10080*sqrt(
1 - x)) - 22*(x + 1)**(9/2)/(315*sqrt(1 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*
(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3 - 25200*sqrt(1 - x)*(x + 1)**2 +
25200*sqrt(1 - x)*(x + 1) - 10080*sqrt(1 - x)) + 99*(x + 1)**(7/2)/(315*sqr
t(1 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x +
1)**3 - 25200*sqrt(1 - x)*(x + 1)**2 + 25200*sqrt(1 - x)*(x + 1) - 10080*sq
rt(1 - x)) - 126*(x + 1)**(5/2)/(315*sqrt(1 - x)*(x + 1)**5 - 3150*sqrt(1 -
 x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3 - 25200*sqrt(1 - x)*(x + 1)**
2 + 25200*sqrt(1 - x)*(x + 1) - 10080*sqrt(1 - x)), True))
```

## 3.1017 $\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}} \, dx$

**Optimal**. Leaf size=81

$$\frac{2(x+1)^{5/2}}{1155(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{231(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{33(1-x)^{9/2}}+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{5/2}}{1155(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{231(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{33(1-x)^{9/2}}+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(5/2)/(11*(1 - x)^(11/2)) + (1 + x)^(5/2)/(33*(1 - x)^(9/2)) + (2*(
1 + x)^(5/2))/(231*(1 - x)^(7/2)) + (2*(1 + x)^(5/2))/(1155*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{3}{11}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2}{33}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2}{231}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{1155(1-x)^{5/2}}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 35, normalized size = 0.43

$$\frac{(x+1)^{5/2}\left(-2x^3+16x^2-61x+152\right)}{1155(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(5/2)*(152 - 61*x + 16*x^2 - 2*x^3))/(1155*(1 - x)^(11/2))
```

**IntegrateAlgebraic** [A] time = 0.08, size = 62, normalized size = 0.77

$$\frac{(x+1)^{11/2}\left(\frac{231(1-x)^3}{(x+1)^3}+\frac{495(1-x)^2}{(x+1)^2}+\frac{385(1-x)}{x+1}+105\right)}{9240(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(11/2)*(105 + (231*(1 - x)^3)/(1 + x)^3 + (495*(1 - x)^2)/(1 + x)^
2 + (385*(1 - x))/(1 + x)))/(9240*(1 - x)^(11/2))
```

**fricas** [A] time = 1.53, size = 101, normalized size = 1.25

$$\frac{152\,x^6-912\,x^5+2280\,x^4-3040\,x^3+2280\,x^2-\left(2\,x^5-12\,x^4+31\,x^3-46\,x^2-243\,x-152\right)\sqrt{x+1}\,\sqrt{-x+1}-912\,x+152}{1155\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(152*x^6 - 912*x^5 + 2280*x^4 - 3040*x^3 + 2280*x^2 - (2*x^5 - 12*x^
4 + 31*x^3 - 46*x^2 - 243*x - 152)*sqrt(x + 1)*sqrt(-x + 1) - 912*x + 152)/
(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

**giac [A]** time = 1.17, size = 35, normalized size = 0.43

$$-\frac{((2(x+1)(x-10)+99)(x+1)-231)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{1155(x-1)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] -1/1155*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1)^(5/2)*sqrt(-x + 1
)/(x - 1)^6
```

**maple [A]** time = 0.00, size = 30, normalized size = 0.37

$$-\frac{(x+1)^{\frac{5}{2}}\left(2x^{3}-16x^{2}+61x-152\right)}{1155(-x+1)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(13/2),x)
```

```
[Out] -1/1155*(x+1)^(5/2)*(2*x^3-16*x^2+61*x-152)/(-x+1)^(11/2)
```

**maxima [B]** time = 1.39, size = 218, normalized size = 2.69

$$-\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{4\left(x^{7}-7x^{6}+21x^{5}-35x^{4}+35x^{3}-21x^{2}+7x-1\right)}-\frac{3\sqrt{-x^{2}+1}}{22\left(x^{6}-6x^{5}+15x^{4}-20x^{3}+15x^{2}-6x+1\right)}-\frac{\sqrt{-x^{2}+1}}{132\left(x^{5}-5x^{4}+10x^{3}-10x^{2}+5x-1\right)}+\frac{\sqrt{-x^{2}+1}}{231\left(x^{4}-4x^{3}+6x^{2}-4x+1\right)}-\frac{\sqrt{-x^{2}+1}}{385\left(x^{3}-3x^{2}+3x-1\right)}+\frac{2\sqrt{-x^{2}+1}}{1155\left(x^{2}-2x+1\right)}-\frac{2\sqrt{-x^{2}+1}}{1155(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*
x - 1) - 3/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x
+ 1) - 1/132*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 1/2
31*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/385*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) + 2/1155*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/1155*sqrt
(-x^2 + 1)/(x - 1)
```

**mupad [B]** time = 0.31, size = 94, normalized size = 1.16

$$\frac{\sqrt{1-x}\left(\frac{81x\sqrt{x+1}}{385}+\frac{152\sqrt{x+1}}{1155}+\frac{46x^{2}\sqrt{x+1}}{1155}-\frac{31x^{3}\sqrt{x+1}}{1155}+\frac{4x^{4}\sqrt{x+1}}{385}-\frac{2x^{5}\sqrt{x+1}}{1155}\right)}{x^{6}-6x^{5}+15x^{4}-20x^{3}+15x^{2}-6x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((81*x*(x + 1)^(1/2))/385 + (152*(x + 1)^(1/2))/1155 + (46*x
^2*(x + 1)^(1/2))/1155 - (31*x^3*(x + 1)^(1/2))/1155 + (4*x^4*(x + 1)^(1/2)
)/385 - (2*x^5*(x + 1)^(1/2))/1155))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 - 6*x^
5 + x^6 + 1)
```

**sympy [B]** time = 132.96, size = 1753, normalized size = 21.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(13/2),x)
```

```
[Out] Piecewise((-2*I*(x + 1)**(17/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x
 - 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x +
1)**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2
069760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x
 - 1)) + 34*I*(x + 1)**(15/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)
**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 206
9760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
 1)) - 255*I*(x + 1)**(13/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)*
*5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069
760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
1)) + 1105*I*(x + 1)**(11/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)*
*5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069
760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
1)) - 2750*I*(x + 1)**(9/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1
)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**
5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 20697
60*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1
)) + 3564*I*(x + 1)**(7/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1)
*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**5
 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 206976
0*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1)
) - 1848*I*(x + 1)**(5/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1)*
(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**5
+ 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069760
*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1))
, Abs(x + 1)/2 > 1), (2*(x + 1)**(17/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 1848
0*sqrt(1 - x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 -
```

x)*(x + 1)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1
)**3 + 2069760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 29568
0*sqrt(1 - x)) - 34*(x + 1)**(15/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sq
rt(1 - x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(
x + 1)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3
 + 2069760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sq
rt(1 - x)) + 255*(x + 1)**(13/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(
1 - x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x +
 1)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 +
2069760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(
1 - x)) - 1105*(x + 1)**(11/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1
 - x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1
)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 20
69760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1
 - x)) + 2750*(x + 1)**(9/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x
)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**
5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 20697
60*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x
)) - 3564*(x + 1)**(7/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x)*(
x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**5 +
 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069760*
sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x))
 + 1848*(x + 1)**(5/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x)*(x +
 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**5 + 12
93600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069760*sqr
t(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x)), Tr
ue))

## 3.1018 $\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}} dx$

**Optimal**. Leaf size=101

$$\frac{8(x+1)^{5/2}}{15015(1-x)^{5/2}}+\frac{8(x+1)^{5/2}}{3003(1-x)^{7/2}}+\frac{4(x+1)^{5/2}}{429(1-x)^{9/2}}+\frac{4(x+1)^{5/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8(x+1)^{5/2}}{15015(1-x)^{5/2}}+\frac{8(x+1)^{5/2}}{3003(1-x)^{7/2}}+\frac{4(x+1)^{5/2}}{429(1-x)^{9/2}}+\frac{4(x+1)^{5/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] (1 + x)^(5/2)/(13*(1 - x)^(13/2)) + (4*(1 + x)^(5/2))/(143*(1 - x)^(11/2))
+ (4*(1 + x)^(5/2))/(429*(1 - x)^(9/2)) + (8*(1 + x)^(5/2))/(3003*(1 - x)^(
7/2)) + (8*(1 + x)^(5/2))/(15015*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx &= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4}{13}\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx \\
&= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{12}{143}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx \\
&= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8}{429}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx \\
&= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}} + \frac{8\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx}{3003} \\
&= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}} + \frac{8(1+x)^{5/2}}{15015(1-x)^{5/2}}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 40, normalized size = 0.40

$$\frac{(x+1)^{5/2}\left(8x^4-72x^3+308x^2-852x+1763\right)}{15015(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] ((1 + x)^(5/2)*(1763 - 852*x + 308*x^2 - 72*x^3 + 8*x^4))/(15015*(1 - x)^(1
3/2))
```

**IntegrateAlgebraic** [A] time = 0.09, size = 76, normalized size = 0.75

$$\frac{(x+1)^{13/2}\left(\frac{3003(1-x)^4}{(x+1)^4}+\frac{8580(1-x)^3}{(x+1)^3}+\frac{10010(1-x)^2}{(x+1)^2}+\frac{5460(1-x)}{x+1}+1155\right)}{240240(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] ((1 + x)^(13/2)*(1155 + (3003*(1 - x)^4)/(1 + x)^4 + (8580*(1 - x)^3)/(1 +
x)^3 + (10010*(1 - x)^2)/(1 + x)^2 + (5460*(1 - x))/(1 + x)))/(240240*(1 -
x)^(13/2))
```

**fricas** [A] time = 1.23, size = 116, normalized size = 1.15

$$\frac{1763\,x^7-12341\,x^6+37023\,x^5-61705\,x^4+61705\,x^3-37023\,x^2-\left(8\,x^6-56\,x^5+172\,x^4-308\,x^3+367\,x^2+2674\,x+1763\right)\sqrt{x+1}\,\sqrt{-x+1}+12341\,x-1763}{15015\left(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1763*x^7 - 12341*x^6 + 37023*x^5 - 61705*x^4 + 61705*x^3 - 37023*x
^2 - (8*x^6 - 56*x^5 + 172*x^4 - 308*x^3 + 367*x^2 + 2674*x + 1763)*sqrt(x
+ 1)*sqrt(-x + 1) + 12341*x - 1763)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3
 - 21*x^2 + 7*x - 1)
```

**giac [A]** time = 1.23, size = 42, normalized size = 0.42

$$-\frac{\left(4\left(\left(2\left(x+1\right)\left(x-12\right)+143\right)\left(x+1\right)-429\right)\left(x+1\right)+3003\right)\left(x+1\right)^{\frac{5}{2}}\sqrt{-x+1}}{15015\left(x-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="giac")
```

```
[Out] -1/15015*(4*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1) + 3003)*(x +
 1)^(5/2)*sqrt(-x + 1)/(x - 1)^7
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.35

$$\frac{\left(x+1\right)^{\frac{5}{2}}\left(8x^{4}-72x^{3}+308x^{2}-852x+1763\right)}{15015\left(-x+1\right)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(15/2),x)
```

```
[Out] 1/15015*(x+1)^(5/2)*(8*x^4-72*x^3+308*x^2-852*x+1763)/(-x+1)^(13/2)
```

**maxima [B]** time = 1.38, size = 269, normalized size = 2.66

$$\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{5\left(x^{8}-8x^{7}+28x^{6}-56x^{5}+70x^{4}-56x^{3}+28x^{2}-8x+1\right)}+\frac{6\sqrt{-x^{2}+1}}{65\left(x^{7}-7x^{6}+21x^{5}-35x^{4}+35x^{3}-21x^{2}+7x-1\right)}+\frac{3\sqrt{-x^{2}+1}}{715\left(x^{6}-6x^{5}+15x^{4}-20x^{3}+15x^{2}-6x+1\right)}-\frac{\sqrt{-x^{2}+1}}{429\left(x^{5}-5x^{4}+10x^{3}-10x^{2}+5x-1\right)}+\frac{4\sqrt{-x^{2}+1}}{3003\left(x^{4}-4x^{3}+6x^{2}-4x+1\right)}-\frac{4\sqrt{-x^{2}+1}}{5005\left(x^{3}-3x^{2}+3x-1\right)}+\frac{8\sqrt{-x^{2}+1}}{15015\left(x^{2}-2x+1\right)}-\frac{8\sqrt{-x^{2}+1}}{15015\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*
x^2 - 8*x + 1) + 6/65*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^
3 - 21*x^2 + 7*x - 1) + 3/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3
 + 15*x^2 - 6*x + 1) - 1/429*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2
+ 5*x - 1) + 4/3003*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4/5005
```

```
*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 8/15015*sqrt(-x^2 + 1)/(x^2 - 2*x
 + 1) - 8/15015*sqrt(-x^2 + 1)/(x - 1)
```

**mupad [B]** time = 0.33, size = 110, normalized size = 1.09

$$-\frac{\sqrt{1-x}\,\left(\frac{382\,x\,\sqrt{x+1}}{2145}+\frac{1763\,\sqrt{x+1}}{15015}+\frac{367\,x^2\,\sqrt{x+1}}{15015}-\frac{4\,x^3\,\sqrt{x+1}}{195}+\frac{172\,x^4\,\sqrt{x+1}}{15015}-\frac{8\,x^5\,\sqrt{x+1}}{2145}+\frac{8\,x^6\,\sqrt{x+1}}{15015}\right)}{x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(15/2),x)
```

```
[Out] -((1 - x)^(1/2)*((382*x*(x + 1)^(1/2))/2145 + (1763*(x + 1)^(1/2))/15015 +
(367*x^2*(x + 1)^(1/2))/15015 - (4*x^3*(x + 1)^(1/2))/195 + (172*x^4*(x + 1
)^(1/2))/15015 - (8*x^5*(x + 1)^(1/2))/2145 + (8*x^6*(x + 1)^(1/2))/15015))
/(7*x - 21*x^2 + 35*x^3 - 35*x^4 + 21*x^5 - 7*x^6 + x^7 - 1)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(15/2),x)
```

```
[Out] Timed out
```

# 3.1019 $\int (1-x)^{11/2}(1+x)^{5/2}\,dx$

**Optimal**. Leaf size=130

$$\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}+\frac{11}{72}(x+1)^{7/2}(1-x)^{9/2}+\frac{11}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{11}{48}x(x+1)^{5/2}(1-x)^{5/2}+\frac{55}{192}x(x+1)^{3/2}(1-x)^{3/2}+\frac{55}{128}x$$

**Rubi [A]** time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}+\frac{11}{72}(x+1)^{7/2}(1-x)^{9/2}+\frac{11}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{11}{48}x(x+1)^{5/2}(1-x)^{5/2}+\frac{55}{192}x(x+1)^{3/2}(1-x)^{3/2}+\frac{55}{128}x\sqrt{x+1}\sqrt{1-x}+\frac{55}{128}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

```
[Out] (55*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (55*(1 - x)^(3/2)*x*(1 + x)^(3/2))/192
 + (11*(1 - x)^(5/2)*x*(1 + x)^(5/2))/48 + (11*(1 - x)^(7/2)*(1 + x)^(7/2))
/56 + (11*(1 - x)^(9/2)*(1 + x)^(7/2))/72 + ((1 - x)^(11/2)*(1 + x)^(7/2))/
9 + (55*ArcSin[x])/128
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{11/2}(1+x)^{5/2}\,dx &= \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{9}\int (1-x)^{9/2}(1+x)^{5/2}\,dx \\
&= \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{8}\int (1-x)^{7/2}(1+x)^{5/2}\,dx \\
&= \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{8}\int (1-x \\
&= \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}(1-x)^{11} \\
&= \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{11}{72}(1- \\
&= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x \\
&= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x \\
&= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 75, normalized size = 0.58

$$\frac{\sqrt{1-x^2}\left(-896x^8+3024x^7-1024x^6-7224x^5+8448x^4+3066x^3-10240x^2+4599x+3712\right)-6930\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{8064}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(3712 + 4599*x - 10240*x^2 + 3066*x^3 + 8448*x^4 - 7224*x^5
- 1024*x^6 + 3024*x^7 - 896*x^8) - 6930*ArcSin[Sqrt[1 - x]/Sqrt[2]])/8064
```

**IntegrateAlgebraic [A]** time = 0.19, size = 205, normalized size = 1.58

$$\frac{-\frac{3465(1-x)^{17/2}}{(x+1)^{17/2}}-\frac{30030(1-x)^{15/2}}{(x+1)^{15/2}}-\frac{115038(1-x)^{13/2}}{(x+1)^{13/2}}+\frac{334602(1-x)^{11/2}}{(x+1)^{11/2}}+\frac{360448(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{255222(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{115038(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{30030(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{3465\sqrt{1-x}}{\sqrt{x+1}}}{4032\left(\frac{1-x}{x+1}+1\right)^9}-\frac{55}{64}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

```
[Out] ((-3465*(1 - x)^(17/2))/(1 + x)^(17/2) - (30030*(1 - x)^(15/2))/(1 + x)^(15
/2) - (115038*(1 - x)^(13/2))/(1 + x)^(13/2) + (334602*(1 - x)^(11/2))/(1 +
```

```
 x)^(11/2) + (360448*(1 - x)^(9/2))/(1 + x)^(9/2) + (255222*(1 - x)^(7/2))/
(1 + x)^(7/2) + (115038*(1 - x)^(5/2))/(1 + x)^(5/2) + (30030*(1 - x)^(3/2)
)/(1 + x)^(3/2) + (3465*Sqrt[1 - x])/Sqrt[1 + x])/(4032*(1 + (1 - x)/(1 + x
))^9) - (55*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/64
```

**fricas [A]** time = 0.77, size = 77, normalized size = 0.59

$$-\frac{1}{8064}\left(896\,x^{8}-3024\,x^{7}+1024\,x^{6}+7224\,x^{5}-8448\,x^{4}-3066\,x^{3}+10240\,x^{2}-4599\,x-3712\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{55}{64}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8064*(896*x^8 - 3024*x^7 + 1024*x^6 + 7224*x^5 - 8448*x^4 - 3066*x^3 + 1
0240*x^2 - 4599*x - 3712)*sqrt(x + 1)*sqrt(-x + 1) - 55/64*arctan((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x)
```

**giac [B]** time = 1.40, size = 323, normalized size = 2.48

$$-\frac{1}{40320}\left(\left(2\left(\left(4\left(5\left(2\left(7\left(8x-65\right)(x+1)+2073\right)(x+1)-9833\right)(x+1)+75293\right)(x+1)-310203\right)(x+1)+216993\right)(x+1)-205275\right)(x+1)+69615\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{6720}\left(\left(2\left(\left(4\left(5\left(6\left(7x-50\right)(x+1)+1219\right)(x+1)-12463\right)(x+1)+64233\right)(x+1)-53963\right)(x+1)+59465\right)(x+1)-23205\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{840}\left(\left(2\left(\left(4\left(5\left(6x-37\right)(x+1)+661\right)(x+1)-4551\right)(x+1)+4781\right)(x+1)-6335\right)(x+1)+2835\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{40}\left(\left(2\left(\left(4\left(5x-26\right)(x+1)+321\right)(x+1)-451\right)(x+1)+745\right)(x+1)-405\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{4}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{3}\left(\left(2x-5\right)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{55}{64}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/40320*((2*((4*(5*(2*(7*(8*x - 65)*(x + 1) + 2073)*(x + 1) - 9833)*(x + 1
) + 75293)*(x + 1) - 310203)*(x + 1) + 216993)*(x + 1) - 205275)*(x + 1) +
69615)*sqrt(x + 1)*sqrt(-x + 1) + 1/6720*((2*((4*(5*(6*(7*x - 50)*(x + 1) +
 1219)*(x + 1) - 12463)*(x + 1) + 64233)*(x + 1) - 53963)*(x + 1) + 59465)*
(x + 1) - 23205)*sqrt(x + 1)*sqrt(-x + 1) + 1/840*((2*((4*(5*(6*x - 37)*(x
+ 1) + 661)*(x + 1) - 4551)*(x + 1) + 4781)*(x + 1) - 6335)*(x + 1) + 2835)
*sqrt(x + 1)*sqrt(-x + 1) - 1/40*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1)
- 451)*(x + 1) + 745)*(x + 1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/4*((2*(3*
x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x -
5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sqrt(-x + 1)
 + sqrt(x + 1)*sqrt(-x + 1) + 55/64*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [A]** time = 0.01, size = 155, normalized size = 1.19

$$\frac{55\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{128\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{11}{2}}(x+1)^{\frac{7}{2}}}{9}+\frac{11(-x+1)^{\frac{9}{2}}(x+1)^{\frac{7}{2}}}{72}+\frac{11(-x+1)^{\frac{7}{2}}(x+1)^{\frac{7}{2}}}{56}+\frac{11(-x+1)^{\frac{5}{2}}(x+1)^{\frac{7}{2}}}{48}+\frac{11(-x+1)^{\frac{3}{2}}(x+1)^{\frac{7}{2}}}{48}+\frac{11\sqrt{-x+1}\,(x+1)^{\frac{7}{2}}}{64}-\frac{11\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{192}-\frac{55\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{384}-\frac{55\sqrt{-x+1}\,\sqrt{x+1}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(11/2)*(x+1)^(5/2),x)
```

```
[Out] 1/9*(-x+1)^(11/2)*(x+1)^(7/2)+11/72*(-x+1)^(9/2)*(x+1)^(7/2)+11/56*(-x+1)^(
7/2)*(x+1)^(7/2)+11/48*(-x+1)^(5/2)*(x+1)^(7/2)+11/48*(-x+1)^(3/2)*(x+1)^(7
```

```
/2)+11/64*(-x+1)^(1/2)*(x+1)^(7/2)-11/192*(-x+1)^(1/2)*(x+1)^(5/2)-55/384*(
-x+1)^(1/2)*(x+1)^(3/2)-55/128*(-x+1)^(1/2)*(x+1)^(1/2)+55/128*((x+1)*(-x+1
))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima** [**A**] time = 3.01, size = 78, normalized size = 0.60

$$\frac{1}{9}\left(-x^2+1\right)^{\frac{7}{2}}x^2-\frac{3}{8}\left(-x^2+1\right)^{\frac{7}{2}}x+\frac{29}{63}\left(-x^2+1\right)^{\frac{7}{2}}+\frac{11}{48}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{55}{192}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{55}{128}\sqrt{-x^2+1}\,x+\frac{55}{128}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(-x^2 + 1)^(7/2)*x^2 - 3/8*(-x^2 + 1)^(7/2)*x + 29/63*(-x^2 + 1)^(7/2)
+ 11/48*(-x^2 + 1)^(5/2)*x + 55/192*(-x^2 + 1)^(3/2)*x + 55/128*sqrt(-x^2 +
 1)*x + 55/128*arcsin(x)
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{11/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(11/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(11/2)*(x + 1)^(5/2), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(11/2)*(1+x)**(5/2),x)
```

```
[Out] Timed out
```

# 3.1020 $\int (1-x)^{9/2}(1+x)^{5/2}\,dx$

**Optimal**. Leaf size=110

$$\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}+\frac{9}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{3}{16}x(x+1)^{5/2}(1-x)^{5/2}+\frac{15}{64}x(x+1)^{3/2}(1-x)^{3/2}+\frac{45}{128}x\sqrt{x+1}\,\sqrt{1-x}+\frac{45}{128}\sin$$

**Rubi [A]** time = 0.02, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}+\frac{9}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{3}{16}x(x+1)^{5/2}(1-x)^{5/2}+\frac{15}{64}x(x+1)^{3/2}(1-x)^{3/2}+\frac{45}{128}x\sqrt{x+1}\,\sqrt{1-x}+\frac{45}{128}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

```
[Out] (45*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (15*(1 - x)^(3/2)*x*(1 + x)^(3/2))/64
+ (3*(1 - x)^(5/2)*x*(1 + x)^(5/2))/16 + (9*(1 - x)^(7/2)*(1 + x)^(7/2))/56
 + ((1 - x)^(9/2)*(1 + x)^(7/2))/8 + (45*ArcSin[x])/128
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{9/2}(1+x)^{5/2}\,dx = \frac{1}{8}(1-x)^{9/2}(1+x)^{7/2} + \frac{9}{8}\int (1-x)^{7/2}(1+x)^{5/2}\,dx$$
$$= \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{1}{8}(1-x)^{9/2}(1+x)^{7/2} + \frac{9}{8}\int (1-x)^{5/2}(1+x)^{5/2}\,dx$$
$$= \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2} + \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{1}{8}(1-x)^{9/2}(1+x)^{7/2} + \frac{15}{16}\int (1-x)$$
$$= \frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2} + \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{1}{8}(1-x)^{9/}$$
$$= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2} + \frac{9}{56}(1-x)^{7/}$$
$$= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2} + \frac{9}{56}(1-x)^{7/}$$
$$= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2} + \frac{9}{56}(1-x)^{7/}$$

**Mathematica [A]** time = 0.06, size = 70, normalized size = 0.64

$$\frac{1}{896}\left(\sqrt{1-x^2}\,\left(112x^7-256x^6-168x^5+768x^4-210x^3-768x^2+581x+256\right)-630\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(256 + 581*x - 768*x^2 - 210*x^3 + 768*x^4 - 168*x^5 - 256*x
^6 + 112*x^7) - 630*ArcSin[Sqrt[1 - x]/Sqrt[2]])/896
```

**IntegrateAlgebraic [A]** time = 0.16, size = 187, normalized size = 1.70

$$\frac{-\frac{315(1-x)^{15/2}}{(x+1)^{15/2}}-\frac{2415(1-x)^{13/2}}{(x+1)^{13/2}}-\frac{8043(1-x)^{11/2}}{(x+1)^{11/2}}+\frac{17609(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{15159(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{8043(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{2415(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{315\sqrt{1-x}}{\sqrt{x+1}}}{448\left(\frac{1-x}{x+1}+1\right)^8}-\frac{45}{64}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

```
[Out] ((-315*(1 - x)^(15/2))/(1 + x)^(15/2) - (2415*(1 - x)^(13/2))/(1 + x)^(13/2
) - (8043*(1 - x)^(11/2))/(1 + x)^(11/2) + (17609*(1 - x)^(9/2))/(1 + x)^(9
/2) + (15159*(1 - x)^(7/2))/(1 + x)^(7/2) + (8043*(1 - x)^(5/2))/(1 + x)^(5
```

```
/2) + (2415*(1 - x)^(3/2))/(1 + x)^(3/2) + (315*Sqrt[1 - x])/Sqrt[1 + x])/(
448*(1 + (1 - x)/(1 + x))^8) - (45*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/64
```

**fricas [A]** time = 1.16, size = 72, normalized size = 0.65

$$\frac{1}{896}\left(112\,x^7-256\,x^6-168\,x^5+768\,x^4-210\,x^3-768\,x^2+581\,x+256\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{45}{64}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/896*(112*x^7 - 256*x^6 - 168*x^5 + 768*x^4 - 210*x^3 - 768*x^2 + 581*x +
256)*sqrt(x + 1)*sqrt(-x + 1) - 45/64*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

**giac [B]** time = 1.49, size = 296, normalized size = 2.69

$$\frac{1}{13440}\left(\left(2\left(\left(4\left(5\left(6\left(7\,x-50\right)(x+1)+1219\right)(x+1)-12463\right)(x+1)+64233\right)(x+1)-53963\right)(x+1)+59465\right)(x+1)-23205\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{1680}\left(\left(2\left(\left(4\left(5\left(6\,x-37\right)(x+1)+661\right)(x+1)-4551\right)(x+1)+4781\right)(x+1)-6335\right)(x+1)+2835\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{80}\left(\left(2\left(\left(4\left(5\,x-26\right)(x+1)+321\right)(x+1)-451\right)(x+1)+745\right)(x+1)-405\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{40}\left(\left(2\left(3\left(4\,x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{8}\left(\left(2\left(3\,x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\left(\left(2\,x-5\right)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\,\sqrt{x+1}(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{45}{64}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/13440*((2*((4*(5*(6*(7*x - 50)*(x + 1) + 1219)*(x + 1) - 12463)*(x + 1) +
 64233)*(x + 1) - 53963)*(x + 1) + 59465)*(x + 1) - 23205)*sqrt(x + 1)*sqrt
(-x + 1) - 1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(x +
 1) + 4781)*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) - 1/80
*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x + 1) -
405)*sqrt(x + 1)*sqrt(-x + 1) + 1/40*((2*(3*(4*x - 17)*(x + 1) + 133)*(x +
1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/8*((2*(3*x - 10)*(x +
 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*((2*x - 5)*(x + 1) +
 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(
x + 1)*sqrt(-x + 1) + 45/64*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [A]** time = 0.00, size = 141, normalized size = 1.28

$$\frac{45\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{128\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{9}{2}}\,(x+1)^{\frac{7}{2}}}{8}+\frac{9\,(-x+1)^{\frac{7}{2}}\,(x+1)^{\frac{7}{2}}}{56}+\frac{3\,(-x+1)^{\frac{5}{2}}\,(x+1)^{\frac{7}{2}}}{16}+\frac{3\,(-x+1)^{\frac{3}{2}}\,(x+1)^{\frac{7}{2}}}{16}+\frac{9\sqrt{-x+1}\,\,(x+1)^{\frac{7}{2}}}{64}-\frac{3\sqrt{-x+1}\,\,(x+1)^{\frac{5}{2}}}{64}-\frac{15\sqrt{-x+1}\,\,(x+1)^{\frac{3}{2}}}{128}-\frac{45\sqrt{-x+1}\,\sqrt{x+1}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(9/2)*(x+1)^(5/2),x)
```

```
[Out] 1/8*(-x+1)^(9/2)*(x+1)^(7/2)+9/56*(-x+1)^(7/2)*(x+1)^(7/2)+3/16*(-x+1)^(5/2
)*(x+1)^(7/2)+3/16*(-x+1)^(3/2)*(x+1)^(7/2)+9/64*(-x+1)^(1/2)*(x+1)^(7/2)-3
/64*(-x+1)^(1/2)*(x+1)^(5/2)-15/128*(-x+1)^(1/2)*(x+1)^(3/2)-45/128*(-x+1)^
(1/2)*(x+1)^(1/2)+45/128*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcs
in(x)
```

**maxima** [A] time = 2.97, size = 64, normalized size = 0.58

$$-\frac{1}{8}\left(-x^2+1\right)^{\frac{7}{2}}x+\frac{2}{7}\left(-x^2+1\right)^{\frac{7}{2}}+\frac{3}{16}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{15}{64}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{45}{128}\sqrt{-x^2+1}\,x+\frac{45}{128}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(-x^2 + 1)^(7/2)*x + 2/7*(-x^2 + 1)^(7/2) + 3/16*(-x^2 + 1)^(5/2)*x +
15/64*(-x^2 + 1)^(3/2)*x + 45/128*sqrt(-x^2 + 1)*x + 45/128*arcsin(x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2}(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(9/2)*(x + 1)^(5/2), x)
```

**sympy** [A] time = 117.57, size = 360, normalized size = 3.27

$$\begin{cases} -\frac{45 i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{64}+\frac{i(x+1)^{\frac{17}{2}}}{8\sqrt{x-1}}-\frac{79 i(x+1)^{\frac{15}{2}}}{56\sqrt{x-1}}+\frac{725 i(x+1)^{\frac{13}{2}}}{112\sqrt{x-1}}-\frac{1699 i(x+1)^{\frac{11}{2}}}{112\sqrt{x-1}}+\frac{8191 i(x+1)^{\frac{9}{2}}}{448\sqrt{x-1}}-\frac{4099 i(x+1)^{\frac{7}{2}}}{448\sqrt{x-1}}-\frac{3 i(x+1)^{\frac{5}{2}}}{128\sqrt{x-1}}-\frac{15 i(x+1)^{\frac{3}{2}}}{128\sqrt{x-1}}+\frac{45 i\sqrt{x+1}}{64\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{45 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{64}-\frac{(x+1)^{\frac{17}{2}}}{8\sqrt{1-x}}+\frac{79(x+1)^{\frac{15}{2}}}{56\sqrt{1-x}}-\frac{725(x+1)^{\frac{13}{2}}}{112\sqrt{1-x}}+\frac{1699(x+1)^{\frac{11}{2}}}{112\sqrt{1-x}}-\frac{8191(x+1)^{\frac{9}{2}}}{448\sqrt{1-x}}+\frac{4099(x+1)^{\frac{7}{2}}}{448\sqrt{1-x}}+\frac{3(x+1)^{\frac{5}{2}}}{128\sqrt{1-x}}+\frac{15(x+1)^{\frac{3}{2}}}{128\sqrt{1-x}}-\frac{45\sqrt{x+1}}{64\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-45*I*acosh(sqrt(2)*sqrt(x + 1)/2)/64 + I*(x + 1)**(17/2)/(8*sqr
t(x - 1)) - 79*I*(x + 1)**(15/2)/(56*sqrt(x - 1)) + 725*I*(x + 1)**(13/2)/(
112*sqrt(x - 1)) - 1699*I*(x + 1)**(11/2)/(112*sqrt(x - 1)) + 8191*I*(x + 1
)**(9/2)/(448*sqrt(x - 1)) - 4099*I*(x + 1)**(7/2)/(448*sqrt(x - 1)) - 3*I*
(x + 1)**(5/2)/(128*sqrt(x - 1)) - 15*I*(x + 1)**(3/2)/(128*sqrt(x - 1)) +
45*I*sqrt(x + 1)/(64*sqrt(x - 1)), Abs(x + 1)/2 > 1), (45*asin(sqrt(2)*sqrt
(x + 1)/2)/64 - (x + 1)**(17/2)/(8*sqrt(1 - x)) + 79*(x + 1)**(15/2)/(56*sq
rt(1 - x)) - 725*(x + 1)**(13/2)/(112*sqrt(1 - x)) + 1699*(x + 1)**(11/2)/(
112*sqrt(1 - x)) - 8191*(x + 1)**(9/2)/(448*sqrt(1 - x)) + 4099*(x + 1)**(7
/2)/(448*sqrt(1 - x)) + 3*(x + 1)**(5/2)/(128*sqrt(1 - x)) + 15*(x + 1)**(3
/2)/(128*sqrt(1 - x)) - 45*sqrt(x + 1)/(64*sqrt(1 - x)), True))
```

## 3.1021 $\int (1-x)^{7/2}(1+x)^{5/2}\,dx$

**Optimal**. Leaf size=90

$$\frac{1}{7}(1-x)^{7/2}(x+1)^{7/2}+\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{7}(1-x)^{7/2}(x+1)^{7/2}+\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + ((1 - x)^(7/2)*(1 + x)^(7/2))/7 + (5*Arc
Sin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{7/2}(1+x)^{5/2}\,dx = \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2} + \int (1-x)^{5/2}(1+x)^{5/2}\,dx$$
$$= \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2} + \frac{5}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx$$
$$= \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2} + \frac{5}{8}\int \sqrt{1-x}$$
$$= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1$$
$$= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1$$
$$= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1$$

**Mathematica [A]** time = 0.06, size = 66, normalized size = 0.73

$$\frac{1}{336}\sqrt{1-x^2}\left(-48x^6+56x^5+144x^4-182x^3-144x^2+231x+48\right)-\frac{5}{8}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(48 + 231*x - 144*x^2 - 182*x^3 + 144*x^4 + 56*x^5 - 48*x^6)
)/336 - (5*ArcSin[Sqrt[1 - x]/Sqrt[2]])/8
```

**IntegrateAlgebraic [A]** time = 0.14, size = 169, normalized size = 1.88

$$\frac{-\frac{105(1-x)^{13/2}}{(x+1)^{13/2}}-\frac{700(1-x)^{11/2}}{(x+1)^{11/2}}-\frac{1981(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{3072(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{1981(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{700(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{105\sqrt{1-x}}{\sqrt{x+1}}}{168\left(\frac{1-x}{x+1}+1\right)^7}-\frac{5}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

```
[Out] ((-105*(1 - x)^(13/2))/(1 + x)^(13/2) - (700*(1 - x)^(11/2))/(1 + x)^(11/2)
 - (1981*(1 - x)^(9/2))/(1 + x)^(9/2) + (3072*(1 - x)^(7/2))/(1 + x)^(7/2)
+ (1981*(1 - x)^(5/2))/(1 + x)^(5/2) + (700*(1 - x)^(3/2))/(1 + x)^(3/2) +
(105*Sqrt[1 - x])/Sqrt[1 + x])/(168*(1 + (1 - x)/(1 + x))^7) - (5*ArcTan[Sq
rt[1 - x]/Sqrt[1 + x]])/8
```

**fricas [A]** time = 1.32, size = 67, normalized size = 0.74

$$-\frac{1}{336}\left(48\,x^{6}-56\,x^{5}-144\,x^{4}+182\,x^{3}+144\,x^{2}-231\,x-48\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{8}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/336*(48*x^6 - 56*x^5 - 144*x^4 + 182*x^3 + 144*x^2 - 231*x - 48)*sqrt(x
+ 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac [B]** time = 1.14, size = 143, normalized size = 1.59

$$-\frac{1}{1680}\left(\left(2\left(\left(4\left(5\left(6x-37\right)(x+1)+661\right)(x+1)-4551\right)(x+1)+4781\right)(x+1)-6335\right)(x+1)+2835\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{40}\left(\left(2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\left(\left(2x-5\right)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{5}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(x + 1) + 4781
)*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) + 1/40*((2*(3*(4
*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x +
 1) - 1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*sq
rt(-x + 1) + 5/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [A]** time = 0.00, size = 127, normalized size = 1.41

$$\frac{5\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{16\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{7}{2}}(x+1)^{\frac{7}{2}}}{7}+\frac{(-x+1)^{\frac{5}{2}}(x+1)^{\frac{7}{2}}}{6}+\frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{7}{2}}}{6}+\frac{\sqrt{-x+1}\,(x+1)^{\frac{7}{2}}}{8}-\frac{\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{24}-\frac{5\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{48}-\frac{5\sqrt{-x+1}\,\sqrt{x+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)*(x+1)^(5/2),x)
```

```
[Out] 1/7*(-x+1)^(7/2)*(x+1)^(7/2)+1/6*(-x+1)^(5/2)*(x+1)^(7/2)+1/6*(-x+1)^(3/2)*
(x+1)^(7/2)+1/8*(-x+1)^(1/2)*(x+1)^(7/2)-1/24*(-x+1)^(1/2)*(x+1)^(5/2)-5/48
*(-x+1)^(1/2)*(x+1)^(3/2)-5/16*(-x+1)^(1/2)*(x+1)^(1/2)+5/16*((x+1)*(-x+1))
^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 3.01, size = 52, normalized size = 0.58

$$\frac{1}{7}\left(-x^{2}+1\right)^{\frac{7}{2}}+\frac{1}{6}\left(-x^{2}+1\right)^{\frac{5}{2}}x+\frac{5}{24}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{5}{16}\sqrt{-x^{2}+1}\,x+\frac{5}{16}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="maxima")
```

[Out] 1/7*(-x^2 + 1)^(7/2) + 1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x + 5
/16*sqrt(-x^2 + 1)*x + 5/16*arcsin(x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x)^(7/2)*(x + 1)^(5/2),x)

[Out] int((1 - x)^(7/2)*(x + 1)^(5/2), x)

**sympy [A]** time = 53.58, size = 321, normalized size = 3.57

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{i(x+1)^{\frac{15}{2}}}{7\sqrt{x-1}} + \frac{55i(x+1)^{\frac{13}{2}}}{42\sqrt{x-1}} - \frac{193i(x+1)^{\frac{11}{2}}}{42\sqrt{x-1}} + \frac{1237i(x+1)^{\frac{9}{2}}}{168\sqrt{x-1}} - \frac{769i(x+1)^{\frac{7}{2}}}{168\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{48\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{(x+1)^{\frac{15}{2}}}{7\sqrt{1-x}} - \frac{55(x+1)^{\frac{13}{2}}}{42\sqrt{1-x}} + \frac{193(x+1)^{\frac{11}{2}}}{42\sqrt{1-x}} - \frac{1237(x+1)^{\frac{9}{2}}}{168\sqrt{1-x}} + \frac{769(x+1)^{\frac{7}{2}}}{168\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{48\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}} - \frac{5\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)**(7/2)*(1+x)**(5/2),x)

[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 - I*(x + 1)**(15/2)/(7*sqrt(
x - 1)) + 55*I*(x + 1)**(13/2)/(42*sqrt(x - 1)) - 193*I*(x + 1)**(11/2)/(42
*sqrt(x - 1)) + 1237*I*(x + 1)**(9/2)/(168*sqrt(x - 1)) - 769*I*(x + 1)**(7
/2)/(168*sqrt(x - 1)) - I*(x + 1)**(5/2)/(48*sqrt(x - 1)) - 5*I*(x + 1)**(3
/2)/(48*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(8*sqrt(x - 1)), Abs(x + 1)/2 > 1),
(5*asin(sqrt(2)*sqrt(x + 1)/2)/8 + (x + 1)**(15/2)/(7*sqrt(1 - x)) - 55*(x
+ 1)**(13/2)/(42*sqrt(1 - x)) + 193*(x + 1)**(11/2)/(42*sqrt(1 - x)) - 1237
*(x + 1)**(9/2)/(168*sqrt(1 - x)) + 769*(x + 1)**(7/2)/(168*sqrt(1 - x)) +
(x + 1)**(5/2)/(48*sqrt(1 - x)) + 5*(x + 1)**(3/2)/(48*sqrt(1 - x)) - 5*sqr
t(x + 1)/(8*sqrt(1 - x)), True))

## 3.1022 $\int (1-x)^{5/2}(1+x)^{5/2}\,dx$

**Optimal**. Leaf size=70

$$\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {38, 41, 216}

$$\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + (5*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{5/2}(1+x)^{5/2}\,dx &= \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{5}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx \\ &= \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{5}{8}\int \sqrt{1-x}\sqrt{1+x}\,dx \\ &= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{5}{16}\int \frac{1}{\sqrt{1-x}\ldots} \\ &= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{5}{16}\int \frac{1}{\sqrt{1-x^2}\ldots} \\ &= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{5}{16}\sin^{-1}(x)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 34, normalized size = 0.49

$$\frac{1}{48}\left(x\sqrt{1-x^2}\left(8x^4-26x^2+33\right)+15\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*(1 + x)^(5/2),x]
[Out] (x*Sqrt[1 - x^2]*(33 - 26*x^2 + 8*x^4) + 15*ArcSin[x])/48
```

**IntegrateAlgebraic [B]** time = 0.12, size = 151, normalized size = 2.16

$$\frac{-\frac{15(1-x)^{11/2}}{(x+1)^{11/2}}-\frac{85(1-x)^{9/2}}{(x+1)^{9/2}}-\frac{198(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{198(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{85(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{15\sqrt{1-x}}{\sqrt{x+1}}}{24\left(\frac{1-x}{x+1}+1\right)^6}-\frac{5}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)*(1 + x)^(5/2),x]
[Out] ((-15*(1 - x)^(11/2))/(1 + x)^(11/2) - (85*(1 - x)^(9/2))/(1 + x)^(9/2) - (
198*(1 - x)^(7/2))/(1 + x)^(7/2) + (198*(1 - x)^(5/2))/(1 + x)^(5/2) + (85*
(1 - x)^(3/2))/(1 + x)^(3/2) + (15*Sqrt[1 - x])/Sqrt[1 + x])/(24*(1 + (1 -
x)/(1 + x))^6) - (5*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/8
```

**fricas [A]** time = 1.29, size = 51, normalized size = 0.73

$$\frac{1}{48}\left(8\,x^5-26\,x^3+33\,x\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{8}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^5 - 26*x^3 + 33*x)*sqrt(x + 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x)
```

**giac [B]** time = 1.31, size = 185, normalized size = 2.64

$$\frac{1}{240}\left(\left(2\left(\left(4(5x-26)(x+1)+321\right)(x+1)-451\right)(x+1)+745\right)(x+1)-405\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{120}\left(\left(2\left(3(4x-17)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{12}\left(\left(2(3x-10)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{5}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/120*((2*(3*(4*x - 17)*(x + 1) + 133)
*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 1
0)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x - 5)*(x
 + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1)
+ sqrt(x + 1)*sqrt(-x + 1) + 5/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [B]** time = 0.00, size = 113, normalized size = 1.61

$$\frac{5\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{16\sqrt{x+1}\ \sqrt{-x+1}}+\frac{(-x+1)^{\frac{5}{2}}(x+1)^{\frac{7}{2}}}{6}+\frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{7}{2}}}{6}+\frac{\sqrt{-x+1}\ (x+1)^{\frac{7}{2}}}{8}-\frac{\sqrt{-x+1}\ (x+1)^{\frac{5}{2}}}{24}-\frac{5\sqrt{-x+1}\ (x+1)^{\frac{3}{2}}}{48}-\frac{5\sqrt{-x+1}\ \sqrt{x+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)*(x+1)^(5/2),x)
```

```
[Out] 1/6*(-x+1)^(5/2)*(x+1)^(7/2)+1/6*(-x+1)^(3/2)*(x+1)^(7/2)+1/8*(-x+1)^(1/2)*
(x+1)^(7/2)-1/24*(-x+1)^(1/2)*(x+1)^(5/2)-5/48*(-x+1)^(1/2)*(x+1)^(3/2)-5/1
6*(-x+1)^(1/2)*(x+1)^(1/2)+5/16*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/
2)*arcsin(x)
```

**maxima [A]** time = 3.10, size = 41, normalized size = 0.59

$$\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{5}{24}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{5}{16}\sqrt{-x^2+1}\,x+\frac{5}{16}\ \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x + 5/16*sqrt(-x^2 + 1)*x +
5/16*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(5/2), x)
```

**sympy [B]** time = 25.76, size = 286, normalized size = 4.09

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{8} + \frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}} - \frac{7i(x+1)^{\frac{11}{2}}}{6\sqrt{x-1}} + \frac{67i(x+1)^{\frac{9}{2}}}{24\sqrt{x-1}} - \frac{55i(x+1)^{\frac{7}{2}}}{24\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{48\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{8} - \frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}} + \frac{7(x+1)^{\frac{11}{2}}}{6\sqrt{1-x}} - \frac{67(x+1)^{\frac{9}{2}}}{24\sqrt{1-x}} + \frac{55(x+1)^{\frac{7}{2}}}{24\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{48\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}} - \frac{5\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(13/2)/(6*sqrt(
x - 1)) - 7*I*(x + 1)**(11/2)/(6*sqrt(x - 1)) + 67*I*(x + 1)**(9/2)/(24*sqr
t(x - 1)) - 55*I*(x + 1)**(7/2)/(24*sqrt(x - 1)) - I*(x + 1)**(5/2)/(48*sqr
t(x - 1)) - 5*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(8*sqrt(x
 - 1)), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x + 1)/2)/8 - (x + 1)**(13/
2)/(6*sqrt(1 - x)) + 7*(x + 1)**(11/2)/(6*sqrt(1 - x)) - 67*(x + 1)**(9/2)/
(24*sqrt(1 - x)) + 55*(x + 1)**(7/2)/(24*sqrt(1 - x)) + (x + 1)**(5/2)/(48*
sqrt(1 - x)) + 5*(x + 1)**(3/2)/(48*sqrt(1 - x)) - 5*sqrt(x + 1)/(8*sqrt(1
- x)), True))
```

## 3.1023 $\int (1-x)^{3/2}(1+x)^{5/2}\,dx$

**Optimal**. Leaf size=69

$$-\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$-\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 - ((1 -
 x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{3/2}(1+x)^{5/2}\,dx = -\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \int (1-x)^{3/2}(1+x)^{3/2}\,dx$$
$$= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} - \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{4}\int \sqrt{1-x}\sqrt{1+x}\,dx$$
$$= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} - \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}$$
$$= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} - \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} - \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{8}\sin^{-1}(x)$$

**Mathematica [A]** time = 0.04, size = 55, normalized size = 0.80

$$\frac{1}{40}\left(\sqrt{1-x^2}\left(-8x^4-10x^3+16x^2+25x-8\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(-8 + 25*x + 16*x^2 - 10*x^3 - 8*x^4) - 30*ArcSin[Sqrt[1 - x
]/Sqrt[2]])/40
```

**IntegrateAlgebraic [A]** time = 0.10, size = 133, normalized size = 1.93

$$\frac{-\frac{15(1-x)^{9/2}}{(x+1)^{9/2}}-\frac{70(1-x)^{7/2}}{(x+1)^{7/2}}-\frac{128(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{70(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{15\sqrt{1-x}}{\sqrt{x+1}}}{20\left(\frac{1-x}{x+1}+1\right)^5}-\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] ((-15*(1 - x)^(9/2))/(1 + x)^(9/2) - (70*(1 - x)^(7/2))/(1 + x)^(7/2) - (12
8*(1 - x)^(5/2))/(1 + x)^(5/2) + (70*(1 - x)^(3/2))/(1 + x)^(3/2) + (15*Sqr
t[1 - x])/Sqrt[1 + x])/(20*(1 + (1 - x)/(1 + x))^5) - (3*ArcTan[Sqrt[1 - x]
/Sqrt[1 + x]])/4
```

**fricas [A]** time = 1.06, size = 57, normalized size = 0.83

$$-\frac{1}{40}\left(8x^4+10x^3-16x^2-25x+8\right)\sqrt{x+1}\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/40*(8*x^4 + 10*x^3 - 16*x^2 - 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*a
rctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac** [B] time = 1.18, size = 114, normalized size = 1.65

$$-\frac{1}{120}\left((2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{12}\left((2\left(3x-10\right)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}+\sqrt{x+1}(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{3}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(
x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(
x + 1)*sqrt(-x + 1) + sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-
x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple** [A] time = 0.00, size = 99, normalized size = 1.43

$$\frac{3\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{7}{2}}}{5}+\frac{3\sqrt{-x+1}\,(x+1)^{\frac{7}{2}}}{20}-\frac{\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{20}-\frac{\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{8}-\frac{3\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)*(x+1)^(5/2),x)
```

```
[Out] 1/5*(-x+1)^(3/2)*(x+1)^(7/2)+3/20*(-x+1)^(1/2)*(x+1)^(7/2)-1/20*(-x+1)^(1/2
)*(x+1)^(5/2)-1/8*(-x+1)^(1/2)*(x+1)^(3/2)-3/8*(-x+1)^(1/2)*(x+1)^(1/2)+3/8
*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima** [A] time = 3.05, size = 40, normalized size = 0.58

$$-\frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}\,x+\frac{3}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8
*arcsin(x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{3/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(3/2)*(x + 1)^(5/2), x)
```

**sympy [B]** time = 16.53, size = 246, normalized size = 3.57

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}} + \frac{19i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}} - \frac{23i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}} - \frac{19(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}} + \frac{23(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}} - \frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) + 19*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 23*I*(x + 1)**(7/2)/(20*sq
rt(x - 1)) - I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8*sqrt(x
 - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), (3*asin(sqrt(2
)*sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 19*(x + 1)**(9/2)/(2
0*sqrt(1 - x)) + 23*(x + 1)**(7/2)/(20*sqrt(1 - x)) + (x + 1)**(5/2)/(40*sq
rt(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1 - x))
, True))
```

## 3.1024 $\int \sqrt{1-x}\,(1+x)^{5/2}\,dx$

**Optimal**. Leaf size=68

$$-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}-\frac{5}{12}(1-x)^{3/2}(x+1)^{3/2}+\frac{5}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{8}\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}-\frac{5}{12}(1-x)^{3/2}(x+1)^{3/2}+\frac{5}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 - (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 - ((1
- x)^(3/2)*(1 + x)^(5/2))/4 + (5*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1-x}(1+x)^{5/2}\,dx &= -\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{4}\int \sqrt{1-x}(1+x)^{3/2}\,dx\\
&= -\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{4}\int \sqrt{1-x}\sqrt{1+x}\,dx\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\sin^{-1}(x)
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 50, normalized size = 0.74

$$\frac{1}{24}\left(\sqrt{1-x^2}\left(6x^3+16x^2+9x-16\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(-16 + 9*x + 16*x^2 + 6*x^3) - 30*ArcSin[Sqrt[1 - x]/Sqrt[2]
])/24
```

**IntegrateAlgebraic [A]** time = 0.08, size = 100, normalized size = 1.47

$$-\frac{\sqrt{1-x}\left(\frac{15(1-x)^3}{(x+1)^3}+\frac{55(1-x)^2}{(x+1)^2}+\frac{73(1-x)}{x+1}-15\right)}{12\sqrt{x+1}\left(\frac{1-x}{x+1}+1\right)^4}-\frac{5}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] -1/12*(Sqrt[1 - x]*(-15 + (15*(1 - x)^3)/(1 + x)^3 + (55*(1 - x)^2)/(1 + x)
^2 + (73*(1 - x))/(1 + x)))/(Sqrt[1 + x]*(1 + (1 - x)/(1 + x))^4) - (5*ArcT
an[Sqrt[1 - x]/Sqrt[1 + x]])/4
```

**fricas [A]** time = 1.13, size = 52, normalized size = 0.76

$$\frac{1}{24}\left(6x^3+16x^2+9x-16\right)\sqrt{x+1}\sqrt{-x+1}-\frac{5}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 + 16*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sqr
t(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac [B]** time = 1.05, size = 101, normalized size = 1.49

$$\frac{1}{24}\left(\left(2\left(3x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\left(\left(2x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\sqrt{-x+1}+\frac{3}{2}\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{5}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) +
1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 3/2*sqrt(x + 1)*(x -
 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5/4*arcsin(1/2*sqrt(2)*sqrt(x
 + 1))
```

**maple [A]** time = 0.00, size = 85, normalized size = 1.25

$$\frac{5\sqrt{\left(x+1\right)\left(-x+1\right)}\,\arcsin\left(x\right)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{\sqrt{-x+1}\,\left(x+1\right)^{\frac{7}{2}}}{4}-\frac{\sqrt{-x+1}\,\left(x+1\right)^{\frac{5}{2}}}{12}-\frac{5\sqrt{-x+1}\,\left(x+1\right)^{\frac{3}{2}}}{24}-\frac{5\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)*(x+1)^(5/2),x)
```

```
[Out] 1/4*(-x+1)^(1/2)*(x+1)^(7/2)-1/12*(-x+1)^(1/2)*(x+1)^(5/2)-5/24*(-x+1)^(1/2
)*(x+1)^(3/2)-5/8*(-x+1)^(1/2)*(x+1)^(1/2)+5/8*((x+1)*(-x+1))^(1/2)/(x+1)^(
1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 3.06, size = 40, normalized size = 0.59

$$-\frac{1}{4}\left(-x^{2}+1\right)^{\frac{3}{2}}x-\frac{2}{3}\left(-x^{2}+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^{2}+1}\,x+\frac{5}{8}\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)*x - 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/8
*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{1-x}\,\left(x+1\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(1/2)*(x + 1)^(5/2), x)
```

**sympy [A]** time = 9.89, size = 214, normalized size = 3.15

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}} - \frac{7i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}} + \frac{7(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}} - \frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) - 7*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) - I*(x + 1)**(5/2)/(24*sqrt(x
- 1)) - 5*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*sqrt(x - 1
)), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(9/2)/(4
*sqrt(1 - x)) + 7*(x + 1)**(7/2)/(12*sqrt(1 - x)) + (x + 1)**(5/2)/(24*sqrt
(1 - x)) + 5*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x + 1)/(4*sqrt(1 - x)
), True))
```

## 3.1025 $\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx$

**Optimal**. Leaf size=67

$$-\frac{1}{3}\sqrt{1-x}\,(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}\,(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\,\sqrt{x+1}\,+\frac{5}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$-\frac{1}{3}\sqrt{1-x}\,(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}\,(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\,\sqrt{x+1}\,+\frac{5}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/Sqrt[1 - x],x]
```

```
[Out] (-5*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/6 - (Sqrt[1
 - x]*(1 + x)^(5/2))/3 + (5*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx &= -\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{3}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx \\
&= -\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\sin^{-1}(x)
\end{aligned}
$$

**Mathematica** [A] time = 0.02, size = 44, normalized size = 0.66

$$
-\frac{1}{6}\sqrt{1-x^2}\,\left(2x^2+9x+22\right)-5\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/Sqrt[1 - x],x]
[Out] -1/6*(Sqrt[1 - x^2]*(22 + 9*x + 2*x^2)) - 5*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

**IntegrateAlgebraic** [A] time = 0.07, size = 84, normalized size = 1.25

$$
-\frac{\sqrt{1-x}\,\left(\frac{15(1-x)^2}{(x+1)^2}+\frac{40(1-x)}{x+1}+33\right)}{3\sqrt{x+1}\,\left(\frac{1-x}{x+1}+1\right)^3}-5\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/Sqrt[1 - x],x]
[Out] -1/3*(Sqrt[1 - x]*(33 + (15*(1 - x)^2)/(1 + x)^2 + (40*(1 - x))/(1 + x)))/(
Sqrt[1 + x]*(1 + (1 - x)/(1 + x))^3) - 5*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

**fricas** [A] time = 1.50, size = 47, normalized size = 0.70

$$
-\frac{1}{6}\left(2\,x^2+9\,x+22\right)\sqrt{x+1}\,\sqrt{-x+1}\,-5\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}\,-1}{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^2 + 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*sq
rt(-x + 1) - 1)/x)
```

**giac [A]** time = 1.05, size = 39, normalized size = 0.58

$$-\frac{1}{6}\left((2x+7)(x+1)+15\right)\sqrt{x+1}\,\sqrt{-x+1}+5\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*x + 7)*(x + 1) + 15)*sqrt(x + 1)*sqrt(-x + 1) + 5*arcsin(1/2*sqrt(
2)*sqrt(x + 1))
```

**maple [A]** time = 0.00, size = 71, normalized size = 1.06

$$\frac{5\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{2\sqrt{x+1}\,\sqrt{-x+1}}-\frac{\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{3}-\frac{5\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{6}-\frac{5\sqrt{-x+1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(1/2),x)
```

```
[Out] -1/3*(-x+1)^(1/2)*(x+1)^(5/2)-5/6*(-x+1)^(1/2)*(x+1)^(3/2)-5/2*(-x+1)^(1/2)
*(x+1)^(1/2)+5/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 2.93, size = 42, normalized size = 0.63

$$-\frac{1}{3}\,\sqrt{-x^2+1}\,x^2-\frac{3}{2}\,\sqrt{-x^2+1}\,x-\frac{11}{3}\,\sqrt{-x^2+1}+\frac{5}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x - 11/3*sqrt(-x^2 + 1) + 5/2*
arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^{5/2}}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(1/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(1/2), x)
```

**sympy [A]** time = 7.50, size = 172, normalized size = 2.57

$$\begin{cases} -5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ 5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) - I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(6*sqrt(x - 1)
) + 5*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x
+ 1)/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) + (x + 1)**(5/2)/(6*sqrt(1 - x)) +
 5*(x + 1)**(3/2)/(6*sqrt(1 - x)) - 5*sqrt(x + 1)/sqrt(1 - x), True))
```

## 3.1026 $\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx$

**Optimal**. Leaf size=65

$$\frac{2(x+1)^{5/2}}{\sqrt{1-x}}+\frac{5}{2}\sqrt{1-x}\,(x+1)^{3/2}+\frac{15}{2}\sqrt{1-x}\,\sqrt{x+1}-\frac{15}{2}\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$\frac{2(x+1)^{5/2}}{\sqrt{1-x}}+\frac{5}{2}\sqrt{1-x}\,(x+1)^{3/2}+\frac{15}{2}\sqrt{1-x}\,\sqrt{x+1}-\frac{15}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(3/2),x]
```

```
[Out] (15*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*Sqrt[1 - x]*(1 + x)^(3/2))/2 + (2*(1 + 
x)^(5/2))/Sqrt[1 - x] - (15*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, 
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n 
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx &= \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - 5\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx \\
&= \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\sin^{-1}(x)
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 35, normalized size = 0.54

$$
\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{1-x}{2}\right)}{\sqrt{1-x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(3/2),x]
[Out] (8*Sqrt[2]*Hypergeometric2F1[-5/2, -1/2, 1/2, (1 - x)/2])/Sqrt[1 - x]
```

**IntegrateAlgebraic [A]** time = 0.08, size = 81, normalized size = 1.25

$$
\frac{\sqrt{x+1}\left(\frac{15(1-x)^2}{(x+1)^2}+\frac{25(1-x)}{x+1}+8\right)}{\sqrt{1-x}\left(\frac{1-x}{x+1}+1\right)^2}+15\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(3/2),x]
```

```
[Out] (Sqrt[1 + x]*(8 + (15*(1 - x)^2)/(1 + x)^2 + (25*(1 - x))/(1 + x)))/(Sqrt[1
 - x]*(1 + (1 - x)/(1 + x))^2) + 15*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

**fricas** [A] time = 1.11, size = 58, normalized size = 0.89

$$\frac{\left(x^2+7x-24\right)\sqrt{x+1}\,\sqrt{-x+1}+30\,(x-1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+24\,x-24}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x - 1)*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x) + 24*x - 24)/(x - 1)
```

**giac** [A] time = 1.01, size = 42, normalized size = 0.65

$$\frac{((x+6)(x+1)-30)\sqrt{x+1}\,\sqrt{-x+1}}{2\,(x-1)}-15\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((x + 6)*(x + 1) - 30)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 15*arcsin(1/2
*sqrt(2)*sqrt(x + 1))
```

**maple** [A] time = 0.02, size = 77, normalized size = 1.18

$$-\frac{15\sqrt{(x+1)\,(-x+1)}\,\arcsin(x)}{2\sqrt{x+1}\,\sqrt{-x+1}}-\frac{\left(x^3+8x^2-17x-24\right)\sqrt{(x+1)\,(-x+1)}}{2\sqrt{-\,(x+1)\,(x-1)}\,\sqrt{-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(3/2),x)
```

```
[Out] -1/2*(x^3+8*x^2-17*x-24)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(
1/2)/(x+1)^(1/2)-15/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(
x)
```

**maxima** [A] time = 2.97, size = 56, normalized size = 0.86

$$-\frac{x^3}{2\,\sqrt{-x^2+1}}-\frac{4\,x^2}{\sqrt{-x^2+1}}+\frac{17\,x}{2\,\sqrt{-x^2+1}}+\frac{12}{\sqrt{-x^2+1}}-\frac{15}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^3/sqrt(-x^2 + 1) - 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) + 12
/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(3/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(3/2), x)
```

**sympy [A]** time = 7.76, size = 139, normalized size = 2.14

$$\begin{cases} 15i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} + \frac{5i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} - \frac{15i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -15\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} - \frac{5(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} + \frac{15\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) + 5*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 15*I*sqrt(x + 1)/sqrt(x - 1), A
bs(x + 1)/2 > 1), (-15*asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqrt
(1 - x)) - 5*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 15*sqrt(x + 1)/sqrt(1 - x), T
rue))
```

## 3.1027 $\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}} \, dx$

**Optimal**. Leaf size=63

$$\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}}-\frac{10(x+1)^{3/2}}{3\sqrt{1-x}}-5\sqrt{1-x}\,\sqrt{x+1}\,+5\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}}-\frac{10(x+1)^{3/2}}{3\sqrt{1-x}}-5\sqrt{1-x}\,\sqrt{x+1}\,+5\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

```
[Out] -5*Sqrt[1 - x]*Sqrt[1 + x] - (10*(1 + x)^(3/2))/(3*Sqrt[1 - x]) + (2*(1 + x
)^(5/2))/(3*(1 - x)^(3/2)) + 5*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx &= \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} - \frac{5}{3}\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx \\
&= -\frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= -5\sqrt{1-x}\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\
&= -5\sqrt{1-x}\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -5\sqrt{1-x}\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\sin^{-1}(x)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 37, normalized size = 0.59

$$\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{1-x}{2}\right)}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

```
[Out] (8*Sqrt[2]*Hypergeometric2F1[-5/2, -3/2, -1/2, (1 - x)/2])/(3*(1 - x)^(3/2)
)
```

**IntegrateAlgebraic [C]** time = 0.18, size = 73, normalized size = 1.16

$$\frac{\sqrt{1-x}\left(-3(x+1)^{5/2}+40(x+1)^{3/2}-60\sqrt{x+1}\right)}{3(x-1)^2}+10i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(-60*Sqrt[1 + x] + 40*(1 + x)^(3/2) - 3*(1 + x)^(5/2)))/(3*(-1
 + x)^2) + (10*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

**fricas** [A] time = 0.71, size = 75, normalized size = 1.19

$$-\frac{23\,x^2+\left(3\,x^2-34\,x+23\right)\sqrt{x+1}\,\sqrt{-x+1}+30\left(x^2-2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)-46\,x+23}{3\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(23*x^2 + (3*x^2 - 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x^2 - 2*x
 + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 46*x + 23)/(x^2 - 2*x + 1)
```

**giac** [A] time = 0.98, size = 44, normalized size = 0.70

$$-\frac{((3\,x-37)(x+1)+60)\sqrt{x+1}\,\sqrt{-x+1}}{3\,(x-1)^2}+10\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((3*x - 37)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 10*arcs
in(1/2*sqrt(2)*sqrt(x + 1))
```

**maple** [A] time = 0.02, size = 84, normalized size = 1.33

$$\frac{5\sqrt{(x+1)\,(-x+1)}\;\arcsin(x)}{\sqrt{x+1}\;\sqrt{-x+1}}+\frac{\left(3x^3-31x^2-11x+23\right)\sqrt{(x+1)\,(-x+1)}}{3\,(x-1)\,\sqrt{-\,(x+1)\,(x-1)}\;\sqrt{-x+1}\;\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(5/2),x)
```

```
[Out] 1/3*(3*x^3-31*x^2-11*x+23)/(x-1)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/
(-x+1)^(1/2)/(x+1)^(1/2)+5*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*ar
csin(x)
```

**maxima** [B] time = 2.97, size = 99, normalized size = 1.57

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{x^4-4\,x^3+6\,x^2-4\,x+1}-\frac{5\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3-3\,x^2+3\,x-1\right)}+\frac{10\,\sqrt{-x^2+1}}{3\left(x^2-2\,x+1\right)}+\frac{35\,\sqrt{-x^2+1}}{3\,(x-1)}+5\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="maxima")
```

[Out] -(-x^2 + 1)^(5/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(x
^3 - 3*x^2 + 3*x - 1) + 10/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 35/3*sqrt(-x^
2 + 1)/(x - 1) + 5*arcsin(x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)^(5/2)/(1 - x)^(5/2),x)

[Out] int((x + 1)^(5/2)/(1 - x)^(5/2), x)

**sympy [B]** time = 7.47, size = 576, normalized size = 9.14

$$\begin{cases} \frac{30i\sqrt{x-1}(x+1)^{\frac{27}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}} - \frac{15\pi\sqrt{x-1}(x+1)^{\frac{27}{2}}}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}} - \frac{60i\sqrt{x-1}(x+1)^{\frac{25}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}} + \frac{30\pi\sqrt{x-1}(x+1)^{\frac{25}{2}}}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}} + \frac{3i(x+1)^{15}}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}} - \frac{40i(x+1)^{14}}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}} + \frac{60i(x+1)^{13}}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{30\sqrt{1-x}(x+1)^{\frac{27}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}} - \frac{60\sqrt{1-x}(x+1)^{\frac{25}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}} + \frac{3(x+1)^{15}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}} - \frac{40(x+1)^{14}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}} + \frac{60(x+1)^{13}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)**(5/2)/(1-x)**(5/2),x)

[Out] Piecewise((30*I*sqrt(x - 1)*(x + 1)**(27/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(-
3*sqrt(x - 1)*(x + 1)**(27/2) + 6*sqrt(x - 1)*(x + 1)**(25/2)) - 15*pi*sqrt
(x - 1)*(x + 1)**(27/2)/(-3*sqrt(x - 1)*(x + 1)**(27/2) + 6*sqrt(x - 1)*(x
+ 1)**(25/2)) - 60*I*sqrt(x - 1)*(x + 1)**(25/2)*acosh(sqrt(2)*sqrt(x + 1)/
2)/(-3*sqrt(x - 1)*(x + 1)**(27/2) + 6*sqrt(x - 1)*(x + 1)**(25/2)) + 30*pi
*sqrt(x - 1)*(x + 1)**(25/2)/(-3*sqrt(x - 1)*(x + 1)**(27/2) + 6*sqrt(x - 1
)*(x + 1)**(25/2)) + 3*I*(x + 1)**15/(-3*sqrt(x - 1)*(x + 1)**(27/2) + 6*sq
rt(x - 1)*(x + 1)**(25/2)) - 40*I*(x + 1)**14/(-3*sqrt(x - 1)*(x + 1)**(27/
2) + 6*sqrt(x - 1)*(x + 1)**(25/2)) + 60*I*(x + 1)**13/(-3*sqrt(x - 1)*(x +
 1)**(27/2) + 6*sqrt(x - 1)*(x + 1)**(25/2)), Abs(x + 1)/2 > 1), (30*sqrt(1
 - x)*(x + 1)**(27/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(
27/2) - 6*sqrt(1 - x)*(x + 1)**(25/2)) - 60*sqrt(1 - x)*(x + 1)**(25/2)*asi
n(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x)*(x
+ 1)**(25/2)) + 3*(x + 1)**15/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x
)*(x + 1)**(25/2)) - 40*(x + 1)**14/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt
(1 - x)*(x + 1)**(25/2)) + 60*(x + 1)**13/(3*sqrt(1 - x)*(x + 1)**(27/2) -
6*sqrt(1 - x)*(x + 1)**(25/2)), True))

## 3.1028 $\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}} \, dx$

**Optimal**. Leaf size=63

$$\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + (2*(1 +
 x)^(5/2))/(5*(1 - x)^(5/2)) - ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx &= \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx\\ &= -\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} + \int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx\\ &= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \sin^{-1}(x)\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 37, normalized size = 0.59

$$\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{1-x}{2}\right)}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (8*Sqrt[2]*Hypergeometric2F1[-5/2, -5/2, -3/2, (1 - x)/2])/(5*(1 - x)^(5/2)
)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 69, normalized size = 1.10

$$\frac{2\left(\frac{15(1-x)^2}{(x+1)^2}-\frac{5(1-x)}{x+1}+3\right)(x+1)^{5/2}}{15(1-x)^{5/2}}+2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (2*(1 + x)^(5/2)*(3 + (15*(1 - x)^2)/(1 + x)^2 - (5*(1 - x))/(1 + x)))/(15*
(1 - x)^(5/2)) + 2*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

**fricas [A]** time = 1.18, size = 91, normalized size = 1.44

$$\frac{2\left(13\,x^3-39\,x^2-\left(23\,x^2-24\,x+13\right)\sqrt{x+1}\,\sqrt{-x+1}+15\left(x^3-3\,x^2+3\,x-1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+39\,x-13\right)}{15\left(x^3-3\,x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(13*x^3 - 39*x^2 - (23*x^2 - 24*x + 13)*sqrt(x + 1)*sqrt(-x + 1) + 15*
(x^3 - 3*x^2 + 3*x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 39*x - 1
3)/(x^3 - 3*x^2 + 3*x - 1)
```

**giac [A]** time = 0.98, size = 44, normalized size = 0.70

$$-\frac{2\,((23\,x-47)(x+1)+60)\sqrt{x+1}\,\sqrt{-x+1}}{15\,(x-1)^3}-2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*((23*x - 47)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3 - 2*arc
sin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [A]** time = 0.02, size = 84, normalized size = 1.33

$$-\frac{\sqrt{(x+1)\,(-x+1)}\;\arcsin(x)}{\sqrt{x+1}\;\sqrt{-x+1}}+\frac{2\left(23x^3-x^2-11x+13\right)\sqrt{(x+1)\,(-x+1)}}{15\,(x-1)^2\,\sqrt{-\,(x+1)\,(x-1)}\;\sqrt{-x+1}\;\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(7/2),x)
```

```
[Out] 2/15*(23*x^3-x^2-11*x+13)/(x-1)^2/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)
/(-x+1)^(1/2)/(x+1)^(1/2)-((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arc
sin(x)
```

**maxima [B]** time = 3.07, size = 160, normalized size = 2.54

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{5\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^4-4\,x^3+6\,x^2-4\,x+1}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3-3\,x^2+3\,x-1\right)}+\frac{6\,\sqrt{-x^2+1}}{5\left(x^3-3\,x^2+3\,x-1\right)}-\frac{7\,\sqrt{-x^2+1}}{15\left(x^2-2\,x+1\right)}-\frac{38\,\sqrt{-x^2+1}}{15\,(x-1)}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + (-x^2 + 1
)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x^2
 + 3*x - 1) + 6/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 7/15*sqrt(-x^2 +
 1)/(x^2 - 2*x + 1) - 38/15*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(7/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(7/2), x)
```

**sympy [B]** time = 11.21, size = 1608, normalized size = 25.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((30*I*sqrt(x - 1)*(x + 1)**(35/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(1
5*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x
 - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 15*pi*sqrt(x - 1
)*(x + 1)**(35/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)*
*(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)
) - 180*I*sqrt(x - 1)*(x + 1)**(33/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt
(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*
(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 90*pi*sqrt(x - 1)*(x +
 1)**(33/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2
) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 36
0*I*sqrt(x - 1)*(x + 1)**(31/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(x - 1
)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*(x + 1
)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 180*pi*sqrt(x - 1)*(x + 1)**
(31/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 1
80*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 240*I*s
qrt(x - 1)*(x + 1)**(29/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(x - 1)*(x
+ 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*(x + 1)**(3
1/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 120*pi*sqrt(x - 1)*(x + 1)**(29/2
)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sq
rt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 46*I*(x + 1)
**18/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180
*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 232*I*(x
+ 1)**17/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) +
 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 400*I
*(x + 1)**16/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/
2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 2
40*I*(x + 1)**15/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**
(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2))
```

```
, Abs(x + 1)/2 > 1), (-30*sqrt(1 - x)*(x + 1)**(35/2)*asin(sqrt(2)*sqrt(x +
 1)/2)/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 1
80*sqrt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 180*sqr
t(1 - x)*(x + 1)**(33/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(1 - x)*(x + 1
)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 - x)*(x + 1)**(31/2
) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 360*sqrt(1 - x)*(x + 1)**(31/2)*asin
(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x
 + 1)**(33/2) + 180*sqrt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**
(29/2)) + 240*sqrt(1 - x)*(x + 1)**(29/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(15*s
qrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 -
x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 46*(x + 1)**18/(15*
sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 -
 x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 232*(x + 1)**17/(1
5*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1
 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 400*(x + 1)**16/
(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt
(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 240*(x + 1)**1
5/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sq
rt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)), True))
```

## 3.1029 $\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}} dx$

**Optimal**. Leaf size=20

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

**Rubi [A]** time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(9/2),x]
[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}} dx = \frac{(1+x)^{7/2}}{7(1-x)^{7/2}}$$

**Mathematica [A]** time = 0.01, size = 20, normalized size = 1.00

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(9/2),x]
[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 20, normalized size = 1.00

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

**fricas [B]** time = 1.30, size = 66, normalized size = 3.30

$$\frac{x^4-4x^3+6x^2+\left(x^3+3x^2+3x+1\right)\sqrt{x+1}\sqrt{-x+1}-4x+1}{7\left(x^4-4x^3+6x^2-4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7*(x^4 - 4*x^3 + 6*x^2 + (x^3 + 3*x^2 + 3*x + 1)*sqrt(x + 1)*sqrt(-x + 1)
 - 4*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

**giac [A]** time = 1.11, size = 19, normalized size = 0.95

$$\frac{(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{7(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^4
```

**maple [A]** time = 0.00, size = 15, normalized size = 0.75

$$\frac{(x+1)^{\frac{7}{2}}}{7(-x+1)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(9/2),x)
```

```
[Out] 1/7*(x+1)^(7/2)/(-x+1)^(7/2)
```

**maxima [B]** time = 1.36, size = 171, normalized size = 8.55

$$\frac{\left(-x^{2}+1\right)^{\frac{5}{2}}}{x^{6}-6 x^{5}+15 x^{4}-20 x^{3}+15 x^{2}-6 x+1}+\frac{5\left(-x^{2}+1\right)^{\frac{3}{2}}}{2\left(x^{5}-5 x^{4}+10 x^{3}-10 x^{2}+5 x-1\right)}+\frac{15 \sqrt{-x^{2}+1}}{7\left(x^{4}-4 x^{3}+6 x^{2}-4 x+1\right)}+\frac{3 \sqrt{-x^{2}+1}}{14\left(x^{3}-3 x^{2}+3 x-1\right)}-\frac{\sqrt{-x^{2}+1}}{7\left(x^{2}-2 x+1\right)}+\frac{\sqrt{-x^{2}+1}}{7(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(5/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/2*(
-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 15/7*sqrt(-x^2
+ 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 3/14*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3
*x - 1) - 1/7*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/7*sqrt(-x^2 + 1)/(x - 1)
```

**mupad [B]** time = 0.28, size = 64, normalized size = 3.20

$$\frac{\sqrt{1-x}\,\left(\frac{3\,x\,\sqrt{x+1}}{7}+\frac{\sqrt{x+1}}{7}+\frac{3\,x^2\,\sqrt{x+1}}{7}+\frac{x^3\,\sqrt{x+1}}{7}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((3*x*(x + 1)^(1/2))/7 + (x + 1)^(1/2)/7 + (3*x^2*(x + 1)^(1
/2))/7 + (x^3*(x + 1)^(1/2))/7))/(6*x^2 - 4*x - 4*x^3 + x^4 + 1)
```

**sympy [B]** time = 19.49, size = 116, normalized size = 5.80

$$\begin{cases} \frac{i(x+1)^{\frac{7}{2}}}{7\sqrt{x-1}\,(x+1)^3-42\sqrt{x-1}\,(x+1)^2+84\sqrt{x-1}\,(x+1)-56\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -\frac{(x+1)^{\frac{7}{2}}}{7\sqrt{1-x}\,(x+1)^3-42\sqrt{1-x}\,(x+1)^2+84\sqrt{1-x}\,(x+1)-56\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(7/2)/(7*sqrt(x - 1)*(x + 1)**3 - 42*sqrt(x - 1)*(x +
 1)**2 + 84*sqrt(x - 1)*(x + 1) - 56*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-(x
+ 1)**(7/2)/(7*sqrt(1 - x)*(x + 1)**3 - 42*sqrt(1 - x)*(x + 1)**2 + 84*sqrt
(1 - x)*(x + 1) - 56*sqrt(1 - x)), True))
```

## 3.1030 $\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}} dx$

**Optimal**. Leaf size=41

$$\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(11/2),x]
```

```
[Out] (1 + x)^(7/2)/(9*(1 - x)^(9/2)) + (1 + x)^(7/2)/(63*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}} dx &= \frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{1}{9}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}} dx\\ &= \frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{7/2}}{63(1-x)^{7/2}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.56

$$-\frac{(x-8)(x+1)^{7/2}}{63(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(11/2),x]
[Out] -1/63*((-8 + x)*(1 + x)^(7/2))/(1 - x)^(9/2)
```

**IntegrateAlgebraic** [A] time = 0.07, size = 34, normalized size = 0.83

$$\frac{(x+1)^{9/2}\left(\frac{9(1-x)}{x+1}+7\right)}{126(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(11/2),x]
[Out] ((1 + x)^(9/2)*(7 + (9*(1 - x))/(1 + x)))/(126*(1 - x)^(9/2))
```

**fricas** [B] time = 1.06, size = 83, normalized size = 2.02

$$\frac{8\,x^5-40\,x^4+80\,x^3-80\,x^2+\left(x^4-5\,x^3-21\,x^2-23\,x-8\right)\sqrt{x+1}\,\sqrt{-x+1}\,+40\,x-8}{63\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="fricas")
[Out] 1/63*(8*x^5 - 40*x^4 + 80*x^3 - 80*x^2 + (x^4 - 5*x^3 - 21*x^2 - 23*x - 8)*
sqrt(x + 1)*sqrt(-x + 1) + 40*x - 8)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x -
 1)
```

**giac** [A] time = 1.23, size = 22, normalized size = 0.54

$$\frac{(x+1)^{\frac{7}{2}}(x-8)\sqrt{-x+1}}{63\,(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="giac")
[Out] 1/63*(x + 1)^(7/2)*(x - 8)*sqrt(-x + 1)/(x - 1)^5
```

**maple [A]** time = 0.00, size = 18, normalized size = 0.44

$$-\frac{(x+1)^{\frac{7}{2}}(x-8)}{63(-x+1)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(11/2),x)
```

```
[Out] -1/63*(x+1)^(7/2)*(x-8)/(-x+1)^(9/2)
```

**maxima [B]** time = 1.40, size = 218, normalized size = 5.32

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{2(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1)}-\frac{5(-x^2+1)^{\frac{3}{2}}}{6(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)}-\frac{5\sqrt{-x^2+1}}{9(x^5-5x^4+10x^3-10x^2+5x-1)}-\frac{5\sqrt{-x^2+1}}{126(x^4-4x^3+6x^2-4x+1)}+\frac{\sqrt{-x^2+1}}{42(x^3-3x^2+3x-1)}-\frac{\sqrt{-x^2+1}}{63(x^2-2x+1)}+\frac{\sqrt{-x^2+1}}{63(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/2*(-x^2 + 1)^(5/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*
x - 1) - 5/6*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x
 + 1) - 5/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5/12
6*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/42*sqrt(-x^2 + 1)/(x^3
 - 3*x^2 + 3*x - 1) - 1/63*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/63*sqrt(-x^2
+ 1)/(x - 1)
```

**mupad [B]** time = 0.30, size = 80, normalized size = 1.95

$$-\frac{\sqrt{1-x}\left(\frac{23x\sqrt{x+1}}{63}+\frac{8\sqrt{x+1}}{63}+\frac{x^2\sqrt{x+1}}{3}+\frac{5x^3\sqrt{x+1}}{63}-\frac{x^4\sqrt{x+1}}{63}\right)}{x^5-5x^4+10x^3-10x^2+5x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(11/2),x)
```

```
[Out] -((1 - x)^(1/2)*((23*x*(x + 1)^(1/2))/63 + (8*(x + 1)^(1/2))/63 + (x^2*(x +
 1)^(1/2))/3 + (5*x^3*(x + 1)^(1/2))/63 - (x^4*(x + 1)^(1/2))/63))/(5*x - 1
0*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

**sympy [B]** time = 53.14, size = 282, normalized size = 6.88

$$\begin{cases} \frac{i(x+1)^{\frac{9}{2}}}{63\sqrt{x-1}(x+1)^4-504\sqrt{x-1}(x+1)^3+1512\sqrt{x-1}(x+1)^2-2016\sqrt{x-1}(x+1)+1008\sqrt{x-1}} - \frac{9i(x+1)^{\frac{7}{2}}}{63\sqrt{x-1}(x+1)^4-504\sqrt{x-1}(x+1)^3+1512\sqrt{x-1}(x+1)^2-2016\sqrt{x-1}(x+1)+1008\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1 \\ -\frac{(x+1)^{\frac{9}{2}}}{63\sqrt{1-x}(x+1)^4-504\sqrt{1-x}(x+1)^3+1512\sqrt{1-x}(x+1)^2-2016\sqrt{1-x}(x+1)+1008\sqrt{1-x}} + \frac{9(x+1)^{\frac{7}{2}}}{63\sqrt{1-x}(x+1)^4-504\sqrt{1-x}(x+1)^3+1512\sqrt{1-x}(x+1)^2-2016\sqrt{1-x}(x+1)+1008\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(11/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(9/2)/(63*sqrt(x - 1)*(x + 1)**4 - 504*sqrt(x - 1)*(x
 + 1)**3 + 1512*sqrt(x - 1)*(x + 1)**2 - 2016*sqrt(x - 1)*(x + 1) + 1008*sq
rt(x - 1)) - 9*I*(x + 1)**(7/2)/(63*sqrt(x - 1)*(x + 1)**4 - 504*sqrt(x - 1
)*(x + 1)**3 + 1512*sqrt(x - 1)*(x + 1)**2 - 2016*sqrt(x - 1)*(x + 1) + 100
8*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-(x + 1)**(9/2)/(63*sqrt(1 - x)*(x + 1)
**4 - 504*sqrt(1 - x)*(x + 1)**3 + 1512*sqrt(1 - x)*(x + 1)**2 - 2016*sqrt(
1 - x)*(x + 1) + 1008*sqrt(1 - x)) + 9*(x + 1)**(7/2)/(63*sqrt(1 - x)*(x +
1)**4 - 504*sqrt(1 - x)*(x + 1)**3 + 1512*sqrt(1 - x)*(x + 1)**2 - 2016*sqr
t(1 - x)*(x + 1) + 1008*sqrt(1 - x)), True))
```

## 3.1031 $\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx$

**Optimal**. Leaf size=61

$$\frac{2(x+1)^{7/2}}{693(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{7/2}}{693(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(7/2)/(11*(1 - x)^(11/2)) + (2*(1 + x)^(7/2))/(99*(1 - x)^(9/2)) +
(2*(1 + x)^(7/2))/(693*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx = \frac{(1+x)^{7/2}}{11(1-x)^{11/2}} + \frac{2}{11}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{99(1-x)^{9/2}} + \frac{2}{99}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{99(1-x)^{9/2}} + \frac{2(1+x)^{7/2}}{693(1-x)^{7/2}}$$

**Mathematica [A]** time = 0.02, size = 30, normalized size = 0.49

$$\frac{(x+1)^{7/2}\left(2x^2-18x+79\right)}{693(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(7/2)*(79 - 18*x + 2*x^2))/(693*(1 - x)^(11/2))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 48, normalized size = 0.79

$$\frac{(x+1)^{11/2}\left(\frac{99(1-x)^2}{(x+1)^2}+\frac{154(1-x)}{x+1}+63\right)}{2772(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(11/2)*(63 + (99*(1 - x)^2)/(1 + x)^2 + (154*(1 - x))/(1 + x)))/(2
772*(1 - x)^(11/2))
```

**fricas [B]** time = 0.97, size = 100, normalized size = 1.64

$$\frac{79\,x^6-474\,x^5+1185\,x^4-1580\,x^3+1185\,x^2+\left(2\,x^5-12\,x^4+31\,x^3+185\,x^2+219\,x+79\right)\sqrt{x+1}\,\sqrt{-x+1}-474\,x+79}{693\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/693*(79*x^6 - 474*x^5 + 1185*x^4 - 1580*x^3 + 1185*x^2 + (2*x^5 - 12*x^4
+ 31*x^3 + 185*x^2 + 219*x + 79)*sqrt(x + 1)*sqrt(-x + 1) - 474*x + 79)/(x^
6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

**giac** [A] time = 1.27, size = 29, normalized size = 0.48

$$\frac{(2(x+1)(x-10)+99)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{693(x-1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] 1/693*(2*(x + 1)*(x - 10) + 99)*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^6
```

**maple** [A] time = 0.00, size = 25, normalized size = 0.41

$$\frac{(x+1)^{\frac{7}{2}}\left(2x^2-18x+79\right)}{693(-x+1)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(13/2),x)
```

```
[Out] 1/693*(x+1)^(7/2)*(2*x^2-18*x+79)/(-x+1)^(11/2)
```

**maxima** [B] time = 1.42, size = 269, normalized size = 4.41

$$\frac{(-x^2+1)^{\frac{5}{2}}}{3(x^8-8x^7+28x^6-56x^5+70x^4-56x^3+28x^2-8x+1)}+\frac{5(-x^2+1)^{\frac{3}{2}}}{12(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1)}+\frac{5\sqrt{-x^2+1}}{22(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)}+\frac{5\sqrt{-x^2+1}}{396(x^5-5x^4+10x^3-10x^2+5x-1)}-\frac{5\sqrt{-x^2+1}}{693(x^4-4x^3+6x^2-4x+1)}+\frac{\sqrt{-x^2+1}}{231(x^3-3x^2+3x-1)}-\frac{2\sqrt{-x^2+1}}{693(x^2-2x+1)}+\frac{2\sqrt{-x^2+1}}{693(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(5/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*
x^2 - 8*x + 1) + 5/12*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*
x^3 - 21*x^2 + 7*x - 1) + 5/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^
3 + 15*x^2 - 6*x + 1) + 5/396*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2
 + 5*x - 1) - 5/693*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/231*
sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/693*sqrt(-x^2 + 1)/(x^2 - 2*x +
1) + 2/693*sqrt(-x^2 + 1)/(x - 1)
```

**mupad** [B] time = 0.31, size = 94, normalized size = 1.54

$$\frac{\sqrt{1-x}\left(\frac{73x\sqrt{x+1}}{231}+\frac{79\sqrt{x+1}}{693}+\frac{185x^2\sqrt{x+1}}{693}+\frac{31x^3\sqrt{x+1}}{693}-\frac{4x^4\sqrt{x+1}}{231}+\frac{2x^5\sqrt{x+1}}{693}\right)}{x^6-6x^5+15x^4-20x^3+15x^2-6x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((73*x*(x + 1)^(1/2))/231 + (79*(x + 1)^(1/2))/693 + (185*x^
2*(x + 1)^(1/2))/693 + (31*x^3*(x + 1)^(1/2))/693 - (4*x^4*(x + 1)^(1/2))/2
31 + (2*x^5*(x + 1)^(1/2))/693))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 - 6*x^5 +
x^6 + 1)
```

**sympy [B]** time = 133.94, size = 785, normalized size = 12.87

$$\begin{cases} \frac{2i(x+1)^{\frac{13}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} - \frac{26i(x+1)^{\frac{11}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} + \frac{143i(x+1)^{\frac{9}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} - \frac{198i(x+1)^{\frac{7}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} & \text{for}\: \frac{|x+1|}{2} > 1 \\ -\frac{2(x+1)^{\frac{13}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} + \frac{26(x+1)^{\frac{11}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} - \frac{143(x+1)^{\frac{9}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} + \frac{198(x+1)^{\frac{7}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(13/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**(13/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x -
1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1)**
3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*sqrt
(x - 1)) - 26*I*(x + 1)**(11/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x -
 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1)*
*3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*sqr
t(x - 1)) + 143*I*(x + 1)**(9/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x
- 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1)
**3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*sq
rt(x - 1)) - 198*I*(x + 1)**(7/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x
 - 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1
)**3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*s
qrt(x - 1)), Abs(x + 1)/2 > 1), (-2*(x + 1)**(13/2)/(693*sqrt(1 - x)*(x + 1
)**6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*
sqrt(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)
*(x + 1) + 44352*sqrt(1 - x)) + 26*(x + 1)**(11/2)/(693*sqrt(1 - x)*(x + 1)
**6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*s
qrt(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)*
(x + 1) + 44352*sqrt(1 - x)) - 143*(x + 1)**(9/2)/(693*sqrt(1 - x)*(x + 1)*
*6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*sq
rt(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)*(
x + 1) + 44352*sqrt(1 - x)) + 198*(x + 1)**(7/2)/(693*sqrt(1 - x)*(x + 1)**
6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*sqr
t(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)*(x
 + 1) + 44352*sqrt(1 - x)), True))
```

## 3.1032 $\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}} dx$

**Optimal**. Leaf size=81

$$\frac{2(x+1)^{7/2}}{3003(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{429(1-x)^{9/2}}+\frac{3(x+1)^{7/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{7/2}}{3003(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{429(1-x)^{9/2}}+\frac{3(x+1)^{7/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(15/2),x]
```

```
[Out] (1 + x)^(7/2)/(13*(1 - x)^(13/2)) + (3*(1 + x)^(7/2))/(143*(1 - x)^(11/2))
+ (2*(1 + x)^(7/2))/(429*(1 - x)^(9/2)) + (2*(1 + x)^(7/2))/(3003*(1 - x)^(
7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx = \frac{(1+x)^{7/2}}{13(1-x)^{13/2}} + \frac{3}{13}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}} + \frac{3(1+x)^{7/2}}{143(1-x)^{11/2}} + \frac{6}{143}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}} + \frac{3(1+x)^{7/2}}{143(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{429(1-x)^{9/2}} + \frac{2}{429}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}} + \frac{3(1+x)^{7/2}}{143(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{429(1-x)^{9/2}} + \frac{2(1+x)^{7/2}}{3003(1-x)^{7/2}}$$

**Mathematica** [A] time = 0.02, size = 35, normalized size = 0.43

$$\frac{(x+1)^{7/2}\left(-2x^3+20x^2-97x+310\right)}{3003(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(15/2),x]
[Out] ((1 + x)^(7/2)*(310 - 97*x + 20*x^2 - 2*x^3))/(3003*(1 - x)^(13/2))
```

**IntegrateAlgebraic** [A] time = 0.08, size = 62, normalized size = 0.77

$$\frac{(x+1)^{13/2}\left(\frac{429(1-x)^3}{(x+1)^3}+\frac{1001(1-x)^2}{(x+1)^2}+\frac{819(1-x)}{x+1}+231\right)}{24024(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(15/2),x]
[Out] ((1 + x)^(13/2)*(231 + (429*(1 - x)^3)/(1 + x)^3 + (1001*(1 - x)^2)/(1 + x)
^2 + (819*(1 - x))/(1 + x)))/(24024*(1 - x)^(13/2))
```

**fricas** [B] time = 1.31, size = 115, normalized size = 1.42

$$\frac{310\,x^7-2170\,x^6+6510\,x^5-10850\,x^4+10850\,x^3-6510\,x^2+\left(2\,x^6-14\,x^5+43\,x^4-77\,x^3-659\,x^2-833\,x-310\right)\sqrt{x+1}\,\sqrt{-x+1}+2170\,x-310}{3003\left(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="fricas")
```

```
[Out] 1/3003*(310*x^7 - 2170*x^6 + 6510*x^5 - 10850*x^4 + 10850*x^3 - 6510*x^2 +
(2*x^6 - 14*x^5 + 43*x^4 - 77*x^3 - 659*x^2 - 833*x - 310)*sqrt(x + 1)*sqrt
(-x + 1) + 2170*x - 310)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 +
 7*x - 1)
```

**giac** [A] time = 0.99, size = 35, normalized size = 0.43

$$\frac{((2(x+1)(x-12)+143)(x+1)-429)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{3003(x-1)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="giac")
```

```
[Out] 1/3003*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1)^(7/2)*sqrt(-x + 1
)/(x - 1)^7
```

**maple** [A] time = 0.00, size = 30, normalized size = 0.37

$$-\frac{(x+1)^{\frac{7}{2}}\left(2x^3-20x^2+97x-310\right)}{3003(-x+1)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(15/2),x)
```

```
[Out] -1/3003*(x+1)^(7/2)*(2*x^3-20*x^2+97*x-310)/(-x+1)^(13/2)
```

**maxima** [B] time = 1.46, size = 325, normalized size = 4.01

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{4(x^9-9x^8+36x^7-84x^6+126x^5-126x^4+84x^3-36x^2+9x-1)}-\frac{(-x^2+1)^{\frac{3}{2}}}{4(x^8-8x^7+28x^6-56x^5+70x^4-56x^3+28x^2-8x+1)}-\frac{3\sqrt{-x^2+1}}{26(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1)}-\frac{3\sqrt{-x^2+1}}{572(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)}+\frac{5\sqrt{-x^2+1}}{1716(x^5-5x^4+10x^3-10x^2+5x-1)}-\frac{5\sqrt{-x^2+1}}{3003(x^4-4x^3+6x^2-4x+1)}+\frac{\sqrt{-x^2+1}}{1001(x^3-3x^2+3x-1)}-\frac{2\sqrt{-x^2+1}}{3003(x^2-2x+1)}+\frac{2\sqrt{-x^2+1}}{3003(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(5/2)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 +
84*x^3 - 36*x^2 + 9*x - 1) - 1/4*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 - 5
6*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1) - 3/26*sqrt(-x^2 + 1)/(x^7 - 7*
x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*x - 1) - 3/572*sqrt(-x^2 + 1)/(
x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/1716*sqrt(-x^2 + 1)/(
x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5/3003*sqrt(-x^2 + 1)/(x^4 - 4*x
^3 + 6*x^2 - 4*x + 1) + 1/1001*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/3
003*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/3003*sqrt(-x^2 + 1)/(x - 1)
```

**mupad [B]** time = 0.31, size = 110, normalized size = 1.36

$$-\frac{\sqrt{1-x}\,\left(\frac{119\,x\,\sqrt{x+1}}{429}+\frac{310\,\sqrt{x+1}}{3003}+\frac{659\,x^2\,\sqrt{x+1}}{3003}+\frac{x^3\,\sqrt{x+1}}{39}-\frac{43\,x^4\,\sqrt{x+1}}{3003}+\frac{2\,x^5\,\sqrt{x+1}}{429}-\frac{2\,x^6\,\sqrt{x+1}}{3003}\right)}{x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(15/2),x)
```

```
[Out] -((1 - x)^(1/2)*((119*x*(x + 1)^(1/2))/429 + (310*(x + 1)^(1/2))/3003 + (65
9*x^2*(x + 1)^(1/2))/3003 + (x^3*(x + 1)^(1/2))/39 - (43*x^4*(x + 1)^(1/2))
/3003 + (2*x^5*(x + 1)^(1/2))/429 - (2*x^6*(x + 1)^(1/2))/3003))/(7*x - 21*
x^2 + 35*x^3 - 35*x^4 + 21*x^5 - 7*x^6 + x^7 - 1)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(15/2),x)
```

```
[Out] Timed out
```

## 3.1033 $\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx$

**Optimal**. Leaf size=101

$$\frac{8(x+1)^{7/2}}{45045(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{6435(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{715(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{195(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8(x+1)^{7/2}}{45045(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{6435(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{715(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{195(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

```
[Out] (1 + x)^(7/2)/(15*(1 - x)^(15/2)) + (4*(1 + x)^(7/2))/(195*(1 - x)^(13/2))
+ (4*(1 + x)^(7/2))/(715*(1 - x)^(11/2)) + (8*(1 + x)^(7/2))/(6435*(1 - x)^
(9/2)) + (8*(1 + x)^(7/2))/(45045*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4}{15}\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx \\
&= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4}{65}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx \\
&= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{715(1-x)^{11/2}}+\frac{8}{715}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx \\
&= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{715(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}}+\frac{8\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx}{6435} \\
&= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{715(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}}+\frac{8(1+x)^{7/2}}{45045(1-x)^{7/2}}
\end{aligned}
$$

**Mathematica** [A] time = 0.02, size = 40, normalized size = 0.40

$$
\frac{(x+1)^{7/2}\left(8x^4-88x^3+468x^2-1628x+4243\right)}{45045(1-x)^{15/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

```
[Out] ((1 + x)^(7/2)*(4243 - 1628*x + 468*x^2 - 88*x^3 + 8*x^4))/(45045*(1 - x)^(
15/2))
```

**IntegrateAlgebraic** [A] time = 0.08, size = 76, normalized size = 0.75

$$
\frac{(x+1)^{15/2}\left(\frac{6435(1-x)^4}{(x+1)^4}+\frac{20020(1-x)^3}{(x+1)^3}+\frac{24570(1-x)^2}{(x+1)^2}+\frac{13860(1-x)}{x+1}+3003\right)}{720720(1-x)^{15/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

```
[Out] ((1 + x)^(15/2)*(3003 + (6435*(1 - x)^4)/(1 + x)^4 + (20020*(1 - x)^3)/(1 +
 x)^3 + (24570*(1 - x)^2)/(1 + x)^2 + (13860*(1 - x))/(1 + x)))/(720720*(1
- x)^(15/2))
```

**fricas** [A] time = 1.36, size = 130, normalized size = 1.29

$$
\frac{4243\,x^{8}-33944\,x^{7}+118804\,x^{6}-237608\,x^{5}+297010\,x^{4}-237608\,x^{3}+118804\,x^{2}+\left(8\,x^{7}-64\,x^{6}+228\,x^{5}-480\,x^{4}+675\,x^{3}+8313\,x^{2}+11101\,x+4243\right)\sqrt{x+1}\,\sqrt{-x+1}-33944\,x+4243}{45045\,\left(x^{8}-8\,x^{7}+28\,x^{6}-56\,x^{5}+70\,x^{4}-56\,x^{3}+28\,x^{2}-8\,x+1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="fricas")
```

```
[Out] 1/45045*(4243*x^8 - 33944*x^7 + 118804*x^6 - 237608*x^5 + 297010*x^4 - 2376
08*x^3 + 118804*x^2 + (8*x^7 - 64*x^6 + 228*x^5 - 480*x^4 + 675*x^3 + 8313*
x^2 + 11101*x + 4243)*sqrt(x + 1)*sqrt(-x + 1) - 33944*x + 4243)/(x^8 - 8*x
^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1)
```

**giac [A]** time = 0.88, size = 42, normalized size = 0.42

$$\frac{(4\,((2\,(x+1)(x-14)+195)(x+1)-715)(x+1)+6435)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{45045\,(x-1)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="giac")
```

```
[Out] 1/45045*(4*((2*(x + 1)*(x - 14) + 195)*(x + 1) - 715)*(x + 1) + 6435)*(x +
1)^(7/2)*sqrt(-x + 1)/(x - 1)^8
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.35

$$\frac{(x+1)^{\frac{7}{2}}\left(8x^{4}-88x^{3}+468x^{2}-1628x+4243\right)}{45045\,(-x+1)^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(17/2),x)
```

```
[Out] 1/45045*(x+1)^(7/2)*(8*x^4-88*x^3+468*x^2-1628*x+4243)/(-x+1)^(15/2)
```

**maxima [B]** time = 1.39, size = 386, normalized size = 3.82

$$\frac{\left(-x^{2}+1\right)^{\frac{5}{2}}}{5\left(x^{10}-10\,x^{9}+45\,x^{8}-120\,x^{7}+210\,x^{6}-252\,x^{5}+210\,x^{4}-120\,x^{3}+45\,x^{2}-10\,x+1\right)}+\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{6\left(x^{9}-9\,x^{8}+36\,x^{7}-84\,x^{6}+126\,x^{5}-126\,x^{4}+84\,x^{3}-36\,x^{2}+9\,x-1\right)}+\frac{\sqrt{-x^{2}+1}}{15\left(x^{8}-8\,x^{7}+28\,x^{6}-56\,x^{5}+70\,x^{4}-56\,x^{3}+28\,x^{2}-8\,x+1\right)}+\frac{\sqrt{-x^{2}+1}}{390\left(x^{7}-7\,x^{6}+21\,x^{5}-35\,x^{4}+35\,x^{3}-21\,x^{2}+7\,x-1\right)}-\frac{\sqrt{-x^{2}+1}}{715\left(x^{6}-6\,x^{5}+15\,x^{4}-20\,x^{3}+15\,x^{2}-6\,x+1\right)}+\frac{\sqrt{-x^{2}+1}}{1287\left(x^{5}-5\,x^{4}+10\,x^{3}-10\,x^{2}+5\,x-1\right)}-\frac{4\sqrt{-x^{2}+1}}{9009\left(x^{4}-4\,x^{3}+6\,x^{2}-4\,x+1\right)}+\frac{4\sqrt{-x^{2}+1}}{15015\left(x^{3}-3\,x^{2}+3\,x-1\right)}-\frac{8\sqrt{-x^{2}+1}}{45045\left(x^{2}-2\,x+1\right)}+\frac{8\sqrt{-x^{2}+1}}{45045\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(5/2)/(x^10 - 10*x^9 + 45*x^8 - 120*x^7 + 210*x^6 - 252*x^5
+ 210*x^4 - 120*x^3 + 45*x^2 - 10*x + 1) + 1/6*(-x^2 + 1)^(3/2)/(x^9 - 9*x^
8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 + 84*x^3 - 36*x^2 + 9*x - 1) + 1/15
*sqrt(-x^2 + 1)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 -
 8*x + 1) + 1/390*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 -
21*x^2 + 7*x - 1) - 1/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 1
```

```
5*x^2 - 6*x + 1) + 1/1287*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5
*x - 1) - 4/9009*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/15015*s
qrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 8/45045*sqrt(-x^2 + 1)/(x^2 - 2*x +
 1) + 8/45045*sqrt(-x^2 + 1)/(x - 1)
```

**mupad [B]** time = 0.35, size = 124, normalized size = 1.23

$$\frac{\sqrt{1-x}\,\left(\frac{11101\,x\,\sqrt{x+1}}{45045}+\frac{4243\,\sqrt{x+1}}{45045}+\frac{2771\,x^2\,\sqrt{x+1}}{15015}+\frac{15\,x^3\,\sqrt{x+1}}{1001}-\frac{32\,x^4\,\sqrt{x+1}}{3003}+\frac{76\,x^5\,\sqrt{x+1}}{15015}-\frac{64\,x^6\,\sqrt{x+1}}{45045}+\frac{8\,x^7\,\sqrt{x+1}}{45045}\right)}{x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(17/2),x)
```

```
[Out] ((1 - x)^(1/2)*((11101*x*(x + 1)^(1/2))/45045 + (4243*(x + 1)^(1/2))/45045
+ (2771*x^2*(x + 1)^(1/2))/15015 + (15*x^3*(x + 1)^(1/2))/1001 - (32*x^4*(x
 + 1)^(1/2))/3003 + (76*x^5*(x + 1)^(1/2))/15015 - (64*x^6*(x + 1)^(1/2))/4
5045 + (8*x^7*(x + 1)^(1/2))/45045))/(28*x^2 - 8*x - 56*x^3 + 70*x^4 - 56*x
^5 + 28*x^6 - 8*x^7 + x^8 + 1)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(17/2),x)
```

```
[Out] Timed out
```

## 3.1034 $\int \frac{(1+x)^{5/2}}{(1-x)^{19/2}}\,dx$

**Optimal**. Leaf size=121

$$\frac{8(x+1)^{7/2}}{153153(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{21879(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{2431(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{663(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{51(1-x)^{15/2}}+\frac{(x+1)^{7/2}}{17(1-x)^{17/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8(x+1)^{7/2}}{153153(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{21879(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{2431(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{663(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{51(1-x)^{15/2}}+\frac{(x+1)^{7/2}}{17(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] (1 + x)^(7/2)/(17*(1 - x)^(17/2)) + (1 + x)^(7/2)/(51*(1 - x)^(15/2)) + (4*
(1 + x)^(7/2))/(663*(1 - x)^(13/2)) + (4*(1 + x)^(7/2))/(2431*(1 - x)^(11/2
)) + (8*(1 + x)^(7/2))/(21879*(1 - x)^(9/2)) + (8*(1 + x)^(7/2))/(153153*(1
 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{19/2}}\,dx = \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{5}{17}\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx$$

$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4}{51}\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx$$

$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4}{221}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx$$

$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}} + \frac{8\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx}{2431}$$

$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}} + \frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}} + \frac{8\int \frac{(1+\cdots}{(1-\cdots}}{218\cdots}$$

$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}} + \frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}} + \frac{8(1\cdots}{153153\cdots}$$

**Mathematica** [A] time = 0.02, size = 45, normalized size = 0.37

$$\frac{(x+1)^{7/2}\left(-8x^5+96x^4-556x^3+2096x^2-5871x+13252\right)}{153153(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] ((1 + x)^(7/2)*(13252 - 5871*x + 2096*x^2 - 556*x^3 + 96*x^4 - 8*x^5))/(153
153*(1 - x)^(17/2))
```

**IntegrateAlgebraic** [A] time = 0.09, size = 90, normalized size = 0.74

$$\frac{(x+1)^{17/2}\left(\frac{21879(1-x)^5}{(x+1)^5}+\frac{85085(1-x)^4}{(x+1)^4}+\frac{139230(1-x)^3}{(x+1)^3}+\frac{117810(1-x)^2}{(x+1)^2}+\frac{51051(1-x)}{x+1}+9009\right)}{4900896(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] ((1 + x)^(17/2)*(9009 + (21879*(1 - x)^5)/(1 + x)^5 + (85085*(1 - x)^4)/(1
+ x)^4 + (139230*(1 - x)^3)/(1 + x)^3 + (117810*(1 - x)^2)/(1 + x)^2 + (510
51*(1 - x))/(1 + x)))/(4900896*(1 - x)^(17/2))
```

**fricas** [A] time = 1.16, size = 145, normalized size = 1.20

$$\frac{13252 \, x^{9} - 119268 \, x^{8} + 477072 \, x^{7} - 1113168 \, x^{6} + 1669752 \, x^{5} - 1669752 \, x^{4} + 1113168 \, x^{3} - 477072 \, x^{2} + {\left(8 \, x^{8} - 72 \, x^{7} + 292 \, x^{6} - 708 \, x^{5} + 1155 \, x^{4} - 1371 \, x^{3} - 24239 \, x^{2} - 33885 \, x - 13252\right)} \sqrt{x + 1} \sqrt{-x + 1} + 119268 \, x - 13252}{153153 \, {\left(x^{9} - 9 \, x^{8} + 36 \, x^{7} - 84 \, x^{6} + 126 \, x^{5} - 126 \, x^{4} + 84 \, x^{3} - 36 \, x^{2} + 9 \, x - 1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="fricas")
```

```
[Out] 1/153153*(13252*x^9 - 119268*x^8 + 477072*x^7 - 1113168*x^6 + 1669752*x^5 -
 1669752*x^4 + 1113168*x^3 - 477072*x^2 + (8*x^8 - 72*x^7 + 292*x^6 - 708*x
^5 + 1155*x^4 - 1371*x^3 - 24239*x^2 - 33885*x - 13252)*sqrt(x + 1)*sqrt(-x
 + 1) + 119268*x - 13252)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^
4 + 84*x^3 - 36*x^2 + 9*x - 1)
```

**giac** [A] time = 0.86, size = 48, normalized size = 0.40

$$\frac{{\left({\left(4 \, {\left({\left(2 \, {\left(x + 1\right)} {\left(x - 16\right)} + 255\right)} {\left(x + 1\right)} - 1105\right)} {\left(x + 1\right)} + 12155\right)} {\left(x + 1\right)} - 21879\right)} {\left(x + 1\right)}^{\frac{7}{2}} \sqrt{-x + 1}}{153153 \, {\left(x - 1\right)}^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="giac")
```

```
[Out] 1/153153*((4*((2*(x + 1)*(x - 16) + 255)*(x + 1) - 1105)*(x + 1) + 12155)*(
x + 1) - 21879)*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^9
```

**maple** [A] time = 0.00, size = 40, normalized size = 0.33

$$-\frac{\left(x +1\right)^{\frac{7}{2}} \left(8 x^{5}-96 x^{4}+556 x^{3}-2096 x^{2}+5871 x -13252\right)}{153153 \left(-x +1\right)^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(19/2),x)
```

```
[Out] -1/153153*(x+1)^(7/2)*(8*x^5-96*x^4+556*x^3-2096*x^2+5871*x-13252)/(-x+1)^(
17/2)
```

**maxima** [B] time = 1.38, size = 452, normalized size = 3.74

$$-\frac{{\left(-x^{2} + 1\right)}^{\frac{5}{2}}}{6 \, {\left(x^{11} - 11 \, x^{10} + 55 \, x^{9} - 165 \, x^{8} + 330 \, x^{7} - 462 \, x^{6} + 462 \, x^{5} - 330 \, x^{4} + 165 \, x^{3} - 55 \, x^{2} + 11 \, x - 1\right)}} - \frac{5 \, {\left(-x^{2} + 1\right)}^{\frac{3}{2}}}{42 \, {\left(x^{10} - 10 \, x^{9} + 45 \, x^{8} - 120 \, x^{7} + 210 \, x^{6} - 252 \, x^{5} + 210 \, x^{4} - 120 \, x^{3} + 45 \, x^{2} - 10 \, x + 1\right)}} - \frac{5 \, \sqrt{-x^{2} + 1}}{119 \, {\left(x^{9} - 9 \, x^{8} + 36 \, x^{7} - 84 \, x^{6} + 126 \, x^{5} - 126 \, x^{4} + 84 \, x^{3} - 36 \, x^{2} + 9 \, x - 1\right)}} - \frac{\sqrt{-x^{2} + 1}}{714 \, {\left(x^{8} - 8 \, x^{7} + 28 \, x^{6} - 56 \, x^{5} + 70 \, x^{4} - 56 \, x^{3} + 28 \, x^{2} - 8 \, x + 1\right)}} + \frac{\sqrt{-x^{2} + 1}}{1326 \, {\left(x^{7} - 7 \, x^{6} + 21 \, x^{5} - 35 \, x^{4} + 35 \, x^{3} - 21 \, x^{2} + 7 \, x - 1\right)}} - \frac{\sqrt{-x^{2} + 1}}{2431 \, {\left(x^{6} - 6 \, x^{5} + 15 \, x^{4} - 20 \, x^{3} + 15 \, x^{2} - 6 \, x + 1\right)}} + \frac{5 \, \sqrt{-x^{2} + 1}}{21879 \, {\left(x^{5} - 5 \, x^{4} + 10 \, x^{3} - 10 \, x^{2} + 5 \, x - 1\right)}} - \frac{20 \, \sqrt{-x^{2} + 1}}{153153 \, {\left(x^{4} - 4 \, x^{3} + 6 \, x^{2} - 4 \, x + 1\right)}} + \frac{4 \, \sqrt{-x^{2} + 1}}{51051 \, {\left(x^{3} - 3 \, x^{2} + 3 \, x - 1\right)}} - \frac{8 \, \sqrt{-x^{2} + 1}}{153153 \, {\left(x^{2} - 2 \, x + 1\right)}} + \frac{8 \, \sqrt{-x^{2} + 1}}{153153 \, {\left(x - 1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(5/2)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 + 330*x^7 - 462*x^
6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1) - 5/42*(-x^2 + 1)^(3/2
)/(x^10 - 10*x^9 + 45*x^8 - 120*x^7 + 210*x^6 - 252*x^5 + 210*x^4 - 120*x^3
 + 45*x^2 - 10*x + 1) - 5/119*sqrt(-x^2 + 1)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6
 + 126*x^5 - 126*x^4 + 84*x^3 - 36*x^2 + 9*x - 1) - 1/714*sqrt(-x^2 + 1)/(x
^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1) + 1/1326
*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*x - 1)
 - 1/2431*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
 + 5/21879*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 20/15
3153*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/51051*sqrt(-x^2 + 1
)/(x^3 - 3*x^2 + 3*x - 1) - 8/153153*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 8/153
153*sqrt(-x^2 + 1)/(x - 1)
```

**mupad [B]** time = 0.37, size = 140, normalized size = 1.16

$$-\frac{\sqrt{1-x}\,\left(\frac{3765\,x\,\sqrt{x+1}}{17017}+\frac{13252\,\sqrt{x+1}}{153153}+\frac{24239\,x^2\,\sqrt{x+1}}{153153}+\frac{457\,x^3\,\sqrt{x+1}}{51051}-\frac{5\,x^4\,\sqrt{x+1}}{663}+\frac{236\,x^5\,\sqrt{x+1}}{51051}-\frac{292\,x^6\,\sqrt{x+1}}{153153}+\frac{8\,x^7\,\sqrt{x+1}}{17017}-\frac{8\,x^8\,\sqrt{x+1}}{153153}\right)}{x^9-9\,x^8+36\,x^7-84\,x^6+126\,x^5-126\,x^4+84\,x^3-36\,x^2+9\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(19/2),x)
```

```
[Out] -((1 - x)^(1/2)*((3765*x*(x + 1)^(1/2))/17017 + (13252*(x + 1)^(1/2))/15315
3 + (24239*x^2*(x + 1)^(1/2))/153153 + (457*x^3*(x + 1)^(1/2))/51051 - (5*x
^4*(x + 1)^(1/2))/663 + (236*x^5*(x + 1)^(1/2))/51051 - (292*x^6*(x + 1)^(1
/2))/153153 + (8*x^7*(x + 1)^(1/2))/17017 - (8*x^8*(x + 1)^(1/2))/153153))/
(9*x - 36*x^2 + 84*x^3 - 126*x^4 + 126*x^5 - 84*x^6 + 36*x^7 - 9*x^8 + x^9
- 1)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(19/2),x)
```

```
[Out] Timed out
```

## 3.1035 $\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx$

**Optimal**. Leaf size=64

$$-\frac{\sqrt{1-ax}\,(ax+1)^{3/2}}{2a}-\frac{3\sqrt{1-ax}\,\sqrt{ax+1}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

**Rubi** [A] time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {50, 41, 216}

$$-\frac{\sqrt{1-ax}\,(ax+1)^{3/2}}{2a}-\frac{3\sqrt{1-ax}\,\sqrt{ax+1}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
```

```
[Out] (-3*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(2*a) - (Sqrt[1 - a*x]*(1 + a*x)^(3/2))/(2
*a) + (3*ArcSin[a*x])/(2*a)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx &= -\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a}+\frac{3}{2}\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx \\
&= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a}-\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a}+\frac{3}{2}\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx \\
&= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a}-\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a}+\frac{3}{2}\int \frac{1}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a}-\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a}+\frac{3\sin^{-1}(ax)}{2a}
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 47, normalized size = 0.73

$$
-\frac{\sqrt{1-a^2x^2}\,(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{2a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
[Out] -1/2*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/a
```

**IntegrateAlgebraic [A]** time = 0.08, size = 86, normalized size = 1.34

$$
-\frac{\sqrt{1-ax}\left(\frac{3(1-ax)}{ax+1}+5\right)}{a\sqrt{ax+1}\left(\frac{1-ax}{ax+1}+1\right)^2}-\frac{3\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
[Out] -((Sqrt[1 - a*x]*(5 + (3*(1 - a*x))/(1 + a*x)))/(a*Sqrt[1 + a*x]*(1 + (1 -
a*x)/(1 + a*x))^2)) - (3*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/a
```

**fricas [A]** time = 1.27, size = 55, normalized size = 0.86

$$
-\frac{(ax+4)\sqrt{ax+1}\,\sqrt{-ax+1}\,+6\,\arctan\left(\frac{\sqrt{ax+1}\,\sqrt{-ax+1}-1}{ax}\right)}{2\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) + 6*arctan((sqrt(a*x + 1)*sqrt
(-a*x + 1) - 1)/(a*x)))/a
```

**giac** [A] time = 0.70, size = 42, normalized size = 0.66

$$-\frac{(ax+4)\sqrt{ax+1}\,\sqrt{-ax+1}-6\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{ax+1}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 6*arcsin(1/2*sqrt(2)*sqrt(a*
x + 1)))/a
```

**maple** [A] time = 0.01, size = 98, normalized size = 1.53

$$\frac{3\sqrt{(ax+1)(-ax+1)}\,\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{ax+1}\,\sqrt{-ax+1}\,\sqrt{a^2}}-\frac{(ax+1)^{\frac{3}{2}}\sqrt{-ax+1}}{2a}-\frac{3\sqrt{-ax+1}\,\sqrt{ax+1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(3/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/2*(a*x+1)^(3/2)*(-a*x+1)^(1/2)/a-3/2*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a+3/2*
((a*x+1)*(-a*x+1))^(1/2)/(a*x+1)^(1/2)/(-a*x+1)^(1/2)/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

**maxima** [A] time = 2.99, size = 42, normalized size = 0.66

$$-\frac{1}{2}\,\sqrt{-a^2x^2+1}\,x+\frac{3\,\arcsin\left(ax\right)}{2\,a}-\frac{2\,\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a\,x+1)^{3/2}}{\sqrt{1-a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^(3/2)/(1 - a*x)^(1/2),x)
```

```
[Out] int((a*x + 1)^(3/2)/(1 - a*x)^(1/2), x)
```

**sympy [A]** time = 33.75, size = 75, normalized size = 1.17

$$\begin{cases} \frac{2\left(\begin{cases} -\frac{ax\sqrt{-ax+1}\sqrt{ax+1}}{4} - \sqrt{-ax+1}\sqrt{ax+1} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{ax+1}}{2}\right)}{2} & \text{for } ax-1 \geq -2 \wedge ax-1 < 0 \end{cases}\right)}{a} & \text{for } a \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(3/2)/(-a*x+1)**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((-a*x*sqrt(-a*x + 1)*sqrt(a*x + 1)/4 - sqrt(-a*x + 1
)*sqrt(a*x + 1) + 3*asin(sqrt(2)*sqrt(a*x + 1)/2)/2, (a*x - 1 >= -2) & (a*x
 - 1 < 0)))/a, Ne(a, 0)), (x, True))
```

## 3.1036 $\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx$

**Optimal**. Leaf size=62

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

**Rubi [A]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {795, 665, 216}

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*a) - (1 - a^2*x^2)^(3/2)/(2*a*(1 - a*x)) + (3*Arc
Sin[a*x])/(2*a)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rubi steps

$$\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx = -\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)} + \frac{3}{2}\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx$$
$$= -\frac{3\sqrt{1-a^2x^2}}{2a} - \frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)} + \frac{3}{2}\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{3\sqrt{1-a^2x^2}}{2a} - \frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)} + \frac{3\sin^{-1}(ax)}{2a}$$

**Mathematica [A]** time = 0.10, size = 91, normalized size = 1.47

$$\frac{\sqrt{1-a^2x^2}\left(6\sqrt{ax+1}\,\sin^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)-\sqrt{1-ax}\left(a^2x^2+5ax+4\right)\right)}{2a\sqrt{1-ax}\,(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-(Sqrt[1 - a*x]*(4 + 5*a*x + a^2*x^2)) + 6*Sqrt[1 + a*x
]*ArcSin[Sqrt[1 + a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*(1 + a*x))
```

**IntegrateAlgebraic [A]** time = 0.29, size = 72, normalized size = 1.16

$$\frac{\sqrt{1-a^2x^2}\,(-ax-4)}{2a}+\frac{3\sqrt{-a^2}\,\log\left(\sqrt{1-a^2x^2}-\sqrt{-a^2}\,x\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

```
[Out] ((-4 - a*x)*Sqrt[1 - a^2*x^2])/(2*a) + (3*Sqrt[-a^2]*Log[-(Sqrt[-a^2]*x) +
Sqrt[1 - a^2*x^2]])/(2*a^2)
```

**fricas [A]** time = 1.32, size = 48, normalized size = 0.77

$$-\frac{\sqrt{-a^2x^2+1}\,(ax+4)+6\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-a^2*x^2 + 1)*(a*x + 4) + 6*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)))/a
```

**giac [A]** time = 0.70, size = 34, normalized size = 0.55

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\left(x+\frac{4}{a}\right)+\frac{3\arcsin(ax)\operatorname{sgn}(a)}{2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x + 4/a) + 3/2*arcsin(a*x)*sgn(a)/abs(a)
```

**maple [B]** time = 0.01, size = 118, normalized size = 1.90

$$-\frac{\sqrt{-a^2x^2+1}\,x}{2}+\frac{2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}}\right)}{\sqrt{a^2}}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}-\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x)
```

```
[Out] -1/2*x*(-a^2*x^2+1)^(1/2)-1/2/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*x^2+1)^(
1/2)*x)-2/a*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)+2/(a^2)^(1/2)*arctan((a^2)^(
1/2)*x/(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2))
```

**maxima [A]** time = 3.03, size = 42, normalized size = 0.68

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\,x+\frac{3\arcsin(ax)}{2a}-\frac{2\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

**mupad [B]** time = 0.15, size = 55, normalized size = 0.89

$$\frac{\frac{3\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{2}+\sqrt{1-a^2x^2}\left(\frac{2a}{\sqrt{-a^2}}-\frac{x\sqrt{-a^2}}{2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1 - a^2*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] ((3*asinh(x*(-a^2)^(1/2)))/2 + (1 - a^2*x^2)^(1/2)*((2*a)/(-a^2)^(1/2) - (x
*(-a^2)^(1/2))/2))/(-a^2)^(1/2)
```

**sympy [A]** time = 7.08, size = 76, normalized size = 1.23

$$-\begin{cases} -\frac{-\sqrt{-a^2x^2+1}+\operatorname{asin}(ax)}{a} & \text{for } ax > -1 \wedge ax < 1 \end{cases} - \begin{cases} -\frac{-\frac{ax\sqrt{-a^2x^2+1}}{2}-\sqrt{-a^2x^2+1}+\frac{\operatorname{asin}(ax)}{2}}{a} & \text{for } ax > -1 \wedge ax < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*x+1),x)
```

```
[Out] -Piecewise((-(-sqrt(-a**2*x**2 + 1) + asin(a*x))/a, (a*x > -1) & (a*x < 1))
) - Piecewise((-(-a*x*sqrt(-a**2*x**2 + 1)/2 - sqrt(-a**2*x**2 + 1) + asin(
a*x)/2)/a, (a*x > -1) & (a*x < 1)))
```

## 3.1037 $\int \frac{(1-x)^{7/2}}{\sqrt{1+x}} \, dx$

**Optimal**. Leaf size=87

$$\frac{1}{4}\sqrt{x+1}\,(1-x)^{7/2}+\frac{7}{12}\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{24}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{8}\sqrt{x+1}\,\sqrt{1-x}+\frac{35}{8}\sin^{-1}(x)$$

**Rubi** [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\frac{1}{4}\sqrt{x+1}\,(1-x)^{7/2}+\frac{7}{12}\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{24}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{8}\sqrt{x+1}\,\sqrt{1-x}+\frac{35}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

```
[Out] (35*Sqrt[1 - x]*Sqrt[1 + x])/8 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/24 + (7*(1
- x)^(5/2)*Sqrt[1 + x])/12 + ((1 - x)^(7/2)*Sqrt[1 + x])/4 + (35*ArcSin[x])
/8
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx = \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{7}{4}\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx$$
$$= \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{12}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx$$
$$= \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{8}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx$$
$$= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{8}\int$$
$$= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{8}\int$$
$$= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{8}\sin$$

**Mathematica [A]** time = 0.02, size = 61, normalized size = 0.70

$$\frac{\sqrt{x+1}\left(6x^4-38x^3+113x^2-241x+160\right)}{24\sqrt{1-x}} - \frac{35}{4}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(160 - 241*x + 113*x^2 - 38*x^3 + 6*x^4))/(24*Sqrt[1 - x]) - (
35*ArcSin[Sqrt[1 - x]/Sqrt[2]])/4
```

**IntegrateAlgebraic [A]** time = 0.07, size = 100, normalized size = 1.15

$$\frac{\sqrt{x+1}\left(\frac{105(x+1)^3}{(1-x)^3}+\frac{385(x+1)^2}{(1-x)^2}+\frac{511(x+1)}{1-x}+279\right)}{12\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^4} + \frac{35}{4}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(279 + (511*(1 + x))/(1 - x) + (385*(1 + x)^2)/(1 - x)^2 + (10
5*(1 + x)^3)/(1 - x)^3))/(12*Sqrt[1 - x]*(1 + (1 + x)/(1 - x))^4) + (35*Arc
Tan[Sqrt[1 + x]/Sqrt[1 - x]])/4
```

**fricas [A]** time = 1.29, size = 52, normalized size = 0.60

$$-\frac{1}{24}\left(6x^{3}-32x^{2}+81x-160\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{35}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*x^3 - 32*x^2 + 81*x - 160)*sqrt(x + 1)*sqrt(-x + 1) - 35/4*arctan(
(sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac [A]** time = 0.76, size = 101, normalized size = 1.16

$$-\frac{1}{24}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{2}\left(\left(2x-5\right)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{2}\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{35}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) +
 1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 3/2*sqrt(x + 1)*(x
- 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 35/4*arcsin(1/2*sqrt(2)*sqrt
(x + 1))
```

**maple [A]** time = 0.01, size = 85, normalized size = 0.98

$$\frac{35\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{7}{2}}\sqrt{x+1}}{4}+\frac{7(-x+1)^{\frac{5}{2}}\sqrt{x+1}}{12}+\frac{35(-x+1)^{\frac{3}{2}}\sqrt{x+1}}{24}+\frac{35\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)/(x+1)^(1/2),x)
```

```
[Out] 1/4*(-x+1)^(7/2)*(x+1)^(1/2)+7/12*(-x+1)^(5/2)*(x+1)^(1/2)+35/24*(-x+1)^(3/
2)*(x+1)^(1/2)+35/8*(-x+1)^(1/2)*(x+1)^(1/2)+35/8*((x+1)*(-x+1))^(1/2)/(x+1
)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 2.99, size = 56, normalized size = 0.64

$$-\frac{1}{4}\sqrt{-x^{2}+1}\,x^{3}+\frac{4}{3}\sqrt{-x^{2}+1}\,x^{2}-\frac{27}{8}\sqrt{-x^{2}+1}\,x+\frac{20}{3}\sqrt{-x^{2}+1}+\frac{35}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-x^2 + 1)*x^3 + 4/3*sqrt(-x^2 + 1)*x^2 - 27/8*sqrt(-x^2 + 1)*x +
20/3*sqrt(-x^2 + 1) + 35/8*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(1/2), x)
```

**sympy [A]** time = 14.68, size = 201, normalized size = 2.31

$$\begin{cases} -\frac{35i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}} + \frac{31i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}} - \frac{263i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}} + \frac{605i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}} - \frac{93i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -\frac{\sqrt{1-x}\,(x+1)^{\frac{7}{2}}}{4} + \frac{25\sqrt{1-x}\,(x+1)^{\frac{5}{2}}}{12} - \frac{163\sqrt{1-x}\,(x+1)^{\frac{3}{2}}}{24} + \frac{93\sqrt{1-x}\,\sqrt{x+1}}{8} + \frac{35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-35*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(9/2)/(4*sqrt(
x - 1)) + 31*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) - 263*I*(x + 1)**(5/2)/(24*s
qrt(x - 1)) + 605*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) - 93*I*sqrt(x + 1)/(4*s
qrt(x - 1)), Abs(x + 1)/2 > 1), (-sqrt(1 - x)*(x + 1)**(7/2)/4 + 25*sqrt(1
- x)*(x + 1)**(5/2)/12 - 163*sqrt(1 - x)*(x + 1)**(3/2)/24 + 93*sqrt(1 - x)
*sqrt(x + 1)/8 + 35*asin(sqrt(2)*sqrt(x + 1)/2)/4, True))
```

## 3.1038 $\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=67

$$\frac{1}{3}\sqrt{x+1}\,(1-x)^{5/2}+\frac{5}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{5}{2}\sqrt{x+1}\,\sqrt{1-x}\,+\frac{5}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\frac{1}{3}\sqrt{x+1}\,(1-x)^{5/2}+\frac{5}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{5}{2}\sqrt{x+1}\,\sqrt{1-x}\,+\frac{5}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (5*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*(1 - x)^(3/2)*Sqrt[1 + x])/6 + ((1 - x)^
(5/2)*Sqrt[1 + x])/3 + (5*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx &= \frac{1}{3}(1-x)^{5/2}\sqrt{1+x} + \frac{5}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\ &= \frac{5}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{1}{3}(1-x)^{5/2}\sqrt{1+x} + \frac{5}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\ &= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{1}{3}(1-x)^{5/2}\sqrt{1+x} + \frac{5}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\ &= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{1}{3}(1-x)^{5/2}\sqrt{1+x} + \frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{1}{3}(1-x)^{5/2}\sqrt{1+x} + \frac{5}{2}\sin^{-1}(x)\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 54, normalized size = 0.81

$$\frac{\sqrt{x+1}\left(-2x^3+11x^2-31x+22\right)}{6\sqrt{1-x}} - 5\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(22 - 31*x + 11*x^2 - 2*x^3))/(6*Sqrt[1 - x]) - 5*ArcSin[Sqrt[
1 - x]/Sqrt[2]]
```

**IntegrateAlgebraic** [A] time = 0.07, size = 84, normalized size = 1.25

$$\frac{\sqrt{x+1}\left(\frac{15(x+1)^2}{(1-x)^2}+\frac{40(x+1)}{1-x}+33\right)}{3\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^3} + 5\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(33 + (40*(1 + x))/(1 - x) + (15*(1 + x)^2)/(1 - x)^2))/(3*Sqr
t[1 - x]*(1 + (1 + x)/(1 - x))^3) + 5*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

**fricas** [A] time = 1.26, size = 47, normalized size = 0.70

$$\frac{1}{6}\left(2\,x^2-9\,x+22\right)\sqrt{x+1}\,\sqrt{-x+1} - 5\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 - 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*sqr
t(-x + 1) - 1)/x)
```

**giac** [A] time = 0.70, size = 69, normalized size = 1.03

$$\frac{1}{6}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+5\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*
sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple** [A] time = 0.00, size = 71, normalized size = 1.06

$$\frac{5\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{2\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{5}{2}}\sqrt{x+1}}{3}+\frac{5(-x+1)^{\frac{3}{2}}\sqrt{x+1}}{6}+\frac{5\sqrt{-x+1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)/(x+1)^(1/2),x)
```

```
[Out] 1/3*(-x+1)^(5/2)*(x+1)^(1/2)+5/6*(-x+1)^(3/2)*(x+1)^(1/2)+5/2*(-x+1)^(1/2)*
(x+1)^(1/2)+5/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima** [A] time = 2.90, size = 42, normalized size = 0.63

$$\frac{1}{3}\sqrt{-x^2+1}\,x^2-\frac{3}{2}\sqrt{-x^2+1}\,x+\frac{11}{3}\sqrt{-x^2+1}+\frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x + 11/3*sqrt(-x^2 + 1) + 5/2*a
rcsin(x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{5/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)/(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(5/2)/(x + 1)^(1/2), x)
```

**sympy** **[A]** time = 5.64, size = 175, normalized size = 2.61

$$\begin{cases} -5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} - \frac{17i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} + \frac{59i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} - \frac{11i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ 5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} + \frac{17(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} - \frac{59(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} + \frac{11\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) - 17*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) + 59*I*(x + 1)**(3/2)/(6*sqrt(x
- 1)) - 11*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sq
rt(x + 1)/2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 17*(x + 1)**(5/2)/(6*sqrt(1
 - x)) - 59*(x + 1)**(3/2)/(6*sqrt(1 - x)) + 11*sqrt(x + 1)/sqrt(1 - x), Tr
ue))
```

## 3.1039 $\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=47

$$\frac{1}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{3}{2}\sqrt{x+1}\,\sqrt{1-x}\,+\frac{3}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\frac{1}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{3}{2}\sqrt{x+1}\,\sqrt{1-x}\,+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/Sqrt[1 + x],x]
```

```
[Out] (3*Sqrt[1 - x]*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*ArcSin[x
])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx &= \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\ &= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\ &= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\sin^{-1}(x)\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 47, normalized size = 1.00

$$\frac{\sqrt{x+1}\,(x^2-5x+4)}{2\sqrt{1-x}} - 3\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/Sqrt[1 + x],x]
[Out] (Sqrt[1 + x]*(4 - 5*x + x^2))/(2*Sqrt[1 - x]) - 3*ArcSin[Sqrt[1 - x]/Sqrt[2
]]
```

**IntegrateAlgebraic** [A] time = 0.07, size = 67, normalized size = 1.43

$$\frac{\sqrt{x+1}\left(\frac{3(x+1)}{1-x}+5\right)}{\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^2} + 3\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)/Sqrt[1 + x],x]
[Out] (Sqrt[1 + x]*(5 + (3*(1 + x))/(1 - x)))/(Sqrt[1 - x]*(1 + (1 + x)/(1 - x))^
2) + 3*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

**fricas** [A] time = 1.40, size = 40, normalized size = 0.85

$$-\frac{1}{2}\sqrt{x+1}\,(x-4)\sqrt{-x+1} - 3\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 4)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
- 1)/x)
```

**giac** [A] time = 0.70, size = 44, normalized size = 0.94

$$-\frac{1}{2}\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+3\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 3*arcsin
(1/2*sqrt(2)*sqrt(x + 1))
```

**maple** [A] time = 0.00, size = 57, normalized size = 1.21

$$\frac{3\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{2\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{3}{2}}\sqrt{x+1}}{2}+\frac{3\sqrt{-x+1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)/(x+1)^(1/2),x)
```

```
[Out] 1/2*(-x+1)^(3/2)*(x+1)^(1/2)+3/2*(-x+1)^(1/2)*(x+1)^(1/2)+3/2*((x+1)*(-x+1)
)^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima** [A] time = 3.02, size = 28, normalized size = 0.60

$$-\frac{1}{2}\sqrt{-x^2+1}\,x+2\sqrt{-x^2+1}+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x + 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{3/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(3/2)/(x + 1)^(1/2), x)
```

**sympy [A]** time = 2.59, size = 139, normalized size = 2.96

$$\begin{cases} -3i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} + \frac{7i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} - \frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ 3\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} - \frac{7(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} + \frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) + 7*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 5*I*sqrt(x + 1)/sqrt(x - 1), Ab
s(x + 1)/2 > 1), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1
- x)) - 7*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 5*sqrt(x + 1)/sqrt(1 - x), True)
)
```

# 3.1040 $\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=20

$$\sqrt{1-x}\,\sqrt{x+1}+\sin^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\sqrt{1-x}\,\sqrt{x+1}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 - x]*Sqrt[1 + x] + ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx = \sqrt{1-x}\,\sqrt{1+x} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= \sqrt{1-x}\,\sqrt{1+x} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \sqrt{1-x}\,\sqrt{1+x} + \sin^{-1}(x)$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 1.50

$$\sqrt{1-x^2} - 2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/Sqrt[1 + x],x]
[Out] Sqrt[1 - x^2] - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

**IntegrateAlgebraic** [C] time = 0.09, size = 44, normalized size = 2.20

$$\sqrt{1-x}\,\sqrt{x+1} + 2i\log\left(\sqrt{1-x} - i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]/Sqrt[1 + x],x]
[Out] Sqrt[1 - x]*Sqrt[1 + x] + (2*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

**fricas** [B] time = 1.28, size = 36, normalized size = 1.80

$$\sqrt{x+1}\,\sqrt{-x+1} - 2\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac** [A] time = 0.65, size = 27, normalized size = 1.35

$$\sqrt{x+1}\,\sqrt{-x+1} + 2\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [B]** time = 0.00, size = 41, normalized size = 2.05

$$\frac{\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\ \sqrt{-x+1}}+\sqrt{-x+1}\ \sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(x+1)^(1/2),x)
[Out] (-x+1)^(1/2)*(x+1)^(1/2)+((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcs
in(x)
```

**maxima [A]** time = 3.10, size = 12, normalized size = 0.60

$$\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] sqrt(-x^2 + 1) + arcsin(x)
```

**mupad [B]** time = 0.12, size = 12, normalized size = 0.60

$$\mathrm{asin}(x)+\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(1/2),x)
[Out] asin(x) + (1 - x^2)^(1/2)
```

**sympy [B]** time = 1.55, size = 100, normalized size = 5.00

$$\begin{cases} -2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}}-\frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ 2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1)
 - 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (2*asin(sqrt(2)*sqrt(x +
 1)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 2*sqrt(x + 1)/sqrt(1 - x), True))
```

## 3.1041 $\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=2

$$\sin^{-1}(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {41, 216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx &= \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sin^{-1}(x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 2, normalized size = 1.00

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] ArcSin[x]
```

**IntegrateAlgebraic [B]** time = 0.04, size = 20, normalized size = 10.00

$$-2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
[Out] -2*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

**fricas [B]** time = 0.83, size = 22, normalized size = 11.00

$$-2\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

**giac [B]** time = 0.65, size = 13, normalized size = 6.50

$$2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [B]** time = 0.00, size = 27, normalized size = 13.50

$$\frac{\sqrt{(x+1)(-x+1)}\arcsin(x)}{\sqrt{x+1}\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(1/2)/(x+1)^(1/2),x)
[Out] ((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima** [A] time = 2.95, size = 2, normalized size = 1.00

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] arcsin(x)
```

**mupad** [B] time = 0.08, size = 22, normalized size = 11.00

$$-4\,\mathrm{atan}\left(\frac{\sqrt{1-x}\,-1}{\sqrt{x+1}\,-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(1/2)),x)
[Out] -4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1))
```

**sympy** [B] time = 1.04, size = 41, normalized size = 20.50

$$\begin{cases} -2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) & \text{for}\ \frac{|x+1|}{2} > 1 \\ 2\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2), Abs(x + 1)/2 > 1), (2*asin(sq
rt(2)*sqrt(x + 1)/2), True))
```

## 3.1042 $\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=17

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{\sqrt{1-x}}$$

**Mathematica [A]** time = 0.00, size = 17, normalized size = 1.00

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

**IntegrateAlgebraic [A]** time = 0.02, size = 17, normalized size = 1.00

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

**fricas [A]** time = 1.15, size = 23, normalized size = 1.35

$$\frac{x-\sqrt{x+1}\,\sqrt{-x+1}\,-1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] (x - sqrt(x + 1)*sqrt(-x + 1) - 1)/(x - 1)
```

**giac [A]** time = 0.68, size = 19, normalized size = 1.12

$$-\frac{\sqrt{x+1}\,\sqrt{-x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -sqrt(x + 1)*sqrt(-x + 1)/(x - 1)
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.82

$$\frac{\sqrt{x+1}}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(3/2)/(x+1)^(1/2),x)
[Out] (x+1)^(1/2)/(-x+1)^(1/2)
```

**maxima [A]** time = 2.98, size = 16, normalized size = 0.94

$$-\frac{\sqrt{-x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/(x - 1)
```

**mupad [B]** time = 0.28, size = 13, normalized size = 0.76

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(1/2)),x)
```

```
[Out] (x + 1)^(1/2)/(1 - x)^(1/2)
```

**sympy [A]** time = 0.94, size = 29, normalized size = 1.71

$$\begin{cases} \frac{1}{\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{2}{|x+1|} > 1 \\ -\frac{i}{\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((1/sqrt(-1 + 2/(x + 1)), 2/Abs(x + 1) > 1), (-I/sqrt(1 - 2/(x + 1
)), True))
```

## 3.1043 $\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=41

$$\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(3*(1 - x)^(3/2)) + Sqrt[1 + x]/(3*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{1}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{\sqrt{1+x}}{3\sqrt{1-x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.56

$$-\frac{(x-2)\sqrt{x+1}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
```

```
[Out] -1/3*((-2 + x)*Sqrt[1 + x])/(1 - x)^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.06, size = 33, normalized size = 0.80

$$\frac{\sqrt{x+1}\left(\frac{x+1}{1-x}+3\right)}{6\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(3 + (1 + x)/(1 - x)))/(6*Sqrt[1 - x])
```

**fricas** [A] time = 1.20, size = 39, normalized size = 0.95

$$\frac{2\,x^2-\sqrt{x+1}\,(x-2)\sqrt{-x+1}\,-4\,x+2}{3\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x^2 - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 4*x + 2)/(x^2 - 2*x + 1)
```

**giac** [A] time = 0.64, size = 22, normalized size = 0.54

$$-\frac{\sqrt{x+1}\,(x-2)\sqrt{-x+1}}{3\,(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(x + 1)*(x - 2)*sqrt(-x + 1)/(x - 1)^2
```

**maple [A]** time = 0.00, size = 18, normalized size = 0.44

$$-\frac{\sqrt{x+1}\,\left(x-2\right)}{3\left(-x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(5/2)/(x+1)^(1/2),x)
```

```
[Out] -1/3*(x+1)^(1/2)*(-2+x)/(-x+1)^(3/2)
```

**maxima [A]** time = 3.12, size = 38, normalized size = 0.93

$$\frac{\sqrt{-x^2+1}}{3\left(x^2-2\,x+1\right)}-\frac{\sqrt{-x^2+1}}{3\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x - 1)
```

**mupad [B]** time = 0.31, size = 43, normalized size = 1.05

$$\frac{x\,\sqrt{1-x}+2\,\sqrt{1-x}-x^2\,\sqrt{1-x}}{3\,{\left(x-1\right)}^2\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(1/2)),x)
```

```
[Out] (x*(1 - x)^(1/2) + 2*(1 - x)^(1/2) - x^2*(1 - x)^(1/2))/(3*(x - 1)^2*(x + 1
)^(1/2))
```

**sympy [C]** time = 2.25, size = 139, normalized size = 3.39

$$\begin{cases} \frac{i(x+1)}{3i\sqrt{-1+\frac{2}{x+1}}\,(x+1)-6i\sqrt{-1+\frac{2}{x+1}}}-\frac{3i}{3i\sqrt{-1+\frac{2}{x+1}}\,(x+1)-6i\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{2}{|x+1|}>1 \\ -\frac{x+1}{-3i\sqrt{1-\frac{2}{x+1}}\,(x+1)+6i\sqrt{1-\frac{2}{x+1}}}+\frac{3}{-3i\sqrt{1-\frac{2}{x+1}}\,(x+1)+6i\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((I*(x + 1)/(3*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*I*sqrt(-1 + 2/(x
 + 1))) - 3*I/(3*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*I*sqrt(-1 + 2/(x + 1)))
, 2/Abs(x + 1) > 1), (-(x + 1)/(-3*I*sqrt(1 - 2/(x + 1))*(x + 1) + 6*I*sqrt
(1 - 2/(x + 1))) + 3/(-3*I*sqrt(1 - 2/(x + 1))*(x + 1) + 6*I*sqrt(1 - 2/(x
+ 1))), True))
```

## 3.1044 $\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=61

$$\frac{2\sqrt{x+1}}{15\sqrt{1-x}}+\frac{2\sqrt{x+1}}{15(1-x)^{3/2}}+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2\sqrt{x+1}}{15\sqrt{1-x}}+\frac{2\sqrt{x+1}}{15(1-x)^{3/2}}+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(5*(1 - x)^(5/2)) + (2*Sqrt[1 + x])/(15*(1 - x)^(3/2)) + (2*Sqr
t[1 + x])/(15*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{5(1-x)^{5/2}} + \frac{2}{5}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1+x}}{5(1-x)^{5/2}} + \frac{2\sqrt{1+x}}{15(1-x)^{3/2}} + \frac{2}{15}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1+x}}{5(1-x)^{5/2}} + \frac{2\sqrt{1+x}}{15(1-x)^{3/2}} + \frac{2\sqrt{1+x}}{15\sqrt{1-x}}$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 0.49

$$\frac{\sqrt{x+1}\left(2x^2-6x+7\right)}{15(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(7 - 6*x + 2*x^2))/(15*(1 - x)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.06, size = 48, normalized size = 0.79

$$\frac{\sqrt{x+1}\left(\frac{3(x+1)^2}{(1-x)^2}+\frac{10(x+1)}{1-x}+15\right)}{60\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(15 + (10*(1 + x))/(1 - x) + (3*(1 + x)^2)/(1 - x)^2))/(60*Sqr
t[1 - x])
```

**fricas** [A] time = 1.18, size = 56, normalized size = 0.92

$$\frac{7\,x^3-21\,x^2-\left(2\,x^2-6\,x+7\right)\sqrt{x+1}\,\sqrt{-x+1}+21\,x-7}{15\left(x^3-3\,x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*x^3 - 21*x^2 - (2*x^2 - 6*x + 7)*sqrt(x + 1)*sqrt(-x + 1) + 21*x -
7)/(x^3 - 3*x^2 + 3*x - 1)
```

**giac [A]** time = 0.69, size = 29, normalized size = 0.48

$$-\frac{(2(x+1)(x-4)+15)\sqrt{x+1}\sqrt{-x+1}}{15(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -1/15*(2*(x + 1)*(x - 4) + 15)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.41

$$\frac{\sqrt{x+1}\,(2x^2-6x+7)}{15(-x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(7/2)/(x+1)^(1/2),x)
[Out] 1/15*(x+1)^(1/2)*(2*x^2-6*x+7)/(-x+1)^(5/2)
```

**maxima [A]** time = 3.03, size = 64, normalized size = 1.05

$$-\frac{\sqrt{-x^2+1}}{5(x^3-3x^2+3x-1)}+\frac{2\sqrt{-x^2+1}}{15(x^2-2x+1)}-\frac{2\sqrt{-x^2+1}}{15(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] -1/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 2/15*sqrt(-x^2 + 1)/(x^2 - 2*
x + 1) - 2/15*sqrt(-x^2 + 1)/(x - 1)
```

**mupad [B]** time = 0.32, size = 55, normalized size = 0.90

$$-\frac{x\sqrt{1-x}+7\sqrt{1-x}-4x^2\sqrt{1-x}+2x^3\sqrt{1-x}}{15(x-1)^3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(1/2)),x)
[Out] -(x*(1 - x)^(1/2) + 7*(1 - x)^(1/2) - 4*x^2*(1 - x)^(1/2) + 2*x^3*(1 - x)^(
1/2))/(15*(x - 1)^3*(x + 1)^(1/2))
```

**sympy [C]** time = 7.89, size = 332, normalized size = 5.44

$$\begin{cases} -\frac{2i(x+1)^2}{-15i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+60i\sqrt{-1+\frac{2}{x+1}}(x+1)-60i\sqrt{-1+\frac{2}{x+1}}} + \frac{10i(x+1)}{-15i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+60i\sqrt{-1+\frac{2}{x+1}}(x+1)-60i\sqrt{-1+\frac{2}{x+1}}} - \frac{15i}{-15i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+60i\sqrt{-1+\frac{2}{x+1}}(x+1)-60i\sqrt{-1+\frac{2}{x+1}}} & \text{for } \frac{2}{|x+1|} > 1 \\ \frac{2(x+1)^2}{15i\sqrt{1-\frac{2}{x+1}}(x+1)^2-60i\sqrt{1-\frac{2}{x+1}}(x+1)+60i\sqrt{1-\frac{2}{x+1}}} - \frac{10(x+1)}{15i\sqrt{1-\frac{2}{x+1}}(x+1)^2-60i\sqrt{1-\frac{2}{x+1}}(x+1)+60i\sqrt{1-\frac{2}{x+1}}} + \frac{15}{15i\sqrt{1-\frac{2}{x+1}}(x+1)^2-60i\sqrt{1-\frac{2}{x+1}}(x+1)+60i\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*(x + 1)**2/(-15*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 60*I*sq
rt(-1 + 2/(x + 1))*(x + 1) - 60*I*sqrt(-1 + 2/(x + 1))) + 10*I*(x + 1)/(-15
*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 60*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 60
*I*sqrt(-1 + 2/(x + 1))) - 15*I/(-15*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 60
*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 60*I*sqrt(-1 + 2/(x + 1))), 2/Abs(x + 1)
> 1), (2*(x + 1)**2/(15*I*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 60*I*sqrt(1 - 2/
(x + 1))*(x + 1) + 60*I*sqrt(1 - 2/(x + 1))) - 10*(x + 1)/(15*I*sqrt(1 - 2/
(x + 1))*(x + 1)**2 - 60*I*sqrt(1 - 2/(x + 1))*(x + 1) + 60*I*sqrt(1 - 2/(x
 + 1))) + 15/(15*I*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 60*I*sqrt(1 - 2/(x + 1)
)*(x + 1) + 60*I*sqrt(1 - 2/(x + 1))), True))
```

## 3.1045 $\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=81

$$\frac{2\sqrt{x+1}}{35\sqrt{1-x}}+\frac{2\sqrt{x+1}}{35(1-x)^{3/2}}+\frac{3\sqrt{x+1}}{35(1-x)^{5/2}}+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2\sqrt{x+1}}{35\sqrt{1-x}}+\frac{2\sqrt{x+1}}{35(1-x)^{3/2}}+\frac{3\sqrt{x+1}}{35(1-x)^{5/2}}+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(7*(1 - x)^(7/2)) + (3*Sqrt[1 + x])/(35*(1 - x)^(5/2)) + (2*Sqr
t[1 + x])/(35*(1 - x)^(3/2)) + (2*Sqrt[1 + x])/(35*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{7(1-x)^{7/2}} + \frac{3}{7}\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1+x}}{7(1-x)^{7/2}} + \frac{3\sqrt{1+x}}{35(1-x)^{5/2}} + \frac{6}{35}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1+x}}{7(1-x)^{7/2}} + \frac{3\sqrt{1+x}}{35(1-x)^{5/2}} + \frac{2\sqrt{1+x}}{35(1-x)^{3/2}} + \frac{2}{35}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1+x}}{7(1-x)^{7/2}} + \frac{3\sqrt{1+x}}{35(1-x)^{5/2}} + \frac{2\sqrt{1+x}}{35(1-x)^{3/2}} + \frac{2\sqrt{1+x}}{35\sqrt{1-x}}$$

**Mathematica** [A] time = 0.01, size = 35, normalized size = 0.43

$$\frac{\sqrt{x+1}\left(-2x^3+8x^2-13x+12\right)}{35(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
[Out] (Sqrt[1 + x]*(12 - 13*x + 8*x^2 - 2*x^3))/(35*(1 - x)^(7/2))
```

**IntegrateAlgebraic** [A] time = 0.07, size = 62, normalized size = 0.77

$$\frac{\sqrt{x+1}\left(\frac{5(x+1)^3}{(1-x)^3}+\frac{21(x+1)^2}{(1-x)^2}+\frac{35(x+1)}{1-x}+35\right)}{280\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
[Out] (Sqrt[1 + x]*(35 + (35*(1 + x))/(1 - x) + (21*(1 + x)^2)/(1 - x)^2 + (5*(1
+ x)^3)/(1 - x)^3))/(280*Sqrt[1 - x])
```

**fricas** [A] time = 1.28, size = 71, normalized size = 0.88

$$\frac{12\,x^4-48\,x^3+72\,x^2-\left(2\,x^3-8\,x^2+13\,x-12\right)\sqrt{x+1}\,\sqrt{-x+1}-48\,x+12}{35\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(12*x^4 - 48*x^3 + 72*x^2 - (2*x^3 - 8*x^2 + 13*x - 12)*sqrt(x + 1)*sq
rt(-x + 1) - 48*x + 12)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

**giac [A]** time = 0.66, size = 35, normalized size = 0.43

$$-\frac{((2(x+1)(x-6)+35)(x+1)-35)\sqrt{x+1}\sqrt{-x+1}}{35(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35*((2*(x + 1)*(x - 6) + 35)*(x + 1) - 35)*sqrt(x + 1)*sqrt(-x + 1)/(x -
 1)^4
```

**maple [A]** time = 0.00, size = 30, normalized size = 0.37

$$-\frac{\sqrt{x+1}\,\left(2x^3-8x^2+13x-12\right)}{35(-x+1)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(9/2)/(x+1)^(1/2),x)
```

```
[Out] -1/35*(x+1)^(1/2)*(2*x^3-8*x^2+13*x-12)/(-x+1)^(7/2)
```

**maxima [A]** time = 3.03, size = 95, normalized size = 1.17

$$\frac{\sqrt{-x^2+1}}{7\left(x^4-4x^3+6x^2-4x+1\right)}-\frac{3\sqrt{-x^2+1}}{35\left(x^3-3x^2+3x-1\right)}+\frac{2\sqrt{-x^2+1}}{35\left(x^2-2x+1\right)}-\frac{2\sqrt{-x^2+1}}{35(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/35*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) + 2/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/35*sqrt(-x^
2 + 1)/(x - 1)
```

**mupad [B]** time = 0.34, size = 67, normalized size = 0.83

$$-\frac{x\sqrt{1-x}-12\sqrt{1-x}+5x^2\sqrt{1-x}-6x^3\sqrt{1-x}+2x^4\sqrt{1-x}}{35(x-1)^4\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(9/2)*(x + 1)^(1/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) - 12*(1 - x)^(1/2) + 5*x^2*(1 - x)^(1/2) - 6*x^3*(1 - x)^
(1/2) + 2*x^4*(1 - x)^(1/2))/(35*(x - 1)^4*(x + 1)^(1/2))
```

**sympy [C]** time = 22.13, size = 595, normalized size = 7.35

$$\begin{cases} \frac{2i(x+1)^3}{35i\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420i\sqrt{-1+\frac{2}{x+1}}(x+1)-280i\sqrt{-1+\frac{2}{x+1}}} - \frac{14i(x+1)^2}{35i\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420i\sqrt{-1+\frac{2}{x+1}}(x+1)-280i\sqrt{-1+\frac{2}{x+1}}} + \frac{35i(x+1)}{35i\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420i\sqrt{-1+\frac{2}{x+1}}(x+1)-280i\sqrt{-1+\frac{2}{x+1}}} - \frac{35i}{35i\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420i\sqrt{-1+\frac{2}{x+1}}(x+1)-280i\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{2}{|x+1|} > 1 \\ -\frac{2(x+1)^3}{-35i\sqrt{1-\frac{2}{x+1}}(x+1)^3+210i\sqrt{1-\frac{2}{x+1}}(x+1)^2-420i\sqrt{1-\frac{2}{x+1}}(x+1)+280i\sqrt{1-\frac{2}{x+1}}} + \frac{14(x+1)^2}{-35i\sqrt{1-\frac{2}{x+1}}(x+1)^3+210i\sqrt{1-\frac{2}{x+1}}(x+1)^2-420i\sqrt{1-\frac{2}{x+1}}(x+1)+280i\sqrt{1-\frac{2}{x+1}}} - \frac{35(x+1)}{-35i\sqrt{1-\frac{2}{x+1}}(x+1)^3+210i\sqrt{1-\frac{2}{x+1}}(x+1)^2-420i\sqrt{1-\frac{2}{x+1}}(x+1)+280i\sqrt{1-\frac{2}{x+1}}} + \frac{35}{-35i\sqrt{1-\frac{2}{x+1}}(x+1)^3+210i\sqrt{1-\frac{2}{x+1}}(x+1)^2-420i\sqrt{1-\frac{2}{x+1}}(x+1)+280i\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**3/(35*I*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*I*sqr
t(-1 + 2/(x + 1))*(x + 1)**2 + 420*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*I*s
qrt(-1 + 2/(x + 1))) - 14*I*(x + 1)**2/(35*I*sqrt(-1 + 2/(x + 1))*(x + 1)**
3 - 210*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 420*I*sqrt(-1 + 2/(x + 1))*(x +
 1) - 280*I*sqrt(-1 + 2/(x + 1))) + 35*I*(x + 1)/(35*I*sqrt(-1 + 2/(x + 1))
*(x + 1)**3 - 210*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 420*I*sqrt(-1 + 2/(x
+ 1))*(x + 1) - 280*I*sqrt(-1 + 2/(x + 1))) - 35*I/(35*I*sqrt(-1 + 2/(x + 1
))*(x + 1)**3 - 210*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 420*I*sqrt(-1 + 2/(
x + 1))*(x + 1) - 280*I*sqrt(-1 + 2/(x + 1))), 2/Abs(x + 1) > 1), (-2*(x +
1)**3/(-35*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 210*I*sqrt(1 - 2/(x + 1))*(x
+ 1)**2 - 420*I*sqrt(1 - 2/(x + 1))*(x + 1) + 280*I*sqrt(1 - 2/(x + 1))) +
14*(x + 1)**2/(-35*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 210*I*sqrt(1 - 2/(x +
 1))*(x + 1)**2 - 420*I*sqrt(1 - 2/(x + 1))*(x + 1) + 280*I*sqrt(1 - 2/(x +
 1))) - 35*(x + 1)/(-35*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 210*I*sqrt(1 - 2
/(x + 1))*(x + 1)**2 - 420*I*sqrt(1 - 2/(x + 1))*(x + 1) + 280*I*sqrt(1 - 2
/(x + 1))) + 35/(-35*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 210*I*sqrt(1 - 2/(x
 + 1))*(x + 1)**2 - 420*I*sqrt(1 - 2/(x + 1))*(x + 1) + 280*I*sqrt(1 - 2/(x
 + 1))), True))
```

## 3.1046 $\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx$

**Optimal**. Leaf size=101

$$\frac{8\sqrt{x+1}}{315\sqrt{1-x}}+\frac{8\sqrt{x+1}}{315(1-x)^{3/2}}+\frac{4\sqrt{x+1}}{105(1-x)^{5/2}}+\frac{4\sqrt{x+1}}{63(1-x)^{7/2}}+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8\sqrt{x+1}}{315\sqrt{1-x}}+\frac{8\sqrt{x+1}}{315(1-x)^{3/2}}+\frac{4\sqrt{x+1}}{105(1-x)^{5/2}}+\frac{4\sqrt{x+1}}{63(1-x)^{7/2}}+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(9*(1 - x)^(9/2)) + (4*Sqrt[1 + x])/(63*(1 - x)^(7/2)) + (4*Sqr
t[1 + x])/(105*(1 - x)^(5/2)) + (8*Sqrt[1 + x])/(315*(1 - x)^(3/2)) + (8*Sq
rt[1 + x])/(315*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4}{9}\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4}{21}\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4\sqrt{1+x}}{105(1-x)^{5/2}}+\frac{8}{105}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4\sqrt{1+x}}{105(1-x)^{5/2}}+\frac{8\sqrt{1+x}}{315(1-x)^{3/2}}+\frac{8}{315}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\\ &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4\sqrt{1+x}}{105(1-x)^{5/2}}+\frac{8\sqrt{1+x}}{315(1-x)^{3/2}}+\frac{8\sqrt{1+x}}{315\sqrt{1-x}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.40

$$\frac{\sqrt{x+1}\left(8x^4-40x^3+84x^2-100x+83\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
[Out] (Sqrt[1 + x]*(83 - 100*x + 84*x^2 - 40*x^3 + 8*x^4))/(315*(1 - x)^(9/2))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 76, normalized size = 0.75

$$\frac{\sqrt{x+1}\left(\frac{35(x+1)^4}{(1-x)^4}+\frac{180(x+1)^3}{(1-x)^3}+\frac{378(x+1)^2}{(1-x)^2}+\frac{420(x+1)}{1-x}+315\right)}{5040\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
[Out] (Sqrt[1 + x]*(315 + (420*(1 + x))/(1 - x) + (378*(1 + x)^2)/(1 - x)^2 + (18
0*(1 + x)^3)/(1 - x)^3 + (35*(1 + x)^4)/(1 - x)^4))/(5040*Sqrt[1 - x])
```

**fricas [A]** time = 1.22, size = 86, normalized size = 0.85

$$\frac{83\,x^5-415\,x^4+830\,x^3-830\,x^2-\left(8\,x^4-40\,x^3+84\,x^2-100\,x+83\right)\sqrt{x+1}\,\sqrt{-x+1}+415\,x-83}{315\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(83*x^5 - 415*x^4 + 830*x^3 - 830*x^2 - (8*x^4 - 40*x^3 + 84*x^2 - 10
0*x + 83)*sqrt(x + 1)*sqrt(-x + 1) + 415*x - 83)/(x^5 - 5*x^4 + 10*x^3 - 10
*x^2 + 5*x - 1)
```

**giac [A]** time = 0.67, size = 42, normalized size = 0.42

$$-\frac{(4\left(\left(2\left(x+1\right)\left(x-8\right)+63\right)\left(x+1\right)-105\right)\left(x+1\right)+315)\sqrt{x+1}\,\sqrt{-x+1}}{315\left(x-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/315*(4*((2*(x + 1)*(x - 8) + 63)*(x + 1) - 105)*(x + 1) + 315)*sqrt(x +
1)*sqrt(-x + 1)/(x - 1)^5
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.35

$$\frac{\sqrt{x+1}\,\left(8x^{4}-40x^{3}+84x^{2}-100x+83\right)}{315\left(-x+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(11/2)/(x+1)^(1/2),x)
```

```
[Out] 1/315*(x+1)^(1/2)*(8*x^4-40*x^3+84*x^2-100*x+83)/(-x+1)^(9/2)
```

**maxima [A]** time = 3.08, size = 131, normalized size = 1.30

$$-\frac{\sqrt{-x^{2}+1}}{9\left(x^{5}-5x^{4}+10x^{3}-10x^{2}+5x-1\right)}+\frac{4\sqrt{-x^{2}+1}}{63\left(x^{4}-4x^{3}+6x^{2}-4x+1\right)}-\frac{4\sqrt{-x^{2}+1}}{105\left(x^{3}-3x^{2}+3x-1\right)}+\frac{8\sqrt{-x^{2}+1}}{315\left(x^{2}-2x+1\right)}-\frac{8\sqrt{-x^{2}+1}}{315\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 4/63*sqrt(-
x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4/105*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) + 8/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 8/315*sqrt(-x^2 + 1)/
(x - 1)
```

**mupad [B]** time = 0.36, size = 80, normalized size = 0.79

$$\frac{17\,x\,\sqrt{1-x}\,-83\,\sqrt{1-x}\,+16\,x^{2}\,\sqrt{1-x}\,-44\,x^{3}\,\sqrt{1-x}\,+32\,x^{4}\,\sqrt{1-x}\,-8\,x^{5}\,\sqrt{1-x}}{315\left(x-1\right)^{5}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(1/2)),x)
```

```
[Out] (17*x*(1 - x)^(1/2) - 83*(1 - x)^(1/2) + 16*x^2*(1 - x)^(1/2) - 44*x^3*(1 -
 x)^(1/2) + 32*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/(315*(x - 1)^5*(x +
 1)^(1/2))
```

**sympy [C]** time = 58.39, size = 933, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-8*I*(x + 1)**4/(-315*I*sqrt(-1 + 2/(x + 1))*(x + 1)**4 + 2520*I
*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 7560*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 +
 10080*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 5040*I*sqrt(-1 + 2/(x + 1))) + 72*I
*(x + 1)**3/(-315*I*sqrt(-1 + 2/(x + 1))*(x + 1)**4 + 2520*I*sqrt(-1 + 2/(x
 + 1))*(x + 1)**3 - 7560*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 10080*I*sqrt(-
1 + 2/(x + 1))*(x + 1) - 5040*I*sqrt(-1 + 2/(x + 1))) - 252*I*(x + 1)**2/(-
315*I*sqrt(-1 + 2/(x + 1))*(x + 1)**4 + 2520*I*sqrt(-1 + 2/(x + 1))*(x + 1)
**3 - 7560*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 10080*I*sqrt(-1 + 2/(x + 1))
*(x + 1) - 5040*I*sqrt(-1 + 2/(x + 1))) + 420*I*(x + 1)/(-315*I*sqrt(-1 + 2
/(x + 1))*(x + 1)**4 + 2520*I*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 7560*I*sqrt
(-1 + 2/(x + 1))*(x + 1)**2 + 10080*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 5040*I
*sqrt(-1 + 2/(x + 1))) - 315*I/(-315*I*sqrt(-1 + 2/(x + 1))*(x + 1)**4 + 25
20*I*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 7560*I*sqrt(-1 + 2/(x + 1))*(x + 1)*
*2 + 10080*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 5040*I*sqrt(-1 + 2/(x + 1))), 2
/Abs(x + 1) > 1), (8*(x + 1)**4/(315*I*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 252
0*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*I*sqrt(1 - 2/(x + 1))*(x + 1)**2
- 10080*I*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*I*sqrt(1 - 2/(x + 1))) - 72*(x
 + 1)**3/(315*I*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 2520*I*sqrt(1 - 2/(x + 1))
*(x + 1)**3 + 7560*I*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 10080*I*sqrt(1 - 2/(x
 + 1))*(x + 1) + 5040*I*sqrt(1 - 2/(x + 1))) + 252*(x + 1)**2/(315*I*sqrt(1
 - 2/(x + 1))*(x + 1)**4 - 2520*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*I*s
qrt(1 - 2/(x + 1))*(x + 1)**2 - 10080*I*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*
I*sqrt(1 - 2/(x + 1))) - 420*(x + 1)/(315*I*sqrt(1 - 2/(x + 1))*(x + 1)**4
- 2520*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*I*sqrt(1 - 2/(x + 1))*(x + 1
)**2 - 10080*I*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*I*sqrt(1 - 2/(x + 1))) +
315/(315*I*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 2520*I*sqrt(1 - 2/(x + 1))*(x +
 1)**3 + 7560*I*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 10080*I*sqrt(1 - 2/(x + 1)
)*(x + 1) + 5040*I*sqrt(1 - 2/(x + 1))), True))
```

## 3.1047 $\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx$

**Optimal**. Leaf size=85

$$-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}-\frac{7}{3}\sqrt{x+1}\,(1-x)^{5/2}-\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}-\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}-\frac{35}{2}\sin^{-1}(x)$$

**Rubi** [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}-\frac{7}{3}\sqrt{x+1}\,(1-x)^{5/2}-\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}-\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}-\frac{35}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(7/2))/Sqrt[1 + x] - (35*Sqrt[1 - x]*Sqrt[1 + x])/2 - (35*(1 -
x)^(3/2)*Sqrt[1 + x])/6 - (7*(1 - x)^(5/2)*Sqrt[1 + x])/3 - (35*ArcSin[x])/
2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - 7\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\int \frac{1}{\sqrt{1-x}\,\cdot} \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\int \frac{1}{\sqrt{1-x^2}} \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\sin^{-1}(x)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 37, normalized size = 0.44

$$-\frac{(1-x)^{9/2}\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1-x}{2}\right)}{9\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/(1 + x)^(3/2),x]
[Out] -1/9*((1 - x)^(9/2)*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - x)/2])/Sqrt[2]
```

**IntegrateAlgebraic [A]** time = 0.10, size = 98, normalized size = 1.15

$$35\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right) - \frac{\sqrt{1-x}\left(\frac{48(1-x)^3}{(x+1)^3} + \frac{231(1-x)^2}{(x+1)^2} + \frac{280(1-x)}{x+1} + 105\right)}{3\sqrt{x+1}\left(\frac{1-x}{x+1}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)/(1 + x)^(3/2),x]
```

```
[Out] -1/3*(Sqrt[1 - x]*(105 + (48*(1 - x)^3)/(1 + x)^3 + (231*(1 - x)^2)/(1 + x)
^2 + (280*(1 - x))/(1 + x)))/(Sqrt[1 + x]*(1 + (1 - x)/(1 + x))^3) + 35*Arc
Tan[Sqrt[1 - x]/Sqrt[1 + x]]
```

**fricas [A]** time = 1.04, size = 65, normalized size = 0.76

$$-\frac{\left(2 x^{3}-13 x^{2}+55 x+166\right) \sqrt{x+1} \sqrt{-x+1}-210 (x+1) \arctan\left(\frac{\sqrt{x+1} \sqrt{-x+1}-1}{x}\right)+166 x+166}{6 (x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*((2*x^3 - 13*x^2 + 55*x + 166)*sqrt(x + 1)*sqrt(-x + 1) - 210*(x + 1)*
arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 166*x + 166)/(x + 1)
```

**giac [A]** time = 0.75, size = 81, normalized size = 0.95

$$-\frac{1}{6} ((2 x-17)(x+1)+87)\sqrt{x+1} \sqrt{-x+1}+\frac{8\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{8 \sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-35 \arcsin\left(\frac{1}{2} \sqrt{2} \sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*x - 17)*(x + 1) + 87)*sqrt(x + 1)*sqrt(-x + 1) + 8*(sqrt(2) - sqrt
(-x + 1))/sqrt(x + 1) - 8*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 35*arcsin(
1/2*sqrt(2)*sqrt(x + 1))
```

**maple [A]** time = 0.02, size = 84, normalized size = 0.99

$$-\frac{35\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{2\sqrt{x+1}\ \sqrt{-x+1}}+\frac{\left(2x^{4}-15x^{3}+68x^{2}+111x-166\right)\sqrt{(x+1)(-x+1)}}{6\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}\ \sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)/(x+1)^(3/2),x)
```

```
[Out] 1/6*(2*x^4-15*x^3+68*x^2+111*x-166)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/
2)/(-x+1)^(1/2)/(x+1)^(1/2)-35/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1
/2)*arcsin(x)
```

**maxima** [A] time = 2.84, size = 70, normalized size = 0.82

$$\frac{x^4}{3\sqrt{-x^2+1}} - \frac{5\,x^3}{2\sqrt{-x^2+1}} + \frac{34\,x^2}{3\sqrt{-x^2+1}} + \frac{37\,x}{2\sqrt{-x^2+1}} - \frac{83}{3\sqrt{-x^2+1}} - \frac{35}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^4/sqrt(-x^2 + 1) - 5/2*x^3/sqrt(-x^2 + 1) + 34/3*x^2/sqrt(-x^2 + 1) +
 37/2*x/sqrt(-x^2 + 1) - 83/3/sqrt(-x^2 + 1) - 35/2*arcsin(x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(3/2), x)
```

**sympy** [A] time = 17.48, size = 207, normalized size = 2.44

$$\begin{cases} 35i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} + \frac{23i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{125i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{13i\sqrt{x+1}}{\sqrt{x-1}} + \frac{32i}{\sqrt{x-1}\sqrt{x+1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} - \frac{23(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{125(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{13\sqrt{x+1}}{\sqrt{1-x}} - \frac{32}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((35*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) + 23*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 125*I*(x + 1)**(3/2)/(6*sqrt(x
 - 1)) + 13*I*sqrt(x + 1)/sqrt(x - 1) + 32*I/(sqrt(x - 1)*sqrt(x + 1)), Abs
(x + 1)/2 > 1), (-35*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(7/2)/(3*sqrt(1
 - x)) - 23*(x + 1)**(5/2)/(6*sqrt(1 - x)) + 125*(x + 1)**(3/2)/(6*sqrt(1 -
 x)) - 13*sqrt(x + 1)/sqrt(1 - x) - 32/(sqrt(1 - x)*sqrt(x + 1)), True))
```

## 3.1048 $\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx$

**Optimal**. Leaf size=65

$$-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}-\frac{5}{2}\sqrt{x+1}\,(1-x)^{3/2}-\frac{15}{2}\sqrt{x+1}\,\sqrt{1-x}-\frac{15}{2}\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}-\frac{5}{2}\sqrt{x+1}\,(1-x)^{3/2}-\frac{15}{2}\sqrt{x+1}\,\sqrt{1-x}-\frac{15}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(5/2))/Sqrt[1 + x] - (15*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*(1 - x
)^(3/2)*Sqrt[1 + x])/2 - (15*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}}-5\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}}-\frac{5}{2}(1-x)^{3/2}\sqrt{1+x}-\frac{15}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}}-\frac{15}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{2}(1-x)^{3/2}\sqrt{1+x}-\frac{15}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}}-\frac{15}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{2}(1-x)^{3/2}\sqrt{1+x}-\frac{15}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}}-\frac{15}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{2}(1-x)^{3/2}\sqrt{1+x}-\frac{15}{2}\sin^{-1}(x)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 37, normalized size = 0.57

$$-\frac{(1-x)^{7/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{1-x}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/(1 + x)^(3/2),x]
[Out] -1/7*((1 - x)^(7/2)*Hypergeometric2F1[3/2, 7/2, 9/2, (1 - x)/2])/Sqrt[2]
```

**IntegrateAlgebraic [A]** time = 0.09, size = 82, normalized size = 1.26

$$15\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)-\frac{\sqrt{1-x}\left(\frac{8(1-x)^2}{(x+1)^2}+\frac{25(1-x)}{x+1}+15\right)}{\sqrt{x+1}\left(\frac{1-x}{x+1}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)/(1 + x)^(3/2),x]
```

```
[Out] -((Sqrt[1 - x]*(15 + (8*(1 - x)^2)/(1 + x)^2 + (25*(1 - x))/(1 + x)))/(Sqrt
[1 + x]*(1 + (1 - x)/(1 + x))^2)) + 15*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

**fricas** [A] time = 1.32, size = 58, normalized size = 0.89

$$\frac{\left(x^2-7\,x-24\right)\sqrt{x+1}\,\sqrt{-x+1}+30\,(x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)-24\,x-24}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x + 1)*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x) - 24*x - 24)/(x + 1)
```

**giac** [A] time = 0.79, size = 73, normalized size = 1.12

$$\frac{1}{2}\,\sqrt{x+1}\,(x-8)\sqrt{-x+1}+\frac{4\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{4\,\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-15\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + 1)*(x - 8)*sqrt(-x + 1) + 4*(sqrt(2) - sqrt(-x + 1))/sqrt(x +
1) - 4*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 15*arcsin(1/2*sqrt(2)*sqrt(x
+ 1))
```

**maple** [A] time = 0.02, size = 77, normalized size = 1.18

$$-\frac{15\sqrt{(x+1)\,(-x+1)}\;\arcsin(x)}{2\sqrt{x+1}\,\sqrt{-x+1}}-\frac{\left(x^3-8x^2-17x+24\right)\sqrt{(x+1)\,(-x+1)}}{2\sqrt{-(x+1)\,(x-1)}\,\sqrt{-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)/(x+1)^(3/2),x)
```

```
[Out] -1/2*(x^3-8*x^2-17*x+24)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(
1/2)/(x+1)^(1/2)-15/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(
x)
```

**maxima** [A] time = 2.99, size = 56, normalized size = 0.86

$$-\frac{x^3}{2\,\sqrt{-x^2+1}}+\frac{4\,x^2}{\sqrt{-x^2+1}}+\frac{17\,x}{2\,\sqrt{-x^2+1}}-\frac{12}{\sqrt{-x^2+1}}-\frac{15}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^3/sqrt(-x^2 + 1) + 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) - 12
/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{5/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(5/2)/(x + 1)^(3/2), x)
```

**sympy [A]** time = 6.99, size = 168, normalized size = 2.58

$$\begin{cases} 15i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}-\frac{11i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}}+\frac{16i}{\sqrt{x-1}\,\sqrt{x+1}} & \text{for } \frac{|x+1|}{2}>1 \\ -15\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{11(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}}-\frac{16}{\sqrt{1-x}\,\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) - 11*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1) + 16
*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1)/2 > 1), (-15*asin(sqrt(2)*sqrt(x +
 1)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x)) + 11*(x + 1)**(3/2)/(2*sqrt(1 - x))
 - sqrt(x + 1)/sqrt(1 - x) - 16/(sqrt(1 - x)*sqrt(x + 1)), True))
```

## 3.1049 $\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}} \, dx$

**Optimal**. Leaf size=41

$$-\frac{2(1-x)^{3/2}}{\sqrt{x+1}}-3\sqrt{x+1}\,\sqrt{1-x}-3\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{3/2}}{\sqrt{x+1}}-3\sqrt{x+1}\,\sqrt{1-x}-3\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(3/2))/Sqrt[1 + x] - 3*Sqrt[1 - x]*Sqrt[1 + x] - 3*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\ &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\,\sqrt{1+x}-3\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\,\sqrt{1+x}-3\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\,\sqrt{1+x}-3\sin^{-1}(x)\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 37, normalized size = 0.90

$$-\frac{(1-x)^{5/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{1-x}{2}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/(1 + x)^(3/2),x]
[Out] -1/5*((1 - x)^(5/2)*Hypergeometric2F1[3/2, 5/2, 7/2, (1 - x)/2])/Sqrt[2]
```

**IntegrateAlgebraic [C]** time = 0.13, size = 49, normalized size = 1.20

$$\frac{(-x-5)\sqrt{1-x}}{\sqrt{x+1}}-6i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)/(1 + x)^(3/2),x]
[Out] ((-5 - x)*Sqrt[1 - x])/Sqrt[1 + x] - (6*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

**fricas [A]** time = 1.39, size = 53, normalized size = 1.29

$$-\frac{(x+5)\sqrt{x+1}\,\sqrt{-x+1}-6\,(x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+5\,x+5}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -((x + 5)*sqrt(x + 1)*sqrt(-x + 1) - 6*(x + 1)*arctan((sqrt(x + 1)*sqrt(-x
+ 1) - 1)/x) + 5*x + 5)/(x + 1)
```

**giac [B]** time = 0.73, size = 70, normalized size = 1.71

$$-\sqrt{x+1}\sqrt{-x+1} + \frac{2\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}} - \frac{2\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1) + 2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 2*sqrt
(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 6*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [B]** time = 0.02, size = 71, normalized size = 1.73

$$-\frac{3\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\ \sqrt{-x+1}} + \frac{\left(x^2+4x-5\right)\sqrt{(x+1)(-x+1)}}{\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}\ \sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)/(x+1)^(3/2),x)
```

```
[Out] (x^2+4*x-5)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)/(x+1)^(1
/2)-3*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 2.86, size = 41, normalized size = 1.00

$$\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2+2x+1} - \frac{6\sqrt{-x^2+1}}{x+1} - 3\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(3/2)/(x^2 + 2*x + 1) - 6*sqrt(-x^2 + 1)/(x + 1) - 3*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{3/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(3/2)/(x + 1)^(3/2), x)
```

**sympy [A]** time = 2.48, size = 133, normalized size = 3.24

$$\begin{cases} 6i\,\text{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} + \frac{8i}{\sqrt{x-1}\sqrt{x+1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -6\,\text{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} + \frac{2\sqrt{x+1}}{\sqrt{1-x}} - \frac{8}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1)
- 2*I*sqrt(x + 1)/sqrt(x - 1) + 8*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1)/2
 > 1), (-6*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(3/2)/sqrt(1 - x) + 2*sqr
t(x + 1)/sqrt(1 - x) - 8/(sqrt(1 - x)*sqrt(x + 1)), True))
```

# 3.1050 $\int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx$

**Optimal**. Leaf size=23

$$-\frac{2\sqrt{1-x}}{\sqrt{x+1}}-\sin^{-1}(x)$$

**Rubi** [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$-\frac{2\sqrt{1-x}}{\sqrt{x+1}}-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + x)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - x])/Sqrt[1 + x] - ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx = -\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= -\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \sin^{-1}(x)$$

**Mathematica [A]** time = 0.04, size = 34, normalized size = 1.48

$$2\left(\frac{x-1}{\sqrt{1-x^2}} + \sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + x)^(3/2),x]
[Out] 2*((-1 + x)/Sqrt[1 - x^2] + ArcSin[Sqrt[1 - x]/Sqrt[2]])
```

**IntegrateAlgebraic [A]** time = 0.04, size = 39, normalized size = 1.70

$$2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right) - \frac{2\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]/(1 + x)^(3/2),x]
[Out] (-2*Sqrt[1 - x])/Sqrt[1 + x] + 2*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

**fricas [B]** time = 0.77, size = 50, normalized size = 2.17

$$\frac{2\left((x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right) - x - \sqrt{x+1}\,\sqrt{-x+1} - 1\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
[Out] 2*((x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - x - sqrt(x + 1)*sqrt(
-x + 1) - 1)/(x + 1)
```

**giac [B]** time = 0.69, size = 55, normalized size = 2.39

$$\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
 - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [B]** time = 0.02, size = 67, normalized size = 2.91

$$-\frac{\sqrt{(x+1)(-x+1)}\arcsin(x)}{\sqrt{x+1}\sqrt{-x+1}}+\frac{2(x-1)\sqrt{(x+1)(-x+1)}}{\sqrt{-(x+1)(x-1)}\sqrt{-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(x+1)^(3/2),x)
```

```
[Out] 2*(x-1)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)/(x+1)^(1/2)-
((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 2.86, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{-x^2+1}}{x+1}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/(x + 1) - arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-x}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(1/2)/(x + 1)^(3/2), x)
```

**sympy [B]** time = 1.54, size = 104, normalized size = 4.52

$$\begin{cases} 2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} + \frac{4i}{\sqrt{x-1}\,\sqrt{x+1}} & \text{for } \frac{|x+1|}{2} > 1 \\ -2\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) + \frac{2\sqrt{x+1}}{\sqrt{1-x}} - \frac{4}{\sqrt{1-x}\,\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1) +
 4*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1)/2 > 1), (-2*asin(sqrt(2)*sqrt(x
+ 1)/2) + 2*sqrt(x + 1)/sqrt(1 - x) - 4/(sqrt(1 - x)*sqrt(x + 1)), True))
```

## 3.1051 $\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}}\,dx$

**Optimal**. Leaf size=18

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

**Rubi** [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx = -\frac{\sqrt{1-x}}{\sqrt{1+x}}$$

**Mathematica** [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

**IntegrateAlgebraic [A]** time = 0.02, size = 18, normalized size = 1.00

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

**fricas [A]** time = 1.02, size = 23, normalized size = 1.28

$$-\frac{x+\sqrt{x+1}\,\sqrt{-x+1}\,+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -(x + sqrt(x + 1)*sqrt(-x + 1) + 1)/(x + 1)
```

**giac [B]** time = 0.65, size = 43, normalized size = 2.39

$$\frac{\sqrt{2}-\sqrt{-x+1}}{2\sqrt{x+1}}-\frac{\sqrt{x+1}}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/2*sqrt(x + 1)/(sqrt(2) - sqrt(
-x + 1))
```

**maple [A]** time = 0.00, size = 15, normalized size = 0.83

$$-\frac{\sqrt{-x+1}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(1/2)/(x+1)^(3/2),x)
```

```
[Out] -(-x+1)^(1/2)/(x+1)^(1/2)
```

**maxima** [A] time = 2.94, size = 16, normalized size = 0.89

$$-\frac{\sqrt{-x^2+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/(x + 1)
```

**mupad** [B] time = 0.36, size = 14, normalized size = 0.78

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(3/2)),x)
```

```
[Out] -(1 - x)^(1/2)/(x + 1)^(1/2)
```

**sympy** [A] time = 1.20, size = 29, normalized size = 1.61

$$\begin{cases} -\sqrt{-1+\frac{2}{x+1}} & \text{for } \frac{2}{|x+1|} > 1 \\ -i\sqrt{1-\frac{2}{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1)), 2/Abs(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1)
), True))
```

# 3.1052 $\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx$

**Optimal**. Leaf size=18

$$\frac{x}{\sqrt{1-x}\,\sqrt{x+1}}$$

**Rubi** [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {39}

$$\frac{x}{\sqrt{1-x}\,\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] x/(Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = \frac{x}{\sqrt{1-x}\,\sqrt{1+x}}$$

**Mathematica** [A] time = 0.00, size = 13, normalized size = 0.72

$$\frac{x}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] x/Sqrt[1 - x^2]
```

**IntegrateAlgebraic** [A] time = 0.06, size = 34, normalized size = 1.89

$$\frac{\sqrt{x+1}\left(1-\frac{1-x}{x+1}\right)}{2\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] (Sqrt[1 + x]*(1 - (1 - x)/(1 + x)))/(2*Sqrt[1 - x])
```

**fricas [A]** time = 1.24, size = 22, normalized size = 1.22

$$-\frac{\sqrt{x+1}\,x\sqrt{-x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1)/(x^2 - 1)
```

**giac [B]** time = 0.72, size = 62, normalized size = 3.44

$$\frac{\sqrt{2}-\sqrt{-x+1}}{4\sqrt{x+1}}-\frac{\sqrt{x+1}\sqrt{-x+1}}{2(x-1)}-\frac{\sqrt{x+1}}{4\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/2*sqrt(x + 1)*sqrt(-x + 1)/(x
- 1) - 1/4*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
```

**maple [A]** time = 0.00, size = 15, normalized size = 0.83

$$\frac{x}{\sqrt{-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(3/2)/(x+1)^(3/2),x)
```

```
[Out] x/(-x+1)^(1/2)/(x+1)^(1/2)
```

**maxima [A]** time = 1.34, size = 11, normalized size = 0.61

$$\frac{x}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(-x^2 + 1)
```

**mupad [B]** time = 0.31, size = 14, normalized size = 0.78

$$\frac{x}{\sqrt{1-x}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(3/2)),x)
```

```
[Out] x/((1 - x)^(1/2)*(x + 1)^(1/2))
```

**sympy [A]** time = 1.86, size = 65, normalized size = 3.61

$$\begin{cases} \frac{1}{\sqrt{-1+\frac{2}{x+1}}} - \frac{1}{\sqrt{-1+\frac{2}{x+1}}\,(x+1)} & \text{for } \frac{2}{|x+1|} > 1 \\ -\frac{i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{x-1} + \frac{i\sqrt{1-\frac{2}{x+1}}}{x-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((1/sqrt(-1 + 2/(x + 1)) - 1/(sqrt(-1 + 2/(x + 1))*(x + 1)), 2/Abs
(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1))*(x + 1)/(x - 1) + I*sqrt(1 - 2/(x + 1
))/(x - 1), True))
```

## 3.1053 $\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx$

**Optimal**. Leaf size=42

$$\frac{2x}{3\sqrt{1-x}\,\sqrt{x+1}}+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}$$

**Rubi [A]** time = 0.00, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{2x}{3\sqrt{1-x}\,\sqrt{x+1}}+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(3*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx &= \frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\ &= \frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{3\sqrt{1-x}\,\sqrt{1+x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 0.71

$$\frac{2x^2-2x-1}{3(x-1)\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

```
[Out] (-1 - 2*x + 2*x^2)/(3*(-1 + x)*Sqrt[1 - x^2])
```

**IntegrateAlgebraic** [A] time = 0.07, size = 48, normalized size = 1.14

$$\frac{(x+1)^{3/2}\left(-\frac{3(1-x)^2}{(x+1)^2}+\frac{6(1-x)}{x+1}+1\right)}{12(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

```
[Out] ((1 + x)^(3/2)*(1 - (3*(1 - x)^2)/(1 + x)^2 + (6*(1 - x))/(1 + x)))/(12*(1
- x)^(3/2))
```

**fricas** [A] time = 1.22, size = 54, normalized size = 1.29

$$\frac{x^3-x^2-\left(2x^2-2x-1\right)\sqrt{x+1}\sqrt{-x+1}-x+1}{3\left(x^3-x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 - x^2 - (2*x^2 - 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x + 1)/(x^3 -
 x^2 - x + 1)
```

**giac** [B] time = 0.70, size = 67, normalized size = 1.60

$$\frac{\sqrt{2}-\sqrt{-x+1}}{8\sqrt{x+1}}-\frac{(5x-7)\sqrt{x+1}\sqrt{-x+1}}{12(x-1)^2}-\frac{\sqrt{x+1}}{8\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/12*(5*x - 7)*sqrt(x + 1)*sqrt(
-x + 1)/(x - 1)^2 - 1/8*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.60

$$-\frac{2x^{2}-2x-1}{3\sqrt{x+1}\,\left(-x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(5/2)/(x+1)^(3/2),x)
```

```
[Out] -1/3*(2*x^2-2*x-1)/(x+1)^(1/2)/(-x+1)^(3/2)
```

**maxima [A]** time = 1.42, size = 40, normalized size = 0.95

$$\frac{2\,x}{3\,\sqrt{-x^{2}+1}}-\frac{1}{3\left(\sqrt{-x^{2}+1}\,x-\sqrt{-x^{2}+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

**mupad [B]** time = 0.32, size = 42, normalized size = 1.00

$$\frac{2\,x\,\sqrt{1-x}+\sqrt{1-x}-2\,x^{2}\,\sqrt{1-x}}{3\,\left(x-1\right)^{2}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(3/2)),x)
```

```
[Out] (2*x*(1 - x)^(1/2) + (1 - x)^(1/2) - 2*x^2*(1 - x)^(1/2))/(3*(x - 1)^2*(x +
 1)^(1/2))
```

**sympy [B]** time = 5.28, size = 158, normalized size = 3.76

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)^{2}}{-12x+3(x+1)^{2}}+\frac{6\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-12x+3(x+1)^{2}}-\frac{3\sqrt{-1+\frac{2}{x+1}}}{-12x+3(x+1)^{2}} & \text{for}\ \frac{2}{|x+1|}>1 \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)^{2}}{-12x+3(x+1)^{2}}+\frac{6i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-12x+3(x+1)^{2}}-\frac{3i\sqrt{1-\frac{2}{x+1}}}{-12x+3(x+1)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-12*x + 3*(x + 1)**2) + 6*sq
rt(-1 + 2/(x + 1))*(x + 1)/(-12*x + 3*(x + 1)**2) - 3*sqrt(-1 + 2/(x + 1))/
(-12*x + 3*(x + 1)**2), 2/Abs(x + 1) > 1), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1
)**2/(-12*x + 3*(x + 1)**2) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-12*x + 3*(x
 + 1)**2) - 3*I*sqrt(1 - 2/(x + 1))/(-12*x + 3*(x + 1)**2), True))
```

## 3.1054 $\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}} dx$

**Optimal**. Leaf size=62

$$\frac{2x}{5\sqrt{1-x}\sqrt{x+1}}+\frac{1}{5(1-x)^{3/2}\sqrt{x+1}}+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

**Rubi** [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{2x}{5\sqrt{1-x}\sqrt{x+1}}+\frac{1}{5(1-x)^{3/2}\sqrt{x+1}}+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(5*(1 - x)^(5/2)*Sqrt[1 + x]) + 1/(5*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(
5*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx = \frac{1}{5(1-x)^{5/2}\sqrt{1+x}} + \frac{3}{5}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{5(1-x)^{5/2}\sqrt{1+x}} + \frac{1}{5(1-x)^{3/2}\sqrt{1+x}} + \frac{2}{5}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{5(1-x)^{5/2}\sqrt{1+x}} + \frac{1}{5(1-x)^{3/2}\sqrt{1+x}} + \frac{2x}{5\sqrt{1-x}\,\sqrt{1+x}}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 0.53

$$\frac{2x^3-4x^2+x+2}{5(x-1)^2\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] (2 + x - 4*x^2 + 2*x^3)/(5*(-1 + x)^2*Sqrt[1 - x^2])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 62, normalized size = 1.00

$$\frac{(x+1)^{5/2}\left(-\frac{5(1-x)^3}{(x+1)^3}+\frac{15(1-x)^2}{(x+1)^2}+\frac{5(1-x)}{x+1}+1\right)}{40(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] ((1 + x)^(5/2)*(1 - (5*(1 - x)^3)/(1 + x)^3 + (15*(1 - x)^2)/(1 + x)^2 + (5
*(1 - x))/(1 + x)))/(40*(1 - x)^(5/2))
```

**fricas [A]** time = 1.04, size = 59, normalized size = 0.95

$$\frac{2\,x^4-4\,x^3-\left(2\,x^3-4\,x^2+x+2\right)\sqrt{x+1}\,\sqrt{-x+1}+4\,x-2}{5\left(x^4-2\,x^3+2\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*x^4 - 4*x^3 - (2*x^3 - 4*x^2 + x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*x
 - 2)/(x^4 - 2*x^3 + 2*x - 1)
```

**giac** [A] time = 0.66, size = 73, normalized size = 1.18

$$\frac{\sqrt{2}-\sqrt{-x+1}}{16\sqrt{x+1}}-\frac{\sqrt{x+1}}{16\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{((11\,x-39)(x+1)+60)\sqrt{x+1}\,\sqrt{-x+1}}{40\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/16*sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)) - 1/40*((11*x - 39)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x -
1)^3
```

**maple** [A] time = 0.00, size = 28, normalized size = 0.45

$$\frac{2x^3-4x^2+x+2}{5\sqrt{x+1}\,\left(-x+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(7/2)/(x+1)^(3/2),x)
```

```
[Out] 1/5*(2*x^3-4*x^2+x+2)/(x+1)^(1/2)/(-x+1)^(5/2)
```

**maxima** [A] time = 1.35, size = 79, normalized size = 1.27

$$\frac{2\,x}{5\,\sqrt{-x^2+1}}+\frac{1}{5\left(\sqrt{-x^2+1}\,x^2-2\,\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{1}{5\left(\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x/sqrt(-x^2 + 1) + 1/5/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt(
-x^2 + 1)) - 1/5/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

**mupad** [B] time = 0.34, size = 55, normalized size = 0.89

$$-\frac{x\,\sqrt{1-x}\,+2\,\sqrt{1-x}\,-4\,x^2\,\sqrt{1-x}\,+2\,x^3\,\sqrt{1-x}}{5\,(x-1)^3\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(3/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) + 2*(1 - x)^(1/2) - 4*x^2*(1 - x)^(1/2) + 2*x^3*(1 - x)^(
1/2))/(5*(x - 1)^3*(x + 1)^(1/2))
```

**sympy [B]** time = 16.83, size = 282, normalized size = 4.55

$$\begin{cases} \frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{-60x-5(x+1)^3+30(x+1)^2-20} - \frac{10\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{-60x-5(x+1)^3+30(x+1)^2-20} + \frac{15\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-60x-5(x+1)^3+30(x+1)^2-20} - \frac{5\sqrt{-1+\frac{2}{x+1}}}{-60x-5(x+1)^3+30(x+1)^2-20} & \text{for } \frac{2}{|x+1|} > 1 \\ \frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{-60x-5(x+1)^3+30(x+1)^2-20} - \frac{10i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-60x-5(x+1)^3+30(x+1)^2-20} + \frac{15i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-60x-5(x+1)^3+30(x+1)^2-20} - \frac{5i\sqrt{1-\frac{2}{x+1}}}{-60x-5(x+1)^3+30(x+1)^2-20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-60*x - 5*(x + 1)**3 + 30*(x
+ 1)**2 - 20) - 10*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-60*x - 5*(x + 1)**3 +
30*(x + 1)**2 - 20) + 15*sqrt(-1 + 2/(x + 1))*(x + 1)/(-60*x - 5*(x + 1)**3
 + 30*(x + 1)**2 - 20) - 5*sqrt(-1 + 2/(x + 1))/(-60*x - 5*(x + 1)**3 + 30*
(x + 1)**2 - 20), 2/Abs(x + 1) > 1), (2*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-
60*x - 5*(x + 1)**3 + 30*(x + 1)**2 - 20) - 10*I*sqrt(1 - 2/(x + 1))*(x + 1
)**2/(-60*x - 5*(x + 1)**3 + 30*(x + 1)**2 - 20) + 15*I*sqrt(1 - 2/(x + 1))
*(x + 1)/(-60*x - 5*(x + 1)**3 + 30*(x + 1)**2 - 20) - 5*I*sqrt(1 - 2/(x +
1))/(-60*x - 5*(x + 1)**3 + 30*(x + 1)**2 - 20), True))
```

## 3.1055 $\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}} \, dx$

**Optimal**. Leaf size=82

$$\frac{8x}{35\sqrt{1-x}\sqrt{x+1}}+\frac{4}{35(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{35(1-x)^{5/2}\sqrt{x+1}}+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

**Rubi [A]** time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{8x}{35\sqrt{1-x}\sqrt{x+1}}+\frac{4}{35(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{35(1-x)^{5/2}\sqrt{x+1}}+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(7*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(35*(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(35*
(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(35*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx = \frac{1}{7(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{7}\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{5/2}\sqrt{1+x}} + \frac{12}{35}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{5/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{3/2}\sqrt{1+x}} + \frac{8}{35}\int \frac{1}{(1-x)^{3/2}(1$$
$$= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{5/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{3/2}\sqrt{1+x}} + \frac{8x}{35\sqrt{1-x}\sqrt{1+x}}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.49

$$\frac{8x^4-24x^3+20x^2+4x-13}{35(x-1)^3\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```

```
[Out] (-13 + 4*x + 20*x^2 - 24*x^3 + 8*x^4)/(35*(-1 + x)^3*Sqrt[1 - x^2])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 76, normalized size = 0.93

$$\frac{(x+1)^{7/2}\left(-\frac{35(1-x)^4}{(x+1)^4}+\frac{140(1-x)^3}{(x+1)^3}+\frac{70(1-x)^2}{(x+1)^2}+\frac{28(1-x)}{x+1}+5\right)}{560(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```

```
[Out] ((1 + x)^(7/2)*(5 - (35*(1 - x)^4)/(1 + x)^4 + (140*(1 - x)^3)/(1 + x)^3 +
(70*(1 - x)^2)/(1 + x)^2 + (28*(1 - x))/(1 + x)))/(560*(1 - x)^(7/2))
```

**fricas [A]** time = 1.27, size = 86, normalized size = 1.05

$$\frac{13\,x^5-39\,x^4+26\,x^3+26\,x^2-\left(8\,x^4-24\,x^3+20\,x^2+4\,x-13\right)\sqrt{x+1}\,\sqrt{-x+1}-39\,x+13}{35\left(x^5-3\,x^4+2\,x^3+2\,x^2-3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(13*x^5 - 39*x^4 + 26*x^3 + 26*x^2 - (8*x^4 - 24*x^3 + 20*x^2 + 4*x -
13)*sqrt(x + 1)*sqrt(-x + 1) - 39*x + 13)/(x^5 - 3*x^4 + 2*x^3 + 2*x^2 - 3*
x + 1)
```

**giac** [A] time = 0.70, size = 79, normalized size = 0.96

$$\frac{\sqrt{2}-\sqrt{-x+1}}{32\sqrt{x+1}}-\frac{\sqrt{x+1}}{32\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{\left(\left((93\,x-523)(x+1)+1400\right)(x+1)-1120\right)\sqrt{x+1}\,\sqrt{-x+1}}{560\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/32*sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)) - 1/560*(((93*x - 523)*(x + 1) + 1400)*(x + 1) - 1120)*sqrt(x +
1)*sqrt(-x + 1)/(x - 1)^4
```

**maple** [A] time = 0.00, size = 35, normalized size = 0.43

$$-\frac{8x^4-24x^3+20x^2+4x-13}{35\sqrt{x+1}\,\left(-x+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(9/2)/(x+1)^(3/2),x)
```

```
[Out] -1/35*(8*x^4-24*x^3+20*x^2+4*x-13)/(x+1)^(1/2)/(-x+1)^(7/2)
```

**maxima** [B] time = 1.38, size = 134, normalized size = 1.63

$$\frac{8\,x}{35\,\sqrt{-x^2+1}}-\frac{1}{7\left(\sqrt{-x^2+1}\,x^3-3\,\sqrt{-x^2+1}\,x^2+3\,\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}+\frac{4}{35\left(\sqrt{-x^2+1}\,x^2-2\,\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{4}{35\left(\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/35*x/sqrt(-x^2 + 1) - 1/7/(sqrt(-x^2 + 1)*x^3 - 3*sqrt(-x^2 + 1)*x^2 + 3*
sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1)) + 4/35/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2
 + 1)*x + sqrt(-x^2 + 1)) - 4/35/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

**mupad** [B] time = 0.36, size = 68, normalized size = 0.83

$$-\frac{4\,x\,\sqrt{1-x}-13\,\sqrt{1-x}+20\,x^2\,\sqrt{1-x}-24\,x^3\,\sqrt{1-x}+8\,x^4\,\sqrt{1-x}}{35\,(x-1)^4\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((1 - x)^(9/2)*(x + 1)^(3/2)),x)

[Out] -(4*x*(1 - x)^(1/2) - 13*(1 - x)^(1/2) + 20*x^2*(1 - x)^(1/2) - 24*x^3*(1 - x)^(1/2) + 8*x^4*(1 - x)^(1/2))/(35*(x - 1)^4*(x + 1)^(1/2))

**sympy [B]** time = 44.94, size = 423, normalized size = 5.16

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}(x+1)^4}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} + \frac{56\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} - \frac{140\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} + \frac{140\sqrt{-1+\frac{2}{x+1}}(x+1)}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} - \frac{35\sqrt{-1+\frac{2}{x+1}}}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} & \text{for } \frac{2}{|x+1|} > 1 \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}(x+1)^4}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} + \frac{56i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} - \frac{140i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} + \frac{140i\sqrt{1-\frac{2}{x+1}}(x+1)}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} - \frac{35i\sqrt{1-\frac{2}{x+1}}}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)**(9/2)/(1+x)**(3/2),x)

[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) + 56*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 140*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) + 140*sqrt(-1 + 2/(x + 1))*(x + 1)/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 35*sqrt(-1 + 2/(x + 1))/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560), 2/Abs(x + 1) > 1), (-8*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) + 56*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 140*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) + 140*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 35*I*sqrt(1 - 2/(x + 1))/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560), True))

## 3.1056 $\int \frac{1}{(1-x)^{11/2}(1+x)^{3/2}}\,dx$

**Optimal**. Leaf size=102

$$\frac{8x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{4}{63(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{63(1-x)^{5/2}\sqrt{x+1}}+\frac{5}{63(1-x)^{7/2}\sqrt{x+1}}+\frac{1}{9(1-x)^{9/2}\sqrt{x+1}}$$

**Rubi [A]** time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{8x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{4}{63(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{63(1-x)^{5/2}\sqrt{x+1}}+\frac{5}{63(1-x)^{7/2}\sqrt{x+1}}+\frac{1}{9(1-x)^{9/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(9*(1 - x)^(9/2)*Sqrt[1 + x]) + 5/(63*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(63*
(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(63*(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(63*S
qrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{11/2}(1+x)^{3/2}}\,dx &= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{9}\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx \\ &= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{20}{63}\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx \\ &= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{63(1-x)^{5/2}\sqrt{1+x}} + \frac{4}{21}\int \frac{1}{(1-x)^{5/2}(1 \\ &= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{63(1-x)^{5/2}\sqrt{1+x}} + \frac{4}{63(1-x)^{3/2}\sqrt{1+ \\ &= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{63(1-x)^{5/2}\sqrt{1+x}} + \frac{4}{63(1-x)^{3/2}\sqrt{1+\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 45, normalized size = 0.44

$$\frac{8x^5-32x^4+44x^3-16x^2-17x+20}{63(x-1)^4\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] (20 - 17*x - 16*x^2 + 44*x^3 - 32*x^4 + 8*x^5)/(63*(-1 + x)^4*Sqrt[1 - x^2]
)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 90, normalized size = 0.88

$$\frac{(x+1)^{9/2}\left(-\frac{63(1-x)^5}{(x+1)^5}+\frac{315(1-x)^4}{(x+1)^4}+\frac{210(1-x)^3}{(x+1)^3}+\frac{126(1-x)^2}{(x+1)^2}+\frac{45(1-x)}{x+1}+7\right)}{2016(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] ((1 + x)^(9/2)*(7 - (63*(1 - x)^5)/(1 + x)^5 + (315*(1 - x)^4)/(1 + x)^4 +
(210*(1 - x)^3)/(1 + x)^3 + (126*(1 - x)^2)/(1 + x)^2 + (45*(1 - x))/(1 + x
)))/(2016*(1 - x)^(9/2))
```

**fricas [A]** time = 1.25, size = 91, normalized size = 0.89

$$\frac{20\,x^6-80\,x^5+100\,x^4-100\,x^2-\left(8\,x^5-32\,x^4+44\,x^3-16\,x^2-17\,x+20\right)\sqrt{x+1}\,\sqrt{-x+1}+80\,x-20}{63\left(x^6-4\,x^5+5\,x^4-5\,x^2+4\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/63*(20*x^6 - 80*x^5 + 100*x^4 - 100*x^2 - (8*x^5 - 32*x^4 + 44*x^3 - 16*x
^2 - 17*x + 20)*sqrt(x + 1)*sqrt(-x + 1) + 80*x - 20)/(x^6 - 4*x^5 + 5*x^4
- 5*x^2 + 4*x - 1)
```

**giac [A]** time = 0.68, size = 85, normalized size = 0.83

$$\frac{\sqrt{2}-\sqrt{-x+1}}{64\,\sqrt{x+1}}-\frac{\sqrt{x+1}}{64\,\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{((((193\,x-1481)(x+1)+5544)(x+1)-8400)(x+1)+5040)\sqrt{x+1}\,\sqrt{-x+1}}{2016\,(x-1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/64*sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)) - 1/2016*((((193*x - 1481)*(x + 1) + 5544)*(x + 1) - 8400)*(x +
1) + 5040)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^5
```

**maple [A]** time = 0.00, size = 40, normalized size = 0.39

$$\frac{8x^{5}-32x^{4}+44x^{3}-16x^{2}-17x+20}{63\sqrt{x+1}\,\left(-x+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(11/2)/(x+1)^(3/2),x)
```

```
[Out] 1/63*(8*x^5-32*x^4+44*x^3-16*x^2-17*x+20)/(x+1)^(1/2)/(-x+1)^(9/2)
```

**maxima [B]** time = 1.37, size = 201, normalized size = 1.97

$$\frac{8\,x}{63\,\sqrt{-x^{2}+1}}+\frac{1}{9\,\left(\sqrt{-x^{2}+1}\,x^{4}-4\,\sqrt{-x^{2}+1}\,x^{3}+6\,\sqrt{-x^{2}+1}\,x^{2}-4\,\sqrt{-x^{2}+1}\,x+\sqrt{-x^{2}+1}\right)}-\frac{5}{63\,\left(\sqrt{-x^{2}+1}\,x^{3}-3\,\sqrt{-x^{2}+1}\,x^{2}+3\,\sqrt{-x^{2}+1}\,x-\sqrt{-x^{2}+1}\right)}+\frac{4}{63\,\left(\sqrt{-x^{2}+1}\,x^{2}-2\,\sqrt{-x^{2}+1}\,x+\sqrt{-x^{2}+1}\right)}-\frac{4}{63\,\left(\sqrt{-x^{2}+1}\,x-\sqrt{-x^{2}+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/63*x/sqrt(-x^2 + 1) + 1/9/(sqrt(-x^2 + 1)*x^4 - 4*sqrt(-x^2 + 1)*x^3 + 6*
sqrt(-x^2 + 1)*x^2 - 4*sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1)) - 5/63/(sqrt(-x^2
 + 1)*x^3 - 3*sqrt(-x^2 + 1)*x^2 + 3*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1)) + 4
/63/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1)) - 4/63/(sqrt
(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

**mupad [B]** time = 0.36, size = 80, normalized size = 0.78

$$\frac{17\,x\,\sqrt{1-x}-20\,\sqrt{1-x}+16\,x^2\,\sqrt{1-x}-44\,x^3\,\sqrt{1-x}+32\,x^4\,\sqrt{1-x}-8\,x^5\,\sqrt{1-x}}{63\,{\left(x-1\right)}^5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(3/2)),x)
```

```
[Out] (17*x*(1 - x)^(1/2) - 20*(1 - x)^(1/2) + 16*x^2*(1 - x)^(1/2) - 44*x^3*(1 -
 x)^(1/2) + 32*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/(63*(x - 1)^5*(x +
1)^(1/2))
```

**sympy [B]** time = 113.61, size = 592, normalized size = 5.80

$$\begin{cases} \frac{8\sqrt{-1+\frac{2}{x+1}}(x+1)^5}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{72\sqrt{-1+\frac{2}{x+1}}(x+1)^4}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} + \frac{252\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{420\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} + \frac{315\sqrt{-1+\frac{2}{x+1}}(x+1)}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{63\sqrt{-1+\frac{2}{x+1}}}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} & \text{for}\ \frac{2}{|x+1|}>1 \\ \frac{8i\sqrt{1-\frac{2}{x+1}}(x+1)^5}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{72i\sqrt{1-\frac{2}{x+1}}(x+1)^4}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} + \frac{252i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{420i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} + \frac{315i\sqrt{1-\frac{2}{x+1}}(x+1)}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{63i\sqrt{1-\frac{2}{x+1}}}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((8*sqrt(-1 + 2/(x + 1))*(x + 1)**5/(-5040*x - 63*(x + 1)**5 + 630
*(x + 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024) - 72*sqrt(-1 + 2/(x
 + 1))*(x + 1)**4/(-5040*x - 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)*
*3 + 5040*(x + 1)**2 - 3024) + 252*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-5040*x
 - 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 302
4) - 420*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-5040*x - 63*(x + 1)**5 + 630*(x
+ 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024) + 315*sqrt(-1 + 2/(x +
1))*(x + 1)/(-5040*x - 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)**3 + 5
040*(x + 1)**2 - 3024) - 63*sqrt(-1 + 2/(x + 1))/(-5040*x - 63*(x + 1)**5 +
 630*(x + 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024), 2/Abs(x + 1) >
 1), (8*I*sqrt(1 - 2/(x + 1))*(x + 1)**5/(-5040*x - 63*(x + 1)**5 + 630*(x
+ 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024) - 72*I*sqrt(1 - 2/(x +
1))*(x + 1)**4/(-5040*x - 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)**3
+ 5040*(x + 1)**2 - 3024) + 252*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-5040*x -
 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024)
 - 420*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(-5040*x - 63*(x + 1)**5 + 630*(x +
 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024) + 315*I*sqrt(1 - 2/(x +
1))*(x + 1)/(-5040*x - 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)**3 + 5
040*(x + 1)**2 - 3024) - 63*I*sqrt(1 - 2/(x + 1))/(-5040*x - 63*(x + 1)**5
+ 630*(x + 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024), True))
```

## 3.1057 $\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx$

**Optimal**. Leaf size=103

$$-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{x+1}}+7\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{105}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{105}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{x+1}}+7\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{105}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{105}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(9/2))/(3*(1 + x)^(3/2)) + (6*(1 - x)^(7/2))/Sqrt[1 + x] + (105
*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/2 + 7*(1 - x)^
(5/2)*Sqrt[1 + x] + (105*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} - 3\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + 21\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + 7(1-x)^{5/2}\sqrt{1+x} + 35\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + \frac{105}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} +
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 37, normalized size = 0.36

$$-\frac{(1-x)^{11/2}\,{}_2F_1\left(\frac{5}{2},\frac{11}{2};\frac{13}{2};\frac{1-x}{2}\right)}{22\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] -1/22*((1 - x)^(11/2)*Hypergeometric2F1[5/2, 11/2, 13/2, (1 - x)/2])/Sqrt[2
]
```

**IntegrateAlgebraic [A]** time = 0.10, size = 131, normalized size = 1.27

$$\frac{-\frac{16(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{144(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{693(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{840(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{315\sqrt{1-x}}{\sqrt{x+1}}}{3\left(\frac{1-x}{x+1}+1\right)^3}-105\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] ((-16*(1 - x)^(9/2))/(1 + x)^(9/2) + (144*(1 - x)^(7/2))/(1 + x)^(7/2) + (6
93*(1 - x)^(5/2))/(1 + x)^(5/2) + (840*(1 - x)^(3/2))/(1 + x)^(3/2) + (315*
Sqrt[1 - x])/Sqrt[1 + x])/(3*(1 + (1 - x)/(1 + x))^3) - 105*ArcTan[Sqrt[1 -
 x]/Sqrt[1 + x]]
```

**fricas [A]** time = 1.50, size = 85, normalized size = 0.83

$$\frac{494\,x^2+\left(2\,x^4-17\,x^3+102\,x^2+679\,x+494\right)\sqrt{x+1}\,\sqrt{-x+1}-630\left(x^2+2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+988\,x+494}{6\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(494*x^2 + (2*x^4 - 17*x^3 + 102*x^2 + 679*x + 494)*sqrt(x + 1)*sqrt(-x
 + 1) - 630*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 988*
x + 494)/(x^2 + 2*x + 1)
```

**giac [A]** time = 0.89, size = 127, normalized size = 1.23

$$\frac{1}{6}\left((2\,x-23)(x+1)+165\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{2\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{3\,(x+1)^{\frac{3}{2}}}-\frac{34\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}+\frac{2\,(x+1)^{\frac{3}{2}}\left(\frac{51\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{3\left(\sqrt{2}-\sqrt{-x+1}\right)^3}+105\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x - 23)*(x + 1) + 165)*sqrt(x + 1)*sqrt(-x + 1) + 2/3*(sqrt(2) - sq
rt(-x + 1))^3/(x + 1)^(3/2) - 34*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 2/3
*(x + 1)^(3/2)*(51*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(
-x + 1))^3 + 105*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [A]** time = 0.02, size = 89, normalized size = 0.86

$$\frac{105\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{2\sqrt{x+1}\ \sqrt{-x+1}}-\frac{\left(2x^5-19x^4+119x^3+577x^2-185x-494\right)\sqrt{(x+1)(-x+1)}}{6\,(x+1)^{\frac{3}{2}}\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(9/2)/(x+1)^(5/2),x)
```

```
[Out] -1/6*(2*x^5-19*x^4+119*x^3+577*x^2-185*x-494)/(x+1)^(3/2)/(-(x+1)*(x-1))^(1
/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)+105/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2
)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [A]** time = 3.01, size = 125, normalized size = 1.21

$$\frac{x^6}{3\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{7\,x^5}{2\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{23\,x^4}{\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{35}{2}\,x\left(\frac{3\,x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{143\,x}{6\,\sqrt{-x^2+1}}-\frac{127\,x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{22\,x}{3\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{247}{3\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{105}{2}\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^6/(-x^2 + 1)^(3/2) - 7/2*x^5/(-x^2 + 1)^(3/2) + 23*x^4/(-x^2 + 1)^(3/
2) + 35/2*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 143/6*x/sqrt(-x
^2 + 1) - 127*x^2/(-x^2 + 1)^(3/2) + 22/3*x/(-x^2 + 1)^(3/2) + 247/3/(-x^2
+ 1)^(3/2) + 105/2*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{9/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(9/2)/(x + 1)^(5/2), x)
```

**sympy [A]** time = 45.20, size = 250, normalized size = 2.43

$$\begin{cases} -105i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{29i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}+\frac{215i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{43i\sqrt{x+1}}{3\sqrt{x-1}}-\frac{448i}{3\sqrt{x-1}\,\sqrt{x+1}}+\frac{64i}{3\sqrt{x-1}\,(x+1)^{\frac{3}{2}}} & \text{for}\ \frac{|x+1|}{2}>1 \\ 105\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{29(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}-\frac{215(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{43\sqrt{x+1}}{3\sqrt{1-x}}+\frac{448}{3\sqrt{1-x}\,\sqrt{x+1}}-\frac{64}{3\sqrt{1-x}\,(x+1)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-105*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x
 - 1)) - 29*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) + 215*I*(x + 1)**(3/2)/(6*sqrt
```

```
(x - 1)) + 43*I*sqrt(x + 1)/(3*sqrt(x - 1)) - 448*I/(3*sqrt(x - 1)*sqrt(x +
 1)) + 64*I/(3*sqrt(x - 1)*(x + 1)**(3/2)), Abs(x + 1)/2 > 1), (105*asin(sq
rt(2)*sqrt(x + 1)/2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 29*(x + 1)**(5/2)/(
6*sqrt(1 - x)) - 215*(x + 1)**(3/2)/(6*sqrt(1 - x)) - 43*sqrt(x + 1)/(3*sqr
t(1 - x)) + 448/(3*sqrt(1 - x)*sqrt(x + 1)) - 64/(3*sqrt(1 - x)*(x + 1)**(3
/2)), True))
```

## 3.1058 $\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx$

**Optimal**. Leaf size=87

$$-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{x+1}}+\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{35}{2}\sin^{-1}(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{x+1}}+\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{35}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(7/2))/(3*(1 + x)^(3/2)) + (14*(1 - x)^(5/2))/(3*Sqrt[1 + x]) +
 (35*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/6 + (35*Ar
cSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} - \frac{7}{3}\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}} \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\sin^{-1}(x)
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 37, normalized size = 0.43

$$-\frac{(1-x)^{9/2}\,{}_2F_1\left(\frac{5}{2},\frac{9}{2};\frac{11}{2};\frac{1-x}{2}\right)}{18\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] -1/18*((1 - x)^(9/2)*Hypergeometric2F1[5/2, 9/2, 11/2, (1 - x)/2])/Sqrt[2]
```

**IntegrateAlgebraic [A]** time = 0.09, size = 113, normalized size = 1.30

$$\frac{-\frac{8(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{56(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{175(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{105\sqrt{1-x}}{\sqrt{x+1}}}{3\left(\frac{1-x}{x+1}+1\right)^2}-35\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] ((-8*(1 - x)^(7/2))/(1 + x)^(7/2) + (56*(1 - x)^(5/2))/(1 + x)^(5/2) + (175
*(1 - x)^(3/2))/(1 + x)^(3/2) + (105*Sqrt[1 - x])/Sqrt[1 + x])/(3*(1 + (1 -
 x)/(1 + x))^2) - 35*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

**fricas [A]** time = 1.27, size = 81, normalized size = 0.93

$$\frac{164\,x^{2}-\left(3\,x^{3}-30\,x^{2}-229\,x-164\right)\sqrt{x+1}\,\sqrt{-x+1}-210\,\left(x^{2}+2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+328\,x+164}{6\,\left(x^{2}+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(164*x^2 - (3*x^3 - 30*x^2 - 229*x - 164)*sqrt(x + 1)*sqrt(-x + 1) - 21
0*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 328*x + 164)/(
x^2 + 2*x + 1)
```

**giac [A]** time = 0.80, size = 119, normalized size = 1.37

$$-\frac{1}{2}\,\sqrt{x+1}\,(x-12)\sqrt{-x+1}+\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{3\,(x+1)^{\frac{3}{2}}}-\frac{13\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{39\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}-1\right)}{3\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}+35\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 12)*sqrt(-x + 1) + 1/3*(sqrt(2) - sqrt(-x + 1))^3/(x
+ 1)^(3/2) - 13*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 1/3*(x + 1)^(3/2)*(3
9*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))^3 + 35*a
rcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [A]** time = 0.02, size = 84, normalized size = 0.97

$$\frac{35\sqrt{(x+1)\,(-x+1)}\;\arcsin(x)}{2\sqrt{x+1}\;\sqrt{-x+1}}+\frac{\left(3x^{4}-33x^{3}-199x^{2}+65x+164\right)\sqrt{(x+1)\,(-x+1)}}{6\,(x+1)^{\frac{3}{2}}\;\sqrt{-(x+1)\,(x-1)}\;\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)/(x+1)^(5/2),x)
```

```
[Out] 1/6*(3*x^4-33*x^3-199*x^2+65*x+164)/(x+1)^(3/2)/(-(x+1)*(x-1))^(1/2)*((x+1)
*(-x+1))^(1/2)/(-x+1)^(1/2)+35/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1
/2)*arcsin(x)
```

**maxima [A]** time = 3.00, size = 111, normalized size = 1.28

$$-\frac{x^5}{2\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{6\,x^4}{\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{35}{6}\,x\left(\frac{3\,x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{61\,x}{6\,\sqrt{-x^2+1}}-\frac{44\,x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{16\,x}{3\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{82}{3\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{35}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^5/(-x^2 + 1)^(3/2) + 6*x^4/(-x^2 + 1)^(3/2) + 35/6*x*(3*x^2/(-x^2 +
1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 61/6*x/sqrt(-x^2 + 1) - 44*x^2/(-x^2 + 1)^
(3/2) + 16/3*x/(-x^2 + 1)^(3/2) + 82/3/(-x^2 + 1)^(3/2) + 35/2*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(5/2), x)
```

**sympy [C]** time = 17.50, size = 207, normalized size = 2.38

$$\begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{2}+\frac{13\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{2}+\frac{80\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{16\sqrt{-1+\frac{2}{x+1}}}{3(x+1)}+\frac{35i\log\left(\frac{1}{x+1}\right)}{2}+\frac{35i\log(x+1)}{2}+35\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) & \text{for } \frac{2}{|x+1|}>1 \\ -\frac{i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{2}+\frac{13i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{2}+\frac{80i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{16i\sqrt{1-\frac{2}{x+1}}}{3(x+1)}+\frac{35i\log\left(\frac{1}{x+1}\right)}{2}-35i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1))*(x + 1)**2/2 + 13*sqrt(-1 + 2/(x + 1))*(x
+ 1)/2 + 80*sqrt(-1 + 2/(x + 1))/3 - 16*sqrt(-1 + 2/(x + 1))/(3*(x + 1)) +
35*I*log(1/(x + 1))/2 + 35*I*log(x + 1)/2 + 35*asin(sqrt(2)*sqrt(x + 1)/2),
 2/Abs(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1))*(x + 1)**2/2 + 13*I*sqrt(1 - 2/
(x + 1))*(x + 1)/2 + 80*I*sqrt(1 - 2/(x + 1))/3 - 16*I*sqrt(1 - 2/(x + 1))/
(3*(x + 1)) + 35*I*log(1/(x + 1))/2 - 35*I*log(sqrt(1 - 2/(x + 1)) + 1), Tr
ue))
```

## 3.1059 $\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}} dx$

**Optimal**. Leaf size=63

$$-\frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{x+1}}+5\sqrt{x+1}\sqrt{1-x}+5\sin^{-1}(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{x+1}}+5\sqrt{x+1}\sqrt{1-x}+5\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(5/2))/(3*(1 + x)^(3/2)) + (10*(1 - x)^(3/2))/(3*Sqrt[1 + x]) +
 5*Sqrt[1 - x]*Sqrt[1 + x] + 5*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} - \frac{5}{3}\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\sqrt{1-x}\,\sqrt{1+x} + 5\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\sqrt{1-x}\,\sqrt{1+x} + 5\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\sqrt{1-x}\,\sqrt{1+x} + 5\sin^{-1}(x)
\end{aligned}$$

**Mathematica** [C] time = 0.01, size = 37, normalized size = 0.59

$$-\frac{(1-x)^{7/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{1-x}{2}\right)}{14\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```

```
[Out] -1/14*((1 - x)^(7/2)*Hypergeometric2F1[5/2, 7/2, 9/2, (1 - x)/2])/Sqrt[2]
```

**IntegrateAlgebraic** [C] time = 0.17, size = 61, normalized size = 0.97

$$\frac{\sqrt{1-x}\,\left(3(x+1)^2+28(x+1)-8\right)}{3(x+1)^{3/2}}+10i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(-8 + 28*(1 + x) + 3*(1 + x)^2))/(3*(1 + x)^(3/2)) + (10*I)*Lo
g[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

**fricas [A]** time = 1.26, size = 75, normalized size = 1.19

$$\frac{23\,x^2+\left(3\,x^2+34\,x+23\right)\sqrt{x+1}\,\sqrt{-x+1}-30\,\left(x^2+2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+46\,x+23}{3\,\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(23*x^2 + (3*x^2 + 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) - 30*(x^2 + 2*x
+ 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 46*x + 23)/(x^2 + 2*x + 1)
```

**giac [B]** time = 0.75, size = 115, normalized size = 1.83

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{6\,(x+1)^{\frac{3}{2}}}+\sqrt{x+1}\,\sqrt{-x+1}-\frac{9\left(\sqrt{2}-\sqrt{-x+1}\right)}{2\,\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{27\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{6\left(\sqrt{2}-\sqrt{-x+1}\right)^3}+10\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + sqrt(x + 1)*sqrt(-x + 1) - 9
/2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 1/6*(x + 1)^(3/2)*(27*(sqrt(2) -
sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))^3 + 10*arcsin(1/2*sqr
t(2)*sqrt(x + 1))
```

**maple [A]** time = 0.02, size = 79, normalized size = 1.25

$$\frac{5\sqrt{(x+1)\,(-x+1)}\;\arcsin(x)}{\sqrt{x+1}\;\sqrt{-x+1}}-\frac{\left(3x^3+31x^2-11x-23\right)\sqrt{(x+1)\,(-x+1)}}{3\,(x+1)^{\frac{3}{2}}\,\sqrt{-(x+1)\,(x-1)}\;\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)/(x+1)^(5/2),x)
```

```
[Out] -1/3*(3*x^3+31*x^2-11*x-23)/(x+1)^(3/2)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))
^(1/2)/(-x+1)^(1/2)+5*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(
x)
```

**maxima [B]** time = 2.97, size = 98, normalized size = 1.56

$$\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{x^4+4\,x^3+6\,x^2+4\,x+1}-\frac{5\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3+3\,x^2+3\,x+1\right)}-\frac{10\,\sqrt{-x^2+1}}{3\left(x^2+2\,x+1\right)}+\frac{35\,\sqrt{-x^2+1}}{3\,(x+1)}+5\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(5/2)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(x^
3 + 3*x^2 + 3*x + 1) - 10/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 35/3*sqrt(-x^2
 + 1)/(x + 1) + 5*arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{5/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(5/2)/(x + 1)^(5/2), x)
```

**sympy [C]** time = 6.46, size = 160, normalized size = 2.54

$$\begin{cases} \sqrt{-1+\frac{2}{x+1}}\,(x+1)+\frac{28\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{8\sqrt{-1+\frac{2}{x+1}}}{3(x+1)}+5i\log\left(\frac{1}{x+1}\right)+5i\log(x+1)+10\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) & \text{for}\ \frac{2}{|x+1|}>1 \\ i\sqrt{1-\frac{2}{x+1}}\,(x+1)+\frac{28i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{8i\sqrt{1-\frac{2}{x+1}}}{3(x+1)}+5i\log\left(\frac{1}{x+1}\right)-10i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((sqrt(-1 + 2/(x + 1))*(x + 1) + 28*sqrt(-1 + 2/(x + 1))/3 - 8*sqr
t(-1 + 2/(x + 1))/(3*(x + 1)) + 5*I*log(1/(x + 1)) + 5*I*log(x + 1) + 10*as
in(sqrt(2)*sqrt(x + 1)/2), 2/Abs(x + 1) > 1), (I*sqrt(1 - 2/(x + 1))*(x + 1
) + 28*I*sqrt(1 - 2/(x + 1))/3 - 8*I*sqrt(1 - 2/(x + 1))/(3*(x + 1)) + 5*I*
log(1/(x + 1)) - 10*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

## 3.1060 $\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}} dx$

**Optimal**. Leaf size=41

$$-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{x+1}}+\sin^{-1}(x)$$

**Rubi [A]** time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{x+1}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(3/2))/(3*(1 + x)^(3/2)) + (2*Sqrt[1 - x])/Sqrt[1 + x] + ArcSin
[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx = -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} - \int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx$$
$$= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \sin^{-1}(x)$$

**Mathematica** [A] time = 0.06, size = 49, normalized size = 1.20

$$\frac{-8x^2+4x+4}{3\sqrt{1-x}\,(x+1)^{3/2}} - 2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

```
[Out] (4 + 4*x - 8*x^2)/(3*Sqrt[1 - x]*(1 + x)^(3/2)) - 2*ArcSin[Sqrt[1 - x]/Sqrt
[2]]
```

**IntegrateAlgebraic** [A] time = 0.06, size = 54, normalized size = 1.32

$$-\frac{2\sqrt{1-x}\left(\frac{1-x}{x+1}-3\right)}{3\sqrt{x+1}} - 2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*Sqrt[1 - x]*(-3 + (1 - x)/(1 + x)))/(3*Sqrt[1 + x]) - 2*ArcTan[Sqrt[1 -
 x]/Sqrt[1 + x]]
```

**fricas** [B] time = 1.19, size = 71, normalized size = 1.73

$$\frac{2\left(2\,x^2+2\,(2\,x+1)\sqrt{x+1}\,\sqrt{-x+1}-3\left(x^2+2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+4\,x+2\right)}{3\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x^2 + 2*(2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) - 3*(x^2 + 2*x + 1)*arcta
n((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 4*x + 2)/(x^2 + 2*x + 1)
```

**giac [B]** time = 0.70, size = 102, normalized size = 2.49

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{12\,(x+1)^{\frac{3}{2}}}-\frac{5\left(\sqrt{2}-\sqrt{-x+1}\right)}{4\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{15\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}-1\right)}{12\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}+2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) - 5/4*(sqrt(2) - sqrt(-x + 1)
)/sqrt(x + 1) + 1/12*(x + 1)^(3/2)*(15*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) -
 1)/(sqrt(2) - sqrt(-x + 1))^3 + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

**maple [B]** time = 0.02, size = 73, normalized size = 1.78

$$\frac{\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\ \sqrt{-x+1}}-\frac{4\left(2x^{2}-x-1\right)\sqrt{(x+1)(-x+1)}}{3\,(x+1)^{\frac{3}{2}}\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)/(x+1)^(5/2),x)
```

```
[Out] -4/3*(2*x^2-x-1)/(x+1)^(3/2)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+
1)^(1/2)+((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

**maxima [B]** time = 3.01, size = 66, normalized size = 1.61

$$-\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{3\left(x^{3}+3\,x^{2}+3\,x+1\right)}-\frac{2\sqrt{-x^{2}+1}}{3\left(x^{2}+2\,x+1\right)}+\frac{7\sqrt{-x^{2}+1}}{3\,(x+1)}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)/(x^3 + 3*x^2 + 3*x + 1) - 2/3*sqrt(-x^2 + 1)/(x^2 + 2
*x + 1) + 7/3*sqrt(-x^2 + 1)/(x + 1) + arcsin(x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{3/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(3/2)/(x + 1)^(5/2), x)
```

**sympy [C]** time = 3.28, size = 126, normalized size = 3.07

$$\begin{cases} \frac{8\sqrt{-1+\frac{2}{x+1}}}{3} - \frac{4\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} + i\log\left(\frac{1}{x+1}\right) + i\log(x+1) + 2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } \frac{2}{|x+1|} > 1 \\ \frac{8i\sqrt{1-\frac{2}{x+1}}}{3} - \frac{4i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} + i\log\left(\frac{1}{x+1}\right) - 2i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((8*sqrt(-1 + 2/(x + 1))/3 - 4*sqrt(-1 + 2/(x + 1))/(3*(x + 1)) +
I*log(1/(x + 1)) + I*log(x + 1) + 2*asin(sqrt(2)*sqrt(x + 1)/2), 2/Abs(x +
1) > 1), (8*I*sqrt(1 - 2/(x + 1))/3 - 4*I*sqrt(1 - 2/(x + 1))/(3*(x + 1)) +
 I*log(1/(x + 1)) - 2*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

## 3.1061 $\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\, dx$

**Optimal**. Leaf size=20

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {37}

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + x)^(5/2),x]
```

```
[Out] -(1 - x)^(3/2)/(3*(1 + x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\, dx = -\frac{(1-x)^{3/2}}{3(1+x)^{3/2}}$$

**Mathematica** [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + x)^(5/2),x]
```

```
[Out] -1/3*(1 - x)^(3/2)/(1 + x)^(3/2)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 20, normalized size = 1.00

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]/(1 + x)^(5/2),x]
[Out] -1/3*(1 - x)^(3/2)/(1 + x)^(3/2)
```

**fricas [B]** time = 1.30, size = 37, normalized size = 1.85

$$-\frac{x^2-\sqrt{x+1}\,(x-1)\sqrt{-x+1}+2\,x+1}{3\,\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
[Out] -1/3*(x^2 - sqrt(x + 1)*(x - 1)*sqrt(-x + 1) + 2*x + 1)/(x^2 + 2*x + 1)
```

**giac [B]** time = 0.71, size = 89, normalized size = 4.45

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{24\,(x+1)^{\frac{3}{2}}}-\frac{\sqrt{2}-\sqrt{-x+1}}{8\,\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{3\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{24\left(\sqrt{2}-\sqrt{-x+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="giac")
[Out] 1/24*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) - 1/8*(sqrt(2) - sqrt(-x + 1)
)/sqrt(x + 1) + 1/24*(x + 1)^(3/2)*(3*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) -
1)/(sqrt(2) - sqrt(-x + 1))^3
```

**maple [A]** time = 0.00, size = 15, normalized size = 0.75

$$-\frac{(-x+1)^{\frac{3}{2}}}{3\,(x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(x+1)^(5/2),x)
```

```
[Out] -1/3*(-x+1)^(3/2)/(x+1)^(3/2)
```

**maxima [B]** time = 1.32, size = 38, normalized size = 1.90

$$-\frac{2\sqrt{-x^2+1}}{3\left(x^2+2x+1\right)}+\frac{\sqrt{-x^2+1}}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x + 1)
```

**mupad [B]** time = 0.26, size = 32, normalized size = 1.60

$$\frac{x\sqrt{1-x}-\sqrt{1-x}}{(3x+3)\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(5/2),x)
```

```
[Out] (x*(1 - x)^(1/2) - (1 - x)^(1/2))/((3*x + 3)*(x + 1)^(1/2))
```

**sympy [A]** time = 1.69, size = 65, normalized size = 3.25

$$\begin{cases} \frac{\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{2\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} & \text{for } \frac{2}{|x+1|}>1 \\ \frac{i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{2i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((sqrt(-1 + 2/(x + 1))/3 - 2*sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 2/A
bs(x + 1) > 1), (I*sqrt(1 - 2/(x + 1))/3 - 2*I*sqrt(1 - 2/(x + 1))/(3*(x +
1)), True))
```

## 3.1062 $\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx$

**Optimal**. Leaf size=41

$$-\frac{\sqrt{1-x}}{3\sqrt{x+1}}-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$-\frac{\sqrt{1-x}}{3\sqrt{x+1}}-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
```

```
[Out] -Sqrt[1 - x]/(3*(1 + x)^(3/2)) - Sqrt[1 - x]/(3*Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx &= -\frac{\sqrt{1-x}}{3(1+x)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx\\ &= -\frac{\sqrt{1-x}}{3(1+x)^{3/2}}-\frac{\sqrt{1-x}}{3\sqrt{1+x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.56

$$-\frac{\sqrt{1-x}(x+2)}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
[Out] -1/3*(Sqrt[1 - x]*(2 + x))/(1 + x)^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.05, size = 33, normalized size = 0.80

$$-\frac{\sqrt{1-x}\left(\frac{1-x}{x+1}+3\right)}{6\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
[Out] -1/6*(Sqrt[1 - x]*(3 + (1 - x)/(1 + x)))/Sqrt[1 + x]
```

**fricas** [A] time = 1.35, size = 38, normalized size = 0.93

$$-\frac{2\,x^2+(x+2)\sqrt{x+1}\,\sqrt{-x+1}\,+4\,x+2}{3\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
[Out] -1/3*(2*x^2 + (x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*x + 2)/(x^2 + 2*x + 1)
```

**giac** [B] time = 0.67, size = 89, normalized size = 2.17

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{48\,(x+1)^{\frac{3}{2}}}+\frac{3\left(\sqrt{2}-\sqrt{-x+1}\right)}{16\,\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{9\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{48\left(\sqrt{2}-\sqrt{-x+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="giac")
[Out] 1/48*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 3/16*(sqrt(2) - sqrt(-x + 1
))/sqrt(x + 1) - 1/48*(x + 1)^(3/2)*(9*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) +
 1)/(sqrt(2) - sqrt(-x + 1))^3
```

**maple [A]** time = 0.00, size = 18, normalized size = 0.44

$$-\frac{(x+2)\sqrt{-x+1}}{3(x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(1/2)/(x+1)^(5/2),x)
```

```
[Out] -1/3*(2+x)/(x+1)^(3/2)*(-x+1)^(1/2)
```

**maxima [A]** time = 2.93, size = 38, normalized size = 0.93

$$-\frac{\sqrt{-x^2+1}}{3\left(x^2+2x+1\right)}-\frac{\sqrt{-x^2+1}}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x + 1)
```

**mupad [B]** time = 0.31, size = 33, normalized size = 0.80

$$-\frac{x\sqrt{1-x}+2\sqrt{1-x}}{(3x+3)\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) + 2*(1 - x)^(1/2))/((3*x + 3)*(x + 1)^(1/2))
```

**sympy [A]** time = 2.35, size = 65, normalized size = 1.59

$$\begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} & \text{for}\ \frac{2}{|x+1|}>1 \\ -\frac{i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1))/3 - sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 2/Ab
s(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1))/3 - I*sqrt(1 - 2/(x + 1))/(3*(x + 1)
), True))
```

## 3.1063 $\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx$

**Optimal**. Leaf size=58

$$-\frac{2\sqrt{1-x}}{3\sqrt{x+1}}-\frac{2\sqrt{1-x}}{3(x+1)^{3/2}}+\frac{1}{(x+1)^{3/2}\sqrt{1-x}}$$

**Rubi [A]** time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$-\frac{2\sqrt{1-x}}{3\sqrt{x+1}}-\frac{2\sqrt{1-x}}{3(x+1)^{3/2}}+\frac{1}{(x+1)^{3/2}\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(Sqrt[1 - x]*(1 + x)^(3/2)) - (2*Sqrt[1 - x])/(3*(1 + x)^(3/2)) - (2*Sqrt
[1 - x])/(3*Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx &= \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}} + 2\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx \\
&= \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}} - \frac{2\sqrt{1-x}}{3(1+x)^{3/2}} + \frac{2}{3}\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx \\
&= \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}} - \frac{2\sqrt{1-x}}{3(1+x)^{3/2}} - \frac{2\sqrt{1-x}}{3\sqrt{1+x}}
\end{aligned}
$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 0.52

$$
\frac{2x^2+2x-1}{3\sqrt{1-x}\,(x+1)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

```
[Out] (-1 + 2*x + 2*x^2)/(3*Sqrt[1 - x]*(1 + x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.07, size = 48, normalized size = 0.83

$$
\frac{\sqrt{x+1}\left(-\frac{(1-x)^2}{(x+1)^2}-\frac{6(1-x)}{x+1}+3\right)}{12\sqrt{1-x}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

```
[Out] (Sqrt[1 + x]*(3 - (1 - x)^2/(1 + x)^2 - (6*(1 - x))/(1 + x)))/(12*Sqrt[1 -
x])
```

**fricas** [A] time = 1.20, size = 49, normalized size = 0.84

$$
-\frac{x^3+x^2+\left(2\,x^2+2\,x-1\right)\sqrt{x+1}\,\sqrt{-x+1}-x-1}{3\left(x^3+x^2-x-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(x^3 + x^2 + (2*x^2 + 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x - 1)/(x^3
+ x^2 - x - 1)
```

**giac [B]** time = 0.68, size = 108, normalized size = 1.86

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{96\,(x+1)^{\frac{3}{2}}}+\frac{7\left(\sqrt{2}-\sqrt{-x+1}\right)}{32\,\sqrt{x+1}}-\frac{\sqrt{x+1}\,\sqrt{-x+1}}{4\,(x-1)}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{21\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{96\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/96*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 7/32*(sqrt(2) - sqrt(-x + 1
))/sqrt(x + 1) - 1/4*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 1/96*(x + 1)^(3/2)*
(21*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.43

$$\frac{2x^{2}+2x-1}{3\,(x+1)^{\frac{3}{2}}\,\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(3/2)/(x+1)^(5/2),x)
```

```
[Out] 1/3*(2*x^2+2*x-1)/(x+1)^(3/2)/(-x+1)^(1/2)
```

**maxima [A]** time = 1.35, size = 38, normalized size = 0.66

$$\frac{2\,x}{3\,\sqrt{-x^{2}+1}}-\frac{1}{3\left(\sqrt{-x^{2}+1}\,x+\sqrt{-x^{2}+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1))
```

**mupad [B]** time = 0.34, size = 48, normalized size = 0.83

$$-\frac{2\,x\,\sqrt{1-x}-\sqrt{1-x}+2\,x^{2}\,\sqrt{1-x}}{\left(3\,x^{2}-3\right)\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(2*x*(1 - x)^(1/2) - (1 - x)^(1/2) + 2*x^2*(1 - x)^(1/2))/((3*x^2 - 3)*(x
+ 1)^(1/2))
```

**sympy [A]** time = 5.40, size = 165, normalized size = 2.84

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-6x+3(x+1)^2-6} + \frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)}{-6x+3(x+1)^2-6} + \frac{\sqrt{-1+\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{for } \frac{2}{|x+1|} > 1 \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-6x+3(x+1)^2-6} + \frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)}{-6x+3(x+1)^2-6} + \frac{i\sqrt{1-\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) + 2
*sqrt(-1 + 2/(x + 1))*(x + 1)/(-6*x + 3*(x + 1)**2 - 6) + sqrt(-1 + 2/(x +
1))/(-6*x + 3*(x + 1)**2 - 6), 2/Abs(x + 1) > 1), (-2*I*sqrt(1 - 2/(x + 1))
*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) + 2*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-6
*x + 3*(x + 1)**2 - 6) + I*sqrt(1 - 2/(x + 1))/(-6*x + 3*(x + 1)**2 - 6), T
rue))
```

## 3.1064 $\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx$

**Optimal**. Leaf size=43

$$\frac{2x}{3\sqrt{1-x}\sqrt{x+1}}+\frac{x}{3(1-x)^{3/2}(x+1)^{3/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {40, 39}

$$\frac{2x}{3\sqrt{1-x}\sqrt{x+1}}+\frac{x}{3(1-x)^{3/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

```
[Out] x/(3*(1 - x)^(3/2)*(1 + x)^(3/2)) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx &= \frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\ &= \frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2x}{3\sqrt{1-x}\sqrt{1+x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 0.53

$$-\frac{x\left(2x^2-3\right)}{3\left(1-x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

```
[Out] -1/3*(x*(-3 + 2*x^2))/(1 - x^2)^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.07, size = 62, normalized size = 1.44

$$\frac{(x+1)^{3/2}\left(-\frac{(1-x)^3}{(x+1)^3}-\frac{9(1-x)^2}{(x+1)^2}+\frac{9(1-x)}{x+1}+1\right)}{24(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

```
[Out] ((1 + x)^(3/2)*(1 - (1 - x)^3/(1 + x)^3 - (9*(1 - x)^2)/(1 + x)^2 + (9*(1 -
 x))/(1 + x)))/(24*(1 - x)^(3/2))
```

**fricas** [A] time = 1.52, size = 35, normalized size = 0.81

$$-\frac{\left(2\,x^3-3\,x\right)\sqrt{x+1}\,\sqrt{-x+1}}{3\left(x^4-2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^3 - 3*x)*sqrt(x + 1)*sqrt(-x + 1)/(x^4 - 2*x^2 + 1)
```

**giac** [B] time = 0.68, size = 113, normalized size = 2.63

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{192\,(x+1)^{\frac{3}{2}}}+\frac{11\left(\sqrt{2}-\sqrt{-x+1}\right)}{64\,\sqrt{x+1}}-\frac{(4\,x-5)\sqrt{x+1}\,\sqrt{-x+1}}{12\,(x-1)^2}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{192\left(\sqrt{2}-\sqrt{-x+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 11/64*(sqrt(2) - sqrt(-x +
 1))/sqrt(x + 1) - 1/12*(4*x - 5)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 - 1/19
2*(x + 1)^(3/2)*(33*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) + 1)/(sqrt(2) - sqrt
(-x + 1))^3
```

**maple [A]** time = 0.00, size = 23, normalized size = 0.53

$$-\frac{\left(2x^{2}-3\right)x}{3\left(x+1\right)^{\frac{3}{2}}\left(-x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(5/2)/(x+1)^(5/2),x)
```

```
[Out] -1/3*x*(2*x^2-3)/(x+1)^(3/2)/(-x+1)^(3/2)
```

**maxima [A]** time = 1.39, size = 25, normalized size = 0.58

$$\frac{2x}{3\sqrt{-x^{2}+1}}+\frac{x}{3\left(-x^{2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(-x^2 + 1) + 1/3*x/(-x^2 + 1)^(3/2)
```

**mupad [B]** time = 0.37, size = 41, normalized size = 0.95

$$\frac{3x\sqrt{1-x}-2x^{3}\sqrt{1-x}}{\left(3x+3\right)\left(x-1\right)^{2}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(5/2)),x)
```

```
[Out] (3*x*(1 - x)^(1/2) - 2*x^3*(1 - x)^(1/2))/((3*x + 3)*(x - 1)^2*(x + 1)^(1/2
))
```

**sympy [B]** time = 9.61, size = 279, normalized size = 6.49

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}\left(x+1\right)^{3}}{12x+3\left(x+1\right)^{3}-12\left(x+1\right)^{2}+12}+\frac{6\sqrt{-1+\frac{2}{x+1}}\left(x+1\right)^{2}}{12x+3\left(x+1\right)^{3}-12\left(x+1\right)^{2}+12}-\frac{3\sqrt{-1+\frac{2}{x+1}}\left(x+1\right)}{12x+3\left(x+1\right)^{3}-12\left(x+1\right)^{2}+12}-\frac{\sqrt{-1+\frac{2}{x+1}}}{12x+3\left(x+1\right)^{3}-12\left(x+1\right)^{2}+12} & \text{for}\: \frac{2}{\left|x+1\right|}>1 \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}\left(x+1\right)^{3}}{12x+3\left(x+1\right)^{3}-12\left(x+1\right)^{2}+12}+\frac{6i\sqrt{1-\frac{2}{x+1}}\left(x+1\right)^{2}}{12x+3\left(x+1\right)^{3}-12\left(x+1\right)^{2}+12}-\frac{3i\sqrt{1-\frac{2}{x+1}}\left(x+1\right)}{12x+3\left(x+1\right)^{3}-12\left(x+1\right)^{2}+12}-\frac{i\sqrt{1-\frac{2}{x+1}}}{12x+3\left(x+1\right)^{3}-12\left(x+1\right)^{2}+12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(12*x + 3*(x + 1)**3 - 12*(x
+ 1)**2 + 12) + 6*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(12*x + 3*(x + 1)**3 - 12
*(x + 1)**2 + 12) - 3*sqrt(-1 + 2/(x + 1))*(x + 1)/(12*x + 3*(x + 1)**3 - 1
2*(x + 1)**2 + 12) - sqrt(-1 + 2/(x + 1))/(12*x + 3*(x + 1)**3 - 12*(x + 1)
**2 + 12), 2/Abs(x + 1) > 1), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(12*x +
3*(x + 1)**3 - 12*(x + 1)**2 + 12) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(12
*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - 3*I*sqrt(1 - 2/(x + 1))*(x + 1)/(
12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - I*sqrt(1 - 2/(x + 1))/(12*x + 3
*(x + 1)**3 - 12*(x + 1)**2 + 12), True))
```

## 3.1065 $\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx$

**Optimal**. Leaf size=63

$$\frac{8x}{15\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{15(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {45, 40, 39}

$$\frac{8x}{15\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{15(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(5*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*x)/(15*(1 - x)^(3/2)*(1 + x)^(3/2))
+ (8*x)/(15*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx = \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}} + \frac{4}{5}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx$$
$$= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}} + \frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}} + \frac{8}{15}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}} + \frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}} + \frac{8x}{15\sqrt{1-x}\sqrt{1+x}}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 0.63

$$\frac{8x^4-8x^3-12x^2+12x+3}{15(1-x)^{5/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] (3 + 12*x - 12*x^2 - 8*x^3 + 8*x^4)/(15*(1 - x)^(5/2)*(1 + x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.08, size = 76, normalized size = 1.21

$$\frac{(x+1)^{5/2}\left(-\frac{5(1-x)^4}{(x+1)^4}-\frac{60(1-x)^3}{(x+1)^3}+\frac{90(1-x)^2}{(x+1)^2}+\frac{20(1-x)}{x+1}+3\right)}{240(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] ((1 + x)^(5/2)*(3 - (5*(1 - x)^4)/(1 + x)^4 - (60*(1 - x)^3)/(1 + x)^3 + (9
0*(1 - x)^2)/(1 + x)^2 + (20*(1 - x))/(1 + x)))/(240*(1 - x)^(5/2))
```

**fricas** [A] time = 1.27, size = 84, normalized size = 1.33

$$\frac{3\,x^5-3\,x^4-6\,x^3+6\,x^2-\left(8\,x^4-8\,x^3-12\,x^2+12\,x+3\right)\sqrt{x+1}\,\sqrt{-x+1}\,+3\,x-3}{15\left(x^5-x^4-2\,x^3+2\,x^2+x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^5 - 3*x^4 - 6*x^3 + 6*x^2 - (8*x^4 - 8*x^3 - 12*x^2 + 12*x + 3)*s
qrt(x + 1)*sqrt(-x + 1) + 3*x - 3)/(x^5 - x^4 - 2*x^3 + 2*x^2 + x - 1)
```

**giac [B]** time = 0.71, size = 119, normalized size = 1.89

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{384\,(x+1)^{\frac{3}{2}}}+\frac{15\left(\sqrt{2}-\sqrt{-x+1}\right)}{128\,\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{45\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{384\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}-\frac{((73\,x-247)(x+1)+360)\sqrt{x+1}\,\sqrt{-x+1}}{240\,(x-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 15/128*(sqrt(2) - sqrt(-x
+ 1))/sqrt(x + 1) - 1/384*(x + 1)^(3/2)*(45*(sqrt(2) - sqrt(-x + 1))^2/(x +
 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/240*((73*x - 247)*(x + 1) + 360)*sq
rt(x + 1)*sqrt(-x + 1)/(x - 1)^3
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.56

$$\frac{8x^{4}-8x^{3}-12x^{2}+12x+3}{15\,(x+1)^{\frac{3}{2}}\,(-x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(7/2)/(x+1)^(5/2),x)
```

```
[Out] 1/15*(8*x^4-8*x^3-12*x^2+12*x+3)/(x+1)^(3/2)/(-x+1)^(5/2)
```

**maxima [A]** time = 1.39, size = 52, normalized size = 0.83

$$\frac{8\,x}{15\,\sqrt{-x^{2}+1}}+\frac{4\,x}{15\left(-x^{2}+1\right)^{\frac{3}{2}}}-\frac{1}{5\left(\left(-x^{2}+1\right)^{\frac{3}{2}}x-\left(-x^{2}+1\right)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/15*x/sqrt(-x^2 + 1) + 4/15*x/(-x^2 + 1)^(3/2) - 1/5/((-x^2 + 1)^(3/2)*x -
 (-x^2 + 1)^(3/2))
```

**mupad [B]** time = 0.38, size = 75, normalized size = 1.19

$$-\frac{12\,x\,\sqrt{1-x}+3\,\sqrt{1-x}-12\,x^{2}\,\sqrt{1-x}-8\,x^{3}\,\sqrt{1-x}+8\,x^{4}\,\sqrt{1-x}}{(15\,x+15)\,(x-1)^{3}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(12*x*(1 - x)^(1/2) + 3*(1 - x)^(1/2) - 12*x^2*(1 - x)^(1/2) - 8*x^3*(1 -
x)^(1/2) + 8*x^4*(1 - x)^(1/2))/((15*x + 15)*(x - 1)^3*(x + 1)^(1/2))
```

**sympy [B]** time = 27.61, size = 423, normalized size = 6.71

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{40\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} - \frac{60\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{20\sqrt{-1+\frac{2}{x+1}}(x+1)}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{5\sqrt{-1+\frac{2}{x+1}}}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} & \text{for } \frac{2}{|x+1|} > 1 \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{40i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} - \frac{60i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{20i\sqrt{1-\frac{2}{x+1}}(x+1)}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{5i\sqrt{1-\frac{2}{x+1}}}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90*
(x + 1)**3 + 180*(x + 1)**2 - 120) + 40*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-1
20*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*sqrt(-1 +
 2/(x + 1))*(x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1
)**2 - 120) + 20*sqrt(-1 + 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4 - 90*
(x + 1)**3 + 180*(x + 1)**2 - 120) + 5*sqrt(-1 + 2/(x + 1))/(-120*x + 15*(x
 + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120), 2/Abs(x + 1) > 1), (-8*I*
sqrt(1 - 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 18
0*(x + 1)**2 - 120) + 40*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-120*x + 15*(x +
 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*I*sqrt(1 - 2/(x + 1))*(
x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) +
 20*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 +
 180*(x + 1)**2 - 120) + 5*I*sqrt(1 - 2/(x + 1))/(-120*x + 15*(x + 1)**4 -
90*(x + 1)**3 + 180*(x + 1)**2 - 120), True))
```

## 3.1066 $\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx$

**Optimal**. Leaf size=83

$$\frac{8x}{21\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{21(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{5/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {45, 40, 39}

$$\frac{8x}{21\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{21(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{5/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(7*(1 - x)^(7/2)*(1 + x)^(3/2)) + 1/(7*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*
x)/(21*(1 - x)^(3/2)*(1 + x)^(3/2)) + (8*x)/(21*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx = \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}} + \frac{5}{7}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx$$
$$= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}} + \frac{1}{7(1-x)^{5/2}(1+x)^{3/2}} + \frac{4}{7}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx$$
$$= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}} + \frac{1}{7(1-x)^{5/2}(1+x)^{3/2}} + \frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}} + \frac{8}{21}\int \frac{1}{(1-x)^{3/2}}$$
$$= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}} + \frac{1}{7(1-x)^{5/2}(1+x)^{3/2}} + \frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}} + \frac{8x}{21\sqrt{1-x}\sqrt{1+}}$$

**Mathematica [A]** time = 0.01, size = 45, normalized size = 0.54

$$\frac{-8x^5+16x^4+4x^3-24x^2+9x+6}{21(1-x)^{7/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] (6 + 9*x - 24*x^2 + 4*x^3 + 16*x^4 - 8*x^5)/(21*(1 - x)^(7/2)*(1 + x)^(3/2)
)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 90, normalized size = 1.08

$$\frac{(x+1)^{7/2}\left(-\frac{7(1-x)^5}{(x+1)^5}-\frac{105(1-x)^4}{(x+1)^4}+\frac{210(1-x)^3}{(x+1)^3}+\frac{70(1-x)^2}{(x+1)^2}+\frac{21(1-x)}{x+1}+3\right)}{672(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] ((1 + x)^(7/2)*(3 - (7*(1 - x)^5)/(1 + x)^5 - (105*(1 - x)^4)/(1 + x)^4 + (
210*(1 - x)^3)/(1 + x)^3 + (70*(1 - x)^2)/(1 + x)^2 + (21*(1 - x))/(1 + x))
)/(672*(1 - x)^(7/2))
```

**fricas [A]** time = 1.33, size = 101, normalized size = 1.22

$$\frac{6\,x^6-12\,x^5-6\,x^4+24\,x^3-6\,x^2-\left(8\,x^5-16\,x^4-4\,x^3+24\,x^2-9\,x-6\right)\sqrt{x+1}\,\sqrt{-x+1}-12\,x+6}{21\left(x^6-2\,x^5-x^4+4\,x^3-x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

[Out] 1/21*(6*x^6 - 12*x^5 - 6*x^4 + 24*x^3 - 6*x^2 - (8*x^5 - 16*x^4 - 4*x^3 + 24*x^2 - 9*x - 6)*sqrt(x + 1)*sqrt(-x + 1) - 12*x + 6)/(x^6 - 2*x^5 - x^4 + 4*x^3 - x^2 - 2*x + 1)

**giac [B]** time = 0.69, size = 125, normalized size = 1.51

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{768\,(x+1)^{\frac{3}{2}}}+\frac{19\left(\sqrt{2}-\sqrt{-x+1}\right)}{256\,\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{57\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{768\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}-\frac{(((79\,x-432)(x+1)+1120)(x+1)-840)\sqrt{x+1}\,\sqrt{-x+1}}{336\,(x-1)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="giac")

[Out] 1/768*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 19/256*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/768*(x + 1)^(3/2)*(57*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/336*(((79*x - 432)*(x + 1) + 1120)*(x + 1) - 840)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^4

**maple [A]** time = 0.00, size = 40, normalized size = 0.48

$$-\frac{8x^{5}-16x^{4}-4x^{3}+24x^{2}-9x-6}{21\,(x+1)^{\frac{3}{2}}\,(-x+1)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x+1)^(9/2)/(x+1)^(5/2),x)

[Out] -1/21*(8*x^5-16*x^4-4*x^3+24*x^2-9*x-6)/(x+1)^(3/2)/(-x+1)^(7/2)

**maxima [A]** time = 1.34, size = 91, normalized size = 1.10

$$\frac{8\,x}{21\,\sqrt{-x^{2}+1}}+\frac{4\,x}{21\left(-x^{2}+1\right)^{\frac{3}{2}}}+\frac{1}{7\left(\left(-x^{2}+1\right)^{\frac{3}{2}}x^{2}-2\left(-x^{2}+1\right)^{\frac{3}{2}}x+\left(-x^{2}+1\right)^{\frac{3}{2}}\right)}-\frac{1}{7\left(\left(-x^{2}+1\right)^{\frac{3}{2}}x-\left(-x^{2}+1\right)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")

[Out] 8/21*x/sqrt(-x^2 + 1) + 4/21*x/(-x^2 + 1)^(3/2) + 1/7/((-x^2 + 1)^(3/2)*x^2 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 1/7/((-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2))

**mupad [B]** time = 0.41, size = 86, normalized size = 1.04

$$\frac{9\,x\,\sqrt{1-x}\,+6\,\sqrt{1-x}\,-24\,x^{2}\,\sqrt{1-x}\,+4\,x^{3}\,\sqrt{1-x}\,+16\,x^{4}\,\sqrt{1-x}\,-8\,x^{5}\,\sqrt{1-x}}{(21\,x+21)\,\left(x-1\right)^{4}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(9/2)*(x + 1)^(5/2)),x)
```

```
[Out] (9*x*(1 - x)^(1/2) + 6*(1 - x)^(1/2) - 24*x^2*(1 - x)^(1/2) + 4*x^3*(1 - x)
^(1/2) + 16*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/((21*x + 21)*(x - 1)^4
*(x + 1)^(1/2))
```

**sympy [B]** time = 71.01, size = 592, normalized size = 7.13

$$\begin{cases} \frac{8\sqrt{-1+\frac{2}{x+1}}\,(x+1)^5}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} - \frac{56\sqrt{-1+\frac{2}{x+1}}\,(x+1)^4}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} + \frac{140\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} - \frac{140\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} + \frac{35\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} + \frac{7\sqrt{-1+\frac{2}{x+1}}}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} & \text{for}\ \frac{2}{|x+1|} > 1 \\ \frac{8i\sqrt{1-\frac{2}{x+1}}\,(x+1)^5}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} - \frac{56i\sqrt{1-\frac{2}{x+1}}\,(x+1)^4}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} + \frac{140i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} - \frac{140i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} + \frac{35i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} + \frac{7i\sqrt{1-\frac{2}{x+1}}}{-336x-21(x+1)^5+168(x+1)^4-504(x+1)^3+672(x+1)^2-336} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((8*sqrt(-1 + 2/(x + 1))*(x + 1)**5/(-336*x - 21*(x + 1)**5 + 168*
(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**2 - 336) - 56*sqrt(-1 + 2/(x + 1
))*(x + 1)**4/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 6
72*(x + 1)**2 - 336) + 140*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-336*x - 21*(x
+ 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**2 - 336) - 140*sqr
t(-1 + 2/(x + 1))*(x + 1)**2/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504
*(x + 1)**3 + 672*(x + 1)**2 - 336) + 35*sqrt(-1 + 2/(x + 1))*(x + 1)/(-336
*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**2 - 336
) + 7*sqrt(-1 + 2/(x + 1))/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(
x + 1)**3 + 672*(x + 1)**2 - 336), 2/Abs(x + 1) > 1), (8*I*sqrt(1 - 2/(x +
1))*(x + 1)**5/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 +
672*(x + 1)**2 - 336) - 56*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(-336*x - 21*(x
 + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**2 - 336) + 140*I*
sqrt(1 - 2/(x + 1))*(x + 1)**3/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 5
04*(x + 1)**3 + 672*(x + 1)**2 - 336) - 140*I*sqrt(1 - 2/(x + 1))*(x + 1)**
2/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**
2 - 336) + 35*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-336*x - 21*(x + 1)**5 + 168*(
x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**2 - 336) + 7*I*sqrt(1 - 2/(x + 1)
)/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**
2 - 336), True))
```

## 3.1067 $\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}} \, dx$

**Optimal**. Leaf size=103

$$\frac{16x}{63\sqrt{1-x}\,\sqrt{x+1}}+\frac{8x}{63(1-x)^{3/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{5/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{7/2}(x+1)^{3/2}}+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {45, 40, 39}

$$\frac{16x}{63\sqrt{1-x}\,\sqrt{x+1}}+\frac{8x}{63(1-x)^{3/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{5/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{7/2}(x+1)^{3/2}}+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(9*(1 - x)^(9/2)*(1 + x)^(3/2)) + 2/(21*(1 - x)^(7/2)*(1 + x)^(3/2)) + 2/
(21*(1 - x)^(5/2)*(1 + x)^(3/2)) + (8*x)/(63*(1 - x)^(3/2)*(1 + x)^(3/2)) +
 (16*x)/(63*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{3}\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{10}{21}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{8}{21}\int \frac{1}{(1-x)^{5}}\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{8x}{63(1-x)^{3/2}(1}\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{8x}{63(1-x)^{3/2}(1}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 50, normalized size = 0.49

$$\frac{16x^6-48x^5+24x^4+56x^3-66x^2+6x+19}{63(1-x)^{9/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] (19 + 6*x - 66*x^2 + 56*x^3 + 24*x^4 - 48*x^5 + 16*x^6)/(63*(1 - x)^(9/2)*(
1 + x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 104, normalized size = 1.01

$$\frac{(x+1)^{9/2}\left(-\frac{21(1-x)^6}{(x+1)^6}-\frac{378(1-x)^5}{(x+1)^5}+\frac{945(1-x)^4}{(x+1)^4}+\frac{420(1-x)^3}{(x+1)^3}+\frac{189(1-x)^2}{(x+1)^2}+\frac{54(1-x)}{x+1}+7\right)}{4032(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] ((1 + x)^(9/2)*(7 - (21*(1 - x)^6)/(1 + x)^6 - (378*(1 - x)^5)/(1 + x)^5 +
(945*(1 - x)^4)/(1 + x)^4 + (420*(1 - x)^3)/(1 + x)^3 + (189*(1 - x)^2)/(1
+ x)^2 + (54*(1 - x))/(1 + x)))/(4032*(1 - x)^(9/2))
```

**fricas [A]** time = 0.88, size = 114, normalized size = 1.11

$$\frac{19\,x^{7}-57\,x^{6}+19\,x^{5}+95\,x^{4}-95\,x^{3}-19\,x^{2}-\left(16\,x^{6}-48\,x^{5}+24\,x^{4}+56\,x^{3}-66\,x^{2}+6\,x+19\right)\sqrt{x+1}\,\sqrt{-x+1}+57\,x-19}{63\,\left(x^{7}-3\,x^{6}+x^{5}+5\,x^{4}-5\,x^{3}-x^{2}+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/63*(19*x^7 - 57*x^6 + 19*x^5 + 95*x^4 - 95*x^3 - 19*x^2 - (16*x^6 - 48*x^
5 + 24*x^4 + 56*x^3 - 66*x^2 + 6*x + 19)*sqrt(x + 1)*sqrt(-x + 1) + 57*x -
19)/(x^7 - 3*x^6 + x^5 + 5*x^4 - 5*x^3 - x^2 + 3*x - 1)
```

**giac [A]** time = 0.74, size = 131, normalized size = 1.27

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{1536\,(x+1)^{\frac{3}{2}}}+\frac{23\left(\sqrt{2}-\sqrt{-x+1}\right)}{512\,\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{69\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{1536\left(\sqrt{2}-\sqrt{-x+1}\right)^3}-\frac{((((667\,x-5021)(x+1)+18396)(x+1)-26880)(x+1)+15120)\sqrt{x+1}\,\sqrt{-x+1}}{4032\,(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 23/512*(sqrt(2) - sqrt(-x
 + 1))/sqrt(x + 1) - 1/1536*(x + 1)^(3/2)*(69*(sqrt(2) - sqrt(-x + 1))^2/(x
 + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/4032*((((667*x - 5021)*(x + 1) +
18396)*(x + 1) - 26880)*(x + 1) + 15120)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^5
```

**maple [A]** time = 0.00, size = 45, normalized size = 0.44

$$\frac{16x^6-48x^5+24x^4+56x^3-66x^2+6x+19}{63\,(x+1)^{\frac{3}{2}}\,(-x+1)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(11/2)/(x+1)^(5/2),x)
```

```
[Out] 1/63*(16*x^6-48*x^5+24*x^4+56*x^3-66*x^2+6*x+19)/(x+1)^(3/2)/(-x+1)^(9/2)
```

**maxima [A]** time = 1.42, size = 146, normalized size = 1.42

$$\frac{16\,x}{63\,\sqrt{-x^2+1}}+\frac{8\,x}{63\,(-x^2+1)^{\frac{3}{2}}}-\frac{1}{9\left((-x^2+1)^{\frac{3}{2}}x^3-3\,(-x^2+1)^{\frac{3}{2}}x^2+3\,(-x^2+1)^{\frac{3}{2}}x-(-x^2+1)^{\frac{3}{2}}\right)}+\frac{2}{21\left((-x^2+1)^{\frac{3}{2}}x^2-2\,(-x^2+1)^{\frac{3}{2}}x+(-x^2+1)^{\frac{3}{2}}\right)}-\frac{2}{21\left((-x^2+1)^{\frac{3}{2}}x-(-x^2+1)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/63*x/sqrt(-x^2 + 1) + 8/63*x/(-x^2 + 1)^(3/2) - 1/9/((-x^2 + 1)^(3/2)*x^
3 - 3*(-x^2 + 1)^(3/2)*x^2 + 3*(-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2)) + 2/2
1/((-x^2 + 1)^(3/2)*x^2 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 2/21/(
(-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2))
```

**mupad [B]** time = 0.42, size = 99, normalized size = 0.96

$$-\frac{6\,x\,\sqrt{1-x}\,+19\,\sqrt{1-x}\,-66\,x^2\,\sqrt{1-x}\,+56\,x^3\,\sqrt{1-x}\,+24\,x^4\,\sqrt{1-x}\,-48\,x^5\,\sqrt{1-x}\,+16\,x^6\,\sqrt{1-x}}{(63\,x+63)\,\left(x-1\right)^5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(6*x*(1 - x)^(1/2) + 19*(1 - x)^(1/2) - 66*x^2*(1 - x)^(1/2) + 56*x^3*(1 -
 x)^(1/2) + 24*x^4*(1 - x)^(1/2) - 48*x^5*(1 - x)^(1/2) + 16*x^6*(1 - x)^(1
/2))/((63*x + 63)*(x - 1)^5*(x + 1)^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(5/2),x)
```

```
[Out] Timed out
```

## 3.1068 $\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx$

**Optimal**. Leaf size=126

$$\frac{5}{8}a^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{5}{16}a^2c^2x\sqrt{ax+a}\sqrt{c-cx}+\frac{5}{24}acx(ax+a)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

**Rubi** [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {38, 63, 217, 203}

$$\frac{5}{16}a^2c^2x\sqrt{ax+a}\sqrt{c-cx}+\frac{5}{8}a^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{5}{24}acx(ax+a)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```

```
[Out] (5*a^2*c^2*x*Sqrt[a + a*x]*Sqrt[c - c*x])/16 + (5*a*c*x*(a + a*x)^(3/2)*(c
- c*x)^(3/2))/24 + (x*(a + a*x)^(5/2)*(c - c*x)^(5/2))/6 + (5*a^(5/2)*c^(5/
2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx = \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{1}{6}(5ac)\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx$$

$$= \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{1}{8}\left(5a^2c^2\right)\int \sqrt{a+ax}\,\sqrt{c-c$$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} +$$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} +$$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} +$$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} +$$

**Mathematica [A]** time = 0.10, size = 114, normalized size = 0.90

$$\frac{c^{3/2}(a(x+1))^{5/2}\sqrt{c-cx}\left(\sqrt{c}\,x\sqrt{x+1}\left(8x^5-8x^4-26x^3+26x^2+33x-33\right)+30\sqrt{c-cx}\,\sin^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{2}\,\sqrt{c}}\right)\right)}{48(x-1)(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```

```
[Out] (c^(3/2)*(a*(1 + x))^(5/2)*Sqrt[c - c*x]*(Sqrt[c]*x*Sqrt[1 + x]*(-33 + 33*x
 + 26*x^2 - 26*x^3 - 8*x^4 + 8*x^5) + 30*Sqrt[c - c*x]*ArcSin[Sqrt[c - c*x]
/(Sqrt[2]*Sqrt[c])]))/(48*(-1 + x)*(1 + x)^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.38, size = 206, normalized size = 1.63

$$-\frac{5}{8}a^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-cx}}{\sqrt{c}\,\sqrt{ax+a}}\right)-\frac{a^3c^3\sqrt{c-cx}\left(\frac{15a^5(c-cx)^5}{(ax+a)^5}+\frac{85a^4c(c-cx)^4}{(ax+a)^4}+\frac{198a^3c^2(c-cx)^3}{(ax+a)^3}-\frac{198a^2c^3(c-cx)^2}{(ax+a)^2}-\frac{85ac^4(c-cx)}{ax+a}-15c^5\right)}{24\sqrt{ax+a}\left(\frac{a(c-cx)}{ax+a}+c\right)^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```

```
[Out] -1/24*(a^3*c^3*Sqrt[c - c*x]*(-15*c^5 - (85*a*c^4*(c - c*x))/(a + a*x) - (1
98*a^2*c^3*(c - c*x)^2)/(a + a*x)^2 + (198*a^3*c^2*(c - c*x)^3)/(a + a*x)^3
 + (85*a^4*c*(c - c*x)^4)/(a + a*x)^4 + (15*a^5*(c - c*x)^5)/(a + a*x)^5))/
(Sqrt[a + a*x]*(c + (a*(c - c*x))/(a + a*x))^6) - (5*a^(5/2)*c^(5/2)*ArcTan
[(Sqrt[a]*Sqrt[c - c*x])/(Sqrt[c]*Sqrt[a + a*x])])/8
```

**fricas [A]** time = 1.61, size = 201, normalized size = 1.60

$$\left[\frac{5}{32}\sqrt{-ac}\,a^2c^2\log\left(2\,acx^2+2\,\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x-ac\right)+\frac{1}{48}\left(8\,a^2c^2x^5-26\,a^2c^2x^3+33\,a^2c^2x\right)\sqrt{ax+a}\,\sqrt{-cx+c},-\frac{5}{16}\sqrt{ac}\,a^2c^2\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)+\frac{1}{48}\left(8\,a^2c^2x^5-26\,a^2c^2x^3+33\,a^2c^2x\right)\sqrt{ax+a}\,\sqrt{-cx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [5/32*sqrt(-a*c)*a^2*c^2*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c
*x + c)*x - a*c) + 1/48*(8*a^2*c^2*x^5 - 26*a^2*c^2*x^3 + 33*a^2*c^2*x)*sqr
t(a*x + a)*sqrt(-c*x + c), -5/16*sqrt(a*c)*a^2*c^2*arctan(sqrt(a*c)*sqrt(a*
x + a)*sqrt(-c*x + c)*x/(a*c*x^2 - a*c)) + 1/48*(8*a^2*c^2*x^5 - 26*a^2*c^2
*x^3 + 33*a^2*c^2*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

**giac [B]** time = 1.57, size = 679, normalized size = 5.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/240*(150*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c +
2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((2*((a*x + a)*(4*(a
*x + a)*(5*(a*x + a)/a^5 - 31/a^4) + 321/a^3) - 451/a^2)*(a*x + a) + 745/a)
*(a*x + a) - 405)*sqrt(a*x + a))*c^2*abs(a) - 1/120*(90*a^2*c*log(abs(-sqrt
(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-
(a*x + a)*a*c + 2*a^2*c)*((2*(a*x + a)*(3*(a*x + a)*(4*(a*x + a)/a^4 - 21/a
^3) + 133/a^2) - 295/a)*(a*x + a) + 195)*sqrt(a*x + a))*c^2*abs(a) - 1/12*(
18*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)
))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*(2*(a*x + a)*(3*(
a*x + a)/a^3 - 13/a^2) + 43/a) - 39)*sqrt(a*x + a))*c^2*abs(a) + 1/3*(6*a^2
*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqr
t(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(2*(a*x +
 a)/a^2 - 7/a) + 9))*c^2*abs(a) - (2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a
) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a
^2*c)*sqrt(a*x + a))*c^2*abs(a) + 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x
 + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c +
 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*a))*c^2*abs(a)/a
```

**maple [B]** time = 0.01, size = 193, normalized size = 1.53

$$\frac{5\sqrt{\left(-cx+c\right)\left(ax+a\right)}\, a^{3} c^{3} \arctan\left(\frac{\sqrt{ac}\, x}{\sqrt{-ac\,x^{2}+ac}}\right)}{16\sqrt{-cx+c}\, \sqrt{ax+a}\, \sqrt{ac}}+\frac{5\sqrt{-cx+c}\, \sqrt{ax+a}\, a^{2} c^{2}}{16}+\frac{5\left(-cx+c\right)^{\frac{3}{2}} \sqrt{ax+a}\, a^{2} c}{48}+\frac{\left(-cx+c\right)^{\frac{5}{2}} \sqrt{ax+a}\, a^{2}}{24}-\frac{\sqrt{ax+a}\, \left(-cx+c\right)^{\frac{7}{2}} a^{2}}{8c}-\frac{\left(ax+a\right)^{\frac{3}{2}} \left(-cx+c\right)^{\frac{7}{2}} a}{6c}-\frac{\left(ax+a\right)^{\frac{5}{2}} \left(-cx+c\right)^{\frac{7}{2}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(5/2)*(-c*x+c)^(5/2),x)
```

```
[Out] -1/6/c*(a*x+a)^(5/2)*(-c*x+c)^(7/2)-1/6*a/c*(a*x+a)^(3/2)*(-c*x+c)^(7/2)-1/
8*a^2/c*(a*x+a)^(1/2)*(-c*x+c)^(7/2)+1/24*a^2*(-c*x+c)^(5/2)*(a*x+a)^(1/2)+
5/48*a^2*c*(-c*x+c)^(3/2)*(a*x+a)^(1/2)+5/16*a^2*c^2*(-c*x+c)^(1/2)*(a*x+a)
^(1/2)+5/16*a^3*c^3*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(
a*c)^(1/2)*arctan((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))
```

**maxima [A]** time = 3.02, size = 72, normalized size = 0.57

$$\frac{5\,a^{3} c^{3} \arcsin\left(x\right)}{16\,\sqrt{ac}}+\frac{5}{16}\,\sqrt{-acx^{2}+ac}\,a^{2} c^{2} x+\frac{5}{24}\,{\left(-acx^{2}+ac\right)}^{\frac{3}{2}} acx+\frac{1}{6}\,{\left(-acx^{2}+ac\right)}^{\frac{5}{2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^3*c^3*arcsin(x)/sqrt(a*c) + 5/16*sqrt(-a*c*x^2 + a*c)*a^2*c^2*x + 5/
24*(-a*c*x^2 + a*c)^(3/2)*a*c*x + 1/6*(-a*c*x^2 + a*c)^(5/2)*x
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,x\right)}^{5/2}\,{\left(c-c\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(5/2)*(c - c*x)^(5/2),x)
```

```
[Out] int((a + a*x)^(5/2)*(c - c*x)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(x + 1\right)\right)^{\frac{5}{2}} \left(- c \left(x - 1\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(5/2)*(-c*x+c)**(5/2),x)
```

```
[Out] Integral((a*(x + 1))**(5/2)*(-c*(x - 1))**(5/2), x)
```

## 3.1069 $\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx$

**Optimal**. Leaf size=96

$$\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{ax+a}}{\sqrt{a}\,\sqrt{c-cx}}\right)+\frac{3}{8}acx\sqrt{ax+a}\,\sqrt{c-cx}\,+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {38, 63, 217, 203}

$$\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{ax+a}}{\sqrt{a}\,\sqrt{c-cx}}\right)+\frac{3}{8}acx\sqrt{ax+a}\,\sqrt{c-cx}\,+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] (3*a*c*x*Sqrt[a + a*x]*Sqrt[c - c*x])/8 + (x*(a + a*x)^(3/2)*(c - c*x)^(3/2
))/4 + (3*a^(3/2)*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c -
c*x])])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx &= \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{4}(3ac)\int \sqrt{a+ax}\,\sqrt{c-cx}\,dx\\ &= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{8}\left(3a^2c^2\right)\int \frac{1}{\sqrt{a+ax}\,\sqrt{c-cx}}\,dx\\ &= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{4}\left(3ac^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx\right.\\ &= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{4}\left(3ac^2\right)\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx,x,\right.\\ &= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ax}}{\sqrt{a}\,\sqrt{c-cx}}\right)\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 104, normalized size = 1.08

$$\frac{\sqrt{c}\,(a(x+1))^{3/2}\sqrt{c-cx}\left(\sqrt{c}\,x\sqrt{x+1}\left(-2x^3+2x^2+5x-5\right)+6\sqrt{c-cx}\,\sin^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{2}\,\sqrt{c}}\right)\right)}{8(x-1)(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] (Sqrt[c]*(a*(1 + x))^(3/2)*Sqrt[c - c*x]*(Sqrt[c]*x*Sqrt[1 + x]*(-5 + 5*x +
 2*x^2 - 2*x^3) + 6*Sqrt[c - c*x]*ArcSin[Sqrt[c - c*x]/(Sqrt[2]*Sqrt[c])]))
/(8*(-1 + x)*(1 + x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.28, size = 160, normalized size = 1.67

$$-\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-cx}}{\sqrt{c}\,\sqrt{ax+a}}\right)-\frac{a^2c^2\sqrt{c-cx}\left(\frac{3a^3(c-cx)^3}{(ax+a)^3}+\frac{11a^2c(c-cx)^2}{(ax+a)^2}-\frac{11ac^2(c-cx)}{ax+a}-3c^3\right)}{4\sqrt{ax+a}\left(\frac{a(c-cx)}{ax+a}+c\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] -1/4*(a^2*c^2*Sqrt[c - c*x]*(-3*c^3 - (11*a*c^2*(c - c*x))/(a + a*x) + (11*
a^2*c*(c - c*x)^2)/(a + a*x)^2 + (3*a^3*(c - c*x)^3)/(a + a*x)^3))/(Sqrt[a
+ a*x]*(c + (a*(c - c*x))/(a + a*x))^4) - (3*a^(3/2)*c^(3/2)*ArcTan[(Sqrt[a
]*Sqrt[c - c*x])/(Sqrt[c]*Sqrt[a + a*x])])/4
```

**fricas [A]** time = 1.40, size = 155, normalized size = 1.61

$$\left[\frac{3}{16}\sqrt{-ac}\,ac\log\left(2\,acx^2+2\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}\,x-ac\right)-\frac{1}{8}\left(2\,acx^3-5\,acx\right)\sqrt{ax+a}\sqrt{-cx+c}\,,-\frac{3}{8}\sqrt{ac}\,ac\arctan\left(\frac{\sqrt{ac}\sqrt{ax+a}\sqrt{-cx+c}\,x}{acx^2-ac}\right)-\frac{1}{8}\left(2\,acx^3-5\,acx\right)\sqrt{ax+a}\sqrt{-cx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [3/16*sqrt(-a*c)*a*c*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x +
 c)*x - a*c) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c), -3/8
*sqrt(a*c)*a*c*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a*c*x^2 - a
*c)) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

**giac [B]** time = 1.25, size = 403, normalized size = 4.20

$$-\frac{\left(\frac{18\,a^2c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}}+\sqrt{-(ax+a)ac+2\,a^2c}\left((ax+a)\left(2\,(ax+a)\left(\frac{3\,(ax+a)}{a^3}-\frac{13}{a^2}\right)+\frac{43}{a}\right)-39\right)\sqrt{ax+a}\right)c|a|}{24\,a}+\frac{\left(\frac{6\,a^2c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\left((ax+a)\left(\frac{2\,(ax+a)}{a^2}-\frac{7}{a}\right)+9\right)\right)c|a|}{6\,a}-\frac{\left(\frac{2\,a^2c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\right)c|a|}{a}+\frac{\left(\frac{2\,a^3c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}}+\sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\,(ax-2\,a)\right)c|a|}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(18*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2
*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*(2*(a*x +
a)*(3*(a*x + a)/a^3 - 13/a^2) + 43/a) - 39)*sqrt(a*x + a))*c*abs(a)/a + 1/6
*(6*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c
)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(2
*(a*x + a)/a^2 - 7/a) + 9))*c*abs(a)/a - (2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(
a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*
c + 2*a^2*c)*sqrt(a*x + a))*c*abs(a)/a + 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*s
qrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a
)*a*c + 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*a))*c*abs(a)/a^2
```

**maple [B]** time = 0.00, size = 143, normalized size = 1.49

$$\frac{3\sqrt{(-cx+c)\,(ax+a)}\;a^2c^2\arctan\left(\frac{\sqrt{ac}\;x}{\sqrt{-ac\,x^2+ac}}\right)}{8\sqrt{-cx+c}\;\sqrt{ax+a}\;\sqrt{ac}}+\frac{3\sqrt{-cx+c}\;\sqrt{ax+a}\;ac}{8}+\frac{\sqrt{ax+a}\;\left(-cx+c\right)^{\frac{3}{2}}a}{8}-\frac{\sqrt{ax+a}\;\left(-cx+c\right)^{\frac{5}{2}}a}{4c}-\frac{\left(ax+a\right)^{\frac{3}{2}}\left(-cx+c\right)^{\frac{5}{2}}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(3/2)*(-c*x+c)^(3/2),x)
```

```
[Out] -1/4/c*(a*x+a)^(3/2)*(-c*x+c)^(5/2)-1/4*a/c*(a*x+a)^(1/2)*(-c*x+c)^(5/2)+1/
8*(a*x+a)^(1/2)*(-c*x+c)^(3/2)*a+3/8*a*c*(-c*x+c)^(1/2)*(a*x+a)^(1/2)+3/8*a
^2*c^2*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(a*c)^(1/2)*ar
ctan((a*c)^(1/2)/(-a*c*x^2+a*c)^(1/2)*x)
```

**maxima [A]** time = 3.09, size = 50, normalized size = 0.52

$$\frac{3\,a^2c^2\arcsin\left(x\right)}{8\,\sqrt{ac}}+\frac{3}{8}\,\sqrt{-acx^2+ac}\,acx+\frac{1}{4}\,\left(-acx^2+ac\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^2*c^2*arcsin(x)/sqrt(a*c) + 3/8*sqrt(-a*c*x^2 + a*c)*a*c*x + 1/4*(-a*
c*x^2 + a*c)^(3/2)*x
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,x\right)}^{3/2}\,{\left(c-c\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(3/2)*(c - c*x)^(3/2),x)
```

```
[Out] int((a + a*x)^(3/2)*(c - c*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(x + 1\right)\right)^{\frac{3}{2}} \left(- c \left(x - 1\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(3/2)*(-c*x+c)**(3/2),x)
```

```
[Out] Integral((a*(x + 1))**(3/2)*(-c*(x - 1))**(3/2), x)
```

## 3.1070 $\int \sqrt{a+ax}\sqrt{c-cx}\,dx$

**Optimal**. Leaf size=67

$$\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}+\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {38, 63, 217, 203}

$$\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}+\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

```
[Out] (x*Sqrt[a + a*x]*Sqrt[c - c*x])/2 + Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
+ a*x])/(Sqrt[a]*Sqrt[c - c*x])]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+ax}\sqrt{c-cx}\,dx &= \frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx}+\frac{1}{2}(ac)\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx \\
&= \frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx}+c\,\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx, x, \sqrt{a+ax}\right) \\
&= \frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx}+c\,\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx, x, \frac{\sqrt{a+ax}}{\sqrt{c-cx}}\right) \\
&= \frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx}+\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 69, normalized size = 1.03

$$\frac{\sqrt{a(x+1)}\left(x\sqrt{x+1}\sqrt{c-cx}-2\sqrt{c}\sin^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{2}\sqrt{c}}\right)\right)}{2\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

```
[Out] (Sqrt[a*(1 + x)]*(x*Sqrt[1 + x]*Sqrt[c - c*x] - 2*Sqrt[c]*ArcSin[Sqrt[c - c
*x]/(Sqrt[2]*Sqrt[c])]))/(2*Sqrt[1 + x])
```

**IntegrateAlgebraic [A]** time = 0.19, size = 105, normalized size = 1.57

$$-\frac{ac\sqrt{c-cx}\left(\frac{a(c-cx)}{ax+a}-c\right)}{\sqrt{ax+a}\left(\frac{a(c-cx)}{ax+a}+c\right)^2}-\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-cx}}{\sqrt{c}\sqrt{ax+a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

```
[Out] -((a*c*Sqrt[c - c*x]*(-c + (a*(c - c*x))/(a + a*x)))/(Sqrt[a + a*x]*(c + (a
*(c - c*x))/(a + a*x))^2)) - Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c - c*x])
/(Sqrt[c]*Sqrt[a + a*x])]
```

**fricas [A]** time = 1.60, size = 127, normalized size = 1.90

$$\left[\frac{1}{2}\sqrt{ax+a}\sqrt{-cx+c}\,x+\frac{1}{4}\sqrt{-ac}\log\left(2\,acx^2+2\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}\,x-ac\right),\frac{1}{2}\sqrt{ax+a}\sqrt{-cx+c}\,x-\frac{1}{2}\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{ax+a}\sqrt{-cx+c}\,x}{acx^2-ac}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a*x + a)*sqrt(-c*x + c)*x + 1/4*sqrt(-a*c)*log(2*a*c*x^2 + 2*sqrt
(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x - a*c), 1/2*sqrt(a*x + a)*sqrt(-c*x +
 c)*x - 1/2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a*c*
x^2 - a*c))]
```

**giac [B]** time = 0.90, size = 173, normalized size = 2.58

$$-\frac{\left(\frac{2\,a^2c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\right)|a|}{a^2}+\frac{\left(\frac{2\,a^3c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}}+\sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\,(ax-2\,a)\right)|a|}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c
)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a))*abs(a)/a^2 +
 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a
^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*
a))*abs(a)/a^3
```

**maple [A]** time = 0.01, size = 98, normalized size = 1.46

$$\frac{\sqrt{(-cx+c)(ax+a)}\;ac\arctan\left(\frac{\sqrt{ac}\;x}{\sqrt{-ac\,x^2+ac}}\right)}{2\sqrt{-cx+c}\;\sqrt{ax+a}\;\sqrt{ac}}-\frac{\sqrt{ax+a}\;(-cx+c)^{\frac{3}{2}}}{2c}+\frac{\sqrt{ax+a}\;\sqrt{-cx+c}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(1/2)*(-c*x+c)^(1/2),x)
```

```
[Out] -1/2/c*(a*x+a)^(1/2)*(-c*x+c)^(3/2)+1/2*(a*x+a)^(1/2)*(-c*x+c)^(1/2)+1/2*a*
c*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(a*c)^(1/2)*arctan(
(a*c)^(1/2)/(-a*c*x^2+a*c)^(1/2)*x)
```

**maxima [A]** time = 3.08, size = 28, normalized size = 0.42

$$\frac{ac\arcsin(x)}{2\sqrt{ac}}+\frac{1}{2}\sqrt{-acx^2+ac}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*c*arcsin(x)/sqrt(a*c) + 1/2*sqrt(-a*c*x^2 + a*c)*x
```

**mupad [B]** time = 0.30, size = 59, normalized size = 0.88

$$\frac{x\sqrt{a+a\,x}\,\sqrt{c-c\,x}}{2}-\frac{\sqrt{a}\,\sqrt{-c}\,\ln\left(\sqrt{-c}\,\sqrt{a\,(x+1)}\,\sqrt{-c\,(x-1)}-\sqrt{a}\,c\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(1/2)*(c - c*x)^(1/2),x)
```

```
[Out] (x*(a + a*x)^(1/2)*(c - c*x)^(1/2))/2 - (a^(1/2)*(-c)^(1/2)*log((-c)^(1/2)*
(a*(x + 1))^(1/2)*(-c*(x - 1))^(1/2) - a^(1/2)*c*x))/2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\,(x+1)}\,\sqrt{-c\,(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(1/2)*(-c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(x + 1))*sqrt(-c*(x - 1)), x)
```

# 3.1071 $\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx$

**Optimal**. Leaf size=43

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

**Rubi [A]** time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {63, 217, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/(Sqrt[a]*Sqrt[c
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx, x, \sqrt{a+ax}\right)}{a}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx, x, \frac{\sqrt{a+ax}}{\sqrt{c-cx}}\right)}{a}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

**Mathematica** [A] time = 0.02, size = 47, normalized size = 1.09

$$\frac{2\sqrt{x+1}\,\tan^{-1}\left(\frac{\sqrt{c}\sqrt{x+1}}{\sqrt{c-cx}}\right)}{\sqrt{c}\sqrt{a(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

```
[Out] (2*Sqrt[1 + x]*ArcTan[(Sqrt[c]*Sqrt[1 + x])/Sqrt[c - c*x]])/(Sqrt[c]*Sqrt[a
*(1 + x)])
```

**IntegrateAlgebraic** [A] time = 0.08, size = 43, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-cx}}{\sqrt{c}\sqrt{ax+a}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Sqrt[c - c*x])/(Sqrt[c]*Sqrt[a + a*x])])/(Sqrt[a]*Sqrt[
c])
```

**fricas** [A] time = 1.64, size = 101, normalized size = 2.35

$$\left[-\frac{\sqrt{-ac}\,\log\left(2\,acx^2 - 2\,\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x - ac\right)}{2\,ac}, -\frac{\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(2*a*c*x^2 - 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*
x - a*c)/(a*c), -sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/
(a*c*x^2 - a*c))/(a*c)]
```

**giac** [A] time = 0.76, size = 49, normalized size = 1.14

$$-\frac{2\,a\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/(
sqrt(-a*c)*abs(a))
```

**maple** [A] time = 0.00, size = 57, normalized size = 1.33

$$\frac{\sqrt{(-cx+c)\,(ax+a)}\,\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)}{\sqrt{ax+a}\,\sqrt{-cx+c}\,\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x)
```

```
[Out] ((-c*x+c)*(a*x+a))^(1/2)/(a*x+a)^(1/2)/(-c*x+c)^(1/2)/(a*c)^(1/2)*arctan((a
*c)^(1/2)/(-a*c*x^2+a*c)^(1/2)*x)
```

**maxima** [A] time = 2.99, size = 8, normalized size = 0.19

$$\frac{\arcsin(x)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)/sqrt(a*c)
```

**mupad** [B] time = 0.18, size = 44, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{a\,\left(\sqrt{c-c\,x}-\sqrt{c}\right)}{\sqrt{a\,c}\,\left(\sqrt{a+a\,x}-\sqrt{a}\right)}\right)}{\sqrt{a\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(1/2)*(c - c*x)^(1/2)),x)
```

```
[Out] -(4*atan((a*((c - c*x)^(1/2) - c^(1/2)))/((a*c)^(1/2)*((a + a*x)^(1/2) - a^
(1/2)))))/(a*c)^(1/2)
```

**sympy [C]** time = 3.95, size = 85, normalized size = 1.98

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\sqrt{c}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(1/2)/(-c*x+c)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
x**(-2))/(4*pi**(3/2)*sqrt(a)*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4, 1/2,
 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/x**2)/(4*pi**(
3/2)*sqrt(a)*sqrt(c))
```

## 3.1072 $\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}} dx$

**Optimal**. Leaf size=27

$$\frac{x}{ac\sqrt{ax+a}\sqrt{c-cx}}$$

**Rubi [A]** time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {39}

$$\frac{x}{ac\sqrt{ax+a}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

```
[Out] x/(a*c*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}} dx = \frac{x}{ac\sqrt{a+ax}\sqrt{c-cx}}$$

**Mathematica [A]** time = 0.02, size = 27, normalized size = 1.00

$$\frac{x(x+1)}{c(a(x+1))^{3/2}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

```
[Out] (x*(1 + x))/(c*(a*(1 + x))^(3/2)*Sqrt[c - c*x])
```

**IntegrateAlgebraic [A]** time = 0.10, size = 47, normalized size = 1.74

$$\frac{\sqrt{ax+a}\left(c-\frac{a(c-cx)}{ax+a}\right)}{2a^2c^2\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
[Out] (Sqrt[a + a*x]*(c - (a*(c - c*x))/(a + a*x)))/(2*a^2*c^2*Sqrt[c - c*x])
```

**fricas [A]** time = 1.48, size = 39, normalized size = 1.44

$$-\frac{\sqrt{ax+a}\sqrt{-cx+c}\,x}{a^2c^2x^2-a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="fricas")
[Out] -sqrt(a*x + a)*sqrt(-c*x + c)*x/(a^2*c^2*x^2 - a^2*c^2)
```

**giac [B]** time = 0.70, size = 116, normalized size = 4.30

$$-\frac{2\sqrt{-ac}\,a}{\left(2\,a^2c-\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2\right)c|a|}-\frac{\sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}}{2\left((ax+a)ac-2\,a^2c\right)c|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="giac")
[Out] -2*sqrt(-a*c)*a/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^2)*c*abs(a)) - 1/2*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a
)/(((a*x + a)*a*c - 2*a^2*c)*c*abs(a))
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.93

$$-\frac{(x+1)(x-1)\,x}{(ax+a)^{\frac{3}{2}}\,(-cx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x)
[Out] -(x+1)*(x-1)*x/(a*x+a)^(3/2)/(-c*x+c)^(3/2)
```

**maxima** [A] time = 1.31, size = 21, normalized size = 0.78

$$\frac{x}{\sqrt{-acx^2+ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-a*c*x^2 + a*c)*a*c)
```

**mupad** [B] time = 0.39, size = 23, normalized size = 0.85

$$\frac{x}{a\,c\,\sqrt{a+a\,x}\,\sqrt{c-c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x)
```

```
[Out] x/(a*c*(a + a*x)^(1/2)*(c - c*x)^(1/2))
```

**sympy** [C] time = 4.44, size = 82, normalized size = 3.04

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2 \\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0 \end{matrix}\middle|\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0 \end{matrix}\middle|\frac{e^{-2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(3/2)/(-c*x+c)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), x
**(-2))/(2*pi**(3/2)*a**(3/2)*c**(3/2)) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4,
 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), exp_polar(-2*I*pi)/x**2)/(2*pi**(3
/2)*a**(3/2)*c**(3/2))
```

## 3.1073 $\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx$

**Optimal**. Leaf size=61

$$\frac{2x}{3a^2c^2\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {40, 39}

$$\frac{2x}{3a^2c^2\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

```
[Out] x/(3*a*c*(a + a*x)^(3/2)*(c - c*x)^(3/2)) + (2*x)/(3*a^2*c^2*Sqrt[a + a*x]*
Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx &= \frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx}{3ac}\\ &= \frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2x}{3a^2c^2\sqrt{a+ax}\,\sqrt{c-cx}}\end{aligned}$$

**Mathematica** [A]  time = 0.03, size = 42, normalized size = 0.69

$$\frac{x(x+1)\left(2x^2-3\right)}{3c^2(x-1)(a(x+1))^{5/2}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

```
[Out] (x*(1 + x)*(-3 + 2*x^2))/(3*c^2*(-1 + x)*(a*(1 + x))^(5/2)*Sqrt[c - c*x])
```

**IntegrateAlgebraic** [A]  time = 0.12, size = 93, normalized size = 1.52

$$\frac{(ax+a)^{3/2}\left(-\frac{a^3(c-cx)^3}{(ax+a)^3}-\frac{9a^2c(c-cx)^2}{(ax+a)^2}+\frac{9ac^2(c-cx)}{ax+a}+c^3\right)}{24a^4c^4(c-cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

```
[Out] ((a + a*x)^(3/2)*(c^3 + (9*a*c^2*(c - c*x))/(a + a*x) - (9*a^2*c*(c - c*x)^
2)/(a + a*x)^2 - (a^3*(c - c*x)^3)/(a + a*x)^3))/(24*a^4*c^4*(c - c*x)^(3/2
))
```

**fricas** [A]  time = 1.08, size = 57, normalized size = 0.93

$$-\frac{\left(2\,x^3-3\,x\right)\sqrt{ax+a}\,\sqrt{-cx+c}}{3\left(a^3c^3x^4-2\,a^3c^3x^2+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^3 - 3*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^3*c^3*x^4 - 2*a^3*c^3*x^
2 + a^3*c^3)
```

**giac** [B]  time = 0.81, size = 237, normalized size = 3.89

$$-\frac{\sqrt{-(ax+a)ac+2\,a^2c}\,\sqrt{ax+a}\left(\frac{4\,(ax+a)|a|}{a^2c}-\frac{9\,|a|}{ac}\right)}{12\left((ax+a)ac-2\,a^2c\right)^2}-\frac{16\,\sqrt{-ac}\,a^4c^2-18\,\sqrt{-ac}\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2a^2c+3\,\sqrt{-ac}\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^4}{3\left(2\,a^2c-\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2\right)^3c^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(4*(a*x + a)*abs(a)/(a^2
*c) - 9*abs(a)/(a*c))/((a*x + a)*a*c - 2*a^2*c)^2 - 1/3*(16*sqrt(-a*c)*a^4*
c^2 - 18*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2
*c))^2*a^2*c + 3*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^4)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^2)^3*c^2*abs(a))
```

**maple [A]** time = 0.00, size = 32, normalized size = 0.52

$$\frac{(x+1)\,(x-1)\left(2x^{2}-3\right)x}{3\,(ax+a)^{\frac{5}{2}}\,(-cx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x)
```

```
[Out] 1/3*(x+1)*(x-1)*x*(2*x^2-3)/(a*x+a)^(5/2)/(-c*x+c)^(5/2)
```

**maxima [A]** time = 1.35, size = 45, normalized size = 0.74

$$\frac{x}{3\left(-acx^{2}+ac\right)^{\frac{3}{2}}ac}+\frac{2\,x}{3\,\sqrt{-acx^{2}+ac}\,a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/((-a*c*x^2 + a*c)^(3/2)*a*c) + 2/3*x/(sqrt(-a*c*x^2 + a*c)*a^2*c^2)
```

**mupad [B]** time = 0.41, size = 62, normalized size = 1.02

$$-\frac{3\,x\,\sqrt{c-c\,x}\,-2\,x^{3}\,\sqrt{c-c\,x}}{\sqrt{a+a\,x}\,\left(c-c\,x\right)^{2}\,\left(3\,a^{2}\,\left(c-c\,x\right)-6\,a^{2}\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x)
```

```
[Out] -(3*x*(c - c*x)^(1/2) - 2*x^3*(c - c*x)^(1/2))/((a + a*x)^(1/2)*(c - c*x)^2
*(3*a^2*(c - c*x) - 6*a^2*c))
```

**sympy [C]** time = 13.69, size = 82, normalized size = 1.34

$$\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{1}{2},\frac{5}{2},3\\ \frac{5}{4},\frac{7}{4},2,\frac{5}{2},3 & 0\end{matrix}\middle|\frac{1}{x^{2}}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{3}{4},\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & -\frac{1}{2},0,2,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^{2}}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(5/2)/(-c*x+c)**(5/2),x)
```

```
[Out] I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), x*
*(-2))/(3*pi**(3/2)*a**(5/2)*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3/4, 5/4,
1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), exp_polar(-2*I*pi)/x**2)/(3*pi**(3/
2)*a**(5/2)*c**(5/2))
```

## 3.1074 $\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx$

**Optimal**. Leaf size=91

$$\frac{8x}{15a^3c^3\sqrt{ax+a}\sqrt{c-cx}}+\frac{4x}{15a^2c^2(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {40, 39}

$$\frac{8x}{15a^3c^3\sqrt{ax+a}\sqrt{c-cx}}+\frac{4x}{15a^2c^2(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] x/(5*a*c*(a + a*x)^(5/2)*(c - c*x)^(5/2)) + (4*x)/(15*a^2*c^2*(a + a*x)^(3/
2)*(c - c*x)^(3/2)) + (8*x)/(15*a^3*c^3*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx}{5ac}\\ &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx}{15a^2c^2}\\ &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8x}{15a^3c^3\sqrt{a+ax}\sqrt{c-cx}}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 49, normalized size = 0.54

$$\frac{x\left(8x^4-20x^2+15\right)}{15a^3c^3\left(x^2-1\right)^2\sqrt{a(x+1)}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] (x*(15 - 20*x^2 + 8*x^4))/(15*a^3*c^3*Sqrt[a*(1 + x)]*Sqrt[c - c*x]*(-1 + x
^2)^2)
```

**IntegrateAlgebraic** [A] time = 0.13, size = 141, normalized size = 1.55

$$\frac{(ax+a)^{5/2}\left(-\frac{3a^5(c-cx)^5}{(ax+a)^5}-\frac{25a^4c(c-cx)^4}{(ax+a)^4}-\frac{150a^3c^2(c-cx)^3}{(ax+a)^3}+\frac{150a^2c^3(c-cx)^2}{(ax+a)^2}+\frac{25ac^4(c-cx)}{ax+a}+3c^5\right)}{480a^6c^6(c-cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] ((a + a*x)^(5/2)*(3*c^5 + (25*a*c^4*(c - c*x))/(a + a*x) + (150*a^2*c^3*(c
- c*x)^2)/(a + a*x)^2 - (150*a^3*c^2*(c - c*x)^3)/(a + a*x)^3 - (25*a^4*c*(
c - c*x)^4)/(a + a*x)^4 - (3*a^5*(c - c*x)^5)/(a + a*x)^5))/(480*a^6*c^6*(c
 - c*x)^(5/2))
```

**fricas** [A] time = 1.12, size = 74, normalized size = 0.81

$$-\frac{\left(8\,x^5-20\,x^3+15\,x\right)\sqrt{ax+a}\,\sqrt{-cx+c}}{15\left(a^4c^4x^6-3\,a^4c^4x^4+3\,a^4c^4x^2-a^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*x^5 - 20*x^3 + 15*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^4*c^4*x^6 - 3
*a^4*c^4*x^4 + 3*a^4*c^4*x^2 - a^4*c^4)
```

**giac** [B] time = 1.06, size = 333, normalized size = 3.66

$$-\frac{\sqrt{-(ax+a)ac+2\,a^2c}\,\sqrt{ax+a}\left((ax+a)\left(\frac{64\,(ax+a)}{c|a|}-\frac{275\,a}{c|a|}\right)+\frac{300\,a^2}{c|a|}\right)}{240\left((ax+a)ac-2\,a^2c\right)^3}+\frac{1024\,a^8c^4-2200\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2a^6c^3+1660\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^4a^4c^2-450\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^6a^2c+45\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^8}{60\left(2\,a^2c-\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2\right)^5\sqrt{-ac}\,c^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/240*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(64*(a*x + a
)/(c*abs(a)) - 275*a/(c*abs(a))) + 300*a^2/(c*abs(a)))/((a*x + a)*a*c - 2*a
^2*c)^3 + 1/60*(1024*a^8*c^4 - 2200*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
+ a)*a*c + 2*a^2*c))^2*a^6*c^3 + 1660*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*
x + a)*a*c + 2*a^2*c))^4*a^4*c^2 - 450*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a
*x + a)*a*c + 2*a^2*c))^6*a^2*c + 45*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
 + a)*a*c + 2*a^2*c))^8)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
 + a)*a*c + 2*a^2*c))^2)^5*sqrt(-a*c)*c^2*abs(a))
```

**maple [A]** time = 0.00, size = 37, normalized size = 0.41

$$-\frac{(x+1)(x-1)\left(8x^4-20x^2+15\right)x}{15\,(ax+a)^{\frac{7}{2}}\,(-cx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x)
```

```
[Out] -1/15*(x+1)*(x-1)*x*(8*x^4-20*x^2+15)/(a*x+a)^(7/2)/(-c*x+c)^(7/2)
```

**maxima [A]** time = 1.36, size = 67, normalized size = 0.74

$$\frac{x}{5\left(-acx^2+ac\right)^{\frac{5}{2}}ac}+\frac{4\,x}{15\left(-acx^2+ac\right)^{\frac{3}{2}}a^2c^2}+\frac{8\,x}{15\,\sqrt{-acx^2+ac}\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-a*c*x^2 + a*c)^(5/2)*a*c) + 4/15*x/((-a*c*x^2 + a*c)^(3/2)*a^2*c^2
) + 8/15*x/(sqrt(-a*c*x^2 + a*c)*a^3*c^3)
```

**mupad [B]** time = 0.44, size = 50, normalized size = 0.55

$$\frac{x\,\left(8\,x^4-20\,x^2+15\right)}{15\,a^3\,\sqrt{a+a\,x}\,{\left(c-c\,x\right)}^{5/2}\,\left(c+3\,c\,x-x\,\left(c-c\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x)
```

```
[Out] (x*(8*x^4 - 20*x^2 + 15))/(15*a^3*(a + a*x)^(1/2)*(c - c*x)^(5/2)*(c + 3*c*
x - x*(c - c*x)))
```

**sympy [C]** time = 55.15, size = 85, normalized size = 0.93

$$-\frac{2iG_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 & \frac{1}{2}, \frac{7}{2}, 4 \\ \frac{7}{4}, \frac{9}{4}, 3, \frac{7}{2}, 4 & 0 \end{matrix} \middle| \frac{1}{x^2}\right)}{15\pi^{\frac{3}{2}}a^{\frac{7}{2}}c^{\frac{7}{2}}} + \frac{2G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, \frac{5}{4}, \frac{7}{4}, 1 & \\ \frac{5}{4}, \frac{7}{4} & -\frac{1}{2}, 0, 3, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{x^2}\right)}{15\pi^{\frac{3}{2}}a^{\frac{7}{2}}c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(7/2)/(-c*x+c)**(7/2),x)
```

```
[Out] -2*I*meijerg(((7/4, 9/4, 1), (1/2, 7/2, 4)), ((7/4, 9/4, 3, 7/2, 4), (0,)),
 x**(-2))/(15*pi**(3/2)*a**(7/2)*c**(7/2)) + 2*meijerg(((-1/2, 0, 1/2, 5/4,
 7/4, 1), ()), ((5/4, 7/4), (-1/2, 0, 3, 0)), exp_polar(-2*I*pi)/x**2)/(15*
pi**(3/2)*a**(7/2)*c**(7/2))
```

## 3.1075 $\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx$

**Optimal**. Leaf size=121

$$\frac{16x}{35a^4c^4\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{8x}{35a^3c^3(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{6x}{35a^2c^2(ax+a)^{5/2}(c-cx)^{5/2}}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {40, 39}

$$\frac{16x}{35a^4c^4\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{8x}{35a^3c^3(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{6x}{35a^2c^2(ax+a)^{5/2}(c-cx)^{5/2}}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] x/(7*a*c*(a + a*x)^(7/2)*(c - c*x)^(7/2)) + (6*x)/(35*a^2*c^2*(a + a*x)^(5/
2)*(c - c*x)^(5/2)) + (8*x)/(35*a^3*c^3*(a + a*x)^(3/2)*(c - c*x)^(3/2)) +
(16*x)/(35*a^4*c^4*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx = \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx}{7ac}$$
$$= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}} + \frac{24\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx}{35a^2c^2}$$
$$= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}} + \frac{8x}{35a^3c^3(a+ax)^{3/2}(c-cx}$$
$$= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}} + \frac{8x}{35a^3c^3(a+ax)^{3/2}(c-cx}$$

**Mathematica** [A] time = 0.04, size = 54, normalized size = 0.45

$$\frac{x\left(16x^6-56x^4+70x^2-35\right)}{35a^4c^4\left(x^2-1\right)^3\sqrt{a(x+1)}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] (x*(-35 + 70*x^2 - 56*x^4 + 16*x^6))/(35*a^4*c^4*Sqrt[a*(1 + x)]*Sqrt[c - c
*x]*(-1 + x^2)^3)
```

**IntegrateAlgebraic** [A] time = 0.14, size = 187, normalized size = 1.55

$$\frac{(ax+a)^{7/2}\left(-\frac{5a^7(c-cx)^7}{(ax+a)^7}-\frac{49a^6c(c-cx)^6}{(ax+a)^6}-\frac{245a^5c^2(c-cx)^5}{(ax+a)^5}-\frac{1225a^4c^3(c-cx)^4}{(ax+a)^4}+\frac{1225a^3c^4(c-cx)^3}{(ax+a)^3}+\frac{245a^2c^5(c-cx)^2}{(ax+a)^2}+\frac{49ac^6(c-cx)}{ax+a}+5c^7\right)}{4480a^8c^8(c-cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] ((a + a*x)^(7/2)*(5*c^7 + (49*a*c^6*(c - c*x))/(a + a*x) + (245*a^2*c^5*(c
- c*x)^2)/(a + a*x)^2 + (1225*a^3*c^4*(c - c*x)^3)/(a + a*x)^3 - (1225*a^4*
c^3*(c - c*x)^4)/(a + a*x)^4 - (245*a^5*c^2*(c - c*x)^5)/(a + a*x)^5 - (49*
a^6*c*(c - c*x)^6)/(a + a*x)^6 - (5*a^7*(c - c*x)^7)/(a + a*x)^7))/(4480*a^
8*c^8*(c - c*x)^(7/2))
```

**fricas** [A] time = 1.26, size = 89, normalized size = 0.74

$$-\frac{\left(16\,x^7-56\,x^5+70\,x^3-35\,x\right)\sqrt{ax+a}\,\sqrt{-cx+c}}{35\left(a^5c^5x^8-4\,a^5c^5x^6+6\,a^5c^5x^4-4\,a^5c^5x^2+a^5c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*x^7 - 56*x^5 + 70*x^3 - 35*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^5*c
^5*x^8 - 4*a^5*c^5*x^6 + 6*a^5*c^5*x^4 - 4*a^5*c^5*x^2 + a^5*c^5)
```

**giac [B]** time = 1.56, size = 437, normalized size = 3.61

$$-\frac{\sqrt{-(ax+a)ac+2a^2c}\left((ax+a)\left((ax+a)\left(\frac{256(ax+a)|a|}{a^2c}-\frac{1617|a|}{ac}\right)+\frac{3430|a|}{c}\right)-\frac{2450a|a|}{c}\right)\sqrt{ax+a}}{1120\left((ax+a)ac-2a^2c\right)^4}+\frac{16384a^{12}c^6-51744\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^2a^{10}c^5+66416\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^4a^8c^4-43120\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^6a^6c^3+14280\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^8a^4c^2-2450\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^{10}a^2c+175\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^{12}}{280\left(2a^2c-\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^2\right)^7\sqrt{-ac}ac^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -1/1120*sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*((a*x + a)*(256*(a*x + a)
*abs(a)/(a^2*c) - 1617*abs(a)/(a*c)) + 3430*abs(a)/c) - 2450*a*abs(a)/c)*sq
rt(a*x + a)/((a*x + a)*a*c - 2*a^2*c)^4 + 1/280*(16384*a^12*c^6 - 51744*(sq
rt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^2*a^10*c^5 + 66416
*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^4*a^8*c^4 - 43
120*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^6*a^6*c^3 +
 14280*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^8*a^4*c^
2 - 2450*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^10*a^2
*c + 175*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^12)/((
2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^2)^7*
sqrt(-a*c)*a*c^3*abs(a))
```

**maple [A]** time = 0.00, size = 42, normalized size = 0.35

$$\frac{(x+1)(x-1)\left(16x^6-56x^4+70x^2-35\right)x}{35(ax+a)^{\frac{9}{2}}(-cx+c)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x)
```

```
[Out] 1/35*(x+1)*(x-1)*x*(16*x^6-56*x^4+70*x^2-35)/(a*x+a)^(9/2)/(-c*x+c)^(9/2)
```

**maxima [A]** time = 1.40, size = 89, normalized size = 0.74

$$\frac{x}{7\left(-acx^2+ac\right)^{\frac{7}{2}}ac}+\frac{6x}{35\left(-acx^2+ac\right)^{\frac{5}{2}}a^2c^2}+\frac{8x}{35\left(-acx^2+ac\right)^{\frac{3}{2}}a^3c^3}+\frac{16x}{35\sqrt{-acx^2+ac}\,a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*x/((-a*c*x^2 + a*c)^(7/2)*a*c) + 6/35*x/((-a*c*x^2 + a*c)^(5/2)*a^2*c^2
) + 8/35*x/((-a*c*x^2 + a*c)^(3/2)*a^3*c^3) + 16/35*x/(sqrt(-a*c*x^2 + a*c)
*a^4*c^4)
```

**mupad [B]** time = 0.48, size = 66, normalized size = 0.55

$$-\frac{x\left(16x^6-56x^4+70x^2-35\right)}{35a^4\sqrt{a+ax}\,(c-cx)^{7/2}\left(c-x^2\,(c-cx)+7cx-4x\,(c-cx)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x)
```

```
[Out] -(x*(70*x^2 - 56*x^4 + 16*x^6 - 35))/(35*a^4*(a + a*x)^(1/2)*(c - c*x)^(7/2
)*(c - x^2*(c - c*x) + 7*c*x - 4*x*(c - c*x)))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(9/2)/(-c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

# 3.1076 $\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx$

**Optimal**. Leaf size=135

$$\frac{5a^6c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{8b}+\frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

**Rubi [A]** time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {38, 63, 217, 203}

$$\frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{5a^6c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{8b}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] (5*a^4*c^2*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/16 + (5*a^2*c*x*(a + b*x)^(3/
2)*(a*c - b*c*x)^(3/2))/24 + (x*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2))/6 + (5
*a^6*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(8*b)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx = \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}+\frac{1}{6}\left(5a^2c\right)\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx$$
$$= \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}+\frac{1}{8}\left(5a^4c^2\right)\int \sqrt{a+$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac$$

**Mathematica [A]** time = 0.15, size = 120, normalized size = 0.89

$$\frac{c^3\left(-30a^{13/2}\sqrt{a-bx}\,\sqrt{\frac{bx}{a}+1}\,\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\,\sqrt{a}}\right)+33a^6bx-59a^4b^3x^3+34a^2b^5x^5-8b^7x^7\right)}{48b\sqrt{a+bx}\,\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] (c^3*(33*a^6*b*x - 59*a^4*b^3*x^3 + 34*a^2*b^5*x^5 - 8*b^7*x^7 - 30*a^(13/2
)*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))
/(48*b*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.33, size = 215, normalized size = 1.59

$$\frac{a^6c^3\sqrt{ac-bcx}\left(\frac{85c^4(ac-bcx)}{a+bx}+\frac{198c^3(ac-bcx)^2}{(a+bx)^2}-\frac{198c^2(ac-bcx)^3}{(a+bx)^3}-\frac{85c(ac-bcx)^4}{(a+bx)^4}-\frac{15(ac-bcx)^5}{(a+bx)^5}+15c^5\right)}{24b\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)^6}-\frac{5a^6c^{5/2}\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\,\sqrt{a+bx}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] (a^6*c^3*Sqrt[a*c - b*c*x]*(15*c^5 + (85*c^4*(a*c - b*c*x))/(a + b*x) + (19
8*c^3*(a*c - b*c*x)^2)/(a + b*x)^2 - (198*c^2*(a*c - b*c*x)^3)/(a + b*x)^3
- (85*c*(a*c - b*c*x)^4)/(a + b*x)^4 - (15*(a*c - b*c*x)^5)/(a + b*x)^5))/(
24*b*Sqrt[a + b*x]*(c + (a*c - b*c*x)/(a + b*x))^6) - (5*a^6*c^(5/2)*ArcTan
[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/(8*b)
```

**fricas [A]** time = 1.16, size = 232, normalized size = 1.72

$$\left[\frac{15\,a^{6}\sqrt{-c}\,c^{2}\log\left(2\,b^{2}cx^{2}+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^{2}c\right)+2\left(8\,b^{5}c^{2}x^{5}-26\,a^{2}b^{3}c^{2}x^{3}+33\,a^{4}bc^{2}x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{96\,b},-\frac{15\,a^{6}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^{2}cx^{2}-a^{2}c}\right)-\left(8\,b^{5}c^{2}x^{5}-26\,a^{2}b^{3}c^{2}x^{3}+33\,a^{4}bc^{2}x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{48\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^6*sqrt(-c)*c^2*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x
+ a)*b*sqrt(-c)*x - a^2*c) + 2*(8*b^5*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a^4
*b*c^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b, -1/48*(15*a^6*c^(5/2)*arctan
(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (8*b^5
*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a^4*b*c^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x
 + a))/b]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.01, size = 243, normalized size = 1.80

$$\frac{5\sqrt{(bx+a)(-bcx+ac)}\;a^{6}c^{3}\arctan\left(\frac{\sqrt{b^{2}c}\;x}{\sqrt{-b^{2}c\,x^{2}+a^{2}c}}\right)}{16\sqrt{-bcx+ac}\;\sqrt{bx+a}\;\sqrt{b^{2}c}}+\frac{5\sqrt{-bcx+ac}\;\sqrt{bx+a}\;a^{5}c^{2}}{16b}+\frac{5\,(-bcx+ac)^{\frac{3}{2}}\sqrt{bx+a}\;a^{4}c}{48b}+\frac{(-bcx+ac)^{\frac{5}{2}}\sqrt{bx+a}\;a^{3}}{24b}-\frac{\sqrt{bx+a}\;(-bcx+ac)^{\frac{7}{2}}a^{2}}{8bc}-\frac{(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{7}{2}}a}{6bc}-\frac{(bx+a)^{\frac{5}{2}}(-bcx+ac)^{\frac{7}{2}}}{6bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x)
```

```
[Out] -1/6/b/c*(b*x+a)^(5/2)*(-b*c*x+a*c)^(7/2)-1/6*a/b/c*(b*x+a)^(3/2)*(-b*c*x+a
*c)^(7/2)-1/8*a^2/b/c*(b*x+a)^(1/2)*(-b*c*x+a*c)^(7/2)+1/24*a^3/b*(-b*c*x+a
*c)^(5/2)*(b*x+a)^(1/2)+5/48*a^4*c/b*(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+5/16*
a^5*c^2/b*(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)+5/16*a^6*c^3*((b*x+a)*(-b*c*x+a*
c))^(1/2)/(-b*c*x+a*c)^(1/2)/(b*x+a)^(1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/
2)*x/(-b^2*c*x^2+a^2*c)^(1/2))
```

**maxima [A]** time = 3.12, size = 89, normalized size = 0.66

$$\frac{5\,a^{6}c^{\frac{5}{2}}\arcsin\left(\frac{bx}{a}\right)}{16\,b}+\frac{5}{16}\,\sqrt{-b^{2}cx^{2}+a^{2}c}\,a^{4}c^{2}x+\frac{5}{24}\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}a^{2}cx+\frac{1}{6}\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^6*c^(5/2)*arcsin(b*x/a)/b + 5/16*sqrt(-b^2*c*x^2 + a^2*c)*a^4*c^2*x
+ 5/24*(-b^2*c*x^2 + a^2*c)^(3/2)*a^2*c*x + 1/6*(-b^2*c*x^2 + a^2*c)^(5/2)*
x
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,c-b\,c\,x)^{5/2}\,(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2),x)
```

```
[Out] int((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (-c\,(-a+bx))^{\frac{5}{2}}\,(a+bx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(-b*c*x+a*c)**(5/2),x)
```

```
[Out] Integral((-c*(-a + b*x))**(5/2)*(a + b*x)**(5/2), x)
```

## 3.1077 $\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx$

**Optimal**. Leaf size=102

$$\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{4b}+\frac{3}{8}a^2cx\sqrt{a+bx}\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {38, 63, 217, 203}

$$\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{4b}+\frac{3}{8}a^2cx\sqrt{a+bx}\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

```
[Out] (3*a^2*c*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 + (x*(a + b*x)^(3/2)*(a*c - b
*c*x)^(3/2))/4 + (3*a^4*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a -
b*x)]])/(4*b)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx = \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{4}\left(3a^2c\right)\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,dx$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{8}\left(3a^4c^2\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{\phantom{x}}}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{\left(3a^4c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2ac-}}\right.}{4b}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{\left(3a^4c^2\right)\text{Subst}\left(\int \frac{1}{1+cx^2}\right.}{4b}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right.}{4b}$$

**Mathematica [A]** time = 0.13, size = 109, normalized size = 1.07

$$\frac{c^2\left(-6a^{9/2}\sqrt{a-bx}\,\sqrt{\frac{bx}{a}+1}\,\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\,\sqrt{a}}\right)+5a^4bx-7a^2b^3x^3+2b^5x^5\right)}{8b\sqrt{a+bx}\,\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

```
[Out] (c^2*(5*a^4*b*x - 7*a^2*b^3*x^3 + 2*b^5*x^5 - 6*a^(9/2)*Sqrt[a - b*x]*Sqrt[
1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(8*b*Sqrt[c*(a - b*x
)]*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.24, size = 169, normalized size = 1.66

$$\frac{a^4c^2\sqrt{ac-bcx}\left(\frac{11c^2(ac-bcx)}{a+bx}-\frac{11c(ac-bcx)^2}{(a+bx)^2}-\frac{3(ac-bcx)^3}{(a+bx)^3}+3c^3\right)}{4b\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)^4}-\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\,\sqrt{a+bx}}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

```
[Out] (a^4*c^2*Sqrt[a*c - b*c*x]*(3*c^3 + (11*c^2*(a*c - b*c*x))/(a + b*x) - (11*
c*(a*c - b*c*x)^2)/(a + b*x)^2 - (3*(a*c - b*c*x)^3)/(a + b*x)^3))/(4*b*Sqr
t[a + b*x]*(c + (a*c - b*c*x)/(a + b*x))^4) - (3*a^4*c^(3/2)*ArcTan[Sqrt[a*
c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/(4*b)
```

**fricas [A]** time = 1.43, size = 193, normalized size = 1.89

$$\left[\frac{3\,a^{4}\sqrt{-c}\,c\log\left(2\,b^{2}cx^{2}+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^{2}c\right)-2\,\left(2\,b^{3}cx^{3}-5\,a^{2}bcx\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{16\,b},\,-\frac{3\,a^{4}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^{2}cx^{2}-a^{2}c}\right)+\left(2\,b^{3}cx^{3}-5\,a^{2}bcx\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^4*sqrt(-c)*c*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a
)*b*sqrt(-c)*x - a^2*c) - 2*(2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(-b*c*x + a*c)*
sqrt(b*x + a))/b, -1/8*(3*a^4*c^(3/2)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(-b*c
*x + a*c)*sqrt(b*x + a))/b]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.01, size = 185, normalized size = 1.81

$$\frac{3\sqrt{(bx+a)(-bcx+ac)}\,a^{4}c^{2}\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-b^{2}c\,x^{2}+a^{2}c}}\right)}{8\sqrt{-bcx+ac}\,\sqrt{bx+a}\,\sqrt{b^{2}c}}+\frac{3\sqrt{-bcx+ac}\,\sqrt{bx+a}\,a^{3}c}{8b}+\frac{(-bcx+ac)^{\frac{3}{2}}\sqrt{bx+a}\,a^{2}}{8b}-\frac{\sqrt{bx+a}\,(-bcx+ac)^{\frac{5}{2}}a}{4bc}-\frac{(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{5}{2}}}{4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x)
```

```
[Out] -1/4/b/c*(b*x+a)^(3/2)*(-b*c*x+a*c)^(5/2)-1/4*a/b/c*(b*x+a)^(1/2)*(-b*c*x+a
*c)^(5/2)+1/8*a^2/b*(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+3/8*a^3*c/b*(-b*c*x+a*
c)^(1/2)*(b*x+a)^(1/2)+3/8*a^4*c^2*((b*x+a)*(-b*c*x+a*c))^(1/2)/(-b*c*x+a*c
)^(1/2)/(b*x+a)^(1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)/(-b^2*c*x^2+a^2*c)
^(1/2)*x)
```

**maxima [A]** time = 3.02, size = 63, normalized size = 0.62

$$\frac{3\,a^{4}c^{\frac{3}{2}}\arcsin\left(\frac{bx}{a}\right)}{8\,b}+\frac{3}{8}\,\sqrt{-b^{2}cx^{2}+a^{2}c}\,a^{2}cx+\frac{1}{4}\,\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^4*c^(3/2)*arcsin(b*x/a)/b + 3/8*sqrt(-b^2*c*x^2 + a^2*c)*a^2*c*x + 1/
4*(-b^2*c*x^2 + a^2*c)^(3/2)*x
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,c-b\,c\,x)^{3/2}\,(a+b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2),x)
```

```
[Out] int((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (-c\,(-a+bx))^{\frac{3}{2}}\,(a+bx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(-b*c*x+a*c)**(3/2),x)
```

```
[Out] Integral((-c*(-a + b*x))**(3/2)*(a + b*x)**(3/2), x)
```

## 3.1078 $\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx$

**Optimal**. Leaf size=68

$$\frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}$$

**Rubi** [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {38, 63, 217, 203}

$$\frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

```
[Out] (x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/2 + (a^2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
 + b*x])/Sqrt[c*(a - b*x)]])/b
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx &= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{1}{2}\left(a^2c\right)\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx\\ &= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{\sqrt{2ac-cx^2}}\,dx,x,\sqrt{a+bx}\right)}{b}\\ &= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}\\ &= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 95, normalized size = 1.40

$$\frac{c\left(-2a^{5/2}\sqrt{a-bx}\sqrt{\frac{bx}{a}+1}\,\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)+a^2bx-b^3x^3\right)}{2b\sqrt{a+bx}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

```
[Out] (c*(a^2*b*x - b^3*x^3 - 2*a^(5/2)*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sq
rt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(2*b*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.16, size = 114, normalized size = 1.68

$$\frac{a^2c\sqrt{ac-bcx}\left(c-\frac{ac-bcx}{a+bx}\right)}{b\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)^2}-\frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\sqrt{a+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

```
[Out] (a^2*c*Sqrt[a*c - b*c*x]*(c - (a*c - b*c*x)/(a + b*x)))/(b*Sqrt[a + b*x]*(c
 + (a*c - b*c*x)/(a + b*x))^2) - (a^2*Sqrt[c]*ArcTan[Sqrt[a*c - b*c*x]/(Sqr
t[c]*Sqrt[a + b*x])])/b
```

**fricas** [A] time = 1.40, size = 159, normalized size = 2.34

$$\left[\frac{a^2\sqrt{-c}\,\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,bx}{4\,b},\ -\frac{a^2\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)-\sqrt{-bcx+ac}\,\sqrt{bx+a}\,bx}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a^2*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*s
qrt(-c)*x - a^2*c) + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b, -1/2*(a^2*s
qrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2
*c)) - sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b]
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [B] time = 0.01, size = 127, normalized size = 1.87

$$\frac{\sqrt{(bx+a)(-bcx+ac)}\,a^2c\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)}{2\sqrt{-bcx+ac}\,\sqrt{bx+a}\,\sqrt{b^2c}}+\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,a}{2b}-\frac{\sqrt{bx+a}\,(-bcx+ac)^{\frac{3}{2}}}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] -1/2/b/c*(b*x+a)^(1/2)*(-b*c*x+a*c)^(3/2)+1/2*a/b*(-b*c*x+a*c)^(1/2)*(b*x+a
)^(1/2)+1/2*a^2*c*((b*x+a)*(-b*c*x+a*c))^(1/2)/(-b*c*x+a*c)^(1/2)/(b*x+a)^(
1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2)*x)
```

**maxima** [A] time = 3.09, size = 39, normalized size = 0.57

$$\frac{a^2\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\,\sqrt{-b^2cx^2+a^2c}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*sqrt(c)*arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*x
```

**mupad [B]** time = 0.20, size = 72, normalized size = 1.06

$$\frac{x\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{2}-\frac{a^2\,\sqrt{b}\,c^2\,\ln\left(\sqrt{-b\,c}\,\sqrt{c\,\left(a-b\,x\right)}\,\sqrt{a+b\,x}-b^{3/2}\,c\,x\right)}{2\,{\left(-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2),x)
[Out] (x*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/2 - (a^2*b^(1/2)*c^2*log((-b*c)^(1/
2)*(c*(a - b*x))^(1/2)*(a + b*x)^(1/2) - b^(3/2)*c*x))/(2*(-b*c)^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-c\,(-a+bx)}\,\sqrt{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x), x)
```

## 3.1079 $\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

**Optimal**. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

**Rubi** [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {63, 217, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(b*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2ac-cx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

**Mathematica** [A] time = 0.01, size = 48, normalized size = 1.26

$$-\frac{2\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)}{b\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
[Out] (-2*Sqrt[a - b*x]*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x]])/(b*Sqrt[c*(a - b*x)]
)
```

**IntegrateAlgebraic** [A] time = 0.08, size = 39, normalized size = 1.03

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\sqrt{a+bx}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
[Out] (-2*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/(b*Sqrt[c])
```

**fricas** [A] time = 0.83, size = 108, normalized size = 2.84

$$\left[-\frac{\sqrt{-c}\log\left(2\,b^2cx^2 - 2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x - a^2c\right)}{2\,bc}, -\frac{\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)}{b\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(
-c)*x - a^2*c)/(b*c), -arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/
(b^2*c*x^2 - a^2*c))/(b*sqrt(c))]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.00, size = 71, normalized size = 1.87

$$\frac{\sqrt{(bx+a)(-bcx+ac)}\ \arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)}{\sqrt{bx+a}\ \sqrt{-bcx+ac}\ \sqrt{b^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] ((b*x+a)*(-b*c*x+a*c))^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(b^2*c)^(1/2)
*arctan((b^2*c)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2)*x)
```

**maxima [A]** time = 2.93, size = 14, normalized size = 0.37

$$\frac{\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(b*x/a)/(b*sqrt(c))
```

**mupad [B]** time = 0.18, size = 53, normalized size = 1.39

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^2\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^
(1/2) - a^(1/2)))))/(b^2*c)^(1/2)
```

**sympy [C]** time = 4.69, size = 90, normalized size = 2.37

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle| \frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
a**2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4, 1
/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(-2*I*pi)/(b**2*
x**2))/(4*pi**(3/2)*b*sqrt(c))
```

## 3.1080 $\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}} \, dx$

**Optimal**. Leaf size=30

$$\frac{x}{a^2c\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

**Rubi** [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {39}

$$\frac{x}{a^2c\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

```
[Out] x/(a^2*c*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}} \, dx = \frac{x}{a^2c\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

**Mathematica** [A] time = 0.01, size = 29, normalized size = 0.97

$$\frac{x}{a^2c\sqrt{a+bx}\,\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

```
[Out] x/(a^2*c*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

**IntegrateAlgebraic** [A] time = 0.12, size = 55, normalized size = 1.83

$$\frac{\sqrt{a+bx}\left(c-\frac{ac-bcx}{a+bx}\right)}{2a^2bc^2\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

```
[Out] (Sqrt[a + b*x]*(c - (a*c - b*c*x)/(a + b*x)))/(2*a^2*b*c^2*Sqrt[a*c - b*c*x
])
```

**fricas** [A] time = 0.96, size = 45, normalized size = 1.50

$$-\frac{\sqrt{-bcx+ac}\sqrt{bx+a}\,x}{a^2b^2c^2x^2-a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*c*x + a*c)*sqrt(b*x + a)*x/(a^2*b^2*c^2*x^2 - a^4*c^2)
```

**giac** [B] time = 1.86, size = 115, normalized size = 3.83

$$\frac{2\sqrt{-c}\,c}{\left(2\,ac^2-\left(\sqrt{-bcx+ac}\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2\right)ab|c|}-\frac{\sqrt{-bcx+ac}}{2\sqrt{2\,ac^2+(bcx-ac)c}\,a^2b|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(-c)*c/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c
*x - a*c)*c))^2)*a*b*abs(c)) - 1/2*sqrt(-b*c*x + a*c)/(sqrt(2*a*c^2 + (b*c*
x - a*c)*c)*a^2*b*abs(c))
```

**maple** [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{(-bx+a)\,x}{\sqrt{bx+a}\,\left(-bcx+ac\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x)
```

```
[Out] 1/(b*x+a)^(1/2)*(-b*x+a)/a^2*x/(-b*c*x+a*c)^(3/2)
```

**maxima [A]** time = 1.40, size = 25, normalized size = 0.83

$$\frac{x}{\sqrt{-b^2cx^2 + a^2c}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-b^2*c*x^2 + a^2*c)*a^2*c)
```

**mupad [B]** time = 0.50, size = 26, normalized size = 0.87

$$\frac{x}{a^2\,c\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2)),x)
```

```
[Out] x/(a^2*c*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))
```

**sympy [C]** time = 5.18, size = 94, normalized size = 3.13

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2 \\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0 \end{matrix}\middle|{\frac{a^2}{b^2x^2}}\right)}{2\pi^{\frac{3}{2}}a^2bc^{\frac{3}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0 \end{matrix}\middle|{\frac{a^2e^{-2i\pi}}{b^2x^2}}\right)}{2\pi^{\frac{3}{2}}a^2bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(-b*c*x+a*c)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), a
**2/(b**2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2)) + meijerg(((-1/2, 0, 1/4, 1/
2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), a**2*exp_polar(-2*I*pi)/(b*
*2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2))
```

## 3.1081 $\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}} dx$

**Optimal**. Leaf size=67

$$\frac{2x}{3a^4c^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {40, 39}

$$\frac{2x}{3a^4c^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] x/(3*a^2*c*(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)) + (2*x)/(3*a^4*c^2*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx&=\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx}{3a^2c}\\&=\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2x}{3a^4c^2\sqrt{a+bx}\sqrt{ac-bcx}}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 46, normalized size = 0.69

$$\frac{3a^2x-2b^2x^3}{3a^4c(a+bx)^{3/2}(c(a-bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] (3*a^2*x - 2*b^2*x^3)/(3*a^4*c*(c*(a - b*x))^(3/2)*(a + b*x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.13, size = 101, normalized size = 1.51

$$\frac{(a+bx)^{3/2}\left(\frac{9c^2(ac-bcx)}{a+bx}-\frac{9c(ac-bcx)^2}{(a+bx)^2}-\frac{(ac-bcx)^3}{(a+bx)^3}+c^3\right)}{24a^4bc^4(ac-bcx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] ((a + b*x)^(3/2)*(c^3 + (9*c^2*(a*c - b*c*x))/(a + b*x) - (9*c*(a*c - b*c*x
)^2)/(a + b*x)^2 - (a*c - b*c*x)^3/(a + b*x)^3))/(24*a^4*b*c^4*(a*c - b*c*x
)^(3/2))
```

**fricas** [A] time = 1.23, size = 72, normalized size = 1.07

$$-\frac{\left(2\,b^2x^3-3\,a^2x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{3\left(a^4b^4c^3x^4-2\,a^6b^2c^3x^2+a^8c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^3 - 3*a^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^4*b^4*c^3*x^
4 - 2*a^6*b^2*c^3*x^2 + a^8*c^3)
```

**giac** [B] time = 2.38, size = 251, normalized size = 3.75

$$-\frac{\sqrt{-bcx+ac}\left(\frac{9\,|c|}{a^3bc}+\frac{4\,(bcx-ac)|c|}{a^4bc^2}\right)}{12\left(2\,ac^2+(bcx-ac)c\right)^{\frac{3}{2}}}+\frac{16\,a^2\sqrt{-c}\,c^4-18\,a\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2\sqrt{-c}\,c^2+3\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^4\sqrt{-c}}{3\left(2\,ac^2-\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2\right)^3a^3b|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(-b*c*x + a*c)*(9*abs(c)/(a^3*b*c) + 4*(b*c*x - a*c)*abs(c)/(a^4*
b*c^2))/(2*a*c^2 + (b*c*x - a*c)*c)^(3/2) + 1/3*(16*a^2*sqrt(-c)*c^4 - 18*a
*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)
*c^2 + 3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*
sqrt(-c))/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x
- a*c)*c))^2)^3*a^3*b*abs(c))
```

**maple** [A] time = 0.00, size = 45, normalized size = 0.67

$$\frac{(-bx+a)\left(-2b^{2}x^{2}+3a^{2}\right)x}{3\,(bx+a)^{\frac{3}{2}}\,(-bcx+ac)^{\frac{5}{2}}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x)
```

```
[Out] 1/3*(-b*x+a)*x*(-2*b^2*x^2+3*a^2)/(b*x+a)^(3/2)/a^4/(-b*c*x+a*c)^(5/2)
```

**maxima** [A] time = 1.43, size = 53, normalized size = 0.79

$$\frac{x}{3\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}a^{2}c}+\frac{2\,x}{3\,\sqrt{-b^{2}cx^{2}+a^{2}c}\,a^{4}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^2*c) + 2/3*x/(sqrt(-b^2*c*x^2 + a^2*c)*
a^4*c^2)
```

**mupad** [B] time = 0.58, size = 80, normalized size = 1.19

$$-\frac{3\,a^2\,x\,\sqrt{a\,c-b\,c\,x}-2\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}}{{\left(a\,c-b\,c\,x\right)}^2\,\left(3\,a^4\,\left(a\,c-b\,c\,x\right)-6\,a^5\,c\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2)),x)
```

```
[Out] -(3*a^2*x*(a*c - b*c*x)^(1/2) - 2*b^2*x^3*(a*c - b*c*x)^(1/2))/((a*c - b*c*
x)^2*(3*a^4*(a*c - b*c*x) - 6*a^5*c)*(a + b*x)^(1/2))
```

**sympy [C]** time = 15.85, size = 94, normalized size = 1.40

$$\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{1}{2},\frac{5}{2},3 \\ \frac{5}{4},\frac{7}{4},2,\frac{5}{2},3 & 0 \end{matrix}\middle|\frac{a^2}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{2},\frac{3}{4},\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & -\frac{1}{2},0,2,0 \end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(-b*c*x+a*c)**(5/2),x)
```

```
[Out] I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), a*
*2/(b**2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3/4
, 5/4, 1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), a**2*exp_polar(-2*I*pi)/(b**
2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2))
```

## 3.1082 $\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}} dx$

**Optimal**. Leaf size=100

$$\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}$$

**Rubi** [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {40, 39}

$$\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] x/(5*a^2*c*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)) + (4*x)/(15*a^4*c^2*(a + b*
x)^(3/2)*(a*c - b*c*x)^(3/2)) + (8*x)/(15*a^6*c^3*Sqrt[a + b*x]*Sqrt[a*c -
b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx = \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{4\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx}{5a^2c}$$
$$= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}} + \frac{8\int \frac{1}{(a+bx)^{3/2}(ac-bcx)}}{15a^4c^2}$$
$$= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}} + \frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{}}$$

**Mathematica** [A] time = 0.03, size = 57, normalized size = 0.57

$$\frac{15a^4x-20a^2b^2x^3+8b^4x^5}{15a^6c(a+bx)^{5/2}(c(a-bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] (15*a^4*x - 20*a^2*b^2*x^3 + 8*b^4*x^5)/(15*a^6*c*(c*(a - b*x))^(5/2)*(a +
b*x)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.14, size = 149, normalized size = 1.49

$$\frac{(a+bx)^{5/2}\left(\frac{25c^4(ac-bcx)}{a+bx}+\frac{150c^3(ac-bcx)^2}{(a+bx)^2}-\frac{150c^2(ac-bcx)^3}{(a+bx)^3}-\frac{25c(ac-bcx)^4}{(a+bx)^4}-\frac{3(ac-bcx)^5}{(a+bx)^5}+3c^5\right)}{480a^6bc^6(ac-bcx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] ((a + b*x)^(5/2)*(3*c^5 + (25*c^4*(a*c - b*c*x))/(a + b*x) + (150*c^3*(a*c
- b*c*x)^2)/(a + b*x)^2 - (150*c^2*(a*c - b*c*x)^3)/(a + b*x)^3 - (25*c*(a*
c - b*c*x)^4)/(a + b*x)^4 - (3*(a*c - b*c*x)^5)/(a + b*x)^5))/(480*a^6*b*c^
6*(a*c - b*c*x)^(5/2))
```

**fricas** [A] time = 1.40, size = 98, normalized size = 0.98

$$-\frac{\left(8\,b^4x^5-20\,a^2b^2x^3+15\,a^4x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{15\left(a^6b^6c^4x^6-3\,a^8b^4c^4x^4+3\,a^{10}b^2c^4x^2-a^{12}c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="fricas")

[Out] -1/15*(8*b^4*x^5 - 20*a^2*b^2*x^3 + 15*a^4*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^6*b^6*c^4*x^6 - 3*a^8*b^4*c^4*x^4 + 3*a^10*b^2*c^4*x^2 - a^12*c^4)

**giac [B]** time = 2.57, size = 366, normalized size = 3.66

$$-\frac{\sqrt{-bcx+ac}\left((bcx-ac)\left(\frac{275\,c}{a^5b|c|}+\frac{64\,(bcx-ac)}{a^6b|c|}\right)+\frac{300\,c^2}{a^4b|c|}\right)}{240\left(2\,ac^2+(bcx-ac)c\right)^{\frac{5}{2}}}-\frac{1024\,a^4c^8-2200\,a^3\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2c^6+1660\,a^2\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^4c^4-450\,a\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^6c^2+45\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^8}{60\left(2\,ac^2-\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2\right)^5a^5b\sqrt{-c}\,|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="giac")

[Out] -1/240*sqrt(-b*c*x + a*c)*((b*c*x - a*c)*(275*c/(a^5*b*abs(c)) + 64*(b*c*x - a*c)/(a^6*b*abs(c))) + 300*c^2/(a^4*b*abs(c)))/(2*a*c^2 + (b*c*x - a*c)*c)^(5/2) - 1/60*(1024*a^4*c^8 - 2200*a^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*c^6 + 1660*a^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*c^4 - 450*a*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*c^2 + 45*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^8)/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2)^5*a^5*b*sqrt(-c)*abs(c))

**maple [A]** time = 0.00, size = 56, normalized size = 0.56

$$\frac{(-bx+a)\left(8b^4x^4-20a^2b^2x^2+15a^4\right)x}{15\,(bx+a)^{\frac{5}{2}}\,(-bcx+ac)^{\frac{7}{2}}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x)

[Out] 1/15*(-b*x+a)*x*(8*b^4*x^4-20*a^2*b^2*x^2+15*a^4)/(b*x+a)^(5/2)/a^6/(-b*c*x+a*c)^(7/2)

**maxima [A]** time = 1.32, size = 79, normalized size = 0.79

$$\frac{x}{5\left(-b^2cx^2+a^2c\right)^{\frac{5}{2}}a^2c}+\frac{4\,x}{15\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^4c^2}+\frac{8\,x}{15\,\sqrt{-b^2cx^2+a^2c}\,a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="maxima")

[Out] 1/5*x/((-b^2*c*x^2 + a^2*c)^(5/2)*a^2*c) + 4/15*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^4*c^2) + 8/15*x/(sqrt(-b^2*c*x^2 + a^2*c)*a^6*c^3)

**mupad [B]** time = 0.65, size = 111, normalized size = 1.11

$$\frac{15\,a^4\,x\,\sqrt{a\,c-b\,c\,x}\,+8\,b^4\,x^5\,\sqrt{a\,c-b\,c\,x}\,-20\,a^2\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}}{{\left(a\,c-b\,c\,x\right)}^3\,\left(60\,a^8\,c-\left(a\,c-b\,c\,x\right)\,\left(45\,a^7+15\,b\,x\,a^6\right)\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(7/2)*(a + b*x)^(7/2)),x)
```

```
[Out] (15*a^4*x*(a*c - b*c*x)^(1/2) + 8*b^4*x^5*(a*c - b*c*x)^(1/2) - 20*a^2*b^2*
x^3*(a*c - b*c*x)^(1/2))/((a*c - b*c*x)^3*(60*a^8*c - (a*c - b*c*x)*(45*a^7
 + 15*a^6*b*x))*(a + b*x)^(1/2))
```

**sympy [C]** time = 59.50, size = 97, normalized size = 0.97

$$-\frac{2iG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & \frac{1}{2},\frac{7}{2},4\\ \frac{7}{4},\frac{9}{4},3,\frac{7}{2},4 & 0\end{matrix}\middle|\frac{a^2}{b^2x^2}\right)}{15\pi^{\frac{3}{2}}a^6bc^{\frac{7}{2}}}+\frac{2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{5}{4},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & -\frac{1}{2},0,3,0\end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{15\pi^{\frac{3}{2}}a^6bc^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(-b*c*x+a*c)**(7/2),x)
```

```
[Out] -2*I*meijerg(((7/4, 9/4, 1), (1/2, 7/2, 4)), ((7/4, 9/4, 3, 7/2, 4), (0,)),
 a**2/(b**2*x**2))/(15*pi**(3/2)*a**6*b*c**(7/2)) + 2*meijerg(((-1/2, 0, 1/
2, 5/4, 7/4, 1), ()), ((5/4, 7/4), (-1/2, 0, 3, 0)), a**2*exp_polar(-2*I*pi
)/(b**2*x**2))/(15*pi**(3/2)*a**6*b*c**(7/2))
```

## 3.1083 $\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}} \, dx$

**Optimal**. Leaf size=133

$$\frac{16x}{35a^8c^4\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{x}{7a^2c(a+bx)^{7/2}(ac-$$

**Rubi** [A] time = 0.03, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {40, 39}

$$\frac{16x}{35a^8c^4\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

```
[Out] x/(7*a^2*c*(a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)) + (6*x)/(35*a^4*c^2*(a + b*
x)^(5/2)*(a*c - b*c*x)^(5/2)) + (8*x)/(35*a^6*c^3*(a + b*x)^(3/2)*(a*c - b*
c*x)^(3/2)) + (16*x)/(35*a^8*c^4*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx = \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx}{7a^2c}$$
$$= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{24\int \frac{1}{(a+bx)^{5/2}(ac-bc}}{35a^4c^2}$$
$$= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{8x}{35a^6c^3(a+bx)^{3/2}}$$
$$= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{8x}{35a^6c^3(a+bx)^{3/2}}$$

**Mathematica** [A] time = 0.04, size = 76, normalized size = 0.57

$$\frac{x\left(35a^6-70a^4b^2x^2+56a^2b^4x^4-16b^6x^6\right)\sqrt{c(a-bx)}}{35a^8c^5(a-bx)^4(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

```
[Out] (x*Sqrt[c*(a - b*x)]*(35*a^6 - 70*a^4*b^2*x^2 + 56*a^2*b^4*x^4 - 16*b^6*x^6
))/(35*a^8*c^5*(a - b*x)^4*(a + b*x)^(7/2))
```

**IntegrateAlgebraic** [A] time = 0.15, size = 195, normalized size = 1.47

$$\frac{(a+bx)^{7/2}\left(\frac{49c^6(ac-bcx)}{a+bx}+\frac{245c^5(ac-bcx)^2}{(a+bx)^2}+\frac{1225c^4(ac-bcx)^3}{(a+bx)^3}-\frac{1225c^3(ac-bcx)^4}{(a+bx)^4}-\frac{245c^2(ac-bcx)^5}{(a+bx)^5}-\frac{49c(ac-bcx)^6}{(a+bx)^6}-\frac{5(ac-bcx)^7}{(a+bx)^7}+5c^7\right)}{4480a^8bc^8(ac-bcx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

```
[Out] ((a + b*x)^(7/2)*(5*c^7 + (49*c^6*(a*c - b*c*x))/(a + b*x) + (245*c^5*(a*c
- b*c*x)^2)/(a + b*x)^2 + (1225*c^4*(a*c - b*c*x)^3)/(a + b*x)^3 - (1225*c^
3*(a*c - b*c*x)^4)/(a + b*x)^4 - (245*c^2*(a*c - b*c*x)^5)/(a + b*x)^5 - (4
9*c*(a*c - b*c*x)^6)/(a + b*x)^6 - (5*(a*c - b*c*x)^7)/(a + b*x)^7))/(4480*
a^8*b*c^8*(a*c - b*c*x)^(7/2))
```

**fricas** [A] time = 1.60, size = 122, normalized size = 0.92

$$-\frac{\left(16\,b^6x^7-56\,a^2b^4x^5+70\,a^4b^2x^3-35\,a^6x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{35\left(a^8b^8c^5x^8-4\,a^{10}b^6c^5x^6+6\,a^{12}b^4c^5x^4-4\,a^{14}b^2c^5x^2+a^{16}c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*b^6*x^7 - 56*a^2*b^4*x^5 + 70*a^4*b^2*x^3 - 35*a^6*x)*sqrt(-b*c*x
 + a*c)*sqrt(b*x + a)/(a^8*b^8*c^5*x^8 - 4*a^10*b^6*c^5*x^6 + 6*a^12*b^4*c^
5*x^4 - 4*a^14*b^2*c^5*x^2 + a^16*c^5)
```

**giac [B]** time = 3.30, size = 487, normalized size = 3.66

$$-\frac{\sqrt{-bcx+ac}\left((bcx-ac)\left((bcx-ac)\left(\frac{1617\,|c|}{a^7bc}+\frac{256\,(bcx-ac)|c|}{a^8bc^2}\right)+\frac{3430\,|c|}{a^6b}\right)+\frac{2450\,c|c|}{a^5b}\right)}{1120\left(2\,ac^2+(bcx-ac)c\right)^{\frac{7}{2}}}-\frac{16384\,a^6c^{12}-51744\,a^5\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2c^{10}+66416\,a^4\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^4c^8-43120\,a^3\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^6c^6+14280\,a^2\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^8c^4-2450\,a\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^{10}c^2+175\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^{12}}{280\left(2\,ac^2-\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2\right)^7a^7b\sqrt{-c}\,c|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="giac")
```

```
[Out] -1/1120*sqrt(-b*c*x + a*c)*((b*c*x - a*c)*((b*c*x - a*c)*(1617*abs(c)/(a^7*
b*c) + 256*(b*c*x - a*c)*abs(c)/(a^8*b*c^2)) + 3430*abs(c)/(a^6*b)) + 2450*
c*abs(c)/(a^5*b))/(2*a*c^2 + (b*c*x - a*c)*c)^(7/2) - 1/280*(16384*a^6*c^12
 - 51744*a^5*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c)
)^2*c^10 + 66416*a^4*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x -
 a*c)*c))^4*c^8 - 43120*a^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (
b*c*x - a*c)*c))^6*c^6 + 14280*a^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*
c^2 + (b*c*x - a*c)*c))^8*c^4 - 2450*a*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(
2*a*c^2 + (b*c*x - a*c)*c))^10*c^2 + 175*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqr
t(2*a*c^2 + (b*c*x - a*c)*c))^12)/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sqrt(-c)
- sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2)^7*a^7*b*sqrt(-c)*c*abs(c))
```

**maple [A]** time = 0.00, size = 67, normalized size = 0.50

$$\frac{(-bx+a)\left(-16b^6x^6+56b^4x^4a^2-70b^2x^2a^4+35a^6\right)x}{35\,(bx+a)^{\frac{7}{2}}\,(-bcx+ac)^{\frac{9}{2}}\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x)
```

```
[Out] 1/35*(-b*x+a)*x*(-16*b^6*x^6+56*a^2*b^4*x^4-70*a^4*b^2*x^2+35*a^6)/(b*x+a)^
(7/2)/a^8/(-b*c*x+a*c)^(9/2)
```

**maxima [A]** time = 1.29, size = 105, normalized size = 0.79

$$\frac{x}{7\left(-b^2cx^2+a^2c\right)^{\frac{7}{2}}a^2c}+\frac{6\,x}{35\left(-b^2cx^2+a^2c\right)^{\frac{5}{2}}a^4c^2}+\frac{8\,x}{35\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^6c^3}+\frac{16\,x}{35\sqrt{-b^2cx^2+a^2c}\,a^8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*x/((-b^2*c*x^2 + a^2*c)^(7/2)*a^2*c) + 6/35*x/((-b^2*c*x^2 + a^2*c)^(5/
2)*a^4*c^2) + 8/35*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^6*c^3) + 16/35*x/(sqrt(-
b^2*c*x^2 + a^2*c)*a^8*c^4)
```

**mupad [B]** time = 0.71, size = 170, normalized size = 1.28

$$-\frac{35\,a^6\,x\,\sqrt{a\,c-b\,c\,x}-16\,b^6\,x^7\,\sqrt{a\,c-b\,c\,x}-70\,a^4\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}+56\,a^2\,b^4\,x^5\,\sqrt{a\,c-b\,c\,x}}{\left(\left(70\,a^9\,{\left(a\,c-b\,c\,x\right)}^5+35\,a^8\,{\left(a\,c-b\,c\,x\right)}^5\,\left(a+b\,x\right)\right)\,\left(a+b\,x\right)+{\left(a\,c-b\,c\,x\right)}^4\,\left(140\,a^{10}\,\left(a\,c-b\,c\,x\right)-280\,a^{11}\,c\right)\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(9/2)*(a + b*x)^(9/2)),x)
```

```
[Out] -(35*a^6*x*(a*c - b*c*x)^(1/2) - 16*b^6*x^7*(a*c - b*c*x)^(1/2) - 70*a^4*b^
2*x^3*(a*c - b*c*x)^(1/2) + 56*a^2*b^4*x^5*(a*c - b*c*x)^(1/2))/(((70*a^9*(
a*c - b*c*x)^5 + 35*a^8*(a*c - b*c*x)^5*(a + b*x))*(a + b*x) + (a*c - b*c*x
)^4*(140*a^10*(a*c - b*c*x) - 280*a^11*c))*(a + b*x)^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(-b*c*x+a*c)**(9/2),x)
```

```
[Out] Timed out
```

## 3.1084 $\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx$

**Optimal**. Leaf size=100

$$6\sqrt{6}\,(1-2x)^{5/2}x(2x+1)^{5/2}+15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{2x+1}+\frac{45}{4}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)$$

**Rubi** [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {38, 41, 216}

$$6\sqrt{6}\,(1-2x)^{5/2}x(2x+1)^{5/2}+15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{2x+1}+\frac{45}{4}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (45*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 15*Sqrt[3/2]*(1 - 2*x)^(3/
2)*x*(1 + 2*x)^(3/2) + 6*Sqrt[6]*(1 - 2*x)^(5/2)*x*(1 + 2*x)^(5/2) + (45*Sq
rt[3/2]*ArcSin[2*x])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = 6\sqrt{6}\,(1-2x)^{5/2}x(1+2x)^{5/2} + 5\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx$$
$$= 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2x)^{5/2}x(1+2x)^{5/2} + \frac{45}{2}\int \sqrt{3-6x}\,\sqrt{2+4$$
$$= \frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2x)^{5/2}x(1+2$$
$$= \frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2x)^{5/2}x(1+2$$
$$= \frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2x)^{5/2}x(1+2$$

**Mathematica [A]** time = 0.03, size = 44, normalized size = 0.44

$$\frac{3}{4}\sqrt{\frac{3}{2}}\left(2x\sqrt{1-4x^2}\left(128x^4-104x^2+33\right)+15\sin^{-1}(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (3*Sqrt[3/2]*(2*x*Sqrt[1 - 4*x^2]*(33 - 104*x^2 + 128*x^4) + 15*ArcSin[2*x]
))/4
```

**IntegrateAlgebraic [B]** time = 0.97, size = 229, normalized size = 2.29

$$\frac{48\sqrt{6}\sqrt{1-2x}\,x\sqrt{2x+1}\left(128x^4-104x^2+33\right)\left(-352x^5-6160x^4-26224x^3-41096x^2-26158x-5741\right)+48\sqrt{3}\sqrt{1-2x}\,x\left(128x^4-104x^2+33\right)\left(64x^6+3712x^5+30160x^4+80768x^3+91052x^2+45112x+8119\right)}{-22528x^5-394240x^4-1678336x^3-2630144x^2+\sqrt{2}\sqrt{2x+1}\left(1024x^5+58880x^4+453120x^3+1065728x^2+923968x+259808\right)-1674112x-367424}+45\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{2x+1}-\sqrt{2}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (48*Sqrt[6]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x]*(33 - 104*x^2 + 128*x^4)*(-5741 -
 26158*x - 41096*x^2 - 26224*x^3 - 6160*x^4 - 352*x^5) + 48*Sqrt[3]*Sqrt[1
- 2*x]*x*(33 - 104*x^2 + 128*x^4)*(8119 + 45112*x + 91052*x^2 + 80768*x^3 +
 30160*x^4 + 3712*x^5 + 64*x^6))/(-367424 - 1674112*x - 2630144*x^2 - 16783
36*x^3 - 394240*x^4 - 22528*x^5 + Sqrt[2]*Sqrt[1 + 2*x]*(259808 + 923968*x
+ 1065728*x^2 + 453120*x^3 + 58880*x^4 + 1024*x^5)) + 45*Sqrt[3/2]*ArcTan[(
-Sqrt[2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

**fricas [A]** time = 1.21, size = 65, normalized size = 0.65

$$\frac{3}{4}\left(128\,x^5-104\,x^3+33\,x\right)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}-\frac{45}{8}\,\sqrt{3}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{12\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 3/4*(128*x^5 - 104*x^3 + 33*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 45/8*sqrt(3)*
sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

**giac [B]** time = 1.24, size = 227, normalized size = 2.27

$$\frac{3}{40}\sqrt{3}\sqrt{2}\left(\left(\left(2\left(\left(8\left(5x-13\right)\left(2x+1\right)+321\right)\left(2x+1\right)-451\right)\left(2x+1\right)+745\right)\left(2x+1\right)-405\right)\sqrt{2x+1}\sqrt{-2x+1}+2\left(\left(2\left(3\left(8x-17\right)\left(2x+1\right)+133\right)\left(2x+1\right)-295\right)\left(2x+1\right)+195\right)\sqrt{2x+1}\sqrt{-2x+1}-20\left(\left(4\left(3x-5\right)\left(2x+1\right)+43\right)\left(2x+1\right)-39\right)\sqrt{2x+1}\sqrt{-2x+1}-80\left(\left(4x-5\right)\left(2x+1\right)+9\right)\sqrt{2x+1}\sqrt{-2x+1}+240\sqrt{2x+1}\left(x-1\right)\sqrt{-2x+1}+240\sqrt{2x+1}\sqrt{-2x+1}+150\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{2x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(3)*sqrt(2)*(((2*((8*(5*x - 13)*(2*x + 1) + 321)*(2*x + 1) - 451)*
(2*x + 1) + 745)*(2*x + 1) - 405)*sqrt(2*x + 1)*sqrt(-2*x + 1) + 2*((2*(3*(
8*x - 17)*(2*x + 1) + 133)*(2*x + 1) - 295)*(2*x + 1) + 195)*sqrt(2*x + 1)*
sqrt(-2*x + 1) - 20*((4*(3*x - 5)*(2*x + 1) + 43)*(2*x + 1) - 39)*sqrt(2*x
+ 1)*sqrt(-2*x + 1) - 80*((4*x - 5)*(2*x + 1) + 9)*sqrt(2*x + 1)*sqrt(-2*x
+ 1) + 240*sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) + 240*sqrt(2*x + 1)*sqrt(-2
*x + 1) + 150*arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

**maple [A]** time = 0.01, size = 134, normalized size = 1.34

$$\frac{45\sqrt{\left(4x+2\right)\left(-6x+3\right)}\,\sqrt{6}\,\arcsin\left(2x\right)}{8\sqrt{4x+2}\,\sqrt{-6x+3}}+\frac{\left(-6x+3\right)^{\frac{5}{2}}\left(4x+2\right)^{\frac{7}{2}}}{24}+\frac{\left(-6x+3\right)^{\frac{3}{2}}\left(4x+2\right)^{\frac{7}{2}}}{8}+\frac{9\sqrt{-6x+3}\,\left(4x+2\right)^{\frac{7}{2}}}{32}-\frac{3\left(4x+2\right)^{\frac{5}{2}}\sqrt{-6x+3}}{16}-\frac{15\left(4x+2\right)^{\frac{3}{2}}\sqrt{-6x+3}}{16}-\frac{45\sqrt{-6x+3}\,\sqrt{4x+2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^(5/2)*(2+4*x)^(5/2),x)
```

```
[Out] 1/24*(3-6*x)^(5/2)*(2+4*x)^(7/2)+1/8*(3-6*x)^(3/2)*(2+4*x)^(7/2)+9/32*(3-6*
x)^(1/2)*(2+4*x)^(7/2)-3/16*(2+4*x)^(5/2)*(3-6*x)^(1/2)-15/16*(2+4*x)^(3/2)
*(3-6*x)^(1/2)-45/8*(3-6*x)^(1/2)*(2+4*x)^(1/2)+45/8*((2+4*x)*(3-6*x))^(1/2
)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*6^(1/2)*arcsin(2*x)
```

**maxima [A]** time = 2.86, size = 46, normalized size = 0.46

$$\frac{1}{6}\left(-24\,x^{2}+6\right)^{\frac{5}{2}}x+\frac{5}{4}\left(-24\,x^{2}+6\right)^{\frac{3}{2}}x+\frac{45}{4}\sqrt{-24\,x^{2}+6}\,x+\frac{45}{8}\sqrt{6}\,\arcsin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-24*x^2 + 6)^(5/2)*x + 5/4*(-24*x^2 + 6)^(3/2)*x + 45/4*sqrt(-24*x^2 +
 6)*x + 45/8*sqrt(6)*arcsin(2*x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (4x+2)^{5/2}(3-6x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(5/2)*(3 - 6*x)^(5/2),x)
[Out] int((4*x + 2)^(5/2)*(3 - 6*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(5/2)*(4*x+2)**(5/2),x)
[Out] Timed out
```

## 3.1085 $\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx$

**Optimal**. Leaf size=74

$$3\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{9}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{2x+1}\,+\frac{9}{4}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)$$

**Rubi [A]** time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {38, 41, 216}

$$3\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{9}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{2x+1}\,+\frac{9}{4}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (9*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 3*Sqrt[3/2]*(1 - 2*x)^(3/2)
*x*(1 + 2*x)^(3/2) + (9*Sqrt[3/2]*ArcSin[2*x])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx = 3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{9}{2}\int \sqrt{3-6x}\,\sqrt{2+4x}\,dx$$
$$= \frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x} + 3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{27}{2}\int \frac{1}{\sqrt{3-6x}\,\sqrt{2+4x}}$$
$$= \frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x} + 3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{27}{2}\int \frac{1}{\sqrt{6-24x^2}}\,dx$$
$$= \frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x} + 3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{9}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

**Mathematica** [**A**] time = 0.04, size = 39, normalized size = 0.53

$$\frac{3}{4}\sqrt{\frac{3}{2}}\left(2x\sqrt{1-4x^2}\left(5-8x^2\right)+3\sin^{-1}(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (3*Sqrt[3/2]*(2*x*(5 - 8*x^2)*Sqrt[1 - 4*x^2] + 3*ArcSin[2*x]))/4
```

**IntegrateAlgebraic** [**B**] time = 0.84, size = 179, normalized size = 2.42

$$\frac{-12\sqrt{6}\sqrt{1-2x}\,x\sqrt{2x+1}\left(8x^2-5\right)\left(-56x^3-364x^2-490x-169\right)-12\sqrt{3}\sqrt{1-2x}\,x\left(8x^2-5\right)\left(16x^4+368x^3+1088x^2+932x+239\right)}{-896x^3-5824x^2+\sqrt{2}\sqrt{2x+1}\left(64x^3+1440x^2+3632x+1912\right)-7840x-2704}+9\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{2x+1}-\sqrt{2}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (-12*Sqrt[6]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x]*(-5 + 8*x^2)*(-169 - 490*x - 364
*x^2 - 56*x^3) - 12*Sqrt[3]*Sqrt[1 - 2*x]*x*(-5 + 8*x^2)*(239 + 932*x + 108
8*x^2 + 368*x^3 + 16*x^4))/(-2704 - 7840*x - 5824*x^2 - 896*x^3 + Sqrt[2]*S
qrt[1 + 2*x]*(1912 + 3632*x + 1440*x^2 + 64*x^3)) + 9*Sqrt[3/2]*ArcTan[(-Sq
rt[2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

**fricas** [**A**] time = 0.87, size = 60, normalized size = 0.81

$$-\frac{3}{4}\left(8x^3-5x\right)\sqrt{4x+2}\,\sqrt{-6x+3}-\frac{9}{8}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}\sqrt{4x+2}\,\sqrt{-6x+3}}{12x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -3/4*(8*x^3 - 5*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 9/8*sqrt(3)*sqrt(2)*arcta
n(1/12*sqrt(3)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

**giac [B]** time = 0.98, size = 125, normalized size = 1.69

$$-\frac{1}{8}\sqrt{3}\sqrt{2}\left(((4(3x-5)(2x+1)+43)(2x+1)-39)\sqrt{2x+1}\sqrt{-2x+1}+4((4x-5)(2x+1)+9)\sqrt{2x+1}\sqrt{-2x+1}-24\sqrt{2x+1}(x-1)\sqrt{-2x+1}-24\sqrt{2x+1}\sqrt{-2x+1}-18\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{2x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(3)*sqrt(2)*(((4*(3*x - 5)*(2*x + 1) + 43)*(2*x + 1) - 39)*sqrt(2*
x + 1)*sqrt(-2*x + 1) + 4*((4*x - 5)*(2*x + 1) + 9)*sqrt(2*x + 1)*sqrt(-2*x
 + 1) - 24*sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) - 24*sqrt(2*x + 1)*sqrt(-2*
x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

**maple [B]** time = 0.00, size = 102, normalized size = 1.38

$$\frac{9\sqrt{(4x+2)(-6x+3)}\,\sqrt{6}\,\arcsin(2x)}{8\sqrt{4x+2}\,\sqrt{-6x+3}}+\frac{(-6x+3)^{\frac{3}{2}}(4x+2)^{\frac{5}{2}}}{16}+\frac{3(4x+2)^{\frac{5}{2}}\sqrt{-6x+3}}{16}-\frac{3(4x+2)^{\frac{3}{2}}\sqrt{-6x+3}}{16}-\frac{9\sqrt{-6x+3}\,\sqrt{4x+2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-6*x+3)^(3/2)*(4*x+2)^(3/2),x)
```

```
[Out] 1/16*(-6*x+3)^(3/2)*(4*x+2)^(5/2)+3/16*(4*x+2)^(5/2)*(-6*x+3)^(1/2)-3/16*(4
*x+2)^(3/2)*(-6*x+3)^(1/2)-9/8*(-6*x+3)^(1/2)*(4*x+2)^(1/2)+9/8*((4*x+2)*(-
6*x+3))^(1/2)/(4*x+2)^(1/2)/(-6*x+3)^(1/2)*6^(1/2)*arcsin(2*x)
```

**maxima [A]** time = 2.86, size = 34, normalized size = 0.46

$$\frac{1}{4}\left(-24x^2+6\right)^{\frac{3}{2}}x+\frac{9}{4}\sqrt{-24x^2+6}\,x+\frac{9}{8}\sqrt{6}\arcsin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-24*x^2 + 6)^(3/2)*x + 9/4*sqrt(-24*x^2 + 6)*x + 9/8*sqrt(6)*arcsin(2*
x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (4x+2)^{3/2}\,(3-6x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(3/2)*(3 - 6*x)^(3/2),x)
```

```
[Out] int((4*x + 2)^(3/2)*(3 - 6*x)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(3/2)*(4*x+2)**(3/2),x)
```

```
[Out] Timed out
```

## 3.1086 $\int \sqrt{3-6x}\sqrt{2+4x}\,dx$

**Optimal**. Leaf size=43

$$\sqrt{\frac{3}{2}}\sqrt{1-2x}\sqrt{2x+1}\,x+\frac{1}{2}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

**Rubi [A]** time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {38, 41, 216}

$$\sqrt{\frac{3}{2}}\sqrt{1-2x}\sqrt{2x+1}\,x+\frac{1}{2}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

```
[Out] Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x] + (Sqrt[3/2]*ArcSin[2*x])/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{3-6x}\,\sqrt{2+4x}\,dx = \sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 3\int \frac{1}{\sqrt{3-6x}\,\sqrt{2+4x}}\,dx$$
$$= \sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 3\int \frac{1}{\sqrt{6-24x^2}}\,dx$$
$$= \sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + \frac{1}{2}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 0.70

$$\frac{1}{2}\sqrt{\frac{3}{2}}\left(2\sqrt{1-4x^2}\,x+\sin^{-1}(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

```
[Out] (Sqrt[3/2]*(2*x*Sqrt[1 - 4*x^2] + ArcSin[2*x]))/2
```

**IntegrateAlgebraic [B]** time = 0.68, size = 122, normalized size = 2.84

$$\frac{2\sqrt{3}\,\sqrt{1-2x}\,\left(4x^2+16x+7\right)x+2\sqrt{6}\,(-6x-5)\sqrt{1-2x}\,\sqrt{2x+1}\,x}{-24x+\sqrt{2}\,\sqrt{2x+1}\,(4x+14)-20}+\sqrt{6}\,\tan^{-1}\left(\frac{\sqrt{2x+1}-\sqrt{2}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

```
[Out] (2*Sqrt[6]*(-5 - 6*x)*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x] + 2*Sqrt[3]*Sqrt[1 - 2*
x]*x*(7 + 16*x + 4*x^2))/(-20 - 24*x + Sqrt[2]*Sqrt[1 + 2*x]*(14 + 4*x)) +
Sqrt[6]*ArcTan[(-Sqrt[2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

**fricas [A]** time = 1.13, size = 52, normalized size = 1.21

$$\frac{1}{2}\,\sqrt{4\,x+2}\,x\sqrt{-6\,x+3}-\frac{1}{4}\,\sqrt{3}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{12\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*x + 2)*x*sqrt(-6*x + 3) - 1/4*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3
)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

**giac** [A] time = 1.07, size = 55, normalized size = 1.28

$$\frac{1}{2}\,\sqrt{3}\,\sqrt{2}\left(\sqrt{2\,x+1}\,(x-1)\sqrt{-2\,x+1}\,+\sqrt{2\,x+1}\,\sqrt{-2\,x+1}\,+\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{2\,x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*sqrt(2)*(sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) + sqrt(2*x + 1)*s
qrt(-2*x + 1) + arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

**maple** [B] time = 0.00, size = 70, normalized size = 1.63

$$\frac{\sqrt{(4x+2)\,(-6x+3)}\,\sqrt{6}\,\arcsin\,(2x)}{4\sqrt{4x+2}\,\sqrt{-6x+3}}-\frac{\sqrt{4x+2}\,\,(-6x+3)^{\frac{3}{2}}}{12}+\frac{\sqrt{-6x+3}\,\sqrt{4x+2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-6*x+3)^(1/2)*(4*x+2)^(1/2),x)
```

```
[Out] -1/12*(4*x+2)^(1/2)*(-6*x+3)^(3/2)+1/4*(-6*x+3)^(1/2)*(4*x+2)^(1/2)+1/4*((4
*x+2)*(-6*x+3))^(1/2)/(4*x+2)^(1/2)/(-6*x+3)^(1/2)*6^(1/2)*arcsin(2*x)
```

**maxima** [A] time = 3.07, size = 22, normalized size = 0.51

$$\frac{1}{2}\,\sqrt{-24\,x^2+6}\,x+\frac{1}{4}\,\sqrt{6}\,\arcsin\,(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-24*x^2 + 6)*x + 1/4*sqrt(6)*arcsin(2*x)
```

**mupad** [B] time = 0.26, size = 44, normalized size = 1.02

$$\frac{x\,\sqrt{4\,x+2}\,\sqrt{3-6\,x}}{2}-\frac{\sqrt{6}\,\ln\left(x-\frac{\sqrt{1-2\,x}\,\sqrt{2\,x+1}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(1/2)*(3 - 6*x)^(1/2),x)
```

```
[Out] (x*(4*x + 2)^(1/2)*(3 - 6*x)^(1/2))/2 - (6^(1/2)*log(x - ((1 - 2*x)^(1/2)*(
2*x + 1)^(1/2)*1i)/2)*1i)/4
```

**sympy [B]** time = 4.74, size = 187, normalized size = 4.35

$$\begin{cases} -\frac{\sqrt{6}\, i \operatorname{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{2} + \frac{\sqrt{6}\, i\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{x-\frac{1}{2}}} - \frac{3\sqrt{6}\, i\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{x-\frac{1}{2}}} + \frac{\sqrt{6}\, i\sqrt{x+\frac{1}{2}}}{2\sqrt{x-\frac{1}{2}}} & \text{for } \left|x+\frac{1}{2}\right| > 1 \\ \frac{\sqrt{6} \operatorname{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{2} - \frac{\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{\frac{1}{2}-x}} + \frac{3\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{\frac{1}{2}-x}} - \frac{\sqrt{6}\sqrt{x+\frac{1}{2}}}{2\sqrt{\frac{1}{2}-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(1/2)*(4*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/2 + sqrt(6)*I*(x + 1/2)**(5/2)/s
qrt(x - 1/2) - 3*sqrt(6)*I*(x + 1/2)**(3/2)/(2*sqrt(x - 1/2)) + sqrt(6)*I*s
qrt(x + 1/2)/(2*sqrt(x - 1/2)), Abs(x + 1/2) > 1), (sqrt(6)*asin(sqrt(x + 1
/2))/2 - sqrt(6)*(x + 1/2)**(5/2)/sqrt(1/2 - x) + 3*sqrt(6)*(x + 1/2)**(3/2
)/(2*sqrt(1/2 - x)) - sqrt(6)*sqrt(x + 1/2)/(2*sqrt(1/2 - x)), True))
```

## 3.1087 $\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx$

**Optimal**. Leaf size=13

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

**Rubi** [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {41, 216}

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

```
[Out] ArcSin[2*x]/(2*Sqrt[6])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx &= \int \frac{1}{\sqrt{6-24x^2}}\,dx\\ &= \frac{\sin^{-1}(2x)}{2\sqrt{6}}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

```
[Out] ArcSin[2*x]/(2*Sqrt[6])
```

**IntegrateAlgebraic [B]** time = 0.59, size = 36, normalized size = 2.77

$$\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{2x+1}-\sqrt{2}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

```
[Out] Sqrt[2/3]*ArcTan[(-Sqrt[2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

**fricas [B]** time = 1.09, size = 28, normalized size = 2.15

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{4x+2}\sqrt{-6x+3}}{12x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

**giac [A]** time = 0.88, size = 15, normalized size = 1.15

$$\frac{1}{6}\sqrt{6}\arcsin\left(\frac{1}{2}\sqrt{4x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*arcsin(1/2*sqrt(4*x + 2))
```

**maple [B]** time = 0.00, size = 37, normalized size = 2.85

$$\frac{\sqrt{(4x+2)(-6x+3)}\,\sqrt{6}\,\arcsin(2x)}{12\sqrt{4x+2}\,\sqrt{-6x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-6*x+3)^(1/2)/(4*x+2)^(1/2),x)
```

[Out] 1/12*((4*x+2)*(-6*x+3))^(1/2)/(4*x+2)^(1/2)/(-6*x+3)^(1/2)*6^(1/2)*arcsin(2*x)

**maxima [A]** time = 2.94, size = 9, normalized size = 0.69

$$\frac{1}{12}\sqrt{6}\arcsin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="maxima")

[Out] 1/12*sqrt(6)*arcsin(2*x)

**mupad [B]** time = 0.05, size = 40, normalized size = 3.08

$$-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{24}\left(\sqrt{3}-\sqrt{3-6x}\right)}{6\left(\sqrt{2}-\sqrt{4x+2}\right)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((4*x + 2)^(1/2)*(3 - 6*x)^(1/2)),x)

[Out] -(6^(1/2)*atan((24^(1/2)*(3^(1/2) - (3 - 6*x)^(1/2)))/(6*(2^(1/2) - (4*x + 2)^(1/2)))))/3

**sympy [A]** time = 3.35, size = 41, normalized size = 3.15

$$\begin{cases} -\frac{\sqrt{6}\,i\,\mathrm{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{for}\ \left|x+\frac{1}{2}\right|>1 \\ \frac{\sqrt{6}\,\mathrm{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-6*x)**(1/2)/(4*x+2)**(1/2),x)

[Out] Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/6, Abs(x + 1/2) > 1), (sqrt(6)*asin(sqrt(x + 1/2))/6, True))

## 3.1088 $\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx$

**Optimal**. Leaf size=28

$$\frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}$$

**Rubi [A]** time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {39}

$$\frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]
```

```
[Out] x/(6*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = \frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{1+2x}}$$

**Mathematica [A]** time = 0.02, size = 16, normalized size = 0.57

$$\frac{x}{6\sqrt{6-24x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]
```

```
[Out] x/(6*Sqrt[6 - 24*x^2])
```

**IntegrateAlgebraic [B]** time = 0.73, size = 80, normalized size = 2.86

$$\frac{x(2x+3)-2\sqrt{2}\,x\sqrt{2x+1}}{6\sqrt{3}\sqrt{1-2x}\,(-8x-4)+6\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}\,(2x+3)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[2]*x*Sqrt[1 + 2*x] + x*(3 + 2*x))/(6*Sqrt[3]*(-4 - 8*x)*Sqrt[1 - 2
*x] + 6*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x]*(3 + 2*x))
```

**fricas [A]** time = 1.39, size = 26, normalized size = 0.93

$$-\frac{\sqrt{4\,x+2}\,x\sqrt{-6\,x+3}}{36\,\left(4\,x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(4*x + 2)*x*sqrt(-6*x + 3)/(4*x^2 - 1)
```

**giac [B]** time = 1.06, size = 71, normalized size = 2.54

$$-\frac{\sqrt{6}\left(\sqrt{-4\,x+2}-2\right)}{288\,\sqrt{4\,x+2}}-\frac{\sqrt{6}\,\sqrt{4\,x+2}\,\sqrt{-4\,x+2}}{288\,(2\,x-1)}+\frac{\sqrt{6}\,\sqrt{4\,x+2}}{288\left(\sqrt{-4\,x+2}-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/288*sqrt(6)*(sqrt(-4*x + 2) - 2)/sqrt(4*x + 2) - 1/288*sqrt(6)*sqrt(4*x
+ 2)*sqrt(-4*x + 2)/(2*x - 1) + 1/288*sqrt(6)*sqrt(4*x + 2)/(sqrt(-4*x + 2)
 - 2)
```

**maple [A]** time = 0.00, size = 28, normalized size = 1.00

$$-\frac{(2x-1)\,(2x+1)\,x}{(-6x+3)^{\frac{3}{2}}\,(4x+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-6*x+3)^(3/2)/(4*x+2)^(3/2),x)
```

```
[Out] -(2*x-1)*(1+2*x)*x/(-6*x+3)^(3/2)/(4*x+2)^(3/2)
```

**maxima [A]** time = 1.36, size = 12, normalized size = 0.43

$$\frac{x}{6\,\sqrt{-24\,x^{2}+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*x/sqrt(-24*x^2 + 6)
```

**mupad [B]** time = 0.46, size = 24, normalized size = 0.86

$$-\frac{x\sqrt{3-6x}}{\sqrt{4x+2}\,(36x-18)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(3/2)*(3 - 6*x)^(3/2)),x)
```

```
[Out] -(x*(3 - 6*x)^(1/2))/((4*x + 2)^(1/2)*(36*x - 18))
```

**sympy [B]** time = 85.28, size = 156, normalized size = 5.57

$$\begin{cases} -\frac{2\sqrt{6}i\sqrt{x-\frac{1}{2}}\left(x+\frac{1}{2}\right)}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}}+\frac{\sqrt{6}i\sqrt{x-\frac{1}{2}}}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}} & \text{for}\ \left|x+\frac{1}{2}\right|>1 \\ -\frac{2\sqrt{6}\sqrt{\frac{1}{2}-x}\left(x+\frac{1}{2}\right)}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}}+\frac{\sqrt{6}\sqrt{\frac{1}{2}-x}}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(3/2)/(4*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(6)*I*sqrt(x - 1/2)*(x + 1/2)/(144*(x + 1/2)**(3/2) - 144
*sqrt(x + 1/2)) + sqrt(6)*I*sqrt(x - 1/2)/(144*(x + 1/2)**(3/2) - 144*sqrt(
x + 1/2)), Abs(x + 1/2) > 1), (-2*sqrt(6)*sqrt(1/2 - x)*(x + 1/2)/(144*(x +
 1/2)**(3/2) - 144*sqrt(x + 1/2)) + sqrt(6)*sqrt(1/2 - x)/(144*(x + 1/2)**(
3/2) - 144*sqrt(x + 1/2)), True))
```

## 3.1089 $\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx$

**Optimal**. Leaf size=57

$$\frac{x}{54\sqrt{6}\,\sqrt{1-2x}\,\sqrt{2x+1}}+\frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(2x+1)^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {40, 39}

$$\frac{x}{54\sqrt{6}\,\sqrt{1-2x}\,\sqrt{2x+1}}+\frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(2x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] x/(108*Sqrt[6]*(1 - 2*x)^(3/2)*(1 + 2*x)^(3/2)) + x/(54*Sqrt[6]*Sqrt[1 - 2*
x]*Sqrt[1 + 2*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx &= \frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{1}{9}\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx\\ &= \frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{54\sqrt{6}\,\sqrt{1-2x}\,\sqrt{1+2x}}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 37, normalized size = 0.65

$$\frac{x\left(8x^2-3\right)}{108\sqrt{6-12x}\,(2x-1)(2x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] (x*(-3 + 8*x^2))/(108*Sqrt[6 - 12*x]*(-1 + 2*x)*(1 + 2*x)^(3/2))
```

**IntegrateAlgebraic** [B] time = 0.85, size = 334, normalized size = 5.86

$$\frac{\left(\sqrt{2}\sqrt{2x+1}-2\right)^9\left(\frac{91\left(4x^2-4x+1\right)}{294912\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^4}+\frac{35\left(8x^3-12x^2+6x-1\right)}{36864\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^6}+\frac{91\left(16x^4-32x^3+24x^2-8x+1\right)}{73728\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^8}+\frac{5\left(32x^5-80x^4+80x^3-40x^2+10x-1\right)}{18432\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{10}}-\frac{64x^6-192x^5+240x^4-160x^3+60x^2-12x+1}{55296\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{12}}+\frac{5(2x-1)}{294912\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^2}-\frac{1}{3538944\sqrt{3}}\right)}{(1-2x)^{3/2}\left(-2x+\sqrt{2}\sqrt{2x+1}-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] ((-2 + Sqrt[2]*Sqrt[1 + 2*x])^9*(-1/3538944*1/Sqrt[3] - (1 - 12*x + 60*x^2
- 160*x^3 + 240*x^4 - 192*x^5 + 64*x^6)/(55296*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1
 + 2*x])^12) + (5*(-1 + 10*x - 40*x^2 + 80*x^3 - 80*x^4 + 32*x^5))/(18432*S
qrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^10) + (91*(1 - 8*x + 24*x^2 - 32*x^3 +
16*x^4))/(73728*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^8) + (35*(-1 + 6*x - 1
2*x^2 + 8*x^3))/(36864*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^6) + (91*(1 - 4
*x + 4*x^2))/(294912*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^4) + (5*(-1 + 2*x
))/(294912*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^2)))/((1 - 2*x)^(3/2)*(-1 -
 2*x + Sqrt[2]*Sqrt[1 + 2*x])^3)
```

**fricas** [A] time = 1.31, size = 39, normalized size = 0.68

$$-\frac{\left(8\,x^3-3\,x\right)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{648\left(16\,x^4-8\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/648*(8*x^3 - 3*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(16*x^4 - 8*x^2 + 1)
```

**giac** [B] time = 1.02, size = 128, normalized size = 2.25

$$-\frac{1}{82944}\sqrt{6}\left(\frac{\left(\sqrt{-4\,x+2}-2\right)^3}{(4\,x+2)^{\frac{3}{2}}}+\frac{33\left(\sqrt{-4\,x+2}-2\right)}{\sqrt{4\,x+2}}\right)-\frac{\left(4\sqrt{6}\,(2\,x+1)-9\sqrt{6}\right)\sqrt{4\,x+2}\,\sqrt{-4\,x+2}}{10368\,(2\,x-1)^2}+\frac{\sqrt{6}\,(4\,x+2)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{-4\,x+2}-2\right)^2}{2\,x+1}+2\right)}{165888\left(\sqrt{-4\,x+2}-2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/82944*sqrt(6)*((sqrt(-4*x + 2) - 2)^3/(4*x + 2)^(3/2) + 33*(sqrt(-4*x +
2) - 2)/sqrt(4*x + 2)) - 1/10368*(4*sqrt(6)*(2*x + 1) - 9*sqrt(6))*sqrt(4*x
 + 2)*sqrt(-4*x + 2)/(2*x - 1)^2 + 1/165888*sqrt(6)*(4*x + 2)^(3/2)*(33*(sq
rt(-4*x + 2) - 2)^2/(2*x + 1) + 2)/(sqrt(-4*x + 2) - 2)^3
```

**maple** [A] time = 0.00, size = 35, normalized size = 0.61

$$\frac{(2x-1)(2x+1)\left(8x^2-3\right)x}{3\left(-6x+3\right)^{\frac{5}{2}}\left(4x+2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-6*x+3)^(5/2)/(4*x+2)^(5/2),x)
```

```
[Out] 1/3*(2*x-1)*(2*x+1)*x*(8*x^2-3)/(-6*x+3)^(5/2)/(4*x+2)^(5/2)
```

**maxima** [A] time = 1.28, size = 25, normalized size = 0.44

$$\frac{x}{54\sqrt{-24x^2+6}}+\frac{x}{18\left(-24x^2+6\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/54*x/sqrt(-24*x^2 + 6) + 1/18*x/(-24*x^2 + 6)^(3/2)
```

**mupad** [B] time = 0.31, size = 49, normalized size = 0.86

$$-\frac{3x\sqrt{3-6x}-8x^3\sqrt{3-6x}}{\sqrt{4x+2}\left(-2592x^3+1296x^2+648x-324\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(5/2)*(3 - 6*x)^(5/2)),x)
```

```
[Out] -(3*x*(3 - 6*x)^(1/2) - 8*x^3*(3 - 6*x)^(1/2))/((4*x + 2)^(1/2)*(648*x + 12
96*x^2 - 2592*x^3 - 324))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(5/2)/(4*x+2)**(5/2),x)
```

```
[Out] Timed out
```

## 3.1090 $\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx$

**Optimal**. Leaf size=85

$$\frac{x}{405\sqrt{6}\,\sqrt{1-2x}\,\sqrt{2x+1}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(2x+1)^{3/2}}+\frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(2x+1)^{5/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {40, 39}

$$\frac{x}{405\sqrt{6}\,\sqrt{1-2x}\,\sqrt{2x+1}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(2x+1)^{3/2}}+\frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(2x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] x/(1080*Sqrt[6]*(1 - 2*x)^(5/2)*(1 + 2*x)^(5/2)) + x/(810*Sqrt[6]*(1 - 2*x)
^(3/2)*(1 + 2*x)^(3/2)) + x/(405*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx &= \frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{2}{15}\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx\\ &= \frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{2}{135}\int \frac{}{(3-6x)^{3/}}\\ &= \frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{405\sqrt{6}\,\sqrt{1-2x}\,}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 42, normalized size = 0.49

$$\frac{x\left(128x^4-80x^2+15\right)}{3240\sqrt{6-12x}\,(1-2x)^2(2x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] (x*(15 - 80*x^2 + 128*x^4))/(3240*Sqrt[6 - 12*x]*(1 - 2*x)^2*(1 + 2*x)^(5/2
))
```

**IntegrateAlgebraic [B]** time = 1.25, size = 616, normalized size = 7.25

$$\frac{\left(\sqrt{2}\sqrt{2x+1}-2\right)^{15}\left(-\frac{347\left(4x^2-4x+1\right)}{2717908992\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^4}-\frac{539\left(8x^3-12x^2+6x-1\right)}{169869312\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^6}-\frac{2101\left(16x^4-32x^3+24x^2-8x+1\right)}{113246208\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^8}-\frac{7469\left(32x^5-80x^4+80x^3-40x^2+10x-1\right)}{141557760\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{10}}-\frac{2101\left(64x^6-192x^5+240x^4-160x^3+60x^2-12x+1\right)}{28311552\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{12}}-\frac{539\left(128x^7-448x^6+672x^5-560x^4+280x^3-84x^2+14x-1\right)}{10616832\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{14}}-\frac{347\left(256x^8-1024x^7+1792x^6-1792x^5+1120x^4-448x^3+112x^2-16x+1\right)}{42467328\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{16}}+\frac{7\left(512x^9-2304x^8+4608x^7-5376x^6+4032x^5-2016x^4+672x^3-144x^2+18x-1\right)}{10616832\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{18}}-\frac{1024x^{10}-5120x^9+11520x^8-15360x^7+13440x^6-8064x^5+3360x^4-960x^3+180x^2-20x+1}{17694720\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{20}}+\frac{7(2x-1)}{2717908992\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^2}-\frac{1}{18119393280\sqrt{3}}\right)}{(1-2x)^{5/2}\left(-2x+\sqrt{2}\sqrt{2x+1}-1\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] ((-2 + Sqrt[2]*Sqrt[1 + 2*x])^15*(-1/18119393280*1/Sqrt[3] - (1 - 20*x + 18
0*x^2 - 960*x^3 + 3360*x^4 - 8064*x^5 + 13440*x^6 - 15360*x^7 + 11520*x^8 -
 5120*x^9 + 1024*x^10)/(17694720*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^20) +
 (7*(-1 + 18*x - 144*x^2 + 672*x^3 - 2016*x^4 + 4032*x^5 - 5376*x^6 + 4608*
x^7 - 2304*x^8 + 512*x^9))/(10616832*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^1
8) - (347*(1 - 16*x + 112*x^2 - 448*x^3 + 1120*x^4 - 1792*x^5 + 1792*x^6 -
1024*x^7 + 256*x^8))/(42467328*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^16) - (
539*(-1 + 14*x - 84*x^2 + 280*x^3 - 560*x^4 + 672*x^5 - 448*x^6 + 128*x^7))
/(10616832*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^14) - (2101*(1 - 12*x + 60*
x^2 - 160*x^3 + 240*x^4 - 192*x^5 + 64*x^6))/(28311552*Sqrt[3]*(-2 + Sqrt[2
]*Sqrt[1 + 2*x])^12) - (7469*(-1 + 10*x - 40*x^2 + 80*x^3 - 80*x^4 + 32*x^5
))/(141557760*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^10) - (2101*(1 - 8*x + 2
4*x^2 - 32*x^3 + 16*x^4))/(113246208*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^8
) - (539*(-1 + 6*x - 12*x^2 + 8*x^3))/(169869312*Sqrt[3]*(-2 + Sqrt[2]*Sqrt
[1 + 2*x])^6) - (347*(1 - 4*x + 4*x^2))/(2717908992*Sqrt[3]*(-2 + Sqrt[2]*S
qrt[1 + 2*x])^4) + (7*(-1 + 2*x))/(2717908992*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1
+ 2*x])^2)))/((1 - 2*x)^(5/2)*(-1 - 2*x + Sqrt[2]*Sqrt[1 + 2*x])^5)
```

**fricas [A]** time = 1.26, size = 49, normalized size = 0.58

$$-\frac{\left(128\,x^5-80\,x^3+15\,x\right)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{19440\left(64\,x^6-48\,x^4+12\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/19440*(128*x^5 - 80*x^3 + 15*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(64*x^6 - 4
8*x^4 + 12*x^2 - 1)
```

**giac [B]** time = 1.02, size = 181, normalized size = 2.13

$$-\frac{1}{39813120}\sqrt{6}\left(\frac{3\left(\sqrt{-4x+2}-2\right)^{5}}{(4x+2)^{\frac{5}{2}}}+\frac{85\left(\sqrt{-4x+2}-2\right)^{3}}{(4x+2)^{\frac{3}{2}}}+\frac{2130\left(\sqrt{-4x+2}-2\right)}{\sqrt{4x+2}}\right)-\frac{\left(\left(64\sqrt{6}(2x+1)-275\sqrt{6}\right)(2x+1)+300\sqrt{6}\right)\sqrt{4x+2}\sqrt{-4x+2}}{1244160(2x-1)^{3}}+\frac{\sqrt{6}\left(\frac{1065\left(\sqrt{-4x+2}-2\right)^{4}}{(2x+1)^{2}}+\frac{85\left(\sqrt{-4x+2}-2\right)^{2}}{2x+1}+6\right)(4x+2)^{\frac{5}{2}}}{79626240\left(\sqrt{-4x+2}-2\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/39813120*sqrt(6)*(3*(sqrt(-4*x + 2) - 2)^5/(4*x + 2)^(5/2) + 85*(sqrt(-4
*x + 2) - 2)^3/(4*x + 2)^(3/2) + 2130*(sqrt(-4*x + 2) - 2)/sqrt(4*x + 2)) -
 1/1244160*((64*sqrt(6)*(2*x + 1) - 275*sqrt(6))*(2*x + 1) + 300*sqrt(6))*s
qrt(4*x + 2)*sqrt(-4*x + 2)/(2*x - 1)^3 + 1/79626240*sqrt(6)*(1065*(sqrt(-4
*x + 2) - 2)^4/(2*x + 1)^2 + 85*(sqrt(-4*x + 2) - 2)^2/(2*x + 1) + 6)*(4*x
+ 2)^(5/2)/(sqrt(-4*x + 2) - 2)^5
```

**maple [A]** time = 0.00, size = 40, normalized size = 0.47

$$-\frac{(2x-1)(2x+1)\left(128x^{4}-80x^{2}+15\right)x}{15(-6x+3)^{\frac{7}{2}}(4x+2)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-6*x+3)^(7/2)/(4*x+2)^(7/2),x)
```

```
[Out] -1/15*(2*x-1)*(2*x+1)*x*(128*x^4-80*x^2+15)/(-6*x+3)^(7/2)/(4*x+2)^(7/2)
```

**maxima [A]** time = 1.31, size = 37, normalized size = 0.44

$$\frac{x}{405\sqrt{-24x^{2}+6}}+\frac{x}{135\left(-24x^{2}+6\right)^{\frac{3}{2}}}+\frac{x}{30\left(-24x^{2}+6\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/405*x/sqrt(-24*x^2 + 6) + 1/135*x/(-24*x^2 + 6)^(3/2) + 1/30*x/(-24*x^2 +
 6)^(5/2)
```

**mupad [B]** time = 0.45, size = 66, normalized size = 0.78

$$-\frac{15x\sqrt{3-6x}-80x^{3}\sqrt{3-6x}+128x^{5}\sqrt{3-6x}}{((6x-3)(240x+360)+1440)\sqrt{4x+2}(6x-3)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(7/2)*(3 - 6*x)^(7/2)),x)
```

```
[Out] -(15*x*(3 - 6*x)^(1/2) - 80*x^3*(3 - 6*x)^(1/2) + 128*x^5*(3 - 6*x)^(1/2))/
(((6*x - 3)*(240*x + 360) + 1440)*(4*x + 2)^(1/2)*(6*x - 3)^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(7/2)/(4*x+2)**(7/2),x)
```

```
[Out] Timed out
```

## 3.1091 $\int (3-x)^{3/2}(-2+x)^{3/2}\,dx$

**Optimal**. Leaf size=91

$$-\frac{1}{4}(x-2)^{3/2}(3-x)^{5/2}-\frac{1}{8}\sqrt{x-2}\,(3-x)^{5/2}+\frac{1}{32}\sqrt{x-2}\,(3-x)^{3/2}+\frac{3}{64}\sqrt{x-2}\,\sqrt{3-x}-\frac{3}{128}\sin^{-1}(5-2x)$$

**Rubi [A]** time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {50, 53, 619, 216}

$$-\frac{1}{4}(x-2)^{3/2}(3-x)^{5/2}-\frac{1}{8}\sqrt{x-2}\,(3-x)^{5/2}+\frac{1}{32}\sqrt{x-2}\,(3-x)^{3/2}+\frac{3}{64}\sqrt{x-2}\,\sqrt{3-x}-\frac{3}{128}\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x)^(3/2)*(-2 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[3 - x]*Sqrt[-2 + x])/64 + ((3 - x)^(3/2)*Sqrt[-2 + x])/32 - ((3 - x
)^(5/2)*Sqrt[-2 + x])/8 - ((3 - x)^(5/2)*(-2 + x)^(3/2))/4 - (3*ArcSin[5 -
2*x])/128
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int (3-x)^{3/2}(-2+x)^{3/2}\,dx &= -\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{3}{8}\int (3-x)^{3/2}\sqrt{-2+x}\,dx\\
&= -\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{1}{16}\int \frac{(3-x)^{3/2}}{\sqrt{-2+x}}\,dx\\
&= \frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{3}{64}\int \frac{\sqrt{3-}}{\sqrt{-2+}}\\
&= \frac{3}{64}\sqrt{3-x}\,\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2\\
&= \frac{3}{64}\sqrt{3-x}\,\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2\\
&= \frac{3}{64}\sqrt{3-x}\,\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2\\
&= \frac{3}{64}\sqrt{3-x}\,\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 80, normalized size = 0.88

$$\frac{\sqrt{-x^2+5x-6}\left(\sqrt{x-2}\left(-16x^4+168x^3-650x^2+1095x-675\right)+3\sqrt{3-x}\,\sin^{-1}\left(\sqrt{3-x}\right)\right)}{64(x-3)\sqrt{x-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x)^(3/2)*(-2 + x)^(3/2),x]
```

```
[Out] (Sqrt[-6 + 5*x - x^2]*(Sqrt[-2 + x]*(-675 + 1095*x - 650*x^2 + 168*x^3 - 16
*x^4) + 3*Sqrt[3 - x]*ArcSin[Sqrt[3 - x]]))/(64*(-3 + x)*Sqrt[-2 + x])
```

**IntegrateAlgebraic [A]** time = 0.09, size = 115, normalized size = 1.26

$$\frac{-\frac{3(3-x)^{7/2}}{(x-2)^{7/2}}-\frac{11(3-x)^{5/2}}{(x-2)^{5/2}}+\frac{11(3-x)^{3/2}}{(x-2)^{3/2}}+\frac{3\sqrt{3-x}}{\sqrt{x-2}}}{64\left(\frac{3-x}{x-2}+1\right)^4}-\frac{3}{64}\tan^{-1}\left(\frac{\sqrt{3-x}}{\sqrt{x-2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x)^(3/2)*(-2 + x)^(3/2),x]
```

```
[Out] ((-3*(3 - x)^(7/2))/(-2 + x)^(7/2) - (11*(3 - x)^(5/2))/(-2 + x)^(5/2) + (1
1*(3 - x)^(3/2))/(-2 + x)^(3/2) + (3*Sqrt[3 - x])/Sqrt[-2 + x])/(64*(1 + (3
 - x)/(-2 + x))^4) - (3*ArcTan[Sqrt[3 - x]/Sqrt[-2 + x]])/64
```

**fricas [A]** time = 1.10, size = 62, normalized size = 0.68

$$-\frac{1}{64}\left(16\,x^{3}-120\,x^{2}+290\,x-225\right)\sqrt{x-2}\,\sqrt{-x+3}-\frac{3}{128}\,\arctan\left(\frac{(2\,x-5)\sqrt{x-2}\,\sqrt{-x+3}}{2\left(x^{2}-5\,x+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(16*x^3 - 120*x^2 + 290*x - 225)*sqrt(x - 2)*sqrt(-x + 3) - 3/128*arc
tan(1/2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

**giac [A]** time = 0.88, size = 101, normalized size = 1.11

$$-\frac{1}{192}\left(2\left(4\left(6\,x+35\right)\left(x-2\right)+523\right)\left(x-2\right)+801\right)\sqrt{x-2}\,\sqrt{-x+3}+\frac{7}{24}\left(2\left(4\,x+15\right)\left(x-2\right)+69\right)\sqrt{x-2}\,\sqrt{-x+3}-4\left(2\,x+3\right)\sqrt{x-2}\,\sqrt{-x+3}+12\,\sqrt{x-2}\,\sqrt{-x+3}+\frac{3}{64}\,\arcsin\left(\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/192*(2*(4*(6*x + 35)*(x - 2) + 523)*(x - 2) + 801)*sqrt(x - 2)*sqrt(-x +
 3) + 7/24*(2*(4*x + 15)*(x - 2) + 69)*sqrt(x - 2)*sqrt(-x + 3) - 4*(2*x +
3)*sqrt(x - 2)*sqrt(-x + 3) + 12*sqrt(x - 2)*sqrt(-x + 3) + 3/64*arcsin(sqr
t(x - 2))
```

**maple [A]** time = 0.01, size = 89, normalized size = 0.98

$$\frac{3\sqrt{(x-2)\,(-x+3)}\;\arcsin\left(2x-5\right)}{128\sqrt{x-2}\;\sqrt{-x+3}}+\frac{\left(-x+3\right)^{\frac{3}{2}}\left(x-2\right)^{\frac{5}{2}}}{4}+\frac{\sqrt{-x+3}\;\left(x-2\right)^{\frac{5}{2}}}{8}-\frac{\sqrt{-x+3}\;\left(x-2\right)^{\frac{3}{2}}}{32}-\frac{3\sqrt{-x+3}\;\sqrt{x-2}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)^(3/2)*(x-2)^(3/2),x)
```

```
[Out] 1/4*(3-x)^(3/2)*(x-2)^(5/2)+1/8*(3-x)^(1/2)*(x-2)^(5/2)-1/32*(3-x)^(1/2)*(x
-2)^(3/2)-3/64*(3-x)^(1/2)*(x-2)^(1/2)+3/128*((x-2)*(3-x))^(1/2)/(x-2)^(1/2
)/(3-x)^(1/2)*arcsin(-5+2*x)
```

**maxima [A]** time = 2.97, size = 67, normalized size = 0.74

$$\frac{1}{4}\left(-x^{2}+5\,x-6\right)^{\frac{3}{2}}x-\frac{5}{8}\left(-x^{2}+5\,x-6\right)^{\frac{3}{2}}+\frac{3}{32}\,\sqrt{-x^{2}+5\,x-6}\,x-\frac{15}{64}\,\sqrt{-x^{2}+5\,x-6}+\frac{3}{128}\,\arcsin\left(2\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + 5*x - 6)^(3/2)*x - 5/8*(-x^2 + 5*x - 6)^(3/2) + 3/32*sqrt(-x^2
+ 5*x - 6)*x - 15/64*sqrt(-x^2 + 5*x - 6) + 3/128*arcsin(2*x - 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (x-2)^{3/2} (3-x)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)^(3/2)*(3 - x)^(3/2),x)
```

```
[Out] int((x - 2)^(3/2)*(3 - x)^(3/2), x)
```

**sympy [A]** time = 7.48, size = 199, normalized size = 2.19

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\sqrt{x-2}\right)}{64} - \frac{i(x-2)^{\frac{9}{2}}}{4\sqrt{x-3}} + \frac{5i(x-2)^{\frac{7}{2}}}{8\sqrt{x-3}} - \frac{13i(x-2)^{\frac{5}{2}}}{32\sqrt{x-3}} - \frac{i(x-2)^{\frac{3}{2}}}{64\sqrt{x-3}} + \frac{3i\sqrt{x-2}}{64\sqrt{x-3}} & \text{for}\ |x-2| > 1 \\ \frac{3\operatorname{asin}\left(\sqrt{x-2}\right)}{64} + \frac{(x-2)^{\frac{9}{2}}}{4\sqrt{3-x}} - \frac{5(x-2)^{\frac{7}{2}}}{8\sqrt{3-x}} + \frac{13(x-2)^{\frac{5}{2}}}{32\sqrt{3-x}} + \frac{(x-2)^{\frac{3}{2}}}{64\sqrt{3-x}} - \frac{3\sqrt{x-2}}{64\sqrt{3-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)**(3/2)*(-2+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(x - 2))/64 - I*(x - 2)**(9/2)/(4*sqrt(x - 3)) +
5*I*(x - 2)**(7/2)/(8*sqrt(x - 3)) - 13*I*(x - 2)**(5/2)/(32*sqrt(x - 3)) -
 I*(x - 2)**(3/2)/(64*sqrt(x - 3)) + 3*I*sqrt(x - 2)/(64*sqrt(x - 3)), Abs(
x - 2) > 1), (3*asin(sqrt(x - 2))/64 + (x - 2)**(9/2)/(4*sqrt(3 - x)) - 5*(
x - 2)**(7/2)/(8*sqrt(3 - x)) + 13*(x - 2)**(5/2)/(32*sqrt(3 - x)) + (x - 2
)**(3/2)/(64*sqrt(3 - x)) - 3*sqrt(x - 2)/(64*sqrt(3 - x)), True))
```

# 3.1092 $\int \sqrt{3-x}\,\sqrt{-2+x}\,dx$

**Optimal**. Leaf size=51

$$-\frac{1}{2}\sqrt{x-2}\,(3-x)^{3/2}+\frac{1}{4}\sqrt{x-2}\,\sqrt{3-x}-\frac{1}{8}\sin^{-1}(5-2x)$$

**Rubi [A]** time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {50, 53, 619, 216}

$$-\frac{1}{2}\sqrt{x-2}\,(3-x)^{3/2}+\frac{1}{4}\sqrt{x-2}\,\sqrt{3-x}-\frac{1}{8}\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x]*Sqrt[-2 + x],x]
```

```
[Out] (Sqrt[3 - x]*Sqrt[-2 + x])/4 - ((3 - x)^(3/2)*Sqrt[-2 + x])/2 - ArcSin[5 -
2*x]/8
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-x}\,\sqrt{-2+x}\,dx &= -\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{4}\int \frac{\sqrt{3-x}}{\sqrt{-2+x}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{8}\int \frac{1}{\sqrt{3-x}\,\sqrt{-2+x}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{8}\int \frac{1}{\sqrt{-6+5x-x^2}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,5-2x\right)\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}\sin^{-1}(5-2x)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 69, normalized size = 1.35

$$\frac{\sqrt{-x^2+5x-6}\,\left(\sqrt{x-2}\,\left(2x^2-11x+15\right)+\sqrt{3-x}\,\sin^{-1}\left(\sqrt{3-x}\right)\right)}{4(x-3)\sqrt{x-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x]*Sqrt[-2 + x],x]
[Out] (Sqrt[-6 + 5*x - x^2]*(Sqrt[-2 + x]*(15 - 11*x + 2*x^2) + Sqrt[3 - x]*ArcSi
n[Sqrt[3 - x]]))/(4*(-3 + x)*Sqrt[-2 + x])
```

**IntegrateAlgebraic [A]** time = 0.06, size = 78, normalized size = 1.53

$$\frac{\frac{\sqrt{3-x}}{\sqrt{x-2}}-\frac{(3-x)^{3/2}}{(x-2)^{3/2}}}{4\left(\frac{3-x}{x-2}+1\right)^2}-\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{3-x}}{\sqrt{x-2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - x]*Sqrt[-2 + x],x]
[Out] (-((3 - x)^(3/2)/(-2 + x)^(3/2)) + Sqrt[3 - x]/Sqrt[-2 + x])/(4*(1 + (3 - x
)/(-2 + x))^2) - ArcTan[Sqrt[3 - x]/Sqrt[-2 + x]]/4
```

**fricas** [A] time = 1.29, size = 52, normalized size = 1.02

$$\frac{1}{4}(2x-5)\sqrt{x-2}\sqrt{-x+3}-\frac{1}{8}\arctan\left(\frac{(2x-5)\sqrt{x-2}\sqrt{-x+3}}{2(x^2-5x+6)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3) - 1/8*arctan(1/2*(2*x - 5)*sqrt(x -
2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

**giac** [A] time = 1.02, size = 42, normalized size = 0.82

$$\frac{1}{4}(2x+3)\sqrt{x-2}\sqrt{-x+3}-2\sqrt{x-2}\sqrt{-x+3}+\frac{1}{4}\arcsin\left(\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*x + 3)*sqrt(x - 2)*sqrt(-x + 3) - 2*sqrt(x - 2)*sqrt(-x + 3) + 1/4*a
rcsin(sqrt(x - 2))
```

**maple** [A] time = 0.01, size = 61, normalized size = 1.20

$$\frac{\sqrt{(x-2)(-x+3)}\,\arcsin(2x-5)}{8\sqrt{x-2}\,\sqrt{-x+3}}-\frac{(-x+3)^{\frac{3}{2}}\sqrt{x-2}}{2}+\frac{\sqrt{-x+3}\,\sqrt{x-2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+3)^(1/2)*(x-2)^(1/2),x)
```

```
[Out] -1/2*(-x+3)^(3/2)*(x-2)^(1/2)+1/4*(-x+3)^(1/2)*(x-2)^(1/2)+1/8*((x-2)*(-x+3
))^(1/2)/(x-2)^(1/2)/(-x+3)^(1/2)*arcsin(2*x-5)
```

**maxima** [A] time = 2.96, size = 38, normalized size = 0.75

$$\frac{1}{2}\sqrt{-x^2+5x-6}\,x-\frac{5}{4}\sqrt{-x^2+5x-6}+\frac{1}{8}\arcsin(2x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 5*x - 6)*x - 5/4*sqrt(-x^2 + 5*x - 6) + 1/8*arcsin(2*x - 5)
```

**mupad [B]** time = 0.21, size = 41, normalized size = 0.80

$$\left(\frac{x}{2}-\frac{5}{4}\right)\sqrt{x-2}\,\sqrt{3-x}-\frac{\ln\left(x-\frac{5}{2}-\sqrt{x-2}\,\sqrt{3-x}\,1\mathrm{i}\right)1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)^(1/2)*(3 - x)^(1/2),x)
```

```
[Out] (x/2 - 5/4)*(x - 2)^(1/2)*(3 - x)^(1/2) - (log(x - (x - 2)^(1/2)*(3 - x)^(1
/2)*1i - 5/2)*1i)/8
```

**sympy [A]** time = 3.01, size = 124, normalized size = 2.43

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\sqrt{x-2}\right)}{4}+\frac{i(x-2)^{\frac{5}{2}}}{2\sqrt{x-3}}-\frac{3i(x-2)^{\frac{3}{2}}}{4\sqrt{x-3}}+\frac{i\sqrt{x-2}}{4\sqrt{x-3}} & \text{for } |x-2|>1 \\ \frac{\operatorname{asin}\left(\sqrt{x-2}\right)}{4}-\frac{(x-2)^{\frac{5}{2}}}{2\sqrt{3-x}}+\frac{3(x-2)^{\frac{3}{2}}}{4\sqrt{3-x}}-\frac{\sqrt{x-2}}{4\sqrt{3-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)**(1/2)*(-2+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(x - 2))/4 + I*(x - 2)**(5/2)/(2*sqrt(x - 3)) - 3*I
*(x - 2)**(3/2)/(4*sqrt(x - 3)) + I*sqrt(x - 2)/(4*sqrt(x - 3)), Abs(x - 2)
 > 1), (asin(sqrt(x - 2))/4 - (x - 2)**(5/2)/(2*sqrt(3 - x)) + 3*(x - 2)**(
3/2)/(4*sqrt(3 - x)) - sqrt(x - 2)/(4*sqrt(3 - x)), True))
```

## 3.1093 $\int \frac{1}{\sqrt{3-x}\,\sqrt{-2+x}}\,dx$

**Optimal**. Leaf size=8

$$-\sin^{-1}(5-2x)$$

**Rubi [A]** time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {53, 619, 216}

$$-\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]
```

```
[Out] -ArcSin[5 - 2*x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-x}\,\sqrt{-2+x}}\,dx &= \int \frac{1}{\sqrt{-6+5x-x^2}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 5-2x\right)\\
&= -\sin^{-1}(5-2x)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 12, normalized size = 1.50

$$-2\sin^{-1}\left(\sqrt{3-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]
[Out] -2*ArcSin[Sqrt[3 - x]]
```

**IntegrateAlgebraic [B]** time = 0.04, size = 20, normalized size = 2.50

$$-2\tan^{-1}\left(\frac{\sqrt{3-x}}{\sqrt{x-2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]
[Out] -2*ArcTan[Sqrt[3 - x]/Sqrt[-2 + x]]
```

**fricas [B]** time = 1.35, size = 32, normalized size = 4.00

$$-\arctan\left(\frac{(2x-5)\sqrt{x-2}\sqrt{-x+3}}{2\left(x^2-5x+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

**giac [A]** time = 1.02, size = 8, normalized size = 1.00

$$2\arcsin\left(\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(sqrt(x - 2))
```

**maple [B]** time = 0.00, size = 31, normalized size = 3.88

$$\frac{\sqrt{(x-2)(-x+3)}\ \arcsin(2x-5)}{\sqrt{x-2}\ \sqrt{-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+3)^(1/2)/(x-2)^(1/2),x)
```

```
[Out] ((x-2)*(-x+3))^(1/2)/(x-2)^(1/2)/(-x+3)^(1/2)*arcsin(2*x-5)
```

**maxima** [A] time = 3.00, size = 6, normalized size = 0.75

$$\arcsin(2x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(2*x - 5)
```

**mupad** [B] time = 0.18, size = 31, normalized size = 3.88

$$-4\,\mathrm{atan}\left(\frac{\sqrt{x-2}-\sqrt{2}\,1\mathrm{i}}{\sqrt{3}-\sqrt{3-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(1/2)*(3 - x)^(1/2)),x)
```

```
[Out] -4*atan(((x - 2)^(1/2) - 2^(1/2)*1i)/(3^(1/2) - (3 - x)^(1/2)))
```

**sympy** [A] time = 1.61, size = 26, normalized size = 3.25

$$\begin{cases} -2i\,\mathrm{acosh}\left(\sqrt{x-2}\right) & \text{for } |x-2|>1 \\ 2\,\mathrm{asin}\left(\sqrt{x-2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(1/2)/(-2+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(x - 2)), Abs(x - 2) > 1), (2*asin(sqrt(x - 2)),
True))
```

## 3.1094 $\int \frac{1}{(3-x)^{3/2}(-2+x)^{3/2}}\,dx$

**Optimal**. Leaf size=37

$$\frac{2}{\sqrt{3-x}\,\sqrt{x-2}}-\frac{4\sqrt{3-x}}{\sqrt{x-2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2}{\sqrt{3-x}\,\sqrt{x-2}}-\frac{4\sqrt{3-x}}{\sqrt{x-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]
```

```
[Out] 2/(Sqrt[3 - x]*Sqrt[-2 + x]) - (4*Sqrt[3 - x])/Sqrt[-2 + x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-x)^{3/2}(-2+x)^{3/2}}\,dx &= \frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}+2\int \frac{1}{\sqrt{3-x}\,(-2+x)^{3/2}}\,dx\\ &= \frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}-\frac{4\sqrt{3-x}}{\sqrt{-2+x}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 21, normalized size = 0.57

$$\frac{2(2x-5)}{\sqrt{-x^2+5x-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]
[Out] (2*(-5 + 2*x))/Sqrt[-6 + 5*x - x^2]
```

**IntegrateAlgebraic** [A] time = 0.05, size = 31, normalized size = 0.84

$$-\frac{2\left(\frac{3-x}{x-2}-1\right)\sqrt{x-2}}{\sqrt{3-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]
[Out] (-2*(-1 + (3 - x)/(-2 + x))*Sqrt[-2 + x])/Sqrt[3 - x]
```

**fricas** [A] time = 1.27, size = 29, normalized size = 0.78

$$-\frac{2\,(2\,x-5)\sqrt{x-2}\,\sqrt{-x+3}}{x^2-5\,x+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="fricas")
[Out] -2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6)
```

**giac** [A] time = 0.85, size = 53, normalized size = 1.43

$$-\frac{\sqrt{-x+3}-1}{\sqrt{x-2}}-\frac{2\,\sqrt{x-2}\,\sqrt{-x+3}}{x-3}+\frac{\sqrt{x-2}}{\sqrt{-x+3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="giac")
[Out] -(sqrt(-x + 3) - 1)/sqrt(x - 2) - 2*sqrt(x - 2)*sqrt(-x + 3)/(x - 3) + sqrt
(x - 2)/(sqrt(-x + 3) - 1)
```

**maple** [A] time = 0.00, size = 20, normalized size = 0.54

$$\frac{-10+4x}{\sqrt{-x+3}\,\sqrt{x-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+3)^(3/2)/(x-2)^(3/2),x)
```

```
[Out] 2*(2*x-5)/(x-2)^(1/2)/(-x+3)^(1/2)
```

**maxima** [A] time = 1.32, size = 30, normalized size = 0.81

$$\frac{4\,x}{\sqrt{-x^2+5\,x-6}}-\frac{10}{\sqrt{-x^2+5\,x-6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 5*x - 6) - 10/sqrt(-x^2 + 5*x - 6)
```

**mupad** [B] time = 0.25, size = 32, normalized size = 0.86

$$-\frac{4\,x\,\sqrt{3-x}\,-10\,\sqrt{3-x}}{\sqrt{x-2}\,\left(x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(3/2)*(3 - x)^(3/2)),x)
```

```
[Out] -(4*x*(3 - x)^(1/2) - 10*(3 - x)^(1/2))/((x - 2)^(1/2)*(x - 3))
```

**sympy** [A] time = 2.30, size = 100, normalized size = 2.70

$$\begin{cases} -\frac{4i\sqrt{x-3}\,(x-2)}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}}+\frac{2i\sqrt{x-3}}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} & \text{for}\ |x-2|>1 \\ -\frac{4\sqrt{3-x}\,(x-2)}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}}+\frac{2\sqrt{3-x}}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(3/2)/(-2+x)**(3/2),x)
```

```
[Out] Piecewise((-4*I*sqrt(x - 3)*(x - 2)/((x - 2)**(3/2) - sqrt(x - 2)) + 2*I*sq
rt(x - 3)/((x - 2)**(3/2) - sqrt(x - 2)), Abs(x - 2) > 1), (-4*sqrt(3 - x)*
(x - 2)/((x - 2)**(3/2) - sqrt(x - 2)) + 2*sqrt(3 - x)/((x - 2)**(3/2) - sq
rt(x - 2)), True))
```

## 3.1095 $\int \frac{1}{(3-x)^{5/2}(-2+x)^{5/2}}\,dx$

**Optimal**. Leaf size=79

$$-\frac{32\sqrt{3-x}}{3\sqrt{x-2}}-\frac{16\sqrt{3-x}}{3(x-2)^{3/2}}+\frac{4}{(x-2)^{3/2}\sqrt{3-x}}+\frac{2}{3(x-2)^{3/2}(3-x)^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$-\frac{32\sqrt{3-x}}{3\sqrt{x-2}}-\frac{16\sqrt{3-x}}{3(x-2)^{3/2}}+\frac{4}{(x-2)^{3/2}\sqrt{3-x}}+\frac{2}{3(x-2)^{3/2}(3-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] 2/(3*(3 - x)^(3/2)*(-2 + x)^(3/2)) + 4/(Sqrt[3 - x]*(-2 + x)^(3/2)) - (16*S
qrt[3 - x])/(3*(-2 + x)^(3/2)) - (32*Sqrt[3 - x])/(3*Sqrt[-2 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(3-x)^{5/2}(-2+x)^{5/2}}\,dx &= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + 2\int \frac{1}{(3-x)^{3/2}(-2+x)^{5/2}}\,dx \\
&= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + \frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}} + 8\int \frac{1}{\sqrt{3-x}\,(-2+x)^{5/2}}\,dx \\
&= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + \frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}} - \frac{16\sqrt{3-x}}{3(-2+x)^{3/2}} + \frac{16}{3}\int \frac{1}{\sqrt{3-x}\,(-2+x)^{3/2}}\,dx \\
&= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + \frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}} - \frac{16\sqrt{3-x}}{3(-2+x)^{3/2}} - \frac{32\sqrt{3-x}}{3\sqrt{-2+x}}
\end{aligned}
$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 0.42

$$\frac{-32x^3+240x^2-588x+470}{3\left(-x^2+5x-6\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] (470 - 588*x + 240*x^2 - 32*x^3)/(3*(-6 + 5*x - x^2)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 61, normalized size = 0.77

$$-\frac{2\left(\frac{(3-x)^3}{(x-2)^3}+\frac{9(3-x)^2}{(x-2)^2}-\frac{9(3-x)}{x-2}-1\right)(x-2)^{3/2}}{3(3-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] (-2*(-1 + (3 - x)^3/(-2 + x)^3 + (9*(3 - x)^2)/(-2 + x)^2 - (9*(3 - x))/(-2
 + x))*(-2 + x)^(3/2))/(3*(3 - x)^(3/2))
```

**fricas [A]** time = 1.22, size = 49, normalized size = 0.62

$$-\frac{2\left(16\,x^3-120\,x^2+294\,x-235\right)\sqrt{x-2}\,\sqrt{-x+3}}{3\left(x^4-10\,x^3+37\,x^2-60\,x+36\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*x^3 - 120*x^2 + 294*x - 235)*sqrt(x - 2)*sqrt(-x + 3)/(x^4 - 10*x^
3 + 37*x^2 - 60*x + 36)
```

**giac** [A] time = 1.10, size = 97, normalized size = 1.23

$$-\frac{\left(\sqrt{-x+3}-1\right)^3}{12\,(x-2)^{\frac{3}{2}}}-\frac{11\left(\sqrt{-x+3}-1\right)}{4\sqrt{x-2}}-\frac{2\,(8\,x-25)\sqrt{x-2}\,\sqrt{-x+3}}{3\,(x-3)^2}+\frac{(x-2)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{-x+3}-1\right)^2}{x-2}+1\right)}{12\left(\sqrt{-x+3}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(-x + 3) - 1)^3/(x - 2)^(3/2) - 11/4*(sqrt(-x + 3) - 1)/sqrt(x -
 2) - 2/3*(8*x - 25)*sqrt(x - 2)*sqrt(-x + 3)/(x - 3)^2 + 1/12*(x - 2)^(3/2
)*(33*(sqrt(-x + 3) - 1)^2/(x - 2) + 1)/(sqrt(-x + 3) - 1)^3
```

**maple** [A] time = 0.00, size = 30, normalized size = 0.38

$$-\frac{2\left(16x^3-120x^2+294x-235\right)}{3\,(x-2)^{\frac{3}{2}}\,(-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+3)^(5/2)/(x-2)^(5/2),x)
```

```
[Out] -2/3*(16*x^3-120*x^2+294*x-235)/(x-2)^(3/2)/(-x+3)^(3/2)
```

**maxima** [A] time = 1.34, size = 59, normalized size = 0.75

$$\frac{32\,x}{3\sqrt{-x^2+5\,x-6}}-\frac{80}{3\sqrt{-x^2+5\,x-6}}+\frac{4\,x}{3\left(-x^2+5\,x-6\right)^{\frac{3}{2}}}-\frac{10}{3\left(-x^2+5\,x-6\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x/sqrt(-x^2 + 5*x - 6) - 80/3/sqrt(-x^2 + 5*x - 6) + 4/3*x/(-x^2 + 5*x
 - 6)^(3/2) - 10/3/(-x^2 + 5*x - 6)^(3/2)
```

**mupad** [B] time = 0.37, size = 69, normalized size = 0.87

$$-\frac{32\,(x-2)^3\,\sqrt{3-x}-48\,(x-2)^2\,\sqrt{3-x}+2\,\sqrt{3-x}+12\,(x-2)\,\sqrt{3-x}}{(3\,x-6)\,\sqrt{x-2}\,(x-3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(5/2)*(3 - x)^(5/2)),x)
```

```
[Out] -(32*(x - 2)^3*(3 - x)^(1/2) - 48*(x - 2)^2*(3 - x)^(1/2) + 2*(3 - x)^(1/2)
 + 12*(x - 2)*(3 - x)^(1/2))/((3*x - 6)*(x - 2)^(1/2)*(x - 3)^2)
```

**sympy [B]** time = 9.85, size = 282, normalized size = 3.57

$$\begin{cases} -\frac{32\sqrt{-1+\frac{1}{x-2}}\,(x-2)^3}{3x+3(x-2)^3-6(x-2)^2-6} + \frac{48\sqrt{-1+\frac{1}{x-2}}\,(x-2)^2}{3x+3(x-2)^3-6(x-2)^2-6} - \frac{12\sqrt{-1+\frac{1}{x-2}}\,(x-2)}{3x+3(x-2)^3-6(x-2)^2-6} - \frac{2\sqrt{-1+\frac{1}{x-2}}}{3x+3(x-2)^3-6(x-2)^2-6} & \text{for } \frac{1}{|x-2|} > 1 \\ -\frac{32i\sqrt{1-\frac{1}{x-2}}\,(x-2)^3}{3x+3(x-2)^3-6(x-2)^2-6} + \frac{48i\sqrt{1-\frac{1}{x-2}}\,(x-2)^2}{3x+3(x-2)^3-6(x-2)^2-6} - \frac{12i\sqrt{1-\frac{1}{x-2}}\,(x-2)}{3x+3(x-2)^3-6(x-2)^2-6} - \frac{2i\sqrt{1-\frac{1}{x-2}}}{3x+3(x-2)^3-6(x-2)^2-6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(5/2)/(-2+x)**(5/2),x)
```

```
[Out] Piecewise((-32*sqrt(-1 + 1/(x - 2))*(x - 2)**3/(3*x + 3*(x - 2)**3 - 6*(x -
 2)**2 - 6) + 48*sqrt(-1 + 1/(x - 2))*(x - 2)**2/(3*x + 3*(x - 2)**3 - 6*(x
 - 2)**2 - 6) - 12*sqrt(-1 + 1/(x - 2))*(x - 2)/(3*x + 3*(x - 2)**3 - 6*(x
- 2)**2 - 6) - 2*sqrt(-1 + 1/(x - 2))/(3*x + 3*(x - 2)**3 - 6*(x - 2)**2 -
6), 1/Abs(x - 2) > 1), (-32*I*sqrt(1 - 1/(x - 2))*(x - 2)**3/(3*x + 3*(x -
2)**3 - 6*(x - 2)**2 - 6) + 48*I*sqrt(1 - 1/(x - 2))*(x - 2)**2/(3*x + 3*(x
 - 2)**3 - 6*(x - 2)**2 - 6) - 12*I*sqrt(1 - 1/(x - 2))*(x - 2)/(3*x + 3*(x
 - 2)**3 - 6*(x - 2)**2 - 6) - 2*I*sqrt(1 - 1/(x - 2))/(3*x + 3*(x - 2)**3
- 6*(x - 2)**2 - 6), True))
```

## 3.1096 $\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx$

**Optimal**. Leaf size=21

$$\frac{x}{9\sqrt{3-x}\,\sqrt{x+3}}$$

**Rubi [A]** time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {39}

$$\frac{x}{9\sqrt{3-x}\,\sqrt{x+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[3 - x]*Sqrt[3 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{9\sqrt{3-x}\,\sqrt{3+x}}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 0.76

$$\frac{x}{9\sqrt{9-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[9 - x^2])
```

**IntegrateAlgebraic [A]** time = 0.06, size = 34, normalized size = 1.62

$$\frac{\sqrt{x+3}\left(1-\frac{3-x}{x+3}\right)}{18\sqrt{3-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] (Sqrt[3 + x]*(1 - (3 - x)/(3 + x)))/(18*Sqrt[3 - x])
```

**fricas [A]** time = 1.56, size = 22, normalized size = 1.05

$$-\frac{\sqrt{x+3}\,x\sqrt{-x+3}}{9\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(x + 3)*x*sqrt(-x + 3)/(x^2 - 9)
```

**giac [B]** time = 0.90, size = 62, normalized size = 2.95

$$\frac{\sqrt{6}-\sqrt{-x+3}}{36\sqrt{x+3}}-\frac{\sqrt{x+3}\sqrt{-x+3}}{18\,(x-3)}-\frac{\sqrt{x+3}}{36\left(\sqrt{6}-\sqrt{-x+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/36*(sqrt(6) - sqrt(-x + 3))/sqrt(x + 3) - 1/18*sqrt(x + 3)*sqrt(-x + 3)/(
x - 3) - 1/36*sqrt(x + 3)/(sqrt(6) - sqrt(-x + 3))
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.76

$$\frac{x}{9\sqrt{-x+3}\,\sqrt{x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+3)^(3/2)/(3+x)^(3/2),x)
```

```
[Out] 1/9*x/(-x+3)^(1/2)/(3+x)^(1/2)
```

**maxima [A]** time = 1.37, size = 12, normalized size = 0.57

$$\frac{x}{9\sqrt{-x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*x/sqrt(-x^2 + 9)
```

**mupad [B]** time = 0.36, size = 22, normalized size = 1.05

$$-\frac{x\,\sqrt{3-x}}{(9\,x-27)\,\sqrt{x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - x)^(3/2)*(x + 3)^(3/2)),x)
```

```
[Out] -(x*(3 - x)^(1/2))/((9*x - 27)*(x + 3)^(1/2))
```

**sympy [A]** time = 1.80, size = 73, normalized size = 3.48

$$\begin{cases} \frac{1}{9\sqrt{-1+\frac{6}{x+3}}} - \frac{1}{3\sqrt{-1+\frac{6}{x+3}}\,(x+3)} & \text{for}\ \frac{6}{|x+3|} > 1 \\ \frac{i\sqrt{1-\frac{6}{x+3}}\,(x+3)}{27-9x} - \frac{3i\sqrt{1-\frac{6}{x+3}}}{27-9x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(3/2)/(3+x)**(3/2),x)
```

```
[Out] Piecewise((1/(9*sqrt(-1 + 6/(x + 3))) - 1/(3*sqrt(-1 + 6/(x + 3))*(x + 3)),
 6/Abs(x + 3) > 1), (I*sqrt(1 - 6/(x + 3))*(x + 3)/(27 - 9*x) - 3*I*sqrt(1
- 6/(x + 3))/(27 - 9*x), True))
```

## 3.1097 $\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=24

$$\frac{x}{9\sqrt{3-bx}\,\sqrt{bx+3}}$$

**Rubi [A]** time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {39}

$$\frac{x}{9\sqrt{3-bx}\,\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[3 - b*x]*Sqrt[3 + b*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}}\,dx=\frac{x}{9\sqrt{3-bx}\,\sqrt{3+bx}}$$

**Mathematica [A]** time = 0.01, size = 19, normalized size = 0.79

$$\frac{x}{9\sqrt{9-b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[9 - b^2*x^2])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 43, normalized size = 1.79

$$\frac{\sqrt{bx+3}\left(1-\frac{3-bx}{bx+3}\right)}{18b\sqrt{3-bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] (Sqrt[3 + b*x]*(1 - (3 - b*x)/(3 + b*x)))/(18*b*Sqrt[3 - b*x])
```

**fricas [A]** time = 1.53, size = 29, normalized size = 1.21

$$-\frac{\sqrt{bx+3}\,\sqrt{-bx+3}\,x}{9\left(b^2x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(b*x + 3)*sqrt(-b*x + 3)*x/(b^2*x^2 - 9)
```

**giac [B]** time = 1.10, size = 82, normalized size = 3.42

$$\frac{\sqrt{6}-\sqrt{-bx+3}}{36\sqrt{bx+3}\,b}-\frac{\sqrt{bx+3}\,\sqrt{-bx+3}}{18\,(bx-3)b}-\frac{\sqrt{bx+3}}{36\,b\left(\sqrt{6}-\sqrt{-bx+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/36*(sqrt(6) - sqrt(-b*x + 3))/(sqrt(b*x + 3)*b) - 1/18*sqrt(b*x + 3)*sqrt
(-b*x + 3)/((b*x - 3)*b) - 1/36*sqrt(b*x + 3)/(b*(sqrt(6) - sqrt(-b*x + 3))
)
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.79

$$\frac{x}{9\sqrt{-bx+3}\,\sqrt{bx+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x)
```

```
[Out] 1/9*x/(-b*x+3)^(1/2)/(b*x+3)^(1/2)
```

**maxima [A]** time = 1.37, size = 15, normalized size = 0.62

$$\frac{x}{9\sqrt{-b^2x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*x/sqrt(-b^2*x^2 + 9)
```

**mupad [B]** time = 0.46, size = 26, normalized size = 1.08

$$-\frac{x\sqrt{3-b\,x}}{\sqrt{b\,x+3}\,\left(9\,b\,x-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - b*x)^(3/2)*(b*x + 3)^(3/2)),x)
```

```
[Out] -(x*(3 - b*x)^(1/2))/((b*x + 3)^(1/2)*(9*b*x - 27))
```

**sympy [C]** time = 5.16, size = 73, normalized size = 3.04

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{9}{b^{2}x^{2}}\right)}{18\pi^{\frac{3}{2}}b}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{9e^{-2i\pi}}{b^{2}x^{2}}\right)}{18\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)**(3/2)/(b*x+3)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), 9
/(b**2*x**2))/(18*pi**(3/2)*b) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4, 1), ()),
 ((1/4, 3/4), (-1/2, 0, 1, 0)), 9*exp_polar(-2*I*pi)/(b**2*x**2))/(18*pi**(
3/2)*b)
```

## 3.1098 $\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx$

**Optimal**. Leaf size=26

$$\frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{x+3}}$$

**Rubi [A]** time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {39}

$$\frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{x+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[2]*Sqrt[3 - x]*Sqrt[3 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{3+x}}$$

**Mathematica [A]** time = 0.01, size = 21, normalized size = 0.81

$$\frac{x}{18\sqrt{6-2x}\sqrt{x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[6 - 2*x]*Sqrt[3 + x])
```

**IntegrateAlgebraic [F]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)), x]
```

**fricas [A]** time = 1.29, size = 22, normalized size = 0.85

$$-\frac{\sqrt{x+3}\,x\sqrt{-2\,x+6}}{36\,\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(x + 3)*x*sqrt(-2*x + 6)/(x^2 - 9)
```

**giac [B]** time = 1.04, size = 71, normalized size = 2.73

$$\frac{\sqrt{2}\left(\sqrt{6}-\sqrt{-x+3}\right)}{144\,\sqrt{x+3}}-\frac{\sqrt{2}\,\sqrt{x+3}\,\sqrt{-x+3}}{72\,(x-3)}-\frac{\sqrt{2}\,\sqrt{x+3}}{144\left(\sqrt{6}-\sqrt{-x+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/144*sqrt(2)*(sqrt(6) - sqrt(-x + 3))/sqrt(x + 3) - 1/72*sqrt(2)*sqrt(x +
3)*sqrt(-x + 3)/(x - 3) - 1/144*sqrt(2)*sqrt(x + 3)/(sqrt(6) - sqrt(-x + 3)
)
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.73

$$-\frac{(x-3)\,x}{9\sqrt{x+3}\,\left(-2x+6\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6-2*x)^(3/2)/(x+3)^(3/2),x)
```

```
[Out] -1/9*(-3+x)/(x+3)^(1/2)*x/(6-2*x)^(3/2)
```

**maxima [A]** time = 1.33, size = 12, normalized size = 0.46

$$\frac{x}{18\,\sqrt{-2\,x^2+18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*x/sqrt(-2*x^2 + 18)
```

**mupad [B]** time = 0.37, size = 22, normalized size = 0.85

$$-\frac{x\sqrt{6-2\,x}}{(36\,x-108)\,\sqrt{x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((6 - 2*x)^(3/2)*(x + 3)^(3/2)),x)
```

```
[Out] -(x*(6 - 2*x)^(1/2))/((36*x - 108)*(x + 3)^(1/2))
```

**sympy [A]** time = 20.45, size = 90, normalized size = 3.46

$$\begin{cases} \frac{\sqrt{2}}{36\sqrt{-1+\frac{6}{x+3}}}-\frac{\sqrt{2}}{12\sqrt{-1+\frac{6}{x+3}}\,(x+3)} & \text{for } \frac{6}{|x+3|}>1 \\ \frac{\sqrt{2}\,i\sqrt{1-\frac{6}{x+3}}\,(x+3)}{108-36x}-\frac{3\sqrt{2}\,i\sqrt{1-\frac{6}{x+3}}}{108-36x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)**(3/2)/(3+x)**(3/2),x)
```

```
[Out] Piecewise((sqrt(2)/(36*sqrt(-1 + 6/(x + 3))) - sqrt(2)/(12*sqrt(-1 + 6/(x +
 3))*(x + 3)), 6/Abs(x + 3) > 1), (sqrt(2)*I*sqrt(1 - 6/(x + 3))*(x + 3)/(1
08 - 36*x) - 3*sqrt(2)*I*sqrt(1 - 6/(x + 3))/(108 - 36*x), True))
```

## 3.1099 $\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=29

$$\frac{x}{18\sqrt{2}\,\sqrt{3-bx}\,\sqrt{bx+3}}$$

**Rubi [A]** time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {39}

$$\frac{x}{18\sqrt{2}\,\sqrt{3-bx}\,\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[2]*Sqrt[3 - b*x]*Sqrt[3 + b*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx = \frac{x}{18\sqrt{2}\,\sqrt{3-bx}\,\sqrt{3+bx}}$$

**Mathematica [A]** time = 0.02, size = 19, normalized size = 0.66

$$\frac{x}{18\sqrt{18-2b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[18 - 2*b^2*x^2])
```

**IntegrateAlgebraic [F]** time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)), x]
```

**fricas [A]** time = 1.21, size = 29, normalized size = 1.00

$$-\frac{\sqrt{bx+3}\,\sqrt{-2\,bx+6}\,x}{36\,\left(b^2x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(b*x + 3)*sqrt(-2*b*x + 6)*x/(b^2*x^2 - 9)
```

**giac [B]** time = 1.14, size = 91, normalized size = 3.14

$$\frac{\sqrt{2}\left(\sqrt{6}-\sqrt{-bx+3}\right)}{144\,\sqrt{bx+3}\,b}-\frac{\sqrt{2}\,\sqrt{bx+3}\,\sqrt{-bx+3}}{72\,(bx-3)b}-\frac{\sqrt{2}\,\sqrt{bx+3}}{144\,b\left(\sqrt{6}-\sqrt{-bx+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/144*sqrt(2)*(sqrt(6) - sqrt(-b*x + 3))/(sqrt(b*x + 3)*b) - 1/72*sqrt(2)*s
qrt(b*x + 3)*sqrt(-b*x + 3)/((b*x - 3)*b) - 1/144*sqrt(2)*sqrt(b*x + 3)/(b*
(sqrt(6) - sqrt(-b*x + 3)))
```

**maple [A]** time = 0.00, size = 24, normalized size = 0.83

$$-\frac{(bx-3)\,x}{9\sqrt{bx+3}\;(-2bx+6)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x)
```

```
[Out] -1/9*(b*x-3)/(b*x+3)^(1/2)*x/(-2*b*x+6)^(3/2)
```

**maxima [A]** time = 1.25, size = 15, normalized size = 0.52

$$\frac{x}{18\,\sqrt{-2\,b^2x^2+18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*x/sqrt(-2*b^2*x^2 + 18)
```

**mupad [B]** time = 0.32, size = 26, normalized size = 0.90

$$-\frac{x\,\sqrt{6-2\,b\,x}}{\sqrt{b\,x+3}\,\,\left(36\,b\,x-108\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 3)^(3/2)*(6 - 2*b*x)^(3/2)),x)
```

```
[Out] -(x*(6 - 2*b*x)^(1/2))/((b*x + 3)^(1/2)*(36*b*x - 108))
```

**sympy [C]** time = 31.50, size = 83, normalized size = 2.86

$$-\frac{\sqrt{2}\,iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2 \\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0 \end{matrix}\middle|\frac{9}{b^{2}x^{2}}\right)}{72\pi^{\frac{3}{2}}b}+\frac{\sqrt{2}\,G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0 \end{matrix}\middle|\frac{9e^{-2i\pi}}{b^{2}x^{2}}\right)}{72\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)**(3/2)/(b*x+3)**(3/2),x)
```

```
[Out] -sqrt(2)*I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2),
(0,)), 9/(b**2*x**2))/(72*pi**(3/2)*b) + sqrt(2)*meijerg(((-1/2, 0, 1/4, 1/
2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), 9*exp_polar(-2*I*pi)/(b**2*
x**2))/(72*pi**(3/2)*b)
```

## 3.1100 $\int \frac{1}{\sqrt{a+bx}\,\sqrt{-ad+bdx}}\,dx$

**Optimal**. Leaf size=39

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

**Rubi [A]** time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(a*d) + b*d*x]])/(b*Sqrt[d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{-2ad+dx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b\sqrt{d}}$$

**Mathematica [A]** time = 0.02, size = 39, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(a*d) + b*d*x]])/(b*Sqrt[d])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 39, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bdx-ad}}{\sqrt{d}\sqrt{a+bx}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
[Out] (2*ArcTanh[Sqrt[-(a*d) + b*d*x]/(Sqrt[d]*Sqrt[a + b*x])])/(b*Sqrt[d])
```

**fricas [A]** time = 1.27, size = 108, normalized size = 2.77

$$\left[\frac{\log\left(2\,b^2dx^2 + 2\sqrt{bdx-ad}\sqrt{bx+a}\,b\sqrt{d}\,x - a^2d\right)}{2\,b\sqrt{d}}, -\frac{\sqrt{-d}\,\arctan\left(\frac{\sqrt{bdx-ad}\sqrt{bx+a}\,b\sqrt{-d}\,x}{b^2dx^2-a^2d}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b^2*d*x^2 + 2*sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(d)*x - a^2*
d)/(b*sqrt(d)), -sqrt(-d)*arctan(sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(-d)
*x/(b^2*d*x^2 - a^2*d))/(b*d)]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.01, size = 76, normalized size = 1.95

$$\frac{\sqrt{(bx+a)\,(bdx-ad)}\;\ln\left(\frac{b^2dx}{\sqrt{b^2d}}+\sqrt{b^2d\,x^2-a^2d}\right)}{\sqrt{bx+a}\;\sqrt{bdx-ad}\;\sqrt{b^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x)
```

```
[Out] ((b*x+a)*(b*d*x-a*d))^(1/2)/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2)*ln(b^2*d*x/(b^2
*d)^(1/2)+(b^2*d*x^2-a^2*d)^(1/2))/(b^2*d)^(1/2)
```

**maxima [A]** time = 1.43, size = 39, normalized size = 1.00

$$\frac{\log\left(2\,b^2dx+2\,\sqrt{b^2dx^2-a^2d}\,b\sqrt{d}\right)}{b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*d*x + 2*sqrt(b^2*d*x^2 - a^2*d)*b*sqrt(d))/(b*sqrt(d))
```

**mupad [B]** time = 0.22, size = 56, normalized size = 1.44

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,d\,x-a\,d}-\sqrt{-a\,d}\right)}{\sqrt{-b^2\,d}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b^2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*d*x - a*d)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((b*d*x - a*d)^(1/2) - (-a*d)^(1/2)))/((-b^2*d)^(1/2)*((a + b*x
)^(1/2) - a^(1/2)))))/(-b^2*d)^(1/2)
```

**sympy [C]** time = 4.77, size = 88, normalized size = 2.26

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| \frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{d}} - \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| \frac{a^2e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(b*d*x-a*d)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), a**
2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(d)) - I*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(2*I*pi)/(b**2*x*
*2))/(4*pi**(3/2)*b*sqrt(d))
```

## 3.1101 $\int \frac{1}{\sqrt[4]{6-3ex}\,(2+ex)^{3/4}}\,dx$

**Optimal**. Leaf size=241

$$-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e}-\frac{\sqrt{2}}{}$$

**Rubi [A]** time = 0.25, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt[4]{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e)
 - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)
*e) - Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt
[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]/(Sqrt[2]*3^(1/4)*e) + Log[(Sqrt[6 - 3*e*x
] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2
 + e*x]]/(Sqrt[2]*3^(1/4)*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{6-3ex}\,(2+ex)^{3/4}}\,dx = -\frac{4\,\text{Subst}\left(\int \frac{x^2}{\left(4-\frac{x^4}{3}\right)^{3/4}}\,dx, x, \sqrt[4]{6-3ex}\right)}{3e}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}-\sqrt{2}\sqrt[4]{3}\,x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}+\sqrt{2}\sqrt[4]{3}\,x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e} -$$

$$= -\frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}\,e} + \frac{\log\left(\frac{\sqrt{2-ex}+\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}\,e} - \frac{\sqrt{2}\,\text{Su}}{}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}\,e} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}\,e} - \frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}}{\sqrt{2+ex}}\right.}{\sqrt{2}\sqrt[4]{3}\,e}$$

**Mathematica [C]** time = 0.02, size = 42, normalized size = 0.17

$$-\frac{\sqrt{2}\,(6-3ex)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{12}(6-3ex)\right)}{9e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

```
[Out] -1/9*(Sqrt[2]*(6 - 3*e*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (6 - 3*e*x
)/12])/e
```

**IntegrateAlgebraic [F]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{6-3ex}\,(2+ex)^{3/4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)), x]
```

**fricas [B]** time = 1.41, size = 505, normalized size = 2.10

$$2\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\frac{1}{e^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e^3\left(\frac{1}{e^4}\right)^{\frac{3}{4}}-\sqrt{3}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(e^4x-2e^3\right)\sqrt{\frac{\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e\left(\frac{1}{e^4}\right)^{\frac{1}{4}}+3\sqrt{\frac{1}{3}}\left(e^3x-2e^2\right)\sqrt{\frac{1}{e^4}}-\sqrt{ex+2}\sqrt{-3ex+6}}{ex-2}}\left(\frac{1}{e^4}\right)^{\frac{3}{4}}+ex-2}{ex-2}\right)+2\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\frac{1}{e^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e^3\left(\frac{1}{e^4}\right)^{\frac{3}{4}}-\sqrt{3}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(e^4x-2e^3\right)\sqrt{-\frac{\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e\left(\frac{1}{e^4}\right)^{\frac{1}{4}}-3\sqrt{\frac{1}{3}}\left(e^3x-2e^2\right)\sqrt{\frac{1}{e^4}}+\sqrt{ex+2}\sqrt{-3ex+6}}{ex-2}}\left(\frac{1}{e^4}\right)^{\frac{3}{4}}-ex+2}{ex-2}\right)-\frac{1}{2}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\frac{1}{e^4}\right)^{\frac{1}{4}}\log\left(\frac{3\left(\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e\left(\frac{1}{e^4}\right)^{\frac{1}{4}}+3\sqrt{\frac{1}{3}}\left(e^3x-2e^2\right)\sqrt{\frac{1}{e^4}}-\sqrt{ex+2}\sqrt{-3ex+6}\right)}{ex-2}\right)+\frac{1}{2}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\frac{1}{e^4}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e\left(\frac{1}{e^4}\right)^{\frac{1}{4}}-3\sqrt{\frac{1}{3}}\left(e^3x-2e^2\right)\sqrt{\frac{1}{e^4}}+\sqrt{ex+2}\sqrt{-3ex+6}\right)}{ex-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*arctan(-(sqrt(2)*(1/3)^(3/4)*(e*x + 2)
^(1/4)*(-3*e*x + 6)^(3/4)*e^3*(e^(-4))^(3/4) - sqrt(3)*sqrt(2)*(1/3)^(3/4)*
(e^4*x - 2*e^3)*sqrt((sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*(-3*e*x + 6)^(3/4
)*e*(e^(-4))^(1/4) + 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-4)) - sqrt(e*x +
2)*sqrt(-3*e*x + 6))/(e*x - 2))*(e^(-4))^(3/4) + e*x - 2)/(e*x - 2)) + 2*sq
rt(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*arctan(-(sqrt(2)*(1/3)^(3/4)*(e*x + 2)^(1/
4)*(-3*e*x + 6)^(3/4)*e^3*(e^(-4))^(3/4) - sqrt(3)*sqrt(2)*(1/3)^(3/4)*(e^4
*x - 2*e^3)*sqrt(-(sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*(-3*e*x + 6)^(3/4)*e
*(e^(-4))^(1/4) - 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-4)) + sqrt(e*x + 2)*
sqrt(-3*e*x + 6))/(e*x - 2))*(e^(-4))^(3/4) - e*x + 2)/(e*x - 2)) - 1/2*sqr
t(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*log(3*(sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*
(-3*e*x + 6)^(3/4)*e*(e^(-4))^(1/4) + 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-
4)) - sqrt(e*x + 2)*sqrt(-3*e*x + 6))/(e*x - 2)) + 1/2*sqrt(2)*(1/3)^(1/4)*
(e^(-4))^(1/4)*log(-3*(sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*(-3*e*x + 6)^(3/
4)*e*(e^(-4))^(1/4) - 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-4)) + sqrt(e*x +
 2)*sqrt(-3*e*x + 6))/(e*x - 2))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+2)^{\frac{3}{4}}(-3\,ex+6)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + 2)^(3/4)*(-3*e*x + 6)^(1/4)), x)
```

**maple [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(-3ex+6)^{\frac{1}{4}}\,(ex+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
[Out] int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+2)^{\frac{3}{4}}(-3\,ex+6)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((e*x + 2)^(3/4)*(-3*e*x + 6)^(1/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x+2)^{3/4}\,(6-3\,e\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x + 2)^(3/4)*(6 - 3*e*x)^(1/4)),x)
[Out] int(1/((e*x + 2)^(3/4)*(6 - 3*e*x)^(1/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{3^{\frac{3}{4}} \int \frac{1}{\sqrt[4]{-ex+2}\,(ex+2)^{\frac{3}{4}}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)**(1/4)/(e*x+2)**(3/4),x)
[Out] 3**(3/4)*Integral(1/((-e*x + 2)**(1/4)*(e*x + 2)**(3/4)), x)/3
```

## 3.1102 $\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}} \, dx$

**Optimal**. Leaf size=256

$$-\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+$$

**Rubi [A]** time = 0.17, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] ((-I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))/a - (I*ArcTan[1 - (Sqrt[2]*(a -
I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (I*ArcTan[1 + (Sqrt[2]*(a - I*a
*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - ((I/2)*Log[1 + Sqrt[a - I*a*x]/Sqr
t[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((
I/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/
(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + \frac{1}{2}a\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + 2i\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + 2i\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) + i\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-}}{\sqrt[4]{a+}}\right.$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + \frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) + \frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}}\right.$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}} + \frac{iS}{}$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - \frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} + \frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}\right.}{2\sqrt{2}}$$

**Mathematica [C]** time = 0.02, size = 70, normalized size = 0.27

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{5/4}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (((2*I)/5)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[1/4,
 5/4, 9/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

**IntegrateAlgebraic [A]** time = 0.47, size = 126, normalized size = 0.49

$$\frac{\sqrt[4]{-1}\,\sqrt[4]{x-i}\,\sqrt[4]{a-iax}\left(-(-1)^{3/4}(x-i)^{3/4}\sqrt[4]{x+i}+\sqrt[4]{-1}\,\tan^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)+\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)\right)}{\sqrt[4]{x+i}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] ((-1)^(1/4)*(-I + x)^(1/4)*(a - I*a*x)^(1/4)*(-((-1)^(3/4)*(-I + x)^(3/4)*(
I + x)^(1/4)) + (-1)^(1/4)*ArcTan[(I + x)^(1/4)/(-I + x)^(1/4)] + (-1)^(1/4
)*ArcTanh[(I + x)^(1/4)/(-I + x)^(1/4)]))/((I + x)^(1/4)*(a + I*a*x)^(1/4))
```

**fricas [A]** time = 1.49, size = 194, normalized size = 0.76

$$\frac{\sqrt{i}\,a\log\left(\frac{\sqrt{i}\,(ax-i\,a)+(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{x-i}\right)-\sqrt{i}\,a\log\left(-\frac{\sqrt{i}\,(ax-i\,a)-(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{x-i}\right)+\sqrt{-i}\,a\log\left(\frac{\sqrt{-i}\,(ax-i\,a)+(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{x-i}\right)-\sqrt{-i}\,a\log\left(-\frac{\sqrt{-i}\,(ax-i\,a)-(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{x-i}\right)-2i\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(I)*a*log((sqrt(I)*(a*x - I*a) + (I*a*x + a)^(3/4)*(-I*a*x + a)^(1
/4))/(x - I)) - sqrt(I)*a*log(-(sqrt(I)*(a*x - I*a) - (I*a*x + a)^(3/4)*(-I
*a*x + a)^(1/4))/(x - I)) + sqrt(-I)*a*log((sqrt(-I)*(a*x - I*a) + (I*a*x +
 a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(-I)*a*log(-(sqrt(-I)*(a*x - I
*a) - (I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 2*I*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(1/4))/a
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4), x)
```

**maple [C]** time = 2.28, size = 477, normalized size = 1.86

$$\frac{i\,(x-i)\,(x+i)\,(-(ix-1)\,a)^{\frac{1}{4}}}{(ix-1)\,((ix+1)\,a)^{\frac{1}{4}}}-\frac{\left(-\frac{\operatorname{RootOf}(\_Z^2-i)\ln\left(\frac{-x^3+(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}x^2\operatorname{RootOf}(\_Z^2-i)-2ix^2+2i(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}x\operatorname{RootOf}(\_Z^2-i)-i\sqrt{-x^4-2ix^3-2ix+1}\,x+x+i(-x^4-2ix^3-2ix+1)^{\frac{3}{4}}\operatorname{RootOf}(\_Z^2-i)-(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}\operatorname{RootOf}(\_Z^2-i)+\sqrt{-x^4-2ix^3-2ix+1}}{(ix-1)^2}\right)}{2}-\frac{i\operatorname{RootOf}(\_Z^2-i)\ln\left(\frac{-x^3+i(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}x^2\operatorname{RootOf}(\_Z^2-i)-2ix^2-2(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}x\operatorname{RootOf}(\_Z^2-i)+i\sqrt{-x^4-2ix^3-2ix+1}\,x+x+(-x^4-2ix^3-2ix+1)^{\frac{3}{4}}\operatorname{RootOf}(\_Z^2-i)-i(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}\operatorname{RootOf}(\_Z^2-i)-\sqrt{-x^4-2ix^3-2ix+1}}{(ix-1)^2}\right)}{2}\right)(-(ix-1)\,a)^{\frac{1}{4}}\left(-(ix-1)^3(ix+1)\right)^{\frac{1}{4}}}{(ix-1)\,((ix+1)\,a)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(1/4)/(I*a*x+a)^(1/4),x)
```

```
[Out] I*(x-I)*(x+I)*(-(I*x-1)*a)^(1/4)/(I*x-1)/((I*x+1)*a)^(1/4)-(-1/2*RootOf(_Z^
2-I)*ln((RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3-x^4)^(1/4)*x^2+I*RootOf(_Z^2-I)*(1
-2*I*x-2*I*x^3-x^4)^(3/4)-x^3+2*I*RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3-x^4)^(1/4
)*x-I*(1-2*I*x-2*I*x^3-x^4)^(1/2)*x-2*I*x^2-RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3
-x^4)^(1/4)+(1-2*I*x-2*I*x^3-x^4)^(1/2)+x)/(I*x-1)^2)-1/2*I*RootOf(_Z^2-I)*
ln((I*RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3-x^4)^(1/4)*x^2-2*RootOf(_Z^2-I)*(1-2*
I*x-2*I*x^3-x^4)^(1/4)*x-x^3+RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3-x^4)^(3/4)+I*(
1-2*I*x-2*I*x^3-x^4)^(1/2)*x-I*RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3-x^4)^(1/4)-2
*I*x^2-(1-2*I*x-2*I*x^3-x^4)^(1/2)+x)/(I*x-1)^2))*(-(I*x-1)*a)^(1/4)/(I*x-1
)*(-(I*x-1)^3*(I*x+1))^(1/4)/((I*x+1)*a)^(1/4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{1/4}}{(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(1/4),x)
```

```
[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(1/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{\sqrt[4]{ia\,(x-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(1/4), x)
```

## 3.1103 $\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$

**Optimal**. Leaf size=233

$$-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

**Rubi [A]** time = 0.13, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a -
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx = \frac{(4i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{(4i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\,\text{Subs}\cdots}{\cdots}$$

$$= -\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} + \frac{\left(i\sqrt{2}\right)\text{Subst}\cdots}{\cdots}$$

$$= -\frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\cdots\right.}{\sqrt{2}\,a}$$

**Mathematica [C]** time = 0.02, size = 68, normalized size = 0.29

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}\,\sqrt[4]{a-iax}\;{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((2*I)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4
, 5/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

**IntegrateAlgebraic [A]** time = 0.20, size = 83, normalized size = 0.36

$$\frac{2\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\sqrt[4]{-1}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{2(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(-1)^(1/4)*ArcTanh[((-1)^(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
- (2*(-1)^(3/4)*ArcTanh[((-1)^(3/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/
a
```

**fricas [A]** time = 1.50, size = 227, normalized size = 0.97

$$\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{(a^2x-ia^2)\sqrt{\frac{4i}{a^2}}+2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)-\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{(a^2x-ia^2)\sqrt{\frac{4i}{a^2}}-2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)+\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(\frac{(a^2x-ia^2)\sqrt{-\frac{4i}{a^2}}+2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)-\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{(a^2x-ia^2)\sqrt{-\frac{4i}{a^2}}-2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(3/4)*
(-I*a*x + a)^(1/4))/(2*x - 2*I)) - 1/2*sqrt(4*I/a^2)*log(-((a^2*x - I*a^2)*
sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) + 1/2*
sqrt(-4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-
I*a*x + a)^(1/4))/(2*x - 2*I)) - 1/2*sqrt(-4*I/a^2)*log(-((a^2*x - I*a^2)*s
qrt(-4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-i,[1,1]%%%}+%%%{-
1,[1,0]%%%}] at parameters values [-27,-87]ext_reduce Error: Bad Argument T
ypeintegrate((-4*i)/a/4*i*a*(-4*i)/a*((i*a*x+a)^(1/4))^2/((-((i*a*x+a)^(1/4
))^4+2*a)^(1/4))^3/4*i*a*((i*a*x+a)^(1/4))^-3,x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{3}{4}}\,(iax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(3/4)/(I*a*x+a)^(1/4),x)
```

```
[Out] int(1/(-I*a*x+a)^(3/4)/(I*a*x+a)^(1/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{3/4}\,(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(1/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,(-ia\,(x+i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(3/4)), x)
```

## 3.1104 $\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx$

**Optimal**. Leaf size=33

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

**Rubi** [A] time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {37}

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

**IntegrateAlgebraic** [A] time = 0.11, size = 33, normalized size = 1.00

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

**fricas** [A] time = 1.71, size = 32, normalized size = 0.97

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\,a^3x+3i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(3*a^3*x + 3*I*a^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(7/4)), x)
```

**maple** [A] time = 0.05, size = 31, normalized size = 0.94

$$\frac{\frac{2x}{3}-\frac{2i}{3}}{(-(ix-1)\,a)^{\frac{3}{4}}\,((ix+1)\,a)^{\frac{1}{4}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(7/4)/(I*a*x+a)^(1/4),x)
[Out] 2/3/a/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(x-I)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(7/4)), x)
```

**mupad [B]** time = 0.55, size = 38, normalized size = 1.15

$$-\frac{2\,\left(x-\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{3\,a^2\,\left(-1+x\,1\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] -(2*(x - 1i)*(-a*(x*1i - 1))^(1/4))/(3*a^2*(x*1i - 1)*(a*(x*1i + 1))^(1/4))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\left(x-i\right)}\,\left(-ia\left(x+i\right)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(7/4)), x)
```

## 3.1105 $\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx$

**Optimal**. Leaf size=67

$$-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}$$

**Rubi [A]** time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/7)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(7/4)) - (((4*I)/21)*(a + I
*a*x)^(3/4))/(a^3*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}} + \frac{2\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{7a}$$
$$= -\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}} - \frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}$$

**Mathematica** [A] time = 0.02, size = 45, normalized size = 0.67

$$\frac{2(5-2ix)(a+iax)^{3/4}}{21a^3(x+i)(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
[Out] (2*(5 - (2*I)*x)*(a + I*a*x)^(3/4))/(21*a^3*(I + x)*(a - I*a*x)^(3/4))
```

**IntegrateAlgebraic** [A] time = 0.12, size = 55, normalized size = 0.82

$$-\frac{i(a+iax)^{7/4}\left(3+\frac{7(a-iax)}{a+iax}\right)}{21a^3(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
[Out] ((-1/21*I)*(a + I*a*x)^(7/4)*(3 + (7*(a - I*a*x))/(a + I*a*x)))/(a^3*(a - I
*a*x)^(7/4))
```

**fricas** [A] time = 1.57, size = 44, normalized size = 0.66

$$\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(4\,x+10i)}{21\left(a^4x^2+2i\,a^4x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
[Out] 1/21*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(4*x + 10*I)/(a^4*x^2 + 2*I*a^4*x
 - a^4)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(11/4)), x)
```

**maple [A]** time = 0.05, size = 44, normalized size = 0.66

$$\frac{\frac{4}{21}x^2+\frac{2}{7}ix+\frac{10}{21}}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x+i\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(11/4)/(I*a*x+a)^(1/4),x)
[Out] 2/21/a^2/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(2*x^2+5+3*I*x)/(x+I)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(11/4)), x)
```

**mupad [B]** time = 0.67, size = 46, normalized size = 0.69

$$-\frac{\left(-a\,\left(-1+x\,1{}\mathrm{i}\right)\right)^{1/4}\,\left(2\,x^2+x\,3{}\mathrm{i}+5\right)\,2{}\mathrm{i}}{21\,a^3\,{\left(-1+x\,1{}\mathrm{i}\right)}^2\,{\left(a\,\left(1+x\,1{}\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(1/4)),x)
[Out] -((-a*(x*1i - 1))^(1/4)*(x*3i + 2*x^2 + 5)*2i)/(21*a^3*(x*1i - 1)^2*(a*(x*1
i + 1))^(1/4))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,(-ia\,(x+i))^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(11/4)), x)
```

## 3.1106 $\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx$

**Optimal**. Leaf size=100

$$-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}$$

**Rubi** [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/11)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(11/4)) - (((8*I)/77)*(a +
 I*a*x)^(3/4))/(a^3*(a - I*a*x)^(7/4)) - (((16*I)/231)*(a + I*a*x)^(3/4))/(
a^4*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}} + \frac{4\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx}{11a}$$

$$= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}} - \frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}} + \frac{8\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{77a^2}$$

$$= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}} - \frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}} - \frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}$$

**Mathematica [A]** time = 0.02, size = 52, normalized size = 0.52

$$\frac{2\left(-8ix^2+28x+41i\right)(a+iax)^{3/4}}{231a^4(x+i)^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(41*I + 28*x - (8*I)*x^2))/(231*a^4*(I + x)^2*(a - I*a
*x)^(3/4))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 77, normalized size = 0.77

$$-\frac{i(a+iax)^{11/4}\left(\frac{77(a-iax)^2}{(a+iax)^2}+\frac{66(a-iax)}{a+iax}+21\right)}{462a^4(a-iax)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-1/462*I)*(a + I*a*x)^(11/4)*(21 + (77*(a - I*a*x)^2)/(a + I*a*x)^2 + (66
*(a - I*a*x))/(a + I*a*x)))/(a^4*(a - I*a*x)^(11/4))
```

**fricas [A]** time = 1.42, size = 58, normalized size = 0.58

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}\left(8\,x^2+28i\,x-41\right)}{231\,a^5x^3+693i\,a^5x^2-693\,a^5x-231i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 28*I*x - 41)/(231*a^5*x^3 +
 693*I*a^5*x^2 - 693*a^5*x - 231*I*a^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{15}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(15/4)), x)
```

**maple [A]** time = 0.05, size = 50, normalized size = 0.50

$$\frac{\frac{16}{231}x^3+\frac{40}{231}ix^2-\frac{26}{231}x+\frac{82}{231}i}{(-(ix-1)\,a)^{\frac{3}{4}}\,((ix+1)\,a)^{\frac{1}{4}}\,(x+i)^2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(15/4)/(I*a*x+a)^(1/4),x)
```

```
[Out] 2/231/a^3/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(20*I*x^2+8*x^3-13*x+41*I)/(
x+I)^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{15}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(15/4)), x)
```

**mupad [B]** time = 0.75, size = 51, normalized size = 0.51

$$\frac{(x-\mathrm{i})^4\,(-a\,(-1+x\,1\mathrm{i}))^{1/4}\,\left(8\,x^2+x\,28\mathrm{i}-41\right)\,2\mathrm{i}}{231\,a^4\,\left(x^2+1\right)^3\,(a\,(1+x\,1\mathrm{i}))^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(15/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] ((x - 1i)^4*(-a*(x*1i - 1))^(1/4)*(x*28i + 8*x^2 - 41)*2i)/(231*a^4*(x^2 +
1)^3*(a*(x*1i + 1))^(1/4))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(15/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Timed out
```

## 3.1107 $\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx$

**Optimal**. Leaf size=133

$$-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

**Rubi** [A] time = 0.03, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/15)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(15/4)) - (((4*I)/55)*(a +
 I*a*x)^(3/4))/(a^3*(a - I*a*x)^(11/4)) - (((16*I)/385)*(a + I*a*x)^(3/4))/
(a^4*(a - I*a*x)^(7/4)) - (((32*I)/1155)*(a + I*a*x)^(3/4))/(a^5*(a - I*a*x
)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} + \frac{2\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx}{5a}$$

$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} + \frac{8\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx}{55a^2}$$

$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} - \frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}} + \frac{16\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,d}{385a^3}$$

$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} - \frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}} - \frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}$$

**Mathematica** [A] time = 0.03, size = 57, normalized size = 0.43

$$\frac{2\left(-16ix^3+72x^2+138ix-159\right)(a+iax)^{3/4}}{1155a^5(x+i)^3(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(-159 + (138*I)*x + 72*x^2 - (16*I)*x^3))/(1155*a^5*(I
 + x)^3*(a - I*a*x)^(3/4))
```

**IntegrateAlgebraic** [A] time = 0.12, size = 99, normalized size = 0.74

$$-\frac{i(a+iax)^{15/4}\left(\frac{385(a-iax)^3}{(a+iax)^3}+\frac{495(a-iax)^2}{(a+iax)^2}+\frac{315(a-iax)}{a+iax}+77\right)}{4620a^5(a-iax)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-1/4620*I)*(a + I*a*x)^(15/4)*(77 + (385*(a - I*a*x)^3)/(a + I*a*x)^3 + (
495*(a - I*a*x)^2)/(a + I*a*x)^2 + (315*(a - I*a*x))/(a + I*a*x)))/(a^5*(a
- I*a*x)^(15/4))
```

**fricas** [A] time = 1.47, size = 70, normalized size = 0.53

$$\frac{\left(32\,x^3+144i\,x^2-276\,x-318i\right)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{1155\,a^6x^4+4620i\,a^6x^3-6930\,a^6x^2-4620i\,a^6x+1155\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] (32*x^3 + 144*I*x^2 - 276*x - 318*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(
1155*a^6*x^4 + 4620*I*a^6*x^3 - 6930*a^6*x^2 - 4620*I*a^6*x + 1155*a^6)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{19}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(19/4)), x)
```

**maple [A]** time = 0.06, size = 55, normalized size = 0.41

$$\frac{\frac{32}{1155}x^4+\frac{16}{165}ix^3-\frac{4}{35}x^2-\frac{2}{55}ix-\frac{106}{385}}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x+i\right)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(19/4)/(I*a*x+a)^(1/4),x)
```

```
[Out] 2/1155/a^4/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(56*I*x^3+16*x^4-21*I*x-159
-66*x^2)/(x+I)^3
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{19}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(19/4)), x)
```

**mupad [B]** time = 0.79, size = 57, normalized size = 0.43

$$-\frac{(x-\mathrm{i})^5\,(-a\,(-1+x\,1\mathrm{i}))^{1/4}\,\left(-16\,x^3-x^2\,72\mathrm{i}+138\,x+159\mathrm{i}\right)\,2\mathrm{i}}{1155\,a^5\,\left(x^2+1\right)^4\,(a\,(1+x\,1\mathrm{i}))^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(19/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] -((x - 1i)^5*(-a*(x*1i - 1))^(1/4)*(138*x - x^2*72i - 16*x^3 + 159i)*2i)/(1
155*a^5*(x^2 + 1)^4*(a*(x*1i + 1))^(1/4))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(19/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Timed out
```

## 3.1108 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx$

**Optimal**. Leaf size=256

$$-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-}}{\sqrt[4]{a+ia}}\right.}{\sqrt{2}}$$

**Rubi [A]** time = 0.16, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-I)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(
a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((3*I)*ArcTan[1 + (Sqrt[2]*
(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((3*I)/2)*Log[1 + Sqrt[a
- I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/
Sqrt[2] - (((3*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a
- I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx = -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{1}{2}(3a)\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+6i\,\text{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx,x,\sqrt[4]{a-iax}\right)$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+6i\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-3i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+3i\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\right.$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3}{2}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+\frac{3}{2}i\,\text{Subst}\left(\int \frac{}{1+}\right.$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}-\frac{3i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right.}{2\sqrt{2}}$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\log\left(1+\right.}{}$$

**Mathematica [C]** time = 0.03, size = 70, normalized size = 0.27

$$\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{7/4}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] (((2*I)/7)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[3/4,
 7/4, 11/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

**IntegrateAlgebraic [A]** time = 0.60, size = 128, normalized size = 0.50

$$\frac{(-1)^{3/4}(x-i)^{3/4}(a-iax)^{3/4}\left(-\sqrt[4]{-1}\,\sqrt[4]{x-i}\,(x+i)^{3/4}+3(-1)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)-3(-1)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)\right)}{(x+i)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-1)^(3/4)*(-I + x)^(3/4)*(a - I*a*x)^(3/4)*(-((-1)^(1/4)*(-I + x)^(1/4)*(
I + x)^(3/4)) + 3*(-1)^(3/4)*ArcTan[(I + x)^(1/4)/(-I + x)^(1/4)] - 3*(-1)^
(3/4)*ArcTanh[(I + x)^(1/4)/(-I + x)^(1/4)]))/((I + x)^(3/4)*(a + I*a*x)^(3
/4))
```

**fricas [A]** time = 1.48, size = 204, normalized size = 0.80

$$\frac{\sqrt{9i}\,a\log\left(\frac{\sqrt{9i}\,(ax+ia)+3\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3x+3i}\right)-\sqrt{9i}\,a\log\left(-\frac{\sqrt{9i}\,(ax+ia)-3\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3x+3i}\right)+\sqrt{-9i}\,a\log\left(\frac{\sqrt{-9i}\,(ax+ia)+3\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3x+3i}\right)-\sqrt{-9i}\,a\log\left(-\frac{\sqrt{-9i}\,(ax+ia)-3\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3x+3i}\right)-2i\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(9*I)*a*log((sqrt(9*I)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*a*x +
 a)^(3/4))/(3*x + 3*I)) - sqrt(9*I)*a*log(-(sqrt(9*I)*(a*x + I*a) - 3*(I*a*
x + a)^(1/4)*(-I*a*x + a)^(3/4))/(3*x + 3*I)) + sqrt(-9*I)*a*log((sqrt(-9*I
)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(3*x + 3*I)) - sqrt
(-9*I)*a*log(-(sqrt(-9*I)*(a*x + I*a) - 3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3
/4))/(3*x + 3*I)) - 2*I*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/a
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(3/4), x)
```

**maple [C]** time = 2.14, size = 464, normalized size = 1.81

$$-\frac{i\,(x-i)\,(x+i)\,a}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}}+\frac{\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(-\frac{x^3-\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+i\right)-2ix^2+2i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^2+i\right)-i\sqrt{-x^4+2ix^3+2ix+1}\,x-x+i\left(-x^4+2ix^3+2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)+\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)-\sqrt{-x^4+2ix^3+2ix+1}}{(ix+1)^2}\right)}{2}-\frac{3i\,\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(-\frac{x^3-i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+i\right)-2ix^2-2\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^2+i\right)+i\sqrt{-x^4+2ix^3+2ix+1}\,x-x+\left(-x^4+2ix^3+2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)+i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)+\sqrt{-x^4+2ix^3+2ix+1}}{(ix+1)^2}\right)}{2}\right)\left(-(ix-1)\,(ix+1)^3\right)^{\frac{1}{4}}a}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(3/4)/(I*a*x+a)^(3/4),x)
```

```
[Out] -I*(x-I)*(x+I)/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*a+(-3/2*RootOf(_Z^2+I)*
ln(-(-RootOf(_Z^2+I)*(1+2*I*x+2*I*x^3-x^4)^(1/4)*x^2+x^3+I*RootOf(_Z^2+I)*(
1+2*I*x+2*I*x^3-x^4)^(3/4)+2*I*RootOf(_Z^2+I)*(1+2*I*x+2*I*x^3-x^4)^(1/4)*x
-I*(1+2*I*x+2*I*x^3-x^4)^(1/2)*x-2*I*x^2+(1+2*I*x+2*I*x^3-x^4)^(1/4)*RootOf
(_Z^2+I)-(1+2*I*x+2*I*x^3-x^4)^(1/2)-x)/(I*x+1)^2)-3/2*I*RootOf(_Z^2+I)*ln(
-(-I*(1+2*I*x+2*I*x^3-x^4)^(1/4)*RootOf(_Z^2+I)*x^2-2*RootOf(_Z^2+I)*(1+2*I
*x+2*I*x^3-x^4)^(1/4)*x+x^3+I*(1+2*I*x+2*I*x^3-x^4)^(1/2)*x+RootOf(_Z^2+I)*
(1+2*I*x+2*I*x^3-x^4)^(3/4)+I*RootOf(_Z^2+I)*(1+2*I*x+2*I*x^3-x^4)^(1/4)-2*
I*x^2+(1+2*I*x+2*I*x^3-x^4)^(1/2)-x)/(I*x+1)^2))/((I*x+1)*a)^(3/4)*(-(I*x-1
)*(I*x+1)^3)^(1/4)/(-(I*x-1)*a)^(1/4)*a
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(3/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(3/4),x)
```

```
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(3/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia\,(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(3/4), x)
```

## 3.1109 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$

**Optimal**. Leaf size=233

$$\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

**Rubi [A]** time = 0.14, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a +
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx = \frac{(4i)\,\text{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{(4i)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= -\frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\Big(}{}$$

$$= \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} + \frac{\left(i\sqrt{2}\right)\text{Subst}\Big(\int}{}$$

$$= -\frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\log\Big(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}}{\sqrt[4]{}}}{\sqrt{2}\,a}$$

**Mathematica [C]** time = 0.02, size = 70, normalized size = 0.30

$$\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((2*I)/3)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4,
 3/4, 7/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

**IntegrateAlgebraic [A]** time = 0.15, size = 83, normalized size = 0.36

$$\frac{2(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a} - \frac{2\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\sqrt[4]{-1}\,\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (-2*(-1)^(1/4)*ArcTanh[((-1)^(1/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)])/a
 + (2*(-1)^(3/4)*ArcTanh[((-1)^(3/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)])
/a
```

**fricas [A]** time = 1.41, size = 227, normalized size = 0.97

$$\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{(a^2x+ia^2)\sqrt{\frac{4i}{a^2}}+2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)-\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{(a^2x+ia^2)\sqrt{\frac{4i}{a^2}}-2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)+\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(\frac{(a^2x+ia^2)\sqrt{-\frac{4i}{a^2}}+2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)-\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{(a^2x+ia^2)\sqrt{-\frac{4i}{a^2}}-2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I/a^2)*log(((a^2*x + I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(1/4)*
(-I*a*x + a)^(3/4))/(2*x + 2*I)) - 1/2*sqrt(4*I/a^2)*log(-((a^2*x + I*a^2)*
sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I)) + 1/2*
sqrt(-4*I/a^2)*log(((a^2*x + I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(1/4)*(-
I*a*x + a)^(3/4))/(2*x + 2*I)) - 1/2*sqrt(-4*I/a^2)*log(-((a^2*x + I*a^2)*s
qrt(-4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate((-4*i)/a/4*i
*a*(-4*i)/a/(-((i*a*x+a)^(1/4))^4+2*a)^(1/4)/4*i*a*((i*a*x+a)^(1/4))^-3,x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{1}{4}}\,(iax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(1/4)/(I*a*x+a)^(3/4),x)
```

```
[Out] int(1/(-I*a*x+a)^(1/4)/(I*a*x+a)^(3/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(3/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,\sqrt[4]{-ia\,(x+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(1/4)), x)
```

## 3.1110 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx$

**Optimal**. Leaf size=31

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

**Rubi** [A] time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {37}

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

**Mathematica** [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 31, normalized size = 1.00

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

**fricas [A]** time = 1.42, size = 31, normalized size = 1.00

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{a^3x+i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(a^3*x + I*a^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(5/4)), x)
```

**maple [A]** time = 0.04, size = 31, normalized size = 1.00

$$\frac{2x-2i}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(5/4)/(I*a*x+a)^(3/4),x)
[Out] 2/a/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(x-I)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(5/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{5/4}\,(a + a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(3/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x - i))^{\frac{3}{4}}\,(-ia\,(x + i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(5/4)), x)
```

## 3.1111 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx$

**Optimal**. Leaf size=67

$$-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}$$

**Rubi [A]** time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/5)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(5/4)) - (((4*I)/5)*(a + I*
a*x)^(1/4))/(a^3*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}} + \frac{2\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx}{5a}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}} - \frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}$$

**Mathematica [A]** time = 0.02, size = 45, normalized size = 0.67

$$\frac{2(3-2ix)\sqrt[4]{a+iax}}{5a^3(x+i)\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (2*(3 - (2*I)*x)*(a + I*a*x)^(1/4))/(5*a^3*(I + x)*(a - I*a*x)^(1/4))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 54, normalized size = 0.81

$$-\frac{i\sqrt[4]{a+iax}\left(5+\frac{a+iax}{a-iax}\right)}{5a^3\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-1/5*I)*(a + I*a*x)^(1/4)*(5 + (a + I*a*x)/(a - I*a*x)))/(a^3*(a - I*a*x)
^(1/4))
```

**fricas [A]** time = 1.16, size = 44, normalized size = 0.66

$$\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(4\,x+6i)}{5\left(a^4x^2+2i\,a^4x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/5*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(4*x + 6*I)/(a^4*x^2 + 2*I*a^4*x -
 a^4)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(9/4)), x)
```

**maple [A]** time = 0.05, size = 44, normalized size = 0.66

$$\frac{\frac{4}{5}x^2+\frac{2}{5}ix+\frac{6}{5}}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}\,(x+i)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(9/4)/(I*a*x+a)^(3/4),x)
[Out] 2/5/a^2/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(2*x^2+3+I*x)/(x+I)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(9/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{9/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(3/4)),x)
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(3/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,(-ia\,(x+i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(9/4)), x)
```

## 3.1112 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx$

**Optimal**. Leaf size=100

$$-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}$$

**Rubi [A]** time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/9)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(9/4)) - (((8*I)/45)*(a + I
*a*x)^(1/4))/(a^3*(a - I*a*x)^(5/4)) - (((16*I)/45)*(a + I*a*x)^(1/4))/(a^4
*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}} + \frac{4\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx}{9a}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}} - \frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}} + \frac{8\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx}{45a^2}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}} - \frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}} - \frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}$$

**Mathematica** [A] time = 0.02, size = 52, normalized size = 0.52

$$\frac{2\left(-8ix^2+20x+17i\right)\sqrt[4]{a+iax}}{45a^4(x+i)^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (2*(a + I*a*x)^(1/4)*(17*I + 20*x - (8*I)*x^2))/(45*a^4*(I + x)^2*(a - I*a*
x)^(1/4))
```

**IntegrateAlgebraic** [A] time = 0.12, size = 77, normalized size = 0.77

$$-\frac{i\sqrt[4]{a+iax}\left(\frac{5(a+iax)^2}{(a-iax)^2}+\frac{18(a+iax)}{a-iax}+45\right)}{90a^4\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-1/90*I)*(a + I*a*x)^(1/4)*(45 + (18*(a + I*a*x))/(a - I*a*x) + (5*(a + I
*a*x)^2)/(a - I*a*x)^2))/(a^4*(a - I*a*x)^(1/4))
```

**fricas** [A] time = 1.40, size = 58, normalized size = 0.58

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}\left(8\,x^2+20i\,x-17\right)}{45\,a^5x^3+135i\,a^5x^2-135\,a^5x-45i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 20*I*x - 17)/(45*a^5*x^3 +
135*I*a^5*x^2 - 135*a^5*x - 45*I*a^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(13/4)), x)
```

**maple [A]** time = 0.06, size = 50, normalized size = 0.50

$$\frac{\frac{16}{45}x^3+\frac{8}{15}ix^2+\frac{2}{15}x+\frac{34}{45}i}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}\,(x+i)^2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(13/4)/(I*a*x+a)^(3/4),x)
```

```
[Out] 2/45/a^3/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(12*I*x^2+8*x^3+3*x+17*I)/(x+
I)^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(13/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{13/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(3/4)), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Timed out
```

## 3.1113 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx$

**Optimal**. Leaf size=291

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i\sqrt[4]{a+iax}\,(a-iax)^{3/4}}{3a}-\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\tan}{}$$

**Rubi [A]** time = 0.18, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {47, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i\sqrt[4]{a+iax}\,(a-iax)^{3/4}}{3a}-\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{7i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(3/4)) + (((7*I)/3)*(a - I*a*x
)^(3/4)*(a + I*a*x)^(1/4))/a + ((7*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4)
)/(a + I*a*x)^(1/4)])/Sqrt[2] - ((7*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((7*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a
+ I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((7*I
)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/
(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} - \frac{7}{3}\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{1}{2}(7a)\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - 14i\,\text{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - 14i\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} + 7i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - 7i\,\text{Subst}\Bigg(\int$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{7}{2}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - \frac{7}{2}i$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{7i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}} + \frac{7i\log\left(1+\frac{\sqrt{a-}}{\sqrt{a+}}\right.}{2}$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} + \frac{7i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \frac{7i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right.}{\sqrt{2}}$$

**Mathematica [C]** time = 0.03, size = 70, normalized size = 0.24

$$\frac{i\sqrt[4]{2}(1+ix)^{3/4}(a-iax)^{11/4}\,{}_2F_1\left(\frac{7}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/11)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[7/4, 1
1/4, 15/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

**IntegrateAlgebraic [A]** time = 0.90, size = 150, normalized size = 0.52

$$-\frac{(-1)^{3/4}(x-i)^{7/4}(a-iax)^{7/4}\left(\frac{3\sqrt[4]{-1}\,(x+i)^{7/4}-14(-1)^{3/4}(x+i)^{3/4}}{3(x-i)^{3/4}}-7(-1)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)+7(-1)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)\right)}{(x+i)^{7/4}(a+iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] -(((-1)^(3/4)*(-I + x)^(7/4)*(a - I*a*x)^(7/4)*((-14*(-1)^(3/4)*(I + x)^(3/
4) + 3*(-1)^(1/4)*(I + x)^(7/4))/(3*(-I + x)^(3/4)) - 7*(-1)^(3/4)*ArcTan[(
I + x)^(1/4)/(-I + x)^(1/4)] + 7*(-1)^(3/4)*ArcTanh[(I + x)^(1/4)/(-I + x)^
(1/4)]))/((I + x)^(7/4)*(a + I*a*x)^(7/4)))
```

**fricas [A]** time = 1.43, size = 244, normalized size = 0.84

$$-\frac{\sqrt{49i}\,(3ax-3ia)\log\left(\frac{\sqrt{49i}\,(ax+ia)+7(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7x+7i}\right)-\sqrt{49i}\,(3ax-3ia)\log\left(-\frac{\sqrt{49i}\,(ax+ia)-7(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7x+7i}\right)+\sqrt{-49i}\,(3ax-3ia)\log\left(\frac{\sqrt{-49i}\,(ax+ia)+7(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7x+7i}\right)-\sqrt{-49i}\,(3ax-3ia)\log\left(-\frac{\sqrt{-49i}\,(ax+ia)-7(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7x+7i}\right)+2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}(-3ix-11)}{6ax-6ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] -(sqrt(49*I)*(3*a*x - 3*I*a)*log((sqrt(49*I)*(a*x + I*a) + 7*(I*a*x + a)^(1
/4)*(-I*a*x + a)^(3/4))/(7*x + 7*I)) - sqrt(49*I)*(3*a*x - 3*I*a)*log(-(sqr
t(49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(7*x + 7*I))
+ sqrt(-49*I)*(3*a*x - 3*I*a)*log((sqrt(-49*I)*(a*x + I*a) + 7*(I*a*x + a)^
(1/4)*(-I*a*x + a)^(3/4))/(7*x + 7*I)) - sqrt(-49*I)*(3*a*x - 3*I*a)*log(-(
sqrt(-49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(7*x + 7*
I)) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(-3*I*x - 11))/(6*a*x - 6*I*a)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-2,[1,0]%%%}+%%%{1
,[0,4]%%%}] at parameters values [-64,-30]Warning, choosing root of [1,0,0,
0,%%%{-2,[1,0]%%%}+%%%{1,[0,4]%%%}] at parameters values [70,22]Warning, ch
oosing root of [1,0,0,0,%%%{-2,[1,0]%%%}+%%%{1,[0,4]%%%}] at parameters val
ues [42,56]Warning, choosing root of [1,0,0,0,%%%{-2,[1,0]%%%}+%%%{1,[0,4]%
%%}] at parameters values [-9,-13]Warning, choosing root of [1,0,0,0,%%%{-2
,[1,0]%%%}+%%%{1,[0,4]%%%}] at parameters values [46,24]ext_reduce Error: B
ad Argument TypeEvaluation time: 0.61integrate(i/4*a/a^2*(16*((i*a*x+a)^(1/
4))^4*((-((i*a*x+a)^(1/4))^4+2*a)^(1/4))^3-32*a*((-((i*a*x+a)^(1/4))^4+2*a)
^(1/4))^3)/((i*a*x+a)^(1/4))^4/4*i*a*((i*a*x+a)^(1/4))^-3,x)
```

**maple [C]** time = 2.01, size = 469, normalized size = 1.61

$$\frac{i\left(3x^{2}-8ix+11\right)a}{3\left((ix+1)a\right)^{\frac{3}{4}}\left(-(ix-1)a\right)^{\frac{1}{4}}}+\frac{\left(-\frac{7\,\mathrm{RootOf}\left(\_Z^{2}-i\right)\ln\left(\frac{-x^{3}-\left(-x^{4}+2ix^{3}+2ix+1\right)^{\frac{1}{4}}x^{2}\,\mathrm{RootOf}\left(\_Z^{2}-i\right)+2ix^{2}+2i\left(-x^{4}+2ix^{3}+2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^{2}-i\right)-i\sqrt{-x^{4}+2ix^{3}+2ix+1}\,x+x-i\left(-x^{4}+2ix^{3}+2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)+\left(-x^{4}+2ix^{3}+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)-\sqrt{-x^{4}+2ix^{3}+2ix+1}}{(ix+1)^{2}}\right)}{2}+\frac{7i\,\mathrm{RootOf}\left(\_Z^{2}-i\right)\ln\left(-\frac{x^{3}-i\left(-x^{4}+2ix^{3}+2ix+1\right)^{\frac{1}{4}}x^{2}\,\mathrm{RootOf}\left(\_Z^{2}-i\right)-2ix^{2}-2\left(-x^{4}+2ix^{3}+2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^{2}-i\right)-i\sqrt{-x^{4}+2ix^{3}+2ix+1}\,x-x-\left(-x^{4}+2ix^{3}+2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)+i\left(-x^{4}+2ix^{3}+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)-\sqrt{-x^{4}+2ix^{3}+2ix+1}}{(ix+1)^{2}}\right)}{2}\right)\left(-(ix-1)(ix+1)^{3}\right)^{\frac{1}{4}}a}{\left((ix+1)a\right)^{\frac{3}{4}}\left(-(ix-1)a\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(7/4)/(I*a*x+a)^(7/4),x)
```

```
[Out] 1/3*I*(-8*I*x+3*x^2+11)/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*a+(-7/2*RootOf
(_Z^2-I)*ln((-(-x^4+2*I*x^3+2*I*x+1)^(1/4)*RootOf(_Z^2-I)*x^2-I*RootOf(_Z^2
-I)*(-x^4+2*I*x^3+2*I*x+1)^(3/4)-x^3+2*I*RootOf(_Z^2-I)*(-x^4+2*I*x^3+2*I*x
+1)^(1/4)*x-I*(-x^4+2*I*x^3+2*I*x+1)^(1/2)*x+2*I*x^2+RootOf(_Z^2-I)*(-x^4+2
*I*x^3+2*I*x+1)^(1/4)-(-x^4+2*I*x^3+2*I*x+1)^(1/2)+x)/(I*x+1)^2)+7/2*I*Root
Of(_Z^2-I)*ln(-(-I*(-x^4+2*I*x^3+2*I*x+1)^(1/4)*RootOf(_Z^2-I)*x^2-2*RootOf
(_Z^2-I)*(-x^4+2*I*x^3+2*I*x+1)^(1/4)*x+x^3-RootOf(_Z^2-I)*(-x^4+2*I*x^3+2*
I*x+1)^(3/4)-I*(-x^4+2*I*x^3+2*I*x+1)^(1/2)*x+I*RootOf(_Z^2-I)*(-x^4+2*I*x^
3+2*I*x+1)^(1/4)-2*I*x^2-(-x^4+2*I*x^3+2*I*x+1)^(1/2)-x)/(I*x+1)^2))/((I*x+
1)*a)^(3/4)*(-(I*x-1)*(I*x+1)^3)^(1/4)/(-(I*x-1)*a)^(1/4)*a
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{7}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(7/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a - a\,x\,1\mathrm{i})^{7/4}}{(a + a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(7/4),x)
```

```
[Out] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(7/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia\,(x+i))^{\frac{7}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(7/4), x)
```

## 3.1114 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}} dx$

**Optimal**. Leaf size=266

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\cdots}{}$$

**Rubi [A]** time = 0.14, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {47, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(3/4)) + (I*Sqrt[2]*ArcTan[1 -
 (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 +
(Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a
*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt
[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^
(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{(4i)\,\text{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{(4i)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} + \frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \cdots$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} - \frac{\left(i\sqrt{2}\right)\text{Su}\cdots}{\cdots}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} + \frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}\cdots\right.}{\sqrt{2}\,a\cdots}$$

**Mathematica [C]** time = 0.02, size = 70, normalized size = 0.26

$$\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{7/4}\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/7)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[7/4, 7/4
, 11/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

**IntegrateAlgebraic [A]** time = 0.19, size = 116, normalized size = 0.44

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}+\frac{2\sqrt[4]{-1}\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a}-\frac{2(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(3/4)) + (2*(-1)^(1/4)*ArcTanh
[((-1)^(1/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)])/a - (2*(-1)^(3/4)*ArcTa
nh[((-1)^(3/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)])/a
```

**fricas [A]** time = 0.84, size = 307, normalized size = 1.15

$$-\frac{\left(3a^2x-3ia^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+ia^2\right)\sqrt{\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2x+2i}\right)-\left(3a^2x-3ia^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+ia^2\right)\sqrt{\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2x+2i}\right)+\left(3a^2x-3ia^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+ia^2\right)\sqrt{-\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2x+2i}\right)-\left(3a^2x-3ia^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+ia^2\right)\sqrt{-\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2x+2i}\right)-8\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{6a^2x-6ia^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] -((3*a^2*x - 3*I*a^2)*sqrt(4*I/a^2)*log(((a^2*x + I*a^2)*sqrt(4*I/a^2) + 2*
(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I)) - (3*a^2*x - 3*I*a^2)*sq
rt(4*I/a^2)*log(-((a^2*x + I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a
*x + a)^(3/4))/(2*x + 2*I)) + (3*a^2*x - 3*I*a^2)*sqrt(-4*I/a^2)*log(((a^2*
x + I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x +
2*I)) - (3*a^2*x - 3*I*a^2)*sqrt(-4*I/a^2)*log(-((a^2*x + I*a^2)*sqrt(-4*I/
a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I)) - 8*(I*a*x + a)
^(1/4)*(-I*a*x + a)^(3/4))/(6*a^2*x - 6*I*a^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-i\,ax+a\right)^{\frac{3}{4}}}{\left(i\,ax+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(7/4), x)
```

**maple [C]** time = 1.88, size = 459, normalized size = 1.73

$$\frac{\frac{4x}{3}+\frac{4i}{3}}{\left((ix+1)a\right)^{\frac{3}{4}}\left(-(ix-1)a\right)^{\frac{1}{4}}}-\frac{\left(i\operatorname{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-x^3-i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x^2\operatorname{RootOf}\left(\_Z^2+i\right)+2ix^2-2\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x\operatorname{RootOf}\left(\_Z^2+i\right)-i\sqrt{-x^4+2ix^3+2ix+1}\,x+x+\left(-x^4+2ix^3+2ix+1\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^2+i\right)+i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+i\right)-\sqrt{-x^4+2ix^3+2ix+1}}{(ix+1)^2}\right)-\operatorname{RootOf}\left(\_Z^2+i\right)\ln\left(-\frac{x^3-\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x^2\operatorname{RootOf}\left(\_Z^2+i\right)-2ix^2+2i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x\operatorname{RootOf}\left(\_Z^2+i\right)-i\sqrt{-x^4+2ix^3+2ix+1}\,x-x+i\left(-x^4+2ix^3+2ix+1\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^2+i\right)+\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+i\right)-\sqrt{-x^4+2ix^3+2ix+1}}{(ix+1)^2}\right)\right)\left(-(ix-1)(ix+1)^3\right)^{\frac{1}{4}}}{\left((ix+1)a\right)^{\frac{3}{4}}\left(-(ix-1)a\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(3/4)/(I*a*x+a)^(7/4),x)
```

```
[Out] 4/3*(x+I)/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)-(-RootOf(_Z^2+I)*ln(-(-RootO
f(_Z^2+I)*(-x^4+2*I*x^3+2*I*x+1)^(1/4)*x^2+x^3+I*RootOf(_Z^2+I)*(-x^4+2*I*x
^3+2*I*x+1)^(3/4)+2*I*RootOf(_Z^2+I)*(-x^4+2*I*x^3+2*I*x+1)^(1/4)*x-I*(-x^4
+2*I*x^3+2*I*x+1)^(1/2)*x-2*I*x^2+(-x^4+2*I*x^3+2*I*x+1)^(1/4)*RootOf(_Z^2+
I)-(-x^4+2*I*x^3+2*I*x+1)^(1/2)-x)/(I*x+1)^2)+I*RootOf(_Z^2+I)*ln((-I*(-x^4
+2*I*x^3+2*I*x+1)^(1/4)*RootOf(_Z^2+I)*x^2-2*RootOf(_Z^2+I)*(-x^4+2*I*x^3+2
*I*x+1)^(1/4)*x-x^3-I*(-x^4+2*I*x^3+2*I*x+1)^(1/2)*x+RootOf(_Z^2+I)*(-x^4+2
*I*x^3+2*I*x+1)^(3/4)+I*RootOf(_Z^2+I)*(-x^4+2*I*x^3+2*I*x+1)^(1/4)+2*I*x^2
-(-x^4+2*I*x^3+2*I*x+1)^(1/2)+x)/(I*x+1)^2))/((I*x+1)*a)^(3/4)*(-(I*x-1)*(I
*x+1)^3)^(1/4)/(-(I*x-1)*a)^(1/4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(7/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(7/4),x)
```

```
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(7/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia\,(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(7/4), x)
```

## 3.1115 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx$

**Optimal**. Leaf size=33

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

**Rubi [A]** time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {37}

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx = \frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

**fricas [A]** time = 1.51, size = 32, normalized size = 0.97

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\,a^3x-3i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(3*a^3*x - 3*I*a^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(1/4)), x)
```

**maple [A]** time = 0.04, size = 31, normalized size = 0.94

$$\frac{\frac{2x}{3}+\frac{2i}{3}}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(1/4)/(I*a*x+a)^(7/4),x)
[Out] 2/3/a/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(x+I)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(1/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{1/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(7/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(i a \left(x - i\right)\right)^{\frac{7}{4}} \sqrt[4]{- i a \left(x + i\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(1/4)), x)
```

## 3.1116 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx$

**Optimal**. Leaf size=65

$$\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}-\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}$$

**Rubi [A]** time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}-\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (-2*I)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((4*I)/3)*(a - I*a*x)^(
3/4))/(a^3*(a + I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{2\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx}{a}$$
$$= -\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}$$

**Mathematica** [A] time = 0.02, size = 38, normalized size = 0.58

$$\frac{4x-2i}{3a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
[Out] (-2*I + 4*x)/(3*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))
```

**IntegrateAlgebraic** [A] time = 0.10, size = 55, normalized size = 0.85

$$-\frac{i(a-iax)^{3/4}\left(-1+\frac{3(a+iax)}{a-iax}\right)}{3a^3(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
[Out] ((-1/3*I)*(a - I*a*x)^(3/4)*(-1 + (3*(a + I*a*x))/(a - I*a*x)))/(a^3*(a + I
*a*x)^(3/4))
```

**fricas** [A] time = 1.10, size = 36, normalized size = 0.55

$$\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(4\,x-2i)}{3\left(a^4x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
[Out] 1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(4*x - 2*I)/(a^4*x^2 + a^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(5/4)), x)
```

**maple [A]** time = 0.05, size = 33, normalized size = 0.51

$$\frac{\frac{4x}{3}-\frac{2i}{3}}{\left(\left(ix+1\right)a\right)^{\frac{3}{4}}\left(-\left(ix-1\right)a\right)^{\frac{1}{4}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(5/4)/(I*a*x+a)^(7/4),x)
```

```
[Out] 2/3/a^2/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(2*x-I)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(i\,ax+a\right)^{\frac{7}{4}}\left(-i\,ax+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(5/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(7/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{7}{4}}\left(-ia\left(x+i\right)\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(5/4)), x)
```

## 3.1117 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx$

**Optimal**. Leaf size=100

$$\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}-\frac{8i}{5a^3(a+iax)^{3/4}\sqrt[4]{a-iax}}-\frac{2i}{5a^2(a+iax)^{3/4}(a-iax)^{5/4}}$$

**Rubi** [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}-\frac{8i}{5a^3(a+iax)^{3/4}\sqrt[4]{a-iax}}-\frac{2i}{5a^2(a+iax)^{3/4}(a-iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)) - ((8*I)/5)/(a^3*(a -
I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((16*I)/15)*(a - I*a*x)^(3/4))/(a^4*(a +
 I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}} + \frac{4\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx}{5a}$$

$$= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}} - \frac{8i}{5a^3\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{8\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx}{5a^2}$$

$$= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}} - \frac{8i}{5a^3\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}$$

**Mathematica** [A] time = 0.02, size = 50, normalized size = 0.50

$$\frac{2\left(8x^2+4ix+7\right)}{15a^3(x+i)\sqrt[4]{a-iax}\,(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (2*(7 + (4*I)*x + 8*x^2))/(15*a^3*(I + x)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/
4))
```

**IntegrateAlgebraic** [A] time = 0.11, size = 77, normalized size = 0.77

$$-\frac{i(a-iax)^{3/4}\left(\frac{3(a+iax)^2}{(a-iax)^2}+\frac{30(a+iax)}{a-iax}-5\right)}{30a^4(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-1/30*I)*(a - I*a*x)^(3/4)*(-5 + (30*(a + I*a*x))/(a - I*a*x) + (3*(a + I
*a*x)^2)/(a - I*a*x)^2))/(a^4*(a + I*a*x)^(3/4))
```

**fricas** [A] time = 1.51, size = 58, normalized size = 0.58

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}\left(8\,x^2+4i\,x+7\right)}{15\,a^5x^3+15i\,a^5x^2+15\,a^5x+15i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 4*I*x + 7)/(15*a^5*x^3 + 15*I*a^5*x^2 + 15*a^5*x + 15*I*a^5)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="giac")

[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(9/4)), x)

**maple [A]** time = 0.06, size = 44, normalized size = 0.44

$$\frac{\frac{16}{15}x^2+\frac{8}{15}ix+\frac{14}{15}}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}\,(x+i)\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-I*a*x+a)^(9/4)/(I*a*x+a)^(7/4),x)

[Out] 2/15/a^3/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(8*x^2+4*I*x+7)/(x+I)

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{9/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(7/4)),x)

[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(7/4)), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{7}{4}}\left(-ia\left(x+i\right)\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(9/4)), x)
```

# 3.1118 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx$

**Optimal**. Leaf size=287

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}+\frac{5i(a+iax)^{3/4}\sqrt[4]{a-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{5i\tan}{}$$

**Rubi** [A] time = 0.18, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {47, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}+\frac{5i(a+iax)^{3/4}\sqrt[4]{a-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{5i\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((4*I)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(1/4)) + ((5*I)*(a - I*a*x)^(1/4)*
(a + I*a*x)^(3/4))/a + ((5*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I
*a*x)^(1/4)])/Sqrt[2] - ((5*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a +
I*a*x)^(1/4)])/Sqrt[2] + (((5*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((5*I)/2)*Log
[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a
*x)^(1/4)])/Sqrt[2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx = \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} - 5\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - \frac{1}{2}(5a)\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 10i\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 10i\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 5i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - 5i\,\text{Subst}\Bigg($$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - \frac{5}{2}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - \frac{5}{2}$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + \frac{5i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}} - \frac{5i\log\Bigg(1+\frac{\sqrt{a-}}{\sqrt{a-}}}{2}$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + \frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \frac{5i\tan^{-1}\Bigg(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}}{\sqrt{2}}$$

**Mathematica [C]** time = 0.03, size = 70, normalized size = 0.24

$$\frac{i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{9/4}\,{}_2F_1\left(\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((I/9)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[5/4, 9/4
, 13/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

**IntegrateAlgebraic [A]** time = 0.71, size = 146, normalized size = 0.51

$$-\frac{\sqrt[4]{-1}\,(x-i)^{5/4}(a-iax)^{5/4}\left(\frac{(-1)^{3/4}(x+i)^{5/4}+10\sqrt[4]{-1}\,\sqrt[4]{x+i}}{\sqrt[4]{x-i}}-5\sqrt[4]{-1}\,\tan^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)-5\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)\right)}{(x+i)^{5/4}(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] -(((-1)^(1/4)*(-I + x)^(5/4)*(a - I*a*x)^(5/4)*((10*(-1)^(1/4)*(I + x)^(1/4
) + (-1)^(3/4)*(I + x)^(5/4))/(-I + x)^(1/4) - 5*(-1)^(1/4)*ArcTan[(I + x)^
(1/4)/(-I + x)^(1/4)] - 5*(-1)^(1/4)*ArcTanh[(I + x)^(1/4)/(-I + x)^(1/4)])
)/((I + x)^(5/4)*(a + I*a*x)^(5/4)))
```

**fricas [A]** time = 1.28, size = 240, normalized size = 0.84

$$-\frac{\sqrt{25i}\,(ax-ia)\log\left(\frac{\sqrt{25i}\,(ax-ia)+5\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{5x-5i}\right)-\sqrt{25i}\,(ax-ia)\log\left(-\frac{\sqrt{25i}\,(ax-ia)-5\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{5x-5i}\right)+\sqrt{-25i}\,(ax-ia)\log\left(\frac{\sqrt{-25i}\,(ax-ia)+5\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{5x-5i}\right)-\sqrt{-25i}\,(ax-ia)\log\left(-\frac{\sqrt{-25i}\,(ax-ia)-5\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{5x-5i}\right)+2\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}(-ix-9)}{2\,ax-2i\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] -(sqrt(25*I)*(a*x - I*a)*log((sqrt(25*I)*(a*x - I*a) + 5*(I*a*x + a)^(3/4)*
(-I*a*x + a)^(1/4))/(5*x - 5*I)) - sqrt(25*I)*(a*x - I*a)*log(-(sqrt(25*I)*
(a*x - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(5*x - 5*I)) + sqrt(-
25*I)*(a*x - I*a)*log((sqrt(-25*I)*(a*x - I*a) + 5*(I*a*x + a)^(3/4)*(-I*a*
x + a)^(1/4))/(5*x - 5*I)) - sqrt(-25*I)*(a*x - I*a)*log(-(sqrt(-25*I)*(a*x
 - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(5*x - 5*I)) + 2*(I*a*x +
 a)^(3/4)*(-I*a*x + a)^(1/4)*(-I*x - 9))/(2*a*x - 2*I*a)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(i/4*a/a^2*(16*((i*a*x+a)^(1/4))^4*(-((i*a*x+a)^(1/4))^4+2*a)^(1/4)-32*a*(-((i*a*x+a)^(1/4))^4+2*a)^(1/4))/((i*a*x+a)^(1/4))^2/4*i*a*((i*a*x+a)^(1/4))^-3,x)

**maple [C]** time = 2.04, size = 481, normalized size = 1.68

$$-\frac{i\left(x^{2}-8ix+9\right)\left(-\left(ix-1\right)a\right)^{\frac{1}{4}}}{\left(ix-1\right)\left(\left(ix+1\right)a\right)^{\frac{1}{4}}}-\frac{\left(\frac{5i\,\mathrm{RootOf}\left(\_Z^{2}-i\right)\ln\left(\frac{-x^{3}+i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x^{2}\mathrm{RootOf}\left(\_Z^{2}-i\right)-2ix^{2}-2\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^{2}-i\right)+i\sqrt{-x^{4}-2ix^{3}-2ix+1}\,x+x+\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)-i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)-\sqrt{-x^{4}-2ix^{3}-2ix+1}}{\left(ix-1\right)^{2}}\right)}{2}-\frac{5\,\mathrm{RootOf}\left(\_Z^{2}-i\right)\ln\left(-\frac{x^{3}+\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x^{2}\mathrm{RootOf}\left(\_Z^{2}-i\right)+2ix^{2}+2i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^{2}-i\right)+i\sqrt{-x^{4}-2ix^{3}-2ix+1}\,x-x+i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)-\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)-\sqrt{-x^{4}-2ix^{3}-2ix+1}}{\left(ix-1\right)^{2}}\right)}{2}\right)\left(-\left(ix-1\right)a\right)^{\frac{1}{4}}\left(-\left(ix-1\right)^{3}\left(ix+1\right)\right)^{\frac{1}{4}}}{\left(ix-1\right)\left(\left(ix+1\right)a\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-I*a*x+a)^(5/4)/(I*a*x+a)^(5/4),x)

[Out] -I*(x^2+9-8*I*x)*(-(I*x-1)*a)^(1/4)/(I*x-1)/((I*x+1)*a)^(1/4)-(-5/2*RootOf(_Z^2-I)*ln(-(RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2+I*RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x+1)^(3/4)+x^3+2*I*RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x+I*(-x^4-2*I*x^3-2*I*x+1)^(1/2)*x+2*I*x^2-RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)-(-x^4-2*I*x^3-2*I*x+1)^(1/2)-x)/(I*x-1)^2)+5/2*I*RootOf(_Z^2-I)*ln((I*RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2-2*RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x-x^3+RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x+1)^(3/4)+I*(-x^4-2*I*x^3-2*I*x+1)^(1/2)*x-I*RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)-2*I*x^2-(-x^4-2*I*x^3-2*I*x+1)^(1/2)+x)/(I*x-1)^2))*(-(I*x-1)*a)^(1/4)/(I*x-1)*(-(I*x-1)^3*(I*x+1))^(1/4)/((I*x+1)*a)^(1/4)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-i\,ax+a\right)^{\frac{5}{4}}}{\left(i\,ax+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")

[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(5/4), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(5/4),x)
```

```
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(5/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{5}{4}}}{(ia(x-i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(5/4), x)
```

## 3.1119 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx$

**Optimal**. Leaf size=264

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\,\mathrm{t}}{}$$

**Rubi** [A] time = 0.14, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {47, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((4*I)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(1/4)) + (I*Sqrt[2]*ArcTan[1 - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 + (Sqr
t[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a + (I*Log[1 + Sqrt[a - I*a*x]/
Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*
a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx = \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(4i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(4i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \cdots$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} - \frac{\left(i\sqrt{2}\right)\text{Sub}\cdots}{\cdots}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}\cdots\right.}{\sqrt{2}\,a\cdots}$$

**Mathematica [C]** time = 0.02, size = 70, normalized size = 0.27

$$\frac{i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((I/5)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[5/4, 5/4
, 9/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

**IntegrateAlgebraic [A]** time = 0.19, size = 114, normalized size = 0.43

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{2\sqrt[4]{-1}\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{2(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((4*I)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(1/4)) - (2*(-1)^(1/4)*ArcTanh[((-
1)^(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a + (2*(-1)^(3/4)*ArcTanh[(
(-1)^(3/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
```

**fricas [A]** time = 0.93, size = 303, normalized size = 1.15

$$-\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right)-\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right)+\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right)-\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right)-8\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2a^2x-2ia^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] -((a^2*x - I*a^2)*sqrt(4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(4*I/a^2) + 2*(I*a
*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (a^2*x - I*a^2)*sqrt(4*I/a
^2)*log(-((a^2*x - I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^
(1/4))/(2*x - 2*I)) + (a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(((a^2*x - I*a^2)*s
qrt(-4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (a^2
*x - I*a^2)*sqrt(-4*I/a^2)*log(-((a^2*x - I*a^2)*sqrt(-4*I/a^2) - 2*(I*a*x
+ a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - 8*(I*a*x + a)^(3/4)*(-I*a*x +
 a)^(1/4))/(2*a^2*x - 2*I*a^2)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

**maple [C]** time = 2.01, size = 476, normalized size = 1.80

$$-\frac{4\left(x+i\right)\left(-\left(ix-1\right)a\right)^{\frac{1}{4}}}{\left(ix-1\right)\left(\left(ix+1\right)a\right)^{\frac{1}{4}}a}+\frac{\left(\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-x^3-\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+i\right)-2ix^2-2i\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^2+i\right)+i\sqrt{-x^4-2ix^3-2ix+1}\,x+x+i\left(-x^4-2ix^3-2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)+\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)-\sqrt{-x^4-2ix^3-2ix+1}}{\left(ix-1\right)^2}\right)+i\,\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-x^3-i\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+i\right)-2ix^2+2\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^2+i\right)-i\sqrt{-x^4-2ix^3-2ix+1}\,x+x+\left(-x^4-2ix^3-2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)+i\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)+\sqrt{-x^4-2ix^3-2ix+1}}{\left(ix-1\right)^2}\right)\right)\left(-\left(ix-1\right)a\right)^{\frac{1}{4}}\left(-\left(ix-1\right)^3\left(ix+1\right)\right)^{\frac{1}{4}}}{\left(ix-1\right)\left(\left(ix+1\right)a\right)^{\frac{1}{4}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(1/4)/(I*a*x+a)^(5/4),x)
```

```
[Out] -4*(x+I)/a*(-(I*x-1)*a)^(1/4)/(I*x-1)/((I*x+1)*a)^(1/4)+(RootOf(_Z^2+I)*ln(
(-RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2+I*RootOf(_Z^2+I)*(-x^4-2*
I*x^3-2*I*x+1)^(3/4)-x^3-2*I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x+
I*(-x^4-2*I*x^3-2*I*x+1)^(1/2)*x-2*I*x^2+RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x
+1)^(1/4)-(-x^4-2*I*x^3-2*I*x+1)^(1/2)+x)/(I*x-1)^2)+I*RootOf(_Z^2+I)*ln((-
I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2+2*RootOf(_Z^2+I)*(-x^4-2*
I*x^3-2*I*x+1)^(1/4)*x-x^3+RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(3/4)-I*(-
x^4-2*I*x^3-2*I*x+1)^(1/2)*x+I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)-
2*I*x^2+(-x^4-2*I*x^3-2*I*x+1)^(1/2)+x)/(I*x-1)^2))/a*(-(I*x-1)*a)^(1/4)/(I
*x-1)*(-(I*x-1)^3*(I*x+1))^(1/4)/((I*x+1)*a)^(1/4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax + a)^{\frac{1}{4}}}{(i\,ax + a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(5/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a - a\,x\,1\mathrm{i})^{1/4}}{(a + a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(5/4),x)
```

```
[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(5/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(5/4), x)
```

## 3.1120 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx$

**Optimal**. Leaf size=31

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

**Rubi** [A] time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {37}

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx = \frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

**Mathematica** [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

**IntegrateAlgebraic** [A] time = 0.07, size = 31, normalized size = 1.00

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

**fricas** [A] time = 0.97, size = 31, normalized size = 1.00

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{a^3x-i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(a^3*x - I*a^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(3/4)), x)
```

**maple** [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{2x+2i}{(-(ix-1)\,a)^{\frac{3}{4}}\,((ix+1)\,a)^{\frac{1}{4}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(3/4)/(I*a*x+a)^(5/4),x)
[Out] 2/a/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(x+I)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(3/4)), x)
```

**mupad [B]** time = 1.16, size = 27, normalized size = 0.87

$$\frac{(-a\,(-1+x\,1\mathrm{i}))^{1/4}\,2\mathrm{i}}{a^2\,(a\,(1+x\,1\mathrm{i}))^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] ((-a*(x*1i - 1))^(1/4)*2i)/(a^2*(a*(x*1i + 1))^(1/4))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{5}{4}}\,(-ia\,(x+i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(3/4)), x)
```

## 3.1121 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}} \, dx$

**Optimal**. Leaf size=67

$$\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

**Rubi [A]** time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((4*I)/3)*(a - I*a*
x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx = -\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{2\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{3a}$$
$$= -\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}$$

**Mathematica** [A] time = 0.02, size = 38, normalized size = 0.57

$$\frac{4x+2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (2*I + 4*x)/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4))
```

**IntegrateAlgebraic** [A] time = 0.13, size = 55, normalized size = 0.82

$$\frac{i(a+iax)^{3/4}\left(-1+\frac{3(a-iax)}{a+iax}\right)}{3a^3(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((I/3)*(a + I*a*x)^(3/4)*(-1 + (3*(a - I*a*x))/(a + I*a*x)))/(a^3*(a - I*a*
x)^(3/4))
```

**fricas** [A] time = 1.50, size = 36, normalized size = 0.54

$$\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(4\,x+2i)}{3\left(a^4x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/3*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(4*x + 2*I)/(a^4*x^2 + a^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(7/4)), x)
```

**maple [A]** time = 0.05, size = 33, normalized size = 0.49

$$\frac{\frac{4x}{3}+\frac{2i}{3}}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(7/4)/(I*a*x+a)^(5/4),x)
```

```
[Out] 2/3/a^2/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(2*x+I)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(i\,ax+a\right)^{\frac{5}{4}}\left(-i\,ax+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(7/4)), x)
```

**mupad [B]** time = 0.60, size = 40, normalized size = 0.60

$$-\frac{2\,\left(2\,x+1{}\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1{}\mathrm{i}\right)\right)}^{1/4}}{3\,a^3\,\left(-1+x\,1{}\mathrm{i}\right)\,{\left(a\,\left(1+x\,1{}\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] -(2*(2*x + 1i)*(-a*(x*1i - 1))^(1/4))/(3*a^3*(x*1i - 1)*(a*(x*1i + 1))^(1/4
))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ia\left(x-i\right)\right)^{\frac{5}{4}}\left(-ia\left(x+i\right)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(7/4)), x)
```

## 3.1122 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx$

**Optimal**. Leaf size=100

$$\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}-\frac{8i}{21a^3\sqrt[4]{a+iax}\,(a-iax)^{3/4}}-\frac{2i}{7a^2\sqrt[4]{a+iax}\,(a-iax)^{7/4}}$$

**Rubi [A]** time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}-\frac{8i}{21a^3\sqrt[4]{a+iax}\,(a-iax)^{3/4}}-\frac{2i}{7a^2\sqrt[4]{a+iax}\,(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)) - ((8*I)/21)/(a^3*(a -
 I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((16*I)/21)*(a - I*a*x)^(1/4))/(a^4*(a
+ I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx = -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}} + \frac{4\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx}{7a}$$
$$= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}} - \frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{8\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{21a^2}$$
$$= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}} - \frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}$$

**Mathematica** [A] time = 0.02, size = 50, normalized size = 0.50

$$\frac{16x^2+24ix-2}{21a^3(x+i)(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
[Out] (-2 + (24*I)*x + 16*x^2)/(21*a^3*(I + x)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4
))
```

**IntegrateAlgebraic** [A] time = 0.14, size = 77, normalized size = 0.77

$$\frac{i(a+iax)^{7/4}\left(\frac{21(a-iax)^2}{(a+iax)^2}-\frac{14(a-iax)}{a+iax}-3\right)}{42a^4(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
[Out] ((I/42)*(a + I*a*x)^(7/4)*(-3 + (21*(a - I*a*x)^2)/(a + I*a*x)^2 - (14*(a -
 I*a*x))/(a + I*a*x)))/(a^4*(a - I*a*x)^(7/4))
```

**fricas** [A] time = 0.88, size = 58, normalized size = 0.58

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}\left(8\,x^2+12i\,x-1\right)}{21\,a^5x^3+21i\,a^5x^2+21\,a^5x+21i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 12*I*x - 1)/(21*a^5*x^3 + 2
1*I*a^5*x^2 + 21*a^5*x + 21*I*a^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(11/4)), x)
```

**maple [A]** time = 0.06, size = 44, normalized size = 0.44

$$\frac{\frac{16}{21}x^2+\frac{8}{7}ix-\frac{2}{21}}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x+i\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(11/4)/(I*a*x+a)^(5/4),x)
```

```
[Out] 2/21/a^3/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(8*x^2+12*I*x-1)/(x+I)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**mupad [B]** time = 0.76, size = 46, normalized size = 0.46

$$-\frac{\left(-a\,\left(-1+x\,1{}\mathrm{i}\right)\right)^{1/4}\,\left(8\,x^2+x\,12{}\mathrm{i}-1\right)\,2{}\mathrm{i}}{21\,a^4\,{\left(-1+x\,1{}\mathrm{i}\right)}^2\,{\left(a\,\left(1+x\,1{}\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] -((-a*(x*1i - 1))^(1/4)*(x*12i + 8*x^2 - 1)*2i)/(21*a^4*(x*1i - 1)^2*(a*(x*
1i + 1))^(1/4))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(x-i))^{\frac{5}{4}}(-ia(x+i))^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(11/4)), x)
```

## 3.1123 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}} \, dx$

**Optimal**. Leaf size=297

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{a}}\right.}{a}$$

**Rubi [A]** time = 0.14, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {47, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((4*I)/5)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(5/4)) - ((4*I)*(a - I*a*x)^(1
/4))/(a*(a + I*a*x)^(1/4)) - (I*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/
4))/(a + I*a*x)^(1/4)])/a + (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[
a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]
)/(Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx = \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx$$

$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(4i)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(4i)\,\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(2i)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \ldots\right.}{a}$$

$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, \ldots\right.}{a}$$

$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right.}{\sqrt{2}a}$$

$$= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\,\mathrm{l}\ldots}{\ldots}$$

**Mathematica [C]** time = 0.03, size = 70, normalized size = 0.24

$$\frac{i\sqrt[4]{1+ix}\,(a-iax)^{9/4}\,{}_2F_1\left(\frac{9}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9\sqrt[4]{2}\,a^3\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] ((I/9)*(1 + I*x)^(1/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[9/4, 9/4, 13/4,
1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

**IntegrateAlgebraic [A]** time = 0.22, size = 137, normalized size = 0.46

$$\frac{4i\sqrt[4]{a-iax}\left(-5+\frac{a-iax}{a+iax}\right)}{5a\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{-1}\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{2(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((4*I)/5)*(a - I*a*x)^(1/4)*(-5 + (a - I*a*x)/(a + I*a*x)))/(a*(a + I*a*x)
^(1/4)) + (2*(-1)^(1/4)*ArcTanh[((-1)^(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^
(1/4)])/a - (2*(-1)^(3/4)*ArcTanh[((-1)^(3/4)*(a - I*a*x)^(1/4))/(a + I*a*x
)^(1/4)])/a
```

**fricas [A]** time = 1.49, size = 351, normalized size = 1.18

$$\frac{\left(5a^2x^2-10ia^2x-5a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}+2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)-\left(5a^2x^2-10ia^2x-5a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}-2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)+\left(5a^2x^2-10ia^2x-5a^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}+2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)-\left(5a^2x^2-10ia^2x-5a^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}-2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)-(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}(48x-32i)}{10a^2x^2-20ia^2x-10a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] ((5*a^2*x^2 - 10*I*a^2*x - 5*a^2)*sqrt(4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(4
*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (5*a^2*x^2
 - 10*I*a^2*x - 5*a^2)*sqrt(4*I/a^2)*log(-((a^2*x - I*a^2)*sqrt(4*I/a^2) -
2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) + (5*a^2*x^2 - 10*I*a^
2*x - 5*a^2)*sqrt(-4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x
+ a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (5*a^2*x^2 - 10*I*a^2*x - 5*a
^2)*sqrt(-4*I/a^2)*log(-((a^2*x - I*a^2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(3/
4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*
(48*x - 32*I))/(10*a^2*x^2 - 20*I*a^2*x - 10*a^2)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(i/4*a/a^2*(1
```

```
6*((i*a*x+a)^(1/4))^4*(-((i*a*x+a)^(1/4))^4+2*a)^(1/4)-32*a*(-((i*a*x+a)^(1
/4))^4+2*a)^(1/4))/((i*a*x+a)^(1/4))^6/4*i*a*((i*a*x+a)^(1/4))^-3,x)
```

**maple [C]** time = 0.06, size = 490, normalized size = 1.65

$$\frac{8\left(3x^2+ix+2\right)\left(-(ix-1)a\right)^{\frac{1}{4}}}{5(x-i)(ix-1)\left((ix+1)a\right)^{\frac{1}{4}}a}-\frac{\left(\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-x^3-\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+i\right)-2ix^2-2i\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^2+i\right)+i\sqrt{-x^4-2ix^3-2ix+1}\,x+x+i\left(-x^4-2ix^3-2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)+\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)-\sqrt{-x^4-2ix^3-2ix+1}}{(ix-1)^2}\right)+i\,\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-x^3-i\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+i\right)-2ix^2+2\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^2+i\right)-i\sqrt{-x^4-2ix^3-2ix+1}\,x+x+\left(-x^4-2ix^3-2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)+i\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)+\sqrt{-x^4-2ix^3-2ix+1}}{(ix-1)^2}\right)\right)\left(-(ix-1)a\right)^{\frac{1}{4}}\left(-(ix-1)^3(ix+1)\right)^{\frac{1}{4}}}{(ix-1)\left((ix+1)a\right)^{\frac{1}{4}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(5/4)/(I*a*x+a)^(9/4),x)
```

```
[Out] 8/5*(3*x^2+2+I*x)/(x-I)/a*(-(I*x-1)*a)^(1/4)/(I*x-1)/((I*x+1)*a)^(1/4)-(Roo
tOf(_Z^2+I)*ln((-RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2+I*RootOf(_
Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(3/4)-x^3-2*I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*
I*x+1)^(1/4)*x+I*(-x^4-2*I*x^3-2*I*x+1)^(1/2)*x-2*I*x^2+RootOf(_Z^2+I)*(-x^
4-2*I*x^3-2*I*x+1)^(1/4)-(-x^4-2*I*x^3-2*I*x+1)^(1/2)+x)/(I*x-1)^2)+I*RootO
f(_Z^2+I)*ln((-I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2+2*RootOf(_
Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x-x^3+RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*
x+1)^(3/4)-I*(-x^4-2*I*x^3-2*I*x+1)^(1/2)*x+I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-
2*I*x+1)^(1/4)-2*I*x^2+(-x^4-2*I*x^3-2*I*x+1)^(1/2)+x)/(I*x-1)^2))/a*(-(I*x
-1)*a)^(1/4)/(I*x-1)*(-(I*x-1)^3*(I*x+1))^(1/4)/((I*x+1)*a)^(1/4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(9/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(9/4),x)
```

```
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(9/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-ia\left(x+i\right)\right)^{\frac{5}{4}}}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(9/4), x)
```

## 3.1124 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx$

**Optimal**. Leaf size=33

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

**Rubi** [A] time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {37}

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx = \frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

**fricas [B]** time = 1.43, size = 45, normalized size = 1.36

$$-\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x+2i)}{5\,a^3x^2-10i\,a^3x-5\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] -(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x + 2*I)/(5*a^3*x^2 - 10*I*a^3*x -
 5*a^3)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

**maple [B]** time = 0.04, size = 50, normalized size = 1.52

$$\frac{2\,(-(ix-1)\,a)^{\frac{1}{4}}\left(x^2+2ix-1\right)}{5\,(ix-1)\,((ix+1)\,a)^{\frac{1}{4}}\,(x-i)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(1/4)/(I*a*x+a)^(9/4),x)
```

```
[Out] 2/5/a^2*(-(I*x-1)*a)^(1/4)/(I*x-1)/((I*x+1)*a)^(1/4)*(2*I*x+x^2-1)/(x-I)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax + a)^{\frac{1}{4}}}{(i\,ax + a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(9/4), x)
```

**mupad [B]** time = 0.55, size = 38, normalized size = 1.15

$$-\frac{2\,(-1+x\,1\mathrm{i})\,\left(-a\,(-1+x\,1\mathrm{i})\right)^{1/4}}{5\,a^2\,(x-\mathrm{i})\,\left(a\,(1+x\,1\mathrm{i})\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(9/4),x)
[Out] -(2*(x*1i - 1)*(-a*(x*1i - 1))^(1/4))/(5*a^2*(x - 1i)*(a*(x*1i + 1))^(1/4))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(9/4),x)
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(9/4), x)
```

## 3.1125 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx$

**Optimal**. Leaf size=67

$$\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}$$

**Rubi** [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(5/4)) + (((4*I)/5)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx = \frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}} + \frac{2\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{5a}$$
$$= \frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}} + \frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}$$

**Mathematica** [A] time = 0.02, size = 45, normalized size = 0.67

$$\frac{2(3+2ix)\sqrt[4]{a-iax}}{5a^3(x-i)\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*(3 + (2*I)*x)*(a - I*a*x)^(1/4))/(5*a^3*(-I + x)*(a + I*a*x)^(1/4))
```

**IntegrateAlgebraic** [A] time = 0.14, size = 54, normalized size = 0.81

$$\frac{i\sqrt[4]{a-iax}\left(5+\frac{a-iax}{a+iax}\right)}{5a^3\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((I/5)*(a - I*a*x)^(1/4)*(5 + (a - I*a*x)/(a + I*a*x)))/(a^3*(a + I*a*x)^(1
/4))
```

**fricas** [A] time = 1.36, size = 44, normalized size = 0.66

$$\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(4\,x-6i)}{5\left(a^4x^2-2i\,a^4x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(4*x - 6*I)/(a^4*x^2 - 2*I*a^4*x -
 a^4)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(3/4)), x)
```

**maple [A]** time = 0.04, size = 44, normalized size = 0.66

$$\frac{\frac{4}{5}x^2-\frac{2}{5}ix+\frac{6}{5}}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x-i\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(3/4)/(I*a*x+a)^(9/4),x)
[Out] 2/5/a^2/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(2*x^2+3-I*x)/(x-I)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(3/4)), x)
```

**mupad [B]** time = 0.63, size = 38, normalized size = 0.57

$$\frac{2\,\left(3+x\,2\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{5\,a^3\,\left(x-\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(9/4)),x)
[Out] (2*(x*2i + 3)*(-a*(x*1i - 1))^(1/4))/(5*a^3*(x - 1i)*(a*(x*1i + 1))^(1/4))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}\left(-ia\left(x+i\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(9/4),x)
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(3/4)), x)
```

## 3.1126 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx$

**Optimal**. Leaf size=100

$$\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}-\frac{2i}{3a^2(a+iax)^{5/4}(a-iax)^{3/4}}$$

**Rubi** [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}-\frac{2i}{3a^2(a+iax)^{5/4}(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((8*I)/15)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(5/4)) + (((16*I)/15)*(a - I*a*x)^(1/4))/(a^4*(
a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{4\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx}{3a}$$
$$= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}} + \frac{8\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{15a^2}$$
$$= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}} + \frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}$$

**Mathematica [A]** time = 0.02, size = 50, normalized size = 0.50

$$\frac{2\left(8x^2-4ix+7\right)}{15a^3(x-i)(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*(7 - (4*I)*x + 8*x^2))/(15*a^3*(-I + x)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1
/4))
```

**IntegrateAlgebraic [A]** time = 0.14, size = 77, normalized size = 0.77

$$\frac{i(a+iax)^{3/4}\left(\frac{3(a-iax)^2}{(a+iax)^2}+\frac{30(a-iax)}{a+iax}-5\right)}{30a^4(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((I/30)*(a + I*a*x)^(3/4)*(-5 + (3*(a - I*a*x)^2)/(a + I*a*x)^2 + (30*(a -
I*a*x))/(a + I*a*x)))/(a^4*(a - I*a*x)^(3/4))
```

**fricas [A]** time = 1.27, size = 58, normalized size = 0.58

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}\left(8\,x^2-4i\,x+7\right)}{15\,a^5x^3-15i\,a^5x^2+15\,a^5x-15i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 - 4*I*x + 7)/(15*a^5*x^3 - 15
*I*a^5*x^2 + 15*a^5*x - 15*I*a^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(7/4)), x)
```

**maple [A]** time = 0.06, size = 44, normalized size = 0.44

$$\frac{\frac{16}{15}x^2-\frac{8}{15}ix+\frac{14}{15}}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x-i\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(7/4)/(I*a*x+a)^(9/4),x)
```

```
[Out] 2/15/a^3/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(8*x^2-4*I*x+7)/(x-I)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

**mupad [B]** time = 0.53, size = 45, normalized size = 0.45

$$\frac{2\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)^{1/4}\,\left(x^2\,8\mathrm{i}+4\,x+7\mathrm{i}\right)}{15\,a^4\,\left(x^2+1\right)\,\left(a\,\left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] (2*(-a*(x*1i - 1))^(1/4)*(4*x + x^2*8i + 7i))/(15*a^4*(x^2 + 1)*(a*(x*1i +
1))^(1/4))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}\left(-ia\left(x+i\right)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(7/4)), x)
```

## 3.1127 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx$

**Optimal**. Leaf size=133

$$\frac{32i\sqrt[4]{a-iax}}{35a^5\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}-\frac{4i}{7a^3(a+iax)^{5/4}(a-iax)^{3/4}}-\frac{2i}{7a^2(a+iax)^{5/4}(a-iax)^{7/4}}$$

**Rubi** [A] time = 0.03, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{32i\sqrt[4]{a-iax}}{35a^5\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}-\frac{4i}{7a^3(a+iax)^{5/4}(a-iax)^{3/4}}-\frac{2i}{7a^2(a+iax)^{5/4}(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)) - ((4*I)/7)/(a^3*(a -
I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((16*I)/35)*(a - I*a*x)^(1/4))/(a^4*(a +
 I*a*x)^(5/4)) + (((32*I)/35)*(a - I*a*x)^(1/4))/(a^5*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} + \frac{6\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx}{7a}$$

$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} - \frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{8\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,d}{7a^2}$$

$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} - \frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}} + \frac{16}{}$$

$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} - \frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}} + \frac{3}{35}$$

**Mathematica** [A] time = 0.03, size = 57, normalized size = 0.43

$$\frac{2\left(16x^3+8ix^2+22x+9i\right)}{35a^4\left(x^2+1\right)(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*(9*I + 22*x + (8*I)*x^2 + 16*x^3))/(35*a^4*(a - I*a*x)^(3/4)*(a + I*a*x)
^(1/4)*(1 + x^2))
```

**IntegrateAlgebraic** [A] time = 0.14, size = 99, normalized size = 0.74

$$\frac{i(a+iax)^{7/4}\left(\frac{7(a-iax)^3}{(a+iax)^3}+\frac{105(a-iax)^2}{(a+iax)^2}-\frac{35(a-iax)}{a+iax}-5\right)}{140a^5(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((I/140)*(a + I*a*x)^(7/4)*(-5 + (7*(a - I*a*x)^3)/(a + I*a*x)^3 + (105*(a
- I*a*x)^2)/(a + I*a*x)^2 - (35*(a - I*a*x))/(a + I*a*x)))/(a^5*(a - I*a*x)
^(7/4))
```

**fricas** [A] time = 1.49, size = 54, normalized size = 0.41

$$\frac{\left(32\,x^3+16i\,x^2+44\,x+18i\right)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{35\left(a^6x^4+2\,a^6x^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/35*(32*x^3 + 16*I*x^2 + 44*x + 18*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)
/(a^6*x^4 + 2*a^6*x^2 + a^6)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(11/4)), x)
```

**maple [A]** time = 0.06, size = 56, normalized size = 0.42

$$\frac{\frac{32}{35}x^3+\frac{16}{35}ix^2+\frac{44}{35}x+\frac{18}{35}i}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x-i\right)\left(x+i\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(11/4)/(I*a*x+a)^(9/4),x)
```

```
[Out] 2/35/a^4/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(16*x^3+8*I*x^2+22*x+9*I)/(x-
I)/(x+I)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(11/4)), x)
```

**mupad [B]** time = 0.69, size = 56, normalized size = 0.42

$$\frac{2\left(-a\ \left(-1+x\,1\mathrm{i}\right)\right)^{1/4}\ \left(x^4\,16\mathrm{i}+8\,x^3+x^2\,30\mathrm{i}+13\,x+9\mathrm{i}\right)}{35\,a^5\,{\left(x^2+1\right)}^2\,\left(a\ \left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] (2*(-a*(x*1i - 1))^(1/4)*(13*x + x^2*30i + 8*x^3 + x^4*16i + 9i))/(35*a^5*(
x^2 + 1)^2*(a*(x*1i + 1))^(1/4))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

## 3.1128 $\int (a+bx)^2(ac-bcx)^n\,dx$

**Optimal**. Leaf size=83

$$-\frac{4a^2(ac-bcx)^{n+1}}{bc(n+1)}-\frac{(ac-bcx)^{n+3}}{bc^3(n+3)}+\frac{4a(ac-bcx)^{n+2}}{bc^2(n+2)}$$

**Rubi [A]** time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{4a^2(ac-bcx)^{n+1}}{bc(n+1)}+\frac{4a(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{(ac-bcx)^{n+3}}{bc^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] (-4*a^2*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (4*a*(a*c - b*c*x)^(2 + n))/
(b*c^2*(2 + n)) - (a*c - b*c*x)^(3 + n)/(b*c^3*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^n\,dx &= \int \left(4a^2(ac-bcx)^n-\frac{4a(ac-bcx)^{1+n}}{c}+\frac{(ac-bcx)^{2+n}}{c^2}\right)dx\\ &= -\frac{4a^2(ac-bcx)^{1+n}}{bc(1+n)}+\frac{4a(ac-bcx)^{2+n}}{bc^2(2+n)}-\frac{(ac-bcx)^{3+n}}{bc^3(3+n)}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 77, normalized size = 0.93

$$\frac{(bx-a)\left(a^2\left(n^2+7n+14\right)+2ab\left(n^2+5n+4\right)x+b^2\left(n^2+3n+2\right)x^2\right)(c(a-bx))^n}{b(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] ((c*(a - b*x))^n*(-a + b*x)*(a^2*(14 + 7*n + n^2) + 2*a*b*(4 + 5*n + n^2)*x
 + b^2*(2 + 3*n + n^2)*x^2))/(b*(1 + n)*(2 + n)*(3 + n))
```

**IntegrateAlgebraic [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int (a+bx)^2(ac-bcx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^2*(a*c - b*c*x)^n, x]
```

**fricas [A]** time = 1.25, size = 128, normalized size = 1.54

$$-\frac{\left(a^3n^2+7\,a^3n-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3+14\,a^3-\left(ab^2n^2+7\,ab^2n+6\,ab^2\right)x^2+\left(a^2bn^2+3\,a^2bn-6\,a^2b\right)x\right)(-bcx+ac)^n}{bn^3+6\,bn^2+11\,bn+6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] -(a^3*n^2 + 7*a^3*n - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 + 14*a^3 - (a*b^2*n^2
 + 7*a*b^2*n + 6*a*b^2)*x^2 + (a^2*b*n^2 + 3*a^2*b*n - 6*a^2*b)*x)*(-b*c*x
+ a*c)^n/(b*n^3 + 6*b*n^2 + 11*b*n + 6*b)
```

**giac [B]** time = 1.15, size = 256, normalized size = 3.08

$$\frac{(-bcx+ac)^n b^3n^2x^3+(-bcx+ac)^n ab^2n^2x^2+3\,(-bcx+ac)^n b^3nx^3-(-bcx+ac)^n a^2bn^2x+7\,(-bcx+ac)^n ab^2nx^2+2\,(-bcx+ac)^n b^3x^3-(-bcx+ac)^n a^3n^2-3\,(-bcx+ac)^n a^2bnx+6\,(-bcx+ac)^n ab^2x^2-7\,(-bcx+ac)^n a^3n+6\,(-bcx+ac)^n a^2bx-14\,(-bcx+ac)^n a^3}{bn^3+6\,bn^2+11\,bn+6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((-b*c*x + a*c)^n*b^3*n^2*x^3 + (-b*c*x + a*c)^n*a*b^2*n^2*x^2 + 3*(-b*c*x
+ a*c)^n*b^3*n*x^3 - (-b*c*x + a*c)^n*a^2*b*n^2*x + 7*(-b*c*x + a*c)^n*a*b^
2*n*x^2 + 2*(-b*c*x + a*c)^n*b^3*x^3 - (-b*c*x + a*c)^n*a^3*n^2 - 3*(-b*c*x
 + a*c)^n*a^2*b*n*x + 6*(-b*c*x + a*c)^n*a*b^2*x^2 - 7*(-b*c*x + a*c)^n*a^3
*n + 6*(-b*c*x + a*c)^n*a^2*b*x - 14*(-b*c*x + a*c)^n*a^3)/(b*n^3 + 6*b*n^2
 + 11*b*n + 6*b)
```

**maple [A]** time = 0.01, size = 103, normalized size = 1.24

$$-\frac{(-bx+a)\left(b^2n^2x^2+2ab\,n^2x+3b^2n\,x^2+a^2n^2+10abnx+2b^2x^2+7a^2n+8abx+14a^2\right)(-bcx+ac)^n}{\left(n^3+6n^2+11n+6\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^n,x)
```

```
[Out] -(-b*x+a)*(b^2*n^2*x^2+2*a*b*n^2*x+3*b^2*n*x^2+a^2*n^2+10*a*b*n*x+2*b^2*x^2
+7*a^2*n+8*a*b*x+14*a^2)*(-b*c*x+a*c)^n/b/(n^3+6*n^2+11*n+6)
```

**maxima [B]** time = 1.53, size = 167, normalized size = 2.01

$$\frac{2\left(b^2c^n(n+1)x^2 - abc^n nx - a^2c^n\right)(-bx+a)^n a}{\left(n^2+3n+2\right)b} + \frac{\left(\left(n^2+3n+2\right)b^3c^nx^3 - \left(n^2+n\right)ab^2c^nx^2 - 2a^2bc^nnx - 2a^3c^n\right)(-bx+a)^n}{\left(n^3+6n^2+11n+6\right)b} - \frac{(-bcx+ac)^{n+1}a^2}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] 2*(b^2*c^n*(n + 1)*x^2 - a*b*c^n*n*x - a^2*c^n)*(-b*x + a)^n*a/((n^2 + 3*n
+ 2)*b) + ((n^2 + 3*n + 2)*b^3*c^n*x^3 - (n^2 + n)*a*b^2*c^n*x^2 - 2*a^2*b*
c^n*n*x - 2*a^3*c^n)*(-b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b) - (-b*c*x +
a*c)^(n + 1)*a^2/(b*c*(n + 1))
```

**mupad [B]** time = 0.49, size = 133, normalized size = 1.60

$$-(a\,c-b\,c\,x)^n\,\left(\frac{a^2\,x\,\left(n^2+3\,n-6\right)}{n^3+6\,n^2+11\,n+6}+\frac{a^3\,\left(n^2+7\,n+14\right)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{b^2\,x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}-\frac{a\,b\,x^2\,\left(n^2+7\,n+6\right)}{n^3+6\,n^2+11\,n+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n*(a + b*x)^2,x)
```

```
[Out] -(a*c - b*c*x)^n*((a^2*x*(3*n + n^2 - 6))/(11*n + 6*n^2 + n^3 + 6) + (a^3*(
7*n + n^2 + 14))/(b*(11*n + 6*n^2 + n^3 + 6)) - (b^2*x^3*(3*n + n^2 + 2))/(
11*n + 6*n^2 + n^3 + 6) - (a*b*x^2*(7*n + n^2 + 6))/(11*n + 6*n^2 + n^3 + 6
))
```

**sympy [A]** time = 1.30, size = 819, normalized size = 9.87

$$\begin{cases} a^2x(ac)^n & \text{for } b=0 \\ -\frac{a^2\log\left(-\frac{a}{b}+x\right)}{a^2bc^3-2ab^2c^3x+b^3c^3x^2} - \frac{2a^2}{a^2bc^3-2ab^2c^3x+b^3c^3x^2} + \frac{2abx\log\left(-\frac{a}{b}+x\right)}{a^2bc^3-2ab^2c^3x+b^3c^3x^2} + \frac{4abx}{a^2bc^3-2ab^2c^3x+b^3c^3x^2} - \frac{b^2x^2\log\left(-\frac{a}{b}+x\right)}{a^2bc^3-2ab^2c^3x+b^3c^3x^2} & \text{for } n=-3 \\ -\frac{4a^2\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} - \frac{5a^2}{-abc^2+b^2c^2x} + \frac{4abx\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} + \frac{b^2x^2}{-abc^2+b^2c^2x} & \text{for } n=-2 \\ -\frac{4a^2\log\left(-\frac{a}{b}+x\right)}{bc} - \frac{3ax}{c} - \frac{bx^2}{2c} & \text{for } n=-1 \\ -\frac{a^3n^2(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} - \frac{7a^3n(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} - \frac{14a^3(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} - \frac{a^2bn^2x(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} - \frac{3a^2bnx(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{6a^2bx(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{ab^2n^2x^2(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{7ab^2nx^2(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{6ab^2x^2(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{b^3n^2x^3(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{3b^3nx^3(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{2b^3x^3(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((a**2*x*(a*c)**n, Eq(b, 0)), (-a**2*log(-a/b + x)/(a**2*b*c**3 -
2*a*b**2*c**3*x + b**3*c**3*x**2) - 2*a**2/(a**2*b*c**3 - 2*a*b**2*c**3*x +
```

```
 b**3*c**3*x**2) + 2*a*b*x*log(-a/b + x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b
**3*c**3*x**2) + 4*a*b*x/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2) -
 b**2*x**2*log(-a/b + x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2),
Eq(n, -3)), (-4*a**2*log(-a/b + x)/(-a*b*c**2 + b**2*c**2*x) - 5*a**2/(-a*b
*c**2 + b**2*c**2*x) + 4*a*b*x*log(-a/b + x)/(-a*b*c**2 + b**2*c**2*x) + b*
*2*x**2/(-a*b*c**2 + b**2*c**2*x), Eq(n, -2)), (-4*a**2*log(-a/b + x)/(b*c)
 - 3*a*x/c - b*x**2/(2*c), Eq(n, -1)), (-a**3*n**2*(a*c - b*c*x)**n/(b*n**3
 + 6*b*n**2 + 11*b*n + 6*b) - 7*a**3*n*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2
+ 11*b*n + 6*b) - 14*a**3*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*
b) - a**2*b*n**2*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) - 3*
a**2*b*n*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 6*a**2*b*x
*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + a*b**2*n**2*x**2*(a*
c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 7*a*b**2*n*x**2*(a*c - b
*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 6*a*b**2*x**2*(a*c - b*c*x)**
n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + b**3*n**2*x**3*(a*c - b*c*x)**n/(b*n
**3 + 6*b*n**2 + 11*b*n + 6*b) + 3*b**3*n*x**3*(a*c - b*c*x)**n/(b*n**3 + 6
*b*n**2 + 11*b*n + 6*b) + 2*b**3*x**3*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 +
 11*b*n + 6*b), True))
```

## 3.1129 $\int (a+bx)(ac-bcx)^n\,dx$

**Optimal**. Leaf size=53

$$\frac{(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{2a(ac-bcx)^{n+1}}{bc(n+1)}$$

**Rubi** [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{2a(ac-bcx)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] (-2*a*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (a*c - b*c*x)^(2 + n)/(b*c^2*(
2 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^n\,dx &= \int \left(2a(ac-bcx)^n-\frac{(ac-bcx)^{1+n}}{c}\right)dx\\ &= -\frac{2a(ac-bcx)^{1+n}}{bc(1+n)}+\frac{(ac-bcx)^{2+n}}{bc^2(2+n)}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 43, normalized size = 0.81

$$\frac{(bx-a)(a(n+3)+b(n+1)x)(c(a-bx))^n}{b(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] ((c*(a - b*x))^n*(-a + b*x)*(a*(3 + n) + b*(1 + n)*x))/(b*(1 + n)*(2 + n))
```

**IntegrateAlgebraic [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int (a+bx)(ac-bcx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)*(a*c - b*c*x)^n, x]
```

**fricas [A]** time = 1.34, size = 58, normalized size = 1.09

$$-\frac{\left(a^2n-2\,abx-\left(b^2n+b^2\right)x^2+3\,a^2\right)(-bcx+ac)^n}{bn^2+3\,bn+2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] -(a^2*n - 2*a*b*x - (b^2*n + b^2)*x^2 + 3*a^2)*(-b*c*x + a*c)^n/(b*n^2 + 3*
b*n + 2*b)
```

**giac [A]** time = 1.05, size = 103, normalized size = 1.94

$$\frac{(-bcx+ac)^nb^2nx^2+(-bcx+ac)^nb^2x^2-(-bcx+ac)^na^2n+2\,(-bcx+ac)^nabx-3\,(-bcx+ac)^na^2}{bn^2+3\,bn+2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((-b*c*x + a*c)^n*b^2*n*x^2 + (-b*c*x + a*c)^n*b^2*x^2 - (-b*c*x + a*c)^n*a
^2*n + 2*(-b*c*x + a*c)^n*a*b*x - 3*(-b*c*x + a*c)^n*a^2)/(b*n^2 + 3*b*n +
2*b)
```

**maple [A]** time = 0.00, size = 47, normalized size = 0.89

$$-\frac{(bnx+an+bx+3a)\,(-bx+a)\,(-bcx+ac)^n}{\left(n^2+3n+2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^n,x)
```

```
[Out] -(-b*c*x+a*c)^n*(b*n*x+a*n+b*x+3*a)*(-b*x+a)/b/(n^2+3*n+2)
```

**maxima** [A] time = 1.40, size = 81, normalized size = 1.53

$$\frac{\left(b^2c^n(n+1)x^2 - abc^n nx - a^2c^n\right)(-bx+a)^n}{\left(n^2+3\,n+2\right)b} - \frac{(-bcx+ac)^{n+1}a}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] (b^2*c^n*(n + 1)*x^2 - a*b*c^n*n*x - a^2*c^n)*(-b*x + a)^n/((n^2 + 3*n + 2)
*b) - (-b*c*x + a*c)^(n + 1)*a/(b*c*(n + 1))
```

**mupad** [B] time = 0.32, size = 66, normalized size = 1.25

$$(a\,c-b\,c\,x)^n\left(\frac{2\,a\,x}{n^2+3\,n+2}-\frac{a^2\,(n+3)}{b\,\left(n^2+3\,n+2\right)}+\frac{b\,x^2\,(n+1)}{n^2+3\,n+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n*(a + b*x),x)
```

```
[Out] (a*c - b*c*x)^n*((2*a*x)/(3*n + n^2 + 2) - (a^2*(n + 3))/(b*(3*n + n^2 + 2)
) + (b*x^2*(n + 1))/(3*n + n^2 + 2))
```

**sympy** [A] time = 0.70, size = 245, normalized size = 4.62

$$\begin{cases} ax\,(ac)^n & \text{for } b=0 \\ -\frac{a\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} - \frac{2a}{-abc^2+b^2c^2x} + \frac{bx\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} & \text{for } n=-2 \\ -\frac{2a\log\left(-\frac{a}{b}+x\right)}{bc} - \frac{x}{c} & \text{for } n=-1 \\ -\frac{a^2n(ac-bcx)^n}{bn^2+3bn+2b} - \frac{3a^2(ac-bcx)^n}{bn^2+3bn+2b} + \frac{2abx(ac-bcx)^n}{bn^2+3bn+2b} + \frac{b^2nx^2(ac-bcx)^n}{bn^2+3bn+2b} + \frac{b^2x^2(ac-bcx)^n}{bn^2+3bn+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((a*x*(a*c)**n, Eq(b, 0)), (-a*log(-a/b + x)/(-a*b*c**2 + b**2*c**
2*x) - 2*a/(-a*b*c**2 + b**2*c**2*x) + b*x*log(-a/b + x)/(-a*b*c**2 + b**2*
c**2*x), Eq(n, -2)), (-2*a*log(-a/b + x)/(b*c) - x/c, Eq(n, -1)), (-a**2*n*
(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) - 3*a**2*(a*c - b*c*x)**n/(b*n**2 +
 3*b*n + 2*b) + 2*a*b*x*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2*n*x*
*2*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2*x**2*(a*c - b*c*x)**n/(b*
n**2 + 3*b*n + 2*b), True))
```

## 3.1130 $\int (a+bx)^4(c+dx)\,dx$

**Optimal**. Leaf size=38

$$\frac{(a+bx)^5(bc-ad)}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(a+bx)^5(bc-ad)}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^5)/(5*b^2) + (d*(a + b*x)^6)/(6*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)\,dx&=\int \left(\frac{(bc-ad)(a+bx)^4}{b}+\frac{d(a+bx)^5}{b}\right)dx\\&=\frac{(bc-ad)(a+bx)^5}{5b^2}+\frac{d(a+bx)^6}{6b^2}\end{aligned}$$

**Mathematica [B]** time = 0.02, size = 84, normalized size = 2.21

$$\frac{1}{30}x\left(15a^4(2c+dx)+20a^3bx(3c+2dx)+15a^2b^2x^2(4c+3dx)+6ab^3x^3(5c+4dx)+b^4x^4(6c+5dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x),x]
```

```
[Out] (x*(15*a^4*(2*c + d*x) + 20*a^3*b*x*(3*c + 2*d*x) + 15*a^2*b^2*x^2*(4*c + 3
*d*x) + 6*a*b^3*x^3*(5*c + 4*d*x) + b^4*x^4*(6*c + 5*d*x)))/30
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^4(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4*(c + d*x), x]
```

**fricas [B]** time = 1.20, size = 97, normalized size = 2.55

$$\frac{1}{6}x^6db^4 + \frac{1}{5}x^5cb^4 + \frac{4}{5}x^5db^3a + x^4cb^3a + \frac{3}{2}x^4db^2a^2 + 2x^3cb^2a^2 + \frac{4}{3}x^3dba^3 + 2x^2cba^3 + \frac{1}{2}x^2da^4 + xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d*b^4 + 1/5*x^5*c*b^4 + 4/5*x^5*d*b^3*a + x^4*c*b^3*a + 3/2*x^4*d*b
^2*a^2 + 2*x^3*c*b^2*a^2 + 4/3*x^3*d*b*a^3 + 2*x^2*c*b*a^3 + 1/2*x^2*d*a^4
+ x*c*a^4
```

**giac [B]** time = 1.03, size = 97, normalized size = 2.55

$$\frac{1}{6}\,b^4dx^6 + \frac{1}{5}\,b^4cx^5 + \frac{4}{5}\,ab^3dx^5 + ab^3cx^4 + \frac{3}{2}\,a^2b^2dx^4 + 2\,a^2b^2cx^3 + \frac{4}{3}\,a^3bdx^3 + 2\,a^3bcx^2 + \frac{1}{2}\,a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*b^4*d*x^6 + 1/5*b^4*c*x^5 + 4/5*a*b^3*d*x^5 + a*b^3*c*x^4 + 3/2*a^2*b^2
*d*x^4 + 2*a^2*b^2*c*x^3 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + 1/2*a^4*d*x^2
+ a^4*c*x
```

**maple [B]** time = 0.00, size = 97, normalized size = 2.55

$$\frac{b^4d\,x^6}{6} + a^4cx + \frac{\left(4a\,b^3d + b^4c\right)x^5}{5} + \frac{\left(6a^2b^2d + 4a\,b^3c\right)x^4}{4} + \frac{\left(4a^3bd + 6a^2b^2c\right)x^3}{3} + \frac{\left(a^4d + 4a^3bc\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c),x)
```

```
[Out] 1/6*b^4*d*x^6+1/5*(4*a*b^3*d+b^4*c)*x^5+1/4*(6*a^2*b^2*d+4*a*b^3*c)*x^4+1/3
*(4*a^3*b*d+6*a^2*b^2*c)*x^3+1/2*(a^4*d+4*a^3*b*c)*x^2+a^4*c*x
```

**maxima [B]** time = 1.38, size = 96, normalized size = 2.53

$$\frac{1}{6}\,b^4dx^6 + a^4cx + \frac{1}{5}\left(b^4c + 4\,ab^3d\right)x^5 + \frac{1}{2}\left(2\,ab^3c + 3\,a^2b^2d\right)x^4 + \frac{2}{3}\left(3\,a^2b^2c + 2\,a^3bd\right)x^3 + \frac{1}{2}\left(4\,a^3bc + a^4d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*b^4*d*x^6 + a^4*c*x + 1/5*(b^4*c + 4*a*b^3*d)*x^5 + 1/2*(2*a*b^3*c + 3*
a^2*b^2*d)*x^4 + 2/3*(3*a^2*b^2*c + 2*a^3*b*d)*x^3 + 1/2*(4*a^3*b*c + a^4*d
)*x^2
```

**mupad [B]** time = 0.19, size = 88, normalized size = 2.32

$$x^5\left(\frac{c\,b^4}{5}+\frac{4\,a\,d\,b^3}{5}\right)+x^2\left(\frac{d\,a^4}{2}+2\,b\,c\,a^3\right)+\frac{b^4\,d\,x^6}{6}+a^4\,c\,x+\frac{2\,a^2\,b\,x^3\,\left(2\,a\,d+3\,b\,c\right)}{3}+\frac{a\,b^2\,x^4\,\left(3\,a\,d+2\,b\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x),x)
```

```
[Out] x^5*((b^4*c)/5 + (4*a*b^3*d)/5) + x^2*((a^4*d)/2 + 2*a^3*b*c) + (b^4*d*x^6)
/6 + a^4*c*x + (2*a^2*b*x^3*(2*a*d + 3*b*c))/3 + (a*b^2*x^4*(3*a*d + 2*b*c)
)/2
```

**sympy [B]** time = 0.08, size = 100, normalized size = 2.63

$$a^4cx + \frac{b^4dx^6}{6} + x^5\left(\frac{4ab^3d}{5} + \frac{b^4c}{5}\right) + x^4\left(\frac{3a^2b^2d}{2} + ab^3c\right) + x^3\left(\frac{4a^3bd}{3} + 2a^2b^2c\right) + x^2\left(\frac{a^4d}{2} + 2a^3bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c),x)
```

```
[Out] a**4*c*x + b**4*d*x**6/6 + x**5*(4*a*b**3*d/5 + b**4*c/5) + x**4*(3*a**2*b*
*2*d/2 + a*b**3*c) + x**3*(4*a**3*b*d/3 + 2*a**2*b**2*c) + x**2*(a**4*d/2 +
 2*a**3*b*c)
```

## 3.1131 $\int (a+bx)^3(c+dx)\,dx$

**Optimal**. Leaf size=38

$$\frac{(a+bx)^4(bc-ad)}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(a+bx)^4(bc-ad)}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^4)/(4*b^2) + (d*(a + b*x)^5)/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^3}{b}+\frac{d(a+bx)^4}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^4}{4b^2}+\frac{d(a+bx)^5}{5b^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 67, normalized size = 1.76

$$a^3cx+\frac{1}{2}a^2x^2(ad+3bc)+\frac{1}{4}b^2x^4(3ad+bc)+abx^3(ad+bc)+\frac{1}{5}b^3dx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^2)/2 + a*b*(b*c + a*d)*x^3 + (b^2*(b*c + 3*a
*d)*x^4)/4 + (b^3*d*x^5)/5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3*(c + d*x), x]
```

**fricas [B]** time = 1.41, size = 72, normalized size = 1.89

$$\frac{1}{5}x^5db^3+\frac{1}{4}x^4cb^3+\frac{3}{4}x^4db^2a+x^3cb^2a+x^3dba^2+\frac{3}{2}x^2cba^2+\frac{1}{2}x^2da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d*b^3 + 1/4*x^4*c*b^3 + 3/4*x^4*d*b^2*a + x^3*c*b^2*a + x^3*d*b*a^2
 + 3/2*x^2*c*b*a^2 + 1/2*x^2*d*a^3 + x*c*a^3
```

**giac [B]** time = 1.14, size = 72, normalized size = 1.89

$$\frac{1}{5}\,b^3dx^5+\frac{1}{4}\,b^3cx^4+\frac{3}{4}\,ab^2dx^4+ab^2cx^3+a^2bdx^3+\frac{3}{2}\,a^2bcx^2+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/5*b^3*d*x^5 + 1/4*b^3*c*x^4 + 3/4*a*b^2*d*x^4 + a*b^2*c*x^3 + a^2*b*d*x^3
 + 3/2*a^2*b*c*x^2 + 1/2*a^3*d*x^2 + a^3*c*x
```

**maple [B]** time = 0.00, size = 73, normalized size = 1.92

$$\frac{b^3d\,x^5}{5}+a^3cx+\frac{\left(3a\,b^2d+b^3c\right)x^4}{4}+\frac{\left(3a^2bd+3a\,b^2c\right)x^3}{3}+\frac{\left(a^3d+3a^2bc\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c),x)
```

```
[Out] 1/5*b^3*d*x^5+1/4*(3*a*b^2*d+b^3*c)*x^4+1/3*(3*a^2*b*d+3*a*b^2*c)*x^3+1/2*(
a^3*d+3*a^2*b*c)*x^2+a^3*c*x
```

**maxima [B]** time = 1.41, size = 69, normalized size = 1.82

$$\frac{1}{5}b^3dx^5 + a^3cx + \frac{1}{4}\left(b^3c + 3\,ab^2d\right)x^4 + \left(ab^2c + a^2bd\right)x^3 + \frac{1}{2}\left(3\,a^2bc + a^3d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/5*b^3*d*x^5 + a^3*c*x + 1/4*(b^3*c + 3*a*b^2*d)*x^4 + (a*b^2*c + a^2*b*d)
*x^3 + 1/2*(3*a^2*b*c + a^3*d)*x^2
```

**mupad [B]** time = 0.16, size = 65, normalized size = 1.71

$$x^4\left(\frac{c\,b^3}{4} + \frac{3\,a\,d\,b^2}{4}\right) + x^2\left(\frac{d\,a^3}{2} + \frac{3\,b\,c\,a^2}{2}\right) + \frac{b^3\,d\,x^5}{5} + a^3\,c\,x + a\,b\,x^3\,\left(a\,d + b\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x),x)
```

```
[Out] x^4*((b^3*c)/4 + (3*a*b^2*d)/4) + x^2*((a^3*d)/2 + (3*a^2*b*c)/2) + (b^3*d*
x^5)/5 + a^3*c*x + a*b*x^3*(a*d + b*c)
```

**sympy [B]** time = 0.08, size = 73, normalized size = 1.92

$$a^3cx + \frac{b^3dx^5}{5} + x^4\left(\frac{3ab^2d}{4} + \frac{b^3c}{4}\right) + x^3\left(a^2bd + ab^2c\right) + x^2\left(\frac{a^3d}{2} + \frac{3a^2bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c),x)
```

```
[Out] a**3*c*x + b**3*d*x**5/5 + x**4*(3*a*b**2*d/4 + b**3*c/4) + x**3*(a**2*b*d
+ a*b**2*c) + x**2*(a**3*d/2 + 3*a**2*b*c/2)
```

## 3.1132 $\int (a+bx)^2(c+dx)\,dx$

**Optimal**. Leaf size=38

$$\frac{(a+bx)^3(bc-ad)}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(a+bx)^3(bc-ad)}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^3)/(3*b^2) + (d*(a + b*x)^4)/(4*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^2}{b}+\frac{d(a+bx)^3}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^3}{3b^2}+\frac{d(a+bx)^4}{4b^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 46, normalized size = 1.21

$$\frac{1}{12}x\left(6a^2(2c+dx)+4abx(3c+2dx)+b^2x^2(4c+3dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x),x]
```

```
[Out] (x*(6*a^2*(2*c + d*x) + 4*a*b*x*(3*c + 2*d*x) + b^2*x^2*(4*c + 3*d*x)))/12
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(c + d*x), x]
```

**fricas [A]** time = 1.73, size = 49, normalized size = 1.29

$$\frac{1}{4}x^4db^2+\frac{1}{3}x^3cb^2+\frac{2}{3}x^3dba+x^2cba+\frac{1}{2}x^2da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*d*b^2 + 1/3*x^3*c*b^2 + 2/3*x^3*d*b*a + x^2*c*b*a + 1/2*x^2*d*a^2 +
 x*c*a^2
```

**giac [A]** time = 0.99, size = 49, normalized size = 1.29

$$\frac{1}{4}\,b^2dx^4+\frac{1}{3}\,b^2cx^3+\frac{2}{3}\,abdx^3+abcx^2+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*b^2*d*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + a*b*c*x^2 + 1/2*a^2*d*x^2 +
 a^2*c*x
```

**maple [A]** time = 0.00, size = 49, normalized size = 1.29

$$\frac{b^2d\,x^4}{4}+a^2cx+\frac{\left(2abd+b^2c\right)x^3}{3}+\frac{\left(a^2d+2abc\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c),x)
```

```
[Out] 1/4*b^2*d*x^4+1/3*(2*a*b*d+b^2*c)*x^3+1/2*(a^2*d+2*a*b*c)*x^2+a^2*c*x
```

**maxima [A]** time = 1.30, size = 48, normalized size = 1.26

$$\frac{1}{4}\,b^2dx^4+a^2cx+\frac{1}{3}\,\left(b^2c+2\,abd\right)x^3+\frac{1}{2}\,\left(2\,abc+a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d*x^4 + a^2*c*x + 1/3*(b^2*c + 2*a*b*d)*x^3 + 1/2*(2*a*b*c + a^2*d)
*x^2
```

**mupad [B]** time = 0.05, size = 47, normalized size = 1.24

$$x^2\left(\frac{d\,a^2}{2}+b\,c\,a\right)+x^3\left(\frac{c\,b^2}{3}+\frac{2\,a\,d\,b}{3}\right)+\frac{b^2\,d\,x^4}{4}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x),x)
```

```
[Out] x^2*((a^2*d)/2 + a*b*c) + x^3*((b^2*c)/3 + (2*a*b*d)/3) + (b^2*d*x^4)/4 + a
^2*c*x
```

**sympy [A]** time = 0.07, size = 49, normalized size = 1.29

$$a^2cx+\frac{b^2dx^4}{4}+x^3\left(\frac{2abd}{3}+\frac{b^2c}{3}\right)+x^2\left(\frac{a^2d}{2}+abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**4/4 + x**3*(2*a*b*d/3 + b**2*c/3) + x**2*(a**2*d/2 + a
*b*c)
```

## 3.1133 $\int (a+bx)(c+dx)\,dx$

**Optimal**. Leaf size=28

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

**Rubi** [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)\,dx &= \int \left(ac+(bc+ad)x+bdx^2\right)\,dx\\ &= acx+\frac{1}{2}(bc+ad)x^2+\frac{1}{3}bdx^3\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x),x]
[Out] IntegrateAlgebraic[(a + b*x)*(c + d*x), x]
```

**fricas [A]** time = 1.56, size = 26, normalized size = 0.93

$$\frac{1}{3}x^3db + \frac{1}{2}x^2cb + \frac{1}{2}x^2da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="fricas")
[Out] 1/3*x^3*d*b + 1/2*x^2*c*b + 1/2*x^2*d*a + x*c*a
```

**giac [A]** time = 0.99, size = 26, normalized size = 0.93

$$\frac{1}{3}\,bdx^3 + \frac{1}{2}\,bcx^2 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="giac")
[Out] 1/3*b*d*x^3 + 1/2*b*c*x^2 + 1/2*a*d*x^2 + a*c*x
```

**maple [A]** time = 0.00, size = 25, normalized size = 0.89

$$\frac{bd\,x^3}{3} + acx + \frac{(ad+bc)\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c),x)
[Out] a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

**maxima [A]** time = 1.35, size = 24, normalized size = 0.86

$$\frac{1}{3}\,bdx^3 + acx + \frac{1}{2}\,(bc+ad)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

**mupad [B]** time = 0.03, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^3}{3}+\left(\frac{a\,d}{2}+\frac{b\,c}{2}\right)\,x^2+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x),x)
```

```
[Out] x^2*((a*d)/2 + (b*c)/2) + a*c*x + (b*d*x^3)/3
```

**sympy [A]** time = 0.06, size = 26, normalized size = 0.93

$$acx+\frac{bdx^3}{3}+x^2\left(\frac{ad}{2}+\frac{bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x)
```

```
[Out] a*c*x + b*d*x**3/3 + x**2*(a*d/2 + b*c/2)
```

## 3.1134 $\int (c+dx)\,dx$

**Optimal**. Leaf size=12

$$cx+\frac{dx^2}{2}$$

**Rubi [A]** time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[c + d*x,x]
```

```
[Out] c*x + (d*x^2)/2
```

Rubi steps

$$\int (c+dx)\,dx=cx+\frac{dx^2}{2}$$

**Mathematica [A]** time = 0.00, size = 12, normalized size = 1.00

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[c + d*x,x]
```

```
[Out] c*x + (d*x^2)/2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[c + d*x,x]
```

```
[Out] IntegrateAlgebraic[c + d*x, x]
```

**fricas** [**A**] time = 1.46, size = 10, normalized size = 0.83

$$\frac{1}{2}x^2d + xc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="fricas")
[Out] 1/2*x^2*d + x*c
```

**giac** [**A**] time = 0.90, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="giac")
[Out] 1/2*d*x^2 + c*x
```

**maple** [**A**] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2}d\,x^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x+c,x)
[Out] 1/2*d*x^2+c*x
```

**maxima** [**A**] time = 1.35, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="maxima")
[Out] 1/2*d*x^2 + c*x
```

**mupad** [**B**] time = 0.02, size = 10, normalized size = 0.83

$$\frac{d\,x^2}{2} + c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c + d*x,x)
```

```
[Out] c*x + (d*x^2)/2
```

**sympy** [**A**] time = 0.06, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x)
```

```
[Out] c*x + d*x**2/2
```

## 3.1135 $\int \frac{c+dx}{a+bx} \, dx$

**Optimal**. Leaf size=25

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

**Rubi** [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a+bx} \, dx &= \int \left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right) dx\\ &= \frac{dx}{b}+\frac{(bc-ad)\log(a+bx)}{b^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x), x]
```

**fricas [A]** time = 1.70, size = 24, normalized size = 0.96

$$\frac{bdx+(bc-ad)\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*d*x + (b*c - a*d)*log(b*x + a))/b^2
```

**giac [A]** time = 0.96, size = 26, normalized size = 1.04

$$\frac{dx}{b}+\frac{(bc-ad)\log(|bx+a|)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] d*x/b + (b*c - a*d)*log(abs(b*x + a))/b^2
```

**maple [A]** time = 0.00, size = 32, normalized size = 1.28

$$-\frac{ad\ln(bx+a)}{b^2}+\frac{c\ln(bx+a)}{b}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a),x)
```

```
[Out] d*x/b-1/b^2*ln(b*x+a)*a*d+1/b*c*ln(b*x+a)
```

**maxima [A]** time = 1.25, size = 25, normalized size = 1.00

$$\frac{dx}{b}+\frac{(bc-ad)\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] d*x/b + (b*c - a*d)*log(b*x + a)/b^2
```

**mupad [B]** time = 0.05, size = 26, normalized size = 1.04

$$\frac{d\,x}{b}-\frac{\ln\left(a+b\,x\right)\,\left(a\,d-b\,c\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x),x)
```

```
[Out] (d*x)/b - (log(a + b*x)*(a*d - b*c))/b^2
```

**sympy [A]** time = 0.15, size = 20, normalized size = 0.80

$$\frac{dx}{b}-\frac{(ad-bc)\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x)
```

```
[Out] d*x/b - (a*d - b*c)*log(a + b*x)/b**2
```

## 3.1136 $\int \frac{c+dx}{(a+bx)^2} \, dx$

**Optimal**. Leaf size=32

$$\frac{d\log(a+bx)}{b^2} - \frac{bc-ad}{b^2(a+bx)}$$

**Rubi [A]** time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{d\log(a+bx)}{b^2} - \frac{bc-ad}{b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^2,x]

[Out] -((b*c - a*d)/(b^2*(a + b*x))) + (d*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^2}\,dx &= \int \left(\frac{bc-ad}{b(a+bx)^2} + \frac{d}{b(a+bx)}\right)dx\\ &= -\frac{bc-ad}{b^2(a+bx)} + \frac{d\log(a+bx)}{b^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 0.97

$$\frac{ad-bc}{b^2(a+bx)} + \frac{d\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^2,x]

[Out] (-(b*c) + a*d)/(b^2*(a + b*x)) + (d*Log[a + b*x])/b^2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x)^2,x]
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x)^2, x]
```

**fricas [A]** time = 1.68, size = 39, normalized size = 1.22

$$-\frac{bc-ad-(bdx+ad)\log(bx+a)}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="fricas")
[Out] -(b*c - a*d - (b*d*x + a*d)*log(b*x + a))/(b^3*x + a*b^2)
```

**giac [A]** time = 1.02, size = 57, normalized size = 1.78

$$-\frac{d\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{a}{(bx+a)b}\right)}{b}-\frac{c}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="giac")
[Out] -d*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b))/b - c/((b*x
 + a)*b)
```

**maple [A]** time = 0.00, size = 39, normalized size = 1.22

$$\frac{ad}{(bx+a)\,b^2}-\frac{c}{(bx+a)\,b}+\frac{d\ln(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^2,x)
[Out] d*ln(b*x+a)/b^2+1/b^2/(b*x+a)*a*d-1/b/(b*x+a)*c
```

**maxima** [A] time = 1.34, size = 35, normalized size = 1.09

$$-\frac{bc - ad}{b^3x + ab^2} + \frac{d\log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)/(b^3*x + a*b^2) + d*log(b*x + a)/b^2
```

**mupad** [B] time = 0.17, size = 31, normalized size = 0.97

$$\frac{a\,d - b\,c}{b^2\,(a + b\,x)} + \frac{d\,\ln(a + b\,x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^2,x)
```

```
[Out] (a*d - b*c)/(b^2*(a + b*x)) + (d*log(a + b*x))/b^2
```

**sympy** [A] time = 0.19, size = 27, normalized size = 0.84

$$\frac{ad - bc}{ab^2 + b^3x} + \frac{d\log(a + bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**2,x)
```

```
[Out] (a*d - b*c)/(a*b**2 + b**3*x) + d*log(a + b*x)/b**2
```

## 3.1137 $\int \frac{c+dx}{(a+bx)^3} \, dx$

**Optimal**. Leaf size=28

$$-\frac{(c+dx)^2}{2(a+bx)^2(bc-ad)}$$

**Rubi** [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {37}

$$-\frac{(c+dx)^2}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^3,x]
```

```
[Out] -(c + d*x)^2/(2*(b*c - a*d)*(a + b*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{c+dx}{(a+bx)^3} \, dx = -\frac{(c+dx)^2}{2(bc-ad)(a+bx)^2}$$

**Mathematica** [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^3,x]
```

```
[Out] -1/2*(a*d + b*(c + 2*d*x))/(b^2*(a + b*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x)^3,x]
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x)^3, x]
```

**fricas [A]** time = 1.75, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="fricas")
[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

**giac [A]** time = 1.06, size = 24, normalized size = 0.86

$$-\frac{2\,bdx+bc+ad}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="giac")
[Out] -1/2*(2*b*d*x + b*c + a*d)/((b*x + a)^2*b^2)
```

**maple [A]** time = 0.01, size = 35, normalized size = 1.25

$$-\frac{d}{(bx+a)\,b^2}-\frac{-ad+bc}{2\,(bx+a)^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^3,x)
[Out] -1/2*(-a*d+b*c)/b^2/(b*x+a)^2-d/b^2/(b*x+a)
```

**maxima [A]** time = 1.36, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

**mupad [B]** time = 0.16, size = 39, normalized size = 1.39

$$-\frac{\frac{a\,d+b\,c}{2\,b^2}+\frac{d\,x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^3,x)
```

```
[Out] -((a*d + b*c)/(2*b^2) + (d*x)/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

**sympy [A]** time = 0.26, size = 39, normalized size = 1.39

$$\frac{-ad-bc-2bdx}{2a^2b^2+4ab^3x+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**3,x)
```

```
[Out] (-a*d - b*c - 2*b*d*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

## 3.1138 $\int \frac{c+dx}{(a+bx)^4} dx$

**Optimal**. Leaf size=38

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^4,x]
```

```
[Out] -(b*c - a*d)/(3*b^2*(a + b*x)^3) - d/(2*b^2*(a + b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^4}\,dx&=\int \left(\frac{bc-ad}{b(a+bx)^4}+\frac{d}{b(a+bx)^3}\right)dx\\&=-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{ad+2bc+3bdx}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^4,x]
```

```
[Out] -1/6*(2*b*c + a*d + 3*b*d*x)/(b^2*(a + b*x)^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x)^4, x]
```

**fricas [A]** time = 1.75, size = 50, normalized size = 1.32

$$-\frac{3\,bdx+2\,bc+ad}{6\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

**giac [A]** time = 0.75, size = 25, normalized size = 0.66

$$-\frac{3\,bdx+2\,bc+ad}{6\,(bx+a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/((b*x + a)^3*b^2)
```

**maple [A]** time = 0.01, size = 35, normalized size = 0.92

$$-\frac{d}{2\,(bx+a)^2\,b^2}-\frac{-ad+bc}{3\,(bx+a)^3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^4,x)
```

```
[Out] -1/3*(-a*d+b*c)/b^2/(b*x+a)^3-1/2*d/b^2/(b*x+a)^2
```

**maxima [A]** time = 1.31, size = 50, normalized size = 1.32

$$-\frac{3\,bdx+2\,bc+ad}{6\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

**mupad [B]** time = 0.17, size = 52, normalized size = 1.37

$$-\frac{\frac{a\,d+2\,b\,c}{6\,b^2}+\frac{d\,x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^4,x)
```

```
[Out] -((a*d + 2*b*c)/(6*b^2) + (d*x)/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2
*b*x)
```

**sympy [A]** time = 0.34, size = 53, normalized size = 1.39

$$\frac{-ad-2bc-3bdx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**4,x)
```

```
[Out] (-a*d - 2*b*c - 3*b*d*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6
*b**5*x**3)
```

## 3.1139 $\int \frac{c+dx}{(a+bx)^5} \, dx$

**Optimal**. Leaf size=38

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

**Rubi [A]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] -(b*c - a*d)/(4*b^2*(a + b*x)^4) - d/(3*b^2*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^5}\,dx &= \int \left(\frac{bc-ad}{b(a+bx)^5}+\frac{d}{b(a+bx)^4}\right)dx\\ &= -\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 27, normalized size = 0.71

$$-\frac{ad+3bc+4bdx}{12b^2(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] -1/12*(3*b*c + a*d + 4*b*d*x)/(b^2*(a + b*x)^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{(a+bx)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x)^5, x]
```

**fricas [A]** time = 1.20, size = 61, normalized size = 1.61

$$-\frac{4\,bdx + 3\,bc + ad}{12\,\left(b^6x^4 + 4\,ab^5x^3 + 6\,a^2b^4x^2 + 4\,a^3b^3x + a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

**giac [A]** time = 0.92, size = 41, normalized size = 1.08

$$-\frac{c}{4\,(bx+a)^4b} - \frac{d}{3\,(bx+a)^3b^2} + \frac{ad}{4\,(bx+a)^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*c/((b*x + a)^4*b) - 1/3*d/((b*x + a)^3*b^2) + 1/4*a*d/((b*x + a)^4*b^2
)
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.92

$$-\frac{d}{3\,(bx+a)^3\,b^2} - \frac{-ad+bc}{4\,(bx+a)^4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^5,x)
```

```
[Out] -1/3*d/b^2/(b*x+a)^3-1/4*(-a*d+b*c)/b^2/(b*x+a)^4
```

**maxima** [A] time = 1.39, size = 61, normalized size = 1.61

$$-\frac{4\,bdx + 3\,bc + ad}{12\,\left(b^{6}x^{4} + 4\,ab^{5}x^{3} + 6\,a^{2}b^{4}x^{2} + 4\,a^{3}b^{3}x + a^{4}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

**mupad** [B] time = 0.04, size = 63, normalized size = 1.66

$$-\frac{\frac{a\,d+3\,b\,c}{12\,b^2} + \frac{d\,x}{3\,b}}{a^4 + 4\,a^3\,b\,x + 6\,a^2\,b^2\,x^2 + 4\,a\,b^3\,x^3 + b^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^5,x)
```

```
[Out] -((a*d + 3*b*c)/(12*b^2) + (d*x)/(3*b))/(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^
2*b^2*x^2 + 4*a^3*b*x)
```

**sympy** [B] time = 0.43, size = 65, normalized size = 1.71

$$\frac{-ad - 3bc - 4bdx}{12a^{4}b^{2} + 48a^{3}b^{3}x + 72a^{2}b^{4}x^{2} + 48ab^{5}x^{3} + 12b^{6}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**5,x)
```

```
[Out] (-a*d - 3*b*c - 4*b*d*x)/(12*a**4*b**2 + 48*a**3*b**3*x + 72*a**2*b**4*x**2
 + 48*a*b**5*x**3 + 12*b**6*x**4)
```

## 3.1140 $\int (a+bx)^4(c+dx)^2\,dx$

**Optimal**. Leaf size=65

$$\frac{d(a+bx)^6(bc-ad)}{3b^3}+\frac{(a+bx)^5(bc-ad)^2}{5b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

**Rubi [A]** time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d(a+bx)^6(bc-ad)}{3b^3}+\frac{(a+bx)^5(bc-ad)^2}{5b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^5)/(5*b^3) + (d*(b*c - a*d)*(a + b*x)^6)/(3*b^3) +
 (d^2*(a + b*x)^7)/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^4}{b^2}+\frac{2d(bc-ad)(a+bx)^5}{b^2}+\frac{d^2(a+bx)^6}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^5}{5b^3}+\frac{d(bc-ad)(a+bx)^6}{3b^3}+\frac{d^2(a+bx)^7}{7b^3}\end{aligned}$$

**Mathematica [B]** time = 0.03, size = 148, normalized size = 2.28

$$a^4c^2x+a^3cx^2(ad+2bc)+\frac{1}{5}b^2x^5\left(6a^2d^2+8abcd+b^2c^2\right)+abx^4\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{3}a^2x^3\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{1}{3}b^3dx^6(2ad+bc)+\frac{1}{7}b^4d^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] a^4*c^2*x + a^3*c*(2*b*c + a*d)*x^2 + (a^2*(6*b^2*c^2 + 8*a*b*c*d + a^2*d^2
)*x^3)/3 + a*b*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^4 + (b^2*(b^2*c^2 + 8*a*b*
c*d + 6*a^2*d^2)*x^5)/5 + (b^3*d*(b*c + 2*a*d)*x^6)/3 + (b^4*d^2*x^7)/7
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^4(c+dx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^2, x]
```

**fricas [B]** time = 1.11, size = 170, normalized size = 2.62

$$\frac{1}{7}x^7d^2b^4+\frac{1}{3}x^6dcb^4+\frac{2}{3}x^6d^2b^3a+\frac{1}{5}x^5c^2b^4+\frac{8}{5}x^5dcb^3a+\frac{6}{5}x^5d^2b^2a^2+x^4c^2b^3a+3x^4dcb^2a^2+x^4d^2ba^3+2x^3c^2b^2a^2+\frac{8}{3}x^3dcba^3+\frac{1}{3}x^3d^2a^4+2x^2c^2ba^3+x^2dca^4+xc^2a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^2*b^4 + 1/3*x^6*d*c*b^4 + 2/3*x^6*d^2*b^3*a + 1/5*x^5*c^2*b^4 + 8
/5*x^5*d*c*b^3*a + 6/5*x^5*d^2*b^2*a^2 + x^4*c^2*b^3*a + 3*x^4*d*c*b^2*a^2
+ x^4*d^2*b*a^3 + 2*x^3*c^2*b^2*a^2 + 8/3*x^3*d*c*b*a^3 + 1/3*x^3*d^2*a^4 +
 2*x^2*c^2*b*a^3 + x^2*d*c*a^4 + x*c^2*a^4
```

**giac [B]** time = 1.13, size = 170, normalized size = 2.62

$$\frac{1}{7}\,b^4d^2x^7+\frac{1}{3}\,b^4cdx^6+\frac{2}{3}\,ab^3d^2x^6+\frac{1}{5}\,b^4c^2x^5+\frac{8}{5}\,ab^3cdx^5+\frac{6}{5}\,a^2b^2d^2x^5+ab^3c^2x^4+3\,a^2b^2cdx^4+a^3bd^2x^4+2\,a^2b^2c^2x^3+\frac{8}{3}\,a^3bcdx^3+\frac{1}{3}\,a^4d^2x^3+2\,a^3bc^2x^2+a^4cdx^2+a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^4*d^2*x^7 + 1/3*b^4*c*d*x^6 + 2/3*a*b^3*d^2*x^6 + 1/5*b^4*c^2*x^5 + 8
/5*a*b^3*c*d*x^5 + 6/5*a^2*b^2*d^2*x^5 + a*b^3*c^2*x^4 + 3*a^2*b^2*c*d*x^4
+ a^3*b*d^2*x^4 + 2*a^2*b^2*c^2*x^3 + 8/3*a^3*b*c*d*x^3 + 1/3*a^4*d^2*x^3 +
 2*a^3*b*c^2*x^2 + a^4*c*d*x^2 + a^4*c^2*x
```

**maple [B]** time = 0.00, size = 163, normalized size = 2.51

$$\frac{b^4d^2x^7}{7}+a^4c^2x+\frac{\left(4a\,b^3d^2+2b^4cd\right)x^6}{6}+\frac{\left(6a^2b^2d^2+8a\,b^3cd+b^4c^2\right)x^5}{5}+\frac{\left(4a^3b\,d^2+12a^2b^2cd+4a\,b^3c^2\right)x^4}{4}+\frac{\left(a^4d^2+8a^3bcd+6a^2b^2c^2\right)x^3}{3}+\frac{\left(2a^4cd+4a^3b\,c^2\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^2,x)
```

```
[Out] 1/7*b^4*d^2*x^7+1/6*(4*a*b^3*d^2+2*b^4*c*d)*x^6+1/5*(6*a^2*b^2*d^2+8*a*b^3*
c*d+b^4*c^2)*x^5+1/4*(4*a^3*b*d^2+12*a^2*b^2*c*d+4*a*b^3*c^2)*x^4+1/3*(a^4*
d^2+8*a^3*b*c*d+6*a^2*b^2*c^2)*x^3+1/2*(2*a^4*c*d+4*a^3*b*c^2)*x^2+a^4*c^2*
x
```

**maxima [B]** time = 1.36, size = 156, normalized size = 2.40

$$\frac{1}{7}b^4d^2x^7 + a^4c^2x + \frac{1}{3}\left(b^4cd + 2\,ab^3d^2\right)x^6 + \frac{1}{5}\left(b^4c^2 + 8\,ab^3cd + 6\,a^2b^2d^2\right)x^5 + \left(ab^3c^2 + 3\,a^2b^2cd + a^3bd^2\right)x^4 + \frac{1}{3}\left(6\,a^2b^2c^2 + 8\,a^3bcd + a^4d^2\right)x^3 + \left(2\,a^3bc^2 + a^4cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^4*d^2*x^7 + a^4*c^2*x + 1/3*(b^4*c*d + 2*a*b^3*d^2)*x^6 + 1/5*(b^4*c^
2 + 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x^5 + (a*b^3*c^2 + 3*a^2*b^2*c*d + a^3*b*d
^2)*x^4 + 1/3*(6*a^2*b^2*c^2 + 8*a^3*b*c*d + a^4*d^2)*x^3 + (2*a^3*b*c^2 +
a^4*c*d)*x^2
```

**mupad [B]** time = 0.07, size = 144, normalized size = 2.22

$$x^3\left(\frac{a^4\,d^2}{3}+\frac{8\,a^3\,b\,c\,d}{3}+2\,a^2\,b^2\,c^2\right)+x^5\left(\frac{6\,a^2\,b^2\,d^2}{5}+\frac{8\,a\,b^3\,c\,d}{5}+\frac{b^4\,c^2}{5}\right)+a^4\,c^2\,x+\frac{b^4\,d^2\,x^7}{7}+a^3\,c\,x^2\,\left(a\,d+2\,b\,c\right)+\frac{b^3\,d\,x^6\,\left(2\,a\,d+b\,c\right)}{3}+a\,b\,x^4\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^2,x)
```

```
[Out] x^3*((a^4*d^2)/3 + 2*a^2*b^2*c^2 + (8*a^3*b*c*d)/3) + x^5*((b^4*c^2)/5 + (6
*a^2*b^2*d^2)/5 + (8*a*b^3*c*d)/5) + a^4*c^2*x + (b^4*d^2*x^7)/7 + a^3*c*x^
2*(a*d + 2*b*c) + (b^3*d*x^6*(2*a*d + b*c))/3 + a*b*x^4*(a^2*d^2 + b^2*c^2
+ 3*a*b*c*d)
```

**sympy [B]** time = 0.10, size = 168, normalized size = 2.58

$$a^{4}c^{2}x + \frac{b^{4}d^{2}x^{7}}{7} + x^{6}\left(\frac{2ab^{3}d^{2}}{3} + \frac{b^{4}cd}{3}\right) + x^{5}\left(\frac{6a^{2}b^{2}d^{2}}{5} + \frac{8ab^{3}cd}{5} + \frac{b^{4}c^{2}}{5}\right) + x^{4}\left(a^{3}bd^{2} + 3a^{2}b^{2}cd + ab^{3}c^{2}\right) + x^{3}\left(\frac{a^{4}d^{2}}{3} + \frac{8a^{3}bcd}{3} + 2a^{2}b^{2}c^{2}\right) + x^{2}\left(a^{4}cd + 2a^{3}bc^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**2,x)
```

```
[Out] a**4*c**2*x + b**4*d**2*x**7/7 + x**6*(2*a*b**3*d**2/3 + b**4*c*d/3) + x**5
*(6*a**2*b**2*d**2/5 + 8*a*b**3*c*d/5 + b**4*c**2/5) + x**4*(a**3*b*d**2 +
3*a**2*b**2*c*d + a*b**3*c**2) + x**3*(a**4*d**2/3 + 8*a**3*b*c*d/3 + 2*a**
2*b**2*c**2) + x**2*(a**4*c*d + 2*a**3*b*c**2)
```

# 3.1141 $\int (a+bx)^3(c+dx)^2\,dx$

**Optimal**. Leaf size=65

$$\frac{2d(a+bx)^5(bc-ad)}{5b^3}+\frac{(a+bx)^4(bc-ad)^2}{4b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

**Rubi** [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2d(a+bx)^5(bc-ad)}{5b^3}+\frac{(a+bx)^4(bc-ad)^2}{4b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^4)/(4*b^3) + (2*d*(b*c - a*d)*(a + b*x)^5)/(5*b^3)
 + (d^2*(a + b*x)^6)/(6*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^3}{b^2}+\frac{2d(bc-ad)(a+bx)^4}{b^2}+\frac{d^2(a+bx)^5}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^4}{4b^3}+\frac{2d(bc-ad)(a+bx)^5}{5b^3}+\frac{d^2(a+bx)^6}{6b^3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 122, normalized size = 1.88

$$a^3c^2x+\frac{1}{4}bx^4\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}ax^3\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{2}a^2cx^2(2ad+3bc)+\frac{1}{5}b^2dx^5(3ad+2bc)+\frac{1}{6}b^3d^2x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^2)/2 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^3)/3 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4 + (b^2*d*(2*b*c
+ 3*a*d)*x^5)/5 + (b^3*d^2*x^6)/6
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^2, x]
```

**fricas [B]** time = 1.56, size = 130, normalized size = 2.00

$$\frac{1}{6}x^6d^2b^3+\frac{2}{5}x^5dcb^3+\frac{3}{5}x^5d^2b^2a+\frac{1}{4}x^4c^2b^3+\frac{3}{2}x^4dcb^2a+\frac{3}{4}x^4d^2ba^2+x^3c^2b^2a+2x^3dcba^2+\frac{1}{3}x^3d^2a^3+\frac{3}{2}x^2c^2ba^2+x^2dca^3+xc^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d^2*b^3 + 2/5*x^5*d*c*b^3 + 3/5*x^5*d^2*b^2*a + 1/4*x^4*c^2*b^3 + 3
/2*x^4*d*c*b^2*a + 3/4*x^4*d^2*b*a^2 + x^3*c^2*b^2*a + 2*x^3*d*c*b*a^2 + 1/
3*x^3*d^2*a^3 + 3/2*x^2*c^2*b*a^2 + x^2*d*c*a^3 + x*c^2*a^3
```

**giac [B]** time = 1.13, size = 130, normalized size = 2.00

$$\frac{1}{6}\,b^3d^2x^6+\frac{2}{5}\,b^3cdx^5+\frac{3}{5}\,ab^2d^2x^5+\frac{1}{4}\,b^3c^2x^4+\frac{3}{2}\,ab^2cdx^4+\frac{3}{4}\,a^2bd^2x^4+ab^2c^2x^3+2\,a^2bcdx^3+\frac{1}{3}\,a^3d^2x^3+\frac{3}{2}\,a^2bc^2x^2+a^3cdx^2+a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*b^3*d^2*x^6 + 2/5*b^3*c*d*x^5 + 3/5*a*b^2*d^2*x^5 + 1/4*b^3*c^2*x^4 + 3
/2*a*b^2*c*d*x^4 + 3/4*a^2*b*d^2*x^4 + a*b^2*c^2*x^3 + 2*a^2*b*c*d*x^3 + 1/
3*a^3*d^2*x^3 + 3/2*a^2*b*c^2*x^2 + a^3*c*d*x^2 + a^3*c^2*x
```

**maple [B]** time = 0.00, size = 125, normalized size = 1.92

$$\frac{b^3d^2x^6}{6}+a^3c^2x+\frac{\left(3a\,b^2d^2+2b^3cd\right)x^5}{5}+\frac{\left(3a^2b\,d^2+6a\,b^2cd+b^3c^2\right)x^4}{4}+\frac{\left(a^3d^2+6a^2bcd+3a\,b^2c^2\right)x^3}{3}+\frac{\left(2a^3cd+3a^2b\,c^2\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^2,x)
```

```
[Out] 1/6*b^3*d^2*x^6+1/5*(3*a*b^2*d^2+2*b^3*c*d)*x^5+1/4*(3*a^2*b*d^2+6*a*b^2*c*
d+b^3*c^2)*x^4+1/3*(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2)*x^3+1/2*(2*a^3*c*d+3*a
^2*b*c^2)*x^2+a^3*c^2*x
```

**maxima** [B] time = 1.34, size = 124, normalized size = 1.91

$$\frac{1}{6}b^3d^2x^6 + a^3c^2x + \frac{1}{5}\left(2b^3cd + 3ab^2d^2\right)x^5 + \frac{1}{4}\left(b^3c^2 + 6ab^2cd + 3a^2bd^2\right)x^4 + \frac{1}{3}\left(3ab^2c^2 + 6a^2bcd + a^3d^2\right)x^3 + \frac{1}{2}\left(3a^2bc^2 + 2a^3cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*d^2*x^6 + a^3*c^2*x + 1/5*(2*b^3*c*d + 3*a*b^2*d^2)*x^5 + 1/4*(b^3*
c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^4 + 1/3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3
*d^2)*x^3 + 1/2*(3*a^2*b*c^2 + 2*a^3*c*d)*x^2
```

**mupad** [B] time = 0.05, size = 115, normalized size = 1.77

$$x^3\left(\frac{a^3d^2}{3} + 2a^2bcd + ab^2c^2\right) + x^4\left(\frac{3a^2bd^2}{4} + \frac{3ab^2cd}{2} + \frac{b^3c^2}{4}\right) + a^3c^2x + \frac{b^3d^2x^6}{6} + \frac{a^2cx^2\,(2ad + 3bc)}{2} + \frac{b^2dx^5\,(3ad + 2bc)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] x^3*((a^3*d^2)/3 + a*b^2*c^2 + 2*a^2*b*c*d) + x^4*((b^3*c^2)/4 + (3*a^2*b*d
^2)/4 + (3*a*b^2*c*d)/2) + a^3*c^2*x + (b^3*d^2*x^6)/6 + (a^2*c*x^2*(2*a*d
+ 3*b*c))/2 + (b^2*d*x^5*(3*a*d + 2*b*c))/5
```

**sympy** [B] time = 0.09, size = 133, normalized size = 2.05

$$a^3c^2x + \frac{b^3d^2x^6}{6} + x^5\left(\frac{3ab^2d^2}{5} + \frac{2b^3cd}{5}\right) + x^4\left(\frac{3a^2bd^2}{4} + \frac{3ab^2cd}{2} + \frac{b^3c^2}{4}\right) + x^3\left(\frac{a^3d^2}{3} + 2a^2bcd + ab^2c^2\right) + x^2\left(a^3cd + \frac{3a^2bc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**2,x)
```

```
[Out] a**3*c**2*x + b**3*d**2*x**6/6 + x**5*(3*a*b**2*d**2/5 + 2*b**3*c*d/5) + x*
*4*(3*a**2*b*d**2/4 + 3*a*b**2*c*d/2 + b**3*c**2/4) + x**3*(a**3*d**2/3 + 2
*a**2*b*c*d + a*b**2*c**2) + x**2*(a**3*c*d + 3*a**2*b*c**2/2)
```

## 3.1142 $\int (a+bx)^2(c+dx)^2\,dx$

**Optimal**. Leaf size=65

$$\frac{d(a+bx)^4(bc-ad)}{2b^3}+\frac{(a+bx)^3(bc-ad)^2}{3b^3}+\frac{d^2(a+bx)^5}{5b^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d(a+bx)^4(bc-ad)}{2b^3}+\frac{(a+bx)^3(bc-ad)^2}{3b^3}+\frac{d^2(a+bx)^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^3)/(3*b^3) + (d*(b*c - a*d)*(a + b*x)^4)/(2*b^3) +
 (d^2*(a + b*x)^5)/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^2}{b^2}+\frac{2d(bc-ad)(a+bx)^3}{b^2}+\frac{d^2(a+bx)^4}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^3}{3b^3}+\frac{d(bc-ad)(a+bx)^4}{2b^3}+\frac{d^2(a+bx)^5}{5b^3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 79, normalized size = 1.22

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{2}bdx^4(ad+bc)+acx^2(ad+bc)+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] a^2*c^2*x + a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3)/3 +
 (b*d*(b*c + a*d)*x^4)/2 + (b^2*d^2*x^5)/5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^2, x]
```

**fricas [A]** time = 1.33, size = 89, normalized size = 1.37

$$\frac{1}{5}x^5d^2b^2+\frac{1}{2}x^4dcb^2+\frac{1}{2}x^4d^2ba+\frac{1}{3}x^3c^2b^2+\frac{4}{3}x^3dcba+\frac{1}{3}x^3d^2a^2+x^2c^2ba+x^2dca^2+xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d^2*b^2 + 1/2*x^4*d*c*b^2 + 1/2*x^4*d^2*b*a + 1/3*x^3*c^2*b^2 + 4/3
*x^3*d*c*b*a + 1/3*x^3*d^2*a^2 + x^2*c^2*b*a + x^2*d*c*a^2 + x*c^2*a^2
```

**giac [A]** time = 0.80, size = 89, normalized size = 1.37

$$\frac{1}{5}\,b^2d^2x^5+\frac{1}{2}\,b^2cdx^4+\frac{1}{2}\,abd^2x^4+\frac{1}{3}\,b^2c^2x^3+\frac{4}{3}\,abcdx^3+\frac{1}{3}\,a^2d^2x^3+abc^2x^2+a^2cdx^2+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d^2*x^5 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/3*b^2*c^2*x^3 + 4/3
*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + a*b*c^2*x^2 + a^2*c*d*x^2 + a^2*c^2*x
```

**maple [A]** time = 0.00, size = 87, normalized size = 1.34

$$\frac{b^2d^2x^5}{5}+a^2c^2x+\frac{\left(2ab\,d^2+2b^2cd\right)x^4}{4}+\frac{\left(a^2d^2+4abcd+b^2c^2\right)x^3}{3}+\frac{\left(2a^2cd+2ab\,c^2\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^2,x)
```

```
[Out] 1/5*b^2*d^2*x^5+1/4*(2*a*b*d^2+2*b^2*c*d)*x^4+1/3*(a^2*d^2+4*a*b*c*d+b^2*c^
2)*x^3+1/2*(2*a^2*c*d+2*a*b*c^2)*x^2+a^2*c^2*x
```

**maxima** [A] time = 1.34, size = 81, normalized size = 1.25

$$\frac{1}{5}\,b^2d^2x^5 + a^2c^2x + \frac{1}{2}\left(b^2cd + abd^2\right)x^4 + \frac{1}{3}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^3 + \left(abc^2 + a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d^2*x^5 + a^2*c^2*x + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/3*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^3 + (a*b*c^2 + a^2*c*d)*x^2
```

**mupad** [B] time = 0.17, size = 74, normalized size = 1.14

$$x^3\left(\frac{a^2\,d^2}{3}+\frac{4\,a\,b\,c\,d}{3}+\frac{b^2\,c^2}{3}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^5}{5}+a\,c\,x^2\,\left(a\,d+b\,c\right)+\frac{b\,d\,x^4\,\left(a\,d+b\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] x^3*((a^2*d^2)/3 + (b^2*c^2)/3 + (4*a*b*c*d)/3) + a^2*c^2*x + (b^2*d^2*x^5)
/5 + a*c*x^2*(a*d + b*c) + (b*d*x^4*(a*d + b*c))/2
```

**sympy** [A] time = 0.08, size = 87, normalized size = 1.34

$$a^2c^2x + \frac{b^2d^2x^5}{5} + x^4\left(\frac{abd^2}{2} + \frac{b^2cd}{2}\right) + x^3\left(\frac{a^2d^2}{3} + \frac{4abcd}{3} + \frac{b^2c^2}{3}\right) + x^2\left(a^2cd + abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**5/5 + x**4*(a*b*d**2/2 + b**2*c*d/2) + x**3*(a**
2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x**2*(a**2*c*d + a*b*c**2)
```

## 3.1143 $\int (a+bx)(c+dx)^2\,dx$

**Optimal**. Leaf size=38

$$\frac{b(c+dx)^4}{4d^2}-\frac{(c+dx)^3(bc-ad)}{3d^2}$$

**Rubi [A]** time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^4}{4d^2}-\frac{(c+dx)^3(bc-ad)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^2,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^3)/(3*d^2) + (b*(c + d*x)^4)/(4*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^2\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^2}{d}+\frac{b(c+dx)^3}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^3}{3d^2}+\frac{b(c+dx)^4}{4d^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 47, normalized size = 1.24

$$\frac{1}{12}x\left(4dx^2(ad+2bc)+6cx(2ad+bc)+12ac^2+3bd^2x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^2,x]
```

```
[Out] (x*(12*a*c^2 + 6*c*(b*c + 2*a*d)*x + 4*d*(2*b*c + a*d)*x^2 + 3*b*d^2*x^3))/
12
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(c + d*x)^2, x]
```

**fricas [A]** time = 1.18, size = 49, normalized size = 1.29

$$\frac{1}{4}x^4d^2b+\frac{2}{3}x^3dcb+\frac{1}{3}x^3d^2a+\frac{1}{2}x^2c^2b+x^2dca+xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*d^2*b + 2/3*x^3*d*c*b + 1/3*x^3*d^2*a + 1/2*x^2*c^2*b + x^2*d*c*a +
 x*c^2*a
```

**giac [A]** time = 1.03, size = 49, normalized size = 1.29

$$\frac{1}{4}\,bd^2x^4+\frac{2}{3}\,bcdx^3+\frac{1}{3}\,ad^2x^3+\frac{1}{2}\,bc^2x^2+acdx^2+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*b*d^2*x^4 + 2/3*b*c*d*x^3 + 1/3*a*d^2*x^3 + 1/2*b*c^2*x^2 + a*c*d*x^2 +
 a*c^2*x
```

**maple [A]** time = 0.00, size = 49, normalized size = 1.29

$$\frac{b\,d^2x^4}{4}+a\,c^2x+\frac{\left(a\,d^2+2bcd\right)x^3}{3}+\frac{\left(2acd+b\,c^2\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^2,x)
```

```
[Out] 1/4*b*d^2*x^4+1/3*(a*d^2+2*b*c*d)*x^3+1/2*(2*a*c*d+b*c^2)*x^2+a*c^2*x
```

**maxima [A]** time = 1.31, size = 48, normalized size = 1.26

$$\frac{1}{4}\,bd^2x^4+ac^2x+\frac{1}{3}\left(2\,bcd+ad^2\right)x^3+\frac{1}{2}\left(bc^2+2\,acd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*d^2*x^4 + a*c^2*x + 1/3*(2*b*c*d + a*d^2)*x^3 + 1/2*(b*c^2 + 2*a*c*d)
*x^2
```

**mupad [B]** time = 0.04, size = 47, normalized size = 1.24

$$x^2\left(\frac{b\,c^2}{2}+a\,d\,c\right)+x^3\left(\frac{a\,d^2}{3}+\frac{2\,b\,c\,d}{3}\right)+\frac{b\,d^2\,x^4}{4}+a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^2,x)
```

```
[Out] x^2*((b*c^2)/2 + a*c*d) + x^3*((a*d^2)/3 + (2*b*c*d)/3) + (b*d^2*x^4)/4 + a
*c^2*x
```

**sympy [A]** time = 0.07, size = 49, normalized size = 1.29

$$ac^2x+\frac{bd^2x^4}{4}+x^3\left(\frac{ad^2}{3}+\frac{2bcd}{3}\right)+x^2\left(acd+\frac{bc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**4/4 + x**3*(a*d**2/3 + 2*b*c*d/3) + x**2*(a*c*d + b*c*
*2/2)
```

## 3.1144 $\int (c+dx)^2\,dx$

**Optimal**. Leaf size=14

$$\frac{(c+dx)^3}{3d}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2,x]
```

```
[Out] (c + d*x)^3/(3*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^2\,dx = \frac{(c+dx)^3}{3d}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2,x]
```

```
[Out] (c + d*x)^3/(3*d)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2, x]
```

**fricas [A]** time = 1.25, size = 20, normalized size = 1.43

$$\frac{1}{3}x^3d^2 + x^2dc + xc^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*d^2 + x^2*d*c + x*c^2
```

**giac [A]** time = 0.96, size = 12, normalized size = 0.86

$$\frac{(dx + c)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(d*x + c)^3/d
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{(dx + c)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2,x)
```

```
[Out] 1/3*(d*x+c)^3/d
```

**maxima [A]** time = 1.31, size = 20, normalized size = 1.43

$$\frac{1}{3}\,d^2x^3 + cdx^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x
```

**mupad [B]** time = 0.03, size = 20, normalized size = 1.43

$$c^2\,x+c\,d\,x^2+\frac{d^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2,x)
```

```
[Out] c^2*x + (d^2*x^3)/3 + c*d*x^2
```

**sympy [B]** time = 0.06, size = 19, normalized size = 1.36

$$c^2x+cdx^2+\frac{d^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2,x)
```

```
[Out] c**2*x + c*d*x**2 + d**2*x**3/3
```

## 3.1145 $\int \frac{(c+dx)^2}{a+bx}\,dx$

**Optimal**. Leaf size=49

$$\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{dx(bc-ad)}{b^2}+\frac{(c+dx)^2}{2b}$$

**Rubi [A]** time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{dx(bc-ad)}{b^2}+\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{(c+dx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b*c - a*d)^2*Log[a + b*x])/b^
3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx\\ &= \frac{d(bc-ad)x}{b^2}+\frac{(c+dx)^2}{2b}+\frac{(bc-ad)^2\log(a+bx)}{b^3}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 43, normalized size = 0.88

$$\frac{bdx(-2ad+4bc+bdx)+2(bc-ad)^2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x),x]
```

```
[Out] (b*d*x*(4*b*c - 2*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[a + b*x])/(2*b^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x), x]
```

**fricas [A]** time = 0.84, size = 63, normalized size = 1.29

$$\frac{b^2d^2x^2 + 2\left(2\,b^2cd - abd^2\right)x + 2\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(bx+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*(2*b^2*c*d - a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(b*x + a))/b^3
```

**giac [A]** time = 1.20, size = 60, normalized size = 1.22

$$\frac{bd^2x^2 + 4\,bcdx - 2\,ad^2x}{2\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(|bx+a|\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2 + 4*b*c*d*x - 2*a*d^2*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(b*x + a))/b^3
```

**maple [A]** time = 0.00, size = 74, normalized size = 1.51

$$\frac{d^2x^2}{2b} + \frac{a^2d^2\ln\left(bx+a\right)}{b^3} - \frac{2acd\ln\left(bx+a\right)}{b^2} - \frac{a\,d^2x}{b^2} + \frac{c^2\ln\left(bx+a\right)}{b} + \frac{2cdx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a),x)
```

```
[Out] 1/2*d^2/b*x^2-d^2/b^2*a*x+2*d/b*x*c+1/b^3*ln(b*x+a)*a^2*d^2-2/b^2*ln(b*x+a)
*a*c*d+1/b*ln(b*x+a)*c^2
```

**maxima [A]** time = 1.36, size = 61, normalized size = 1.24

$$\frac{bd^2x^2 + 2\left(2\,bcd - ad^2\right)x}{2\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*d^2*x^2 + 2*(2*b*c*d - a*d^2)*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(b*x + a)/b^3
```

**mupad [B]** time = 0.19, size = 62, normalized size = 1.27

$$\frac{\ln\left(a + b\,x\right)\,\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{b^3} - x\,\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right) + \frac{d^2\,x^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x),x)
```

```
[Out] (log(a + b*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/b^3 - x*((a*d^2)/b^2 - (2*c*
d)/b) + (d^2*x^2)/(2*b)
```

**sympy [A]** time = 0.22, size = 44, normalized size = 0.90

$$x\left(-\frac{ad^2}{b^2} + \frac{2cd}{b}\right) + \frac{d^2x^2}{2b} + \frac{\left(ad - bc\right)^2\log\left(a + bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + d**2*x**2/(2*b) + (a*d - b*c)**2*log(a + b*x)/
b**3
```

## 3.1146 $\int \frac{(c+dx)^2}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=51

$$-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}+\frac{d^2x}{b^2}$$

**Rubi [A]** time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] (d^2*x)/b^2 - (b*c - a*d)^2/(b^3*(a + b*x)) + (2*d*(b*c - a*d)*Log[a + b*x]
)/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^2}\,dx&=\int \left(\frac{d^2}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)^2}+\frac{2d(bc-ad)}{b^2(a+bx)}\right)dx\\&=\frac{d^2x}{b^2}-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 47, normalized size = 0.92

$$\frac{-\frac{(bc-ad)^2}{a+bx}+2d(bc-ad)\log(a+bx)+bd^2x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] (b*d^2*x - (b*c - a*d)^2/(a + b*x) + 2*d*(b*c - a*d)*Log[a + b*x])/b^3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^2, x]
```

**fricas [A]** time = 1.42, size = 92, normalized size = 1.80

$$\frac{b^2d^2x^2 + abd^2x - b^2c^2 + 2\,abcd - a^2d^2 + 2\left(abcd - a^2d^2 + \left(b^2cd - abd^2\right)x\right)\log\left(bx+a\right)}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + a*b*d^2*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 + 2*(a*b*c*d - a^2
*d^2 + (b^2*c*d - a*b*d^2)*x)*log(b*x + a))/(b^4*x + a*b^3)
```

**giac [A]** time = 0.94, size = 98, normalized size = 1.92

$$\frac{(bx+a)d^2}{b^3} - \frac{2\left(bcd - ad^2\right)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3} - \frac{\frac{b^3c^2}{bx+a} - \frac{2\,ab^2cd}{bx+a} + \frac{a^2bd^2}{bx+a}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)*d^2/b^3 - 2*(b*c*d - a*d^2)*log(abs(b*x + a)/((b*x + a)^2*abs(b))
)/b^3 - (b^3*c^2/(b*x + a) - 2*a*b^2*c*d/(b*x + a) + a^2*b*d^2/(b*x + a))/b
^4
```

**maple [A]** time = 0.01, size = 86, normalized size = 1.69

$$-\frac{a^2d^2}{(bx+a)\,b^3} + \frac{2acd}{(bx+a)\,b^2} - \frac{2a\,d^2\ln\left(bx+a\right)}{b^3} - \frac{c^2}{(bx+a)\,b} + \frac{2cd\ln\left(bx+a\right)}{b^2} + \frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^2,x)
```

```
[Out] d^2*x/b^2-2/b^3*d^2*ln(b*x+a)*a+2/b^2*d*ln(b*x+a)*c-1/b^3/(b*x+a)*a^2*d^2+2
/b^2/(b*x+a)*a*c*d-1/b/(b*x+a)*c^2
```

**maxima [A]** time = 1.37, size = 67, normalized size = 1.31

$$\frac{d^2x}{b^2} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{b^4x + ab^3} + \frac{2\left(bcd - ad^2\right)\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] d^2*x/b^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x + a*b^3) + 2*(b*c*d - a*
d^2)*log(b*x + a)/b^3
```

**mupad [B]** time = 0.20, size = 71, normalized size = 1.39

$$\frac{d^2\,x}{b^2} - \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{b\,\left(x\,b^3 + a\,b^2\right)} - \frac{\ln\left(a + b\,x\right)\,\left(2\,a\,d^2 - 2\,b\,c\,d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^2,x)
```

```
[Out] (d^2*x)/b^2 - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(b*(a*b^2 + b^3*x)) - (log(a
+ b*x)*(2*a*d^2 - 2*b*c*d))/b^3
```

**sympy [A]** time = 0.34, size = 60, normalized size = 1.18

$$\frac{-a^2d^2 + 2abcd - b^2c^2}{ab^3 + b^4x} + \frac{d^2x}{b^2} - \frac{2d\left(ad - bc\right)\log\left(a + bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**2,x)
```

```
[Out] (-a**2*d**2 + 2*a*b*c*d - b**2*c**2)/(a*b**3 + b**4*x) + d**2*x/b**2 - 2*d*
(a*d - b*c)*log(a + b*x)/b**3
```

## 3.1147 $\int \frac{(c+dx)^2}{(a+bx)^3} dx$

**Optimal**. Leaf size=59

$$-\frac{2d(bc-ad)}{b^3(a+bx)}-\frac{(bc-ad)^2}{2b^3(a+bx)^2}+\frac{d^2\log(a+bx)}{b^3}$$

**Rubi [A]** time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2d(bc-ad)}{b^3(a+bx)}-\frac{(bc-ad)^2}{2b^3(a+bx)^2}+\frac{d^2\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^3,x]
```

```
[Out] -(b*c - a*d)^2/(2*b^3*(a + b*x)^2) - (2*d*(b*c - a*d))/(b^3*(a + b*x)) + (d
^2*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^3}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^3}+\frac{2d(bc-ad)}{b^2(a+bx)^2}+\frac{d^2}{b^2(a+bx)}\right) dx\\ &= -\frac{(bc-ad)^2}{2b^3(a+bx)^2}-\frac{2d(bc-ad)}{b^3(a+bx)}+\frac{d^2\log(a+bx)}{b^3}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 0.83

$$\frac{2d^2\log(a+bx)-\frac{(bc-ad)(3ad+b(c+4dx))}{(a+bx)^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^3,x]
```

```
[Out] (-(((b*c - a*d)*(3*a*d + b*(c + 4*d*x)))/(a + b*x)^2) + 2*d^2*Log[a + b*x])
/(2*b^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^3, x]
```

**fricas [A]** time = 1.14, size = 99, normalized size = 1.68

$$-\frac{b^2c^2 + 2\,abcd - 3\,a^2d^2 + 4\left(b^2cd - abd^2\right)x - 2\left(b^2d^2x^2 + 2\,abd^2x + a^2d^2\right)\log\left(bx+a\right)}{2\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3
)
```

**giac [A]** time = 1.03, size = 68, normalized size = 1.15

$$\frac{d^2\log\left(|bx+a|\right)}{b^3} - \frac{4\left(bcd - ad^2\right)x + \frac{b^2c^2 + 2\,abcd - 3\,a^2d^2}{b}}{2\left(bx+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] d^2*log(abs(b*x + a))/b^3 - 1/2*(4*(b*c*d - a*d^2)*x + (b^2*c^2 + 2*a*b*c*d
 - 3*a^2*d^2)/b)/((b*x + a)^2*b^2)
```

**maple [A]** time = 0.01, size = 92, normalized size = 1.56

$$-\frac{a^2d^2}{2\left(bx+a\right)^2b^3} + \frac{acd}{\left(bx+a\right)^2b^2} - \frac{c^2}{2\left(bx+a\right)^2b} + \frac{2a\,d^2}{\left(bx+a\right)b^3} - \frac{2cd}{\left(bx+a\right)b^2} + \frac{d^2\ln\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^3,x)
```

```
[Out] -1/2/b^3/(b*x+a)^2*a^2*d^2+1/b^2/(b*x+a)^2*a*c*d-1/2/b/(b*x+a)^2*c^2+d^2*ln
(b*x+a)/b^3+2/b^3*d^2/(b*x+a)*a-2/b^2*d/(b*x+a)*c
```

**maxima [A]** time = 1.30, size = 79, normalized size = 1.34

$$-\frac{b^2c^2 + 2\,abcd - 3\,a^2d^2 + 4\left(b^2cd - abd^2\right)x}{2\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)} + \frac{d^2\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3) + d^2*log(b*x + a)/b^3
```

**mupad [B]** time = 0.20, size = 77, normalized size = 1.31

$$\frac{d^2\,\ln\left(a+b\,x\right)}{b^3}-\frac{\frac{-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2}{2\,b^3}-\frac{2\,d\,x\,\left(a\,d-b\,c\right)}{b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^3,x)
```

```
[Out] (d^2*log(a + b*x))/b^3 - ((b^2*c^2 - 3*a^2*d^2 + 2*a*b*c*d)/(2*b^3) - (2*d*
x*(a*d - b*c))/b^2)/(a^2 + b^2*x^2 + 2*a*b*x)
```

**sympy [A]** time = 0.45, size = 80, normalized size = 1.36

$$\frac{3a^2d^2 - 2abcd - b^2c^2 + x\left(4abd^2 - 4b^2cd\right)}{2a^2b^3 + 4ab^4x + 2b^5x^2} + \frac{d^2\log\left(a + bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**3,x)
```

```
[Out] (3*a**2*d**2 - 2*a*b*c*d - b**2*c**2 + x*(4*a*b*d**2 - 4*b**2*c*d))/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + d**2*log(a + b*x)/b**3
```

## 3.1148 $\int \frac{(c+dx)^2}{(a+bx)^4}\,dx$

**Optimal**. Leaf size=28

$$-\frac{(c+dx)^3}{3(a+bx)^3(bc-ad)}$$

**Rubi** [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^3}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] -(c + d*x)^3/(3*(b*c - a*d)*(a + b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+bx)^4}\,dx = -\frac{(c+dx)^3}{3(bc-ad)(a+bx)^3}$$

**Mathematica** [A] time = 0.02, size = 53, normalized size = 1.89

$$-\frac{a^2d^2+abd(c+3dx)+b^2\left(c^2+3cdx+3d^2x^2\right)}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] -1/3*(a^2*d^2 + a*b*d*(c + 3*d*x) + b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))/(b^3*(
a + b*x)^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^4, x]
```

**fricas [B]** time = 1.39, size = 84, normalized size = 3.00

$$-\frac{3\,b^2d^2x^2+b^2c^2+abcd+a^2d^2+3\left(b^2cd+abd^2\right)x}{3\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

**giac [B]** time = 0.96, size = 59, normalized size = 2.11

$$-\frac{3\,b^2d^2x^2+3\,b^2cdx+3\,abd^2x+b^2c^2+abcd+a^2d^2}{3\,(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + 3*b^2*c*d*x + 3*a*b*d^2*x + b^2*c^2 + a*b*c*d + a^2*d
^2)/((b*x + a)^3*b^3)
```

**maple [B]** time = 0.01, size = 70, normalized size = 2.50

$$-\frac{d^2}{(bx+a)\,b^3}+\frac{(ad-bc)\,d}{(bx+a)^2\,b^3}-\frac{a^2d^2-2abcd+b^2c^2}{3\,(bx+a)^3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^4,x)
```

```
[Out] -1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^3+d*(a*d-b*c)/b^3/(b*x+a)^2-d^
2/b^3/(b*x+a)
```

**maxima [B]** time = 1.37, size = 84, normalized size = 3.00

$$-\frac{3\,b^2d^2x^2+b^2c^2+abcd+a^2d^2+3\left(b^2cd+abd^2\right)x}{3\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

**mupad [B]** time = 0.04, size = 80, normalized size = 2.86

$$-\frac{\frac{a^2\,d^2+a\,b\,c\,d+b^2\,c^2}{3\,b^3}+\frac{d^2\,x^2}{b}+\frac{d\,x\,(a\,d+b\,c)}{b^2}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^4,x)
```

```
[Out] -((a^2*d^2 + b^2*c^2 + a*b*c*d)/(3*b^3) + (d^2*x^2)/b + (d*x*(a*d + b*c))/b
^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

**sympy [B]** time = 0.60, size = 88, normalized size = 3.14

$$\frac{-a^2d^2-abcd-b^2c^2-3b^2d^2x^2+x\left(-3abd^2-3b^2cd\right)}{3a^3b^3+9a^2b^4x+9ab^5x^2+3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**4,x)
```

```
[Out] (-a**2*d**2 - a*b*c*d - b**2*c**2 - 3*b**2*d**2*x**2 + x*(-3*a*b*d**2 - 3*b
**2*c*d))/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**2 + 3*b**6*x**3)
```

# 3.1149 $\int \frac{(c+dx)^2}{(a+bx)^5} dx$

**Optimal**. Leaf size=65

$$-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{d^2}{2b^3(a+bx)^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{d^2}{2b^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] -(b*c - a*d)^2/(4*b^3*(a + b*x)^4) - (2*d*(b*c - a*d))/(3*b^3*(a + b*x)^3)
- d^2/(2*b^3*(a + b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^5}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^5}+\frac{2d(bc-ad)}{b^2(a+bx)^4}+\frac{d^2}{b^2(a+bx)^3}\right)dx\\ &= -\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{d^2}{2b^3(a+bx)^2}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 56, normalized size = 0.86

$$-\frac{a^2d^2+2abd(c+2dx)+b^2\left(3c^2+8cdx+6d^2x^2\right)}{12b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] -1/12*(a^2*d^2 + 2*a*b*d*(c + 2*d*x) + b^2*(3*c^2 + 8*c*d*x + 6*d^2*x^2))/(
b^3*(a + b*x)^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^5, x]
```

**fricas [A]** time = 1.49, size = 98, normalized size = 1.51

$$-\frac{6\,b^2d^2x^2+3\,b^2c^2+2\,abcd+a^2d^2+4\left(2\,b^2cd+abd^2\right)x}{12\left(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

**giac [A]** time = 1.13, size = 96, normalized size = 1.48

$$-\frac{\frac{3\,c^2}{(bx+a)^4}+\frac{8\,cd}{(bx+a)^3b}-\frac{6\,acd}{(bx+a)^4b}+\frac{6\,d^2}{(bx+a)^2b^2}-\frac{8\,ad^2}{(bx+a)^3b^2}+\frac{3\,a^2d^2}{(bx+a)^4b^2}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/12*(3*c^2/(b*x + a)^4 + 8*c*d/((b*x + a)^3*b) - 6*a*c*d/((b*x + a)^4*b)
+ 6*d^2/((b*x + a)^2*b^2) - 8*a*d^2/((b*x + a)^3*b^2) + 3*a^2*d^2/((b*x + a
)^4*b^2))/b
```

**maple [A]** time = 0.01, size = 71, normalized size = 1.09

$$-\frac{d^2}{2\,(bx+a)^2\,b^3}+\frac{2\,(ad-bc)\,d}{3\,(bx+a)^3\,b^3}-\frac{a^2d^2-2abcd+b^2c^2}{4\,(bx+a)^4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^5,x)
```

```
[Out] 2/3*d*(a*d-b*c)/b^3/(b*x+a)^3-1/2*d^2/b^3/(b*x+a)^2-1/4*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/b^3/(b*x+a)^4
```

**maxima [A]** time = 1.33, size = 98, normalized size = 1.51

$$-\frac{6\,b^2d^2x^2+3\,b^2c^2+2\,abcd+a^2d^2+4\left(2\,b^2cd+abd^2\right)x}{12\left(b^7x^4+4\,ab^6x^3+6\,a^2b^5x^2+4\,a^3b^4x+a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

**mupad [B]** time = 0.19, size = 39, normalized size = 0.60

$$\frac{{\left(c+d\,x\right)}^3\,\left(4\,a\,d-3\,b\,c+b\,d\,x\right)}{12\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^5,x)
```

```
[Out] ((c + d*x)^3*(4*a*d - 3*b*c + b*d*x))/(12*(a*d - b*c)^2*(a + b*x)^4)
```

**sympy [A]** time = 0.76, size = 104, normalized size = 1.60

$$\frac{-a^2d^2-2abcd-3b^2c^2-6b^2d^2x^2+x\left(-4abd^2-8b^2cd\right)}{12a^4b^3+48a^3b^4x+72a^2b^5x^2+48ab^6x^3+12b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**5,x)
```

```
[Out] (-a**2*d**2 - 2*a*b*c*d - 3*b**2*c**2 - 6*b**2*d**2*x**2 + x*(-4*a*b*d**2 -
 8*b**2*c*d))/(12*a**4*b**3 + 48*a**3*b**4*x + 72*a**2*b**5*x**2 + 48*a*b**
6*x**3 + 12*b**7*x**4)
```

## 3.1150 $\int \frac{(c+dx)^2}{(a+bx)^6} dx$

**Optimal**. Leaf size=65

$$-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d^2}{3b^3(a+bx)^3}$$

**Rubi** [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] -(b*c - a*d)^2/(5*b^3*(a + b*x)^5) - (d*(b*c - a*d))/(2*b^3*(a + b*x)^4) -
d^2/(3*b^3*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^6} dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^6}+\frac{2d(bc-ad)}{b^2(a+bx)^5}+\frac{d^2}{b^2(a+bx)^4}\right) dx\\ &= -\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{d^2}{3b^3(a+bx)^3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 57, normalized size = 0.88

$$-\frac{a^2d^2+abd(3c+5dx)+b^2\left(6c^2+15cdx+10d^2x^2\right)}{30b^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] -1/30*(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^2*(6*c^2 + 15*c*d*x + 10*d^2*x^2))
/(b^3*(a + b*x)^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^6, x]
```

**fricas [A]** time = 1.34, size = 109, normalized size = 1.68

$$-\frac{10\,b^2d^2x^2+6\,b^2c^2+3\,abcd+a^2d^2+5\left(3\,b^2cd+abd^2\right)x}{30\left(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

**giac [A]** time = 0.87, size = 61, normalized size = 0.94

$$-\frac{10\,b^2d^2x^2+15\,b^2cdx+5\,abd^2x+6\,b^2c^2+3\,abcd+a^2d^2}{30\,(bx+a)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 15*b^2*c*d*x + 5*a*b*d^2*x + 6*b^2*c^2 + 3*a*b*c*d
+ a^2*d^2)/((b*x + a)^5*b^3)
```

**maple [A]** time = 0.01, size = 71, normalized size = 1.09

$$-\frac{d^2}{3\,(bx+a)^3\,b^3}+\frac{(ad-bc)\,d}{2\,(bx+a)^4\,b^3}-\frac{a^2d^2-2abcd+b^2c^2}{5\,(bx+a)^5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^6,x)
```

```
[Out] -1/3*d^2/b^3/(b*x+a)^3-1/5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5+1/2*d*
(a*d-b*c)/b^3/(b*x+a)^4
```

**maxima [A]** time = 1.35, size = 109, normalized size = 1.68

$$-\frac{10\,b^2d^2x^2+6\,b^2c^2+3\,abcd+a^2d^2+5\left(3\,b^2cd+abd^2\right)x}{30\left(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

**mupad [B]** time = 0.20, size = 107, normalized size = 1.65

$$-\frac{\frac{a^2\,d^2+3\,a\,b\,c\,d+6\,b^2\,c^2}{30\,b^3}+\frac{d^2\,x^2}{3\,b}+\frac{d\,x\,\left(a\,d+3\,b\,c\right)}{6\,b^2}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^6,x)
```

```
[Out] -((a^2*d^2 + 6*b^2*c^2 + 3*a*b*c*d)/(30*b^3) + (d^2*x^2)/(3*b) + (d*x*(a*d
+ 3*b*c))/(6*b^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b
^3*x^3 + 5*a^4*b*x)
```

**sympy [B]** time = 0.96, size = 116, normalized size = 1.78

$$\frac{-a^2d^2-3abcd-6b^2c^2-10b^2d^2x^2+x\left(-5abd^2-15b^2cd\right)}{30a^5b^3+150a^4b^4x+300a^3b^5x^2+300a^2b^6x^3+150ab^7x^4+30b^8x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**6,x)
```

```
[Out] (-a**2*d**2 - 3*a*b*c*d - 6*b**2*c**2 - 10*b**2*d**2*x**2 + x*(-5*a*b*d**2
- 15*b**2*c*d))/(30*a**5*b**3 + 150*a**4*b**4*x + 300*a**3*b**5*x**2 + 300*
a**2*b**6*x**3 + 150*a*b**7*x**4 + 30*b**8*x**5)
```

## 3.1151 $\int \frac{(c+dx)^2}{(a+bx)^7} dx$

**Optimal**. Leaf size=65

$$-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{d^2}{4b^3(a+bx)^4}$$

**Rubi** [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{d^2}{4b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] -(b*c - a*d)^2/(6*b^3*(a + b*x)^6) - (2*d*(b*c - a*d))/(5*b^3*(a + b*x)^5)
- d^2/(4*b^3*(a + b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^7} dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^7}+\frac{2d(bc-ad)}{b^2(a+bx)^6}+\frac{d^2}{b^2(a+bx)^5}\right) dx\\ &= -\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{d^2}{4b^3(a+bx)^4}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 58, normalized size = 0.89

$$-\frac{a^2d^2+2abd(2c+3dx)+b^2\left(10c^2+24cdx+15d^2x^2\right)}{60b^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^2*d^2 + 2*a*b*d*(2*c + 3*d*x) + b^2*(10*c^2 + 24*c*d*x + 15*d^2*x^
2))/(b^3*(a + b*x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^7, x]
```

**fricas [B]** time = 1.33, size = 120, normalized size = 1.85

$$-\frac{15\,b^2d^2x^2+10\,b^2c^2+4\,abcd+a^2d^2+6\left(4\,b^2cd+abd^2\right)x}{60\left(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

**giac [A]** time = 1.04, size = 61, normalized size = 0.94

$$-\frac{15\,b^2d^2x^2+24\,b^2cdx+6\,abd^2x+10\,b^2c^2+4\,abcd+a^2d^2}{60\,(bx+a)^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 24*b^2*c*d*x + 6*a*b*d^2*x + 10*b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)/((b*x + a)^6*b^3)
```

**maple [A]** time = 0.00, size = 71, normalized size = 1.09

$$-\frac{d^2}{4\,(bx+a)^4\,b^3}+\frac{2\,(ad-bc)\,d}{5\,(bx+a)^5\,b^3}-\frac{a^2d^2-2abcd+b^2c^2}{6\,(bx+a)^6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^7,x)
```

```
[Out] 2/5*d*(a*d-b*c)/b^3/(b*x+a)^5-1/4*d^2/b^3/(b*x+a)^4-1/6*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/b^3/(b*x+a)^6
```

**maxima [B]** time = 1.39, size = 120, normalized size = 1.85

$$-\frac{15\,b^2d^2x^2+10\,b^2c^2+4\,abcd+a^2d^2+6\left(4\,b^2cd+abd^2\right)x}{60\left(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

**mupad [B]** time = 0.09, size = 118, normalized size = 1.82

$$-\frac{\frac{a^2\,d^2+4\,a\,b\,c\,d+10\,b^2\,c^2}{60\,b^3}+\frac{d^2\,x^2}{4\,b}+\frac{d\,x\,\left(a\,d+4\,b\,c\right)}{10\,b^2}}{a^6+6\,a^5\,b\,x+15\,a^4\,b^2\,x^2+20\,a^3\,b^3\,x^3+15\,a^2\,b^4\,x^4+6\,a\,b^5\,x^5+b^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^7,x)
```

```
[Out] -((a^2*d^2 + 10*b^2*c^2 + 4*a*b*c*d)/(60*b^3) + (d^2*x^2)/(4*b) + (d*x*(a*d
 + 4*b*c))/(10*b^2))/(a^6 + b^6*x^6 + 6*a*b^5*x^5 + 15*a^4*b^2*x^2 + 20*a^3
*b^3*x^3 + 15*a^2*b^4*x^4 + 6*a^5*b*x)
```

**sympy [B]** time = 1.16, size = 128, normalized size = 1.97

$$\frac{-a^2d^2-4abcd-10b^2c^2-15b^2d^2x^2+x\left(-6abd^2-24b^2cd\right)}{60a^6b^3+360a^5b^4x+900a^4b^5x^2+1200a^3b^6x^3+900a^2b^7x^4+360ab^8x^5+60b^9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**7,x)
```

```
[Out] (-a**2*d**2 - 4*a*b*c*d - 10*b**2*c**2 - 15*b**2*d**2*x**2 + x*(-6*a*b*d**2
 - 24*b**2*c*d))/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4*b**5*x**2 + 120
0*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 60*b**9*x**6)
```

## 3.1152 $\int (a+bx)^5(c+dx)^3\,dx$

**Optimal**. Leaf size=92

$$\frac{3d^2(a+bx)^8(bc-ad)}{8b^4}+\frac{3d(a+bx)^7(bc-ad)^2}{7b^4}+\frac{(a+bx)^6(bc-ad)^3}{6b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

**Rubi [A]** time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3d^2(a+bx)^8(bc-ad)}{8b^4}+\frac{3d(a+bx)^7(bc-ad)^2}{7b^4}+\frac{(a+bx)^6(bc-ad)^3}{6b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^6)/(6*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^7)/(7*b^
4) + (3*d^2*(b*c - a*d)*(a + b*x)^8)/(8*b^4) + (d^3*(a + b*x)^9)/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^5}{b^3}+\frac{3d(bc-ad)^2(a+bx)^6}{b^3}+\frac{3d^2(bc-ad)(a+bx)^7}{b^3}+\frac{d^3(a+bx)^8}{b^3}\right)dx\\ &= \frac{(bc-ad)^3(a+bx)^6}{6b^4}+\frac{3d(bc-ad)^2(a+bx)^7}{7b^4}+\frac{3d^2(bc-ad)(a+bx)^8}{8b^4}+\frac{d^3(a+bx)^9}{9b^4}\end{aligned}$$

**Mathematica [B]** time = 0.08, size = 235, normalized size = 2.55

$$\frac{1}{504}x\left(126a^5\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)+126a^4bx\left(10c^3+20c^2dx+15cd^2x^2+4d^3x^3\right)+84a^3b^2x^2\left(20c^3+45c^2dx+36cd^2x^2+10d^3x^3\right)+36a^2b^3x^3\left(35c^3+84c^2dx+70cd^2x^2+20d^3x^3\right)+9ab^4x^4\left(56c^3+140c^2dx+120cd^2x^2+35d^3x^3\right)+b^5x^5\left(84c^3+216c^2dx+189cd^2x^2+56d^3x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^3,x]
```

```
[Out] (x*(126*a^5*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 126*a^4*b*x*(10*c
^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d^3*x^3) + 84*a^3*b^2*x^2*(20*c^3 + 45*c
^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 36*a^2*b^3*x^3*(35*c^3 + 84*c^2*d*x +
 70*c*d^2*x^2 + 20*d^3*x^3) + 9*a*b^4*x^4*(56*c^3 + 140*c^2*d*x + 120*c*d^2
*x^2 + 35*d^3*x^3) + b^5*x^5*(84*c^3 + 216*c^2*d*x + 189*c*d^2*x^2 + 56*d^3
*x^3)))/504
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^5 (c + dx)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^3, x]
```

**fricas [B]** time = 1.14, size = 303, normalized size = 3.29

$$\frac{1}{9}x^9d^3b^5 + \frac{3}{8}x^8d^2cb^5 + \frac{5}{8}x^8d^3b^4a + \frac{3}{7}x^7dc^2b^5 + \frac{15}{7}x^7d^2cb^4a + \frac{10}{7}x^7d^3b^3a^2 + \frac{1}{6}x^6c^3b^5 + \frac{5}{2}x^6dc^2b^4a + 5x^6d^2cb^3a^2 + \frac{5}{3}x^6d^3b^2a^3 + x^5c^3b^4a + 6x^5dc^2b^3a^2 + 6x^5d^2cb^2a^3 + x^5d^3ba^4 + \frac{5}{2}x^4c^3b^3a^2 + \frac{15}{2}x^4dc^2b^2a^3 + \frac{15}{4}x^4d^2cba^4 + \frac{1}{4}x^4d^3a^5 + \frac{10}{3}x^3c^3b^2a^3 + 5x^3dc^2ba^4 + x^3d^2ca^5 + \frac{5}{2}x^2c^3ba^4 + \frac{3}{2}x^2dc^2a^5 + xc^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^3*b^5 + 3/8*x^8*d^2*c*b^5 + 5/8*x^8*d^3*b^4*a + 3/7*x^7*d*c^2*b^5
 + 15/7*x^7*d^2*c*b^4*a + 10/7*x^7*d^3*b^3*a^2 + 1/6*x^6*c^3*b^5 + 5/2*x^6*
d*c^2*b^4*a + 5*x^6*d^2*c*b^3*a^2 + 5/3*x^6*d^3*b^2*a^3 + x^5*c^3*b^4*a + 6
*x^5*d*c^2*b^3*a^2 + 6*x^5*d^2*c*b^2*a^3 + x^5*d^3*b*a^4 + 5/2*x^4*c^3*b^3*
a^2 + 15/2*x^4*d*c^2*b^2*a^3 + 15/4*x^4*d^2*c*b*a^4 + 1/4*x^4*d^3*a^5 + 10/
3*x^3*c^3*b^2*a^3 + 5*x^3*d*c^2*b*a^4 + x^3*d^2*c*a^5 + 5/2*x^2*c^3*b*a^4 +
 3/2*x^2*d*c^2*a^5 + x*c^3*a^5
```

**giac [B]** time = 1.01, size = 303, normalized size = 3.29

$$\frac{1}{9}b^5d^3x^9 + \frac{3}{8}b^5cd^2x^8 + \frac{5}{8}ab^4d^3x^8 + \frac{3}{7}b^5c^2dx^7 + \frac{15}{7}ab^4cd^2x^7 + \frac{10}{7}a^2b^3d^3x^7 + \frac{1}{6}b^5c^3x^6 + \frac{5}{2}ab^4c^2dx^6 + 5a^2b^3cd^2x^6 + \frac{5}{3}a^3b^2d^3x^6 + ab^4c^3x^5 + 6a^2b^3c^2dx^5 + 6a^3b^2cd^2x^5 + a^4bd^3x^5 + \frac{5}{2}a^2b^3c^3x^4 + \frac{15}{2}a^3b^2c^2dx^4 + \frac{15}{4}a^4bcd^2x^4 + \frac{1}{4}a^5d^3x^4 + \frac{10}{3}a^3b^2c^3x^3 + 5a^4bc^2dx^3 + a^5cd^2x^3 + \frac{5}{2}a^4bc^3x^2 + \frac{3}{2}a^5c^2dx^2 + a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^5*d^3*x^9 + 3/8*b^5*c*d^2*x^8 + 5/8*a*b^4*d^3*x^8 + 3/7*b^5*c^2*d*x^7
 + 15/7*a*b^4*c*d^2*x^7 + 10/7*a^2*b^3*d^3*x^7 + 1/6*b^5*c^3*x^6 + 5/2*a*b^
4*c^2*d*x^6 + 5*a^2*b^3*c*d^2*x^6 + 5/3*a^3*b^2*d^3*x^6 + a*b^4*c^3*x^5 + 6
*a^2*b^3*c^2*d*x^5 + 6*a^3*b^2*c*d^2*x^5 + a^4*b*d^3*x^5 + 5/2*a^2*b^3*c^3*
x^4 + 15/2*a^3*b^2*c^2*d*x^4 + 15/4*a^4*b*c*d^2*x^4 + 1/4*a^5*d^3*x^4 + 10/
```

```
3*a^3*b^2*c^3*x^3 + 5*a^4*b*c^2*d*x^3 + a^5*c*d^2*x^3 + 5/2*a^4*b*c^3*x^2 +
 3/2*a^5*c^2*d*x^2 + a^5*c^3*x
```

**maple [B]** time = 0.00, size = 281, normalized size = 3.05

$$\frac{b^5d^3x^9}{9}+a^5c^3x+\frac{\left(5a\,b^4d^3+3b^5c\,d^2\right)x^8}{8}+\frac{\left(10a^2b^3d^3+15a\,b^4c\,d^2+3b^5c^2d\right)x^7}{7}+\frac{\left(10a^3b^2d^3+30a^2b^3c\,d^2+15a\,b^4c^2d+b^5c^3\right)x^6}{6}+\frac{\left(5a^4b\,d^3+30a^3b^2c\,d^2+30a^2b^3c^2d+5a\,b^4c^3\right)x^5}{5}+\frac{\left(a^5d^3+15a^4bc\,d^2+30a^3b^2c^2d+10a^2b^3c^3\right)x^4}{4}+\frac{\left(3a^5c\,d^2+15a^4b\,c^2d+10a^3b^2c^3\right)x^3}{3}+\frac{\left(3a^5c^2d+5a^4b\,c^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^3,x)
```

```
[Out] 1/9*b^5*d^3*x^9+1/8*(5*a*b^4*d^3+3*b^5*c*d^2)*x^8+1/7*(10*a^2*b^3*d^3+15*a*
b^4*c*d^2+3*b^5*c^2*d)*x^7+1/6*(10*a^3*b^2*d^3+30*a^2*b^3*c*d^2+15*a*b^4*c^
2*d+b^5*c^3)*x^6+1/5*(5*a^4*b*d^3+30*a^3*b^2*c*d^2+30*a^2*b^3*c^2*d+5*a*b^4
*c^3)*x^5+1/4*(a^5*d^3+15*a^4*b*c*d^2+30*a^3*b^2*c^2*d+10*a^2*b^3*c^3)*x^4+
1/3*(3*a^5*c*d^2+15*a^4*b*c^2*d+10*a^3*b^2*c^3)*x^3+1/2*(3*a^5*c^2*d+5*a^4*
b*c^3)*x^2+a^5*c^3*x
```

**maxima [B]** time = 1.38, size = 277, normalized size = 3.01

$$\frac{1}{9}\,b^5d^3x^9+a^5c^3x+\frac{1}{8}\left(3\,b^5cd^2+5\,ab^4d^3\right)x^8+\frac{1}{7}\left(3\,b^5c^2d+15\,ab^4cd^2+10\,a^2b^3d^3\right)x^7+\frac{1}{6}\left(b^5c^3+15\,ab^4c^2d+30\,a^2b^3cd^2+10\,a^3b^2d^3\right)x^6+\left(ab^4c^3+6\,a^2b^3c^2d+6\,a^3b^2cd^2+a^4bd^3\right)x^5+\frac{1}{4}\left(10\,a^2b^3c^3+30\,a^3b^2c^2d+15\,a^4bcd^2+a^5d^3\right)x^4+\frac{1}{3}\left(10\,a^3b^2c^3+15\,a^4bc^2d+3\,a^5cd^2\right)x^3+\frac{1}{2}\left(5\,a^4bc^3+3\,a^5c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*d^3*x^9 + a^5*c^3*x + 1/8*(3*b^5*c*d^2 + 5*a*b^4*d^3)*x^8 + 1/7*(3*
b^5*c^2*d + 15*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^7 + 1/6*(b^5*c^3 + 15*a*b^4*
c^2*d + 30*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^6 + (a*b^4*c^3 + 6*a^2*b^3*c^2
*d + 6*a^3*b^2*c*d^2 + a^4*b*d^3)*x^5 + 1/4*(10*a^2*b^3*c^3 + 30*a^3*b^2*c^
2*d + 15*a^4*b*c*d^2 + a^5*d^3)*x^4 + 1/3*(10*a^3*b^2*c^3 + 15*a^4*b*c^2*d
+ 3*a^5*c*d^2)*x^3 + 1/2*(5*a^4*b*c^3 + 3*a^5*c^2*d)*x^2
```

**mupad [B]** time = 0.24, size = 261, normalized size = 2.84

$$x^5\,\left(a^4\,b\,d^3+6\,a^3\,b^2\,c\,d^2+6\,a^2\,b^3\,c^2\,d+a\,b^4\,c^3\right)+x^4\,\left(\frac{a^5\,d^3}{4}+\frac{15\,a^4\,b\,c\,d^2}{4}+\frac{15\,a^3\,b^2\,c^2\,d}{2}+\frac{5\,a^2\,b^3\,c^3}{2}\right)+x^6\,\left(\frac{5\,a^3\,b^2\,d^3}{3}+5\,a^2\,b^3\,c\,d^2+\frac{5\,a\,b^4\,c^2\,d}{2}+\frac{b^5\,c^3}{6}\right)+a^5\,c^3\,x+\frac{b^5\,d^3\,x^9}{9}+\frac{a^4\,c^2\,x^2\,\left(3\,a\,d+5\,b\,c\right)}{2}+\frac{b^4\,d^2\,x^8\,\left(5\,a\,d+3\,b\,c\right)}{8}+\frac{a^3\,c\,x^3\,\left(3\,a^2\,d^2+15\,a\,b\,c\,d+10\,b^2\,c^2\right)}{3}+\frac{b^3\,d\,x^7\,\left(10\,a^2\,d^2+15\,a\,b\,c\,d+3\,b^2\,c^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^3,x)
```

```
[Out] x^5*(a*b^4*c^3 + a^4*b*d^3 + 6*a^2*b^3*c^2*d + 6*a^3*b^2*c*d^2) + x^4*((a^5
*d^3)/4 + (5*a^2*b^3*c^3)/2 + (15*a^3*b^2*c^2*d)/2 + (15*a^4*b*c*d^2)/4) +
x^6*((b^5*c^3)/6 + (5*a^3*b^2*d^3)/3 + 5*a^2*b^3*c*d^2 + (5*a*b^4*c^2*d)/2)
 + a^5*c^3*x + (b^5*d^3*x^9)/9 + (a^4*c^2*x^2*(3*a*d + 5*b*c))/2 + (b^4*d^2
*x^8*(5*a*d + 3*b*c))/8 + (a^3*c*x^3*(3*a^2*d^2 + 10*b^2*c^2 + 15*a*b*c*d))
/3 + (b^3*d*x^7*(10*a^2*d^2 + 3*b^2*c^2 + 15*a*b*c*d))/7
```

**sympy [B]** time = 0.12, size = 308, normalized size = 3.35

$$a^{5}c^{3}x + \frac{b^{5}d^{3}x^{9}}{9} + x^{8}\left(\frac{5ab^{4}d^{3}}{8} + \frac{3b^{5}cd^{2}}{8}\right) + x^{7}\left(\frac{10a^{2}b^{3}d^{3}}{7} + \frac{15ab^{4}cd^{2}}{7} + \frac{3b^{5}c^{2}d}{7}\right) + x^{6}\left(\frac{5a^{3}b^{2}d^{3}}{3} + 5a^{2}b^{3}cd^{2} + \frac{5ab^{4}c^{2}d}{2} + \frac{b^{5}c^{3}}{6}\right) + x^{5}\left(a^{4}bd^{3} + 6a^{3}b^{2}cd^{2} + 6a^{2}b^{3}c^{2}d + ab^{4}c^{3}\right) + x^{4}\left(\frac{a^{5}d^{3}}{4} + \frac{15a^{4}bcd^{2}}{4} + \frac{15a^{3}b^{2}c^{2}d}{2} + \frac{5a^{2}b^{3}c^{3}}{2}\right) + x^{3}\left(a^{5}cd^{2} + 5a^{4}bc^{2}d + \frac{10a^{3}b^{2}c^{3}}{3}\right) + x^{2}\left(\frac{3a^{5}c^{2}d}{2} + \frac{5a^{4}bc^{3}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**3,x)
```

```
[Out] a**5*c**3*x + b**5*d**3*x**9/9 + x**8*(5*a*b**4*d**3/8 + 3*b**5*c*d**2/8) +
 x**7*(10*a**2*b**3*d**3/7 + 15*a*b**4*c*d**2/7 + 3*b**5*c**2*d/7) + x**6*(
5*a**3*b**2*d**3/3 + 5*a**2*b**3*c*d**2 + 5*a*b**4*c**2*d/2 + b**5*c**3/6)
+ x**5*(a**4*b*d**3 + 6*a**3*b**2*c*d**2 + 6*a**2*b**3*c**2*d + a*b**4*c**3
) + x**4*(a**5*d**3/4 + 15*a**4*b*c*d**2/4 + 15*a**3*b**2*c**2*d/2 + 5*a**2
*b**3*c**3/2) + x**3*(a**5*c*d**2 + 5*a**4*b*c**2*d + 10*a**3*b**2*c**3/3)
+ x**2*(3*a**5*c**2*d/2 + 5*a**4*b*c**3/2)
```

## 3.1153 $\int (a+bx)^4(c+dx)^3\,dx$

**Optimal**. Leaf size=92

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

**Rubi [A]** time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4)
 + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^4}{b^3}+\frac{3d(bc-ad)^2(a+bx)^5}{b^3}+\frac{3d^2(bc-ad)(a+bx)^6}{b^3}+\frac{d^3(a+bx)^7}{b^3}\right)dx\\ &= \frac{(bc-ad)^3(a+bx)^5}{5b^4}+\frac{d(bc-ad)^2(a+bx)^6}{2b^4}+\frac{3d^2(bc-ad)(a+bx)^7}{7b^4}+\frac{d^3(a+bx)^8}{8b^4}\end{aligned}$$

**Mathematica [B]** time = 0.03, size = 217, normalized size = 2.36

$$a^4c^3x+\frac{1}{2}a^3c^2x^2(3ad+4bc)+\frac{1}{2}b^2dx^6\left(2a^2d^2+4abcd+b^2c^2\right)+a^2cx^3\left(a^2d^2+4abcd+2b^2c^2\right)+\frac{1}{5}bx^5\left(4a^3d^3+18a^2bcd^2+12ab^2c^2d+b^3c^3\right)+\frac{1}{4}ax^4\left(a^3d^3+12a^2bcd^2+18ab^2c^2d+4b^3c^3\right)+\frac{1}{7}b^3d^2x^7(4ad+3bc)+\frac{1}{8}b^4d^3x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] a^4*c^3*x + (a^3*c^2*(4*b*c + 3*a*d)*x^2)/2 + a^2*c*(2*b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^3 + (a*(4*b^3*c^3 + 18*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^3)
*x^4)/4 + (b*(b^3*c^3 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 4*a^3*d^3)*x^5)/5
 + (b^2*d*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^6)/2 + (b^3*d^2*(3*b*c + 4*a*
d)*x^7)/7 + (b^4*d^3*x^8)/8
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^4(c+dx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^3, x]
```

**fricas [B]** time = 1.20, size = 245, normalized size = 2.66

$$\frac{1}{8}x^8d^3b^4 + \frac{3}{7}x^7d^2cb^4 + \frac{4}{7}x^7d^3b^3a + \frac{1}{2}x^6dc^2b^4 + 2x^6d^2cb^3a + x^6d^3b^2a^2 + \frac{1}{5}x^5c^3b^4 + \frac{12}{5}x^5dc^2b^3a + \frac{18}{5}x^5d^2cb^2a^2 + \frac{4}{5}x^5d^3ba^3 + x^4c^3b^3a + \frac{9}{2}x^4dc^2b^2a^2 + 3x^4d^2cba^3 + \frac{1}{4}x^4d^3a^4 + 2x^3c^3b^2a^2 + 4x^3dc^2ba^3 + x^3d^2ca^4 + 2x^2c^3ba^3 + \frac{3}{2}x^2dc^2a^4 + xc^3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d^3*b^4 + 3/7*x^7*d^2*c*b^4 + 4/7*x^7*d^3*b^3*a + 1/2*x^6*d*c^2*b^4
 + 2*x^6*d^2*c*b^3*a + x^6*d^3*b^2*a^2 + 1/5*x^5*c^3*b^4 + 12/5*x^5*d*c^2*b
^3*a + 18/5*x^5*d^2*c*b^2*a^2 + 4/5*x^5*d^3*b*a^3 + x^4*c^3*b^3*a + 9/2*x^4
*d*c^2*b^2*a^2 + 3*x^4*d^2*c*b*a^3 + 1/4*x^4*d^3*a^4 + 2*x^3*c^3*b^2*a^2 +
4*x^3*d*c^2*b*a^3 + x^3*d^2*c*a^4 + 2*x^2*c^3*b*a^3 + 3/2*x^2*d*c^2*a^4 + x
*c^3*a^4
```

**giac [B]** time = 1.12, size = 245, normalized size = 2.66

$$\frac{1}{8}\,b^4d^3x^8 + \frac{3}{7}\,b^4cd^2x^7 + \frac{4}{7}\,ab^3d^3x^7 + \frac{1}{2}\,b^4c^2dx^6 + 2\,ab^3cd^2x^6 + a^2b^2d^3x^6 + \frac{1}{5}\,b^4c^3x^5 + \frac{12}{5}\,ab^3c^2dx^5 + \frac{18}{5}\,a^2b^2cd^2x^5 + \frac{4}{5}\,a^3bd^3x^5 + ab^3c^3x^4 + \frac{9}{2}\,a^2b^2c^2dx^4 + 3\,a^3bcd^2x^4 + \frac{1}{4}\,a^4d^3x^4 + 2\,a^2b^2c^3x^3 + 4\,a^3bc^2dx^3 + a^4cd^2x^3 + 2\,a^3bc^3x^2 + \frac{3}{2}\,a^4c^2dx^2 + a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 4/7*a*b^3*d^3*x^7 + 1/2*b^4*c^2*d*x^6
 + 2*a*b^3*c*d^2*x^6 + a^2*b^2*d^3*x^6 + 1/5*b^4*c^3*x^5 + 12/5*a*b^3*c^2*d
*x^5 + 18/5*a^2*b^2*c*d^2*x^5 + 4/5*a^3*b*d^3*x^5 + a*b^3*c^3*x^4 + 9/2*a^2
*b^2*c^2*d*x^4 + 3*a^3*b*c*d^2*x^4 + 1/4*a^4*d^3*x^4 + 2*a^2*b^2*c^3*x^3 +
4*a^3*b*c^2*d*x^3 + a^4*c*d^2*x^3 + 2*a^3*b*c^3*x^2 + 3/2*a^4*c^2*d*x^2 + a
^4*c^3*x
```

**maple [B]** time = 0.00, size = 229, normalized size = 2.49

$$\frac{b^4d^3x^8}{8}+a^4c^3x+\frac{\left(4a\,b^3d^3+3b^4c\,d^2\right)x^7}{7}+\frac{\left(6a^2b^2d^3+12a\,b^3c\,d^2+3b^4c^2d\right)x^6}{6}+\frac{\left(4a^3b\,d^3+18a^2b^2c\,d^2+12a\,b^3c^2d+b^4c^3\right)x^5}{5}+\frac{\left(a^4d^3+12a^3bc\,d^2+18a^2b^2c^2d+4a\,b^3c^3\right)x^4}{4}+\frac{\left(3a^4c\,d^2+12a^3b\,c^2d+6a^2b^2c^3\right)x^3}{3}+\frac{\left(3a^4c^2d+4a^3b\,c^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^3,x)
```

```
[Out] 1/8*b^4*d^3*x^8+1/7*(4*a*b^3*d^3+3*b^4*c*d^2)*x^7+1/6*(6*a^2*b^2*d^3+12*a*b
^3*c*d^2+3*b^4*c^2*d)*x^6+1/5*(4*a^3*b*d^3+18*a^2*b^2*c*d^2+12*a*b^3*c^2*d+
b^4*c^3)*x^5+1/4*(a^4*d^3+12*a^3*b*c*d^2+18*a^2*b^2*c^2*d+4*a*b^3*c^3)*x^4+
1/3*(3*a^4*c*d^2+12*a^3*b*c^2*d+6*a^2*b^2*c^3)*x^3+1/2*(3*a^4*c^2*d+4*a^3*b
*c^3)*x^2+a^4*c^3*x
```

**maxima [B]** time = 1.32, size = 225, normalized size = 2.45

$$\frac{1}{8}\,b^4d^3x^8+a^4c^3x+\frac{1}{7}\left(3\,b^4cd^2+4\,ab^3d^3\right)x^7+\frac{1}{2}\left(b^4c^2d+4\,ab^3cd^2+2\,a^2b^2d^3\right)x^6+\frac{1}{5}\left(b^4c^3+12\,ab^3c^2d+18\,a^2b^2cd^2+4\,a^3bd^3\right)x^5+\frac{1}{4}\left(4\,ab^3c^3+18\,a^2b^2c^2d+12\,a^3bcd^2+a^4d^3\right)x^4+\left(2\,a^2b^2c^3+4\,a^3bc^2d+a^4cd^2\right)x^3+\frac{1}{2}\left(4\,a^3bc^3+3\,a^4c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b^
4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^2*
d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*c^2
*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a^4*c
*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

**mupad [B]** time = 0.21, size = 208, normalized size = 2.26

$$x^4\left(\frac{a^4\,d^3}{4}+3\,a^3\,b\,c\,d^2+\frac{9\,a^2\,b^2\,c^2\,d}{2}+a\,b^3\,c^3\right)+x^5\left(\frac{4\,a^3\,b\,d^3}{5}+\frac{18\,a^2\,b^2\,c\,d^2}{5}+\frac{12\,a\,b^3\,c^2\,d}{5}+\frac{b^4\,c^3}{5}\right)+a^4\,c^3\,x+\frac{b^4\,d^3\,x^8}{8}+\frac{a^3\,c^2\,x^2\,\left(3\,a\,d+4\,b\,c\right)}{2}+\frac{b^3\,d^2\,x^7\,\left(4\,a\,d+3\,b\,c\right)}{7}+a^2\,c\,x^3\,\left(a^2\,d^2+4\,a\,b\,c\,d+2\,b^2\,c^2\right)+\frac{b^2\,d\,x^6\,\left(2\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^3,x)
```

```
[Out] x^4*((a^4*d^3)/4 + a*b^3*c^3 + (9*a^2*b^2*c^2*d)/2 + 3*a^3*b*c*d^2) + x^5*(
(b^4*c^3)/5 + (4*a^3*b*d^3)/5 + (18*a^2*b^2*c*d^2)/5 + (12*a*b^3*c^2*d)/5)
+ a^4*c^3*x + (b^4*d^3*x^8)/8 + (a^3*c^2*x^2*(3*a*d + 4*b*c))/2 + (b^3*d^2*
x^7*(4*a*d + 3*b*c))/7 + a^2*c*x^3*(a^2*d^2 + 2*b^2*c^2 + 4*a*b*c*d) + (b^2
*d*x^6*(2*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/2
```

**sympy [B]** time = 0.11, size = 243, normalized size = 2.64

$$a^4c^3x+\frac{b^4d^3x^8}{8}+x^7\left(\frac{4ab^3d^3}{7}+\frac{3b^4cd^2}{7}\right)+x^6\left(a^2b^2d^3+2ab^3cd^2+\frac{b^4c^2d}{2}\right)+x^5\left(\frac{4a^3bd^3}{5}+\frac{18a^2b^2cd^2}{5}+\frac{12ab^3c^2d}{5}+\frac{b^4c^3}{5}\right)+x^4\left(\frac{a^4d^3}{4}+3a^3bcd^2+\frac{9a^2b^2c^2d}{2}+ab^3c^3\right)+x^3\left(a^4cd^2+4a^3bc^2d+2a^2b^2c^3\right)+x^2\left(\frac{3a^4c^2d}{2}+2a^3bc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**3,x)
```

```
[Out] a**4*c**3*x + b**4*d**3*x**8/8 + x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7) +
 x**6*(a**2*b**2*d**3 + 2*a*b**3*c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b*d
**3/5 + 18*a**2*b**2*c*d**2/5 + 12*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4*(a
**4*d**3/4 + 3*a**3*b*c*d**2 + 9*a**2*b**2*c**2*d/2 + a*b**3*c**3) + x**3*(
a**4*c*d**2 + 4*a**3*b*c**2*d + 2*a**2*b**2*c**3) + x**2*(3*a**4*c**2*d/2 +
 2*a**3*b*c**3)
```

## 3.1154 $\int (a+bx)^3(c+dx)^3\,dx$

**Optimal**. Leaf size=92

$$\frac{d^2(a+bx)^6(bc-ad)}{2b^4}+\frac{3d(a+bx)^5(bc-ad)^2}{5b^4}+\frac{(a+bx)^4(bc-ad)^3}{4b^4}+\frac{d^3(a+bx)^7}{7b^4}$$

**Rubi** [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^2(a+bx)^6(bc-ad)}{2b^4}+\frac{3d(a+bx)^5(bc-ad)^2}{5b^4}+\frac{(a+bx)^4(bc-ad)^3}{4b^4}+\frac{d^3(a+bx)^7}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^4)/(4*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^5)/(5*b^
4) + (d^2*(b*c - a*d)*(a + b*x)^6)/(2*b^4) + (d^3*(a + b*x)^7)/(7*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^3}{b^3}+\frac{3d(bc-ad)^2(a+bx)^4}{b^3}+\frac{3d^2(bc-ad)(a+bx)^5}{b^3}+\frac{d^3(a+bx)^6}{b^3}\right)dx\\ &= \frac{(bc-ad)^3(a+bx)^4}{4b^4}+\frac{3d(bc-ad)^2(a+bx)^5}{5b^4}+\frac{d^2(bc-ad)(a+bx)^6}{2b^4}+\frac{d^3(a+bx)^7}{7b^4}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 161, normalized size = 1.75

$$a^3c^3x+\frac{3}{5}bdx^5\left(a^2d^2+3abcd+b^2c^2\right)+acx^3\left(a^2d^2+3abcd+b^2c^2\right)+\frac{3}{2}a^2c^2x^2(ad+bc)+\frac{1}{4}x^4\left(a^3d^3+9a^2bcd^2+9ab^2c^2d+b^3c^3\right)+\frac{1}{2}b^2d^2x^6(ad+bc)+\frac{1}{7}b^3d^3x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] a^3*c^3*x + (3*a^2*c^2*(b*c + a*d)*x^2)/2 + a*c*(b^2*c^2 + 3*a*b*c*d + a^2*
d^2)*x^3 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^4)/4 + (3
*b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5)/5 + (b^2*d^2*(b*c + a*d)*x^6)/2 +
 (b^3*d^3*x^7)/7
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^3, x]
```

**fricas [B]** time = 1.09, size = 188, normalized size = 2.04

$$\frac{1}{7}x^7d^3b^3 + \frac{1}{2}x^6d^2cb^3 + \frac{1}{2}x^6d^3b^2a + \frac{3}{5}x^5dc^2b^3 + \frac{9}{5}x^5d^2cb^2a + \frac{3}{5}x^5d^3ba^2 + \frac{1}{4}x^4c^3b^3 + \frac{9}{4}x^4dc^2b^2a + \frac{9}{4}x^4d^2cba^2 + \frac{1}{4}x^4d^3a^3 + x^3c^3b^2a + 3x^3dc^2ba^2 + x^3d^2ca^3 + \frac{3}{2}x^2c^3ba^2 + \frac{3}{2}x^2dc^2a^3 + xc^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^3*b^3 + 1/2*x^6*d^2*c*b^3 + 1/2*x^6*d^3*b^2*a + 3/5*x^5*d*c^2*b^3
 + 9/5*x^5*d^2*c*b^2*a + 3/5*x^5*d^3*b*a^2 + 1/4*x^4*c^3*b^3 + 9/4*x^4*d*c^
2*b^2*a + 9/4*x^4*d^2*c*b*a^2 + 1/4*x^4*d^3*a^3 + x^3*c^3*b^2*a + 3*x^3*d*c
^2*b*a^2 + x^3*d^2*c*a^3 + 3/2*x^2*c^3*b*a^2 + 3/2*x^2*d*c^2*a^3 + x*c^3*a^
3
```

**giac [B]** time = 0.97, size = 188, normalized size = 2.04

$$\frac{1}{7}\,b^3d^3x^7 + \frac{1}{2}\,b^3cd^2x^6 + \frac{1}{2}\,ab^2d^3x^6 + \frac{3}{5}\,b^3c^2dx^5 + \frac{9}{5}\,ab^2cd^2x^5 + \frac{3}{5}\,a^2bd^3x^5 + \frac{1}{4}\,b^3c^3x^4 + \frac{9}{4}\,ab^2c^2dx^4 + \frac{9}{4}\,a^2bcd^2x^4 + \frac{1}{4}\,a^3d^3x^4 + ab^2c^3x^3 + 3\,a^2bc^2dx^3 + a^3cd^2x^3 + \frac{3}{2}\,a^2bc^3x^2 + \frac{3}{2}\,a^3c^2dx^2 + a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*d^3*x^7 + 1/2*b^3*c*d^2*x^6 + 1/2*a*b^2*d^3*x^6 + 3/5*b^3*c^2*d*x^5
 + 9/5*a*b^2*c*d^2*x^5 + 3/5*a^2*b*d^3*x^5 + 1/4*b^3*c^3*x^4 + 9/4*a*b^2*c^
2*d*x^4 + 9/4*a^2*b*c*d^2*x^4 + 1/4*a^3*d^3*x^4 + a*b^2*c^3*x^3 + 3*a^2*b*c
^2*d*x^3 + a^3*c*d^2*x^3 + 3/2*a^2*b*c^3*x^2 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*
x
```

**maple [B]** time = 0.00, size = 177, normalized size = 1.92

$$\frac{b^3d^3x^7}{7} + a^3c^3x + \frac{\left(3a\,b^2d^3 + 3b^3c\,d^2\right)x^6}{6} + \frac{\left(3a^2b\,d^3 + 9a\,b^2c\,d^2 + 3b^3c^2d\right)x^5}{5} + \frac{\left(a^3d^3 + 9a^2bc\,d^2 + 9a\,b^2c^2d + b^3c^3\right)x^4}{4} + \frac{\left(3a^3c\,d^2 + 9a^2b\,c^2d + 3a\,b^2c^3\right)x^3}{3} + \frac{\left(3a^3c^2d + 3a^2b\,c^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^3,x)
```

```
[Out] 1/7*b^3*d^3*x^7+1/6*(3*a*b^2*d^3+3*b^3*c*d^2)*x^6+1/5*(3*a^2*b*d^3+9*a*b^2*
c*d^2+3*b^3*c^2*d)*x^5+1/4*(a^3*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)*x^
4+1/3*(3*a^3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*c^3)*x^3+1/2*(3*a^3*c^2*d+3*a^2*b*
c^3)*x^2+a^3*c^3*x
```

**maxima** [A] time = 1.34, size = 167, normalized size = 1.82

$$\frac{1}{7}b^3d^3x^7 + a^3c^3x + \frac{1}{2}\left(b^3cd^2 + ab^2d^3\right)x^6 + \frac{3}{5}\left(b^3c^2d + 3\,ab^2cd^2 + a^2bd^3\right)x^5 + \frac{1}{4}\left(b^3c^3 + 9\,ab^2c^2d + 9\,a^2bcd^2 + a^3d^3\right)x^4 + \left(ab^2c^3 + 3\,a^2bc^2d + a^3cd^2\right)x^3 + \frac{3}{2}\left(a^2bc^3 + a^3c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*d^3*x^7 + a^3*c^3*x + 1/2*(b^3*c*d^2 + a*b^2*d^3)*x^6 + 3/5*(b^3*c^
2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^5 + 1/4*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2
*b*c*d^2 + a^3*d^3)*x^4 + (a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^3 + 3/2
*(a^2*b*c^3 + a^3*c^2*d)*x^2
```

**mupad** [B] time = 0.06, size = 152, normalized size = 1.65

$$x^4\left(\frac{a^3\,d^3}{4}+\frac{9\,a^2\,b\,c\,d^2}{4}+\frac{9\,a\,b^2\,c^2\,d}{4}+\frac{b^3\,c^3}{4}\right)+a^3\,c^3\,x+\frac{b^3\,d^3\,x^7}{7}+a\,c\,x^3\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)+\frac{3\,b\,d\,x^5\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{5}+\frac{3\,a^2\,c^2\,x^2\,\left(a\,d+b\,c\right)}{2}+\frac{b^2\,d^2\,x^6\,\left(a\,d+b\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] x^4*((a^3*d^3)/4 + (b^3*c^3)/4 + (9*a*b^2*c^2*d)/4 + (9*a^2*b*c*d^2)/4) + a
^3*c^3*x + (b^3*d^3*x^7)/7 + a*c*x^3*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d) + (3*b
*d*x^5*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/5 + (3*a^2*c^2*x^2*(a*d + b*c))/2 +
 (b^2*d^2*x^6*(a*d + b*c))/2
```

**sympy** [B] time = 0.10, size = 190, normalized size = 2.07

$$a^3c^3x + \frac{b^3d^3x^7}{7} + x^6\left(\frac{ab^2d^3}{2} + \frac{b^3cd^2}{2}\right) + x^5\left(\frac{3a^2bd^3}{5} + \frac{9ab^2cd^2}{5} + \frac{3b^3c^2d}{5}\right) + x^4\left(\frac{a^3d^3}{4} + \frac{9a^2bcd^2}{4} + \frac{9ab^2c^2d}{4} + \frac{b^3c^3}{4}\right) + x^3\left(a^3cd^2 + 3a^2bc^2d + ab^2c^3\right) + x^2\left(\frac{3a^3c^2d}{2} + \frac{3a^2bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**3,x)
```

```
[Out] a**3*c**3*x + b**3*d**3*x**7/7 + x**6*(a*b**2*d**3/2 + b**3*c*d**2/2) + x**
5*(3*a**2*b*d**3/5 + 9*a*b**2*c*d**2/5 + 3*b**3*c**2*d/5) + x**4*(a**3*d**3
/4 + 9*a**2*b*c*d**2/4 + 9*a*b**2*c**2*d/4 + b**3*c**3/4) + x**3*(a**3*c*d*
*2 + 3*a**2*b*c**2*d + a*b**2*c**3) + x**2*(3*a**3*c**2*d/2 + 3*a**2*b*c**3
/2)
```

## 3.1155 $\int (a+bx)^2(c+dx)^3\,dx$

**Optimal**. Leaf size=65

$$-\frac{2b(c+dx)^5(bc-ad)}{5d^3}+\frac{(c+dx)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

**Rubi [A]** time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b(c+dx)^5(bc-ad)}{5d^3}+\frac{(c+dx)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^4)/(4*d^3) - (2*b*(b*c - a*d)*(c + d*x)^5)/(5*d^3)
 + (b^2*(c + d*x)^6)/(6*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^3\,dx&=\int\left(\frac{(-bc+ad)^2(c+dx)^3}{d^2}-\frac{2b(bc-ad)(c+dx)^4}{d^2}+\frac{b^2(c+dx)^5}{d^2}\right)dx\\&=\frac{(bc-ad)^2(c+dx)^4}{4d^3}-\frac{2b(bc-ad)(c+dx)^5}{5d^3}+\frac{b^2(c+dx)^6}{6d^3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 122, normalized size = 1.88

$$\frac{1}{4}dx^4\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{6}b^2d^3x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c
+ 2*a*d)*x^5)/5 + (b^2*d^3*x^6)/6
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^3, x]
```

**fricas [B]** time = 1.27, size = 130, normalized size = 2.00

$$\frac{1}{6}x^6d^3b^2 + \frac{3}{5}x^5d^2cb^2 + \frac{2}{5}x^5d^3ba + \frac{3}{4}x^4dc^2b^2 + \frac{3}{2}x^4d^2cba + \frac{1}{4}x^4d^3a^2 + \frac{1}{3}x^3c^3b^2 + 2x^3dc^2ba + x^3d^2ca^2 + x^2c^3ba + \frac{3}{2}x^2dc^2a^2 + xc^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d^3*b^2 + 3/5*x^5*d^2*c*b^2 + 2/5*x^5*d^3*b*a + 3/4*x^4*d*c^2*b^2 +
 3/2*x^4*d^2*c*b*a + 1/4*x^4*d^3*a^2 + 1/3*x^3*c^3*b^2 + 2*x^3*d*c^2*b*a +
x^3*d^2*c*a^2 + x^2*c^3*b*a + 3/2*x^2*d*c^2*a^2 + x*c^3*a^2
```

**giac [B]** time = 1.04, size = 130, normalized size = 2.00

$$\frac{1}{6}b^2d^3x^6 + \frac{3}{5}b^2cd^2x^5 + \frac{2}{5}abd^3x^5 + \frac{3}{4}b^2c^2dx^4 + \frac{3}{2}abcd^2x^4 + \frac{1}{4}a^2d^3x^4 + \frac{1}{3}b^2c^3x^3 + 2abc^2dx^3 + a^2cd^2x^3 + abc^3x^2 + \frac{3}{2}a^2c^2dx^2 + a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d^3*x^6 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + 3/4*b^2*c^2*d*x^4 +
 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/3*b^2*c^3*x^3 + 2*a*b*c^2*d*x^3 +
a^2*c*d^2*x^3 + a*b*c^3*x^2 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x
```

**maple [B]** time = 0.00, size = 125, normalized size = 1.92

$$\frac{b^2d^3x^6}{6} + a^2c^3x + \frac{\left(2abd^3 + 3b^2cd^2\right)x^5}{5} + \frac{\left(a^2d^3 + 6abcd^2 + 3b^2c^2d\right)x^4}{4} + \frac{\left(3a^2cd^2 + 6abc^2d + b^2c^3\right)x^3}{3} + \frac{\left(3a^2c^2d + 2abc^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^3,x)
```

```
[Out] 1/6*b^2*d^3*x^6+1/5*(2*a*b*d^3+3*b^2*c*d^2)*x^5+1/4*(a^2*d^3+6*a*b*c*d^2+3*
b^2*c^2*d)*x^4+1/3*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^3+1/2*(3*a^2*c^2*d+2
*a*b*c^3)*x^2+a^2*c^3*x
```

**maxima** [B] time = 1.34, size = 124, normalized size = 1.91

$$\frac{1}{6}b^2d^3x^6 + a^2c^3x + \frac{1}{5}\left(3\,b^2cd^2 + 2\,abd^3\right)x^5 + \frac{1}{4}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^4 + \frac{1}{3}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^3 + \frac{1}{2}\left(2\,abc^3 + 3\,a^2c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d^3*x^6 + a^2*c^3*x + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/4*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

**mupad** [B] time = 0.05, size = 115, normalized size = 1.77

$$x^3\left(a^2\,c\,d^2 + 2\,a\,b\,c^2\,d + \frac{b^2\,c^3}{3}\right) + x^4\left(\frac{a^2\,d^3}{4} + \frac{3\,a\,b\,c\,d^2}{2} + \frac{3\,b^2\,c^2\,d}{4}\right) + a^2\,c^3\,x + \frac{b^2\,d^3\,x^6}{6} + \frac{a\,c^2\,x^2\,\left(3\,a\,d + 2\,b\,c\right)}{2} + \frac{b\,d^2\,x^5\,\left(2\,a\,d + 3\,b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] x^3*((b^2*c^3)/3 + a^2*c*d^2 + 2*a*b*c^2*d) + x^4*((a^2*d^3)/4 + (3*b^2*c^2
*d)/4 + (3*a*b*c*d^2)/2) + a^2*c^3*x + (b^2*d^3*x^6)/6 + (a*c^2*x^2*(3*a*d
+ 2*b*c))/2 + (b*d^2*x^5*(2*a*d + 3*b*c))/5
```

**sympy** [B] time = 0.09, size = 133, normalized size = 2.05

$$a^2c^3x + \frac{b^2d^3x^6}{6} + x^5\left(\frac{2abd^3}{5} + \frac{3b^2cd^2}{5}\right) + x^4\left(\frac{a^2d^3}{4} + \frac{3abcd^2}{2} + \frac{3b^2c^2d}{4}\right) + x^3\left(a^2cd^2 + 2abc^2d + \frac{b^2c^3}{3}\right) + x^2\left(\frac{3a^2c^2d}{2} + abc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**6/6 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x*
*4*(a**2*d**3/4 + 3*a*b*c*d**2/2 + 3*b**2*c**2*d/4) + x**3*(a**2*c*d**2 + 2
*a*b*c**2*d + b**2*c**3/3) + x**2*(3*a**2*c**2*d/2 + a*b*c**3)
```

## 3.1156 $\int (a+bx)(c+dx)^3\,dx$

**Optimal**. Leaf size=38

$$\frac{b(c+dx)^5}{5d^2}-\frac{(c+dx)^4(bc-ad)}{4d^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^5}{5d^2}-\frac{(c+dx)^4(bc-ad)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^4)/(4*d^2) + (b*(c + d*x)^5)/(5*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^3\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^3}{d}+\frac{b(c+dx)^4}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^4}{4d^2}+\frac{b(c+dx)^5}{5d^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 67, normalized size = 1.76

$$\frac{1}{2}c^2x^2(3ad+bc)+\frac{1}{4}d^2x^4(ad+3bc)+cdx^3(ad+bc)+ac^3x+\frac{1}{5}bd^3x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^2)/2 + c*d*(b*c + a*d)*x^3 + (d^2*(3*b*c + a
*d)*x^4)/4 + (b*d^3*x^5)/5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(c + d*x)^3, x]
```

**fricas [B]** time = 1.09, size = 72, normalized size = 1.89

$$\frac{1}{5}x^5d^3b+\frac{3}{4}x^4d^2cb+\frac{1}{4}x^4d^3a+x^3dc^2b+x^3d^2ca+\frac{1}{2}x^2c^3b+\frac{3}{2}x^2dc^2a+xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d^3*b + 3/4*x^4*d^2*c*b + 1/4*x^4*d^3*a + x^3*d*c^2*b + x^3*d^2*c*a
 + 1/2*x^2*c^3*b + 3/2*x^2*d*c^2*a + x*c^3*a
```

**giac [B]** time = 0.88, size = 72, normalized size = 1.89

$$\frac{1}{5}\,bd^3x^5+\frac{3}{4}\,bcd^2x^4+\frac{1}{4}\,ad^3x^4+bc^2dx^3+acd^2x^3+\frac{1}{2}\,bc^3x^2+\frac{3}{2}\,ac^2dx^2+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/5*b*d^3*x^5 + 3/4*b*c*d^2*x^4 + 1/4*a*d^3*x^4 + b*c^2*d*x^3 + a*c*d^2*x^3
 + 1/2*b*c^3*x^2 + 3/2*a*c^2*d*x^2 + a*c^3*x
```

**maple [B]** time = 0.00, size = 73, normalized size = 1.92

$$\frac{b\,d^3x^5}{5}+a\,c^3x+\frac{\left(a\,d^3+3bc\,d^2\right)x^4}{4}+\frac{\left(3ac\,d^2+3b\,c^2d\right)x^3}{3}+\frac{\left(3a\,c^2d+b\,c^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^3,x)
```

```
[Out] 1/5*b*d^3*x^5+1/4*(a*d^3+3*b*c*d^2)*x^4+1/3*(3*a*c*d^2+3*b*c^2*d)*x^3+1/2*(
3*a*c^2*d+b*c^3)*x^2+a*c^3*x
```

**maximà** [B] time = 1.39, size = 69, normalized size = 1.82

$$\frac{1}{5}\,bd^3x^5 + ac^3x + \frac{1}{4}\left(3\,bcd^2 + ad^3\right)x^4 + \left(bc^2d + acd^2\right)x^3 + \frac{1}{2}\left(bc^3 + 3\,ac^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/5*b*d^3*x^5 + a*c^3*x + 1/4*(3*b*c*d^2 + a*d^3)*x^4 + (b*c^2*d + a*c*d^2)
*x^3 + 1/2*(b*c^3 + 3*a*c^2*d)*x^2
```

**mupad** [B] time = 0.03, size = 65, normalized size = 1.71

$$x^2\left(\frac{b\,c^3}{2}+\frac{3\,a\,d\,c^2}{2}\right)+x^4\left(\frac{a\,d^3}{4}+\frac{3\,b\,c\,d^2}{4}\right)+\frac{b\,d^3\,x^5}{5}+a\,c^3\,x+c\,d\,x^3\,\left(a\,d+b\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^3,x)
```

```
[Out] x^2*((b*c^3)/2 + (3*a*c^2*d)/2) + x^4*((a*d^3)/4 + (3*b*c*d^2)/4) + (b*d^3*
x^5)/5 + a*c^3*x + c*d*x^3*(a*d + b*c)
```

**sympy** [B] time = 0.08, size = 73, normalized size = 1.92

$$ac^3x + \frac{bd^3x^5}{5} + x^4\left(\frac{ad^3}{4} + \frac{3bcd^2}{4}\right) + x^3\left(acd^2 + bc^2d\right) + x^2\left(\frac{3ac^2d}{2} + \frac{bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**5/5 + x**4*(a*d**3/4 + 3*b*c*d**2/4) + x**3*(a*c*d**2
+ b*c**2*d) + x**2*(3*a*c**2*d/2 + b*c**3/2)
```

## 3.1157 $\int (c+dx)^3 \, dx$

**Optimal**. Leaf size=14

$$\frac{(c+dx)^4}{4d}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3,x]
```

```
[Out] (c + d*x)^4/(4*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^3 \, dx = \frac{(c+dx)^4}{4d}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3,x]
```

```
[Out] (c + d*x)^4/(4*d)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3, x]
```

**fricas [B]** time = 1.18, size = 31, normalized size = 2.21

$$\frac{1}{4}x^4d^3 + x^3d^2c + \frac{3}{2}x^2dc^2 + xc^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*d^3 + x^3*d^2*c + 3/2*x^2*d*c^2 + x*c^3
```

**giac [A]** time = 1.00, size = 12, normalized size = 0.86

$$\frac{(dx + c)^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(d*x + c)^4/d
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{(dx + c)^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3,x)
```

```
[Out] 1/4*(d*x+c)^4/d
```

**maxima [B]** time = 1.35, size = 31, normalized size = 2.21

$$\frac{1}{4}\,d^3x^4 + cd^2x^3 + \frac{3}{2}\,c^2dx^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

**mupad [B]** time = 0.04, size = 31, normalized size = 2.21

$$c^3\,x+\frac{3\,c^2\,d\,x^2}{2}+c\,d^2\,x^3+\frac{d^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3,x)
[Out] c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3
```

**sympy [B]** time = 0.06, size = 32, normalized size = 2.29

$$c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3,x)
[Out] c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
```

## 3.1158 $\int \frac{(c+dx)^3}{a+bx}\,dx$

**Optimal**. Leaf size=73

$$\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(c+dx)^3}{3b}$$

**Rubi [A]** time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{(c+dx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x)^2)/(2*b^2) + (c + d*x)^3/(
3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{a+bx}\,dx&=\int\left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx\\&=\frac{d(bc-ad)^2x}{b^3}+\frac{(bc-ad)(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{3b}+\frac{(bc-ad)^3\log(a+bx)}{b^4}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 74, normalized size = 1.01

$$\frac{bdx\left(6a^2d^2-3abd(6c+dx)+b^2\left(18c^2+9cdx+2d^2x^2\right)\right)+6(bc-ad)^3\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x),x]
```

```
[Out] (b*d*x*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x) + b^2*(18*c^2 + 9*c*d*x + 2*d^2*x^2
)) + 6*(b*c - a*d)^3*Log[a + b*x])/(6*b^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x), x]
```

**fricas [A]** time = 1.79, size = 116, normalized size = 1.59

$$\frac{2\,b^3d^3x^3+3\left(3\,b^3cd^2-ab^2d^3\right)x^2+6\left(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3\right)x+6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(bx+a\right)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(3*b^3*c^2*d - 3*a
*b^2*c*d^2 + a^2*b*d^3)*x + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(b*x + a))/b^4
```

**giac [A]** time = 1.03, size = 115, normalized size = 1.58

$$\frac{2\,b^2d^3x^3+9\,b^2cd^2x^2-3\,abd^3x^2+18\,b^2c^2dx-18\,abcd^2x+6\,a^2d^3x}{6\,b^3}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(|bx+a|\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 9*b^2*c*d^2*x^2 - 3*a*b*d^3*x^2 + 18*b^2*c^2*d*x - 18*
a*b*c*d^2*x + 6*a^2*d^3*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(b*x + a))/b^4
```

**maple [A]** time = 0.00, size = 133, normalized size = 1.82

$$\frac{d^3x^3}{3b}-\frac{a\,d^3x^2}{2b^2}+\frac{3c\,d^2x^2}{2b}-\frac{a^3d^3\ln\left(bx+a\right)}{b^4}+\frac{3a^2c\,d^2\ln\left(bx+a\right)}{b^3}+\frac{a^2d^3x}{b^3}-\frac{3a\,c^2d\ln\left(bx+a\right)}{b^2}-\frac{3ac\,d^2x}{b^2}+\frac{c^3\ln\left(bx+a\right)}{b}+\frac{3c^2dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a),x)
```

```
[Out] 1/3*d^3/b*x^3-1/2*d^3/b^2*x^2*a+3/2*d^2/b*x^2*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c
*x+3*d/b*c^2*x-1/b^4*ln(b*x+a)*a^3*d^3+3/b^3*ln(b*x+a)*a^2*c*d^2-3/b^2*ln(b
*x+a)*a*c^2*d+1/b*ln(b*x+a)*c^3
```

**maxima** [A] time = 1.33, size = 114, normalized size = 1.56

$$\frac{2\,b^2d^3x^3+3\left(3\,b^2cd^2-abd^3\right)x^2+6\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)x}{6\,b^3}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^2 + 6*(3*b^2*c^2*d - 3*a*b
*c*d^2 + a^2*d^3)*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(b*x + a)/b^4
```

**mupad** [B] time = 0.20, size = 118, normalized size = 1.62

$$x\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^2\,\left(\frac{a\,d^3}{2\,b^2}-\frac{3\,c\,d^2}{2\,b}\right)+\frac{d^3\,x^3}{3\,b}-\frac{\ln\left(a+b\,x\right)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^2*((a*d^3)/(2*b^2)
- (3*c*d^2)/(2*b)) + (d^3*x^3)/(3*b) - (log(a + b*x)*(a^3*d^3 - b^3*c^3 + 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2))/b^4
```

**sympy** [A] time = 0.30, size = 83, normalized size = 1.14

$$x^2\left(-\frac{ad^3}{2b^2}+\frac{3cd^2}{2b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)+\frac{d^3x^3}{3b}-\frac{\left(ad-bc\right)^3\log\left(a+bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a),x)
```

```
[Out] x**2*(-a*d**3/(2*b**2) + 3*c*d**2/(2*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + d**3*x**3/(3*b) - (a*d - b*c)**3*log(a + b*x)/b**4
```

## 3.1159 $\int \frac{(c+dx)^3}{(a+bx)^2} dx$

**Optimal**. Leaf size=75

$$-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{d^3x^2}{2b^2}$$

**Rubi [A]** time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^2x(3bc-2ad)}{b^3}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}+\frac{d^3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^2)/(2*b^2) - (b*c - a*d)^3/(b^4*(a + b
*x)) + (3*d*(b*c - a*d)^2*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^2}\,dx &= \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x}{b^2}+\frac{(bc-ad)^3}{b^3(a+bx)^2}+\frac{3d(bc-ad)^2}{b^3(a+bx)}\right)dx\\ &= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^2}{2b^2}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 72, normalized size = 0.96

$$\frac{2bd^2x(3bc-2ad)-\frac{2(bc-ad)^3}{a+bx}+6d(bc-ad)^2\log(a+bx)+b^2d^3x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] (2*b*d^2*(3*b*c - 2*a*d)*x + b^2*d^3*x^2 - (2*(b*c - a*d)^3)/(a + b*x) + 6*
d*(b*c - a*d)^2*Log[a + b*x])/(2*b^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^2, x]
```

**fricas [B]** time = 1.42, size = 173, normalized size = 2.31

$$\frac{b^3d^3x^3-2\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+3\left(2\,b^3cd^2-ab^2d^3\right)x^2+2\left(3\,ab^2cd^2-2\,a^2bd^3\right)x+6\left(ab^2c^2d-2\,a^2bcd^2+a^3d^3+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x\right)\log\left(bx+a\right)}{2\left(b^5x+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
3*(2*b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(3*a*b^2*c*d^2 - 2*a^2*b*d^3)*x + 6*(a*
b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(b*x + a))/(b^5*x + a*b^4)
```

**giac [B]** time = 0.99, size = 167, normalized size = 2.23

$$\frac{\left(d^3+\frac{6\left(b^2cd^2-abd^3\right)}{(bx+a)b}\right)(bx+a)^2}{2\,b^4}-\frac{3\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4}-\frac{\frac{b^5c^3}{bx+a}-\frac{3\,ab^4c^2d}{bx+a}+\frac{3\,a^2b^3cd^2}{bx+a}-\frac{a^3b^2d^3}{bx+a}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d^3 + 6*(b^2*c*d^2 - a*b*d^3)/((b*x + a)*b))*(b*x + a)^2/b^4 - 3*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^4 -
 (b^5*c^3/(b*x + a) - 3*a*b^4*c^2*d/(b*x + a) + 3*a^2*b^3*c*d^2/(b*x + a) -
 a^3*b^2*d^3/(b*x + a))/b^6
```

**maple [B]** time = 0.01, size = 149, normalized size = 1.99

$$\frac{d^3x^2}{2b^2}+\frac{a^3d^3}{(bx+a)\,b^4}-\frac{3a^2c\,d^2}{(bx+a)\,b^3}+\frac{3a^2d^3\ln(bx+a)}{b^4}+\frac{3a\,c^2d}{(bx+a)\,b^2}-\frac{6ac\,d^2\ln(bx+a)}{b^3}-\frac{2a\,d^3x}{b^3}-\frac{c^3}{(bx+a)\,b}+\frac{3c^2d\ln(bx+a)}{b^2}+\frac{3c\,d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^2,x)
```

```
[Out] 1/2*d^3*x^2/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c+3/b^4*d^3*ln(b*x+a)*a^2-6/b^3*d
^2*ln(b*x+a)*a*c+3/b^2*d*ln(b*x+a)*c^2+1/b^4/(b*x+a)*a^3*d^3-3/b^3/(b*x+a)*
a^2*c*d^2+3/b^2/(b*x+a)*a*c^2*d-1/b/(b*x+a)*c^3
```

**maxima [A]** time = 1.32, size = 118, normalized size = 1.57

$$-\frac{b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3}{b^5x + ab^4} + \frac{bd^3x^2 + 2\left(3\,bcd^2 - 2\,ad^3\right)x}{2\,b^3} + \frac{3\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\log\left(bx + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x + a*b^4) + 1/2*
(b*d^3*x^2 + 2*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*log(b*x + a)/b^4
```

**mupad [B]** time = 0.21, size = 123, normalized size = 1.64

$$\frac{\ln\left(a+b\,x\right)\,\left(3\,a^2\,d^3-6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{b^4}-x\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)+\frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{b\,\left(x\,b^4+a\,b^3\right)}+\frac{d^3\,x^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^2,x)
```

```
[Out] (log(a + b*x)*(3*a^2*d^3 + 3*b^2*c^2*d - 6*a*b*c*d^2))/b^4 - x*((2*a*d^3)/b
^3 - (3*c*d^2)/b^2) + (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)/(
b*(a*b^3 + b^4*x)) + (d^3*x^2)/(2*b^2)
```

**sympy [A]** time = 0.50, size = 102, normalized size = 1.36

$$x\left(-\frac{2ad^3}{b^3} + \frac{3cd^2}{b^2}\right) + \frac{a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3}{ab^4 + b^5x} + \frac{d^3x^2}{2b^2} + \frac{3d\left(ad - bc\right)^2\log\left(a + bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + (a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**
2*c**2*d - b**3*c**3)/(a*b**4 + b**5*x) + d**3*x**2/(2*b**2) + 3*d*(a*d - b
*c)**2*log(a + b*x)/b**4
```

## 3.1160 $\int \frac{(c+dx)^3}{(a+bx)^3}\,dx$

**Optimal**. Leaf size=78

$$\frac{3d^2(bc-ad)\log(a+bx)}{b^4}-\frac{3d(bc-ad)^2}{b^4(a+bx)}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}+\frac{d^3x}{b^3}$$

**Rubi** [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3d^2(bc-ad)\log(a+bx)}{b^4}-\frac{3d(bc-ad)^2}{b^4(a+bx)}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] (d^3*x)/b^3 - (b*c - a*d)^3/(2*b^4*(a + b*x)^2) - (3*d*(b*c - a*d)^2)/(b^4*
(a + b*x)) + (3*d^2*(b*c - a*d)*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^3}\,dx &= \int \left(\frac{d^3}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)^3}+\frac{3d(bc-ad)^2}{b^3(a+bx)^2}+\frac{3d^2(bc-ad)}{b^3(a+bx)}\right)dx\\ &= \frac{d^3x}{b^3}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}-\frac{3d(bc-ad)^2}{b^4(a+bx)}+\frac{3d^2(bc-ad)\log(a+bx)}{b^4}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 114, normalized size = 1.46

$$\frac{-5a^3d^3+a^2bd^2(9c-4dx)+ab^2d\left(-3c^2+12cdx+4d^2x^2\right)-6d^2(a+bx)^2(ad-bc)\log(a+bx)-\left(b^3\left(c^3+6c^2dx-2d^3x^3\right)\right)}{2b^4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] (-5*a^3*d^3 + a^2*b*d^2*(9*c - 4*d*x) + a*b^2*d*(-3*c^2 + 12*c*d*x + 4*d^2*
x^2) - b^3*(c^3 + 6*c^2*d*x - 2*d^3*x^3) - 6*d^2*(-(b*c) + a*d)*(a + b*x)^2
*Log[a + b*x])/(2*b^4*(a + b*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^3, x]
```

**fricas [B]** time = 1.34, size = 188, normalized size = 2.41

$$\frac{2\,b^{3}d^{3}x^{3} + 4\,ab^{2}d^{3}x^{2} - b^{3}c^{3} - 3\,ab^{2}c^{2}d + 9\,a^{2}bcd^{2} - 5\,a^{3}d^{3} - 2\left(3\,b^{3}c^{2}d - 6\,ab^{2}cd^{2} + 2\,a^{2}bd^{3}\right)x + 6\left(a^{2}bcd^{2} - a^{3}d^{3} + \left(b^{3}cd^{2} - ab^{2}d^{3}\right)x^{2} + 2\left(ab^{2}cd^{2} - a^{2}bd^{3}\right)x\right)\log\left(bx+a\right)}{2\left(b^{6}x^{2} + 2\,ab^{5}x + a^{2}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 4*a*b^2*d^3*x^2 - b^3*c^3 - 3*a*b^2*c^2*d + 9*a^2*b*c*
d^2 - 5*a^3*d^3 - 2*(3*b^3*c^2*d - 6*a*b^2*c*d^2 + 2*a^2*b*d^3)*x + 6*(a^2*
b*c*d^2 - a^3*d^3 + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(a*b^2*c*d^2 - a^2*b*d^
3)*x)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

**giac [A]** time = 0.95, size = 112, normalized size = 1.44

$$\frac{d^{3}x}{b^{3}} + \frac{3\left(bcd^{2} - ad^{3}\right)\log\left(\left|bx+a\right|\right)}{b^{4}} - \frac{b^{3}c^{3} + 3\,ab^{2}c^{2}d - 9\,a^{2}bcd^{2} + 5\,a^{3}d^{3} + 6\left(b^{3}c^{2}d - 2\,ab^{2}cd^{2} + a^{2}bd^{3}\right)x}{2\left(bx+a\right)^{2}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] d^3*x/b^3 + 3*(b*c*d^2 - a*d^3)*log(abs(b*x + a))/b^4 - 1/2*(b^3*c^3 + 3*a*
b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/((b*x + a)^2*b^4)
```

**maple [B]** time = 0.01, size = 160, normalized size = 2.05

$$\frac{a^{3}d^{3}}{2\left(bx+a\right)^{2}b^{4}} - \frac{3a^{2}c\,d^{2}}{2\left(bx+a\right)^{2}b^{3}} + \frac{3a\,c^{2}d}{2\left(bx+a\right)^{2}b^{2}} - \frac{c^{3}}{2\left(bx+a\right)^{2}b} - \frac{3a^{2}d^{3}}{\left(bx+a\right)b^{4}} + \frac{6ac\,d^{2}}{\left(bx+a\right)b^{3}} - \frac{3a\,d^{3}\ln\left(bx+a\right)}{b^{4}} - \frac{3c^{2}d}{\left(bx+a\right)b^{2}} + \frac{3c\,d^{2}\ln\left(bx+a\right)}{b^{3}} + \frac{d^{3}x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^3,x)
```

```
[Out] d^3*x/b^3+1/2/b^4/(b*x+a)^2*a^3*d^3-3/2/b^3/(b*x+a)^2*a^2*c*d^2+3/2/b^2/(b*
x+a)^2*a*c^2*d-1/2/b/(b*x+a)^2*c^3-3/b^4*d^3*ln(b*x+a)*a+3/b^3*d^2*ln(b*x+a
)*c-3/b^4*d^3/(b*x+a)*a^2+6/b^3*d^2/(b*x+a)*a*c-3/b^2*d/(b*x+a)*c^2
```

**maxima [A]** time = 1.37, size = 125, normalized size = 1.60

$$\frac{d^3x}{b^3} - \frac{b^3c^3 + 3\,ab^2c^2d - 9\,a^2bcd^2 + 5\,a^3d^3 + 6\left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x}{2\left(b^6x^2 + 2\,ab^5x + a^2b^4\right)} + \frac{3\left(bcd^2 - ad^3\right)\log\left(bx + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] d^3*x/b^3 - 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) +
3*(b*c*d^2 - a*d^3)*log(b*x + a)/b^4
```

**mupad [B]** time = 0.82, size = 130, normalized size = 1.67

$$\frac{d^3\,x}{b^3} - \frac{\ln\left(a + b\,x\right)\,\left(3\,a\,d^3 - 3\,b\,c\,d^2\right)}{b^4} - \frac{\frac{5\,a^3\,d^3 - 9\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d + b^3\,c^3}{2\,b} + x\,\left(3\,a^2\,d^3 - 6\,a\,b\,c\,d^2 + 3\,b^2\,c^2\,d\right)}{a^2\,b^3 + 2\,a\,b^4\,x + b^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^3,x)
```

```
[Out] (d^3*x)/b^3 - (log(a + b*x)*(3*a*d^3 - 3*b*c*d^2))/b^4 - ((5*a^3*d^3 + b^3*
c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2)/(2*b) + x*(3*a^2*d^3 + 3*b^2*c^2*d - 6
*a*b*c*d^2))/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x)
```

**sympy [A]** time = 0.82, size = 128, normalized size = 1.64

$$\frac{-5a^3d^3 + 9a^2bcd^2 - 3ab^2c^2d - b^3c^3 + x\left(-6a^2bd^3 + 12ab^2cd^2 - 6b^3c^2d\right)}{2a^2b^4 + 4ab^5x + 2b^6x^2} + \frac{d^3x}{b^3} - \frac{3d^2\left(ad - bc\right)\log\left(a + bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**3,x)
```

```
[Out] (-5*a**3*d**3 + 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - b**3*c**3 + x*(-6*a**2*
b*d**3 + 12*a*b**2*c*d**2 - 6*b**3*c**2*d))/(2*a**2*b**4 + 4*a*b**5*x + 2*b
**6*x**2) + d**3*x/b**3 - 3*d**2*(a*d - b*c)*log(a + b*x)/b**4
```

# 3.1161 $\int \frac{(c+dx)^3}{(a+bx)^4} dx$

**Optimal**. Leaf size=86

$$-\frac{3d^2(bc-ad)}{b^4(a+bx)}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{(bc-ad)^3}{3b^4(a+bx)^3}+\frac{d^3\log(a+bx)}{b^4}$$

**Rubi** [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3d^2(bc-ad)}{b^4(a+bx)}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{(bc-ad)^3}{3b^4(a+bx)^3}+\frac{d^3\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^4,x]
```

```
[Out] -(b*c - a*d)^3/(3*b^4*(a + b*x)^3) - (3*d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^2
) - (3*d^2*(b*c - a*d))/(b^4*(a + b*x)) + (d^3*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^4}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^4}+\frac{3d(bc-ad)^2}{b^3(a+bx)^3}+\frac{3d^2(bc-ad)}{b^3(a+bx)^2}+\frac{d^3}{b^3(a+bx)}\right)dx\\ &= -\frac{(bc-ad)^3}{3b^4(a+bx)^3}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{3d^2(bc-ad)}{b^4(a+bx)}+\frac{d^3\log(a+bx)}{b^4}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 80, normalized size = 0.93

$$\frac{6d^3\log(a+bx)-\frac{(bc-ad)\left(11a^2d^2+abd(5c+27dx)+b^2\left(2c^2+9cdx+18d^2x^2\right)\right)}{(a+bx)^3}}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^4,x]
```

```
[Out] (-(((b*c - a*d)*(11*a^2*d^2 + a*b*d*(5*c + 27*d*x) + b^2*(2*c^2 + 9*c*d*x +
 18*d^2*x^2)))/(a + b*x)^3) + 6*d^3*Log[a + b*x])/(6*b^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^4, x]
```

**fricas [B]** time = 1.51, size = 176, normalized size = 2.05

$$-\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3+18\left(b^3cd^2-ab^2d^3\right)x^2+9\left(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3\right)x-6\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)\log\left(bx+a\right)}{6\left(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3
*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

**giac [A]** time = 0.94, size = 118, normalized size = 1.37

$$\frac{d^3\log\left(|bx+a|\right)}{b^4}-\frac{18\left(b^2cd^2-abd^3\right)x^2+9\left(b^2c^2d+2\,abcd^2-3\,a^2d^3\right)x+\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3}{b}}{6\,(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] d^3*log(abs(b*x + a))/b^4 - 1/6*(18*(b^2*c*d^2 - a*b*d^3)*x^2 + 9*(b^2*c^2*
d + 2*a*b*c*d^2 - 3*a^2*d^3)*x + (2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2
 - 11*a^3*d^3)/b)/((b*x + a)^3*b^3)
```

**maple [B]** time = 0.01, size = 166, normalized size = 1.93

$$\frac{a^3d^3}{3\,(bx+a)^3\,b^4}-\frac{a^2c\,d^2}{(bx+a)^3\,b^3}+\frac{a\,c^2d}{(bx+a)^3\,b^2}-\frac{c^3}{3\,(bx+a)^3\,b}-\frac{3a^2d^3}{2\,(bx+a)^2\,b^4}+\frac{3ac\,d^2}{(bx+a)^2\,b^3}-\frac{3c^2d}{2\,(bx+a)^2\,b^2}+\frac{3a\,d^3}{(bx+a)\,b^4}-\frac{3c\,d^2}{(bx+a)\,b^3}+\frac{d^3\ln\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^4,x)
```

```
[Out] 1/3/b^4/(b*x+a)^3*a^3*d^3-1/b^3/(b*x+a)^3*a^2*c*d^2+1/b^2/(b*x+a)^3*a*c^2*d
-1/3/b/(b*x+a)^3*c^3-3/2*d^3/b^4/(b*x+a)^2*a^2+3*d^2/b^3/(b*x+a)^2*a*c-3/2*
d/b^2/(b*x+a)^2*c^2+d^3*ln(b*x+a)/b^4+3/b^4*d^3/(b*x+a)*a-3/b^3*d^2/(b*x+a)
*c
```

**maxima** [A] time = 1.35, size = 142, normalized size = 1.65

$$-\frac{2\,b^3c^3 + 3\,ab^2c^2d + 6\,a^2bcd^2 - 11\,a^3d^3 + 18\left(b^3cd^2 - ab^2d^3\right)x^2 + 9\left(b^3c^2d + 2\,ab^2cd^2 - 3\,a^2bd^3\right)x}{6\left(b^7x^3 + 3\,ab^6x^2 + 3\,a^2b^5x + a^3b^4\right)} + \frac{d^3\log\left(bx + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x)/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4) + d^3*log(b*x + a)/b^4
```

**mupad** [B] time = 0.25, size = 138, normalized size = 1.60

$$\frac{d^3\,\ln\left(a+b\,x\right)}{b^4}-\frac{\frac{-11\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+2\,b^3\,c^3}{6\,b^4}+\frac{3\,x\,\left(-3\,a^2\,d^3+2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{2\,b^3}-\frac{3\,d^2\,x^2\,\left(a\,d-b\,c\right)}{b^2}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^4,x)
```

```
[Out] (d^3*log(a + b*x))/b^4 - ((2*b^3*c^3 - 11*a^3*d^3 + 3*a*b^2*c^2*d + 6*a^2*b
*c*d^2)/(6*b^4) + (3*x*(b^2*c^2*d - 3*a^2*d^3 + 2*a*b*c*d^2))/(2*b^3) - (3*
d^2*x^2*(a*d - b*c))/b^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

**sympy** [A] time = 1.13, size = 148, normalized size = 1.72

$$\frac{11a^3d^3 - 6a^2bcd^2 - 3ab^2c^2d - 2b^3c^3 + x^2\left(18ab^2d^3 - 18b^3cd^2\right) + x\left(27a^2bd^3 - 18ab^2cd^2 - 9b^3c^2d\right)}{6a^3b^4 + 18a^2b^5x + 18ab^6x^2 + 6b^7x^3} + \frac{d^3\log\left(a + bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**4,x)
```

```
[Out] (11*a**3*d**3 - 6*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 2*b**3*c**3 + x**2*(18*
a*b**2*d**3 - 18*b**3*c*d**2) + x*(27*a**2*b*d**3 - 18*a*b**2*c*d**2 - 9*b*
*3*c**2*d))/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 d**3*log(a + b*x)/b**4
```

## 3.1162 $\int \frac{(c+dx)^3}{(a+bx)^5}\,dx$

**Optimal**. Leaf size=28

$$-\frac{(c+dx)^4}{4(a+bx)^4(bc-ad)}$$

**Rubi** [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^4}{4(a+bx)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] -(c + d*x)^4/(4*(b*c - a*d)*(a + b*x)^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+bx)^5}\,dx = -\frac{(c+dx)^4}{4(bc-ad)(a+bx)^4}$$

**Mathematica** [B] time = 0.03, size = 91, normalized size = 3.25

$$-\frac{a^3d^3+a^2bd^2(c+4dx)+ab^2d\left(c^2+4cdx+6d^2x^2\right)+b^3\left(c^3+4c^2dx+6cd^2x^2+4d^3x^3\right)}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] -1/4*(a^3*d^3 + a^2*b*d^2*(c + 4*d*x) + a*b^2*d*(c^2 + 4*c*d*x + 6*d^2*x^2)
 + b^3*(c^3 + 4*c^2*d*x + 6*c*d^2*x^2 + 4*d^3*x^3))/(b^4*(a + b*x)^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^5, x]
```

**fricas [B]** time = 1.54, size = 143, normalized size = 5.11

$$-\frac{4\,b^3d^3x^3+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3+6\left(b^3cd^2+ab^2d^3\right)x^2+4\left(b^3c^2d+ab^2cd^2+a^2bd^3\right)x}{4\left(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

**giac [B]** time = 0.97, size = 159, normalized size = 5.68

$$-\frac{\frac{b^2c^3}{(bx+a)^4}+\frac{4\,bc^2d}{(bx+a)^3}-\frac{3\,abc^2d}{(bx+a)^4}+\frac{6\,cd^2}{(bx+a)^2}-\frac{8\,acd^2}{(bx+a)^3}+\frac{3\,a^2cd^2}{(bx+a)^4}+\frac{4\,d^3}{(bx+a)b}-\frac{6\,ad^3}{(bx+a)^2b}+\frac{4\,a^2d^3}{(bx+a)^3b}-\frac{a^3d^3}{(bx+a)^4b}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*(b^2*c^3/(b*x + a)^4 + 4*b*c^2*d/(b*x + a)^3 - 3*a*b*c^2*d/(b*x + a)^4
 + 6*c*d^2/(b*x + a)^2 - 8*a*c*d^2/(b*x + a)^3 + 3*a^2*c*d^2/(b*x + a)^4 +
4*d^3/((b*x + a)*b) - 6*a*d^3/((b*x + a)^2*b) + 4*a^2*d^3/((b*x + a)^3*b) -
 a^3*d^3/((b*x + a)^4*b))/b^3
```

**maple [B]** time = 0.01, size = 122, normalized size = 4.36

$$-\frac{d^3}{(bx+a)\,b^4}+\frac{3\,(ad-bc)\,d^2}{2\,(bx+a)^2\,b^4}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)d}{(bx+a)^3\,b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{4\,(bx+a)^4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^5,x)
```

```
[Out] -d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^3+3/2*d^2*(a*d-b*c)/b^4/(b*x+a)^
2-1/4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^4-d^3/b^4/
(b*x+a)
```

**maxima [B]** time = 1.39, size = 143, normalized size = 5.11

$$-\frac{4\,b^3d^3x^3 + b^3c^3 + ab^2c^2d + a^2bcd^2 + a^3d^3 + 6\left(b^3cd^2 + ab^2d^3\right)x^2 + 4\left(b^3c^2d + ab^2cd^2 + a^2bd^3\right)x}{4\left(b^8x^4 + 4\,ab^7x^3 + 6\,a^2b^6x^2 + 4\,a^3b^5x + a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

**mupad [B]** time = 0.07, size = 135, normalized size = 4.82

$$-\frac{\frac{a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3}{4\,b^4}+\frac{d^3\,x^3}{b}+\frac{d\,x\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{b^3}+\frac{3\,d^2\,x^2\,\left(a\,d+b\,c\right)}{2\,b^2}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^5,x)
```

```
[Out] -((a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2)/(4*b^4) + (d^3*x^3)/b + (
d*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/b^3 + (3*d^2*x^2*(a*d + b*c))/(2*b^2))/(
a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)
```

**sympy [B]** time = 1.50, size = 155, normalized size = 5.54

$$\frac{-a^3d^3 - a^2bcd^2 - ab^2c^2d - b^3c^3 - 4b^3d^3x^3 + x^2\left(-6ab^2d^3 - 6b^3cd^2\right) + x\left(-4a^2bd^3 - 4ab^2cd^2 - 4b^3c^2d\right)}{4a^4b^4 + 16a^3b^5x + 24a^2b^6x^2 + 16ab^7x^3 + 4b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**5,x)
```

```
[Out] (-a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d - b**3*c**3 - 4*b**3*d**3*x**3
+ x**2*(-6*a*b**2*d**3 - 6*b**3*c*d**2) + x*(-4*a**2*b*d**3 - 4*a*b**2*c*d*
*2 - 4*b**3*c**2*d))/(4*a**4*b**4 + 16*a**3*b**5*x + 24*a**2*b**6*x**2 + 16
*a*b**7*x**3 + 4*b**8*x**4)
```

## 3.1163 $\int \frac{(c+dx)^3}{(a+bx)^6} dx$

**Optimal**. Leaf size=58

$$\frac{d(c+dx)^4}{20(a+bx)^4(bc-ad)^2} - \frac{(c+dx)^4}{5(a+bx)^5(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d(c+dx)^4}{20(a+bx)^4(bc-ad)^2} - \frac{(c+dx)^4}{5(a+bx)^5(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] -(c + d*x)^4/(5*(b*c - a*d)*(a + b*x)^5) + (d*(c + d*x)^4)/(20*(b*c - a*d)^
2*(a + b*x)^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+bx)^6}\,dx = -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5} - \frac{d\int \frac{(c+dx)^3}{(a+bx)^5}\,dx}{5(bc-ad)}$$
$$= -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5} + \frac{d(c+dx)^4}{20(bc-ad)^2(a+bx)^4}$$

**Mathematica [A]** time = 0.03, size = 97, normalized size = 1.67

$$-\frac{a^3d^3 + a^2bd^2(2c+5dx) + ab^2d\left(3c^2+10cdx+10d^2x^2\right) + b^3\left(4c^3+15c^2dx+20cd^2x^2+10d^3x^3\right)}{20b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] -1/20*(a^3*d^3 + a^2*b*d^2*(2*c + 5*d*x) + a*b^2*d*(3*c^2 + 10*c*d*x + 10*d
^2*x^2) + b^3*(4*c^3 + 15*c^2*d*x + 20*c*d^2*x^2 + 10*d^3*x^3))/(b^4*(a + b
*x)^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^6, x]
```

**fricas [B]** time = 1.65, size = 160, normalized size = 2.76

$$-\frac{10\,b^3d^3x^3 + 4\,b^3c^3 + 3\,ab^2c^2d + 2\,a^2bcd^2 + a^3d^3 + 10\left(2\,b^3cd^2 + ab^2d^3\right)x^2 + 5\left(3\,b^3c^2d + 2\,ab^2cd^2 + a^2bd^3\right)x}{20\left(b^9x^5 + 5\,ab^8x^4 + 10\,a^2b^7x^3 + 10\,a^3b^6x^2 + 5\,a^4b^5x + a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

**giac [B]** time = 1.00, size = 114, normalized size = 1.97

$$-\frac{10\,b^{3}d^{3}x^{3}+20\,b^{3}cd^{2}x^{2}+10\,ab^{2}d^{3}x^{2}+15\,b^{3}c^{2}dx+10\,ab^{2}cd^{2}x+5\,a^{2}bd^{3}x+4\,b^{3}c^{3}+3\,ab^{2}c^{2}d+2\,a^{2}bcd^{2}+a^{3}d^{3}}{20\,(bx+a)^{5}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 20*b^3*c*d^2*x^2 + 10*a*b^2*d^3*x^2 + 15*b^3*c^2*d*
x + 10*a*b^2*c*d^2*x + 5*a^2*b*d^3*x + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*
c*d^2 + a^3*d^3)/((b*x + a)^5*b^4)
```

**maple [B]** time = 0.01, size = 121, normalized size = 2.09

$$-\frac{d^{3}}{2\,(bx+a)^{2}\,b^{4}}+\frac{(ad-bc)\,d^{2}}{(bx+a)^{3}\,b^{4}}-\frac{3\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)d}{4\,(bx+a)^{4}\,b^{4}}-\frac{-a^{3}d^{3}+3a^{2}bc\,d^{2}-3a\,b^{2}c^{2}d+b^{3}c^{3}}{5\,(bx+a)^{5}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^6,x)
```

```
[Out] d^2*(a*d-b*c)/b^4/(b*x+a)^3-1/5*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c
^3)/b^4/(b*x+a)^5-1/2*d^3/b^4/(b*x+a)^2-3/4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b
^4/(b*x+a)^4
```

**maxima [B]** time = 1.47, size = 160, normalized size = 2.76

$$-\frac{10\,b^{3}d^{3}x^{3}+4\,b^{3}c^{3}+3\,ab^{2}c^{2}d+2\,a^{2}bcd^{2}+a^{3}d^{3}+10\left(2\,b^{3}cd^{2}+ab^{2}d^{3}\right)x^{2}+5\left(3\,b^{3}c^{2}d+2\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x}{20\left(b^{9}x^{5}+5\,ab^{8}x^{4}+10\,a^{2}b^{7}x^{3}+10\,a^{3}b^{6}x^{2}+5\,a^{4}b^{5}x+a^{5}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

**mupad [B]** time = 0.08, size = 39, normalized size = 0.67

$$\frac{(c+d\,x)^{4}\,\left(5\,a\,d-4\,b\,c+b\,d\,x\right)}{20\,(a\,d-b\,c)^{2}\,(a+b\,x)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^6,x)
```

```
[Out] ((c + d*x)^4*(5*a*d - 4*b*c + b*d*x))/(20*(a*d - b*c)^2*(a + b*x)^5)
```

**sympy [B]** time = 1.96, size = 172, normalized size = 2.97

$$\frac{-a^3d^3 - 2a^2bcd^2 - 3ab^2c^2d - 4b^3c^3 - 10b^3d^3x^3 + x^2\left(-10ab^2d^3 - 20b^3cd^2\right) + x\left(-5a^2bd^3 - 10ab^2cd^2 - 15b^3c^2d\right)}{20a^5b^4 + 100a^4b^5x + 200a^3b^6x^2 + 200a^2b^7x^3 + 100ab^8x^4 + 20b^9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**6,x)
```

```
[Out] (-a**3*d**3 - 2*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 4*b**3*c**3 - 10*b**3*d**
3*x**3 + x**2*(-10*a*b**2*d**3 - 20*b**3*c*d**2) + x*(-5*a**2*b*d**3 - 10*a
*b**2*c*d**2 - 15*b**3*c**2*d))/(20*a**5*b**4 + 100*a**4*b**5*x + 200*a**3*
b**6*x**2 + 200*a**2*b**7*x**3 + 100*a*b**8*x**4 + 20*b**9*x**5)
```

## 3.1164 $\int \frac{(c+dx)^3}{(a+bx)^7}\,dx$

**Optimal**. Leaf size=92

$$-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{d^3}{3b^4(a+bx)^3}$$

**Rubi** [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{d^3}{3b^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^7,x]
```

```
[Out] -(b*c - a*d)^3/(6*b^4*(a + b*x)^6) - (3*d*(b*c - a*d)^2)/(5*b^4*(a + b*x)^5
) - (3*d^2*(b*c - a*d))/(4*b^4*(a + b*x)^4) - d^3/(3*b^4*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^7}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^7}+\frac{3d(bc-ad)^2}{b^3(a+bx)^6}+\frac{3d^2(bc-ad)}{b^3(a+bx)^5}+\frac{d^3}{b^3(a+bx)^4}\right)dx\\ &= -\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{d^3}{3b^4(a+bx)^3}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+3a^2bd^2(c+2dx)+3ab^2d\left(2c^2+6cdx+5d^2x^2\right)+b^3\left(10c^3+36c^2dx+45cd^2x^2+20d^3x^3\right)}{60b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^3*d^3 + 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*d*(2*c^2 + 6*c*d*x + 5*d
^2*x^2) + b^3*(10*c^3 + 36*c^2*d*x + 45*c*d^2*x^2 + 20*d^3*x^3))/(b^4*(a +
b*x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^7} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^7, x]
```

**fricas [B]** time = 1.49, size = 171, normalized size = 1.86

$$-\frac{20\,b^3d^3x^3+10\,b^3c^3+6\,ab^2c^2d+3\,a^2bcd^2+a^3d^3+15\left(3\,b^3cd^2+ab^2d^3\right)x^2+6\left(6\,b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x}{60\left(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

**giac [A]** time = 0.98, size = 114, normalized size = 1.24

$$-\frac{20\,b^3d^3x^3+45\,b^3cd^2x^2+15\,ab^2d^3x^2+36\,b^3c^2dx+18\,ab^2cd^2x+6\,a^2bd^3x+10\,b^3c^3+6\,ab^2c^2d+3\,a^2bcd^2+a^3d^3}{60\,(bx+a)^6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 45*b^3*c*d^2*x^2 + 15*a*b^2*d^3*x^2 + 36*b^3*c^2*d*
x + 18*a*b^2*c*d^2*x + 6*a^2*b*d^3*x + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b
*c*d^2 + a^3*d^3)/((b*x + a)^6*b^4)
```

**maple [A]** time = 0.01, size = 122, normalized size = 1.33

$$-\frac{d^3}{3\,(bx+a)^3\,b^4}+\frac{3\,(ad-bc)\,d^2}{4\,(bx+a)^4\,b^4}-\frac{3\left(a^2d^2-2abcd+b^2c^2\right)d}{5\,(bx+a)^5\,b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{6\,(bx+a)^6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^7,x)
```

```
[Out] -1/3*d^3/b^4/(b*x+a)^3-3/5*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^5+3/4*
d^2*(a*d-b*c)/b^4/(b*x+a)^4-1/6*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c
^3)/b^4/(b*x+a)^6
```

**maxima [B]** time = 1.44, size = 171, normalized size = 1.86

$$-\frac{20\,b^{3}d^{3}x^{3}+10\,b^{3}c^{3}+6\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}+a^{3}d^{3}+15\left(3\,b^{3}cd^{2}+ab^{2}d^{3}\right)x^{2}+6\left(6\,b^{3}c^{2}d+3\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x}{60\left(b^{10}x^{6}+6\,ab^{9}x^{5}+15\,a^{2}b^{8}x^{4}+20\,a^{3}b^{7}x^{3}+15\,a^{4}b^{6}x^{2}+6\,a^{5}b^{5}x+a^{6}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

**mupad [B]** time = 0.22, size = 165, normalized size = 1.79

$$-\frac{\frac{a^{3}\,d^{3}+3\,a^{2}\,b\,c\,d^{2}+6\,a\,b^{2}\,c^{2}\,d+10\,b^{3}\,c^{3}}{60\,b^{4}}+\frac{d^{3}\,x^{3}}{3\,b}+\frac{d\,x\,\left(a^{2}\,d^{2}+3\,a\,b\,c\,d+6\,b^{2}\,c^{2}\right)}{10\,b^{3}}+\frac{d^{2}\,x^{2}\,\left(a\,d+3\,b\,c\right)}{4\,b^{2}}}{a^{6}+6\,a^{5}\,b\,x+15\,a^{4}\,b^{2}\,x^{2}+20\,a^{3}\,b^{3}\,x^{3}+15\,a^{2}\,b^{4}\,x^{4}+6\,a\,b^{5}\,x^{5}+b^{6}\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^7,x)
```

```
[Out] -((a^3*d^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(60*b^4) + (d^3*x^
3)/(3*b) + (d*x*(a^2*d^2 + 6*b^2*c^2 + 3*a*b*c*d))/(10*b^3) + (d^2*x^2*(a*d
 + 3*b*c))/(4*b^2))/(a^6 + b^6*x^6 + 6*a*b^5*x^5 + 15*a^4*b^2*x^2 + 20*a^3*
b^3*x^3 + 15*a^2*b^4*x^4 + 6*a^5*b*x)
```

**sympy [B]** time = 2.54, size = 184, normalized size = 2.00

$$\frac{-a^{3}d^{3}-3a^{2}bcd^{2}-6ab^{2}c^{2}d-10b^{3}c^{3}-20b^{3}d^{3}x^{3}+x^{2}\left(-15ab^{2}d^{3}-45b^{3}cd^{2}\right)+x\left(-6a^{2}bd^{3}-18ab^{2}cd^{2}-36b^{3}c^{2}d\right)}{60a^{6}b^{4}+360a^{5}b^{5}x+900a^{4}b^{6}x^{2}+1200a^{3}b^{7}x^{3}+900a^{2}b^{8}x^{4}+360ab^{9}x^{5}+60b^{10}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**7,x)
```

```
[Out] (-a**3*d**3 - 3*a**2*b*c*d**2 - 6*a*b**2*c**2*d - 10*b**3*c**3 - 20*b**3*d*
*3*x**3 + x**2*(-15*a*b**2*d**3 - 45*b**3*c*d**2) + x*(-6*a**2*b*d**3 - 18*
```

```
a*b**2*c*d**2 - 36*b**3*c**2*d))/(60*a**6*b**4 + 360*a**5*b**5*x + 900*a**4
*b**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 + 360*a*b**9*x**5 + 6
0*b**10*x**6)
```

## 3.1165 $\int \frac{(c+dx)^3}{(a+bx)^8} dx$

**Optimal**. Leaf size=92

$$-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d^3}{4b^4(a+bx)^4}$$

**Rubi** [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d^3}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] -(b*c - a*d)^3/(7*b^4*(a + b*x)^7) - (d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^6)
 - (3*d^2*(b*c - a*d))/(5*b^4*(a + b*x)^5) - d^3/(4*b^4*(a + b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^8}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^8}+\frac{3d(bc-ad)^2}{b^3(a+bx)^7}+\frac{3d^2(bc-ad)}{b^3(a+bx)^6}+\frac{d^3}{b^3(a+bx)^5}\right)dx\\ &= -\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d^3}{4b^4(a+bx)^4}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+a^2bd^2(4c+7dx)+ab^2d\left(10c^2+28cdx+21d^2x^2\right)+b^3\left(20c^3+70c^2dx+84cd^2x^2+35d^3x^3\right)}{140b^4(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] -1/140*(a^3*d^3 + a^2*b*d^2*(4*c + 7*d*x) + a*b^2*d*(10*c^2 + 28*c*d*x + 21
*d^2*x^2) + b^3*(20*c^3 + 70*c^2*d*x + 84*c*d^2*x^2 + 35*d^3*x^3))/(b^4*(a
+ b*x)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^8, x]
```

**fricas [B]** time = 1.46, size = 182, normalized size = 1.98

$$-\frac{35\,b^3d^3x^3+20\,b^3c^3+10\,ab^2c^2d+4\,a^2bcd^2+a^3d^3+21\left(4\,b^3cd^2+ab^2d^3\right)x^2+7\left(10\,b^3c^2d+4\,ab^2cd^2+a^2bd^3\right)x}{140\left(b^{11}x^7+7\,ab^{10}x^6+21\,a^2b^9x^5+35\,a^3b^8x^4+35\,a^4b^7x^3+21\,a^5b^6x^2+7\,a^6b^5x+a^7b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

**giac [A]** time = 0.95, size = 114, normalized size = 1.24

$$-\frac{35\,b^3d^3x^3+84\,b^3cd^2x^2+21\,ab^2d^3x^2+70\,b^3c^2dx+28\,ab^2cd^2x+7\,a^2bd^3x+20\,b^3c^3+10\,ab^2c^2d+4\,a^2bcd^2+a^3d^3}{140\,(bx+a)^7b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 84*b^3*c*d^2*x^2 + 21*a*b^2*d^3*x^2 + 70*b^3*c^2*d
*x + 28*a*b^2*c*d^2*x + 7*a^2*b*d^3*x + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2
*b*c*d^2 + a^3*d^3)/((b*x + a)^7*b^4)
```

**maple [A]** time = 0.01, size = 122, normalized size = 1.33

$$-\frac{d^3}{4\,(bx+a)^4\,b^4}+\frac{3\,(ad-bc)\,d^2}{5\,(bx+a)^5\,b^4}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)d}{2\,(bx+a)^6\,b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{7\,(bx+a)^7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^8,x)
```

```
[Out] -1/7*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^7+3/5*d^2*(
a*d-b*c)/b^4/(b*x+a)^5-1/4*d^3/b^4/(b*x+a)^4-1/2*d*(a^2*d^2-2*a*b*c*d+b^2*c
^2)/b^4/(b*x+a)^6
```

**maxima [B]** time = 1.45, size = 182, normalized size = 1.98

$$-\frac{35 \, b^{3} d^{3} x^{3} + 20 \, b^{3} c^{3} + 10 \, a b^{2} c^{2} d + 4 \, a^{2} b c d^{2} + a^{3} d^{3} + 21 \, {\left(4 \, b^{3} c d^{2} + a b^{2} d^{3}\right)} x^{2} + 7 \, {\left(10 \, b^{3} c^{2} d + 4 \, a b^{2} c d^{2} + a^{2} b d^{3}\right)} x}{140 \, {\left(b^{11} x^{7} + 7 \, a b^{10} x^{6} + 21 \, a^{2} b^{9} x^{5} + 35 \, a^{3} b^{8} x^{4} + 35 \, a^{4} b^{7} x^{3} + 21 \, a^{5} b^{6} x^{2} + 7 \, a^{6} b^{5} x + a^{7} b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

**mupad [B]** time = 0.11, size = 176, normalized size = 1.91

$$-\frac{\frac{a^3\,d^3+4\,a^2\,b\,c\,d^2+10\,a\,b^2\,c^2\,d+20\,b^3\,c^3}{140\,b^4}+\frac{d^3\,x^3}{4\,b}+\frac{d\,x\,\left(a^2\,d^2+4\,a\,b\,c\,d+10\,b^2\,c^2\right)}{20\,b^3}+\frac{3\,d^2\,x^2\,\left(a\,d+4\,b\,c\right)}{20\,b^2}}{a^7+7\,a^6\,b\,x+21\,a^5\,b^2\,x^2+35\,a^4\,b^3\,x^3+35\,a^3\,b^4\,x^4+21\,a^2\,b^5\,x^5+7\,a\,b^6\,x^6+b^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^8,x)
```

```
[Out] -((a^3*d^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2)/(140*b^4) + (d^3*
x^3)/(4*b) + (d*x*(a^2*d^2 + 10*b^2*c^2 + 4*a*b*c*d))/(20*b^3) + (3*d^2*x^2
*(a*d + 4*b*c))/(20*b^2))/(a^7 + b^7*x^7 + 7*a*b^6*x^6 + 21*a^5*b^2*x^2 + 3
5*a^4*b^3*x^3 + 35*a^3*b^4*x^4 + 21*a^2*b^5*x^5 + 7*a^6*b*x)
```

**sympy [B]** time = 3.12, size = 196, normalized size = 2.13

$$\frac{- a^{3} d^{3} - 4 a^{2} b c d^{2} - 10 a b^{2} c^{2} d - 20 b^{3} c^{3} - 35 b^{3} d^{3} x^{3} + x^{2} \left(- 21 a b^{2} d^{3} - 84 b^{3} c d^{2}\right) + x \left(- 7 a^{2} b d^{3} - 28 a b^{2} c d^{2} - 70 b^{3} c^{2} d\right)}{140 a^{7} b^{4} + 980 a^{6} b^{5} x + 2940 a^{5} b^{6} x^{2} + 4900 a^{4} b^{7} x^{3} + 4900 a^{3} b^{8} x^{4} + 2940 a^{2} b^{9} x^{5} + 980 a b^{10} x^{6} + 140 b^{11} x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**8,x)
```

```
[Out] (-a**3*d**3 - 4*a**2*b*c*d**2 - 10*a*b**2*c**2*d - 20*b**3*c**3 - 35*b**3*d
**3*x**3 + x**2*(-21*a*b**2*d**3 - 84*b**3*c*d**2) + x*(-7*a**2*b*d**3 - 28
```

```
*a*b**2*c*d**2 - 70*b**3*c**2*d))/(140*a**7*b**4 + 980*a**6*b**5*x + 2940*a
**5*b**6*x**2 + 4900*a**4*b**7*x**3 + 4900*a**3*b**8*x**4 + 2940*a**2*b**9*
x**5 + 980*a*b**10*x**6 + 140*b**11*x**7)
```

## 3.1166 $\int \frac{(c+dx)^3}{(a+bx)^9} \, dx$

**Optimal**. Leaf size=92

$$-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{d^3}{5b^4(a+bx)^5}$$

**Rubi** [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{d^3}{5b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] -(b*c - a*d)^3/(8*b^4*(a + b*x)^8) - (3*d*(b*c - a*d)^2)/(7*b^4*(a + b*x)^7
) - (d^2*(b*c - a*d))/(2*b^4*(a + b*x)^6) - d^3/(5*b^4*(a + b*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{(a+bx)^9} \, dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^9}+\frac{3d(bc-ad)^2}{b^3(a+bx)^8}+\frac{3d^2(bc-ad)}{b^3(a+bx)^7}+\frac{d^3}{b^3(a+bx)^6}\right) dx\\
&= -\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{d^3}{5b^4(a+bx)^5}
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+a^2bd^2(5c+8dx)+ab^2d\left(15c^2+40cdx+28d^2x^2\right)+b^3\left(35c^3+120c^2dx+140cd^2x^2+56d^3x^3\right)}{280b^4(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] -1/280*(a^3*d^3 + a^2*b*d^2*(5*c + 8*d*x) + a*b^2*d*(15*c^2 + 40*c*d*x + 28
*d^2*x^2) + b^3*(35*c^3 + 120*c^2*d*x + 140*c*d^2*x^2 + 56*d^3*x^3))/(b^4*(
a + b*x)^8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^9, x]
```

**fricas [B]** time = 1.44, size = 193, normalized size = 2.10

$$-\frac{56\,b^3d^3x^3+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3+28\left(5\,b^3cd^2+ab^2d^3\right)x^2+8\left(15\,b^3c^2d+5\,ab^2cd^2+a^2bd^3\right)x}{280\left(b^{12}x^8+8\,ab^{11}x^7+28\,a^2b^{10}x^6+56\,a^3b^9x^5+70\,a^4b^8x^4+56\,a^5b^7x^3+28\,a^6b^6x^2+8\,a^7b^5x+a^8b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

**giac [A]** time = 0.86, size = 114, normalized size = 1.24

$$-\frac{56\,b^3d^3x^3+140\,b^3cd^2x^2+28\,ab^2d^3x^2+120\,b^3c^2dx+40\,ab^2cd^2x+8\,a^2bd^3x+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3}{280\,(bx+a)^8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 140*b^3*c*d^2*x^2 + 28*a*b^2*d^3*x^2 + 120*b^3*c^2
*d*x + 40*a*b^2*c*d^2*x + 8*a^2*b*d^3*x + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a
^2*b*c*d^2 + a^3*d^3)/((b*x + a)^8*b^4)
```

**maple [A]** time = 0.01, size = 122, normalized size = 1.33

$$-\frac{d^3}{5\,(bx+a)^5\,b^4}+\frac{(ad-bc)\,d^2}{2\,(bx+a)^6\,b^4}-\frac{3\left(a^2d^2-2abcd+b^2c^2\right)d}{7\,(bx+a)^7\,b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{8\,(bx+a)^8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^9,x)
```

```
[Out] -1/8*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^8-3/7*d*(a^
2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^7-1/5*d^3/b^4/(b*x+a)^5+1/2*d^2*(a*d-b
*c)/b^4/(b*x+a)^6
```

**maxima [B]** time = 1.52, size = 193, normalized size = 2.10

$$-\frac{56\,b^3d^3x^3+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3+28\left(5\,b^3cd^2+ab^2d^3\right)x^2+8\left(15\,b^3c^2d+5\,ab^2cd^2+a^2bd^3\right)x}{280\left(b^{12}x^8+8\,ab^{11}x^7+28\,a^2b^{10}x^6+56\,a^3b^9x^5+70\,a^4b^8x^4+56\,a^5b^7x^3+28\,a^6b^6x^2+8\,a^7b^5x+a^8b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

**mupad [B]** time = 0.23, size = 187, normalized size = 2.03

$$-\frac{\frac{a^3\,d^3+5\,a^2\,b\,c\,d^2+15\,a\,b^2\,c^2\,d+35\,b^3\,c^3}{280\,b^4}+\frac{d^3\,x^3}{5\,b}+\frac{d\,x\,\left(a^2\,d^2+5\,a\,b\,c\,d+15\,b^2\,c^2\right)}{35\,b^3}+\frac{d^2\,x^2\,\left(a\,d+5\,b\,c\right)}{10\,b^2}}{a^8+8\,a^7\,b\,x+28\,a^6\,b^2\,x^2+56\,a^5\,b^3\,x^3+70\,a^4\,b^4\,x^4+56\,a^3\,b^5\,x^5+28\,a^2\,b^6\,x^6+8\,a\,b^7\,x^7+b^8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^9,x)
```

```
[Out] -((a^3*d^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2)/(280*b^4) + (d^3*
x^3)/(5*b) + (d*x*(a^2*d^2 + 15*b^2*c^2 + 5*a*b*c*d))/(35*b^3) + (d^2*x^2*(
a*d + 5*b*c))/(10*b^2))/(a^8 + b^8*x^8 + 8*a*b^7*x^7 + 28*a^6*b^2*x^2 + 56*
a^5*b^3*x^3 + 70*a^4*b^4*x^4 + 56*a^3*b^5*x^5 + 28*a^2*b^6*x^6 + 8*a^7*b*x)
```

**sympy [B]** time = 3.95, size = 207, normalized size = 2.25

$$\frac{-a^3d^3-5a^2bcd^2-15ab^2c^2d-35b^3c^3-56b^3d^3x^3+x^2\left(-28ab^2d^3-140b^3cd^2\right)+x\left(-8a^2bd^3-40ab^2cd^2-120b^3c^2d\right)}{280a^8b^4+2240a^7b^5x+7840a^6b^6x^2+15680a^5b^7x^3+19600a^4b^8x^4+15680a^3b^9x^5+7840a^2b^{10}x^6+2240ab^{11}x^7+280b^{12}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**9,x)
```

```
[Out] (-a**3*d**3 - 5*a**2*b*c*d**2 - 15*a*b**2*c**2*d - 35*b**3*c**3 - 56*b**3*d
**3*x**3 + x**2*(-28*a*b**2*d**3 - 140*b**3*c*d**2) + x*(-8*a**2*b*d**3 - 4
0*a*b**2*c*d**2 - 120*b**3*c**2*d))/(280*a**8*b**4 + 2240*a**7*b**5*x + 784
0*a**6*b**6*x**2 + 15680*a**5*b**7*x**3 + 19600*a**4*b**8*x**4 + 15680*a**3
*b**9*x**5 + 7840*a**2*b**10*x**6 + 2240*a*b**11*x**7 + 280*b**12*x**8)
```

## 3.1167 $\int (a+bx)^9(c+dx)^7\,dx$

**Optimal**. Leaf size=200

$$\frac{7d^6(a+bx)^{16}(bc-ad)}{16b^8}+\frac{7d^5(a+bx)^{15}(bc-ad)^2}{5b^8}+\frac{5d^4(a+bx)^{14}(bc-ad)^3}{2b^8}+\frac{35d^3(a+bx)^{13}(bc-ad)^4}{13b^8}+\frac{7d^2(a+bx}{4}$$

**Rubi [A]** time = 0.68, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^{16}(bc-ad)}{16b^8}+\frac{7d^5(a+bx)^{15}(bc-ad)^2}{5b^8}+\frac{5d^4(a+bx)^{14}(bc-ad)^3}{2b^8}+\frac{35d^3(a+bx)^{13}(bc-ad)^4}{13b^8}+\frac{7d^2(a+bx)^{12}(bc-ad)^5}{4b^8}+\frac{7d(a+bx)^{11}(bc-ad)^6}{11b^8}+\frac{(a+bx)^{10}(bc-ad)^7}{10b^8}+\frac{d^7(a+bx)^{17}}{17b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^10)/(10*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^11)/(1
1*b^8) + (7*d^2*(b*c - a*d)^5*(a + b*x)^12)/(4*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^13)/(13*b^8) + (5*d^4*(b*c - a*d)^3*(a + b*x)^14)/(2*b^8) + (7*d
^5*(b*c - a*d)^2*(a + b*x)^15)/(5*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^16)/(
16*b^8) + (d^7*(a + b*x)^17)/(17*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^9(c+dx)^7\,dx &= \int \left(\frac{(bc-ad)^7(a+bx)^9}{b^7}+\frac{7d(bc-ad)^6(a+bx)^{10}}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{b^7}+\frac{35d^3(b}{}\right.\\ &= \frac{(bc-ad)^7(a+bx)^{10}}{10b^8}+\frac{7d(bc-ad)^6(a+bx)^{11}}{11b^8}+\frac{7d^2(bc-ad)^5(a+bx)^{12}}{4b^8}+\frac{35d^3(bc-a}{1}\end{aligned}$$

**Mathematica [B]** time = 0.15, size = 993, normalized size = 4.96

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9*(c + d*x)^7,x]
```

```
[Out] a^9*c^7*x + (a^8*c^6*(9*b*c + 7*a*d)*x^2)/2 + a^7*c^5*(12*b^2*c^2 + 21*a*b*
c*d + 7*a^2*d^2)*x^3 + (7*a^6*c^4*(12*b^3*c^3 + 36*a*b^2*c^2*d + 27*a^2*b*c
*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^5*c^3*(18*b^4*c^4 + 84*a*b^3*c^3*d + 108*a^
2*b^2*c^2*d^2 + 45*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^4*c^2*(6*b^5*c^5
+ 42*a*b^4*c^4*d + 84*a^2*b^3*c^3*d^2 + 60*a^3*b^2*c^2*d^3 + 15*a^4*b*c*d^4
 + a^5*d^5)*x^6)/2 + a^3*c*(12*b^6*c^6 + 126*a*b^5*c^5*d + 378*a^2*b^4*c^4*
d^2 + 420*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^4 + 27*a^5*b*c*d^5 + a^6*d^6)
*x^7 + (a^2*(36*b^7*c^7 + 588*a*b^6*c^6*d + 2646*a^2*b^5*c^5*d^2 + 4410*a^3
*b^4*c^4*d^3 + 2940*a^4*b^3*c^3*d^4 + 756*a^5*b^2*c^2*d^5 + 63*a^6*b*c*d^6
+ a^7*d^7)*x^8)/8 + a*b*(b^7*c^7 + 28*a*b^6*c^6*d + 196*a^2*b^5*c^5*d^2 + 4
90*a^3*b^4*c^4*d^3 + 490*a^4*b^3*c^3*d^4 + 196*a^5*b^2*c^2*d^5 + 28*a^6*b*c
*d^6 + a^7*d^7)*x^9 + (b^2*(b^7*c^7 + 63*a*b^6*c^6*d + 756*a^2*b^5*c^5*d^2
+ 2940*a^3*b^4*c^4*d^3 + 4410*a^4*b^3*c^3*d^4 + 2646*a^5*b^2*c^2*d^5 + 588*
a^6*b*c*d^6 + 36*a^7*d^7)*x^10)/10 + (7*b^3*d*(b^6*c^6 + 27*a*b^5*c^5*d + 1
80*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d^3 + 378*a^4*b^2*c^2*d^4 + 126*a^5*b*
c*d^5 + 12*a^6*d^6)*x^11)/11 + (7*b^4*d^2*(b^5*c^5 + 15*a*b^4*c^4*d + 60*a^
2*b^3*c^3*d^2 + 84*a^3*b^2*c^2*d^3 + 42*a^4*b*c*d^4 + 6*a^5*d^5)*x^12)/4 +
(7*b^5*d^3*(5*b^4*c^4 + 45*a*b^3*c^3*d + 108*a^2*b^2*c^2*d^2 + 84*a^3*b*c*d
^3 + 18*a^4*d^4)*x^13)/13 + (b^6*d^4*(5*b^3*c^3 + 27*a*b^2*c^2*d + 36*a^2*b
*c*d^2 + 12*a^3*d^3)*x^14)/2 + (b^7*d^5*(7*b^2*c^2 + 21*a*b*c*d + 12*a^2*d^
2)*x^15)/5 + (b^8*d^6*(7*b*c + 9*a*d)*x^16)/16 + (b^9*d^7*x^17)/17
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^9(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^9*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^9*(c + d*x)^7, x]
```

**fricas [B]** time = 0.86, size = 1175, normalized size = 5.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^7*b^9 + 7/16*x^16*d^6*c*b^9 + 9/16*x^16*d^7*b^8*a + 7/5*x^15*d^
5*c^2*b^9 + 21/5*x^15*d^6*c*b^8*a + 12/5*x^15*d^7*b^7*a^2 + 5/2*x^14*d^4*c^
3*b^9 + 27/2*x^14*d^5*c^2*b^8*a + 18*x^14*d^6*c*b^7*a^2 + 6*x^14*d^7*b^6*a^
3 + 35/13*x^13*d^3*c^4*b^9 + 315/13*x^13*d^4*c^3*b^8*a + 756/13*x^13*d^5*c^
```

```
2*b^7*a^2 + 588/13*x^13*d^6*c*b^6*a^3 + 126/13*x^13*d^7*b^5*a^4 + 7/4*x^12*
d^2*c^5*b^9 + 105/4*x^12*d^3*c^4*b^8*a + 105*x^12*d^4*c^3*b^7*a^2 + 147*x^1
2*d^5*c^2*b^6*a^3 + 147/2*x^12*d^6*c*b^5*a^4 + 21/2*x^12*d^7*b^4*a^5 + 7/11
*x^11*d*c^6*b^9 + 189/11*x^11*d^2*c^5*b^8*a + 1260/11*x^11*d^3*c^4*b^7*a^2
+ 2940/11*x^11*d^4*c^3*b^6*a^3 + 2646/11*x^11*d^5*c^2*b^5*a^4 + 882/11*x^11
*d^6*c*b^4*a^5 + 84/11*x^11*d^7*b^3*a^6 + 1/10*x^10*c^7*b^9 + 63/10*x^10*d*
c^6*b^8*a + 378/5*x^10*d^2*c^5*b^7*a^2 + 294*x^10*d^3*c^4*b^6*a^3 + 441*x^1
0*d^4*c^3*b^5*a^4 + 1323/5*x^10*d^5*c^2*b^4*a^5 + 294/5*x^10*d^6*c*b^3*a^6
+ 18/5*x^10*d^7*b^2*a^7 + x^9*c^7*b^8*a + 28*x^9*d*c^6*b^7*a^2 + 196*x^9*d^
2*c^5*b^6*a^3 + 490*x^9*d^3*c^4*b^5*a^4 + 490*x^9*d^4*c^3*b^4*a^5 + 196*x^9
*d^5*c^2*b^3*a^6 + 28*x^9*d^6*c*b^2*a^7 + x^9*d^7*b*a^8 + 9/2*x^8*c^7*b^7*a
^2 + 147/2*x^8*d*c^6*b^6*a^3 + 1323/4*x^8*d^2*c^5*b^5*a^4 + 2205/4*x^8*d^3*
c^4*b^4*a^5 + 735/2*x^8*d^4*c^3*b^3*a^6 + 189/2*x^8*d^5*c^2*b^2*a^7 + 63/8*
x^8*d^6*c*b*a^8 + 1/8*x^8*d^7*a^9 + 12*x^7*c^7*b^6*a^3 + 126*x^7*d*c^6*b^5*
a^4 + 378*x^7*d^2*c^5*b^4*a^5 + 420*x^7*d^3*c^4*b^3*a^6 + 180*x^7*d^4*c^3*b
^2*a^7 + 27*x^7*d^5*c^2*b*a^8 + x^7*d^6*c*a^9 + 21*x^6*c^7*b^5*a^4 + 147*x^
6*d*c^6*b^4*a^5 + 294*x^6*d^2*c^5*b^3*a^6 + 210*x^6*d^3*c^4*b^2*a^7 + 105/2
*x^6*d^4*c^3*b*a^8 + 7/2*x^6*d^5*c^2*a^9 + 126/5*x^5*c^7*b^4*a^5 + 588/5*x^
5*d*c^6*b^3*a^6 + 756/5*x^5*d^2*c^5*b^2*a^7 + 63*x^5*d^3*c^4*b*a^8 + 7*x^5*
d^4*c^3*a^9 + 21*x^4*c^7*b^3*a^6 + 63*x^4*d*c^6*b^2*a^7 + 189/4*x^4*d^2*c^5
*b*a^8 + 35/4*x^4*d^3*c^4*a^9 + 12*x^3*c^7*b^2*a^7 + 21*x^3*d*c^6*b*a^8 + 7
*x^3*d^2*c^5*a^9 + 9/2*x^2*c^7*b*a^8 + 7/2*x^2*d*c^6*a^9 + x*c^7*a^9
```

**giac [B]** time = 1.05, size = 1175, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/17*b^9*d^7*x^17 + 7/16*b^9*c*d^6*x^16 + 9/16*a*b^8*d^7*x^16 + 7/5*b^9*c^2
*d^5*x^15 + 21/5*a*b^8*c*d^6*x^15 + 12/5*a^2*b^7*d^7*x^15 + 5/2*b^9*c^3*d^4
*x^14 + 27/2*a*b^8*c^2*d^5*x^14 + 18*a^2*b^7*c*d^6*x^14 + 6*a^3*b^6*d^7*x^1
4 + 35/13*b^9*c^4*d^3*x^13 + 315/13*a*b^8*c^3*d^4*x^13 + 756/13*a^2*b^7*c^2
*d^5*x^13 + 588/13*a^3*b^6*c*d^6*x^13 + 126/13*a^4*b^5*d^7*x^13 + 7/4*b^9*c
^5*d^2*x^12 + 105/4*a*b^8*c^4*d^3*x^12 + 105*a^2*b^7*c^3*d^4*x^12 + 147*a^3
*b^6*c^2*d^5*x^12 + 147/2*a^4*b^5*c*d^6*x^12 + 21/2*a^5*b^4*d^7*x^12 + 7/11
*b^9*c^6*d*x^11 + 189/11*a*b^8*c^5*d^2*x^11 + 1260/11*a^2*b^7*c^4*d^3*x^11
+ 2940/11*a^3*b^6*c^3*d^4*x^11 + 2646/11*a^4*b^5*c^2*d^5*x^11 + 882/11*a^5*
b^4*c*d^6*x^11 + 84/11*a^6*b^3*d^7*x^11 + 1/10*b^9*c^7*x^10 + 63/10*a*b^8*c
^6*d*x^10 + 378/5*a^2*b^7*c^5*d^2*x^10 + 294*a^3*b^6*c^4*d^3*x^10 + 441*a^4
*b^5*c^3*d^4*x^10 + 1323/5*a^5*b^4*c^2*d^5*x^10 + 294/5*a^6*b^3*c*d^6*x^10
+ 18/5*a^7*b^2*d^7*x^10 + a*b^8*c^7*x^9 + 28*a^2*b^7*c^6*d*x^9 + 196*a^3*b^
6*c^5*d^2*x^9 + 490*a^4*b^5*c^4*d^3*x^9 + 490*a^5*b^4*c^3*d^4*x^9 + 196*a^6
*b^3*c^2*d^5*x^9 + 28*a^7*b^2*c*d^6*x^9 + a^8*b*d^7*x^9 + 9/2*a^2*b^7*c^7*x
```

```
^8 + 147/2*a^3*b^6*c^6*d*x^8 + 1323/4*a^4*b^5*c^5*d^2*x^8 + 2205/4*a^5*b^4*
c^4*d^3*x^8 + 735/2*a^6*b^3*c^3*d^4*x^8 + 189/2*a^7*b^2*c^2*d^5*x^8 + 63/8*
a^8*b*c*d^6*x^8 + 1/8*a^9*d^7*x^8 + 12*a^3*b^6*c^7*x^7 + 126*a^4*b^5*c^6*d*
x^7 + 378*a^5*b^4*c^5*d^2*x^7 + 420*a^6*b^3*c^4*d^3*x^7 + 180*a^7*b^2*c^3*d
^4*x^7 + 27*a^8*b*c^2*d^5*x^7 + a^9*c*d^6*x^7 + 21*a^4*b^5*c^7*x^6 + 147*a^
5*b^4*c^6*d*x^6 + 294*a^6*b^3*c^5*d^2*x^6 + 210*a^7*b^2*c^4*d^3*x^6 + 105/2
*a^8*b*c^3*d^4*x^6 + 7/2*a^9*c^2*d^5*x^6 + 126/5*a^5*b^4*c^7*x^5 + 588/5*a^
6*b^3*c^6*d*x^5 + 756/5*a^7*b^2*c^5*d^2*x^5 + 63*a^8*b*c^4*d^3*x^5 + 7*a^9*
c^3*d^4*x^5 + 21*a^6*b^3*c^7*x^4 + 63*a^7*b^2*c^6*d*x^4 + 189/4*a^8*b*c^5*d
^2*x^4 + 35/4*a^9*c^4*d^3*x^4 + 12*a^7*b^2*c^7*x^3 + 21*a^8*b*c^6*d*x^3 + 7
*a^9*c^5*d^2*x^3 + 9/2*a^8*b*c^7*x^2 + 7/2*a^9*c^6*d*x^2 + a^9*c^7*x
```

**maple [B]** time = 0.00, size = 1033, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9*(d*x+c)^7,x)
```

```
[Out] 1/17*b^9*d^7*x^17+1/16*(9*a*b^8*d^7+7*b^9*c*d^6)*x^16+1/15*(36*a^2*b^7*d^7+
63*a*b^8*c*d^6+21*b^9*c^2*d^5)*x^15+1/14*(84*a^3*b^6*d^7+252*a^2*b^7*c*d^6+
189*a*b^8*c^2*d^5+35*b^9*c^3*d^4)*x^14+1/13*(126*a^4*b^5*d^7+588*a^3*b^6*c*
d^6+756*a^2*b^7*c^2*d^5+315*a*b^8*c^3*d^4+35*b^9*c^4*d^3)*x^13+1/12*(126*a^
5*b^4*d^7+882*a^4*b^5*c*d^6+1764*a^3*b^6*c^2*d^5+1260*a^2*b^7*c^3*d^4+315*a
*b^8*c^4*d^3+21*b^9*c^5*d^2)*x^12+1/11*(84*a^6*b^3*d^7+882*a^5*b^4*c*d^6+26
46*a^4*b^5*c^2*d^5+2940*a^3*b^6*c^3*d^4+1260*a^2*b^7*c^4*d^3+189*a*b^8*c^5*
d^2+7*b^9*c^6*d)*x^11+1/10*(36*a^7*b^2*d^7+588*a^6*b^3*c*d^6+2646*a^5*b^4*c
^2*d^5+4410*a^4*b^5*c^3*d^4+2940*a^3*b^6*c^4*d^3+756*a^2*b^7*c^5*d^2+63*a*b
^8*c^6*d+b^9*c^7)*x^10+1/9*(9*a^8*b*d^7+252*a^7*b^2*c*d^6+1764*a^6*b^3*c^2*
d^5+4410*a^5*b^4*c^3*d^4+4410*a^4*b^5*c^4*d^3+1764*a^3*b^6*c^5*d^2+252*a^2*
b^7*c^6*d+9*a*b^8*c^7)*x^9+1/8*(a^9*d^7+63*a^8*b*c*d^6+756*a^7*b^2*c^2*d^5+
2940*a^6*b^3*c^3*d^4+4410*a^5*b^4*c^4*d^3+2646*a^4*b^5*c^5*d^2+588*a^3*b^6*
c^6*d+36*a^2*b^7*c^7)*x^8+1/7*(7*a^9*c*d^6+189*a^8*b*c^2*d^5+1260*a^7*b^2*c
^3*d^4+2940*a^6*b^3*c^4*d^3+2646*a^5*b^4*c^5*d^2+882*a^4*b^5*c^6*d+84*a^3*b
^6*c^7)*x^7+1/6*(21*a^9*c^2*d^5+315*a^8*b*c^3*d^4+1260*a^7*b^2*c^4*d^3+1764
*a^6*b^3*c^5*d^2+882*a^5*b^4*c^6*d+126*a^4*b^5*c^7)*x^6+1/5*(35*a^9*c^3*d^4
+315*a^8*b*c^4*d^3+756*a^7*b^2*c^5*d^2+588*a^6*b^3*c^6*d+126*a^5*b^4*c^7)*x
^5+1/4*(35*a^9*c^4*d^3+189*a^8*b*c^5*d^2+252*a^7*b^2*c^6*d+84*a^6*b^3*c^7)*
x^4+1/3*(21*a^9*c^5*d^2+63*a^8*b*c^6*d+36*a^7*b^2*c^7)*x^3+1/2*(7*a^9*c^6*d
+9*a^8*b*c^7)*x^2+a^9*c^7*x
```

**maxima [B]** time = 1.43, size = 1023, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/17*b^9*d^7*x^17 + a^9*c^7*x + 1/16*(7*b^9*c*d^6 + 9*a*b^8*d^7)*x^16 + 1/5
*(7*b^9*c^2*d^5 + 21*a*b^8*c*d^6 + 12*a^2*b^7*d^7)*x^15 + 1/2*(5*b^9*c^3*d^
4 + 27*a*b^8*c^2*d^5 + 36*a^2*b^7*c*d^6 + 12*a^3*b^6*d^7)*x^14 + 7/13*(5*b^
9*c^4*d^3 + 45*a*b^8*c^3*d^4 + 108*a^2*b^7*c^2*d^5 + 84*a^3*b^6*c*d^6 + 18*
a^4*b^5*d^7)*x^13 + 7/4*(b^9*c^5*d^2 + 15*a*b^8*c^4*d^3 + 60*a^2*b^7*c^3*d^
4 + 84*a^3*b^6*c^2*d^5 + 42*a^4*b^5*c*d^6 + 6*a^5*b^4*d^7)*x^12 + 7/11*(b^9
*c^6*d + 27*a*b^8*c^5*d^2 + 180*a^2*b^7*c^4*d^3 + 420*a^3*b^6*c^3*d^4 + 378
*a^4*b^5*c^2*d^5 + 126*a^5*b^4*c*d^6 + 12*a^6*b^3*d^7)*x^11 + 1/10*(b^9*c^7
 + 63*a*b^8*c^6*d + 756*a^2*b^7*c^5*d^2 + 2940*a^3*b^6*c^4*d^3 + 4410*a^4*b
^5*c^3*d^4 + 2646*a^5*b^4*c^2*d^5 + 588*a^6*b^3*c*d^6 + 36*a^7*b^2*d^7)*x^1
0 + (a*b^8*c^7 + 28*a^2*b^7*c^6*d + 196*a^3*b^6*c^5*d^2 + 490*a^4*b^5*c^4*d
^3 + 490*a^5*b^4*c^3*d^4 + 196*a^6*b^3*c^2*d^5 + 28*a^7*b^2*c*d^6 + a^8*b*d
^7)*x^9 + 1/8*(36*a^2*b^7*c^7 + 588*a^3*b^6*c^6*d + 2646*a^4*b^5*c^5*d^2 +
4410*a^5*b^4*c^4*d^3 + 2940*a^6*b^3*c^3*d^4 + 756*a^7*b^2*c^2*d^5 + 63*a^8*
b*c*d^6 + a^9*d^7)*x^8 + (12*a^3*b^6*c^7 + 126*a^4*b^5*c^6*d + 378*a^5*b^4*
c^5*d^2 + 420*a^6*b^3*c^4*d^3 + 180*a^7*b^2*c^3*d^4 + 27*a^8*b*c^2*d^5 + a^
9*c*d^6)*x^7 + 7/2*(6*a^4*b^5*c^7 + 42*a^5*b^4*c^6*d + 84*a^6*b^3*c^5*d^2 +
 60*a^7*b^2*c^4*d^3 + 15*a^8*b*c^3*d^4 + a^9*c^2*d^5)*x^6 + 7/5*(18*a^5*b^4
*c^7 + 84*a^6*b^3*c^6*d + 108*a^7*b^2*c^5*d^2 + 45*a^8*b*c^4*d^3 + 5*a^9*c^
3*d^4)*x^5 + 7/4*(12*a^6*b^3*c^7 + 36*a^7*b^2*c^6*d + 27*a^8*b*c^5*d^2 + 5*
a^9*c^4*d^3)*x^4 + (12*a^7*b^2*c^7 + 21*a^8*b*c^6*d + 7*a^9*c^5*d^2)*x^3 +
1/2*(9*a^8*b*c^7 + 7*a^9*c^6*d)*x^2
```

**mupad [B]** time = 0.55, size = 997, normalized size = 4.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9*(c + d*x)^7,x)
```

```
[Out] x^5*((126*a^5*b^4*c^7)/5 + 7*a^9*c^3*d^4 + (588*a^6*b^3*c^6*d)/5 + 63*a^8*b
*c^4*d^3 + (756*a^7*b^2*c^5*d^2)/5) + x^13*((126*a^4*b^5*d^7)/13 + (35*b^9*
c^4*d^3)/13 + (315*a*b^8*c^3*d^4)/13 + (588*a^3*b^6*c*d^6)/13 + (756*a^2*b^
7*c^2*d^5)/13) + x^8*((a^9*d^7)/8 + (9*a^2*b^7*c^7)/2 + (147*a^3*b^6*c^6*d)
/2 + (1323*a^4*b^5*c^5*d^2)/4 + (2205*a^5*b^4*c^4*d^3)/4 + (735*a^6*b^3*c^3
*d^4)/2 + (189*a^7*b^2*c^2*d^5)/2 + (63*a^8*b*c*d^6)/8) + x^10*((b^9*c^7)/1
0 + (18*a^7*b^2*d^7)/5 + (294*a^6*b^3*c*d^6)/5 + (378*a^2*b^7*c^5*d^2)/5 +
294*a^3*b^6*c^4*d^3 + 441*a^4*b^5*c^3*d^4 + (1323*a^5*b^4*c^2*d^5)/5 + (63*
a*b^8*c^6*d)/10) + x^6*(21*a^4*b^5*c^7 + (7*a^9*c^2*d^5)/2 + 147*a^5*b^4*c^
6*d + (105*a^8*b*c^3*d^4)/2 + 294*a^6*b^3*c^5*d^2 + 210*a^7*b^2*c^4*d^3) +
x^12*((21*a^5*b^4*d^7)/2 + (7*b^9*c^5*d^2)/4 + (105*a*b^8*c^4*d^3)/4 + (147
*a^4*b^5*c*d^6)/2 + 105*a^2*b^7*c^3*d^4 + 147*a^3*b^6*c^2*d^5) + x^7*(a^9*c
*d^6 + 12*a^3*b^6*c^7 + 126*a^4*b^5*c^6*d + 27*a^8*b*c^2*d^5 + 378*a^5*b^4*
```

```
c^5*d^2 + 420*a^6*b^3*c^4*d^3 + 180*a^7*b^2*c^3*d^4) + x^11*((7*b^9*c^6*d)/
11 + (84*a^6*b^3*d^7)/11 + (189*a*b^8*c^5*d^2)/11 + (882*a^5*b^4*c*d^6)/11
+ (1260*a^2*b^7*c^4*d^3)/11 + (2940*a^3*b^6*c^3*d^4)/11 + (2646*a^4*b^5*c^2
*d^5)/11) + x^9*(a*b^8*c^7 + a^8*b*d^7 + 28*a^2*b^7*c^6*d + 28*a^7*b^2*c*d^
6 + 196*a^3*b^6*c^5*d^2 + 490*a^4*b^5*c^4*d^3 + 490*a^5*b^4*c^3*d^4 + 196*a
^6*b^3*c^2*d^5) + a^9*c^7*x + (b^9*d^7*x^17)/17 + (7*a^6*c^4*x^4*(5*a^3*d^3
 + 12*b^3*c^3 + 36*a*b^2*c^2*d + 27*a^2*b*c*d^2))/4 + (b^6*d^4*x^14*(12*a^3
*d^3 + 5*b^3*c^3 + 27*a*b^2*c^2*d + 36*a^2*b*c*d^2))/2 + (a^8*c^6*x^2*(7*a*
d + 9*b*c))/2 + (b^8*d^6*x^16*(9*a*d + 7*b*c))/16 + a^7*c^5*x^3*(7*a^2*d^2
+ 12*b^2*c^2 + 21*a*b*c*d) + (b^7*d^5*x^15*(12*a^2*d^2 + 7*b^2*c^2 + 21*a*b
*c*d))/5
```

**sympy [B]** time = 0.23, size = 1163, normalized size = 5.82

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9*(d*x+c)**7,x)
```

```
[Out] a**9*c**7*x + b**9*d**7*x**17/17 + x**16*(9*a*b**8*d**7/16 + 7*b**9*c*d**6/
16) + x**15*(12*a**2*b**7*d**7/5 + 21*a*b**8*c*d**6/5 + 7*b**9*c**2*d**5/5)
 + x**14*(6*a**3*b**6*d**7 + 18*a**2*b**7*c*d**6 + 27*a*b**8*c**2*d**5/2 +
5*b**9*c**3*d**4/2) + x**13*(126*a**4*b**5*d**7/13 + 588*a**3*b**6*c*d**6/1
3 + 756*a**2*b**7*c**2*d**5/13 + 315*a*b**8*c**3*d**4/13 + 35*b**9*c**4*d**
3/13) + x**12*(21*a**5*b**4*d**7/2 + 147*a**4*b**5*c*d**6/2 + 147*a**3*b**6
*c**2*d**5 + 105*a**2*b**7*c**3*d**4 + 105*a*b**8*c**4*d**3/4 + 7*b**9*c**5
*d**2/4) + x**11*(84*a**6*b**3*d**7/11 + 882*a**5*b**4*c*d**6/11 + 2646*a**
4*b**5*c**2*d**5/11 + 2940*a**3*b**6*c**3*d**4/11 + 1260*a**2*b**7*c**4*d**
3/11 + 189*a*b**8*c**5*d**2/11 + 7*b**9*c**6*d/11) + x**10*(18*a**7*b**2*d*
*7/5 + 294*a**6*b**3*c*d**6/5 + 1323*a**5*b**4*c**2*d**5/5 + 441*a**4*b**5*
c**3*d**4 + 294*a**3*b**6*c**4*d**3 + 378*a**2*b**7*c**5*d**2/5 + 63*a*b**8
*c**6*d/10 + b**9*c**7/10) + x**9*(a**8*b*d**7 + 28*a**7*b**2*c*d**6 + 196*
a**6*b**3*c**2*d**5 + 490*a**5*b**4*c**3*d**4 + 490*a**4*b**5*c**4*d**3 + 1
96*a**3*b**6*c**5*d**2 + 28*a**2*b**7*c**6*d + a*b**8*c**7) + x**8*(a**9*d*
*7/8 + 63*a**8*b*c*d**6/8 + 189*a**7*b**2*c**2*d**5/2 + 735*a**6*b**3*c**3*
d**4/2 + 2205*a**5*b**4*c**4*d**3/4 + 1323*a**4*b**5*c**5*d**2/4 + 147*a**3
*b**6*c**6*d/2 + 9*a**2*b**7*c**7/2) + x**7*(a**9*c*d**6 + 27*a**8*b*c**2*d
**5 + 180*a**7*b**2*c**3*d**4 + 420*a**6*b**3*c**4*d**3 + 378*a**5*b**4*c**
5*d**2 + 126*a**4*b**5*c**6*d + 12*a**3*b**6*c**7) + x**6*(7*a**9*c**2*d**5
/2 + 105*a**8*b*c**3*d**4/2 + 210*a**7*b**2*c**4*d**3 + 294*a**6*b**3*c**5*
d**2 + 147*a**5*b**4*c**6*d + 21*a**4*b**5*c**7) + x**5*(7*a**9*c**3*d**4 +
 63*a**8*b*c**4*d**3 + 756*a**7*b**2*c**5*d**2/5 + 588*a**6*b**3*c**6*d/5 +
 126*a**5*b**4*c**7/5) + x**4*(35*a**9*c**4*d**3/4 + 189*a**8*b*c**5*d**2/4
 + 63*a**7*b**2*c**6*d + 21*a**6*b**3*c**7) + x**3*(7*a**9*c**5*d**2 + 21*a
```

```
**8*b*c**6*d + 12*a**7*b**2*c**7) + x**2*(7*a**9*c**6*d/2 + 9*a**8*b*c**7/2
)
```

## 3.1168 $\int (a+bx)^8(c+dx)^7\,dx$

**Optimal**. Leaf size=200

$$\frac{7d^6(a+bx)^{15}(bc-ad)}{15b^8}+\frac{3d^5(a+bx)^{14}(bc-ad)^2}{2b^8}+\frac{35d^4(a+bx)^{13}(bc-ad)^3}{13b^8}+\frac{35d^3(a+bx)^{12}(bc-ad)^4}{12b^8}+\frac{21d^2(a+}{}$$

**Rubi [A]** time = 0.57, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^{15}(bc-ad)}{15b^8}+\frac{3d^5(a+bx)^{14}(bc-ad)^2}{2b^8}+\frac{35d^4(a+bx)^{13}(bc-ad)^3}{13b^8}+\frac{35d^3(a+bx)^{12}(bc-ad)^4}{12b^8}+\frac{21d^2(a+bx)^{11}(bc-ad)^5}{11b^8}+\frac{7d(a+bx)^{10}(bc-ad)^6}{10b^8}+\frac{(a+bx)^9(bc-ad)^7}{9b^8}+\frac{d^7(a+bx)^{16}}{16b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^9)/(9*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^10)/(10*
b^8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^11)/(11*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^12)/(12*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^13)/(13*b^8) + (3
*d^5*(b*c - a*d)^2*(a + b*x)^14)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^15)
/(15*b^8) + (d^7*(a + b*x)^16)/(16*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^8(c+dx)^7\,dx=\int\left(\frac{(bc-ad)^7(a+bx)^8}{b^7}+\frac{7d(bc-ad)^6(a+bx)^9}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{b^7}+\frac{35d^3(}{}\right.$$

$$=\frac{(bc-ad)^7(a+bx)^9}{9b^8}+\frac{7d(bc-ad)^6(a+bx)^{10}}{10b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{11b^8}+\frac{35d^3(bc-}{}$$

**Mathematica [B]** time = 0.11, size = 897, normalized size = 4.48

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8*(c + d*x)^7,x]
```

```
[Out] a^8*c^7*x + (a^7*c^6*(8*b*c + 7*a*d)*x^2)/2 + (7*a^6*c^5*(4*b^2*c^2 + 8*a*b
*c*d + 3*a^2*d^2)*x^3)/3 + (7*a^5*c^4*(8*b^3*c^3 + 28*a*b^2*c^2*d + 24*a^2*
b*c*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^4*c^3*(10*b^4*c^4 + 56*a*b^3*c^3*d + 84*
a^2*b^2*c^2*d^2 + 40*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^3*c^2*(8*b^5*c^
5 + 70*a*b^4*c^4*d + 168*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 40*a^4*b*c
*d^4 + 3*a^5*d^5)*x^6)/6 + a^2*c*(4*b^6*c^6 + 56*a*b^5*c^5*d + 210*a^2*b^4*
c^4*d^2 + 280*a^3*b^3*c^3*d^3 + 140*a^4*b^2*c^2*d^4 + 24*a^5*b*c*d^5 + a^6*
d^6)*x^7 + (a*(8*b^7*c^7 + 196*a*b^6*c^6*d + 1176*a^2*b^5*c^5*d^2 + 2450*a^
3*b^4*c^4*d^3 + 1960*a^4*b^3*c^3*d^4 + 588*a^5*b^2*c^2*d^5 + 56*a^6*b*c*d^6
 + a^7*d^7)*x^8)/8 + (b*(b^7*c^7 + 56*a*b^6*c^6*d + 588*a^2*b^5*c^5*d^2 + 1
960*a^3*b^4*c^4*d^3 + 2450*a^4*b^3*c^3*d^4 + 1176*a^5*b^2*c^2*d^5 + 196*a^6
*b*c*d^6 + 8*a^7*d^7)*x^9)/9 + (7*b^2*d*(b^6*c^6 + 24*a*b^5*c^5*d + 140*a^2
*b^4*c^4*d^2 + 280*a^3*b^3*c^3*d^3 + 210*a^4*b^2*c^2*d^4 + 56*a^5*b*c*d^5 +
 4*a^6*d^6)*x^10)/10 + (7*b^3*d^2*(3*b^5*c^5 + 40*a*b^4*c^4*d + 140*a^2*b^3
*c^3*d^2 + 168*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 8*a^5*d^5)*x^11)/11 + (7*
b^4*d^3*(5*b^4*c^4 + 40*a*b^3*c^3*d + 84*a^2*b^2*c^2*d^2 + 56*a^3*b*c*d^3 +
 10*a^4*d^4)*x^12)/12 + (7*b^5*d^4*(5*b^3*c^3 + 24*a*b^2*c^2*d + 28*a^2*b*c
*d^2 + 8*a^3*d^3)*x^13)/13 + (b^6*d^5*(3*b^2*c^2 + 8*a*b*c*d + 4*a^2*d^2)*x
^14)/2 + (b^7*d^6*(7*b*c + 8*a*d)*x^15)/15 + (b^8*d^7*x^16)/16
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^8(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^8*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^8*(c + d*x)^7, x]
```

**fricas [B]** time = 1.28, size = 1050, normalized size = 5.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^7*b^8 + 7/15*x^15*d^6*c*b^8 + 8/15*x^15*d^7*b^7*a + 3/2*x^14*d^
5*c^2*b^8 + 4*x^14*d^6*c*b^7*a + 2*x^14*d^7*b^6*a^2 + 35/13*x^13*d^4*c^3*b^
8 + 168/13*x^13*d^5*c^2*b^7*a + 196/13*x^13*d^6*c*b^6*a^2 + 56/13*x^13*d^7*
b^5*a^3 + 35/12*x^12*d^3*c^4*b^8 + 70/3*x^12*d^4*c^3*b^7*a + 49*x^12*d^5*c^
2*b^6*a^2 + 98/3*x^12*d^6*c*b^5*a^3 + 35/6*x^12*d^7*b^4*a^4 + 21/11*x^11*d^
2*c^5*b^8 + 280/11*x^11*d^3*c^4*b^7*a + 980/11*x^11*d^4*c^3*b^6*a^2 + 1176/
```

```
11*x^11*d^5*c^2*b^5*a^3 + 490/11*x^11*d^6*c*b^4*a^4 + 56/11*x^11*d^7*b^3*a^
5 + 7/10*x^10*d*c^6*b^8 + 84/5*x^10*d^2*c^5*b^7*a + 98*x^10*d^3*c^4*b^6*a^2
 + 196*x^10*d^4*c^3*b^5*a^3 + 147*x^10*d^5*c^2*b^4*a^4 + 196/5*x^10*d^6*c*b
^3*a^5 + 14/5*x^10*d^7*b^2*a^6 + 1/9*x^9*c^7*b^8 + 56/9*x^9*d*c^6*b^7*a + 1
96/3*x^9*d^2*c^5*b^6*a^2 + 1960/9*x^9*d^3*c^4*b^5*a^3 + 2450/9*x^9*d^4*c^3*
b^4*a^4 + 392/3*x^9*d^5*c^2*b^3*a^5 + 196/9*x^9*d^6*c*b^2*a^6 + 8/9*x^9*d^7
*b*a^7 + x^8*c^7*b^7*a + 49/2*x^8*d*c^6*b^6*a^2 + 147*x^8*d^2*c^5*b^5*a^3 +
 1225/4*x^8*d^3*c^4*b^4*a^4 + 245*x^8*d^4*c^3*b^3*a^5 + 147/2*x^8*d^5*c^2*b
^2*a^6 + 7*x^8*d^6*c*b*a^7 + 1/8*x^8*d^7*a^8 + 4*x^7*c^7*b^6*a^2 + 56*x^7*d
*c^6*b^5*a^3 + 210*x^7*d^2*c^5*b^4*a^4 + 280*x^7*d^3*c^4*b^3*a^5 + 140*x^7*
d^4*c^3*b^2*a^6 + 24*x^7*d^5*c^2*b*a^7 + x^7*d^6*c*a^8 + 28/3*x^6*c^7*b^5*a
^3 + 245/3*x^6*d*c^6*b^4*a^4 + 196*x^6*d^2*c^5*b^3*a^5 + 490/3*x^6*d^3*c^4*
b^2*a^6 + 140/3*x^6*d^4*c^3*b*a^7 + 7/2*x^6*d^5*c^2*a^8 + 14*x^5*c^7*b^4*a^
4 + 392/5*x^5*d*c^6*b^3*a^5 + 588/5*x^5*d^2*c^5*b^2*a^6 + 56*x^5*d^3*c^4*b*
a^7 + 7*x^5*d^4*c^3*a^8 + 14*x^4*c^7*b^3*a^5 + 49*x^4*d*c^6*b^2*a^6 + 42*x^
4*d^2*c^5*b*a^7 + 35/4*x^4*d^3*c^4*a^8 + 28/3*x^3*c^7*b^2*a^6 + 56/3*x^3*d*
c^6*b*a^7 + 7*x^3*d^2*c^5*a^8 + 4*x^2*c^7*b*a^7 + 7/2*x^2*d*c^6*a^8 + x*c^7
*a^8
```

**giac [B]** time = 1.01, size = 1050, normalized size = 5.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/16*b^8*d^7*x^16 + 7/15*b^8*c*d^6*x^15 + 8/15*a*b^7*d^7*x^15 + 3/2*b^8*c^2
*d^5*x^14 + 4*a*b^7*c*d^6*x^14 + 2*a^2*b^6*d^7*x^14 + 35/13*b^8*c^3*d^4*x^1
3 + 168/13*a*b^7*c^2*d^5*x^13 + 196/13*a^2*b^6*c*d^6*x^13 + 56/13*a^3*b^5*d
^7*x^13 + 35/12*b^8*c^4*d^3*x^12 + 70/3*a*b^7*c^3*d^4*x^12 + 49*a^2*b^6*c^2
*d^5*x^12 + 98/3*a^3*b^5*c*d^6*x^12 + 35/6*a^4*b^4*d^7*x^12 + 21/11*b^8*c^5
*d^2*x^11 + 280/11*a*b^7*c^4*d^3*x^11 + 980/11*a^2*b^6*c^3*d^4*x^11 + 1176/
11*a^3*b^5*c^2*d^5*x^11 + 490/11*a^4*b^4*c*d^6*x^11 + 56/11*a^5*b^3*d^7*x^1
1 + 7/10*b^8*c^6*d*x^10 + 84/5*a*b^7*c^5*d^2*x^10 + 98*a^2*b^6*c^4*d^3*x^10
 + 196*a^3*b^5*c^3*d^4*x^10 + 147*a^4*b^4*c^2*d^5*x^10 + 196/5*a^5*b^3*c*d^
6*x^10 + 14/5*a^6*b^2*d^7*x^10 + 1/9*b^8*c^7*x^9 + 56/9*a*b^7*c^6*d*x^9 + 1
96/3*a^2*b^6*c^5*d^2*x^9 + 1960/9*a^3*b^5*c^4*d^3*x^9 + 2450/9*a^4*b^4*c^3*
d^4*x^9 + 392/3*a^5*b^3*c^2*d^5*x^9 + 196/9*a^6*b^2*c*d^6*x^9 + 8/9*a^7*b*d
^7*x^9 + a*b^7*c^7*x^8 + 49/2*a^2*b^6*c^6*d*x^8 + 147*a^3*b^5*c^5*d^2*x^8 +
 1225/4*a^4*b^4*c^4*d^3*x^8 + 245*a^5*b^3*c^3*d^4*x^8 + 147/2*a^6*b^2*c^2*d
^5*x^8 + 7*a^7*b*c*d^6*x^8 + 1/8*a^8*d^7*x^8 + 4*a^2*b^6*c^7*x^7 + 56*a^3*b
^5*c^6*d*x^7 + 210*a^4*b^4*c^5*d^2*x^7 + 280*a^5*b^3*c^4*d^3*x^7 + 140*a^6*
b^2*c^3*d^4*x^7 + 24*a^7*b*c^2*d^5*x^7 + a^8*c*d^6*x^7 + 28/3*a^3*b^5*c^7*x
^6 + 245/3*a^4*b^4*c^6*d*x^6 + 196*a^5*b^3*c^5*d^2*x^6 + 490/3*a^6*b^2*c^4*
d^3*x^6 + 140/3*a^7*b*c^3*d^4*x^6 + 7/2*a^8*c^2*d^5*x^6 + 14*a^4*b^4*c^7*x^
```

```
5 + 392/5*a^5*b^3*c^6*d*x^5 + 588/5*a^6*b^2*c^5*d^2*x^5 + 56*a^7*b*c^4*d^3*
x^5 + 7*a^8*c^3*d^4*x^5 + 14*a^5*b^3*c^7*x^4 + 49*a^6*b^2*c^6*d*x^4 + 42*a^
7*b*c^5*d^2*x^4 + 35/4*a^8*c^4*d^3*x^4 + 28/3*a^6*b^2*c^7*x^3 + 56/3*a^7*b*
c^6*d*x^3 + 7*a^8*c^5*d^2*x^3 + 4*a^7*b*c^7*x^2 + 7/2*a^8*c^6*d*x^2 + a^8*c
^7*x
```

**maple [B]** time = 0.00, size = 925, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8*(d*x+c)^7,x)
```

```
[Out] 1/16*b^8*d^7*x^16+1/15*(8*a*b^7*d^7+7*b^8*c*d^6)*x^15+1/14*(28*a^2*b^6*d^7+
56*a*b^7*c*d^6+21*b^8*c^2*d^5)*x^14+1/13*(56*a^3*b^5*d^7+196*a^2*b^6*c*d^6+
168*a*b^7*c^2*d^5+35*b^8*c^3*d^4)*x^13+1/12*(70*a^4*b^4*d^7+392*a^3*b^5*c*d
^6+588*a^2*b^6*c^2*d^5+280*a*b^7*c^3*d^4+35*b^8*c^4*d^3)*x^12+1/11*(56*a^5*
b^3*d^7+490*a^4*b^4*c*d^6+1176*a^3*b^5*c^2*d^5+980*a^2*b^6*c^3*d^4+280*a*b^
7*c^4*d^3+21*b^8*c^5*d^2)*x^11+1/10*(28*a^6*b^2*d^7+392*a^5*b^3*c*d^6+1470*
a^4*b^4*c^2*d^5+1960*a^3*b^5*c^3*d^4+980*a^2*b^6*c^4*d^3+168*a*b^7*c^5*d^2+
7*b^8*c^6*d)*x^10+1/9*(8*a^7*b*d^7+196*a^6*b^2*c*d^6+1176*a^5*b^3*c^2*d^5+2
450*a^4*b^4*c^3*d^4+1960*a^3*b^5*c^4*d^3+588*a^2*b^6*c^5*d^2+56*a*b^7*c^6*d
+b^8*c^7)*x^9+1/8*(a^8*d^7+56*a^7*b*c*d^6+588*a^6*b^2*c^2*d^5+1960*a^5*b^3*
c^3*d^4+2450*a^4*b^4*c^4*d^3+1176*a^3*b^5*c^5*d^2+196*a^2*b^6*c^6*d+8*a*b^7
*c^7)*x^8+1/7*(7*a^8*c*d^6+168*a^7*b*c^2*d^5+980*a^6*b^2*c^3*d^4+1960*a^5*b
^3*c^4*d^3+1470*a^4*b^4*c^5*d^2+392*a^3*b^5*c^6*d+28*a^2*b^6*c^7)*x^7+1/6*(
21*a^8*c^2*d^5+280*a^7*b*c^3*d^4+980*a^6*b^2*c^4*d^3+1176*a^5*b^3*c^5*d^2+4
90*a^4*b^4*c^6*d+56*a^3*b^5*c^7)*x^6+1/5*(35*a^8*c^3*d^4+280*a^7*b*c^4*d^3+
588*a^6*b^2*c^5*d^2+392*a^5*b^3*c^6*d+70*a^4*b^4*c^7)*x^5+1/4*(35*a^8*c^4*d
^3+168*a^7*b*c^5*d^2+196*a^6*b^2*c^6*d+56*a^5*b^3*c^7)*x^4+1/3*(21*a^8*c^5*
d^2+56*a^7*b*c^6*d+28*a^6*b^2*c^7)*x^3+1/2*(7*a^8*c^6*d+8*a^7*b*c^7)*x^2+a^
8*c^7*x
```

**maxima [B]** time = 1.40, size = 921, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^8*d^7*x^16 + a^8*c^7*x + 1/15*(7*b^8*c*d^6 + 8*a*b^7*d^7)*x^15 + 1/2
*(3*b^8*c^2*d^5 + 8*a*b^7*c*d^6 + 4*a^2*b^6*d^7)*x^14 + 7/13*(5*b^8*c^3*d^4
 + 24*a*b^7*c^2*d^5 + 28*a^2*b^6*c*d^6 + 8*a^3*b^5*d^7)*x^13 + 7/12*(5*b^8*
c^4*d^3 + 40*a*b^7*c^3*d^4 + 84*a^2*b^6*c^2*d^5 + 56*a^3*b^5*c*d^6 + 10*a^4
*b^4*d^7)*x^12 + 7/11*(3*b^8*c^5*d^2 + 40*a*b^7*c^4*d^3 + 140*a^2*b^6*c^3*d
```

```
^4 + 168*a^3*b^5*c^2*d^5 + 70*a^4*b^4*c*d^6 + 8*a^5*b^3*d^7)*x^11 + 7/10*(b
^8*c^6*d + 24*a*b^7*c^5*d^2 + 140*a^2*b^6*c^4*d^3 + 280*a^3*b^5*c^3*d^4 + 2
10*a^4*b^4*c^2*d^5 + 56*a^5*b^3*c*d^6 + 4*a^6*b^2*d^7)*x^10 + 1/9*(b^8*c^7
+ 56*a*b^7*c^6*d + 588*a^2*b^6*c^5*d^2 + 1960*a^3*b^5*c^4*d^3 + 2450*a^4*b^
4*c^3*d^4 + 1176*a^5*b^3*c^2*d^5 + 196*a^6*b^2*c*d^6 + 8*a^7*b*d^7)*x^9 + 1
/8*(8*a*b^7*c^7 + 196*a^2*b^6*c^6*d + 1176*a^3*b^5*c^5*d^2 + 2450*a^4*b^4*c
^4*d^3 + 1960*a^5*b^3*c^3*d^4 + 588*a^6*b^2*c^2*d^5 + 56*a^7*b*c*d^6 + a^8*
d^7)*x^8 + (4*a^2*b^6*c^7 + 56*a^3*b^5*c^6*d + 210*a^4*b^4*c^5*d^2 + 280*a^
5*b^3*c^4*d^3 + 140*a^6*b^2*c^3*d^4 + 24*a^7*b*c^2*d^5 + a^8*c*d^6)*x^7 + 7
/6*(8*a^3*b^5*c^7 + 70*a^4*b^4*c^6*d + 168*a^5*b^3*c^5*d^2 + 140*a^6*b^2*c^
4*d^3 + 40*a^7*b*c^3*d^4 + 3*a^8*c^2*d^5)*x^6 + 7/5*(10*a^4*b^4*c^7 + 56*a^
5*b^3*c^6*d + 84*a^6*b^2*c^5*d^2 + 40*a^7*b*c^4*d^3 + 5*a^8*c^3*d^4)*x^5 +
7/4*(8*a^5*b^3*c^7 + 28*a^6*b^2*c^6*d + 24*a^7*b*c^5*d^2 + 5*a^8*c^4*d^3)*x
^4 + 7/3*(4*a^6*b^2*c^7 + 8*a^7*b*c^6*d + 3*a^8*c^5*d^2)*x^3 + 1/2*(8*a^7*b
*c^7 + 7*a^8*c^6*d)*x^2
```

**mupad [B]** time = 0.36, size = 892, normalized size = 4.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8*(c + d*x)^7,x)
```

```
[Out] x^8*((a^8*d^7)/8 + a*b^7*c^7 + (49*a^2*b^6*c^6*d)/2 + 147*a^3*b^5*c^5*d^2 +
 (1225*a^4*b^4*c^4*d^3)/4 + 245*a^5*b^3*c^3*d^4 + (147*a^6*b^2*c^2*d^5)/2 +
 7*a^7*b*c*d^6) + x^9*((b^8*c^7)/9 + (8*a^7*b*d^7)/9 + (196*a^6*b^2*c*d^6)/
9 + (196*a^2*b^6*c^5*d^2)/3 + (1960*a^3*b^5*c^4*d^3)/9 + (2450*a^4*b^4*c^3*
d^4)/9 + (392*a^5*b^3*c^2*d^5)/3 + (56*a*b^7*c^6*d)/9) + x^5*(14*a^4*b^4*c^
7 + 7*a^8*c^3*d^4 + (392*a^5*b^3*c^6*d)/5 + 56*a^7*b*c^4*d^3 + (588*a^6*b^2
*c^5*d^2)/5) + x^12*((35*a^4*b^4*d^7)/6 + (35*b^8*c^4*d^3)/12 + (70*a*b^7*c
^3*d^4)/3 + (98*a^3*b^5*c*d^6)/3 + 49*a^2*b^6*c^2*d^5) + x^6*((28*a^3*b^5*c
^7)/3 + (7*a^8*c^2*d^5)/2 + (245*a^4*b^4*c^6*d)/3 + (140*a^7*b*c^3*d^4)/3 +
 196*a^5*b^3*c^5*d^2 + (490*a^6*b^2*c^4*d^3)/3) + x^11*((56*a^5*b^3*d^7)/11
 + (21*b^8*c^5*d^2)/11 + (280*a*b^7*c^4*d^3)/11 + (490*a^4*b^4*c*d^6)/11 +
(980*a^2*b^6*c^3*d^4)/11 + (1176*a^3*b^5*c^2*d^5)/11) + x^7*(a^8*c*d^6 + 4*
a^2*b^6*c^7 + 56*a^3*b^5*c^6*d + 24*a^7*b*c^2*d^5 + 210*a^4*b^4*c^5*d^2 + 2
80*a^5*b^3*c^4*d^3 + 140*a^6*b^2*c^3*d^4) + x^10*((7*b^8*c^6*d)/10 + (14*a^
6*b^2*d^7)/5 + (84*a*b^7*c^5*d^2)/5 + (196*a^5*b^3*c*d^6)/5 + 98*a^2*b^6*c^
4*d^3 + 196*a^3*b^5*c^3*d^4 + 147*a^4*b^4*c^2*d^5) + a^8*c^7*x + (b^8*d^7*x
^16)/16 + (7*a^5*c^4*x^4*(5*a^3*d^3 + 8*b^3*c^3 + 28*a*b^2*c^2*d + 24*a^2*b
*c*d^2))/4 + (7*b^5*d^4*x^13*(8*a^3*d^3 + 5*b^3*c^3 + 24*a*b^2*c^2*d + 28*a
^2*b*c*d^2))/13 + (a^7*c^6*x^2*(7*a*d + 8*b*c))/2 + (b^7*d^6*x^15*(8*a*d +
7*b*c))/15 + (7*a^6*c^5*x^3*(3*a^2*d^2 + 4*b^2*c^2 + 8*a*b*c*d))/3 + (b^6*d
^5*x^14*(4*a^2*d^2 + 3*b^2*c^2 + 8*a*b*c*d))/2
```

**sympy [B]** time = 0.21, size = 1046, normalized size = 5.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8*(d*x+c)**7,x)
```

```
[Out] a**8*c**7*x + b**8*d**7*x**16/16 + x**15*(8*a*b**7*d**7/15 + 7*b**8*c*d**6/
15) + x**14*(2*a**2*b**6*d**7 + 4*a*b**7*c*d**6 + 3*b**8*c**2*d**5/2) + x**
13*(56*a**3*b**5*d**7/13 + 196*a**2*b**6*c*d**6/13 + 168*a*b**7*c**2*d**5/1
3 + 35*b**8*c**3*d**4/13) + x**12*(35*a**4*b**4*d**7/6 + 98*a**3*b**5*c*d**
6/3 + 49*a**2*b**6*c**2*d**5 + 70*a*b**7*c**3*d**4/3 + 35*b**8*c**4*d**3/12
) + x**11*(56*a**5*b**3*d**7/11 + 490*a**4*b**4*c*d**6/11 + 1176*a**3*b**5*
c**2*d**5/11 + 980*a**2*b**6*c**3*d**4/11 + 280*a*b**7*c**4*d**3/11 + 21*b*
*8*c**5*d**2/11) + x**10*(14*a**6*b**2*d**7/5 + 196*a**5*b**3*c*d**6/5 + 14
7*a**4*b**4*c**2*d**5 + 196*a**3*b**5*c**3*d**4 + 98*a**2*b**6*c**4*d**3 +
84*a*b**7*c**5*d**2/5 + 7*b**8*c**6*d/10) + x**9*(8*a**7*b*d**7/9 + 196*a**
6*b**2*c*d**6/9 + 392*a**5*b**3*c**2*d**5/3 + 2450*a**4*b**4*c**3*d**4/9 +
1960*a**3*b**5*c**4*d**3/9 + 196*a**2*b**6*c**5*d**2/3 + 56*a*b**7*c**6*d/9
 + b**8*c**7/9) + x**8*(a**8*d**7/8 + 7*a**7*b*c*d**6 + 147*a**6*b**2*c**2*
d**5/2 + 245*a**5*b**3*c**3*d**4 + 1225*a**4*b**4*c**4*d**3/4 + 147*a**3*b*
*5*c**5*d**2 + 49*a**2*b**6*c**6*d/2 + a*b**7*c**7) + x**7*(a**8*c*d**6 + 2
4*a**7*b*c**2*d**5 + 140*a**6*b**2*c**3*d**4 + 280*a**5*b**3*c**4*d**3 + 21
0*a**4*b**4*c**5*d**2 + 56*a**3*b**5*c**6*d + 4*a**2*b**6*c**7) + x**6*(7*a
**8*c**2*d**5/2 + 140*a**7*b*c**3*d**4/3 + 490*a**6*b**2*c**4*d**3/3 + 196*
a**5*b**3*c**5*d**2 + 245*a**4*b**4*c**6*d/3 + 28*a**3*b**5*c**7/3) + x**5*
(7*a**8*c**3*d**4 + 56*a**7*b*c**4*d**3 + 588*a**6*b**2*c**5*d**2/5 + 392*a
**5*b**3*c**6*d/5 + 14*a**4*b**4*c**7) + x**4*(35*a**8*c**4*d**3/4 + 42*a**
7*b*c**5*d**2 + 49*a**6*b**2*c**6*d + 14*a**5*b**3*c**7) + x**3*(7*a**8*c**
5*d**2 + 56*a**7*b*c**6*d/3 + 28*a**6*b**2*c**7/3) + x**2*(7*a**8*c**6*d/2
+ 4*a**7*b*c**7)
```

# 3.1169 $\int (a+bx)^7 (c+dx)^7 \, dx$

**Optimal**. Leaf size=200

$$\frac{d^6(a+bx)^{14}(bc-ad)}{2b^8}+\frac{21d^5(a+bx)^{13}(bc-ad)^2}{13b^8}+\frac{35d^4(a+bx)^{12}(bc-ad)^3}{12b^8}+\frac{35d^3(a+bx)^{11}(bc-ad)^4}{11b^8}+\frac{21d^2(a\ldots}{}$$

**Rubi [A]** time = 0.45, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^6(a+bx)^{14}(bc-ad)}{2b^8}+\frac{21d^5(a+bx)^{13}(bc-ad)^2}{13b^8}+\frac{35d^4(a+bx)^{12}(bc-ad)^3}{12b^8}+\frac{35d^3(a+bx)^{11}(bc-ad)^4}{11b^8}+\frac{21d^2(a+bx)^{10}(bc-ad)^5}{10b^8}+\frac{7d(a+bx)^9(bc-ad)^6}{9b^8}+\frac{(a+bx)^8(bc-ad)^7}{8b^8}+\frac{d^7(a+bx)^{15}}{15b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^8)/(8*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^9)/(9*b^
8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^10)/(10*b^8) + (35*d^3*(b*c - a*d)^4*(
a + b*x)^11)/(11*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^12)/(12*b^8) + (21*
d^5*(b*c - a*d)^2*(a + b*x)^13)/(13*b^8) + (d^6*(b*c - a*d)*(a + b*x)^14)/(
2*b^8) + (d^7*(a + b*x)^15)/(15*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^7(c+dx)^7\,dx=\int\left(\frac{(bc-ad)^7(a+bx)^7}{b^7}+\frac{7d(bc-ad)^6(a+bx)^8}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^9}{b^7}+\frac{35d^3(b\ldots}{}\right.$$
$$=\frac{(bc-ad)^7(a+bx)^8}{8b^8}+\frac{7d(bc-ad)^6(a+bx)^9}{9b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{10b^8}+\frac{35d^3(bc-\ldots}{}$$

**Mathematica [B]** time = 0.09, size = 785, normalized size = 3.92

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7*(c + d*x)^7,x]
```

```
[Out] a^7*c^7*x + (7*a^6*c^6*(b*c + a*d)*x^2)/2 + (7*a^5*c^5*(3*b^2*c^2 + 7*a*b*c
*d + 3*a^2*d^2)*x^3)/3 + (7*a^4*c^4*(5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*
c*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^3*c^3*(5*b^4*c^4 + 35*a*b^3*c^3*d + 63*a^2
*b^2*c^2*d^2 + 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^2*c^2*(3*b^5*c^5 +
 35*a*b^4*c^4*d + 105*a^2*b^3*c^3*d^2 + 105*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^
4 + 3*a^5*d^5)*x^6)/6 + a*c*(b^6*c^6 + 21*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2
 + 175*a^3*b^3*c^3*d^3 + 105*a^4*b^2*c^2*d^4 + 21*a^5*b*c*d^5 + a^6*d^6)*x^
7 + ((b^7*c^7 + 49*a*b^6*c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3
 + 1225*a^4*b^3*c^3*d^4 + 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x
^8)/8 + (7*b*d*(b^6*c^6 + 21*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2 + 175*a^3*b^
3*c^3*d^3 + 105*a^4*b^2*c^2*d^4 + 21*a^5*b*c*d^5 + a^6*d^6)*x^9)/9 + (7*b^2
*d^2*(3*b^5*c^5 + 35*a*b^4*c^4*d + 105*a^2*b^3*c^3*d^2 + 105*a^3*b^2*c^2*d^
3 + 35*a^4*b*c*d^4 + 3*a^5*d^5)*x^10)/10 + (7*b^3*d^3*(5*b^4*c^4 + 35*a*b^3
*c^3*d + 63*a^2*b^2*c^2*d^2 + 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^11)/11 + (7*b^4
*d^4*(5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2 + 5*a^3*d^3)*x^12)/12 + (
7*b^5*d^5*(3*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*x^13)/13 + (b^6*d^6*(b*c + a*
d)*x^14)/2 + (b^7*d^7*x^15)/15
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^7(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7*(c + d*x)^7, x]
```

**fricas [B]** time = 1.25, size = 924, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*d^7*b^7 + 1/2*x^14*d^6*c*b^7 + 1/2*x^14*d^7*b^6*a + 21/13*x^13*d^
5*c^2*b^7 + 49/13*x^13*d^6*c*b^6*a + 21/13*x^13*d^7*b^5*a^2 + 35/12*x^12*d^
4*c^3*b^7 + 49/4*x^12*d^5*c^2*b^6*a + 49/4*x^12*d^6*c*b^5*a^2 + 35/12*x^12*
d^7*b^4*a^3 + 35/11*x^11*d^3*c^4*b^7 + 245/11*x^11*d^4*c^3*b^6*a + 441/11*x
^11*d^5*c^2*b^5*a^2 + 245/11*x^11*d^6*c*b^4*a^3 + 35/11*x^11*d^7*b^3*a^4 +
21/10*x^10*d^2*c^5*b^7 + 49/2*x^10*d^3*c^4*b^6*a + 147/2*x^10*d^4*c^3*b^5*a
^2 + 147/2*x^10*d^5*c^2*b^4*a^3 + 49/2*x^10*d^6*c*b^3*a^4 + 21/10*x^10*d^7*
b^2*a^5 + 7/9*x^9*d*c^6*b^7 + 49/3*x^9*d^2*c^5*b^6*a + 245/3*x^9*d^3*c^4*b^
```

```
5*a^2 + 1225/9*x^9*d^4*c^3*b^4*a^3 + 245/3*x^9*d^5*c^2*b^3*a^4 + 49/3*x^9*d
^6*c*b^2*a^5 + 7/9*x^9*d^7*b*a^6 + 1/8*x^8*c^7*b^7 + 49/8*x^8*d*c^6*b^6*a +
 441/8*x^8*d^2*c^5*b^5*a^2 + 1225/8*x^8*d^3*c^4*b^4*a^3 + 1225/8*x^8*d^4*c^
3*b^3*a^4 + 441/8*x^8*d^5*c^2*b^2*a^5 + 49/8*x^8*d^6*c*b*a^6 + 1/8*x^8*d^7*
a^7 + x^7*c^7*b^6*a + 21*x^7*d*c^6*b^5*a^2 + 105*x^7*d^2*c^5*b^4*a^3 + 175*
x^7*d^3*c^4*b^3*a^4 + 105*x^7*d^4*c^3*b^2*a^5 + 21*x^7*d^5*c^2*b*a^6 + x^7*
d^6*c*a^7 + 7/2*x^6*c^7*b^5*a^2 + 245/6*x^6*d*c^6*b^4*a^3 + 245/2*x^6*d^2*c
^5*b^3*a^4 + 245/2*x^6*d^3*c^4*b^2*a^5 + 245/6*x^6*d^4*c^3*b*a^6 + 7/2*x^6*
d^5*c^2*a^7 + 7*x^5*c^7*b^4*a^3 + 49*x^5*d*c^6*b^3*a^4 + 441/5*x^5*d^2*c^5*
b^2*a^5 + 49*x^5*d^3*c^4*b*a^6 + 7*x^5*d^4*c^3*a^7 + 35/4*x^4*c^7*b^3*a^4 +
 147/4*x^4*d*c^6*b^2*a^5 + 147/4*x^4*d^2*c^5*b*a^6 + 35/4*x^4*d^3*c^4*a^7 +
 7*x^3*c^7*b^2*a^5 + 49/3*x^3*d*c^6*b*a^6 + 7*x^3*d^2*c^5*a^7 + 7/2*x^2*c^7
*b*a^6 + 7/2*x^2*d*c^6*a^7 + x*c^7*a^7
```

**giac [B]** time = 1.01, size = 924, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/15*b^7*d^7*x^15 + 1/2*b^7*c*d^6*x^14 + 1/2*a*b^6*d^7*x^14 + 21/13*b^7*c^2
*d^5*x^13 + 49/13*a*b^6*c*d^6*x^13 + 21/13*a^2*b^5*d^7*x^13 + 35/12*b^7*c^3
*d^4*x^12 + 49/4*a*b^6*c^2*d^5*x^12 + 49/4*a^2*b^5*c*d^6*x^12 + 35/12*a^3*b
^4*d^7*x^12 + 35/11*b^7*c^4*d^3*x^11 + 245/11*a*b^6*c^3*d^4*x^11 + 441/11*a
^2*b^5*c^2*d^5*x^11 + 245/11*a^3*b^4*c*d^6*x^11 + 35/11*a^4*b^3*d^7*x^11 +
21/10*b^7*c^5*d^2*x^10 + 49/2*a*b^6*c^4*d^3*x^10 + 147/2*a^2*b^5*c^3*d^4*x^
10 + 147/2*a^3*b^4*c^2*d^5*x^10 + 49/2*a^4*b^3*c*d^6*x^10 + 21/10*a^5*b^2*d
^7*x^10 + 7/9*b^7*c^6*d*x^9 + 49/3*a*b^6*c^5*d^2*x^9 + 245/3*a^2*b^5*c^4*d^
3*x^9 + 1225/9*a^3*b^4*c^3*d^4*x^9 + 245/3*a^4*b^3*c^2*d^5*x^9 + 49/3*a^5*b
^2*c*d^6*x^9 + 7/9*a^6*b*d^7*x^9 + 1/8*b^7*c^7*x^8 + 49/8*a*b^6*c^6*d*x^8 +
 441/8*a^2*b^5*c^5*d^2*x^8 + 1225/8*a^3*b^4*c^4*d^3*x^8 + 1225/8*a^4*b^3*c^
3*d^4*x^8 + 441/8*a^5*b^2*c^2*d^5*x^8 + 49/8*a^6*b*c*d^6*x^8 + 1/8*a^7*d^7*
x^8 + a*b^6*c^7*x^7 + 21*a^2*b^5*c^6*d*x^7 + 105*a^3*b^4*c^5*d^2*x^7 + 175*
a^4*b^3*c^4*d^3*x^7 + 105*a^5*b^2*c^3*d^4*x^7 + 21*a^6*b*c^2*d^5*x^7 + a^7*
c*d^6*x^7 + 7/2*a^2*b^5*c^7*x^6 + 245/6*a^3*b^4*c^6*d*x^6 + 245/2*a^4*b^3*c
^5*d^2*x^6 + 245/2*a^5*b^2*c^4*d^3*x^6 + 245/6*a^6*b*c^3*d^4*x^6 + 7/2*a^7*
c^2*d^5*x^6 + 7*a^3*b^4*c^7*x^5 + 49*a^4*b^3*c^6*d*x^5 + 441/5*a^5*b^2*c^5*
d^2*x^5 + 49*a^6*b*c^4*d^3*x^5 + 7*a^7*c^3*d^4*x^5 + 35/4*a^4*b^3*c^7*x^4 +
 147/4*a^5*b^2*c^6*d*x^4 + 147/4*a^6*b*c^5*d^2*x^4 + 35/4*a^7*c^4*d^3*x^4 +
 7*a^5*b^2*c^7*x^3 + 49/3*a^6*b*c^6*d*x^3 + 7*a^7*c^5*d^2*x^3 + 7/2*a^6*b*c
^7*x^2 + 7/2*a^7*c^6*d*x^2 + a^7*c^7*x
```

**maple [B]** time = 0.00, size = 817, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7*(d*x+c)^7,x)
```

```
[Out] 1/15*b^7*d^7*x^15+1/14*(7*a*b^6*d^7+7*b^7*c*d^6)*x^14+1/13*(21*a^2*b^5*d^7+
49*a*b^6*c*d^6+21*b^7*c^2*d^5)*x^13+1/12*(35*a^3*b^4*d^7+147*a^2*b^5*c*d^6+
147*a*b^6*c^2*d^5+35*b^7*c^3*d^4)*x^12+1/11*(35*a^4*b^3*d^7+245*a^3*b^4*c*d
^6+441*a^2*b^5*c^2*d^5+245*a*b^6*c^3*d^4+35*b^7*c^4*d^3)*x^11+1/10*(21*a^5*
b^2*d^7+245*a^4*b^3*c*d^6+735*a^3*b^4*c^2*d^5+735*a^2*b^5*c^3*d^4+245*a*b^6
*c^4*d^3+21*b^7*c^5*d^2)*x^10+1/9*(7*a^6*b*d^7+147*a^5*b^2*c*d^6+735*a^4*b^
3*c^2*d^5+1225*a^3*b^4*c^3*d^4+735*a^2*b^5*c^4*d^3+147*a*b^6*c^5*d^2+7*b^7*
c^6*d)*x^9+1/8*(a^7*d^7+49*a^6*b*c*d^6+441*a^5*b^2*c^2*d^5+1225*a^4*b^3*c^3
*d^4+1225*a^3*b^4*c^4*d^3+441*a^2*b^5*c^5*d^2+49*a*b^6*c^6*d+b^7*c^7)*x^8+1
/7*(7*a^7*c*d^6+147*a^6*b*c^2*d^5+735*a^5*b^2*c^3*d^4+1225*a^4*b^3*c^4*d^3+
735*a^3*b^4*c^5*d^2+147*a^2*b^5*c^6*d+7*a*b^6*c^7)*x^7+1/6*(21*a^7*c^2*d^5+
245*a^6*b*c^3*d^4+735*a^5*b^2*c^4*d^3+735*a^4*b^3*c^5*d^2+245*a^3*b^4*c^6*d
+21*a^2*b^5*c^7)*x^6+1/5*(35*a^7*c^3*d^4+245*a^6*b*c^4*d^3+441*a^5*b^2*c^5*
d^2+245*a^4*b^3*c^6*d+35*a^3*b^4*c^7)*x^5+1/4*(35*a^7*c^4*d^3+147*a^6*b*c^5
*d^2+147*a^5*b^2*c^6*d+35*a^4*b^3*c^7)*x^4+1/3*(21*a^7*c^5*d^2+49*a^6*b*c^6
*d+21*a^5*b^2*c^7)*x^3+1/2*(7*a^7*c^6*d+7*a^6*b*c^7)*x^2+a^7*c^7*x
```

**maxima** [B] time = 1.33, size = 807, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/15*b^7*d^7*x^15 + a^7*c^7*x + 1/2*(b^7*c*d^6 + a*b^6*d^7)*x^14 + 7/13*(3*
b^7*c^2*d^5 + 7*a*b^6*c*d^6 + 3*a^2*b^5*d^7)*x^13 + 7/12*(5*b^7*c^3*d^4 + 2
1*a*b^6*c^2*d^5 + 21*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^12 + 7/11*(5*b^7*c^4*
d^3 + 35*a*b^6*c^3*d^4 + 63*a^2*b^5*c^2*d^5 + 35*a^3*b^4*c*d^6 + 5*a^4*b^3*
d^7)*x^11 + 7/10*(3*b^7*c^5*d^2 + 35*a*b^6*c^4*d^3 + 105*a^2*b^5*c^3*d^4 +
105*a^3*b^4*c^2*d^5 + 35*a^4*b^3*c*d^6 + 3*a^5*b^2*d^7)*x^10 + 7/9*(b^7*c^6
*d + 21*a*b^6*c^5*d^2 + 105*a^2*b^5*c^4*d^3 + 175*a^3*b^4*c^3*d^4 + 105*a^4
*b^3*c^2*d^5 + 21*a^5*b^2*c*d^6 + a^6*b*d^7)*x^9 + 1/8*(b^7*c^7 + 49*a*b^6*
c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3 + 1225*a^4*b^3*c^3*d^4 +
 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x^8 + (a*b^6*c^7 + 21*a^2*
b^5*c^6*d + 105*a^3*b^4*c^5*d^2 + 175*a^4*b^3*c^4*d^3 + 105*a^5*b^2*c^3*d^4
 + 21*a^6*b*c^2*d^5 + a^7*c*d^6)*x^7 + 7/6*(3*a^2*b^5*c^7 + 35*a^3*b^4*c^6*
d + 105*a^4*b^3*c^5*d^2 + 105*a^5*b^2*c^4*d^3 + 35*a^6*b*c^3*d^4 + 3*a^7*c^
2*d^5)*x^6 + 7/5*(5*a^3*b^4*c^7 + 35*a^4*b^3*c^6*d + 63*a^5*b^2*c^5*d^2 + 3
5*a^6*b*c^4*d^3 + 5*a^7*c^3*d^4)*x^5 + 7/4*(5*a^4*b^3*c^7 + 21*a^5*b^2*c^6*
d + 21*a^6*b*c^5*d^2 + 5*a^7*c^4*d^3)*x^4 + 7/3*(3*a^5*b^2*c^7 + 7*a^6*b*c^
6*d + 3*a^7*c^5*d^2)*x^3 + 7/2*(a^6*b*c^7 + a^7*c^6*d)*x^2
```

**mupad [B]** time = 0.40, size = 781, normalized size = 3.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7*(c + d*x)^7,x)
```

```
[Out] x^8*((a^7*d^7)/8 + (b^7*c^7)/8 + (441*a^2*b^5*c^5*d^2)/8 + (1225*a^3*b^4*c^
4*d^3)/8 + (1225*a^4*b^3*c^3*d^4)/8 + (441*a^5*b^2*c^2*d^5)/8 + (49*a*b^6*c
^6*d)/8 + (49*a^6*b*c*d^6)/8) + x^5*(7*a^3*b^4*c^7 + 7*a^7*c^3*d^4 + 49*a^4
*b^3*c^6*d + 49*a^6*b*c^4*d^3 + (441*a^5*b^2*c^5*d^2)/5) + x^11*((35*a^4*b^
3*d^7)/11 + (35*b^7*c^4*d^3)/11 + (245*a*b^6*c^3*d^4)/11 + (245*a^3*b^4*c*d
^6)/11 + (441*a^2*b^5*c^2*d^5)/11) + x^7*(a*b^6*c^7 + a^7*c*d^6 + 21*a^2*b^
5*c^6*d + 21*a^6*b*c^2*d^5 + 105*a^3*b^4*c^5*d^2 + 175*a^4*b^3*c^4*d^3 + 10
5*a^5*b^2*c^3*d^4) + x^9*((7*a^6*b*d^7)/9 + (7*b^7*c^6*d)/9 + (49*a*b^6*c^5
*d^2)/3 + (49*a^5*b^2*c*d^6)/3 + (245*a^2*b^5*c^4*d^3)/3 + (1225*a^3*b^4*c^
3*d^4)/9 + (245*a^4*b^3*c^2*d^5)/3) + x^6*((7*a^2*b^5*c^7)/2 + (7*a^7*c^2*d
^5)/2 + (245*a^3*b^4*c^6*d)/6 + (245*a^6*b*c^3*d^4)/6 + (245*a^4*b^3*c^5*d^
2)/2 + (245*a^5*b^2*c^4*d^3)/2) + x^10*((21*a^5*b^2*d^7)/10 + (21*b^7*c^5*d
^2)/10 + (49*a*b^6*c^4*d^3)/2 + (49*a^4*b^3*c*d^6)/2 + (147*a^2*b^5*c^3*d^4
)/2 + (147*a^3*b^4*c^2*d^5)/2) + a^7*c^7*x + (b^7*d^7*x^15)/15 + (7*a^4*c^4
*x^4*(5*a^3*d^3 + 5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2))/4 + (7*b^4*
d^4*x^12*(5*a^3*d^3 + 5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2))/12 + (7
*a^6*c^6*x^2*(a*d + b*c))/2 + (b^6*d^6*x^14*(a*d + b*c))/2 + (7*a^5*c^5*x^3
*(3*a^2*d^2 + 3*b^2*c^2 + 7*a*b*c*d))/3 + (7*b^5*d^5*x^13*(3*a^2*d^2 + 3*b^
2*c^2 + 7*a*b*c*d))/13
```

**sympy [B]** time = 0.19, size = 935, normalized size = 4.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7*(d*x+c)**7,x)
```

```
[Out] a**7*c**7*x + b**7*d**7*x**15/15 + x**14*(a*b**6*d**7/2 + b**7*c*d**6/2) +
x**13*(21*a**2*b**5*d**7/13 + 49*a*b**6*c*d**6/13 + 21*b**7*c**2*d**5/13) +
 x**12*(35*a**3*b**4*d**7/12 + 49*a**2*b**5*c*d**6/4 + 49*a*b**6*c**2*d**5/
4 + 35*b**7*c**3*d**4/12) + x**11*(35*a**4*b**3*d**7/11 + 245*a**3*b**4*c*d
**6/11 + 441*a**2*b**5*c**2*d**5/11 + 245*a*b**6*c**3*d**4/11 + 35*b**7*c**
4*d**3/11) + x**10*(21*a**5*b**2*d**7/10 + 49*a**4*b**3*c*d**6/2 + 147*a**3
*b**4*c**2*d**5/2 + 147*a**2*b**5*c**3*d**4/2 + 49*a*b**6*c**4*d**3/2 + 21*
b**7*c**5*d**2/10) + x**9*(7*a**6*b*d**7/9 + 49*a**5*b**2*c*d**6/3 + 245*a*
*4*b**3*c**2*d**5/3 + 1225*a**3*b**4*c**3*d**4/9 + 245*a**2*b**5*c**4*d**3/
3 + 49*a*b**6*c**5*d**2/3 + 7*b**7*c**6*d/9) + x**8*(a**7*d**7/8 + 49*a**6*
```

```
b*c*d**6/8 + 441*a**5*b**2*c**2*d**5/8 + 1225*a**4*b**3*c**3*d**4/8 + 1225*
a**3*b**4*c**4*d**3/8 + 441*a**2*b**5*c**5*d**2/8 + 49*a*b**6*c**6*d/8 + b*
*7*c**7/8) + x**7*(a**7*c*d**6 + 21*a**6*b*c**2*d**5 + 105*a**5*b**2*c**3*d
**4 + 175*a**4*b**3*c**4*d**3 + 105*a**3*b**4*c**5*d**2 + 21*a**2*b**5*c**6
*d + a*b**6*c**7) + x**6*(7*a**7*c**2*d**5/2 + 245*a**6*b*c**3*d**4/6 + 245
*a**5*b**2*c**4*d**3/2 + 245*a**4*b**3*c**5*d**2/2 + 245*a**3*b**4*c**6*d/6
 + 7*a**2*b**5*c**7/2) + x**5*(7*a**7*c**3*d**4 + 49*a**6*b*c**4*d**3 + 441
*a**5*b**2*c**5*d**2/5 + 49*a**4*b**3*c**6*d + 7*a**3*b**4*c**7) + x**4*(35
*a**7*c**4*d**3/4 + 147*a**6*b*c**5*d**2/4 + 147*a**5*b**2*c**6*d/4 + 35*a*
*4*b**3*c**7/4) + x**3*(7*a**7*c**5*d**2 + 49*a**6*b*c**6*d/3 + 7*a**5*b**2
*c**7) + x**2*(7*a**7*c**6*d/2 + 7*a**6*b*c**7/2)
```

## 3.1170 $\int (a+bx)^6(c+dx)^7\,dx$

**Optimal**. Leaf size=173

$$-\frac{6b^5(c+dx)^{13}(bc-ad)}{13d^7}+\frac{5b^4(c+dx)^{12}(bc-ad)^2}{4d^7}-\frac{20b^3(c+dx)^{11}(bc-ad)^3}{11d^7}+\frac{3b^2(c+dx)^{10}(bc-ad)^4}{2d^7}-\frac{2b(c+d}{}$$

**Rubi [A]** time = 0.43, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{6b^5(c+dx)^{13}(bc-ad)}{13d^7}+\frac{5b^4(c+dx)^{12}(bc-ad)^2}{4d^7}-\frac{20b^3(c+dx)^{11}(bc-ad)^3}{11d^7}+\frac{3b^2(c+dx)^{10}(bc-ad)^4}{2d^7}-\frac{2b(c+dx)^9(bc-ad)^5}{3d^7}+\frac{(c+dx)^8(bc-ad)^6}{8d^7}+\frac{b^6(c+dx)^{14}}{14d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^6*(c + d*x)^8)/(8*d^7) - (2*b*(b*c - a*d)^5*(c + d*x)^9)/(3*d^
7) + (3*b^2*(b*c - a*d)^4*(c + d*x)^10)/(2*d^7) - (20*b^3*(b*c - a*d)^3*(c
+ d*x)^11)/(11*d^7) + (5*b^4*(b*c - a*d)^2*(c + d*x)^12)/(4*d^7) - (6*b^5*(
b*c - a*d)*(c + d*x)^13)/(13*d^7) + (b^6*(c + d*x)^14)/(14*d^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^6(c+dx)^7\,dx=\int\left(\frac{(-bc+ad)^6(c+dx)^7}{d^6}-\frac{6b(bc-ad)^5(c+dx)^8}{d^6}+\frac{15b^2(bc-ad)^4(c+dx)^9}{d^6}-\frac{20b^3(b}{}\right.$$
$$=\frac{(bc-ad)^6(c+dx)^8}{8d^7}-\frac{2b(bc-ad)^5(c+dx)^9}{3d^7}+\frac{3b^2(bc-ad)^4(c+dx)^{10}}{2d^7}-\frac{20b^3(bc-a}{11}$$

**Mathematica [B]** time = 0.08, size = 684, normalized size = 3.95

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6*(c + d*x)^7,x]
```

```
[Out] a^6*c^7*x + (a^5*c^6*(6*b*c + 7*a*d)*x^2)/2 + a^4*c^5*(5*b^2*c^2 + 14*a*b*c
*d + 7*a^2*d^2)*x^3 + (a^3*c^4*(20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*
d^2 + 35*a^3*d^3)*x^4)/4 + a^2*c^3*(3*b^4*c^4 + 28*a*b^3*c^3*d + 63*a^2*b^2
*c^2*d^2 + 42*a^3*b*c*d^3 + 7*a^4*d^4)*x^5 + (a*c^2*(2*b^5*c^5 + 35*a*b^4*c
^4*d + 140*a^2*b^3*c^3*d^2 + 175*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 7*a^5*d
^5)*x^6)/2 + (c*(b^6*c^6 + 42*a*b^5*c^5*d + 315*a^2*b^4*c^4*d^2 + 700*a^3*b
^3*c^3*d^3 + 525*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 + 7*a^6*d^6)*x^7)/7 + (d
*(7*b^6*c^6 + 126*a*b^5*c^5*d + 525*a^2*b^4*c^4*d^2 + 700*a^3*b^3*c^3*d^3 +
 315*a^4*b^2*c^2*d^4 + 42*a^5*b*c*d^5 + a^6*d^6)*x^8)/8 + (b*d^2*(7*b^5*c^5
 + 70*a*b^4*c^4*d + 175*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 35*a^4*b*c*
d^4 + 2*a^5*d^5)*x^9)/3 + (b^2*d^3*(7*b^4*c^4 + 42*a*b^3*c^3*d + 63*a^2*b^2
*c^2*d^2 + 28*a^3*b*c*d^3 + 3*a^4*d^4)*x^10)/2 + (b^3*d^4*(35*b^3*c^3 + 126
*a*b^2*c^2*d + 105*a^2*b*c*d^2 + 20*a^3*d^3)*x^11)/11 + (b^4*d^5*(7*b^2*c^2
 + 14*a*b*c*d + 5*a^2*d^2)*x^12)/4 + (b^5*d^6*(7*b*c + 6*a*d)*x^13)/13 + (b
^6*d^7*x^14)/14
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^6(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^6*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^6*(c + d*x)^7, x]
```

**fricas [B]** time = 1.20, size = 798, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d^7*b^6 + 7/13*x^13*d^6*c*b^6 + 6/13*x^13*d^7*b^5*a + 7/4*x^12*d^
5*c^2*b^6 + 7/2*x^12*d^6*c*b^5*a + 5/4*x^12*d^7*b^4*a^2 + 35/11*x^11*d^4*c^
3*b^6 + 126/11*x^11*d^5*c^2*b^5*a + 105/11*x^11*d^6*c*b^4*a^2 + 20/11*x^11*
d^7*b^3*a^3 + 7/2*x^10*d^3*c^4*b^6 + 21*x^10*d^4*c^3*b^5*a + 63/2*x^10*d^5*
c^2*b^4*a^2 + 14*x^10*d^6*c*b^3*a^3 + 3/2*x^10*d^7*b^2*a^4 + 7/3*x^9*d^2*c^
5*b^6 + 70/3*x^9*d^3*c^4*b^5*a + 175/3*x^9*d^4*c^3*b^4*a^2 + 140/3*x^9*d^5*
c^2*b^3*a^3 + 35/3*x^9*d^6*c*b^2*a^4 + 2/3*x^9*d^7*b*a^5 + 7/8*x^8*d*c^6*b^
6 + 63/4*x^8*d^2*c^5*b^5*a + 525/8*x^8*d^3*c^4*b^4*a^2 + 175/2*x^8*d^4*c^3*
b^3*a^3 + 315/8*x^8*d^5*c^2*b^2*a^4 + 21/4*x^8*d^6*c*b*a^5 + 1/8*x^8*d^7*a^
6 + 1/7*x^7*c^7*b^6 + 6*x^7*d*c^6*b^5*a + 45*x^7*d^2*c^5*b^4*a^2 + 100*x^7*
d^3*c^4*b^3*a^3 + 75*x^7*d^4*c^3*b^2*a^4 + 18*x^7*d^5*c^2*b*a^5 + x^7*d^6*c
*a^6 + x^6*c^7*b^5*a + 35/2*x^6*d*c^6*b^4*a^2 + 70*x^6*d^2*c^5*b^3*a^3 + 17
```

```
5/2*x^6*d^3*c^4*b^2*a^4 + 35*x^6*d^4*c^3*b*a^5 + 7/2*x^6*d^5*c^2*a^6 + 3*x^
5*c^7*b^4*a^2 + 28*x^5*d*c^6*b^3*a^3 + 63*x^5*d^2*c^5*b^2*a^4 + 42*x^5*d^3*
c^4*b*a^5 + 7*x^5*d^4*c^3*a^6 + 5*x^4*c^7*b^3*a^3 + 105/4*x^4*d*c^6*b^2*a^4
 + 63/2*x^4*d^2*c^5*b*a^5 + 35/4*x^4*d^3*c^4*a^6 + 5*x^3*c^7*b^2*a^4 + 14*x
^3*d*c^6*b*a^5 + 7*x^3*d^2*c^5*a^6 + 3*x^2*c^7*b*a^5 + 7/2*x^2*d*c^6*a^6 +
x*c^7*a^6
```

**giac [B]** time = 0.95, size = 798, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/14*b^6*d^7*x^14 + 7/13*b^6*c*d^6*x^13 + 6/13*a*b^5*d^7*x^13 + 7/4*b^6*c^2
*d^5*x^12 + 7/2*a*b^5*c*d^6*x^12 + 5/4*a^2*b^4*d^7*x^12 + 35/11*b^6*c^3*d^4
*x^11 + 126/11*a*b^5*c^2*d^5*x^11 + 105/11*a^2*b^4*c*d^6*x^11 + 20/11*a^3*b
^3*d^7*x^11 + 7/2*b^6*c^4*d^3*x^10 + 21*a*b^5*c^3*d^4*x^10 + 63/2*a^2*b^4*c
^2*d^5*x^10 + 14*a^3*b^3*c*d^6*x^10 + 3/2*a^4*b^2*d^7*x^10 + 7/3*b^6*c^5*d^
2*x^9 + 70/3*a*b^5*c^4*d^3*x^9 + 175/3*a^2*b^4*c^3*d^4*x^9 + 140/3*a^3*b^3*
c^2*d^5*x^9 + 35/3*a^4*b^2*c*d^6*x^9 + 2/3*a^5*b*d^7*x^9 + 7/8*b^6*c^6*d*x^
8 + 63/4*a*b^5*c^5*d^2*x^8 + 525/8*a^2*b^4*c^4*d^3*x^8 + 175/2*a^3*b^3*c^3*
d^4*x^8 + 315/8*a^4*b^2*c^2*d^5*x^8 + 21/4*a^5*b*c*d^6*x^8 + 1/8*a^6*d^7*x^
8 + 1/7*b^6*c^7*x^7 + 6*a*b^5*c^6*d*x^7 + 45*a^2*b^4*c^5*d^2*x^7 + 100*a^3*
b^3*c^4*d^3*x^7 + 75*a^4*b^2*c^3*d^4*x^7 + 18*a^5*b*c^2*d^5*x^7 + a^6*c*d^6
*x^7 + a*b^5*c^7*x^6 + 35/2*a^2*b^4*c^6*d*x^6 + 70*a^3*b^3*c^5*d^2*x^6 + 17
5/2*a^4*b^2*c^4*d^3*x^6 + 35*a^5*b*c^3*d^4*x^6 + 7/2*a^6*c^2*d^5*x^6 + 3*a^
2*b^4*c^7*x^5 + 28*a^3*b^3*c^6*d*x^5 + 63*a^4*b^2*c^5*d^2*x^5 + 42*a^5*b*c^
4*d^3*x^5 + 7*a^6*c^3*d^4*x^5 + 5*a^3*b^3*c^7*x^4 + 105/4*a^4*b^2*c^6*d*x^4
 + 63/2*a^5*b*c^5*d^2*x^4 + 35/4*a^6*c^4*d^3*x^4 + 5*a^4*b^2*c^7*x^3 + 14*a
^5*b*c^6*d*x^3 + 7*a^6*c^5*d^2*x^3 + 3*a^5*b*c^7*x^2 + 7/2*a^6*c^6*d*x^2 +
a^6*c^7*x
```

**maple [B]** time = 0.00, size = 709, normalized size = 4.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6*(d*x+c)^7,x)
```

```
[Out] 1/14*b^6*d^7*x^14+1/13*(6*a*b^5*d^7+7*b^6*c*d^6)*x^13+1/12*(15*a^2*b^4*d^7+
42*a*b^5*c*d^6+21*b^6*c^2*d^5)*x^12+1/11*(20*a^3*b^3*d^7+105*a^2*b^4*c*d^6+
126*a*b^5*c^2*d^5+35*b^6*c^3*d^4)*x^11+1/10*(15*a^4*b^2*d^7+140*a^3*b^3*c*d
^6+315*a^2*b^4*c^2*d^5+210*a*b^5*c^3*d^4+35*b^6*c^4*d^3)*x^10+1/9*(6*a^5*b*
```

```
d^7+105*a^4*b^2*c*d^6+420*a^3*b^3*c^2*d^5+525*a^2*b^4*c^3*d^4+210*a*b^5*c^4
*d^3+21*b^6*c^5*d^2)*x^9+1/8*(a^6*d^7+42*a^5*b*c*d^6+315*a^4*b^2*c^2*d^5+70
0*a^3*b^3*c^3*d^4+525*a^2*b^4*c^4*d^3+126*a*b^5*c^5*d^2+7*b^6*c^6*d)*x^8+1/
7*(7*a^6*c*d^6+126*a^5*b*c^2*d^5+525*a^4*b^2*c^3*d^4+700*a^3*b^3*c^4*d^3+31
5*a^2*b^4*c^5*d^2+42*a*b^5*c^6*d+b^6*c^7)*x^7+1/6*(21*a^6*c^2*d^5+210*a^5*b
*c^3*d^4+525*a^4*b^2*c^4*d^3+420*a^3*b^3*c^5*d^2+105*a^2*b^4*c^6*d+6*a*b^5*
c^7)*x^6+1/5*(35*a^6*c^3*d^4+210*a^5*b*c^4*d^3+315*a^4*b^2*c^5*d^2+140*a^3*
b^3*c^6*d+15*a^2*b^4*c^7)*x^5+1/4*(35*a^6*c^4*d^3+126*a^5*b*c^5*d^2+105*a^4
*b^2*c^6*d+20*a^3*b^3*c^7)*x^4+1/3*(21*a^6*c^5*d^2+42*a^5*b*c^6*d+15*a^4*b^
2*c^7)*x^3+1/2*(7*a^6*c^6*d+6*a^5*b*c^7)*x^2+a^6*c^7*x
```

**maxima** [B] time = 1.32, size = 706, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/14*b^6*d^7*x^14 + a^6*c^7*x + 1/13*(7*b^6*c*d^6 + 6*a*b^5*d^7)*x^13 + 1/4
*(7*b^6*c^2*d^5 + 14*a*b^5*c*d^6 + 5*a^2*b^4*d^7)*x^12 + 1/11*(35*b^6*c^3*d
^4 + 126*a*b^5*c^2*d^5 + 105*a^2*b^4*c*d^6 + 20*a^3*b^3*d^7)*x^11 + 1/2*(7*
b^6*c^4*d^3 + 42*a*b^5*c^3*d^4 + 63*a^2*b^4*c^2*d^5 + 28*a^3*b^3*c*d^6 + 3*
a^4*b^2*d^7)*x^10 + 1/3*(7*b^6*c^5*d^2 + 70*a*b^5*c^4*d^3 + 175*a^2*b^4*c^3
*d^4 + 140*a^3*b^3*c^2*d^5 + 35*a^4*b^2*c*d^6 + 2*a^5*b*d^7)*x^9 + 1/8*(7*b
^6*c^6*d + 126*a*b^5*c^5*d^2 + 525*a^2*b^4*c^4*d^3 + 700*a^3*b^3*c^3*d^4 +
315*a^4*b^2*c^2*d^5 + 42*a^5*b*c*d^6 + a^6*d^7)*x^8 + 1/7*(b^6*c^7 + 42*a*b
^5*c^6*d + 315*a^2*b^4*c^5*d^2 + 700*a^3*b^3*c^4*d^3 + 525*a^4*b^2*c^3*d^4
+ 126*a^5*b*c^2*d^5 + 7*a^6*c*d^6)*x^7 + 1/2*(2*a*b^5*c^7 + 35*a^2*b^4*c^6*
d + 140*a^3*b^3*c^5*d^2 + 175*a^4*b^2*c^4*d^3 + 70*a^5*b*c^3*d^4 + 7*a^6*c^
2*d^5)*x^6 + (3*a^2*b^4*c^7 + 28*a^3*b^3*c^6*d + 63*a^4*b^2*c^5*d^2 + 42*a^
5*b*c^4*d^3 + 7*a^6*c^3*d^4)*x^5 + 1/4*(20*a^3*b^3*c^7 + 105*a^4*b^2*c^6*d
+ 126*a^5*b*c^5*d^2 + 35*a^6*c^4*d^3)*x^4 + (5*a^4*b^2*c^7 + 14*a^5*b*c^6*d
 + 7*a^6*c^5*d^2)*x^3 + 1/2*(6*a^5*b*c^7 + 7*a^6*c^6*d)*x^2
```

**mupad** [B] time = 0.26, size = 683, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6*(c + d*x)^7,x)
```

```
[Out] x^5*(3*a^2*b^4*c^7 + 7*a^6*c^3*d^4 + 28*a^3*b^3*c^6*d + 42*a^5*b*c^4*d^3 +
63*a^4*b^2*c^5*d^2) + x^10*((3*a^4*b^2*d^7)/2 + (7*b^6*c^4*d^3)/2 + 21*a*b^
5*c^3*d^4 + 14*a^3*b^3*c*d^6 + (63*a^2*b^4*c^2*d^5)/2) + x^6*(a*b^5*c^7 + (
7*a^6*c^2*d^5)/2 + (35*a^2*b^4*c^6*d)/2 + 35*a^5*b*c^3*d^4 + 70*a^3*b^3*c^5
```

```
*d^2 + (175*a^4*b^2*c^4*d^3)/2) + x^9*((2*a^5*b*d^7)/3 + (7*b^6*c^5*d^2)/3
+ (70*a*b^5*c^4*d^3)/3 + (35*a^4*b^2*c*d^6)/3 + (175*a^2*b^4*c^3*d^4)/3 + (
140*a^3*b^3*c^2*d^5)/3) + x^7*((b^6*c^7)/7 + a^6*c*d^6 + 18*a^5*b*c^2*d^5 +
 45*a^2*b^4*c^5*d^2 + 100*a^3*b^3*c^4*d^3 + 75*a^4*b^2*c^3*d^4 + 6*a*b^5*c^
6*d) + x^8*((a^6*d^7)/8 + (7*b^6*c^6*d)/8 + (63*a*b^5*c^5*d^2)/4 + (525*a^2
*b^4*c^4*d^3)/8 + (175*a^3*b^3*c^3*d^4)/2 + (315*a^4*b^2*c^2*d^5)/8 + (21*a
^5*b*c*d^6)/4) + x^4*(5*a^3*b^3*c^7 + (35*a^6*c^4*d^3)/4 + (105*a^4*b^2*c^6
*d)/4 + (63*a^5*b*c^5*d^2)/2) + x^11*((20*a^3*b^3*d^7)/11 + (35*b^6*c^3*d^4
)/11 + (126*a*b^5*c^2*d^5)/11 + (105*a^2*b^4*c*d^6)/11) + a^6*c^7*x + (b^6*
d^7*x^14)/14 + (a^5*c^6*x^2*(7*a*d + 6*b*c))/2 + (b^5*d^6*x^13*(6*a*d + 7*b
*c))/13 + a^4*c^5*x^3*(7*a^2*d^2 + 5*b^2*c^2 + 14*a*b*c*d) + (b^4*d^5*x^12*
(5*a^2*d^2 + 7*b^2*c^2 + 14*a*b*c*d))/4
```

**sympy [B]** time = 0.18, size = 796, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6*(d*x+c)**7,x)
```

```
[Out] a**6*c**7*x + b**6*d**7*x**14/14 + x**13*(6*a*b**5*d**7/13 + 7*b**6*c*d**6/
13) + x**12*(5*a**2*b**4*d**7/4 + 7*a*b**5*c*d**6/2 + 7*b**6*c**2*d**5/4) +
 x**11*(20*a**3*b**3*d**7/11 + 105*a**2*b**4*c*d**6/11 + 126*a*b**5*c**2*d*
*5/11 + 35*b**6*c**3*d**4/11) + x**10*(3*a**4*b**2*d**7/2 + 14*a**3*b**3*c*
d**6 + 63*a**2*b**4*c**2*d**5/2 + 21*a*b**5*c**3*d**4 + 7*b**6*c**4*d**3/2)
 + x**9*(2*a**5*b*d**7/3 + 35*a**4*b**2*c*d**6/3 + 140*a**3*b**3*c**2*d**5/
3 + 175*a**2*b**4*c**3*d**4/3 + 70*a*b**5*c**4*d**3/3 + 7*b**6*c**5*d**2/3)
 + x**8*(a**6*d**7/8 + 21*a**5*b*c*d**6/4 + 315*a**4*b**2*c**2*d**5/8 + 175
*a**3*b**3*c**3*d**4/2 + 525*a**2*b**4*c**4*d**3/8 + 63*a*b**5*c**5*d**2/4
+ 7*b**6*c**6*d/8) + x**7*(a**6*c*d**6 + 18*a**5*b*c**2*d**5 + 75*a**4*b**2
*c**3*d**4 + 100*a**3*b**3*c**4*d**3 + 45*a**2*b**4*c**5*d**2 + 6*a*b**5*c*
*6*d + b**6*c**7/7) + x**6*(7*a**6*c**2*d**5/2 + 35*a**5*b*c**3*d**4 + 175*
a**4*b**2*c**4*d**3/2 + 70*a**3*b**3*c**5*d**2 + 35*a**2*b**4*c**6*d/2 + a*
b**5*c**7) + x**5*(7*a**6*c**3*d**4 + 42*a**5*b*c**4*d**3 + 63*a**4*b**2*c*
*5*d**2 + 28*a**3*b**3*c**6*d + 3*a**2*b**4*c**7) + x**4*(35*a**6*c**4*d**3
/4 + 63*a**5*b*c**5*d**2/2 + 105*a**4*b**2*c**6*d/4 + 5*a**3*b**3*c**7) + x
**3*(7*a**6*c**5*d**2 + 14*a**5*b*c**6*d + 5*a**4*b**2*c**7) + x**2*(7*a**6
*c**6*d/2 + 3*a**5*b*c**7)
```

## 3.1171 $\int (a+bx)^5(c+dx)^7\,dx$

**Optimal**. Leaf size=144

$$-\frac{5b^4(c+dx)^{12}(bc-ad)}{12d^6}+\frac{10b^3(c+dx)^{11}(bc-ad)^2}{11d^6}-\frac{b^2(c+dx)^{10}(bc-ad)^3}{d^6}+\frac{5b(c+dx)^9(bc-ad)^4}{9d^6}-\frac{(c+dx)^8(bc-}{8d^6}$$

**Rubi [A]** time = 0.36, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{5b^4(c+dx)^{12}(bc-ad)}{12d^6}+\frac{10b^3(c+dx)^{11}(bc-ad)^2}{11d^6}-\frac{b^2(c+dx)^{10}(bc-ad)^3}{d^6}+\frac{5b(c+dx)^9(bc-ad)^4}{9d^6}-\frac{(c+dx)^8(bc-ad)^5}{8d^6}+\frac{b^5(c+dx)^{13}}{13d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] -((b*c - a*d)^5*(c + d*x)^8)/(8*d^6) + (5*b*(b*c - a*d)^4*(c + d*x)^9)/(9*d
^6) - (b^2*(b*c - a*d)^3*(c + d*x)^10)/d^6 + (10*b^3*(b*c - a*d)^2*(c + d*x
)^11)/(11*d^6) - (5*b^4*(b*c - a*d)*(c + d*x)^12)/(12*d^6) + (b^5*(c + d*x)
^13)/(13*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^7\,dx=\int\left(\frac{(-bc+ad)^5(c+dx)^7}{d^5}+\frac{5b(bc-ad)^4(c+dx)^8}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^9}{d^5}+\frac{10b^3(bc\right.$$
$$=-\frac{(bc-ad)^5(c+dx)^8}{8d^6}+\frac{5b(bc-ad)^4(c+dx)^9}{9d^6}-\frac{b^2(bc-ad)^3(c+dx)^{10}}{d^6}+\frac{10b^3(bc-ad}{11d}$$

**Mathematica [B]** time = 0.08, size = 574, normalized size = 3.99

$$a^5c^7x+\frac{1}{2}a^4c^6x^2(7ad+5bc)+\frac{1}{11}b^3d^5x^{11}\left(10a^2d^2+35abcd+21b^2c^2\right)+\frac{1}{3}a^3c^5x^3\left(21a^2d^2+35abcd+10b^2c^2\right)+\frac{1}{2}b^2d^4x^{10}\left(2a^3d^3+14a^2bcd^2+21ab^2c^2d+7b^3c^3\right)+\frac{5}{4}a^2c^4x^4\left(7a^3d^3+21a^2bcd^2+14ab^2c^2d+2b^3c^3\right)+\frac{5}{9}bd^3x^9\left(a^4d^4+14a^3bcd^3+42a^2b^2c^2d^2+35ab^3c^3d+7b^4c^4\right)+ac^3x^5\left(7a^4d^4+35a^3bcd^3+42a^2b^2c^2d^2+14ab^3c^3d+b^4c^4\right)+\frac{1}{8}d^2x^8\left(a^5d^5+35a^4bcd^4+210a^3b^2c^2d^3+350a^2b^3c^3d^2+175ab^4c^4d+21b^5c^5\right)+cdx^7\left(a^5d^5+15a^4bcd^4+50a^3b^2c^2d^3+50a^2b^3c^3d^2+15ab^4c^4d+b^5c^5\right)+\frac{1}{6}c^2x^6\left(21a^5d^5+175a^4bcd^4+350a^3b^2c^2d^3+210a^2b^3c^3d^2+35ab^4c^4d+b^5c^5\right)+\frac{1}{12}b^4d^6x^{12}(5ad+7bc)+\frac{1}{13}b^5d^7x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] a^5*c^7*x + (a^4*c^6*(5*b*c + 7*a*d)*x^2)/2 + (a^3*c^5*(10*b^2*c^2 + 35*a*b
*c*d + 21*a^2*d^2)*x^3)/3 + (5*a^2*c^4*(2*b^3*c^3 + 14*a*b^2*c^2*d + 21*a^2
*b*c*d^2 + 7*a^3*d^3)*x^4)/4 + a*c^3*(b^4*c^4 + 14*a*b^3*c^3*d + 42*a^2*b^2
*c^2*d^2 + 35*a^3*b*c*d^3 + 7*a^4*d^4)*x^5 + (c^2*(b^5*c^5 + 35*a*b^4*c^4*d
 + 210*a^2*b^3*c^3*d^2 + 350*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d^4 + 21*a^5*d^5
)*x^6)/6 + c*d*(b^5*c^5 + 15*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 + 50*a^3*b^2*
c^2*d^3 + 15*a^4*b*c*d^4 + a^5*d^5)*x^7 + (d^2*(21*b^5*c^5 + 175*a*b^4*c^4*
d + 350*a^2*b^3*c^3*d^2 + 210*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 + a^5*d^5)*x
^8)/8 + (5*b*d^3*(7*b^4*c^4 + 35*a*b^3*c^3*d + 42*a^2*b^2*c^2*d^2 + 14*a^3*
b*c*d^3 + a^4*d^4)*x^9)/9 + (b^2*d^4*(7*b^3*c^3 + 21*a*b^2*c^2*d + 14*a^2*b
*c*d^2 + 2*a^3*d^3)*x^10)/2 + (b^3*d^5*(21*b^2*c^2 + 35*a*b*c*d + 10*a^2*d^
2)*x^11)/11 + (b^4*d^6*(7*b*c + 5*a*d)*x^12)/12 + (b^5*d^7*x^13)/13
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^5 (c + dx)^7 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^7, x]
```

**fricas [B]** time = 1.09, size = 670, normalized size = 4.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^7*b^5 + 7/12*x^12*d^6*c*b^5 + 5/12*x^12*d^7*b^4*a + 21/11*x^11*
d^5*c^2*b^5 + 35/11*x^11*d^6*c*b^4*a + 10/11*x^11*d^7*b^3*a^2 + 7/2*x^10*d^
4*c^3*b^5 + 21/2*x^10*d^5*c^2*b^4*a + 7*x^10*d^6*c*b^3*a^2 + x^10*d^7*b^2*a
^3 + 35/9*x^9*d^3*c^4*b^5 + 175/9*x^9*d^4*c^3*b^4*a + 70/3*x^9*d^5*c^2*b^3*
a^2 + 70/9*x^9*d^6*c*b^2*a^3 + 5/9*x^9*d^7*b*a^4 + 21/8*x^8*d^2*c^5*b^5 + 1
75/8*x^8*d^3*c^4*b^4*a + 175/4*x^8*d^4*c^3*b^3*a^2 + 105/4*x^8*d^5*c^2*b^2*
a^3 + 35/8*x^8*d^6*c*b*a^4 + 1/8*x^8*d^7*a^5 + x^7*d*c^6*b^5 + 15*x^7*d^2*c
^5*b^4*a + 50*x^7*d^3*c^4*b^3*a^2 + 50*x^7*d^4*c^3*b^2*a^3 + 15*x^7*d^5*c^2
*b*a^4 + x^7*d^6*c*a^5 + 1/6*x^6*c^7*b^5 + 35/6*x^6*d*c^6*b^4*a + 35*x^6*d^
2*c^5*b^3*a^2 + 175/3*x^6*d^3*c^4*b^2*a^3 + 175/6*x^6*d^4*c^3*b*a^4 + 7/2*x
^6*d^5*c^2*a^5 + x^5*c^7*b^4*a + 14*x^5*d*c^6*b^3*a^2 + 42*x^5*d^2*c^5*b^2*
a^3 + 35*x^5*d^3*c^4*b*a^4 + 7*x^5*d^4*c^3*a^5 + 5/2*x^4*c^7*b^3*a^2 + 35/2
*x^4*d*c^6*b^2*a^3 + 105/4*x^4*d^2*c^5*b*a^4 + 35/4*x^4*d^3*c^4*a^5 + 10/3*
```

```
x^3*c^7*b^2*a^3 + 35/3*x^3*d*c^6*b*a^4 + 7*x^3*d^2*c^5*a^5 + 5/2*x^2*c^7*b*
a^4 + 7/2*x^2*d*c^6*a^5 + x*c^7*a^5
```

**giac [B]** time = 1.28, size = 670, normalized size = 4.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/13*b^5*d^7*x^13 + 7/12*b^5*c*d^6*x^12 + 5/12*a*b^4*d^7*x^12 + 21/11*b^5*c
^2*d^5*x^11 + 35/11*a*b^4*c*d^6*x^11 + 10/11*a^2*b^3*d^7*x^11 + 7/2*b^5*c^3
*d^4*x^10 + 21/2*a*b^4*c^2*d^5*x^10 + 7*a^2*b^3*c*d^6*x^10 + a^3*b^2*d^7*x^
10 + 35/9*b^5*c^4*d^3*x^9 + 175/9*a*b^4*c^3*d^4*x^9 + 70/3*a^2*b^3*c^2*d^5*
x^9 + 70/9*a^3*b^2*c*d^6*x^9 + 5/9*a^4*b*d^7*x^9 + 21/8*b^5*c^5*d^2*x^8 + 1
75/8*a*b^4*c^4*d^3*x^8 + 175/4*a^2*b^3*c^3*d^4*x^8 + 105/4*a^3*b^2*c^2*d^5*
x^8 + 35/8*a^4*b*c*d^6*x^8 + 1/8*a^5*d^7*x^8 + b^5*c^6*d*x^7 + 15*a*b^4*c^5
*d^2*x^7 + 50*a^2*b^3*c^4*d^3*x^7 + 50*a^3*b^2*c^3*d^4*x^7 + 15*a^4*b*c^2*d
^5*x^7 + a^5*c*d^6*x^7 + 1/6*b^5*c^7*x^6 + 35/6*a*b^4*c^6*d*x^6 + 35*a^2*b^
3*c^5*d^2*x^6 + 175/3*a^3*b^2*c^4*d^3*x^6 + 175/6*a^4*b*c^3*d^4*x^6 + 7/2*a
^5*c^2*d^5*x^6 + a*b^4*c^7*x^5 + 14*a^2*b^3*c^6*d*x^5 + 42*a^3*b^2*c^5*d^2*
x^5 + 35*a^4*b*c^4*d^3*x^5 + 7*a^5*c^3*d^4*x^5 + 5/2*a^2*b^3*c^7*x^4 + 35/2
*a^3*b^2*c^6*d*x^4 + 105/4*a^4*b*c^5*d^2*x^4 + 35/4*a^5*c^4*d^3*x^4 + 10/3*
a^3*b^2*c^7*x^3 + 35/3*a^4*b*c^6*d*x^3 + 7*a^5*c^5*d^2*x^3 + 5/2*a^4*b*c^7*
x^2 + 7/2*a^5*c^6*d*x^2 + a^5*c^7*x
```

**maple [B]** time = 0.00, size = 601, normalized size = 4.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^7,x)
```

```
[Out] 1/13*b^5*d^7*x^13+1/12*(5*a*b^4*d^7+7*b^5*c*d^6)*x^12+1/11*(10*a^2*b^3*d^7+
35*a*b^4*c*d^6+21*b^5*c^2*d^5)*x^11+1/10*(10*a^3*b^2*d^7+70*a^2*b^3*c*d^6+1
05*a*b^4*c^2*d^5+35*b^5*c^3*d^4)*x^10+1/9*(5*a^4*b*d^7+70*a^3*b^2*c*d^6+210
*a^2*b^3*c^2*d^5+175*a*b^4*c^3*d^4+35*b^5*c^4*d^3)*x^9+1/8*(a^5*d^7+35*a^4*
b*c*d^6+210*a^3*b^2*c^2*d^5+350*a^2*b^3*c^3*d^4+175*a*b^4*c^4*d^3+21*b^5*c^
5*d^2)*x^8+1/7*(7*a^5*c*d^6+105*a^4*b*c^2*d^5+350*a^3*b^2*c^3*d^4+350*a^2*b
^3*c^4*d^3+105*a*b^4*c^5*d^2+7*b^5*c^6*d)*x^7+1/6*(21*a^5*c^2*d^5+175*a^4*b
*c^3*d^4+350*a^3*b^2*c^4*d^3+210*a^2*b^3*c^5*d^2+35*a*b^4*c^6*d+b^5*c^7)*x^
6+1/5*(35*a^5*c^3*d^4+175*a^4*b*c^4*d^3+210*a^3*b^2*c^5*d^2+70*a^2*b^3*c^6*
d+5*a*b^4*c^7)*x^5+1/4*(35*a^5*c^4*d^3+105*a^4*b*c^5*d^2+70*a^3*b^2*c^6*d+1
0*a^2*b^3*c^7)*x^4+1/3*(21*a^5*c^5*d^2+35*a^4*b*c^6*d+10*a^3*b^2*c^7)*x^3+1
/2*(7*a^5*c^6*d+5*a^4*b*c^7)*x^2+a^5*c^7*x
```

**maxima [B]** time = 1.38, size = 594, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/13*b^5*d^7*x^13 + a^5*c^7*x + 1/12*(7*b^5*c*d^6 + 5*a*b^4*d^7)*x^12 + 1/1
1*(21*b^5*c^2*d^5 + 35*a*b^4*c*d^6 + 10*a^2*b^3*d^7)*x^11 + 1/2*(7*b^5*c^3*
d^4 + 21*a*b^4*c^2*d^5 + 14*a^2*b^3*c*d^6 + 2*a^3*b^2*d^7)*x^10 + 5/9*(7*b^
5*c^4*d^3 + 35*a*b^4*c^3*d^4 + 42*a^2*b^3*c^2*d^5 + 14*a^3*b^2*c*d^6 + a^4*
b*d^7)*x^9 + 1/8*(21*b^5*c^5*d^2 + 175*a*b^4*c^4*d^3 + 350*a^2*b^3*c^3*d^4
+ 210*a^3*b^2*c^2*d^5 + 35*a^4*b*c*d^6 + a^5*d^7)*x^8 + (b^5*c^6*d + 15*a*b
^4*c^5*d^2 + 50*a^2*b^3*c^4*d^3 + 50*a^3*b^2*c^3*d^4 + 15*a^4*b*c^2*d^5 + a
^5*c*d^6)*x^7 + 1/6*(b^5*c^7 + 35*a*b^4*c^6*d + 210*a^2*b^3*c^5*d^2 + 350*a
^3*b^2*c^4*d^3 + 175*a^4*b*c^3*d^4 + 21*a^5*c^2*d^5)*x^6 + (a*b^4*c^7 + 14*
a^2*b^3*c^6*d + 42*a^3*b^2*c^5*d^2 + 35*a^4*b*c^4*d^3 + 7*a^5*c^3*d^4)*x^5
+ 5/4*(2*a^2*b^3*c^7 + 14*a^3*b^2*c^6*d + 21*a^4*b*c^5*d^2 + 7*a^5*c^4*d^3)
*x^4 + 1/3*(10*a^3*b^2*c^7 + 35*a^4*b*c^6*d + 21*a^5*c^5*d^2)*x^3 + 1/2*(5*
a^4*b*c^7 + 7*a^5*c^6*d)*x^2
```

**mupad [B]** time = 0.21, size = 570, normalized size = 3.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^7,x)
```

```
[Out] x^7*(a^5*c*d^6 + b^5*c^6*d + 15*a*b^4*c^5*d^2 + 15*a^4*b*c^2*d^5 + 50*a^2*b
^3*c^4*d^3 + 50*a^3*b^2*c^3*d^4) + x^6*((b^5*c^7)/6 + (7*a^5*c^2*d^5)/2 + (
175*a^4*b*c^3*d^4)/6 + 35*a^2*b^3*c^5*d^2 + (175*a^3*b^2*c^4*d^3)/3 + (35*a
*b^4*c^6*d)/6) + x^8*((a^5*d^7)/8 + (21*b^5*c^5*d^2)/8 + (175*a*b^4*c^4*d^3
)/8 + (175*a^2*b^3*c^3*d^4)/4 + (105*a^3*b^2*c^2*d^5)/4 + (35*a^4*b*c*d^6)/
8) + x^5*(a*b^4*c^7 + 7*a^5*c^3*d^4 + 14*a^2*b^3*c^6*d + 35*a^4*b*c^4*d^3 +
 42*a^3*b^2*c^5*d^2) + x^9*((5*a^4*b*d^7)/9 + (35*b^5*c^4*d^3)/9 + (175*a*b
^4*c^3*d^4)/9 + (70*a^3*b^2*c*d^6)/9 + (70*a^2*b^3*c^2*d^5)/3) + a^5*c^7*x
+ (b^5*d^7*x^13)/13 + (5*a^2*c^4*x^4*(7*a^3*d^3 + 2*b^3*c^3 + 14*a*b^2*c^2*
d + 21*a^2*b*c*d^2))/4 + (b^2*d^4*x^10*(2*a^3*d^3 + 7*b^3*c^3 + 21*a*b^2*c^
2*d + 14*a^2*b*c*d^2))/2 + (a^4*c^6*x^2*(7*a*d + 5*b*c))/2 + (b^4*d^6*x^12*
(5*a*d + 7*b*c))/12 + (a^3*c^5*x^3*(21*a^2*d^2 + 10*b^2*c^2 + 35*a*b*c*d))/
3 + (b^3*d^5*x^11*(10*a^2*d^2 + 21*b^2*c^2 + 35*a*b*c*d))/11
```

**sympy [B]** time = 0.16, size = 673, normalized size = 4.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**7,x)
```

```
[Out] a**5*c**7*x + b**5*d**7*x**13/13 + x**12*(5*a*b**4*d**7/12 + 7*b**5*c*d**6/
12) + x**11*(10*a**2*b**3*d**7/11 + 35*a*b**4*c*d**6/11 + 21*b**5*c**2*d**5
/11) + x**10*(a**3*b**2*d**7 + 7*a**2*b**3*c*d**6 + 21*a*b**4*c**2*d**5/2 +
 7*b**5*c**3*d**4/2) + x**9*(5*a**4*b*d**7/9 + 70*a**3*b**2*c*d**6/9 + 70*a
**2*b**3*c**2*d**5/3 + 175*a*b**4*c**3*d**4/9 + 35*b**5*c**4*d**3/9) + x**8
*(a**5*d**7/8 + 35*a**4*b*c*d**6/8 + 105*a**3*b**2*c**2*d**5/4 + 175*a**2*b
**3*c**3*d**4/4 + 175*a*b**4*c**4*d**3/8 + 21*b**5*c**5*d**2/8) + x**7*(a**
5*c*d**6 + 15*a**4*b*c**2*d**5 + 50*a**3*b**2*c**3*d**4 + 50*a**2*b**3*c**4
*d**3 + 15*a*b**4*c**5*d**2 + b**5*c**6*d) + x**6*(7*a**5*c**2*d**5/2 + 175
*a**4*b*c**3*d**4/6 + 175*a**3*b**2*c**4*d**3/3 + 35*a**2*b**3*c**5*d**2 +
35*a*b**4*c**6*d/6 + b**5*c**7/6) + x**5*(7*a**5*c**3*d**4 + 35*a**4*b*c**4
*d**3 + 42*a**3*b**2*c**5*d**2 + 14*a**2*b**3*c**6*d + a*b**4*c**7) + x**4*
(35*a**5*c**4*d**3/4 + 105*a**4*b*c**5*d**2/4 + 35*a**3*b**2*c**6*d/2 + 5*a
**2*b**3*c**7/2) + x**3*(7*a**5*c**5*d**2 + 35*a**4*b*c**6*d/3 + 10*a**3*b*
*2*c**7/3) + x**2*(7*a**5*c**6*d/2 + 5*a**4*b*c**7/2)
```

## 3.1172 $\int (a+bx)^4 (c+dx)^7 \, dx$

**Optimal**. Leaf size=119

$$-\frac{4b^3(c+dx)^{11}(bc-ad)}{11d^5}+\frac{3b^2(c+dx)^{10}(bc-ad)^2}{5d^5}-\frac{4b(c+dx)^9(bc-ad)^3}{9d^5}+\frac{(c+dx)^8(bc-ad)^4}{8d^5}+\frac{b^4(c+dx)^{12}}{12d^5}$$

**Rubi [A]** time = 0.28, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{4b^3(c+dx)^{11}(bc-ad)}{11d^5}+\frac{3b^2(c+dx)^{10}(bc-ad)^2}{5d^5}-\frac{4b(c+dx)^9(bc-ad)^3}{9d^5}+\frac{(c+dx)^8(bc-ad)^4}{8d^5}+\frac{b^4(c+dx)^{12}}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^4*(c + d*x)^8)/(8*d^5) - (4*b*(b*c - a*d)^3*(c + d*x)^9)/(9*d^
5) + (3*b^2*(b*c - a*d)^2*(c + d*x)^10)/(5*d^5) - (4*b^3*(b*c - a*d)*(c + d
*x)^11)/(11*d^5) + (b^4*(c + d*x)^12)/(12*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4(c+dx)^7\,dx = \int \left(\frac{(-bc+ad)^4(c+dx)^7}{d^4}-\frac{4b(bc-ad)^3(c+dx)^8}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^9}{d^4}-\frac{4b^3(bc\right.$$

$$=\frac{(bc-ad)^4(c+dx)^8}{8d^5}-\frac{4b(bc-ad)^3(c+dx)^9}{9d^5}+\frac{3b^2(bc-ad)^2(c+dx)^{10}}{5d^5}-\frac{4b^3(bc-ad}{11d}$$

**Mathematica [B]** time = 0.05, size = 473, normalized size = 3.97

$$a^4c^7x+\frac{1}{2}a^3c^6x^2(7ad+4bc)+\frac{1}{10}b^2d^5x^{10}\left(6a^2d^2+28abcd+21b^2c^2\right)+\frac{1}{3}a^2c^5x^3\left(21a^2d^2+28abcd+6b^2c^2\right)+\frac{1}{9}bd^4x^9\left(4a^3d^3+42a^2bcd^2+84ab^2c^2d+35b^3c^3\right)+\frac{1}{4}ac^4x^4\left(35a^3d^3+84a^2bcd^2+42ab^2c^2d+4b^3c^3\right)+\frac{1}{8}d^3x^8\left(a^4d^4+28a^3bcd^3+126a^2b^2c^2d^2+140ab^3c^3d+35b^4c^4\right)+cd^2x^7\left(a^4d^4+12a^3bcd^3+30a^2b^2c^2d^2+20ab^3c^3d+3b^4c^4\right)+\frac{7}{6}c^2dx^6\left(3a^4d^4+20a^3bcd^3+30a^2b^2c^2d^2+12ab^3c^3d+b^4c^4\right)+\frac{1}{5}c^3x^5\left(35a^4d^4+140a^3bcd^3+126a^2b^2c^2d^2+28ab^3c^3d+b^4c^4\right)+\frac{1}{11}b^3d^6x^{11}(4ad+7bc)+\frac{1}{12}b^4d^7x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] a^4*c^7*x + (a^3*c^6*(4*b*c + 7*a*d)*x^2)/2 + (a^2*c^5*(6*b^2*c^2 + 28*a*b*
c*d + 21*a^2*d^2)*x^3)/3 + (a*c^4*(4*b^3*c^3 + 42*a*b^2*c^2*d + 84*a^2*b*c*
d^2 + 35*a^3*d^3)*x^4)/4 + (c^3*(b^4*c^4 + 28*a*b^3*c^3*d + 126*a^2*b^2*c^2
*d^2 + 140*a^3*b*c*d^3 + 35*a^4*d^4)*x^5)/5 + (7*c^2*d*(b^4*c^4 + 12*a*b^3*
c^3*d + 30*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 + 3*a^4*d^4)*x^6)/6 + c*d^2*(3*
b^4*c^4 + 20*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)*x
^7 + (d^3*(35*b^4*c^4 + 140*a*b^3*c^3*d + 126*a^2*b^2*c^2*d^2 + 28*a^3*b*c*
d^3 + a^4*d^4)*x^8)/8 + (b*d^4*(35*b^3*c^3 + 84*a*b^2*c^2*d + 42*a^2*b*c*d^
2 + 4*a^3*d^3)*x^9)/9 + (b^2*d^5*(21*b^2*c^2 + 28*a*b*c*d + 6*a^2*d^2)*x^10
)/10 + (b^3*d^6*(7*b*c + 4*a*d)*x^11)/11 + (b^4*d^7*x^12)/12
```

**IntegrateAlgebraic [F]**   time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^4(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^7, x]
```

**fricas [B]**   time = 1.31, size = 546, normalized size = 4.59



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^7*b^4 + 7/11*x^11*d^6*c*b^4 + 4/11*x^11*d^7*b^3*a + 21/10*x^10*
d^5*c^2*b^4 + 14/5*x^10*d^6*c*b^3*a + 3/5*x^10*d^7*b^2*a^2 + 35/9*x^9*d^4*c
^3*b^4 + 28/3*x^9*d^5*c^2*b^3*a + 14/3*x^9*d^6*c*b^2*a^2 + 4/9*x^9*d^7*b*a^
3 + 35/8*x^8*d^3*c^4*b^4 + 35/2*x^8*d^4*c^3*b^3*a + 63/4*x^8*d^5*c^2*b^2*a^
2 + 7/2*x^8*d^6*c*b*a^3 + 1/8*x^8*d^7*a^4 + 3*x^7*d^2*c^5*b^4 + 20*x^7*d^3*
c^4*b^3*a + 30*x^7*d^4*c^3*b^2*a^2 + 12*x^7*d^5*c^2*b*a^3 + x^7*d^6*c*a^4 +
 7/6*x^6*d*c^6*b^4 + 14*x^6*d^2*c^5*b^3*a + 35*x^6*d^3*c^4*b^2*a^2 + 70/3*x
^6*d^4*c^3*b*a^3 + 7/2*x^6*d^5*c^2*a^4 + 1/5*x^5*c^7*b^4 + 28/5*x^5*d*c^6*b
^3*a + 126/5*x^5*d^2*c^5*b^2*a^2 + 28*x^5*d^3*c^4*b*a^3 + 7*x^5*d^4*c^3*a^4
 + x^4*c^7*b^3*a + 21/2*x^4*d*c^6*b^2*a^2 + 21*x^4*d^2*c^5*b*a^3 + 35/4*x^4
*d^3*c^4*a^4 + 2*x^3*c^7*b^2*a^2 + 28/3*x^3*d*c^6*b*a^3 + 7*x^3*d^2*c^5*a^4
 + 2*x^2*c^7*b*a^3 + 7/2*x^2*d*c^6*a^4 + x*c^7*a^4
```

**giac [B]**   time = 1.21, size = 546, normalized size = 4.59



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/12*b^4*d^7*x^12 + 7/11*b^4*c*d^6*x^11 + 4/11*a*b^3*d^7*x^11 + 21/10*b^4*c
^2*d^5*x^10 + 14/5*a*b^3*c*d^6*x^10 + 3/5*a^2*b^2*d^7*x^10 + 35/9*b^4*c^3*d
^4*x^9 + 28/3*a*b^3*c^2*d^5*x^9 + 14/3*a^2*b^2*c*d^6*x^9 + 4/9*a^3*b*d^7*x^
9 + 35/8*b^4*c^4*d^3*x^8 + 35/2*a*b^3*c^3*d^4*x^8 + 63/4*a^2*b^2*c^2*d^5*x^
8 + 7/2*a^3*b*c*d^6*x^8 + 1/8*a^4*d^7*x^8 + 3*b^4*c^5*d^2*x^7 + 20*a*b^3*c^
4*d^3*x^7 + 30*a^2*b^2*c^3*d^4*x^7 + 12*a^3*b*c^2*d^5*x^7 + a^4*c*d^6*x^7 +
 7/6*b^4*c^6*d*x^6 + 14*a*b^3*c^5*d^2*x^6 + 35*a^2*b^2*c^4*d^3*x^6 + 70/3*a
^3*b*c^3*d^4*x^6 + 7/2*a^4*c^2*d^5*x^6 + 1/5*b^4*c^7*x^5 + 28/5*a*b^3*c^6*d
*x^5 + 126/5*a^2*b^2*c^5*d^2*x^5 + 28*a^3*b*c^4*d^3*x^5 + 7*a^4*c^3*d^4*x^5
 + a*b^3*c^7*x^4 + 21/2*a^2*b^2*c^6*d*x^4 + 21*a^3*b*c^5*d^2*x^4 + 35/4*a^4
*c^4*d^3*x^4 + 2*a^2*b^2*c^7*x^3 + 28/3*a^3*b*c^6*d*x^3 + 7*a^4*c^5*d^2*x^3
 + 2*a^3*b*c^7*x^2 + 7/2*a^4*c^6*d*x^2 + a^4*c^7*x
```

<u>**maple**</u> [**B**] time = 0.00, size = 493, normalized size = 4.14

$$\frac{b^4d^7x^{12}}{12}+a^4c^7x+\frac{\left(4ab^3d^7+7b^4cd^6\right)x^{11}}{11}+\frac{\left(6a^2b^2d^7+28ab^3cd^6+21b^4c^2d^5\right)x^{10}}{10}+\frac{\left(4a^3bd^7+42a^2b^2cd^6+84ab^3c^2d^5+35b^4c^3d^4\right)x^9}{9}+\frac{\left(a^4d^7+28a^3bcd^6+126a^2b^2c^2d^5+140ab^3c^3d^4+35b^4c^4d^3\right)x^8}{8}+\frac{\left(7a^4cd^6+84a^3bc^2d^5+210a^2b^2c^3d^4+140ab^3c^4d^3+21b^4c^5d^2\right)x^7}{7}+\frac{\left(21a^4c^2d^5+140a^3bc^3d^4+210a^2b^2c^4d^3+84ab^3c^5d^2+7b^4c^6d\right)x^6}{6}+\frac{\left(35a^4c^3d^4+140a^3bc^4d^3+126a^2b^2c^5d^2+28ab^3c^6d+b^4c^7\right)x^5}{5}+\frac{\left(35a^4c^4d^3+84a^3bc^5d^2+42a^2b^2c^6d+4ab^3c^7\right)x^4}{4}+\frac{\left(21a^4c^5d^2+28a^3bc^6d+6a^2b^2c^7\right)x^3}{3}+\frac{\left(7a^4c^6d+4a^3bc^7\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^7,x)
```

```
[Out] 1/12*b^4*d^7*x^12+1/11*(4*a*b^3*d^7+7*b^4*c*d^6)*x^11+1/10*(6*a^2*b^2*d^7+2
8*a*b^3*c*d^6+21*b^4*c^2*d^5)*x^10+1/9*(4*a^3*b*d^7+42*a^2*b^2*c*d^6+84*a*b
^3*c^2*d^5+35*b^4*c^3*d^4)*x^9+1/8*(a^4*d^7+28*a^3*b*c*d^6+126*a^2*b^2*c^2*
d^5+140*a*b^3*c^3*d^4+35*b^4*c^4*d^3)*x^8+1/7*(7*a^4*c*d^6+84*a^3*b*c^2*d^5
+210*a^2*b^2*c^3*d^4+140*a*b^3*c^4*d^3+21*b^4*c^5*d^2)*x^7+1/6*(21*a^4*c^2*
d^5+140*a^3*b*c^3*d^4+210*a^2*b^2*c^4*d^3+84*a*b^3*c^5*d^2+7*b^4*c^6*d)*x^6
+1/5*(35*a^4*c^3*d^4+140*a^3*b*c^4*d^3+126*a^2*b^2*c^5*d^2+28*a*b^3*c^6*d+b
^4*c^7)*x^5+1/4*(35*a^4*c^4*d^3+84*a^3*b*c^5*d^2+42*a^2*b^2*c^6*d+4*a*b^3*c
^7)*x^4+1/3*(21*a^4*c^5*d^2+28*a^3*b*c^6*d+6*a^2*b^2*c^7)*x^3+1/2*(7*a^4*c^
6*d+4*a^3*b*c^7)*x^2+a^4*c^7*x
```

<u>**maxima**</u> [**B**] time = 1.48, size = 489, normalized size = 4.11

$$\frac{1}{12}\,b^4d^7x^{12}+a^4c^7x+\frac{1}{11}\left(7\,b^4cd^6+4\,ab^3d^7\right)x^{11}+\frac{1}{10}\left(21\,b^4c^2d^5+28\,ab^3cd^6+6\,a^2b^2d^7\right)x^{10}+\frac{1}{9}\left(35\,b^4c^3d^4+84\,ab^3c^2d^5+42\,a^2b^2cd^6+4\,a^3bd^7\right)x^9+\frac{1}{8}\left(35\,b^4c^4d^3+140\,ab^3c^3d^4+126\,a^2b^2c^2d^5+28\,a^3bcd^6+a^4d^7\right)x^8+\left(3\,b^4c^5d^2+20\,ab^3c^4d^3+30\,a^2b^2c^3d^4+12\,a^3bc^2d^5+a^4cd^6\right)x^7+\frac{7}{6}\left(b^4c^6d+12\,ab^3c^5d^2+30\,a^2b^2c^4d^3+20\,a^3bc^3d^4+3\,a^4c^2d^5\right)x^6+\frac{1}{5}\left(b^4c^7+28\,ab^3c^6d+126\,a^2b^2c^5d^2+140\,a^3bc^4d^3+35\,a^4c^3d^4\right)x^5+\frac{1}{4}\left(4\,ab^3c^7+42\,a^2b^2c^6d+84\,a^3bc^5d^2+35\,a^4c^4d^3\right)x^4+\frac{1}{3}\left(6\,a^2b^2c^7+28\,a^3bc^6d+21\,a^4c^5d^2\right)x^3+\frac{1}{2}\left(4\,a^3bc^7+7\,a^4c^6d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*d^7*x^12 + a^4*c^7*x + 1/11*(7*b^4*c*d^6 + 4*a*b^3*d^7)*x^11 + 1/1
0*(21*b^4*c^2*d^5 + 28*a*b^3*c*d^6 + 6*a^2*b^2*d^7)*x^10 + 1/9*(35*b^4*c^3*
d^4 + 84*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 + 4*a^3*b*d^7)*x^9 + 1/8*(35*b^4*
c^4*d^3 + 140*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^
7)*x^8 + (3*b^4*c^5*d^2 + 20*a*b^3*c^4*d^3 + 30*a^2*b^2*c^3*d^4 + 12*a^3*b*
```

```
c^2*d^5 + a^4*c*d^6)*x^7 + 7/6*(b^4*c^6*d + 12*a*b^3*c^5*d^2 + 30*a^2*b^2*c
^4*d^3 + 20*a^3*b*c^3*d^4 + 3*a^4*c^2*d^5)*x^6 + 1/5*(b^4*c^7 + 28*a*b^3*c^
6*d + 126*a^2*b^2*c^5*d^2 + 140*a^3*b*c^4*d^3 + 35*a^4*c^3*d^4)*x^5 + 1/4*(
4*a*b^3*c^7 + 42*a^2*b^2*c^6*d + 84*a^3*b*c^5*d^2 + 35*a^4*c^4*d^3)*x^4 + 1
/3*(6*a^2*b^2*c^7 + 28*a^3*b*c^6*d + 21*a^4*c^5*d^2)*x^3 + 1/2*(4*a^3*b*c^7
 + 7*a^4*c^6*d)*x^2
```

**mupad [B]** time = 0.31, size = 470, normalized size = 3.95

$$x^5\left(7a^4c^3d^4+28a^3bc^4d^3+\frac{126a^2b^2c^5d^2}{5}+\frac{28ab^3c^6d}{5}+\frac{b^4c^7}{5}\right)+x^8\left(\frac{a^4d^7}{8}+\frac{7a^3bcd^6}{2}+\frac{63a^2b^2c^2d^5}{4}+\frac{35ab^3c^3d^4}{2}+\frac{35b^4c^4d^3}{8}\right)+x^4\left(\frac{35a^4c^4d^3}{4}+21a^3bc^5d^2+\frac{21a^2b^2c^6d}{2}+ab^3c^7\right)+x^9\left(\frac{4a^3bd^7}{9}+\frac{14a^2b^2cd^6}{3}+\frac{28ab^3c^2d^5}{3}+\frac{35b^4c^3d^4}{9}\right)+x^7\left(a^4cd^6+12a^3bc^2d^5+30a^2b^2c^3d^4+20ab^3c^4d^3+3b^4c^5d^2\right)+x^6\left(\frac{7a^4c^2d^5}{2}+\frac{70a^3bc^3d^4}{3}+35a^2b^2c^4d^3+14ab^3c^5d^2+\frac{7b^4c^6d}{6}\right)+a^4c^7x+\frac{b^4d^7x^{12}}{12}+\frac{a^3c^6x^2(7ad+4bc)}{2}+\frac{b^3d^6x^{11}(4ad+7bc)}{11}+\frac{a^2c^5x^3(21a^2d^2+28abcd+6b^2c^2)}{3}+\frac{b^2d^5x^{10}(6a^2d^2+28abcd+21b^2c^2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^7,x)
```

```
[Out] x^5*((b^4*c^7)/5 + 7*a^4*c^3*d^4 + 28*a^3*b*c^4*d^3 + (126*a^2*b^2*c^5*d^2)
/5 + (28*a*b^3*c^6*d)/5) + x^8*((a^4*d^7)/8 + (35*b^4*c^4*d^3)/8 + (35*a*b^
3*c^3*d^4)/2 + (63*a^2*b^2*c^2*d^5)/4 + (7*a^3*b*c*d^6)/2) + x^4*(a*b^3*c^7
 + (35*a^4*c^4*d^3)/4 + (21*a^2*b^2*c^6*d)/2 + 21*a^3*b*c^5*d^2) + x^9*((4*
a^3*b*d^7)/9 + (35*b^4*c^3*d^4)/9 + (28*a*b^3*c^2*d^5)/3 + (14*a^2*b^2*c*d^
6)/3) + x^7*(a^4*c*d^6 + 3*b^4*c^5*d^2 + 20*a*b^3*c^4*d^3 + 12*a^3*b*c^2*d^
5 + 30*a^2*b^2*c^3*d^4) + x^6*((7*b^4*c^6*d)/6 + (7*a^4*c^2*d^5)/2 + 14*a*b
^3*c^5*d^2 + (70*a^3*b*c^3*d^4)/3 + 35*a^2*b^2*c^4*d^3) + a^4*c^7*x + (b^4*
d^7*x^12)/12 + (a^3*c^6*x^2*(7*a*d + 4*b*c))/2 + (b^3*d^6*x^11*(4*a*d + 7*b
*c))/11 + (a^2*c^5*x^3*(21*a^2*d^2 + 6*b^2*c^2 + 28*a*b*c*d))/3 + (b^2*d^5*
x^10*(6*a^2*d^2 + 21*b^2*c^2 + 28*a*b*c*d))/10
```

**sympy [B]** time = 0.15, size = 549, normalized size = 4.61

$$a^4c^7x+\frac{b^4d^7x^{12}}{12}+x^{11}\left(\frac{4ab^3d^7}{11}+\frac{7b^4cd^6}{11}\right)+x^{10}\left(\frac{3a^2b^2d^7}{5}+\frac{14ab^3cd^6}{5}+\frac{21b^4c^2d^5}{10}\right)+x^9\left(\frac{4a^3bd^7}{9}+\frac{14a^2b^2cd^6}{3}+\frac{28ab^3c^2d^5}{3}+\frac{35b^4c^3d^4}{9}\right)+x^8\left(\frac{a^4d^7}{8}+\frac{7a^3bcd^6}{2}+\frac{63a^2b^2c^2d^5}{4}+\frac{35ab^3c^3d^4}{2}+\frac{35b^4c^4d^3}{8}\right)+x^7\left(a^4cd^6+12a^3bc^2d^5+30a^2b^2c^3d^4+20ab^3c^4d^3+3b^4c^5d^2\right)+x^6\left(\frac{7a^4c^2d^5}{2}+\frac{70a^3bc^3d^4}{3}+35a^2b^2c^4d^3+14ab^3c^5d^2+\frac{7b^4c^6d}{6}\right)+x^5\left(7a^4c^3d^4+28a^3bc^4d^3+\frac{126a^2b^2c^5d^2}{5}+\frac{28ab^3c^6d}{5}+\frac{b^4c^7}{5}\right)+x^4\left(\frac{35a^4c^4d^3}{4}+21a^3bc^5d^2+\frac{21a^2b^2c^6d}{2}+ab^3c^7\right)+x^3\left(7a^4c^5d^2+\frac{28a^3bc^6d}{3}+2a^2b^2c^7\right)+x^2\left(\frac{7a^4c^6d}{2}+2a^3bc^7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**7,x)
```

```
[Out] a**4*c**7*x + b**4*d**7*x**12/12 + x**11*(4*a*b**3*d**7/11 + 7*b**4*c*d**6/
11) + x**10*(3*a**2*b**2*d**7/5 + 14*a*b**3*c*d**6/5 + 21*b**4*c**2*d**5/10
) + x**9*(4*a**3*b*d**7/9 + 14*a**2*b**2*c*d**6/3 + 28*a*b**3*c**2*d**5/3 +
 35*b**4*c**3*d**4/9) + x**8*(a**4*d**7/8 + 7*a**3*b*c*d**6/2 + 63*a**2*b**
2*c**2*d**5/4 + 35*a*b**3*c**3*d**4/2 + 35*b**4*c**4*d**3/8) + x**7*(a**4*c
*d**6 + 12*a**3*b*c**2*d**5 + 30*a**2*b**2*c**3*d**4 + 20*a*b**3*c**4*d**3
+ 3*b**4*c**5*d**2) + x**6*(7*a**4*c**2*d**5/2 + 70*a**3*b*c**3*d**4/3 + 35
*a**2*b**2*c**4*d**3 + 14*a*b**3*c**5*d**2 + 7*b**4*c**6*d/6) + x**5*(7*a**
4*c**3*d**4 + 28*a**3*b*c**4*d**3 + 126*a**2*b**2*c**5*d**2/5 + 28*a*b**3*c
**6*d/5 + b**4*c**7/5) + x**4*(35*a**4*c**4*d**3/4 + 21*a**3*b*c**5*d**2 +
21*a**2*b**2*c**6*d/2 + a*b**3*c**7) + x**3*(7*a**4*c**5*d**2 + 28*a**3*b*c
**6*d/3 + 2*a**2*b**2*c**7) + x**2*(7*a**4*c**6*d/2 + 2*a**3*b*c**7)
```

## 3.1173 $\int (a+bx)^3(c+dx)^7\,dx$

**Optimal**. Leaf size=92

$$-\frac{3b^2(c+dx)^{10}(bc-ad)}{10d^4}+\frac{b(c+dx)^9(bc-ad)^2}{3d^4}-\frac{(c+dx)^8(bc-ad)^3}{8d^4}+\frac{b^3(c+dx)^{11}}{11d^4}$$

**Rubi [A]** time = 0.22, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^2(c+dx)^{10}(bc-ad)}{10d^4}+\frac{b(c+dx)^9(bc-ad)^2}{3d^4}-\frac{(c+dx)^8(bc-ad)^3}{8d^4}+\frac{b^3(c+dx)^{11}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] -((b*c - a*d)^3*(c + d*x)^8)/(8*d^4) + (b*(b*c - a*d)^2*(c + d*x)^9)/(3*d^4
) - (3*b^2*(b*c - a*d)*(c + d*x)^10)/(10*d^4) + (b^3*(c + d*x)^11)/(11*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^7}{d^3}+\frac{3b(bc-ad)^2(c+dx)^8}{d^3}-\frac{3b^2(bc-ad)(c+dx)^9}{d^3}+\frac{b^3(c+d}{d^3}\right.\\ &= -\frac{(bc-ad)^3(c+dx)^8}{8d^4}+\frac{b(bc-ad)^2(c+dx)^9}{3d^4}-\frac{3b^2(bc-ad)(c+dx)^{10}}{10d^4}+\frac{b^3(c+dx)^{11}}{11d^4}\end{aligned}$$

**Mathematica [B]** time = 0.04, size = 360, normalized size = 3.91

$$a^3c^7x+\frac{1}{3}bd^5x^9\left(a^2d^2+7abcd+7b^2c^2\right)+ac^5x^3\left(7a^2d^2+7abcd+b^2c^2\right)+\frac{1}{2}a^2c^6x^2(7ad+3bc)+cd^3x^7\left(a^3d^3+9a^2bcd^2+15ab^2c^2d+5b^3c^3\right)+\frac{7}{2}c^2d^2x^6\left(a^3d^3+5a^2bcd^2+5ab^2c^2d+b^3c^3\right)+\frac{7}{5}c^3dx^5\left(5a^3d^3+15a^2bcd^2+9ab^2c^2d+b^3c^3\right)+\frac{1}{8}d^4x^8\left(a^3d^3+21a^2bcd^2+63ab^2c^2d+35b^3c^3\right)+\frac{1}{4}c^4x^4\left(35a^3d^3+63a^2bcd^2+21ab^2c^2d+b^3c^3\right)+\frac{1}{10}b^2d^6x^{10}(3ad+7bc)+\frac{1}{11}b^3d^7x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] a^3*c^7*x + (a^2*c^6*(3*b*c + 7*a*d)*x^2)/2 + a*c^5*(b^2*c^2 + 7*a*b*c*d +
7*a^2*d^2)*x^3 + (c^4*(b^3*c^3 + 21*a*b^2*c^2*d + 63*a^2*b*c*d^2 + 35*a^3*d
^3)*x^4)/4 + (7*c^3*d*(b^3*c^3 + 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 5*a^3*d^3
)*x^5)/5 + (7*c^2*d^2*(b^3*c^3 + 5*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*d^3)*x
^6)/2 + c*d^3*(5*b^3*c^3 + 15*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^7 +
(d^4*(35*b^3*c^3 + 63*a*b^2*c^2*d + 21*a^2*b*c*d^2 + a^3*d^3)*x^8)/8 + (b*d
^5*(7*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*x^9)/3 + (b^2*d^6*(7*b*c + 3*a*d)*x^10
)/10 + (b^3*d^7*x^11)/11
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^7, x]
```

**fricas [B]** time = 1.25, size = 420, normalized size = 4.57

$$\frac{1}{11}x^{11}d^7b^3+\frac{7}{10}x^{10}d^6cb^3+\frac{3}{10}x^{10}d^7b^2a+\frac{7}{3}x^9d^5c^2b^3+\frac{7}{3}x^9d^6cb^2a+\frac{1}{3}x^9d^7ba^2+\frac{35}{8}x^8d^4c^3b^3+\frac{63}{8}x^8d^5c^2b^2a+\frac{21}{8}x^8d^6cba^2+\frac{1}{8}x^8d^7a^3+5x^7d^3c^4b^3+15x^7d^4c^3b^2a+9x^7d^5c^2ba^2+x^7d^6ca^3+\frac{7}{2}x^6d^2c^5b^3+\frac{35}{2}x^6d^3c^4b^2a+\frac{35}{2}x^6d^4c^3ba^2+\frac{7}{2}x^6d^5c^2a^3+\frac{7}{5}x^5dc^6b^3+\frac{63}{5}x^5d^2c^5b^2a+21x^5d^3c^4ba^2+7x^5d^4c^3a^3+\frac{1}{4}x^4c^7b^3+\frac{21}{4}x^4dc^6b^2a+\frac{63}{4}x^4d^2c^5ba^2+\frac{35}{4}x^4d^3c^4a^3+x^3c^7b^2a+7x^3dc^6ba^2+7x^3d^2c^5a^3+\frac{3}{2}x^2c^7ba^2+\frac{7}{2}x^2dc^6a^3+xc^7a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^7*b^3 + 7/10*x^10*d^6*c*b^3 + 3/10*x^10*d^7*b^2*a + 7/3*x^9*d^5
*c^2*b^3 + 7/3*x^9*d^6*c*b^2*a + 1/3*x^9*d^7*b*a^2 + 35/8*x^8*d^4*c^3*b^3 +
 63/8*x^8*d^5*c^2*b^2*a + 21/8*x^8*d^6*c*b*a^2 + 1/8*x^8*d^7*a^3 + 5*x^7*d^
3*c^4*b^3 + 15*x^7*d^4*c^3*b^2*a + 9*x^7*d^5*c^2*b*a^2 + x^7*d^6*c*a^3 + 7/
2*x^6*d^2*c^5*b^3 + 35/2*x^6*d^3*c^4*b^2*a + 35/2*x^6*d^4*c^3*b*a^2 + 7/2*x
^6*d^5*c^2*a^3 + 7/5*x^5*d*c^6*b^3 + 63/5*x^5*d^2*c^5*b^2*a + 21*x^5*d^3*c^
4*b*a^2 + 7*x^5*d^4*c^3*a^3 + 1/4*x^4*c^7*b^3 + 21/4*x^4*d*c^6*b^2*a + 63/4
*x^4*d^2*c^5*b*a^2 + 35/4*x^4*d^3*c^4*a^3 + x^3*c^7*b^2*a + 7*x^3*d*c^6*b*a
^2 + 7*x^3*d^2*c^5*a^3 + 3/2*x^2*c^7*b*a^2 + 7/2*x^2*d*c^6*a^3 + x*c^7*a^3
```

**giac [B]** time = 1.26, size = 420, normalized size = 4.57

$$\frac{1}{11}b^3d^7x^{11}+\frac{7}{10}b^3cd^6x^{10}+\frac{3}{10}ab^2d^7x^{10}+\frac{7}{3}b^3c^2d^5x^9+\frac{7}{3}ab^2cd^6x^9+\frac{1}{3}a^2bd^7x^9+\frac{35}{8}b^3c^3d^4x^8+\frac{63}{8}ab^2c^2d^5x^8+\frac{21}{8}a^2bcd^6x^8+\frac{1}{8}a^3d^7x^8+5\,b^3c^4d^3x^7+15\,ab^2c^3d^4x^7+9\,a^2bc^2d^5x^7+a^3cd^6x^7+\frac{7}{2}b^3c^5d^2x^6+\frac{35}{2}ab^2c^4d^3x^6+\frac{35}{2}a^2bc^3d^4x^6+\frac{7}{2}a^3c^2d^5x^6+\frac{7}{5}b^3c^6dx^5+\frac{63}{5}ab^2c^5d^2x^5+21\,a^2bc^4d^3x^5+7\,a^3c^3d^4x^5+\frac{1}{4}b^3c^7x^4+\frac{21}{4}ab^2c^6dx^4+\frac{63}{4}a^2bc^5d^2x^4+\frac{35}{4}a^3c^4d^3x^4+ab^2c^7x^3+7\,a^2bc^6dx^3+7\,a^3c^5d^2x^3+\frac{3}{2}a^2bc^7x^2+\frac{7}{2}a^3c^6dx^2+a^3c^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/11*b^3*d^7*x^11 + 7/10*b^3*c*d^6*x^10 + 3/10*a*b^2*d^7*x^10 + 7/3*b^3*c^2
*d^5*x^9 + 7/3*a*b^2*c*d^6*x^9 + 1/3*a^2*b*d^7*x^9 + 35/8*b^3*c^3*d^4*x^8 +
```

```
 63/8*a*b^2*c^2*d^5*x^8 + 21/8*a^2*b*c*d^6*x^8 + 1/8*a^3*d^7*x^8 + 5*b^3*c^
4*d^3*x^7 + 15*a*b^2*c^3*d^4*x^7 + 9*a^2*b*c^2*d^5*x^7 + a^3*c*d^6*x^7 + 7/
2*b^3*c^5*d^2*x^6 + 35/2*a*b^2*c^4*d^3*x^6 + 35/2*a^2*b*c^3*d^4*x^6 + 7/2*a
^3*c^2*d^5*x^6 + 7/5*b^3*c^6*d*x^5 + 63/5*a*b^2*c^5*d^2*x^5 + 21*a^2*b*c^4*
d^3*x^5 + 7*a^3*c^3*d^4*x^5 + 1/4*b^3*c^7*x^4 + 21/4*a*b^2*c^6*d*x^4 + 63/4
*a^2*b*c^5*d^2*x^4 + 35/4*a^3*c^4*d^3*x^4 + a*b^2*c^7*x^3 + 7*a^2*b*c^6*d*x
^3 + 7*a^3*c^5*d^2*x^3 + 3/2*a^2*b*c^7*x^2 + 7/2*a^3*c^6*d*x^2 + a^3*c^7*x
```

**maple [B]** time = 0.00, size = 385, normalized size = 4.18

$$\frac{b^3d^7x^{11}}{11}+a^3c^7x+\frac{\left(3ab^2d^7+7b^3cd^6\right)x^{10}}{10}+\frac{\left(3a^2bd^7+21ab^2cd^6+21b^3c^2d^5\right)x^9}{9}+\frac{\left(a^3d^7+21a^2bcd^6+63ab^2c^2d^5+35b^3c^3d^4\right)x^8}{8}+\frac{\left(7a^3cd^6+63a^2bc^2d^5+105ab^2c^3d^4+35b^3c^4d^3\right)x^7}{7}+\frac{\left(21a^3c^2d^5+105a^2bc^3d^4+105ab^2c^4d^3+21b^3c^5d^2\right)x^6}{6}+\frac{\left(35a^3c^3d^4+105a^2bc^4d^3+63ab^2c^5d^2+7b^3c^6d\right)x^5}{5}+\frac{\left(35a^3c^4d^3+63a^2bc^5d^2+21ab^2c^6d+b^3c^7\right)x^4}{4}+\frac{\left(21a^3c^5d^2+21a^2bc^6d+3ab^2c^7\right)x^3}{3}+\frac{\left(7a^3c^6d+3a^2bc^7\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^7,x)
```

```
[Out] 1/11*b^3*d^7*x^11+1/10*(3*a*b^2*d^7+7*b^3*c*d^6)*x^10+1/9*(3*a^2*b*d^7+21*a
*b^2*c*d^6+21*b^3*c^2*d^5)*x^9+1/8*(a^3*d^7+21*a^2*b*c*d^6+63*a*b^2*c^2*d^5
+35*b^3*c^3*d^4)*x^8+1/7*(7*a^3*c*d^6+63*a^2*b*c^2*d^5+105*a*b^2*c^3*d^4+35
*b^3*c^4*d^3)*x^7+1/6*(21*a^3*c^2*d^5+105*a^2*b*c^3*d^4+105*a*b^2*c^4*d^3+2
1*b^3*c^5*d^2)*x^6+1/5*(35*a^3*c^3*d^4+105*a^2*b*c^4*d^3+63*a*b^2*c^5*d^2+7
*b^3*c^6*d)*x^5+1/4*(35*a^3*c^4*d^3+63*a^2*b*c^5*d^2+21*a*b^2*c^6*d+b^3*c^7
)*x^4+1/3*(21*a^3*c^5*d^2+21*a^2*b*c^6*d+3*a*b^2*c^7)*x^3+1/2*(7*a^3*c^6*d+
3*a^2*b*c^7)*x^2+a^3*c^7*x
```

**maxima [B]** time = 1.37, size = 376, normalized size = 4.09

$$\frac{1}{11}b^3d^7x^{11}+a^3c^7x+\frac{1}{10}\left(7b^3cd^6+3ab^2d^7\right)x^{10}+\frac{1}{3}\left(7b^3c^2d^5+7ab^2cd^6+a^2bd^7\right)x^9+\frac{1}{8}\left(35b^3c^3d^4+63ab^2c^2d^5+21a^2bcd^6+a^3d^7\right)x^8+\left(5b^3c^4d^3+15ab^2c^3d^4+9a^2bc^2d^5+a^3cd^6\right)x^7+\frac{7}{2}\left(b^3c^5d^2+5ab^2c^4d^3+5a^2bc^3d^4+a^3c^2d^5\right)x^6+\frac{7}{5}\left(b^3c^6d+9ab^2c^5d^2+15a^2bc^4d^3+5a^3c^3d^4\right)x^5+\frac{1}{4}\left(b^3c^7+21ab^2c^6d+63a^2bc^5d^2+35a^3c^4d^3\right)x^4+\left(ab^2c^7+7a^2bc^6d+7a^3c^5d^2\right)x^3+\frac{1}{2}\left(3a^2bc^7+7a^3c^6d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d^7*x^11 + a^3*c^7*x + 1/10*(7*b^3*c*d^6 + 3*a*b^2*d^7)*x^10 + 1/3
*(7*b^3*c^2*d^5 + 7*a*b^2*c*d^6 + a^2*b*d^7)*x^9 + 1/8*(35*b^3*c^3*d^4 + 63
*a*b^2*c^2*d^5 + 21*a^2*b*c*d^6 + a^3*d^7)*x^8 + (5*b^3*c^4*d^3 + 15*a*b^2*
c^3*d^4 + 9*a^2*b*c^2*d^5 + a^3*c*d^6)*x^7 + 7/2*(b^3*c^5*d^2 + 5*a*b^2*c^4
*d^3 + 5*a^2*b*c^3*d^4 + a^3*c^2*d^5)*x^6 + 7/5*(b^3*c^6*d + 9*a*b^2*c^5*d^
2 + 15*a^2*b*c^4*d^3 + 5*a^3*c^3*d^4)*x^5 + 1/4*(b^3*c^7 + 21*a*b^2*c^6*d +
 63*a^2*b*c^5*d^2 + 35*a^3*c^4*d^3)*x^4 + (a*b^2*c^7 + 7*a^2*b*c^6*d + 7*a^
3*c^5*d^2)*x^3 + 1/2*(3*a^2*b*c^7 + 7*a^3*c^6*d)*x^2
```

**mupad [B]** time = 0.27, size = 356, normalized size = 3.87

$$x^7\left(a^3cd^6+9a^2bc^2d^5+15ab^2c^3d^4+5b^3c^4d^3\right)+x^5\left(7a^3c^3d^4+21a^2bc^4d^3+\frac{63ab^2c^5d^2}{5}+\frac{7b^3c^6d}{5}\right)+x^4\left(\frac{35a^3c^4d^3}{4}+\frac{63a^2bc^5d^2}{4}+\frac{21ab^2c^6d}{4}+\frac{b^3c^7}{4}\right)+x^8\left(\frac{a^3d^7}{8}+\frac{21a^2bcd^6}{8}+\frac{63ab^2c^2d^5}{8}+\frac{35b^3c^3d^4}{8}\right)+a^3c^7x+\frac{b^3d^7x^{11}}{11}+\frac{7c^2d^2x^6\left(a^3d^3+5a^2bcd^2+5ab^2c^2d+b^3c^3\right)}{2}+\frac{a^2c^6x^2\left(7ad+3bc\right)}{2}+\frac{b^2d^6x^{10}\left(3ad+7bc\right)}{10}+ac^5x^3\left(7a^2d^2+7abcd+b^2c^2\right)+\frac{bd^5x^9\left(a^2d^2+7abcd+7b^2c^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^7,x)
```

```
[Out] x^7*(a^3*c*d^6 + 5*b^3*c^4*d^3 + 15*a*b^2*c^3*d^4 + 9*a^2*b*c^2*d^5) + x^5*
((7*b^3*c^6*d)/5 + 7*a^3*c^3*d^4 + (63*a*b^2*c^5*d^2)/5 + 21*a^2*b*c^4*d^3)
 + x^4*((b^3*c^7)/4 + (35*a^3*c^4*d^3)/4 + (63*a^2*b*c^5*d^2)/4 + (21*a*b^2
*c^6*d)/4) + x^8*((a^3*d^7)/8 + (35*b^3*c^3*d^4)/8 + (63*a*b^2*c^2*d^5)/8 +
 (21*a^2*b*c*d^6)/8) + a^3*c^7*x + (b^3*d^7*x^11)/11 + (7*c^2*d^2*x^6*(a^3*
d^3 + b^3*c^3 + 5*a*b^2*c^2*d + 5*a^2*b*c*d^2))/2 + (a^2*c^6*x^2*(7*a*d + 3
*b*c))/2 + (b^2*d^6*x^10*(3*a*d + 7*b*c))/10 + a*c^5*x^3*(7*a^2*d^2 + b^2*c
^2 + 7*a*b*c*d) + (b*d^5*x^9*(a^2*d^2 + 7*b^2*c^2 + 7*a*b*c*d))/3
```

**sympy [B]** time = 0.13, size = 427, normalized size = 4.64

$$a^{3}c^{7}x + \frac{b^{3}d^{7}x^{11}}{11} + x^{10}\left(\frac{3ab^{2}d^{7}}{10} + \frac{7b^{3}cd^{6}}{10}\right) + x^{9}\left(\frac{a^{2}bd^{7}}{3} + \frac{7ab^{2}cd^{6}}{3} + \frac{7b^{3}c^{2}d^{5}}{3}\right) + x^{8}\left(\frac{a^{3}d^{7}}{8} + \frac{21a^{2}bcd^{6}}{8} + \frac{63ab^{2}c^{2}d^{5}}{8} + \frac{35b^{3}c^{3}d^{4}}{8}\right) + x^{7}\left(a^{3}cd^{6} + 9a^{2}bc^{2}d^{5} + 15ab^{2}c^{3}d^{4} + 5b^{3}c^{4}d^{3}\right) + x^{6}\left(\frac{7a^{3}c^{2}d^{5}}{2} + \frac{35a^{2}bc^{3}d^{4}}{2} + \frac{35ab^{2}c^{4}d^{3}}{2} + \frac{7b^{3}c^{5}d^{2}}{2}\right) + x^{5}\left(7a^{3}c^{3}d^{4} + 21a^{2}bc^{4}d^{3} + \frac{63ab^{2}c^{5}d^{2}}{5} + \frac{7b^{3}c^{6}d}{5}\right) + x^{4}\left(\frac{35a^{3}c^{4}d^{3}}{4} + \frac{63a^{2}bc^{5}d^{2}}{4} + \frac{21ab^{2}c^{6}d}{4} + \frac{b^{3}c^{7}}{4}\right) + x^{3}\left(7a^{3}c^{5}d^{2} + 7a^{2}bc^{6}d + ab^{2}c^{7}\right) + x^{2}\left(\frac{7a^{3}c^{6}d}{2} + \frac{3a^{2}bc^{7}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**7,x)
```

```
[Out] a**3*c**7*x + b**3*d**7*x**11/11 + x**10*(3*a*b**2*d**7/10 + 7*b**3*c*d**6/
10) + x**9*(a**2*b*d**7/3 + 7*a*b**2*c*d**6/3 + 7*b**3*c**2*d**5/3) + x**8*
(a**3*d**7/8 + 21*a**2*b*c*d**6/8 + 63*a*b**2*c**2*d**5/8 + 35*b**3*c**3*d*
*4/8) + x**7*(a**3*c*d**6 + 9*a**2*b*c**2*d**5 + 15*a*b**2*c**3*d**4 + 5*b*
*3*c**4*d**3) + x**6*(7*a**3*c**2*d**5/2 + 35*a**2*b*c**3*d**4/2 + 35*a*b**
2*c**4*d**3/2 + 7*b**3*c**5*d**2/2) + x**5*(7*a**3*c**3*d**4 + 21*a**2*b*c*
*4*d**3 + 63*a*b**2*c**5*d**2/5 + 7*b**3*c**6*d/5) + x**4*(35*a**3*c**4*d**
3/4 + 63*a**2*b*c**5*d**2/4 + 21*a*b**2*c**6*d/4 + b**3*c**7/4) + x**3*(7*a
**3*c**5*d**2 + 7*a**2*b*c**6*d + a*b**2*c**7) + x**2*(7*a**3*c**6*d/2 + 3*
a**2*b*c**7/2)
```

## 3.1174 $\int (a+bx)^2(c+dx)^7\,dx$

**Optimal**. Leaf size=65

$$-\frac{2b(c+dx)^9(bc-ad)}{9d^3}+\frac{(c+dx)^8(bc-ad)^2}{8d^3}+\frac{b^2(c+dx)^{10}}{10d^3}$$

**Rubi [A]** time = 0.16, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b(c+dx)^9(bc-ad)}{9d^3}+\frac{(c+dx)^8(bc-ad)^2}{8d^3}+\frac{b^2(c+dx)^{10}}{10d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^8)/(8*d^3) - (2*b*(b*c - a*d)*(c + d*x)^9)/(9*d^3)
 + (b^2*(c + d*x)^10)/(10*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^7}{d^2}-\frac{2b(bc-ad)(c+dx)^8}{d^2}+\frac{b^2(c+dx)^9}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^8}{8d^3}-\frac{2b(bc-ad)(c+dx)^9}{9d^3}+\frac{b^2(c+dx)^{10}}{10d^3}\end{aligned}$$

**Mathematica [B]** time = 0.03, size = 261, normalized size = 4.02

$$\frac{1}{8}d^5x^8\left(a^2d^2+14abcd+21b^2c^2\right)+cd^4x^7\left(a^2d^2+6abcd+5b^2c^2\right)+\frac{7}{6}c^2d^3x^6\left(3a^2d^2+10abcd+5b^2c^2\right)+\frac{1}{3}c^5x^3\left(21a^2d^2+14abcd+b^2c^2\right)+\frac{7}{4}c^4dx^4\left(5a^2d^2+6abcd+b^2c^2\right)+\frac{7}{5}c^3d^2x^5\left(5a^2d^2+10abcd+3b^2c^2\right)+a^2c^7x+\frac{1}{2}ac^6x^2(7ad+2bc)+\frac{1}{9}bd^6x^9(2ad+7bc)+\frac{1}{10}b^2d^7x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] a^2*c^7*x + (a*c^6*(2*b*c + 7*a*d)*x^2)/2 + (c^5*(b^2*c^2 + 14*a*b*c*d + 21
*a^2*d^2)*x^3)/3 + (7*c^4*d*(b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*x^4)/4 + (7*c
^3*d^2*(3*b^2*c^2 + 10*a*b*c*d + 5*a^2*d^2)*x^5)/5 + (7*c^2*d^3*(5*b^2*c^2
+ 10*a*b*c*d + 3*a^2*d^2)*x^6)/6 + c*d^4*(5*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
x^7 + (d^5*(21*b^2*c^2 + 14*a*b*c*d + a^2*d^2)*x^8)/8 + (b*d^6*(7*b*c + 2*a
*d)*x^9)/9 + (b^2*d^7*x^10)/10
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^7, x]
```

**fricas [B]** time = 1.21, size = 294, normalized size = 4.52

$$\frac{1}{10}x^{10}d^7b^2 + \frac{7}{9}x^9d^6cb^2 + \frac{2}{9}x^9d^7ba + \frac{21}{8}x^8d^5c^2b^2 + \frac{7}{4}x^8d^6cba + \frac{1}{8}x^8d^7a^2 + 5x^7d^4c^3b^2 + 6x^7d^5c^2ba + x^7d^6ca^2 + \frac{35}{6}x^6d^3c^4b^2 + \frac{35}{3}x^6d^4c^3ba + \frac{7}{2}x^6d^5c^2a^2 + \frac{21}{5}x^5d^2c^5b^2 + 14x^5d^3c^4ba + 7x^5d^4c^3a^2 + \frac{7}{4}x^4dc^6b^2 + \frac{21}{2}x^4d^2c^5ba + \frac{35}{4}x^4d^3c^4a^2 + \frac{1}{3}x^3c^7b^2 + \frac{14}{3}x^3dc^6ba + 7x^3d^2c^5a^2 + x^2c^7ba + \frac{7}{2}x^2dc^6a^2 + xc^7a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^7*b^2 + 7/9*x^9*d^6*c*b^2 + 2/9*x^9*d^7*b*a + 21/8*x^8*d^5*c^2*
b^2 + 7/4*x^8*d^6*c*b*a + 1/8*x^8*d^7*a^2 + 5*x^7*d^4*c^3*b^2 + 6*x^7*d^5*c
^2*b*a + x^7*d^6*c*a^2 + 35/6*x^6*d^3*c^4*b^2 + 35/3*x^6*d^4*c^3*b*a + 7/2*
x^6*d^5*c^2*a^2 + 21/5*x^5*d^2*c^5*b^2 + 14*x^5*d^3*c^4*b*a + 7*x^5*d^4*c^3
*a^2 + 7/4*x^4*d*c^6*b^2 + 21/2*x^4*d^2*c^5*b*a + 35/4*x^4*d^3*c^4*a^2 + 1/
3*x^3*c^7*b^2 + 14/3*x^3*d*c^6*b*a + 7*x^3*d^2*c^5*a^2 + x^2*c^7*b*a + 7/2*
x^2*d*c^6*a^2 + x*c^7*a^2
```

**giac [B]** time = 1.24, size = 294, normalized size = 4.52

$$\frac{1}{10}\,b^2d^7x^{10} + \frac{7}{9}\,b^2cd^6x^9 + \frac{2}{9}\,abd^7x^9 + \frac{21}{8}\,b^2c^2d^5x^8 + \frac{7}{4}\,abcd^6x^8 + \frac{1}{8}\,a^2d^7x^8 + 5\,b^2c^3d^4x^7 + 6\,abc^2d^5x^7 + a^2cd^6x^7 + \frac{35}{6}\,b^2c^4d^3x^6 + \frac{35}{3}\,abc^3d^4x^6 + \frac{7}{2}\,a^2c^2d^5x^6 + \frac{21}{5}\,b^2c^5d^2x^5 + 14\,abc^4d^3x^5 + 7\,a^2c^3d^4x^5 + \frac{7}{4}\,b^2c^6dx^4 + \frac{21}{2}\,abc^5d^2x^4 + \frac{35}{4}\,a^2c^4d^3x^4 + \frac{1}{3}\,b^2c^7x^3 + \frac{14}{3}\,abc^6dx^3 + 7\,a^2c^5d^2x^3 + abc^7x^2 + \frac{7}{2}\,a^2c^6dx^2 + a^2c^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^7*x^10 + 7/9*b^2*c*d^6*x^9 + 2/9*a*b*d^7*x^9 + 21/8*b^2*c^2*d^5*
x^8 + 7/4*a*b*c*d^6*x^8 + 1/8*a^2*d^7*x^8 + 5*b^2*c^3*d^4*x^7 + 6*a*b*c^2*d
^5*x^7 + a^2*c*d^6*x^7 + 35/6*b^2*c^4*d^3*x^6 + 35/3*a*b*c^3*d^4*x^6 + 7/2*
a^2*c^2*d^5*x^6 + 21/5*b^2*c^5*d^2*x^5 + 14*a*b*c^4*d^3*x^5 + 7*a^2*c^3*d^4
*x^5 + 7/4*b^2*c^6*d*x^4 + 21/2*a*b*c^5*d^2*x^4 + 35/4*a^2*c^4*d^3*x^4 + 1/
```

```
3*b^2*c^7*x^3 + 14/3*a*b*c^6*d*x^3 + 7*a^2*c^5*d^2*x^3 + a*b*c^7*x^2 + 7/2*
a^2*c^6*d*x^2 + a^2*c^7*x
```

**maple** [**B**] time = 0.00, size = 277, normalized size = 4.26

$$\frac{b^2d^7x^{10}}{10}+a^2c^7x+\frac{(2abd^7+7b^2cd^6)x^9}{9}+\frac{(a^2d^7+14abcd^6+21b^2c^2d^5)x^8}{8}+\frac{(7a^2cd^6+42abc^2d^5+35b^2c^3d^4)x^7}{7}+\frac{(21a^2c^2d^5+70abc^3d^4+35b^2c^4d^3)x^6}{6}+\frac{(35a^2c^3d^4+70abc^4d^3+21b^2c^5d^2)x^5}{5}+\frac{(35a^2c^4d^3+42abc^5d^2+7b^2c^6d)x^4}{4}+\frac{(21a^2c^5d^2+14abc^6d+b^2c^7)x^3}{3}+\frac{(7a^2c^6d+2abc^7)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^7,x)
```

```
[Out] 1/10*b^2*d^7*x^10+1/9*(2*a*b*d^7+7*b^2*c*d^6)*x^9+1/8*(a^2*d^7+14*a*b*c*d^6
+21*b^2*c^2*d^5)*x^8+1/7*(7*a^2*c*d^6+42*a*b*c^2*d^5+35*b^2*c^3*d^4)*x^7+1/
6*(21*a^2*c^2*d^5+70*a*b*c^3*d^4+35*b^2*c^4*d^3)*x^6+1/5*(35*a^2*c^3*d^4+70
*a*b*c^4*d^3+21*b^2*c^5*d^2)*x^5+1/4*(35*a^2*c^4*d^3+42*a*b*c^5*d^2+7*b^2*c
^6*d)*x^4+1/3*(21*a^2*c^5*d^2+14*a*b*c^6*d+b^2*c^7)*x^3+1/2*(7*a^2*c^6*d+2*
a*b*c^7)*x^2+a^2*c^7*x
```

**maxima** [**B**] time = 1.36, size = 273, normalized size = 4.20

$$\frac{1}{10}b^2d^7x^{10}+a^2c^7x+\frac{1}{9}(7b^2cd^6+2abd^7)x^9+\frac{1}{8}(21b^2c^2d^5+14abcd^6+a^2d^7)x^8+(5b^2c^3d^4+6abc^2d^5+a^2cd^6)x^7+\frac{7}{6}(5b^2c^4d^3+10abc^3d^4+3a^2c^2d^5)x^6+\frac{7}{5}(3b^2c^5d^2+10abc^4d^3+5a^2c^3d^4)x^5+\frac{7}{4}(b^2c^6d+6abc^5d^2+5a^2c^4d^3)x^4+\frac{1}{3}(b^2c^7+14abc^6d+21a^2c^5d^2)x^3+\frac{1}{2}(2abc^7+7a^2c^6d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^7*x^10 + a^2*c^7*x + 1/9*(7*b^2*c*d^6 + 2*a*b*d^7)*x^9 + 1/8*(21
*b^2*c^2*d^5 + 14*a*b*c*d^6 + a^2*d^7)*x^8 + (5*b^2*c^3*d^4 + 6*a*b*c^2*d^5
 + a^2*c*d^6)*x^7 + 7/6*(5*b^2*c^4*d^3 + 10*a*b*c^3*d^4 + 3*a^2*c^2*d^5)*x^
6 + 7/5*(3*b^2*c^5*d^2 + 10*a*b*c^4*d^3 + 5*a^2*c^3*d^4)*x^5 + 7/4*(b^2*c^6
*d + 6*a*b*c^5*d^2 + 5*a^2*c^4*d^3)*x^4 + 1/3*(b^2*c^7 + 14*a*b*c^6*d + 21*
a^2*c^5*d^2)*x^3 + 1/2*(2*a*b*c^7 + 7*a^2*c^6*d)*x^2
```

**mupad** [**B**] time = 0.11, size = 249, normalized size = 3.83

$$x^3\left(7a^2c^5d^2+\frac{14abc^6d}{3}+\frac{b^2c^7}{3}\right)+x^8\left(\frac{a^2d^7}{8}+\frac{7abcd^6}{4}+\frac{21b^2c^2d^5}{8}\right)+a^2c^7x+\frac{b^2d^7x^{10}}{10}+\frac{ac^6x^2(7ad+2bc)}{2}+\frac{bd^6x^9(2ad+7bc)}{9}+\frac{7c^4dx^4(5a^2d^2+6abcd+b^2c^2)}{4}+cd^4x^7(a^2d^2+6abcd+5b^2c^2)+\frac{7c^3d^2x^5(5a^2d^2+10abcd+3b^2c^2)}{5}+\frac{7c^2d^3x^6(3a^2d^2+10abcd+5b^2c^2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^7,x)
```

```
[Out] x^3*((b^2*c^7)/3 + 7*a^2*c^5*d^2 + (14*a*b*c^6*d)/3) + x^8*((a^2*d^7)/8 + (
21*b^2*c^2*d^5)/8 + (7*a*b*c*d^6)/4) + a^2*c^7*x + (b^2*d^7*x^10)/10 + (a*c
^6*x^2*(7*a*d + 2*b*c))/2 + (b*d^6*x^9*(2*a*d + 7*b*c))/9 + (7*c^4*d*x^4*(5
*a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/4 + c*d^4*x^7*(a^2*d^2 + 5*b^2*c^2 + 6*a*b
*c*d) + (7*c^3*d^2*x^5*(5*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/5 + (7*c^2*d^3
*x^6*(3*a^2*d^2 + 5*b^2*c^2 + 10*a*b*c*d))/6
```

**sympy [B]** time = 0.12, size = 303, normalized size = 4.66

$$a^{2}c^{7}x+\frac{b^{2}d^{7}x^{10}}{10}+x^{9}\left(\frac{2abd^{7}}{9}+\frac{7b^{2}cd^{6}}{9}\right)+x^{8}\left(\frac{a^{2}d^{7}}{8}+\frac{7abcd^{6}}{4}+\frac{21b^{2}c^{2}d^{5}}{8}\right)+x^{7}\left(a^{2}cd^{6}+6abc^{2}d^{5}+5b^{2}c^{3}d^{4}\right)+x^{6}\left(\frac{7a^{2}c^{2}d^{5}}{2}+\frac{35abc^{3}d^{4}}{3}+\frac{35b^{2}c^{4}d^{3}}{6}\right)+x^{5}\left(7a^{2}c^{3}d^{4}+14abc^{4}d^{3}+\frac{21b^{2}c^{5}d^{2}}{5}\right)+x^{4}\left(\frac{35a^{2}c^{4}d^{3}}{4}+\frac{21abc^{5}d^{2}}{2}+\frac{7b^{2}c^{6}d}{4}\right)+x^{3}\left(7a^{2}c^{5}d^{2}+\frac{14abc^{6}d}{3}+\frac{b^{2}c^{7}}{3}\right)+x^{2}\left(\frac{7a^{2}c^{6}d}{2}+abc^{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**7,x)
```

```
[Out] a**2*c**7*x + b**2*d**7*x**10/10 + x**9*(2*a*b*d**7/9 + 7*b**2*c*d**6/9) +
x**8*(a**2*d**7/8 + 7*a*b*c*d**6/4 + 21*b**2*c**2*d**5/8) + x**7*(a**2*c*d*
*6 + 6*a*b*c**2*d**5 + 5*b**2*c**3*d**4) + x**6*(7*a**2*c**2*d**5/2 + 35*a*
b*c**3*d**4/3 + 35*b**2*c**4*d**3/6) + x**5*(7*a**2*c**3*d**4 + 14*a*b*c**4
*d**3 + 21*b**2*c**5*d**2/5) + x**4*(35*a**2*c**4*d**3/4 + 21*a*b*c**5*d**2
/2 + 7*b**2*c**6*d/4) + x**3*(7*a**2*c**5*d**2 + 14*a*b*c**6*d/3 + b**2*c**
7/3) + x**2*(7*a**2*c**6*d/2 + a*b*c**7)
```

## 3.1175 $\int (a+bx)(c+dx)^7\,dx$

**Optimal**. Leaf size=38

$$\frac{b(c+dx)^9}{9d^2}-\frac{(c+dx)^8(bc-ad)}{8d^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^9}{9d^2}-\frac{(c+dx)^8(bc-ad)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^8)/(8*d^2) + (b*(c + d*x)^9)/(9*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^7}{d}+\frac{b(c+dx)^8}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^8}{8d^2}+\frac{b(c+dx)^9}{9d^2}\end{aligned}$$

**Mathematica [B]** time = 0.02, size = 151, normalized size = 3.97

$$\frac{1}{2}c^6x^2(7ad+bc)+\frac{7}{3}c^5dx^3(3ad+bc)+\frac{7}{4}c^4d^2x^4(5ad+3bc)+7c^3d^3x^5(ad+bc)+\frac{7}{6}c^2d^4x^6(3ad+5bc)+\frac{1}{8}d^6x^8(ad+7bc)+cd^5x^7(ad+3bc)+ac^7x+\frac{1}{9}bd^7x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] a*c^7*x + (c^6*(b*c + 7*a*d)*x^2)/2 + (7*c^5*d*(b*c + 3*a*d)*x^3)/3 + (7*c^
4*d^2*(3*b*c + 5*a*d)*x^4)/4 + 7*c^3*d^3*(b*c + a*d)*x^5 + (7*c^2*d^4*(5*b*
```

```
c + 3*a*d)*x^6)/6 + c*d^5*(3*b*c + a*d)*x^7 + (d^6*(7*b*c + a*d)*x^8)/8 + (
b*d^7*x^9)/9
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(c + d*x)^7, x]
```

**fricas [B]** time = 1.57, size = 169, normalized size = 4.45

$$\frac{1}{9}x^9d^7b+\frac{7}{8}x^8d^6cb+\frac{1}{8}x^8d^7a+3x^7d^5c^2b+x^7d^6ca+\frac{35}{6}x^6d^4c^3b+\frac{7}{2}x^6d^5c^2a+7x^5d^3c^4b+7x^5d^4c^3a+\frac{21}{4}x^4d^2c^5b+\frac{35}{4}x^4d^3c^4a+\frac{7}{3}x^3dc^6b+7x^3d^2c^5a+\frac{1}{2}x^2c^7b+\frac{7}{2}x^2dc^6a+xc^7a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^7*b + 7/8*x^8*d^6*c*b + 1/8*x^8*d^7*a + 3*x^7*d^5*c^2*b + x^7*d^6
*c*a + 35/6*x^6*d^4*c^3*b + 7/2*x^6*d^5*c^2*a + 7*x^5*d^3*c^4*b + 7*x^5*d^4
*c^3*a + 21/4*x^4*d^2*c^5*b + 35/4*x^4*d^3*c^4*a + 7/3*x^3*d*c^6*b + 7*x^3*
d^2*c^5*a + 1/2*x^2*c^7*b + 7/2*x^2*d*c^6*a + x*c^7*a
```

**giac [B]** time = 1.30, size = 169, normalized size = 4.45

$$\frac{1}{9}\,bd^7x^9+\frac{7}{8}\,bcd^6x^8+\frac{1}{8}\,ad^7x^8+3\,bc^2d^5x^7+acd^6x^7+\frac{35}{6}\,bc^3d^4x^6+\frac{7}{2}\,ac^2d^5x^6+7\,bc^4d^3x^5+7\,ac^3d^4x^5+\frac{21}{4}\,bc^5d^2x^4+\frac{35}{4}\,ac^4d^3x^4+\frac{7}{3}\,bc^6dx^3+7\,ac^5d^2x^3+\frac{1}{2}\,bc^7x^2+\frac{7}{2}\,ac^6dx^2+ac^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/9*b*d^7*x^9 + 7/8*b*c*d^6*x^8 + 1/8*a*d^7*x^8 + 3*b*c^2*d^5*x^7 + a*c*d^6
*x^7 + 35/6*b*c^3*d^4*x^6 + 7/2*a*c^2*d^5*x^6 + 7*b*c^4*d^3*x^5 + 7*a*c^3*d
^4*x^5 + 21/4*b*c^5*d^2*x^4 + 35/4*a*c^4*d^3*x^4 + 7/3*b*c^6*d*x^3 + 7*a*c^
5*d^2*x^3 + 1/2*b*c^7*x^2 + 7/2*a*c^6*d*x^2 + a*c^7*x
```

**maple [B]** time = 0.00, size = 169, normalized size = 4.45

$$\frac{b\,d^7x^9}{9}+a\,c^7x+\frac{\left(a\,d^7+7bc\,d^6\right)x^8}{8}+\frac{\left(7ac\,d^6+21b\,c^2d^5\right)x^7}{7}+\frac{\left(21a\,c^2d^5+35b\,c^3d^4\right)x^6}{6}+\frac{\left(35a\,c^3d^4+35b\,c^4d^3\right)x^5}{5}+\frac{\left(35a\,c^4d^3+21b\,c^5d^2\right)x^4}{4}+\frac{\left(21a\,c^5d^2+7b\,c^6d\right)x^3}{3}+\frac{\left(7a\,c^6d+b\,c^7\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^7,x)
```

```
[Out] 1/9*b*d^7*x^9+1/8*(a*d^7+7*b*c*d^6)*x^8+1/7*(7*a*c*d^6+21*b*c^2*d^5)*x^7+1/
6*(21*a*c^2*d^5+35*b*c^3*d^4)*x^6+1/5*(35*a*c^3*d^4+35*b*c^4*d^3)*x^5+1/4*(
35*a*c^4*d^3+21*b*c^5*d^2)*x^4+1/3*(21*a*c^5*d^2+7*b*c^6*d)*x^3+1/2*(7*a*c^
6*d+b*c^7)*x^2+a*c^7*x
```

**maxima [B]** time = 1.38, size = 163, normalized size = 4.29

$$\frac{1}{9}bd^7x^9 + ac^7x + \frac{1}{8}\left(7\,bcd^6 + ad^7\right)x^8 + \left(3\,bc^2d^5 + acd^6\right)x^7 + \frac{7}{6}\left(5\,bc^3d^4 + 3\,ac^2d^5\right)x^6 + 7\left(bc^4d^3 + ac^3d^4\right)x^5 + \frac{7}{4}\left(3\,bc^5d^2 + 5\,ac^4d^3\right)x^4 + \frac{7}{3}\left(bc^6d + 3\,ac^5d^2\right)x^3 + \frac{1}{2}\left(bc^7 + 7\,ac^6d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^7*x^9 + a*c^7*x + 1/8*(7*b*c*d^6 + a*d^7)*x^8 + (3*b*c^2*d^5 + a*c*
d^6)*x^7 + 7/6*(5*b*c^3*d^4 + 3*a*c^2*d^5)*x^6 + 7*(b*c^4*d^3 + a*c^3*d^4)*
x^5 + 7/4*(3*b*c^5*d^2 + 5*a*c^4*d^3)*x^4 + 7/3*(b*c^6*d + 3*a*c^5*d^2)*x^3
 + 1/2*(b*c^7 + 7*a*c^6*d)*x^2
```

**mupad [B]** time = 0.08, size = 143, normalized size = 3.76

$$x^2\left(\frac{b\,c^7}{2}+\frac{7\,a\,d\,c^6}{2}\right)+x^8\left(\frac{a\,d^7}{8}+\frac{7\,b\,c\,d^6}{8}\right)+\frac{b\,d^7\,x^9}{9}+a\,c^7\,x+\frac{7\,c^5\,d\,x^3\,\left(3\,a\,d+b\,c\right)}{3}+c\,d^5\,x^7\,\left(a\,d+3\,b\,c\right)+7\,c^3\,d^3\,x^5\,\left(a\,d+b\,c\right)+\frac{7\,c^4\,d^2\,x^4\,\left(5\,a\,d+3\,b\,c\right)}{4}+\frac{7\,c^2\,d^4\,x^6\,\left(3\,a\,d+5\,b\,c\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^7,x)
```

```
[Out] x^2*((b*c^7)/2 + (7*a*c^6*d)/2) + x^8*((a*d^7)/8 + (7*b*c*d^6)/8) + (b*d^7*
x^9)/9 + a*c^7*x + (7*c^5*d*x^3*(3*a*d + b*c))/3 + c*d^5*x^7*(a*d + 3*b*c)
+ 7*c^3*d^3*x^5*(a*d + b*c) + (7*c^4*d^2*x^4*(5*a*d + 3*b*c))/4 + (7*c^2*d^
4*x^6*(3*a*d + 5*b*c))/6
```

**sympy [B]** time = 0.10, size = 178, normalized size = 4.68

$$ac^7x + \frac{bd^7x^9}{9} + x^8\left(\frac{ad^7}{8} + \frac{7bcd^6}{8}\right) + x^7\left(acd^6 + 3bc^2d^5\right) + x^6\left(\frac{7ac^2d^5}{2} + \frac{35bc^3d^4}{6}\right) + x^5\left(7ac^3d^4 + 7bc^4d^3\right) + x^4\left(\frac{35ac^4d^3}{4} + \frac{21bc^5d^2}{4}\right) + x^3\left(7ac^5d^2 + \frac{7bc^6d}{3}\right) + x^2\left(\frac{7ac^6d}{2} + \frac{bc^7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**7,x)
```

```
[Out] a*c**7*x + b*d**7*x**9/9 + x**8*(a*d**7/8 + 7*b*c*d**6/8) + x**7*(a*c*d**6
+ 3*b*c**2*d**5) + x**6*(7*a*c**2*d**5/2 + 35*b*c**3*d**4/6) + x**5*(7*a*c*
*3*d**4 + 7*b*c**4*d**3) + x**4*(35*a*c**4*d**3/4 + 21*b*c**5*d**2/4) + x**
3*(7*a*c**5*d**2 + 7*b*c**6*d/3) + x**2*(7*a*c**6*d/2 + b*c**7/2)
```

## 3.1176 $\int (c+dx)^7 \, dx$

**Optimal**. Leaf size=14

$$\frac{(c+dx)^8}{8d}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7,x]
```

```
[Out] (c + d*x)^8/(8*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^7 \, dx = \frac{(c+dx)^8}{8d}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7,x]
```

```
[Out] (c + d*x)^8/(8*d)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^7 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7, x]
```

**fricas [B]** time = 1.29, size = 75, normalized size = 5.36

$$\frac{1}{8}x^8d^7 + x^7d^6c + \frac{7}{2}x^6d^5c^2 + 7x^5d^4c^3 + \frac{35}{4}x^4d^3c^4 + 7x^3d^2c^5 + \frac{7}{2}x^2dc^6 + xc^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d^7 + x^7*d^6*c + 7/2*x^6*d^5*c^2 + 7*x^5*d^4*c^3 + 35/4*x^4*d^3*c^
4 + 7*x^3*d^2*c^5 + 7/2*x^2*d*c^6 + x*c^7
```

**giac [A]** time = 1.30, size = 12, normalized size = 0.86

$$\frac{(dx + c)^8}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x + c)^8/d
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{(dx + c)^8}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7,x)
```

```
[Out] 1/8*(d*x+c)^8/d
```

**maxima [A]** time = 1.32, size = 12, normalized size = 0.86

$$\frac{(dx + c)^8}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x + c)^8/d
```

**mupad [B]** time = 0.06, size = 75, normalized size = 5.36

$$c^7\,x+\frac{7\,c^6\,d\,x^2}{2}+7\,c^5\,d^2\,x^3+\frac{35\,c^4\,d^3\,x^4}{4}+7\,c^3\,d^4\,x^5+\frac{7\,c^2\,d^5\,x^6}{2}+c\,d^6\,x^7+\frac{d^7\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7,x)
```

```
[Out] c^7*x + (d^7*x^8)/8 + (7*c^6*d*x^2)/2 + c*d^6*x^7 + 7*c^5*d^2*x^3 + (35*c^4
*d^3*x^4)/4 + 7*c^3*d^4*x^5 + (7*c^2*d^5*x^6)/2
```

**sympy [B]** time = 0.08, size = 83, normalized size = 5.93

$$c^7x+\frac{7c^6dx^2}{2}+7c^5d^2x^3+\frac{35c^4d^3x^4}{4}+7c^3d^4x^5+\frac{7c^2d^5x^6}{2}+cd^6x^7+\frac{d^7x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7,x)
```

```
[Out] c**7*x + 7*c**6*d*x**2/2 + 7*c**5*d**2*x**3 + 35*c**4*d**3*x**4/4 + 7*c**3*
d**4*x**5 + 7*c**2*d**5*x**6/2 + c*d**6*x**7 + d**7*x**8/8
```

# 3.1177 $\int \frac{(c+dx)^7}{a+bx} \, dx$

**Optimal**. Leaf size=169

$$\frac{(bc-ad)^7 \log(a+bx)}{b^8}+\frac{dx(bc-ad)^6}{b^7}+\frac{(c+dx)^2(bc-ad)^5}{2b^6}+\frac{(c+dx)^3(bc-ad)^4}{3b^5}+\frac{(c+dx)^4(bc-ad)^3}{4b^4}+\frac{(c+dx)^5(}{5b}$$

**Rubi [A]** time = 0.07, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{dx(bc-ad)^6}{b^7}+\frac{(c+dx)^2(bc-ad)^5}{2b^6}+\frac{(c+dx)^3(bc-ad)^4}{3b^5}+\frac{(c+dx)^4(bc-ad)^3}{4b^4}+\frac{(c+dx)^5(bc-ad)^2}{5b^3}+\frac{(c+dx)^6(bc-ad)}{6b^2}+\frac{(bc-ad)^7 \log(a+bx)}{b^8}+\frac{(c+dx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^6*x)/b^7 + ((b*c - a*d)^5*(c + d*x)^2)/(2*b^6) + ((b*c - a*d
)^4*(c + d*x)^3)/(3*b^5) + ((b*c - a*d)^3*(c + d*x)^4)/(4*b^4) + ((b*c - a*
d)^2*(c + d*x)^5)/(5*b^3) + ((b*c - a*d)*(c + d*x)^6)/(6*b^2) + (c + d*x)^7
/(7*b) + ((b*c - a*d)^7*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{a+bx} \, dx &= \int \left(\frac{d(bc-ad)^6}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)}+\frac{d(bc-ad)^5(c+dx)}{b^6}+\frac{d(bc-ad)^4(c+dx)^2}{b^5}+\frac{d(bc-ad)^3(c+}{b^4}\right.\\ &= \frac{d(bc-ad)^6x}{b^7}+\frac{(bc-ad)^5(c+dx)^2}{2b^6}+\frac{(bc-ad)^4(c+dx)^3}{3b^5}+\frac{(bc-ad)^3(c+dx)^4}{4b^4}+\frac{(bc-ad)^2(}{5b^3}\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 304, normalized size = 1.80

$$\frac{dx\left(420a^6d^6-210a^5bd^5(14c+dx)+70a^4b^2d^4\left(126c^2+21cdx+2d^2x^2\right)-35a^3b^3d^3\left(420c^3+126c^2dx+28cd^2x^2+3d^3x^3\right)+21a^2b^4d^2\left(700c^4+350c^3dx+140c^2d^2x^2+35cd^3x^3+4d^4x^4\right)-7ab^5d\left(1260c^5+1050c^4dx+700c^3d^2x^2+315c^2d^3x^3+84cd^4x^4+10d^5x^5\right)+b^6\left(2940c^6+4410c^5dx+4900c^4d^2x^2+3675c^3d^3x^3+1764c^2d^4x^4+490cd^5x^5+60d^6x^6\right)\right)}{420b^7}+\frac{(bc-ad)^7\log(a+bx)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x),x]
```

```
[Out] (d*x*(420*a^6*d^6 - 210*a^5*b*d^5*(14*c + d*x) + 70*a^4*b^2*d^4*(126*c^2 +
21*c*d*x + 2*d^2*x^2) - 35*a^3*b^3*d^3*(420*c^3 + 126*c^2*d*x + 28*c*d^2*x^
2 + 3*d^3*x^3) + 21*a^2*b^4*d^2*(700*c^4 + 350*c^3*d*x + 140*c^2*d^2*x^2 +
35*c*d^3*x^3 + 4*d^4*x^4) - 7*a*b^5*d*(1260*c^5 + 1050*c^4*d*x + 700*c^3*d^
2*x^2 + 315*c^2*d^3*x^3 + 84*c*d^4*x^4 + 10*d^5*x^5) + b^6*(2940*c^6 + 4410
*c^5*d*x + 4900*c^4*d^2*x^2 + 3675*c^3*d^3*x^3 + 1764*c^2*d^4*x^4 + 490*c*d
^5*x^5 + 60*d^6*x^6)))/(420*b^7) + ((b*c - a*d)^7*Log[a + b*x])/b^8
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x), x]
```

**fricas [B]** time = 1.17, size = 462, normalized size = 2.73

$$\frac{60 b^{7} d^{7} x^{7} + 70 \left(7 b^{7} c d^{6} - a b^{6} d^{7}\right) x^{6} + 84 \left(21 b^{7} c^{2} d^{5} - 7 a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right) x^{5} + 105 \left(35 b^{7} c^{3} d^{4} - 21 a b^{6} c^{2} d^{5} + 7 a^{2} b^{5} c d^{6} - a^{3} b^{4} d^{7}\right) x^{4} + 140 \left(35 b^{7} c^{4} d^{3} - 35 a b^{6} c^{3} d^{4} + 21 a^{2} b^{5} c^{2} d^{5} - 7 a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right) x^{3} + 210 \left(21 b^{7} c^{5} d^{2} - 35 a b^{6} c^{4} d^{3} + 35 a^{2} b^{5} c^{3} d^{4} - 21 a^{3} b^{4} c^{2} d^{5} + 7 a^{4} b^{3} c d^{6} - a^{5} b^{2} d^{7}\right) x^{2} + 420 \left(7 b^{7} c^{6} d - 21 a b^{6} c^{5} d^{2} + 35 a^{2} b^{5} c^{4} d^{3} - 35 a^{3} b^{4} c^{3} d^{4} + 21 a^{4} b^{3} c^{2} d^{5} - 7 a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right) x + 420 \left(b^{7} c^{7} - 7 a b^{6} c^{6} d + 21 a^{2} b^{5} c^{5} d^{2} - 35 a^{3} b^{4} c^{4} d^{3} + 35 a^{4} b^{3} c^{3} d^{4} - 21 a^{5} b^{2} c^{2} d^{5} + 7 a^{6} b c d^{6} - a^{7} d^{7}\right) \log\left(b x + a\right)}{420 b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/420*(60*b^7*d^7*x^7 + 70*(7*b^7*c*d^6 - a*b^6*d^7)*x^6 + 84*(21*b^7*c^2*d
^5 - 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 105*(35*b^7*c^3*d^4 - 21*a*b^6*c^2*
d^5 + 7*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 140*(35*b^7*c^4*d^3 - 35*a*b^6*c
^3*d^4 + 21*a^2*b^5*c^2*d^5 - 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 210*(21*
b^7*c^5*d^2 - 35*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 - 21*a^3*b^4*c^2*d^5 +
7*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 420*(7*b^7*c^6*d - 21*a*b^6*c^5*d^2 +
35*a^2*b^5*c^4*d^3 - 35*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 - 7*a^5*b^2*c*
d^6 + a^6*b*d^7)*x + 420*(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35
*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6
- a^7*d^7)*log(b*x + a))/b^8
```

**giac [B]** time = 1.30, size = 497, normalized size = 2.94

$$\frac{60 b^{6} d^{7} x^{7} + 490 b^{6} c d^{6} x^{6} - 70 a b^{5} d^{7} x^{6} + 1764 b^{6} c^{2} d^{5} x^{5} - 588 a b^{5} c d^{6} x^{5} + 84 a^{2} b^{4} d^{7} x^{5} + 3675 b^{6} c^{3} d^{4} x^{4} - 2205 a b^{5} c^{2} d^{5} x^{4} + 735 a^{2} b^{4} c d^{6} x^{4} - 105 a^{3} b^{3} d^{7} x^{4} + 4900 b^{6} c^{4} d^{3} x^{3} - 4900 a b^{5} c^{3} d^{4} x^{3} + 2940 a^{2} b^{4} c^{2} d^{5} x^{3} - 980 a^{3} b^{3} c d^{6} x^{3} + 140 a^{4} b^{2} d^{7} x^{3} + 4410 b^{6} c^{5} d^{2} x^{2} - 7350 a b^{5} c^{4} d^{3} x^{2} + 7350 a^{2} b^{4} c^{3} d^{4} x^{2} - 4410 a^{3} b^{3} c^{2} d^{5} x^{2} + 1470 a^{4} b^{2} c d^{6} x^{2} - 210 a^{5} b d^{7} x^{2} + 2940 b^{6} c^{6} d x - 8820 a b^{5} c^{5} d^{2} x + 14700 a^{2} b^{4} c^{4} d^{3} x - 14700 a^{3} b^{3} c^{3} d^{4} x + 8820 a^{4} b^{2} c^{2} d^{5} x - 2940 a^{5} b c d^{6} x + 420 a^{6} d^{7} x}{420 b^{7}} + \frac{\left(b^{7} c^{7} - 7 a b^{6} c^{6} d + 21 a^{2} b^{5} c^{5} d^{2} - 35 a^{3} b^{4} c^{4} d^{3} + 35 a^{4} b^{3} c^{3} d^{4} - 21 a^{5} b^{2} c^{2} d^{5} + 7 a^{6} b c d^{6} - a^{7} d^{7}\right) \log\left({\left| b x + a \right|}\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/420*(60*b^6*d^7*x^7 + 490*b^6*c*d^6*x^6 - 70*a*b^5*d^7*x^6 + 1764*b^6*c^2
*d^5*x^5 - 588*a*b^5*c*d^6*x^5 + 84*a^2*b^4*d^7*x^5 + 3675*b^6*c^3*d^4*x^4
- 2205*a*b^5*c^2*d^5*x^4 + 735*a^2*b^4*c*d^6*x^4 - 105*a^3*b^3*d^7*x^4 + 49
00*b^6*c^4*d^3*x^3 - 4900*a*b^5*c^3*d^4*x^3 + 2940*a^2*b^4*c^2*d^5*x^3 - 98
0*a^3*b^3*c*d^6*x^3 + 140*a^4*b^2*d^7*x^3 + 4410*b^6*c^5*d^2*x^2 - 7350*a*b
^5*c^4*d^3*x^2 + 7350*a^2*b^4*c^3*d^4*x^2 - 4410*a^3*b^3*c^2*d^5*x^2 + 1470
*a^4*b^2*c*d^6*x^2 - 210*a^5*b*d^7*x^2 + 2940*b^6*c^6*d*x - 8820*a*b^5*c^5*
d^2*x + 14700*a^2*b^4*c^4*d^3*x - 14700*a^3*b^3*c^3*d^4*x + 8820*a^4*b^2*c^
2*d^5*x - 2940*a^5*b*c*d^6*x + 420*a^6*d^7*x)/b^7 + (b^7*c^7 - 7*a*b^6*c^6*
d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b
^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)*log(abs(b*x + a))/b^8
```

**maple [B]** time = 0.01, size = 539, normalized size = 3.19

$$\frac{d^7x^7}{7b}-\frac{a\,d^7x^6}{6b^2}+\frac{7c\,d^6x^6}{6b}+\frac{a^2d^7x^5}{5b^3}-\frac{7ac\,d^6x^5}{5b^2}+\frac{21c^2d^5x^5}{5b}-\frac{a^3d^7x^4}{4b^4}+\frac{7a^2c\,d^6x^4}{4b^3}-\frac{21a\,c^2d^5x^4}{4b^2}+\frac{35c^3d^4x^4}{4b}+\frac{a^4d^7x^3}{3b^5}-\frac{7a^3c\,d^6x^3}{3b^4}+\frac{7a^2c^2d^5x^3}{b^3}-\frac{35a\,c^3d^4x^3}{3b^2}+\frac{35c^4d^3x^3}{3b}-\frac{a^5d^7x^2}{2b^6}+\frac{7a^4c\,d^6x^2}{2b^5}-\frac{21a^3c^2d^5x^2}{2b^4}+\frac{35a^2c^3d^4x^2}{2b^3}-\frac{35a\,c^4d^3x^2}{2b^2}+\frac{21c^5d^2x^2}{2b}-\frac{a^7d^7\ln(bx+a)}{b^8}+\frac{7a^6c\,d^6\ln(bx+a)}{b^7}+\frac{a^6d^7x}{b^7}-\frac{21a^5c^2d^5\ln(bx+a)}{b^6}-\frac{7a^5c\,d^6x}{b^6}+\frac{35a^4c^3d^4\ln(bx+a)}{b^5}+\frac{21a^4c^2d^5x}{b^5}-\frac{35a^3c^4d^3\ln(bx+a)}{b^4}-\frac{35a^3c^3d^4x}{b^4}+\frac{21a^2c^5d^2\ln(bx+a)}{b^3}+\frac{35a^2c^4d^3x}{b^3}-\frac{7a\,c^6d\ln(bx+a)}{b^2}-\frac{21a\,c^5d^2x}{b^2}+\frac{c^7\ln(bx+a)}{b}+\frac{7c^6dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a),x)
```

```
[Out] 7/6*d^6/b*x^6*c+1/5*d^7/b^3*x^5*a^2+7*d/b*c^6*x+d^7/b^7*a^6*x-1/b^8*ln(b*x+
a)*a^7*d^7+21/5*d^5/b*x^5*c^2-1/4*d^7/b^4*x^4*a^3+35/4*d^4/b*x^4*c^3+1/3*d^
7/b^5*x^3*a^4+35/3*d^3/b*x^3*c^4-1/2*d^7/b^6*x^2*a^5+21/2*d^2/b*x^2*c^5-1/6
*d^7/b^2*x^6*a+35*d^3/b^3*a^2*c^4*x-21*d^2/b^2*a*c^5*x-21/2*d^5/b^4*x^2*a^3
*c^2+35/2*d^4/b^3*x^2*a^2*c^3+7/4*d^6/b^3*x^4*a^2*c-21/4*d^5/b^2*x^4*a*c^2-
7/b^2*ln(b*x+a)*a*c^6*d-35/2*d^3/b^2*x^2*a*c^4+7/2*d^6/b^5*x^2*a^4*c+7*d^5/
b^3*x^3*a^2*c^2-35/3*d^4/b^2*x^3*a*c^3+7/b^7*ln(b*x+a)*a^6*c*d^6-21/b^6*ln(
b*x+a)*a^5*c^2*d^5+35/b^5*ln(b*x+a)*a^4*c^3*d^4-7/3*d^6/b^4*x^3*a^3*c-7/5*d
^6/b^2*x^5*a*c-7*d^6/b^6*a^5*c*x+21*d^5/b^5*a^4*c^2*x-35*d^4/b^4*a^3*c^3*x-
35/b^4*ln(b*x+a)*a^3*c^4*d^3+21/b^3*ln(b*x+a)*a^2*c^5*d^2+1/b*ln(b*x+a)*c^7
+1/7*d^7/b*x^7
```

**maxima [B]** time = 1.39, size = 460, normalized size = 2.72

$$\frac{60\,b^6d^7x^7+70\left(7\,b^6cd^6-ab^5d^7\right)x^6+84\left(21\,b^6c^2d^5-7\,ab^5cd^6+a^2b^4d^7\right)x^5+105\left(35\,b^6c^3d^4-21\,ab^5c^2d^5+7\,a^2b^4cd^6-a^3b^3d^7\right)x^4+140\left(35\,b^6c^4d^3-35\,ab^5c^3d^4+21\,a^2b^4c^2d^5-7\,a^3b^3cd^6+a^4b^2d^7\right)x^3+210\left(21\,b^6c^5d^2-35\,ab^5c^4d^3+35\,a^2b^4c^3d^4-21\,a^3b^3c^2d^5+7\,a^4b^2cd^6-a^5bd^7\right)x^2+420\left(7\,b^6c^6d-21\,ab^5c^5d^2+35\,a^2b^4c^4d^3-35\,a^3b^3c^3d^4+21\,a^4b^2c^2d^5-7\,a^5bcd^6+a^6d^7\right)x}{420\,b^7}+\frac{\left(b^7c^7-7\,ab^6c^6d+21\,a^2b^5c^5d^2-35\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4-21\,a^5b^2c^2d^5+7\,a^6bcd^6-a^7d^7\right)\log\left(bx+a\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/420*(60*b^6*d^7*x^7 + 70*(7*b^6*c*d^6 - a*b^5*d^7)*x^6 + 84*(21*b^6*c^2*d
^5 - 7*a*b^5*c*d^6 + a^2*b^4*d^7)*x^5 + 105*(35*b^6*c^3*d^4 - 21*a*b^5*c^2*
d^5 + 7*a^2*b^4*c*d^6 - a^3*b^3*d^7)*x^4 + 140*(35*b^6*c^4*d^3 - 35*a*b^5*c
^3*d^4 + 21*a^2*b^4*c^2*d^5 - 7*a^3*b^3*c*d^6 + a^4*b^2*d^7)*x^3 + 210*(21*
b^6*c^5*d^2 - 35*a*b^5*c^4*d^3 + 35*a^2*b^4*c^3*d^4 - 21*a^3*b^3*c^2*d^5 +
7*a^4*b^2*c*d^6 - a^5*b*d^7)*x^2 + 420*(7*b^6*c^6*d - 21*a*b^5*c^5*d^2 + 35
*a^2*b^4*c^4*d^3 - 35*a^3*b^3*c^3*d^4 + 21*a^4*b^2*c^2*d^5 - 7*a^5*b*c*d^6
```

```
+ a^6*d^7)*x)/b^7 + (b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*
b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7
*d^7)*log(b*x + a)/b^8
```

**mupad [B]** time = 0.22, size = 509, normalized size = 3.01

$$x\left(\frac{7c^6d}{b}-\frac{a\left(\frac{a\left(\frac{a\left(\frac{35c^3d^4}{b}-\frac{a\left(\frac{a\left(\frac{ad^7}{b^2}-\frac{7cd^6}{b}\right)}{b}+\frac{21c^2d^5}{b}\right)}{b}\right)}{b}-\frac{35c^4d^3}{b}\right)}{b}+\frac{21c^5d^2}{b}\right)}{b}\right)-x^6\left(\frac{ad^7}{6b^2}-\frac{7cd^6}{6b}\right)+x^4\left(\frac{35c^3d^4}{4b}-\frac{a\left(\frac{a\left(\frac{ad^7}{b^2}-\frac{7cd^6}{b}\right)}{b}+\frac{21c^2d^5}{b}\right)}{4b}\right)+x^2\left(\frac{a\left(\frac{a\left(\frac{35c^3d^4}{b}-\frac{a\left(\frac{a\left(\frac{ad^7}{b^2}-\frac{7cd^6}{b}\right)}{b}+\frac{21c^2d^5}{b}\right)}{b}\right)}{b}-\frac{35c^4d^3}{b}\right)}{2b}+\frac{21c^5d^2}{2b}\right)+x^5\left(\frac{a\left(\frac{ad^7}{b^2}-\frac{7cd^6}{b}\right)}{5b}+\frac{21c^2d^5}{5b}\right)-x^3\left(\frac{a\left(\frac{35c^3d^4}{b}-\frac{a\left(\frac{a\left(\frac{ad^7}{b^2}-\frac{7cd^6}{b}\right)}{b}+\frac{21c^2d^5}{b}\right)}{b}\right)}{3b}-\frac{35c^4d^3}{3b}\right)-\frac{\ln(a+bx)\left(a^7d^7-7a^6bcd^6+21a^5b^2c^2d^5-35a^4b^3c^3d^4+35a^3b^4c^4d^3-21a^2b^5c^5d^2+7ab^6c^6d-b^7c^7\right)}{b^8}+\frac{d^7x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x),x)
```

```
[Out] x*((7*c^6*d)/b - (a*((a*((a*((35*c^3*d^4)/b - (a*((a*((a*d^7)/b^2 - (7*c*d^
6)/b))/b + (21*c^2*d^5)/b))/b))/b - (35*c^4*d^3)/b))/b + (21*c^5*d^2)/b))/b
) - x^6*((a*d^7)/(6*b^2) - (7*c*d^6)/(6*b)) + x^4*((35*c^3*d^4)/(4*b) - (a*
((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/(4*b)) + x^2*((a*((a*
((35*c^3*d^4)/b - (a*((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/
b))/b - (35*c^4*d^3)/b))/(2*b) + (21*c^5*d^2)/(2*b)) + x^5*((a*((a*d^7)/b^2
 - (7*c*d^6)/b))/(5*b) + (21*c^2*d^5)/(5*b)) - x^3*((a*((35*c^3*d^4)/b - (a
*((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/b))/(3*b) - (35*c^4*
d^3)/(3*b)) - (log(a + b*x)*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^
3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7
*a^6*b*c*d^6))/b^8 + (d^7*x^7)/(7*b)
```

**sympy [B]** time = 0.80, size = 408, normalized size = 2.41

$$x^6\left(-\frac{ad^7}{6b^2}+\frac{7cd^6}{6b}\right)+x^5\left(\frac{a^2d^7}{5b^3}-\frac{7acd^6}{5b^2}+\frac{21c^2d^5}{5b}\right)+x^4\left(-\frac{a^3d^7}{4b^4}+\frac{7a^2cd^6}{4b^3}-\frac{21ac^2d^5}{4b^2}+\frac{35c^3d^4}{4b}\right)+x^3\left(\frac{a^4d^7}{3b^5}-\frac{7a^3cd^6}{3b^4}+\frac{7a^2c^2d^5}{b^3}-\frac{35ac^3d^4}{3b^2}+\frac{35c^4d^3}{3b}\right)+x^2\left(-\frac{a^5d^7}{2b^6}+\frac{7a^4cd^6}{2b^5}-\frac{21a^3c^2d^5}{2b^4}+\frac{35a^2c^3d^4}{2b^3}-\frac{35ac^4d^3}{2b^2}+\frac{21c^5d^2}{2b}\right)+x\left(\frac{a^6d^7}{b^7}-\frac{7a^5cd^6}{b^6}+\frac{21a^4c^2d^5}{b^5}-\frac{35a^3c^3d^4}{b^4}+\frac{35a^2c^4d^3}{b^3}-\frac{21ac^5d^2}{b^2}+\frac{7c^6d}{b}\right)+\frac{d^7x^7}{7b}-\frac{(ad-bc)^7\log(a+bx)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a),x)
```

```
[Out] x**6*(-a*d**7/(6*b**2) + 7*c*d**6/(6*b)) + x**5*(a**2*d**7/(5*b**3) - 7*a*c
*d**6/(5*b**2) + 21*c**2*d**5/(5*b)) + x**4*(-a**3*d**7/(4*b**4) + 7*a**2*c
*d**6/(4*b**3) - 21*a*c**2*d**5/(4*b**2) + 35*c**3*d**4/(4*b)) + x**3*(a**4
*d**7/(3*b**5) - 7*a**3*c*d**6/(3*b**4) + 7*a**2*c**2*d**5/b**3 - 35*a*c**3
*d**4/(3*b**2) + 35*c**4*d**3/(3*b)) + x**2*(-a**5*d**7/(2*b**6) + 7*a**4*c
```

```
*d**6/(2*b**5) - 21*a**3*c**2*d**5/(2*b**4) + 35*a**2*c**3*d**4/(2*b**3) -
35*a*c**4*d**3/(2*b**2) + 21*c**5*d**2/(2*b)) + x*(a**6*d**7/b**7 - 7*a**5*
c*d**6/b**6 + 21*a**4*c**2*d**5/b**5 - 35*a**3*c**3*d**4/b**4 + 35*a**2*c**
4*d**3/b**3 - 21*a*c**5*d**2/b**2 + 7*c**6*d/b) + d**7*x**7/(7*b) - (a*d -
b*c)**7*log(a + b*x)/b**8
```

## 3.1178 $\int \frac{(c+dx)^7}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=187

$$\frac{7d^6(a+bx)^5(bc-ad)}{5b^8}+\frac{21d^5(a+bx)^4(bc-ad)^2}{4b^8}+\frac{35d^4(a+bx)^3(bc-ad)^3}{3b^8}+\frac{35d^3(a+bx)^2(bc-ad)^4}{2b^8}-\frac{(bc-ad)^7}{b^8(a+bx)}+$$

**Rubi [A]** time = 0.23, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^5(bc-ad)}{5b^8}+\frac{21d^5(a+bx)^4(bc-ad)^2}{4b^8}+\frac{35d^4(a+bx)^3(bc-ad)^3}{3b^8}+\frac{35d^3(a+bx)^2(bc-ad)^4}{2b^8}+\frac{21d^2x(bc-ad)^5}{b^7}-\frac{(bc-ad)^7}{b^8(a+bx)}+\frac{7d(bc-ad)^6\log(a+bx)}{b^8}+\frac{d^7(a+bx)^6}{6b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^2,x]
```

```
[Out] (21*d^2*(b*c - a*d)^5*x)/b^7 - (b*c - a*d)^7/(b^8*(a + b*x)) + (35*d^3*(b*c
 - a*d)^4*(a + b*x)^2)/(2*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^3)/(3*b^8)
 + (21*d^5*(b*c - a*d)^2*(a + b*x)^4)/(4*b^8) + (7*d^6*(b*c - a*d)*(a + b*x
)^5)/(5*b^8) + (d^7*(a + b*x)^6)/(6*b^8) + (7*d*(b*c - a*d)^6*Log[a + b*x])
/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^2}\,dx &= \int \left(\frac{21d^2(bc-ad)^5}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^2}+\frac{7d(bc-ad)^6}{b^7(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)}{b^7}+\frac{35d^4(bc-ad)^3(a}{b^7}\right.\\ &= \frac{21d^2(bc-ad)^5x}{b^7}-\frac{(bc-ad)^7}{b^8(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)^2}{2b^8}+\frac{35d^4(bc-ad)^3(a+bx)^3}{3b^8}+\frac{21d^5(bc}{}\end{aligned}$$

**Mathematica [B]** time = 0.12, size = 388, normalized size = 2.07

$$\frac{60a^7d^7-60a^6bd^6(7c+6dx)+210a^5b^2d^5\left(6c^2+10cdx-d^2x^2\right)+70a^4b^3d^4\left(-30c^3-72c^2dx+18cd^2x^2+d^3x^3\right)-35a^3b^4d^3\left(-60c^4-180c^3dx+90c^2d^2x^2+12cd^3x^3+d^4x^4\right)+21a^2b^5d^2\left(-60c^5-200c^4dx+200c^3d^2x^2+50c^2d^3x^3+10cd^4x^4+d^5x^5\right)-7ab^6d\left(-60c^6-180c^5dx+450c^4d^2x^2+200c^3d^3x^3+75c^2d^4x^4+18cd^5x^5+2d^6x^6\right)+420d(a+bx)(bc-ad)^6\log(a+bx)+b^7\left(-60c^7+1260c^5d^2x^2+1050c^4d^3x^3+700c^3d^4x^4+315c^2d^5x^5+84cd^6x^6+10d^7x^7\right)}{60b^8(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^2,x]
```

```
[Out] (60*a^7*d^7 - 60*a^6*b*d^6*(7*c + 6*d*x) + 210*a^5*b^2*d^5*(6*c^2 + 10*c*d*
x - d^2*x^2) + 70*a^4*b^3*d^4*(-30*c^3 - 72*c^2*d*x + 18*c*d^2*x^2 + d^3*x^
3) - 35*a^3*b^4*d^3*(-60*c^4 - 180*c^3*d*x + 90*c^2*d^2*x^2 + 12*c*d^3*x^3
+ d^4*x^4) + 21*a^2*b^5*d^2*(-60*c^5 - 200*c^4*d*x + 200*c^3*d^2*x^2 + 50*c
^2*d^3*x^3 + 10*c*d^4*x^4 + d^5*x^5) - 7*a*b^6*d*(-60*c^6 - 180*c^5*d*x + 4
50*c^4*d^2*x^2 + 200*c^3*d^3*x^3 + 75*c^2*d^4*x^4 + 18*c*d^5*x^5 + 2*d^6*x^
6) + b^7*(-60*c^7 + 1260*c^5*d^2*x^2 + 1050*c^4*d^3*x^3 + 700*c^3*d^4*x^4 +
 315*c^2*d^5*x^5 + 84*c*d^6*x^6 + 10*d^7*x^7) + 420*d*(b*c - a*d)^6*(a + b*
x)*Log[a + b*x])/(60*b^8*(a + b*x))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^2, x]
```

**fricas [B]** time = 1.58, size = 632, normalized size = 3.38

$$\frac{10\,b^7d^7x^7 - 60\,b^7c^7 + 420\,ab^6c^6d - 1260\,a^2b^5c^5d^2 + 2100\,a^3b^4c^4d^3 - 2100\,a^4b^3c^3d^4 + 1260\,a^5b^2c^2d^5 - 420\,a^6bcd^6 + 60\,a^7d^7 + 14\left(6\,b^7cd^6 - ab^6d^7\right)x^6 + 21\left(15\,b^7c^2d^5 - 6\,ab^6cd^6 + a^2b^5d^7\right)x^5 + 35\left(20\,b^7c^3d^4 - 15\,ab^6c^2d^5 + 6\,a^2b^5cd^6 - a^3b^4d^7\right)x^4 + 70\left(15\,b^7c^4d^3 - 20\,ab^6c^3d^4 + 15\,a^2b^5c^2d^5 - 6\,a^3b^4cd^6 + a^4b^3d^7\right)x^3 + 210\left(6\,b^7c^5d^2 - 15\,ab^6c^4d^3 + 20\,a^2b^5c^3d^4 - 15\,a^3b^4c^2d^5 + 6\,a^4b^3cd^6 - a^5b^2d^7\right)x^2 + 60\left(21\,ab^6c^5d^2 - 70\,a^2b^5c^4d^3 + 105\,a^3b^4c^3d^4 - 84\,a^4b^3c^2d^5 + 35\,a^5b^2cd^6 - 6\,a^6bd^7\right)x + 420\left(ab^6c^6d - 6\,a^2b^5c^5d^2 + 15\,a^3b^4c^4d^3 - 20\,a^4b^3c^3d^4 + 15\,a^5b^2c^2d^5 - 6\,a^6bcd^6 + a^7d^7 + \left(b^7c^6d - 6\,ab^6c^5d^2 + 15\,a^2b^5c^4d^3 - 20\,a^3b^4c^3d^4 + 15\,a^4b^3c^2d^5 - 6\,a^5b^2cd^6 + a^6bd^7\right)x\right)\log\left(bx + a\right)}{60\left(b^9x + ab^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^7*d^7*x^7 - 60*b^7*c^7 + 420*a*b^6*c^6*d - 1260*a^2*b^5*c^5*d^2
+ 2100*a^3*b^4*c^4*d^3 - 2100*a^4*b^3*c^3*d^4 + 1260*a^5*b^2*c^2*d^5 - 420*
a^6*b*c*d^6 + 60*a^7*d^7 + 14*(6*b^7*c*d^6 - a*b^6*d^7)*x^6 + 21*(15*b^7*c^
2*d^5 - 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 35*(20*b^7*c^3*d^4 - 15*a*b^6*c^
2*d^5 + 6*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 70*(15*b^7*c^4*d^3 - 20*a*b^6*
c^3*d^4 + 15*a^2*b^5*c^2*d^5 - 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 210*(6*
b^7*c^5*d^2 - 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 - 15*a^3*b^4*c^2*d^5 +
6*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 60*(21*a*b^6*c^5*d^2 - 70*a^2*b^5*c^4*
d^3 + 105*a^3*b^4*c^3*d^4 - 84*a^4*b^3*c^2*d^5 + 35*a^5*b^2*c*d^6 - 6*a^6*b
*d^7)*x + 420*(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^
4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d -
 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2
*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)*log(b*x + a))/(b^9*x + a*b^8)
```

**giac [B]** time = 1.28, size = 567, normalized size = 3.03

$$\frac{\left(10\,d^7 + \frac{84\left(b^2cd^6-abd^7\right)}{(bx+a)b} + \frac{315\left(b^4c^2d^5-2\,ab^3cd^6+a^2b^2d^7\right)}{(bx+a)^2b^2} + \frac{700\left(b^6c^3d^4-3\,ab^5c^2d^5+3\,a^2b^4cd^6-a^3b^3d^7\right)}{(bx+a)^3b^3} + \frac{1050\left(b^8c^4d^3-4\,ab^7c^3d^4+6\,a^2b^6c^2d^5-4\,a^3b^5cd^6+a^4b^4d^7\right)}{(bx+a)^4b^4} + \frac{1260\left(b^{10}c^5d^2-5\,ab^9c^4d^3+10\,a^2b^8c^3d^4-10\,a^3b^7c^2d^5+5\,a^4b^6cd^6-a^5b^5d^7\right)}{(bx+a)^5b^5}\right)(bx+a)^6}{60\,b^8} - \frac{7\left(b^6c^6d - 6\,ab^5c^5d^2 + 15\,a^2b^4c^4d^3 - 20\,a^3b^3c^3d^4 + 15\,a^4b^2c^2d^5 - 6\,a^5bcd^6 + a^6d^7\right)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^8} - \frac{\frac{b^{13}c^7}{bx+a} - \frac{7\,ab^{12}c^6d}{bx+a} + \frac{21\,a^2b^{11}c^5d^2}{bx+a} - \frac{35\,a^3b^{10}c^4d^3}{bx+a} + \frac{35\,a^4b^9c^3d^4}{bx+a} - \frac{21\,a^5b^8c^2d^5}{bx+a} + \frac{7\,a^6b^7cd^6}{bx+a} - \frac{a^7b^6d^7}{bx+a}}{b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="giac")

[Out] 1/60*(10*d^7 + 84*(b^2*c*d^6 - a*b*d^7)/((b*x + a)*b) + 315*(b^4*c^2*d^5 - 2*a*b^3*c*d^6 + a^2*b^2*d^7)/((b*x + a)^2*b^2) + 700*(b^6*c^3*d^4 - 3*a*b^5*c^2*d^5 + 3*a^2*b^4*c*d^6 - a^3*b^3*d^7)/((b*x + a)^3*b^3) + 1050*(b^8*c^4*d^3 - 4*a*b^7*c^3*d^4 + 6*a^2*b^6*c^2*d^5 - 4*a^3*b^5*c*d^6 + a^4*b^4*d^7)/((b*x + a)^4*b^4) + 1260*(b^10*c^5*d^2 - 5*a*b^9*c^4*d^3 + 10*a^2*b^8*c^3*d^4 - 10*a^3*b^7*c^2*d^5 + 5*a^4*b^6*c*d^6 - a^5*b^5*d^7)/((b*x + a)^5*b^5))*(b*x + a)^6/b^8 - 7*(b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^8 - (b^13*c^7/(b*x + a) - 7*a*b^12*c^6*d/(b*x + a) + 21*a^2*b^11*c^5*d^2/(b*x + a) - 35*a^3*b^10*c^4*d^3/(b*x + a) + 35*a^4*b^9*c^3*d^4/(b*x + a) - 21*a^5*b^8*c^2*d^5/(b*x + a) + 7*a^6*b^7*c*d^6/(b*x + a) - a^7*b^6*d^7/(b*x + a))/b^14

**maple [B]** time = 0.01, size = 571, normalized size = 3.05

$$\frac{d^7x^6}{6b^2}-\frac{2ad^7x^5}{5b^3}+\frac{7cd^6x^5}{5b^2}+\frac{3a^2d^7x^4}{4b^4}-\frac{7acd^6x^4}{2b^3}+\frac{21c^2d^5x^4}{4b^2}-\frac{4a^3d^7x^3}{3b^5}+\frac{7a^2cd^6x^3}{b^4}-\frac{14ac^2d^5x^3}{b^3}+\frac{35c^3d^4x^3}{3b^2}+\frac{5a^4d^7x^2}{2b^6}-\frac{14a^3cd^6x^2}{b^5}+\frac{63a^2c^2d^5x^2}{2b^4}-\frac{35ac^3d^4x^2}{b^3}+\frac{35c^4d^3x^2}{2b^2}+\frac{a^7d^7}{(bx+a)b^8}-\frac{7a^6cd^6}{(bx+a)b^7}+\frac{7a^6d^7\ln(bx+a)}{b^8}+\frac{21a^5c^2d^5}{(bx+a)b^6}-\frac{42a^5cd^6\ln(bx+a)}{b^7}-\frac{6a^5d^7x}{b^7}-\frac{35a^4c^3d^4}{(bx+a)b^5}+\frac{105a^4c^2d^5\ln(bx+a)}{b^6}+\frac{35a^4cd^6x}{b^6}+\frac{35a^3c^4d^3}{(bx+a)b^4}-\frac{140a^3c^3d^4\ln(bx+a)}{b^5}-\frac{84a^3c^2d^5x}{b^5}-\frac{21a^2c^5d^2}{(bx+a)b^3}+\frac{105a^2c^4d^3\ln(bx+a)}{b^4}+\frac{105a^2c^3d^4x}{b^4}+\frac{7ac^6d}{(bx+a)b^2}-\frac{42ac^5d^2\ln(bx+a)}{b^3}-\frac{70ac^4d^3x}{b^3}-\frac{c^7}{(bx+a)b}+\frac{7c^6d\ln(bx+a)}{b^2}+\frac{21c^5d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^7/(b*x+a)^2,x)

[Out] 35/3*d^4/b^2*x^3*c^3+5/2*d^7/b^6*x^2*a^4+35/2*d^3/b^2*x^2*c^4-6*d^7/b^7*a^5*x+21*d^2/b^2*c^5*x+7/b^8*d^7*ln(b*x+a)*a^6+7/b^2*d*ln(b*x+a)*c^6+1/b^8/(b*x+a)*a^7*d^7-2/5*d^7/b^3*x^5*a+7/5*d^6/b^2*x^5*c+3/4*d^7/b^4*x^4*a^2+21/4*d^5/b^2*x^4*c^2-4/3*d^7/b^5*x^3*a^3-21/b^3/(b*x+a)*a^2*c^5*d^2+7/b^2/(b*x+a)*a*c^6*d-14*d^5/b^3*x^3*a*c^2-14*d^6/b^5*x^2*a^3*c+63/2*d^5/b^4*x^2*a^2*c^2-35*d^4/b^3*x^2*a*c^3+35*d^6/b^6*a^4*c*x-7/2*d^6/b^3*x^4*a*c+7*d^6/b^4*x^3*a^2*c-70*d^3/b^3*a*c^4*x-42/b^7*d^6*ln(b*x+a)*a^5*c+105/b^6*d^5*ln(b*x+a)*a^4*c^2-140/b^5*d^4*ln(b*x+a)*a^3*c^3+105/b^4*d^3*ln(b*x+a)*a^2*c^4-42/b^3*d^2*ln(b*x+a)*a*c^5-7/b^7/(b*x+a)*a^6*c*d^6+21/b^6/(b*x+a)*a^5*c^2*d^5-35/b^5/(b*x+a)*a^4*c^3*d^4+35/b^4/(b*x+a)*a^3*c^4*d^3-84*d^5/b^5*a^3*c^2*x+105*d^4/b^4*a^2*c^3*x-1/b/(b*x+a)*c^7+1/6*d^7/b^2*x^6

**maxima [B]** time = 1.40, size = 467, normalized size = 2.50

$$-\frac{b^7c^7-7ab^6c^6d+21a^2b^5c^5d^2-35a^3b^4c^4d^3+35a^4b^3c^3d^4-21a^5b^2c^2d^5+7a^6bcd^6-a^7d^7}{b^9x+ab^8}+\frac{10b^5d^7x^6+12\left(7b^5cd^6-2ab^4d^7\right)x^5+15\left(21b^5c^2d^5-14ab^4cd^6+3a^2b^3d^7\right)x^4+20\left(35b^5c^3d^4-42ab^4c^2d^5+21a^2b^3cd^6-4a^3b^2d^7\right)x^3+30\left(35b^5c^4d^3-70ab^4c^3d^4+63a^2b^3c^2d^5-28a^3b^2cd^6+5a^4bd^7\right)x^2+60\left(21b^5c^5d^2-70ab^4c^4d^3+105a^2b^3c^3d^4-84a^3b^2c^2d^5+35a^4bcd^6-6a^5d^7\right)x}{60b^7}+\frac{7\left(b^6c^6d-6ab^5c^5d^2+15a^2b^4c^4d^3-20a^3b^3c^3d^4+15a^4b^2c^2d^5-6a^5bcd^6+a^6d^7\right)\log\left(bx+a\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="maxima")

[Out] -(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(b^9*x + a*b^

```
8) + 1/60*(10*b^5*d^7*x^6 + 12*(7*b^5*c*d^6 - 2*a*b^4*d^7)*x^5 + 15*(21*b^5
*c^2*d^5 - 14*a*b^4*c*d^6 + 3*a^2*b^3*d^7)*x^4 + 20*(35*b^5*c^3*d^4 - 42*a*
b^4*c^2*d^5 + 21*a^2*b^3*c*d^6 - 4*a^3*b^2*d^7)*x^3 + 30*(35*b^5*c^4*d^3 -
70*a*b^4*c^3*d^4 + 63*a^2*b^3*c^2*d^5 - 28*a^3*b^2*c*d^6 + 5*a^4*b*d^7)*x^2
 + 60*(21*b^5*c^5*d^2 - 70*a*b^4*c^4*d^3 + 105*a^2*b^3*c^3*d^4 - 84*a^3*b^2
*c^2*d^5 + 35*a^4*b*c*d^6 - 6*a^5*d^7)*x)/b^7 + 7*(b^6*c^6*d - 6*a*b^5*c^5*
d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*
b*c*d^6 + a^6*d^7)*log(b*x + a)/b^8
```

**mupad** [**B**] time = 0.24, size = 841, normalized size = 4.50

$$x^4\left(\frac{a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{2b}-\frac{a^2d^7}{4b^4}+\frac{21c^2d^5}{4b^2}\right)-x^2\left(\frac{a\left(\frac{35c^3d^4}{b^2}-\frac{2a\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{b}+\frac{a^2\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b^2}\right)}{b}-\frac{35c^4d^3}{2b^2}+\frac{a^2\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{2b^2}\right)-x^5\left(\frac{2ad^7}{5b^3}-\frac{7cd^6}{5b^2}\right)+x\left(\frac{2a\left(\frac{2a\left(\frac{35c^3d^4}{b^2}-\frac{2a\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{b}+\frac{a^2\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b^2}\right)}{b}-\frac{35c^4d^3}{b^2}+\frac{a^2\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{b^2}\right)}{b}-\frac{a^2\left(\frac{35c^3d^4}{b^2}-\frac{2a\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{b}+\frac{a^2\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b^2}\right)}{b^2}+\frac{21c^5d^2}{b^2}\right)+x^3\left(\frac{35c^3d^4}{3b^2}-\frac{2a\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{3b}+\frac{a^2\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{3b^2}\right)+\frac{a^7d^7-b^7c^7-21a^2b^5c^5d^2+35a^3b^4c^4d^3-35a^4b^3c^3d^4+21a^5b^2c^2d^5+7ab^6c^6d-7a^6bcd^6}{b\left(ab^7+b^8x\right)}+\frac{d^7x^6}{6b^2}+\frac{\ln\left(a+bx\right)\left(7a^6d^7+7b^6c^6d-42ab^5c^5d^2+105a^2b^4c^4d^3-140a^3b^3c^3d^4+105a^4b^2c^2d^5-42a^5bcd^6\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^2,x)
```

```
[Out] x^4*((a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/(2*b) - (a^2*d^7)/(4*b^4) + (21*c^
2*d^5)/(4*b^2)) - x^2*((a*((35*c^3*d^4)/b^2 - (2*a*((2*a*((2*a*d^7)/b^3 - (
7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/b + (a^2*((2*a*d^7)/b
^3 - (7*c*d^6)/b^2))/b^2))/b - (35*c^4*d^3)/(2*b^2) + (a^2*((2*a*((2*a*d^7)
/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/(2*b^2)) - x^
5*((2*a*d^7)/(5*b^3) - (7*c*d^6)/(5*b^2)) + x*((2*a*((2*a*((35*c^3*d^4)/b^2
 - (2*a*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*
d^5)/b^2))/b + (a^2*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b^2))/b - (35*c^4*d^3)
/b^2 + (a^2*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*
c^2*d^5)/b^2))/b^2))/b - (a^2*((35*c^3*d^4)/b^2 - (2*a*((2*a*((2*a*d^7)/b^3
 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/b + (a^2*((2*a*d^
7)/b^3 - (7*c*d^6)/b^2))/b^2))/b^2 + (21*c^5*d^2)/b^2) + x^3*((35*c^3*d^4)/
(3*b^2) - (2*a*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (
21*c^2*d^5)/b^2))/(3*b) + (a^2*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/(3*b^2)) +
(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c
^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)/(b*(a*b^7 + b^
8*x)) + (d^7*x^6)/(6*b^2) + (log(a + b*x)*(7*a^6*d^7 + 7*b^6*c^6*d - 42*a*b
^5*c^5*d^2 + 105*a^2*b^4*c^4*d^3 - 140*a^3*b^3*c^3*d^4 + 105*a^4*b^2*c^2*d^
5 - 42*a^5*b*c*d^6))/b^8
```

**sympy** [**B**] time = 1.44, size = 428, normalized size = 2.29

$$x^{5}\left(-\frac{2ad^{7}}{5b^{3}}+\frac{7cd^{6}}{5b^{2}}\right)+x^{4}\left(\frac{3a^{2}d^{7}}{4b^{4}}-\frac{7acd^{6}}{2b^{3}}+\frac{21c^{2}d^{5}}{4b^{2}}\right)+x^{3}\left(-\frac{4a^{3}d^{7}}{3b^{5}}+\frac{7a^{2}cd^{6}}{b^{4}}-\frac{14ac^{2}d^{5}}{b^{3}}+\frac{35c^{3}d^{4}}{3b^{2}}\right)+x^{2}\left(\frac{5a^{4}d^{7}}{2b^{6}}-\frac{14a^{3}cd^{6}}{b^{5}}+\frac{63a^{2}c^{2}d^{5}}{2b^{4}}-\frac{35ac^{3}d^{4}}{b^{3}}+\frac{35c^{4}d^{3}}{2b^{2}}\right)+x\left(-\frac{6a^{5}d^{7}}{b^{7}}+\frac{35a^{4}cd^{6}}{b^{6}}-\frac{84a^{3}c^{2}d^{5}}{b^{5}}+\frac{105a^{2}c^{3}d^{4}}{b^{4}}-\frac{70ac^{4}d^{3}}{b^{3}}+\frac{21c^{5}d^{2}}{b^{2}}\right)+\frac{a^{7}d^{7}-7a^{6}bcd^{6}+21a^{5}b^{2}c^{2}d^{5}-35a^{4}b^{3}c^{3}d^{4}+35a^{3}b^{4}c^{4}d^{3}-21a^{2}b^{5}c^{5}d^{2}+7ab^{6}c^{6}d-b^{7}c^{7}}{ab^{8}+b^{9}x}+\frac{d^{7}x^{6}}{6b^{2}}+\frac{7d\left(ad-bc\right)^{6}\log\left(a+bx\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**2,x)
```

```
[Out] x**5*(-2*a*d**7/(5*b**3) + 7*c*d**6/(5*b**2)) + x**4*(3*a**2*d**7/(4*b**4)
- 7*a*c*d**6/(2*b**3) + 21*c**2*d**5/(4*b**2)) + x**3*(-4*a**3*d**7/(3*b**5
) + 7*a**2*c*d**6/b**4 - 14*a*c**2*d**5/b**3 + 35*c**3*d**4/(3*b**2)) + x**
2*(5*a**4*d**7/(2*b**6) - 14*a**3*c*d**6/b**5 + 63*a**2*c**2*d**5/(2*b**4)
- 35*a*c**3*d**4/b**3 + 35*c**4*d**3/(2*b**2)) + x*(-6*a**5*d**7/b**7 + 35*
a**4*c*d**6/b**6 - 84*a**3*c**2*d**5/b**5 + 105*a**2*c**3*d**4/b**4 - 70*a*
c**4*d**3/b**3 + 21*c**5*d**2/b**2) + (a**7*d**7 - 7*a**6*b*c*d**6 + 21*a**
5*b**2*c**2*d**5 - 35*a**4*b**3*c**3*d**4 + 35*a**3*b**4*c**4*d**3 - 21*a**
2*b**5*c**5*d**2 + 7*a*b**6*c**6*d - b**7*c**7)/(a*b**8 + b**9*x) + d**7*x*
*6/(6*b**2) + 7*d*(a*d - b*c)**6*log(a + b*x)/b**8
```

# 3.1179 $\int \frac{(c+dx)^7}{(a+bx)^3} \, dx$

**Optimal**. Leaf size=185

$$\frac{7d^6(a+bx)^4(bc-ad)}{4b^8}+\frac{7d^5(a+bx)^3(bc-ad)^2}{b^8}+\frac{35d^4(a+bx)^2(bc-ad)^3}{2b^8}+\frac{21d^2(bc-ad)^5\log(a+bx)}{b^8}-\frac{7d(bc-a}{b^8(a+b}$$

**Rubi** [A] time = 0.22, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^4(bc-ad)}{4b^8}+\frac{7d^5(a+bx)^3(bc-ad)^2}{b^8}+\frac{35d^4(a+bx)^2(bc-ad)^3}{2b^8}+\frac{35d^3x(bc-ad)^4}{b^7}+\frac{21d^2(bc-ad)^5\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{b^8(a+bx)}-\frac{(bc-ad)^7}{2b^8(a+bx)^2}+\frac{d^7(a+bx)^5}{5b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^3,x]
```

```
[Out] (35*d^3*(b*c - a*d)^4*x)/b^7 - (b*c - a*d)^7/(2*b^8*(a + b*x)^2) - (7*d*(b*
c - a*d)^6)/(b^8*(a + b*x)) + (35*d^4*(b*c - a*d)^3*(a + b*x)^2)/(2*b^8) +
(7*d^5*(b*c - a*d)^2*(a + b*x)^3)/b^8 + (7*d^6*(b*c - a*d)*(a + b*x)^4)/(4*
b^8) + (d^7*(a + b*x)^5)/(5*b^8) + (21*d^2*(b*c - a*d)^5*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^3}\,dx &= \int \left(\frac{35d^3(bc-ad)^4}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^3}+\frac{7d(bc-ad)^6}{b^7(a+bx)^2}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)}+\frac{35d^4(bc-ad)^3(a+bx)}{b^7}\right.\\ &= \frac{35d^3(bc-ad)^4x}{b^7}-\frac{(bc-ad)^7}{2b^8(a+bx)^2}-\frac{7d(bc-ad)^6}{b^8(a+bx)}+\frac{35d^4(bc-ad)^3(a+bx)^2}{2b^8}+\frac{7d^5(bc-ad)^2(a}{b^8}\end{aligned}$$

**Mathematica** [B] time = 0.13, size = 389, normalized size = 2.10

$$\frac{-130a^7d^7+10a^6bd^6(77c+16dx)+10a^5b^2d^5\left(-189c^2-56cdx+50d^2x^2\right)+70a^4b^3d^4\left(35c^3+6c^2dx-34cd^2x^2+2d^3x^3\right)-35a^3b^4d^3\left(50c^4-20c^3dx-126c^2d^2x^2+20cd^3x^3+d^4x^4\right)+7a^2b^5d^2\left(90c^5-200c^4dx-550c^3d^2x^2+200c^2d^3x^3+25cd^4x^4+2d^5x^5\right)-7ab^6d\left(10c^6-120c^5dx-200c^4d^2x^2+200c^3d^3x^3+50c^2d^4x^4+10cd^5x^5+d^6x^6\right)-420d^2(a+bx)^2(ad-bc)^5\log(a+bx)+b^7\left(-10c^7-140c^6dx+700c^4d^3x^3+350c^3d^4x^4+140c^2d^5x^5+35cd^6x^6+4d^7x^7\right)}{20b^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^3,x]
```

```
[Out] (-130*a^7*d^7 + 10*a^6*b*d^6*(77*c + 16*d*x) + 10*a^5*b^2*d^5*(-189*c^2 - 5
6*c*d*x + 50*d^2*x^2) + 70*a^4*b^3*d^4*(35*c^3 + 6*c^2*d*x - 34*c*d^2*x^2 +
 2*d^3*x^3) - 35*a^3*b^4*d^3*(50*c^4 - 20*c^3*d*x - 126*c^2*d^2*x^2 + 20*c*
d^3*x^3 + d^4*x^4) + 7*a^2*b^5*d^2*(90*c^5 - 200*c^4*d*x - 550*c^3*d^2*x^2
+ 200*c^2*d^3*x^3 + 25*c*d^4*x^4 + 2*d^5*x^5) - 7*a*b^6*d*(10*c^6 - 120*c^5
*d*x - 200*c^4*d^2*x^2 + 200*c^3*d^3*x^3 + 50*c^2*d^4*x^4 + 10*c*d^5*x^5 +
d^6*x^6) + b^7*(-10*c^7 - 140*c^6*d*x + 700*c^4*d^3*x^3 + 350*c^3*d^4*x^4 +
 140*c^2*d^5*x^5 + 35*c*d^6*x^6 + 4*d^7*x^7) - 420*d^2*(-(b*c) + a*d)^5*(a
+ b*x)^2*Log[a + b*x])/(20*b^8*(a + b*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^3, x]
```

**fricas [B]** time = 1.20, size = 703, normalized size = 3.80

$$\frac{4\,b^{7}d^{7}x^{7} - 10\,b^{7}c^{7} - 70\,ab^{6}c^{6}d + 630\,a^{2}b^{5}c^{5}d^{2} - 1750\,a^{3}b^{4}c^{4}d^{3} + 2450\,a^{4}b^{3}c^{3}d^{4} - 1890\,a^{5}b^{2}c^{2}d^{5} + 770\,a^{6}bcd^{6} - 130\,a^{7}d^{7} + 7\left(5\,b^{7}cd^{6} - ab^{6}d^{7}\right)x^{6} + 14\left(10\,b^{7}c^{2}d^{5} - 5\,ab^{6}cd^{6} + a^{2}b^{5}d^{7}\right)x^{5} + 35\left(10\,b^{7}c^{3}d^{4} - 10\,ab^{6}c^{2}d^{5} + 5\,a^{2}b^{5}cd^{6} - a^{3}b^{4}d^{7}\right)x^{4} + 140\left(5\,b^{7}c^{4}d^{3} - 10\,ab^{6}c^{3}d^{4} + 10\,a^{2}b^{5}c^{2}d^{5} - 5\,a^{3}b^{4}cd^{6} + a^{4}b^{3}d^{7}\right)x^{3} + 10\left(140\,ab^{6}c^{4}d^{3} - 385\,a^{2}b^{5}c^{3}d^{4} + 441\,a^{3}b^{4}c^{2}d^{5} - 238\,a^{4}b^{3}cd^{6} + 50\,a^{5}b^{2}d^{7}\right)x^{2} - 20\left(7\,b^{7}c^{6}d - 42\,ab^{6}c^{5}d^{2} + 70\,a^{2}b^{5}c^{4}d^{3} - 35\,a^{3}b^{4}c^{3}d^{4} - 21\,a^{4}b^{3}c^{2}d^{5} + 28\,a^{5}b^{2}cd^{6} - 8\,a^{6}bd^{7}\right)x + 420\left(a^{2}b^{5}c^{5}d^{2} - 5\,a^{3}b^{4}c^{4}d^{3} + 10\,a^{4}b^{3}c^{3}d^{4} - 10\,a^{5}b^{2}c^{2}d^{5} + 5\,a^{6}bcd^{6} - a^{7}d^{7} + \left(b^{7}c^{5}d^{2} - 5\,ab^{6}c^{4}d^{3} + 10\,a^{2}b^{5}c^{3}d^{4} - 10\,a^{3}b^{4}c^{2}d^{5} + 5\,a^{4}b^{3}cd^{6} - a^{5}b^{2}d^{7}\right)x^{2} + 2\left(ab^{6}c^{5}d^{2} - 5\,a^{2}b^{5}c^{4}d^{3} + 10\,a^{3}b^{4}c^{3}d^{4} - 10\,a^{4}b^{3}c^{2}d^{5} + 5\,a^{5}b^{2}cd^{6} - a^{6}bd^{7}\right)x\right)\log\left(bx + a\right)}{20\left(b^{10}x^{2} + 2\,ab^{9}x + a^{2}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*d^7*x^7 - 10*b^7*c^7 - 70*a*b^6*c^6*d + 630*a^2*b^5*c^5*d^2 - 1
750*a^3*b^4*c^4*d^3 + 2450*a^4*b^3*c^3*d^4 - 1890*a^5*b^2*c^2*d^5 + 770*a^6
*b*c*d^6 - 130*a^7*d^7 + 7*(5*b^7*c*d^6 - a*b^6*d^7)*x^6 + 14*(10*b^7*c^2*d
^5 - 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 35*(10*b^7*c^3*d^4 - 10*a*b^6*c^2*d
^5 + 5*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 140*(5*b^7*c^4*d^3 - 10*a*b^6*c^3
*d^4 + 10*a^2*b^5*c^2*d^5 - 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 10*(140*a*
b^6*c^4*d^3 - 385*a^2*b^5*c^3*d^4 + 441*a^3*b^4*c^2*d^5 - 238*a^4*b^3*c*d^6
 + 50*a^5*b^2*d^7)*x^2 - 20*(7*b^7*c^6*d - 42*a*b^6*c^5*d^2 + 70*a^2*b^5*c^
4*d^3 - 35*a^3*b^4*c^3*d^4 - 21*a^4*b^3*c^2*d^5 + 28*a^5*b^2*c*d^6 - 8*a^6*
b*d^7)*x + 420*(a^2*b^5*c^5*d^2 - 5*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 -
10*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 - a^7*d^7 + (b^7*c^5*d^2 - 5*a*b^6*c^4*d
^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^
7)*x^2 + 2*(a*b^6*c^5*d^2 - 5*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 - 10*a^4
*b^3*c^2*d^5 + 5*a^5*b^2*c*d^6 - a^6*b*d^7)*x)*log(b*x + a))/(b^10*x^2 + 2*
a*b^9*x + a^2*b^8)
```

**giac [B]** time = 1.26, size = 477, normalized size = 2.58

$$\frac{21\left(b^{5}c^{5}d^{2}-5ab^{4}c^{4}d^{3}+10a^{2}b^{3}c^{3}d^{4}-10a^{3}b^{2}c^{2}d^{5}+5a^{4}bcd^{6}-a^{5}d^{7}\right)\log\left(|bx+a|\right)}{b^{8}}-\frac{b^{7}c^{7}+7ab^{6}c^{6}d-63a^{2}b^{5}c^{5}d^{2}+175a^{3}b^{4}c^{4}d^{3}-245a^{4}b^{3}c^{3}d^{4}+189a^{5}b^{2}c^{2}d^{5}-77a^{6}bcd^{6}+13a^{7}d^{7}+14\left(b^{7}c^{6}d-6ab^{6}c^{5}d^{2}+15a^{2}b^{5}c^{4}d^{3}-20a^{3}b^{4}c^{3}d^{4}+15a^{4}b^{3}c^{2}d^{5}-6a^{5}b^{2}cd^{6}+a^{6}bd^{7}\right)x}{2\left(bx+a\right)^{2}b^{8}}+\frac{4b^{12}d^{7}x^{5}+35b^{12}cd^{6}x^{4}-15ab^{11}d^{7}x^{4}+140b^{12}c^{2}d^{5}x^{3}-140ab^{11}cd^{6}x^{3}+40a^{2}b^{10}d^{7}x^{3}+350b^{12}c^{3}d^{4}x^{2}-630ab^{11}c^{2}d^{5}x^{2}+420a^{2}b^{10}cd^{6}x^{2}-100a^{3}b^{9}d^{7}x^{2}+700b^{12}c^{4}d^{3}x-2100ab^{11}c^{3}d^{4}x+2520a^{2}b^{10}c^{2}d^{5}x-1400a^{3}b^{9}cd^{6}x+300a^{4}b^{8}d^{7}x}{20b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 21*(b^5*c^5*d^2 - 5*a*b^4*c^4*d^3 + 10*a^2*b^3*c^3*d^4 - 10*a^3*b^2*c^2*d^5
 + 5*a^4*b*c*d^6 - a^5*d^7)*log(abs(b*x + a))/b^8 - 1/2*(b^7*c^7 + 7*a*b^6*
c^6*d - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 - 245*a^4*b^3*c^3*d^4 + 18
9*a^5*b^2*c^2*d^5 - 77*a^6*b*c*d^6 + 13*a^7*d^7 + 14*(b^7*c^6*d - 6*a*b^6*c
^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a
^5*b^2*c*d^6 + a^6*b*d^7)*x)/((b*x + a)^2*b^8) + 1/20*(4*b^12*d^7*x^5 + 35*
b^12*c*d^6*x^4 - 15*a*b^11*d^7*x^4 + 140*b^12*c^2*d^5*x^3 - 140*a*b^11*c*d^
6*x^3 + 40*a^2*b^10*d^7*x^3 + 350*b^12*c^3*d^4*x^2 - 630*a*b^11*c^2*d^5*x^2
 + 420*a^2*b^10*c*d^6*x^2 - 100*a^3*b^9*d^7*x^2 + 700*b^12*c^4*d^3*x - 2100
*a*b^11*c^3*d^4*x + 2520*a^2*b^10*c^2*d^5*x - 1400*a^3*b^9*c*d^6*x + 300*a^
4*b^8*d^7*x)/b^15
```

**maple [B]** time = 0.01, size = 599, normalized size = 3.24

$$\frac{d^{7}x^{5}}{5b^{3}}-\frac{3ad^{7}x^{4}}{4b^{4}}+\frac{7cd^{6}x^{4}}{4b^{3}}+\frac{2a^{2}d^{7}x^{3}}{b^{5}}-\frac{7acd^{6}x^{3}}{b^{4}}+\frac{7c^{2}d^{5}x^{3}}{b^{3}}+\frac{a^{7}d^{7}}{2\left(bx+a\right)^{2}b^{8}}-\frac{7a^{6}cd^{6}}{2\left(bx+a\right)^{2}b^{7}}+\frac{21a^{5}c^{2}d^{5}}{2\left(bx+a\right)^{2}b^{6}}-\frac{35a^{4}c^{3}d^{4}}{2\left(bx+a\right)^{2}b^{5}}+\frac{35a^{3}c^{4}d^{3}}{2\left(bx+a\right)^{2}b^{4}}-\frac{5a^{3}d^{7}x^{2}}{b^{6}}-\frac{21a^{2}c^{5}d^{2}}{2\left(bx+a\right)^{2}b^{3}}+\frac{21a^{2}cd^{6}x^{2}}{b^{5}}+\frac{7ac^{6}d}{2\left(bx+a\right)^{2}b^{2}}-\frac{63ac^{2}d^{5}x^{2}}{2b^{4}}-\frac{c^{7}}{2\left(bx+a\right)^{2}b}+\frac{35c^{3}d^{4}x^{2}}{2b^{3}}-\frac{7a^{6}d^{7}}{\left(bx+a\right)b^{8}}+\frac{42a^{5}cd^{6}}{\left(bx+a\right)b^{7}}-\frac{21a^{5}d^{7}\ln\left(bx+a\right)}{b^{8}}-\frac{105a^{4}c^{2}d^{5}}{\left(bx+a\right)b^{6}}+\frac{105a^{4}cd^{6}\ln\left(bx+a\right)}{b^{7}}+\frac{15a^{4}d^{7}x}{b^{7}}+\frac{140a^{3}c^{3}d^{4}}{\left(bx+a\right)b^{5}}-\frac{210a^{3}c^{2}d^{5}\ln\left(bx+a\right)}{b^{6}}-\frac{70a^{3}cd^{6}x}{b^{6}}-\frac{105a^{2}c^{4}d^{3}}{\left(bx+a\right)b^{4}}+\frac{210a^{2}c^{3}d^{4}\ln\left(bx+a\right)}{b^{5}}+\frac{126a^{2}c^{2}d^{5}x}{b^{5}}+\frac{42ac^{5}d^{2}}{\left(bx+a\right)b^{3}}-\frac{105ac^{4}d^{3}\ln\left(bx+a\right)}{b^{4}}-\frac{105ac^{3}d^{4}x}{b^{4}}-\frac{7c^{6}d}{\left(bx+a\right)b^{2}}+\frac{21c^{5}d^{2}\ln\left(bx+a\right)}{b^{3}}+\frac{35c^{4}d^{3}x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^3,x)
```

```
[Out] 2*d^7/b^5*x^3*a^2+7*d^5/b^3*x^3*c^2-5*d^7/b^6*x^2*a^3+35/2*d^4/b^3*x^2*c^3+
15*d^7/b^7*a^4*x+35*d^3/b^3*c^4*x+1/2/b^8/(b*x+a)^2*a^7*d^7-21/b^8*d^7*ln(b
*x+a)*a^5+21/b^3*d^2*ln(b*x+a)*c^5-7/b^8*d^7/(b*x+a)*a^6-7/b^2*d/(b*x+a)*c^
6-3/4*d^7/b^4*x^4*a+7/4*d^6/b^3*x^4*c+210/b^5*d^4*ln(b*x+a)*a^2*c^3-105/b^4
*d^3*ln(b*x+a)*a*c^4+42/b^7*d^6/(b*x+a)*a^5*c-105/b^6*d^5/(b*x+a)*a^4*c^2+1
40/b^5*d^4/(b*x+a)*a^3*c^3-105/b^4*d^3/(b*x+a)*a^2*c^4+42/b^3*d^2/(b*x+a)*a
*c^5-63/2*d^5/b^4*x^2*a*c^2-70*d^6/b^6*a^3*c*x+126*d^5/b^5*a^2*c^2*x-105*d^
4/b^4*a*c^3*x-7/2/b^7/(b*x+a)^2*a^6*c*d^6+21/2/b^6/(b*x+a)^2*a^5*c^2*d^5-35
/2/b^5/(b*x+a)^2*a^4*c^3*d^4+35/2/b^4/(b*x+a)^2*a^3*c^4*d^3-21/2/b^3/(b*x+a
)^2*a^2*c^5*d^2+7/2/b^2/(b*x+a)^2*a*c^6*d+105/b^7*d^6*ln(b*x+a)*a^4*c-210/b
^6*d^5*ln(b*x+a)*a^3*c^2-7*d^6/b^4*x^3*a*c+21*d^6/b^5*x^2*a^2*c-1/2/b/(b*x+
a)^2*c^7+1/5*d^7/b^3*x^5
```

**maxima [B]** time = 1.54, size = 473, normalized size = 2.56

$$-\frac{b^{7}c^{7}+7ab^{6}c^{6}d-63a^{2}b^{5}c^{5}d^{2}+175a^{3}b^{4}c^{4}d^{3}-245a^{4}b^{3}c^{3}d^{4}+189a^{5}b^{2}c^{2}d^{5}-77a^{6}bcd^{6}+13a^{7}d^{7}+14\left(b^{7}c^{6}d-6ab^{6}c^{5}d^{2}+15a^{2}b^{5}c^{4}d^{3}-20a^{3}b^{4}c^{3}d^{4}+15a^{4}b^{3}c^{2}d^{5}-6a^{5}b^{2}cd^{6}+a^{6}bd^{7}\right)x}{2\left(b^{10}x^{2}+2ab^{9}x+a^{2}b^{8}\right)}+\frac{4b^{4}d^{7}x^{5}+5\left(7b^{4}cd^{6}-3ab^{3}d^{7}\right)x^{4}+20\left(7b^{4}c^{2}d^{5}-7ab^{3}cd^{6}+2a^{2}b^{2}d^{7}\right)x^{3}+10\left(35b^{4}c^{3}d^{4}-63ab^{3}c^{2}d^{5}+42a^{2}b^{2}cd^{6}-10a^{3}bd^{7}\right)x^{2}+20\left(35b^{4}c^{4}d^{3}-105ab^{3}c^{3}d^{4}+126a^{2}b^{2}c^{2}d^{5}-70a^{3}bcd^{6}+15a^{4}d^{7}\right)x}{20b^{7}}+\frac{21\left(b^{5}c^{5}d^{2}-5ab^{4}c^{4}d^{3}+10a^{2}b^{3}c^{3}d^{4}-10a^{3}b^{2}c^{2}d^{5}+5a^{4}bcd^{6}-a^{5}d^{7}\right)\log\left(bx+a\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^7*c^7 + 7*a*b^6*c^6*d - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 -
245*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 77*a^6*b*c*d^6 + 13*a^7*d^7 + 1
4*(b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 +
15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^10*x^2 + 2*a*b^9*x
+ a^2*b^8) + 1/20*(4*b^4*d^7*x^5 + 5*(7*b^4*c*d^6 - 3*a*b^3*d^7)*x^4 + 20*(
7*b^4*c^2*d^5 - 7*a*b^3*c*d^6 + 2*a^2*b^2*d^7)*x^3 + 10*(35*b^4*c^3*d^4 - 6
3*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 - 10*a^3*b*d^7)*x^2 + 20*(35*b^4*c^4*d^3
 - 105*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 - 70*a^3*b*c*d^6 + 15*a^4*d^7)*x
)/b^7 + 21*(b^5*c^5*d^2 - 5*a*b^4*c^4*d^3 + 10*a^2*b^3*c^3*d^4 - 10*a^3*b^2
*c^2*d^5 + 5*a^4*b*c*d^6 - a^5*d^7)*log(b*x + a)/b^8
```

**mupad [B]** time = 0.27, size = 690, normalized size = 3.73

$$x\left(\frac{3a\left(\frac{3a\left(\frac{3a\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b}-\frac{3a^2d^7}{b^5}+\frac{21c^2d^5}{b^3}\right)}{b}+\frac{a^3d^7}{b^6}-\frac{35c^3d^4}{b^3}-\frac{3a^2\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b^2}\right)}{b}+\frac{35c^4d^3}{b^3}+\frac{a^3\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b^3}-\frac{3a^2\left(\frac{3a\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b}-\frac{3a^2d^7}{b^5}+\frac{21c^2d^5}{b^3}\right)}{b^2}\right)-x^4\left(\frac{3ad^7}{4b^4}-\frac{7cd^6}{4b^3}\right)-x^2\left(\frac{3a\left(\frac{3a\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b}-\frac{3a^2d^7}{b^5}+\frac{21c^2d^5}{b^3}\right)}{2b}+\frac{a^3d^7}{2b^6}-\frac{35c^3d^4}{2b^3}-\frac{3a^2\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{2b^2}\right)+x^3\left(\frac{a\left(\frac{3ad^7}{b^4}-\frac{7cd^6}{b^3}\right)}{b}-\frac{a^2d^7}{b^5}+\frac{7c^2d^5}{b^3}\right)-\frac{\frac{13a^7d^7-77a^6bcd^6+189a^5b^2c^2d^5-245a^4b^3c^3d^4+175a^3b^4c^4d^3-63a^2b^5c^5d^2+7ab^6c^6d+b^7c^7}{2b}+x\left(7a^6d^7-42a^5bcd^6+105a^4b^2c^2d^5-140a^3b^3c^3d^4+105a^2b^4c^4d^3-42ab^5c^5d^2+7b^6c^6d\right)}{a^2b^7+2ab^8x+b^9x^2}+\frac{d^7x^5}{5b^3}-\frac{\ln(a+bx)\left(21a^5d^7-105a^4bcd^6+210a^3b^2c^2d^5-210a^2b^3c^3d^4+105ab^4c^4d^3-21b^5c^5d^2\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^3,x)
```

```
[Out] x*((3*a*((3*a*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a^2*d^7)/b^5 +
(21*c^2*d^5)/b^3))/b + (a^3*d^7)/b^6 - (35*c^3*d^4)/b^3 - (3*a^2*((3*a*d^7)
/b^4 - (7*c*d^6)/b^3))/b^2))/b + (35*c^4*d^3)/b^3 + (a^3*((3*a*d^7)/b^4 - (
7*c*d^6)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a
^2*d^7)/b^5 + (21*c^2*d^5)/b^3))/b^2) - x^4*((3*a*d^7)/(4*b^4) - (7*c*d^6)/
(4*b^3)) - x^2*((3*a*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a^2*d^7)
/b^5 + (21*c^2*d^5)/b^3))/(2*b) + (a^3*d^7)/(2*b^6) - (35*c^3*d^4)/(2*b^3)
- (3*a^2*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/(2*b^2)) + x^3*((a*((3*a*d^7)/b^4
 - (7*c*d^6)/b^3))/b - (a^2*d^7)/b^5 + (7*c^2*d^5)/b^3) - ((13*a^7*d^7 + b^
7*c^7 - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 - 245*a^4*b^3*c^3*d^4 + 18
9*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 77*a^6*b*c*d^6)/(2*b) + x*(7*a^6*d^7 +
7*b^6*c^6*d - 42*a*b^5*c^5*d^2 + 105*a^2*b^4*c^4*d^3 - 140*a^3*b^3*c^3*d^4
+ 105*a^4*b^2*c^2*d^5 - 42*a^5*b*c*d^6))/(a^2*b^7 + b^9*x^2 + 2*a*b^8*x) +
(d^7*x^5)/(5*b^3) - (log(a + b*x)*(21*a^5*d^7 - 21*b^5*c^5*d^2 + 105*a*b^4*
c^4*d^3 - 210*a^2*b^3*c^3*d^4 + 210*a^3*b^2*c^2*d^5 - 105*a^4*b*c*d^6))/b^8
```

**sympy [B]** time = 2.95, size = 447, normalized size = 2.42

$$x^4\left(-\frac{3ad^7}{4b^4}+\frac{7cd^6}{4b^3}\right)+x^3\left(\frac{2a^2d^7}{b^5}-\frac{7acd^6}{b^4}+\frac{7c^2d^5}{b^3}\right)+x^2\left(-\frac{5a^3d^7}{b^6}+\frac{21a^2cd^6}{b^5}-\frac{63ac^2d^5}{2b^4}+\frac{35c^3d^4}{2b^3}\right)+x\left(\frac{15a^4d^7}{b^7}-\frac{70a^3cd^6}{b^6}+\frac{126a^2c^2d^5}{b^5}-\frac{105ac^3d^4}{b^4}+\frac{35c^4d^3}{b^3}\right)+\frac{-13a^7d^7+77a^6bcd^6-189a^5b^2c^2d^5+245a^4b^3c^3d^4-175a^3b^4c^4d^3+63a^2b^5c^5d^2-7ab^6c^6d-b^7c^7+x\left(-14a^6bd^7+84a^5b^2cd^6-210a^4b^3c^2d^5+280a^3b^4c^3d^4-210a^2b^5c^4d^3+84ab^6c^5d^2-14b^7c^6d\right)}{2a^2b^8+4ab^9x+2b^{10}x^2}+\frac{d^7x^5}{5b^3}-\frac{21d^2(ad-bc)^5\log(a+bx)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**3,x)
```

```
[Out] x**4*(-3*a*d**7/(4*b**4) + 7*c*d**6/(4*b**3)) + x**3*(2*a**2*d**7/b**5 - 7*
a*c*d**6/b**4 + 7*c**2*d**5/b**3) + x**2*(-5*a**3*d**7/b**6 + 21*a**2*c*d**
```

```
6/b**5 - 63*a*c**2*d**5/(2*b**4) + 35*c**3*d**4/(2*b**3)) + x*(15*a**4*d**7
/b**7 - 70*a**3*c*d**6/b**6 + 126*a**2*c**2*d**5/b**5 - 105*a*c**3*d**4/b**
4 + 35*c**4*d**3/b**3) + (-13*a**7*d**7 + 77*a**6*b*c*d**6 - 189*a**5*b**2*
c**2*d**5 + 245*a**4*b**3*c**3*d**4 - 175*a**3*b**4*c**4*d**3 + 63*a**2*b**
5*c**5*d**2 - 7*a*b**6*c**6*d - b**7*c**7 + x*(-14*a**6*b*d**7 + 84*a**5*b*
*2*c*d**6 - 210*a**4*b**3*c**2*d**5 + 280*a**3*b**4*c**3*d**4 - 210*a**2*b*
*5*c**4*d**3 + 84*a*b**6*c**5*d**2 - 14*b**7*c**6*d))/(2*a**2*b**8 + 4*a*b*
*9*x + 2*b**10*x**2) + d**7*x**5/(5*b**3) - 21*d**2*(a*d - b*c)**5*log(a +
b*x)/b**8
```

## 3.1180 $\int \frac{(c+dx)^7}{(a+bx)^4} \, dx$

**Optimal**. Leaf size=187

$$\frac{7d^6(a+bx)^3(bc-ad)}{3b^8}+\frac{21d^5(a+bx)^2(bc-ad)^2}{2b^8}+\frac{35d^3(bc-ad)^4\log(a+bx)}{b^8}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}-\frac{7d(bc-ad)^6}{2b^8(a+bx)^2}-\frac{(bc}{3b^8(}$$

**Rubi** [A] time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^3(bc-ad)}{3b^8}+\frac{21d^5(a+bx)^2(bc-ad)^2}{2b^8}+\frac{35d^4x(bc-ad)^3}{b^7}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}+\frac{35d^3(bc-ad)^4\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{2b^8(a+bx)^2}-\frac{(bc-ad)^7}{3b^8(a+bx)^3}+\frac{d^7(a+bx)^4}{4b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^4,x]
```

```
[Out] (35*d^4*(b*c - a*d)^3*x)/b^7 - (b*c - a*d)^7/(3*b^8*(a + b*x)^3) - (7*d*(b*
c - a*d)^6)/(2*b^8*(a + b*x)^2) - (21*d^2*(b*c - a*d)^5)/(b^8*(a + b*x)) +
(21*d^5*(b*c - a*d)^2*(a + b*x)^2)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^3
)/(3*b^8) + (d^7*(a + b*x)^4)/(4*b^8) + (35*d^3*(b*c - a*d)^4*Log[a + b*x])
/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^4}\,dx &= \int \left(\frac{35d^4(bc-ad)^3}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^4}+\frac{7d(bc-ad)^6}{b^7(a+bx)^3}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^2}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)}+\frac{21d^5(b}{}\right.\\ &= \frac{35d^4(bc-ad)^3x}{b^7}-\frac{(bc-ad)^7}{3b^8(a+bx)^3}-\frac{7d(bc-ad)^6}{2b^8(a+bx)^2}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}+\frac{21d^5(bc-ad)^2(a+bx)^2}{2b^8}+\end{aligned}$$

**Mathematica** [A] time = 0.11, size = 199, normalized size = 1.06

$$\frac{6b^2d^5x^2\left(10a^2d^2-28abcd+21b^2c^2\right)+12bd^4x\left(-20a^3d^3+70a^2bcd^2-84ab^2c^2d+35b^3c^3\right)+4b^3d^6x^3(7bc-4ad)+420d^3(bc-ad)^4\log(a+bx)+\frac{252d^2(ad-bc)^5}{a+bx}-\frac{42d(bc-ad)^6}{(a+bx)^2}-\frac{4(bc-ad)^7}{(a+bx)^3}+3b^4d^7x^4}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^4,x]
```

```
[Out] (12*b*d^4*(35*b^3*c^3 - 84*a*b^2*c^2*d + 70*a^2*b*c*d^2 - 20*a^3*d^3)*x + 6
*b^2*d^5*(21*b^2*c^2 - 28*a*b*c*d + 10*a^2*d^2)*x^2 + 4*b^3*d^6*(7*b*c - 4*
a*d)*x^3 + 3*b^4*d^7*x^4 - (4*(b*c - a*d)^7)/(a + b*x)^3 - (42*d*(b*c - a*d
)^6)/(a + b*x)^2 + (252*d^2*(-(b*c) + a*d)^5)/(a + b*x) + 420*d^3*(b*c - a*
d)^4*Log[a + b*x])/(12*b^8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^4, x]
```

**fricas [B]** time = 1.51, size = 739, normalized size = 3.95

$$\frac{3 b^7 d^7 x^7 - 4 b^7 c^7 - 14 a b^6 c^6 d - 84 a^2 b^5 c^5 d^2 + 770 a^3 b^4 c^4 d^3 - 1820 a^4 b^3 c^3 d^4 + 1974 a^5 b^2 c^2 d^5 - 1036 a^6 b c d^6 + 214 a^7 d^7 + 7 \left(4 b^7 c d^6 - a b^6 d^7\right) x^6 + 21 \left(6 b^7 c^2 d^5 - 4 a b^6 c d^6 + a^2 b^5 d^7\right) x^5 + 105 \left(4 b^7 c^3 d^4 - 6 a b^6 c^2 d^5 + 4 a^2 b^5 c d^6 - a^3 b^4 d^7\right) x^4 + 2 \left(630 a b^6 c^3 d^4 - 1323 a^2 b^5 c^2 d^5 + 1022 a^3 b^4 c d^6 - 278 a^4 b^3 d^7\right) x^3 - 6 \left(42 b^7 c^5 d^2 - 210 a b^6 c^4 d^3 + 210 a^2 b^5 c^3 d^4 + 63 a^3 b^4 c^2 d^5 - 182 a^4 b^3 c d^6 + 68 a^5 b^2 d^7\right) x^2 - 6 \left(7 b^7 c^6 d + 42 a b^6 c^5 d^2 - 315 a^2 b^5 c^4 d^3 + 630 a^3 b^4 c^3 d^4 - 567 a^4 b^3 c^2 d^5 + 238 a^5 b^2 c d^6 - 37 a^6 b d^7\right) x + 420 \left(a^3 b^4 c^4 d^3 - 4 a^4 b^3 c^3 d^4 + 6 a^5 b^2 c^2 d^5 - 4 a^6 b c d^6 + a^7 d^7 + \left(b^7 c^4 d^3 - 4 a b^6 c^3 d^4 + 6 a^2 b^5 c^2 d^5 - 4 a^3 b^4 c d^6 + a^4 b^3 d^7\right) x^3 + 3 \left(a b^6 c^4 d^3 - 4 a^2 b^5 c^3 d^4 + 6 a^3 b^4 c^2 d^5 - 4 a^4 b^3 c d^6 + a^5 b^2 d^7\right) x^2 + 3 \left(a^2 b^5 c^4 d^3 - 4 a^3 b^4 c^3 d^4 + 6 a^4 b^3 c^2 d^5 - 4 a^5 b^2 c d^6 + a^6 b d^7\right) x\right) \log\left(b x + a\right)}{12 \left(b^{11} x^3 + 3 a b^{10} x^2 + 3 a^2 b^9 x + a^3 b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*d^7*x^7 - 4*b^7*c^7 - 14*a*b^6*c^6*d - 84*a^2*b^5*c^5*d^2 + 770
*a^3*b^4*c^4*d^3 - 1820*a^4*b^3*c^3*d^4 + 1974*a^5*b^2*c^2*d^5 - 1036*a^6*b
*c*d^6 + 214*a^7*d^7 + 7*(4*b^7*c*d^6 - a*b^6*d^7)*x^6 + 21*(6*b^7*c^2*d^5
- 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 105*(4*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 +
 4*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 2*(630*a*b^6*c^3*d^4 - 1323*a^2*b^5*c
^2*d^5 + 1022*a^3*b^4*c*d^6 - 278*a^4*b^3*d^7)*x^3 - 6*(42*b^7*c^5*d^2 - 21
0*a*b^6*c^4*d^3 + 210*a^2*b^5*c^3*d^4 + 63*a^3*b^4*c^2*d^5 - 182*a^4*b^3*c*
d^6 + 68*a^5*b^2*d^7)*x^2 - 6*(7*b^7*c^6*d + 42*a*b^6*c^5*d^2 - 315*a^2*b^5
*c^4*d^3 + 630*a^3*b^4*c^3*d^4 - 567*a^4*b^3*c^2*d^5 + 238*a^5*b^2*c*d^6 -
37*a^6*b*d^7)*x + 420*(a^3*b^4*c^4*d^3 - 4*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*
d^5 - 4*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*
c^2*d^5 - 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 3*(a*b^6*c^4*d^3 - 4*a^2*b^5
*c^3*d^4 + 6*a^3*b^4*c^2*d^5 - 4*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 3*(a^2*
b^5*c^4*d^3 - 4*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 - 4*a^5*b^2*c*d^6 + a^6
*b*d^7)*x)*log(b*x + a))/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2*b^9*x + a^3*b^8)
```

**giac [B]** time = 1.32, size = 470, normalized size = 2.51

$$\frac{35 \left(b^4 c^4 d^3 - 4 a b^3 c^3 d^4 + 6 a^2 b^2 c^2 d^5 - 4 a^3 b c d^6 + a^4 d^7\right) \log\left(|bx + a|\right)}{b^8} - \frac{2 b^7 c^7 + 7 a b^6 c^6 d + 42 a^2 b^5 c^5 d^2 - 385 a^3 b^4 c^4 d^3 + 910 a^4 b^3 c^3 d^4 - 987 a^5 b^2 c^2 d^5 + 518 a^6 b c d^6 - 107 a^7 d^7 + 126 \left(b^7 c^5 d^2 - 5 a b^6 c^4 d^3 + 10 a^2 b^5 c^3 d^4 - 10 a^3 b^4 c^2 d^5 + 5 a^4 b^3 c d^6 - a^5 b^2 d^7\right) x^2 + 21 \left(b^7 c^6 d + 6 a b^6 c^5 d^2 - 45 a^2 b^5 c^4 d^3 + 100 a^3 b^4 c^3 d^4 - 105 a^4 b^3 c^2 d^5 + 54 a^5 b^2 c d^6 - 11 a^6 b d^7\right) x}{6 (bx + a)^3 b^8} + \frac{3 b^{12} d^7 x^4 + 28 b^{12} c d^6 x^3 - 16 a b^{11} d^7 x^3 + 126 b^{12} c^2 d^5 x^2 - 168 a b^{11} c d^6 x^2 + 60 a^2 b^{10} d^7 x^2 + 420 b^{12} c^3 d^4 x - 1008 a b^{11} c^2 d^5 x + 840 a^2 b^{10} c d^6 x - 240 a^3 b^9 d^7 x}{12 b^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 35*(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4
*d^7)*log(abs(b*x + a))/b^8 - 1/6*(2*b^7*c^7 + 7*a*b^6*c^6*d + 42*a^2*b^5*c
^5*d^2 - 385*a^3*b^4*c^4*d^3 + 910*a^4*b^3*c^3*d^4 - 987*a^5*b^2*c^2*d^5 +
518*a^6*b*c*d^6 - 107*a^7*d^7 + 126*(b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2
*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 21
*(b^7*c^6*d + 6*a*b^6*c^5*d^2 - 45*a^2*b^5*c^4*d^3 + 100*a^3*b^4*c^3*d^4 -
105*a^4*b^3*c^2*d^5 + 54*a^5*b^2*c*d^6 - 11*a^6*b*d^7)*x)/((b*x + a)^3*b^8)
 + 1/12*(3*b^12*d^7*x^4 + 28*b^12*c*d^6*x^3 - 16*a*b^11*d^7*x^3 + 126*b^12*
c^2*d^5*x^2 - 168*a*b^11*c*d^6*x^2 + 60*a^2*b^10*d^7*x^2 + 420*b^12*c^3*d^4
*x - 1008*a*b^11*c^2*d^5*x + 840*a^2*b^10*c*d^6*x - 240*a^3*b^9*d^7*x)/b^16
```

**maple [B]** time = 0.01, size = 622, normalized size = 3.33

$$\frac{d^7x^4}{4b^4}+\frac{a^7d^7}{3(bx+a)^3b^8}-\frac{7a^6cd^6}{3(bx+a)^3b^7}+\frac{7a^5c^2d^5}{(bx+a)^3b^6}-\frac{35a^4c^3d^4}{3(bx+a)^3b^5}+\frac{35a^3c^4d^3}{3(bx+a)^3b^4}-\frac{7a^2c^5d^2}{(bx+a)^3b^3}+\frac{7ac^6d}{3(bx+a)^3b^2}-\frac{4ad^7x^3}{3b^5}-\frac{c^7}{3(bx+a)^3b}+\frac{7cd^6x^3}{3b^4}-\frac{7a^6d^7}{2(bx+a)^2b^8}+\frac{21a^5cd^6}{(bx+a)^2b^7}-\frac{105a^4c^2d^5}{2(bx+a)^2b^6}+\frac{70a^3c^3d^4}{(bx+a)^2b^5}-\frac{105a^2c^4d^3}{2(bx+a)^2b^4}+\frac{5a^2d^7x^2}{b^6}+\frac{21ac^5d^2}{(bx+a)^2b^3}-\frac{14acd^6x^2}{b^5}-\frac{7c^6d}{2(bx+a)^2b^2}+\frac{21c^2d^5x^2}{2b^4}+\frac{21a^5d^7}{(bx+a)b^8}-\frac{105a^4cd^6}{(bx+a)b^7}+\frac{35a^4d^7\ln(bx+a)}{b^8}+\frac{210a^3c^2d^5}{(bx+a)b^6}-\frac{140a^3cd^6\ln(bx+a)}{b^7}-\frac{20a^3d^7x}{b^7}-\frac{210a^2c^3d^4}{(bx+a)b^5}+\frac{210a^2c^2d^5\ln(bx+a)}{b^6}+\frac{70a^2cd^6x}{b^6}+\frac{105ac^4d^3}{(bx+a)b^4}-\frac{140ac^3d^4\ln(bx+a)}{b^5}-\frac{84ac^2d^5x}{b^5}-\frac{21c^5d^2}{(bx+a)b^3}+\frac{35c^4d^3\ln(bx+a)}{b^4}+\frac{35c^3d^4x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^4,x)
```

```
[Out] 210/b^6*d^5/(b*x+a)*a^3*c^2-210/b^5*d^4/(b*x+a)*a^2*c^3+105/b^4*d^3/(b*x+a)
*a*c^4-7/2/b^8*d^7/(b*x+a)^2*a^6-7/2/b^2*d/(b*x+a)^2*c^6+35/b^8*d^7*ln(b*x+
a)*a^4+35/b^4*d^3*ln(b*x+a)*c^4+21/b^8*d^7/(b*x+a)*a^5-21/b^3*d^2/(b*x+a)*c
^5-4/3*d^7/b^5*x^3*a+7/3*d^6/b^4*x^3*c+5*d^7/b^6*x^2*a^2+21/2*d^5/b^4*x^2*c
^2-20*d^7/b^7*a^3*x+35*d^4/b^4*c^3*x+1/3/b^8/(b*x+a)^3*a^7*d^7+1/4*d^7/b^4*
x^4-1/3/b/(b*x+a)^3*c^7-84*d^5/b^5*a*c^2*x-7/3/b^7/(b*x+a)^3*a^6*c*d^6+7/b^
6/(b*x+a)^3*a^5*c^2*d^5-35/3/b^5/(b*x+a)^3*a^4*c^3*d^4+35/3/b^4/(b*x+a)^3*a
^3*c^4*d^3-7/b^3/(b*x+a)^3*a^2*c^5*d^2+7/3/b^2/(b*x+a)^3*a*c^6*d+21/b^7*d^6
/(b*x+a)^2*a^5*c-105/2/b^6*d^5/(b*x+a)^2*a^4*c^2+70/b^5*d^4/(b*x+a)^2*a^3*c
^3-105/2/b^4*d^3/(b*x+a)^2*a^2*c^4+21/b^3*d^2/(b*x+a)^2*a*c^5-14*d^6/b^5*x^
2*a*c+70*d^6/b^6*a^2*c*x-140/b^7*d^6*ln(b*x+a)*a^3*c+210/b^6*d^5*ln(b*x+a)*
a^2*c^2-140/b^5*d^4*ln(b*x+a)*a*c^3-105/b^7*d^6/(b*x+a)*a^4*c
```

**maxima [B]** time = 1.63, size = 484, normalized size = 2.59

$$-\frac{2b^7c^7+7ab^6c^6d+42a^2b^5c^5d^2-385a^3b^4c^4d^3+910a^4b^3c^3d^4-987a^5b^2c^2d^5+518a^6bcd^6-107a^7d^7+126\left(b^7c^5d^2-5ab^6c^4d^3+10a^2b^5c^3d^4-10a^3b^4c^2d^5+5a^4b^3cd^6-a^5b^2d^7\right)x^2+21\left(b^7c^6d+6ab^6c^5d^2-45a^2b^5c^4d^3+100a^3b^4c^3d^4-105a^4b^3c^2d^5+54a^5b^2cd^6-11a^6bd^7\right)x}{6\left(b^{11}x^3+3ab^{10}x^2+3a^2b^9x+a^3b^8\right)}+\frac{3b^3d^7x^4+4\left(7b^3cd^6-4ab^2d^7\right)x^3+6\left(21b^3c^2d^5-28ab^2cd^6+10a^2bd^7\right)x^2+12\left(35b^3c^3d^4-84ab^2c^2d^5+70a^2bcd^6-20a^3d^7\right)x}{12b^7}+\frac{35\left(b^4c^4d^3-4ab^3c^3d^4+6a^2b^2c^2d^5-4a^3bcd^6+a^4d^7\right)\log(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^7*c^7 + 7*a*b^6*c^6*d + 42*a^2*b^5*c^5*d^2 - 385*a^3*b^4*c^4*d^3
+ 910*a^4*b^3*c^3*d^4 - 987*a^5*b^2*c^2*d^5 + 518*a^6*b*c*d^6 - 107*a^7*d^7
 + 126*(b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2
```

```
*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 21*(b^7*c^6*d + 6*a*b^6*c^5*d^2
 - 45*a^2*b^5*c^4*d^3 + 100*a^3*b^4*c^3*d^4 - 105*a^4*b^3*c^2*d^5 + 54*a^5*
b^2*c*d^6 - 11*a^6*b*d^7)*x)/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2*b^9*x + a^3*b
^8) + 1/12*(3*b^3*d^7*x^4 + 4*(7*b^3*c*d^6 - 4*a*b^2*d^7)*x^3 + 6*(21*b^3*c
^2*d^5 - 28*a*b^2*c*d^6 + 10*a^2*b*d^7)*x^2 + 12*(35*b^3*c^3*d^4 - 84*a*b^2
*c^2*d^5 + 70*a^2*b*c*d^6 - 20*a^3*d^7)*x)/b^7 + 35*(b^4*c^4*d^3 - 4*a*b^3*
c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7)*log(b*x + a)/b^8
```

**mupad [B]** time = 0.29, size = 559, normalized size = 2.99

$$x^2\left(\frac{2a\left(\frac{4ad^7}{b^5}-\frac{7cd^6}{b^4}\right)}{b}-\frac{3a^2d^7}{b^6}+\frac{21c^2d^5}{2b^4}\right)-x^3\left(\frac{4ad^7}{3b^5}-\frac{7cd^6}{3b^4}\right)-\frac{\frac{-107a^7d^7+518a^6bcd^6-987a^5b^2c^2d^5+910a^4b^3c^3d^4-385a^3b^4c^4d^3+42a^2b^5c^5d^2+7ab^6c^6d+2b^7c^7}{6b}+x\left(-\frac{77a^6d^7}{2}+189a^5bcd^6-\frac{735a^4b^2c^2d^5}{2}+350a^3b^3c^3d^4-\frac{315a^2b^4c^4d^3}{2}+21ab^5c^5d^2+\frac{7b^6c^6d}{2}\right)-x^2\left(21a^5bd^7-105a^4b^2cd^6+210a^3b^3c^2d^5-210a^2b^4c^3d^4+105ab^5c^4d^3-21b^6c^5d^2\right)}{a^3b^7+3a^2b^8x+3ab^9x^2+b^{10}x^3}-x\left(\frac{4a\left(\frac{4a\left(\frac{4ad^7}{b^5}-\frac{7cd^6}{b^4}\right)}{b}-\frac{6a^2d^7}{b^6}+\frac{21c^2d^5}{b^4}\right)}{b}+\frac{4a^3d^7}{b^7}-\frac{35c^3d^4}{b^4}-\frac{6a^2\left(\frac{4ad^7}{b^5}-\frac{7cd^6}{b^4}\right)}{b^2}\right)+\frac{\ln(a+bx)\left(35a^4d^7-140a^3bcd^6+210a^2b^2c^2d^5-140ab^3c^3d^4+35b^4c^4d^3\right)}{b^8}+\frac{d^7x^4}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^4,x)
```

```
[Out] x^2*((2*a*((4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b - (3*a^2*d^7)/b^6 + (21*c^2*d^
5)/(2*b^4)) - x^3*((4*a*d^7)/(3*b^5) - (7*c*d^6)/(3*b^4)) - ((2*b^7*c^7 - 1
07*a^7*d^7 + 42*a^2*b^5*c^5*d^2 - 385*a^3*b^4*c^4*d^3 + 910*a^4*b^3*c^3*d^4
 - 987*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d + 518*a^6*b*c*d^6)/(6*b) + x*((7*b^6
*c^6*d)/2 - (77*a^6*d^7)/2 + 21*a*b^5*c^5*d^2 - (315*a^2*b^4*c^4*d^3)/2 + 3
50*a^3*b^3*c^3*d^4 - (735*a^4*b^2*c^2*d^5)/2 + 189*a^5*b*c*d^6) - x^2*(21*a
^5*b*d^7 - 21*b^6*c^5*d^2 + 105*a*b^5*c^4*d^3 - 105*a^4*b^2*c*d^6 - 210*a^2
*b^4*c^3*d^4 + 210*a^3*b^3*c^2*d^5))/(a^3*b^7 + b^10*x^3 + 3*a^2*b^8*x + 3*
a*b^9*x^2) - x*((4*a*((4*a*((4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b - (6*a^2*d^7)
/b^6 + (21*c^2*d^5)/b^4))/b + (4*a^3*d^7)/b^7 - (35*c^3*d^4)/b^4 - (6*a^2*(
(4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b^2) + (log(a + b*x)*(35*a^4*d^7 + 35*b^4*c
^4*d^3 - 140*a*b^3*c^3*d^4 + 210*a^2*b^2*c^2*d^5 - 140*a^3*b*c*d^6))/b^8 +
(d^7*x^4)/(4*b^4)
```

**sympy [B]** time = 6.12, size = 474, normalized size = 2.53

$$x^3\left(-\frac{4ad^7}{3b^5}+\frac{7cd^6}{3b^4}\right)+x^2\left(\frac{5a^2d^7}{b^6}-\frac{14acd^6}{b^5}+\frac{21c^2d^5}{2b^4}\right)+x\left(-\frac{20a^3d^7}{b^7}+\frac{70a^2cd^6}{b^6}-\frac{84ac^2d^5}{b^5}+\frac{35c^3d^4}{b^4}\right)+\frac{107a^7d^7-518a^6bcd^6+987a^5b^2c^2d^5-910a^4b^3c^3d^4+385a^3b^4c^4d^3-42a^2b^5c^5d^2-7ab^6c^6d-2b^7c^7+x^2\left(126a^5b^2d^7-630a^4b^3cd^6+1260a^3b^4c^2d^5-1260a^2b^5c^3d^4+630ab^6c^4d^3-126b^7c^5d^2\right)+x\left(231a^6bd^7-1134a^5b^2cd^6+2205a^4b^3c^2d^5-2100a^3b^4c^3d^4+945a^2b^5c^4d^3-126ab^6c^5d^2-21b^7c^6d\right)}{6a^3b^8+18a^2b^9x+18ab^{10}x^2+6b^{11}x^3}+\frac{d^7x^4}{4b^4}+\frac{35d^3(ad-bc)^4\log(a+bx)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**4,x)
```

```
[Out] x**3*(-4*a*d**7/(3*b**5) + 7*c*d**6/(3*b**4)) + x**2*(5*a**2*d**7/b**6 - 14
*a*c*d**6/b**5 + 21*c**2*d**5/(2*b**4)) + x*(-20*a**3*d**7/b**7 + 70*a**2*c
*d**6/b**6 - 84*a*c**2*d**5/b**5 + 35*c**3*d**4/b**4) + (107*a**7*d**7 - 51
8*a**6*b*c*d**6 + 987*a**5*b**2*c**2*d**5 - 910*a**4*b**3*c**3*d**4 + 385*a
**3*b**4*c**4*d**3 - 42*a**2*b**5*c**5*d**2 - 7*a*b**6*c**6*d - 2*b**7*c**7
 + x**2*(126*a**5*b**2*d**7 - 630*a**4*b**3*c*d**6 + 1260*a**3*b**4*c**2*d*
*5 - 1260*a**2*b**5*c**3*d**4 + 630*a*b**6*c**4*d**3 - 126*b**7*c**5*d**2)
+ x*(231*a**6*b*d**7 - 1134*a**5*b**2*c*d**6 + 2205*a**4*b**3*c**2*d**5 - 2
```

```
100*a**3*b**4*c**3*d**4 + 945*a**2*b**5*c**4*d**3 - 126*a*b**6*c**5*d**2 -
21*b**7*c**6*d))/(6*a**3*b**8 + 18*a**2*b**9*x + 18*a*b**10*x**2 + 6*b**11*
x**3) + d**7*x**4/(4*b**4) + 35*d**3*(a*d - b*c)**4*log(a + b*x)/b**8
```

## 3.1181 $\int \frac{(c+dx)^7}{(a+bx)^5} dx$

**Optimal**. Leaf size=187

$$\frac{7d^6(a+bx)^2(bc-ad)}{2b^8}+\frac{35d^4(bc-ad)^3\log(a+bx)}{b^8}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{(bc-ad)^7}{4b^8(a+bx)}$$

**Rubi** [A] time = 0.20, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^2(bc-ad)}{2b^8}+\frac{21d^5x(bc-ad)^2}{b^7}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}+\frac{35d^4(bc-ad)^3\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}+\frac{d^7(a+bx)^3}{3b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] (21*d^5*(b*c - a*d)^2*x)/b^7 - (b*c - a*d)^7/(4*b^8*(a + b*x)^4) - (7*d*(b*
c - a*d)^6)/(3*b^8*(a + b*x)^3) - (21*d^2*(b*c - a*d)^5)/(2*b^8*(a + b*x)^2
) - (35*d^3*(b*c - a*d)^4)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*(a + b*x)^2
)/(2*b^8) + (d^7*(a + b*x)^3)/(3*b^8) + (35*d^4*(b*c - a*d)^3*Log[a + b*x])
/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^5}\,dx &= \int \left(\frac{21d^5(bc-ad)^2}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^5}+\frac{7d(bc-ad)^6}{b^7(a+bx)^4}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^3}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^2}+\frac{35d^4}{b^7(}\right.\\ &= \frac{21d^5(bc-ad)^2x}{b^7}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}+\frac{7d^6(bc}{}\end{aligned}$$

**Mathematica** [A] time = 0.11, size = 173, normalized size = 0.93

$$\frac{12bd^5x\left(15a^2d^2-35abcd+21b^2c^2\right)+6b^2d^6x^2(7bc-5ad)+420d^4(bc-ad)^3\log(a+bx)-\frac{420d^3(bc-ad)^4}{a+bx}+\frac{126d^2(ad-bc)^5}{(a+bx)^2}-\frac{28d(bc-ad)^6}{(a+bx)^3}-\frac{3(bc-ad)^7}{(a+bx)^4}+4b^3d^7x^3}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] (12*b*d^5*(21*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2)*x + 6*b^2*d^6*(7*b*c - 5*a
*d)*x^2 + 4*b^3*d^7*x^3 - (3*(b*c - a*d)^7)/(a + b*x)^4 - (28*d*(b*c - a*d)
^6)/(a + b*x)^3 + (126*d^2*(-(b*c) + a*d)^5)/(a + b*x)^2 - (420*d^3*(b*c -
a*d)^4)/(a + b*x) + 420*d^4*(b*c - a*d)^3*Log[a + b*x])/(12*b^8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^5, x]
```

**fricas [B]** time = 1.58, size = 754, normalized size = 4.03

$$\frac{4 b^7 d^7 x^7 - 3 b^7 c^7 - 7 a b^6 c^6 d - 21 a^2 b^5 c^5 d^2 - 105 a^3 b^4 c^4 d^3 + 875 a^4 b^3 c^3 d^4 - 1617 a^5 b^2 c^2 d^5 + 1197 a^6 b c d^6 - 319 a^7 d^7 + 14 \left(3 b^7 c d^6 - a b^6 d^7\right) x^6 + 84 \left(3 b^7 c^2 d^5 - 3 a b^6 c d^6 + a^2 b^5 d^7\right) x^5 + 4 \left(252 a b^6 c^2 d^5 - 357 a^2 b^5 c d^6 + 139 a^3 b^4 d^7\right) x^4 - 4 \left(105 b^7 c^4 d^3 - 420 a b^6 c^3 d^4 + 252 a^2 b^5 c^2 d^5 + 168 a^3 b^4 c d^6 - 136 a^4 b^3 d^7\right) x^3 - 6 \left(21 b^7 c^5 d^2 + 105 a b^6 c^4 d^3 - 630 a^2 b^5 c^3 d^4 + 882 a^3 b^4 c^2 d^5 - 462 a^4 b^3 c d^6 + 74 a^5 b^2 d^7\right) x^2 - 4 \left(7 b^7 c^6 d + 21 a b^6 c^5 d^2 + 105 a^2 b^5 c^4 d^3 - 770 a^3 b^4 c^3 d^4 + 1302 a^4 b^3 c^2 d^5 - 882 a^5 b^2 c d^6 + 214 a^6 b d^7\right) x + 420 \left(a^4 b^3 c^3 d^4 - 3 a^5 b^2 c^2 d^5 + 3 a^6 b c d^6 - a^7 d^7 + \left(b^7 c^3 d^4 - 3 a b^6 c^2 d^5 + 3 a^2 b^5 c d^6 - a^3 b^4 d^7\right) x^4 + 4 \left(a b^6 c^3 d^4 - 3 a^2 b^5 c^2 d^5 + 3 a^3 b^4 c d^6 - a^4 b^3 d^7\right) x^3 + 6 \left(a^2 b^5 c^3 d^4 - 3 a^3 b^4 c^2 d^5 + 3 a^4 b^3 c d^6 - a^5 b^2 d^7\right) x^2 + 4 \left(a^3 b^4 c^3 d^4 - 3 a^4 b^3 c^2 d^5 + 3 a^5 b^2 c d^6 - a^6 b d^7\right) x\right) \log\left(b x + a\right)}{12 \left(b^{12} x^4 + 4 a b^{11} x^3 + 6 a^2 b^{10} x^2 + 4 a^3 b^9 x + a^4 b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^7*d^7*x^7 - 3*b^7*c^7 - 7*a*b^6*c^6*d - 21*a^2*b^5*c^5*d^2 - 105*
a^3*b^4*c^4*d^3 + 875*a^4*b^3*c^3*d^4 - 1617*a^5*b^2*c^2*d^5 + 1197*a^6*b*c
*d^6 - 319*a^7*d^7 + 14*(3*b^7*c*d^6 - a*b^6*d^7)*x^6 + 84*(3*b^7*c^2*d^5 -
 3*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 4*(252*a*b^6*c^2*d^5 - 357*a^2*b^5*c*d^
6 + 139*a^3*b^4*d^7)*x^4 - 4*(105*b^7*c^4*d^3 - 420*a*b^6*c^3*d^4 + 252*a^2
*b^5*c^2*d^5 + 168*a^3*b^4*c*d^6 - 136*a^4*b^3*d^7)*x^3 - 6*(21*b^7*c^5*d^2
 + 105*a*b^6*c^4*d^3 - 630*a^2*b^5*c^3*d^4 + 882*a^3*b^4*c^2*d^5 - 462*a^4*
b^3*c*d^6 + 74*a^5*b^2*d^7)*x^2 - 4*(7*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 105*a
^2*b^5*c^4*d^3 - 770*a^3*b^4*c^3*d^4 + 1302*a^4*b^3*c^2*d^5 - 882*a^5*b^2*c
*d^6 + 214*a^6*b*d^7)*x + 420*(a^4*b^3*c^3*d^4 - 3*a^5*b^2*c^2*d^5 + 3*a^6*
b*c*d^6 - a^7*d^7 + (b^7*c^3*d^4 - 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*
b^4*d^7)*x^4 + 4*(a*b^6*c^3*d^4 - 3*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 - a^4
*b^3*d^7)*x^3 + 6*(a^2*b^5*c^3*d^4 - 3*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*d^6 -
a^5*b^2*d^7)*x^2 + 4*(a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6
 - a^6*b*d^7)*x)*log(b*x + a))/(b^12*x^4 + 4*a*b^11*x^3 + 6*a^2*b^10*x^2 +
4*a^3*b^9*x + a^4*b^8)
```

**giac [B]** time = 1.29, size = 660, normalized size = 3.53

$$\frac{\left(2 d^7 + \frac{21 \left(b^2 c d^6 - a b d^7\right)}{(bx+a) b} + \frac{126 \left(b^4 c^2 d^5 - 2 a b^3 c d^6 + a^2 b^2 d^7\right)}{(bx+a)^2 b^2}\right) (bx+a)^3}{6 b^8} - \frac{35 \left(b^3 c^3 d^4 - 3 a b^2 c^2 d^5 + 3 a^2 b c d^6 - a^3 d^7\right) \log\left(\frac{|bx+a|}{(bx+a)^2 |b|}\right)}{b^8} - \frac{\frac{3 b^{43} c^7}{(bx+a)^4} + \frac{28 b^{42} c^6 d}{(bx+a)^3} - \frac{21 a b^{42} c^6 d}{(bx+a)^4} + \frac{126 b^{41} c^5 d^2}{(bx+a)^2} - \frac{168 a b^{41} c^5 d^2}{(bx+a)^3} + \frac{63 a^2 b^{41} c^5 d^2}{(bx+a)^4} + \frac{420 b^{40} c^4 d^3}{bx+a} - \frac{630 a b^{40} c^4 d^3}{(bx+a)^2} + \frac{420 a^2 b^{40} c^4 d^3}{(bx+a)^3} - \frac{105 a^3 b^{40} c^4 d^3}{(bx+a)^4} - \frac{1680 a b^{39} c^3 d^4}{bx+a} + \frac{1260 a^2 b^{39} c^3 d^4}{(bx+a)^2} - \frac{560 a^3 b^{39} c^3 d^4}{(bx+a)^3} + \frac{105 a^4 b^{39} c^3 d^4}{(bx+a)^4} + \frac{2520 a^2 b^{38} c^2 d^5}{bx+a} - \frac{1260 a^3 b^{38} c^2 d^5}{(bx+a)^2} + \frac{420 a^4 b^{38} c^2 d^5}{(bx+a)^3} - \frac{63 a^5 b^{38} c^2 d^5}{(bx+a)^4} - \frac{1680 a^3 b^{37} c d^6}{bx+a} + \frac{630 a^4 b^{37} c d^6}{(bx+a)^2} - \frac{168 a^5 b^{37} c d^6}{(bx+a)^3} + \frac{21 a^6 b^{37} c d^6}{(bx+a)^4} + \frac{420 a^4 b^{36} d^7}{bx+a} - \frac{126 a^5 b^{36} d^7}{(bx+a)^2} + \frac{28 a^6 b^{36} d^7}{(bx+a)^3} - \frac{3 a^7 b^{36} d^7}{(bx+a)^4}}{12 b^{44}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="giac")

[Out] 1/6*(2*d^7 + 21*(b^2*c*d^6 - a*b*d^7)/((b*x + a)*b) + 126*(b^4*c^2*d^5 - 2*a*b^3*c*d^6 + a^2*b^2*d^7)/((b*x + a)^2*b^2))*(b*x + a)^3/b^8 - 35*(b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^8 - 1/12*(3*b^43*c^7/(b*x + a)^4 + 28*b^42*c^6*d/(b*x + a)^3 - 21*a*b^42*c^6*d/(b*x + a)^4 + 126*b^41*c^5*d^2/(b*x + a)^2 - 168*a*b^41*c^5*d^2/(b*x + a)^3 + 63*a^2*b^41*c^5*d^2/(b*x + a)^4 + 420*b^40*c^4*d^3/(b*x + a) - 630*a*b^40*c^4*d^3/(b*x + a)^2 + 420*a^2*b^40*c^4*d^3/(b*x + a)^3 - 105*a^3*b^40*c^4*d^3/(b*x + a)^4 - 1680*a*b^39*c^3*d^4/(b*x + a) + 1260*a^2*b^39*c^3*d^4/(b*x + a)^2 - 560*a^3*b^39*c^3*d^4/(b*x + a)^3 + 105*a^4*b^39*c^3*d^4/(b*x + a)^4 + 2520*a^2*b^38*c^2*d^5/(b*x + a) - 1260*a^3*b^38*c^2*d^5/(b*x + a)^2 + 420*a^4*b^38*c^2*d^5/(b*x + a)^3 - 63*a^5*b^38*c^2*d^5/(b*x + a)^4 - 1680*a^3*b^37*c*d^6/(b*x + a) + 630*a^4*b^37*c*d^6/(b*x + a)^2 - 168*a^5*b^37*c*d^6/(b*x + a)^3 + 21*a^6*b^37*c*d^6/(b*x + a)^4 + 420*a^4*b^36*d^7/(b*x + a) - 126*a^5*b^36*d^7/(b*x + a)^2 + 28*a^6*b^36*d^7/(b*x + a)^3 - 3*a^7*b^36*d^7/(b*x + a)^4)/b^44

**maple** [B] time = 0.02, size = 641, normalized size = 3.43

$$\frac{a^7 d^7}{4 (bx+a)^4 b^8}-\frac{7 a^6 c d^6}{4 (bx+a)^4 b^7}+\frac{21 a^5 c^2 d^5}{4 (bx+a)^4 b^6}-\frac{35 a^4 c^3 d^4}{4 (bx+a)^4 b^5}+\frac{35 a^3 c^4 d^3}{4 (bx+a)^4 b^4}-\frac{21 a^2 c^5 d^2}{4 (bx+a)^4 b^3}+\frac{7 a c^6 d}{4 (bx+a)^4 b^2}-\frac{c^7}{4 (bx+a)^4 b}-\frac{7 a^6 d^7}{3 (bx+a)^3 b^8}+\frac{14 a^5 c d^6}{(bx+a)^3 b^7}-\frac{35 a^4 c^2 d^5}{(bx+a)^3 b^6}+\frac{140 a^3 c^3 d^4}{3 (bx+a)^3 b^5}-\frac{35 a^2 c^4 d^3}{(bx+a)^3 b^4}+\frac{14 a c^5 d^2}{(bx+a)^3 b^3}-\frac{7 c^6 d}{3 (bx+a)^3 b^2}+\frac{d^7 x^3}{3 b^5}+\frac{21 a^5 d^7}{2 (bx+a)^2 b^8}-\frac{105 a^4 c d^6}{2 (bx+a)^2 b^7}+\frac{105 a^3 c^2 d^5}{(bx+a)^2 b^6}-\frac{105 a^2 c^3 d^4}{(bx+a)^2 b^5}+\frac{105 a c^4 d^3}{2 (bx+a)^2 b^4}-\frac{5 a d^7 x^2}{2 b^6}-\frac{21 c^5 d^2}{2 (bx+a)^2 b^3}+\frac{7 c d^6 x^2}{2 b^5}-\frac{35 a^4 d^7}{(bx+a) b^8}+\frac{140 a^3 c d^6}{(bx+a) b^7}-\frac{35 a^3 d^7 \ln (bx+a)}{b^8}-\frac{210 a^2 c^2 d^5}{(bx+a) b^6}+\frac{105 a^2 c d^6 \ln (bx+a)}{b^7}+\frac{15 a^2 d^7 x}{b^7}+\frac{140 a c^3 d^4}{(bx+a) b^5}-\frac{105 a c^2 d^5 \ln (bx+a)}{b^6}-\frac{35 a c d^6 x}{b^6}-\frac{35 c^4 d^3}{(bx+a) b^4}+\frac{35 c^3 d^4 \ln (bx+a)}{b^5}+\frac{21 c^2 d^5 x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^7/(b*x+a)^5,x)

[Out] 21/2/b^8*d^7/(b*x+a)^2*a^5-21/2/b^3*d^2/(b*x+a)^2*c^5-35/b^8*d^7*ln(b*x+a)*a^3+35/b^5*d^4*ln(b*x+a)*c^3+1/4/b^8/(b*x+a)^4*a^7*d^7-35/b^8*d^7/(b*x+a)*a^4-35/b^4*d^3/(b*x+a)*c^4-5/2*d^7/b^6*x^2*a+7/2*d^6/b^5*x^2*c+15*d^7/b^7*a^2*x+21*d^5/b^5*c^2*x-7/3/b^8*d^7/(b*x+a)^3*a^6-7/3/b^2*d/(b*x+a)^3*c^6+1/3*d^7/b^5*x^3-1/4/b/(b*x+a)^4*c^7-35*d^6/b^6*a*c*x+14/b^7*d^6/(b*x+a)^3*a^5*c-35/b^6*d^5/(b*x+a)^3*a^4*c^2+140/3/b^5*d^4/(b*x+a)^3*a^3*c^3-35/b^4*d^3/(b*x+a)^3*a^2*c^4+105/b^7*d^6*ln(b*x+a)*a^2*c-105/b^6*d^5*ln(b*x+a)*a*c^2-7/4/b^7/(b*x+a)^4*a^6*c*d^6+21/4/b^6/(b*x+a)^4*a^5*c^2*d^5-35/4/b^5/(b*x+a)^4*a^4*c^3*d^4+35/4/b^4/(b*x+a)^4*a^3*c^4*d^3-21/4/b^3/(b*x+a)^4*a^2*c^5*d^2+7/4/b^2/(b*x+a)^4*a*c^6*d+140/b^7*d^6/(b*x+a)*a^3*c-210/b^6*d^5/(b*x+a)*a^2*c^2+140/b^5*d^4/(b*x+a)*a*c^3+14/b^3*d^2/(b*x+a)^3*a*c^5-105/2/b^7*d^6/(b*x+a)^2*a^4*c+105/b^6*d^5/(b*x+a)^2*a^3*c^2-105/b^5*d^4/(b*x+a)^2*a^2*c^3+105/2/b^4*d^3/(b*x+a)^2*a*c^4

**maxima** [B] time = 1.73, size = 494, normalized size = 2.64

$$-\frac{3 b^7 c^7+7 a b^6 c^6 d+21 a^2 b^5 c^5 d^2+105 a^3 b^4 c^4 d^3-875 a^4 b^3 c^3 d^4+1617 a^5 b^2 c^2 d^5-1197 a^6 b c d^6+319 a^7 d^7+420 \left(b^7 c^4 d^3-4 a b^6 c^3 d^4+6 a^2 b^5 c^2 d^5-4 a^3 b^4 c d^6+a^4 b^3 d^7\right) x^3+126 \left(b^7 c^5 d^2+5 a b^6 c^4 d^3-30 a^2 b^5 c^3 d^4+50 a^3 b^4 c^2 d^5-35 a^4 b^3 c d^6+9 a^5 b^2 d^7\right) x^2+28 \left(b^7 c^6 d+3 a b^6 c^5 d^2+15 a^2 b^5 c^4 d^3-110 a^3 b^4 c^3 d^4+195 a^4 b^3 c^2 d^5-141 a^5 b^2 c d^6+37 a^6 b d^7\right) x}{12 \left(b^{12} x^4+4 a b^{11} x^3+6 a^2 b^{10} x^2+4 a^3 b^9 x+a^4 b^8\right)}+\frac{2 b^2 d^7 x^3+3 \left(7 b^2 c d^6-5 a b d^7\right) x^2+6 \left(21 b^2 c^2 d^5-35 a b c d^6+15 a^2 d^7\right) x}{6 b^7}+\frac{35 \left(b^3 c^3 d^4-3 a b^2 c^2 d^5+3 a^2 b c d^6-a^3 d^7\right) \log (bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^7*c^7 + 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3
 - 875*a^4*b^3*c^3*d^4 + 1617*a^5*b^2*c^2*d^5 - 1197*a^6*b*c*d^6 + 319*a^7*
d^7 + 420*(b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 4*a^3*b^4*c*
d^6 + a^4*b^3*d^7)*x^3 + 126*(b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 - 30*a^2*b^5*c^
3*d^4 + 50*a^3*b^4*c^2*d^5 - 35*a^4*b^3*c*d^6 + 9*a^5*b^2*d^7)*x^2 + 28*(b^
7*c^6*d + 3*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 110*a^3*b^4*c^3*d^4 + 195*
a^4*b^3*c^2*d^5 - 141*a^5*b^2*c*d^6 + 37*a^6*b*d^7)*x)/(b^12*x^4 + 4*a*b^11
*x^3 + 6*a^2*b^10*x^2 + 4*a^3*b^9*x + a^4*b^8) + 1/6*(2*b^2*d^7*x^3 + 3*(7*
b^2*c*d^6 - 5*a*b*d^7)*x^2 + 6*(21*b^2*c^2*d^5 - 35*a*b*c*d^6 + 15*a^2*d^7)
*x)/b^7 + 35*(b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*log(
b*x + a)/b^8
```

**mupad [B]** time = 0.77, size = 512, normalized size = 2.74

$$x\left(\frac{5a\left(\frac{5ad^7}{b^6}-\frac{7cd^6}{b^5}\right)}{b}-\frac{10a^2d^7}{b^7}+\frac{21c^2d^5}{b^5}\right)-x^2\left(\frac{5ad^7}{2b^6}-\frac{7cd^6}{2b^5}\right)-\frac{\frac{319a^7d^7-1197a^6bcd^6+1617a^5b^2c^2d^5-875a^4b^3c^3d^4+105a^3b^4c^4d^3+21a^2b^5c^5d^2+7ab^6c^6d+3b^7c^7}{12b}+x\left(\frac{259a^6d^7}{3}-329a^5bcd^6+455a^4b^2c^2d^5-\frac{770a^3b^3c^3d^4}{3}+35a^2b^4c^4d^3+7ab^5c^5d^2+\frac{7b^6c^6d}{3}\right)+x^3\left(35a^4b^2d^7-140a^3b^3cd^6+210a^2b^4c^2d^5-140ab^5c^3d^4+35b^6c^4d^3\right)+x^2\left(\frac{189a^5bd^7}{2}-\frac{735a^4b^2cd^6}{2}+525a^3b^3c^2d^5-315a^2b^4c^3d^4+\frac{105ab^5c^4d^3}{2}+\frac{21b^6c^5d^2}{2}\right)}{a^4b^7+4a^3b^8x+6a^2b^9x^2+4ab^{10}x^3+b^{11}x^4}-\frac{\ln(a+bx)\left(35a^3d^7-105a^2bcd^6+105ab^2c^2d^5-35b^3c^3d^4\right)}{b^8}+\frac{d^7x^3}{3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^5,x)
```

```
[Out] x*((5*a*((5*a*d^7)/b^6 - (7*c*d^6)/b^5))/b - (10*a^2*d^7)/b^7 + (21*c^2*d^5
)/b^5) - x^2*((5*a*d^7)/(2*b^6) - (7*c*d^6)/(2*b^5)) - ((319*a^7*d^7 + 3*b^
7*c^7 + 21*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3 - 875*a^4*b^3*c^3*d^4 + 16
17*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 1197*a^6*b*c*d^6)/(12*b) + x*((259*a^6
*d^7)/3 + (7*b^6*c^6*d)/3 + 7*a*b^5*c^5*d^2 + 35*a^2*b^4*c^4*d^3 - (770*a^3
*b^3*c^3*d^4)/3 + 455*a^4*b^2*c^2*d^5 - 329*a^5*b*c*d^6) + x^3*(35*a^4*b^2*
d^7 + 35*b^6*c^4*d^3 - 140*a*b^5*c^3*d^4 - 140*a^3*b^3*c*d^6 + 210*a^2*b^4*
c^2*d^5) + x^2*((189*a^5*b*d^7)/2 + (21*b^6*c^5*d^2)/2 + (105*a*b^5*c^4*d^3
)/2 - (735*a^4*b^2*c*d^6)/2 - 315*a^2*b^4*c^3*d^4 + 525*a^3*b^3*c^2*d^5))/(
a^4*b^7 + b^11*x^4 + 4*a^3*b^8*x + 4*a*b^10*x^3 + 6*a^2*b^9*x^2) - (log(a +
 b*x)*(35*a^3*d^7 - 35*b^3*c^3*d^4 + 105*a*b^2*c^2*d^5 - 105*a^2*b*c*d^6))/
b^8 + (d^7*x^3)/(3*b^5)
```

**sympy [B]** time = 22.44, size = 500, normalized size = 2.67

$$x^2\left(-\frac{5ad^7}{2b^6}+\frac{7cd^6}{2b^5}\right)+x\left(\frac{15a^2d^7}{b^7}-\frac{35acd^6}{b^6}+\frac{21c^2d^5}{b^5}\right)+\frac{-319a^7d^7+1197a^6bcd^6-1617a^5b^2c^2d^5+875a^4b^3c^3d^4-105a^3b^4c^4d^3-21a^2b^5c^5d^2-7ab^6c^6d-3b^7c^7+x^3\left(-420a^4b^3d^7+1680a^3b^4cd^6-2520a^2b^5c^2d^5+1680ab^6c^3d^4-420b^7c^4d^3\right)+x^2\left(-1134a^5b^2d^7+4410a^4b^3cd^6-6300a^3b^4c^2d^5+3780a^2b^5c^3d^4-630ab^6c^4d^3-126b^7c^5d^2\right)+x\left(-1036a^6bd^7+3948a^5b^2cd^6-5460a^4b^3c^2d^5+3080a^3b^4c^3d^4-420a^2b^5c^4d^3-84ab^6c^5d^2-28b^7c^6d\right)}{12a^4b^8+48a^3b^9x+72a^2b^{10}x^2+48ab^{11}x^3+12b^{12}x^4}+\frac{d^7x^3}{3b^5}-\frac{35d^4(ad-bc)^3\log(a+bx)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**5,x)
```

```
[Out] x**2*(-5*a*d**7/(2*b**6) + 7*c*d**6/(2*b**5)) + x*(15*a**2*d**7/b**7 - 35*a
*c*d**6/b**6 + 21*c**2*d**5/b**5) + (-319*a**7*d**7 + 1197*a**6*b*c*d**6 -
1617*a**5*b**2*c**2*d**5 + 875*a**4*b**3*c**3*d**4 - 105*a**3*b**4*c**4*d**
3 - 21*a**2*b**5*c**5*d**2 - 7*a*b**6*c**6*d - 3*b**7*c**7 + x**3*(-420*a**
```

```
4*b**3*d**7 + 1680*a**3*b**4*c*d**6 - 2520*a**2*b**5*c**2*d**5 + 1680*a*b**
6*c**3*d**4 - 420*b**7*c**4*d**3) + x**2*(-1134*a**5*b**2*d**7 + 4410*a**4*
b**3*c*d**6 - 6300*a**3*b**4*c**2*d**5 + 3780*a**2*b**5*c**3*d**4 - 630*a*b
**6*c**4*d**3 - 126*b**7*c**5*d**2) + x*(-1036*a**6*b*d**7 + 3948*a**5*b**2
*c*d**6 - 5460*a**4*b**3*c**2*d**5 + 3080*a**3*b**4*c**3*d**4 - 420*a**2*b*
*5*c**4*d**3 - 84*a*b**6*c**5*d**2 - 28*b**7*c**6*d))/(12*a**4*b**8 + 48*a*
*3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) + d**7*x*
*3/(3*b**5) - 35*d**4*(a*d - b*c)**3*log(a + b*x)/b**8
```

## 3.1182 $\int \frac{(c+dx)^7}{(a+bx)^6}\,dx$

**Optimal**. Leaf size=181

$$\frac{21d^5(bc-ad)^2\log(a+bx)}{b^8}-\frac{35d^4(bc-ad)^3}{b^8(a+bx)}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}+\frac{d^6x(7b\cdots}{\cdots}$$

**Rubi [A]** time = 0.19, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^6x(7bc-6ad)}{b^7}-\frac{35d^4(bc-ad)^3}{b^8(a+bx)}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}+\frac{21d^5(bc-ad)^2\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}+\frac{d^7x^2}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] (d^6*(7*b*c - 6*a*d)*x)/b^7 + (d^7*x^2)/(2*b^6) - (b*c - a*d)^7/(5*b^8*(a +
 b*x)^5) - (7*d*(b*c - a*d)^6)/(4*b^8*(a + b*x)^4) - (7*d^2*(b*c - a*d)^5)/
(b^8*(a + b*x)^3) - (35*d^3*(b*c - a*d)^4)/(2*b^8*(a + b*x)^2) - (35*d^4*(b
*c - a*d)^3)/(b^8*(a + b*x)) + (21*d^5*(b*c - a*d)^2*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^6}\,dx &= \int \left(\frac{d^6(7bc-6ad)}{b^7}+\frac{d^7x}{b^6}+\frac{(bc-ad)^7}{b^7(a+bx)^6}+\frac{7d(bc-ad)^6}{b^7(a+bx)^5}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^4}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^3}+\cdots\right.\\ &= \frac{d^6(7bc-6ad)x}{b^7}+\frac{d^7x^2}{2b^6}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{35\cdots}{b\cdots}\end{aligned}$$

**Mathematica [B]** time = 0.15, size = 389, normalized size = 2.15

$$\frac{459a^7d^7+3a^6bd^6(625dx-406c)+a^5b^2d^5\left(959c^2-5250cdx+2700d^2x^2\right)+5a^4b^3d^4\left(-28c^3+875c^2dx-1680cd^2x^2+260d^3x^3\right)-5a^3b^4d^3\left(7c^4+140c^3dx-1540c^2d^2x^2+1120cd^3x^3+80d^4x^4\right)-a^2b^5d^2\left(14c^5+175c^4dx+1400c^3d^2x^2-6300c^2d^3x^3+700cd^4x^4+500d^5x^5\right)-7ab^6d\left(c^6+10c^5dx+50c^4d^2x^2+200c^3d^3x^3-300c^2d^4x^4-100cd^5x^5+10d^6x^6\right)+420d^5(a+bx)^5(bc-ad)^2\log(a+bx)-\left(b^7\left(4c^7+35c^6dx+140c^5d^2x^2+350c^4d^3x^3+700c^3d^4x^4-140cd^6x^6-10d^7x^7\right)\right)}{20b^8(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] (459*a^7*d^7 + 3*a^6*b*d^6*(-406*c + 625*d*x) + a^5*b^2*d^5*(959*c^2 - 5250
*c*d*x + 2700*d^2*x^2) + 5*a^4*b^3*d^4*(-28*c^3 + 875*c^2*d*x - 1680*c*d^2*
x^2 + 260*d^3*x^3) - 5*a^3*b^4*d^3*(7*c^4 + 140*c^3*d*x - 1540*c^2*d^2*x^2
+ 1120*c*d^3*x^3 + 80*d^4*x^4) - a^2*b^5*d^2*(14*c^5 + 175*c^4*d*x + 1400*c
^3*d^2*x^2 - 6300*c^2*d^3*x^3 + 700*c*d^4*x^4 + 500*d^5*x^5) - 7*a*b^6*d*(c
^6 + 10*c^5*d*x + 50*c^4*d^2*x^2 + 200*c^3*d^3*x^3 - 300*c^2*d^4*x^4 - 100*
c*d^5*x^5 + 10*d^6*x^6) - b^7*(4*c^7 + 35*c^6*d*x + 140*c^5*d^2*x^2 + 350*c
^4*d^3*x^3 + 700*c^3*d^4*x^4 - 140*c*d^6*x^6 - 10*d^7*x^7) + 420*d^5*(b*c -
 a*d)^2*(a + b*x)^5*Log[a + b*x])/(20*b^8*(a + b*x)^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^6, x]
```

**fricas [B]** time = 1.40, size = 732, normalized size = 4.04

$$\frac{10 b^7 d^7 x^7 - 4 b^7 c^7 - 7 a b^6 c^6 d - 14 a^2 b^5 c^5 d^2 - 35 a^3 b^4 c^4 d^3 - 140 a^4 b^3 c^3 d^4 + 959 a^5 b^2 c^2 d^5 - 1218 a^6 b c d^6 + 459 a^7 d^7 + 70 \left(2 b^7 c d^6 - a b^6 d^7\right) x^6 + 100 \left(7 a b^6 c d^6 - 5 a^2 b^5 d^7\right) x^5 - 100 \left(7 b^7 c^3 d^4 - 21 a b^6 c^2 d^5 + 7 a^2 b^5 c d^6 + 4 a^3 b^4 d^7\right) x^4 - 50 \left(7 b^7 c^4 d^3 + 28 a b^6 c^3 d^4 - 126 a^2 b^5 c^2 d^5 + 112 a^3 b^4 c d^6 - 26 a^4 b^3 d^7\right) x^3 - 10 \left(14 b^7 c^5 d^2 + 35 a b^6 c^4 d^3 + 140 a^2 b^5 c^3 d^4 - 770 a^3 b^4 c^2 d^5 + 840 a^4 b^3 c d^6 - 270 a^5 b^2 d^7\right) x^2 - 5 \left(7 b^7 c^6 d + 14 a b^6 c^5 d^2 + 35 a^2 b^5 c^4 d^3 + 140 a^3 b^4 c^3 d^4 - 875 a^4 b^3 c^2 d^5 + 1050 a^5 b^2 c d^6 - 375 a^6 b d^7\right) x + 420 \left(a^5 b^2 c^2 d^5 - 2 a^6 b c d^6 + a^7 d^7 + \left(b^7 c^2 d^5 - 2 a b^6 c d^6 + a^2 b^5 d^7\right) x^5 + 5 \left(a b^6 c^2 d^5 - 2 a^2 b^5 c d^6 + a^3 b^4 d^7\right) x^4 + 10 \left(a^2 b^5 c^2 d^5 - 2 a^3 b^4 c d^6 + a^4 b^3 d^7\right) x^3 + 10 \left(a^3 b^4 c^2 d^5 - 2 a^4 b^3 c d^6 + a^5 b^2 d^7\right) x^2 + 5 \left(a^4 b^3 c^2 d^5 - 2 a^5 b^2 c d^6 + a^6 b d^7\right) x\right) \log\left(b x + a\right)}{20 \left(b^{13} x^5 + 5 a b^{12} x^4 + 10 a^2 b^{11} x^3 + 10 a^3 b^{10} x^2 + 5 a^4 b^9 x + a^5 b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^7*d^7*x^7 - 4*b^7*c^7 - 7*a*b^6*c^6*d - 14*a^2*b^5*c^5*d^2 - 35*
a^3*b^4*c^4*d^3 - 140*a^4*b^3*c^3*d^4 + 959*a^5*b^2*c^2*d^5 - 1218*a^6*b*c*
d^6 + 459*a^7*d^7 + 70*(2*b^7*c*d^6 - a*b^6*d^7)*x^6 + 100*(7*a*b^6*c*d^6 -
 5*a^2*b^5*d^7)*x^5 - 100*(7*b^7*c^3*d^4 - 21*a*b^6*c^2*d^5 + 7*a^2*b^5*c*d
^6 + 4*a^3*b^4*d^7)*x^4 - 50*(7*b^7*c^4*d^3 + 28*a*b^6*c^3*d^4 - 126*a^2*b^
5*c^2*d^5 + 112*a^3*b^4*c*d^6 - 26*a^4*b^3*d^7)*x^3 - 10*(14*b^7*c^5*d^2 +
35*a*b^6*c^4*d^3 + 140*a^2*b^5*c^3*d^4 - 770*a^3*b^4*c^2*d^5 + 840*a^4*b^3*
c*d^6 - 270*a^5*b^2*d^7)*x^2 - 5*(7*b^7*c^6*d + 14*a*b^6*c^5*d^2 + 35*a^2*b
^5*c^4*d^3 + 140*a^3*b^4*c^3*d^4 - 875*a^4*b^3*c^2*d^5 + 1050*a^5*b^2*c*d^6
 - 375*a^6*b*d^7)*x + 420*(a^5*b^2*c^2*d^5 - 2*a^6*b*c*d^6 + a^7*d^7 + (b^7
*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 5*(a*b^6*c^2*d^5 - 2*a^2*b^5*
c*d^6 + a^3*b^4*d^7)*x^4 + 10*(a^2*b^5*c^2*d^5 - 2*a^3*b^4*c*d^6 + a^4*b^3*
d^7)*x^3 + 10*(a^3*b^4*c^2*d^5 - 2*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 5*(a^
4*b^3*c^2*d^5 - 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)*log(b*x + a))/(b^13*x^5 + 5
*a*b^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8)
```

**giac** [B] time = 1.38, size = 463, normalized size = 2.56

$$\frac{21\left(b^{2}c^{2}d^{5}-2abcd^{6}+a^{2}d^{7}\right)\log\left(|bx+a|\right)}{b^{8}}+\frac{b^{6}d^{7}x^{2}+14b^{6}cd^{6}x-12ab^{5}d^{7}x}{2b^{12}}-\frac{4b^{7}c^{7}+7ab^{6}c^{6}d+14a^{2}b^{5}c^{5}d^{2}+35a^{3}b^{4}c^{4}d^{3}+140a^{4}b^{3}c^{3}d^{4}-959a^{5}b^{2}c^{2}d^{5}+1218a^{6}bcd^{6}-459a^{7}d^{7}+700\left(b^{7}c^{3}d^{4}-3ab^{6}c^{2}d^{5}+3a^{2}b^{5}cd^{6}-a^{3}b^{4}d^{7}\right)x^{4}+350\left(b^{7}c^{4}d^{3}+4ab^{6}c^{3}d^{4}-18a^{2}b^{5}c^{2}d^{5}+20a^{3}b^{4}cd^{6}-7a^{4}b^{3}d^{7}\right)x^{3}+70\left(2b^{7}c^{5}d^{2}+5ab^{6}c^{4}d^{3}+20a^{2}b^{5}c^{3}d^{4}-110a^{3}b^{4}c^{2}d^{5}+130a^{4}b^{3}cd^{6}-47a^{5}b^{2}d^{7}\right)x^{2}+35\left(b^{7}c^{6}d+2ab^{6}c^{5}d^{2}+5a^{2}b^{5}c^{4}d^{3}+20a^{3}b^{4}c^{3}d^{4}-125a^{4}b^{3}c^{2}d^{5}+154a^{5}b^{2}cd^{6}-57a^{6}bd^{7}\right)x}{20\left(bx+a\right)^{5}b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 21*(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7)*log(abs(b*x + a))/b^8 + 1/2*(b^6*d
^7*x^2 + 14*b^6*c*d^6*x - 12*a*b^5*d^7*x)/b^12 - 1/20*(4*b^7*c^7 + 7*a*b^6*
c^6*d + 14*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 - 959
*a^5*b^2*c^2*d^5 + 1218*a^6*b*c*d^6 - 459*a^7*d^7 + 700*(b^7*c^3*d^4 - 3*a*
b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 350*(b^7*c^4*d^3 + 4*a*b
^6*c^3*d^4 - 18*a^2*b^5*c^2*d^5 + 20*a^3*b^4*c*d^6 - 7*a^4*b^3*d^7)*x^3 + 7
0*(2*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 - 110*a^3*b^4*c^2*d
^5 + 130*a^4*b^3*c*d^6 - 47*a^5*b^2*d^7)*x^2 + 35*(b^7*c^6*d + 2*a*b^6*c^5*
d^2 + 5*a^2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 - 125*a^4*b^3*c^2*d^5 + 154*a^
5*b^2*c*d^6 - 57*a^6*b*d^7)*x)/((b*x + a)^5*b^8)
```

**maple** [B] time = 0.01, size = 656, normalized size = 3.62

$$\frac{a^{7}d^{7}}{5\left(bx+a\right)^{5}b^{8}}-\frac{7a^{6}cd^{6}}{5\left(bx+a\right)^{5}b^{7}}+\frac{21a^{5}c^{2}d^{5}}{5\left(bx+a\right)^{5}b^{6}}-\frac{7a^{4}c^{3}d^{4}}{\left(bx+a\right)^{5}b^{5}}+\frac{7a^{3}c^{4}d^{3}}{\left(bx+a\right)^{5}b^{4}}-\frac{21a^{2}c^{5}d^{2}}{5\left(bx+a\right)^{5}b^{3}}+\frac{7ac^{6}d}{5\left(bx+a\right)^{5}b^{2}}-\frac{c^{7}}{5\left(bx+a\right)^{5}b}-\frac{7a^{6}d^{7}}{4\left(bx+a\right)^{4}b^{8}}+\frac{21a^{5}cd^{6}}{2\left(bx+a\right)^{4}b^{7}}-\frac{105a^{4}c^{2}d^{5}}{4\left(bx+a\right)^{4}b^{6}}+\frac{35a^{3}c^{3}d^{4}}{\left(bx+a\right)^{4}b^{5}}-\frac{105a^{2}c^{4}d^{3}}{4\left(bx+a\right)^{4}b^{4}}+\frac{21ac^{5}d^{2}}{2\left(bx+a\right)^{4}b^{3}}-\frac{7c^{6}d}{4\left(bx+a\right)^{4}b^{2}}+\frac{7a^{5}d^{7}}{\left(bx+a\right)^{3}b^{8}}-\frac{35a^{4}cd^{6}}{\left(bx+a\right)^{3}b^{7}}+\frac{70a^{3}c^{2}d^{5}}{\left(bx+a\right)^{3}b^{6}}-\frac{70a^{2}c^{3}d^{4}}{\left(bx+a\right)^{3}b^{5}}+\frac{35ac^{4}d^{3}}{\left(bx+a\right)^{3}b^{4}}-\frac{7c^{5}d^{2}}{\left(bx+a\right)^{3}b^{3}}-\frac{35a^{4}d^{7}}{2\left(bx+a\right)^{2}b^{8}}+\frac{70a^{3}cd^{6}}{\left(bx+a\right)^{2}b^{7}}-\frac{105a^{2}c^{2}d^{5}}{\left(bx+a\right)^{2}b^{6}}+\frac{70ac^{3}d^{4}}{\left(bx+a\right)^{2}b^{5}}-\frac{35c^{4}d^{3}}{2\left(bx+a\right)^{2}b^{4}}+\frac{d^{7}x^{2}}{2b^{6}}+\frac{35a^{3}d^{7}}{\left(bx+a\right)b^{8}}-\frac{105a^{2}cd^{6}}{\left(bx+a\right)b^{7}}+\frac{21a^{2}d^{7}\ln\left(bx+a\right)}{b^{8}}+\frac{105ac^{2}d^{5}}{\left(bx+a\right)b^{6}}-\frac{42acd^{6}\ln\left(bx+a\right)}{b^{7}}-\frac{6ad^{7}x}{b^{7}}-\frac{35c^{3}d^{4}}{\left(bx+a\right)b^{5}}+\frac{21c^{2}d^{5}\ln\left(bx+a\right)}{b^{6}}+\frac{7cd^{6}x}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^6,x)
```

```
[Out] -105/b^7*d^6/(b*x+a)*a^2*c+105/b^6*d^5/(b*x+a)*a*c^2+1/2*d^7*x^2/b^6-35/2/b
^4*d^3/(b*x+a)^2*c^4-6*d^7/b^7*a*x+7*d^6/b^6*x*c+35/b^8*d^7/(b*x+a)*a^3-35/
b^5*d^4/(b*x+a)*c^3+21/b^8*d^7*ln(b*x+a)*a^2+21/b^6*d^5*ln(b*x+a)*c^2-7/4/b
^8*d^7/(b*x+a)^4*a^6-7/4/b^2*d/(b*x+a)^4*c^6+7/b^8*d^7/(b*x+a)^3*a^5-7/b^3*
d^2/(b*x+a)^3*c^5+1/5/b^8/(b*x+a)^5*a^7*d^7-35/2/b^8*d^7/(b*x+a)^2*a^4+7/b^
4/(b*x+a)^5*a^3*c^4*d^3-21/5/b^3/(b*x+a)^5*a^2*c^5*d^2-1/5/b/(b*x+a)^5*c^7-
35/b^7*d^6/(b*x+a)^3*a^4*c+70/b^6*d^5/(b*x+a)^3*a^3*c^2-70/b^5*d^4/(b*x+a)^
3*a^2*c^3+35/b^4*d^3/(b*x+a)^3*a*c^4-7/5/b^7/(b*x+a)^5*a^6*c*d^6+21/5/b^6/(
b*x+a)^5*a^5*c^2*d^5-7/b^5/(b*x+a)^5*a^4*c^3*d^4-105/b^6*d^5/(b*x+a)^2*a^2*
c^2+70/b^5*d^4/(b*x+a)^2*a*c^3-42/b^7*d^6*ln(b*x+a)*a*c+21/2/b^7*d^6/(b*x+a
)^4*a^5*c+7/5/b^2/(b*x+a)^5*a*c^6*d+70/b^7*d^6/(b*x+a)^2*a^3*c-105/4/b^6*d^
5/(b*x+a)^4*a^4*c^2+35/b^5*d^4/(b*x+a)^4*a^3*c^3-105/4/b^4*d^3/(b*x+a)^4*a^
2*c^4+21/2/b^3*d^2/(b*x+a)^4*a*c^5
```

**maxima** [B] time = 1.82, size = 504, normalized size = 2.78

$$-\frac{4b^{7}c^{7}+7ab^{6}c^{6}d+14a^{2}b^{5}c^{5}d^{2}+35a^{3}b^{4}c^{4}d^{3}+140a^{4}b^{3}c^{3}d^{4}-959a^{5}b^{2}c^{2}d^{5}+1218a^{6}bcd^{6}-459a^{7}d^{7}+700\left(b^{7}c^{3}d^{4}-3ab^{6}c^{2}d^{5}+3a^{2}b^{5}cd^{6}-a^{3}b^{4}d^{7}\right)x^{4}+350\left(b^{7}c^{4}d^{3}+4ab^{6}c^{3}d^{4}-18a^{2}b^{5}c^{2}d^{5}+20a^{3}b^{4}cd^{6}-7a^{4}b^{3}d^{7}\right)x^{3}+70\left(2b^{7}c^{5}d^{2}+5ab^{6}c^{4}d^{3}+20a^{2}b^{5}c^{3}d^{4}-110a^{3}b^{4}c^{2}d^{5}+130a^{4}b^{3}cd^{6}-47a^{5}b^{2}d^{7}\right)x^{2}+35\left(b^{7}c^{6}d+2ab^{6}c^{5}d^{2}+5a^{2}b^{5}c^{4}d^{3}+20a^{3}b^{4}c^{3}d^{4}-125a^{4}b^{3}c^{2}d^{5}+154a^{5}b^{2}cd^{6}-57a^{6}bd^{7}\right)x}{20\left(b^{13}x^{5}+5ab^{12}x^{4}+10a^{2}b^{11}x^{3}+10a^{3}b^{10}x^{2}+5a^{4}b^{9}x+a^{5}b^{8}\right)}+\frac{bd^{7}x^{2}+2\left(7bcd^{6}-6ad^{7}\right)x}{2b^{7}}+\frac{21\left(b^{2}c^{2}d^{5}-2abcd^{6}+a^{2}d^{7}\right)\log\left(bx+a\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(4*b^7*c^7 + 7*a*b^6*c^6*d + 14*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3
+ 140*a^4*b^3*c^3*d^4 - 959*a^5*b^2*c^2*d^5 + 1218*a^6*b*c*d^6 - 459*a^7*d^
7 + 700*(b^7*c^3*d^4 - 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4
 + 350*(b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 - 18*a^2*b^5*c^2*d^5 + 20*a^3*b^4*c*d
^6 - 7*a^4*b^3*d^7)*x^3 + 70*(2*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 20*a^2*b^5*
c^3*d^4 - 110*a^3*b^4*c^2*d^5 + 130*a^4*b^3*c*d^6 - 47*a^5*b^2*d^7)*x^2 + 3
5*(b^7*c^6*d + 2*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 - 1
25*a^4*b^3*c^2*d^5 + 154*a^5*b^2*c*d^6 - 57*a^6*b*d^7)*x)/(b^13*x^5 + 5*a*b
^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8) + 1/2*
(b*d^7*x^2 + 2*(7*b*c*d^6 - 6*a*d^7)*x)/b^7 + 21*(b^2*c^2*d^5 - 2*a*b*c*d^6
 + a^2*d^7)*log(b*x + a)/b^8
```

**mupad** [**B**] time = 0.34, size = 508, normalized size = 2.81

$$\frac{\ln\left(a+b\,x\right)\,\left(21\,a^2\,d^7-42\,a\,b\,c\,d^6+21\,b^2\,c^2\,d^5\right)}{b^8}-x\,\left(\frac{6\,a\,d^7}{b^7}-\frac{7\,c\,d^6}{b^6}\right)-\frac{\frac{-459\,a^7\,d^7+1218\,a^6\,b\,c\,d^6-959\,a^5\,b^2\,c^2\,d^5+140\,a^4\,b^3\,c^3\,d^4+35\,a^3\,b^4\,c^4\,d^3+14\,a^2\,b^5\,c^5\,d^2+7\,a\,b^6\,c^6\,d+4\,b^7\,c^7}{20\,b}+x\,\left(-\frac{399\,a^6\,d^7}{4}+\frac{539\,a^5\,b\,c\,d^6}{2}-\frac{875\,a^4\,b^2\,c^2\,d^5}{4}+35\,a^3\,b^3\,c^3\,d^4+\frac{35\,a^2\,b^4\,c^4\,d^3}{4}+\frac{7\,a\,b^5\,c^5\,d^2}{2}+\frac{7\,b^6\,c^6\,d}{4}\right)+x^3\,\left(-\frac{245\,a^4\,b^2\,d^7}{2}+350\,a^3\,b^3\,c\,d^6-315\,a^2\,b^4\,c^2\,d^5+70\,a\,b^5\,c^3\,d^4+\frac{35\,b^6\,c^4\,d^3}{2}\right)+x^2\,\left(-\frac{329\,a^5\,b\,d^7}{2}+455\,a^4\,b^2\,c\,d^6-385\,a^3\,b^3\,c^2\,d^5+70\,a^2\,b^4\,c^3\,d^4+\frac{35\,a\,b^5\,c^4\,d^3}{2}+7\,b^6\,c^5\,d^2\right)-x^4\,\left(35\,a^3\,b^3\,d^7-105\,a^2\,b^4\,c\,d^6+105\,a\,b^5\,c^2\,d^5-35\,b^6\,c^3\,d^4\right)}{a^5\,b^7+5\,a^4\,b^8\,x+10\,a^3\,b^9\,x^2+10\,a^2\,b^{10}\,x^3+5\,a\,b^{11}\,x^4+b^{12}\,x^5}+\frac{d^7\,x^2}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^6,x)
```

```
[Out] (log(a + b*x)*(21*a^2*d^7 + 21*b^2*c^2*d^5 - 42*a*b*c*d^6))/b^8 - x*((6*a*d
^7)/b^7 - (7*c*d^6)/b^6) - ((4*b^7*c^7 - 459*a^7*d^7 + 14*a^2*b^5*c^5*d^2 +
 35*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 - 959*a^5*b^2*c^2*d^5 + 7*a*b^6*c
^6*d + 1218*a^6*b*c*d^6)/(20*b) + x*((7*b^6*c^6*d)/4 - (399*a^6*d^7)/4 + (7
*a*b^5*c^5*d^2)/2 + (35*a^2*b^4*c^4*d^3)/4 + 35*a^3*b^3*c^3*d^4 - (875*a^4*
b^2*c^2*d^5)/4 + (539*a^5*b*c*d^6)/2) + x^3*((35*b^6*c^4*d^3)/2 - (245*a^4*
b^2*d^7)/2 + 70*a*b^5*c^3*d^4 + 350*a^3*b^3*c*d^6 - 315*a^2*b^4*c^2*d^5) +
x^2*(7*b^6*c^5*d^2 - (329*a^5*b*d^7)/2 + (35*a*b^5*c^4*d^3)/2 + 455*a^4*b^2
*c*d^6 + 70*a^2*b^4*c^3*d^4 - 385*a^3*b^3*c^2*d^5) - x^4*(35*a^3*b^3*d^7 -
35*b^6*c^3*d^4 + 105*a*b^5*c^2*d^5 - 105*a^2*b^4*c*d^6))/(a^5*b^7 + b^12*x^
5 + 5*a^4*b^8*x + 5*a*b^11*x^4 + 10*a^3*b^9*x^2 + 10*a^2*b^10*x^3) + (d^7*x
^2)/(2*b^6)
```

**sympy** [**B**] time = 97.19, size = 524, normalized size = 2.90

$$x\left(-\frac{6ad^7}{b^7}+\frac{7cd^6}{b^6}\right)+\frac{459a^7d^7-1218a^6bcd^6+959a^5b^2c^2d^5-140a^4b^3c^3d^4-35a^3b^4c^4d^3-14a^2b^5c^5d^2-7ab^6c^6d-4b^7c^7+x^4\left(700a^3b^4d^7-2100a^2b^5cd^6+2100ab^6c^2d^5-700b^7c^3d^4\right)+x^3\left(2450a^4b^3d^7-7000a^3b^4cd^6+6300a^2b^5c^2d^5-1400ab^6c^3d^4-350b^7c^4d^3\right)+x^2\left(3290a^5b^2d^7-9100a^4b^3cd^6+7700a^3b^4c^2d^5-1400a^2b^5c^3d^4-350ab^6c^4d^3-140b^7c^5d^2\right)+x\left(1995a^6bd^7-5390a^5b^2cd^6+4375a^4b^3c^2d^5-700a^3b^4c^3d^4-175a^2b^5c^4d^3-70ab^6c^5d^2-35b^7c^6d\right)}{20a^5b^8+100a^4b^9x+200a^3b^{10}x^2+200a^2b^{11}x^3+100ab^{12}x^4+20b^{13}x^5}+\frac{d^7x^2}{2b^6}+\frac{21d^5\left(ad-bc\right)^2\log\left(a+bx\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**6,x)
```

```
[Out] x*(-6*a*d**7/b**7 + 7*c*d**6/b**6) + (459*a**7*d**7 - 1218*a**6*b*c*d**6 +
959*a**5*b**2*c**2*d**5 - 140*a**4*b**3*c**3*d**4 - 35*a**3*b**4*c**4*d**3
- 14*a**2*b**5*c**5*d**2 - 7*a*b**6*c**6*d - 4*b**7*c**7 + x**4*(700*a**3*b
**4*d**7 - 2100*a**2*b**5*c*d**6 + 2100*a*b**6*c**2*d**5 - 700*b**7*c**3*d*
```

```
*4) + x**3*(2450*a**4*b**3*d**7 - 7000*a**3*b**4*c*d**6 + 6300*a**2*b**5*c*
*2*d**5 - 1400*a*b**6*c**3*d**4 - 350*b**7*c**4*d**3) + x**2*(3290*a**5*b**
2*d**7 - 9100*a**4*b**3*c*d**6 + 7700*a**3*b**4*c**2*d**5 - 1400*a**2*b**5*
c**3*d**4 - 350*a*b**6*c**4*d**3 - 140*b**7*c**5*d**2) + x*(1995*a**6*b*d**
7 - 5390*a**5*b**2*c*d**6 + 4375*a**4*b**3*c**2*d**5 - 700*a**3*b**4*c**3*d
**4 - 175*a**2*b**5*c**4*d**3 - 70*a*b**6*c**5*d**2 - 35*b**7*c**6*d))/(20*
a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 1
00*a*b**12*x**4 + 20*b**13*x**5) + d**7*x**2/(2*b**6) + 21*d**5*(a*d - b*c)
**2*log(a + b*x)/b**8
```

## 3.1183 $\int \frac{(c+dx)^7}{(a+bx)^7} dx$

**Optimal**. Leaf size=186

$$\frac{7d^6(bc-ad)\log(a+bx)}{b^8}-\frac{21d^5(bc-ad)^2}{b^8(a+bx)}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{(bc}{6b^8(}$$

**Rubi** [A] time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{21d^5(bc-ad)^2}{b^8(a+bx)}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}+\frac{7d^6(bc-ad)\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{(bc-ad)^7}{6b^8(a+bx)^6}+\frac{d^7x}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] (d^7*x)/b^7 - (b*c - a*d)^7/(6*b^8*(a + b*x)^6) - (7*d*(b*c - a*d)^6)/(5*b^
8*(a + b*x)^5) - (21*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^4) - (35*d^3*(b*c
- a*d)^4)/(3*b^8*(a + b*x)^3) - (35*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^2)
- (21*d^5*(b*c - a*d)^2)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*Log[a + b*x])
/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^7}\,dx &= \int \left(\frac{d^7}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^7}+\frac{7d(bc-ad)^6}{b^7(a+bx)^6}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^5}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^4}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^3}+\right.\\ &= \frac{d^7x}{b^7}-\frac{(bc-ad)^7}{6b^8(a+bx)^6}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-21\end{aligned}$$

**Mathematica** [B] time = 0.20, size = 390, normalized size = 2.10

$$-\frac{669a^7d^7+3a^6bd^6(1198dx-343c)+3a^5b^2d^5\left(70c^2-1918cdx+2575d^2x^2\right)+5a^4b^3d^4\left(14c^3+252c^2dx-2625cd^2x^2+1640d^3x^3\right)+5a^3b^4d^3\left(7c^4+84c^3dx+630c^2d^2x^2-3080cd^3x^3+810d^4x^4\right)+3a^2b^5d^2\left(7c^5+70c^4dx+350c^3d^2x^2+1400c^2d^3x^3-3150cd^4x^4+120d^5x^5\right)+ab^6d\left(14c^6+126c^5dx+525c^4d^2x^2+1400c^3d^3x^3+3150c^2d^4x^4-2520cd^5x^5-360d^6x^6\right)+420d^6(a+bx)^6(ad-bc)\log(a+bx)+b^7\left(10c^7+84c^6dx+315c^5d^2x^2+700c^4d^3x^3+1050c^3d^4x^4+1260c^2d^5x^5-60d^7x^7\right)}{60b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] -1/60*(669*a^7*d^7 + 3*a^6*b*d^6*(-343*c + 1198*d*x) + 3*a^5*b^2*d^5*(70*c^
2 - 1918*c*d*x + 2575*d^2*x^2) + 5*a^4*b^3*d^4*(14*c^3 + 252*c^2*d*x - 2625
*c*d^2*x^2 + 1640*d^3*x^3) + 5*a^3*b^4*d^3*(7*c^4 + 84*c^3*d*x + 630*c^2*d^
2*x^2 - 3080*c*d^3*x^3 + 810*d^4*x^4) + 3*a^2*b^5*d^2*(7*c^5 + 70*c^4*d*x +
 350*c^3*d^2*x^2 + 1400*c^2*d^3*x^3 - 3150*c*d^4*x^4 + 120*d^5*x^5) + a*b^6
*d*(14*c^6 + 126*c^5*d*x + 525*c^4*d^2*x^2 + 1400*c^3*d^3*x^3 + 3150*c^2*d^
4*x^4 - 2520*c*d^5*x^5 - 360*d^6*x^6) + b^7*(10*c^7 + 84*c^6*d*x + 315*c^5*
d^2*x^2 + 700*c^4*d^3*x^3 + 1050*c^3*d^4*x^4 + 1260*c^2*d^5*x^5 - 60*d^7*x^
7) + 420*d^6*(-(b*c) + a*d)*(a + b*x)^6*Log[a + b*x])/(b^8*(a + b*x)^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^7, x]
```

**fricas [B]** time = 1.51, size = 692, normalized size = 3.72

$$\frac{60\,b^{7}d^{7}x^{7} + 360\,ab^{6}d^{7}x^{6} - 10\,b^{7}c^{7} - 14\,ab^{6}c^{6}d - 21\,a^{2}b^{5}c^{5}d^{2} - 35\,a^{3}b^{4}c^{4}d^{3} - 70\,a^{4}b^{3}c^{3}d^{4} - 210\,a^{5}b^{2}c^{2}d^{5} + 1029\,a^{6}bcd^{6} - 669\,a^{7}d^{7} - 180\,(7\,b^{7}c^{2}d^{5} - 14\,ab^{6}cd^{6} + 2\,a^{2}b^{5}d^{7})x^{5} - 150\,(7\,b^{7}c^{3}d^{4} + 21\,ab^{6}c^{2}d^{5} - 63\,a^{2}b^{5}cd^{6} + 27\,a^{3}b^{4}d^{7})x^{4} - 100\,(7\,b^{7}c^{4}d^{3} + 14\,ab^{6}c^{3}d^{4} + 42\,a^{2}b^{5}c^{2}d^{5} - 154\,a^{3}b^{4}cd^{6} + 82\,a^{4}b^{3}d^{7})x^{3} - 15\,(21\,b^{7}c^{5}d^{2} + 35\,ab^{6}c^{4}d^{3} + 70\,a^{2}b^{5}c^{3}d^{4} + 210\,a^{3}b^{4}c^{2}d^{5} - 875\,a^{4}b^{3}cd^{6} + 515\,a^{5}b^{2}d^{7})x^{2} - 6\,(14\,b^{7}c^{6}d + 21\,ab^{6}c^{5}d^{2} + 35\,a^{2}b^{5}c^{4}d^{3} + 70\,a^{3}b^{4}c^{3}d^{4} + 210\,a^{4}b^{3}c^{2}d^{5} - 959\,a^{5}b^{2}cd^{6} + 599\,a^{6}bd^{7})x + 420\,(a^{6}bcd^{6} - a^{7}d^{7} + (b^{7}cd^{6} - ab^{6}d^{7})x^{6} + 6\,(ab^{6}cd^{6} - a^{2}b^{5}d^{7})x^{5} + 15\,(a^{2}b^{5}cd^{6} - a^{3}b^{4}d^{7})x^{4} + 20\,(a^{3}b^{4}cd^{6} - a^{4}b^{3}d^{7})x^{3} + 15\,(a^{4}b^{3}cd^{6} - a^{5}b^{2}d^{7})x^{2} + 6\,(a^{5}b^{2}cd^{6} - a^{6}bd^{7})x)\log(bx+a)}{60\,(b^{14}x^{6} + 6\,ab^{13}x^{5} + 15\,a^{2}b^{12}x^{4} + 20\,a^{3}b^{11}x^{3} + 15\,a^{4}b^{10}x^{2} + 6\,a^{5}b^{9}x + a^{6}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*d^7*x^7 + 360*a*b^6*d^7*x^6 - 10*b^7*c^7 - 14*a*b^6*c^6*d - 21
*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 - 70*a^4*b^3*c^3*d^4 - 210*a^5*b^2*c^
2*d^5 + 1029*a^6*b*c*d^6 - 669*a^7*d^7 - 180*(7*b^7*c^2*d^5 - 14*a*b^6*c*d^
6 + 2*a^2*b^5*d^7)*x^5 - 150*(7*b^7*c^3*d^4 + 21*a*b^6*c^2*d^5 - 63*a^2*b^5
*c*d^6 + 27*a^3*b^4*d^7)*x^4 - 100*(7*b^7*c^4*d^3 + 14*a*b^6*c^3*d^4 + 42*a
^2*b^5*c^2*d^5 - 154*a^3*b^4*c*d^6 + 82*a^4*b^3*d^7)*x^3 - 15*(21*b^7*c^5*d
^2 + 35*a*b^6*c^4*d^3 + 70*a^2*b^5*c^3*d^4 + 210*a^3*b^4*c^2*d^5 - 875*a^4*
b^3*c*d^6 + 515*a^5*b^2*d^7)*x^2 - 6*(14*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 35*
a^2*b^5*c^4*d^3 + 70*a^3*b^4*c^3*d^4 + 210*a^4*b^3*c^2*d^5 - 959*a^5*b^2*c*
d^6 + 599*a^6*b*d^7)*x + 420*(a^6*b*c*d^6 - a^7*d^7 + (b^7*c*d^6 - a*b^6*d^
7)*x^6 + 6*(a*b^6*c*d^6 - a^2*b^5*d^7)*x^5 + 15*(a^2*b^5*c*d^6 - a^3*b^4*d^
7)*x^4 + 20*(a^3*b^4*c*d^6 - a^4*b^3*d^7)*x^3 + 15*(a^4*b^3*c*d^6 - a^5*b^2
*d^7)*x^2 + 6*(a^5*b^2*c*d^6 - a^6*b*d^7)*x)*log(b*x + a))/(b^14*x^6 + 6*a*
```

```
b^13*x^5 + 15*a^2*b^12*x^4 + 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*
x + a^6*b^8)
```

**giac [B]** time = 1.30, size = 459, normalized size = 2.47

$$\frac{d^7x}{b^7}+\frac{7\left(bcd^6-ad^7\right)\log\left(|bx+a|\right)}{b^8}-\frac{10\,b^7c^7+14\,ab^6c^6d+21\,a^2b^5c^5d^2+35\,a^3b^4c^4d^3+70\,a^4b^3c^3d^4+210\,a^5b^2c^2d^5-1029\,a^6bcd^6+669\,a^7d^7+1260\left(b^7c^2d^5-2\,ab^6cd^6+a^2b^5d^7\right)x^5+1050\left(b^7c^3d^4+3\,ab^6c^2d^5-9\,a^2b^5cd^6+5\,a^3b^4d^7\right)x^4+700\left(b^7c^4d^3+2\,ab^6c^3d^4+6\,a^2b^5c^2d^5-22\,a^3b^4cd^6+13\,a^4b^3d^7\right)x^3+105\left(3\,b^7c^5d^2+5\,ab^6c^4d^3+10\,a^2b^5c^3d^4+30\,a^3b^4c^2d^5-125\,a^4b^3cd^6+77\,a^5b^2d^7\right)x^2+42\left(2\,b^7c^6d+3\,ab^6c^5d^2+5\,a^2b^5c^4d^3+10\,a^3b^4c^3d^4+30\,a^4b^3c^2d^5-137\,a^5b^2cd^6+87\,a^6bd^7\right)x}{60\left(bx+a\right)^6b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] d^7*x/b^7 + 7*(b*c*d^6 - a*d^7)*log(abs(b*x + a))/b^8 - 1/60*(10*b^7*c^7 +
14*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d
^4 + 210*a^5*b^2*c^2*d^5 - 1029*a^6*b*c*d^6 + 669*a^7*d^7 + 1260*(b^7*c^2*d
^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1050*(b^7*c^3*d^4 + 3*a*b^6*c^2*d^5
 - 9*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^4 + 700*(b^7*c^4*d^3 + 2*a*b^6*c^3*d^
4 + 6*a^2*b^5*c^2*d^5 - 22*a^3*b^4*c*d^6 + 13*a^4*b^3*d^7)*x^3 + 105*(3*b^7
*c^5*d^2 + 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 - 125*
a^4*b^3*c*d^6 + 77*a^5*b^2*d^7)*x^2 + 42*(2*b^7*c^6*d + 3*a*b^6*c^5*d^2 + 5
*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 - 137*a^5*b^2*c*
d^6 + 87*a^6*b*d^7)*x)/((b*x + a)^6*b^8)
```

**maple [B]** time = 0.01, size = 666, normalized size = 3.58

$$\frac{a^7d^7}{6\left(bx+a\right)^6b^8}-\frac{7a^6c\,d^6}{6\left(bx+a\right)^6b^7}+\frac{7a^5c^2d^5}{2\left(bx+a\right)^6b^6}-\frac{35a^4c^3d^4}{6\left(bx+a\right)^6b^5}+\frac{35a^3c^4d^3}{6\left(bx+a\right)^6b^4}-\frac{7a^2c^5d^2}{2\left(bx+a\right)^6b^3}+\frac{7a\,c^6d}{6\left(bx+a\right)^6b^2}-\frac{c^7}{6\left(bx+a\right)^6b}-\frac{7a^6d^7}{5\left(bx+a\right)^5b^8}+\frac{42a^5c\,d^6}{5\left(bx+a\right)^5b^7}-\frac{21a^4c^2d^5}{\left(bx+a\right)^5b^6}+\frac{28a^3c^3d^4}{\left(bx+a\right)^5b^5}-\frac{21a^2c^4d^3}{\left(bx+a\right)^5b^4}+\frac{42a\,c^5d^2}{5\left(bx+a\right)^5b^3}-\frac{7c^6d}{5\left(bx+a\right)^5b^2}+\frac{21a^5d^7}{4\left(bx+a\right)^4b^8}-\frac{105a^4c\,d^6}{4\left(bx+a\right)^4b^7}+\frac{105a^3c^2d^5}{2\left(bx+a\right)^4b^6}-\frac{105a^2c^3d^4}{2\left(bx+a\right)^4b^5}+\frac{105a\,c^4d^3}{4\left(bx+a\right)^4b^4}-\frac{21c^5d^2}{4\left(bx+a\right)^4b^3}-\frac{35a^4d^7}{3\left(bx+a\right)^3b^8}+\frac{140a^3c\,d^6}{3\left(bx+a\right)^3b^7}-\frac{70a^2c^2d^5}{\left(bx+a\right)^3b^6}+\frac{140a\,c^3d^4}{3\left(bx+a\right)^3b^5}-\frac{35c^4d^3}{3\left(bx+a\right)^3b^4}+\frac{35a^3d^7}{2\left(bx+a\right)^2b^8}-\frac{105a^2c\,d^6}{2\left(bx+a\right)^2b^7}+\frac{105a\,c^2d^5}{2\left(bx+a\right)^2b^6}-\frac{35c^3d^4}{2\left(bx+a\right)^2b^5}-\frac{21a^2d^7}{\left(bx+a\right)b^8}+\frac{42ac\,d^6}{\left(bx+a\right)b^7}-\frac{7a\,d^7\ln\left(bx+a\right)}{b^8}-\frac{21c^2d^5}{\left(bx+a\right)b^6}+\frac{7c\,d^6\ln\left(bx+a\right)}{b^7}+\frac{d^7x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^7,x)
```

```
[Out] 42/5/b^3*d^2/(b*x+a)^5*a*c^5-105/2/b^7*d^6/(b*x+a)^2*a^2*c+7/b^7*d^6*ln(b*x
+a)*c-7/5/b^2*d/(b*x+a)^5*c^6+35/2/b^8*d^7/(b*x+a)^2*a^3-35/2/b^5*d^4/(b*x+
a)^2*c^3-7/b^8*d^7*ln(b*x+a)*a+1/6/b^8/(b*x+a)^6*a^7*d^7+21/4/b^8*d^7/(b*x+
a)^4*a^5-21/4/b^3*d^2/(b*x+a)^4*c^5-21/b^8*d^7/(b*x+a)*a^2-21/b^6*d^5/(b*x+
a)*c^2-35/3/b^8*d^7/(b*x+a)^3*a^4+d^7*x/b^7-35/3/b^4*d^3/(b*x+a)^3*c^4-7/5/
b^8*d^7/(b*x+a)^5*a^6+42/b^7*d^6/(b*x+a)*a*c-7/6/b^7/(b*x+a)^6*a^6*c*d^6-1/
6/b/(b*x+a)^6*c^7+105/2/b^6*d^5/(b*x+a)^2*a*c^2+140/3/b^7*d^6/(b*x+a)^3*a^3
*c-70/b^6*d^5/(b*x+a)^3*a^2*c^2+140/3/b^5*d^4/(b*x+a)^3*a*c^3+35/6/b^4/(b*x
+a)^6*a^3*c^4*d^3-7/2/b^3/(b*x+a)^6*a^2*c^5*d^2+7/6/b^2/(b*x+a)^6*a*c^6*d+7
/2/b^6/(b*x+a)^6*a^5*c^2*d^5-35/6/b^5/(b*x+a)^6*a^4*c^3*d^4-105/4/b^7*d^6/(
b*x+a)^4*a^4*c+105/2/b^6*d^5/(b*x+a)^4*a^3*c^2-105/2/b^5*d^4/(b*x+a)^4*a^2*
c^3+105/4/b^4*d^3/(b*x+a)^4*a*c^4+42/5/b^7*d^6/(b*x+a)^5*a^5*c-21/b^6*d^5/(
b*x+a)^5*a^4*c^2+28/b^5*d^4/(b*x+a)^5*a^3*c^3-21/b^4*d^3/(b*x+a)^5*a^2*c^4
```

**maxima [B]** time = 1.80, size = 516, normalized size = 2.77

$$\frac{d^7x}{b^7}-\frac{10\,b^7c^7+14\,ab^6c^6d+21\,a^2b^5c^5d^2+35\,a^3b^4c^4d^3+70\,a^4b^3c^3d^4+210\,a^5b^2c^2d^5-1029\,a^6bcd^6+669\,a^7d^7+1260\left(b^7c^2d^5-2\,ab^6cd^6+a^2b^5d^7\right)x^5+1050\left(b^7c^3d^4+3\,ab^6c^2d^5-9\,a^2b^5cd^6+5\,a^3b^4d^7\right)x^4+700\left(b^7c^4d^3+2\,ab^6c^3d^4+6\,a^2b^5c^2d^5-22\,a^3b^4cd^6+13\,a^4b^3d^7\right)x^3+105\left(3\,b^7c^5d^2+5\,ab^6c^4d^3+10\,a^2b^5c^3d^4+30\,a^3b^4c^2d^5-125\,a^4b^3cd^6+77\,a^5b^2d^7\right)x^2+42\left(2\,b^7c^6d+3\,ab^6c^5d^2+5\,a^2b^5c^4d^3+10\,a^3b^4c^3d^4+30\,a^4b^3c^2d^5-137\,a^5b^2cd^6+87\,a^6bd^7\right)x}{60\left(b^{14}x^6+6\,ab^{13}x^5+15\,a^2b^{12}x^4+20\,a^3b^{11}x^3+15\,a^4b^{10}x^2+6\,a^5b^9x+a^6b^8\right)}+\frac{7\left(bcd^6-ad^7\right)\log\left(bx+a\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] d^7*x/b^7 - 1/60*(10*b^7*c^7 + 14*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 35*a^3
*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 - 1029*a^6*b*c*d^6
+ 669*a^7*d^7 + 1260*(b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1050
*(b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 - 9*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^4 + 70
0*(b^7*c^4*d^3 + 2*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 22*a^3*b^4*c*d^6 + 1
3*a^4*b^3*d^7)*x^3 + 105*(3*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*
d^4 + 30*a^3*b^4*c^2*d^5 - 125*a^4*b^3*c*d^6 + 77*a^5*b^2*d^7)*x^2 + 42*(2*
b^7*c^6*d + 3*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 + 30*a
^4*b^3*c^2*d^5 - 137*a^5*b^2*c*d^6 + 87*a^6*b*d^7)*x)/(b^14*x^6 + 6*a*b^13*
x^5 + 15*a^2*b^12*x^4 + 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a
^6*b^8) + 7*(b*c*d^6 - a*d^7)*log(b*x + a)/b^8
```

**mupad [B]** time = 0.37, size = 517, normalized size = 2.78

$$\frac{d^7\,x}{b^7}-\frac{\ln\left(a+b\,x\right)\,\left(7\,a\,d^7-7\,b\,c\,d^6\right)}{b^8}-\frac{\frac{669\,a^7\,d^7-1029\,a^6\,b\,c\,d^6+210\,a^5\,b^2\,c^2\,d^5+70\,a^4\,b^3\,c^3\,d^4+35\,a^3\,b^4\,c^4\,d^3+21\,a^2\,b^5\,c^5\,d^2+14\,a\,b^6\,c^6\,d+10\,b^7\,c^7}{60\,b}+x\,\left(\frac{609\,a^6\,d^7}{10}-\frac{959\,a^5\,b\,c\,d^6}{10}+21\,a^4\,b^2\,c^2\,d^5+7\,a^3\,b^3\,c^3\,d^4+\frac{7\,a^2\,b^4\,c^4\,d^3}{2}+\frac{21\,a\,b^5\,c^5\,d^2}{10}+\frac{7\,b^6\,c^6\,d}{5}\right)+x^3\,\left(\frac{455\,a^4\,b^2\,d^7}{3}-\frac{770\,a^3\,b^3\,c\,d^6}{3}+70\,a^2\,b^4\,c^2\,d^5+\frac{70\,a\,b^5\,c^3\,d^4}{3}+\frac{35\,b^6\,c^4\,d^3}{3}\right)+x^2\,\left(\frac{539\,a^5\,b\,d^7}{4}-\frac{875\,a^4\,b^2\,c\,d^6}{4}+\frac{105\,a^3\,b^3\,c^2\,d^5}{2}+\frac{35\,a^2\,b^4\,c^3\,d^4}{2}+\frac{35\,a\,b^5\,c^4\,d^3}{4}+\frac{21\,b^6\,c^5\,d^2}{4}\right)+x^5\,\left(21\,a^2\,b^4\,d^7-42\,a\,b^5\,c\,d^6+21\,b^6\,c^2\,d^5\right)+x^4\,\left(\frac{175\,a^3\,b^3\,d^7}{2}-\frac{315\,a^2\,b^4\,c\,d^6}{2}+\frac{105\,a\,b^5\,c^2\,d^5}{2}+\frac{35\,b^6\,c^3\,d^4}{2}\right)}{a^6\,b^7+6\,a^5\,b^8\,x+15\,a^4\,b^9\,x^2+20\,a^3\,b^{10}\,x^3+15\,a^2\,b^{11}\,x^4+6\,a\,b^{12}\,x^5+b^{13}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^7,x)
```

```
[Out] (d^7*x)/b^7 - (log(a + b*x)*(7*a*d^7 - 7*b*c*d^6))/b^8 - ((669*a^7*d^7 + 10
*b^7*c^7 + 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d^4 + 2
10*a^5*b^2*c^2*d^5 + 14*a*b^6*c^6*d - 1029*a^6*b*c*d^6)/(60*b) + x*((609*a^
6*d^7)/10 + (7*b^6*c^6*d)/5 + (21*a*b^5*c^5*d^2)/10 + (7*a^2*b^4*c^4*d^3)/2
 + 7*a^3*b^3*c^3*d^4 + 21*a^4*b^2*c^2*d^5 - (959*a^5*b*c*d^6)/10) + x^3*((4
55*a^4*b^2*d^7)/3 + (35*b^6*c^4*d^3)/3 + (70*a*b^5*c^3*d^4)/3 - (770*a^3*b^
3*c*d^6)/3 + 70*a^2*b^4*c^2*d^5) + x^2*((539*a^5*b*d^7)/4 + (21*b^6*c^5*d^2
)/4 + (35*a*b^5*c^4*d^3)/4 - (875*a^4*b^2*c*d^6)/4 + (35*a^2*b^4*c^3*d^4)/2
 + (105*a^3*b^3*c^2*d^5)/2) + x^5*(21*a^2*b^4*d^7 + 21*b^6*c^2*d^5 - 42*a*b
^5*c*d^6) + x^4*((175*a^3*b^3*d^7)/2 + (35*b^6*c^3*d^4)/2 + (105*a*b^5*c^2*
d^5)/2 - (315*a^2*b^4*c*d^6)/2))/(a^6*b^7 + b^13*x^6 + 6*a^5*b^8*x + 6*a*b^
12*x^5 + 15*a^4*b^9*x^2 + 20*a^3*b^10*x^3 + 15*a^2*b^11*x^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**7,x)
```

```
[Out] Timed out
```

## 3.1184 $\int \frac{(c+dx)^7}{(a+bx)^8}\,dx$

**Optimal**. Leaf size=194

$$-\frac{7d^6(bc-ad)}{b^8(a+bx)}-\frac{21d^5(bc-ad)^2}{2b^8(a+bx)^2}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{(bc-ad)^7}{7b^8(a+bx)^7}+$$

**Rubi** [A] time = 0.16, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{7d^6(bc-ad)}{b^8(a+bx)}-\frac{21d^5(bc-ad)^2}{2b^8(a+bx)^2}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{(bc-ad)^7}{7b^8(a+bx)^7}+\frac{d^7\log(a+bx)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] -(b*c - a*d)^7/(7*b^8*(a + b*x)^7) - (7*d*(b*c - a*d)^6)/(6*b^8*(a + b*x)^6
) - (21*d^2*(b*c - a*d)^5)/(5*b^8*(a + b*x)^5) - (35*d^3*(b*c - a*d)^4)/(4*
b^8*(a + b*x)^4) - (35*d^4*(b*c - a*d)^3)/(3*b^8*(a + b*x)^3) - (21*d^5*(b*
c - a*d)^2)/(2*b^8*(a + b*x)^2) - (7*d^6*(b*c - a*d))/(b^8*(a + b*x)) + (d^
7*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^8}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^8}+\frac{7d(bc-ad)^6}{b^7(a+bx)^7}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^6}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^5}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^4}+\frac{21d^5}{b^7(}\right.\\ &= -\frac{(bc-ad)^7}{7b^8(a+bx)^7}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{21d^5(b}{2b^8(a}\end{aligned}$$

**Mathematica** [A] time = 0.16, size = 308, normalized size = 1.59

$$\frac{d^7\log(a+bx)}{b^8}-\frac{(bc-ad)\left(1089a^6d^6+3a^5bd^5(223c+2401dx)+3a^4b^2d^4\left(153c^2+1421cdx+6713d^2x^2\right)+a^3b^3d^3\left(319c^3+2793c^2dx+11319cd^2x^2+30625d^3x^3\right)+a^2b^4d^2\left(214c^4+1813c^3dx+6909c^2d^2x^2+15925cd^3x^3+26950d^4x^4\right)+ab^5d\left(130c^5+1078c^4dx+3969c^3d^2x^2+8575c^2d^3x^3+12250cd^4x^4+13230d^5x^5\right)+b^6\left(60c^6+490c^5dx+1764c^4d^2x^2+3675c^3d^3x^3+4900c^2d^4x^4+4410cd^5x^5+2940d^6x^6\right)\right)}{420b^8(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] -1/420*((b*c - a*d)*(1089*a^6*d^6 + 3*a^5*b*d^5*(223*c + 2401*d*x) + 3*a^4*
b^2*d^4*(153*c^2 + 1421*c*d*x + 6713*d^2*x^2) + a^3*b^3*d^3*(319*c^3 + 2793
*c^2*d*x + 11319*c*d^2*x^2 + 30625*d^3*x^3) + a^2*b^4*d^2*(214*c^4 + 1813*c
^3*d*x + 6909*c^2*d^2*x^2 + 15925*c*d^3*x^3 + 26950*d^4*x^4) + a*b^5*d*(130
*c^5 + 1078*c^4*d*x + 3969*c^3*d^2*x^2 + 8575*c^2*d^3*x^3 + 12250*c*d^4*x^4
 + 13230*d^5*x^5) + b^6*(60*c^6 + 490*c^5*d*x + 1764*c^4*d^2*x^2 + 3675*c^3
*d^3*x^3 + 4900*c^2*d^4*x^4 + 4410*c*d^5*x^5 + 2940*d^6*x^6)))/(b^8*(a + b*
x)^7) + (d^7*Log[a + b*x])/b^8
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^8, x]
```

**fricas [B]** time = 1.42, size = 624, normalized size = 3.22

$$-\frac{60\,b^7c^7 + 70\,ab^6c^6d + 84\,a^2b^5c^5d^2 + 105\,a^3b^4c^4d^3 + 140\,a^4b^3c^3d^4 + 210\,a^5b^2c^2d^5 + 420\,a^6bcd^6 - 1089\,a^7d^7 + 2940\left(b^7cd^6 - ab^6d^7\right)x^6 + 4410\left(b^7c^2d^5 + 2\,ab^6cd^6 - 3\,a^2b^5d^7\right)x^5 + 2450\left(2\,b^7c^3d^4 + 3\,ab^6c^2d^5 + 6\,a^2b^5cd^6 - 11\,a^3b^4d^7\right)x^4 + 1225\left(3\,b^7c^4d^3 + 4\,ab^6c^3d^4 + 6\,a^2b^5c^2d^5 + 12\,a^3b^4cd^6 - 25\,a^4b^3d^7\right)x^3 + 147\left(12\,b^7c^5d^2 + 15\,ab^6c^4d^3 + 20\,a^2b^5c^3d^4 + 30\,a^3b^4c^2d^5 + 60\,a^4b^3cd^6 - 137\,a^5b^2d^7\right)x^2 + 49\left(10\,b^7c^6d + 12\,ab^6c^5d^2 + 15\,a^2b^5c^4d^3 + 20\,a^3b^4c^3d^4 + 30\,a^4b^3c^2d^5 + 60\,a^5b^2cd^6 - 147\,a^6bd^7\right)x - 420\left(b^7d^7x^7 + 7\,ab^6d^7x^6 + 21\,a^2b^5d^7x^5 + 35\,a^3b^4d^7x^4 + 35\,a^4b^3d^7x^3 + 21\,a^5b^2d^7x^2 + 7\,a^6bd^7x + a^7d^7\right)\log\left(bx+a\right)}{420\left(b^{15}x^7 + 7\,ab^{14}x^6 + 21\,a^2b^{13}x^5 + 35\,a^3b^{12}x^4 + 35\,a^4b^{11}x^3 + 21\,a^5b^{10}x^2 + 7\,a^6b^9x + a^7b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/420*(60*b^7*c^7 + 70*a*b^6*c^6*d + 84*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*
d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^
7*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(b^7*c^2*d^5 + 2*a*b^6*c*d^
6 - 3*a^2*b^5*d^7)*x^5 + 2450*(2*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 6*a^2*b^5*
c*d^6 - 11*a^3*b^4*d^7)*x^4 + 1225*(3*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 6*a^2
*b^5*c^2*d^5 + 12*a^3*b^4*c*d^6 - 25*a^4*b^3*d^7)*x^3 + 147*(12*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 + 60*a^4*b^3*
c*d^6 - 137*a^5*b^2*d^7)*x^2 + 49*(10*b^7*c^6*d + 12*a*b^6*c^5*d^2 + 15*a^2
*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 + 60*a^5*b^2*c*d^6 -
 147*a^6*b*d^7)*x - 420*(b^7*d^7*x^7 + 7*a*b^6*d^7*x^6 + 21*a^2*b^5*d^7*x^5
 + 35*a^3*b^4*d^7*x^4 + 35*a^4*b^3*d^7*x^3 + 21*a^5*b^2*d^7*x^2 + 7*a^6*b*d
^7*x + a^7*d^7)*log(b*x + a))/(b^15*x^7 + 7*a*b^14*x^6 + 21*a^2*b^13*x^5 +
35*a^3*b^12*x^4 + 35*a^4*b^11*x^3 + 21*a^5*b^10*x^2 + 7*a^6*b^9*x + a^7*b^8
)
```

**giac [B]** time = 1.28, size = 466, normalized size = 2.40

$$\frac{d^{7}\log\left(|bx+a|\right)}{b^{8}} - \frac{2940\left(b^{6}cd^{6} - ab^{5}d^{7}\right)x^{6} + 4410\left(b^{6}c^{2}d^{5} + 2ab^{5}cd^{6} - 3a^{2}b^{4}d^{7}\right)x^{5} + 2450\left(2b^{6}c^{3}d^{4} + 3ab^{5}c^{2}d^{5} + 6a^{2}b^{4}cd^{6} - 11a^{3}b^{3}d^{7}\right)x^{4} + 1225\left(3b^{6}c^{4}d^{3} + 4ab^{5}c^{3}d^{4} + 6a^{2}b^{4}c^{2}d^{5} + 12a^{3}b^{3}cd^{6} - 25a^{4}b^{2}d^{7}\right)x^{3} + 147\left(12b^{6}c^{5}d^{2} + 15ab^{5}c^{4}d^{3} + 20a^{2}b^{4}c^{3}d^{4} + 30a^{3}b^{3}c^{2}d^{5} + 60a^{4}b^{2}cd^{6} - 137a^{5}bd^{7}\right)x^{2} + 49\left(10b^{6}c^{6}d + 12ab^{5}c^{5}d^{2} + 15a^{2}b^{4}c^{4}d^{3} + 20a^{3}b^{3}c^{3}d^{4} + 30a^{4}b^{2}c^{2}d^{5} + 60a^{5}bcd^{6} - 147a^{6}d^{7}\right)x + \frac{60b^{7}c^{7} + 70ab^{6}c^{6}d + 84a^{2}b^{5}c^{5}d^{2} + 105a^{3}b^{4}c^{4}d^{3} + 140a^{4}b^{3}c^{3}d^{4} + 210a^{5}b^{2}c^{2}d^{5} + 420a^{6}bcd^{6} - 1089a^{7}d^{7}}{b}}{420\,(bx+a)^{7}b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] d^7*log(abs(b*x + a))/b^8 - 1/420*(2940*(b^6*c*d^6 - a*b^5*d^7)*x^6 + 4410*
(b^6*c^2*d^5 + 2*a*b^5*c*d^6 - 3*a^2*b^4*d^7)*x^5 + 2450*(2*b^6*c^3*d^4 + 3
*a*b^5*c^2*d^5 + 6*a^2*b^4*c*d^6 - 11*a^3*b^3*d^7)*x^4 + 1225*(3*b^6*c^4*d^
3 + 4*a*b^5*c^3*d^4 + 6*a^2*b^4*c^2*d^5 + 12*a^3*b^3*c*d^6 - 25*a^4*b^2*d^7
)*x^3 + 147*(12*b^6*c^5*d^2 + 15*a*b^5*c^4*d^3 + 20*a^2*b^4*c^3*d^4 + 30*a^
3*b^3*c^2*d^5 + 60*a^4*b^2*c*d^6 - 137*a^5*b*d^7)*x^2 + 49*(10*b^6*c^6*d +
12*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 + 20*a^3*b^3*c^3*d^4 + 30*a^4*b^2*c^2
*d^5 + 60*a^5*b*c*d^6 - 147*a^6*d^7)*x + (60*b^7*c^7 + 70*a*b^6*c^6*d + 84*
a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c
^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^7*d^7)/b)/((b*x + a)^7*b^7)
```

**maple [B]** time = 0.01, size = 672, normalized size = 3.46

$$\frac{a^{7}d^{7}}{7(bx+a)^{7}b^{8}} - \frac{a^{6}cd^{6}}{(bx+a)^{7}b^{7}} + \frac{3a^{5}c^{2}d^{5}}{(bx+a)^{7}b^{6}} - \frac{5a^{4}c^{3}d^{4}}{(bx+a)^{7}b^{5}} + \frac{5a^{3}c^{4}d^{3}}{(bx+a)^{7}b^{4}} - \frac{3a^{2}c^{5}d^{2}}{(bx+a)^{7}b^{3}} + \frac{ac^{6}d}{(bx+a)^{7}b^{2}} - \frac{c^{7}}{7(bx+a)^{7}b} - \frac{7a^{6}d^{7}}{6(bx+a)^{6}b^{8}} + \frac{7a^{5}cd^{6}}{(bx+a)^{6}b^{7}} - \frac{35a^{4}c^{2}d^{5}}{2(bx+a)^{6}b^{6}} + \frac{70a^{3}c^{3}d^{4}}{3(bx+a)^{6}b^{5}} - \frac{35a^{2}c^{4}d^{3}}{2(bx+a)^{6}b^{4}} + \frac{7ac^{5}d^{2}}{(bx+a)^{6}b^{3}} - \frac{7c^{6}d}{6(bx+a)^{6}b^{2}} + \frac{21a^{5}d^{7}}{5(bx+a)^{5}b^{8}} - \frac{21a^{4}cd^{6}}{(bx+a)^{5}b^{7}} + \frac{42a^{3}c^{2}d^{5}}{(bx+a)^{5}b^{6}} - \frac{42a^{2}c^{3}d^{4}}{(bx+a)^{5}b^{5}} + \frac{21ac^{4}d^{3}}{(bx+a)^{5}b^{4}} - \frac{21c^{5}d^{2}}{5(bx+a)^{5}b^{3}} - \frac{35a^{4}d^{7}}{4(bx+a)^{4}b^{8}} + \frac{35a^{3}cd^{6}}{(bx+a)^{4}b^{7}} - \frac{105a^{2}c^{2}d^{5}}{2(bx+a)^{4}b^{6}} + \frac{35ac^{3}d^{4}}{(bx+a)^{4}b^{5}} - \frac{35c^{4}d^{3}}{4(bx+a)^{4}b^{4}} + \frac{35a^{3}d^{7}}{3(bx+a)^{3}b^{8}} - \frac{35a^{2}cd^{6}}{(bx+a)^{3}b^{7}} + \frac{35ac^{2}d^{5}}{(bx+a)^{3}b^{6}} - \frac{35c^{3}d^{4}}{3(bx+a)^{3}b^{5}} - \frac{21a^{2}d^{7}}{2(bx+a)^{2}b^{8}} + \frac{21acd^{6}}{(bx+a)^{2}b^{7}} - \frac{21c^{2}d^{5}}{2(bx+a)^{2}b^{6}} + \frac{7ad^{7}}{(bx+a)b^{8}} - \frac{7cd^{6}}{(bx+a)b^{7}} + \frac{d^{7}\ln(bx+a)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^8,x)
```

```
[Out] 7/b^8*d^7/(b*x+a)*a-7/b^7*d^6/(b*x+a)*c+35/3*d^7/b^8/(b*x+a)^3*a^3-35/3*d^4
/b^5/(b*x+a)^3*c^3+1/7/b^8/(b*x+a)^7*a^7*d^7+21/5*d^7/b^8/(b*x+a)^5*a^5-21/
5*d^2/b^3/(b*x+a)^5*c^5-7/6*d^7/b^8/(b*x+a)^6*a^6-7/6*d/b^2/(b*x+a)^6*c^6-2
1/2*d^7/b^8/(b*x+a)^2*a^2-21/2*d^5/b^6/(b*x+a)^2*c^2-35/4*d^7/b^8/(b*x+a)^4
*a^4-35/4*d^3/b^4/(b*x+a)^4*c^4+d^7*ln(b*x+a)/b^8+42*d^5/b^6/(b*x+a)^5*a^3*
c^2-21*d^6/b^7/(b*x+a)^5*a^4*c-1/7/b/(b*x+a)^7*c^7-42*d^4/b^5/(b*x+a)^5*a^2
*c^3+1/b^2/(b*x+a)^7*a*c^6*d-35*d^6/b^7/(b*x+a)^3*a^2*c+35*d^5/b^6/(b*x+a)^
3*a*c^2-1/b^7/(b*x+a)^7*a^6*c*d^6+3/b^6/(b*x+a)^7*a^5*c^2*d^5-5/b^5/(b*x+a)
^7*a^4*c^3*d^4+5/b^4/(b*x+a)^7*a^3*c^4*d^3-3/b^3/(b*x+a)^7*a^2*c^5*d^2+21*d
^3/b^4/(b*x+a)^5*a*c^4+21*d^6/b^7/(b*x+a)^2*a*c+35*d^6/b^7/(b*x+a)^4*a^3*c-
105/2*d^5/b^6/(b*x+a)^4*a^2*c^2+35*d^4/b^5/(b*x+a)^4*a*c^3+7*d^6/b^7/(b*x+a
)^6*a^5*c-35/2*d^5/b^6/(b*x+a)^6*a^4*c^2+70/3*d^4/b^5/(b*x+a)^6*a^3*c^3-35/
2*d^3/b^4/(b*x+a)^6*a^2*c^4+7*d^2/b^3/(b*x+a)^6*a*c^5
```

**maxima [B]** time = 1.65, size = 534, normalized size = 2.75

$$-\frac{60b^{7}c^{7} + 70ab^{6}c^{6}d + 84a^{2}b^{5}c^{5}d^{2} + 105a^{3}b^{4}c^{4}d^{3} + 140a^{4}b^{3}c^{3}d^{4} + 210a^{5}b^{2}c^{2}d^{5} + 420a^{6}bcd^{6} - 1089a^{7}d^{7} + 2940\left(b^{7}cd^{6} - ab^{6}d^{7}\right)x^{6} + 4410\left(b^{7}c^{2}d^{5} + 2ab^{6}cd^{6} - 3a^{2}b^{5}d^{7}\right)x^{5} + 2450\left(2b^{7}c^{3}d^{4} + 3ab^{6}c^{2}d^{5} + 6a^{2}b^{5}cd^{6} - 11a^{3}b^{4}d^{7}\right)x^{4} + 1225\left(3b^{7}c^{4}d^{3} + 4ab^{6}c^{3}d^{4} + 6a^{2}b^{5}c^{2}d^{5} + 12a^{3}b^{4}cd^{6} - 25a^{4}b^{3}d^{7}\right)x^{3} + 147\left(12b^{7}c^{5}d^{2} + 15ab^{6}c^{4}d^{3} + 20a^{2}b^{5}c^{3}d^{4} + 30a^{3}b^{4}c^{2}d^{5} + 60a^{4}b^{3}cd^{6} - 137a^{5}b^{2}d^{7}\right)x^{2} + 49\left(10b^{7}c^{6}d + 12ab^{6}c^{5}d^{2} + 15a^{2}b^{5}c^{4}d^{3} + 20a^{3}b^{4}c^{3}d^{4} + 30a^{4}b^{3}c^{2}d^{5} + 60a^{5}b^{2}cd^{6} - 147a^{6}bd^{7}\right)x}{420\left(b^{15}x^{7} + 7ab^{14}x^{6} + 21a^{2}b^{13}x^{5} + 35a^{3}b^{12}x^{4} + 35a^{4}b^{11}x^{3} + 21a^{5}b^{10}x^{2} + 7a^{6}b^{9}x + a^{7}b^{8}\right)} + \frac{d^{7}\log\left(bx+a\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/420*(60*b^7*c^7 + 70*a*b^6*c^6*d + 84*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*
d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^
7*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(b^7*c^2*d^5 + 2*a*b^6*c*d^
6 - 3*a^2*b^5*d^7)*x^5 + 2450*(2*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 6*a^2*b^5*
c*d^6 - 11*a^3*b^4*d^7)*x^4 + 1225*(3*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 6*a^2
*b^5*c^2*d^5 + 12*a^3*b^4*c*d^6 - 25*a^4*b^3*d^7)*x^3 + 147*(12*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 + 60*a^4*b^3*
c*d^6 - 137*a^5*b^2*d^7)*x^2 + 49*(10*b^7*c^6*d + 12*a*b^6*c^5*d^2 + 15*a^2
*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 + 60*a^5*b^2*c*d^6 -
 147*a^6*b*d^7)*x)/(b^15*x^7 + 7*a*b^14*x^6 + 21*a^2*b^13*x^5 + 35*a^3*b^12
*x^4 + 35*a^4*b^11*x^3 + 21*a^5*b^10*x^2 + 7*a^6*b^9*x + a^7*b^8) + d^7*log
(b*x + a)/b^8
```

**mupad [B]** time = 0.35, size = 461, normalized size = 2.38

$$\frac{d^7\,\ln\left(a+b\,x\right)}{b^8}-\frac{x\,\left(-\frac{343\,a^6\,b\,d^7}{20}+7\,a^5\,b^2\,c\,d^6+\frac{7\,a^4\,b^3\,c^2\,d^5}{2}+\frac{7\,a^3\,b^4\,c^3\,d^4}{3}+\frac{7\,a^2\,b^5\,c^4\,d^3}{4}+\frac{7\,a\,b^6\,c^5\,d^2}{5}+\frac{7\,b^7\,c^6\,d}{6}\right)-x^6\,\left(7\,a\,b^6\,d^7-7\,b^7\,c\,d^6\right)+x^3\,\left(-\frac{875\,a^4\,b^3\,d^7}{12}+35\,a^3\,b^4\,c\,d^6+\frac{35\,a^2\,b^5\,c^2\,d^5}{2}+\frac{35\,a\,b^6\,c^3\,d^4}{3}+\frac{35\,b^7\,c^4\,d^3}{4}\right)+x^5\,\left(-\frac{63\,a^2\,b^5\,d^7}{2}+21\,a\,b^6\,c\,d^6+\frac{21\,b^7\,c^2\,d^5}{2}\right)+x^2\,\left(-\frac{959\,a^5\,b^2\,d^7}{20}+21\,a^4\,b^3\,c\,d^6+\frac{21\,a^3\,b^4\,c^2\,d^5}{2}+7\,a^2\,b^5\,c^3\,d^4+\frac{21\,a\,b^6\,c^4\,d^3}{4}+\frac{21\,b^7\,c^5\,d^2}{5}\right)-\frac{363\,a^7\,d^7}{140}+\frac{b^7\,c^7}{7}+x^4\,\left(-\frac{385\,a^3\,b^4\,d^7}{6}+35\,a^2\,b^5\,c\,d^6+\frac{35\,a\,b^6\,c^2\,d^5}{2}+\frac{35\,b^7\,c^3\,d^4}{3}\right)+\frac{a^2\,b^5\,c^5\,d^2}{5}+\frac{a^3\,b^4\,c^4\,d^3}{4}+\frac{a^4\,b^3\,c^3\,d^4}{3}+\frac{a^5\,b^2\,c^2\,d^5}{2}+\frac{a\,b^6\,c^6\,d}{6}+a^6\,b\,c\,d^6}{b^8\,{\left(a+b\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^8,x)
```

```
[Out] (d^7*log(a + b*x))/b^8 - (x*((7*b^7*c^6*d)/6 - (343*a^6*b*d^7)/20 + (7*a*b^
6*c^5*d^2)/5 + 7*a^5*b^2*c*d^6 + (7*a^2*b^5*c^4*d^3)/4 + (7*a^3*b^4*c^3*d^4
)/3 + (7*a^4*b^3*c^2*d^5)/2) - x^6*(7*a*b^6*d^7 - 7*b^7*c*d^6) + x^3*((35*b
^7*c^4*d^3)/4 - (875*a^4*b^3*d^7)/12 + (35*a*b^6*c^3*d^4)/3 + 35*a^3*b^4*c*
d^6 + (35*a^2*b^5*c^2*d^5)/2) + x^5*((21*b^7*c^2*d^5)/2 - (63*a^2*b^5*d^7)/
2 + 21*a*b^6*c*d^6) + x^2*((21*b^7*c^5*d^2)/5 - (959*a^5*b^2*d^7)/20 + (21*
a*b^6*c^4*d^3)/4 + 21*a^4*b^3*c*d^6 + 7*a^2*b^5*c^3*d^4 + (21*a^3*b^4*c^2*d
^5)/2) - (363*a^7*d^7)/140 + (b^7*c^7)/7 + x^4*((35*b^7*c^3*d^4)/3 - (385*a
^3*b^4*d^7)/6 + (35*a*b^6*c^2*d^5)/2 + 35*a^2*b^5*c*d^6) + (a^2*b^5*c^5*d^2
)/5 + (a^3*b^4*c^4*d^3)/4 + (a^4*b^3*c^3*d^4)/3 + (a^5*b^2*c^2*d^5)/2 + (a*
b^6*c^6*d)/6 + a^6*b*c*d^6)/(b^8*(a + b*x)^7)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**8,x)
```

```
[Out] Timed out
```

## 3.1185 $\int \frac{(c+dx)^7}{(a+bx)^9}\,dx$

**Optimal**. Leaf size=28

$$-\frac{(c+dx)^8}{8(a+bx)^8(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^8}{8(a+bx)^8(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] -(c + d*x)^8/(8*(b*c - a*d)*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^9}\,dx = -\frac{(c+dx)^8}{8(bc-ad)(a+bx)^8}$$

**Mathematica [B]** time = 0.13, size = 353, normalized size = 12.61

$$-\frac{a^7d^7+a^6bd^6(c+8dx)+a^5b^2d^5\left(c^2+8cdx+28d^2x^2\right)+a^4b^3d^4\left(c^3+8c^2dx+28cd^2x^2+56d^3x^3\right)+a^3b^4d^3\left(c^4+8c^3dx+28c^2d^2x^2+56cd^3x^3+70d^4x^4\right)+a^2b^5d^2\left(c^5+8c^4dx+28c^3d^2x^2+56c^2d^3x^3+70cd^4x^4+56d^5x^5\right)+ab^6d\left(c^6+8c^5dx+28c^4d^2x^2+56c^3d^3x^3+70c^2d^4x^4+56cd^5x^5+28d^6x^6\right)+b^7\left(c^7+8c^6dx+28c^5d^2x^2+56c^4d^3x^3+70c^3d^4x^4+56c^2d^5x^5+28cd^6x^6+8d^7x^7\right)}{8b^8(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] -1/8*(a^7*d^7 + a^6*b*d^6*(c + 8*d*x) + a^5*b^2*d^5*(c^2 + 8*c*d*x + 28*d^2
*x^2) + a^4*b^3*d^4*(c^3 + 8*c^2*d*x + 28*c*d^2*x^2 + 56*d^3*x^3) + a^3*b^4
*d^3*(c^4 + 8*c^3*d*x + 28*c^2*d^2*x^2 + 56*c*d^3*x^3 + 70*d^4*x^4) + a^2*b
```

```
^5*d^2*(c^5 + 8*c^4*d*x + 28*c^3*d^2*x^2 + 56*c^2*d^3*x^3 + 70*c*d^4*x^4 +
56*d^5*x^5) + a*b^6*d*(c^6 + 8*c^5*d*x + 28*c^4*d^2*x^2 + 56*c^3*d^3*x^3 +
70*c^2*d^4*x^4 + 56*c*d^5*x^5 + 28*d^6*x^6) + b^7*(c^7 + 8*c^6*d*x + 28*c^5
*d^2*x^2 + 56*c^4*d^3*x^3 + 70*c^3*d^4*x^4 + 56*c^2*d^5*x^5 + 28*c*d^6*x^6
+ 8*d^7*x^7))/(b^8*(a + b*x)^8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^9, x]
```

**fricas [B]** time = 1.39, size = 509, normalized size = 18.18

$$-\frac{8 b^{7} d^{7} x^{7} + b^{7} c^{7} + a b^{6} c^{6} d + a^{2} b^{5} c^{5} d^{2} + a^{3} b^{4} c^{4} d^{3} + a^{4} b^{3} c^{3} d^{4} + a^{5} b^{2} c^{2} d^{5} + a^{6} b c d^{6} + a^{7} d^{7} + 28 \left(b^{7} c d^{6} + a b^{6} d^{7}\right) x^{6} + 56 \left(b^{7} c^{2} d^{5} + a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right) x^{5} + 70 \left(b^{7} c^{3} d^{4} + a b^{6} c^{2} d^{5} + a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}\right) x^{4} + 56 \left(b^{7} c^{4} d^{3} + a b^{6} c^{3} d^{4} + a^{2} b^{5} c^{2} d^{5} + a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right) x^{3} + 28 \left(b^{7} c^{5} d^{2} + a b^{6} c^{4} d^{3} + a^{2} b^{5} c^{3} d^{4} + a^{3} b^{4} c^{2} d^{5} + a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}\right) x^{2} + 8 \left(b^{7} c^{6} d + a b^{6} c^{5} d^{2} + a^{2} b^{5} c^{4} d^{3} + a^{3} b^{4} c^{3} d^{4} + a^{4} b^{3} c^{2} d^{5} + a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right) x}{8 \left(b^{16} x^{8} + 8 a b^{15} x^{7} + 28 a^{2} b^{14} x^{6} + 56 a^{3} b^{13} x^{5} + 70 a^{4} b^{12} x^{4} + 56 a^{5} b^{11} x^{3} + 28 a^{6} b^{10} x^{2} + 8 a^{7} b^{9} x + a^{8} b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4
*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7 + 28*(b^7*
c*d^6 + a*b^6*d^7)*x^6 + 56*(b^7*c^2*d^5 + a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 +
 70*(b^7*c^3*d^4 + a*b^6*c^2*d^5 + a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 56*(b
^7*c^4*d^3 + a*b^6*c^3*d^4 + a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + a^4*b^3*d^7)
*x^3 + 28*(b^7*c^5*d^2 + a*b^6*c^4*d^3 + a^2*b^5*c^3*d^4 + a^3*b^4*c^2*d^5
+ a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 8*(b^7*c^6*d + a*b^6*c^5*d^2 + a^2*b^5
*c^4*d^3 + a^3*b^4*c^3*d^4 + a^4*b^3*c^2*d^5 + a^5*b^2*c*d^6 + a^6*b*d^7)*x
)/(b^16*x^8 + 8*a*b^15*x^7 + 28*a^2*b^14*x^6 + 56*a^3*b^13*x^5 + 70*a^4*b^1
2*x^4 + 56*a^5*b^11*x^3 + 28*a^6*b^10*x^2 + 8*a^7*b^9*x + a^8*b^8)
```

**giac [B]** time = 1.29, size = 489, normalized size = 17.46

$$-\frac{8 b^{7} d^{7} x^{7} + 28 b^{7} c d^{6} x^{6} + 28 a b^{6} d^{7} x^{6} + 56 b^{7} c^{2} d^{5} x^{5} + 56 a b^{6} c d^{6} x^{5} + 56 a^{2} b^{5} d^{7} x^{5} + 70 b^{7} c^{3} d^{4} x^{4} + 70 a b^{6} c^{2} d^{5} x^{4} + 70 a^{2} b^{5} c d^{6} x^{4} + 70 a^{3} b^{4} d^{7} x^{4} + 56 b^{7} c^{4} d^{3} x^{3} + 56 a b^{6} c^{3} d^{4} x^{3} + 56 a^{2} b^{5} c^{2} d^{5} x^{3} + 56 a^{3} b^{4} c d^{6} x^{3} + 56 a^{4} b^{3} d^{7} x^{3} + 28 b^{7} c^{5} d^{2} x^{2} + 28 a b^{6} c^{4} d^{3} x^{2} + 28 a^{2} b^{5} c^{3} d^{4} x^{2} + 28 a^{3} b^{4} c^{2} d^{5} x^{2} + 28 a^{4} b^{3} c d^{6} x^{2} + 28 a^{5} b^{2} d^{7} x^{2} + 8 b^{7} c^{6} d x + 8 a b^{6} c^{5} d^{2} x + 8 a^{2} b^{5} c^{4} d^{3} x + 8 a^{3} b^{4} c^{3} d^{4} x + 8 a^{4} b^{3} c^{2} d^{5} x + 8 a^{5} b^{2} c d^{6} x + 8 a^{6} b d^{7} x + b^{7} c^{7} + a b^{6} c^{6} d + a^{2} b^{5} c^{5} d^{2} + a^{3} b^{4} c^{4} d^{3} + a^{4} b^{3} c^{3} d^{4} + a^{5} b^{2} c^{2} d^{5} + a^{6} b c d^{6} + a^{7} d^{7}}{8 \, (b x + a)^{8} b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + 28*b^7*c*d^6*x^6 + 28*a*b^6*d^7*x^6 + 56*b^7*c^2*d^5*
x^5 + 56*a*b^6*c*d^6*x^5 + 56*a^2*b^5*d^7*x^5 + 70*b^7*c^3*d^4*x^4 + 70*a*b
^6*c^2*d^5*x^4 + 70*a^2*b^5*c*d^6*x^4 + 70*a^3*b^4*d^7*x^4 + 56*b^7*c^4*d^3
```

```
*x^3 + 56*a*b^6*c^3*d^4*x^3 + 56*a^2*b^5*c^2*d^5*x^3 + 56*a^3*b^4*c*d^6*x^3
 + 56*a^4*b^3*d^7*x^3 + 28*b^7*c^5*d^2*x^2 + 28*a*b^6*c^4*d^3*x^2 + 28*a^2*
b^5*c^3*d^4*x^2 + 28*a^3*b^4*c^2*d^5*x^2 + 28*a^4*b^3*c*d^6*x^2 + 28*a^5*b^
2*d^7*x^2 + 8*b^7*c^6*d*x + 8*a*b^6*c^5*d^2*x + 8*a^2*b^5*c^4*d^3*x + 8*a^3
*b^4*c^3*d^4*x + 8*a^4*b^3*c^2*d^5*x + 8*a^5*b^2*c*d^6*x + 8*a^6*b*d^7*x +
b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4
 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^8*b^8)
```

**maple [B]** time = 0.01, size = 464, normalized size = 16.57

$$-\frac{d^7}{(bx+a)\,b^8}+\frac{7(ad-bc)\,d^6}{2(bx+a)^2\,b^8}-\frac{7\left(a^2d^2-2abcd+b^2c^2\right)d^5}{(bx+a)^3\,b^8}+\frac{35\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)d^4}{4(bx+a)^4\,b^8}-\frac{7\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)d^3}{(bx+a)^5\,b^8}+\frac{7\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)d^2}{2(bx+a)^6\,b^8}-\frac{\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)d}{(bx+a)^7\,b^8}-\frac{-a^7d^7+7a^6bcd^6-21a^5b^2c^2d^5+35a^4b^3c^3d^4-35a^3b^4c^4d^3+21a^2b^5c^5d^2-7ab^6c^6d+b^7c^7}{8(bx+a)^8\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^9,x)
```

```
[Out] -1/8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b
^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^8-7*d^5*(a
^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^3-d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2
*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(
b*x+a)^7-7*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c
^4)/b^8/(b*x+a)^5+7/2*d^6*(a*d-b*c)/b^8/(b*x+a)^2+35/4*d^4*(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^4-d^7/b^8/(b*x+a)+7/2*d^2*(a^5*d^
5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5
)/b^8/(b*x+a)^6
```

**maxima [B]** time = 1.65, size = 509, normalized size = 18.18

$$-\frac{8b^7d^7x^7+b^7c^7+ab^6c^6d+a^2b^5c^5d^2+a^3b^4c^4d^3+a^4b^3c^3d^4+a^5b^2c^2d^5+a^6bcd^6+a^7d^7+28\left(b^7cd^6+ab^6d^7\right)x^6+56\left(b^7c^2d^5+ab^6cd^6+a^2b^5d^7\right)x^5+70\left(b^7c^3d^4+ab^6c^2d^5+a^2b^5cd^6+a^3b^4d^7\right)x^4+56\left(b^7c^4d^3+ab^6c^3d^4+a^2b^5c^2d^5+a^3b^4cd^6+a^4b^3d^7\right)x^3+28\left(b^7c^5d^2+ab^6c^4d^3+a^2b^5c^3d^4+a^3b^4c^2d^5+a^4b^3cd^6+a^5b^2d^7\right)x^2+8\left(b^7c^6d+ab^6c^5d^2+a^2b^5c^4d^3+a^3b^4c^3d^4+a^4b^3c^2d^5+a^5b^2cd^6+a^6bd^7\right)x}{8\left(b^{16}x^8+8ab^{15}x^7+28a^2b^{14}x^6+56a^3b^{13}x^5+70a^4b^{12}x^4+56a^5b^{11}x^3+28a^6b^{10}x^2+8a^7b^9x+a^8b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4
*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7 + 28*(b^7*
c*d^6 + a*b^6*d^7)*x^6 + 56*(b^7*c^2*d^5 + a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 +
 70*(b^7*c^3*d^4 + a*b^6*c^2*d^5 + a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 56*(b
^7*c^4*d^3 + a*b^6*c^3*d^4 + a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + a^4*b^3*d^7)
*x^3 + 28*(b^7*c^5*d^2 + a*b^6*c^4*d^3 + a^2*b^5*c^3*d^4 + a^3*b^4*c^2*d^5
+ a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 8*(b^7*c^6*d + a*b^6*c^5*d^2 + a^2*b^5
*c^4*d^3 + a^3*b^4*c^3*d^4 + a^4*b^3*c^2*d^5 + a^5*b^2*c*d^6 + a^6*b*d^7)*x
)/(b^16*x^8 + 8*a*b^15*x^7 + 28*a^2*b^14*x^6 + 56*a^3*b^13*x^5 + 70*a^4*b^1
2*x^4 + 56*a^5*b^11*x^3 + 28*a^6*b^10*x^2 + 8*a^7*b^9*x + a^8*b^8)
```

**mupad [B]** time = 0.17, size = 571, normalized size = 20.39

$$-\frac{a^7d^7+a^6bcd^6+8a^6bd^7x+a^5b^2c^2d^5+8a^5b^2cd^6x+28a^5b^2d^7x^2+a^4b^3c^3d^4+8a^4b^3c^2d^5x+28a^4b^3cd^6x^2+56a^4b^3d^7x^3+a^3b^4c^4d^3+8a^3b^4c^3d^4x+28a^3b^4c^2d^5x^2+56a^3b^4cd^6x^3+70a^3b^4d^7x^4+a^2b^5c^5d^2+8a^2b^5c^4d^3x+28a^2b^5c^3d^4x^2+56a^2b^5c^2d^5x^3+70a^2b^5cd^6x^4+56a^2b^5d^7x^5+ab^6c^6d+8ab^6c^5d^2x+28ab^6c^4d^3x^2+56ab^6c^3d^4x^3+70ab^6c^2d^5x^4+56ab^6cd^6x^5+28ab^6d^7x^6+b^7c^7+8b^7c^6dx+28b^7c^5d^2x^2+56b^7c^4d^3x^3+70b^7c^3d^4x^4+56b^7c^2d^5x^5+28b^7cd^6x^6+8b^7d^7x^7}{8a^8b^8+64a^7b^9x+224a^6b^{10}x^2+448a^5b^{11}x^3+560a^4b^{12}x^4+448a^3b^{13}x^5+224a^2b^{14}x^6+64ab^{15}x^7+8b^{16}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^9,x)
```

```
[Out] -(a^7*d^7 + b^7*c^7 + 8*b^7*d^7*x^7 + 28*a*b^6*d^7*x^6 + 28*b^7*c*d^6*x^6 +
 a^2*b^5*c^5*d^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + 28
*a^5*b^2*d^7*x^2 + 56*a^4*b^3*d^7*x^3 + 70*a^3*b^4*d^7*x^4 + 56*a^2*b^5*d^7
*x^5 + 28*b^7*c^5*d^2*x^2 + 56*b^7*c^4*d^3*x^3 + 70*b^7*c^3*d^4*x^4 + 56*b^
7*c^2*d^5*x^5 + a*b^6*c^6*d + a^6*b*c*d^6 + 8*a^6*b*d^7*x + 8*b^7*c^6*d*x +
 28*a^2*b^5*c^3*d^4*x^2 + 28*a^3*b^4*c^2*d^5*x^2 + 56*a^2*b^5*c^2*d^5*x^3 +
 8*a*b^6*c^5*d^2*x + 8*a^5*b^2*c*d^6*x + 56*a*b^6*c*d^6*x^5 + 8*a^2*b^5*c^4
*d^3*x + 8*a^3*b^4*c^3*d^4*x + 8*a^4*b^3*c^2*d^5*x + 28*a*b^6*c^4*d^3*x^2 +
 28*a^4*b^3*c*d^6*x^2 + 56*a*b^6*c^3*d^4*x^3 + 56*a^3*b^4*c*d^6*x^3 + 70*a*
b^6*c^2*d^5*x^4 + 70*a^2*b^5*c*d^6*x^4)/(8*a^8*b^8 + 8*b^16*x^8 + 64*a^7*b^
9*x + 64*a*b^15*x^7 + 224*a^6*b^10*x^2 + 448*a^5*b^11*x^3 + 560*a^4*b^12*x^
4 + 448*a^3*b^13*x^5 + 224*a^2*b^14*x^6)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**9,x)
```

```
[Out] Timed out
```

# 3.1186 $\int \frac{(c+dx)^7}{(a+bx)^{10}} dx$

**Optimal**. Leaf size=58

$$\frac{d(c+dx)^8}{72(a+bx)^8(bc-ad)^2} - \frac{(c+dx)^8}{9(a+bx)^9(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d(c+dx)^8}{72(a+bx)^8(bc-ad)^2} - \frac{(c+dx)^8}{9(a+bx)^9(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^10,x]
```

```
[Out] -(c + d*x)^8/(9*(b*c - a*d)*(a + b*x)^9) + (d*(c + d*x)^8)/(72*(b*c - a*d)^
2*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx = -\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9} - \frac{d\int \frac{(c+dx)^7}{(a+bx)^9}\,dx}{9(bc-ad)}$$
$$= -\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9} + \frac{d(c+dx)^8}{72(bc-ad)^2(a+bx)^8}$$

**Mathematica [B]** time = 0.13, size = 367, normalized size = 6.33

$$-\frac{a^7d^7+a^6bd^6(2c+9dx)+3a^5b^2d^5\left(c^2+6cdx+12d^2x^2\right)+a^4b^3d^4\left(4c^3+27c^2dx+72cd^2x^2+84d^3x^3\right)+a^3b^4d^3\left(5c^4+36c^3dx+108c^2d^2x^2+168cd^3x^3+126d^4x^4\right)+3a^2b^5d^2\left(2c^5+15c^4dx+48c^3d^2x^2+84c^2d^3x^3+84cd^4x^4+42d^5x^5\right)+ab^6d\left(7c^6+54c^5dx+180c^4d^2x^2+336c^3d^3x^3+378c^2d^4x^4+252cd^5x^5+84d^6x^6\right)+b^7\left(8c^7+63c^6dx+216c^5d^2x^2+420c^4d^3x^3+504c^3d^4x^4+378c^2d^5x^5+168cd^6x^6+36d^7x^7\right)}{72b^8(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^10,x]
```

```
[Out] -1/72*(a^7*d^7 + a^6*b*d^6*(2*c + 9*d*x) + 3*a^5*b^2*d^5*(c^2 + 6*c*d*x + 1
2*d^2*x^2) + a^4*b^3*d^4*(4*c^3 + 27*c^2*d*x + 72*c*d^2*x^2 + 84*d^3*x^3) +
 a^3*b^4*d^3*(5*c^4 + 36*c^3*d*x + 108*c^2*d^2*x^2 + 168*c*d^3*x^3 + 126*d^
4*x^4) + 3*a^2*b^5*d^2*(2*c^5 + 15*c^4*d*x + 48*c^3*d^2*x^2 + 84*c^2*d^3*x^
3 + 84*c*d^4*x^4 + 42*d^5*x^5) + a*b^6*d*(7*c^6 + 54*c^5*d*x + 180*c^4*d^2*
x^2 + 336*c^3*d^3*x^3 + 378*c^2*d^4*x^4 + 252*c*d^5*x^5 + 84*d^6*x^6) + b^7
*(8*c^7 + 63*c^6*d*x + 216*c^5*d^2*x^2 + 420*c^4*d^3*x^3 + 504*c^3*d^4*x^4
+ 378*c^2*d^5*x^5 + 168*c*d^6*x^6 + 36*d^7*x^7))/(b^8*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^10, x]
```

**fricas [B]** time = 1.41, size = 548, normalized size = 9.45

$$-\frac{36b^7d^7x^7+8b^7c^7+7ab^6c^6d+6a^2b^5c^5d^2+5a^3b^4c^4d^3+4a^4b^3c^3d^4+3a^5b^2c^2d^5+2a^6bcd^6+a^7d^7+84\left(2b^7cd^6+ab^6d^7\right)x^6+126\left(3b^7c^2d^5+2ab^6cd^6+a^2b^5d^7\right)x^5+126\left(4b^7c^3d^4+3ab^6c^2d^5+2a^2b^5cd^6+a^3b^4d^7\right)x^4+84\left(5b^7c^4d^3+4ab^6c^3d^4+3a^2b^5c^2d^5+2a^3b^4cd^6+a^4b^3d^7\right)x^3+36\left(6b^7c^5d^2+5ab^6c^4d^3+4a^2b^5c^3d^4+3a^3b^4c^2d^5+2a^4b^3cd^6+a^5b^2d^7\right)x^2+9\left(7b^7c^6d+6ab^6c^5d^2+5a^2b^5c^4d^3+4a^3b^4c^3d^4+3a^4b^3c^2d^5+2a^5b^2cd^6+a^6bd^7\right)x}{72\left(b^{17}x^9+9ab^{16}x^8+36a^2b^{15}x^7+84a^3b^{14}x^6+126a^4b^{13}x^5+126a^5b^{12}x^4+84a^6b^{11}x^3+36a^7b^{10}x^2+9a^8b^9x+a^9b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
```

```
7*d^7 + 84*(2*b^7*c*d^6 + a*b^6*d^7)*x^6 + 126*(3*b^7*c^2*d^5 + 2*a*b^6*c*d
^6 + a^2*b^5*d^7)*x^5 + 126*(4*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 2*a^2*b^5*c*
d^6 + a^3*b^4*d^7)*x^4 + 84*(5*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 3*a^2*b^5*c^
2*d^5 + 2*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 36*(6*b^7*c^5*d^2 + 5*a*b^6*c^
4*d^3 + 4*a^2*b^5*c^3*d^4 + 3*a^3*b^4*c^2*d^5 + 2*a^4*b^3*c*d^6 + a^5*b^2*d
^7)*x^2 + 9*(7*b^7*c^6*d + 6*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 4*a^3*b^4*
c^3*d^4 + 3*a^4*b^3*c^2*d^5 + 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^17*x^9 + 9
*a*b^16*x^8 + 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^
5*b^12*x^4 + 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

**giac [B]** time = 1.27, size = 496, normalized size = 8.55

$$-\frac{36 b^7 d^7 x^7 + 168 b^7 c d^6 x^6 + 84 a b^6 d^7 x^6 + 378 b^7 c^2 d^5 x^5 + 252 a b^6 c d^6 x^5 + 126 a^2 b^5 d^7 x^5 + 504 b^7 c^3 d^4 x^4 + 378 a b^6 c^2 d^5 x^4 + 252 a^2 b^5 c d^6 x^4 + 126 a^3 b^4 d^7 x^4 + 420 b^7 c^4 d^3 x^3 + 336 a b^6 c^3 d^4 x^3 + 252 a^2 b^5 c^2 d^5 x^3 + 168 a^3 b^4 c d^6 x^3 + 84 a^4 b^3 d^7 x^3 + 216 b^7 c^5 d^2 x^2 + 180 a b^6 c^4 d^3 x^2 + 144 a^2 b^5 c^3 d^4 x^2 + 108 a^3 b^4 c^2 d^5 x^2 + 72 a^4 b^3 c d^6 x^2 + 36 a^5 b^2 d^7 x^2 + 63 b^7 c^6 d x + 54 a b^6 c^5 d^2 x + 45 a^2 b^5 c^4 d^3 x + 36 a^3 b^4 c^3 d^4 x + 27 a^4 b^3 c^2 d^5 x + 18 a^5 b^2 c d^6 x + 9 a^6 b d^7 x + 8 b^7 c^7 + 7 a b^6 c^6 d + 6 a^2 b^5 c^5 d^2 + 5 a^3 b^4 c^4 d^3 + 4 a^4 b^3 c^3 d^4 + 3 a^5 b^2 c^2 d^5 + 2 a^6 b c d^6 + a^7 d^7}{72 (bx + a)^9 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 168*b^7*c*d^6*x^6 + 84*a*b^6*d^7*x^6 + 378*b^7*c^2*
d^5*x^5 + 252*a*b^6*c*d^6*x^5 + 126*a^2*b^5*d^7*x^5 + 504*b^7*c^3*d^4*x^4 +
 378*a*b^6*c^2*d^5*x^4 + 252*a^2*b^5*c*d^6*x^4 + 126*a^3*b^4*d^7*x^4 + 420*
b^7*c^4*d^3*x^3 + 336*a*b^6*c^3*d^4*x^3 + 252*a^2*b^5*c^2*d^5*x^3 + 168*a^3
*b^4*c*d^6*x^3 + 84*a^4*b^3*d^7*x^3 + 216*b^7*c^5*d^2*x^2 + 180*a*b^6*c^4*d
^3*x^2 + 144*a^2*b^5*c^3*d^4*x^2 + 108*a^3*b^4*c^2*d^5*x^2 + 72*a^4*b^3*c*d
^6*x^2 + 36*a^5*b^2*d^7*x^2 + 63*b^7*c^6*d*x + 54*a*b^6*c^5*d^2*x + 45*a^2*
b^5*c^4*d^3*x + 36*a^3*b^4*c^3*d^4*x + 27*a^4*b^3*c^2*d^5*x + 18*a^5*b^2*c*
d^6*x + 9*a^6*b*d^7*x + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7)/((b*x + a)^9*b^8)
```

**maple [B]** time = 0.01, size = 464, normalized size = 8.00

$$-\frac{d^7}{2 (bx+a)^2 b^8} + \frac{7 (ad - bc) d^6}{3 (bx+a)^3 b^8} - \frac{21 \left(a^2 d^2 - 2abcd + b^2 c^2\right) d^5}{4 (bx+a)^4 b^8} + \frac{7 \left(a^3 d^3 - 3a^2 bc d^2 + 3a b^2 c^2 d - b^3 c^3\right) d^4}{(bx+a)^5 b^8} - \frac{35 \left(a^4 d^4 - 4a^3 bc d^3 + 6a^2 b^2 c^2 d^2 - 4a b^3 c^3 d + b^4 c^4\right) d^3}{6 (bx+a)^6 b^8} + \frac{3 \left(a^5 d^5 - 5a^4 bc d^4 + 10a^3 b^2 c^2 d^3 - 10a^2 b^3 c^3 d^2 + 5a b^4 c^4 d - b^5 c^5\right) d^2}{(bx+a)^7 b^8} - \frac{7 \left(a^6 d^6 - 6a^5 bc d^5 + 15a^4 b^2 c^2 d^4 - 20a^3 b^3 c^3 d^3 + 15a^2 b^4 c^4 d^2 - 6a b^5 c^5 d + b^6 c^6\right) d}{8 (bx+a)^8 b^8} - \frac{-a^7 d^7 + 7a^6 bc d^6 - 21a^5 b^2 c^2 d^5 + 35a^4 c^3 d^4 b^3 - 35a^3 b^4 c^4 d^3 + 21a^2 c^5 d^2 b^5 - 7a b^6 c^6 d + b^7 c^7}{9 (bx+a)^9 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^10,x)
```

```
[Out] -7/8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*
b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^8+7/3*d^6*(a*d-b*c)/b^8/(b*x
+a)^3+3*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*
a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^7-1/9*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c
^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6
*d+b^7*c^7)/b^8/(b*x+a)^9+7*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)/b^8/(b*x+a)^5-1/2*d^7/b^8/(b*x+a)^2-21/4*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)
/b^8/(b*x+a)^4-35/6*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^
3*d+b^4*c^4)/b^8/(b*x+a)^6
```

**maxima [B]** time = 1.71, size = 548, normalized size = 9.45

$$-\frac{36 b^{7} d^{7} x^{7} + 8 b^{7} c^{7} + 7 a b^{6} c^{6} d + 6 a^{2} b^{5} c^{5} d^{2} + 5 a^{3} b^{4} c^{4} d^{3} + 4 a^{4} b^{3} c^{3} d^{4} + 3 a^{5} b^{2} c^{2} d^{5} + 2 a^{6} b c d^{6} + a^{7} d^{7} + 84 \left(2 b^{7} c d^{6} + a b^{6} d^{7}\right) x^{6} + 126 \left(3 b^{7} c^{2} d^{5} + 2 a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right) x^{5} + 126 \left(4 b^{7} c^{3} d^{4} + 3 a b^{6} c^{2} d^{5} + 2 a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}\right) x^{4} + 84 \left(5 b^{7} c^{4} d^{3} + 4 a b^{6} c^{3} d^{4} + 3 a^{2} b^{5} c^{2} d^{5} + 2 a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right) x^{3} + 36 \left(6 b^{7} c^{5} d^{2} + 5 a b^{6} c^{4} d^{3} + 4 a^{2} b^{5} c^{3} d^{4} + 3 a^{3} b^{4} c^{2} d^{5} + 2 a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}\right) x^{2} + 9 \left(7 b^{7} c^{6} d + 6 a b^{6} c^{5} d^{2} + 5 a^{2} b^{5} c^{4} d^{3} + 4 a^{3} b^{4} c^{3} d^{4} + 3 a^{4} b^{3} c^{2} d^{5} + 2 a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right) x}{72 \left(b^{17} x^{9} + 9 a b^{16} x^{8} + 36 a^{2} b^{15} x^{7} + 84 a^{3} b^{14} x^{6} + 126 a^{4} b^{13} x^{5} + 126 a^{5} b^{12} x^{4} + 84 a^{6} b^{11} x^{3} + 36 a^{7} b^{10} x^{2} + 9 a^{8} b^{9} x + a^{9} b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7 + 84*(2*b^7*c*d^6 + a*b^6*d^7)*x^6 + 126*(3*b^7*c^2*d^5 + 2*a*b^6*c*d
^6 + a^2*b^5*d^7)*x^5 + 126*(4*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 2*a^2*b^5*c*
d^6 + a^3*b^4*d^7)*x^4 + 84*(5*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 3*a^2*b^5*c^
2*d^5 + 2*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 36*(6*b^7*c^5*d^2 + 5*a*b^6*c^
4*d^3 + 4*a^2*b^5*c^3*d^4 + 3*a^3*b^4*c^2*d^5 + 2*a^4*b^3*c*d^6 + a^5*b^2*d
^7)*x^2 + 9*(7*b^7*c^6*d + 6*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 4*a^3*b^4*
c^3*d^4 + 3*a^4*b^3*c^2*d^5 + 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^17*x^9 + 9
*a*b^16*x^8 + 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^
5*b^12*x^4 + 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

**mupad [B]** time = 0.15, size = 39, normalized size = 0.67

$$\frac{{\left(c+d\,x\right)}^8\,\left(9\,a\,d-8\,b\,c+b\,d\,x\right)}{72\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^10,x)
```

```
[Out] ((c + d*x)^8*(9*a*d - 8*b*c + b*d*x))/(72*(a*d - b*c)^2*(a + b*x)^9)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**10,x)
```

```
[Out] Timed out
```

## 3.1187 $\int \frac{(c+dx)^7}{(a+bx)^{11}} \, dx$

**Optimal**. Leaf size=89

$$-\frac{d^2(c+dx)^8}{360(a+bx)^8(bc-ad)^3}+\frac{d(c+dx)^8}{45(a+bx)^9(bc-ad)^2}-\frac{(c+dx)^8}{10(a+bx)^{10}(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^2(c+dx)^8}{360(a+bx)^8(bc-ad)^3}+\frac{d(c+dx)^8}{45(a+bx)^9(bc-ad)^2}-\frac{(c+dx)^8}{10(a+bx)^{10}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^11,x]
```

```
[Out] -(c + d*x)^8/(10*(b*c - a*d)*(a + b*x)^10) + (d*(c + d*x)^8)/(45*(b*c - a*d
)^2*(a + b*x)^9) - (d^2*(c + d*x)^8)/(360*(b*c - a*d)^3*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx = -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}} - \frac{d\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx}{5(bc-ad)}$$

$$= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}} + \frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9} + \frac{d^2\int \frac{(c+dx)^7}{(a+bx)^9}\,dx}{45(bc-ad)^2}$$

$$= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}} + \frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9} - \frac{d^2(c+dx)^8}{360(bc-ad)^3(a+bx)^8}$$

**Mathematica [B]** time = 0.12, size = 371, normalized size = 4.17

$$-\frac{a^7d^7 + a^6bd^6(3c+10dx) + 3a^5b^2d^5(2c^2+10cdx+15d^2x^2) + 5a^4b^3d^4(2c^3+12c^2dx+27cd^2x^2+24d^3x^3) + 5a^3b^4d^3(3c^4+20c^3dx+54c^2d^2x^2+72cd^3x^3+42d^4x^4) + 3a^2b^5d^2(7c^5+50c^4dx+150c^3d^2x^2+240c^2d^3x^3+210cd^4x^4+84d^5x^5) + ab^6d(28c^6+210c^5dx+675c^4d^2x^2+1200c^3d^3x^3+1260c^2d^4x^4+756cd^5x^5+210d^6x^6) + b^7(36c^7+280c^6dx+945c^5d^2x^2+1800c^4d^3x^3+2100c^3d^4x^4+1512c^2d^5x^5+630cd^6x^6+120d^7x^7)}{360b^8(a+bx)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^11,x]
```

```
[Out] -1/360*(a^7*d^7 + a^6*b*d^6*(3*c + 10*d*x) + 3*a^5*b^2*d^5*(2*c^2 + 10*c*d*
x + 15*d^2*x^2) + 5*a^4*b^3*d^4*(2*c^3 + 12*c^2*d*x + 27*c*d^2*x^2 + 24*d^3
*x^3) + 5*a^3*b^4*d^3*(3*c^4 + 20*c^3*d*x + 54*c^2*d^2*x^2 + 72*c*d^3*x^3 +
 42*d^4*x^4) + 3*a^2*b^5*d^2*(7*c^5 + 50*c^4*d*x + 150*c^3*d^2*x^2 + 240*c^
2*d^3*x^3 + 210*c*d^4*x^4 + 84*d^5*x^5) + a*b^6*d*(28*c^6 + 210*c^5*d*x + 6
75*c^4*d^2*x^2 + 1200*c^3*d^3*x^3 + 1260*c^2*d^4*x^4 + 756*c*d^5*x^5 + 210*
d^6*x^6) + b^7*(36*c^7 + 280*c^6*d*x + 945*c^5*d^2*x^2 + 1800*c^4*d^3*x^3 +
 2100*c^3*d^4*x^4 + 1512*c^2*d^5*x^5 + 630*c*d^6*x^6 + 120*d^7*x^7))/(b^8*(
a + b*x)^10)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^11,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^11, x]
```

**fricas [B]** time = 1.50, size = 559, normalized size = 6.28

$$-\frac{120\,b^7d^7x^7 + 36\,b^7c^7 + 28\,ab^6c^6d + 21\,a^2b^5c^5d^2 + 15\,a^3b^4c^4d^3 + 10\,a^4b^3c^3d^4 + 6\,a^5b^2c^2d^5 + 3\,a^6bcd^6 + a^7d^7 + 210\left(3\,b^7cd^6 + ab^6d^7\right)x^6 + 252\left(6\,b^7c^2d^5 + 3\,ab^6cd^6 + a^2b^5d^7\right)x^5 + 210\left(10\,b^7c^3d^4 + 6\,ab^6c^2d^5 + 3\,a^2b^5cd^6 + a^3b^4d^7\right)x^4 + 120\left(15\,b^7c^4d^3 + 10\,ab^6c^3d^4 + 6\,a^2b^5c^2d^5 + 3\,a^3b^4cd^6 + a^4b^3d^7\right)x^3 + 45\left(21\,b^7c^5d^2 + 15\,ab^6c^4d^3 + 10\,a^2b^5c^3d^4 + 6\,a^3b^4c^2d^5 + 3\,a^4b^3cd^6 + a^5b^2d^7\right)x^2 + 10\left(28\,b^7c^6d + 21\,ab^6c^5d^2 + 15\,a^2b^5c^4d^3 + 10\,a^3b^4c^3d^4 + 6\,a^4b^3c^2d^5 + 3\,a^5b^2cd^6 + a^6bd^7\right)x}{360\left(b^{18}x^{10} + 10\,ab^{17}x^9 + 45\,a^2b^{16}x^8 + 120\,a^3b^{15}x^7 + 210\,a^4b^{14}x^6 + 252\,a^5b^{13}x^5 + 210\,a^6b^{12}x^4 + 120\,a^7b^{11}x^3 + 45\,a^8b^{10}x^2 + 10\,a^9b^9x + a^{10}b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="fricas")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2
+ 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + a^7*d^7 + 210*(3*b^7*c*d^6 + a*b^6*d^7)*x^6 + 252*(6*b^7*c^2*d^5 + 3*a
*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 210*(10*b^7*c^3*d^4 + 6*a*b^6*c^2*d^5 + 3*a
^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 120*(15*b^7*c^4*d^3 + 10*a*b^6*c^3*d^4 +
6*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 45*(21*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 6*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*
d^6 + a^5*b^2*d^7)*x^2 + 10*(28*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 15*a^2*b^5*c
^4*d^3 + 10*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6 + a^6*b*d
^7)*x)/(b^18*x^10 + 10*a*b^17*x^9 + 45*a^2*b^16*x^8 + 120*a^3*b^15*x^7 + 21
0*a^4*b^14*x^6 + 252*a^5*b^13*x^5 + 210*a^6*b^12*x^4 + 120*a^7*b^11*x^3 + 4
5*a^8*b^10*x^2 + 10*a^9*b^9*x + a^10*b^8)
```

**giac [B]** time = 1.31, size = 496, normalized size = 5.57

$$-\frac{120 b^7 d^7 x^7 + 630 b^7 c d^6 x^6 + 210 a b^6 d^7 x^6 + 1512 b^7 c^2 d^5 x^5 + 756 a b^6 c d^6 x^5 + 252 a^2 b^5 d^7 x^5 + 2100 b^7 c^3 d^4 x^4 + 1260 a b^6 c^2 d^5 x^4 + 630 a^2 b^5 c d^6 x^4 + 210 a^3 b^4 d^7 x^4 + 1800 b^7 c^4 d^3 x^3 + 1200 a b^6 c^3 d^4 x^3 + 720 a^2 b^5 c^2 d^5 x^3 + 360 a^3 b^4 c d^6 x^3 + 120 a^4 b^3 d^7 x^3 + 945 b^7 c^5 d^2 x^2 + 675 a b^6 c^4 d^3 x^2 + 450 a^2 b^5 c^3 d^4 x^2 + 270 a^3 b^4 c^2 d^5 x^2 + 135 a^4 b^3 c d^6 x^2 + 45 a^5 b^2 d^7 x^2 + 280 b^7 c^6 d x + 210 a b^6 c^5 d^2 x + 150 a^2 b^5 c^4 d^3 x + 100 a^3 b^4 c^3 d^4 x + 60 a^4 b^3 c^2 d^5 x + 30 a^5 b^2 c d^6 x + 10 a^6 b d^7 x + 36 b^7 c^7 + 28 a b^6 c^6 d + 21 a^2 b^5 c^5 d^2 + 15 a^3 b^4 c^4 d^3 + 10 a^4 b^3 c^3 d^4 + 6 a^5 b^2 c^2 d^5 + 3 a^6 b c d^6 + a^7 d^7}{360 (bx + a)^{10} b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="giac")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 630*b^7*c*d^6*x^6 + 210*a*b^6*d^7*x^6 + 1512*b^7*
c^2*d^5*x^5 + 756*a*b^6*c*d^6*x^5 + 252*a^2*b^5*d^7*x^5 + 2100*b^7*c^3*d^4*
x^4 + 1260*a*b^6*c^2*d^5*x^4 + 630*a^2*b^5*c*d^6*x^4 + 210*a^3*b^4*d^7*x^4
+ 1800*b^7*c^4*d^3*x^3 + 1200*a*b^6*c^3*d^4*x^3 + 720*a^2*b^5*c^2*d^5*x^3 +
 360*a^3*b^4*c*d^6*x^3 + 120*a^4*b^3*d^7*x^3 + 945*b^7*c^5*d^2*x^2 + 675*a*
b^6*c^4*d^3*x^2 + 450*a^2*b^5*c^3*d^4*x^2 + 270*a^3*b^4*c^2*d^5*x^2 + 135*a
^4*b^3*c*d^6*x^2 + 45*a^5*b^2*d^7*x^2 + 280*b^7*c^6*d*x + 210*a*b^6*c^5*d^2
*x + 150*a^2*b^5*c^4*d^3*x + 100*a^3*b^4*c^3*d^4*x + 60*a^4*b^3*c^2*d^5*x +
 30*a^5*b^2*c*d^6*x + 10*a^6*b*d^7*x + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2
*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5
+ 3*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^10*b^8)
```

**maple [B]** time = 0.01, size = 464, normalized size = 5.21

$$-\frac{d^7}{3 (bx + a)^3 b^8} + \frac{7 (ad - bc) d^6}{4 (bx + a)^4 b^8} - \frac{21 \left(a^2 d^2 - 2abcd + b^2 c^2\right) d^5}{5 (bx + a)^5 b^8} + \frac{35 \left(a^3 d^3 - 3a^2 bc d^2 + 3a b^2 c^2 d - b^3 c^3\right) d^4}{6 (bx + a)^6 b^8} - \frac{5 \left(a^4 d^4 - 4a^3 bc d^3 + 6a^2 b^2 c^2 d^2 - 4a b^3 c^3 d + b^4 c^4\right) d^3}{(bx + a)^7 b^8} + \frac{21 \left(a^5 d^5 - 5a^4 bc d^4 + 10a^3 b^2 c^2 d^3 - 10a^2 b^3 c^3 d^2 + 5a b^4 c^4 d - b^5 c^5\right) d^2}{8 (bx + a)^8 b^8} - \frac{7 \left(a^6 d^6 - 6a^5 bc d^5 + 15a^4 b^2 c^2 d^4 - 20a^3 b^3 c^3 d^3 + 15a^2 b^4 c^4 d^2 - 6a b^5 c^5 d + b^6 c^6\right) d}{9 (bx + a)^9 b^8} - \frac{-a^7 d^7 + 7a^6 bc d^6 - 21a^5 b^2 c^2 d^5 + 35a^4 c^3 d^4 b^3 - 35a^3 b^4 c^4 d^3 + 21a^2 c^5 d^2 b^5 - 7a b^6 c^6 d + b^7 c^7}{10 (bx + a)^{10} b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^11,x)
```

```
[Out] 21/8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b
^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^8-1/3*d^7/b^8/(b*x+a)^3-5*d^3*(a^4*d^4-4*a^3*
b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^7-7/9*d*(a^6*d
^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6
```

```
*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^9-21/5*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^
8/(b*x+a)^5+7/4*d^6*(a*d-b*c)/b^8/(b*x+a)^4+35/6*d^4*(a^3*d^3-3*a^2*b*c*d^2
+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^6-1/10*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b
^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6
*c^6*d+b^7*c^7)/b^8/(b*x+a)^10
```

**maxima [B]** time = 1.73, size = 559, normalized size = 6.28

$$-\frac{120\,b^{7}d^{7}x^{7}+36\,b^{7}c^{7}+28\,ab^{6}c^{6}d+21\,a^{2}b^{5}c^{5}d^{2}+15\,a^{3}b^{4}c^{4}d^{3}+10\,a^{4}b^{3}c^{3}d^{4}+6\,a^{5}b^{2}c^{2}d^{5}+3\,a^{6}bcd^{6}+a^{7}d^{7}+210\left(3\,b^{7}cd^{6}+ab^{6}d^{7}\right)x^{6}+252\left(6\,b^{7}c^{2}d^{5}+3\,ab^{6}cd^{6}+a^{2}b^{5}d^{7}\right)x^{5}+210\left(10\,b^{7}c^{3}d^{4}+6\,ab^{6}c^{2}d^{5}+3\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7}\right)x^{4}+120\left(15\,b^{7}c^{4}d^{3}+10\,ab^{6}c^{3}d^{4}+6\,a^{2}b^{5}c^{2}d^{5}+3\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7}\right)x^{3}+45\left(21\,b^{7}c^{5}d^{2}+15\,ab^{6}c^{4}d^{3}+10\,a^{2}b^{5}c^{3}d^{4}+6\,a^{3}b^{4}c^{2}d^{5}+3\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7}\right)x^{2}+10\left(28\,b^{7}c^{6}d+21\,ab^{6}c^{5}d^{2}+15\,a^{2}b^{5}c^{4}d^{3}+10\,a^{3}b^{4}c^{3}d^{4}+6\,a^{4}b^{3}c^{2}d^{5}+3\,a^{5}b^{2}cd^{6}+a^{6}bd^{7}\right)x}{360\left(b^{18}x^{10}+10\,ab^{17}x^{9}+45\,a^{2}b^{16}x^{8}+120\,a^{3}b^{15}x^{7}+210\,a^{4}b^{14}x^{6}+252\,a^{5}b^{13}x^{5}+210\,a^{6}b^{12}x^{4}+120\,a^{7}b^{11}x^{3}+45\,a^{8}b^{10}x^{2}+10\,a^{9}b^{9}x+a^{10}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="maxima")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2
+ 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + a^7*d^7 + 210*(3*b^7*c*d^6 + a*b^6*d^7)*x^6 + 252*(6*b^7*c^2*d^5 + 3*a
*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 210*(10*b^7*c^3*d^4 + 6*a*b^6*c^2*d^5 + 3*a
^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 120*(15*b^7*c^4*d^3 + 10*a*b^6*c^3*d^4 +
6*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 45*(21*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 6*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*
d^6 + a^5*b^2*d^7)*x^2 + 10*(28*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 15*a^2*b^5*c
^4*d^3 + 10*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6 + a^6*b*d
^7)*x)/(b^18*x^10 + 10*a*b^17*x^9 + 45*a^2*b^16*x^8 + 120*a^3*b^15*x^7 + 21
0*a^4*b^14*x^6 + 252*a^5*b^13*x^5 + 210*a^6*b^12*x^4 + 120*a^7*b^11*x^3 + 4
5*a^8*b^10*x^2 + 10*a^9*b^9*x + a^10*b^8)
```

**mupad [B]** time = 0.45, size = 600, normalized size = 6.74

$$-\frac{a^{7}d^{7}+3\,a^{6}bcd^{6}+10\,a^{6}bd^{7}x+6\,a^{5}b^{2}c^{2}d^{5}+30\,a^{5}b^{2}cd^{6}x+45\,a^{5}b^{2}d^{7}x^{2}+10\,a^{4}b^{3}c^{3}d^{4}+60\,a^{4}b^{3}c^{2}d^{5}x+135\,a^{4}b^{3}cd^{6}x^{2}+120\,a^{4}b^{3}d^{7}x^{3}+15\,a^{3}b^{4}c^{4}d^{3}+100\,a^{3}b^{4}c^{3}d^{4}x+270\,a^{3}b^{4}c^{2}d^{5}x^{2}+360\,a^{3}b^{4}cd^{6}x^{3}+210\,a^{3}b^{4}d^{7}x^{4}+21\,a^{2}b^{5}c^{5}d^{2}+150\,a^{2}b^{5}c^{4}d^{3}x+450\,a^{2}b^{5}c^{3}d^{4}x^{2}+720\,a^{2}b^{5}c^{2}d^{5}x^{3}+630\,a^{2}b^{5}cd^{6}x^{4}+252\,a^{2}b^{5}d^{7}x^{5}+28\,ab^{6}c^{6}d+210\,ab^{6}c^{5}d^{2}x+675\,ab^{6}c^{4}d^{3}x^{2}+1200\,ab^{6}c^{3}d^{4}x^{3}+1260\,ab^{6}c^{2}d^{5}x^{4}+756\,ab^{6}cd^{6}x^{5}+210\,ab^{6}d^{7}x^{6}+36\,b^{7}c^{7}+280\,b^{7}c^{6}dx+945\,b^{7}c^{5}d^{2}x^{2}+1800\,b^{7}c^{4}d^{3}x^{3}+2100\,b^{7}c^{3}d^{4}x^{4}+1512\,b^{7}c^{2}d^{5}x^{5}+630\,b^{7}cd^{6}x^{6}+120\,b^{7}d^{7}x^{7}}{360\,a^{10}b^{8}+3600\,a^{9}b^{9}x+16200\,a^{8}b^{10}x^{2}+43200\,a^{7}b^{11}x^{3}+75600\,a^{6}b^{12}x^{4}+90720\,a^{5}b^{13}x^{5}+75600\,a^{4}b^{14}x^{6}+43200\,a^{3}b^{15}x^{7}+16200\,a^{2}b^{16}x^{8}+3600\,ab^{17}x^{9}+360\,b^{18}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^11,x)
```

```
[Out] -(a^7*d^7 + 36*b^7*c^7 + 120*b^7*d^7*x^7 + 210*a*b^6*d^7*x^6 + 630*b^7*c*d^
6*x^6 + 21*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^
5*b^2*c^2*d^5 + 45*a^5*b^2*d^7*x^2 + 120*a^4*b^3*d^7*x^3 + 210*a^3*b^4*d^7*
x^4 + 252*a^2*b^5*d^7*x^5 + 945*b^7*c^5*d^2*x^2 + 1800*b^7*c^4*d^3*x^3 + 21
00*b^7*c^3*d^4*x^4 + 1512*b^7*c^2*d^5*x^5 + 28*a*b^6*c^6*d + 3*a^6*b*c*d^6
+ 10*a^6*b*d^7*x + 280*b^7*c^6*d*x + 450*a^2*b^5*c^3*d^4*x^2 + 270*a^3*b^4*
c^2*d^5*x^2 + 720*a^2*b^5*c^2*d^5*x^3 + 210*a*b^6*c^5*d^2*x + 30*a^5*b^2*c*
d^6*x + 756*a*b^6*c*d^6*x^5 + 150*a^2*b^5*c^4*d^3*x + 100*a^3*b^4*c^3*d^4*x
 + 60*a^4*b^3*c^2*d^5*x + 675*a*b^6*c^4*d^3*x^2 + 135*a^4*b^3*c*d^6*x^2 + 1
200*a*b^6*c^3*d^4*x^3 + 360*a^3*b^4*c*d^6*x^3 + 1260*a*b^6*c^2*d^5*x^4 + 63
0*a^2*b^5*c*d^6*x^4)/(360*a^10*b^8 + 360*b^18*x^10 + 3600*a^9*b^9*x + 3600*
a*b^17*x^9 + 16200*a^8*b^10*x^2 + 43200*a^7*b^11*x^3 + 75600*a^6*b^12*x^4 +
```

```
 90720*a^5*b^13*x^5 + 75600*a^4*b^14*x^6 + 43200*a^3*b^15*x^7 + 16200*a^2*b
^16*x^8)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**11,x)
[Out] Timed out
```

## 3.1188 $\int \frac{(c+dx)^7}{(a+bx)^{12}}\,dx$

**Optimal**. Leaf size=120

$$\frac{d^3(c+dx)^8}{1320(a+bx)^8(bc-ad)^4}-\frac{d^2(c+dx)^8}{165(a+bx)^9(bc-ad)^3}+\frac{3d(c+dx)^8}{110(a+bx)^{10}(bc-ad)^2}-\frac{(c+dx)^8}{11(a+bx)^{11}(bc-ad)}$$

**Rubi [A]** time = 0.03, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d^3(c+dx)^8}{1320(a+bx)^8(bc-ad)^4}-\frac{d^2(c+dx)^8}{165(a+bx)^9(bc-ad)^3}+\frac{3d(c+dx)^8}{110(a+bx)^{10}(bc-ad)^2}-\frac{(c+dx)^8}{11(a+bx)^{11}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] -(c + d*x)^8/(11*(b*c - a*d)*(a + b*x)^11) + (3*d*(c + d*x)^8)/(110*(b*c -
a*d)^2*(a + b*x)^10) - (d^2*(c + d*x)^8)/(165*(b*c - a*d)^3*(a + b*x)^9) +
(d^3*(c + d*x)^8)/(1320*(b*c - a*d)^4*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{12}}\,dx = -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} - \frac{(3d)\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx}{11(bc-ad)}$$

$$= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} + \frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}} + \frac{\left(3d^2\right)\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx}{55(bc-ad)^2}$$

$$= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} + \frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}} - \frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9} - \frac{d^3\int \frac{(c+dx)^7}{(a+bx)^9}\,dx}{165(bc-ad)^3 \ldots}$$

$$= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} + \frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}} - \frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9} + \frac{d^3(c+\ldots}{1320(bc-ad)\ldots}$$

**Mathematica [B]** time = 0.12, size = 369, normalized size = 3.08

$$-\frac{a^7d^7+a^6bd^6(4c+11dx)+a^5b^2d^5\left(10c^2+44cdx+55d^2x^2\right)+5a^4b^3d^4\left(4c^3+22c^2dx+44cd^2x^2+33d^3x^3\right)+5a^3b^4d^3\left(7c^4+44c^3dx+110c^2d^2x^2+132cd^3x^3+66d^4x^4\right)+a^2b^5d^2\left(56c^5+385c^4dx+1100c^3d^2x^2+1650c^2d^3x^3+1320cd^4x^4+462d^5x^5\right)+ab^6d\left(84c^6+616c^5dx+1925c^4d^2x^2+3300c^3d^3x^3+3300c^2d^4x^4+1848cd^5x^5+462d^6x^6\right)+b^7\left(120c^7+924c^6dx+3080c^5d^2x^2+5775c^4d^3x^3+6600c^3d^4x^4+4620c^2d^5x^5+1848cd^6x^6+330d^7x^7\right)}{1320b^8(a+bx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] -1/1320*(a^7*d^7 + a^6*b*d^6*(4*c + 11*d*x) + a^5*b^2*d^5*(10*c^2 + 44*c*d*
x + 55*d^2*x^2) + 5*a^4*b^3*d^4*(4*c^3 + 22*c^2*d*x + 44*c*d^2*x^2 + 33*d^3
*x^3) + 5*a^3*b^4*d^3*(7*c^4 + 44*c^3*d*x + 110*c^2*d^2*x^2 + 132*c*d^3*x^3
 + 66*d^4*x^4) + a^2*b^5*d^2*(56*c^5 + 385*c^4*d*x + 1100*c^3*d^2*x^2 + 165
0*c^2*d^3*x^3 + 1320*c*d^4*x^4 + 462*d^5*x^5) + a*b^6*d*(84*c^6 + 616*c^5*d
*x + 1925*c^4*d^2*x^2 + 3300*c^3*d^3*x^3 + 3300*c^2*d^4*x^4 + 1848*c*d^5*x^
5 + 462*d^6*x^6) + b^7*(120*c^7 + 924*c^6*d*x + 3080*c^5*d^2*x^2 + 5775*c^4
*d^3*x^3 + 6600*c^3*d^4*x^4 + 4620*c^2*d^5*x^5 + 1848*c*d^6*x^6 + 330*d^7*x
^7))/(b^8*(a + b*x)^11)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^12, x]
```

**fricas [B]** time = 0.71, size = 570, normalized size = 4.75

$$-\frac{330\,b^7d^7x^7+120\,b^7c^7+84\,ab^6c^6d+56\,a^2b^5c^5d^2+35\,a^3b^4c^4d^3+20\,a^4b^3c^3d^4+10\,a^5b^2c^2d^5+4\,a^6bcd^6+a^7d^7+462\left(4\,b^7cd^6+ab^6d^7\right)x^6+462\left(10\,b^7c^2d^5+4\,ab^6cd^6+a^2b^5d^7\right)x^5+330\left(20\,b^7c^3d^4+10\,ab^6c^2d^5+4\,a^2b^5cd^6+a^3b^4d^7\right)x^4+165\left(35\,b^7c^4d^3+20\,ab^6c^3d^4+10\,a^2b^5c^2d^5+4\,a^3b^4cd^6+a^4b^3d^7\right)x^3+55\left(56\,b^7c^5d^2+35\,ab^6c^4d^3+20\,a^2b^5c^3d^4+10\,a^3b^4c^2d^5+4\,a^4b^3cd^6+a^5b^2d^7\right)x^2+11\left(84\,b^7c^6d+56\,ab^6c^5d^2+35\,a^2b^5c^4d^3+20\,a^3b^4c^3d^4+10\,a^4b^3c^2d^5+4\,a^5b^2cd^6+a^6bd^7\right)x}{1320\left(b^{19}x^{11}+11\,ab^{18}x^{10}+55\,a^2b^{17}x^9+165\,a^3b^{16}x^8+330\,a^4b^{15}x^7+462\,a^5b^{14}x^6+462\,a^6b^{13}x^5+330\,a^7b^{12}x^4+165\,a^8b^{11}x^3+55\,a^9b^{10}x^2+11\,a^{10}b^9x+a^{11}b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 120*b^7*c^7 + 84*a*b^6*c^6*d + 56*a^2*b^5*c^5*d^
2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 4*a^6*b*
c*d^6 + a^7*d^7 + 462*(4*b^7*c*d^6 + a*b^6*d^7)*x^6 + 462*(10*b^7*c^2*d^5 +
 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 330*(20*b^7*c^3*d^4 + 10*a*b^6*c^2*d^5
+ 4*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 165*(35*b^7*c^4*d^3 + 20*a*b^6*c^3*d
^4 + 10*a^2*b^5*c^2*d^5 + 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 55*(56*b^7*c
^5*d^2 + 35*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 10*a^3*b^4*c^2*d^5 + 4*a^4
*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 11*(84*b^7*c^6*d + 56*a*b^6*c^5*d^2 + 35*a^
2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 10*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 +
 a^6*b*d^7)*x)/(b^19*x^11 + 11*a*b^18*x^10 + 55*a^2*b^17*x^9 + 165*a^3*b^16
*x^8 + 330*a^4*b^15*x^7 + 462*a^5*b^14*x^6 + 462*a^6*b^13*x^5 + 330*a^7*b^1
2*x^4 + 165*a^8*b^11*x^3 + 55*a^9*b^10*x^2 + 11*a^10*b^9*x + a^11*b^8)
```

**giac [B]** time = 1.30, size = 496, normalized size = 4.13

$$-\frac{330\,b^{7}d^{7}x^{7} + 1848\,b^{7}cd^{6}x^{6} + 462\,ab^{6}d^{7}x^{6} + 4620\,b^{7}c^{2}d^{5}x^{5} + 1848\,ab^{6}cd^{6}x^{5} + 462\,a^{2}b^{5}d^{7}x^{5} + 6600\,b^{7}c^{3}d^{4}x^{4} + 3300\,ab^{6}c^{2}d^{5}x^{4} + 1320\,a^{2}b^{5}cd^{6}x^{4} + 330\,a^{3}b^{4}d^{7}x^{4} + 5775\,b^{7}c^{4}d^{3}x^{3} + 3300\,ab^{6}c^{3}d^{4}x^{3} + 1650\,a^{2}b^{5}c^{2}d^{5}x^{3} + 660\,a^{3}b^{4}cd^{6}x^{3} + 165\,a^{4}b^{3}d^{7}x^{3} + 3080\,b^{7}c^{5}d^{2}x^{2} + 1925\,ab^{6}c^{4}d^{3}x^{2} + 1100\,a^{2}b^{5}c^{3}d^{4}x^{2} + 550\,a^{3}b^{4}c^{2}d^{5}x^{2} + 220\,a^{4}b^{3}cd^{6}x^{2} + 55\,a^{5}b^{2}d^{7}x^{2} + 924\,b^{7}c^{6}dx + 616\,ab^{6}c^{5}d^{2}x + 385\,a^{2}b^{5}c^{4}d^{3}x + 220\,a^{3}b^{4}c^{3}d^{4}x + 110\,a^{4}b^{3}c^{2}d^{5}x + 44\,a^{5}b^{2}cd^{6}x + 11\,a^{6}bd^{7}x + 120\,b^{7}c^{7} + 84\,ab^{6}c^{6}d + 56\,a^{2}b^{5}c^{5}d^{2} + 35\,a^{3}b^{4}c^{4}d^{3} + 20\,a^{4}b^{3}c^{3}d^{4} + 10\,a^{5}b^{2}c^{2}d^{5} + 4\,a^{6}bcd^{6} + a^{7}d^{7}}{1320\,(bx+a)^{11}b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="giac")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 1848*b^7*c*d^6*x^6 + 462*a*b^6*d^7*x^6 + 4620*b^
7*c^2*d^5*x^5 + 1848*a*b^6*c*d^6*x^5 + 462*a^2*b^5*d^7*x^5 + 6600*b^7*c^3*d
^4*x^4 + 3300*a*b^6*c^2*d^5*x^4 + 1320*a^2*b^5*c*d^6*x^4 + 330*a^3*b^4*d^7*
x^4 + 5775*b^7*c^4*d^3*x^3 + 3300*a*b^6*c^3*d^4*x^3 + 1650*a^2*b^5*c^2*d^5*
x^3 + 660*a^3*b^4*c*d^6*x^3 + 165*a^4*b^3*d^7*x^3 + 3080*b^7*c^5*d^2*x^2 +
1925*a*b^6*c^4*d^3*x^2 + 1100*a^2*b^5*c^3*d^4*x^2 + 550*a^3*b^4*c^2*d^5*x^2
 + 220*a^4*b^3*c*d^6*x^2 + 55*a^5*b^2*d^7*x^2 + 924*b^7*c^6*d*x + 616*a*b^6
*c^5*d^2*x + 385*a^2*b^5*c^4*d^3*x + 220*a^3*b^4*c^3*d^4*x + 110*a^4*b^3*c^
2*d^5*x + 44*a^5*b^2*c*d^6*x + 11*a^6*b*d^7*x + 120*b^7*c^7 + 84*a*b^6*c^6*
d + 56*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b
^2*c^2*d^5 + 4*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^11*b^8)
```

**maple [B]** time = 0.00, size = 464, normalized size = 3.87

$$-\frac{d^{7}}{4\,(bx+a)^{4}\,b^{8}}+\frac{7\,(ad-bc)\,d^{6}}{5\,(bx+a)^{5}\,b^{8}}-\frac{7\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)d^{5}}{2\,(bx+a)^{6}\,b^{8}}+\frac{5\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)d^{4}}{(bx+a)^{7}\,b^{8}}-\frac{35\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)d^{3}}{8\,(bx+a)^{8}\,b^{8}}+\frac{7\left(a^{5}d^{5}-5a^{4}bc\,d^{4}+10a^{3}b^{2}c^{2}d^{3}-10a^{2}b^{3}c^{3}d^{2}+5a\,b^{4}c^{4}d-b^{5}c^{5}\right)d^{2}}{3\,(bx+a)^{9}\,b^{8}}-\frac{7\left(a^{6}d^{6}-6a^{5}bc\,d^{5}+15a^{4}b^{2}c^{2}d^{4}-20a^{3}b^{3}c^{3}d^{3}+15a^{2}b^{4}c^{4}d^{2}-6a\,b^{5}c^{5}d+b^{6}c^{6}\right)d}{10\,(bx+a)^{10}\,b^{8}}-\frac{-a^{7}d^{7}+7a^{6}bc\,d^{6}-21a^{5}b^{2}c^{2}d^{5}+35a^{4}c^{3}d^{4}b^{3}-35a^{3}b^{4}c^{4}d^{3}+21a^{2}c^{5}d^{2}b^{5}-7a\,b^{6}c^{6}d+b^{7}c^{7}}{11\,(bx+a)^{11}\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^12,x)
```

```
[Out] -35/8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b
^8/(b*x+a)^8+5*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a
```

```
)^7+7/3*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*
a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^9+7/5*d^6*(a*d-b*c)/b^8/(b*x+a)^5-1/4*d^7/
b^8/(b*x+a)^4-1/11*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^
3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x
+a)^11-7/2*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^6-7/10*d*(a^6*d^6-6*
a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^
5*c^5*d+b^6*c^6)/b^8/(b*x+a)^10
```

**maxima [B]** time = 1.81, size = 570, normalized size = 4.75

$$-\frac{330 b^7 d^7 x^7 + 120 b^7 c^7 + 84 a b^6 c^6 d + 56 a^2 b^5 c^5 d^2 + 35 a^3 b^4 c^4 d^3 + 20 a^4 b^3 c^3 d^4 + 10 a^5 b^2 c^2 d^5 + 4 a^6 b c d^6 + a^7 d^7 + 462 \left(4 b^7 c d^6 + a b^6 d^7\right) x^6 + 462 \left(10 b^7 c^2 d^5 + 4 a b^6 c d^6 + a^2 b^5 d^7\right) x^5 + 330 \left(20 b^7 c^3 d^4 + 10 a b^6 c^2 d^5 + 4 a^2 b^5 c d^6 + a^3 b^4 d^7\right) x^4 + 165 \left(35 b^7 c^4 d^3 + 20 a b^6 c^3 d^4 + 10 a^2 b^5 c^2 d^5 + 4 a^3 b^4 c d^6 + a^4 b^3 d^7\right) x^3 + 55 \left(56 b^7 c^5 d^2 + 35 a b^6 c^4 d^3 + 20 a^2 b^5 c^3 d^4 + 10 a^3 b^4 c^2 d^5 + 4 a^4 b^3 c d^6 + a^5 b^2 d^7\right) x^2 + 11 \left(84 b^7 c^6 d + 56 a b^6 c^5 d^2 + 35 a^2 b^5 c^4 d^3 + 20 a^3 b^4 c^3 d^4 + 10 a^4 b^3 c^2 d^5 + 4 a^5 b^2 c d^6 + a^6 b d^7\right) x}{1320 \left(b^{19} x^{11} + 11 a b^{18} x^{10} + 55 a^2 b^{17} x^9 + 165 a^3 b^{16} x^8 + 330 a^4 b^{15} x^7 + 462 a^5 b^{14} x^6 + 462 a^6 b^{13} x^5 + 330 a^7 b^{12} x^4 + 165 a^8 b^{11} x^3 + 55 a^9 b^{10} x^2 + 11 a^{10} b^9 x + a^{11} b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 120*b^7*c^7 + 84*a*b^6*c^6*d + 56*a^2*b^5*c^5*d^
2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 4*a^6*b*
c*d^6 + a^7*d^7 + 462*(4*b^7*c*d^6 + a*b^6*d^7)*x^6 + 462*(10*b^7*c^2*d^5 +
 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 330*(20*b^7*c^3*d^4 + 10*a*b^6*c^2*d^5
+ 4*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 165*(35*b^7*c^4*d^3 + 20*a*b^6*c^3*d
^4 + 10*a^2*b^5*c^2*d^5 + 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 55*(56*b^7*c
^5*d^2 + 35*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 10*a^3*b^4*c^2*d^5 + 4*a^4
*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 11*(84*b^7*c^6*d + 56*a*b^6*c^5*d^2 + 35*a^
2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 10*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 +
 a^6*b*d^7)*x)/(b^19*x^11 + 11*a*b^18*x^10 + 55*a^2*b^17*x^9 + 165*a^3*b^16
*x^8 + 330*a^4*b^15*x^7 + 462*a^5*b^14*x^6 + 462*a^6*b^13*x^5 + 330*a^7*b^1
2*x^4 + 165*a^8*b^11*x^3 + 55*a^9*b^10*x^2 + 11*a^10*b^9*x + a^11*b^8)
```

**mupad [B]** time = 0.52, size = 548, normalized size = 4.57

$$-\frac{\frac{a^7 d^7+4 a^6 b c d^6+10 a^5 b^2 c^2 d^5+20 a^4 b^3 c^3 d^4+35 a^3 b^4 c^4 d^3+56 a^2 b^5 c^5 d^2+84 a b^6 c^6 d+120 b^7 c^7}{1320 b^8}+\frac{d^7 x^7}{4 b}+\frac{d^2 x^2 \left(a^5 d^5+4 a^4 b c d^4+10 a^3 b^2 c^2 d^3+20 a^2 b^3 c^3 d^2+35 a b^4 c^4 d+56 b^5 c^5\right)}{24 b^6}+\frac{d^4 x^4 \left(a^3 d^3+4 a^2 b c d^2+10 a b^2 c^2 d+20 b^3 c^3\right)}{4 b^4}+\frac{7 d^6 x^6 (a d+4 b c)}{20 b^2}+\frac{d^3 x^3 \left(a^4 d^4+4 a^3 b c d^3+10 a^2 b^2 c^2 d^2+20 a b^3 c^3 d+35 b^4 c^4\right)}{8 b^5}+\frac{d x \left(a^6 d^6+4 a^5 b c d^5+10 a^4 b^2 c^2 d^4+20 a^3 b^3 c^3 d^3+35 a^2 b^4 c^4 d^2+56 a b^5 c^5 d+84 b^6 c^6\right)}{120 b^7}+\frac{7 d^5 x^5 \left(a^2 d^2+4 a b c d+10 b^2 c^2\right)}{20 b^3}}{a^{11}+11 a^{10} b x+55 a^9 b^2 x^2+165 a^8 b^3 x^3+330 a^7 b^4 x^4+462 a^6 b^5 x^5+462 a^5 b^6 x^6+330 a^4 b^7 x^7+165 a^3 b^8 x^8+55 a^2 b^9 x^9+11 a b^{10} x^{10}+b^{11} x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^12,x)
```

```
[Out] -((a^7*d^7 + 120*b^7*c^7 + 56*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 20*a^4
*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 84*a*b^6*c^6*d + 4*a^6*b*c*d^6)/(1320*b
^8) + (d^7*x^7)/(4*b) + (d^2*x^2*(a^5*d^5 + 56*b^5*c^5 + 20*a^2*b^3*c^3*d^2
 + 10*a^3*b^2*c^2*d^3 + 35*a*b^4*c^4*d + 4*a^4*b*c*d^4))/(24*b^6) + (d^4*x^
4*(a^3*d^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2))/(4*b^4) + (7*d^6
*x^6*(a*d + 4*b*c))/(20*b^2) + (d^3*x^3*(a^4*d^4 + 35*b^4*c^4 + 10*a^2*b^2*
c^2*d^2 + 20*a*b^3*c^3*d + 4*a^3*b*c*d^3))/(8*b^5) + (d*x*(a^6*d^6 + 84*b^6
*c^6 + 35*a^2*b^4*c^4*d^2 + 20*a^3*b^3*c^3*d^3 + 10*a^4*b^2*c^2*d^4 + 56*a*
b^5*c^5*d + 4*a^5*b*c*d^5))/(120*b^7) + (7*d^5*x^5*(a^2*d^2 + 10*b^2*c^2 +
4*a*b*c*d))/(20*b^3))/(a^11 + b^11*x^11 + 11*a*b^10*x^10 + 55*a^9*b^2*x^2 +
```

```
 165*a^8*b^3*x^3 + 330*a^7*b^4*x^4 + 462*a^6*b^5*x^5 + 462*a^5*b^6*x^6 + 33
0*a^4*b^7*x^7 + 165*a^3*b^8*x^8 + 55*a^2*b^9*x^9 + 11*a^10*b*x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**12,x)
```

```
[Out] Timed out
```

# 3.1189 $\int \frac{(c+dx)^7}{(a+bx)^{13}} dx$

**Optimal**. Leaf size=151

$$-\frac{d^4(c+dx)^8}{3960(a+bx)^8(bc-ad)^5}+\frac{d^3(c+dx)^8}{495(a+bx)^9(bc-ad)^4}-\frac{d^2(c+dx)^8}{110(a+bx)^{10}(bc-ad)^3}+\frac{d(c+dx)^8}{33(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^8}{12(a+bx)^{12}(bc-ad)}$$

**Rubi** [A] time = 0.05, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^4(c+dx)^8}{3960(a+bx)^8(bc-ad)^5}+\frac{d^3(c+dx)^8}{495(a+bx)^9(bc-ad)^4}-\frac{d^2(c+dx)^8}{110(a+bx)^{10}(bc-ad)^3}+\frac{d(c+dx)^8}{33(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^8}{12(a+bx)^{12}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^13,x]
```

```
[Out] -(c + d*x)^8/(12*(b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^8)/(33*(b*c - a*d
)^2*(a + b*x)^11) - (d^2*(c + d*x)^8)/(110*(b*c - a*d)^3*(a + b*x)^10) + (d
^3*(c + d*x)^8)/(495*(b*c - a*d)^4*(a + b*x)^9) - (d^4*(c + d*x)^8)/(3960*(
b*c - a*d)^5*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{13}}\,dx = -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} - \frac{d\int \frac{(c+dx)^7}{(a+bx)^{12}}\,dx}{3(bc-ad)}$$

$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} + \frac{d^2\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx}{11(bc-ad)^2}$$

$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} - \frac{d^3\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx}{55(bc-ad)^3}$$

$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} + \frac{d^3(c+dx}{495(bc-ad)^4(a}$$

$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} + \frac{d^3(c+dx}{495(bc-ad)^4(a}$$

**Mathematica [B]** time = 0.13, size = 371, normalized size = 2.46

$$-\frac{a^7d^7+a^6bd^6(5c+12dx)+3a^5b^2d^5\left(5c^2+20cdx+22d^2x^2\right)+5a^4b^3d^4\left(7c^3+36c^2dx+66cd^2x^2+44d^3x^3\right)+5a^3b^4d^3\left(14c^4+84c^3dx+198c^2d^2x^2+220cd^3x^3+99d^4x^4\right)+3a^2b^5d^2\left(42c^5+280c^4dx+770c^3d^2x^2+1100c^2d^3x^3+825cd^4x^4+264d^5x^5\right)+ab^6d\left(210c^6+1512c^5dx+4620c^4d^2x^2+7700c^3d^3x^3+7425c^2d^4x^4+3960cd^5x^5+924d^6x^6\right)+b^7\left(330c^7+2520c^6dx+8316c^5d^2x^2+15400c^4d^3x^3+17325c^3d^4x^4+11880c^2d^5x^5+4620cd^6x^6+792d^7x^7\right)}{3960b^8(a+bx)^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^13,x]
```

```
[Out] -1/3960*(a^7*d^7 + a^6*b*d^6*(5*c + 12*d*x) + 3*a^5*b^2*d^5*(5*c^2 + 20*c*d
*x + 22*d^2*x^2) + 5*a^4*b^3*d^4*(7*c^3 + 36*c^2*d*x + 66*c*d^2*x^2 + 44*d^
3*x^3) + 5*a^3*b^4*d^3*(14*c^4 + 84*c^3*d*x + 198*c^2*d^2*x^2 + 220*c*d^3*x
^3 + 99*d^4*x^4) + 3*a^2*b^5*d^2*(42*c^5 + 280*c^4*d*x + 770*c^3*d^2*x^2 +
1100*c^2*d^3*x^3 + 825*c*d^4*x^4 + 264*d^5*x^5) + a*b^6*d*(210*c^6 + 1512*c
^5*d*x + 4620*c^4*d^2*x^2 + 7700*c^3*d^3*x^3 + 7425*c^2*d^4*x^4 + 3960*c*d^
5*x^5 + 924*d^6*x^6) + b^7*(330*c^7 + 2520*c^6*d*x + 8316*c^5*d^2*x^2 + 154
00*c^4*d^3*x^3 + 17325*c^3*d^4*x^4 + 11880*c^2*d^5*x^5 + 4620*c*d^6*x^6 + 7
92*d^7*x^7))/(b^8*(a + b*x)^12)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^13,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^13, x]
```

**fricas** [B] time = 1.42, size = 581, normalized size = 3.85

$$-\frac{792\,b^{7}d^{7}x^{7} + 330\,b^{7}c^{7} + 210\,ab^{6}c^{6}d + 126\,a^{2}b^{5}c^{5}d^{2} + 70\,a^{3}b^{4}c^{4}d^{3} + 35\,a^{4}b^{3}c^{3}d^{4} + 15\,a^{5}b^{2}c^{2}d^{5} + 5\,a^{6}bcd^{6} + a^{7}d^{7} + 924\,(5\,b^{7}cd^{6} + ab^{6}d^{7})x^{6} + 792\,(15\,b^{7}c^{2}d^{5} + 5\,ab^{6}cd^{6} + a^{2}b^{5}d^{7})x^{5} + 495\,(35\,b^{7}c^{3}d^{4} + 15\,ab^{6}c^{2}d^{5} + 5\,a^{2}b^{5}cd^{6} + a^{3}b^{4}d^{7})x^{4} + 220\,(70\,b^{7}c^{4}d^{3} + 35\,ab^{6}c^{3}d^{4} + 15\,a^{2}b^{5}c^{2}d^{5} + 5\,a^{3}b^{4}cd^{6} + a^{4}b^{3}d^{7})x^{3} + 66\,(126\,b^{7}c^{5}d^{2} + 70\,ab^{6}c^{4}d^{3} + 35\,a^{2}b^{5}c^{3}d^{4} + 15\,a^{3}b^{4}c^{2}d^{5} + 5\,a^{4}b^{3}cd^{6} + a^{5}b^{2}d^{7})x^{2} + 12\,(210\,b^{7}c^{6}d + 126\,ab^{6}c^{5}d^{2} + 70\,a^{2}b^{5}c^{4}d^{3} + 35\,a^{3}b^{4}c^{3}d^{4} + 15\,a^{4}b^{3}c^{2}d^{5} + 5\,a^{5}b^{2}cd^{6} + a^{6}bd^{7})x}{3960\,(b^{20}x^{12} + 12\,ab^{19}x^{11} + 66\,a^{2}b^{18}x^{10} + 220\,a^{3}b^{17}x^{9} + 495\,a^{4}b^{16}x^{8} + 792\,a^{5}b^{15}x^{7} + 924\,a^{6}b^{14}x^{6} + 792\,a^{7}b^{13}x^{5} + 495\,a^{8}b^{12}x^{4} + 220\,a^{9}b^{11}x^{3} + 66\,a^{10}b^{10}x^{2} + 12\,a^{11}b^{9}x + a^{12}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="fricas")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 330*b^7*c^7 + 210*a*b^6*c^6*d + 126*a^2*b^5*c^5*
d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 5*a^6*
b*c*d^6 + a^7*d^7 + 924*(5*b^7*c*d^6 + a*b^6*d^7)*x^6 + 792*(15*b^7*c^2*d^5
 + 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 495*(35*b^7*c^3*d^4 + 15*a*b^6*c^2*d^
5 + 5*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 220*(70*b^7*c^4*d^3 + 35*a*b^6*c^3
*d^4 + 15*a^2*b^5*c^2*d^5 + 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 66*(126*b^
7*c^5*d^2 + 70*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 + 15*a^3*b^4*c^2*d^5 + 5*
a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 12*(210*b^7*c^6*d + 126*a*b^6*c^5*d^2 +
70*a^2*b^5*c^4*d^3 + 35*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 + 5*a^5*b^2*c*
d^6 + a^6*b*d^7)*x)/(b^20*x^12 + 12*a*b^19*x^11 + 66*a^2*b^18*x^10 + 220*a^
3*b^17*x^9 + 495*a^4*b^16*x^8 + 792*a^5*b^15*x^7 + 924*a^6*b^14*x^6 + 792*a
^7*b^13*x^5 + 495*a^8*b^12*x^4 + 220*a^9*b^11*x^3 + 66*a^10*b^10*x^2 + 12*a
^11*b^9*x + a^12*b^8)
```

**giac** [B] time = 1.26, size = 496, normalized size = 3.28

$$-\frac{792\,b^{7}d^{7}x^{7} + 4620\,b^{7}cd^{6}x^{6} + 924\,ab^{6}d^{7}x^{6} + 11880\,b^{7}c^{2}d^{5}x^{5} + 3960\,ab^{6}cd^{6}x^{5} + 792\,a^{2}b^{5}d^{7}x^{5} + 17325\,b^{7}c^{3}d^{4}x^{4} + 7425\,ab^{6}c^{2}d^{5}x^{4} + 2475\,a^{2}b^{5}cd^{6}x^{4} + 495\,a^{3}b^{4}d^{7}x^{4} + 15400\,b^{7}c^{4}d^{3}x^{3} + 7700\,ab^{6}c^{3}d^{4}x^{3} + 3300\,a^{2}b^{5}c^{2}d^{5}x^{3} + 1100\,a^{3}b^{4}cd^{6}x^{3} + 220\,a^{4}b^{3}d^{7}x^{3} + 8316\,b^{7}c^{5}d^{2}x^{2} + 4620\,ab^{6}c^{4}d^{3}x^{2} + 2310\,a^{2}b^{5}c^{3}d^{4}x^{2} + 990\,a^{3}b^{4}c^{2}d^{5}x^{2} + 330\,a^{4}b^{3}cd^{6}x^{2} + 66\,a^{5}b^{2}d^{7}x^{2} + 2520\,b^{7}c^{6}dx + 1512\,ab^{6}c^{5}d^{2}x + 840\,a^{2}b^{5}c^{4}d^{3}x + 420\,a^{3}b^{4}c^{3}d^{4}x + 180\,a^{4}b^{3}c^{2}d^{5}x + 60\,a^{5}b^{2}cd^{6}x + 12\,a^{6}bd^{7}x + 330\,b^{7}c^{7} + 210\,ab^{6}c^{6}d + 126\,a^{2}b^{5}c^{5}d^{2} + 70\,a^{3}b^{4}c^{4}d^{3} + 35\,a^{4}b^{3}c^{3}d^{4} + 15\,a^{5}b^{2}c^{2}d^{5} + 5\,a^{6}bcd^{6} + a^{7}d^{7}}{3960\,(bx + a)^{12}b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="giac")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 4620*b^7*c*d^6*x^6 + 924*a*b^6*d^7*x^6 + 11880*b
^7*c^2*d^5*x^5 + 3960*a*b^6*c*d^6*x^5 + 792*a^2*b^5*d^7*x^5 + 17325*b^7*c^3
*d^4*x^4 + 7425*a*b^6*c^2*d^5*x^4 + 2475*a^2*b^5*c*d^6*x^4 + 495*a^3*b^4*d^
7*x^4 + 15400*b^7*c^4*d^3*x^3 + 7700*a*b^6*c^3*d^4*x^3 + 3300*a^2*b^5*c^2*d
^5*x^3 + 1100*a^3*b^4*c*d^6*x^3 + 220*a^4*b^3*d^7*x^3 + 8316*b^7*c^5*d^2*x^
2 + 4620*a*b^6*c^4*d^3*x^2 + 2310*a^2*b^5*c^3*d^4*x^2 + 990*a^3*b^4*c^2*d^5
*x^2 + 330*a^4*b^3*c*d^6*x^2 + 66*a^5*b^2*d^7*x^2 + 2520*b^7*c^6*d*x + 1512
*a*b^6*c^5*d^2*x + 840*a^2*b^5*c^4*d^3*x + 420*a^3*b^4*c^3*d^4*x + 180*a^4*
b^3*c^2*d^5*x + 60*a^5*b^2*c*d^6*x + 12*a^6*b*d^7*x + 330*b^7*c^7 + 210*a*b
^6*c^6*d + 126*a^2*b^5*c^5*d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 +
15*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^12*b^8)
```

**maple** [B] time = 0.01, size = 464, normalized size = 3.07

$$-\frac{d^{7}}{5\,(bx+a)^{5}b^{8}} + \frac{7\,(ad-bc)\,d^{6}}{6\,(bx+a)^{6}b^{8}} - \frac{3\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)d^{5}}{(bx+a)^{7}b^{8}} + \frac{35\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)d^{4}}{8\,(bx+a)^{8}b^{8}} - \frac{35\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)d^{3}}{9\,(bx+a)^{9}b^{8}} + \frac{21\left(a^{5}d^{5}-5a^{4}bc\,d^{4}+10a^{3}b^{2}c^{2}d^{3}-10a^{2}b^{3}c^{3}d^{2}+5a\,b^{4}c^{4}d-b^{5}c^{5}\right)d^{2}}{10\,(bx+a)^{10}b^{8}} - \frac{7\left(a^{6}d^{6}-6a^{5}bc\,d^{5}+15a^{4}b^{2}c^{2}d^{4}-20a^{3}b^{3}c^{3}d^{3}+15a^{2}b^{4}c^{4}d^{2}-6a\,b^{5}c^{5}d+b^{6}c^{6}\right)d}{11\,(bx+a)^{11}b^{8}} - \frac{-a^{7}d^{7}+7a^{6}bc\,d^{6}-21a^{5}b^{2}c^{2}d^{5}+35a^{4}c^{3}d^{4}b^{3}-35a^{3}b^{4}c^{4}d^{3}+21a^{2}c^{5}d^{2}b^{5}-7a\,b^{6}c^{6}d+b^{7}c^{7}}{12\,(bx+a)^{12}b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^13,x)
```

```
[Out] 35/8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^8-3*d^5*
(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^7-35/9*d^3*(a^4*d^4-4*a^3*b*c*d^3+6
*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^9-1/5*d^7/b^8/(b*x+a)^5
-7/11*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2
*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^11-1/12*(-a^7*d^7+7*a^6*b*c
*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^
5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^12+7/6*d^6*(a*d-b*c)/b^8/(b*x+a)^6
+21/10*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a
*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^10
```

**maxima [B]** time = 1.76, size = 581, normalized size = 3.85

$$-\frac{792 \, b^{7} d^{7} x^{7} + 330 \, b^{7} c^{7} + 210 \, a b^{6} c^{6} d + 126 \, a^{2} b^{5} c^{5} d^{2} + 70 \, a^{3} b^{4} c^{4} d^{3} + 35 \, a^{4} b^{3} c^{3} d^{4} + 15 \, a^{5} b^{2} c^{2} d^{5} + 5 \, a^{6} b c d^{6} + a^{7} d^{7} + 924 \, {\left(5 \, b^{7} c d^{6} + a b^{6} d^{7}\right)} x^{6} + 792 \, {\left(15 \, b^{7} c^{2} d^{5} + 5 \, a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right)} x^{5} + 495 \, {\left(35 \, b^{7} c^{3} d^{4} + 15 \, a b^{6} c^{2} d^{5} + 5 \, a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}\right)} x^{4} + 220 \, {\left(70 \, b^{7} c^{4} d^{3} + 35 \, a b^{6} c^{3} d^{4} + 15 \, a^{2} b^{5} c^{2} d^{5} + 5 \, a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right)} x^{3} + 66 \, {\left(126 \, b^{7} c^{5} d^{2} + 70 \, a b^{6} c^{4} d^{3} + 35 \, a^{2} b^{5} c^{3} d^{4} + 15 \, a^{3} b^{4} c^{2} d^{5} + 5 \, a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}\right)} x^{2} + 12 \, {\left(210 \, b^{7} c^{6} d + 126 \, a b^{6} c^{5} d^{2} + 70 \, a^{2} b^{5} c^{4} d^{3} + 35 \, a^{3} b^{4} c^{3} d^{4} + 15 \, a^{4} b^{3} c^{2} d^{5} + 5 \, a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right)} x}{3960 \, {\left(b^{20} x^{12} + 12 \, a b^{19} x^{11} + 66 \, a^{2} b^{18} x^{10} + 220 \, a^{3} b^{17} x^{9} + 495 \, a^{4} b^{16} x^{8} + 792 \, a^{5} b^{15} x^{7} + 924 \, a^{6} b^{14} x^{6} + 792 \, a^{7} b^{13} x^{5} + 495 \, a^{8} b^{12} x^{4} + 220 \, a^{9} b^{11} x^{3} + 66 \, a^{10} b^{10} x^{2} + 12 \, a^{11} b^{9} x + a^{12} b^{8}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="maxima")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 330*b^7*c^7 + 210*a*b^6*c^6*d + 126*a^2*b^5*c^5*
d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 5*a^6*
b*c*d^6 + a^7*d^7 + 924*(5*b^7*c*d^6 + a*b^6*d^7)*x^6 + 792*(15*b^7*c^2*d^5
 + 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 495*(35*b^7*c^3*d^4 + 15*a*b^6*c^2*d^
5 + 5*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 220*(70*b^7*c^4*d^3 + 35*a*b^6*c^3
*d^4 + 15*a^2*b^5*c^2*d^5 + 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 66*(126*b^
7*c^5*d^2 + 70*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 + 15*a^3*b^4*c^2*d^5 + 5*
a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 12*(210*b^7*c^6*d + 126*a*b^6*c^5*d^2 +
70*a^2*b^5*c^4*d^3 + 35*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 + 5*a^5*b^2*c*
d^6 + a^6*b*d^7)*x)/(b^20*x^12 + 12*a*b^19*x^11 + 66*a^2*b^18*x^10 + 220*a^
3*b^17*x^9 + 495*a^4*b^16*x^8 + 792*a^5*b^15*x^7 + 924*a^6*b^14*x^6 + 792*a
^7*b^13*x^5 + 495*a^8*b^12*x^4 + 220*a^9*b^11*x^3 + 66*a^10*b^10*x^2 + 12*a
^11*b^9*x + a^12*b^8)
```

**mupad [B]** time = 0.23, size = 559, normalized size = 3.70

$$-\frac{\frac{a^7\,d^7+5\,a^6\,b\,c\,d^6+15\,a^5\,b^2\,c^2\,d^5+35\,a^4\,b^3\,c^3\,d^4+70\,a^3\,b^4\,c^4\,d^3+126\,a^2\,b^5\,c^5\,d^2+210\,a\,b^6\,c^6\,d+330\,b^7\,c^7}{3960\,b^8}+\frac{d^7\,x^7}{5\,b}+\frac{d^2\,x^2\,\left(a^5\,d^5+5\,a^4\,b\,c\,d^4+15\,a^3\,b^2\,c^2\,d^3+35\,a^2\,b^3\,c^3\,d^2+70\,a\,b^4\,c^4\,d+126\,b^5\,c^5\right)}{60\,b^6}+\frac{d^4\,x^4\,\left(a^3\,d^3+5\,a^2\,b\,c\,d^2+15\,a\,b^2\,c^2\,d+35\,b^3\,c^3\right)}{8\,b^4}+\frac{7\,d^6\,x^6\,\left(a\,d+5\,b\,c\right)}{30\,b^2}+\frac{d^3\,x^3\,\left(a^4\,d^4+5\,a^3\,b\,c\,d^3+15\,a^2\,b^2\,c^2\,d^2+35\,a\,b^3\,c^3\,d+70\,b^4\,c^4\right)}{18\,b^5}+\frac{d\,x\,\left(a^6\,d^6+5\,a^5\,b\,c\,d^5+15\,a^4\,b^2\,c^2\,d^4+35\,a^3\,b^3\,c^3\,d^3+70\,a^2\,b^4\,c^4\,d^2+126\,a\,b^5\,c^5\,d+210\,b^6\,c^6\right)}{330\,b^7}+\frac{d^5\,x^5\,\left(a^2\,d^2+5\,a\,b\,c\,d+15\,b^2\,c^2\right)}{5\,b^3}}{a^{12}+12\,a^{11}\,b\,x+66\,a^{10}\,b^2\,x^2+220\,a^9\,b^3\,x^3+495\,a^8\,b^4\,x^4+792\,a^7\,b^5\,x^5+924\,a^6\,b^6\,x^6+792\,a^5\,b^7\,x^7+495\,a^4\,b^8\,x^8+220\,a^3\,b^9\,x^9+66\,a^2\,b^{10}\,x^{10}+12\,a\,b^{11}\,x^{11}+b^{12}\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^13,x)
```

```
[Out] -((a^7*d^7 + 330*b^7*c^7 + 126*a^2*b^5*c^5*d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^
4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 210*a*b^6*c^6*d + 5*a^6*b*c*d^6)/(3960
*b^8) + (d^7*x^7)/(5*b) + (d^2*x^2*(a^5*d^5 + 126*b^5*c^5 + 35*a^2*b^3*c^3*
d^2 + 15*a^3*b^2*c^2*d^3 + 70*a*b^4*c^4*d + 5*a^4*b*c*d^4))/(60*b^6) + (d^4
*x^4*(a^3*d^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2))/(8*b^4) + (7*
```

```
d^6*x^6*(a*d + 5*b*c))/(30*b^2) + (d^3*x^3*(a^4*d^4 + 70*b^4*c^4 + 15*a^2*b
^2*c^2*d^2 + 35*a*b^3*c^3*d + 5*a^3*b*c*d^3))/(18*b^5) + (d*x*(a^6*d^6 + 21
0*b^6*c^6 + 70*a^2*b^4*c^4*d^2 + 35*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 +
126*a*b^5*c^5*d + 5*a^5*b*c*d^5))/(330*b^7) + (d^5*x^5*(a^2*d^2 + 15*b^2*c^
2 + 5*a*b*c*d))/(5*b^3))/(a^12 + b^12*x^12 + 12*a*b^11*x^11 + 66*a^10*b^2*x
^2 + 220*a^9*b^3*x^3 + 495*a^8*b^4*x^4 + 792*a^7*b^5*x^5 + 924*a^6*b^6*x^6
+ 792*a^5*b^7*x^7 + 495*a^4*b^8*x^8 + 220*a^3*b^9*x^9 + 66*a^2*b^10*x^10 +
12*a^11*b*x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**13,x)
```

```
[Out] Timed out
```

# 3.1190 $\int \frac{(c+dx)^7}{(a+bx)^{14}}\,dx$

**Optimal**. Leaf size=198

$$-\frac{d^6(bc-ad)}{b^8(a+bx)^7}-\frac{21d^5(bc-ad)^2}{8b^8(a+bx)^8}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-$$

**Rubi** [A] time = 0.15, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{d^6(bc-ad)}{b^8(a+bx)^7}-\frac{21d^5(bc-ad)^2}{8b^8(a+bx)^8}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-\frac{d^7}{6b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^14,x]
```

```
[Out] -(b*c - a*d)^7/(13*b^8*(a + b*x)^13) - (7*d*(b*c - a*d)^6)/(12*b^8*(a + b*x
)^12) - (21*d^2*(b*c - a*d)^5)/(11*b^8*(a + b*x)^11) - (7*d^3*(b*c - a*d)^4
)/(2*b^8*(a + b*x)^10) - (35*d^4*(b*c - a*d)^3)/(9*b^8*(a + b*x)^9) - (21*d
^5*(b*c - a*d)^2)/(8*b^8*(a + b*x)^8) - (d^6*(b*c - a*d))/(b^8*(a + b*x)^7)
 - d^7/(6*b^8*(a + b*x)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{14}}\,dx = \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{14}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{13}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{12}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{11}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{10}}+\frac{21d^5}{b^7(}\right.$$

$$= -\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{21d^5}{8b^8}$$

**Mathematica** [A] time = 0.13, size = 369, normalized size = 1.86

$$-\frac{a^7d^7+a^6bd^6(6c+13dx)+3a^5b^2d^5\left(7c^2+26cdx+26d^2x^2\right)+a^4b^3d^4\left(56c^3+273c^2dx+468cd^2x^2+286d^3x^3\right)+a^3b^4d^3\left(126c^4+728c^3dx+1638c^2d^2x^2+1716cd^3x^3+715d^4x^4\right)+3a^2b^5d^2\left(84c^5+546c^4dx+1456c^3d^2x^2+2002c^2d^3x^3+1430cd^4x^4+429d^5x^5\right)+ab^6d\left(462c^6+3276c^5dx+9828c^4d^2x^2+16016c^3d^3x^3+15015c^2d^4x^4+7722cd^5x^5+1716d^6x^6\right)+b^7\left(792c^7+6006c^6dx+19656c^5d^2x^2+36036c^4d^3x^3+40040c^3d^4x^4+27027c^2d^5x^5+10296cd^6x^6+1716d^7x^7\right)}{10296b^8(a+bx)^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^14,x]
```

```
[Out] -1/10296*(a^7*d^7 + a^6*b*d^6*(6*c + 13*d*x) + 3*a^5*b^2*d^5*(7*c^2 + 26*c*
d*x + 26*d^2*x^2) + a^4*b^3*d^4*(56*c^3 + 273*c^2*d*x + 468*c*d^2*x^2 + 286
*d^3*x^3) + a^3*b^4*d^3*(126*c^4 + 728*c^3*d*x + 1638*c^2*d^2*x^2 + 1716*c*
d^3*x^3 + 715*d^4*x^4) + 3*a^2*b^5*d^2*(84*c^5 + 546*c^4*d*x + 1456*c^3*d^2
*x^2 + 2002*c^2*d^3*x^3 + 1430*c*d^4*x^4 + 429*d^5*x^5) + a*b^6*d*(462*c^6
+ 3276*c^5*d*x + 9828*c^4*d^2*x^2 + 16016*c^3*d^3*x^3 + 15015*c^2*d^4*x^4 +
 7722*c*d^5*x^5 + 1716*d^6*x^6) + b^7*(792*c^7 + 6006*c^6*d*x + 19656*c^5*d
^2*x^2 + 36036*c^4*d^3*x^3 + 40040*c^3*d^4*x^4 + 27027*c^2*d^5*x^5 + 10296*
c*d^6*x^6 + 1716*d^7*x^7))/(b^8*(a + b*x)^13)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^14,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^14, x]
```

**fricas [B]** time = 1.23, size = 592, normalized size = 2.99

$$-\frac{1716\,b^{7}d^{7}x^{7} + 792\,b^{7}c^{7} + 462\,ab^{6}c^{6}d + 252\,a^{2}b^{5}c^{5}d^{2} + 126\,a^{3}b^{4}c^{4}d^{3} + 56\,a^{4}b^{3}c^{3}d^{4} + 21\,a^{5}b^{2}c^{2}d^{5} + 6\,a^{6}bcd^{6} + a^{7}d^{7} + 1716\left(6\,b^{7}cd^{6} + ab^{6}d^{7}\right)x^{6} + 1287\left(21\,b^{7}c^{2}d^{5} + 6\,ab^{6}cd^{6} + a^{2}b^{5}d^{7}\right)x^{5} + 715\left(56\,b^{7}c^{3}d^{4} + 21\,ab^{6}c^{2}d^{5} + 6\,a^{2}b^{5}cd^{6} + a^{3}b^{4}d^{7}\right)x^{4} + 286\left(126\,b^{7}c^{4}d^{3} + 56\,ab^{6}c^{3}d^{4} + 21\,a^{2}b^{5}c^{2}d^{5} + 6\,a^{3}b^{4}cd^{6} + a^{4}b^{3}d^{7}\right)x^{3} + 78\left(252\,b^{7}c^{5}d^{2} + 126\,ab^{6}c^{4}d^{3} + 56\,a^{2}b^{5}c^{3}d^{4} + 21\,a^{3}b^{4}c^{2}d^{5} + 6\,a^{4}b^{3}cd^{6} + a^{5}b^{2}d^{7}\right)x^{2} + 13\left(462\,b^{7}c^{6}d + 252\,ab^{6}c^{5}d^{2} + 126\,a^{2}b^{5}c^{4}d^{3} + 56\,a^{3}b^{4}c^{3}d^{4} + 21\,a^{4}b^{3}c^{2}d^{5} + 6\,a^{5}b^{2}cd^{6} + a^{6}bd^{7}\right)x}{10296\left(b^{21}x^{13} + 13\,ab^{20}x^{12} + 78\,a^{2}b^{19}x^{11} + 286\,a^{3}b^{18}x^{10} + 715\,a^{4}b^{17}x^{9} + 1287\,a^{5}b^{16}x^{8} + 1716\,a^{6}b^{15}x^{7} + 1716\,a^{7}b^{14}x^{6} + 1287\,a^{8}b^{13}x^{5} + 715\,a^{9}b^{12}x^{4} + 286\,a^{10}b^{11}x^{3} + 78\,a^{11}b^{10}x^{2} + 13\,a^{12}b^{9}x + a^{13}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 792*b^7*c^7 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^
5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a
^6*b*c*d^6 + a^7*d^7 + 1716*(6*b^7*c*d^6 + a*b^6*d^7)*x^6 + 1287*(21*b^7*c^
2*d^5 + 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 715*(56*b^7*c^3*d^4 + 21*a*b^6*c
^2*d^5 + 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 286*(126*b^7*c^4*d^3 + 56*a*b
^6*c^3*d^4 + 21*a^2*b^5*c^2*d^5 + 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 78*(
252*b^7*c^5*d^2 + 126*a*b^6*c^4*d^3 + 56*a^2*b^5*c^3*d^4 + 21*a^3*b^4*c^2*d
^5 + 6*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 13*(462*b^7*c^6*d + 252*a*b^6*c^5
*d^2 + 126*a^2*b^5*c^4*d^3 + 56*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 + 6*a^
5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^21*x^13 + 13*a*b^20*x^12 + 78*a^2*b^19*x^11
+ 286*a^3*b^18*x^10 + 715*a^4*b^17*x^9 + 1287*a^5*b^16*x^8 + 1716*a^6*b^15*
x^7 + 1716*a^7*b^14*x^6 + 1287*a^8*b^13*x^5 + 715*a^9*b^12*x^4 + 286*a^10*b
^11*x^3 + 78*a^11*b^10*x^2 + 13*a^12*b^9*x + a^13*b^8)
```

**giac [B]** time = 1.24, size = 496, normalized size = 2.51

$$-\frac{1716\,b^{7} d^{7} x^{7} + 10296\,b^{7} c d^{6} x^{6} + 1716\,a b^{6} d^{7} x^{6} + 27027\,b^{7} c^{2} d^{5} x^{5} + 7722\,a b^{6} c d^{6} x^{5} + 1287\,a^{2} b^{5} d^{7} x^{5} + 40040\,b^{7} c^{3} d^{4} x^{4} + 15015\,a b^{6} c^{2} d^{5} x^{4} + 4290\,a^{2} b^{5} c d^{6} x^{4} + 715\,a^{3} b^{4} d^{7} x^{4} + 36036\,b^{7} c^{4} d^{3} x^{3} + 16016\,a b^{6} c^{3} d^{4} x^{3} + 6006\,a^{2} b^{5} c^{2} d^{5} x^{3} + 1716\,a^{3} b^{4} c d^{6} x^{3} + 286\,a^{4} b^{3} d^{7} x^{3} + 19656\,b^{7} c^{5} d^{2} x^{2} + 9828\,a b^{6} c^{4} d^{3} x^{2} + 4368\,a^{2} b^{5} c^{3} d^{4} x^{2} + 1638\,a^{3} b^{4} c^{2} d^{5} x^{2} + 468\,a^{4} b^{3} c d^{6} x^{2} + 78\,a^{5} b^{2} d^{7} x^{2} + 6006\,b^{7} c^{6} d x + 3276\,a b^{6} c^{5} d^{2} x + 1638\,a^{2} b^{5} c^{4} d^{3} x + 728\,a^{3} b^{4} c^{3} d^{4} x + 273\,a^{4} b^{3} c^{2} d^{5} x + 78\,a^{5} b^{2} c d^{6} x + 13\,a^{6} b d^{7} x + 792\,b^{7} c^{7} + 462\,a b^{6} c^{6} d + 252\,a^{2} b^{5} c^{5} d^{2} + 126\,a^{3} b^{4} c^{4} d^{3} + 56\,a^{4} b^{3} c^{3} d^{4} + 21\,a^{5} b^{2} c^{2} d^{5} + 6\,a^{6} b c d^{6} + a^{7} d^{7}}{10296\,{\left(b x + a\right)}^{13} b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 10296*b^7*c*d^6*x^6 + 1716*a*b^6*d^7*x^6 + 270
27*b^7*c^2*d^5*x^5 + 7722*a*b^6*c*d^6*x^5 + 1287*a^2*b^5*d^7*x^5 + 40040*b^
7*c^3*d^4*x^4 + 15015*a*b^6*c^2*d^5*x^4 + 4290*a^2*b^5*c*d^6*x^4 + 715*a^3*
b^4*d^7*x^4 + 36036*b^7*c^4*d^3*x^3 + 16016*a*b^6*c^3*d^4*x^3 + 6006*a^2*b^
5*c^2*d^5*x^3 + 1716*a^3*b^4*c*d^6*x^3 + 286*a^4*b^3*d^7*x^3 + 19656*b^7*c^
5*d^2*x^2 + 9828*a*b^6*c^4*d^3*x^2 + 4368*a^2*b^5*c^3*d^4*x^2 + 1638*a^3*b^
4*c^2*d^5*x^2 + 468*a^4*b^3*c*d^6*x^2 + 78*a^5*b^2*d^7*x^2 + 6006*b^7*c^6*d
*x + 3276*a*b^6*c^5*d^2*x + 1638*a^2*b^5*c^4*d^3*x + 728*a^3*b^4*c^3*d^4*x
+ 273*a^4*b^3*c^2*d^5*x + 78*a^5*b^2*c*d^6*x + 13*a^6*b*d^7*x + 792*b^7*c^7
 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3
*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^13*b^8)
```

**maple [B]** time = 0.01, size = 463, normalized size = 2.34

$$-\frac{d^{7}}{6 \left(b x +a \right)^{6} b^{8}}+\frac{\left(a d -b c \right) d^{6}}{\left(b x +a \right)^{7} b^{8}}-\frac{21 \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) d^{5}}{8 \left(b x +a \right)^{8} b^{8}}+\frac{35 \left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right) d^{4}}{9 \left(b x +a \right)^{9} b^{8}}-\frac{7 \left(a^{4} d^{4}-4 a^{3} b c \,d^{3}+6 a^{2} b^{2} c^{2} d^{2}-4 a \,b^{3} c^{3} d +b^{4} c^{4}\right) d^{3}}{2 \left(b x +a \right)^{10} b^{8}}+\frac{21 \left(a^{5} d^{5}-5 a^{4} b c \,d^{4}+10 a^{3} b^{2} c^{2} d^{3}-10 a^{2} b^{3} c^{3} d^{2}+5 a \,b^{4} c^{4} d -b^{5} c^{5}\right) d^{2}}{11 \left(b x +a \right)^{11} b^{8}}-\frac{7 \left(a^{6} d^{6}-6 a^{5} b c \,d^{5}+15 a^{4} b^{2} c^{2} d^{4}-20 a^{3} b^{3} c^{3} d^{3}+15 a^{2} b^{4} c^{4} d^{2}-6 a \,b^{5} c^{5} d +b^{6} c^{6}\right) d}{12 \left(b x +a \right)^{12} b^{8}}-\frac{-a^{7} d^{7}+7 a^{6} b c \,d^{6}-21 a^{5} b^{2} c^{2} d^{5}+35 a^{4} b^{3} c^{3} d^{4}-35 a^{3} b^{4} c^{4} d^{3}+21 a^{2} b^{5} c^{5} d^{2}-7 a \,b^{6} c^{6} d +b^{7} c^{7}}{13 \left(b x +a \right)^{13} b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^14,x)
```

```
[Out] -21/8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^8-1/13*(-a^7*d^7+7*a^6*b*
c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c
^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^13+d^6*(a*d-b*c)/b^8/(b*x+a)^7+35
/9*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^9+21/11*d^
2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*
d-b^5*c^5)/b^8/(b*x+a)^11-7/12*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-
20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^12
-1/6*d^7/b^8/(b*x+a)^6-7/2*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^10
```

**maxima [B]** time = 1.74, size = 592, normalized size = 2.99

$$-\frac{1716\,b^{7} d^{7} x^{7} + 792\,b^{7} c^{7} + 462\,a b^{6} c^{6} d + 252\,a^{2} b^{5} c^{5} d^{2} + 126\,a^{3} b^{4} c^{4} d^{3} + 56\,a^{4} b^{3} c^{3} d^{4} + 21\,a^{5} b^{2} c^{2} d^{5} + 6\,a^{6} b c d^{6} + a^{7} d^{7} + 1716\,{\left(6\,b^{7} c d^{6} + a b^{6} d^{7}\right)} x^{6} + 1287\,{\left(21\,b^{7} c^{2} d^{5} + 6\,a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right)} x^{5} + 715\,{\left(56\,b^{7} c^{3} d^{4} + 21\,a b^{6} c^{2} d^{5} + 6\,a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}\right)} x^{4} + 286\,{\left(126\,b^{7} c^{4} d^{3} + 56\,a b^{6} c^{3} d^{4} + 21\,a^{2} b^{5} c^{2} d^{5} + 6\,a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right)} x^{3} + 78\,{\left(252\,b^{7} c^{5} d^{2} + 126\,a b^{6} c^{4} d^{3} + 56\,a^{2} b^{5} c^{3} d^{4} + 21\,a^{3} b^{4} c^{2} d^{5} + 6\,a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}\right)} x^{2} + 13\,{\left(462\,b^{7} c^{6} d + 252\,a b^{6} c^{5} d^{2} + 126\,a^{2} b^{5} c^{4} d^{3} + 56\,a^{3} b^{4} c^{3} d^{4} + 21\,a^{4} b^{3} c^{2} d^{5} + 6\,a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right)} x}{10296\,{\left(b^{21} x^{13} + 13\,a b^{20} x^{12} + 78\,a^{2} b^{19} x^{11} + 286\,a^{3} b^{18} x^{10} + 715\,a^{4} b^{17} x^{9} + 1287\,a^{5} b^{16} x^{8} + 1716\,a^{6} b^{15} x^{7} + 1716\,a^{7} b^{14} x^{6} + 1287\,a^{8} b^{13} x^{5} + 715\,a^{9} b^{12} x^{4} + 286\,a^{10} b^{11} x^{3} + 78\,a^{11} b^{10} x^{2} + 13\,a^{12} b^{9} x + a^{13} b^{8}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 792*b^7*c^7 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^
5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a
^6*b*c*d^6 + a^7*d^7 + 1716*(6*b^7*c*d^6 + a*b^6*d^7)*x^6 + 1287*(21*b^7*c^
2*d^5 + 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 715*(56*b^7*c^3*d^4 + 21*a*b^6*c
^2*d^5 + 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 286*(126*b^7*c^4*d^3 + 56*a*b
^6*c^3*d^4 + 21*a^2*b^5*c^2*d^5 + 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 78*(
252*b^7*c^5*d^2 + 126*a*b^6*c^4*d^3 + 56*a^2*b^5*c^3*d^4 + 21*a^3*b^4*c^2*d
^5 + 6*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 13*(462*b^7*c^6*d + 252*a*b^6*c^5
*d^2 + 126*a^2*b^5*c^4*d^3 + 56*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 + 6*a^
5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^21*x^13 + 13*a*b^20*x^12 + 78*a^2*b^19*x^11
+ 286*a^3*b^18*x^10 + 715*a^4*b^17*x^9 + 1287*a^5*b^16*x^8 + 1716*a^6*b^15*
x^7 + 1716*a^7*b^14*x^6 + 1287*a^8*b^13*x^5 + 715*a^9*b^12*x^4 + 286*a^10*b
^11*x^3 + 78*a^11*b^10*x^2 + 13*a^12*b^9*x + a^13*b^8)
```

**mupad [B]** time = 0.40, size = 570, normalized size = 2.88

$$-\frac{\frac{a^7 d^7+6 a^6 b c d^6+21 a^5 b^2 c^2 d^5+56 a^4 b^3 c^3 d^4+126 a^3 b^4 c^4 d^3+252 a^2 b^5 c^5 d^2+462 a b^6 c^6 d+792 b^7 c^7}{10296 b^8}+\frac{d^7 x^7}{6 b}+\frac{d^2 x^2 \left(a^5 d^5+6 a^4 b c d^4+21 a^3 b^2 c^2 d^3+56 a^2 b^3 c^3 d^2+126 a b^4 c^4 d+252 b^5 c^5\right)}{132 b^6}+\frac{5 d^4 x^4 \left(a^3 d^3+6 a^2 b c d^2+21 a b^2 c^2 d+56 b^3 c^3\right)}{72 b^4}+\frac{d^6 x^6 (a d+6 b c)}{6 b^2}+\frac{d^3 x^3 \left(a^4 d^4+6 a^3 b c d^3+21 a^2 b^2 c^2 d^2+56 a b^3 c^3 d+126 b^4 c^4\right)}{36 b^5}+\frac{d x \left(a^6 d^6+6 a^5 b c d^5+21 a^4 b^2 c^2 d^4+56 a^3 b^3 c^3 d^3+126 a^2 b^4 c^4 d^2+252 a b^5 c^5 d+462 b^6 c^6\right)}{792 b^7}+\frac{d^5 x^5 \left(a^2 d^2+6 a b c d+21 b^2 c^2\right)}{8 b^3}}{a^{13}+13 a^{12} b x+78 a^{11} b^2 x^2+286 a^{10} b^3 x^3+715 a^9 b^4 x^4+1287 a^8 b^5 x^5+1716 a^7 b^6 x^6+1716 a^6 b^7 x^7+1287 a^5 b^8 x^8+715 a^4 b^9 x^9+286 a^3 b^{10} x^{10}+78 a^2 b^{11} x^{11}+13 a b^{12} x^{12}+b^{13} x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^14,x)
```

```
[Out] -((a^7*d^7 + 792*b^7*c^7 + 252*a^2*b^5*c^5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a
^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 462*a*b^6*c^6*d + 6*a^6*b*c*d^6)/(102
96*b^8) + (d^7*x^7)/(6*b) + (d^2*x^2*(a^5*d^5 + 252*b^5*c^5 + 56*a^2*b^3*c^
3*d^2 + 21*a^3*b^2*c^2*d^3 + 126*a*b^4*c^4*d + 6*a^4*b*c*d^4))/(132*b^6) +
(5*d^4*x^4*(a^3*d^3 + 56*b^3*c^3 + 21*a*b^2*c^2*d + 6*a^2*b*c*d^2))/(72*b^4
) + (d^6*x^6*(a*d + 6*b*c))/(6*b^2) + (d^3*x^3*(a^4*d^4 + 126*b^4*c^4 + 21*
a^2*b^2*c^2*d^2 + 56*a*b^3*c^3*d + 6*a^3*b*c*d^3))/(36*b^5) + (d*x*(a^6*d^6
 + 462*b^6*c^6 + 126*a^2*b^4*c^4*d^2 + 56*a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*
d^4 + 252*a*b^5*c^5*d + 6*a^5*b*c*d^5))/(792*b^7) + (d^5*x^5*(a^2*d^2 + 21*
b^2*c^2 + 6*a*b*c*d))/(8*b^3))/(a^13 + b^13*x^13 + 13*a*b^12*x^12 + 78*a^11
*b^2*x^2 + 286*a^10*b^3*x^3 + 715*a^9*b^4*x^4 + 1287*a^8*b^5*x^5 + 1716*a^7
*b^6*x^6 + 1716*a^6*b^7*x^7 + 1287*a^5*b^8*x^8 + 715*a^4*b^9*x^9 + 286*a^3*
b^10*x^10 + 78*a^2*b^11*x^11 + 13*a^12*b*x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**14,x)
```

```
[Out] Timed out
```

## 3.1191 $\int \frac{(c+dx)^7}{(a+bx)^{15}} \, dx$

**Optimal**. Leaf size=200

$$-\frac{7d^6(bc-ad)}{8b^8(a+bx)^8}-\frac{7d^5(bc-ad)^2}{3b^8(a+bx)^9}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{7}{}$$

**Rubi** [A] time = 0.14, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{7d^6(bc-ad)}{8b^8(a+bx)^8}-\frac{7d^5(bc-ad)^2}{3b^8(a+bx)^9}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{d^7}{7b^8(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^15,x]
```

```
[Out] -(b*c - a*d)^7/(14*b^8*(a + b*x)^14) - (7*d*(b*c - a*d)^6)/(13*b^8*(a + b*x
)^13) - (7*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^12) - (35*d^3*(b*c - a*d)^4)
/(11*b^8*(a + b*x)^11) - (7*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^10) - (7*d^
5*(b*c - a*d)^2)/(3*b^8*(a + b*x)^9) - (7*d^6*(b*c - a*d))/(8*b^8*(a + b*x)
^8) - d^7/(7*b^8*(a + b*x)^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^{15}}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{15}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{14}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{13}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{12}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{11}}+\frac{21d^5}{b^7(}\right.\\ &= -\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{7d^5(b}{3b^8(a}\end{aligned}$$

**Mathematica** [A] time = 0.13, size = 371, normalized size = 1.86

$$-\frac{a^7d^7+7a^6bd^6(c+2dx)+7a^5b^2d^5\left(4c^2+14cdx+13d^2x^2\right)+7a^4b^3d^4\left(12c^3+56c^2dx+91cd^2x^2+52d^3x^3\right)+7a^3b^4d^3\left(30c^4+168c^3dx+364c^2d^2x^2+364cd^3x^3+143d^4x^4\right)+7a^2b^5d^2\left(66c^5+420c^4dx+1092c^3d^2x^2+1456c^2d^3x^3+1001cd^4x^4+286d^5x^5\right)+7ab^6d\left(132c^6+924c^5dx+2730c^4d^2x^2+4368c^3d^3x^3+4004c^2d^4x^4+2002cd^5x^5+429d^6x^6\right)+b^7\left(1716c^7+12936c^6dx+42042c^5d^2x^2+76440c^4d^3x^3+84084c^3d^4x^4+56056c^2d^5x^5+21021cd^6x^6+3432d^7x^7\right)}{24024b^8(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^15,x]
```

```
[Out] -1/24024*(a^7*d^7 + 7*a^6*b*d^6*(c + 2*d*x) + 7*a^5*b^2*d^5*(4*c^2 + 14*c*d
*x + 13*d^2*x^2) + 7*a^4*b^3*d^4*(12*c^3 + 56*c^2*d*x + 91*c*d^2*x^2 + 52*d
^3*x^3) + 7*a^3*b^4*d^3*(30*c^4 + 168*c^3*d*x + 364*c^2*d^2*x^2 + 364*c*d^3
*x^3 + 143*d^4*x^4) + 7*a^2*b^5*d^2*(66*c^5 + 420*c^4*d*x + 1092*c^3*d^2*x^
2 + 1456*c^2*d^3*x^3 + 1001*c*d^4*x^4 + 286*d^5*x^5) + 7*a*b^6*d*(132*c^6 +
 924*c^5*d*x + 2730*c^4*d^2*x^2 + 4368*c^3*d^3*x^3 + 4004*c^2*d^4*x^4 + 200
2*c*d^5*x^5 + 429*d^6*x^6) + b^7*(1716*c^7 + 12936*c^6*d*x + 42042*c^5*d^2*
x^2 + 76440*c^4*d^3*x^3 + 84084*c^3*d^4*x^4 + 56056*c^2*d^5*x^5 + 21021*c*d
^6*x^6 + 3432*d^7*x^7))/(b^8*(a + b*x)^14)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^15,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^15, x]
```

**fricas [B]** time = 1.35, size = 603, normalized size = 3.02

$$-\frac{3432\,b^{7}d^{7}x^{7} + 1716\,b^{7}c^{7} + 924\,ab^{6}c^{6}d + 462\,a^{2}b^{5}c^{5}d^{2} + 210\,a^{3}b^{4}c^{4}d^{3} + 84\,a^{4}b^{3}c^{3}d^{4} + 28\,a^{5}b^{2}c^{2}d^{5} + 7\,a^{6}bcd^{6} + a^{7}d^{7} + 3003\,\left(7\,b^{7}cd^{6} + ab^{6}d^{7}\right)x^{6} + 2002\,\left(28\,b^{7}c^{2}d^{5} + 7\,ab^{6}cd^{6} + a^{2}b^{5}d^{7}\right)x^{5} + 1001\,\left(84\,b^{7}c^{3}d^{4} + 28\,ab^{6}c^{2}d^{5} + 7\,a^{2}b^{5}cd^{6} + a^{3}b^{4}d^{7}\right)x^{4} + 364\,\left(210\,b^{7}c^{4}d^{3} + 84\,ab^{6}c^{3}d^{4} + 28\,a^{2}b^{5}c^{2}d^{5} + 7\,a^{3}b^{4}cd^{6} + a^{4}b^{3}d^{7}\right)x^{3} + 91\,\left(462\,b^{7}c^{5}d^{2} + 210\,ab^{6}c^{4}d^{3} + 84\,a^{2}b^{5}c^{3}d^{4} + 28\,a^{3}b^{4}c^{2}d^{5} + 7\,a^{4}b^{3}cd^{6} + a^{5}b^{2}d^{7}\right)x^{2} + 14\,\left(924\,b^{7}c^{6}d + 462\,ab^{6}c^{5}d^{2} + 210\,a^{2}b^{5}c^{4}d^{3} + 84\,a^{3}b^{4}c^{3}d^{4} + 28\,a^{4}b^{3}c^{2}d^{5} + 7\,a^{5}b^{2}cd^{6} + a^{6}bd^{7}\right)x}{24024\,\left(b^{22}x^{14} + 14\,ab^{21}x^{13} + 91\,a^{2}b^{20}x^{12} + 364\,a^{3}b^{19}x^{11} + 1001\,a^{4}b^{18}x^{10} + 2002\,a^{5}b^{17}x^{9} + 3003\,a^{6}b^{16}x^{8} + 3432\,a^{7}b^{15}x^{7} + 3003\,a^{8}b^{14}x^{6} + 2002\,a^{9}b^{13}x^{5} + 1001\,a^{10}b^{12}x^{4} + 364\,a^{11}b^{11}x^{3} + 91\,a^{12}b^{10}x^{2} + 14\,a^{13}b^{9}x + a^{14}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="fricas")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 1716*b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c
^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*
a^6*b*c*d^6 + a^7*d^7 + 3003*(7*b^7*c*d^6 + a*b^6*d^7)*x^6 + 2002*(28*b^7*c
^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1001*(84*b^7*c^3*d^4 + 28*a*b^6
*c^2*d^5 + 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 364*(210*b^7*c^4*d^3 + 84*a
*b^6*c^3*d^4 + 28*a^2*b^5*c^2*d^5 + 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 91
*(462*b^7*c^5*d^2 + 210*a*b^6*c^4*d^3 + 84*a^2*b^5*c^3*d^4 + 28*a^3*b^4*c^2
*d^5 + 7*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 14*(924*b^7*c^6*d + 462*a*b^6*c
^5*d^2 + 210*a^2*b^5*c^4*d^3 + 84*a^3*b^4*c^3*d^4 + 28*a^4*b^3*c^2*d^5 + 7*
a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^22*x^14 + 14*a*b^21*x^13 + 91*a^2*b^20*x^1
2 + 364*a^3*b^19*x^11 + 1001*a^4*b^18*x^10 + 2002*a^5*b^17*x^9 + 3003*a^6*b
^16*x^8 + 3432*a^7*b^15*x^7 + 3003*a^8*b^14*x^6 + 2002*a^9*b^13*x^5 + 1001*
a^10*b^12*x^4 + 364*a^11*b^11*x^3 + 91*a^12*b^10*x^2 + 14*a^13*b^9*x + a^14
*b^8)
```

**giac [B]** time = 1.28, size = 496, normalized size = 2.48

$$-\frac{3432 \, b^{7} d^{7} x^{7} + 21021 \, b^{7} c d^{6} x^{6} + 3003 \, a b^{6} d^{7} x^{6} + 56056 \, b^{7} c^{2} d^{5} x^{5} + 14014 \, a b^{6} c d^{6} x^{5} + 2002 \, a^{2} b^{5} d^{7} x^{5} + 84084 \, b^{7} c^{3} d^{4} x^{4} + 28028 \, a b^{6} c^{2} d^{5} x^{4} + 7007 \, a^{2} b^{5} c d^{6} x^{4} + 1001 \, a^{3} b^{4} d^{7} x^{4} + 76440 \, b^{7} c^{4} d^{3} x^{3} + 30576 \, a b^{6} c^{3} d^{4} x^{3} + 10192 \, a^{2} b^{5} c^{2} d^{5} x^{3} + 2548 \, a^{3} b^{4} c d^{6} x^{3} + 364 \, a^{4} b^{3} d^{7} x^{3} + 42042 \, b^{7} c^{5} d^{2} x^{2} + 19110 \, a b^{6} c^{4} d^{3} x^{2} + 7644 \, a^{2} b^{5} c^{3} d^{4} x^{2} + 2548 \, a^{3} b^{4} c^{2} d^{5} x^{2} + 637 \, a^{4} b^{3} c d^{6} x^{2} + 91 \, a^{5} b^{2} d^{7} x^{2} + 12936 \, b^{7} c^{6} d x + 6468 \, a b^{6} c^{5} d^{2} x + 2940 \, a^{2} b^{5} c^{4} d^{3} x + 1176 \, a^{3} b^{4} c^{3} d^{4} x + 392 \, a^{4} b^{3} c^{2} d^{5} x + 98 \, a^{5} b^{2} c d^{6} x + 14 \, a^{6} b d^{7} x + 1716 \, b^{7} c^{7} + 924 \, a b^{6} c^{6} d + 462 \, a^{2} b^{5} c^{5} d^{2} + 210 \, a^{3} b^{4} c^{4} d^{3} + 84 \, a^{4} b^{3} c^{3} d^{4} + 28 \, a^{5} b^{2} c^{2} d^{5} + 7 \, a^{6} b c d^{6} + a^{7} d^{7}}{24024 \, {\left(b x + a\right)}^{14} b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="giac")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 21021*b^7*c*d^6*x^6 + 3003*a*b^6*d^7*x^6 + 560
56*b^7*c^2*d^5*x^5 + 14014*a*b^6*c*d^6*x^5 + 2002*a^2*b^5*d^7*x^5 + 84084*b
^7*c^3*d^4*x^4 + 28028*a*b^6*c^2*d^5*x^4 + 7007*a^2*b^5*c*d^6*x^4 + 1001*a^
3*b^4*d^7*x^4 + 76440*b^7*c^4*d^3*x^3 + 30576*a*b^6*c^3*d^4*x^3 + 10192*a^2
*b^5*c^2*d^5*x^3 + 2548*a^3*b^4*c*d^6*x^3 + 364*a^4*b^3*d^7*x^3 + 42042*b^7
*c^5*d^2*x^2 + 19110*a*b^6*c^4*d^3*x^2 + 7644*a^2*b^5*c^3*d^4*x^2 + 2548*a^
3*b^4*c^2*d^5*x^2 + 637*a^4*b^3*c*d^6*x^2 + 91*a^5*b^2*d^7*x^2 + 12936*b^7*
c^6*d*x + 6468*a*b^6*c^5*d^2*x + 2940*a^2*b^5*c^4*d^3*x + 1176*a^3*b^4*c^3*
d^4*x + 392*a^4*b^3*c^2*d^5*x + 98*a^5*b^2*c*d^6*x + 14*a^6*b*d^7*x + 1716*
b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*
a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^
14*b^8)
```

**maple [B]** time = 0.01, size = 464, normalized size = 2.32

$$-\frac{d^{7}}{7 \left(b x +a \right)^{7} b^{8}}+\frac{7 \left(a d -b c \right) d^{6}}{8 \left(b x +a \right)^{8} b^{8}}-\frac{7 \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) d^{5}}{3 \left(b x +a \right)^{9} b^{8}}+\frac{7 \left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right) d^{4}}{2 \left(b x +a \right)^{10} b^{8}}-\frac{35 \left(a^{4} d^{4}-4 a^{3} b c \,d^{3}+6 a^{2} b^{2} c^{2} d^{2}-4 a \,b^{3} c^{3} d +b^{4} c^{4}\right) d^{3}}{11 \left(b x +a \right)^{11} b^{8}}+\frac{7 \left(a^{5} d^{5}-5 a^{4} b c \,d^{4}+10 a^{3} b^{2} c^{2} d^{3}-10 a^{2} b^{3} c^{3} d^{2}+5 a \,b^{4} c^{4} d -b^{5} c^{5}\right) d^{2}}{4 \left(b x +a \right)^{12} b^{8}}-\frac{7 \left(a^{6} d^{6}-6 a^{5} b c \,d^{5}+15 a^{4} b^{2} c^{2} d^{4}-20 a^{3} b^{3} c^{3} d^{3}+15 a^{2} b^{4} c^{4} d^{2}-6 a \,b^{5} c^{5} d +b^{6} c^{6}\right) d}{13 \left(b x +a \right)^{13} b^{8}}-\frac{-a^{7} d^{7}+7 a^{6} b c \,d^{6}-21 a^{5} b^{2} c^{2} d^{5}+35 a^{4} c^{3} d^{4} b^{3}-35 a^{3} b^{4} c^{4} d^{3}+21 a^{2} c^{5} d^{2} b^{5}-7 a \,b^{6} c^{6} d +b^{7} c^{7}}{14 \left(b x +a \right)^{14} b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^15,x)
```

```
[Out] 7/8*d^6*(a*d-b*c)/b^8/(b*x+a)^8-7/13*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^
2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x
+a)^13-1/7*d^7/b^8/(b*x+a)^7-7/3*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a
)^9-35/11*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^
4)/b^8/(b*x+a)^11+7/4*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*
b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^12+7/2*d^4*(a^3*d^3-3*a^2*b*
c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^10-1/14*(-a^7*d^7+7*a^6*b*c*d^6-21
*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7
*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^14
```

**maxima [B]** time = 1.83, size = 603, normalized size = 3.02

$$-\frac{3432 \, b^{7} d^{7} x^{7} + 1716 \, b^{7} c^{7} + 924 \, a b^{6} c^{6} d + 462 \, a^{2} b^{5} c^{5} d^{2} + 210 \, a^{3} b^{4} c^{4} d^{3} + 84 \, a^{4} b^{3} c^{3} d^{4} + 28 \, a^{5} b^{2} c^{2} d^{5} + 7 \, a^{6} b c d^{6} + a^{7} d^{7} + 3003 \, {\left(7 \, b^{7} c d^{6} + a b^{6} d^{7}\right)} x^{6} + 2002 \, {\left(28 \, b^{7} c^{2} d^{5} + 7 \, a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right)} x^{5} + 1001 \, {\left(84 \, b^{7} c^{3} d^{4} + 28 \, a b^{6} c^{2} d^{5} + 7 \, a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}\right)} x^{4} + 364 \, {\left(210 \, b^{7} c^{4} d^{3} + 84 \, a b^{6} c^{3} d^{4} + 28 \, a^{2} b^{5} c^{2} d^{5} + 7 \, a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right)} x^{3} + 91 \, {\left(462 \, b^{7} c^{5} d^{2} + 210 \, a b^{6} c^{4} d^{3} + 84 \, a^{2} b^{5} c^{3} d^{4} + 28 \, a^{3} b^{4} c^{2} d^{5} + 7 \, a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}\right)} x^{2} + 14 \, {\left(924 \, b^{7} c^{6} d + 462 \, a b^{6} c^{5} d^{2} + 210 \, a^{2} b^{5} c^{4} d^{3} + 84 \, a^{3} b^{4} c^{3} d^{4} + 28 \, a^{4} b^{3} c^{2} d^{5} + 7 \, a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right)} x}{24024 \, {\left(b^{22} x^{14} + 14 \, a b^{21} x^{13} + 91 \, a^{2} b^{20} x^{12} + 364 \, a^{3} b^{19} x^{11} + 1001 \, a^{4} b^{18} x^{10} + 2002 \, a^{5} b^{17} x^{9} + 3003 \, a^{6} b^{16} x^{8} + 3432 \, a^{7} b^{15} x^{7} + 3003 \, a^{8} b^{14} x^{6} + 2002 \, a^{9} b^{13} x^{5} + 1001 \, a^{10} b^{12} x^{4} + 364 \, a^{11} b^{11} x^{3} + 91 \, a^{12} b^{10} x^{2} + 14 \, a^{13} b^{9} x + a^{14} b^{8}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="maxima")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 1716*b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c
^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*
a^6*b*c*d^6 + a^7*d^7 + 3003*(7*b^7*c*d^6 + a*b^6*d^7)*x^6 + 2002*(28*b^7*c
^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1001*(84*b^7*c^3*d^4 + 28*a*b^6
*c^2*d^5 + 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 364*(210*b^7*c^4*d^3 + 84*a
*b^6*c^3*d^4 + 28*a^2*b^5*c^2*d^5 + 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 91
*(462*b^7*c^5*d^2 + 210*a*b^6*c^4*d^3 + 84*a^2*b^5*c^3*d^4 + 28*a^3*b^4*c^2
*d^5 + 7*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 14*(924*b^7*c^6*d + 462*a*b^6*c
^5*d^2 + 210*a^2*b^5*c^4*d^3 + 84*a^3*b^4*c^3*d^4 + 28*a^4*b^3*c^2*d^5 + 7*
a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^22*x^14 + 14*a*b^21*x^13 + 91*a^2*b^20*x^1
2 + 364*a^3*b^19*x^11 + 1001*a^4*b^18*x^10 + 2002*a^5*b^17*x^9 + 3003*a^6*b
^16*x^8 + 3432*a^7*b^15*x^7 + 3003*a^8*b^14*x^6 + 2002*a^9*b^13*x^5 + 1001*
a^10*b^12*x^4 + 364*a^11*b^11*x^3 + 91*a^12*b^10*x^2 + 14*a^13*b^9*x + a^14
*b^8)
```

**mupad** [**B**] time = 1.24, size = 581, normalized size = 2.90

$$-\frac{\frac{a^7 d^7+7 a^6 b c d^6+28 a^5 b^2 c^2 d^5+84 a^4 b^3 c^3 d^4+210 a^3 b^4 c^4 d^3+462 a^2 b^5 c^5 d^2+924 a b^6 c^6 d+1716 b^7 c^7}{24024 b^8}+\frac{d^7 x^7}{7 b}+\frac{d^2 x^2 \left(a^5 d^5+7 a^4 b c d^4+28 a^3 b^2 c^2 d^3+84 a^2 b^3 c^3 d^2+210 a b^4 c^4 d+462 b^5 c^5\right)}{264 b^6}+\frac{d^4 x^4 \left(a^3 d^3+7 a^2 b c d^2+28 a b^2 c^2 d+84 b^3 c^3\right)}{24 b^4}+\frac{d^6 x^6 (a d+7 b c)}{8 b^2}+\frac{d^3 x^3 \left(a^4 d^4+7 a^3 b c d^3+28 a^2 b^2 c^2 d^2+84 a b^3 c^3 d+210 b^4 c^4\right)}{66 b^5}+\frac{d x \left(a^6 d^6+7 a^5 b c d^5+28 a^4 b^2 c^2 d^4+84 a^3 b^3 c^3 d^3+210 a^2 b^4 c^4 d^2+462 a b^5 c^5 d+924 b^6 c^6\right)}{1716 b^7}+\frac{d^5 x^5 \left(a^2 d^2+7 a b c d+28 b^2 c^2\right)}{12 b^3}}{a^{14}+14 a^{13} b x+91 a^{12} b^2 x^2+364 a^{11} b^3 x^3+1001 a^{10} b^4 x^4+2002 a^9 b^5 x^5+3003 a^8 b^6 x^6+3432 a^7 b^7 x^7+3003 a^6 b^8 x^8+2002 a^5 b^9 x^9+1001 a^4 b^{10} x^{10}+364 a^3 b^{11} x^{11}+91 a^2 b^{12} x^{12}+14 a b^{13} x^{13}+b^{14} x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^15,x)
```

```
[Out] -((a^7*d^7 + 1716*b^7*c^7 + 462*a^2*b^5*c^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*
a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 924*a*b^6*c^6*d + 7*a^6*b*c*d^6)/(24
024*b^8) + (d^7*x^7)/(7*b) + (d^2*x^2*(a^5*d^5 + 462*b^5*c^5 + 84*a^2*b^3*c
^3*d^2 + 28*a^3*b^2*c^2*d^3 + 210*a*b^4*c^4*d + 7*a^4*b*c*d^4))/(264*b^6) +
 (d^4*x^4*(a^3*d^3 + 84*b^3*c^3 + 28*a*b^2*c^2*d + 7*a^2*b*c*d^2))/(24*b^4)
 + (d^6*x^6*(a*d + 7*b*c))/(8*b^2) + (d^3*x^3*(a^4*d^4 + 210*b^4*c^4 + 28*a
^2*b^2*c^2*d^2 + 84*a*b^3*c^3*d + 7*a^3*b*c*d^3))/(66*b^5) + (d*x*(a^6*d^6
+ 924*b^6*c^6 + 210*a^2*b^4*c^4*d^2 + 84*a^3*b^3*c^3*d^3 + 28*a^4*b^2*c^2*d
^4 + 462*a*b^5*c^5*d + 7*a^5*b*c*d^5))/(1716*b^7) + (d^5*x^5*(a^2*d^2 + 28*
b^2*c^2 + 7*a*b*c*d))/(12*b^3))/(a^14 + b^14*x^14 + 14*a*b^13*x^13 + 91*a^1
2*b^2*x^2 + 364*a^11*b^3*x^3 + 1001*a^10*b^4*x^4 + 2002*a^9*b^5*x^5 + 3003*
a^8*b^6*x^6 + 3432*a^7*b^7*x^7 + 3003*a^6*b^8*x^8 + 2002*a^5*b^9*x^9 + 1001
*a^4*b^10*x^10 + 364*a^3*b^11*x^11 + 91*a^2*b^12*x^12 + 14*a^13*b*x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**15,x)
```

```
[Out] Timed out
```

## 3.1192 $\int \frac{(c+dx)^7}{(a+bx)^{16}}\,dx$

**Optimal**. Leaf size=200

$$-\frac{7d^6(bc-ad)}{9b^8(a+bx)^9}-\frac{21d^5(bc-ad)^2}{10b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}$$

**Rubi** [A] time = 0.14, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{7d^6(bc-ad)}{9b^8(a+bx)^9}-\frac{21d^5(bc-ad)^2}{10b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}-\frac{d^7}{8b^8(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] -(b*c - a*d)^7/(15*b^8*(a + b*x)^15) - (d*(b*c - a*d)^6)/(2*b^8*(a + b*x)^1
4) - (21*d^2*(b*c - a*d)^5)/(13*b^8*(a + b*x)^13) - (35*d^3*(b*c - a*d)^4)/
(12*b^8*(a + b*x)^12) - (35*d^4*(b*c - a*d)^3)/(11*b^8*(a + b*x)^11) - (21*
d^5*(b*c - a*d)^2)/(10*b^8*(a + b*x)^10) - (7*d^6*(b*c - a*d))/(9*b^8*(a +
b*x)^9) - d^7/(8*b^8*(a + b*x)^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^{16}}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{16}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{15}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{14}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{13}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{12}}+\ldots\right.\\ &= -\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\ldots\end{aligned}$$

**Mathematica** [A] time = 0.13, size = 371, normalized size = 1.86

$$-\frac{a^7d^7+a^6bd^6(8c+15dx)+3a^5b^2d^5\left(12c^2+40cdx+35d^2x^2\right)+5a^4b^3d^4\left(24c^3+108c^2dx+168cd^2x^2+91d^3x^3\right)+5a^3b^4d^3\left(66c^4+360c^3dx+756c^2d^2x^2+728cd^3x^3+273d^4x^4\right)+3a^2b^5d^2\left(264c^5+1650c^4dx+4200c^3d^2x^2+5460c^2d^3x^3+3640cd^4x^4+1001d^5x^5\right)+ab^6d\left(1716c^6+11880c^5dx+34650c^4d^2x^2+54600c^3d^3x^3+49140c^2d^4x^4+24024cd^5x^5+5005d^6x^6\right)+b^7\left(3432c^7+25740c^6dx+83160c^5d^2x^2+150150c^4d^3x^3+163800c^3d^4x^4+108108c^2d^5x^5+40040cd^6x^6+6435d^7x^7\right)}{51480b^8(a+bx)^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] -1/51480*(a^7*d^7 + a^6*b*d^6*(8*c + 15*d*x) + 3*a^5*b^2*d^5*(12*c^2 + 40*c
*d*x + 35*d^2*x^2) + 5*a^4*b^3*d^4*(24*c^3 + 108*c^2*d*x + 168*c*d^2*x^2 +
91*d^3*x^3) + 5*a^3*b^4*d^3*(66*c^4 + 360*c^3*d*x + 756*c^2*d^2*x^2 + 728*c
*d^3*x^3 + 273*d^4*x^4) + 3*a^2*b^5*d^2*(264*c^5 + 1650*c^4*d*x + 4200*c^3*
d^2*x^2 + 5460*c^2*d^3*x^3 + 3640*c*d^4*x^4 + 1001*d^5*x^5) + a*b^6*d*(1716
*c^6 + 11880*c^5*d*x + 34650*c^4*d^2*x^2 + 54600*c^3*d^3*x^3 + 49140*c^2*d^
4*x^4 + 24024*c*d^5*x^5 + 5005*d^6*x^6) + b^7*(3432*c^7 + 25740*c^6*d*x + 8
3160*c^5*d^2*x^2 + 150150*c^4*d^3*x^3 + 163800*c^3*d^4*x^4 + 108108*c^2*d^5
*x^5 + 40040*c*d^6*x^6 + 6435*d^7*x^7))/(b^8*(a + b*x)^15)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^16, x]
```

**fricas [B]** time = 1.10, size = 614, normalized size = 3.07

$$-\frac{6435\,b^{7}d^{7}x^{7}+3432\,b^{7}c^{7}+1716\,ab^{6}c^{6}d+792\,a^{2}b^{5}c^{5}d^{2}+330\,a^{3}b^{4}c^{4}d^{3}+120\,a^{4}b^{3}c^{3}d^{4}+36\,a^{5}b^{2}c^{2}d^{5}+8\,a^{6}bcd^{6}+a^{7}d^{7}+5005\left(8\,b^{7}cd^{6}+ab^{6}d^{7}\right)x^{6}+3003\left(36\,b^{7}c^{2}d^{5}+8\,ab^{6}cd^{6}+a^{2}b^{5}d^{7}\right)x^{5}+1365\left(120\,b^{7}c^{3}d^{4}+36\,ab^{6}c^{2}d^{5}+8\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7}\right)x^{4}+455\left(330\,b^{7}c^{4}d^{3}+120\,ab^{6}c^{3}d^{4}+36\,a^{2}b^{5}c^{2}d^{5}+8\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7}\right)x^{3}+105\left(792\,b^{7}c^{5}d^{2}+330\,ab^{6}c^{4}d^{3}+120\,a^{2}b^{5}c^{3}d^{4}+36\,a^{3}b^{4}c^{2}d^{5}+8\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7}\right)x^{2}+15\left(1716\,b^{7}c^{6}d+792\,ab^{6}c^{5}d^{2}+330\,a^{2}b^{5}c^{4}d^{3}+120\,a^{3}b^{4}c^{3}d^{4}+36\,a^{4}b^{3}c^{2}d^{5}+8\,a^{5}b^{2}cd^{6}+a^{6}bd^{7}\right)x}{51480\left(b^{23}x^{15}+15\,ab^{22}x^{14}+105\,a^{2}b^{21}x^{13}+455\,a^{3}b^{20}x^{12}+1365\,a^{4}b^{19}x^{11}+3003\,a^{5}b^{18}x^{10}+5005\,a^{6}b^{17}x^{9}+6435\,a^{7}b^{16}x^{8}+6435\,a^{8}b^{15}x^{7}+5005\,a^{9}b^{14}x^{6}+3003\,a^{10}b^{13}x^{5}+1365\,a^{11}b^{12}x^{4}+455\,a^{12}b^{11}x^{3}+105\,a^{13}b^{10}x^{2}+15\,a^{14}b^{9}x+a^{15}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="fricas")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*
c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 +
8*a^6*b*c*d^6 + a^7*d^7 + 5005*(8*b^7*c*d^6 + a*b^6*d^7)*x^6 + 3003*(36*b^7
*c^2*d^5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1365*(120*b^7*c^3*d^4 + 36*a*
b^6*c^2*d^5 + 8*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 455*(330*b^7*c^4*d^3 + 1
20*a*b^6*c^3*d^4 + 36*a^2*b^5*c^2*d^5 + 8*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3
+ 105*(792*b^7*c^5*d^2 + 330*a*b^6*c^4*d^3 + 120*a^2*b^5*c^3*d^4 + 36*a^3*b
^4*c^2*d^5 + 8*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 15*(1716*b^7*c^6*d + 792*
a*b^6*c^5*d^2 + 330*a^2*b^5*c^4*d^3 + 120*a^3*b^4*c^3*d^4 + 36*a^4*b^3*c^2*
d^5 + 8*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^23*x^15 + 15*a*b^22*x^14 + 105*a^2
*b^21*x^13 + 455*a^3*b^20*x^12 + 1365*a^4*b^19*x^11 + 3003*a^5*b^18*x^10 +
5005*a^6*b^17*x^9 + 6435*a^7*b^16*x^8 + 6435*a^8*b^15*x^7 + 5005*a^9*b^14*x
^6 + 3003*a^10*b^13*x^5 + 1365*a^11*b^12*x^4 + 455*a^12*b^11*x^3 + 105*a^13
*b^10*x^2 + 15*a^14*b^9*x + a^15*b^8)
```

**giac [B]** time = 1.29, size = 496, normalized size = 2.48

$$-\frac{6435 \, b^{7} d^{7} x^{7} + 40040 \, b^{7} c d^{6} x^{6} + 5005 \, a b^{6} d^{7} x^{6} + 108108 \, b^{7} c^{2} d^{5} x^{5} + 24024 \, a b^{6} c d^{6} x^{5} + 3003 \, a^{2} b^{5} d^{7} x^{5} + 163800 \, b^{7} c^{3} d^{4} x^{4} + 49140 \, a b^{6} c^{2} d^{5} x^{4} + 10920 \, a^{2} b^{5} c d^{6} x^{4} + 1365 \, a^{3} b^{4} d^{7} x^{4} + 150150 \, b^{7} c^{4} d^{3} x^{3} + 54600 \, a b^{6} c^{3} d^{4} x^{3} + 16380 \, a^{2} b^{5} c^{2} d^{5} x^{3} + 3640 \, a^{3} b^{4} c d^{6} x^{3} + 455 \, a^{4} b^{3} d^{7} x^{3} + 83160 \, b^{7} c^{5} d^{2} x^{2} + 34650 \, a b^{6} c^{4} d^{3} x^{2} + 12600 \, a^{2} b^{5} c^{3} d^{4} x^{2} + 3780 \, a^{3} b^{4} c^{2} d^{5} x^{2} + 840 \, a^{4} b^{3} c d^{6} x^{2} + 105 \, a^{5} b^{2} d^{7} x^{2} + 25740 \, b^{7} c^{6} d x + 11880 \, a b^{6} c^{5} d^{2} x + 4950 \, a^{2} b^{5} c^{4} d^{3} x + 1800 \, a^{3} b^{4} c^{3} d^{4} x + 540 \, a^{4} b^{3} c^{2} d^{5} x + 120 \, a^{5} b^{2} c d^{6} x + 15 \, a^{6} b d^{7} x + 3432 \, b^{7} c^{7} + 1716 \, a b^{6} c^{6} d + 792 \, a^{2} b^{5} c^{5} d^{2} + 330 \, a^{3} b^{4} c^{4} d^{3} + 120 \, a^{4} b^{3} c^{3} d^{4} + 36 \, a^{5} b^{2} c^{2} d^{5} + 8 \, a^{6} b c d^{6} + a^{7} d^{7}}{51480 \, {\left(b x + a\right)}^{15} b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="giac")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 40040*b^7*c*d^6*x^6 + 5005*a*b^6*d^7*x^6 + 108
108*b^7*c^2*d^5*x^5 + 24024*a*b^6*c*d^6*x^5 + 3003*a^2*b^5*d^7*x^5 + 163800
*b^7*c^3*d^4*x^4 + 49140*a*b^6*c^2*d^5*x^4 + 10920*a^2*b^5*c*d^6*x^4 + 1365
*a^3*b^4*d^7*x^4 + 150150*b^7*c^4*d^3*x^3 + 54600*a*b^6*c^3*d^4*x^3 + 16380
*a^2*b^5*c^2*d^5*x^3 + 3640*a^3*b^4*c*d^6*x^3 + 455*a^4*b^3*d^7*x^3 + 83160
*b^7*c^5*d^2*x^2 + 34650*a*b^6*c^4*d^3*x^2 + 12600*a^2*b^5*c^3*d^4*x^2 + 37
80*a^3*b^4*c^2*d^5*x^2 + 840*a^4*b^3*c*d^6*x^2 + 105*a^5*b^2*d^7*x^2 + 2574
0*b^7*c^6*d*x + 11880*a*b^6*c^5*d^2*x + 4950*a^2*b^5*c^4*d^3*x + 1800*a^3*b
^4*c^3*d^4*x + 540*a^4*b^3*c^2*d^5*x + 120*a^5*b^2*c*d^6*x + 15*a^6*b*d^7*x
 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*c^5*d^2 + 330*a^3*b^4*c^4*
d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 + 8*a^6*b*c*d^6 + a^7*d^7)/(
(b*x + a)^15*b^8)
```

**maple [B]** time = 0.01, size = 464, normalized size = 2.32

$$-\frac{d^{7}}{8 \left(b x +a \right)^{8} b^{8}}+\frac{7 \left(a d -b c \right) d^{6}}{9 \left(b x +a \right)^{9} b^{8}}-\frac{21 \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) d^{5}}{10 \left(b x +a \right)^{10} b^{8}}+\frac{35 \left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right) d^{4}}{11 \left(b x +a \right)^{11} b^{8}}-\frac{35 \left(a^{4} d^{4}-4 a^{3} b c \,d^{3}+6 a^{2} b^{2} c^{2} d^{2}-4 a \,b^{3} c^{3} d +b^{4} c^{4}\right) d^{3}}{12 \left(b x +a \right)^{12} b^{8}}+\frac{21 \left(a^{5} d^{5}-5 a^{4} b c \,d^{4}+10 a^{3} b^{2} c^{2} d^{3}-10 a^{2} b^{3} c^{3} d^{2}+5 a \,b^{4} c^{4} d -b^{5} c^{5}\right) d^{2}}{13 \left(b x +a \right)^{13} b^{8}}-\frac{\left(a^{6} d^{6}-6 a^{5} b c \,d^{5}+15 a^{4} b^{2} c^{2} d^{4}-20 a^{3} b^{3} c^{3} d^{3}+15 a^{2} b^{4} c^{4} d^{2}-6 a \,b^{5} c^{5} d +b^{6} c^{6}\right) d}{2 \left(b x +a \right)^{14} b^{8}}-\frac{-a^{7} d^{7}+7 a^{6} b c \,d^{6}-21 a^{5} b^{2} c^{2} d^{5}+35 a^{4} c^{3} d^{4} b^{3}-35 a^{3} b^{4} c^{4} d^{3}+21 a^{2} c^{5} d^{2} b^{5}-7 a \,b^{6} c^{6} d +b^{7} c^{7}}{15 \left(b x +a \right)^{15} b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^16,x)
```

```
[Out] -1/8*d^7/b^8/(b*x+a)^8+21/13*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-
10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^13+7/9*d^6*(a*d-b*c)/
b^8/(b*x+a)^9+35/11*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(
b*x+a)^11-1/15*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^
4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^
15-35/12*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4
)/b^8/(b*x+a)^12-21/10*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^10-1/2*d
*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^
4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^14
```

**maxima [B]** time = 1.91, size = 614, normalized size = 3.07

$$-\frac{6435 \, b^{7} d^{7} x^{7} + 3432 \, b^{7} c^{7} + 1716 \, a b^{6} c^{6} d + 792 \, a^{2} b^{5} c^{5} d^{2} + 330 \, a^{3} b^{4} c^{4} d^{3} + 120 \, a^{4} b^{3} c^{3} d^{4} + 36 \, a^{5} b^{2} c^{2} d^{5} + 8 \, a^{6} b c d^{6} + a^{7} d^{7} + 5005 \, {\left(8 \, b^{7} c d^{6} + a b^{6} d^{7}\right)} x^{6} + 3003 \, {\left(36 \, b^{7} c^{2} d^{5} + 8 \, a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right)} x^{5} + 1365 \, {\left(120 \, b^{7} c^{3} d^{4} + 36 \, a b^{6} c^{2} d^{5} + 8 \, a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}\right)} x^{4} + 455 \, {\left(330 \, b^{7} c^{4} d^{3} + 120 \, a b^{6} c^{3} d^{4} + 36 \, a^{2} b^{5} c^{2} d^{5} + 8 \, a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right)} x^{3} + 105 \, {\left(792 \, b^{7} c^{5} d^{2} + 330 \, a b^{6} c^{4} d^{3} + 120 \, a^{2} b^{5} c^{3} d^{4} + 36 \, a^{3} b^{4} c^{2} d^{5} + 8 \, a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}\right)} x^{2} + 15 \, {\left(1716 \, b^{7} c^{6} d + 792 \, a b^{6} c^{5} d^{2} + 330 \, a^{2} b^{5} c^{4} d^{3} + 120 \, a^{3} b^{4} c^{3} d^{4} + 36 \, a^{4} b^{3} c^{2} d^{5} + 8 \, a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right)} x}{51480 \, {\left(b^{23} x^{15} + 15 \, a b^{22} x^{14} + 105 \, a^{2} b^{21} x^{13} + 455 \, a^{3} b^{20} x^{12} + 1365 \, a^{4} b^{19} x^{11} + 3003 \, a^{5} b^{18} x^{10} + 5005 \, a^{6} b^{17} x^{9} + 6435 \, a^{7} b^{16} x^{8} + 6435 \, a^{8} b^{15} x^{7} + 5005 \, a^{9} b^{14} x^{6} + 3003 \, a^{10} b^{13} x^{5} + 1365 \, a^{11} b^{12} x^{4} + 455 \, a^{12} b^{11} x^{3} + 105 \, a^{13} b^{10} x^{2} + 15 \, a^{14} b^{9} x + a^{15} b^{8}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="maxima")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*
c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 +
8*a^6*b*c*d^6 + a^7*d^7 + 5005*(8*b^7*c*d^6 + a*b^6*d^7)*x^6 + 3003*(36*b^7
*c^2*d^5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1365*(120*b^7*c^3*d^4 + 36*a*
b^6*c^2*d^5 + 8*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 455*(330*b^7*c^4*d^3 + 1
20*a*b^6*c^3*d^4 + 36*a^2*b^5*c^2*d^5 + 8*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3
+ 105*(792*b^7*c^5*d^2 + 330*a*b^6*c^4*d^3 + 120*a^2*b^5*c^3*d^4 + 36*a^3*b
^4*c^2*d^5 + 8*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 15*(1716*b^7*c^6*d + 792*
a*b^6*c^5*d^2 + 330*a^2*b^5*c^4*d^3 + 120*a^3*b^4*c^3*d^4 + 36*a^4*b^3*c^2*
d^5 + 8*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^23*x^15 + 15*a*b^22*x^14 + 105*a^2
*b^21*x^13 + 455*a^3*b^20*x^12 + 1365*a^4*b^19*x^11 + 3003*a^5*b^18*x^10 +
5005*a^6*b^17*x^9 + 6435*a^7*b^16*x^8 + 6435*a^8*b^15*x^7 + 5005*a^9*b^14*x
^6 + 3003*a^10*b^13*x^5 + 1365*a^11*b^12*x^4 + 455*a^12*b^11*x^3 + 105*a^13
*b^10*x^2 + 15*a^14*b^9*x + a^15*b^8)
```

**mupad** [B] time = 2.20, size = 592, normalized size = 2.96

$$-\frac{\frac{a^7 d^7+8 a^6 b c d^6+36 a^5 b^2 c^2 d^5+120 a^4 b^3 c^3 d^4+330 a^3 b^4 c^4 d^3+792 a^2 b^5 c^5 d^2+1716 a b^6 c^6 d+3432 b^7 c^7}{51480 b^8}+\frac{d^7 x^7}{8 b}+\frac{7 d^2 x^2 \left(a^5 d^5+8 a^4 b c d^4+36 a^3 b^2 c^2 d^3+120 a^2 b^3 c^3 d^2+330 a b^4 c^4 d+792 b^5 c^5\right)}{3432 b^6}+\frac{7 d^4 x^4 \left(a^3 d^3+8 a^2 b c d^2+36 a b^2 c^2 d+120 b^3 c^3\right)}{264 b^4}+\frac{7 d^6 x^6 (a d+8 b c)}{72 b^2}+\frac{7 d^3 x^3 \left(a^4 d^4+8 a^3 b c d^3+36 a^2 b^2 c^2 d^2+120 a b^3 c^3 d+330 b^4 c^4\right)}{792 b^5}+\frac{d x \left(a^6 d^6+8 a^5 b c d^5+36 a^4 b^2 c^2 d^4+120 a^3 b^3 c^3 d^3+330 a^2 b^4 c^4 d^2+792 a b^5 c^5 d+1716 b^6 c^6\right)}{3432 b^7}+\frac{7 d^5 x^5 \left(a^2 d^2+8 a b c d+36 b^2 c^2\right)}{120 b^3}}{a^{15}+15 a^{14} b x+105 a^{13} b^2 x^2+455 a^{12} b^3 x^3+1365 a^{11} b^4 x^4+3003 a^{10} b^5 x^5+5005 a^9 b^6 x^6+6435 a^8 b^7 x^7+6435 a^7 b^8 x^8+5005 a^6 b^9 x^9+3003 a^5 b^{10} x^{10}+1365 a^4 b^{11} x^{11}+455 a^3 b^{12} x^{12}+105 a^2 b^{13} x^{13}+15 a b^{14} x^{14}+b^{15} x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^16,x)
```

```
[Out] -((a^7*d^7 + 3432*b^7*c^7 + 792*a^2*b^5*c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120
*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 + 1716*a*b^6*c^6*d + 8*a^6*b*c*d^6)/(
51480*b^8) + (d^7*x^7)/(8*b) + (7*d^2*x^2*(a^5*d^5 + 792*b^5*c^5 + 120*a^2*
b^3*c^3*d^2 + 36*a^3*b^2*c^2*d^3 + 330*a*b^4*c^4*d + 8*a^4*b*c*d^4))/(3432*
b^6) + (7*d^4*x^4*(a^3*d^3 + 120*b^3*c^3 + 36*a*b^2*c^2*d + 8*a^2*b*c*d^2))
/(264*b^4) + (7*d^6*x^6*(a*d + 8*b*c))/(72*b^2) + (7*d^3*x^3*(a^4*d^4 + 330
*b^4*c^4 + 36*a^2*b^2*c^2*d^2 + 120*a*b^3*c^3*d + 8*a^3*b*c*d^3))/(792*b^5)
 + (d*x*(a^6*d^6 + 1716*b^6*c^6 + 330*a^2*b^4*c^4*d^2 + 120*a^3*b^3*c^3*d^3
 + 36*a^4*b^2*c^2*d^4 + 792*a*b^5*c^5*d + 8*a^5*b*c*d^5))/(3432*b^7) + (7*d
^5*x^5*(a^2*d^2 + 36*b^2*c^2 + 8*a*b*c*d))/(120*b^3))/(a^15 + b^15*x^15 + 1
5*a*b^14*x^14 + 105*a^13*b^2*x^2 + 455*a^12*b^3*x^3 + 1365*a^11*b^4*x^4 + 3
003*a^10*b^5*x^5 + 5005*a^9*b^6*x^6 + 6435*a^8*b^7*x^7 + 6435*a^7*b^8*x^8 +
 5005*a^6*b^9*x^9 + 3003*a^5*b^10*x^10 + 1365*a^4*b^11*x^11 + 455*a^3*b^12*
x^12 + 105*a^2*b^13*x^13 + 15*a^14*b*x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**16,x)
```

```
[Out] Timed out
```

## 3.1193 $\int (a+bx)^{12}(c+dx)^{10}\,dx$

**Optimal**. Leaf size=275

$$\frac{5d^9(a+bx)^{22}(bc-ad)}{11b^{11}}+\frac{15d^8(a+bx)^{21}(bc-ad)^2}{7b^{11}}+\frac{6d^7(a+bx)^{20}(bc-ad)^3}{b^{11}}+\frac{210d^6(a+bx)^{19}(bc-ad)^4}{19b^{11}}+\frac{14d^5(a+$$

**Rubi [A]** time = 1.47, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^{22}(bc-ad)}{11b^{11}}+\frac{15d^8(a+bx)^{21}(bc-ad)^2}{7b^{11}}+\frac{6d^7(a+bx)^{20}(bc-ad)^3}{b^{11}}+\frac{210d^6(a+bx)^{19}(bc-ad)^4}{19b^{11}}+\frac{14d^5(a+bx)^{18}(bc-ad)^5}{b^{11}}+\frac{210d^4(a+bx)^{17}(bc-ad)^6}{17b^{11}}+\frac{15d^3(a+bx)^{16}(bc-ad)^7}{2b^{11}}+\frac{3d^2(a+bx)^{15}(bc-ad)^8}{b^{11}}+\frac{5d(a+bx)^{14}(bc-ad)^9}{7b^{11}}+\frac{(a+bx)^{13}(bc-ad)^{10}}{13b^{11}}+\frac{d^{10}(a+bx)^{23}}{23b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^12*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^13)/(13*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^14)/
(7*b^11) + (3*d^2*(b*c - a*d)^8*(a + b*x)^15)/b^11 + (15*d^3*(b*c - a*d)^7*
(a + b*x)^16)/(2*b^11) + (210*d^4*(b*c - a*d)^6*(a + b*x)^17)/(17*b^11) + (
14*d^5*(b*c - a*d)^5*(a + b*x)^18)/b^11 + (210*d^6*(b*c - a*d)^4*(a + b*x)^
19)/(19*b^11) + (6*d^7*(b*c - a*d)^3*(a + b*x)^20)/b^11 + (15*d^8*(b*c - a*
d)^2*(a + b*x)^21)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^22)/(11*b^11) +
(d^10*(a + b*x)^23)/(23*b^11)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{12}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{12}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)^{14}}{b^{10}}+\frac{12\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{13}}{13b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{14}}{7b^{11}}+\frac{3d^2(bc-ad)^8(a+bx)^{15}}{b^{11}}+\frac{15d^3(bc\end{aligned}$$

**Mathematica [B]** time = 0.29, size = 1817, normalized size = 6.61

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^12*(c + d*x)^10,x]

[Out] a^12*c^10*x + a^11*c^9*(6*b*c + 5*a*d)*x^2 + a^10*c^8*(22*b^2*c^2 + 40*a*b*
c*d + 15*a^2*d^2)*x^3 + 5*a^9*c^7*(11*b^3*c^3 + 33*a*b^2*c^2*d + 27*a^2*b*c
*d^2 + 6*a^3*d^3)*x^4 + a^8*c^6*(99*b^4*c^4 + 440*a*b^3*c^3*d + 594*a^2*b^2
*c^2*d^2 + 288*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + 3*a^7*c^5*(44*b^5*c^5 + 275*
a*b^4*c^4*d + 550*a^2*b^3*c^3*d^2 + 440*a^3*b^2*c^2*d^3 + 140*a^4*b*c*d^4 +
 14*a^5*d^5)*x^6 + (3*a^6*c^4*(308*b^6*c^6 + 2640*a*b^5*c^5*d + 7425*a^2*b^
4*c^4*d^2 + 8800*a^3*b^3*c^3*d^3 + 4620*a^4*b^2*c^2*d^4 + 1008*a^5*b*c*d^5
+ 70*a^6*d^6)*x^7)/7 + 3*a^5*c^3*(33*b^7*c^7 + 385*a*b^6*c^6*d + 1485*a^2*b
^5*c^5*d^2 + 2475*a^3*b^4*c^4*d^3 + 1925*a^4*b^3*c^3*d^4 + 693*a^5*b^2*c^2*
d^5 + 105*a^6*b*c*d^6 + 5*a^7*d^7)*x^8 + 5*a^4*c^2*(11*b^8*c^8 + 176*a*b^7*
c^7*d + 924*a^2*b^6*c^6*d^2 + 2112*a^3*b^5*c^5*d^3 + 2310*a^4*b^4*c^4*d^4 +
 1232*a^5*b^3*c^3*d^5 + 308*a^6*b^2*c^2*d^6 + 32*a^7*b*c*d^7 + a^8*d^8)*x^9
 + a^3*c*(22*b^9*c^9 + 495*a*b^8*c^8*d + 3564*a^2*b^7*c^7*d^2 + 11088*a^3*b
^6*c^6*d^3 + 16632*a^4*b^5*c^5*d^4 + 12474*a^5*b^4*c^4*d^5 + 4620*a^6*b^3*c
^3*d^6 + 792*a^7*b^2*c^2*d^7 + 54*a^8*b*c*d^8 + a^9*d^9)*x^10 + (a^2*(66*b^
10*c^10 + 2200*a*b^9*c^9*d + 22275*a^2*b^8*c^8*d^2 + 95040*a^3*b^7*c^7*d^3
+ 194040*a^4*b^6*c^6*d^4 + 199584*a^5*b^5*c^5*d^5 + 103950*a^6*b^4*c^4*d^6
+ 26400*a^7*b^3*c^3*d^7 + 2970*a^8*b^2*c^2*d^8 + 120*a^9*b*c*d^9 + a^10*d^1
0)*x^11)/11 + a*b*(b^10*c^10 + 55*a*b^9*c^9*d + 825*a^2*b^8*c^8*d^2 + 4950*
a^3*b^7*c^7*d^3 + 13860*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 + 13860*a^6
*b^4*c^4*d^6 + 4950*a^7*b^3*c^3*d^7 + 825*a^8*b^2*c^2*d^8 + 55*a^9*b*c*d^9
+ a^10*d^10)*x^12 + (b^2*(b^10*c^10 + 120*a*b^9*c^9*d + 2970*a^2*b^8*c^8*d^
2 + 26400*a^3*b^7*c^7*d^3 + 103950*a^4*b^6*c^6*d^4 + 199584*a^5*b^5*c^5*d^5
 + 194040*a^6*b^4*c^4*d^6 + 95040*a^7*b^3*c^3*d^7 + 22275*a^8*b^2*c^2*d^8 +
 2200*a^9*b*c*d^9 + 66*a^10*d^10)*x^13)/13 + (5*b^3*d*(b^9*c^9 + 54*a*b^8*c
^8*d + 792*a^2*b^7*c^7*d^2 + 4620*a^3*b^6*c^6*d^3 + 12474*a^4*b^5*c^5*d^4 +
 16632*a^5*b^4*c^4*d^5 + 11088*a^6*b^3*c^3*d^6 + 3564*a^7*b^2*c^2*d^7 + 495
*a^8*b*c*d^8 + 22*a^9*d^9)*x^14)/7 + 3*b^4*d^2*(b^8*c^8 + 32*a*b^7*c^7*d +
308*a^2*b^6*c^6*d^2 + 1232*a^3*b^5*c^5*d^3 + 2310*a^4*b^4*c^4*d^4 + 2112*a^
5*b^3*c^3*d^5 + 924*a^6*b^2*c^2*d^6 + 176*a^7*b*c*d^7 + 11*a^8*d^8)*x^15 +
(3*b^5*d^3*(5*b^7*c^7 + 105*a*b^6*c^6*d + 693*a^2*b^5*c^5*d^2 + 1925*a^3*b^
4*c^4*d^3 + 2475*a^4*b^3*c^3*d^4 + 1485*a^5*b^2*c^2*d^5 + 385*a^6*b*c*d^6 +
 33*a^7*d^7)*x^16)/2 + (3*b^6*d^4*(70*b^6*c^6 + 1008*a*b^5*c^5*d + 4620*a^2
*b^4*c^4*d^2 + 8800*a^3*b^3*c^3*d^3 + 7425*a^4*b^2*c^2*d^4 + 2640*a^5*b*c*d
^5 + 308*a^6*d^6)*x^17)/17 + b^7*d^5*(14*b^5*c^5 + 140*a*b^4*c^4*d + 440*a^
2*b^3*c^3*d^2 + 550*a^3*b^2*c^2*d^3 + 275*a^4*b*c*d^4 + 44*a^5*d^5)*x^18 +
(5*b^8*d^6*(42*b^4*c^4 + 288*a*b^3*c^3*d + 594*a^2*b^2*c^2*d^2 + 440*a^3*b*
c*d^3 + 99*a^4*d^4)*x^19)/19 + b^9*d^7*(6*b^3*c^3 + 27*a*b^2*c^2*d + 33*a^2
*b*c*d^2 + 11*a^3*d^3)*x^20 + (b^10*d^8*(15*b^2*c^2 + 40*a*b*c*d + 22*a^2*d
^2)*x^21)/7 + (b^11*d^9*(5*b*c + 6*a*d)*x^22)/11 + (b^12*d^10*x^23)/23
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{12}(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^12*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^12*(c + d*x)^10, x]
```

**fricas [B]** time = 1.33, size = 2186, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*d^10*b^12 + 5/11*x^22*d^9*c*b^12 + 6/11*x^22*d^10*b^11*a + 15/7*x
^21*d^8*c^2*b^12 + 40/7*x^21*d^9*c*b^11*a + 22/7*x^21*d^10*b^10*a^2 + 6*x^2
0*d^7*c^3*b^12 + 27*x^20*d^8*c^2*b^11*a + 33*x^20*d^9*c*b^10*a^2 + 11*x^20*
d^10*b^9*a^3 + 210/19*x^19*d^6*c^4*b^12 + 1440/19*x^19*d^7*c^3*b^11*a + 297
0/19*x^19*d^8*c^2*b^10*a^2 + 2200/19*x^19*d^9*c*b^9*a^3 + 495/19*x^19*d^10*
b^8*a^4 + 14*x^18*d^5*c^5*b^12 + 140*x^18*d^6*c^4*b^11*a + 440*x^18*d^7*c^3
*b^10*a^2 + 550*x^18*d^8*c^2*b^9*a^3 + 275*x^18*d^9*c*b^8*a^4 + 44*x^18*d^1
0*b^7*a^5 + 210/17*x^17*d^4*c^6*b^12 + 3024/17*x^17*d^5*c^5*b^11*a + 13860/
17*x^17*d^6*c^4*b^10*a^2 + 26400/17*x^17*d^7*c^3*b^9*a^3 + 22275/17*x^17*d^
8*c^2*b^8*a^4 + 7920/17*x^17*d^9*c*b^7*a^5 + 924/17*x^17*d^10*b^6*a^6 + 15/
2*x^16*d^3*c^7*b^12 + 315/2*x^16*d^4*c^6*b^11*a + 2079/2*x^16*d^5*c^5*b^10*
a^2 + 5775/2*x^16*d^6*c^4*b^9*a^3 + 7425/2*x^16*d^7*c^3*b^8*a^4 + 4455/2*x^
16*d^8*c^2*b^7*a^5 + 1155/2*x^16*d^9*c*b^6*a^6 + 99/2*x^16*d^10*b^5*a^7 + 3
*x^15*d^2*c^8*b^12 + 96*x^15*d^3*c^7*b^11*a + 924*x^15*d^4*c^6*b^10*a^2 + 3
696*x^15*d^5*c^5*b^9*a^3 + 6930*x^15*d^6*c^4*b^8*a^4 + 6336*x^15*d^7*c^3*b^
7*a^5 + 2772*x^15*d^8*c^2*b^6*a^6 + 528*x^15*d^9*c*b^5*a^7 + 33*x^15*d^10*b
^4*a^8 + 5/7*x^14*d*c^9*b^12 + 270/7*x^14*d^2*c^8*b^11*a + 3960/7*x^14*d^3*
c^7*b^10*a^2 + 3300*x^14*d^4*c^6*b^9*a^3 + 8910*x^14*d^5*c^5*b^8*a^4 + 1188
0*x^14*d^6*c^4*b^7*a^5 + 7920*x^14*d^7*c^3*b^6*a^6 + 17820/7*x^14*d^8*c^2*b
^5*a^7 + 2475/7*x^14*d^9*c*b^4*a^8 + 110/7*x^14*d^10*b^3*a^9 + 1/13*x^13*c^
10*b^12 + 120/13*x^13*d*c^9*b^11*a + 2970/13*x^13*d^2*c^8*b^10*a^2 + 26400/
13*x^13*d^3*c^7*b^9*a^3 + 103950/13*x^13*d^4*c^6*b^8*a^4 + 199584/13*x^13*d
^5*c^5*b^7*a^5 + 194040/13*x^13*d^6*c^4*b^6*a^6 + 95040/13*x^13*d^7*c^3*b^5
*a^7 + 22275/13*x^13*d^8*c^2*b^4*a^8 + 2200/13*x^13*d^9*c*b^3*a^9 + 66/13*x
^13*d^10*b^2*a^10 + x^12*c^10*b^11*a + 55*x^12*d*c^9*b^10*a^2 + 825*x^12*d^
2*c^8*b^9*a^3 + 4950*x^12*d^3*c^7*b^8*a^4 + 13860*x^12*d^4*c^6*b^7*a^5 + 19
404*x^12*d^5*c^5*b^6*a^6 + 13860*x^12*d^6*c^4*b^5*a^7 + 4950*x^12*d^7*c^3*b
^4*a^8 + 825*x^12*d^8*c^2*b^3*a^9 + 55*x^12*d^9*c*b^2*a^10 + x^12*d^10*b*a^
```

```
11 + 6*x^11*c^10*b^10*a^2 + 200*x^11*d*c^9*b^9*a^3 + 2025*x^11*d^2*c^8*b^8*
a^4 + 8640*x^11*d^3*c^7*b^7*a^5 + 17640*x^11*d^4*c^6*b^6*a^6 + 18144*x^11*d
^5*c^5*b^5*a^7 + 9450*x^11*d^6*c^4*b^4*a^8 + 2400*x^11*d^7*c^3*b^3*a^9 + 27
0*x^11*d^8*c^2*b^2*a^10 + 120/11*x^11*d^9*c*b*a^11 + 1/11*x^11*d^10*a^12 +
22*x^10*c^10*b^9*a^3 + 495*x^10*d*c^9*b^8*a^4 + 3564*x^10*d^2*c^8*b^7*a^5 +
 11088*x^10*d^3*c^7*b^6*a^6 + 16632*x^10*d^4*c^6*b^5*a^7 + 12474*x^10*d^5*c
^5*b^4*a^8 + 4620*x^10*d^6*c^4*b^3*a^9 + 792*x^10*d^7*c^3*b^2*a^10 + 54*x^1
0*d^8*c^2*b*a^11 + x^10*d^9*c*a^12 + 55*x^9*c^10*b^8*a^4 + 880*x^9*d*c^9*b^
7*a^5 + 4620*x^9*d^2*c^8*b^6*a^6 + 10560*x^9*d^3*c^7*b^5*a^7 + 11550*x^9*d^
4*c^6*b^4*a^8 + 6160*x^9*d^5*c^5*b^3*a^9 + 1540*x^9*d^6*c^4*b^2*a^10 + 160*
x^9*d^7*c^3*b*a^11 + 5*x^9*d^8*c^2*a^12 + 99*x^8*c^10*b^7*a^5 + 1155*x^8*d*
c^9*b^6*a^6 + 4455*x^8*d^2*c^8*b^5*a^7 + 7425*x^8*d^3*c^7*b^4*a^8 + 5775*x^
8*d^4*c^6*b^3*a^9 + 2079*x^8*d^5*c^5*b^2*a^10 + 315*x^8*d^6*c^4*b*a^11 + 15
*x^8*d^7*c^3*a^12 + 132*x^7*c^10*b^6*a^6 + 7920/7*x^7*d*c^9*b^5*a^7 + 22275
/7*x^7*d^2*c^8*b^4*a^8 + 26400/7*x^7*d^3*c^7*b^3*a^9 + 1980*x^7*d^4*c^6*b^2
*a^10 + 432*x^7*d^5*c^5*b*a^11 + 30*x^7*d^6*c^4*a^12 + 132*x^6*c^10*b^5*a^7
 + 825*x^6*d*c^9*b^4*a^8 + 1650*x^6*d^2*c^8*b^3*a^9 + 1320*x^6*d^3*c^7*b^2*
a^10 + 420*x^6*d^4*c^6*b*a^11 + 42*x^6*d^5*c^5*a^12 + 99*x^5*c^10*b^4*a^8 +
 440*x^5*d*c^9*b^3*a^9 + 594*x^5*d^2*c^8*b^2*a^10 + 288*x^5*d^3*c^7*b*a^11
+ 42*x^5*d^4*c^6*a^12 + 55*x^4*c^10*b^3*a^9 + 165*x^4*d*c^9*b^2*a^10 + 135*
x^4*d^2*c^8*b*a^11 + 30*x^4*d^3*c^7*a^12 + 22*x^3*c^10*b^2*a^10 + 40*x^3*d*
c^9*b*a^11 + 15*x^3*d^2*c^8*a^12 + 6*x^2*c^10*b*a^11 + 5*x^2*d*c^9*a^12 + x
*c^10*a^12
```

**giac [B]** time = 1.31, size = 2186, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/23*b^12*d^10*x^23 + 5/11*b^12*c*d^9*x^22 + 6/11*a*b^11*d^10*x^22 + 15/7*b
^12*c^2*d^8*x^21 + 40/7*a*b^11*c*d^9*x^21 + 22/7*a^2*b^10*d^10*x^21 + 6*b^1
2*c^3*d^7*x^20 + 27*a*b^11*c^2*d^8*x^20 + 33*a^2*b^10*c*d^9*x^20 + 11*a^3*b
^9*d^10*x^20 + 210/19*b^12*c^4*d^6*x^19 + 1440/19*a*b^11*c^3*d^7*x^19 + 297
0/19*a^2*b^10*c^2*d^8*x^19 + 2200/19*a^3*b^9*c*d^9*x^19 + 495/19*a^4*b^8*d^
10*x^19 + 14*b^12*c^5*d^5*x^18 + 140*a*b^11*c^4*d^6*x^18 + 440*a^2*b^10*c^3
*d^7*x^18 + 550*a^3*b^9*c^2*d^8*x^18 + 275*a^4*b^8*c*d^9*x^18 + 44*a^5*b^7*
d^10*x^18 + 210/17*b^12*c^6*d^4*x^17 + 3024/17*a*b^11*c^5*d^5*x^17 + 13860/
17*a^2*b^10*c^4*d^6*x^17 + 26400/17*a^3*b^9*c^3*d^7*x^17 + 22275/17*a^4*b^8
*c^2*d^8*x^17 + 7920/17*a^5*b^7*c*d^9*x^17 + 924/17*a^6*b^6*d^10*x^17 + 15/
2*b^12*c^7*d^3*x^16 + 315/2*a*b^11*c^6*d^4*x^16 + 2079/2*a^2*b^10*c^5*d^5*x
^16 + 5775/2*a^3*b^9*c^4*d^6*x^16 + 7425/2*a^4*b^8*c^3*d^7*x^16 + 4455/2*a^
5*b^7*c^2*d^8*x^16 + 1155/2*a^6*b^6*c*d^9*x^16 + 99/2*a^7*b^5*d^10*x^16 + 3
*b^12*c^8*d^2*x^15 + 96*a*b^11*c^7*d^3*x^15 + 924*a^2*b^10*c^6*d^4*x^15 + 3
```

```
696*a^3*b^9*c^5*d^5*x^15 + 6930*a^4*b^8*c^4*d^6*x^15 + 6336*a^5*b^7*c^3*d^7
*x^15 + 2772*a^6*b^6*c^2*d^8*x^15 + 528*a^7*b^5*c*d^9*x^15 + 33*a^8*b^4*d^1
0*x^15 + 5/7*b^12*c^9*d*x^14 + 270/7*a*b^11*c^8*d^2*x^14 + 3960/7*a^2*b^10*
c^7*d^3*x^14 + 3300*a^3*b^9*c^6*d^4*x^14 + 8910*a^4*b^8*c^5*d^5*x^14 + 1188
0*a^5*b^7*c^4*d^6*x^14 + 7920*a^6*b^6*c^3*d^7*x^14 + 17820/7*a^7*b^5*c^2*d^
8*x^14 + 2475/7*a^8*b^4*c*d^9*x^14 + 110/7*a^9*b^3*d^10*x^14 + 1/13*b^12*c^
10*x^13 + 120/13*a*b^11*c^9*d*x^13 + 2970/13*a^2*b^10*c^8*d^2*x^13 + 26400/
13*a^3*b^9*c^7*d^3*x^13 + 103950/13*a^4*b^8*c^6*d^4*x^13 + 199584/13*a^5*b^
7*c^5*d^5*x^13 + 194040/13*a^6*b^6*c^4*d^6*x^13 + 95040/13*a^7*b^5*c^3*d^7*
x^13 + 22275/13*a^8*b^4*c^2*d^8*x^13 + 2200/13*a^9*b^3*c*d^9*x^13 + 66/13*a
^10*b^2*d^10*x^13 + a*b^11*c^10*x^12 + 55*a^2*b^10*c^9*d*x^12 + 825*a^3*b^9
*c^8*d^2*x^12 + 4950*a^4*b^8*c^7*d^3*x^12 + 13860*a^5*b^7*c^6*d^4*x^12 + 19
404*a^6*b^6*c^5*d^5*x^12 + 13860*a^7*b^5*c^4*d^6*x^12 + 4950*a^8*b^4*c^3*d^
7*x^12 + 825*a^9*b^3*c^2*d^8*x^12 + 55*a^10*b^2*c*d^9*x^12 + a^11*b*d^10*x^
12 + 6*a^2*b^10*c^10*x^11 + 200*a^3*b^9*c^9*d*x^11 + 2025*a^4*b^8*c^8*d^2*x
^11 + 8640*a^5*b^7*c^7*d^3*x^11 + 17640*a^6*b^6*c^6*d^4*x^11 + 18144*a^7*b^
5*c^5*d^5*x^11 + 9450*a^8*b^4*c^4*d^6*x^11 + 2400*a^9*b^3*c^3*d^7*x^11 + 27
0*a^10*b^2*c^2*d^8*x^11 + 120/11*a^11*b*c*d^9*x^11 + 1/11*a^12*d^10*x^11 +
22*a^3*b^9*c^10*x^10 + 495*a^4*b^8*c^9*d*x^10 + 3564*a^5*b^7*c^8*d^2*x^10 +
 11088*a^6*b^6*c^7*d^3*x^10 + 16632*a^7*b^5*c^6*d^4*x^10 + 12474*a^8*b^4*c^
5*d^5*x^10 + 4620*a^9*b^3*c^4*d^6*x^10 + 792*a^10*b^2*c^3*d^7*x^10 + 54*a^1
1*b*c^2*d^8*x^10 + a^12*c*d^9*x^10 + 55*a^4*b^8*c^10*x^9 + 880*a^5*b^7*c^9*
d*x^9 + 4620*a^6*b^6*c^8*d^2*x^9 + 10560*a^7*b^5*c^7*d^3*x^9 + 11550*a^8*b^
4*c^6*d^4*x^9 + 6160*a^9*b^3*c^5*d^5*x^9 + 1540*a^10*b^2*c^4*d^6*x^9 + 160*
a^11*b*c^3*d^7*x^9 + 5*a^12*c^2*d^8*x^9 + 99*a^5*b^7*c^10*x^8 + 1155*a^6*b^
6*c^9*d*x^8 + 4455*a^7*b^5*c^8*d^2*x^8 + 7425*a^8*b^4*c^7*d^3*x^8 + 5775*a^
9*b^3*c^6*d^4*x^8 + 2079*a^10*b^2*c^5*d^5*x^8 + 315*a^11*b*c^4*d^6*x^8 + 15
*a^12*c^3*d^7*x^8 + 132*a^6*b^6*c^10*x^7 + 7920/7*a^7*b^5*c^9*d*x^7 + 22275
/7*a^8*b^4*c^8*d^2*x^7 + 26400/7*a^9*b^3*c^7*d^3*x^7 + 1980*a^10*b^2*c^6*d^
4*x^7 + 432*a^11*b*c^5*d^5*x^7 + 30*a^12*c^4*d^6*x^7 + 132*a^7*b^5*c^10*x^6
 + 825*a^8*b^4*c^9*d*x^6 + 1650*a^9*b^3*c^8*d^2*x^6 + 1320*a^10*b^2*c^7*d^3
*x^6 + 420*a^11*b*c^6*d^4*x^6 + 42*a^12*c^5*d^5*x^6 + 99*a^8*b^4*c^10*x^5 +
 440*a^9*b^3*c^9*d*x^5 + 594*a^10*b^2*c^8*d^2*x^5 + 288*a^11*b*c^7*d^3*x^5
+ 42*a^12*c^6*d^4*x^5 + 55*a^9*b^3*c^10*x^4 + 165*a^10*b^2*c^9*d*x^4 + 135*
a^11*b*c^8*d^2*x^4 + 30*a^12*c^7*d^3*x^4 + 22*a^10*b^2*c^10*x^3 + 40*a^11*b
*c^9*d*x^3 + 15*a^12*c^8*d^2*x^3 + 6*a^11*b*c^10*x^2 + 5*a^12*c^9*d*x^2 + a
^12*c^10*x
```

**maple [B]** time = 0.00, size = 1891, normalized size = 6.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^12*(d*x+c)^10,x)
```

```
[Out] 1/23*b^12*d^10*x^23+1/22*(12*a*b^11*d^10+10*b^12*c*d^9)*x^22+1/21*(66*a^2*b
^10*d^10+120*a*b^11*c*d^9+45*b^12*c^2*d^8)*x^21+1/20*(220*a^3*b^9*d^10+660*
a^2*b^10*c*d^9+540*a*b^11*c^2*d^8+120*b^12*c^3*d^7)*x^20+1/19*(495*a^4*b^8*
d^10+2200*a^3*b^9*c*d^9+2970*a^2*b^10*c^2*d^8+1440*a*b^11*c^3*d^7+210*b^12*
c^4*d^6)*x^19+1/18*(792*a^5*b^7*d^10+4950*a^4*b^8*c*d^9+9900*a^3*b^9*c^2*d^
8+7920*a^2*b^10*c^3*d^7+2520*a*b^11*c^4*d^6+252*b^12*c^5*d^5)*x^18+1/17*(92
4*a^6*b^6*d^10+7920*a^5*b^7*c*d^9+22275*a^4*b^8*c^2*d^8+26400*a^3*b^9*c^3*d
^7+13860*a^2*b^10*c^4*d^6+3024*a*b^11*c^5*d^5+210*b^12*c^6*d^4)*x^17+1/16*(
792*a^7*b^5*d^10+9240*a^6*b^6*c*d^9+35640*a^5*b^7*c^2*d^8+59400*a^4*b^8*c^3
*d^7+46200*a^3*b^9*c^4*d^6+16632*a^2*b^10*c^5*d^5+2520*a*b^11*c^6*d^4+120*b
^12*c^7*d^3)*x^16+1/15*(495*a^8*b^4*d^10+7920*a^7*b^5*c*d^9+41580*a^6*b^6*c
^2*d^8+95040*a^5*b^7*c^3*d^7+103950*a^4*b^8*c^4*d^6+55440*a^3*b^9*c^5*d^5+1
3860*a^2*b^10*c^6*d^4+1440*a*b^11*c^7*d^3+45*b^12*c^8*d^2)*x^15+1/14*(220*a
^9*b^3*d^10+4950*a^8*b^4*c*d^9+35640*a^7*b^5*c^2*d^8+110880*a^6*b^6*c^3*d^7
+166320*a^5*b^7*c^4*d^6+124740*a^4*b^8*c^5*d^5+46200*a^3*b^9*c^6*d^4+7920*a
^2*b^10*c^7*d^3+540*a*b^11*c^8*d^2+10*b^12*c^9*d)*x^14+1/13*(66*a^10*b^2*d^
10+2200*a^9*b^3*c*d^9+22275*a^8*b^4*c^2*d^8+95040*a^7*b^5*c^3*d^7+194040*a^
6*b^6*c^4*d^6+199584*a^5*b^7*c^5*d^5+103950*a^4*b^8*c^6*d^4+26400*a^3*b^9*c
^7*d^3+2970*a^2*b^10*c^8*d^2+120*a*b^11*c^9*d+b^12*c^10)*x^13+1/12*(12*a^11
*b*d^10+660*a^10*b^2*c*d^9+9900*a^9*b^3*c^2*d^8+59400*a^8*b^4*c^3*d^7+16632
0*a^7*b^5*c^4*d^6+232848*a^6*b^6*c^5*d^5+166320*a^5*b^7*c^6*d^4+59400*a^4*b
^8*c^7*d^3+9900*a^3*b^9*c^8*d^2+660*a^2*b^10*c^9*d+12*a*b^11*c^10)*x^12+1/1
1*(a^12*d^10+120*a^11*b*c*d^9+2970*a^10*b^2*c^2*d^8+26400*a^9*b^3*c^3*d^7+1
03950*a^8*b^4*c^4*d^6+199584*a^7*b^5*c^5*d^5+194040*a^6*b^6*c^6*d^4+95040*a
^5*b^7*c^7*d^3+22275*a^4*b^8*c^8*d^2+2200*a^3*b^9*c^9*d+66*a^2*b^10*c^10)*x
^11+1/10*(10*a^12*c*d^9+540*a^11*b*c^2*d^8+7920*a^10*b^2*c^3*d^7+46200*a^9*
b^3*c^4*d^6+124740*a^8*b^4*c^5*d^5+166320*a^7*b^5*c^6*d^4+110880*a^6*b^6*c^
7*d^3+35640*a^5*b^7*c^8*d^2+4950*a^4*b^8*c^9*d+220*a^3*b^9*c^10)*x^10+1/9*(
45*a^12*c^2*d^8+1440*a^11*b*c^3*d^7+13860*a^10*b^2*c^4*d^6+55440*a^9*b^3*c^
5*d^5+103950*a^8*b^4*c^6*d^4+95040*a^7*b^5*c^7*d^3+41580*a^6*b^6*c^8*d^2+79
20*a^5*b^7*c^9*d+495*a^4*b^8*c^10)*x^9+1/8*(120*a^12*c^3*d^7+2520*a^11*b*c^
4*d^6+16632*a^10*b^2*c^5*d^5+46200*a^9*b^3*c^6*d^4+59400*a^8*b^4*c^7*d^3+35
640*a^7*b^5*c^8*d^2+9240*a^6*b^6*c^9*d+792*a^5*b^7*c^10)*x^8+1/7*(210*a^12*
c^4*d^6+3024*a^11*b*c^5*d^5+13860*a^10*b^2*c^6*d^4+26400*a^9*b^3*c^7*d^3+22
275*a^8*b^4*c^8*d^2+7920*a^7*b^5*c^9*d+924*a^6*b^6*c^10)*x^7+1/6*(252*a^12*
c^5*d^5+2520*a^11*b*c^6*d^4+7920*a^10*b^2*c^7*d^3+9900*a^9*b^3*c^8*d^2+4950
*a^8*b^4*c^9*d+792*a^7*b^5*c^10)*x^6+1/5*(210*a^12*c^6*d^4+1440*a^11*b*c^7*
d^3+2970*a^10*b^2*c^8*d^2+2200*a^9*b^3*c^9*d+495*a^8*b^4*c^10)*x^5+1/4*(120
*a^12*c^7*d^3+540*a^11*b*c^8*d^2+660*a^10*b^2*c^9*d+220*a^9*b^3*c^10)*x^4+1
/3*(45*a^12*c^8*d^2+120*a^11*b*c^9*d+66*a^10*b^2*c^10)*x^3+1/2*(10*a^12*c^9
*d+12*a^11*b*c^10)*x^2+a^12*c^10*x
```

**maxima [B]** time = 1.55, size = 1877, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/23*b^12*d^10*x^23 + a^12*c^10*x + 1/11*(5*b^12*c*d^9 + 6*a*b^11*d^10)*x^2
2 + 1/7*(15*b^12*c^2*d^8 + 40*a*b^11*c*d^9 + 22*a^2*b^10*d^10)*x^21 + (6*b^
12*c^3*d^7 + 27*a*b^11*c^2*d^8 + 33*a^2*b^10*c*d^9 + 11*a^3*b^9*d^10)*x^20
+ 5/19*(42*b^12*c^4*d^6 + 288*a*b^11*c^3*d^7 + 594*a^2*b^10*c^2*d^8 + 440*a
^3*b^9*c*d^9 + 99*a^4*b^8*d^10)*x^19 + (14*b^12*c^5*d^5 + 140*a*b^11*c^4*d^
6 + 440*a^2*b^10*c^3*d^7 + 550*a^3*b^9*c^2*d^8 + 275*a^4*b^8*c*d^9 + 44*a^5
*b^7*d^10)*x^18 + 3/17*(70*b^12*c^6*d^4 + 1008*a*b^11*c^5*d^5 + 4620*a^2*b^
10*c^4*d^6 + 8800*a^3*b^9*c^3*d^7 + 7425*a^4*b^8*c^2*d^8 + 2640*a^5*b^7*c*d
^9 + 308*a^6*b^6*d^10)*x^17 + 3/2*(5*b^12*c^7*d^3 + 105*a*b^11*c^6*d^4 + 69
3*a^2*b^10*c^5*d^5 + 1925*a^3*b^9*c^4*d^6 + 2475*a^4*b^8*c^3*d^7 + 1485*a^5
*b^7*c^2*d^8 + 385*a^6*b^6*c*d^9 + 33*a^7*b^5*d^10)*x^16 + 3*(b^12*c^8*d^2
+ 32*a*b^11*c^7*d^3 + 308*a^2*b^10*c^6*d^4 + 1232*a^3*b^9*c^5*d^5 + 2310*a^
4*b^8*c^4*d^6 + 2112*a^5*b^7*c^3*d^7 + 924*a^6*b^6*c^2*d^8 + 176*a^7*b^5*c*
d^9 + 11*a^8*b^4*d^10)*x^15 + 5/7*(b^12*c^9*d + 54*a*b^11*c^8*d^2 + 792*a^2
*b^10*c^7*d^3 + 4620*a^3*b^9*c^6*d^4 + 12474*a^4*b^8*c^5*d^5 + 16632*a^5*b^
7*c^4*d^6 + 11088*a^6*b^6*c^3*d^7 + 3564*a^7*b^5*c^2*d^8 + 495*a^8*b^4*c*d^
9 + 22*a^9*b^3*d^10)*x^14 + 1/13*(b^12*c^10 + 120*a*b^11*c^9*d + 2970*a^2*b
^10*c^8*d^2 + 26400*a^3*b^9*c^7*d^3 + 103950*a^4*b^8*c^6*d^4 + 199584*a^5*b
^7*c^5*d^5 + 194040*a^6*b^6*c^4*d^6 + 95040*a^7*b^5*c^3*d^7 + 22275*a^8*b^4
*c^2*d^8 + 2200*a^9*b^3*c*d^9 + 66*a^10*b^2*d^10)*x^13 + (a*b^11*c^10 + 55*
a^2*b^10*c^9*d + 825*a^3*b^9*c^8*d^2 + 4950*a^4*b^8*c^7*d^3 + 13860*a^5*b^7
*c^6*d^4 + 19404*a^6*b^6*c^5*d^5 + 13860*a^7*b^5*c^4*d^6 + 4950*a^8*b^4*c^3
*d^7 + 825*a^9*b^3*c^2*d^8 + 55*a^10*b^2*c*d^9 + a^11*b*d^10)*x^12 + 1/11*(
66*a^2*b^10*c^10 + 2200*a^3*b^9*c^9*d + 22275*a^4*b^8*c^8*d^2 + 95040*a^5*b
^7*c^7*d^3 + 194040*a^6*b^6*c^6*d^4 + 199584*a^7*b^5*c^5*d^5 + 103950*a^8*b
^4*c^4*d^6 + 26400*a^9*b^3*c^3*d^7 + 2970*a^10*b^2*c^2*d^8 + 120*a^11*b*c*d
^9 + a^12*d^10)*x^11 + (22*a^3*b^9*c^10 + 495*a^4*b^8*c^9*d + 3564*a^5*b^7*
c^8*d^2 + 11088*a^6*b^6*c^7*d^3 + 16632*a^7*b^5*c^6*d^4 + 12474*a^8*b^4*c^5
*d^5 + 4620*a^9*b^3*c^4*d^6 + 792*a^10*b^2*c^3*d^7 + 54*a^11*b*c^2*d^8 + a^
12*c*d^9)*x^10 + 5*(11*a^4*b^8*c^10 + 176*a^5*b^7*c^9*d + 924*a^6*b^6*c^8*d
^2 + 2112*a^7*b^5*c^7*d^3 + 2310*a^8*b^4*c^6*d^4 + 1232*a^9*b^3*c^5*d^5 + 3
08*a^10*b^2*c^4*d^6 + 32*a^11*b*c^3*d^7 + a^12*c^2*d^8)*x^9 + 3*(33*a^5*b^7
*c^10 + 385*a^6*b^6*c^9*d + 1485*a^7*b^5*c^8*d^2 + 2475*a^8*b^4*c^7*d^3 + 1
925*a^9*b^3*c^6*d^4 + 693*a^10*b^2*c^5*d^5 + 105*a^11*b*c^4*d^6 + 5*a^12*c^
3*d^7)*x^8 + 3/7*(308*a^6*b^6*c^10 + 2640*a^7*b^5*c^9*d + 7425*a^8*b^4*c^8*
d^2 + 8800*a^9*b^3*c^7*d^3 + 4620*a^10*b^2*c^6*d^4 + 1008*a^11*b*c^5*d^5 +
70*a^12*c^4*d^6)*x^7 + 3*(44*a^7*b^5*c^10 + 275*a^8*b^4*c^9*d + 550*a^9*b^3
*c^8*d^2 + 440*a^10*b^2*c^7*d^3 + 140*a^11*b*c^6*d^4 + 14*a^12*c^5*d^5)*x^6
 + (99*a^8*b^4*c^10 + 440*a^9*b^3*c^9*d + 594*a^10*b^2*c^8*d^2 + 288*a^11*b
*c^7*d^3 + 42*a^12*c^6*d^4)*x^5 + 5*(11*a^9*b^3*c^10 + 33*a^10*b^2*c^9*d +
27*a^11*b*c^8*d^2 + 6*a^12*c^7*d^3)*x^4 + (22*a^10*b^2*c^10 + 40*a^11*b*c^9
```

```
*d + 15*a^12*c^8*d^2)*x^3 + (6*a^11*b*c^10 + 5*a^12*c^9*d)*x^2
```

**mupad [B]** time = 0.98, size = 1847, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^12*(c + d*x)^10,x)
```

```
[Out] x^12*(a*b^11*c^10 + a^11*b*d^10 + 55*a^2*b^10*c^9*d + 55*a^10*b^2*c*d^9 + 8
25*a^3*b^9*c^8*d^2 + 4950*a^4*b^8*c^7*d^3 + 13860*a^5*b^7*c^6*d^4 + 19404*a
^6*b^6*c^5*d^5 + 13860*a^7*b^5*c^4*d^6 + 4950*a^8*b^4*c^3*d^7 + 825*a^9*b^3
*c^2*d^8) + x^7*(132*a^6*b^6*c^10 + 30*a^12*c^4*d^6 + (7920*a^7*b^5*c^9*d)/
7 + 432*a^11*b*c^5*d^5 + (22275*a^8*b^4*c^8*d^2)/7 + (26400*a^9*b^3*c^7*d^3
)/7 + 1980*a^10*b^2*c^6*d^4) + x^17*((924*a^6*b^6*d^10)/17 + (210*b^12*c^6*
d^4)/17 + (3024*a*b^11*c^5*d^5)/17 + (7920*a^5*b^7*c*d^9)/17 + (13860*a^2*b
^10*c^4*d^6)/17 + (26400*a^3*b^9*c^3*d^7)/17 + (22275*a^4*b^8*c^2*d^8)/17)
+ x^5*(99*a^8*b^4*c^10 + 42*a^12*c^6*d^4 + 440*a^9*b^3*c^9*d + 288*a^11*b*c
^7*d^3 + 594*a^10*b^2*c^8*d^2) + x^19*((495*a^4*b^8*d^10)/19 + (210*b^12*c^
4*d^6)/19 + (1440*a*b^11*c^3*d^7)/19 + (2200*a^3*b^9*c*d^9)/19 + (2970*a^2*
b^10*c^2*d^8)/19) + x^8*(99*a^5*b^7*c^10 + 15*a^12*c^3*d^7 + 1155*a^6*b^6*c
^9*d + 315*a^11*b*c^4*d^6 + 4455*a^7*b^5*c^8*d^2 + 7425*a^8*b^4*c^7*d^3 + 5
775*a^9*b^3*c^6*d^4 + 2079*a^10*b^2*c^5*d^5) + x^16*((99*a^7*b^5*d^10)/2 +
(15*b^12*c^7*d^3)/2 + (315*a*b^11*c^6*d^4)/2 + (1155*a^6*b^6*c*d^9)/2 + (20
79*a^2*b^10*c^5*d^5)/2 + (5775*a^3*b^9*c^4*d^6)/2 + (7425*a^4*b^8*c^3*d^7)/
2 + (4455*a^5*b^7*c^2*d^8)/2) + x^11*((a^12*d^10)/11 + 6*a^2*b^10*c^10 + 20
0*a^3*b^9*c^9*d + 2025*a^4*b^8*c^8*d^2 + 8640*a^5*b^7*c^7*d^3 + 17640*a^6*b
^6*c^6*d^4 + 18144*a^7*b^5*c^5*d^5 + 9450*a^8*b^4*c^4*d^6 + 2400*a^9*b^3*c^
3*d^7 + 270*a^10*b^2*c^2*d^8 + (120*a^11*b*c*d^9)/11) + x^13*((b^12*c^10)/1
3 + (66*a^10*b^2*d^10)/13 + (2200*a^9*b^3*c*d^9)/13 + (2970*a^2*b^10*c^8*d^
2)/13 + (26400*a^3*b^9*c^7*d^3)/13 + (103950*a^4*b^8*c^6*d^4)/13 + (199584*
a^5*b^7*c^5*d^5)/13 + (194040*a^6*b^6*c^4*d^6)/13 + (95040*a^7*b^5*c^3*d^7)
/13 + (22275*a^8*b^4*c^2*d^8)/13 + (120*a*b^11*c^9*d)/13) + x^6*(132*a^7*b^
5*c^10 + 42*a^12*c^5*d^5 + 825*a^8*b^4*c^9*d + 420*a^11*b*c^6*d^4 + 1650*a^
9*b^3*c^8*d^2 + 1320*a^10*b^2*c^7*d^3) + x^18*(44*a^5*b^7*d^10 + 14*b^12*c^
5*d^5 + 140*a*b^11*c^4*d^6 + 275*a^4*b^8*c*d^9 + 440*a^2*b^10*c^3*d^7 + 550
*a^3*b^9*c^2*d^8) + x^9*(55*a^4*b^8*c^10 + 5*a^12*c^2*d^8 + 880*a^5*b^7*c^9
*d + 160*a^11*b*c^3*d^7 + 4620*a^6*b^6*c^8*d^2 + 10560*a^7*b^5*c^7*d^3 + 11
550*a^8*b^4*c^6*d^4 + 6160*a^9*b^3*c^5*d^5 + 1540*a^10*b^2*c^4*d^6) + x^15*
(33*a^8*b^4*d^10 + 3*b^12*c^8*d^2 + 96*a*b^11*c^7*d^3 + 528*a^7*b^5*c*d^9 +
 924*a^2*b^10*c^6*d^4 + 3696*a^3*b^9*c^5*d^5 + 6930*a^4*b^8*c^4*d^6 + 6336*
a^5*b^7*c^3*d^7 + 2772*a^6*b^6*c^2*d^8) + x^10*(a^12*c*d^9 + 22*a^3*b^9*c^1
0 + 495*a^4*b^8*c^9*d + 54*a^11*b*c^2*d^8 + 3564*a^5*b^7*c^8*d^2 + 11088*a^
6*b^6*c^7*d^3 + 16632*a^7*b^5*c^6*d^4 + 12474*a^8*b^4*c^5*d^5 + 4620*a^9*b^
3*c^4*d^6 + 792*a^10*b^2*c^3*d^7) + x^14*((5*b^12*c^9*d)/7 + (110*a^9*b^3*d
```

```
^10)/7 + (270*a*b^11*c^8*d^2)/7 + (2475*a^8*b^4*c*d^9)/7 + (3960*a^2*b^10*c
^7*d^3)/7 + 3300*a^3*b^9*c^6*d^4 + 8910*a^4*b^8*c^5*d^5 + 11880*a^5*b^7*c^4
*d^6 + 7920*a^6*b^6*c^3*d^7 + (17820*a^7*b^5*c^2*d^8)/7) + a^12*c^10*x + (b
^12*d^10*x^23)/23 + 5*a^9*c^7*x^4*(6*a^3*d^3 + 11*b^3*c^3 + 33*a*b^2*c^2*d
+ 27*a^2*b*c*d^2) + b^9*d^7*x^20*(11*a^3*d^3 + 6*b^3*c^3 + 27*a*b^2*c^2*d +
 33*a^2*b*c*d^2) + a^11*c^9*x^2*(5*a*d + 6*b*c) + (b^11*d^9*x^22*(6*a*d + 5
*b*c))/11 + a^10*c^8*x^3*(15*a^2*d^2 + 22*b^2*c^2 + 40*a*b*c*d) + (b^10*d^8
*x^21*(22*a^2*d^2 + 15*b^2*c^2 + 40*a*b*c*d))/7
```

**sympy [B]** time = 0.37, size = 2088, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**12*(d*x+c)**10,x)
```

```
[Out] a**12*c**10*x + b**12*d**10*x**23/23 + x**22*(6*a*b**11*d**10/11 + 5*b**12*
c*d**9/11) + x**21*(22*a**2*b**10*d**10/7 + 40*a*b**11*c*d**9/7 + 15*b**12*
c**2*d**8/7) + x**20*(11*a**3*b**9*d**10 + 33*a**2*b**10*c*d**9 + 27*a*b**1
1*c**2*d**8 + 6*b**12*c**3*d**7) + x**19*(495*a**4*b**8*d**10/19 + 2200*a**
3*b**9*c*d**9/19 + 2970*a**2*b**10*c**2*d**8/19 + 1440*a*b**11*c**3*d**7/19
 + 210*b**12*c**4*d**6/19) + x**18*(44*a**5*b**7*d**10 + 275*a**4*b**8*c*d*
*9 + 550*a**3*b**9*c**2*d**8 + 440*a**2*b**10*c**3*d**7 + 140*a*b**11*c**4*
d**6 + 14*b**12*c**5*d**5) + x**17*(924*a**6*b**6*d**10/17 + 7920*a**5*b**7
*c*d**9/17 + 22275*a**4*b**8*c**2*d**8/17 + 26400*a**3*b**9*c**3*d**7/17 +
13860*a**2*b**10*c**4*d**6/17 + 3024*a*b**11*c**5*d**5/17 + 210*b**12*c**6*
d**4/17) + x**16*(99*a**7*b**5*d**10/2 + 1155*a**6*b**6*c*d**9/2 + 4455*a**
5*b**7*c**2*d**8/2 + 7425*a**4*b**8*c**3*d**7/2 + 5775*a**3*b**9*c**4*d**6/
2 + 2079*a**2*b**10*c**5*d**5/2 + 315*a*b**11*c**6*d**4/2 + 15*b**12*c**7*d
**3/2) + x**15*(33*a**8*b**4*d**10 + 528*a**7*b**5*c*d**9 + 2772*a**6*b**6*
c**2*d**8 + 6336*a**5*b**7*c**3*d**7 + 6930*a**4*b**8*c**4*d**6 + 3696*a**3
*b**9*c**5*d**5 + 924*a**2*b**10*c**6*d**4 + 96*a*b**11*c**7*d**3 + 3*b**12
*c**8*d**2) + x**14*(110*a**9*b**3*d**10/7 + 2475*a**8*b**4*c*d**9/7 + 1782
0*a**7*b**5*c**2*d**8/7 + 7920*a**6*b**6*c**3*d**7 + 11880*a**5*b**7*c**4*d
**6 + 8910*a**4*b**8*c**5*d**5 + 3300*a**3*b**9*c**6*d**4 + 3960*a**2*b**10
*c**7*d**3/7 + 270*a*b**11*c**8*d**2/7 + 5*b**12*c**9*d/7) + x**13*(66*a**1
0*b**2*d**10/13 + 2200*a**9*b**3*c*d**9/13 + 22275*a**8*b**4*c**2*d**8/13 +
 95040*a**7*b**5*c**3*d**7/13 + 194040*a**6*b**6*c**4*d**6/13 + 199584*a**5
*b**7*c**5*d**5/13 + 103950*a**4*b**8*c**6*d**4/13 + 26400*a**3*b**9*c**7*d
**3/13 + 2970*a**2*b**10*c**8*d**2/13 + 120*a*b**11*c**9*d/13 + b**12*c**10
/13) + x**12*(a**11*b*d**10 + 55*a**10*b**2*c*d**9 + 825*a**9*b**3*c**2*d**
8 + 4950*a**8*b**4*c**3*d**7 + 13860*a**7*b**5*c**4*d**6 + 19404*a**6*b**6*
c**5*d**5 + 13860*a**5*b**7*c**6*d**4 + 4950*a**4*b**8*c**7*d**3 + 825*a**3
*b**9*c**8*d**2 + 55*a**2*b**10*c**9*d + a*b**11*c**10) + x**11*(a**12*d**1
0/11 + 120*a**11*b*c*d**9/11 + 270*a**10*b**2*c**2*d**8 + 2400*a**9*b**3*c*
```

```
*3*d**7 + 9450*a**8*b**4*c**4*d**6 + 18144*a**7*b**5*c**5*d**5 + 17640*a**6
*b**6*c**6*d**4 + 8640*a**5*b**7*c**7*d**3 + 2025*a**4*b**8*c**8*d**2 + 200
*a**3*b**9*c**9*d + 6*a**2*b**10*c**10) + x**10*(a**12*c*d**9 + 54*a**11*b*
c**2*d**8 + 792*a**10*b**2*c**3*d**7 + 4620*a**9*b**3*c**4*d**6 + 12474*a**
8*b**4*c**5*d**5 + 16632*a**7*b**5*c**6*d**4 + 11088*a**6*b**6*c**7*d**3 +
3564*a**5*b**7*c**8*d**2 + 495*a**4*b**8*c**9*d + 22*a**3*b**9*c**10) + x**
9*(5*a**12*c**2*d**8 + 160*a**11*b*c**3*d**7 + 1540*a**10*b**2*c**4*d**6 +
6160*a**9*b**3*c**5*d**5 + 11550*a**8*b**4*c**6*d**4 + 10560*a**7*b**5*c**7
*d**3 + 4620*a**6*b**6*c**8*d**2 + 880*a**5*b**7*c**9*d + 55*a**4*b**8*c**1
0) + x**8*(15*a**12*c**3*d**7 + 315*a**11*b*c**4*d**6 + 2079*a**10*b**2*c**
5*d**5 + 5775*a**9*b**3*c**6*d**4 + 7425*a**8*b**4*c**7*d**3 + 4455*a**7*b*
*5*c**8*d**2 + 1155*a**6*b**6*c**9*d + 99*a**5*b**7*c**10) + x**7*(30*a**12
*c**4*d**6 + 432*a**11*b*c**5*d**5 + 1980*a**10*b**2*c**6*d**4 + 26400*a**9
*b**3*c**7*d**3/7 + 22275*a**8*b**4*c**8*d**2/7 + 7920*a**7*b**5*c**9*d/7 +
 132*a**6*b**6*c**10) + x**6*(42*a**12*c**5*d**5 + 420*a**11*b*c**6*d**4 +
1320*a**10*b**2*c**7*d**3 + 1650*a**9*b**3*c**8*d**2 + 825*a**8*b**4*c**9*d
 + 132*a**7*b**5*c**10) + x**5*(42*a**12*c**6*d**4 + 288*a**11*b*c**7*d**3
+ 594*a**10*b**2*c**8*d**2 + 440*a**9*b**3*c**9*d + 99*a**8*b**4*c**10) + x
**4*(30*a**12*c**7*d**3 + 135*a**11*b*c**8*d**2 + 165*a**10*b**2*c**9*d + 5
5*a**9*b**3*c**10) + x**3*(15*a**12*c**8*d**2 + 40*a**11*b*c**9*d + 22*a**1
0*b**2*c**10) + x**2*(5*a**12*c**9*d + 6*a**11*b*c**10)
```

## 3.1194 $\int (a+bx)^{11}(c+dx)^{10}\,dx$

**Optimal**. Leaf size=279

$$\frac{10d^9(a+bx)^{21}(bc-ad)}{21b^{11}}+\frac{9d^8(a+bx)^{20}(bc-ad)^2}{4b^{11}}+\frac{120d^7(a+bx)^{19}(bc-ad)^3}{19b^{11}}+\frac{35d^6(a+bx)^{18}(bc-ad)^4}{3b^{11}}+\frac{252d^5(\cdots}{}$$

**Rubi [A]** time = 1.28, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{10d^9(a+bx)^{21}(bc-ad)}{21b^{11}}+\frac{9d^8(a+bx)^{20}(bc-ad)^2}{4b^{11}}+\frac{120d^7(a+bx)^{19}(bc-ad)^3}{19b^{11}}+\frac{35d^6(a+bx)^{18}(bc-ad)^4}{3b^{11}}+\frac{252d^5(a+bx)^{17}(bc-ad)^5}{17b^{11}}+\frac{105d^4(a+bx)^{16}(bc-ad)^6}{8b^{11}}+\frac{8d^3(a+bx)^{15}(bc-ad)^7}{b^{11}}+\frac{45d^2(a+bx)^{14}(bc-ad)^8}{14b^{11}}+\frac{10d(a+bx)^{13}(bc-ad)^9}{13b^{11}}+\frac{(a+bx)^{12}(bc-ad)^{10}}{12b^{11}}+\frac{d^{10}(a+bx)^{22}}{22b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^12)/(12*b^11) + (10*d*(b*c - a*d)^9*(a + b*x)^13)
/(13*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^14)/(14*b^11) + (8*d^3*(b*c -
a*d)^7*(a + b*x)^15)/b^11 + (105*d^4*(b*c - a*d)^6*(a + b*x)^16)/(8*b^11) +
 (252*d^5*(b*c - a*d)^5*(a + b*x)^17)/(17*b^11) + (35*d^6*(b*c - a*d)^4*(a
+ b*x)^18)/(3*b^11) + (120*d^7*(b*c - a*d)^3*(a + b*x)^19)/(19*b^11) + (9*d
^8*(b*c - a*d)^2*(a + b*x)^20)/(4*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^21)
/(21*b^11) + (d^10*(a + b*x)^22)/(22*b^11)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^{11}(c+dx)^{10}\,dx=\int \left(\frac{(bc-ad)^{10}(a+bx)^{11}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{12}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)^{13}}{b^{10}}+\cdots\right.$$

$$=\frac{(bc-ad)^{10}(a+bx)^{12}}{12b^{11}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{13b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{14}}{14b^{11}}+\frac{8d^3(b\cdots}{}$$

**Mathematica [B]** time = 0.23, size = 1702, normalized size = 6.10

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] a^11*c^10*x + (a^10*c^9*(11*b*c + 10*a*d)*x^2)/2 + (5*a^9*c^8*(11*b^2*c^2 +
 22*a*b*c*d + 9*a^2*d^2)*x^3)/3 + (5*a^8*c^7*(33*b^3*c^3 + 110*a*b^2*c^2*d
+ 99*a^2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + 3*a^7*c^6*(22*b^4*c^4 + 110*a*b^3*c
^3*d + 165*a^2*b^2*c^2*d^2 + 88*a^3*b*c*d^3 + 14*a^4*d^4)*x^5 + (a^6*c^5*(1
54*b^5*c^5 + 1100*a*b^4*c^4*d + 2475*a^2*b^3*c^3*d^2 + 2200*a^3*b^2*c^2*d^3
 + 770*a^4*b*c*d^4 + 84*a^5*d^5)*x^6)/2 + (6*a^5*c^4*(77*b^6*c^6 + 770*a*b^
5*c^5*d + 2475*a^2*b^4*c^4*d^2 + 3300*a^3*b^3*c^3*d^3 + 1925*a^4*b^2*c^2*d^
4 + 462*a^5*b*c*d^5 + 35*a^6*d^6)*x^7)/7 + (15*a^4*c^3*(11*b^7*c^7 + 154*a*
b^6*c^6*d + 693*a^2*b^5*c^5*d^2 + 1320*a^3*b^4*c^4*d^3 + 1155*a^4*b^3*c^3*d
^4 + 462*a^5*b^2*c^2*d^5 + 77*a^6*b*c*d^6 + 4*a^7*d^7)*x^8)/4 + (5*a^3*c^2*
(11*b^8*c^8 + 220*a*b^7*c^7*d + 1386*a^2*b^6*c^6*d^2 + 3696*a^3*b^5*c^5*d^3
 + 4620*a^4*b^4*c^4*d^4 + 2772*a^5*b^3*c^3*d^5 + 770*a^6*b^2*c^2*d^6 + 88*a
^7*b*c*d^7 + 3*a^8*d^8)*x^9)/3 + (a^2*c*(11*b^9*c^9 + 330*a*b^8*c^8*d + 297
0*a^2*b^7*c^7*d^2 + 11088*a^3*b^6*c^6*d^3 + 19404*a^4*b^5*c^5*d^4 + 16632*a
^5*b^4*c^4*d^5 + 6930*a^6*b^3*c^3*d^6 + 1320*a^7*b^2*c^2*d^7 + 99*a^8*b*c*d
^8 + 2*a^9*d^9)*x^10)/2 + (a*(11*b^10*c^10 + 550*a*b^9*c^9*d + 7425*a^2*b^8
*c^8*d^2 + 39600*a^3*b^7*c^7*d^3 + 97020*a^4*b^6*c^6*d^4 + 116424*a^5*b^5*c
^5*d^5 + 69300*a^6*b^4*c^4*d^6 + 19800*a^7*b^3*c^3*d^7 + 2475*a^8*b^2*c^2*d
^8 + 110*a^9*b*c*d^9 + a^10*d^10)*x^11)/11 + (b*(b^10*c^10 + 110*a*b^9*c^9*
d + 2475*a^2*b^8*c^8*d^2 + 19800*a^3*b^7*c^7*d^3 + 69300*a^4*b^6*c^6*d^4 +
116424*a^5*b^5*c^5*d^5 + 97020*a^6*b^4*c^4*d^6 + 39600*a^7*b^3*c^3*d^7 + 74
25*a^8*b^2*c^2*d^8 + 550*a^9*b*c*d^9 + 11*a^10*d^10)*x^12)/12 + (5*b^2*d*(2
*b^9*c^9 + 99*a*b^8*c^8*d + 1320*a^2*b^7*c^7*d^2 + 6930*a^3*b^6*c^6*d^3 + 1
6632*a^4*b^5*c^5*d^4 + 19404*a^5*b^4*c^4*d^5 + 11088*a^6*b^3*c^3*d^6 + 2970
*a^7*b^2*c^2*d^7 + 330*a^8*b*c*d^8 + 11*a^9*d^9)*x^13)/13 + (15*b^3*d^2*(3*
b^8*c^8 + 88*a*b^7*c^7*d + 770*a^2*b^6*c^6*d^2 + 2772*a^3*b^5*c^5*d^3 + 462
0*a^4*b^4*c^4*d^4 + 3696*a^5*b^3*c^3*d^5 + 1386*a^6*b^2*c^2*d^6 + 220*a^7*b
*c*d^7 + 11*a^8*d^8)*x^14)/14 + 2*b^4*d^3*(4*b^7*c^7 + 77*a*b^6*c^6*d + 462
*a^2*b^5*c^5*d^2 + 1155*a^3*b^4*c^4*d^3 + 1320*a^4*b^3*c^3*d^4 + 693*a^5*b^
2*c^2*d^5 + 154*a^6*b*c*d^6 + 11*a^7*d^7)*x^15 + (3*b^5*d^4*(35*b^6*c^6 + 4
62*a*b^5*c^5*d + 1925*a^2*b^4*c^4*d^2 + 3300*a^3*b^3*c^3*d^3 + 2475*a^4*b^2
*c^2*d^4 + 770*a^5*b*c*d^5 + 77*a^6*d^6)*x^16)/8 + (3*b^6*d^5*(84*b^5*c^5 +
 770*a*b^4*c^4*d + 2200*a^2*b^3*c^3*d^2 + 2475*a^3*b^2*c^2*d^3 + 1100*a^4*b
*c*d^4 + 154*a^5*d^5)*x^17)/17 + (5*b^7*d^6*(14*b^4*c^4 + 88*a*b^3*c^3*d +
165*a^2*b^2*c^2*d^2 + 110*a^3*b*c*d^3 + 22*a^4*d^4)*x^18)/6 + (5*b^8*d^7*(2
4*b^3*c^3 + 99*a*b^2*c^2*d + 110*a^2*b*c*d^2 + 33*a^3*d^3)*x^19)/19 + (b^9*
d^8*(9*b^2*c^2 + 22*a*b*c*d + 11*a^2*d^2)*x^20)/4 + (b^10*d^9*(10*b*c + 11*
a*d)*x^21)/21 + (b^11*d^10*x^22)/22
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{11}(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^11*(c + d*x)^10, x]
```

**fricas [B]** time = 1.11, size = 2010, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*d^10*b^11 + 10/21*x^21*d^9*c*b^11 + 11/21*x^21*d^10*b^10*a + 9/4*
x^20*d^8*c^2*b^11 + 11/2*x^20*d^9*c*b^10*a + 11/4*x^20*d^10*b^9*a^2 + 120/1
9*x^19*d^7*c^3*b^11 + 495/19*x^19*d^8*c^2*b^10*a + 550/19*x^19*d^9*c*b^9*a^
2 + 165/19*x^19*d^10*b^8*a^3 + 35/3*x^18*d^6*c^4*b^11 + 220/3*x^18*d^7*c^3*
b^10*a + 275/2*x^18*d^8*c^2*b^9*a^2 + 275/3*x^18*d^9*c*b^8*a^3 + 55/3*x^18*
d^10*b^7*a^4 + 252/17*x^17*d^5*c^5*b^11 + 2310/17*x^17*d^6*c^4*b^10*a + 660
0/17*x^17*d^7*c^3*b^9*a^2 + 7425/17*x^17*d^8*c^2*b^8*a^3 + 3300/17*x^17*d^9
*c*b^7*a^4 + 462/17*x^17*d^10*b^6*a^5 + 105/8*x^16*d^4*c^6*b^11 + 693/4*x^1
6*d^5*c^5*b^10*a + 5775/8*x^16*d^6*c^4*b^9*a^2 + 2475/2*x^16*d^7*c^3*b^8*a^
3 + 7425/8*x^16*d^8*c^2*b^7*a^4 + 1155/4*x^16*d^9*c*b^6*a^5 + 231/8*x^16*d^
10*b^5*a^6 + 8*x^15*d^3*c^7*b^11 + 154*x^15*d^4*c^6*b^10*a + 924*x^15*d^5*c
^5*b^9*a^2 + 2310*x^15*d^6*c^4*b^8*a^3 + 2640*x^15*d^7*c^3*b^7*a^4 + 1386*x
^15*d^8*c^2*b^6*a^5 + 308*x^15*d^9*c*b^5*a^6 + 22*x^15*d^10*b^4*a^7 + 45/14
*x^14*d^2*c^8*b^11 + 660/7*x^14*d^3*c^7*b^10*a + 825*x^14*d^4*c^6*b^9*a^2 +
 2970*x^14*d^5*c^5*b^8*a^3 + 4950*x^14*d^6*c^4*b^7*a^4 + 3960*x^14*d^7*c^3*
b^6*a^5 + 1485*x^14*d^8*c^2*b^5*a^6 + 1650/7*x^14*d^9*c*b^4*a^7 + 165/14*x^
14*d^10*b^3*a^8 + 10/13*x^13*d*c^9*b^11 + 495/13*x^13*d^2*c^8*b^10*a + 6600
/13*x^13*d^3*c^7*b^9*a^2 + 34650/13*x^13*d^4*c^6*b^8*a^3 + 83160/13*x^13*d^
5*c^5*b^7*a^4 + 97020/13*x^13*d^6*c^4*b^6*a^5 + 55440/13*x^13*d^7*c^3*b^5*a
^6 + 14850/13*x^13*d^8*c^2*b^4*a^7 + 1650/13*x^13*d^9*c*b^3*a^8 + 55/13*x^1
3*d^10*b^2*a^9 + 1/12*x^12*c^10*b^11 + 55/6*x^12*d*c^9*b^10*a + 825/4*x^12*
d^2*c^8*b^9*a^2 + 1650*x^12*d^3*c^7*b^8*a^3 + 5775*x^12*d^4*c^6*b^7*a^4 + 9
702*x^12*d^5*c^5*b^6*a^5 + 8085*x^12*d^6*c^4*b^5*a^6 + 3300*x^12*d^7*c^3*b^
4*a^7 + 2475/4*x^12*d^8*c^2*b^3*a^8 + 275/6*x^12*d^9*c*b^2*a^9 + 11/12*x^12
*d^10*b*a^10 + x^11*c^10*b^10*a + 50*x^11*d*c^9*b^9*a^2 + 675*x^11*d^2*c^8*
b^8*a^3 + 3600*x^11*d^3*c^7*b^7*a^4 + 8820*x^11*d^4*c^6*b^6*a^5 + 10584*x^1
1*d^5*c^5*b^5*a^6 + 6300*x^11*d^6*c^4*b^4*a^7 + 1800*x^11*d^7*c^3*b^3*a^8 +
 225*x^11*d^8*c^2*b^2*a^9 + 10*x^11*d^9*c*b*a^10 + 1/11*x^11*d^10*a^11 + 11
```

```
/2*x^10*c^10*b^9*a^2 + 165*x^10*d*c^9*b^8*a^3 + 1485*x^10*d^2*c^8*b^7*a^4 +
 5544*x^10*d^3*c^7*b^6*a^5 + 9702*x^10*d^4*c^6*b^5*a^6 + 8316*x^10*d^5*c^5*
b^4*a^7 + 3465*x^10*d^6*c^4*b^3*a^8 + 660*x^10*d^7*c^3*b^2*a^9 + 99/2*x^10*
d^8*c^2*b*a^10 + x^10*d^9*c*a^11 + 55/3*x^9*c^10*b^8*a^3 + 1100/3*x^9*d*c^9
*b^7*a^4 + 2310*x^9*d^2*c^8*b^6*a^5 + 6160*x^9*d^3*c^7*b^5*a^6 + 7700*x^9*d
^4*c^6*b^4*a^7 + 4620*x^9*d^5*c^5*b^3*a^8 + 3850/3*x^9*d^6*c^4*b^2*a^9 + 44
0/3*x^9*d^7*c^3*b*a^10 + 5*x^9*d^8*c^2*a^11 + 165/4*x^8*c^10*b^7*a^4 + 1155
/2*x^8*d*c^9*b^6*a^5 + 10395/4*x^8*d^2*c^8*b^5*a^6 + 4950*x^8*d^3*c^7*b^4*a
^7 + 17325/4*x^8*d^4*c^6*b^3*a^8 + 3465/2*x^8*d^5*c^5*b^2*a^9 + 1155/4*x^8*
d^6*c^4*b*a^10 + 15*x^8*d^7*c^3*a^11 + 66*x^7*c^10*b^6*a^5 + 660*x^7*d*c^9*
b^5*a^6 + 14850/7*x^7*d^2*c^8*b^4*a^7 + 19800/7*x^7*d^3*c^7*b^3*a^8 + 1650*
x^7*d^4*c^6*b^2*a^9 + 396*x^7*d^5*c^5*b*a^10 + 30*x^7*d^6*c^4*a^11 + 77*x^6
*c^10*b^5*a^6 + 550*x^6*d*c^9*b^4*a^7 + 2475/2*x^6*d^2*c^8*b^3*a^8 + 1100*x
^6*d^3*c^7*b^2*a^9 + 385*x^6*d^4*c^6*b*a^10 + 42*x^6*d^5*c^5*a^11 + 66*x^5*
c^10*b^4*a^7 + 330*x^5*d*c^9*b^3*a^8 + 495*x^5*d^2*c^8*b^2*a^9 + 264*x^5*d^
3*c^7*b*a^10 + 42*x^5*d^4*c^6*a^11 + 165/4*x^4*c^10*b^3*a^8 + 275/2*x^4*d*c
^9*b^2*a^9 + 495/4*x^4*d^2*c^8*b*a^10 + 30*x^4*d^3*c^7*a^11 + 55/3*x^3*c^10
*b^2*a^9 + 110/3*x^3*d*c^9*b*a^10 + 15*x^3*d^2*c^8*a^11 + 11/2*x^2*c^10*b*a
^10 + 5*x^2*d*c^9*a^11 + x*c^10*a^11
```

**giac [B]** time = 1.35, size = 2010, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/22*b^11*d^10*x^22 + 10/21*b^11*c*d^9*x^21 + 11/21*a*b^10*d^10*x^21 + 9/4*
b^11*c^2*d^8*x^20 + 11/2*a*b^10*c*d^9*x^20 + 11/4*a^2*b^9*d^10*x^20 + 120/1
9*b^11*c^3*d^7*x^19 + 495/19*a*b^10*c^2*d^8*x^19 + 550/19*a^2*b^9*c*d^9*x^1
9 + 165/19*a^3*b^8*d^10*x^19 + 35/3*b^11*c^4*d^6*x^18 + 220/3*a*b^10*c^3*d^
7*x^18 + 275/2*a^2*b^9*c^2*d^8*x^18 + 275/3*a^3*b^8*c*d^9*x^18 + 55/3*a^4*b
^7*d^10*x^18 + 252/17*b^11*c^5*d^5*x^17 + 2310/17*a*b^10*c^4*d^6*x^17 + 660
0/17*a^2*b^9*c^3*d^7*x^17 + 7425/17*a^3*b^8*c^2*d^8*x^17 + 3300/17*a^4*b^7*
c*d^9*x^17 + 462/17*a^5*b^6*d^10*x^17 + 105/8*b^11*c^6*d^4*x^16 + 693/4*a*b
^10*c^5*d^5*x^16 + 5775/8*a^2*b^9*c^4*d^6*x^16 + 2475/2*a^3*b^8*c^3*d^7*x^1
6 + 7425/8*a^4*b^7*c^2*d^8*x^16 + 1155/4*a^5*b^6*c*d^9*x^16 + 231/8*a^6*b^5
*d^10*x^16 + 8*b^11*c^7*d^3*x^15 + 154*a*b^10*c^6*d^4*x^15 + 924*a^2*b^9*c^
5*d^5*x^15 + 2310*a^3*b^8*c^4*d^6*x^15 + 2640*a^4*b^7*c^3*d^7*x^15 + 1386*a
^5*b^6*c^2*d^8*x^15 + 308*a^6*b^5*c*d^9*x^15 + 22*a^7*b^4*d^10*x^15 + 45/14
*b^11*c^8*d^2*x^14 + 660/7*a*b^10*c^7*d^3*x^14 + 825*a^2*b^9*c^6*d^4*x^14 +
 2970*a^3*b^8*c^5*d^5*x^14 + 4950*a^4*b^7*c^4*d^6*x^14 + 3960*a^5*b^6*c^3*d
^7*x^14 + 1485*a^6*b^5*c^2*d^8*x^14 + 1650/7*a^7*b^4*c*d^9*x^14 + 165/14*a^
8*b^3*d^10*x^14 + 10/13*b^11*c^9*d*x^13 + 495/13*a*b^10*c^8*d^2*x^13 + 6600
/13*a^2*b^9*c^7*d^3*x^13 + 34650/13*a^3*b^8*c^6*d^4*x^13 + 83160/13*a^4*b^7
```

```
*c^5*d^5*x^13 + 97020/13*a^5*b^6*c^4*d^6*x^13 + 55440/13*a^6*b^5*c^3*d^7*x^
13 + 14850/13*a^7*b^4*c^2*d^8*x^13 + 1650/13*a^8*b^3*c*d^9*x^13 + 55/13*a^9
*b^2*d^10*x^13 + 1/12*b^11*c^10*x^12 + 55/6*a*b^10*c^9*d*x^12 + 825/4*a^2*b
^9*c^8*d^2*x^12 + 1650*a^3*b^8*c^7*d^3*x^12 + 5775*a^4*b^7*c^6*d^4*x^12 + 9
702*a^5*b^6*c^5*d^5*x^12 + 8085*a^6*b^5*c^4*d^6*x^12 + 3300*a^7*b^4*c^3*d^7
*x^12 + 2475/4*a^8*b^3*c^2*d^8*x^12 + 275/6*a^9*b^2*c*d^9*x^12 + 11/12*a^10
*b*d^10*x^12 + a*b^10*c^10*x^11 + 50*a^2*b^9*c^9*d*x^11 + 675*a^3*b^8*c^8*d
^2*x^11 + 3600*a^4*b^7*c^7*d^3*x^11 + 8820*a^5*b^6*c^6*d^4*x^11 + 10584*a^6
*b^5*c^5*d^5*x^11 + 6300*a^7*b^4*c^4*d^6*x^11 + 1800*a^8*b^3*c^3*d^7*x^11 +
 225*a^9*b^2*c^2*d^8*x^11 + 10*a^10*b*c*d^9*x^11 + 1/11*a^11*d^10*x^11 + 11
/2*a^2*b^9*c^10*x^10 + 165*a^3*b^8*c^9*d*x^10 + 1485*a^4*b^7*c^8*d^2*x^10 +
 5544*a^5*b^6*c^7*d^3*x^10 + 9702*a^6*b^5*c^6*d^4*x^10 + 8316*a^7*b^4*c^5*d
^5*x^10 + 3465*a^8*b^3*c^4*d^6*x^10 + 660*a^9*b^2*c^3*d^7*x^10 + 99/2*a^10*
b*c^2*d^8*x^10 + a^11*c*d^9*x^10 + 55/3*a^3*b^8*c^10*x^9 + 1100/3*a^4*b^7*c
^9*d*x^9 + 2310*a^5*b^6*c^8*d^2*x^9 + 6160*a^6*b^5*c^7*d^3*x^9 + 7700*a^7*b
^4*c^6*d^4*x^9 + 4620*a^8*b^3*c^5*d^5*x^9 + 3850/3*a^9*b^2*c^4*d^6*x^9 + 44
0/3*a^10*b*c^3*d^7*x^9 + 5*a^11*c^2*d^8*x^9 + 165/4*a^4*b^7*c^10*x^8 + 1155
/2*a^5*b^6*c^9*d*x^8 + 10395/4*a^6*b^5*c^8*d^2*x^8 + 4950*a^7*b^4*c^7*d^3*x
^8 + 17325/4*a^8*b^3*c^6*d^4*x^8 + 3465/2*a^9*b^2*c^5*d^5*x^8 + 1155/4*a^10
*b*c^4*d^6*x^8 + 15*a^11*c^3*d^7*x^8 + 66*a^5*b^6*c^10*x^7 + 660*a^6*b^5*c^
9*d*x^7 + 14850/7*a^7*b^4*c^8*d^2*x^7 + 19800/7*a^8*b^3*c^7*d^3*x^7 + 1650*
a^9*b^2*c^6*d^4*x^7 + 396*a^10*b*c^5*d^5*x^7 + 30*a^11*c^4*d^6*x^7 + 77*a^6
*b^5*c^10*x^6 + 550*a^7*b^4*c^9*d*x^6 + 2475/2*a^8*b^3*c^8*d^2*x^6 + 1100*a
^9*b^2*c^7*d^3*x^6 + 385*a^10*b*c^6*d^4*x^6 + 42*a^11*c^5*d^5*x^6 + 66*a^7*
b^4*c^10*x^5 + 330*a^8*b^3*c^9*d*x^5 + 495*a^9*b^2*c^8*d^2*x^5 + 264*a^10*b
*c^7*d^3*x^5 + 42*a^11*c^6*d^4*x^5 + 165/4*a^8*b^3*c^10*x^4 + 275/2*a^9*b^2
*c^9*d*x^4 + 495/4*a^10*b*c^8*d^2*x^4 + 30*a^11*c^7*d^3*x^4 + 55/3*a^9*b^2*
c^10*x^3 + 110/3*a^10*b*c^9*d*x^3 + 15*a^11*c^8*d^2*x^3 + 11/2*a^10*b*c^10*
x^2 + 5*a^11*c^9*d*x^2 + a^11*c^10*x
```

**maple [B]** time = 0.00, size = 1741, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^11*(d*x+c)^10,x)
```

```
[Out] 1/22*b^11*d^10*x^22+1/21*(11*a*b^10*d^10+10*b^11*c*d^9)*x^21+1/20*(55*a^2*b
^9*d^10+110*a*b^10*c*d^9+45*b^11*c^2*d^8)*x^20+1/19*(165*a^3*b^8*d^10+550*a
^2*b^9*c*d^9+495*a*b^10*c^2*d^8+120*b^11*c^3*d^7)*x^19+1/18*(330*a^4*b^7*d^
10+1650*a^3*b^8*c*d^9+2475*a^2*b^9*c^2*d^8+1320*a*b^10*c^3*d^7+210*b^11*c^4
*d^6)*x^18+1/17*(462*a^5*b^6*d^10+3300*a^4*b^7*c*d^9+7425*a^3*b^8*c^2*d^8+6
600*a^2*b^9*c^3*d^7+2310*a*b^10*c^4*d^6+252*b^11*c^5*d^5)*x^17+1/16*(462*a^
6*b^5*d^10+4620*a^5*b^6*c*d^9+14850*a^4*b^7*c^2*d^8+19800*a^3*b^8*c^3*d^7+1
1550*a^2*b^9*c^4*d^6+2772*a*b^10*c^5*d^5+210*b^11*c^6*d^4)*x^16+1/15*(330*a
```

```
^7*b^4*d^10+4620*a^6*b^5*c*d^9+20790*a^5*b^6*c^2*d^8+39600*a^4*b^7*c^3*d^7+
34650*a^3*b^8*c^4*d^6+13860*a^2*b^9*c^5*d^5+2310*a*b^10*c^6*d^4+120*b^11*c^
7*d^3)*x^15+1/14*(165*a^8*b^3*d^10+3300*a^7*b^4*c*d^9+20790*a^6*b^5*c^2*d^8
+55440*a^5*b^6*c^3*d^7+69300*a^4*b^7*c^4*d^6+41580*a^3*b^8*c^5*d^5+11550*a^
2*b^9*c^6*d^4+1320*a*b^10*c^7*d^3+45*b^11*c^8*d^2)*x^14+1/13*(55*a^9*b^2*d^
10+1650*a^8*b^3*c*d^9+14850*a^7*b^4*c^2*d^8+55440*a^6*b^5*c^3*d^7+97020*a^5
*b^6*c^4*d^6+83160*a^4*b^7*c^5*d^5+34650*a^3*b^8*c^6*d^4+6600*a^2*b^9*c^7*d
^3+495*a*b^10*c^8*d^2+10*b^11*c^9*d)*x^13+1/12*(11*a^10*b*d^10+550*a^9*b^2*
c*d^9+7425*a^8*b^3*c^2*d^8+39600*a^7*b^4*c^3*d^7+97020*a^6*b^5*c^4*d^6+1164
24*a^5*b^6*c^5*d^5+69300*a^4*b^7*c^6*d^4+19800*a^3*b^8*c^7*d^3+2475*a^2*b^9
*c^8*d^2+110*a*b^10*c^9*d+b^11*c^10)*x^12+1/11*(a^11*d^10+110*a^10*b*c*d^9+
2475*a^9*b^2*c^2*d^8+19800*a^8*b^3*c^3*d^7+69300*a^7*b^4*c^4*d^6+116424*a^6
*b^5*c^5*d^5+97020*a^5*b^6*c^6*d^4+39600*a^4*b^7*c^7*d^3+7425*a^3*b^8*c^8*d
^2+550*a^2*b^9*c^9*d+11*a*b^10*c^10)*x^11+1/10*(10*a^11*c*d^9+495*a^10*b*c^
2*d^8+6600*a^9*b^2*c^3*d^7+34650*a^8*b^3*c^4*d^6+83160*a^7*b^4*c^5*d^5+9702
0*a^6*b^5*c^6*d^4+55440*a^5*b^6*c^7*d^3+14850*a^4*b^7*c^8*d^2+1650*a^3*b^8*
c^9*d+55*a^2*b^9*c^10)*x^10+1/9*(45*a^11*c^2*d^8+1320*a^10*b*c^3*d^7+11550*
a^9*b^2*c^4*d^6+41580*a^8*b^3*c^5*d^5+69300*a^7*b^4*c^6*d^4+55440*a^6*b^5*c
^7*d^3+20790*a^5*b^6*c^8*d^2+3300*a^4*b^7*c^9*d+165*a^3*b^8*c^10)*x^9+1/8*(
120*a^11*c^3*d^7+2310*a^10*b*c^4*d^6+13860*a^9*b^2*c^5*d^5+34650*a^8*b^3*c^
6*d^4+39600*a^7*b^4*c^7*d^3+20790*a^6*b^5*c^8*d^2+4620*a^5*b^6*c^9*d+330*a^
4*b^7*c^10)*x^8+1/7*(210*a^11*c^4*d^6+2772*a^10*b*c^5*d^5+11550*a^9*b^2*c^6
*d^4+19800*a^8*b^3*c^7*d^3+14850*a^7*b^4*c^8*d^2+4620*a^6*b^5*c^9*d+462*a^5
*b^6*c^10)*x^7+1/6*(252*a^11*c^5*d^5+2310*a^10*b*c^6*d^4+6600*a^9*b^2*c^7*d
^3+7425*a^8*b^3*c^8*d^2+3300*a^7*b^4*c^9*d+462*a^6*b^5*c^10)*x^6+1/5*(210*a
^11*c^6*d^4+1320*a^10*b*c^7*d^3+2475*a^9*b^2*c^8*d^2+1650*a^8*b^3*c^9*d+330
*a^7*b^4*c^10)*x^5+1/4*(120*a^11*c^7*d^3+495*a^10*b*c^8*d^2+550*a^9*b^2*c^9
*d+165*a^8*b^3*c^10)*x^4+1/3*(45*a^11*c^8*d^2+110*a^10*b*c^9*d+55*a^9*b^2*c
^10)*x^3+1/2*(10*a^11*c^9*d+11*a^10*b*c^10)*x^2+a^11*c^10*x
```

**maxima [B]** time = 1.58, size = 1740, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/22*b^11*d^10*x^22 + a^11*c^10*x + 1/21*(10*b^11*c*d^9 + 11*a*b^10*d^10)*x
^21 + 1/4*(9*b^11*c^2*d^8 + 22*a*b^10*c*d^9 + 11*a^2*b^9*d^10)*x^20 + 5/19*
(24*b^11*c^3*d^7 + 99*a*b^10*c^2*d^8 + 110*a^2*b^9*c*d^9 + 33*a^3*b^8*d^10)
*x^19 + 5/6*(14*b^11*c^4*d^6 + 88*a*b^10*c^3*d^7 + 165*a^2*b^9*c^2*d^8 + 11
0*a^3*b^8*c*d^9 + 22*a^4*b^7*d^10)*x^18 + 3/17*(84*b^11*c^5*d^5 + 770*a*b^1
0*c^4*d^6 + 2200*a^2*b^9*c^3*d^7 + 2475*a^3*b^8*c^2*d^8 + 1100*a^4*b^7*c*d^
9 + 154*a^5*b^6*d^10)*x^17 + 3/8*(35*b^11*c^6*d^4 + 462*a*b^10*c^5*d^5 + 19
25*a^2*b^9*c^4*d^6 + 3300*a^3*b^8*c^3*d^7 + 2475*a^4*b^7*c^2*d^8 + 770*a^5*
b^6*c*d^9 + 77*a^6*b^5*d^10)*x^16 + 2*(4*b^11*c^7*d^3 + 77*a*b^10*c^6*d^4 +
```

```
 462*a^2*b^9*c^5*d^5 + 1155*a^3*b^8*c^4*d^6 + 1320*a^4*b^7*c^3*d^7 + 693*a^
5*b^6*c^2*d^8 + 154*a^6*b^5*c*d^9 + 11*a^7*b^4*d^10)*x^15 + 15/14*(3*b^11*c
^8*d^2 + 88*a*b^10*c^7*d^3 + 770*a^2*b^9*c^6*d^4 + 2772*a^3*b^8*c^5*d^5 + 4
620*a^4*b^7*c^4*d^6 + 3696*a^5*b^6*c^3*d^7 + 1386*a^6*b^5*c^2*d^8 + 220*a^7
*b^4*c*d^9 + 11*a^8*b^3*d^10)*x^14 + 5/13*(2*b^11*c^9*d + 99*a*b^10*c^8*d^2
 + 1320*a^2*b^9*c^7*d^3 + 6930*a^3*b^8*c^6*d^4 + 16632*a^4*b^7*c^5*d^5 + 19
404*a^5*b^6*c^4*d^6 + 11088*a^6*b^5*c^3*d^7 + 2970*a^7*b^4*c^2*d^8 + 330*a^
8*b^3*c*d^9 + 11*a^9*b^2*d^10)*x^13 + 1/12*(b^11*c^10 + 110*a*b^10*c^9*d +
2475*a^2*b^9*c^8*d^2 + 19800*a^3*b^8*c^7*d^3 + 69300*a^4*b^7*c^6*d^4 + 1164
24*a^5*b^6*c^5*d^5 + 97020*a^6*b^5*c^4*d^6 + 39600*a^7*b^4*c^3*d^7 + 7425*a
^8*b^3*c^2*d^8 + 550*a^9*b^2*c*d^9 + 11*a^10*b*d^10)*x^12 + 1/11*(11*a*b^10
*c^10 + 550*a^2*b^9*c^9*d + 7425*a^3*b^8*c^8*d^2 + 39600*a^4*b^7*c^7*d^3 +
97020*a^5*b^6*c^6*d^4 + 116424*a^6*b^5*c^5*d^5 + 69300*a^7*b^4*c^4*d^6 + 19
800*a^8*b^3*c^3*d^7 + 2475*a^9*b^2*c^2*d^8 + 110*a^10*b*c*d^9 + a^11*d^10)*
x^11 + 1/2*(11*a^2*b^9*c^10 + 330*a^3*b^8*c^9*d + 2970*a^4*b^7*c^8*d^2 + 11
088*a^5*b^6*c^7*d^3 + 19404*a^6*b^5*c^6*d^4 + 16632*a^7*b^4*c^5*d^5 + 6930*
a^8*b^3*c^4*d^6 + 1320*a^9*b^2*c^3*d^7 + 99*a^10*b*c^2*d^8 + 2*a^11*c*d^9)*
x^10 + 5/3*(11*a^3*b^8*c^10 + 220*a^4*b^7*c^9*d + 1386*a^5*b^6*c^8*d^2 + 36
96*a^6*b^5*c^7*d^3 + 4620*a^7*b^4*c^6*d^4 + 2772*a^8*b^3*c^5*d^5 + 770*a^9*
b^2*c^4*d^6 + 88*a^10*b*c^3*d^7 + 3*a^11*c^2*d^8)*x^9 + 15/4*(11*a^4*b^7*c^
10 + 154*a^5*b^6*c^9*d + 693*a^6*b^5*c^8*d^2 + 1320*a^7*b^4*c^7*d^3 + 1155*
a^8*b^3*c^6*d^4 + 462*a^9*b^2*c^5*d^5 + 77*a^10*b*c^4*d^6 + 4*a^11*c^3*d^7)
*x^8 + 6/7*(77*a^5*b^6*c^10 + 770*a^6*b^5*c^9*d + 2475*a^7*b^4*c^8*d^2 + 33
00*a^8*b^3*c^7*d^3 + 1925*a^9*b^2*c^6*d^4 + 462*a^10*b*c^5*d^5 + 35*a^11*c^
4*d^6)*x^7 + 1/2*(154*a^6*b^5*c^10 + 1100*a^7*b^4*c^9*d + 2475*a^8*b^3*c^8*
d^2 + 2200*a^9*b^2*c^7*d^3 + 770*a^10*b*c^6*d^4 + 84*a^11*c^5*d^5)*x^6 + 3*
(22*a^7*b^4*c^10 + 110*a^8*b^3*c^9*d + 165*a^9*b^2*c^8*d^2 + 88*a^10*b*c^7*
d^3 + 14*a^11*c^6*d^4)*x^5 + 5/4*(33*a^8*b^3*c^10 + 110*a^9*b^2*c^9*d + 99*
a^10*b*c^8*d^2 + 24*a^11*c^7*d^3)*x^4 + 5/3*(11*a^9*b^2*c^10 + 22*a^10*b*c^
9*d + 9*a^11*c^8*d^2)*x^3 + 1/2*(11*a^10*b*c^10 + 10*a^11*c^9*d)*x^2
```

**mupad [B]** time = 1.03, size = 1702, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^11*(c + d*x)^10,x)
```

```
[Out] x^7*(66*a^5*b^6*c^10 + 30*a^11*c^4*d^6 + 660*a^6*b^5*c^9*d + 396*a^10*b*c^5
*d^5 + (14850*a^7*b^4*c^8*d^2)/7 + (19800*a^8*b^3*c^7*d^3)/7 + 1650*a^9*b^2
*c^6*d^4) + x^16*((231*a^6*b^5*d^10)/8 + (105*b^11*c^6*d^4)/8 + (693*a*b^10
*c^5*d^5)/4 + (1155*a^5*b^6*c*d^9)/4 + (5775*a^2*b^9*c^4*d^6)/8 + (2475*a^3
*b^8*c^3*d^7)/2 + (7425*a^4*b^7*c^2*d^8)/8) + x^11*((a^11*d^10)/11 + a*b^10
*c^10 + 50*a^2*b^9*c^9*d + 675*a^3*b^8*c^8*d^2 + 3600*a^4*b^7*c^7*d^3 + 882
0*a^5*b^6*c^6*d^4 + 10584*a^6*b^5*c^5*d^5 + 6300*a^7*b^4*c^4*d^6 + 1800*a^8
```

```
*b^3*c^3*d^7 + 225*a^9*b^2*c^2*d^8 + 10*a^10*b*c*d^9) + x^12*((b^11*c^10)/1
2 + (11*a^10*b*d^10)/12 + (275*a^9*b^2*c*d^9)/6 + (825*a^2*b^9*c^8*d^2)/4 +
 1650*a^3*b^8*c^7*d^3 + 5775*a^4*b^7*c^6*d^4 + 9702*a^5*b^6*c^5*d^5 + 8085*
a^6*b^5*c^4*d^6 + 3300*a^7*b^4*c^3*d^7 + (2475*a^8*b^3*c^2*d^8)/4 + (55*a*b
^10*c^9*d)/6) + x^5*(66*a^7*b^4*c^10 + 42*a^11*c^6*d^4 + 330*a^8*b^3*c^9*d
+ 264*a^10*b*c^7*d^3 + 495*a^9*b^2*c^8*d^2) + x^18*((55*a^4*b^7*d^10)/3 + (
35*b^11*c^4*d^6)/3 + (220*a*b^10*c^3*d^7)/3 + (275*a^3*b^8*c*d^9)/3 + (275*
a^2*b^9*c^2*d^8)/2) + x^8*((165*a^4*b^7*c^10)/4 + 15*a^11*c^3*d^7 + (1155*a
^5*b^6*c^9*d)/2 + (1155*a^10*b*c^4*d^6)/4 + (10395*a^6*b^5*c^8*d^2)/4 + 495
0*a^7*b^4*c^7*d^3 + (17325*a^8*b^3*c^6*d^4)/4 + (3465*a^9*b^2*c^5*d^5)/2) +
 x^15*(22*a^7*b^4*d^10 + 8*b^11*c^7*d^3 + 154*a*b^10*c^6*d^4 + 308*a^6*b^5*
c*d^9 + 924*a^2*b^9*c^5*d^5 + 2310*a^3*b^8*c^4*d^6 + 2640*a^4*b^7*c^3*d^7 +
 1386*a^5*b^6*c^2*d^8) + x^6*(77*a^6*b^5*c^10 + 42*a^11*c^5*d^5 + 550*a^7*b
^4*c^9*d + 385*a^10*b*c^6*d^4 + (2475*a^8*b^3*c^8*d^2)/2 + 1100*a^9*b^2*c^7
*d^3) + x^17*((462*a^5*b^6*d^10)/17 + (252*b^11*c^5*d^5)/17 + (2310*a*b^10*
c^4*d^6)/17 + (3300*a^4*b^7*c*d^9)/17 + (6600*a^2*b^9*c^3*d^7)/17 + (7425*a
^3*b^8*c^2*d^8)/17) + x^9*((55*a^3*b^8*c^10)/3 + 5*a^11*c^2*d^8 + (1100*a^4
*b^7*c^9*d)/3 + (440*a^10*b*c^3*d^7)/3 + 2310*a^5*b^6*c^8*d^2 + 6160*a^6*b^
5*c^7*d^3 + 7700*a^7*b^4*c^6*d^4 + 4620*a^8*b^3*c^5*d^5 + (3850*a^9*b^2*c^4
*d^6)/3) + x^14*((165*a^8*b^3*d^10)/14 + (45*b^11*c^8*d^2)/14 + (660*a*b^10
*c^7*d^3)/7 + (1650*a^7*b^4*c*d^9)/7 + 825*a^2*b^9*c^6*d^4 + 2970*a^3*b^8*c
^5*d^5 + 4950*a^4*b^7*c^4*d^6 + 3960*a^5*b^6*c^3*d^7 + 1485*a^6*b^5*c^2*d^8
) + x^10*(a^11*c*d^9 + (11*a^2*b^9*c^10)/2 + 165*a^3*b^8*c^9*d + (99*a^10*b
*c^2*d^8)/2 + 1485*a^4*b^7*c^8*d^2 + 5544*a^5*b^6*c^7*d^3 + 9702*a^6*b^5*c^
6*d^4 + 8316*a^7*b^4*c^5*d^5 + 3465*a^8*b^3*c^4*d^6 + 660*a^9*b^2*c^3*d^7)
+ x^13*((10*b^11*c^9*d)/13 + (55*a^9*b^2*d^10)/13 + (495*a*b^10*c^8*d^2)/13
 + (1650*a^8*b^3*c*d^9)/13 + (6600*a^2*b^9*c^7*d^3)/13 + (34650*a^3*b^8*c^6
*d^4)/13 + (83160*a^4*b^7*c^5*d^5)/13 + (97020*a^5*b^6*c^4*d^6)/13 + (55440
*a^6*b^5*c^3*d^7)/13 + (14850*a^7*b^4*c^2*d^8)/13) + a^11*c^10*x + (b^11*d^
10*x^22)/22 + (5*a^8*c^7*x^4*(24*a^3*d^3 + 33*b^3*c^3 + 110*a*b^2*c^2*d + 9
9*a^2*b*c*d^2))/4 + (5*b^8*d^7*x^19*(33*a^3*d^3 + 24*b^3*c^3 + 99*a*b^2*c^2
*d + 110*a^2*b*c*d^2))/19 + (a^10*c^9*x^2*(10*a*d + 11*b*c))/2 + (b^10*d^9*
x^21*(11*a*d + 10*b*c))/21 + (5*a^9*c^8*x^3*(9*a^2*d^2 + 11*b^2*c^2 + 22*a*
b*c*d))/3 + (b^9*d^8*x^20*(11*a^2*d^2 + 9*b^2*c^2 + 22*a*b*c*d))/4
```

**sympy [B]** time = 0.34, size = 1965, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**11*(d*x+c)**10,x)
```

```
[Out] a**11*c**10*x + b**11*d**10*x**22/22 + x**21*(11*a*b**10*d**10/21 + 10*b**1
1*c*d**9/21) + x**20*(11*a**2*b**9*d**10/4 + 11*a*b**10*c*d**9/2 + 9*b**11*
c**2*d**8/4) + x**19*(165*a**3*b**8*d**10/19 + 550*a**2*b**9*c*d**9/19 + 49
```

5*a*b**10*c**2*d**8/19 + 120*b**11*c**3*d**7/19) + x**18*(55*a**4*b**7*d**1
0/3 + 275*a**3*b**8*c*d**9/3 + 275*a**2*b**9*c**2*d**8/2 + 220*a*b**10*c**3
*d**7/3 + 35*b**11*c**4*d**6/3) + x**17*(462*a**5*b**6*d**10/17 + 3300*a**4
*b**7*c*d**9/17 + 7425*a**3*b**8*c**2*d**8/17 + 6600*a**2*b**9*c**3*d**7/17
 + 2310*a*b**10*c**4*d**6/17 + 252*b**11*c**5*d**5/17) + x**16*(231*a**6*b*
*5*d**10/8 + 1155*a**5*b**6*c*d**9/4 + 7425*a**4*b**7*c**2*d**8/8 + 2475*a*
*3*b**8*c**3*d**7/2 + 5775*a**2*b**9*c**4*d**6/8 + 693*a*b**10*c**5*d**5/4
+ 105*b**11*c**6*d**4/8) + x**15*(22*a**7*b**4*d**10 + 308*a**6*b**5*c*d**9
 + 1386*a**5*b**6*c**2*d**8 + 2640*a**4*b**7*c**3*d**7 + 2310*a**3*b**8*c**
4*d**6 + 924*a**2*b**9*c**5*d**5 + 154*a*b**10*c**6*d**4 + 8*b**11*c**7*d**
3) + x**14*(165*a**8*b**3*d**10/14 + 1650*a**7*b**4*c*d**9/7 + 1485*a**6*b*
*5*c**2*d**8 + 3960*a**5*b**6*c**3*d**7 + 4950*a**4*b**7*c**4*d**6 + 2970*a
**3*b**8*c**5*d**5 + 825*a**2*b**9*c**6*d**4 + 660*a*b**10*c**7*d**3/7 + 45
*b**11*c**8*d**2/14) + x**13*(55*a**9*b**2*d**10/13 + 1650*a**8*b**3*c*d**9
/13 + 14850*a**7*b**4*c**2*d**8/13 + 55440*a**6*b**5*c**3*d**7/13 + 97020*a
**5*b**6*c**4*d**6/13 + 83160*a**4*b**7*c**5*d**5/13 + 34650*a**3*b**8*c**6
*d**4/13 + 6600*a**2*b**9*c**7*d**3/13 + 495*a*b**10*c**8*d**2/13 + 10*b**1
1*c**9*d/13) + x**12*(11*a**10*b*d**10/12 + 275*a**9*b**2*c*d**9/6 + 2475*a
**8*b**3*c**2*d**8/4 + 3300*a**7*b**4*c**3*d**7 + 8085*a**6*b**5*c**4*d**6
+ 9702*a**5*b**6*c**5*d**5 + 5775*a**4*b**7*c**6*d**4 + 1650*a**3*b**8*c**7
*d**3 + 825*a**2*b**9*c**8*d**2/4 + 55*a*b**10*c**9*d/6 + b**11*c**10/12) +
 x**11*(a**11*d**10/11 + 10*a**10*b*c*d**9 + 225*a**9*b**2*c**2*d**8 + 1800
*a**8*b**3*c**3*d**7 + 6300*a**7*b**4*c**4*d**6 + 10584*a**6*b**5*c**5*d**5
 + 8820*a**5*b**6*c**6*d**4 + 3600*a**4*b**7*c**7*d**3 + 675*a**3*b**8*c**8
*d**2 + 50*a**2*b**9*c**9*d + a*b**10*c**10) + x**10*(a**11*c*d**9 + 99*a**
10*b*c**2*d**8/2 + 660*a**9*b**2*c**3*d**7 + 3465*a**8*b**3*c**4*d**6 + 831
6*a**7*b**4*c**5*d**5 + 9702*a**6*b**5*c**6*d**4 + 5544*a**5*b**6*c**7*d**3
 + 1485*a**4*b**7*c**8*d**2 + 165*a**3*b**8*c**9*d + 11*a**2*b**9*c**10/2)
+ x**9*(5*a**11*c**2*d**8 + 440*a**10*b*c**3*d**7/3 + 3850*a**9*b**2*c**4*d
**6/3 + 4620*a**8*b**3*c**5*d**5 + 7700*a**7*b**4*c**6*d**4 + 6160*a**6*b**
5*c**7*d**3 + 2310*a**5*b**6*c**8*d**2 + 1100*a**4*b**7*c**9*d/3 + 55*a**3*
b**8*c**10/3) + x**8*(15*a**11*c**3*d**7 + 1155*a**10*b*c**4*d**6/4 + 3465*
a**9*b**2*c**5*d**5/2 + 17325*a**8*b**3*c**6*d**4/4 + 4950*a**7*b**4*c**7*d
**3 + 10395*a**6*b**5*c**8*d**2/4 + 1155*a**5*b**6*c**9*d/2 + 165*a**4*b**7
*c**10/4) + x**7*(30*a**11*c**4*d**6 + 396*a**10*b*c**5*d**5 + 1650*a**9*b*
*2*c**6*d**4 + 19800*a**8*b**3*c**7*d**3/7 + 14850*a**7*b**4*c**8*d**2/7 +
660*a**6*b**5*c**9*d + 66*a**5*b**6*c**10) + x**6*(42*a**11*c**5*d**5 + 385
*a**10*b*c**6*d**4 + 1100*a**9*b**2*c**7*d**3 + 2475*a**8*b**3*c**8*d**2/2
+ 550*a**7*b**4*c**9*d + 77*a**6*b**5*c**10) + x**5*(42*a**11*c**6*d**4 + 2
64*a**10*b*c**7*d**3 + 495*a**9*b**2*c**8*d**2 + 330*a**8*b**3*c**9*d + 66*
a**7*b**4*c**10) + x**4*(30*a**11*c**7*d**3 + 495*a**10*b*c**8*d**2/4 + 275
*a**9*b**2*c**9*d/2 + 165*a**8*b**3*c**10/4) + x**3*(15*a**11*c**8*d**2 + 1
10*a**10*b*c**9*d/3 + 55*a**9*b**2*c**10/3) + x**2*(5*a**11*c**9*d + 11*a**
10*b*c**10/2)

## 3.1195 $\int (a+bx)^{10}(c+dx)^{10}\,dx$

**Optimal**. Leaf size=279

$$\frac{d^9(a+bx)^{20}(bc-ad)}{2b^{11}}+\frac{45d^8(a+bx)^{19}(bc-ad)^2}{19b^{11}}+\frac{20d^7(a+bx)^{18}(bc-ad)^3}{3b^{11}}+\frac{210d^6(a+bx)^{17}(bc-ad)^4}{17b^{11}}+\frac{63d^5(a$$

**Rubi [A]** time = 1.11, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^9(a+bx)^{20}(bc-ad)}{2b^{11}}+\frac{45d^8(a+bx)^{19}(bc-ad)^2}{19b^{11}}+\frac{20d^7(a+bx)^{18}(bc-ad)^3}{3b^{11}}+\frac{210d^6(a+bx)^{17}(bc-ad)^4}{17b^{11}}+\frac{63d^5(a+bx)^{16}(bc-ad)^5}{4b^{11}}+\frac{14d^4(a+bx)^{15}(bc-ad)^6}{b^{11}}+\frac{60d^3(a+bx)^{14}(bc-ad)^7}{7b^{11}}+\frac{45d^2(a+bx)^{13}(bc-ad)^8}{13b^{11}}+\frac{5d(a+bx)^{12}(bc-ad)^9}{6b^{11}}+\frac{(a+bx)^{11}(bc-ad)^{10}}{11b^{11}}+\frac{d^{10}(a+bx)^{21}}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^11)/(11*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^12)/
(6*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^13)/(13*b^11) + (60*d^3*(b*c - a
*d)^7*(a + b*x)^14)/(7*b^11) + (14*d^4*(b*c - a*d)^6*(a + b*x)^15)/b^11 + (
63*d^5*(b*c - a*d)^5*(a + b*x)^16)/(4*b^11) + (210*d^6*(b*c - a*d)^4*(a + b
*x)^17)/(17*b^11) + (20*d^7*(b*c - a*d)^3*(a + b*x)^18)/(3*b^11) + (45*d^8*
(b*c - a*d)^2*(a + b*x)^19)/(19*b^11) + (d^9*(b*c - a*d)*(a + b*x)^20)/(2*b
^11) + (d^10*(a + b*x)^21)/(21*b^11)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{10}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{10}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{11}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)^{12}}{b^{10}}+\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{11}}{11b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{12}}{6b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{13}}{13b^{11}}+\frac{60d^3(\end{aligned}$$

**Mathematica [B]** time = 0.17, size = 1539, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10*(c + d*x)^10,x]
```

```
[Out] a^10*c^10*x + 5*a^9*c^9*(b*c + a*d)*x^2 + (5*a^8*c^8*(9*b^2*c^2 + 20*a*b*c*
d + 9*a^2*d^2)*x^3)/3 + (15*a^7*c^7*(4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*
c*d^2 + 4*a^3*d^3)*x^4)/2 + 3*a^6*c^6*(14*b^4*c^4 + 80*a*b^3*c^3*d + 135*a^
2*b^2*c^2*d^2 + 80*a^3*b*c*d^3 + 14*a^4*d^4)*x^5 + 2*a^5*c^5*(21*b^5*c^5 +
175*a*b^4*c^4*d + 450*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d
^4 + 21*a^5*d^5)*x^6 + (30*a^4*c^4*(7*b^6*c^6 + 84*a*b^5*c^5*d + 315*a^2*b^
4*c^4*d^2 + 480*a^3*b^3*c^3*d^3 + 315*a^4*b^2*c^2*d^4 + 84*a^5*b*c*d^5 + 7*
a^6*d^6)*x^7)/7 + (15*a^3*c^3*(2*b^7*c^7 + 35*a*b^6*c^6*d + 189*a^2*b^5*c^5
*d^2 + 420*a^3*b^4*c^4*d^3 + 420*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 + 35
*a^6*b*c*d^6 + 2*a^7*d^7)*x^8)/2 + (5*a^2*c^2*(3*b^8*c^8 + 80*a*b^7*c^7*d +
 630*a^2*b^6*c^6*d^2 + 2016*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4*d^4 + 2016*a
^5*b^3*c^3*d^5 + 630*a^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + 3*a^8*d^8)*x^9)/3 +
 a*c*(b^9*c^9 + 45*a*b^8*c^8*d + 540*a^2*b^7*c^7*d^2 + 2520*a^3*b^6*c^6*d^3
 + 5292*a^4*b^5*c^5*d^4 + 5292*a^5*b^4*c^4*d^5 + 2520*a^6*b^3*c^3*d^6 + 540
*a^7*b^2*c^2*d^7 + 45*a^8*b*c*d^8 + a^9*d^9)*x^10 + ((b^10*c^10 + 100*a*b^9
*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100*a^4*b^6*c^6*d
^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^7*b^3*c^3*d^7
+ 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11)/11 + (5*b*d*(b^
9*c^9 + 45*a*b^8*c^8*d + 540*a^2*b^7*c^7*d^2 + 2520*a^3*b^6*c^6*d^3 + 5292*
a^4*b^5*c^5*d^4 + 5292*a^5*b^4*c^4*d^5 + 2520*a^6*b^3*c^3*d^6 + 540*a^7*b^2
*c^2*d^7 + 45*a^8*b*c*d^8 + a^9*d^9)*x^12)/6 + (15*b^2*d^2*(3*b^8*c^8 + 80*
a*b^7*c^7*d + 630*a^2*b^6*c^6*d^2 + 2016*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4
*d^4 + 2016*a^5*b^3*c^3*d^5 + 630*a^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + 3*a^8*
d^8)*x^13)/13 + (30*b^3*d^3*(2*b^7*c^7 + 35*a*b^6*c^6*d + 189*a^2*b^5*c^5*d
^2 + 420*a^3*b^4*c^4*d^3 + 420*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 + 35*a
^6*b*c*d^6 + 2*a^7*d^7)*x^14)/7 + 2*b^4*d^4*(7*b^6*c^6 + 84*a*b^5*c^5*d + 3
15*a^2*b^4*c^4*d^2 + 480*a^3*b^3*c^3*d^3 + 315*a^4*b^2*c^2*d^4 + 84*a^5*b*c
*d^5 + 7*a^6*d^6)*x^15 + (3*b^5*d^5*(21*b^5*c^5 + 175*a*b^4*c^4*d + 450*a^2
*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d^4 + 21*a^5*d^5)*x^16)/4
+ (15*b^6*d^6*(14*b^4*c^4 + 80*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 + 80*a^3*b
*c*d^3 + 14*a^4*d^4)*x^17)/17 + (5*b^7*d^7*(4*b^3*c^3 + 15*a*b^2*c^2*d + 15
*a^2*b*c*d^2 + 4*a^3*d^3)*x^18)/3 + (5*b^8*d^8*(9*b^2*c^2 + 20*a*b*c*d + 9*
a^2*d^2)*x^19)/19 + (b^9*d^9*(b*c + a*d)*x^20)/2 + (b^10*d^10*x^21)/21
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{10}(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10*(c + d*x)^10,x]
```

[Out] IntegrateAlgebraic[(a + b*x)^10*(c + d*x)^10, x]

**fricas [B]** time = 0.95, size = 1833, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="fricas")

[Out] 1/21*x^21*d^10*b^10 + 1/2*x^20*d^9*c*b^10 + 1/2*x^20*d^10*b^9*a + 45/19*x^19*d^8*c^2*b^10 + 100/19*x^19*d^9*c*b^9*a + 45/19*x^19*d^10*b^8*a^2 + 20/3*x^18*d^7*c^3*b^10 + 25*x^18*d^8*c^2*b^9*a + 25*x^18*d^9*c*b^8*a^2 + 20/3*x^18*d^10*b^7*a^3 + 210/17*x^17*d^6*c^4*b^10 + 1200/17*x^17*d^7*c^3*b^9*a + 2025/17*x^17*d^8*c^2*b^8*a^2 + 1200/17*x^17*d^9*c*b^7*a^3 + 210/17*x^17*d^10*b^6*a^4 + 63/4*x^16*d^5*c^5*b^10 + 525/4*x^16*d^6*c^4*b^9*a + 675/2*x^16*d^7*c^3*b^8*a^2 + 675/2*x^16*d^8*c^2*b^7*a^3 + 525/4*x^16*d^9*c*b^6*a^4 + 63/4*x^16*d^10*b^5*a^5 + 14*x^15*d^4*c^6*b^10 + 168*x^15*d^5*c^5*b^9*a + 630*x^15*d^6*c^4*b^8*a^2 + 960*x^15*d^7*c^3*b^7*a^3 + 630*x^15*d^8*c^2*b^6*a^4 + 168*x^15*d^9*c*b^5*a^5 + 14*x^15*d^10*b^4*a^6 + 60/7*x^14*d^3*c^7*b^10 + 150*x^14*d^4*c^6*b^9*a + 810*x^14*d^5*c^5*b^8*a^2 + 1800*x^14*d^6*c^4*b^7*a^3 + 1800*x^14*d^7*c^3*b^6*a^4 + 810*x^14*d^8*c^2*b^5*a^5 + 150*x^14*d^9*c*b^4*a^6 + 60/7*x^14*d^10*b^3*a^7 + 45/13*x^13*d^2*c^8*b^10 + 1200/13*x^13*d^3*c^7*b^9*a + 9450/13*x^13*d^4*c^6*b^8*a^2 + 30240/13*x^13*d^5*c^5*b^7*a^3 + 44100/13*x^13*d^6*c^4*b^6*a^4 + 30240/13*x^13*d^7*c^3*b^5*a^5 + 9450/13*x^13*d^8*c^2*b^4*a^6 + 1200/13*x^13*d^9*c*b^3*a^7 + 45/13*x^13*d^10*b^2*a^8 + 5/6*x^12*d*c^9*b^10 + 75/2*x^12*d^2*c^8*b^9*a + 450*x^12*d^3*c^7*b^8*a^2 + 2100*x^12*d^4*c^6*b^7*a^3 + 4410*x^12*d^5*c^5*b^6*a^4 + 4410*x^12*d^6*c^4*b^5*a^5 + 2100*x^12*d^7*c^3*b^4*a^6 + 450*x^12*d^8*c^2*b^3*a^7 + 75/2*x^12*d^9*c*b^2*a^8 + 5/6*x^12*d^10*b*a^9 + 1/11*x^11*c^10*b^10 + 100/11*x^11*d*c^9*b^9*a + 2025/11*x^11*d^2*c^8*b^8*a^2 + 14400/11*x^11*d^3*c^7*b^7*a^3 + 44100/11*x^11*d^4*c^6*b^6*a^4 + 63504/11*x^11*d^5*c^5*b^5*a^5 + 44100/11*x^11*d^6*c^4*b^4*a^6 + 14400/11*x^11*d^7*c^3*b^3*a^7 + 2025/11*x^11*d^8*c^2*b^2*a^8 + 100/11*x^11*d^9*c*b*a^9 + 1/11*x^11*d^10*a^10 + x^10*c^10*b^9*a + 45*x^10*d*c^9*b^8*a^2 + 540*x^10*d^2*c^8*b^7*a^3 + 2520*x^10*d^3*c^7*b^6*a^4 + 5292*x^10*d^4*c^6*b^5*a^5 + 5292*x^10*d^5*c^5*b^4*a^6 + 2520*x^10*d^6*c^4*b^3*a^7 + 540*x^10*d^7*c^3*b^2*a^8 + 45*x^10*d^8*c^2*b*a^9 + x^10*d^9*c*a^10 + 5*x^9*c^10*b^8*a^2 + 400/3*x^9*d*c^9*b^7*a^3 + 1050*x^9*d^2*c^8*b^6*a^4 + 3360*x^9*d^3*c^7*b^5*a^5 + 4900*x^9*d^4*c^6*b^4*a^6 + 3360*x^9*d^5*c^5*b^3*a^7 + 1050*x^9*d^6*c^4*b^2*a^8 + 400/3*x^9*d^7*c^3*b*a^9 + 5*x^9*d^8*c^2*a^10 + 15*x^8*c^10*b^7*a^3 + 525/2*x^8*d*c^9*b^6*a^4 + 2835/2*x^8*d^2*c^8*b^5*a^5 + 3150*x^8*d^3*c^7*b^4*a^6 + 3150*x^8*d^4*c^6*b^3*a^7 + 2835/2*x^8*d^5*c^5*b^2*a^8 + 525/2*x^8*d^6*c^4*b*a^9 + 15*x^8*d^7*c^3*a^10 + 30*x^7*c^10*b^6*a^4 + 360*x^7*d*c^9*b^5*a^5 + 1350*x^7*d^2*c^8*b^4*a^6 + 14400/7*x^7*d^3*c^7*b^3*a^7 + 1350*x^7*d^4*c^6*b^2*a^8 + 360*x^7*d^5*c^5*b*a^9 + 30*x^7*d^6*c^4*a^10 + 42*x^6*c^10*b^5*a^5 + 350*x^6*d*c^9*b^4*a^6 + 900*x^6*d^2*c^8*b^3*a^7 + 900*x^6*d^3*c^7*b^2*a^8 + 350*x^6*d^4*c^6*b*a^9 + 42*x^6*d

```
^5*c^5*a^10 + 42*x^5*c^10*b^4*a^6 + 240*x^5*d*c^9*b^3*a^7 + 405*x^5*d^2*c^8
*b^2*a^8 + 240*x^5*d^3*c^7*b*a^9 + 42*x^5*d^4*c^6*a^10 + 30*x^4*c^10*b^3*a^
7 + 225/2*x^4*d*c^9*b^2*a^8 + 225/2*x^4*d^2*c^8*b*a^9 + 30*x^4*d^3*c^7*a^10
 + 15*x^3*c^10*b^2*a^8 + 100/3*x^3*d*c^9*b*a^9 + 15*x^3*d^2*c^8*a^10 + 5*x^
2*c^10*b*a^9 + 5*x^2*d*c^9*a^10 + x*c^10*a^10
```

**giac [B]** time = 1.34, size = 1833, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/21*b^10*d^10*x^21 + 1/2*b^10*c*d^9*x^20 + 1/2*a*b^9*d^10*x^20 + 45/19*b^1
0*c^2*d^8*x^19 + 100/19*a*b^9*c*d^9*x^19 + 45/19*a^2*b^8*d^10*x^19 + 20/3*b
^10*c^3*d^7*x^18 + 25*a*b^9*c^2*d^8*x^18 + 25*a^2*b^8*c*d^9*x^18 + 20/3*a^3
*b^7*d^10*x^18 + 210/17*b^10*c^4*d^6*x^17 + 1200/17*a*b^9*c^3*d^7*x^17 + 20
25/17*a^2*b^8*c^2*d^8*x^17 + 1200/17*a^3*b^7*c*d^9*x^17 + 210/17*a^4*b^6*d^
10*x^17 + 63/4*b^10*c^5*d^5*x^16 + 525/4*a*b^9*c^4*d^6*x^16 + 675/2*a^2*b^8
*c^3*d^7*x^16 + 675/2*a^3*b^7*c^2*d^8*x^16 + 525/4*a^4*b^6*c*d^9*x^16 + 63/
4*a^5*b^5*d^10*x^16 + 14*b^10*c^6*d^4*x^15 + 168*a*b^9*c^5*d^5*x^15 + 630*a
^2*b^8*c^4*d^6*x^15 + 960*a^3*b^7*c^3*d^7*x^15 + 630*a^4*b^6*c^2*d^8*x^15 +
 168*a^5*b^5*c*d^9*x^15 + 14*a^6*b^4*d^10*x^15 + 60/7*b^10*c^7*d^3*x^14 + 1
50*a*b^9*c^6*d^4*x^14 + 810*a^2*b^8*c^5*d^5*x^14 + 1800*a^3*b^7*c^4*d^6*x^1
4 + 1800*a^4*b^6*c^3*d^7*x^14 + 810*a^5*b^5*c^2*d^8*x^14 + 150*a^6*b^4*c*d^
9*x^14 + 60/7*a^7*b^3*d^10*x^14 + 45/13*b^10*c^8*d^2*x^13 + 1200/13*a*b^9*c
^7*d^3*x^13 + 9450/13*a^2*b^8*c^6*d^4*x^13 + 30240/13*a^3*b^7*c^5*d^5*x^13
+ 44100/13*a^4*b^6*c^4*d^6*x^13 + 30240/13*a^5*b^5*c^3*d^7*x^13 + 9450/13*a
^6*b^4*c^2*d^8*x^13 + 1200/13*a^7*b^3*c*d^9*x^13 + 45/13*a^8*b^2*d^10*x^13
+ 5/6*b^10*c^9*d*x^12 + 75/2*a*b^9*c^8*d^2*x^12 + 450*a^2*b^8*c^7*d^3*x^12
+ 2100*a^3*b^7*c^6*d^4*x^12 + 4410*a^4*b^6*c^5*d^5*x^12 + 4410*a^5*b^5*c^4*
d^6*x^12 + 2100*a^6*b^4*c^3*d^7*x^12 + 450*a^7*b^3*c^2*d^8*x^12 + 75/2*a^8*
b^2*c*d^9*x^12 + 5/6*a^9*b*d^10*x^12 + 1/11*b^10*c^10*x^11 + 100/11*a*b^9*c
^9*d*x^11 + 2025/11*a^2*b^8*c^8*d^2*x^11 + 14400/11*a^3*b^7*c^7*d^3*x^11 +
44100/11*a^4*b^6*c^6*d^4*x^11 + 63504/11*a^5*b^5*c^5*d^5*x^11 + 44100/11*a^
6*b^4*c^4*d^6*x^11 + 14400/11*a^7*b^3*c^3*d^7*x^11 + 2025/11*a^8*b^2*c^2*d^
8*x^11 + 100/11*a^9*b*c*d^9*x^11 + 1/11*a^10*d^10*x^11 + a*b^9*c^10*x^10 +
45*a^2*b^8*c^9*d*x^10 + 540*a^3*b^7*c^8*d^2*x^10 + 2520*a^4*b^6*c^7*d^3*x^1
0 + 5292*a^5*b^5*c^6*d^4*x^10 + 5292*a^6*b^4*c^5*d^5*x^10 + 2520*a^7*b^3*c^
4*d^6*x^10 + 540*a^8*b^2*c^3*d^7*x^10 + 45*a^9*b*c^2*d^8*x^10 + a^10*c*d^9*
x^10 + 5*a^2*b^8*c^10*x^9 + 400/3*a^3*b^7*c^9*d*x^9 + 1050*a^4*b^6*c^8*d^2*
x^9 + 3360*a^5*b^5*c^7*d^3*x^9 + 4900*a^6*b^4*c^6*d^4*x^9 + 3360*a^7*b^3*c^
5*d^5*x^9 + 1050*a^8*b^2*c^4*d^6*x^9 + 400/3*a^9*b*c^3*d^7*x^9 + 5*a^10*c^2
*d^8*x^9 + 15*a^3*b^7*c^10*x^8 + 525/2*a^4*b^6*c^9*d*x^8 + 2835/2*a^5*b^5*c
^8*d^2*x^8 + 3150*a^6*b^4*c^7*d^3*x^8 + 3150*a^7*b^3*c^6*d^4*x^8 + 2835/2*a
```

```
^8*b^2*c^5*d^5*x^8 + 525/2*a^9*b*c^4*d^6*x^8 + 15*a^10*c^3*d^7*x^8 + 30*a^4
*b^6*c^10*x^7 + 360*a^5*b^5*c^9*d*x^7 + 1350*a^6*b^4*c^8*d^2*x^7 + 14400/7*
a^7*b^3*c^7*d^3*x^7 + 1350*a^8*b^2*c^6*d^4*x^7 + 360*a^9*b*c^5*d^5*x^7 + 30
*a^10*c^4*d^6*x^7 + 42*a^5*b^5*c^10*x^6 + 350*a^6*b^4*c^9*d*x^6 + 900*a^7*b
^3*c^8*d^2*x^6 + 900*a^8*b^2*c^7*d^3*x^6 + 350*a^9*b*c^6*d^4*x^6 + 42*a^10*
c^5*d^5*x^6 + 42*a^6*b^4*c^10*x^5 + 240*a^7*b^3*c^9*d*x^5 + 405*a^8*b^2*c^8
*d^2*x^5 + 240*a^9*b*c^7*d^3*x^5 + 42*a^10*c^6*d^4*x^5 + 30*a^7*b^3*c^10*x^
4 + 225/2*a^8*b^2*c^9*d*x^4 + 225/2*a^9*b*c^8*d^2*x^4 + 30*a^10*c^7*d^3*x^4
 + 15*a^8*b^2*c^10*x^3 + 100/3*a^9*b*c^9*d*x^3 + 15*a^10*c^8*d^2*x^3 + 5*a^
9*b*c^10*x^2 + 5*a^10*c^9*d*x^2 + a^10*c^10*x
```

**maple [B]** time = 0.00, size = 1591, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10*(d*x+c)^10,x)
```

```
[Out] 1/21*b^10*d^10*x^21+1/20*(10*a*b^9*d^10+10*b^10*c*d^9)*x^20+1/19*(45*a^2*b^
8*d^10+100*a*b^9*c*d^9+45*b^10*c^2*d^8)*x^19+1/18*(120*a^3*b^7*d^10+450*a^2
*b^8*c*d^9+450*a*b^9*c^2*d^8+120*b^10*c^3*d^7)*x^18+1/17*(210*a^4*b^6*d^10+
1200*a^3*b^7*c*d^9+2025*a^2*b^8*c^2*d^8+1200*a*b^9*c^3*d^7+210*b^10*c^4*d^6
)*x^17+1/16*(252*a^5*b^5*d^10+2100*a^4*b^6*c*d^9+5400*a^3*b^7*c^2*d^8+5400*
a^2*b^8*c^3*d^7+2100*a*b^9*c^4*d^6+252*b^10*c^5*d^5)*x^16+1/15*(210*a^6*b^4
*d^10+2520*a^5*b^5*c*d^9+9450*a^4*b^6*c^2*d^8+14400*a^3*b^7*c^3*d^7+9450*a^
2*b^8*c^4*d^6+2520*a*b^9*c^5*d^5+210*b^10*c^6*d^4)*x^15+1/14*(120*a^7*b^3*d
^10+2100*a^6*b^4*c*d^9+11340*a^5*b^5*c^2*d^8+25200*a^4*b^6*c^3*d^7+25200*a^
3*b^7*c^4*d^6+11340*a^2*b^8*c^5*d^5+2100*a*b^9*c^6*d^4+120*b^10*c^7*d^3)*x^
14+1/13*(45*a^8*b^2*d^10+1200*a^7*b^3*c*d^9+9450*a^6*b^4*c^2*d^8+30240*a^5*
b^5*c^3*d^7+44100*a^4*b^6*c^4*d^6+30240*a^3*b^7*c^5*d^5+9450*a^2*b^8*c^6*d^
4+1200*a*b^9*c^7*d^3+45*b^10*c^8*d^2)*x^13+1/12*(10*a^9*b*d^10+450*a^8*b^2*
c*d^9+5400*a^7*b^3*c^2*d^8+25200*a^6*b^4*c^3*d^7+52920*a^5*b^5*c^4*d^6+5292
0*a^4*b^6*c^5*d^5+25200*a^3*b^7*c^6*d^4+5400*a^2*b^8*c^7*d^3+450*a*b^9*c^8*
d^2+10*b^10*c^9*d)*x^12+1/11*(a^10*d^10+100*a^9*b*c*d^9+2025*a^8*b^2*c^2*d^
8+14400*a^7*b^3*c^3*d^7+44100*a^6*b^4*c^4*d^6+63504*a^5*b^5*c^5*d^5+44100*a
^4*b^6*c^6*d^4+14400*a^3*b^7*c^7*d^3+2025*a^2*b^8*c^8*d^2+100*a*b^9*c^9*d+b
^10*c^10)*x^11+1/10*(10*a^10*c*d^9+450*a^9*b*c^2*d^8+5400*a^8*b^2*c^3*d^7+2
5200*a^7*b^3*c^4*d^6+52920*a^6*b^4*c^5*d^5+52920*a^5*b^5*c^6*d^4+25200*a^4*
b^6*c^7*d^3+5400*a^3*b^7*c^8*d^2+450*a^2*b^8*c^9*d+10*a*b^9*c^10)*x^10+1/9*
(45*a^10*c^2*d^8+1200*a^9*b*c^3*d^7+9450*a^8*b^2*c^4*d^6+30240*a^7*b^3*c^5*
d^5+44100*a^6*b^4*c^6*d^4+30240*a^5*b^5*c^7*d^3+9450*a^4*b^6*c^8*d^2+1200*a
^3*b^7*c^9*d+45*a^2*b^8*c^10)*x^9+1/8*(120*a^10*c^3*d^7+2100*a^9*b*c^4*d^6+
11340*a^8*b^2*c^5*d^5+25200*a^7*b^3*c^6*d^4+25200*a^6*b^4*c^7*d^3+11340*a^5
*b^5*c^8*d^2+2100*a^4*b^6*c^9*d+120*a^3*b^7*c^10)*x^8+1/7*(210*a^10*c^4*d^6
+2520*a^9*b*c^5*d^5+9450*a^8*b^2*c^6*d^4+14400*a^7*b^3*c^7*d^3+9450*a^6*b^4
```

```
*c^8*d^2+2520*a^5*b^5*c^9*d+210*a^4*b^6*c^10)*x^7+1/6*(252*a^10*c^5*d^5+210
0*a^9*b*c^6*d^4+5400*a^8*b^2*c^7*d^3+5400*a^7*b^3*c^8*d^2+2100*a^6*b^4*c^9*
d+252*a^5*b^5*c^10)*x^6+1/5*(210*a^10*c^6*d^4+1200*a^9*b*c^7*d^3+2025*a^8*b
^2*c^8*d^2+1200*a^7*b^3*c^9*d+210*a^6*b^4*c^10)*x^5+1/4*(120*a^10*c^7*d^3+4
50*a^9*b*c^8*d^2+450*a^8*b^2*c^9*d+120*a^7*b^3*c^10)*x^4+1/3*(45*a^10*c^8*d
^2+100*a^9*b*c^9*d+45*a^8*b^2*c^10)*x^3+1/2*(10*a^10*c^9*d+10*a^9*b*c^10)*x
^2+a^10*c^10*x
```

**maxima [B]** time = 1.56, size = 1581, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/21*b^10*d^10*x^21 + a^10*c^10*x + 1/2*(b^10*c*d^9 + a*b^9*d^10)*x^20 + 5/
19*(9*b^10*c^2*d^8 + 20*a*b^9*c*d^9 + 9*a^2*b^8*d^10)*x^19 + 5/3*(4*b^10*c^
3*d^7 + 15*a*b^9*c^2*d^8 + 15*a^2*b^8*c*d^9 + 4*a^3*b^7*d^10)*x^18 + 15/17*
(14*b^10*c^4*d^6 + 80*a*b^9*c^3*d^7 + 135*a^2*b^8*c^2*d^8 + 80*a^3*b^7*c*d^
9 + 14*a^4*b^6*d^10)*x^17 + 3/4*(21*b^10*c^5*d^5 + 175*a*b^9*c^4*d^6 + 450*
a^2*b^8*c^3*d^7 + 450*a^3*b^7*c^2*d^8 + 175*a^4*b^6*c*d^9 + 21*a^5*b^5*d^10
)*x^16 + 2*(7*b^10*c^6*d^4 + 84*a*b^9*c^5*d^5 + 315*a^2*b^8*c^4*d^6 + 480*a
^3*b^7*c^3*d^7 + 315*a^4*b^6*c^2*d^8 + 84*a^5*b^5*c*d^9 + 7*a^6*b^4*d^10)*x
^15 + 30/7*(2*b^10*c^7*d^3 + 35*a*b^9*c^6*d^4 + 189*a^2*b^8*c^5*d^5 + 420*a
^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 + 35*a^6*b^4*c*d
^9 + 2*a^7*b^3*d^10)*x^14 + 15/13*(3*b^10*c^8*d^2 + 80*a*b^9*c^7*d^3 + 630*
a^2*b^8*c^6*d^4 + 2016*a^3*b^7*c^5*d^5 + 2940*a^4*b^6*c^4*d^6 + 2016*a^5*b^
5*c^3*d^7 + 630*a^6*b^4*c^2*d^8 + 80*a^7*b^3*c*d^9 + 3*a^8*b^2*d^10)*x^13 +
 5/6*(b^10*c^9*d + 45*a*b^9*c^8*d^2 + 540*a^2*b^8*c^7*d^3 + 2520*a^3*b^7*c^
6*d^4 + 5292*a^4*b^6*c^5*d^5 + 5292*a^5*b^5*c^4*d^6 + 2520*a^6*b^4*c^3*d^7
+ 540*a^7*b^3*c^2*d^8 + 45*a^8*b^2*c*d^9 + a^9*b*d^10)*x^12 + 1/11*(b^10*c^
10 + 100*a*b^9*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100
*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^
7*b^3*c^3*d^7 + 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11 +
(a*b^9*c^10 + 45*a^2*b^8*c^9*d + 540*a^3*b^7*c^8*d^2 + 2520*a^4*b^6*c^7*d^3
 + 5292*a^5*b^5*c^6*d^4 + 5292*a^6*b^4*c^5*d^5 + 2520*a^7*b^3*c^4*d^6 + 540
*a^8*b^2*c^3*d^7 + 45*a^9*b*c^2*d^8 + a^10*c*d^9)*x^10 + 5/3*(3*a^2*b^8*c^1
0 + 80*a^3*b^7*c^9*d + 630*a^4*b^6*c^8*d^2 + 2016*a^5*b^5*c^7*d^3 + 2940*a^
6*b^4*c^6*d^4 + 2016*a^7*b^3*c^5*d^5 + 630*a^8*b^2*c^4*d^6 + 80*a^9*b*c^3*d
^7 + 3*a^10*c^2*d^8)*x^9 + 15/2*(2*a^3*b^7*c^10 + 35*a^4*b^6*c^9*d + 189*a^
5*b^5*c^8*d^2 + 420*a^6*b^4*c^7*d^3 + 420*a^7*b^3*c^6*d^4 + 189*a^8*b^2*c^5
*d^5 + 35*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*x^8 + 30/7*(7*a^4*b^6*c^10 + 84*a
^5*b^5*c^9*d + 315*a^6*b^4*c^8*d^2 + 480*a^7*b^3*c^7*d^3 + 315*a^8*b^2*c^6*
d^4 + 84*a^9*b*c^5*d^5 + 7*a^10*c^4*d^6)*x^7 + 2*(21*a^5*b^5*c^10 + 175*a^6
*b^4*c^9*d + 450*a^7*b^3*c^8*d^2 + 450*a^8*b^2*c^7*d^3 + 175*a^9*b*c^6*d^4
+ 21*a^10*c^5*d^5)*x^6 + 3*(14*a^6*b^4*c^10 + 80*a^7*b^3*c^9*d + 135*a^8*b^
```

```
2*c^8*d^2 + 80*a^9*b*c^7*d^3 + 14*a^10*c^6*d^4)*x^5 + 15/2*(4*a^7*b^3*c^10
+ 15*a^8*b^2*c^9*d + 15*a^9*b*c^8*d^2 + 4*a^10*c^7*d^3)*x^4 + 5/3*(9*a^8*b^
2*c^10 + 20*a^9*b*c^9*d + 9*a^10*c^8*d^2)*x^3 + 5*(a^9*b*c^10 + a^10*c^9*d)
*x^2
```

**mupad [B]** time = 0.69, size = 1549, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10*(c + d*x)^10,x)
```

```
[Out] x^7*(30*a^4*b^6*c^10 + 30*a^10*c^4*d^6 + 360*a^5*b^5*c^9*d + 360*a^9*b*c^5*
d^5 + 1350*a^6*b^4*c^8*d^2 + (14400*a^7*b^3*c^7*d^3)/7 + 1350*a^8*b^2*c^6*d
^4) + x^15*(14*a^6*b^4*d^10 + 14*b^10*c^6*d^4 + 168*a*b^9*c^5*d^5 + 168*a^5
*b^5*c*d^9 + 630*a^2*b^8*c^4*d^6 + 960*a^3*b^7*c^3*d^7 + 630*a^4*b^6*c^2*d^
8) + x^5*(42*a^6*b^4*c^10 + 42*a^10*c^6*d^4 + 240*a^7*b^3*c^9*d + 240*a^9*b
*c^7*d^3 + 405*a^8*b^2*c^8*d^2) + x^17*((210*a^4*b^6*d^10)/17 + (210*b^10*c
^4*d^6)/17 + (1200*a*b^9*c^3*d^7)/17 + (1200*a^3*b^7*c*d^9)/17 + (2025*a^2*
b^8*c^2*d^8)/17) + x^11*((a^10*d^10)/11 + (b^10*c^10)/11 + (2025*a^2*b^8*c^
8*d^2)/11 + (14400*a^3*b^7*c^7*d^3)/11 + (44100*a^4*b^6*c^6*d^4)/11 + (6350
4*a^5*b^5*c^5*d^5)/11 + (44100*a^6*b^4*c^4*d^6)/11 + (14400*a^7*b^3*c^3*d^7
)/11 + (2025*a^8*b^2*c^2*d^8)/11 + (100*a*b^9*c^9*d)/11 + (100*a^9*b*c*d^9)
/11) + x^8*(15*a^3*b^7*c^10 + 15*a^10*c^3*d^7 + (525*a^4*b^6*c^9*d)/2 + (52
5*a^9*b*c^4*d^6)/2 + (2835*a^5*b^5*c^8*d^2)/2 + 3150*a^6*b^4*c^7*d^3 + 3150
*a^7*b^3*c^6*d^4 + (2835*a^8*b^2*c^5*d^5)/2) + x^14*((60*a^7*b^3*d^10)/7 +
(60*b^10*c^7*d^3)/7 + 150*a*b^9*c^6*d^4 + 150*a^6*b^4*c*d^9 + 810*a^2*b^8*c
^5*d^5 + 1800*a^3*b^7*c^4*d^6 + 1800*a^4*b^6*c^3*d^7 + 810*a^5*b^5*c^2*d^8)
 + x^10*(a*b^9*c^10 + a^10*c*d^9 + 45*a^2*b^8*c^9*d + 45*a^9*b*c^2*d^8 + 54
0*a^3*b^7*c^8*d^2 + 2520*a^4*b^6*c^7*d^3 + 5292*a^5*b^5*c^6*d^4 + 5292*a^6*
b^4*c^5*d^5 + 2520*a^7*b^3*c^4*d^6 + 540*a^8*b^2*c^3*d^7) + x^12*((5*a^9*b*
d^10)/6 + (5*b^10*c^9*d)/6 + (75*a*b^9*c^8*d^2)/2 + (75*a^8*b^2*c*d^9)/2 +
450*a^2*b^8*c^7*d^3 + 2100*a^3*b^7*c^6*d^4 + 4410*a^4*b^6*c^5*d^5 + 4410*a^
5*b^5*c^4*d^6 + 2100*a^6*b^4*c^3*d^7 + 450*a^7*b^3*c^2*d^8) + x^6*(42*a^5*b
^5*c^10 + 42*a^10*c^5*d^5 + 350*a^6*b^4*c^9*d + 350*a^9*b*c^6*d^4 + 900*a^7
*b^3*c^8*d^2 + 900*a^8*b^2*c^7*d^3) + x^16*((63*a^5*b^5*d^10)/4 + (63*b^10*
c^5*d^5)/4 + (525*a*b^9*c^4*d^6)/4 + (525*a^4*b^6*c*d^9)/4 + (675*a^2*b^8*c
^3*d^7)/2 + (675*a^3*b^7*c^2*d^8)/2) + x^9*(5*a^2*b^8*c^10 + 5*a^10*c^2*d^8
 + (400*a^3*b^7*c^9*d)/3 + (400*a^9*b*c^3*d^7)/3 + 1050*a^4*b^6*c^8*d^2 + 3
360*a^5*b^5*c^7*d^3 + 4900*a^6*b^4*c^6*d^4 + 3360*a^7*b^3*c^5*d^5 + 1050*a^
8*b^2*c^4*d^6) + x^13*((45*a^8*b^2*d^10)/13 + (45*b^10*c^8*d^2)/13 + (1200*
a*b^9*c^7*d^3)/13 + (1200*a^7*b^3*c*d^9)/13 + (9450*a^2*b^8*c^6*d^4)/13 + (
30240*a^3*b^7*c^5*d^5)/13 + (44100*a^4*b^6*c^4*d^6)/13 + (30240*a^5*b^5*c^3
*d^7)/13 + (9450*a^6*b^4*c^2*d^8)/13) + a^10*c^10*x + (b^10*d^10*x^21)/21 +
 (15*a^7*c^7*x^4*(4*a^3*d^3 + 4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*c*d^2))
```

```
/2 + (5*b^7*d^7*x^18*(4*a^3*d^3 + 4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*c*d
^2))/3 + 5*a^9*c^9*x^2*(a*d + b*c) + (b^9*d^9*x^20*(a*d + b*c))/2 + (5*a^8*
c^8*x^3*(9*a^2*d^2 + 9*b^2*c^2 + 20*a*b*c*d))/3 + (5*b^8*d^8*x^19*(9*a^2*d^
2 + 9*b^2*c^2 + 20*a*b*c*d))/19
```

**sympy [B]** time = 0.31, size = 1775, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10*(d*x+c)**10,x)
```

```
[Out] a**10*c**10*x + b**10*d**10*x**21/21 + x**20*(a*b**9*d**10/2 + b**10*c*d**9
/2) + x**19*(45*a**2*b**8*d**10/19 + 100*a*b**9*c*d**9/19 + 45*b**10*c**2*d
**8/19) + x**18*(20*a**3*b**7*d**10/3 + 25*a**2*b**8*c*d**9 + 25*a*b**9*c**
2*d**8 + 20*b**10*c**3*d**7/3) + x**17*(210*a**4*b**6*d**10/17 + 1200*a**3*
b**7*c*d**9/17 + 2025*a**2*b**8*c**2*d**8/17 + 1200*a*b**9*c**3*d**7/17 + 2
10*b**10*c**4*d**6/17) + x**16*(63*a**5*b**5*d**10/4 + 525*a**4*b**6*c*d**9
/4 + 675*a**3*b**7*c**2*d**8/2 + 675*a**2*b**8*c**3*d**7/2 + 525*a*b**9*c**
4*d**6/4 + 63*b**10*c**5*d**5/4) + x**15*(14*a**6*b**4*d**10 + 168*a**5*b**
5*c*d**9 + 630*a**4*b**6*c**2*d**8 + 960*a**3*b**7*c**3*d**7 + 630*a**2*b**
8*c**4*d**6 + 168*a*b**9*c**5*d**5 + 14*b**10*c**6*d**4) + x**14*(60*a**7*b
**3*d**10/7 + 150*a**6*b**4*c*d**9 + 810*a**5*b**5*c**2*d**8 + 1800*a**4*b*
*6*c**3*d**7 + 1800*a**3*b**7*c**4*d**6 + 810*a**2*b**8*c**5*d**5 + 150*a*b
**9*c**6*d**4 + 60*b**10*c**7*d**3/7) + x**13*(45*a**8*b**2*d**10/13 + 1200
*a**7*b**3*c*d**9/13 + 9450*a**6*b**4*c**2*d**8/13 + 30240*a**5*b**5*c**3*d
**7/13 + 44100*a**4*b**6*c**4*d**6/13 + 30240*a**3*b**7*c**5*d**5/13 + 9450
*a**2*b**8*c**6*d**4/13 + 1200*a*b**9*c**7*d**3/13 + 45*b**10*c**8*d**2/13)
 + x**12*(5*a**9*b*d**10/6 + 75*a**8*b**2*c*d**9/2 + 450*a**7*b**3*c**2*d**
8 + 2100*a**6*b**4*c**3*d**7 + 4410*a**5*b**5*c**4*d**6 + 4410*a**4*b**6*c*
*5*d**5 + 2100*a**3*b**7*c**6*d**4 + 450*a**2*b**8*c**7*d**3 + 75*a*b**9*c*
*8*d**2/2 + 5*b**10*c**9*d/6) + x**11*(a**10*d**10/11 + 100*a**9*b*c*d**9/1
1 + 2025*a**8*b**2*c**2*d**8/11 + 14400*a**7*b**3*c**3*d**7/11 + 44100*a**6
*b**4*c**4*d**6/11 + 63504*a**5*b**5*c**5*d**5/11 + 44100*a**4*b**6*c**6*d*
*4/11 + 14400*a**3*b**7*c**7*d**3/11 + 2025*a**2*b**8*c**8*d**2/11 + 100*a*
b**9*c**9*d/11 + b**10*c**10/11) + x**10*(a**10*c*d**9 + 45*a**9*b*c**2*d**
8 + 540*a**8*b**2*c**3*d**7 + 2520*a**7*b**3*c**4*d**6 + 5292*a**6*b**4*c**
5*d**5 + 5292*a**5*b**5*c**6*d**4 + 2520*a**4*b**6*c**7*d**3 + 540*a**3*b**
7*c**8*d**2 + 45*a**2*b**8*c**9*d + a*b**9*c**10) + x**9*(5*a**10*c**2*d**8
 + 400*a**9*b*c**3*d**7/3 + 1050*a**8*b**2*c**4*d**6 + 3360*a**7*b**3*c**5*
d**5 + 4900*a**6*b**4*c**6*d**4 + 3360*a**5*b**5*c**7*d**3 + 1050*a**4*b**6
*c**8*d**2 + 400*a**3*b**7*c**9*d/3 + 5*a**2*b**8*c**10) + x**8*(15*a**10*c
**3*d**7 + 525*a**9*b*c**4*d**6/2 + 2835*a**8*b**2*c**5*d**5/2 + 3150*a**7*
b**3*c**6*d**4 + 3150*a**6*b**4*c**7*d**3 + 2835*a**5*b**5*c**8*d**2/2 + 52
5*a**4*b**6*c**9*d/2 + 15*a**3*b**7*c**10) + x**7*(30*a**10*c**4*d**6 + 360
```

```
*a**9*b*c**5*d**5 + 1350*a**8*b**2*c**6*d**4 + 14400*a**7*b**3*c**7*d**3/7
+ 1350*a**6*b**4*c**8*d**2 + 360*a**5*b**5*c**9*d + 30*a**4*b**6*c**10) + x
**6*(42*a**10*c**5*d**5 + 350*a**9*b*c**6*d**4 + 900*a**8*b**2*c**7*d**3 +
900*a**7*b**3*c**8*d**2 + 350*a**6*b**4*c**9*d + 42*a**5*b**5*c**10) + x**5
*(42*a**10*c**6*d**4 + 240*a**9*b*c**7*d**3 + 405*a**8*b**2*c**8*d**2 + 240
*a**7*b**3*c**9*d + 42*a**6*b**4*c**10) + x**4*(30*a**10*c**7*d**3 + 225*a*
*9*b*c**8*d**2/2 + 225*a**8*b**2*c**9*d/2 + 30*a**7*b**3*c**10) + x**3*(15*
a**10*c**8*d**2 + 100*a**9*b*c**9*d/3 + 15*a**8*b**2*c**10) + x**2*(5*a**10
*c**9*d + 5*a**9*b*c**10)
```

## 3.1196 $\int (a+bx)^9(c+dx)^{10}\,dx$

**Optimal**. Leaf size=250

$$-\frac{9b^8(c+dx)^{19}(bc-ad)}{19d^{10}}+\frac{2b^7(c+dx)^{18}(bc-ad)^2}{d^{10}}-\frac{84b^6(c+dx)^{17}(bc-ad)^3}{17d^{10}}+\frac{63b^5(c+dx)^{16}(bc-ad)^4}{8d^{10}}-\frac{42b^4(c+}{}$$

**Rubi [A]** time = 1.04, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{9b^8(c+dx)^{19}(bc-ad)}{19d^{10}}+\frac{2b^7(c+dx)^{18}(bc-ad)^2}{d^{10}}-\frac{84b^6(c+dx)^{17}(bc-ad)^3}{17d^{10}}+\frac{63b^5(c+dx)^{16}(bc-ad)^4}{8d^{10}}-\frac{42b^4(c+dx)^{15}(bc-ad)^5}{5d^{10}}+\frac{6b^3(c+dx)^{14}(bc-ad)^6}{d^{10}}-\frac{36b^2(c+dx)^{13}(bc-ad)^7}{13d^{10}}+\frac{3b(c+dx)^{12}(bc-ad)^8}{4d^{10}}-\frac{(c+dx)^{11}(bc-ad)^9}{11d^{10}}+\frac{b^9(c+dx)^{20}}{20d^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)^9*(c + d*x)^11)/(11*d^10) + (3*b*(b*c - a*d)^8*(c + d*x)^12)/
(4*d^10) - (36*b^2*(b*c - a*d)^7*(c + d*x)^13)/(13*d^10) + (6*b^3*(b*c - a*
d)^6*(c + d*x)^14)/d^10 - (42*b^4*(b*c - a*d)^5*(c + d*x)^15)/(5*d^10) + (6
3*b^5*(b*c - a*d)^4*(c + d*x)^16)/(8*d^10) - (84*b^6*(b*c - a*d)^3*(c + d*x
)^17)/(17*d^10) + (2*b^7*(b*c - a*d)^2*(c + d*x)^18)/d^10 - (9*b^8*(b*c - a
*d)*(c + d*x)^19)/(19*d^10) + (b^9*(c + d*x)^20)/(20*d^10)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^9(c+dx)^{10}\,dx=\int \left(\frac{(-bc+ad)^9(c+dx)^{10}}{d^9}+\frac{9b(bc-ad)^8(c+dx)^{11}}{d^9}-\frac{36b^2(bc-ad)^7(c+dx)^{12}}{d^9}+\frac{84b}{}\right.$$

$$=-\frac{(bc-ad)^9(c+dx)^{11}}{11d^{10}}+\frac{3b(bc-ad)^8(c+dx)^{12}}{4d^{10}}-\frac{36b^2(bc-ad)^7(c+dx)^{13}}{13d^{10}}+\frac{6b^3(bc-}{}$$

**Mathematica [B]** time = 0.19, size = 1397, normalized size = 5.59

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9*(c + d*x)^10,x]
```

```
[Out] a^9*c^10*x + (a^8*c^9*(9*b*c + 10*a*d)*x^2)/2 + 3*a^7*c^8*(4*b^2*c^2 + 10*a
*b*c*d + 5*a^2*d^2)*x^3 + (3*a^6*c^7*(28*b^3*c^3 + 120*a*b^2*c^2*d + 135*a^
2*b*c*d^2 + 40*a^3*d^3)*x^4)/4 + (6*a^5*c^6*(21*b^4*c^4 + 140*a*b^3*c^3*d +
 270*a^2*b^2*c^2*d^2 + 180*a^3*b*c*d^3 + 35*a^4*d^4)*x^5)/5 + 3*a^4*c^5*(7*
b^5*c^5 + 70*a*b^4*c^4*d + 210*a^2*b^3*c^3*d^2 + 240*a^3*b^2*c^2*d^3 + 105*
a^4*b*c*d^4 + 14*a^5*d^5)*x^6 + 6*a^3*c^4*(2*b^6*c^6 + 30*a*b^5*c^5*d + 135
*a^2*b^4*c^4*d^2 + 240*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^4 + 54*a^5*b*c*d
^5 + 5*a^6*d^6)*x^7 + (3*a^2*c^3*(6*b^7*c^7 + 140*a*b^6*c^6*d + 945*a^2*b^5
*c^5*d^2 + 2520*a^3*b^4*c^4*d^3 + 2940*a^4*b^3*c^3*d^4 + 1512*a^5*b^2*c^2*d
^5 + 315*a^6*b*c*d^6 + 20*a^7*d^7)*x^8)/4 + a*c^2*(b^8*c^8 + 40*a*b^7*c^7*d
 + 420*a^2*b^6*c^6*d^2 + 1680*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4*d^4 + 2352
*a^5*b^3*c^3*d^5 + 840*a^6*b^2*c^2*d^6 + 120*a^7*b*c*d^7 + 5*a^8*d^8)*x^9 +
 (c*(b^9*c^9 + 90*a*b^8*c^8*d + 1620*a^2*b^7*c^7*d^2 + 10080*a^3*b^6*c^6*d^
3 + 26460*a^4*b^5*c^5*d^4 + 31752*a^5*b^4*c^4*d^5 + 17640*a^6*b^3*c^3*d^6 +
 4320*a^7*b^2*c^2*d^7 + 405*a^8*b*c*d^8 + 10*a^9*d^9)*x^10)/10 + (d*(10*b^9
*c^9 + 405*a*b^8*c^8*d + 4320*a^2*b^7*c^7*d^2 + 17640*a^3*b^6*c^6*d^3 + 317
52*a^4*b^5*c^5*d^4 + 26460*a^5*b^4*c^4*d^5 + 10080*a^6*b^3*c^3*d^6 + 1620*a
^7*b^2*c^2*d^7 + 90*a^8*b*c*d^8 + a^9*d^9)*x^11)/11 + (3*b*d^2*(5*b^8*c^8 +
 120*a*b^7*c^7*d + 840*a^2*b^6*c^6*d^2 + 2352*a^3*b^5*c^5*d^3 + 2940*a^4*b^
4*c^4*d^4 + 1680*a^5*b^3*c^3*d^5 + 420*a^6*b^2*c^2*d^6 + 40*a^7*b*c*d^7 + a
^8*d^8)*x^12)/4 + (6*b^2*d^3*(20*b^7*c^7 + 315*a*b^6*c^6*d + 1512*a^2*b^5*c
^5*d^2 + 2940*a^3*b^4*c^4*d^3 + 2520*a^4*b^3*c^3*d^4 + 945*a^5*b^2*c^2*d^5
+ 140*a^6*b*c*d^6 + 6*a^7*d^7)*x^13)/13 + 3*b^3*d^4*(5*b^6*c^6 + 54*a*b^5*c
^5*d + 180*a^2*b^4*c^4*d^2 + 240*a^3*b^3*c^3*d^3 + 135*a^4*b^2*c^2*d^4 + 30
*a^5*b*c*d^5 + 2*a^6*d^6)*x^14 + (6*b^4*d^5*(14*b^5*c^5 + 105*a*b^4*c^4*d +
 240*a^2*b^3*c^3*d^2 + 210*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 7*a^5*d^5)*x^
15)/5 + (3*b^5*d^6*(35*b^4*c^4 + 180*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 14
0*a^3*b*c*d^3 + 21*a^4*d^4)*x^16)/8 + (3*b^6*d^7*(40*b^3*c^3 + 135*a*b^2*c^
2*d + 120*a^2*b*c*d^2 + 28*a^3*d^3)*x^17)/17 + (b^7*d^8*(5*b^2*c^2 + 10*a*b
*c*d + 4*a^2*d^2)*x^18)/2 + (b^8*d^9*(10*b*c + 9*a*d)*x^19)/19 + (b^9*d^10*
x^20)/20
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^9 (c + dx)^{10} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^9*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^9*(c + d*x)^10, x]
```

**fricas [B]** time = 1.17, size = 1656, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*d^10*b^9 + 10/19*x^19*d^9*c*b^9 + 9/19*x^19*d^10*b^8*a + 5/2*x^18
*d^8*c^2*b^9 + 5*x^18*d^9*c*b^8*a + 2*x^18*d^10*b^7*a^2 + 120/17*x^17*d^7*c
^3*b^9 + 405/17*x^17*d^8*c^2*b^8*a + 360/17*x^17*d^9*c*b^7*a^2 + 84/17*x^17
*d^10*b^6*a^3 + 105/8*x^16*d^6*c^4*b^9 + 135/2*x^16*d^7*c^3*b^8*a + 405/4*x
^16*d^8*c^2*b^7*a^2 + 105/2*x^16*d^9*c*b^6*a^3 + 63/8*x^16*d^10*b^5*a^4 + 8
4/5*x^15*d^5*c^5*b^9 + 126*x^15*d^6*c^4*b^8*a + 288*x^15*d^7*c^3*b^7*a^2 +
252*x^15*d^8*c^2*b^6*a^3 + 84*x^15*d^9*c*b^5*a^4 + 42/5*x^15*d^10*b^4*a^5 +
 15*x^14*d^4*c^6*b^9 + 162*x^14*d^5*c^5*b^8*a + 540*x^14*d^6*c^4*b^7*a^2 +
720*x^14*d^7*c^3*b^6*a^3 + 405*x^14*d^8*c^2*b^5*a^4 + 90*x^14*d^9*c*b^4*a^5
 + 6*x^14*d^10*b^3*a^6 + 120/13*x^13*d^3*c^7*b^9 + 1890/13*x^13*d^4*c^6*b^8
*a + 9072/13*x^13*d^5*c^5*b^7*a^2 + 17640/13*x^13*d^6*c^4*b^6*a^3 + 15120/1
3*x^13*d^7*c^3*b^5*a^4 + 5670/13*x^13*d^8*c^2*b^4*a^5 + 840/13*x^13*d^9*c*b
^3*a^6 + 36/13*x^13*d^10*b^2*a^7 + 15/4*x^12*d^2*c^8*b^9 + 90*x^12*d^3*c^7*
b^8*a + 630*x^12*d^4*c^6*b^7*a^2 + 1764*x^12*d^5*c^5*b^6*a^3 + 2205*x^12*d^
6*c^4*b^5*a^4 + 1260*x^12*d^7*c^3*b^4*a^5 + 315*x^12*d^8*c^2*b^3*a^6 + 30*x
^12*d^9*c*b^2*a^7 + 3/4*x^12*d^10*b*a^8 + 10/11*x^11*d*c^9*b^9 + 405/11*x^1
1*d^2*c^8*b^8*a + 4320/11*x^11*d^3*c^7*b^7*a^2 + 17640/11*x^11*d^4*c^6*b^6*
a^3 + 31752/11*x^11*d^5*c^5*b^5*a^4 + 26460/11*x^11*d^6*c^4*b^4*a^5 + 10080
/11*x^11*d^7*c^3*b^3*a^6 + 1620/11*x^11*d^8*c^2*b^2*a^7 + 90/11*x^11*d^9*c*
b*a^8 + 1/11*x^11*d^10*a^9 + 1/10*x^10*c^10*b^9 + 9*x^10*d*c^9*b^8*a + 162*
x^10*d^2*c^8*b^7*a^2 + 1008*x^10*d^3*c^7*b^6*a^3 + 2646*x^10*d^4*c^6*b^5*a^
4 + 15876/5*x^10*d^5*c^5*b^4*a^5 + 1764*x^10*d^6*c^4*b^3*a^6 + 432*x^10*d^7
*c^3*b^2*a^7 + 81/2*x^10*d^8*c^2*b*a^8 + x^10*d^9*c*a^9 + x^9*c^10*b^8*a +
40*x^9*d*c^9*b^7*a^2 + 420*x^9*d^2*c^8*b^6*a^3 + 1680*x^9*d^3*c^7*b^5*a^4 +
 2940*x^9*d^4*c^6*b^4*a^5 + 2352*x^9*d^5*c^5*b^3*a^6 + 840*x^9*d^6*c^4*b^2*
a^7 + 120*x^9*d^7*c^3*b*a^8 + 5*x^9*d^8*c^2*a^9 + 9/2*x^8*c^10*b^7*a^2 + 10
5*x^8*d*c^9*b^6*a^3 + 2835/4*x^8*d^2*c^8*b^5*a^4 + 1890*x^8*d^3*c^7*b^4*a^5
 + 2205*x^8*d^4*c^6*b^3*a^6 + 1134*x^8*d^5*c^5*b^2*a^7 + 945/4*x^8*d^6*c^4*
b*a^8 + 15*x^8*d^7*c^3*a^9 + 12*x^7*c^10*b^6*a^3 + 180*x^7*d*c^9*b^5*a^4 +
810*x^7*d^2*c^8*b^4*a^5 + 1440*x^7*d^3*c^7*b^3*a^6 + 1080*x^7*d^4*c^6*b^2*a
^7 + 324*x^7*d^5*c^5*b*a^8 + 30*x^7*d^6*c^4*a^9 + 21*x^6*c^10*b^5*a^4 + 210
*x^6*d*c^9*b^4*a^5 + 630*x^6*d^2*c^8*b^3*a^6 + 720*x^6*d^3*c^7*b^2*a^7 + 31
5*x^6*d^4*c^6*b*a^8 + 42*x^6*d^5*c^5*a^9 + 126/5*x^5*c^10*b^4*a^5 + 168*x^5
*d*c^9*b^3*a^6 + 324*x^5*d^2*c^8*b^2*a^7 + 216*x^5*d^3*c^7*b*a^8 + 42*x^5*d
^4*c^6*a^9 + 21*x^4*c^10*b^3*a^6 + 90*x^4*d*c^9*b^2*a^7 + 405/4*x^4*d^2*c^8
*b*a^8 + 30*x^4*d^3*c^7*a^9 + 12*x^3*c^10*b^2*a^7 + 30*x^3*d*c^9*b*a^8 + 15
*x^3*d^2*c^8*a^9 + 9/2*x^2*c^10*b*a^8 + 5*x^2*d*c^9*a^9 + x*c^10*a^9
```

**giac [B]** time = 1.30, size = 1656, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/20*b^9*d^10*x^20 + 10/19*b^9*c*d^9*x^19 + 9/19*a*b^8*d^10*x^19 + 5/2*b^9*
c^2*d^8*x^18 + 5*a*b^8*c*d^9*x^18 + 2*a^2*b^7*d^10*x^18 + 120/17*b^9*c^3*d^
7*x^17 + 405/17*a*b^8*c^2*d^8*x^17 + 360/17*a^2*b^7*c*d^9*x^17 + 84/17*a^3*
b^6*d^10*x^17 + 105/8*b^9*c^4*d^6*x^16 + 135/2*a*b^8*c^3*d^7*x^16 + 405/4*a
^2*b^7*c^2*d^8*x^16 + 105/2*a^3*b^6*c*d^9*x^16 + 63/8*a^4*b^5*d^10*x^16 + 8
4/5*b^9*c^5*d^5*x^15 + 126*a*b^8*c^4*d^6*x^15 + 288*a^2*b^7*c^3*d^7*x^15 +
252*a^3*b^6*c^2*d^8*x^15 + 84*a^4*b^5*c*d^9*x^15 + 42/5*a^5*b^4*d^10*x^15 +
 15*b^9*c^6*d^4*x^14 + 162*a*b^8*c^5*d^5*x^14 + 540*a^2*b^7*c^4*d^6*x^14 +
720*a^3*b^6*c^3*d^7*x^14 + 405*a^4*b^5*c^2*d^8*x^14 + 90*a^5*b^4*c*d^9*x^14
 + 6*a^6*b^3*d^10*x^14 + 120/13*b^9*c^7*d^3*x^13 + 1890/13*a*b^8*c^6*d^4*x^
13 + 9072/13*a^2*b^7*c^5*d^5*x^13 + 17640/13*a^3*b^6*c^4*d^6*x^13 + 15120/1
3*a^4*b^5*c^3*d^7*x^13 + 5670/13*a^5*b^4*c^2*d^8*x^13 + 840/13*a^6*b^3*c*d^
9*x^13 + 36/13*a^7*b^2*d^10*x^13 + 15/4*b^9*c^8*d^2*x^12 + 90*a*b^8*c^7*d^3
*x^12 + 630*a^2*b^7*c^6*d^4*x^12 + 1764*a^3*b^6*c^5*d^5*x^12 + 2205*a^4*b^5
*c^4*d^6*x^12 + 1260*a^5*b^4*c^3*d^7*x^12 + 315*a^6*b^3*c^2*d^8*x^12 + 30*a
^7*b^2*c*d^9*x^12 + 3/4*a^8*b*d^10*x^12 + 10/11*b^9*c^9*d*x^11 + 405/11*a*b
^8*c^8*d^2*x^11 + 4320/11*a^2*b^7*c^7*d^3*x^11 + 17640/11*a^3*b^6*c^6*d^4*x
^11 + 31752/11*a^4*b^5*c^5*d^5*x^11 + 26460/11*a^5*b^4*c^4*d^6*x^11 + 10080
/11*a^6*b^3*c^3*d^7*x^11 + 1620/11*a^7*b^2*c^2*d^8*x^11 + 90/11*a^8*b*c*d^9
*x^11 + 1/11*a^9*d^10*x^11 + 1/10*b^9*c^10*x^10 + 9*a*b^8*c^9*d*x^10 + 162*
a^2*b^7*c^8*d^2*x^10 + 1008*a^3*b^6*c^7*d^3*x^10 + 2646*a^4*b^5*c^6*d^4*x^1
0 + 15876/5*a^5*b^4*c^5*d^5*x^10 + 1764*a^6*b^3*c^4*d^6*x^10 + 432*a^7*b^2*
c^3*d^7*x^10 + 81/2*a^8*b*c^2*d^8*x^10 + a^9*c*d^9*x^10 + a*b^8*c^10*x^9 +
40*a^2*b^7*c^9*d*x^9 + 420*a^3*b^6*c^8*d^2*x^9 + 1680*a^4*b^5*c^7*d^3*x^9 +
 2940*a^5*b^4*c^6*d^4*x^9 + 2352*a^6*b^3*c^5*d^5*x^9 + 840*a^7*b^2*c^4*d^6*
x^9 + 120*a^8*b*c^3*d^7*x^9 + 5*a^9*c^2*d^8*x^9 + 9/2*a^2*b^7*c^10*x^8 + 10
5*a^3*b^6*c^9*d*x^8 + 2835/4*a^4*b^5*c^8*d^2*x^8 + 1890*a^5*b^4*c^7*d^3*x^8
 + 2205*a^6*b^3*c^6*d^4*x^8 + 1134*a^7*b^2*c^5*d^5*x^8 + 945/4*a^8*b*c^4*d^
6*x^8 + 15*a^9*c^3*d^7*x^8 + 12*a^3*b^6*c^10*x^7 + 180*a^4*b^5*c^9*d*x^7 +
810*a^5*b^4*c^8*d^2*x^7 + 1440*a^6*b^3*c^7*d^3*x^7 + 1080*a^7*b^2*c^6*d^4*x
^7 + 324*a^8*b*c^5*d^5*x^7 + 30*a^9*c^4*d^6*x^7 + 21*a^4*b^5*c^10*x^6 + 210
*a^5*b^4*c^9*d*x^6 + 630*a^6*b^3*c^8*d^2*x^6 + 720*a^7*b^2*c^7*d^3*x^6 + 31
5*a^8*b*c^6*d^4*x^6 + 42*a^9*c^5*d^5*x^6 + 126/5*a^5*b^4*c^10*x^5 + 168*a^6
*b^3*c^9*d*x^5 + 324*a^7*b^2*c^8*d^2*x^5 + 216*a^8*b*c^7*d^3*x^5 + 42*a^9*c
^6*d^4*x^5 + 21*a^6*b^3*c^10*x^4 + 90*a^7*b^2*c^9*d*x^4 + 405/4*a^8*b*c^8*d
^2*x^4 + 30*a^9*c^7*d^3*x^4 + 12*a^7*b^2*c^10*x^3 + 30*a^8*b*c^9*d*x^3 + 15
*a^9*c^8*d^2*x^3 + 9/2*a^8*b*c^10*x^2 + 5*a^9*c^9*d*x^2 + a^9*c^10*x
```

**maple [B]** time = 0.00, size = 1441, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^9*(d*x+c)^10,x)

[Out] 1/20*b^9*d^10*x^20+1/19*(9*a*b^8*d^10+10*b^9*c*d^9)*x^19+1/18*(36*a^2*b^7*d^10+90*a*b^8*c*d^9+45*b^9*c^2*d^8)*x^18+1/17*(84*a^3*b^6*d^10+360*a^2*b^7*c*d^9+405*a*b^8*c^2*d^8+120*b^9*c^3*d^7)*x^17+1/16*(126*a^4*b^5*d^10+840*a^3*b^6*c*d^9+1620*a^2*b^7*c^2*d^8+1080*a*b^8*c^3*d^7+210*b^9*c^4*d^6)*x^16+1/15*(126*a^5*b^4*d^10+1260*a^4*b^5*c*d^9+3780*a^3*b^6*c^2*d^8+4320*a^2*b^7*c^3*d^7+1890*a*b^8*c^4*d^6+252*b^9*c^5*d^5)*x^15+1/14*(84*a^6*b^3*d^10+1260*a^5*b^4*c*d^9+5670*a^4*b^5*c^2*d^8+10080*a^3*b^6*c^3*d^7+7560*a^2*b^7*c^4*d^6+2268*a*b^8*c^5*d^5+210*b^9*c^6*d^4)*x^14+1/13*(36*a^7*b^2*d^10+840*a^6*b^3*c*d^9+5670*a^5*b^4*c^2*d^8+15120*a^4*b^5*c^3*d^7+17640*a^3*b^6*c^4*d^6+9072*a^2*b^7*c^5*d^5+1890*a*b^8*c^6*d^4+120*b^9*c^7*d^3)*x^13+1/12*(9*a^8*b*d^10+360*a^7*b^2*c*d^9+3780*a^6*b^3*c^2*d^8+15120*a^5*b^4*c^3*d^7+26460*a^4*b^5*c^4*d^6+21168*a^3*b^6*c^5*d^5+7560*a^2*b^7*c^6*d^4+1080*a*b^8*c^7*d^3+45*b^9*c^8*d^2)*x^12+1/11*(a^9*d^10+90*a^8*b*c*d^9+1620*a^7*b^2*c^2*d^8+10080*a^6*b^3*c^3*d^7+26460*a^5*b^4*c^4*d^6+31752*a^4*b^5*c^5*d^5+17640*a^3*b^6*c^6*d^4+4320*a^2*b^7*c^7*d^3+405*a*b^8*c^8*d^2+10*b^9*c^9*d)*x^11+1/10*(10*a^9*c*d^9+405*a^8*b*c^2*d^8+4320*a^7*b^2*c^3*d^7+17640*a^6*b^3*c^4*d^6+31752*a^5*b^4*c^5*d^5+26460*a^4*b^5*c^6*d^4+10080*a^3*b^6*c^7*d^3+1620*a^2*b^7*c^8*d^2+90*a*b^8*c^9*d+b^9*c^10)*x^10+1/9*(45*a^9*c^2*d^8+1080*a^8*b*c^3*d^7+7560*a^7*b^2*c^4*d^6+21168*a^6*b^3*c^5*d^5+26460*a^5*b^4*c^6*d^4+15120*a^4*b^5*c^7*d^3+3780*a^3*b^6*c^8*d^2+360*a^2*b^7*c^9*d+9*a*b^8*c^10)*x^9+1/8*(120*a^9*c^3*d^7+1890*a^8*b*c^4*d^6+9072*a^7*b^2*c^5*d^5+17640*a^6*b^3*c^6*d^4+15120*a^5*b^4*c^7*d^3+5670*a^4*b^5*c^8*d^2+840*a^3*b^6*c^9*d+36*a^2*b^7*c^10)*x^8+1/7*(210*a^9*c^4*d^6+2268*a^8*b*c^5*d^5+7560*a^7*b^2*c^6*d^4+10080*a^6*b^3*c^7*d^3+5670*a^5*b^4*c^8*d^2+1260*a^4*b^5*c^9*d+84*a^3*b^6*c^10)*x^7+1/6*(252*a^9*c^5*d^5+1890*a^8*b*c^6*d^4+4320*a^7*b^2*c^7*d^3+3780*a^6*b^3*c^8*d^2+1260*a^5*b^4*c^9*d+126*a^4*b^5*c^10)*x^6+1/5*(210*a^9*c^6*d^4+1080*a^8*b*c^7*d^3+1620*a^7*b^2*c^8*d^2+840*a^6*b^3*c^9*d+126*a^5*b^4*c^10)*x^5+1/4*(120*a^9*c^7*d^3+405*a^8*b*c^8*d^2+360*a^7*b^2*c^9*d+84*a^6*b^3*c^10)*x^4+1/3*(45*a^9*c^8*d^2+90*a^8*b*c^9*d+36*a^7*b^2*c^10)*x^3+1/2*(10*a^9*c^9*d+9*a^8*b*c^10)*x^2+a^9*c^10*x

**maxima [B]** time = 1.51, size = 1437, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="maxima")

[Out] 1/20*b^9*d^10*x^20 + a^9*c^10*x + 1/19*(10*b^9*c*d^9 + 9*a*b^8*d^10)*x^19 + 1/2*(5*b^9*c^2*d^8 + 10*a*b^8*c*d^9 + 4*a^2*b^7*d^10)*x^18 + 3/17*(40*b^9*c^3*d^7 + 135*a*b^8*c^2*d^8 + 120*a^2*b^7*c*d^9 + 28*a^3*b^6*d^10)*x^17 + 3/8*(35*b^9*c^4*d^6 + 180*a*b^8*c^3*d^7 + 270*a^2*b^7*c^2*d^8 + 140*a^3*b^6*

```
c*d^9 + 21*a^4*b^5*d^10)*x^16 + 6/5*(14*b^9*c^5*d^5 + 105*a*b^8*c^4*d^6 + 2
40*a^2*b^7*c^3*d^7 + 210*a^3*b^6*c^2*d^8 + 70*a^4*b^5*c*d^9 + 7*a^5*b^4*d^1
0)*x^15 + 3*(5*b^9*c^6*d^4 + 54*a*b^8*c^5*d^5 + 180*a^2*b^7*c^4*d^6 + 240*a
^3*b^6*c^3*d^7 + 135*a^4*b^5*c^2*d^8 + 30*a^5*b^4*c*d^9 + 2*a^6*b^3*d^10)*x
^14 + 6/13*(20*b^9*c^7*d^3 + 315*a*b^8*c^6*d^4 + 1512*a^2*b^7*c^5*d^5 + 294
0*a^3*b^6*c^4*d^6 + 2520*a^4*b^5*c^3*d^7 + 945*a^5*b^4*c^2*d^8 + 140*a^6*b^
3*c*d^9 + 6*a^7*b^2*d^10)*x^13 + 3/4*(5*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 8
40*a^2*b^7*c^6*d^4 + 2352*a^3*b^6*c^5*d^5 + 2940*a^4*b^5*c^4*d^6 + 1680*a^5
*b^4*c^3*d^7 + 420*a^6*b^3*c^2*d^8 + 40*a^7*b^2*c*d^9 + a^8*b*d^10)*x^12 +
1/11*(10*b^9*c^9*d + 405*a*b^8*c^8*d^2 + 4320*a^2*b^7*c^7*d^3 + 17640*a^3*b
^6*c^6*d^4 + 31752*a^4*b^5*c^5*d^5 + 26460*a^5*b^4*c^4*d^6 + 10080*a^6*b^3*
c^3*d^7 + 1620*a^7*b^2*c^2*d^8 + 90*a^8*b*c*d^9 + a^9*d^10)*x^11 + 1/10*(b^
9*c^10 + 90*a*b^8*c^9*d + 1620*a^2*b^7*c^8*d^2 + 10080*a^3*b^6*c^7*d^3 + 26
460*a^4*b^5*c^6*d^4 + 31752*a^5*b^4*c^5*d^5 + 17640*a^6*b^3*c^4*d^6 + 4320*
a^7*b^2*c^3*d^7 + 405*a^8*b*c^2*d^8 + 10*a^9*c*d^9)*x^10 + (a*b^8*c^10 + 40
*a^2*b^7*c^9*d + 420*a^3*b^6*c^8*d^2 + 1680*a^4*b^5*c^7*d^3 + 2940*a^5*b^4*
c^6*d^4 + 2352*a^6*b^3*c^5*d^5 + 840*a^7*b^2*c^4*d^6 + 120*a^8*b*c^3*d^7 +
5*a^9*c^2*d^8)*x^9 + 3/4*(6*a^2*b^7*c^10 + 140*a^3*b^6*c^9*d + 945*a^4*b^5*
c^8*d^2 + 2520*a^5*b^4*c^7*d^3 + 2940*a^6*b^3*c^6*d^4 + 1512*a^7*b^2*c^5*d^
5 + 315*a^8*b*c^4*d^6 + 20*a^9*c^3*d^7)*x^8 + 6*(2*a^3*b^6*c^10 + 30*a^4*b^
5*c^9*d + 135*a^5*b^4*c^8*d^2 + 240*a^6*b^3*c^7*d^3 + 180*a^7*b^2*c^6*d^4 +
 54*a^8*b*c^5*d^5 + 5*a^9*c^4*d^6)*x^7 + 3*(7*a^4*b^5*c^10 + 70*a^5*b^4*c^9
*d + 210*a^6*b^3*c^8*d^2 + 240*a^7*b^2*c^7*d^3 + 105*a^8*b*c^6*d^4 + 14*a^9
*c^5*d^5)*x^6 + 6/5*(21*a^5*b^4*c^10 + 140*a^6*b^3*c^9*d + 270*a^7*b^2*c^8*
d^2 + 180*a^8*b*c^7*d^3 + 35*a^9*c^6*d^4)*x^5 + 3/4*(28*a^6*b^3*c^10 + 120*
a^7*b^2*c^9*d + 135*a^8*b*c^8*d^2 + 40*a^9*c^7*d^3)*x^4 + 3*(4*a^7*b^2*c^10
 + 10*a^8*b*c^9*d + 5*a^9*c^8*d^2)*x^3 + 1/2*(9*a^8*b*c^10 + 10*a^9*c^9*d)*
x^2
```

**mupad [B]** time = 0.79, size = 1404, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9*(c + d*x)^10,x)
```

```
[Out] x^7*(12*a^3*b^6*c^10 + 30*a^9*c^4*d^6 + 180*a^4*b^5*c^9*d + 324*a^8*b*c^5*d
^5 + 810*a^5*b^4*c^8*d^2 + 1440*a^6*b^3*c^7*d^3 + 1080*a^7*b^2*c^6*d^4) + x
^14*(6*a^6*b^3*d^10 + 15*b^9*c^6*d^4 + 162*a*b^8*c^5*d^5 + 90*a^5*b^4*c*d^9
 + 540*a^2*b^7*c^4*d^6 + 720*a^3*b^6*c^3*d^7 + 405*a^4*b^5*c^2*d^8) + x^5*(
(126*a^5*b^4*c^10)/5 + 42*a^9*c^6*d^4 + 168*a^6*b^3*c^9*d + 216*a^8*b*c^7*d
^3 + 324*a^7*b^2*c^8*d^2) + x^16*((63*a^4*b^5*d^10)/8 + (105*b^9*c^4*d^6)/8
 + (135*a*b^8*c^3*d^7)/2 + (105*a^3*b^6*c*d^9)/2 + (405*a^2*b^7*c^2*d^8)/4)
 + x^8*((9*a^2*b^7*c^10)/2 + 15*a^9*c^3*d^7 + 105*a^3*b^6*c^9*d + (945*a^8*
b*c^4*d^6)/4 + (2835*a^4*b^5*c^8*d^2)/4 + 1890*a^5*b^4*c^7*d^3 + 2205*a^6*b
```

```
^3*c^6*d^4 + 1134*a^7*b^2*c^5*d^5) + x^13*((36*a^7*b^2*d^10)/13 + (120*b^9*
c^7*d^3)/13 + (1890*a*b^8*c^6*d^4)/13 + (840*a^6*b^3*c*d^9)/13 + (9072*a^2*
b^7*c^5*d^5)/13 + (17640*a^3*b^6*c^4*d^6)/13 + (15120*a^4*b^5*c^3*d^7)/13 +
 (5670*a^5*b^4*c^2*d^8)/13) + x^9*(a*b^8*c^10 + 5*a^9*c^2*d^8 + 40*a^2*b^7*
c^9*d + 120*a^8*b*c^3*d^7 + 420*a^3*b^6*c^8*d^2 + 1680*a^4*b^5*c^7*d^3 + 29
40*a^5*b^4*c^6*d^4 + 2352*a^6*b^3*c^5*d^5 + 840*a^7*b^2*c^4*d^6) + x^12*((3
*a^8*b*d^10)/4 + (15*b^9*c^8*d^2)/4 + 90*a*b^8*c^7*d^3 + 30*a^7*b^2*c*d^9 +
 630*a^2*b^7*c^6*d^4 + 1764*a^3*b^6*c^5*d^5 + 2205*a^4*b^5*c^4*d^6 + 1260*a
^5*b^4*c^3*d^7 + 315*a^6*b^3*c^2*d^8) + x^6*(21*a^4*b^5*c^10 + 42*a^9*c^5*d
^5 + 210*a^5*b^4*c^9*d + 315*a^8*b*c^6*d^4 + 630*a^6*b^3*c^8*d^2 + 720*a^7*
b^2*c^7*d^3) + x^15*((42*a^5*b^4*d^10)/5 + (84*b^9*c^5*d^5)/5 + 126*a*b^8*c
^4*d^6 + 84*a^4*b^5*c*d^9 + 288*a^2*b^7*c^3*d^7 + 252*a^3*b^6*c^2*d^8) + x^
10*((b^9*c^10)/10 + a^9*c*d^9 + (81*a^8*b*c^2*d^8)/2 + 162*a^2*b^7*c^8*d^2
+ 1008*a^3*b^6*c^7*d^3 + 2646*a^4*b^5*c^6*d^4 + (15876*a^5*b^4*c^5*d^5)/5 +
 1764*a^6*b^3*c^4*d^6 + 432*a^7*b^2*c^3*d^7 + 9*a*b^8*c^9*d) + x^11*((a^9*d
^10)/11 + (10*b^9*c^9*d)/11 + (405*a*b^8*c^8*d^2)/11 + (4320*a^2*b^7*c^7*d^
3)/11 + (17640*a^3*b^6*c^6*d^4)/11 + (31752*a^4*b^5*c^5*d^5)/11 + (26460*a^
5*b^4*c^4*d^6)/11 + (10080*a^6*b^3*c^3*d^7)/11 + (1620*a^7*b^2*c^2*d^8)/11
+ (90*a^8*b*c*d^9)/11) + a^9*c^10*x + (b^9*d^10*x^20)/20 + (3*a^6*c^7*x^4*(
40*a^3*d^3 + 28*b^3*c^3 + 120*a*b^2*c^2*d + 135*a^2*b*c*d^2))/4 + (3*b^6*d^
7*x^17*(28*a^3*d^3 + 40*b^3*c^3 + 135*a*b^2*c^2*d + 120*a^2*b*c*d^2))/17 +
(a^8*c^9*x^2*(10*a*d + 9*b*c))/2 + (b^8*d^9*x^19*(9*a*d + 10*b*c))/19 + 3*a
^7*c^8*x^3*(5*a^2*d^2 + 4*b^2*c^2 + 10*a*b*c*d) + (b^7*d^8*x^18*(4*a^2*d^2
+ 5*b^2*c^2 + 10*a*b*c*d))/2
```

**sympy [B]** time = 0.30, size = 1598, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9*(d*x+c)**10,x)
```

```
[Out] a**9*c**10*x + b**9*d**10*x**20/20 + x**19*(9*a*b**8*d**10/19 + 10*b**9*c*d
**9/19) + x**18*(2*a**2*b**7*d**10 + 5*a*b**8*c*d**9 + 5*b**9*c**2*d**8/2)
+ x**17*(84*a**3*b**6*d**10/17 + 360*a**2*b**7*c*d**9/17 + 405*a*b**8*c**2*
d**8/17 + 120*b**9*c**3*d**7/17) + x**16*(63*a**4*b**5*d**10/8 + 105*a**3*b
**6*c*d**9/2 + 405*a**2*b**7*c**2*d**8/4 + 135*a*b**8*c**3*d**7/2 + 105*b**
9*c**4*d**6/8) + x**15*(42*a**5*b**4*d**10/5 + 84*a**4*b**5*c*d**9 + 252*a*
*3*b**6*c**2*d**8 + 288*a**2*b**7*c**3*d**7 + 126*a*b**8*c**4*d**6 + 84*b**
9*c**5*d**5/5) + x**14*(6*a**6*b**3*d**10 + 90*a**5*b**4*c*d**9 + 405*a**4*
b**5*c**2*d**8 + 720*a**3*b**6*c**3*d**7 + 540*a**2*b**7*c**4*d**6 + 162*a*
b**8*c**5*d**5 + 15*b**9*c**6*d**4) + x**13*(36*a**7*b**2*d**10/13 + 840*a*
*6*b**3*c*d**9/13 + 5670*a**5*b**4*c**2*d**8/13 + 15120*a**4*b**5*c**3*d**7
/13 + 17640*a**3*b**6*c**4*d**6/13 + 9072*a**2*b**7*c**5*d**5/13 + 1890*a*b
**8*c**6*d**4/13 + 120*b**9*c**7*d**3/13) + x**12*(3*a**8*b*d**10/4 + 30*a*
```

```
*7*b**2*c*d**9 + 315*a**6*b**3*c**2*d**8 + 1260*a**5*b**4*c**3*d**7 + 2205*
a**4*b**5*c**4*d**6 + 1764*a**3*b**6*c**5*d**5 + 630*a**2*b**7*c**6*d**4 +
90*a*b**8*c**7*d**3 + 15*b**9*c**8*d**2/4) + x**11*(a**9*d**10/11 + 90*a**8
*b*c*d**9/11 + 1620*a**7*b**2*c**2*d**8/11 + 10080*a**6*b**3*c**3*d**7/11 +
 26460*a**5*b**4*c**4*d**6/11 + 31752*a**4*b**5*c**5*d**5/11 + 17640*a**3*b
**6*c**6*d**4/11 + 4320*a**2*b**7*c**7*d**3/11 + 405*a*b**8*c**8*d**2/11 +
10*b**9*c**9*d/11) + x**10*(a**9*c*d**9 + 81*a**8*b*c**2*d**8/2 + 432*a**7*
b**2*c**3*d**7 + 1764*a**6*b**3*c**4*d**6 + 15876*a**5*b**4*c**5*d**5/5 + 2
646*a**4*b**5*c**6*d**4 + 1008*a**3*b**6*c**7*d**3 + 162*a**2*b**7*c**8*d**
2 + 9*a*b**8*c**9*d + b**9*c**10/10) + x**9*(5*a**9*c**2*d**8 + 120*a**8*b*
c**3*d**7 + 840*a**7*b**2*c**4*d**6 + 2352*a**6*b**3*c**5*d**5 + 2940*a**5*
b**4*c**6*d**4 + 1680*a**4*b**5*c**7*d**3 + 420*a**3*b**6*c**8*d**2 + 40*a*
*2*b**7*c**9*d + a*b**8*c**10) + x**8*(15*a**9*c**3*d**7 + 945*a**8*b*c**4*
d**6/4 + 1134*a**7*b**2*c**5*d**5 + 2205*a**6*b**3*c**6*d**4 + 1890*a**5*b*
*4*c**7*d**3 + 2835*a**4*b**5*c**8*d**2/4 + 105*a**3*b**6*c**9*d + 9*a**2*b
**7*c**10/2) + x**7*(30*a**9*c**4*d**6 + 324*a**8*b*c**5*d**5 + 1080*a**7*b
**2*c**6*d**4 + 1440*a**6*b**3*c**7*d**3 + 810*a**5*b**4*c**8*d**2 + 180*a*
*4*b**5*c**9*d + 12*a**3*b**6*c**10) + x**6*(42*a**9*c**5*d**5 + 315*a**8*b
*c**6*d**4 + 720*a**7*b**2*c**7*d**3 + 630*a**6*b**3*c**8*d**2 + 210*a**5*b
**4*c**9*d + 21*a**4*b**5*c**10) + x**5*(42*a**9*c**6*d**4 + 216*a**8*b*c**
7*d**3 + 324*a**7*b**2*c**8*d**2 + 168*a**6*b**3*c**9*d + 126*a**5*b**4*c**
10/5) + x**4*(30*a**9*c**7*d**3 + 405*a**8*b*c**8*d**2/4 + 90*a**7*b**2*c**
9*d + 21*a**6*b**3*c**10) + x**3*(15*a**9*c**8*d**2 + 30*a**8*b*c**9*d + 12
*a**7*b**2*c**10) + x**2*(5*a**9*c**9*d + 9*a**8*b*c**10/2)
```

## 3.1197 $\int (a+bx)^8(c+dx)^{10}\,dx$

**Optimal**. Leaf size=225

$$-\frac{4b^7(c+dx)^{18}(bc-ad)}{9d^9}+\frac{28b^6(c+dx)^{17}(bc-ad)^2}{17d^9}-\frac{7b^5(c+dx)^{16}(bc-ad)^3}{2d^9}+\frac{14b^4(c+dx)^{15}(bc-ad)^4}{3d^9}-\frac{4b^3(c+d}{}$$

**Rubi [A]** time = 0.90, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{4b^7(c+dx)^{18}(bc-ad)}{9d^9}+\frac{28b^6(c+dx)^{17}(bc-ad)^2}{17d^9}-\frac{7b^5(c+dx)^{16}(bc-ad)^3}{2d^9}+\frac{14b^4(c+dx)^{15}(bc-ad)^4}{3d^9}-\frac{4b^3(c+dx)^{14}(bc-ad)^5}{d^9}+\frac{28b^2(c+dx)^{13}(bc-ad)^6}{13d^9}-\frac{2b(c+dx)^{12}(bc-ad)^7}{3d^9}+\frac{(c+dx)^{11}(bc-ad)^8}{11d^9}+\frac{b^8(c+dx)^{19}}{19d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^8*(c + d*x)^11)/(11*d^9) - (2*b*(b*c - a*d)^7*(c + d*x)^12)/(3
*d^9) + (28*b^2*(b*c - a*d)^6*(c + d*x)^13)/(13*d^9) - (4*b^3*(b*c - a*d)^5
*(c + d*x)^14)/d^9 + (14*b^4*(b*c - a*d)^4*(c + d*x)^15)/(3*d^9) - (7*b^5*(
b*c - a*d)^3*(c + d*x)^16)/(2*d^9) + (28*b^6*(b*c - a*d)^2*(c + d*x)^17)/(1
7*d^9) - (4*b^7*(b*c - a*d)*(c + d*x)^18)/(9*d^9) + (b^8*(c + d*x)^19)/(19*
d^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^8(c+dx)^{10}\,dx=\int \left(\frac{(-bc+ad)^8(c+dx)^{10}}{d^8}-\frac{8b(bc-ad)^7(c+dx)^{11}}{d^8}+\frac{28b^2(bc-ad)^6(c+dx)^{12}}{d^8}-\frac{56b}{}\right.$$
$$=\frac{(bc-ad)^8(c+dx)^{11}}{11d^9}-\frac{2b(bc-ad)^7(c+dx)^{12}}{3d^9}+\frac{28b^2(bc-ad)^6(c+dx)^{13}}{13d^9}-\frac{4b^3(bc-a}{}$$

**Mathematica [B]** time = 0.16, size = 1241, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8*(c + d*x)^10,x]
```

```
[Out] a^8*c^10*x + a^7*c^9*(4*b*c + 5*a*d)*x^2 + (a^6*c^8*(28*b^2*c^2 + 80*a*b*c*
d + 45*a^2*d^2)*x^3)/3 + 2*a^5*c^7*(7*b^3*c^3 + 35*a*b^2*c^2*d + 45*a^2*b*c
*d^2 + 15*a^3*d^3)*x^4 + 2*a^4*c^6*(7*b^4*c^4 + 56*a*b^3*c^3*d + 126*a^2*b^
2*c^2*d^2 + 96*a^3*b*c*d^3 + 21*a^4*d^4)*x^5 + (14*a^3*c^5*(2*b^5*c^5 + 25*
a*b^4*c^4*d + 90*a^2*b^3*c^3*d^2 + 120*a^3*b^2*c^2*d^3 + 60*a^4*b*c*d^4 + 9
*a^5*d^5)*x^6)/3 + 2*a^2*c^4*(2*b^6*c^6 + 40*a*b^5*c^5*d + 225*a^2*b^4*c^4*
d^2 + 480*a^3*b^3*c^3*d^3 + 420*a^4*b^2*c^2*d^4 + 144*a^5*b*c*d^5 + 15*a^6*
d^6)*x^7 + a*c^3*(b^7*c^7 + 35*a*b^6*c^6*d + 315*a^2*b^5*c^5*d^2 + 1050*a^3
*b^4*c^4*d^3 + 1470*a^4*b^3*c^3*d^4 + 882*a^5*b^2*c^2*d^5 + 210*a^6*b*c*d^6
 + 15*a^7*d^7)*x^8 + (c^2*(b^8*c^8 + 80*a*b^7*c^7*d + 1260*a^2*b^6*c^6*d^2
+ 6720*a^3*b^5*c^5*d^3 + 14700*a^4*b^4*c^4*d^4 + 14112*a^5*b^3*c^3*d^5 + 58
80*a^6*b^2*c^2*d^6 + 960*a^7*b*c*d^7 + 45*a^8*d^8)*x^9)/9 + c*d*(b^8*c^8 +
36*a*b^7*c^7*d + 336*a^2*b^6*c^6*d^2 + 1176*a^3*b^5*c^5*d^3 + 1764*a^4*b^4*
c^4*d^4 + 1176*a^5*b^3*c^3*d^5 + 336*a^6*b^2*c^2*d^6 + 36*a^7*b*c*d^7 + a^8
*d^8)*x^10 + (d^2*(45*b^8*c^8 + 960*a*b^7*c^7*d + 5880*a^2*b^6*c^6*d^2 + 14
112*a^3*b^5*c^5*d^3 + 14700*a^4*b^4*c^4*d^4 + 6720*a^5*b^3*c^3*d^5 + 1260*a
^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + a^8*d^8)*x^11)/11 + (2*b*d^3*(15*b^7*c^7
+ 210*a*b^6*c^6*d + 882*a^2*b^5*c^5*d^2 + 1470*a^3*b^4*c^4*d^3 + 1050*a^4*b
^3*c^3*d^4 + 315*a^5*b^2*c^2*d^5 + 35*a^6*b*c*d^6 + a^7*d^7)*x^12)/3 + (14*
b^2*d^4*(15*b^6*c^6 + 144*a*b^5*c^5*d + 420*a^2*b^4*c^4*d^2 + 480*a^3*b^3*c
^3*d^3 + 225*a^4*b^2*c^2*d^4 + 40*a^5*b*c*d^5 + 2*a^6*d^6)*x^13)/13 + 2*b^3
*d^5*(9*b^5*c^5 + 60*a*b^4*c^4*d + 120*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3
 + 25*a^4*b*c*d^4 + 2*a^5*d^5)*x^14 + (2*b^4*d^6*(21*b^4*c^4 + 96*a*b^3*c^3
*d + 126*a^2*b^2*c^2*d^2 + 56*a^3*b*c*d^3 + 7*a^4*d^4)*x^15)/3 + (b^5*d^7*(
15*b^3*c^3 + 45*a*b^2*c^2*d + 35*a^2*b*c*d^2 + 7*a^3*d^3)*x^16)/2 + (b^6*d^
8*(45*b^2*c^2 + 80*a*b*c*d + 28*a^2*d^2)*x^17)/17 + (b^7*d^9*(5*b*c + 4*a*d
)*x^18)/9 + (b^8*d^10*x^19)/19
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^8(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^8*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^8*(c + d*x)^10, x]
```

**fricas [B]** time = 1.17, size = 1478, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*d^10*b^8 + 5/9*x^18*d^9*c*b^8 + 4/9*x^18*d^10*b^7*a + 45/17*x^17*
d^8*c^2*b^8 + 80/17*x^17*d^9*c*b^7*a + 28/17*x^17*d^10*b^6*a^2 + 15/2*x^16*
d^7*c^3*b^8 + 45/2*x^16*d^8*c^2*b^7*a + 35/2*x^16*d^9*c*b^6*a^2 + 7/2*x^16*
d^10*b^5*a^3 + 14*x^15*d^6*c^4*b^8 + 64*x^15*d^7*c^3*b^7*a + 84*x^15*d^8*c^
2*b^6*a^2 + 112/3*x^15*d^9*c*b^5*a^3 + 14/3*x^15*d^10*b^4*a^4 + 18*x^14*d^5
*c^5*b^8 + 120*x^14*d^6*c^4*b^7*a + 240*x^14*d^7*c^3*b^6*a^2 + 180*x^14*d^8
*c^2*b^5*a^3 + 50*x^14*d^9*c*b^4*a^4 + 4*x^14*d^10*b^3*a^5 + 210/13*x^13*d^
4*c^6*b^8 + 2016/13*x^13*d^5*c^5*b^7*a + 5880/13*x^13*d^6*c^4*b^6*a^2 + 672
0/13*x^13*d^7*c^3*b^5*a^3 + 3150/13*x^13*d^8*c^2*b^4*a^4 + 560/13*x^13*d^9*
c*b^3*a^5 + 28/13*x^13*d^10*b^2*a^6 + 10*x^12*d^3*c^7*b^8 + 140*x^12*d^4*c^
6*b^7*a + 588*x^12*d^5*c^5*b^6*a^2 + 980*x^12*d^6*c^4*b^5*a^3 + 700*x^12*d^
7*c^3*b^4*a^4 + 210*x^12*d^8*c^2*b^3*a^5 + 70/3*x^12*d^9*c*b^2*a^6 + 2/3*x^
12*d^10*b*a^7 + 45/11*x^11*d^2*c^8*b^8 + 960/11*x^11*d^3*c^7*b^7*a + 5880/1
1*x^11*d^4*c^6*b^6*a^2 + 14112/11*x^11*d^5*c^5*b^5*a^3 + 14700/11*x^11*d^6*
c^4*b^4*a^4 + 6720/11*x^11*d^7*c^3*b^3*a^5 + 1260/11*x^11*d^8*c^2*b^2*a^6 +
 80/11*x^11*d^9*c*b*a^7 + 1/11*x^11*d^10*a^8 + x^10*d*c^9*b^8 + 36*x^10*d^2
*c^8*b^7*a + 336*x^10*d^3*c^7*b^6*a^2 + 1176*x^10*d^4*c^6*b^5*a^3 + 1764*x^
10*d^5*c^5*b^4*a^4 + 1176*x^10*d^6*c^4*b^3*a^5 + 336*x^10*d^7*c^3*b^2*a^6 +
 36*x^10*d^8*c^2*b*a^7 + x^10*d^9*c*a^8 + 1/9*x^9*c^10*b^8 + 80/9*x^9*d*c^9
*b^7*a + 140*x^9*d^2*c^8*b^6*a^2 + 2240/3*x^9*d^3*c^7*b^5*a^3 + 4900/3*x^9*
d^4*c^6*b^4*a^4 + 1568*x^9*d^5*c^5*b^3*a^5 + 1960/3*x^9*d^6*c^4*b^2*a^6 + 3
20/3*x^9*d^7*c^3*b*a^7 + 5*x^9*d^8*c^2*a^8 + x^8*c^10*b^7*a + 35*x^8*d*c^9*
b^6*a^2 + 315*x^8*d^2*c^8*b^5*a^3 + 1050*x^8*d^3*c^7*b^4*a^4 + 1470*x^8*d^4
*c^6*b^3*a^5 + 882*x^8*d^5*c^5*b^2*a^6 + 210*x^8*d^6*c^4*b*a^7 + 15*x^8*d^7
*c^3*a^8 + 4*x^7*c^10*b^6*a^2 + 80*x^7*d*c^9*b^5*a^3 + 450*x^7*d^2*c^8*b^4*
a^4 + 960*x^7*d^3*c^7*b^3*a^5 + 840*x^7*d^4*c^6*b^2*a^6 + 288*x^7*d^5*c^5*b
*a^7 + 30*x^7*d^6*c^4*a^8 + 28/3*x^6*c^10*b^5*a^3 + 350/3*x^6*d*c^9*b^4*a^4
 + 420*x^6*d^2*c^8*b^3*a^5 + 560*x^6*d^3*c^7*b^2*a^6 + 280*x^6*d^4*c^6*b*a^
7 + 42*x^6*d^5*c^5*a^8 + 14*x^5*c^10*b^4*a^4 + 112*x^5*d*c^9*b^3*a^5 + 252*
x^5*d^2*c^8*b^2*a^6 + 192*x^5*d^3*c^7*b*a^7 + 42*x^5*d^4*c^6*a^8 + 14*x^4*c
^10*b^3*a^5 + 70*x^4*d*c^9*b^2*a^6 + 90*x^4*d^2*c^8*b*a^7 + 30*x^4*d^3*c^7*
a^8 + 28/3*x^3*c^10*b^2*a^6 + 80/3*x^3*d*c^9*b*a^7 + 15*x^3*d^2*c^8*a^8 + 4
*x^2*c^10*b*a^7 + 5*x^2*d*c^9*a^8 + x*c^10*a^8
```

**giac [B]** time = 1.29, size = 1478, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/19*b^8*d^10*x^19 + 5/9*b^8*c*d^9*x^18 + 4/9*a*b^7*d^10*x^18 + 45/17*b^8*c
^2*d^8*x^17 + 80/17*a*b^7*c*d^9*x^17 + 28/17*a^2*b^6*d^10*x^17 + 15/2*b^8*c
^3*d^7*x^16 + 45/2*a*b^7*c^2*d^8*x^16 + 35/2*a^2*b^6*c*d^9*x^16 + 7/2*a^3*b
^5*d^10*x^16 + 14*b^8*c^4*d^6*x^15 + 64*a*b^7*c^3*d^7*x^15 + 84*a^2*b^6*c^2
```

```
*d^8*x^15 + 112/3*a^3*b^5*c*d^9*x^15 + 14/3*a^4*b^4*d^10*x^15 + 18*b^8*c^5*
d^5*x^14 + 120*a*b^7*c^4*d^6*x^14 + 240*a^2*b^6*c^3*d^7*x^14 + 180*a^3*b^5*
c^2*d^8*x^14 + 50*a^4*b^4*c*d^9*x^14 + 4*a^5*b^3*d^10*x^14 + 210/13*b^8*c^6
*d^4*x^13 + 2016/13*a*b^7*c^5*d^5*x^13 + 5880/13*a^2*b^6*c^4*d^6*x^13 + 672
0/13*a^3*b^5*c^3*d^7*x^13 + 3150/13*a^4*b^4*c^2*d^8*x^13 + 560/13*a^5*b^3*c
*d^9*x^13 + 28/13*a^6*b^2*d^10*x^13 + 10*b^8*c^7*d^3*x^12 + 140*a*b^7*c^6*d
^4*x^12 + 588*a^2*b^6*c^5*d^5*x^12 + 980*a^3*b^5*c^4*d^6*x^12 + 700*a^4*b^4
*c^3*d^7*x^12 + 210*a^5*b^3*c^2*d^8*x^12 + 70/3*a^6*b^2*c*d^9*x^12 + 2/3*a^
7*b*d^10*x^12 + 45/11*b^8*c^8*d^2*x^11 + 960/11*a*b^7*c^7*d^3*x^11 + 5880/1
1*a^2*b^6*c^6*d^4*x^11 + 14112/11*a^3*b^5*c^5*d^5*x^11 + 14700/11*a^4*b^4*c
^4*d^6*x^11 + 6720/11*a^5*b^3*c^3*d^7*x^11 + 1260/11*a^6*b^2*c^2*d^8*x^11 +
 80/11*a^7*b*c*d^9*x^11 + 1/11*a^8*d^10*x^11 + b^8*c^9*d*x^10 + 36*a*b^7*c^
8*d^2*x^10 + 336*a^2*b^6*c^7*d^3*x^10 + 1176*a^3*b^5*c^6*d^4*x^10 + 1764*a^
4*b^4*c^5*d^5*x^10 + 1176*a^5*b^3*c^4*d^6*x^10 + 336*a^6*b^2*c^3*d^7*x^10 +
 36*a^7*b*c^2*d^8*x^10 + a^8*c*d^9*x^10 + 1/9*b^8*c^10*x^9 + 80/9*a*b^7*c^9
*d*x^9 + 140*a^2*b^6*c^8*d^2*x^9 + 2240/3*a^3*b^5*c^7*d^3*x^9 + 4900/3*a^4*
b^4*c^6*d^4*x^9 + 1568*a^5*b^3*c^5*d^5*x^9 + 1960/3*a^6*b^2*c^4*d^6*x^9 + 3
20/3*a^7*b*c^3*d^7*x^9 + 5*a^8*c^2*d^8*x^9 + a*b^7*c^10*x^8 + 35*a^2*b^6*c^
9*d*x^8 + 315*a^3*b^5*c^8*d^2*x^8 + 1050*a^4*b^4*c^7*d^3*x^8 + 1470*a^5*b^3
*c^6*d^4*x^8 + 882*a^6*b^2*c^5*d^5*x^8 + 210*a^7*b*c^4*d^6*x^8 + 15*a^8*c^3
*d^7*x^8 + 4*a^2*b^6*c^10*x^7 + 80*a^3*b^5*c^9*d*x^7 + 450*a^4*b^4*c^8*d^2*
x^7 + 960*a^5*b^3*c^7*d^3*x^7 + 840*a^6*b^2*c^6*d^4*x^7 + 288*a^7*b*c^5*d^5
*x^7 + 30*a^8*c^4*d^6*x^7 + 28/3*a^3*b^5*c^10*x^6 + 350/3*a^4*b^4*c^9*d*x^6
 + 420*a^5*b^3*c^8*d^2*x^6 + 560*a^6*b^2*c^7*d^3*x^6 + 280*a^7*b*c^6*d^4*x^
6 + 42*a^8*c^5*d^5*x^6 + 14*a^4*b^4*c^10*x^5 + 112*a^5*b^3*c^9*d*x^5 + 252*
a^6*b^2*c^8*d^2*x^5 + 192*a^7*b*c^7*d^3*x^5 + 42*a^8*c^6*d^4*x^5 + 14*a^5*b
^3*c^10*x^4 + 70*a^6*b^2*c^9*d*x^4 + 90*a^7*b*c^8*d^2*x^4 + 30*a^8*c^7*d^3*
x^4 + 28/3*a^6*b^2*c^10*x^3 + 80/3*a^7*b*c^9*d*x^3 + 15*a^8*c^8*d^2*x^3 + 4
*a^7*b*c^10*x^2 + 5*a^8*c^9*d*x^2 + a^8*c^10*x
```

**maple [B]** time = 0.00, size = 1291, normalized size = 5.74

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8*(d*x+c)^10,x)
```

```
[Out] 1/19*b^8*d^10*x^19+1/18*(8*a*b^7*d^10+10*b^8*c*d^9)*x^18+1/17*(28*a^2*b^6*d
^10+80*a*b^7*c*d^9+45*b^8*c^2*d^8)*x^17+1/16*(56*a^3*b^5*d^10+280*a^2*b^6*c
*d^9+360*a*b^7*c^2*d^8+120*b^8*c^3*d^7)*x^16+1/15*(70*a^4*b^4*d^10+560*a^3*
b^5*c*d^9+1260*a^2*b^6*c^2*d^8+960*a*b^7*c^3*d^7+210*b^8*c^4*d^6)*x^15+1/14
*(56*a^5*b^3*d^10+700*a^4*b^4*c*d^9+2520*a^3*b^5*c^2*d^8+3360*a^2*b^6*c^3*d
^7+1680*a*b^7*c^4*d^6+252*b^8*c^5*d^5)*x^14+1/13*(28*a^6*b^2*d^10+560*a^5*b
^3*c*d^9+3150*a^4*b^4*c^2*d^8+6720*a^3*b^5*c^3*d^7+5880*a^2*b^6*c^4*d^6+201
6*a*b^7*c^5*d^5+210*b^8*c^6*d^4)*x^13+1/12*(8*a^7*b*d^10+280*a^6*b^2*c*d^9+
```

```
2520*a^5*b^3*c^2*d^8+8400*a^4*b^4*c^3*d^7+11760*a^3*b^5*c^4*d^6+7056*a^2*b^
6*c^5*d^5+1680*a*b^7*c^6*d^4+120*b^8*c^7*d^3)*x^12+1/11*(a^8*d^10+80*a^7*b*
c*d^9+1260*a^6*b^2*c^2*d^8+6720*a^5*b^3*c^3*d^7+14700*a^4*b^4*c^4*d^6+14112
*a^3*b^5*c^5*d^5+5880*a^2*b^6*c^6*d^4+960*a*b^7*c^7*d^3+45*b^8*c^8*d^2)*x^1
1+1/10*(10*a^8*c*d^9+360*a^7*b*c^2*d^8+3360*a^6*b^2*c^3*d^7+11760*a^5*b^3*c
^4*d^6+17640*a^4*b^4*c^5*d^5+11760*a^3*b^5*c^6*d^4+3360*a^2*b^6*c^7*d^3+360
*a*b^7*c^8*d^2+10*b^8*c^9*d)*x^10+1/9*(45*a^8*c^2*d^8+960*a^7*b*c^3*d^7+588
0*a^6*b^2*c^4*d^6+14112*a^5*b^3*c^5*d^5+14700*a^4*b^4*c^6*d^4+6720*a^3*b^5*
c^7*d^3+1260*a^2*b^6*c^8*d^2+80*a*b^7*c^9*d+b^8*c^10)*x^9+1/8*(120*a^8*c^3*
d^7+1680*a^7*b*c^4*d^6+7056*a^6*b^2*c^5*d^5+11760*a^5*b^3*c^6*d^4+8400*a^4*
b^4*c^7*d^3+2520*a^3*b^5*c^8*d^2+280*a^2*b^6*c^9*d+8*a*b^7*c^10)*x^8+1/7*(2
10*a^8*c^4*d^6+2016*a^7*b*c^5*d^5+5880*a^6*b^2*c^6*d^4+6720*a^5*b^3*c^7*d^3
+3150*a^4*b^4*c^8*d^2+560*a^3*b^5*c^9*d+28*a^2*b^6*c^10)*x^7+1/6*(252*a^8*c
^5*d^5+1680*a^7*b*c^6*d^4+3360*a^6*b^2*c^7*d^3+2520*a^5*b^3*c^8*d^2+700*a^4
*b^4*c^9*d+56*a^3*b^5*c^10)*x^6+1/5*(210*a^8*c^6*d^4+960*a^7*b*c^7*d^3+1260
*a^6*b^2*c^8*d^2+560*a^5*b^3*c^9*d+70*a^4*b^4*c^10)*x^5+1/4*(120*a^8*c^7*d^
3+360*a^7*b*c^8*d^2+280*a^6*b^2*c^9*d+56*a^5*b^3*c^10)*x^4+1/3*(45*a^8*c^8*
d^2+80*a^7*b*c^9*d+28*a^6*b^2*c^10)*x^3+1/2*(10*a^8*c^9*d+8*a^7*b*c^10)*x^2
+a^8*c^10*x
```

**maxima [B]** time = 1.51, size = 1283, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/19*b^8*d^10*x^19 + a^8*c^10*x + 1/9*(5*b^8*c*d^9 + 4*a*b^7*d^10)*x^18 + 1
/17*(45*b^8*c^2*d^8 + 80*a*b^7*c*d^9 + 28*a^2*b^6*d^10)*x^17 + 1/2*(15*b^8*
c^3*d^7 + 45*a*b^7*c^2*d^8 + 35*a^2*b^6*c*d^9 + 7*a^3*b^5*d^10)*x^16 + 2/3*
(21*b^8*c^4*d^6 + 96*a*b^7*c^3*d^7 + 126*a^2*b^6*c^2*d^8 + 56*a^3*b^5*c*d^9
 + 7*a^4*b^4*d^10)*x^15 + 2*(9*b^8*c^5*d^5 + 60*a*b^7*c^4*d^6 + 120*a^2*b^6
*c^3*d^7 + 90*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 + 2*a^5*b^3*d^10)*x^14 + 1
4/13*(15*b^8*c^6*d^4 + 144*a*b^7*c^5*d^5 + 420*a^2*b^6*c^4*d^6 + 480*a^3*b^
5*c^3*d^7 + 225*a^4*b^4*c^2*d^8 + 40*a^5*b^3*c*d^9 + 2*a^6*b^2*d^10)*x^13 +
 2/3*(15*b^8*c^7*d^3 + 210*a*b^7*c^6*d^4 + 882*a^2*b^6*c^5*d^5 + 1470*a^3*b
^5*c^4*d^6 + 1050*a^4*b^4*c^3*d^7 + 315*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9
+ a^7*b*d^10)*x^12 + 1/11*(45*b^8*c^8*d^2 + 960*a*b^7*c^7*d^3 + 5880*a^2*b^
6*c^6*d^4 + 14112*a^3*b^5*c^5*d^5 + 14700*a^4*b^4*c^4*d^6 + 6720*a^5*b^3*c^
3*d^7 + 1260*a^6*b^2*c^2*d^8 + 80*a^7*b*c*d^9 + a^8*d^10)*x^11 + (b^8*c^9*d
 + 36*a*b^7*c^8*d^2 + 336*a^2*b^6*c^7*d^3 + 1176*a^3*b^5*c^6*d^4 + 1764*a^4
*b^4*c^5*d^5 + 1176*a^5*b^3*c^4*d^6 + 336*a^6*b^2*c^3*d^7 + 36*a^7*b*c^2*d^
8 + a^8*c*d^9)*x^10 + 1/9*(b^8*c^10 + 80*a*b^7*c^9*d + 1260*a^2*b^6*c^8*d^2
 + 6720*a^3*b^5*c^7*d^3 + 14700*a^4*b^4*c^6*d^4 + 14112*a^5*b^3*c^5*d^5 + 5
880*a^6*b^2*c^4*d^6 + 960*a^7*b*c^3*d^7 + 45*a^8*c^2*d^8)*x^9 + (a*b^7*c^10
 + 35*a^2*b^6*c^9*d + 315*a^3*b^5*c^8*d^2 + 1050*a^4*b^4*c^7*d^3 + 1470*a^5
```

```
*b^3*c^6*d^4 + 882*a^6*b^2*c^5*d^5 + 210*a^7*b*c^4*d^6 + 15*a^8*c^3*d^7)*x^
8 + 2*(2*a^2*b^6*c^10 + 40*a^3*b^5*c^9*d + 225*a^4*b^4*c^8*d^2 + 480*a^5*b^
3*c^7*d^3 + 420*a^6*b^2*c^6*d^4 + 144*a^7*b*c^5*d^5 + 15*a^8*c^4*d^6)*x^7 +
 14/3*(2*a^3*b^5*c^10 + 25*a^4*b^4*c^9*d + 90*a^5*b^3*c^8*d^2 + 120*a^6*b^2
*c^7*d^3 + 60*a^7*b*c^6*d^4 + 9*a^8*c^5*d^5)*x^6 + 2*(7*a^4*b^4*c^10 + 56*a
^5*b^3*c^9*d + 126*a^6*b^2*c^8*d^2 + 96*a^7*b*c^7*d^3 + 21*a^8*c^6*d^4)*x^5
 + 2*(7*a^5*b^3*c^10 + 35*a^6*b^2*c^9*d + 45*a^7*b*c^8*d^2 + 15*a^8*c^7*d^3
)*x^4 + 1/3*(28*a^6*b^2*c^10 + 80*a^7*b*c^9*d + 45*a^8*c^8*d^2)*x^3 + (4*a^
7*b*c^10 + 5*a^8*c^9*d)*x^2
```

**mupad [B]** time = 0.71, size = 1253, normalized size = 5.57

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8*(c + d*x)^10,x)
```

```
[Out] x^7*(4*a^2*b^6*c^10 + 30*a^8*c^4*d^6 + 80*a^3*b^5*c^9*d + 288*a^7*b*c^5*d^5
 + 450*a^4*b^4*c^8*d^2 + 960*a^5*b^3*c^7*d^3 + 840*a^6*b^2*c^6*d^4) + x^13*
((28*a^6*b^2*d^10)/13 + (210*b^8*c^6*d^4)/13 + (2016*a*b^7*c^5*d^5)/13 + (5
60*a^5*b^3*c*d^9)/13 + (5880*a^2*b^6*c^4*d^6)/13 + (6720*a^3*b^5*c^3*d^7)/1
3 + (3150*a^4*b^4*c^2*d^8)/13) + x^8*(a*b^7*c^10 + 15*a^8*c^3*d^7 + 35*a^2*
b^6*c^9*d + 210*a^7*b*c^4*d^6 + 315*a^3*b^5*c^8*d^2 + 1050*a^4*b^4*c^7*d^3
+ 1470*a^5*b^3*c^6*d^4 + 882*a^6*b^2*c^5*d^5) + x^12*((2*a^7*b*d^10)/3 + 10
*b^8*c^7*d^3 + 140*a*b^7*c^6*d^4 + (70*a^6*b^2*c*d^9)/3 + 588*a^2*b^6*c^5*d
^5 + 980*a^3*b^5*c^4*d^6 + 700*a^4*b^4*c^3*d^7 + 210*a^5*b^3*c^2*d^8) + x^1
0*(a^8*c*d^9 + b^8*c^9*d + 36*a*b^7*c^8*d^2 + 36*a^7*b*c^2*d^8 + 336*a^2*b^
6*c^7*d^3 + 1176*a^3*b^5*c^6*d^4 + 1764*a^4*b^4*c^5*d^5 + 1176*a^5*b^3*c^4*
d^6 + 336*a^6*b^2*c^3*d^7) + x^5*(14*a^4*b^4*c^10 + 42*a^8*c^6*d^4 + 112*a^
5*b^3*c^9*d + 192*a^7*b*c^7*d^3 + 252*a^6*b^2*c^8*d^2) + x^15*((14*a^4*b^4*
d^10)/3 + 14*b^8*c^4*d^6 + 64*a*b^7*c^3*d^7 + (112*a^3*b^5*c*d^9)/3 + 84*a^
2*b^6*c^2*d^8) + x^6*((28*a^3*b^5*c^10)/3 + 42*a^8*c^5*d^5 + (350*a^4*b^4*c
^9*d)/3 + 280*a^7*b*c^6*d^4 + 420*a^5*b^3*c^8*d^2 + 560*a^6*b^2*c^7*d^3) +
x^14*(4*a^5*b^3*d^10 + 18*b^8*c^5*d^5 + 120*a*b^7*c^4*d^6 + 50*a^4*b^4*c*d^
9 + 240*a^2*b^6*c^3*d^7 + 180*a^3*b^5*c^2*d^8) + x^9*((b^8*c^10)/9 + 5*a^8*
c^2*d^8 + (320*a^7*b*c^3*d^7)/3 + 140*a^2*b^6*c^8*d^2 + (2240*a^3*b^5*c^7*d
^3)/3 + (4900*a^4*b^4*c^6*d^4)/3 + 1568*a^5*b^3*c^5*d^5 + (1960*a^6*b^2*c^4
*d^6)/3 + (80*a*b^7*c^9*d)/9) + x^11*((a^8*d^10)/11 + (45*b^8*c^8*d^2)/11 +
 (960*a*b^7*c^7*d^3)/11 + (5880*a^2*b^6*c^6*d^4)/11 + (14112*a^3*b^5*c^5*d^
5)/11 + (14700*a^4*b^4*c^4*d^6)/11 + (6720*a^5*b^3*c^3*d^7)/11 + (1260*a^6*
b^2*c^2*d^8)/11 + (80*a^7*b*c*d^9)/11) + a^8*c^10*x + (b^8*d^10*x^19)/19 +
2*a^5*c^7*x^4*(15*a^3*d^3 + 7*b^3*c^3 + 35*a*b^2*c^2*d + 45*a^2*b*c*d^2) +
(b^5*d^7*x^16*(7*a^3*d^3 + 15*b^3*c^3 + 45*a*b^2*c^2*d + 35*a^2*b*c*d^2))/2
 + a^7*c^9*x^2*(5*a*d + 4*b*c) + (b^7*d^9*x^18*(4*a*d + 5*b*c))/9 + (a^6*c^
```

```
8*x^3*(45*a^2*d^2 + 28*b^2*c^2 + 80*a*b*c*d))/3 + (b^6*d^8*x^17*(28*a^2*d^2
 + 45*b^2*c^2 + 80*a*b*c*d))/17
```

**sympy [B]** time = 0.27, size = 1428, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8*(d*x+c)**10,x)
```

```
[Out] a**8*c**10*x + b**8*d**10*x**19/19 + x**18*(4*a*b**7*d**10/9 + 5*b**8*c*d**
9/9) + x**17*(28*a**2*b**6*d**10/17 + 80*a*b**7*c*d**9/17 + 45*b**8*c**2*d*
*8/17) + x**16*(7*a**3*b**5*d**10/2 + 35*a**2*b**6*c*d**9/2 + 45*a*b**7*c**
2*d**8/2 + 15*b**8*c**3*d**7/2) + x**15*(14*a**4*b**4*d**10/3 + 112*a**3*b*
*5*c*d**9/3 + 84*a**2*b**6*c**2*d**8 + 64*a*b**7*c**3*d**7 + 14*b**8*c**4*d
**6) + x**14*(4*a**5*b**3*d**10 + 50*a**4*b**4*c*d**9 + 180*a**3*b**5*c**2*
d**8 + 240*a**2*b**6*c**3*d**7 + 120*a*b**7*c**4*d**6 + 18*b**8*c**5*d**5)
+ x**13*(28*a**6*b**2*d**10/13 + 560*a**5*b**3*c*d**9/13 + 3150*a**4*b**4*c
**2*d**8/13 + 6720*a**3*b**5*c**3*d**7/13 + 5880*a**2*b**6*c**4*d**6/13 + 2
016*a*b**7*c**5*d**5/13 + 210*b**8*c**6*d**4/13) + x**12*(2*a**7*b*d**10/3
+ 70*a**6*b**2*c*d**9/3 + 210*a**5*b**3*c**2*d**8 + 700*a**4*b**4*c**3*d**7
 + 980*a**3*b**5*c**4*d**6 + 588*a**2*b**6*c**5*d**5 + 140*a*b**7*c**6*d**4
 + 10*b**8*c**7*d**3) + x**11*(a**8*d**10/11 + 80*a**7*b*c*d**9/11 + 1260*a
**6*b**2*c**2*d**8/11 + 6720*a**5*b**3*c**3*d**7/11 + 14700*a**4*b**4*c**4*
d**6/11 + 14112*a**3*b**5*c**5*d**5/11 + 5880*a**2*b**6*c**6*d**4/11 + 960*
a*b**7*c**7*d**3/11 + 45*b**8*c**8*d**2/11) + x**10*(a**8*c*d**9 + 36*a**7*
b*c**2*d**8 + 336*a**6*b**2*c**3*d**7 + 1176*a**5*b**3*c**4*d**6 + 1764*a**
4*b**4*c**5*d**5 + 1176*a**3*b**5*c**6*d**4 + 336*a**2*b**6*c**7*d**3 + 36*
a*b**7*c**8*d**2 + b**8*c**9*d) + x**9*(5*a**8*c**2*d**8 + 320*a**7*b*c**3*
d**7/3 + 1960*a**6*b**2*c**4*d**6/3 + 1568*a**5*b**3*c**5*d**5 + 4900*a**4*
b**4*c**6*d**4/3 + 2240*a**3*b**5*c**7*d**3/3 + 140*a**2*b**6*c**8*d**2 + 8
0*a*b**7*c**9*d/9 + b**8*c**10/9) + x**8*(15*a**8*c**3*d**7 + 210*a**7*b*c*
*4*d**6 + 882*a**6*b**2*c**5*d**5 + 1470*a**5*b**3*c**6*d**4 + 1050*a**4*b*
*4*c**7*d**3 + 315*a**3*b**5*c**8*d**2 + 35*a**2*b**6*c**9*d + a*b**7*c**10
) + x**7*(30*a**8*c**4*d**6 + 288*a**7*b*c**5*d**5 + 840*a**6*b**2*c**6*d**
4 + 960*a**5*b**3*c**7*d**3 + 450*a**4*b**4*c**8*d**2 + 80*a**3*b**5*c**9*d
 + 4*a**2*b**6*c**10) + x**6*(42*a**8*c**5*d**5 + 280*a**7*b*c**6*d**4 + 56
0*a**6*b**2*c**7*d**3 + 420*a**5*b**3*c**8*d**2 + 350*a**4*b**4*c**9*d/3 +
28*a**3*b**5*c**10/3) + x**5*(42*a**8*c**6*d**4 + 192*a**7*b*c**7*d**3 + 25
2*a**6*b**2*c**8*d**2 + 112*a**5*b**3*c**9*d + 14*a**4*b**4*c**10) + x**4*(
30*a**8*c**7*d**3 + 90*a**7*b*c**8*d**2 + 70*a**6*b**2*c**9*d + 14*a**5*b**
3*c**10) + x**3*(15*a**8*c**8*d**2 + 80*a**7*b*c**9*d/3 + 28*a**6*b**2*c**1
0/3) + x**2*(5*a**8*c**9*d + 4*a**7*b*c**10)
```

## 3.1198 $\int (a+bx)^7(c+dx)^{10}\,dx$

**Optimal**. Leaf size=200

$$-\frac{7b^6(c+dx)^{17}(bc-ad)}{17d^8}+\frac{21b^5(c+dx)^{16}(bc-ad)^2}{16d^8}-\frac{7b^4(c+dx)^{15}(bc-ad)^3}{3d^8}+\frac{5b^3(c+dx)^{14}(bc-ad)^4}{2d^8}-\frac{21b^2(c+$$

**Rubi** [A] time = 0.77, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{7b^6(c+dx)^{17}(bc-ad)}{17d^8}+\frac{21b^5(c+dx)^{16}(bc-ad)^2}{16d^8}-\frac{7b^4(c+dx)^{15}(bc-ad)^3}{3d^8}+\frac{5b^3(c+dx)^{14}(bc-ad)^4}{2d^8}-\frac{21b^2(c+dx)^{13}(bc-ad)^5}{13d^8}+\frac{7b(c+dx)^{12}(bc-ad)^6}{12d^8}-\frac{(c+dx)^{11}(bc-ad)^7}{11d^8}+\frac{b^7(c+dx)^{18}}{18d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)^7*(c + d*x)^11)/(11*d^8) + (7*b*(b*c - a*d)^6*(c + d*x)^12)/(
12*d^8) - (21*b^2*(b*c - a*d)^5*(c + d*x)^13)/(13*d^8) + (5*b^3*(b*c - a*d)
^4*(c + d*x)^14)/(2*d^8) - (7*b^4*(b*c - a*d)^3*(c + d*x)^15)/(3*d^8) + (21
*b^5*(b*c - a*d)^2*(c + d*x)^16)/(16*d^8) - (7*b^6*(b*c - a*d)*(c + d*x)^17
)/(17*d^8) + (b^7*(c + d*x)^18)/(18*d^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^7(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^7(c+dx)^{10}}{d^7}+\frac{7b(bc-ad)^6(c+dx)^{11}}{d^7}-\frac{21b^2(bc-ad)^5(c+dx)^{12}}{d^7}+\frac{35\right.\\ &= -\frac{(bc-ad)^7(c+dx)^{11}}{11d^8}+\frac{7b(bc-ad)^6(c+dx)^{12}}{12d^8}-\frac{21b^2(bc-ad)^5(c+dx)^{13}}{13d^8}+\frac{5b^3(bc\end{aligned}$$

**Mathematica** [B] time = 0.14, size = 1105, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] a^7*c^10*x + (a^6*c^9*(7*b*c + 10*a*d)*x^2)/2 + (a^5*c^8*(21*b^2*c^2 + 70*a
*b*c*d + 45*a^2*d^2)*x^3)/3 + (5*a^4*c^7*(7*b^3*c^3 + 42*a*b^2*c^2*d + 63*a
^2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + 7*a^3*c^6*(b^4*c^4 + 10*a*b^3*c^3*d + 27*
a^2*b^2*c^2*d^2 + 24*a^3*b*c*d^3 + 6*a^4*d^4)*x^5 + (7*a^2*c^5*(3*b^5*c^5 +
 50*a*b^4*c^4*d + 225*a^2*b^3*c^3*d^2 + 360*a^3*b^2*c^2*d^3 + 210*a^4*b*c*d
^4 + 36*a^5*d^5)*x^6)/6 + a*c^4*(b^6*c^6 + 30*a*b^5*c^5*d + 225*a^2*b^4*c^4
*d^2 + 600*a^3*b^3*c^3*d^3 + 630*a^4*b^2*c^2*d^4 + 252*a^5*b*c*d^5 + 30*a^6
*d^6)*x^7 + (c^3*(b^7*c^7 + 70*a*b^6*c^6*d + 945*a^2*b^5*c^5*d^2 + 4200*a^3
*b^4*c^4*d^3 + 7350*a^4*b^3*c^3*d^4 + 5292*a^5*b^2*c^2*d^5 + 1470*a^6*b*c*d
^6 + 120*a^7*d^7)*x^8)/8 + (5*c^2*d*(2*b^7*c^7 + 63*a*b^6*c^6*d + 504*a^2*b
^5*c^5*d^2 + 1470*a^3*b^4*c^4*d^3 + 1764*a^4*b^3*c^3*d^4 + 882*a^5*b^2*c^2*
d^5 + 168*a^6*b*c*d^6 + 9*a^7*d^7)*x^9)/9 + (c*d^2*(9*b^7*c^7 + 168*a*b^6*c
^6*d + 882*a^2*b^5*c^5*d^2 + 1764*a^3*b^4*c^4*d^3 + 1470*a^4*b^3*c^3*d^4 +
504*a^5*b^2*c^2*d^5 + 63*a^6*b*c*d^6 + 2*a^7*d^7)*x^10)/2 + (d^3*(120*b^7*c
^7 + 1470*a*b^6*c^6*d + 5292*a^2*b^5*c^5*d^2 + 7350*a^3*b^4*c^4*d^3 + 4200*
a^4*b^3*c^3*d^4 + 945*a^5*b^2*c^2*d^5 + 70*a^6*b*c*d^6 + a^7*d^7)*x^11)/11
+ (7*b*d^4*(30*b^6*c^6 + 252*a*b^5*c^5*d + 630*a^2*b^4*c^4*d^2 + 600*a^3*b^
3*c^3*d^3 + 225*a^4*b^2*c^2*d^4 + 30*a^5*b*c*d^5 + a^6*d^6)*x^12)/12 + (7*b
^2*d^5*(36*b^5*c^5 + 210*a*b^4*c^4*d + 360*a^2*b^3*c^3*d^2 + 225*a^3*b^2*c^
2*d^3 + 50*a^4*b*c*d^4 + 3*a^5*d^5)*x^13)/13 + (5*b^3*d^6*(6*b^4*c^4 + 24*a
*b^3*c^3*d + 27*a^2*b^2*c^2*d^2 + 10*a^3*b*c*d^3 + a^4*d^4)*x^14)/2 + (b^4*
d^7*(24*b^3*c^3 + 63*a*b^2*c^2*d + 42*a^2*b*c*d^2 + 7*a^3*d^3)*x^15)/3 + (b
^5*d^8*(45*b^2*c^2 + 70*a*b*c*d + 21*a^2*d^2)*x^16)/16 + (b^6*d^9*(10*b*c +
 7*a*d)*x^17)/17 + (b^7*d^10*x^18)/18
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^7(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7*(c + d*x)^10, x]
```

**fricas [B]** time = 1.08, size = 1302, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*d^10*b^7 + 10/17*x^17*d^9*c*b^7 + 7/17*x^17*d^10*b^6*a + 45/16*x^
16*d^8*c^2*b^7 + 35/8*x^16*d^9*c*b^6*a + 21/16*x^16*d^10*b^5*a^2 + 8*x^15*d
```

```
^7*c^3*b^7 + 21*x^15*d^8*c^2*b^6*a + 14*x^15*d^9*c*b^5*a^2 + 7/3*x^15*d^10*
b^4*a^3 + 15*x^14*d^6*c^4*b^7 + 60*x^14*d^7*c^3*b^6*a + 135/2*x^14*d^8*c^2*
b^5*a^2 + 25*x^14*d^9*c*b^4*a^3 + 5/2*x^14*d^10*b^3*a^4 + 252/13*x^13*d^5*c
^5*b^7 + 1470/13*x^13*d^6*c^4*b^6*a + 2520/13*x^13*d^7*c^3*b^5*a^2 + 1575/1
3*x^13*d^8*c^2*b^4*a^3 + 350/13*x^13*d^9*c*b^3*a^4 + 21/13*x^13*d^10*b^2*a^
5 + 35/2*x^12*d^4*c^6*b^7 + 147*x^12*d^5*c^5*b^6*a + 735/2*x^12*d^6*c^4*b^5
*a^2 + 350*x^12*d^7*c^3*b^4*a^3 + 525/4*x^12*d^8*c^2*b^3*a^4 + 35/2*x^12*d^
9*c*b^2*a^5 + 7/12*x^12*d^10*b*a^6 + 120/11*x^11*d^3*c^7*b^7 + 1470/11*x^11
*d^4*c^6*b^6*a + 5292/11*x^11*d^5*c^5*b^5*a^2 + 7350/11*x^11*d^6*c^4*b^4*a^
3 + 4200/11*x^11*d^7*c^3*b^3*a^4 + 945/11*x^11*d^8*c^2*b^2*a^5 + 70/11*x^11
*d^9*c*b*a^6 + 1/11*x^11*d^10*a^7 + 9/2*x^10*d^2*c^8*b^7 + 84*x^10*d^3*c^7*
b^6*a + 441*x^10*d^4*c^6*b^5*a^2 + 882*x^10*d^5*c^5*b^4*a^3 + 735*x^10*d^6*
c^4*b^3*a^4 + 252*x^10*d^7*c^3*b^2*a^5 + 63/2*x^10*d^8*c^2*b*a^6 + x^10*d^9
*c*a^7 + 10/9*x^9*d*c^9*b^7 + 35*x^9*d^2*c^8*b^6*a + 280*x^9*d^3*c^7*b^5*a^
2 + 2450/3*x^9*d^4*c^6*b^4*a^3 + 980*x^9*d^5*c^5*b^3*a^4 + 490*x^9*d^6*c^4*
b^2*a^5 + 280/3*x^9*d^7*c^3*b*a^6 + 5*x^9*d^8*c^2*a^7 + 1/8*x^8*c^10*b^7 +
35/4*x^8*d*c^9*b^6*a + 945/8*x^8*d^2*c^8*b^5*a^2 + 525*x^8*d^3*c^7*b^4*a^3
+ 3675/4*x^8*d^4*c^6*b^3*a^4 + 1323/2*x^8*d^5*c^5*b^2*a^5 + 735/4*x^8*d^6*c
^4*b*a^6 + 15*x^8*d^7*c^3*a^7 + x^7*c^10*b^6*a + 30*x^7*d*c^9*b^5*a^2 + 225
*x^7*d^2*c^8*b^4*a^3 + 600*x^7*d^3*c^7*b^3*a^4 + 630*x^7*d^4*c^6*b^2*a^5 +
252*x^7*d^5*c^5*b*a^6 + 30*x^7*d^6*c^4*a^7 + 7/2*x^6*c^10*b^5*a^2 + 175/3*x
^6*d*c^9*b^4*a^3 + 525/2*x^6*d^2*c^8*b^3*a^4 + 420*x^6*d^3*c^7*b^2*a^5 + 24
5*x^6*d^4*c^6*b*a^6 + 42*x^6*d^5*c^5*a^7 + 7*x^5*c^10*b^4*a^3 + 70*x^5*d*c^
9*b^3*a^4 + 189*x^5*d^2*c^8*b^2*a^5 + 168*x^5*d^3*c^7*b*a^6 + 42*x^5*d^4*c^
6*a^7 + 35/4*x^4*c^10*b^3*a^4 + 105/2*x^4*d*c^9*b^2*a^5 + 315/4*x^4*d^2*c^8
*b*a^6 + 30*x^4*d^3*c^7*a^7 + 7*x^3*c^10*b^2*a^5 + 70/3*x^3*d*c^9*b*a^6 + 1
5*x^3*d^2*c^8*a^7 + 7/2*x^2*c^10*b*a^6 + 5*x^2*d*c^9*a^7 + x*c^10*a^7
```

**giac [B]** time = 1.26, size = 1302, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/18*b^7*d^10*x^18 + 10/17*b^7*c*d^9*x^17 + 7/17*a*b^6*d^10*x^17 + 45/16*b^
7*c^2*d^8*x^16 + 35/8*a*b^6*c*d^9*x^16 + 21/16*a^2*b^5*d^10*x^16 + 8*b^7*c^
3*d^7*x^15 + 21*a*b^6*c^2*d^8*x^15 + 14*a^2*b^5*c*d^9*x^15 + 7/3*a^3*b^4*d^
10*x^15 + 15*b^7*c^4*d^6*x^14 + 60*a*b^6*c^3*d^7*x^14 + 135/2*a^2*b^5*c^2*d
^8*x^14 + 25*a^3*b^4*c*d^9*x^14 + 5/2*a^4*b^3*d^10*x^14 + 252/13*b^7*c^5*d^
5*x^13 + 1470/13*a*b^6*c^4*d^6*x^13 + 2520/13*a^2*b^5*c^3*d^7*x^13 + 1575/1
3*a^3*b^4*c^2*d^8*x^13 + 350/13*a^4*b^3*c*d^9*x^13 + 21/13*a^5*b^2*d^10*x^1
3 + 35/2*b^7*c^6*d^4*x^12 + 147*a*b^6*c^5*d^5*x^12 + 735/2*a^2*b^5*c^4*d^6*
x^12 + 350*a^3*b^4*c^3*d^7*x^12 + 525/4*a^4*b^3*c^2*d^8*x^12 + 35/2*a^5*b^2
*c*d^9*x^12 + 7/12*a^6*b*d^10*x^12 + 120/11*b^7*c^7*d^3*x^11 + 1470/11*a*b^
```

```
6*c^6*d^4*x^11 + 5292/11*a^2*b^5*c^5*d^5*x^11 + 7350/11*a^3*b^4*c^4*d^6*x^1
1 + 4200/11*a^4*b^3*c^3*d^7*x^11 + 945/11*a^5*b^2*c^2*d^8*x^11 + 70/11*a^6*
b*c*d^9*x^11 + 1/11*a^7*d^10*x^11 + 9/2*b^7*c^8*d^2*x^10 + 84*a*b^6*c^7*d^3
*x^10 + 441*a^2*b^5*c^6*d^4*x^10 + 882*a^3*b^4*c^5*d^5*x^10 + 735*a^4*b^3*c
^4*d^6*x^10 + 252*a^5*b^2*c^3*d^7*x^10 + 63/2*a^6*b*c^2*d^8*x^10 + a^7*c*d^
9*x^10 + 10/9*b^7*c^9*d*x^9 + 35*a*b^6*c^8*d^2*x^9 + 280*a^2*b^5*c^7*d^3*x^
9 + 2450/3*a^3*b^4*c^6*d^4*x^9 + 980*a^4*b^3*c^5*d^5*x^9 + 490*a^5*b^2*c^4*
d^6*x^9 + 280/3*a^6*b*c^3*d^7*x^9 + 5*a^7*c^2*d^8*x^9 + 1/8*b^7*c^10*x^8 +
35/4*a*b^6*c^9*d*x^8 + 945/8*a^2*b^5*c^8*d^2*x^8 + 525*a^3*b^4*c^7*d^3*x^8
+ 3675/4*a^4*b^3*c^6*d^4*x^8 + 1323/2*a^5*b^2*c^5*d^5*x^8 + 735/4*a^6*b*c^4
*d^6*x^8 + 15*a^7*c^3*d^7*x^8 + a*b^6*c^10*x^7 + 30*a^2*b^5*c^9*d*x^7 + 225
*a^3*b^4*c^8*d^2*x^7 + 600*a^4*b^3*c^7*d^3*x^7 + 630*a^5*b^2*c^6*d^4*x^7 +
252*a^6*b*c^5*d^5*x^7 + 30*a^7*c^4*d^6*x^7 + 7/2*a^2*b^5*c^10*x^6 + 175/3*a
^3*b^4*c^9*d*x^6 + 525/2*a^4*b^3*c^8*d^2*x^6 + 420*a^5*b^2*c^7*d^3*x^6 + 24
5*a^6*b*c^6*d^4*x^6 + 42*a^7*c^5*d^5*x^6 + 7*a^3*b^4*c^10*x^5 + 70*a^4*b^3*
c^9*d*x^5 + 189*a^5*b^2*c^8*d^2*x^5 + 168*a^6*b*c^7*d^3*x^5 + 42*a^7*c^6*d^
4*x^5 + 35/4*a^4*b^3*c^10*x^4 + 105/2*a^5*b^2*c^9*d*x^4 + 315/4*a^6*b*c^8*d
^2*x^4 + 30*a^7*c^7*d^3*x^4 + 7*a^5*b^2*c^10*x^3 + 70/3*a^6*b*c^9*d*x^3 + 1
5*a^7*c^8*d^2*x^3 + 7/2*a^6*b*c^10*x^2 + 5*a^7*c^9*d*x^2 + a^7*c^10*x
```

**maple [B]** time = 0.00, size = 1141, normalized size = 5.70

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7*(d*x+c)^10,x)
```

```
[Out] 1/18*b^7*d^10*x^18+1/17*(7*a*b^6*d^10+10*b^7*c*d^9)*x^17+1/16*(21*a^2*b^5*d
^10+70*a*b^6*c*d^9+45*b^7*c^2*d^8)*x^16+1/15*(35*a^3*b^4*d^10+210*a^2*b^5*c
*d^9+315*a*b^6*c^2*d^8+120*b^7*c^3*d^7)*x^15+1/14*(35*a^4*b^3*d^10+350*a^3*
b^4*c*d^9+945*a^2*b^5*c^2*d^8+840*a*b^6*c^3*d^7+210*b^7*c^4*d^6)*x^14+1/13*
(21*a^5*b^2*d^10+350*a^4*b^3*c*d^9+1575*a^3*b^4*c^2*d^8+2520*a^2*b^5*c^3*d^
7+1470*a*b^6*c^4*d^6+252*b^7*c^5*d^5)*x^13+1/12*(7*a^6*b*d^10+210*a^5*b^2*c
*d^9+1575*a^4*b^3*c^2*d^8+4200*a^3*b^4*c^3*d^7+4410*a^2*b^5*c^4*d^6+1764*a*
b^6*c^5*d^5+210*b^7*c^6*d^4)*x^12+1/11*(a^7*d^10+70*a^6*b*c*d^9+945*a^5*b^2
*c^2*d^8+4200*a^4*b^3*c^3*d^7+7350*a^3*b^4*c^4*d^6+5292*a^2*b^5*c^5*d^5+147
0*a*b^6*c^6*d^4+120*b^7*c^7*d^3)*x^11+1/10*(10*a^7*c*d^9+315*a^6*b*c^2*d^8+
2520*a^5*b^2*c^3*d^7+7350*a^4*b^3*c^4*d^6+8820*a^3*b^4*c^5*d^5+4410*a^2*b^5
*c^6*d^4+840*a*b^6*c^7*d^3+45*b^7*c^8*d^2)*x^10+1/9*(45*a^7*c^2*d^8+840*a^6
*b*c^3*d^7+4410*a^5*b^2*c^4*d^6+8820*a^4*b^3*c^5*d^5+7350*a^3*b^4*c^6*d^4+2
520*a^2*b^5*c^7*d^3+315*a*b^6*c^8*d^2+10*b^7*c^9*d)*x^9+1/8*(120*a^7*c^3*d^
7+1470*a^6*b*c^4*d^6+5292*a^5*b^2*c^5*d^5+7350*a^4*b^3*c^6*d^4+4200*a^3*b^4
*c^7*d^3+945*a^2*b^5*c^8*d^2+70*a*b^6*c^9*d+b^7*c^10)*x^8+1/7*(210*a^7*c^4*
d^6+1764*a^6*b*c^5*d^5+4410*a^5*b^2*c^6*d^4+4200*a^4*b^3*c^7*d^3+1575*a^3*b
^4*c^8*d^2+210*a^2*b^5*c^9*d+7*a*b^6*c^10)*x^7+1/6*(252*a^7*c^5*d^5+1470*a^
```

```
6*b*c^6*d^4+2520*a^5*b^2*c^7*d^3+1575*a^4*b^3*c^8*d^2+350*a^3*b^4*c^9*d+21*
a^2*b^5*c^10)*x^6+1/5*(210*a^7*c^6*d^4+840*a^6*b*c^7*d^3+945*a^5*b^2*c^8*d^
2+350*a^4*b^3*c^9*d+35*a^3*b^4*c^10)*x^5+1/4*(120*a^7*c^7*d^3+315*a^6*b*c^8
*d^2+210*a^5*b^2*c^9*d+35*a^4*b^3*c^10)*x^4+1/3*(45*a^7*c^8*d^2+70*a^6*b*c^
9*d+21*a^5*b^2*c^10)*x^3+1/2*(10*a^7*c^9*d+7*a^6*b*c^10)*x^2+a^7*c^10*x
```

**maxima** [B] time = 1.52, size = 1135, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/18*b^7*d^10*x^18 + a^7*c^10*x + 1/17*(10*b^7*c*d^9 + 7*a*b^6*d^10)*x^17 +
 1/16*(45*b^7*c^2*d^8 + 70*a*b^6*c*d^9 + 21*a^2*b^5*d^10)*x^16 + 1/3*(24*b^
7*c^3*d^7 + 63*a*b^6*c^2*d^8 + 42*a^2*b^5*c*d^9 + 7*a^3*b^4*d^10)*x^15 + 5/
2*(6*b^7*c^4*d^6 + 24*a*b^6*c^3*d^7 + 27*a^2*b^5*c^2*d^8 + 10*a^3*b^4*c*d^9
 + a^4*b^3*d^10)*x^14 + 7/13*(36*b^7*c^5*d^5 + 210*a*b^6*c^4*d^6 + 360*a^2*
b^5*c^3*d^7 + 225*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 + 3*a^5*b^2*d^10)*x^13
 + 7/12*(30*b^7*c^6*d^4 + 252*a*b^6*c^5*d^5 + 630*a^2*b^5*c^4*d^6 + 600*a^3
*b^4*c^3*d^7 + 225*a^4*b^3*c^2*d^8 + 30*a^5*b^2*c*d^9 + a^6*b*d^10)*x^12 +
1/11*(120*b^7*c^7*d^3 + 1470*a*b^6*c^6*d^4 + 5292*a^2*b^5*c^5*d^5 + 7350*a^
3*b^4*c^4*d^6 + 4200*a^4*b^3*c^3*d^7 + 945*a^5*b^2*c^2*d^8 + 70*a^6*b*c*d^9
 + a^7*d^10)*x^11 + 1/2*(9*b^7*c^8*d^2 + 168*a*b^6*c^7*d^3 + 882*a^2*b^5*c^
6*d^4 + 1764*a^3*b^4*c^5*d^5 + 1470*a^4*b^3*c^4*d^6 + 504*a^5*b^2*c^3*d^7 +
 63*a^6*b*c^2*d^8 + 2*a^7*c*d^9)*x^10 + 5/9*(2*b^7*c^9*d + 63*a*b^6*c^8*d^2
 + 504*a^2*b^5*c^7*d^3 + 1470*a^3*b^4*c^6*d^4 + 1764*a^4*b^3*c^5*d^5 + 882*
a^5*b^2*c^4*d^6 + 168*a^6*b*c^3*d^7 + 9*a^7*c^2*d^8)*x^9 + 1/8*(b^7*c^10 +
70*a*b^6*c^9*d + 945*a^2*b^5*c^8*d^2 + 4200*a^3*b^4*c^7*d^3 + 7350*a^4*b^3*
c^6*d^4 + 5292*a^5*b^2*c^5*d^5 + 1470*a^6*b*c^4*d^6 + 120*a^7*c^3*d^7)*x^8
+ (a*b^6*c^10 + 30*a^2*b^5*c^9*d + 225*a^3*b^4*c^8*d^2 + 600*a^4*b^3*c^7*d^
3 + 630*a^5*b^2*c^6*d^4 + 252*a^6*b*c^5*d^5 + 30*a^7*c^4*d^6)*x^7 + 7/6*(3*
a^2*b^5*c^10 + 50*a^3*b^4*c^9*d + 225*a^4*b^3*c^8*d^2 + 360*a^5*b^2*c^7*d^3
 + 210*a^6*b*c^6*d^4 + 36*a^7*c^5*d^5)*x^6 + 7*(a^3*b^4*c^10 + 10*a^4*b^3*c
^9*d + 27*a^5*b^2*c^8*d^2 + 24*a^6*b*c^7*d^3 + 6*a^7*c^6*d^4)*x^5 + 5/4*(7*
a^4*b^3*c^10 + 42*a^5*b^2*c^9*d + 63*a^6*b*c^8*d^2 + 24*a^7*c^7*d^3)*x^4 +
1/3*(21*a^5*b^2*c^10 + 70*a^6*b*c^9*d + 45*a^7*c^8*d^2)*x^3 + 1/2*(7*a^6*b*
c^10 + 10*a^7*c^9*d)*x^2
```

**mupad** [B] time = 0.61, size = 1106, normalized size = 5.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7*(c + d*x)^10,x)
```

```
[Out] x^10*(a^7*c*d^9 + (9*b^7*c^8*d^2)/2 + 84*a*b^6*c^7*d^3 + (63*a^6*b*c^2*d^8)
/2 + 441*a^2*b^5*c^6*d^4 + 882*a^3*b^4*c^5*d^5 + 735*a^4*b^3*c^4*d^6 + 252*
a^5*b^2*c^3*d^7) + x^9*((10*b^7*c^9*d)/9 + 5*a^7*c^2*d^8 + 35*a*b^6*c^8*d^2
 + (280*a^6*b*c^3*d^7)/3 + 280*a^2*b^5*c^7*d^3 + (2450*a^3*b^4*c^6*d^4)/3 +
 980*a^4*b^3*c^5*d^5 + 490*a^5*b^2*c^4*d^6) + x^5*(7*a^3*b^4*c^10 + 42*a^7*
c^6*d^4 + 70*a^4*b^3*c^9*d + 168*a^6*b*c^7*d^3 + 189*a^5*b^2*c^8*d^2) + x^1
4*((5*a^4*b^3*d^10)/2 + 15*b^7*c^4*d^6 + 60*a*b^6*c^3*d^7 + 25*a^3*b^4*c*d^
9 + (135*a^2*b^5*c^2*d^8)/2) + x^8*((b^7*c^10)/8 + 15*a^7*c^3*d^7 + (735*a^
6*b*c^4*d^6)/4 + (945*a^2*b^5*c^8*d^2)/8 + 525*a^3*b^4*c^7*d^3 + (3675*a^4*
b^3*c^6*d^4)/4 + (1323*a^5*b^2*c^5*d^5)/2 + (35*a*b^6*c^9*d)/4) + x^11*((a^
7*d^10)/11 + (120*b^7*c^7*d^3)/11 + (1470*a*b^6*c^6*d^4)/11 + (5292*a^2*b^5
*c^5*d^5)/11 + (7350*a^3*b^4*c^4*d^6)/11 + (4200*a^4*b^3*c^3*d^7)/11 + (945
*a^5*b^2*c^2*d^8)/11 + (70*a^6*b*c*d^9)/11) + x^6*((7*a^2*b^5*c^10)/2 + 42*
a^7*c^5*d^5 + (175*a^3*b^4*c^9*d)/3 + 245*a^6*b*c^6*d^4 + (525*a^4*b^3*c^8*
d^2)/2 + 420*a^5*b^2*c^7*d^3) + x^13*((21*a^5*b^2*d^10)/13 + (252*b^7*c^5*d
^5)/13 + (1470*a*b^6*c^4*d^6)/13 + (350*a^4*b^3*c*d^9)/13 + (2520*a^2*b^5*c
^3*d^7)/13 + (1575*a^3*b^4*c^2*d^8)/13) + x^7*(a*b^6*c^10 + 30*a^7*c^4*d^6
+ 30*a^2*b^5*c^9*d + 252*a^6*b*c^5*d^5 + 225*a^3*b^4*c^8*d^2 + 600*a^4*b^3*
c^7*d^3 + 630*a^5*b^2*c^6*d^4) + x^12*((7*a^6*b*d^10)/12 + (35*b^7*c^6*d^4)
/2 + 147*a*b^6*c^5*d^5 + (35*a^5*b^2*c*d^9)/2 + (735*a^2*b^5*c^4*d^6)/2 + 3
50*a^3*b^4*c^3*d^7 + (525*a^4*b^3*c^2*d^8)/4) + a^7*c^10*x + (b^7*d^10*x^18
)/18 + (5*a^4*c^7*x^4*(24*a^3*d^3 + 7*b^3*c^3 + 42*a*b^2*c^2*d + 63*a^2*b*c
*d^2))/4 + (b^4*d^7*x^15*(7*a^3*d^3 + 24*b^3*c^3 + 63*a*b^2*c^2*d + 42*a^2*
b*c*d^2))/3 + (a^6*c^9*x^2*(10*a*d + 7*b*c))/2 + (b^6*d^9*x^17*(7*a*d + 10*
b*c))/17 + (a^5*c^8*x^3*(45*a^2*d^2 + 21*b^2*c^2 + 70*a*b*c*d))/3 + (b^5*d^
8*x^16*(21*a^2*d^2 + 45*b^2*c^2 + 70*a*b*c*d))/16
```

**sympy [B]** time = 0.25, size = 1280, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7*(d*x+c)**10,x)
```

```
[Out] a**7*c**10*x + b**7*d**10*x**18/18 + x**17*(7*a*b**6*d**10/17 + 10*b**7*c*d
**9/17) + x**16*(21*a**2*b**5*d**10/16 + 35*a*b**6*c*d**9/8 + 45*b**7*c**2*
d**8/16) + x**15*(7*a**3*b**4*d**10/3 + 14*a**2*b**5*c*d**9 + 21*a*b**6*c**
2*d**8 + 8*b**7*c**3*d**7) + x**14*(5*a**4*b**3*d**10/2 + 25*a**3*b**4*c*d*
*9 + 135*a**2*b**5*c**2*d**8/2 + 60*a*b**6*c**3*d**7 + 15*b**7*c**4*d**6) +
 x**13*(21*a**5*b**2*d**10/13 + 350*a**4*b**3*c*d**9/13 + 1575*a**3*b**4*c*
*2*d**8/13 + 2520*a**2*b**5*c**3*d**7/13 + 1470*a*b**6*c**4*d**6/13 + 252*b
**7*c**5*d**5/13) + x**12*(7*a**6*b*d**10/12 + 35*a**5*b**2*c*d**9/2 + 525*
a**4*b**3*c**2*d**8/4 + 350*a**3*b**4*c**3*d**7 + 735*a**2*b**5*c**4*d**6/2
 + 147*a*b**6*c**5*d**5 + 35*b**7*c**6*d**4/2) + x**11*(a**7*d**10/11 + 70*
a**6*b*c*d**9/11 + 945*a**5*b**2*c**2*d**8/11 + 4200*a**4*b**3*c**3*d**7/11
```

```
 + 7350*a**3*b**4*c**4*d**6/11 + 5292*a**2*b**5*c**5*d**5/11 + 1470*a*b**6*
c**6*d**4/11 + 120*b**7*c**7*d**3/11) + x**10*(a**7*c*d**9 + 63*a**6*b*c**2
*d**8/2 + 252*a**5*b**2*c**3*d**7 + 735*a**4*b**3*c**4*d**6 + 882*a**3*b**4
*c**5*d**5 + 441*a**2*b**5*c**6*d**4 + 84*a*b**6*c**7*d**3 + 9*b**7*c**8*d*
*2/2) + x**9*(5*a**7*c**2*d**8 + 280*a**6*b*c**3*d**7/3 + 490*a**5*b**2*c**
4*d**6 + 980*a**4*b**3*c**5*d**5 + 2450*a**3*b**4*c**6*d**4/3 + 280*a**2*b*
*5*c**7*d**3 + 35*a*b**6*c**8*d**2 + 10*b**7*c**9*d/9) + x**8*(15*a**7*c**3
*d**7 + 735*a**6*b*c**4*d**6/4 + 1323*a**5*b**2*c**5*d**5/2 + 3675*a**4*b**
3*c**6*d**4/4 + 525*a**3*b**4*c**7*d**3 + 945*a**2*b**5*c**8*d**2/8 + 35*a*
b**6*c**9*d/4 + b**7*c**10/8) + x**7*(30*a**7*c**4*d**6 + 252*a**6*b*c**5*d
**5 + 630*a**5*b**2*c**6*d**4 + 600*a**4*b**3*c**7*d**3 + 225*a**3*b**4*c**
8*d**2 + 30*a**2*b**5*c**9*d + a*b**6*c**10) + x**6*(42*a**7*c**5*d**5 + 24
5*a**6*b*c**6*d**4 + 420*a**5*b**2*c**7*d**3 + 525*a**4*b**3*c**8*d**2/2 +
175*a**3*b**4*c**9*d/3 + 7*a**2*b**5*c**10/2) + x**5*(42*a**7*c**6*d**4 + 1
68*a**6*b*c**7*d**3 + 189*a**5*b**2*c**8*d**2 + 70*a**4*b**3*c**9*d + 7*a**
3*b**4*c**10) + x**4*(30*a**7*c**7*d**3 + 315*a**6*b*c**8*d**2/4 + 105*a**5
*b**2*c**9*d/2 + 35*a**4*b**3*c**10/4) + x**3*(15*a**7*c**8*d**2 + 70*a**6*
b*c**9*d/3 + 7*a**5*b**2*c**10) + x**2*(5*a**7*c**9*d + 7*a**6*b*c**10/2)
```

## 3.1199 $\int (a+bx)^6(c+dx)^{10}\,dx$

**Optimal**. Leaf size=170

$$-\frac{3b^5(c+dx)^{16}(bc-ad)}{8d^7}+\frac{b^4(c+dx)^{15}(bc-ad)^2}{d^7}-\frac{10b^3(c+dx)^{14}(bc-ad)^3}{7d^7}+\frac{15b^2(c+dx)^{13}(bc-ad)^4}{13d^7}-\frac{b(c+dx)^1}{2}$$

**Rubi** [A] time = 0.67, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^5(c+dx)^{16}(bc-ad)}{8d^7}+\frac{b^4(c+dx)^{15}(bc-ad)^2}{d^7}-\frac{10b^3(c+dx)^{14}(bc-ad)^3}{7d^7}+\frac{15b^2(c+dx)^{13}(bc-ad)^4}{13d^7}-\frac{b(c+dx)^{12}(bc-ad)^5}{2d^7}+\frac{(c+dx)^{11}(bc-ad)^6}{11d^7}+\frac{b^6(c+dx)^{17}}{17d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^6*(c + d*x)^11)/(11*d^7) - (b*(b*c - a*d)^5*(c + d*x)^12)/(2*d
^7) + (15*b^2*(b*c - a*d)^4*(c + d*x)^13)/(13*d^7) - (10*b^3*(b*c - a*d)^3*
(c + d*x)^14)/(7*d^7) + (b^4*(b*c - a*d)^2*(c + d*x)^15)/d^7 - (3*b^5*(b*c
- a*d)*(c + d*x)^16)/(8*d^7) + (b^6*(c + d*x)^17)/(17*d^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^6(c+dx)^{10}\,dx=\int \left(\frac{(-bc+ad)^6(c+dx)^{10}}{d^6}-\frac{6b(bc-ad)^5(c+dx)^{11}}{d^6}+\frac{15b^2(bc-ad)^4(c+dx)^{12}}{d^6}-\frac{20b}{}\right.$$

$$=\frac{(bc-ad)^6(c+dx)^{11}}{11d^7}-\frac{b(bc-ad)^5(c+dx)^{12}}{2d^7}+\frac{15b^2(bc-ad)^4(c+dx)^{13}}{13d^7}-\frac{10b^3(bc-a}{7}$$

**Mathematica** [B] time = 0.12, size = 939, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] a^6*c^10*x + a^5*c^9*(3*b*c + 5*a*d)*x^2 + 5*a^4*c^8*(b^2*c^2 + 4*a*b*c*d +
 3*a^2*d^2)*x^3 + (5*a^3*c^7*(2*b^3*c^3 + 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 +
 12*a^3*d^3)*x^4)/2 + a^2*c^6*(3*b^4*c^4 + 40*a*b^3*c^3*d + 135*a^2*b^2*c^2
*d^2 + 144*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + a*c^5*(b^5*c^5 + 25*a*b^4*c^4*d
+ 150*a^2*b^3*c^3*d^2 + 300*a^3*b^2*c^2*d^3 + 210*a^4*b*c*d^4 + 42*a^5*d^5)
*x^6 + (c^4*(b^6*c^6 + 60*a*b^5*c^5*d + 675*a^2*b^4*c^4*d^2 + 2400*a^3*b^3*
c^3*d^3 + 3150*a^4*b^2*c^2*d^4 + 1512*a^5*b*c*d^5 + 210*a^6*d^6)*x^7)/7 + (
5*c^3*d*(b^6*c^6 + 27*a*b^5*c^5*d + 180*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d
^3 + 378*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 + 12*a^6*d^6)*x^8)/4 + 5*c^2*d^2
*(b^6*c^6 + 16*a*b^5*c^5*d + 70*a^2*b^4*c^4*d^2 + 112*a^3*b^3*c^3*d^3 + 70*
a^4*b^2*c^2*d^4 + 16*a^5*b*c*d^5 + a^6*d^6)*x^9 + c*d^3*(12*b^6*c^6 + 126*a
*b^5*c^5*d + 378*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^
4 + 27*a^5*b*c*d^5 + a^6*d^6)*x^10 + (d^4*(210*b^6*c^6 + 1512*a*b^5*c^5*d +
 3150*a^2*b^4*c^4*d^2 + 2400*a^3*b^3*c^3*d^3 + 675*a^4*b^2*c^2*d^4 + 60*a^5
*b*c*d^5 + a^6*d^6)*x^11)/11 + (b*d^5*(42*b^5*c^5 + 210*a*b^4*c^4*d + 300*a
^2*b^3*c^3*d^2 + 150*a^3*b^2*c^2*d^3 + 25*a^4*b*c*d^4 + a^5*d^5)*x^12)/2 +
(5*b^2*d^6*(42*b^4*c^4 + 144*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 + 40*a^3*b*c
*d^3 + 3*a^4*d^4)*x^13)/13 + (5*b^3*d^7*(12*b^3*c^3 + 27*a*b^2*c^2*d + 15*a
^2*b*c*d^2 + 2*a^3*d^3)*x^14)/7 + b^4*d^8*(3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)
*x^15 + (b^5*d^9*(5*b*c + 3*a*d)*x^16)/8 + (b^6*d^10*x^17)/17
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^6(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^6*(c + d*x)^10, x]
```

**fricas [B]** time = 1.06, size = 1124, normalized size = 6.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^10*b^6 + 5/8*x^16*d^9*c*b^6 + 3/8*x^16*d^10*b^5*a + 3*x^15*d^8*
c^2*b^6 + 4*x^15*d^9*c*b^5*a + x^15*d^10*b^4*a^2 + 60/7*x^14*d^7*c^3*b^6 +
135/7*x^14*d^8*c^2*b^5*a + 75/7*x^14*d^9*c*b^4*a^2 + 10/7*x^14*d^10*b^3*a^3
 + 210/13*x^13*d^6*c^4*b^6 + 720/13*x^13*d^7*c^3*b^5*a + 675/13*x^13*d^8*c^
2*b^4*a^2 + 200/13*x^13*d^9*c*b^3*a^3 + 15/13*x^13*d^10*b^2*a^4 + 21*x^12*d
^5*c^5*b^6 + 105*x^12*d^6*c^4*b^5*a + 150*x^12*d^7*c^3*b^4*a^2 + 75*x^12*d^
8*c^2*b^3*a^3 + 25/2*x^12*d^9*c*b^2*a^4 + 1/2*x^12*d^10*b*a^5 + 210/11*x^11
```

```
*d^4*c^6*b^6 + 1512/11*x^11*d^5*c^5*b^5*a + 3150/11*x^11*d^6*c^4*b^4*a^2 +
2400/11*x^11*d^7*c^3*b^3*a^3 + 675/11*x^11*d^8*c^2*b^2*a^4 + 60/11*x^11*d^9
*c*b*a^5 + 1/11*x^11*d^10*a^6 + 12*x^10*d^3*c^7*b^6 + 126*x^10*d^4*c^6*b^5*
a + 378*x^10*d^5*c^5*b^4*a^2 + 420*x^10*d^6*c^4*b^3*a^3 + 180*x^10*d^7*c^3*
b^2*a^4 + 27*x^10*d^8*c^2*b*a^5 + x^10*d^9*c*a^6 + 5*x^9*d^2*c^8*b^6 + 80*x
^9*d^3*c^7*b^5*a + 350*x^9*d^4*c^6*b^4*a^2 + 560*x^9*d^5*c^5*b^3*a^3 + 350*
x^9*d^6*c^4*b^2*a^4 + 80*x^9*d^7*c^3*b*a^5 + 5*x^9*d^8*c^2*a^6 + 5/4*x^8*d*
c^9*b^6 + 135/4*x^8*d^2*c^8*b^5*a + 225*x^8*d^3*c^7*b^4*a^2 + 525*x^8*d^4*c
^6*b^3*a^3 + 945/2*x^8*d^5*c^5*b^2*a^4 + 315/2*x^8*d^6*c^4*b*a^5 + 15*x^8*d
^7*c^3*a^6 + 1/7*x^7*c^10*b^6 + 60/7*x^7*d*c^9*b^5*a + 675/7*x^7*d^2*c^8*b^
4*a^2 + 2400/7*x^7*d^3*c^7*b^3*a^3 + 450*x^7*d^4*c^6*b^2*a^4 + 216*x^7*d^5*
c^5*b*a^5 + 30*x^7*d^6*c^4*a^6 + x^6*c^10*b^5*a + 25*x^6*d*c^9*b^4*a^2 + 15
0*x^6*d^2*c^8*b^3*a^3 + 300*x^6*d^3*c^7*b^2*a^4 + 210*x^6*d^4*c^6*b*a^5 + 4
2*x^6*d^5*c^5*a^6 + 3*x^5*c^10*b^4*a^2 + 40*x^5*d*c^9*b^3*a^3 + 135*x^5*d^2
*c^8*b^2*a^4 + 144*x^5*d^3*c^7*b*a^5 + 42*x^5*d^4*c^6*a^6 + 5*x^4*c^10*b^3*
a^3 + 75/2*x^4*d*c^9*b^2*a^4 + 135/2*x^4*d^2*c^8*b*a^5 + 30*x^4*d^3*c^7*a^6
 + 5*x^3*c^10*b^2*a^4 + 20*x^3*d*c^9*b*a^5 + 15*x^3*d^2*c^8*a^6 + 3*x^2*c^1
0*b*a^5 + 5*x^2*d*c^9*a^6 + x*c^10*a^6
```

**giac [B]** time = 1.30, size = 1124, normalized size = 6.61

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/17*b^6*d^10*x^17 + 5/8*b^6*c*d^9*x^16 + 3/8*a*b^5*d^10*x^16 + 3*b^6*c^2*d
^8*x^15 + 4*a*b^5*c*d^9*x^15 + a^2*b^4*d^10*x^15 + 60/7*b^6*c^3*d^7*x^14 +
135/7*a*b^5*c^2*d^8*x^14 + 75/7*a^2*b^4*c*d^9*x^14 + 10/7*a^3*b^3*d^10*x^14
 + 210/13*b^6*c^4*d^6*x^13 + 720/13*a*b^5*c^3*d^7*x^13 + 675/13*a^2*b^4*c^2
*d^8*x^13 + 200/13*a^3*b^3*c*d^9*x^13 + 15/13*a^4*b^2*d^10*x^13 + 21*b^6*c^
5*d^5*x^12 + 105*a*b^5*c^4*d^6*x^12 + 150*a^2*b^4*c^3*d^7*x^12 + 75*a^3*b^3
*c^2*d^8*x^12 + 25/2*a^4*b^2*c*d^9*x^12 + 1/2*a^5*b*d^10*x^12 + 210/11*b^6*
c^6*d^4*x^11 + 1512/11*a*b^5*c^5*d^5*x^11 + 3150/11*a^2*b^4*c^4*d^6*x^11 +
2400/11*a^3*b^3*c^3*d^7*x^11 + 675/11*a^4*b^2*c^2*d^8*x^11 + 60/11*a^5*b*c*
d^9*x^11 + 1/11*a^6*d^10*x^11 + 12*b^6*c^7*d^3*x^10 + 126*a*b^5*c^6*d^4*x^1
0 + 378*a^2*b^4*c^5*d^5*x^10 + 420*a^3*b^3*c^4*d^6*x^10 + 180*a^4*b^2*c^3*d
^7*x^10 + 27*a^5*b*c^2*d^8*x^10 + a^6*c*d^9*x^10 + 5*b^6*c^8*d^2*x^9 + 80*a
*b^5*c^7*d^3*x^9 + 350*a^2*b^4*c^6*d^4*x^9 + 560*a^3*b^3*c^5*d^5*x^9 + 350*
a^4*b^2*c^4*d^6*x^9 + 80*a^5*b*c^3*d^7*x^9 + 5*a^6*c^2*d^8*x^9 + 5/4*b^6*c^
9*d*x^8 + 135/4*a*b^5*c^8*d^2*x^8 + 225*a^2*b^4*c^7*d^3*x^8 + 525*a^3*b^3*c
^6*d^4*x^8 + 945/2*a^4*b^2*c^5*d^5*x^8 + 315/2*a^5*b*c^4*d^6*x^8 + 15*a^6*c
^3*d^7*x^8 + 1/7*b^6*c^10*x^7 + 60/7*a*b^5*c^9*d*x^7 + 675/7*a^2*b^4*c^8*d^
2*x^7 + 2400/7*a^3*b^3*c^7*d^3*x^7 + 450*a^4*b^2*c^6*d^4*x^7 + 216*a^5*b*c^
5*d^5*x^7 + 30*a^6*c^4*d^6*x^7 + a*b^5*c^10*x^6 + 25*a^2*b^4*c^9*d*x^6 + 15
```

```
0*a^3*b^3*c^8*d^2*x^6 + 300*a^4*b^2*c^7*d^3*x^6 + 210*a^5*b*c^6*d^4*x^6 + 4
2*a^6*c^5*d^5*x^6 + 3*a^2*b^4*c^10*x^5 + 40*a^3*b^3*c^9*d*x^5 + 135*a^4*b^2
*c^8*d^2*x^5 + 144*a^5*b*c^7*d^3*x^5 + 42*a^6*c^6*d^4*x^5 + 5*a^3*b^3*c^10*
x^4 + 75/2*a^4*b^2*c^9*d*x^4 + 135/2*a^5*b*c^8*d^2*x^4 + 30*a^6*c^7*d^3*x^4
 + 5*a^4*b^2*c^10*x^3 + 20*a^5*b*c^9*d*x^3 + 15*a^6*c^8*d^2*x^3 + 3*a^5*b*c
^10*x^2 + 5*a^6*c^9*d*x^2 + a^6*c^10*x
```

**maple [B]** time = 0.00, size = 991, normalized size = 5.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6*(d*x+c)^10,x)
```

```
[Out] 1/17*b^6*d^10*x^17+1/16*(6*a*b^5*d^10+10*b^6*c*d^9)*x^16+1/15*(15*a^2*b^4*d
^10+60*a*b^5*c*d^9+45*b^6*c^2*d^8)*x^15+1/14*(20*a^3*b^3*d^10+150*a^2*b^4*c
*d^9+270*a*b^5*c^2*d^8+120*b^6*c^3*d^7)*x^14+1/13*(15*a^4*b^2*d^10+200*a^3*
b^3*c*d^9+675*a^2*b^4*c^2*d^8+720*a*b^5*c^3*d^7+210*b^6*c^4*d^6)*x^13+1/12*
(6*a^5*b*d^10+150*a^4*b^2*c*d^9+900*a^3*b^3*c^2*d^8+1800*a^2*b^4*c^3*d^7+12
60*a*b^5*c^4*d^6+252*b^6*c^5*d^5)*x^12+1/11*(a^6*d^10+60*a^5*b*c*d^9+675*a^
4*b^2*c^2*d^8+2400*a^3*b^3*c^3*d^7+3150*a^2*b^4*c^4*d^6+1512*a*b^5*c^5*d^5+
210*b^6*c^6*d^4)*x^11+1/10*(10*a^6*c*d^9+270*a^5*b*c^2*d^8+1800*a^4*b^2*c^3
*d^7+4200*a^3*b^3*c^4*d^6+3780*a^2*b^4*c^5*d^5+1260*a*b^5*c^6*d^4+120*b^6*c
^7*d^3)*x^10+1/9*(45*a^6*c^2*d^8+720*a^5*b*c^3*d^7+3150*a^4*b^2*c^4*d^6+504
0*a^3*b^3*c^5*d^5+3150*a^2*b^4*c^6*d^4+720*a*b^5*c^7*d^3+45*b^6*c^8*d^2)*x^
9+1/8*(120*a^6*c^3*d^7+1260*a^5*b*c^4*d^6+3780*a^4*b^2*c^5*d^5+4200*a^3*b^3
*c^6*d^4+1800*a^2*b^4*c^7*d^3+270*a*b^5*c^8*d^2+10*b^6*c^9*d)*x^8+1/7*(210*
a^6*c^4*d^6+1512*a^5*b*c^5*d^5+3150*a^4*b^2*c^6*d^4+2400*a^3*b^3*c^7*d^3+67
5*a^2*b^4*c^8*d^2+60*a*b^5*c^9*d+b^6*c^10)*x^7+1/6*(252*a^6*c^5*d^5+1260*a^
5*b*c^6*d^4+1800*a^4*b^2*c^7*d^3+900*a^3*b^3*c^8*d^2+150*a^2*b^4*c^9*d+6*a*
b^5*c^10)*x^6+1/5*(210*a^6*c^6*d^4+720*a^5*b*c^7*d^3+675*a^4*b^2*c^8*d^2+20
0*a^3*b^3*c^9*d+15*a^2*b^4*c^10)*x^5+1/4*(120*a^6*c^7*d^3+270*a^5*b*c^8*d^2
+150*a^4*b^2*c^9*d+20*a^3*b^3*c^10)*x^4+1/3*(45*a^6*c^8*d^2+60*a^5*b*c^9*d+
15*a^4*b^2*c^10)*x^3+1/2*(10*a^6*c^9*d+6*a^5*b*c^10)*x^2+a^6*c^10*x
```

**maxima [B]** time = 1.44, size = 977, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/17*b^6*d^10*x^17 + a^6*c^10*x + 1/8*(5*b^6*c*d^9 + 3*a*b^5*d^10)*x^16 + (
3*b^6*c^2*d^8 + 4*a*b^5*c*d^9 + a^2*b^4*d^10)*x^15 + 5/7*(12*b^6*c^3*d^7 +
27*a*b^5*c^2*d^8 + 15*a^2*b^4*c*d^9 + 2*a^3*b^3*d^10)*x^14 + 5/13*(42*b^6*c
```

```
^4*d^6 + 144*a*b^5*c^3*d^7 + 135*a^2*b^4*c^2*d^8 + 40*a^3*b^3*c*d^9 + 3*a^4
*b^2*d^10)*x^13 + 1/2*(42*b^6*c^5*d^5 + 210*a*b^5*c^4*d^6 + 300*a^2*b^4*c^3
*d^7 + 150*a^3*b^3*c^2*d^8 + 25*a^4*b^2*c*d^9 + a^5*b*d^10)*x^12 + 1/11*(21
0*b^6*c^6*d^4 + 1512*a*b^5*c^5*d^5 + 3150*a^2*b^4*c^4*d^6 + 2400*a^3*b^3*c^
3*d^7 + 675*a^4*b^2*c^2*d^8 + 60*a^5*b*c*d^9 + a^6*d^10)*x^11 + (12*b^6*c^7
*d^3 + 126*a*b^5*c^6*d^4 + 378*a^2*b^4*c^5*d^5 + 420*a^3*b^3*c^4*d^6 + 180*
a^4*b^2*c^3*d^7 + 27*a^5*b*c^2*d^8 + a^6*c*d^9)*x^10 + 5*(b^6*c^8*d^2 + 16*
a*b^5*c^7*d^3 + 70*a^2*b^4*c^6*d^4 + 112*a^3*b^3*c^5*d^5 + 70*a^4*b^2*c^4*d
^6 + 16*a^5*b*c^3*d^7 + a^6*c^2*d^8)*x^9 + 5/4*(b^6*c^9*d + 27*a*b^5*c^8*d^
2 + 180*a^2*b^4*c^7*d^3 + 420*a^3*b^3*c^6*d^4 + 378*a^4*b^2*c^5*d^5 + 126*a
^5*b*c^4*d^6 + 12*a^6*c^3*d^7)*x^8 + 1/7*(b^6*c^10 + 60*a*b^5*c^9*d + 675*a
^2*b^4*c^8*d^2 + 2400*a^3*b^3*c^7*d^3 + 3150*a^4*b^2*c^6*d^4 + 1512*a^5*b*c
^5*d^5 + 210*a^6*c^4*d^6)*x^7 + (a*b^5*c^10 + 25*a^2*b^4*c^9*d + 150*a^3*b^
3*c^8*d^2 + 300*a^4*b^2*c^7*d^3 + 210*a^5*b*c^6*d^4 + 42*a^6*c^5*d^5)*x^6 +
 (3*a^2*b^4*c^10 + 40*a^3*b^3*c^9*d + 135*a^4*b^2*c^8*d^2 + 144*a^5*b*c^7*d
^3 + 42*a^6*c^6*d^4)*x^5 + 5/2*(2*a^3*b^3*c^10 + 15*a^4*b^2*c^9*d + 27*a^5*
b*c^8*d^2 + 12*a^6*c^7*d^3)*x^4 + 5*(a^4*b^2*c^10 + 4*a^5*b*c^9*d + 3*a^6*c
^8*d^2)*x^3 + (3*a^5*b*c^10 + 5*a^6*c^9*d)*x^2
```

**mupad [B]** time = 0.53, size = 953, normalized size = 5.61

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6*(c + d*x)^10,x)
```

```
[Out] x^7*((b^6*c^10)/7 + 30*a^6*c^4*d^6 + 216*a^5*b*c^5*d^5 + (675*a^2*b^4*c^8*d
^2)/7 + (2400*a^3*b^3*c^7*d^3)/7 + 450*a^4*b^2*c^6*d^4 + (60*a*b^5*c^9*d)/7
) + x^11*((a^6*d^10)/11 + (210*b^6*c^6*d^4)/11 + (1512*a*b^5*c^5*d^5)/11 +
(3150*a^2*b^4*c^4*d^6)/11 + (2400*a^3*b^3*c^3*d^7)/11 + (675*a^4*b^2*c^2*d^
8)/11 + (60*a^5*b*c*d^9)/11) + x^9*(5*a^6*c^2*d^8 + 5*b^6*c^8*d^2 + 80*a*b^
5*c^7*d^3 + 80*a^5*b*c^3*d^7 + 350*a^2*b^4*c^6*d^4 + 560*a^3*b^3*c^5*d^5 +
350*a^4*b^2*c^4*d^6) + x^5*(3*a^2*b^4*c^10 + 42*a^6*c^6*d^4 + 40*a^3*b^3*c^
9*d + 144*a^5*b*c^7*d^3 + 135*a^4*b^2*c^8*d^2) + x^13*((15*a^4*b^2*d^10)/13
 + (210*b^6*c^4*d^6)/13 + (720*a*b^5*c^3*d^7)/13 + (200*a^3*b^3*c*d^9)/13 +
 (675*a^2*b^4*c^2*d^8)/13) + x^6*(a*b^5*c^10 + 42*a^6*c^5*d^5 + 25*a^2*b^4*
c^9*d + 210*a^5*b*c^6*d^4 + 150*a^3*b^3*c^8*d^2 + 300*a^4*b^2*c^7*d^3) + x^
12*((a^5*b*d^10)/2 + 21*b^6*c^5*d^5 + 105*a*b^5*c^4*d^6 + (25*a^4*b^2*c*d^9
)/2 + 150*a^2*b^4*c^3*d^7 + 75*a^3*b^3*c^2*d^8) + x^10*(a^6*c*d^9 + 12*b^6*
c^7*d^3 + 126*a*b^5*c^6*d^4 + 27*a^5*b*c^2*d^8 + 378*a^2*b^4*c^5*d^5 + 420*
a^3*b^3*c^4*d^6 + 180*a^4*b^2*c^3*d^7) + x^8*((5*b^6*c^9*d)/4 + 15*a^6*c^3*
d^7 + (135*a*b^5*c^8*d^2)/4 + (315*a^5*b*c^4*d^6)/2 + 225*a^2*b^4*c^7*d^3 +
 525*a^3*b^3*c^6*d^4 + (945*a^4*b^2*c^5*d^5)/2) + a^6*c^10*x + (b^6*d^10*x^
17)/17 + (5*a^3*c^7*x^4*(12*a^3*d^3 + 2*b^3*c^3 + 15*a*b^2*c^2*d + 27*a^2*b
*c*d^2))/2 + (5*b^3*d^7*x^14*(2*a^3*d^3 + 12*b^3*c^3 + 27*a*b^2*c^2*d + 15*
```

```
a^2*b*c*d^2))/7 + a^5*c^9*x^2*(5*a*d + 3*b*c) + (b^5*d^9*x^16*(3*a*d + 5*b*
c))/8 + 5*a^4*c^8*x^3*(3*a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + b^4*d^8*x^15*(a^2
*d^2 + 3*b^2*c^2 + 4*a*b*c*d)
```

**sympy [B]** time = 0.23, size = 1088, normalized size = 6.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6*(d*x+c)**10,x)
```

```
[Out] a**6*c**10*x + b**6*d**10*x**17/17 + x**16*(3*a*b**5*d**10/8 + 5*b**6*c*d**
9/8) + x**15*(a**2*b**4*d**10 + 4*a*b**5*c*d**9 + 3*b**6*c**2*d**8) + x**14
*(10*a**3*b**3*d**10/7 + 75*a**2*b**4*c*d**9/7 + 135*a*b**5*c**2*d**8/7 + 6
0*b**6*c**3*d**7/7) + x**13*(15*a**4*b**2*d**10/13 + 200*a**3*b**3*c*d**9/1
3 + 675*a**2*b**4*c**2*d**8/13 + 720*a*b**5*c**3*d**7/13 + 210*b**6*c**4*d*
*6/13) + x**12*(a**5*b*d**10/2 + 25*a**4*b**2*c*d**9/2 + 75*a**3*b**3*c**2*
d**8 + 150*a**2*b**4*c**3*d**7 + 105*a*b**5*c**4*d**6 + 21*b**6*c**5*d**5)
+ x**11*(a**6*d**10/11 + 60*a**5*b*c*d**9/11 + 675*a**4*b**2*c**2*d**8/11 +
 2400*a**3*b**3*c**3*d**7/11 + 3150*a**2*b**4*c**4*d**6/11 + 1512*a*b**5*c*
*5*d**5/11 + 210*b**6*c**6*d**4/11) + x**10*(a**6*c*d**9 + 27*a**5*b*c**2*d
**8 + 180*a**4*b**2*c**3*d**7 + 420*a**3*b**3*c**4*d**6 + 378*a**2*b**4*c**
5*d**5 + 126*a*b**5*c**6*d**4 + 12*b**6*c**7*d**3) + x**9*(5*a**6*c**2*d**8
 + 80*a**5*b*c**3*d**7 + 350*a**4*b**2*c**4*d**6 + 560*a**3*b**3*c**5*d**5
+ 350*a**2*b**4*c**6*d**4 + 80*a*b**5*c**7*d**3 + 5*b**6*c**8*d**2) + x**8*
(15*a**6*c**3*d**7 + 315*a**5*b*c**4*d**6/2 + 945*a**4*b**2*c**5*d**5/2 + 5
25*a**3*b**3*c**6*d**4 + 225*a**2*b**4*c**7*d**3 + 135*a*b**5*c**8*d**2/4 +
 5*b**6*c**9*d/4) + x**7*(30*a**6*c**4*d**6 + 216*a**5*b*c**5*d**5 + 450*a*
*4*b**2*c**6*d**4 + 2400*a**3*b**3*c**7*d**3/7 + 675*a**2*b**4*c**8*d**2/7
+ 60*a*b**5*c**9*d/7 + b**6*c**10/7) + x**6*(42*a**6*c**5*d**5 + 210*a**5*b
*c**6*d**4 + 300*a**4*b**2*c**7*d**3 + 150*a**3*b**3*c**8*d**2 + 25*a**2*b*
*4*c**9*d + a*b**5*c**10) + x**5*(42*a**6*c**6*d**4 + 144*a**5*b*c**7*d**3
+ 135*a**4*b**2*c**8*d**2 + 40*a**3*b**3*c**9*d + 3*a**2*b**4*c**10) + x**4
*(30*a**6*c**7*d**3 + 135*a**5*b*c**8*d**2/2 + 75*a**4*b**2*c**9*d/2 + 5*a*
*3*b**3*c**10) + x**3*(15*a**6*c**8*d**2 + 20*a**5*b*c**9*d + 5*a**4*b**2*c
**10) + x**2*(5*a**6*c**9*d + 3*a**5*b*c**10)
```

## 3.1200 $\int (a+bx)^5 (c+dx)^{10}\,dx$

**Optimal**. Leaf size=146

$$-\frac{b^4(c+dx)^{15}(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^{14}(bc-ad)^2}{7d^6}-\frac{10b^2(c+dx)^{13}(bc-ad)^3}{13d^6}+\frac{5b(c+dx)^{12}(bc-ad)^4}{12d^6}-\frac{(c+dx)^{11}(b\ldots}{11d^6}$$

**Rubi [A]** time = 0.53, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b^4(c+dx)^{15}(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^{14}(bc-ad)^2}{7d^6}-\frac{10b^2(c+dx)^{13}(bc-ad)^3}{13d^6}+\frac{5b(c+dx)^{12}(bc-ad)^4}{12d^6}-\frac{(c+dx)^{11}(bc-ad)^5}{11d^6}+\frac{b^5(c+dx)^{16}}{16d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)^5*(c + d*x)^11)/(11*d^6) + (5*b*(b*c - a*d)^4*(c + d*x)^12)/(
12*d^6) - (10*b^2*(b*c - a*d)^3*(c + d*x)^13)/(13*d^6) + (5*b^3*(b*c - a*d)
^2*(c + d*x)^14)/(7*d^6) - (b^4*(b*c - a*d)*(c + d*x)^15)/(3*d^6) + (b^5*(c
 + d*x)^16)/(16*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^{10}\,dx=\int \left(\frac{(-bc+ad)^5(c+dx)^{10}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{11}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{12}}{d^5}+\frac{10b\ldots}{}\right.$$

$$=-\frac{(bc-ad)^5(c+dx)^{11}}{11d^6}+\frac{5b(bc-ad)^4(c+dx)^{12}}{12d^6}-\frac{10b^2(bc-ad)^3(c+dx)^{13}}{13d^6}+\frac{5b^3(bc-\ldots}{}$$

**Mathematica [B]** time = 0.09, size = 811, normalized size = 5.55

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^10,x]
```

```
[Out] a^5*c^10*x + (5*a^4*c^9*(b*c + 2*a*d)*x^2)/2 + (5*a^3*c^8*(2*b^2*c^2 + 10*a
*b*c*d + 9*a^2*d^2)*x^3)/3 + (5*a^2*c^7*(2*b^3*c^3 + 20*a*b^2*c^2*d + 45*a^
2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + a*c^6*(b^4*c^4 + 20*a*b^3*c^3*d + 90*a^2*b
^2*c^2*d^2 + 120*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + (c^5*(b^5*c^5 + 50*a*b^4*c
^4*d + 450*a^2*b^3*c^3*d^2 + 1200*a^3*b^2*c^2*d^3 + 1050*a^4*b*c*d^4 + 252*
a^5*d^5)*x^6)/6 + (5*c^4*d*(2*b^5*c^5 + 45*a*b^4*c^4*d + 240*a^2*b^3*c^3*d^
2 + 420*a^3*b^2*c^2*d^3 + 252*a^4*b*c*d^4 + 42*a^5*d^5)*x^7)/7 + (15*c^3*d^
2*(3*b^5*c^5 + 40*a*b^4*c^4*d + 140*a^2*b^3*c^3*d^2 + 168*a^3*b^2*c^2*d^3 +
 70*a^4*b*c*d^4 + 8*a^5*d^5)*x^8)/8 + (5*c^2*d^3*(8*b^5*c^5 + 70*a*b^4*c^4*
d + 168*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 40*a^4*b*c*d^4 + 3*a^5*d^5)
*x^9)/3 + (c*d^4*(42*b^5*c^5 + 252*a*b^4*c^4*d + 420*a^2*b^3*c^3*d^2 + 240*
a^3*b^2*c^2*d^3 + 45*a^4*b*c*d^4 + 2*a^5*d^5)*x^10)/2 + (d^5*(252*b^5*c^5 +
 1050*a*b^4*c^4*d + 1200*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 50*a^4*b*c
*d^4 + a^5*d^5)*x^11)/11 + (5*b*d^6*(42*b^4*c^4 + 120*a*b^3*c^3*d + 90*a^2*
b^2*c^2*d^2 + 20*a^3*b*c*d^3 + a^4*d^4)*x^12)/12 + (5*b^2*d^7*(24*b^3*c^3 +
 45*a*b^2*c^2*d + 20*a^2*b*c*d^2 + 2*a^3*d^3)*x^13)/13 + (5*b^3*d^8*(9*b^2*
c^2 + 10*a*b*c*d + 2*a^2*d^2)*x^14)/14 + (b^4*d^9*(2*b*c + a*d)*x^15)/3 + (
b^5*d^10*x^16)/16
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^10, x]
```

**fricas [B]** time = 0.78, size = 948, normalized size = 6.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^10*b^5 + 2/3*x^15*d^9*c*b^5 + 1/3*x^15*d^10*b^4*a + 45/14*x^14*
d^8*c^2*b^5 + 25/7*x^14*d^9*c*b^4*a + 5/7*x^14*d^10*b^3*a^2 + 120/13*x^13*d
^7*c^3*b^5 + 225/13*x^13*d^8*c^2*b^4*a + 100/13*x^13*d^9*c*b^3*a^2 + 10/13*
x^13*d^10*b^2*a^3 + 35/2*x^12*d^6*c^4*b^5 + 50*x^12*d^7*c^3*b^4*a + 75/2*x^
12*d^8*c^2*b^3*a^2 + 25/3*x^12*d^9*c*b^2*a^3 + 5/12*x^12*d^10*b*a^4 + 252/1
1*x^11*d^5*c^5*b^5 + 1050/11*x^11*d^6*c^4*b^4*a + 1200/11*x^11*d^7*c^3*b^3*
a^2 + 450/11*x^11*d^8*c^2*b^2*a^3 + 50/11*x^11*d^9*c*b*a^4 + 1/11*x^11*d^10
```

```
*a^5 + 21*x^10*d^4*c^6*b^5 + 126*x^10*d^5*c^5*b^4*a + 210*x^10*d^6*c^4*b^3*
a^2 + 120*x^10*d^7*c^3*b^2*a^3 + 45/2*x^10*d^8*c^2*b*a^4 + x^10*d^9*c*a^5 +
 40/3*x^9*d^3*c^7*b^5 + 350/3*x^9*d^4*c^6*b^4*a + 280*x^9*d^5*c^5*b^3*a^2 +
 700/3*x^9*d^6*c^4*b^2*a^3 + 200/3*x^9*d^7*c^3*b*a^4 + 5*x^9*d^8*c^2*a^5 +
45/8*x^8*d^2*c^8*b^5 + 75*x^8*d^3*c^7*b^4*a + 525/2*x^8*d^4*c^6*b^3*a^2 + 3
15*x^8*d^5*c^5*b^2*a^3 + 525/4*x^8*d^6*c^4*b*a^4 + 15*x^8*d^7*c^3*a^5 + 10/
7*x^7*d*c^9*b^5 + 225/7*x^7*d^2*c^8*b^4*a + 1200/7*x^7*d^3*c^7*b^3*a^2 + 30
0*x^7*d^4*c^6*b^2*a^3 + 180*x^7*d^5*c^5*b*a^4 + 30*x^7*d^6*c^4*a^5 + 1/6*x^
6*c^10*b^5 + 25/3*x^6*d*c^9*b^4*a + 75*x^6*d^2*c^8*b^3*a^2 + 200*x^6*d^3*c^
7*b^2*a^3 + 175*x^6*d^4*c^6*b*a^4 + 42*x^6*d^5*c^5*a^5 + x^5*c^10*b^4*a + 2
0*x^5*d*c^9*b^3*a^2 + 90*x^5*d^2*c^8*b^2*a^3 + 120*x^5*d^3*c^7*b*a^4 + 42*x
^5*d^4*c^6*a^5 + 5/2*x^4*c^10*b^3*a^2 + 25*x^4*d*c^9*b^2*a^3 + 225/4*x^4*d^
2*c^8*b*a^4 + 30*x^4*d^3*c^7*a^5 + 10/3*x^3*c^10*b^2*a^3 + 50/3*x^3*d*c^9*b
*a^4 + 15*x^3*d^2*c^8*a^5 + 5/2*x^2*c^10*b*a^4 + 5*x^2*d*c^9*a^5 + x*c^10*a
^5
```

**giac [B]** time = 1.32, size = 948, normalized size = 6.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/16*b^5*d^10*x^16 + 2/3*b^5*c*d^9*x^15 + 1/3*a*b^4*d^10*x^15 + 45/14*b^5*c
^2*d^8*x^14 + 25/7*a*b^4*c*d^9*x^14 + 5/7*a^2*b^3*d^10*x^14 + 120/13*b^5*c^
3*d^7*x^13 + 225/13*a*b^4*c^2*d^8*x^13 + 100/13*a^2*b^3*c*d^9*x^13 + 10/13*
a^3*b^2*d^10*x^13 + 35/2*b^5*c^4*d^6*x^12 + 50*a*b^4*c^3*d^7*x^12 + 75/2*a^
2*b^3*c^2*d^8*x^12 + 25/3*a^3*b^2*c*d^9*x^12 + 5/12*a^4*b*d^10*x^12 + 252/1
1*b^5*c^5*d^5*x^11 + 1050/11*a*b^4*c^4*d^6*x^11 + 1200/11*a^2*b^3*c^3*d^7*x
^11 + 450/11*a^3*b^2*c^2*d^8*x^11 + 50/11*a^4*b*c*d^9*x^11 + 1/11*a^5*d^10*
x^11 + 21*b^5*c^6*d^4*x^10 + 126*a*b^4*c^5*d^5*x^10 + 210*a^2*b^3*c^4*d^6*x
^10 + 120*a^3*b^2*c^3*d^7*x^10 + 45/2*a^4*b*c^2*d^8*x^10 + a^5*c*d^9*x^10 +
 40/3*b^5*c^7*d^3*x^9 + 350/3*a*b^4*c^6*d^4*x^9 + 280*a^2*b^3*c^5*d^5*x^9 +
 700/3*a^3*b^2*c^4*d^6*x^9 + 200/3*a^4*b*c^3*d^7*x^9 + 5*a^5*c^2*d^8*x^9 +
45/8*b^5*c^8*d^2*x^8 + 75*a*b^4*c^7*d^3*x^8 + 525/2*a^2*b^3*c^6*d^4*x^8 + 3
15*a^3*b^2*c^5*d^5*x^8 + 525/4*a^4*b*c^4*d^6*x^8 + 15*a^5*c^3*d^7*x^8 + 10/
7*b^5*c^9*d*x^7 + 225/7*a*b^4*c^8*d^2*x^7 + 1200/7*a^2*b^3*c^7*d^3*x^7 + 30
0*a^3*b^2*c^6*d^4*x^7 + 180*a^4*b*c^5*d^5*x^7 + 30*a^5*c^4*d^6*x^7 + 1/6*b^
5*c^10*x^6 + 25/3*a*b^4*c^9*d*x^6 + 75*a^2*b^3*c^8*d^2*x^6 + 200*a^3*b^2*c^
7*d^3*x^6 + 175*a^4*b*c^6*d^4*x^6 + 42*a^5*c^5*d^5*x^6 + a*b^4*c^10*x^5 + 2
0*a^2*b^3*c^9*d*x^5 + 90*a^3*b^2*c^8*d^2*x^5 + 120*a^4*b*c^7*d^3*x^5 + 42*a
^5*c^6*d^4*x^5 + 5/2*a^2*b^3*c^10*x^4 + 25*a^3*b^2*c^9*d*x^4 + 225/4*a^4*b*
c^8*d^2*x^4 + 30*a^5*c^7*d^3*x^4 + 10/3*a^3*b^2*c^10*x^3 + 50/3*a^4*b*c^9*d
*x^3 + 15*a^5*c^8*d^2*x^3 + 5/2*a^4*b*c^10*x^2 + 5*a^5*c^9*d*x^2 + a^5*c^10
*x
```

**maple [B]** time = 0.00, size = 841, normalized size = 5.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^10,x)
```

```
[Out] 1/16*b^5*d^10*x^16+1/15*(5*a*b^4*d^10+10*b^5*c*d^9)*x^15+1/14*(10*a^2*b^3*d
^10+50*a*b^4*c*d^9+45*b^5*c^2*d^8)*x^14+1/13*(10*a^3*b^2*d^10+100*a^2*b^3*c
*d^9+225*a*b^4*c^2*d^8+120*b^5*c^3*d^7)*x^13+1/12*(5*a^4*b*d^10+100*a^3*b^2
*c*d^9+450*a^2*b^3*c^2*d^8+600*a*b^4*c^3*d^7+210*b^5*c^4*d^6)*x^12+1/11*(a^
5*d^10+50*a^4*b*c*d^9+450*a^3*b^2*c^2*d^8+1200*a^2*b^3*c^3*d^7+1050*a*b^4*c
^4*d^6+252*b^5*c^5*d^5)*x^11+1/10*(10*a^5*c*d^9+225*a^4*b*c^2*d^8+1200*a^3*
b^2*c^3*d^7+2100*a^2*b^3*c^4*d^6+1260*a*b^4*c^5*d^5+210*b^5*c^6*d^4)*x^10+1
/9*(45*a^5*c^2*d^8+600*a^4*b*c^3*d^7+2100*a^3*b^2*c^4*d^6+2520*a^2*b^3*c^5*
d^5+1050*a*b^4*c^6*d^4+120*b^5*c^7*d^3)*x^9+1/8*(120*a^5*c^3*d^7+1050*a^4*b
*c^4*d^6+2520*a^3*b^2*c^5*d^5+2100*a^2*b^3*c^6*d^4+600*a*b^4*c^7*d^3+45*b^5
*c^8*d^2)*x^8+1/7*(210*a^5*c^4*d^6+1260*a^4*b*c^5*d^5+2100*a^3*b^2*c^6*d^4+
1200*a^2*b^3*c^7*d^3+225*a*b^4*c^8*d^2+10*b^5*c^9*d)*x^7+1/6*(252*a^5*c^5*d
^5+1050*a^4*b*c^6*d^4+1200*a^3*b^2*c^7*d^3+450*a^2*b^3*c^8*d^2+50*a*b^4*c^9
*d+b^5*c^10)*x^6+1/5*(210*a^5*c^6*d^4+600*a^4*b*c^7*d^3+450*a^3*b^2*c^8*d^2
+100*a^2*b^3*c^9*d+5*a*b^4*c^10)*x^5+1/4*(120*a^5*c^7*d^3+225*a^4*b*c^8*d^2
+100*a^3*b^2*c^9*d+10*a^2*b^3*c^10)*x^4+1/3*(45*a^5*c^8*d^2+50*a^4*b*c^9*d+
10*a^3*b^2*c^10)*x^3+1/2*(10*a^5*c^9*d+5*a^4*b*c^10)*x^2+a^5*c^10*x
```

**maxima [B]** time = 1.46, size = 835, normalized size = 5.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/16*b^5*d^10*x^16 + a^5*c^10*x + 1/3*(2*b^5*c*d^9 + a*b^4*d^10)*x^15 + 5/1
4*(9*b^5*c^2*d^8 + 10*a*b^4*c*d^9 + 2*a^2*b^3*d^10)*x^14 + 5/13*(24*b^5*c^3
*d^7 + 45*a*b^4*c^2*d^8 + 20*a^2*b^3*c*d^9 + 2*a^3*b^2*d^10)*x^13 + 5/12*(4
2*b^5*c^4*d^6 + 120*a*b^4*c^3*d^7 + 90*a^2*b^3*c^2*d^8 + 20*a^3*b^2*c*d^9 +
 a^4*b*d^10)*x^12 + 1/11*(252*b^5*c^5*d^5 + 1050*a*b^4*c^4*d^6 + 1200*a^2*b
^3*c^3*d^7 + 450*a^3*b^2*c^2*d^8 + 50*a^4*b*c*d^9 + a^5*d^10)*x^11 + 1/2*(4
2*b^5*c^6*d^4 + 252*a*b^4*c^5*d^5 + 420*a^2*b^3*c^4*d^6 + 240*a^3*b^2*c^3*d
^7 + 45*a^4*b*c^2*d^8 + 2*a^5*c*d^9)*x^10 + 5/3*(8*b^5*c^7*d^3 + 70*a*b^4*c
^6*d^4 + 168*a^2*b^3*c^5*d^5 + 140*a^3*b^2*c^4*d^6 + 40*a^4*b*c^3*d^7 + 3*a
^5*c^2*d^8)*x^9 + 15/8*(3*b^5*c^8*d^2 + 40*a*b^4*c^7*d^3 + 140*a^2*b^3*c^6*
d^4 + 168*a^3*b^2*c^5*d^5 + 70*a^4*b*c^4*d^6 + 8*a^5*c^3*d^7)*x^8 + 5/7*(2*
b^5*c^9*d + 45*a*b^4*c^8*d^2 + 240*a^2*b^3*c^7*d^3 + 420*a^3*b^2*c^6*d^4 +
```

```
252*a^4*b*c^5*d^5 + 42*a^5*c^4*d^6)*x^7 + 1/6*(b^5*c^10 + 50*a*b^4*c^9*d +
450*a^2*b^3*c^8*d^2 + 1200*a^3*b^2*c^7*d^3 + 1050*a^4*b*c^6*d^4 + 252*a^5*c
^5*d^5)*x^6 + (a*b^4*c^10 + 20*a^2*b^3*c^9*d + 90*a^3*b^2*c^8*d^2 + 120*a^4
*b*c^7*d^3 + 42*a^5*c^6*d^4)*x^5 + 5/4*(2*a^2*b^3*c^10 + 20*a^3*b^2*c^9*d +
 45*a^4*b*c^8*d^2 + 24*a^5*c^7*d^3)*x^4 + 5/3*(2*a^3*b^2*c^10 + 10*a^4*b*c^
9*d + 9*a^5*c^8*d^2)*x^3 + 5/2*(a^4*b*c^10 + 2*a^5*c^9*d)*x^2
```

**mupad [B]** time = 0.34, size = 806, normalized size = 5.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^10,x)
```

```
[Out] x^10*(a^5*c*d^9 + 21*b^5*c^6*d^4 + 126*a*b^4*c^5*d^5 + (45*a^4*b*c^2*d^8)/2
 + 210*a^2*b^3*c^4*d^6 + 120*a^3*b^2*c^3*d^7) + x^7*((10*b^5*c^9*d)/7 + 30*
a^5*c^4*d^6 + (225*a*b^4*c^8*d^2)/7 + 180*a^4*b*c^5*d^5 + (1200*a^2*b^3*c^7
*d^3)/7 + 300*a^3*b^2*c^6*d^4) + x^6*((b^5*c^10)/6 + 42*a^5*c^5*d^5 + 175*a
^4*b*c^6*d^4 + 75*a^2*b^3*c^8*d^2 + 200*a^3*b^2*c^7*d^3 + (25*a*b^4*c^9*d)/
3) + x^11*((a^5*d^10)/11 + (252*b^5*c^5*d^5)/11 + (1050*a*b^4*c^4*d^6)/11 +
 (1200*a^2*b^3*c^3*d^7)/11 + (450*a^3*b^2*c^2*d^8)/11 + (50*a^4*b*c*d^9)/11
) + x^8*(15*a^5*c^3*d^7 + (45*b^5*c^8*d^2)/8 + 75*a*b^4*c^7*d^3 + (525*a^4*
b*c^4*d^6)/4 + (525*a^2*b^3*c^6*d^4)/2 + 315*a^3*b^2*c^5*d^5) + x^9*(5*a^5*
c^2*d^8 + (40*b^5*c^7*d^3)/3 + (350*a*b^4*c^6*d^4)/3 + (200*a^4*b*c^3*d^7)/
3 + 280*a^2*b^3*c^5*d^5 + (700*a^3*b^2*c^4*d^6)/3) + x^5*(a*b^4*c^10 + 42*a
^5*c^6*d^4 + 20*a^2*b^3*c^9*d + 120*a^4*b*c^7*d^3 + 90*a^3*b^2*c^8*d^2) + x
^12*((5*a^4*b*d^10)/12 + (35*b^5*c^4*d^6)/2 + 50*a*b^4*c^3*d^7 + (25*a^3*b^
2*c*d^9)/3 + (75*a^2*b^3*c^2*d^8)/2) + a^5*c^10*x + (b^5*d^10*x^16)/16 + (5
*a^2*c^7*x^4*(24*a^3*d^3 + 2*b^3*c^3 + 20*a*b^2*c^2*d + 45*a^2*b*c*d^2))/4
+ (5*b^2*d^7*x^13*(2*a^3*d^3 + 24*b^3*c^3 + 45*a*b^2*c^2*d + 20*a^2*b*c*d^2
))/13 + (5*a^4*c^9*x^2*(2*a*d + b*c))/2 + (b^4*d^9*x^15*(a*d + 2*b*c))/3 +
(5*a^3*c^8*x^3*(9*a^2*d^2 + 2*b^2*c^2 + 10*a*b*c*d))/3 + (5*b^3*d^8*x^14*(2
*a^2*d^2 + 9*b^2*c^2 + 10*a*b*c*d))/14
```

**sympy [B]** time = 0.21, size = 940, normalized size = 6.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**10,x)
```

```
[Out] a**5*c**10*x + b**5*d**10*x**16/16 + x**15*(a*b**4*d**10/3 + 2*b**5*c*d**9/
3) + x**14*(5*a**2*b**3*d**10/7 + 25*a*b**4*c*d**9/7 + 45*b**5*c**2*d**8/14
) + x**13*(10*a**3*b**2*d**10/13 + 100*a**2*b**3*c*d**9/13 + 225*a*b**4*c**
```

```
2*d**8/13 + 120*b**5*c**3*d**7/13) + x**12*(5*a**4*b*d**10/12 + 25*a**3*b**
2*c*d**9/3 + 75*a**2*b**3*c**2*d**8/2 + 50*a*b**4*c**3*d**7 + 35*b**5*c**4*
d**6/2) + x**11*(a**5*d**10/11 + 50*a**4*b*c*d**9/11 + 450*a**3*b**2*c**2*d
**8/11 + 1200*a**2*b**3*c**3*d**7/11 + 1050*a*b**4*c**4*d**6/11 + 252*b**5*
c**5*d**5/11) + x**10*(a**5*c*d**9 + 45*a**4*b*c**2*d**8/2 + 120*a**3*b**2*
c**3*d**7 + 210*a**2*b**3*c**4*d**6 + 126*a*b**4*c**5*d**5 + 21*b**5*c**6*d
**4) + x**9*(5*a**5*c**2*d**8 + 200*a**4*b*c**3*d**7/3 + 700*a**3*b**2*c**4
*d**6/3 + 280*a**2*b**3*c**5*d**5 + 350*a*b**4*c**6*d**4/3 + 40*b**5*c**7*d
**3/3) + x**8*(15*a**5*c**3*d**7 + 525*a**4*b*c**4*d**6/4 + 315*a**3*b**2*c
**5*d**5 + 525*a**2*b**3*c**6*d**4/2 + 75*a*b**4*c**7*d**3 + 45*b**5*c**8*d
**2/8) + x**7*(30*a**5*c**4*d**6 + 180*a**4*b*c**5*d**5 + 300*a**3*b**2*c**
6*d**4 + 1200*a**2*b**3*c**7*d**3/7 + 225*a*b**4*c**8*d**2/7 + 10*b**5*c**9
*d/7) + x**6*(42*a**5*c**5*d**5 + 175*a**4*b*c**6*d**4 + 200*a**3*b**2*c**7
*d**3 + 75*a**2*b**3*c**8*d**2 + 25*a*b**4*c**9*d/3 + b**5*c**10/6) + x**5*
(42*a**5*c**6*d**4 + 120*a**4*b*c**7*d**3 + 90*a**3*b**2*c**8*d**2 + 20*a**
2*b**3*c**9*d + a*b**4*c**10) + x**4*(30*a**5*c**7*d**3 + 225*a**4*b*c**8*d
**2/4 + 25*a**3*b**2*c**9*d + 5*a**2*b**3*c**10/2) + x**3*(15*a**5*c**8*d**
2 + 50*a**4*b*c**9*d/3 + 10*a**3*b**2*c**10/3) + x**2*(5*a**5*c**9*d + 5*a*
*4*b*c**10/2)
```

## 3.1201 $\int (a+bx)^4(c+dx)^{10}\,dx$

**Optimal**. Leaf size=119

$$-\frac{2b^3(c+dx)^{14}(bc-ad)}{7d^5}+\frac{6b^2(c+dx)^{13}(bc-ad)^2}{13d^5}-\frac{b(c+dx)^{12}(bc-ad)^3}{3d^5}+\frac{(c+dx)^{11}(bc-ad)^4}{11d^5}+\frac{b^4(c+dx)^{15}}{15d^5}$$

**Rubi [A]** time = 0.44, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b^3(c+dx)^{14}(bc-ad)}{7d^5}+\frac{6b^2(c+dx)^{13}(bc-ad)^2}{13d^5}-\frac{b(c+dx)^{12}(bc-ad)^3}{3d^5}+\frac{(c+dx)^{11}(bc-ad)^4}{11d^5}+\frac{b^4(c+dx)^{15}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^4*(c + d*x)^11)/(11*d^5) - (b*(b*c - a*d)^3*(c + d*x)^12)/(3*d
^5) + (6*b^2*(b*c - a*d)^2*(c + d*x)^13)/(13*d^5) - (2*b^3*(b*c - a*d)*(c +
 d*x)^14)/(7*d^5) + (b^4*(c + d*x)^15)/(15*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^4(c+dx)^{10}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{11}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{12}}{d^4}-4b^3(b$$

$$=\frac{(bc-ad)^4(c+dx)^{11}}{11d^5}-\frac{b(bc-ad)^3(c+dx)^{12}}{3d^5}+\frac{6b^2(bc-ad)^2(c+dx)^{13}}{13d^5}-\frac{2b^3(bc-ad}{7d}$$

**Mathematica [B]** time = 0.08, size = 660, normalized size = 5.55

$$a^4c^{10}x+a^3c^9x^2(5ad+2bc)+\frac{1}{13}b^2d^8x^{13}\left(6a^2d^2+40abcd+45b^2c^2\right)+\frac{1}{3}a^2c^8x^3\left(45a^2d^2+40abcd+6b^2c^2\right)+\frac{1}{3}bd^7x^{12}\left(a^3d^3+15a^2bcd^2+45ab^2c^2d+30b^3c^3\right)+ac^7x^4\left(30a^3d^3+45a^2bcd^2+15ab^2c^2d+b^3c^3\right)+\frac{1}{3}c^2d^4x^9\left(15a^4d^4+160a^3bcd^3+420a^2b^2c^2d^2+336ab^3c^3d+70b^4c^4\right)+3c^3d^3x^8\left(5a^4d^4+35a^3bcd^3+63a^2b^2c^2d^2+35ab^3c^3d+5b^4c^4\right)+\frac{3}{7}c^4d^2x^7\left(70a^4d^4+336a^3bcd^3+420a^2b^2c^2d^2+160ab^3c^3d+15b^4c^4\right)+\frac{1}{11}d^6x^{11}\left(a^4d^4+40a^3bcd^3+270a^2b^2c^2d^2+480ab^3c^3d+210b^4c^4\right)+\frac{1}{5}cd^5x^{10}\left(5a^4d^4+90a^3bcd^3+360a^2b^2c^2d^2+420ab^3c^3d+126b^4c^4\right)+\frac{1}{5}c^6x^5\left(210a^4d^4+480a^3bcd^3+270a^2b^2c^2d^2+40ab^3c^3d+b^4c^4\right)+\frac{1}{3}c^5dx^6\left(126a^4d^4+420a^3bcd^3+360a^2b^2c^2d^2+90ab^3c^3d+5b^4c^4\right)+\frac{1}{7}b^3d^9x^{14}(2ad+5bc)+\frac{1}{15}b^4d^{10}x^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] a^4*c^10*x + a^3*c^9*(2*b*c + 5*a*d)*x^2 + (a^2*c^8*(6*b^2*c^2 + 40*a*b*c*d
 + 45*a^2*d^2)*x^3)/3 + a*c^7*(b^3*c^3 + 15*a*b^2*c^2*d + 45*a^2*b*c*d^2 +
30*a^3*d^3)*x^4 + (c^6*(b^4*c^4 + 40*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 48
0*a^3*b*c*d^3 + 210*a^4*d^4)*x^5)/5 + (c^5*d*(5*b^4*c^4 + 90*a*b^3*c^3*d +
360*a^2*b^2*c^2*d^2 + 420*a^3*b*c*d^3 + 126*a^4*d^4)*x^6)/3 + (3*c^4*d^2*(1
5*b^4*c^4 + 160*a*b^3*c^3*d + 420*a^2*b^2*c^2*d^2 + 336*a^3*b*c*d^3 + 70*a^
4*d^4)*x^7)/7 + 3*c^3*d^3*(5*b^4*c^4 + 35*a*b^3*c^3*d + 63*a^2*b^2*c^2*d^2
+ 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^8 + (c^2*d^4*(70*b^4*c^4 + 336*a*b^3*c^3*d
+ 420*a^2*b^2*c^2*d^2 + 160*a^3*b*c*d^3 + 15*a^4*d^4)*x^9)/3 + (c*d^5*(126*
b^4*c^4 + 420*a*b^3*c^3*d + 360*a^2*b^2*c^2*d^2 + 90*a^3*b*c*d^3 + 5*a^4*d^
4)*x^10)/5 + (d^6*(210*b^4*c^4 + 480*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 40
*a^3*b*c*d^3 + a^4*d^4)*x^11)/11 + (b*d^7*(30*b^3*c^3 + 45*a*b^2*c^2*d + 15
*a^2*b*c*d^2 + a^3*d^3)*x^12)/3 + (b^2*d^8*(45*b^2*c^2 + 40*a*b*c*d + 6*a^2
*d^2)*x^13)/13 + (b^3*d^9*(5*b*c + 2*a*d)*x^14)/7 + (b^4*d^10*x^15)/15
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^4(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^10, x]
```

**fricas [B]** time = 1.10, size = 771, normalized size = 6.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*d^10*b^4 + 5/7*x^14*d^9*c*b^4 + 2/7*x^14*d^10*b^3*a + 45/13*x^13*
d^8*c^2*b^4 + 40/13*x^13*d^9*c*b^3*a + 6/13*x^13*d^10*b^2*a^2 + 10*x^12*d^7
*c^3*b^4 + 15*x^12*d^8*c^2*b^3*a + 5*x^12*d^9*c*b^2*a^2 + 1/3*x^12*d^10*b*a
^3 + 210/11*x^11*d^6*c^4*b^4 + 480/11*x^11*d^7*c^3*b^3*a + 270/11*x^11*d^8*
c^2*b^2*a^2 + 40/11*x^11*d^9*c*b*a^3 + 1/11*x^11*d^10*a^4 + 126/5*x^10*d^5*
c^5*b^4 + 84*x^10*d^6*c^4*b^3*a + 72*x^10*d^7*c^3*b^2*a^2 + 18*x^10*d^8*c^2
*b*a^3 + x^10*d^9*c*a^4 + 70/3*x^9*d^4*c^6*b^4 + 112*x^9*d^5*c^5*b^3*a + 14
0*x^9*d^6*c^4*b^2*a^2 + 160/3*x^9*d^7*c^3*b*a^3 + 5*x^9*d^8*c^2*a^4 + 15*x^
8*d^3*c^7*b^4 + 105*x^8*d^4*c^6*b^3*a + 189*x^8*d^5*c^5*b^2*a^2 + 105*x^8*d
^6*c^4*b*a^3 + 15*x^8*d^7*c^3*a^4 + 45/7*x^7*d^2*c^8*b^4 + 480/7*x^7*d^3*c^
7*b^3*a + 180*x^7*d^4*c^6*b^2*a^2 + 144*x^7*d^5*c^5*b*a^3 + 30*x^7*d^6*c^4*
a^4 + 5/3*x^6*d*c^9*b^4 + 30*x^6*d^2*c^8*b^3*a + 120*x^6*d^3*c^7*b^2*a^2 +
140*x^6*d^4*c^6*b*a^3 + 42*x^6*d^5*c^5*a^4 + 1/5*x^5*c^10*b^4 + 8*x^5*d*c^9
```

```
*b^3*a + 54*x^5*d^2*c^8*b^2*a^2 + 96*x^5*d^3*c^7*b*a^3 + 42*x^5*d^4*c^6*a^4
 + x^4*c^10*b^3*a + 15*x^4*d*c^9*b^2*a^2 + 45*x^4*d^2*c^8*b*a^3 + 30*x^4*d^
3*c^7*a^4 + 2*x^3*c^10*b^2*a^2 + 40/3*x^3*d*c^9*b*a^3 + 15*x^3*d^2*c^8*a^4
+ 2*x^2*c^10*b*a^3 + 5*x^2*d*c^9*a^4 + x*c^10*a^4
```

**giac [B]** time = 1.27, size = 771, normalized size = 6.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/15*b^4*d^10*x^15 + 5/7*b^4*c*d^9*x^14 + 2/7*a*b^3*d^10*x^14 + 45/13*b^4*c
^2*d^8*x^13 + 40/13*a*b^3*c*d^9*x^13 + 6/13*a^2*b^2*d^10*x^13 + 10*b^4*c^3*
d^7*x^12 + 15*a*b^3*c^2*d^8*x^12 + 5*a^2*b^2*c*d^9*x^12 + 1/3*a^3*b*d^10*x^
12 + 210/11*b^4*c^4*d^6*x^11 + 480/11*a*b^3*c^3*d^7*x^11 + 270/11*a^2*b^2*c
^2*d^8*x^11 + 40/11*a^3*b*c*d^9*x^11 + 1/11*a^4*d^10*x^11 + 126/5*b^4*c^5*d
^5*x^10 + 84*a*b^3*c^4*d^6*x^10 + 72*a^2*b^2*c^3*d^7*x^10 + 18*a^3*b*c^2*d^
8*x^10 + a^4*c*d^9*x^10 + 70/3*b^4*c^6*d^4*x^9 + 112*a*b^3*c^5*d^5*x^9 + 14
0*a^2*b^2*c^4*d^6*x^9 + 160/3*a^3*b*c^3*d^7*x^9 + 5*a^4*c^2*d^8*x^9 + 15*b^
4*c^7*d^3*x^8 + 105*a*b^3*c^6*d^4*x^8 + 189*a^2*b^2*c^5*d^5*x^8 + 105*a^3*b
*c^4*d^6*x^8 + 15*a^4*c^3*d^7*x^8 + 45/7*b^4*c^8*d^2*x^7 + 480/7*a*b^3*c^7*
d^3*x^7 + 180*a^2*b^2*c^6*d^4*x^7 + 144*a^3*b*c^5*d^5*x^7 + 30*a^4*c^4*d^6*
x^7 + 5/3*b^4*c^9*d*x^6 + 30*a*b^3*c^8*d^2*x^6 + 120*a^2*b^2*c^7*d^3*x^6 +
140*a^3*b*c^6*d^4*x^6 + 42*a^4*c^5*d^5*x^6 + 1/5*b^4*c^10*x^5 + 8*a*b^3*c^9
*d*x^5 + 54*a^2*b^2*c^8*d^2*x^5 + 96*a^3*b*c^7*d^3*x^5 + 42*a^4*c^6*d^4*x^5
 + a*b^3*c^10*x^4 + 15*a^2*b^2*c^9*d*x^4 + 45*a^3*b*c^8*d^2*x^4 + 30*a^4*c^
7*d^3*x^4 + 2*a^2*b^2*c^10*x^3 + 40/3*a^3*b*c^9*d*x^3 + 15*a^4*c^8*d^2*x^3
+ 2*a^3*b*c^10*x^2 + 5*a^4*c^9*d*x^2 + a^4*c^10*x
```

**maple [B]** time = 0.00, size = 691, normalized size = 5.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^10,x)
```

```
[Out] 1/15*b^4*d^10*x^15+1/14*(4*a*b^3*d^10+10*b^4*c*d^9)*x^14+1/13*(6*a^2*b^2*d^
10+40*a*b^3*c*d^9+45*b^4*c^2*d^8)*x^13+1/12*(4*a^3*b*d^10+60*a^2*b^2*c*d^9+
180*a*b^3*c^2*d^8+120*b^4*c^3*d^7)*x^12+1/11*(a^4*d^10+40*a^3*b*c*d^9+270*a
^2*b^2*c^2*d^8+480*a*b^3*c^3*d^7+210*b^4*c^4*d^6)*x^11+1/10*(10*a^4*c*d^9+1
80*a^3*b*c^2*d^8+720*a^2*b^2*c^3*d^7+840*a*b^3*c^4*d^6+252*b^4*c^5*d^5)*x^1
0+1/9*(45*a^4*c^2*d^8+480*a^3*b*c^3*d^7+1260*a^2*b^2*c^4*d^6+1008*a*b^3*c^5
*d^5+210*b^4*c^6*d^4)*x^9+1/8*(120*a^4*c^3*d^7+840*a^3*b*c^4*d^6+1512*a^2*b
```

```
^2*c^5*d^5+840*a*b^3*c^6*d^4+120*b^4*c^7*d^3)*x^8+1/7*(210*a^4*c^4*d^6+1008
*a^3*b*c^5*d^5+1260*a^2*b^2*c^6*d^4+480*a*b^3*c^7*d^3+45*b^4*c^8*d^2)*x^7+1
/6*(252*a^4*c^5*d^5+840*a^3*b*c^6*d^4+720*a^2*b^2*c^7*d^3+180*a*b^3*c^8*d^2
+10*b^4*c^9*d)*x^6+1/5*(210*a^4*c^6*d^4+480*a^3*b*c^7*d^3+270*a^2*b^2*c^8*d
^2+40*a*b^3*c^9*d+b^4*c^10)*x^5+1/4*(120*a^4*c^7*d^3+180*a^3*b*c^8*d^2+60*a
^2*b^2*c^9*d+4*a*b^3*c^10)*x^4+1/3*(45*a^4*c^8*d^2+40*a^3*b*c^9*d+6*a^2*b^2
*c^10)*x^3+1/2*(10*a^4*c^9*d+4*a^3*b*c^10)*x^2+a^4*c^10*x
```

**maxima** [B] time = 1.42, size = 686, normalized size = 5.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*d^10*x^15 + a^4*c^10*x + 1/7*(5*b^4*c*d^9 + 2*a*b^3*d^10)*x^14 + 1
/13*(45*b^4*c^2*d^8 + 40*a*b^3*c*d^9 + 6*a^2*b^2*d^10)*x^13 + 1/3*(30*b^4*c
^3*d^7 + 45*a*b^3*c^2*d^8 + 15*a^2*b^2*c*d^9 + a^3*b*d^10)*x^12 + 1/11*(210
*b^4*c^4*d^6 + 480*a*b^3*c^3*d^7 + 270*a^2*b^2*c^2*d^8 + 40*a^3*b*c*d^9 + a
^4*d^10)*x^11 + 1/5*(126*b^4*c^5*d^5 + 420*a*b^3*c^4*d^6 + 360*a^2*b^2*c^3*
d^7 + 90*a^3*b*c^2*d^8 + 5*a^4*c*d^9)*x^10 + 1/3*(70*b^4*c^6*d^4 + 336*a*b^
3*c^5*d^5 + 420*a^2*b^2*c^4*d^6 + 160*a^3*b*c^3*d^7 + 15*a^4*c^2*d^8)*x^9 +
 3*(5*b^4*c^7*d^3 + 35*a*b^3*c^6*d^4 + 63*a^2*b^2*c^5*d^5 + 35*a^3*b*c^4*d^
6 + 5*a^4*c^3*d^7)*x^8 + 3/7*(15*b^4*c^8*d^2 + 160*a*b^3*c^7*d^3 + 420*a^2*
b^2*c^6*d^4 + 336*a^3*b*c^5*d^5 + 70*a^4*c^4*d^6)*x^7 + 1/3*(5*b^4*c^9*d +
90*a*b^3*c^8*d^2 + 360*a^2*b^2*c^7*d^3 + 420*a^3*b*c^6*d^4 + 126*a^4*c^5*d^
5)*x^6 + 1/5*(b^4*c^10 + 40*a*b^3*c^9*d + 270*a^2*b^2*c^8*d^2 + 480*a^3*b*c
^7*d^3 + 210*a^4*c^6*d^4)*x^5 + (a*b^3*c^10 + 15*a^2*b^2*c^9*d + 45*a^3*b*c
^8*d^2 + 30*a^4*c^7*d^3)*x^4 + 1/3*(6*a^2*b^2*c^10 + 40*a^3*b*c^9*d + 45*a^
4*c^8*d^2)*x^3 + (2*a^3*b*c^10 + 5*a^4*c^9*d)*x^2
```

**mupad** [B] time = 0.43, size = 664, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^10,x)
```

```
[Out] x^5*((b^4*c^10)/5 + 42*a^4*c^6*d^4 + 96*a^3*b*c^7*d^3 + 54*a^2*b^2*c^8*d^2
+ 8*a*b^3*c^9*d) + x^11*((a^4*d^10)/11 + (210*b^4*c^4*d^6)/11 + (480*a*b^3*
c^3*d^7)/11 + (270*a^2*b^2*c^2*d^8)/11 + (40*a^3*b*c*d^9)/11) + x^8*(15*a^4
*c^3*d^7 + 15*b^4*c^7*d^3 + 105*a*b^3*c^6*d^4 + 105*a^3*b*c^4*d^6 + 189*a^2
*b^2*c^5*d^5) + x^9*(5*a^4*c^2*d^8 + (70*b^4*c^6*d^4)/3 + 112*a*b^3*c^5*d^5
 + (160*a^3*b*c^3*d^7)/3 + 140*a^2*b^2*c^4*d^6) + x^7*(30*a^4*c^4*d^6 + (45
*b^4*c^8*d^2)/7 + (480*a*b^3*c^7*d^3)/7 + 144*a^3*b*c^5*d^5 + 180*a^2*b^2*c
```

```
^6*d^4) + x^4*(a*b^3*c^10 + 30*a^4*c^7*d^3 + 15*a^2*b^2*c^9*d + 45*a^3*b*c^
8*d^2) + x^12*((a^3*b*d^10)/3 + 10*b^4*c^3*d^7 + 15*a*b^3*c^2*d^8 + 5*a^2*b
^2*c*d^9) + x^10*(a^4*c*d^9 + (126*b^4*c^5*d^5)/5 + 84*a*b^3*c^4*d^6 + 18*a
^3*b*c^2*d^8 + 72*a^2*b^2*c^3*d^7) + x^6*((5*b^4*c^9*d)/3 + 42*a^4*c^5*d^5
+ 30*a*b^3*c^8*d^2 + 140*a^3*b*c^6*d^4 + 120*a^2*b^2*c^7*d^3) + a^4*c^10*x
+ (b^4*d^10*x^15)/15 + a^3*c^9*x^2*(5*a*d + 2*b*c) + (b^3*d^9*x^14*(2*a*d +
 5*b*c))/7 + (a^2*c^8*x^3*(45*a^2*d^2 + 6*b^2*c^2 + 40*a*b*c*d))/3 + (b^2*d
^8*x^13*(6*a^2*d^2 + 45*b^2*c^2 + 40*a*b*c*d))/13
```

**sympy [B]** time = 0.18, size = 748, normalized size = 6.29

$$a^{4}c^{10}x + \frac{b^{4}d^{10}x^{15}}{15} + x^{14}\left(\frac{2ab^{3}d^{10}}{7} + \frac{5b^{4}cd^{9}}{7}\right) + x^{13}\left(\frac{6a^{2}b^{2}d^{10}}{13} + \frac{40ab^{3}cd^{9}}{13} + \frac{45b^{4}c^{2}d^{8}}{13}\right) + x^{12}\left(\frac{a^{3}bd^{10}}{3} + 5a^{2}b^{2}cd^{9} + 15ab^{3}c^{2}d^{8} + 10b^{4}c^{3}d^{7}\right) + x^{11}\left(\frac{a^{4}d^{10}}{11} + \frac{40a^{3}bcd^{9}}{11} + \frac{270a^{2}b^{2}c^{2}d^{8}}{11} + \frac{480ab^{3}c^{3}d^{7}}{11} + \frac{210b^{4}c^{4}d^{6}}{11}\right) + x^{10}\left(a^{4}cd^{9} + 18a^{3}bc^{2}d^{8} + 72a^{2}b^{2}c^{3}d^{7} + 84ab^{3}c^{4}d^{6} + \frac{126b^{4}c^{5}d^{5}}{5}\right) + x^{9}\left(5a^{4}c^{2}d^{8} + \frac{160a^{3}bc^{3}d^{7}}{3} + 140a^{2}b^{2}c^{4}d^{6} + 112ab^{3}c^{5}d^{5} + \frac{70b^{4}c^{6}d^{4}}{3}\right) + x^{8}\left(15a^{4}c^{3}d^{7} + 105a^{3}bc^{4}d^{6} + 189a^{2}b^{2}c^{5}d^{5} + 105ab^{3}c^{6}d^{4} + 15b^{4}c^{7}d^{3}\right) + x^{7}\left(30a^{4}c^{4}d^{6} + 144a^{3}bc^{5}d^{5} + 180a^{2}b^{2}c^{6}d^{4} + \frac{480ab^{3}c^{7}d^{3}}{7} + \frac{45b^{4}c^{8}d^{2}}{7}\right) + x^{6}\left(42a^{4}c^{5}d^{5} + 140a^{3}bc^{6}d^{4} + 120a^{2}b^{2}c^{7}d^{3} + 30ab^{3}c^{8}d^{2} + \frac{5b^{4}c^{9}d}{3}\right) + x^{5}\left(42a^{4}c^{6}d^{4} + 96a^{3}bc^{7}d^{3} + 54a^{2}b^{2}c^{8}d^{2} + 8ab^{3}c^{9}d + \frac{b^{4}c^{10}}{5}\right) + x^{4}\left(30a^{4}c^{7}d^{3} + 45a^{3}bc^{8}d^{2} + 15a^{2}b^{2}c^{9}d + ab^{3}c^{10}\right) + x^{3}\left(15a^{4}c^{8}d^{2} + \frac{40a^{3}bc^{9}d}{3} + 2a^{2}b^{2}c^{10}\right) + x^{2}\left(5a^{4}c^{9}d + 2a^{3}bc^{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**10,x)
```

```
[Out] a**4*c**10*x + b**4*d**10*x**15/15 + x**14*(2*a*b**3*d**10/7 + 5*b**4*c*d**
9/7) + x**13*(6*a**2*b**2*d**10/13 + 40*a*b**3*c*d**9/13 + 45*b**4*c**2*d**
8/13) + x**12*(a**3*b*d**10/3 + 5*a**2*b**2*c*d**9 + 15*a*b**3*c**2*d**8 +
10*b**4*c**3*d**7) + x**11*(a**4*d**10/11 + 40*a**3*b*c*d**9/11 + 270*a**2*
b**2*c**2*d**8/11 + 480*a*b**3*c**3*d**7/11 + 210*b**4*c**4*d**6/11) + x**1
0*(a**4*c*d**9 + 18*a**3*b*c**2*d**8 + 72*a**2*b**2*c**3*d**7 + 84*a*b**3*c
**4*d**6 + 126*b**4*c**5*d**5/5) + x**9*(5*a**4*c**2*d**8 + 160*a**3*b*c**3
*d**7/3 + 140*a**2*b**2*c**4*d**6 + 112*a*b**3*c**5*d**5 + 70*b**4*c**6*d**
4/3) + x**8*(15*a**4*c**3*d**7 + 105*a**3*b*c**4*d**6 + 189*a**2*b**2*c**5*
d**5 + 105*a*b**3*c**6*d**4 + 15*b**4*c**7*d**3) + x**7*(30*a**4*c**4*d**6
+ 144*a**3*b*c**5*d**5 + 180*a**2*b**2*c**6*d**4 + 480*a*b**3*c**7*d**3/7 +
 45*b**4*c**8*d**2/7) + x**6*(42*a**4*c**5*d**5 + 140*a**3*b*c**6*d**4 + 12
0*a**2*b**2*c**7*d**3 + 30*a*b**3*c**8*d**2 + 5*b**4*c**9*d/3) + x**5*(42*a
**4*c**6*d**4 + 96*a**3*b*c**7*d**3 + 54*a**2*b**2*c**8*d**2 + 8*a*b**3*c**
9*d + b**4*c**10/5) + x**4*(30*a**4*c**7*d**3 + 45*a**3*b*c**8*d**2 + 15*a*
*2*b**2*c**9*d + a*b**3*c**10) + x**3*(15*a**4*c**8*d**2 + 40*a**3*b*c**9*d
/3 + 2*a**2*b**2*c**10) + x**2*(5*a**4*c**9*d + 2*a**3*b*c**10)
```

## 3.1202 $\int (a+bx)^3(c+dx)^{10}\,dx$

**Optimal**. Leaf size=92

$$-\frac{3b^2(c+dx)^{13}(bc-ad)}{13d^4}+\frac{b(c+dx)^{12}(bc-ad)^2}{4d^4}-\frac{(c+dx)^{11}(bc-ad)^3}{11d^4}+\frac{b^3(c+dx)^{14}}{14d^4}$$

**Rubi [A]** time = 0.35, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^2(c+dx)^{13}(bc-ad)}{13d^4}+\frac{b(c+dx)^{12}(bc-ad)^2}{4d^4}-\frac{(c+dx)^{11}(bc-ad)^3}{11d^4}+\frac{b^3(c+dx)^{14}}{14d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)^3*(c + d*x)^11)/(11*d^4) + (b*(b*c - a*d)^2*(c + d*x)^12)/(4*
d^4) - (3*b^2*(b*c - a*d)*(c + d*x)^13)/(13*d^4) + (b^3*(c + d*x)^14)/(14*d
^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^{10}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{11}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{12}}{d^3}+\frac{b^3(c\right.\\ &= -\frac{(bc-ad)^3(c+dx)^{11}}{11d^4}+\frac{b(bc-ad)^2(c+dx)^{12}}{4d^4}-\frac{3b^2(bc-ad)(c+dx)^{13}}{13d^4}+\frac{b^3(c+dx}{14d^4}\end{aligned}$$

**Mathematica [B]** time = 0.07, size = 511, normalized size = 5.55

$$a^3c^{10}x+\frac{1}{4}bd^8x^{12}\left(a^2d^2+10abcd+15b^2c^2\right)+ac^8x^3\left(15a^2d^2+10abcd+b^2c^2\right)+\frac{1}{2}a^2c^9x^2(10ad+3bc)+\frac{1}{11}d^7x^{11}\left(a^3d^3+30a^2bcd^2+135ab^2c^2d+120b^3c^3\right)+\frac{1}{2}cd^6x^{10}\left(2a^3d^3+27a^2bcd^2+72ab^2c^2d+42b^3c^3\right)+c^2d^5x^9\left(5a^3d^3+40a^2bcd^2+70ab^2c^2d+28b^3c^3\right)+\frac{3}{4}c^3d^4x^8\left(20a^3d^3+105a^2bcd^2+126ab^2c^2d+35b^3c^3\right)+\frac{1}{4}c^7x^4\left(120a^3d^3+135a^2bcd^2+30ab^2c^2d+b^3c^3\right)+c^6dx^5\left(42a^3d^3+72a^2bcd^2+27ab^2c^2d+2b^3c^3\right)+\frac{3}{2}c^5d^2x^6\left(28a^3d^3+70a^2bcd^2+40ab^2c^2d+5b^3c^3\right)+\frac{6}{7}c^4d^3x^7\left(35a^3d^3+126a^2bcd^2+105ab^2c^2d+20b^3c^3\right)+\frac{1}{13}b^2d^9x^{13}(3ad+10bc)+\frac{1}{14}b^3d^{10}x^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] a^3*c^10*x + (a^2*c^9*(3*b*c + 10*a*d)*x^2)/2 + a*c^8*(b^2*c^2 + 10*a*b*c*d
 + 15*a^2*d^2)*x^3 + (c^7*(b^3*c^3 + 30*a*b^2*c^2*d + 135*a^2*b*c*d^2 + 120
*a^3*d^3)*x^4)/4 + c^6*d*(2*b^3*c^3 + 27*a*b^2*c^2*d + 72*a^2*b*c*d^2 + 42*
a^3*d^3)*x^5 + (3*c^5*d^2*(5*b^3*c^3 + 40*a*b^2*c^2*d + 70*a^2*b*c*d^2 + 28
*a^3*d^3)*x^6)/2 + (6*c^4*d^3*(20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*d
^2 + 35*a^3*d^3)*x^7)/7 + (3*c^3*d^4*(35*b^3*c^3 + 126*a*b^2*c^2*d + 105*a^
2*b*c*d^2 + 20*a^3*d^3)*x^8)/4 + c^2*d^5*(28*b^3*c^3 + 70*a*b^2*c^2*d + 40*
a^2*b*c*d^2 + 5*a^3*d^3)*x^9 + (c*d^6*(42*b^3*c^3 + 72*a*b^2*c^2*d + 27*a^2
*b*c*d^2 + 2*a^3*d^3)*x^10)/2 + (d^7*(120*b^3*c^3 + 135*a*b^2*c^2*d + 30*a^
2*b*c*d^2 + a^3*d^3)*x^11)/11 + (b*d^8*(15*b^2*c^2 + 10*a*b*c*d + a^2*d^2)*
x^12)/4 + (b^2*d^9*(10*b*c + 3*a*d)*x^13)/13 + (b^3*d^10*x^14)/14
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^10, x]
```

**fricas [B]** time = 1.05, size = 594, normalized size = 6.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d^10*b^3 + 10/13*x^13*d^9*c*b^3 + 3/13*x^13*d^10*b^2*a + 15/4*x^1
2*d^8*c^2*b^3 + 5/2*x^12*d^9*c*b^2*a + 1/4*x^12*d^10*b*a^2 + 120/11*x^11*d^
7*c^3*b^3 + 135/11*x^11*d^8*c^2*b^2*a + 30/11*x^11*d^9*c*b*a^2 + 1/11*x^11*
d^10*a^3 + 21*x^10*d^6*c^4*b^3 + 36*x^10*d^7*c^3*b^2*a + 27/2*x^10*d^8*c^2*
b*a^2 + x^10*d^9*c*a^3 + 28*x^9*d^5*c^5*b^3 + 70*x^9*d^6*c^4*b^2*a + 40*x^9
*d^7*c^3*b*a^2 + 5*x^9*d^8*c^2*a^3 + 105/4*x^8*d^4*c^6*b^3 + 189/2*x^8*d^5*
c^5*b^2*a + 315/4*x^8*d^6*c^4*b*a^2 + 15*x^8*d^7*c^3*a^3 + 120/7*x^7*d^3*c^
7*b^3 + 90*x^7*d^4*c^6*b^2*a + 108*x^7*d^5*c^5*b*a^2 + 30*x^7*d^6*c^4*a^3 +
 15/2*x^6*d^2*c^8*b^3 + 60*x^6*d^3*c^7*b^2*a + 105*x^6*d^4*c^6*b*a^2 + 42*x
^6*d^5*c^5*a^3 + 2*x^5*d*c^9*b^3 + 27*x^5*d^2*c^8*b^2*a + 72*x^5*d^3*c^7*b*
a^2 + 42*x^5*d^4*c^6*a^3 + 1/4*x^4*c^10*b^3 + 15/2*x^4*d*c^9*b^2*a + 135/4*
x^4*d^2*c^8*b*a^2 + 30*x^4*d^3*c^7*a^3 + x^3*c^10*b^2*a + 10*x^3*d*c^9*b*a^
2 + 15*x^3*d^2*c^8*a^3 + 3/2*x^2*c^10*b*a^2 + 5*x^2*d*c^9*a^3 + x*c^10*a^3
```

**giac [B]** time = 1.28, size = 594, normalized size = 6.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/14*b^3*d^10*x^14 + 10/13*b^3*c*d^9*x^13 + 3/13*a*b^2*d^10*x^13 + 15/4*b^3
*c^2*d^8*x^12 + 5/2*a*b^2*c*d^9*x^12 + 1/4*a^2*b*d^10*x^12 + 120/11*b^3*c^3
*d^7*x^11 + 135/11*a*b^2*c^2*d^8*x^11 + 30/11*a^2*b*c*d^9*x^11 + 1/11*a^3*d
^10*x^11 + 21*b^3*c^4*d^6*x^10 + 36*a*b^2*c^3*d^7*x^10 + 27/2*a^2*b*c^2*d^8
*x^10 + a^3*c*d^9*x^10 + 28*b^3*c^5*d^5*x^9 + 70*a*b^2*c^4*d^6*x^9 + 40*a^2
*b*c^3*d^7*x^9 + 5*a^3*c^2*d^8*x^9 + 105/4*b^3*c^6*d^4*x^8 + 189/2*a*b^2*c^
5*d^5*x^8 + 315/4*a^2*b*c^4*d^6*x^8 + 15*a^3*c^3*d^7*x^8 + 120/7*b^3*c^7*d^
3*x^7 + 90*a*b^2*c^6*d^4*x^7 + 108*a^2*b*c^5*d^5*x^7 + 30*a^3*c^4*d^6*x^7 +
 15/2*b^3*c^8*d^2*x^6 + 60*a*b^2*c^7*d^3*x^6 + 105*a^2*b*c^6*d^4*x^6 + 42*a
^3*c^5*d^5*x^6 + 2*b^3*c^9*d*x^5 + 27*a*b^2*c^8*d^2*x^5 + 72*a^2*b*c^7*d^3*
x^5 + 42*a^3*c^6*d^4*x^5 + 1/4*b^3*c^10*x^4 + 15/2*a*b^2*c^9*d*x^4 + 135/4*
a^2*b*c^8*d^2*x^4 + 30*a^3*c^7*d^3*x^4 + a*b^2*c^10*x^3 + 10*a^2*b*c^9*d*x^
3 + 15*a^3*c^8*d^2*x^3 + 3/2*a^2*b*c^10*x^2 + 5*a^3*c^9*d*x^2 + a^3*c^10*x
```

**maple [B]** time = 0.00, size = 541, normalized size = 5.88

$$\frac{b^3d^{10}x^{14}}{14}+a^3c^{10}x+\frac{\left(3ab^2d^{10}+10b^3cd^9\right)x^{13}}{13}+\frac{\left(3a^2bd^{10}+30ab^2cd^9+45b^3c^2d^8\right)x^{12}}{12}+\frac{\left(a^3d^{10}+30a^2bcd^9+135ab^2c^2d^8+120b^3c^3d^7\right)x^{11}}{11}+\frac{\left(10a^3cd^9+135a^2bc^2d^8+360ab^2c^3d^7+210b^3c^4d^6\right)x^{10}}{10}+\frac{\left(45a^3c^2d^8+360a^2bc^3d^7+630ab^2c^4d^6+252b^3c^5d^5\right)x^9}{9}+\frac{\left(120a^3c^3d^7+630a^2bc^4d^6+756ab^2c^5d^5+210b^3c^6d^4\right)x^8}{8}+\frac{\left(210a^3c^4d^6+756a^2bc^5d^5+630ab^2c^6d^4+120b^3c^7d^3\right)x^7}{7}+\frac{\left(252a^3c^5d^5+630a^2bc^6d^4+360ab^2c^7d^3+45b^3c^8d^2\right)x^6}{6}+\frac{\left(210a^3c^6d^4+360a^2bc^7d^3+135ab^2c^8d^2+10b^3c^9d\right)x^5}{5}+\frac{\left(120a^3c^7d^3+135a^2bc^8d^2+30ab^2c^9d+b^3c^{10}\right)x^4}{4}+\frac{\left(45a^3c^8d^2+30a^2bc^9d+3ab^2c^{10}\right)x^3}{3}+\frac{\left(10a^3c^9d+3a^2bc^{10}\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^10,x)
```

```
[Out] 1/14*b^3*d^10*x^14+1/13*(3*a*b^2*d^10+10*b^3*c*d^9)*x^13+1/12*(3*a^2*b*d^10
+30*a*b^2*c*d^9+45*b^3*c^2*d^8)*x^12+1/11*(a^3*d^10+30*a^2*b*c*d^9+135*a*b^
2*c^2*d^8+120*b^3*c^3*d^7)*x^11+1/10*(10*a^3*c*d^9+135*a^2*b*c^2*d^8+360*a*
b^2*c^3*d^7+210*b^3*c^4*d^6)*x^10+1/9*(45*a^3*c^2*d^8+360*a^2*b*c^3*d^7+630
*a*b^2*c^4*d^6+252*b^3*c^5*d^5)*x^9+1/8*(120*a^3*c^3*d^7+630*a^2*b*c^4*d^6+
756*a*b^2*c^5*d^5+210*b^3*c^6*d^4)*x^8+1/7*(210*a^3*c^4*d^6+756*a^2*b*c^5*d
^5+630*a*b^2*c^6*d^4+120*b^3*c^7*d^3)*x^7+1/6*(252*a^3*c^5*d^5+630*a^2*b*c^
6*d^4+360*a*b^2*c^7*d^3+45*b^3*c^8*d^2)*x^6+1/5*(210*a^3*c^6*d^4+360*a^2*b*
c^7*d^3+135*a*b^2*c^8*d^2+10*b^3*c^9*d)*x^5+1/4*(120*a^3*c^7*d^3+135*a^2*b*
c^8*d^2+30*a*b^2*c^9*d+b^3*c^10)*x^4+1/3*(45*a^3*c^8*d^2+30*a^2*b*c^9*d+3*a
*b^2*c^10)*x^3+1/2*(10*a^3*c^9*d+3*a^2*b*c^10)*x^2+a^3*c^10*x
```

**maxima [B]** time = 1.32, size = 535, normalized size = 5.82

$$\frac{1}{14}b^3d^{10}x^{14}+a^3c^{10}x+\frac{1}{13}\left(10b^3cd^9+3ab^2d^{10}\right)x^{13}+\frac{1}{4}\left(15b^3c^2d^8+10ab^2cd^9+a^2bd^{10}\right)x^{12}+\frac{1}{11}\left(120b^3c^3d^7+135ab^2c^2d^8+30a^2bcd^9+a^3d^{10}\right)x^{11}+\frac{1}{2}\left(42b^3c^4d^6+72ab^2c^3d^7+27a^2bc^2d^8+2a^3cd^9\right)x^{10}+\left(28b^3c^5d^5+70ab^2c^4d^6+40a^2bc^3d^7+5a^3c^2d^8\right)x^9+\frac{3}{4}\left(35b^3c^6d^4+126ab^2c^5d^5+105a^2bc^4d^6+20a^3c^3d^7\right)x^8+\frac{6}{7}\left(20b^3c^7d^3+105ab^2c^6d^4+126a^2bc^5d^5+35a^3c^4d^6\right)x^7+\frac{3}{2}\left(5b^3c^8d^2+40ab^2c^7d^3+70a^2bc^6d^4+28a^3c^5d^5\right)x^6+\left(2b^3c^9d+27ab^2c^8d^2+72a^2bc^7d^3+42a^3c^6d^4\right)x^5+\frac{1}{4}\left(b^3c^{10}+30ab^2c^9d+135a^2bc^8d^2+120a^3c^7d^3\right)x^4+\left(ab^2c^{10}+10a^2bc^9d+15a^3c^8d^2\right)x^3+\frac{1}{2}\left(3a^2bc^{10}+10a^3c^9d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*d^10*x^14 + a^3*c^10*x + 1/13*(10*b^3*c*d^9 + 3*a*b^2*d^10)*x^13 +
 1/4*(15*b^3*c^2*d^8 + 10*a*b^2*c*d^9 + a^2*b*d^10)*x^12 + 1/11*(120*b^3*c^
```

```
3*d^7 + 135*a*b^2*c^2*d^8 + 30*a^2*b*c*d^9 + a^3*d^10)*x^11 + 1/2*(42*b^3*c
^4*d^6 + 72*a*b^2*c^3*d^7 + 27*a^2*b*c^2*d^8 + 2*a^3*c*d^9)*x^10 + (28*b^3*
c^5*d^5 + 70*a*b^2*c^4*d^6 + 40*a^2*b*c^3*d^7 + 5*a^3*c^2*d^8)*x^9 + 3/4*(3
5*b^3*c^6*d^4 + 126*a*b^2*c^5*d^5 + 105*a^2*b*c^4*d^6 + 20*a^3*c^3*d^7)*x^8
 + 6/7*(20*b^3*c^7*d^3 + 105*a*b^2*c^6*d^4 + 126*a^2*b*c^5*d^5 + 35*a^3*c^4
*d^6)*x^7 + 3/2*(5*b^3*c^8*d^2 + 40*a*b^2*c^7*d^3 + 70*a^2*b*c^6*d^4 + 28*a
^3*c^5*d^5)*x^6 + (2*b^3*c^9*d + 27*a*b^2*c^8*d^2 + 72*a^2*b*c^7*d^3 + 42*a
^3*c^6*d^4)*x^5 + 1/4*(b^3*c^10 + 30*a*b^2*c^9*d + 135*a^2*b*c^8*d^2 + 120*
a^3*c^7*d^3)*x^4 + (a*b^2*c^10 + 10*a^2*b*c^9*d + 15*a^3*c^8*d^2)*x^3 + 1/2
*(3*a^2*b*c^10 + 10*a^3*c^9*d)*x^2
```

**mupad [B]** time = 0.23, size = 495, normalized size = 5.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^10,x)
```

```
[Out] x^4*((b^3*c^10)/4 + 30*a^3*c^7*d^3 + (135*a^2*b*c^8*d^2)/4 + (15*a*b^2*c^9*
d)/2) + x^11*((a^3*d^10)/11 + (120*b^3*c^3*d^7)/11 + (135*a*b^2*c^2*d^8)/11
 + (30*a^2*b*c*d^9)/11) + a^3*c^10*x + (b^3*d^10*x^14)/14 + (3*c^5*d^2*x^6*
(28*a^3*d^3 + 5*b^3*c^3 + 40*a*b^2*c^2*d + 70*a^2*b*c*d^2))/2 + c^2*d^5*x^9
*(5*a^3*d^3 + 28*b^3*c^3 + 70*a*b^2*c^2*d + 40*a^2*b*c*d^2) + (6*c^4*d^3*x^
7*(35*a^3*d^3 + 20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*d^2))/7 + (3*c^3
*d^4*x^8*(20*a^3*d^3 + 35*b^3*c^3 + 126*a*b^2*c^2*d + 105*a^2*b*c*d^2))/4 +
 (a^2*c^9*x^2*(10*a*d + 3*b*c))/2 + (b^2*d^9*x^13*(3*a*d + 10*b*c))/13 + a*
c^8*x^3*(15*a^2*d^2 + b^2*c^2 + 10*a*b*c*d) + (b*d^8*x^12*(a^2*d^2 + 15*b^2
*c^2 + 10*a*b*c*d))/4 + c^6*d*x^5*(42*a^3*d^3 + 2*b^3*c^3 + 27*a*b^2*c^2*d
+ 72*a^2*b*c*d^2) + (c*d^6*x^10*(2*a^3*d^3 + 42*b^3*c^3 + 72*a*b^2*c^2*d +
27*a^2*b*c*d^2))/2
```

**sympy [B]** time = 0.16, size = 586, normalized size = 6.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**10,x)
```

```
[Out] a**3*c**10*x + b**3*d**10*x**14/14 + x**13*(3*a*b**2*d**10/13 + 10*b**3*c*d
**9/13) + x**12*(a**2*b*d**10/4 + 5*a*b**2*c*d**9/2 + 15*b**3*c**2*d**8/4)
+ x**11*(a**3*d**10/11 + 30*a**2*b*c*d**9/11 + 135*a*b**2*c**2*d**8/11 + 12
0*b**3*c**3*d**7/11) + x**10*(a**3*c*d**9 + 27*a**2*b*c**2*d**8/2 + 36*a*b*
*2*c**3*d**7 + 21*b**3*c**4*d**6) + x**9*(5*a**3*c**2*d**8 + 40*a**2*b*c**3
*d**7 + 70*a*b**2*c**4*d**6 + 28*b**3*c**5*d**5) + x**8*(15*a**3*c**3*d**7
```

```
+ 315*a**2*b*c**4*d**6/4 + 189*a*b**2*c**5*d**5/2 + 105*b**3*c**6*d**4/4) +
 x**7*(30*a**3*c**4*d**6 + 108*a**2*b*c**5*d**5 + 90*a*b**2*c**6*d**4 + 120
*b**3*c**7*d**3/7) + x**6*(42*a**3*c**5*d**5 + 105*a**2*b*c**6*d**4 + 60*a*
b**2*c**7*d**3 + 15*b**3*c**8*d**2/2) + x**5*(42*a**3*c**6*d**4 + 72*a**2*b
*c**7*d**3 + 27*a*b**2*c**8*d**2 + 2*b**3*c**9*d) + x**4*(30*a**3*c**7*d**3
 + 135*a**2*b*c**8*d**2/4 + 15*a*b**2*c**9*d/2 + b**3*c**10/4) + x**3*(15*a
**3*c**8*d**2 + 10*a**2*b*c**9*d + a*b**2*c**10) + x**2*(5*a**3*c**9*d + 3*
a**2*b*c**10/2)
```

## 3.1203 $\int (a+bx)^2(c+dx)^{10}\,dx$

**Optimal**. Leaf size=65

$$-\frac{b(c+dx)^{12}(bc-ad)}{6d^3}+\frac{(c+dx)^{11}(bc-ad)^2}{11d^3}+\frac{b^2(c+dx)^{13}}{13d^3}$$

**Rubi [A]** time = 0.25, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b(c+dx)^{12}(bc-ad)}{6d^3}+\frac{(c+dx)^{11}(bc-ad)^2}{11d^3}+\frac{b^2(c+dx)^{13}}{13d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^11)/(11*d^3) - (b*(b*c - a*d)*(c + d*x)^12)/(6*d^3
) + (b^2*(c + d*x)^13)/(13*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{10}}{d^2}-\frac{2b(bc-ad)(c+dx)^{11}}{d^2}+\frac{b^2(c+dx)^{12}}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^{11}}{11d^3}-\frac{b(bc-ad)(c+dx)^{12}}{6d^3}+\frac{b^2(c+dx)^{13}}{13d^3}\end{aligned}$$

**Mathematica [B]** time = 0.05, size = 358, normalized size = 5.51

$$\frac{1}{11}d^8x^{11}\left(a^2d^2+20abcd+45b^2c^2\right)+cd^7x^{10}\left(a^2d^2+9abcd+12b^2c^2\right)+\frac{5}{3}c^2d^6x^9\left(3a^2d^2+16abcd+14b^2c^2\right)+\frac{1}{3}c^8x^3\left(45a^2d^2+20abcd+b^2c^2\right)+\frac{5}{2}c^7dx^4\left(12a^2d^2+9abcd+b^2c^2\right)+3c^6d^2x^5\left(14a^2d^2+16abcd+3b^2c^2\right)+2c^5d^3x^6\left(21a^2d^2+35abcd+10b^2c^2\right)+6c^4d^4x^7\left(5a^2d^2+12abcd+5b^2c^2\right)+\frac{3}{2}c^3d^5x^8\left(10a^2d^2+35abcd+21b^2c^2\right)+a^2c^{10}x+ac^9x^2(5ad+bc)+\frac{1}{6}bd^9x^{12}(ad+5bc)+\frac{1}{13}b^2d^{10}x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] a^2*c^10*x + a*c^9*(b*c + 5*a*d)*x^2 + (c^8*(b^2*c^2 + 20*a*b*c*d + 45*a^2*
d^2)*x^3)/3 + (5*c^7*d*(b^2*c^2 + 9*a*b*c*d + 12*a^2*d^2)*x^4)/2 + 3*c^6*d^
2*(3*b^2*c^2 + 16*a*b*c*d + 14*a^2*d^2)*x^5 + 2*c^5*d^3*(10*b^2*c^2 + 35*a*
b*c*d + 21*a^2*d^2)*x^6 + 6*c^4*d^4*(5*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*x^
7 + (3*c^3*d^5*(21*b^2*c^2 + 35*a*b*c*d + 10*a^2*d^2)*x^8)/2 + (5*c^2*d^6*(
14*b^2*c^2 + 16*a*b*c*d + 3*a^2*d^2)*x^9)/3 + c*d^7*(12*b^2*c^2 + 9*a*b*c*d
 + a^2*d^2)*x^10 + (d^8*(45*b^2*c^2 + 20*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d
^9*(5*b*c + a*d)*x^12)/6 + (b^2*d^10*x^13)/13
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^10, x]
```

**fricas [B]** time = 0.94, size = 417, normalized size = 6.42

$$\frac{1}{13}x^{13}d^{10}b^2+\frac{5}{6}x^{12}d^9cb^2+\frac{1}{6}x^{12}d^{10}ba+\frac{45}{11}x^{11}d^8c^2b^2+\frac{20}{11}x^{11}d^9cba+\frac{1}{11}x^{11}d^{10}a^2+12x^{10}d^7c^3b^2+9x^{10}d^8c^2ba+x^{10}d^9ca^2+\frac{70}{3}x^9d^6c^4b^2+\frac{80}{3}x^9d^7c^3ba+5x^9d^8c^2a^2+\frac{63}{2}x^8d^5c^5b^2+\frac{105}{2}x^8d^6c^4ba+15x^8d^7c^3a^2+30x^7d^4c^6b^2+72x^7d^5c^5ba+30x^7d^6c^4a^2+20x^6d^3c^7b^2+70x^6d^4c^6ba+42x^6d^5c^5a^2+9x^5d^2c^8b^2+48x^5d^3c^7ba+42x^5d^4c^6a^2+\frac{5}{2}x^4dc^9b^2+\frac{45}{2}x^4d^2c^8ba+30x^4d^3c^7a^2+\frac{1}{3}x^3c^{10}b^2+\frac{20}{3}x^3dc^9ba+15x^3d^2c^8a^2+x^2c^{10}ba+5x^2dc^9a^2+xc^{10}a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^10*b^2 + 5/6*x^12*d^9*c*b^2 + 1/6*x^12*d^10*b*a + 45/11*x^11*d^
8*c^2*b^2 + 20/11*x^11*d^9*c*b*a + 1/11*x^11*d^10*a^2 + 12*x^10*d^7*c^3*b^2
 + 9*x^10*d^8*c^2*b*a + x^10*d^9*c*a^2 + 70/3*x^9*d^6*c^4*b^2 + 80/3*x^9*d^
7*c^3*b*a + 5*x^9*d^8*c^2*a^2 + 63/2*x^8*d^5*c^5*b^2 + 105/2*x^8*d^6*c^4*b*
a + 15*x^8*d^7*c^3*a^2 + 30*x^7*d^4*c^6*b^2 + 72*x^7*d^5*c^5*b*a + 30*x^7*d
^6*c^4*a^2 + 20*x^6*d^3*c^7*b^2 + 70*x^6*d^4*c^6*b*a + 42*x^6*d^5*c^5*a^2 +
 9*x^5*d^2*c^8*b^2 + 48*x^5*d^3*c^7*b*a + 42*x^5*d^4*c^6*a^2 + 5/2*x^4*d*c^
9*b^2 + 45/2*x^4*d^2*c^8*b*a + 30*x^4*d^3*c^7*a^2 + 1/3*x^3*c^10*b^2 + 20/3
*x^3*d*c^9*b*a + 15*x^3*d^2*c^8*a^2 + x^2*c^10*b*a + 5*x^2*d*c^9*a^2 + x*c^
10*a^2
```

**giac [B]** time = 1.26, size = 417, normalized size = 6.42

$$\frac{1}{13}b^2d^{10}x^{13}+\frac{5}{6}b^2cd^9x^{12}+\frac{1}{6}abd^{10}x^{12}+\frac{45}{11}b^2c^2d^8x^{11}+\frac{20}{11}abcd^9x^{11}+\frac{1}{11}a^2d^{10}x^{11}+12\,b^2c^3d^7x^{10}+9\,abc^2d^8x^{10}+a^2cd^9x^{10}+\frac{70}{3}b^2c^4d^6x^9+\frac{80}{3}abc^3d^7x^9+5\,a^2c^2d^8x^9+\frac{63}{2}b^2c^5d^5x^8+\frac{105}{2}abc^4d^6x^8+15\,a^2c^3d^7x^8+30\,b^2c^6d^4x^7+72\,abc^5d^5x^7+30\,a^2c^4d^6x^7+20\,b^2c^7d^3x^6+70\,abc^6d^4x^6+42\,a^2c^5d^5x^6+9\,b^2c^8d^2x^5+48\,abc^7d^3x^5+42\,a^2c^6d^4x^5+\frac{5}{2}b^2c^9dx^4+\frac{45}{2}abc^8d^2x^4+30\,a^2c^7d^3x^4+\frac{1}{3}b^2c^{10}x^3+\frac{20}{3}abc^9dx^3+15\,a^2c^8d^2x^3+abc^{10}x^2+5\,a^2c^9dx^2+a^2c^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^10*x^13 + 5/6*b^2*c*d^9*x^12 + 1/6*a*b*d^10*x^12 + 45/11*b^2*c^2
*d^8*x^11 + 20/11*a*b*c*d^9*x^11 + 1/11*a^2*d^10*x^11 + 12*b^2*c^3*d^7*x^10
 + 9*a*b*c^2*d^8*x^10 + a^2*c*d^9*x^10 + 70/3*b^2*c^4*d^6*x^9 + 80/3*a*b*c^
3*d^7*x^9 + 5*a^2*c^2*d^8*x^9 + 63/2*b^2*c^5*d^5*x^8 + 105/2*a*b*c^4*d^6*x^
8 + 15*a^2*c^3*d^7*x^8 + 30*b^2*c^6*d^4*x^7 + 72*a*b*c^5*d^5*x^7 + 30*a^2*c
^4*d^6*x^7 + 20*b^2*c^7*d^3*x^6 + 70*a*b*c^6*d^4*x^6 + 42*a^2*c^5*d^5*x^6 +
 9*b^2*c^8*d^2*x^5 + 48*a*b*c^7*d^3*x^5 + 42*a^2*c^6*d^4*x^5 + 5/2*b^2*c^9*
d*x^4 + 45/2*a*b*c^8*d^2*x^4 + 30*a^2*c^7*d^3*x^4 + 1/3*b^2*c^10*x^3 + 20/3
*a*b*c^9*d*x^3 + 15*a^2*c^8*d^2*x^3 + a*b*c^10*x^2 + 5*a^2*c^9*d*x^2 + a^2*
c^10*x
```

**maple** [B] time = 0.00, size = 391, normalized size = 6.02

$$\frac{b^2d^{10}x^{13}}{13}+a^2c^{10}x+\frac{\left(2ab\,d^{10}+10b^2c\,d^9\right)x^{12}}{12}+\frac{\left(a^2d^{10}+20abc\,d^9+45b^2c^2d^8\right)x^{11}}{11}+\frac{\left(10a^2c\,d^9+90ab\,c^2d^8+120b^2c^3d^7\right)x^{10}}{10}+\frac{\left(45a^2c^2d^8+240ab\,c^3d^7+210b^2c^4d^6\right)x^9}{9}+\frac{\left(120a^2c^3d^7+420ab\,c^4d^6+252b^2c^5d^5\right)x^8}{8}+\frac{\left(210a^2c^4d^6+504ab\,c^5d^5+210b^2c^6d^4\right)x^7}{7}+\frac{\left(252a^2c^5d^5+420ab\,c^6d^4+120b^2c^7d^3\right)x^6}{6}+\frac{\left(210a^2c^6d^4+240ab\,c^7d^3+45b^2c^8d^2\right)x^5}{5}+\frac{\left(120a^2c^7d^3+90ab\,c^8d^2+10b^2c^9d\right)x^4}{4}+\frac{\left(45a^2c^8d^2+20ab\,c^9d+b^2c^{10}\right)x^3}{3}+\frac{\left(10a^2c^9d+2ab\,c^{10}\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^10,x)
```

```
[Out] 1/13*b^2*d^10*x^13+1/12*(2*a*b*d^10+10*b^2*c*d^9)*x^12+1/11*(a^2*d^10+20*a*
b*c*d^9+45*b^2*c^2*d^8)*x^11+1/10*(10*a^2*c*d^9+90*a*b*c^2*d^8+120*b^2*c^3*
d^7)*x^10+1/9*(45*a^2*c^2*d^8+240*a*b*c^3*d^7+210*b^2*c^4*d^6)*x^9+1/8*(120
*a^2*c^3*d^7+420*a*b*c^4*d^6+252*b^2*c^5*d^5)*x^8+1/7*(210*a^2*c^4*d^6+504*
a*b*c^5*d^5+210*b^2*c^6*d^4)*x^7+1/6*(252*a^2*c^5*d^5+420*a*b*c^6*d^4+120*b
^2*c^7*d^3)*x^6+1/5*(210*a^2*c^6*d^4+240*a*b*c^7*d^3+45*b^2*c^8*d^2)*x^5+1/
4*(120*a^2*c^7*d^3+90*a*b*c^8*d^2+10*b^2*c^9*d)*x^4+1/3*(45*a^2*c^8*d^2+20*
a*b*c^9*d+b^2*c^10)*x^3+1/2*(10*a^2*c^9*d+2*a*b*c^10)*x^2+a^2*c^10*x
```

**maxima** [B] time = 1.33, size = 384, normalized size = 5.91

$$\frac{1}{13}\,b^2d^{10}x^{13}+a^2c^{10}x+\frac{1}{6}\left(5\,b^2cd^9+abd^{10}\right)x^{12}+\frac{1}{11}\left(45\,b^2c^2d^8+20\,abcd^9+a^2d^{10}\right)x^{11}+\left(12\,b^2c^3d^7+9\,abc^2d^8+a^2cd^9\right)x^{10}+\frac{5}{3}\left(14\,b^2c^4d^6+16\,abc^3d^7+3\,a^2c^2d^8\right)x^9+\frac{3}{2}\left(21\,b^2c^5d^5+35\,abc^4d^6+10\,a^2c^3d^7\right)x^8+6\left(5\,b^2c^6d^4+12\,abc^5d^5+5\,a^2c^4d^6\right)x^7+2\left(10\,b^2c^7d^3+35\,abc^6d^4+21\,a^2c^5d^5\right)x^6+3\left(3\,b^2c^8d^2+16\,abc^7d^3+14\,a^2c^6d^4\right)x^5+\frac{5}{2}\left(b^2c^9d+9\,abc^8d^2+12\,a^2c^7d^3\right)x^4+\frac{1}{3}\left(b^2c^{10}+20\,abc^9d+45\,a^2c^8d^2\right)x^3+\left(abc^{10}+5\,a^2c^9d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^10*x^13 + a^2*c^10*x + 1/6*(5*b^2*c*d^9 + a*b*d^10)*x^12 + 1/11*
(45*b^2*c^2*d^8 + 20*a*b*c*d^9 + a^2*d^10)*x^11 + (12*b^2*c^3*d^7 + 9*a*b*c
^2*d^8 + a^2*c*d^9)*x^10 + 5/3*(14*b^2*c^4*d^6 + 16*a*b*c^3*d^7 + 3*a^2*c^2
*d^8)*x^9 + 3/2*(21*b^2*c^5*d^5 + 35*a*b*c^4*d^6 + 10*a^2*c^3*d^7)*x^8 + 6*
(5*b^2*c^6*d^4 + 12*a*b*c^5*d^5 + 5*a^2*c^4*d^6)*x^7 + 2*(10*b^2*c^7*d^3 +
35*a*b*c^6*d^4 + 21*a^2*c^5*d^5)*x^6 + 3*(3*b^2*c^8*d^2 + 16*a*b*c^7*d^3 +
14*a^2*c^6*d^4)*x^5 + 5/2*(b^2*c^9*d + 9*a*b*c^8*d^2 + 12*a^2*c^7*d^3)*x^4
+ 1/3*(b^2*c^10 + 20*a*b*c^9*d + 45*a^2*c^8*d^2)*x^3 + (a*b*c^10 + 5*a^2*c^
9*d)*x^2
```

**mupad [B]** time = 0.32, size = 348, normalized size = 5.35

$$x^3\left(15a^2c^8d^2+\frac{20abc^9d}{3}+\frac{b^2c^{10}}{3}\right)+x^{11}\left(\frac{a^2d^{10}}{11}+\frac{20abcd^9}{11}+\frac{45b^2c^2d^8}{11}\right)+a^2c^{10}x+\frac{b^2d^{10}x^{13}}{13}+ac^9x^2(5ad+bc)+\frac{bd^9x^{12}(ad+5bc)}{6}+\frac{5c^7dx^4\left(12a^2d^2+9abcd+b^2c^2\right)}{2}+cd^7x^{10}\left(a^2d^2+9abcd+12b^2c^2\right)+6c^4d^4x^7\left(5a^2d^2+12abcd+5b^2c^2\right)+3c^6d^2x^5\left(14a^2d^2+16abcd+3b^2c^2\right)+\frac{5c^2d^6x^9\left(3a^2d^2+16abcd+14b^2c^2\right)}{3}+2c^5d^3x^6\left(21a^2d^2+35abcd+10b^2c^2\right)+\frac{3c^3d^5x^8\left(10a^2d^2+35abcd+21b^2c^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^10,x)
```

```
[Out] x^3*((b^2*c^10)/3 + 15*a^2*c^8*d^2 + (20*a*b*c^9*d)/3) + x^11*((a^2*d^10)/1
1 + (45*b^2*c^2*d^8)/11 + (20*a*b*c*d^9)/11) + a^2*c^10*x + (b^2*d^10*x^13)
/13 + a*c^9*x^2*(5*a*d + b*c) + (b*d^9*x^12*(a*d + 5*b*c))/6 + (5*c^7*d*x^4
*(12*a^2*d^2 + b^2*c^2 + 9*a*b*c*d))/2 + c*d^7*x^10*(a^2*d^2 + 12*b^2*c^2 +
 9*a*b*c*d) + 6*c^4*d^4*x^7*(5*a^2*d^2 + 5*b^2*c^2 + 12*a*b*c*d) + 3*c^6*d^
2*x^5*(14*a^2*d^2 + 3*b^2*c^2 + 16*a*b*c*d) + (5*c^2*d^6*x^9*(3*a^2*d^2 + 1
4*b^2*c^2 + 16*a*b*c*d))/3 + 2*c^5*d^3*x^6*(21*a^2*d^2 + 10*b^2*c^2 + 35*a*
b*c*d) + (3*c^3*d^5*x^8*(10*a^2*d^2 + 21*b^2*c^2 + 35*a*b*c*d))/2
```

**sympy [B]** time = 0.14, size = 415, normalized size = 6.38

$$a^2c^{10}x+\frac{b^2d^{10}x^{13}}{13}+x^{12}\left(\frac{abd^{10}}{6}+\frac{5b^2cd^9}{6}\right)+x^{11}\left(\frac{a^2d^{10}}{11}+\frac{20abcd^9}{11}+\frac{45b^2c^2d^8}{11}\right)+x^{10}\left(a^2cd^9+9abc^2d^8+12b^2c^3d^7\right)+x^9\left(5a^2c^2d^8+\frac{80abc^3d^7}{3}+\frac{70b^2c^4d^6}{3}\right)+x^8\left(15a^2c^3d^7+\frac{105abc^4d^6}{2}+\frac{63b^2c^5d^5}{2}\right)+x^7\left(30a^2c^4d^6+72abc^5d^5+30b^2c^6d^4\right)+x^6\left(42a^2c^5d^5+70abc^6d^4+20b^2c^7d^3\right)+x^5\left(42a^2c^6d^4+48abc^7d^3+9b^2c^8d^2\right)+x^4\left(30a^2c^7d^3+\frac{45abc^8d^2}{2}+\frac{5b^2c^9d}{2}\right)+x^3\left(15a^2c^8d^2+\frac{20abc^9d}{3}+\frac{b^2c^{10}}{3}\right)+x^2\left(5a^2c^9d+abc^{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**10,x)
```

```
[Out] a**2*c**10*x + b**2*d**10*x**13/13 + x**12*(a*b*d**10/6 + 5*b**2*c*d**9/6)
+ x**11*(a**2*d**10/11 + 20*a*b*c*d**9/11 + 45*b**2*c**2*d**8/11) + x**10*(
a**2*c*d**9 + 9*a*b*c**2*d**8 + 12*b**2*c**3*d**7) + x**9*(5*a**2*c**2*d**8
 + 80*a*b*c**3*d**7/3 + 70*b**2*c**4*d**6/3) + x**8*(15*a**2*c**3*d**7 + 10
5*a*b*c**4*d**6/2 + 63*b**2*c**5*d**5/2) + x**7*(30*a**2*c**4*d**6 + 72*a*b
*c**5*d**5 + 30*b**2*c**6*d**4) + x**6*(42*a**2*c**5*d**5 + 70*a*b*c**6*d**
4 + 20*b**2*c**7*d**3) + x**5*(42*a**2*c**6*d**4 + 48*a*b*c**7*d**3 + 9*b**
2*c**8*d**2) + x**4*(30*a**2*c**7*d**3 + 45*a*b*c**8*d**2/2 + 5*b**2*c**9*d
/2) + x**3*(15*a**2*c**8*d**2 + 20*a*b*c**9*d/3 + b**2*c**10/3) + x**2*(5*a
**2*c**9*d + a*b*c**10)
```

## 3.1204 $\int (a+bx)(c+dx)^{10}\,dx$

**Optimal**. Leaf size=38

$$\frac{b(c+dx)^{12}}{12d^2}-\frac{(c+dx)^{11}(bc-ad)}{11d^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^{12}}{12d^2}-\frac{(c+dx)^{11}(bc-ad)}{11d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^11)/(11*d^2) + (b*(c + d*x)^12)/(12*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^{10}}{d}+\frac{b(c+dx)^{11}}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^{11}}{11d^2}+\frac{b(c+dx)^{12}}{12d^2}\end{aligned}$$

**Mathematica** [B] time = 0.03, size = 220, normalized size = 5.79

$$\frac{1}{2}c^9x^2(10ad+bc)+\frac{5}{3}c^8dx^3(9ad+2bc)+\frac{15}{4}c^7d^2x^4(8ad+3bc)+6c^6d^3x^5(7ad+4bc)+7c^5d^4x^6(6ad+5bc)+6c^4d^5x^7(5ad+6bc)+\frac{15}{4}c^3d^6x^8(4ad+7bc)+\frac{5}{3}c^2d^7x^9(3ad+8bc)+\frac{1}{11}d^9x^{11}(ad+10bc)+\frac{1}{2}cd^8x^{10}(2ad+9bc)+ac^{10}x+\frac{1}{12}bd^{10}x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] a*c^10*x + (c^9*(b*c + 10*a*d)*x^2)/2 + (5*c^8*d*(2*b*c + 9*a*d)*x^3)/3 + (
15*c^7*d^2*(3*b*c + 8*a*d)*x^4)/4 + 6*c^6*d^3*(4*b*c + 7*a*d)*x^5 + 7*c^5*d
```

```
^4*(5*b*c + 6*a*d)*x^6 + 6*c^4*d^5*(6*b*c + 5*a*d)*x^7 + (15*c^3*d^6*(7*b*c
 + 4*a*d)*x^8)/4 + (5*c^2*d^7*(8*b*c + 3*a*d)*x^9)/3 + (c*d^8*(9*b*c + 2*a*
d)*x^10)/2 + (d^9*(10*b*c + a*d)*x^11)/11 + (b*d^10*x^12)/12
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(c + d*x)^10, x]
```

**fricas [B]** time = 1.16, size = 241, normalized size = 6.34

$$\frac{1}{12}x^{12}d^{10}b+\frac{10}{11}x^{11}d^{9}cb+\frac{1}{11}x^{11}d^{10}a+\frac{9}{2}x^{10}d^{8}c^{2}b+x^{10}d^{9}ca+\frac{40}{3}x^{9}d^{7}c^{3}b+5x^{9}d^{8}c^{2}a+\frac{105}{4}x^{8}d^{6}c^{4}b+15x^{8}d^{7}c^{3}a+36x^{7}d^{5}c^{5}b+30x^{7}d^{6}c^{4}a+35x^{6}d^{4}c^{6}b+42x^{6}d^{5}c^{5}a+24x^{5}d^{3}c^{7}b+42x^{5}d^{4}c^{6}a+\frac{45}{4}x^{4}d^{2}c^{8}b+30x^{4}d^{3}c^{7}a+\frac{10}{3}x^{3}dc^{9}b+15x^{3}d^{2}c^{8}a+\frac{1}{2}x^{2}c^{10}b+5x^{2}dc^{9}a+xc^{10}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^10*b + 10/11*x^11*d^9*c*b + 1/11*x^11*d^10*a + 9/2*x^10*d^8*c^2
*b + x^10*d^9*c*a + 40/3*x^9*d^7*c^3*b + 5*x^9*d^8*c^2*a + 105/4*x^8*d^6*c^
4*b + 15*x^8*d^7*c^3*a + 36*x^7*d^5*c^5*b + 30*x^7*d^6*c^4*a + 35*x^6*d^4*c
^6*b + 42*x^6*d^5*c^5*a + 24*x^5*d^3*c^7*b + 42*x^5*d^4*c^6*a + 45/4*x^4*d^
2*c^8*b + 30*x^4*d^3*c^7*a + 10/3*x^3*d*c^9*b + 15*x^3*d^2*c^8*a + 1/2*x^2*
c^10*b + 5*x^2*d*c^9*a + x*c^10*a
```

**giac [B]** time = 1.26, size = 241, normalized size = 6.34

$$\frac{1}{12}bd^{10}x^{12}+\frac{10}{11}bcd^{9}x^{11}+\frac{1}{11}ad^{10}x^{11}+\frac{9}{2}bc^{2}d^{8}x^{10}+acd^{9}x^{10}+\frac{40}{3}bc^{3}d^{7}x^{9}+5ac^{2}d^{8}x^{9}+\frac{105}{4}bc^{4}d^{6}x^{8}+15ac^{3}d^{7}x^{8}+36bc^{5}d^{5}x^{7}+30ac^{4}d^{6}x^{7}+35bc^{6}d^{4}x^{6}+42ac^{5}d^{5}x^{6}+24bc^{7}d^{3}x^{5}+42ac^{6}d^{4}x^{5}+\frac{45}{4}bc^{8}d^{2}x^{4}+30ac^{7}d^{3}x^{4}+\frac{10}{3}bc^{9}dx^{3}+15ac^{8}d^{2}x^{3}+\frac{1}{2}bc^{10}x^{2}+5ac^{9}dx^{2}+ac^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/12*b*d^10*x^12 + 10/11*b*c*d^9*x^11 + 1/11*a*d^10*x^11 + 9/2*b*c^2*d^8*x^
10 + a*c*d^9*x^10 + 40/3*b*c^3*d^7*x^9 + 5*a*c^2*d^8*x^9 + 105/4*b*c^4*d^6*
x^8 + 15*a*c^3*d^7*x^8 + 36*b*c^5*d^5*x^7 + 30*a*c^4*d^6*x^7 + 35*b*c^6*d^4
*x^6 + 42*a*c^5*d^5*x^6 + 24*b*c^7*d^3*x^5 + 42*a*c^6*d^4*x^5 + 45/4*b*c^8*
d^2*x^4 + 30*a*c^7*d^3*x^4 + 10/3*b*c^9*d*x^3 + 15*a*c^8*d^2*x^3 + 1/2*b*c^
10*x^2 + 5*a*c^9*d*x^2 + a*c^10*x
```

**maple [B]** time = 0.00, size = 241, normalized size = 6.34

$$\frac{b\,d^{10}x^{12}}{12}+a\,c^{10}x+\frac{\left(a\,d^{10}+10bc\,d^{9}\right)x^{11}}{11}+\frac{\left(10ac\,d^{9}+45b\,c^{2}d^{8}\right)x^{10}}{10}+\frac{\left(45a\,c^{2}d^{8}+120b\,c^{3}d^{7}\right)x^{9}}{9}+\frac{\left(120a\,c^{3}d^{7}+210b\,c^{4}d^{6}\right)x^{8}}{8}+\frac{\left(210a\,c^{4}d^{6}+252b\,c^{5}d^{5}\right)x^{7}}{7}+\frac{\left(252a\,c^{5}d^{5}+210b\,c^{6}d^{4}\right)x^{6}}{6}+\frac{\left(210a\,c^{6}d^{4}+120b\,c^{7}d^{3}\right)x^{5}}{5}+\frac{\left(120a\,c^{7}d^{3}+45b\,c^{8}d^{2}\right)x^{4}}{4}+\frac{\left(45a\,c^{8}d^{2}+10b\,c^{9}d\right)x^{3}}{3}+\frac{\left(10a\,c^{9}d+b\,c^{10}\right)x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^10,x)
```

```
[Out] 1/12*b*d^10*x^12+1/11*(a*d^10+10*b*c*d^9)*x^11+1/10*(10*a*c*d^9+45*b*c^2*d^
8)*x^10+1/9*(45*a*c^2*d^8+120*b*c^3*d^7)*x^9+1/8*(120*a*c^3*d^7+210*b*c^4*d
^6)*x^8+1/7*(210*a*c^4*d^6+252*b*c^5*d^5)*x^7+1/6*(252*a*c^5*d^5+210*b*c^6*
d^4)*x^6+1/5*(210*a*c^6*d^4+120*b*c^7*d^3)*x^5+1/4*(120*a*c^7*d^3+45*b*c^8*
d^2)*x^4+1/3*(45*a*c^8*d^2+10*b*c^9*d)*x^3+1/2*(10*a*c^9*d+b*c^10)*x^2+a*c^
10*x
```

**maxima [B]** time = 1.43, size = 240, normalized size = 6.32

$$\frac{1}{12}\,bd^{10}x^{12} + ac^{10}x + \frac{1}{11}\left(10\,bcd^{9} + ad^{10}\right)x^{11} + \frac{1}{2}\left(9\,bc^{2}d^{8} + 2\,acd^{9}\right)x^{10} + \frac{5}{3}\left(8\,bc^{3}d^{7} + 3\,ac^{2}d^{8}\right)x^{9} + \frac{15}{4}\left(7\,bc^{4}d^{6} + 4\,ac^{3}d^{7}\right)x^{8} + 6\left(6\,bc^{5}d^{5} + 5\,ac^{4}d^{6}\right)x^{7} + 7\left(5\,bc^{6}d^{4} + 6\,ac^{5}d^{5}\right)x^{6} + 6\left(4\,bc^{7}d^{3} + 7\,ac^{6}d^{4}\right)x^{5} + \frac{15}{4}\left(3\,bc^{8}d^{2} + 8\,ac^{7}d^{3}\right)x^{4} + \frac{5}{3}\left(2\,bc^{9}d + 9\,ac^{8}d^{2}\right)x^{3} + \frac{1}{2}\left(bc^{10} + 10\,ac^{9}d\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/12*b*d^10*x^12 + a*c^10*x + 1/11*(10*b*c*d^9 + a*d^10)*x^11 + 1/2*(9*b*c^
2*d^8 + 2*a*c*d^9)*x^10 + 5/3*(8*b*c^3*d^7 + 3*a*c^2*d^8)*x^9 + 15/4*(7*b*c
^4*d^6 + 4*a*c^3*d^7)*x^8 + 6*(6*b*c^5*d^5 + 5*a*c^4*d^6)*x^7 + 7*(5*b*c^6*
d^4 + 6*a*c^5*d^5)*x^6 + 6*(4*b*c^7*d^3 + 7*a*c^6*d^4)*x^5 + 15/4*(3*b*c^8*
d^2 + 8*a*c^7*d^3)*x^4 + 5/3*(2*b*c^9*d + 9*a*c^8*d^2)*x^3 + 1/2*(b*c^10 +
10*a*c^9*d)*x^2
```

**mupad [B]** time = 0.13, size = 208, normalized size = 5.47

$$x^2\,\left(\frac{b\,c^{10}}{2}+5\,a\,d\,c^9\right)+x^{11}\,\left(\frac{a\,d^{10}}{11}+\frac{10\,b\,c\,d^9}{11}\right)+\frac{b\,d^{10}\,x^{12}}{12}+a\,c^{10}\,x+\frac{5\,c^8\,d\,x^3\,\left(9\,a\,d+2\,b\,c\right)}{3}+\frac{c\,d^8\,x^{10}\,\left(2\,a\,d+9\,b\,c\right)}{2}+\frac{15\,c^7\,d^2\,x^4\,\left(8\,a\,d+3\,b\,c\right)}{4}+6\,c^6\,d^3\,x^5\,\left(7\,a\,d+4\,b\,c\right)+7\,c^5\,d^4\,x^6\,\left(6\,a\,d+5\,b\,c\right)+6\,c^4\,d^5\,x^7\,\left(5\,a\,d+6\,b\,c\right)+\frac{15\,c^3\,d^6\,x^8\,\left(4\,a\,d+7\,b\,c\right)}{4}+\frac{5\,c^2\,d^7\,x^9\,\left(3\,a\,d+8\,b\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^10,x)
```

```
[Out] x^2*((b*c^10)/2 + 5*a*c^9*d) + x^11*((a*d^10)/11 + (10*b*c*d^9)/11) + (b*d^
10*x^12)/12 + a*c^10*x + (5*c^8*d*x^3*(9*a*d + 2*b*c))/3 + (c*d^8*x^10*(2*a
*d + 9*b*c))/2 + (15*c^7*d^2*x^4*(8*a*d + 3*b*c))/4 + 6*c^6*d^3*x^5*(7*a*d
+ 4*b*c) + 7*c^5*d^4*x^6*(6*a*d + 5*b*c) + 6*c^4*d^5*x^7*(5*a*d + 6*b*c) +
(15*c^3*d^6*x^8*(4*a*d + 7*b*c))/4 + (5*c^2*d^7*x^9*(3*a*d + 8*b*c))/3
```

**sympy [B]** time = 0.12, size = 248, normalized size = 6.53

$$ac^{10}x + \frac{bd^{10}x^{12}}{12} + x^{11}\left(\frac{ad^{10}}{11} + \frac{10bcd^{9}}{11}\right) + x^{10}\left(acd^{9} + \frac{9bc^{2}d^{8}}{2}\right) + x^{9}\left(5ac^{2}d^{8} + \frac{40bc^{3}d^{7}}{3}\right) + x^{8}\left(15ac^{3}d^{7} + \frac{105bc^{4}d^{6}}{4}\right) + x^{7}\left(30ac^{4}d^{6} + 36bc^{5}d^{5}\right) + x^{6}\left(42ac^{5}d^{5} + 35bc^{6}d^{4}\right) + x^{5}\left(42ac^{6}d^{4} + 24bc^{7}d^{3}\right) + x^{4}\left(30ac^{7}d^{3} + \frac{45bc^{8}d^{2}}{4}\right) + x^{3}\left(15ac^{8}d^{2} + \frac{10bc^{9}d}{3}\right) + x^{2}\left(5ac^{9}d + \frac{bc^{10}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**10,x)
```

```
[Out] a*c**10*x + b*d**10*x**12/12 + x**11*(a*d**10/11 + 10*b*c*d**9/11) + x**10*
(a*c*d**9 + 9*b*c**2*d**8/2) + x**9*(5*a*c**2*d**8 + 40*b*c**3*d**7/3) + x*
*8*(15*a*c**3*d**7 + 105*b*c**4*d**6/4) + x**7*(30*a*c**4*d**6 + 36*b*c**5*
d**5) + x**6*(42*a*c**5*d**5 + 35*b*c**6*d**4) + x**5*(42*a*c**6*d**4 + 24*
b*c**7*d**3) + x**4*(30*a*c**7*d**3 + 45*b*c**8*d**2/4) + x**3*(15*a*c**8*d
**2 + 10*b*c**9*d/3) + x**2*(5*a*c**9*d + b*c**10/2)
```

## 3.1205 $\int (c+dx)^{10}\,dx$

**Optimal**. Leaf size=14

$$\frac{(c+dx)^{11}}{11d}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^{11}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10,x]
```

```
[Out] (c + d*x)^11/(11*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{10}\,dx = \frac{(c+dx)^{11}}{11d}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^{11}}{11d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10,x]
```

```
[Out] (c + d*x)^11/(11*d)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10, x]
```

**fricas [B]** time = 0.98, size = 108, normalized size = 7.71

$$\frac{1}{11}x^{11}d^{10} + x^{10}d^9c + 5x^9d^8c^2 + 15x^8d^7c^3 + 30x^7d^6c^4 + 42x^6d^5c^5 + 42x^5d^4c^6 + 30x^4d^3c^7 + 15x^3d^2c^8 + 5x^2dc^9 + xc^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^10 + x^10*d^9*c + 5*x^9*d^8*c^2 + 15*x^8*d^7*c^3 + 30*x^7*d^6*c
^4 + 42*x^6*d^5*c^5 + 42*x^5*d^4*c^6 + 30*x^4*d^3*c^7 + 15*x^3*d^2*c^8 + 5*
x^2*d*c^9 + x*c^10
```

**giac [A]** time = 1.28, size = 12, normalized size = 0.86

$$\frac{(dx + c)^{11}}{11\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/11*(d*x + c)^11/d
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{(dx + c)^{11}}{11d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10,x)
```

```
[Out] 1/11*(d*x+c)^11/d
```

**maxima [A]** time = 1.37, size = 12, normalized size = 0.86

$$\frac{(dx + c)^{11}}{11\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/11*(d*x + c)^11/d
```

**mupad [B]** time = 0.08, size = 108, normalized size = 7.71

$$c^{10}\,x+5\,c^9\,d\,x^2+15\,c^8\,d^2\,x^3+30\,c^7\,d^3\,x^4+42\,c^6\,d^4\,x^5+42\,c^5\,d^5\,x^6+30\,c^4\,d^6\,x^7+15\,c^3\,d^7\,x^8+5\,c^2\,d^8\,x^9+c\,d^9\,x^{10}+\frac{d^{10}\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10,x)
```

```
[Out] c^10*x + (d^10*x^11)/11 + 5*c^9*d*x^2 + c*d^9*x^10 + 15*c^8*d^2*x^3 + 30*c^
7*d^3*x^4 + 42*c^6*d^4*x^5 + 42*c^5*d^5*x^6 + 30*c^4*d^6*x^7 + 15*c^3*d^7*x
^8 + 5*c^2*d^8*x^9
```

**sympy [B]** time = 0.09, size = 114, normalized size = 8.14

$$c^{10}x+5c^9dx^2+15c^8d^2x^3+30c^7d^3x^4+42c^6d^4x^5+42c^5d^5x^6+30c^4d^6x^7+15c^3d^7x^8+5c^2d^8x^9+cd^9x^{10}+\frac{d^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10,x)
```

```
[Out] c**10*x + 5*c**9*d*x**2 + 15*c**8*d**2*x**3 + 30*c**7*d**3*x**4 + 42*c**6*d
**4*x**5 + 42*c**5*d**5*x**6 + 30*c**4*d**6*x**7 + 15*c**3*d**7*x**8 + 5*c*
*2*d**8*x**9 + c*d**9*x**10 + d**10*x**11/11
```

# 3.1206 $\int \frac{(c+dx)^{10}}{a+bx} \, dx$

**Optimal**. Leaf size=241

$$\frac{(bc-ad)^{10}\log(a+bx)}{b^{11}}+\frac{dx(bc-ad)^9}{b^{10}}+\frac{(c+dx)^2(bc-ad)^8}{2b^9}+\frac{(c+dx)^3(bc-ad)^7}{3b^8}+\frac{(c+dx)^4(bc-ad)^6}{4b^7}+\frac{(c+dx)^5}{5}$$

**Rubi** [A] time = 0.10, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{dx(bc-ad)^9}{b^{10}}+\frac{(c+dx)^2(bc-ad)^8}{2b^9}+\frac{(c+dx)^3(bc-ad)^7}{3b^8}+\frac{(c+dx)^4(bc-ad)^6}{4b^7}+\frac{(c+dx)^5(bc-ad)^5}{5b^6}+\frac{(c+dx)^6(bc-ad)^4}{6b^5}+\frac{(c+dx)^7(bc-ad)^3}{7b^4}+\frac{(c+dx)^8(bc-ad)^2}{8b^3}+\frac{(c+dx)^9(bc-ad)}{9b^2}+\frac{(bc-ad)^{10}\log(a+bx)}{b^{11}}+\frac{(c+dx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^9*x)/b^10 + ((b*c - a*d)^8*(c + d*x)^2)/(2*b^9) + ((b*c - a*
d)^7*(c + d*x)^3)/(3*b^8) + ((b*c - a*d)^6*(c + d*x)^4)/(4*b^7) + ((b*c - a
*d)^5*(c + d*x)^5)/(5*b^6) + ((b*c - a*d)^4*(c + d*x)^6)/(6*b^5) + ((b*c -
a*d)^3*(c + d*x)^7)/(7*b^4) + ((b*c - a*d)^2*(c + d*x)^8)/(8*b^3) + ((b*c -
 a*d)*(c + d*x)^9)/(9*b^2) + (c + d*x)^10/(10*b) + ((b*c - a*d)^10*Log[a +
b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{a+bx}\,dx=\int\left(\frac{d(bc-ad)^9}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)}+\frac{d(bc-ad)^8(c+dx)}{b^9}+\frac{d(bc-ad)^7(c+dx)^2}{b^8}+\frac{d(bc-ad)^6(c}{b^7}\right.$$
$$=\frac{d(bc-ad)^9x}{b^{10}}+\frac{(bc-ad)^8(c+dx)^2}{2b^9}+\frac{(bc-ad)^7(c+dx)^3}{3b^8}+\frac{(bc-ad)^6(c+dx)^4}{4b^7}+\frac{(bc-ad)^5}{5b}$$

**Mathematica** [B] time = 0.34, size = 591, normalized size = 2.45

$$\frac{dx\left(-2520a^9d^9+1260a^8bd^8(20c+dx)-840a^7b^2d^7\left(135c^2+15cdx+d^2x^2\right)+210a^6b^3d^6\left(1440c^3+270c^2dx+40cd^2x^2+3d^3x^3\right)-252a^5b^4d^5\left(2100c^4+600c^3dx+150c^2d^2x^2+25cd^3x^3+2d^4x^4\right)+210a^4b^5d^4\left(3024c^5+1260c^4dx+480c^3d^2x^2+135c^2d^3x^3+24cd^4x^4+2d^5x^5\right)-120a^3b^6d^3\left(4410c^6+2646c^5dx+1470c^4d^2x^2+630c^3d^3x^3+189c^2d^4x^4+35cd^5x^5+3d^6x^6\right)+45a^2b^7d^2\left(6720c^7+5880c^6dx+4704c^5d^2x^2+2940c^4d^3x^3+1344c^3d^4x^4+420c^2d^5x^5+80cd^6x^6+7d^7x^7\right)-10ab^8d\left(11340c^8+15120c^7dx+17640c^6d^2x^2+15876c^5d^3x^3+10584c^4d^4x^4+5040c^3d^5x^5+1620c^2d^6x^6+315cd^7x^7+28d^8x^8\right)+b^9\left(25200c^9+56700c^8dx+100800c^7d^2x^2+132300c^6d^3x^3+127008c^5d^4x^4+88200c^4d^5x^5+43200c^3d^6x^6+14175c^2d^7x^7+2800cd^8x^8+252d^9x^9\right)\right)}{2520b^{10}}+\frac{(bc-ad)^{10}\log(a+bx)}{b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x),x]
```

```
[Out] (d*x*(-2520*a^9*d^9 + 1260*a^8*b*d^8*(20*c + d*x) - 840*a^7*b^2*d^7*(135*c^
2 + 15*c*d*x + d^2*x^2) + 210*a^6*b^3*d^6*(1440*c^3 + 270*c^2*d*x + 40*c*d^
2*x^2 + 3*d^3*x^3) - 252*a^5*b^4*d^5*(2100*c^4 + 600*c^3*d*x + 150*c^2*d^2*
x^2 + 25*c*d^3*x^3 + 2*d^4*x^4) + 210*a^4*b^5*d^4*(3024*c^5 + 1260*c^4*d*x
+ 480*c^3*d^2*x^2 + 135*c^2*d^3*x^3 + 24*c*d^4*x^4 + 2*d^5*x^5) - 120*a^3*b
^6*d^3*(4410*c^6 + 2646*c^5*d*x + 1470*c^4*d^2*x^2 + 630*c^3*d^3*x^3 + 189*
c^2*d^4*x^4 + 35*c*d^5*x^5 + 3*d^6*x^6) + 45*a^2*b^7*d^2*(6720*c^7 + 5880*c
^6*d*x + 4704*c^5*d^2*x^2 + 2940*c^4*d^3*x^3 + 1344*c^3*d^4*x^4 + 420*c^2*d
^5*x^5 + 80*c*d^6*x^6 + 7*d^7*x^7) - 10*a*b^8*d*(11340*c^8 + 15120*c^7*d*x
+ 17640*c^6*d^2*x^2 + 15876*c^5*d^3*x^3 + 10584*c^4*d^4*x^4 + 5040*c^3*d^5*
x^5 + 1620*c^2*d^6*x^6 + 315*c*d^7*x^7 + 28*d^8*x^8) + b^9*(25200*c^9 + 567
00*c^8*d*x + 100800*c^7*d^2*x^2 + 132300*c^6*d^3*x^3 + 127008*c^5*d^4*x^4 +
 88200*c^4*d^5*x^5 + 43200*c^3*d^6*x^6 + 14175*c^2*d^7*x^7 + 2800*c*d^8*x^8
 + 252*d^9*x^9)))/(2520*b^10) + ((b*c - a*d)^10*Log[a + b*x])/b^11
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x), x]
```

**fricas [B]** time = 1.28, size = 868, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2520*(252*b^10*d^10*x^10 + 280*(10*b^10*c*d^9 - a*b^9*d^10)*x^9 + 315*(45
*b^10*c^2*d^8 - 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 360*(120*b^10*c^3*d^7
- 45*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 420*(210*b^10*c
^4*d^6 - 120*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 - 10*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 504*(252*b^10*c^5*d^5 - 210*a*b^9*c^4*d^6 + 120*a^2*b^8*c^3*d
^7 - 45*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 630*(210*b
^10*c^6*d^4 - 252*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 - 120*a^3*b^7*c^3*d^7
 + 45*a^4*b^6*c^2*d^8 - 10*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 840*(120*b^1
0*c^7*d^3 - 210*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 - 210*a^3*b^7*c^4*d^6 +
```

```
 120*a^4*b^6*c^3*d^7 - 45*a^5*b^5*c^2*d^8 + 10*a^6*b^4*c*d^9 - a^7*b^3*d^10
)*x^3 + 1260*(45*b^10*c^8*d^2 - 120*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 - 2
52*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 120*a^5*b^5*c^3*d^7 + 45*a^6*b^4
*c^2*d^8 - 10*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 2520*(10*b^10*c^9*d - 45*
a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d^3 - 210*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5
*d^5 - 210*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 - 45*a^7*b^3*c^2*d^8 + 10*
a^8*b^2*c*d^9 - a^9*b*d^10)*x + 2520*(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b
^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^
5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9
*b*c*d^9 + a^10*d^10)*log(b*x + a))/b^11
```

**giac [B]** time = 1.35, size = 961, normalized size = 3.99

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2520*(252*b^9*d^10*x^10 + 2800*b^9*c*d^9*x^9 - 280*a*b^8*d^10*x^9 + 14175
*b^9*c^2*d^8*x^8 - 3150*a*b^8*c*d^9*x^8 + 315*a^2*b^7*d^10*x^8 + 43200*b^9*
c^3*d^7*x^7 - 16200*a*b^8*c^2*d^8*x^7 + 3600*a^2*b^7*c*d^9*x^7 - 360*a^3*b^
6*d^10*x^7 + 88200*b^9*c^4*d^6*x^6 - 50400*a*b^8*c^3*d^7*x^6 + 18900*a^2*b^
7*c^2*d^8*x^6 - 4200*a^3*b^6*c*d^9*x^6 + 420*a^4*b^5*d^10*x^6 + 127008*b^9*
c^5*d^5*x^5 - 105840*a*b^8*c^4*d^6*x^5 + 60480*a^2*b^7*c^3*d^7*x^5 - 22680*
a^3*b^6*c^2*d^8*x^5 + 5040*a^4*b^5*c*d^9*x^5 - 504*a^5*b^4*d^10*x^5 + 13230
0*b^9*c^6*d^4*x^4 - 158760*a*b^8*c^5*d^5*x^4 + 132300*a^2*b^7*c^4*d^6*x^4 -
 75600*a^3*b^6*c^3*d^7*x^4 + 28350*a^4*b^5*c^2*d^8*x^4 - 6300*a^5*b^4*c*d^9
*x^4 + 630*a^6*b^3*d^10*x^4 + 100800*b^9*c^7*d^3*x^3 - 176400*a*b^8*c^6*d^4
*x^3 + 211680*a^2*b^7*c^5*d^5*x^3 - 176400*a^3*b^6*c^4*d^6*x^3 + 100800*a^4
*b^5*c^3*d^7*x^3 - 37800*a^5*b^4*c^2*d^8*x^3 + 8400*a^6*b^3*c*d^9*x^3 - 840
*a^7*b^2*d^10*x^3 + 56700*b^9*c^8*d^2*x^2 - 151200*a*b^8*c^7*d^3*x^2 + 2646
00*a^2*b^7*c^6*d^4*x^2 - 317520*a^3*b^6*c^5*d^5*x^2 + 264600*a^4*b^5*c^4*d^
6*x^2 - 151200*a^5*b^4*c^3*d^7*x^2 + 56700*a^6*b^3*c^2*d^8*x^2 - 12600*a^7*
b^2*c*d^9*x^2 + 1260*a^8*b*d^10*x^2 + 25200*b^9*c^9*d*x - 113400*a*b^8*c^8*
d^2*x + 302400*a^2*b^7*c^7*d^3*x - 529200*a^3*b^6*c^6*d^4*x + 635040*a^4*b^
5*c^5*d^5*x - 529200*a^5*b^4*c^4*d^6*x + 302400*a^6*b^3*c^3*d^7*x - 113400*
a^7*b^2*c^2*d^8*x + 25200*a^8*b*c*d^9*x - 2520*a^9*d^10*x)/b^10 + (b^10*c^1
0 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6
*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7
+ 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)*log(abs(b*x + a))/b^11
```

**maple [B]** time = 0.01, size = 1022, normalized size = 4.24

---

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^10/(b*x+a),x)

[Out] 105/2*d^4/b*x^4*c^6-1/3*d^10/b^8*x^3*a^7+40*d^3/b*x^3*c^7+1/2*d^10/b^9*x^2*a^8+45/2*d^2/b*x^2*c^8+1/8*d^10/b^3*x^8*a^2+45/8*d^8/b*x^8*c^2-1/7*d^10/b^4*x^7*a^3+120/7*d^7/b*x^7*c^3+1/6*d^10/b^5*x^6*a^4+35*d^6/b*x^6*c^4-1/5*d^10/b^6*x^5*a^5+252/5*d^5/b*x^5*c^5+1/4*d^10/b^7*x^4*a^6-1/9*d^10/b^2*x^9*a+10/9*d^9/b*x^9*c+1/b^11*ln(b*x+a)*a^10*d^10+10*d/b*c^9*x-d^10/b^10*a^9*x-42*d^6/b^2*x^5*a*c^4-5/2*d^9/b^6*x^4*a^5*c+10/3*d^9/b^7*x^3*a^6*c-15*d^8/b^6*x^3*a^5*c^2+24*d^7/b^3*x^5*a^2*c^3+40*d^7/b^5*x^3*a^4*c^3-30*d^7/b^4*x^4*a^3*c^3+105/2*d^6/b^3*x^4*a^2*c^4-63*d^5/b^2*x^4*a*c^5-5/4*d^9/b^2*x^8*a*c+2*d^9/b^5*x^5*a^4*c-9*d^8/b^4*x^5*a^3*c^2-5/3*d^9/b^4*x^6*a^3*c+15/2*d^8/b^3*x^6*a^2*c^2-20*d^7/b^2*x^6*a*c^3+10/7*d^9/b^3*x^7*a^2*c-45/7*d^8/b^2*x^7*a*c^2-120/b^8*ln(b*x+a)*a^7*c^3*d^7+210/b^7*ln(b*x+a)*a^6*c^4*d^6-252/b^6*ln(b*x+a)*a^5*c^5*d^5+210/b^5*ln(b*x+a)*a^4*c^6*d^4-120/b^4*ln(b*x+a)*a^3*c^7*d^3+45/b^3*ln(b*x+a)*a^2*c^8*d^2-10/b^2*ln(b*x+a)*a*c^9*d-10/b^10*ln(b*x+a)*a^9*c*d^9+45/b^9*ln(b*x+a)*a^8*c^2*d^8+1/10*d^10/b*x^10+1/b*ln(b*x+a)*c^10+45/4*d^8/b^5*x^4*a^4*c^2+120*d^3/b^3*a^2*c^7*x-45*d^2/b^2*a*c^8*x-70*d^4/b^2*x^3*a*c^6-5*d^9/b^8*x^2*a^7*c+45/2*d^8/b^7*x^2*a^6*c^2-60*d^7/b^6*x^2*a^5*c^3+105*d^6/b^5*x^2*a^4*c^4-126*d^5/b^4*x^2*a^3*c^5+105*d^4/b^3*x^2*a^2*c^6-60*d^3/b^2*x^2*a*c^7+10*d^9/b^9*a^8*c*x-45*d^8/b^8*a^7*c^2*x+120*d^7/b^7*a^6*c^3*x-210*d^6/b^6*a^5*c^4*x-70*d^6/b^4*x^3*a^3*c^4+84*d^5/b^3*x^3*a^2*c^5+252*d^5/b^5*a^4*c^5*x-210*d^4/b^4*a^3*c^6*x

**maxima** [B] time = 1.52, size = 866, normalized size = 3.59

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="maxima")

[Out] 1/2520*(252*b^9*d^10*x^10 + 280*(10*b^9*c*d^9 - a*b^8*d^10)*x^9 + 315*(45*b^9*c^2*d^8 - 10*a*b^8*c*d^9 + a^2*b^7*d^10)*x^8 + 360*(120*b^9*c^3*d^7 - 45*a*b^8*c^2*d^8 + 10*a^2*b^7*c*d^9 - a^3*b^6*d^10)*x^7 + 420*(210*b^9*c^4*d^6 - 120*a*b^8*c^3*d^7 + 45*a^2*b^7*c^2*d^8 - 10*a^3*b^6*c*d^9 + a^4*b^5*d^10)*x^6 + 504*(252*b^9*c^5*d^5 - 210*a*b^8*c^4*d^6 + 120*a^2*b^7*c^3*d^7 - 45*a^3*b^6*c^2*d^8 + 10*a^4*b^5*c*d^9 - a^5*b^4*d^10)*x^5 + 630*(210*b^9*c^6*d^4 - 252*a*b^8*c^5*d^5 + 210*a^2*b^7*c^4*d^6 - 120*a^3*b^6*c^3*d^7 + 45*a^4*b^5*c^2*d^8 - 10*a^5*b^4*c*d^9 + a^6*b^3*d^10)*x^4 + 840*(120*b^9*c^7*d^3 - 210*a*b^8*c^6*d^4 + 252*a^2*b^7*c^5*d^5 - 210*a^3*b^6*c^4*d^6 + 120*a^4*b^5*c^3*d^7 - 45*a^5*b^4*c^2*d^8 + 10*a^6*b^3*c*d^9 - a^7*b^2*d^10)*x^3 + 1260*(45*b^9*c^8*d^2 - 120*a*b^8*c^7*d^3 + 210*a^2*b^7*c^6*d^4 - 252*a^3*b^6*c^5*d^5 + 210*a^4*b^5*c^4*d^6 - 120*a^5*b^4*c^3*d^7 + 45*a^6*b^3*c^2*d^8 - 10*a^7*b^2*c*d^9 + a^8*b*d^10)*x^2 + 2520*(10*b^9*c^9*d - 45*a*b^8*c^8*d^

```
2 + 120*a^2*b^7*c^7*d^3 - 210*a^3*b^6*c^6*d^4 + 252*a^4*b^5*c^5*d^5 - 210*a
^5*b^4*c^4*d^6 + 120*a^6*b^3*c^3*d^7 - 45*a^7*b^2*c^2*d^8 + 10*a^8*b*c*d^9
- a^9*d^10)*x)/b^10 + (b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 12
0*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4
*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10
*d^10)*log(b*x + a)/b^11
```

**mupad [B]** time = 0.13, size = 979, normalized size = 4.06

$$[illegible] - \frac{45 c^8 d^2}{2 b} + \frac{d^{10} x^{10}}{10 b} + \frac{\ln(a + b x) \left(a^{10} d^{10} - 10 a^9 b c d^9 + 45 a^8 b^2 c^2 d^8 - 120 a^7 b^3 c^3 d^7 + 210 a^6 b^4 c^4 d^6 - 252 a^5 b^5 c^5 d^5 + 210 a^4 b^6 c^6 d^4 - 120 a^3 b^7 c^7 d^3 + 45 a^2 b^8 c^8 d^2 - 10 a b^9 c^9 d + b^{10} c^{10}\right)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x),x)
```

```
[Out] x^7*((120*c^3*d^7)/(7*b) - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^
2*d^8)/b))/(7*b)) - x^9*((a*d^10)/(9*b^2) - (10*c*d^9)/(9*b)) + x^5*((a*((a
*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/
b))/b))/b - (210*c^4*d^6)/b))/(5*b) + (252*c^5*d^5)/(5*b)) + x^3*((a*((a*((
a*((a*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*
d^8)/b))/b))/b - (210*c^4*d^6)/b))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/
b))/(3*b) + (40*c^7*d^3)/b) + x*((a*((a*((a*((a*((a*((a*((120*c^3*d^7)/b -
(a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/b - (210*c^4
*d^6)/b))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/b))/b + (120*c^7*d^3)/b))
/b - (45*c^8*d^2)/b))/b + (10*c^9*d)/b) + x^8*((a*((a*d^10)/b^2 - (10*c*d^9
)/b))/(8*b) + (45*c^2*d^8)/(8*b)) - x^6*((a*((120*c^3*d^7)/b - (a*((a*((a*d
^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/(6*b) - (35*c^4*d^6)/b)
- x^4*((a*((a*((a*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/
b + (45*c^2*d^8)/b))/b))/b - (210*c^4*d^6)/b))/b + (252*c^5*d^5)/b))/(4*b)
- (105*c^6*d^4)/(2*b)) - x^2*((a*((a*((a*((a*((a*((120*c^3*d^7)/b - (a*((a*
((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/b - (210*c^4*d^6)/b
))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/b))/b + (120*c^7*d^3)/b))/(2*b)
- (45*c^8*d^2)/(2*b)) + (d^10*x^10)/(10*b) + (log(a + b*x)*(a^10*d^10 + b^1
0*c^10 + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 2
52*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2
*c^2*d^8 - 10*a*b^9*c^9*d - 10*a^9*b*c*d^9))/b^11
```

**sympy [B]** time = 1.42, size = 799, normalized size = 3.32

$$x^{9}\left(-\frac{ad^{10}}{9b^{2}}+\frac{10cd^{9}}{9b}\right)+x^{8}\left(\frac{a^{2}d^{10}}{8b^{3}}-\frac{5acd^{9}}{4b^{2}}+\frac{45c^{2}d^{8}}{8b}\right)+x^{7}\left(-\frac{a^{3}d^{10}}{7b^{4}}+\frac{10a^{2}cd^{9}}{7b^{3}}-\frac{45ac^{2}d^{8}}{7b^{2}}+\frac{120c^{3}d^{7}}{7b}\right)+x^{6}\left(\frac{a^{4}d^{10}}{6b^{5}}-\frac{5a^{3}cd^{9}}{3b^{4}}+\frac{15a^{2}c^{2}d^{8}}{2b^{3}}-\frac{20ac^{3}d^{7}}{b^{2}}+\frac{35c^{4}d^{6}}{b}\right)+x^{5}\left(-\frac{a^{5}d^{10}}{5b^{6}}+\frac{2a^{4}cd^{9}}{b^{5}}-\frac{9a^{3}c^{2}d^{8}}{b^{4}}+\frac{24a^{2}c^{3}d^{7}}{b^{3}}-\frac{42ac^{4}d^{6}}{b^{2}}+\frac{252c^{5}d^{5}}{5b}\right)+x^{4}\left(\frac{a^{6}d^{10}}{4b^{7}}-\frac{5a^{5}cd^{9}}{2b^{6}}+\frac{45a^{4}c^{2}d^{8}}{4b^{5}}-\frac{30a^{3}c^{3}d^{7}}{b^{4}}+\frac{105a^{2}c^{4}d^{6}}{2b^{3}}-\frac{63ac^{5}d^{5}}{b^{2}}+\frac{105c^{6}d^{4}}{2b}\right)+x^{3}\left(-\frac{a^{7}d^{10}}{3b^{8}}+\frac{10a^{6}cd^{9}}{3b^{7}}-\frac{15a^{5}c^{2}d^{8}}{b^{6}}+\frac{40a^{4}c^{3}d^{7}}{b^{5}}-\frac{70a^{3}c^{4}d^{6}}{b^{4}}+\frac{84a^{2}c^{5}d^{5}}{b^{3}}-\frac{70ac^{6}d^{4}}{b^{2}}+\frac{40c^{7}d^{3}}{b}\right)+x^{2}\left(\frac{a^{8}d^{10}}{2b^{9}}-\frac{5a^{7}cd^{9}}{b^{8}}+\frac{45a^{6}c^{2}d^{8}}{2b^{7}}-\frac{60a^{5}c^{3}d^{7}}{b^{6}}+\frac{105a^{4}c^{4}d^{6}}{b^{5}}-\frac{126a^{3}c^{5}d^{5}}{b^{4}}+\frac{105a^{2}c^{6}d^{4}}{b^{3}}-\frac{60ac^{7}d^{3}}{b^{2}}+\frac{45c^{8}d^{2}}{2b}\right)+x\left(-\frac{a^{9}d^{10}}{b^{10}}+\frac{10a^{8}cd^{9}}{b^{9}}-\frac{45a^{7}c^{2}d^{8}}{b^{8}}+\frac{120a^{6}c^{3}d^{7}}{b^{7}}-\frac{210a^{5}c^{4}d^{6}}{b^{6}}+\frac{252a^{4}c^{5}d^{5}}{b^{5}}-\frac{210a^{3}c^{6}d^{4}}{b^{4}}+\frac{120a^{2}c^{7}d^{3}}{b^{3}}-\frac{45ac^{8}d^{2}}{b^{2}}+\frac{10c^{9}d}{b}\right)+\frac{d^{10}x^{10}}{10b}+\frac{(ad-bc)^{10}\log(a+bx)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a),x)
```

```
[Out] x**9*(-a*d**10/(9*b**2) + 10*c*d**9/(9*b)) + x**8*(a**2*d**10/(8*b**3) - 5*
a*c*d**9/(4*b**2) + 45*c**2*d**8/(8*b)) + x**7*(-a**3*d**10/(7*b**4) + 10*a
**2*c*d**9/(7*b**3) - 45*a*c**2*d**8/(7*b**2) + 120*c**3*d**7/(7*b)) + x**6
*(a**4*d**10/(6*b**5) - 5*a**3*c*d**9/(3*b**4) + 15*a**2*c**2*d**8/(2*b**3)
 - 20*a*c**3*d**7/b**2 + 35*c**4*d**6/b) + x**5*(-a**5*d**10/(5*b**6) + 2*a
**4*c*d**9/b**5 - 9*a**3*c**2*d**8/b**4 + 24*a**2*c**3*d**7/b**3 - 42*a*c**
4*d**6/b**2 + 252*c**5*d**5/(5*b)) + x**4*(a**6*d**10/(4*b**7) - 5*a**5*c*d
**9/(2*b**6) + 45*a**4*c**2*d**8/(4*b**5) - 30*a**3*c**3*d**7/b**4 + 105*a*
*2*c**4*d**6/(2*b**3) - 63*a*c**5*d**5/b**2 + 105*c**6*d**4/(2*b)) + x**3*(
-a**7*d**10/(3*b**8) + 10*a**6*c*d**9/(3*b**7) - 15*a**5*c**2*d**8/b**6 + 4
0*a**4*c**3*d**7/b**5 - 70*a**3*c**4*d**6/b**4 + 84*a**2*c**5*d**5/b**3 - 7
0*a*c**6*d**4/b**2 + 40*c**7*d**3/b) + x**2*(a**8*d**10/(2*b**9) - 5*a**7*c
```

```
*d**9/b**8 + 45*a**6*c**2*d**8/(2*b**7) - 60*a**5*c**3*d**7/b**6 + 105*a**4
*c**4*d**6/b**5 - 126*a**3*c**5*d**5/b**4 + 105*a**2*c**6*d**4/b**3 - 60*a*
c**7*d**3/b**2 + 45*c**8*d**2/(2*b)) + x*(-a**9*d**10/b**10 + 10*a**8*c*d**
9/b**9 - 45*a**7*c**2*d**8/b**8 + 120*a**6*c**3*d**7/b**7 - 210*a**5*c**4*d
**6/b**6 + 252*a**4*c**5*d**5/b**5 - 210*a**3*c**6*d**4/b**4 + 120*a**2*c**
7*d**3/b**3 - 45*a*c**8*d**2/b**2 + 10*c**9*d/b) + d**10*x**10/(10*b) + (a*
d - b*c)**10*log(a + b*x)/b**11
```

## 3.1207 $\int \frac{(c+dx)^{10}}{(a+bx)^2} dx$

**Optimal**. Leaf size=258

$$\frac{5d^9(a+bx)^8(bc-ad)}{4b^{11}}+\frac{45d^8(a+bx)^7(bc-ad)^2}{7b^{11}}+\frac{20d^7(a+bx)^6(bc-ad)^3}{b^{11}}+\frac{42d^6(a+bx)^5(bc-ad)^4}{b^{11}}+\frac{63d^5(a+b}{}$$

**Rubi [A]** time = 0.47, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^8(bc-ad)}{4b^{11}}+\frac{45d^8(a+bx)^7(bc-ad)^2}{7b^{11}}+\frac{20d^7(a+bx)^6(bc-ad)^3}{b^{11}}+\frac{42d^6(a+bx)^5(bc-ad)^4}{b^{11}}+\frac{63d^5(a+bx)^4(bc-ad)^5}{b^{11}}+\frac{70d^4(a+bx)^3(bc-ad)^6}{b^{11}}+\frac{60d^3(a+bx)^2(bc-ad)^7}{b^{11}}+\frac{45d^2x(bc-ad)^8}{b^{10}}-\frac{(bc-ad)^{10}}{b^{11}(a+bx)}+\frac{10d(bc-ad)^9\log(a+bx)}{b^{11}}+\frac{d^{10}(a+bx)^9}{9b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^2,x]
```

```
[Out] (45*d^2*(b*c - a*d)^8*x)/b^10 - (b*c - a*d)^10/(b^11*(a + b*x)) + (60*d^3*(
b*c - a*d)^7*(a + b*x)^2)/b^11 + (70*d^4*(b*c - a*d)^6*(a + b*x)^3)/b^11 +
(63*d^5*(b*c - a*d)^5*(a + b*x)^4)/b^11 + (42*d^6*(b*c - a*d)^4*(a + b*x)^5
)/b^11 + (20*d^7*(b*c - a*d)^3*(a + b*x)^6)/b^11 + (45*d^8*(b*c - a*d)^2*(a
 + b*x)^7)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^8)/(4*b^11) + (d^10*(a +
 b*x)^9)/(9*b^11) + (10*d*(b*c - a*d)^9*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^2}\,dx = \int \left(\frac{45d^2(bc-ad)^8}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^2}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)}+\frac{120d^3(bc-ad)^7(a+bx)}{b^{10}}+\frac{210d^4(bc-}{}\right.$$

$$= \frac{45d^2(bc-ad)^8x}{b^{10}}-\frac{(bc-ad)^{10}}{b^{11}(a+bx)}+\frac{60d^3(bc-ad)^7(a+bx)^2}{b^{11}}+\frac{70d^4(bc-ad)^6(a+bx)^3}{b^{11}}+\frac{63d^5}{}$$

**Mathematica [B]** time = 0.24, size = 708, normalized size = 2.74

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^2,x]
```

```
[Out] (-252*a^10*d^10 + 252*a^9*b*d^9*(10*c + 9*d*x) + 1260*a^8*b^2*d^8*(-9*c^2 -
 16*c*d*x + d^2*x^2) - 420*a^7*b^3*d^7*(-72*c^3 - 189*c^2*d*x + 27*c*d^2*x^
2 + d^3*x^3) + 210*a^6*b^4*d^6*(-252*c^4 - 864*c^3*d*x + 216*c^2*d^2*x^2 +
18*c*d^3*x^3 + d^4*x^4) - 126*a^5*b^5*d^5*(-504*c^5 - 2100*c^4*d*x + 840*c^
3*d^2*x^2 + 120*c^2*d^3*x^3 + 15*c*d^4*x^4 + d^5*x^5) + 42*a^4*b^6*d^4*(-12
60*c^6 - 6048*c^5*d*x + 3780*c^4*d^2*x^2 + 840*c^3*d^3*x^3 + 180*c^2*d^4*x^
4 + 27*c*d^5*x^5 + 2*d^6*x^6) - 12*a^3*b^7*d^3*(-2520*c^7 - 13230*c^6*d*x +
 13230*c^5*d^2*x^2 + 4410*c^4*d^3*x^3 + 1470*c^3*d^4*x^4 + 378*c^2*d^5*x^5
+ 63*c*d^6*x^6 + 5*d^7*x^7) + 9*a^2*b^8*d^2*(-1260*c^8 - 6720*c^7*d*x + 117
60*c^6*d^2*x^2 + 5880*c^5*d^3*x^3 + 2940*c^4*d^4*x^4 + 1176*c^3*d^5*x^5 + 3
36*c^2*d^6*x^6 + 60*c*d^7*x^7 + 5*d^8*x^8) - a*b^9*d*(-2520*c^9 - 11340*c^8
*d*x + 45360*c^7*d^2*x^2 + 35280*c^6*d^3*x^3 + 26460*c^5*d^4*x^4 + 15876*c^
4*d^5*x^5 + 7056*c^3*d^6*x^6 + 2160*c^2*d^7*x^7 + 405*c*d^8*x^8 + 35*d^9*x^
9) + b^10*(-252*c^10 + 11340*c^8*d^2*x^2 + 15120*c^7*d^3*x^3 + 17640*c^6*d^
4*x^4 + 15876*c^5*d^5*x^5 + 10584*c^4*d^6*x^6 + 5040*c^3*d^7*x^7 + 1620*c^2
*d^8*x^8 + 315*c*d^9*x^9 + 28*d^10*x^10) - 2520*d*(-(b*c) + a*d)^9*(a + b*x
)*Log[a + b*x])/(252*b^11*(a + b*x))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^2, x]
```

**fricas [B]** time = 1.31, size = 1124, normalized size = 4.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^10*d^10*x^10 - 252*b^10*c^10 + 2520*a*b^9*c^9*d - 11340*a^2*b^8
*c^8*d^2 + 30240*a^3*b^7*c^7*d^3 - 52920*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^
5*d^5 - 52920*a^6*b^4*c^4*d^6 + 30240*a^7*b^3*c^3*d^7 - 11340*a^8*b^2*c^2*d
^8 + 2520*a^9*b*c*d^9 - 252*a^10*d^10 + 35*(9*b^10*c*d^9 - a*b^9*d^10)*x^9
+ 45*(36*b^10*c^2*d^8 - 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 60*(84*b^10*c^3
*d^7 - 36*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 84*(126*b^1
```

```
0*c^4*d^6 - 84*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 - 9*a^3*b^7*c*d^9 + a^4*b
^6*d^10)*x^6 + 126*(126*b^10*c^5*d^5 - 126*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 - 36*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 210*(84*b^1
0*c^6*d^4 - 126*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 - 84*a^3*b^7*c^3*d^7 +
36*a^4*b^6*c^2*d^8 - 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 420*(36*b^10*c^7
*d^3 - 84*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 - 126*a^3*b^7*c^4*d^6 + 84*a^
4*b^6*c^3*d^7 - 36*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 +
1260*(9*b^10*c^8*d^2 - 36*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 - 126*a^3*b^7*
c^5*d^5 + 126*a^4*b^6*c^4*d^6 - 84*a^5*b^5*c^3*d^7 + 36*a^6*b^4*c^2*d^8 - 9
*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 252*(45*a*b^9*c^8*d^2 - 240*a^2*b^8*c^
7*d^3 + 630*a^3*b^7*c^6*d^4 - 1008*a^4*b^6*c^5*d^5 + 1050*a^5*b^5*c^4*d^6 -
 720*a^6*b^4*c^3*d^7 + 315*a^7*b^3*c^2*d^8 - 80*a^8*b^2*c*d^9 + 9*a^9*b*d^1
0)*x + 2520*(a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 + 36*a^3*b^7*c^7*d^3 - 84*a^4*
b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 - 126*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^
7 - 36*a^8*b^2*c^2*d^8 + 9*a^9*b*c*d^9 - a^10*d^10 + (b^10*c^9*d - 9*a*b^9*
c^8*d^2 + 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 - 1
26*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*
d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^12*x + a*b^11)
```

**giac [B]** time = 1.27, size = 1012, normalized size = 3.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/252*(28*d^10 + 315*(b^2*c*d^9 - a*b*d^10)/((b*x + a)*b) + 1620*(b^4*c^2*d
^8 - 2*a*b^3*c*d^9 + a^2*b^2*d^10)/((b*x + a)^2*b^2) + 5040*(b^6*c^3*d^7 -
3*a*b^5*c^2*d^8 + 3*a^2*b^4*c*d^9 - a^3*b^3*d^10)/((b*x + a)^3*b^3) + 10584
*(b^8*c^4*d^6 - 4*a*b^7*c^3*d^7 + 6*a^2*b^6*c^2*d^8 - 4*a^3*b^5*c*d^9 + a^4
*b^4*d^10)/((b*x + a)^4*b^4) + 15876*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a
^2*b^8*c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)/((b*x
 + a)^5*b^5) + 17640*(b^12*c^6*d^4 - 6*a*b^11*c^5*d^5 + 15*a^2*b^10*c^4*d^6
 - 20*a^3*b^9*c^3*d^7 + 15*a^4*b^8*c^2*d^8 - 6*a^5*b^7*c*d^9 + a^6*b^6*d^10
)/((b*x + a)^6*b^6) + 15120*(b^14*c^7*d^3 - 7*a*b^13*c^6*d^4 + 21*a^2*b^12*
c^5*d^5 - 35*a^3*b^11*c^4*d^6 + 35*a^4*b^10*c^3*d^7 - 21*a^5*b^9*c^2*d^8 +
7*a^6*b^8*c*d^9 - a^7*b^7*d^10)/((b*x + a)^7*b^7) + 11340*(b^16*c^8*d^2 - 8
*a*b^15*c^7*d^3 + 28*a^2*b^14*c^6*d^4 - 56*a^3*b^13*c^5*d^5 + 70*a^4*b^12*c
^4*d^6 - 56*a^5*b^11*c^3*d^7 + 28*a^6*b^10*c^2*d^8 - 8*a^7*b^9*c*d^9 + a^8*
b^8*d^10)/((b*x + a)^8*b^8))*(b*x + a)^9/b^11 - 10*(b^9*c^9*d - 9*a*b^8*c^8
*d^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*
a^5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 -
 a^9*d^10)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^11 - (b^19*c^10/(b*x +
a) - 10*a*b^18*c^9*d/(b*x + a) + 45*a^2*b^17*c^8*d^2/(b*x + a) - 120*a^3*b^
16*c^7*d^3/(b*x + a) + 210*a^4*b^15*c^6*d^4/(b*x + a) - 252*a^5*b^14*c^5*d^
```

```
5/(b*x + a) + 210*a^6*b^13*c^4*d^6/(b*x + a) - 120*a^7*b^12*c^3*d^7/(b*x +
a) + 45*a^8*b^11*c^2*d^8/(b*x + a) - 10*a^9*b^10*c*d^9/(b*x + a) + a^10*b^9
*d^10/(b*x + a))/b^20
```

**maple [B]** time = 0.02, size = 1066, normalized size = 4.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^2,x)
```

```
[Out] 45*d^2/b^2*c^8*x+9*d^10/b^10*a^8*x-10/b^11*d^10*ln(b*x+a)*a^9+10/b^2*d*ln(b
*x+a)*c^9-1/b^11/(b*x+a)*a^10*d^10-4*d^10/b^9*x^2*a^7+60*d^3/b^2*x^2*c^7+3/
7*d^10/b^4*x^7*a^2+45/7*d^8/b^2*x^7*c^2-2/3*d^10/b^5*x^6*a^3+20*d^7/b^2*x^6
*c^3+42*d^6/b^2*x^5*c^4-3/2*d^10/b^7*x^4*a^5+63*d^5/b^2*x^4*c^5+7/3*d^10/b^
8*x^3*a^6+70*d^4/b^2*x^3*c^6-1/4*d^10/b^3*x^8*a+5/4*d^9/b^2*x^8*c+d^10/b^6*
x^5*a^4+1/9*d^10/b^2*x^9-1/b/(b*x+a)*c^10+10/b^2/(b*x+a)*a*c^9*d-1008*d^5/b
^5*a^3*c^5*x+630*d^4/b^4*a^2*c^6*x-240*d^3/b^3*a*c^7*x+5*d^9/b^4*x^6*a^2*c-
15*d^8/b^3*x^6*a*c^2-8*d^9/b^5*x^5*a^3*c+27*d^8/b^4*x^5*a^2*c^2-48*d^7/b^3*
x^5*a*c^3+25/2*d^9/b^6*x^4*a^4*c+300*d^7/b^6*x^2*a^4*c^3-420*d^6/b^5*x^2*a^
3*c^4+378*d^5/b^4*x^2*a^2*c^5-210*d^4/b^3*x^2*a*c^6-80*d^9/b^9*a^7*c*x+315*
d^8/b^8*a^6*c^2*x-720*d^7/b^7*a^5*c^3*x+1050*d^6/b^6*a^4*c^4*x+35*d^9/b^8*x
^2*a^6*c-135*d^8/b^7*x^2*a^5*c^2-90/b^3*d^2*ln(b*x+a)*a*c^8+10/b^10/(b*x+a)
*a^9*c*d^9-45/b^9/(b*x+a)*a^8*c^2*d^8+120/b^8/(b*x+a)*a^7*c^3*d^7-210/b^7/(
b*x+a)*a^6*c^4*d^6+252/b^6/(b*x+a)*a^5*c^5*d^5-210/b^5/(b*x+a)*a^4*c^6*d^4+
120/b^4/(b*x+a)*a^3*c^7*d^3-45/b^3/(b*x+a)*a^2*c^8*d^2-840/b^5*d^4*ln(b*x+a
)*a^3*c^6+360/b^4*d^3*ln(b*x+a)*a^2*c^7-45*d^8/b^5*x^4*a^3*c^2+90*d^7/b^4*x
^4*a^2*c^3-105*d^6/b^3*x^4*a*c^4-20*d^9/b^7*x^3*a^5*c+75*d^8/b^6*x^3*a^4*c^
2-160*d^7/b^5*x^3*a^3*c^3+210*d^6/b^4*x^3*a^2*c^4-168*d^5/b^3*x^3*a*c^5-20/
7*d^9/b^3*x^7*a*c+90/b^10*d^9*ln(b*x+a)*a^8*c-360/b^9*d^8*ln(b*x+a)*a^7*c^2
+840/b^8*d^7*ln(b*x+a)*a^6*c^3-1260/b^7*d^6*ln(b*x+a)*a^5*c^4+1260/b^6*d^5*
ln(b*x+a)*a^4*c^5
```

**maxima [B]** time = 1.39, size = 874, normalized size = 3.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 2
10*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^
3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)/(b^12*x + a*b^
11) + 1/252*(28*b^8*d^10*x^9 + 63*(5*b^8*c*d^9 - a*b^7*d^10)*x^8 + 36*(45*b
^8*c^2*d^8 - 20*a*b^7*c*d^9 + 3*a^2*b^6*d^10)*x^7 + 84*(60*b^8*c^3*d^7 - 45
```

```
*a*b^7*c^2*d^8 + 15*a^2*b^6*c*d^9 - 2*a^3*b^5*d^10)*x^6 + 252*(42*b^8*c^4*d
^6 - 48*a*b^7*c^3*d^7 + 27*a^2*b^6*c^2*d^8 - 8*a^3*b^5*c*d^9 + a^4*b^4*d^10
)*x^5 + 126*(126*b^8*c^5*d^5 - 210*a*b^7*c^4*d^6 + 180*a^2*b^6*c^3*d^7 - 90
*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 - 3*a^5*b^3*d^10)*x^4 + 84*(210*b^8*c^6
*d^4 - 504*a*b^7*c^5*d^5 + 630*a^2*b^6*c^4*d^6 - 480*a^3*b^5*c^3*d^7 + 225*
a^4*b^4*c^2*d^8 - 60*a^5*b^3*c*d^9 + 7*a^6*b^2*d^10)*x^3 + 252*(60*b^8*c^7*
d^3 - 210*a*b^7*c^6*d^4 + 378*a^2*b^6*c^5*d^5 - 420*a^3*b^5*c^4*d^6 + 300*a
^4*b^4*c^3*d^7 - 135*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9 - 4*a^7*b*d^10)*x^2
 + 252*(45*b^8*c^8*d^2 - 240*a*b^7*c^7*d^3 + 630*a^2*b^6*c^6*d^4 - 1008*a^3
*b^5*c^5*d^5 + 1050*a^4*b^4*c^4*d^6 - 720*a^5*b^3*c^3*d^7 + 315*a^6*b^2*c^2
*d^8 - 80*a^7*b*c*d^9 + 9*a^8*d^10)*x)/b^10 + 10*(b^9*c^9*d - 9*a*b^8*c^8*d
^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*a^
5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 - a
^9*d^10)*log(b*x + a)/b^11
```

**mupad [B]** time = 0.35, size = 3475, normalized size = 13.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^2,x)
```

```
[Out] x^7*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/(7*b) - (a^2*d^10)/(7*b^4) + (
45*c^2*d^8)/(7*b^2)) - x^5*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10
)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*(
(2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/(5*b) - (42*c^4*d^6)/b^2 + (a^2*((2
*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2
))/(5*b^2)) - x^8*((a*d^10)/(4*b^3) - (5*c*d^9)/(4*b^2)) + x^3*((70*c^6*d^4
)/b^2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 -
(10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^1
0)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^
10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b
- (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b
 - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9
)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/(3*b) + (a^2*((2*a*((120*c^3*d^7)/b
^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45
*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*
c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)
/b^4 + (45*c^2*d^8)/b^2))/b^2))/(3*b^2)) - x^2*((a*((210*c^6*d^4)/b^2 - (2*
a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/
b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (1
0*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (
10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((12
0*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^1
0)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2
```

))/b^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b^2))/b - (60*c^7*d^3)/b^2 + (a^2*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/(2*b^2)) + x^6*((20*c^3*d^7)/b^2 - (a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/(3*b) + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/(6*b^2)) + x*((45*c^8*d^2)/b^2 - (a^2*((210*c^6*d^4)/b^2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b^2))/b^2 + (2*a*((2*a*((210*c^6*d^4)/b^2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b^2))/b - (120*c^7*d^3)/b^2 + (a^2*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/b^2))/b) + x^4*((a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^

```
10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/(2
*b) - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2
))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c
*d^9)/b^2))/b^2))/(4*b^2) + (63*c^5*d^5)/b^2) - (log(a + b*x)*(10*a^9*d^10
- 10*b^9*c^9*d + 90*a*b^8*c^8*d^2 - 360*a^2*b^7*c^7*d^3 + 840*a^3*b^6*c^6*d
^4 - 1260*a^4*b^5*c^5*d^5 + 1260*a^5*b^4*c^4*d^6 - 840*a^6*b^3*c^3*d^7 + 36
0*a^7*b^2*c^2*d^8 - 90*a^8*b*c*d^9))/b^11 - (a^10*d^10 + b^10*c^10 + 45*a^2
*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*
d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a
*b^9*c^9*d - 10*a^9*b*c*d^9)/(b*(a*b^10 + b^11*x)) + (d^10*x^9)/(9*b^2)
```

**sympy [B]** time = 2.65, size = 816, normalized size = 3.16

$$x^8\left(-\frac{ad^{10}}{4b^3}+\frac{5cd^9}{4b^2}\right)+x^7\left(\frac{3a^2d^{10}}{7b^4}-\frac{20acd^9}{7b^3}+\frac{45c^2d^8}{7b^2}\right)+x^6\left(-\frac{2a^3d^{10}}{3b^5}+\frac{5a^2cd^9}{b^4}-\frac{15ac^2d^8}{b^3}+\frac{20c^3d^7}{b^2}\right)+x^5\left(\frac{a^4d^{10}}{b^6}-\frac{8a^3cd^9}{b^5}+\frac{27a^2c^2d^8}{b^4}-\frac{48ac^3d^7}{b^3}+\frac{42c^4d^6}{b^2}\right)+x^4\left(-\frac{3a^5d^{10}}{2b^7}+\frac{25a^4cd^9}{2b^6}-\frac{45a^3c^2d^8}{b^5}+\frac{90a^2c^3d^7}{b^4}-\frac{105ac^4d^6}{b^3}+\frac{63c^5d^5}{b^2}\right)+x^3\left(\frac{7a^6d^{10}}{3b^8}-\frac{20a^5cd^9}{b^7}+\frac{75a^4c^2d^8}{b^6}-\frac{160a^3c^3d^7}{b^5}+\frac{210a^2c^4d^6}{b^4}-\frac{168ac^5d^5}{b^3}+\frac{70c^6d^4}{b^2}\right)+x^2\left(-\frac{4a^7d^{10}}{b^9}+\frac{35a^6cd^9}{b^8}-\frac{135a^5c^2d^8}{b^7}+\frac{300a^4c^3d^7}{b^6}-\frac{420a^3c^4d^6}{b^5}+\frac{378a^2c^5d^5}{b^4}-\frac{210ac^6d^4}{b^3}+\frac{60c^7d^3}{b^2}\right)+x\left(\frac{9a^8d^{10}}{b^{10}}-\frac{80a^7cd^9}{b^9}+\frac{315a^6c^2d^8}{b^8}-\frac{720a^5c^3d^7}{b^7}+\frac{1050a^4c^4d^6}{b^6}-\frac{1008a^3c^5d^5}{b^5}+\frac{630a^2c^6d^4}{b^4}-\frac{240ac^7d^3}{b^3}+\frac{45c^8d^2}{b^2}\right)+\frac{-a^{10}d^{10}+10a^9bcd^9-45a^8b^2c^2d^8+120a^7b^3c^3d^7-210a^6b^4c^4d^6+252a^5b^5c^5d^5-210a^4b^6c^6d^4+120a^3b^7c^7d^3-45a^2b^8c^8d^2+10ab^9c^9d-b^{10}c^{10}}{ab^{11}+b^{12}x}+\frac{d^{10}x^9}{9b^2}-\frac{10d(ad-bc)^9\log(a+bx)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**2,x)
```

```
[Out] x**8*(-a*d**10/(4*b**3) + 5*c*d**9/(4*b**2)) + x**7*(3*a**2*d**10/(7*b**4)
- 20*a*c*d**9/(7*b**3) + 45*c**2*d**8/(7*b**2)) + x**6*(-2*a**3*d**10/(3*b*
*5) + 5*a**2*c*d**9/b**4 - 15*a*c**2*d**8/b**3 + 20*c**3*d**7/b**2) + x**5*
(a**4*d**10/b**6 - 8*a**3*c*d**9/b**5 + 27*a**2*c**2*d**8/b**4 - 48*a*c**3*
d**7/b**3 + 42*c**4*d**6/b**2) + x**4*(-3*a**5*d**10/(2*b**7) + 25*a**4*c*d
**9/(2*b**6) - 45*a**3*c**2*d**8/b**5 + 90*a**2*c**3*d**7/b**4 - 105*a*c**4
*d**6/b**3 + 63*c**5*d**5/b**2) + x**3*(7*a**6*d**10/(3*b**8) - 20*a**5*c*d
**9/b**7 + 75*a**4*c**2*d**8/b**6 - 160*a**3*c**3*d**7/b**5 + 210*a**2*c**4
*d**6/b**4 - 168*a*c**5*d**5/b**3 + 70*c**6*d**4/b**2) + x**2*(-4*a**7*d**1
0/b**9 + 35*a**6*c*d**9/b**8 - 135*a**5*c**2*d**8/b**7 + 300*a**4*c**3*d**7
/b**6 - 420*a**3*c**4*d**6/b**5 + 378*a**2*c**5*d**5/b**4 - 210*a*c**6*d**4
/b**3 + 60*c**7*d**3/b**2) + x*(9*a**8*d**10/b**10 - 80*a**7*c*d**9/b**9 +
315*a**6*c**2*d**8/b**8 - 720*a**5*c**3*d**7/b**7 + 1050*a**4*c**4*d**6/b**
6 - 1008*a**3*c**5*d**5/b**5 + 630*a**2*c**6*d**4/b**4 - 240*a*c**7*d**3/b*
*3 + 45*c**8*d**2/b**2) + (-a**10*d**10 + 10*a**9*b*c*d**9 - 45*a**8*b**2*c
**2*d**8 + 120*a**7*b**3*c**3*d**7 - 210*a**6*b**4*c**4*d**6 + 252*a**5*b**
5*c**5*d**5 - 210*a**4*b**6*c**6*d**4 + 120*a**3*b**7*c**7*d**3 - 45*a**2*b
**8*c**8*d**2 + 10*a*b**9*c**9*d - b**10*c**10)/(a*b**11 + b**12*x) + d**10
*x**9/(9*b**2) - 10*d*(a*d - b*c)**9*log(a + b*x)/b**11
```

## 3.1208 $\int \frac{(c+dx)^{10}}{(a+bx)^3}\, dx$

**Optimal**. Leaf size=262

$$\frac{10d^9(a+bx)^7(bc-ad)}{7b^{11}}+\frac{15d^8(a+bx)^6(bc-ad)^2}{2b^{11}}+\frac{24d^7(a+bx)^5(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^4(bc-ad)^4}{2b^{11}}+\frac{84d^5(a+\cdots}{}$$

**Rubi [A]** time = 0.44, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{10d^9(a+bx)^7(bc-ad)}{7b^{11}}+\frac{15d^8(a+bx)^6(bc-ad)^2}{2b^{11}}+\frac{24d^7(a+bx)^5(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^4(bc-ad)^4}{2b^{11}}+\frac{84d^5(a+bx)^3(bc-ad)^5}{b^{11}}+\frac{105d^4(a+bx)^2(bc-ad)^6}{b^{11}}+\frac{120d^3x(bc-ad)^7}{b^{10}}+\frac{45d^2(bc-ad)^8\log(a+bx)}{b^{11}}-\frac{10d(bc-ad)^9}{b^{11}(a+bx)}-\frac{(bc-ad)^{10}}{2b^{11}(a+bx)^2}+\frac{d^{10}(a+bx)^8}{8b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] (120*d^3*(b*c - a*d)^7*x)/b^10 - (b*c - a*d)^10/(2*b^11*(a + b*x)^2) - (10*
d*(b*c - a*d)^9)/(b^11*(a + b*x)) + (105*d^4*(b*c - a*d)^6*(a + b*x)^2)/b^1
1 + (84*d^5*(b*c - a*d)^5*(a + b*x)^3)/b^11 + (105*d^6*(b*c - a*d)^4*(a + b
*x)^4)/(2*b^11) + (24*d^7*(b*c - a*d)^3*(a + b*x)^5)/b^11 + (15*d^8*(b*c -
a*d)^2*(a + b*x)^6)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^7)/(7*b^11) +
(d^10*(a + b*x)^8)/(8*b^11) + (45*d^2*(b*c - a*d)^8*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^3}\,dx = \int \Bigg(\frac{120d^3(bc-ad)^7}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^3}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^2}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)}+\frac{210d^4(bc-ad)^6(a+\cdots}{b^{10}}$$

$$=\frac{120d^3(bc-ad)^7x}{b^{10}}-\frac{(bc-ad)^{10}}{2b^{11}(a+bx)^2}-\frac{10d(bc-ad)^9}{b^{11}(a+bx)}+\frac{105d^4(bc-ad)^6(a+bx)^2}{b^{11}}+\frac{84d^5(bc-a\cdots}{b^{1\cdots}}$$

**Mathematica [B]** time = 0.24, size = 708, normalized size = 2.70

$$\frac{532a^{10}d^{10}-56a^9bd^9(85c+26dx)+28a^8b^2d^8\left(675c^2+380cdx-116d^2x^2\right)-280a^7b^3d^7\left(156c^3+117c^2dx-91cd^2x^2+3d^3x^3\right)+210a^6b^4d^6\left(308c^4+256c^3dx-414c^2d^2x^2+32cd^3x^3+d^4x^4\right)-84a^5b^5d^5\left(756c^5+560c^4dx-2000c^3d^2x^2+280c^2d^3x^3+20cd^4x^4+d^5x^5\right)+42a^4b^6d^4\left(980c^6+336c^5dx-4760c^4d^2x^2+1120c^3d^3x^3+140c^2d^4x^4+16cd^5x^5+d^6x^6\right)-24a^3b^7d^3\left(700c^7-490c^6dx-6174c^5d^2x^2+2450c^4d^3x^3+490c^3d^4x^4+98c^2d^5x^5+14cd^6x^6+d^7x^7\right)+3a^2b^8d^2\left(1260c^8-4480c^7dx-21560c^6d^2x^2+15680c^5d^3x^3+4900c^4d^4x^4+1568c^3d^5x^5+392c^2d^6x^6+64cd^7x^7+5d^8x^8\right)-2ab^9d\left(140c^9-2520c^8dx-6720c^7d^2x^2+11760c^6d^3x^3+5880c^5d^4x^4+2940c^4d^5x^5+1176c^3d^6x^6+336c^2d^7x^7+60cd^8x^8+5d^9x^9\right)+2520d^2(a+bx)^2(bc-ad)^8\log(a+bx)+b^{10}\left(-28c^{10}-560c^9dx+6720c^7d^3x^3+5880c^6d^4x^4+4704c^5d^5x^5+2940c^4d^6x^6+1344c^3d^7x^7+420c^2d^8x^8+80cd^9x^9+7d^{10}x^{10}\right)}{56b^{11}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] (532*a^10*d^10 - 56*a^9*b*d^9*(85*c + 26*d*x) + 28*a^8*b^2*d^8*(675*c^2 + 3
80*c*d*x - 116*d^2*x^2) - 280*a^7*b^3*d^7*(156*c^3 + 117*c^2*d*x - 91*c*d^2
*x^2 + 3*d^3*x^3) + 210*a^6*b^4*d^6*(308*c^4 + 256*c^3*d*x - 414*c^2*d^2*x^
2 + 32*c*d^3*x^3 + d^4*x^4) - 84*a^5*b^5*d^5*(756*c^5 + 560*c^4*d*x - 2000*
c^3*d^2*x^2 + 280*c^2*d^3*x^3 + 20*c*d^4*x^4 + d^5*x^5) + 42*a^4*b^6*d^4*(9
80*c^6 + 336*c^5*d*x - 4760*c^4*d^2*x^2 + 1120*c^3*d^3*x^3 + 140*c^2*d^4*x^
4 + 16*c*d^5*x^5 + d^6*x^6) - 24*a^3*b^7*d^3*(700*c^7 - 490*c^6*d*x - 6174*
c^5*d^2*x^2 + 2450*c^4*d^3*x^3 + 490*c^3*d^4*x^4 + 98*c^2*d^5*x^5 + 14*c*d^
6*x^6 + d^7*x^7) + 3*a^2*b^8*d^2*(1260*c^8 - 4480*c^7*d*x - 21560*c^6*d^2*x
^2 + 15680*c^5*d^3*x^3 + 4900*c^4*d^4*x^4 + 1568*c^3*d^5*x^5 + 392*c^2*d^6*
x^6 + 64*c*d^7*x^7 + 5*d^8*x^8) - 2*a*b^9*d*(140*c^9 - 2520*c^8*d*x - 6720*
c^7*d^2*x^2 + 11760*c^6*d^3*x^3 + 5880*c^5*d^4*x^4 + 2940*c^4*d^5*x^5 + 117
6*c^3*d^6*x^6 + 336*c^2*d^7*x^7 + 60*c*d^8*x^8 + 5*d^9*x^9) + b^10*(-28*c^1
0 - 560*c^9*d*x + 6720*c^7*d^3*x^3 + 5880*c^6*d^4*x^4 + 4704*c^5*d^5*x^5 +
2940*c^4*d^6*x^6 + 1344*c^3*d^7*x^7 + 420*c^2*d^8*x^8 + 80*c*d^9*x^9 + 7*d^
10*x^10) + 2520*d^2*(b*c - a*d)^8*(a + b*x)^2*Log[a + b*x])/(56*b^11*(a + b
*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^3, x]
```

**fricas [B]** time = 1.26, size = 1233, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^10*d^10*x^10 - 28*b^10*c^10 - 280*a*b^9*c^9*d + 3780*a^2*b^8*c^8*
d^2 - 16800*a^3*b^7*c^7*d^3 + 41160*a^4*b^6*c^6*d^4 - 63504*a^5*b^5*c^5*d^5
 + 64680*a^6*b^4*c^4*d^6 - 43680*a^7*b^3*c^3*d^7 + 18900*a^8*b^2*c^2*d^8 -
4760*a^9*b*c*d^9 + 532*a^10*d^10 + 10*(8*b^10*c*d^9 - a*b^9*d^10)*x^9 + 15*
(28*b^10*c^2*d^8 - 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 24*(56*b^10*c^3*d^7
- 28*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 42*(70*b^10*c^4*
d^6 - 56*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 - 8*a^3*b^7*c*d^9 + a^4*b^6*d^1
```

```
0)*x^6 + 84*(56*b^10*c^5*d^5 - 70*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 - 28*a
^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 210*(28*b^10*c^6*d^4
 - 56*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 - 56*a^3*b^7*c^3*d^7 + 28*a^4*b^6*
c^2*d^8 - 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 840*(8*b^10*c^7*d^3 - 28*a*
b^9*c^6*d^4 + 56*a^2*b^8*c^5*d^5 - 70*a^3*b^7*c^4*d^6 + 56*a^4*b^6*c^3*d^7
- 28*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 28*(480*a*b^9*
c^7*d^3 - 2310*a^2*b^8*c^6*d^4 + 5292*a^3*b^7*c^5*d^5 - 7140*a^4*b^6*c^4*d^
6 + 6000*a^5*b^5*c^3*d^7 - 3105*a^6*b^4*c^2*d^8 + 910*a^7*b^3*c*d^9 - 116*a
^8*b^2*d^10)*x^2 - 56*(10*b^10*c^9*d - 90*a*b^9*c^8*d^2 + 240*a^2*b^8*c^7*d
^3 - 210*a^3*b^7*c^6*d^4 - 252*a^4*b^6*c^5*d^5 + 840*a^5*b^5*c^4*d^6 - 960*
a^6*b^4*c^3*d^7 + 585*a^7*b^3*c^2*d^8 - 190*a^8*b^2*c*d^9 + 26*a^9*b*d^10)*
x + 2520*(a^2*b^8*c^8*d^2 - 8*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 - 56*a^5
*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 - 56*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8
 - 8*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8
*c^6*d^4 - 56*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 2
8*a^6*b^4*c^2*d^8 - 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 2*(a*b^9*c^8*d^2
- 8*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^4 - 56*a^4*b^6*c^5*d^5 + 70*a^5*b^5*
c^4*d^6 - 56*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 - 8*a^8*b^2*c*d^9 + a^9*b
*d^10)*x)*log(b*x + a))/(b^13*x^2 + 2*a*b^12*x + a^2*b^11)
```

**giac [B]** time = 1.25, size = 924, normalized size = 3.53

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 45*(b^8*c^8*d^2 - 8*a*b^7*c^7*d^3 + 28*a^2*b^6*c^6*d^4 - 56*a^3*b^5*c^5*d^5
 + 70*a^4*b^4*c^4*d^6 - 56*a^5*b^3*c^3*d^7 + 28*a^6*b^2*c^2*d^8 - 8*a^7*b*c
*d^9 + a^8*d^10)*log(abs(b*x + a))/b^11 - 1/2*(b^10*c^10 + 10*a*b^9*c^9*d -
 135*a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^
5*b^5*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 1560*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c
^2*d^8 + 170*a^9*b*c*d^9 - 19*a^10*d^10 + 20*(b^10*c^9*d - 9*a*b^9*c^8*d^2
+ 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 - 126*a^5*b
^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 - a^
9*b*d^10)*x)/((b*x + a)^2*b^11) + 1/56*(7*b^21*d^10*x^8 + 80*b^21*c*d^9*x^7
 - 24*a*b^20*d^10*x^7 + 420*b^21*c^2*d^8*x^6 - 280*a*b^20*c*d^9*x^6 + 56*a^
2*b^19*d^10*x^6 + 1344*b^21*c^3*d^7*x^5 - 1512*a*b^20*c^2*d^8*x^5 + 672*a^2
*b^19*c*d^9*x^5 - 112*a^3*b^18*d^10*x^5 + 2940*b^21*c^4*d^6*x^4 - 5040*a*b^
20*c^3*d^7*x^4 + 3780*a^2*b^19*c^2*d^8*x^4 - 1400*a^3*b^18*c*d^9*x^4 + 210*
a^4*b^17*d^10*x^4 + 4704*b^21*c^5*d^5*x^3 - 11760*a*b^20*c^4*d^6*x^3 + 1344
0*a^2*b^19*c^3*d^7*x^3 - 8400*a^3*b^18*c^2*d^8*x^3 + 2800*a^4*b^17*c*d^9*x^
3 - 392*a^5*b^16*d^10*x^3 + 5880*b^21*c^6*d^4*x^2 - 21168*a*b^20*c^5*d^5*x^
2 + 35280*a^2*b^19*c^4*d^6*x^2 - 33600*a^3*b^18*c^3*d^7*x^2 + 18900*a^4*b^1
7*c^2*d^8*x^2 - 5880*a^5*b^16*c*d^9*x^2 + 784*a^6*b^15*d^10*x^2 + 6720*b^21
```

```
*c^7*d^3*x - 35280*a*b^20*c^6*d^4*x + 84672*a^2*b^19*c^5*d^5*x - 117600*a^3
*b^18*c^4*d^6*x + 100800*a^4*b^17*c^3*d^7*x - 52920*a^5*b^16*c^2*d^8*x + 15
680*a^6*b^15*c*d^9*x - 2016*a^7*b^14*d^10*x)/b^24
```

**maple [B]** time = 0.02, size = 1105, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^3,x)
```

```
[Out] -1/2/b^11/(b*x+a)^2*a^10*d^10+45/b^11*d^10*ln(b*x+a)*a^8+45/b^3*d^2*ln(b*x+
a)*c^8+10/b^11*d^10/(b*x+a)*a^9-10/b^2*d/(b*x+a)*c^9+105/2*d^6/b^3*x^4*c^4-
7*d^10/b^8*x^3*a^5+84*d^5/b^3*x^3*c^5+14*d^10/b^9*x^2*a^6+105*d^4/b^3*x^2*c
^6-3/7*d^10/b^4*x^7*a+10/7*d^9/b^3*x^7*c+15/2*d^8/b^3*x^6*c^2-2*d^10/b^6*x^
5*a^3+24*d^7/b^3*x^5*c^3+15/4*d^10/b^7*x^4*a^4+d^10/b^5*x^6*a^2-36*d^10/b^1
0*a^7*x+120*d^3/b^3*c^7*x-105/b^5/(b*x+a)^2*a^4*c^6*d^4+60/b^4/(b*x+a)^2*a^
3*c^7*d^3-45/2/b^3/(b*x+a)^2*a^2*c^8*d^2+5/b^2/(b*x+a)^2*a*c^9*d-360/b^10*d
^9*ln(b*x+a)*a^7*c+1/8*d^10/b^3*x^8-1/2/b/(b*x+a)^2*c^10+1260/b^9*d^8*ln(b*
x+a)*a^6*c^2-2520/b^8*d^7*ln(b*x+a)*a^5*c^3+3150/b^7*d^6*ln(b*x+a)*a^4*c^4-
2520/b^6*d^5*ln(b*x+a)*a^3*c^5+1260/b^5*d^4*ln(b*x+a)*a^2*c^6-360/b^4*d^3*l
n(b*x+a)*a*c^7-90/b^10*d^9/(b*x+a)*a^8*c+360/b^9*d^8/(b*x+a)*a^7*c^2-840/b^
8*d^7/(b*x+a)*a^6*c^3+1260/b^7*d^6/(b*x+a)*a^5*c^4-1260/b^6*d^5/(b*x+a)*a^4
*c^5+840/b^5*d^4/(b*x+a)*a^3*c^6-360/b^4*d^3/(b*x+a)*a^2*c^7+90/b^3*d^2/(b*
x+a)*a*c^8+675/2*d^8/b^7*x^2*a^4*c^2-600*d^7/b^6*x^2*a^3*c^3+630*d^6/b^5*x^
2*a^2*c^4-378*d^5/b^4*x^2*a*c^5+280*d^9/b^9*a^6*c*x-945*d^8/b^8*a^5*c^2*x+1
800*d^7/b^7*a^4*c^3*x-2100*d^6/b^6*a^3*c^4*x+1512*d^5/b^5*a^2*c^5*x-630*d^4
/b^4*a*c^6*x-27*d^8/b^4*x^5*a*c^2-25*d^9/b^6*x^4*a^3*c+135/2*d^8/b^5*x^4*a^
2*c^2-90*d^7/b^4*x^4*a*c^3+50*d^9/b^7*x^3*a^4*c-150*d^8/b^6*x^3*a^3*c^2+240
*d^7/b^5*x^3*a^2*c^3-210*d^6/b^4*x^3*a*c^4-105*d^9/b^8*x^2*a^5*c-5*d^9/b^4*
x^6*a*c+12*d^9/b^5*x^5*a^2*c+5/b^10/(b*x+a)^2*a^9*c*d^9-45/2/b^9/(b*x+a)^2*
a^8*c^2*d^8+60/b^8/(b*x+a)^2*a^7*c^3*d^7-105/b^7/(b*x+a)^2*a^6*c^4*d^6+126/
b^6/(b*x+a)^2*a^5*c^5*d^5
```

**maxima [B]** time = 1.62, size = 881, normalized size = 3.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^10*c^10 + 10*a*b^9*c^9*d - 135*a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^
3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^5*b^5*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 15
60*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c^2*d^8 + 170*a^9*b*c*d^9 - 19*a^10*d^10 +
 20*(b^10*c^9*d - 9*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4
 + 126*a^4*b^6*c^5*d^5 - 126*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*
```

```
b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 - a^9*b*d^10)*x)/(b^13*x^2 + 2*a*b^12*x + a^2
*b^11) + 1/56*(7*b^7*d^10*x^8 + 8*(10*b^7*c*d^9 - 3*a*b^6*d^10)*x^7 + 28*(1
5*b^7*c^2*d^8 - 10*a*b^6*c*d^9 + 2*a^2*b^5*d^10)*x^6 + 56*(24*b^7*c^3*d^7 -
 27*a*b^6*c^2*d^8 + 12*a^2*b^5*c*d^9 - 2*a^3*b^4*d^10)*x^5 + 70*(42*b^7*c^4
*d^6 - 72*a*b^6*c^3*d^7 + 54*a^2*b^5*c^2*d^8 - 20*a^3*b^4*c*d^9 + 3*a^4*b^3
*d^10)*x^4 + 56*(84*b^7*c^5*d^5 - 210*a*b^6*c^4*d^6 + 240*a^2*b^5*c^3*d^7 -
 150*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 - 7*a^5*b^2*d^10)*x^3 + 28*(210*b^7
*c^6*d^4 - 756*a*b^6*c^5*d^5 + 1260*a^2*b^5*c^4*d^6 - 1200*a^3*b^4*c^3*d^7
+ 675*a^4*b^3*c^2*d^8 - 210*a^5*b^2*c*d^9 + 28*a^6*b*d^10)*x^2 + 56*(120*b^
7*c^7*d^3 - 630*a*b^6*c^6*d^4 + 1512*a^2*b^5*c^5*d^5 - 2100*a^3*b^4*c^4*d^6
 + 1800*a^4*b^3*c^3*d^7 - 945*a^5*b^2*c^2*d^8 + 280*a^6*b*c*d^9 - 36*a^7*d^
10)*x)/b^10 + 45*(b^8*c^8*d^2 - 8*a*b^7*c^7*d^3 + 28*a^2*b^6*c^6*d^4 - 56*a
^3*b^5*c^5*d^5 + 70*a^4*b^4*c^4*d^6 - 56*a^5*b^3*c^3*d^7 + 28*a^6*b^2*c^2*d
^8 - 8*a^7*b*c*d^9 + a^8*d^10)*log(b*x + a)/b^11
```

**mupad [B]** time = 0.38, size = 3299, normalized size = 12.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^3,x)
```

```
[Out] x^3*((84*c^5*d^5)/b^3 - (a*((3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b
^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^
3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d
^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*
d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2)
)/b + (a^2*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/
b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((
3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (
10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/(3*b^3)) - x^7*((
3*a*d^10)/(7*b^4) - (10*c*d^9)/(7*b^3)) - ((b^10*c^10 - 19*a^10*d^10 - 135*
a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^5*b^5
*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 1560*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c^2*d^
8 + 10*a*b^9*c^9*d + 170*a^9*b*c*d^9)/(2*b) - x*(10*a^9*d^10 - 10*b^9*c^9*d
 + 90*a*b^8*c^8*d^2 - 360*a^2*b^7*c^7*d^3 + 840*a^3*b^6*c^6*d^4 - 1260*a^4*
b^5*c^5*d^5 + 1260*a^5*b^4*c^4*d^6 - 840*a^6*b^3*c^3*d^7 + 360*a^7*b^2*c^2*
d^8 - 90*a^8*b*c*d^9))/(a^2*b^10 + b^12*x^2 + 2*a*b^11*x) - x^5*((3*a*((3*a
*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3
))/(5*b) + (a^3*d^10)/(5*b^6) - (24*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 -
 (10*c*d^9)/b^3))/(5*b^2)) + x^6*((a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/(2*
b) - (a^2*d^10)/(2*b^5) + (15*c^2*d^8)/(2*b^3)) + x^4*((3*a*((3*a*((3*a*((3
*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b
 + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)
/b^3))/b^2))/(4*b) + (105*c^4*d^6)/(2*b^3) + (a^3*((3*a*d^10)/b^4 - (10*c*d
```

^9)/b^3))/(4*b^3) - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*
a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/(4*b^2)) - x^2*((3*a^2*((3*a*((3*a*((3*a
*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3
))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*
d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/
b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10
)/b^5 + (45*c^2*d^8)/b^3))/b^2))/(2*b^2) + (3*a*((252*c^5*d^5)/b^3 - (3*a*(
(3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 +
(45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^
10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b
^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))
/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (3*a^2*((3*a*((3*a*((3
*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b
 + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)
/b^3))/b^2))/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2
*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^3))/(2*b) - (105*c^6*d^4)/b^3 - (a^3*((3*
a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d
^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 -
 (10*c*d^9)/b^3))/b^2))/(2*b^3)) + x*((3*a*((3*a^2*((3*a*((3*a*((3*a*((3*a*
d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b +
(a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^
3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b
^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 +
 (45*c^2*d^8)/b^3))/b^2))/b^2 + (3*a*((252*c^5*d^5)/b^3 - (3*a*((3*a*((3*a*
((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8
)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (
10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*
d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2
*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (3*a^2*((3*a*((3*a*((3*a*d^10)/b^
4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^1
0)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2)
)/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5
+ (45*c^2*d^8)/b^3))/b^3))/b - (210*c^6*d^4)/b^3 - (a^3*((3*a*((3*a*((3*a*d
^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (
a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3
))/b^2))/b^3))/b - (a^3*((3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3)
)/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d
^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)
/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^1
0)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b
^3 + (120*c^7*d^3)/b^3 - (3*a^2*((252*c^5*d^5)/b^3 - (3*a*((3*a*((3*a*((3*a
*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3
))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*
d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/
b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10

```
)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (3*a^2*((3*a*((3*a*((3*a*d^10)/b^4 - (
10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^
6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b^2
 - (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45
*c^2*d^8)/b^3))/b^3))/b^2) + (log(a + b*x)*(45*a^8*d^10 + 45*b^8*c^8*d^2 -
360*a*b^7*c^7*d^3 + 1260*a^2*b^6*c^6*d^4 - 2520*a^3*b^5*c^5*d^5 + 3150*a^4*
b^4*c^4*d^6 - 2520*a^5*b^3*c^3*d^7 + 1260*a^6*b^2*c^2*d^8 - 360*a^7*b*c*d^9
))/b^11 + (d^10*x^8)/(8*b^3)
```

**sympy [B]** time = 5.67, size = 843, normalized size = 3.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**3,x)
```

```
[Out] x**7*(-3*a*d**10/(7*b**4) + 10*c*d**9/(7*b**3)) + x**6*(a**2*d**10/b**5 - 5
*a*c*d**9/b**4 + 15*c**2*d**8/(2*b**3)) + x**5*(-2*a**3*d**10/b**6 + 12*a**
2*c*d**9/b**5 - 27*a*c**2*d**8/b**4 + 24*c**3*d**7/b**3) + x**4*(15*a**4*d*
*10/(4*b**7) - 25*a**3*c*d**9/b**6 + 135*a**2*c**2*d**8/(2*b**5) - 90*a*c**
3*d**7/b**4 + 105*c**4*d**6/(2*b**3)) + x**3*(-7*a**5*d**10/b**8 + 50*a**4*
c*d**9/b**7 - 150*a**3*c**2*d**8/b**6 + 240*a**2*c**3*d**7/b**5 - 210*a*c**
4*d**6/b**4 + 84*c**5*d**5/b**3) + x**2*(14*a**6*d**10/b**9 - 105*a**5*c*d*
*9/b**8 + 675*a**4*c**2*d**8/(2*b**7) - 600*a**3*c**3*d**7/b**6 + 630*a**2*
c**4*d**6/b**5 - 378*a*c**5*d**5/b**4 + 105*c**6*d**4/b**3) + x*(-36*a**7*d
**10/b**10 + 280*a**6*c*d**9/b**9 - 945*a**5*c**2*d**8/b**8 + 1800*a**4*c**
3*d**7/b**7 - 2100*a**3*c**4*d**6/b**6 + 1512*a**2*c**5*d**5/b**5 - 630*a*c
**6*d**4/b**4 + 120*c**7*d**3/b**3) + (19*a**10*d**10 - 170*a**9*b*c*d**9 +
 675*a**8*b**2*c**2*d**8 - 1560*a**7*b**3*c**3*d**7 + 2310*a**6*b**4*c**4*d
**6 - 2268*a**5*b**5*c**5*d**5 + 1470*a**4*b**6*c**6*d**4 - 600*a**3*b**7*c
**7*d**3 + 135*a**2*b**8*c**8*d**2 - 10*a*b**9*c**9*d - b**10*c**10 + x*(20
*a**9*b*d**10 - 180*a**8*b**2*c*d**9 + 720*a**7*b**3*c**2*d**8 - 1680*a**6*
b**4*c**3*d**7 + 2520*a**5*b**5*c**4*d**6 - 2520*a**4*b**6*c**5*d**5 + 1680
*a**3*b**7*c**6*d**4 - 720*a**2*b**8*c**7*d**3 + 180*a*b**9*c**8*d**2 - 20*
b**10*c**9*d))/(2*a**2*b**11 + 4*a*b**12*x + 2*b**13*x**2) + d**10*x**8/(8*
b**3) + 45*d**2*(a*d - b*c)**8*log(a + b*x)/b**11
```

## 3.1209 $\int \frac{(c+dx)^{10}}{(a+bx)^4}\,dx$

**Optimal**. Leaf size=258

$$\frac{5d^9(a+bx)^6(bc-ad)}{3b^{11}}+\frac{9d^8(a+bx)^5(bc-ad)^2}{b^{11}}+\frac{30d^7(a+bx)^4(bc-ad)^3}{b^{11}}+\frac{70d^6(a+bx)^3(bc-ad)^4}{b^{11}}+\frac{126d^5(a+b}{b}$$

**Rubi** [A] time = 0.44, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^6(bc-ad)}{3b^{11}}+\frac{9d^8(a+bx)^5(bc-ad)^2}{b^{11}}+\frac{30d^7(a+bx)^4(bc-ad)^3}{b^{11}}+\frac{70d^6(a+bx)^3(bc-ad)^4}{b^{11}}+\frac{126d^5(a+bx)^2(bc-ad)^5}{b^{11}}+\frac{210d^4x(bc-ad)^6}{b^{10}}-\frac{45d^2(bc-ad)^8}{b^{11}(a+bx)}+\frac{120d^3(bc-ad)^7\log(a+bx)}{b^{11}}-\frac{5d(bc-ad)^9}{b^{11}(a+bx)^2}-\frac{(bc-ad)^{10}}{3b^{11}(a+bx)^3}+\frac{d^{10}(a+bx)^7}{7b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^4,x]
```

```
[Out] (210*d^4*(b*c - a*d)^6*x)/b^10 - (b*c - a*d)^10/(3*b^11*(a + b*x)^3) - (5*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^2) - (45*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
) + (126*d^5*(b*c - a*d)^5*(a + b*x)^2)/b^11 + (70*d^6*(b*c - a*d)^4*(a + b
*x)^3)/b^11 + (30*d^7*(b*c - a*d)^3*(a + b*x)^4)/b^11 + (9*d^8*(b*c - a*d)^
2*(a + b*x)^5)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^6)/(3*b^11) + (d^10*(a +
 b*x)^7)/(7*b^11) + (120*d^3*(b*c - a*d)^7*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^4}\,dx=\int \left(\frac{210d^4(bc-ad)^6}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^4}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^3}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^2}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)}+\right.$$

$$=\frac{210d^4(bc-ad)^6x}{b^{10}}-\frac{(bc-ad)^{10}}{3b^{11}(a+bx)^3}-\frac{5d(bc-ad)^9}{b^{11}(a+bx)^2}-\frac{45d^2(bc-ad)^8}{b^{11}(a+bx)}+\frac{126d^5(bc-ad)^5(a+b}{b^{11}}$$

**Mathematica** [A] time = 0.18, size = 427, normalized size = 1.66

$$\frac{21b^5d^8x^5\left(2a^2d^2-8abcd+9b^2c^2\right)+105b^4d^7x^4\left(-a^3d^3+5a^2bcd^2-9ab^2c^2d+6b^3c^3\right)+35b^3d^6x^3\left(7a^4d^4-40a^3bcd^3+90a^2b^2c^2d^2-96ab^3c^3d+42b^4c^4\right)+21b^2d^5x^2\left(-28a^5d^5+175a^4bcd^4-450a^3b^2c^2d^3+600a^2b^3c^3d^2-420ab^4c^4d+126b^5c^5\right)+21bd^4x\left(84a^6d^6-560a^5bcd^5+1575a^4b^2c^2d^4-2400a^3b^3c^3d^3+2100a^2b^4c^4d^2-1008ab^5c^5d+210b^6c^6\right)+7b^6d^9x^6(5bc-2ad)+2520d^3(bc-ad)^7\log(a+bx)-\frac{945d^2(bc-ad)^8}{a+bx}+\frac{105d(ad-bc)^9}{(a+bx)^2}-\frac{7(bc-ad)^{10}}{(a+bx)^3}+3b^7d^{10}x^7}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^4,x]
```

```
[Out] (21*b*d^4*(210*b^6*c^6 - 1008*a*b^5*c^5*d + 2100*a^2*b^4*c^4*d^2 - 2400*a^3
*b^3*c^3*d^3 + 1575*a^4*b^2*c^2*d^4 - 560*a^5*b*c*d^5 + 84*a^6*d^6)*x + 21*
b^2*d^5*(126*b^5*c^5 - 420*a*b^4*c^4*d + 600*a^2*b^3*c^3*d^2 - 450*a^3*b^2*
c^2*d^3 + 175*a^4*b*c*d^4 - 28*a^5*d^5)*x^2 + 35*b^3*d^6*(42*b^4*c^4 - 96*a
*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 - 40*a^3*b*c*d^3 + 7*a^4*d^4)*x^3 + 105*b^4
*d^7*(6*b^3*c^3 - 9*a*b^2*c^2*d + 5*a^2*b*c*d^2 - a^3*d^3)*x^4 + 21*b^5*d^8
*(9*b^2*c^2 - 8*a*b*c*d + 2*a^2*d^2)*x^5 + 7*b^6*d^9*(5*b*c - 2*a*d)*x^6 +
3*b^7*d^10*x^7 - (7*(b*c - a*d)^10)/(a + b*x)^3 + (105*d*(-(b*c) + a*d)^9)/
(a + b*x)^2 - (945*d^2*(b*c - a*d)^8)/(a + b*x) + 2520*d^3*(b*c - a*d)^7*Lo
g[a + b*x])/(21*b^11)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^4, x]
```

**fricas [B]** time = 1.25, size = 1316, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^10*d^10*x^10 - 7*b^10*c^10 - 35*a*b^9*c^9*d - 315*a^2*b^8*c^8*d^2
 + 4620*a^3*b^7*c^7*d^3 - 19110*a^4*b^6*c^6*d^4 + 41454*a^5*b^5*c^5*d^5 - 5
4390*a^6*b^4*c^4*d^6 + 44940*a^7*b^3*c^3*d^7 - 22995*a^8*b^2*c^2*d^8 + 6685
*a^9*b*c*d^9 - 847*a^10*d^10 + 5*(7*b^10*c*d^9 - a*b^9*d^10)*x^9 + 9*(21*b^
10*c^2*d^8 - 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 18*(35*b^10*c^3*d^7 - 21*a
*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 42*(35*b^10*c^4*d^6 -
35*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 - 7*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 126*(21*b^10*c^5*d^5 - 35*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 - 21*a^3*b^
7*c^2*d^8 + 7*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 630*(7*b^10*c^6*d^4 - 21*
a*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 - 35*a^3*b^7*c^3*d^7 + 21*a^4*b^6*c^2*d^
8 - 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 7*(1890*a*b^9*c^6*d^4 - 7938*a^2*
b^8*c^5*d^5 + 15330*a^3*b^7*c^4*d^6 - 16680*a^4*b^6*c^3*d^7 + 10575*a^5*b^5
*c^2*d^8 - 3665*a^6*b^4*c*d^9 + 539*a^7*b^3*d^10)*x^3 - 21*(45*b^10*c^8*d^2
 - 360*a*b^9*c^7*d^3 + 630*a^2*b^8*c^6*d^4 + 378*a^3*b^7*c^5*d^5 - 2730*a^4
```

```
*b^6*c^4*d^6 + 4080*a^5*b^5*c^3*d^7 - 3015*a^6*b^4*c^2*d^8 + 1145*a^7*b^3*c
*d^9 - 179*a^8*b^2*d^10)*x^2 - 21*(5*b^10*c^9*d + 45*a*b^9*c^8*d^2 - 540*a^
2*b^8*c^7*d^3 + 1890*a^3*b^7*c^6*d^4 - 3402*a^4*b^6*c^5*d^5 + 3570*a^5*b^5*
c^4*d^6 - 2220*a^6*b^4*c^3*d^7 + 765*a^7*b^3*c^2*d^8 - 115*a^8*b^2*c*d^9 +
a^9*b*d^10)*x + 2520*(a^3*b^7*c^7*d^3 - 7*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*
d^5 - 35*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 - 21*a^8*b^2*c^2*d^8 + 7*a^9*
b*c*d^9 - a^10*d^10 + (b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5
- 35*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*
c*d^9 - a^7*b^3*d^10)*x^3 + 3*(a*b^9*c^7*d^3 - 7*a^2*b^8*c^6*d^4 + 21*a^3*b
^7*c^5*d^5 - 35*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 - 21*a^6*b^4*c^2*d^8 +
 7*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 3*(a^2*b^8*c^7*d^3 - 7*a^3*b^7*c^6*d
^4 + 21*a^4*b^6*c^5*d^5 - 35*a^5*b^5*c^4*d^6 + 35*a^6*b^4*c^3*d^7 - 21*a^7*
b^3*c^2*d^8 + 7*a^8*b^2*c*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^14*x^3 + 3*
a*b^13*x^2 + 3*a^2*b^12*x + a^3*b^11)
```

**giac** [B] time = 1.26, size = 907, normalized size = 3.52

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 120*(b^7*c^7*d^3 - 7*a*b^6*c^6*d^4 + 21*a^2*b^5*c^5*d^5 - 35*a^3*b^4*c^4*d^
6 + 35*a^4*b^3*c^3*d^7 - 21*a^5*b^2*c^2*d^8 + 7*a^6*b*c*d^9 - a^7*d^10)*log
(abs(b*x + a))/b^11 - 1/3*(b^10*c^10 + 5*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 -
 660*a^3*b^7*c^7*d^3 + 2730*a^4*b^6*c^6*d^4 - 5922*a^5*b^5*c^5*d^5 + 7770*a
^6*b^4*c^4*d^6 - 6420*a^7*b^3*c^3*d^7 + 3285*a^8*b^2*c^2*d^8 - 955*a^9*b*c*
d^9 + 121*a^10*d^10 + 135*(b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*
d^4 - 56*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 28*a^6
*b^4*c^2*d^8 - 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(b^10*c^9*d + 9*a*b
^9*c^8*d^2 - 108*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 882*a^4*b^6*c^5*d^
5 + 1134*a^5*b^5*c^4*d^6 - 924*a^6*b^4*c^3*d^7 + 468*a^7*b^3*c^2*d^8 - 135*
a^8*b^2*c*d^9 + 17*a^9*b*d^10)*x)/((b*x + a)^3*b^11) + 1/21*(3*b^24*d^10*x^
7 + 35*b^24*c*d^9*x^6 - 14*a*b^23*d^10*x^6 + 189*b^24*c^2*d^8*x^5 - 168*a*b
^23*c*d^9*x^5 + 42*a^2*b^22*d^10*x^5 + 630*b^24*c^3*d^7*x^4 - 945*a*b^23*c^
2*d^8*x^4 + 525*a^2*b^22*c*d^9*x^4 - 105*a^3*b^21*d^10*x^4 + 1470*b^24*c^4*
d^6*x^3 - 3360*a*b^23*c^3*d^7*x^3 + 3150*a^2*b^22*c^2*d^8*x^3 - 1400*a^3*b^
21*c*d^9*x^3 + 245*a^4*b^20*d^10*x^3 + 2646*b^24*c^5*d^5*x^2 - 8820*a*b^23*
c^4*d^6*x^2 + 12600*a^2*b^22*c^3*d^7*x^2 - 9450*a^3*b^21*c^2*d^8*x^2 + 3675
*a^4*b^20*c*d^9*x^2 - 588*a^5*b^19*d^10*x^2 + 4410*b^24*c^6*d^4*x - 21168*a
*b^23*c^5*d^5*x + 44100*a^2*b^22*c^4*d^6*x - 50400*a^3*b^21*c^3*d^7*x + 330
75*a^4*b^20*c^2*d^8*x - 11760*a^5*b^19*c*d^9*x + 1764*a^6*b^18*d^10*x)/b^28
```

**maple** [B] time = 0.02, size = 1141, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^4,x)
```

```
[Out] -1/3/b^11/(b*x+a)^3*a^10*d^10+5/b^11*d^10/(b*x+a)^2*a^9-5/b^2*d/(b*x+a)^2*c
^9-120/b^11*d^10*ln(b*x+a)*a^7+120/b^4*d^3*ln(b*x+a)*c^7-45/b^11*d^10/(b*x+
a)*a^8-45/b^3*d^2/(b*x+a)*c^8+9*d^8/b^4*x^5*c^2-5*d^10/b^7*x^4*a^3+30*d^7/b
^4*x^4*c^3+35/3*d^10/b^8*x^3*a^4+70*d^6/b^4*x^3*c^4-28*d^10/b^9*x^2*a^5+126
*d^5/b^4*x^2*c^5-2/3*d^10/b^5*x^6*a+5/3*d^9/b^4*x^6*c+2*d^10/b^6*x^5*a^2+21
0*d^4/b^4*c^6*x+84*d^10/b^10*a^6*x+1/7*d^10/b^4*x^7-1/3/b/(b*x+a)^3*c^10-70
/b^5/(b*x+a)^3*a^4*c^6*d^4+40/b^4/(b*x+a)^3*a^3*c^7*d^3-15/b^3/(b*x+a)^3*a^
2*c^8*d^2+10/3/b^2/(b*x+a)^3*a*c^9*d-45/b^10*d^9/(b*x+a)^2*a^8*c+180/b^9*d^
8/(b*x+a)^2*a^7*c^2-420/b^8*d^7/(b*x+a)^2*a^6*c^3+630/b^7*d^6/(b*x+a)^2*a^5
*c^4-630/b^6*d^5/(b*x+a)^2*a^4*c^5+420/b^5*d^4/(b*x+a)^2*a^3*c^6-180/b^4*d^
3/(b*x+a)^2*a^2*c^7+45/b^3*d^2/(b*x+a)^2*a*c^8+840/b^10*d^9*ln(b*x+a)*a^6*c
-2520/b^9*d^8*ln(b*x+a)*a^5*c^2+4200/b^8*d^7*ln(b*x+a)*a^4*c^3-4200/b^7*d^6
*ln(b*x+a)*a^3*c^4+2520/b^6*d^5*ln(b*x+a)*a^2*c^5-840/b^5*d^4*ln(b*x+a)*a*c
^6+360/b^10*d^9/(b*x+a)*a^7*c-1260/b^9*d^8/(b*x+a)*a^6*c^2+2520/b^8*d^7/(b*
x+a)*a^5*c^3-3150/b^7*d^6/(b*x+a)*a^4*c^4+2520/b^6*d^5/(b*x+a)*a^3*c^5-1260
/b^5*d^4/(b*x+a)*a^2*c^6+360/b^4*d^3/(b*x+a)*a*c^7-200/3*d^9/b^7*x^3*a^3*c-
8*d^9/b^5*x^5*a*c-560*d^9/b^9*a^5*c*x+1575*d^8/b^8*a^4*c^2*x-2400*d^7/b^7*a
^3*c^3*x+2100*d^6/b^6*a^2*c^4*x-1008*d^5/b^5*a*c^5*x-450*d^8/b^7*x^2*a^3*c^
2+600*d^7/b^6*x^2*a^2*c^3-420*d^6/b^5*x^2*a*c^4+175*d^9/b^8*x^2*a^4*c+150*d
^8/b^6*x^3*a^2*c^2-160*d^7/b^5*x^3*a*c^3+10/3/b^10/(b*x+a)^3*a^9*c*d^9-15/b
^9/(b*x+a)^3*a^8*c^2*d^8+40/b^8/(b*x+a)^3*a^7*c^3*d^7+25*d^9/b^6*x^4*a^2*c-
45*d^8/b^5*x^4*a*c^2-70/b^7/(b*x+a)^3*a^6*c^4*d^6+84/b^6/(b*x+a)^3*a^5*c^5*
d^5
```

**maxima [B]** time = 1.73, size = 891, normalized size = 3.45

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(b^10*c^10 + 5*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 660*a^3*b^7*c^7*d^3
+ 2730*a^4*b^6*c^6*d^4 - 5922*a^5*b^5*c^5*d^5 + 7770*a^6*b^4*c^4*d^6 - 6420
*a^7*b^3*c^3*d^7 + 3285*a^8*b^2*c^2*d^8 - 955*a^9*b*c*d^9 + 121*a^10*d^10 +
 135*(b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 56*a^3*b^7*c^5*
d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c^2*d^8 - 8*a^7*
b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(b^10*c^9*d + 9*a*b^9*c^8*d^2 - 108*a^2*
b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 882*a^4*b^6*c^5*d^5 + 1134*a^5*b^5*c^4*
d^6 - 924*a^6*b^4*c^3*d^7 + 468*a^7*b^3*c^2*d^8 - 135*a^8*b^2*c*d^9 + 17*a^
9*b*d^10)*x)/(b^14*x^3 + 3*a*b^13*x^2 + 3*a^2*b^12*x + a^3*b^11) + 1/21*(3*
b^6*d^10*x^7 + 7*(5*b^6*c*d^9 - 2*a*b^5*d^10)*x^6 + 21*(9*b^6*c^2*d^8 - 8*a
```

```
*b^5*c*d^9 + 2*a^2*b^4*d^10)*x^5 + 105*(6*b^6*c^3*d^7 - 9*a*b^5*c^2*d^8 + 5
*a^2*b^4*c*d^9 - a^3*b^3*d^10)*x^4 + 35*(42*b^6*c^4*d^6 - 96*a*b^5*c^3*d^7
+ 90*a^2*b^4*c^2*d^8 - 40*a^3*b^3*c*d^9 + 7*a^4*b^2*d^10)*x^3 + 21*(126*b^6
*c^5*d^5 - 420*a*b^5*c^4*d^6 + 600*a^2*b^4*c^3*d^7 - 450*a^3*b^3*c^2*d^8 +
175*a^4*b^2*c*d^9 - 28*a^5*b*d^10)*x^2 + 21*(210*b^6*c^6*d^4 - 1008*a*b^5*c
^5*d^5 + 2100*a^2*b^4*c^4*d^6 - 2400*a^3*b^3*c^3*d^7 + 1575*a^4*b^2*c^2*d^8
 - 560*a^5*b*c*d^9 + 84*a^6*d^10)*x)/b^10 + 120*(b^7*c^7*d^3 - 7*a*b^6*c^6*
d^4 + 21*a^2*b^5*c^5*d^5 - 35*a^3*b^4*c^4*d^6 + 35*a^4*b^3*c^3*d^7 - 21*a^5
*b^2*c^2*d^8 + 7*a^6*b*c*d^9 - a^7*d^10)*log(b*x + a)/b^11
```

**mupad [B]** time = 0.39, size = 2219, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^4,x)
```

```
[Out] x^3*((4*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b
^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*(
(4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^2))/(3*b) - (a^4*d^10)/(3*b^8) + (70*c^
4*d^6)/b^4 + (4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(3*b^3) - (2*a^2*((4
*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b
^4))/b^2) - x^6*((2*a*d^10)/(3*b^5) - (5*c*d^9)/(3*b^4)) - x^4*((a*((4*a*((
4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/
b + (a^3*d^10)/b^7 - (30*c^3*d^7)/b^4 - (3*a^2*((4*a*d^10)/b^5 - (10*c*d^9)
/b^4))/(2*b^2)) + x^5*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(5*b) - (6*a
^2*d^10)/(5*b^6) + (9*c^2*d^8)/b^4) - x*((4*a*((252*c^5*d^5)/b^4 - (4*a*((4
*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (4
5*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^
10)/b^5 - (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d^6)/b^4 + (
4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*a*d^10)/b^5
 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^2))/b + (a^
4*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^4 + (6*a^2*((4*a*((4*a*((4*a*d^10)/b
^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*
d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b
^2))/b^2 - (4*a^3*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)
/b^6 + (45*c^2*d^8)/b^4))/b^3))/b - (210*c^6*d^4)/b^4 + (6*a^2*((4*a*((4*a*
((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8
)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 -
 (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d^6)/b^4 + (4*a^3*((4
*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*a*d^10)/b^5 - (10*c*
d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^2))/b^2 - (4*a^3*((4
*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*
d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^
5 - (10*c*d^9)/b^4))/b^2))/b^3 + (a^4*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^
```

```
4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^4) + x^2*((126*c^5*d^5)/b^4
 - (2*a*((4*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^1
0)/b^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a
^2*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d
^6)/b^4 + (4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*
a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^
2))/b + (a^4*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(2*b^4) + (3*a^2*((4*a*((4*
a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^
4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 - (10
*c*d^9)/b^4))/b^2))/b^2 - (2*a^3*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b
 - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^3) - ((121*a^10*d^10 + b^10*c^10
 + 45*a^2*b^8*c^8*d^2 - 660*a^3*b^7*c^7*d^3 + 2730*a^4*b^6*c^6*d^4 - 5922*a
^5*b^5*c^5*d^5 + 7770*a^6*b^4*c^4*d^6 - 6420*a^7*b^3*c^3*d^7 + 3285*a^8*b^2
*c^2*d^8 + 5*a*b^9*c^9*d - 955*a^9*b*c*d^9)/(3*b) + x*(85*a^9*d^10 + 5*b^9*
c^9*d + 45*a*b^8*c^8*d^2 - 540*a^2*b^7*c^7*d^3 + 2100*a^3*b^6*c^6*d^4 - 441
0*a^4*b^5*c^5*d^5 + 5670*a^5*b^4*c^4*d^6 - 4620*a^6*b^3*c^3*d^7 + 2340*a^7*
b^2*c^2*d^8 - 675*a^8*b*c*d^9) + x^2*(45*a^8*b*d^10 + 45*b^9*c^8*d^2 - 360*
a*b^8*c^7*d^3 - 360*a^7*b^2*c*d^9 + 1260*a^2*b^7*c^6*d^4 - 2520*a^3*b^6*c^5
*d^5 + 3150*a^4*b^5*c^4*d^6 - 2520*a^5*b^4*c^3*d^7 + 1260*a^6*b^3*c^2*d^8))
/(a^3*b^10 + b^13*x^3 + 3*a^2*b^11*x + 3*a*b^12*x^2) + (d^10*x^7)/(7*b^4) -
 (log(a + b*x)*(120*a^7*d^10 - 120*b^7*c^7*d^3 + 840*a*b^6*c^6*d^4 - 2520*a
^2*b^5*c^5*d^5 + 4200*a^3*b^4*c^4*d^6 - 4200*a^4*b^3*c^3*d^7 + 2520*a^5*b^2
*c^2*d^8 - 840*a^6*b*c*d^9))/b^11
```

**sympy** [**B**] time = 32.53, size = 867, normalized size = 3.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**4,x)
```

```
[Out] x**6*(-2*a*d**10/(3*b**5) + 5*c*d**9/(3*b**4)) + x**5*(2*a**2*d**10/b**6 -
8*a*c*d**9/b**5 + 9*c**2*d**8/b**4) + x**4*(-5*a**3*d**10/b**7 + 25*a**2*c*
d**9/b**6 - 45*a*c**2*d**8/b**5 + 30*c**3*d**7/b**4) + x**3*(35*a**4*d**10/
(3*b**8) - 200*a**3*c*d**9/(3*b**7) + 150*a**2*c**2*d**8/b**6 - 160*a*c**3*
d**7/b**5 + 70*c**4*d**6/b**4) + x**2*(-28*a**5*d**10/b**9 + 175*a**4*c*d**
9/b**8 - 450*a**3*c**2*d**8/b**7 + 600*a**2*c**3*d**7/b**6 - 420*a*c**4*d**
6/b**5 + 126*c**5*d**5/b**4) + x*(84*a**6*d**10/b**10 - 560*a**5*c*d**9/b**
9 + 1575*a**4*c**2*d**8/b**8 - 2400*a**3*c**3*d**7/b**7 + 2100*a**2*c**4*d*
*6/b**6 - 1008*a*c**5*d**5/b**5 + 210*c**6*d**4/b**4) + (-121*a**10*d**10 +
 955*a**9*b*c*d**9 - 3285*a**8*b**2*c**2*d**8 + 6420*a**7*b**3*c**3*d**7 -
7770*a**6*b**4*c**4*d**6 + 5922*a**5*b**5*c**5*d**5 - 2730*a**4*b**6*c**6*d
**4 + 660*a**3*b**7*c**7*d**3 - 45*a**2*b**8*c**8*d**2 - 5*a*b**9*c**9*d -
b**10*c**10 + x**2*(-135*a**8*b**2*d**10 + 1080*a**7*b**3*c*d**9 - 3780*a**
6*b**4*c**2*d**8 + 7560*a**5*b**5*c**3*d**7 - 9450*a**4*b**6*c**4*d**6 + 75
```

```
60*a**3*b**7*c**5*d**5 - 3780*a**2*b**8*c**6*d**4 + 1080*a*b**9*c**7*d**3 -
 135*b**10*c**8*d**2) + x*(-255*a**9*b*d**10 + 2025*a**8*b**2*c*d**9 - 7020
*a**7*b**3*c**2*d**8 + 13860*a**6*b**4*c**3*d**7 - 17010*a**5*b**5*c**4*d**
6 + 13230*a**4*b**6*c**5*d**5 - 6300*a**3*b**7*c**6*d**4 + 1620*a**2*b**8*c
**7*d**3 - 135*a*b**9*c**8*d**2 - 15*b**10*c**9*d))/(3*a**3*b**11 + 9*a**2*
b**12*x + 9*a*b**13*x**2 + 3*b**14*x**3) + d**10*x**7/(7*b**4) - 120*d**3*(
a*d - b*c)**7*log(a + b*x)/b**11
```

## 3.1210 $\int \frac{(c+dx)^{10}}{(a+bx)^5}\,dx$

**Optimal**. Leaf size=262

$$\frac{2d^9(a+bx)^5(bc-ad)}{b^{11}}+\frac{45d^8(a+bx)^4(bc-ad)^2}{4b^{11}}+\frac{40d^7(a+bx)^3(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^2(bc-ad)^4}{b^{11}}+\frac{210d^4(bc-}{}$$

**Rubi** [A] time = 0.42, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2d^9(a+bx)^5(bc-ad)}{b^{11}}+\frac{45d^8(a+bx)^4(bc-ad)^2}{4b^{11}}+\frac{40d^7(a+bx)^3(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^2(bc-ad)^4}{b^{11}}+\frac{252d^5x(bc-ad)^5}{b^{10}}-\frac{120d^3(bc-ad)^7}{b^{11}(a+bx)}-\frac{45d^2(bc-ad)^8}{2b^{11}(a+bx)^2}+\frac{210d^4(bc-ad)^6\log(a+bx)}{b^{11}}-\frac{10d(bc-ad)^9}{3b^{11}(a+bx)^3}-\frac{(bc-ad)^{10}}{4b^{11}(a+bx)^4}+\frac{d^{10}(a+bx)^6}{6b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] (252*d^5*(b*c - a*d)^5*x)/b^10 - (b*c - a*d)^10/(4*b^11*(a + b*x)^4) - (10*
d*(b*c - a*d)^9)/(3*b^11*(a + b*x)^3) - (45*d^2*(b*c - a*d)^8)/(2*b^11*(a +
 b*x)^2) - (120*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)) + (105*d^6*(b*c - a*d)^
4*(a + b*x)^2)/b^11 + (40*d^7*(b*c - a*d)^3*(a + b*x)^3)/b^11 + (45*d^8*(b*
c - a*d)^2*(a + b*x)^4)/(4*b^11) + (2*d^9*(b*c - a*d)*(a + b*x)^5)/b^11 + (
d^10*(a + b*x)^6)/(6*b^11) + (210*d^4*(b*c - a*d)^6*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^5}\,dx &= \int \left(\frac{252d^5(bc-ad)^5}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^5}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^4}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^3}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^2}+\frac{2}{}\right.\\ &= \frac{252d^5(bc-ad)^5x}{b^{10}}-\frac{(bc-ad)^{10}}{4b^{11}(a+bx)^4}-\frac{10d(bc-ad)^9}{3b^{11}(a+bx)^3}-\frac{45d^2(bc-ad)^8}{2b^{11}(a+bx)^2}-\frac{120d^3(bc-ad)^7}{b^{11}(a+bx)}+\frac{105}{}\end{aligned}$$

**Mathematica** [A] time = 0.20, size = 359, normalized size = 1.37

$$\frac{15b^4d^8x^4\left(3a^2d^2-10abcd+9b^2c^2\right)+20b^3d^7x^3\left(-7a^3d^3+30a^2bcd^2-45ab^2c^2d+24b^3c^3\right)+30b^2d^6x^2\left(14a^4d^4-70a^3bcd^3+135a^2b^2c^2d^2-120ab^3c^3d+42b^4c^4\right)+12bd^5x\left(-126a^5d^5+700a^4bcd^4-1575a^3b^2c^2d^3+1800a^2b^3c^3d^2-1050ab^4c^4d+252b^5c^5\right)+12b^5d^9x^5(2bc-ad)+2520d^4(bc-ad)^6\log(a+bx)+\frac{1440d^3(ad-bc)^7}{a+bx}-\frac{270d^2(bc-ad)^8}{(a+bx)^2}+\frac{40d(ad-bc)^9}{(a+bx)^3}-\frac{3(bc-ad)^{10}}{(a+bx)^4}+2b^6d^{10}x^6}{12b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] (12*b*d^5*(252*b^5*c^5 - 1050*a*b^4*c^4*d + 1800*a^2*b^3*c^3*d^2 - 1575*a^3
*b^2*c^2*d^3 + 700*a^4*b*c*d^4 - 126*a^5*d^5)*x + 30*b^2*d^6*(42*b^4*c^4 -
120*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 - 70*a^3*b*c*d^3 + 14*a^4*d^4)*x^2 +
20*b^3*d^7*(24*b^3*c^3 - 45*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 7*a^3*d^3)*x^3 +
 15*b^4*d^8*(9*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 12*b^5*d^9*(2*b*c -
a*d)*x^5 + 2*b^6*d^10*x^6 - (3*(b*c - a*d)^10)/(a + b*x)^4 + (40*d*(-(b*c)
+ a*d)^9)/(a + b*x)^3 - (270*d^2*(b*c - a*d)^8)/(a + b*x)^2 + (1440*d^3*(-(
b*c) + a*d)^7)/(a + b*x) + 2520*d^4*(b*c - a*d)^6*Log[a + b*x])/(12*b^11)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^5, x]
```

**fricas [B]** time = 1.25, size = 1365, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^10*d^10*x^10 - 3*b^10*c^10 - 10*a*b^9*c^9*d - 45*a^2*b^8*c^8*d^2
- 360*a^3*b^7*c^7*d^3 + 5250*a^4*b^6*c^6*d^4 - 19404*a^5*b^5*c^5*d^5 + 3591
0*a^6*b^4*c^4*d^6 - 38280*a^7*b^3*c^3*d^7 + 23985*a^8*b^2*c^2*d^8 - 8250*a^
9*b*c*d^9 + 1207*a^10*d^10 + 4*(6*b^10*c*d^9 - a*b^9*d^10)*x^9 + 9*(15*b^10
*c^2*d^8 - 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 24*(20*b^10*c^3*d^7 - 15*a*b
^9*c^2*d^8 + 6*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 84*(15*b^10*c^4*d^6 - 20
*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 - 6*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 +
 504*(6*b^10*c^5*d^5 - 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 - 15*a^3*b^7*c
^2*d^8 + 6*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + (12096*a*b^9*c^5*d^5 - 42840
*a^2*b^8*c^4*d^6 + 66720*a^3*b^7*c^3*d^7 - 54765*a^4*b^6*c^2*d^8 + 23250*a^
5*b^5*c*d^9 - 4043*a^6*b^4*d^10)*x^4 - 4*(360*b^10*c^7*d^3 - 2520*a*b^9*c^6
*d^4 + 3024*a^2*b^8*c^5*d^5 + 5040*a^3*b^7*c^4*d^6 - 16320*a^4*b^6*c^3*d^7
+ 16965*a^5*b^5*c^2*d^8 - 8130*a^6*b^4*c*d^9 + 1523*a^7*b^3*d^10)*x^3 - 6*(
45*b^10*c^8*d^2 + 360*a*b^9*c^7*d^3 - 3780*a^2*b^8*c^6*d^4 + 10584*a^3*b^7*
c^5*d^5 - 13860*a^4*b^6*c^4*d^6 + 8880*a^5*b^5*c^3*d^7 - 1935*a^6*b^4*c^2*d
^8 - 570*a^7*b^3*c*d^9 + 263*a^8*b^2*d^10)*x^2 - 4*(10*b^10*c^9*d + 45*a*b^
```

```
9*c^8*d^2 + 360*a^2*b^8*c^7*d^3 - 4620*a^3*b^7*c^6*d^4 + 15624*a^4*b^6*c^5*
d^5 - 26460*a^5*b^5*c^4*d^6 + 25680*a^6*b^4*c^3*d^7 - 14535*a^7*b^3*c^2*d^8
 + 4470*a^8*b^2*c*d^9 - 577*a^9*b*d^10)*x + 2520*(a^4*b^6*c^6*d^4 - 6*a^5*b
^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 - 20*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 -
 6*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^6*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c
^4*d^6 - 20*a^3*b^7*c^3*d^7 + 15*a^4*b^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^
4*d^10)*x^4 + 4*(a*b^9*c^6*d^4 - 6*a^2*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 - 2
0*a^4*b^6*c^3*d^7 + 15*a^5*b^5*c^2*d^8 - 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^
3 + 6*(a^2*b^8*c^6*d^4 - 6*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 - 20*a^5*b^
5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 - 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 4*(a
^3*b^7*c^6*d^4 - 6*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 - 20*a^6*b^4*c^3*d^
7 + 15*a^7*b^3*c^2*d^8 - 6*a^8*b^2*c*d^9 + a^9*b*d^10)*x)*log(b*x + a))/(b^
15*x^4 + 4*a*b^14*x^3 + 6*a^2*b^13*x^2 + 4*a^3*b^12*x + a^4*b^11)
```

**giac [B]** time = 1.38, size = 1168, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*(2*d^10 + 24*(b^2*c*d^9 - a*b*d^10)/((b*x + a)*b) + 135*(b^4*c^2*d^8 -
 2*a*b^3*c*d^9 + a^2*b^2*d^10)/((b*x + a)^2*b^2) + 480*(b^6*c^3*d^7 - 3*a*b
^5*c^2*d^8 + 3*a^2*b^4*c*d^9 - a^3*b^3*d^10)/((b*x + a)^3*b^3) + 1260*(b^8*
c^4*d^6 - 4*a*b^7*c^3*d^7 + 6*a^2*b^6*c^2*d^8 - 4*a^3*b^5*c*d^9 + a^4*b^4*d
^10)/((b*x + a)^4*b^4) + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*
c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)/((b*x + a)^5
*b^5))*(b*x + a)^6/b^11 - 210*(b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4*c
^4*d^6 - 20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^10
)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^11 - 1/12*(3*b^67*c^10/(b*x + a)
^4 + 40*b^66*c^9*d/(b*x + a)^3 - 30*a*b^66*c^9*d/(b*x + a)^4 + 270*b^65*c^8
*d^2/(b*x + a)^2 - 360*a*b^65*c^8*d^2/(b*x + a)^3 + 135*a^2*b^65*c^8*d^2/(b
*x + a)^4 + 1440*b^64*c^7*d^3/(b*x + a) - 2160*a*b^64*c^7*d^3/(b*x + a)^2 +
 1440*a^2*b^64*c^7*d^3/(b*x + a)^3 - 360*a^3*b^64*c^7*d^3/(b*x + a)^4 - 100
80*a*b^63*c^6*d^4/(b*x + a) + 7560*a^2*b^63*c^6*d^4/(b*x + a)^2 - 3360*a^3*
b^63*c^6*d^4/(b*x + a)^3 + 630*a^4*b^63*c^6*d^4/(b*x + a)^4 + 30240*a^2*b^6
2*c^5*d^5/(b*x + a) - 15120*a^3*b^62*c^5*d^5/(b*x + a)^2 + 5040*a^4*b^62*c^
5*d^5/(b*x + a)^3 - 756*a^5*b^62*c^5*d^5/(b*x + a)^4 - 50400*a^3*b^61*c^4*d
^6/(b*x + a) + 18900*a^4*b^61*c^4*d^6/(b*x + a)^2 - 5040*a^5*b^61*c^4*d^6/(
b*x + a)^3 + 630*a^6*b^61*c^4*d^6/(b*x + a)^4 + 50400*a^4*b^60*c^3*d^7/(b*x
 + a) - 15120*a^5*b^60*c^3*d^7/(b*x + a)^2 + 3360*a^6*b^60*c^3*d^7/(b*x + a
)^3 - 360*a^7*b^60*c^3*d^7/(b*x + a)^4 - 30240*a^5*b^59*c^2*d^8/(b*x + a) +
 7560*a^6*b^59*c^2*d^8/(b*x + a)^2 - 1440*a^7*b^59*c^2*d^8/(b*x + a)^3 + 13
5*a^8*b^59*c^2*d^8/(b*x + a)^4 + 10080*a^6*b^58*c*d^9/(b*x + a) - 2160*a^7*
b^58*c*d^9/(b*x + a)^2 + 360*a^8*b^58*c*d^9/(b*x + a)^3 - 30*a^9*b^58*c*d^9
```

```
/(b*x + a)^4 - 1440*a^7*b^57*d^10/(b*x + a) + 270*a^8*b^57*d^10/(b*x + a)^2
 - 40*a^9*b^57*d^10/(b*x + a)^3 + 3*a^10*b^57*d^10/(b*x + a)^4)/b^68
```

**maple [B]** time = 0.02, size = 1172, normalized size = 4.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^5,x)
```

```
[Out] 10/3/b^11*d^10/(b*x+a)^3*a^9-10/3/b^2*d/(b*x+a)^3*c^9-45/2/b^11*d^10/(b*x+a
)^2*a^8-45/2/b^3*d^2/(b*x+a)^2*c^8+210/b^11*d^10*ln(b*x+a)*a^6+210/b^5*d^4*
ln(b*x+a)*c^6-1/4/b^11/(b*x+a)^4*a^10*d^10+120/b^11*d^10/(b*x+a)*a^7-120/b^
4*d^3/(b*x+a)*c^7-d^10/b^6*x^5*a+2*d^9/b^5*x^5*c+15/4*d^10/b^7*x^4*a^2+45/4
*d^8/b^5*x^4*c^2-35/3*d^10/b^8*x^3*a^3+40*d^7/b^5*x^3*c^3+35*d^10/b^9*x^2*a
^4+105*d^6/b^5*x^2*c^4-126*d^10/b^10*a^5*x+252*d^5/b^5*c^5*x+1/6*d^10/b^5*x
^6-1/4/b/(b*x+a)^4*c^10-25/2*d^9/b^6*x^4*a*c+50*d^9/b^7*x^3*a^2*c-1260/b^10
*d^9*ln(b*x+a)*a^5*c+3150/b^9*d^8*ln(b*x+a)*a^4*c^2-4200/b^8*d^7*ln(b*x+a)*
a^3*c^3+3150/b^7*d^6*ln(b*x+a)*a^2*c^4-1260/b^6*d^5*ln(b*x+a)*a*c^5+5/2/b^1
0/(b*x+a)^4*a^9*c*d^9-45/4/b^9/(b*x+a)^4*a^8*c^2*d^8+30/b^8/(b*x+a)^4*a^7*c
^3*d^7-105/2/b^7/(b*x+a)^4*a^6*c^4*d^6+63/b^6/(b*x+a)^4*a^5*c^5*d^5-105/2/b
^5/(b*x+a)^4*a^4*c^6*d^4+30/b^4/(b*x+a)^4*a^3*c^7*d^3-45/4/b^3/(b*x+a)^4*a^
2*c^8*d^2+5/2/b^2/(b*x+a)^4*a*c^9*d-840/b^10*d^9/(b*x+a)*a^6*c+2520/b^9*d^8
/(b*x+a)*a^5*c^2-4200/b^8*d^7/(b*x+a)*a^4*c^3+4200/b^7*d^6/(b*x+a)*a^3*c^4-
2520/b^6*d^5/(b*x+a)*a^2*c^5+840/b^5*d^4/(b*x+a)*a*c^6+700*d^9/b^9*a^4*c*x-
1575*d^8/b^8*a^3*c^2*x+1800*d^7/b^7*a^2*c^3*x-1050*d^6/b^6*a*c^4*x-30/b^10*
d^9/(b*x+a)^3*a^8*c+120/b^9*d^8/(b*x+a)^3*a^7*c^2-280/b^8*d^7/(b*x+a)^3*a^6
*c^3+420/b^7*d^6/(b*x+a)^3*a^5*c^4-420/b^6*d^5/(b*x+a)^3*a^4*c^5+280/b^5*d^
4/(b*x+a)^3*a^3*c^6-120/b^4*d^3/(b*x+a)^3*a^2*c^7+30/b^3*d^2/(b*x+a)^3*a*c^
8+180/b^10*d^9/(b*x+a)^2*a^7*c-630/b^9*d^8/(b*x+a)^2*a^6*c^2+1260/b^8*d^7/(
b*x+a)^2*a^5*c^3-1575/b^7*d^6/(b*x+a)^2*a^4*c^4+1260/b^6*d^5/(b*x+a)^2*a^3*
c^5-630/b^5*d^4/(b*x+a)^2*a^2*c^6+180/b^4*d^3/(b*x+a)^2*a*c^7-75*d^8/b^6*x^
3*a*c^2-175*d^9/b^8*x^2*a^3*c+675/2*d^8/b^7*x^2*a^2*c^2-300*d^7/b^6*x^2*a*c
^3
```

**maxima [B]** time = 1.93, size = 903, normalized size = 3.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^10*c^10 + 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*
d^3 - 5250*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 - 35910*a^6*b^4*c^4*d^6
+ 38280*a^7*b^3*c^3*d^7 - 23985*a^8*b^2*c^2*d^8 + 8250*a^9*b*c*d^9 - 1207*a
^10*d^10 + 1440*(b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 - 35*a
```

```
^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9
- a^7*b^3*d^10)*x^3 + 270*(b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 - 84*a^2*b^8*c^6*
d^4 + 280*a^3*b^7*c^5*d^5 - 490*a^4*b^6*c^4*d^6 + 504*a^5*b^5*c^3*d^7 - 308
*a^6*b^4*c^2*d^8 + 104*a^7*b^3*c*d^9 - 15*a^8*b^2*d^10)*x^2 + 20*(2*b^10*c^
9*d + 9*a*b^9*c^8*d^2 + 72*a^2*b^8*c^7*d^3 - 924*a^3*b^7*c^6*d^4 + 3276*a^4
*b^6*c^5*d^5 - 5922*a^5*b^5*c^4*d^6 + 6216*a^6*b^4*c^3*d^7 - 3852*a^7*b^3*c
^2*d^8 + 1314*a^8*b^2*c*d^9 - 191*a^9*b*d^10)*x)/(b^15*x^4 + 4*a*b^14*x^3 +
 6*a^2*b^13*x^2 + 4*a^3*b^12*x + a^4*b^11) + 1/12*(2*b^5*d^10*x^6 + 12*(2*b
^5*c*d^9 - a*b^4*d^10)*x^5 + 15*(9*b^5*c^2*d^8 - 10*a*b^4*c*d^9 + 3*a^2*b^3
*d^10)*x^4 + 20*(24*b^5*c^3*d^7 - 45*a*b^4*c^2*d^8 + 30*a^2*b^3*c*d^9 - 7*a
^3*b^2*d^10)*x^3 + 30*(42*b^5*c^4*d^6 - 120*a*b^4*c^3*d^7 + 135*a^2*b^3*c^2
*d^8 - 70*a^3*b^2*c*d^9 + 14*a^4*b*d^10)*x^2 + 12*(252*b^5*c^5*d^5 - 1050*a
*b^4*c^4*d^6 + 1800*a^2*b^3*c^3*d^7 - 1575*a^3*b^2*c^2*d^8 + 700*a^4*b*c*d^
9 - 126*a^5*d^10)*x)/b^10 + 210*(b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4
*c^4*d^6 - 20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^
10)*log(b*x + a)/b^11
```

**mupad [B]** time = 0.38, size = 1494, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^5,x)
```

```
[Out] x^2*((5*a*((5*a*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/
b^7 + (45*c^2*d^8)/b^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a^
2*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^2))/(2*b) - (5*a^4*d^10)/(2*b^9) + (
105*c^4*d^6)/b^5 + (5*a^3*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^3 - (5*a^2*(
(5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8
)/b^5))/b^2) - x^5*((a*d^10)/b^6 - (2*c*d^9)/b^5) - x^3*((5*a*((5*a*((5*a*d
^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/(3*b
) + (10*a^3*d^10)/(3*b^8) - (40*c^3*d^7)/b^5 - (10*a^2*((5*a*d^10)/b^6 - (1
0*c*d^9)/b^5))/(3*b^2)) + x^4*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/(4*b
) - (5*a^2*d^10)/(2*b^7) + (45*c^2*d^8)/(4*b^5)) - x*((5*a*((5*a*((5*a*((5*
a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b
^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a^2*((5*a*d^10)/b^6 -
(10*c*d^9)/b^5))/b^2))/b - (5*a^4*d^10)/b^9 + (210*c^4*d^6)/b^5 + (10*a^3*(
(5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^3 - (10*a^2*((5*a*((5*a*d^10)/b^6 - (10
*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/b^2))/b + (a^5*d^1
0)/b^10 - (252*c^5*d^5)/b^5 - (5*a^4*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^4
 - (10*a^2*((5*a*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)
/b^7 + (45*c^2*d^8)/b^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a
^2*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^2))/b^2 + (10*a^3*((5*a*((5*a*d^10)
/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/b^3) - (
(3*b^10*c^10 - 1207*a^10*d^10 + 45*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*d^3 -
```

```
5250*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 - 35910*a^6*b^4*c^4*d^6 + 3828
0*a^7*b^3*c^3*d^7 - 23985*a^8*b^2*c^2*d^8 + 10*a*b^9*c^9*d + 8250*a^9*b*c*d
^9)/(12*b) + x*((10*b^9*c^9*d)/3 - (955*a^9*d^10)/3 + 15*a*b^8*c^8*d^2 + 12
0*a^2*b^7*c^7*d^3 - 1540*a^3*b^6*c^6*d^4 + 5460*a^4*b^5*c^5*d^5 - 9870*a^5*
b^4*c^4*d^6 + 10360*a^6*b^3*c^3*d^7 - 6420*a^7*b^2*c^2*d^8 + 2190*a^8*b*c*d
^9) - x^3*(120*a^7*b^2*d^10 - 120*b^9*c^7*d^3 + 840*a*b^8*c^6*d^4 - 840*a^6
*b^3*c*d^9 - 2520*a^2*b^7*c^5*d^5 + 4200*a^3*b^6*c^4*d^6 - 4200*a^4*b^5*c^3
*d^7 + 2520*a^5*b^4*c^2*d^8) + x^2*((45*b^9*c^8*d^2)/2 - (675*a^8*b*d^10)/2
 + 180*a*b^8*c^7*d^3 + 2340*a^7*b^2*c*d^9 - 1890*a^2*b^7*c^6*d^4 + 6300*a^3
*b^6*c^5*d^5 - 11025*a^4*b^5*c^4*d^6 + 11340*a^5*b^4*c^3*d^7 - 6930*a^6*b^3
*c^2*d^8))/(a^4*b^10 + b^14*x^4 + 4*a^3*b^11*x + 4*a*b^13*x^3 + 6*a^2*b^12*
x^2) + (log(a + b*x)*(210*a^6*d^10 + 210*b^6*c^6*d^4 - 1260*a*b^5*c^5*d^5 +
 3150*a^2*b^4*c^4*d^6 - 4200*a^3*b^3*c^3*d^7 + 3150*a^4*b^2*c^2*d^8 - 1260*
a^5*b*c*d^9))/b^11 + (d^10*x^6)/(6*b^5)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**5,x)
```

```
[Out] Timed out
```

## 3.1211 $\int \frac{(c+dx)^{10}}{(a+bx)^6} \, dx$

**Optimal**. Leaf size=260

$$\frac{5d^9(a+bx)^4(bc-ad)}{2b^{11}}+\frac{15d^8(a+bx)^3(bc-ad)^2}{b^{11}}+\frac{60d^7(a+bx)^2(bc-ad)^3}{b^{11}}+\frac{252d^5(bc-ad)^5\log(a+bx)}{b^{11}}-\frac{210d^4(b}{b^{11}(a}$$

**Rubi [A]** time = 0.42, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^4(bc-ad)}{2b^{11}}+\frac{15d^8(a+bx)^3(bc-ad)^2}{b^{11}}+\frac{60d^7(a+bx)^2(bc-ad)^3}{b^{11}}+\frac{210d^6x(bc-ad)^4}{b^{10}}-\frac{210d^4(bc-ad)^6}{b^{11}(a+bx)}-\frac{60d^3(bc-ad)^7}{b^{11}(a+bx)^2}-\frac{15d^2(bc-ad)^8}{b^{11}(a+bx)^3}+\frac{252d^5(bc-ad)^5\log(a+bx)}{b^{11}}-\frac{5d(bc-ad)^9}{2b^{11}(a+bx)^4}-\frac{(bc-ad)^{10}}{5b^{11}(a+bx)^5}+\frac{d^{10}(a+bx)^5}{5b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] (210*d^6*(b*c - a*d)^4*x)/b^10 - (b*c - a*d)^10/(5*b^11*(a + b*x)^5) - (5*d
*(b*c - a*d)^9)/(2*b^11*(a + b*x)^4) - (15*d^2*(b*c - a*d)^8)/(b^11*(a + b*
x)^3) - (60*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^2) - (210*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)) + (60*d^7*(b*c - a*d)^3*(a + b*x)^2)/b^11 + (15*d^8*(b*c
- a*d)^2*(a + b*x)^3)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^4)/(2*b^11) + (d^
10*(a + b*x)^5)/(5*b^11) + (252*d^5*(b*c - a*d)^5*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^6}\,dx &= \int \left(\frac{210d^6(bc-ad)^4}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^6}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^5}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^4}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^3}+\frac{2}{}\right.\\ &= \frac{210d^6(bc-ad)^4x}{b^{10}}-\frac{(bc-ad)^{10}}{5b^{11}(a+bx)^5}-\frac{5d(bc-ad)^9}{2b^{11}(a+bx)^4}-\frac{15d^2(bc-ad)^8}{b^{11}(a+bx)^3}-\frac{60d^3(bc-ad)^7}{b^{11}(a+bx)^2}-\frac{210d}{b^{11}}\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 305, normalized size = 1.17

$$\frac{10b^3d^8x^3\left(7a^2d^2-20abcd+15b^2c^2\right)+10b^2d^7x^2\left(-28a^3d^3+105a^2bcd^2-135ab^2c^2d+60b^3c^3\right)+10bd^6x\left(126a^4d^4-560a^3bcd^3+945a^2b^2c^2d^2-720ab^3c^3d+210b^4c^4\right)+5b^4d^9x^4(5bc-3ad)+2520d^5(bc-ad)^5\log(a+bx)-\frac{2100d^4(bc-ad)^6}{a+bx}+\frac{600d^3(ad-bc)^7}{(a+bx)^2}-\frac{150d^2(bc-ad)^8}{(a+bx)^3}+\frac{25d(ad-bc)^9}{(a+bx)^4}-\frac{2(bc-ad)^{10}}{(a+bx)^5}+2b^5d^{10}x^5}{10b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] (10*b*d^6*(210*b^4*c^4 - 720*a*b^3*c^3*d + 945*a^2*b^2*c^2*d^2 - 560*a^3*b*
c*d^3 + 126*a^4*d^4)*x + 10*b^2*d^7*(60*b^3*c^3 - 135*a*b^2*c^2*d + 105*a^2
*b*c*d^2 - 28*a^3*d^3)*x^2 + 10*b^3*d^8*(15*b^2*c^2 - 20*a*b*c*d + 7*a^2*d^
2)*x^3 + 5*b^4*d^9*(5*b*c - 3*a*d)*x^4 + 2*b^5*d^10*x^5 - (2*(b*c - a*d)^10
)/(a + b*x)^5 + (25*d*(-(b*c) + a*d)^9)/(a + b*x)^4 - (150*d^2*(b*c - a*d)^
8)/(a + b*x)^3 + (600*d^3*(-(b*c) + a*d)^7)/(a + b*x)^2 - (2100*d^4*(b*c -
a*d)^6)/(a + b*x) + 2520*d^5*(b*c - a*d)^5*Log[a + b*x])/(10*b^11)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^6, x]
```

**fricas [B]** time = 1.27, size = 1395, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^10*d^10*x^10 - 2*b^10*c^10 - 5*a*b^9*c^9*d - 15*a^2*b^8*c^8*d^2 -
 60*a^3*b^7*c^7*d^3 - 420*a^4*b^6*c^6*d^4 + 5754*a^5*b^5*c^5*d^5 - 18270*a^
6*b^4*c^4*d^6 + 27540*a^7*b^3*c^3*d^7 - 22290*a^8*b^2*c^2*d^8 + 9395*a^9*b*
c*d^9 - 1627*a^10*d^10 + 5*(5*b^10*c*d^9 - a*b^9*d^10)*x^9 + 15*(10*b^10*c^
2*d^8 - 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 60*(10*b^10*c^3*d^7 - 10*a*b^9*
c^2*d^8 + 5*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 420*(5*b^10*c^4*d^6 - 10*a*
b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 - 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + (1
0500*a*b^9*c^4*d^6 - 30000*a^2*b^8*c^3*d^7 + 35250*a^3*b^7*c^2*d^8 - 19375*
a^4*b^6*c*d^9 + 4127*a^5*b^5*d^10)*x^5 - 5*(420*b^10*c^6*d^4 - 2520*a*b^9*c
^5*d^5 + 2100*a^2*b^8*c^4*d^6 + 4800*a^3*b^7*c^3*d^7 - 10050*a^4*b^6*c^2*d^
8 + 6775*a^5*b^5*c*d^9 - 1607*a^6*b^4*d^10)*x^4 - 10*(60*b^10*c^7*d^3 + 420
*a*b^9*c^6*d^4 - 3780*a^2*b^8*c^5*d^5 + 8400*a^3*b^7*c^4*d^6 - 7800*a^4*b^6
*c^3*d^7 + 2550*a^5*b^5*c^2*d^8 + 475*a^6*b^4*c*d^9 - 347*a^7*b^3*d^10)*x^3
 - 10*(15*b^10*c^8*d^2 + 60*a*b^9*c^7*d^3 + 420*a^2*b^8*c^6*d^4 - 4620*a^3*
b^7*c^5*d^5 + 12600*a^4*b^6*c^4*d^6 - 16200*a^5*b^5*c^3*d^7 + 10950*a^6*b^4
*c^2*d^8 - 3725*a^7*b^3*c*d^9 + 493*a^8*b^2*d^10)*x^2 - 5*(5*b^10*c^9*d + 1
5*a*b^9*c^8*d^2 + 60*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 5250*a^4*b^6*c
```

```
^5*d^5 + 15750*a^5*b^5*c^4*d^6 - 22500*a^6*b^4*c^3*d^7 + 17250*a^7*b^3*c^2*
d^8 - 6875*a^8*b^2*c*d^9 + 1123*a^9*b*d^10)*x + 2520*(a^5*b^5*c^5*d^5 - 5*a
^6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 - 10*a^8*b^2*c^2*d^8 + 5*a^9*b*c*d^9 -
a^10*d^10 + (b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 - 10*a^3*b
^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 5*(a*b^9*c^5*d^5 - 5*a^2
*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 - 10*a^4*b^6*c^2*d^8 + 5*a^5*b^5*c*d^9 -
a^6*b^4*d^10)*x^4 + 10*(a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^
3*d^7 - 10*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 10*(a^3*
b^7*c^5*d^5 - 5*a^4*b^6*c^4*d^6 + 10*a^5*b^5*c^3*d^7 - 10*a^6*b^4*c^2*d^8 +
 5*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 5*(a^4*b^6*c^5*d^5 - 5*a^5*b^5*c^4*d
^6 + 10*a^6*b^4*c^3*d^7 - 10*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 - a^9*b*d^10
)*x)*log(b*x + a))/(b^16*x^5 + 5*a*b^15*x^4 + 10*a^2*b^14*x^3 + 10*a^3*b^13
*x^2 + 5*a^4*b^12*x + a^5*b^11)
```

**giac [B]** time = 1.34, size = 883, normalized size = 3.40

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 252*(b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d^
8 + 5*a^4*b*c*d^9 - a^5*d^10)*log(abs(b*x + a))/b^11 - 1/10*(2*b^10*c^10 +
5*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d
^4 - 5754*a^5*b^5*c^5*d^5 + 18270*a^6*b^4*c^4*d^6 - 27540*a^7*b^3*c^3*d^7 +
 22290*a^8*b^2*c^2*d^8 - 9395*a^9*b*c*d^9 + 1627*a^10*d^10 + 2100*(b^10*c^6
*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 20*a^3*b^7*c^3*d^7 + 15*a^4*b
^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 600*(b^10*c^7*d^3 + 7*a*
b^9*c^6*d^4 - 63*a^2*b^8*c^5*d^5 + 175*a^3*b^7*c^4*d^6 - 245*a^4*b^6*c^3*d^
7 + 189*a^5*b^5*c^2*d^8 - 77*a^6*b^4*c*d^9 + 13*a^7*b^3*d^10)*x^3 + 150*(b^
10*c^8*d^2 + 4*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 308*a^3*b^7*c^5*d^5 + 9
10*a^4*b^6*c^4*d^6 - 1316*a^5*b^5*c^3*d^7 + 1036*a^6*b^4*c^2*d^8 - 428*a^7*
b^3*c*d^9 + 73*a^8*b^2*d^10)*x^2 + 25*(b^10*c^9*d + 3*a*b^9*c^8*d^2 + 12*a^
2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 - 1050*a^4*b^6*c^5*d^5 + 3234*a^5*b^5*c^
4*d^6 - 4788*a^6*b^4*c^3*d^7 + 3828*a^7*b^3*c^2*d^8 - 1599*a^8*b^2*c*d^9 +
275*a^9*b*d^10)*x)/((b*x + a)^5*b^11) + 1/10*(2*b^24*d^10*x^5 + 25*b^24*c*d
^9*x^4 - 15*a*b^23*d^10*x^4 + 150*b^24*c^2*d^8*x^3 - 200*a*b^23*c*d^9*x^3 +
 70*a^2*b^22*d^10*x^3 + 600*b^24*c^3*d^7*x^2 - 1350*a*b^23*c^2*d^8*x^2 + 10
50*a^2*b^22*c*d^9*x^2 - 280*a^3*b^21*d^10*x^2 + 2100*b^24*c^4*d^6*x - 7200*
a*b^23*c^3*d^7*x + 9450*a^2*b^22*c^2*d^8*x - 5600*a^3*b^21*c*d^9*x + 1260*a
^4*b^20*d^10*x)/b^30
```

**maple [B]** time = 0.02, size = 1199, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^6,x)
```

```
[Out] -252/b^11*d^10*ln(b*x+a)*a^5+252/b^6*d^5*ln(b*x+a)*c^5+5/2/b^11*d^10/(b*x+a
)^4*a^9-5/2/b^2*d/(b*x+a)^4*c^9-210/b^11*d^10/(b*x+a)*a^6-210/b^5*d^4/(b*x+
a)*c^6-3/2*d^10/b^7*x^4*a+5/2*d^9/b^6*x^4*c+7*d^10/b^8*x^3*a^2+15*d^8/b^6*x
^3*c^2-28*d^10/b^9*x^2*a^3+60*d^7/b^6*x^2*c^3+126*d^10/b^10*a^4*x+210*d^6/b
^6*c^4*x-15/b^11*d^10/(b*x+a)^3*a^8-15/b^3*d^2/(b*x+a)^3*c^8-1/5/b^11/(b*x+
a)^5*a^10*d^10+60/b^11*d^10/(b*x+a)^2*a^7-60/b^4*d^3/(b*x+a)^2*c^7+1260/b^1
0*d^9*ln(b*x+a)*a^4*c-2520/b^9*d^8*ln(b*x+a)*a^3*c^2-1260/b^6*d^5/(b*x+a)^2
*a^2*c^5+420/b^5*d^4/(b*x+a)^2*a*c^6-135*d^8/b^7*x^2*a*c^2-560*d^9/b^9*a^3*
c*x+945*d^8/b^8*a^2*c^2*x-720*d^7/b^7*a*c^3*x+120/b^10*d^9/(b*x+a)^3*a^7*c-
420/b^9*d^8/(b*x+a)^3*a^6*c^2+840/b^8*d^7/(b*x+a)^3*a^5*c^3-1050/b^7*d^6/(b
*x+a)^3*a^4*c^4+840/b^6*d^5/(b*x+a)^3*a^3*c^5-420/b^5*d^4/(b*x+a)^3*a^2*c^6
+120/b^4*d^3/(b*x+a)^3*a*c^7+2/b^10/(b*x+a)^5*a^9*c*d^9-9/b^9/(b*x+a)^5*a^8
*c^2*d^8+24/b^8/(b*x+a)^5*a^7*c^3*d^7-42/b^7/(b*x+a)^5*a^6*c^4*d^6+252/5/b^
6/(b*x+a)^5*a^5*c^5*d^5-42/b^5/(b*x+a)^5*a^4*c^6*d^4+24/b^4/(b*x+a)^5*a^3*c
^7*d^3-9/b^3/(b*x+a)^5*a^2*c^8*d^2+2520/b^8*d^7*ln(b*x+a)*a^2*c^3-1260/b^7*
d^6*ln(b*x+a)*a*c^4-20*d^9/b^7*x^3*a*c+105*d^9/b^8*x^2*a^2*c-45/2/b^10*d^9/
(b*x+a)^4*a^8*c+90/b^9*d^8/(b*x+a)^4*a^7*c^2-210/b^8*d^7/(b*x+a)^4*a^6*c^3+
315/b^7*d^6/(b*x+a)^4*a^5*c^4-315/b^6*d^5/(b*x+a)^4*a^4*c^5+210/b^5*d^4/(b*
x+a)^4*a^3*c^6-90/b^4*d^3/(b*x+a)^4*a^2*c^7+45/2/b^3*d^2/(b*x+a)^4*a*c^8+12
60/b^10*d^9/(b*x+a)*a^5*c-3150/b^9*d^8/(b*x+a)*a^4*c^2+4200/b^8*d^7/(b*x+a)
*a^3*c^3-3150/b^7*d^6/(b*x+a)*a^2*c^4+1260/b^6*d^5/(b*x+a)*a*c^5+2/b^2/(b*x
+a)^5*a*c^9*d-420/b^10*d^9/(b*x+a)^2*a^6*c+1260/b^9*d^8/(b*x+a)^2*a^5*c^2-2
100/b^8*d^7/(b*x+a)^2*a^4*c^3+2100/b^7*d^6/(b*x+a)^2*a^3*c^4+1/5*d^10/b^6*x
^5-1/5/b/(b*x+a)^5*c^10
```

**maxima [B]** time = 2.25, size = 912, normalized size = 3.51

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/10*(2*b^10*c^10 + 5*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^
3 + 420*a^4*b^6*c^6*d^4 - 5754*a^5*b^5*c^5*d^5 + 18270*a^6*b^4*c^4*d^6 - 27
540*a^7*b^3*c^3*d^7 + 22290*a^8*b^2*c^2*d^8 - 9395*a^9*b*c*d^9 + 1627*a^10*
d^10 + 2100*(b^10*c^6*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 20*a^3*b
^7*c^3*d^7 + 15*a^4*b^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 600
*(b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 - 63*a^2*b^8*c^5*d^5 + 175*a^3*b^7*c^4*d^6
 - 245*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 - 77*a^6*b^4*c*d^9 + 13*a^7*b^
3*d^10)*x^3 + 150*(b^10*c^8*d^2 + 4*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 30
8*a^3*b^7*c^5*d^5 + 910*a^4*b^6*c^4*d^6 - 1316*a^5*b^5*c^3*d^7 + 1036*a^6*b
```

```
^4*c^2*d^8 - 428*a^7*b^3*c*d^9 + 73*a^8*b^2*d^10)*x^2 + 25*(b^10*c^9*d + 3*
a*b^9*c^8*d^2 + 12*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 - 1050*a^4*b^6*c^5*
d^5 + 3234*a^5*b^5*c^4*d^6 - 4788*a^6*b^4*c^3*d^7 + 3828*a^7*b^3*c^2*d^8 -
1599*a^8*b^2*c*d^9 + 275*a^9*b*d^10)*x)/(b^16*x^5 + 5*a*b^15*x^4 + 10*a^2*b
^14*x^3 + 10*a^3*b^13*x^2 + 5*a^4*b^12*x + a^5*b^11) + 1/10*(2*b^4*d^10*x^5
 + 5*(5*b^4*c*d^9 - 3*a*b^3*d^10)*x^4 + 10*(15*b^4*c^2*d^8 - 20*a*b^3*c*d^9
 + 7*a^2*b^2*d^10)*x^3 + 10*(60*b^4*c^3*d^7 - 135*a*b^3*c^2*d^8 + 105*a^2*b
^2*c*d^9 - 28*a^3*b*d^10)*x^2 + 10*(210*b^4*c^4*d^6 - 720*a*b^3*c^3*d^7 + 9
45*a^2*b^2*c^2*d^8 - 560*a^3*b*c*d^9 + 126*a^4*d^10)*x)/b^10 + 252*(b^5*c^5
*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d^8 + 5*a^4*b*
c*d^9 - a^5*d^10)*log(b*x + a)/b^11
```

**mupad [B]** time = 0.40, size = 1141, normalized size = 4.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^6,x)
```

```
[Out] x^3*((2*a*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (5*a^2*d^10)/b^8 + (15*c^2
*d^8)/b^6) - x^2*((3*a*((6*a*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (15*a^2
*d^10)/b^8 + (45*c^2*d^8)/b^6))/b + (10*a^3*d^10)/b^9 - (60*c^3*d^7)/b^6 -
(15*a^2*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/(2*b^2)) - x^4*((3*a*d^10)/(2*b^
7) - (5*c*d^9)/(2*b^6)) - (x^4*(210*a^6*b^3*d^10 + 210*b^9*c^6*d^4 - 1260*a
*b^8*c^5*d^5 - 1260*a^5*b^4*c*d^9 + 3150*a^2*b^7*c^4*d^6 - 4200*a^3*b^6*c^3
*d^7 + 3150*a^4*b^5*c^2*d^8) + (1627*a^10*d^10 + 2*b^10*c^10 + 15*a^2*b^8*c
^8*d^2 + 60*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 - 5754*a^5*b^5*c^5*d^5 +
18270*a^6*b^4*c^4*d^6 - 27540*a^7*b^3*c^3*d^7 + 22290*a^8*b^2*c^2*d^8 + 5*a
*b^9*c^9*d - 9395*a^9*b*c*d^9)/(10*b) + x*((1375*a^9*d^10)/2 + (5*b^9*c^9*d
)/2 + (15*a*b^8*c^8*d^2)/2 + 30*a^2*b^7*c^7*d^3 + 210*a^3*b^6*c^6*d^4 - 262
5*a^4*b^5*c^5*d^5 + 8085*a^5*b^4*c^4*d^6 - 11970*a^6*b^3*c^3*d^7 + 9570*a^7
*b^2*c^2*d^8 - (7995*a^8*b*c*d^9)/2) + x^3*(780*a^7*b^2*d^10 + 60*b^9*c^7*d
^3 + 420*a*b^8*c^6*d^4 - 4620*a^6*b^3*c*d^9 - 3780*a^2*b^7*c^5*d^5 + 10500*
a^3*b^6*c^4*d^6 - 14700*a^4*b^5*c^3*d^7 + 11340*a^5*b^4*c^2*d^8) + x^2*(109
5*a^8*b*d^10 + 15*b^9*c^8*d^2 + 60*a*b^8*c^7*d^3 - 6420*a^7*b^2*c*d^9 + 420
*a^2*b^7*c^6*d^4 - 4620*a^3*b^6*c^5*d^5 + 13650*a^4*b^5*c^4*d^6 - 19740*a^5
*b^4*c^3*d^7 + 15540*a^6*b^3*c^2*d^8))/(a^5*b^10 + b^15*x^5 + 5*a^4*b^11*x
+ 5*a*b^14*x^4 + 10*a^3*b^12*x^2 + 10*a^2*b^13*x^3) + x*((6*a*((6*a*((6*a*(
(6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (15*a^2*d^10)/b^8 + (45*c^2*d^8)/b^6)
)/b + (20*a^3*d^10)/b^9 - (120*c^3*d^7)/b^6 - (15*a^2*((6*a*d^10)/b^7 - (10
*c*d^9)/b^6))/b^2))/b - (15*a^4*d^10)/b^10 + (210*c^4*d^6)/b^6 + (20*a^3*((
6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b^3 - (15*a^2*((6*a*((6*a*d^10)/b^7 - (10*
c*d^9)/b^6))/b - (15*a^2*d^10)/b^8 + (45*c^2*d^8)/b^6))/b^2) + (d^10*x^5)/(
5*b^6) - (log(a + b*x)*(252*a^5*d^10 - 252*b^5*c^5*d^5 + 1260*a*b^4*c^4*d^6
 - 2520*a^2*b^3*c^3*d^7 + 2520*a^3*b^2*c^2*d^8 - 1260*a^4*b*c*d^9))/b^11
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**6,x)
```

```
[Out] Timed out
```

# 3.1212 $\int \frac{(c+dx)^{10}}{(a+bx)^7} \, dx$

**Optimal**. Leaf size=262

$$\frac{10d^9(a+bx)^3(bc-ad)}{3b^{11}}+\frac{45d^8(a+bx)^2(bc-ad)^2}{2b^{11}}+\frac{210d^6(bc-ad)^4\log(a+bx)}{b^{11}}-\frac{252d^5(bc-ad)^5}{b^{11}(a+bx)}-\frac{105d^4(bc-ad)}{b^{11}(a+bx)^2}$$

**Rubi** [A] time = 0.39, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{10d^9(a+bx)^3(bc-ad)}{3b^{11}}+\frac{45d^8(a+bx)^2(bc-ad)^2}{2b^{11}}+\frac{120d^7x(bc-ad)^3}{b^{10}}-\frac{252d^5(bc-ad)^5}{b^{11}(a+bx)}-\frac{105d^4(bc-ad)^6}{b^{11}(a+bx)^2}-\frac{40d^3(bc-ad)^7}{b^{11}(a+bx)^3}-\frac{45d^2(bc-ad)^8}{4b^{11}(a+bx)^4}+\frac{210d^6(bc-ad)^4\log(a+bx)}{b^{11}}-\frac{2d(bc-ad)^9}{b^{11}(a+bx)^5}-\frac{(bc-ad)^{10}}{6b^{11}(a+bx)^6}+\frac{d^{10}(a+bx)^4}{4b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] (120*d^7*(b*c - a*d)^3*x)/b^10 - (b*c - a*d)^10/(6*b^11*(a + b*x)^6) - (2*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^5) - (45*d^2*(b*c - a*d)^8)/(4*b^11*(a + b*
x)^4) - (40*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^3) - (105*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)^2) - (252*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)) + (45*d^8*(b
*c - a*d)^2*(a + b*x)^2)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^3)/(3*b^1
1) + (d^10*(a + b*x)^4)/(4*b^11) + (210*d^6*(b*c - a*d)^4*Log[a + b*x])/b^1
1
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^7}\,dx &= \int \left(\frac{120d^7(bc-ad)^3}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^7}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^6}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^5}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^4}+\ldots\right.\\ &= \frac{120d^7(bc-ad)^3x}{b^{10}}-\frac{(bc-ad)^{10}}{6b^{11}(a+bx)^6}-\frac{2d(bc-ad)^9}{b^{11}(a+bx)^5}-\frac{45d^2(bc-ad)^8}{4b^{11}(a+bx)^4}-\frac{40d^3(bc-ad)^7}{b^{11}(a+bx)^3}-\frac{105d^4}{b^{11}(\ldots}\end{aligned}$$

**Mathematica** [A] time = 0.22, size = 265, normalized size = 1.01

$$\frac{6b^2d^8x^2\left(28a^2d^2-70abcd+45b^2c^2\right)+12bd^7x\left(-84a^3d^3+280a^2bcd^2-315ab^2c^2d+120b^3c^3\right)+4b^3d^9x^3(10bc-7ad)+2520d^6(bc-ad)^4\log(a+bx)+\frac{3024d^5(ad-bc)^5}{a+bx}-\frac{1260d^4(bc-ad)^6}{(a+bx)^2}+\frac{480d^3(ad-bc)^7}{(a+bx)^3}-\frac{135d^2(bc-ad)^8}{(a+bx)^4}+\frac{24d(ad-bc)^9}{(a+bx)^5}-\frac{2(bc-ad)^{10}}{(a+bx)^6}+3b^4d^{10}x^4}{12b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] (12*b*d^7*(120*b^3*c^3 - 315*a*b^2*c^2*d + 280*a^2*b*c*d^2 - 84*a^3*d^3)*x
+ 6*b^2*d^8*(45*b^2*c^2 - 70*a*b*c*d + 28*a^2*d^2)*x^2 + 4*b^3*d^9*(10*b*c
- 7*a*d)*x^3 + 3*b^4*d^10*x^4 - (2*(b*c - a*d)^10)/(a + b*x)^6 + (24*d*(-(b
*c) + a*d)^9)/(a + b*x)^5 - (135*d^2*(b*c - a*d)^8)/(a + b*x)^4 + (480*d^3*
(-(b*c) + a*d)^7)/(a + b*x)^3 - (1260*d^4*(b*c - a*d)^6)/(a + b*x)^2 + (302
4*d^5*(-(b*c) + a*d)^5)/(a + b*x) + 2520*d^6*(b*c - a*d)^4*Log[a + b*x])/(1
2*b^11)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^7, x]
```

**fricas [B]** time = 1.23, size = 1386, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*d^10*x^10 - 2*b^10*c^10 - 4*a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 -
24*a^3*b^7*c^7*d^3 - 84*a^4*b^6*c^6*d^4 - 504*a^5*b^5*c^5*d^5 + 6174*a^6*b^
4*c^4*d^6 - 16056*a^7*b^3*c^3*d^7 + 18414*a^8*b^2*c^2*d^8 - 10036*a^9*b*c*d
^9 + 2131*a^10*d^10 + 10*(4*b^10*c*d^9 - a*b^9*d^10)*x^9 + 45*(6*b^10*c^2*d
^8 - 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 360*(4*b^10*c^3*d^7 - 6*a*b^9*c^2*
d^8 + 4*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + (8640*a*b^9*c^3*d^7 - 18630*a^2
*b^8*c^2*d^8 + 14660*a^3*b^7*c*d^9 - 4043*a^4*b^6*d^10)*x^6 - 6*(504*b^10*c
^5*d^5 - 2520*a*b^9*c^4*d^6 + 1440*a^2*b^8*c^3*d^7 + 3510*a^3*b^7*c^2*d^8 -
 4580*a^4*b^6*c*d^9 + 1523*a^5*b^5*d^10)*x^5 - 15*(84*b^10*c^6*d^4 + 504*a*
b^9*c^5*d^5 - 3780*a^2*b^8*c^4*d^6 + 6480*a^3*b^7*c^3*d^7 - 4050*a^4*b^6*c^
2*d^8 + 460*a^5*b^5*c*d^9 + 263*a^6*b^4*d^10)*x^4 - 20*(24*b^10*c^7*d^3 + 8
4*a*b^9*c^6*d^4 + 504*a^2*b^8*c^5*d^5 - 4620*a^3*b^7*c^4*d^6 + 9840*a^4*b^6
*c^3*d^7 - 9090*a^5*b^5*c^2*d^8 + 3820*a^6*b^4*c*d^9 - 577*a^7*b^3*d^10)*x^
3 - 15*(9*b^10*c^8*d^2 + 24*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 504*a^3*b^
7*c^5*d^5 - 5250*a^4*b^6*c^4*d^6 + 12360*a^5*b^5*c^3*d^7 - 12870*a^6*b^4*c^
2*d^8 + 6340*a^7*b^3*c*d^9 - 1207*a^8*b^2*d^10)*x^2 - 6*(4*b^10*c^9*d + 9*a
*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^
```

```
5 - 5754*a^5*b^5*c^4*d^6 + 14376*a^6*b^4*c^3*d^7 - 15894*a^7*b^3*c^2*d^8 +
8356*a^8*b^2*c*d^9 - 1711*a^9*b*d^10)*x + 2520*(a^6*b^4*c^4*d^6 - 4*a^7*b^3
*c^3*d^7 + 6*a^8*b^2*c^2*d^8 - 4*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^4*d^6 -
4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 +
 6*(a*b^9*c^4*d^6 - 4*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 - 4*a^4*b^6*c*d^9
 + a^5*b^5*d^10)*x^5 + 15*(a^2*b^8*c^4*d^6 - 4*a^3*b^7*c^3*d^7 + 6*a^4*b^6*
c^2*d^8 - 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 20*(a^3*b^7*c^4*d^6 - 4*a^4
*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8 - 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 15
*(a^4*b^6*c^4*d^6 - 4*a^5*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 - 4*a^7*b^3*c*d^9
 + a^8*b^2*d^10)*x^2 + 6*(a^5*b^5*c^4*d^6 - 4*a^6*b^4*c^3*d^7 + 6*a^7*b^3*c
^2*d^8 - 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)*log(b*x + a))/(b^17*x^6 + 6*a*b^1
6*x^5 + 15*a^2*b^15*x^4 + 20*a^3*b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x
+ a^6*b^11)
```

**giac [B]** time = 1.29, size = 878, normalized size = 3.35

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 210*(b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2*b^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^
4*d^10)*log(abs(b*x + a))/b^11 - 1/12*(2*b^10*c^10 + 4*a*b^9*c^9*d + 9*a^2*
b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5
 - 6174*a^6*b^4*c^4*d^6 + 16056*a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 1
0036*a^9*b*c*d^9 - 2131*a^10*d^10 + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 +
10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x
^5 + 1260*(b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 - 45*a^2*b^8*c^4*d^6 + 100*a^3*b^
7*c^3*d^7 - 105*a^4*b^6*c^2*d^8 + 54*a^5*b^5*c*d^9 - 11*a^6*b^4*d^10)*x^4 +
 240*(2*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 42*a^2*b^8*c^5*d^5 - 385*a^3*b^7*c
^4*d^6 + 910*a^4*b^6*c^3*d^7 - 987*a^5*b^5*c^2*d^8 + 518*a^6*b^4*c*d^9 - 10
7*a^7*b^3*d^10)*x^3 + 45*(3*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6
*d^4 + 168*a^3*b^7*c^5*d^5 - 1750*a^4*b^6*c^4*d^6 + 4312*a^5*b^5*c^3*d^7 -
4788*a^6*b^4*c^2*d^8 + 2552*a^7*b^3*c*d^9 - 533*a^8*b^2*d^10)*x^2 + 6*(4*b^
10*c^9*d + 9*a*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*
a^4*b^6*c^5*d^5 - 5754*a^5*b^5*c^4*d^6 + 14616*a^6*b^4*c^3*d^7 - 16524*a^7*
b^3*c^2*d^8 + 8916*a^8*b^2*c*d^9 - 1879*a^9*b*d^10)*x)/((b*x + a)^6*b^11) +
 1/12*(3*b^21*d^10*x^4 + 40*b^21*c*d^9*x^3 - 28*a*b^20*d^10*x^3 + 270*b^21*
c^2*d^8*x^2 - 420*a*b^20*c*d^9*x^2 + 168*a^2*b^19*d^10*x^2 + 1440*b^21*c^3*
d^7*x - 3780*a*b^20*c^2*d^8*x + 3360*a^2*b^19*c*d^9*x - 1008*a^3*b^18*d^10*
x)/b^28
```

**maple [B]** time = 0.02, size = 1222, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^10/(b*x+a)^7,x)

[Out] 252/b^11*d^10/(b*x+a)*a^5-252/b^6*d^5/(b*x+a)*c^5-1/6/b^11/(b*x+a)^6*a^10*d
^10+2/b^11*d^10/(b*x+a)^5*a^9-2/b^2*d/(b*x+a)^5*c^9-105/b^11*d^10/(b*x+a)^2
*a^6-105/b^5*d^4/(b*x+a)^2*c^6+210/b^11*d^10*ln(b*x+a)*a^4+210/b^7*d^6*ln(b
*x+a)*c^4-45/4/b^11*d^10/(b*x+a)^4*a^8-45/4/b^3*d^2/(b*x+a)^4*c^8+45/2*d^8/
b^7*x^2*c^2-84*d^10/b^10*a^3*x+120*d^7/b^7*c^3*x+40/b^11*d^10/(b*x+a)^3*a^7
-40/b^4*d^3/(b*x+a)^3*c^7-7/3*d^10/b^8*x^3*a+10/3*d^9/b^7*x^3*c+14*d^10/b^9
*x^2*a^2-315*d^8/b^8*a*c^2*x-280/b^10*d^9/(b*x+a)^3*a^6*c+2100/b^8*d^7/(b*x
+a)^2*a^3*c^3-1575/b^7*d^6/(b*x+a)^2*a^2*c^4+72/b^9*d^8/(b*x+a)^5*a^7*c^2-1
68/b^8*d^7/(b*x+a)^5*a^6*c^3+252/b^7*d^6/(b*x+a)^5*a^5*c^4-252/b^6*d^5/(b*x
+a)^5*a^4*c^5+168/b^5*d^4/(b*x+a)^5*a^3*c^6-72/b^4*d^3/(b*x+a)^5*a^2*c^7+18
/b^3*d^2/(b*x+a)^5*a*c^8+840/b^9*d^8/(b*x+a)^3*a^5*c^2-1400/b^8*d^7/(b*x+a)
^3*a^4*c^3+1400/b^7*d^6/(b*x+a)^3*a^3*c^4-840/b^6*d^5/(b*x+a)^3*a^2*c^5+280
/b^5*d^4/(b*x+a)^3*a*c^6+630/b^6*d^5/(b*x+a)^2*a*c^5-35*d^9/b^8*x^2*a*c+280
*d^9/b^9*a^2*c*x-18/b^10*d^9/(b*x+a)^5*a^8*c+1260/b^7*d^6/(b*x+a)*a*c^4+5/3
/b^10/(b*x+a)^6*a^9*c*d^9-15/2/b^9/(b*x+a)^6*a^8*c^2*d^8+20/b^8/(b*x+a)^6*a
^7*c^3*d^7-35/b^7/(b*x+a)^6*a^6*c^4*d^6+42/b^6/(b*x+a)^6*a^5*c^5*d^5-35/b^5
/(b*x+a)^6*a^4*c^6*d^4+20/b^4/(b*x+a)^6*a^3*c^7*d^3-15/2/b^3/(b*x+a)^6*a^2*
c^8*d^2+5/3/b^2/(b*x+a)^6*a*c^9*d+90/b^10*d^9/(b*x+a)^4*a^7*c-315/b^9*d^8/(
b*x+a)^4*a^6*c^2+630/b^8*d^7/(b*x+a)^4*a^5*c^3-1575/2/b^7*d^6/(b*x+a)^4*a^4
*c^4+630/b^6*d^5/(b*x+a)^4*a^3*c^5-315/b^5*d^4/(b*x+a)^4*a^2*c^6+90/b^4*d^3
/(b*x+a)^4*a*c^7-1260/b^10*d^9/(b*x+a)*a^4*c+2520/b^9*d^8/(b*x+a)*a^3*c^2-2
520/b^8*d^7/(b*x+a)*a^2*c^3+1260/b^9*d^8*ln(b*x+a)*a^2*c^2-840/b^8*d^7*ln(b
*x+a)*a*c^3-840/b^10*d^9*ln(b*x+a)*a^3*c+630/b^10*d^9/(b*x+a)^2*a^5*c-1575/
b^9*d^8/(b*x+a)^2*a^4*c^2+1/4*d^10/b^7*x^4-1/6/b/(b*x+a)^6*c^10

**maxima [B]** time = 2.45, size = 925, normalized size = 3.53

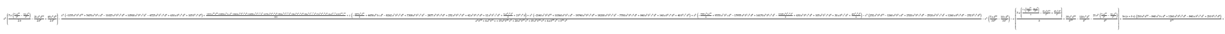
Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="maxima")

[Out] -1/12*(2*b^10*c^10 + 4*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3
 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 - 6174*a^6*b^4*c^4*d^6 + 16056*
a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 10036*a^9*b*c*d^9 - 2131*a^10*d^1
0 + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*
c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 1260*(b^10*c^6*d^4 + 6*a*b^
9*c^5*d^5 - 45*a^2*b^8*c^4*d^6 + 100*a^3*b^7*c^3*d^7 - 105*a^4*b^6*c^2*d^8
+ 54*a^5*b^5*c*d^9 - 11*a^6*b^4*d^10)*x^4 + 240*(2*b^10*c^7*d^3 + 7*a*b^9*c
^6*d^4 + 42*a^2*b^8*c^5*d^5 - 385*a^3*b^7*c^4*d^6 + 910*a^4*b^6*c^3*d^7 - 9
87*a^5*b^5*c^2*d^8 + 518*a^6*b^4*c*d^9 - 107*a^7*b^3*d^10)*x^3 + 45*(3*b^10

```
*c^8*d^2 + 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c^5*d^5 - 175
0*a^4*b^6*c^4*d^6 + 4312*a^5*b^5*c^3*d^7 - 4788*a^6*b^4*c^2*d^8 + 2552*a^7*
b^3*c*d^9 - 533*a^8*b^2*d^10)*x^2 + 6*(4*b^10*c^9*d + 9*a*b^9*c^8*d^2 + 24*
a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5 - 5754*a^5*b^5*c
^4*d^6 + 14616*a^6*b^4*c^3*d^7 - 16524*a^7*b^3*c^2*d^8 + 8916*a^8*b^2*c*d^9
 - 1879*a^9*b*d^10)*x)/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15*x^4 + 20*a^3*
b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11) + 1/12*(3*b^3*d^10*x^
4 + 4*(10*b^3*c*d^9 - 7*a*b^2*d^10)*x^3 + 6*(45*b^3*c^2*d^8 - 70*a*b^2*c*d^
9 + 28*a^2*b*d^10)*x^2 + 12*(120*b^3*c^3*d^7 - 315*a*b^2*c^2*d^8 + 280*a^2*
b*c*d^9 - 84*a^3*d^10)*x)/b^10 + 210*(b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2
*b^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^4*d^10)*log(b*x + a)/b^11
```

**mupad [B]** time = 0.42, size = 997, normalized size = 3.81



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^7,x)
```

```
[Out] x^2*((7*a*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/(2*b) - (21*a^2*d^10)/(2*b^9)
+ (45*c^2*d^8)/(2*b^7)) - (x^4*(105*b^9*c^6*d^4 - 1155*a^6*b^3*d^10 + 630*a
*b^8*c^5*d^5 + 5670*a^5*b^4*c*d^9 - 4725*a^2*b^7*c^4*d^6 + 10500*a^3*b^6*c^
3*d^7 - 11025*a^4*b^5*c^2*d^8) + (2*b^10*c^10 - 2131*a^10*d^10 + 9*a^2*b^8*
c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 - 6
174*a^6*b^4*c^4*d^6 + 16056*a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 4*a*b
^9*c^9*d + 10036*a^9*b*c*d^9)/(12*b) + x*(2*b^9*c^9*d - (1879*a^9*d^10)/2 +
 (9*a*b^8*c^8*d^2)/2 + 12*a^2*b^7*c^7*d^3 + 42*a^3*b^6*c^6*d^4 + 252*a^4*b^
5*c^5*d^5 - 2877*a^5*b^4*c^4*d^6 + 7308*a^6*b^3*c^3*d^7 - 8262*a^7*b^2*c^2*
d^8 + 4458*a^8*b*c*d^9) + x^3*(40*b^9*c^7*d^3 - 2140*a^7*b^2*d^10 + 140*a*b
^8*c^6*d^4 + 10360*a^6*b^3*c*d^9 + 840*a^2*b^7*c^5*d^5 - 7700*a^3*b^6*c^4*d
^6 + 18200*a^4*b^5*c^3*d^7 - 19740*a^5*b^4*c^2*d^8) + x^2*((45*b^9*c^8*d^2)
/4 - (7995*a^8*b*d^10)/4 + 30*a*b^8*c^7*d^3 + 9570*a^7*b^2*c*d^9 + 105*a^2*
b^7*c^6*d^4 + 630*a^3*b^6*c^5*d^5 - (13125*a^4*b^5*c^4*d^6)/2 + 16170*a^5*b
^4*c^3*d^7 - 17955*a^6*b^3*c^2*d^8) - x^5*(252*a^5*b^4*d^10 - 252*b^9*c^5*d
^5 + 1260*a*b^8*c^4*d^6 - 1260*a^4*b^5*c*d^9 - 2520*a^2*b^7*c^3*d^7 + 2520*
a^3*b^6*c^2*d^8))/(a^6*b^10 + b^16*x^6 + 6*a^5*b^11*x + 6*a*b^15*x^5 + 15*a
^4*b^12*x^2 + 20*a^3*b^13*x^3 + 15*a^2*b^14*x^4) - x^3*((7*a*d^10)/(3*b^8)
- (10*c*d^9)/(3*b^7)) - x*((7*a*((7*a*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/b
- (21*a^2*d^10)/b^9 + (45*c^2*d^8)/b^7))/b + (35*a^3*d^10)/b^10 - (120*c^3*
d^7)/b^7 - (21*a^2*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/b^2) + (log(a + b*x)*
(210*a^4*d^10 + 210*b^4*c^4*d^6 - 840*a*b^3*c^3*d^7 + 1260*a^2*b^2*c^2*d^8
- 840*a^3*b*c*d^9))/b^11 + (d^10*x^4)/(4*b^7)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**7,x)
```

```
[Out] Timed out
```

## 3.1213 $\int \frac{(c+dx)^{10}}{(a+bx)^8} \, dx$

**Optimal**. Leaf size=258

$$\frac{5d^9(a+bx)^2(bc-ad)}{b^{11}}+\frac{120d^7(bc-ad)^3\log(a+bx)}{b^{11}}-\frac{210d^6(bc-ad)^4}{b^{11}(a+bx)}-\frac{126d^5(bc-ad)^5}{b^{11}(a+bx)^2}-\frac{70d^4(bc-ad)^6}{b^{11}(a+bx)^3}-\frac{30d^3(bc\cdots}{b^{11}(a\cdots}$$

**Rubi** [A] time = 0.36, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^2(bc-ad)}{b^{11}}+\frac{45d^8x(bc-ad)^2}{b^{10}}-\frac{210d^6(bc-ad)^4}{b^{11}(a+bx)}-\frac{126d^5(bc-ad)^5}{b^{11}(a+bx)^2}-\frac{70d^4(bc-ad)^6}{b^{11}(a+bx)^3}-\frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4}-\frac{9d^2(bc-ad)^8}{b^{11}(a+bx)^5}+\frac{120d^7(bc-ad)^3\log(a+bx)}{b^{11}}-\frac{5d(bc-ad)^9}{3b^{11}(a+bx)^6}-\frac{(bc-ad)^{10}}{7b^{11}(a+bx)^7}+\frac{d^{10}(a+bx)^3}{3b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] (45*d^8*(b*c - a*d)^2*x)/b^10 - (b*c - a*d)^10/(7*b^11*(a + b*x)^7) - (5*d*
(b*c - a*d)^9)/(3*b^11*(a + b*x)^6) - (9*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
^5) - (30*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^4) - (70*d^4*(b*c - a*d)^6)/(b
^11*(a + b*x)^3) - (126*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^2) - (210*d^6*(b
*c - a*d)^4)/(b^11*(a + b*x)) + (5*d^9*(b*c - a*d)*(a + b*x)^2)/b^11 + (d^1
0*(a + b*x)^3)/(3*b^11) + (120*d^7*(b*c - a*d)^3*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^8} \, dx &= \int \left(\frac{45d^8(bc-ad)^2}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^8}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^7}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^6}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^5}+\frac{21\cdots}{b\cdots}\right.\\ &= \frac{45d^8(bc-ad)^2x}{b^{10}}-\frac{(bc-ad)^{10}}{7b^{11}(a+bx)^7}-\frac{5d(bc-ad)^9}{3b^{11}(a+bx)^6}-\frac{9d^2(bc-ad)^8}{b^{11}(a+bx)^5}-\frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4}-\frac{70d^4(b\cdots}{b^{11}(a\cdots}\end{aligned}$$

**Mathematica** [A] time = 0.25, size = 239, normalized size = 0.93

$$\frac{21bd^8x\left(36a^2d^2-80abcd+45b^2c^2\right)+21b^2d^9x^2(5bc-4ad)+2520d^7(bc-ad)^3\log(a+bx)-\frac{4410d^6(bc-ad)^4}{a+bx}+\frac{2646d^5(ad-bc)^5}{(a+bx)^2}-\frac{1470d^4(bc-ad)^6}{(a+bx)^3}+\frac{630d^3(ad-bc)^7}{(a+bx)^4}-\frac{189d^2(bc-ad)^8}{(a+bx)^5}+\frac{35d(ad-bc)^9}{(a+bx)^6}-\frac{3(bc-ad)^{10}}{(a+bx)^7}+7b^3d^{10}x^3}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] (21*b*d^8*(45*b^2*c^2 - 80*a*b*c*d + 36*a^2*d^2)*x + 21*b^2*d^9*(5*b*c - 4*
a*d)*x^2 + 7*b^3*d^10*x^3 - (3*(b*c - a*d)^10)/(a + b*x)^7 + (35*d*(-(b*c)
+ a*d)^9)/(a + b*x)^6 - (189*d^2*(b*c - a*d)^8)/(a + b*x)^5 + (630*d^3*(-(b
*c) + a*d)^7)/(a + b*x)^4 - (1470*d^4*(b*c - a*d)^6)/(a + b*x)^3 + (2646*d^
5*(-(b*c) + a*d)^5)/(a + b*x)^2 - (4410*d^6*(b*c - a*d)^4)/(a + b*x) + 2520
*d^7*(b*c - a*d)^3*Log[a + b*x])/(21*b^11)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^8, x]
```

**fricas [B]** time = 1.40, size = 1362, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^10*d^10*x^10 - 3*b^10*c^10 - 5*a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 -
18*a^3*b^7*c^7*d^3 - 42*a^4*b^6*c^6*d^4 - 126*a^5*b^5*c^5*d^5 - 630*a^6*b^4
*c^4*d^6 + 6534*a^7*b^3*c^3*d^7 - 12987*a^8*b^2*c^2*d^8 + 10047*a^9*b*c*d^9
 - 2761*a^10*d^10 + 35*(3*b^10*c*d^9 - a*b^9*d^10)*x^9 + 315*(3*b^10*c^2*d^
8 - 3*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 49*(135*a*b^9*c^2*d^8 - 195*a^2*b^8
*c*d^9 + 77*a^3*b^7*d^10)*x^7 - 49*(90*b^10*c^4*d^6 - 360*a*b^9*c^3*d^7 + 1
35*a^2*b^8*c^2*d^8 + 285*a^3*b^7*c*d^9 - 179*a^4*b^6*d^10)*x^6 - 147*(18*b^
10*c^5*d^5 + 90*a*b^9*c^4*d^6 - 540*a^2*b^8*c^3*d^7 + 675*a^3*b^7*c^2*d^8 -
 255*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 - 245*(6*b^10*c^6*d^4 + 18*a*b^9*c^5
*d^5 + 90*a^2*b^8*c^4*d^6 - 660*a^3*b^7*c^3*d^7 + 1035*a^4*b^6*c^2*d^8 - 61
5*a^5*b^5*c*d^9 + 121*a^6*b^4*d^10)*x^4 - 35*(18*b^10*c^7*d^3 + 42*a*b^9*c^
6*d^4 + 126*a^2*b^8*c^5*d^5 + 630*a^3*b^7*c^4*d^6 - 5250*a^4*b^6*c^3*d^7 +
9135*a^5*b^5*c^2*d^8 - 6195*a^6*b^4*c*d^9 + 1477*a^7*b^3*d^10)*x^3 - 21*(9*
b^10*c^8*d^2 + 18*a*b^9*c^7*d^3 + 42*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
+ 630*a^4*b^6*c^4*d^6 - 5754*a^5*b^5*c^3*d^7 + 10647*a^6*b^4*c^2*d^8 - 7707
*a^7*b^3*c*d^9 + 1981*a^8*b^2*d^10)*x^2 - 7*(5*b^10*c^9*d + 9*a*b^9*c^8*d^2
 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 630*a^5*
b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 11907*a^7*b^3*c^2*d^8 - 8967*a^8*b^2*c
```

```
*d^9 + 2401*a^9*b*d^10)*x + 2520*(a^7*b^3*c^3*d^7 - 3*a^8*b^2*c^2*d^8 + 3*a
^9*b*c*d^9 - a^10*d^10 + (b^10*c^3*d^7 - 3*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9
- a^3*b^7*d^10)*x^7 + 7*(a*b^9*c^3*d^7 - 3*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^
9 - a^4*b^6*d^10)*x^6 + 21*(a^2*b^8*c^3*d^7 - 3*a^3*b^7*c^2*d^8 + 3*a^4*b^6
*c*d^9 - a^5*b^5*d^10)*x^5 + 35*(a^3*b^7*c^3*d^7 - 3*a^4*b^6*c^2*d^8 + 3*a^
5*b^5*c*d^9 - a^6*b^4*d^10)*x^4 + 35*(a^4*b^6*c^3*d^7 - 3*a^5*b^5*c^2*d^8 +
 3*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 21*(a^5*b^5*c^3*d^7 - 3*a^6*b^4*c^2*
d^8 + 3*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 7*(a^6*b^4*c^3*d^7 - 3*a^7*b^3*
c^2*d^8 + 3*a^8*b^2*c*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^18*x^7 + 7*a*b^
17*x^6 + 21*a^2*b^16*x^5 + 35*a^3*b^15*x^4 + 35*a^4*b^14*x^3 + 21*a^5*b^13*
x^2 + 7*a^6*b^12*x + a^7*b^11)
```

**giac [B]** time = 1.26, size = 872, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 120*(b^3*c^3*d^7 - 3*a*b^2*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10)*log(abs(b*x
+ a))/b^11 - 1/21*(3*b^10*c^10 + 5*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 18*a^3
*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d
^6 - 6534*a^7*b^3*c^3*d^7 + 12987*a^8*b^2*c^2*d^8 - 10047*a^9*b*c*d^9 + 276
1*a^10*d^10 + 4410*(b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2646*(b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 -
30*a^2*b^8*c^3*d^7 + 50*a^3*b^7*c^2*d^8 - 35*a^4*b^6*c*d^9 + 9*a^5*b^5*d^10
)*x^5 + 1470*(b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 110*a^3
*b^7*c^3*d^7 + 195*a^4*b^6*c^2*d^8 - 141*a^5*b^5*c*d^9 + 37*a^6*b^4*d^10)*x
^4 + 210*(3*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 105*a^3*b
^7*c^4*d^6 - 875*a^4*b^6*c^3*d^7 + 1617*a^5*b^5*c^2*d^8 - 1197*a^6*b^4*c*d^
9 + 319*a^7*b^3*d^10)*x^3 + 63*(3*b^10*c^8*d^2 + 6*a*b^9*c^7*d^3 + 14*a^2*b
^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 1918*a^5*b^5*c^3*d^
7 + 3654*a^6*b^4*c^2*d^8 - 2754*a^7*b^3*c*d^9 + 743*a^8*b^2*d^10)*x^2 + 7*(
5*b^10*c^9*d + 9*a*b^9*c^8*d^2 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 +
126*a^4*b^6*c^5*d^5 + 630*a^5*b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 12042*a^
7*b^3*c^2*d^8 - 9207*a^8*b^2*c*d^9 + 2509*a^9*b*d^10)*x)/((b*x + a)^7*b^11)
 + 1/3*(b^16*d^10*x^3 + 15*b^16*c*d^9*x^2 - 12*a*b^15*d^10*x^2 + 135*b^16*c
^2*d^8*x - 240*a*b^15*c*d^9*x + 108*a^2*b^14*d^10*x)/b^24
```

**maple [B]** time = 0.02, size = 1241, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^8,x)
```

```
[Out] -30/b^4*d^3/(b*x+a)^4*c^7-210/b^11*d^10/(b*x+a)*a^4-210/b^7*d^6/(b*x+a)*c^4
+5/3/b^11*d^10/(b*x+a)^6*a^9-5/3/b^2*d/(b*x+a)^6*c^9+120/b^8*d^7*ln(b*x+a)*
c^3+30/b^11*d^10/(b*x+a)^4*a^7-9/b^11*d^10/(b*x+a)^5*a^8-9/b^3*d^2/(b*x+a)^
5*c^8+126/b^11*d^10/(b*x+a)^2*a^5-126/b^6*d^5/(b*x+a)^2*c^5-120/b^11*d^10*l
n(b*x+a)*a^3-70/b^5*d^4/(b*x+a)^3*c^6-1/7/b^11/(b*x+a)^7*a^10*d^10-4*d^10/b
^9*x^2*a+5*d^9/b^8*x^2*c+36*d^10/b^10*a^2*x+45*d^8/b^8*c^2*x-70/b^11*d^10/(
b*x+a)^3*a^6+360/b^10*d^9*ln(b*x+a)*a^2*c-360/b^9*d^8*ln(b*x+a)*a*c^2-210/b
^10*d^9/(b*x+a)^4*a^6*c+630/b^9*d^8/(b*x+a)^4*a^5*c^2-1050/b^8*d^7/(b*x+a)^
4*a^4*c^3+1050/b^7*d^6/(b*x+a)^4*a^3*c^4-630/b^6*d^5/(b*x+a)^4*a^2*c^5+420/
b^6*d^5/(b*x+a)^3*a*c^5+10/7/b^10/(b*x+a)^7*a^9*c*d^9-45/7/b^9/(b*x+a)^7*a^
8*c^2*d^8+120/7/b^8/(b*x+a)^7*a^7*c^3*d^7-30/b^7/(b*x+a)^7*a^6*c^4*d^6+36/b
^6/(b*x+a)^7*a^5*c^5*d^5-30/b^5/(b*x+a)^7*a^4*c^6*d^4+120/7/b^4/(b*x+a)^7*a
^3*c^7*d^3-45/7/b^3/(b*x+a)^7*a^2*c^8*d^2+10/7/b^2/(b*x+a)^7*a*c^9*d+72/b^1
0*d^9/(b*x+a)^5*a^7*c-252/b^9*d^8/(b*x+a)^5*a^6*c^2+504/b^8*d^7/(b*x+a)^5*a
^5*c^3-630/b^7*d^6/(b*x+a)^5*a^4*c^4+504/b^6*d^5/(b*x+a)^5*a^3*c^5-252/b^5*
d^4/(b*x+a)^5*a^2*c^6+72/b^4*d^3/(b*x+a)^5*a*c^7-630/b^10*d^9/(b*x+a)^2*a^4
*c+1260/b^9*d^8/(b*x+a)^2*a^3*c^2-1260/b^8*d^7/(b*x+a)^2*a^2*c^3+630/b^7*d^
6/(b*x+a)^2*a*c^4-80*d^9/b^9*a*c*x+140/b^5*d^4/(b*x+a)^6*a^3*c^6-60/b^4*d^3
/(b*x+a)^6*a^2*c^7+15/b^3*d^2/(b*x+a)^6*a*c^8-15/b^10*d^9/(b*x+a)^6*a^8*c+6
0/b^9*d^8/(b*x+a)^6*a^7*c^2+420/b^10*d^9/(b*x+a)^3*a^5*c-1050/b^9*d^8/(b*x+
a)^3*a^4*c^2+1400/b^8*d^7/(b*x+a)^3*a^3*c^3-1050/b^7*d^6/(b*x+a)^3*a^2*c^4+
210/b^7*d^6/(b*x+a)^6*a^5*c^4-210/b^6*d^5/(b*x+a)^6*a^4*c^5+210/b^5*d^4/(b*
x+a)^4*a*c^6+840/b^10*d^9/(b*x+a)*a^3*c-1260/b^9*d^8/(b*x+a)*a^2*c^2+840/b^
8*d^7/(b*x+a)*a*c^3-140/b^8*d^7/(b*x+a)^6*a^6*c^3+1/3*d^10/b^8*x^3-1/7/b/(b
*x+a)^7*c^10
```

**maxima** [**B**] time = 2.44, size = 934, normalized size = 3.62

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/21*(3*b^10*c^10 + 5*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 18*a^3*b^7*c^7*d^3
 + 42*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 - 6534*a^
7*b^3*c^3*d^7 + 12987*a^8*b^2*c^2*d^8 - 10047*a^9*b*c*d^9 + 2761*a^10*d^10
+ 4410*(b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^
9 + a^4*b^6*d^10)*x^6 + 2646*(b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 - 30*a^2*b^8*c
^3*d^7 + 50*a^3*b^7*c^2*d^8 - 35*a^4*b^6*c*d^9 + 9*a^5*b^5*d^10)*x^5 + 1470
*(b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 110*a^3*b^7*c^3*d^7
 + 195*a^4*b^6*c^2*d^8 - 141*a^5*b^5*c*d^9 + 37*a^6*b^4*d^10)*x^4 + 210*(3*
b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 -
 875*a^4*b^6*c^3*d^7 + 1617*a^5*b^5*c^2*d^8 - 1197*a^6*b^4*c*d^9 + 319*a^7*
b^3*d^10)*x^3 + 63*(3*b^10*c^8*d^2 + 6*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*d^4 +
 42*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 1918*a^5*b^5*c^3*d^7 + 3654*a^6
```

```
*b^4*c^2*d^8 - 2754*a^7*b^3*c*d^9 + 743*a^8*b^2*d^10)*x^2 + 7*(5*b^10*c^9*d
 + 9*a*b^9*c^8*d^2 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 126*a^4*b^6*
c^5*d^5 + 630*a^5*b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 12042*a^7*b^3*c^2*d^
8 - 9207*a^8*b^2*c*d^9 + 2509*a^9*b*d^10)*x)/(b^18*x^7 + 7*a*b^17*x^6 + 21*
a^2*b^16*x^5 + 35*a^3*b^15*x^4 + 35*a^4*b^14*x^3 + 21*a^5*b^13*x^2 + 7*a^6*
b^12*x + a^7*b^11) + 1/3*(b^2*d^10*x^3 + 3*(5*b^2*c*d^9 - 4*a*b*d^10)*x^2 +
 3*(45*b^2*c^2*d^8 - 80*a*b*c*d^9 + 36*a^2*d^10)*x)/b^10 + 120*(b^3*c^3*d^7
 - 3*a*b^2*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10)*log(b*x + a)/b^11
```

**mupad [B]** time = 0.43, size = 950, normalized size = 3.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^8,x)
```

```
[Out] x*((8*a*((8*a*d^10)/b^9 - (10*c*d^9)/b^8))/b - (28*a^2*d^10)/b^10 + (45*c^2
*d^8)/b^8) - (x^4*(2590*a^6*b^3*d^10 + 70*b^9*c^6*d^4 + 210*a*b^8*c^5*d^5 -
 9870*a^5*b^4*c*d^9 + 1050*a^2*b^7*c^4*d^6 - 7700*a^3*b^6*c^3*d^7 + 13650*a
^4*b^5*c^2*d^8) + x^6*(210*a^4*b^5*d^10 + 210*b^9*c^4*d^6 - 840*a*b^8*c^3*d
^7 - 840*a^3*b^6*c*d^9 + 1260*a^2*b^7*c^2*d^8) + (2761*a^10*d^10 + 3*b^10*c
^10 + 9*a^2*b^8*c^8*d^2 + 18*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 126*a^5
*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 - 6534*a^7*b^3*c^3*d^7 + 12987*a^8*b^2*c
^2*d^8 + 5*a*b^9*c^9*d - 10047*a^9*b*c*d^9)/(21*b) + x*((2509*a^9*d^10)/3 +
 (5*b^9*c^9*d)/3 + 3*a*b^8*c^8*d^2 + 6*a^2*b^7*c^7*d^3 + 14*a^3*b^6*c^6*d^4
 + 42*a^4*b^5*c^5*d^5 + 210*a^5*b^4*c^4*d^6 - 2058*a^6*b^3*c^3*d^7 + 4014*a
^7*b^2*c^2*d^8 - 3069*a^8*b*c*d^9) + x^3*(3190*a^7*b^2*d^10 + 30*b^9*c^7*d^
3 + 70*a*b^8*c^6*d^4 - 11970*a^6*b^3*c*d^9 + 210*a^2*b^7*c^5*d^5 + 1050*a^3
*b^6*c^4*d^6 - 8750*a^4*b^5*c^3*d^7 + 16170*a^5*b^4*c^2*d^8) + x^2*(2229*a^
8*b*d^10 + 9*b^9*c^8*d^2 + 18*a*b^8*c^7*d^3 - 8262*a^7*b^2*c*d^9 + 42*a^2*b
^7*c^6*d^4 + 126*a^3*b^6*c^5*d^5 + 630*a^4*b^5*c^4*d^6 - 5754*a^5*b^4*c^3*d
^7 + 10962*a^6*b^3*c^2*d^8) + x^5*(1134*a^5*b^4*d^10 + 126*b^9*c^5*d^5 + 63
0*a*b^8*c^4*d^6 - 4410*a^4*b^5*c*d^9 - 3780*a^2*b^7*c^3*d^7 + 6300*a^3*b^6*
c^2*d^8))/(a^7*b^10 + b^17*x^7 + 7*a^6*b^11*x + 7*a*b^16*x^6 + 21*a^5*b^12*
x^2 + 35*a^4*b^13*x^3 + 35*a^3*b^14*x^4 + 21*a^2*b^15*x^5) - x^2*((4*a*d^10
)/b^9 - (5*c*d^9)/b^8) - (log(a + b*x)*(120*a^3*d^10 - 120*b^3*c^3*d^7 + 36
0*a*b^2*c^2*d^8 - 360*a^2*b*c*d^9))/b^11 + (d^10*x^3)/(3*b^8)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**8,x)
```

```
[Out] Timed out
```

## 3.1214 $\int \frac{(c+dx)^{10}}{(a+bx)^9}\,dx$

**Optimal**. Leaf size=258

$$\frac{45d^8(bc-ad)^2\log(a+bx)}{b^{11}}-\frac{120d^7(bc-ad)^3}{b^{11}(a+bx)}-\frac{105d^6(bc-ad)^4}{b^{11}(a+bx)^2}-\frac{84d^5(bc-ad)^5}{b^{11}(a+bx)^3}-\frac{105d^4(bc-ad)^6}{2b^{11}(a+bx)^4}-\frac{24d^3(bc-ad}{b^{11}(a+bx)}$$

**Rubi** [A] time = 0.34, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^9x(10bc-9ad)}{b^{10}}-\frac{120d^7(bc-ad)^3}{b^{11}(a+bx)}-\frac{105d^6(bc-ad)^4}{b^{11}(a+bx)^2}-\frac{84d^5(bc-ad)^5}{b^{11}(a+bx)^3}-\frac{105d^4(bc-ad)^6}{2b^{11}(a+bx)^4}-\frac{24d^3(bc-ad)^7}{b^{11}(a+bx)^5}-\frac{15d^2(bc-ad)^8}{2b^{11}(a+bx)^6}+\frac{45d^8(bc-ad)^2\log(a+bx)}{b^{11}}-\frac{10d(bc-ad)^9}{7b^{11}(a+bx)^7}-\frac{(bc-ad)^{10}}{8b^{11}(a+bx)^8}+\frac{d^{10}x^2}{2b^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] (d^9*(10*b*c - 9*a*d)*x)/b^10 + (d^10*x^2)/(2*b^9) - (b*c - a*d)^10/(8*b^11
*(a + b*x)^8) - (10*d*(b*c - a*d)^9)/(7*b^11*(a + b*x)^7) - (15*d^2*(b*c -
a*d)^8)/(2*b^11*(a + b*x)^6) - (24*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^5) -
(105*d^4*(b*c - a*d)^6)/(2*b^11*(a + b*x)^4) - (84*d^5*(b*c - a*d)^5)/(b^11
*(a + b*x)^3) - (105*d^6*(b*c - a*d)^4)/(b^11*(a + b*x)^2) - (120*d^7*(b*c
- a*d)^3)/(b^11*(a + b*x)) + (45*d^8*(b*c - a*d)^2*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^9}\,dx=\int\left(\frac{d^9(10bc-9ad)}{b^{10}}+\frac{d^{10}x}{b^9}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^9}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^8}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^7}+\frac{120d^3(bc-}{b^{10}(a+bx}\right.$$
$$=\frac{d^9(10bc-9ad)x}{b^{10}}+\frac{d^{10}x^2}{2b^9}-\frac{(bc-ad)^{10}}{8b^{11}(a+bx)^8}-\frac{10d(bc-ad)^9}{7b^{11}(a+bx)^7}-\frac{15d^2(bc-ad)^8}{2b^{11}(a+bx)^6}-\frac{24d^3(bc-ad)}{b^{11}(a+bx)^5}$$

**Mathematica** [B] time = 0.32, size = 712, normalized size = 2.76

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] (3601*a^10*d^10 + 2*a^9*b*d^9*(-4609*c + 13144*d*x) + a^8*b^2*d^8*(6849*c^2
 - 68704*c*d*x + 81928*d^2*x^2) + 8*a^7*b^3*d^7*(-105*c^3 + 6534*c^2*d*x -
27538*c*d^2*x^2 + 17542*d^3*x^3) + 14*a^6*b^4*d^6*(-15*c^4 - 480*c^3*d*x +
12348*c^2*d^2*x^2 - 28112*c*d^3*x^3 + 10010*d^4*x^4) - 28*a^5*b^5*d^5*(3*c^
5 + 60*c^4*d*x + 840*c^3*d^2*x^2 - 11508*c^2*d^3*x^3 + 15050*c*d^4*x^4 - 27
44*d^5*x^5) - 14*a^4*b^6*d^4*(3*c^6 + 48*c^5*d*x + 420*c^4*d^2*x^2 + 3360*c
^3*d^3*x^3 - 26250*c^2*d^4*x^4 + 19040*c*d^5*x^5 - 1064*d^6*x^6) - 8*a^3*b^
7*d^3*(3*c^7 + 42*c^6*d*x + 294*c^5*d^2*x^2 + 1470*c^4*d^3*x^3 + 7350*c^3*d
^4*x^4 - 32340*c^2*d^5*x^5 + 10780*c*d^6*x^6 + 728*d^7*x^7) - a^2*b^8*d^2*(
15*c^8 + 192*c^7*d*x + 1176*c^6*d^2*x^2 + 4704*c^5*d^3*x^3 + 14700*c^4*d^4*
x^4 + 47040*c^3*d^5*x^5 - 105840*c^2*d^6*x^6 + 4480*c*d^7*x^7 + 3248*d^8*x^
8) - 2*a*b^9*d*(5*c^9 + 60*c^8*d*x + 336*c^7*d^2*x^2 + 1176*c^6*d^3*x^3 + 2
940*c^5*d^4*x^4 + 5880*c^4*d^5*x^5 + 11760*c^3*d^6*x^6 - 10080*c^2*d^7*x^7
- 2240*c*d^8*x^8 + 140*d^9*x^9) - b^10*(7*c^10 + 80*c^9*d*x + 420*c^8*d^2*x
^2 + 1344*c^7*d^3*x^3 + 2940*c^6*d^4*x^4 + 4704*c^5*d^5*x^5 + 5880*c^4*d^6*
x^6 + 6720*c^3*d^7*x^7 - 560*c*d^9*x^9 - 28*d^10*x^10) + 2520*d^8*(b*c - a*
d)^2*(a + b*x)^8*Log[a + b*x])/(56*b^11*(a + b*x)^8)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^9, x]
```

**fricas [B]** time = 1.39, size = 1296, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] 1/56*(28*b^10*d^10*x^10 - 7*b^10*c^10 - 10*a*b^9*c^9*d - 15*a^2*b^8*c^8*d^2
 - 24*a^3*b^7*c^7*d^3 - 42*a^4*b^6*c^6*d^4 - 84*a^5*b^5*c^5*d^5 - 210*a^6*b
^4*c^4*d^6 - 840*a^7*b^3*c^3*d^7 + 6849*a^8*b^2*c^2*d^8 - 9218*a^9*b*c*d^9
+ 3601*a^10*d^10 + 280*(2*b^10*c*d^9 - a*b^9*d^10)*x^9 + 112*(40*a*b^9*c*d^
9 - 29*a^2*b^8*d^10)*x^8 - 448*(15*b^10*c^3*d^7 - 45*a*b^9*c^2*d^8 + 10*a^2
*b^8*c*d^9 + 13*a^3*b^7*d^10)*x^7 - 392*(15*b^10*c^4*d^6 + 60*a*b^9*c^3*d^7
 - 270*a^2*b^8*c^2*d^8 + 220*a^3*b^7*c*d^9 - 38*a^4*b^6*d^10)*x^6 - 784*(6*
```

```
b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 60*a^2*b^8*c^3*d^7 - 330*a^3*b^7*c^2*d^8
+ 340*a^4*b^6*c*d^9 - 98*a^5*b^5*d^10)*x^5 - 980*(3*b^10*c^6*d^4 + 6*a*b^9*
c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 60*a^3*b^7*c^3*d^7 - 375*a^4*b^6*c^2*d^8 + 4
30*a^5*b^5*c*d^9 - 143*a^6*b^4*d^10)*x^4 - 112*(12*b^10*c^7*d^3 + 21*a*b^9*
c^6*d^4 + 42*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 -
2877*a^5*b^5*c^2*d^8 + 3514*a^6*b^4*c*d^9 - 1253*a^7*b^3*d^10)*x^3 - 28*(15
*b^10*c^8*d^2 + 24*a*b^9*c^7*d^3 + 42*a^2*b^8*c^6*d^4 + 84*a^3*b^7*c^5*d^5
+ 210*a^4*b^6*c^4*d^6 + 840*a^5*b^5*c^3*d^7 - 6174*a^6*b^4*c^2*d^8 + 7868*a
^7*b^3*c*d^9 - 2926*a^8*b^2*d^10)*x^2 - 8*(10*b^10*c^9*d + 15*a*b^9*c^8*d^2
 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4*b^6*c^5*d^5 + 210*a^5*b
^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2*d^8 + 8588*a^8*b^2*c*d^
9 - 3286*a^9*b*d^10)*x + 2520*(a^8*b^2*c^2*d^8 - 2*a^9*b*c*d^9 + a^10*d^10
+ (b^10*c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 8*(a*b^9*c^2*d^8 - 2*
a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 28*(a^2*b^8*c^2*d^8 - 2*a^3*b^7*c*d^9 +
 a^4*b^6*d^10)*x^6 + 56*(a^3*b^7*c^2*d^8 - 2*a^4*b^6*c*d^9 + a^5*b^5*d^10)*
x^5 + 70*(a^4*b^6*c^2*d^8 - 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 56*(a^5*b
^5*c^2*d^8 - 2*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 28*(a^6*b^4*c^2*d^8 - 2*
a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 8*(a^7*b^3*c^2*d^8 - 2*a^8*b^2*c*d^9 +
a^9*b*d^10)*x)*log(b*x + a))/(b^19*x^8 + 8*a*b^18*x^7 + 28*a^2*b^17*x^6 + 5
6*a^3*b^16*x^5 + 70*a^4*b^15*x^4 + 56*a^5*b^14*x^3 + 28*a^6*b^13*x^2 + 8*a^
7*b^12*x + a^8*b^11)
```

**giac [B]** time = 1.29, size = 871, normalized size = 3.38

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] 45*(b^2*c^2*d^8 - 2*a*b*c*d^9 + a^2*d^10)*log(abs(b*x + a))/b^11 + 1/2*(b^9
*d^10*x^2 + 20*b^9*c*d^9*x - 18*a*b^8*d^10*x)/b^18 - 1/56*(7*b^10*c^10 + 10
*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4
 + 84*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 - 6849*a^
8*b^2*c^2*d^8 + 9218*a^9*b*c*d^9 - 3601*a^10*d^10 + 6720*(b^10*c^3*d^7 - 3*
a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 5880*(b^10*c^4*d^6 +
4*a*b^9*c^3*d^7 - 18*a^2*b^8*c^2*d^8 + 20*a^3*b^7*c*d^9 - 7*a^4*b^6*d^10)*x
^6 + 2352*(2*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 - 110*a^3*
b^7*c^2*d^8 + 130*a^4*b^6*c*d^9 - 47*a^5*b^5*d^10)*x^5 + 2940*(b^10*c^6*d^4
 + 2*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 - 125*a^4*b^6*c
^2*d^8 + 154*a^5*b^5*c*d^9 - 57*a^6*b^4*d^10)*x^4 + 336*(4*b^10*c^7*d^3 + 7
*a*b^9*c^6*d^4 + 14*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*
d^7 - 959*a^5*b^5*c^2*d^8 + 1218*a^6*b^4*c*d^9 - 459*a^7*b^3*d^10)*x^3 + 84
*(5*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*d^4 + 28*a^3*b^7*c^5*d^
5 + 70*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 - 2058*a^6*b^4*c^2*d^8 + 2676*
a^7*b^3*c*d^9 - 1023*a^8*b^2*d^10)*x^2 + 8*(10*b^10*c^9*d + 15*a*b^9*c^8*d^
```

```
2 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4*b^6*c^5*d^5 + 210*a^5*
b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2*d^8 + 8658*a^8*b^2*c*d
^9 - 3349*a^9*b*d^10)*x)/((b*x + a)^8*b^11)
```

**maple [B]** time = 0.02, size = 1256, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^9,x)
```

```
[Out] 1/2*d^10*x^2/b^9-9*d^10/b^10*a*x+10*d^9/b^9*x*c-15/2/b^11*d^10/(b*x+a)^6*a^
8-15/2/b^3*d^2/(b*x+a)^6*c^8-105/b^7*d^6/(b*x+a)^2*c^4+45/b^11*d^10*ln(b*x+
a)*a^2+45/b^9*d^8*ln(b*x+a)*c^2-105/2/b^11*d^10/(b*x+a)^4*a^6-105/2/b^5*d^4
/(b*x+a)^4*c^6+120/b^11*d^10/(b*x+a)*a^3-120/b^8*d^7/(b*x+a)*c^3-1/8/b^11/(
b*x+a)^8*a^10*d^10+84/b^11*d^10/(b*x+a)^3*a^5-84/b^6*d^5/(b*x+a)^3*c^5+10/7
/b^11*d^10/(b*x+a)^7*a^9-10/7/b^2*d/(b*x+a)^7*c^9+24/b^11*d^10/(b*x+a)^5*a^
7-24/b^4*d^3/(b*x+a)^5*c^7-105/b^11*d^10/(b*x+a)^2*a^4-1/8/b/(b*x+a)^8*c^10
-360/7/b^4*d^3/(b*x+a)^7*a^2*c^7+90/7/b^3*d^2/(b*x+a)^7*a*c^8-168/b^10*d^9/
(b*x+a)^5*a^6*c+504/b^9*d^8/(b*x+a)^5*a^5*c^2+180/b^7*d^6/(b*x+a)^7*a^5*c^4
-180/b^6*d^5/(b*x+a)^7*a^4*c^5+120/b^5*d^4/(b*x+a)^7*a^3*c^6-840/b^8*d^7/(b
*x+a)^5*a^4*c^3+840/b^7*d^6/(b*x+a)^5*a^3*c^4-504/b^6*d^5/(b*x+a)^5*a^2*c^5
-210/b^9*d^8/(b*x+a)^6*a^6*c^2+420/b^8*d^7/(b*x+a)^6*a^5*c^3-525/b^7*d^6/(b
*x+a)^6*a^4*c^4+420/b^6*d^5/(b*x+a)^6*a^3*c^5-210/b^5*d^4/(b*x+a)^6*a^2*c^6
+60/b^4*d^3/(b*x+a)^6*a*c^7+168/b^5*d^4/(b*x+a)^5*a*c^6+315/b^10*d^9/(b*x+a
)^4*a^5*c-1575/2/b^9*d^8/(b*x+a)^4*a^4*c^2+1050/b^8*d^7/(b*x+a)^4*a^3*c^3-1
575/2/b^7*d^6/(b*x+a)^4*a^2*c^4+315/b^6*d^5/(b*x+a)^4*a*c^5-360/b^10*d^9/(b
*x+a)*a^2*c+360/b^9*d^8/(b*x+a)*a*c^2+60/b^10*d^9/(b*x+a)^6*a^7*c+420/b^10*
d^9/(b*x+a)^2*a^3*c-630/b^9*d^8/(b*x+a)^2*a^2*c^2+420/b^8*d^7/(b*x+a)^2*a*c
^3-90/b^10*d^9*ln(b*x+a)*a*c+5/4/b^10/(b*x+a)^8*a^9*c*d^9-45/8/b^9/(b*x+a)^
8*a^8*c^2*d^8+15/b^8/(b*x+a)^8*a^7*c^3*d^7-105/4/b^7/(b*x+a)^8*a^6*c^4*d^6+
63/2/b^6/(b*x+a)^8*a^5*c^5*d^5-105/4/b^5/(b*x+a)^8*a^4*c^6*d^4+15/b^4/(b*x+
a)^8*a^3*c^7*d^3-45/8/b^3/(b*x+a)^8*a^2*c^8*d^2+5/4/b^2/(b*x+a)^8*a*c^9*d-4
20/b^10*d^9/(b*x+a)^3*a^4*c+840/b^9*d^8/(b*x+a)^3*a^3*c^2-840/b^8*d^7/(b*x+
a)^3*a^2*c^3+420/b^7*d^6/(b*x+a)^3*a*c^4-90/7/b^10*d^9/(b*x+a)^7*a^8*c+360/
7/b^9*d^8/(b*x+a)^7*a^7*c^2-120/b^8*d^7/(b*x+a)^7*a^6*c^3
```

**maxima [B]** time = 2.58, size = 945, normalized size = 3.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/56*(7*b^10*c^10 + 10*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d
^3 + 42*a^4*b^6*c^6*d^4 + 84*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^
```

```
7*b^3*c^3*d^7 - 6849*a^8*b^2*c^2*d^8 + 9218*a^9*b*c*d^9 - 3601*a^10*d^10 +
6720*(b^10*c^3*d^7 - 3*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7
+ 5880*(b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 - 18*a^2*b^8*c^2*d^8 + 20*a^3*b^7*c*
d^9 - 7*a^4*b^6*d^10)*x^6 + 2352*(2*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 20*a^2
*b^8*c^3*d^7 - 110*a^3*b^7*c^2*d^8 + 130*a^4*b^6*c*d^9 - 47*a^5*b^5*d^10)*x
^5 + 2940*(b^10*c^6*d^4 + 2*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 20*a^3*b^7*
c^3*d^7 - 125*a^4*b^6*c^2*d^8 + 154*a^5*b^5*c*d^9 - 57*a^6*b^4*d^10)*x^4 +
336*(4*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 14*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4
*d^6 + 140*a^4*b^6*c^3*d^7 - 959*a^5*b^5*c^2*d^8 + 1218*a^6*b^4*c*d^9 - 459
*a^7*b^3*d^10)*x^3 + 84*(5*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*
d^4 + 28*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 - 2058*
a^6*b^4*c^2*d^8 + 2676*a^7*b^3*c*d^9 - 1023*a^8*b^2*d^10)*x^2 + 8*(10*b^10*
c^9*d + 15*a*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4
*b^6*c^5*d^5 + 210*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2
*d^8 + 8658*a^8*b^2*c*d^9 - 3349*a^9*b*d^10)*x)/(b^19*x^8 + 8*a*b^18*x^7 +
28*a^2*b^17*x^6 + 56*a^3*b^16*x^5 + 70*a^4*b^15*x^4 + 56*a^5*b^14*x^3 + 28*
a^6*b^13*x^2 + 8*a^7*b^12*x + a^8*b^11) + 1/2*(b*d^10*x^2 + 2*(10*b*c*d^9 -
 9*a*d^10)*x)/b^10 + 45*(b^2*c^2*d^8 - 2*a*b*c*d^9 + a^2*d^10)*log(b*x + a)
/b^11
```

**mupad [B]** time = 0.26, size = 946, normalized size = 3.67

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^9,x)
```

```
[Out] (log(a + b*x)*(45*a^2*d^10 + 45*b^2*c^2*d^8 - 90*a*b*c*d^9))/b^11 - (x^4*((
105*b^9*c^6*d^4)/2 - (5985*a^6*b^3*d^10)/2 + 105*a*b^8*c^5*d^5 + 8085*a^5*b
^4*c*d^9 + (525*a^2*b^7*c^4*d^6)/2 + 1050*a^3*b^6*c^3*d^7 - (13125*a^4*b^5*
c^2*d^8)/2) + x^6*(105*b^9*c^4*d^6 - 735*a^4*b^5*d^10 + 420*a*b^8*c^3*d^7 +
 2100*a^3*b^6*c*d^9 - 1890*a^2*b^7*c^2*d^8) + (7*b^10*c^10 - 3601*a^10*d^10
 + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 84*a^5*b^
5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 - 6849*a^8*b^2*c^2*d^
8 + 10*a*b^9*c^9*d + 9218*a^9*b*c*d^9)/(56*b) + x*((10*b^9*c^9*d)/7 - (3349
*a^9*d^10)/7 + (15*a*b^8*c^8*d^2)/7 + (24*a^2*b^7*c^7*d^3)/7 + 6*a^3*b^6*c^
6*d^4 + 12*a^4*b^5*c^5*d^5 + 30*a^5*b^4*c^4*d^6 + 120*a^6*b^3*c^3*d^7 - (65
34*a^7*b^2*c^2*d^8)/7 + (8658*a^8*b*c*d^9)/7) + x^3*(24*b^9*c^7*d^3 - 2754*
a^7*b^2*d^10 + 42*a*b^8*c^6*d^4 + 7308*a^6*b^3*c*d^9 + 84*a^2*b^7*c^5*d^5 +
 210*a^3*b^6*c^4*d^6 + 840*a^4*b^5*c^3*d^7 - 5754*a^5*b^4*c^2*d^8) + x^2*((
15*b^9*c^8*d^2)/2 - (3069*a^8*b*d^10)/2 + 12*a*b^8*c^7*d^3 + 4014*a^7*b^2*c
*d^9 + 21*a^2*b^7*c^6*d^4 + 42*a^3*b^6*c^5*d^5 + 105*a^4*b^5*c^4*d^6 + 420*
a^5*b^4*c^3*d^7 - 3087*a^6*b^3*c^2*d^8) + x^5*(84*b^9*c^5*d^5 - 1974*a^5*b^
4*d^10 + 210*a*b^8*c^4*d^6 + 5460*a^4*b^5*c*d^9 + 840*a^2*b^7*c^3*d^7 - 462
0*a^3*b^6*c^2*d^8) - x^7*(120*a^3*b^6*d^10 - 120*b^9*c^3*d^7 + 360*a*b^8*c^
```

```
2*d^8 - 360*a^2*b^7*c*d^9))/(a^8*b^10 + b^18*x^8 + 8*a^7*b^11*x + 8*a*b^17*
x^7 + 28*a^6*b^12*x^2 + 56*a^5*b^13*x^3 + 70*a^4*b^14*x^4 + 56*a^3*b^15*x^5
 + 28*a^2*b^16*x^6) - x*((9*a*d^10)/b^10 - (10*c*d^9)/b^9) + (d^10*x^2)/(2*
b^9)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**9,x)
```

```
[Out] Timed out
```

## 3.1215 $\int \frac{(c+dx)^{10}}{(a+bx)^{10}}\,dx$

**Optimal**. Leaf size=257

$$\frac{10d^9(bc-ad)\log(a+bx)}{b^{11}}-\frac{45d^8(bc-ad)^2}{b^{11}(a+bx)}-\frac{60d^7(bc-ad)^3}{b^{11}(a+bx)^2}-\frac{70d^6(bc-ad)^4}{b^{11}(a+bx)^3}-\frac{63d^5(bc-ad)^5}{b^{11}(a+bx)^4}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-$$

**Rubi [A]** time = 0.31, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{45d^8(bc-ad)^2}{b^{11}(a+bx)}-\frac{60d^7(bc-ad)^3}{b^{11}(a+bx)^2}-\frac{70d^6(bc-ad)^4}{b^{11}(a+bx)^3}-\frac{63d^5(bc-ad)^5}{b^{11}(a+bx)^4}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{45d^2(bc-ad)^8}{7b^{11}(a+bx)^7}+\frac{10d^9(bc-ad)\log(a+bx)}{b^{11}}-\frac{5d(bc-ad)^9}{4b^{11}(a+bx)^8}-\frac{(bc-ad)^{10}}{9b^{11}(a+bx)^9}+\frac{d^{10}x}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] (d^10*x)/b^10 - (b*c - a*d)^10/(9*b^11*(a + b*x)^9) - (5*d*(b*c - a*d)^9)/(
4*b^11*(a + b*x)^8) - (45*d^2*(b*c - a*d)^8)/(7*b^11*(a + b*x)^7) - (20*d^3
*(b*c - a*d)^7)/(b^11*(a + b*x)^6) - (42*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)
^5) - (63*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^4) - (70*d^6*(b*c - a*d)^4)/(b
^11*(a + b*x)^3) - (60*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^2) - (45*d^8*(b*c
 - a*d)^2)/(b^11*(a + b*x)) + (10*d^9*(b*c - a*d)*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{10}}\,dx &= \int \left(\frac{d^{10}}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{10}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^9}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^8}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^7}+\frac{210d^4(bc-}{b^{10}(a+b}\right.\\ &= \frac{d^{10}x}{b^{10}}-\frac{(bc-ad)^{10}}{9b^{11}(a+bx)^9}-\frac{5d(bc-ad)^9}{4b^{11}(a+bx)^8}-\frac{45d^2(bc-ad)^8}{7b^{11}(a+bx)^7}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}\end{aligned}$$

**Mathematica [B]** time = 0.42, size = 708, normalized size = 2.75

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] -1/252*(4861*a^10*d^10 + a^9*b*d^9*(-7129*c + 41229*d*x) + 9*a^8*b^2*d^8*(1
40*c^2 - 6849*c*d*x + 17064*d^2*x^2) + 12*a^7*b^3*d^7*(35*c^3 + 945*c^2*d*x
 - 19602*c*d^2*x^2 + 27342*d^3*x^3) + 42*a^6*b^4*d^6*(5*c^4 + 90*c^3*d*x +
1080*c^2*d^2*x^2 - 12348*c*d^3*x^3 + 10458*d^4*x^4) + 126*a^5*b^5*d^5*(c^5
+ 15*c^4*d*x + 120*c^3*d^2*x^2 + 840*c^2*d^3*x^3 - 5754*c*d^4*x^4 + 2982*d^
5*x^5) + 42*a^4*b^6*d^4*(2*c^6 + 27*c^5*d*x + 180*c^4*d^2*x^2 + 840*c^3*d^3
*x^3 + 3780*c^2*d^4*x^4 - 15750*c*d^5*x^5 + 4704*d^6*x^6) + 12*a^3*b^7*d^3*
(5*c^7 + 63*c^6*d*x + 378*c^5*d^2*x^2 + 1470*c^4*d^3*x^3 + 4410*c^3*d^4*x^4
 + 13230*c^2*d^5*x^5 - 32340*c*d^6*x^6 + 4536*d^7*x^7) + 9*a^2*b^8*d^2*(5*c
^8 + 60*c^7*d*x + 336*c^6*d^2*x^2 + 1176*c^5*d^3*x^3 + 2940*c^4*d^4*x^4 + 5
880*c^3*d^5*x^5 + 11760*c^2*d^6*x^6 - 15120*c*d^7*x^7 + 252*d^8*x^8) + a*b^
9*d*(35*c^9 + 405*c^8*d*x + 2160*c^7*d^2*x^2 + 7056*c^6*d^3*x^3 + 15876*c^5
*d^4*x^4 + 26460*c^4*d^5*x^5 + 35280*c^3*d^6*x^6 + 45360*c^2*d^7*x^7 - 2268
0*c*d^8*x^8 - 2268*d^9*x^9) + b^10*(28*c^10 + 315*c^9*d*x + 1620*c^8*d^2*x^
2 + 5040*c^7*d^3*x^3 + 10584*c^6*d^4*x^4 + 15876*c^5*d^5*x^5 + 17640*c^4*d^
6*x^6 + 15120*c^3*d^7*x^7 + 11340*c^2*d^8*x^8 - 252*d^10*x^10) + 2520*d^9*(
-(b*c) + a*d)*(a + b*x)^9*Log[a + b*x])/(b^11*(a + b*x)^9)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^10, x]
```

**fricas [B]** time = 1.23, size = 1216, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*d^10*x^10 + 2268*a*b^9*d^10*x^9 - 28*b^10*c^10 - 35*a*b^9*c
^9*d - 45*a^2*b^8*c^8*d^2 - 60*a^3*b^7*c^7*d^3 - 84*a^4*b^6*c^6*d^4 - 126*a
^5*b^5*c^5*d^5 - 210*a^6*b^4*c^4*d^6 - 420*a^7*b^3*c^3*d^7 - 1260*a^8*b^2*c
^2*d^8 + 7129*a^9*b*c*d^9 - 4861*a^10*d^10 - 2268*(5*b^10*c^2*d^8 - 10*a*b^
9*c*d^9 + a^2*b^8*d^10)*x^8 - 3024*(5*b^10*c^3*d^7 + 15*a*b^9*c^2*d^8 - 45*
a^2*b^8*c*d^9 + 18*a^3*b^7*d^10)*x^7 - 3528*(5*b^10*c^4*d^6 + 10*a*b^9*c^3*
d^7 + 30*a^2*b^8*c^2*d^8 - 110*a^3*b^7*c*d^9 + 56*a^4*b^6*d^10)*x^6 - 5292*
```

```
(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d^8
 - 125*a^4*b^6*c*d^9 + 71*a^5*b^5*d^10)*x^5 - 5292*(2*b^10*c^6*d^4 + 3*a*b^
9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^8 - 1
37*a^5*b^5*c*d^9 + 83*a^6*b^4*d^10)*x^4 - 504*(10*b^10*c^7*d^3 + 14*a*b^9*c
^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 70*a^4*b^6*c^3*d^7 + 210
*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 + 651*a^7*b^3*d^10)*x^3 - 108*(15*b^1
0*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 70
*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 - 2178*a^7*b^3
*c*d^9 + 1422*a^8*b^2*d^10)*x^2 - 9*(35*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 60*
a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 210*a^5*b^5*c^
4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 - 6849*a^8*b^2*c*d^9 + 4
581*a^9*b*d^10)*x + 2520*(a^9*b*c*d^9 - a^10*d^10 + (b^10*c*d^9 - a*b^9*d^1
0)*x^9 + 9*(a*b^9*c*d^9 - a^2*b^8*d^10)*x^8 + 36*(a^2*b^8*c*d^9 - a^3*b^7*d
^10)*x^7 + 84*(a^3*b^7*c*d^9 - a^4*b^6*d^10)*x^6 + 126*(a^4*b^6*c*d^9 - a^5
*b^5*d^10)*x^5 + 126*(a^5*b^5*c*d^9 - a^6*b^4*d^10)*x^4 + 84*(a^6*b^4*c*d^9
 - a^7*b^3*d^10)*x^3 + 36*(a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 9*(a^8*b^2*c
*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*
x^7 + 84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x
^3 + 36*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11)
```

**giac [B]** time = 1.25, size = 867, normalized size = 3.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] d^10*x/b^10 + 10*(b*c*d^9 - a*d^10)*log(abs(b*x + a))/b^11 - 1/252*(28*b^10
*c^10 + 35*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^3 + 84*a^4*b
^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 420*a^7*b^3*c^3*d^
7 + 1260*a^8*b^2*c^2*d^8 - 7129*a^9*b*c*d^9 + 4861*a^10*d^10 + 11340*(b^10*
c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 15120*(b^10*c^3*d^7 + 3*a*b^9
*c^2*d^8 - 9*a^2*b^8*c*d^9 + 5*a^3*b^7*d^10)*x^7 + 17640*(b^10*c^4*d^6 + 2*
a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 22*a^3*b^7*c*d^9 + 13*a^4*b^6*d^10)*x^6
 + 5292*(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 30*a^3*b^7
*c^2*d^8 - 125*a^4*b^6*c*d^9 + 77*a^5*b^5*d^10)*x^5 + 5292*(2*b^10*c^6*d^4
+ 3*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2
*d^8 - 137*a^5*b^5*c*d^9 + 87*a^6*b^4*d^10)*x^4 + 504*(10*b^10*c^7*d^3 + 14
*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 70*a^4*b^6*c^3*d
^7 + 210*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 + 669*a^7*b^3*d^10)*x^3 + 108
*(15*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 42*a^3*b^7*c^5*
d^5 + 70*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 - 2178
*a^7*b^3*c*d^9 + 1443*a^8*b^2*d^10)*x^2 + 9*(35*b^10*c^9*d + 45*a*b^9*c^8*d
^2 + 60*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 210*a^
```

```
5*b^5*c^4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 - 6849*a^8*b^2*c
*d^9 + 4609*a^9*b*d^10)*x)/((b*x + a)^9*b^11)
```

**maple [B]** time = 0.02, size = 1266, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^10,x)
```

```
[Out] 60/b^11*d^10/(b*x+a)^2*a^3-60/b^8*d^7/(b*x+a)^2*c^3-10/b^11*d^10*ln(b*x+a)*
a+10/b^10*d^9*ln(b*x+a)*c+63/b^11*d^10/(b*x+a)^4*a^5-63/b^6*d^5/(b*x+a)^4*c
^5-45/b^11*d^10/(b*x+a)*a^2-45/b^9*d^8/(b*x+a)*c^2+20/b^11*d^10/(b*x+a)^6*a
^7-20/b^4*d^3/(b*x+a)^6*c^7-45/7/b^11*d^10/(b*x+a)^7*a^8-45/7/b^3*d^2/(b*x+
a)^7*c^8-1/9/b^11/(b*x+a)^9*a^10*d^10+5/4/b^11*d^10/(b*x+a)^8*a^9-5/4/b^2*d
/(b*x+a)^8*c^9-70/b^11*d^10/(b*x+a)^3*a^4-70/b^7*d^6/(b*x+a)^3*c^4-42/b^11*
d^10/(b*x+a)^5*a^6-42/b^5*d^4/(b*x+a)^5*c^6+d^10*x/b^10-1/9/b/(b*x+a)^9*c^1
0-140/b^10*d^9/(b*x+a)^6*a^6*c+420/b^9*d^8/(b*x+a)^6*a^5*c^2+360/7/b^4*d^3/
(b*x+a)^7*a*c^7+10/9/b^10/(b*x+a)^9*a^9*c*d^9-5/b^9/(b*x+a)^9*a^8*c^2*d^8+4
0/3/b^8/(b*x+a)^9*a^7*c^3*d^7+700/b^7*d^6/(b*x+a)^6*a^3*c^4-420/b^6*d^5/(b*
x+a)^6*a^2*c^5+140/b^5*d^4/(b*x+a)^6*a*c^6-70/3/b^7/(b*x+a)^9*a^6*c^4*d^6+2
8/b^6/(b*x+a)^9*a^5*c^5*d^5-70/3/b^5/(b*x+a)^9*a^4*c^6*d^4+40/3/b^4/(b*x+a)
^9*a^3*c^7*d^3-5/b^3/(b*x+a)^9*a^2*c^8*d^2+10/9/b^2/(b*x+a)^9*a*c^9*d+252/b
^10*d^9/(b*x+a)^5*a^5*c-630/b^9*d^8/(b*x+a)^5*a^4*c^2+840/b^8*d^7/(b*x+a)^5
*a^3*c^3-630/b^7*d^6/(b*x+a)^5*a^2*c^4+252/b^6*d^5/(b*x+a)^5*a*c^5-180/b^10
*d^9/(b*x+a)^2*a^2*c+180/b^9*d^8/(b*x+a)^2*a*c^2-315/b^10*d^9/(b*x+a)^4*a^4
*c+630/b^9*d^8/(b*x+a)^4*a^3*c^2-630/b^8*d^7/(b*x+a)^4*a^2*c^3+315/b^7*d^6/
(b*x+a)^4*a*c^4+90/b^10*d^9/(b*x+a)*a*c+280/b^8*d^7/(b*x+a)^3*a*c^3+360/7/b
^10*d^9/(b*x+a)^7*a^7*c-180/b^9*d^8/(b*x+a)^7*a^6*c^2+360/b^8*d^7/(b*x+a)^7
*a^5*c^3-450/b^7*d^6/(b*x+a)^7*a^4*c^4+360/b^6*d^5/(b*x+a)^7*a^3*c^5-180/b^
5*d^4/(b*x+a)^7*a^2*c^6-700/b^8*d^7/(b*x+a)^6*a^4*c^3-45/4/b^10*d^9/(b*x+a)
^8*a^8*c+45/b^9*d^8/(b*x+a)^8*a^7*c^2-105/b^8*d^7/(b*x+a)^8*a^6*c^3+315/2/b
^7*d^6/(b*x+a)^8*a^5*c^4-315/2/b^6*d^5/(b*x+a)^8*a^4*c^5+105/b^5*d^4/(b*x+a
)^8*a^3*c^6-45/b^4*d^3/(b*x+a)^8*a^2*c^7+45/4/b^3*d^2/(b*x+a)^8*a*c^8+280/b
^10*d^9/(b*x+a)^3*a^3*c-420/b^9*d^8/(b*x+a)^3*a^2*c^2
```

**maxima [B]** time = 2.36, size = 957, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] d^10*x/b^10 - 1/252*(28*b^10*c^10 + 35*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 6
0*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*
c^4*d^6 + 420*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 - 7129*a^9*b*c*d^9 + 4
```

```
861*a^10*d^10 + 11340*(b^10*c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1
5120*(b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 - 9*a^2*b^8*c*d^9 + 5*a^3*b^7*d^10)*x^
7 + 17640*(b^10*c^4*d^6 + 2*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 22*a^3*b^7*
c*d^9 + 13*a^4*b^6*d^10)*x^6 + 5292*(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*
a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d^8 - 125*a^4*b^6*c*d^9 + 77*a^5*b^5*d^10)
*x^5 + 5292*(2*b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*
b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^8 - 137*a^5*b^5*c*d^9 + 87*a^6*b^4*d^10)*x^4
 + 504*(10*b^10*c^7*d^3 + 14*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^
7*c^4*d^6 + 70*a^4*b^6*c^3*d^7 + 210*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 +
 669*a^7*b^3*d^10)*x^3 + 108*(15*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b
^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7
+ 420*a^6*b^4*c^2*d^8 - 2178*a^7*b^3*c*d^9 + 1443*a^8*b^2*d^10)*x^2 + 9*(35
*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 60*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 +
126*a^4*b^6*c^5*d^5 + 210*a^5*b^5*c^4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*
b^3*c^2*d^8 - 6849*a^8*b^2*c*d^9 + 4609*a^9*b*d^10)*x)/(b^20*x^9 + 9*a*b^19
*x^8 + 36*a^2*b^18*x^7 + 84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*
x^4 + 84*a^6*b^14*x^3 + 36*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11) + 10*(b*
c*d^9 - a*d^10)*log(b*x + a)/b^11
```

**mupad [B]** time = 0.50, size = 955, normalized size = 3.72

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^10,x)
```

```
[Out] (d^10*x)/b^10 - (log(a + b*x)*(10*a*d^10 - 10*b*c*d^9))/b^11 - (x^4*(1827*a
^6*b^3*d^10 + 42*b^9*c^6*d^4 + 63*a*b^8*c^5*d^5 - 2877*a^5*b^4*c*d^9 + 105*
a^2*b^7*c^4*d^6 + 210*a^3*b^6*c^3*d^7 + 630*a^4*b^5*c^2*d^8) + x^6*(910*a^4
*b^5*d^10 + 70*b^9*c^4*d^6 + 140*a*b^8*c^3*d^7 - 1540*a^3*b^6*c*d^9 + 420*a
^2*b^7*c^2*d^8) + (4861*a^10*d^10 + 28*b^10*c^10 + 45*a^2*b^8*c^8*d^2 + 60*
a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^
4*d^6 + 420*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 35*a*b^9*c^9*d - 7129*
a^9*b*c*d^9)/(252*b) + x*((4609*a^9*d^10)/28 + (5*b^9*c^9*d)/4 + (45*a*b^8*
c^8*d^2)/28 + (15*a^2*b^7*c^7*d^3)/7 + 3*a^3*b^6*c^6*d^4 + (9*a^4*b^5*c^5*d
^5)/2 + (15*a^5*b^4*c^4*d^6)/2 + 15*a^6*b^3*c^3*d^7 + 45*a^7*b^2*c^2*d^8 -
(6849*a^8*b*c*d^9)/28) + x^8*(45*a^2*b^7*d^10 + 45*b^9*c^2*d^8 - 90*a*b^8*c
*d^9) + x^3*(1338*a^7*b^2*d^10 + 20*b^9*c^7*d^3 + 28*a*b^8*c^6*d^4 - 2058*a
^6*b^3*c*d^9 + 42*a^2*b^7*c^5*d^5 + 70*a^3*b^6*c^4*d^6 + 140*a^4*b^5*c^3*d^
7 + 420*a^5*b^4*c^2*d^8) + x^2*((4329*a^8*b*d^10)/7 + (45*b^9*c^8*d^2)/7 +
(60*a*b^8*c^7*d^3)/7 - (6534*a^7*b^2*c*d^9)/7 + 12*a^2*b^7*c^6*d^4 + 18*a^3
*b^6*c^5*d^5 + 30*a^4*b^5*c^4*d^6 + 60*a^5*b^4*c^3*d^7 + 180*a^6*b^3*c^2*d^
8) + x^5*(1617*a^5*b^4*d^10 + 63*b^9*c^5*d^5 + 105*a*b^8*c^4*d^6 - 2625*a^4
*b^5*c*d^9 + 210*a^2*b^7*c^3*d^7 + 630*a^3*b^6*c^2*d^8) + x^7*(300*a^3*b^6*
d^10 + 60*b^9*c^3*d^7 + 180*a*b^8*c^2*d^8 - 540*a^2*b^7*c*d^9))/(a^9*b^10 +
```

```
b^19*x^9 + 9*a^8*b^11*x + 9*a*b^18*x^8 + 36*a^7*b^12*x^2 + 84*a^6*b^13*x^3
+ 126*a^5*b^14*x^4 + 126*a^4*b^15*x^5 + 84*a^3*b^16*x^6 + 36*a^2*b^17*x^7)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**10,x)
```

```
[Out] Timed out
```

## 3.1216 $\int \frac{(c+dx)^{10}}{(a+bx)^{11}}\,dx$

**Optimal**. Leaf size=271

$$-\frac{10d^9(bc-ad)}{b^{11}(a+bx)}-\frac{45d^8(bc-ad)^2}{2b^{11}(a+bx)^2}-\frac{40d^7(bc-ad)^3}{b^{11}(a+bx)^3}-\frac{105d^6(bc-ad)^4}{2b^{11}(a+bx)^4}-\frac{252d^5(bc-ad)^5}{5b^{11}(a+bx)^5}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\frac{120d^3(bc\ldots}{7b^{11}(a\ldots}$$

**Rubi [A]** time = 0.29, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{10d^9(bc-ad)}{b^{11}(a+bx)}-\frac{45d^8(bc-ad)^2}{2b^{11}(a+bx)^2}-\frac{40d^7(bc-ad)^3}{b^{11}(a+bx)^3}-\frac{105d^6(bc-ad)^4}{2b^{11}(a+bx)^4}-\frac{252d^5(bc-ad)^5}{5b^{11}(a+bx)^5}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\frac{120d^3(bc-ad)^7}{7b^{11}(a+bx)^7}-\frac{45d^2(bc-ad)^8}{8b^{11}(a+bx)^8}-\frac{10d(bc-ad)^9}{9b^{11}(a+bx)^9}-\frac{(bc-ad)^{10}}{10b^{11}(a+bx)^{10}}+\frac{d^{10}\log(a+bx)}{b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^11,x]
```

```
[Out] -(b*c - a*d)^10/(10*b^11*(a + b*x)^10) - (10*d*(b*c - a*d)^9)/(9*b^11*(a +
b*x)^9) - (45*d^2*(b*c - a*d)^8)/(8*b^11*(a + b*x)^8) - (120*d^3*(b*c - a*d
)^7)/(7*b^11*(a + b*x)^7) - (35*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^6) - (25
2*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^5) - (105*d^6*(b*c - a*d)^4)/(2*b^11
*(a + b*x)^4) - (40*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^3) - (45*d^8*(b*c -
a*d)^2)/(2*b^11*(a + b*x)^2) - (10*d^9*(b*c - a*d))/(b^11*(a + b*x)) + (d^1
0*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{11}}\,dx &= \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{11}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{10}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^9}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^8}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^7}+\ldots\right.\\ &= -\frac{(bc-ad)^{10}}{10b^{11}(a+bx)^{10}}-\frac{10d(bc-ad)^9}{9b^{11}(a+bx)^9}-\frac{45d^2(bc-ad)^8}{8b^{11}(a+bx)^8}-\frac{120d^3(bc-ad)^7}{7b^{11}(a+bx)^7}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\ldots\end{aligned}$$

**Mathematica [B]** time = 0.36, size = 591, normalized size = 2.18

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^11,x]
```

```
[Out] -1/2520*((b*c - a*d)*(7381*a^9*d^9 + a^8*b*d^8*(4861*c + 71290*d*x) + a^7*b
^2*d^7*(3601*c^2 + 46090*c*d*x + 308205*d^2*x^2) + a^6*b^3*d^6*(2761*c^3 +
33490*c^2*d*x + 194805*c*d^2*x^2 + 784080*d^3*x^3) + a^5*b^4*d^5*(2131*c^4
+ 25090*c^3*d*x + 138105*c^2*d^2*x^2 + 481680*c*d^3*x^3 + 1296540*d^4*x^4)
+ a^4*b^5*d^4*(1627*c^5 + 18790*c^4*d*x + 100305*c^3*d^2*x^2 + 330480*c^2*d
^3*x^3 + 767340*c*d^4*x^4 + 1450008*d^5*x^5) + a^3*b^6*d^3*(1207*c^6 + 1375
0*c^5*d*x + 71955*c^4*d^2*x^2 + 229680*c^3*d^3*x^3 + 502740*c^2*d^4*x^4 + 8
14968*c*d^5*x^5 + 1102500*d^6*x^6) + a^2*b^7*d^2*(847*c^7 + 9550*c^6*d*x +
49275*c^5*d^2*x^2 + 154080*c^4*d^3*x^3 + 326340*c^3*d^4*x^4 + 497448*c^2*d^
5*x^5 + 573300*c*d^6*x^6 + 554400*d^7*x^7) + a*b^8*d*(532*c^8 + 5950*c^7*d*
x + 30375*c^6*d^2*x^2 + 93600*c^5*d^3*x^3 + 194040*c^4*d^4*x^4 + 285768*c^3
*d^5*x^5 + 308700*c^2*d^6*x^6 + 252000*c*d^7*x^7 + 170100*d^8*x^8) + b^9*(2
52*c^9 + 2800*c^8*d*x + 14175*c^7*d^2*x^2 + 43200*c^6*d^3*x^3 + 88200*c^5*d
^4*x^4 + 127008*c^4*d^5*x^5 + 132300*c^3*d^6*x^6 + 100800*c^2*d^7*x^7 + 567
00*c*d^8*x^8 + 25200*d^9*x^9)))/(b^11*(a + b*x)^10) + (d^10*Log[a + b*x])/b
^11
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^11,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^11, x]
```

**fricas [B]** time = 1.22, size = 1107, normalized size = 4.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^10*c^10 + 280*a*b^9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^
7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6
 + 840*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^1
0*d^10 + 25200*(b^10*c*d^9 - a*b^9*d^10)*x^9 + 56700*(b^10*c^2*d^8 + 2*a*b^
9*c*d^9 - 3*a^2*b^8*d^10)*x^8 + 50400*(2*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 6
*a^2*b^8*c*d^9 - 11*a^3*b^7*d^10)*x^7 + 44100*(3*b^10*c^4*d^6 + 4*a*b^9*c^3
*d^7 + 6*a^2*b^8*c^2*d^8 + 12*a^3*b^7*c*d^9 - 25*a^4*b^6*d^10)*x^6 + 10584*
```

```
(12*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d
^8 + 60*a^4*b^6*c*d^9 - 137*a^5*b^5*d^10)*x^5 + 8820*(10*b^10*c^6*d^4 + 12*
a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^
8 + 60*a^5*b^5*c*d^9 - 147*a^6*b^4*d^10)*x^4 + 720*(60*b^10*c^7*d^3 + 70*a*
b^9*c^6*d^4 + 84*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*d^
7 + 210*a^5*b^5*c^2*d^8 + 420*a^6*b^4*c*d^9 - 1089*a^7*b^3*d^10)*x^3 + 135*
(105*b^10*c^8*d^2 + 120*a*b^9*c^7*d^3 + 140*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c
^5*d^5 + 210*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 +
840*a^7*b^3*c*d^9 - 2283*a^8*b^2*d^10)*x^2 + 10*(280*b^10*c^9*d + 315*a*b^9
*c^8*d^2 + 360*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5
+ 630*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 + 2520*a
^8*b^2*c*d^9 - 7129*a^9*b*d^10)*x - 2520*(b^10*d^10*x^10 + 10*a*b^9*d^10*x^
9 + 45*a^2*b^8*d^10*x^8 + 120*a^3*b^7*d^10*x^7 + 210*a^4*b^6*d^10*x^6 + 252
*a^5*b^5*d^10*x^5 + 210*a^6*b^4*d^10*x^4 + 120*a^7*b^3*d^10*x^3 + 45*a^8*b^
2*d^10*x^2 + 10*a^9*b*d^10*x + a^10*d^10)*log(b*x + a))/(b^21*x^10 + 10*a*b
^20*x^9 + 45*a^2*b^19*x^8 + 120*a^3*b^18*x^7 + 210*a^4*b^17*x^6 + 252*a^5*b
^16*x^5 + 210*a^6*b^15*x^4 + 120*a^7*b^14*x^3 + 45*a^8*b^13*x^2 + 10*a^9*b^
12*x + a^10*b^11)
```

**giac [B]** time = 1.36, size = 874, normalized size = 3.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="giac")
```

```
[Out] d^10*log(abs(b*x + a))/b^11 - 1/2520*(25200*(b^9*c*d^9 - a*b^8*d^10)*x^9 +
56700*(b^9*c^2*d^8 + 2*a*b^8*c*d^9 - 3*a^2*b^7*d^10)*x^8 + 50400*(2*b^9*c^3
*d^7 + 3*a*b^8*c^2*d^8 + 6*a^2*b^7*c*d^9 - 11*a^3*b^6*d^10)*x^7 + 44100*(3*
b^9*c^4*d^6 + 4*a*b^8*c^3*d^7 + 6*a^2*b^7*c^2*d^8 + 12*a^3*b^6*c*d^9 - 25*a
^4*b^5*d^10)*x^6 + 10584*(12*b^9*c^5*d^5 + 15*a*b^8*c^4*d^6 + 20*a^2*b^7*c^
3*d^7 + 30*a^3*b^6*c^2*d^8 + 60*a^4*b^5*c*d^9 - 137*a^5*b^4*d^10)*x^5 + 882
0*(10*b^9*c^6*d^4 + 12*a*b^8*c^5*d^5 + 15*a^2*b^7*c^4*d^6 + 20*a^3*b^6*c^3*
d^7 + 30*a^4*b^5*c^2*d^8 + 60*a^5*b^4*c*d^9 - 147*a^6*b^3*d^10)*x^4 + 720*(
60*b^9*c^7*d^3 + 70*a*b^8*c^6*d^4 + 84*a^2*b^7*c^5*d^5 + 105*a^3*b^6*c^4*d^
6 + 140*a^4*b^5*c^3*d^7 + 210*a^5*b^4*c^2*d^8 + 420*a^6*b^3*c*d^9 - 1089*a^
7*b^2*d^10)*x^3 + 135*(105*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 140*a^2*b^7*c^
6*d^4 + 168*a^3*b^6*c^5*d^5 + 210*a^4*b^5*c^4*d^6 + 280*a^5*b^4*c^3*d^7 + 4
20*a^6*b^3*c^2*d^8 + 840*a^7*b^2*c*d^9 - 2283*a^8*b*d^10)*x^2 + 10*(280*b^9
*c^9*d + 315*a*b^8*c^8*d^2 + 360*a^2*b^7*c^7*d^3 + 420*a^3*b^6*c^6*d^4 + 50
4*a^4*b^5*c^5*d^5 + 630*a^5*b^4*c^4*d^6 + 840*a^6*b^3*c^3*d^7 + 1260*a^7*b^
2*c^2*d^8 + 2520*a^8*b*c*d^9 - 7129*a^9*d^10)*x + (252*b^10*c^10 + 280*a*b^
9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 +
 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 + 1260*a^8
*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^10*d^10)/b)/((b*x + a)^10*b^10)
```

**maple [B]** time = 0.01, size = 1271, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^11,x)
```

```
[Out] -35*d^10/b^11/(b*x+a)^6*a^6-35*d^4/b^5/(b*x+a)^6*c^6-1/10/b^11/(b*x+a)^10*a
^10*d^10-120/7*d^3/b^4/(b*x+a)^7*c^7+10/9*d^10/b^11/(b*x+a)^9*a^9-10/9*d/b^
2/(b*x+a)^9*c^9+252/5*d^10/b^11/(b*x+a)^5*a^5-252/5*d^5/b^6/(b*x+a)^5*c^5-4
5/2*d^10/b^11/(b*x+a)^2*a^2-45/2*d^8/b^9/(b*x+a)^2*c^2-105/2*d^10/b^11/(b*x
+a)^4*a^4-105/2*d^6/b^7/(b*x+a)^4*c^4+10/b^11*d^10/(b*x+a)*a-10/b^10*d^9/(b
*x+a)*c-45/8*d^10/b^11/(b*x+a)^8*a^8-45/8*d^2/b^3/(b*x+a)^8*c^8+40*d^10/b^1
1/(b*x+a)^3*a^3-40*d^7/b^8/(b*x+a)^3*c^3+120/7*d^10/b^11/(b*x+a)^7*a^7-315/
2*d^4/b^5/(b*x+a)^8*a^2*c^6+45*d^3/b^4/(b*x+a)^8*a*c^7-120*d^9/b^10/(b*x+a)
^3*a^2*c+120*d^8/b^9/(b*x+a)^3*a*c^2-120*d^9/b^10/(b*x+a)^7*a^6*c+360*d^8/b
^9/(b*x+a)^7*a^5*c^2-600*d^7/b^8/(b*x+a)^7*a^4*c^3+600*d^6/b^7/(b*x+a)^7*a^
3*c^4-360*d^5/b^6/(b*x+a)^7*a^2*c^5+120*d^4/b^5/(b*x+a)^7*a*c^6-10*d^9/b^10
/(b*x+a)^9*a^8*c+40*d^8/b^9/(b*x+a)^9*a^7*c^2-280/3*d^7/b^8/(b*x+a)^9*a^6*c
^3+140*d^6/b^7/(b*x+a)^9*a^5*c^4-140*d^5/b^6/(b*x+a)^9*a^4*c^5+280/3*d^4/b^
5/(b*x+a)^9*a^3*c^6-40*d^3/b^4/(b*x+a)^9*a^2*c^7+10*d^2/b^3/(b*x+a)^9*a*c^8
-252*d^9/b^10/(b*x+a)^5*a^4*c+504*d^8/b^9/(b*x+a)^5*a^3*c^2-504*d^7/b^8/(b*
x+a)^5*a^2*c^3+252*d^6/b^7/(b*x+a)^5*a*c^4+45*d^9/b^10/(b*x+a)^2*a*c+210*d^
9/b^10/(b*x+a)^4*a^3*c-315*d^8/b^9/(b*x+a)^4*a^2*c^2+210*d^7/b^8/(b*x+a)^4*
a*c^3+210*d^9/b^10/(b*x+a)^6*a^5*c-525*d^8/b^9/(b*x+a)^6*a^4*c^2+700*d^7/b^
8/(b*x+a)^6*a^3*c^3-525*d^6/b^7/(b*x+a)^6*a^2*c^4+210*d^5/b^6/(b*x+a)^6*a*c
^5+1/b^10/(b*x+a)^10*a^9*c*d^9-9/2/b^9/(b*x+a)^10*a^8*c^2*d^8+12/b^8/(b*x+a
)^10*a^7*c^3*d^7-21/b^7/(b*x+a)^10*a^6*c^4*d^6+126/5/b^6/(b*x+a)^10*a^5*c^5
*d^5-21/b^5/(b*x+a)^10*a^4*c^6*d^4+12/b^4/(b*x+a)^10*a^3*c^7*d^3-9/2/b^3/(b
*x+a)^10*a^2*c^8*d^2+1/b^2/(b*x+a)^10*a*c^9*d+45*d^9/b^10/(b*x+a)^8*a^7*c-3
15/2*d^8/b^9/(b*x+a)^8*a^6*c^2+315*d^5/b^6/(b*x+a)^8*a^3*c^5+d^10*ln(b*x+a)
/b^11-1/10/b/(b*x+a)^10*c^10+315*d^7/b^8/(b*x+a)^8*a^5*c^3-1575/4*d^6/b^7/(
b*x+a)^8*a^4*c^4
```

**maxima [B]** time = 2.01, size = 975, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="maxima")
```

```
[Out] -1/2520*(252*b^10*c^10 + 280*a*b^9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^
7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6
 + 840*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^1
0*d^10 + 25200*(b^10*c*d^9 - a*b^9*d^10)*x^9 + 56700*(b^10*c^2*d^8 + 2*a*b^
```

```
9*c*d^9 - 3*a^2*b^8*d^10)*x^8 + 50400*(2*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 6
*a^2*b^8*c*d^9 - 11*a^3*b^7*d^10)*x^7 + 44100*(3*b^10*c^4*d^6 + 4*a*b^9*c^3
*d^7 + 6*a^2*b^8*c^2*d^8 + 12*a^3*b^7*c*d^9 - 25*a^4*b^6*d^10)*x^6 + 10584*
(12*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d
^8 + 60*a^4*b^6*c*d^9 - 137*a^5*b^5*d^10)*x^5 + 8820*(10*b^10*c^6*d^4 + 12*
a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^
8 + 60*a^5*b^5*c*d^9 - 147*a^6*b^4*d^10)*x^4 + 720*(60*b^10*c^7*d^3 + 70*a*
b^9*c^6*d^4 + 84*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*d^
7 + 210*a^5*b^5*c^2*d^8 + 420*a^6*b^4*c*d^9 - 1089*a^7*b^3*d^10)*x^3 + 135*
(105*b^10*c^8*d^2 + 120*a*b^9*c^7*d^3 + 140*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c
^5*d^5 + 210*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 +
840*a^7*b^3*c*d^9 - 2283*a^8*b^2*d^10)*x^2 + 10*(280*b^10*c^9*d + 315*a*b^9
*c^8*d^2 + 360*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5
+ 630*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 + 2520*a
^8*b^2*c*d^9 - 7129*a^9*b*d^10)*x)/(b^21*x^10 + 10*a*b^20*x^9 + 45*a^2*b^19
*x^8 + 120*a^3*b^18*x^7 + 210*a^4*b^17*x^6 + 252*a^5*b^16*x^5 + 210*a^6*b^1
5*x^4 + 120*a^7*b^14*x^3 + 45*a^8*b^13*x^2 + 10*a^9*b^12*x + a^10*b^11) + d
^10*log(b*x + a)/b^11
```

**mupad [B]** time = 0.56, size = 866, normalized size = 3.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^11,x)
```

```
[Out] (d^10*log(a + b*x))/b^11 - (x^4*(35*b^10*c^6*d^4 - (1029*a^6*b^4*d^10)/2 +
42*a*b^9*c^5*d^5 + 210*a^5*b^5*c*d^9 + (105*a^2*b^8*c^4*d^6)/2 + 70*a^3*b^7
*c^3*d^7 + 105*a^4*b^6*c^2*d^8) - x^9*(10*a*b^9*d^10 - 10*b^10*c*d^9) + x*(
(10*b^10*c^9*d)/9 - (7129*a^9*b*d^10)/252 + (5*a*b^9*c^8*d^2)/4 + 10*a^8*b^
2*c*d^9 + (10*a^2*b^8*c^7*d^3)/7 + (5*a^3*b^7*c^6*d^4)/3 + 2*a^4*b^6*c^5*d^
5 + (5*a^5*b^5*c^4*d^6)/2 + (10*a^6*b^4*c^3*d^7)/3 + 5*a^7*b^3*c^2*d^8) + x
^6*((105*b^10*c^4*d^6)/2 - (875*a^4*b^6*d^10)/2 + 70*a*b^9*c^3*d^7 + 210*a^
3*b^7*c*d^9 + 105*a^2*b^8*c^2*d^8) + x^8*((45*b^10*c^2*d^8)/2 - (135*a^2*b^
8*d^10)/2 + 45*a*b^9*c*d^9) + x^3*((120*b^10*c^7*d^3)/7 - (2178*a^7*b^3*d^1
0)/7 + 20*a*b^9*c^6*d^4 + 120*a^6*b^4*c*d^9 + 24*a^2*b^8*c^5*d^5 + 30*a^3*b
^7*c^4*d^6 + 40*a^4*b^6*c^3*d^7 + 60*a^5*b^5*c^2*d^8) + x^5*((252*b^10*c^5*
d^5)/5 - (2877*a^5*b^5*d^10)/5 + 63*a*b^9*c^4*d^6 + 252*a^4*b^6*c*d^9 + 84*
a^2*b^8*c^3*d^7 + 126*a^3*b^7*c^2*d^8) - (7381*a^10*d^10)/2520 + (b^10*c^10
)/10 + x^7*(40*b^10*c^3*d^7 - 220*a^3*b^7*d^10 + 60*a*b^9*c^2*d^8 + 120*a^2
*b^8*c*d^9) + x^2*((45*b^10*c^8*d^2)/8 - (6849*a^8*b^2*d^10)/56 + (45*a*b^9
*c^7*d^3)/7 + 45*a^7*b^3*c*d^9 + (15*a^2*b^8*c^6*d^4)/2 + 9*a^3*b^7*c^5*d^5
 + (45*a^4*b^6*c^4*d^6)/4 + 15*a^5*b^5*c^3*d^7 + (45*a^6*b^4*c^2*d^8)/2) +
(a^2*b^8*c^8*d^2)/8 + (a^3*b^7*c^7*d^3)/7 + (a^4*b^6*c^6*d^4)/6 + (a^5*b^5*
```

```
c^5*d^5)/5 + (a^6*b^4*c^4*d^6)/4 + (a^7*b^3*c^3*d^7)/3 + (a^8*b^2*c^2*d^8)/
2 + (a*b^9*c^9*d)/9 + a^9*b*c*d^9)/(b^11*(a + b*x)^10)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**11,x)
```

```
[Out] Timed out
```

## 3.1217 $\int \frac{(c+dx)^{10}}{(a+bx)^{12}} \, dx$

**Optimal**. Leaf size=28

$$-\frac{(c+dx)^{11}}{11(a+bx)^{11}(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^{11}}{11(a+bx)^{11}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] -(c + d*x)^11/(11*(b*c - a*d)*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{12}} \, dx = -\frac{(c+dx)^{11}}{11(bc-ad)(a+bx)^{11}}$$

**Mathematica [B]** time = 0.28, size = 665, normalized size = 23.75

$$-\frac{a^{10}d^{10}+a^9bd^9(c+11dx)+a^8b^2d^8(c^2+11cdx+55d^2x^2)+a^7b^3d^7(c^3+11c^2dx+55cd^2x^2+165d^3x^3)+a^6b^4d^6(c^4+11c^3dx+55c^2d^2x^2+165cd^3x^3+330d^4x^4)+a^5b^5d^5(c^5+11c^4dx+55c^3d^2x^2+165c^2d^3x^3+330cd^4x^4+462d^5x^5)+a^4b^6d^4(c^6+11c^5dx+55c^4d^2x^2+165c^3d^3x^3+330c^2d^4x^4+462cd^5x^5+462d^6x^6)+a^3b^7d^3(c^7+11c^6dx+55c^5d^2x^2+165c^4d^3x^3+330c^3d^4x^4+462c^2d^5x^5+462cd^6x^6+330d^7x^7)+a^2b^8d^2(c^8+11c^7dx+55c^6d^2x^2+165c^5d^3x^3+330c^4d^4x^4+462c^3d^5x^5+462c^2d^6x^6+330cd^7x^7+165d^8x^8)+ab^9d(c^9+11c^8dx+55c^7d^2x^2+165c^6d^3x^3+330c^5d^4x^4+462c^4d^5x^5+462c^3d^6x^6+330c^2d^7x^7+165cd^8x^8+55d^9x^9)+b^{10}(c^{10}+11c^9dx+55c^8d^2x^2+165c^7d^3x^3+330c^6d^4x^4+462c^5d^5x^5+462c^4d^6x^6+330c^3d^7x^7+165c^2d^8x^8+55cd^9x^9+11d^{10}x^{10})}{11b^{11}(a+bx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] -1/11*(a^10*d^10 + a^9*b*d^9*(c + 11*d*x) + a^8*b^2*d^8*(c^2 + 11*c*d*x + 5
5*d^2*x^2) + a^7*b^3*d^7*(c^3 + 11*c^2*d*x + 55*c*d^2*x^2 + 165*d^3*x^3) +
a^6*b^4*d^6*(c^4 + 11*c^3*d*x + 55*c^2*d^2*x^2 + 165*c*d^3*x^3 + 330*d^4*x^
```

```
4) + a^5*b^5*d^5*(c^5 + 11*c^4*d*x + 55*c^3*d^2*x^2 + 165*c^2*d^3*x^3 + 330
*c*d^4*x^4 + 462*d^5*x^5) + a^4*b^6*d^4*(c^6 + 11*c^5*d*x + 55*c^4*d^2*x^2
+ 165*c^3*d^3*x^3 + 330*c^2*d^4*x^4 + 462*c*d^5*x^5 + 462*d^6*x^6) + a^3*b^
7*d^3*(c^7 + 11*c^6*d*x + 55*c^5*d^2*x^2 + 165*c^4*d^3*x^3 + 330*c^3*d^4*x^
4 + 462*c^2*d^5*x^5 + 462*c*d^6*x^6 + 330*d^7*x^7) + a^2*b^8*d^2*(c^8 + 11*
c^7*d*x + 55*c^6*d^2*x^2 + 165*c^5*d^3*x^3 + 330*c^4*d^4*x^4 + 462*c^3*d^5*
x^5 + 462*c^2*d^6*x^6 + 330*c*d^7*x^7 + 165*d^8*x^8) + a*b^9*d*(c^9 + 11*c^
8*d*x + 55*c^7*d^2*x^2 + 165*c^6*d^3*x^3 + 330*c^5*d^4*x^4 + 462*c^4*d^5*x^
5 + 462*c^3*d^6*x^6 + 330*c^2*d^7*x^7 + 165*c*d^8*x^8 + 55*d^9*x^9) + b^10*
(c^10 + 11*c^9*d*x + 55*c^8*d^2*x^2 + 165*c^7*d^3*x^3 + 330*c^6*d^4*x^4 + 4
62*c^5*d^5*x^5 + 462*c^4*d^6*x^6 + 330*c^3*d^7*x^7 + 165*c^2*d^8*x^8 + 55*c
*d^9*x^9 + 11*d^10*x^10))/(b^11*(a + b*x)^11)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^12, x]
```

**fricas [B]** time = 1.35, size = 920, normalized size = 32.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="fricas")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*
b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3
*c^3*d^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10 + 55*(b^10*c*d^9 + a*b
^9*d^10)*x^9 + 165*(b^10*c^2*d^8 + a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 330*(b
^10*c^3*d^7 + a*b^9*c^2*d^8 + a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 462*(b^10
*c^4*d^6 + a*b^9*c^3*d^7 + a^2*b^8*c^2*d^8 + a^3*b^7*c*d^9 + a^4*b^6*d^10)*
x^6 + 462*(b^10*c^5*d^5 + a*b^9*c^4*d^6 + a^2*b^8*c^3*d^7 + a^3*b^7*c^2*d^8
 + a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 330*(b^10*c^6*d^4 + a*b^9*c^5*d^5 +
a^2*b^8*c^4*d^6 + a^3*b^7*c^3*d^7 + a^4*b^6*c^2*d^8 + a^5*b^5*c*d^9 + a^6*b
^4*d^10)*x^4 + 165*(b^10*c^7*d^3 + a*b^9*c^6*d^4 + a^2*b^8*c^5*d^5 + a^3*b^
7*c^4*d^6 + a^4*b^6*c^3*d^7 + a^5*b^5*c^2*d^8 + a^6*b^4*c*d^9 + a^7*b^3*d^1
0)*x^3 + 55*(b^10*c^8*d^2 + a*b^9*c^7*d^3 + a^2*b^8*c^6*d^4 + a^3*b^7*c^5*d
^5 + a^4*b^6*c^4*d^6 + a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8 + a^7*b^3*c*d^9 +
a^8*b^2*d^10)*x^2 + 11*(b^10*c^9*d + a*b^9*c^8*d^2 + a^2*b^8*c^7*d^3 + a^3*
```

b^7*c^6*d^4 + a^4*b^6*c^5*d^5 + a^5*b^5*c^4*d^6 + a^6*b^4*c^3*d^7 + a^7*b^3
*c^2*d^8 + a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^22*x^11 + 11*a*b^21*x^10 + 55*
a^2*b^20*x^9 + 165*a^3*b^19*x^8 + 330*a^4*b^18*x^7 + 462*a^5*b^17*x^6 + 462
*a^6*b^16*x^5 + 330*a^7*b^15*x^4 + 165*a^8*b^14*x^3 + 55*a^9*b^13*x^2 + 11*
a^10*b^12*x + a^11*b^11)

**giac [B]** time = 1.36, size = 951, normalized size = 33.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="giac")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + 55*b^10*c*d^9*x^9 + 55*a*b^9*d^10*x^9 + 165*b^10
*c^2*d^8*x^8 + 165*a*b^9*c*d^9*x^8 + 165*a^2*b^8*d^10*x^8 + 330*b^10*c^3*d^
7*x^7 + 330*a*b^9*c^2*d^8*x^7 + 330*a^2*b^8*c*d^9*x^7 + 330*a^3*b^7*d^10*x^
7 + 462*b^10*c^4*d^6*x^6 + 462*a*b^9*c^3*d^7*x^6 + 462*a^2*b^8*c^2*d^8*x^6
+ 462*a^3*b^7*c*d^9*x^6 + 462*a^4*b^6*d^10*x^6 + 462*b^10*c^5*d^5*x^5 + 462
*a*b^9*c^4*d^6*x^5 + 462*a^2*b^8*c^3*d^7*x^5 + 462*a^3*b^7*c^2*d^8*x^5 + 46
2*a^4*b^6*c*d^9*x^5 + 462*a^5*b^5*d^10*x^5 + 330*b^10*c^6*d^4*x^4 + 330*a*b
^9*c^5*d^5*x^4 + 330*a^2*b^8*c^4*d^6*x^4 + 330*a^3*b^7*c^3*d^7*x^4 + 330*a^
4*b^6*c^2*d^8*x^4 + 330*a^5*b^5*c*d^9*x^4 + 330*a^6*b^4*d^10*x^4 + 165*b^10
*c^7*d^3*x^3 + 165*a*b^9*c^6*d^4*x^3 + 165*a^2*b^8*c^5*d^5*x^3 + 165*a^3*b^
7*c^4*d^6*x^3 + 165*a^4*b^6*c^3*d^7*x^3 + 165*a^5*b^5*c^2*d^8*x^3 + 165*a^6
*b^4*c*d^9*x^3 + 165*a^7*b^3*d^10*x^3 + 55*b^10*c^8*d^2*x^2 + 55*a*b^9*c^7*
d^3*x^2 + 55*a^2*b^8*c^6*d^4*x^2 + 55*a^3*b^7*c^5*d^5*x^2 + 55*a^4*b^6*c^4*
d^6*x^2 + 55*a^5*b^5*c^3*d^7*x^2 + 55*a^6*b^4*c^2*d^8*x^2 + 55*a^7*b^3*c*d^
9*x^2 + 55*a^8*b^2*d^10*x^2 + 11*b^10*c^9*d*x + 11*a*b^9*c^8*d^2*x + 11*a^2
*b^8*c^7*d^3*x + 11*a^3*b^7*c^6*d^4*x + 11*a^4*b^6*c^5*d^5*x + 11*a^5*b^5*c
^4*d^6*x + 11*a^6*b^4*c^3*d^7*x + 11*a^7*b^3*c^2*d^8*x + 11*a^8*b^2*c*d^9*x
 + 11*a^9*b*d^10*x + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*b^7*c^
7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3*c^3*d
^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^11*b^11)
```

**maple [B]** time = 0.01, size = 866, normalized size = 30.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^12,x)
```

```
[Out] 15*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*
b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^8-15*d^8
*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^3-30*d^4*(a^6*d^6-6*a^5*b*c*d^5+1
```

```
5*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c
^6)/b^11/(b*x+a)^7-5*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b
^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7
*c^7*d+b^8*c^8)/b^11/(b*x+a)^9-42*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*
d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^5+5*d^9*(a*d-b*c)/b^11/(b*x+a)^2+30
*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^4-1/11*(a^1
0*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^
4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^
8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^11-d^10/b^11/(b*x+a)+42*d^
5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*
d-b^5*c^5)/b^11/(b*x+a)^6+d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^
6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36
*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^10
```

**maxima [B]** time = 2.13, size = 920, normalized size = 32.86

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="maxima")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*
b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3
*c^3*d^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10 + 55*(b^10*c*d^9 + a*b
^9*d^10)*x^9 + 165*(b^10*c^2*d^8 + a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 330*(b
^10*c^3*d^7 + a*b^9*c^2*d^8 + a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 462*(b^10
*c^4*d^6 + a*b^9*c^3*d^7 + a^2*b^8*c^2*d^8 + a^3*b^7*c*d^9 + a^4*b^6*d^10)*
x^6 + 462*(b^10*c^5*d^5 + a*b^9*c^4*d^6 + a^2*b^8*c^3*d^7 + a^3*b^7*c^2*d^8
 + a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 330*(b^10*c^6*d^4 + a*b^9*c^5*d^5 +
a^2*b^8*c^4*d^6 + a^3*b^7*c^3*d^7 + a^4*b^6*c^2*d^8 + a^5*b^5*c*d^9 + a^6*b
^4*d^10)*x^4 + 165*(b^10*c^7*d^3 + a*b^9*c^6*d^4 + a^2*b^8*c^5*d^5 + a^3*b^
7*c^4*d^6 + a^4*b^6*c^3*d^7 + a^5*b^5*c^2*d^8 + a^6*b^4*c*d^9 + a^7*b^3*d^1
0)*x^3 + 55*(b^10*c^8*d^2 + a*b^9*c^7*d^3 + a^2*b^8*c^6*d^4 + a^3*b^7*c^5*d
^5 + a^4*b^6*c^4*d^6 + a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8 + a^7*b^3*c*d^9 +
a^8*b^2*d^10)*x^2 + 11*(b^10*c^9*d + a*b^9*c^8*d^2 + a^2*b^8*c^7*d^3 + a^3*
b^7*c^6*d^4 + a^4*b^6*c^5*d^5 + a^5*b^5*c^4*d^6 + a^6*b^4*c^3*d^7 + a^7*b^3
*c^2*d^8 + a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^22*x^11 + 11*a*b^21*x^10 + 55*
a^2*b^20*x^9 + 165*a^3*b^19*x^8 + 330*a^4*b^18*x^7 + 462*a^5*b^17*x^6 + 462
*a^6*b^16*x^5 + 330*a^7*b^15*x^4 + 165*a^8*b^14*x^3 + 55*a^9*b^13*x^2 + 11*
a^10*b^12*x + a^11*b^11)
```

**mupad [B]** time = 0.46, size = 1066, normalized size = 38.07

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^12,x)
```

```
[Out] -(a^10*d^10 + b^10*c^10 + 11*b^10*d^10*x^10 + 55*a*b^9*d^10*x^9 + 55*b^10*c
*d^9*x^9 + a^2*b^8*c^8*d^2 + a^3*b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^
5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3*c^3*d^7 + a^8*b^2*c^2*d^8 + 55*a^8*b^2*d^
10*x^2 + 165*a^7*b^3*d^10*x^3 + 330*a^6*b^4*d^10*x^4 + 462*a^5*b^5*d^10*x^5
 + 462*a^4*b^6*d^10*x^6 + 330*a^3*b^7*d^10*x^7 + 165*a^2*b^8*d^10*x^8 + 55*
b^10*c^8*d^2*x^2 + 165*b^10*c^7*d^3*x^3 + 330*b^10*c^6*d^4*x^4 + 462*b^10*c
^5*d^5*x^5 + 462*b^10*c^4*d^6*x^6 + 330*b^10*c^3*d^7*x^7 + 165*b^10*c^2*d^8
*x^8 + a*b^9*c^9*d + a^9*b*c*d^9 + 11*a^9*b*d^10*x + 11*b^10*c^9*d*x + 55*a
^2*b^8*c^6*d^4*x^2 + 55*a^3*b^7*c^5*d^5*x^2 + 55*a^4*b^6*c^4*d^6*x^2 + 55*a
^5*b^5*c^3*d^7*x^2 + 55*a^6*b^4*c^2*d^8*x^2 + 165*a^2*b^8*c^5*d^5*x^3 + 165
*a^3*b^7*c^4*d^6*x^3 + 165*a^4*b^6*c^3*d^7*x^3 + 165*a^5*b^5*c^2*d^8*x^3 +
330*a^2*b^8*c^4*d^6*x^4 + 330*a^3*b^7*c^3*d^7*x^4 + 330*a^4*b^6*c^2*d^8*x^4
 + 462*a^2*b^8*c^3*d^7*x^5 + 462*a^3*b^7*c^2*d^8*x^5 + 462*a^2*b^8*c^2*d^8*
x^6 + 11*a*b^9*c^8*d^2*x + 11*a^8*b^2*c*d^9*x + 165*a*b^9*c*d^9*x^8 + 11*a^
2*b^8*c^7*d^3*x + 11*a^3*b^7*c^6*d^4*x + 11*a^4*b^6*c^5*d^5*x + 11*a^5*b^5*
c^4*d^6*x + 11*a^6*b^4*c^3*d^7*x + 11*a^7*b^3*c^2*d^8*x + 55*a*b^9*c^7*d^3*
x^2 + 55*a^7*b^3*c*d^9*x^2 + 165*a*b^9*c^6*d^4*x^3 + 165*a^6*b^4*c*d^9*x^3
+ 330*a*b^9*c^5*d^5*x^4 + 330*a^5*b^5*c*d^9*x^4 + 462*a*b^9*c^4*d^6*x^5 + 4
62*a^4*b^6*c*d^9*x^5 + 462*a*b^9*c^3*d^7*x^6 + 462*a^3*b^7*c*d^9*x^6 + 330*
a*b^9*c^2*d^8*x^7 + 330*a^2*b^8*c*d^9*x^7)/(11*a^11*b^11 + 11*b^22*x^11 + 1
21*a^10*b^12*x + 121*a*b^21*x^10 + 605*a^9*b^13*x^2 + 1815*a^8*b^14*x^3 + 3
630*a^7*b^15*x^4 + 5082*a^6*b^16*x^5 + 5082*a^5*b^17*x^6 + 3630*a^4*b^18*x^
7 + 1815*a^3*b^19*x^8 + 605*a^2*b^20*x^9)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**12,x)
```

```
[Out] Timed out
```

## 3.1218 $\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\, dx$

**Optimal**. Leaf size=58

$$\frac{d(c+dx)^{11}}{132(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^{11}}{12(a+bx)^{12}(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d(c+dx)^{11}}{132(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^{11}}{12(a+bx)^{12}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] -(c + d*x)^11/(12*(b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^11)/(132*(b*c -
a*d)^2*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx = -\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}} - \frac{d\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx}{12(bc-ad)}$$
$$= -\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^{11}}{132(bc-ad)^2(a+bx)^{11}}$$

**Mathematica** [**B**] time = 0.28, size = 684, normalized size = 11.79

$$-\frac{a^{10}d^{10} + 2a^9bd^9(c+6dx) + 3a^8b^2d^8\left(c^2+8cdx+22d^2x^2\right) + 4a^7b^3d^7\left(c^3+9c^2dx+33cd^2x^2+55d^3x^3\right) + a^6b^4d^6\left(5c^4+48c^3dx+198c^2d^2x^2+440cd^3x^3+495d^4x^4\right) + 6a^5b^5d^5\left(c^5+10c^4dx+44c^3d^2x^2+110c^2d^3x^3+165cd^4x^4+132d^5x^5\right) + a^4b^6d^4\left(7c^6+72c^5dx+330c^4d^2x^2+880c^3d^3x^3+1485c^2d^4x^4+1584cd^5x^5+924d^6x^6\right) + 4a^3b^7d^3\left(2c^7+21c^6dx+99c^5d^2x^2+275c^4d^3x^3+495c^3d^4x^4+594c^2d^5x^5+462cd^6x^6+198d^7x^7\right) + 3a^2b^8d^2\left(3c^8+32c^7dx+154c^6d^2x^2+440c^5d^3x^3+825c^4d^4x^4+1056c^3d^5x^5+924c^2d^6x^6+528cd^7x^7+165d^8x^8\right) + 2ab^9d\left(5c^9+54c^8dx+264c^7d^2x^2+770c^6d^3x^3+1485c^5d^4x^4+1980c^4d^5x^5+1848c^3d^6x^6+1188c^2d^7x^7+495cd^8x^8+110d^9x^9\right) + b^{10}\left(11c^{10}+120c^9dx+594c^8d^2x^2+1760c^7d^3x^3+3465c^6d^4x^4+4752c^5d^5x^5+4620c^4d^6x^6+3168c^3d^7x^7+1485c^2d^8x^8+440cd^9x^9+66d^{10}x^{10}\right)}{132b^{11}(a+bx)^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] -1/132*(a^10*d^10 + 2*a^9*b*d^9*(c + 6*d*x) + 3*a^8*b^2*d^8*(c^2 + 8*c*d*x
+ 22*d^2*x^2) + 4*a^7*b^3*d^7*(c^3 + 9*c^2*d*x + 33*c*d^2*x^2 + 55*d^3*x^3)
 + a^6*b^4*d^6*(5*c^4 + 48*c^3*d*x + 198*c^2*d^2*x^2 + 440*c*d^3*x^3 + 495*
d^4*x^4) + 6*a^5*b^5*d^5*(c^5 + 10*c^4*d*x + 44*c^3*d^2*x^2 + 110*c^2*d^3*x
^3 + 165*c*d^4*x^4 + 132*d^5*x^5) + a^4*b^6*d^4*(7*c^6 + 72*c^5*d*x + 330*c
^4*d^2*x^2 + 880*c^3*d^3*x^3 + 1485*c^2*d^4*x^4 + 1584*c*d^5*x^5 + 924*d^6*
x^6) + 4*a^3*b^7*d^3*(2*c^7 + 21*c^6*d*x + 99*c^5*d^2*x^2 + 275*c^4*d^3*x^3
 + 495*c^3*d^4*x^4 + 594*c^2*d^5*x^5 + 462*c*d^6*x^6 + 198*d^7*x^7) + 3*a^2
*b^8*d^2*(3*c^8 + 32*c^7*d*x + 154*c^6*d^2*x^2 + 440*c^5*d^3*x^3 + 825*c^4*
d^4*x^4 + 1056*c^3*d^5*x^5 + 924*c^2*d^6*x^6 + 528*c*d^7*x^7 + 165*d^8*x^8)
 + 2*a*b^9*d*(5*c^9 + 54*c^8*d*x + 264*c^7*d^2*x^2 + 770*c^6*d^3*x^3 + 1485
*c^5*d^4*x^4 + 1980*c^4*d^5*x^5 + 1848*c^3*d^6*x^6 + 1188*c^2*d^7*x^7 + 495
*c*d^8*x^8 + 110*d^9*x^9) + b^10*(11*c^10 + 120*c^9*d*x + 594*c^8*d^2*x^2 +
 1760*c^7*d^3*x^3 + 3465*c^6*d^4*x^4 + 4752*c^5*d^5*x^5 + 4620*c^4*d^6*x^6
+ 3168*c^3*d^7*x^7 + 1485*c^2*d^8*x^8 + 440*c*d^9*x^9 + 66*d^10*x^10))/(b^1
1*(a + b*x)^12)
```

**IntegrateAlgebraic** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^13, x]
```

**fricas** [**B**] time = 1.29, size = 986, normalized size = 17.00

$$-\frac{66b^{10}d^{10}x^{10} + 11b^{10}c^{10} + 10ab^9c^9d + 9a^2b^8c^8d^2 + 8a^3b^7c^7d^3 + 7a^4b^6c^6d^4 + 6a^5b^5c^5d^5 + 5a^6b^4c^4d^6 + 4a^7b^3c^3d^7 + 3a^8b^2c^2d^8 + 2a^9bcd^9 + a^{10}d^{10} + 220\left(2b^{10}cd^9 + ab^9d^{10}\right)x^9 + 495\left(3b^{10}c^2d^8 + 2ab^9cd^9 + a^2b^8d^{10}\right)x^8 + 792\left(4b^{10}c^3d^7 + 3ab^9c^2d^8 + 2a^2b^8cd^9 + a^3b^7d^{10}\right)x^7 + 924\left(5b^{10}c^4d^6 + 4ab^9c^3d^7 + 3a^2b^8c^2d^8 + 2a^3b^7cd^9 + a^4b^6d^{10}\right)x^6 + 792\left(6b^{10}c^5d^5 + 5ab^9c^4d^6 + 4a^2b^8c^3d^7 + 3a^3b^7c^2d^8 + 2a^4b^6cd^9 + a^5b^5d^{10}\right)x^5 + 495\left(7b^{10}c^6d^4 + 6ab^9c^5d^5 + 5a^2b^8c^4d^6 + 4a^3b^7c^3d^7 + 3a^4b^6c^2d^8 + 2a^5b^5cd^9 + a^6b^4d^{10}\right)x^4 + 220\left(8b^{10}c^7d^3 + 7ab^9c^6d^4 + 6a^2b^8c^5d^5 + 5a^3b^7c^4d^6 + 4a^4b^6c^3d^7 + 3a^5b^5c^2d^8 + 2a^6b^4cd^9 + a^7b^3d^{10}\right)x^3 + 66\left(9b^{10}c^8d^2 + 8ab^9c^7d^3 + 7a^2b^8c^6d^4 + 6a^3b^7c^5d^5 + 5a^4b^6c^4d^6 + 4a^5b^5c^3d^7 + 3a^6b^4c^2d^8 + 2a^7b^3cd^9 + a^8b^2d^{10}\right)x^2 + 12\left(10b^{10}c^9d + 9ab^9c^8d^2 + 8a^2b^8c^7d^3 + 7a^3b^7c^6d^4 + 6a^4b^6c^5d^5 + 5a^5b^5c^4d^6 + 4a^6b^4c^3d^7 + 3a^7b^3c^2d^8 + 2a^8b^2cd^9 + a^9bd^{10}\right)x}{132\left(b^{23}x^{12} + 12ab^{22}x^{11} + 66a^2b^{21}x^{10} + 220a^3b^{20}x^9 + 495a^4b^{19}x^8 + 792a^5b^{18}x^7 + 924a^6b^{17}x^6 + 792a^7b^{16}x^5 + 495a^8b^{15}x^4 + 220a^9b^{14}x^3 + 66a^{10}b^{13}x^2 + 12a^{11}b^{12}x + a^{12}b^{11}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="fricas")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 11*b^10*c^10 + 10*a*b^9*c^9*d + 9*a^2*b^8*c^8*d
^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*a^5*b^5*c^5*d^5 + 5*a^6*b^4*
c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9 + a^10*d^10
 + 220*(2*b^10*c*d^9 + a*b^9*d^10)*x^9 + 495*(3*b^10*c^2*d^8 + 2*a*b^9*c*d^
9 + a^2*b^8*d^10)*x^8 + 792*(4*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 2*a^2*b^8*c
*d^9 + a^3*b^7*d^10)*x^7 + 924*(5*b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 + 3*a^2*b^
8*c^2*d^8 + 2*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 792*(6*b^10*c^5*d^5 + 5*a
*b^9*c^4*d^6 + 4*a^2*b^8*c^3*d^7 + 3*a^3*b^7*c^2*d^8 + 2*a^4*b^6*c*d^9 + a^
5*b^5*d^10)*x^5 + 495*(7*b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
 + 4*a^3*b^7*c^3*d^7 + 3*a^4*b^6*c^2*d^8 + 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*
x^4 + 220*(8*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 6*a^2*b^8*c^5*d^5 + 5*a^3*b^7
*c^4*d^6 + 4*a^4*b^6*c^3*d^7 + 3*a^5*b^5*c^2*d^8 + 2*a^6*b^4*c*d^9 + a^7*b^
3*d^10)*x^3 + 66*(9*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 7*a^2*b^8*c^6*d^4 + 6*
a^3*b^7*c^5*d^5 + 5*a^4*b^6*c^4*d^6 + 4*a^5*b^5*c^3*d^7 + 3*a^6*b^4*c^2*d^8
 + 2*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 12*(10*b^10*c^9*d + 9*a*b^9*c^8*d^
2 + 8*a^2*b^8*c^7*d^3 + 7*a^3*b^7*c^6*d^4 + 6*a^4*b^6*c^5*d^5 + 5*a^5*b^5*c
^4*d^6 + 4*a^6*b^4*c^3*d^7 + 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^23*x^12 + 12*a*b^22*x^11 + 66*a^2*b^21*x^10 + 220*a^3*b^20*x^9 +
495*a^4*b^19*x^8 + 792*a^5*b^18*x^7 + 924*a^6*b^17*x^6 + 792*a^7*b^16*x^5 +
 495*a^8*b^15*x^4 + 220*a^9*b^14*x^3 + 66*a^10*b^13*x^2 + 12*a^11*b^12*x +
a^12*b^11)
```

**giac [B]** time = 1.32, size = 961, normalized size = 16.57

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="giac")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 440*b^10*c*d^9*x^9 + 220*a*b^9*d^10*x^9 + 1485*
b^10*c^2*d^8*x^8 + 990*a*b^9*c*d^9*x^8 + 495*a^2*b^8*d^10*x^8 + 3168*b^10*c
^3*d^7*x^7 + 2376*a*b^9*c^2*d^8*x^7 + 1584*a^2*b^8*c*d^9*x^7 + 792*a^3*b^7*
d^10*x^7 + 4620*b^10*c^4*d^6*x^6 + 3696*a*b^9*c^3*d^7*x^6 + 2772*a^2*b^8*c^
2*d^8*x^6 + 1848*a^3*b^7*c*d^9*x^6 + 924*a^4*b^6*d^10*x^6 + 4752*b^10*c^5*d
^5*x^5 + 3960*a*b^9*c^4*d^6*x^5 + 3168*a^2*b^8*c^3*d^7*x^5 + 2376*a^3*b^7*c
^2*d^8*x^5 + 1584*a^4*b^6*c*d^9*x^5 + 792*a^5*b^5*d^10*x^5 + 3465*b^10*c^6*
d^4*x^4 + 2970*a*b^9*c^5*d^5*x^4 + 2475*a^2*b^8*c^4*d^6*x^4 + 1980*a^3*b^7*
c^3*d^7*x^4 + 1485*a^4*b^6*c^2*d^8*x^4 + 990*a^5*b^5*c*d^9*x^4 + 495*a^6*b^
4*d^10*x^4 + 1760*b^10*c^7*d^3*x^3 + 1540*a*b^9*c^6*d^4*x^3 + 1320*a^2*b^8*
c^5*d^5*x^3 + 1100*a^3*b^7*c^4*d^6*x^3 + 880*a^4*b^6*c^3*d^7*x^3 + 660*a^5*
b^5*c^2*d^8*x^3 + 440*a^6*b^4*c*d^9*x^3 + 220*a^7*b^3*d^10*x^3 + 594*b^10*c
```

```
^8*d^2*x^2 + 528*a*b^9*c^7*d^3*x^2 + 462*a^2*b^8*c^6*d^4*x^2 + 396*a^3*b^7*
c^5*d^5*x^2 + 330*a^4*b^6*c^4*d^6*x^2 + 264*a^5*b^5*c^3*d^7*x^2 + 198*a^6*b
^4*c^2*d^8*x^2 + 132*a^7*b^3*c*d^9*x^2 + 66*a^8*b^2*d^10*x^2 + 120*b^10*c^9
*d*x + 108*a*b^9*c^8*d^2*x + 96*a^2*b^8*c^7*d^3*x + 84*a^3*b^7*c^6*d^4*x +
72*a^4*b^6*c^5*d^5*x + 60*a^5*b^5*c^4*d^6*x + 48*a^6*b^4*c^3*d^7*x + 36*a^7
*b^3*c^2*d^8*x + 24*a^8*b^2*c*d^9*x + 12*a^9*b*d^10*x + 11*b^10*c^10 + 10*a
*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*
a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8
 + 2*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^12*b^11)
```

**maple [B]** time = 0.01, size = 867, normalized size = 14.95

$$-\frac{d^{10}}{2\left(bx+a\right)^{2}b^{11}}+\frac{10\left(ad-bc\right)d^{9}}{3\left(bx+a\right)^{3}b^{11}}-\frac{45\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)d^{8}}{4\left(bx+a\right)^{4}b^{11}}+\frac{24\left(a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}\right)d^{7}}{\left(bx+a\right)^{5}b^{11}}-\frac{35\left(a^{4}d^{4}-4a^{3}bcd^{3}+6a^{2}b^{2}c^{2}d^{2}-4ab^{3}c^{3}d+b^{4}c^{4}\right)d^{6}}{\left(bx+a\right)^{6}b^{11}}+\frac{36\left(a^{5}d^{5}-5a^{4}bcd^{4}+10a^{3}b^{2}c^{2}d^{3}-10a^{2}b^{3}c^{3}d^{2}+5ab^{4}c^{4}d-b^{5}c^{5}\right)d^{5}}{\left(bx+a\right)^{7}b^{11}}-\frac{105\left(a^{6}d^{6}-6a^{5}bcd^{5}+15a^{4}b^{2}c^{2}d^{4}-20a^{3}b^{3}c^{3}d^{3}+15a^{2}b^{4}c^{4}d^{2}-6ab^{5}c^{5}d+b^{6}c^{6}\right)d^{4}}{4\left(bx+a\right)^{8}b^{11}}+\frac{40\left(a^{7}d^{7}-7a^{6}bcd^{6}+21a^{5}b^{2}c^{2}d^{5}-35a^{4}b^{3}c^{3}d^{4}+35a^{3}b^{4}c^{4}d^{3}-21a^{2}b^{5}c^{5}d^{2}+7ab^{6}c^{6}d-b^{7}c^{7}\right)d^{3}}{3\left(bx+a\right)^{9}b^{11}}-\frac{9\left(a^{8}d^{8}-8a^{7}bcd^{7}+28a^{6}b^{2}c^{2}d^{6}-56a^{5}b^{3}c^{3}d^{5}+70a^{4}b^{4}c^{4}d^{4}-56a^{3}b^{5}c^{5}d^{3}+28a^{2}b^{6}c^{6}d^{2}-8ab^{7}c^{7}d+b^{8}c^{8}\right)d^{2}}{2\left(bx+a\right)^{10}b^{11}}+\frac{10\left(a^{9}d^{9}-9a^{8}bcd^{8}+36a^{7}b^{2}c^{2}d^{7}-84a^{6}b^{3}c^{3}d^{6}+126a^{5}b^{4}c^{4}d^{5}-126a^{4}b^{5}c^{5}d^{4}+84a^{3}b^{6}c^{6}d^{3}-36a^{2}b^{7}c^{7}d^{2}+9ab^{8}c^{8}d-b^{9}c^{9}\right)d}{11\left(bx+a\right)^{11}b^{11}}-\frac{d^{10}a^{10}-10cd^{9}a^{9}b+45c^{2}d^{8}a^{8}b^{2}-120a^{7}c^{3}d^{7}b^{3}+210c^{4}d^{6}a^{6}b^{4}-252a^{5}c^{5}d^{5}b^{5}+210a^{4}c^{6}d^{4}b^{6}-120a^{3}c^{7}d^{3}b^{7}+45a^{2}c^{8}d^{2}b^{8}-10ab^{9}c^{9}d+c^{10}b^{10}}{12\left(bx+a\right)^{12}b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^13,x)
```

```
[Out] -105/4*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*
a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^8+10/3*d^9*(a*d-b*c)/b^
11/(b*x+a)^3+36*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^
3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^7+40/3*d^3*(a^7*d^7-7*a^6*b*c*d^6
+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^
2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^9+24*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b
^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^5-1/2*d^10/b^11/(b*x+a)^2-45/4*d^8*(a^2*d^2-
2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^4+10/11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2
*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*
b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^11-1/12*
(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^
4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^
2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^12-35*d^6*(a^4*d^4-4*a
^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^6-9/2*d^2*
(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4
*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x
+a)^10
```

**maxima [B]** time = 2.17, size = 986, normalized size = 17.00

$$-\frac{66b^{10}d^{10}x^{10}+11b^{10}c^{10}+10ab^{9}c^{9}d+9a^{2}b^{8}c^{8}d^{2}+8a^{3}b^{7}c^{7}d^{3}+7a^{4}b^{6}c^{6}d^{4}+6a^{5}b^{5}c^{5}d^{5}+5a^{6}b^{4}c^{4}d^{6}+4a^{7}b^{3}c^{3}d^{7}+3a^{8}b^{2}c^{2}d^{8}+2a^{9}bcd^{9}+a^{10}d^{10}+220\left(2b^{10}cd^{9}+ab^{9}d^{10}\right)x^{9}+495\left(3b^{10}c^{2}d^{8}+2ab^{9}cd^{9}+a^{2}b^{8}d^{10}\right)x^{8}+792\left(4b^{10}c^{3}d^{7}+3ab^{9}c^{2}d^{8}+2a^{2}b^{8}cd^{9}+a^{3}b^{7}d^{10}\right)x^{7}+924\left(5b^{10}c^{4}d^{6}+4ab^{9}c^{3}d^{7}+3a^{2}b^{8}c^{2}d^{8}+2a^{3}b^{7}cd^{9}+a^{4}b^{6}d^{10}\right)x^{6}+792\left(6b^{10}c^{5}d^{5}+5ab^{9}c^{4}d^{6}+4a^{2}b^{8}c^{3}d^{7}+3a^{3}b^{7}c^{2}d^{8}+2a^{4}b^{6}cd^{9}+a^{5}b^{5}d^{10}\right)x^{5}+495\left(7b^{10}c^{6}d^{4}+6ab^{9}c^{5}d^{5}+5a^{2}b^{8}c^{4}d^{6}+4a^{3}b^{7}c^{3}d^{7}+3a^{4}b^{6}c^{2}d^{8}+2a^{5}b^{5}cd^{9}+a^{6}b^{4}d^{10}\right)x^{4}+220\left(8b^{10}c^{7}d^{3}+7ab^{9}c^{6}d^{4}+6a^{2}b^{8}c^{5}d^{5}+5a^{3}b^{7}c^{4}d^{6}+4a^{4}b^{6}c^{3}d^{7}+3a^{5}b^{5}c^{2}d^{8}+2a^{6}b^{4}cd^{9}+a^{7}b^{3}d^{10}\right)x^{3}+66\left(9b^{10}c^{8}d^{2}+8ab^{9}c^{7}d^{3}+7a^{2}b^{8}c^{6}d^{4}+6a^{3}b^{7}c^{5}d^{5}+5a^{4}b^{6}c^{4}d^{6}+4a^{5}b^{5}c^{3}d^{7}+3a^{6}b^{4}c^{2}d^{8}+2a^{7}b^{3}cd^{9}+a^{8}b^{2}d^{10}\right)x^{2}+12\left(10b^{10}c^{9}d+9ab^{9}c^{8}d^{2}+8a^{2}b^{8}c^{7}d^{3}+7a^{3}b^{7}c^{6}d^{4}+6a^{4}b^{6}c^{5}d^{5}+5a^{5}b^{5}c^{4}d^{6}+4a^{6}b^{4}c^{3}d^{7}+3a^{7}b^{3}c^{2}d^{8}+2a^{8}b^{2}cd^{9}+a^{9}bd^{10}\right)x}{132\left(b^{23}x^{12}+12ab^{22}x^{11}+66a^{2}b^{21}x^{10}+220a^{3}b^{20}x^{9}+495a^{4}b^{19}x^{8}+792a^{5}b^{18}x^{7}+924a^{6}b^{17}x^{6}+792a^{7}b^{16}x^{5}+495a^{8}b^{15}x^{4}+220a^{9}b^{14}x^{3}+66a^{10}b^{13}x^{2}+12a^{11}b^{12}x+a^{12}b^{11}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="maxima")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 11*b^10*c^10 + 10*a*b^9*c^9*d + 9*a^2*b^8*c^8*d
^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*a^5*b^5*c^5*d^5 + 5*a^6*b^4*
c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9 + a^10*d^10
```

```
 + 220*(2*b^10*c*d^9 + a*b^9*d^10)*x^9 + 495*(3*b^10*c^2*d^8 + 2*a*b^9*c*d^
9 + a^2*b^8*d^10)*x^8 + 792*(4*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 2*a^2*b^8*c
*d^9 + a^3*b^7*d^10)*x^7 + 924*(5*b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 + 3*a^2*b^
8*c^2*d^8 + 2*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 792*(6*b^10*c^5*d^5 + 5*a
*b^9*c^4*d^6 + 4*a^2*b^8*c^3*d^7 + 3*a^3*b^7*c^2*d^8 + 2*a^4*b^6*c*d^9 + a^
5*b^5*d^10)*x^5 + 495*(7*b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
 + 4*a^3*b^7*c^3*d^7 + 3*a^4*b^6*c^2*d^8 + 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*
x^4 + 220*(8*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 6*a^2*b^8*c^5*d^5 + 5*a^3*b^7
*c^4*d^6 + 4*a^4*b^6*c^3*d^7 + 3*a^5*b^5*c^2*d^8 + 2*a^6*b^4*c*d^9 + a^7*b^
3*d^10)*x^3 + 66*(9*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 7*a^2*b^8*c^6*d^4 + 6*
a^3*b^7*c^5*d^5 + 5*a^4*b^6*c^4*d^6 + 4*a^5*b^5*c^3*d^7 + 3*a^6*b^4*c^2*d^8
 + 2*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 12*(10*b^10*c^9*d + 9*a*b^9*c^8*d^
2 + 8*a^2*b^8*c^7*d^3 + 7*a^3*b^7*c^6*d^4 + 6*a^4*b^6*c^5*d^5 + 5*a^5*b^5*c
^4*d^6 + 4*a^6*b^4*c^3*d^7 + 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^23*x^12 + 12*a*b^22*x^11 + 66*a^2*b^21*x^10 + 220*a^3*b^20*x^9 +
495*a^4*b^19*x^8 + 792*a^5*b^18*x^7 + 924*a^6*b^17*x^6 + 792*a^7*b^16*x^5 +
 495*a^8*b^15*x^4 + 220*a^9*b^14*x^3 + 66*a^10*b^13*x^2 + 12*a^11*b^12*x +
a^12*b^11)
```

**mupad [B]** time = 0.39, size = 39, normalized size = 0.67

$$\frac{(c+d\,x)^{11}\,\left(12\,a\,d-11\,b\,c+b\,d\,x\right)}{132\,(a\,d-b\,c)^2\,(a+b\,x)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^13,x)
[Out] ((c + d*x)^11*(12*a*d - 11*b*c + b*d*x))/(132*(a*d - b*c)^2*(a + b*x)^12)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**13,x)
[Out] Timed out
```

## 3.1219 $\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx$

**Optimal**. Leaf size=89

$$-\frac{d^2(c+dx)^{11}}{858(a+bx)^{11}(bc-ad)^3}+\frac{d(c+dx)^{11}}{78(a+bx)^{12}(bc-ad)^2}-\frac{(c+dx)^{11}}{13(a+bx)^{13}(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^2(c+dx)^{11}}{858(a+bx)^{11}(bc-ad)^3}+\frac{d(c+dx)^{11}}{78(a+bx)^{12}(bc-ad)^2}-\frac{(c+dx)^{11}}{13(a+bx)^{13}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^14,x]
```

```
[Out] -(c + d*x)^11/(13*(b*c - a*d)*(a + b*x)^13) + (d*(c + d*x)^11)/(78*(b*c - a
*d)^2*(a + b*x)^12) - (d^2*(c + d*x)^11)/(858*(b*c - a*d)^3*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx = -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}} - \frac{(2d)\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx}{13(bc-ad)}$$
$$= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}} + \frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}} + \frac{d^2\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx}{78(bc-ad)^2}$$
$$= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}} + \frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}} - \frac{d^2(c+dx)^{11}}{858(bc-ad)^3(a+bx)^{11}}$$

**Mathematica [B]** time = 0.29, size = 690, normalized size = 7.75

---

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^14,x]
```

```
[Out] -1/858*(a^10*d^10 + a^9*b*d^9*(3*c + 13*d*x) + 3*a^8*b^2*d^8*(2*c^2 + 13*c*
d*x + 26*d^2*x^2) + 2*a^7*b^3*d^7*(5*c^3 + 39*c^2*d*x + 117*c*d^2*x^2 + 143
*d^3*x^3) + a^6*b^4*d^6*(15*c^4 + 130*c^3*d*x + 468*c^2*d^2*x^2 + 858*c*d^3
*x^3 + 715*d^4*x^4) + 3*a^5*b^5*d^5*(7*c^5 + 65*c^4*d*x + 260*c^3*d^2*x^2 +
 572*c^2*d^3*x^3 + 715*c*d^4*x^4 + 429*d^5*x^5) + a^4*b^6*d^4*(28*c^6 + 273
*c^5*d*x + 1170*c^4*d^2*x^2 + 2860*c^3*d^3*x^3 + 4290*c^2*d^4*x^4 + 3861*c*
d^5*x^5 + 1716*d^6*x^6) + 2*a^3*b^7*d^3*(18*c^7 + 182*c^6*d*x + 819*c^5*d^2
*x^2 + 2145*c^4*d^3*x^3 + 3575*c^3*d^4*x^4 + 3861*c^2*d^5*x^5 + 2574*c*d^6*
x^6 + 858*d^7*x^7) + 3*a^2*b^8*d^2*(15*c^8 + 156*c^7*d*x + 728*c^6*d^2*x^2
+ 2002*c^5*d^3*x^3 + 3575*c^4*d^4*x^4 + 4290*c^3*d^5*x^5 + 3432*c^2*d^6*x^6
 + 1716*c*d^7*x^7 + 429*d^8*x^8) + a*b^9*d*(55*c^9 + 585*c^8*d*x + 2808*c^7
*d^2*x^2 + 8008*c^6*d^3*x^3 + 15015*c^5*d^4*x^4 + 19305*c^4*d^5*x^5 + 17160
*c^3*d^6*x^6 + 10296*c^2*d^7*x^7 + 3861*c*d^8*x^8 + 715*d^9*x^9) + b^10*(66
*c^10 + 715*c^9*d*x + 3510*c^8*d^2*x^2 + 10296*c^7*d^3*x^3 + 20020*c^6*d^4*
x^4 + 27027*c^5*d^5*x^5 + 25740*c^4*d^6*x^6 + 17160*c^3*d^7*x^7 + 7722*c^2*
d^8*x^8 + 2145*c*d^9*x^9 + 286*d^10*x^10))/(b^11*(a + b*x)^13)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^14,x]
```

[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^14, x]

**fricas [B]** time = 1.26, size = 997, normalized size = 11.20

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="fricas")

[Out] -1/858*(286*b^10*d^10*x^10 + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^10*d^10 + 715*(3*b^10*c*d^9 + a*b^9*d^10)*x^9 + 1287*(6*b^10*c^2*d^8 + 3*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1716*(10*b^10*c^3*d^7 + 6*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 1716*(15*b^10*c^4*d^6 + 10*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 1287*(21*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 + 3*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 715*(28*b^10*c^6*d^4 + 21*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 6*a^4*b^6*c^2*d^8 + 3*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 286*(36*b^10*c^7*d^3 + 28*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8 + 3*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 78*(45*b^10*c^8*d^2 + 36*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 21*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 + 10*a^5*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 + 3*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 13*(55*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^4 + 21*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 + 10*a^6*b^4*c^3*d^7 + 6*a^7*b^3*c^2*d^8 + 3*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^24*x^13 + 13*a*b^23*x^12 + 78*a^2*b^22*x^11 + 286*a^3*b^21*x^10 + 715*a^4*b^20*x^9 + 1287*a^5*b^19*x^8 + 1716*a^6*b^18*x^7 + 1716*a^7*b^17*x^6 + 1287*a^8*b^16*x^5 + 715*a^9*b^15*x^4 + 286*a^10*b^14*x^3 + 78*a^11*b^13*x^2 + 13*a^12*b^12*x + a^13*b^11)

**giac [B]** time = 1.28, size = 961, normalized size = 10.80

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="giac")

[Out] -1/858*(286*b^10*d^10*x^10 + 2145*b^10*c*d^9*x^9 + 715*a*b^9*d^10*x^9 + 7722*b^10*c^2*d^8*x^8 + 3861*a*b^9*c*d^9*x^8 + 1287*a^2*b^8*d^10*x^8 + 17160*b^10*c^3*d^7*x^7 + 10296*a*b^9*c^2*d^8*x^7 + 5148*a^2*b^8*c*d^9*x^7 + 1716*a^3*b^7*d^10*x^7 + 25740*b^10*c^4*d^6*x^6 + 17160*a*b^9*c^3*d^7*x^6 + 10296*a^2*b^8*c^2*d^8*x^6 + 5148*a^3*b^7*c*d^9*x^6 + 1716*a^4*b^6*d^10*x^6 + 27027*b^10*c^5*d^5*x^5 + 19305*a*b^9*c^4*d^6*x^5 + 12870*a^2*b^8*c^3*d^7*x^5 + 7722*a^3*b^7*c^2*d^8*x^5 + 3861*a^4*b^6*c*d^9*x^5 + 1287*a^5*b^5*d^10*x^5 +

```
 20020*b^10*c^6*d^4*x^4 + 15015*a*b^9*c^5*d^5*x^4 + 10725*a^2*b^8*c^4*d^6*x
^4 + 7150*a^3*b^7*c^3*d^7*x^4 + 4290*a^4*b^6*c^2*d^8*x^4 + 2145*a^5*b^5*c*d
^9*x^4 + 715*a^6*b^4*d^10*x^4 + 10296*b^10*c^7*d^3*x^3 + 8008*a*b^9*c^6*d^4
*x^3 + 6006*a^2*b^8*c^5*d^5*x^3 + 4290*a^3*b^7*c^4*d^6*x^3 + 2860*a^4*b^6*c
^3*d^7*x^3 + 1716*a^5*b^5*c^2*d^8*x^3 + 858*a^6*b^4*c*d^9*x^3 + 286*a^7*b^3
*d^10*x^3 + 3510*b^10*c^8*d^2*x^2 + 2808*a*b^9*c^7*d^3*x^2 + 2184*a^2*b^8*c
^6*d^4*x^2 + 1638*a^3*b^7*c^5*d^5*x^2 + 1170*a^4*b^6*c^4*d^6*x^2 + 780*a^5*
b^5*c^3*d^7*x^2 + 468*a^6*b^4*c^2*d^8*x^2 + 234*a^7*b^3*c*d^9*x^2 + 78*a^8*
b^2*d^10*x^2 + 715*b^10*c^9*d*x + 585*a*b^9*c^8*d^2*x + 468*a^2*b^8*c^7*d^3
*x + 364*a^3*b^7*c^6*d^4*x + 273*a^4*b^6*c^5*d^5*x + 195*a^5*b^5*c^4*d^6*x
+ 130*a^6*b^4*c^3*d^7*x + 78*a^7*b^3*c^2*d^8*x + 39*a^8*b^2*c*d^9*x + 13*a^
9*b*d^10*x + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 36*a^3*b^
7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 +
10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^10*d^10)/((b*x +
 a)^13*b^11)
```

**maple [B]** time = 0.01, size = 867, normalized size = 9.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^14,x)
```

```
[Out] 63/2*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b
^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^8-1/13*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*
c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4
*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^1
0)/b^11/(b*x+a)^13-1/3*d^10/b^11/(b*x+a)^3-30*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*
a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^7-70/3*d^4*(a^6*d^6-6*a
^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5
*c^5*d+b^6*c^6)/b^11/(b*x+a)^9-9*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+
a)^5+5/2*d^9*(a*d-b*c)/b^11/(b*x+a)^4-45/11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a
^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*
a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^11+5/6*d*(a^9*d^9-9*a^8
*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*
b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^
11/(b*x+a)^12+20*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*
x+a)^6+12*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+
35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^1
0
```

**maxima [B]** time = 2.21, size = 997, normalized size = 11.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="maxima")
```

```
[Out] -1/858*(286*b^10*d^10*x^10 + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8
*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^
6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^
10*d^10 + 715*(3*b^10*c*d^9 + a*b^9*d^10)*x^9 + 1287*(6*b^10*c^2*d^8 + 3*a*
b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1716*(10*b^10*c^3*d^7 + 6*a*b^9*c^2*d^8 + 3
*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 1716*(15*b^10*c^4*d^6 + 10*a*b^9*c^3*d
^7 + 6*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 1287*(21*b^1
0*c^5*d^5 + 15*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 + 3*a
^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 715*(28*b^10*c^6*d^4 + 21*a*b^9*c^5*d^5
+ 15*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 6*a^4*b^6*c^2*d^8 + 3*a^5*b^5*c
*d^9 + a^6*b^4*d^10)*x^4 + 286*(36*b^10*c^7*d^3 + 28*a*b^9*c^6*d^4 + 21*a^2
*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8
+ 3*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 78*(45*b^10*c^8*d^2 + 36*a*b^9*c^7*
d^3 + 28*a^2*b^8*c^6*d^4 + 21*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 + 10*a^5
*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 + 3*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 13
*(55*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^
4 + 21*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 + 10*a^6*b^4*c^3*d^7 + 6*a^7*b^
3*c^2*d^8 + 3*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^24*x^13 + 13*a*b^23*x^12 +
78*a^2*b^22*x^11 + 286*a^3*b^21*x^10 + 715*a^4*b^20*x^9 + 1287*a^5*b^19*x^8
 + 1716*a^6*b^18*x^7 + 1716*a^7*b^17*x^6 + 1287*a^8*b^16*x^5 + 715*a^9*b^15
*x^4 + 286*a^10*b^14*x^3 + 78*a^11*b^13*x^2 + 13*a^12*b^12*x + a^13*b^11)
```

**mupad [B]** time = 0.48, size = 1098, normalized size = 12.34

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^14,x)
```

```
[Out] -(a^10*d^10 + 66*b^10*c^10 + 286*b^10*d^10*x^10 + 715*a*b^9*d^10*x^9 + 2145
*b^10*c*d^9*x^9 + 45*a^2*b^8*c^8*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*
d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*
b^2*c^2*d^8 + 78*a^8*b^2*d^10*x^2 + 286*a^7*b^3*d^10*x^3 + 715*a^6*b^4*d^10
*x^4 + 1287*a^5*b^5*d^10*x^5 + 1716*a^4*b^6*d^10*x^6 + 1716*a^3*b^7*d^10*x^
7 + 1287*a^2*b^8*d^10*x^8 + 3510*b^10*c^8*d^2*x^2 + 10296*b^10*c^7*d^3*x^3
+ 20020*b^10*c^6*d^4*x^4 + 27027*b^10*c^5*d^5*x^5 + 25740*b^10*c^4*d^6*x^6
+ 17160*b^10*c^3*d^7*x^7 + 7722*b^10*c^2*d^8*x^8 + 55*a*b^9*c^9*d + 3*a^9*b
*c*d^9 + 13*a^9*b*d^10*x + 715*b^10*c^9*d*x + 2184*a^2*b^8*c^6*d^4*x^2 + 16
38*a^3*b^7*c^5*d^5*x^2 + 1170*a^4*b^6*c^4*d^6*x^2 + 780*a^5*b^5*c^3*d^7*x^2
 + 468*a^6*b^4*c^2*d^8*x^2 + 6006*a^2*b^8*c^5*d^5*x^3 + 4290*a^3*b^7*c^4*d^
6*x^3 + 2860*a^4*b^6*c^3*d^7*x^3 + 1716*a^5*b^5*c^2*d^8*x^3 + 10725*a^2*b^8
*c^4*d^6*x^4 + 7150*a^3*b^7*c^3*d^7*x^4 + 4290*a^4*b^6*c^2*d^8*x^4 + 12870*
a^2*b^8*c^3*d^7*x^5 + 7722*a^3*b^7*c^2*d^8*x^5 + 10296*a^2*b^8*c^2*d^8*x^6
```

```
+ 585*a*b^9*c^8*d^2*x + 39*a^8*b^2*c*d^9*x + 3861*a*b^9*c*d^9*x^8 + 468*a^2
*b^8*c^7*d^3*x + 364*a^3*b^7*c^6*d^4*x + 273*a^4*b^6*c^5*d^5*x + 195*a^5*b^
5*c^4*d^6*x + 130*a^6*b^4*c^3*d^7*x + 78*a^7*b^3*c^2*d^8*x + 2808*a*b^9*c^7
*d^3*x^2 + 234*a^7*b^3*c*d^9*x^2 + 8008*a*b^9*c^6*d^4*x^3 + 858*a^6*b^4*c*d
^9*x^3 + 15015*a*b^9*c^5*d^5*x^4 + 2145*a^5*b^5*c*d^9*x^4 + 19305*a*b^9*c^4
*d^6*x^5 + 3861*a^4*b^6*c*d^9*x^5 + 17160*a*b^9*c^3*d^7*x^6 + 5148*a^3*b^7*
c*d^9*x^6 + 10296*a*b^9*c^2*d^8*x^7 + 5148*a^2*b^8*c*d^9*x^7)/(858*a^13*b^1
1 + 858*b^24*x^13 + 11154*a^12*b^12*x + 11154*a*b^23*x^12 + 66924*a^11*b^13
*x^2 + 245388*a^10*b^14*x^3 + 613470*a^9*b^15*x^4 + 1104246*a^8*b^16*x^5 +
1472328*a^7*b^17*x^6 + 1472328*a^6*b^18*x^7 + 1104246*a^5*b^19*x^8 + 613470
*a^4*b^20*x^9 + 245388*a^3*b^21*x^10 + 66924*a^2*b^22*x^11)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**14,x)
```

```
[Out] Timed out
```

## 3.1220 $\int \frac{(c+dx)^{10}}{(a+bx)^{15}} dx$

**Optimal**. Leaf size=120

$$\frac{d^3(c+dx)^{11}}{4004(a+bx)^{11}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{364(a+bx)^{12}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{182(a+bx)^{13}(bc-ad)^2}-\frac{(c+dx)^{11}}{14(a+bx)^{14}(bc-ad)}$$

**Rubi [A]** time = 0.03, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d^3(c+dx)^{11}}{4004(a+bx)^{11}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{364(a+bx)^{12}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{182(a+bx)^{13}(bc-ad)^2}-\frac{(c+dx)^{11}}{14(a+bx)^{14}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] -(c + d*x)^11/(14*(b*c - a*d)*(a + b*x)^14) + (3*d*(c + d*x)^11)/(182*(b*c
- a*d)^2*(a + b*x)^13) - (d^2*(c + d*x)^11)/(364*(b*c - a*d)^3*(a + b*x)^12
) + (d^3*(c + d*x)^11)/(4004*(b*c - a*d)^4*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx = -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} - \frac{(3d)\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{14(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} + \frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}} + \frac{\left(3d^2\right)\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx}{91(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} + \frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}} - \frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}} - \frac{d^3\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx}{364(bc-ad)^3}$$

$$= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} + \frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}} - \frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}} + \frac{d^3(c+}{4004(bc-ad)}$$

**Mathematica** [B] time = 0.29, size = 692, normalized size = 5.77

$$-\frac{a^{10}d^{10} + 2a^9bd^9(2c+7dx) + a^8b^2d^8(10c^2+56cdx+91d^2x^2) + 4a^7b^3d^7(5c^3+35c^2dx+91cd^2x^2+91d^3x^3) + 7a^6b^4d^6(5c^4+40c^3dx+130c^2d^2x^2+208cd^3x^3+143d^4x^4) + 14a^5b^5d^5(4c^5+35c^4dx+130c^3d^2x^2+260c^2d^3x^3+286cd^4x^4+143d^5x^5) + 7a^4b^6d^4(12c^6+112c^5dx+455c^4d^2x^2+1040c^3d^3x^3+1430c^2d^4x^4+1144cd^5x^5+429d^6x^6) + 4a^3b^7d^3(30c^7+294c^6dx+1274c^5d^2x^2+3185c^4d^3x^3+5005c^3d^4x^4+5005c^2d^5x^5+3003cd^6x^6+858d^7x^7) + a^2b^8d^2(165c^8+1680c^7dx+7644c^6d^2x^2+20384c^5d^3x^3+35035c^4d^4x^4+40040c^3d^5x^5+30030c^2d^6x^6+13728cd^7x^7+3003d^8x^8) + 2ab^9d(110c^9+1155c^8dx+5460c^7d^2x^2+15288c^6d^3x^3+28028c^5d^4x^4+35035c^4d^5x^5+30030c^3d^6x^6+17160c^2d^7x^7+6006cd^8x^8+1001d^9x^9) + b^{10}(286c^{10}+3080c^9dx+15015c^8d^2x^2+43680c^7d^3x^3+84084c^6d^4x^4+112112c^5d^5x^5+105105c^4d^6x^6+68640c^3d^7x^7+30030c^2d^8x^8+8008cd^9x^9+1001d^{10}x^{10})}{4004b^{11}(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] -1/4004*(a^10*d^10 + 2*a^9*b*d^9*(2*c + 7*d*x) + a^8*b^2*d^8*(10*c^2 + 56*c
*d*x + 91*d^2*x^2) + 4*a^7*b^3*d^7*(5*c^3 + 35*c^2*d*x + 91*c*d^2*x^2 + 91*
d^3*x^3) + 7*a^6*b^4*d^6*(5*c^4 + 40*c^3*d*x + 130*c^2*d^2*x^2 + 208*c*d^3*
x^3 + 143*d^4*x^4) + 14*a^5*b^5*d^5*(4*c^5 + 35*c^4*d*x + 130*c^3*d^2*x^2 +
 260*c^2*d^3*x^3 + 286*c*d^4*x^4 + 143*d^5*x^5) + 7*a^4*b^6*d^4*(12*c^6 + 1
12*c^5*d*x + 455*c^4*d^2*x^2 + 1040*c^3*d^3*x^3 + 1430*c^2*d^4*x^4 + 1144*c
*d^5*x^5 + 429*d^6*x^6) + 4*a^3*b^7*d^3*(30*c^7 + 294*c^6*d*x + 1274*c^5*d^
2*x^2 + 3185*c^4*d^3*x^3 + 5005*c^3*d^4*x^4 + 5005*c^2*d^5*x^5 + 3003*c*d^6
*x^6 + 858*d^7*x^7) + a^2*b^8*d^2*(165*c^8 + 1680*c^7*d*x + 7644*c^6*d^2*x^
2 + 20384*c^5*d^3*x^3 + 35035*c^4*d^4*x^4 + 40040*c^3*d^5*x^5 + 30030*c^2*d
^6*x^6 + 13728*c*d^7*x^7 + 3003*d^8*x^8) + 2*a*b^9*d*(110*c^9 + 1155*c^8*d*
x + 5460*c^7*d^2*x^2 + 15288*c^6*d^3*x^3 + 28028*c^5*d^4*x^4 + 35035*c^4*d^
5*x^5 + 30030*c^3*d^6*x^6 + 17160*c^2*d^7*x^7 + 6006*c*d^8*x^8 + 1001*d^9*x
^9) + b^10*(286*c^10 + 3080*c^9*d*x + 15015*c^8*d^2*x^2 + 43680*c^7*d^3*x^3
 + 84084*c^6*d^4*x^4 + 112112*c^5*d^5*x^5 + 105105*c^4*d^6*x^6 + 68640*c^3*
d^7*x^7 + 30030*c^2*d^8*x^8 + 8008*c*d^9*x^9 + 1001*d^10*x^10))/(b^11*(a +
b*x)^14)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^15, x]
```

**fricas [B]** time = 1.32, size = 1008, normalized size = 8.40

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="fricas")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165*a^2*b^
8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^5*d^5 +
 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^9*b*c*d
^9 + a^10*d^10 + 2002*(4*b^10*c*d^9 + a*b^9*d^10)*x^9 + 3003*(10*b^10*c^2*d
^8 + 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 3432*(20*b^10*c^3*d^7 + 10*a*b^9*c
^2*d^8 + 4*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 3003*(35*b^10*c^4*d^6 + 20*a
*b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 + 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2
002*(56*b^10*c^5*d^5 + 35*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 10*a^3*b^7*c
^2*d^8 + 4*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1001*(84*b^10*c^6*d^4 + 56*a
*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 10*a^4*b^6*c^2*d^8
 + 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 364*(120*b^10*c^7*d^3 + 84*a*b^9*c
^6*d^4 + 56*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 20*a^4*b^6*c^3*d^7 + 10*
a^5*b^5*c^2*d^8 + 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 91*(165*b^10*c^8*d^
2 + 120*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 56*a^3*b^7*c^5*d^5 + 35*a^4*b^
6*c^4*d^6 + 20*a^5*b^5*c^3*d^7 + 10*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9 + a^8
*b^2*d^10)*x^2 + 14*(220*b^10*c^9*d + 165*a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d
^3 + 84*a^3*b^7*c^6*d^4 + 56*a^4*b^6*c^5*d^5 + 35*a^5*b^5*c^4*d^6 + 20*a^6*
b^4*c^3*d^7 + 10*a^7*b^3*c^2*d^8 + 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^25*x
^14 + 14*a*b^24*x^13 + 91*a^2*b^23*x^12 + 364*a^3*b^22*x^11 + 1001*a^4*b^21
*x^10 + 2002*a^5*b^20*x^9 + 3003*a^6*b^19*x^8 + 3432*a^7*b^18*x^7 + 3003*a^
8*b^17*x^6 + 2002*a^9*b^16*x^5 + 1001*a^10*b^15*x^4 + 364*a^11*b^14*x^3 + 9
1*a^12*b^13*x^2 + 14*a^13*b^12*x + a^14*b^11)
```

**giac [B]** time = 1.39, size = 961, normalized size = 8.01

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="giac")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 8008*b^10*c*d^9*x^9 + 2002*a*b^9*d^10*x^9 +
30030*b^10*c^2*d^8*x^8 + 12012*a*b^9*c*d^9*x^8 + 3003*a^2*b^8*d^10*x^8 + 68
```

```
640*b^10*c^3*d^7*x^7 + 34320*a*b^9*c^2*d^8*x^7 + 13728*a^2*b^8*c*d^9*x^7 +
3432*a^3*b^7*d^10*x^7 + 105105*b^10*c^4*d^6*x^6 + 60060*a*b^9*c^3*d^7*x^6 +
 30030*a^2*b^8*c^2*d^8*x^6 + 12012*a^3*b^7*c*d^9*x^6 + 3003*a^4*b^6*d^10*x^
6 + 112112*b^10*c^5*d^5*x^5 + 70070*a*b^9*c^4*d^6*x^5 + 40040*a^2*b^8*c^3*d
^7*x^5 + 20020*a^3*b^7*c^2*d^8*x^5 + 8008*a^4*b^6*c*d^9*x^5 + 2002*a^5*b^5*
d^10*x^5 + 84084*b^10*c^6*d^4*x^4 + 56056*a*b^9*c^5*d^5*x^4 + 35035*a^2*b^8
*c^4*d^6*x^4 + 20020*a^3*b^7*c^3*d^7*x^4 + 10010*a^4*b^6*c^2*d^8*x^4 + 4004
*a^5*b^5*c*d^9*x^4 + 1001*a^6*b^4*d^10*x^4 + 43680*b^10*c^7*d^3*x^3 + 30576
*a*b^9*c^6*d^4*x^3 + 20384*a^2*b^8*c^5*d^5*x^3 + 12740*a^3*b^7*c^4*d^6*x^3
+ 7280*a^4*b^6*c^3*d^7*x^3 + 3640*a^5*b^5*c^2*d^8*x^3 + 1456*a^6*b^4*c*d^9*
x^3 + 364*a^7*b^3*d^10*x^3 + 15015*b^10*c^8*d^2*x^2 + 10920*a*b^9*c^7*d^3*x
^2 + 7644*a^2*b^8*c^6*d^4*x^2 + 5096*a^3*b^7*c^5*d^5*x^2 + 3185*a^4*b^6*c^4
*d^6*x^2 + 1820*a^5*b^5*c^3*d^7*x^2 + 910*a^6*b^4*c^2*d^8*x^2 + 364*a^7*b^3
*c*d^9*x^2 + 91*a^8*b^2*d^10*x^2 + 3080*b^10*c^9*d*x + 2310*a*b^9*c^8*d^2*x
 + 1680*a^2*b^8*c^7*d^3*x + 1176*a^3*b^7*c^6*d^4*x + 784*a^4*b^6*c^5*d^5*x
+ 490*a^5*b^5*c^4*d^6*x + 280*a^6*b^4*c^3*d^7*x + 140*a^7*b^3*c^2*d^8*x + 5
6*a^8*b^2*c*d^9*x + 14*a^9*b*d^10*x + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165
*a^2*b^8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^
5*d^5 + 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^
9*b*c*d^9 + a^10*d^10)/((b*x + a)^14*b^11)
```

**maple [B]** time = 0.01, size = 867, normalized size = 7.22

$$-\frac{d^{10}}{4\left(bx+a\right)^{4}b^{11}}+\frac{2\left(ad-bc\right)d^{9}}{\left(bx+a\right)^{5}b^{11}}-\frac{15\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)d^{8}}{2\left(bx+a\right)^{6}b^{11}}+\frac{120\left(a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}\right)d^{7}}{7\left(bx+a\right)^{7}b^{11}}-\frac{105\left(a^{4}d^{4}-4a^{3}bcd^{3}+6a^{2}b^{2}c^{2}d^{2}-4ab^{3}c^{3}d+b^{4}c^{4}\right)d^{6}}{4\left(bx+a\right)^{8}b^{11}}+\frac{28\left(a^{5}d^{5}-5a^{4}bcd^{4}+10a^{3}b^{2}c^{2}d^{3}-10a^{2}b^{3}c^{3}d^{2}+5ab^{4}c^{4}d-b^{5}c^{5}\right)d^{5}}{\left(bx+a\right)^{9}b^{11}}-\frac{21\left(a^{6}d^{6}-6a^{5}bcd^{5}+15a^{4}b^{2}c^{2}d^{4}-20a^{3}b^{3}c^{3}d^{3}+15a^{2}b^{4}c^{4}d^{2}-6ab^{5}c^{5}d+b^{6}c^{6}\right)d^{4}}{\left(bx+a\right)^{10}b^{11}}+\frac{120\left(a^{7}d^{7}-7a^{6}bcd^{6}+21a^{5}b^{2}c^{2}d^{5}-35a^{4}b^{3}c^{3}d^{4}+35a^{3}b^{4}c^{4}d^{3}-21a^{2}b^{5}c^{5}d^{2}+7ab^{6}c^{6}d-b^{7}c^{7}\right)d^{3}}{11\left(bx+a\right)^{11}b^{11}}-\frac{15\left(a^{8}d^{8}-8a^{7}bcd^{7}+28a^{6}b^{2}c^{2}d^{6}-56a^{5}b^{3}c^{3}d^{5}+70a^{4}b^{4}c^{4}d^{4}-56a^{3}b^{5}c^{5}d^{3}+28a^{2}b^{6}c^{6}d^{2}-8ab^{7}c^{7}d+b^{8}c^{8}\right)d^{2}}{4\left(bx+a\right)^{12}b^{11}}+\frac{10\left(a^{9}d^{9}-9a^{8}bcd^{8}+36a^{7}b^{2}c^{2}d^{7}-84a^{6}b^{3}c^{3}d^{6}+126a^{5}b^{4}c^{4}d^{5}-126a^{4}b^{5}c^{5}d^{4}+84a^{3}b^{6}c^{6}d^{3}-36a^{2}b^{7}c^{7}d^{2}+9ab^{8}c^{8}d-b^{9}c^{9}\right)d}{13\left(bx+a\right)^{13}b^{11}}-\frac{d^{10}a^{10}-10cd^{9}a^{9}b+45c^{2}d^{8}a^{8}b^{2}-120a^{7}c^{3}d^{7}b^{3}+210c^{4}d^{6}a^{6}b^{4}-252a^{5}c^{5}d^{5}b^{5}+210a^{4}c^{6}d^{4}b^{6}-120a^{3}c^{7}d^{3}b^{7}+45a^{2}c^{8}d^{2}b^{8}-10ab^{9}c^{9}d+c^{10}b^{10}}{14\left(bx+a\right)^{14}b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^15,x)
```

```
[Out] -105/4*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/
b^11/(b*x+a)^8+10/13*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3
*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*
b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^13+120/7*d^7*(a^3*d^3-3*a^2
*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^7+28*d^5*(a^5*d^5-5*a^4*b*c*d^
4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)
^9+2*d^9*(a*d-b*c)/b^11/(b*x+a)^5-1/4*d^10/b^11/(b*x+a)^4+120/11*d^3*(a^7*d
^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-2
1*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^11-15/4*d^2*(a^8*d^8-
8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a
^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^12-15
/2*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^6-21*d^4*(a^6*d^6-6*a^5*b*c
*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d
+b^6*c^6)/b^11/(b*x+a)^10-1/14*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8
-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^
6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11
/(b*x+a)^14
```

**maxima [B]** time = 2.16, size = 1008, normalized size = 8.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="maxima")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165*a^2*b^
8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^5*d^5 +
 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^9*b*c*d
^9 + a^10*d^10 + 2002*(4*b^10*c*d^9 + a*b^9*d^10)*x^9 + 3003*(10*b^10*c^2*d
^8 + 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 3432*(20*b^10*c^3*d^7 + 10*a*b^9*c
^2*d^8 + 4*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 3003*(35*b^10*c^4*d^6 + 20*a
*b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 + 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2
002*(56*b^10*c^5*d^5 + 35*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 10*a^3*b^7*c
^2*d^8 + 4*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1001*(84*b^10*c^6*d^4 + 56*a
*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 10*a^4*b^6*c^2*d^8
 + 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 364*(120*b^10*c^7*d^3 + 84*a*b^9*c
^6*d^4 + 56*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 20*a^4*b^6*c^3*d^7 + 10*
a^5*b^5*c^2*d^8 + 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 91*(165*b^10*c^8*d^
2 + 120*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 56*a^3*b^7*c^5*d^5 + 35*a^4*b^
6*c^4*d^6 + 20*a^5*b^5*c^3*d^7 + 10*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9 + a^8
*b^2*d^10)*x^2 + 14*(220*b^10*c^9*d + 165*a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d
^3 + 84*a^3*b^7*c^6*d^4 + 56*a^4*b^6*c^5*d^5 + 35*a^5*b^5*c^4*d^6 + 20*a^6*
b^4*c^3*d^7 + 10*a^7*b^3*c^2*d^8 + 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^25*x
^14 + 14*a*b^24*x^13 + 91*a^2*b^23*x^12 + 364*a^3*b^22*x^11 + 1001*a^4*b^21
*x^10 + 2002*a^5*b^20*x^9 + 3003*a^6*b^19*x^8 + 3432*a^7*b^18*x^7 + 3003*a^
8*b^17*x^6 + 2002*a^9*b^16*x^5 + 1001*a^10*b^15*x^4 + 364*a^11*b^14*x^3 + 9
1*a^12*b^13*x^2 + 14*a^13*b^12*x + a^14*b^11)
```

**mupad [B]** time = 1.30, size = 1109, normalized size = 9.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^15,x)
```

```
[Out] -(a^10*d^10 + 286*b^10*c^10 + 1001*b^10*d^10*x^10 + 2002*a*b^9*d^10*x^9 + 8
008*b^10*c*d^9*x^9 + 165*a^2*b^8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6
*c^6*d^4 + 56*a^5*b^5*c^5*d^5 + 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 1
0*a^8*b^2*c^2*d^8 + 91*a^8*b^2*d^10*x^2 + 364*a^7*b^3*d^10*x^3 + 1001*a^6*b
^4*d^10*x^4 + 2002*a^5*b^5*d^10*x^5 + 3003*a^4*b^6*d^10*x^6 + 3432*a^3*b^7*
d^10*x^7 + 3003*a^2*b^8*d^10*x^8 + 15015*b^10*c^8*d^2*x^2 + 43680*b^10*c^7*
d^3*x^3 + 84084*b^10*c^6*d^4*x^4 + 112112*b^10*c^5*d^5*x^5 + 105105*b^10*c^
```

```
4*d^6*x^6 + 68640*b^10*c^3*d^7*x^7 + 30030*b^10*c^2*d^8*x^8 + 220*a*b^9*c^9
*d + 4*a^9*b*c*d^9 + 14*a^9*b*d^10*x + 3080*b^10*c^9*d*x + 7644*a^2*b^8*c^6
*d^4*x^2 + 5096*a^3*b^7*c^5*d^5*x^2 + 3185*a^4*b^6*c^4*d^6*x^2 + 1820*a^5*b
^5*c^3*d^7*x^2 + 910*a^6*b^4*c^2*d^8*x^2 + 20384*a^2*b^8*c^5*d^5*x^3 + 1274
0*a^3*b^7*c^4*d^6*x^3 + 7280*a^4*b^6*c^3*d^7*x^3 + 3640*a^5*b^5*c^2*d^8*x^3
 + 35035*a^2*b^8*c^4*d^6*x^4 + 20020*a^3*b^7*c^3*d^7*x^4 + 10010*a^4*b^6*c^
2*d^8*x^4 + 40040*a^2*b^8*c^3*d^7*x^5 + 20020*a^3*b^7*c^2*d^8*x^5 + 30030*a
^2*b^8*c^2*d^8*x^6 + 2310*a*b^9*c^8*d^2*x + 56*a^8*b^2*c*d^9*x + 12012*a*b^
9*c*d^9*x^8 + 1680*a^2*b^8*c^7*d^3*x + 1176*a^3*b^7*c^6*d^4*x + 784*a^4*b^6
*c^5*d^5*x + 490*a^5*b^5*c^4*d^6*x + 280*a^6*b^4*c^3*d^7*x + 140*a^7*b^3*c^
2*d^8*x + 10920*a*b^9*c^7*d^3*x^2 + 364*a^7*b^3*c*d^9*x^2 + 30576*a*b^9*c^6
*d^4*x^3 + 1456*a^6*b^4*c*d^9*x^3 + 56056*a*b^9*c^5*d^5*x^4 + 4004*a^5*b^5*
c*d^9*x^4 + 70070*a*b^9*c^4*d^6*x^5 + 8008*a^4*b^6*c*d^9*x^5 + 60060*a*b^9*
c^3*d^7*x^6 + 12012*a^3*b^7*c*d^9*x^6 + 34320*a*b^9*c^2*d^8*x^7 + 13728*a^2
*b^8*c*d^9*x^7)/(4004*a^14*b^11 + 4004*b^25*x^14 + 56056*a^13*b^12*x + 5605
6*a*b^24*x^13 + 364364*a^12*b^13*x^2 + 1457456*a^11*b^14*x^3 + 4008004*a^10
*b^15*x^4 + 8016008*a^9*b^16*x^5 + 12024012*a^8*b^17*x^6 + 13741728*a^7*b^1
8*x^7 + 12024012*a^6*b^19*x^8 + 8016008*a^5*b^20*x^9 + 4008004*a^4*b^21*x^1
0 + 1457456*a^3*b^22*x^11 + 364364*a^2*b^23*x^12)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**15,x)
```

```
[Out] Timed out
```

# 3.1221 $\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx$

**Optimal**. Leaf size=151

$$-\frac{d^4(c+dx)^{11}}{15015(a+bx)^{11}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{1365(a+bx)^{12}(bc-ad)^4}-\frac{2d^2(c+dx)^{11}}{455(a+bx)^{13}(bc-ad)^3}+\frac{2d(c+dx)^{11}}{105(a+bx)^{14}(bc-ad)^2}-15($$

**Rubi [A]** time = 0.04, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^4(c+dx)^{11}}{15015(a+bx)^{11}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{1365(a+bx)^{12}(bc-ad)^4}-\frac{2d^2(c+dx)^{11}}{455(a+bx)^{13}(bc-ad)^3}+\frac{2d(c+dx)^{11}}{105(a+bx)^{14}(bc-ad)^2}-\frac{(c+dx)^{11}}{15(a+bx)^{15}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] -(c + d*x)^11/(15*(b*c - a*d)*(a + b*x)^15) + (2*d*(c + d*x)^11)/(105*(b*c
- a*d)^2*(a + b*x)^14) - (2*d^2*(c + d*x)^11)/(455*(b*c - a*d)^3*(a + b*x)^
13) + (d^3*(c + d*x)^11)/(1365*(b*c - a*d)^4*(a + b*x)^12) - (d^4*(c + d*x)
^11)/(15015*(b*c - a*d)^5*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx = -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} - \frac{(4d)\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{15(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} + \frac{\left(2d^2\right)\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{35(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} - \frac{\left(4d^3\right)\int \frac{(c+dx)^{1}}{(a+bx)^{1}}}{455(bc-ad}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} + \frac{d^3(c+}{1365(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} + \frac{d^3(c+}{1365(bc-ad)}$$

**Mathematica [B]** time = 0.29, size = 690, normalized size = 4.57

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] -1/15015*(a^10*d^10 + 5*a^9*b*d^9*(c + 3*d*x) + 15*a^8*b^2*d^8*(c^2 + 5*c*d
*x + 7*d^2*x^2) + 5*a^7*b^3*d^7*(7*c^3 + 45*c^2*d*x + 105*c*d^2*x^2 + 91*d^
3*x^3) + 35*a^6*b^4*d^6*(2*c^4 + 15*c^3*d*x + 45*c^2*d^2*x^2 + 65*c*d^3*x^3
 + 39*d^4*x^4) + 21*a^5*b^5*d^5*(6*c^5 + 50*c^4*d*x + 175*c^3*d^2*x^2 + 325
*c^2*d^3*x^3 + 325*c*d^4*x^4 + 143*d^5*x^5) + 35*a^4*b^6*d^4*(6*c^6 + 54*c^
5*d*x + 210*c^4*d^2*x^2 + 455*c^3*d^3*x^3 + 585*c^2*d^4*x^4 + 429*c*d^5*x^5
 + 143*d^6*x^6) + 5*a^3*b^7*d^3*(66*c^7 + 630*c^6*d*x + 2646*c^5*d^2*x^2 +
6370*c^4*d^3*x^3 + 9555*c^3*d^4*x^4 + 9009*c^2*d^5*x^5 + 5005*c*d^6*x^6 + 1
287*d^7*x^7) + 15*a^2*b^8*d^2*(33*c^8 + 330*c^7*d*x + 1470*c^6*d^2*x^2 + 38
22*c^5*d^3*x^3 + 6370*c^4*d^4*x^4 + 7007*c^3*d^5*x^5 + 5005*c^2*d^6*x^6 + 2
145*c*d^7*x^7 + 429*d^8*x^8) + 5*a*b^9*d*(143*c^9 + 1485*c^8*d*x + 6930*c^7
*d^2*x^2 + 19110*c^6*d^3*x^3 + 34398*c^5*d^4*x^4 + 42042*c^4*d^5*x^5 + 3503
5*c^3*d^6*x^6 + 19305*c^2*d^7*x^7 + 6435*c*d^8*x^8 + 1001*d^9*x^9) + b^10*(
1001*c^10 + 10725*c^9*d*x + 51975*c^8*d^2*x^2 + 150150*c^7*d^3*x^3 + 286650
*c^6*d^4*x^4 + 378378*c^5*d^5*x^5 + 350350*c^4*d^6*x^6 + 225225*c^3*d^7*x^7
 + 96525*c^2*d^8*x^8 + 25025*c*d^9*x^9 + 3003*d^10*x^10))/(b^11*(a + b*x)^1
5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^16, x]
```

**fricas [B]** time = 1.12, size = 1019, normalized size = 6.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 1001*b^10*c^10 + 715*a*b^9*c^9*d + 495*a^2*
b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d
^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 + 5*a^9*b
*c*d^9 + a^10*d^10 + 5005*(5*b^10*c*d^9 + a*b^9*d^10)*x^9 + 6435*(15*b^10*c
^2*d^8 + 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 6435*(35*b^10*c^3*d^7 + 15*a*b
^9*c^2*d^8 + 5*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 5005*(70*b^10*c^4*d^6 +
35*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 + 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 3003*(126*b^10*c^5*d^5 + 70*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 + 15*a^3*
b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1365*(210*b^10*c^6*d^4
+ 126*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 + 35*a^3*b^7*c^3*d^7 + 15*a^4*b^6*
c^2*d^8 + 5*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 455*(330*b^10*c^7*d^3 + 210
*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 + 70*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*
d^7 + 15*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 105*(495*b
^10*c^8*d^2 + 330*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
 + 70*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 + 5*a^7*b^3
*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(715*b^10*c^9*d + 495*a*b^9*c^8*d^2 + 330*a
^2*b^8*c^7*d^3 + 210*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 70*a^5*b^5*c^4
*d^6 + 35*a^6*b^4*c^3*d^7 + 15*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^26*x^15 + 15*a*b^25*x^14 + 105*a^2*b^24*x^13 + 455*a^3*b^23*x^12
+ 1365*a^4*b^22*x^11 + 3003*a^5*b^21*x^10 + 5005*a^6*b^20*x^9 + 6435*a^7*b^
19*x^8 + 6435*a^8*b^18*x^7 + 5005*a^9*b^17*x^6 + 3003*a^10*b^16*x^5 + 1365*
a^11*b^15*x^4 + 455*a^12*b^14*x^3 + 105*a^13*b^13*x^2 + 15*a^14*b^12*x + a^
15*b^11)
```

**giac [B]** time = 1.32, size = 961, normalized size = 6.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="giac")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 25025*b^10*c*d^9*x^9 + 5005*a*b^9*d^10*x^9
+ 96525*b^10*c^2*d^8*x^8 + 32175*a*b^9*c*d^9*x^8 + 6435*a^2*b^8*d^10*x^8 +
225225*b^10*c^3*d^7*x^7 + 96525*a*b^9*c^2*d^8*x^7 + 32175*a^2*b^8*c*d^9*x^7
 + 6435*a^3*b^7*d^10*x^7 + 350350*b^10*c^4*d^6*x^6 + 175175*a*b^9*c^3*d^7*x
^6 + 75075*a^2*b^8*c^2*d^8*x^6 + 25025*a^3*b^7*c*d^9*x^6 + 5005*a^4*b^6*d^1
0*x^6 + 378378*b^10*c^5*d^5*x^5 + 210210*a*b^9*c^4*d^6*x^5 + 105105*a^2*b^8
*c^3*d^7*x^5 + 45045*a^3*b^7*c^2*d^8*x^5 + 15015*a^4*b^6*c*d^9*x^5 + 3003*a
^5*b^5*d^10*x^5 + 286650*b^10*c^6*d^4*x^4 + 171990*a*b^9*c^5*d^5*x^4 + 9555
0*a^2*b^8*c^4*d^6*x^4 + 47775*a^3*b^7*c^3*d^7*x^4 + 20475*a^4*b^6*c^2*d^8*x
^4 + 6825*a^5*b^5*c*d^9*x^4 + 1365*a^6*b^4*d^10*x^4 + 150150*b^10*c^7*d^3*x
^3 + 95550*a*b^9*c^6*d^4*x^3 + 57330*a^2*b^8*c^5*d^5*x^3 + 31850*a^3*b^7*c^
4*d^6*x^3 + 15925*a^4*b^6*c^3*d^7*x^3 + 6825*a^5*b^5*c^2*d^8*x^3 + 2275*a^6
*b^4*c*d^9*x^3 + 455*a^7*b^3*d^10*x^3 + 51975*b^10*c^8*d^2*x^2 + 34650*a*b^
9*c^7*d^3*x^2 + 22050*a^2*b^8*c^6*d^4*x^2 + 13230*a^3*b^7*c^5*d^5*x^2 + 735
0*a^4*b^6*c^4*d^6*x^2 + 3675*a^5*b^5*c^3*d^7*x^2 + 1575*a^6*b^4*c^2*d^8*x^2
 + 525*a^7*b^3*c*d^9*x^2 + 105*a^8*b^2*d^10*x^2 + 10725*b^10*c^9*d*x + 7425
*a*b^9*c^8*d^2*x + 4950*a^2*b^8*c^7*d^3*x + 3150*a^3*b^7*c^6*d^4*x + 1890*a
^4*b^6*c^5*d^5*x + 1050*a^5*b^5*c^4*d^6*x + 525*a^6*b^4*c^3*d^7*x + 225*a^7
*b^3*c^2*d^8*x + 75*a^8*b^2*c*d^9*x + 15*a^9*b*d^10*x + 1001*b^10*c^10 + 71
5*a*b^9*c^9*d + 495*a^2*b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6
*d^4 + 126*a^5*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a
^8*b^2*c^2*d^8 + 5*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^15*b^11)
```

**maple [B]** time = 0.01, size = 867, normalized size = 5.74

$$-\frac{d^{10}}{5(bx+a)^5 b^{11}}+\frac{5(ad-bc)d^9}{3(bx+a)^6 b^{11}}-\frac{45\left(a^2d^2-2abcd+b^2c^2\right)d^8}{7(bx+a)^7 b^{11}}+\frac{15\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)d^7}{(bx+a)^8 b^{11}}-\frac{70\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)d^6}{3(bx+a)^9 b^{11}}+\frac{126\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)d^5}{5(bx+a)^{10} b^{11}}-\frac{210\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)d^4}{11(bx+a)^{11} b^{11}}+\frac{10\left(a^7d^7-7a^6bcd^6+21a^5b^2c^2d^5-35a^4b^3c^3d^4+35a^3b^4c^4d^3-21a^2b^5c^5d^2+7ab^6c^6d-b^7c^7\right)d^3}{(bx+a)^{12} b^{11}}-\frac{45\left(a^8d^8-8a^7bcd^7+28a^6b^2c^2d^6-56a^5b^3c^3d^5+70a^4b^4c^4d^4-56a^3b^5c^5d^3+28a^2b^6c^6d^2-8ab^7c^7d+b^8c^8\right)d^2}{13(bx+a)^{13} b^{11}}+\frac{5\left(a^9d^9-9a^8bcd^8+36a^7b^2c^2d^7-84a^6b^3c^3d^6+126a^5b^4c^4d^5-126a^4b^5c^5d^4+84a^3b^6c^6d^3-36a^2b^7c^7d^2+9ab^8c^8d-b^9c^9\right)d}{7(bx+a)^{14} b^{11}}-\frac{d^{10}a^{10}-10cd^9a^9b+45c^2d^8a^8b^2-120a^7c^3d^7b^3+210c^4d^6a^6b^4-252a^5c^5d^5b^5+210a^4c^6d^4b^6-120a^3c^7d^3b^7+45a^2c^8d^2b^8-10ab^9c^9d+c^{10}b^{10}}{15(bx+a)^{15} b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^16,x)
```

```
[Out] 15*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^8-45/13*d
^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*
c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(
b*x+a)^13-45/7*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^7-70/3*d^6*(a^4
*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^9-
1/5*d^10/b^11/(b*x+a)^5-210/11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^
4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)
^11-1/15*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+2
10*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*
d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^15+10*d^3*(a^
7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^
```

```
3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^12+5/3*d^9*(a*d-b*
c)/b^11/(b*x+a)^6+126/5*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^
2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^10+5/7*d*(a^9*d^9-9*a^8*b
*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^
5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11
/(b*x+a)^14
```

**maxima [B]** time = 2.26, size = 1019, normalized size = 6.75

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 1001*b^10*c^10 + 715*a*b^9*c^9*d + 495*a^2*
b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d
^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 + 5*a^9*b
*c*d^9 + a^10*d^10 + 5005*(5*b^10*c*d^9 + a*b^9*d^10)*x^9 + 6435*(15*b^10*c
^2*d^8 + 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 6435*(35*b^10*c^3*d^7 + 15*a*b
^9*c^2*d^8 + 5*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 5005*(70*b^10*c^4*d^6 +
35*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 + 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 3003*(126*b^10*c^5*d^5 + 70*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 + 15*a^3*
b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1365*(210*b^10*c^6*d^4
+ 126*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 + 35*a^3*b^7*c^3*d^7 + 15*a^4*b^6*
c^2*d^8 + 5*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 455*(330*b^10*c^7*d^3 + 210
*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 + 70*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*
d^7 + 15*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 105*(495*b
^10*c^8*d^2 + 330*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
 + 70*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 + 5*a^7*b^3
*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(715*b^10*c^9*d + 495*a*b^9*c^8*d^2 + 330*a
^2*b^8*c^7*d^3 + 210*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 70*a^5*b^5*c^4
*d^6 + 35*a^6*b^4*c^3*d^7 + 15*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^26*x^15 + 15*a*b^25*x^14 + 105*a^2*b^24*x^13 + 455*a^3*b^23*x^12
+ 1365*a^4*b^22*x^11 + 3003*a^5*b^21*x^10 + 5005*a^6*b^20*x^9 + 6435*a^7*b^
19*x^8 + 6435*a^8*b^18*x^7 + 5005*a^9*b^17*x^6 + 3003*a^10*b^16*x^5 + 1365*
a^11*b^15*x^4 + 455*a^12*b^14*x^3 + 105*a^13*b^13*x^2 + 15*a^14*b^12*x + a^
15*b^11)
```

**mupad [B]** time = 2.28, size = 1120, normalized size = 7.42

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^16,x)
```

```
[Out] -(a^10*d^10 + 1001*b^10*c^10 + 3003*b^10*d^10*x^10 + 5005*a*b^9*d^10*x^9 +
25025*b^10*c*d^9*x^9 + 495*a^2*b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*
b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7
 + 15*a^8*b^2*c^2*d^8 + 105*a^8*b^2*d^10*x^2 + 455*a^7*b^3*d^10*x^3 + 1365*
a^6*b^4*d^10*x^4 + 3003*a^5*b^5*d^10*x^5 + 5005*a^4*b^6*d^10*x^6 + 6435*a^3
*b^7*d^10*x^7 + 6435*a^2*b^8*d^10*x^8 + 51975*b^10*c^8*d^2*x^2 + 150150*b^1
0*c^7*d^3*x^3 + 286650*b^10*c^6*d^4*x^4 + 378378*b^10*c^5*d^5*x^5 + 350350*
b^10*c^4*d^6*x^6 + 225225*b^10*c^3*d^7*x^7 + 96525*b^10*c^2*d^8*x^8 + 715*a
*b^9*c^9*d + 5*a^9*b*c*d^9 + 15*a^9*b*d^10*x + 10725*b^10*c^9*d*x + 22050*a
^2*b^8*c^6*d^4*x^2 + 13230*a^3*b^7*c^5*d^5*x^2 + 7350*a^4*b^6*c^4*d^6*x^2 +
 3675*a^5*b^5*c^3*d^7*x^2 + 1575*a^6*b^4*c^2*d^8*x^2 + 57330*a^2*b^8*c^5*d^
5*x^3 + 31850*a^3*b^7*c^4*d^6*x^3 + 15925*a^4*b^6*c^3*d^7*x^3 + 6825*a^5*b^
5*c^2*d^8*x^3 + 95550*a^2*b^8*c^4*d^6*x^4 + 47775*a^3*b^7*c^3*d^7*x^4 + 204
75*a^4*b^6*c^2*d^8*x^4 + 105105*a^2*b^8*c^3*d^7*x^5 + 45045*a^3*b^7*c^2*d^8
*x^5 + 75075*a^2*b^8*c^2*d^8*x^6 + 7425*a*b^9*c^8*d^2*x + 75*a^8*b^2*c*d^9*
x + 32175*a*b^9*c*d^9*x^8 + 4950*a^2*b^8*c^7*d^3*x + 3150*a^3*b^7*c^6*d^4*x
 + 1890*a^4*b^6*c^5*d^5*x + 1050*a^5*b^5*c^4*d^6*x + 525*a^6*b^4*c^3*d^7*x
+ 225*a^7*b^3*c^2*d^8*x + 34650*a*b^9*c^7*d^3*x^2 + 525*a^7*b^3*c*d^9*x^2 +
 95550*a*b^9*c^6*d^4*x^3 + 2275*a^6*b^4*c*d^9*x^3 + 171990*a*b^9*c^5*d^5*x^
4 + 6825*a^5*b^5*c*d^9*x^4 + 210210*a*b^9*c^4*d^6*x^5 + 15015*a^4*b^6*c*d^9
*x^5 + 175175*a*b^9*c^3*d^7*x^6 + 25025*a^3*b^7*c*d^9*x^6 + 96525*a*b^9*c^2
*d^8*x^7 + 32175*a^2*b^8*c*d^9*x^7)/(15015*a^15*b^11 + 15015*b^26*x^15 + 22
5225*a^14*b^12*x + 225225*a*b^25*x^14 + 1576575*a^13*b^13*x^2 + 6831825*a^1
2*b^14*x^3 + 20495475*a^11*b^15*x^4 + 45090045*a^10*b^16*x^5 + 75150075*a^9
*b^17*x^6 + 96621525*a^8*b^18*x^7 + 96621525*a^7*b^19*x^8 + 75150075*a^6*b^
20*x^9 + 45090045*a^5*b^21*x^10 + 20495475*a^4*b^22*x^11 + 6831825*a^3*b^23
*x^12 + 1576575*a^2*b^24*x^13)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**16,x)
```

```
[Out] Timed out
```

# 3.1222 $\int \frac{(c+dx)^{10}}{(a+bx)^{17}} \, dx$

**Optimal**. Leaf size=182

$$\frac{d^5(c+dx)^{11}}{48048(a+bx)^{11}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{4368(a+bx)^{12}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{728(a+bx)^{13}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{168(a+bx)^{14}(bc-ad)^3}+\frac{}{48(a}$$

**Rubi [A]** time = 0.06, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d^5(c+dx)^{11}}{48048(a+bx)^{11}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{4368(a+bx)^{12}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{728(a+bx)^{13}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{168(a+bx)^{14}(bc-ad)^3}+\frac{d(c+dx)^{11}}{48(a+bx)^{15}(bc-ad)^2}-\frac{(c+dx)^{11}}{16(a+bx)^{16}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^17,x]
```

```
[Out] -(c + d*x)^11/(16*(b*c - a*d)*(a + b*x)^16) + (d*(c + d*x)^11)/(48*(b*c - a
*d)^2*(a + b*x)^15) - (d^2*(c + d*x)^11)/(168*(b*c - a*d)^3*(a + b*x)^14) +
 (d^3*(c + d*x)^11)/(728*(b*c - a*d)^4*(a + b*x)^13) - (d^4*(c + d*x)^11)/(
4368*(b*c - a*d)^5*(a + b*x)^12) + (d^5*(c + d*x)^11)/(48048*(b*c - a*d)^6*
(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx = -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} - \frac{(5d)\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx}{16(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} + \frac{d^2\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{12(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} - \frac{d^3\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{56(bc-ad)^3}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d^3(c+dx}{728(bc-ad)^4(a}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d^3(c+dx}{728(bc-ad)^4(a}$$

$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d^3(c+dx}{728(bc-ad)^4(a}$$

**Mathematica [B]** time = 0.28, size = 694, normalized size = 3.81

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^17,x]
```

```
[Out] -1/48048*(a^10*d^10 + 2*a^9*b*d^9*(3*c + 8*d*x) + 3*a^8*b^2*d^8*(7*c^2 + 32
*c*d*x + 40*d^2*x^2) + 8*a^7*b^3*d^7*(7*c^3 + 42*c^2*d*x + 90*c*d^2*x^2 + 7
0*d^3*x^3) + 14*a^6*b^4*d^6*(9*c^4 + 64*c^3*d*x + 180*c^2*d^2*x^2 + 240*c*d
^3*x^3 + 130*d^4*x^4) + 84*a^5*b^5*d^5*(3*c^5 + 24*c^4*d*x + 80*c^3*d^2*x^2
 + 140*c^2*d^3*x^3 + 130*c*d^4*x^4 + 52*d^5*x^5) + 14*a^4*b^6*d^4*(33*c^6 +
 288*c^5*d*x + 1080*c^4*d^2*x^2 + 2240*c^3*d^3*x^3 + 2730*c^2*d^4*x^4 + 187
2*c*d^5*x^5 + 572*d^6*x^6) + 8*a^3*b^7*d^3*(99*c^7 + 924*c^6*d*x + 3780*c^5
*d^2*x^2 + 8820*c^4*d^3*x^3 + 12740*c^3*d^4*x^4 + 11466*c^2*d^5*x^5 + 6006*
c*d^6*x^6 + 1430*d^7*x^7) + 3*a^2*b^8*d^2*(429*c^8 + 4224*c^7*d*x + 18480*c
^6*d^2*x^2 + 47040*c^5*d^3*x^3 + 76440*c^4*d^4*x^4 + 81536*c^3*d^5*x^5 + 56
056*c^2*d^6*x^6 + 22880*c*d^7*x^7 + 4290*d^8*x^8) + 2*a*b^9*d*(1001*c^9 + 1
0296*c^8*d*x + 47520*c^7*d^2*x^2 + 129360*c^6*d^3*x^3 + 229320*c^5*d^4*x^4
+ 275184*c^4*d^5*x^5 + 224224*c^3*d^6*x^6 + 120120*c^2*d^7*x^7 + 38610*c*d^
8*x^8 + 5720*d^9*x^9) + b^10*(3003*c^10 + 32032*c^9*d*x + 154440*c^8*d^2*x^
2 + 443520*c^7*d^3*x^3 + 840840*c^6*d^4*x^4 + 1100736*c^5*d^5*x^5 + 1009008
*c^4*d^6*x^6 + 640640*c^3*d^7*x^7 + 270270*c^2*d^8*x^8 + 68640*c*d^9*x^9 +
8008*d^10*x^10))/(b^11*(a + b*x)^16)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^17,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^17, x]
```

**fricas [B]** time = 1.26, size = 1030, normalized size = 5.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="fricas")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 3003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^
2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5
*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^
9*b*c*d^9 + a^10*d^10 + 11440*(6*b^10*c*d^9 + a*b^9*d^10)*x^9 + 12870*(21*b
^10*c^2*d^8 + 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 11440*(56*b^10*c^3*d^7 +
21*a*b^9*c^2*d^8 + 6*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 8008*(126*b^10*c^4
*d^6 + 56*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 + 6*a^3*b^7*c*d^9 + a^4*b^6*d^
10)*x^6 + 4368*(252*b^10*c^5*d^5 + 126*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 +
 21*a^3*b^7*c^2*d^8 + 6*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1820*(462*b^10*
c^6*d^4 + 252*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 + 56*a^3*b^7*c^3*d^7 + 21
*a^4*b^6*c^2*d^8 + 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 560*(792*b^10*c^7*
d^3 + 462*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 + 126*a^3*b^7*c^4*d^6 + 56*a^
4*b^6*c^3*d^7 + 21*a^5*b^5*c^2*d^8 + 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 +
120*(1287*b^10*c^8*d^2 + 792*a*b^9*c^7*d^3 + 462*a^2*b^8*c^6*d^4 + 252*a^3*
b^7*c^5*d^5 + 126*a^4*b^6*c^4*d^6 + 56*a^5*b^5*c^3*d^7 + 21*a^6*b^4*c^2*d^8
 + 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 16*(2002*b^10*c^9*d + 1287*a*b^9*c
^8*d^2 + 792*a^2*b^8*c^7*d^3 + 462*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5*d^5 +
126*a^5*b^5*c^4*d^6 + 56*a^6*b^4*c^3*d^7 + 21*a^7*b^3*c^2*d^8 + 6*a^8*b^2*c
*d^9 + a^9*b*d^10)*x)/(b^27*x^16 + 16*a*b^26*x^15 + 120*a^2*b^25*x^14 + 560
*a^3*b^24*x^13 + 1820*a^4*b^23*x^12 + 4368*a^5*b^22*x^11 + 8008*a^6*b^21*x^
10 + 11440*a^7*b^20*x^9 + 12870*a^8*b^19*x^8 + 11440*a^9*b^18*x^7 + 8008*a^
10*b^17*x^6 + 4368*a^11*b^16*x^5 + 1820*a^12*b^15*x^4 + 560*a^13*b^14*x^3 +
 120*a^14*b^13*x^2 + 16*a^15*b^12*x + a^16*b^11)
```

**giac [B]** time = 1.31, size = 961, normalized size = 5.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="giac")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 68640*b^10*c*d^9*x^9 + 11440*a*b^9*d^10*x^9
 + 270270*b^10*c^2*d^8*x^8 + 77220*a*b^9*c*d^9*x^8 + 12870*a^2*b^8*d^10*x^8
 + 640640*b^10*c^3*d^7*x^7 + 240240*a*b^9*c^2*d^8*x^7 + 68640*a^2*b^8*c*d^9
*x^7 + 11440*a^3*b^7*d^10*x^7 + 1009008*b^10*c^4*d^6*x^6 + 448448*a*b^9*c^3
*d^7*x^6 + 168168*a^2*b^8*c^2*d^8*x^6 + 48048*a^3*b^7*c*d^9*x^6 + 8008*a^4*
b^6*d^10*x^6 + 1100736*b^10*c^5*d^5*x^5 + 550368*a*b^9*c^4*d^6*x^5 + 244608
*a^2*b^8*c^3*d^7*x^5 + 91728*a^3*b^7*c^2*d^8*x^5 + 26208*a^4*b^6*c*d^9*x^5
+ 4368*a^5*b^5*d^10*x^5 + 840840*b^10*c^6*d^4*x^4 + 458640*a*b^9*c^5*d^5*x^
4 + 229320*a^2*b^8*c^4*d^6*x^4 + 101920*a^3*b^7*c^3*d^7*x^4 + 38220*a^4*b^6
*c^2*d^8*x^4 + 10920*a^5*b^5*c*d^9*x^4 + 1820*a^6*b^4*d^10*x^4 + 443520*b^1
0*c^7*d^3*x^3 + 258720*a*b^9*c^6*d^4*x^3 + 141120*a^2*b^8*c^5*d^5*x^3 + 705
60*a^3*b^7*c^4*d^6*x^3 + 31360*a^4*b^6*c^3*d^7*x^3 + 11760*a^5*b^5*c^2*d^8*
x^3 + 3360*a^6*b^4*c*d^9*x^3 + 560*a^7*b^3*d^10*x^3 + 154440*b^10*c^8*d^2*x
^2 + 95040*a*b^9*c^7*d^3*x^2 + 55440*a^2*b^8*c^6*d^4*x^2 + 30240*a^3*b^7*c^
5*d^5*x^2 + 15120*a^4*b^6*c^4*d^6*x^2 + 6720*a^5*b^5*c^3*d^7*x^2 + 2520*a^6
*b^4*c^2*d^8*x^2 + 720*a^7*b^3*c*d^9*x^2 + 120*a^8*b^2*d^10*x^2 + 32032*b^1
0*c^9*d*x + 20592*a*b^9*c^8*d^2*x + 12672*a^2*b^8*c^7*d^3*x + 7392*a^3*b^7*
c^6*d^4*x + 4032*a^4*b^6*c^5*d^5*x + 2016*a^5*b^5*c^4*d^6*x + 896*a^6*b^4*c
^3*d^7*x + 336*a^7*b^3*c^2*d^8*x + 96*a^8*b^2*c*d^9*x + 16*a^9*b*d^10*x + 3
003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d
^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a
^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)
^16*b^11)
```

**maple [B]** time = 0.01, size = 867, normalized size = 4.76

$$-\frac{d^{10}}{6 (bx+a)^{6} b^{11}}+\frac{10 (ad-bc) d^{9}}{7 (bx+a)^{7} b^{11}}-\frac{45 \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) d^{8}}{8 (bx+a)^{8} b^{11}}+\frac{40 \left(a^{3} d^{3}-3 a^{2} b c d^{2}+3 a b^{2} c^{2} d-b^{3} c^{3}\right) d^{7}}{3 (bx+a)^{9} b^{11}}-\frac{21 \left(a^{4} d^{4}-4 a^{3} b c d^{3}+6 a^{2} b^{2} c^{2} d^{2}-4 a b^{3} c^{3} d+b^{4} c^{4}\right) d^{6}}{(bx+a)^{10} b^{11}}+\frac{252 \left(a^{5} d^{5}-5 a^{4} b c d^{4}+10 a^{3} b^{2} c^{2} d^{3}-10 a^{2} b^{3} c^{3} d^{2}+5 a b^{4} c^{4} d-b^{5} c^{5}\right) d^{5}}{11 (bx+a)^{11} b^{11}}-\frac{35 \left(a^{6} d^{6}-6 a^{5} b c d^{5}+15 a^{4} b^{2} c^{2} d^{4}-20 a^{3} b^{3} c^{3} d^{3}+15 a^{2} b^{4} c^{4} d^{2}-6 a b^{5} c^{5} d+b^{6} c^{6}\right) d^{4}}{2 (bx+a)^{12} b^{11}}+\frac{120 \left(a^{7} d^{7}-7 a^{6} b c d^{6}+21 a^{5} b^{2} c^{2} d^{5}-35 a^{4} b^{3} c^{3} d^{4}+35 a^{3} b^{4} c^{4} d^{3}-21 a^{2} b^{5} c^{5} d^{2}+7 a b^{6} c^{6} d-b^{7} c^{7}\right) d^{3}}{13 (bx+a)^{13} b^{11}}-\frac{45 \left(a^{8} d^{8}-8 a^{7} b c d^{7}+28 a^{6} b^{2} c^{2} d^{6}-56 a^{5} b^{3} c^{3} d^{5}+70 a^{4} b^{4} c^{4} d^{4}-56 a^{3} b^{5} c^{5} d^{3}+28 a^{2} b^{6} c^{6} d^{2}-8 a b^{7} c^{7} d+b^{8} c^{8}\right) d^{2}}{14 (bx+a)^{14} b^{11}}+\frac{2 \left(a^{9} d^{9}-9 a^{8} b c d^{8}+36 a^{7} b^{2} c^{2} d^{7}-84 a^{6} b^{3} c^{3} d^{6}+126 a^{5} b^{4} c^{4} d^{5}-126 a^{4} b^{5} c^{5} d^{4}+84 a^{3} b^{6} c^{6} d^{3}-36 a^{2} b^{7} c^{7} d^{2}+9 a b^{8} c^{8} d-b^{9} c^{9}\right) d}{3 (bx+a)^{15} b^{11}}-\frac{d^{10} a^{10}-10 c d^{9} a^{9} b+45 c^{2} d^{8} a^{8} b^{2}-120 a^{7} c^{3} d^{7} b^{3}+210 c^{4} d^{6} a^{6} b^{4}-252 a^{5} c^{5} d^{5} b^{5}+210 a^{4} c^{6} d^{4} b^{6}-120 a^{3} c^{7} d^{3} b^{7}+45 a^{2} c^{8} d^{2} b^{8}-10 a b^{9} c^{9} d+c^{10} b^{10}}{16 (bx+a)^{16} b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^17,x)
```

```
[Out] -45/8*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^8+120/13*d^3*(a^7*d^7-7*
a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2
*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^13+10/7*d^9*(a*d-b*c)/b^11
/(b*x+a)^7+40/3*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x
+a)^9+252/11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d
^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^11+2/3*d*(a^9*d^9-9*a^8*b*c*d^8+36*a
^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+8
4*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^15
-35/2*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a
^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^12-1/6*d^10/b^11/(b*x+a)
```

```
^6-21*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b
^11/(b*x+a)^10-45/14*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b
^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7
*c^7*d+b^8*c^8)/b^11/(b*x+a)^14-1/16*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c
^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*
b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10
)/b^11/(b*x+a)^16
```

**maxima** [B] time = 2.27, size = 1030, normalized size = 5.66

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="maxima")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 3003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^
2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5
*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^
9*b*c*d^9 + a^10*d^10 + 11440*(6*b^10*c*d^9 + a*b^9*d^10)*x^9 + 12870*(21*b
^10*c^2*d^8 + 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 11440*(56*b^10*c^3*d^7 +
21*a*b^9*c^2*d^8 + 6*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 8008*(126*b^10*c^4
*d^6 + 56*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 + 6*a^3*b^7*c*d^9 + a^4*b^6*d^
10)*x^6 + 4368*(252*b^10*c^5*d^5 + 126*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 +
 21*a^3*b^7*c^2*d^8 + 6*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1820*(462*b^10*
c^6*d^4 + 252*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 + 56*a^3*b^7*c^3*d^7 + 21
*a^4*b^6*c^2*d^8 + 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 560*(792*b^10*c^7*
d^3 + 462*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 + 126*a^3*b^7*c^4*d^6 + 56*a^
4*b^6*c^3*d^7 + 21*a^5*b^5*c^2*d^8 + 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 +
120*(1287*b^10*c^8*d^2 + 792*a*b^9*c^7*d^3 + 462*a^2*b^8*c^6*d^4 + 252*a^3*
b^7*c^5*d^5 + 126*a^4*b^6*c^4*d^6 + 56*a^5*b^5*c^3*d^7 + 21*a^6*b^4*c^2*d^8
 + 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 16*(2002*b^10*c^9*d + 1287*a*b^9*c
^8*d^2 + 792*a^2*b^8*c^7*d^3 + 462*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5*d^5 +
126*a^5*b^5*c^4*d^6 + 56*a^6*b^4*c^3*d^7 + 21*a^7*b^3*c^2*d^8 + 6*a^8*b^2*c
*d^9 + a^9*b*d^10)*x)/(b^27*x^16 + 16*a*b^26*x^15 + 120*a^2*b^25*x^14 + 560
*a^3*b^24*x^13 + 1820*a^4*b^23*x^12 + 4368*a^5*b^22*x^11 + 8008*a^6*b^21*x^
10 + 11440*a^7*b^20*x^9 + 12870*a^8*b^19*x^8 + 11440*a^9*b^18*x^7 + 8008*a^
10*b^17*x^6 + 4368*a^11*b^16*x^5 + 1820*a^12*b^15*x^4 + 560*a^13*b^14*x^3 +
 120*a^14*b^13*x^2 + 16*a^15*b^12*x + a^16*b^11)
```

**mupad** [B] time = 0.58, size = 1131, normalized size = 6.21

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^17,x)
```

```
[Out] -(a^10*d^10 + 3003*b^10*c^10 + 8008*b^10*d^10*x^10 + 11440*a*b^9*d^10*x^9 +
 68640*b^10*c*d^9*x^9 + 1287*a^2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^
4*b^6*c^6*d^4 + 252*a^5*b^5*c^5*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*
d^7 + 21*a^8*b^2*c^2*d^8 + 120*a^8*b^2*d^10*x^2 + 560*a^7*b^3*d^10*x^3 + 18
20*a^6*b^4*d^10*x^4 + 4368*a^5*b^5*d^10*x^5 + 8008*a^4*b^6*d^10*x^6 + 11440
*a^3*b^7*d^10*x^7 + 12870*a^2*b^8*d^10*x^8 + 154440*b^10*c^8*d^2*x^2 + 4435
20*b^10*c^7*d^3*x^3 + 840840*b^10*c^6*d^4*x^4 + 1100736*b^10*c^5*d^5*x^5 +
1009008*b^10*c^4*d^6*x^6 + 640640*b^10*c^3*d^7*x^7 + 270270*b^10*c^2*d^8*x^
8 + 2002*a*b^9*c^9*d + 6*a^9*b*c*d^9 + 16*a^9*b*d^10*x + 32032*b^10*c^9*d*x
 + 55440*a^2*b^8*c^6*d^4*x^2 + 30240*a^3*b^7*c^5*d^5*x^2 + 15120*a^4*b^6*c^
4*d^6*x^2 + 6720*a^5*b^5*c^3*d^7*x^2 + 2520*a^6*b^4*c^2*d^8*x^2 + 141120*a^
2*b^8*c^5*d^5*x^3 + 70560*a^3*b^7*c^4*d^6*x^3 + 31360*a^4*b^6*c^3*d^7*x^3 +
 11760*a^5*b^5*c^2*d^8*x^3 + 229320*a^2*b^8*c^4*d^6*x^4 + 101920*a^3*b^7*c^
3*d^7*x^4 + 38220*a^4*b^6*c^2*d^8*x^4 + 244608*a^2*b^8*c^3*d^7*x^5 + 91728*
a^3*b^7*c^2*d^8*x^5 + 168168*a^2*b^8*c^2*d^8*x^6 + 20592*a*b^9*c^8*d^2*x +
96*a^8*b^2*c*d^9*x + 77220*a*b^9*c*d^9*x^8 + 12672*a^2*b^8*c^7*d^3*x + 7392
*a^3*b^7*c^6*d^4*x + 4032*a^4*b^6*c^5*d^5*x + 2016*a^5*b^5*c^4*d^6*x + 896*
a^6*b^4*c^3*d^7*x + 336*a^7*b^3*c^2*d^8*x + 95040*a*b^9*c^7*d^3*x^2 + 720*a
^7*b^3*c*d^9*x^2 + 258720*a*b^9*c^6*d^4*x^3 + 3360*a^6*b^4*c*d^9*x^3 + 4586
40*a*b^9*c^5*d^5*x^4 + 10920*a^5*b^5*c*d^9*x^4 + 550368*a*b^9*c^4*d^6*x^5 +
 26208*a^4*b^6*c*d^9*x^5 + 448448*a*b^9*c^3*d^7*x^6 + 48048*a^3*b^7*c*d^9*x
^6 + 240240*a*b^9*c^2*d^8*x^7 + 68640*a^2*b^8*c*d^9*x^7)/(48048*a^16*b^11 +
 48048*b^27*x^16 + 768768*a^15*b^12*x + 768768*a*b^26*x^15 + 5765760*a^14*b
^13*x^2 + 26906880*a^13*b^14*x^3 + 87447360*a^12*b^15*x^4 + 209873664*a^11*
b^16*x^5 + 384768384*a^10*b^17*x^6 + 549669120*a^9*b^18*x^7 + 618377760*a^8
*b^19*x^8 + 549669120*a^7*b^20*x^9 + 384768384*a^6*b^21*x^10 + 209873664*a^
5*b^22*x^11 + 87447360*a^4*b^23*x^12 + 26906880*a^3*b^24*x^13 + 5765760*a^2
*b^25*x^14)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**17,x)
```

```
[Out] Timed out
```

# 3.1223 $\int \frac{(c+dx)^{10}}{(a+bx)^{18}} dx$

**Optimal**. Leaf size=213

$$-\frac{d^6(c+dx)^{11}}{136136(a+bx)^{11}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{12376(a+bx)^{12}(bc-ad)^6}-\frac{3d^4(c+dx)^{11}}{6188(a+bx)^{13}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{476(a+bx)^{14}(bc-ad)^4}-$$

**Rubi** [A] time = 0.08, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^6(c+dx)^{11}}{136136(a+bx)^{11}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{12376(a+bx)^{12}(bc-ad)^6}-\frac{3d^4(c+dx)^{11}}{6188(a+bx)^{13}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{476(a+bx)^{14}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{136(a+bx)^{15}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{136(a+bx)^{16}(bc-ad)^2}-\frac{(c+dx)^{11}}{17(a+bx)^{17}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^18,x]
```

```
[Out] -(c + d*x)^11/(17*(b*c - a*d)*(a + b*x)^17) + (3*d*(c + d*x)^11)/(136*(b*c
- a*d)^2*(a + b*x)^16) - (d^2*(c + d*x)^11)/(136*(b*c - a*d)^3*(a + b*x)^15
) + (d^3*(c + d*x)^11)/(476*(b*c - a*d)^4*(a + b*x)^14) - (3*d^4*(c + d*x)^
11)/(6188*(b*c - a*d)^5*(a + b*x)^13) + (d^5*(c + d*x)^11)/(12376*(b*c - a*
d)^6*(a + b*x)^12) - (d^6*(c + d*x)^11)/(136136*(b*c - a*d)^7*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx = -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} - \frac{(6d)\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx}{17(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} + \frac{\left(15d^2\right)\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx}{136(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} - \frac{d^3\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{34(bc-ad)^3}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+d}{476(bc-ad)^4(}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+d}{476(bc-ad)^4(}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+d}{476(bc-ad)^4(}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+d}{476(bc-ad)^4(}$$

**Mathematica [B]** time = 0.32, size = 690, normalized size = 3.24

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^18,x]
```

```
[Out] -1/136136*(a^10*d^10 + a^9*b*d^9*(7*c + 17*d*x) + a^8*b^2*d^8*(28*c^2 + 119
*c*d*x + 136*d^2*x^2) + 4*a^7*b^3*d^7*(21*c^3 + 119*c^2*d*x + 238*c*d^2*x^2
 + 170*d^3*x^3) + 14*a^6*b^4*d^6*(15*c^4 + 102*c^3*d*x + 272*c^2*d^2*x^2 +
340*c*d^3*x^3 + 170*d^4*x^4) + 14*a^5*b^5*d^5*(33*c^5 + 255*c^4*d*x + 816*c
^3*d^2*x^2 + 1360*c^2*d^3*x^3 + 1190*c*d^4*x^4 + 442*d^5*x^5) + 14*a^4*b^6*
d^4*(66*c^6 + 561*c^5*d*x + 2040*c^4*d^2*x^2 + 4080*c^3*d^3*x^3 + 4760*c^2*
d^4*x^4 + 3094*c*d^5*x^5 + 884*d^6*x^6) + 4*a^3*b^7*d^3*(429*c^7 + 3927*c^6
*d*x + 15708*c^5*d^2*x^2 + 35700*c^4*d^3*x^3 + 49980*c^3*d^4*x^4 + 43316*c^
2*d^5*x^5 + 21658*c*d^6*x^6 + 4862*d^7*x^7) + a^2*b^8*d^2*(3003*c^8 + 29172
*c^7*d*x + 125664*c^6*d^2*x^2 + 314160*c^5*d^3*x^3 + 499800*c^4*d^4*x^4 + 5
19792*c^3*d^5*x^5 + 346528*c^2*d^6*x^6 + 136136*c*d^7*x^7 + 24310*d^8*x^8)
+ a*b^9*d*(5005*c^9 + 51051*c^8*d*x + 233376*c^7*d^2*x^2 + 628320*c^6*d^3*x
^3 + 1099560*c^5*d^4*x^4 + 1299480*c^4*d^5*x^5 + 1039584*c^3*d^6*x^6 + 5445
```

```
44*c^2*d^7*x^7 + 170170*c*d^8*x^8 + 24310*d^9*x^9) + b^10*(8008*c^10 + 8508
5*c^9*d*x + 408408*c^8*d^2*x^2 + 1166880*c^7*d^3*x^3 + 2199120*c^6*d^4*x^4
+ 2858856*c^5*d^5*x^5 + 2598960*c^4*d^6*x^6 + 1633632*c^3*d^7*x^7 + 680680*
c^2*d^8*x^8 + 170170*c*d^9*x^9 + 19448*d^10*x^10))/(b^11*(a + b*x)^17)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^18,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^18, x]
```

**fricas [B]** time = 1.31, size = 1041, normalized size = 4.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="fricas")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*
a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*
c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7
*a^9*b*c*d^9 + a^10*d^10 + 24310*(7*b^10*c*d^9 + a*b^9*d^10)*x^9 + 24310*(2
8*b^10*c^2*d^8 + 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 19448*(84*b^10*c^3*d^7
 + 28*a*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 12376*(210*b^10
*c^4*d^6 + 84*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 + 7*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 6188*(462*b^10*c^5*d^5 + 210*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 + 28*a^3*b^7*c^2*d^8 + 7*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 2380*(924*b
^10*c^6*d^4 + 462*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 + 84*a^3*b^7*c^3*d^7
+ 28*a^4*b^6*c^2*d^8 + 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 680*(1716*b^10
*c^7*d^3 + 924*a*b^9*c^6*d^4 + 462*a^2*b^8*c^5*d^5 + 210*a^3*b^7*c^4*d^6 +
84*a^4*b^6*c^3*d^7 + 28*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x
^3 + 136*(3003*b^10*c^8*d^2 + 1716*a*b^9*c^7*d^3 + 924*a^2*b^8*c^6*d^4 + 46
2*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 + 84*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c
^2*d^8 + 7*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 17*(5005*b^10*c^9*d + 3003*a
*b^9*c^8*d^2 + 1716*a^2*b^8*c^7*d^3 + 924*a^3*b^7*c^6*d^4 + 462*a^4*b^6*c^5
*d^5 + 210*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 + 7*a^
8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^28*x^17 + 17*a*b^27*x^16 + 136*a^2*b^26*x^1
5 + 680*a^3*b^25*x^14 + 2380*a^4*b^24*x^13 + 6188*a^5*b^23*x^12 + 12376*a^6
*b^22*x^11 + 19448*a^7*b^21*x^10 + 24310*a^8*b^20*x^9 + 24310*a^9*b^19*x^8
+ 19448*a^10*b^18*x^7 + 12376*a^11*b^17*x^6 + 6188*a^12*b^16*x^5 + 2380*a^1
```

3*b^15*x^4 + 680*a^14*b^14*x^3 + 136*a^15*b^13*x^2 + 17*a^16*b^12*x + a^17*
b^11)

**giac [B]** time = 1.32, size = 961, normalized size = 4.51

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="giac")

[Out] -1/136136*(19448*b^10*d^10*x^10 + 170170*b^10*c*d^9*x^9 + 24310*a*b^9*d^10*
x^9 + 680680*b^10*c^2*d^8*x^8 + 170170*a*b^9*c*d^9*x^8 + 24310*a^2*b^8*d^10
*x^8 + 1633632*b^10*c^3*d^7*x^7 + 544544*a*b^9*c^2*d^8*x^7 + 136136*a^2*b^8
*c*d^9*x^7 + 19448*a^3*b^7*d^10*x^7 + 2598960*b^10*c^4*d^6*x^6 + 1039584*a*
b^9*c^3*d^7*x^6 + 346528*a^2*b^8*c^2*d^8*x^6 + 86632*a^3*b^7*c*d^9*x^6 + 12
376*a^4*b^6*d^10*x^6 + 2858856*b^10*c^5*d^5*x^5 + 1299480*a*b^9*c^4*d^6*x^5
 + 519792*a^2*b^8*c^3*d^7*x^5 + 173264*a^3*b^7*c^2*d^8*x^5 + 43316*a^4*b^6*
c*d^9*x^5 + 6188*a^5*b^5*d^10*x^5 + 2199120*b^10*c^6*d^4*x^4 + 1099560*a*b^
9*c^5*d^5*x^4 + 499800*a^2*b^8*c^4*d^6*x^4 + 199920*a^3*b^7*c^3*d^7*x^4 + 6
6640*a^4*b^6*c^2*d^8*x^4 + 16660*a^5*b^5*c*d^9*x^4 + 2380*a^6*b^4*d^10*x^4
+ 1166880*b^10*c^7*d^3*x^3 + 628320*a*b^9*c^6*d^4*x^3 + 314160*a^2*b^8*c^5*
d^5*x^3 + 142800*a^3*b^7*c^4*d^6*x^3 + 57120*a^4*b^6*c^3*d^7*x^3 + 19040*a^
5*b^5*c^2*d^8*x^3 + 4760*a^6*b^4*c*d^9*x^3 + 680*a^7*b^3*d^10*x^3 + 408408*
b^10*c^8*d^2*x^2 + 233376*a*b^9*c^7*d^3*x^2 + 125664*a^2*b^8*c^6*d^4*x^2 +
62832*a^3*b^7*c^5*d^5*x^2 + 28560*a^4*b^6*c^4*d^6*x^2 + 11424*a^5*b^5*c^3*d
^7*x^2 + 3808*a^6*b^4*c^2*d^8*x^2 + 952*a^7*b^3*c*d^9*x^2 + 136*a^8*b^2*d^1
0*x^2 + 85085*b^10*c^9*d*x + 51051*a*b^9*c^8*d^2*x + 29172*a^2*b^8*c^7*d^3*
x + 15708*a^3*b^7*c^6*d^4*x + 7854*a^4*b^6*c^5*d^5*x + 3570*a^5*b^5*c^4*d^6
*x + 1428*a^6*b^4*c^3*d^7*x + 476*a^7*b^3*c^2*d^8*x + 119*a^8*b^2*c*d^9*x +
 17*a^9*b*d^10*x + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*a^2*b^8*c^8*d^2
 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*c^5*d^5 + 210*a
^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7*a^9*b*c*d^9 +
a^10*d^10)/((b*x + a)^17*b^11)

**maple [B]** time = 0.01, size = 867, normalized size = 4.07

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^10/(b*x+a)^18,x)

[Out] 5/4*d^9*(a*d-b*c)/b^11/(b*x+a)^8-210/13*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b
^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^1
1/(b*x+a)^13-1/7*d^10/b^11/(b*x+a)^7-5*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11

```
/(b*x+a)^9-1/17*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^
3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b
^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^17-210
/11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^1
1/(b*x+a)^11-3*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3
*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d
+b^8*c^8)/b^11/(b*x+a)^15+21*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-
10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^12+12*d^7*(a^3*d^3-3
*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^10+60/7*d^3*(a^7*d^7-7*a^6
*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^
5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^14+5/8*d*(a^9*d^9-9*a^8*b*c*d
^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^
5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*
x+a)^16
```

**maxima [B]** time = 2.29, size = 1041, normalized size = 4.89

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="maxima")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*
a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*
c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7
*a^9*b*c*d^9 + a^10*d^10 + 24310*(7*b^10*c*d^9 + a*b^9*d^10)*x^9 + 24310*(2
8*b^10*c^2*d^8 + 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 19448*(84*b^10*c^3*d^7
 + 28*a*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 12376*(210*b^10
*c^4*d^6 + 84*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 + 7*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 6188*(462*b^10*c^5*d^5 + 210*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 + 28*a^3*b^7*c^2*d^8 + 7*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 2380*(924*b
^10*c^6*d^4 + 462*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 + 84*a^3*b^7*c^3*d^7
+ 28*a^4*b^6*c^2*d^8 + 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 680*(1716*b^10
*c^7*d^3 + 924*a*b^9*c^6*d^4 + 462*a^2*b^8*c^5*d^5 + 210*a^3*b^7*c^4*d^6 +
84*a^4*b^6*c^3*d^7 + 28*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x
^3 + 136*(3003*b^10*c^8*d^2 + 1716*a*b^9*c^7*d^3 + 924*a^2*b^8*c^6*d^4 + 46
2*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 + 84*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c
^2*d^8 + 7*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 17*(5005*b^10*c^9*d + 3003*a
*b^9*c^8*d^2 + 1716*a^2*b^8*c^7*d^3 + 924*a^3*b^7*c^6*d^4 + 462*a^4*b^6*c^5
*d^5 + 210*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 + 7*a^
8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^28*x^17 + 17*a*b^27*x^16 + 136*a^2*b^26*x^1
5 + 680*a^3*b^25*x^14 + 2380*a^4*b^24*x^13 + 6188*a^5*b^23*x^12 + 12376*a^6
*b^22*x^11 + 19448*a^7*b^21*x^10 + 24310*a^8*b^20*x^9 + 24310*a^9*b^19*x^8
+ 19448*a^10*b^18*x^7 + 12376*a^11*b^17*x^6 + 6188*a^12*b^16*x^5 + 2380*a^1
```

```
3*b^15*x^4 + 680*a^14*b^14*x^3 + 136*a^15*b^13*x^2 + 17*a^16*b^12*x + a^17*
b^11)
```

**mupad [B]** time = 0.66, size = 1142, normalized size = 5.36

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^18,x)
```

```
[Out] -(a^10*d^10 + 8008*b^10*c^10 + 19448*b^10*d^10*x^10 + 24310*a*b^9*d^10*x^9
+ 170170*b^10*c*d^9*x^9 + 3003*a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924
*a^4*b^6*c^6*d^4 + 462*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c
^3*d^7 + 28*a^8*b^2*c^2*d^8 + 136*a^8*b^2*d^10*x^2 + 680*a^7*b^3*d^10*x^3 +
 2380*a^6*b^4*d^10*x^4 + 6188*a^5*b^5*d^10*x^5 + 12376*a^4*b^6*d^10*x^6 + 1
9448*a^3*b^7*d^10*x^7 + 24310*a^2*b^8*d^10*x^8 + 408408*b^10*c^8*d^2*x^2 +
1166880*b^10*c^7*d^3*x^3 + 2199120*b^10*c^6*d^4*x^4 + 2858856*b^10*c^5*d^5*
x^5 + 2598960*b^10*c^4*d^6*x^6 + 1633632*b^10*c^3*d^7*x^7 + 680680*b^10*c^2
*d^8*x^8 + 5005*a*b^9*c^9*d + 7*a^9*b*c*d^9 + 17*a^9*b*d^10*x + 85085*b^10*
c^9*d*x + 125664*a^2*b^8*c^6*d^4*x^2 + 62832*a^3*b^7*c^5*d^5*x^2 + 28560*a^
4*b^6*c^4*d^6*x^2 + 11424*a^5*b^5*c^3*d^7*x^2 + 3808*a^6*b^4*c^2*d^8*x^2 +
314160*a^2*b^8*c^5*d^5*x^3 + 142800*a^3*b^7*c^4*d^6*x^3 + 57120*a^4*b^6*c^3
*d^7*x^3 + 19040*a^5*b^5*c^2*d^8*x^3 + 499800*a^2*b^8*c^4*d^6*x^4 + 199920*
a^3*b^7*c^3*d^7*x^4 + 66640*a^4*b^6*c^2*d^8*x^4 + 519792*a^2*b^8*c^3*d^7*x^
5 + 173264*a^3*b^7*c^2*d^8*x^5 + 346528*a^2*b^8*c^2*d^8*x^6 + 51051*a*b^9*c
^8*d^2*x + 119*a^8*b^2*c*d^9*x + 170170*a*b^9*c*d^9*x^8 + 29172*a^2*b^8*c^7
*d^3*x + 15708*a^3*b^7*c^6*d^4*x + 7854*a^4*b^6*c^5*d^5*x + 3570*a^5*b^5*c^
4*d^6*x + 1428*a^6*b^4*c^3*d^7*x + 476*a^7*b^3*c^2*d^8*x + 233376*a*b^9*c^7
*d^3*x^2 + 952*a^7*b^3*c*d^9*x^2 + 628320*a*b^9*c^6*d^4*x^3 + 4760*a^6*b^4*
c*d^9*x^3 + 1099560*a*b^9*c^5*d^5*x^4 + 16660*a^5*b^5*c*d^9*x^4 + 1299480*a
*b^9*c^4*d^6*x^5 + 43316*a^4*b^6*c*d^9*x^5 + 1039584*a*b^9*c^3*d^7*x^6 + 86
632*a^3*b^7*c*d^9*x^6 + 544544*a*b^9*c^2*d^8*x^7 + 136136*a^2*b^8*c*d^9*x^7
)/(136136*a^17*b^11 + 136136*b^28*x^17 + 2314312*a^16*b^12*x + 2314312*a*b^
27*x^16 + 18514496*a^15*b^13*x^2 + 92572480*a^14*b^14*x^3 + 324003680*a^13*
b^15*x^4 + 842409568*a^12*b^16*x^5 + 1684819136*a^11*b^17*x^6 + 2647572928*
a^10*b^18*x^7 + 3309466160*a^9*b^19*x^8 + 3309466160*a^8*b^20*x^9 + 2647572
928*a^7*b^21*x^10 + 1684819136*a^6*b^22*x^11 + 842409568*a^5*b^23*x^12 + 32
4003680*a^4*b^24*x^13 + 92572480*a^3*b^25*x^14 + 18514496*a^2*b^26*x^15)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**18,x)
```

```
[Out] Timed out
```

## 3.1224 $\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx$

**Optimal**. Leaf size=244

$$\frac{d^7(c+dx)^{11}}{350064(a+bx)^{11}(bc-ad)^8}-\frac{d^6(c+dx)^{11}}{31824(a+bx)^{12}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{5304(a+bx)^{13}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{1224(a+bx)^{14}(bc-ad)^5}+ 24$$

**Rubi [A]** time = 0.10, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d^7(c+dx)^{11}}{350064(a+bx)^{11}(bc-ad)^8}-\frac{d^6(c+dx)^{11}}{31824(a+bx)^{12}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{5304(a+bx)^{13}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{1224(a+bx)^{14}(bc-ad)^5}+\frac{7d^3(c+dx)^{11}}{2448(a+bx)^{15}(bc-ad)^4}-\frac{7d^2(c+dx)^{11}}{816(a+bx)^{16}(bc-ad)^3}+\frac{7d(c+dx)^{11}}{306(a+bx)^{17}(bc-ad)^2}-\frac{(c+dx)^{11}}{18(a+bx)^{18}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^19,x]
```

```
[Out] -(c + d*x)^11/(18*(b*c - a*d)*(a + b*x)^18) + (7*d*(c + d*x)^11)/(306*(b*c
- a*d)^2*(a + b*x)^17) - (7*d^2*(c + d*x)^11)/(816*(b*c - a*d)^3*(a + b*x)^
16) + (7*d^3*(c + d*x)^11)/(2448*(b*c - a*d)^4*(a + b*x)^15) - (d^4*(c + d*
x)^11)/(1224*(b*c - a*d)^5*(a + b*x)^14) + (d^5*(c + d*x)^11)/(5304*(b*c -
a*d)^6*(a + b*x)^13) - (d^6*(c + d*x)^11)/(31824*(b*c - a*d)^7*(a + b*x)^12
) + (d^7*(c + d*x)^11)/(350064*(b*c - a*d)^8*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx = -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} - \frac{(7d)\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx}{18(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} + \frac{\left(7d^2\right)\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx}{51(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} - \frac{\left(35d^3\right)\int \frac{(c+}{(a+}}{816(bc-}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c-}{2448(bc-a}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c-}{2448(bc-a}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c-}{2448(bc-a}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c-}{2448(bc-a}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c-}{2448(bc-a}$$

**Mathematica [B]** time = 0.28, size = 694, normalized size = 2.84

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^19,x]
```

```
[Out] -1/350064*(a^10*d^10 + 2*a^9*b*d^9*(4*c + 9*d*x) + 9*a^8*b^2*d^8*(4*c^2 + 1
6*c*d*x + 17*d^2*x^2) + 24*a^7*b^3*d^7*(5*c^3 + 27*c^2*d*x + 51*c*d^2*x^2 +
 34*d^3*x^3) + 6*a^6*b^4*d^6*(55*c^4 + 360*c^3*d*x + 918*c^2*d^2*x^2 + 1088
*c*d^3*x^3 + 510*d^4*x^4) + 36*a^5*b^5*d^5*(22*c^5 + 165*c^4*d*x + 510*c^3*
d^2*x^2 + 816*c^2*d^3*x^3 + 680*c*d^4*x^4 + 238*d^5*x^5) + 6*a^4*b^6*d^4*(2
86*c^6 + 2376*c^5*d*x + 8415*c^4*d^2*x^2 + 16320*c^3*d^3*x^3 + 18360*c^2*d^
4*x^4 + 11424*c*d^5*x^5 + 3094*d^6*x^6) + 24*a^3*b^7*d^3*(143*c^7 + 1287*c^
6*d*x + 5049*c^5*d^2*x^2 + 11220*c^4*d^3*x^3 + 15300*c^3*d^4*x^4 + 12852*c^
2*d^5*x^5 + 6188*c*d^6*x^6 + 1326*d^7*x^7) + 9*a^2*b^8*d^2*(715*c^8 + 6864*
c^7*d*x + 29172*c^6*d^2*x^2 + 71808*c^5*d^3*x^3 + 112200*c^4*d^4*x^4 + 1142
```

```
40*c^3*d^5*x^5 + 74256*c^2*d^6*x^6 + 28288*c*d^7*x^7 + 4862*d^8*x^8) + 2*a*
b^9*d*(5720*c^9 + 57915*c^8*d*x + 262548*c^7*d^2*x^2 + 700128*c^6*d^3*x^3 +
 1211760*c^5*d^4*x^4 + 1413720*c^4*d^5*x^5 + 1113840*c^3*d^6*x^6 + 572832*c
^2*d^7*x^7 + 175032*c*d^8*x^8 + 24310*d^9*x^9) + b^10*(19448*c^10 + 205920*
c^9*d*x + 984555*c^8*d^2*x^2 + 2800512*c^7*d^3*x^3 + 5250960*c^6*d^4*x^4 +
6785856*c^5*d^5*x^5 + 6126120*c^4*d^6*x^6 + 3818880*c^3*d^7*x^7 + 1575288*c
^2*d^8*x^8 + 388960*c*d^9*x^9 + 43758*d^10*x^10))/(b^11*(a + b*x)^18)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^19,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^19, x]
```

**fricas [B]** time = 1.19, size = 1052, normalized size = 4.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="fricas")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 643
5*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b
^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8
 + 8*a^9*b*c*d^9 + a^10*d^10 + 48620*(8*b^10*c*d^9 + a*b^9*d^10)*x^9 + 4375
8*(36*b^10*c^2*d^8 + 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 31824*(120*b^10*c^
3*d^7 + 36*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 18564*(330
*b^10*c^4*d^6 + 120*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 + 8*a^3*b^7*c*d^9 +
a^4*b^6*d^10)*x^6 + 8568*(792*b^10*c^5*d^5 + 330*a*b^9*c^4*d^6 + 120*a^2*b^
8*c^3*d^7 + 36*a^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 3060
*(1716*b^10*c^6*d^4 + 792*a*b^9*c^5*d^5 + 330*a^2*b^8*c^4*d^6 + 120*a^3*b^7
*c^3*d^7 + 36*a^4*b^6*c^2*d^8 + 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 816*(
3432*b^10*c^7*d^3 + 1716*a*b^9*c^6*d^4 + 792*a^2*b^8*c^5*d^5 + 330*a^3*b^7*
c^4*d^6 + 120*a^4*b^6*c^3*d^7 + 36*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 + a^7*
b^3*d^10)*x^3 + 153*(6435*b^10*c^8*d^2 + 3432*a*b^9*c^7*d^3 + 1716*a^2*b^8*
c^6*d^4 + 792*a^3*b^7*c^5*d^5 + 330*a^4*b^6*c^4*d^6 + 120*a^5*b^5*c^3*d^7 +
 36*a^6*b^4*c^2*d^8 + 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 18*(11440*b^10*
c^9*d + 6435*a*b^9*c^8*d^2 + 3432*a^2*b^8*c^7*d^3 + 1716*a^3*b^7*c^6*d^4 +
792*a^4*b^6*c^5*d^5 + 330*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 + 36*a^7*b^
3*c^2*d^8 + 8*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^29*x^18 + 18*a*b^28*x^17 +
```

```
153*a^2*b^27*x^16 + 816*a^3*b^26*x^15 + 3060*a^4*b^25*x^14 + 8568*a^5*b^24*
x^13 + 18564*a^6*b^23*x^12 + 31824*a^7*b^22*x^11 + 43758*a^8*b^21*x^10 + 48
620*a^9*b^20*x^9 + 43758*a^10*b^19*x^8 + 31824*a^11*b^18*x^7 + 18564*a^12*b
^17*x^6 + 8568*a^13*b^16*x^5 + 3060*a^14*b^15*x^4 + 816*a^15*b^14*x^3 + 153
*a^16*b^13*x^2 + 18*a^17*b^12*x + a^18*b^11)
```

**giac [B]** time = 1.40, size = 961, normalized size = 3.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="giac")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 388960*b^10*c*d^9*x^9 + 48620*a*b^9*d^10*
x^9 + 1575288*b^10*c^2*d^8*x^8 + 350064*a*b^9*c*d^9*x^8 + 43758*a^2*b^8*d^1
0*x^8 + 3818880*b^10*c^3*d^7*x^7 + 1145664*a*b^9*c^2*d^8*x^7 + 254592*a^2*b
^8*c*d^9*x^7 + 31824*a^3*b^7*d^10*x^7 + 6126120*b^10*c^4*d^6*x^6 + 2227680*
a*b^9*c^3*d^7*x^6 + 668304*a^2*b^8*c^2*d^8*x^6 + 148512*a^3*b^7*c*d^9*x^6 +
 18564*a^4*b^6*d^10*x^6 + 6785856*b^10*c^5*d^5*x^5 + 2827440*a*b^9*c^4*d^6*
x^5 + 1028160*a^2*b^8*c^3*d^7*x^5 + 308448*a^3*b^7*c^2*d^8*x^5 + 68544*a^4*
b^6*c*d^9*x^5 + 8568*a^5*b^5*d^10*x^5 + 5250960*b^10*c^6*d^4*x^4 + 2423520*
a*b^9*c^5*d^5*x^4 + 1009800*a^2*b^8*c^4*d^6*x^4 + 367200*a^3*b^7*c^3*d^7*x^
4 + 110160*a^4*b^6*c^2*d^8*x^4 + 24480*a^5*b^5*c*d^9*x^4 + 3060*a^6*b^4*d^1
0*x^4 + 2800512*b^10*c^7*d^3*x^3 + 1400256*a*b^9*c^6*d^4*x^3 + 646272*a^2*b
^8*c^5*d^5*x^3 + 269280*a^3*b^7*c^4*d^6*x^3 + 97920*a^4*b^6*c^3*d^7*x^3 + 2
9376*a^5*b^5*c^2*d^8*x^3 + 6528*a^6*b^4*c*d^9*x^3 + 816*a^7*b^3*d^10*x^3 +
984555*b^10*c^8*d^2*x^2 + 525096*a*b^9*c^7*d^3*x^2 + 262548*a^2*b^8*c^6*d^4
*x^2 + 121176*a^3*b^7*c^5*d^5*x^2 + 50490*a^4*b^6*c^4*d^6*x^2 + 18360*a^5*b
^5*c^3*d^7*x^2 + 5508*a^6*b^4*c^2*d^8*x^2 + 1224*a^7*b^3*c*d^9*x^2 + 153*a^
8*b^2*d^10*x^2 + 205920*b^10*c^9*d*x + 115830*a*b^9*c^8*d^2*x + 61776*a^2*b
^8*c^7*d^3*x + 30888*a^3*b^7*c^6*d^4*x + 14256*a^4*b^6*c^5*d^5*x + 5940*a^5
*b^5*c^4*d^6*x + 2160*a^6*b^4*c^3*d^7*x + 648*a^7*b^3*c^2*d^8*x + 144*a^8*b
^2*c*d^9*x + 18*a^9*b*d^10*x + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 6435*a
^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b^5*
c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8 +
8*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^18*b^11)
```

**maple [B]** time = 0.01, size = 867, normalized size = 3.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^19,x)
```

```
[Out] -1/8*d^10/b^11/(b*x+a)^8+252/13*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d
^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^13+10/9*d^9*(a*d-
b*c)/b^11/(b*x+a)^9+10/17*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^
6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36
*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^17+120/11*d^7*(a^3*d^3
-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^11+8*d^3*(a^7*d^7-7*a^6*
b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5
*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^15-35/2*d^6*(a^4*d^4-4*a^3*b*c
*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^12-1/18*(a^10*d^
10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^
6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^
8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^18-9/2*d^8*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/b^11/(b*x+a)^10-15*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-2
0*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^14
-45/16*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*
a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8
)/b^11/(b*x+a)^16
```

**maxima [B]** time = 2.54, size = 1052, normalized size = 4.31

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="maxima")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 643
5*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b
^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8
 + 8*a^9*b*c*d^9 + a^10*d^10 + 48620*(8*b^10*c*d^9 + a*b^9*d^10)*x^9 + 4375
8*(36*b^10*c^2*d^8 + 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 31824*(120*b^10*c^
3*d^7 + 36*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 18564*(330
*b^10*c^4*d^6 + 120*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 + 8*a^3*b^7*c*d^9 +
a^4*b^6*d^10)*x^6 + 8568*(792*b^10*c^5*d^5 + 330*a*b^9*c^4*d^6 + 120*a^2*b^
8*c^3*d^7 + 36*a^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 3060
*(1716*b^10*c^6*d^4 + 792*a*b^9*c^5*d^5 + 330*a^2*b^8*c^4*d^6 + 120*a^3*b^7
*c^3*d^7 + 36*a^4*b^6*c^2*d^8 + 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 816*(
3432*b^10*c^7*d^3 + 1716*a*b^9*c^6*d^4 + 792*a^2*b^8*c^5*d^5 + 330*a^3*b^7*
c^4*d^6 + 120*a^4*b^6*c^3*d^7 + 36*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 + a^7*
b^3*d^10)*x^3 + 153*(6435*b^10*c^8*d^2 + 3432*a*b^9*c^7*d^3 + 1716*a^2*b^8*
c^6*d^4 + 792*a^3*b^7*c^5*d^5 + 330*a^4*b^6*c^4*d^6 + 120*a^5*b^5*c^3*d^7 +
 36*a^6*b^4*c^2*d^8 + 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 18*(11440*b^10*
c^9*d + 6435*a*b^9*c^8*d^2 + 3432*a^2*b^8*c^7*d^3 + 1716*a^3*b^7*c^6*d^4 +
792*a^4*b^6*c^5*d^5 + 330*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 + 36*a^7*b^
3*c^2*d^8 + 8*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^29*x^18 + 18*a*b^28*x^17 +
153*a^2*b^27*x^16 + 816*a^3*b^26*x^15 + 3060*a^4*b^25*x^14 + 8568*a^5*b^24*
```

```
x^13 + 18564*a^6*b^23*x^12 + 31824*a^7*b^22*x^11 + 43758*a^8*b^21*x^10 + 48
620*a^9*b^20*x^9 + 43758*a^10*b^19*x^8 + 31824*a^11*b^18*x^7 + 18564*a^12*b
^17*x^6 + 8568*a^13*b^16*x^5 + 3060*a^14*b^15*x^4 + 816*a^15*b^14*x^3 + 153
*a^16*b^13*x^2 + 18*a^17*b^12*x + a^18*b^11)
```

**mupad** [B] time = 12.02, size = 1153, normalized size = 4.73

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^19,x)
```

```
[Out] -(a^10*d^10 + 19448*b^10*c^10 + 43758*b^10*d^10*x^10 + 48620*a*b^9*d^10*x^9
 + 388960*b^10*c*d^9*x^9 + 6435*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 17
16*a^4*b^6*c^6*d^4 + 792*a^5*b^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^
3*c^3*d^7 + 36*a^8*b^2*c^2*d^8 + 153*a^8*b^2*d^10*x^2 + 816*a^7*b^3*d^10*x^
3 + 3060*a^6*b^4*d^10*x^4 + 8568*a^5*b^5*d^10*x^5 + 18564*a^4*b^6*d^10*x^6
+ 31824*a^3*b^7*d^10*x^7 + 43758*a^2*b^8*d^10*x^8 + 984555*b^10*c^8*d^2*x^2
 + 2800512*b^10*c^7*d^3*x^3 + 5250960*b^10*c^6*d^4*x^4 + 6785856*b^10*c^5*d
^5*x^5 + 6126120*b^10*c^4*d^6*x^6 + 3818880*b^10*c^3*d^7*x^7 + 1575288*b^10
*c^2*d^8*x^8 + 11440*a*b^9*c^9*d + 8*a^9*b*c*d^9 + 18*a^9*b*d^10*x + 205920
*b^10*c^9*d*x + 262548*a^2*b^8*c^6*d^4*x^2 + 121176*a^3*b^7*c^5*d^5*x^2 + 5
0490*a^4*b^6*c^4*d^6*x^2 + 18360*a^5*b^5*c^3*d^7*x^2 + 5508*a^6*b^4*c^2*d^8
*x^2 + 646272*a^2*b^8*c^5*d^5*x^3 + 269280*a^3*b^7*c^4*d^6*x^3 + 97920*a^4*
b^6*c^3*d^7*x^3 + 29376*a^5*b^5*c^2*d^8*x^3 + 1009800*a^2*b^8*c^4*d^6*x^4 +
 367200*a^3*b^7*c^3*d^7*x^4 + 110160*a^4*b^6*c^2*d^8*x^4 + 1028160*a^2*b^8*
c^3*d^7*x^5 + 308448*a^3*b^7*c^2*d^8*x^5 + 668304*a^2*b^8*c^2*d^8*x^6 + 115
830*a*b^9*c^8*d^2*x + 144*a^8*b^2*c*d^9*x + 350064*a*b^9*c*d^9*x^8 + 61776*
a^2*b^8*c^7*d^3*x + 30888*a^3*b^7*c^6*d^4*x + 14256*a^4*b^6*c^5*d^5*x + 594
0*a^5*b^5*c^4*d^6*x + 2160*a^6*b^4*c^3*d^7*x + 648*a^7*b^3*c^2*d^8*x + 5250
96*a*b^9*c^7*d^3*x^2 + 1224*a^7*b^3*c*d^9*x^2 + 1400256*a*b^9*c^6*d^4*x^3 +
 6528*a^6*b^4*c*d^9*x^3 + 2423520*a*b^9*c^5*d^5*x^4 + 24480*a^5*b^5*c*d^9*x
^4 + 2827440*a*b^9*c^4*d^6*x^5 + 68544*a^4*b^6*c*d^9*x^5 + 2227680*a*b^9*c^
3*d^7*x^6 + 148512*a^3*b^7*c*d^9*x^6 + 1145664*a*b^9*c^2*d^8*x^7 + 254592*a
^2*b^8*c*d^9*x^7)/(350064*a^18*b^11 + 350064*b^29*x^18 + 6301152*a^17*b^12*
x + 6301152*a*b^28*x^17 + 53559792*a^16*b^13*x^2 + 285652224*a^15*b^14*x^3
+ 1071195840*a^14*b^15*x^4 + 2999348352*a^13*b^16*x^5 + 6498588096*a^12*b^1
7*x^6 + 11140436736*a^11*b^18*x^7 + 15318100512*a^10*b^19*x^8 + 17020111680
*a^9*b^20*x^9 + 15318100512*a^8*b^21*x^10 + 11140436736*a^7*b^22*x^11 + 649
8588096*a^6*b^23*x^12 + 2999348352*a^5*b^24*x^13 + 1071195840*a^4*b^25*x^14
 + 285652224*a^3*b^26*x^15 + 53559792*a^2*b^27*x^16)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**19,x)
```

```
[Out] Timed out
```

# 3.1225 $\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx$

**Optimal**. Leaf size=273

$$-\frac{d^9(bc-ad)}{b^{11}(a+bx)^{10}}-\frac{45d^8(bc-ad)^2}{11b^{11}(a+bx)^{11}}-\frac{10d^7(bc-ad)^3}{b^{11}(a+bx)^{12}}-\frac{210d^6(bc-ad)^4}{13b^{11}(a+bx)^{13}}-\frac{18d^5(bc-ad)^5}{b^{11}(a+bx)^{14}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-\frac{15d^3(bc-}{2b^{11}(a+b}$$

**Rubi** [A] time = 0.28, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{d^9(bc-ad)}{b^{11}(a+bx)^{10}}-\frac{45d^8(bc-ad)^2}{11b^{11}(a+bx)^{11}}-\frac{10d^7(bc-ad)^3}{b^{11}(a+bx)^{12}}-\frac{210d^6(bc-ad)^4}{13b^{11}(a+bx)^{13}}-\frac{18d^5(bc-ad)^5}{b^{11}(a+bx)^{14}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-\frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}}-\frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}}-\frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}}-\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}}-\frac{d^{10}}{9b^{11}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] -(b*c - a*d)^10/(19*b^11*(a + b*x)^19) - (5*d*(b*c - a*d)^9)/(9*b^11*(a + b
*x)^18) - (45*d^2*(b*c - a*d)^8)/(17*b^11*(a + b*x)^17) - (15*d^3*(b*c - a*
d)^7)/(2*b^11*(a + b*x)^16) - (14*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^15) -
(18*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^14) - (210*d^6*(b*c - a*d)^4)/(13*b^
11*(a + b*x)^13) - (10*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^12) - (45*d^8*(b*
c - a*d)^2)/(11*b^11*(a + b*x)^11) - (d^9*(b*c - a*d))/(b^11*(a + b*x)^10)
- d^10/(9*b^11*(a + b*x)^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx = \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{20}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{19}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{18}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{17}}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^{16}}+\right.$$

$$= -\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}}-\frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}}-\frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}}-\frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-$$

**Mathematica** [B] time = 0.28, size = 692, normalized size = 2.53

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] -1/831402*(a^10*d^10 + a^9*b*d^9*(9*c + 19*d*x) + 9*a^8*b^2*d^8*(5*c^2 + 19
*c*d*x + 19*d^2*x^2) + 3*a^7*b^3*d^7*(55*c^3 + 285*c^2*d*x + 513*c*d^2*x^2
+ 323*d^3*x^3) + 3*a^6*b^4*d^6*(165*c^4 + 1045*c^3*d*x + 2565*c^2*d^2*x^2 +
 2907*c*d^3*x^3 + 1292*d^4*x^4) + 9*a^5*b^5*d^5*(143*c^5 + 1045*c^4*d*x + 3
135*c^3*d^2*x^2 + 4845*c^2*d^3*x^3 + 3876*c*d^4*x^4 + 1292*d^5*x^5) + 3*a^4
*b^6*d^4*(1001*c^6 + 8151*c^5*d*x + 28215*c^4*d^2*x^2 + 53295*c^3*d^3*x^3 +
 58140*c^2*d^4*x^4 + 34884*c*d^5*x^5 + 9044*d^6*x^6) + 3*a^3*b^7*d^3*(2145*
c^7 + 19019*c^6*d*x + 73359*c^5*d^2*x^2 + 159885*c^4*d^3*x^3 + 213180*c^3*d
^4*x^4 + 174420*c^2*d^5*x^5 + 81396*c*d^6*x^6 + 16796*d^7*x^7) + 9*a^2*b^8*
d^2*(1430*c^8 + 13585*c^7*d*x + 57057*c^6*d^2*x^2 + 138567*c^5*d^3*x^3 + 21
3180*c^4*d^4*x^4 + 213180*c^3*d^5*x^5 + 135660*c^2*d^6*x^6 + 50388*c*d^7*x^
7 + 8398*d^8*x^8) + a*b^9*d*(24310*c^9 + 244530*c^8*d*x + 1100385*c^7*d^2*x
^2 + 2909907*c^6*d^3*x^3 + 4988412*c^5*d^4*x^4 + 5755860*c^4*d^5*x^5 + 4476
780*c^3*d^6*x^6 + 2267460*c^2*d^7*x^7 + 680238*c*d^8*x^8 + 92378*d^9*x^9) +
 b^10*(43758*c^10 + 461890*c^9*d*x + 2200770*c^8*d^2*x^2 + 6235515*c^7*d^3*
x^3 + 11639628*c^6*d^4*x^4 + 14965236*c^5*d^5*x^5 + 13430340*c^4*d^6*x^6 +
8314020*c^3*d^7*x^7 + 3401190*c^2*d^8*x^8 + 831402*c*d^9*x^9 + 92378*d^10*x
^10))/(b^11*(a + b*x)^19)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^20, x]
```

**fricas [B]** time = 1.30, size = 1063, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="fricas")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 43758*b^10*c^10 + 24310*a*b^9*c^9*d + 128
70*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 + 1287*a^5
*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d
^8 + 9*a^9*b*c*d^9 + a^10*d^10 + 92378*(9*b^10*c*d^9 + a*b^9*d^10)*x^9 + 75
582*(45*b^10*c^2*d^8 + 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 50388*(165*b^10*
```

```
c^3*d^7 + 45*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 27132*(4
95*b^10*c^4*d^6 + 165*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 + 9*a^3*b^7*c*d^9
+ a^4*b^6*d^10)*x^6 + 11628*(1287*b^10*c^5*d^5 + 495*a*b^9*c^4*d^6 + 165*a^
2*b^8*c^3*d^7 + 45*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 +
3876*(3003*b^10*c^6*d^4 + 1287*a*b^9*c^5*d^5 + 495*a^2*b^8*c^4*d^6 + 165*a^
3*b^7*c^3*d^7 + 45*a^4*b^6*c^2*d^8 + 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 +
969*(6435*b^10*c^7*d^3 + 3003*a*b^9*c^6*d^4 + 1287*a^2*b^8*c^5*d^5 + 495*a^
3*b^7*c^4*d^6 + 165*a^4*b^6*c^3*d^7 + 45*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9
+ a^7*b^3*d^10)*x^3 + 171*(12870*b^10*c^8*d^2 + 6435*a*b^9*c^7*d^3 + 3003*a
^2*b^8*c^6*d^4 + 1287*a^3*b^7*c^5*d^5 + 495*a^4*b^6*c^4*d^6 + 165*a^5*b^5*c
^3*d^7 + 45*a^6*b^4*c^2*d^8 + 9*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 19*(243
10*b^10*c^9*d + 12870*a*b^9*c^8*d^2 + 6435*a^2*b^8*c^7*d^3 + 3003*a^3*b^7*c
^6*d^4 + 1287*a^4*b^6*c^5*d^5 + 495*a^5*b^5*c^4*d^6 + 165*a^6*b^4*c^3*d^7 +
 45*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^30*x^19 + 19*a*b^
29*x^18 + 171*a^2*b^28*x^17 + 969*a^3*b^27*x^16 + 3876*a^4*b^26*x^15 + 1162
8*a^5*b^25*x^14 + 27132*a^6*b^24*x^13 + 50388*a^7*b^23*x^12 + 75582*a^8*b^2
2*x^11 + 92378*a^9*b^21*x^10 + 92378*a^10*b^20*x^9 + 75582*a^11*b^19*x^8 +
50388*a^12*b^18*x^7 + 27132*a^13*b^17*x^6 + 11628*a^14*b^16*x^5 + 3876*a^15
*b^15*x^4 + 969*a^16*b^14*x^3 + 171*a^17*b^13*x^2 + 19*a^18*b^12*x + a^19*b
^11)
```

**giac [B]** time = 1.31, size = 961, normalized size = 3.52

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="giac")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 831402*b^10*c*d^9*x^9 + 92378*a*b^9*d^10*
x^9 + 3401190*b^10*c^2*d^8*x^8 + 680238*a*b^9*c*d^9*x^8 + 75582*a^2*b^8*d^1
0*x^8 + 8314020*b^10*c^3*d^7*x^7 + 2267460*a*b^9*c^2*d^8*x^7 + 453492*a^2*b
^8*c*d^9*x^7 + 50388*a^3*b^7*d^10*x^7 + 13430340*b^10*c^4*d^6*x^6 + 4476780
*a*b^9*c^3*d^7*x^6 + 1220940*a^2*b^8*c^2*d^8*x^6 + 244188*a^3*b^7*c*d^9*x^6
 + 27132*a^4*b^6*d^10*x^6 + 14965236*b^10*c^5*d^5*x^5 + 5755860*a*b^9*c^4*d
^6*x^5 + 1918620*a^2*b^8*c^3*d^7*x^5 + 523260*a^3*b^7*c^2*d^8*x^5 + 104652*
a^4*b^6*c*d^9*x^5 + 11628*a^5*b^5*d^10*x^5 + 11639628*b^10*c^6*d^4*x^4 + 49
88412*a*b^9*c^5*d^5*x^4 + 1918620*a^2*b^8*c^4*d^6*x^4 + 639540*a^3*b^7*c^3*
d^7*x^4 + 174420*a^4*b^6*c^2*d^8*x^4 + 34884*a^5*b^5*c*d^9*x^4 + 3876*a^6*b
^4*d^10*x^4 + 6235515*b^10*c^7*d^3*x^3 + 2909907*a*b^9*c^6*d^4*x^3 + 124710
3*a^2*b^8*c^5*d^5*x^3 + 479655*a^3*b^7*c^4*d^6*x^3 + 159885*a^4*b^6*c^3*d^7
*x^3 + 43605*a^5*b^5*c^2*d^8*x^3 + 8721*a^6*b^4*c*d^9*x^3 + 969*a^7*b^3*d^1
0*x^3 + 2200770*b^10*c^8*d^2*x^2 + 1100385*a*b^9*c^7*d^3*x^2 + 513513*a^2*b
^8*c^6*d^4*x^2 + 220077*a^3*b^7*c^5*d^5*x^2 + 84645*a^4*b^6*c^4*d^6*x^2 + 2
8215*a^5*b^5*c^3*d^7*x^2 + 7695*a^6*b^4*c^2*d^8*x^2 + 1539*a^7*b^3*c*d^9*x^
2 + 171*a^8*b^2*d^10*x^2 + 461890*b^10*c^9*d*x + 244530*a*b^9*c^8*d^2*x + 1
```

```
22265*a^2*b^8*c^7*d^3*x + 57057*a^3*b^7*c^6*d^4*x + 24453*a^4*b^6*c^5*d^5*x
 + 9405*a^5*b^5*c^4*d^6*x + 3135*a^6*b^4*c^3*d^7*x + 855*a^7*b^3*c^2*d^8*x
+ 171*a^8*b^2*c*d^9*x + 19*a^9*b*d^10*x + 43758*b^10*c^10 + 24310*a*b^9*c^9
*d + 12870*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 +
1287*a^5*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b
^2*c^2*d^8 + 9*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^19*b^11)
```

**maple [B]** time = 0.01, size = 866, normalized size = 3.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^20,x)
```

```
[Out] -210/13*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
/b^11/(b*x+a)^13-1/9*d^10/b^11/(b*x+a)^9-45/17*d^2*(a^8*d^8-8*a^7*b*c*d^7+2
8*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+
28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^17-45/11*d^8*(a^2*d^
2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^11-14*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b
^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^1
1/(b*x+a)^15+10*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x
+a)^12+5/9*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+1
26*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^
2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^18-1/19*(a^10*d^10-10*a^9*b*c*d^9+45*
a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5
+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+
b^10*c^10)/b^11/(b*x+a)^19+d^9*(a*d-b*c)/b^11/(b*x+a)^10+18*d^5*(a^5*d^5-5*
a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^
11/(b*x+a)^14+15/2*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3
*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/
(b*x+a)^16
```

**maxima [B]** time = 2.45, size = 1063, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="maxima")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 43758*b^10*c^10 + 24310*a*b^9*c^9*d + 128
70*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 + 1287*a^5
*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d
^8 + 9*a^9*b*c*d^9 + a^10*d^10 + 92378*(9*b^10*c*d^9 + a*b^9*d^10)*x^9 + 75
582*(45*b^10*c^2*d^8 + 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 50388*(165*b^10*
c^3*d^7 + 45*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 27132*(4
```

```
95*b^10*c^4*d^6 + 165*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 + 9*a^3*b^7*c*d^9
+ a^4*b^6*d^10)*x^6 + 11628*(1287*b^10*c^5*d^5 + 495*a*b^9*c^4*d^6 + 165*a^
2*b^8*c^3*d^7 + 45*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 +
3876*(3003*b^10*c^6*d^4 + 1287*a*b^9*c^5*d^5 + 495*a^2*b^8*c^4*d^6 + 165*a^
3*b^7*c^3*d^7 + 45*a^4*b^6*c^2*d^8 + 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 +
969*(6435*b^10*c^7*d^3 + 3003*a*b^9*c^6*d^4 + 1287*a^2*b^8*c^5*d^5 + 495*a^
3*b^7*c^4*d^6 + 165*a^4*b^6*c^3*d^7 + 45*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9
+ a^7*b^3*d^10)*x^3 + 171*(12870*b^10*c^8*d^2 + 6435*a*b^9*c^7*d^3 + 3003*a
^2*b^8*c^6*d^4 + 1287*a^3*b^7*c^5*d^5 + 495*a^4*b^6*c^4*d^6 + 165*a^5*b^5*c
^3*d^7 + 45*a^6*b^4*c^2*d^8 + 9*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 19*(243
10*b^10*c^9*d + 12870*a*b^9*c^8*d^2 + 6435*a^2*b^8*c^7*d^3 + 3003*a^3*b^7*c
^6*d^4 + 1287*a^4*b^6*c^5*d^5 + 495*a^5*b^5*c^4*d^6 + 165*a^6*b^4*c^3*d^7 +
 45*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^30*x^19 + 19*a*b^
29*x^18 + 171*a^2*b^28*x^17 + 969*a^3*b^27*x^16 + 3876*a^4*b^26*x^15 + 1162
8*a^5*b^25*x^14 + 27132*a^6*b^24*x^13 + 50388*a^7*b^23*x^12 + 75582*a^8*b^2
2*x^11 + 92378*a^9*b^21*x^10 + 92378*a^10*b^20*x^9 + 75582*a^11*b^19*x^8 +
50388*a^12*b^18*x^7 + 27132*a^13*b^17*x^6 + 11628*a^14*b^16*x^5 + 3876*a^15
*b^15*x^4 + 969*a^16*b^14*x^3 + 171*a^17*b^13*x^2 + 19*a^18*b^12*x + a^19*b
^11)
```

**mupad [B]** time = 25.72, size = 1164, normalized size = 4.26

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^20,x)
```

```
[Out] -(a^10*d^10 + 43758*b^10*c^10 + 92378*b^10*d^10*x^10 + 92378*a*b^9*d^10*x^9
 + 831402*b^10*c*d^9*x^9 + 12870*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3
003*a^4*b^6*c^6*d^4 + 1287*a^5*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*
b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 + 171*a^8*b^2*d^10*x^2 + 969*a^7*b^3*d^10*
x^3 + 3876*a^6*b^4*d^10*x^4 + 11628*a^5*b^5*d^10*x^5 + 27132*a^4*b^6*d^10*x
^6 + 50388*a^3*b^7*d^10*x^7 + 75582*a^2*b^8*d^10*x^8 + 2200770*b^10*c^8*d^2
*x^2 + 6235515*b^10*c^7*d^3*x^3 + 11639628*b^10*c^6*d^4*x^4 + 14965236*b^10
*c^5*d^5*x^5 + 13430340*b^10*c^4*d^6*x^6 + 8314020*b^10*c^3*d^7*x^7 + 34011
90*b^10*c^2*d^8*x^8 + 24310*a*b^9*c^9*d + 9*a^9*b*c*d^9 + 19*a^9*b*d^10*x +
 461890*b^10*c^9*d*x + 513513*a^2*b^8*c^6*d^4*x^2 + 220077*a^3*b^7*c^5*d^5*
x^2 + 84645*a^4*b^6*c^4*d^6*x^2 + 28215*a^5*b^5*c^3*d^7*x^2 + 7695*a^6*b^4*
c^2*d^8*x^2 + 1247103*a^2*b^8*c^5*d^5*x^3 + 479655*a^3*b^7*c^4*d^6*x^3 + 15
9885*a^4*b^6*c^3*d^7*x^3 + 43605*a^5*b^5*c^2*d^8*x^3 + 1918620*a^2*b^8*c^4*
d^6*x^4 + 639540*a^3*b^7*c^3*d^7*x^4 + 174420*a^4*b^6*c^2*d^8*x^4 + 1918620
*a^2*b^8*c^3*d^7*x^5 + 523260*a^3*b^7*c^2*d^8*x^5 + 1220940*a^2*b^8*c^2*d^8
*x^6 + 244530*a*b^9*c^8*d^2*x + 171*a^8*b^2*c*d^9*x + 680238*a*b^9*c*d^9*x^
8 + 122265*a^2*b^8*c^7*d^3*x + 57057*a^3*b^7*c^6*d^4*x + 24453*a^4*b^6*c^5*
d^5*x + 9405*a^5*b^5*c^4*d^6*x + 3135*a^6*b^4*c^3*d^7*x + 855*a^7*b^3*c^2*d
```

```
^8*x + 1100385*a*b^9*c^7*d^3*x^2 + 1539*a^7*b^3*c*d^9*x^2 + 2909907*a*b^9*c
^6*d^4*x^3 + 8721*a^6*b^4*c*d^9*x^3 + 4988412*a*b^9*c^5*d^5*x^4 + 34884*a^5
*b^5*c*d^9*x^4 + 5755860*a*b^9*c^4*d^6*x^5 + 104652*a^4*b^6*c*d^9*x^5 + 447
6780*a*b^9*c^3*d^7*x^6 + 244188*a^3*b^7*c*d^9*x^6 + 2267460*a*b^9*c^2*d^8*x
^7 + 453492*a^2*b^8*c*d^9*x^7)/(831402*a^19*b^11 + 831402*b^30*x^19 + 15796
638*a^18*b^12*x + 15796638*a*b^29*x^18 + 142169742*a^17*b^13*x^2 + 80562853
8*a^16*b^14*x^3 + 3222514152*a^15*b^15*x^4 + 9667542456*a^14*b^16*x^5 + 225
57599064*a^13*b^17*x^6 + 41892683976*a^12*b^18*x^7 + 62839025964*a^11*b^19*
x^8 + 76803253956*a^10*b^20*x^9 + 76803253956*a^9*b^21*x^10 + 62839025964*a
^8*b^22*x^11 + 41892683976*a^7*b^23*x^12 + 22557599064*a^6*b^24*x^13 + 9667
542456*a^5*b^25*x^14 + 3222514152*a^4*b^26*x^15 + 805628538*a^3*b^27*x^16 +
 142169742*a^2*b^28*x^17)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**20,x)
```

```
[Out] Timed out
```

## 3.1226 $\int \frac{(c+dx)^{10}}{(a+bx)^{21}}\,dx$

**Optimal**. Leaf size=279

$$-\frac{10d^9(bc-ad)}{11b^{11}(a+bx)^{11}}-\frac{15d^8(bc-ad)^2}{4b^{11}(a+bx)^{12}}-\frac{120d^7(bc-ad)^3}{13b^{11}(a+bx)^{13}}-\frac{15d^6(bc-ad)^4}{b^{11}(a+bx)^{14}}-\frac{84d^5(bc-ad)^5}{5b^{11}(a+bx)^{15}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\frac{120d^3(b}{17b^{11}(a}$$

**Rubi** [A] time = 0.27, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{10d^9(bc-ad)}{11b^{11}(a+bx)^{11}}-\frac{15d^8(bc-ad)^2}{4b^{11}(a+bx)^{12}}-\frac{120d^7(bc-ad)^3}{13b^{11}(a+bx)^{13}}-\frac{15d^6(bc-ad)^4}{b^{11}(a+bx)^{14}}-\frac{84d^5(bc-ad)^5}{5b^{11}(a+bx)^{15}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\frac{120d^3(bc-ad)^7}{17b^{11}(a+bx)^{17}}-\frac{5d^2(bc-ad)^8}{2b^{11}(a+bx)^{18}}-\frac{10d(bc-ad)^9}{19b^{11}(a+bx)^{19}}-\frac{(bc-ad)^{10}}{20b^{11}(a+bx)^{20}}-\frac{d^{10}}{10b^{11}(a+bx)^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^21,x]
```

```
[Out] -(b*c - a*d)^10/(20*b^11*(a + b*x)^20) - (10*d*(b*c - a*d)^9)/(19*b^11*(a +
 b*x)^19) - (5*d^2*(b*c - a*d)^8)/(2*b^11*(a + b*x)^18) - (120*d^3*(b*c - a
*d)^7)/(17*b^11*(a + b*x)^17) - (105*d^4*(b*c - a*d)^6)/(8*b^11*(a + b*x)^1
6) - (84*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^15) - (15*d^6*(b*c - a*d)^4)/
(b^11*(a + b*x)^14) - (120*d^7*(b*c - a*d)^3)/(13*b^11*(a + b*x)^13) - (15*
d^8*(b*c - a*d)^2)/(4*b^11*(a + b*x)^12) - (10*d^9*(b*c - a*d))/(11*b^11*(a
 + b*x)^11) - d^10/(10*b^11*(a + b*x)^10)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{21}}\,dx = \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{21}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{20}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{19}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{18}}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^{17}}+\right.$$

$$= -\frac{(bc-ad)^{10}}{20b^{11}(a+bx)^{20}}-\frac{10d(bc-ad)^9}{19b^{11}(a+bx)^{19}}-\frac{5d^2(bc-ad)^8}{2b^{11}(a+bx)^{18}}-\frac{120d^3(bc-ad)^7}{17b^{11}(a+bx)^{17}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}$$

**Mathematica** [B] time = 0.29, size = 692, normalized size = 2.48

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^21,x]
```

```
[Out] -1/1847560*(a^10*d^10 + 10*a^9*b*d^9*(c + 2*d*x) + 5*a^8*b^2*d^8*(11*c^2 +
40*c*d*x + 38*d^2*x^2) + 20*a^7*b^3*d^7*(11*c^3 + 55*c^2*d*x + 95*c*d^2*x^2
 + 57*d^3*x^3) + 5*a^6*b^4*d^6*(143*c^4 + 880*c^3*d*x + 2090*c^2*d^2*x^2 +
2280*c*d^3*x^3 + 969*d^4*x^4) + 2*a^5*b^5*d^5*(1001*c^5 + 7150*c^4*d*x + 20
900*c^3*d^2*x^2 + 31350*c^2*d^3*x^3 + 24225*c*d^4*x^4 + 7752*d^5*x^5) + 5*a
^4*b^6*d^4*(1001*c^6 + 8008*c^5*d*x + 27170*c^4*d^2*x^2 + 50160*c^3*d^3*x^3
 + 53295*c^2*d^4*x^4 + 31008*c*d^5*x^5 + 7752*d^6*x^6) + 20*a^3*b^7*d^3*(57
2*c^7 + 5005*c^6*d*x + 19019*c^5*d^2*x^2 + 40755*c^4*d^3*x^3 + 53295*c^3*d^
4*x^4 + 42636*c^2*d^5*x^5 + 19380*c*d^6*x^6 + 3876*d^7*x^7) + 5*a^2*b^8*d^2
*(4862*c^8 + 45760*c^7*d*x + 190190*c^6*d^2*x^2 + 456456*c^5*d^3*x^3 + 6928
35*c^4*d^4*x^4 + 682176*c^3*d^5*x^5 + 426360*c^2*d^6*x^6 + 155040*c*d^7*x^7
 + 25194*d^8*x^8) + 10*a*b^9*d*(4862*c^9 + 48620*c^8*d*x + 217360*c^7*d^2*x
^2 + 570570*c^6*d^3*x^3 + 969969*c^5*d^4*x^4 + 1108536*c^4*d^5*x^5 + 852720
*c^3*d^6*x^6 + 426360*c^2*d^7*x^7 + 125970*c*d^8*x^8 + 16796*d^9*x^9) + b^1
0*(92378*c^10 + 972400*c^9*d*x + 4618900*c^8*d^2*x^2 + 13041600*c^7*d^3*x^3
 + 24249225*c^6*d^4*x^4 + 31039008*c^5*d^5*x^5 + 27713400*c^4*d^6*x^6 + 170
54400*c^3*d^7*x^7 + 6928350*c^2*d^8*x^8 + 1679600*c*d^9*x^9 + 184756*d^10*x
^10))/(b^11*(a + b*x)^20)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{21}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^21,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^21, x]
```

**fricas [B]** time = 1.23, size = 1074, normalized size = 3.85

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="fricas")
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 92378*b^10*c^10 + 48620*a*b^9*c^9*d + 2
4310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*a^4*b^6*c^6*d^4 + 2002*
a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^
2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10 + 167960*(10*b^10*c*d^9 + a*b^9*d^10)*x^
9 + 125970*(55*b^10*c^2*d^8 + 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 77520*(2
```

```
20*b^10*c^3*d^7 + 55*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 +
 38760*(715*b^10*c^4*d^6 + 220*a*b^9*c^3*d^7 + 55*a^2*b^8*c^2*d^8 + 10*a^3*
b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 15504*(2002*b^10*c^5*d^5 + 715*a*b^9*c^4*d^
6 + 220*a^2*b^8*c^3*d^7 + 55*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 + a^5*b^5*d
^10)*x^5 + 4845*(5005*b^10*c^6*d^4 + 2002*a*b^9*c^5*d^5 + 715*a^2*b^8*c^4*d
^6 + 220*a^3*b^7*c^3*d^7 + 55*a^4*b^6*c^2*d^8 + 10*a^5*b^5*c*d^9 + a^6*b^4*
d^10)*x^4 + 1140*(11440*b^10*c^7*d^3 + 5005*a*b^9*c^6*d^4 + 2002*a^2*b^8*c^
5*d^5 + 715*a^3*b^7*c^4*d^6 + 220*a^4*b^6*c^3*d^7 + 55*a^5*b^5*c^2*d^8 + 10
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 190*(24310*b^10*c^8*d^2 + 11440*a*b^9*
c^7*d^3 + 5005*a^2*b^8*c^6*d^4 + 2002*a^3*b^7*c^5*d^5 + 715*a^4*b^6*c^4*d^6
 + 220*a^5*b^5*c^3*d^7 + 55*a^6*b^4*c^2*d^8 + 10*a^7*b^3*c*d^9 + a^8*b^2*d^
10)*x^2 + 20*(48620*b^10*c^9*d + 24310*a*b^9*c^8*d^2 + 11440*a^2*b^8*c^7*d^
3 + 5005*a^3*b^7*c^6*d^4 + 2002*a^4*b^6*c^5*d^5 + 715*a^5*b^5*c^4*d^6 + 220
*a^6*b^4*c^3*d^7 + 55*a^7*b^3*c^2*d^8 + 10*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(
b^31*x^20 + 20*a*b^30*x^19 + 190*a^2*b^29*x^18 + 1140*a^3*b^28*x^17 + 4845*
a^4*b^27*x^16 + 15504*a^5*b^26*x^15 + 38760*a^6*b^25*x^14 + 77520*a^7*b^24*
x^13 + 125970*a^8*b^23*x^12 + 167960*a^9*b^22*x^11 + 184756*a^10*b^21*x^10
+ 167960*a^11*b^20*x^9 + 125970*a^12*b^19*x^8 + 77520*a^13*b^18*x^7 + 38760
*a^14*b^17*x^6 + 15504*a^15*b^16*x^5 + 4845*a^16*b^15*x^4 + 1140*a^17*b^14*
x^3 + 190*a^18*b^13*x^2 + 20*a^19*b^12*x + a^20*b^11)
```

**giac [B]** time = 1.31, size = 961, normalized size = 3.44

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="giac")
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 1679600*b^10*c*d^9*x^9 + 167960*a*b^9*d
^10*x^9 + 6928350*b^10*c^2*d^8*x^8 + 1259700*a*b^9*c*d^9*x^8 + 125970*a^2*b
^8*d^10*x^8 + 17054400*b^10*c^3*d^7*x^7 + 4263600*a*b^9*c^2*d^8*x^7 + 77520
0*a^2*b^8*c*d^9*x^7 + 77520*a^3*b^7*d^10*x^7 + 27713400*b^10*c^4*d^6*x^6 +
8527200*a*b^9*c^3*d^7*x^6 + 2131800*a^2*b^8*c^2*d^8*x^6 + 387600*a^3*b^7*c*
d^9*x^6 + 38760*a^4*b^6*d^10*x^6 + 31039008*b^10*c^5*d^5*x^5 + 11085360*a*b
^9*c^4*d^6*x^5 + 3410880*a^2*b^8*c^3*d^7*x^5 + 852720*a^3*b^7*c^2*d^8*x^5 +
 155040*a^4*b^6*c*d^9*x^5 + 15504*a^5*b^5*d^10*x^5 + 24249225*b^10*c^6*d^4*
x^4 + 9699690*a*b^9*c^5*d^5*x^4 + 3464175*a^2*b^8*c^4*d^6*x^4 + 1065900*a^3
*b^7*c^3*d^7*x^4 + 266475*a^4*b^6*c^2*d^8*x^4 + 48450*a^5*b^5*c*d^9*x^4 + 4
845*a^6*b^4*d^10*x^4 + 13041600*b^10*c^7*d^3*x^3 + 5705700*a*b^9*c^6*d^4*x^
3 + 2282280*a^2*b^8*c^5*d^5*x^3 + 815100*a^3*b^7*c^4*d^6*x^3 + 250800*a^4*b
^6*c^3*d^7*x^3 + 62700*a^5*b^5*c^2*d^8*x^3 + 11400*a^6*b^4*c*d^9*x^3 + 1140
*a^7*b^3*d^10*x^3 + 4618900*b^10*c^8*d^2*x^2 + 2173600*a*b^9*c^7*d^3*x^2 +
950950*a^2*b^8*c^6*d^4*x^2 + 380380*a^3*b^7*c^5*d^5*x^2 + 135850*a^4*b^6*c^
4*d^6*x^2 + 41800*a^5*b^5*c^3*d^7*x^2 + 10450*a^6*b^4*c^2*d^8*x^2 + 1900*a^
7*b^3*c*d^9*x^2 + 190*a^8*b^2*d^10*x^2 + 972400*b^10*c^9*d*x + 486200*a*b^9
```

```
*c^8*d^2*x + 228800*a^2*b^8*c^7*d^3*x + 100100*a^3*b^7*c^6*d^4*x + 40040*a^
4*b^6*c^5*d^5*x + 14300*a^5*b^5*c^4*d^6*x + 4400*a^6*b^4*c^3*d^7*x + 1100*a
^7*b^3*c^2*d^8*x + 200*a^8*b^2*c*d^9*x + 20*a^9*b*d^10*x + 92378*b^10*c^10
+ 48620*a*b^9*c^9*d + 24310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*
a^4*b^6*c^6*d^4 + 2002*a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*
c^3*d^7 + 55*a^8*b^2*c^2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^20*b^
11)
```

**maple [B]** time = 0.01, size = 867, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^21,x)
```

```
[Out] 120/13*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^13-1/
20*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6
*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45
*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^20+120/17*d^3*(a^7*
d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-
21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^17+10/11*d^9*(a*d-b*
c)/b^11/(b*x+a)^11+84/5*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^
2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^15-15/4*d^8*(a^2*d^2-2*a*
b*c*d+b^2*c^2)/b^11/(b*x+a)^12-5/2*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^
2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c
^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^18+10/19*d*(a^9*d^9-9*a^8*b*c*d^
8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5
*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x
+a)^19-1/10*d^10/b^11/(b*x+a)^10-15*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^
2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^14-105/8*d^4*(a^6*d^6-6*a^5*b*c*d
^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b
^6*c^6)/b^11/(b*x+a)^16
```

**maxima [B]** time = 2.51, size = 1074, normalized size = 3.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="maxima")
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 92378*b^10*c^10 + 48620*a*b^9*c^9*d + 2
4310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*a^4*b^6*c^6*d^4 + 2002*
a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^
2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10 + 167960*(10*b^10*c*d^9 + a*b^9*d^10)*x^
9 + 125970*(55*b^10*c^2*d^8 + 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 77520*(2
```

```
20*b^10*c^3*d^7 + 55*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 +
 38760*(715*b^10*c^4*d^6 + 220*a*b^9*c^3*d^7 + 55*a^2*b^8*c^2*d^8 + 10*a^3*
b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 15504*(2002*b^10*c^5*d^5 + 715*a*b^9*c^4*d^
6 + 220*a^2*b^8*c^3*d^7 + 55*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 + a^5*b^5*d
^10)*x^5 + 4845*(5005*b^10*c^6*d^4 + 2002*a*b^9*c^5*d^5 + 715*a^2*b^8*c^4*d
^6 + 220*a^3*b^7*c^3*d^7 + 55*a^4*b^6*c^2*d^8 + 10*a^5*b^5*c*d^9 + a^6*b^4*
d^10)*x^4 + 1140*(11440*b^10*c^7*d^3 + 5005*a*b^9*c^6*d^4 + 2002*a^2*b^8*c^
5*d^5 + 715*a^3*b^7*c^4*d^6 + 220*a^4*b^6*c^3*d^7 + 55*a^5*b^5*c^2*d^8 + 10
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 190*(24310*b^10*c^8*d^2 + 11440*a*b^9*
c^7*d^3 + 5005*a^2*b^8*c^6*d^4 + 2002*a^3*b^7*c^5*d^5 + 715*a^4*b^6*c^4*d^6
 + 220*a^5*b^5*c^3*d^7 + 55*a^6*b^4*c^2*d^8 + 10*a^7*b^3*c*d^9 + a^8*b^2*d^
10)*x^2 + 20*(48620*b^10*c^9*d + 24310*a*b^9*c^8*d^2 + 11440*a^2*b^8*c^7*d^
3 + 5005*a^3*b^7*c^6*d^4 + 2002*a^4*b^6*c^5*d^5 + 715*a^5*b^5*c^4*d^6 + 220
*a^6*b^4*c^3*d^7 + 55*a^7*b^3*c^2*d^8 + 10*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(
b^31*x^20 + 20*a*b^30*x^19 + 190*a^2*b^29*x^18 + 1140*a^3*b^28*x^17 + 4845*
a^4*b^27*x^16 + 15504*a^5*b^26*x^15 + 38760*a^6*b^25*x^14 + 77520*a^7*b^24*
x^13 + 125970*a^8*b^23*x^12 + 167960*a^9*b^22*x^11 + 184756*a^10*b^21*x^10
+ 167960*a^11*b^20*x^9 + 125970*a^12*b^19*x^8 + 77520*a^13*b^18*x^7 + 38760
*a^14*b^17*x^6 + 15504*a^15*b^16*x^5 + 4845*a^16*b^15*x^4 + 1140*a^17*b^14*
x^3 + 190*a^18*b^13*x^2 + 20*a^19*b^12*x + a^20*b^11)
```

**mupad [B]** time = 0.80, size = 1175, normalized size = 4.21

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^21,x)
```

```
[Out] -(a^10*d^10 + 92378*b^10*c^10 + 184756*b^10*d^10*x^10 + 167960*a*b^9*d^10*x
^9 + 1679600*b^10*c*d^9*x^9 + 24310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3
 + 5005*a^4*b^6*c^6*d^4 + 2002*a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*
a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^2*d^8 + 190*a^8*b^2*d^10*x^2 + 1140*a^7*b^3*
d^10*x^3 + 4845*a^6*b^4*d^10*x^4 + 15504*a^5*b^5*d^10*x^5 + 38760*a^4*b^6*d
^10*x^6 + 77520*a^3*b^7*d^10*x^7 + 125970*a^2*b^8*d^10*x^8 + 4618900*b^10*c
^8*d^2*x^2 + 13041600*b^10*c^7*d^3*x^3 + 24249225*b^10*c^6*d^4*x^4 + 310390
08*b^10*c^5*d^5*x^5 + 27713400*b^10*c^4*d^6*x^6 + 17054400*b^10*c^3*d^7*x^7
 + 6928350*b^10*c^2*d^8*x^8 + 48620*a*b^9*c^9*d + 10*a^9*b*c*d^9 + 20*a^9*b
*d^10*x + 972400*b^10*c^9*d*x + 950950*a^2*b^8*c^6*d^4*x^2 + 380380*a^3*b^7
*c^5*d^5*x^2 + 135850*a^4*b^6*c^4*d^6*x^2 + 41800*a^5*b^5*c^3*d^7*x^2 + 104
50*a^6*b^4*c^2*d^8*x^2 + 2282280*a^2*b^8*c^5*d^5*x^3 + 815100*a^3*b^7*c^4*d
^6*x^3 + 250800*a^4*b^6*c^3*d^7*x^3 + 62700*a^5*b^5*c^2*d^8*x^3 + 3464175*a
^2*b^8*c^4*d^6*x^4 + 1065900*a^3*b^7*c^3*d^7*x^4 + 266475*a^4*b^6*c^2*d^8*x
^4 + 3410880*a^2*b^8*c^3*d^7*x^5 + 852720*a^3*b^7*c^2*d^8*x^5 + 2131800*a^2
*b^8*c^2*d^8*x^6 + 486200*a*b^9*c^8*d^2*x + 200*a^8*b^2*c*d^9*x + 1259700*a
*b^9*c*d^9*x^8 + 228800*a^2*b^8*c^7*d^3*x + 100100*a^3*b^7*c^6*d^4*x + 4004
```

```
0*a^4*b^6*c^5*d^5*x + 14300*a^5*b^5*c^4*d^6*x + 4400*a^6*b^4*c^3*d^7*x + 11
00*a^7*b^3*c^2*d^8*x + 2173600*a*b^9*c^7*d^3*x^2 + 1900*a^7*b^3*c*d^9*x^2 +
 5705700*a*b^9*c^6*d^4*x^3 + 11400*a^6*b^4*c*d^9*x^3 + 9699690*a*b^9*c^5*d^
5*x^4 + 48450*a^5*b^5*c*d^9*x^4 + 11085360*a*b^9*c^4*d^6*x^5 + 155040*a^4*b
^6*c*d^9*x^5 + 8527200*a*b^9*c^3*d^7*x^6 + 387600*a^3*b^7*c*d^9*x^6 + 42636
00*a*b^9*c^2*d^8*x^7 + 775200*a^2*b^8*c*d^9*x^7)/(1847560*a^20*b^11 + 18475
60*b^31*x^20 + 36951200*a^19*b^12*x + 36951200*a*b^30*x^19 + 351036400*a^18
*b^13*x^2 + 2106218400*a^17*b^14*x^3 + 8951428200*a^16*b^15*x^4 + 286445702
40*a^15*b^16*x^5 + 71611425600*a^14*b^17*x^6 + 143222851200*a^13*b^18*x^7 +
 232737133200*a^12*b^19*x^8 + 310316177600*a^11*b^20*x^9 + 341347795360*a^1
0*b^21*x^10 + 310316177600*a^9*b^22*x^11 + 232737133200*a^8*b^23*x^12 + 143
222851200*a^7*b^24*x^13 + 71611425600*a^6*b^25*x^14 + 28644570240*a^5*b^26*
x^15 + 8951428200*a^4*b^27*x^16 + 2106218400*a^3*b^28*x^17 + 351036400*a^2*
b^29*x^18)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**21,x)
```

```
[Out] Timed out
```

## 3.1227 $\int \frac{(c+dx)^{10}}{(a+bx)^{22}}\,dx$

**Optimal**. Leaf size=279

$$-\frac{5d^9(bc-ad)}{6b^{11}(a+bx)^{12}}-\frac{45d^8(bc-ad)^2}{13b^{11}(a+bx)^{13}}-\frac{60d^7(bc-ad)^3}{7b^{11}(a+bx)^{14}}-\frac{14d^6(bc-ad)^4}{b^{11}(a+bx)^{15}}-\frac{63d^5(bc-ad)^5}{4b^{11}(a+bx)^{16}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-\frac{20d^3(bc}{3b^{11}(a+}$$

**Rubi [A]** time = 0.27, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{5d^9(bc-ad)}{6b^{11}(a+bx)^{12}}-\frac{45d^8(bc-ad)^2}{13b^{11}(a+bx)^{13}}-\frac{60d^7(bc-ad)^3}{7b^{11}(a+bx)^{14}}-\frac{14d^6(bc-ad)^4}{b^{11}(a+bx)^{15}}-\frac{63d^5(bc-ad)^5}{4b^{11}(a+bx)^{16}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-\frac{20d^3(bc-ad)^7}{3b^{11}(a+bx)^{18}}-\frac{45d^2(bc-ad)^8}{19b^{11}(a+bx)^{19}}-\frac{d(bc-ad)^9}{2b^{11}(a+bx)^{20}}-\frac{(bc-ad)^{10}}{21b^{11}(a+bx)^{21}}-\frac{d^{10}}{11b^{11}(a+bx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^22,x]
```

```
[Out] -(b*c - a*d)^10/(21*b^11*(a + b*x)^21) - (d*(b*c - a*d)^9)/(2*b^11*(a + b*x
)^20) - (45*d^2*(b*c - a*d)^8)/(19*b^11*(a + b*x)^19) - (20*d^3*(b*c - a*d)
^7)/(3*b^11*(a + b*x)^18) - (210*d^4*(b*c - a*d)^6)/(17*b^11*(a + b*x)^17)
- (63*d^5*(b*c - a*d)^5)/(4*b^11*(a + b*x)^16) - (14*d^6*(b*c - a*d)^4)/(b^
11*(a + b*x)^15) - (60*d^7*(b*c - a*d)^3)/(7*b^11*(a + b*x)^14) - (45*d^8*(
b*c - a*d)^2)/(13*b^11*(a + b*x)^13) - (5*d^9*(b*c - a*d))/(6*b^11*(a + b*x
)^12) - d^10/(11*b^11*(a + b*x)^11)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{22}}\,dx = \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{22}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{21}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{20}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{19}}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^{18}}+\right.$$

$$= -\frac{(bc-ad)^{10}}{21b^{11}(a+bx)^{21}}-\frac{d(bc-ad)^9}{2b^{11}(a+bx)^{20}}-\frac{45d^2(bc-ad)^8}{19b^{11}(a+bx)^{19}}-\frac{20d^3(bc-ad)^7}{3b^{11}(a+bx)^{18}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-$$

**Mathematica [B]** time = 0.30, size = 692, normalized size = 2.48

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^22,x]
```

```
[Out] -1/3879876*(a^10*d^10 + a^9*b*d^9*(11*c + 21*d*x) + 3*a^8*b^2*d^8*(22*c^2 +
 77*c*d*x + 70*d^2*x^2) + 2*a^7*b^3*d^7*(143*c^3 + 693*c^2*d*x + 1155*c*d^2
*x^2 + 665*d^3*x^3) + 7*a^6*b^4*d^6*(143*c^4 + 858*c^3*d*x + 1980*c^2*d^2*x
^2 + 2090*c*d^3*x^3 + 855*d^4*x^4) + 21*a^5*b^5*d^5*(143*c^5 + 1001*c^4*d*x
 + 2860*c^3*d^2*x^2 + 4180*c^2*d^3*x^3 + 3135*c*d^4*x^4 + 969*d^5*x^5) + 7*
a^4*b^6*d^4*(1144*c^6 + 9009*c^5*d*x + 30030*c^4*d^2*x^2 + 54340*c^3*d^3*x^
3 + 56430*c^2*d^4*x^4 + 31977*c*d^5*x^5 + 7752*d^6*x^6) + 2*a^3*b^7*d^3*(97
24*c^7 + 84084*c^6*d*x + 315315*c^5*d^2*x^2 + 665665*c^4*d^3*x^3 + 855855*c
^3*d^4*x^4 + 671517*c^2*d^5*x^5 + 298452*c*d^6*x^6 + 58140*d^7*x^7) + 3*a^2
*b^8*d^2*(14586*c^8 + 136136*c^7*d*x + 560560*c^6*d^2*x^2 + 1331330*c^5*d^3
*x^3 + 1996995*c^4*d^4*x^4 + 1939938*c^3*d^5*x^5 + 1193808*c^2*d^6*x^6 + 42
6360*c*d^7*x^7 + 67830*d^8*x^8) + a*b^9*d*(92378*c^9 + 918918*c^8*d*x + 408
4080*c^7*d^2*x^2 + 10650640*c^6*d^3*x^3 + 17972955*c^5*d^4*x^4 + 20369349*c
^4*d^5*x^5 + 15519504*c^3*d^6*x^6 + 7674480*c^2*d^7*x^7 + 2238390*c*d^8*x^8
 + 293930*d^9*x^9) + b^10*(184756*c^10 + 1939938*c^9*d*x + 9189180*c^8*d^2*
x^2 + 25865840*c^7*d^3*x^3 + 47927880*c^6*d^4*x^4 + 61108047*c^5*d^5*x^5 +
54318264*c^4*d^6*x^6 + 33256080*c^3*d^7*x^7 + 13430340*c^2*d^8*x^8 + 323323
0*c*d^9*x^9 + 352716*d^10*x^10))/(b^11*(a + b*x)^21)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{22}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^22,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^22, x]
```

**fricas [B]** time = 1.29, size = 1085, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="fricas")
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 184756*b^10*c^10 + 92378*a*b^9*c^9*d +
43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003
*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*
c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10 + 293930*(11*b^10*c*d^9 + a*b^9*d^10)*
x^9 + 203490*(66*b^10*c^2*d^8 + 11*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 116280
```

```
*(286*b^10*c^3*d^7 + 66*a*b^9*c^2*d^8 + 11*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^
7 + 54264*(1001*b^10*c^4*d^6 + 286*a*b^9*c^3*d^7 + 66*a^2*b^8*c^2*d^8 + 11*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 20349*(3003*b^10*c^5*d^5 + 1001*a*b^9*c
^4*d^6 + 286*a^2*b^8*c^3*d^7 + 66*a^3*b^7*c^2*d^8 + 11*a^4*b^6*c*d^9 + a^5*
b^5*d^10)*x^5 + 5985*(8008*b^10*c^6*d^4 + 3003*a*b^9*c^5*d^5 + 1001*a^2*b^8
*c^4*d^6 + 286*a^3*b^7*c^3*d^7 + 66*a^4*b^6*c^2*d^8 + 11*a^5*b^5*c*d^9 + a^
6*b^4*d^10)*x^4 + 1330*(19448*b^10*c^7*d^3 + 8008*a*b^9*c^6*d^4 + 3003*a^2*
b^8*c^5*d^5 + 1001*a^3*b^7*c^4*d^6 + 286*a^4*b^6*c^3*d^7 + 66*a^5*b^5*c^2*d
^8 + 11*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 210*(43758*b^10*c^8*d^2 + 19448
*a*b^9*c^7*d^3 + 8008*a^2*b^8*c^6*d^4 + 3003*a^3*b^7*c^5*d^5 + 1001*a^4*b^6
*c^4*d^6 + 286*a^5*b^5*c^3*d^7 + 66*a^6*b^4*c^2*d^8 + 11*a^7*b^3*c*d^9 + a^
8*b^2*d^10)*x^2 + 21*(92378*b^10*c^9*d + 43758*a*b^9*c^8*d^2 + 19448*a^2*b^
8*c^7*d^3 + 8008*a^3*b^7*c^6*d^4 + 3003*a^4*b^6*c^5*d^5 + 1001*a^5*b^5*c^4*
d^6 + 286*a^6*b^4*c^3*d^7 + 66*a^7*b^3*c^2*d^8 + 11*a^8*b^2*c*d^9 + a^9*b*d
^10)*x)/(b^32*x^21 + 21*a*b^31*x^20 + 210*a^2*b^30*x^19 + 1330*a^3*b^29*x^1
8 + 5985*a^4*b^28*x^17 + 20349*a^5*b^27*x^16 + 54264*a^6*b^26*x^15 + 116280
*a^7*b^25*x^14 + 203490*a^8*b^24*x^13 + 293930*a^9*b^23*x^12 + 352716*a^10*
b^22*x^11 + 352716*a^11*b^21*x^10 + 293930*a^12*b^20*x^9 + 203490*a^13*b^19
*x^8 + 116280*a^14*b^18*x^7 + 54264*a^15*b^17*x^6 + 20349*a^16*b^16*x^5 + 5
985*a^17*b^15*x^4 + 1330*a^18*b^14*x^3 + 210*a^19*b^13*x^2 + 21*a^20*b^12*x
 + a^21*b^11)
```

**giac [B]** time = 1.30, size = 961, normalized size = 3.44

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="giac")
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 3233230*b^10*c*d^9*x^9 + 293930*a*b^9*d
^10*x^9 + 13430340*b^10*c^2*d^8*x^8 + 2238390*a*b^9*c*d^9*x^8 + 203490*a^2*
b^8*d^10*x^8 + 33256080*b^10*c^3*d^7*x^7 + 7674480*a*b^9*c^2*d^8*x^7 + 1279
080*a^2*b^8*c*d^9*x^7 + 116280*a^3*b^7*d^10*x^7 + 54318264*b^10*c^4*d^6*x^6
 + 15519504*a*b^9*c^3*d^7*x^6 + 3581424*a^2*b^8*c^2*d^8*x^6 + 596904*a^3*b^
7*c*d^9*x^6 + 54264*a^4*b^6*d^10*x^6 + 61108047*b^10*c^5*d^5*x^5 + 20369349
*a*b^9*c^4*d^6*x^5 + 5819814*a^2*b^8*c^3*d^7*x^5 + 1343034*a^3*b^7*c^2*d^8*
x^5 + 223839*a^4*b^6*c*d^9*x^5 + 20349*a^5*b^5*d^10*x^5 + 47927880*b^10*c^6
*d^4*x^4 + 17972955*a*b^9*c^5*d^5*x^4 + 5990985*a^2*b^8*c^4*d^6*x^4 + 17117
10*a^3*b^7*c^3*d^7*x^4 + 395010*a^4*b^6*c^2*d^8*x^4 + 65835*a^5*b^5*c*d^9*x
^4 + 5985*a^6*b^4*d^10*x^4 + 25865840*b^10*c^7*d^3*x^3 + 10650640*a*b^9*c^6
*d^4*x^3 + 3993990*a^2*b^8*c^5*d^5*x^3 + 1331330*a^3*b^7*c^4*d^6*x^3 + 3803
80*a^4*b^6*c^3*d^7*x^3 + 87780*a^5*b^5*c^2*d^8*x^3 + 14630*a^6*b^4*c*d^9*x^
3 + 1330*a^7*b^3*d^10*x^3 + 9189180*b^10*c^8*d^2*x^2 + 4084080*a*b^9*c^7*d^
3*x^2 + 1681680*a^2*b^8*c^6*d^4*x^2 + 630630*a^3*b^7*c^5*d^5*x^2 + 210210*a
^4*b^6*c^4*d^6*x^2 + 60060*a^5*b^5*c^3*d^7*x^2 + 13860*a^6*b^4*c^2*d^8*x^2
```

```
+ 2310*a^7*b^3*c*d^9*x^2 + 210*a^8*b^2*d^10*x^2 + 1939938*b^10*c^9*d*x + 91
8918*a*b^9*c^8*d^2*x + 408408*a^2*b^8*c^7*d^3*x + 168168*a^3*b^7*c^6*d^4*x
+ 63063*a^4*b^6*c^5*d^5*x + 21021*a^5*b^5*c^4*d^6*x + 6006*a^6*b^4*c^3*d^7*
x + 1386*a^7*b^3*c^2*d^8*x + 231*a^8*b^2*c*d^9*x + 21*a^9*b*d^10*x + 184756
*b^10*c^10 + 92378*a*b^9*c^9*d + 43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*
d^3 + 8008*a^4*b^6*c^6*d^4 + 3003*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 +
286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10)/((b*
x + a)^21*b^11)
```

**maple [B]** time = 0.01, size = 867, normalized size = 3.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^22,x)
```

```
[Out] -45/13*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^13+1/2*d*(a^9*d^9-9*a^8
*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*
b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^
11/(b*x+a)^20-1/21*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3
*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^
3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^21-
210/17*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*
a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^17-1/11*d^10/b^11/(b*x+
a)^11-14*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4
)/b^11/(b*x+a)^15+5/6*d^9*(a*d-b*c)/b^11/(b*x+a)^12+20/3*d^3*(a^7*d^7-7*a^6
*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^
5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^18-45/19*d^2*(a^8*d^8-8*a^7*b
*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*
c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^19+60/7*d^7*
(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^14+63/4*d^5*(a^5
*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*
c^5)/b^11/(b*x+a)^16
```

**maxima [B]** time = 2.46, size = 1085, normalized size = 3.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="maxima")
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 184756*b^10*c^10 + 92378*a*b^9*c^9*d +
43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003
*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*
c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10 + 293930*(11*b^10*c*d^9 + a*b^9*d^10)*
```

```
x^9 + 203490*(66*b^10*c^2*d^8 + 11*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 116280
*(286*b^10*c^3*d^7 + 66*a*b^9*c^2*d^8 + 11*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^
7 + 54264*(1001*b^10*c^4*d^6 + 286*a*b^9*c^3*d^7 + 66*a^2*b^8*c^2*d^8 + 11*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 20349*(3003*b^10*c^5*d^5 + 1001*a*b^9*c
^4*d^6 + 286*a^2*b^8*c^3*d^7 + 66*a^3*b^7*c^2*d^8 + 11*a^4*b^6*c*d^9 + a^5*
b^5*d^10)*x^5 + 5985*(8008*b^10*c^6*d^4 + 3003*a*b^9*c^5*d^5 + 1001*a^2*b^8
*c^4*d^6 + 286*a^3*b^7*c^3*d^7 + 66*a^4*b^6*c^2*d^8 + 11*a^5*b^5*c*d^9 + a^
6*b^4*d^10)*x^4 + 1330*(19448*b^10*c^7*d^3 + 8008*a*b^9*c^6*d^4 + 3003*a^2*
b^8*c^5*d^5 + 1001*a^3*b^7*c^4*d^6 + 286*a^4*b^6*c^3*d^7 + 66*a^5*b^5*c^2*d
^8 + 11*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 210*(43758*b^10*c^8*d^2 + 19448
*a*b^9*c^7*d^3 + 8008*a^2*b^8*c^6*d^4 + 3003*a^3*b^7*c^5*d^5 + 1001*a^4*b^6
*c^4*d^6 + 286*a^5*b^5*c^3*d^7 + 66*a^6*b^4*c^2*d^8 + 11*a^7*b^3*c*d^9 + a^
8*b^2*d^10)*x^2 + 21*(92378*b^10*c^9*d + 43758*a*b^9*c^8*d^2 + 19448*a^2*b^
8*c^7*d^3 + 8008*a^3*b^7*c^6*d^4 + 3003*a^4*b^6*c^5*d^5 + 1001*a^5*b^5*c^4*
d^6 + 286*a^6*b^4*c^3*d^7 + 66*a^7*b^3*c^2*d^8 + 11*a^8*b^2*c*d^9 + a^9*b*d
^10)*x)/(b^32*x^21 + 21*a*b^31*x^20 + 210*a^2*b^30*x^19 + 1330*a^3*b^29*x^1
8 + 5985*a^4*b^28*x^17 + 20349*a^5*b^27*x^16 + 54264*a^6*b^26*x^15 + 116280
*a^7*b^25*x^14 + 203490*a^8*b^24*x^13 + 293930*a^9*b^23*x^12 + 352716*a^10*
b^22*x^11 + 352716*a^11*b^21*x^10 + 293930*a^12*b^20*x^9 + 203490*a^13*b^19
*x^8 + 116280*a^14*b^18*x^7 + 54264*a^15*b^17*x^6 + 20349*a^16*b^16*x^5 + 5
985*a^17*b^15*x^4 + 1330*a^18*b^14*x^3 + 210*a^19*b^13*x^2 + 21*a^20*b^12*x
 + a^21*b^11)
```

**mupad [B]** time = 1.04, size = 1186, normalized size = 4.25

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^22,x)
```

```
[Out] -(a^10*d^10 + 184756*b^10*c^10 + 352716*b^10*d^10*x^10 + 293930*a*b^9*d^10*
x^9 + 3233230*b^10*c*d^9*x^9 + 43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^
3 + 8008*a^4*b^6*c^6*d^4 + 3003*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 28
6*a^7*b^3*c^3*d^7 + 66*a^8*b^2*c^2*d^8 + 210*a^8*b^2*d^10*x^2 + 1330*a^7*b^
3*d^10*x^3 + 5985*a^6*b^4*d^10*x^4 + 20349*a^5*b^5*d^10*x^5 + 54264*a^4*b^6
*d^10*x^6 + 116280*a^3*b^7*d^10*x^7 + 203490*a^2*b^8*d^10*x^8 + 9189180*b^1
0*c^8*d^2*x^2 + 25865840*b^10*c^7*d^3*x^3 + 47927880*b^10*c^6*d^4*x^4 + 611
08047*b^10*c^5*d^5*x^5 + 54318264*b^10*c^4*d^6*x^6 + 33256080*b^10*c^3*d^7*
x^7 + 13430340*b^10*c^2*d^8*x^8 + 92378*a*b^9*c^9*d + 11*a^9*b*c*d^9 + 21*a
^9*b*d^10*x + 1939938*b^10*c^9*d*x + 1681680*a^2*b^8*c^6*d^4*x^2 + 630630*a
^3*b^7*c^5*d^5*x^2 + 210210*a^4*b^6*c^4*d^6*x^2 + 60060*a^5*b^5*c^3*d^7*x^2
 + 13860*a^6*b^4*c^2*d^8*x^2 + 3993990*a^2*b^8*c^5*d^5*x^3 + 1331330*a^3*b^
7*c^4*d^6*x^3 + 380380*a^4*b^6*c^3*d^7*x^3 + 87780*a^5*b^5*c^2*d^8*x^3 + 59
90985*a^2*b^8*c^4*d^6*x^4 + 1711710*a^3*b^7*c^3*d^7*x^4 + 395010*a^4*b^6*c^
2*d^8*x^4 + 5819814*a^2*b^8*c^3*d^7*x^5 + 1343034*a^3*b^7*c^2*d^8*x^5 + 358
```

```
1424*a^2*b^8*c^2*d^8*x^6 + 918918*a*b^9*c^8*d^2*x + 231*a^8*b^2*c*d^9*x + 2
238390*a*b^9*c*d^9*x^8 + 408408*a^2*b^8*c^7*d^3*x + 168168*a^3*b^7*c^6*d^4*
x + 63063*a^4*b^6*c^5*d^5*x + 21021*a^5*b^5*c^4*d^6*x + 6006*a^6*b^4*c^3*d^
7*x + 1386*a^7*b^3*c^2*d^8*x + 4084080*a*b^9*c^7*d^3*x^2 + 2310*a^7*b^3*c*d
^9*x^2 + 10650640*a*b^9*c^6*d^4*x^3 + 14630*a^6*b^4*c*d^9*x^3 + 17972955*a*
b^9*c^5*d^5*x^4 + 65835*a^5*b^5*c*d^9*x^4 + 20369349*a*b^9*c^4*d^6*x^5 + 22
3839*a^4*b^6*c*d^9*x^5 + 15519504*a*b^9*c^3*d^7*x^6 + 596904*a^3*b^7*c*d^9*
x^6 + 7674480*a*b^9*c^2*d^8*x^7 + 1279080*a^2*b^8*c*d^9*x^7)/(3879876*a^21*
b^11 + 3879876*b^32*x^21 + 81477396*a^20*b^12*x + 81477396*a*b^31*x^20 + 81
4773960*a^19*b^13*x^2 + 5160235080*a^18*b^14*x^3 + 23221057860*a^17*b^15*x^
4 + 78951596724*a^16*b^16*x^5 + 210537591264*a^15*b^17*x^6 + 451151981280*a
^14*b^18*x^7 + 789515967240*a^13*b^19*x^8 + 1140411952680*a^12*b^20*x^9 + 1
368494343216*a^11*b^21*x^10 + 1368494343216*a^10*b^22*x^11 + 1140411952680*
a^9*b^23*x^12 + 789515967240*a^8*b^24*x^13 + 451151981280*a^7*b^25*x^14 + 2
10537591264*a^6*b^26*x^15 + 78951596724*a^5*b^27*x^16 + 23221057860*a^4*b^2
8*x^17 + 5160235080*a^3*b^29*x^18 + 814773960*a^2*b^30*x^19)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**22,x)
```

```
[Out] Timed out
```

## 3.1228 $\int \frac{(a+bx)^5}{c+dx}\,dx$

**Optimal**. Leaf size=122

$$-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}+\frac{(a+bx)^5}{5d}$$

**Rubi [A]** time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{(a+bx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x),x]
```

```
[Out] (b*(b*c - a*d)^4*x)/d^5 - ((b*c - a*d)^3*(a + b*x)^2)/(2*d^4) + ((b*c - a*d
)^2*(a + b*x)^3)/(3*d^3) - ((b*c - a*d)*(a + b*x)^4)/(4*d^2) + (a + b*x)^5/
(5*d) - ((b*c - a*d)^5*Log[c + d*x])/d^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{c+dx}\,dx &= \int \left(\frac{b(bc-ad)^4}{d^5}-\frac{b(bc-ad)^3(a+bx)}{d^4}+\frac{b(bc-ad)^2(a+bx)^2}{d^3}-\frac{b(bc-ad)(a+bx)^3}{d^2}+\frac{b(a+b}{d}\right.\\ &= \frac{b(bc-ad)^4x}{d^5}-\frac{(bc-ad)^3(a+bx)^2}{2d^4}+\frac{(bc-ad)^2(a+bx)^3}{3d^3}-\frac{(bc-ad)(a+bx)^4}{4d^2}+\frac{(a+bx)^5}{5d}-\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 167, normalized size = 1.37

$$\frac{bdx\left(300a^4d^4+300a^3bd^3(dx-2c)+100a^2b^2d^2\left(6c^2-3cdx+2d^2x^2\right)+25ab^3d\left(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3\right)+b^4\left(60c^4-30c^3dx+20c^2d^2x^2-15cd^3x^3+12d^4x^4\right)\right)-60(bc-ad)^5\log(c+dx)}{60d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x),x]
```

```
[Out] (b*d*x*(300*a^4*d^4 + 300*a^3*b*d^3*(-2*c + d*x) + 100*a^2*b^2*d^2*(6*c^2 -
 3*c*d*x + 2*d^2*x^2) + 25*a*b^3*d*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d
^3*x^3) + b^4*(60*c^4 - 30*c^3*d*x + 20*c^2*d^2*x^2 - 15*c*d^3*x^3 + 12*d^4
*x^4)) - 60*(b*c - a*d)^5*Log[c + d*x])/(60*d^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(c + d*x), x]
```

**fricas [B]** time = 1.09, size = 259, normalized size = 2.12

$$\frac{12\,b^5d^5x^5 - 15\left(b^5cd^4 - 5\,ab^4d^5\right)x^4 + 20\left(b^5c^2d^3 - 5\,ab^4cd^4 + 10\,a^2b^3d^5\right)x^3 - 30\left(b^5c^3d^2 - 5\,ab^4c^2d^3 + 10\,a^2b^3cd^4 - 10\,a^3b^2d^5\right)x^2 + 60\left(b^5c^4d - 5\,ab^4c^3d^2 + 10\,a^2b^3c^2d^3 - 10\,a^3b^2cd^4 + 5\,a^4bd^5\right)x - 60\left(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5\right)\log\left(dx+c\right)}{60\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*d^5*x^5 - 15*(b^5*c*d^4 - 5*a*b^4*d^5)*x^4 + 20*(b^5*c^2*d^3 -
 5*a*b^4*c*d^4 + 10*a^2*b^3*d^5)*x^3 - 30*(b^5*c^3*d^2 - 5*a*b^4*c^2*d^3 +
10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*x^2 + 60*(b^5*c^4*d - 5*a*b^4*c^3*d^2 +
10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x - 60*(b^5*c^5 - 5*a*
b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d
^5)*log(d*x + c))/d^6
```

**giac [B]** time = 1.26, size = 273, normalized size = 2.24

$$\frac{12\,b^5d^4x^5 - 15\,b^5cd^3x^4 + 75\,ab^4d^4x^4 + 20\,b^5c^2d^2x^3 - 100\,ab^4cd^3x^3 + 200\,a^2b^3d^4x^3 - 30\,b^5c^3dx^2 + 150\,ab^4c^2d^2x^2 - 300\,a^2b^3cd^3x^2 + 300\,a^3b^2d^4x^2 + 60\,b^5c^4x - 300\,ab^4c^3dx + 600\,a^2b^3c^2d^2x - 600\,a^3b^2cd^3x + 300\,a^4bd^4x}{60\,d^5} - \frac{\left(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5\right)\log\left(|dx+c|\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*b^5*c*d^3*x^4 + 75*a*b^4*d^4*x^4 + 20*b^5*c^2*d^2
*x^3 - 100*a*b^4*c*d^3*x^3 + 200*a^2*b^3*d^4*x^3 - 30*b^5*c^3*d*x^2 + 150*a
*b^4*c^2*d^2*x^2 - 300*a^2*b^3*c*d^3*x^2 + 300*a^3*b^2*d^4*x^2 + 60*b^5*c^4
*x - 300*a*b^4*c^3*d*x + 600*a^2*b^3*c^2*d^2*x - 600*a^3*b^2*c*d^3*x + 300*
a^4*b*d^4*x)/d^5 - (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b
^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*log(abs(d*x + c))/d^6
```

**maple [B]** time = 0.00, size = 302, normalized size = 2.48

$$\frac{b^5x^5}{5d} + \frac{5a\,b^4x^4}{4d} - \frac{b^5c\,x^4}{4d^2} + \frac{10a^2b^3x^3}{3d} - \frac{5a\,b^4c\,x^3}{3d^2} + \frac{b^5c^2x^3}{3d^3} + \frac{5a^3b^2x^2}{d} - \frac{5a^2b^3c\,x^2}{d^2} + \frac{5a\,b^4c^2x^2}{2d^3} - \frac{b^5c^3x^2}{2d^4} + \frac{a^5\ln\left(dx+c\right)}{d} - \frac{5a^4bc\ln\left(dx+c\right)}{d^2} + \frac{5a^4bx}{d} + \frac{10a^3b^2c^2\ln\left(dx+c\right)}{d^3} - \frac{10a^3b^2cx}{d^2} - \frac{10a^2b^3c^3\ln\left(dx+c\right)}{d^4} + \frac{10a^2b^3c^2x}{d^3} + \frac{5a\,b^4c^4\ln\left(dx+c\right)}{d^5} - \frac{5a\,b^4c^3x}{d^4} - \frac{b^5c^5\ln\left(dx+c\right)}{d^6} + \frac{b^5c^4x}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c),x)
```

```
[Out] 1/5*b^5/d*x^5+5/4*b^4/d*x^4*a-1/4*b^5/d^2*x^4*c+10/3*b^3/d*x^3*a^2-5/3*b^4/
d^2*x^3*a*c+1/3*b^5/d^3*x^3*c^2+5*b^2/d*x^2*a^3-5*b^3/d^2*x^2*a^2*c+5/2*b^4
/d^3*x^2*a*c^2-1/2*b^5/d^4*x^2*c^3+5*b/d*a^4*x-10*b^2/d^2*a^3*c*x+10*b^3/d^
3*a^2*c^2*x-5*b^4/d^4*a*c^3*x+b^5/d^5*c^4*x+1/d*ln(d*x+c)*a^5-5/d^2*ln(d*x+
c)*a^4*b*c+10/d^3*ln(d*x+c)*a^3*b^2*c^2-10/d^4*ln(d*x+c)*a^2*b^3*c^3+5/d^5*
ln(d*x+c)*a*b^4*c^4-1/d^6*ln(d*x+c)*b^5*c^5
```

**maxima [B]** time = 1.35, size = 258, normalized size = 2.11

$$\frac{12\,b^5d^4x^5 - 15\,(b^5cd^3 - 5\,ab^4d^4)x^4 + 20\,(b^5c^2d^2 - 5\,ab^4cd^3 + 10\,a^2b^3d^4)x^3 - 30\,(b^5c^3d - 5\,ab^4c^2d^2 + 10\,a^2b^3cd^3 - 10\,a^3b^2d^4)x^2 + 60\,(b^5c^4 - 5\,ab^4c^3d + 10\,a^2b^3c^2d^2 - 10\,a^3b^2cd^3 + 5\,a^4bd^4)x}{60\,d^5} - \frac{(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)\log(dx + c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*(b^5*c*d^3 - 5*a*b^4*d^4)*x^4 + 20*(b^5*c^2*d^2 -
 5*a*b^4*c*d^3 + 10*a^2*b^3*d^4)*x^3 - 30*(b^5*c^3*d - 5*a*b^4*c^2*d^2 + 10
*a^2*b^3*c*d^3 - 10*a^3*b^2*d^4)*x^2 + 60*(b^5*c^4 - 5*a*b^4*c^3*d + 10*a^2
*b^3*c^2*d^2 - 10*a^3*b^2*c*d^3 + 5*a^4*b*d^4)*x)/d^5 - (b^5*c^5 - 5*a*b^4*
c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*
log(d*x + c)/d^6
```

**mupad [B]** time = 0.07, size = 280, normalized size = 2.30

$$x\left(\frac{5\,a^4\,b}{d} - \frac{c\left(\frac{10\,a^3\,b^2}{d} + \frac{c\left(\frac{c\left(\frac{5\,a\,b^4}{d} - \frac{b^5\,c}{d^2}\right)}{d} - \frac{10\,a^2\,b^3}{d}\right)}{d}\right)}{d}\right) + x^4\left(\frac{5\,a\,b^4}{4\,d} - \frac{b^5\,c}{4\,d^2}\right) + x^2\left(\frac{5\,a^3\,b^2}{d} + \frac{c\left(\frac{c\left(\frac{5\,a\,b^4}{d} - \frac{b^5\,c}{d^2}\right)}{d} - \frac{10\,a^2\,b^3}{d}\right)}{2\,d}\right) - x^3\left(\frac{c\left(\frac{5\,a\,b^4}{d} - \frac{b^5\,c}{d^2}\right)}{3\,d} - \frac{10\,a^2\,b^3}{3\,d}\right) + \frac{b^5\,x^5}{5\,d} + \frac{\ln(c + d\,x)\,\left(a^5\,d^5 - 5\,a^4\,b\,c\,d^4 + 10\,a^3\,b^2\,c^2\,d^3 - 10\,a^2\,b^3\,c^3\,d^2 + 5\,a\,b^4\,c^4\,d - b^5\,c^5\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x),x)
```

```
[Out] x*((5*a^4*b)/d - (c*((10*a^3*b^2)/d + (c*((c*((5*a*b^4)/d - (b^5*c)/d^2))/d
 - (10*a^2*b^3)/d))/d))/d) + x^4*((5*a*b^4)/(4*d) - (b^5*c)/(4*d^2)) + x^2*
((5*a^3*b^2)/d + (c*((c*((5*a*b^4)/d - (b^5*c)/d^2))/d - (10*a^2*b^3)/d))/(
2*d)) - x^3*((c*((5*a*b^4)/d - (b^5*c)/d^2))/(3*d) - (10*a^2*b^3)/(3*d)) +
(b^5*x^5)/(5*d) + (log(c + d*x)*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 1
0*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/d^6
```

**sympy [B]** time = 0.50, size = 209, normalized size = 1.71

$$\frac{b^5x^5}{5d} + x^4\left(\frac{5ab^4}{4d} - \frac{b^5c}{4d^2}\right) + x^3\left(\frac{10a^2b^3}{3d} - \frac{5ab^4c}{3d^2} + \frac{b^5c^2}{3d^3}\right) + x^2\left(\frac{5a^3b^2}{d} - \frac{5a^2b^3c}{d^2} + \frac{5ab^4c^2}{2d^3} - \frac{b^5c^3}{2d^4}\right) + x\left(\frac{5a^4b}{d} - \frac{10a^3b^2c}{d^2} + \frac{10a^2b^3c^2}{d^3} - \frac{5ab^4c^3}{d^4} + \frac{b^5c^4}{d^5}\right) + \frac{(ad - bc)^5\log(c + dx)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c),x)
```

```
[Out] b**5*x**5/(5*d) + x**4*(5*a*b**4/(4*d) - b**5*c/(4*d**2)) + x**3*(10*a**2*b
**3/(3*d) - 5*a*b**4*c/(3*d**2) + b**5*c**2/(3*d**3)) + x**2*(5*a**3*b**2/d
 - 5*a**2*b**3*c/d**2 + 5*a*b**4*c**2/(2*d**3) - b**5*c**3/(2*d**4)) + x*(5
*a**4*b/d - 10*a**3*b**2*c/d**2 + 10*a**2*b**3*c**2/d**3 - 5*a*b**4*c**3/d*
*4 + b**5*c**4/d**5) + (a*d - b*c)**5*log(c + d*x)/d**6
```

# 3.1229 $\int \frac{(a+bx)^4}{c+dx}\,dx$

**Optimal**. Leaf size=98

$$\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}$$

**Rubi** [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(bc-ad)^4\log(c+dx)}{d^5}+\frac{(a+bx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x),x]
```

```
[Out] -((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d^3) - ((b*c -
a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)^4*Log[c + d*x]
)/d^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{c+dx}\,dx &= \int \left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)^4}{d^4(c+dx)}\right) d\\ &= -\frac{b(bc-ad)^3x}{d^4}+\frac{(bc-ad)^2(a+bx)^2}{2d^3}-\frac{(bc-ad)(a+bx)^3}{3d^2}+\frac{(a+bx)^4}{4d}+\frac{(bc-ad)^4\log(c+d\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 115, normalized size = 1.17

$$\frac{bdx\left(48a^3d^3+36a^2bd^2(dx-2c)+8ab^2d\left(6c^2-3cdx+2d^2x^2\right)+b^3\left(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3\right)\right)+12(bc-ad)^4\log(c+dx)}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x),x]
```

```
[Out] (b*d*x*(48*a^3*d^3 + 36*a^2*b*d^2*(-2*c + d*x) + 8*a*b^2*d*(6*c^2 - 3*c*d*x
 + 2*d^2*x^2) + b^3*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^3*x^3)) + 12*(
b*c - a*d)^4*Log[c + d*x])/(12*d^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4/(c + d*x), x]
```

**fricas [A]** time = 1.19, size = 179, normalized size = 1.83

$$\frac{3\,b^4d^4x^4-4\left(b^4cd^3-4\,ab^3d^4\right)x^3+6\left(b^4c^2d^2-4\,ab^3cd^3+6\,a^2b^2d^4\right)x^2-12\left(b^4c^3d-4\,ab^3c^2d^2+6\,a^2b^2cd^3-4\,a^3bd^4\right)x+12\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\log\left(dx+c\right)}{12\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*d^4*x^4 - 4*(b^4*c*d^3 - 4*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 4*
a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 - 12*(b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6*a^2*
b^2*c*d^3 - 4*a^3*b*d^4)*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c))/d^5
```

**giac [A]** time = 1.23, size = 184, normalized size = 1.88

$$\frac{3\,b^4d^3x^4-4\,b^4cd^2x^3+16\,ab^3d^3x^3+6\,b^4c^2dx^2-24\,ab^3cd^2x^2+36\,a^2b^2d^3x^2-12\,b^4c^3x+48\,ab^3c^2dx-72\,a^2b^2cd^2x+48\,a^3bd^3x}{12\,d^4}+\frac{\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\log\left(|dx+c|\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*b^4*c*d^2*x^3 + 16*a*b^3*d^3*x^3 + 6*b^4*c^2*d*x^2
- 24*a*b^3*c*d^2*x^2 + 36*a^2*b^2*d^3*x^2 - 12*b^4*c^3*x + 48*a*b^3*c^2*d*x
 - 72*a^2*b^2*c*d^2*x + 48*a^3*b*d^3*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(d*x + c))/d^5
```

**maple [B]** time = 0.00, size = 209, normalized size = 2.13

$$\frac{b^4x^4}{4d}+\frac{4a\,b^3x^3}{3d}-\frac{b^4c\,x^3}{3d^2}+\frac{3a^2b^2x^2}{d}-\frac{2a\,b^3c\,x^2}{d^2}+\frac{b^4c^2x^2}{2d^3}+\frac{a^4\ln\left(dx+c\right)}{d}-\frac{4a^3bc\ln\left(dx+c\right)}{d^2}+\frac{4a^3bx}{d}+\frac{6a^2b^2c^2\ln\left(dx+c\right)}{d^3}-\frac{6a^2b^2cx}{d^2}-\frac{4a\,b^3c^3\ln\left(dx+c\right)}{d^4}+\frac{4a\,b^3c^2x}{d^3}+\frac{b^4c^4\ln\left(dx+c\right)}{d^5}-\frac{b^4c^3x}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c),x)
```

```
[Out] 1/4*b^4/d*x^4+4/3*b^3/d*x^3*a-1/3*b^4/d^2*x^3*c+3*b^2/d*x^2*a^2-2*b^3/d^2*x
^2*a*c+1/2*b^4/d^3*x^2*c^2+4*b/d*a^3*x-6*b^2/d^2*a^2*c*x+4*b^3/d^3*a*c^2*x-
b^4/d^4*c^3*x+1/d*ln(d*x+c)*a^4-4/d^2*ln(d*x+c)*a^3*b*c+6/d^3*ln(d*x+c)*a^2
*b^2*c^2-4/d^4*ln(d*x+c)*a*b^3*c^3+1/d^5*ln(d*x+c)*b^4*c^4
```

**maxima** [A] time = 1.41, size = 177, normalized size = 1.81

$$\frac{3\,b^{4}d^{3}x^{4}-4\left(b^{4}cd^{2}-4\,ab^{3}d^{3}\right)x^{3}+6\left(b^{4}c^{2}d-4\,ab^{3}cd^{2}+6\,a^{2}b^{2}d^{3}\right)x^{2}-12\left(b^{4}c^{3}-4\,ab^{3}c^{2}d+6\,a^{2}b^{2}cd^{2}-4\,a^{3}bd^{3}\right)x}{12\,d^{4}}+\frac{\left(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}\right)\log\left(dx+c\right)}{d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*(b^4*c*d^2 - 4*a*b^3*d^3)*x^3 + 6*(b^4*c^2*d - 4*a*
b^3*c*d^2 + 6*a^2*b^2*d^3)*x^2 - 12*(b^4*c^3 - 4*a*b^3*c^2*d + 6*a^2*b^2*c*
d^2 - 4*a^3*b*d^3)*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c)/d^5
```

**mupad** [B] time = 0.22, size = 189, normalized size = 1.93

$$x^{3}\left(\frac{4\,a\,b^{3}}{3\,d}-\frac{b^{4}\,c}{3\,d^{2}}\right)+x\left(\frac{4\,a^{3}\,b}{d}+\frac{c\left(\frac{c\left(\frac{4\,a\,b^{3}}{d}-\frac{b^{4}\,c}{d^{2}}\right)}{d}-\frac{6\,a^{2}\,b^{2}}{d}\right)}{d}\right)-x^{2}\left(\frac{c\left(\frac{4\,a\,b^{3}}{d}-\frac{b^{4}\,c}{d^{2}}\right)}{2\,d}-\frac{3\,a^{2}\,b^{2}}{d}\right)+\frac{\ln\left(c+d\,x\right)\,\left(a^{4}\,d^{4}-4\,a^{3}\,b\,c\,d^{3}+6\,a^{2}\,b^{2}\,c^{2}\,d^{2}-4\,a\,b^{3}\,c^{3}\,d+b^{4}\,c^{4}\right)}{d^{5}}+\frac{b^{4}\,x^{4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x),x)
```

```
[Out] x^3*((4*a*b^3)/(3*d) - (b^4*c)/(3*d^2)) + x*((4*a^3*b)/d + (c*((c*((4*a*b^3
)/d - (b^4*c)/d^2))/d - (6*a^2*b^2)/d))/d) - x^2*((c*((4*a*b^3)/d - (b^4*c)
/d^2))/(2*d) - (3*a^2*b^2)/d) + (log(c + d*x)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d^5 + (b^4*x^4)/(4*d)
```

**sympy** [A] time = 0.39, size = 136, normalized size = 1.39

$$\frac{b^{4}x^{4}}{4d}+x^{3}\left(\frac{4ab^{3}}{3d}-\frac{b^{4}c}{3d^{2}}\right)+x^{2}\left(\frac{3a^{2}b^{2}}{d}-\frac{2ab^{3}c}{d^{2}}+\frac{b^{4}c^{2}}{2d^{3}}\right)+x\left(\frac{4a^{3}b}{d}-\frac{6a^{2}b^{2}c}{d^{2}}+\frac{4ab^{3}c^{2}}{d^{3}}-\frac{b^{4}c^{3}}{d^{4}}\right)+\frac{\left(ad-bc\right)^{4}\log\left(c+dx\right)}{d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c),x)
```

```
[Out] b**4*x**4/(4*d) + x**3*(4*a*b**3/(3*d) - b**4*c/(3*d**2)) + x**2*(3*a**2*b*
*2/d - 2*a*b**3*c/d**2 + b**4*c**2/(2*d**3)) + x*(4*a**3*b/d - 6*a**2*b**2*
c/d**2 + 4*a*b**3*c**2/d**3 - b**4*c**3/d**4) + (a*d - b*c)**4*log(c + d*x)
/d**5
```

## 3.1230 $\int \frac{(a+bx)^3}{c+dx} \, dx$

**Optimal**. Leaf size=74

$$-\frac{(bc-ad)^3 \log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}$$

**Rubi [A]** time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}-\frac{(bc-ad)^3 \log(c+dx)}{d^4}+\frac{(a+bx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x),x]
```

```
[Out] (b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) + (a + b*x)^3/(
3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{c+dx} \, dx &= \int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right) dx\\ &= \frac{b(bc-ad)^2x}{d^3}-\frac{(bc-ad)(a+bx)^2}{2d^2}+\frac{(a+bx)^3}{3d}-\frac{(bc-ad)^3 \log(c+dx)}{d^4}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 74, normalized size = 1.00

$$\frac{bdx\left(18a^2d^2+9abd(dx-2c)+b^2\left(6c^2-3cdx+2d^2x^2\right)\right)-6(bc-ad)^3 \log(c+dx)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x),x]
```

```
[Out] (b*d*x*(18*a^2*d^2 + 9*a*b*d*(-2*c + d*x) + b^2*(6*c^2 - 3*c*d*x + 2*d^2*x^
2)) - 6*(b*c - a*d)^3*Log[c + d*x])/(6*d^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/(c + d*x), x]
```

**fricas [A]** time = 0.67, size = 115, normalized size = 1.55

$$\frac{2\,b^3d^3x^3 - 3\left(b^3cd^2 - 3\,ab^2d^3\right)x^2 + 6\left(b^3c^2d - 3\,ab^2cd^2 + 3\,a^2bd^3\right)x - 6\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(dx + c\right)}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 - 3*(b^3*c*d^2 - 3*a*b^2*d^3)*x^2 + 6*(b^3*c^2*d - 3*a*b
^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(d*x + c))/d^4
```

**giac [A]** time = 1.20, size = 116, normalized size = 1.57

$$\frac{2\,b^3d^2x^3 - 3\,b^3cdx^2 + 9\,ab^2d^2x^2 + 6\,b^3c^2x - 18\,ab^2cdx + 18\,a^2bd^2x}{6\,d^3} - \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(|dx + c|\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*b^3*c*d*x^2 + 9*a*b^2*d^2*x^2 + 6*b^3*c^2*x - 18*a*b
^2*c*d*x + 18*a^2*b*d^2*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(d*x + c))/d^4
```

**maple [A]** time = 0.00, size = 133, normalized size = 1.80

$$\frac{b^3x^3}{3d} + \frac{3a\,b^2x^2}{2d} - \frac{b^3c\,x^2}{2d^2} + \frac{a^3\ln\left(dx + c\right)}{d} - \frac{3a^2bc\ln\left(dx + c\right)}{d^2} + \frac{3a^2bx}{d} + \frac{3a\,b^2c^2\ln\left(dx + c\right)}{d^3} - \frac{3a\,b^2cx}{d^2} - \frac{b^3c^3\ln\left(dx + c\right)}{d^4} + \frac{b^3c^2x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/3*b^3/d*x^3+3/2*b^2/d*x^2*a-1/2*b^3/d^2*x^2*c+3*b/d*a^2*x-3*b^2/d^2*a*c*x
+b^3/d^3*c^2*x+1/d*ln(d*x+c)*a^3-3/d^2*ln(d*x+c)*a^2*b*c+3/d^3*ln(d*x+c)*a*
b^2*c^2-1/d^4*ln(d*x+c)*b^3*c^3
```

**maxima** [A] time = 1.30, size = 114, normalized size = 1.54

$$\frac{2\,b^3d^2x^3 - 3\left(b^3cd - 3\,ab^2d^2\right)x^2 + 6\left(b^3c^2 - 3\,ab^2cd + 3\,a^2bd^2\right)x}{6\,d^3} - \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(dx + c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*x^2 + 6*(b^3*c^2 - 3*a*b^2*c
*d + 3*a^2*b*d^2)*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(d*x + c)/d^4
```

**mupad** [B] time = 0.07, size = 118, normalized size = 1.59

$$x^2\left(\frac{3\,a\,b^2}{2\,d} - \frac{b^3\,c}{2\,d^2}\right) + x\left(\frac{3\,a^2\,b}{d} - \frac{c\,\left(\frac{3\,a\,b^2}{d} - \frac{b^3\,c}{d^2}\right)}{d}\right) + \frac{\ln\left(c + d\,x\right)\,\left(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3\right)}{d^4} + \frac{b^3\,x^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x),x)
```

```
[Out] x^2*((3*a*b^2)/(2*d) - (b^3*c)/(2*d^2)) + x*((3*a^2*b)/d - (c*((3*a*b^2)/d
- (b^3*c)/d^2))/d) + (log(c + d*x)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/d^4 + (b^3*x^3)/(3*d)
```

**sympy** [A] time = 0.30, size = 83, normalized size = 1.12

$$\frac{b^3x^3}{3d} + x^2\left(\frac{3ab^2}{2d} - \frac{b^3c}{2d^2}\right) + x\left(\frac{3a^2b}{d} - \frac{3ab^2c}{d^2} + \frac{b^3c^2}{d^3}\right) + \frac{\left(ad - bc\right)^3\log\left(c + dx\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c),x)
```

```
[Out] b**3*x**3/(3*d) + x**2*(3*a*b**2/(2*d) - b**3*c/(2*d**2)) + x*(3*a**2*b/d -
 3*a*b**2*c/d**2 + b**3*c**2/d**3) + (a*d - b*c)**3*log(c + d*x)/d**4
```

## 3.1231 $\int \frac{(a+bx)^2}{c+dx}\,dx$

**Optimal**. Leaf size=50

$$\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}$$

**Rubi** [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{bx(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+dx)}{d^3}+\frac{(a+bx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x),x]
```

```
[Out] -((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2*Log[c + d*x])
/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{c+dx}\,dx &= \int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx\\ &= -\frac{b(bc-ad)x}{d^2}+\frac{(a+bx)^2}{2d}+\frac{(bc-ad)^2\log(c+dx)}{d^3}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 43, normalized size = 0.86

$$\frac{bdx(4ad-2bc+bdx)+2(bc-ad)^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x),x]
```

```
[Out] (b*d*x*(-2*b*c + 4*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[c + d*x])/(2*d^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(c + d*x), x]
```

**fricas [A]** time = 1.48, size = 62, normalized size = 1.24

$$\frac{b^2d^2x^2 - 2\left(b^2cd - 2\,abd^2\right)x + 2\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(dx + c\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 - 2*(b^2*c*d - 2*a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(d*x + c))/d^3
```

**giac [A]** time = 1.21, size = 60, normalized size = 1.20

$$\frac{b^2dx^2 - 2\,b^2cx + 4\,abdx}{2\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(|dx + c|\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*x^2 - 2*b^2*c*x + 4*a*b*d*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(d*x + c))/d^3
```

**maple [A]** time = 0.00, size = 74, normalized size = 1.48

$$\frac{b^2x^2}{2d} + \frac{a^2\ln\left(dx + c\right)}{d} - \frac{2abc\ln\left(dx + c\right)}{d^2} + \frac{2abx}{d} + \frac{b^2c^2\ln\left(dx + c\right)}{d^3} - \frac{b^2cx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/2*b^2/d*x^2+2*b/d*a*x-b^2/d^2*x*c+1/d*ln(d*x+c)*a^2-2/d^2*ln(d*x+c)*a*b*c
+1/d^3*ln(d*x+c)*b^2*c^2
```

**maxima [A]** time = 1.35, size = 60, normalized size = 1.20

$$\frac{b^2dx^2 - 2\left(b^2c - 2\,abd\right)x}{2\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*d*x^2 - 2*(b^2*c - 2*a*b*d)*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(d*x + c)/d^3
```

**mupad [B]** time = 0.22, size = 62, normalized size = 1.24

$$\frac{\ln\left(c + d\,x\right)\,\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{d^3} - x\,\left(\frac{b^2\,c}{d^2} - \frac{2\,a\,b}{d}\right) + \frac{b^2\,x^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x),x)
```

```
[Out] (log(c + d*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/d^3 - x*((b^2*c)/d^2 - (2*a*
b)/d) + (b^2*x^2)/(2*d)
```

**sympy [A]** time = 0.22, size = 44, normalized size = 0.88

$$\frac{b^2x^2}{2d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right) + \frac{(ad - bc)^2\log\left(c + dx\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c),x)
```

```
[Out] b**2*x**2/(2*d) + x*(2*a*b/d - b**2*c/d**2) + (a*d - b*c)**2*log(c + d*x)/d
**3
```

## 3.1232 $\int \frac{a+bx}{c+dx} \, dx$

**Optimal**. Leaf size=26

$$\frac{bx}{d} - \frac{(bc - ad)\log(c + dx)}{d^2}$$

**Rubi** [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{bx}{d} - \frac{(bc - ad)\log(c + dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a + bx}{c + dx} \, dx &= \int \left(\frac{b}{d} + \frac{-bc + ad}{d(c + dx)}\right) dx \\ &= \frac{bx}{d} - \frac{(bc - ad)\log(c + dx)}{d^2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 25, normalized size = 0.96

$$\frac{(ad - bc)\log(c + dx)}{d^2} + \frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x),x]
```

```
[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x),x]
[Out] IntegrateAlgebraic[(a + b*x)/(c + d*x), x]
```

**fricas** [A] time = 0.93, size = 25, normalized size = 0.96

$$\frac{bdx-(bc-ad)\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

**giac** [A] time = 1.20, size = 27, normalized size = 1.04

$$\frac{bx}{d}-\frac{(bc-ad)\log(|dx+c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="giac")
[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

**maple** [A] time = 0.00, size = 32, normalized size = 1.23

$$\frac{a\ln(dx+c)}{d}-\frac{bc\ln(dx+c)}{d^2}+\frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c),x)
[Out] b*x/d+1/d*ln(d*x+c)*a-1/d^2*ln(d*x+c)*b*c
```

**maxima** [A] time = 1.32, size = 26, normalized size = 1.00

$$\frac{bx}{d}-\frac{(bc-ad)\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

**mupad [B]** time = 0.20, size = 25, normalized size = 0.96

$$\frac{\ln(c+d\,x)\,(a\,d-b\,c)}{d^2}+\frac{b\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x),x)
```

```
[Out] (log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d
```

**sympy [A]** time = 0.15, size = 20, normalized size = 0.77

$$\frac{bx}{d}+\frac{(ad-bc)\log(c+dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x)
```

```
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

## 3.1233 $\int \frac{1}{c+dx}\,dx$

**Optimal**. Leaf size=10

$$\frac{\log(c+dx)}{d}$$

**Rubi [A]** time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-1),x]
```

```
[Out] Log[c + d*x]/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{c+dx}\,dx = \frac{\log(c+dx)}{d}$$

**Mathematica [A]** time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-1),x]
```

```
[Out] Log[c + d*x]/d
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^(-1), x]
```

**fricas [A]** time = 0.90, size = 10, normalized size = 1.00

$$\frac{\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="fricas")
```

```
[Out] log(d*x + c)/d
```

**giac [A]** time = 1.26, size = 11, normalized size = 1.10

$$\frac{\log(|dx+c|)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="giac")
```

```
[Out] log(abs(d*x + c))/d
```

**maple [A]** time = 0.00, size = 11, normalized size = 1.10

$$\frac{\ln(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c),x)
```

```
[Out] ln(d*x+c)/d
```

**maxima [A]** time = 1.30, size = 10, normalized size = 1.00

$$\frac{\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="maxima")
```

```
[Out] log(d*x + c)/d
```

**mupad** [B] time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x),x)
```

```
[Out] log(c + d*x)/d
```

**sympy** [A] time = 0.06, size = 7, normalized size = 0.70

$$\frac{\log\left(c+dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x)
```

```
[Out] log(c + d*x)/d
```

## 3.1234 $\int \frac{1}{(a+bx)(c+dx)} \, dx$

**Optimal**. Leaf size=36

$$\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}$$

**Rubi [A]** time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {36, 31}

$$\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)),x]
```

```
[Out] Log[a + b*x]/(b*c - a*d) - Log[c + d*x]/(b*c - a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)} \, dx &= \frac{b \int \frac{1}{a+bx} \, dx}{bc-ad}-\frac{d \int \frac{1}{c+dx} \, dx}{bc-ad}\\ &= \frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 0.72

$$\frac{\log(a+bx)-\log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[a + b*x] - Log[c + d*x])/(b*c - a*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)), x]
```

**fricas [A]** time = 1.33, size = 26, normalized size = 0.72

$$\frac{\log(bx+a)-\log(dx+c)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (log(b*x + a) - log(d*x + c))/(b*c - a*d)
```

**giac [A]** time = 1.24, size = 46, normalized size = 1.28

$$\frac{b\log(|bx+a|)}{b^2c-abd}-\frac{d\log(|dx+c|)}{bcd-ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/(b^2*c - a*b*d) - d*log(abs(d*x + c))/(b*c*d - a*d^2)
```

**maple [A]** time = 0.01, size = 37, normalized size = 1.03

$$-\frac{\ln(bx+a)}{ad-bc}+\frac{\ln(dx+c)}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c),x)
```

```
[Out] 1/(a*d-b*c)*ln(d*x+c)-1/(a*d-b*c)*ln(b*x+a)
```

**maxima** [A] time = 1.36, size = 36, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{bc-ad}-\frac{\log\left(dx+c\right)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)
```

**mupad** [B] time = 0.26, size = 25, normalized size = 0.69

$$\frac{\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{a\,d-b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)),x)
```

```
[Out] log((c + d*x)/(a + b*x))/(a*d - b*c)
```

**sympy** [B] time = 0.33, size = 128, normalized size = 3.56

$$\frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}-\frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x)
```

```
[Out] log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**2/(
a*d - b*c) + b*c)/(2*b*d))/(a*d - b*c) - log(x + (a**2*d**2/(a*d - b*c) - 2
*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(a*d - b
*c)
```

## 3.1235 $\int \frac{1}{(a+bx)^2(c+dx)}\,dx$

**Optimal**. Leaf size=57

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)}\,dx &= \int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx\\ &= -\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 53, normalized size = 0.93

$$\frac{d(a+bx)\log(c+dx)-d(a+bx)\log(a+bx)+ad-bc}{(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
- a*d)^2*(a + b*x))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)), x]
```

**fricas [A]** time = 1.27, size = 93, normalized size = 1.63

$$-\frac{bc-ad+(bdx+ad)\log(bx+a)-(bdx+ad)\log(dx+c)}{ab^2c^2-2\,a^2bcd+a^3d^2+\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

**giac [A]** time = 1.32, size = 78, normalized size = 1.37

$$\frac{bd\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{b}{\left(b^2c-abd\right)(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] b*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^3*c^2 - 2*a*b^2*c*d + a^
2*b*d^2) - b/((b^2*c - a*b*d)*(b*x + a))
```

**maple [A]** time = 0.01, size = 57, normalized size = 1.00

$$-\frac{d\ln(bx+a)}{(ad-bc)^2}+\frac{d\ln(dx+c)}{(ad-bc)^2}+\frac{1}{(ad-bc)\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c),x)
```

```
[Out] d/(a*d-b*c)^2*ln(d*x+c)+1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)
```

**maxima** [A] time = 1.36, size = 92, normalized size = 1.61

$$-\frac{d\log\left(bx+a\right)}{b^2c^2-2\,abcd+a^2d^2}+\frac{d\log\left(dx+c\right)}{b^2c^2-2\,abcd+a^2d^2}-\frac{1}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

**mupad** [B] time = 0.14, size = 46, normalized size = 0.81

$$\frac{1}{(a\,d-b\,c)\,(a+b\,x)}-\frac{d\,\ln\left(\frac{a+b\,x}{c+d\,x}\right)}{(a\,d-b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)),x)
```

```
[Out] 1/((a*d - b*c)*(a + b*x)) - (d*log((a + b*x)/(c + d*x)))/(a*d - b*c)^2
```

**sympy** [B] time = 0.68, size = 233, normalized size = 4.09

$$\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}-\frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}+\frac{1}{a^2d-abc+x\left(abd-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c),x)
```

```
[Out] d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a
*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*
d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**
2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b*
*3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2*d -
a*b*c + x*(a*b*d - b**2*c))
```

## 3.1236 $\int \frac{1}{(a+bx)^3(c+dx)}\,dx$

**Optimal**. Leaf size=82

$$\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}$$

**Rubi [A]** time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] -1/(2*(b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) + (d^2*Log[a +
 b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3(c+dx)}\,dx &= \int \left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)}\right)dx\\ &= -\frac{1}{2(bc-ad)(a+bx)^2}+\frac{d}{(bc-ad)^2(a+bx)}+\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 67, normalized size = 0.82

$$\frac{\frac{(bc-ad)(3ad-bc+2bdx)}{(a+bx)^2}+2d^2\log(a+bx)-2d^2\log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] (((b*c - a*d)*(-(b*c) + 3*a*d + 2*b*d*x))/(a + b*x)^2 + 2*d^2*Log[a + b*x]
- 2*d^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3(c+dx)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)), x]
```

**fricas [B]** time = 1.36, size = 242, normalized size = 2.95

$$-\frac{b^2c^2 - 4\,abcd + 3\,a^2d^2 - 2\,\left(b^2cd - abd^2\right)x - 2\,\left(b^2d^2x^2 + 2\,abd^2x + a^2d^2\right)\log\left(bx + a\right) + 2\,\left(b^2d^2x^2 + 2\,abd^2x + a^2d^2\right)\log\left(dx + c\right)}{2\,\left(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3 + \left(b^5c^3 - 3\,ab^4c^2d + 3\,a^2b^3cd^2 - a^3b^2d^3\right)x^2 + 2\,\left(ab^4c^3 - 3\,a^2b^3c^2d + 3\,a^3b^2cd^2 - a^4bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x
+ a^2*d^2)*log(d*x + c))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
^5*d^3 + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^2 + 2*
(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x)
```

**giac [B]** time = 1.29, size = 165, normalized size = 2.01

$$\frac{bd^2\log\left(|bx + a|\right)}{b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3} - \frac{d^3\log\left(|dx + c|\right)}{b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4} - \frac{b^2c^2 - 4\,abcd + 3\,a^2d^2 - 2\,\left(b^2cd - abd^2\right)x}{2\,(bc - ad)^3(bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] b*d^2*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3) - d^3*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4) - 1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x)
/((b*c - a*d)^3*(b*x + a)^2)
```

**maple [A]** time = 0.01, size = 81, normalized size = 0.99

$$-\frac{d^2\ln\left(bx + a\right)}{\left(ad - bc\right)^3} + \frac{d^2\ln\left(dx + c\right)}{\left(ad - bc\right)^3} + \frac{d}{\left(ad - bc\right)^2\left(bx + a\right)} + \frac{1}{2\left(ad - bc\right)\left(bx + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c),x)
```

```
[Out] d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2/(b*x+a)-d^2
/(a*d-b*c)^3*ln(b*x+a)
```

**maxima** [B] time = 1.41, size = 202, normalized size = 2.46

$$\frac{d^2\log(bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{d^2\log(dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{2\,bdx-bc+3\,ad}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - d^2*
log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b
*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b
^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

**mupad** [B] time = 0.16, size = 182, normalized size = 2.22

$$\frac{\frac{3\,a\,d-b\,c}{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{a^2+2\,a\,b\,x+b^2\,x^2}-\frac{2\,d^2\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{{\left(a\,d-b\,c\right)}^3}+\frac{2\,b\,d\,x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3}\right)}{{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)),x)
```

```
[Out] ((3*a*d - b*c)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x)/(a^2*d^2 + b^2
*c^2 - 2*a*b*c*d))/(a^2 + b^2*x^2 + 2*a*b*x) - (2*d^2*atanh((a^3*d^3 + b^3*
c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3 + (2*b*d*x*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
```

**sympy** [B] time = 1.06, size = 381, normalized size = 4.65

$$\frac{d^2\log\left(x+\frac{-\frac{a^4d^6}{(ad-bc)^3}+\frac{4a^3bcd^5}{(ad-bc)^3}-\frac{6a^2b^2c^2d^4}{(ad-bc)^3}+\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3-\frac{b^4c^4d^2}{(ad-bc)^3}+bcd^2}{2bd^3}\right)}{(ad-bc)^3}-\frac{d^2\log\left(x+\frac{\frac{a^4d^6}{(ad-bc)^3}-\frac{4a^3bcd^5}{(ad-bc)^3}+\frac{6a^2b^2c^2d^4}{(ad-bc)^3}-\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3+\frac{b^4c^4d^2}{(ad-bc)^3}+bcd^2}{2bd^3}\right)}{(ad-bc)^3}+\frac{3ad-bc+2bdx}{2a^4d^2-4a^3bcd+2a^2b^2c^2+x^2\left(2a^2b^2d^2-4ab^3cd+2b^4c^2\right)+x\left(4a^3bd^2-8a^2b^2cd+4ab^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c),x)
```

```
[Out] d**2*log(x + (-a**4*d**6/(a*d - b*c)**3 + 4*a**3*b*c*d**5/(a*d - b*c)**3 -
6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 + 4*a*b**3*c**3*d**3/(a*d - b*c)**3 +
a*d**3 - b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b*c)*
```

```
*3 - d**2*log(x + (a**4*d**6/(a*d - b*c)**3 - 4*a**3*b*c*d**5/(a*d - b*c)**
3 + 6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 - 4*a*b**3*c**3*d**3/(a*d - b*c)**
3 + a*d**3 + b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b
*c)**3 + (3*a*d - b*c + 2*b*d*x)/(2*a**4*d**2 - 4*a**3*b*c*d + 2*a**2*b**2*
c**2 + x**2*(2*a**2*b**2*d**2 - 4*a*b**3*c*d + 2*b**4*c**2) + x*(4*a**3*b*d
**2 - 8*a**2*b**2*c*d + 4*a*b**3*c**2))
```

## 3.1237 $\int \frac{(a+bx)^5}{(c+dx)^2} \, dx$

**Optimal**. Leaf size=130

$$-\frac{5b^4(c+dx)^3(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^2(bc-ad)^2}{d^6}-\frac{10b^2x(bc-ad)^3}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b(bc-ad)^4\log(c+dx)}{d^6}+\frac{b^5(c+}{4d^6}$$

**Rubi [A]** time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{5b^4(c+dx)^3(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^2(bc-ad)^2}{d^6}-\frac{10b^2x(bc-ad)^3}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b(bc-ad)^4\log(c+dx)}{d^6}+\frac{b^5(c+dx)^4}{4d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] (-10*b^2*(b*c - a*d)^3*x)/d^5 + (b*c - a*d)^5/(d^6*(c + d*x)) + (5*b^3*(b*c
 - a*d)^2*(c + d*x)^2)/d^6 - (5*b^4*(b*c - a*d)*(c + d*x)^3)/(3*d^6) + (b^5
*(c + d*x)^4)/(4*d^6) + (5*b*(b*c - a*d)^4*Log[c + d*x])/d^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(c+dx)^2} \, dx &= \int \left(-\frac{10b^2(bc-ad)^3}{d^5}+\frac{(-bc+ad)^5}{d^5(c+dx)^2}+\frac{5b(bc-ad)^4}{d^5(c+dx)}+\frac{10b^3(bc-ad)^2(c+dx)}{d^5}-\frac{5b^4(bc-ad)(c+}{d^5}\right.\\ &= -\frac{10b^2(bc-ad)^3x}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b^3(bc-ad)^2(c+dx)^2}{d^6}-\frac{5b^4(bc-ad)(c+dx)^3}{3d^6}+\frac{b^5(c+dx)^4}{4d^6}\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 228, normalized size = 1.75

$$\frac{-12a^5d^5+60a^4bcd^4+120a^3b^2d^3\left(-c^2+cdx+d^2x^2\right)+60a^2b^3d^2\left(2c^3-4c^2dx-3cd^2x^2+d^3x^3\right)+20ab^4d\left(-3c^4+9c^3dx+6c^2d^2x^2-2cd^3x^3+d^4x^4\right)+60b(c+dx)(bc-ad)^4\log(c+dx)+b^5\left(12c^5-48c^4dx-30c^3d^2x^2+10c^2d^3x^3-5cd^4x^4+3d^5x^5\right)}{12d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] (60*a^4*b*c*d^4 - 12*a^5*d^5 + 120*a^3*b^2*d^3*(-c^2 + c*d*x + d^2*x^2) + 6
0*a^2*b^3*d^2*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + 20*a*b^4*d*(-3*
c^4 + 9*c^3*d*x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) + b^5*(12*c^5 - 48
*c^4*d*x - 30*c^3*d^2*x^2 + 10*c^2*d^3*x^3 - 5*c*d^4*x^4 + 3*d^5*x^5) + 60*
b*(b*c - a*d)^4*(c + d*x)*Log[c + d*x])/(12*d^6*(c + d*x))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^2, x]
```

**fricas [B]** time = 1.21, size = 373, normalized size = 2.87

$$\frac{3\,b^5d^5x^5+12\,b^5c^5-60\,ab^4c^4d+120\,a^2b^3c^3d^2-120\,a^3b^2c^2d^3+60\,a^4bcd^4-12\,a^5d^5-5\left(b^5cd^4-4\,ab^4d^5\right)x^4+10\left(b^5c^2d^3-4\,ab^4cd^4+6\,a^2b^3d^5\right)x^3-30\left(b^5c^3d^2-4\,ab^4c^2d^3+6\,a^2b^3cd^4-4\,a^3b^2d^5\right)x^2-12\left(4\,b^5c^4d-15\,ab^4c^3d^2+20\,a^2b^3c^2d^3-10\,a^3b^2cd^4\right)x+60\left(b^5c^5-4\,ab^4c^4d+6\,a^2b^3c^3d^2-4\,a^3b^2c^2d^3+a^4bcd^4+\left(b^5c^4d-4\,ab^4c^3d^2+6\,a^2b^3c^2d^3-4\,a^3b^2cd^4+a^4bd^5\right)x\right)\log\left(dx+c\right)}{12\left(d^7x+cd^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*d^5*x^5 + 12*b^5*c^5 - 60*a*b^4*c^4*d + 120*a^2*b^3*c^3*d^2 - 1
20*a^3*b^2*c^2*d^3 + 60*a^4*b*c*d^4 - 12*a^5*d^5 - 5*(b^5*c*d^4 - 4*a*b^4*d
^5)*x^4 + 10*(b^5*c^2*d^3 - 4*a*b^4*c*d^4 + 6*a^2*b^3*d^5)*x^3 - 30*(b^5*c^
3*d^2 - 4*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 - 4*a^3*b^2*d^5)*x^2 - 12*(4*b^5*
c^4*d - 15*a*b^4*c^3*d^2 + 20*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4)*x + 60*(b
^5*c^5 - 4*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^2*c^2*d^3 + a^4*b*c*d^
4 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^
4*b*d^5)*x)*log(d*x + c))/(d^7*x + c*d^6)
```

**giac [B]** time = 1.27, size = 339, normalized size = 2.61

$$\frac{\left(3\,b^5-\frac{20\left(b^5cd-ab^4d^2\right)}{(dx+c)d}+\frac{60\left(b^5c^2d^2-2\,ab^4cd^3+a^2b^3d^4\right)}{(dx+c)^2d^2}-\frac{120\left(b^5c^3d^3-3\,ab^4c^2d^4+3\,a^2b^3cd^5-a^3b^2d^6\right)}{(dx+c)^3d^3}\right)(dx+c)^4}{12\,d^6}-\frac{5\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^6}+\frac{\frac{b^5c^5d^4}{dx+c}-\frac{5\,ab^4c^4d^5}{dx+c}+\frac{10\,a^2b^3c^3d^6}{dx+c}-\frac{10\,a^3b^2c^2d^7}{dx+c}+\frac{5\,a^4bcd^8}{dx+c}-\frac{a^5d^9}{dx+c}}{d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^5 - 20*(b^5*c*d - a*b^4*d^2)/((d*x + c)*d) + 60*(b^5*c^2*d^2 - 2*
a*b^4*c*d^3 + a^2*b^3*d^4)/((d*x + c)^2*d^2) - 120*(b^5*c^3*d^3 - 3*a*b^4*c
^2*d^4 + 3*a^2*b^3*c*d^5 - a^3*b^2*d^6)/((d*x + c)^3*d^3))*(d*x + c)^4/d^6
```

```
- 5*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4)*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 + (b^5*c^5*d^4/(d*x + c) -
5*a*b^4*c^4*d^5/(d*x + c) + 10*a^2*b^3*c^3*d^6/(d*x + c) - 10*a^3*b^2*c^2*d
^7/(d*x + c) + 5*a^4*b*c*d^8/(d*x + c) - a^5*d^9/(d*x + c))/d^10
```

**maple [B]** time = 0.01, size = 326, normalized size = 2.51

$$\frac{b^5x^4}{4d^2}+\frac{5a\,b^4x^3}{3d^2}-\frac{2b^5c\,x^3}{3d^3}+\frac{5a^2b^3x^2}{d^2}-\frac{5a\,b^4c\,x^2}{d^3}+\frac{3b^5c^2x^2}{2d^4}-\frac{a^5}{(dx+c)\,d}+\frac{5a^4bc}{(dx+c)\,d^2}+\frac{5a^4b\ln(dx+c)}{d^2}-\frac{10a^3b^2c^2}{(dx+c)\,d^3}-\frac{20a^3b^2c\ln(dx+c)}{d^3}+\frac{10a^3b^2x}{d^2}+\frac{10a^2b^3c^3}{(dx+c)\,d^4}+\frac{30a^2b^3c^2\ln(dx+c)}{d^4}-\frac{20a^2b^3cx}{d^3}-\frac{5a\,b^4c^4}{(dx+c)\,d^5}-\frac{20a\,b^4c^3\ln(dx+c)}{d^5}+\frac{15a\,b^4c^2x}{d^4}+\frac{b^5c^5}{(dx+c)\,d^6}+\frac{5b^5c^4\ln(dx+c)}{d^6}-\frac{4b^5c^3x}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^2,x)
```

```
[Out] 1/4*b^5/d^2*x^4+5/3*b^4/d^2*x^3*a-2/3*b^5/d^3*x^3*c+5*b^3/d^2*x^2*a^2-5*b^4
/d^3*x^2*a*c+3/2*b^5/d^4*x^2*c^2+10*b^2/d^2*a^3*x-20*b^3/d^3*a^2*c*x+15*b^4
/d^4*a*c^2*x-4*b^5/d^5*c^3*x-1/d/(d*x+c)*a^5+5/d^2/(d*x+c)*a^4*b*c-10/d^3/(
d*x+c)*a^3*b^2*c^2+10/d^4/(d*x+c)*a^2*b^3*c^3-5/d^5/(d*x+c)*a*b^4*c^4+1/d^6
/(d*x+c)*b^5*c^5+5*b/d^2*ln(d*x+c)*a^4-20*b^2/d^3*ln(d*x+c)*a^3*c+30*b^3/d^
4*ln(d*x+c)*a^2*c^2-20*b^4/d^5*ln(d*x+c)*a*c^3+5*b^5/d^6*ln(d*x+c)*c^4
```

**maxima [B]** time = 1.40, size = 264, normalized size = 2.03

$$\frac{b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5}{d^7x+cd^6}+\frac{3\,b^5d^3x^4-4\left(2\,b^5cd^2-5\,ab^4d^3\right)x^3+6\left(3\,b^5c^2d-10\,ab^4cd^2+10\,a^2b^3d^3\right)x^2-12\left(4\,b^5c^3-15\,ab^4c^2d+20\,a^2b^3cd^2-10\,a^3b^2d^3\right)x}{12\,d^5}+\frac{5\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)\log(dx+c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*
b*c*d^4 - a^5*d^5)/(d^7*x + c*d^6) + 1/12*(3*b^5*d^3*x^4 - 4*(2*b^5*c*d^2 -
 5*a*b^4*d^3)*x^3 + 6*(3*b^5*c^2*d - 10*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^2 -
 12*(4*b^5*c^3 - 15*a*b^4*c^2*d + 20*a^2*b^3*c*d^2 - 10*a^3*b^2*d^3)*x)/d^5
 + 5*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b
*d^4)*log(d*x + c)/d^6
```

**mupad [B]** time = 0.25, size = 327, normalized size = 2.52

$$x^3\left(\frac{5\,a\,b^4}{3\,d^2}-\frac{2\,b^5\,c}{3\,d^3}\right)+x\left(\frac{2\,c\left(\frac{2\,c\left(\frac{5\,a\,b^4}{d^2}-\frac{2\,b^5\,c}{d^3}\right)}{d}-\frac{10\,a^2\,b^3}{d^2}+\frac{b^5\,c^2}{d^4}\right)}{d}+\frac{10\,a^3\,b^2}{d^2}-\frac{c^2\left(\frac{5\,a\,b^4}{d^2}-\frac{2\,b^5\,c}{d^3}\right)}{d^2}\right)-x^2\left(\frac{c\left(\frac{5\,a\,b^4}{d^2}-\frac{2\,b^5\,c}{d^3}\right)}{d}-\frac{5\,a^2\,b^3}{d^2}+\frac{b^5\,c^2}{2\,d^4}\right)+\frac{\ln(c+d\,x)\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)}{d^6}-\frac{a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5}{d\left(x\,d^6+c\,d^5\right)}+\frac{b^5\,x^4}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^2,x)
```

```
[Out] x^3*((5*a*b^4)/(3*d^2) - (2*b^5*c)/(3*d^3)) + x*((2*c*((2*c*((5*a*b^4)/d^2
- (2*b^5*c)/d^3))/d - (10*a^2*b^3)/d^2 + (b^5*c^2)/d^4))/d + (10*a^3*b^2)/d
^2 - (c^2*((5*a*b^4)/d^2 - (2*b^5*c)/d^3))/d^2) - x^2*((c*((5*a*b^4)/d^2 -
```

```
(2*b^5*c)/d^3))/d - (5*a^2*b^3)/d^2 + (b^5*c^2)/(2*d^4)) + (log(c + d*x)*(5
*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^4*c
^3*d))/d^6 - (a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 +
 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)/(d*(c*d^5 + d^6*x)) + (b^5*x^4)/(4*d^2)
```

**sympy [A]** time = 0.89, size = 231, normalized size = 1.78

$$\frac{b^5x^4}{4d^2}+\frac{5b\,(ad-bc)^4\log{(c+dx)}}{d^6}+x^3\left(\frac{5ab^4}{3d^2}-\frac{2b^5c}{3d^3}\right)+x^2\left(\frac{5a^2b^3}{d^2}-\frac{5ab^4c}{d^3}+\frac{3b^5c^2}{2d^4}\right)+x\left(\frac{10a^3b^2}{d^2}-\frac{20a^2b^3c}{d^3}+\frac{15ab^4c^2}{d^4}-\frac{4b^5c^3}{d^5}\right)+\frac{-a^5d^5+5a^4bcd^4-10a^3b^2c^2d^3+10a^2b^3c^3d^2-5ab^4c^4d+b^5c^5}{cd^6+d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**2,x)
```

```
[Out] b**5*x**4/(4*d**2) + 5*b*(a*d - b*c)**4*log(c + d*x)/d**6 + x**3*(5*a*b**4/
(3*d**2) - 2*b**5*c/(3*d**3)) + x**2*(5*a**2*b**3/d**2 - 5*a*b**4*c/d**3 +
3*b**5*c**2/(2*d**4)) + x*(10*a**3*b**2/d**2 - 20*a**2*b**3*c/d**3 + 15*a*b
**4*c**2/d**4 - 4*b**5*c**3/d**5) + (-a**5*d**5 + 5*a**4*b*c*d**4 - 10*a**3
*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b**4*c**4*d + b**5*c**5)/(c*
d**6 + d**7*x)
```

## 3.1238 $\int \frac{(a+bx)^4}{(c+dx)^2} \, dx$

**Optimal**. Leaf size=104

$$-\frac{2b^3(c+dx)^2(bc-ad)}{d^5}+\frac{6b^2x(bc-ad)^2}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}+\frac{b^4(c+dx)^3}{3d^5}$$

**Rubi [A]** time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b^3(c+dx)^2(bc-ad)}{d^5}+\frac{6b^2x(bc-ad)^2}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}+\frac{b^4(c+dx)^3}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] (6*b^2*(b*c - a*d)^2*x)/d^4 - (b*c - a*d)^4/(d^5*(c + d*x)) - (2*b^3*(b*c -
 a*d)*(c + d*x)^2)/d^5 + (b^4*(c + d*x)^3)/(3*d^5) - (4*b*(b*c - a*d)^3*Log
[c + d*x])/d^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^2}\,dx &= \int \left(\frac{6b^2(bc-ad)^2}{d^4}+\frac{(-bc+ad)^4}{d^4(c+dx)^2}-\frac{4b(bc-ad)^3}{d^4(c+dx)}-\frac{4b^3(bc-ad)(c+dx)}{d^4}+\frac{b^4(c+dx)^2}{d^4}\right)dx\\ &= \frac{6b^2(bc-ad)^2x}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{2b^3(bc-ad)(c+dx)^2}{d^5}+\frac{b^4(c+dx)^3}{3d^5}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 165, normalized size = 1.59

$$\frac{-3a^4d^4+12a^3bcd^3+18a^2b^2d^2\left(-c^2+cdx+d^2x^2\right)+6ab^3d\left(2c^3-4c^2dx-3cd^2x^2+d^3x^3\right)-12b(c+dx)(bc-ad)^3\log(c+dx)+b^4\left(-3c^4+9c^3dx+6c^2d^2x^2-2cd^3x^3+d^4x^4\right)}{3d^5(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] (12*a^3*b*c*d^3 - 3*a^4*d^4 + 18*a^2*b^2*d^2*(-c^2 + c*d*x + d^2*x^2) + 6*a
*b^3*d*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + b^4*(-3*c^4 + 9*c^3*d*
x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) - 12*b*(b*c - a*d)^3*(c + d*x)*L
og[c + d*x])/(3*d^5*(c + d*x))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^2, x]
```

**fricas [B]** time = 0.90, size = 267, normalized size = 2.57

$$\frac{b^4d^4x^4-3\,b^4c^4+12\,ab^3c^3d-18\,a^2b^2c^2d^2+12\,a^3bcd^3-3\,a^4d^4-2\left(b^4cd^3-3\,ab^3d^4\right)x^3+6\left(b^4c^2d^2-3\,ab^3cd^3+3\,a^2b^2d^4\right)x^2+3\left(3\,b^4c^3d-8\,ab^3c^2d^2+6\,a^2b^2cd^3\right)x-12\left(b^4c^4-3\,ab^3c^3d+3\,a^2b^2c^2d^2-a^3bcd^3+\left(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4\right)x\right)\log\left(dx+c\right)}{3\left(d^6x+cd^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*d^4*x^4 - 3*b^4*c^4 + 12*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 12*a^3
*b*c*d^3 - 3*a^4*d^4 - 2*(b^4*c*d^3 - 3*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 3
*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2 + 3*(3*b^4*c^3*d - 8*a*b^3*c^2*d^2 + 6*a^
2*b^2*c*d^3)*x - 12*(b^4*c^4 - 3*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 - a^3*b*c*
d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)*log(d*
x + c))/(d^6*x + c*d^5)
```

**giac [B]** time = 1.27, size = 245, normalized size = 2.36

$$\frac{\left(b^4-\frac{6\left(b^4cd-ab^3d^2\right)}{(dx+c)d}+\frac{18\left(b^4c^2d^2-2\,ab^3cd^3+a^2b^2d^4\right)}{(dx+c)^2d^2}\right)(dx+c)^3}{3\,d^5}+\frac{4\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5}-\frac{\frac{b^4c^4d^3}{dx+c}-\frac{4\,ab^3c^3d^4}{dx+c}+\frac{6\,a^2b^2c^2d^5}{dx+c}-\frac{4\,a^3bcd^6}{dx+c}+\frac{a^4d^7}{dx+c}}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^4 - 6*(b^4*c*d - a*b^3*d^2)/((d*x + c)*d) + 18*(b^4*c^2*d^2 - 2*a*b^
3*c*d^3 + a^2*b^2*d^4)/((d*x + c)^2*d^2))*(d*x + c)^3/d^5 + 4*(b^4*c^3 - 3*
a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*log(abs(d*x + c)/((d*x + c)^2*ab
s(d)))/d^5 - (b^4*c^4*d^3/(d*x + c) - 4*a*b^3*c^3*d^4/(d*x + c) + 6*a^2*b^2
*c^2*d^5/(d*x + c) - 4*a^3*b*c*d^6/(d*x + c) + a^4*d^7/(d*x + c))/d^8
```

**maple [B]** time = 0.01, size = 230, normalized size = 2.21

$$\frac{b^4x^3}{3d^2}+\frac{2a\,b^3x^2}{d^2}-\frac{b^4c\,x^2}{d^3}-\frac{a^4}{(dx+c)\,d}+\frac{4a^3bc}{(dx+c)\,d^2}+\frac{4a^3b\ln(dx+c)}{d^2}-\frac{6a^2b^2c^2}{(dx+c)\,d^3}-\frac{12a^2b^2c\ln(dx+c)}{d^3}+\frac{6a^2b^2x}{d^2}+\frac{4a\,b^3c^3}{(dx+c)\,d^4}+\frac{12a\,b^3c^2\ln(dx+c)}{d^4}-\frac{8a\,b^3cx}{d^3}-\frac{b^4c^4}{(dx+c)\,d^5}-\frac{4b^4c^3\ln(dx+c)}{d^5}+\frac{3b^4c^2x}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^2,x)
```

```
[Out] 1/3*b^4/d^2*x^3+2*b^3/d^2*x^2*a-b^4/d^3*x^2*c+6*b^2/d^2*a^2*x-8*b^3/d^3*a*c
*x+3*b^4/d^4*c^2*x-1/d/(d*x+c)*a^4+4/d^2/(d*x+c)*a^3*b*c-6/d^3/(d*x+c)*a^2*
b^2*c^2+4/d^4/(d*x+c)*a*b^3*c^3-1/d^5/(d*x+c)*b^4*c^4+4*b/d^2*ln(d*x+c)*a^3
-12*b^2/d^3*ln(d*x+c)*a^2*c+12*b^3/d^4*ln(d*x+c)*a*c^2-4*b^4/d^5*ln(d*x+c)*
c^3
```

**maxima [A]** time = 1.36, size = 183, normalized size = 1.76

$$-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{d^6x+cd^5}+\frac{b^4d^2x^3-3\left(b^4cd-2\,ab^3d^2\right)x^2+3\left(3\,b^4c^2-8\,ab^3cd+6\,a^2b^2d^2\right)x}{3\,d^4}-\frac{4\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)\log\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(d
^6*x + c*d^5) + 1/3*(b^4*d^2*x^3 - 3*(b^4*c*d - 2*a*b^3*d^2)*x^2 + 3*(3*b^4
*c^2 - 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x)/d^4 - 4*(b^4*c^3 - 3*a*b^3*c^2*d + 3
*a^2*b^2*c*d^2 - a^3*b*d^3)*log(d*x + c)/d^5
```

**mupad [B]** time = 0.07, size = 203, normalized size = 1.95

$$x^2\left(\frac{2\,a\,b^3}{d^2}-\frac{b^4\,c}{d^3}\right)-x\left(\frac{2\,c\left(\frac{4\,a\,b^3}{d^2}-\frac{2\,b^4\,c}{d^3}\right)}{d}-\frac{6\,a^2\,b^2}{d^2}+\frac{b^4\,c^2}{d^4}\right)+\frac{b^4\,x^3}{3\,d^2}-\frac{\ln\left(c+d\,x\right)\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)}{d^5}-\frac{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}{d\left(x\,d^5+c\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^2,x)
```

```
[Out] x^2*((2*a*b^3)/d^2 - (b^4*c)/d^3) - x*((2*c*((4*a*b^3)/d^2 - (2*b^4*c)/d^3)
)/d - (6*a^2*b^2)/d^2 + (b^4*c^2)/d^4) + (b^4*x^3)/(3*d^2) - (log(c + d*x)*
(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/d^5 - (a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(d*(c*d^4
 + d^5*x))
```

**sympy [A]** time = 0.68, size = 155, normalized size = 1.49

$$\frac{b^4x^3}{3d^2}+\frac{4b\left(ad-bc\right)^3\log\left(c+dx\right)}{d^5}+x^2\left(\frac{2ab^3}{d^2}-\frac{b^4c}{d^3}\right)+x\left(\frac{6a^2b^2}{d^2}-\frac{8ab^3c}{d^3}+\frac{3b^4c^2}{d^4}\right)+\frac{-a^4d^4+4a^3bcd^3-6a^2b^2c^2d^2+4ab^3c^3d-b^4c^4}{cd^5+d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**2,x)
```

```
[Out] b**4*x**3/(3*d**2) + 4*b*(a*d - b*c)**3*log(c + d*x)/d**5 + x**2*(2*a*b**3/
d**2 - b**4*c/d**3) + x*(6*a**2*b**2/d**2 - 8*a*b**3*c/d**3 + 3*b**4*c**2/d
**4) + (-a**4*d**4 + 4*a**3*b*c*d**3 - 6*a**2*b**2*c**2*d**2 + 4*a*b**3*c**
3*d - b**4*c**4)/(c*d**5 + d**6*x)
```

## 3.1239 $\int \frac{(a+bx)^3}{(c+dx)^2} dx$

**Optimal**. Leaf size=75

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}+\frac{b^3x^2}{2d^2}$$

**Rubi** [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}+\frac{b^3x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b*c - a*d)^3/(d^4*(c
+ d*x)) + (3*b*(b*c - a*d)^2*Log[c + d*x])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^2}\,dx &= \int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{b^3x}{d^2}+\frac{(-bc+ad)^3}{d^3(c+dx)^2}+\frac{3b(bc-ad)^2}{d^3(c+dx)}\right)dx\\ &= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^2}{2d^2}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 114, normalized size = 1.52

$$\frac{3\left(a^2bd^2-2ab^2cd+b^3c^2\right)\log(c+dx)}{d^4}+\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{d^4(c+dx)}-\frac{b^2x(2bc-3ad)}{d^3}+\frac{b^3x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^4*(c + d*x)) + (3*(b^3*c^2 - 2*a*b^2*c*d +
 a^2*b*d^2)*Log[c + d*x])/d^4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^2, x]
```

**fricas [B]** time = 1.82, size = 172, normalized size = 2.29

$$\frac{b^3d^3x^3 + 2\,b^3c^3 - 6\,ab^2c^2d + 6\,a^2bcd^2 - 2\,a^3d^3 - 3\left(b^3cd^2 - 2\,ab^2d^3\right)x^2 - 2\left(2\,b^3c^2d - 3\,ab^2cd^2\right)x + 6\left(b^3c^3 - 2\,ab^2c^2d + a^2bcd^2 + \left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x\right)\log\left(dx+c\right)}{2\left(d^5x + cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 -
3*(b^3*c*d^2 - 2*a*b^2*d^3)*x^2 - 2*(2*b^3*c^2*d - 3*a*b^2*c*d^2)*x + 6*(b^
3*c^3 - 2*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(d*x + c))/(d^5*x + c*d^4)
```

**giac [B]** time = 1.26, size = 166, normalized size = 2.21

$$\frac{\left(b^3 - \frac{6\left(b^3cd - ab^2d^2\right)}{(dx+c)d}\right)(dx+c)^2}{2\,d^4} - \frac{3\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^4} + \frac{\frac{b^3c^3d^2}{dx+c} - \frac{3\,ab^2c^2d^3}{dx+c} + \frac{3\,a^2bcd^4}{dx+c} - \frac{a^3d^5}{dx+c}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 6*(b^3*c*d - a*b^2*d^2)/((d*x + c)*d))*(d*x + c)^2/d^4 - 3*(b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 +
 (b^3*c^3*d^2/(d*x + c) - 3*a*b^2*c^2*d^3/(d*x + c) + 3*a^2*b*c*d^4/(d*x +
c) - a^3*d^5/(d*x + c))/d^6
```

**maple [B]** time = 0.01, size = 149, normalized size = 1.99

$$\frac{b^3x^2}{2d^2} - \frac{a^3}{(dx+c)\,d} + \frac{3a^2bc}{(dx+c)\,d^2} + \frac{3a^2b\ln(dx+c)}{d^2} - \frac{3a\,b^2c^2}{(dx+c)\,d^3} - \frac{6a\,b^2c\ln(dx+c)}{d^3} + \frac{3a\,b^2x}{d^2} + \frac{b^3c^3}{(dx+c)\,d^4} + \frac{3b^3c^2\ln(dx+c)}{d^4} - \frac{2b^3cx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/2*b^3*x^2/d^2+3*b^2/d^2*a*x-2*b^3/d^3*x*c-1/d/(d*x+c)*a^3+3/d^2/(d*x+c)*a
^2*b*c-3/d^3/(d*x+c)*a*b^2*c^2+1/d^4/(d*x+c)*b^3*c^3+3*b/d^2*ln(d*x+c)*a^2-
6*b^2/d^3*ln(d*x+c)*a*c+3*b^3/d^4*ln(d*x+c)*c^2
```

**maxima [A]** time = 1.36, size = 117, normalized size = 1.56

$$\frac{b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3}{d^5x + cd^4} + \frac{b^3dx^2 - 2\left(2\,b^3c - 3\,ab^2d\right)x}{2\,d^3} + \frac{3\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)\log\left(dx + c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^5*x + c*d^4) + 1/2*(
b^3*d*x^2 - 2*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 3*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*log(d*x + c)/d^4
```

**mupad [B]** time = 0.08, size = 123, normalized size = 1.64

$$x\left(\frac{3\,a\,b^2}{d^2} - \frac{2\,b^3\,c}{d^3}\right) + \frac{\ln\left(c + d\,x\right)\,\left(3\,a^2\,b\,d^2 - 6\,a\,b^2\,c\,d + 3\,b^3\,c^2\right)}{d^4} - \frac{a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3}{d\,\left(x\,d^4 + c\,d^3\right)} + \frac{b^3\,x^2}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^2,x)
```

```
[Out] x*((3*a*b^2)/d^2 - (2*b^3*c)/d^3) + (log(c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2
- 6*a*b^2*c*d))/d^4 - (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)/(
d*(c*d^3 + d^4*x)) + (b^3*x^2)/(2*d^2)
```

**sympy [A]** time = 0.51, size = 102, normalized size = 1.36

$$\frac{b^3x^2}{2d^2} + \frac{3b\left(ad - bc\right)^2\log\left(c + dx\right)}{d^4} + x\left(\frac{3ab^2}{d^2} - \frac{2b^3c}{d^3}\right) + \frac{-a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3}{cd^4 + d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] b**3*x**2/(2*d**2) + 3*b*(a*d - b*c)**2*log(c + d*x)/d**4 + x*(3*a*b**2/d**
2 - 2*b**3*c/d**3) + (-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + b**3
*c**3)/(c*d**4 + d**5*x)
```

# 3.1240 $\int \frac{(a+bx)^2}{(c+dx)^2}\,dx$

**Optimal**. Leaf size=51

$$-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}+\frac{b^2x}{d^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^2,x]
```

```
[Out] (b^2*x)/d^2 - (b*c - a*d)^2/(d^3*(c + d*x)) - (2*b*(b*c - a*d)*Log[c + d*x]
)/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^2}\,dx &= \int \left(\frac{b^2}{d^2}+\frac{(-bc+ad)^2}{d^2(c+dx)^2}-\frac{2b(bc-ad)}{d^2(c+dx)}\right)dx\\ &= \frac{b^2x}{d^2}-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 47, normalized size = 0.92

$$\frac{-\frac{(bc-ad)^2}{c+dx}+2b(ad-bc)\log(c+dx)+b^2dx}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^2,x]
```

```
[Out] (b^2*d*x - (b*c - a*d)^2/(c + d*x) + 2*b*(-(b*c) + a*d)*Log[c + d*x])/d^3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^2, x]
```

**fricas [A]** time = 1.16, size = 92, normalized size = 1.80

$$\frac{b^2d^2x^2 + b^2cdx - b^2c^2 + 2\,abcd - a^2d^2 - 2\left(b^2c^2 - abcd + \left(b^2cd - abd^2\right)x\right)\log\left(dx + c\right)}{d^4x + cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + b^2*c*d*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 - 2*(b^2*c^2 - a*b
*c*d + (b^2*c*d - a*b*d^2)*x)*log(d*x + c))/(d^4*x + c*d^3)
```

**giac [A]** time = 1.25, size = 98, normalized size = 1.92

$$\frac{(dx+c)b^2}{d^3} + \frac{2\left(b^2c - abd\right)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^3} - \frac{\frac{b^2c^2d}{dx+c} - \frac{2\,abcd^2}{dx+c} + \frac{a^2d^3}{dx+c}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (d*x + c)*b^2/d^3 + 2*(b^2*c - a*b*d)*log(abs(d*x + c)/((d*x + c)^2*abs(d))
)/d^3 - (b^2*c^2*d/(d*x + c) - 2*a*b*c*d^2/(d*x + c) + a^2*d^3/(d*x + c))/d
^4
```

**maple [A]** time = 0.01, size = 86, normalized size = 1.69

$$-\frac{a^2}{(dx+c)\,d} + \frac{2abc}{(dx+c)\,d^2} + \frac{2ab\ln\left(dx+c\right)}{d^2} - \frac{b^2c^2}{(dx+c)\,d^3} - \frac{2b^2c\ln\left(dx+c\right)}{d^3} + \frac{b^2x}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] b^2*x/d^2-1/d/(d*x+c)*a^2+2/d^2/(d*x+c)*a*b*c-1/d^3/(d*x+c)*b^2*c^2+2*b/d^2
*ln(d*x+c)*a-2*b^2/d^3*ln(d*x+c)*c
```

**maxima [A]** time = 1.35, size = 67, normalized size = 1.31

$$\frac{b^2x}{d^2} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{d^4x + cd^3} - \frac{2\left(b^2c - abd\right)\log\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] b^2*x/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x + c*d^3) - 2*(b^2*c - a*
b*d)*log(d*x + c)/d^3
```

**mupad [B]** time = 0.24, size = 71, normalized size = 1.39

$$\frac{b^2\,x}{d^2} - \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{d\,\left(x\,d^3 + c\,d^2\right)} - \frac{\ln\left(c + d\,x\right)\,\left(2\,b^2\,c - 2\,a\,b\,d\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] (b^2*x)/d^2 - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(d*(c*d^2 + d^3*x)) - (log(c
+ d*x)*(2*b^2*c - 2*a*b*d))/d^3
```

**sympy [A]** time = 0.34, size = 60, normalized size = 1.18

$$\frac{b^2x}{d^2} + \frac{2b\left(ad - bc\right)\log\left(c + dx\right)}{d^3} + \frac{-a^2d^2 + 2abcd - b^2c^2}{cd^3 + d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] b**2*x/d**2 + 2*b*(a*d - b*c)*log(c + d*x)/d**3 + (-a**2*d**2 + 2*a*b*c*d -
 b**2*c**2)/(c*d**3 + d**4*x)
```

## 3.1241 $\int \frac{a+bx}{(c+dx)^2} dx$

**Optimal**. Leaf size=31

$$\frac{bc - ad}{d^2(c + dx)} + \frac{b\log(c + dx)}{d^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{bc - ad}{d^2(c + dx)} + \frac{b\log(c + dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^2,x]
```

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a + bx}{(c + dx)^2}\,dx &= \int \left(\frac{-bc + ad}{d(c + dx)^2} + \frac{b}{d(c + dx)}\right)dx\\ &= \frac{bc - ad}{d^2(c + dx)} + \frac{b\log(c + dx)}{d^2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 1.00

$$\frac{bc - ad}{d^2(c + dx)} + \frac{b\log(c + dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^2,x]
```

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(c + d*x)^2, x]
```

**fricas [A]** time = 1.31, size = 37, normalized size = 1.19

$$\frac{bc-ad+(bdx+bc)\log(dx+c)}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b*c - a*d + (b*d*x + b*c)*log(d*x + c))/(d^3*x + c*d^2)
```

**giac [A]** time = 1.26, size = 57, normalized size = 1.84

$$-\frac{b\left(\frac{\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d}-\frac{c}{(dx+c)d}\right)}{d}-\frac{a}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -b*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d))/d - a/((d*x
 + c)*d)
```

**maple [A]** time = 0.01, size = 39, normalized size = 1.26

$$-\frac{a}{(dx+c)\,d}+\frac{bc}{(dx+c)\,d^2}+\frac{b\ln(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^2,x)
```

```
[Out] -1/d/(d*x+c)*a+1/d^2/(d*x+c)*b*c+b*ln(d*x+c)/d^2
```

**maxima** [A] time = 1.34, size = 34, normalized size = 1.10

$$\frac{bc-ad}{d^3x+cd^2}+\frac{b\log\left(dx+c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b*c - a*d)/(d^3*x + c*d^2) + b*log(d*x + c)/d^2
```

**mupad** [B] time = 0.04, size = 32, normalized size = 1.03

$$\frac{b\,\ln\left(c+d\,x\right)}{d^2}-\frac{a\,d-b\,c}{d^2\,\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^2,x)
```

```
[Out] (b*log(c + d*x))/d^2 - (a*d - b*c)/(d^2*(c + d*x))
```

**sympy** [A] time = 0.19, size = 27, normalized size = 0.87

$$\frac{b\log\left(c+dx\right)}{d^2}+\frac{-ad+bc}{cd^2+d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**2,x)
```

```
[Out] b*log(c + d*x)/d**2 + (-a*d + b*c)/(c*d**2 + d**3*x)
```

## 3.1242 $\int \frac{1}{(c+dx)^2} \, dx$

**Optimal**. Leaf size=12

$$-\frac{1}{d(c+dx)}$$

**Rubi** [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-2),x]
```

```
[Out] -(1/(d*(c + d*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2} \, dx = -\frac{1}{d(c+dx)}$$

**Mathematica** [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-2),x]
```

```
[Out] -(1/(d*(c + d*x)))
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^(-2), x]
```

**fricas [A]** time = 1.15, size = 13, normalized size = 1.08

$$-\frac{1}{d^2x + cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/(d^2*x + c*d)
```

**giac [A]** time = 1.23, size = 12, normalized size = 1.00

$$-\frac{1}{(dx + c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/((d*x + c)*d)
```

**maple [A]** time = 0.00, size = 13, normalized size = 1.08

$$-\frac{1}{(dx + c)\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2,x)
```

```
[Out] -1/d/(d*x+c)
```

**maxima [A]** time = 1.28, size = 12, normalized size = 1.00

$$-\frac{1}{(dx + c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/((d*x + c)*d)
```

**mupad** [B] time = 0.19, size = 12, normalized size = 1.00

$$-\frac{1}{d\,\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^2,x)
[Out] -1/(d*(c + d*x))
```

**sympy** [A] time = 0.13, size = 10, normalized size = 0.83

$$-\frac{1}{cd+d^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2,x)
[Out] -1/(c*d + d**2*x)
```

## 3.1243 $\int \frac{1}{(a+bx)(c+dx)^2}\,dx$

**Optimal**. Leaf size=56

$$\frac{1}{(c+dx)(bc-ad)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{1}{(c+dx)(bc-ad)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^2),x]
```

```
[Out] 1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[c + d*x
])/(b*c - a*d)^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^2}\,dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)(c+dx)^2}-\frac{bd}{(bc-ad)^2(c+dx)}\right)dx\\ &= \frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 53, normalized size = 0.95

$$\frac{b(c+dx)\log(a+bx)-ad-b(c+dx)\log(c+dx)+bc}{(c+dx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^2),x]
```

```
[Out] (b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x])/((b*c - a
*d)^2*(c + d*x))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^2), x]
```

**fricas [A]** time = 1.21, size = 92, normalized size = 1.64

$$\frac{bc-ad+(bdx+bc)\log(bx+a)-(bdx+bc)\log(dx+c)}{b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b*c - a*d + (b*d*x + b*c)*log(b*x + a) - (b*d*x + b*c)*log(d*x + c))/(b^2*
c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)
```

**giac [A]** time = 1.34, size = 77, normalized size = 1.38

$$\frac{bd\log\left(\left|b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right|\right)}{b^2c^2d-2\,abcd^2+a^2d^3}+\frac{d}{\left(bcd-ad^2\right)(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] b*d*log(abs(b - b*c/(d*x + c) + a*d/(d*x + c)))/(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3) + d/((b*c*d - a*d^2)*(d*x + c))
```

**maple [A]** time = 0.01, size = 58, normalized size = 1.04

$$\frac{b\ln(bx+a)}{(ad-bc)^2}-\frac{b\ln(dx+c)}{(ad-bc)^2}-\frac{1}{(ad-bc)(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^2,x)
```

```
[Out] -1/(a*d-b*c)/(d*x+c)-b/(a*d-b*c)^2*ln(d*x+c)+b/(a*d-b*c)^2*ln(b*x+a)
```

**maxima [A]** time = 1.32, size = 90, normalized size = 1.61

$$\frac{b\log\left(bx+a\right)}{b^2c^2-2\,abcd+a^2d^2}-\frac{b\log\left(dx+c\right)}{b^2c^2-2\,abcd+a^2d^2}+\frac{1}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - b*log(d*x + c)/(b^2*c^2 -
2*a*b*c*d + a^2*d^2) + 1/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

**mupad [B]** time = 0.29, size = 47, normalized size = 0.84

$$-\frac{1}{\left(a\,d-b\,c\right)\,\left(c+d\,x\right)}-\frac{b\,\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{{\left(a\,d-b\,c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^2),x)
```

```
[Out] - 1/((a*d - b*c)*(c + d*x)) - (b*log((c + d*x)/(a + b*x)))/(a*d - b*c)^2
```

**sympy [B]** time = 0.68, size = 233, normalized size = 4.16

$$-\frac{b\log\left(x+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{\left(ad-bc\right)^2}+\frac{b\log\left(x+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{\left(ad-bc\right)^2}-\frac{1}{acd-bc^2+x\left(ad^2-bcd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**2,x)
```

```
[Out] -b*log(x + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)**2
 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b**2
*c)/(2*b**2*d))/(a*d - b*c)**2 + b*log(x + (a**3*b*d**3/(a*d - b*c)**2 - 3*
a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d -
b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(a*d - b*c)**2 - 1/(a*c*d -
b*c**2 + x*(a*d**2 - b*c*d))
```

## 3.1244 $\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx$

**Optimal**. Leaf size=81

$$-\frac{b}{(a+bx)(bc-ad)^2}-\frac{d}{(c+dx)(bc-ad)^2}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

**Rubi** [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{b}{(a+bx)(bc-ad)^2}-\frac{d}{(c+dx)(bc-ad)^2}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] -(b/((b*c - a*d)^2*(a + b*x))) - d/((b*c - a*d)^2*(c + d*x)) - (2*b*d*Log[a
 + b*x])/(b*c - a*d)^3 + (2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx=\int \left(\frac{b^2}{(bc-ad)^2(a+bx)^2}-\frac{2b^2d}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^2}+\frac{2bd^2}{(bc-ad)^3(c+\right.$$
$$=-\frac{b}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)^2(c+dx)}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

**Mathematica** [A] time = 0.07, size = 66, normalized size = 0.81

$$\frac{\frac{b(ad-bc)}{a+bx}+\frac{d(ad-bc)}{c+dx}-2bd\log(a+bx)+2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] ((b*(-(b*c) + a*d))/(a + b*x) + (d*(-(b*c) + a*d))/(c + d*x) - 2*b*d*Log[a
+ b*x] + 2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^2), x]
```

**fricas [B]** time = 1.24, size = 241, normalized size = 2.98

$$-\frac{b^2c^2 - a^2d^2 + 2\left(b^2cd - abd^2\right)x + 2\left(b^2d^2x^2 + abcd + \left(b^2cd + abd^2\right)x\right)\log\left(bx+a\right) - 2\left(b^2d^2x^2 + abcd + \left(b^2cd + abd^2\right)x\right)\log\left(dx+c\right)}{ab^3c^4 - 3\,a^2b^2c^3d + 3\,a^3bc^2d^2 - a^4cd^3 + \left(b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4\right)x^2 + \left(b^4c^4 - 2\,ab^3c^3d + 2\,a^3bcd^3 - a^4d^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2*x^2 + a*b*c*d +
(b^2*c*d + a*b*d^2)*x)*log(b*x + a) - 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x)*log(d*x + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 -
 a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^
2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x)
```

**giac [A]** time = 1.21, size = 153, normalized size = 1.89

$$\frac{2\,b^2d\log\left(\left|\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right|\right)}{b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3} - \frac{b^3}{\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)(bx+a)} + \frac{bd^2}{(bc-ad)^3\left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 2*b^2*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^4*c^3 - 3*a*b^3*c^2*
d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^
2)*(b*x + a)) + b*d^2/((b*c - a*d)^3*(b*c/(b*x + a) - a*d/(b*x + a) + d))
```

**maple [A]** time = 0.01, size = 82, normalized size = 1.01

$$\frac{2bd\ln\left(bx+a\right)}{\left(ad-bc\right)^3} - \frac{2bd\ln\left(dx+c\right)}{\left(ad-bc\right)^3} - \frac{b}{\left(ad-bc\right)^2\left(bx+a\right)} - \frac{d}{\left(ad-bc\right)^2\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] -d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b*c)^2/(b*x+a)+2*
d/(a*d-b*c)^3*b*ln(b*x+a)
```

**maxima [B]** time = 1.43, size = 208, normalized size = 2.57

$$-\frac{2\,bd\log\left(bx+a\right)}{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}+\frac{2\,bd\log\left(dx+c\right)}{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}-\frac{2\,bdx+bc+ad}{ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}+\left(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{2}+\left(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -2*b*d*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 2
*b*d*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - (2*
b*d*x + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*
a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x)
```

**mupad [B]** time = 0.33, size = 74, normalized size = 0.91

$$\frac{1}{\left(a\,d-b\,c\right)\,\left(a+b\,x\right)\,\left(c+d\,x\right)}-\frac{2\,d}{{\left(a\,d-b\,c\right)}^2\,\left(c+d\,x\right)}-\frac{2\,b\,d\,\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] 1/((a*d - b*c)*(a + b*x)*(c + d*x)) - (2*d)/((a*d - b*c)^2*(c + d*x)) - (2*
b*d*log((c + d*x)/(a + b*x)))/(a*d - b*c)^3
```

**sympy [B]** time = 1.11, size = 406, normalized size = 5.01

$$-\frac{2bd\log\left(x+\frac{-\frac{2a^{4}bd^{5}}{(ad-bc)^{3}}+\frac{8a^{3}b^{2}cd^{4}}{(ad-bc)^{3}}-\frac{12a^{2}b^{3}c^{2}d^{3}}{(ad-bc)^{3}}+\frac{8ab^{4}c^{3}d^{2}}{(ad-bc)^{3}}+2abd^{2}-\frac{2b^{5}c^{4}d}{(ad-bc)^{3}}+2b^{2}cd}{4b^{2}d^{2}}\right)}{(ad-bc)^{3}}+\frac{2bd\log\left(x+\frac{\frac{2a^{4}bd^{5}}{(ad-bc)^{3}}-\frac{8a^{3}b^{2}cd^{4}}{(ad-bc)^{3}}+\frac{12a^{2}b^{3}c^{2}d^{3}}{(ad-bc)^{3}}-\frac{8ab^{4}c^{3}d^{2}}{(ad-bc)^{3}}+2abd^{2}+\frac{2b^{5}c^{4}d}{(ad-bc)^{3}}+2b^{2}cd}{4b^{2}d^{2}}\right)}{(ad-bc)^{3}}+\frac{-ad-bc-2bdx}{a^{3}cd^{2}-2a^{2}bc^{2}d+ab^{2}c^{3}+x^{2}\left(a^{2}bd^{3}-2ab^{2}cd^{2}+b^{3}c^{2}d\right)+x\left(a^{3}d^{3}-a^{2}bcd^{2}-ab^{2}c^{2}d+b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] -2*b*d*log(x + (-2*a**4*b*d**5/(a*d - b*c)**3 + 8*a**3*b**2*c*d**4/(a*d - b
*c)**3 - 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 8*a*b**4*c**3*d**2/(a*d -
b*c)**3 + 2*a*b*d**2 - 2*b**5*c**4*d/(a*d - b*c)**3 + 2*b**2*c*d)/(4*b**2*d
**2))/(a*d - b*c)**3 + 2*b*d*log(x + (2*a**4*b*d**5/(a*d - b*c)**3 - 8*a**3
```

```
*b**2*c*d**4/(a*d - b*c)**3 + 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 8*a*b
**4*c**3*d**2/(a*d - b*c)**3 + 2*a*b*d**2 + 2*b**5*c**4*d/(a*d - b*c)**3 +
2*b**2*c*d)/(4*b**2*d**2))/(a*d - b*c)**3 + (-a*d - b*c - 2*b*d*x)/(a**3*c*
d**2 - 2*a**2*b*c**2*d + a*b**2*c**3 + x**2*(a**2*b*d**3 - 2*a*b**2*c*d**2
+ b**3*c**2*d) + x*(a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d + b**3*c**3))
```

## 3.1245 $\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx$

**Optimal**. Leaf size=109

$$\frac{d^2}{(c+dx)(bc-ad)^3}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}+\frac{2bd}{(a+bx)(bc-ad)^3}-\frac{b}{2(a+bx)^2(bc-ad)^2}$$

**Rubi** [A] time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{d^2}{(c+dx)(bc-ad)^3}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}+\frac{2bd}{(a+bx)(bc-ad)^3}-\frac{b}{2(a+bx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] -b/(2*(b*c - a*d)^2*(a + b*x)^2) + (2*b*d)/((b*c - a*d)^3*(a + b*x)) + d^2/
((b*c - a*d)^3*(c + d*x)) + (3*b*d^2*Log[a + b*x])/(b*c - a*d)^4 - (3*b*d^2
*Log[c + d*x])/(b*c - a*d)^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx = \int \left(\frac{b^2}{(bc-ad)^2(a+bx)^3}-\frac{2b^2d}{(bc-ad)^3(a+bx)^2}+\frac{3b^2d^2}{(bc-ad)^4(a+bx)}-\frac{d^3}{(bc-ad)^3(c+\right.$$
$$= -\frac{b}{2(bc-ad)^2(a+bx)^2}+\frac{2bd}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^3(c+dx)}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}$$

**Mathematica** [A] time = 0.07, size = 98, normalized size = 0.90

$$\frac{\frac{2d^2(bc-ad)}{c+dx}+\frac{4bd(bc-ad)}{a+bx}-\frac{b(bc-ad)^2}{(a+bx)^2}+6bd^2\log(a+bx)-6bd^2\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] (-((b*(b*c - a*d)^2)/(a + b*x)^2) + (4*b*d*(b*c - a*d))/(a + b*x) + (2*d^2*
(b*c - a*d))/(c + d*x) + 6*b*d^2*Log[a + b*x] - 6*b*d^2*Log[c + d*x])/(2*(b
*c - a*d)^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^2), x]
```

**fricas [B]** time = 1.23, size = 494, normalized size = 4.53

$$-\frac{b^3c^3 - 6\,ab^2c^2d + 3\,a^2bcd^2 + 2\,a^3d^3 - 6\left(b^3cd^2 - ab^2d^3\right)x^2 - 3\left(b^3c^2d + 2\,ab^2cd^2 - 3\,a^2bd^3\right)x - 6\left(b^3d^3x^3 + a^2bcd^2 + \left(b^3cd^2 + 2\,ab^2d^3\right)x^2 + \left(2\,ab^2cd^2 + a^2bd^3\right)x\right)\log\left(bx+a\right) + 6\left(b^3d^3x^3 + a^2bcd^2 + \left(b^3cd^2 + 2\,ab^2d^3\right)x^2 + \left(2\,ab^2cd^2 + a^2bd^3\right)x\right)\log\left(dx+c\right)}{2\left(a^2b^4c^5 - 4\,a^3b^3c^4d + 6\,a^4b^2c^3d^2 - 4\,a^5bc^2d^3 + a^6cd^4 + \left(b^6c^4d - 4\,ab^5c^3d^2 + 6\,a^2b^4c^2d^3 - 4\,a^3b^3cd^4 + a^4b^2d^5\right)x^3 + \left(b^6c^5 - 2\,ab^5c^4d - 2\,a^2b^4c^3d^2 + 8\,a^3b^3c^2d^3 - 7\,a^4b^2cd^4 + 2\,a^5bd^5\right)x^2 + \left(2\,ab^5c^5 - 7\,a^2b^4c^4d + 8\,a^3b^3c^3d^2 - 2\,a^4b^2c^2d^3 - 2\,a^5bcd^4 + a^6d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*c^3 - 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3 - 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^3
*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b
*d^3)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2
*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c))/(a^2*b^4*c^5 - 4*a
^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4 + (b^6*c^4*d
 - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4*b^2*d^5)*x^3
 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*a^3*b^3*c^2*d^3 - 7*a^4
*b^2*c*d^4 + 2*a^5*b*d^5)*x^2 + (2*a*b^5*c^5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*
c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^4 + a^6*d^5)*x)
```

**giac [B]** time = 1.35, size = 216, normalized size = 1.98

$$\frac{3\,bd^3\log\left(\left|b - \frac{bc}{dx+c} + \frac{ad}{dx+c}\right|\right)}{b^4c^4d - 4\,ab^3c^3d^2 + 6\,a^2b^2c^2d^3 - 4\,a^3bcd^4 + a^4d^5} + \frac{d^5}{\left(b^3c^3d^3 - 3\,ab^2c^2d^4 + 3\,a^2bcd^5 - a^3d^6\right)(dx+c)} + \frac{5\,b^3d^2 - \frac{6\left(b^3cd^3 - ab^2d^4\right)}{(dx+c)d}}{2\,(bc-ad)^4\left(b - \frac{bc}{dx+c} + \frac{ad}{dx+c}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

[Out] 3*b*d^3*log(abs(b - b*c/(d*x + c) + a*d/(d*x + c)))/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) + d^5/((b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*(d*x + c)) + 1/2*(5*b^3*d^2 - 6*(b^3*c*d^3 - a*b^2*d^4)/((d*x + c)*d))/((b*c - a*d)^4*(b - b*c/(d*x + c) + a*d/(d*x + c))^2)

**maple [A]** time = 0.01, size = 109, normalized size = 1.00

$$\frac{3b\,d^{2} \ln \left(bx+a\right)}{\left(ad-bc\right)^{4}}-\frac{3b\,d^{2} \ln \left(dx+c\right)}{\left(ad-bc\right)^{4}}-\frac{2bd}{\left(ad-bc\right)^{3} \left(bx+a\right)}-\frac{d^{2}}{\left(ad-bc\right)^{3} \left(dx+c\right)}-\frac{b}{2 \left(ad-bc\right)^{2} \left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^3/(d*x+c)^2,x)

[Out] -d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/(a*d-b*c)^2/(b*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x+a)

**maxima [B]** time = 1.55, size = 386, normalized size = 3.54

$$\frac{3\,bd^{2}\log\left(bx+a\right)}{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}-\frac{3\,bd^{2}\log\left(dx+c\right)}{b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}}+\frac{6\,b^{2}d^{2}x^{2}-b^{2}c^{2}+5\,abcd+2\,a^{2}d^{2}+3\left(b^{2}cd+3\,abd^{2}\right)x}{2\left(a^{2}b^{3}c^{4}-3\,a^{3}b^{2}c^{3}d+3\,a^{4}bc^{2}d^{2}-a^{5}cd^{3}+\left(b^{5}c^{3}d-3\,ab^{4}c^{2}d^{2}+3\,a^{2}b^{3}cd^{3}-a^{3}b^{2}d^{4}\right)x^{3}+\left(b^{5}c^{4}-ab^{4}c^{3}d-3\,a^{2}b^{3}c^{2}d^{2}+5\,a^{3}b^{2}cd^{3}-2\,a^{4}bd^{4}\right)x^{2}+\left(2\,ab^{4}c^{4}-5\,a^{2}b^{3}c^{3}d+3\,a^{3}b^{2}c^{2}d^{2}+a^{4}bcd^{3}-a^{5}d^{4}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")

[Out] 3*b*d^2*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 3*b*d^2*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/2*(6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)

**mupad [B]** time = 0.40, size = 330, normalized size = 3.03

$$\frac{6\,b\,d^2\,\mathrm{atanh}\left(\frac{a^4\,d^4-2\,a^3\,b\,c\,d^3+2\,a\,b^3\,c^3\,d-b^4\,c^4}{{\left(a\,d-b\,c\right)}^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{{\left(a\,d-b\,c\right)}^4}-\frac{\frac{2\,a^2\,d^2+5\,a\,b\,c\,d-b^2\,c^2}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,d\,x\,\left(c\,b^2+3\,a\,d\,b\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,b^2\,d^2\,x^2}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{x\,\left(d\,a^2+2\,b\,c\,a\right)+a^2\,c+x^2\,\left(c\,b^2+2\,a\,d\,b\right)+b^2\,d\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x)^3*(c + d*x)^2),x)

[Out] (6*b*d^2*atanh((a^4*d^4 - b^4*c^4 + 2*a*b^3*c^3*d - 2*a^3*b*c*d^3)/(a*d - b*c)^4 + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^4))/(a*d - b*c)^4 - ((2*a^2*d^2 - b^2*c^2 + 5*a*b*c*d)/(2*(a^3*d^3 -

```
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (3*d*x*(b^2*c + 3*a*b*d))/(2*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (3*b^2*d^2*x^2)/(a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(x*(a^2*d + 2*a*b*c) + a^2*
c + x^2*(b^2*c + 2*a*b*d) + b^2*d*x^3)
```

**sympy [B]** time = 1.72, size = 634, normalized size = 5.82

$$-\frac{3bd^2\log\left(x+\frac{-\frac{3a^5bd^7}{(ad-bc)^4}+\frac{15a^4b^2cd^6}{(ad-bc)^4}-\frac{30a^3b^3c^2d^5}{(ad-bc)^4}+\frac{30a^2b^4c^3d^4}{(ad-bc)^4}-\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3+\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}+\frac{3bd^2\log\left(x+\frac{\frac{3a^5bd^7}{(ad-bc)^4}-\frac{15a^4b^2cd^6}{(ad-bc)^4}+\frac{30a^3b^3c^2d^5}{(ad-bc)^4}-\frac{30a^2b^4c^3d^4}{(ad-bc)^4}+\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3-\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}+\frac{-2a^2d^2-5abcd+b^2c^2-6b^2d^2x^2+x\left(-9abd^2-3b^2cd\right)}{2a^5cd^3-6a^4bc^2d^2+6a^3b^2c^3d-2a^2b^3c^4+x^3\left(2a^3b^2d^4-6a^2b^3cd^3+6ab^4c^2d^2-2b^5c^3d\right)+x^2\left(4a^4bd^4-10a^3b^2cd^3+6a^2b^3c^2d^2+2ab^4c^3d-2b^5c^4\right)+x\left(2a^5d^4-2a^4bcd^3-6a^3b^2c^2d^2+10a^2b^3c^3d-4ab^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] -3*b*d**2*log(x + (-3*a**5*b*d**7/(a*d - b*c)**4 + 15*a**4*b**2*c*d**6/(a*d
 - b*c)**4 - 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 + 30*a**2*b**4*c**3*d**4
/(a*d - b*c)**4 - 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 + 3*b**6*
c**5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 + 3
*b*d**2*log(x + (3*a**5*b*d**7/(a*d - b*c)**4 - 15*a**4*b**2*c*d**6/(a*d -
b*c)**4 + 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 - 30*a**2*b**4*c**3*d**4/(a
*d - b*c)**4 + 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 - 3*b**6*c**
5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 + (-2*
a**2*d**2 - 5*a*b*c*d + b**2*c**2 - 6*b**2*d**2*x**2 + x*(-9*a*b*d**2 - 3*b
**2*c*d))/(2*a**5*c*d**3 - 6*a**4*b*c**2*d**2 + 6*a**3*b**2*c**3*d - 2*a**2
*b**3*c**4 + x**3*(2*a**3*b**2*d**4 - 6*a**2*b**3*c*d**3 + 6*a*b**4*c**2*d*
*2 - 2*b**5*c**3*d) + x**2*(4*a**4*b*d**4 - 10*a**3*b**2*c*d**3 + 6*a**2*b*
*3*c**2*d**2 + 2*a*b**4*c**3*d - 2*b**5*c**4) + x*(2*a**5*d**4 - 2*a**4*b*c
*d**3 - 6*a**3*b**2*c**2*d**2 + 10*a**2*b**3*c**3*d - 4*a*b**4*c**4))
```

## 3.1246 $\int \frac{(a+bx)^6}{(c+dx)^3} \, dx$

**Optimal**. Leaf size=158

$$-\frac{2b^5(c+dx)^3(bc-ad)}{d^7}+\frac{15b^4(c+dx)^2(bc-ad)^2}{2d^7}-\frac{20b^3x(bc-ad)^3}{d^6}+\frac{15b^2(bc-ad)^4\log(c+dx)}{d^7}+\frac{6b(bc-ad)^5}{d^7(c+dx)}-$$

**Rubi** [A] time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b^5(c+dx)^3(bc-ad)}{d^7}+\frac{15b^4(c+dx)^2(bc-ad)^2}{2d^7}-\frac{20b^3x(bc-ad)^3}{d^6}+\frac{15b^2(bc-ad)^4\log(c+dx)}{d^7}+\frac{6b(bc-ad)^5}{d^7(c+dx)}-\frac{(bc-ad)^6}{2d^7(c+dx)^2}+\frac{b^6(c+dx)^4}{4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(c + d*x)^3,x]
```

```
[Out] (-20*b^3*(b*c - a*d)^3*x)/d^6 - (b*c - a*d)^6/(2*d^7*(c + d*x)^2) + (6*b*(b
*c - a*d)^5)/(d^7*(c + d*x)) + (15*b^4*(b*c - a*d)^2*(c + d*x)^2)/(2*d^7) -
 (2*b^5*(b*c - a*d)*(c + d*x)^3)/d^7 + (b^6*(c + d*x)^4)/(4*d^7) + (15*b^2*
(b*c - a*d)^4*Log[c + d*x])/d^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^6}{(c+dx)^3}\,dx = \int \left(-\frac{20b^3(bc-ad)^3}{d^6}+\frac{(-bc+ad)^6}{d^6(c+dx)^3}-\frac{6b(bc-ad)^5}{d^6(c+dx)^2}+\frac{15b^2(bc-ad)^4}{d^6(c+dx)}+\frac{15b^4(bc-ad)^2(c+dx}{d^6}\right.$$

$$=-\frac{20b^3(bc-ad)^3x}{d^6}-\frac{(bc-ad)^6}{2d^7(c+dx)^2}+\frac{6b(bc-ad)^5}{d^7(c+dx)}+\frac{15b^4(bc-ad)^2(c+dx)^2}{2d^7}-\frac{2b^5(bc-ad)(c}{d^7}$$

**Mathematica** [A] time = 0.11, size = 303, normalized size = 1.92

$$\frac{-2a^6d^6-12a^5bd^5(c+2dx)+30a^4b^2cd^4(3c+4dx)+40a^3b^3d^3\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)+30a^2b^4d^2\left(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4\right)+4ab^5d\left(-27c^5+6c^4dx+63c^3d^2x^2+20c^2d^3x^3-5cd^4x^4+2d^5x^5\right)+60b^2(c+dx)^2(bc-ad)^4\log(c+dx)+b^6\left(22c^6-16c^5dx-68c^4d^2x^2-20c^3d^3x^3+5c^2d^4x^4-2cd^5x^5+d^6x^6\right)}{4d^7(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(c + d*x)^3,x]
```

```
[Out] (-2*a^6*d^6 - 12*a^5*b*d^5*(c + 2*d*x) + 30*a^4*b^2*c*d^4*(3*c + 4*d*x) + 4
0*a^3*b^3*d^3*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 30*a^2*b^4*d
^2*(7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 4*a*b^5*d
*(-27*c^5 + 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d
^5*x^5) + b^6*(22*c^6 - 16*c^5*d*x - 68*c^4*d^2*x^2 - 20*c^3*d^3*x^3 + 5*c^
2*d^4*x^4 - 2*c*d^5*x^5 + d^6*x^6) + 60*b^2*(b*c - a*d)^4*(c + d*x)^2*Log[c
 + d*x])/(4*d^7*(c + d*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^6}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^6/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^6/(c + d*x)^3, x]
```

**fricas [B]** time = 1.26, size = 548, normalized size = 3.47

$$\frac{b^6d^6x^6+22\,b^6c^6-108\,ab^5c^5d+210\,a^2b^4c^4d^2-200\,a^3b^3c^3d^3+90\,a^4b^2c^2d^4-12\,a^5bcd^5-2\,a^6d^6-2\left(b^6cd^5-4\,ab^5d^6\right)x^5+5\left(b^6c^2d^4-4\,ab^5cd^5+6\,a^2b^4d^6\right)x^4-20\left(b^6c^3d^3-4\,ab^5c^2d^4+6\,a^2b^4cd^5-4\,a^3b^3d^6\right)x^3-2\left(34\,b^6c^4d^2-126\,ab^5c^3d^3+165\,a^2b^4c^2d^4-80\,a^3b^3cd^5\right)x^2-4\left(4\,b^6c^5d-6\,ab^5c^4d^2-15\,a^2b^4c^3d^3+40\,a^3b^3c^2d^4-30\,a^4b^2cd^5+6\,a^5bd^6\right)x+60\left(b^6c^6-4\,ab^5c^5d+6\,a^2b^4c^4d^2-4\,a^3b^3c^3d^3+a^4b^2c^2d^4+\left(b^6c^4d^2-4\,ab^5c^3d^3+6\,a^2b^4c^2d^4-4\,a^3b^3cd^5+a^4b^2d^6\right)x^2+2\left(b^6c^5d-4\,ab^5c^4d^2+6\,a^2b^4c^3d^3-4\,a^3b^3c^2d^4+a^4b^2cd^5\right)x\right)\log\left(dx+c\right)}{4\left(d^9x^2+2\,cd^8x+c^2d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^6*d^6*x^6 + 22*b^6*c^6 - 108*a*b^5*c^5*d + 210*a^2*b^4*c^4*d^2 - 200
*a^3*b^3*c^3*d^3 + 90*a^4*b^2*c^2*d^4 - 12*a^5*b*c*d^5 - 2*a^6*d^6 - 2*(b^6
*c*d^5 - 4*a*b^5*d^6)*x^5 + 5*(b^6*c^2*d^4 - 4*a*b^5*c*d^5 + 6*a^2*b^4*d^6)
*x^4 - 20*(b^6*c^3*d^3 - 4*a*b^5*c^2*d^4 + 6*a^2*b^4*c*d^5 - 4*a^3*b^3*d^6)
*x^3 - 2*(34*b^6*c^4*d^2 - 126*a*b^5*c^3*d^3 + 165*a^2*b^4*c^2*d^4 - 80*a^3
*b^3*c*d^5)*x^2 - 4*(4*b^6*c^5*d - 6*a*b^5*c^4*d^2 - 15*a^2*b^4*c^3*d^3 + 4
0*a^3*b^3*c^2*d^4 - 30*a^4*b^2*c*d^5 + 6*a^5*b*d^6)*x + 60*(b^6*c^6 - 4*a*b
^5*c^5*d + 6*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4 + (b^6*c
^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^
6)*x^2 + 2*(b^6*c^5*d - 4*a*b^5*c^4*d^2 + 6*a^2*b^4*c^3*d^3 - 4*a^3*b^3*c^2
*d^4 + a^4*b^2*c*d^5)*x)*log(d*x + c))/(d^9*x^2 + 2*c*d^8*x + c^2*d^7)
```

**giac [B]** time = 1.28, size = 362, normalized size = 2.29

$$\frac{15\left(b^6c^4-4\,ab^5c^3d+6\,a^2b^4c^2d^2-4\,a^3b^3cd^3+a^4b^2d^4\right)\log\left(|dx+c|\right)}{d^7}+\frac{11\,b^6c^6-54\,ab^5c^5d+105\,a^2b^4c^4d^2-100\,a^3b^3c^3d^3+45\,a^4b^2c^2d^4-6\,a^5bcd^5-a^6d^6+12\left(b^6c^5d-5\,ab^5c^4d^2+10\,a^2b^4c^3d^3-10\,a^3b^3c^2d^4+5\,a^4b^2cd^5-a^5bd^6\right)x}{2\left(dx+c\right)^2d^7}+\frac{b^6d^9x^4-4\,b^6cd^8x^3+8\,ab^5d^9x^3+12\,b^6c^2d^7x^2-36\,ab^5cd^8x^2+30\,a^2b^4d^9x^2-40\,b^6c^3d^6x+144\,ab^5c^2d^7x-180\,a^2b^4cd^8x+80\,a^3b^3d^9x}{4\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 15*(b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2
*d^4)*log(abs(d*x + c))/d^7 + 1/2*(11*b^6*c^6 - 54*a*b^5*c^5*d + 105*a^2*b^
4*c^4*d^2 - 100*a^3*b^3*c^3*d^3 + 45*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 - a^6*
d^6 + 12*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 10*a^2*b^4*c^3*d^3 - 10*a^3*b^3*c^2
*d^4 + 5*a^4*b^2*c*d^5 - a^5*b*d^6)*x)/((d*x + c)^2*d^7) + 1/4*(b^6*d^9*x^4
 - 4*b^6*c*d^8*x^3 + 8*a*b^5*d^9*x^3 + 12*b^6*c^2*d^7*x^2 - 36*a*b^5*c*d^8*
x^2 + 30*a^2*b^4*d^9*x^2 - 40*b^6*c^3*d^6*x + 144*a*b^5*c^2*d^7*x - 180*a^2
*b^4*c*d^8*x + 80*a^3*b^3*d^9*x)/d^12
```

**maple [B]** time = 0.01, size = 464, normalized size = 2.94

$$\frac{b^6x^4}{4d^3}+\frac{2ab^5x^3}{d^3}-\frac{b^6cx^3}{d^4}-\frac{a^6}{2(dx+c)^2d}+\frac{3a^5bc}{(dx+c)^2d^2}-\frac{15a^4b^2c^2}{2(dx+c)^2d^3}+\frac{10a^3b^3c^3}{(dx+c)^2d^4}-\frac{15a^2b^4c^4}{2(dx+c)^2d^5}+\frac{15a^2b^4x^2}{2d^3}+\frac{3ab^5c^5}{(dx+c)^2d^6}-\frac{9ab^5cx^2}{d^4}-\frac{b^6c^6}{2(dx+c)^2d^7}+\frac{3b^6c^2x^2}{d^5}-\frac{6a^5b}{(dx+c)d^2}+\frac{30a^4b^2c}{(dx+c)d^3}+\frac{15a^4b^2\ln(dx+c)}{d^3}-\frac{60a^3b^3c^2}{(dx+c)d^4}-\frac{60a^3b^3c\ln(dx+c)}{d^4}+\frac{20a^3b^3x}{d^3}+\frac{60a^2b^4c^3}{(dx+c)d^5}+\frac{90a^2b^4c^2\ln(dx+c)}{d^5}-\frac{45a^2b^4cx}{d^4}-\frac{30ab^5c^4}{(dx+c)d^6}-\frac{60ab^5c^3\ln(dx+c)}{d^6}+\frac{36ab^5c^2x}{d^5}+\frac{6b^6c^5}{(dx+c)d^7}+\frac{15b^6c^4\ln(dx+c)}{d^7}-\frac{10b^6c^3x}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(d*x+c)^3,x)
```

```
[Out] -1/2/d^7/(d*x+c)^2*b^6*c^6-6*b/d^2/(d*x+c)*a^5+6*b^6/d^7/(d*x+c)*c^5+15*b^2
/d^3*ln(d*x+c)*a^4+15*b^6/d^7*ln(d*x+c)*c^4+20*b^3/d^3*a^3*x-10*b^6/d^6*c^3
*x+3*b^6/d^5*x^2*c^2+15/2*b^4/d^3*x^2*a^2-b^6/d^4*x^3*c+2*b^5/d^3*x^3*a+3/d
^2/(d*x+c)^2*a^5*b*c-15/2/d^3/(d*x+c)^2*a^4*b^2*c^2+10/d^4/(d*x+c)^2*a^3*b^
3*c^3-15/2/d^5/(d*x+c)^2*a^2*b^4*c^4+3/d^6/(d*x+c)^2*a*b^5*c^5+36*b^5/d^5*a
*c^2*x+30*b^2/d^3/(d*x+c)*a^4*c-60*b^3/d^4/(d*x+c)*a^3*c^2+60*b^4/d^5/(d*x+
c)*a^2*c^3-30*b^5/d^6/(d*x+c)*a*c^4-9*b^5/d^4*x^2*a*c-45*b^4/d^4*a^2*c*x+1/
4*b^6/d^3*x^4-1/2/d/(d*x+c)^2*a^6-60*b^5/d^6*ln(d*x+c)*a*c^3-60*b^3/d^4*ln(
d*x+c)*a^3*c+90*b^4/d^5*ln(d*x+c)*a^2*c^2
```

**maxima [B]** time = 1.47, size = 364, normalized size = 2.30

$$\frac{11\,b^6c^6-54\,ab^5c^5d+105\,a^2b^4c^4d^2-100\,a^3b^3c^3d^3+45\,a^4b^2c^2d^4-6\,a^5bcd^5-a^6d^6+12\left(b^6c^5d-5\,ab^5c^4d^2+10\,a^2b^4c^3d^3-10\,a^3b^3c^2d^4+5\,a^4b^2cd^5-a^5bd^6\right)x}{2\left(d^9x^2+2\,cd^8x+c^2d^7\right)}+\frac{b^6d^3x^4-4\left(b^6cd^2-2\,ab^5d^3\right)x^3+6\left(2\,b^6c^2d-6\,ab^5cd^2+5\,a^2b^4d^3\right)x^2-4\left(10\,b^6c^3-36\,ab^5c^2d+45\,a^2b^4cd^2-20\,a^3b^3d^3\right)x}{4\,d^6}+\frac{15\left(b^6c^4-4\,ab^5c^3d+6\,a^2b^4c^2d^2-4\,a^3b^3cd^3+a^4b^2d^4\right)\log(dx+c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*b^6*c^6 - 54*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2 - 100*a^3*b^3*c^3*d^
3 + 45*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 - a^6*d^6 + 12*(b^6*c^5*d - 5*a*b^5*
c^4*d^2 + 10*a^2*b^4*c^3*d^3 - 10*a^3*b^3*c^2*d^4 + 5*a^4*b^2*c*d^5 - a^5*b
*d^6)*x)/(d^9*x^2 + 2*c*d^8*x + c^2*d^7) + 1/4*(b^6*d^3*x^4 - 4*(b^6*c*d^2
- 2*a*b^5*d^3)*x^3 + 6*(2*b^6*c^2*d - 6*a*b^5*c*d^2 + 5*a^2*b^4*d^3)*x^2 -
4*(10*b^6*c^3 - 36*a*b^5*c^2*d + 45*a^2*b^4*c*d^2 - 20*a^3*b^3*d^3)*x)/d^6
+ 15*(b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b
^2*d^4)*log(d*x + c)/d^7
```

**mupad [B]** time = 0.27, size = 441, normalized size = 2.79

$$x^3\left(\frac{2\,a\,b^5}{d^3}-\frac{b^6\,c}{d^4}\right)-\frac{\frac{a^6\,d^6+6\,a^5\,b\,c\,d^5-45\,a^4\,b^2\,c^2\,d^4+100\,a^3\,b^3\,c^3\,d^3-105\,a^2\,b^4\,c^4\,d^2+54\,a\,b^5\,c^5\,d-11\,b^6\,c^6}{2\,d}-x\,\left(-6\,a^5\,b\,d^5+30\,a^4\,b^2\,c\,d^4-60\,a^3\,b^3\,c^2\,d^3+60\,a^2\,b^4\,c^3\,d^2-30\,a\,b^5\,c^4\,d+6\,b^6\,c^5\right)}{c^2\,d^6+2\,c\,d^7\,x+d^8\,x^2}-x^2\,\left(\frac{3\,c\,\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{2\,d}-\frac{15\,a^2\,b^4}{2\,d^3}+\frac{3\,b^6\,c^2}{2\,d^5}\right)+x\,\left(\frac{3\,c\,\left(\frac{3\,c\,\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{d}-\frac{15\,a^2\,b^4}{d^3}+\frac{3\,b^6\,c^2}{d^5}\right)}{d}+\frac{20\,a^3\,b^3}{d^3}-\frac{b^6\,c^3}{d^6}-\frac{3\,c^2\,\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{d^2}\right)+\frac{\ln\left(c+d\,x\right)\,\left(15\,a^4\,b^2\,d^4-60\,a^3\,b^3\,c\,d^3+90\,a^2\,b^4\,c^2\,d^2-60\,a\,b^5\,c^3\,d+15\,b^6\,c^4\right)}{d^7}+\frac{b^6\,x^4}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6/(c + d*x)^3,x)
```

```
[Out] x^3*((2*a*b^5)/d^3 - (b^6*c)/d^4) - ((a^6*d^6 - 11*b^6*c^6 - 105*a^2*b^4*c^
4*d^2 + 100*a^3*b^3*c^3*d^3 - 45*a^4*b^2*c^2*d^4 + 54*a*b^5*c^5*d + 6*a^5*b
*c*d^5)/(2*d) - x*(6*b^6*c^5 - 6*a^5*b*d^5 + 30*a^4*b^2*c*d^4 + 60*a^2*b^4*
c^3*d^2 - 60*a^3*b^3*c^2*d^3 - 30*a*b^5*c^4*d))/(c^2*d^6 + d^8*x^2 + 2*c*d^
7*x) - x^2*((3*c*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))/(2*d) - (15*a^2*b^4)/(2*d
^3) + (3*b^6*c^2)/(2*d^5)) + x*((3*c*((3*c*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))
/d - (15*a^2*b^4)/d^3 + (3*b^6*c^2)/d^5))/d + (20*a^3*b^3)/d^3 - (b^6*c^3)/
d^6 - (3*c^2*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))/d^2) + (log(c + d*x)*(15*b^6*
c^4 + 15*a^4*b^2*d^4 - 60*a^3*b^3*c*d^3 + 90*a^2*b^4*c^2*d^2 - 60*a*b^5*c^3
*d))/d^7 + (b^6*x^4)/(4*d^3)
```

**sympy [B]** time = 2.15, size = 340, normalized size = 2.15

$$\frac{b^6x^4}{4d^3} + \frac{15b^2\left(ad-bc\right)^4\log\left(c+dx\right)}{d^7} + x^3\left(\frac{2ab^5}{d^3} - \frac{b^6c}{d^4}\right) + x^2\left(\frac{15a^2b^4}{2d^3} - \frac{9ab^5c}{d^4} + \frac{3b^6c^2}{d^5}\right) + x\left(\frac{20a^3b^3}{d^3} - \frac{45a^2b^4c}{d^4} + \frac{36ab^5c^2}{d^5} - \frac{10b^6c^3}{d^6}\right) + \frac{-a^6d^6 - 6a^5bcd^5 + 45a^4b^2c^2d^4 - 100a^3b^3c^3d^3 + 105a^2b^4c^4d^2 - 54ab^5c^5d + 11b^6c^6 + x\left(-12a^5bd^6 + 60a^4b^2cd^5 - 120a^3b^3c^2d^4 + 120a^2b^4c^3d^3 - 60ab^5c^4d^2 + 12b^6c^5d\right)}{2c^2d^7 + 4cd^8x + 2d^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(d*x+c)**3,x)
```

```
[Out] b**6*x**4/(4*d**3) + 15*b**2*(a*d - b*c)**4*log(c + d*x)/d**7 + x**3*(2*a*b
**5/d**3 - b**6*c/d**4) + x**2*(15*a**2*b**4/(2*d**3) - 9*a*b**5*c/d**4 + 3
*b**6*c**2/d**5) + x*(20*a**3*b**3/d**3 - 45*a**2*b**4*c/d**4 + 36*a*b**5*c
**2/d**5 - 10*b**6*c**3/d**6) + (-a**6*d**6 - 6*a**5*b*c*d**5 + 45*a**4*b**
2*c**2*d**4 - 100*a**3*b**3*c**3*d**3 + 105*a**2*b**4*c**4*d**2 - 54*a*b**5
*c**5*d + 11*b**6*c**6 + x*(-12*a**5*b*d**6 + 60*a**4*b**2*c*d**5 - 120*a**
3*b**3*c**2*d**4 + 120*a**2*b**4*c**3*d**3 - 60*a*b**5*c**4*d**2 + 12*b**6*
c**5*d))/(2*c**2*d**7 + 4*c*d**8*x + 2*d**9*x**2)
```

# 3.1247 $\int \frac{(a+bx)^5}{(c+dx)^3} \, dx$

**Optimal**. Leaf size=133

$$-\frac{5b^4(c+dx)^2(bc-ad)}{2d^6}+\frac{10b^3x(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}-\frac{5b(bc-ad)^4}{d^6(c+dx)}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}+\frac{b^5(c+dx)^3}{3d^6}$$

**Rubi** [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{5b^4(c+dx)^2(bc-ad)}{2d^6}+\frac{10b^3x(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}-\frac{5b(bc-ad)^4}{d^6(c+dx)}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}+\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (10*b^3*(b*c - a*d)^2*x)/d^5 + (b*c - a*d)^5/(2*d^6*(c + d*x)^2) - (5*b*(b*
c - a*d)^4)/(d^6*(c + d*x)) - (5*b^4*(b*c - a*d)*(c + d*x)^2)/(2*d^6) + (b^
5*(c + d*x)^3)/(3*d^6) - (10*b^2*(b*c - a*d)^3*Log[c + d*x])/d^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^3}\,dx = \int \left(\frac{10b^3(bc-ad)^2}{d^5}+\frac{(-bc+ad)^5}{d^5(c+dx)^3}+\frac{5b(bc-ad)^4}{d^5(c+dx)^2}-\frac{10b^2(bc-ad)^3}{d^5(c+dx)}-\frac{5b^4(bc-ad)(c+dx)}{d^5}+\right.$$

$$=\frac{10b^3(bc-ad)^2x}{d^5}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}-\frac{5b(bc-ad)^4}{d^6(c+dx)}-\frac{5b^4(bc-ad)(c+dx)^2}{2d^6}+\frac{b^5(c+dx)^3}{3d^6}-10b^2$$

**Mathematica** [A] time = 0.07, size = 230, normalized size = 1.73

$$\frac{-3a^5d^5-15a^4bd^4(c+2dx)+30a^3b^2cd^3(3c+4dx)+30a^2b^3d^2\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)+15ab^4d\left(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4\right)-60b^2(c+dx)^2(bc-ad)^3\log(c+dx)+b^5\left(-27c^5+6c^4dx+63c^3d^2x^2+20c^2d^3x^3-5cd^4x^4+2d^5x^5\right)}{6d^6(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (-3*a^5*d^5 - 15*a^4*b*d^4*(c + 2*d*x) + 30*a^3*b^2*c*d^3*(3*c + 4*d*x) + 3
0*a^2*b^3*d^2*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 15*a*b^4*d*(
7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + b^5*(-27*c^5
+ 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d^5*x^5) -
60*b^2*(b*c - a*d)^3*(c + d*x)^2*Log[c + d*x])/(6*d^6*(c + d*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^3, x]
```

**fricas [B]** time = 1.20, size = 416, normalized size = 3.13

$$\frac{2\,b^5d^5x^5 - 27\,b^5c^5 + 105\,ab^4c^4d - 150\,a^2b^3c^3d^2 + 90\,a^3b^2c^2d^3 - 15\,a^4bcd^4 - 3\,a^5d^5 - 5\left(b^5cd^4 - 3\,ab^4d^5\right)x^4 + 20\left(b^5c^2d^3 - 3\,ab^4cd^4 + 3\,a^2b^3d^5\right)x^3 + 3\left(21\,b^5c^3d^2 - 55\,ab^4c^2d^3 + 40\,a^2b^3cd^4\right)x^2 + 6\left(b^5c^4d + 5\,ab^4c^3d^2 - 20\,a^2b^3c^2d^3 + 20\,a^3b^2cd^4 - 5\,a^4bd^5\right)x - 60\left(b^5c^5 - 3\,ab^4c^4d + 3\,a^2b^3c^3d^2 - a^3b^2c^2d^3 + \left(b^5c^3d^2 - 3\,ab^4c^2d^3 + 3\,a^2b^3cd^4 - a^3b^2d^5\right)x^2 + 2\left(b^5c^4d - 3\,ab^4c^3d^2 + 3\,a^2b^3c^2d^3 - a^3b^2cd^4\right)x\right)\log\left(dx + c\right)}{6\left(d^8x^2 + 2\,cd^7x + c^2d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*d^5*x^5 - 27*b^5*c^5 + 105*a*b^4*c^4*d - 150*a^2*b^3*c^3*d^2 + 9
0*a^3*b^2*c^2*d^3 - 15*a^4*b*c*d^4 - 3*a^5*d^5 - 5*(b^5*c*d^4 - 3*a*b^4*d^5
)*x^4 + 20*(b^5*c^2*d^3 - 3*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 3*(21*b^5*c^
3*d^2 - 55*a*b^4*c^2*d^3 + 40*a^2*b^3*c*d^4)*x^2 + 6*(b^5*c^4*d + 5*a*b^4*c
^3*d^2 - 20*a^2*b^3*c^2*d^3 + 20*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x - 60*(b^5*c
^5 - 3*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + (b^5*c^3*d^2 - 3
*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 - a^3*b^2*d^5)*x^2 + 2*(b^5*c^4*d - 3*a*b^
4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 - a^3*b^2*c*d^4)*x)*log(d*x + c))/(d^8*x^2 +
2*c*d^7*x + c^2*d^6)
```

**giac [B]** time = 1.29, size = 264, normalized size = 1.98

$$-\frac{10\left(b^5c^3 - 3\,ab^4c^2d + 3\,a^2b^3cd^2 - a^3b^2d^3\right)\log\left(|dx + c|\right)}{d^6} - \frac{9\,b^5c^5 - 35\,ab^4c^4d + 50\,a^2b^3c^3d^2 - 30\,a^3b^2c^2d^3 + 5\,a^4bcd^4 + a^5d^5 + 10\left(b^5c^4d - 4\,ab^4c^3d^2 + 6\,a^2b^3c^2d^3 - 4\,a^3b^2cd^4 + a^4bd^5\right)x}{2\left(dx + c\right)^2d^6} + \frac{2\,b^5d^6x^3 - 9\,b^5cd^5x^2 + 15\,ab^4d^6x^2 + 36\,b^5c^2d^4x - 90\,ab^4cd^5x + 60\,a^2b^3d^6x}{6\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*log(abs(d*x +
 c))/d^6 - 1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^
2*c^2*d^3 + 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a
```

```
^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/((d*x + c)^2*d^6) + 1/6*(2
*b^5*d^6*x^3 - 9*b^5*c*d^5*x^2 + 15*a*b^4*d^6*x^2 + 36*b^5*c^2*d^4*x - 90*a
*b^4*c*d^5*x + 60*a^2*b^3*d^6*x)/d^9
```

**maple [B]** time = 0.01, size = 346, normalized size = 2.60

$$\frac{b^5x^3}{3d^3}-\frac{a^5}{2\left(dx+c\right)^2d}+\frac{5a^4bc}{2\left(dx+c\right)^2d^2}-\frac{5a^3b^2c^2}{\left(dx+c\right)^2d^3}+\frac{5a^2b^3c^3}{\left(dx+c\right)^2d^4}-\frac{5a\,b^4c^4}{2\left(dx+c\right)^2d^5}+\frac{5a\,b^4x^2}{2d^3}+\frac{b^5c^5}{2\left(dx+c\right)^2d^6}-\frac{3b^5c\,x^2}{2d^4}-\frac{5a^4b}{\left(dx+c\right)d^2}+\frac{20a^3b^2c}{\left(dx+c\right)d^3}+\frac{10a^3b^2\ln\left(dx+c\right)}{d^3}-\frac{30a^2b^3c^2}{\left(dx+c\right)d^4}-\frac{30a^2b^3c\ln\left(dx+c\right)}{d^4}+\frac{10a^2b^3x}{d^3}+\frac{20a\,b^4c^3}{\left(dx+c\right)d^5}+\frac{30a\,b^4c^2\ln\left(dx+c\right)}{d^5}-\frac{15a\,b^4cx}{d^4}-\frac{5b^5c^4}{\left(dx+c\right)d^6}-\frac{10b^5c^3\ln\left(dx+c\right)}{d^6}+\frac{6b^5c^2x}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^3,x)
```

```
[Out] 1/3*b^5/d^3*x^3+5/2*b^4/d^3*x^2*a-3/2*b^5/d^4*x^2*c+10*b^3/d^3*a^2*x-15*b^4
/d^4*a*c*x+6*b^5/d^5*c^2*x-5*b/d^2/(d*x+c)*a^4+20*b^2/d^3/(d*x+c)*a^3*c-30*
b^3/d^4/(d*x+c)*a^2*c^2+20*b^4/d^5/(d*x+c)*a*c^3-5*b^5/d^6/(d*x+c)*c^4+10*b
^2/d^3*ln(d*x+c)*a^3-30*b^3/d^4*ln(d*x+c)*a^2*c+30*b^4/d^5*ln(d*x+c)*a*c^2-
10*b^5/d^6*ln(d*x+c)*c^3-1/2/d/(d*x+c)^2*a^5+5/2/d^2/(d*x+c)^2*a^4*b*c-5/d^
3/(d*x+c)^2*a^3*b^2*c^2+5/d^4/(d*x+c)^2*a^2*b^3*c^3-5/2/d^5/(d*x+c)^2*a*b^4
*c^4+1/2/d^6/(d*x+c)^2*b^5*c^5
```

**maxima [B]** time = 1.48, size = 271, normalized size = 2.04

$$-\frac{9\,b^5c^5-35\,ab^4c^4d+50\,a^2b^3c^3d^2-30\,a^3b^2c^2d^3+5\,a^4bcd^4+a^5d^5+10\left(b^5c^4d-4\,ab^4c^3d^2+6\,a^2b^3c^2d^3-4\,a^3b^2cd^4+a^4bd^5\right)x}{2\left(d^8x^2+2\,cd^7x+c^2d^6\right)}+\frac{2\,b^5d^2x^3-3\left(3\,b^5cd-5\,ab^4d^2\right)x^2+6\left(6\,b^5c^2-15\,ab^4cd+10\,a^2b^3d^2\right)x}{6\,d^5}-\frac{10\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)\log\left(dx+c\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3
+ 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2
*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6) + 1/
6*(2*b^5*d^2*x^3 - 3*(3*b^5*c*d - 5*a*b^4*d^2)*x^2 + 6*(6*b^5*c^2 - 15*a*b^
4*c*d + 10*a^2*b^3*d^2)*x)/d^5 - 10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*
d^2 - a^3*b^2*d^3)*log(d*x + c)/d^6
```

**mupad [B]** time = 0.10, size = 291, normalized size = 2.19

$$x^2\left(\frac{5\,a\,b^4}{2\,d^3}-\frac{3\,b^5\,c}{2\,d^4}\right)-\frac{\frac{a^5\,d^5+5\,a^4\,b\,c\,d^4-30\,a^3\,b^2\,c^2\,d^3+50\,a^2\,b^3\,c^3\,d^2-35\,a\,b^4\,c^4\,d+9\,b^5\,c^5}{2\,d}+x\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)}{c^2\,d^5+2\,c\,d^6\,x+d^7\,x^2}-x\,\left(\frac{3\,c\,\left(\frac{5\,a\,b^4}{d^3}-\frac{3\,b^5\,c}{d^4}\right)}{d}-\frac{10\,a^2\,b^3}{d^3}+\frac{3\,b^5\,c^2}{d^5}\right)-\frac{\ln\left(c+d\,x\right)\,\left(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3\right)}{d^6}+\frac{b^5\,x^3}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^3,x)
```

```
[Out] x^2*((5*a*b^4)/(2*d^3) - (3*b^5*c)/(2*d^4)) - ((a^5*d^5 + 9*b^5*c^5 + 50*a^
2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 35*a*b^4*c^4*d + 5*a^4*b*c*d^4)/(2*d)
+ x*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a
*b^4*c^3*d))/(c^2*d^5 + d^7*x^2 + 2*c*d^6*x) - x*((3*c*((5*a*b^4)/d^3 - (3*
```

```
b^5*c)/d^4))/d - (10*a^2*b^3)/d^3 + (3*b^5*c^2)/d^5) - (log(c + d*x)*(10*b^
5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 30*a*b^4*c^2*d))/d^6 + (b^5*x^3
)/(3*d^3)
```

**sympy [B]** time = 1.65, size = 258, normalized size = 1.94

$$\frac{b^5x^3}{3d^3}+\frac{10b^2\left(ad-bc\right)^3\log\left(c+dx\right)}{d^6}+x^2\left(\frac{5ab^4}{2d^3}-\frac{3b^5c}{2d^4}\right)+x\left(\frac{10a^2b^3}{d^3}-\frac{15ab^4c}{d^4}+\frac{6b^5c^2}{d^5}\right)+\frac{-a^5d^5-5a^4bcd^4+30a^3b^2c^2d^3-50a^2b^3c^3d^2+35ab^4c^4d-9b^5c^5+x\left(-10a^4bd^5+40a^3b^2cd^4-60a^2b^3c^2d^3+40ab^4c^3d^2-10b^5c^4d\right)}{2c^2d^6+4cd^7x+2d^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**3,x)
```

```
[Out] b**5*x**3/(3*d**3) + 10*b**2*(a*d - b*c)**3*log(c + d*x)/d**6 + x**2*(5*a*b
**4/(2*d**3) - 3*b**5*c/(2*d**4)) + x*(10*a**2*b**3/d**3 - 15*a*b**4*c/d**4
 + 6*b**5*c**2/d**5) + (-a**5*d**5 - 5*a**4*b*c*d**4 + 30*a**3*b**2*c**2*d*
*3 - 50*a**2*b**3*c**3*d**2 + 35*a*b**4*c**4*d - 9*b**5*c**5 + x*(-10*a**4*
b*d**5 + 40*a**3*b**2*c*d**4 - 60*a**2*b**3*c**2*d**3 + 40*a*b**4*c**3*d**2
 - 10*b**5*c**4*d))/(2*c**2*d**6 + 4*c*d**7*x + 2*d**8*x**2)
```

## 3.1248 $\int \frac{(a+bx)^4}{(c+dx)^3} dx$

**Optimal**. Leaf size=103

$$-\frac{b^3x(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}+\frac{4b(bc-ad)^3}{d^5(c+dx)}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{b^4x^2}{2d^3}$$

**Rubi** [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b^3x(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}+\frac{4b(bc-ad)^3}{d^5(c+dx)}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^3,x]
```

```
[Out] -((b^3*(3*b*c - 4*a*d)*x)/d^4) + (b^4*x^2)/(2*d^3) - (b*c - a*d)^4/(2*d^5*(
c + d*x)^2) + (4*b*(b*c - a*d)^3)/(d^5*(c + d*x)) + (6*b^2*(b*c - a*d)^2*Lo
g[c + d*x])/d^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^3}\,dx &= \int \left(-\frac{b^3(3bc-4ad)}{d^4}+\frac{b^4x}{d^3}+\frac{(-bc+ad)^4}{d^4(c+dx)^3}-\frac{4b(bc-ad)^3}{d^4(c+dx)^2}+\frac{6b^2(bc-ad)^2}{d^4(c+dx)}\right)dx\\ &= -\frac{b^3(3bc-4ad)x}{d^4}+\frac{b^4x^2}{2d^3}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{4b(bc-ad)^3}{d^5(c+dx)}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 167, normalized size = 1.62

$$\frac{-a^4d^4-4a^3bd^3(c+2dx)+6a^2b^2cd^2(3c+4dx)+4ab^3d\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)+12b^2(c+dx)^2(bc-ad)^2\log(c+dx)+b^4\left(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4\right)}{2d^5(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (-(a^4*d^4) - 4*a^3*b*d^3*(c + 2*d*x) + 6*a^2*b^2*c*d^2*(3*c + 4*d*x) + 4*a
*b^3*d*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + b^4*(7*c^4 + 2*c^3*
d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 12*b^2*(b*c - a*d)^2*(c + d
*x)^2*Log[c + d*x])/(2*d^5*(c + d*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^3, x]
```

**fricas [B]** time = 0.77, size = 291, normalized size = 2.83

$$\frac{b^4d^4x^4+7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4-4\left(b^4cd^3-2\,ab^3d^4\right)x^3-\left(11\,b^4c^2d^2-16\,ab^3cd^3\right)x^2+2\left(b^4c^3d-8\,ab^3c^2d^2+12\,a^2b^2cd^3-4\,a^3bd^4\right)x+12\left(b^4c^4-2\,ab^3c^3d+a^2b^2c^2d^2+\left(b^4c^2d^2-2\,ab^3cd^3+a^2b^2d^4\right)x^2+2\left(b^4c^3d-2\,ab^3c^2d^2+a^2b^2cd^3\right)x\right)\log\left(dx+c\right)}{2\left(d^7x^2+2\,cd^6x+c^2d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 - a^4*d^4 - 4*(b^4*c*d^3 - 2*a*b^3*d^4)*x^3 - (11*b^4*c^2*d^2 - 16*
a*b^3*c*d^3)*x^2 + 2*(b^4*c^3*d - 8*a*b^3*c^2*d^2 + 12*a^2*b^2*c*d^3 - 4*a^
3*b*d^4)*x + 12*(b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2 + (b^4*c^2*d^2 -
 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(b^4*c^3*d - 2*a*b^3*c^2*d^2 + a^2*b^
2*c*d^3)*x)*log(d*x + c))/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

**giac [A]** time = 1.35, size = 183, normalized size = 1.78

$$\frac{6\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\log\left(|dx+c|\right)}{d^5}+\frac{b^4d^3x^2-6\,b^4cd^2x+8\,ab^3d^3x}{2\,d^6}+\frac{7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4+8\left(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4\right)x}{2\,(dx+c)^2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(abs(d*x + c))/d^5 + 1/2*(b^4*d^
3*x^2 - 6*b^4*c*d^2*x + 8*a*b^3*d^3*x)/d^6 + 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*
d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4 + 8*(b^4*c^3*d - 3*a*b^3*c
^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/((d*x + c)^2*d^5)
```

**maple [B]** time = 0.01, size = 245, normalized size = 2.38

$$-\frac{a^4}{2\,(dx+c)^2\,d}+\frac{2a^3bc}{(dx+c)^2\,d^2}-\frac{3a^2b^2c^2}{(dx+c)^2\,d^3}+\frac{2a\,b^3c^3}{(dx+c)^2\,d^4}-\frac{b^4c^4}{2\,(dx+c)^2\,d^5}+\frac{b^4x^2}{2d^3}-\frac{4a^3b}{(dx+c)\,d^2}+\frac{12a^2b^2c}{(dx+c)\,d^3}+\frac{6a^2b^2\ln(dx+c)}{d^3}-\frac{12a\,b^3c^2}{(dx+c)\,d^4}-\frac{12a\,b^3c\ln(dx+c)}{d^4}+\frac{4a\,b^3x}{d^3}+\frac{4b^4c^3}{(dx+c)\,d^5}+\frac{6b^4c^2\ln(dx+c)}{d^5}-\frac{3b^4cx}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^3,x)
```

```
[Out] 1/2*b^4*x^2/d^3+4*a*b^3*x/d^3-3*b^4*c*x/d^4-4*b/d^2/(d*x+c)*a^3+12*b^2/d^3/
(d*x+c)*a^2*c-12*b^3/d^4/(d*x+c)*a*c^2+4*b^4/d^5/(d*x+c)*c^3+6*b^2/d^3*ln(d
*x+c)*a^2-12*b^3/d^4*ln(d*x+c)*a*c+6*b^4/d^5*ln(d*x+c)*c^2-1/2/d/(d*x+c)^2*
a^4+2/d^2/(d*x+c)^2*a^3*b*c-3/d^3/(d*x+c)^2*a^2*b^2*c^2+2/d^4/(d*x+c)^2*a*b
^3*c^3-1/2/d^5/(d*x+c)^2*b^4*c^4
```

**maxima** [A] time = 1.39, size = 191, normalized size = 1.85

$$\frac{7\,b^4c^4 - 20\,ab^3c^3d + 18\,a^2b^2c^2d^2 - 4\,a^3bcd^3 - a^4d^4 + 8\,\left(b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4\right)x}{2\,\left(d^7x^2 + 2\,cd^6x + c^2d^5\right)} + \frac{b^4dx^2 - 2\,\left(3\,b^4c - 4\,ab^3d\right)x}{2\,d^4} + \frac{6\,\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)\log\left(dx + c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*
d^4 + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/(d^7
*x^2 + 2*c*d^6*x + c^2*d^5) + 1/2*(b^4*d*x^2 - 2*(3*b^4*c - 4*a*b^3*d)*x)/d
^4 + 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(d*x + c)/d^5
```

**mupad** [B] time = 0.10, size = 196, normalized size = 1.90

$$x\left(\frac{4\,a\,b^3}{d^3} - \frac{3\,b^4\,c}{d^4}\right) - \frac{\frac{a^4\,d^4+4\,a^3\,b\,c\,d^3-18\,a^2\,b^2\,c^2\,d^2+20\,a\,b^3\,c^3\,d-7\,b^4\,c^4}{2\,d} - x\,\left(-4\,a^3\,b\,d^3 + 12\,a^2\,b^2\,c\,d^2 - 12\,a\,b^3\,c^2\,d + 4\,b^4\,c^3\right)}{c^2\,d^4 + 2\,c\,d^5\,x + d^6\,x^2} + \frac{b^4\,x^2}{2\,d^3} + \frac{\ln\left(c + d\,x\right)\,\left(6\,a^2\,b^2\,d^2 - 12\,a\,b^3\,c\,d + 6\,b^4\,c^2\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^3,x)
```

```
[Out] x*((4*a*b^3)/d^3 - (3*b^4*c)/d^4) - ((a^4*d^4 - 7*b^4*c^4 - 18*a^2*b^2*c^2*
d^2 + 20*a*b^3*c^3*d + 4*a^3*b*c*d^3)/(2*d) - x*(4*b^4*c^3 - 4*a^3*b*d^3 +
12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/(c^2*d^4 + d^6*x^2 + 2*c*d^5*x) + (b^4*
x^2)/(2*d^3) + (log(c + d*x)*(6*b^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*c*d))/d^
5
```

**sympy** [A] time = 1.25, size = 185, normalized size = 1.80

$$\frac{b^4x^2}{2d^3} + \frac{6b^2\left(ad - bc\right)^2\log\left(c + dx\right)}{d^5} + x\left(\frac{4ab^3}{d^3} - \frac{3b^4c}{d^4}\right) + \frac{-a^4d^4 - 4a^3bcd^3 + 18a^2b^2c^2d^2 - 20ab^3c^3d + 7b^4c^4 + x\left(-8a^3bd^4 + 24a^2b^2cd^3 - 24ab^3c^2d^2 + 8b^4c^3d\right)}{2c^2d^5 + 4cd^6x + 2d^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**3,x)
```

```
[Out] b**4*x**2/(2*d**3) + 6*b**2*(a*d - b*c)**2*log(c + d*x)/d**5 + x*(4*a*b**3/
d**3 - 3*b**4*c/d**4) + (-a**4*d**4 - 4*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d
```

```
**2 - 20*a*b**3*c**3*d + 7*b**4*c**4 + x*(-8*a**3*b*d**4 + 24*a**2*b**2*c*d
**3 - 24*a*b**3*c**2*d**2 + 8*b**4*c**3*d))/(2*c**2*d**5 + 4*c*d**6*x + 2*d
**7*x**2)
```

## 3.1249 $\int \frac{(a+bx)^3}{(c+dx)^3} dx$

**Optimal**. Leaf size=78

$$-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}-\frac{3b(bc-ad)^2}{d^4(c+dx)}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}+\frac{b^3x}{d^3}$$

**Rubi** [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}-\frac{3b(bc-ad)^2}{d^4(c+dx)}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3 + (b*c - a*d)^3/(2*d^4*(c + d*x)^2) - (3*b*(b*c - a*d)^2)/(d^4*
(c + d*x)) - (3*b^2*(b*c - a*d)*Log[c + d*x])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^3}\,dx &= \int \left(\frac{b^3}{d^3}+\frac{(-bc+ad)^3}{d^3(c+dx)^3}+\frac{3b(bc-ad)^2}{d^3(c+dx)^2}-\frac{3b^2(bc-ad)}{d^3(c+dx)}\right)dx\\ &= \frac{b^3x}{d^3}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}-\frac{3b(bc-ad)^2}{d^4(c+dx)}-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 114, normalized size = 1.46

$$\frac{-a^3d^3-3a^2bd^2(c+2dx)+3ab^2cd(3c+4dx)-6b^2(c+dx)^2(bc-ad)\log(c+dx)+b^3\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)}{2d^4(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (-(a^3*d^3) - 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*c*d*(3*c + 4*d*x) + b^3*(-5
*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) - 6*b^2*(b*c - a*d)*(c + d*x)^2
*Log[c + d*x])/(2*d^4*(c + d*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^3, x]
```

**fricas [B]** time = 1.40, size = 188, normalized size = 2.41

$$\frac{2\,b^3d^3x^3+4\,b^3cd^2x^2-5\,b^3c^3+9\,ab^2c^2d-3\,a^2bcd^2-a^3d^3-2\left(2\,b^3c^2d-6\,ab^2cd^2+3\,a^2bd^3\right)x-6\left(b^3c^3-ab^2c^2d+\left(b^3cd^2-ab^2d^3\right)x^2+2\left(b^3c^2d-ab^2cd^2\right)x\right)\log\left(dx+c\right)}{2\left(d^6x^2+2\,cd^5x+c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 4*b^3*c*d^2*x^2 - 5*b^3*c^3 + 9*a*b^2*c^2*d - 3*a^2*b*
c*d^2 - a^3*d^3 - 2*(2*b^3*c^2*d - 6*a*b^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*
c^3 - a*b^2*c^2*d + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(b^3*c^2*d - a*b^2*c*d^
2)*x)*log(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

**giac [A]** time = 1.28, size = 112, normalized size = 1.44

$$\frac{b^3x}{d^3}-\frac{3\left(b^3c-ab^2d\right)\log\left(|dx+c|\right)}{d^4}-\frac{5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3+6\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x}{2\left(dx+c\right)^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3 - 3*(b^3*c - a*b^2*d)*log(abs(d*x + c))/d^4 - 1/2*(5*b^3*c^3 - 9*
a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/((d*x + c)^2*d^4)
```

**maple [B]** time = 0.01, size = 160, normalized size = 2.05

$$-\frac{a^3}{2\left(dx+c\right)^2d}+\frac{3a^2bc}{2\left(dx+c\right)^2d^2}-\frac{3a\,b^2c^2}{2\left(dx+c\right)^2d^3}+\frac{b^3c^3}{2\left(dx+c\right)^2d^4}-\frac{3a^2b}{\left(dx+c\right)d^2}+\frac{6a\,b^2c}{\left(dx+c\right)d^3}+\frac{3a\,b^2\ln\left(dx+c\right)}{d^3}-\frac{3b^3c^2}{\left(dx+c\right)d^4}-\frac{3b^3c\ln\left(dx+c\right)}{d^4}+\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] b^3/d^3*x-3*b/d^2/(d*x+c)*a^2+6*b^2/d^3/(d*x+c)*a*c-3*b^3/d^4/(d*x+c)*c^2+3
*b^2/d^3*ln(d*x+c)*a-3*b^3/d^4*ln(d*x+c)*c-1/2/d/(d*x+c)^2*a^3+3/2/d^2/(d*x
+c)^2*a^2*b*c-3/2/d^3/(d*x+c)^2*a*b^2*c^2+1/2/d^4/(d*x+c)^2*b^3*c^3
```

**maxima [A]** time = 1.34, size = 125, normalized size = 1.60

$$\frac{b^3x}{d^3} - \frac{5\,b^3c^3 - 9\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3 + 6\left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x}{2\left(d^6x^2 + 2\,cd^5x + c^2d^4\right)} - \frac{3\left(b^3c - ab^2d\right)\log\left(dx + c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3 - 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4) -
3*(b^3*c - a*b^2*d)*log(d*x + c)/d^4
```

**mupad [B]** time = 0.11, size = 130, normalized size = 1.67

$$\frac{b^3\,x}{d^3} - \frac{\ln\left(c + d\,x\right)\,\left(3\,b^3\,c - 3\,a\,b^2\,d\right)}{d^4} - \frac{\frac{a^3\,d^3 + 3\,a^2\,b\,c\,d^2 - 9\,a\,b^2\,c^2\,d + 5\,b^3\,c^3}{2\,d} + x\,\left(3\,a^2\,b\,d^2 - 6\,a\,b^2\,c\,d + 3\,b^3\,c^2\right)}{c^2\,d^3 + 2\,c\,d^4\,x + d^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^3,x)
```

```
[Out] (b^3*x)/d^3 - (log(c + d*x)*(3*b^3*c - 3*a*b^2*d))/d^4 - ((a^3*d^3 + 5*b^3*
c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(2*d) + x*(3*b^3*c^2 + 3*a^2*b*d^2 - 6
*a*b^2*c*d))/(c^2*d^3 + d^5*x^2 + 2*c*d^4*x)
```

**sympy [A]** time = 0.83, size = 128, normalized size = 1.64

$$\frac{b^3x}{d^3} + \frac{3b^2\left(ad - bc\right)\log\left(c + dx\right)}{d^4} + \frac{-a^3d^3 - 3a^2bcd^2 + 9ab^2c^2d - 5b^3c^3 + x\left(-6a^2bd^3 + 12ab^2cd^2 - 6b^3c^2d\right)}{2c^2d^4 + 4cd^5x + 2d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] b**3*x/d**3 + 3*b**2*(a*d - b*c)*log(c + d*x)/d**4 + (-a**3*d**3 - 3*a**2*b
*c*d**2 + 9*a*b**2*c**2*d - 5*b**3*c**3 + x*(-6*a**2*b*d**3 + 12*a*b**2*c*d
**2 - 6*b**3*c**2*d))/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2)
```

## 3.1250 $\int \frac{(a+bx)^2}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=59

$$\frac{2b(bc-ad)}{d^3(c+dx)}-\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

**Rubi [A]** time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(bc-ad)}{d^3(c+dx)}-\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] -(b*c - a*d)^2/(2*d^3*(c + d*x)^2) + (2*b*(b*c - a*d))/(d^3*(c + d*x)) + (b
^2*Log[c + d*x])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^3}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^3}-\frac{2b(bc-ad)}{d^2(c+dx)^2}+\frac{b^2}{d^2(c+dx)}\right)dx\\ &= -\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{2b(bc-ad)}{d^3(c+dx)}+\frac{b^2\log(c+dx)}{d^3}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 48, normalized size = 0.81

$$\frac{\frac{(bc-ad)(ad+3bc+4bdx)}{(c+dx)^2}+2b^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (((b*c - a*d)*(3*b*c + a*d + 4*b*d*x))/(c + d*x)^2 + 2*b^2*Log[c + d*x])/(2
*d^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^3, x]
```

**fricas [A]** time = 1.44, size = 100, normalized size = 1.69

$$\frac{3\,b^2c^2-2\,abcd-a^2d^2+4\left(b^2cd-abd^2\right)x+2\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\log\left(dx+c\right)}{2\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**giac [A]** time = 1.30, size = 69, normalized size = 1.17

$$\frac{b^2\log\left(|dx+c|\right)}{d^3}+\frac{4\left(b^2c-abd\right)x+\frac{3\,b^2c^2-2\,abcd-a^2d^2}{d}}{2\,(dx+c)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(d*x + c))/d^3 + 1/2*(4*(b^2*c - a*b*d)*x + (3*b^2*c^2 - 2*a*b*c
*d - a^2*d^2)/d)/((d*x + c)^2*d^2)
```

**maple [A]** time = 0.01, size = 92, normalized size = 1.56

$$-\frac{a^2}{2\,(dx+c)^2\,d}+\frac{abc}{(dx+c)^2\,d^2}-\frac{b^2c^2}{2\,(dx+c)^2\,d^3}-\frac{2ab}{(dx+c)\,d^2}+\frac{2b^2c}{(dx+c)\,d^3}+\frac{b^2\ln\left(dx+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] -2*b/d^2/(d*x+c)*a+2*b^2/d^3/(d*x+c)*c+b^2/d^3*ln(d*x+c)-1/2/d/(d*x+c)^2*a^
2+1/d^2/(d*x+c)^2*a*b*c-1/2/d^3/(d*x+c)^2*b^2*c^2
```

**maxima [A]** time = 1.33, size = 80, normalized size = 1.36

$$\frac{3\,b^2c^2 - 2\,abcd - a^2d^2 + 4\left(b^2cd - abd^2\right)x}{2\left(d^5x^2 + 2\,cd^4x + c^2d^3\right)} + \frac{b^2\log\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(d^5*x^2 +
2*c*d^4*x + c^2*d^3) + b^2*log(d*x + c)/d^3
```

**mupad [B]** time = 0.23, size = 77, normalized size = 1.31

$$\frac{b^2\,\ln\left(c + d\,x\right)}{d^3} - \frac{\frac{a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2}{2\,d^3} + \frac{2\,b\,x\,\left(a\,d-b\,c\right)}{d^2}}{c^2 + 2\,c\,d\,x + d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^3,x)
```

```
[Out] (b^2*log(c + d*x))/d^3 - ((a^2*d^2 - 3*b^2*c^2 + 2*a*b*c*d)/(2*d^3) + (2*b*
x*(a*d - b*c))/d^2)/(c^2 + d^2*x^2 + 2*c*d*x)
```

**sympy [A]** time = 0.45, size = 80, normalized size = 1.36

$$\frac{b^2\log\left(c + dx\right)}{d^3} + \frac{-a^2d^2 - 2abcd + 3b^2c^2 + x\left(-4abd^2 + 4b^2cd\right)}{2c^2d^3 + 4cd^4x + 2d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] b**2*log(c + d*x)/d**3 + (-a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2 + x*(-4*a*b*
d**2 + 4*b**2*c*d))/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2)
```

## 3.1251 $\int \frac{a+bx}{(c+dx)^3}\,dx$

**Optimal**. Leaf size=28

$$\frac{(a+bx)^2}{2(c+dx)^2(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {37}

$$\frac{(a+bx)^2}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^3,x]
```

```
[Out] (a + b*x)^2/(2*(b*c - a*d)*(c + d*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx}{(c+dx)^3}\,dx = \frac{(a+bx)^2}{2(bc-ad)(c+dx)^2}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2d^2(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^3,x]
```

```
[Out] -1/2*(a*d + b*(c + 2*d*x))/(d^2*(c + d*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(c + d*x)^3, x]
```

**fricas [A]** time = 0.99, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\left(d^4x^2+2\,cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

**giac [A]** time = 1.25, size = 24, normalized size = 0.86

$$-\frac{2\,bdx+bc+ad}{2\,(dx+c)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/((d*x + c)^2*d^2)
```

**maple [A]** time = 0.00, size = 35, normalized size = 1.25

$$-\frac{b}{(dx+c)\,d^2}-\frac{ad-bc}{2\,(dx+c)^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^3,x)
```

```
[Out] -1/(d*x+c)*b/d^2-1/2*(a*d-b*c)/d^2/(d*x+c)^2
```

**maxima [A]** time = 1.36, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\left(d^4x^2+2\,cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

**mupad [B]** time = 0.03, size = 39, normalized size = 1.39

$$-\frac{\frac{a\,d+b\,c}{2\,d^2}+\frac{b\,x}{d}}{c^2+2\,c\,d\,x+d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^3,x)
```

```
[Out] -((a*d + b*c)/(2*d^2) + (b*x)/d)/(c^2 + d^2*x^2 + 2*c*d*x)
```

**sympy [A]** time = 0.26, size = 39, normalized size = 1.39

$$\frac{-ad-bc-2bdx}{2c^2d^2+4cd^3x+2d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**3,x)
```

```
[Out] (-a*d - b*c - 2*b*d*x)/(2*c**2*d**2 + 4*c*d**3*x + 2*d**4*x**2)
```

## 3.1252 $\int \frac{1}{(c+dx)^3} \, dx$

**Optimal**. Leaf size=14

$$-\frac{1}{2d(c+dx)^2}$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-3),x]
```

```
[Out] -1/(2*d*(c + d*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3} \, dx = -\frac{1}{2d(c+dx)^2}$$

**Mathematica [A]** time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-3),x]
```

```
[Out] -1/2*1/(d*(c + d*x)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^(-3), x]
```

**fricas** [A] time = 1.24, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

**giac** [A] time = 1.20, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2/((d*x + c)^2*d)
```

**maple** [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{2\,(dx+c)^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3,x)
```

```
[Out] -1/2/d/(d*x+c)^2
```

**maxima** [A] time = 1.34, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((d*x + c)^2*d)
```

**mupad [B]** time = 0.02, size = 26, normalized size = 1.86

$$-\frac{1}{2\,c^2\,d+4\,c\,d^2\,x+2\,d^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^3,x)
```

```
[Out] -1/(2*c^2*d + 2*d^3*x^2 + 4*c*d^2*x)
```

**sympy [B]** time = 0.18, size = 26, normalized size = 1.86

$$-\frac{1}{2c^2d+4cd^2x+2d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3,x)
```

```
[Out] -1/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2)
```

## 3.1253 $\int \frac{1}{(a+bx)(c+dx)^3}\,dx$

**Optimal**. Leaf size=82

$$\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}$$

**Rubi** [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^3),x]
```

```
[Out] 1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b^2*Log[a +
b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^3}\,dx &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{bd}{(bc-ad)^2(c+dx)^2}-\frac{b^2d}{(bc-ad)^3(c+dx}\right.\\ &= \frac{1}{2(bc-ad)(c+dx)^2}+\frac{b}{(bc-ad)^2(c+dx)}+\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 67, normalized size = 0.82

$$\frac{2b^2\log(a+bx)+\frac{(bc-ad)(-ad+3bc+2bdx)}{(c+dx)^2}-2b^2\log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^3),x]
```

```
[Out] (((b*c - a*d)*(3*b*c - a*d + 2*b*d*x))/(c + d*x)^2 + 2*b^2*Log[a + b*x] - 2
*b^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^3), x]
```

**fricas [B]** time = 1.20, size = 242, normalized size = 2.95

$$\frac{3\,b^2c^2 - 4\,abcd + a^2d^2 + 2\left(b^2cd - abd^2\right)x + 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\log\left(bx + a\right) - 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\log\left(dx + c\right)}{2\left(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3 + \left(b^3c^3d^2 - 3\,ab^2c^2d^3 + 3\,a^2bcd^4 - a^3d^5\right)x^2 + 2\left(b^3c^4d - 3\,ab^2c^3d^2 + 3\,a^2bc^2d^3 - a^3cd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(d*x + c))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^
2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^2 + 2*(
b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)
```

**giac [B]** time = 1.36, size = 165, normalized size = 2.01

$$\frac{b^3\log\left(|bx + a|\right)}{b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3} - \frac{b^2d\log\left(|dx + c|\right)}{b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4} + \frac{3\,b^2c^2 - 4\,abcd + a^2d^2 + 2\left(b^2cd - abd^2\right)x}{2\left(bc - ad\right)^3\left(dx + c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^
3) - b^2*d*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4) + 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x)
/((b*c - a*d)^3*(d*x + c)^2)
```

**maple [A]** time = 0.01, size = 81, normalized size = 0.99

$$-\frac{b^2\ln\left(bx + a\right)}{\left(ad - bc\right)^3} + \frac{b^2\ln\left(dx + c\right)}{\left(ad - bc\right)^3} + \frac{b}{\left(ad - bc\right)^2\left(dx + c\right)} - \frac{1}{2\left(ad - bc\right)\left(dx + c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^3,x)
```

```
[Out] -1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c)^2/(d*x+c)-b^
2/(a*d-b*c)^3*ln(b*x+a)
```

**maxima [B]** time = 1.45, size = 202, normalized size = 2.46

$$\frac{b^2 \log(bx+a)}{b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3} - \frac{b^2 \log(dx+c)}{b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3} + \frac{2bdx + 3bc - ad}{2\left(b^2c^4 - 2abc^3d + a^2c^2d^2 + \left(b^2c^2d^2 - 2abcd^3 + a^2d^4\right)x^2 + 2\left(b^2c^3d - 2abc^2d^2 + a^2cd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - b^2*
log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b
*d*x + 3*b*c - a*d)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

**mupad [B]** time = 0.30, size = 183, normalized size = 2.23

$$-\frac{\frac{a\,d-3\,b\,c}{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{c^2+2\,c\,d\,x+d^2\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{{\left(a\,d-b\,c\right)}^3}+\frac{2\,b\,d\,x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3}\right)}{{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^3),x)
```

```
[Out] - ((a*d - 3*b*c)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (b*d*x)/(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))/(c^2 + d^2*x^2 + 2*c*d*x) - (2*b^2*atanh((a^3*d^3 + b^
3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3 + (2*b*d*x*(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
```

**sympy [B]** time = 1.07, size = 381, normalized size = 4.65

$$\frac{b^2 \log\left(x + \frac{-\frac{a^4b^2d^4}{(ad-bc)^3} + \frac{4a^3b^3cd^3}{(ad-bc)^3} - \frac{6a^2b^4c^2d^2}{(ad-bc)^3} + \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d - \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{(ad-bc)^3} - \frac{b^2 \log\left(x + \frac{\frac{a^4b^2d^4}{(ad-bc)^3} - \frac{4a^3b^3cd^3}{(ad-bc)^3} + \frac{6a^2b^4c^2d^2}{(ad-bc)^3} - \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d + \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{(ad-bc)^3} + \frac{-ad + 3bc + 2bdx}{2a^2c^2d^2 - 4abc^3d + 2b^2c^4 + x^2\left(2a^2d^4 - 4abcd^3 + 2b^2c^2d^2\right) + x\left(4a^2cd^3 - 8abc^2d^2 + 4b^2c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**3,x)
```

```
[Out] b**2*log(x + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3/(a*d - b*
c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b**5*c**3*d/(a*d - b*c)*
*3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b*c)*
```

```
*3 - b**2*log(x + (a**4*b**2*d**4/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3/(a*d
- b*c)**3 + 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*c**3*d/(a*d - b
*c)**3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b
*c)**3 + (-a*d + 3*b*c + 2*b*d*x)/(2*a**2*c**2*d**2 - 4*a*b*c**3*d + 2*b**2
*c**4 + x**2*(2*a**2*d**4 - 4*a*b*c*d**3 + 2*b**2*c**2*d**2) + x*(4*a**2*c*
d**3 - 8*a*b*c**2*d**2 + 4*b**2*c**3*d))
```

## 3.1254 $\int \frac{1}{(a+bx)^2(c+dx)^3}\,dx$

**Optimal**. Leaf size=110

$$-\frac{b^2}{(a+bx)(bc-ad)^3}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}-\frac{2bd}{(c+dx)(bc-ad)^3}-\frac{d}{2(c+dx)^2(bc-ad)^2}$$

**Rubi** [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{b^2}{(a+bx)(bc-ad)^3}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}-\frac{2bd}{(c+dx)(bc-ad)^3}-\frac{d}{2(c+dx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -(b^2/((b*c - a*d)^3*(a + b*x))) - d/(2*(b*c - a*d)^2*(c + d*x)^2) - (2*b*d
)/((b*c - a*d)^3*(c + d*x)) - (3*b^2*d*Log[a + b*x])/(b*c - a*d)^4 + (3*b^2
*d*Log[c + d*x])/(b*c - a*d)^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)^3}\,dx &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)^2}-\frac{3b^3d}{(bc-ad)^4(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^3}+\frac{2bd^2}{(bc-ad)^3(c+\right.\\ &= -\frac{b^2}{(bc-ad)^3(a+bx)}-\frac{d}{2(bc-ad)^2(c+dx)^2}-\frac{2bd}{(bc-ad)^3(c+dx)}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}\end{aligned}$$

**Mathematica** [A] time = 0.10, size = 97, normalized size = 0.88

$$-\frac{\frac{2b^2(bc-ad)}{a+bx}+6b^2d\log(a+bx)+\frac{4bd(bc-ad)}{c+dx}+\frac{d(bc-ad)^2}{(c+dx)^2}-6b^2d\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -1/2*((2*b^2*(b*c - a*d))/(a + b*x) + (d*(b*c - a*d)^2)/(c + d*x)^2 + (4*b*
d*(b*c - a*d))/(c + d*x) + 6*b^2*d*Log[a + b*x] - 6*b^2*d*Log[c + d*x])/(b*
c - a*d)^4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^3), x]
```

**fricas [B]** time = 1.61, size = 495, normalized size = 4.50

$$-\frac{2\,b^3c^3+3\,ab^2c^2d-6\,a^2bcd^2+a^3d^3+6\left(b^3cd^2-ab^2d^3\right)x^2+3\left(3\,b^3c^2d-2\,ab^2cd^2-a^2bd^3\right)x+6\left(b^3d^3x^3+ab^2c^2d+\left(2\,b^3cd^2+ab^2d^3\right)x^2+\left(b^3c^2d+2\,ab^2cd^2\right)x\right)\log\left(bx+a\right)-6\left(b^3d^3x^3+ab^2c^2d+\left(2\,b^3cd^2+ab^2d^3\right)x^2+\left(b^3c^2d+2\,ab^2cd^2\right)x\right)\log\left(dx+c\right)}{2\left(ab^4c^6-4\,a^2b^3c^5d+6\,a^3b^2c^4d^2-4\,a^4bc^3d^3+a^5c^2d^4+\left(b^5c^4d^2-4\,ab^4c^3d^3+6\,a^2b^3c^2d^4-4\,a^3b^2cd^5+a^4bd^6\right)x^3+\left(2\,b^5c^5d-7\,ab^4c^4d^2+8\,a^2b^3c^3d^3-2\,a^3b^2c^2d^4-2\,a^4bcd^5+a^5d^6\right)x^2+\left(b^5c^6-2\,ab^4c^5d-2\,a^2b^3c^4d^2+8\,a^3b^2c^3d^3-7\,a^4bc^2d^4+2\,a^5cd^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x + 6*(b^3*d^3
*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c
*d^2)*x)*log(b*x + a) - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2
*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c))/(a*b^4*c^6 - 4*a^2
*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^4*d
^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^3
 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^4 -
 2*a^4*b*c*d^5 + a^5*d^6)*x^2 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^
2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x)
```

**giac [B]** time = 1.27, size = 217, normalized size = 1.97

$$\frac{3\,b^3d\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}-\frac{b^5}{\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)(bx+a)}+\frac{5\,b^2d^3+\frac{6\left(b^4cd^2-ab^3d^3\right)}{(bx+a)b}}{2\,(bc-ad)^4\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 3*b^3*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^5*c^4 - 4*a*b^4*c^3*
d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - b^5/((b^6*c^3 - 3*a*
b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b*x + a)) + 1/2*(5*b^2*d^3 + 6*
(b^4*c*d^2 - a*b^3*d^3)/((b*x + a)*b))/((b*c - a*d)^4*(b*c/(b*x + a) - a*d/
(b*x + a) + d)^2)
```

**maple** [A] time = 0.01, size = 108, normalized size = 0.98

$$-\frac{3b^2d\ln(bx+a)}{(ad-bc)^4}+\frac{3b^2d\ln(dx+c)}{(ad-bc)^4}+\frac{b^2}{(ad-bc)^3(bx+a)}+\frac{2bd}{(ad-bc)^3(dx+c)}-\frac{d}{2(ad-bc)^2(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] -1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+2*d/(a*d-b*c)^3*
b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b*x+a)
```

**maxima** [B] time = 1.59, size = 386, normalized size = 3.51

$$-\frac{3b^2d\log(bx+a)}{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4}+\frac{3b^2d\log(dx+c)}{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4}-\frac{6b^2d^2x^2+2b^2c^2+5abcd-a^2d^2+3(3b^2cd+abd^2)x}{2\left(ab^3c^5-3a^2b^2c^4d+3a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3ab^3c^2d^3+3a^2b^2cd^4-a^3bd^5)x^3+(2b^4c^4d-5ab^3c^3d^2+3a^2b^2c^2d^3+a^3bcd^4-a^4d^5)x^2+(b^4c^5-ab^3c^4d-3a^2b^2c^3d^2+5a^3bc^2d^3-2a^4cd^4)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -3*b^2*d*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) + 3*b^2*d*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(6*b^2*d^2*x^2 + 2*b^2*c^2 + 5
*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)/(a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*
b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*
d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

**mupad** [B] time = 0.40, size = 329, normalized size = 2.99

$$\frac{\frac{-a^2d^2+5abcd+2b^2c^2}{2(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}+\frac{3bx(ad^2+3bcd)}{2(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}+\frac{3b^2d^2x^2}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}}{x(bc^2+2adc)+ac^2+x^2(ad^2+2bcd)+bd^2x^3}-\frac{6b^2d\,\mathrm{atanh}\left(\frac{a^4d^4-2a^3bcd^3+2ab^3c^3d-b^4c^4}{(ad-bc)^4}+\frac{2bdx(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}{(ad-bc)^4}\right)}{(ad-bc)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^3),x)
```

```
[Out] ((2*b^2*c^2 - a^2*d^2 + 5*a*b*c*d)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) + (3*b*x*(a*d^2 + 3*b*c*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2)) + (3*b^2*d^2*x^2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
```

```
*d - 3*a^2*b*c*d^2))/(x*(b*c^2 + 2*a*c*d) + a*c^2 + x^2*(a*d^2 + 2*b*c*d) +
 b*d^2*x^3) - (6*b^2*d*atanh((a^4*d^4 - b^4*c^4 + 2*a*b^3*c^3*d - 2*a^3*b*c
*d^3)/(a*d - b*c)^4 + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2))/(a*d - b*c)^4))/(a*d - b*c)^4
```

**sympy [B]** time = 1.72, size = 632, normalized size = 5.75

$$\frac{3b^2d\log\left(x+\frac{-\frac{3a^5b^2d^6}{(ad-bc)^4}+\frac{15a^4b^3cd^5}{(ad-bc)^4}-\frac{30a^3b^4c^2d^4}{(ad-bc)^4}+\frac{30a^2b^5c^3d^3}{(ad-bc)^4}-\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2+\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{(ad-bc)^4}-\frac{3b^2d\log\left(x+\frac{\frac{3a^5b^2d^6}{(ad-bc)^4}-\frac{15a^4b^3cd^5}{(ad-bc)^4}+\frac{30a^3b^4c^2d^4}{(ad-bc)^4}-\frac{30a^2b^5c^3d^3}{(ad-bc)^4}+\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2-\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{(ad-bc)^4}+\frac{-a^2d^2+5abcd+2b^2c^2+6b^2d^2x^2+x\left(3abd^2+9b^2cd\right)}{2a^4c^2d^3-6a^3bc^3d^2+6a^2b^2c^4d-2ab^3c^5+x^3\left(2a^3bd^5-6a^2b^2cd^4+6ab^3c^2d^3-2b^4c^3d^2\right)+x^2\left(2a^4d^5-2a^3bcd^4-6a^2b^2c^2d^3+10ab^3c^3d^2-4b^4c^4d\right)+x\left(4a^4cd^4-10a^3bc^2d^3+6a^2b^2c^3d^2+2ab^3c^4d-2b^4c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] 3*b**2*d*log(x + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5/(a
*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 + 30*a**2*b**5*c**3*d*
*3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 + 3*
b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 - 3*
b**2*d*log(x + (3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b**3*c*d**5/(a*d
- b*c)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2*b**5*c**3*d**3/
(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 - 3*b**
7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 + (-a**
2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**2 + x*(3*a*b*d**2 + 9*b**
2*c*d))/(2*a**4*c**2*d**3 - 6*a**3*b*c**3*d**2 + 6*a**2*b**2*c**4*d - 2*a*b
**3*c**5 + x**3*(2*a**3*b*d**5 - 6*a**2*b**2*c*d**4 + 6*a*b**3*c**2*d**3 -
2*b**4*c**3*d**2) + x**2*(2*a**4*d**5 - 2*a**3*b*c*d**4 - 6*a**2*b**2*c**2*
d**3 + 10*a*b**3*c**3*d**2 - 4*b**4*c**4*d) + x*(4*a**4*c*d**4 - 10*a**3*b*
c**2*d**3 + 6*a**2*b**2*c**3*d**2 + 2*a*b**3*c**4*d - 2*b**4*c**5))
```

## 3.1255 $\int \frac{1}{(a+bx)^3(c+dx)^3}\,dx$

**Optimal**. Leaf size=143

$$\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}+\frac{3b^2d}{(a+bx)(bc-ad)^4}-\frac{b^2}{2(a+bx)^2(bc-ad)^3}+\frac{3bd^2}{(c+dx)(bc-ad)^4}+\frac{d^2}{2(c+dx)^2(bc-ad)^3}$$

**Rubi [A]** time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}+\frac{3b^2d}{(a+bx)(bc-ad)^4}-\frac{b^2}{2(a+bx)^2(bc-ad)^3}+\frac{3bd^2}{(c+dx)(bc-ad)^4}+\frac{d^2}{2(c+dx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -b^2/(2*(b*c - a*d)^3*(a + b*x)^2) + (3*b^2*d)/((b*c - a*d)^4*(a + b*x)) +
d^2/(2*(b*c - a*d)^3*(c + d*x)^2) + (3*b*d^2)/((b*c - a*d)^4*(c + d*x)) + (
6*b^2*d^2*Log[a + b*x])/(b*c - a*d)^5 - (6*b^2*d^2*Log[c + d*x])/(b*c - a*d
)^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^3}\,dx=\int \left(\frac{b^3}{(bc-ad)^3(a+bx)^3}-\frac{3b^3d}{(bc-ad)^4(a+bx)^2}+\frac{6b^3d^2}{(bc-ad)^5(a+bx)}-\frac{d^3}{(bc-ad)^3(c+\ldots}\right.$$
$$=-\frac{b^2}{2(bc-ad)^3(a+bx)^2}+\frac{3b^2d}{(bc-ad)^4(a+bx)}+\frac{d^2}{2(bc-ad)^3(c+dx)^2}+\frac{3bd^2}{(bc-ad)^4(c+\ldots}$$

**Mathematica [A]** time = 0.12, size = 128, normalized size = 0.90

$$\frac{\frac{6b^2d(bc-ad)}{a+bx}-\frac{b^2(bc-ad)^2}{(a+bx)^2}+12b^2d^2\log(a+bx)+\frac{6bd^2(bc-ad)}{c+dx}+\frac{d^2(bc-ad)^2}{(c+dx)^2}-12b^2d^2\log(c+dx)}{2(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] (-((b^2*(b*c - a*d)^2)/(a + b*x)^2) + (6*b^2*d*(b*c - a*d))/(a + b*x) + (d^
2*(b*c - a*d)^2)/(c + d*x)^2 + (6*b*d^2*(b*c - a*d))/(c + d*x) + 12*b^2*d^2
*Log[a + b*x] - 12*b^2*d^2*Log[c + d*x])/(2*(b*c - a*d)^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^3), x]
```

**fricas [B]** time = 1.18, size = 760, normalized size = 5.31

$$-\frac{b^4c^4 - 8\,ab^3c^3d + 8\,a^3bcd^3 - a^4d^4 - 12\left(b^4cd^3 - ab^3d^4\right)x^3 - 18\left(b^4c^2d^2 - a^2b^2d^4\right)x^2 - 4\left(b^4c^3d + 6\,ab^3c^2d^2 - 6\,a^2b^2cd^3 - a^3bd^4\right)x - 12\left(b^4d^4x^4 + a^2b^2c^2d^2 + 2\left(b^4cd^3 + ab^3d^4\right)x^3 + \left(b^4c^2d^2 + 4\,ab^3cd^3 + a^2b^2d^4\right)x^2 + 2\left(ab^3c^2d^2 + a^2b^2cd^3\right)x\right)\log\left(bx + a\right) + 12\left(b^4d^4x^4 + a^2b^2c^2d^2 + 2\left(b^4cd^3 + ab^3d^4\right)x^3 + \left(b^4c^2d^2 + 4\,ab^3cd^3 + a^2b^2d^4\right)x^2 + 2\left(ab^3c^2d^2 + a^2b^2cd^3\right)x\right)\log\left(dx + c\right)}{2\left(a^2b^5c^7 - 5\,a^3b^4c^6d + 10\,a^4b^3c^5d^2 - 10\,a^5b^2c^4d^3 + 5\,a^6bc^3d^4 - a^7c^2d^5 + \left(b^7c^5d^2 - 5\,ab^6c^4d^3 + 10\,a^2b^5c^3d^4 - 10\,a^3b^4c^2d^5 + 5\,a^4b^3cd^6 - a^5b^2d^7\right)x^4 + 2\left(b^7c^6d - 4\,ab^6c^5d^2 + 5\,a^2b^5c^4d^3 - 5\,a^4b^3c^2d^5 + 4\,a^5b^2cd^6 - a^6bd^7\right)x^3 + \left(b^7c^7 - ab^6c^6d - 9\,a^2b^5c^5d^2 + 25\,a^3b^4c^4d^3 - 25\,a^4b^3c^3d^4 + 9\,a^5b^2c^2d^5 + a^6bcd^6 - a^7d^7\right)x^2 + 2\left(ab^6c^7 - 4\,a^2b^5c^6d + 5\,a^3b^4c^5d^2 - 5\,a^5b^2c^3d^4 + 4\,a^6bc^2d^5 - a^7cd^6\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*c^4 - 8*a*b^3*c^3*d + 8*a^3*b*c*d^3 - a^4*d^4 - 12*(b^4*c*d^3 - a
*b^3*d^4)*x^3 - 18*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d + 6*a*b^3
*c^2*d^2 - 6*a^2*b^2*c*d^3 - a^3*b*d^4)*x - 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d
^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2
*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a) + 12*(b^4*d^4
*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a
*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*
x + c))/(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^
4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5 + (b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10
*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^4
+ 2*(b^7*c^6*d - 4*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 +
4*a^5*b^2*c*d^6 - a^6*b*d^7)*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d
^2 + 25*a^3*b^4*c^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*
d^6 - a^7*d^7)*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 - 5
*a^5*b^2*c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*x)
```

**giac [B]** time = 1.28, size = 345, normalized size = 2.41

$$\frac{6\,b^3d^2\log\left(\left|bx + a\right|\right)}{b^6c^5 - 5\,ab^5c^4d + 10\,a^2b^4c^3d^2 - 10\,a^3b^3c^2d^3 + 5\,a^4b^2cd^4 - a^5bd^5} - \frac{6\,b^2d^3\log\left(\left|dx + c\right|\right)}{b^5c^5d - 5\,ab^4c^4d^2 + 10\,a^2b^3c^3d^3 - 10\,a^3b^2c^2d^4 + 5\,a^4bcd^5 - a^5d^6} + \frac{12\,b^3d^3x^3 + 18\,b^3cd^2x^2 + 18\,ab^2d^3x^2 + 4\,b^3c^2dx + 28\,ab^2cd^2x + 4\,a^2bd^3x - b^3c^3 + 7\,ab^2c^2d + 7\,a^2bcd^2 - a^3d^3}{2\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)\left(bdx^2 + bcx + adx + ac\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 6*b^3*d^2*log(abs(b*x + a))/(b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 -
 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5) - 6*b^2*d^3*log(abs(d*x
+ c))/(b^5*c^5*d - 5*a*b^4*c^4*d^2 + 10*a^2*b^3*c^3*d^3 - 10*a^3*b^2*c^2*d^
4 + 5*a^4*b*c*d^5 - a^5*d^6) + 1/2*(12*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 18*
a*b^2*d^3*x^2 + 4*b^3*c^2*d*x + 28*a*b^2*c*d^2*x + 4*a^2*b*d^3*x - b^3*c^3
+ 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*d*x^2 + b*c*x + a*d*x + a*c)^2)
```

**maple [A]** time = 0.01, size = 140, normalized size = 0.98

$$-\frac{6b^2d^2\ln(bx+a)}{(ad-bc)^5}+\frac{6b^2d^2\ln(dx+c)}{(ad-bc)^5}+\frac{3b^2d}{(ad-bc)^4(bx+a)}+\frac{3bd^2}{(ad-bc)^4(dx+c)}+\frac{b^2}{2(ad-bc)^3(bx+a)^2}-\frac{d^2}{2(ad-bc)^3(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c)+3*d^2/(a*d-b
*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c)^5*b^2*ln(b*x+
a)+3*b^2/(a*d-b*c)^4*d/(b*x+a)
```

**maxima [B]** time = 1.55, size = 594, normalized size = 4.15

$$\frac{6b^2d^2\log(bx+a)}{b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5}-\frac{6b^2d^2\log(dx+c)}{b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5}+\frac{12b^3d^3x^3-b^3c^3+7ab^2c^2d+7a^2bcd^2-a^3d^3+18\left(b^3cd^2+ab^2d^3\right)x^2+4\left(b^3c^2d+7ab^2cd^2+a^2bd^3\right)x}{2\left(a^2b^4c^6-4a^3b^3c^5d+6a^4b^2c^4d^2-4a^5bc^3d^3+a^6c^2d^4+\left(b^6c^4d^2-4ab^5c^3d^3+6a^2b^4c^2d^4-4a^3b^3cd^5+a^4b^2d^6\right)x^4+2\left(b^6c^5d-3ab^5c^4d^2+2a^2b^4c^3d^3+2a^3b^3c^2d^4-3a^4b^2cd^5+a^5bd^6\right)x^3+\left(b^6c^6-9a^2b^4c^4d^2+16a^3b^3c^3d^3-9a^4b^2c^2d^4+a^6d^6\right)x^2+2\left(ab^5c^6-3a^2b^4c^5d+2a^3b^3c^4d^2+2a^4b^2c^3d^3-3a^5bc^2d^4+a^6cd^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 6*b^2*d^2*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a
^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - 6*b^2*d^2*log(d*x + c)/(b^5*c^5
 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
- a^5*d^5) + 1/2*(12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2
- a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 +
 a^2*b*d^3)*x)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b
*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4
 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*
a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b
^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d
^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^
3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

**mupad [B]** time = 0.53, size = 542, normalized size = 3.79

$$\frac{\frac{6b^3d^3x^3}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}-\frac{a^3d^3-7a^2bcd^2-7ab^2c^2d+b^3c^3}{2\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}+\frac{9bdx^2\left(cb^2d+abd^2\right)}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+\frac{2bdx\left(a^2d^2+7abcd+b^2c^2\right)}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}}{x\left(2da^2c+2bac^2\right)+x^2\left(a^2d^2+4abcd+b^2c^2\right)+x^3\left(2cb^2d+2abd^2\right)+a^2c^2+b^2d^2x^4}-\frac{12b^2d^2\,\mathrm{atanh}\left(\frac{a^5d^5-3a^4bcd^4+2a^3b^2c^2d^3+2a^2b^3c^3d^2-3ab^4c^4d+b^5c^5}{(ad-bc)^5}+\frac{2bdx\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{(ad-bc)^5}\right)}{(ad-bc)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^3),x)
```

```
[Out] ((6*b^3*d^3*x^3)/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3) - (a^3*d^3 + b^3*c^3 - 7*a*b^2*c^2*d - 7*a^2*b*c*d^2)/(2*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (9*b
*d*x^2*(a*b*d^2 + b^2*c*d))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3) + (2*b*d*x*(a^2*d^2 + b^2*c^2 + 7*a*b*c*d))/(a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(x*(2*a*
b*c^2 + 2*a^2*c*d) + x^2*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + x^3*(2*a*b*d^2 +
 2*b^2*c*d) + a^2*c^2 + b^2*d^2*x^4) - (12*b^2*d^2*atanh((a^5*d^5 + b^5*c^5
 + 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - 3*a*b^4*c^4*d - 3*a^4*b*c*d^4)/(
a*d - b*c)^5 + (2*b*d*x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^
3*d - 4*a^3*b*c*d^3))/(a*d - b*c)^5))/(a*d - b*c)^5
```

**sympy [B]** time = 2.42, size = 881, normalized size = 6.16

$$\frac{6b^2d^2\log\left(x+\frac{-\frac{6a^6b^2d^8}{(ad-bc)^5}+\frac{36a^5b^3cd^7}{(ad-bc)^5}-\frac{90a^4b^4c^2d^6}{(ad-bc)^5}+\frac{120a^3b^5c^3d^5}{(ad-bc)^5}-\frac{90a^2b^6c^4d^4}{(ad-bc)^5}+\frac{36ab^7c^5d^3}{(ad-bc)^5}+6ab^2d^3-\frac{6b^8c^6d^2}{(ad-bc)^5}+6b^3cd^2}{12b^3d^3}\right)}{(ad-bc)^5}-\frac{6b^2d^2\log\left(x+\frac{\frac{6a^6b^2d^8}{(ad-bc)^5}-\frac{36a^5b^3cd^7}{(ad-bc)^5}+\frac{90a^4b^4c^2d^6}{(ad-bc)^5}-\frac{120a^3b^5c^3d^5}{(ad-bc)^5}+\frac{90a^2b^6c^4d^4}{(ad-bc)^5}-\frac{36ab^7c^5d^3}{(ad-bc)^5}+6ab^2d^3+\frac{6b^8c^6d^2}{(ad-bc)^5}+6b^3cd^2}{12b^3d^3}\right)}{(ad-bc)^5}+\frac{-a^3d^3+7a^2bcd^2+7ab^2c^2d-b^3c^3+12b^3d^3x^3+x^2\left(18ab^2d^3+18b^3cd^2\right)+x\left(4a^2bd^3+28ab^2cd^2+4b^3c^2d\right)}{2a^6c^2d^4-8a^5bc^3d^3+12a^4b^2c^4d^2-8a^3b^3c^5d+2a^2b^4c^6+x^4\left(2a^4b^2d^6-8a^3b^3cd^5+12a^2b^4c^2d^4-8ab^5c^3d^3+2b^6c^4d^2\right)+x^3\left(4a^5bd^6-12a^4b^2cd^5+8a^3b^3c^2d^4+8a^2b^4c^3d^3-12ab^5c^4d^2+4b^6c^5d\right)+x^2\left(2a^6d^6-18a^4b^2c^2d^4+32a^3b^3c^3d^3-18a^2b^4c^4d^2+2b^6c^6\right)+x\left(4a^6cd^5-12a^5bc^2d^4+8a^4b^2c^3d^3+8a^3b^3c^4d^2-12a^2b^4c^5d+4ab^5c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] 6*b**2*d**2*log(x + (-6*a**6*b**2*d**8/(a*d - b*c)**5 + 36*a**5*b**3*c*d**7
/(a*d - b*c)**5 - 90*a**4*b**4*c**2*d**6/(a*d - b*c)**5 + 120*a**3*b**5*c**
3*d**5/(a*d - b*c)**5 - 90*a**2*b**6*c**4*d**4/(a*d - b*c)**5 + 36*a*b**7*c
**5*d**3/(a*d - b*c)**5 + 6*a*b**2*d**3 - 6*b**8*c**6*d**2/(a*d - b*c)**5 +
 6*b**3*c*d**2)/(12*b**3*d**3))/(a*d - b*c)**5 - 6*b**2*d**2*log(x + (6*a**
6*b**2*d**8/(a*d - b*c)**5 - 36*a**5*b**3*c*d**7/(a*d - b*c)**5 + 90*a**4*b
**4*c**2*d**6/(a*d - b*c)**5 - 120*a**3*b**5*c**3*d**5/(a*d - b*c)**5 + 90*
a**2*b**6*c**4*d**4/(a*d - b*c)**5 - 36*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6
*a*b**2*d**3 + 6*b**8*c**6*d**2/(a*d - b*c)**5 + 6*b**3*c*d**2)/(12*b**3*d*
*3))/(a*d - b*c)**5 + (-a**3*d**3 + 7*a**2*b*c*d**2 + 7*a*b**2*c**2*d - b**
3*c**3 + 12*b**3*d**3*x**3 + x**2*(18*a*b**2*d**3 + 18*b**3*c*d**2) + x*(4*
a**2*b*d**3 + 28*a*b**2*c*d**2 + 4*b**3*c**2*d))/(2*a**6*c**2*d**4 - 8*a**5
*b*c**3*d**3 + 12*a**4*b**2*c**4*d**2 - 8*a**3*b**3*c**5*d + 2*a**2*b**4*c*
*6 + x**4*(2*a**4*b**2*d**6 - 8*a**3*b**3*c*d**5 + 12*a**2*b**4*c**2*d**4 -
 8*a*b**5*c**3*d**3 + 2*b**6*c**4*d**2) + x**3*(4*a**5*b*d**6 - 12*a**4*b**
2*c*d**5 + 8*a**3*b**3*c**2*d**4 + 8*a**2*b**4*c**3*d**3 - 12*a*b**5*c**4*d
**2 + 4*b**6*c**5*d) + x**2*(2*a**6*d**6 - 18*a**4*b**2*c**2*d**4 + 32*a**3
*b**3*c**3*d**3 - 18*a**2*b**4*c**4*d**2 + 2*b**6*c**6) + x*(4*a**6*c*d**5
- 12*a**5*b*c**2*d**4 + 8*a**4*b**2*c**3*d**3 + 8*a**3*b**3*c**4*d**2 - 12*
a**2*b**4*c**5*d + 4*a*b**5*c**6))
```

## 3.1256 $\int \frac{(a+bx)^9}{(c+dx)^8} dx$

**Optimal**. Leaf size=232

$$-\frac{b^8x(8bc-9ad)}{d^9}+\frac{36b^7(bc-ad)^2\log(c+dx)}{d^{10}}+\frac{84b^6(bc-ad)^3}{d^{10}(c+dx)}-\frac{63b^5(bc-ad)^4}{d^{10}(c+dx)^2}+\frac{42b^4(bc-ad)^5}{d^{10}(c+dx)^3}-\frac{21b^3(bc-ad)^6}{d^{10}(c+dx)^4}$$

**Rubi** [A] time = 0.36, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b^8x(8bc-9ad)}{d^9}+\frac{84b^6(bc-ad)^3}{d^{10}(c+dx)}-\frac{63b^5(bc-ad)^4}{d^{10}(c+dx)^2}+\frac{42b^4(bc-ad)^5}{d^{10}(c+dx)^3}-\frac{21b^3(bc-ad)^6}{d^{10}(c+dx)^4}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}+\frac{36b^7(bc-ad)^2\log(c+dx)}{d^{10}}-\frac{3b(bc-ad)^8}{2d^{10}(c+dx)^6}+\frac{(bc-ad)^9}{7d^{10}(c+dx)^7}+\frac{b^9x^2}{2d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9/(c + d*x)^8,x]
```

```
[Out] -((b^8*(8*b*c - 9*a*d)*x)/d^9) + (b^9*x^2)/(2*d^8) + (b*c - a*d)^9/(7*d^10*
(c + d*x)^7) - (3*b*(b*c - a*d)^8)/(2*d^10*(c + d*x)^6) + (36*b^2*(b*c - a*
d)^7)/(5*d^10*(c + d*x)^5) - (21*b^3*(b*c - a*d)^6)/(d^10*(c + d*x)^4) + (4
2*b^4*(b*c - a*d)^5)/(d^10*(c + d*x)^3) - (63*b^5*(b*c - a*d)^4)/(d^10*(c +
 d*x)^2) + (84*b^6*(b*c - a*d)^3)/(d^10*(c + d*x)) + (36*b^7*(b*c - a*d)^2*
Log[c + d*x])/d^10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^9}{(c+dx)^8} dx = \int \left(-\frac{b^8(8bc-9ad)}{d^9}+\frac{b^9x}{d^8}+\frac{(-bc+ad)^9}{d^9(c+dx)^8}+\frac{9b(bc-ad)^8}{d^9(c+dx)^7}-\frac{36b^2(bc-ad)^7}{d^9(c+dx)^6}+\frac{84b^3(bc-ad)^6}{d^9(c+dx)^5}\right.$$

$$= -\frac{b^8(8bc-9ad)x}{d^9}+\frac{b^9x^2}{2d^8}+\frac{(bc-ad)^9}{7d^{10}(c+dx)^7}-\frac{3b(bc-ad)^8}{2d^{10}(c+dx)^6}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}-\frac{21b^3(bc-ad)^6}{d^{10}(c+dx)^4}$$

**Mathematica** [B] time = 0.27, size = 584, normalized size = 2.52

$$-\frac{10a^9d^9+15a^8bd^8(c+7dx)+24a^7b^2d^7\left(c^2+7cdx+21d^2x^2\right)+42a^6b^3d^6\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right)+84a^5b^4d^5\left(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4\right)+210a^4b^5d^4\left(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5\right)+840a^3b^6d^3\left(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6\right)-6a^2b^7cd^2\left(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6\right)+6ab^8d\left(1443c^8+9261c^7dx+24843c^6d^2x^2+35525c^5d^3x^3+28175c^4d^4x^4+11025c^3d^5x^5+735c^2d^6x^6-735cd^7x^7-105d^8x^8\right)-2520b^7(c+dx)^7(bc-ad)^2\log(c+dx)-\left(b^9\left(3349c^9+20923c^8dx+53949c^7d^2x^2+72275c^6d^3x^3+50225c^5d^4x^4+12495c^4d^5x^5-4655c^3d^6x^6-3185c^2d^7x^7-315cd^8x^8+35d^9x^9\right)\right)}{70d^{10}(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9/(c + d*x)^8,x]
```

```
[Out] -1/70*(10*a^9*d^9 + 15*a^8*b*d^8*(c + 7*d*x) + 24*a^7*b^2*d^7*(c^2 + 7*c*d*
x + 21*d^2*x^2) + 42*a^6*b^3*d^6*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x
^3) + 84*a^5*b^4*d^5*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*
d^4*x^4) + 210*a^4*b^5*d^4*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x
^3 + 35*c*d^4*x^4 + 21*d^5*x^5) + 840*a^3*b^6*d^3*(c^6 + 7*c^5*d*x + 21*c^4
*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6) - 6*
a^2*b^7*c*d^2*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*
x^3 + 26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6) + 6*a*b^8*d*(1443
*c^8 + 9261*c^7*d*x + 24843*c^6*d^2*x^2 + 35525*c^5*d^3*x^3 + 28175*c^4*d^4
*x^4 + 11025*c^3*d^5*x^5 + 735*c^2*d^6*x^6 - 735*c*d^7*x^7 - 105*d^8*x^8) -
 b^9*(3349*c^9 + 20923*c^8*d*x + 53949*c^7*d^2*x^2 + 72275*c^6*d^3*x^3 + 50
225*c^5*d^4*x^4 + 12495*c^4*d^5*x^5 - 4655*c^3*d^6*x^6 - 3185*c^2*d^7*x^7 -
 315*c*d^8*x^8 + 35*d^9*x^9) - 2520*b^7*(b*c - a*d)^2*(c + d*x)^7*Log[c + d
*x])/(d^10*(c + d*x)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^9}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^9/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^9/(c + d*x)^8, x]
```

**fricas [B]** time = 0.96, size = 1093, normalized size = 4.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/70*(35*b^9*d^9*x^9 + 3349*b^9*c^9 - 8658*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d
^2 - 840*a^3*b^6*c^6*d^3 - 210*a^4*b^5*c^5*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^
6*b^3*c^3*d^6 - 24*a^7*b^2*c^2*d^7 - 15*a^8*b*c*d^8 - 10*a^9*d^9 - 315*(b^9
*c*d^8 - 2*a*b^8*d^9)*x^8 - 245*(13*b^9*c^2*d^7 - 18*a*b^8*c*d^8)*x^7 - 245
*(19*b^9*c^3*d^6 + 18*a*b^8*c^2*d^7 - 72*a^2*b^7*c*d^8 + 24*a^3*b^6*d^9)*x^
6 + 735*(17*b^9*c^4*d^5 - 90*a*b^8*c^3*d^6 + 108*a^2*b^7*c^2*d^7 - 24*a^3*b
^6*c*d^8 - 6*a^4*b^5*d^9)*x^5 + 245*(205*b^9*c^5*d^4 - 690*a*b^8*c^4*d^5 +
660*a^2*b^7*c^3*d^6 - 120*a^3*b^6*c^2*d^7 - 30*a^4*b^5*c*d^8 - 12*a^5*b^4*d
^9)*x^4 + 245*(295*b^9*c^6*d^3 - 870*a*b^8*c^5*d^4 + 750*a^2*b^7*c^4*d^5 -
```

```
120*a^3*b^6*c^3*d^6 - 30*a^4*b^5*c^2*d^7 - 12*a^5*b^4*c*d^8 - 6*a^6*b^3*d^9
)*x^3 + 21*(2569*b^9*c^7*d^2 - 7098*a*b^8*c^6*d^3 + 5754*a^2*b^7*c^5*d^4 -
840*a^3*b^6*c^4*d^5 - 210*a^4*b^5*c^3*d^6 - 84*a^5*b^4*c^2*d^7 - 42*a^6*b^3
*c*d^8 - 24*a^7*b^2*d^9)*x^2 + 7*(2989*b^9*c^8*d - 7938*a*b^8*c^7*d^2 + 617
4*a^2*b^7*c^6*d^3 - 840*a^3*b^6*c^5*d^4 - 210*a^4*b^5*c^4*d^5 - 84*a^5*b^4*
c^3*d^6 - 42*a^6*b^3*c^2*d^7 - 24*a^7*b^2*c*d^8 - 15*a^8*b*d^9)*x + 2520*(b
^9*c^9 - 2*a*b^8*c^8*d + a^2*b^7*c^7*d^2 + (b^9*c^2*d^7 - 2*a*b^8*c*d^8 + a
^2*b^7*d^9)*x^7 + 7*(b^9*c^3*d^6 - 2*a*b^8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 2
1*(b^9*c^4*d^5 - 2*a*b^8*c^3*d^6 + a^2*b^7*c^2*d^7)*x^5 + 35*(b^9*c^5*d^4 -
 2*a*b^8*c^4*d^5 + a^2*b^7*c^3*d^6)*x^4 + 35*(b^9*c^6*d^3 - 2*a*b^8*c^5*d^4
 + a^2*b^7*c^4*d^5)*x^3 + 21*(b^9*c^7*d^2 - 2*a*b^8*c^6*d^3 + a^2*b^7*c^5*d
^4)*x^2 + 7*(b^9*c^8*d - 2*a*b^8*c^7*d^2 + a^2*b^7*c^6*d^3)*x)*log(d*x + c)
)/(d^17*x^7 + 7*c*d^16*x^6 + 21*c^2*d^15*x^5 + 35*c^3*d^14*x^4 + 35*c^4*d^1
3*x^3 + 21*c^5*d^12*x^2 + 7*c^6*d^11*x + c^7*d^10)
```

**giac** [**B**] time = 1.32, size = 723, normalized size = 3.12

$$\frac{36\,(b^9c^2-2\,ab^8cd+a^2b^7d^2)\log\left(|dx+c|\right)}{d^{10}}+\frac{b^9d^8x^2-16\,b^9cd^7x+18\,ab^8d^8x}{2\,d^{16}}+\frac{3349\,b^9c^9-8658\,ab^8c^8d+6534\,a^2b^7c^7d^2-840\,a^3b^6c^6d^3-210\,a^4b^5c^5d^4-84\,a^5b^4c^4d^5-42\,a^6b^3c^3d^6-24\,a^7b^2c^2d^7-15\,a^8bcd^8-10\,a^9d^9+5880\,(b^9c^3d^6-3\,ab^8c^2d^7+3\,a^2b^7cd^8-a^3b^6d^9)x^6+4410\,(7\,b^9c^4d^5-20\,ab^8c^3d^6+18\,a^2b^7c^2d^7-4\,a^3b^6cd^8-a^4b^5d^9)x^5+1470\,(47\,b^9c^5d^4-130\,ab^8c^4d^5+110\,a^2b^7c^3d^6-20\,a^3b^6c^2d^7-5\,a^4b^5cd^8-2\,a^5b^4d^9)x^4+1470\,(57\,b^9c^6d^3-154\,ab^8c^5d^4+125\,a^2b^7c^4d^5-20\,a^3b^6c^3d^6-5\,a^4b^5c^2d^7-2\,a^5b^4cd^8-a^6b^3d^9)x^3+126\,(459\,b^9c^7d^2-1218\,ab^8c^6d^3+959\,a^2b^7c^5d^4-140\,a^3b^6c^4d^5-35\,a^4b^5c^3d^6-14\,a^5b^4c^2d^7-7\,a^6b^3cd^8-4\,a^7b^2d^9)x^2+21\,(1023\,b^9c^8d-2676\,ab^8c^7d^2+2058\,a^2b^7c^6d^3-280\,a^3b^6c^5d^4-70\,a^4b^5c^4d^5-28\,a^5b^4c^3d^6-14\,a^6b^3c^2d^7-8\,a^7b^2cd^8-5\,a^8bd^9)x}{70\,(dx+c)^7d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 36*(b^9*c^2 - 2*a*b^8*c*d + a^2*b^7*d^2)*log(abs(d*x + c))/d^10 + 1/2*(b^9*
d^8*x^2 - 16*b^9*c*d^7*x + 18*a*b^8*d^8*x)/d^16 + 1/70*(3349*b^9*c^9 - 8658
*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d^2 - 840*a^3*b^6*c^6*d^3 - 210*a^4*b^5*c^5
*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^6*b^3*c^3*d^6 - 24*a^7*b^2*c^2*d^7 - 15*a^
8*b*c*d^8 - 10*a^9*d^9 + 5880*(b^9*c^3*d^6 - 3*a*b^8*c^2*d^7 + 3*a^2*b^7*c*
d^8 - a^3*b^6*d^9)*x^6 + 4410*(7*b^9*c^4*d^5 - 20*a*b^8*c^3*d^6 + 18*a^2*b^
7*c^2*d^7 - 4*a^3*b^6*c*d^8 - a^4*b^5*d^9)*x^5 + 1470*(47*b^9*c^5*d^4 - 130
*a*b^8*c^4*d^5 + 110*a^2*b^7*c^3*d^6 - 20*a^3*b^6*c^2*d^7 - 5*a^4*b^5*c*d^8
 - 2*a^5*b^4*d^9)*x^4 + 1470*(57*b^9*c^6*d^3 - 154*a*b^8*c^5*d^4 + 125*a^2*
b^7*c^4*d^5 - 20*a^3*b^6*c^3*d^6 - 5*a^4*b^5*c^2*d^7 - 2*a^5*b^4*c*d^8 - a^
6*b^3*d^9)*x^3 + 126*(459*b^9*c^7*d^2 - 1218*a*b^8*c^6*d^3 + 959*a^2*b^7*c^
5*d^4 - 140*a^3*b^6*c^4*d^5 - 35*a^4*b^5*c^3*d^6 - 14*a^5*b^4*c^2*d^7 - 7*a
^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 + 21*(1023*b^9*c^8*d - 2676*a*b^8*c^7*d^2
 + 2058*a^2*b^7*c^6*d^3 - 280*a^3*b^6*c^5*d^4 - 70*a^4*b^5*c^4*d^5 - 28*a^5
*b^4*c^3*d^6 - 14*a^6*b^3*c^2*d^7 - 8*a^7*b^2*c*d^8 - 5*a^8*b*d^9)*x)/((d*x
 + c)^7*d^10)
```

**maple** [**B**] time = 0.02, size = 1035, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9/(d*x+c)^8,x)
```

```
[Out] 1/2*b^9*x^2/d^8-21*b^9/d^10/(d*x+c)^4*c^6+36/5*b^9/d^10/(d*x+c)^5*c^7+1/7/d
^10/(d*x+c)^7*b^9*c^9-36/5*b^2/d^3/(d*x+c)^5*a^7+9*b^8/d^8*a*x-8*b^9/d^9*x*
c-84*b^6/d^7/(d*x+c)*a^3+84*b^9/d^10/(d*x+c)*c^3+36*b^7/d^8*ln(d*x+c)*a^2+3
6*b^9/d^10*ln(d*x+c)*c^2-3/2*b/d^2/(d*x+c)^6*a^8-3/2*b^9/d^10/(d*x+c)^6*c^8
-42*b^4/d^5/(d*x+c)^3*a^5+42*b^9/d^10/(d*x+c)^3*c^5-63*b^5/d^6/(d*x+c)^2*a^
4-63*b^9/d^10/(d*x+c)^2*c^4-21*b^3/d^4/(d*x+c)^4*a^6-1/7/d/(d*x+c)^7*a^9+25
2*b^6/d^7/(d*x+c)^2*a^3*c-378*b^7/d^8/(d*x+c)^2*a^2*c^2+252*b^8/d^9/(d*x+c)
^2*a*c^3-36/7/d^3/(d*x+c)^7*a^7*b^2*c^2+126*b^4/d^5/(d*x+c)^4*a^5*c-315*b^5
/d^6/(d*x+c)^4*a^4*c^2+420*b^6/d^7/(d*x+c)^4*a^3*c^3-315*b^7/d^8/(d*x+c)^4*
a^2*c^4+126*b^8/d^9/(d*x+c)^4*a*c^5+9/7/d^2/(d*x+c)^7*a^8*b*c+252/5*b^3/d^4
/(d*x+c)^5*a^6*c-756/5*b^4/d^5/(d*x+c)^5*a^5*c^2+252*b^5/d^6/(d*x+c)^5*a^4*
c^3-252*b^6/d^7/(d*x+c)^5*a^3*c^4+756/5*b^7/d^8/(d*x+c)^5*a^2*c^5-252/5*b^8
/d^9/(d*x+c)^5*a*c^6+252*b^7/d^8/(d*x+c)*a^2*c-252*b^8/d^9/(d*x+c)*a*c^2-72
*b^8/d^9*ln(d*x+c)*a*c+12*b^2/d^3/(d*x+c)^6*a^7*c-42*b^3/d^4/(d*x+c)^6*a^6*
c^2+84*b^4/d^5/(d*x+c)^6*a^5*c^3-105*b^5/d^6/(d*x+c)^6*a^4*c^4+84*b^6/d^7/(
d*x+c)^6*a^3*c^5+12/d^4/(d*x+c)^7*a^6*b^3*c^3+18/d^6/(d*x+c)^7*a^4*b^5*c^5-
12/d^7/(d*x+c)^7*a^3*b^6*c^6+36/7/d^8/(d*x+c)^7*a^2*b^7*c^7-9/7/d^9/(d*x+c)
^7*a*b^8*c^8-42*b^7/d^8/(d*x+c)^6*a^2*c^6+12*b^8/d^9/(d*x+c)^6*a*c^7+210*b^
5/d^6/(d*x+c)^3*a^4*c-420*b^6/d^7/(d*x+c)^3*a^3*c^2+420*b^7/d^8/(d*x+c)^3*a
^2*c^3-210*b^8/d^9/(d*x+c)^3*a*c^4-18/d^5/(d*x+c)^7*a^5*b^4*c^4
```

**maxima [B]** time = 2.20, size = 786, normalized size = 3.39

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/70*(3349*b^9*c^9 - 8658*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d^2 - 840*a^3*b^6*
c^6*d^3 - 210*a^4*b^5*c^5*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^6*b^3*c^3*d^6 - 2
4*a^7*b^2*c^2*d^7 - 15*a^8*b*c*d^8 - 10*a^9*d^9 + 5880*(b^9*c^3*d^6 - 3*a*b
^8*c^2*d^7 + 3*a^2*b^7*c*d^8 - a^3*b^6*d^9)*x^6 + 4410*(7*b^9*c^4*d^5 - 20*
a*b^8*c^3*d^6 + 18*a^2*b^7*c^2*d^7 - 4*a^3*b^6*c*d^8 - a^4*b^5*d^9)*x^5 + 1
470*(47*b^9*c^5*d^4 - 130*a*b^8*c^4*d^5 + 110*a^2*b^7*c^3*d^6 - 20*a^3*b^6*
c^2*d^7 - 5*a^4*b^5*c*d^8 - 2*a^5*b^4*d^9)*x^4 + 1470*(57*b^9*c^6*d^3 - 154
*a*b^8*c^5*d^4 + 125*a^2*b^7*c^4*d^5 - 20*a^3*b^6*c^3*d^6 - 5*a^4*b^5*c^2*d
^7 - 2*a^5*b^4*c*d^8 - a^6*b^3*d^9)*x^3 + 126*(459*b^9*c^7*d^2 - 1218*a*b^8
*c^6*d^3 + 959*a^2*b^7*c^5*d^4 - 140*a^3*b^6*c^4*d^5 - 35*a^4*b^5*c^3*d^6 -
 14*a^5*b^4*c^2*d^7 - 7*a^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 + 21*(1023*b^9*c
^8*d - 2676*a*b^8*c^7*d^2 + 2058*a^2*b^7*c^6*d^3 - 280*a^3*b^6*c^5*d^4 - 70
*a^4*b^5*c^4*d^5 - 28*a^5*b^4*c^3*d^6 - 14*a^6*b^3*c^2*d^7 - 8*a^7*b^2*c*d^
8 - 5*a^8*b*d^9)*x)/(d^17*x^7 + 7*c*d^16*x^6 + 21*c^2*d^15*x^5 + 35*c^3*d^1
4*x^4 + 35*c^4*d^13*x^3 + 21*c^5*d^12*x^2 + 7*c^6*d^11*x + c^7*d^10) + 1/2*
(b^9*d*x^2 - 2*(8*b^9*c - 9*a*b^8*d)*x)/d^9 + 36*(b^9*c^2 - 2*a*b^8*c*d + a
^2*b^7*d^2)*log(d*x + c)/d^10
```

**mupad** [B] time = 0.26, size = 784, normalized size = 3.38

$$x\left(\frac{9ab^8}{d^8}-\frac{8b^9c}{d^9}\right)-\frac{10a^9d^9+15a^8bcd^8+24a^7b^2c^2d^7+42a^6b^3c^3d^6+84a^5b^4c^4d^5+210a^4b^5c^5d^4+840a^3b^6c^6d^3-6534a^2b^7c^7d^2+8658ab^8c^8d-3349b^9c^9}{70d}+x\left(\frac{3a^8bd^8}{2}+\frac{12a^7b^2cd^7}{5}+\frac{21a^6b^3c^2d^6}{5}+\frac{42a^5b^4c^3d^5}{5}+21a^4b^5c^4d^4+84a^3b^6c^5d^3-\frac{3087a^2b^7c^6d^2}{5}+\frac{4014ab^8c^7d}{5}-\frac{3069b^9c^8}{10}\right)+x^3\left(21a^6b^3d^8+42a^5b^4cd^7+105a^4b^5c^2d^6+420a^3b^6c^3d^5 \ldots\right.$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9/(c + d*x)^8,x)
```

```
[Out] x*((9*a*b^8)/d^8 - (8*b^9*c)/d^9) - ((10*a^9*d^9 - 3349*b^9*c^9 - 6534*a^2*
b^7*c^7*d^2 + 840*a^3*b^6*c^6*d^3 + 210*a^4*b^5*c^5*d^4 + 84*a^5*b^4*c^4*d^
5 + 42*a^6*b^3*c^3*d^6 + 24*a^7*b^2*c^2*d^7 + 8658*a*b^8*c^8*d + 15*a^8*b*c
*d^8)/(70*d) + x*((3*a^8*b*d^8)/2 - (3069*b^9*c^8)/10 + (12*a^7*b^2*c*d^7)/
5 - (3087*a^2*b^7*c^6*d^2)/5 + 84*a^3*b^6*c^5*d^3 + 21*a^4*b^5*c^4*d^4 + (4
2*a^5*b^4*c^3*d^5)/5 + (21*a^6*b^3*c^2*d^6)/5 + (4014*a*b^8*c^7*d)/5) + x^3
*(21*a^6*b^3*d^8 - 1197*b^9*c^6*d^2 + 3234*a*b^8*c^5*d^3 + 42*a^5*b^4*c*d^7
 - 2625*a^2*b^7*c^4*d^4 + 420*a^3*b^6*c^3*d^5 + 105*a^4*b^5*c^2*d^6) + x^2*
((36*a^7*b^2*d^8)/5 - (4131*b^9*c^7*d)/5 + (10962*a*b^8*c^6*d^2)/5 + (63*a^
6*b^3*c*d^7)/5 - (8631*a^2*b^7*c^5*d^3)/5 + 252*a^3*b^6*c^4*d^4 + 63*a^4*b^
5*c^3*d^5 + (126*a^5*b^4*c^2*d^6)/5) + x^5*(63*a^4*b^5*d^8 - 441*b^9*c^4*d^
4 + 1260*a*b^8*c^3*d^5 + 252*a^3*b^6*c*d^7 - 1134*a^2*b^7*c^2*d^6) + x^4*(4
2*a^5*b^4*d^8 - 987*b^9*c^5*d^3 + 2730*a*b^8*c^4*d^4 + 105*a^4*b^5*c*d^7 -
2310*a^2*b^7*c^3*d^5 + 420*a^3*b^6*c^2*d^6) + x^6*(84*a^3*b^6*d^8 - 84*b^9*
c^3*d^5 + 252*a*b^8*c^2*d^6 - 252*a^2*b^7*c*d^7))/(c^7*d^9 + d^16*x^7 + 7*c
^6*d^10*x + 7*c*d^15*x^6 + 21*c^5*d^11*x^2 + 35*c^4*d^12*x^3 + 35*c^3*d^13*
x^4 + 21*c^2*d^14*x^5) + (b^9*x^2)/(2*d^8) + (log(c + d*x)*(36*b^9*c^2 + 36
*a^2*b^7*d^2 - 72*a*b^8*c*d))/d^10
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9/(d*x+c)**8,x)
```

```
[Out] Timed out
```

## 3.1257 $\int \frac{(a+bx)^8}{(c+dx)^8} \, dx$

**Optimal**. Leaf size=209

$$-\frac{8b^7(bc-ad)\log(c+dx)}{d^9}-\frac{28b^6(bc-ad)^2}{d^9(c+dx)}+\frac{28b^5(bc-ad)^3}{d^9(c+dx)^2}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}+\frac{4b}{3d}$$

**Rubi** [A] time = 0.28, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{28b^6(bc-ad)^2}{d^9(c+dx)}+\frac{28b^5(bc-ad)^3}{d^9(c+dx)^2}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}-\frac{8b^7(bc-ad)\log(c+dx)}{d^9}+\frac{4b(bc-ad)^7}{3d^9(c+dx)^6}-\frac{(bc-ad)^8}{7d^9(c+dx)^7}+\frac{b^8x}{d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] (b^8*x)/d^8 - (b*c - a*d)^8/(7*d^9*(c + d*x)^7) + (4*b*(b*c - a*d)^7)/(3*d^
9*(c + d*x)^6) - (28*b^2*(b*c - a*d)^6)/(5*d^9*(c + d*x)^5) + (14*b^3*(b*c
- a*d)^5)/(d^9*(c + d*x)^4) - (70*b^4*(b*c - a*d)^4)/(3*d^9*(c + d*x)^3) +
(28*b^5*(b*c - a*d)^3)/(d^9*(c + d*x)^2) - (28*b^6*(b*c - a*d)^2)/(d^9*(c +
 d*x)) - (8*b^7*(b*c - a*d)*Log[c + d*x])/d^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^8}{(c+dx)^8} \, dx = \int \left(\frac{b^8}{d^8}+\frac{(-bc+ad)^8}{d^8(c+dx)^8}-\frac{8b(bc-ad)^7}{d^8(c+dx)^7}+\frac{28b^2(bc-ad)^6}{d^8(c+dx)^6}-\frac{56b^3(bc-ad)^5}{d^8(c+dx)^5}+\frac{70b^4(bc-ad)^4}{d^8(c+dx)^4}-\frac{5}{}\right.$$

$$= \frac{b^8x}{d^8}-\frac{(bc-ad)^8}{7d^9(c+dx)^7}+\frac{4b(bc-ad)^7}{3d^9(c+dx)^6}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{28b}{d}$$

**Mathematica** [B] time = 0.20, size = 474, normalized size = 2.27

$$-\frac{15a^8d^8+20a^7bd^7(c+7dx)+28a^6b^2d^6\left(c^2+7cdx+21d^2x^2\right)+42a^5b^3d^5\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right)+70a^4b^4d^4\left(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4\right)+140a^3b^5d^3\left(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5\right)+420a^2b^6d^2\left(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6\right)-2ab^7cd\left(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6\right)+840b^7(c+dx)^7(bc-ad)\log(c+dx)+b^8\left(1443c^8+9261c^7dx+24843c^6d^2x^2+35525c^5d^3x^3+28175c^4d^4x^4+11025c^3d^5x^5+735c^2d^6x^6-735cd^7x^7-105d^8x^8\right)}{105d^9(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^8*d^8 + 20*a^7*b*d^7*(c + 7*d*x) + 28*a^6*b^2*d^6*(c^2 + 7*c*d
*x + 21*d^2*x^2) + 42*a^5*b^3*d^5*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*
x^3) + 70*a^4*b^4*d^4*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35
*d^4*x^4) + 140*a^3*b^5*d^3*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*
x^3 + 35*c*d^4*x^4 + 21*d^5*x^5) + 420*a^2*b^6*d^2*(c^6 + 7*c^5*d*x + 21*c^
4*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6) - 2
*a*b^7*c*d*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*x^3
 + 26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6) + b^8*(1443*c^8 + 92
61*c^7*d*x + 24843*c^6*d^2*x^2 + 35525*c^5*d^3*x^3 + 28175*c^4*d^4*x^4 + 11
025*c^3*d^5*x^5 + 735*c^2*d^6*x^6 - 735*c*d^7*x^7 - 105*d^8*x^8) + 840*b^7*
(b*c - a*d)*(c + d*x)^7*Log[c + d*x])/(d^9*(c + d*x)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^8}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^8/(c + d*x)^8, x]
```

**fricas [B]** time = 0.97, size = 852, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*b^8*d^8*x^8 + 735*b^8*c*d^7*x^7 - 1443*b^8*c^8 + 2178*a*b^7*c^7*
d - 420*a^2*b^6*c^6*d^2 - 140*a^3*b^5*c^5*d^3 - 70*a^4*b^4*c^4*d^4 - 42*a^5
*b^3*c^3*d^5 - 28*a^6*b^2*c^2*d^6 - 20*a^7*b*c*d^7 - 15*a^8*d^8 - 735*(b^8*
c^2*d^6 - 8*a*b^7*c*d^7 + 4*a^2*b^6*d^8)*x^6 - 735*(15*b^8*c^3*d^5 - 36*a*b
^7*c^2*d^6 + 12*a^2*b^6*c*d^7 + 4*a^3*b^5*d^8)*x^5 - 1225*(23*b^8*c^4*d^4 -
 44*a*b^7*c^3*d^5 + 12*a^2*b^6*c^2*d^6 + 4*a^3*b^5*c*d^7 + 2*a^4*b^4*d^8)*x
^4 - 245*(145*b^8*c^5*d^3 - 250*a*b^7*c^4*d^4 + 60*a^2*b^6*c^3*d^5 + 20*a^3
*b^5*c^2*d^6 + 10*a^4*b^4*c*d^7 + 6*a^5*b^3*d^8)*x^3 - 147*(169*b^8*c^6*d^2
 - 274*a*b^7*c^5*d^3 + 60*a^2*b^6*c^4*d^4 + 20*a^3*b^5*c^3*d^5 + 10*a^4*b^4
*c^2*d^6 + 6*a^5*b^3*c*d^7 + 4*a^6*b^2*d^8)*x^2 - 7*(1323*b^8*c^7*d - 2058*
a*b^7*c^6*d^2 + 420*a^2*b^6*c^5*d^3 + 140*a^3*b^5*c^4*d^4 + 70*a^4*b^4*c^3*
d^5 + 42*a^5*b^3*c^2*d^6 + 28*a^6*b^2*c*d^7 + 20*a^7*b*d^8)*x - 840*(b^8*c^
```

```
8 - a*b^7*c^7*d + (b^8*c*d^7 - a*b^7*d^8)*x^7 + 7*(b^8*c^2*d^6 - a*b^7*c*d^
7)*x^6 + 21*(b^8*c^3*d^5 - a*b^7*c^2*d^6)*x^5 + 35*(b^8*c^4*d^4 - a*b^7*c^3
*d^5)*x^4 + 35*(b^8*c^5*d^3 - a*b^7*c^4*d^4)*x^3 + 21*(b^8*c^6*d^2 - a*b^7*
c^5*d^3)*x^2 + 7*(b^8*c^7*d - a*b^7*c^6*d^2)*x)*log(d*x + c))/(d^16*x^7 + 7
*c*d^15*x^6 + 21*c^2*d^14*x^5 + 35*c^3*d^13*x^4 + 35*c^4*d^12*x^3 + 21*c^5*
d^11*x^2 + 7*c^6*d^10*x + c^7*d^9)
```

**giac [B]** time = 1.27, size = 581, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^8*x/d^8 - 8*(b^8*c - a*b^7*d)*log(abs(d*x + c))/d^9 - 1/105*(1443*b^8*c^8
 - 2178*a*b^7*c^7*d + 420*a^2*b^6*c^6*d^2 + 140*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 + 20*a^7*b*c*d^7 + 15*a
^8*d^8 + 2940*(b^8*c^2*d^6 - 2*a*b^7*c*d^7 + a^2*b^6*d^8)*x^6 + 2940*(5*b^8
*c^3*d^5 - 9*a*b^7*c^2*d^6 + 3*a^2*b^6*c*d^7 + a^3*b^5*d^8)*x^5 + 2450*(13*
b^8*c^4*d^4 - 22*a*b^7*c^3*d^5 + 6*a^2*b^6*c^2*d^6 + 2*a^3*b^5*c*d^7 + a^4*
b^4*d^8)*x^4 + 490*(77*b^8*c^5*d^3 - 125*a*b^7*c^4*d^4 + 30*a^2*b^6*c^3*d^5
 + 10*a^3*b^5*c^2*d^6 + 5*a^4*b^4*c*d^7 + 3*a^5*b^3*d^8)*x^3 + 294*(87*b^8*
c^6*d^2 - 137*a*b^7*c^5*d^3 + 30*a^2*b^6*c^4*d^4 + 10*a^3*b^5*c^3*d^5 + 5*a
^4*b^4*c^2*d^6 + 3*a^5*b^3*c*d^7 + 2*a^6*b^2*d^8)*x^2 + 14*(669*b^8*c^7*d -
 1029*a*b^7*c^6*d^2 + 210*a^2*b^6*c^5*d^3 + 70*a^3*b^5*c^4*d^4 + 35*a^4*b^4
*c^3*d^5 + 21*a^5*b^3*c^2*d^6 + 14*a^6*b^2*c*d^7 + 10*a^7*b*d^8)*x)/((d*x +
 c)^7*d^9)
```

**maple [B]** time = 0.01, size = 845, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/(d*x+c)^8,x)
```

```
[Out] -14*b^3/d^4/(d*x+c)^4*a^5+14*b^8/d^9/(d*x+c)^4*c^5-28*b^6/d^7/(d*x+c)*a^2-2
8*b^8/d^9/(d*x+c)*c^2+8*b^7/d^8*ln(d*x+c)*a-8*b^8/d^9*ln(d*x+c)*c-28/5*b^2/
d^3/(d*x+c)^5*a^6-28/5*b^8/d^9/(d*x+c)^5*c^6-28*b^5/d^6/(d*x+c)^2*a^3+28*b^
8/d^9/(d*x+c)^2*c^3-1/7/d^9/(d*x+c)^7*b^8*c^8-4/3*b/d^2/(d*x+c)^6*a^7+4/3*b
^8/d^9/(d*x+c)^6*c^7-70/3*b^4/d^5/(d*x+c)^3*a^4-70/3*b^8/d^9/(d*x+c)^3*c^4+
b^8*x/d^8+8/7/d^2/(d*x+c)^7*a^7*b*c-4/d^3/(d*x+c)^7*a^6*b^2*c^2-1/7/d/(d*x+
c)^7*a^8+8/d^4/(d*x+c)^7*a^5*b^3*c^3-10/d^5/(d*x+c)^7*a^4*b^4*c^4+8/d^6/(d*
x+c)^7*a^3*b^5*c^5-4/d^7/(d*x+c)^7*a^2*b^6*c^6+8/7/d^8/(d*x+c)^7*a*b^7*c^7+
140/3*b^4/d^5/(d*x+c)^6*a^4*c^3-140/3*b^5/d^6/(d*x+c)^6*a^3*c^4+28*b^6/d^7/
```

(d*x+c)^6*a^2*c^5-28/3*b^7/d^8/(d*x+c)^6*a*c^6+280/3*b^5/d^6/(d*x+c)^3*a^3*
c-140*b^6/d^7/(d*x+c)^3*a^2*c^2+280/3*b^7/d^8/(d*x+c)^3*a*c^3+70*b^4/d^5/(d
*x+c)^4*a^4*c-140*b^5/d^6/(d*x+c)^4*a^3*c^2+140*b^6/d^7/(d*x+c)^4*a^2*c^3-7
0*b^7/d^8/(d*x+c)^4*a*c^4+56*b^7/d^8/(d*x+c)*a*c+168/5*b^3/d^4/(d*x+c)^5*a^
5*c-84*b^4/d^5/(d*x+c)^5*a^4*c^2+112*b^5/d^6/(d*x+c)^5*a^3*c^3-84*b^6/d^7/(
d*x+c)^5*a^2*c^4+168/5*b^7/d^8/(d*x+c)^5*a*c^5+84*b^6/d^7/(d*x+c)^2*a^2*c-8
4*b^7/d^8/(d*x+c)^2*a*c^2+28/3*b^2/d^3/(d*x+c)^6*a^6*c-28*b^3/d^4/(d*x+c)^6
*a^5*c^2

**maxima [B]** time = 1.96, size = 649, normalized size = 3.11

$$\frac{b^8x}{d^8} - \frac{1443\,b^8c^8 - 2178\,ab^7c^7d + 420\,a^2b^6c^6d^2 + 140\,a^3b^5c^5d^3 + 70\,a^4b^4c^4d^4 + 42\,a^5b^3c^3d^5 + 28\,a^6b^2c^2d^6 + 20\,a^7bcd^7 + 15\,a^8d^8 + 2940\left(b^8c^2d^6 - 2\,ab^7cd^7 + a^2b^6d^8\right)x^6 + 2940\left(5\,b^8c^3d^5 - 9\,ab^7c^2d^6 + 3\,a^2b^6cd^7 + a^3b^5d^8\right)x^5 + 2450\left(13\,b^8c^4d^4 - 22\,ab^7c^3d^5 + 6\,a^2b^6c^2d^6 + 2\,a^3b^5cd^7 + a^4b^4d^8\right)x^4 + 490\left(77\,b^8c^5d^3 - 125\,ab^7c^4d^4 + 30\,a^2b^6c^3d^5 + 10\,a^3b^5c^2d^6 + 5\,a^4b^4cd^7 + 3\,a^5b^3d^8\right)x^3 + 294\left(87\,b^8c^6d^2 - 137\,ab^7c^5d^3 + 30\,a^2b^6c^4d^4 + 10\,a^3b^5c^3d^5 + 5\,a^4b^4c^2d^6 + 3\,a^5b^3cd^7 + 2\,a^6b^2d^8\right)x^2 + 14\left(669\,b^8c^7d - 1029\,ab^7c^6d^2 + 210\,a^2b^6c^5d^3 + 70\,a^3b^5c^4d^4 + 35\,a^4b^4c^3d^5 + 21\,a^5b^3c^2d^6 + 14\,a^6b^2cd^7 + 10\,a^7bd^8\right)x}{105\left(d^{16}x^7 + 7\,cd^{15}x^6 + 21\,c^2d^{14}x^5 + 35\,c^3d^{13}x^4 + 35\,c^4d^{12}x^3 + 21\,c^5d^{11}x^2 + 7\,c^6d^{10}x + c^7d^9\right)} - \frac{8\left(b^8c - ab^7d\right)\log\left(dx + c\right)}{d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] b^8*x/d^8 - 1/105*(1443*b^8*c^8 - 2178*a*b^7*c^7*d + 420*a^2*b^6*c^6*d^2 +
140*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 + 20*a^7*b*c*d^7 + 15*a^8*d^8 + 2940*(b^8*c^2*d^6 - 2*a*b^7*c*d^7 +
 a^2*b^6*d^8)*x^6 + 2940*(5*b^8*c^3*d^5 - 9*a*b^7*c^2*d^6 + 3*a^2*b^6*c*d^7
 + a^3*b^5*d^8)*x^5 + 2450*(13*b^8*c^4*d^4 - 22*a*b^7*c^3*d^5 + 6*a^2*b^6*c
^2*d^6 + 2*a^3*b^5*c*d^7 + a^4*b^4*d^8)*x^4 + 490*(77*b^8*c^5*d^3 - 125*a*b
^7*c^4*d^4 + 30*a^2*b^6*c^3*d^5 + 10*a^3*b^5*c^2*d^6 + 5*a^4*b^4*c*d^7 + 3*
a^5*b^3*d^8)*x^3 + 294*(87*b^8*c^6*d^2 - 137*a*b^7*c^5*d^3 + 30*a^2*b^6*c^4
*d^4 + 10*a^3*b^5*c^3*d^5 + 5*a^4*b^4*c^2*d^6 + 3*a^5*b^3*c*d^7 + 2*a^6*b^2
*d^8)*x^2 + 14*(669*b^8*c^7*d - 1029*a*b^7*c^6*d^2 + 210*a^2*b^6*c^5*d^3 +
70*a^3*b^5*c^4*d^4 + 35*a^4*b^4*c^3*d^5 + 21*a^5*b^3*c^2*d^6 + 14*a^6*b^2*c
*d^7 + 10*a^7*b*d^8)*x)/(d^16*x^7 + 7*c*d^15*x^6 + 21*c^2*d^14*x^5 + 35*c^3
*d^13*x^4 + 35*c^4*d^12*x^3 + 21*c^5*d^11*x^2 + 7*c^6*d^10*x + c^7*d^9) - 8
*(b^8*c - a*b^7*d)*log(d*x + c)/d^9
```

**mupad [B]** time = 0.43, size = 649, normalized size = 3.11

$$\frac{b^8x}{d^8} - \frac{\ln(c + d\,x)\left(8\,b^8c - 8\,a\,b^7d\right)}{d^9} - \frac{x^4\left(\frac{70\,a^4b^4d^7}{3} + \frac{140\,a^3b^5c\,d^6}{3} + 140\,a^2b^6c^2d^5 - \frac{1540\,a\,b^7c^3d^4}{3} + \frac{910\,b^8c^4d^3}{3}\right) + x^6\left(28\,a^2b^6d^7 - 56\,a\,b^7c\,d^6 + 28\,b^8c^2d^5\right) + \frac{15\,a^8d^8 + 20\,a^7b\,c\,d^7 + 28\,a^6b^2c^2d^6 + 42\,a^5b^3c^3d^5 + 70\,a^4b^4c^4d^4 + 140\,a^3b^5c^5d^3 + 420\,a^2b^6c^6d^2 - 2178\,a\,b^7c^7d + 1443\,b^8c^8}{105\,d} + x\left(\frac{4\,a^7b\,d^7}{3} + \frac{28\,a^6b^2c\,d^6}{15} + \frac{14\,a^5b^3c^2d^5}{5} + \frac{14\,a^4b^4c^3d^4}{3} + \frac{28\,a^3b^5c^4d^3}{3} + 28\,a^2b^6c^5d^2 - \frac{686\,a\,b^7c^6d}{5} + \frac{446\,b^8c^7}{5}\right) + x^3\left(14\,a^5b^3d^7 + \frac{70\,a^4b^4c\,d^6}{3} + \frac{140\,a^3b^5c^2d^5}{3} + 140\,a^2b^6c^3d^4 - \frac{1750\,a\,b^7c^4d^3}{3} + \frac{1078\,b^8c^5d^2}{3}\right) + x^2\left(\frac{28\,a^6b^2d^7}{5} + \frac{42\,a^5b^3c\,d^6}{5} + 14\,a^4b^4c^2d^5 + 28\,a^3b^5c^3d^4 + 84\,a^2b^6c^4d^3 - \frac{1918\,a\,b^7c^5d^2}{5} + \frac{1218\,b^8c^6d}{5}\right) + x^5\left(28\,a^3b^5d^7 + 84\,a^2b^6c\,d^6 - 252\,a\,b^7c^2d^5 + 140\,b^8c^3d^4\right)}{c^7d^8 + 7\,c^6d^9x + 21\,c^5d^{10}x^2 + 35\,c^4d^{11}x^3 + 35\,c^3d^{12}x^4 + 21\,c^2d^{13}x^5 + 7\,c\,d^{14}x^6 + d^{15}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8/(c + d*x)^8,x)
```

```
[Out] (b^8*x)/d^8 - (log(c + d*x)*(8*b^8*c - 8*a*b^7*d))/d^9 - (x^4*((70*a^4*b^4*
d^7)/3 + (910*b^8*c^4*d^3)/3 - (1540*a*b^7*c^3*d^4)/3 + (140*a^3*b^5*c*d^6)
/3 + 140*a^2*b^6*c^2*d^5) + x^6*(28*a^2*b^6*d^7 + 28*b^8*c^2*d^5 - 56*a*b^7
*c*d^6) + (15*a^8*d^8 + 1443*b^8*c^8 + 420*a^2*b^6*c^6*d^2 + 140*a^3*b^5*c^
5*d^3 + 70*a^4*b^4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 2178
*a*b^7*c^7*d + 20*a^7*b*c*d^7)/(105*d) + x*((446*b^8*c^7)/5 + (4*a^7*b*d^7)
/3 + (28*a^6*b^2*c*d^6)/15 + 28*a^2*b^6*c^5*d^2 + (28*a^3*b^5*c^4*d^3)/3 +
```

```
(14*a^4*b^4*c^3*d^4)/3 + (14*a^5*b^3*c^2*d^5)/5 - (686*a*b^7*c^6*d)/5) + x^
3*(14*a^5*b^3*d^7 + (1078*b^8*c^5*d^2)/3 - (1750*a*b^7*c^4*d^3)/3 + (70*a^4
*b^4*c*d^6)/3 + 140*a^2*b^6*c^3*d^4 + (140*a^3*b^5*c^2*d^5)/3) + x^2*((1218
*b^8*c^6*d)/5 + (28*a^6*b^2*d^7)/5 - (1918*a*b^7*c^5*d^2)/5 + (42*a^5*b^3*c
*d^6)/5 + 84*a^2*b^6*c^4*d^3 + 28*a^3*b^5*c^3*d^4 + 14*a^4*b^4*c^2*d^5) + x
^5*(28*a^3*b^5*d^7 + 140*b^8*c^3*d^4 - 252*a*b^7*c^2*d^5 + 84*a^2*b^6*c*d^6
))/(c^7*d^8 + d^15*x^7 + 7*c^6*d^9*x + 7*c*d^14*x^6 + 21*c^5*d^10*x^2 + 35*
c^4*d^11*x^3 + 35*c^3*d^12*x^4 + 21*c^2*d^13*x^5)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8/(d*x+c)**8,x)
```

```
[Out] Timed out
```

## 3.1258 $\int \frac{(a+bx)^7}{(c+dx)^8}\,dx$

**Optimal**. Leaf size=194

$$\frac{7b^6(bc-ad)}{d^8(c+dx)}-\frac{21b^5(bc-ad)^2}{2d^8(c+dx)^2}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{(bc-ad)^7}{7d^8(c+dx)^7}+\frac{b}{}$$

**Rubi [A]** time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7b^6(bc-ad)}{d^8(c+dx)}-\frac{21b^5(bc-ad)^2}{2d^8(c+dx)^2}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{(bc-ad)^7}{7d^8(c+dx)^7}+\frac{b^7\log(c+dx)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/(c + d*x)^8,x]

[Out] (b*c - a*d)^7/(7*d^8*(c + d*x)^7) - (7*b*(b*c - a*d)^6)/(6*d^8*(c + d*x)^6)
 + (21*b^2*(b*c - a*d)^5)/(5*d^8*(c + d*x)^5) - (35*b^3*(b*c - a*d)^4)/(4*d
^8*(c + d*x)^4) + (35*b^4*(b*c - a*d)^3)/(3*d^8*(c + d*x)^3) - (21*b^5*(b*c
 - a*d)^2)/(2*d^8*(c + d*x)^2) + (7*b^6*(b*c - a*d))/(d^8*(c + d*x)) + (b^7
*Log[c + d*x])/d^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{(c+dx)^8}\,dx &= \int \left(\frac{(-bc+ad)^7}{d^7(c+dx)^8}+\frac{7b(bc-ad)^6}{d^7(c+dx)^7}-\frac{21b^2(bc-ad)^5}{d^7(c+dx)^6}+\frac{35b^3(bc-ad)^4}{d^7(c+dx)^5}-\frac{35b^4(bc-ad)^3}{d^7(c+dx)^4}+\frac{21b^5(}{d^7(}\right.\\ &= \frac{(bc-ad)^7}{7d^8(c+dx)^7}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\frac{21b^5(bc-}{2d^8(c+}\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 308, normalized size = 1.59

$$\frac{(bc-ad)\left(60a^6d^6+10a^5bd^5(13c+49dx)+2a^4b^2d^4\left(107c^2+539cdx+882d^2x^2\right)+a^3b^3d^3\left(319c^3+1813c^2dx+3969cd^2x^2+3675d^3x^3\right)+a^2b^4d^2\left(459c^4+2793c^3dx+6909c^2d^2x^2+8575cd^3x^3+4900d^4x^4\right)+ab^5d\left(669c^5+4263c^4dx+11319c^3d^2x^2+15925c^2d^3x^3+12250cd^4x^4+4410d^5x^5\right)+b^6\left(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6\right)\right)}{420d^8(c+dx)^7}+\frac{b^7\log(c+dx)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/(c + d*x)^8,x]
```

```
[Out] ((b*c - a*d)*(60*a^6*d^6 + 10*a^5*b*d^5*(13*c + 49*d*x) + 2*a^4*b^2*d^4*(10
7*c^2 + 539*c*d*x + 882*d^2*x^2) + a^3*b^3*d^3*(319*c^3 + 1813*c^2*d*x + 39
69*c*d^2*x^2 + 3675*d^3*x^3) + a^2*b^4*d^2*(459*c^4 + 2793*c^3*d*x + 6909*c
^2*d^2*x^2 + 8575*c*d^3*x^3 + 4900*d^4*x^4) + a*b^5*d*(669*c^5 + 4263*c^4*d
*x + 11319*c^3*d^2*x^2 + 15925*c^2*d^3*x^3 + 12250*c*d^4*x^4 + 4410*d^5*x^5
) + b^6*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*x^3 +
26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6)))/(420*d^8*(c + d*x)^7)
 + (b^7*Log[c + d*x])/d^8
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/(c + d*x)^8, x]
```

**fricas [B]** time = 1.12, size = 625, normalized size = 3.22

$$\frac{1089\,b^{7}c^{7} - 420\,ab^{6}c^{6}d - 210\,a^{2}b^{5}c^{5}d^{2} - 140\,a^{3}b^{4}c^{4}d^{3} - 105\,a^{4}b^{3}c^{3}d^{4} - 84\,a^{5}b^{2}c^{2}d^{5} - 70\,a^{6}bcd^{6} - 60\,a^{7}d^{7} + 2940\,(b^{7}cd^{6} - ab^{6}d^{7})x^{6} + 4410\,(3\,b^{7}c^{2}d^{5} - 2\,ab^{6}cd^{6} - a^{2}b^{5}d^{7})x^{5} + 2450\,(11\,b^{7}c^{3}d^{4} - 6\,ab^{6}c^{2}d^{5} - 3\,a^{2}b^{5}cd^{6} - 2\,a^{3}b^{4}d^{7})x^{4} + 1225\,(25\,b^{7}c^{4}d^{3} - 12\,ab^{6}c^{3}d^{4} - 6\,a^{2}b^{5}c^{2}d^{5} - 4\,a^{3}b^{4}cd^{6} - 3\,a^{4}b^{3}d^{7})x^{3} + 147\,(137\,b^{7}c^{5}d^{2} - 60\,ab^{6}c^{4}d^{3} - 30\,a^{2}b^{5}c^{3}d^{4} - 20\,a^{3}b^{4}c^{2}d^{5} - 15\,a^{4}b^{3}cd^{6} - 12\,a^{5}b^{2}d^{7})x^{2} + 49\,(147\,b^{7}c^{6}d - 60\,ab^{6}c^{5}d^{2} - 30\,a^{2}b^{5}c^{4}d^{3} - 20\,a^{3}b^{4}c^{3}d^{4} - 15\,a^{4}b^{3}c^{2}d^{5} - 12\,a^{5}b^{2}cd^{6} - 10\,a^{6}bd^{7})x + 420\,(b^{7}d^{7}x^{7} + 7\,b^{7}cd^{6}x^{6} + 21\,b^{7}c^{2}d^{5}x^{5} + 35\,b^{7}c^{3}d^{4}x^{4} + 35\,b^{7}c^{4}d^{3}x^{3} + 21\,b^{7}c^{5}d^{2}x^{2} + 7\,b^{7}c^{6}dx + b^{7}c^{7})\log(dx+c)}{420\,(d^{15}x^{7} + 7\,cd^{14}x^{6} + 21\,c^{2}d^{13}x^{5} + 35\,c^{3}d^{12}x^{4} + 35\,c^{4}d^{11}x^{3} + 21\,c^{5}d^{10}x^{2} + 7\,c^{6}d^{9}x + c^{7}d^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/420*(1089*b^7*c^7 - 420*a*b^6*c^6*d - 210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c
^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7
*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(3*b^7*c^2*d^5 - 2*a*b^6*c*d
^6 - a^2*b^5*d^7)*x^5 + 2450*(11*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 - 3*a^2*b^5*
c*d^6 - 2*a^3*b^4*d^7)*x^4 + 1225*(25*b^7*c^4*d^3 - 12*a*b^6*c^3*d^4 - 6*a^
2*b^5*c^2*d^5 - 4*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 147*(137*b^7*c^5*d^2
 - 60*a*b^6*c^4*d^3 - 30*a^2*b^5*c^3*d^4 - 20*a^3*b^4*c^2*d^5 - 15*a^4*b^3*
c*d^6 - 12*a^5*b^2*d^7)*x^2 + 49*(147*b^7*c^6*d - 60*a*b^6*c^5*d^2 - 30*a^2
*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 - 15*a^4*b^3*c^2*d^5 - 12*a^5*b^2*c*d^6 -
 10*a^6*b*d^7)*x + 420*(b^7*d^7*x^7 + 7*b^7*c*d^6*x^6 + 21*b^7*c^2*d^5*x^5
+ 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21*b^7*c^5*d^2*x^2 + 7*b^7*c^6*
d*x + b^7*c^7)*log(d*x + c))/(d^15*x^7 + 7*c*d^14*x^6 + 21*c^2*d^13*x^5 + 3
5*c^3*d^12*x^4 + 35*c^4*d^11*x^3 + 21*c^5*d^10*x^2 + 7*c^6*d^9*x + c^7*d^8)
```

**giac [B]** time = 1.30, size = 467, normalized size = 2.41

$$\frac{b^{7}\log(|dx+c|)}{d^{8}} + \frac{2940\,(b^{7}cd^{5} - ab^{6}d^{6})x^{6} + 4410\,(3\,b^{7}c^{2}d^{4} - 2\,ab^{6}cd^{5} - a^{2}b^{5}d^{6})x^{5} + 2450\,(11\,b^{7}c^{3}d^{3} - 6\,ab^{6}c^{2}d^{4} - 3\,a^{2}b^{5}cd^{5} - 2\,a^{3}b^{4}d^{6})x^{4} + 1225\,(25\,b^{7}c^{4}d^{2} - 12\,ab^{6}c^{3}d^{3} - 6\,a^{2}b^{5}c^{2}d^{4} - 4\,a^{3}b^{4}cd^{5} - 3\,a^{4}b^{3}d^{6})x^{3} + 147\,(137\,b^{7}c^{5}d - 60\,ab^{6}c^{4}d^{2} - 30\,a^{2}b^{5}c^{3}d^{3} - 20\,a^{3}b^{4}c^{2}d^{4} - 15\,a^{4}b^{3}cd^{5} - 12\,a^{5}b^{2}d^{6})x^{2} + 49\,(147\,b^{7}c^{6} - 60\,ab^{6}c^{5}d - 30\,a^{2}b^{5}c^{4}d^{2} - 20\,a^{3}b^{4}c^{3}d^{3} - 15\,a^{4}b^{3}c^{2}d^{4} - 12\,a^{5}b^{2}cd^{5} - 10\,a^{6}bd^{6})x + \frac{1089\,b^{7}c^{7} - 420\,ab^{6}c^{6}d - 210\,a^{2}b^{5}c^{5}d^{2} - 140\,a^{3}b^{4}c^{4}d^{3} - 105\,a^{4}b^{3}c^{3}d^{4} - 84\,a^{5}b^{2}c^{2}d^{5} - 70\,a^{6}bcd^{6} - 60\,a^{7}d^{7}}{d}}{420\,(dx+c)^{7}d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^7*log(abs(d*x + c))/d^8 + 1/420*(2940*(b^7*c*d^5 - a*b^6*d^6)*x^6 + 4410*
(3*b^7*c^2*d^4 - 2*a*b^6*c*d^5 - a^2*b^5*d^6)*x^5 + 2450*(11*b^7*c^3*d^3 -
6*a*b^6*c^2*d^4 - 3*a^2*b^5*c*d^5 - 2*a^3*b^4*d^6)*x^4 + 1225*(25*b^7*c^4*d
^2 - 12*a*b^6*c^3*d^3 - 6*a^2*b^5*c^2*d^4 - 4*a^3*b^4*c*d^5 - 3*a^4*b^3*d^6
)*x^3 + 147*(137*b^7*c^5*d - 60*a*b^6*c^4*d^2 - 30*a^2*b^5*c^3*d^3 - 20*a^3
*b^4*c^2*d^4 - 15*a^4*b^3*c*d^5 - 12*a^5*b^2*d^6)*x^2 + 49*(147*b^7*c^6 - 6
0*a*b^6*c^5*d - 30*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 - 15*a^4*b^3*c^2*d^
4 - 12*a^5*b^2*c*d^5 - 10*a^6*b*d^6)*x + (1089*b^7*c^7 - 420*a*b^6*c^6*d -
210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^
2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7*d^7)/d)/((d*x + c)^7*d^7)
```

**maple** [B] time = 0.01, size = 672, normalized size = 3.46

$$-\frac{a^7}{7(dx+c)^7 d}+\frac{a^6bc}{(dx+c)^7 d^2}-\frac{3a^5b^2c^2}{(dx+c)^7 d^3}+\frac{5a^4b^3c^3}{(dx+c)^7 d^4}-\frac{5a^3b^4c^4}{(dx+c)^7 d^5}+\frac{3a^2b^5c^5}{(dx+c)^7 d^6}-\frac{a\,b^6c^6}{(dx+c)^7 d^7}+\frac{b^7c^7}{7(dx+c)^7 d^8}-\frac{7a^6b}{6(dx+c)^6 d^2}+\frac{7a^5b^2c}{(dx+c)^6 d^3}-\frac{35a^4b^3c^2}{2(dx+c)^6 d^4}+\frac{70a^3b^4c^3}{3(dx+c)^6 d^5}-\frac{35a^2b^5c^4}{2(dx+c)^6 d^6}+\frac{7a\,b^6c^5}{(dx+c)^6 d^7}-\frac{7b^7c^6}{6(dx+c)^6 d^8}-\frac{21a^5b^2}{5(dx+c)^5 d^3}+\frac{21a^4b^3c}{(dx+c)^5 d^4}-\frac{42a^3b^4c^2}{(dx+c)^5 d^5}+\frac{42a^2b^5c^3}{(dx+c)^5 d^6}-\frac{21a\,b^6c^4}{(dx+c)^5 d^7}+\frac{21b^7c^5}{5(dx+c)^5 d^8}-\frac{35a^4b^3}{4(dx+c)^4 d^4}+\frac{35a^3b^4c}{(dx+c)^4 d^5}-\frac{105a^2b^5c^2}{2(dx+c)^4 d^6}+\frac{35a\,b^6c^3}{(dx+c)^4 d^7}-\frac{35b^7c^4}{4(dx+c)^4 d^8}-\frac{35a^3b^4}{3(dx+c)^3 d^5}+\frac{35a^2b^5c}{(dx+c)^3 d^6}-\frac{35a\,b^6c^2}{(dx+c)^3 d^7}+\frac{35b^7c^3}{3(dx+c)^3 d^8}-\frac{21a^2b^5}{2(dx+c)^2 d^6}+\frac{21a\,b^6c}{(dx+c)^2 d^7}-\frac{21b^7c^2}{2(dx+c)^2 d^8}-\frac{7a\,b^6}{(dx+c)d^7}+\frac{7b^7c}{(dx+c)d^8}+\frac{b^7\ln(dx+c)}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/(d*x+c)^8,x)
```

```
[Out] -35/3*b^4/d^5/(d*x+c)^3*a^3+35/3*b^7/d^8/(d*x+c)^3*c^3-21/2*b^5/d^6/(d*x+c)
^2*a^2-21/2*b^7/d^8/(d*x+c)^2*c^2+1/7/d^8/(d*x+c)^7*b^7*c^7-7*b^6/d^7/(d*x+
c)*a+7*b^7/d^8/(d*x+c)*c-21/5*b^2/d^3/(d*x+c)^5*a^5+21/5*b^7/d^8/(d*x+c)^5*
c^5-7/6*b/d^2/(d*x+c)^6*a^6-7/6*b^7/d^8/(d*x+c)^6*c^6-35/4*b^3/d^4/(d*x+c)^
4*a^4-35/4*b^7/d^8/(d*x+c)^4*c^4-21*b^6/d^7/(d*x+c)^5*a*c^4+35*b^6/d^7/(d*x
+c)^4*a*c^3+35*b^5/d^6/(d*x+c)^3*a^2*c-35*b^6/d^7/(d*x+c)^3*a*c^2+21*b^6/d^
7/(d*x+c)^2*a*c+21*b^3/d^4/(d*x+c)^5*a^4*c-42*b^4/d^5/(d*x+c)^5*a^3*c^2+42*
b^5/d^6/(d*x+c)^5*a^2*c^3+b^7*ln(d*x+c)/d^8-1/7/d/(d*x+c)^7*a^7-3/d^3/(d*x+
c)^7*a^5*b^2*c^2+5/d^4/(d*x+c)^7*a^4*b^3*c^3-5/d^5/(d*x+c)^7*a^3*b^4*c^4+3/
d^6/(d*x+c)^7*a^2*b^5*c^5-1/d^7/(d*x+c)^7*a*b^6*c^6+7*b^2/d^3/(d*x+c)^6*a^5
*c-35/2*b^3/d^4/(d*x+c)^6*a^4*c^2+70/3*b^4/d^5/(d*x+c)^6*a^3*c^3-35/2*b^5/d
^6/(d*x+c)^6*a^2*c^4+7*b^6/d^7/(d*x+c)^6*a*c^5+35*b^4/d^5/(d*x+c)^4*a^3*c-1
05/2*b^5/d^6/(d*x+c)^4*a^2*c^2+1/d^2/(d*x+c)^7*a^6*b*c
```

**maxima** [B] time = 1.66, size = 535, normalized size = 2.76

$$\frac{1089\,b^7c^7-420\,ab^6c^6d-210\,a^2b^5c^5d^2-140\,a^3b^4c^4d^3-105\,a^4b^3c^3d^4-84\,a^5b^2c^2d^5-70\,a^6bcd^6-60\,a^7d^7+2940\left(b^7cd^6-ab^6d^7\right)x^6+4410\left(3\,b^7c^2d^5-2\,ab^6cd^6-a^2b^5d^7\right)x^5+2450\left(11\,b^7c^3d^4-6\,ab^6c^2d^5-3\,a^2b^5cd^6-2\,a^3b^4d^7\right)x^4+1225\left(25\,b^7c^4d^3-12\,ab^6c^3d^4-6\,a^2b^5c^2d^5-4\,a^3b^4cd^6-3\,a^4b^3d^7\right)x^3+147\left(137\,b^7c^5d^2-60\,ab^6c^4d^3-30\,a^2b^5c^3d^4-20\,a^3b^4c^2d^5-15\,a^4b^3cd^6-12\,a^5b^2d^7\right)x^2+49\left(147\,b^7c^6d-60\,ab^6c^5d^2-30\,a^2b^5c^4d^3-20\,a^3b^4c^3d^4-15\,a^4b^3c^2d^5-12\,a^5b^2cd^6-10\,a^6bd^7\right)x}{420\left(d^{15}x^7+7\,cd^{14}x^6+21\,c^2d^{13}x^5+35\,c^3d^{12}x^4+35\,c^4d^{11}x^3+21\,c^5d^{10}x^2+7\,c^6d^9x+c^7d^8\right)}+\frac{b^7\log\left(dx+c\right)}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/420*(1089*b^7*c^7 - 420*a*b^6*c^6*d - 210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c
^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7
```

```
*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(3*b^7*c^2*d^5 - 2*a*b^6*c*d
^6 - a^2*b^5*d^7)*x^5 + 2450*(11*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 - 3*a^2*b^5*
c*d^6 - 2*a^3*b^4*d^7)*x^4 + 1225*(25*b^7*c^4*d^3 - 12*a*b^6*c^3*d^4 - 6*a^
2*b^5*c^2*d^5 - 4*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 147*(137*b^7*c^5*d^2
 - 60*a*b^6*c^4*d^3 - 30*a^2*b^5*c^3*d^4 - 20*a^3*b^4*c^2*d^5 - 15*a^4*b^3*
c*d^6 - 12*a^5*b^2*d^7)*x^2 + 49*(147*b^7*c^6*d - 60*a*b^6*c^5*d^2 - 30*a^2
*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 - 15*a^4*b^3*c^2*d^5 - 12*a^5*b^2*c*d^6 -
 10*a^6*b*d^7)*x)/(d^15*x^7 + 7*c*d^14*x^6 + 21*c^2*d^13*x^5 + 35*c^3*d^12*
x^4 + 35*c^4*d^11*x^3 + 21*c^5*d^10*x^2 + 7*c^6*d^9*x + c^7*d^8) + b^7*log(
d*x + c)/d^8
```

**mupad** [**B**] time = 0.38, size = 460, normalized size = 2.37

$$\frac{b^7\ln(c+d\,x)}{d^8}-\frac{x\left(\frac{7\,a^6\,b\,d^7}{6}+\frac{7\,a^5\,b^2\,c\,d^6}{5}+\frac{7\,a^4\,b^3\,c^2\,d^5}{4}+\frac{7\,a^3\,b^4\,c^3\,d^4}{3}+\frac{7\,a^2\,b^5\,c^4\,d^3}{2}+7\,a\,b^6\,c^5\,d^2-\frac{343\,b^7\,c^6\,d}{20}\right)+x^6\left(7\,a\,b^6\,d^7-7\,b^7\,c\,d^6\right)+x^3\left(\frac{35\,a^4\,b^3\,d^7}{4}+\frac{35\,a^3\,b^4\,c\,d^6}{3}+\frac{35\,a^2\,b^5\,c^2\,d^5}{2}+35\,a\,b^6\,c^3\,d^4-\frac{875\,b^7\,c^4\,d^3}{12}\right)+x^5\left(\frac{21\,a^2\,b^5\,d^7}{2}+21\,a\,b^6\,c\,d^6-\frac{63\,b^7\,c^2\,d^5}{2}\right)+x^2\left(\frac{21\,a^5\,b^2\,d^7}{5}+\frac{21\,a^4\,b^3\,c\,d^6}{4}+7\,a^3\,b^4\,c^2\,d^5+\frac{21\,a^2\,b^5\,c^3\,d^4}{2}+21\,a\,b^6\,c^4\,d^3-\frac{959\,b^7\,c^5\,d^2}{20}\right)+\frac{a^7\,d^7}{7}-\frac{363\,b^7\,c^7}{140}+x^4\left(\frac{35\,a^3\,b^4\,d^7}{3}+\frac{35\,a^2\,b^5\,c\,d^6}{2}+35\,a\,b^6\,c^2\,d^5-\frac{385\,b^7\,c^3\,d^4}{6}\right)+\frac{a^2\,b^5\,c^5\,d^2}{2}+\frac{a^3\,b^4\,c^4\,d^3}{3}+\frac{a^4\,b^3\,c^3\,d^4}{4}+\frac{a^5\,b^2\,c^2\,d^5}{5}+a\,b^6\,c^6\,d+\frac{a^6\,b\,c\,d^6}{6}}{d^8\,{\left(c+d\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/(c + d*x)^8,x)
```

```
[Out] (b^7*log(c + d*x))/d^8 - (x*((7*a^6*b*d^7)/6 - (343*b^7*c^6*d)/20 + 7*a*b^6
*c^5*d^2 + (7*a^5*b^2*c*d^6)/5 + (7*a^2*b^5*c^4*d^3)/2 + (7*a^3*b^4*c^3*d^4
)/3 + (7*a^4*b^3*c^2*d^5)/4) + x^6*(7*a*b^6*d^7 - 7*b^7*c*d^6) + x^3*((35*a
^4*b^3*d^7)/4 - (875*b^7*c^4*d^3)/12 + 35*a*b^6*c^3*d^4 + (35*a^3*b^4*c*d^6
)/3 + (35*a^2*b^5*c^2*d^5)/2) + x^5*((21*a^2*b^5*d^7)/2 - (63*b^7*c^2*d^5)/
2 + 21*a*b^6*c*d^6) + x^2*((21*a^5*b^2*d^7)/5 - (959*b^7*c^5*d^2)/20 + 21*a
*b^6*c^4*d^3 + (21*a^4*b^3*c*d^6)/4 + (21*a^2*b^5*c^3*d^4)/2 + 7*a^3*b^4*c^
2*d^5) + (a^7*d^7)/7 - (363*b^7*c^7)/140 + x^4*((35*a^3*b^4*d^7)/3 - (385*b
^7*c^3*d^4)/6 + 35*a*b^6*c^2*d^5 + (35*a^2*b^5*c*d^6)/2) + (a^2*b^5*c^5*d^2
)/2 + (a^3*b^4*c^4*d^3)/3 + (a^4*b^3*c^3*d^4)/4 + (a^5*b^2*c^2*d^5)/5 + a*b
^6*c^6*d + (a^6*b*c*d^6)/6)/(d^8*(c + d*x)^7)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/(d*x+c)**8,x)
```

```
[Out] Timed out
```

## 3.1259 $\int \frac{(a+bx)^6}{(c+dx)^8} \, dx$

**Optimal**. Leaf size=28

$$\frac{(a+bx)^7}{7(c+dx)^7(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{(a+bx)^7}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^7/(7*(b*c - a*d)*(c + d*x)^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^6}{(c+dx)^8} \, dx = \frac{(a+bx)^7}{7(bc-ad)(c+dx)^7}$$

**Mathematica [B]** time = 0.09, size = 271, normalized size = 9.68

$$-\frac{a^6d^6 + a^5bd^5(c+7dx) + a^4b^2d^4\left(c^2+7cdx+21d^2x^2\right) + a^3b^3d^3\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right) + a^2b^4d^2\left(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4\right) + ab^5d\left(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5\right) + b^6\left(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6\right)}{7d^7(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] -1/7*(a^6*d^6 + a^5*b*d^5*(c + 7*d*x) + a^4*b^2*d^4*(c^2 + 7*c*d*x + 21*d^2
*x^2) + a^3*b^3*d^3*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) + a^2*b^4
*d^2*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4) + a*b^5
```

```
*d*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 + 35*c*d^4*x^4 + 21*d
^5*x^5) + b^6*(c^6 + 7*c^5*d*x + 21*c^4*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d
^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6))/(d^7*(c + d*x)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^6}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^6/(c + d*x)^8, x]
```

**fricas [B]** time = 1.38, size = 398, normalized size = 14.21

$$-\frac{7\,b^{6}d^{6}x^{6}+b^{6}c^{6}+ab^{5}c^{5}d+a^{2}b^{4}c^{4}d^{2}+a^{3}b^{3}c^{3}d^{3}+a^{4}b^{2}c^{2}d^{4}+a^{5}bcd^{5}+a^{6}d^{6}+21\left(b^{6}cd^{5}+ab^{5}d^{6}\right)x^{5}+35\left(b^{6}c^{2}d^{4}+ab^{5}cd^{5}+a^{2}b^{4}d^{6}\right)x^{4}+35\left(b^{6}c^{3}d^{3}+ab^{5}c^{2}d^{4}+a^{2}b^{4}cd^{5}+a^{3}b^{3}d^{6}\right)x^{3}+21\left(b^{6}c^{4}d^{2}+ab^{5}c^{3}d^{3}+a^{2}b^{4}c^{2}d^{4}+a^{3}b^{3}cd^{5}+a^{4}b^{2}d^{6}\right)x^{2}+7\left(b^{6}c^{5}d+ab^{5}c^{4}d^{2}+a^{2}b^{4}c^{3}d^{3}+a^{3}b^{3}c^{2}d^{4}+a^{4}b^{2}cd^{5}+a^{5}bd^{6}\right)x}{7\left(d^{14}x^{7}+7\,cd^{13}x^{6}+21\,c^{2}d^{12}x^{5}+35\,c^{3}d^{11}x^{4}+35\,c^{4}d^{10}x^{3}+21\,c^{5}d^{9}x^{2}+7\,c^{6}d^{8}x+c^{7}d^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + b^6*c^6 + a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3
*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c*d^5 + a^6*d^6 + 21*(b^6*c*d^5 + a*b^5*d^6)
*x^5 + 35*(b^6*c^2*d^4 + a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + 35*(b^6*c^3*d^3 +
 a*b^5*c^2*d^4 + a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 21*(b^6*c^4*d^2 + a*b^5
*c^3*d^3 + a^2*b^4*c^2*d^4 + a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^2 + 7*(b^6*c^5*
d + a*b^5*c^4*d^2 + a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4 + a^4*b^2*c*d^5 + a^5
*b*d^6)*x)/(d^14*x^7 + 7*c*d^13*x^6 + 21*c^2*d^12*x^5 + 35*c^3*d^11*x^4 + 3
5*c^4*d^10*x^3 + 21*c^5*d^9*x^2 + 7*c^6*d^8*x + c^7*d^7)
```

**giac [B]** time = 1.28, size = 369, normalized size = 13.18

$$-\frac{7\,b^{6}d^{6}x^{6}+21\,b^{6}cd^{5}x^{5}+21\,ab^{5}d^{6}x^{5}+35\,b^{6}c^{2}d^{4}x^{4}+35\,ab^{5}cd^{5}x^{4}+35\,a^{2}b^{4}d^{6}x^{4}+35\,b^{6}c^{3}d^{3}x^{3}+35\,ab^{5}c^{2}d^{4}x^{3}+35\,a^{2}b^{4}cd^{5}x^{3}+35\,a^{3}b^{3}d^{6}x^{3}+21\,b^{6}c^{4}d^{2}x^{2}+21\,ab^{5}c^{3}d^{3}x^{2}+21\,a^{2}b^{4}c^{2}d^{4}x^{2}+21\,a^{3}b^{3}cd^{5}x^{2}+21\,a^{4}b^{2}d^{6}x^{2}+7\,b^{6}c^{5}dx+7\,ab^{5}c^{4}d^{2}x+7\,a^{2}b^{4}c^{3}d^{3}x+7\,a^{3}b^{3}c^{2}d^{4}x+7\,a^{4}b^{2}cd^{5}x+7\,a^{5}bd^{6}x+b^{6}c^{6}+ab^{5}c^{5}d+a^{2}b^{4}c^{4}d^{2}+a^{3}b^{3}c^{3}d^{3}+a^{4}b^{2}c^{2}d^{4}+a^{5}bcd^{5}+a^{6}d^{6}}{7\,(dx+c)^{7}d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + 21*b^6*c*d^5*x^5 + 21*a*b^5*d^6*x^5 + 35*b^6*c^2*d^4*
x^4 + 35*a*b^5*c*d^5*x^4 + 35*a^2*b^4*d^6*x^4 + 35*b^6*c^3*d^3*x^3 + 35*a*b
^5*c^2*d^4*x^3 + 35*a^2*b^4*c*d^5*x^3 + 35*a^3*b^3*d^6*x^3 + 21*b^6*c^4*d^2
*x^2 + 21*a*b^5*c^3*d^3*x^2 + 21*a^2*b^4*c^2*d^4*x^2 + 21*a^3*b^3*c*d^5*x^2
 + 21*a^4*b^2*d^6*x^2 + 7*b^6*c^5*d*x + 7*a*b^5*c^4*d^2*x + 7*a^2*b^4*c^3*d
^3*x + 7*a^3*b^3*c^2*d^4*x + 7*a^4*b^2*c*d^5*x + 7*a^5*b*d^6*x + b^6*c^6 +
a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c
*d^5 + a^6*d^6)/((d*x + c)^7*d^7)
```

**maple [B]** time = 0.01, size = 357, normalized size = 12.75

$$-\frac{b^6}{(dx+c)\,d^7}-\frac{3\,(ad-bc)\,b^5}{(dx+c)^2\,d^7}-\frac{5\left(a^2d^2-2abcd+b^2c^2\right)b^4}{(dx+c)^3\,d^7}-\frac{5\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)b^3}{(dx+c)^4\,d^7}-\frac{3\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)b^2}{(dx+c)^5\,d^7}-\frac{\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)b}{(dx+c)^6\,d^7}-\frac{a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6}{7\,(dx+c)^7\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^
4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/d^7/(d*x+c)^7-5*b^4*(a^2*d^2-2*a*b*c*d+b^2
*c^2)/d^7/(d*x+c)^3-3*b^2*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*
c^3*d+b^4*c^4)/d^7/(d*x+c)^5-b^6/d^7/(d*x+c)-b*(a^5*d^5-5*a^4*b*c*d^4+10*a^
3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^7/(d*x+c)^6-5*b^3
*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^7/(d*x+c)^4-3*b^5*(a*d-b*c
)/d^7/(d*x+c)^2
```

**maxima [B]** time = 1.61, size = 398, normalized size = 14.21

$$-\frac{7\,b^6d^6x^6+b^6c^6+ab^5c^5d+a^2b^4c^4d^2+a^3b^3c^3d^3+a^4b^2c^2d^4+a^5bcd^5+a^6d^6+21\left(b^6cd^5+ab^5d^6\right)x^5+35\left(b^6c^2d^4+ab^5cd^5+a^2b^4d^6\right)x^4+35\left(b^6c^3d^3+ab^5c^2d^4+a^2b^4cd^5+a^3b^3d^6\right)x^3+21\left(b^6c^4d^2+ab^5c^3d^3+a^2b^4c^2d^4+a^3b^3cd^5+a^4b^2d^6\right)x^2+7\left(b^6c^5d+ab^5c^4d^2+a^2b^4c^3d^3+a^3b^3c^2d^4+a^4b^2cd^5+a^5bd^6\right)x}{7\left(d^{14}x^7+7\,cd^{13}x^6+21\,c^2d^{12}x^5+35\,c^3d^{11}x^4+35\,c^4d^{10}x^3+21\,c^5d^9x^2+7\,c^6d^8x+c^7d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + b^6*c^6 + a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3
*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c*d^5 + a^6*d^6 + 21*(b^6*c*d^5 + a*b^5*d^6)
*x^5 + 35*(b^6*c^2*d^4 + a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + 35*(b^6*c^3*d^3 +
 a*b^5*c^2*d^4 + a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 21*(b^6*c^4*d^2 + a*b^5
*c^3*d^3 + a^2*b^4*c^2*d^4 + a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^2 + 7*(b^6*c^5*
d + a*b^5*c^4*d^2 + a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4 + a^4*b^2*c*d^5 + a^5
*b*d^6)*x)/(d^14*x^7 + 7*c*d^13*x^6 + 21*c^2*d^12*x^5 + 35*c^3*d^11*x^4 + 3
5*c^4*d^10*x^3 + 21*c^5*d^9*x^2 + 7*c^6*d^8*x + c^7*d^7)
```

**mupad [B]** time = 0.15, size = 378, normalized size = 13.50

$$-\frac{\frac{a^6\,d^6+a^5\,b\,c\,d^5+a^4\,b^2\,c^2\,d^4+a^3\,b^3\,c^3\,d^3+a^2\,b^4\,c^4\,d^2+a\,b^5\,c^5\,d+b^6\,c^6}{7\,d^7}+\frac{b^6\,x^6}{d}+\frac{5\,b^3\,x^3\,\left(a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3\right)}{d^4}+\frac{b\,x\,\left(a^5\,d^5+a^4\,b\,c\,d^4+a^3\,b^2\,c^2\,d^3+a^2\,b^3\,c^3\,d^2+a\,b^4\,c^4\,d+b^5\,c^5\right)}{d^6}+\frac{3\,b^5\,x^5\,\left(a\,d+b\,c\right)}{d^2}+\frac{3\,b^2\,x^2\,\left(a^4\,d^4+a^3\,b\,c\,d^3+a^2\,b^2\,c^2\,d^2+a\,b^3\,c^3\,d+b^4\,c^4\right)}{d^5}+\frac{5\,b^4\,x^4\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{d^3}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6/(c + d*x)^8,x)
```

```
[Out] -((a^6*d^6 + b^6*c^6 + a^2*b^4*c^4*d^2 + a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4
+ a*b^5*c^5*d + a^5*b*c*d^5)/(7*d^7) + (b^6*x^6)/d + (5*b^3*x^3*(a^3*d^3 +
b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2))/d^4 + (b*x*(a^5*d^5 + b^5*c^5 + a^2*b
^3*c^3*d^2 + a^3*b^2*c^2*d^3 + a*b^4*c^4*d + a^4*b*c*d^4))/d^6 + (3*b^5*x^5
*(a*d + b*c))/d^2 + (3*b^2*x^2*(a^4*d^4 + b^4*c^4 + a^2*b^2*c^2*d^2 + a*b^3
```

```
*c^3*d + a^3*b*c*d^3))/d^5 + (5*b^4*x^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))/d^3)
/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^
4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(d*x+c)**8,x)
```

```
[Out] Timed out
```

## 3.1260 $\int \frac{(a+bx)^5}{(c+dx)^8} dx$

**Optimal**. Leaf size=58

$$\frac{b(a+bx)^6}{42(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^6}{7(c+dx)^7(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{b(a+bx)^6}{42(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^6}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^6/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^6)/(42*(b*c - a*d)^2
*(c + d*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^8}\,dx = \frac{(a+bx)^6}{7(bc-ad)(c+dx)^7} + \frac{b\int \frac{(a+bx)^5}{(c+dx)^7}\,dx}{7(bc-ad)}$$
$$= \frac{(a+bx)^6}{7(bc-ad)(c+dx)^7} + \frac{b(a+bx)^6}{42(bc-ad)^2(c+dx)^6}$$

**Mathematica [B]** time = 0.06, size = 205, normalized size = 3.53

$$-\frac{6a^5d^5+5a^4bd^4(c+7dx)+4a^3b^2d^3\left(c^2+7cdx+21d^2x^2\right)+3a^2b^3d^2\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right)+2ab^4d\left(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4\right)+b^5\left(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5\right)}{42d^6(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] -1/42*(6*a^5*d^5 + 5*a^4*b*d^4*(c + 7*d*x) + 4*a^3*b^2*d^3*(c^2 + 7*c*d*x +
 21*d^2*x^2) + 3*a^2*b^3*d^2*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3)
+ 2*a*b^4*d*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4)
+ b^5*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 + 35*c*d^4*x^4 + 2
1*d^5*x^5))/(d^6*(c + d*x)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^8, x]
```

**fricas [B]** time = 1.31, size = 326, normalized size = 5.62

$$-\frac{21\,b^5d^5x^5+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5+35\,\left(b^5cd^4+2\,ab^4d^5\right)x^4+35\,\left(b^5c^2d^3+2\,ab^4cd^4+3\,a^2b^3d^5\right)x^3+21\,\left(b^5c^3d^2+2\,ab^4c^2d^3+3\,a^2b^3cd^4+4\,a^3b^2d^5\right)x^2+7\,\left(b^5c^4d+2\,ab^4c^3d^2+3\,a^2b^3c^2d^3+4\,a^3b^2cd^4+5\,a^4bd^5\right)x}{42\,\left(d^{13}x^7+7\,cd^{12}x^6+21\,c^2d^{11}x^5+35\,c^3d^{10}x^4+35\,c^4d^9x^3+21\,c^5d^8x^2+7\,c^6d^7x+c^7d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 + 6*a^5*d^5 + 35*(b^5*c*d^4 + 2*a*b^4*d^5)*x^4
 + 35*(b^5*c^2*d^3 + 2*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 21*(b^5*c^3*d^2 +
 2*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 + 4*a^3*b^2*d^5)*x^2 + 7*(b^5*c^4*d + 2*
a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 4*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x)/(d^13
```

```
*x^7 + 7*c*d^12*x^6 + 21*c^2*d^11*x^5 + 35*c^3*d^10*x^4 + 35*c^4*d^9*x^3 +
21*c^5*d^8*x^2 + 7*c^6*d^7*x + c^7*d^6)
```

**giac [B]** time = 1.32, size = 271, normalized size = 4.67

$$-\frac{21\,b^5d^5x^5+35\,b^5cd^4x^4+70\,ab^4d^5x^4+35\,b^5c^2d^3x^3+70\,ab^4cd^4x^3+105\,a^2b^3d^5x^3+21\,b^5c^3d^2x^2+42\,ab^4c^2d^3x^2+63\,a^2b^3cd^4x^2+84\,a^3b^2d^5x^2+7\,b^5c^4dx+14\,ab^4c^3d^2x+21\,a^2b^3c^2d^3x+28\,a^3b^2cd^4x+35\,a^4bd^5x+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5}{42\,(dx+c)^7d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + 35*b^5*c*d^4*x^4 + 70*a*b^4*d^5*x^4 + 35*b^5*c^2*d^
3*x^3 + 70*a*b^4*c*d^4*x^3 + 105*a^2*b^3*d^5*x^3 + 21*b^5*c^3*d^2*x^2 + 42*
a*b^4*c^2*d^3*x^2 + 63*a^2*b^3*c*d^4*x^2 + 84*a^3*b^2*d^5*x^2 + 7*b^5*c^4*d
*x + 14*a*b^4*c^3*d^2*x + 21*a^2*b^3*c^2*d^3*x + 28*a^3*b^2*c*d^4*x + 35*a^
4*b*d^5*x + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3*b^2*c^2*d^3
 + 5*a^4*b*c*d^4 + 6*a^5*d^5)/((d*x + c)^7*d^6)
```

**maple [B]** time = 0.01, size = 265, normalized size = 4.57

$$-\frac{b^5}{2\,(dx+c)^2\,d^6}-\frac{5\,(ad-bc)\,b^4}{3\,(dx+c)^3\,d^6}-\frac{5\left(a^2d^2-2abcd+b^2c^2\right)b^3}{2\,(dx+c)^4\,d^6}-\frac{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)b^2}{(dx+c)^5\,d^6}-\frac{5\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)b}{6\,(dx+c)^6\,d^6}-\frac{a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5}{7\,(dx+c)^7\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c
^4*d-b^5*c^5)/d^6/(d*x+c)^7-5/3*b^4*(a*d-b*c)/d^6/(d*x+c)^3-5/6*b*(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d^6/(d*x+c)^6-2*b^2*
(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^6/(d*x+c)^5-5/2*b^3*(a^2*d^
2-2*a*b*c*d+b^2*c^2)/d^6/(d*x+c)^4-1/2*b^5/d^6/(d*x+c)^2
```

**maxima [B]** time = 1.58, size = 326, normalized size = 5.62

$$-\frac{21\,b^5d^5x^5+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5+35\left(b^5cd^4+2\,ab^4d^5\right)x^4+35\left(b^5c^2d^3+2\,ab^4cd^4+3\,a^2b^3d^5\right)x^3+21\left(b^5c^3d^2+2\,ab^4c^2d^3+3\,a^2b^3cd^4+4\,a^3b^2d^5\right)x^2+7\left(b^5c^4d+2\,ab^4c^3d^2+3\,a^2b^3c^2d^3+4\,a^3b^2cd^4+5\,a^4bd^5\right)x}{42\left(d^{13}x^7+7\,cd^{12}x^6+21\,c^2d^{11}x^5+35\,c^3d^{10}x^4+35\,c^4d^9x^3+21\,c^5d^8x^2+7\,c^6d^7x+c^7d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 + 6*a^5*d^5 + 35*(b^5*c*d^4 + 2*a*b^4*d^5)*x^4
 + 35*(b^5*c^2*d^3 + 2*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 21*(b^5*c^3*d^2 +
 2*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 + 4*a^3*b^2*d^5)*x^2 + 7*(b^5*c^4*d + 2*
a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 4*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x)/(d^13
*x^7 + 7*c*d^12*x^6 + 21*c^2*d^11*x^5 + 35*c^3*d^10*x^4 + 35*c^4*d^9*x^3 +
21*c^5*d^8*x^2 + 7*c^6*d^7*x + c^7*d^6)
```

**mupad [B]** time = 0.28, size = 39, normalized size = 0.67

$$\frac{(a+bx)^6\,(7bc-6ad+bdx)}{42\,(ad-bc)^2\,(c+dx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^8,x)
```

```
[Out] ((a + b*x)^6*(7*b*c - 6*a*d + b*d*x))/(42*(a*d - b*c)^2*(c + d*x)^7)
```

**sympy [B]** time = 54.91, size = 354, normalized size = 6.10

$$\frac{-6a^5d^5-5a^4bcd^4-4a^3b^2c^2d^3-3a^2b^3c^3d^2-2ab^4c^4d-b^5c^5-21b^5d^5x^5+x^4\left(-70ab^4d^5-35b^5cd^4\right)+x^3\left(-105a^2b^3d^5-70ab^4cd^4-35b^5c^2d^3\right)+x^2\left(-84a^3b^2d^5-63a^2b^3cd^4-42ab^4c^2d^3-21b^5c^3d^2\right)+x\left(-35a^4bd^5-28a^3b^2cd^4-21a^2b^3c^2d^3-14ab^4c^3d^2-7b^5c^4d\right)}{42c^7d^6+294c^6d^7x+882c^5d^8x^2+1470c^4d^9x^3+1470c^3d^{10}x^4+882c^2d^{11}x^5+294cd^{12}x^6+42d^{13}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**8,x)
```

```
[Out] (-6*a**5*d**5 - 5*a**4*b*c*d**4 - 4*a**3*b**2*c**2*d**3 - 3*a**2*b**3*c**3*
d**2 - 2*a*b**4*c**4*d - b**5*c**5 - 21*b**5*d**5*x**5 + x**4*(-70*a*b**4*d
**5 - 35*b**5*c*d**4) + x**3*(-105*a**2*b**3*d**5 - 70*a*b**4*c*d**4 - 35*b
**5*c**2*d**3) + x**2*(-84*a**3*b**2*d**5 - 63*a**2*b**3*c*d**4 - 42*a*b**4
*c**2*d**3 - 21*b**5*c**3*d**2) + x*(-35*a**4*b*d**5 - 28*a**3*b**2*c*d**4
- 21*a**2*b**3*c**2*d**3 - 14*a*b**4*c**3*d**2 - 7*b**5*c**4*d))/(42*c**7*d
**6 + 294*c**6*d**7*x + 882*c**5*d**8*x**2 + 1470*c**4*d**9*x**3 + 1470*c**
3*d**10*x**4 + 882*c**2*d**11*x**5 + 294*c*d**12*x**6 + 42*d**13*x**7)
```

## 3.1261 $\int \frac{(a+bx)^4}{(c+dx)^8} dx$

**Optimal**. Leaf size=89

$$\frac{b^2(a+bx)^5}{105(c+dx)^5(bc-ad)^3}+\frac{b(a+bx)^5}{21(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^5}{7(c+dx)^7(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{b^2(a+bx)^5}{105(c+dx)^5(bc-ad)^3}+\frac{b(a+bx)^5}{21(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^5}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^5/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^5)/(21*(b*c - a*d)^2
*(c + d*x)^6) + (b^2*(a + b*x)^5)/(105*(b*c - a*d)^3*(c + d*x)^5)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^8}\,dx &= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7}+\frac{(2b)\int \frac{(a+bx)^4}{(c+dx)^7}\,dx}{7(bc-ad)}\\ &= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6}+\frac{b^2\int \frac{(a+bx)^4}{(c+dx)^6}\,dx}{21(bc-ad)^2}\\ &= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6}+\frac{b^2(a+bx)^5}{105(bc-ad)^3(c+dx)^5}\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 144, normalized size = 1.62

$$-\frac{15a^4d^4+10a^3bd^3(c+7dx)+6a^2b^2d^2\left(c^2+7cdx+21d^2x^2\right)+3ab^3d\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right)+b^4\left(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4\right)}{105d^5(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^4*d^4 + 10*a^3*b*d^3*(c + 7*d*x) + 6*a^2*b^2*d^2*(c^2 + 7*c*d*
x + 21*d^2*x^2) + 3*a*b^3*d*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) +
 b^4*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4))/(d^5*(
c + d*x)^7)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^8, x]
```

**fricas** [B] time = 1.06, size = 247, normalized size = 2.78

$$-\frac{35\,b^4d^4x^4+b^4c^4+3\,ab^3c^3d+6\,a^2b^2c^2d^2+10\,a^3bcd^3+15\,a^4d^4+35\left(b^4cd^3+3\,ab^3d^4\right)x^3+21\left(b^4c^2d^2+3\,ab^3cd^3+6\,a^2b^2d^4\right)x^2+7\left(b^4c^3d+3\,ab^3c^2d^2+6\,a^2b^2cd^3+10\,a^3bd^4\right)x}{105\left(d^{12}x^7+7\,cd^{11}x^6+21\,c^2d^{10}x^5+35\,c^3d^9x^4+35\,c^4d^8x^3+21\,c^5d^7x^2+7\,c^6d^6x+c^7d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + b^4*c^4 + 3*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 10*a
^3*b*c*d^3 + 15*a^4*d^4 + 35*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 21*(b^4*c^2*d^
```

```
2 + 3*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 + 7*(b^4*c^3*d + 3*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 + 10*a^3*b*d^4)*x)/(d^12*x^7 + 7*c*d^11*x^6 + 21*c^2*d^10*x^
5 + 35*c^3*d^9*x^4 + 35*c^4*d^8*x^3 + 21*c^5*d^7*x^2 + 7*c^6*d^6*x + c^7*d^
5)
```

**giac [B]** time = 1.29, size = 184, normalized size = 2.07

$$-\frac{35 b^4 d^4 x^4 + 35 b^4 c d^3 x^3 + 105 a b^3 d^4 x^3 + 21 b^4 c^2 d^2 x^2 + 63 a b^3 c d^3 x^2 + 126 a^2 b^2 d^4 x^2 + 7 b^4 c^3 d x + 21 a b^3 c^2 d^2 x + 42 a^2 b^2 c d^3 x + 70 a^3 b d^4 x + b^4 c^4 + 3 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 + 10 a^3 b c d^3 + 15 a^4 d^4}{105 (dx + c)^7 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + 35*b^4*c*d^3*x^3 + 105*a*b^3*d^4*x^3 + 21*b^4*c^2*
d^2*x^2 + 63*a*b^3*c*d^3*x^2 + 126*a^2*b^2*d^4*x^2 + 7*b^4*c^3*d*x + 21*a*b
^3*c^2*d^2*x + 42*a^2*b^2*c*d^3*x + 70*a^3*b*d^4*x + b^4*c^4 + 3*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 + 10*a^3*b*c*d^3 + 15*a^4*d^4)/((d*x + c)^7*d^5)
```

**maple [B]** time = 0.01, size = 186, normalized size = 2.09

$$-\frac{b^4}{3 (dx+c)^3 d^5} - \frac{(ad-bc) b^3}{(dx+c)^4 d^5} - \frac{6 \left(a^2 d^2 - 2abcd + b^2 c^2\right) b^2}{5 (dx+c)^5 d^5} - \frac{2 \left(a^3 d^3 - 3a^2 bc d^2 + 3a b^2 c^2 d - b^3 c^3\right) b}{3 (dx+c)^6 d^5} - \frac{a^4 d^4 - 4a^3 bc d^3 + 6a^2 b^2 c^2 d^2 - 4a b^3 c^3 d + b^4 c^4}{7 (dx+c)^7 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d^5/(d
*x+c)^7-1/3*b^4/d^5/(d*x+c)^3-b^3*(a*d-b*c)/d^5/(d*x+c)^4-2/3*b*(a^3*d^3-3*
a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^5/(d*x+c)^6-6/5*b^2*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/d^5/(d*x+c)^5
```

**maxima [B]** time = 1.52, size = 247, normalized size = 2.78

$$-\frac{35 b^4 d^4 x^4 + b^4 c^4 + 3 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 + 10 a^3 b c d^3 + 15 a^4 d^4 + 35 \left(b^4 c d^3 + 3 a b^3 d^4\right) x^3 + 21 \left(b^4 c^2 d^2 + 3 a b^3 c d^3 + 6 a^2 b^2 d^4\right) x^2 + 7 \left(b^4 c^3 d + 3 a b^3 c^2 d^2 + 6 a^2 b^2 c d^3 + 10 a^3 b d^4\right) x}{105 \left(d^{12} x^7 + 7 c d^{11} x^6 + 21 c^2 d^{10} x^5 + 35 c^3 d^9 x^4 + 35 c^4 d^8 x^3 + 21 c^5 d^7 x^2 + 7 c^6 d^6 x + c^7 d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + b^4*c^4 + 3*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 10*a
^3*b*c*d^3 + 15*a^4*d^4 + 35*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 21*(b^4*c^2*d^
2 + 3*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 + 7*(b^4*c^3*d + 3*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 + 10*a^3*b*d^4)*x)/(d^12*x^7 + 7*c*d^11*x^6 + 21*c^2*d^10*x^
5 + 35*c^3*d^9*x^4 + 35*c^4*d^8*x^3 + 21*c^5*d^7*x^2 + 7*c^6*d^6*x + c^7*d^
5)
```

**mupad [B]** time = 0.11, size = 237, normalized size = 2.66

$$-\frac{\frac{15\,a^4\,d^4+10\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2+3\,a\,b^3\,c^3\,d+b^4\,c^4}{105\,d^5}+\frac{b^4\,x^4}{3\,d}+\frac{b^3\,x^3\,\left(3\,a\,d+b\,c\right)}{3\,d^2}+\frac{b\,x\,\left(10\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3\right)}{15\,d^4}+\frac{b^2\,x^2\,\left(6\,a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{5\,d^3}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^8,x)
```

```
[Out] -((15*a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 + 3*a*b^3*c^3*d + 10*a^3*b*c*d^
3)/(105*d^5) + (b^4*x^4)/(3*d) + (b^3*x^3*(3*a*d + b*c))/(3*d^2) + (b*x*(10
*a^3*d^3 + b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2))/(15*d^4) + (b^2*x^2*(6
*a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/(5*d^3))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21
*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x
)
```

**sympy [B]** time = 9.64, size = 267, normalized size = 3.00

$$\frac{-15a^4d^4-10a^3bcd^3-6a^2b^2c^2d^2-3ab^3c^3d-b^4c^4-35b^4d^4x^4+x^3\left(-105ab^3d^4-35b^4cd^3\right)+x^2\left(-126a^2b^2d^4-63ab^3cd^3-21b^4c^2d^2\right)+x\left(-70a^3bd^4-42a^2b^2cd^3-21ab^3c^2d^2-7b^4c^3d\right)}{105c^7d^5+735c^6d^6x+2205c^5d^7x^2+3675c^4d^8x^3+3675c^3d^9x^4+2205c^2d^{10}x^5+735cd^{11}x^6+105d^{12}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**8,x)
```

```
[Out] (-15*a**4*d**4 - 10*a**3*b*c*d**3 - 6*a**2*b**2*c**2*d**2 - 3*a*b**3*c**3*d
 - b**4*c**4 - 35*b**4*d**4*x**4 + x**3*(-105*a*b**3*d**4 - 35*b**4*c*d**3)
 + x**2*(-126*a**2*b**2*d**4 - 63*a*b**3*c*d**3 - 21*b**4*c**2*d**2) + x*(-
70*a**3*b*d**4 - 42*a**2*b**2*c*d**3 - 21*a*b**3*c**2*d**2 - 7*b**4*c**3*d)
)/(105*c**7*d**5 + 735*c**6*d**6*x + 2205*c**5*d**7*x**2 + 3675*c**4*d**8*x
**3 + 3675*c**3*d**9*x**4 + 2205*c**2*d**10*x**5 + 735*c*d**11*x**6 + 105*d
**12*x**7)
```

## 3.1262 $\int \frac{(a+bx)^3}{(c+dx)^8} dx$

**Optimal**. Leaf size=92

$$\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b^3}{4d^4(c+dx)^4}$$

**Rubi [A]** time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b^3}{4d^4(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)^3/(7*d^4*(c + d*x)^7) - (b*(b*c - a*d)^2)/(2*d^4*(c + d*x)^6) +
 (3*b^2*(b*c - a*d))/(5*d^4*(c + d*x)^5) - b^3/(4*d^4*(c + d*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^8}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^8}+\frac{3b(bc-ad)^2}{d^3(c+dx)^7}-\frac{3b^2(bc-ad)}{d^3(c+dx)^6}+\frac{b^3}{d^3(c+dx)^5}\right)dx\\ &= \frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b^3}{4d^4(c+dx)^4}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 94, normalized size = 1.02

$$-\frac{20a^3d^3+10a^2bd^2(c+7dx)+4ab^2d\left(c^2+7cdx+21d^2x^2\right)+b^3\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right)}{140d^4(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] -1/140*(20*a^3*d^3 + 10*a^2*b*d^2*(c + 7*d*x) + 4*a*b^2*d*(c^2 + 7*c*d*x +
21*d^2*x^2) + b^3*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3))/(d^4*(c +
d*x)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^8, x]
```

**fricas [B]** time = 0.96, size = 182, normalized size = 1.98

$$-\frac{35\,b^3d^3x^3 + b^3c^3 + 4\,ab^2c^2d + 10\,a^2bcd^2 + 20\,a^3d^3 + 21\left(b^3cd^2 + 4\,ab^2d^3\right)x^2 + 7\left(b^3c^2d + 4\,ab^2cd^2 + 10\,a^2bd^3\right)x}{140\left(d^{11}x^7 + 7\,cd^{10}x^6 + 21\,c^2d^9x^5 + 35\,c^3d^8x^4 + 35\,c^4d^7x^3 + 21\,c^5d^6x^2 + 7\,c^6d^5x + c^7d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2 + 20*a^3*
d^3 + 21*(b^3*c*d^2 + 4*a*b^2*d^3)*x^2 + 7*(b^3*c^2*d + 4*a*b^2*c*d^2 + 10*
a^2*b*d^3)*x)/(d^11*x^7 + 7*c*d^10*x^6 + 21*c^2*d^9*x^5 + 35*c^3*d^8*x^4 +
35*c^4*d^7*x^3 + 21*c^5*d^6*x^2 + 7*c^6*d^5*x + c^7*d^4)
```

**giac [A]** time = 1.24, size = 114, normalized size = 1.24

$$-\frac{35\,b^3d^3x^3 + 21\,b^3cd^2x^2 + 84\,ab^2d^3x^2 + 7\,b^3c^2dx + 28\,ab^2cd^2x + 70\,a^2bd^3x + b^3c^3 + 4\,ab^2c^2d + 10\,a^2bcd^2 + 20\,a^3d^3}{140\,(dx+c)^7d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 21*b^3*c*d^2*x^2 + 84*a*b^2*d^3*x^2 + 7*b^3*c^2*d*
x + 28*a*b^2*c*d^2*x + 70*a^2*b*d^3*x + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*
c*d^2 + 20*a^3*d^3)/((d*x + c)^7*d^4)
```

**maple [A]** time = 0.01, size = 122, normalized size = 1.33

$$-\frac{b^3}{4\,(dx+c)^4\,d^4} - \frac{3\,(ad-bc)\,b^2}{5\,(dx+c)^5\,d^4} - \frac{\left(a^2d^2 - 2abcd + b^2c^2\right)b}{2\,(dx+c)^6\,d^4} - \frac{a^3d^3 - 3a^2bc\,d^2 + 3a\,b^2c^2d - b^3c^3}{7\,(dx+c)^7\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4/(d*x+c)^7-1/2*b*(a^2
*d^2-2*a*b*c*d+b^2*c^2)/d^4/(d*x+c)^6-1/4*b^3/d^4/(d*x+c)^4-3/5*b^2*(a*d-b*
c)/d^4/(d*x+c)^5
```

**maxima [B]** time = 1.47, size = 182, normalized size = 1.98

$$-\frac{35\,b^3d^3x^3+b^3c^3+4\,ab^2c^2d+10\,a^2bcd^2+20\,a^3d^3+21\left(b^3cd^2+4\,ab^2d^3\right)x^2+7\left(b^3c^2d+4\,ab^2cd^2+10\,a^2bd^3\right)x}{140\left(d^{11}x^7+7\,cd^{10}x^6+21\,c^2d^9x^5+35\,c^3d^8x^4+35\,c^4d^7x^3+21\,c^5d^6x^2+7\,c^6d^5x+c^7d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2 + 20*a^3*
d^3 + 21*(b^3*c*d^2 + 4*a*b^2*d^3)*x^2 + 7*(b^3*c^2*d + 4*a*b^2*c*d^2 + 10*
a^2*b*d^3)*x)/(d^11*x^7 + 7*c*d^10*x^6 + 21*c^2*d^9*x^5 + 35*c^3*d^8*x^4 +
35*c^4*d^7*x^3 + 21*c^5*d^6*x^2 + 7*c^6*d^5*x + c^7*d^4)
```

**mupad [B]** time = 0.10, size = 176, normalized size = 1.91

$$-\frac{\frac{20\,a^3\,d^3+10\,a^2\,b\,c\,d^2+4\,a\,b^2\,c^2\,d+b^3\,c^3}{140\,d^4}+\frac{b^3\,x^3}{4\,d}+\frac{b\,x\,\left(10\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{20\,d^3}+\frac{3\,b^2\,x^2\,\left(4\,a\,d+b\,c\right)}{20\,d^2}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^8,x)
```

```
[Out] -((20*a^3*d^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2)/(140*d^4) + (b^3*
x^3)/(4*d) + (b*x*(10*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/(20*d^3) + (3*b^2*x^2
*(4*a*d + b*c))/(20*d^2))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 3
5*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

**sympy [B]** time = 3.10, size = 196, normalized size = 2.13

$$\frac{-20a^3d^3-10a^2bcd^2-4ab^2c^2d-b^3c^3-35b^3d^3x^3+x^2\left(-84ab^2d^3-21b^3cd^2\right)+x\left(-70a^2bd^3-28ab^2cd^2-7b^3c^2d\right)}{140c^7d^4+980c^6d^5x+2940c^5d^6x^2+4900c^4d^7x^3+4900c^3d^8x^4+2940c^2d^9x^5+980cd^{10}x^6+140d^{11}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**8,x)
```

```
[Out] (-20*a**3*d**3 - 10*a**2*b*c*d**2 - 4*a*b**2*c**2*d - b**3*c**3 - 35*b**3*d
**3*x**3 + x**2*(-84*a*b**2*d**3 - 21*b**3*c*d**2) + x*(-70*a**2*b*d**3 - 2
```

```
8*a*b**2*c*d**2 - 7*b**3*c**2*d))/(140*c**7*d**4 + 980*c**6*d**5*x + 2940*c
**5*d**6*x**2 + 4900*c**4*d**7*x**3 + 4900*c**3*d**8*x**4 + 2940*c**2*d**9*
x**5 + 980*c*d**10*x**6 + 140*d**11*x**7)
```

## 3.1263 $\int \frac{(a+bx)^2}{(c+dx)^8}\,dx$

**Optimal**. Leaf size=65

$$\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{(bc-ad)^2}{7d^3(c+dx)^7}-\frac{b^2}{5d^3(c+dx)^5}$$

**Rubi** [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{(bc-ad)^2}{7d^3(c+dx)^7}-\frac{b^2}{5d^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] -(b*c - a*d)^2/(7*d^3*(c + d*x)^7) + (b*(b*c - a*d))/(3*d^3*(c + d*x)^6) -
b^2/(5*d^3*(c + d*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^8}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^8}-\frac{2b(bc-ad)}{d^2(c+dx)^7}+\frac{b^2}{d^2(c+dx)^6}\right)dx\\ &= -\frac{(bc-ad)^2}{7d^3(c+dx)^7}+\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{b^2}{5d^3(c+dx)^5}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 55, normalized size = 0.85

$$-\frac{15a^2d^2+5abd(c+7dx)+b^2\left(c^2+7cdx+21d^2x^2\right)}{105d^3(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^2*d^2 + 5*a*b*d*(c + 7*d*x) + b^2*(c^2 + 7*c*d*x + 21*d^2*x^2)
)/(d^3*(c + d*x)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^8, x]
```

**fricas [B]** time = 1.26, size = 131, normalized size = 2.02

$$-\frac{21\,b^2d^2x^2+b^2c^2+5\,abcd+15\,a^2d^2+7\left(b^2cd+5\,abd^2\right)x}{105\left(d^{10}x^7+7\,cd^9x^6+21\,c^2d^8x^5+35\,c^3d^7x^4+35\,c^4d^6x^3+21\,c^5d^5x^2+7\,c^6d^4x+c^7d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + 7*(b^2*c*d + 5*
a*b*d^2)*x)/(d^10*x^7 + 7*c*d^9*x^6 + 21*c^2*d^8*x^5 + 35*c^3*d^7*x^4 + 35*
c^4*d^6*x^3 + 21*c^5*d^5*x^2 + 7*c^6*d^4*x + c^7*d^3)
```

**giac [A]** time = 1.22, size = 61, normalized size = 0.94

$$-\frac{21\,b^2d^2x^2+7\,b^2cdx+35\,abd^2x+b^2c^2+5\,abcd+15\,a^2d^2}{105\,(dx+c)^7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + 7*b^2*c*d*x + 35*a*b*d^2*x + b^2*c^2 + 5*a*b*c*d +
 15*a^2*d^2)/((d*x + c)^7*d^3)
```

**maple [A]** time = 0.00, size = 71, normalized size = 1.09

$$-\frac{b^2}{5\,(dx+c)^5\,d^3}-\frac{(ad-bc)\,b}{3\,(dx+c)^6\,d^3}-\frac{a^2d^2-2abcd+b^2c^2}{7\,(dx+c)^7\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3/(d*x+c)^7-1/5*b^2/d^3/(d*x+c)^5-1/3*b*
(a*d-b*c)/d^3/(d*x+c)^6
```

**maxima [B]** time = 1.46, size = 131, normalized size = 2.02

$$-\frac{21\, b^{2} d^{2} x^{2} + b^{2} c^{2} + 5\, abcd + 15\, a^{2} d^{2} + 7\, \left(b^{2} cd + 5\, abd^{2}\right) x}{105\, \left(d^{10} x^{7} + 7\, cd^{9} x^{6} + 21\, c^{2} d^{8} x^{5} + 35\, c^{3} d^{7} x^{4} + 35\, c^{4} d^{6} x^{3} + 21\, c^{5} d^{5} x^{2} + 7\, c^{6} d^{4} x + c^{7} d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + 7*(b^2*c*d + 5*
a*b*d^2)*x)/(d^10*x^7 + 7*c*d^9*x^6 + 21*c^2*d^8*x^5 + 35*c^3*d^7*x^4 + 35*
c^4*d^6*x^3 + 21*c^5*d^5*x^2 + 7*c^6*d^4*x + c^7*d^3)
```

**mupad [B]** time = 0.09, size = 129, normalized size = 1.98

$$-\frac{\frac{15\,a^2\,d^2+5\,a\,b\,c\,d+b^2\,c^2}{105\,d^3}+\frac{b^2\,x^2}{5\,d}+\frac{b\,x\,\left(5\,a\,d+b\,c\right)}{15\,d^2}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^8,x)
```

```
[Out] -((15*a^2*d^2 + b^2*c^2 + 5*a*b*c*d)/(105*d^3) + (b^2*x^2)/(5*d) + (b*x*(5*
a*d + b*c))/(15*d^2))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^
4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

**sympy [B]** time = 1.39, size = 139, normalized size = 2.14

$$\frac{-15a^{2}d^{2} - 5abcd - b^{2}c^{2} - 21b^{2}d^{2}x^{2} + x \left(-35abd^{2} - 7b^{2}cd\right)}{105c^{7}d^{3} + 735c^{6}d^{4}x + 2205c^{5}d^{5}x^{2} + 3675c^{4}d^{6}x^{3} + 3675c^{3}d^{7}x^{4} + 2205c^{2}d^{8}x^{5} + 735cd^{9}x^{6} + 105d^{10}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**8,x)
```

```
[Out] (-15*a**2*d**2 - 5*a*b*c*d - b**2*c**2 - 21*b**2*d**2*x**2 + x*(-35*a*b*d**
2 - 7*b**2*c*d))/(105*c**7*d**3 + 735*c**6*d**4*x + 2205*c**5*d**5*x**2 + 3
675*c**4*d**6*x**3 + 3675*c**3*d**7*x**4 + 2205*c**2*d**8*x**5 + 735*c*d**9
*x**6 + 105*d**10*x**7)
```

## 3.1264 $\int \frac{a+bx}{(c+dx)^8} dx$

**Optimal**. Leaf size=38

$$\frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}$$

**Rubi [A]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)/(7*d^2*(c + d*x)^7) - b/(6*d^2*(c + d*x)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^8} dx &= \int \left(\frac{-bc+ad}{d(c+dx)^8}+\frac{b}{d(c+dx)^7}\right) dx\\ &= \frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 27, normalized size = 0.71

$$-\frac{6ad+b(c+7dx)}{42d^2(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] -1/42*(6*a*d + b*(c + 7*d*x))/(d^2*(c + d*x)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(c + d*x)^8, x]
```

**fricas [B]** time = 1.20, size = 94, normalized size = 2.47

$$-\frac{7\,bdx+bc+6\,ad}{42\left(d^9x^7+7\,cd^8x^6+21\,c^2d^7x^5+35\,c^3d^6x^4+35\,c^4d^5x^3+21\,c^5d^4x^2+7\,c^6d^3x+c^7d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d^7*x^5 + 35*
c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x + c^7*d^2)
```

**giac [A]** time = 1.31, size = 25, normalized size = 0.66

$$-\frac{7\,bdx+bc+6\,ad}{42\,(dx+c)^7d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/((d*x + c)^7*d^2)
```

**maple [A]** time = 0.01, size = 35, normalized size = 0.92

$$-\frac{b}{6\,(dx+c)^6\,d^2}-\frac{ad-bc}{7\,(dx+c)^7\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^8,x)
```

```
[Out] -1/7*(a*d-b*c)/d^2/(d*x+c)^7-1/6*b/d^2/(d*x+c)^6
```

**maxima [B]** time = 1.38, size = 94, normalized size = 2.47

$$-\frac{7\,bdx+bc+6\,ad}{42\left(d^9x^7+7\,cd^8x^6+21\,c^2d^7x^5+35\,c^3d^6x^4+35\,c^4d^5x^3+21\,c^5d^4x^2+7\,c^6d^3x+c^7d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d^7*x^5 + 35*
c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x + c^7*d^2)
```

**mupad [B]** time = 0.23, size = 96, normalized size = 2.53

$$-\frac{\frac{6\,a\,d+b\,c}{42\,d^2}+\frac{b\,x}{6\,d}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^8,x)
```

```
[Out] -((6*a*d + b*c)/(42*d^2) + (b*x)/(6*d))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c
^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

**sympy [B]** time = 0.73, size = 100, normalized size = 2.63

$$\frac{-6ad-bc-7bdx}{42c^7d^2+294c^6d^3x+882c^5d^4x^2+1470c^4d^5x^3+1470c^3d^6x^4+882c^2d^7x^5+294cd^8x^6+42d^9x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**8,x)
```

```
[Out] (-6*a*d - b*c - 7*b*d*x)/(42*c**7*d**2 + 294*c**6*d**3*x + 882*c**5*d**4*x*
*2 + 1470*c**4*d**5*x**3 + 1470*c**3*d**6*x**4 + 882*c**2*d**7*x**5 + 294*c
*d**8*x**6 + 42*d**9*x**7)
```

## 3.1265 $\int \frac{1}{(c+dx)^8} dx$

**Optimal**. Leaf size=14

$$-\frac{1}{7d(c+dx)^7}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{7d(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-8),x]
```

```
[Out] -1/(7*d*(c + d*x)^7)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^8} dx = -\frac{1}{7d(c+dx)^7}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{7d(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-8),x]
```

```
[Out] -1/7*1/(d*(c + d*x)^7)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(-8),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^(-8), x]
```

**fricas [B]** time = 1.17, size = 79, normalized size = 5.64

$$-\frac{1}{7\left(d^{8}x^{7}+7\,cd^{7}x^{6}+21\,c^{2}d^{6}x^{5}+35\,c^{3}d^{5}x^{4}+35\,c^{4}d^{4}x^{3}+21\,c^{5}d^{3}x^{2}+7\,c^{6}d^{2}x+c^{7}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(d^8*x^7 + 7*c*d^7*x^6 + 21*c^2*d^6*x^5 + 35*c^3*d^5*x^4 + 35*c^4*d^4*
x^3 + 21*c^5*d^3*x^2 + 7*c^6*d^2*x + c^7*d)
```

**giac [A]** time = 1.26, size = 12, normalized size = 0.86

$$-\frac{1}{7\,(dx+c)^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/7/((d*x + c)^7*d)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{7\,(dx+c)^{7}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^8,x)
```

```
[Out] -1/7/d/(d*x+c)^7
```

**maxima [A]** time = 1.36, size = 12, normalized size = 0.86

$$-\frac{1}{7\,(dx+c)^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/7/((d*x + c)^7*d)
```

**mupad [B]** time = 0.22, size = 81, normalized size = 5.79

$$-\frac{1}{7\,c^7\,d+49\,c^6\,d^2\,x+147\,c^5\,d^3\,x^2+245\,c^4\,d^4\,x^3+245\,c^3\,d^5\,x^4+147\,c^2\,d^6\,x^5+49\,c\,d^7\,x^6+7\,d^8\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^8,x)
```

```
[Out] -1/(7*c^7*d + 7*d^8*x^7 + 49*c^6*d^2*x + 49*c*d^7*x^6 + 147*c^5*d^3*x^2 + 2
45*c^4*d^4*x^3 + 245*c^3*d^5*x^4 + 147*c^2*d^6*x^5)
```

**sympy [B]** time = 0.46, size = 85, normalized size = 6.07

$$-\frac{1}{7c^7d+49c^6d^2x+147c^5d^3x^2+245c^4d^4x^3+245c^3d^5x^4+147c^2d^6x^5+49cd^7x^6+7d^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**8,x)
```

```
[Out] -1/(7*c**7*d + 49*c**6*d**2*x + 147*c**5*d**3*x**2 + 245*c**4*d**4*x**3 + 2
45*c**3*d**5*x**4 + 147*c**2*d**6*x**5 + 49*c*d**7*x**6 + 7*d**8*x**7)
```

## 3.1266 $\int \frac{1}{(a+bx)(c+dx)^8} \, dx$

**Optimal**. Leaf size=202

$$\frac{b^7 \log(a+bx)}{(bc-ad)^8} - \frac{b^7 \log(c+dx)}{(bc-ad)^8} + \frac{b^6}{(c+dx)(bc-ad)^7} + \frac{b^5}{2(c+dx)^2(bc-ad)^6} + \frac{b^4}{3(c+dx)^3(bc-ad)^5} + \frac{b^3}{4(c+dx)^4(bc-}$$

**Rubi** [A] time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{b^6}{(c+dx)(bc-ad)^7} + \frac{b^5}{2(c+dx)^2(bc-ad)^6} + \frac{b^4}{3(c+dx)^3(bc-ad)^5} + \frac{b^3}{4(c+dx)^4(bc-ad)^4} + \frac{b^2}{5(c+dx)^5(bc-ad)^3} + \frac{b^7 \log(a+bx)}{(bc-ad)^8} - \frac{b^7 \log(c+dx)}{(bc-ad)^8} + \frac{b}{6(c+dx)^6(bc-ad)^2} + \frac{1}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] 1/(7*(b*c - a*d)*(c + d*x)^7) + b/(6*(b*c - a*d)^2*(c + d*x)^6) + b^2/(5*(b
*c - a*d)^3*(c + d*x)^5) + b^3/(4*(b*c - a*d)^4*(c + d*x)^4) + b^4/(3*(b*c
- a*d)^5*(c + d*x)^3) + b^5/(2*(b*c - a*d)^6*(c + d*x)^2) + b^6/((b*c - a*d
)^7*(c + d*x)) + (b^7*Log[a + b*x])/(b*c - a*d)^8 - (b^7*Log[c + d*x])/(b*c
 - a*d)^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^8} \, dx = \int \left( \frac{b^8}{(bc-ad)^8(a+bx)} - \frac{d}{(bc-ad)(c+dx)^8} - \frac{bd}{(bc-ad)^2(c+dx)^7} - \frac{b^2 d}{(bc-ad)^3(c+dx)^6}\right.$$
$$= \frac{1}{7(bc-ad)(c+dx)^7} + \frac{b}{6(bc-ad)^2(c+dx)^6} + \frac{b^2}{5(bc-ad)^3(c+dx)^5} + \frac{b^3}{4(bc-ad)^4(c+d}$$

**Mathematica** [A] time = 0.10, size = 196, normalized size = 0.97

$$\frac{420b^7(c+dx)^7 \log(a+bx) + 420b^6(c+dx)^6(bc-ad) + 210b^5(c+dx)^5(bc-ad)^2 + 140b^4(c+dx)^4(bc-ad)^3 + 105b^3(c+dx)^3(bc-ad)^4 + 84b^2(c+dx)^2(bc-ad)^5 + 70b(c+dx)(bc-ad)^6 + 60(bc-ad)^7 - 420b^7(c+dx)^7 \log(c+dx)}{420(c+dx)^7(bc-ad)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] (60*(b*c - a*d)^7 + 70*b*(b*c - a*d)^6*(c + d*x) + 84*b^2*(b*c - a*d)^5*(c
+ d*x)^2 + 105*b^3*(b*c - a*d)^4*(c + d*x)^3 + 140*b^4*(b*c - a*d)^3*(c + d
*x)^4 + 210*b^5*(b*c - a*d)^2*(c + d*x)^5 + 420*b^6*(b*c - a*d)*(c + d*x)^6
 + 420*b^7*(c + d*x)^7*Log[a + b*x] - 420*b^7*(c + d*x)^7*Log[c + d*x])/(42
0*(b*c - a*d)^8*(c + d*x)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^8), x]
```

**fricas [B]** time = 1.54, size = 1589, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/420*(1089*b^7*c^7 - 2940*a*b^6*c^6*d + 4410*a^2*b^5*c^5*d^2 - 4900*a^3*b^
4*c^4*d^3 + 3675*a^4*b^3*c^3*d^4 - 1764*a^5*b^2*c^2*d^5 + 490*a^6*b*c*d^6 -
 60*a^7*d^7 + 420*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 210*(13*b^7*c^2*d^5 - 14*a*
b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 70*(107*b^7*c^3*d^4 - 126*a*b^6*c^2*d^5 + 21
*a^2*b^5*c*d^6 - 2*a^3*b^4*d^7)*x^4 + 35*(319*b^7*c^4*d^3 - 420*a*b^6*c^3*d
^4 + 126*a^2*b^5*c^2*d^5 - 28*a^3*b^4*c*d^6 + 3*a^4*b^3*d^7)*x^3 + 21*(459*
b^7*c^5*d^2 - 700*a*b^6*c^4*d^3 + 350*a^2*b^5*c^3*d^4 - 140*a^3*b^4*c^2*d^5
 + 35*a^4*b^3*c*d^6 - 4*a^5*b^2*d^7)*x^2 + 7*(669*b^7*c^6*d - 1260*a*b^6*c^
5*d^2 + 1050*a^2*b^5*c^4*d^3 - 700*a^3*b^4*c^3*d^4 + 315*a^4*b^3*c^2*d^5 -
84*a^5*b^2*c*d^6 + 10*a^6*b*d^7)*x + 420*(b^7*d^7*x^7 + 7*b^7*c*d^6*x^6 + 2
1*b^7*c^2*d^5*x^5 + 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21*b^7*c^5*d^
2*x^2 + 7*b^7*c^6*d*x + b^7*c^7)*log(b*x + a) - 420*(b^7*d^7*x^7 + 7*b^7*c*
d^6*x^6 + 21*b^7*c^2*d^5*x^5 + 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21
*b^7*c^5*d^2*x^2 + 7*b^7*c^6*d*x + b^7*c^7)*log(d*x + c))/(b^8*c^15 - 8*a*b
^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4
 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8
 + (b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^1
0 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7
*b*c*d^14 + a^8*d^15)*x^7 + 7*(b^8*c^9*d^6 - 8*a*b^7*c^8*d^7 + 28*a^2*b^6*c
^7*d^8 - 56*a^3*b^5*c^6*d^9 + 70*a^4*b^4*c^5*d^10 - 56*a^5*b^3*c^4*d^11 + 2
8*a^6*b^2*c^3*d^12 - 8*a^7*b*c^2*d^13 + a^8*c*d^14)*x^6 + 21*(b^8*c^10*d^5
```

```
- 8*a*b^7*c^9*d^6 + 28*a^2*b^6*c^8*d^7 - 56*a^3*b^5*c^7*d^8 + 70*a^4*b^4*c^
6*d^9 - 56*a^5*b^3*c^5*d^10 + 28*a^6*b^2*c^4*d^11 - 8*a^7*b*c^3*d^12 + a^8*
c^2*d^13)*x^5 + 35*(b^8*c^11*d^4 - 8*a*b^7*c^10*d^5 + 28*a^2*b^6*c^9*d^6 -
56*a^3*b^5*c^8*d^7 + 70*a^4*b^4*c^7*d^8 - 56*a^5*b^3*c^6*d^9 + 28*a^6*b^2*c
^5*d^10 - 8*a^7*b*c^4*d^11 + a^8*c^3*d^12)*x^4 + 35*(b^8*c^12*d^3 - 8*a*b^7
*c^11*d^4 + 28*a^2*b^6*c^10*d^5 - 56*a^3*b^5*c^9*d^6 + 70*a^4*b^4*c^8*d^7 -
 56*a^5*b^3*c^7*d^8 + 28*a^6*b^2*c^6*d^9 - 8*a^7*b*c^5*d^10 + a^8*c^4*d^11)
*x^3 + 21*(b^8*c^13*d^2 - 8*a*b^7*c^12*d^3 + 28*a^2*b^6*c^11*d^4 - 56*a^3*b
^5*c^10*d^5 + 70*a^4*b^4*c^9*d^6 - 56*a^5*b^3*c^8*d^7 + 28*a^6*b^2*c^7*d^8
- 8*a^7*b*c^6*d^9 + a^8*c^5*d^10)*x^2 + 7*(b^8*c^14*d - 8*a*b^7*c^13*d^2 +
28*a^2*b^6*c^12*d^3 - 56*a^3*b^5*c^11*d^4 + 70*a^4*b^4*c^10*d^5 - 56*a^5*b^
3*c^9*d^6 + 28*a^6*b^2*c^8*d^7 - 8*a^7*b*c^7*d^8 + a^8*c^6*d^9)*x)
```

**giac [B]** time = 1.33, size = 703, normalized size = 3.48

$$\frac{b^8\log(|bx+a|)}{b^9c^8-8ab^8c^7d+28a^2b^7c^6d^2-56a^3b^6c^5d^3+70a^4b^5c^4d^4-56a^5b^4c^3d^5+28a^6b^3c^2d^6-8a^7b^2cd^7+a^8bd^8}-\frac{b^7d\log(|dx+c|)}{b^8c^8d-8ab^7c^7d^2+28a^2b^6c^6d^3-56a^3b^5c^5d^4+70a^4b^4c^4d^5-56a^5b^3c^3d^6+28a^6b^2c^2d^7-8a^7bcd^8+a^8d^9}+\frac{1089b^7c^7-2940ab^6c^6d+4410a^2b^5c^5d^2-4900a^3b^4c^4d^3+3675a^4b^3c^3d^4-1764a^5b^2c^2d^5+490a^6bcd^6-60a^7d^7+420(b^7cd^6-ab^6d^7)x^6+210(13b^7c^2d^5-14ab^6cd^6+a^2b^5d^7)x^5+70(107b^7c^3d^4-126ab^6c^2d^5+21a^2b^5cd^6-2a^3b^4d^7)x^4+35(319b^7c^4d^3-420ab^6c^3d^4+126a^2b^5c^2d^5-28a^3b^4cd^6+3a^4b^3d^7)x^3+21(459b^7c^5d^2-700ab^6c^4d^3+350a^2b^5c^3d^4-140a^3b^4c^2d^5+35a^4b^3cd^6-4a^5b^2d^7)x^2+7(669b^7c^6d-1260ab^6c^5d^2+1050a^2b^5c^4d^3-700a^3b^4c^3d^4+315a^4b^3c^2d^5-84a^5b^2cd^6+10a^6bd^7)x}{420(bc-ad)^8(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^8*log(abs(b*x + a))/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^
3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^
6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8) - b^7*d*log(abs(d*x + c))/(b^8*c^8*d - 8*a
*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5
 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8 + a^8*d^9) + 1/4
20*(1089*b^7*c^7 - 2940*a*b^6*c^6*d + 4410*a^2*b^5*c^5*d^2 - 4900*a^3*b^4*c
^4*d^3 + 3675*a^4*b^3*c^3*d^4 - 1764*a^5*b^2*c^2*d^5 + 490*a^6*b*c*d^6 - 60
*a^7*d^7 + 420*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 210*(13*b^7*c^2*d^5 - 14*a*b^6
*c*d^6 + a^2*b^5*d^7)*x^5 + 70*(107*b^7*c^3*d^4 - 126*a*b^6*c^2*d^5 + 21*a^
2*b^5*c*d^6 - 2*a^3*b^4*d^7)*x^4 + 35*(319*b^7*c^4*d^3 - 420*a*b^6*c^3*d^4
+ 126*a^2*b^5*c^2*d^5 - 28*a^3*b^4*c*d^6 + 3*a^4*b^3*d^7)*x^3 + 21*(459*b^7
*c^5*d^2 - 700*a*b^6*c^4*d^3 + 350*a^2*b^5*c^3*d^4 - 140*a^3*b^4*c^2*d^5 +
35*a^4*b^3*c*d^6 - 4*a^5*b^2*d^7)*x^2 + 7*(669*b^7*c^6*d - 1260*a*b^6*c^5*d
^2 + 1050*a^2*b^5*c^4*d^3 - 700*a^3*b^4*c^3*d^4 + 315*a^4*b^3*c^2*d^5 - 84*
a^5*b^2*c*d^6 + 10*a^6*b*d^7)*x)/((b*c - a*d)^8*(d*x + c)^7)
```

**maple [A]** time = 0.02, size = 192, normalized size = 0.95

$$\frac{b^7\ln(bx+a)}{(ad-bc)^8}-\frac{b^7\ln(dx+c)}{(ad-bc)^8}-\frac{b^6}{(ad-bc)^7(dx+c)}+\frac{b^5}{2(ad-bc)^6(dx+c)^2}-\frac{b^4}{3(ad-bc)^5(dx+c)^3}+\frac{b^3}{4(ad-bc)^4(dx+c)^4}-\frac{b^2}{5(ad-bc)^3(dx+c)^5}+\frac{b}{6(ad-bc)^2(dx+c)^6}-\frac{1}{7(ad-bc)(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^8,x)
```

```
[Out] -1/7/(a*d-b*c)/(d*x+c)^7-1/5*b^2/(a*d-b*c)^3/(d*x+c)^5-1/3*b^4/(a*d-b*c)^5/
(d*x+c)^3-b^6/(a*d-b*c)^7/(d*x+c)+1/6*b/(a*d-b*c)^2/(d*x+c)^6+1/4*b^3/(a*d-
```

```
b*c)^4/(d*x+c)^4+1/2*b^5/(a*d-b*c)^6/(d*x+c)^2-b^7/(a*d-b*c)^8*ln(d*x+c)+b^
7/(a*d-b*c)^8*ln(b*x+a)
```

**maxima [B]** time = 2.97, size = 1418, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] b^7*log(b*x + a)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8) - b^7*log(d*x + c)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8) + 1/420*(420*b^6*d^6*x^6
+ 1089*b^6*c^6 - 1851*a*b^5*c^5*d + 2559*a^2*b^4*c^4*d^2 - 2341*a^3*b^3*c^3
*d^3 + 1334*a^4*b^2*c^2*d^4 - 430*a^5*b*c*d^5 + 60*a^6*d^6 + 210*(13*b^6*c*
d^5 - a*b^5*d^6)*x^5 + 70*(107*b^6*c^2*d^4 - 19*a*b^5*c*d^5 + 2*a^2*b^4*d^6
)*x^4 + 35*(319*b^6*c^3*d^3 - 101*a*b^5*c^2*d^4 + 25*a^2*b^4*c*d^5 - 3*a^3*
b^3*d^6)*x^3 + 21*(459*b^6*c^4*d^2 - 241*a*b^5*c^3*d^3 + 109*a^2*b^4*c^2*d^
4 - 31*a^3*b^3*c*d^5 + 4*a^4*b^2*d^6)*x^2 + 7*(669*b^6*c^5*d - 591*a*b^5*c^
4*d^2 + 459*a^2*b^4*c^3*d^3 - 241*a^3*b^3*c^2*d^4 + 74*a^4*b^2*c*d^5 - 10*a
^5*b*d^6)*x)/(b^7*c^14 - 7*a*b^6*c^13*d + 21*a^2*b^5*c^12*d^2 - 35*a^3*b^4*
c^11*d^3 + 35*a^4*b^3*c^10*d^4 - 21*a^5*b^2*c^9*d^5 + 7*a^6*b*c^8*d^6 - a^7
*c^7*d^7 + (b^7*c^7*d^7 - 7*a*b^6*c^6*d^8 + 21*a^2*b^5*c^5*d^9 - 35*a^3*b^4
*c^4*d^10 + 35*a^4*b^3*c^3*d^11 - 21*a^5*b^2*c^2*d^12 + 7*a^6*b*c*d^13 - a^
7*d^14)*x^7 + 7*(b^7*c^8*d^6 - 7*a*b^6*c^7*d^7 + 21*a^2*b^5*c^6*d^8 - 35*a^
3*b^4*c^5*d^9 + 35*a^4*b^3*c^4*d^10 - 21*a^5*b^2*c^3*d^11 + 7*a^6*b*c^2*d^1
2 - a^7*c*d^13)*x^6 + 21*(b^7*c^9*d^5 - 7*a*b^6*c^8*d^6 + 21*a^2*b^5*c^7*d^
7 - 35*a^3*b^4*c^6*d^8 + 35*a^4*b^3*c^5*d^9 - 21*a^5*b^2*c^4*d^10 + 7*a^6*b
*c^3*d^11 - a^7*c^2*d^12)*x^5 + 35*(b^7*c^10*d^4 - 7*a*b^6*c^9*d^5 + 21*a^2
*b^5*c^8*d^6 - 35*a^3*b^4*c^7*d^7 + 35*a^4*b^3*c^6*d^8 - 21*a^5*b^2*c^5*d^9
 + 7*a^6*b*c^4*d^10 - a^7*c^3*d^11)*x^4 + 35*(b^7*c^11*d^3 - 7*a*b^6*c^10*d
^4 + 21*a^2*b^5*c^9*d^5 - 35*a^3*b^4*c^8*d^6 + 35*a^4*b^3*c^7*d^7 - 21*a^5*
b^2*c^6*d^8 + 7*a^6*b*c^5*d^9 - a^7*c^4*d^10)*x^3 + 21*(b^7*c^12*d^2 - 7*a*
b^6*c^11*d^3 + 21*a^2*b^5*c^10*d^4 - 35*a^3*b^4*c^9*d^5 + 35*a^4*b^3*c^8*d^
6 - 21*a^5*b^2*c^7*d^7 + 7*a^6*b*c^6*d^8 - a^7*c^5*d^9)*x^2 + 7*(b^7*c^13*d
 - 7*a*b^6*c^12*d^2 + 21*a^2*b^5*c^11*d^3 - 35*a^3*b^4*c^10*d^4 + 35*a^4*b^
3*c^9*d^5 - 21*a^5*b^2*c^8*d^6 + 7*a^6*b*c^7*d^7 - a^7*c^6*d^8)*x)
```

**mupad [B]** time = 0.87, size = 1299, normalized size = 6.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^8),x)
```

```
[Out] (2*b^7*atanh((a^8*d^8 - b^8*c^8 - 14*a^2*b^6*c^6*d^2 + 14*a^3*b^5*c^5*d^3 -
 14*a^5*b^3*c^3*d^5 + 14*a^6*b^2*c^2*d^6 + 6*a*b^7*c^7*d - 6*a^7*b*c*d^7)/(
a*d - b*c)^8 + (2*b*d*x*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^
4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6
*b*c*d^6))/(a*d - b*c)^8))/(a*d - b*c)^8 - ((60*a^6*d^6 + 1089*b^6*c^6 + 25
59*a^2*b^4*c^4*d^2 - 2341*a^3*b^3*c^3*d^3 + 1334*a^4*b^2*c^2*d^4 - 1851*a*b
^5*c^5*d - 430*a^5*b*c*d^5)/(420*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 +
35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*
d - 7*a^6*b*c*d^6)) - (b^3*x^3*(3*a^3*d^6 - 319*b^3*c^3*d^3 + 101*a*b^2*c^2
*d^4 - 25*a^2*b*c*d^5))/(12*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^
3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7
*a^6*b*c*d^6)) + (b^6*d^6*x^6)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35
*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d
- 7*a^6*b*c*d^6) - (b^5*x^5*(a*d^6 - 13*b*c*d^5))/(2*(a^7*d^7 - b^7*c^7 - 2
1*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^
2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)) + (b^2*x^2*(4*a^4*d^6 + 459*b^4*c^4
*d^2 - 241*a*b^3*c^3*d^3 + 109*a^2*b^2*c^2*d^4 - 31*a^3*b*c*d^5))/(20*(a^7*
d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^
4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)) + (b^4*x^4*(2*a^2*
d^6 + 107*b^2*c^2*d^4 - 19*a*b*c*d^5))/(6*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c
^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a
*b^6*c^6*d - 7*a^6*b*c*d^6)) - (b*x*(10*a^5*d^6 - 669*b^5*c^5*d + 591*a*b^4
*c^4*d^2 - 459*a^2*b^3*c^3*d^3 + 241*a^3*b^2*c^2*d^4 - 74*a^4*b*c*d^5))/(60
*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*
c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)))/(c^7 + d^7*
x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c
^2*d^5*x^5 + 7*c^6*d*x)
```

**sympy [B]** time = 4.49, size = 1776, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**8,x)
```

```
[Out] -b**7*log(x + (-a**9*b**7*d**9/(a*d - b*c)**8 + 9*a**8*b**8*c*d**8/(a*d - b
*c)**8 - 36*a**7*b**9*c**2*d**7/(a*d - b*c)**8 + 84*a**6*b**10*c**3*d**6/(a
*d - b*c)**8 - 126*a**5*b**11*c**4*d**5/(a*d - b*c)**8 + 126*a**4*b**12*c**
5*d**4/(a*d - b*c)**8 - 84*a**3*b**13*c**6*d**3/(a*d - b*c)**8 + 36*a**2*b*
*14*c**7*d**2/(a*d - b*c)**8 - 9*a*b**15*c**8*d/(a*d - b*c)**8 + a*b**7*d +
 b**16*c**9/(a*d - b*c)**8 + b**8*c)/(2*b**8*d))/(a*d - b*c)**8 + b**7*log(
x + (a**9*b**7*d**9/(a*d - b*c)**8 - 9*a**8*b**8*c*d**8/(a*d - b*c)**8 + 36
*a**7*b**9*c**2*d**7/(a*d - b*c)**8 - 84*a**6*b**10*c**3*d**6/(a*d - b*c)**
8 + 126*a**5*b**11*c**4*d**5/(a*d - b*c)**8 - 126*a**4*b**12*c**5*d**4/(a*d
```

```
 - b*c)**8 + 84*a**3*b**13*c**6*d**3/(a*d - b*c)**8 - 36*a**2*b**14*c**7*d*
*2/(a*d - b*c)**8 + 9*a*b**15*c**8*d/(a*d - b*c)**8 + a*b**7*d - b**16*c**9
/(a*d - b*c)**8 + b**8*c)/(2*b**8*d))/(a*d - b*c)**8 + (-60*a**6*d**6 + 430
*a**5*b*c*d**5 - 1334*a**4*b**2*c**2*d**4 + 2341*a**3*b**3*c**3*d**3 - 2559
*a**2*b**4*c**4*d**2 + 1851*a*b**5*c**5*d - 1089*b**6*c**6 - 420*b**6*d**6*
x**6 + x**5*(210*a*b**5*d**6 - 2730*b**6*c*d**5) + x**4*(-140*a**2*b**4*d**
6 + 1330*a*b**5*c*d**5 - 7490*b**6*c**2*d**4) + x**3*(105*a**3*b**3*d**6 -
875*a**2*b**4*c*d**5 + 3535*a*b**5*c**2*d**4 - 11165*b**6*c**3*d**3) + x**2
*(-84*a**4*b**2*d**6 + 651*a**3*b**3*c*d**5 - 2289*a**2*b**4*c**2*d**4 + 50
61*a*b**5*c**3*d**3 - 9639*b**6*c**4*d**2) + x*(70*a**5*b*d**6 - 518*a**4*b
**2*c*d**5 + 1687*a**3*b**3*c**2*d**4 - 3213*a**2*b**4*c**3*d**3 + 4137*a*b
**5*c**4*d**2 - 4683*b**6*c**5*d))/(420*a**7*c**7*d**7 - 2940*a**6*b*c**8*d
**6 + 8820*a**5*b**2*c**9*d**5 - 14700*a**4*b**3*c**10*d**4 + 14700*a**3*b*
*4*c**11*d**3 - 8820*a**2*b**5*c**12*d**2 + 2940*a*b**6*c**13*d - 420*b**7*
c**14 + x**7*(420*a**7*d**14 - 2940*a**6*b*c*d**13 + 8820*a**5*b**2*c**2*d*
*12 - 14700*a**4*b**3*c**3*d**11 + 14700*a**3*b**4*c**4*d**10 - 8820*a**2*b
**5*c**5*d**9 + 2940*a*b**6*c**6*d**8 - 420*b**7*c**7*d**7) + x**6*(2940*a*
*7*c*d**13 - 20580*a**6*b*c**2*d**12 + 61740*a**5*b**2*c**3*d**11 - 102900*
a**4*b**3*c**4*d**10 + 102900*a**3*b**4*c**5*d**9 - 61740*a**2*b**5*c**6*d*
*8 + 20580*a*b**6*c**7*d**7 - 2940*b**7*c**8*d**6) + x**5*(8820*a**7*c**2*d
**12 - 61740*a**6*b*c**3*d**11 + 185220*a**5*b**2*c**4*d**10 - 308700*a**4*
b**3*c**5*d**9 + 308700*a**3*b**4*c**6*d**8 - 185220*a**2*b**5*c**7*d**7 +
61740*a*b**6*c**8*d**6 - 8820*b**7*c**9*d**5) + x**4*(14700*a**7*c**3*d**11
 - 102900*a**6*b*c**4*d**10 + 308700*a**5*b**2*c**5*d**9 - 514500*a**4*b**3
*c**6*d**8 + 514500*a**3*b**4*c**7*d**7 - 308700*a**2*b**5*c**8*d**6 + 1029
00*a*b**6*c**9*d**5 - 14700*b**7*c**10*d**4) + x**3*(14700*a**7*c**4*d**10
- 102900*a**6*b*c**5*d**9 + 308700*a**5*b**2*c**6*d**8 - 514500*a**4*b**3*c
**7*d**7 + 514500*a**3*b**4*c**8*d**6 - 308700*a**2*b**5*c**9*d**5 + 102900
*a*b**6*c**10*d**4 - 14700*b**7*c**11*d**3) + x**2*(8820*a**7*c**5*d**9 - 6
1740*a**6*b*c**6*d**8 + 185220*a**5*b**2*c**7*d**7 - 308700*a**4*b**3*c**8*
d**6 + 308700*a**3*b**4*c**9*d**5 - 185220*a**2*b**5*c**10*d**4 + 61740*a*b
**6*c**11*d**3 - 8820*b**7*c**12*d**2) + x*(2940*a**7*c**6*d**8 - 20580*a**
6*b*c**7*d**7 + 61740*a**5*b**2*c**8*d**6 - 102900*a**4*b**3*c**9*d**5 + 10
2900*a**3*b**4*c**10*d**4 - 61740*a**2*b**5*c**11*d**3 + 20580*a*b**6*c**12
*d**2 - 2940*b**7*c**13*d))
```

## 3.1267 $\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx$

**Optimal**. Leaf size=231

$$-\frac{b^7}{(a+bx)(bc-ad)^8}-\frac{8b^7d\log(a+bx)}{(bc-ad)^9}+\frac{8b^7d\log(c+dx)}{(bc-ad)^9}-\frac{7b^6d}{(c+dx)(bc-ad)^8}-\frac{3b^5d}{(c+dx)^2(bc-ad)^7}-\frac{5b^4d}{3(c+dx)^3(bc$$

**Rubi** [A] time = 0.27, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{b^7}{(a+bx)(bc-ad)^8}-\frac{7b^6d}{(c+dx)(bc-ad)^8}-\frac{3b^5d}{(c+dx)^2(bc-ad)^7}-\frac{5b^4d}{3(c+dx)^3(bc-ad)^6}-\frac{b^3d}{(c+dx)^4(bc-ad)^5}-\frac{3b^2d}{5(c+dx)^5(bc-ad)^4}-\frac{8b^7d\log(a+bx)}{(bc-ad)^9}+\frac{8b^7d\log(c+dx)}{(bc-ad)^9}-\frac{bd}{3(c+dx)^6(bc-ad)^3}-\frac{d}{7(c+dx)^7(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^8),x]
```

```
[Out] -(b^7/((b*c - a*d)^8*(a + b*x))) - d/(7*(b*c - a*d)^2*(c + d*x)^7) - (b*d)/
(3*(b*c - a*d)^3*(c + d*x)^6) - (3*b^2*d)/(5*(b*c - a*d)^4*(c + d*x)^5) - (
b^3*d)/((b*c - a*d)^5*(c + d*x)^4) - (5*b^4*d)/(3*(b*c - a*d)^6*(c + d*x)^3
) - (3*b^5*d)/((b*c - a*d)^7*(c + d*x)^2) - (7*b^6*d)/((b*c - a*d)^8*(c + d
*x)) - (8*b^7*d*Log[a + b*x])/(b*c - a*d)^9 + (8*b^7*d*Log[c + d*x])/(b*c -
 a*d)^9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx = \int \left(\frac{b^8}{(bc-ad)^8(a+bx)^2}-\frac{8b^8d}{(bc-ad)^9(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^8}+\frac{2bd^2}{(bc-ad)^3(c+d$$
$$= -\frac{b^7}{(bc-ad)^8(a+bx)}-\frac{d}{7(bc-ad)^2(c+dx)^7}-\frac{bd}{3(bc-ad)^3(c+dx)^6}-\frac{3b^2d}{5(bc-ad)^4(c+$$

**Mathematica** [A] time = 0.24, size = 213, normalized size = 0.92

$$-\frac{\frac{105b^7(bc-ad)}{a+bx}+840b^7d\log(a+bx)+\frac{735b^6d(bc-ad)}{c+dx}+\frac{315b^5d(bc-ad)^2}{(c+dx)^2}+\frac{175b^4d(bc-ad)^3}{(c+dx)^3}+\frac{105b^3d(bc-ad)^4}{(c+dx)^4}+\frac{63b^2d(bc-ad)^5}{(c+dx)^5}+\frac{35bd(bc-ad)^6}{(c+dx)^6}-\frac{15d(ad-bc)^7}{(c+dx)^7}-840b^7d\log(c+dx)}{105(bc-ad)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^8),x]
```

```
[Out] -1/105*((105*b^7*(b*c - a*d))/(a + b*x) - (15*d*(-(b*c) + a*d)^7)/(c + d*x)
^7 + (35*b*d*(b*c - a*d)^6)/(c + d*x)^6 + (63*b^2*d*(b*c - a*d)^5)/(c + d*x
)^5 + (105*b^3*d*(b*c - a*d)^4)/(c + d*x)^4 + (175*b^4*d*(b*c - a*d)^3)/(c
+ d*x)^3 + (315*b^5*d*(b*c - a*d)^2)/(c + d*x)^2 + (735*b^6*d*(b*c - a*d))/
(c + d*x) + 840*b^7*d*Log[a + b*x] - 840*b^7*d*Log[c + d*x])/(b*c - a*d)^9
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^8),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^8), x]
```

**fricas [B]** time = 1.46, size = 2264, normalized size = 9.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(105*b^8*c^8 + 1338*a*b^7*c^7*d - 2940*a^2*b^6*c^6*d^2 + 2940*a^3*b^
5*c^5*d^3 - 2450*a^4*b^4*c^4*d^4 + 1470*a^5*b^3*c^3*d^5 - 588*a^6*b^2*c^2*d
^6 + 140*a^7*b*c*d^7 - 15*a^8*d^8 + 840*(b^8*c*d^7 - a*b^7*d^8)*x^7 + 420*(
13*b^8*c^2*d^6 - 12*a*b^7*c*d^7 - a^2*b^6*d^8)*x^6 + 140*(107*b^8*c^3*d^5 -
 87*a*b^7*c^2*d^6 - 21*a^2*b^6*c*d^7 + a^3*b^5*d^8)*x^5 + 70*(319*b^8*c^4*d
^4 - 206*a*b^7*c^3*d^5 - 126*a^2*b^6*c^2*d^6 + 14*a^3*b^5*c*d^7 - a^4*b^4*d
^8)*x^4 + 14*(1377*b^8*c^5*d^3 - 505*a*b^7*c^4*d^4 - 1050*a^2*b^6*c^3*d^5 +
 210*a^3*b^5*c^2*d^6 - 35*a^4*b^4*c*d^7 + 3*a^5*b^3*d^8)*x^3 + 14*(669*b^8*
c^6*d^2 + 117*a*b^7*c^5*d^3 - 1050*a^2*b^6*c^4*d^4 + 350*a^3*b^5*c^3*d^5 -
105*a^4*b^4*c^2*d^6 + 21*a^5*b^3*c*d^7 - 2*a^6*b^2*d^8)*x^2 + 2*(1089*b^8*c
^7*d + 1743*a*b^7*c^6*d^2 - 4410*a^2*b^6*c^5*d^3 + 2450*a^3*b^5*c^4*d^4 - 1
225*a^4*b^4*c^3*d^5 + 441*a^5*b^3*c^2*d^6 - 98*a^6*b^2*c*d^7 + 10*a^7*b*d^8
)*x + 840*(b^8*d^8*x^8 + a*b^7*c^7*d + (7*b^8*c*d^7 + a*b^7*d^8)*x^7 + 7*(3
*b^8*c^2*d^6 + a*b^7*c*d^7)*x^6 + 7*(5*b^8*c^3*d^5 + 3*a*b^7*c^2*d^6)*x^5 +
 35*(b^8*c^4*d^4 + a*b^7*c^3*d^5)*x^4 + 7*(3*b^8*c^5*d^3 + 5*a*b^7*c^4*d^4)
*x^3 + 7*(b^8*c^6*d^2 + 3*a*b^7*c^5*d^3)*x^2 + (b^8*c^7*d + 7*a*b^7*c^6*d^2
)*x)*log(b*x + a) - 840*(b^8*d^8*x^8 + a*b^7*c^7*d + (7*b^8*c*d^7 + a*b^7*d
^8)*x^7 + 7*(3*b^8*c^2*d^6 + a*b^7*c*d^7)*x^6 + 7*(5*b^8*c^3*d^5 + 3*a*b^7*
c^2*d^6)*x^5 + 35*(b^8*c^4*d^4 + a*b^7*c^3*d^5)*x^4 + 7*(3*b^8*c^5*d^3 + 5*
```

```
a*b^7*c^4*d^4)*x^3 + 7*(b^8*c^6*d^2 + 3*a*b^7*c^5*d^3)*x^2 + (b^8*c^7*d + 7
*a*b^7*c^6*d^2)*x)*log(d*x + c))/(a*b^9*c^16 - 9*a^2*b^8*c^15*d + 36*a^3*b^
7*c^14*d^2 - 84*a^4*b^6*c^13*d^3 + 126*a^5*b^5*c^12*d^4 - 126*a^6*b^4*c^11*
d^5 + 84*a^7*b^3*c^10*d^6 - 36*a^8*b^2*c^9*d^7 + 9*a^9*b*c^8*d^8 - a^10*c^7
*d^9 + (b^10*c^9*d^7 - 9*a*b^9*c^8*d^8 + 36*a^2*b^8*c^7*d^9 - 84*a^3*b^7*c^
6*d^10 + 126*a^4*b^6*c^5*d^11 - 126*a^5*b^5*c^4*d^12 + 84*a^6*b^4*c^3*d^13
- 36*a^7*b^3*c^2*d^14 + 9*a^8*b^2*c*d^15 - a^9*b*d^16)*x^8 + (7*b^10*c^10*d
^6 - 62*a*b^9*c^9*d^7 + 243*a^2*b^8*c^8*d^8 - 552*a^3*b^7*c^7*d^9 + 798*a^4
*b^6*c^6*d^10 - 756*a^5*b^5*c^5*d^11 + 462*a^6*b^4*c^4*d^12 - 168*a^7*b^3*c
^3*d^13 + 27*a^8*b^2*c^2*d^14 + 2*a^9*b*c*d^15 - a^10*d^16)*x^7 + 7*(3*b^10
*c^11*d^5 - 26*a*b^9*c^10*d^6 + 99*a^2*b^8*c^9*d^7 - 216*a^3*b^7*c^8*d^8 +
294*a^4*b^6*c^7*d^9 - 252*a^5*b^5*c^6*d^10 + 126*a^6*b^4*c^5*d^11 - 24*a^7*
b^3*c^4*d^12 - 9*a^8*b^2*c^3*d^13 + 6*a^9*b*c^2*d^14 - a^10*c*d^15)*x^6 + 7
*(5*b^10*c^12*d^4 - 42*a*b^9*c^11*d^5 + 153*a^2*b^8*c^10*d^6 - 312*a^3*b^7*
c^9*d^7 + 378*a^4*b^6*c^8*d^8 - 252*a^5*b^5*c^7*d^9 + 42*a^6*b^4*c^6*d^10 +
 72*a^7*b^3*c^5*d^11 - 63*a^8*b^2*c^4*d^12 + 22*a^9*b*c^3*d^13 - 3*a^10*c^2
*d^14)*x^5 + 35*(b^10*c^13*d^3 - 8*a*b^9*c^12*d^4 + 27*a^2*b^8*c^11*d^5 - 4
8*a^3*b^7*c^10*d^6 + 42*a^4*b^6*c^9*d^7 - 42*a^6*b^4*c^7*d^9 + 48*a^7*b^3*c
^6*d^10 - 27*a^8*b^2*c^5*d^11 + 8*a^9*b*c^4*d^12 - a^10*c^3*d^13)*x^4 + 7*(
3*b^10*c^14*d^2 - 22*a*b^9*c^13*d^3 + 63*a^2*b^8*c^12*d^4 - 72*a^3*b^7*c^11
*d^5 - 42*a^4*b^6*c^10*d^6 + 252*a^5*b^5*c^9*d^7 - 378*a^6*b^4*c^8*d^8 + 31
2*a^7*b^3*c^7*d^9 - 153*a^8*b^2*c^6*d^10 + 42*a^9*b*c^5*d^11 - 5*a^10*c^4*d
^12)*x^3 + 7*(b^10*c^15*d - 6*a*b^9*c^14*d^2 + 9*a^2*b^8*c^13*d^3 + 24*a^3*
b^7*c^12*d^4 - 126*a^4*b^6*c^11*d^5 + 252*a^5*b^5*c^10*d^6 - 294*a^6*b^4*c^
9*d^7 + 216*a^7*b^3*c^8*d^8 - 99*a^8*b^2*c^7*d^9 + 26*a^9*b*c^6*d^10 - 3*a^
10*c^5*d^11)*x^2 + (b^10*c^16 - 2*a*b^9*c^15*d - 27*a^2*b^8*c^14*d^2 + 168*
a^3*b^7*c^13*d^3 - 462*a^4*b^6*c^12*d^4 + 756*a^5*b^5*c^11*d^5 - 798*a^6*b^
4*c^10*d^6 + 552*a^7*b^3*c^9*d^7 - 243*a^8*b^2*c^8*d^8 + 62*a^9*b*c^7*d^9 -
 7*a^10*c^6*d^10)*x)
```

**giac [B]** time = 1.37, size = 714, normalized size = 3.09

$$-\frac{b^{15}}{\left(b^{16}c^{8}-8\,ab^{15}c^{7}d+28\,a^{2}b^{14}c^{6}d^{2}-56\,a^{3}b^{13}c^{5}d^{3}+70\,a^{4}b^{12}c^{4}d^{4}-56\,a^{5}b^{11}c^{3}d^{5}+28\,a^{6}b^{10}c^{2}d^{6}-8\,a^{7}b^{9}cd^{7}+a^{8}b^{8}d^{8}\right)(bx+a)}+\frac{8\,b^{8}d\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^{10}c^{9}-9\,ab^{9}c^{8}d+36\,a^{2}b^{8}c^{7}d^{2}-84\,a^{3}b^{7}c^{6}d^{3}+126\,a^{4}b^{6}c^{5}d^{4}-126\,a^{5}b^{5}c^{4}d^{5}+84\,a^{6}b^{4}c^{3}d^{6}-36\,a^{7}b^{3}c^{2}d^{7}+9\,a^{8}b^{2}cd^{8}-a^{9}bd^{9}}+\frac{1443\,b^{7}d^{8}+\frac{9366\,(b^{9}cd^{7}-ab^{8}d^{8})}{(bx+a)b}+\frac{25578\,(b^{11}c^{2}d^{6}-2\,ab^{10}cd^{7}+a^{2}b^{9}d^{8})}{(bx+a)^{2}b^{2}}+\frac{37730\,(b^{13}c^{3}d^{5}-3\,ab^{12}c^{2}d^{6}+3\,a^{2}b^{11}cd^{7}-a^{3}b^{10}d^{8})}{(bx+a)^{3}b^{3}}+\frac{31850\,(b^{15}c^{4}d^{4}-4\,ab^{14}c^{3}d^{5}+6\,a^{2}b^{13}c^{2}d^{6}-4\,a^{3}b^{12}cd^{7}+a^{4}b^{11}d^{8})}{(bx+a)^{4}b^{4}}+\frac{14700\,(b^{17}c^{5}d^{3}-5\,ab^{16}c^{4}d^{4}+10\,a^{2}b^{15}c^{3}d^{5}-10\,a^{3}b^{14}c^{2}d^{6}+5\,a^{4}b^{13}cd^{7}-a^{5}b^{12}d^{8})}{(bx+a)^{5}b^{5}}+\frac{2940\,(b^{19}c^{6}d^{2}-6\,ab^{18}c^{5}d^{3}+15\,a^{2}b^{17}c^{4}d^{4}-20\,a^{3}b^{16}c^{3}d^{5}+15\,a^{4}b^{15}c^{2}d^{6}-6\,a^{5}b^{14}cd^{7}+a^{6}b^{13}d^{8})}{(bx+a)^{6}b^{6}}}{105\,(bc-ad)^{9}\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -b^15/((b^16*c^8 - 8*a*b^15*c^7*d + 28*a^2*b^14*c^6*d^2 - 56*a^3*b^13*c^5*d
^3 + 70*a^4*b^12*c^4*d^4 - 56*a^5*b^11*c^3*d^5 + 28*a^6*b^10*c^2*d^6 - 8*a^
7*b^9*c*d^7 + a^8*b^8*d^8)*(b*x + a)) + 8*b^8*d*log(abs(b*c/(b*x + a) - a*d
/(b*x + a) + d))/(b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^3*b^
7*c^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^4*c^3*d^6
- 36*a^7*b^3*c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9) + 1/105*(1443*b^7*d^8 +
 9366*(b^9*c*d^7 - a*b^8*d^8)/((b*x + a)*b) + 25578*(b^11*c^2*d^6 - 2*a*b^1
0*c*d^7 + a^2*b^9*d^8)/((b*x + a)^2*b^2) + 37730*(b^13*c^3*d^5 - 3*a*b^12*c
```

```
^2*d^6 + 3*a^2*b^11*c*d^7 - a^3*b^10*d^8)/((b*x + a)^3*b^3) + 31850*(b^15*c
^4*d^4 - 4*a*b^14*c^3*d^5 + 6*a^2*b^13*c^2*d^6 - 4*a^3*b^12*c*d^7 + a^4*b^1
1*d^8)/((b*x + a)^4*b^4) + 14700*(b^17*c^5*d^3 - 5*a*b^16*c^4*d^4 + 10*a^2*
b^15*c^3*d^5 - 10*a^3*b^14*c^2*d^6 + 5*a^4*b^13*c*d^7 - a^5*b^12*d^8)/((b*x
 + a)^5*b^5) + 2940*(b^19*c^6*d^2 - 6*a*b^18*c^5*d^3 + 15*a^2*b^17*c^4*d^4
- 20*a^3*b^16*c^3*d^5 + 15*a^4*b^15*c^2*d^6 - 6*a^5*b^14*c*d^7 + a^6*b^13*d
^8)/((b*x + a)^6*b^6))/((b*c - a*d)^9*(b*c/(b*x + a) - a*d/(b*x + a) + d)^7
)
```

**maple [A]** time = 0.02, size = 223, normalized size = 0.97

$$\frac{8b^7 d\ln(bx+a)}{(ad-bc)^9} - \frac{8b^7 d\ln(dx+c)}{(ad-bc)^9} - \frac{b^7}{(ad-bc)^8(bx+a)} - \frac{7b^6 d}{(ad-bc)^8(dx+c)} + \frac{3b^5 d}{(ad-bc)^7(dx+c)^2} - \frac{5b^4 d}{3(ad-bc)^6(dx+c)^3} + \frac{b^3 d}{(ad-bc)^5(dx+c)^4} - \frac{3b^2 d}{5(ad-bc)^4(dx+c)^5} + \frac{bd}{3(ad-bc)^3(dx+c)^6} - \frac{d}{7(ad-bc)^2(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^8,x)
```

```
[Out] -1/7*d/(a*d-b*c)^2/(d*x+c)^7-8*d/(a*d-b*c)^9*b^7*ln(d*x+c)-7*d/(a*d-b*c)^8*
b^6/(d*x+c)+3*d/(a*d-b*c)^7*b^5/(d*x+c)^2-5/3*d/(a*d-b*c)^6*b^4/(d*x+c)^3+d
/(a*d-b*c)^5*b^3/(d*x+c)^4-3/5*d/(a*d-b*c)^4*b^2/(d*x+c)^5+1/3*d/(a*d-b*c)^
3*b/(d*x+c)^6-b^7/(a*d-b*c)^8/(b*x+a)+8*d/(a*d-b*c)^9*b^7*ln(b*x+a)
```

**maxima [B]** time = 3.88, size = 1881, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -8*b^7*d*log(b*x + a)/(b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^
3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*
d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9) + 8*b^7*d*log(d*x + c)/
(b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^
4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d
^7 + 9*a^8*b*c*d^8 - a^9*d^9) - 1/105*(840*b^7*d^7*x^7 + 105*b^7*c^7 + 1443
*a*b^6*c^6*d - 1497*a^2*b^5*c^5*d^2 + 1443*a^3*b^4*c^4*d^3 - 1007*a^4*b^3*c
^3*d^4 + 463*a^5*b^2*c^2*d^5 - 125*a^6*b*c*d^6 + 15*a^7*d^7 + 420*(13*b^7*c
*d^6 + a*b^6*d^7)*x^6 + 140*(107*b^7*c^2*d^5 + 20*a*b^6*c*d^6 - a^2*b^5*d^7
)*x^5 + 70*(319*b^7*c^3*d^4 + 113*a*b^6*c^2*d^5 - 13*a^2*b^5*c*d^6 + a^3*b^
4*d^7)*x^4 + 14*(1377*b^7*c^4*d^3 + 872*a*b^6*c^3*d^4 - 178*a^2*b^5*c^2*d^5
 + 32*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 14*(669*b^7*c^5*d^2 + 786*a*b^6*
c^4*d^3 - 264*a^2*b^5*c^3*d^4 + 86*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 2*a
^5*b^2*d^7)*x^2 + 2*(1089*b^7*c^6*d + 2832*a*b^6*c^5*d^2 - 1578*a^2*b^5*c^4
*d^3 + 872*a^3*b^4*c^3*d^4 - 353*a^4*b^3*c^2*d^5 + 88*a^5*b^2*c*d^6 - 10*a^
6*b*d^7)*x)/(a*b^8*c^15 - 8*a^2*b^7*c^14*d + 28*a^3*b^6*c^13*d^2 - 56*a^4*b
^5*c^12*d^3 + 70*a^5*b^4*c^11*d^4 - 56*a^6*b^3*c^10*d^5 + 28*a^7*b^2*c^9*d^
```

```
6 - 8*a^8*b*c^8*d^7 + a^9*c^7*d^8 + (b^9*c^8*d^7 - 8*a*b^8*c^7*d^8 + 28*a^2
*b^7*c^6*d^9 - 56*a^3*b^6*c^5*d^10 + 70*a^4*b^5*c^4*d^11 - 56*a^5*b^4*c^3*d
^12 + 28*a^6*b^3*c^2*d^13 - 8*a^7*b^2*c*d^14 + a^8*b*d^15)*x^8 + (7*b^9*c^9
*d^6 - 55*a*b^8*c^8*d^7 + 188*a^2*b^7*c^7*d^8 - 364*a^3*b^6*c^6*d^9 + 434*a
^4*b^5*c^5*d^10 - 322*a^5*b^4*c^4*d^11 + 140*a^6*b^3*c^3*d^12 - 28*a^7*b^2*
c^2*d^13 - a^8*b*c*d^14 + a^9*d^15)*x^7 + 7*(3*b^9*c^10*d^5 - 23*a*b^8*c^9*
d^6 + 76*a^2*b^7*c^8*d^7 - 140*a^3*b^6*c^7*d^8 + 154*a^4*b^5*c^6*d^9 - 98*a
^5*b^4*c^5*d^10 + 28*a^6*b^3*c^4*d^11 + 4*a^7*b^2*c^3*d^12 - 5*a^8*b*c^2*d^
13 + a^9*c*d^14)*x^6 + 7*(5*b^9*c^11*d^4 - 37*a*b^8*c^10*d^5 + 116*a^2*b^7*
c^9*d^6 - 196*a^3*b^6*c^8*d^7 + 182*a^4*b^5*c^7*d^8 - 70*a^5*b^4*c^6*d^9 -
28*a^6*b^3*c^5*d^10 + 44*a^7*b^2*c^4*d^11 - 19*a^8*b*c^3*d^12 + 3*a^9*c^2*d
^13)*x^5 + 35*(b^9*c^12*d^3 - 7*a*b^8*c^11*d^4 + 20*a^2*b^7*c^10*d^5 - 28*a
^3*b^6*c^9*d^6 + 14*a^4*b^5*c^8*d^7 + 14*a^5*b^4*c^7*d^8 - 28*a^6*b^3*c^6*d
^9 + 20*a^7*b^2*c^5*d^10 - 7*a^8*b*c^4*d^11 + a^9*c^3*d^12)*x^4 + 7*(3*b^9*
c^13*d^2 - 19*a*b^8*c^12*d^3 + 44*a^2*b^7*c^11*d^4 - 28*a^3*b^6*c^10*d^5 -
70*a^4*b^5*c^9*d^6 + 182*a^5*b^4*c^8*d^7 - 196*a^6*b^3*c^7*d^8 + 116*a^7*b^
2*c^6*d^9 - 37*a^8*b*c^5*d^10 + 5*a^9*c^4*d^11)*x^3 + 7*(b^9*c^14*d - 5*a*b
^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 + 28*a^3*b^6*c^11*d^4 - 98*a^4*b^5*c^10*d^
5 + 154*a^5*b^4*c^9*d^6 - 140*a^6*b^3*c^8*d^7 + 76*a^7*b^2*c^7*d^8 - 23*a^8
*b*c^6*d^9 + 3*a^9*c^5*d^10)*x^2 + (b^9*c^15 - a*b^8*c^14*d - 28*a^2*b^7*c^
13*d^2 + 140*a^3*b^6*c^12*d^3 - 322*a^4*b^5*c^11*d^4 + 434*a^5*b^4*c^10*d^5
 - 364*a^6*b^3*c^9*d^6 + 188*a^7*b^2*c^8*d^7 - 55*a^8*b*c^7*d^8 + 7*a^9*c^6
*d^9)*x)
```

**mupad [B]** time = 1.39, size = 1738, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^8),x)
```

```
[Out] (16*b^7*d*atanh((a^9*d^9 + b^9*c^9 + 20*a^2*b^7*c^7*d^2 - 28*a^3*b^6*c^6*d^
3 + 14*a^4*b^5*c^5*d^4 + 14*a^5*b^4*c^4*d^5 - 28*a^6*b^3*c^3*d^6 + 20*a^7*b
^2*c^2*d^7 - 7*a*b^8*c^8*d - 7*a^8*b*c*d^8)/(a*d - b*c)^9 + (2*b*d*x*(a^8*d
^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)
)/(a*d - b*c)^9))/(a*d - b*c)^9 - ((15*a^7*d^7 + 105*b^7*c^7 - 1497*a^2*b^5
*c^5*d^2 + 1443*a^3*b^4*c^4*d^3 - 1007*a^4*b^3*c^3*d^4 + 463*a^5*b^2*c^2*d^
5 + 1443*a*b^6*c^6*d - 125*a^6*b*c*d^6)/(105*(a^8*d^8 + b^8*c^8 + 28*a^2*b^
6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 +
28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (4*b^5*x^5*(107*b^2*
c^2*d^5 - a^2*d^7 + 20*a*b*c*d^6))/(3*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d
^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*
b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b^2*x^2*(2*a^5*d^7 + 669
*b^5*c^5*d^2 + 786*a*b^4*c^4*d^3 - 264*a^2*b^3*c^3*d^4 + 86*a^3*b^2*c^2*d^5
```

```
 - 19*a^4*b*c*d^6))/(15*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^
5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b^4*x^4*(a^3*d^7 + 319*b^3*c^3*d^4 + 1
13*a*b^2*c^2*d^5 - 13*a^2*b*c*d^6))/(3*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b*x*(1089*b^6*c^6*d - 1
0*a^6*d^7 + 2832*a*b^5*c^5*d^2 - 1578*a^2*b^4*c^4*d^3 + 872*a^3*b^3*c^3*d^4
 - 353*a^4*b^2*c^2*d^5 + 88*a^5*b*c*d^6))/(105*(a^8*d^8 + b^8*c^8 + 28*a^2*
b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
+ 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (8*b^7*d^7*x^7)/(a
^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*
d^7) + (4*b^6*x^6*(a*d^7 + 13*b*c*d^6))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^
6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) + (2*b^3*x^3*(1377*b^4*c^4*d
^3 - 3*a^4*d^7 + 872*a*b^3*c^3*d^4 - 178*a^2*b^2*c^2*d^5 + 32*a^3*b*c*d^6))
/(15*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a
^7*b*c*d^7)))/(x^7*(a*d^7 + 7*b*c*d^6) + x^3*(35*a*c^4*d^3 + 21*b*c^5*d^2)
+ x^5*(21*a*c^2*d^5 + 35*b*c^3*d^4) + x^4*(35*a*c^3*d^4 + 35*b*c^4*d^3) + a
*c^7 + x*(b*c^7 + 7*a*c^6*d) + x^2*(21*a*c^5*d^2 + 7*b*c^6*d) + x^6*(21*b*c
^2*d^5 + 7*a*c*d^6) + b*d^7*x^8)
```

**sympy [B]** time = 7.75, size = 2336, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**8,x)
```

```
[Out] -8*b**7*d*log(x + (-8*a**10*b**7*d**11/(a*d - b*c)**9 + 80*a**9*b**8*c*d**1
0/(a*d - b*c)**9 - 360*a**8*b**9*c**2*d**9/(a*d - b*c)**9 + 960*a**7*b**10*
c**3*d**8/(a*d - b*c)**9 - 1680*a**6*b**11*c**4*d**7/(a*d - b*c)**9 + 2016*
a**5*b**12*c**5*d**6/(a*d - b*c)**9 - 1680*a**4*b**13*c**6*d**5/(a*d - b*c)
**9 + 960*a**3*b**14*c**7*d**4/(a*d - b*c)**9 - 360*a**2*b**15*c**8*d**3/(a
*d - b*c)**9 + 80*a*b**16*c**9*d**2/(a*d - b*c)**9 + 8*a*b**7*d**2 - 8*b**1
7*c**10*d/(a*d - b*c)**9 + 8*b**8*c*d)/(16*b**8*d**2))/(a*d - b*c)**9 + 8*b
**7*d*log(x + (8*a**10*b**7*d**11/(a*d - b*c)**9 - 80*a**9*b**8*c*d**10/(a*
d - b*c)**9 + 360*a**8*b**9*c**2*d**9/(a*d - b*c)**9 - 960*a**7*b**10*c**3*
d**8/(a*d - b*c)**9 + 1680*a**6*b**11*c**4*d**7/(a*d - b*c)**9 - 2016*a**5*
b**12*c**5*d**6/(a*d - b*c)**9 + 1680*a**4*b**13*c**6*d**5/(a*d - b*c)**9 -
 960*a**3*b**14*c**7*d**4/(a*d - b*c)**9 + 360*a**2*b**15*c**8*d**3/(a*d -
b*c)**9 - 80*a*b**16*c**9*d**2/(a*d - b*c)**9 + 8*a*b**7*d**2 + 8*b**17*c**
10*d/(a*d - b*c)**9 + 8*b**8*c*d)/(16*b**8*d**2))/(a*d - b*c)**9 + (-15*a**
7*d**7 + 125*a**6*b*c*d**6 - 463*a**5*b**2*c**2*d**5 + 1007*a**4*b**3*c**3*
```

d**4 - 1443*a**3*b**4*c**4*d**3 + 1497*a**2*b**5*c**5*d**2 - 1443*a*b**6*c*
*6*d - 105*b**7*c**7 - 840*b**7*d**7*x**7 + x**6*(-420*a*b**6*d**7 - 5460*b
**7*c*d**6) + x**5*(140*a**2*b**5*d**7 - 2800*a*b**6*c*d**6 - 14980*b**7*c*
*2*d**5) + x**4*(-70*a**3*b**4*d**7 + 910*a**2*b**5*c*d**6 - 7910*a*b**6*c*
*2*d**5 - 22330*b**7*c**3*d**4) + x**3*(42*a**4*b**3*d**7 - 448*a**3*b**4*c
*d**6 + 2492*a**2*b**5*c**2*d**5 - 12208*a*b**6*c**3*d**4 - 19278*b**7*c**4
*d**3) + x**2*(-28*a**5*b**2*d**7 + 266*a**4*b**3*c*d**6 - 1204*a**3*b**4*c
**2*d**5 + 3696*a**2*b**5*c**3*d**4 - 11004*a*b**6*c**4*d**3 - 9366*b**7*c*
*5*d**2) + x*(20*a**6*b*d**7 - 176*a**5*b**2*c*d**6 + 706*a**4*b**3*c**2*d*
*5 - 1744*a**3*b**4*c**3*d**4 + 3156*a**2*b**5*c**4*d**3 - 5664*a*b**6*c**5
*d**2 - 2178*b**7*c**6*d))/(105*a**9*c**7*d**8 - 840*a**8*b*c**8*d**7 + 294
0*a**7*b**2*c**9*d**6 - 5880*a**6*b**3*c**10*d**5 + 7350*a**5*b**4*c**11*d*
*4 - 5880*a**4*b**5*c**12*d**3 + 2940*a**3*b**6*c**13*d**2 - 840*a**2*b**7*
c**14*d + 105*a*b**8*c**15 + x**8*(105*a**8*b*d**15 - 840*a**7*b**2*c*d**14
 + 2940*a**6*b**3*c**2*d**13 - 5880*a**5*b**4*c**3*d**12 + 7350*a**4*b**5*c
**4*d**11 - 5880*a**3*b**6*c**5*d**10 + 2940*a**2*b**7*c**6*d**9 - 840*a*b*
*8*c**7*d**8 + 105*b**9*c**8*d**7) + x**7*(105*a**9*d**15 - 105*a**8*b*c*d*
*14 - 2940*a**7*b**2*c**2*d**13 + 14700*a**6*b**3*c**3*d**12 - 33810*a**5*b
**4*c**4*d**11 + 45570*a**4*b**5*c**5*d**10 - 38220*a**3*b**6*c**6*d**9 + 1
9740*a**2*b**7*c**7*d**8 - 5775*a*b**8*c**8*d**7 + 735*b**9*c**9*d**6) + x*
*6*(735*a**9*c*d**14 - 3675*a**8*b*c**2*d**13 + 2940*a**7*b**2*c**3*d**12 +
 20580*a**6*b**3*c**4*d**11 - 72030*a**5*b**4*c**5*d**10 + 113190*a**4*b**5
*c**6*d**9 - 102900*a**3*b**6*c**7*d**8 + 55860*a**2*b**7*c**8*d**7 - 16905
*a*b**8*c**9*d**6 + 2205*b**9*c**10*d**5) + x**5*(2205*a**9*c**2*d**13 - 13
965*a**8*b*c**3*d**12 + 32340*a**7*b**2*c**4*d**11 - 20580*a**6*b**3*c**5*d
**10 - 51450*a**5*b**4*c**6*d**9 + 133770*a**4*b**5*c**7*d**8 - 144060*a**3
*b**6*c**8*d**7 + 85260*a**2*b**7*c**9*d**6 - 27195*a*b**8*c**10*d**5 + 367
5*b**9*c**11*d**4) + x**4*(3675*a**9*c**3*d**12 - 25725*a**8*b*c**4*d**11 +
 73500*a**7*b**2*c**5*d**10 - 102900*a**6*b**3*c**6*d**9 + 51450*a**5*b**4*
c**7*d**8 + 51450*a**4*b**5*c**8*d**7 - 102900*a**3*b**6*c**9*d**6 + 73500*
a**2*b**7*c**10*d**5 - 25725*a*b**8*c**11*d**4 + 3675*b**9*c**12*d**3) + x*
*3*(3675*a**9*c**4*d**11 - 27195*a**8*b*c**5*d**10 + 85260*a**7*b**2*c**6*d
**9 - 144060*a**6*b**3*c**7*d**8 + 133770*a**5*b**4*c**8*d**7 - 51450*a**4*
b**5*c**9*d**6 - 20580*a**3*b**6*c**10*d**5 + 32340*a**2*b**7*c**11*d**4 -
13965*a*b**8*c**12*d**3 + 2205*b**9*c**13*d**2) + x**2*(2205*a**9*c**5*d**1
0 - 16905*a**8*b*c**6*d**9 + 55860*a**7*b**2*c**7*d**8 - 102900*a**6*b**3*c
**8*d**7 + 113190*a**5*b**4*c**9*d**6 - 72030*a**4*b**5*c**10*d**5 + 20580*
a**3*b**6*c**11*d**4 + 2940*a**2*b**7*c**12*d**3 - 3675*a*b**8*c**13*d**2 +
 735*b**9*c**14*d) + x*(735*a**9*c**6*d**9 - 5775*a**8*b*c**7*d**8 + 19740*
a**7*b**2*c**8*d**7 - 38220*a**6*b**3*c**9*d**6 + 45570*a**5*b**4*c**10*d**
5 - 33810*a**4*b**5*c**11*d**4 + 14700*a**3*b**6*c**12*d**3 - 2940*a**2*b**
7*c**13*d**2 - 105*a*b**8*c**14*d + 105*b**9*c**15))

## 3.1268 $\int \frac{1}{(a+bx)^3(c+dx)^8}\,dx$

**Optimal**. Leaf size=276

$$\frac{36b^7d^2\log(a+bx)}{(bc-ad)^{10}}-\frac{36b^7d^2\log(c+dx)}{(bc-ad)^{10}}+\frac{8b^7d}{(a+bx)(bc-ad)^9}-\frac{b^7}{2(a+bx)^2(bc-ad)^8}+\frac{28b^6d^2}{(c+dx)(bc-ad)^9}+\frac{2}{2(c+d}$$

**Rubi** [A] time = 0.36, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{28b^6d^2}{(c+dx)(bc-ad)^9}+\frac{21b^5d^2}{2(c+dx)^2(bc-ad)^8}+\frac{5b^4d^2}{(c+dx)^3(bc-ad)^7}+\frac{5b^3d^2}{2(c+dx)^4(bc-ad)^6}+\frac{6b^2d^2}{5(c+dx)^5(bc-ad)^5}+\frac{36b^7d^2\log(a+bx)}{(bc-ad)^{10}}-\frac{36b^7d^2\log(c+dx)}{(bc-ad)^{10}}+\frac{8b^7d}{(a+bx)(bc-ad)^9}-\frac{b^7}{2(a+bx)^2(bc-ad)^8}+\frac{bd^2}{2(c+dx)^6(bc-ad)^4}+\frac{d^2}{7(c+dx)^7(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^8),x]
```

```
[Out] -b^7/(2*(b*c - a*d)^8*(a + b*x)^2) + (8*b^7*d)/((b*c - a*d)^9*(a + b*x)) +
d^2/(7*(b*c - a*d)^3*(c + d*x)^7) + (b*d^2)/(2*(b*c - a*d)^4*(c + d*x)^6) +
 (6*b^2*d^2)/(5*(b*c - a*d)^5*(c + d*x)^5) + (5*b^3*d^2)/(2*(b*c - a*d)^6*(
c + d*x)^4) + (5*b^4*d^2)/((b*c - a*d)^7*(c + d*x)^3) + (21*b^5*d^2)/(2*(b*
c - a*d)^8*(c + d*x)^2) + (28*b^6*d^2)/((b*c - a*d)^9*(c + d*x)) + (36*b^7*
d^2*Log[a + b*x])/(b*c - a*d)^10 - (36*b^7*d^2*Log[c + d*x])/(b*c - a*d)^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^8}\,dx=\int \left(\frac{b^8}{(bc-ad)^8(a+bx)^3}-\frac{8b^8d}{(bc-ad)^9(a+bx)^2}+\frac{36b^8d^2}{(bc-ad)^{10}(a+bx)}-\frac{d^3}{(bc-ad)^3(c+}\right.$$

$$=-\frac{b^7}{2(bc-ad)^8(a+bx)^2}+\frac{8b^7d}{(bc-ad)^9(a+bx)}+\frac{d^2}{7(bc-ad)^3(c+dx)^7}+\frac{bd^2}{2(bc-ad)^4(c-}$$

**Mathematica** [A] time = 0.20, size = 254, normalized size = 0.92

$$\frac{\frac{560b^7d(bc-ad)}{a+bx}-\frac{35b^7(bc-ad)^2}{(a+bx)^2}+2520b^7d^2\log(a+bx)+\frac{1960b^6d^2(bc-ad)}{c+dx}+\frac{735b^5d^2(bc-ad)^2}{(c+dx)^2}+\frac{350b^4d^2(bc-ad)^3}{(c+dx)^3}+\frac{175b^3d^2(bc-ad)^4}{(c+dx)^4}+\frac{84b^2d^2(bc-ad)^5}{(c+dx)^5}+\frac{35bd^2(bc-ad)^6}{(c+dx)^6}+\frac{10d^2(bc-ad)^7}{(c+dx)^7}-2520b^7d^2\log(c+dx)}{70(bc-ad)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^8),x]
```

```
[Out] ((-35*b^7*(b*c - a*d)^2)/(a + b*x)^2 + (560*b^7*d*(b*c - a*d))/(a + b*x) +
(10*d^2*(b*c - a*d)^7)/(c + d*x)^7 + (35*b*d^2*(b*c - a*d)^6)/(c + d*x)^6 +
 (84*b^2*d^2*(b*c - a*d)^5)/(c + d*x)^5 + (175*b^3*d^2*(b*c - a*d)^4)/(c +
d*x)^4 + (350*b^4*d^2*(b*c - a*d)^3)/(c + d*x)^3 + (735*b^5*d^2*(b*c - a*d)
^2)/(c + d*x)^2 + (1960*b^6*d^2*(b*c - a*d))/(c + d*x) + 2520*b^7*d^2*Log[a
 + b*x] - 2520*b^7*d^2*Log[c + d*x])/(70*(b*c - a*d)^10)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^8),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^8), x]
```

**fricas [B]** time = 1.59, size = 3016, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/70*(35*b^9*c^9 - 630*a*b^8*c^8*d - 2754*a^2*b^7*c^7*d^2 + 5880*a^3*b^6*c
^6*d^3 - 4410*a^4*b^5*c^5*d^4 + 2940*a^5*b^4*c^4*d^5 - 1470*a^6*b^3*c^3*d^6
 + 504*a^7*b^2*c^2*d^7 - 105*a^8*b*c*d^8 + 10*a^9*d^9 - 2520*(b^9*c*d^8 - a
*b^8*d^9)*x^8 - 1260*(13*b^9*c^2*d^7 - 10*a*b^8*c*d^8 - 3*a^2*b^7*d^9)*x^7
- 420*(107*b^9*c^3*d^6 - 48*a*b^8*c^2*d^7 - 57*a^2*b^7*c*d^8 - 2*a^3*b^6*d^
9)*x^6 - 210*(319*b^9*c^4*d^5 + 8*a*b^8*c^3*d^6 - 300*a^2*b^7*c^2*d^7 - 28*
a^3*b^6*c*d^8 + a^4*b^5*d^9)*x^5 - 42*(1377*b^9*c^5*d^4 + 1090*a*b^8*c^4*d^
5 - 2080*a^2*b^7*c^3*d^6 - 420*a^3*b^6*c^2*d^7 + 35*a^4*b^5*c*d^8 - 2*a^5*b
^4*d^9)*x^4 - 42*(669*b^9*c^6*d^3 + 1494*a*b^8*c^5*d^4 - 1555*a^2*b^7*c^4*d
^5 - 700*a^3*b^6*c^3*d^6 + 105*a^4*b^5*c^2*d^7 - 14*a^5*b^4*c*d^8 + a^6*b^3
*d^9)*x^3 - 6*(1089*b^9*c^7*d^2 + 6426*a*b^8*c^6*d^3 - 3591*a^2*b^7*c^5*d^4
 - 4900*a^3*b^6*c^4*d^5 + 1225*a^4*b^5*c^3*d^6 - 294*a^5*b^4*c^2*d^7 + 49*a
^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 - 3*(105*b^9*c^8*d + 3516*a*b^8*c^7*d^2 +
 546*a^2*b^7*c^6*d^3 - 5880*a^3*b^6*c^5*d^4 + 2450*a^4*b^5*c^4*d^5 - 980*a^
5*b^4*c^3*d^6 + 294*a^6*b^3*c^2*d^7 - 56*a^7*b^2*c*d^8 + 5*a^8*b*d^9)*x - 2
520*(b^9*d^9*x^9 + a^2*b^7*c^7*d^2 + (7*b^9*c*d^8 + 2*a*b^8*d^9)*x^8 + (21*
b^9*c^2*d^7 + 14*a*b^8*c*d^8 + a^2*b^7*d^9)*x^7 + 7*(5*b^9*c^3*d^6 + 6*a*b^
8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 7*(5*b^9*c^4*d^5 + 10*a*b^8*c^3*d^6 + 3*a^
```

2*b^7*c^2*d^7)*x^5 + 7*(3*b^9*c^5*d^4 + 10*a*b^8*c^4*d^5 + 5*a^2*b^7*c^3*d^6)*x^4 + 7*(b^9*c^6*d^3 + 6*a*b^8*c^5*d^4 + 5*a^2*b^7*c^4*d^5)*x^3 + (b^9*c^7*d^2 + 14*a*b^8*c^6*d^3 + 21*a^2*b^7*c^5*d^4)*x^2 + (2*a*b^8*c^7*d^2 + 7*a^2*b^7*c^6*d^3)*x)*log(b*x + a) + 2520*(b^9*d^9*x^9 + a^2*b^7*c^7*d^2 + (7*b^9*c*d^8 + 2*a*b^8*d^9)*x^8 + (21*b^9*c^2*d^7 + 14*a*b^8*c*d^8 + a^2*b^7*d^9)*x^7 + 7*(5*b^9*c^3*d^6 + 6*a*b^8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 7*(5*b^9*c^4*d^5 + 10*a*b^8*c^3*d^6 + 3*a^2*b^7*c^2*d^7)*x^5 + 7*(3*b^9*c^5*d^4 + 10*a*b^8*c^4*d^5 + 5*a^2*b^7*c^3*d^6)*x^4 + 7*(b^9*c^6*d^3 + 6*a*b^8*c^5*d^4 + 5*a^2*b^7*c^4*d^5)*x^3 + (b^9*c^7*d^2 + 14*a*b^8*c^6*d^3 + 21*a^2*b^7*c^5*d^4)*x^2 + (2*a*b^8*c^7*d^2 + 7*a^2*b^7*c^6*d^3)*x)*log(d*x + c))/(a^2*b^10*c^17 - 10*a^3*b^9*c^16*d + 45*a^4*b^8*c^15*d^2 - 120*a^5*b^7*c^14*d^3 + 210*a^6*b^6*c^13*d^4 - 252*a^7*b^5*c^12*d^5 + 210*a^8*b^4*c^11*d^6 - 120*a^9*b^3*c^10*d^7 + 45*a^10*b^2*c^9*d^8 - 10*a^11*b*c^8*d^9 + a^12*c^7*d^10 + (b^12*c^10*d^7 - 10*a*b^11*c^9*d^8 + 45*a^2*b^10*c^8*d^9 - 120*a^3*b^9*c^7*d^10 + 210*a^4*b^8*c^6*d^11 - 252*a^5*b^7*c^5*d^12 + 210*a^6*b^6*c^4*d^13 - 120*a^7*b^5*c^3*d^14 + 45*a^8*b^4*c^2*d^15 - 10*a^9*b^3*c*d^16 + a^10*b^2*d^17)*x^9 + (7*b^12*c^11*d^6 - 68*a*b^11*c^10*d^7 + 295*a^2*b^10*c^9*d^8 - 750*a^3*b^9*c^8*d^9 + 1230*a^4*b^8*c^7*d^10 - 1344*a^5*b^7*c^6*d^11 + 966*a^6*b^6*c^5*d^12 - 420*a^7*b^5*c^4*d^13 + 75*a^8*b^4*c^3*d^14 + 20*a^9*b^3*c^2*d^15 - 13*a^10*b^2*c*d^16 + 2*a^11*b*d^17)*x^8 + (21*b^12*c^12*d^5 - 196*a*b^11*c^11*d^6 + 806*a^2*b^10*c^10*d^7 - 1900*a^3*b^9*c^9*d^8 + 2775*a^4*b^8*c^8*d^9 - 2472*a^5*b^7*c^7*d^10 + 1092*a^6*b^6*c^6*d^11 + 168*a^7*b^5*c^5*d^12 - 525*a^8*b^4*c^4*d^13 + 300*a^9*b^3*c^3*d^14 - 74*a^10*b^2*c^2*d^15 + 4*a^11*b*c*d^16 + a^12*d^17)*x^7 + 7*(5*b^12*c^13*d^4 - 44*a*b^11*c^12*d^5 + 166*a^2*b^10*c^11*d^6 - 340*a^3*b^9*c^10*d^7 + 375*a^4*b^8*c^9*d^8 - 120*a^5*b^7*c^8*d^9 - 252*a^6*b^6*c^7*d^10 + 408*a^7*b^5*c^6*d^11 - 285*a^8*b^4*c^5*d^12 + 100*a^9*b^3*c^4*d^13 - 10*a^10*b^2*c^3*d^14 - 4*a^11*b*c^2*d^15 + a^12*c*d^16)*x^6 + 7*(5*b^12*c^14*d^3 - 40*a*b^11*c^13*d^4 + 128*a^2*b^10*c^12*d^5 - 180*a^3*b^9*c^11*d^6 - 15*a^4*b^8*c^10*d^7 + 480*a^5*b^7*c^9*d^8 - 840*a^6*b^6*c^8*d^9 + 744*a^7*b^5*c^7*d^10 - 345*a^8*b^4*c^6*d^11 + 40*a^9*b^3*c^5*d^12 + 40*a^10*b^2*c^4*d^13 - 20*a^11*b*c^3*d^14 + 3*a^12*c^2*d^15)*x^5 + 7*(3*b^12*c^15*d^2 - 20*a*b^11*c^14*d^3 + 40*a^2*b^10*c^13*d^4 + 40*a^3*b^9*c^12*d^5 - 345*a^4*b^8*c^11*d^6 + 744*a^5*b^7*c^10*d^7 - 840*a^6*b^6*c^9*d^8 + 480*a^7*b^5*c^8*d^9 - 15*a^8*b^4*c^7*d^10 - 180*a^9*b^3*c^6*d^11 + 128*a^10*b^2*c^5*d^12 - 40*a^11*b*c^4*d^13 + 5*a^12*c^3*d^14)*x^4 + 7*(b^12*c^16*d - 4*a*b^11*c^15*d^2 - 10*a^2*b^10*c^14*d^3 + 100*a^3*b^9*c^13*d^4 - 285*a^4*b^8*c^12*d^5 + 408*a^5*b^7*c^11*d^6 - 252*a^6*b^6*c^10*d^7 - 120*a^7*b^5*c^9*d^8 + 375*a^8*b^4*c^8*d^9 - 340*a^9*b^3*c^7*d^10 + 166*a^10*b^2*c^6*d^11 - 44*a^11*b*c^5*d^12 + 5*a^12*c^4*d^13)*x^3 + (b^12*c^17 + 4*a*b^11*c^16*d - 74*a^2*b^10*c^15*d^2 + 300*a^3*b^9*c^14*d^3 - 525*a^4*b^8*c^13*d^4 + 168*a^5*b^7*c^12*d^5 + 1092*a^6*b^6*c^11*d^6 - 2472*a^7*b^5*c^10*d^7 + 2775*a^8*b^4*c^9*d^8 - 1900*a^9*b^3*c^8*d^9 + 806*a^10*b^2*c^7*d^10 - 196*a^11*b*c^6*d^11 + 21*a^12*c^5*d^12)*x^2 + (2*a*b^11*c^17 - 13*a^2*b^10*c^16*d + 20*a^3*b^9*c^15*d^2 + 75*a^4*b^8*c^14*d^3 - 420*a^5*b^7*c^13*d^4 + 966*a^6*b^6*c^12*d^5 - 1344*a^7*b^5*c^11*d^6 + 1230*a^8*b^4*c^1

```
0*d^7 - 750*a^9*b^3*c^9*d^8 + 295*a^10*b^2*c^8*d^9 - 68*a^11*b*c^7*d^10 + 7
*a^12*c^6*d^11)*x)
```

**giac [B]** time = 1.49, size = 1029, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 36*b^8*d^2*log(abs(b*x + a))/(b^11*c^10 - 10*a*b^10*c^9*d + 45*a^2*b^9*c^8*
d^2 - 120*a^3*b^8*c^7*d^3 + 210*a^4*b^7*c^6*d^4 - 252*a^5*b^6*c^5*d^5 + 210
*a^6*b^5*c^4*d^6 - 120*a^7*b^4*c^3*d^7 + 45*a^8*b^3*c^2*d^8 - 10*a^9*b^2*c*
d^9 + a^10*b*d^10) - 36*b^7*d^3*log(abs(d*x + c))/(b^10*c^10*d - 10*a*b^9*c
^9*d^2 + 45*a^2*b^8*c^8*d^3 - 120*a^3*b^7*c^7*d^4 + 210*a^4*b^6*c^6*d^5 - 2
52*a^5*b^5*c^5*d^6 + 210*a^6*b^4*c^4*d^7 - 120*a^7*b^3*c^3*d^8 + 45*a^8*b^2
*c^2*d^9 - 10*a^9*b*c*d^10 + a^10*d^11) - 1/70*(35*b^9*c^9 - 630*a*b^8*c^8*
d - 2754*a^2*b^7*c^7*d^2 + 5880*a^3*b^6*c^6*d^3 - 4410*a^4*b^5*c^5*d^4 + 29
40*a^5*b^4*c^4*d^5 - 1470*a^6*b^3*c^3*d^6 + 504*a^7*b^2*c^2*d^7 - 105*a^8*b
*c*d^8 + 10*a^9*d^9 - 2520*(b^9*c*d^8 - a*b^8*d^9)*x^8 - 1260*(13*b^9*c^2*d
^7 - 10*a*b^8*c*d^8 - 3*a^2*b^7*d^9)*x^7 - 420*(107*b^9*c^3*d^6 - 48*a*b^8*
c^2*d^7 - 57*a^2*b^7*c*d^8 - 2*a^3*b^6*d^9)*x^6 - 210*(319*b^9*c^4*d^5 + 8*
a*b^8*c^3*d^6 - 300*a^2*b^7*c^2*d^7 - 28*a^3*b^6*c*d^8 + a^4*b^5*d^9)*x^5 -
 42*(1377*b^9*c^5*d^4 + 1090*a*b^8*c^4*d^5 - 2080*a^2*b^7*c^3*d^6 - 420*a^3
*b^6*c^2*d^7 + 35*a^4*b^5*c*d^8 - 2*a^5*b^4*d^9)*x^4 - 42*(669*b^9*c^6*d^3
+ 1494*a*b^8*c^5*d^4 - 1555*a^2*b^7*c^4*d^5 - 700*a^3*b^6*c^3*d^6 + 105*a^4
*b^5*c^2*d^7 - 14*a^5*b^4*c*d^8 + a^6*b^3*d^9)*x^3 - 6*(1089*b^9*c^7*d^2 +
6426*a*b^8*c^6*d^3 - 3591*a^2*b^7*c^5*d^4 - 4900*a^3*b^6*c^4*d^5 + 1225*a^4
*b^5*c^3*d^6 - 294*a^5*b^4*c^2*d^7 + 49*a^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2
- 3*(105*b^9*c^8*d + 3516*a*b^8*c^7*d^2 + 546*a^2*b^7*c^6*d^3 - 5880*a^3*b^
6*c^5*d^4 + 2450*a^4*b^5*c^4*d^5 - 980*a^5*b^4*c^3*d^6 + 294*a^6*b^3*c^2*d^
7 - 56*a^7*b^2*c*d^8 + 5*a^8*b*d^9)*x)/((b*c - a*d)^10*(b*x + a)^2*(d*x + c
)^7)
```

**maple [A]** time = 0.02, size = 265, normalized size = 0.96

$$\frac{36 b^{7} d^{2} \ln \left(b x+a\right)}{\left(a d-b c\right)^{10}}-\frac{36 b^{7} d^{2} \ln \left(d x+c\right)}{\left(a d-b c\right)^{10}}-\frac{8 b^{7} d}{\left(a d-b c\right)^{9} \left(b x+a\right)}-\frac{28 b^{6} d^{2}}{\left(a d-b c\right)^{9} \left(d x+c\right)}-\frac{b^{7}}{2 \left(a d-b c\right)^{8} \left(b x+a\right)^{2}}+\frac{21 b^{5} d^{2}}{2 \left(a d-b c\right)^{8} \left(d x+c\right)^{2}}-\frac{5 b^{4} d^{2}}{\left(a d-b c\right)^{7} \left(d x+c\right)^{3}}+\frac{5 b^{3} d^{2}}{2 \left(a d-b c\right)^{6} \left(d x+c\right)^{4}}-\frac{6 b^{2} d^{2}}{5 \left(a d-b c\right)^{5} \left(d x+c\right)^{5}}+\frac{b d^{2}}{2 \left(a d-b c\right)^{4} \left(d x+c\right)^{6}}-\frac{d^{2}}{7 \left(a d-b c\right)^{3} \left(d x+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^8,x)
```

```
[Out] -1/7*d^2/(a*d-b*c)^3/(d*x+c)^7-36*d^2/(a*d-b*c)^10*b^7*ln(d*x+c)-28*d^2/(a*
d-b*c)^9*b^6/(d*x+c)+21/2*d^2/(a*d-b*c)^8*b^5/(d*x+c)^2-5*d^2/(a*d-b*c)^7*b
^4/(d*x+c)^3+5/2*d^2/(a*d-b*c)^6*b^3/(d*x+c)^4-6/5*d^2/(a*d-b*c)^5*b^2/(d*x
```

```
+c)^5+1/2*d^2/(a*d-b*c)^4*b/(d*x+c)^6-1/2*b^7/(a*d-b*c)^8/(b*x+a)^2+36*d^2/
(a*d-b*c)^10*b^7*ln(b*x+a)-8*b^7/(a*d-b*c)^9*d/(b*x+a)
```

**maxima [B]** time = 5.22, size = 2399, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 36*b^7*d^2*log(b*x + a)/(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 -
120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b
^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^
10*d^10) - 36*b^7*d^2*log(d*x + c)/(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8
*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5
+ 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b
*c*d^9 + a^10*d^10) + 1/70*(2520*b^8*d^8*x^8 - 35*b^8*c^8 + 595*a*b^7*c^7*d
 + 3349*a^2*b^6*c^6*d^2 - 2531*a^3*b^5*c^5*d^3 + 1879*a^4*b^4*c^4*d^4 - 106
1*a^5*b^3*c^3*d^5 + 409*a^6*b^2*c^2*d^6 - 95*a^7*b*c*d^7 + 10*a^8*d^8 + 126
0*(13*b^8*c*d^7 + 3*a*b^7*d^8)*x^7 + 420*(107*b^8*c^2*d^6 + 59*a*b^7*c*d^7
+ 2*a^2*b^6*d^8)*x^6 + 210*(319*b^8*c^3*d^5 + 327*a*b^7*c^2*d^6 + 27*a^2*b^
6*c*d^7 - a^3*b^5*d^8)*x^5 + 42*(1377*b^8*c^4*d^4 + 2467*a*b^7*c^3*d^5 + 38
7*a^2*b^6*c^2*d^6 - 33*a^3*b^5*c*d^7 + 2*a^4*b^4*d^8)*x^4 + 42*(669*b^8*c^5
*d^3 + 2163*a*b^7*c^4*d^4 + 608*a^2*b^6*c^3*d^5 - 92*a^3*b^5*c^2*d^6 + 13*a
^4*b^4*c*d^7 - a^5*b^3*d^8)*x^3 + 6*(1089*b^8*c^6*d^2 + 7515*a*b^7*c^5*d^3
+ 3924*a^2*b^6*c^4*d^4 - 976*a^3*b^5*c^3*d^5 + 249*a^4*b^4*c^2*d^6 - 45*a^5
*b^3*c*d^7 + 4*a^6*b^2*d^8)*x^2 + 3*(105*b^8*c^7*d + 3621*a*b^7*c^6*d^2 + 4
167*a^2*b^6*c^5*d^3 - 1713*a^3*b^5*c^4*d^4 + 737*a^4*b^4*c^3*d^5 - 243*a^5*
b^3*c^2*d^6 + 51*a^6*b^2*c*d^7 - 5*a^7*b*d^8)*x)/(a^2*b^9*c^16 - 9*a^3*b^8*
c^15*d + 36*a^4*b^7*c^14*d^2 - 84*a^5*b^6*c^13*d^3 + 126*a^6*b^5*c^12*d^4 -
 126*a^7*b^4*c^11*d^5 + 84*a^8*b^3*c^10*d^6 - 36*a^9*b^2*c^9*d^7 + 9*a^10*b
*c^8*d^8 - a^11*c^7*d^9 + (b^11*c^9*d^7 - 9*a*b^10*c^8*d^8 + 36*a^2*b^9*c^7
*d^9 - 84*a^3*b^8*c^6*d^10 + 126*a^4*b^7*c^5*d^11 - 126*a^5*b^6*c^4*d^12 +
84*a^6*b^5*c^3*d^13 - 36*a^7*b^4*c^2*d^14 + 9*a^8*b^3*c*d^15 - a^9*b^2*d^16
)*x^9 + (7*b^11*c^10*d^6 - 61*a*b^10*c^9*d^7 + 234*a^2*b^9*c^8*d^8 - 516*a^
3*b^8*c^7*d^9 + 714*a^4*b^7*c^6*d^10 - 630*a^5*b^6*c^5*d^11 + 336*a^6*b^5*c
^4*d^12 - 84*a^7*b^4*c^3*d^13 - 9*a^8*b^3*c^2*d^14 + 11*a^9*b^2*c*d^15 - 2*
a^10*b*d^16)*x^8 + (21*b^11*c^11*d^5 - 175*a*b^10*c^10*d^6 + 631*a^2*b^9*c^
9*d^7 - 1269*a^3*b^8*c^8*d^8 + 1506*a^4*b^7*c^7*d^9 - 966*a^5*b^6*c^6*d^10
+ 126*a^6*b^5*c^5*d^11 + 294*a^7*b^4*c^4*d^12 - 231*a^8*b^3*c^3*d^13 + 69*a
^9*b^2*c^2*d^14 - 5*a^10*b*c*d^15 - a^11*d^16)*x^7 + 7*(5*b^11*c^12*d^4 - 3
9*a*b^10*c^11*d^5 + 127*a^2*b^9*c^10*d^6 - 213*a^3*b^8*c^9*d^7 + 162*a^4*b^
7*c^8*d^8 + 42*a^5*b^6*c^7*d^9 - 210*a^6*b^5*c^6*d^10 + 198*a^7*b^4*c^5*d^1
1 - 87*a^8*b^3*c^4*d^12 + 13*a^9*b^2*c^3*d^13 + 3*a^10*b*c^2*d^14 - a^11*c*
d^15)*x^6 + 7*(5*b^11*c^13*d^3 - 35*a*b^10*c^12*d^4 + 93*a^2*b^9*c^11*d^5 -
 87*a^3*b^8*c^10*d^6 - 102*a^4*b^7*c^9*d^7 + 378*a^5*b^6*c^8*d^8 - 462*a^6*
```

```
b^5*c^7*d^9 + 282*a^7*b^4*c^6*d^10 - 63*a^8*b^3*c^5*d^11 - 23*a^9*b^2*c^4*d
^12 + 17*a^10*b*c^3*d^13 - 3*a^11*c^2*d^14)*x^5 + 7*(3*b^11*c^14*d^2 - 17*a
*b^10*c^13*d^3 + 23*a^2*b^9*c^12*d^4 + 63*a^3*b^8*c^11*d^5 - 282*a^4*b^7*c^
10*d^6 + 462*a^5*b^6*c^9*d^7 - 378*a^6*b^5*c^8*d^8 + 102*a^7*b^4*c^7*d^9 +
87*a^8*b^3*c^6*d^10 - 93*a^9*b^2*c^5*d^11 + 35*a^10*b*c^4*d^12 - 5*a^11*c^3
*d^13)*x^4 + 7*(b^11*c^15*d - 3*a*b^10*c^14*d^2 - 13*a^2*b^9*c^13*d^3 + 87*
a^3*b^8*c^12*d^4 - 198*a^4*b^7*c^11*d^5 + 210*a^5*b^6*c^10*d^6 - 42*a^6*b^5
*c^9*d^7 - 162*a^7*b^4*c^8*d^8 + 213*a^8*b^3*c^7*d^9 - 127*a^9*b^2*c^6*d^10
 + 39*a^10*b*c^5*d^11 - 5*a^11*c^4*d^12)*x^3 + (b^11*c^16 + 5*a*b^10*c^15*d
 - 69*a^2*b^9*c^14*d^2 + 231*a^3*b^8*c^13*d^3 - 294*a^4*b^7*c^12*d^4 - 126*
a^5*b^6*c^11*d^5 + 966*a^6*b^5*c^10*d^6 - 1506*a^7*b^4*c^9*d^7 + 1269*a^8*b
^3*c^8*d^8 - 631*a^9*b^2*c^7*d^9 + 175*a^10*b*c^6*d^10 - 21*a^11*c^5*d^11)*
x^2 + (2*a*b^10*c^16 - 11*a^2*b^9*c^15*d + 9*a^3*b^8*c^14*d^2 + 84*a^4*b^7*
c^13*d^3 - 336*a^5*b^6*c^12*d^4 + 630*a^6*b^5*c^11*d^5 - 714*a^7*b^4*c^10*d
^6 + 516*a^8*b^3*c^9*d^7 - 234*a^9*b^2*c^8*d^8 + 61*a^10*b*c^7*d^9 - 7*a^11
*c^6*d^10)*x)
```

**mupad [B]** time = 1.91, size = 2224, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^8),x)
```

```
[Out] (72*b^7*d^2*atanh((a^10*d^10 - b^10*c^10 - 27*a^2*b^8*c^8*d^2 + 48*a^3*b^7*
c^7*d^3 - 42*a^4*b^6*c^6*d^4 + 42*a^6*b^4*c^4*d^6 - 48*a^7*b^3*c^3*d^7 + 27
*a^8*b^2*c^2*d^8 + 8*a*b^9*c^9*d - 8*a^9*b*c*d^9)/(a*d - b*c)^10 + (2*b*d*x
*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5
*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 +
9*a*b^8*c^8*d - 9*a^8*b*c*d^8))/(a*d - b*c)^10))/(a*d - b*c)^10 - ((10*a^8*
d^8 - 35*b^8*c^8 + 3349*a^2*b^6*c^6*d^2 - 2531*a^3*b^5*c^5*d^3 + 1879*a^4*b
^4*c^4*d^4 - 1061*a^5*b^3*c^3*d^5 + 409*a^6*b^2*c^2*d^6 + 595*a*b^7*c^7*d -
 95*a^7*b*c*d^7)/(70*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c
^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 3
6*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) + (3*b^2*x^2*(4*a^6*d^8
 + 1089*b^6*c^6*d^2 + 7515*a*b^5*c^5*d^3 + 3924*a^2*b^4*c^4*d^4 - 976*a^3*b
^3*c^3*d^5 + 249*a^4*b^2*c^2*d^6 - 45*a^5*b*c*d^7))/(35*(a^9*d^9 - b^9*c^9
- 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b
^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^
8*b*c*d^8)) + (3*b^4*x^4*(2*a^4*d^8 + 1377*b^4*c^4*d^4 + 2467*a*b^3*c^3*d^5
 + 387*a^2*b^2*c^2*d^6 - 33*a^3*b*c*d^7))/(5*(a^9*d^9 - b^9*c^9 - 36*a^2*b^
7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5
- 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8))
 + (3*b*x*(105*b^7*c^7*d - 5*a^7*d^8 + 3621*a*b^6*c^6*d^2 + 4167*a^2*b^5*c^
5*d^3 - 1713*a^3*b^4*c^4*d^4 + 737*a^4*b^3*c^3*d^5 - 243*a^5*b^2*c^2*d^6 +
```

```
51*a^6*b*c*d^7))/(70*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c
^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 3
6*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) + (6*b^6*x^6*(2*a^2*d^8
 + 107*b^2*c^2*d^6 + 59*a*b*c*d^7))/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2
 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*
b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (3*b^3*
x^3*(669*b^5*c^5*d^3 - a^5*d^8 + 2163*a*b^4*c^4*d^4 + 608*a^2*b^3*c^3*d^5 -
 92*a^3*b^2*c^2*d^6 + 13*a^4*b*c*d^7))/(5*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c
^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 8
4*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) +
(3*b^5*x^5*(319*b^3*c^3*d^5 - a^3*d^8 + 327*a*b^2*c^2*d^6 + 27*a^2*b*c*d^7)
)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^
5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 +
 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (36*b^8*d^8*x^8)/(a^9*d^9 - b^9*c^9 - 36*
a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^
4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c
*d^8) + (18*b^6*d*x^7*(13*b^2*c*d^6 + 3*a*b*d^7))/(a^9*d^9 - b^9*c^9 - 36*a
^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4
*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*
d^8))/(x^3*(7*b^2*c^6*d + 35*a^2*c^4*d^3 + 42*a*b*c^5*d^2) + x^6*(7*a^2*c*d
^6 + 35*b^2*c^3*d^4 + 42*a*b*c^2*d^5) + x*(7*a^2*c^6*d + 2*a*b*c^7) + x^2*(
b^2*c^7 + 21*a^2*c^5*d^2 + 14*a*b*c^6*d) + x^7*(a^2*d^7 + 21*b^2*c^2*d^5 +
14*a*b*c*d^6) + x^4*(35*a^2*c^3*d^4 + 21*b^2*c^5*d^2 + 70*a*b*c^4*d^3) + x^
5*(21*a^2*c^2*d^5 + 35*b^2*c^4*d^3 + 70*a*b*c^3*d^4) + x^8*(7*b^2*c*d^6 + 2
*a*b*d^7) + a^2*c^7 + b^2*d^7*x^9)
```

**sympy [B]** time = 20.66, size = 2917, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**8,x)
```

```
[Out] -36*b**7*d**2*log(x + (-36*a**11*b**7*d**13/(a*d - b*c)**10 + 396*a**10*b**
8*c*d**12/(a*d - b*c)**10 - 1980*a**9*b**9*c**2*d**11/(a*d - b*c)**10 + 594
0*a**8*b**10*c**3*d**10/(a*d - b*c)**10 - 11880*a**7*b**11*c**4*d**9/(a*d -
 b*c)**10 + 16632*a**6*b**12*c**5*d**8/(a*d - b*c)**10 - 16632*a**5*b**13*c
**6*d**7/(a*d - b*c)**10 + 11880*a**4*b**14*c**7*d**6/(a*d - b*c)**10 - 594
0*a**3*b**15*c**8*d**5/(a*d - b*c)**10 + 1980*a**2*b**16*c**9*d**4/(a*d - b
*c)**10 - 396*a*b**17*c**10*d**3/(a*d - b*c)**10 + 36*a*b**7*d**3 + 36*b**1
8*c**11*d**2/(a*d - b*c)**10 + 36*b**8*c*d**2)/(72*b**8*d**3))/(a*d - b*c)*
*10 + 36*b**7*d**2*log(x + (36*a**11*b**7*d**13/(a*d - b*c)**10 - 396*a**10
*b**8*c*d**12/(a*d - b*c)**10 + 1980*a**9*b**9*c**2*d**11/(a*d - b*c)**10 -
 5940*a**8*b**10*c**3*d**10/(a*d - b*c)**10 + 11880*a**7*b**11*c**4*d**9/(a
*d - b*c)**10 - 16632*a**6*b**12*c**5*d**8/(a*d - b*c)**10 + 16632*a**5*b**
```

13*c**6*d**7/(a*d - b*c)**10 - 11880*a**4*b**14*c**7*d**6/(a*d - b*c)**10 +
 5940*a**3*b**15*c**8*d**5/(a*d - b*c)**10 - 1980*a**2*b**16*c**9*d**4/(a*d
 - b*c)**10 + 396*a*b**17*c**10*d**3/(a*d - b*c)**10 + 36*a*b**7*d**3 - 36*
b**18*c**11*d**2/(a*d - b*c)**10 + 36*b**8*c*d**2)/(72*b**8*d**3))/(a*d - b
*c)**10 + (-10*a**8*d**8 + 95*a**7*b*c*d**7 - 409*a**6*b**2*c**2*d**6 + 106
1*a**5*b**3*c**3*d**5 - 1879*a**4*b**4*c**4*d**4 + 2531*a**3*b**5*c**5*d**3
 - 3349*a**2*b**6*c**6*d**2 - 595*a*b**7*c**7*d + 35*b**8*c**8 - 2520*b**8*
d**8*x**8 + x**7*(-3780*a*b**7*d**8 - 16380*b**8*c*d**7) + x**6*(-840*a**2*
b**6*d**8 - 24780*a*b**7*c*d**7 - 44940*b**8*c**2*d**6) + x**5*(210*a**3*b*
*5*d**8 - 5670*a**2*b**6*c*d**7 - 68670*a*b**7*c**2*d**6 - 66990*b**8*c**3*
d**5) + x**4*(-84*a**4*b**4*d**8 + 1386*a**3*b**5*c*d**7 - 16254*a**2*b**6*
c**2*d**6 - 103614*a*b**7*c**3*d**5 - 57834*b**8*c**4*d**4) + x**3*(42*a**5
*b**3*d**8 - 546*a**4*b**4*c*d**7 + 3864*a**3*b**5*c**2*d**6 - 25536*a**2*b
**6*c**3*d**5 - 90846*a*b**7*c**4*d**4 - 28098*b**8*c**5*d**3) + x**2*(-24*
a**6*b**2*d**8 + 270*a**5*b**3*c*d**7 - 1494*a**4*b**4*c**2*d**6 + 5856*a**
3*b**5*c**3*d**5 - 23544*a**2*b**6*c**4*d**4 - 45090*a*b**7*c**5*d**3 - 653
4*b**8*c**6*d**2) + x*(15*a**7*b*d**8 - 153*a**6*b**2*c*d**7 + 729*a**5*b**
3*c**2*d**6 - 2211*a**4*b**4*c**3*d**5 + 5139*a**3*b**5*c**4*d**4 - 12501*a
**2*b**6*c**5*d**3 - 10863*a*b**7*c**6*d**2 - 315*b**8*c**7*d))/(70*a**11*c
**7*d**9 - 630*a**10*b*c**8*d**8 + 2520*a**9*b**2*c**9*d**7 - 5880*a**8*b**
3*c**10*d**6 + 8820*a**7*b**4*c**11*d**5 - 8820*a**6*b**5*c**12*d**4 + 5880
*a**5*b**6*c**13*d**3 - 2520*a**4*b**7*c**14*d**2 + 630*a**3*b**8*c**15*d -
 70*a**2*b**9*c**16 + x**9*(70*a**9*b**2*d**16 - 630*a**8*b**3*c*d**15 + 25
20*a**7*b**4*c**2*d**14 - 5880*a**6*b**5*c**3*d**13 + 8820*a**5*b**6*c**4*d
**12 - 8820*a**4*b**7*c**5*d**11 + 5880*a**3*b**8*c**6*d**10 - 2520*a**2*b*
*9*c**7*d**9 + 630*a*b**10*c**8*d**8 - 70*b**11*c**9*d**7) + x**8*(140*a**1
0*b*d**16 - 770*a**9*b**2*c*d**15 + 630*a**8*b**3*c**2*d**14 + 5880*a**7*b*
*4*c**3*d**13 - 23520*a**6*b**5*c**4*d**12 + 44100*a**5*b**6*c**5*d**11 - 4
9980*a**4*b**7*c**6*d**10 + 36120*a**3*b**8*c**7*d**9 - 16380*a**2*b**9*c**
8*d**8 + 4270*a*b**10*c**9*d**7 - 490*b**11*c**10*d**6) + x**7*(70*a**11*d*
*16 + 350*a**10*b*c*d**15 - 4830*a**9*b**2*c**2*d**14 + 16170*a**8*b**3*c**
3*d**13 - 20580*a**7*b**4*c**4*d**12 - 8820*a**6*b**5*c**5*d**11 + 67620*a*
*5*b**6*c**6*d**10 - 105420*a**4*b**7*c**7*d**9 + 88830*a**3*b**8*c**8*d**8
 - 44170*a**2*b**9*c**9*d**7 + 12250*a*b**10*c**10*d**6 - 1470*b**11*c**11*
d**5) + x**6*(490*a**11*c*d**15 - 1470*a**10*b*c**2*d**14 - 6370*a**9*b**2*
c**3*d**13 + 42630*a**8*b**3*c**4*d**12 - 97020*a**7*b**4*c**5*d**11 + 1029
00*a**6*b**5*c**6*d**10 - 20580*a**5*b**6*c**7*d**9 - 79380*a**4*b**7*c**8*
d**8 + 104370*a**3*b**8*c**9*d**7 - 62230*a**2*b**9*c**10*d**6 + 19110*a*b*
*10*c**11*d**5 - 2450*b**11*c**12*d**4) + x**5*(1470*a**11*c**2*d**14 - 833
0*a**10*b*c**3*d**13 + 11270*a**9*b**2*c**4*d**12 + 30870*a**8*b**3*c**5*d*
*11 - 138180*a**7*b**4*c**6*d**10 + 226380*a**6*b**5*c**7*d**9 - 185220*a**
5*b**6*c**8*d**8 + 49980*a**4*b**7*c**9*d**7 + 42630*a**3*b**8*c**10*d**6 -
 45570*a**2*b**9*c**11*d**5 + 17150*a*b**10*c**12*d**4 - 2450*b**11*c**13*d
**3) + x**4*(2450*a**11*c**3*d**13 - 17150*a**10*b*c**4*d**12 + 45570*a**9*
b**2*c**5*d**11 - 42630*a**8*b**3*c**6*d**10 - 49980*a**7*b**4*c**7*d**9 +

185220*a**6*b**5*c**8*d**8 - 226380*a**5*b**6*c**9*d**7 + 138180*a**4*b**7*c**10*d**6 - 30870*a**3*b**8*c**11*d**5 - 11270*a**2*b**9*c**12*d**4 + 8330*a*b**10*c**13*d**3 - 1470*b**11*c**14*d**2) + x**3*(2450*a**11*c**4*d**12 - 19110*a**10*b*c**5*d**11 + 62230*a**9*b**2*c**6*d**10 - 104370*a**8*b**3*c**7*d**9 + 79380*a**7*b**4*c**8*d**8 + 20580*a**6*b**5*c**9*d**7 - 102900*a**5*b**6*c**10*d**6 + 97020*a**4*b**7*c**11*d**5 - 42630*a**3*b**8*c**12*d**4 + 6370*a**2*b**9*c**13*d**3 + 1470*a*b**10*c**14*d**2 - 490*b**11*c**15*d) + x**2*(1470*a**11*c**5*d**11 - 12250*a**10*b*c**6*d**10 + 44170*a**9*b**2*c**7*d**9 - 88830*a**8*b**3*c**8*d**8 + 105420*a**7*b**4*c**9*d**7 - 67620*a**6*b**5*c**10*d**6 + 8820*a**5*b**6*c**11*d**5 + 20580*a**4*b**7*c**12*d**4 - 16170*a**3*b**8*c**13*d**3 + 4830*a**2*b**9*c**14*d**2 - 350*a*b**10*c**15*d - 70*b**11*c**16) + x*(490*a**11*c**6*d**10 - 4270*a**10*b*c**7*d**9 + 16380*a**9*b**2*c**8*d**8 - 36120*a**8*b**3*c**9*d**7 + 49980*a**7*b**4*c**10*d**6 - 44100*a**6*b**5*c**11*d**5 + 23520*a**5*b**6*c**12*d**4 - 5880*a**4*b**7*c**13*d**3 - 630*a**3*b**8*c**14*d**2 + 770*a**2*b**9*c**15*d - 140*a*b**10*c**16))

## 3.1269 $\int (a+bx)^5\sqrt{c+dx}\,dx$

**Optimal**. Leaf size=156

$$-\frac{10b^4(c+dx)^{11/2}(bc-ad)}{11d^6}+\frac{20b^3(c+dx)^{9/2}(bc-ad)^2}{9d^6}-\frac{20b^2(c+dx)^{7/2}(bc-ad)^3}{7d^6}+\frac{2b(c+dx)^{5/2}(bc-ad)^4}{d^6}-\frac{2(c+$$

**Rubi [A]** time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{11/2}(bc-ad)}{11d^6}+\frac{20b^3(c+dx)^{9/2}(bc-ad)^2}{9d^6}-\frac{20b^2(c+dx)^{7/2}(bc-ad)^3}{7d^6}+\frac{2b(c+dx)^{5/2}(bc-ad)^4}{d^6}-\frac{2(c+dx)^{3/2}(bc-ad)^5}{3d^6}+\frac{2b^5(c+dx)^{13/2}}{13d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(3/2))/(3*d^6) + (2*b*(b*c - a*d)^4*(c + d*x)^(
5/2))/d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^6) + (20*b^3*(b*c -
 a*d)^2*(c + d*x)^(9/2))/(9*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(11/2))/(1
1*d^6) + (2*b^5*(c + d*x)^(13/2))/(13*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5\sqrt{c+dx}\,dx=\int\left(\frac{(-bc+ad)^5\sqrt{c+dx}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{3/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{5/2}}{d^5}+\frac{10b^3(\right.$$

$$=-\frac{2(bc-ad)^5(c+dx)^{3/2}}{3d^6}+\frac{2b(bc-ad)^4(c+dx)^{5/2}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{7/2}}{7d^6}+\frac{20b^3(b$$

**Mathematica [A]** time = 0.15, size = 123, normalized size = 0.79

$$\frac{2(c+dx)^{3/2}\left(-4095b^4(c+dx)^4(bc-ad)+10010b^3(c+dx)^3(bc-ad)^2-12870b^2(c+dx)^2(bc-ad)^3+9009b(c+dx)(bc-ad)^4-3003(bc-ad)^5+693b^5(c+dx)^5\right)}{9009d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-3003*(b*c - a*d)^5 + 9009*b*(b*c - a*d)^4*(c + d*x) -
12870*b^2*(b*c - a*d)^3*(c + d*x)^2 + 10010*b^3*(b*c - a*d)^2*(c + d*x)^3 -
 4095*b^4*(b*c - a*d)*(c + d*x)^4 + 693*b^5*(c + d*x)^5))/(9009*d^6)
```

**IntegrateAlgebraic [B]** time = 0.10, size = 315, normalized size = 2.02

$$\frac{2(c+dx)^{3/2}\left(3003a^5d^5+9009a^4bd^4(c+dx)-15015a^4bcd^4+30030a^3b^2c^2d^3+12870a^3b^2d^3(c+dx)^2-36036a^3b^2cd^3(c+dx)-30030a^2b^3c^3d^2+54054a^2b^3c^2d^2(c+dx)+10010a^2b^3d^2(c+dx)^3-38610a^2b^3cd^2(c+dx)^2+15015ab^4c^4d-36036ab^4c^3d(c+dx)+38610ab^4c^2d(c+dx)^2+4095ab^4d(c+dx)^4-20020ab^4cd(c+dx)^3-3003b^5c^5+9009b^5c^4(c+dx)-12870b^5c^3(c+dx)^2+10010b^5c^2(c+dx)^3+693b^5(c+dx)^5-4095b^5c(c+dx)^4\right)}{9009d^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-3003*b^5*c^5 + 15015*a*b^4*c^4*d - 30030*a^2*b^3*c^3*d
^2 + 30030*a^3*b^2*c^2*d^3 - 15015*a^4*b*c*d^4 + 3003*a^5*d^5 + 9009*b^5*c^
4*(c + d*x) - 36036*a*b^4*c^3*d*(c + d*x) + 54054*a^2*b^3*c^2*d^2*(c + d*x)
 - 36036*a^3*b^2*c*d^3*(c + d*x) + 9009*a^4*b*d^4*(c + d*x) - 12870*b^5*c^3
*(c + d*x)^2 + 38610*a*b^4*c^2*d*(c + d*x)^2 - 38610*a^2*b^3*c*d^2*(c + d*x
)^2 + 12870*a^3*b^2*d^3*(c + d*x)^2 + 10010*b^5*c^2*(c + d*x)^3 - 20020*a*b
^4*c*d*(c + d*x)^3 + 10010*a^2*b^3*d^2*(c + d*x)^3 - 4095*b^5*c*(c + d*x)^4
 + 4095*a*b^4*d*(c + d*x)^4 + 693*b^5*(c + d*x)^5))/(9009*d^6)
```

**fricas [B]** time = 1.21, size = 338, normalized size = 2.17

$$\frac{2\left(693b^5d^6x^6-256b^5c^6+1664ab^4c^5d-4576a^2b^3c^4d^2+6864a^3b^2c^3d^3-6006a^4bc^2d^4+3003a^5cd^5+63\left(b^5cd^5+65ab^4d^6\right)x^5-35\left(2b^5c^2d^4-13ab^4cd^5-286a^2b^3d^6\right)x^4+10\left(8b^5c^3d^3-52ab^4c^2d^4+143a^2b^3cd^5+1287a^3b^2d^6\right)x^3-3\left(32b^5c^4d^2-208ab^4c^3d^3+572a^2b^3c^2d^4-858a^3b^2cd^5-3003a^4bd^6\right)x^2+\left(128b^5c^5d-832ab^4c^4d^2+2288a^2b^3c^3d^3-3432a^3b^2c^2d^4+3003a^4bcd^5+3003a^5d^6\right)x\right)\sqrt{dx+c}}{9009d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9009*(693*b^5*d^6*x^6 - 256*b^5*c^6 + 1664*a*b^4*c^5*d - 4576*a^2*b^3*c^4
*d^2 + 6864*a^3*b^2*c^3*d^3 - 6006*a^4*b*c^2*d^4 + 3003*a^5*c*d^5 + 63*(b^5
*c*d^5 + 65*a*b^4*d^6)*x^5 - 35*(2*b^5*c^2*d^4 - 13*a*b^4*c*d^5 - 286*a^2*b
^3*d^6)*x^4 + 10*(8*b^5*c^3*d^3 - 52*a*b^4*c^2*d^4 + 143*a^2*b^3*c*d^5 + 12
87*a^3*b^2*d^6)*x^3 - 3*(32*b^5*c^4*d^2 - 208*a*b^4*c^3*d^3 + 572*a^2*b^3*c
^2*d^4 - 858*a^3*b^2*c*d^5 - 3003*a^4*b*d^6)*x^2 + (128*b^5*c^5*d - 832*a*b
^4*c^4*d^2 + 2288*a^2*b^3*c^3*d^3 - 3432*a^3*b^2*c^2*d^4 + 3003*a^4*b*c*d^5
 + 3003*a^5*d^6)*x)*sqrt(d*x + c)/d^6
```

**giac [B]** time = 1.38, size = 641, normalized size = 4.11

$$\frac{2\left(9009\sqrt{dx+c}\,a^5c+3003\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^5+\frac{15015\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^4bc}{d}+\frac{6006\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)a^3b^2c}{d^2}+\frac{3003\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)a^4b}{d}+\frac{2574\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)a^2b^3c}{d^3}+\frac{2574\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)a^3b^2}{d^2}+\frac{143\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)ab^4c}{d^4}+\frac{286\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)a^2b^3}{d^3}+\frac{13\left(63(dx+c)^{\frac{11}{2}}-385(dx+c)^{\frac{9}{2}}c+990(dx+c)^{\frac{7}{2}}c^2-1386(dx+c)^{\frac{5}{2}}c^3+1155(dx+c)^{\frac{3}{2}}c^4-693\sqrt{dx+c}\,c^5\right)b^5c}{d^5}+\frac{65\left(63(dx+c)^{\frac{11}{2}}-385(dx+c)^{\frac{9}{2}}c+990(dx+c)^{\frac{7}{2}}c^2-1386(dx+c)^{\frac{5}{2}}c^3+1155(dx+c)^{\frac{3}{2}}c^4-693\sqrt{dx+c}\,c^5\right)ab^4}{d^4}+\frac{3\left(231(dx+c)^{\frac{13}{2}}-1638(dx+c)^{\frac{11}{2}}c+5005(dx+c)^{\frac{9}{2}}c^2-8580(dx+c)^{\frac{7}{2}}c^3+9009(dx+c)^{\frac{5}{2}}c^4-6006(dx+c)^{\frac{3}{2}}c^5+3003\sqrt{dx+c}\,c^6\right)b^5}{d^5}\right)}{9009d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9009*(9009*sqrt(d*x + c)*a^5*c + 3003*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*
c)*a^5 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c/d + 6006*(3*(d
*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3*b^2*c/d^2
+ 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^
4*b/d + 2574*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)
*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3*c/d^3 + 2574*(5*(d*x + c)^(7/2) - 21*(
d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b^2/d
^2 + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*
c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^4*c/d^4 + 286*(3
5*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(
d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b^3/d^3 + 13*(63*(d*x + c)^
(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(
5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5*c/d^5 + 65
*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1
386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)
*a*b^4/d^4 + 3*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x
+ c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006
*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5/d^5)/d
```

**maple [B]** time = 0.01, size = 273, normalized size = 1.75

$$\frac{2 (dx+c)^{\frac{3}{2}} \left(693b^5x^5d^5+4095a\,b^4d^5x^4-630b^5c\,d^4x^4+10010a^2b^3d^5x^3-3640a\,b^4c\,d^4x^3+560b^5c^2d^3x^3+12870a^3b^2d^5x^2-8580a^2b^3c\,d^4x^2+3120a\,b^4c^2d^3x^2-480b^5c^3d^2x^2+9009a^4b\,d^5x-10296a^3b^2c\,d^4x+6864a^2b^3c^2d^3x-2496a\,b^4c^3d^2x+384b^5c^4dx+3003a^5d^5-6006a^4bc\,d^4+6864a^3b^2c^2d^3-4576a^2b^3c^3d^2+1664a\,b^4c^4d-256b^5c^5\right)}{9009d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(1/2),x)
```

```
[Out] 2/9009*(d*x+c)^(3/2)*(693*b^5*d^5*x^5+4095*a*b^4*d^5*x^4-630*b^5*c*d^4*x^4+
10010*a^2*b^3*d^5*x^3-3640*a*b^4*c*d^4*x^3+560*b^5*c^2*d^3*x^3+12870*a^3*b^
2*d^5*x^2-8580*a^2*b^3*c*d^4*x^2+3120*a*b^4*c^2*d^3*x^2-480*b^5*c^3*d^2*x^2
+9009*a^4*b*d^5*x-10296*a^3*b^2*c*d^4*x+6864*a^2*b^3*c^2*d^3*x-2496*a*b^4*c
^3*d^2*x+384*b^5*c^4*d*x+3003*a^5*d^5-6006*a^4*b*c*d^4+6864*a^3*b^2*c^2*d^3
-4576*a^2*b^3*c^3*d^2+1664*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

**maxima [A]** time = 1.42, size = 259, normalized size = 1.66

$$\frac{2 \left(693 (dx+c)^{\frac{13}{2}} b^5-4095 \left(b^5c-ab^4d\right)(dx+c)^{\frac{11}{2}}+10010 \left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)(dx+c)^{\frac{9}{2}}-12870 \left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)(dx+c)^{\frac{7}{2}}+9009 \left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)(dx+c)^{\frac{5}{2}}-3003 \left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5\right)(dx+c)^{\frac{3}{2}}\right)}{9009\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9009*(693*(d*x + c)^(13/2)*b^5 - 4095*(b^5*c - a*b^4*d)*(d*x + c)^(11/2)
+ 10010*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(9/2) - 12870*(b^5*
c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(7/2) + 9009
*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4
```

```
)*(d*x + c)^(5/2) - 3003*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10
*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(3/2))/d^6
```

**mupad [B]** time = 0.08, size = 137, normalized size = 0.88

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{13/2}}{13\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}+\frac{2\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{5/2}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(13/2))/(13*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(11/
2))/(11*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(3/2))/(3*d^6) + (20*b^2*(a*d - b
*c)^3*(c + d*x)^(7/2))/(7*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(9/2))/(9*
d^6) + (2*b*(a*d - b*c)^4*(c + d*x)^(5/2))/d^6
```

**sympy [B]** time = 5.12, size = 314, normalized size = 2.01

$$\frac{2\left(\frac{b^5(c+dx)^{\frac{13}{2}}}{13d^5}+\frac{(c+dx)^{\frac{11}{2}}\left(5ab^4d-5b^5c\right)}{11d^5}+\frac{(c+dx)^{\frac{9}{2}}\left(10a^2b^3d^2-20ab^4cd+10b^5c^2\right)}{9d^5}+\frac{(c+dx)^{\frac{7}{2}}\left(10a^3b^2d^3-30a^2b^3cd^2+30ab^4c^2d-10b^5c^3\right)}{7d^5}+\frac{(c+dx)^{\frac{5}{2}}\left(5a^4bd^4-20a^3b^2cd^3+30a^2b^3c^2d^2-20ab^4c^3d+5b^5c^4\right)}{5d^5}+\frac{(c+dx)^{\frac{3}{2}}\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)}{3d^5}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**5*(c + d*x)**(13/2)/(13*d**5) + (c + d*x)**(11/2)*(5*a*b**4*d - 5*b**
5*c)/(11*d**5) + (c + d*x)**(9/2)*(10*a**2*b**3*d**2 - 20*a*b**4*c*d + 10*b
**5*c**2)/(9*d**5) + (c + d*x)**(7/2)*(10*a**3*b**2*d**3 - 30*a**2*b**3*c*d
**2 + 30*a*b**4*c**2*d - 10*b**5*c**3)/(7*d**5) + (c + d*x)**(5/2)*(5*a**4*
b*d**4 - 20*a**3*b**2*c*d**3 + 30*a**2*b**3*c**2*d**2 - 20*a*b**4*c**3*d +
5*b**5*c**4)/(5*d**5) + (c + d*x)**(3/2)*(a**5*d**5 - 5*a**4*b*c*d**4 + 10*
a**3*b**2*c**2*d**3 - 10*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d - b**5*c**5)
/(3*d**5))/d
```

## 3.1270 $\int (a+bx)^4\sqrt{c+dx}\,dx$

**Optimal**. Leaf size=129

$$-\frac{8b^3(c+dx)^{9/2}(bc-ad)}{9d^5}+\frac{12b^2(c+dx)^{7/2}(bc-ad)^2}{7d^5}-\frac{8b(c+dx)^{5/2}(bc-ad)^3}{5d^5}+\frac{2(c+dx)^{3/2}(bc-ad)^4}{3d^5}+\frac{2b^4(c+dx)^{11/2}}{11d^5}$$

**Rubi [A]** time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{9/2}(bc-ad)}{9d^5}+\frac{12b^2(c+dx)^{7/2}(bc-ad)^2}{7d^5}-\frac{8b(c+dx)^{5/2}(bc-ad)^3}{5d^5}+\frac{2(c+dx)^{3/2}(bc-ad)^4}{3d^5}+\frac{2b^4(c+dx)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(3/2))/(3*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(5
/2))/(5*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^5) - (8*b^3*(b*c
 - a*d)*(c + d*x)^(9/2))/(9*d^5) + (2*b^4*(c + d*x)^(11/2))/(11*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4\sqrt{c+dx}\,dx=\int \left(\frac{(-bc+ad)^4\sqrt{c+dx}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{3/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{5/2}}{d^4}-\frac{4b^3(bc\right.$$
$$=\frac{2(bc-ad)^4(c+dx)^{3/2}}{3d^5}-\frac{8b(bc-ad)^3(c+dx)^{5/2}}{5d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{7/2}}{7d^5}-\frac{8b^3(bc-$$

**Mathematica [A]** time = 0.10, size = 101, normalized size = 0.78

$$\frac{2(c+dx)^{3/2}\left(-1540b^3(c+dx)^3(bc-ad)+2970b^2(c+dx)^2(bc-ad)^2-2772b(c+dx)(bc-ad)^3+1155(bc-ad)^4+315b^4(c+dx)^4\right)}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(1155*(b*c - a*d)^4 - 2772*b*(b*c - a*d)^3*(c + d*x) + 2
970*b^2*(b*c - a*d)^2*(c + d*x)^2 - 1540*b^3*(b*c - a*d)*(c + d*x)^3 + 315*
b^4*(c + d*x)^4))/(3465*d^5)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 213, normalized size = 1.65

$$\frac{2(c+dx)^{3/2}\left(1155a^4d^4+2772a^3bd^3(c+dx)-4620a^3bcd^3+6930a^2b^2c^2d^2+2970a^2b^2d^2(c+dx)^2-8316a^2b^2cd^2(c+dx)-4620ab^3c^3d+8316ab^3c^2d(c+dx)+1540ab^3d(c+dx)^3-5940ab^3cd(c+dx)^2+1155b^4c^4-2772b^4c^3(c+dx)+2970b^4c^2(c+dx)^2+315b^4(c+dx)^4-1540b^4c(c+dx)^3\right)}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(1155*b^4*c^4 - 4620*a*b^3*c^3*d + 6930*a^2*b^2*c^2*d^2
- 4620*a^3*b*c*d^3 + 1155*a^4*d^4 - 2772*b^4*c^3*(c + d*x) + 8316*a*b^3*c^2
*d*(c + d*x) - 8316*a^2*b^2*c*d^2*(c + d*x) + 2772*a^3*b*d^3*(c + d*x) + 29
70*b^4*c^2*(c + d*x)^2 - 5940*a*b^3*c*d*(c + d*x)^2 + 2970*a^2*b^2*d^2*(c +
 d*x)^2 - 1540*b^4*c*(c + d*x)^3 + 1540*a*b^3*d*(c + d*x)^3 + 315*b^4*(c +
d*x)^4))/(3465*d^5)
```

**fricas [B]** time = 1.69, size = 245, normalized size = 1.90

$$\frac{2\left(315\,b^4d^5x^5+128\,b^4c^5-704\,ab^3c^4d+1584\,a^2b^2c^3d^2-1848\,a^3bc^2d^3+1155\,a^4cd^4+35\left(b^4cd^4+44\,ab^3d^5\right)x^4-10\left(4\,b^4c^2d^3-22\,ab^3cd^4-297\,a^2b^2d^5\right)x^3+6\left(8\,b^4c^3d^2-44\,ab^3c^2d^3+99\,a^2b^2cd^4+462\,a^3bd^5\right)x^2-\left(64\,b^4c^4d-352\,ab^3c^3d^2+792\,a^2b^2c^2d^3-924\,a^3bcd^4-1155\,a^4d^5\right)x\right)\sqrt{dx+c}}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*b^4*d^5*x^5 + 128*b^4*c^5 - 704*a*b^3*c^4*d + 1584*a^2*b^2*c^3*
d^2 - 1848*a^3*b*c^2*d^3 + 1155*a^4*c*d^4 + 35*(b^4*c*d^4 + 44*a*b^3*d^5)*x
^4 - 10*(4*b^4*c^2*d^3 - 22*a*b^3*c*d^4 - 297*a^2*b^2*d^5)*x^3 + 6*(8*b^4*c
^3*d^2 - 44*a*b^3*c^2*d^3 + 99*a^2*b^2*c*d^4 + 462*a^3*b*d^5)*x^2 - (64*b^4
*c^4*d - 352*a*b^3*c^3*d^2 + 792*a^2*b^2*c^2*d^3 - 924*a^3*b*c*d^4 - 1155*a
^4*d^5)*x)*sqrt(d*x + c)/d^5
```

**giac [B]** time = 1.25, size = 470, normalized size = 3.64

$$\frac{2\left(3465\,\sqrt{dx+c}\,a^4c+1155\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^4+\frac{4620\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^3bc}{d}+\frac{1386\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^2b^2c}{d^2}+\frac{924\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^3b}{d}+\frac{396\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)ab^3c}{d^3}+\frac{594\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)a^2b^2}{d^2}+\frac{11\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)b^4c}{d^4}+\frac{44\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)ab^3}{d^3}+\frac{5\left(63\,(dx+c)^{\frac{11}{2}}-385\,(dx+c)^{\frac{9}{2}}c+990\,(dx+c)^{\frac{7}{2}}c^2-1386\,(dx+c)^{\frac{5}{2}}c^3+1155\,(dx+c)^{\frac{3}{2}}c^4-693\,\sqrt{dx+c}\,c^5\right)b^4}{d^4}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^4*c + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*
c)*a^4 + 4620*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c/d + 1386*(3*(d*
x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*b^2*c/d^2 +
 924*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3*
```

```
b/d + 396*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*a*b^3*c/d^3 + 594*(5*(d*x + c)^(7/2) - 21*(d*x +
c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^2/d^2 + 1
1*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 4
20*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c/d^4 + 44*(35*(d*x + c
)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(
3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^3/d^3 + 5*(63*(d*x + c)^(11/2) - 385*
(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 11
55*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^4/d^4)/d
```

**maple** [A] time = 0.01, size = 186, normalized size = 1.44

$$\frac{2 (dx+c)^{\frac{3}{2}} \left(315b^4x^4d^4+1540a\,b^3d^4x^3-280b^4c\,d^3x^3+2970a^2b^2d^4x^2-1320a\,b^3c\,d^3x^2+240b^4c^2d^2x^2+2772a^3b\,d^4x-2376a^2b^2c\,d^3x+1056a\,b^3c^2d^2x-192b^4c^3dx+1155a^4d^4-1848a^3bc\,d^3+1584a^2b^2c^2d^2-704a\,b^3c^3d+128b^4c^4\right)}{3465d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(1/2),x)
```

```
[Out] 2/3465*(d*x+c)^(3/2)*(315*b^4*d^4*x^4+1540*a*b^3*d^4*x^3-280*b^4*c*d^3*x^3+
2970*a^2*b^2*d^4*x^2-1320*a*b^3*c*d^3*x^2+240*b^4*c^2*d^2*x^2+2772*a^3*b*d^
4*x-2376*a^2*b^2*c*d^3*x+1056*a*b^3*c^2*d^2*x-192*b^4*c^3*d*x+1155*a^4*d^4-
1848*a^3*b*c*d^3+1584*a^2*b^2*c^2*d^2-704*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

**maxima** [A] time = 1.36, size = 181, normalized size = 1.40

$$\frac{2 \left(315 (dx+c)^{\frac{11}{2}} b^4-1540 \left(b^4c-ab^3d\right)(dx+c)^{\frac{9}{2}}+2970 \left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)(dx+c)^{\frac{7}{2}}-2772 \left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)(dx+c)^{\frac{5}{2}}+1155 \left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)(dx+c)^{\frac{3}{2}}\right)}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(d*x + c)^(11/2)*b^4 - 1540*(b^4*c - a*b^3*d)*(d*x + c)^(9/2) +
 2970*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(7/2) - 2772*(b^4*c^3
 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(5/2) + 1155*(b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*x +
c)^(3/2))/d^5
```

**mupad** [B] time = 0.22, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{11/2}}{11\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{9/2}}{9\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{3/2}}{3\,d^5}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(11/2))/(11*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(9/2))
/(9*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(3/2))/(3*d^5) + (12*b^2*(a*d - b*c)^
2*(c + d*x)^(7/2))/(7*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(5/2))/(5*d^5)
```

**sympy [A]** time = 4.19, size = 223, normalized size = 1.73

$$\frac{2\left(\frac{b^4(c+dx)^{\frac{11}{2}}}{11d^4}+\frac{(c+dx)^{\frac{9}{2}}\left(4ab^3d-4b^4c\right)}{9d^4}+\frac{(c+dx)^{\frac{7}{2}}\left(6a^2b^2d^2-12ab^3cd+6b^4c^2\right)}{7d^4}+\frac{(c+dx)^{\frac{5}{2}}\left(4a^3bd^3-12a^2b^2cd^2+12ab^3c^2d-4b^4c^3\right)}{5d^4}+\frac{(c+dx)^{\frac{3}{2}}\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{3d^4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**4*(c + d*x)**(11/2)/(11*d**4) + (c + d*x)**(9/2)*(4*a*b**3*d - 4*b**4
*c)/(9*d**4) + (c + d*x)**(7/2)*(6*a**2*b**2*d**2 - 12*a*b**3*c*d + 6*b**4*
c**2)/(7*d**4) + (c + d*x)**(5/2)*(4*a**3*b*d**3 - 12*a**2*b**2*c*d**2 + 12
*a*b**3*c**2*d - 4*b**4*c**3)/(5*d**4) + (c + d*x)**(3/2)*(a**4*d**4 - 4*a*
*3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*d**4)
)/d
```

## 3.1271 $\int (a+bx)^3\sqrt{c+dx}\,dx$

**Optimal**. Leaf size=100

$$-\frac{6b^2(c+dx)^{7/2}(bc-ad)}{7d^4}+\frac{6b(c+dx)^{5/2}(bc-ad)^2}{5d^4}-\frac{2(c+dx)^{3/2}(bc-ad)^3}{3d^4}+\frac{2b^3(c+dx)^{9/2}}{9d^4}$$

**Rubi [A]** time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2(c+dx)^{7/2}(bc-ad)}{7d^4}+\frac{6b(c+dx)^{5/2}(bc-ad)^2}{5d^4}-\frac{2(c+dx)^{3/2}(bc-ad)^3}{3d^4}+\frac{2b^3(c+dx)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
5/2))/(5*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^4) + (2*b^3*(c + d
*x)^(9/2))/(9*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)^3\sqrt{c+dx}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{3/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{5/2}}{d^3}+\frac{b^3(c+a}{d^3}\right.\\ &= -\frac{2(bc-ad)^3(c+dx)^{3/2}}{3d^4}+\frac{6b(bc-ad)^2(c+dx)^{5/2}}{5d^4}-\frac{6b^2(bc-ad)(c+dx)^{7/2}}{7d^4}+\frac{2b^3(c+a}{9d^4}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 79, normalized size = 0.79

$$\frac{2(c+dx)^{3/2}\left(-135b^2(c+dx)^2(bc-ad)+189b(c+dx)(bc-ad)^2-105(bc-ad)^3+35b^3(c+dx)^3\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-105*(b*c - a*d)^3 + 189*b*(b*c - a*d)^2*(c + d*x) - 13
5*b^2*(b*c - a*d)*(c + d*x)^2 + 35*b^3*(c + d*x)^3))/(315*d^4)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 132, normalized size = 1.32

$$\frac{2(c+dx)^{3/2}\left(105a^3d^3+189a^2bd^2(c+dx)-315a^2bcd^2+315ab^2c^2d+135ab^2d(c+dx)^2-378ab^2cd(c+dx)-105b^3c^3+189b^3c^2(c+dx)+35b^3(c+dx)^3-135b^3c(c+dx)^2\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-105*b^3*c^3 + 315*a*b^2*c^2*d - 315*a^2*b*c*d^2 + 105*
a^3*d^3 + 189*b^3*c^2*(c + d*x) - 378*a*b^2*c*d*(c + d*x) + 189*a^2*b*d^2*(
c + d*x) - 135*b^3*c*(c + d*x)^2 + 135*a*b^2*d*(c + d*x)^2 + 35*b^3*(c + d*
x)^3))/(315*d^4)
```

**fricas [A]** time = 1.16, size = 164, normalized size = 1.64

$$\frac{2\left(35\,b^3d^4x^4-16\,b^3c^4+72\,ab^2c^3d-126\,a^2bc^2d^2+105\,a^3cd^3+5\left(b^3cd^3+27\,ab^2d^4\right)x^3-3\left(2\,b^3c^2d^2-9\,ab^2cd^3-63\,a^2bd^4\right)x^2+\left(8\,b^3c^3d-36\,ab^2c^2d^2+63\,a^2bcd^3+105\,a^3d^4\right)x\right)\sqrt{dx+c}}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^3*d^4*x^4 - 16*b^3*c^4 + 72*a*b^2*c^3*d - 126*a^2*b*c^2*d^2 + 1
05*a^3*c*d^3 + 5*(b^3*c*d^3 + 27*a*b^2*d^4)*x^3 - 3*(2*b^3*c^2*d^2 - 9*a*b^
2*c*d^3 - 63*a^2*b*d^4)*x^2 + (8*b^3*c^3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*c*
d^3 + 105*a^3*d^4)*x)*sqrt(d*x + c)/d^4
```

**giac [B]** time = 1.27, size = 322, normalized size = 3.22

$$\frac{2\left(315\,\sqrt{dx+c}\,a^3c+105\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^3+\frac{315\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^2bc}{d}+\frac{63\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)ab^2c}{d^2}+\frac{63\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^2b}{d}+\frac{9\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)b^3c}{d^3}+\frac{27\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)ab^2}{d^2}+\frac{\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)b^3}{d^3}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^3*c + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*
a^3 + 315*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c/d + 63*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b^2*c/d^2 + 63*(3*(
d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*b/d + 9*(
5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt
(d*x + c)*c^3)*b^3*c/d^3 + 27*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 3
5*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2/d^2 + (35*(d*x + c)^(9/
```

```
2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*
c^3 + 315*sqrt(d*x + c)*c^4)*b^3/d^3)/d
```

**maple [A]** time = 0.00, size = 116, normalized size = 1.16

$$\frac{2 (dx+c)^{\frac{3}{2}} \left(35b^{3}x^{3}d^{3}+135a\,b^{2}d^{3}x^{2}-30b^{3}c\,d^{2}x^{2}+189a^{2}b\,d^{3}x-108a\,b^{2}c\,d^{2}x+24b^{3}c^{2}dx+105a^{3}d^{3}-126a^{2}bc\,d^{2}+72a\,b^{2}c^{2}d-16b^{3}c^{3}\right)}{315d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(1/2),x)
```

```
[Out] 2/315*(d*x+c)^(3/2)*(35*b^3*d^3*x^3+135*a*b^2*d^3*x^2-30*b^3*c*d^2*x^2+189*
a^2*b*d^3*x-108*a*b^2*c*d^2*x+24*b^3*c^2*d*x+105*a^3*d^3-126*a^2*b*c*d^2+72
*a*b^2*c^2*d-16*b^3*c^3)/d^4
```

**maxima [A]** time = 1.37, size = 118, normalized size = 1.18

$$\frac{2 \left(35 (dx+c)^{\frac{9}{2}}b^{3}-135 \left(b^{3}c-ab^{2}d\right)(dx+c)^{\frac{7}{2}}+189 \left(b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2}\right)(dx+c)^{\frac{5}{2}}-105 \left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)(dx+c)^{\frac{3}{2}}\right)}{315\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(d*x + c)^(9/2)*b^3 - 135*(b^3*c - a*b^2*d)*(d*x + c)^(7/2) + 189
*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(5/2) - 105*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(3/2))/d^4
```

**mupad [B]** time = 0.07, size = 87, normalized size = 0.87

$$\frac{2\,b^{3}\,(c+d\,x)^{9/2}}{9\,d^{4}}-\frac{\left(6\,b^{3}\,c-6\,a\,b^{2}\,d\right)\,(c+d\,x)^{7/2}}{7\,d^{4}}+\frac{2\,(a\,d-b\,c)^{3}\,(c+d\,x)^{3/2}}{3\,d^{4}}+\frac{6\,b\,(a\,d-b\,c)^{2}\,(c+d\,x)^{5/2}}{5\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(9/2))/(9*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(7/2))/(
7*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(3/2))/(3*d^4) + (6*b*(a*d - b*c)^2*(c
+ d*x)^(5/2))/(5*d^4)
```

**sympy [A]** time = 3.34, size = 146, normalized size = 1.46

$$\frac{2 \left(\frac{b^{3}(c+dx)^{\frac{9}{2}}}{9d^{3}}+\frac{(c+dx)^{\frac{7}{2}}\left(3ab^{2}d-3b^{3}c\right)}{7d^{3}}+\frac{(c+dx)^{\frac{5}{2}}\left(3a^{2}bd^{2}-6ab^{2}cd+3b^{3}c^{2}\right)}{5d^{3}}+\frac{(c+dx)^{\frac{3}{2}}\left(a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}\right)}{3d^{3}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**3*(c + d*x)**(9/2)/(9*d**3) + (c + d*x)**(7/2)*(3*a*b**2*d - 3*b**3*c
)/(7*d**3) + (c + d*x)**(5/2)*(3*a**2*b*d**2 - 6*a*b**2*c*d + 3*b**3*c**2)/
(5*d**3) + (c + d*x)**(3/2)*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
- b**3*c**3)/(3*d**3))/d
```

## 3.1272 $\int (a+bx)^2\sqrt{c+dx}\,dx$

**Optimal**. Leaf size=71

$$-\frac{4b(c+dx)^{5/2}(bc-ad)}{5d^3}+\frac{2(c+dx)^{3/2}(bc-ad)^2}{3d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}$$

**Rubi** [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{5/2}(bc-ad)}{5d^3}+\frac{2(c+dx)^{3/2}(bc-ad)^2}{3d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(3/2))/(3*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(5/2
))/(5*d^3) + (2*b^2*(c + d*x)^(7/2))/(7*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)^2\sqrt{c+dx}}{d^2}-\frac{2b(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{b^2(c+dx)^{5/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{3/2}}{3d^3}-\frac{4b(bc-ad)(c+dx)^{5/2}}{5d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{3/2}\left(35a^2d^2+14abd(3dx-2c)+b^2\left(8c^2-12cdx+15d^2x^2\right)\right)}{105d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-2*c + 3*d*x) + b^2*(8*c^2 - 12*
c*d*x + 15*d^2*x^2)))/(105*d^3)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 72, normalized size = 1.01

$$\frac{2(c+dx)^{3/2}\left(35a^2d^2+42abd(c+dx)-70abcd+35b^2c^2+15b^2(c+dx)^2-42b^2c(c+dx)\right)}{105d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(35*b^2*c^2 - 70*a*b*c*d + 35*a^2*d^2 - 42*b^2*c*(c + d*
x) + 42*a*b*d*(c + d*x) + 15*b^2*(c + d*x)^2))/(105*d^3)
```

**fricas [A]** time = 1.39, size = 99, normalized size = 1.39

$$\frac{2\left(15\,b^2d^3x^3+8\,b^2c^3-28\,abc^2d+35\,a^2cd^2+3\left(b^2cd^2+14\,abd^3\right)x^2-\left(4\,b^2c^2d-14\,abcd^2-35\,a^2d^3\right)x\right)\sqrt{dx+c}}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*d^3*x^3 + 8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2 + 3*(b^2*c*
d^2 + 14*a*b*d^3)*x^2 - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*x)*sqrt(d
*x + c)/d^3
```

**giac [B]** time = 1.29, size = 200, normalized size = 2.82

$$\frac{2\left(105\,\sqrt{dx+c}\,a^2c+35\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^2+\frac{70\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)abc}{d}+\frac{7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)b^2c}{d^2}+\frac{14\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)ab}{d}+\frac{3\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)b^2}{d^2}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(d*x + c)*a^2*c + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^2 + 70*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c/d + 7*(3*(d*x + c)^(5/2
) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c/d^2 + 14*(3*(d*x + c
)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b/d + 3*(5*(d*x +
c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)
*c^3)*b^2/d^2)/d
```

**maple** [A] time = 0.01, size = 63, normalized size = 0.89

$$\frac{2 (dx+c)^{\frac{3}{2}} \left(15b^2x^2d^2+42ab\,d^2x-12b^2cdx+35a^2d^2-28abcd+8b^2c^2\right)}{105d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(1/2),x)
```

```
[Out] 2/105*(d*x+c)^(3/2)*(15*b^2*d^2*x^2+42*a*b*d^2*x-12*b^2*c*d*x+35*a^2*d^2-28
*a*b*c*d+8*b^2*c^2)/d^3
```

**maxima** [A] time = 1.35, size = 68, normalized size = 0.96

$$\frac{2 \left(15 (dx+c)^{\frac{7}{2}} b^2 - 42 \left(b^2c - abd\right)(dx+c)^{\frac{5}{2}} + 35 \left(b^2c^2 - 2\,abcd + a^2d^2\right)(dx+c)^{\frac{3}{2}}\right)}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(d*x + c)^(7/2)*b^2 - 42*(b^2*c - a*b*d)*(d*x + c)^(5/2) + 35*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(3/2))/d^3
```

**mupad** [B] time = 0.24, size = 68, normalized size = 0.96

$$\frac{2\,(c+d\,x)^{3/2}\,\left(15\,b^2\,(c+d\,x)^2+35\,a^2\,d^2+35\,b^2\,c^2-42\,b^2\,c\,\,(c+d\,x)+42\,a\,b\,d\,\,(c+d\,x)-70\,a\,b\,c\,d\right)}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(3/2)*(15*b^2*(c + d*x)^2 + 35*a^2*d^2 + 35*b^2*c^2 - 42*b^2*c
*(c + d*x) + 42*a*b*d*(c + d*x) - 70*a*b*c*d))/(105*d^3)
```

**sympy** [A] time = 2.69, size = 85, normalized size = 1.20

$$\frac{2 \left(\frac{b^2(c+dx)^{\frac{7}{2}}}{7d^2} + \frac{(c+dx)^{\frac{5}{2}}\left(2abd-2b^2c\right)}{5d^2} + \frac{(c+dx)^{\frac{3}{2}}\left(a^2d^2-2abcd+b^2c^2\right)}{3d^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**2*(c + d*x)**(7/2)/(7*d**2) + (c + d*x)**(5/2)*(2*a*b*d - 2*b**2*c)/(
5*d**2) + (c + d*x)**(3/2)*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*d**2))/d
```

# 3.1273 $\int (a+bx)\sqrt{c+dx}\,dx$

**Optimal**. Leaf size=42

$$\frac{2b(c+dx)^{5/2}}{5d^2}-\frac{2(c+dx)^{3/2}(bc-ad)}{3d^2}$$

**Rubi** [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{5/2}}{5d^2}-\frac{2(c+dx)^{3/2}(bc-ad)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(3/2))/(3*d^2) + (2*b*(c + d*x)^(5/2))/(5*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\sqrt{c+dx}\,dx&=\int \left(\frac{(-bc+ad)\sqrt{c+dx}}{d}+\frac{b(c+dx)^{3/2}}{d}\right)dx\\&=-\frac{2(bc-ad)(c+dx)^{3/2}}{3d^2}+\frac{2b(c+dx)^{5/2}}{5d^2}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{3/2}(5ad-2bc+3bdx)}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Sqrt[c + d*x],x]
```

[Out] (2*(c + d*x)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x))/(15*d^2)

**IntegrateAlgebraic [A]** time = 0.02, size = 33, normalized size = 0.79

$$\frac{2(c+dx)^{3/2}(5ad+3b(c+dx)-5bc)}{15d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x)*Sqrt[c + d*x],x]

[Out] (2*(c + d*x)^(3/2)*(-5*b*c + 5*a*d + 3*b*(c + d*x)))/(15*d^2)

**fricas [A]** time = 1.43, size = 46, normalized size = 1.10

$$\frac{2\left(3\,bd^2x^2-2\,bc^2+5\,acd+\left(bcd+5\,ad^2\right)x\right)\sqrt{dx+c}}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 2/15*(3*b*d^2*x^2 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x)*sqrt(d*x + c)/d^2

**giac [B]** time = 1.37, size = 100, normalized size = 2.38

$$\frac{2\left(15\,\sqrt{dx+c}\,ac+5\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a+\frac{5\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)bc}{d}+\frac{\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)b}{d}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")

[Out] 2/15*(15*sqrt(d*x + c)*a*c + 5*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a + 5*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c/d + (3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b/d)/d

**maple [A]** time = 0.00, size = 27, normalized size = 0.64

$$\frac{2\,(dx+c)^{\frac{3}{2}}\,(3bdx+5ad-2bc)}{15d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] 2/15*(d*x+c)^(3/2)*(3*b*d*x+5*a*d-2*b*c)/d^2
```

**maxima [A]** time = 1.29, size = 33, normalized size = 0.79

$$\frac{2\left(3\,(dx+c)^{\frac{5}{2}}b-5\,(bc-ad)(dx+c)^{\frac{3}{2}}\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(d*x + c)^(5/2)*b - 5*(b*c - a*d)*(d*x + c)^(3/2))/d^2
```

**mupad [B]** time = 0.04, size = 29, normalized size = 0.69

$$\frac{2\,(c+d\,x)^{3/2}\,\left(5\,a\,d-5\,b\,c+3\,b\,\left(c+d\,x\right)\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(3/2)*(5*a*d - 5*b*c + 3*b*(c + d*x)))/(15*d^2)
```

**sympy [A]** time = 2.12, size = 36, normalized size = 0.86

$$\frac{2\left(\frac{b(c+dx)^{\frac{5}{2}}}{5d}+\frac{(c+dx)^{\frac{3}{2}}(ad-bc)}{3d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b*(c + d*x)**(5/2)/(5*d) + (c + d*x)**(3/2)*(a*d - b*c)/(3*d))/d
```

## 3.1274 $\int \sqrt{c+dx}\,dx$

**Optimal**. Leaf size=16

$$\frac{2(c+dx)^{3/2}}{3d}$$

**Rubi [A]** time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{c+dx}\,dx = \frac{2(c+dx)^{3/2}}{3d}$$

**Mathematica [A]** time = 0.00, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x],x]
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

**fricas [A]** time = 1.28, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*(d*x + c)^(3/2)/d
```

**giac [A]** time = 1.34, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*(d*x + c)^(3/2)/d
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2),x)
[Out] 2/3*(d*x+c)^(3/2)/d
```

**maxima [A]** time = 1.35, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(d*x + c)^(3/2)/d
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.75

$$\frac{2\left(c+dx\right)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2),x)
```

```
[Out] 2*(c + d*x)**(3/2)/(3*d)
```

## 3.1275 $\int \frac{\sqrt{c+dx}}{a+bx}\,dx$

**Optimal**. Leaf size=62

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 63, 208}

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{a+bx}\,dx = \frac{2\sqrt{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b}$$
$$= \frac{2\sqrt{c+dx}}{b} + \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{bd}$$
$$= \frac{2\sqrt{c+dx}}{b} - \frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

**Mathematica** [A] time = 0.04, size = 62, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{b} - \frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/b^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.07, size = 72, normalized size = 1.16

$$\frac{2\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}\,\sqrt{ad-bc}}{bc-ad}\right)}{b^{3/2}} + \frac{2\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x])/b + (2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a
*d]*Sqrt[c + d*x])/(b*c - a*d)])/b^(3/2)
```

**fricas** [A] time = 1.36, size = 143, normalized size = 2.31

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,\sqrt{dx+c}}{b}, -\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx+c}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*
c - a*d)/b))/(b*x + a)) + 2*sqrt(d*x + c))/b, -2*(sqrt(-(b*c - a*d)/b)*arct
an(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqrt(d*x + c))/b]
```

**giac [A]** time = 1.27, size = 62, normalized size = 1.00

$$\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}+\frac{2\,\sqrt{dx+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b*c - a*d)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b) + 2*sqrt(d*x + c)/b
```

**maple [A]** time = 0.01, size = 92, normalized size = 1.48

$$-\frac{2ad\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;b}+\frac{2c\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}}+\frac{2\sqrt{dx+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a),x)
```

```
[Out] 2*(d*x+c)^(1/2)/b-2/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)*b/((a*d-b*c)
*b)^(1/2))*a*d+2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)*b/((a*d-b*c)*b)^(
1/2))*c
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.07, size = 50, normalized size = 0.81

$$\frac{2\sqrt{c+d\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(1/2))/b - (2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)
)*(a*d - b*c)^(1/2))/b^(3/2)
```

**sympy [A]** time = 4.39, size = 61, normalized size = 0.98

$$\frac{2\left(\frac{d\sqrt{c+dx}}{b}-\frac{d(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x)/b - d*(a*d - b*c)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b)
)/(b**2*sqrt((a*d - b*c)/b)))/d
```

# 3.1276 $\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=70

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx}}{b(a+bx)}$$

**Rubi [A]** time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 63, 208}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[c + d*x]/(b*(a + b*x))) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*
c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx = -\frac{\sqrt{c+dx}}{b(a+bx)} + \frac{d\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b}$$
$$= -\frac{\sqrt{c+dx}}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{b}$$
$$= -\frac{\sqrt{c+dx}}{b(a+bx)} - \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}$$

**Mathematica** [A] time = 0.08, size = 69, normalized size = 0.99

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{b^{3/2}\sqrt{ad-bc}} - \frac{\sqrt{c+dx}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[c + d*x]/(b*(a + b*x))) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b
*c) + a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d])
```

**IntegrateAlgebraic** [A] time = 0.22, size = 91, normalized size = 1.30

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{b^{3/2}\sqrt{ad-bc}} - \frac{d\sqrt{c+dx}}{b(ad+b(c+dx)-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -((d*Sqrt[c + d*x])/(b*(-(b*c) + a*d + b*(c + d*x)))) - (d*ArcTan[(Sqrt[b]*
Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(b^(3/2)*Sqrt[-(b*c) + a*d]
)
```

**fricas** [A] time = 1.44, size = 232, normalized size = 3.31

$$\left[\frac{\sqrt{b^2c-abd}\,(bdx+ad)\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\left(b^2c-abd\right)\sqrt{dx+c}}{2\left(ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x\right)},\ \frac{\sqrt{-b^2c+abd}\,(bdx+ad)\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-\left(b^2c-abd\right)\sqrt{dx+c}}{ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2*c - a*b*d)*(b*d*x + a*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b
^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(b^2*c - a*b*d)*sqrt(d*x + c))/
(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x), (sqrt(-b^2*c + a*b*d)*(b*d*x +
 a*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (b^2*c - a
*b*d)*sqrt(d*x + c))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x)]
```

**giac [A]** time = 1.38, size = 72, normalized size = 1.03

$$\frac{d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx+c}\,d}{((dx+c)b-bc+ad)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b) - s
qrt(d*x + c)*d/(((d*x + c)*b - b*c + a*d)*b)
```

**maple [A]** time = 0.01, size = 64, normalized size = 0.91

$$\frac{d\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;b}-\frac{\sqrt{dx+c}\;d}{(bdx+ad)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^2,x)
```

```
[Out] -d/b*(d*x+c)^(1/2)/(b*d*x+a*d)+d/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)
/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 0.24, size = 61, normalized size = 0.87

$$\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{b^{3/2}\,\sqrt{a\,d-b\,c}}-\frac{d\,\sqrt{c+d\,x}}{d\,x\,b^2+a\,d\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^2,x)
```

```
[Out] (d*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(b^(3/2)*(a*d - b*c)^
(1/2)) - (d*(c + d*x)^(1/2))/(a*b*d + b^2*d*x)
```

**sympy** [B] time = 58.58, size = 573, normalized size = 8.19

$$-\frac{2ad^2\sqrt{c+dx}}{2a^2bd^2-2ab^2cd+2ab^2d^2x-2b^3cdx}+\frac{ad^2\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(-a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}+2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}-b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2b}-\frac{ad^2\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}-2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}+b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2b}-\frac{cd\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(-a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}+2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}-b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2}+\frac{cd\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}-2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}+b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2}+\frac{2cd\sqrt{c+dx}}{2a^2d^2-2abcd+2abd^2x-2b^2cdx}+\frac{2d\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad}{b}-c}}\right)}{b^2\sqrt{\frac{ad}{b}-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**2,x)
```

```
[Out] -2*a*d**2*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x - 2
*b**3*c*d*x) + a*d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b
*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(
-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) - a*d**2*sqrt(-1/(b*(a*d - b*
c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a
*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*
b) - c*d*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)*
*3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d -
b*c)**3)) + sqrt(c + d*x))/2 + c*d*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**
2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**
2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + 2*c*d*sqrt(c + d*x)
/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*d*x) + 2*d*atan(sqrt(c
+ d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c))
```

## 3.1277 $\int \frac{\sqrt{c+dx}}{(a+bx)^3} \, dx$

**Optimal**. Leaf size=110

$$\frac{d^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}} - \frac{d\sqrt{c+dx}}{4b(a+bx)(bc-ad)} - \frac{\sqrt{c+dx}}{2b(a+bx)^2}$$

**Rubi** [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$\frac{d^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}} - \frac{d\sqrt{c+dx}}{4b(a+bx)(bc-ad)} - \frac{\sqrt{c+dx}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -Sqrt[c + d*x]/(2*b*(a + b*x)^2) - (d*Sqrt[c + d*x])/(4*b*(b*c - a*d)*(a +
b*x)) + (d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(
b*c - a*d)^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}+\frac{d\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{4b}\\ &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}-\frac{d^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b(bc-ad)}\\ &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}-\frac{d\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4b(bc-ad)}\\ &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 52, normalized size = 0.47

$$\frac{2d^2(c+dx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] (2*d^2*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^3)
```

**IntegrateAlgebraic [A]** time = 0.42, size = 125, normalized size = 1.14

$$-\frac{d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4b^{3/2}(ad-bc)^{3/2}}-\frac{d^2\sqrt{c+dx}\,(-ad+b(c+dx)+bc)}{4b(bc-ad)(-ad-b(c+dx)+bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -1/4*(d^2*Sqrt[c + d*x]*(b*c - a*d + b*(c + d*x)))/(b*(b*c - a*d)*(b*c - a*
d - b*(c + d*x))^2) - (d^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x]
)/(b*c - a*d)])/(4*b^(3/2)*(-(b*c) + a*d)^(3/2))
```

**fricas [B]** time = 1.06, size = 456, normalized size = 4.15

$$\left[-\frac{\left(b^{2} d^{2} x^{2}+2 \, a b d^{2} x+a^{2} d^{2}\right) \sqrt{b^{2} c-a b d} \log \left(\frac{b d x+2 \, b c-a d-2 \, \sqrt{b^{2} c-a b d} \sqrt{d x+c}}{b x+a}\right)+2 \, \left(2 \, b^{3} c^{2}-3 \, a b^{2} c d+a^{2} b d^{2}+\left(b^{3} c d-a b^{2} d^{2}\right) x\right) \sqrt{d x+c}}{8 \, \left(a^{2} b^{4} c^{2}-2 \, a^{3} b^{3} c d+a^{4} b^{2} d^{2}+\left(b^{6} c^{2}-2 \, a b^{5} c d+a^{2} b^{4} d^{2}\right) x^{2}+2 \, \left(a b^{5} c^{2}-2 \, a^{2} b^{4} c d+a^{3} b^{3} d^{2}\right) x\right)}, -\frac{\left(b^{2} d^{2} x^{2}+2 \, a b d^{2} x+a^{2} d^{2}\right) \sqrt{-b^{2} c+a b d} \arctan \left(\frac{\sqrt{-b^{2} c+a b d} \sqrt{d x+c}}{b d x+b c}\right)+\left(2 \, b^{3} c^{2}-3 \, a b^{2} c d+a^{2} b d^{2}+\left(b^{3} c d-a b^{2} d^{2}\right) x\right) \sqrt{d x+c}}{4 \, \left(a^{2} b^{4} c^{2}-2 \, a^{3} b^{3} c d+a^{4} b^{2} d^{2}+\left(b^{6} c^{2}-2 \, a b^{5} c d+a^{2} b^{4} d^{2}\right) x^{2}+2 \, \left(a b^{5} c^{2}-2 \, a^{2} b^{4} c d+a^{3} b^{3} d^{2}\right) x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x
 + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(2*b^3
*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a
^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4
*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x), -1/4*((b^2*d^2*
x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*
d)*sqrt(d*x + c)/(b*d*x + b*c)) + (2*b^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b
^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^
2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^
4*c*d + a^3*b^3*d^2)*x)]
```

**giac [A]** time = 1.33, size = 126, normalized size = 1.15

$$-\frac{d^{2} \arctan \left(\frac{\sqrt{d x+c} b}{\sqrt{-b^{2} c+a b d}}\right)}{4 \, \left(b^{2} c-a b d\right) \sqrt{-b^{2} c+a b d}}-\frac{(d x+c)^{\frac{3}{2}} b d^{2}+\sqrt{d x+c} b c d^{2}-\sqrt{d x+c} a d^{3}}{4 \, \left(b^{2} c-a b d\right) \left((d x+c) b-b c+a d\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqrt
(-b^2*c + a*b*d)) - 1/4*((d*x + c)^(3/2)*b*d^2 + sqrt(d*x + c)*b*c*d^2 - sq
rt(d*x + c)*a*d^3)/((b^2*c - a*b*d)*((d*x + c)*b - b*c + a*d)^2)
```

**maple [A]** time = 0.01, size = 111, normalized size = 1.01

$$\frac{d^{2} \arctan \left(\frac{\sqrt{d x+c}\, b}{\sqrt{\left(a d-b c\right) b}}\right)}{4 \left(a d-b c\right) \sqrt{\left(a d-b c\right) b}\, b}+\frac{\left(d x+c\right)^{\frac{3}{2}} d^{2}}{4 \left(b d x+a d\right)^{2} \left(a d-b c\right)}-\frac{\sqrt{d x+c}\, d^{2}}{4 \left(b d x+a d\right)^{2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^3,x)
```

```
[Out] 1/4*d^2/(b*d*x+a*d)^2/(a*d-b*c)*(d*x+c)^(3/2)-1/4*d^2/(b*d*x+a*d)^2/b*(d*x+
c)^(1/2)+1/4*d^2/(a*d-b*c)/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d
-b*c)*b)^(1/2)*b)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [**B**] time = 0.30, size = 135, normalized size = 1.23

$$\frac{d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{\frac{d^2\,\sqrt{c+d\,x}}{4\,b}-\frac{d^2\,{\left(c+d\,x\right)}^{3/2}}{4\,\left(a\,d-b\,c\right)}}{b^2\,{\left(c+d\,x\right)}^2-\left(2\,b^2\,c-2\,a\,b\,d\right)\,\left(c+d\,x\right)+a^2\,d^2+b^2\,c^2-2\,a\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^3,x)
```

```
[Out] (d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(3/2)*(a*d - b
*c)^(3/2)) - ((d^2*(c + d*x)^(1/2))/(4*b) - (d^2*(c + d*x)^(3/2))/(4*(a*d -
 b*c)))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.1278 $\int \frac{\sqrt{c+dx}}{(a+bx)^4} dx$

**Optimal**. Leaf size=146

$$-\frac{d^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}+\frac{d^2\sqrt{c+dx}}{8b(a+bx)(bc-ad)^2}-\frac{d\sqrt{c+dx}}{12b(a+bx)^2(bc-ad)}-\frac{\sqrt{c+dx}}{3b(a+bx)^3}$$

**Rubi [A]** time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{d^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}+\frac{d^2\sqrt{c+dx}}{8b(a+bx)(bc-ad)^2}-\frac{d\sqrt{c+dx}}{12b(a+bx)^2(bc-ad)}-\frac{\sqrt{c+dx}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] -Sqrt[c + d*x]/(3*b*(a + b*x)^3) - (d*Sqrt[c + d*x])/(12*b*(b*c - a*d)*(a +
 b*x)^2) + (d^2*Sqrt[c + d*x])/(8*b*(b*c - a*d)^2*(a + b*x)) - (d^3*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx &= -\frac{\sqrt{c+dx}}{3b(a+bx)^3} + \frac{d\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{6b} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3} - \frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2} - \frac{d^2\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8b(bc-ad)} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3} - \frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2} + \frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)} + \frac{d^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b(bc-ad)^2} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3} - \frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2} + \frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)} + \frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c
}{8b(bc-ad)^2} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3} - \frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2} + \frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)} - \frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 52, normalized size = 0.36

$$
\frac{2d^3(c+dx)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] (2*d^3*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^4)
```

**IntegrateAlgebraic [A]** time = 0.75, size = 176, normalized size = 1.21

$$\frac{d^3\sqrt{c+dx}\,\left(3a^2d^2-8abd(c+dx)-6abcd+3b^2c^2-3b^2(c+dx)^2+8b^2c(c+dx)\right)}{24b(bc-ad)^2(-ad-b(c+dx)+bc)^3}-\frac{d^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}\,\sqrt{ad-bc}}{bc-ad}\right)}{8b^{3/2}(bc-ad)^2\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] (d^3*Sqrt[c + d*x]*(3*b^2*c^2 - 6*a*b*c*d + 3*a^2*d^2 + 8*b^2*c*(c + d*x) -
 8*a*b*d*(c + d*x) - 3*b^2*(c + d*x)^2))/(24*b*(b*c - a*d)^2*(b*c - a*d - b
*(c + d*x))^3) - (d^3*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*
c - a*d)])/(8*b^(3/2)*(b*c - a*d)^2*Sqrt[-(b*c) + a*d])
```

**fricas [B]** time = 1.09, size = 785, normalized size = 5.38

$$\left[\frac{3\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2bc-ad-2\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\left(8b^4c^3-22ab^3c^2d+17a^2b^2cd^2-3a^3bd^3-3\left(b^4cd^2-ab^3d^3\right)x^2+2\left(b^4c^2d-5ab^3cd^2+4a^2b^2d^3\right)x\right)\sqrt{dx+c}}{48\left(a^3b^5c^3-3a^4b^4c^2d+3a^5b^3cd^2-a^6b^2d^3+\left(b^8c^3-3ab^7c^2d+3a^2b^6cd^2-a^3b^5d^3\right)x^3+3\left(ab^7c^3-3a^2b^6c^2d+3a^3b^5cd^2-a^4b^4d^3\right)x^2+3\left(a^2b^6c^3-3a^3b^5c^2d+3a^4b^4cd^2-a^5b^3d^3\right)x\right)},\ \frac{3\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-\left(8b^4c^3-22ab^3c^2d+17a^2b^2cd^2-3a^3bd^3-3\left(b^4cd^2-ab^3d^3\right)x^2+2\left(b^4c^2d-5ab^3cd^2+4a^2b^2d^3\right)x\right)\sqrt{dx+c}}{24\left(a^3b^5c^3-3a^4b^4c^2d+3a^5b^3cd^2-a^6b^2d^3+\left(b^8c^3-3ab^7c^2d+3a^2b^6cd^2-a^3b^5d^3\right)x^3+3\left(ab^7c^3-3a^2b^6c^2d+3a^3b^5cd^2-a^4b^4d^3\right)x^2+3\left(a^2b^6c^3-3a^3b^5c^2d+3a^4b^4cd^2-a^5b^3d^3\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^2
*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))
/(b*x + a)) - 2*(8*b^4*c^3 - 22*a*b^3*c^2*d + 17*a^2*b^2*c*d^2 - 3*a^3*b*d^
3 - 3*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^
2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
 a^6*b^2*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^
3 + 3*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^2 + 3
*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x), 1/24*(
3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a
*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*b^4*c^3
 - 22*a*b^3*c^2*d + 17*a^2*b^2*c*d^2 - 3*a^3*b*d^3 - 3*(b^4*c*d^2 - a*b^3*d
^3)*x^2 + 2*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(
a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3 + (b^8*c^3 -
3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^3 + 3*(a*b^7*c^3 - 3*a^2*b
^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^2 + 3*(a^2*b^6*c^3 - 3*a^3*b^5*
c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x)]
```

**giac [A]** time = 1.35, size = 207, normalized size = 1.42

$$\frac{d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\sqrt{-b^2c+abd}}+\frac{3\,(dx+c)^{\frac{5}{2}}b^2d^3-8\,(dx+c)^{\frac{3}{2}}b^2cd^3-3\,\sqrt{dx+c}\,b^2c^2d^3+8\,(dx+c)^{\frac{3}{2}}abd^4+6\,\sqrt{dx+c}\,abcd^4-3\,\sqrt{dx+c}\,a^2d^5}{24\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\left((dx+c)b-bc+ad\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^2 - 2*a*b^2*c*
d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) + 1/24*(3*(d*x + c)^(5/2)*b^2*d^3 - 8*
(d*x + c)^(3/2)*b^2*c*d^3 - 3*sqrt(d*x + c)*b^2*c^2*d^3 + 8*(d*x + c)^(3/2)
*a*b*d^4 + 6*sqrt(d*x + c)*a*b*c*d^4 - 3*sqrt(d*x + c)*a^2*d^5)/((b^3*c^2 -
 2*a*b^2*c*d + a^2*b*d^2)*((d*x + c)*b - b*c + a*d)^3)
```

**maple [A]** time = 0.02, size = 170, normalized size = 1.16

$$\frac{(dx+c)^{\frac{5}{2}}\, b\, d^3}{8\,(bdx+ad)^3\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{d^3\arctan\left(\frac{\sqrt{dx+c}\; b}{\sqrt{(ad-bc)b}}\right)}{8\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{(ad-bc)\,b}\; b}+\frac{(dx+c)^{\frac{3}{2}}\, d^3}{3\,(bdx+ad)^3\,(ad-bc)}-\frac{\sqrt{dx+c}\; d^3}{8\,(bdx+ad)^3\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^4,x)
```

```
[Out] 1/8*d^3/(b*d*x+a*d)^3*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+1/3*d^3/(
b*d*x+a*d)^3/(a*d-b*c)*(d*x+c)^(3/2)-1/8*d^3/(b*d*x+a*d)^3/b*(d*x+c)^(1/2)+
1/8*d^3/b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1
/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.37, size = 207, normalized size = 1.42

$$\frac{\frac{d^3\,{\left(c+d\,x\right)}^{3/2}}{3\,\left(a\,d-b\,c\right)}-\frac{d^3\,\sqrt{c+d\,x}}{8\,b}+\frac{b\,d^3\,{\left(c+d\,x\right)}^{5/2}}{8\,{\left(a\,d-b\,c\right)}^2}}{\left(c+d\,x\right)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)+b^3\,{\left(c+d\,x\right)}^3-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^2+a^3\,d^3-b^3\,c^3+3\,a\,b^2\,c^2\,d-3\,a^2\,b\,c\,d^2}+\frac{d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^4,x)
```

```
[Out] ((d^3*(c + d*x)^(3/2))/(3*(a*d - b*c)) - (d^3*(c + d*x)^(1/2))/(8*b) + (b*d
^3*(c + d*x)^(5/2))/(8*(a*d - b*c)^2))/((c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2
- 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^2 + a^3*
```

```
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (d^3*atan((b^(1/2)*(c + d*
x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(3/2)*(a*d - b*c)^(5/2))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

## 3.1279 $\int \frac{\sqrt{c+dx}}{(a+bx)^5} \, dx$

**Optimal**. Leaf size=182

$$\frac{5d^4 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{3/2}(bc-ad)^{7/2}} - \frac{5d^3\sqrt{c+dx}}{64b(a+bx)(bc-ad)^3} + \frac{5d^2\sqrt{c+dx}}{96b(a+bx)^2(bc-ad)^2} - \frac{d\sqrt{c+dx}}{24b(a+bx)^3(bc-ad)} - \frac{\sqrt{c+dx}}{4b(a+bx)^4}$$

**Rubi [A]** time = 0.12, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$\frac{5d^4 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{3/2}(bc-ad)^{7/2}} - \frac{5d^3\sqrt{c+dx}}{64b(a+bx)(bc-ad)^3} + \frac{5d^2\sqrt{c+dx}}{96b(a+bx)^2(bc-ad)^2} - \frac{d\sqrt{c+dx}}{24b(a+bx)^3(bc-ad)} - \frac{\sqrt{c+dx}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] -Sqrt[c + d*x]/(4*b*(a + b*x)^4) - (d*Sqrt[c + d*x])/(24*b*(b*c - a*d)*(a +
 b*x)^3) + (5*d^2*Sqrt[c + d*x])/(96*b*(b*c - a*d)^2*(a + b*x)^2) - (5*d^3*
Sqrt[c + d*x])/(64*b*(b*c - a*d)^3*(a + b*x)) + (5*d^4*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x])/Sqrt[b*c - a*d]])/(64*b^(3/2)*(b*c - a*d)^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx = -\frac{\sqrt{c+dx}}{4b(a+bx)^4}+\frac{d\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{8b}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}-\frac{\left(5d^2\right)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{48b(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}+\frac{\left(5d^3\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx)}-$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx)}-$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx)}+$$

**Mathematica [C]** time = 0.02, size = 52, normalized size = 0.29

$$\frac{2d^4(c+dx)^{3/2}\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] (2*d^4*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^5)
```

**IntegrateAlgebraic [A]** time = 1.07, size = 226, normalized size = 1.24

$$-\frac{d^4\sqrt{c+dx}\left(-15a^3d^3+73a^2bd^2(c+dx)+45a^2bcd^2-45ab^2c^2d+55ab^2d(c+dx)^2-146ab^2cd(c+dx)+15b^3c^3+73b^3c^2(c+dx)+15b^3(c+dx)^3-55b^3c(c+dx)^2\right)}{192b(bc-ad)^3(-ad-b(c+dx)+bc)^4}-\frac{5d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{64b^{3/2}(ad-bc)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] -1/192*(d^4*Sqrt[c + d*x]*(15*b^3*c^3 - 45*a*b^2*c^2*d + 45*a^2*b*c*d^2 - 1
5*a^3*d^3 + 73*b^3*c^2*(c + d*x) - 146*a*b^2*c*d*(c + d*x) + 73*a^2*b*d^2*(
c + d*x) - 55*b^3*c*(c + d*x)^2 + 55*a*b^2*d*(c + d*x)^2 + 15*b^3*(c + d*x)
^3))/(b*(b*c - a*d)^3*(b*c - a*d - b*(c + d*x))^4) - (5*d^4*ArcTan[(Sqrt[b]
*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(64*b^(3/2)*(-(b*c) + a*d)
^(7/2))
```

**fricas [B]** time = 0.89, size = 1176, normalized size = 6.46

$$\left[-\frac{15\left(b^4d^4x^4+4ab^3d^4x^3+6a^2b^2d^4x^2+4a^3bd^4x+a^4d^4\right)\sqrt{b^2c-abd}\log\left(\frac{bdx+2bc-ad-2\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)+2\left(48b^5c^4-184ab^4c^3d+254a^2b^3c^2d^2-133a^3b^2cd^3+15a^4bd^4+15\left(b^5cd^3-ab^4d^4\right)x^3-5\left(2b^5c^2d^2-13ab^4cd^3+11a^2b^3d^4\right)x^2+\left(8b^5c^3d-44ab^4c^2d^2+109a^2b^3cd^3-73a^3b^2d^4\right)x\right)\sqrt{dx+c}}{384\left(a^4b^6c^4-4a^5b^5c^3d+6a^6b^4c^2d^2-4a^7b^3cd^3+a^8b^2d^4+\left(b^{10}c^4-4ab^9c^3d+6a^2b^8c^2d^2-4a^3b^7cd^3+a^4b^6d^4\right)x^4+4\left(ab^9c^4-4a^2b^8c^3d+6a^3b^7c^2d^2-4a^4b^6cd^3+a^5b^5d^4\right)x^3+6\left(a^2b^8c^4-4a^3b^7c^3d+6a^4b^6c^2d^2-4a^5b^5cd^3+a^6b^4d^4\right)x^2+4\left(a^3b^7c^4-4a^4b^6c^3d+6a^5b^5c^2d^2-4a^6b^4cd^3+a^7b^3d^4\right)x\right)},-\frac{15\left(b^4d^4x^4+4ab^3d^4x^3+6a^2b^2d^4x^2+4a^3bd^4x+a^4d^4\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{-b^2c+abd}\sqrt{dx+c}}{bdx+bc}\right)+\left(48b^5c^4-184ab^4c^3d+254a^2b^3c^2d^2-133a^3b^2cd^3+15a^4bd^4+15\left(b^5cd^3-ab^4d^4\right)x^3-5\left(2b^5c^2d^2-13ab^4cd^3+11a^2b^3d^4\right)x^2+\left(8b^5c^3d-44ab^4c^2d^2+109a^2b^3cd^3-73a^3b^2d^4\right)x\right)\sqrt{dx+c}}{192\left(a^4b^6c^4-4a^5b^5c^3d+6a^6b^4c^2d^2-4a^7b^3cd^3+a^8b^2d^4+\left(b^{10}c^4-4ab^9c^3d+6a^2b^8c^2d^2-4a^3b^7cd^3+a^4b^6d^4\right)x^4+4\left(ab^9c^4-4a^2b^8c^3d+6a^3b^7c^2d^2-4a^4b^6cd^3+a^5b^5d^4\right)x^3+6\left(a^2b^8c^4-4a^3b^7c^3d+6a^4b^6c^2d^2-4a^5b^5cd^3+a^6b^4d^4\right)x^2+4\left(a^3b^7c^4-4a^4b^6c^3d+6a^5b^5c^2d^2-4a^6b^4cd^3+a^7b^3d^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(48*b^5*c^4 - 184*a*b^4*c^3*d + 254*
a^2*b^3*c^2*d^2 - 133*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*
d^4)*x^3 - 5*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (8*b^5
*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqrt(d*x
 + c))/(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3
 + a^8*b^2*d^4 + (b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*
c*d^3 + a^4*b^6*d^4)*x^4 + 4*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d
^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*x^3 + 6*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d
+ 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^2 + 4*(a^3*b^7*c^4 -
 4*a^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x), -
1/192*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*
x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)
/(b*d*x + b*c)) + (48*b^5*c^4 - 184*a*b^4*c^3*d + 254*a^2*b^3*c^2*d^2 - 133
*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4)*x^3 - 5*(2*b^5*c
^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (8*b^5*c^3*d - 44*a*b^4*c^2
*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^4 -
 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4 + (b^1
0*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*
```

```
x^4 + 4*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3
+ a^5*b^5*d^4)*x^3 + 6*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 -
 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^2 + 4*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*
a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x)]
```

**giac [B]** time = 1.39, size = 311, normalized size = 1.71

$$-\frac{5\,d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{64\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)\sqrt{-b^2c+abd}}-\frac{15\,(dx+c)^{\frac{7}{2}}b^3d^4-55\,(dx+c)^{\frac{5}{2}}b^3cd^4+73\,(dx+c)^{\frac{3}{2}}b^3c^2d^4+15\,\sqrt{dx+c}\,b^3c^3d^4+55\,(dx+c)^{\frac{5}{2}}ab^2d^5-146\,(dx+c)^{\frac{3}{2}}ab^2cd^5-45\,\sqrt{dx+c}\,ab^2c^2d^5+73\,(dx+c)^{\frac{3}{2}}a^2bd^6+45\,\sqrt{dx+c}\,a^2bcd^6-15\,\sqrt{dx+c}\,a^3d^7}{192\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)\left((dx+c)b-bc+ad\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -5/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^3 - 3*a*b^3*
c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(-b^2*c + a*b*d)) - 1/192*(15*(d*x
 + c)^(7/2)*b^3*d^4 - 55*(d*x + c)^(5/2)*b^3*c*d^4 + 73*(d*x + c)^(3/2)*b^3
*c^2*d^4 + 15*sqrt(d*x + c)*b^3*c^3*d^4 + 55*(d*x + c)^(5/2)*a*b^2*d^5 - 14
6*(d*x + c)^(3/2)*a*b^2*c*d^5 - 45*sqrt(d*x + c)*a*b^2*c^2*d^5 + 73*(d*x +
c)^(3/2)*a^2*b*d^6 + 45*sqrt(d*x + c)*a^2*b*c*d^6 - 15*sqrt(d*x + c)*a^3*d^
7)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*((d*x + c)*b -
b*c + a*d)^4)
```

**maple [A]** time = 0.02, size = 248, normalized size = 1.36

$$\frac{5\,(dx+c)^{\frac{7}{2}}\,b^2d^4}{64\,(bdx+ad)^4\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{55\,(dx+c)^{\frac{5}{2}}\,b\,d^4}{192\,(bdx+ad)^4\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{5d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{64\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\sqrt{(ad-bc)\,b}\;b}+\frac{73\,(dx+c)^{\frac{3}{2}}\,d^4}{192\,(bdx+ad)^4\,(ad-bc)}-\frac{5\sqrt{dx+c}\,d^4}{64\,(bdx+ad)^4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^5,x)
```

```
[Out] 5/64*d^4/(b*d*x+a*d)^4*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(d
*x+c)^(7/2)+55/192*d^4/(b*d*x+a*d)^4*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^
(5/2)+73/192*d^4/(b*d*x+a*d)^4/(a*d-b*c)*(d*x+c)^(3/2)-5/64*d^4/(b*d*x+a*d)
^4/b*(d*x+c)^(1/2)+5/64*d^4/b/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.22, size = 297, normalized size = 1.63

$$\frac{\frac{73\,d^4\,{\left(c+d\,x\right)}^{3/2}}{192\,\left(a\,d-b\,c\right)}-\frac{5\,d^4\,\sqrt{c+d\,x}}{64\,b}+\frac{5\,b^2\,d^4\,{\left(c+d\,x\right)}^{7/2}}{64\,{\left(a\,d-b\,c\right)}^3}+\frac{55\,b\,d^4\,{\left(c+d\,x\right)}^{5/2}}{192\,{\left(a\,d-b\,c\right)}^2}}{b^4\,{\left(c+d\,x\right)}^4-\left(4\,b^4\,c-4\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^3-\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)+a^4\,d^4+b^4\,c^4+{\left(c+d\,x\right)}^2\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}+\frac{5\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^5,x)
```

```
[Out] ((73*d^4*(c + d*x)^(3/2))/(192*(a*d - b*c)) - (5*d^4*(c + d*x)^(1/2))/(64*b
) + (5*b^2*d^4*(c + d*x)^(7/2))/(64*(a*d - b*c)^3) + (55*b*d^4*(c + d*x)^(5
/2))/(192*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c - 4*a*b^3*d)*(c + d*x
)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*
d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*
c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) + (5*d^4*atan((b^
(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(3/2)*(a*d - b*c)^(7/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

## 3.1280 $\int \frac{\sqrt{c+dx}}{(a+bx)^6} dx$

**Optimal**. Leaf size=218

$$-\frac{7d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{3/2}(bc-ad)^{9/2}}+\frac{7d^4\sqrt{c+dx}}{128b(a+bx)(bc-ad)^4}-\frac{7d^3\sqrt{c+dx}}{192b(a+bx)^2(bc-ad)^3}+\frac{7d^2\sqrt{c+dx}}{240b(a+bx)^3(bc-ad)^2}-\frac{d\sqrt{c+dx}}{40b(a+}$$

**Rubi** [A] time = 0.15, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{7d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{3/2}(bc-ad)^{9/2}}+\frac{7d^4\sqrt{c+dx}}{128b(a+bx)(bc-ad)^4}-\frac{7d^3\sqrt{c+dx}}{192b(a+bx)^2(bc-ad)^3}+\frac{7d^2\sqrt{c+dx}}{240b(a+bx)^3(bc-ad)^2}-\frac{d\sqrt{c+dx}}{40b(a+bx)^4(bc-ad)}-\frac{\sqrt{c+dx}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^6,x]
```

```
[Out] -Sqrt[c + d*x]/(5*b*(a + b*x)^5) - (d*Sqrt[c + d*x])/(40*b*(b*c - a*d)*(a +
 b*x)^4) + (7*d^2*Sqrt[c + d*x])/(240*b*(b*c - a*d)^2*(a + b*x)^3) - (7*d^3
*Sqrt[c + d*x])/(192*b*(b*c - a*d)^3*(a + b*x)^2) + (7*d^4*Sqrt[c + d*x])/(
128*b*(b*c - a*d)^4*(a + b*x)) - (7*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqr
t[b*c - a*d]])/(128*b^(3/2)*(b*c - a*d)^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^6}\,dx = -\frac{\sqrt{c+dx}}{5b(a+bx)^5} + \frac{d\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx}{10b}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} - \frac{\left(7d^2\right)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{80b(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} + \frac{\left(7d^3\right)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{96b(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2} -$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2} +$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2} +$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2} +$$

**Mathematica [C]** time = 0.02, size = 52, normalized size = 0.24

$$\frac{2d^5(c+dx)^{3/2}\,{}_2F_1\left(\frac{3}{2},6;\frac{5}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^6,x]
```

```
[Out] (2*d^5*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 6, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^6)
```

**IntegrateAlgebraic [A]** time = 1.52, size = 317, normalized size = 1.45

$$\frac{d^5\sqrt{c+dx}\left(105a^4d^4-790a^3bd^3(c+dx)-420a^3bcd^3+630a^2b^2c^2d^2-896a^2b^2d^2(c+dx)^2+2370a^2b^2cd^2(c+dx)-420ab^3c^3d-2370ab^3c^2d(c+dx)-490ab^3d(c+dx)^3+1792ab^3cd(c+dx)^2+105b^4c^4+790b^4c^3(c+dx)-896b^4c^2(c+dx)^2-105b^4(c+dx)^4+490b^4c(c+dx)^3\right)}{1920b(bc-ad)^4(-ad-b(c+dx)+bc)^5}-\frac{7d^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{128b^{3/2}(bc-ad)^4\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^6,x]
```

```
[Out] (d^5*Sqrt[c + d*x]*(105*b^4*c^4 - 420*a*b^3*c^3*d + 630*a^2*b^2*c^2*d^2 - 4
20*a^3*b*c*d^3 + 105*a^4*d^4 + 790*b^4*c^3*(c + d*x) - 2370*a*b^3*c^2*d*(c
+ d*x) + 2370*a^2*b^2*c*d^2*(c + d*x) - 790*a^3*b*d^3*(c + d*x) - 896*b^4*c
^2*(c + d*x)^2 + 1792*a*b^3*c*d*(c + d*x)^2 - 896*a^2*b^2*d^2*(c + d*x)^2 +
 490*b^4*c*(c + d*x)^3 - 490*a*b^3*d*(c + d*x)^3 - 105*b^4*(c + d*x)^4))/(1
920*b*(b*c - a*d)^4*(b*c - a*d - b*(c + d*x))^5) - (7*d^5*ArcTan[(Sqrt[b]*S
qrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(128*b^(3/2)*(b*c - a*d)^4*S
qrt[-(b*c) + a*d])
```

**fricas [B]** time = 1.34, size = 1673, normalized size = 7.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [1/3840*(105*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b
^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*
c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(384*b^6*c^5
- 1872*a*b^5*c^4*d + 3592*a^2*b^4*c^3*d^2 - 3314*a^3*b^3*c^2*d^3 + 1315*a^4
*b^2*c*d^4 - 105*a^5*b*d^5 - 105*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 70*(b^6*c^2*
d^3 - 8*a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 - 14*(4*b^6*c^3*d^2 - 27*a*b^5*c^2
*d^3 + 87*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(24*b^6*c^4*d - 152*a*b^5
*c^3*d^2 + 417*a^2*b^4*c^2*d^3 - 684*a^3*b^3*c*d^4 + 395*a^4*b^2*d^5)*x)*sq
rt(d*x + c))/(a^5*b^7*c^5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b
^4*c^2*d^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*d^5 + (b^12*c^5 - 5*a*b^11*c^4*d +
10*a^2*b^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*x
^5 + 5*(a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2
*d^3 + 5*a^5*b^7*c*d^4 - a^6*b^6*d^5)*x^4 + 10*(a^2*b^10*c^5 - 5*a^3*b^9*c^
4*d + 10*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d
^5)*x^3 + 10*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b
^6*c^2*d^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*x^2 + 5*(a^4*b^8*c^5 - 5*a^5*b^
7*c^4*d + 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b
^3*d^5)*x), 1/1920*(105*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3
```

```
 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(-b^2*c + a*b*d)*arcta
n(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (384*b^6*c^5 - 1872*a
*b^5*c^4*d + 3592*a^2*b^4*c^3*d^2 - 3314*a^3*b^3*c^2*d^3 + 1315*a^4*b^2*c*d
^4 - 105*a^5*b*d^5 - 105*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 70*(b^6*c^2*d^3 - 8*
a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 - 14*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 8
7*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(24*b^6*c^4*d - 152*a*b^5*c^3*d^2
 + 417*a^2*b^4*c^2*d^3 - 684*a^3*b^3*c*d^4 + 395*a^4*b^2*d^5)*x)*sqrt(d*x +
 c))/(a^5*b^7*c^5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b^4*c^2*d
^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*d^5 + (b^12*c^5 - 5*a*b^11*c^4*d + 10*a^2*b
^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*x^5 + 5*(
a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2*d^3 + 5
*a^5*b^7*c*d^4 - a^6*b^6*d^5)*x^4 + 10*(a^2*b^10*c^5 - 5*a^3*b^9*c^4*d + 10
*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d^5)*x^3
+ 10*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b^6*c^2*d
^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*x^2 + 5*(a^4*b^8*c^5 - 5*a^5*b^7*c^4*d
+ 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b^3*d^5)*
x)]
```

**giac [B]** time = 1.47, size = 432, normalized size = 1.98

$$\frac{7\,d^{5}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{128\,\left(b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4}\right)\sqrt{-b^{2}c+abd}}+\frac{105\,(dx+c)^{\frac{9}{2}}b^{4}d^{5}-490\,(dx+c)^{\frac{7}{2}}b^{4}cd^{5}+896\,(dx+c)^{\frac{5}{2}}b^{4}c^{2}d^{5}-790\,(dx+c)^{\frac{3}{2}}b^{4}c^{3}d^{5}-105\,\sqrt{dx+c}\,b^{4}c^{4}d^{5}+490\,(dx+c)^{\frac{7}{2}}ab^{3}d^{6}-1792\,(dx+c)^{\frac{5}{2}}ab^{3}cd^{6}+2370\,(dx+c)^{\frac{3}{2}}ab^{3}c^{2}d^{6}+420\,\sqrt{dx+c}\,ab^{3}c^{3}d^{6}+896\,(dx+c)^{\frac{5}{2}}a^{2}b^{2}d^{7}-2370\,(dx+c)^{\frac{3}{2}}a^{2}b^{2}cd^{7}-630\,\sqrt{dx+c}\,a^{2}b^{2}c^{2}d^{7}+790\,(dx+c)^{\frac{3}{2}}a^{3}bd^{8}+420\,\sqrt{dx+c}\,a^{3}bcd^{8}-105\,\sqrt{dx+c}\,a^{4}d^{9}}{1920\,\left(b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4}\right)\left((dx+c)b-bc+ad\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 7/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(-b^2*c + a*b*
d)) + 1/1920*(105*(d*x + c)^(9/2)*b^4*d^5 - 490*(d*x + c)^(7/2)*b^4*c*d^5 +
 896*(d*x + c)^(5/2)*b^4*c^2*d^5 - 790*(d*x + c)^(3/2)*b^4*c^3*d^5 - 105*sq
rt(d*x + c)*b^4*c^4*d^5 + 490*(d*x + c)^(7/2)*a*b^3*d^6 - 1792*(d*x + c)^(5
/2)*a*b^3*c*d^6 + 2370*(d*x + c)^(3/2)*a*b^3*c^2*d^6 + 420*sqrt(d*x + c)*a*
b^3*c^3*d^6 + 896*(d*x + c)^(5/2)*a^2*b^2*d^7 - 2370*(d*x + c)^(3/2)*a^2*b^
2*c*d^7 - 630*sqrt(d*x + c)*a^2*b^2*c^2*d^7 + 790*(d*x + c)^(3/2)*a^3*b*d^8
 + 420*sqrt(d*x + c)*a^3*b*c*d^8 - 105*sqrt(d*x + c)*a^4*d^9)/((b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*((d*x + c)*
b - b*c + a*d)^5)
```

**maple [A]** time = 0.02, size = 337, normalized size = 1.55

$$\frac{7\,(dx+c)^{\frac{9}{2}}\,b^{3}d^{5}}{128\,(bdx+ad)^{5}\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)}+\frac{49\,(dx+c)^{\frac{7}{2}}\,b^{2}d^{5}}{192\,(bdx+ad)^{5}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{7\,(dx+c)^{\frac{5}{2}}\,b\,d^{5}}{15\,(bdx+ad)^{5}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}+\frac{7d^{5}\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{128\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)\sqrt{(ad-bc)b}\;b}+\frac{79\,(dx+c)^{\frac{3}{2}}\,d^{5}}{192\,(bdx+ad)^{5}\,(ad-bc)}-\frac{7\sqrt{dx+c}\;d^{5}}{128\,(bdx+ad)^{5}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^6,x)
```

```
[Out] 7/128*d^5/(b*d*x+a*d)^5*b^3/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^
3*c^3*d+b^4*c^4)*(d*x+c)^(9/2)+49/192*d^5/(b*d*x+a*d)^5*b^2/(a^3*d^3-3*a^2*
b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(d*x+c)^(7/2)+7/15*d^5/(b*d*x+a*d)^5*b/(a^2*
d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+79/192*d^5/(b*d*x+a*d)^5/(a*d-b*c)*(d*
x+c)^(3/2)-7/128*d^5/(b*d*x+a*d)^5/b*(d*x+c)^(1/2)+7/128*d^5/b/(a^4*d^4-4*a
^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/((a*d-b*c)*b)^(1/2)*arc
tan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 0.49, size = 401, normalized size = 1.84

$$\frac{\frac{79\,d^5\,{\left(c+d\,x\right)}^{3/2}}{192\,\left(a\,d-b\,c\right)}-\frac{7\,d^5\,\sqrt{c+d\,x}}{128\,b}+\frac{49\,b^2\,d^5\,{\left(c+d\,x\right)}^{7/2}}{192\,{\left(a\,d-b\,c\right)}^3}+\frac{7\,b^3\,d^5\,{\left(c+d\,x\right)}^{9/2}}{128\,{\left(a\,d-b\,c\right)}^4}+\frac{7\,b\,d^5\,{\left(c+d\,x\right)}^{5/2}}{15\,{\left(a\,d-b\,c\right)}^2}}{b^5\,{\left(c+d\,x\right)}^5-{\left(c+d\,x\right)}^2\,\left(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3\right)-\left(5\,b^5\,c-5\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^4+a^5\,d^5-b^5\,c^5+{\left(c+d\,x\right)}^3\,\left(10\,a^2\,b^3\,d^2-20\,a\,b^4\,c\,d+10\,b^5\,c^2\right)+\left(c+d\,x\right)\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)-10\,a^2\,b^3\,c^3\,d^2+10\,a^3\,b^2\,c^2\,d^3+5\,a\,b^4\,c^4\,d-5\,a^4\,b\,c\,d^4}+\frac{7\,d^5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^6,x)
```

```
[Out] ((79*d^5*(c + d*x)^(3/2))/(192*(a*d - b*c)) - (7*d^5*(c + d*x)^(1/2))/(128*
b) + (49*b^2*d^5*(c + d*x)^(7/2))/(192*(a*d - b*c)^3) + (7*b^3*d^5*(c + d*x
)^(9/2))/(128*(a*d - b*c)^4) + (7*b*d^5*(c + d*x)^(5/2))/(15*(a*d - b*c)^2)
)/(b^5*(c + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*
c*d^2 - 30*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(c + d*x)^4 + a^5*d^5 - b^5
*c^5 + (c + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 - 20*a*b^4*c*d) + (c + d*x)
*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^
4*c^3*d) - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*
b*c*d^4) + (7*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b
^(3/2)*(a*d - b*c)^(9/2))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

## 3.1281 $\int (a+bx)^5 (c+dx)^{3/2}\,dx$

**Optimal**. Leaf size=158

$$-\frac{10b^4(c+dx)^{13/2}(bc-ad)}{13d^6}+\frac{20b^3(c+dx)^{11/2}(bc-ad)^2}{11d^6}-\frac{20b^2(c+dx)^{9/2}(bc-ad)^3}{9d^6}+\frac{10b(c+dx)^{7/2}(bc-ad)^4}{7d^6}-\frac{2(c$$

**Rubi [A]** time = 0.05, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{13/2}(bc-ad)}{13d^6}+\frac{20b^3(c+dx)^{11/2}(bc-ad)^2}{11d^6}-\frac{20b^2(c+dx)^{9/2}(bc-ad)^3}{9d^6}+\frac{10b(c+dx)^{7/2}(bc-ad)^4}{7d^6}-\frac{2(c+dx)^{5/2}(bc-ad)^5}{5d^6}+\frac{2b^5(c+dx)^{15/2}}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(5/2))/(5*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(7/2))/(7*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(9/2))/(9*d^6) + (20*b^3*(
b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(13
/2))/(13*d^6) + (2*b^5*(c + d*x)^(15/2))/(15*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^{3/2}\,dx = \int \left(\frac{(-bc+ad)^5(c+dx)^{3/2}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{5/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{7/2}}{d^5}+\frac{1(}{}\right.$$

$$=-\frac{2(bc-ad)^5(c+dx)^{5/2}}{5d^6}+\frac{10b(bc-ad)^4(c+dx)^{7/2}}{7d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{9/2}}{9d^6}+\frac{20b}{}$$

**Mathematica [A]** time = 0.15, size = 123, normalized size = 0.78

$$\frac{2(c+dx)^{5/2}\left(-17325b^4(c+dx)^4(bc-ad)+40950b^3(c+dx)^3(bc-ad)^2-50050b^2(c+dx)^2(bc-ad)^3+32175b(c+dx)(bc-ad)^4-9009(bc-ad)^5+3003b^5(c+dx)^5\right)}{45045d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-9009*(b*c - a*d)^5 + 32175*b*(b*c - a*d)^4*(c + d*x) -
 50050*b^2*(b*c - a*d)^3*(c + d*x)^2 + 40950*b^3*(b*c - a*d)^2*(c + d*x)^3
- 17325*b^4*(b*c - a*d)*(c + d*x)^4 + 3003*b^5*(c + d*x)^5))/(45045*d^6)
```

**IntegrateAlgebraic** [A] time = 0.10, size = 315, normalized size = 1.99

$$\frac{2(c+dx)^{5/2}\left(9009a^5d^5+32175a^4bd^4(c+dx)-45045a^4bcd^4+90090a^3b^2c^2d^3+50050a^3b^2d^3(c+dx)^2-128700a^3b^2cd^3(c+dx)-90090a^2b^3c^3d^2+193050a^2b^3c^2d^2(c+dx)+40950a^2b^3d^2(c+dx)^3-150150a^2b^3cd^2(c+dx)^2+45045ab^4c^4d-128700ab^4c^3d(c+dx)+150150ab^4c^2d(c+dx)^2+17325ab^4d(c+dx)^4-81900ab^4cd(c+dx)^3-9009b^5c^5+32175b^5c^4(c+dx)-50050b^5c^3(c+dx)^2+40950b^5c^2(c+dx)^3+3003b^5(c+dx)^5-17325b^5c(c+dx)^4\right)}{45045d^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-9009*b^5*c^5 + 45045*a*b^4*c^4*d - 90090*a^2*b^3*c^3*d
^2 + 90090*a^3*b^2*c^2*d^3 - 45045*a^4*b*c*d^4 + 9009*a^5*d^5 + 32175*b^5*c
^4*(c + d*x) - 128700*a*b^4*c^3*d*(c + d*x) + 193050*a^2*b^3*c^2*d^2*(c + d
*x) - 128700*a^3*b^2*c*d^3*(c + d*x) + 32175*a^4*b*d^4*(c + d*x) - 50050*b^
5*c^3*(c + d*x)^2 + 150150*a*b^4*c^2*d*(c + d*x)^2 - 150150*a^2*b^3*c*d^2*(
c + d*x)^2 + 50050*a^3*b^2*d^3*(c + d*x)^2 + 40950*b^5*c^2*(c + d*x)^3 - 81
900*a*b^4*c*d*(c + d*x)^3 + 40950*a^2*b^3*d^2*(c + d*x)^3 - 17325*b^5*c*(c
+ d*x)^4 + 17325*a*b^4*d*(c + d*x)^4 + 3003*b^5*(c + d*x)^5))/(45045*d^6)
```

**fricas** [B] time = 1.37, size = 418, normalized size = 2.65

$$\frac{2\left(3003\,b^5d^7x^7-256\,b^5c^7+1920\,ab^4c^6d-6240\,a^2b^3c^5d^2+11440\,a^3b^2c^4d^3-12870\,a^4bc^3d^4+9009\,a^5c^2d^5+231\left(16\,b^5cd^6+75\,ab^4d^7\right)x^6+63\left(b^5c^2d^5+350\,ab^4cd^6+650\,a^2b^3d^7\right)x^5-35\left(2\,b^5c^3d^4-15\,ab^4c^2d^5-1560\,a^2b^3cd^6-1430\,a^3b^2d^7\right)x^4+5\left(16\,b^5c^4d^3-120\,ab^4c^3d^4+390\,a^2b^3c^2d^5+14300\,a^3b^2cd^6+6435\,a^4bd^7\right)x^3-3\left(32\,b^5c^5d^2-240\,ab^4c^4d^3+780\,a^2b^3c^3d^4-1430\,a^3b^2c^2d^5-17160\,a^4bcd^6-3003\,a^5d^7\right)x^2+\left(128\,b^5c^6d-960\,ab^4c^5d^2+3120\,a^2b^3c^4d^3-5720\,a^3b^2c^3d^4+6435\,a^4bc^2d^5+18018\,a^5cd^6\right)x\right)\sqrt{dx+c}}{45045\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^5*d^7*x^7 - 256*b^5*c^7 + 1920*a*b^4*c^6*d - 6240*a^2*b^3*c
^5*d^2 + 11440*a^3*b^2*c^4*d^3 - 12870*a^4*b*c^3*d^4 + 9009*a^5*c^2*d^5 + 2
31*(16*b^5*c*d^6 + 75*a*b^4*d^7)*x^6 + 63*(b^5*c^2*d^5 + 350*a*b^4*c*d^6 +
650*a^2*b^3*d^7)*x^5 - 35*(2*b^5*c^3*d^4 - 15*a*b^4*c^2*d^5 - 1560*a^2*b^3*
c*d^6 - 1430*a^3*b^2*d^7)*x^4 + 5*(16*b^5*c^4*d^3 - 120*a*b^4*c^3*d^4 + 390
*a^2*b^3*c^2*d^5 + 14300*a^3*b^2*c*d^6 + 6435*a^4*b*d^7)*x^3 - 3*(32*b^5*c^
5*d^2 - 240*a*b^4*c^4*d^3 + 780*a^2*b^3*c^3*d^4 - 1430*a^3*b^2*c^2*d^5 - 17
160*a^4*b*c*d^6 - 3003*a^5*d^7)*x^2 + (128*b^5*c^6*d - 960*a*b^4*c^5*d^2 +
3120*a^2*b^3*c^4*d^3 - 5720*a^3*b^2*c^3*d^4 + 6435*a^4*b*c^2*d^5 + 18018*a^
5*c*d^6)*x)*sqrt(d*x + c)/d^6
```

**giac** [B] time = 1.53, size = 1084, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(d*x + c)*a^5*c^2 + 30030*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^5*c + 75075*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c^2/d + 3
003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^5 +
 30030*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^
3*b^2*c^2/d^2 + 30030*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
*x + c)*c^2)*a^4*b*c/d + 12870*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3*c^2/d^3 + 25740*(5*(
d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*
x + c)*c^3)*a^3*b^2*c/d^2 + 6435*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c
+ 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^4*b/d + 715*(35*(d*x + c
)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(
3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^4*c^2/d^4 + 2860*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a^2*b^3*c/d^3 + 1430*(35*(d*x + c)^(9/2) - 180*(d
*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*s
qrt(d*x + c)*c^4)*a^3*b^2/d^2 + 65*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/
2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^
(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5*c^2/d^5 + 650*(63*(d*x + c)^(11/2) -
 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3
 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^4*c/d^4 + 650*(63*
(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(
d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^2*
b^3/d^3 + 30*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x +
c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(
d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5*c/d^5 + 75*(231*(d*x + c)^
(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c
)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sq
rt(d*x + c)*c^6)*a*b^4/d^4 + 7*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2
)*c + 12285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x +
 c)^(7/2)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 643
5*sqrt(d*x + c)*c^7)*b^5/d^5)/d
```

**maple [B]** time = 0.01, size = 273, normalized size = 1.73

$$\frac{2 (dx+c)^{\frac{5}{2}} \left(3003 b^5 x^5 d^5 + 17325 a b^4 d^5 x^4 - 2310 b^5 c d^4 x^4 + 40950 a^2 b^3 d^5 x^3 - 12600 a b^4 c d^4 x^3 + 1680 b^5 c^2 d^3 x^3 + 50050 a^3 b^2 d^5 x^2 - 27300 a^2 b^3 c d^4 x^2 + 8400 a b^4 c^2 d^3 x^2 - 1120 b^5 c^3 d^2 x^2 + 32175 a^4 b d^5 x - 28600 a^3 b^2 c d^4 x + 15600 a^2 b^3 c^2 d^3 x - 4800 a b^4 c^3 d^2 x + 640 b^5 c^4 d x + 9009 a^5 d^5 - 12870 a^4 b c d^4 + 11440 a^3 b^2 c^2 d^3 - 6240 a^2 b^3 c^3 d^2 + 1920 a b^4 c^4 d - 256 b^5 c^5\right)}{45045 d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(3/2),x)
```

```
[Out] 2/45045*(d*x+c)^(5/2)*(3003*b^5*d^5*x^5+17325*a*b^4*d^5*x^4-2310*b^5*c*d^4*
x^4+40950*a^2*b^3*d^5*x^3-12600*a*b^4*c*d^4*x^3+1680*b^5*c^2*d^3*x^3+50050*
a^3*b^2*d^5*x^2-27300*a^2*b^3*c*d^4*x^2+8400*a*b^4*c^2*d^3*x^2-1120*b^5*c^3
*d^2*x^2+32175*a^4*b*d^5*x-28600*a^3*b^2*c*d^4*x+15600*a^2*b^3*c^2*d^3*x-48
00*a*b^4*c^3*d^2*x+640*b^5*c^4*d*x+9009*a^5*d^5-12870*a^4*b*c*d^4+11440*a^3
*b^2*c^2*d^3-6240*a^2*b^3*c^3*d^2+1920*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

**maxima** [A] time = 1.35, size = 259, normalized size = 1.64

$$\frac{2\left(3003\,(dx+c)^{\frac{15}{2}}b^5-17325\left(b^5c-ab^4d\right)(dx+c)^{\frac{13}{2}}+40950\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)(dx+c)^{\frac{11}{2}}-50050\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)(dx+c)^{\frac{9}{2}}+32175\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)(dx+c)^{\frac{7}{2}}-9009\left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5\right)(dx+c)^{\frac{5}{2}}\right)}{45045\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(d*x + c)^(15/2)*b^5 - 17325*(b^5*c - a*b^4*d)*(d*x + c)^(13/
2) + 40950*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(11/2) - 50050*(
b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(9/2) +
32175*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*
b*d^4)*(d*x + c)^(7/2) - 9009*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2
 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(5/2))/d^6
```

**mupad** [B] time = 0.24, size = 137, normalized size = 0.87

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{15/2}}{15\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{13/2}}{13\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,{\left(c+d\,x\right)}^{5/2}}{5\,d^6}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(3/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(15/2))/(15*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(13/
2))/(13*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(5/2))/(5*d^6) + (20*b^2*(a*d - b
*c)^3*(c + d*x)^(9/2))/(9*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(11/2))/(1
1*d^6) + (10*b*(a*d - b*c)^4*(c + d*x)^(7/2))/(7*d^6)
```

**sympy** [A] time = 26.42, size = 763, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(3/2),x)
```

```
[Out] a**5*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**5*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 10*a**4*b*c*(-c*(c
 + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 10*a**4*b*(c**2*(c + d*x)**(3
/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 20*a**3*b**2*c*
(c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**
3 + 20*a**3*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*
c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 20*a**2*b**3*c*(-c**3*(c
+ d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c +
 d*x)**(9/2)/9)/d**4 + 20*a**2*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c +
d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d
*x)**(11/2)/11)/d**4 + 10*a*b**4*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d
*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*
```

```
x)**(11/2)/11)/d**5 + 10*a*b**4*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)*
*(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c +
 d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5 + 2*b**5*c*(-c**5*(c + d*x)**
(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c +
 d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**6 + 2*
b**5*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4*(c + d*
x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2)/11 - 6
*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**6
```

## 3.1282 $\int (a+bx)^4(c+dx)^{3/2}\,dx$

**Optimal**. Leaf size=129

$$-\frac{8b^3(c+dx)^{11/2}(bc-ad)}{11d^5}+\frac{4b^2(c+dx)^{9/2}(bc-ad)^2}{3d^5}-\frac{8b(c+dx)^{7/2}(bc-ad)^3}{7d^5}+\frac{2(c+dx)^{5/2}(bc-ad)^4}{5d^5}+\frac{2b^4(c+d\ldots}{13d^5}$$

**Rubi** [A] time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{11/2}(bc-ad)}{11d^5}+\frac{4b^2(c+dx)^{9/2}(bc-ad)^2}{3d^5}-\frac{8b(c+dx)^{7/2}(bc-ad)^3}{7d^5}+\frac{2(c+dx)^{5/2}(bc-ad)^4}{5d^5}+\frac{2b^4(c+dx)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(5/2))/(5*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(7
/2))/(7*d^5) + (4*b^2*(b*c - a*d)^2*(c + d*x)^(9/2))/(3*d^5) - (8*b^3*(b*c
- a*d)*(c + d*x)^(11/2))/(11*d^5) + (2*b^4*(c + d*x)^(13/2))/(13*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4(c+dx)^{3/2}\,dx=\int\left(\frac{(-bc+ad)^4(c+dx)^{3/2}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{5/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{7/2}}{d^4}-\ldots\right.$$
$$=\frac{2(bc-ad)^4(c+dx)^{5/2}}{5d^5}-\frac{8b(bc-ad)^3(c+dx)^{7/2}}{7d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{9/2}}{3d^5}-\frac{8b^3(b\ldots}{}$$

**Mathematica** [A] time = 0.10, size = 101, normalized size = 0.78

$$\frac{2(c+dx)^{5/2}\left(-5460b^3(c+dx)^3(bc-ad)+10010b^2(c+dx)^2(bc-ad)^2-8580b(c+dx)(bc-ad)^3+3003(bc-ad)^4+1155b^4(c+dx)^4\right)}{15015d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(3003*(b*c - a*d)^4 - 8580*b*(b*c - a*d)^3*(c + d*x) + 1
0010*b^2*(b*c - a*d)^2*(c + d*x)^2 - 5460*b^3*(b*c - a*d)*(c + d*x)^3 + 115
5*b^4*(c + d*x)^4))/(15015*d^5)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 213, normalized size = 1.65

$$\frac{2(c+dx)^{5/2}\left(3003a^4d^4+8580a^3bd^3(c+dx)-12012a^3bcd^3+18018a^2b^2c^2d^2+10010a^2b^2d^2(c+dx)^2-25740a^2b^2cd^2(c+dx)-12012ab^3c^3d+25740ab^3c^2d(c+dx)+5460ab^3d(c+dx)^3-20020ab^3cd(c+dx)^2+3003b^4c^4-8580b^4c^3(c+dx)+10010b^4c^2(c+dx)^2+1155b^4(c+dx)^4-5460b^4c(c+dx)^3\right)}{15015d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(3003*b^4*c^4 - 12012*a*b^3*c^3*d + 18018*a^2*b^2*c^2*d^
2 - 12012*a^3*b*c*d^3 + 3003*a^4*d^4 - 8580*b^4*c^3*(c + d*x) + 25740*a*b^3
*c^2*d*(c + d*x) - 25740*a^2*b^2*c*d^2*(c + d*x) + 8580*a^3*b*d^3*(c + d*x)
 + 10010*b^4*c^2*(c + d*x)^2 - 20020*a*b^3*c*d*(c + d*x)^2 + 10010*a^2*b^2*
d^2*(c + d*x)^2 - 5460*b^4*c*(c + d*x)^3 + 5460*a*b^3*d*(c + d*x)^3 + 1155*
b^4*(c + d*x)^4))/(15015*d^5)
```

**fricas [B]** time = 1.07, size = 311, normalized size = 2.41

$$\frac{2\left(1155\,b^4d^6x^6+128\,b^4c^6-832\,ab^3c^5d+2288\,a^2b^2c^4d^2-3432\,a^3bc^3d^3+3003\,a^4c^2d^4+210\left(7\,b^4cd^5+26\,ab^3d^6\right)x^5+35\left(b^4c^2d^4+208\,ab^3cd^5+286\,a^2b^2d^6\right)x^4-20\left(2\,b^4c^3d^3-13\,ab^3c^2d^4-715\,a^2b^2cd^5-429\,a^3bd^6\right)x^3+3\left(16\,b^4c^4d^2-104\,ab^3c^3d^3+286\,a^2b^2c^2d^4+4576\,a^3bcd^5+1001\,a^4d^6\right)x^2-2\left(32\,b^4c^5d-208\,ab^3c^4d^2+572\,a^2b^2c^3d^3-858\,a^3bc^2d^4-3003\,a^4cd^5\right)x\right)\sqrt{dx+c}}{15015\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^4*d^6*x^6 + 128*b^4*c^6 - 832*a*b^3*c^5*d + 2288*a^2*b^2*c^
4*d^2 - 3432*a^3*b*c^3*d^3 + 3003*a^4*c^2*d^4 + 210*(7*b^4*c*d^5 + 26*a*b^3
*d^6)*x^5 + 35*(b^4*c^2*d^4 + 208*a*b^3*c*d^5 + 286*a^2*b^2*d^6)*x^4 - 20*(
2*b^4*c^3*d^3 - 13*a*b^3*c^2*d^4 - 715*a^2*b^2*c*d^5 - 429*a^3*b*d^6)*x^3 +
 3*(16*b^4*c^4*d^2 - 104*a*b^3*c^3*d^3 + 286*a^2*b^2*c^2*d^4 + 4576*a^3*b*c
*d^5 + 1001*a^4*d^6)*x^2 - 2*(32*b^4*c^5*d - 208*a*b^3*c^4*d^2 + 572*a^2*b^
2*c^3*d^3 - 858*a^3*b*c^2*d^4 - 3003*a^4*c*d^5)*x)*sqrt(d*x + c)/d^5
```

**giac [B]** time = 1.40, size = 807, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(d*x + c)*a^4*c^2 + 30030*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^4*c + 60060*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c^2/d + 3
003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^4 +
 18018*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^
2*b^2*c^2/d^2 + 24024*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
```

```
*x + c)*c^2)*a^3*b*c/d + 5148*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 3
5*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3*c^2/d^3 + 15444*(5*(d*x
 + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x +
 c)*c^3)*a^2*b^2*c/d^2 + 5148*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 3
5*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b/d + 143*(35*(d*x + c)^(
9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2
)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c^2/d^4 + 1144*(35*(d*x + c)^(9/2) - 180
*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 31
5*sqrt(d*x + c)*c^4)*a*b^3*c/d^3 + 858*(35*(d*x + c)^(9/2) - 180*(d*x + c)^
(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x
+ c)*c^4)*a^2*b^2/d^2 + 130*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c +
990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c
^4 - 693*sqrt(d*x + c)*c^5)*b^4*c/d^4 + 260*(63*(d*x + c)^(11/2) - 385*(d*x
 + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(
d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^3/d^3 + 15*(231*(d*x + c)^(
13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)
^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqr
t(d*x + c)*c^6)*b^4/d^4)/d
```

**maple** [A] time = 0.01, size = 186, normalized size = 1.44

$$\frac{2\,(dx+c)^{\frac{5}{2}}\left(1155b^4x^4d^4+5460a\,b^3d^4x^3-840b^4c\,d^3x^3+10010a^2b^2d^4x^2-3640a\,b^3c\,d^3x^2+560b^4c^2d^2x^2+8580a^3b\,d^4x-5720a^2b^2c\,d^3x+2080a\,b^3c^2d^2x-320b^4c^3dx+3003a^4d^4-3432a^3bc\,d^3+2288a^2b^2c^2d^2-832a\,b^3c^3d+128b^4c^4\right)}{15015d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(3/2),x)
```

```
[Out] 2/15015*(d*x+c)^(5/2)*(1155*b^4*d^4*x^4+5460*a*b^3*d^4*x^3-840*b^4*c*d^3*x^
3+10010*a^2*b^2*d^4*x^2-3640*a*b^3*c*d^3*x^2+560*b^4*c^2*d^2*x^2+8580*a^3*b
*d^4*x-5720*a^2*b^2*c*d^3*x+2080*a*b^3*c^2*d^2*x-320*b^4*c^3*d*x+3003*a^4*d
^4-3432*a^3*b*c*d^3+2288*a^2*b^2*c^2*d^2-832*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

**maxima** [A] time = 1.36, size = 181, normalized size = 1.40

$$\frac{2\left(1155\,(dx+c)^{\frac{13}{2}}b^4-5460\left(b^4c-ab^3d\right)(dx+c)^{\frac{11}{2}}+10010\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)(dx+c)^{\frac{9}{2}}-8580\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)(dx+c)^{\frac{7}{2}}+3003\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)(dx+c)^{\frac{5}{2}}\right)}{15015\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*(d*x + c)^(13/2)*b^4 - 5460*(b^4*c - a*b^3*d)*(d*x + c)^(11/2
) + 10010*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(9/2) - 8580*(b^4
*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(7/2) + 3003*
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*
x + c)^(5/2))/d^5
```

**mupad [B]** time = 0.24, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{13/2}}{13\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{11/2}}{11\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}+\frac{4\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{9/2}}{3\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^(3/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(13/2))/(13*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(11/2)
)/(11*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(5/2))/(5*d^5) + (4*b^2*(a*d - b*c)
^2*(c + d*x)^(9/2))/(3*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(7/2))/(7*d^5)
```

**sympy [A]** time = 20.09, size = 559, normalized size = 4.33

$$a^4 c \left(\begin{cases} \sqrt{c}\, x & \text{for}\: d = 0 \\ \frac{2 (c + d x)^{\frac{3}{2}}}{3 d} & \text{otherwise} \end{cases}\right) + \frac{2 a^4 \left(- \frac{c (c + d x)^{\frac{3}{2}}}{3} + \frac{(c + d x)^{\frac{5}{2}}}{5}\right)}{d} + \frac{8 a^3 b c \left(- \frac{c (c + d x)^{\frac{3}{2}}}{3} + \frac{(c + d x)^{\frac{5}{2}}}{5}\right)}{d^2} + \frac{8 a^3 b \left(\frac{c^2 (c + d x)^{\frac{3}{2}}}{3} - \frac{2 c (c + d x)^{\frac{5}{2}}}{5} + \frac{(c + d x)^{\frac{7}{2}}}{7}\right)}{d^2} + \frac{12 a^2 b^2 c \left(\frac{c^2 (c + d x)^{\frac{3}{2}}}{3} - \frac{2 c (c + d x)^{\frac{5}{2}}}{5} + \frac{(c + d x)^{\frac{7}{2}}}{7}\right)}{d^3} + \frac{12 a^2 b^2 \left(- \frac{c^3 (c + d x)^{\frac{3}{2}}}{3} + \frac{3 c^2 (c + d x)^{\frac{5}{2}}}{5} - \frac{3 c (c + d x)^{\frac{7}{2}}}{7} + \frac{(c + d x)^{\frac{9}{2}}}{9}\right)}{d^3} + \frac{8 a b^3 c \left(- \frac{c^3 (c + d x)^{\frac{3}{2}}}{3} + \frac{3 c^2 (c + d x)^{\frac{5}{2}}}{5} - \frac{3 c (c + d x)^{\frac{7}{2}}}{7} + \frac{(c + d x)^{\frac{9}{2}}}{9}\right)}{d^4} + \frac{8 a b^3 \left(\frac{c^4 (c + d x)^{\frac{3}{2}}}{3} - \frac{4 c^3 (c + d x)^{\frac{5}{2}}}{5} + \frac{6 c^2 (c + d x)^{\frac{7}{2}}}{7} - \frac{4 c (c + d x)^{\frac{9}{2}}}{9} + \frac{(c + d x)^{\frac{11}{2}}}{11}\right)}{d^4} + \frac{2 b^4 c \left(\frac{c^4 (c + d x)^{\frac{3}{2}}}{3} - \frac{4 c^3 (c + d x)^{\frac{5}{2}}}{5} + \frac{6 c^2 (c + d x)^{\frac{7}{2}}}{7} - \frac{4 c (c + d x)^{\frac{9}{2}}}{9} + \frac{(c + d x)^{\frac{11}{2}}}{11}\right)}{d^5} + \frac{2 b^4 \left(- \frac{c^5 (c + d x)^{\frac{3}{2}}}{3} + c^4 (c + d x)^{\frac{5}{2}} - \frac{10 c^3 (c + d x)^{\frac{7}{2}}}{7} + \frac{10 c^2 (c + d x)^{\frac{9}{2}}}{9} - \frac{5 c (c + d x)^{\frac{11}{2}}}{11} + \frac{(c + d x)^{\frac{13}{2}}}{13}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(3/2),x)
```

```
[Out] a**4*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**4*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 8*a**3*b*c*(-c*(c
+ d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 8*a**3*b*(c**2*(c + d*x)**(3/2
)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 12*a**2*b**2*c*(c
**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3
+ 12*a**2*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*
(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 8*a*b**3*c*(-c**3*(c + d*x)
**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)*
*(9/2)/9)/d**4 + 8*a*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2
)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2
)/11)/d**4 + 2*b**4*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5
+ 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11
)/d**5 + 2*b**4*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3
*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11
 + (c + d*x)**(13/2)/13)/d**5
```

# 3.1283 $\int (a+bx)^3(c+dx)^{3/2}\,dx$

**Optimal**. Leaf size=100

$$-\frac{2b^2(c+dx)^{9/2}(bc-ad)}{3d^4}+\frac{6b(c+dx)^{7/2}(bc-ad)^2}{7d^4}-\frac{2(c+dx)^{5/2}(bc-ad)^3}{5d^4}+\frac{2b^3(c+dx)^{11/2}}{11d^4}$$

**Rubi** [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2b^2(c+dx)^{9/2}(bc-ad)}{3d^4}+\frac{6b(c+dx)^{7/2}(bc-ad)^2}{7d^4}-\frac{2(c+dx)^{5/2}(bc-ad)^3}{5d^4}+\frac{2b^3(c+dx)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(5/2))/(5*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
7/2))/(7*d^4) - (2*b^2*(b*c - a*d)*(c + d*x)^(9/2))/(3*d^4) + (2*b^3*(c + d
*x)^(11/2))/(11*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{3/2}\,dx=\int\left(\frac{(-bc+ad)^3(c+dx)^{3/2}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{5/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{7/2}}{d^3}+\frac{b^3}{}\right.$$
$$=-\frac{2(bc-ad)^3(c+dx)^{5/2}}{5d^4}+\frac{6b(bc-ad)^2(c+dx)^{7/2}}{7d^4}-\frac{2b^2(bc-ad)(c+dx)^{9/2}}{3d^4}+\frac{2b^3(c}{}$$

**Mathematica** [A] time = 0.07, size = 79, normalized size = 0.79

$$\frac{2(c+dx)^{5/2}\left(-385b^2(c+dx)^2(bc-ad)+495b(c+dx)(bc-ad)^2-231(bc-ad)^3+105b^3(c+dx)^3\right)}{1155d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-231*(b*c - a*d)^3 + 495*b*(b*c - a*d)^2*(c + d*x) - 38
5*b^2*(b*c - a*d)*(c + d*x)^2 + 105*b^3*(c + d*x)^3))/(1155*d^4)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 132, normalized size = 1.32

$$\frac{2(c+dx)^{5/2}\left(231a^3d^3+495a^2bd^2(c+dx)-693a^2bcd^2+693ab^2c^2d+385ab^2d(c+dx)^2-990ab^2cd(c+dx)-231b^3c^3+495b^3c^2(c+dx)+105b^3(c+dx)^3-385b^3c(c+dx)^2\right)}{1155d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-231*b^3*c^3 + 693*a*b^2*c^2*d - 693*a^2*b*c*d^2 + 231*
a^3*d^3 + 495*b^3*c^2*(c + d*x) - 990*a*b^2*c*d*(c + d*x) + 495*a^2*b*d^2*(
c + d*x) - 385*b^3*c*(c + d*x)^2 + 385*a*b^2*d*(c + d*x)^2 + 105*b^3*(c + d
*x)^3))/(1155*d^4)
```

**fricas [B]** time = 0.78, size = 216, normalized size = 2.16

$$\frac{2\left(105\,b^3d^5x^5-16\,b^3c^5+88\,ab^2c^4d-198\,a^2bc^3d^2+231\,a^3c^2d^3+35\left(4\,b^3cd^4+11\,ab^2d^5\right)x^4+5\left(b^3c^2d^3+110\,ab^2cd^4+99\,a^2bd^5\right)x^3-3\left(2\,b^3c^3d^2-11\,ab^2c^2d^3-264\,a^2bcd^4-77\,a^3d^5\right)x^2+\left(8\,b^3c^4d-44\,ab^2c^3d^2+99\,a^2bc^2d^3+462\,a^3cd^4\right)x\right)\sqrt{dx+c}}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*d^5*x^5 - 16*b^3*c^5 + 88*a*b^2*c^4*d - 198*a^2*b*c^3*d^2 +
 231*a^3*c^2*d^3 + 35*(4*b^3*c*d^4 + 11*a*b^2*d^5)*x^4 + 5*(b^3*c^2*d^3 + 1
10*a*b^2*c*d^4 + 99*a^2*b*d^5)*x^3 - 3*(2*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3 -
264*a^2*b*c*d^4 - 77*a^3*d^5)*x^2 + (8*b^3*c^4*d - 44*a*b^2*c^3*d^2 + 99*a^
2*b*c^2*d^3 + 462*a^3*c*d^4)*x)*sqrt(d*x + c)/d^4
```

**giac [B]** time = 1.31, size = 566, normalized size = 5.66

$$\frac{2\left(3465\,\sqrt{dx+c}\,a^3c^2+2310\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^3c+\frac{3465\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^2bc^2}{d}+231\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^3+\frac{693\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)ab^2c^2}{d^2}+\frac{1386\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^2bc}{d}+\frac{99\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)b^3c^2}{d^3}+\frac{594\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)ab^2c}{d^2}+\frac{297\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)a^2b}{d}+\frac{22\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)b^3c}{d^3}+\frac{33\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)ab^2}{d^2}+\frac{5\left(63\,(dx+c)^{\frac{11}{2}}-385\,(dx+c)^{\frac{9}{2}}c+990\,(dx+c)^{\frac{7}{2}}c^2-1386\,(dx+c)^{\frac{5}{2}}c^3+1155\,(dx+c)^{\frac{3}{2}}c^4-693\,\sqrt{dx+c}\,c^5\right)b^3}{d^3}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^3*c^2 + 2310*((d*x + c)^(3/2) - 3*sqrt(d*x + c
)*c)*a^3*c + 3465*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c^2/d + 231*(
3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3 + 693*
(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b^2*c^2
/d^2 + 1386*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b*c/d + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^
(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^3*c^2/d^3 + 594*(5*(d*x + c)^(7/2) - 21
```

```
*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2*c
/d^2 + 297*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c
^2 - 35*sqrt(d*x + c)*c^3)*a^2*b/d + 22*(35*(d*x + c)^(9/2) - 180*(d*x + c)
^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x
 + c)*c^4)*b^3*c/d^3 + 33*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378
*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b
^2/d^2 + 5*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/
2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x
 + c)*c^5)*b^3/d^3)/d
```

**maple** [A] time = 0.01, size = 116, normalized size = 1.16

$$\frac{2\,(dx+c)^{\frac{5}{2}}\left(105b^3x^3d^3+385a\,b^2d^3x^2-70b^3c\,d^2x^2+495a^2b\,d^3x-220a\,b^2c\,d^2x+40b^3c^2dx+231a^3d^3-198a^2bc\,d^2+88a\,b^2c^2d-16b^3c^3\right)}{1155d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(3/2),x)
```

```
[Out] 2/1155*(d*x+c)^(5/2)*(105*b^3*d^3*x^3+385*a*b^2*d^3*x^2-70*b^3*c*d^2*x^2+49
5*a^2*b*d^3*x-220*a*b^2*c*d^2*x+40*b^3*c^2*d*x+231*a^3*d^3-198*a^2*b*c*d^2+
88*a*b^2*c^2*d-16*b^3*c^3)/d^4
```

**maxima** [A] time = 1.36, size = 118, normalized size = 1.18

$$\frac{2\left(105\,(dx+c)^{\frac{11}{2}}b^3-385\left(b^3c-ab^2d\right)(dx+c)^{\frac{9}{2}}+495\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)(dx+c)^{\frac{7}{2}}-231\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)(dx+c)^{\frac{5}{2}}\right)}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*(d*x + c)^(11/2)*b^3 - 385*(b^3*c - a*b^2*d)*(d*x + c)^(9/2) +
495*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(7/2) - 231*(b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(5/2))/d^4
```

**mupad** [B] time = 0.25, size = 87, normalized size = 0.87

$$\frac{2\,b^3\,(c+d\,x)^{11/2}}{11\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,(c+d\,x)^{9/2}}{9\,d^4}+\frac{2\,(a\,d-b\,c)^3\,(c+d\,x)^{5/2}}{5\,d^4}+\frac{6\,b\,(a\,d-b\,c)^2\,(c+d\,x)^{7/2}}{7\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(11/2))/(11*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(9/2))
/(9*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(5/2))/(5*d^4) + (6*b*(a*d - b*c)^2*(
c + d*x)^(7/2))/(7*d^4)
```

**sympy [A]** time = 14.38, size = 386, normalized size = 3.86

$$a^{3} c \left(\begin{cases} \sqrt{c} x & \text{for}\: d = 0 \\ \frac{2 \left(c + d x\right)^{\frac{3}{2}}}{3 d} & \text{otherwise} \end{cases}\right) + \frac{2 a^{3} \left(- \frac{c \left(c + d x\right)^{\frac{3}{2}}}{3} + \frac{\left(c + d x\right)^{\frac{5}{2}}}{5}\right)}{d} + \frac{6 a^{2} b c \left(- \frac{c \left(c + d x\right)^{\frac{3}{2}}}{3} + \frac{\left(c + d x\right)^{\frac{5}{2}}}{5}\right)}{d^{2}} + \frac{6 a^{2} b \left(\frac{c^{2} \left(c + d x\right)^{\frac{3}{2}}}{3} - \frac{2 c \left(c + d x\right)^{\frac{5}{2}}}{5} + \frac{\left(c + d x\right)^{\frac{7}{2}}}{7}\right)}{d^{2}} + \frac{6 a b^{2} c \left(\frac{c^{2} \left(c + d x\right)^{\frac{3}{2}}}{3} - \frac{2 c \left(c + d x\right)^{\frac{5}{2}}}{5} + \frac{\left(c + d x\right)^{\frac{7}{2}}}{7}\right)}{d^{3}} + \frac{6 a b^{2} \left(- \frac{c^{3} \left(c + d x\right)^{\frac{3}{2}}}{3} + \frac{3 c^{2} \left(c + d x\right)^{\frac{5}{2}}}{5} - \frac{3 c \left(c + d x\right)^{\frac{7}{2}}}{7} + \frac{\left(c + d x\right)^{\frac{9}{2}}}{9}\right)}{d^{3}} + \frac{2 b^{3} c \left(- \frac{c^{3} \left(c + d x\right)^{\frac{3}{2}}}{3} + \frac{3 c^{2} \left(c + d x\right)^{\frac{5}{2}}}{5} - \frac{3 c \left(c + d x\right)^{\frac{7}{2}}}{7} + \frac{\left(c + d x\right)^{\frac{9}{2}}}{9}\right)}{d^{4}} + \frac{2 b^{3} \left(\frac{c^{4} \left(c + d x\right)^{\frac{3}{2}}}{3} - \frac{4 c^{3} \left(c + d x\right)^{\frac{5}{2}}}{5} + \frac{6 c^{2} \left(c + d x\right)^{\frac{7}{2}}}{7} - \frac{4 c \left(c + d x\right)^{\frac{9}{2}}}{9} + \frac{\left(c + d x\right)^{\frac{11}{2}}}{11}\right)}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(3/2),x)
```

```
[Out] a**3*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**3*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 6*a**2*b*c*(-c*(c
+ d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 6*a**2*b*(c**2*(c + d*x)**(3/2
)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 6*a*b**2*c*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 6*
a*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x
)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 2*b**3*c*(-c**3*(c + d*x)**(3/2)/3
+ 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/
d**4 + 2*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2
*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11)/d**4
```

## 3.1284 $\int (a+bx)^2(c+dx)^{3/2}\,dx$

**Optimal**. Leaf size=71

$$-\frac{4b(c+dx)^{7/2}(bc-ad)}{7d^3}+\frac{2(c+dx)^{5/2}(bc-ad)^2}{5d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}$$

**Rubi** [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{7/2}(bc-ad)}{7d^3}+\frac{2(c+dx)^{5/2}(bc-ad)^2}{5d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(7/2
))/(7*d^3) + (2*b^2*(c + d*x)^(9/2))/(9*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{3/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{5/2}}{d^2}+\frac{b^2(c+dx)^{7/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{5/2}}{5d^3}-\frac{4b(bc-ad)(c+dx)^{7/2}}{7d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{5/2}\left(63a^2d^2+18abd(5dx-2c)+b^2\left(8c^2-20cdx+35d^2x^2\right)\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-2*c + 5*d*x) + b^2*(8*c^2 - 20*
c*d*x + 35*d^2*x^2)))/(315*d^3)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 72, normalized size = 1.01

$$\frac{2(c+dx)^{5/2}\left(63a^2d^2+90abd(c+dx)-126abcd+63b^2c^2+35b^2(c+dx)^2-90b^2c(c+dx)\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(63*b^2*c^2 - 126*a*b*c*d + 63*a^2*d^2 - 90*b^2*c*(c + d
*x) + 90*a*b*d*(c + d*x) + 35*b^2*(c + d*x)^2))/(315*d^3)
```

**fricas [B]** time = 1.24, size = 137, normalized size = 1.93

$$\frac{2\left(35\,b^2d^4x^4+8\,b^2c^4-36\,abc^3d+63\,a^2c^2d^2+10\left(5\,b^2cd^3+9\,abd^4\right)x^3+3\left(b^2c^2d^2+48\,abcd^3+21\,a^2d^4\right)x^2-2\left(2\,b^2c^3d-9\,abc^2d^2-63\,a^2cd^3\right)x\right)\sqrt{dx+c}}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^2*d^4*x^4 + 8*b^2*c^4 - 36*a*b*c^3*d + 63*a^2*c^2*d^2 + 10*(5*b
^2*c*d^3 + 9*a*b*d^4)*x^3 + 3*(b^2*c^2*d^2 + 48*a*b*c*d^3 + 21*a^2*d^4)*x^2
 - 2*(2*b^2*c^3*d - 9*a*b*c^2*d^2 - 63*a^2*c*d^3)*x)*sqrt(d*x + c)/d^3
```

**giac [B]** time = 1.41, size = 360, normalized size = 5.07

$$\frac{2\left(315\,\sqrt{dx+c}\,a^2c^2+210\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^2c+\frac{210\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)abc^2}{d}+21\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^2+\frac{21\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)b^2c^2}{d^2}+\frac{84\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)abc}{d}+\frac{18\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)b^2c}{d^2}+\frac{18\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)ab}{d}+\frac{\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)b^2}{d^2}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^2*c^2 + 210*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c
)*a^2*c + 210*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c^2/d + 21*(3*(d*x
+ c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2 + 21*(3*(d*x
+ c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c^2/d^2 + 84*
(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b*c/d +
 18*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35
*sqrt(d*x + c)*c^3)*b^2*c/d^2 + 18*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*
c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b/d + (35*(d*x + c)^(9
/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)
*c^3 + 315*sqrt(d*x + c)*c^4)*b^2/d^2)/d
```

**maple** [A]  time = 0.01, size = 63, normalized size = 0.89

$$\frac{2\left(dx+c\right)^{\frac{5}{2}}\left(35b^{2}x^{2}d^{2}+90ab\,d^{2}x-20b^{2}cdx+63a^{2}d^{2}-36abcd+8b^{2}c^{2}\right)}{315d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(3/2),x)
```

```
[Out] 2/315*(d*x+c)^(5/2)*(35*b^2*d^2*x^2+90*a*b*d^2*x-20*b^2*c*d*x+63*a^2*d^2-36
*a*b*c*d+8*b^2*c^2)/d^3
```

**maxima** [A]  time = 1.38, size = 68, normalized size = 0.96

$$\frac{2\left(35\left(dx+c\right)^{\frac{9}{2}}b^{2}-90\left(b^{2}c-abd\right)\left(dx+c\right)^{\frac{7}{2}}+63\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left(dx+c\right)^{\frac{5}{2}}\right)}{315\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(d*x + c)^(9/2)*b^2 - 90*(b^2*c - a*b*d)*(d*x + c)^(7/2) + 63*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(5/2))/d^3
```

**mupad** [B]  time = 0.06, size = 68, normalized size = 0.96

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}\,\left(35\,b^2\,{\left(c+d\,x\right)}^2+63\,a^2\,d^2+63\,b^2\,c^2-90\,b^2\,c\,\left(c+d\,x\right)+90\,a\,b\,d\,\left(c+d\,x\right)-126\,a\,b\,c\,d\right)}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] (2*(c + d*x)^(5/2)*(35*b^2*(c + d*x)^2 + 63*a^2*d^2 + 63*b^2*c^2 - 90*b^2*c
*(c + d*x) + 90*a*b*d*(c + d*x) - 126*a*b*c*d))/(315*d^3)
```

**sympy** [A]  time = 9.61, size = 240, normalized size = 3.38

$$a^{2}c\left(\begin{cases}\sqrt{c}\,x & \text{for}\: d=0\\ \frac{2\left(c+dx\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^{2}\left(-\frac{c\left(c+dx\right)^{\frac{3}{2}}}{3}+\frac{\left(c+dx\right)^{\frac{5}{2}}}{5}\right)}{d}+\frac{4abc\left(-\frac{c\left(c+dx\right)^{\frac{3}{2}}}{3}+\frac{\left(c+dx\right)^{\frac{5}{2}}}{5}\right)}{d^{2}}+\frac{4ab\left(\frac{c^{2}\left(c+dx\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+dx\right)^{\frac{5}{2}}}{5}+\frac{\left(c+dx\right)^{\frac{7}{2}}}{7}\right)}{d^{2}}+\frac{2b^{2}c\left(\frac{c^{2}\left(c+dx\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+dx\right)^{\frac{5}{2}}}{5}+\frac{\left(c+dx\right)^{\frac{7}{2}}}{7}\right)}{d^{3}}+\frac{2b^{2}\left(-\frac{c^{3}\left(c+dx\right)^{\frac{3}{2}}}{3}+\frac{3c^{2}\left(c+dx\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+dx\right)^{\frac{7}{2}}}{7}+\frac{\left(c+dx\right)^{\frac{9}{2}}}{9}\right)}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 4*a*b*c*(-c*(c + d
```

```
*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 4*a*b*(c**2*(c + d*x)**(3/2)/3 -
2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 2*b**2*c*(c**2*(c + d*x
)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 2*b**2*(-c
**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7
 + (c + d*x)**(9/2)/9)/d**3
```

## 3.1285 $\int (a+bx)(c+dx)^{3/2}\,dx$

**Optimal**. Leaf size=42

$$\frac{2b(c+dx)^{7/2}}{7d^2}-\frac{2(c+dx)^{5/2}(bc-ad)}{5d^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{7/2}}{7d^2}-\frac{2(c+dx)^{5/2}(bc-ad)}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^2) + (2*b*(c + d*x)^(7/2))/(7*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^{3/2}}{d}+\frac{b(c+dx)^{5/2}}{d}\right)dx\\ &= -\frac{2(bc-ad)(c+dx)^{5/2}}{5d^2}+\frac{2b(c+dx)^{7/2}}{7d^2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{5/2}(7ad-2bc+5bdx)}{35d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-2*b*c + 7*a*d + 5*b*d*x))/(35*d^2)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 33, normalized size = 0.79

$$\frac{2(c+dx)^{5/2}(7ad+5b(c+dx)-7bc)}{35d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-7*b*c + 7*a*d + 5*b*(c + d*x)))/(35*d^2)
```

**fricas [B]** time = 1.27, size = 69, normalized size = 1.64

$$\frac{2\left(5\,bd^3x^3-2\,bc^3+7\,ac^2d+\left(8\,bcd^2+7\,ad^3\right)x^2+\left(bc^2d+14\,acd^2\right)x\right)\sqrt{dx+c}}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b*d^3*x^3 - 2*b*c^3 + 7*a*c^2*d + (8*b*c*d^2 + 7*a*d^3)*x^2 + (b*c^
2*d + 14*a*c*d^2)*x)*sqrt(d*x + c)/d^2
```

**giac [B]** time = 1.23, size = 192, normalized size = 4.57

$$\frac{2\left(105\,\sqrt{dx+c}\,ac^2+70\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)ac+\frac{35\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)bc^2}{d}+7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a+\frac{14\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)bc}{d}+\frac{3\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)b}{d}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(d*x + c)*a*c^2 + 70*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
*c + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c^2/d + 7*(3*(d*x + c)^(5/2
) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a + 14*(3*(d*x + c)^(5/2)
- 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b*c/d + 3*(5*(d*x + c)^(7/2)
 - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b/
d)/d
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.64

$$\frac{2\,(dx+c)^{\frac{5}{2}}\,(5bdx+7ad-2bc)}{35d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(3/2),x)
```

```
[Out] 2/35*(d*x+c)^(5/2)*(5*b*d*x+7*a*d-2*b*c)/d^2
```

**maxima** [A] time = 1.37, size = 33, normalized size = 0.79

$$\frac{2\left(5\,(dx+c)^{\frac{7}{2}}b-7\,(bc-ad)(dx+c)^{\frac{5}{2}}\right)}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*(d*x + c)^(7/2)*b - 7*(b*c - a*d)*(d*x + c)^(5/2))/d^2
```

**mupad** [B] time = 0.21, size = 29, normalized size = 0.69

$$\frac{2\,(c+d\,x)^{5/2}\,\left(7\,a\,d-7\,b\,c+5\,b\,\left(c+d\,x\right)\right)}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] (2*(c + d*x)^(5/2)*(7*a*d - 7*b*c + 5*b*(c + d*x)))/(35*d^2)
```

**sympy** [A] time = 0.67, size = 146, normalized size = 3.48

$$\begin{cases} \frac{2ac^2\sqrt{c+dx}}{5d}+\frac{4acx\sqrt{c+dx}}{5}+\frac{2adx^2\sqrt{c+dx}}{5}-\frac{4bc^3\sqrt{c+dx}}{35d^2}+\frac{2bc^2x\sqrt{c+dx}}{35d}+\frac{16bcx^2\sqrt{c+dx}}{35}+\frac{2bdx^3\sqrt{c+dx}}{7} & \text{for } d\neq 0 \\ c^{\frac{3}{2}}\left(ax+\frac{bx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(3/2),x)
```

```
[Out] Piecewise((2*a*c**2*sqrt(c + d*x)/(5*d) + 4*a*c*x*sqrt(c + d*x)/5 + 2*a*d*x
**2*sqrt(c + d*x)/5 - 4*b*c**3*sqrt(c + d*x)/(35*d**2) + 2*b*c**2*x*sqrt(c
+ d*x)/(35*d) + 16*b*c*x**2*sqrt(c + d*x)/35 + 2*b*d*x**3*sqrt(c + d*x)/7,
Ne(d, 0)), (c**(3/2)*(a*x + b*x**2/2), True))
```

## 3.1286 $\int (c+dx)^{3/2}\,dx$

**Optimal**. Leaf size=16

$$\frac{2(c+dx)^{5/2}}{5d}$$

**Rubi** [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{3/2}\,dx=\frac{2(c+dx)^{5/2}}{5d}$$

**Mathematica** [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

**fricas [B]** time = 1.20, size = 28, normalized size = 1.75

$$\frac{2\left(d^{2}x^{2}+2\,cdx+c^{2}\right)\sqrt{dx+c}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(d*x + c)/d
```

**giac [B]** time = 1.24, size = 58, normalized size = 3.62

$$\frac{2\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+30\,\sqrt{dx+c}\,c^{2}+10\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 30*sqrt(d*x + c)*c^2 + 10*
((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*c)/d
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(dx+c)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2),x)
```

```
[Out] 2/5*(d*x+c)^(5/2)/d
```

**maxima [A]** time = 1.36, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{5}{2}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(d*x + c)^(5/2)/d
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2),x)
```

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.75

$$\frac{2\left(c+dx\right)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2),x)
```

```
[Out] 2*(c + d*x)**(5/2)/(5*d)
```

## 3.1287 $\int \frac{(c+dx)^{3/2}}{a+bx}\,dx$

**Optimal**. Leaf size=86

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{2\sqrt{c+dx}\,(bc-ad)}{b^2}+\frac{2(c+dx)^{3/2}}{3b}$$

**Rubi** [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 63, 208}

$$\frac{2\sqrt{c+dx}\,(bc-ad)}{b^2}-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{2(c+dx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[c + d*x])/b^2 + (2*(c + d*x)^(3/2))/(3*b) - (2*(b*c - a
*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{3/2}}{a+bx}\,dx &= \frac{2(c+dx)^{3/2}}{3b} + \frac{(bc-ad)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{b} \\
&= \frac{2(bc-ad)\sqrt{c+dx}}{b^2} + \frac{2(c+dx)^{3/2}}{3b} + \frac{(bc-ad)^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b^2} \\
&= \frac{2(bc-ad)\sqrt{c+dx}}{b^2} + \frac{2(c+dx)^{3/2}}{3b} + \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{b^2 d} \\
&= \frac{2(bc-ad)\sqrt{c+dx}}{b^2} + \frac{2(c+dx)^{3/2}}{3b} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.07, size = 77, normalized size = 0.90

$$
\frac{2\sqrt{c+dx}\,(-3ad+4bc+bdx)}{3b^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(4*b*c - 3*a*d + b*d*x))/(3*b^2) - (2*(b*c - a*d)^(3/2)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

**IntegrateAlgebraic [A]** time = 0.12, size = 90, normalized size = 1.05

$$
\frac{2\sqrt{c+dx}\,(-3ad+b(c+dx)+3bc)}{3b^2} - \frac{2(ad-bc)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(3*b*c - 3*a*d + b*(c + d*x)))/(3*b^2) - (2*(-(b*c) + a*d)
^(3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/b^(5
/2)
```

**fricas [A]** time = 1.14, size = 188, normalized size = 2.19

$$
\left[-\frac{3(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad+2\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right) - 2(bdx+4bc-3ad)\sqrt{dx+c}}{3b^2}, -\frac{2\left(3(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right) - (bdx+4bc-3ad)\sqrt{dx+c}\right)}{3b^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d + 2*sqrt(
d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) - 2*(b*d*x + 4*b*c - 3*a*d)*sqrt
(d*x + c))/b^2, -2/3*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x + 4*b*c - 3*a*d)*sqrt(d*x
+ c))/b^2]
```

**giac [A]** time = 1.29, size = 105, normalized size = 1.22

$$\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}+\frac{2\left((dx+c)^{\frac{3}{2}}b^2+3\,\sqrt{dx+c}\,b^2c-3\,\sqrt{dx+c}\,abd\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*
d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/3*((d*x + c)^(3/2)*b^2 + 3*sqrt(d*x + c)
*b^2*c - 3*sqrt(d*x + c)*a*b*d)/b^3
```

**maple [B]** time = 0.01, size = 167, normalized size = 1.94

$$\frac{2a^2d^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b^2}-\frac{4acd\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b}+\frac{2c^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}}-\frac{2\sqrt{dx+c}\,ad}{b^2}+\frac{2\sqrt{dx+c}\,c}{b}+\frac{2\,(dx+c)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a),x)
```

```
[Out] 2/3*(d*x+c)^(3/2)/b-2/b^2*a*d*(d*x+c)^(1/2)+2/b*(d*x+c)^(1/2)*c+2/b^2/((a*d
-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a^2*d^2-4/b/((a*
d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a*c*d+2/((a*d-b
*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*c^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.07, size = 93, normalized size = 1.08

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}}{3\,b}-\frac{2\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{b^2}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}\,\sqrt{c+d\,x}}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}\right)\,{\left(a\,d-b\,c\right)}^{3/2}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(3/2))/(3*b) - (2*(a*d - b*c)*(c + d*x)^(1/2))/b^2 + (2*atan((
b^(1/2)*(a*d - b*c)^(3/2)*(c + d*x)^(1/2))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
*(a*d - b*c)^(3/2))/b^(5/2)
```

**sympy [A]** time = 14.70, size = 82, normalized size = 0.95

$$\frac{2\left(c+dx\right)^{\frac{3}{2}}}{3b}+\frac{\sqrt{c+dx}\,\left(-2ad+2bc\right)}{b^2}+\frac{2\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a),x)
```

```
[Out] 2*(c + d*x)**(3/2)/(3*b) + sqrt(c + d*x)*(-2*a*d + 2*b*c)/b**2 + 2*(a*d - b
*c)**2*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**3*sqrt((a*d - b*c)/b))
```

# 3.1288 $\int \frac{(c+dx)^{3/2}}{(a+bx)^2} dx$

**Optimal**. Leaf size=85

$$-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{3d\sqrt{c+dx}}{b^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 63, 208}

$$-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{3d\sqrt{c+dx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x])/b^2 - (c + d*x)^(3/2)/(b*(a + b*x)) - (3*d*Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx &= -\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3d)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{2b}\\
&= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3d(bc-ad))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b^2}\\
&= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{b^2}\\
&= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 50, normalized size = 0.59

$$\frac{2d(c+dx)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^2,x]
[Out] (2*d*(c + d*x)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(5*(-(b*c) + a*d)^2)
```

**IntegrateAlgebraic [A]** time = 0.26, size = 107, normalized size = 1.26

$$\frac{3d\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}\,\sqrt{ad-bc}}{bc-ad}\right)}{b^{5/2}}+\frac{d\sqrt{c+dx}\,(3ad+2b(c+dx)-3bc)}{b^2(ad+b(c+dx)-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (d*Sqrt[c + d*x]*(-3*b*c + 3*a*d + 2*b*(c + d*x)))/(b^2*(-(b*c) + a*d + b*(
c + d*x))) + (3*d*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqr
t[c + d*x])/(b*c - a*d)])/b^(5/2)
```

**fricas [A]** time = 1.74, size = 210, normalized size = 2.47

$$\left[\frac{3\,(bdx+ad)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,(2\,bdx-bc+3\,ad)\sqrt{dx+c}}{2\,\left(b^3x+ab^2\right)},-\frac{3\,(bdx+ad)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(2\,bdx-bc+3\,ad)\sqrt{dx+c}}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*d*x + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt
(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(2*b*d*x - b*c + 3*a*d)*sqr
t(d*x + c))/(b^3*x + a*b^2), -(3*(b*d*x + a*d)*sqrt(-(b*c - a*d)/b)*arctan(
-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (2*b*d*x - b*c + 3*a*d
)*sqrt(d*x + c))/(b^3*x + a*b^2)]
```

**giac [A]** time = 1.30, size = 113, normalized size = 1.33

$$\frac{2\,\sqrt{dx+c}\,d}{b^2}+\frac{3\left(bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}-\frac{\sqrt{dx+c}\,bcd-\sqrt{dx+c}\,ad^2}{((dx+c)b-bc+ad)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*d/b^2 + 3*(b*c*d - a*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - (sqrt(d*x + c)*b*c*d - sqrt(d*x +
c)*a*d^2)/(((d*x + c)*b - b*c + a*d)*b^2)
```

**maple [B]** time = 0.01, size = 148, normalized size = 1.74

$$-\frac{3a\,d^2\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;b^2}+\frac{3cd\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;b}+\frac{\sqrt{dx+c}\;a\,d^2}{(bdx+ad)\,b^2}-\frac{\sqrt{dx+c}\;cd}{(bdx+ad)\,b}+\frac{2\sqrt{dx+c}\;d}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 2*d*(d*x+c)^(1/2)/b^2+1/b^2*(d*x+c)^(1/2)/(b*d*x+a*d)*a*d^2-d/b*(d*x+c)^(1/
2)/(b*d*x+a*d)*c-3/b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*
b)^(1/2)*b)*a*d^2+3*d/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)
*b)^(1/2)*b)*c
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.11, size = 109, normalized size = 1.28

$$\frac{\left(a\,d^2-b\,c\,d\right)\,\sqrt{c+d\,x}}{b^3\,\left(c+d\,x\right)-b^3\,c+a\,b^2\,d}+\frac{2\,d\,\sqrt{c+d\,x}}{b^2}-\frac{3\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,\sqrt{a\,d-b\,c}\,\sqrt{c+d\,x}}{a\,d^2-b\,c\,d}\right)\,\sqrt{a\,d-b\,c}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^2,x)
```

```
[Out] ((a*d^2 - b*c*d)*(c + d*x)^(1/2))/(b^3*(c + d*x) - b^3*c + a*b^2*d) + (2*d*
(c + d*x)^(1/2))/b^2 - (3*d*atan((b^(1/2)*d*(a*d - b*c)^(1/2)*(c + d*x)^(1/
2))/(a*d^2 - b*c*d))*(a*d - b*c)^(1/2))/b^(5/2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.1289 $\int \frac{(c+dx)^{3/2}}{(a+bx)^3} dx$

**Optimal**. Leaf size=100

$$-\frac{3d^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}$$

**Rubi** [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 63, 208}

$$-\frac{3d^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(4*b^2*(a + b*x)) - (c + d*x)^(3/2)/(2*b*(a + b*x)^2)
- (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[
b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^3}\,dx = -\frac{(c+dx)^{3/2}}{2b(a+bx)^2}+\frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx}{4b}$$
$$= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}+\frac{\left(3d^2\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b^2}$$
$$= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}+\frac{(3d)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4b^2}$$
$$= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

**Mathematica** [A]    time = 0.10, size = 90, normalized size = 0.90

$$\frac{3d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{4b^{5/2}\sqrt{ad-bc}}-\frac{\sqrt{c+dx}\,(3ad+2bc+5bdx)}{4b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[c + d*x]*(2*b*c + 3*a*d + 5*b*d*x))/(b^2*(a + b*x)^2) + (3*d^2*A
rcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(4*b^(5/2)*Sqrt[-(b*c) +
 a*d])
```

**IntegrateAlgebraic** [A]    time = 0.38, size = 116, normalized size = 1.16

$$-\frac{3d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4b^{5/2}\sqrt{ad-bc}}-\frac{d^2\sqrt{c+dx}\,(3ad+5b(c+dx)-3bc)}{4b^2(ad+b(c+dx)-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/4*(d^2*Sqrt[c + d*x]*(-3*b*c + 3*a*d + 5*b*(c + d*x)))/(b^2*(-(b*c) + a*
d + b*(c + d*x))^2) - (3*d^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*
x])/(b*c - a*d)])/(4*b^(5/2)*Sqrt[-(b*c) + a*d])
```

**fricas [B]** time = 1.00, size = 383, normalized size = 3.83

$$\left[\frac{3\left(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2}\right)\sqrt{b^{2}c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^{2}c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\left(2\,b^{3}c^{2}+ab^{2}cd-3\,a^{2}bd^{2}+5\left(b^{3}cd-ab^{2}d^{2}\right)x\right)\sqrt{dx+c}}{8\left(a^{2}b^{4}c-a^{3}b^{3}d+\left(b^{6}c-ab^{5}d\right)x^{2}+2\left(ab^{5}c-a^{2}b^{4}d\right)x\right)},\frac{3\left(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2}\right)\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{-b^{2}c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-\left(2\,b^{3}c^{2}+ab^{2}cd-3\,a^{2}bd^{2}+5\left(b^{3}cd-ab^{2}d^{2}\right)x\right)\sqrt{dx+c}}{4\left(a^{2}b^{4}c-a^{3}b^{3}d+\left(b^{6}c-ab^{5}d\right)x^{2}+2\left(ab^{5}c-a^{2}b^{4}d\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*
x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(2*b^
3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 + 5*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))
/(a^2*b^4*c - a^3*b^3*d + (b^6*c - a*b^5*d)*x^2 + 2*(a*b^5*c - a^2*b^4*d)*x
), 1/4*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (2*b^3*c^2 + a*b^2*c*d
 - 3*a^2*b*d^2 + 5*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c - a^3
*b^3*d + (b^6*c - a*b^5*d)*x^2 + 2*(a*b^5*c - a^2*b^4*d)*x)]
```

**giac [A]** time = 1.36, size = 108, normalized size = 1.08

$$\frac{3\,d^{2}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{4\,\sqrt{-b^{2}c+abd}\,b^{2}}-\frac{5\,(dx+c)^{\frac{3}{2}}bd^{2}-3\,\sqrt{dx+c}\,bcd^{2}+3\,\sqrt{dx+c}\,ad^{3}}{4\,((dx+c)b-bc+ad)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b^2) - 1/4*(5*(d*x + c)^(3/2)*b*d^2 - 3*sqrt(d*x + c)*b*c*d^2 + 3*sqrt(d*x
+ c)*a*d^3)/(((d*x + c)*b - b*c + a*d)^2*b^2)
```

**maple [A]** time = 0.01, size = 121, normalized size = 1.21

$$-\frac{3\sqrt{dx+c}\;a\,d^{3}}{4\left(bdx+ad\right)^{2}b^{2}}+\frac{3\sqrt{dx+c}\;c\,d^{2}}{4\left(bdx+ad\right)^{2}b}-\frac{5\left(dx+c\right)^{\frac{3}{2}}d^{2}}{4\left(bdx+ad\right)^{2}b}+\frac{3d^{2}\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{\left(ad-bc\right)b}}\right)}{4\sqrt{\left(ad-bc\right)b}\;b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^3,x)
```

```
[Out] -5/4*d^2/(b*d*x+a*d)^2/b*(d*x+c)^(3/2)-3/4*d^3/(b*d*x+a*d)^2/b^2*(d*x+c)^(1
/2)*a+3/4*d^2/(b*d*x+a*d)^2/b*(d*x+c)^(1/2)*c+3/4*d^2/b^2/((a*d-b*c)*b)^(1/
2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.28, size = 135, normalized size = 1.35

$$\frac{3\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{5/2}\,\sqrt{a\,d-b\,c}}-\frac{\frac{5\,d^2\,{\left(c+d\,x\right)}^{3/2}}{4\,b}+\frac{3\,d^2\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{4\,b^2}}{b^2\,{\left(c+d\,x\right)}^2-\left(2\,b^2\,c-2\,a\,b\,d\right)\,\left(c+d\,x\right)+a^2\,d^2+b^2\,c^2-2\,a\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^3,x)
```

```
[Out] (3*d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(5/2)*(a*d -
 b*c)^(1/2)) - ((5*d^2*(c + d*x)^(3/2))/(4*b) + (3*d^2*(a*d - b*c)*(c + d*x
)^(1/2))/(4*b^2))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.1290 $\int \frac{(c+dx)^{3/2}}{(a+bx)^4} dx$

**Optimal**. Leaf size=136

$$\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}-\frac{d^2\sqrt{c+dx}}{8b^2(a+bx)(bc-ad)}-\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}$$

**Rubi** [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{d^2\sqrt{c+dx}}{8b^2(a+bx)(bc-ad)}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}-\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^4,x]
```

```
[Out] -(d*Sqrt[c + d*x])/(4*b^2*(a + b*x)^2) - (d^2*Sqrt[c + d*x])/(8*b^2*(b*c -
a*d)*(a + b*x)) - (c + d*x)^(3/2)/(3*b*(a + b*x)^3) + (d^3*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^4}\,dx &= -\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx}{2b}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^2\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8b^2}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}-\frac{d^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b^2(bc-ad)}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}-\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{8b^2(bc-ad)}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 52, normalized size = 0.38

$$\frac{2d^3(c+dx)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^4,x]
```

```
[Out] (2*d^3*(c + d*x)^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(5*(-(b*c) + a*d)^4)
```

**IntegrateAlgebraic [A]** time = 0.62, size = 166, normalized size = 1.22

$$-\frac{d^3\sqrt{c+dx}\left(3a^2d^2+8abd(c+dx)-6abcd+3b^2c^2-3b^2(c+dx)^2-8b^2c(c+dx)\right)}{24b^2(bc-ad)(-ad-b(c+dx)+bc)^3}-\frac{d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{8b^{5/2}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^4,x]
```

```
[Out] -1/24*(d^3*Sqrt[c + d*x]*(3*b^2*c^2 - 6*a*b*c*d + 3*a^2*d^2 - 8*b^2*c*(c +
d*x) + 8*a*b*d*(c + d*x) - 3*b^2*(c + d*x)^2))/(b^2*(b*c - a*d)*(b*c - a*d
- b*(c + d*x))^3) - (d^3*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/
(b*c - a*d)])/(8*b^(5/2)*(-(b*c) + a*d)^(3/2))
```

**fricas [B]** time = 1.37, size = 666, normalized size = 4.90

$$\left[-\frac{3\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{b^2c-abd}\log\left(\frac{bdx+2bc-ad-2\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)+2\left(8b^4c^3-10ab^3c^2d-a^2b^2cd^2+3a^3bd^3+3\left(b^4cd^2-ab^3d^3\right)x^2+2\left(7b^4c^2d-11ab^3cd^2+4a^2b^2d^3\right)x\right)\sqrt{dx+c}}{48\left(a^3b^5c^2-2a^4b^4cd+a^5b^3d^2+\left(b^8c^2-2ab^7cd+a^2b^6d^2\right)x^3+3\left(ab^7c^2-2a^2b^6cd+a^3b^5d^2\right)x^2+3\left(a^2b^6c^2-2a^3b^5cd+a^4b^4d^2\right)x\right)},-\frac{3\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{-b^2c+abd}\sqrt{dx+c}}{bdx+bc}\right)+\left(8b^4c^3-10ab^3c^2d-a^2b^2cd^2+3a^3bd^3+3\left(b^4cd^2-ab^3d^3\right)x^2+2\left(7b^4c^2d-11ab^3cd^2+4a^2b^2d^3\right)x\right)\sqrt{dx+c}}{24\left(a^3b^5c^2-2a^4b^4cd+a^5b^3d^2+\left(b^8c^2-2ab^7cd+a^2b^6d^2\right)x^3+3\left(ab^7c^2-2a^2b^6cd+a^3b^5d^2\right)x^2+3\left(a^2b^6c^2-2a^3b^5cd+a^4b^4d^2\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) + 2*(8*b^4*c^3 - 10*a*b^3*c^2*d - a^2*b^2*c*d^2 + 3*a^3*b*d^3
+ 3*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(7*b^4*c^2*d - 11*a*b^3*c*d^2 + 4*a^2*b
^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2 + (b^8
*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*
b^5*d^2)*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4*d^2)*x), -1/24*(3*(
b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a*b*
d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (8*b^4*c^3 -
10*a*b^3*c^2*d - a^2*b^2*c*d^2 + 3*a^3*b*d^3 + 3*(b^4*c*d^2 - a*b^3*d^3)*x^
2 + 2*(7*b^4*c^2*d - 11*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3
*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2 + (b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d
^2)*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*x^2 + 3*(a^2*b^6*c^2
- 2*a^3*b^5*c*d + a^4*b^4*d^2)*x)]
```

**giac [A]** time = 1.40, size = 185, normalized size = 1.36

$$-\frac{d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}-\frac{3\,(dx+c)^{\frac{5}{2}}b^2d^3+8\,(dx+c)^{\frac{3}{2}}b^2cd^3-3\,\sqrt{dx+c}\,b^2c^2d^3-8\,(dx+c)^{\frac{3}{2}}abd^4+6\,\sqrt{dx+c}\,abcd^4-3\,\sqrt{dx+c}\,a^2d^5}{24\left(b^3c-ab^2d\right)\left((dx+c)b-bc+ad\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c - a*b^2*d)*sq
rt(-b^2*c + a*b*d)) - 1/24*(3*(d*x + c)^(5/2)*b^2*d^3 + 8*(d*x + c)^(3/2)*b
^2*c*d^3 - 3*sqrt(d*x + c)*b^2*c^2*d^3 - 8*(d*x + c)^(3/2)*a*b*d^4 + 6*sqrt
(d*x + c)*a*b*c*d^4 - 3*sqrt(d*x + c)*a^2*d^5)/((b^3*c - a*b^2*d)*((d*x + c
)*b - b*c + a*d)^3)
```

**maple [A]** time = 0.02, size = 163, normalized size = 1.20

$$-\frac{\sqrt{dx+c}\; a\, d^{4}}{8\left(bdx+ad\right)^{3} b^{2}}+\frac{\sqrt{dx+c}\; c\, d^{3}}{8\left(bdx+ad\right)^{3} b}+\frac{\left(dx+c\right)^{\frac{5}{2}} d^{3}}{8\left(bdx+ad\right)^{3} \left(ad-bc\right)}-\frac{\left(dx+c\right)^{\frac{3}{2}} d^{3}}{3\left(bdx+ad\right)^{3} b}+\frac{d^{3} \arctan\left(\frac{\sqrt{dx+c}\; b}{\sqrt{\left(ad-bc\right) b}}\right)}{8\left(ad-bc\right) \sqrt{\left(ad-bc\right) b}\; b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^4,x)
```

```
[Out] 1/8*d^3/(b*d*x+a*d)^3/(a*d-b*c)*(d*x+c)^(5/2)-1/3*d^3/(b*d*x+a*d)^3/b*(d*x+
c)^(3/2)-1/8*d^4/(b*d*x+a*d)^3/b^2*(d*x+c)^(1/2)*a+1/8*d^3/(b*d*x+a*d)^3/b*
(d*x+c)^(1/2)*c+1/8*d^3/(a*d-b*c)/b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1
/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.34, size = 209, normalized size = 1.54

$$\frac{d^{3} \operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,b^{5/2}\,\left(a\,d-b\,c\right)^{3/2}}-\frac{\frac{d^{3}\,\left(c+d\,x\right)^{3/2}}{3\,b}-\frac{d^{3}\,\left(c+d\,x\right)^{5/2}}{8\,\left(a\,d-b\,c\right)}+\frac{d^{3}\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{8\,b^{2}}}{\left(c+d\,x\right)\,\left(3\,a^{2}\,b\,d^{2}-6\,a\,b^{2}\,c\,d+3\,b^{3}\,c^{2}\right)+b^{3}\,\left(c+d\,x\right)^{3}-\left(3\,b^{3}\,c-3\,a\,b^{2}\,d\right)\,\left(c+d\,x\right)^{2}+a^{3}\,d^{3}-b^{3}\,c^{3}+3\,a\,b^{2}\,c^{2}\,d-3\,a^{2}\,b\,c\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^4,x)
```

```
[Out] (d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(5/2)*(a*d - b
*c)^(3/2)) - ((d^3*(c + d*x)^(3/2))/(3*b) - (d^3*(c + d*x)^(5/2))/(8*(a*d -
 b*c)) + (d^3*(a*d - b*c)*(c + d*x)^(1/2))/(8*b^2))/((c + d*x)*(3*b^3*c^2 +
 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^2*d)*(c +
d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

## 3.1291 $\int \frac{(c+dx)^{3/2}}{(a+bx)^5} dx$

**Optimal**. Leaf size=172

$$-\frac{3d^4 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{5/2}(bc-ad)^{5/2}}+\frac{3d^3\sqrt{c+dx}}{64b^2(a+bx)(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{32b^2(a+bx)^2(bc-ad)}-\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}$$

**Rubi [A]** time = 0.07, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$\frac{3d^3\sqrt{c+dx}}{64b^2(a+bx)(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{32b^2(a+bx)^2(bc-ad)}-\frac{3d^4 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{5/2}(bc-ad)^{5/2}}-\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^5,x]
```

```
[Out] -(d*Sqrt[c + d*x])/(8*b^2*(a + b*x)^3) - (d^2*Sqrt[c + d*x])/(32*b^2*(b*c -
 a*d)*(a + b*x)^2) + (3*d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x))
 - (c + d*x)^(3/2)/(4*b*(a + b*x)^4) - (3*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/Sqrt[b*c - a*d]])/(64*b^(5/2)*(b*c - a*d)^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^5}\,dx = -\frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx}{8b}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{d^2\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{16b^2}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} - \frac{\left(3d^3\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b^2(bc-ad)}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{\left(3d^4\right)\int}{128b}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{\left(3d^3\right)\mathrm{Su}}{}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} - \frac{3d^4\tanh}{64b^{5/2}}$$

**Mathematica [C]** time = 0.02, size = 52, normalized size = 0.30

$$\frac{2d^4(c+dx)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^5,x]
```

[Out] (2*d^4*(c + d*x)^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, -((b*(c + d*x))/(-(b*c) + a*d))])/(5*(-(b*c) + a*d)^5)

**IntegrateAlgebraic [A]** time = 1.01, size = 226, normalized size = 1.31

$$\frac{d^4\sqrt{c+dx}\left(-3a^3d^3-11a^2bd^2(c+dx)+9a^2bcd^2-9ab^2c^2d+11ab^2d(c+dx)^2+22ab^2cd(c+dx)+3b^3c^3-11b^3c^2(c+dx)+3b^3(c+dx)^3-11b^3c(c+dx)^2\right)}{64b^2(bc-ad)^2(-ad-b(c+dx)+bc)^4}-\frac{3d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{64b^{5/2}(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^5,x]

[Out] (d^4*Sqrt[c + d*x]*(3*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 3*a^3*d^3 - 11*b^3*c^2*(c + d*x) + 22*a*b^2*c*d*(c + d*x) - 11*a^2*b*d^2*(c + d*x) - 11*b^3*c*(c + d*x)^2 + 11*a*b^2*d*(c + d*x)^2 + 3*b^3*(c + d*x)^3))/(64*b^2*(b*c - a*d)^2*(b*c - a*d - b*(c + d*x))^4) - (3*d^4*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(64*b^(5/2)*(-(b*c) + a*d)^(5/2))

**fricas [B]** time = 1.54, size = 1043, normalized size = 6.06

$$\left[\frac{3\left(b^4d^4x^4+4ab^3d^4x^3+6a^2b^2d^4x^2+4a^3bd^4x+a^4d^4\right)\sqrt{b^2c-abd}\log\left(\frac{bdx+2bc-ad-2\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)-2\left(16b^5c^4-40ab^4c^3d+26a^2b^3c^2d^2+a^3b^2cd^3-3a^4bd^4-3\left(b^5cd^3-ab^4d^4\right)x^3+\left(2b^5c^2d^2-13ab^4cd^3+11a^2b^3d^4\right)x^2+\left(24b^5c^3d-68ab^4c^2d^2+55a^2b^3cd^3-11a^3b^2d^4\right)x\right)\sqrt{dx+c}}{128\left(a^4b^6c^3-3a^5b^5c^2d+3a^6b^4cd^2-a^7b^3d^3+\left(b^{10}c^3-3ab^9c^2d+3a^2b^8cd^2-a^3b^7d^3\right)x^4+4\left(ab^9c^3-3a^2b^8c^2d+3a^3b^7cd^2-a^4b^6d^3\right)x^3+6\left(a^2b^8c^3-3a^3b^7c^2d+3a^4b^6cd^2-a^5b^5d^3\right)x^2+4\left(a^3b^7c^3-3a^4b^6c^2d+3a^5b^5cd^2-a^6b^4d^3\right)x\right)}, \frac{3\left(b^4d^4x^4+4ab^3d^4x^3+6a^2b^2d^4x^2+4a^3bd^4x+a^4d^4\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{-b^2c+abd}\sqrt{dx+c}}{bdx+bc}\right)-\left(16b^5c^4-40ab^4c^3d+26a^2b^3c^2d^2+a^3b^2cd^3-3a^4bd^4-3\left(b^5cd^3-ab^4d^4\right)x^3+\left(2b^5c^2d^2-13ab^4cd^3+11a^2b^3d^4\right)x^2+\left(24b^5c^3d-68ab^4c^2d^2+55a^2b^3cd^3-11a^3b^2d^4\right)x\right)\sqrt{dx+c}}{64\left(a^4b^6c^3-3a^5b^5c^2d+3a^6b^4cd^2-a^7b^3d^3+\left(b^{10}c^3-3ab^9c^2d+3a^2b^8cd^2-a^3b^7d^3\right)x^4+4\left(ab^9c^3-3a^2b^8c^2d+3a^3b^7cd^2-a^4b^6d^3\right)x^3+6\left(a^2b^8c^3-3a^3b^7c^2d+3a^4b^6cd^2-a^5b^5d^3\right)x^2+4\left(a^3b^7c^3-3a^4b^6c^2d+3a^5b^5cd^2-a^6b^4d^3\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="fricas")

[Out] [1/128*(3*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(16*b^5*c^4 - 40*a*b^4*c^3*d + 26*a^2*b^3*c^2*d^2 + a^3*b^2*c*d^3 - 3*a^4*b*d^4 - 3*(b^5*c*d^3 - a*b^4*d^4)*x^3 + (2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (24*b^5*c^3*d - 68*a*b^4*c^2*d^2 + 55*a^2*b^3*c*d^3 - 11*a^3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3 + (b^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*x), 1/64*(3*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (16*b^5*c^4 - 40*a*b^4*c^3*d + 26*a^2*b^3*c^2*d^2 + a^3*b^2*c*d^3 - 3*a^4*b*d^4 - 3*(b^5*c*d^3 - a*b^4*d^4)*x^3 + (2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (24*b^5*c^3*d - 68*a*b^4*c^2*d^2 + 55*a^2*b^3*c*d^3 - 11*a^3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3 + (b^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*x)]

**giac [A]** time = 1.47, size = 285, normalized size = 1.66

$$\frac{3\,d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{64\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\sqrt{-b^2c+abd}}+\frac{3\,(dx+c)^{\frac{7}{2}}b^3d^4-11\,(dx+c)^{\frac{5}{2}}b^3cd^4-11\,(dx+c)^{\frac{3}{2}}b^3c^2d^4+3\,\sqrt{dx+c}\,b^3c^3d^4+11\,(dx+c)^{\frac{5}{2}}ab^2d^5+22\,(dx+c)^{\frac{3}{2}}ab^2cd^5-9\,\sqrt{dx+c}\,ab^2c^2d^5-11\,(dx+c)^{\frac{3}{2}}a^2bd^6+9\,\sqrt{dx+c}\,a^2bcd^6-3\,\sqrt{dx+c}\,a^3d^7}{64\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)((dx+c)b-bc+ad)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 3/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/64*(3*(d*x + c)^(7/2)*b^3*d^4 -
 11*(d*x + c)^(5/2)*b^3*c*d^4 - 11*(d*x + c)^(3/2)*b^3*c^2*d^4 + 3*sqrt(d*x
 + c)*b^3*c^3*d^4 + 11*(d*x + c)^(5/2)*a*b^2*d^5 + 22*(d*x + c)^(3/2)*a*b^2
*c*d^5 - 9*sqrt(d*x + c)*a*b^2*c^2*d^5 - 11*(d*x + c)^(3/2)*a^2*b*d^6 + 9*s
qrt(d*x + c)*a^2*b*c*d^6 - 3*sqrt(d*x + c)*a^3*d^7)/((b^4*c^2 - 2*a*b^3*c*d
 + a^2*b^2*d^2)*((d*x + c)*b - b*c + a*d)^4)
```

**maple [A]** time = 0.02, size = 222, normalized size = 1.29

$$\frac{3\,(dx+c)^{\frac{7}{2}}\,b\,d^4}{64\,(bdx+ad)^4\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{3\sqrt{dx+c}\;a\,d^5}{64\,(bdx+ad)^4\,b^2}+\frac{3\sqrt{dx+c}\;c\,d^4}{64\,(bdx+ad)^4\,b}+\frac{11\,(dx+c)^{\frac{5}{2}}\,d^4}{64\,(bdx+ad)^4\,(ad-bc)}-\frac{11\,(dx+c)^{\frac{3}{2}}\,d^4}{64\,(bdx+ad)^4\,b}+\frac{3d^4\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{64\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{(ad-bc)\,b}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^5,x)
```

```
[Out] 3/64*d^4/(b*d*x+a*d)^4*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(7/2)+11/64*d^
4/(b*d*x+a*d)^4/(a*d-b*c)*(d*x+c)^(5/2)-11/64*d^4/(b*d*x+a*d)^4/b*(d*x+c)^(
3/2)-3/64*d^5/(b*d*x+a*d)^4/b^2*(d*x+c)^(1/2)*a+3/64*d^4/(b*d*x+a*d)^4/b*(d
*x+c)^(1/2)*c+3/64*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/((a*d-b*c)*b)^(1/2)*
arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.37, size = 296, normalized size = 1.72

$$\frac{3\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{11\,d^4\,{\left(c+d\,x\right)}^{3/2}}{64\,b}-\frac{11\,d^4\,{\left(c+d\,x\right)}^{5/2}}{64\,\left(a\,d-b\,c\right)}+\frac{3\,d^4\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{64\,b^2}-\frac{3\,b\,d^4\,{\left(c+d\,x\right)}^{7/2}}{64\,{\left(a\,d-b\,c\right)}^2}}{b^4\,{\left(c+d\,x\right)}^4-\left(4\,b^4\,c-4\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^3-\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)+a^4\,d^4+b^4\,c^4+{\left(c+d\,x\right)}^2\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^5,x)
```

```
[Out] (3*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(5/2)*(a*d
- b*c)^(5/2)) - ((11*d^4*(c + d*x)^(3/2))/(64*b) - (11*d^4*(c + d*x)^(5/2))
/(64*(a*d - b*c)) + (3*d^4*(a*d - b*c)*(c + d*x)^(1/2))/(64*b^2) - (3*b*d^4
*(c + d*x)^(7/2))/(64*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c - 4*a*b^3
*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 1
2*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2*b^2*d^2
 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

## 3.1292 $\int \frac{(c+dx)^{3/2}}{(a+bx)^6} \, dx$

**Optimal**. Leaf size=208

$$\frac{3d^5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{5/2}(bc-ad)^{7/2}} - \frac{3d^4\sqrt{c+dx}}{128b^2(a+bx)(bc-ad)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(a+bx)^2(bc-ad)^2} - \frac{d^2\sqrt{c+dx}}{80b^2(a+bx)^3(bc-ad)} - \frac{3d\sqrt{c+d}}{40b^2(a+b}$$

**Rubi** [A] time = 0.09, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{3d^4\sqrt{c+dx}}{128b^2(a+bx)(bc-ad)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(a+bx)^2(bc-ad)^2} - \frac{d^2\sqrt{c+dx}}{80b^2(a+bx)^3(bc-ad)} + \frac{3d^5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{5/2}(bc-ad)^{7/2}} - \frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{(c+dx)^{3/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(40*b^2*(a + b*x)^4) - (d^2*Sqrt[c + d*x])/(80*b^2*(b*
c - a*d)*(a + b*x)^3) + (d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x)
^2) - (3*d^4*Sqrt[c + d*x])/(128*b^2*(b*c - a*d)^3*(a + b*x)) - (c + d*x)^(
3/2)/(5*b*(a + b*x)^5) + (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c -
a*d]])/(128*b^(5/2)*(b*c - a*d)^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^6}\,dx = -\frac{(c+dx)^{3/2}}{5b(a+bx)^5}+\frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx}{10b}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{(c+dx)^{3/2}}{5b(a+bx)^5}+\frac{\left(3d^2\right)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{80b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}-\frac{(c+dx)^{3/2}}{5b(a+bx)^5}-\frac{d^3\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{32b^2(bc-ad)}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{5b(a+bx)^5}+\frac{\left(3d^4\right)\int}{128}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2}-\frac{3d^4\sqrt{c+dx}}{128b^2(bc-ad)^3(a+b}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2}-\frac{3d^4\sqrt{c+dx}}{128b^2(bc-ad)^3(a+b}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2}-\frac{3d^4\sqrt{c+dx}}{128b^2(bc-ad)^3(a+b}$$

**Mathematica [C]** time = 0.02, size = 52, normalized size = 0.25

$$\frac{2d^5(c+dx)^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] (2*d^5*(c + d*x)^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(5*(-(b*c) + a*d)^6)
```

**IntegrateAlgebraic [A]** time = 1.98, size = 317, normalized size = 1.52

$$\frac{3d^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{128b^{5/2}(bc-ad)^3\sqrt{ad-bc}}-\frac{d^5\sqrt{c+dx}\left(15a^4d^4+70a^3bd^3(c+dx)-60a^3bcd^3+90a^2b^2c^2d^2-128a^2b^2d^2(c+dx)^2-210a^2b^2cd^2(c+dx)-60ab^3c^3d+210ab^3c^2d(c+dx)-70ab^3d(c+dx)^3+256ab^3cd(c+dx)^2+15b^4c^4-70b^4c^3(c+dx)-128b^4c^2(c+dx)^2-15b^4(c+dx)^4+70b^4c(c+dx)^3\right)}{640b^2(bc-ad)^3(-ad-b(c+dx)+bc)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] -1/640*(d^5*Sqrt[c + d*x]*(15*b^4*c^4 - 60*a*b^3*c^3*d + 90*a^2*b^2*c^2*d^2
 - 60*a^3*b*c*d^3 + 15*a^4*d^4 - 70*b^4*c^3*(c + d*x) + 210*a*b^3*c^2*d*(c
+ d*x) - 210*a^2*b^2*c*d^2*(c + d*x) + 70*a^3*b*d^3*(c + d*x) - 128*b^4*c^2
*(c + d*x)^2 + 256*a*b^3*c*d*(c + d*x)^2 - 128*a^2*b^2*d^2*(c + d*x)^2 + 70
*b^4*c*(c + d*x)^3 - 70*a*b^3*d*(c + d*x)^3 - 15*b^4*(c + d*x)^4))/(b^2*(b*
c - a*d)^3*(b*c - a*d - b*(c + d*x))^5) + (3*d^5*ArcTan[(Sqrt[b]*Sqrt[-(b*c
) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(128*b^(5/2)*(b*c - a*d)^3*Sqrt[-(b*c
) + a*d])
```

**fricas [B]** time = 1.31, size = 1492, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b
^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*
c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(128*b^6*c^5
- 464*a*b^5*c^4*d + 584*a^2*b^4*c^3*d^2 - 258*a^3*b^3*c^2*d^3 - 5*a^4*b^2*c
*d^4 + 15*a^5*b*d^5 + 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 - 10*(b^6*c^2*d^3 - 8*
a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 87
*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(88*b^6*c^4*d - 344*a*b^5*c^3*d^2
+ 489*a^2*b^4*c^2*d^3 - 268*a^3*b^3*c*d^4 + 35*a^4*b^2*d^5)*x)*sqrt(d*x + c
))/(a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a
^9*b^3*d^4 + (b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*
d^3 + a^4*b^8*d^4)*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d
^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*
d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*x^3 + 10*(a^3*b^9*c^
4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*x^
2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3
+ a^8*b^4*d^4)*x), -1/640*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d
^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(-b^2*c + a*b*d)
*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (128*b^6*c^5 -
```

```
464*a*b^5*c^4*d + 584*a^2*b^4*c^3*d^2 - 258*a^3*b^3*c^2*d^3 - 5*a^4*b^2*c*d
^4 + 15*a^5*b*d^5 + 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 - 10*(b^6*c^2*d^3 - 8*a*
b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 87*a
^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(88*b^6*c^4*d - 344*a*b^5*c^3*d^2 +
489*a^2*b^4*c^2*d^3 - 268*a^3*b^3*c*d^4 + 35*a^4*b^2*d^5)*x)*sqrt(d*x + c))
/(a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9
*b^3*d^4 + (b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*d^
3 + a^4*b^8*d^4)*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d^2
 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*d
+ 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*x^3 + 10*(a^3*b^9*c^4
- 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*x^2
+ 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 +
a^8*b^4*d^4)*x)]
```

**giac** [B] time = 1.39, size = 410, normalized size = 1.97

$$-\frac{3\,d^{5}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{128\,\left(b^{5}c^{3}-3\,ab^{4}c^{2}d+3\,a^{2}b^{3}cd^{2}-a^{3}b^{2}d^{3}\right)\sqrt{-b^{2}c+abd}}-\frac{15\,(dx+c)^{\frac{9}{2}}b^{4}d^{5}-70\,(dx+c)^{\frac{7}{2}}b^{4}cd^{5}+128\,(dx+c)^{\frac{5}{2}}b^{4}c^{2}d^{5}+70\,(dx+c)^{\frac{3}{2}}b^{4}c^{3}d^{5}-15\,\sqrt{dx+c}\,b^{4}c^{4}d^{5}+70\,(dx+c)^{\frac{7}{2}}ab^{3}d^{6}-256\,(dx+c)^{\frac{5}{2}}ab^{3}cd^{6}-210\,(dx+c)^{\frac{3}{2}}ab^{3}c^{2}d^{6}+60\,\sqrt{dx+c}\,ab^{3}c^{3}d^{6}+128\,(dx+c)^{\frac{5}{2}}a^{2}b^{2}d^{7}+210\,(dx+c)^{\frac{3}{2}}a^{2}b^{2}cd^{7}-90\,\sqrt{dx+c}\,a^{2}b^{2}c^{2}d^{7}-70\,(dx+c)^{\frac{3}{2}}a^{3}bd^{8}+60\,\sqrt{dx+c}\,a^{3}bcd^{8}-15\,\sqrt{dx+c}\,a^{4}d^{9}}{640\,\left(b^{5}c^{3}-3\,ab^{4}c^{2}d+3\,a^{2}b^{3}cd^{2}-a^{3}b^{2}d^{3}\right)\left((dx+c)b-bc+ad\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -3/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^3 - 3*a*b^4
*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt(-b^2*c + a*b*d)) - 1/640*(15*(
d*x + c)^(9/2)*b^4*d^5 - 70*(d*x + c)^(7/2)*b^4*c*d^5 + 128*(d*x + c)^(5/2)
*b^4*c^2*d^5 + 70*(d*x + c)^(3/2)*b^4*c^3*d^5 - 15*sqrt(d*x + c)*b^4*c^4*d^
5 + 70*(d*x + c)^(7/2)*a*b^3*d^6 - 256*(d*x + c)^(5/2)*a*b^3*c*d^6 - 210*(d
*x + c)^(3/2)*a*b^3*c^2*d^6 + 60*sqrt(d*x + c)*a*b^3*c^3*d^6 + 128*(d*x + c
)^(5/2)*a^2*b^2*d^7 + 210*(d*x + c)^(3/2)*a^2*b^2*c*d^7 - 90*sqrt(d*x + c)*
a^2*b^2*c^2*d^7 - 70*(d*x + c)^(3/2)*a^3*b*d^8 + 60*sqrt(d*x + c)*a^3*b*c*d
^8 - 15*sqrt(d*x + c)*a^4*d^9)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2
- a^3*b^2*d^3)*((d*x + c)*b - b*c + a*d)^5)
```

**maple** [A] time = 0.02, size = 300, normalized size = 1.44

$$\frac{3\,(dx+c)^{\frac{9}{2}}\,b^{2}d^{5}}{128\,(bdx+ad)^{5}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{7\,(dx+c)^{\frac{7}{2}}\,b\,d^{5}}{64\,(bdx+ad)^{5}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}-\frac{3\sqrt{dx+c}\;a\,d^{6}}{128\,(bdx+ad)^{5}\,b^{2}}+\frac{3\sqrt{dx+c}\;c\,d^{5}}{128\,(bdx+ad)^{5}\,b}+\frac{(dx+c)^{\frac{5}{2}}\,d^{5}}{5\,(bdx+ad)^{5}\,(ad-bc)}-\frac{7\,(dx+c)^{\frac{3}{2}}\,d^{5}}{64\,(bdx+ad)^{5}\,b}+\frac{3d^{5}\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{128\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)\sqrt{(ad-bc)\,b}\;b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^6,x)
```

```
[Out] 3/128*d^5/(b*d*x+a*d)^5*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(
d*x+c)^(9/2)+7/64*d^5/(b*d*x+a*d)^5*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(
7/2)+1/5*d^5/(b*d*x+a*d)^5/(a*d-b*c)*(d*x+c)^(5/2)-7/64*d^5/(b*d*x+a*d)^5/b
*(d*x+c)^(3/2)-3/128*d^6/(b*d*x+a*d)^5/b^2*(d*x+c)^(1/2)*a+3/128*d^5/(b*d*x
```

```
+a*d)^5/b*(d*x+c)^(1/2)*c+3/128*d^5/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)/b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.47, size = 398, normalized size = 1.91

$$\frac{\frac{d^5\,{\left(c+d\,x\right)}^{5/2}}{5\,\left(a\,d-b\,c\right)}-\frac{7\,d^5\,{\left(c+d\,x\right)}^{3/2}}{64\,b}+\frac{3\,b^2\,d^5\,{\left(c+d\,x\right)}^{9/2}}{128\,{\left(a\,d-b\,c\right)}^3}-\frac{3\,d^5\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{128\,b^2}+\frac{7\,b\,d^5\,{\left(c+d\,x\right)}^{7/2}}{64\,{\left(a\,d-b\,c\right)}^2}}{b^5\,{\left(c+d\,x\right)}^5-{\left(c+d\,x\right)}^2\,\left(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3\right)-\left(5\,b^5\,c-5\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^4+a^5\,d^5-b^5\,c^5+{\left(c+d\,x\right)}^3\,\left(10\,a^2\,b^3\,d^2-20\,a\,b^4\,c\,d+10\,b^5\,c^2\right)+\left(c+d\,x\right)\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)-10\,a^2\,b^3\,c^3\,d^2+10\,a^3\,b^2\,c^2\,d^3+5\,a\,b^4\,c^4\,d-5\,a^4\,b\,c\,d^4}+\frac{3\,d^5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^6,x)
```

```
[Out] ((d^5*(c + d*x)^(5/2))/(5*(a*d - b*c)) - (7*d^5*(c + d*x)^(3/2))/(64*b) + (
3*b^2*d^5*(c + d*x)^(9/2))/(128*(a*d - b*c)^3) - (3*d^5*(a*d - b*c)*(c + d*
x)^(1/2))/(128*b^2) + (7*b*d^5*(c + d*x)^(7/2))/(64*(a*d - b*c)^2))/(b^5*(c
 + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 3
0*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(c + d*x)^4 + a^5*d^5 - b^5*c^5 + (c
 + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 - 20*a*b^4*c*d) + (c + d*x)*(5*b^5*c
^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^4*c^3*d)
- 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)
+ (3*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b^(5/2)*(a
*d - b*c)^(7/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

## 3.1293 $\int (a+bx)^5(c+dx)^{5/2}\,dx$

**Optimal**. Leaf size=158

$$-\frac{2b^4(c+dx)^{15/2}(bc-ad)}{3d^6}+\frac{20b^3(c+dx)^{13/2}(bc-ad)^2}{13d^6}-\frac{20b^2(c+dx)^{11/2}(bc-ad)^3}{11d^6}+\frac{10b(c+dx)^{9/2}(bc-ad)^4}{9d^6}-\frac{2(c$$

**Rubi [A]** time = 0.05, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2b^4(c+dx)^{15/2}(bc-ad)}{3d^6}+\frac{20b^3(c+dx)^{13/2}(bc-ad)^2}{13d^6}-\frac{20b^2(c+dx)^{11/2}(bc-ad)^3}{11d^6}+\frac{10b(c+dx)^{9/2}(bc-ad)^4}{9d^6}-\frac{2(c+dx)^{7/2}(bc-ad)^5}{7d^6}+\frac{2b^5(c+dx)^{17/2}}{17d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(7/2))/(7*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(9/2))/(9*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(11/2))/(11*d^6) + (20*b^3
*(b*c - a*d)^2*(c + d*x)^(13/2))/(13*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(1
5/2))/(3*d^6) + (2*b^5*(c + d*x)^(17/2))/(17*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^{5/2}\,dx=\int \left(\frac{(-bc+ad)^5(c+dx)^{5/2}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{7/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{9/2}}{d^5}+\frac{10}{}\right.$$
$$=-\frac{2(bc-ad)^5(c+dx)^{7/2}}{7d^6}+\frac{10b(bc-ad)^4(c+dx)^{9/2}}{9d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{11/2}}{11d^6}+\frac{20}{}$$

**Mathematica [A]** time = 0.11, size = 123, normalized size = 0.78

$$\frac{2(c+dx)^{7/2}\left(-51051b^4(c+dx)^4(bc-ad)+117810b^3(c+dx)^3(bc-ad)^2-139230b^2(c+dx)^2(bc-ad)^3+85085b(c+dx)(bc-ad)^4-21879(bc-ad)^5+9009b^5(c+dx)^5\right)}{153153d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-21879*(b*c - a*d)^5 + 85085*b*(b*c - a*d)^4*(c + d*x)
- 139230*b^2*(b*c - a*d)^3*(c + d*x)^2 + 117810*b^3*(b*c - a*d)^2*(c + d*x)
^3 - 51051*b^4*(b*c - a*d)*(c + d*x)^4 + 9009*b^5*(c + d*x)^5))/(153153*d^6
)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 315, normalized size = 1.99

$$\frac{2(c+dx)^{7/2}\left(21879a^5d^5+85085a^4bd^4(c+dx)-109395a^4bcd^4+218790a^3b^2c^2d^3+139230a^3b^2d^3(c+dx)^2-340340a^3b^2cd^3(c+dx)-218790a^2b^3c^3d^2+510510a^2b^3c^2d^2(c+dx)+117810a^2b^3d^2(c+dx)^3-417690a^2b^3cd^2(c+dx)^2+109395ab^4c^4d-340340ab^4c^3d(c+dx)+417690ab^4c^2d(c+dx)^2+51051ab^4d(c+dx)^4-235620ab^4cd(c+dx)^3-21879b^5c^5+85085b^5c^4(c+dx)-139230b^5c^3(c+dx)^2+117810b^5c^2(c+dx)^3+9009b^5(c+dx)^5-51051b^5c(c+dx)^4\right)}{153153d^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-21879*b^5*c^5 + 109395*a*b^4*c^4*d - 218790*a^2*b^3*c^
3*d^2 + 218790*a^3*b^2*c^2*d^3 - 109395*a^4*b*c*d^4 + 21879*a^5*d^5 + 85085
*b^5*c^4*(c + d*x) - 340340*a*b^4*c^3*d*(c + d*x) + 510510*a^2*b^3*c^2*d^2*
(c + d*x) - 340340*a^3*b^2*c*d^3*(c + d*x) + 85085*a^4*b*d^4*(c + d*x) - 13
9230*b^5*c^3*(c + d*x)^2 + 417690*a*b^4*c^2*d*(c + d*x)^2 - 417690*a^2*b^3*
c*d^2*(c + d*x)^2 + 139230*a^3*b^2*d^3*(c + d*x)^2 + 117810*b^5*c^2*(c + d*
x)^3 - 235620*a*b^4*c*d*(c + d*x)^3 + 117810*a^2*b^3*d^2*(c + d*x)^3 - 5105
1*b^5*c*(c + d*x)^4 + 51051*a*b^4*d*(c + d*x)^4 + 9009*b^5*(c + d*x)^5))/(1
53153*d^6)
```

**fricas [B]** time = 1.29, size = 497, normalized size = 3.15

$$\frac{2\left(9009b^5d^8x^8-256b^5c^8+2176ab^4c^7d-8160a^2b^3c^6d^2+17680a^3b^2c^5d^3-24310a^4bc^4d^4+21879a^5c^3d^5+3003\left(7b^5cd^7+17ab^4d^8\right)x^7+231\left(55b^5c^2d^6+527ab^4cd^7+510a^2b^3d^8\right)x^6+63\left(b^5c^3d^5+1207ab^4c^2d^6+4590a^2b^3cd^7+2210a^3b^2d^8\right)x^5-35\left(2b^5c^4d^4-17ab^4c^3d^5-5406a^2b^3c^2d^6-10166a^3b^2cd^7-2431a^4bd^8\right)x^4+\left(80b^5c^5d^3-680ab^4c^4d^4+2550a^2b^3c^3d^5+249730a^3b^2c^2d^6+230945a^4bcd^7+21879a^5d^8\right)x^3-3\left(32b^5c^6d^2-272ab^4c^5d^3+1020a^2b^3c^4d^4-2210a^3b^2c^3d^5-60775a^4bc^2d^6-21879a^5cd^7\right)x^2+\left(128b^5c^7d-1088ab^4c^6d^2+4080a^2b^3c^5d^3-8840a^3b^2c^4d^4+12155a^4bc^3d^5+65637a^5c^2d^6\right)x\right)\sqrt{dx+c}}{153153d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/153153*(9009*b^5*d^8*x^8 - 256*b^5*c^8 + 2176*a*b^4*c^7*d - 8160*a^2*b^3*
c^6*d^2 + 17680*a^3*b^2*c^5*d^3 - 24310*a^4*b*c^4*d^4 + 21879*a^5*c^3*d^5 +
 3003*(7*b^5*c*d^7 + 17*a*b^4*d^8)*x^7 + 231*(55*b^5*c^2*d^6 + 527*a*b^4*c*
d^7 + 510*a^2*b^3*d^8)*x^6 + 63*(b^5*c^3*d^5 + 1207*a*b^4*c^2*d^6 + 4590*a^
2*b^3*c*d^7 + 2210*a^3*b^2*d^8)*x^5 - 35*(2*b^5*c^4*d^4 - 17*a*b^4*c^3*d^5
- 5406*a^2*b^3*c^2*d^6 - 10166*a^3*b^2*c*d^7 - 2431*a^4*b*d^8)*x^4 + (80*b^
5*c^5*d^3 - 680*a*b^4*c^4*d^4 + 2550*a^2*b^3*c^3*d^5 + 249730*a^3*b^2*c^2*d
^6 + 230945*a^4*b*c*d^7 + 21879*a^5*d^8)*x^3 - 3*(32*b^5*c^6*d^2 - 272*a*b^
4*c^5*d^3 + 1020*a^2*b^3*c^4*d^4 - 2210*a^3*b^2*c^3*d^5 - 60775*a^4*b*c^2*d
^6 - 21879*a^5*c*d^7)*x^2 + (128*b^5*c^7*d - 1088*a*b^4*c^6*d^2 + 4080*a^2*
b^3*c^5*d^3 - 8840*a^3*b^2*c^4*d^4 + 12155*a^4*b*c^3*d^5 + 65637*a^5*c^2*d^
6)*x)*sqrt(d*x + c)/d^6
```

**giac [B]** time = 1.52, size = 1599, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/765765*(765765*sqrt(d*x + c)*a^5*c^3 + 765765*((d*x + c)^(3/2) - 3*sqrt(d
*x + c)*c)*a^5*c^2 + 1276275*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c^
3/d + 153153*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^5*c + 510510*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x
+ c)*c^2)*a^3*b^2*c^3/d^2 + 765765*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*
c + 15*sqrt(d*x + c)*c^2)*a^4*b*c^2/d + 21879*(5*(d*x + c)^(7/2) - 21*(d*x
+ c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^5 + 218790*
(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqr
t(d*x + c)*c^3)*a^2*b^3*c^3/d^3 + 656370*(5*(d*x + c)^(7/2) - 21*(d*x + c)^
(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b^2*c^2/d^2 +
328185*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 -
 35*sqrt(d*x + c)*c^3)*a^4*b*c/d + 12155*(35*(d*x + c)^(9/2) - 180*(d*x + c
)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*
x + c)*c^4)*a*b^4*c^3/d^4 + 72930*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)
*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*
c^4)*a^2*b^3*c^2/d^3 + 72930*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c +
378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*
a^3*b^2*c/d^2 + 12155*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*
x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^4*b/d
 + 1105*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*
c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x +
c)*c^5)*b^5*c^3/d^5 + 16575*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c +
990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c
^4 - 693*sqrt(d*x + c)*c^5)*a*b^4*c^2/d^4 + 33150*(63*(d*x + c)^(11/2) - 38
5*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 +
1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^2*b^3*c/d^3 + 11050*(63
*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*
(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^3
*b^2/d^2 + 765*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x
+ c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006
*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5*c^2/d^5 + 3825*(231*(d*x
 + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d
*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3
003*sqrt(d*x + c)*c^6)*a*b^4*c/d^4 + 2550*(231*(d*x + c)^(13/2) - 1638*(d*x
 + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009
*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*a
^2*b^3/d^3 + 357*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2)*c + 12285*(d
*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x + c)^(7/2)*c^4
- 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 6435*sqrt(d*x + c
)*c^7)*b^5*c/d^5 + 595*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2)*c + 12
```

```
285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x + c)^(7/2
)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 6435*sqrt(d
*x + c)*c^7)*a*b^4/d^4 + 7*(6435*(d*x + c)^(17/2) - 58344*(d*x + c)^(15/2)*
c + 235620*(d*x + c)^(13/2)*c^2 - 556920*(d*x + c)^(11/2)*c^3 + 850850*(d*x
 + c)^(9/2)*c^4 - 875160*(d*x + c)^(7/2)*c^5 + 612612*(d*x + c)^(5/2)*c^6 -
 291720*(d*x + c)^(3/2)*c^7 + 109395*sqrt(d*x + c)*c^8)*b^5/d^5)/d
```

**maple [B]** time = 0.00, size = 273, normalized size = 1.73

$$\frac{2 (dx+c)^{\frac{7}{2}} \left(9009b^5x^5d^5+51051a\,b^4d^5x^4-6006b^5c\,d^4x^4+117810a^2b^3d^5x^3-31416a\,b^4c\,d^4x^3+3696b^5c^2d^3x^3+139230a^3b^2d^5x^2-64260a^2b^3c\,d^4x^2+17136a\,b^4c^2d^3x^2-2016b^5c^3d^2x^2+85085a^4b\,d^5x-61880a^3b^2c\,d^4x+28560a^2b^3c^2d^3x-7616a\,b^4c^3d^2x+896b^5c^4dx+21879a^5d^5-24310a^4bc\,d^4+17680a^3b^2c^2d^3-8160a^2b^3c^3d^2+2176a\,b^4c^4d-256b^5c^5\right)}{153153d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(5/2),x)
```

```
[Out] 2/153153*(d*x+c)^(7/2)*(9009*b^5*d^5*x^5+51051*a*b^4*d^5*x^4-6006*b^5*c*d^4
*x^4+117810*a^2*b^3*d^5*x^3-31416*a*b^4*c*d^4*x^3+3696*b^5*c^2*d^3*x^3+1392
30*a^3*b^2*d^5*x^2-64260*a^2*b^3*c*d^4*x^2+17136*a*b^4*c^2*d^3*x^2-2016*b^5
*c^3*d^2*x^2+85085*a^4*b*d^5*x-61880*a^3*b^2*c*d^4*x+28560*a^2*b^3*c^2*d^3*
x-7616*a*b^4*c^3*d^2*x+896*b^5*c^4*d*x+21879*a^5*d^5-24310*a^4*b*c*d^4+1768
0*a^3*b^2*c^2*d^3-8160*a^2*b^3*c^3*d^2+2176*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

**maxima [A]** time = 1.36, size = 259, normalized size = 1.64

$$\frac{2 \left(9009 (dx+c)^{\frac{17}{2}} b^5-51051 \left(b^5c-ab^4d\right)(dx+c)^{\frac{15}{2}}+117810 \left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)(dx+c)^{\frac{13}{2}}-139230 \left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)(dx+c)^{\frac{11}{2}}+85085 \left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)(dx+c)^{\frac{9}{2}}-21879 \left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5\right)(dx+c)^{\frac{7}{2}}\right)}{153153\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/153153*(9009*(d*x + c)^(17/2)*b^5 - 51051*(b^5*c - a*b^4*d)*(d*x + c)^(15
/2) + 117810*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(13/2) - 13923
0*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(11/2
) + 85085*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)*(d*x + c)^(9/2) - 21879*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^
3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(7/2))/d^6
```

**mupad [B]** time = 0.27, size = 137, normalized size = 0.87

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{17/2}}{17\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{15/2}}{15\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{13/2}}{13\,d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(5/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(17/2))/(17*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(15/
2))/(15*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(7/2))/(7*d^6) + (20*b^2*(a*d - b
```

```
*c)^3*(c + d*x)^(11/2))/(11*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(13/2))/
(13*d^6) + (10*b*(a*d - b*c)^4*(c + d*x)^(9/2))/(9*d^6)
```

**sympy [A]** time = 43.08, size = 1292, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(5/2),x)

[Out] a**5*c**2*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)
) + 4*a**5*c*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*a**5*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d + 10*a*
*4*b*c**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 20*a**4*b*c*(
c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2
 + 10*a**4*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c
 + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**2 + 20*a**3*b**2*c**2*(c**2*(c +
d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 40*a**3
*b**2*c*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*
x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 20*a**3*b**2*(c**4*(c + d*x)**(3/2
)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)
**(9/2)/9 + (c + d*x)**(11/2)/11)/d**3 + 20*a**2*b**3*c**2*(-c**3*(c + d*x)
**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)*
*(9/2)/9)/d**4 + 40*a**2*b**3*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)
**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)*
*(11/2)/11)/d**4 + 20*a**2*b**3*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)*
*(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c +
 d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**4 + 10*a*b**4*c**2*(c**4*(c + d
*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*
(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11)/d**5 + 20*a*b**4*c*(-c**5*(c + d
*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2
*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5
 + 10*a*b**4*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4
*(c + d*x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2
)/11 - 6*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**5 + 2*b**5*c**2*
(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2
)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(1
3/2)/13)/d**6 + 4*b**5*c*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)
/5 + 15*c**4*(c + d*x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c +
 d*x)**(11/2)/11 - 6*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**6 +
2*b**5*(-c**7*(c + d*x)**(3/2)/3 + 7*c**6*(c + d*x)**(5/2)/5 - 3*c**5*(c +
d*x)**(7/2) + 35*c**4*(c + d*x)**(9/2)/9 - 35*c**3*(c + d*x)**(11/2)/11 + 2
1*c**2*(c + d*x)**(13/2)/13 - 7*c*(c + d*x)**(15/2)/15 + (c + d*x)**(17/2)/
17)/d**6
```

# 3.1294 $\int (a+bx)^4(c+dx)^{5/2}\,dx$

**Optimal**. Leaf size=129

$$-\frac{8b^3(c+dx)^{13/2}(bc-ad)}{13d^5}+\frac{12b^2(c+dx)^{11/2}(bc-ad)^2}{11d^5}-\frac{8b(c+dx)^{9/2}(bc-ad)^3}{9d^5}+\frac{2(c+dx)^{7/2}(bc-ad)^4}{7d^5}+\frac{2b^4(c+\cdots}{15\cdots}$$

**Rubi** [A] time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{13/2}(bc-ad)}{13d^5}+\frac{12b^2(c+dx)^{11/2}(bc-ad)^2}{11d^5}-\frac{8b(c+dx)^{9/2}(bc-ad)^3}{9d^5}+\frac{2(c+dx)^{7/2}(bc-ad)^4}{7d^5}+\frac{2b^4(c+dx)^{15/2}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(7/2))/(7*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(9
/2))/(9*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^5) - (8*b^3*(b
*c - a*d)*(c + d*x)^(13/2))/(13*d^5) + (2*b^4*(c + d*x)^(15/2))/(15*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^4(c+dx)^{5/2}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{7/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{9/2}}{d^4}-\frac{4\cdots}{\cdots}\right.\\ &= \frac{2(bc-ad)^4(c+dx)^{7/2}}{7d^5}-\frac{8b(bc-ad)^3(c+dx)^{9/2}}{9d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{11/2}}{11d^5}-\frac{8b^3\cdots}{\cdots}\end{aligned}$$

**Mathematica** [A] time = 0.11, size = 101, normalized size = 0.78

$$\frac{2(c+dx)^{7/2}\left(-13860b^3(c+dx)^3(bc-ad)+24570b^2(c+dx)^2(bc-ad)^2-20020b(c+dx)(bc-ad)^3+6435(bc-ad)^4+3003b^4(c+dx)^4\right)}{45045d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(6435*(b*c - a*d)^4 - 20020*b*(b*c - a*d)^3*(c + d*x) +
24570*b^2*(b*c - a*d)^2*(c + d*x)^2 - 13860*b^3*(b*c - a*d)*(c + d*x)^3 + 3
003*b^4*(c + d*x)^4))/(45045*d^5)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 213, normalized size = 1.65

$$\frac{2(c+dx)^{7/2}\left(6435a^4d^4+20020a^3bd^3(c+dx)-25740a^3bcd^3+38610a^2b^2c^2d^2+24570a^2b^2d^2(c+dx)^2-60060a^2b^2cd^2(c+dx)-25740ab^3c^3d+60060ab^3c^2d(c+dx)+13860ab^3d(c+dx)^3-49140ab^3cd(c+dx)^2+6435b^4c^4-20020b^4c^3(c+dx)+24570b^4c^2(c+dx)^2+3003b^4(c+dx)^4-13860b^4c(c+dx)^3\right)}{45045d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(6435*b^4*c^4 - 25740*a*b^3*c^3*d + 38610*a^2*b^2*c^2*d^
2 - 25740*a^3*b*c*d^3 + 6435*a^4*d^4 - 20020*b^4*c^3*(c + d*x) + 60060*a*b^
3*c^2*d*(c + d*x) - 60060*a^2*b^2*c*d^2*(c + d*x) + 20020*a^3*b*d^3*(c + d*
x) + 24570*b^4*c^2*(c + d*x)^2 - 49140*a*b^3*c*d*(c + d*x)^2 + 24570*a^2*b^
2*d^2*(c + d*x)^2 - 13860*b^4*c*(c + d*x)^3 + 13860*a*b^3*d*(c + d*x)^3 + 3
003*b^4*(c + d*x)^4))/(45045*d^5)
```

**fricas [B]** time = 1.38, size = 377, normalized size = 2.92

$$\frac{2\left(3003\,b^4d^7x^7+128\,b^4c^7-960\,ab^3c^6d+3120\,a^2b^2c^5d^2-5720\,a^3bc^4d^3+6435\,a^4c^3d^4+231\left(31\,b^4cd^6+60\,ab^3d^7\right)x^6+63\left(71\,b^4c^2d^5+540\,ab^3cd^6+390\,a^2b^2d^7\right)x^5+35\left(b^4c^3d^4+636\,ab^3c^2d^5+1794\,a^2b^2cd^6+572\,a^3bd^7\right)x^4-5\left(8\,b^4c^4d^3-60\,ab^3c^3d^4-8814\,a^2b^2c^2d^5-10868\,a^3bcd^6-1287\,a^4d^7\right)x^3+3\left(16\,b^4c^5d^2-120\,ab^3c^4d^3+390\,a^2b^2c^3d^4+14300\,a^3bc^2d^5+6435\,a^4cd^6\right)x^2-\left(64\,b^4c^6d-480\,ab^3c^5d^2+1560\,a^2b^2c^4d^3-2860\,a^3bc^3d^4-19305\,a^4c^2d^5\right)x\right)\sqrt{dx+c}}{45045\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^4*d^7*x^7 + 128*b^4*c^7 - 960*a*b^3*c^6*d + 3120*a^2*b^2*c^
5*d^2 - 5720*a^3*b*c^4*d^3 + 6435*a^4*c^3*d^4 + 231*(31*b^4*c*d^6 + 60*a*b^
3*d^7)*x^6 + 63*(71*b^4*c^2*d^5 + 540*a*b^3*c*d^6 + 390*a^2*b^2*d^7)*x^5 +
35*(b^4*c^3*d^4 + 636*a*b^3*c^2*d^5 + 1794*a^2*b^2*c*d^6 + 572*a^3*b*d^7)*x
^4 - 5*(8*b^4*c^4*d^3 - 60*a*b^3*c^3*d^4 - 8814*a^2*b^2*c^2*d^5 - 10868*a^3
*b*c*d^6 - 1287*a^4*d^7)*x^3 + 3*(16*b^4*c^5*d^2 - 120*a*b^3*c^4*d^3 + 390*
a^2*b^2*c^3*d^4 + 14300*a^3*b*c^2*d^5 + 6435*a^4*c*d^6)*x^2 - (64*b^4*c^6*d
 - 480*a*b^3*c^5*d^2 + 1560*a^2*b^2*c^4*d^3 - 2860*a^3*b*c^3*d^4 - 19305*a^
4*c^2*d^5)*x)*sqrt(d*x + c)/d^5
```

**giac [B]** time = 1.45, size = 1204, normalized size = 9.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(d*x + c)*a^4*c^3 + 45045*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^4*c^2 + 60060*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c^3/d +
 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^4
```

```
*c + 18018*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2
)*a^2*b^2*c^3/d^2 + 36036*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sq
rt(d*x + c)*c^2)*a^3*b*c^2/d + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)
*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^4 + 5148*(5*(d*x + c)
^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c
^3)*a*b^3*c^3/d^3 + 23166*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d
*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^2*c^2/d^2 + 15444*(5*(d*x +
 c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c
)*c^3)*a^3*b*c/d + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d
*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c^
3/d^4 + 1716*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5
/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^3*c^2/d^3 +
2574*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2
 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b^2*c/d^2 + 572*(35*
(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*
x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^3*b/d + 195*(63*(d*x + c)^(11/2
) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*
c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^4*c^2/d^4 + 780*(
63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 138
6*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a
*b^3*c/d^3 + 390*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x +
c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sq
rt(d*x + c)*c^5)*a^2*b^2/d^2 + 45*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(1
1/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x +
c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^4*c/d^4
 + 60*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2
)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c
)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*a*b^3/d^3 + 7*(429*(d*x + c)^(15/2) -
 3465*(d*x + c)^(13/2)*c + 12285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/
2)*c^3 + 32175*(d*x + c)^(7/2)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x
 + c)^(3/2)*c^6 - 6435*sqrt(d*x + c)*c^7)*b^4/d^4)/d
```

**maple [A]** time = 0.01, size = 186, normalized size = 1.44

$$\frac{2 \left(d x + c\right)^{\frac{7}{2}} \left(3003 b^{4} x^{4} d^{4} + 13860 a\, b^{3} d^{4} x^{3} - 1848 b^{4} c\, d^{3} x^{3} + 24570 a^{2} b^{2} d^{4} x^{2} - 7560 a\, b^{3} c\, d^{3} x^{2} + 1008 b^{4} c^{2} d^{2} x^{2} + 20020 a^{3} b\, d^{4} x - 10920 a^{2} b^{2} c\, d^{3} x + 3360 a\, b^{3} c^{2} d^{2} x - 448 b^{4} c^{3} d x + 6435 a^{4} d^{4} - 5720 a^{3} b c\, d^{3} + 3120 a^{2} b^{2} c^{2} d^{2} - 960 a\, b^{3} c^{3} d + 128 b^{4} c^{4}\right)}{45045 d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(5/2),x)
```

```
[Out] 2/45045*(d*x+c)^(7/2)*(3003*b^4*d^4*x^4+13860*a*b^3*d^4*x^3-1848*b^4*c*d^3*
x^3+24570*a^2*b^2*d^4*x^2-7560*a*b^3*c*d^3*x^2+1008*b^4*c^2*d^2*x^2+20020*a
^3*b*d^4*x-10920*a^2*b^2*c*d^3*x+3360*a*b^3*c^2*d^2*x-448*b^4*c^3*d*x+6435*
a^4*d^4-5720*a^3*b*c*d^3+3120*a^2*b^2*c^2*d^2-960*a*b^3*c^3*d+128*b^4*c^4)/
d^5
```

**maxima** [A] time = 1.33, size = 181, normalized size = 1.40

$$\frac{2\left(3003\,(dx+c)^{\frac{15}{2}}b^4-13860\left(b^4c-ab^3d\right)(dx+c)^{\frac{13}{2}}+24570\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)(dx+c)^{\frac{11}{2}}-20020\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)(dx+c)^{\frac{9}{2}}+6435\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)(dx+c)^{\frac{7}{2}}\right)}{45045\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(d*x + c)^(15/2)*b^4 - 13860*(b^4*c - a*b^3*d)*(d*x + c)^(13/
2) + 24570*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(11/2) - 20020*(
b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(9/2) + 64
35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
(d*x + c)^(7/2))/d^5
```

**mupad** [B] time = 0.23, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{15/2}}{15\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{13/2}}{13\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{11/2}}{11\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{9/2}}{9\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^(5/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(15/2))/(15*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(13/2)
)/(13*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(7/2))/(7*d^5) + (12*b^2*(a*d - b*c
)^2*(c + d*x)^(11/2))/(11*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(9/2))/(9*d^5
)
```

**sympy** [A] time = 33.64, size = 960, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(5/2),x)
```

```
[Out] a**4*c**2*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)
) + 4*a**4*c*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*a**4*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d + 8*a**
3*b*c**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 16*a**3*b*c*(c
**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2
+ 8*a**3*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c +
 d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**2 + 12*a**2*b**2*c**2*(c**2*(c + d*
x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 24*a**2*b
**2*c*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)
**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 12*a**2*b**2*(c**4*(c + d*x)**(3/2)/
3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**
(9/2)/9 + (c + d*x)**(11/2)/11)/d**3 + 8*a*b**3*c**2*(-c**3*(c + d*x)**(3/2
```

```
)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)
/9)/d**4 + 16*a*b**3*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5
 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/1
1)/d**4 + 8*a*b**3*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c
**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)
/11 + (c + d*x)**(13/2)/13)/d**4 + 2*b**4*c**2*(c**4*(c + d*x)**(3/2)/3 - 4
*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)
/9 + (c + d*x)**(11/2)/11)/d**5 + 4*b**4*c*(-c**5*(c + d*x)**(3/2)/3 + c**4
*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9
 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5 + 2*b**4*(c**6*(c
+ d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4*(c + d*x)**(7/2)/7 -
20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2)/11 - 6*c*(c + d*x)**
(13/2)/13 + (c + d*x)**(15/2)/15)/d**5
```

## 3.1295 $\int (a+bx)^3(c+dx)^{5/2}\,dx$

**Optimal**. Leaf size=100

$$-\frac{6b^2(c+dx)^{11/2}(bc-ad)}{11d^4}+\frac{2b(c+dx)^{9/2}(bc-ad)^2}{3d^4}-\frac{2(c+dx)^{7/2}(bc-ad)^3}{7d^4}+\frac{2b^3(c+dx)^{13/2}}{13d^4}$$

**Rubi [A]** time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2(c+dx)^{11/2}(bc-ad)}{11d^4}+\frac{2b(c+dx)^{9/2}(bc-ad)^2}{3d^4}-\frac{2(c+dx)^{7/2}(bc-ad)^3}{7d^4}+\frac{2b^3(c+dx)^{13/2}}{13d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^4) + (2*b*(b*c - a*d)^2*(c + d*x)^(
9/2))/(3*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(11/2))/(11*d^4) + (2*b^3*(c +
 d*x)^(13/2))/(13*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{5/2}\,dx=\int \left(\frac{(-bc+ad)^3(c+dx)^{5/2}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{7/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{9/2}}{d^3}+\frac{b^3(c$$
$$=-\frac{2(bc-ad)^3(c+dx)^{7/2}}{7d^4}+\frac{2b(bc-ad)^2(c+dx)^{9/2}}{3d^4}-\frac{6b^2(bc-ad)(c+dx)^{11/2}}{11d^4}+\frac{2b^3(c$$

**Mathematica [A]** time = 0.07, size = 79, normalized size = 0.79

$$\frac{2(c+dx)^{7/2}\left(-819b^2(c+dx)^2(bc-ad)+1001b(c+dx)(bc-ad)^2-429(bc-ad)^3+231b^3(c+dx)^3\right)}{3003d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-429*(b*c - a*d)^3 + 1001*b*(b*c - a*d)^2*(c + d*x) - 8
19*b^2*(b*c - a*d)*(c + d*x)^2 + 231*b^3*(c + d*x)^3))/(3003*d^4)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 132, normalized size = 1.32

$$\frac{2(c+dx)^{7/2}\left(429a^3d^3+1001a^2bd^2(c+dx)-1287a^2bcd^2+1287ab^2c^2d+819ab^2d(c+dx)^2-2002ab^2cd(c+dx)-429b^3c^3+1001b^3c^2(c+dx)+231b^3(c+dx)^3-819b^3c(c+dx)^2\right)}{3003d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-429*b^3*c^3 + 1287*a*b^2*c^2*d - 1287*a^2*b*c*d^2 + 42
9*a^3*d^3 + 1001*b^3*c^2*(c + d*x) - 2002*a*b^2*c*d*(c + d*x) + 1001*a^2*b*
d^2*(c + d*x) - 819*b^3*c*(c + d*x)^2 + 819*a*b^2*d*(c + d*x)^2 + 231*b^3*(
c + d*x)^3))/(3003*d^4)
```

**fricas [B]** time = 1.54, size = 268, normalized size = 2.68

$$\frac{2\left(231b^3d^6x^6-16b^3c^6+104ab^2c^5d-286a^2bc^4d^2+429a^3c^3d^3+63\left(9b^3cd^5+13ab^2d^6\right)x^5+7\left(53b^3c^2d^4+299ab^2cd^5+143a^2bd^6\right)x^4+\left(5b^3c^3d^3+1469ab^2c^2d^4+2717a^2bcd^5+429a^3d^6\right)x^3-3\left(2b^3c^4d^2-13ab^2c^3d^3-715a^2bc^2d^4-429a^3cd^5\right)x^2+\left(8b^3c^5d-52ab^2c^4d^2+143a^2bc^3d^3+1287a^3c^2d^4\right)x\right)\sqrt{dx+c}}{3003d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*d^6*x^6 - 16*b^3*c^6 + 104*a*b^2*c^5*d - 286*a^2*b*c^4*d^2
+ 429*a^3*c^3*d^3 + 63*(9*b^3*c*d^5 + 13*a*b^2*d^6)*x^5 + 7*(53*b^3*c^2*d^4
 + 299*a*b^2*c*d^5 + 143*a^2*b*d^6)*x^4 + (5*b^3*c^3*d^3 + 1469*a*b^2*c^2*d
^4 + 2717*a^2*b*c*d^5 + 429*a^3*d^6)*x^3 - 3*(2*b^3*c^4*d^2 - 13*a*b^2*c^3*
d^3 - 715*a^2*b*c^2*d^4 - 429*a^3*c*d^5)*x^2 + (8*b^3*c^5*d - 52*a*b^2*c^4*
d^2 + 143*a^2*b*c^3*d^3 + 1287*a^3*c^2*d^4)*x)*sqrt(d*x + c)/d^4
```

**giac [B]** time = 1.58, size = 857, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15015*(15015*sqrt(d*x + c)*a^3*c^3 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^3*c^2 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c^3/d +
 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3
*c + 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)
*a*b^2*c^3/d^2 + 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
*x + c)*c^2)*a^2*b*c^2/d + 429*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3 + 429*(5*(d*x + c)^(7/2)
 - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^
```

```
3*c^3/d^3 + 3861*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(
3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2*c^2/d^2 + 3861*(5*(d*x + c)^(7/2) -
21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b
*c/d + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2
)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^3*c^2/d^3 + 429*
(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420
*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^2*c/d^2 + 143*(35*(d*x +
c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^
(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b/d + 65*(63*(d*x + c)^(11/2) - 385*
(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 11
55*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^3*c/d^3 + 65*(63*(d*x + c
)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)
^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^2/d^2 +
5*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^
2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3
/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^3/d^3)/d
```

**maple** [A] time = 0.01, size = 116, normalized size = 1.16

$$\frac{2\,(dx+c)^{\frac{7}{2}}\left(231b^3x^3d^3+819a\,b^2d^3x^2-126b^3c\,d^2x^2+1001a^2b\,d^3x-364a\,b^2c\,d^2x+56b^3c^2dx+429a^3d^3-286a^2bc\,d^2+104a\,b^2c^2d-16b^3c^3\right)}{3003d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(5/2),x)
```

```
[Out] 2/3003*(d*x+c)^(7/2)*(231*b^3*d^3*x^3+819*a*b^2*d^3*x^2-126*b^3*c*d^2*x^2+1
001*a^2*b*d^3*x-364*a*b^2*c*d^2*x+56*b^3*c^2*d*x+429*a^3*d^3-286*a^2*b*c*d^
2+104*a*b^2*c^2*d-16*b^3*c^3)/d^4
```

**maxima** [A] time = 1.40, size = 118, normalized size = 1.18

$$\frac{2\left(231\,(dx+c)^{\frac{13}{2}}b^3-819\left(b^3c-ab^2d\right)(dx+c)^{\frac{11}{2}}+1001\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)(dx+c)^{\frac{9}{2}}-429\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)(dx+c)^{\frac{7}{2}}\right)}{3003\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(231*(d*x + c)^(13/2)*b^3 - 819*(b^3*c - a*b^2*d)*(d*x + c)^(11/2) +
 1001*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(9/2) - 429*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(7/2))/d^4
```

**mupad** [B] time = 0.08, size = 87, normalized size = 0.87

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{13/2}}{13\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{11/2}}{11\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}+\frac{2\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{9/2}}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(13/2))/(13*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(11/2)
)/(11*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(7/2))/(7*d^4) + (2*b*(a*d - b*c)^2
*(c + d*x)^(9/2))/(3*d^4)
```

**sympy [A]** time = 4.61, size = 549, normalized size = 5.49

$$\begin{cases} \frac{2a^3c^3\sqrt{c+dx}}{7d} + \frac{6a^3c^2x\sqrt{c+dx}}{7} + \frac{6a^3cdx^2\sqrt{c+dx}}{7} + \frac{2a^3d^2x^3\sqrt{c+dx}}{7} - \frac{4a^2bc^4\sqrt{c+dx}}{21d^2} + \frac{2a^2bc^3x\sqrt{c+dx}}{21d} + \frac{10a^2bc^2x^2\sqrt{c+dx}}{7} + \frac{38a^2bcdx^3\sqrt{c+dx}}{21} + \frac{2a^2bd^2x^4\sqrt{c+dx}}{3} + \frac{16ab^2c^5\sqrt{c+dx}}{231d^3} - \frac{8ab^2c^4x\sqrt{c+dx}}{231d^2} + \frac{2ab^2c^3x^2\sqrt{c+dx}}{77d} + \frac{226ab^2c^2x^3\sqrt{c+dx}}{231} + \frac{46ab^2cdx^4\sqrt{c+dx}}{33} + \frac{6ab^2d^2x^5\sqrt{c+dx}}{11} - \frac{32b^3c^6\sqrt{c+dx}}{3003d^4} + \frac{16b^3c^5x\sqrt{c+dx}}{3003d^3} - \frac{4b^3c^4x^2\sqrt{c+dx}}{1001d^2} + \frac{10b^3c^3x^3\sqrt{c+dx}}{3003d} + \frac{106b^3c^2x^4\sqrt{c+dx}}{429} + \frac{54b^3cdx^5\sqrt{c+dx}}{143} + \frac{2b^3d^2x^6\sqrt{c+dx}}{13} & \text{for}\: d \neq 0 \\ c^{\frac{5}{2}}\left(a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a**3*c**3*sqrt(c + d*x)/(7*d) + 6*a**3*c**2*x*sqrt(c + d*x)/7
+ 6*a**3*c*d*x**2*sqrt(c + d*x)/7 + 2*a**3*d**2*x**3*sqrt(c + d*x)/7 - 4*a*
*2*b*c**4*sqrt(c + d*x)/(21*d**2) + 2*a**2*b*c**3*x*sqrt(c + d*x)/(21*d) +
10*a**2*b*c**2*x**2*sqrt(c + d*x)/7 + 38*a**2*b*c*d*x**3*sqrt(c + d*x)/21 +
 2*a**2*b*d**2*x**4*sqrt(c + d*x)/3 + 16*a*b**2*c**5*sqrt(c + d*x)/(231*d**
3) - 8*a*b**2*c**4*x*sqrt(c + d*x)/(231*d**2) + 2*a*b**2*c**3*x**2*sqrt(c +
 d*x)/(77*d) + 226*a*b**2*c**2*x**3*sqrt(c + d*x)/231 + 46*a*b**2*c*d*x**4*
sqrt(c + d*x)/33 + 6*a*b**2*d**2*x**5*sqrt(c + d*x)/11 - 32*b**3*c**6*sqrt(
c + d*x)/(3003*d**4) + 16*b**3*c**5*x*sqrt(c + d*x)/(3003*d**3) - 4*b**3*c*
*4*x**2*sqrt(c + d*x)/(1001*d**2) + 10*b**3*c**3*x**3*sqrt(c + d*x)/(3003*d
) + 106*b**3*c**2*x**4*sqrt(c + d*x)/429 + 54*b**3*c*d*x**5*sqrt(c + d*x)/1
43 + 2*b**3*d**2*x**6*sqrt(c + d*x)/13, Ne(d, 0)), (c**(5/2)*(a**3*x + 3*a*
*2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4), True))
```

## 3.1296 $\int (a+bx)^2(c+dx)^{5/2}\,dx$

**Optimal**. Leaf size=71

$$-\frac{4b(c+dx)^{9/2}(bc-ad)}{9d^3}+\frac{2(c+dx)^{7/2}(bc-ad)^2}{7d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}$$

**Rubi [A]** time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{9/2}(bc-ad)}{9d^3}+\frac{2(c+dx)^{7/2}(bc-ad)^2}{7d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(9/2
))/(9*d^3) + (2*b^2*(c + d*x)^(11/2))/(11*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{5/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{7/2}}{d^2}+\frac{b^2(c+dx)^{9/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{7/2}}{7d^3}-\frac{4b(bc-ad)(c+dx)^{9/2}}{9d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{7/2}\left(99a^2d^2+22abd(7dx-2c)+b^2\left(8c^2-28cdx+63d^2x^2\right)\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-2*c + 7*d*x) + b^2*(8*c^2 - 28*
c*d*x + 63*d^2*x^2)))/(693*d^3)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 72, normalized size = 1.01

$$\frac{2(c+dx)^{7/2}\left(99a^2d^2+154abd(c+dx)-198abcd+99b^2c^2+63b^2(c+dx)^2-154b^2c(c+dx)\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(99*b^2*c^2 - 198*a*b*c*d + 99*a^2*d^2 - 154*b^2*c*(c +
d*x) + 154*a*b*d*(c + d*x) + 63*b^2*(c + d*x)^2))/(693*d^3)
```

**fricas [B]** time = 1.23, size = 174, normalized size = 2.45

$$\frac{2\left(63\,b^2d^5x^5+8\,b^2c^5-44\,abc^4d+99\,a^2c^3d^2+7\left(23\,b^2cd^4+22\,abd^5\right)x^4+\left(113\,b^2c^2d^3+418\,abcd^4+99\,a^2d^5\right)x^3+3\left(b^2c^3d^2+110\,abc^2d^3+99\,a^2cd^4\right)x^2-\left(4\,b^2c^4d-22\,abc^3d^2-297\,a^2c^2d^3\right)x\right)\sqrt{dx+c}}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^2*d^5*x^5 + 8*b^2*c^5 - 44*a*b*c^4*d + 99*a^2*c^3*d^2 + 7*(23*b
^2*c*d^4 + 22*a*b*d^5)*x^4 + (113*b^2*c^2*d^3 + 418*a*b*c*d^4 + 99*a^2*d^5)
*x^3 + 3*(b^2*c^3*d^2 + 110*a*b*c^2*d^3 + 99*a^2*c*d^4)*x^2 - (4*b^2*c^4*d
- 22*a*b*c^3*d^2 - 297*a^2*c^2*d^3)*x)*sqrt(d*x + c)/d^3
```

**giac [B]** time = 1.76, size = 558, normalized size = 7.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^2*c^3 + 3465*((d*x + c)^(3/2) - 3*sqrt(d*x + c
)*c)*a^2*c^2 + 2310*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c^3/d + 693*(
3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*c + 23
1*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c^3
/d^2 + 1386*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a*b*c^2/d + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^
(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2 + 297*(5*(d*x + c)^(7/2) - 21*(d*x +
c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^2*c^2/d^2 + 5
94*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*
sqrt(d*x + c)*c^3)*a*b*c/d + 33*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c
 + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^
```

```
4)*b^2*c/d^2 + 22*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x +
c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b/d + 5*(
63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 138
6*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b
^2/d^2)/d
```

**maple** [A] time = 0.01, size = 63, normalized size = 0.89

$$\frac{2 (dx+c)^{\frac{7}{2}} \left(63b^2x^2d^2+154ab\,d^2x-28b^2cdx+99a^2d^2-44abcd+8b^2c^2\right)}{693d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(5/2),x)
```

```
[Out] 2/693*(d*x+c)^(7/2)*(63*b^2*d^2*x^2+154*a*b*d^2*x-28*b^2*c*d*x+99*a^2*d^2-4
4*a*b*c*d+8*b^2*c^2)/d^3
```

**maxima** [A] time = 1.38, size = 68, normalized size = 0.96

$$\frac{2 \left(63 (dx+c)^{\frac{11}{2}} b^2 - 154 \left(b^2c - abd\right)(dx+c)^{\frac{9}{2}} + 99 \left(b^2c^2 - 2\,abcd + a^2d^2\right)(dx+c)^{\frac{7}{2}}\right)}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(d*x + c)^(11/2)*b^2 - 154*(b^2*c - a*b*d)*(d*x + c)^(9/2) + 99*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(7/2))/d^3
```

**mupad** [B] time = 0.07, size = 68, normalized size = 0.96

$$\frac{2\,(c+d\,x)^{7/2}\,\left(63\,b^2\,(c+d\,x)^2+99\,a^2\,d^2+99\,b^2\,c^2-154\,b^2\,c\,\,(c+d\,x)+154\,a\,b\,d\,\,(c+d\,x)-198\,a\,b\,c\,d\right)}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(7/2)*(63*b^2*(c + d*x)^2 + 99*a^2*d^2 + 99*b^2*c^2 - 154*b^2*
c*(c + d*x) + 154*a*b*d*(c + d*x) - 198*a*b*c*d))/(693*d^3)
```

**sympy** [A] time = 3.58, size = 355, normalized size = 5.00

$$\begin{cases} \frac{2a^2c^3\sqrt{c+dx}}{7d} + \frac{6a^2c^2x\sqrt{c+dx}}{7} + \frac{6a^2cdx^2\sqrt{c+dx}}{7} + \frac{2a^2d^2x^3\sqrt{c+dx}}{7} - \frac{8abc^4\sqrt{c+dx}}{63d^2} + \frac{4abc^3x\sqrt{c+dx}}{63d} + \frac{20abc^2x^2\sqrt{c+dx}}{21} + \frac{76abcdx^3\sqrt{c+dx}}{63} + \frac{4abd^2x^4\sqrt{c+dx}}{9} + \frac{16b^2c^5\sqrt{c+dx}}{693d^3} - \frac{8b^2c^4x\sqrt{c+dx}}{693d^2} + \frac{2b^2c^3x^2\sqrt{c+dx}}{231d} + \frac{226b^2c^2x^3\sqrt{c+dx}}{693} + \frac{46b^2cdx^4\sqrt{c+dx}}{99} + \frac{2b^2d^2x^5\sqrt{c+dx}}{11} & \text{for } d \neq 0 \\ c^{\frac{5}{2}}\left(a^2x + abx^2 + \frac{b^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a**2*c**3*sqrt(c + d*x)/(7*d) + 6*a**2*c**2*x*sqrt(c + d*x)/7
+ 6*a**2*c*d*x**2*sqrt(c + d*x)/7 + 2*a**2*d**2*x**3*sqrt(c + d*x)/7 - 8*a*
b*c**4*sqrt(c + d*x)/(63*d**2) + 4*a*b*c**3*x*sqrt(c + d*x)/(63*d) + 20*a*b
*c**2*x**2*sqrt(c + d*x)/21 + 76*a*b*c*d*x**3*sqrt(c + d*x)/63 + 4*a*b*d**2
*x**4*sqrt(c + d*x)/9 + 16*b**2*c**5*sqrt(c + d*x)/(693*d**3) - 8*b**2*c**4
*x*sqrt(c + d*x)/(693*d**2) + 2*b**2*c**3*x**2*sqrt(c + d*x)/(231*d) + 226*
b**2*c**2*x**3*sqrt(c + d*x)/693 + 46*b**2*c*d*x**4*sqrt(c + d*x)/99 + 2*b*
*2*d**2*x**5*sqrt(c + d*x)/11, Ne(d, 0)), (c**(5/2)*(a**2*x + a*b*x**2 + b*
*2*x**3/3), True))
```

## 3.1297 $\int (a+bx)(c+dx)^{5/2}\,dx$

**Optimal**. Leaf size=42

$$\frac{2b(c+dx)^{9/2}}{9d^2}-\frac{2(c+dx)^{7/2}(bc-ad)}{7d^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{9/2}}{9d^2}-\frac{2(c+dx)^{7/2}(bc-ad)}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^2) + (2*b*(c + d*x)^(9/2))/(9*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^{5/2}}{d}+\frac{b(c+dx)^{7/2}}{d}\right)dx\\ &= -\frac{2(bc-ad)(c+dx)^{7/2}}{7d^2}+\frac{2b(c+dx)^{9/2}}{9d^2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{7/2}(9ad-2bc+7bdx)}{63d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-2*b*c + 9*a*d + 7*b*d*x))/(63*d^2)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 33, normalized size = 0.79

$$\frac{2(c+dx)^{7/2}(9ad+7b(c+dx)-9bc)}{63d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^(5/2),x]
[Out] (2*(c + d*x)^(7/2)*(-9*b*c + 9*a*d + 7*b*(c + d*x)))/(63*d^2)
```

**fricas [B]** time = 1.22, size = 93, normalized size = 2.21

$$\frac{2\left(7\,bd^4x^4-2\,bc^4+9\,ac^3d+\left(19\,bcd^3+9\,ad^4\right)x^3+3\left(5\,bc^2d^2+9\,acd^3\right)x^2+\left(bc^3d+27\,ac^2d^2\right)x\right)\sqrt{dx+c}}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="fricas")
[Out] 2/63*(7*b*d^4*x^4 - 2*b*c^4 + 9*a*c^3*d + (19*b*c*d^3 + 9*a*d^4)*x^3 + 3*(5
*b*c^2*d^2 + 9*a*c*d^3)*x^2 + (b*c^3*d + 27*a*c^2*d^2)*x)*sqrt(d*x + c)/d^2
```

**giac [B]** time = 1.61, size = 306, normalized size = 7.29

$$\frac{2\left(315\,\sqrt{dx+c}\,ac^3+315\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)ac^2+\frac{105\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)bc^3}{d}+63\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)ac+\frac{63\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)bc^2}{d}+9\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)a+\frac{27\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)bc}{d}+\frac{\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)b}{d}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="giac")
[Out] 2/315*(315*sqrt(d*x + c)*a*c^3 + 315*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*
a*c^2 + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c^3/d + 63*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*c + 63*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b*c^2/d + 9*(5*(d*x +
 c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c
)*c^3)*a + 27*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2
)*c^2 - 35*sqrt(d*x + c)*c^3)*b*c/d + (35*(d*x + c)^(9/2) - 180*(d*x + c)^(
7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x +
 c)*c^4)*b/d)/d
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.64

$$\frac{2\,(dx+c)^{\frac{7}{2}}\,(7bdx+9ad-2bc)}{63d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(5/2),x)
```

```
[Out] 2/63*(d*x+c)^(7/2)*(7*b*d*x+9*a*d-2*b*c)/d^2
```

**maxima** [A] time = 1.40, size = 33, normalized size = 0.79

$$\frac{2\left(7\,(dx+c)^{\frac{9}{2}}b-9\,(bc-ad)(dx+c)^{\frac{7}{2}}\right)}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/63*(7*(d*x + c)^(9/2)*b - 9*(b*c - a*d)*(d*x + c)^(7/2))/d^2
```

**mupad** [B] time = 0.05, size = 29, normalized size = 0.69

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}\,\left(9\,a\,d-9\,b\,c+7\,b\,\left(c+d\,x\right)\right)}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(7/2)*(9*a*d - 9*b*c + 7*b*(c + d*x)))/(63*d^2)
```

**sympy** [A] time = 2.36, size = 194, normalized size = 4.62

$$\begin{cases} \frac{2ac^3\sqrt{c+dx}}{7d}+\frac{6ac^2x\sqrt{c+dx}}{7}+\frac{6acdx^2\sqrt{c+dx}}{7}+\frac{2ad^2x^3\sqrt{c+dx}}{7}-\frac{4bc^4\sqrt{c+dx}}{63d^2}+\frac{2bc^3x\sqrt{c+dx}}{63d}+\frac{10bc^2x^2\sqrt{c+dx}}{21}+\frac{38bcdx^3\sqrt{c+dx}}{63}+\frac{2bd^2x^4\sqrt{c+dx}}{9} & \text{for } d\neq 0 \\ c^{\frac{5}{2}}\left(ax+\frac{bx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a*c**3*sqrt(c + d*x)/(7*d) + 6*a*c**2*x*sqrt(c + d*x)/7 + 6*a*
c*d*x**2*sqrt(c + d*x)/7 + 2*a*d**2*x**3*sqrt(c + d*x)/7 - 4*b*c**4*sqrt(c
+ d*x)/(63*d**2) + 2*b*c**3*x*sqrt(c + d*x)/(63*d) + 10*b*c**2*x**2*sqrt(c
+ d*x)/21 + 38*b*c*d*x**3*sqrt(c + d*x)/63 + 2*b*d**2*x**4*sqrt(c + d*x)/9,
 Ne(d, 0)), (c**(5/2)*(a*x + b*x**2/2), True))
```

# 3.1298 $\int (c+dx)^{5/2}\,dx$

**Optimal**. Leaf size=16

$$\frac{2(c+dx)^{7/2}}{7d}$$

**Rubi** [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{5/2}\,dx=\frac{2(c+dx)^{7/2}}{7d}$$

**Mathematica** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

**IntegrateAlgebraic** [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

**fricas** [B] time = 1.22, size = 39, normalized size = 2.44

$$\frac{2\left(d^{3}x^{3}+3\,cd^{2}x^{2}+3\,c^{2}dx+c^{3}\right)\sqrt{dx+c}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sqrt(d*x + c)/d
```

**giac** [B] time = 1.78, size = 95, normalized size = 5.94

$$\frac{2\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}+35\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)c^{2}+7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)c\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 + 3
5*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*c^2 + 7*(3*(d*x + c)^(5/2) - 10*(d*
x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*c)/d
```

**maple** [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(dx+c)^{\frac{7}{2}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2),x)
```

```
[Out] 2/7*(d*x+c)^(7/2)/d
```

**maxima** [A] time = 1.30, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{7}{2}}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="maxima")
[Out] 2/7*(d*x + c)^(7/2)/d
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2),x)
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.75

$$\frac{2\left(c+dx\right)^{\frac{7}{2}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2),x)
[Out] 2*(c + d*x)**(7/2)/(7*d)
```

## 3.1299 $\int \frac{(c+dx)^{5/2}}{a+bx} \, dx$

**Optimal**. Leaf size=112

$$-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{2\sqrt{c+dx}\,(bc-ad)^2}{b^3}+\frac{2(c+dx)^{3/2}(bc-ad)}{3b^2}+\frac{2(c+dx)^{5/2}}{5b}$$

**Rubi** [A] time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 63, 208}

$$\frac{2\sqrt{c+dx}\,(bc-ad)^2}{b^3}+\frac{2(c+dx)^{3/2}(bc-ad)}{3b^2}-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{2(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x),x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/b^3 + (2*(b*c - a*d)*(c + d*x)^(3/2))/(3*b^
2) + (2*(c + d*x)^(5/2))/(5*b) - (2*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{a+bx}\,dx = \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)\int \frac{(c+dx)^{3/2}}{a+bx}\,dx}{b}$$
$$= \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)^2 \int \frac{\sqrt{c+dx}}{a+bx}\,dx}{b^2}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)^3 \int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b^3}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{b}{}}\right.}{b^3 d}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} - \frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right.}{b^{7/2}}$$

**Mathematica [A]** time = 0.16, size = 105, normalized size = 0.94

$$\frac{2(bc-ad)\left(\sqrt{b}\sqrt{c+dx}\,(-3ad+4bc+bdx)-3(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\right)}{3b^{7/2}} + \frac{2(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*b) + (2*(b*c - a*d)*(Sqrt[b]*Sqrt[c + d*x]*(4*b*c -
3*a*d + b*d*x) - 3*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b
*c - a*d]]))/(3*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 130, normalized size = 1.16

$$\frac{2\sqrt{c+dx}\,\left(15a^2d^2-5abd(c+dx)-30abcd+15b^2c^2+3b^2(c+dx)^2+5b^2c(c+dx)\right)}{15b^3} + \frac{2(ad-bc)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(15*b^2*c^2 - 30*a*b*c*d + 15*a^2*d^2 + 5*b^2*c*(c + d*x)
- 5*a*b*d*(c + d*x) + 3*b^2*(c + d*x)^2))/(15*b^3) + (2*(-(b*c) + a*d)^(5/2
)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/b^(7/2)
```

**fricas [A]** time = 1.25, size = 290, normalized size = 2.59

$$\left[\frac{15\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\left(3\,b^{2}d^{2}x^{2}+23\,b^{2}c^{2}-35\,abcd+15\,a^{2}d^{2}+\left(11\,b^{2}cd-5\,abd^{2}\right)x\right)\sqrt{dx+c}}{15\,b^{3}},\ -\frac{2\left(15\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(3\,b^{2}d^{2}x^{2}+23\,b^{2}c^{2}-35\,abcd+15\,a^{2}d^{2}+\left(11\,b^{2}cd-5\,abd^{2}\right)x\right)\sqrt{dx+c}\right)}{15\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x +
2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(3*b^2*
d^2*x^2 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)*x
)*sqrt(d*x + c))/b^3, -2/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^
2*d^2*x^2 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)
*x)*sqrt(d*x + c))/b^3]
```

**giac [A]** time = 1.60, size = 171, normalized size = 1.53

$$\frac{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,b^{3}}+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}b^{4}+5\,(dx+c)^{\frac{3}{2}}b^{4}c+15\,\sqrt{dx+c}\,b^{4}c^{2}-5\,(dx+c)^{\frac{3}{2}}ab^{3}d-30\,\sqrt{dx+c}\,ab^{3}cd+15\,\sqrt{dx+c}\,a^{2}b^{2}d^{2}\right)}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x + c)*
b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/15*(3*(d*x + c)^(5/2
)*b^4 + 5*(d*x + c)^(3/2)*b^4*c + 15*sqrt(d*x + c)*b^4*c^2 - 5*(d*x + c)^(3
/2)*a*b^3*d - 30*sqrt(d*x + c)*a*b^3*c*d + 15*sqrt(d*x + c)*a^2*b^2*d^2)/b^
5
```

**maple [B]** time = 0.01, size = 263, normalized size = 2.35

$$-\frac{2a^{3}d^{3}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b^{3}}+\frac{6a^{2}c\,d^{2}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b^{2}}-\frac{6a\,c^{2}d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b}+\frac{2c^{3}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}}+\frac{2\sqrt{dx+c}\,a^{2}d^{2}}{b^{3}}-\frac{4\sqrt{dx+c}\,acd}{b^{2}}+\frac{2\sqrt{dx+c}\,c^{2}}{b}-\frac{2\,(dx+c)^{\frac{3}{2}}ad}{3b^{2}}+\frac{2\,(dx+c)^{\frac{3}{2}}c}{3b}+\frac{2\,(dx+c)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a),x)
```

```
[Out] 2/5*(d*x+c)^(5/2)/b-2/3/b^2*(d*x+c)^(3/2)*a*d+2/3/b*(d*x+c)^(3/2)*c+2/b^3*a
^2*d^2*(d*x+c)^(1/2)-4/b^2*a*c*d*(d*x+c)^(1/2)+2/b*c^2*(d*x+c)^(1/2)-2/b^3/
((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a^3*d^3+6/b
^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a^2*c*d^
2-6/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a*c^2
*d+2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*c^3
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.08, size = 130, normalized size = 1.16

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,b}-\frac{2\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,b^2}+\frac{2\,{\left(a\,d-b\,c\right)}^2\,\sqrt{c+d\,x}}{b^3}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,{\left(a\,d-b\,c\right)}^{5/2}\,\sqrt{c+d\,x}}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^{5/2}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(5/2))/(5*b) - (2*(a*d - b*c)*(c + d*x)^(3/2))/(3*b^2) + (2*(a
*d - b*c)^2*(c + d*x)^(1/2))/b^3 - (2*atan((b^(1/2)*(a*d - b*c)^(5/2)*(c +
d*x)^(1/2))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(a*d - b*c
)^(5/2))/b^(7/2)
```

**sympy [A]** time = 27.01, size = 121, normalized size = 1.08

$$\frac{2\left(c+dx\right)^{\frac{5}{2}}}{5b}+\frac{\left(c+dx\right)^{\frac{3}{2}}\left(-2ad+2bc\right)}{3b^2}+\frac{\sqrt{c+dx}\,\left(2a^2d^2-4abcd+2b^2c^2\right)}{b^3}-\frac{2\left(ad-bc\right)^3\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^4\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a),x)
```

```
[Out] 2*(c + d*x)**(5/2)/(5*b) + (c + d*x)**(3/2)*(-2*a*d + 2*b*c)/(3*b**2) + sqr
t(c + d*x)*(2*a**2*d**2 - 4*a*b*c*d + 2*b**2*c**2)/b**3 - 2*(a*d - b*c)**3*
atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**4*sqrt((a*d - b*c)/b))
```

## 3.1300 $\int \frac{(c+dx)^{5/2}}{(a+bx)^2}\,dx$

**Optimal**. Leaf size=110

$$-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{5d\sqrt{c+dx}\,(bc-ad)}{b^3}-\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{5d(c+dx)^{3/2}}{3b^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 63, 208}

$$\frac{5d\sqrt{c+dx}\,(bc-ad)}{b^3}-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}-\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{5d(c+dx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x])/b^3 + (5*d*(c + d*x)^(3/2))/(3*b^2) - (c +
d*x)^(5/2)/(b*(a + b*x)) - (5*d*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^2}\,dx &= -\frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{a+bx}\,dx}{2b} \\
&= \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5d(bc-ad))\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{2b^2} \\
&= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{\left(5d(bc-ad)^2\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b^3} \\
&= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \right.}{b^3} \\
&= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} - \frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 50, normalized size = 0.45

$$\frac{2d(c+dx)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] (2*d*(c + d*x)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(7*(-(b*c) + a*d)^2)
```

**IntegrateAlgebraic [A]** time = 0.37, size = 187, normalized size = 1.70

$$\frac{d\sqrt{c+dx}\left(-15a^2d^2-10abd(c+dx)+30abcd-15b^2c^2+2b^2(c+dx)^2+10b^2c(c+dx)\right)}{3b^3(ad+b(c+dx)-bc)}+\frac{5\left(-a^3d^4+3a^2bcd^3-3ab^2c^2d^2+b^3c^3d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{b^{7/2}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] (d*Sqrt[c + d*x]*(-15*b^2*c^2 + 30*a*b*c*d - 15*a^2*d^2 + 10*b^2*c*(c + d*x
) - 10*a*b*d*(c + d*x) + 2*b^2*(c + d*x)^2))/(3*b^3*(-(b*c) + a*d + b*(c +
d*x))) + (5*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*ArcTan[
(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(b^(7/2)*(-(b*c) +
 a*d)^(3/2))
```

**fricas [A]** time = 1.25, size = 330, normalized size = 3.00

$$\left[-\frac{15\left(abcd-a^{2}d^{2}+\left(b^{2}cd-abd^{2}\right)x\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad+2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)-2\left(2\,b^{2}d^{2}x^{2}-3\,b^{2}c^{2}+20\,abcd-15\,a^{2}d^{2}+2\left(7\,b^{2}cd-5\,abd^{2}\right)x\right)\sqrt{dx+c}}{6\left(b^{4}x+ab^{3}\right)},-\frac{15\left(abcd-a^{2}d^{2}+\left(b^{2}cd-abd^{2}\right)x\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(2\,b^{2}d^{2}x^{2}-3\,b^{2}c^{2}+20\,abcd-15\,a^{2}d^{2}+2\left(7\,b^{2}cd-5\,abd^{2}\right)x\right)\sqrt{dx+c}}{3\left(b^{4}x+ab^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x)*sqrt((b*c - a*d)/b)*l
og((b*d*x + 2*b*c - a*d + 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a))
 - 2*(2*b^2*d^2*x^2 - 3*b^2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 2*(7*b^2*c*d -
5*a*b*d^2)*x)*sqrt(d*x + c))/(b^4*x + a*b^3), -1/3*(15*(a*b*c*d - a^2*d^2 +
 (b^2*c*d - a*b*d^2)*x)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-
(b*c - a*d)/b)/(b*c - a*d)) - (2*b^2*d^2*x^2 - 3*b^2*c^2 + 20*a*b*c*d - 15*
a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x)*sqrt(d*x + c))/(b^4*x + a*b^3)]
```

**giac [A]** time = 1.28, size = 181, normalized size = 1.65

$$\frac{5\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,b^{3}}-\frac{\sqrt{dx+c}\,b^{2}c^{2}d-2\,\sqrt{dx+c}\,abcd^{2}+\sqrt{dx+c}\,a^{2}d^{3}}{((dx+c)b-bc+ad)b^{3}}+\frac{2\left((dx+c)^{\frac{3}{2}}b^{4}d+6\,\sqrt{dx+c}\,b^{4}cd-6\,\sqrt{dx+c}\,ab^{3}d^{2}\right)}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - (sqrt(d*x + c)*b^2*c^2*d - 2*sqrt(d*x
+ c)*a*b*c*d^2 + sqrt(d*x + c)*a^2*d^3)/(((d*x + c)*b - b*c + a*d)*b^3) + 2
/3*((d*x + c)^(3/2)*b^4*d + 6*sqrt(d*x + c)*b^4*c*d - 6*sqrt(d*x + c)*a*b^3
*d^2)/b^6
```

**maple [B]** time = 0.01, size = 258, normalized size = 2.35

$$\frac{5a^{2}d^{3}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b^{3}}-\frac{10ac\,d^{2}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b^{2}}+\frac{5c^{2}d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b}-\frac{\sqrt{dx+c}\,a^{2}d^{3}}{(bdx+ad)\,b^{3}}+\frac{2\sqrt{dx+c}\,ac\,d^{2}}{(bdx+ad)\,b^{2}}-\frac{\sqrt{dx+c}\,c^{2}d}{(bdx+ad)\,b}-\frac{4\sqrt{dx+c}\,a\,d^{2}}{b^{3}}+\frac{4\sqrt{dx+c}\,cd}{b^{2}}+\frac{2\,(dx+c)^{\frac{3}{2}}\,d}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^2,x)
```

```
[Out] 2/3*d*(d*x+c)^(3/2)/b^2-4/b^3*a*d^2*(d*x+c)^(1/2)+4*d/b^2*(d*x+c)^(1/2)*c-1
/b^3*(d*x+c)^(1/2)/(b*d*x+a*d)*a^2*d^3+2/b^2*(d*x+c)^(1/2)/(b*d*x+a*d)*a*c*
d^2-d/b*(d*x+c)^(1/2)/(b*d*x+a*d)*c^2+5/b^3/((a*d-b*c)*b)^(1/2)*arctan((d*x
+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a^2*d^3-10/b^2/((a*d-b*c)*b)^(1/2)*arctan(
(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a*c*d^2+5*d/b/((a*d-b*c)*b)^(1/2)*arct
an((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*c^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.12, size = 161, normalized size = 1.46

$$\frac{2\,d\,{\left(c+d\,x\right)}^{3/2}}{3\,b^2}-\frac{\sqrt{c+d\,x}\,\left(a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{b^4\,\left(c+d\,x\right)-b^4\,c+a\,b^3\,d}+\frac{5\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,{\left(a\,d-b\,c\right)}^{3/2}\,\sqrt{c+d\,x}}{a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d}\right)\,{\left(a\,d-b\,c\right)}^{3/2}}{b^{7/2}}+\frac{2\,d\,\left(2\,b^2\,c-2\,a\,b\,d\right)\,\sqrt{c+d\,x}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^2,x)
```

```
[Out] (2*d*(c + d*x)^(3/2))/(3*b^2) - ((c + d*x)^(1/2)*(a^2*d^3 + b^2*c^2*d - 2*a
*b*c*d^2))/(b^4*(c + d*x) - b^4*c + a*b^3*d) + (5*d*atan((b^(1/2)*d*(a*d -
b*c)^(3/2)*(c + d*x)^(1/2))/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))*(a*d - b*c
)^(3/2))/b^(7/2) + (2*d*(2*b^2*c - 2*a*b*d)*(c + d*x)^(1/2))/b^4
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

## 3.1301 $\int \frac{(c+dx)^{5/2}}{(a+bx)^3} dx$

**Optimal**. Leaf size=119

$$-\frac{15d^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{15d^2\sqrt{c+dx}}{4b^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 63, 208}

$$-\frac{15d^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{15d^2\sqrt{c+dx}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x])/(4*b^3) - (5*d*(c + d*x)^(3/2))/(4*b^2*(a + b*x)) -
(c + d*x)^(5/2)/(2*b*(a + b*x)^2) - (15*d^2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^3}\,dx = -\frac{(c+dx)^{5/2}}{2b(a+bx)^2} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx}{4b}$$

$$= -\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} + \frac{\left(15d^2\right)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{8b^2}$$

$$= \frac{15d^2\sqrt{c+dx}}{4b^3} - \frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} + \frac{\left(15d^2(bc-ad)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b^3}$$

$$= \frac{15d^2\sqrt{c+dx}}{4b^3} - \frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} + \frac{(15d(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+\right.}{4b^3}$$

$$= \frac{15d^2\sqrt{c+dx}}{4b^3} - \frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} - \frac{15d^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}$$

**Mathematica [C]** time = 0.02, size = 52, normalized size = 0.44

$$\frac{2d^2(c+dx)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (2*d^2*(c + d*x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(7*(-(b*c) + a*d)^3)
```

**IntegrateAlgebraic [A]** time = 0.48, size = 155, normalized size = 1.30

$$\frac{d^2\sqrt{c+dx}\left(15a^2d^2+25abd(c+dx)-30abcd+15b^2c^2+8b^2(c+dx)^2-25b^2c(c+dx)\right)}{4b^3(ad+b(c+dx)-bc)^2} + \frac{15d^2\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (d^2*Sqrt[c + d*x]*(15*b^2*c^2 - 30*a*b*c*d + 15*a^2*d^2 - 25*b^2*c*(c + d*
x) + 25*a*b*d*(c + d*x) + 8*b^2*(c + d*x)^2))/(4*b^3*(-(b*c) + a*d + b*(c +
 d*x))^2) + (15*d^2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*S
qrt[c + d*x])/(b*c - a*d)])/(4*b^(7/2))
```

**fricas [A]** time = 1.56, size = 344, normalized size = 2.89

$$\left[\frac{15\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\left(8\,b^2d^2x^2-2\,b^2c^2-5\,abcd+15\,a^2d^2-\left(9\,b^2cd-25\,abd^2\right)x\right)\sqrt{dx+c}}{8\left(b^5x^2+2\,ab^4x+a^2b^3\right)}, -\frac{15\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(8\,b^2d^2x^2-2\,b^2c^2-5\,abcd+15\,a^2d^2-\left(9\,b^2cd-25\,abd^2\right)x\right)\sqrt{dx+c}}{4\left(b^5x^2+2\,ab^4x+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d
*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(8
*b^2*d^2*x^2 - 2*b^2*c^2 - 5*a*b*c*d + 15*a^2*d^2 - (9*b^2*c*d - 25*a*b*d^2
)*x)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*d^2*x^2
+ 2*a*b*d^2*x + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(
-(b*c - a*d)/b)/(b*c - a*d)) - (8*b^2*d^2*x^2 - 2*b^2*c^2 - 5*a*b*c*d + 15*
a^2*d^2 - (9*b^2*c*d - 25*a*b*d^2)*x)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x +
 a^2*b^3)]
```

**giac [A]** time = 1.24, size = 171, normalized size = 1.44

$$\frac{2\,\sqrt{dx+c}\,d^2}{b^3}+\frac{15\left(bcd^2-ad^3\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,\sqrt{-b^2c+abd}\,b^3}-\frac{9\,(dx+c)^{\frac{3}{2}}b^2cd^2-7\,\sqrt{dx+c}\,b^2c^2d^2-9\,(dx+c)^{\frac{3}{2}}abd^3+14\,\sqrt{dx+c}\,abcd^3-7\,\sqrt{dx+c}\,a^2d^4}{4\left((dx+c)b-bc+ad\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*d^2/b^3 + 15/4*(b*c*d^2 - a*d^3)*arctan(sqrt(d*x + c)*b/sqr
t(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - 1/4*(9*(d*x + c)^(3/2)*b^2*
c*d^2 - 7*sqrt(d*x + c)*b^2*c^2*d^2 - 9*(d*x + c)^(3/2)*a*b*d^3 + 14*sqrt(d
*x + c)*a*b*c*d^3 - 7*sqrt(d*x + c)*a^2*d^4)/(((d*x + c)*b - b*c + a*d)^2*b
^3)
```

**maple [B]** time = 0.02, size = 238, normalized size = 2.00

$$\frac{7\sqrt{dx+c}\;a^2d^4}{4\left(bdx+ad\right)^2b^3}-\frac{7\sqrt{dx+c}\;ac\,d^3}{2\left(bdx+ad\right)^2b^2}+\frac{7\sqrt{dx+c}\;c^2d^2}{4\left(bdx+ad\right)^2b}+\frac{9\left(dx+c\right)^{\frac{3}{2}}a\,d^3}{4\left(bdx+ad\right)^2b^2}-\frac{15a\,d^3\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{4\sqrt{(ad-bc)\,b}\;b^3}-\frac{9\left(dx+c\right)^{\frac{3}{2}}c\,d^2}{4\left(bdx+ad\right)^2b}+\frac{15c\,d^2\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{4\sqrt{(ad-bc)\,b}\;b^2}+\frac{2\sqrt{dx+c}\;d^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^3,x)
```

```
[Out] 2*d^2*(d*x+c)^(1/2)/b^3+9/4*d^3/b^2/(b*d*x+a*d)^2*(d*x+c)^(3/2)*a-9/4*d^2/b
/(b*d*x+a*d)^2*(d*x+c)^(3/2)*c+7/4*d^4/b^3/(b*d*x+a*d)^2*(d*x+c)^(1/2)*a^2-
7/2*d^3/b^2/(b*d*x+a*d)^2*(d*x+c)^(1/2)*a*c+7/4*d^2/b/(b*d*x+a*d)^2*(d*x+c)
^(1/2)*c^2-15/4*d^3/b^3/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)
*b)^(1/2)*b)*a+15/4*d^2/b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-
b*c)*b)^(1/2)*b)*c
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.16, size = 199, normalized size = 1.67

$$\frac{2\,d^2\,\sqrt{c+d\,x}}{b^3}-\frac{\left(\frac{9\,b^2\,c\,d^2}{4}-\frac{9\,a\,b\,d^3}{4}\right)\,{\left(c+d\,x\right)}^{3/2}-\sqrt{c+d\,x}\,\left(\frac{7\,a^2\,d^4}{4}-\frac{7\,a\,b\,c\,d^3}{2}+\frac{7\,b^2\,c^2\,d^2}{4}\right)}{b^5\,{\left(c+d\,x\right)}^2-\left(2\,b^5\,c-2\,a\,b^4\,d\right)\,\left(c+d\,x\right)+b^5\,c^2+a^2\,b^3\,d^2-2\,a\,b^4\,c\,d}-\frac{15\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,d^2\,\sqrt{a\,d-b\,c}\,\sqrt{c+d\,x}}{a\,d^3-b\,c\,d^2}\right)\,\sqrt{a\,d-b\,c}}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^3,x)
```

```
[Out] (2*d^2*(c + d*x)^(1/2))/b^3 - (((9*b^2*c*d^2)/4 - (9*a*b*d^3)/4)*(c + d*x)^
(3/2) - (c + d*x)^(1/2)*((7*a^2*d^4)/4 + (7*b^2*c^2*d^2)/4 - (7*a*b*c*d^3)/
2))/(b^5*(c + d*x)^2 - (2*b^5*c - 2*a*b^4*d)*(c + d*x) + b^5*c^2 + a^2*b^3*
d^2 - 2*a*b^4*c*d) - (15*d^2*atan((b^(1/2)*d^2*(a*d - b*c)^(1/2)*(c + d*x)^
(1/2))/(a*d^3 - b*c*d^2))*(a*d - b*c)^(1/2))/(4*b^(7/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

## 3.1302 $\int \frac{(c+dx)^{5/2}}{(a+bx)^4}\,dx$

**Optimal**. Leaf size=126

$$-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}-\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 63, 208}

$$-\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] (-5*d^2*Sqrt[c + d*x])/(8*b^3*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(12*b^2*(a
 + b*x)^2) - (c + d*x)^(5/2)/(3*b*(a + b*x)^3) - (5*d^3*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(7/2)*Sqrt[b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^4}\,dx = -\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{(5d)\int\frac{(c+dx)^{3/2}}{(a+bx)^3}\,dx}{6b}$$
$$= -\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{\left(5d^2\right)\int\frac{\sqrt{c+dx}}{(a+bx)^2}\,dx}{8b^2}$$
$$= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{\left(5d^3\right)\int\frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b^3}$$
$$= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{\left(5d^2\right)\text{Subst}\left(\int\frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{8b^3}$$
$$= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}$$

**Mathematica [A]** time = 0.15, size = 119, normalized size = 0.94

$$\frac{5d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{8b^{7/2}\sqrt{ad-bc}}-\frac{\sqrt{c+dx}\left(15a^2d^2+10abd(c+4dx)+b^2\left(8c^2+26cdx+33d^2x^2\right)\right)}{24b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] -1/24*(Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(c + 4*d*x) + b^2*(8*c^2 + 26*c
*d*x + 33*d^2*x^2)))/(b^3*(a + b*x)^3) + (5*d^3*ArcTan[(Sqrt[b]*Sqrt[c + d*
x])/Sqrt[-(b*c) + a*d]])/(8*b^(7/2)*Sqrt[-(b*c) + a*d])
```

**IntegrateAlgebraic [A]** time = 0.62, size = 155, normalized size = 1.23

$$-\frac{d^3\sqrt{c+dx}\left(15a^2d^2+40abd(c+dx)-30abcd+15b^2c^2+33b^2(c+dx)^2-40b^2c(c+dx)\right)}{24b^3(ad+b(c+dx)-bc)^3}-\frac{5d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{8b^{7/2}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] -1/24*(d^3*Sqrt[c + d*x]*(15*b^2*c^2 - 30*a*b*c*d + 15*a^2*d^2 - 40*b^2*c*(
c + d*x) + 40*a*b*d*(c + d*x) + 33*b^2*(c + d*x)^2))/(b^3*(-(b*c) + a*d + b
```

```
*(c + d*x))^3) - (5*d^3*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(
b*c - a*d)])/(8*b^(7/2)*Sqrt[-(b*c) + a*d])
```

**fricas [B]** time = 1.20, size = 563, normalized size = 4.47

$$\left[\frac{15\left(b^{3}d^{3}x^{3}+3\,ab^{2}d^{3}x^{2}+3\,a^{2}bd^{3}x+a^{3}d^{3}\right)\sqrt{b^{2}c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^{2}c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\left(8\,b^{4}c^{3}+2\,ab^{3}c^{2}d+5\,a^{2}b^{2}cd^{2}-15\,a^{3}bd^{3}+33\left(b^{4}cd^{2}-ab^{3}d^{3}\right)x^{2}+2\left(13\,b^{4}c^{2}d+7\,ab^{3}cd^{2}-20\,a^{2}b^{2}d^{3}\right)x\right)\sqrt{dx+c}}{48\left(a^{3}b^{5}c-a^{4}b^{4}d+\left(b^{8}c-ab^{7}d\right)x^{3}+3\left(ab^{7}c-a^{2}b^{6}d\right)x^{2}+3\left(a^{2}b^{6}c-a^{3}b^{5}d\right)x\right)},\ \frac{15\left(b^{3}d^{3}x^{3}+3\,ab^{2}d^{3}x^{2}+3\,a^{2}bd^{3}x+a^{3}d^{3}\right)\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{-b^{2}c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-\left(8\,b^{4}c^{3}+2\,ab^{3}c^{2}d+5\,a^{2}b^{2}cd^{2}-15\,a^{3}bd^{3}+33\left(b^{4}cd^{2}-ab^{3}d^{3}\right)x^{2}+2\left(13\,b^{4}c^{2}d+7\,ab^{3}cd^{2}-20\,a^{2}b^{2}d^{3}\right)x\right)\sqrt{dx+c}}{24\left(a^{3}b^{5}c-a^{4}b^{4}d+\left(b^{8}c-ab^{7}d\right)x^{3}+3\left(ab^{7}c-a^{2}b^{6}d\right)x^{2}+3\left(a^{2}b^{6}c-a^{3}b^{5}d\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) - 2*(8*b^4*c^3 + 2*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 15*a^3*b*d^
3 + 33*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(13*b^4*c^2*d + 7*a*b^3*c*d^2 - 20*a
^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c - a^4*b^4*d + (b^8*c - a*b^7*d)*x^
3 + 3*(a*b^7*c - a^2*b^6*d)*x^2 + 3*(a^2*b^6*c - a^3*b^5*d)*x), 1/24*(15*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a*b*d
)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*b^4*c^3 + 2
*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 15*a^3*b*d^3 + 33*(b^4*c*d^2 - a*b^3*d^3)*
x^2 + 2*(13*b^4*c^2*d + 7*a*b^3*c*d^2 - 20*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(
a^3*b^5*c - a^4*b^4*d + (b^8*c - a*b^7*d)*x^3 + 3*(a*b^7*c - a^2*b^6*d)*x^2
 + 3*(a^2*b^6*c - a^3*b^5*d)*x)]
```

**giac [A]** time = 0.98, size = 161, normalized size = 1.28

$$\frac{5\,d^{3}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{8\,\sqrt{-b^{2}c+abd}\,b^{3}}-\frac{33\,(dx+c)^{\frac{5}{2}}b^{2}d^{3}-40\,(dx+c)^{\frac{3}{2}}b^{2}cd^{3}+15\,\sqrt{dx+c}\,b^{2}c^{2}d^{3}+40\,(dx+c)^{\frac{3}{2}}abd^{4}-30\,\sqrt{dx+c}\,abcd^{4}+15\,\sqrt{dx+c}\,a^{2}d^{5}}{24\,((dx+c)b-bc+ad)^{3}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 5/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b^3) - 1/24*(33*(d*x + c)^(5/2)*b^2*d^3 - 40*(d*x + c)^(3/2)*b^2*c*d^3 + 15
*sqrt(d*x + c)*b^2*c^2*d^3 + 40*(d*x + c)^(3/2)*a*b*d^4 - 30*sqrt(d*x + c)*
a*b*c*d^4 + 15*sqrt(d*x + c)*a^2*d^5)/(((d*x + c)*b - b*c + a*d)^3*b^3)
```

**maple [A]** time = 0.02, size = 204, normalized size = 1.62

$$-\frac{5\sqrt{dx+c}\;a^{2}d^{5}}{8\,(bdx+ad)^{3}\,b^{3}}+\frac{5\sqrt{dx+c}\;ac\,d^{4}}{4\,(bdx+ad)^{3}\,b^{2}}-\frac{5\sqrt{dx+c}\;c^{2}d^{3}}{8\,(bdx+ad)^{3}\,b}-\frac{5\,(dx+c)^{\frac{3}{2}}\,a\,d^{4}}{3\,(bdx+ad)^{3}\,b^{2}}+\frac{5\,(dx+c)^{\frac{3}{2}}\,c\,d^{3}}{3\,(bdx+ad)^{3}\,b}-\frac{11\,(dx+c)^{\frac{5}{2}}\,d^{3}}{8\,(bdx+ad)^{3}\,b}+\frac{5d^{3}\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{8\sqrt{(ad-bc)\,b}\;b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^4,x)
```

```
[Out] -11/8*d^3/(b*d*x+a*d)^3/b*(d*x+c)^(5/2)-5/3*d^4/(b*d*x+a*d)^3/b^2*(d*x+c)^(
3/2)*a+5/3*d^3/(b*d*x+a*d)^3/b*(d*x+c)^(3/2)*c-5/8*d^5/(b*d*x+a*d)^3/b^3*(d
*x+c)^(1/2)*a^2+5/4*d^4/(b*d*x+a*d)^3/b^2*(d*x+c)^(1/2)*a*c-5/8*d^3/(b*d*x+
a*d)^3/b*(d*x+c)^(1/2)*c^2+5/8*d^3/b^3/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(
1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.36, size = 222, normalized size = 1.76

$$\frac{5\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,b^{7/2}\,\sqrt{a\,d-b\,c}}-\frac{\frac{11\,d^3\,{\left(c+d\,x\right)}^{5/2}}{8\,b}+\frac{5\,d^3\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{8\,b^3}+\frac{5\,d^3\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,b^2}}{\left(c+d\,x\right)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)+b^3\,{\left(c+d\,x\right)}^3-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^2+a^3\,d^3-b^3\,c^3+3\,a\,b^2\,c^2\,d-3\,a^2\,b\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^4,x)
```

```
[Out] (5*d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(7/2)*(a*d -
 b*c)^(1/2)) - ((11*d^3*(c + d*x)^(5/2))/(8*b) + (5*d^3*(c + d*x)^(1/2)*(a^
2*d^2 + b^2*c^2 - 2*a*b*c*d))/(8*b^3) + (5*d^3*(a*d - b*c)*(c + d*x)^(3/2))
/(3*b^2))/((c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x
)^3 - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

# 3.1303 $\int \frac{(c+dx)^{5/2}}{(a+bx)^5} dx$

**Optimal**. Leaf size=162

$$\frac{5d^4 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{7/2}(bc-ad)^{3/2}} - \frac{5d^3\sqrt{c+dx}}{64b^3(a+bx)(bc-ad)} - \frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4}$$

**Rubi [A]** time = 0.07, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{5d^3\sqrt{c+dx}}{64b^3(a+bx)(bc-ad)} - \frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} + \frac{5d^4 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{7/2}(bc-ad)^{3/2}} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] (-5*d^2*Sqrt[c + d*x])/(32*b^3*(a + b*x)^2) - (5*d^3*Sqrt[c + d*x])/(64*b^3
*(b*c - a*d)*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(24*b^2*(a + b*x)^3) - (c +
 d*x)^(5/2)/(4*b*(a + b*x)^4) + (5*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[b*c - a*d]])/(64*b^(7/2)*(b*c - a*d)^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^5}\,dx = -\frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^4}\,dx}{8b}$$

$$= -\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{\left(5d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx}{16b^2}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{\left(5d^3\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b^3}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} - \frac{\left(5d^4\right)\int \frac{1}{(a+bx)\sqrt{c+\cdots}}}{128b^3(bc-a\cdots}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} - \frac{\left(5d^3\right)\mathrm{Subst}\left(\int \cdots\right.}{64b\cdots}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{\cdots}}{\sqrt{bc\cdots}}\right.}{64b^{7/2}(bc-ad\cdots}$$

**Mathematica [C]** time = 0.02, size = 52, normalized size = 0.32

$$\frac{2d^4(c+dx)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] (2*d^4*(c + d*x)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(7*(-(b*c) + a*d)^5)
```

**IntegrateAlgebraic [A]** time = 1.08, size = 226, normalized size = 1.40

$$-\frac{d^4\sqrt{c+dx}\left(-15a^3d^3-55a^2bd^2(c+dx)+45a^2bcd^2-45ab^2c^2d-73ab^2d(c+dx)^2+110ab^2cd(c+dx)+15b^3c^3-55b^3c^2(c+dx)+15b^3(c+dx)^3+73b^3c(c+dx)^2\right)}{192b^3(bc-ad)(-ad-b(c+dx)+bc)^4}-\frac{5d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{64b^{7/2}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] -1/192*(d^4*Sqrt[c + d*x]*(15*b^3*c^3 - 45*a*b^2*c^2*d + 45*a^2*b*c*d^2 - 1
5*a^3*d^3 - 55*b^3*c^2*(c + d*x) + 110*a*b^2*c*d*(c + d*x) - 55*a^2*b*d^2*(
c + d*x) + 73*b^3*c*(c + d*x)^2 - 73*a*b^2*d*(c + d*x)^2 + 15*b^3*(c + d*x)
^3))/(b^3*(b*c - a*d)*(b*c - a*d - b*(c + d*x))^4) - (5*d^4*ArcTan[(Sqrt[b]
*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(64*b^(7/2)*(-(b*c) + a*d)
^(3/2))
```

**fricas [B]** time = 1.42, size = 894, normalized size = 5.52

$$\left[-\frac{15\left(b^4d^4x^4+4ab^3d^4x^3+6a^2b^2d^4x^2+4a^3bd^4x+a^4d^4\right)\sqrt{b^2c-abd}\log\left(\frac{bdx+2bc-ad-2\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)+2\left(48b^5c^4-56ab^4c^3d-2a^2b^3c^2d^2-5a^3b^2cd^3+15a^4bd^4+15\left(b^5cd^3-ab^4d^4\right)x^3+\left(118b^5c^2d^2-191ab^4cd^3+73a^2b^3d^4\right)x^2+\left(136b^5c^3d-172ab^4c^2d^2-19a^2b^3cd^3+55a^3b^2d^4\right)x\right)\sqrt{dx+c}}{384\left(a^4b^6c^2-2a^5b^5cd+a^6b^4d^2+\left(b^{10}c^2-2ab^9cd+a^2b^8d^2\right)x^4+4\left(ab^9c^2-2a^2b^8cd+a^3b^7d^2\right)x^3+6\left(a^2b^8c^2-2a^3b^7cd+a^4b^6d^2\right)x^2+4\left(a^3b^7c^2-2a^4b^6cd+a^5b^5d^2\right)x\right)},-\frac{15\left(b^4d^4x^4+4ab^3d^4x^3+6a^2b^2d^4x^2+4a^3bd^4x+a^4d^4\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{-b^2c+abd}\sqrt{dx+c}}{bdx+bc}\right)+\left(48b^5c^4-56ab^4c^3d-2a^2b^3c^2d^2-5a^3b^2cd^3+15a^4bd^4+15\left(b^5cd^3-ab^4d^4\right)x^3+\left(118b^5c^2d^2-191ab^4cd^3+73a^2b^3d^4\right)x^2+\left(136b^5c^3d-172ab^4c^2d^2-19a^2b^3cd^3+55a^3b^2d^4\right)x\right)\sqrt{dx+c}}{192\left(a^4b^6c^2-2a^5b^5cd+a^6b^4d^2+\left(b^{10}c^2-2ab^9cd+a^2b^8d^2\right)x^4+4\left(ab^9c^2-2a^2b^8cd+a^3b^7d^2\right)x^3+6\left(a^2b^8c^2-2a^3b^7cd+a^4b^6d^2\right)x^2+4\left(a^3b^7c^2-2a^4b^6cd+a^5b^5d^2\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(48*b^5*c^4 - 56*a*b^4*c^3*d - 2*a^2
*b^3*c^2*d^2 - 5*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4)*
x^3 + (118*b^5*c^2*d^2 - 191*a*b^4*c*d^3 + 73*a^2*b^3*d^4)*x^2 + (136*b^5*c
^3*d - 172*a*b^4*c^2*d^2 - 19*a^2*b^3*c*d^3 + 55*a^3*b^2*d^4)*x)*sqrt(d*x +
 c))/(a^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2 + (b^10*c^2 - 2*a*b^9*c*d +
 a^2*b^8*d^2)*x^4 + 4*(a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7*d^2)*x^3 + 6*(a^
2*b^8*c^2 - 2*a^3*b^7*c*d + a^4*b^6*d^2)*x^2 + 4*(a^3*b^7*c^2 - 2*a^4*b^6*c
*d + a^5*b^5*d^2)*x), -1/192*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2
*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c
 + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (48*b^5*c^4 - 56*a*b^4*c^3*d - 2*a
^2*b^3*c^2*d^2 - 5*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4
)*x^3 + (118*b^5*c^2*d^2 - 191*a*b^4*c*d^3 + 73*a^2*b^3*d^4)*x^2 + (136*b^5
*c^3*d - 172*a*b^4*c^2*d^2 - 19*a^2*b^3*c*d^3 + 55*a^3*b^2*d^4)*x)*sqrt(d*x
 + c))/(a^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2 + (b^10*c^2 - 2*a*b^9*c*d
 + a^2*b^8*d^2)*x^4 + 4*(a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7*d^2)*x^3 + 6*(
a^2*b^8*c^2 - 2*a^3*b^7*c*d + a^4*b^6*d^2)*x^2 + 4*(a^3*b^7*c^2 - 2*a^4*b^6
*c*d + a^5*b^5*d^2)*x)]
```

**giac [A]** time = 1.09, size = 259, normalized size = 1.60

$$-\frac{5\,d^{4}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{64\,\left(b^{4}c-ab^{3}d\right)\sqrt{-b^{2}c+abd}}-\frac{15\,(dx+c)^{\frac{7}{2}}b^{3}d^{4}+73\,(dx+c)^{\frac{5}{2}}b^{3}cd^{4}-55\,(dx+c)^{\frac{3}{2}}b^{3}c^{2}d^{4}+15\,\sqrt{dx+c}\,b^{3}c^{3}d^{4}-73\,(dx+c)^{\frac{5}{2}}ab^{2}d^{5}+110\,(dx+c)^{\frac{3}{2}}ab^{2}cd^{5}-45\,\sqrt{dx+c}\,ab^{2}c^{2}d^{5}-55\,(dx+c)^{\frac{3}{2}}a^{2}bd^{6}+45\,\sqrt{dx+c}\,a^{2}bcd^{6}-15\,\sqrt{dx+c}\,a^{3}d^{7}}{192\,\left(b^{4}c-ab^{3}d\right)\left((dx+c)b-bc+ad\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -5/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c - a*b^3*d)*s
qrt(-b^2*c + a*b*d)) - 1/192*(15*(d*x + c)^(7/2)*b^3*d^4 + 73*(d*x + c)^(5/
2)*b^3*c*d^4 - 55*(d*x + c)^(3/2)*b^3*c^2*d^4 + 15*sqrt(d*x + c)*b^3*c^3*d^
4 - 73*(d*x + c)^(5/2)*a*b^2*d^5 + 110*(d*x + c)^(3/2)*a*b^2*c*d^5 - 45*sqr
t(d*x + c)*a*b^2*c^2*d^5 - 55*(d*x + c)^(3/2)*a^2*b*d^6 + 45*sqrt(d*x + c)*
a^2*b*c*d^6 - 15*sqrt(d*x + c)*a^3*d^7)/((b^4*c - a*b^3*d)*((d*x + c)*b - b
*c + a*d)^4)
```

**maple [A]** time = 0.02, size = 246, normalized size = 1.52

$$-\frac{5\sqrt{dx+c}\,a^{2}d^{6}}{64\,(bdx+ad)^{4}\,b^{3}}+\frac{5\sqrt{dx+c}\,ac\,d^{5}}{32\,(bdx+ad)^{4}\,b^{2}}-\frac{5\sqrt{dx+c}\,c^{2}d^{4}}{64\,(bdx+ad)^{4}\,b}-\frac{55\,(dx+c)^{\frac{3}{2}}\,a\,d^{5}}{192\,(bdx+ad)^{4}\,b^{2}}+\frac{55\,(dx+c)^{\frac{3}{2}}\,c\,d^{4}}{192\,(bdx+ad)^{4}\,b}+\frac{5\,(dx+c)^{\frac{7}{2}}\,d^{4}}{64\,(bdx+ad)^{4}\,(ad-bc)}-\frac{73\,(dx+c)^{\frac{5}{2}}\,d^{4}}{192\,(bdx+ad)^{4}\,b}+\frac{5d^{4}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{64\,(ad-bc)\,\sqrt{(ad-bc)\,b}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^5,x)
```

```
[Out] 5/64*d^4/(b*d*x+a*d)^4/(a*d-b*c)*(d*x+c)^(7/2)-73/192*d^4/(b*d*x+a*d)^4/b*(
d*x+c)^(5/2)-55/192*d^5/(b*d*x+a*d)^4/b^2*(d*x+c)^(3/2)*a+55/192*d^4/(b*d*x
+a*d)^4/b*(d*x+c)^(3/2)*c-5/64*d^6/(b*d*x+a*d)^4/b^3*(d*x+c)^(1/2)*a^2+5/32
*d^5/(b*d*x+a*d)^4/b^2*(d*x+c)^(1/2)*a*c-5/64*d^4/(b*d*x+a*d)^4/b*(d*x+c)^(
1/2)*c^2+5/64*d^4/(a*d-b*c)/b^3/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((
a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.41, size = 309, normalized size = 1.91

$$\frac{5\,d^{4}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{7/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{\frac{73\,d^{4}\,{\left(c+d\,x\right)}^{5/2}}{192\,b}-\frac{5\,d^{4}\,{\left(c+d\,x\right)}^{7/2}}{64\,\left(a\,d-b\,c\right)}+\frac{5\,d^{4}\,\sqrt{c+d\,x}\,\left(a^{2}\,d^{2}-2\,a\,b\,c\,d+b^{2}\,c^{2}\right)}{64\,b^{3}}+\frac{55\,d^{4}\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{192\,b^{2}}}{b^{4}\,{\left(c+d\,x\right)}^{4}-\left(4\,b^{4}\,c-4\,a\,b^{3}\,d\right)\,{\left(c+d\,x\right)}^{3}-\left(c+d\,x\right)\,\left(-4\,a^{3}\,b\,d^{3}+12\,a^{2}\,b^{2}\,c\,d^{2}-12\,a\,b^{3}\,c^{2}\,d+4\,b^{4}\,c^{3}\right)+a^{4}\,d^{4}+b^{4}\,c^{4}+{\left(c+d\,x\right)}^{2}\,\left(6\,a^{2}\,b^{2}\,d^{2}-12\,a\,b^{3}\,c\,d+6\,b^{4}\,c^{2}\right)+6\,a^{2}\,b^{2}\,c^{2}\,d^{2}-4\,a\,b^{3}\,c^{3}\,d-4\,a^{3}\,b\,c\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^5,x)
```

```
[Out] (5*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(7/2)*(a*d
- b*c)^(3/2)) - ((73*d^4*(c + d*x)^(5/2))/(192*b) - (5*d^4*(c + d*x)^(7/2))
/(64*(a*d - b*c)) + (5*d^4*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
/(64*b^3) + (55*d^4*(a*d - b*c)*(c + d*x)^(3/2))/(192*b^2))/(b^4*(c + d*x)^
4 - (4*b^4*c - 4*a*b^3*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3
+ 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b
^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

## 3.1304 $\int \frac{(c+dx)^{5/2}}{(a+bx)^6} \, dx$

**Optimal**. Leaf size=198

$$-\frac{3d^5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{7/2}(bc-ad)^{5/2}}+\frac{3d^4\sqrt{c+dx}}{128b^3(a+bx)(bc-ad)^2}-\frac{d^3\sqrt{c+dx}}{64b^3(a+bx)^2(bc-ad)}-\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+}{5b(a}$$

**Rubi** [A] time = 0.09, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$\frac{3d^4\sqrt{c+dx}}{128b^3(a+bx)(bc-ad)^2}-\frac{d^3\sqrt{c+dx}}{64b^3(a+bx)^2(bc-ad)}-\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{3d^5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{7/2}(bc-ad)^{5/2}}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] -(d^2*Sqrt[c + d*x])/(16*b^3*(a + b*x)^3) - (d^3*Sqrt[c + d*x])/(64*b^3*(b*
c - a*d)*(a + b*x)^2) + (3*d^4*Sqrt[c + d*x])/(128*b^3*(b*c - a*d)^2*(a + b
*x)) - (d*(c + d*x)^(3/2))/(8*b^2*(a + b*x)^4) - (c + d*x)^(5/2)/(5*b*(a +
b*x)^5) - (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(128*b^(
7/2)*(b*c - a*d)^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^6}\,dx = -\frac{(c+dx)^{5/2}}{5b(a+bx)^5} + \frac{d\int \frac{(c+dx)^{3/2}}{(a+bx)^5}\,dx}{2b}$$

$$= -\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+dx)^{5/2}}{5b(a+bx)^5} + \frac{\left(3d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx}{16b^2}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+dx)^{5/2}}{5b(a+bx)^5} + \frac{d^3\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{32b^3}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+dx)^{5/2}}{5b(a+bx)^5} - \frac{\left(3d^4\right)\int \frac{1}{(a+bx)^2\sqrt{c+d}}}{128b^3(bc-ad)}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} + \frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+d}{5b(a+}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} + \frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+d}{5b(a+}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} + \frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+d}{5b(a+}$$

**Mathematica** [C] time = 0.02, size = 52, normalized size = 0.26

$$\frac{2d^5(c+dx)^{7/2}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] (2*d^5*(c + d*x)^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(7*(-(b*c) + a*d)^6)
```

**IntegrateAlgebraic [A]** time = 1.43, size = 307, normalized size = 1.55

$$\frac{d^5\sqrt{c+dx}\left(15a^4d^4+70a^3bd^3(c+dx)-60a^3bcd^3+90a^2b^2c^2d^2+128a^2b^2d^2(c+dx)^2-210a^2b^2cd^2(c+dx)-60ab^3c^3d+210ab^3c^2d(c+dx)-70ab^3d(c+dx)^3-256ab^3cd(c+dx)^2+15b^4c^4-70b^4c^3(c+dx)+128b^4c^2(c+dx)^2-15b^4(c+dx)^4+70b^4c(c+dx)^3\right)}{640b^3(bc-ad)^2(-ad-b(c+dx)+bc)^5}-\frac{3d^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{128b^{7/2}(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] (d^5*Sqrt[c + d*x]*(15*b^4*c^4 - 60*a*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 - 60*a
^3*b*c*d^3 + 15*a^4*d^4 - 70*b^4*c^3*(c + d*x) + 210*a*b^3*c^2*d*(c + d*x)
- 210*a^2*b^2*c*d^2*(c + d*x) + 70*a^3*b*d^3*(c + d*x) + 128*b^4*c^2*(c + d
*x)^2 - 256*a*b^3*c*d*(c + d*x)^2 + 128*a^2*b^2*d^2*(c + d*x)^2 + 70*b^4*c*
(c + d*x)^3 - 70*a*b^3*d*(c + d*x)^3 - 15*b^4*(c + d*x)^4))/(640*b^3*(b*c -
 a*d)^2*(b*c - a*d - b*(c + d*x))^5) - (3*d^5*ArcTan[(Sqrt[b]*Sqrt[-(b*c) +
 a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(128*b^(7/2)*(-(b*c) + a*d)^(5/2))
```

**fricas [B]** time = 1.49, size = 1337, normalized size = 6.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^
2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c
 - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(128*b^6*c^5 -
 304*a*b^5*c^4*d + 184*a^2*b^4*c^3*d^2 + 2*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^
4 - 15*a^5*b*d^5 - 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 10*(b^6*c^2*d^3 - 8*a*b
^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(124*b^6*c^3*d^2 - 357*a*b^5*c^2*d^3 + 29
7*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(168*b^6*c^4*d - 424*a*b^5*c^3*d^
2 + 279*a^2*b^4*c^2*d^3 + 12*a^3*b^3*c*d^4 - 35*a^4*b^2*d^5)*x)*sqrt(d*x +
c))/(a^5*b^7*c^3 - 3*a^6*b^6*c^2*d + 3*a^7*b^5*c*d^2 - a^8*b^4*d^3 + (b^12*
c^3 - 3*a*b^11*c^2*d + 3*a^2*b^10*c*d^2 - a^3*b^9*d^3)*x^5 + 5*(a*b^11*c^3
- 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - a^4*b^8*d^3)*x^4 + 10*(a^2*b^10*c^3
- 3*a^3*b^9*c^2*d + 3*a^4*b^8*c*d^2 - a^5*b^7*d^3)*x^3 + 10*(a^3*b^9*c^3 -
3*a^4*b^8*c^2*d + 3*a^5*b^7*c*d^2 - a^6*b^6*d^3)*x^2 + 5*(a^4*b^8*c^3 - 3*a
^5*b^7*c^2*d + 3*a^6*b^6*c*d^2 - a^7*b^5*d^3)*x), 1/640*(15*(b^5*d^5*x^5 +
5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x +
 a^5*d^5)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b
*d*x + b*c)) - (128*b^6*c^5 - 304*a*b^5*c^4*d + 184*a^2*b^4*c^3*d^2 + 2*a^3
*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - 15*a^5*b*d^5 - 15*(b^6*c*d^4 - a*b^5*d^5)*
x^4 + 10*(b^6*c^2*d^3 - 8*a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(124*b^6*c^3
```

```
*d^2 - 357*a*b^5*c^2*d^3 + 297*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(168
*b^6*c^4*d - 424*a*b^5*c^3*d^2 + 279*a^2*b^4*c^2*d^3 + 12*a^3*b^3*c*d^4 - 3
5*a^4*b^2*d^5)*x)*sqrt(d*x + c))/(a^5*b^7*c^3 - 3*a^6*b^6*c^2*d + 3*a^7*b^5
*c*d^2 - a^8*b^4*d^3 + (b^12*c^3 - 3*a*b^11*c^2*d + 3*a^2*b^10*c*d^2 - a^3*
b^9*d^3)*x^5 + 5*(a*b^11*c^3 - 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - a^4*b^8
*d^3)*x^4 + 10*(a^2*b^10*c^3 - 3*a^3*b^9*c^2*d + 3*a^4*b^8*c*d^2 - a^5*b^7*
d^3)*x^3 + 10*(a^3*b^9*c^3 - 3*a^4*b^8*c^2*d + 3*a^5*b^7*c*d^2 - a^6*b^6*d^
3)*x^2 + 5*(a^4*b^8*c^3 - 3*a^5*b^7*c^2*d + 3*a^6*b^6*c*d^2 - a^7*b^5*d^3)*
x)]
```

**giac** [**B**] time = 1.25, size = 380, normalized size = 1.92

$$\frac{3\,d^{5}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{128\,\left(b^{5}c^{2}-2\,ab^{4}cd+a^{2}b^{3}d^{2}\right)\sqrt{-b^{2}c+abd}}+\frac{15\,(dx+c)^{\frac{9}{2}}b^{4}d^{5}-70\,(dx+c)^{\frac{7}{2}}b^{4}cd^{5}-128\,(dx+c)^{\frac{5}{2}}b^{4}c^{2}d^{5}+70\,(dx+c)^{\frac{3}{2}}b^{4}c^{3}d^{5}-15\,\sqrt{dx+c}\,b^{4}c^{4}d^{5}+70\,(dx+c)^{\frac{7}{2}}ab^{3}d^{6}+256\,(dx+c)^{\frac{5}{2}}ab^{3}cd^{6}-210\,(dx+c)^{\frac{3}{2}}ab^{3}c^{2}d^{6}+60\,\sqrt{dx+c}\,ab^{3}c^{3}d^{6}-128\,(dx+c)^{\frac{5}{2}}a^{2}b^{2}d^{7}+210\,(dx+c)^{\frac{3}{2}}a^{2}b^{2}cd^{7}-90\,\sqrt{dx+c}\,a^{2}b^{2}c^{2}d^{7}-70\,(dx+c)^{\frac{3}{2}}a^{3}bd^{8}+60\,\sqrt{dx+c}\,a^{3}bcd^{8}-15\,\sqrt{dx+c}\,a^{4}d^{9}}{640\,\left(b^{5}c^{2}-2\,ab^{4}cd+a^{2}b^{3}d^{2}\right)((dx+c)b-bc+ad)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 3/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^2 - 2*a*b^4*
c*d + a^2*b^3*d^2)*sqrt(-b^2*c + a*b*d)) + 1/640*(15*(d*x + c)^(9/2)*b^4*d^
5 - 70*(d*x + c)^(7/2)*b^4*c*d^5 - 128*(d*x + c)^(5/2)*b^4*c^2*d^5 + 70*(d*
x + c)^(3/2)*b^4*c^3*d^5 - 15*sqrt(d*x + c)*b^4*c^4*d^5 + 70*(d*x + c)^(7/2
)*a*b^3*d^6 + 256*(d*x + c)^(5/2)*a*b^3*c*d^6 - 210*(d*x + c)^(3/2)*a*b^3*c
^2*d^6 + 60*sqrt(d*x + c)*a*b^3*c^3*d^6 - 128*(d*x + c)^(5/2)*a^2*b^2*d^7 +
 210*(d*x + c)^(3/2)*a^2*b^2*c*d^7 - 90*sqrt(d*x + c)*a^2*b^2*c^2*d^7 - 70*
(d*x + c)^(3/2)*a^3*b*d^8 + 60*sqrt(d*x + c)*a^3*b*c*d^8 - 15*sqrt(d*x + c)
*a^4*d^9)/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*((d*x + c)*b - b*c + a*d)^
5)
```

**maple** [**A**] time = 0.02, size = 305, normalized size = 1.54

$$-\frac{3\sqrt{dx+c}\;a^{2}d^{7}}{128\,(bdx+ad)^{5}\,b^{3}}+\frac{3\sqrt{dx+c}\;ac\,d^{6}}{64\,(bdx+ad)^{5}\,b^{2}}+\frac{3\,(dx+c)^{\frac{9}{2}}\,b\,d^{5}}{128\,(bdx+ad)^{5}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}-\frac{3\sqrt{dx+c}\;c^{2}d^{5}}{128\,(bdx+ad)^{5}\,b}-\frac{7\,(dx+c)^{\frac{3}{2}}\,a\,d^{6}}{64\,(bdx+ad)^{5}\,b^{2}}+\frac{7\,(dx+c)^{\frac{3}{2}}\,c\,d^{5}}{64\,(bdx+ad)^{5}\,b}+\frac{7\,(dx+c)^{\frac{7}{2}}\,d^{5}}{64\,(bdx+ad)^{5}\,(ad-bc)}-\frac{(dx+c)^{\frac{5}{2}}\,d^{5}}{5\,(bdx+ad)^{5}\,b}+\frac{3d^{5}\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{128\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\sqrt{(ad-bc)\,b}\;b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^6,x)
```

```
[Out] 3/128*d^5/(b*d*x+a*d)^5*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(9/2)+7/64*d^
5/(b*d*x+a*d)^5/(a*d-b*c)*(d*x+c)^(7/2)-1/5*d^5/(b*d*x+a*d)^5/b*(d*x+c)^(5/
2)-7/64*d^6/(b*d*x+a*d)^5/b^2*(d*x+c)^(3/2)*a+7/64*d^5/(b*d*x+a*d)^5/b*(d*x
+c)^(3/2)*c-3/128*d^7/(b*d*x+a*d)^5/b^3*(d*x+c)^(1/2)*a^2+3/64*d^6/(b*d*x+a
*d)^5/b^2*(d*x+c)^(1/2)*a*c-3/128*d^5/(b*d*x+a*d)^5/b*(d*x+c)^(1/2)*c^2+3/1
28*d^5/b^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(
1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 0.50, size = 411, normalized size = 2.08

$$\frac{3\,d^5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{7/2}\,{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{d^5\,{\left(c+d\,x\right)}^{5/2}}{5\,b}-\frac{7\,d^5\,{\left(c+d\,x\right)}^{7/2}}{64\,\left(a\,d-b\,c\right)}+\frac{3\,d^5\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{128\,b^3}+\frac{7\,d^5\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{64\,b^2}-\frac{3\,b\,d^5\,{\left(c+d\,x\right)}^{9/2}}{128\,{\left(a\,d-b\,c\right)}^2}}{b^5\,{\left(c+d\,x\right)}^5-{\left(c+d\,x\right)}^2\,\left(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3\right)-\left(5\,b^5\,c-5\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^4+a^5\,d^5-b^5\,c^5+{\left(c+d\,x\right)}^3\,\left(10\,a^2\,b^3\,d^2-20\,a\,b^4\,c\,d+10\,b^5\,c^2\right)+\left(c+d\,x\right)\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)-10\,a^2\,b^3\,c^3\,d^2+10\,a^3\,b^2\,c^2\,d^3+5\,a\,b^4\,c^4\,d-5\,a^4\,b\,c\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^6,x)
```

```
[Out] (3*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b^(7/2)*(a*d
 - b*c)^(5/2)) - ((d^5*(c + d*x)^(5/2))/(5*b) - (7*d^5*(c + d*x)^(7/2))/(64
*(a*d - b*c)) + (3*d^5*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(12
8*b^3) + (7*d^5*(a*d - b*c)*(c + d*x)^(3/2))/(64*b^2) - (3*b*d^5*(c + d*x)^
(9/2))/(128*(a*d - b*c)^2))/(b^5*(c + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10
*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 30*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(
c + d*x)^4 + a^5*d^5 - b^5*c^5 + (c + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 -
 20*a*b^4*c*d) + (c + d*x)*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30
*a^2*b^3*c^2*d^2 - 20*a*b^4*c^3*d) - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^
3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

## 3.1305 $\int \frac{\sqrt{-1+x}}{(1+x)^2}\,dx$

**Optimal**. Leaf size=35

$$\frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\sqrt{x-1}}{x+1}$$

**Rubi** [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\sqrt{x-1}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x)^2,x]
```

```
[Out] -(Sqrt[-1 + x]/(1 + x)) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/Sqrt[2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-1+x}}{(1+x)^2}\,dx = -\frac{\sqrt{-1+x}}{1+x} + \frac{1}{2}\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx$$
$$= -\frac{\sqrt{-1+x}}{1+x} + \text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{-1+x}\right)$$
$$= -\frac{\sqrt{-1+x}}{1+x} + \frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{\sqrt{2}}$$

**Mathematica** [A] time = 0.03, size = 51, normalized size = 1.46

$$\frac{-2x-\sqrt{2-2x}\,(x+1)\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+2}{2\sqrt{x-1}\,(x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x)^2,x]
```

```
[Out] (2 - 2*x - Sqrt[2 - 2*x]*(1 + x)*ArcTanh[Sqrt[1 - x]/Sqrt[2]])/(2*Sqrt[-1 +
 x]*(1 + x))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 35, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{\sqrt{x-1}}{x+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x]/(1 + x)^2,x]
```

```
[Out] -(Sqrt[-1 + x]/(1 + x)) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/Sqrt[2]
```

**fricas** [A] time = 1.37, size = 33, normalized size = 0.94

$$\frac{\sqrt{2}\,(x+1)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)-2\sqrt{x-1}}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - 2*sqrt(x - 1))/(x +
1)
```

**giac** [A] time = 0.96, size = 29, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - sqrt(x - 1)/(x + 1)
```

**maple** [A] time = 0.01, size = 30, normalized size = 0.86

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{x-1}\,\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/2)/(x+1)^2,x)
```

```
[Out] 1/2*arctan(1/2*(x-1)^(1/2)*2^(1/2))*2^(1/2)-(x-1)^(1/2)/(x+1)
```

**maxima** [A] time = 2.97, size = 29, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - sqrt(x - 1)/(x + 1)
```

**mupad** [B] time = 0.06, size = 29, normalized size = 0.83

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x-1}}{2}\right)}{2}-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x + 1)^2,x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(x - 1)^(1/2))/2))/2 - (x - 1)^(1/2)/(x + 1)
```

**sympy [A]** time = 1.50, size = 104, normalized size = 2.97

$$\begin{cases} \frac{\sqrt{2}\, i \operatorname{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2} + \frac{i}{\sqrt{-1+\frac{2}{x+1}}\,\sqrt{x+1}} - \frac{2i}{\sqrt{-1+\frac{2}{x+1}}\,(x+1)^{\frac{3}{2}}} & \text{for } \frac{2}{|x+1|} > 1 \\ -\frac{\sqrt{1-\frac{2}{x+1}}}{\sqrt{x+1}} - \frac{\sqrt{2}\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x)**2,x)
```

```
[Out] Piecewise((sqrt(2)*I*acosh(sqrt(2)/sqrt(x + 1))/2 + I/(sqrt(-1 + 2/(x + 1))
*sqrt(x + 1)) - 2*I/(sqrt(-1 + 2/(x + 1))*(x + 1)**(3/2)), 2/Abs(x + 1) > 1
), (-sqrt(1 - 2/(x + 1))/sqrt(x + 1) - sqrt(2)*asin(sqrt(2)/sqrt(x + 1))/2,
 True))
```

## 3.1306 $\int \frac{\sqrt{-1+x}}{(1+x)^3} \, dx$

**Optimal**. Leaf size=56

$$\frac{\sqrt{x-1}}{8(x+1)} - \frac{\sqrt{x-1}}{2(x+1)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

**Rubi** [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 203}

$$\frac{\sqrt{x-1}}{8(x+1)} - \frac{\sqrt{x-1}}{2(x+1)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x)^3,x]
```

```
[Out] -Sqrt[-1 + x]/(2*(1 + x)^2) + Sqrt[-1 + x]/(8*(1 + x)) + ArcTan[Sqrt[-1 + x
]/Sqrt[2]]/(8*Sqrt[2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x}}{(1+x)^3}\,dx &= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{1}{4}\int \frac{1}{\sqrt{-1+x}\,(1+x)^2}\,dx \\
&= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{1}{16}\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx \\
&= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\sqrt{-1+x}\right) \\
&= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 28, normalized size = 0.50

$$\frac{1}{12}(x-1)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x)^3,x]
[Out] ((-1 + x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (1 - x)/2])/12
```

**IntegrateAlgebraic [A]** time = 0.06, size = 43, normalized size = 0.77

$$\frac{\sqrt{x-1}\,(x-3)}{8(x+1)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x]/(1 + x)^3,x]
```

```
[Out] ((-3 + x)*Sqrt[-1 + x])/(8*(1 + x)^2) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/(8*Sqr
t[2])
```

**fricas** [A] time = 1.12, size = 46, normalized size = 0.82

$$\frac{\sqrt{2}\left(x^{2}+2\,x+1\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x-1}\right)+2\,\sqrt{x-1}\,(x-3)}{16\left(x^{2}+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*(x^2 + 2*x + 1)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x -
1)*(x - 3))/(x^2 + 2*x + 1)
```

**giac** [A] time = 1.04, size = 37, normalized size = 0.66

$$\frac{1}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x-1}\right)+\frac{(x-1)^{\frac{3}{2}}-2\,\sqrt{x-1}}{8\,(x+1)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 1/8*((x - 1)^(3/2) - 2*sqrt(
x - 1))/(x + 1)^2
```

**maple** [A] time = 0.01, size = 40, normalized size = 0.71

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{x-1}\,\sqrt{2}}{2}\right)}{16}+\frac{\frac{(x-1)^{\frac{3}{2}}}{8}-\frac{\sqrt{x-1}}{4}}{(x+1)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/2)/(x+1)^3,x)
```

```
[Out] 2*(1/16*(x-1)^(3/2)-1/8*(x-1)^(1/2))/(x+1)^2+1/16*2^(1/2)*arctan(1/2*(x-1)^
(1/2)*2^(1/2))
```

**maxima** [A] time = 3.03, size = 43, normalized size = 0.77

$$\frac{1}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x-1}\right)+\frac{(x-1)^{\frac{3}{2}}-2\,\sqrt{x-1}}{8\left((x-1)^{2}+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 1/8*((x - 1)^(3/2) - 2*sqrt(
x - 1))/((x - 1)^2 + 4*x)
```

**mupad** [B] time = 0.04, size = 45, normalized size = 0.80

$$\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \sqrt{x-1}}{2}\right)}{16}-\frac{\frac{\sqrt{x-1}}{4}-\frac{(x-1)^{3/2}}{8}}{4\,x+(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x + 1)^3,x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(x - 1)^(1/2))/2))/16 - ((x - 1)^(1/2)/4 - (x - 1)^(
3/2)/8)/(4*x + (x - 1)^2)
```

**sympy** [A] time = 2.61, size = 167, normalized size = 2.98

$$\begin{cases} \frac{\sqrt{2}\, i \operatorname{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{16} - \frac{i}{8\sqrt{-1+\frac{2}{x+1}}\,\sqrt{x+1}} + \frac{3i}{4\sqrt{-1+\frac{2}{x+1}}\,(x+1)^{\frac{3}{2}}} - \frac{i}{\sqrt{-1+\frac{2}{x+1}}\,(x+1)^{\frac{5}{2}}} & \text{for}\ \frac{2}{|x+1|} > 1 \\ -\frac{\sqrt{2}\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{16} + \frac{1}{8\sqrt{1-\frac{2}{x+1}}\,\sqrt{x+1}} - \frac{3}{4\sqrt{1-\frac{2}{x+1}}\,(x+1)^{\frac{3}{2}}} + \frac{1}{\sqrt{1-\frac{2}{x+1}}\,(x+1)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x)**3,x)
```

```
[Out] Piecewise((sqrt(2)*I*acosh(sqrt(2)/sqrt(x + 1))/16 - I/(8*sqrt(-1 + 2/(x +
1))*sqrt(x + 1)) + 3*I/(4*sqrt(-1 + 2/(x + 1))*(x + 1)**(3/2)) - I/(sqrt(-1
 + 2/(x + 1))*(x + 1)**(5/2)), 2/Abs(x + 1) > 1), (-sqrt(2)*asin(sqrt(2)/sq
rt(x + 1))/16 + 1/(8*sqrt(1 - 2/(x + 1))*sqrt(x + 1)) - 3/(4*sqrt(1 - 2/(x
+ 1))*(x + 1)**(3/2)) + 1/(sqrt(1 - 2/(x + 1))*(x + 1)**(5/2)), True))
```

# 3.1307 $\int \frac{(a+bx)^5}{\sqrt{c+dx}} \, dx$

**Optimal**. Leaf size=154

$$-\frac{10b^4(c+dx)^{9/2}(bc-ad)}{9d^6}+\frac{20b^3(c+dx)^{7/2}(bc-ad)^2}{7d^6}-\frac{4b^2(c+dx)^{5/2}(bc-ad)^3}{d^6}+\frac{10b(c+dx)^{3/2}(bc-ad)^4}{3d^6}-\frac{2\sqrt{c+}}{}$$

**Rubi [A]** time = 0.05, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{9/2}(bc-ad)}{9d^6}+\frac{20b^3(c+dx)^{7/2}(bc-ad)^2}{7d^6}-\frac{4b^2(c+dx)^{5/2}(bc-ad)^3}{d^6}+\frac{10b(c+dx)^{3/2}(bc-ad)^4}{3d^6}-\frac{2\sqrt{c+dx}\,(bc-ad)^5}{d^6}+\frac{2b^5(c+dx)^{11/2}}{11d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^5*Sqrt[c + d*x])/d^6 + (10*b*(b*c - a*d)^4*(c + d*x)^(3/2))
/(3*d^6) - (4*b^2*(b*c - a*d)^3*(c + d*x)^(5/2))/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(7/2))/(7*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(9/2))/(9*d^6) +
 (2*b^5*(c + d*x)^(11/2))/(11*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^5}{d^5\sqrt{c+dx}}+\frac{5b(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{3/2}}{d^5}+\frac{10b^3(bc-ad)^2(c+dx)^5}{d^5}\right.\\ &= -\frac{2(bc-ad)^5\sqrt{c+dx}}{d^6}+\frac{10b(bc-ad)^4(c+dx)^{3/2}}{3d^6}-\frac{4b^2(bc-ad)^3(c+dx)^{5/2}}{d^6}+\frac{20b^3(bc-ad)^2(}{7d^6}\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 123, normalized size = 0.80

$$\frac{2\sqrt{c+dx}\left(-385b^4(c+dx)^4(bc-ad)+990b^3(c+dx)^3(bc-ad)^2-1386b^2(c+dx)^2(bc-ad)^3+1155b(c+dx)(bc-ad)^4-693(bc-ad)^5+63b^5(c+dx)^5\right)}{693d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-693*(b*c - a*d)^5 + 1155*b*(b*c - a*d)^4*(c + d*x) - 138
6*b^2*(b*c - a*d)^3*(c + d*x)^2 + 990*b^3*(b*c - a*d)^2*(c + d*x)^3 - 385*b
^4*(b*c - a*d)*(c + d*x)^4 + 63*b^5*(c + d*x)^5))/(693*d^6)
```

**IntegrateAlgebraic [B]** time = 0.10, size = 315, normalized size = 2.05

$$\frac{2\sqrt{c+dx}\left(693a^5d^5+1155a^4bd^4(c+dx)-3465a^4bcd^4+6930a^3b^2c^2d^3+1386a^3b^2d^3(c+dx)^2-4620a^3b^2cd^3(c+dx)-6930a^2b^3c^3d^2+6930a^2b^3c^2d^2(c+dx)+990a^2b^3d^2(c+dx)^3-4158a^2b^3cd^2(c+dx)^2+3465ab^4c^4d-4620ab^4c^3d(c+dx)+4158ab^4c^2d(c+dx)^2+385ab^4d(c+dx)^4-1980ab^4cd(c+dx)^3-693b^5c^5+1155b^5c^4(c+dx)-1386b^5c^3(c+dx)^2+990b^5c^2(c+dx)^3+63b^5(c+dx)^5-385b^5c(c+dx)^4\right)}{693d^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-693*b^5*c^5 + 3465*a*b^4*c^4*d - 6930*a^2*b^3*c^3*d^2 +
6930*a^3*b^2*c^2*d^3 - 3465*a^4*b*c*d^4 + 693*a^5*d^5 + 1155*b^5*c^4*(c + d
*x) - 4620*a*b^4*c^3*d*(c + d*x) + 6930*a^2*b^3*c^2*d^2*(c + d*x) - 4620*a^
3*b^2*c*d^3*(c + d*x) + 1155*a^4*b*d^4*(c + d*x) - 1386*b^5*c^3*(c + d*x)^2
 + 4158*a*b^4*c^2*d*(c + d*x)^2 - 4158*a^2*b^3*c*d^2*(c + d*x)^2 + 1386*a^3
*b^2*d^3*(c + d*x)^2 + 990*b^5*c^2*(c + d*x)^3 - 1980*a*b^4*c*d*(c + d*x)^3
 + 990*a^2*b^3*d^2*(c + d*x)^3 - 385*b^5*c*(c + d*x)^4 + 385*a*b^4*d*(c + d
*x)^4 + 63*b^5*(c + d*x)^5))/(693*d^6)
```

**fricas [A]** time = 1.26, size = 261, normalized size = 1.69

$$\frac{2\left(63b^5d^5x^5-256b^5c^5+1408ab^4c^4d-3168a^2b^3c^3d^2+3696a^3b^2c^2d^3-2310a^4bcd^4+693a^5d^5-35\left(2b^5cd^4-11ab^4d^5\right)x^4+10\left(8b^5c^2d^3-44ab^4cd^4+99a^2b^3d^5\right)x^3-6\left(16b^5c^3d^2-88ab^4c^2d^3+198a^2b^3cd^4-231a^3b^2d^5\right)x^2+\left(128b^5c^4d-704ab^4c^3d^2+1584a^2b^3c^2d^3-1848a^3b^2cd^4+1155a^4bd^5\right)x\right)\sqrt{dx+c}}{693d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*d^5*x^5 - 256*b^5*c^5 + 1408*a*b^4*c^4*d - 3168*a^2*b^3*c^3*d
^2 + 3696*a^3*b^2*c^2*d^3 - 2310*a^4*b*c*d^4 + 693*a^5*d^5 - 35*(2*b^5*c*d^
4 - 11*a*b^4*d^5)*x^4 + 10*(8*b^5*c^2*d^3 - 44*a*b^4*c*d^4 + 99*a^2*b^3*d^5
)*x^3 - 6*(16*b^5*c^3*d^2 - 88*a*b^4*c^2*d^3 + 198*a^2*b^3*c*d^4 - 231*a^3*
b^2*d^5)*x^2 + (128*b^5*c^4*d - 704*a*b^4*c^3*d^2 + 1584*a^2*b^3*c^2*d^3 -
1848*a^3*b^2*c*d^4 + 1155*a^4*b*d^5)*x)*sqrt(d*x + c)/d^6
```

**giac [B]** time = 1.07, size = 283, normalized size = 1.84

$$\frac{2\left(693\sqrt{dx+c}\,a^5+\frac{1155\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^4b}{d}+\frac{462\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)a^3b^2}{d^2}+\frac{198\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)a^2b^3}{d^3}+\frac{11\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)ab^4}{d^4}+\frac{\left(63(dx+c)^{\frac{11}{2}}-385(dx+c)^{\frac{9}{2}}c+990(dx+c)^{\frac{7}{2}}c^2-1386(dx+c)^{\frac{5}{2}}c^3+1155(dx+c)^{\frac{3}{2}}c^4-693\sqrt{dx+c}\,c^5\right)b^5}{d^5}\right)}{693d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/693*(693*sqrt(d*x + c)*a^5 + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^4*b/d + 462*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
```

```
^2)*a^3*b^2/d^2 + 198*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3/d^3 + 11*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a*b^4/d^4 + (63*(d*x + c)^(11/2) - 385*(d*x + c)^
(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x +
c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5/d^5)/d
```

**maple [B]** time = 0.01, size = 273, normalized size = 1.77

$$\frac{2\sqrt{dx+c}\,\left(63b^5x^5d^5+385a\,b^4d^5x^4-70b^5c\,d^4x^4+990a^2b^3d^5x^3-440a\,b^4c\,d^4x^3+80b^5c^2d^3x^3+1386a^3b^2d^5x^2-1188a^2b^3c\,d^4x^2+528a\,b^4c^2d^3x^2-96b^5c^3d^2x^2+1155a^4b\,d^5x-1848a^3b^2c\,d^4x+1584a^2b^3c^2d^3x-704a\,b^4c^3d^2x+128b^5c^4dx+693a^5d^5-2310a^4bc\,d^4+3696a^3b^2c^2d^3-3168a^2b^3c^3d^2+1408a\,b^4c^4d-256b^5c^5\right)}{693d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(1/2),x)
```

```
[Out] 2/693*(d*x+c)^(1/2)*(63*b^5*d^5*x^5+385*a*b^4*d^5*x^4-70*b^5*c*d^4*x^4+990*
a^2*b^3*d^5*x^3-440*a*b^4*c*d^4*x^3+80*b^5*c^2*d^3*x^3+1386*a^3*b^2*d^5*x^2
-1188*a^2*b^3*c*d^4*x^2+528*a*b^4*c^2*d^3*x^2-96*b^5*c^3*d^2*x^2+1155*a^4*b
*d^5*x-1848*a^3*b^2*c*d^4*x+1584*a^2*b^3*c^2*d^3*x-704*a*b^4*c^3*d^2*x+128*
b^5*c^4*d*x+693*a^5*d^5-2310*a^4*b*c*d^4+3696*a^3*b^2*c^2*d^3-3168*a^2*b^3*
c^3*d^2+1408*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

**maxima [B]** time = 1.38, size = 283, normalized size = 1.84

$$\frac{2\left(693\,\sqrt{dx+c}\,a^5+\frac{1155\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^4b}{d}+\frac{462\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^3b^2}{d^2}+\frac{198\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)a^2b^3}{d^3}+\frac{11\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)ab^4}{d^4}+\frac{\left(63\,(dx+c)^{\frac{11}{2}}-385\,(dx+c)^{\frac{9}{2}}c+990\,(dx+c)^{\frac{7}{2}}c^2-1386\,(dx+c)^{\frac{5}{2}}c^3+1155\,(dx+c)^{\frac{3}{2}}c^4-693\,\sqrt{dx+c}\,c^5\right)b^5}{d^5}\right)}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(693*sqrt(d*x + c)*a^5 + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^4*b/d + 462*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^3*b^2/d^2 + 198*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3/d^3 + 11*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a*b^4/d^4 + (63*(d*x + c)^(11/2) - 385*(d*x + c)^
(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x +
c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5/d^5)/d
```

**mupad [B]** time = 0.07, size = 137, normalized size = 0.89

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,\sqrt{c+d\,x}}{d^6}+\frac{4\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{5/2}}{d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(11/2))/(11*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(9/2
))/(9*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(1/2))/d^6 + (4*b^2*(a*d - b*c)^3*(
c + d*x)^(5/2))/d^6 + (20*b^3*(a*d - b*c)^2*(c + d*x)^(7/2))/(7*d^6) + (10*
b*(a*d - b*c)^4*(c + d*x)^(3/2))/(3*d^6)
```

**sympy [A]** time = 79.91, size = 728, normalized size = 4.73

$$\begin{cases} \frac{-\frac{2a^5c}{\sqrt{c+dx}} - 2a^5\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right) - \frac{10a^4bc\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right)}{d} - \frac{10a^4b\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d} - \frac{20a^3b^2c\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2} - \frac{20a^3b^2\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2} - \frac{20a^2b^3c\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^3} - \frac{20a^2b^3\left(\frac{c^4}{\sqrt{c+dx}} + 4c^3\sqrt{c+dx} - 2c^2(c+dx)^{\frac{3}{2}} + \frac{4c(c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3} - \frac{10ab^4c\left(\frac{c^4}{\sqrt{c+dx}} + 4c^3\sqrt{c+dx} - 2c^2(c+dx)^{\frac{3}{2}} + \frac{4c(c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^4} - \frac{10ab^4\left(-\frac{c^5}{\sqrt{c+dx}} - 5c^4\sqrt{c+dx} + \frac{10c^3(c+dx)^{\frac{3}{2}}}{3} - 2c^2(c+dx)^{\frac{5}{2}} + \frac{5c(c+dx)^{\frac{7}{2}}}{7} - \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4} - \frac{2b^5c\left(-\frac{c^5}{\sqrt{c+dx}} - 5c^4\sqrt{c+dx} + \frac{10c^3(c+dx)^{\frac{3}{2}}}{3} - 2c^2(c+dx)^{\frac{5}{2}} + \frac{5c(c+dx)^{\frac{7}{2}}}{7} - \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^5} - \frac{2b^5\left(\frac{c^6}{\sqrt{c+dx}} + 6c^5\sqrt{c+dx} - 5c^4(c+dx)^{\frac{3}{2}} + 4c^3(c+dx)^{\frac{5}{2}} - \frac{15c^2(c+dx)^{\frac{7}{2}}}{7} + \frac{2c(c+dx)^{\frac{9}{2}}}{3} - \frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^5}}{d} & \text{for}\ d \neq 0 \\ \frac{\begin{cases} a^5x & \text{for}\ b = 0 \\ \frac{(a+bx)^6}{6b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**5*c/sqrt(c + d*x) - 2*a**5*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 10*a**4*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 10*a**4*b*(c**2/s
qrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 20*a**3*b**2*c*(
c**2/sqrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 20*a**3
*b**2*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c
 + d*x)**(5/2)/5)/d**2 - 20*a**2*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(
c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 20*a**2*b**3*(c*
*4/sqrt(c + d*x) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c
+ d*x)**(5/2)/5 - (c + d*x)**(7/2)/7)/d**3 - 10*a*b**4*c*(c**4/sqrt(c + d*x
) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5
 - (c + d*x)**(7/2)/7)/d**4 - 10*a*b**4*(-c**5/sqrt(c + d*x) - 5*c**4*sqrt(
c + d*x) + 10*c**3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2) + 5*c*(c +
d*x)**(7/2)/7 - (c + d*x)**(9/2)/9)/d**4 - 2*b**5*c*(-c**5/sqrt(c + d*x) -
5*c**4*sqrt(c + d*x) + 10*c**3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2)
 + 5*c*(c + d*x)**(7/2)/7 - (c + d*x)**(9/2)/9)/d**5 - 2*b**5*(c**6/sqrt(c
+ d*x) + 6*c**5*sqrt(c + d*x) - 5*c**4*(c + d*x)**(3/2) + 4*c**3*(c + d*x)*
*(5/2) - 15*c**2*(c + d*x)**(7/2)/7 + 2*c*(c + d*x)**(9/2)/3 - (c + d*x)**(
11/2)/11)/d**5)/d, Ne(d, 0)), (Piecewise((a**5*x, Eq(b, 0)), ((a + b*x)**6/
(6*b), True))/sqrt(c), True))
```

## 3.1308 $\int \frac{(a+bx)^4}{\sqrt{c+dx}} \, dx$

**Optimal**. Leaf size=127

$$-\frac{8b^3(c+dx)^{7/2}(bc-ad)}{7d^5}+\frac{12b^2(c+dx)^{5/2}(bc-ad)^2}{5d^5}-\frac{8b(c+dx)^{3/2}(bc-ad)^3}{3d^5}+\frac{2\sqrt{c+dx}\,(bc-ad)^4}{d^5}+\frac{2b^4(c+dx)^{9/}}{9d^5}$$

**Rubi [A]** time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{7/2}(bc-ad)}{7d^5}+\frac{12b^2(c+dx)^{5/2}(bc-ad)^2}{5d^5}-\frac{8b(c+dx)^{3/2}(bc-ad)^3}{3d^5}+\frac{2\sqrt{c+dx}\,(bc-ad)^4}{d^5}+\frac{2b^4(c+dx)^{9/2}}{9d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^4*Sqrt[c + d*x])/d^5 - (8*b*(b*c - a*d)^3*(c + d*x)^(3/2))/(
3*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^5) - (8*b^3*(b*c - a*d
)*(c + d*x)^(7/2))/(7*d^5) + (2*b^4*(c + d*x)^(9/2))/(9*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4\sqrt{c+dx}}-\frac{4b(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{4b^3(bc-ad)(c+dx)^{5/2}}{d^4}+\right.\\ &= \frac{2(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{8b(bc-ad)^3(c+dx)^{3/2}}{3d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{5/2}}{5d^5}-\frac{8b^3(bc-ad)(c+}{7d^5}\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 101, normalized size = 0.80

$$\frac{2\sqrt{c+dx}\left(-180b^3(c+dx)^3(bc-ad)+378b^2(c+dx)^2(bc-ad)^2-420b(c+dx)(bc-ad)^3+315(bc-ad)^4+35b^4(c+dx)^4\right)}{315d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(315*(b*c - a*d)^4 - 420*b*(b*c - a*d)^3*(c + d*x) + 378*b
^2*(b*c - a*d)^2*(c + d*x)^2 - 180*b^3*(b*c - a*d)*(c + d*x)^3 + 35*b^4*(c
+ d*x)^4))/(315*d^5)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 213, normalized size = 1.68

$$\frac{2\sqrt{c+dx}\left(315a^4d^4+420a^3bd^3(c+dx)-1260a^3bcd^3+1890a^2b^2c^2d^2+378a^2b^2d^2(c+dx)^2-1260a^2b^2cd^2(c+dx)-1260ab^3c^3d+1260ab^3c^2d(c+dx)+180ab^3d(c+dx)^3-756ab^3cd(c+dx)^2+315b^4c^4-420b^4c^3(c+dx)+378b^4c^2(c+dx)^2+35b^4(c+dx)^4-180b^4c(c+dx)^3\right)}{315d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(315*b^4*c^4 - 1260*a*b^3*c^3*d + 1890*a^2*b^2*c^2*d^2 - 1
260*a^3*b*c*d^3 + 315*a^4*d^4 - 420*b^4*c^3*(c + d*x) + 1260*a*b^3*c^2*d*(c
 + d*x) - 1260*a^2*b^2*c*d^2*(c + d*x) + 420*a^3*b*d^3*(c + d*x) + 378*b^4*
c^2*(c + d*x)^2 - 756*a*b^3*c*d*(c + d*x)^2 + 378*a^2*b^2*d^2*(c + d*x)^2 -
 180*b^4*c*(c + d*x)^3 + 180*a*b^3*d*(c + d*x)^3 + 35*b^4*(c + d*x)^4))/(31
5*d^5)
```

**fricas [A]** time = 1.18, size = 182, normalized size = 1.43

$$\frac{2\left(35\,b^4d^4x^4+128\,b^4c^4-576\,ab^3c^3d+1008\,a^2b^2c^2d^2-840\,a^3bcd^3+315\,a^4d^4-20\left(2\,b^4cd^3-9\,ab^3d^4\right)x^3+6\left(8\,b^4c^2d^2-36\,ab^3cd^3+63\,a^2b^2d^4\right)x^2-4\left(16\,b^4c^3d-72\,ab^3c^2d^2+126\,a^2b^2cd^3-105\,a^3bd^4\right)x\right)\sqrt{dx+c}}{315\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*d^4*x^4 + 128*b^4*c^4 - 576*a*b^3*c^3*d + 1008*a^2*b^2*c^2*d^
2 - 840*a^3*b*c*d^3 + 315*a^4*d^4 - 20*(2*b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 6*
(8*b^4*c^2*d^2 - 36*a*b^3*c*d^3 + 63*a^2*b^2*d^4)*x^2 - 4*(16*b^4*c^3*d - 7
2*a*b^3*c^2*d^2 + 126*a^2*b^2*c*d^3 - 105*a^3*b*d^4)*x)*sqrt(d*x + c)/d^5
```

**giac [A]** time = 0.96, size = 204, normalized size = 1.61

$$\frac{2\left(315\,\sqrt{dx+c}\,a^4+\frac{420\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^3b}{d}+\frac{126\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^2b^2}{d^2}+\frac{36\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)ab^3}{d^3}+\frac{\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)b^4}{d^4}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^4 + 420*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^
3*b/d + 126*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b^2/d^2 + 36*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3/d^3 + (35*(d*x + c)^(9/2) - 180*(
```

```
d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*
sqrt(d*x + c)*c^4)*b^4/d^4)/d
```

**maple [A]** time = 0.01, size = 186, normalized size = 1.46

$$\frac{2\sqrt{dx+c}\left(35b^4x^4d^4+180a\,b^3d^4x^3-40b^4c\,d^3x^3+378a^2b^2d^4x^2-216a\,b^3c\,d^3x^2+48b^4c^2d^2x^2+420a^3b\,d^4x-504a^2b^2c\,d^3x+288a\,b^3c^2d^2x-64b^4c^3dx+315a^4d^4-840a^3bc\,d^3+1008a^2b^2c^2d^2-576a\,b^3c^3d+128b^4c^4\right)}{315d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(1/2),x)
```

```
[Out] 2/315*(d*x+c)^(1/2)*(35*b^4*d^4*x^4+180*a*b^3*d^4*x^3-40*b^4*c*d^3*x^3+378*
a^2*b^2*d^4*x^2-216*a*b^3*c*d^3*x^2+48*b^4*c^2*d^2*x^2+420*a^3*b*d^4*x-504*
a^2*b^2*c*d^3*x+288*a*b^3*c^2*d^2*x-64*b^4*c^3*d*x+315*a^4*d^4-840*a^3*b*c*
d^3+1008*a^2*b^2*c^2*d^2-576*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

**maxima [A]** time = 1.38, size = 204, normalized size = 1.61

$$\frac{2\left(315\sqrt{dx+c}\,a^4+\frac{420\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^3b}{d}+\frac{126\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)a^2b^2}{d^2}+\frac{36\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)ab^3}{d^3}+\frac{\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)b^4}{d^4}\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^4 + 420*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^
3*b/d + 126*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b^2/d^2 + 36*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3/d^3 + (35*(d*x + c)^(9/2) - 180*(
d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*
sqrt(d*x + c)*c^4)*b^4/d^4)/d
```

**mupad [B]** time = 0.24, size = 112, normalized size = 0.88

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{9/2}}{9\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,\sqrt{c+d\,x}}{d^5}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(9/2))/(9*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(7/2))/(
7*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(1/2))/d^5 + (12*b^2*(a*d - b*c)^2*(c +
 d*x)^(5/2))/(5*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(3/2))/(3*d^5)
```

**sympy [A]** time = 56.90, size = 532, normalized size = 4.19

$$\begin{cases} \frac{-\frac{2a^4c}{\sqrt{c+dx}}-2a^4\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)-\frac{8a^3bc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}-\frac{8a^3b\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}-\frac{12a^2b^2c\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2}-\frac{12a^2b^2\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{8ab^3c\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^3}-\frac{8ab^3\left(\frac{c^4}{\sqrt{c+dx}}+4c^3\sqrt{c+dx}-2c^2(c+dx)^{\frac{3}{2}}+\frac{4c(c+dx)^{\frac{5}{2}}}{5}-\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{2b^4c\left(\frac{c^4}{\sqrt{c+dx}}+4c^3\sqrt{c+dx}-2c^2(c+dx)^{\frac{3}{2}}+\frac{4c(c+dx)^{\frac{5}{2}}}{5}-\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^4}-\frac{2b^4\left(-\frac{c^5}{\sqrt{c+dx}}-5c^4\sqrt{c+dx}+\frac{10c^3(c+dx)^{\frac{3}{2}}}{3}-2c^2(c+dx)^{\frac{5}{2}}+\frac{5c(c+dx)^{\frac{7}{2}}}{7}-\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4}}{d} & \text{for}\ d \neq 0 \\ \frac{\begin{cases} a^4x & \text{for}\ b = 0 \\ \frac{(a+bx)^5}{5b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**4*c/sqrt(c + d*x) - 2*a**4*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 8*a**3*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 8*a**3*b*(c**2/sqr
t(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 12*a**2*b**2*c*(c*
*2/sqrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 12*a**2*b
**2*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c +
 d*x)**(5/2)/5)/d**2 - 8*a*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*
x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 8*a*b**3*(c**4/sqrt(c
+ d*x) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5
/2)/5 - (c + d*x)**(7/2)/7)/d**3 - 2*b**4*c*(c**4/sqrt(c + d*x) + 4*c**3*sq
rt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5 - (c + d*x)*
*(7/2)/7)/d**4 - 2*b**4*(-c**5/sqrt(c + d*x) - 5*c**4*sqrt(c + d*x) + 10*c*
*3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2) + 5*c*(c + d*x)**(7/2)/7 -
(c + d*x)**(9/2)/9)/d**4)/d, Ne(d, 0)), (Piecewise((a**4*x, Eq(b, 0)), ((a
+ b*x)**5/(5*b), True))/sqrt(c), True))
```

## 3.1309 $\int \frac{(a+bx)^3}{\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=96

$$-\frac{6b^2(c+dx)^{5/2}(bc-ad)}{5d^4}+\frac{2b(c+dx)^{3/2}(bc-ad)^2}{d^4}-\frac{2\sqrt{c+dx}\,(bc-ad)^3}{d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}$$

**Rubi** [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2(c+dx)^{5/2}(bc-ad)}{5d^4}+\frac{2b(c+dx)^{3/2}(bc-ad)^2}{d^4}-\frac{2\sqrt{c+dx}\,(bc-ad)^3}{d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^3*Sqrt[c + d*x])/d^4 + (2*b*(b*c - a*d)^2*(c + d*x)^(3/2))/
d^4 - (6*b^2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^4) + (2*b^3*(c + d*x)^(7/2))
/(7*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3\sqrt{c+dx}}+\frac{3b(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{3/2}}{d^3}+\frac{b^3(c+dx)^{5/2}}{d^3}\right)dx\\ &= -\frac{2(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{2b(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{6b^2(bc-ad)(c+dx)^{5/2}}{5d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 79, normalized size = 0.82

$$\frac{2\sqrt{c+dx}\,\left(-21b^2(c+dx)^2(bc-ad)+35b(c+dx)(bc-ad)^2-35(bc-ad)^3+5b^3(c+dx)^3\right)}{35d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-35*(b*c - a*d)^3 + 35*b*(b*c - a*d)^2*(c + d*x) - 21*b^2
*(b*c - a*d)*(c + d*x)^2 + 5*b^3*(c + d*x)^3))/(35*d^4)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 132, normalized size = 1.38

$$\frac{2\sqrt{c+dx}\left(35a^3d^3+35a^2bd^2(c+dx)-105a^2bcd^2+105ab^2c^2d+21ab^2d(c+dx)^2-70ab^2cd(c+dx)-35b^3c^3+35b^3c^2(c+dx)+5b^3(c+dx)^3-21b^3c(c+dx)^2\right)}{35d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-35*b^3*c^3 + 105*a*b^2*c^2*d - 105*a^2*b*c*d^2 + 35*a^3*
d^3 + 35*b^3*c^2*(c + d*x) - 70*a*b^2*c*d*(c + d*x) + 35*a^2*b*d^2*(c + d*x
) - 21*b^3*c*(c + d*x)^2 + 21*a*b^2*d*(c + d*x)^2 + 5*b^3*(c + d*x)^3))/(35
*d^4)
```

**fricas [A]** time = 0.94, size = 115, normalized size = 1.20

$$\frac{2\left(5\,b^3d^3x^3-16\,b^3c^3+56\,ab^2c^2d-70\,a^2bcd^2+35\,a^3d^3-3\left(2\,b^3cd^2-7\,ab^2d^3\right)x^2+\left(8\,b^3c^2d-28\,ab^2cd^2+35\,a^2bd^3\right)x\right)\sqrt{dx+c}}{35\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*d^3*x^3 - 16*b^3*c^3 + 56*a*b^2*c^2*d - 70*a^2*b*c*d^2 + 35*a^3
*d^3 - 3*(2*b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + (8*b^3*c^2*d - 28*a*b^2*c*d^2 +
35*a^2*b*d^3)*x)*sqrt(d*x + c)/d^4
```

**giac [A]** time = 1.01, size = 137, normalized size = 1.43

$$\frac{2\left(35\,\sqrt{dx+c}\,a^3+\frac{35\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^2b}{d}+\frac{7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)ab^2}{d^2}+\frac{\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)b^3}{d^3}\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*sqrt(d*x + c)*a^3 + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b
/d + 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*
b^2/d^2 + (5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*b^3/d^3)/d
```

**maple [A]** time = 0.01, size = 116, normalized size = 1.21

$$\frac{2\sqrt{dx+c}\,\left(5b^3x^3d^3+21a\,b^2d^3x^2-6b^3c\,d^2x^2+35a^2b\,d^3x-28a\,b^2c\,d^2x+8b^3c^2dx+35a^3d^3-70a^2bc\,d^2+56a\,b^2c^2d-16b^3c^3\right)}{35d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] 2/35*(d*x+c)^(1/2)*(5*b^3*d^3*x^3+21*a*b^2*d^3*x^2-6*b^3*c*d^2*x^2+35*a^2*b
*d^3*x-28*a*b^2*c*d^2*x+8*b^3*c^2*d*x+35*a^3*d^3-70*a^2*b*c*d^2+56*a*b^2*c^
2*d-16*b^3*c^3)/d^4
```

**maxima [A]** time = 1.38, size = 137, normalized size = 1.43

$$\frac{2\left(35\sqrt{dx+c}\,a^3+\frac{35\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^2b}{d}+\frac{7\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)ab^2}{d^2}+\frac{\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)b^3}{d^3}\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(d*x + c)*a^3 + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b
/d + 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*
b^2/d^2 + (5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*b^3/d^3)/d
```

**mupad [B]** time = 0.26, size = 87, normalized size = 0.91

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,\sqrt{c+d\,x}}{d^4}+\frac{2\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{3/2}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(7/2))/(7*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(5/2))/(
5*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(1/2))/d^4 + (2*b*(a*d - b*c)^2*(c + d*
x)^(3/2))/d^4
```

**sympy [A]** time = 37.06, size = 366, normalized size = 3.81

$$\begin{cases} \frac{-\frac{2a^3c}{\sqrt{c+dx}}-2a^3\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)-\frac{6a^2bc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}-\frac{6a^2b\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}-\frac{6ab^2c\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2}-\frac{6ab^2\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{2b^3c\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^3}-\frac{2b^3\left(\frac{c^4}{\sqrt{c+dx}}+4c^3\sqrt{c+dx}-2c^2(c+dx)^{\frac{3}{2}}+\frac{4c(c+dx)^{\frac{5}{2}}}{5}-\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}}{d} & \text{for } d \neq 0 \\ \frac{\begin{cases} a^3x & \text{for } b = 0 \\ \frac{(a+bx)^4}{4b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**3*c/sqrt(c + d*x) - 2*a**3*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 6*a**2*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 6*a**2*b*(c**2/sqr
t(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 6*a*b**2*c*(c**2/s
qrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 6*a*b**2*(-c*
*3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(
5/2)/5)/d**2 - 2*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c
+ d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 2*b**3*(c**4/sqrt(c + d*x) + 4*c
**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5 - (c +
 d*x)**(7/2)/7)/d**3)/d, Ne(d, 0)), (Piecewise((a**3*x, Eq(b, 0)), ((a + b*
x)**4/(4*b), True))/sqrt(c), True))
```

## 3.1310 $\int \frac{(a+bx)^2}{\sqrt{c+dx}} dx$

**Optimal**. Leaf size=69

$$-\frac{4b(c+dx)^{3/2}(bc-ad)}{3d^3}+\frac{2\sqrt{c+dx}\,(bc-ad)^2}{d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}$$

**Rubi [A]** time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{3/2}(bc-ad)}{3d^3}+\frac{2\sqrt{c+dx}\,(bc-ad)^2}{d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/d^3 - (4*b*(b*c - a*d)*(c + d*x)^(3/2))/(3*
d^3) + (2*b^2*(c + d*x)^(5/2))/(5*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2\sqrt{c+dx}}-\frac{2b(bc-ad)\sqrt{c+dx}}{d^2}+\frac{b^2(c+dx)^{3/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{4b(bc-ad)(c+dx)^{3/2}}{3d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 0.87

$$\frac{2\sqrt{c+dx}\,\left(15a^2d^2+10abd(dx-2c)+b^2\left(8c^2-4cdx+3d^2x^2\right)\right)}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x) + b^2*(8*c^2 - 4*c*d*x
 + 3*d^2*x^2)))/(15*d^3)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 72, normalized size = 1.04

$$\frac{2\sqrt{c+dx}\left(15a^2d^2+10abd(c+dx)-30abcd+15b^2c^2+3b^2(c+dx)^2-10b^2c(c+dx)\right)}{15d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(15*b^2*c^2 - 30*a*b*c*d + 15*a^2*d^2 - 10*b^2*c*(c + d*x)
 + 10*a*b*d*(c + d*x) + 3*b^2*(c + d*x)^2))/(15*d^3)
```

**fricas [A]** time = 1.46, size = 64, normalized size = 0.93

$$\frac{2\left(3\,b^2d^2x^2+8\,b^2c^2-20\,abcd+15\,a^2d^2-2\left(2\,b^2cd-5\,abd^2\right)x\right)\sqrt{dx+c}}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*d^2*x^2 + 8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 - 2*(2*b^2*c*d -
5*a*b*d^2)*x)*sqrt(d*x + c)/d^3
```

**giac [A]** time = 1.10, size = 82, normalized size = 1.19

$$\frac{2\left(15\,\sqrt{dx+c}\,a^2+\frac{10\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)ab}{d}+\frac{\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)b^2}{d^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b/d
 + (3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2/d^
2)/d
```

**maple [A]** time = 0.00, size = 63, normalized size = 0.91

$$\frac{2\sqrt{dx+c}\,\left(3b^2x^2d^2+10ab\,d^2x-4b^2cdx+15a^2d^2-20abcd+8b^2c^2\right)}{15d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] 2/15*(d*x+c)^(1/2)*(3*b^2*d^2*x^2+10*a*b*d^2*x-4*b^2*c*d*x+15*a^2*d^2-20*a*
b*c*d+8*b^2*c^2)/d^3
```

**maxima** [A] time = 1.37, size = 82, normalized size = 1.19

$$\frac{2\left(15\sqrt{dx+c}\,a^2+\frac{10\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)ab}{d}+\frac{\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)b^2}{d^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b/d
 + (3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2/d^
2)/d
```

**mupad** [B] time = 0.07, size = 68, normalized size = 0.99

$$\frac{2\sqrt{c+d\,x}\,\left(3\,b^2\,(c+d\,x)^2+15\,a^2\,d^2+15\,b^2\,c^2-10\,b^2\,c\,(c+d\,x)+10\,a\,b\,d\,(c+d\,x)-30\,a\,b\,c\,d\right)}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(1/2)*(3*b^2*(c + d*x)^2 + 15*a^2*d^2 + 15*b^2*c^2 - 10*b^2*c*
(c + d*x) + 10*a*b*d*(c + d*x) - 30*a*b*c*d))/(15*d^3)
```

**sympy** [A] time = 20.94, size = 231, normalized size = 3.35

$$\begin{cases} \frac{-\frac{2a^2c}{\sqrt{c+dx}}-2a^2\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)-\frac{4abc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}-\frac{4ab\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}-\frac{2b^2c\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2}-\frac{2b^2\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}}{d} & \text{for}\ d\neq 0 \\ \frac{\begin{cases} a^2x & \text{for}\ b=0 \\ \frac{(a+bx)^3}{3b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**2*c/sqrt(c + d*x) - 2*a**2*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 4*a*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 4*a*b*(c**2/sqrt(c +
d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 2*b**2*c*(c**2/sqrt(c +
d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 2*b**2*(-c**3/sqrt(c
+ d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d*
*2)/d, Ne(d, 0)), (Piecewise((a**2*x, Eq(b, 0)), ((a + b*x)**3/(3*b), True)
)/sqrt(c), True))
```

## 3.1311 $\int \frac{a+bx}{\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=40

$$\frac{2b(c+dx)^{3/2}}{3d^2}-\frac{2\sqrt{c+dx}\,(bc-ad)}{d^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{3/2}}{3d^2}-\frac{2\sqrt{c+dx}\,(bc-ad)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)*Sqrt[c + d*x])/d^2 + (2*b*(c + d*x)^(3/2))/(3*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{c+dx}}\,dx&=\int \left(\frac{-bc+ad}{d\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d}\right)dx\\&=-\frac{2(bc-ad)\sqrt{c+dx}}{d^2}+\frac{2b(c+dx)^{3/2}}{3d^2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 29, normalized size = 0.72

$$\frac{2\sqrt{c+dx}\,(3ad-2bc+bdx)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-2*b*c + 3*a*d + b*d*x))/(3*d^2)
```

**IntegrateAlgebraic [A]** time = 0.02, size = 32, normalized size = 0.80

$$\frac{2\sqrt{c+dx}\,(3ad+b(c+dx)-3bc)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/Sqrt[c + d*x],x]
[Out] (2*Sqrt[c + d*x]*(-3*b*c + 3*a*d + b*(c + d*x)))/(3*d^2)
```

**fricas [A]** time = 1.06, size = 25, normalized size = 0.62

$$\frac{2\,(bdx-2\,bc+3\,ad)\sqrt{dx+c}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*d*x - 2*b*c + 3*a*d)*sqrt(d*x + c)/d^2
```

**giac [A]** time = 0.88, size = 39, normalized size = 0.98

$$\frac{2\left(3\,\sqrt{dx+c}\,a+\frac{\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)b}{d}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*sqrt(d*x + c)*a + ((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b/d)/d
```

**maple [A]** time = 0.00, size = 26, normalized size = 0.65

$$\frac{2\sqrt{dx+c}\;(bdx+3ad-2bc)}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(1/2),x)
[Out] 2/3*(d*x+c)^(1/2)*(b*d*x+3*a*d-2*b*c)/d^2
```

**maxima** [A] time = 1.35, size = 39, normalized size = 0.98

$$\frac{2\left(3\sqrt{dx+c}\,a+\frac{\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)b}{d}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(3*sqrt(d*x + c)*a + ((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b/d)/d
```

**mupad** [B] time = 0.05, size = 28, normalized size = 0.70

$$\frac{2\sqrt{c+d\,x}\,\left(3\,a\,d-3\,b\,c+b\,\left(c+d\,x\right)\right)}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(1/2),x)
[Out] (2*(c + d*x)^(1/2)*(3*a*d - 3*b*c + b*(c + d*x)))/(3*d^2)
```

**sympy** [A] time = 4.78, size = 121, normalized size = 3.02

$$\begin{cases} \frac{-\frac{2ac}{\sqrt{c+dx}}-2a\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)-\frac{2bc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}-\frac{2b\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{for } d\neq 0 \\ \frac{ax+\frac{bx^2}{2}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(1/2),x)
[Out] Piecewise(((-2*a*c/sqrt(c + d*x) - 2*a*(-c/sqrt(c + d*x) - sqrt(c + d*x)) -
 2*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 2*b*(c**2/sqrt(c + d*x) + 2*c
*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d)/d, Ne(d, 0)), ((a*x + b*x**2/2)/sqr
t(c), True))
```

## 3.1312 $\int \frac{1}{\sqrt{c+dx}} dx$

**Optimal**. Leaf size=14

$$\frac{2\sqrt{c+dx}}{d}$$

**Rubi** [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x])/d
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+dx}} dx = \frac{2\sqrt{c+dx}}{d}$$

**Mathematica** [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x])/d
```

**IntegrateAlgebraic** [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[c + d*x],x]
[Out] (2*Sqrt[c + d*x])/d
```

**fricas [A]** time = 1.22, size = 12, normalized size = 0.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(d*x + c)/d
```

**giac [A]** time = 0.91, size = 12, normalized size = 0.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(d*x + c)/d
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(1/2),x)
[Out] 2*(d*x+c)^(1/2)/d
```

**maxima [A]** time = 1.30, size = 12, normalized size = 0.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(d*x + c)/d
```

**mupad** [B] time = 0.02, size = 12, normalized size = 0.86

$$\frac{2\sqrt{c+d\,x}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(1/2),x)
[Out] (2*(c + d*x)^(1/2))/d
```

**sympy** [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{2\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(1/2),x)
[Out] 2*sqrt(c + d*x)/d
```

## 3.1313 $\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

**Rubi** [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,\sqrt{bc-ad}}$$

**Mathematica** [A] time = 0.02, size = 47, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a
*d])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 57, normalized size = 1.21

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}\,\sqrt{ad-bc}}{bc-ad}\right)}{\sqrt{b}\,\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(Sqrt[b
]*Sqrt[-(b*c) + a*d])
```

**fricas** [A] time = 1.20, size = 119, normalized size = 2.53

$$\left[\frac{\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)}{\sqrt{b^2c-abd}}, \frac{2\,\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)}{b^2c-abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a))
/sqrt(b^2*c - a*b*d), 2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sq
rt(d*x + c)/(b*d*x + b*c))/(b^2*c - a*b*d)]
```

**giac [A]** time = 0.88, size = 38, normalized size = 0.81

$$\frac{2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

**maple [A]** time = 0.01, size = 37, normalized size = 0.79

$$\frac{2\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] 2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.27, size = 38, normalized size = 0.81

$$\frac{2\,\mathrm{atan}\left(\frac{b\,\sqrt{c+d\,x}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] (2*atan((b*(c + d*x)^(1/2))/(a*b*d - b^2*c)^(1/2)))/(a*b*d - b^2*c)^(1/2)
```

**sympy [A]** time = 5.41, size = 44, normalized size = 0.94

$$-\frac{2\operatorname{atan}\left(\frac{1}{\sqrt{\frac{b}{ad-bc}}\sqrt{c+dx}}\right)}{\sqrt{\frac{b}{ad-bc}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] -2*atan(1/(sqrt(b/(a*d - b*c))*sqrt(c + d*x)))/(sqrt(b/(a*d - b*c))*(a*d -
b*c))
```

## 3.1314 $\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=76

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx}}{(a+bx)(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx}}{(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*Sqrt[c + d*x]),x]
```

```
[Out] -(Sqrt[c + d*x]/((b*c - a*d)*(a + b*x))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx &= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} - \frac{d\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2(bc-ad)} \\
&= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{bc-ad} \\
&= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.07, size = 76, normalized size = 1.00

$$
\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\,(ad-bc)^{3/2}} - \frac{\sqrt{c+dx}}{(a+bx)(bc-ad)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*Sqrt[c + d*x]),x]
```

```
[Out] -(Sqrt[c + d*x]/((b*c - a*d)*(a + b*x))) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.20, size = 98, normalized size = 1.29

$$
\frac{d\sqrt{c+dx}}{(bc-ad)(-ad-b(c+dx)+bc)} - \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{\sqrt{b}\,(ad-bc)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*Sqrt[c + d*x]),x]
```

```
[Out] (d*Sqrt[c + d*x])/((b*c - a*d)*(b*c - a*d - b*(c + d*x))) - (d*ArcTan[(Sqrt
[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(Sqrt[b]*(-(b*c) + a*d)
^(3/2))
```

**fricas [B]** time = 1.29, size = 280, normalized size = 3.68

$$
\left[-\frac{\sqrt{b^2c-abd}\,(bdx+ad)\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\left(b^2c-abd\right)\sqrt{dx+c}}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x\right)}, -\frac{\sqrt{-b^2c+abd}\,(bdx+ad)\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+\left(b^2c-abd\right)\sqrt{dx+c}}{ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(b^2*c - a*b*d)*(b*d*x + a*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(
b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(b^2*c - a*b*d)*sqrt(d*x + c))
/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*
d^2)*x), -(sqrt(-b^2*c + a*b*d)*(b*d*x + a*d)*arctan(sqrt(-b^2*c + a*b*d)*s
qrt(d*x + c)/(b*d*x + b*c)) + (b^2*c - a*b*d)*sqrt(d*x + c))/(a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x)]
```

**giac** [**A**] time = 1.03, size = 87, normalized size = 1.14

$$-\frac{d \arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx+c}\,d}{((dx+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
- a*d)) - sqrt(d*x + c)*d/(((d*x + c)*b - b*c + a*d)*(b*c - a*d))
```

**maple** [**A**] time = 0.01, size = 77, normalized size = 1.01

$$\frac{d \arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)\sqrt{(ad-bc)\,b}}+\frac{\sqrt{dx+c}\;d}{(ad-bc)\,(bdx+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] d*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)+d/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arcta
n((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.09, size = 74, normalized size = 0.97

$$\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}+\frac{d\,\sqrt{c+d\,x}}{\left(a\,d-b\,c\right)\,\left(a\,d-b\,c+b\,\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(1/2)),x)
```

```
[Out] (d*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(b^(1/2)*(a*d - b*c)^
(3/2)) + (d*(c + d*x)^(1/2))/((a*d - b*c)*(a*d - b*c + b*(c + d*x)))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx\right)^{2}\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**2*sqrt(c + d*x)), x)
```

## 3.1315 $\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=114

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}+\frac{3d\sqrt{c+dx}}{4(a+bx)(bc-ad)^2}-\frac{\sqrt{c+dx}}{2(a+bx)^2(bc-ad)}$$

**Rubi [A]** time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}+\frac{3d\sqrt{c+dx}}{4(a+bx)(bc-ad)^2}-\frac{\sqrt{c+dx}}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*Sqrt[c + d*x]),x]
```

```
[Out] -Sqrt[c + d*x]/(2*(b*c - a*d)*(a + b*x)^2) + (3*d*Sqrt[c + d*x])/(4*(b*c -
a*d)^2*(a + b*x)) - (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]]
)/(4*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx &= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}-\frac{(3d)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{4(bc-ad)}\\
&= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}+\frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)}+\frac{\left(3d^2\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8(bc-ad)^2}\\
&= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}+\frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)}+\frac{(3d)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4(bc-ad)^2}\\
&= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}+\frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.01, size = 50, normalized size = 0.44

$$
\frac{2d^2\sqrt{c+dx}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{b(c+dx)}{ad-bc}\right)}{(ad-bc)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*Sqrt[c + d*x]),x]
```

```
[Out] (2*d^2*Sqrt[c + d*x]*Hypergeometric2F1[1/2, 3, 3/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(-(b*c) + a*d)^3
```

**IntegrateAlgebraic [A]** time = 0.23, size = 124, normalized size = 1.09

$$
\frac{d^2\sqrt{c+dx}\,(5ad+3b(c+dx)-5bc)}{4(bc-ad)^2(-ad-b(c+dx)+bc)^2}-\frac{3d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4\sqrt{b}\,(ad-bc)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*Sqrt[c + d*x]),x]
```

```
[Out] (d^2*Sqrt[c + d*x]*(-5*b*c + 5*a*d + 3*b*(c + d*x)))/(4*(b*c - a*d)^2*(b*c
- a*d - b*(c + d*x))^2) - (3*d^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c
+ d*x])/(b*c - a*d)])/(4*Sqrt[b]*(-(b*c) + a*d)^(5/2))
```

**fricas [B]** time = 1.57, size = 549, normalized size = 4.82

$$\left[\frac{3\left(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2}\right)\sqrt{b^{2}c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^{2}c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\left(2\,b^{3}c^{2}-7\,ab^{2}cd+5\,a^{2}bd^{2}-3\left(b^{3}cd-ab^{2}d^{2}\right)x\right)\sqrt{dx+c}}{8\left(a^{2}b^{4}c^{3}-3\,a^{3}b^{3}c^{2}d+3\,a^{4}b^{2}cd^{2}-a^{5}bd^{3}+\left(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3}\right)x^{2}+2\left(ab^{5}c^{3}-3\,a^{2}b^{4}c^{2}d+3\,a^{3}b^{3}cd^{2}-a^{4}b^{2}d^{3}\right)x\right)},\ \frac{3\left(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2}\right)\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{-b^{2}c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-\left(2\,b^{3}c^{2}-7\,ab^{2}cd+5\,a^{2}bd^{2}-3\left(b^{3}cd-ab^{2}d^{2}\right)x\right)\sqrt{dx+c}}{4\left(a^{2}b^{4}c^{3}-3\,a^{3}b^{3}c^{2}d+3\,a^{4}b^{2}cd^{2}-a^{5}bd^{3}+\left(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3}\right)x^{2}+2\left(ab^{5}c^{3}-3\,a^{2}b^{4}c^{2}d+3\,a^{3}b^{3}cd^{2}-a^{4}b^{2}d^{3}\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*
x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(2*b^
3*c^2 - 7*a*b^2*c*d + 5*a^2*b*d^2 - 3*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c
))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3 + (b^6*c^3
- 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^2 + 2*(a*b^5*c^3 - 3*a^2
*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x), 1/4*(3*(b^2*d^2*x^2 + 2*a*b
*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x
 + c)/(b*d*x + b*c)) - (2*b^3*c^2 - 7*a*b^2*c*d + 5*a^2*b*d^2 - 3*(b^3*c*d
- a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c
*d^2 - a^5*b*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3
)*x^2 + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x)]
```

**giac [A]** time = 0.93, size = 148, normalized size = 1.30

$$\frac{3\,d^{2}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{4\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{-b^{2}c+abd}}+\frac{3\,(dx+c)^{\frac{3}{2}}bd^{2}-5\,\sqrt{dx+c}\,bcd^{2}+5\,\sqrt{dx+c}\,ad^{3}}{4\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left((dx+c)b-bc+ad\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/4*(3*(d*x + c)^(3/2)*b*d^2 - 5*sqrt(d*
x + c)*b*c*d^2 + 5*sqrt(d*x + c)*a*d^3)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((
d*x + c)*b - b*c + a*d)^2)
```

**maple [A]** time = 0.01, size = 115, normalized size = 1.01

$$\frac{3d^{2}\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{4\,(ad-bc)^{2}\,\sqrt{(ad-bc)\,b}}+\frac{\sqrt{dx+c}\;d^{2}}{2\,(ad-bc)\,(bdx+ad)^{2}}+\frac{3\sqrt{dx+c}\;d^{2}}{4\,(ad-bc)^{2}\,(bdx+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] 1/2*d^2*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)^2+3/4*d^2/(a*d-b*c)^2*(d*x+c)^(
1/2)/(b*d*x+a*d)+3/4*d^2/(a*d-b*c)^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/
2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.33, size = 142, normalized size = 1.25

$$\frac{\frac{5\,d^2\,\sqrt{c+d\,x}}{4\,\left(a\,d-b\,c\right)}+\frac{3\,b\,d^2\,{\left(c+d\,x\right)}^{3/2}}{4\,{\left(a\,d-b\,c\right)}^2}}{b^2\,{\left(c+d\,x\right)}^2-\left(2\,b^2\,c-2\,a\,b\,d\right)\,\left(c+d\,x\right)+a^2\,d^2+b^2\,c^2-2\,a\,b\,c\,d}+\frac{3\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(1/2)),x)
```

```
[Out] ((5*d^2*(c + d*x)^(1/2))/(4*(a*d - b*c)) + (3*b*d^2*(c + d*x)^(3/2))/(4*(a*
d - b*c)^2))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b
^2*c^2 - 2*a*b*c*d) + (3*d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/
2)))/(4*b^(1/2)*(a*d - b*c)^(5/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.1316 $\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=147

$$\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{7/2}}-\frac{5d^2\sqrt{c+dx}}{8(a+bx)(bc-ad)^3}+\frac{5d\sqrt{c+dx}}{12(a+bx)^2(bc-ad)^2}-\frac{\sqrt{c+dx}}{3(a+bx)^3(bc-ad)}$$

**Rubi [A]** time = 0.05, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{5d^2\sqrt{c+dx}}{8(a+bx)(bc-ad)^3}+\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{7/2}}+\frac{5d\sqrt{c+dx}}{12(a+bx)^2(bc-ad)^2}-\frac{\sqrt{c+dx}}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```

```
[Out] -Sqrt[c + d*x]/(3*(b*c - a*d)*(a + b*x)^3) + (5*d*Sqrt[c + d*x])/(12*(b*c -
 a*d)^2*(a + b*x)^2) - (5*d^2*Sqrt[c + d*x])/(8*(b*c - a*d)^3*(a + b*x)) +
(5*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c -
a*d)^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} - \frac{(5d)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{6(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} + \frac{\left(5d^2\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} - \frac{\left(5d^3\right)\int \frac{}{(a+bx)}}{16(bc-}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} - \frac{\left(5d^2\right)\text{Subst}\Big(}{}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} + \frac{5d^3\tanh^{-1}\Big(}{8\sqrt{b}\,(bc-}$$

**Mathematica [C]** time = 0.01, size = 50, normalized size = 0.34

$$\frac{2d^3\sqrt{c+dx}\;{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};-\frac{b(c+dx)}{ad-bc}\right)}{(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```

```
[Out] (2*d^3*Sqrt[c + d*x]*Hypergeometric2F1[1/2, 4, 3/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(-(b*c) + a*d)^4
```

**IntegrateAlgebraic [A]** time = 0.27, size = 173, normalized size = 1.18

$$\frac{d^3\sqrt{c+dx}\left(33a^2d^2+40abd(c+dx)-66abcd+33b^2c^2+15b^2(c+dx)^2-40b^2c(c+dx)\right)}{24(bc-ad)^3(-ad-b(c+dx)+bc)^3} + \frac{5d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{8\sqrt{b}\,(bc-ad)^3\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```

```
[Out] (d^3*Sqrt[c + d*x]*(33*b^2*c^2 - 66*a*b*c*d + 33*a^2*d^2 - 40*b^2*c*(c + d*
x) + 40*a*b*d*(c + d*x) + 15*b^2*(c + d*x)^2))/(24*(b*c - a*d)^3*(b*c - a*d
 - b*(c + d*x))^3) + (5*d^3*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x
])/(b*c - a*d)])/(8*Sqrt[b]*(b*c - a*d)^3*Sqrt[-(b*c) + a*d])
```

**fricas** [**B**] time = 1.38, size = 884, normalized size = 6.01

$$\left[-\frac{15\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\left(8\,b^4c^3-34\,ab^3c^2d+59\,a^2b^2cd^2-33\,a^3bd^3+15\left(b^4cd^2-ab^3d^3\right)x^2-10\left(b^4c^2d-5\,ab^3cd^2+4\,a^2b^2d^3\right)x\right)\sqrt{dx+c}}{48\left(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^3+3\left(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4\right)x^2+3\left(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4\right)x\right)},\ -\frac{15\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+\left(8\,b^4c^3-34\,ab^3c^2d+59\,a^2b^2cd^2-33\,a^3bd^3+15\left(b^4cd^2-ab^3d^3\right)x^2-10\left(b^4c^2d-5\,ab^3cd^2+4\,a^2b^2d^3\right)x\right)\sqrt{dx+c}}{24\left(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^3+3\left(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4\right)x^2+3\left(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b
^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c
))/(b*x + a)) + 2*(8*b^4*c^3 - 34*a*b^3*c^2*d + 59*a^2*b^2*c*d^2 - 33*a^3*b
*d^3 + 15*(b^4*c*d^2 - a*b^3*d^3)*x^2 - 10*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a
^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*
c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^3 + 3*(a*b^7*c^4 - 4*a^2*b^6*c^3
*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^2 + 3*(a^2*b^6*c^
4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x)
, -1/24*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) +
(8*b^4*c^3 - 34*a*b^3*c^2*d + 59*a^2*b^2*c*d^2 - 33*a^3*b*d^3 + 15*(b^4*c*d
^2 - a*b^3*d^3)*x^2 - 10*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqr
t(d*x + c))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*
c*d^3 + a^7*b*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^
5*c*d^3 + a^4*b^4*d^4)*x^3 + 3*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2
*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^2 + 3*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*
d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x)]
```

**giac** [**A**] time = 1.02, size = 231, normalized size = 1.57

$$-\frac{5\,d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}-\frac{15\,(dx+c)^{\frac{5}{2}}b^2d^3-40\,(dx+c)^{\frac{3}{2}}b^2cd^3+33\,\sqrt{dx+c}\,b^2c^2d^3+40\,(dx+c)^{\frac{3}{2}}abd^4-66\,\sqrt{dx+c}\,abcd^4+33\,\sqrt{dx+c}\,a^2d^5}{24\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left((dx+c)b-bc+ad\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -5/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - 1/24*(15*(d*x + c)^
(5/2)*b^2*d^3 - 40*(d*x + c)^(3/2)*b^2*c*d^3 + 33*sqrt(d*x + c)*b^2*c^2*d^3
 + 40*(d*x + c)^(3/2)*a*b*d^4 - 66*sqrt(d*x + c)*a*b*c*d^4 + 33*sqrt(d*x +
c)*a^2*d^5)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)
*b - b*c + a*d)^3)
```

**maple** [A]   time = 0.01, size = 147, normalized size = 1.00

$$\frac{5d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{8(ad-bc)^3\sqrt{(ad-bc)\,b}}+\frac{\sqrt{dx+c}\,d^3}{3(ad-bc)(bdx+ad)^3}+\frac{5\sqrt{dx+c}\,d^3}{12(ad-bc)^2(bdx+ad)^2}+\frac{5\sqrt{dx+c}\,d^3}{8(ad-bc)^3(bdx+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*d^3*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)^3+5/12*d^3/(a*d-b*c)^2*(d*x+c)^
(1/2)/(b*d*x+a*d)^2+5/8*d^3/(a*d-b*c)^3*(d*x+c)^(1/2)/(b*d*x+a*d)+5/8*d^3/(
a*d-b*c)^3/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima** [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B]   time = 0.39, size = 218, normalized size = 1.48

$$\frac{\frac{11\,d^3\,\sqrt{c+d\,x}}{8\,(a\,d-b\,c)}+\frac{5\,b^2\,d^3\,(c+d\,x)^{5/2}}{8\,(a\,d-b\,c)^3}+\frac{5\,b\,d^3\,(c+d\,x)^{3/2}}{3\,(a\,d-b\,c)^2}}{(c+d\,x)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)+b^3\,(c+d\,x)^3-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,(c+d\,x)^2+a^3\,d^3-b^3\,c^3+3\,a\,b^2\,c^2\,d-3\,a^2\,b\,c\,d^2}+\frac{5\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,\sqrt{b}\,(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(1/2)),x)
```

```
[Out] ((11*d^3*(c + d*x)^(1/2))/(8*(a*d - b*c)) + (5*b^2*d^3*(c + d*x)^(5/2))/(8*
(a*d - b*c)^3) + (5*b*d^3*(c + d*x)^(3/2))/(3*(a*d - b*c)^2))/((c + d*x)*(3
*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^
2*d)*(c + d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (5*
d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(1/2)*(a*d - b*
c)^(7/2))
```

**sympy** [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.1317 $\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=180

$$-\frac{35d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64\sqrt{b}\,(bc-ad)^{9/2}}+\frac{35d^3\sqrt{c+dx}}{64(a+bx)(bc-ad)^4}-\frac{35d^2\sqrt{c+dx}}{96(a+bx)^2(bc-ad)^3}+\frac{7d\sqrt{c+dx}}{24(a+bx)^3(bc-ad)^2}-\frac{\sqrt{c+dx}}{4(a+bx)^4(bc-ad)}$$

**Rubi [A]** time = 0.06, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{35d^3\sqrt{c+dx}}{64(a+bx)(bc-ad)^4}-\frac{35d^2\sqrt{c+dx}}{96(a+bx)^2(bc-ad)^3}-\frac{35d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64\sqrt{b}\,(bc-ad)^{9/2}}+\frac{7d\sqrt{c+dx}}{24(a+bx)^3(bc-ad)^2}-\frac{\sqrt{c+dx}}{4(a+bx)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```

```
[Out] -Sqrt[c + d*x]/(4*(b*c - a*d)*(a + b*x)^4) + (7*d*Sqrt[c + d*x])/(24*(b*c -
 a*d)^2*(a + b*x)^3) - (35*d^2*Sqrt[c + d*x])/(96*(b*c - a*d)^3*(a + b*x)^2
) + (35*d^3*Sqrt[c + d*x])/(64*(b*c - a*d)^4*(a + b*x)) - (35*d^4*ArcTanh[(
Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(64*Sqrt[b]*(b*c - a*d)^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} - \frac{(7d)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{8(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} + \frac{\left(35d^2\right)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{48(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} - \frac{\left(35d^3\right)\int \frac{}{(a+}}{64(bc-}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35d^3\sqrt{c-}}{64(bc-ad)^4}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35d^3\sqrt{c-}}{64(bc-ad)^4}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35d^3\sqrt{c-}}{64(bc-ad)^4}$$

**Mathematica** [C] time = 0.01, size = 50, normalized size = 0.28

$$\frac{2d^4\sqrt{c+dx}\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};-\frac{b(c+dx)}{ad-bc}\right)}{(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```

```
[Out] (2*d^4*Sqrt[c + d*x]*Hypergeometric2F1[1/2, 5, 3/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(-(b*c) + a*d)^5
```

**IntegrateAlgebraic** [A] time = 0.44, size = 223, normalized size = 1.24

$$\frac{d^4\sqrt{c+dx}\left(279a^3d^3+511a^2bd^2(c+dx)-837a^2bcd^2+837ab^2c^2d+385ab^2d(c+dx)^2-1022ab^2cd(c+dx)-279b^3c^3+511b^3c^2(c+dx)+105b^3(c+dx)^3-385b^3c(c+dx)^2\right)}{192(bc-ad)^4(-ad-b(c+dx)+bc)^4} - \frac{35d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{64\sqrt{b}\,(ad-bc)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```

```
[Out] (d^4*Sqrt[c + d*x]*(-279*b^3*c^3 + 837*a*b^2*c^2*d - 837*a^2*b*c*d^2 + 279*
a^3*d^3 + 511*b^3*c^2*(c + d*x) - 1022*a*b^2*c*d*(c + d*x) + 511*a^2*b*d^2*
(c + d*x) - 385*b^3*c*(c + d*x)^2 + 385*a*b^2*d*(c + d*x)^2 + 105*b^3*(c +
d*x)^3))/(192*(b*c - a*d)^4*(b*c - a*d - b*(c + d*x))^4) - (35*d^4*ArcTan[(
Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(64*Sqrt[b]*(-(b*c)
 + a*d)^(9/2))
```

**fricas [B]** time = 1.17, size = 1325, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(105*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(48*b^5*c^4 - 248*a*b^4*c^3*d + 526*
a^2*b^3*c^2*d^2 - 605*a^3*b^2*c*d^3 + 279*a^4*b*d^4 - 105*(b^5*c*d^3 - a*b^
4*d^4)*x^3 + 35*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 - 7*(
8*b^5*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqr
t(d*x + c))/(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^
3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5 + (b^10*c^5 - 5*a*b^9*c^4*d + 10*a^
2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*x^4 + 4
*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5
*a^5*b^5*c*d^4 - a^6*b^4*d^5)*x^3 + 6*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a
^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*x^2 +
4*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3
+ 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*x), 1/192*(105*(b^4*d^4*x^4 + 4*a*b^3*d^4*
x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arc
tan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (48*b^5*c^4 - 248*a
*b^4*c^3*d + 526*a^2*b^3*c^2*d^2 - 605*a^3*b^2*c*d^3 + 279*a^4*b*d^4 - 105*
(b^5*c*d^3 - a*b^4*d^4)*x^3 + 35*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b
^3*d^4)*x^2 - 7*(8*b^5*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^
3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^
3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5 + (b^10*c^5 - 5*a*
b^9*c^4*d + 10*a^2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5
*b^5*d^5)*x^4 + 4*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^
4*b^6*c^2*d^3 + 5*a^5*b^5*c*d^4 - a^6*b^4*d^5)*x^3 + 6*(a^2*b^8*c^5 - 5*a^3
*b^7*c^4*d + 10*a^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^
7*b^3*d^5)*x^2 + 4*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10
*a^6*b^4*c^2*d^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*x)]
```

**giac [B]** time = 1.13, size = 331, normalized size = 1.84

$$\frac{35\,d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{64\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{-b^2c+abd}}+\frac{105\,(dx+c)^{\frac{7}{2}}b^3d^4-385\,(dx+c)^{\frac{5}{2}}b^3cd^4+511\,(dx+c)^{\frac{3}{2}}b^3c^2d^4-279\,\sqrt{dx+c}\,b^3c^3d^4+385\,(dx+c)^{\frac{5}{2}}ab^2d^5-1022\,(dx+c)^{\frac{3}{2}}ab^2cd^5+837\,\sqrt{dx+c}\,ab^2c^2d^5+511\,(dx+c)^{\frac{3}{2}}a^2bd^6-837\,\sqrt{dx+c}\,a^2bcd^6+279\,\sqrt{dx+c}\,a^3d^7}{192\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\left((dx+c)b-bc+ad\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 35/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d))
+ 1/192*(105*(d*x + c)^(7/2)*b^3*d^4 - 385*(d*x + c)^(5/2)*b^3*c*d^4 + 511*
(d*x + c)^(3/2)*b^3*c^2*d^4 - 279*sqrt(d*x + c)*b^3*c^3*d^4 + 385*(d*x + c)
^(5/2)*a*b^2*d^5 - 1022*(d*x + c)^(3/2)*a*b^2*c*d^5 + 837*sqrt(d*x + c)*a*b
^2*c^2*d^5 + 511*(d*x + c)^(3/2)*a^2*b*d^6 - 837*sqrt(d*x + c)*a^2*b*c*d^6
+ 279*sqrt(d*x + c)*a^3*d^7)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*((d*x + c)*b - b*c + a*d)^4)
```

**maple [A]** time = 0.01, size = 179, normalized size = 0.99

$$\frac{35 d^{4} \arctan\left(\frac{\sqrt{d x+c}\, b}{\sqrt{\left(a d-b c\right) b}}\right)}{64 \left(a d-b c\right)^{4} \sqrt{\left(a d-b c\right) b}}+\frac{\sqrt{d x+c}\, d^{4}}{4 \left(a d-b c\right) \left(b d x+a d\right)^{4}}+\frac{7 \sqrt{d x+c}\, d^{4}}{24 \left(a d-b c\right)^{2} \left(b d x+a d\right)^{3}}+\frac{35 \sqrt{d x+c}\, d^{4}}{96 \left(a d-b c\right)^{3} \left(b d x+a d\right)^{2}}+\frac{35 \sqrt{d x+c}\, d^{4}}{64 \left(a d-b c\right)^{4} \left(b d x+a d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^5/(d*x+c)^(1/2),x)
```

```
[Out] 1/4*d^4*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)^4+7/24*d^4/(a*d-b*c)^2*(d*x+c)^
(1/2)/(b*d*x+a*d)^3+35/96*d^4/(a*d-b*c)^3*(d*x+c)^(1/2)/(b*d*x+a*d)^2+35/64
*d^4/(a*d-b*c)^4*(d*x+c)^(1/2)/(b*d*x+a*d)+35/64*d^4/(a*d-b*c)^4/((a*d-b*c)
*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.46, size = 307, normalized size = 1.71

$$\frac{\frac{93 d^{4} \sqrt{c+d x}}{64 \left(a d-b c\right)}+\frac{385 b^{2} d^{4} \left(c+d x\right)^{5/2}}{192 \left(a d-b c\right)^{3}}+\frac{35 b^{3} d^{4} \left(c+d x\right)^{7/2}}{64 \left(a d-b c\right)^{4}}+\frac{511 b d^{4} \left(c+d x\right)^{3/2}}{192 \left(a d-b c\right)^{2}}}{b^{4} \left(c+d x\right)^{4}-\left(4 b^{4} c-4 a b^{3} d\right) \left(c+d x\right)^{3}-\left(c+d x\right) \left(-4 a^{3} b d^{3}+12 a^{2} b^{2} c d^{2}-12 a b^{3} c^{2} d+4 b^{4} c^{3}\right)+a^{4} d^{4}+b^{4} c^{4}+\left(c+d x\right)^{2} \left(6 a^{2} b^{2} d^{2}-12 a b^{3} c d+6 b^{4} c^{2}\right)+6 a^{2} b^{2} c^{2} d^{2}-4 a b^{3} c^{3} d-4 a^{3} b c d^{3}}+\frac{35 d^{4} \mathrm{atan}\left(\frac{\sqrt{b} \sqrt{c+d x}}{\sqrt{a d-b c}}\right)}{64 \sqrt{b} \left(a d-b c\right)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^5*(c + d*x)^(1/2)),x)
```

```
[Out] ((93*d^4*(c + d*x)^(1/2))/(64*(a*d - b*c)) + (385*b^2*d^4*(c + d*x)^(5/2))/
(192*(a*d - b*c)^3) + (35*b^3*d^4*(c + d*x)^(7/2))/(64*(a*d - b*c)^4) + (51
```

```
1*b*d^4*(c + d*x)^(3/2))/(192*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c -
 4*a*b^3*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c
*d^2 - 12*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2
*b^2*d^2 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^
3) + (35*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(1/2)
*(a*d - b*c)^(9/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**5/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.1318 $\int \frac{(a+bx)^5}{(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=152

$$-\frac{10b^4(c+dx)^{7/2}(bc-ad)}{7d^6}+\frac{4b^3(c+dx)^{5/2}(bc-ad)^2}{d^6}-\frac{20b^2(c+dx)^{3/2}(bc-ad)^3}{3d^6}+\frac{10b\sqrt{c+dx}\,(bc-ad)^4}{d^6}+\frac{2(bc-a}{d^6\sqrt{c+}}$$

**Rubi [A]** time = 0.05, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{7/2}(bc-ad)}{7d^6}+\frac{4b^3(c+dx)^{5/2}(bc-ad)^2}{d^6}-\frac{20b^2(c+dx)^{3/2}(bc-ad)^3}{3d^6}+\frac{10b\sqrt{c+dx}\,(bc-ad)^4}{d^6}+\frac{2(bc-ad)^5}{d^6\sqrt{c+dx}}+\frac{2b^5(c+dx)^{9/2}}{9d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^5)/(d^6*Sqrt[c + d*x]) + (10*b*(b*c - a*d)^4*Sqrt[c + d*x])/
d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^6) + (4*b^3*(b*c - a*d)^2
*(c + d*x)^(5/2))/d^6 - (10*b^4*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^6) + (2*b
^5*(c + d*x)^(9/2))/(9*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^{3/2}}\,dx=\int \left(\frac{(-bc+ad)^5}{d^5(c+dx)^{3/2}}+\frac{5b(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{10b^2(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{10b^3(bc-ad)^2(c+dx)^{3/2}}{d^5}-5b^4\right.$$

$$=\frac{2(bc-ad)^5}{d^6\sqrt{c+dx}}+\frac{10b(bc-ad)^4\sqrt{c+dx}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{3/2}}{3d^6}+\frac{4b^3(bc-ad)^2(c+dx)^{5/2}}{d^6}-$$

**Mathematica [A]** time = 0.12, size = 123, normalized size = 0.81

$$\frac{2\left(-45b^4(c+dx)^4(bc-ad)+126b^3(c+dx)^3(bc-ad)^2-210b^2(c+dx)^2(bc-ad)^3+315b(c+dx)(bc-ad)^4+63(bc-ad)^5+7b^5(c+dx)^5\right)}{63d^6\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^(3/2),x]
```

```
[Out] (2*(63*(b*c - a*d)^5 + 315*b*(b*c - a*d)^4*(c + d*x) - 210*b^2*(b*c - a*d)^
3*(c + d*x)^2 + 126*b^3*(b*c - a*d)^2*(c + d*x)^3 - 45*b^4*(b*c - a*d)*(c +
 d*x)^4 + 7*b^5*(c + d*x)^5))/(63*d^6*Sqrt[c + d*x])
```

**IntegrateAlgebraic [B]** time = 0.07, size = 315, normalized size = 2.07

$$\frac{2\left(-63a^5d^5+315a^4bd^4(c+dx)+315a^4bcd^4-630a^3b^2c^2d^3+210a^3b^2d^3(c+dx)^2-1260a^3b^2cd^3(c+dx)+630a^2b^3c^3d^2+1890a^2b^3c^2d^2(c+dx)+126a^2b^3d^2(c+dx)^3-630a^2b^3cd^2(c+dx)^2-315ab^4c^4d-1260ab^4c^3d(c+dx)+630ab^4c^2d(c+dx)^2+45ab^4d(c+dx)^4-252ab^4cd(c+dx)^3+63b^5c^5+315b^5c^4(c+dx)-210b^5c^3(c+dx)^2+126b^5c^2(c+dx)^3+7b^5(c+dx)^5-45b^5c(c+dx)^4\right)}{63d^6\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^(3/2),x]
```

```
[Out] (2*(63*b^5*c^5 - 315*a*b^4*c^4*d + 630*a^2*b^3*c^3*d^2 - 630*a^3*b^2*c^2*d^
3 + 315*a^4*b*c*d^4 - 63*a^5*d^5 + 315*b^5*c^4*(c + d*x) - 1260*a*b^4*c^3*d
*(c + d*x) + 1890*a^2*b^3*c^2*d^2*(c + d*x) - 1260*a^3*b^2*c*d^3*(c + d*x)
+ 315*a^4*b*d^4*(c + d*x) - 210*b^5*c^3*(c + d*x)^2 + 630*a*b^4*c^2*d*(c +
d*x)^2 - 630*a^2*b^3*c*d^2*(c + d*x)^2 + 210*a^3*b^2*d^3*(c + d*x)^2 + 126*
b^5*c^2*(c + d*x)^3 - 252*a*b^4*c*d*(c + d*x)^3 + 126*a^2*b^3*d^2*(c + d*x)
^3 - 45*b^5*c*(c + d*x)^4 + 45*a*b^4*d*(c + d*x)^4 + 7*b^5*(c + d*x)^5))/(6
3*d^6*Sqrt[c + d*x])
```

**fricas [B]** time = 1.09, size = 271, normalized size = 1.78

$$\frac{2\left(7\,b^5d^5x^5+256\,b^5c^5-1152\,ab^4c^4d+2016\,a^2b^3c^3d^2-1680\,a^3b^2c^2d^3+630\,a^4bcd^4-63\,a^5d^5-5\left(2\,b^5cd^4-9\,ab^4d^5\right)x^4+2\left(8\,b^5c^2d^3-36\,ab^4cd^4+63\,a^2b^3d^5\right)x^3-2\left(16\,b^5c^3d^2-72\,ab^4c^2d^3+126\,a^2b^3cd^4-105\,a^3b^2d^5\right)x^2+\left(128\,b^5c^4d-576\,ab^4c^3d^2+1008\,a^2b^3c^2d^3-840\,a^3b^2cd^4+315\,a^4bd^5\right)x\right)\sqrt{dx+c}}{63\left(d^7x+cd^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^5*d^5*x^5 + 256*b^5*c^5 - 1152*a*b^4*c^4*d + 2016*a^2*b^3*c^3*d^2
 - 1680*a^3*b^2*c^2*d^3 + 630*a^4*b*c*d^4 - 63*a^5*d^5 - 5*(2*b^5*c*d^4 - 9
*a*b^4*d^5)*x^4 + 2*(8*b^5*c^2*d^3 - 36*a*b^4*c*d^4 + 63*a^2*b^3*d^5)*x^3 -
 2*(16*b^5*c^3*d^2 - 72*a*b^4*c^2*d^3 + 126*a^2*b^3*c*d^4 - 105*a^3*b^2*d^5
)*x^2 + (128*b^5*c^4*d - 576*a*b^4*c^3*d^2 + 1008*a^2*b^3*c^2*d^3 - 840*a^3
*b^2*c*d^4 + 315*a^4*b*d^5)*x)*sqrt(d*x + c)/(d^7*x + c*d^6)
```

**giac [B]** time = 1.01, size = 350, normalized size = 2.30

$$\frac{2\left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5\right)}{\sqrt{dx+c}\,d^6}+\frac{2\left(7\,(dx+c)^{\frac{9}{2}}b^5d^{48}-45\,(dx+c)^{\frac{7}{2}}b^5cd^{48}+126\,(dx+c)^{\frac{5}{2}}b^5c^2d^{48}-210\,(dx+c)^{\frac{3}{2}}b^5c^3d^{48}+315\,\sqrt{dx+c}\,b^5c^4d^{48}+45\,(dx+c)^{\frac{7}{2}}ab^4d^{49}-252\,(dx+c)^{\frac{5}{2}}ab^4cd^{49}+630\,(dx+c)^{\frac{3}{2}}ab^4c^2d^{49}-1260\,\sqrt{dx+c}\,ab^4c^3d^{49}+126\,(dx+c)^{\frac{5}{2}}a^2b^3d^{50}-630\,(dx+c)^{\frac{3}{2}}a^2b^3cd^{50}+1890\,\sqrt{dx+c}\,a^2b^3c^2d^{50}+210\,(dx+c)^{\frac{3}{2}}a^3b^2d^{51}-1260\,\sqrt{dx+c}\,a^3b^2cd^{51}+315\,\sqrt{dx+c}\,a^4bd^{52}\right)}{63\,d^{54}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^
4*b*c*d^4 - a^5*d^5)/(sqrt(d*x + c)*d^6) + 2/63*(7*(d*x + c)^(9/2)*b^5*d^48
 - 45*(d*x + c)^(7/2)*b^5*c*d^48 + 126*(d*x + c)^(5/2)*b^5*c^2*d^48 - 210*(
d*x + c)^(3/2)*b^5*c^3*d^48 + 315*sqrt(d*x + c)*b^5*c^4*d^48 + 45*(d*x + c)
^(7/2)*a*b^4*d^49 - 252*(d*x + c)^(5/2)*a*b^4*c*d^49 + 630*(d*x + c)^(3/2)*
a*b^4*c^2*d^49 - 1260*sqrt(d*x + c)*a*b^4*c^3*d^49 + 126*(d*x + c)^(5/2)*a^
2*b^3*d^50 - 630*(d*x + c)^(3/2)*a^2*b^3*c*d^50 + 1890*sqrt(d*x + c)*a^2*b^
3*c^2*d^50 + 210*(d*x + c)^(3/2)*a^3*b^2*d^51 - 1260*sqrt(d*x + c)*a^3*b^2*
c*d^51 + 315*sqrt(d*x + c)*a^4*b*d^52)/d^54
```

**maple [B]** time = 0.01, size = 273, normalized size = 1.80

$$-\frac{2\left(-7b^5x^5d^5-45a\,b^4d^5x^4+10b^5c\,d^4x^4-126a^2b^3d^5x^3+72a\,b^4c\,d^4x^3-16b^5c^2d^3x^3-210a^3b^2d^5x^2+252a^2b^3c\,d^4x^2-144a\,b^4c^2d^3x^2+32b^5c^3d^2x^2-315a^4b\,d^5x+840a^3b^2c\,d^4x-1008a^2b^3c^2d^3x+576a\,b^4c^3d^2x-128b^5c^4dx+63a^5d^5-630a^4bc\,d^4+1680a^3b^2c^2d^3-2016a^2b^3c^3d^2+1152a\,b^4c^4d-256b^5c^5\right)}{63\sqrt{dx+c}\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(3/2),x)
```

```
[Out] -2/63/(d*x+c)^(1/2)*(-7*b^5*d^5*x^5-45*a*b^4*d^5*x^4+10*b^5*c*d^4*x^4-126*a
^2*b^3*d^5*x^3+72*a*b^4*c*d^4*x^3-16*b^5*c^2*d^3*x^3-210*a^3*b^2*d^5*x^2+25
2*a^2*b^3*c*d^4*x^2-144*a*b^4*c^2*d^3*x^2+32*b^5*c^3*d^2*x^2-315*a^4*b*d^5*
x+840*a^3*b^2*c*d^4*x-1008*a^2*b^3*c^2*d^3*x+576*a*b^4*c^3*d^2*x-128*b^5*c^
4*d*x+63*a^5*d^5-630*a^4*b*c*d^4+1680*a^3*b^2*c^2*d^3-2016*a^2*b^3*c^3*d^2+
1152*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

**maxima [A]** time = 1.56, size = 267, normalized size = 1.76

$$\frac{2\left(\frac{7\,(dx+c)^{\frac{9}{2}}b^5-45\left(b^5c-ab^4d\right)(dx+c)^{\frac{7}{2}}+126\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)(dx+c)^{\frac{5}{2}}-210\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)(dx+c)^{\frac{3}{2}}+315\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)\sqrt{dx+c}}{d^5}+\frac{63\left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5\right)}{\sqrt{dx+c}\,d^5}\right)}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/63*((7*(d*x + c)^(9/2)*b^5 - 45*(b^5*c - a*b^4*d)*(d*x + c)^(7/2) + 126*(
b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(5/2) - 210*(b^5*c^3 - 3*a*b
^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(3/2) + 315*(b^5*c^4 -
4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(d*x +
 c))/d^5 + 63*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^
2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)/(sqrt(d*x + c)*d^5))/d
```

**mupad [B]** time = 0.08, size = 192, normalized size = 1.26

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}-\frac{2\,a^5\,d^5-10\,a^4\,b\,c\,d^4+20\,a^3\,b^2\,c^2\,d^3-20\,a^2\,b^3\,c^3\,d^2+10\,a\,b^4\,c^4\,d-2\,b^5\,c^5}{d^6\,\sqrt{c+d\,x}}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}+\frac{4\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{5/2}}{d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,\sqrt{c+d\,x}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(9/2))/(9*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(7/2))
/(7*d^6) - (2*a^5*d^5 - 2*b^5*c^5 - 20*a^2*b^3*c^3*d^2 + 20*a^3*b^2*c^2*d^3
 + 10*a*b^4*c^4*d - 10*a^4*b*c*d^4)/(d^6*(c + d*x)^(1/2)) + (20*b^2*(a*d -
b*c)^3*(c + d*x)^(3/2))/(3*d^6) + (4*b^3*(a*d - b*c)^2*(c + d*x)^(5/2))/d^6
 + (10*b*(a*d - b*c)^4*(c + d*x)^(1/2))/d^6
```

**sympy** [A] time = 47.94, size = 243, normalized size = 1.60

$$\frac{2b^5(c+dx)^{\frac{9}{2}}}{9d^6}+\frac{(c+dx)^{\frac{7}{2}}\left(10ab^4d-10b^5c\right)}{7d^6}+\frac{(c+dx)^{\frac{5}{2}}\left(20a^2b^3d^2-40ab^4cd+20b^5c^2\right)}{5d^6}+\frac{(c+dx)^{\frac{3}{2}}\left(20a^3b^2d^3-60a^2b^3cd^2+60ab^4c^2d-20b^5c^3\right)}{3d^6}+\frac{\sqrt{c+dx}\left(10a^4bd^4-40a^3b^2cd^3+60a^2b^3c^2d^2-40ab^4c^3d+10b^5c^4\right)}{d^6}-\frac{2(ad-bc)^5}{d^6\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**5*(c + d*x)**(9/2)/(9*d**6) + (c + d*x)**(7/2)*(10*a*b**4*d - 10*b**5*
c)/(7*d**6) + (c + d*x)**(5/2)*(20*a**2*b**3*d**2 - 40*a*b**4*c*d + 20*b**5
*c**2)/(5*d**6) + (c + d*x)**(3/2)*(20*a**3*b**2*d**3 - 60*a**2*b**3*c*d**2
 + 60*a*b**4*c**2*d - 20*b**5*c**3)/(3*d**6) + sqrt(c + d*x)*(10*a**4*b*d**
4 - 40*a**3*b**2*c*d**3 + 60*a**2*b**3*c**2*d**2 - 40*a*b**4*c**3*d + 10*b*
*5*c**4)/d**6 - 2*(a*d - b*c)**5/(d**6*sqrt(c + d*x))
```

## 3.1319 $\int \frac{(a+bx)^4}{(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=123

$$-\frac{8b^3(c+dx)^{5/2}(bc-ad)}{5d^5}+\frac{4b^2(c+dx)^{3/2}(bc-ad)^2}{d^5}-\frac{8b\sqrt{c+dx}\,(bc-ad)^3}{d^5}-\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}+\frac{2b^4(c+dx)^{7/2}}{7d^5}$$

**Rubi [A]** time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{5/2}(bc-ad)}{5d^5}+\frac{4b^2(c+dx)^{3/2}(bc-ad)^2}{d^5}-\frac{8b\sqrt{c+dx}\,(bc-ad)^3}{d^5}-\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}+\frac{2b^4(c+dx)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^4)/(d^5*Sqrt[c + d*x]) - (8*b*(b*c - a*d)^3*Sqrt[c + d*x])/
d^5 + (4*b^2*(b*c - a*d)^2*(c + d*x)^(3/2))/d^5 - (8*b^3*(b*c - a*d)*(c + d
*x)^(5/2))/(5*d^5) + (2*b^4*(c + d*x)^(7/2))/(7*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^4}{(c+dx)^{3/2}}\,dx = \int \left(\frac{(-bc+ad)^4}{d^4(c+dx)^{3/2}}-\frac{4b(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b^2(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{4b^3(bc-ad)(c+dx)^{3/2}}{d^4}+\frac{b^4(c+\ldots}{d\ldots}\right.$$

$$= -\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{8b(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{3/2}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{5/2}}{5d^5}+\ldots$$

**Mathematica [A]** time = 0.08, size = 101, normalized size = 0.82

$$\frac{2\left(-28b^3(c+dx)^3(bc-ad)+70b^2(c+dx)^2(bc-ad)^2-140b(c+dx)(bc-ad)^3-35(bc-ad)^4+5b^4(c+dx)^4\right)}{35d^5\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-35*(b*c - a*d)^4 - 140*b*(b*c - a*d)^3*(c + d*x) + 70*b^2*(b*c - a*d)^
2*(c + d*x)^2 - 28*b^3*(b*c - a*d)*(c + d*x)^3 + 5*b^4*(c + d*x)^4))/(35*d^
5*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.07, size = 213, normalized size = 1.73

$$\frac{2\left(-35a^4d^4+140a^3bd^3(c+dx)+140a^3bcd^3-210a^2b^2c^2d^2+70a^2b^2d^2(c+dx)^2-420a^2b^2cd^2(c+dx)+140ab^3c^3d+420ab^3c^2d(c+dx)+28ab^3d(c+dx)^3-140ab^3cd(c+dx)^2-35b^4c^4-140b^4c^3(c+dx)+70b^4c^2(c+dx)^2+5b^4(c+dx)^4-28b^4c(c+dx)^3\right)}{35d^5\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-35*b^4*c^4 + 140*a*b^3*c^3*d - 210*a^2*b^2*c^2*d^2 + 140*a^3*b*c*d^3 -
 35*a^4*d^4 - 140*b^4*c^3*(c + d*x) + 420*a*b^3*c^2*d*(c + d*x) - 420*a^2*b
^2*c*d^2*(c + d*x) + 140*a^3*b*d^3*(c + d*x) + 70*b^4*c^2*(c + d*x)^2 - 140
*a*b^3*c*d*(c + d*x)^2 + 70*a^2*b^2*d^2*(c + d*x)^2 - 28*b^4*c*(c + d*x)^3
+ 28*a*b^3*d*(c + d*x)^3 + 5*b^4*(c + d*x)^4))/(35*d^5*Sqrt[c + d*x])
```

**fricas [A]** time = 1.54, size = 192, normalized size = 1.56

$$\frac{2\left(5b^4d^4x^4-128b^4c^4+448ab^3c^3d-560a^2b^2c^2d^2+280a^3bcd^3-35a^4d^4-4\left(2b^4cd^3-7ab^3d^4\right)x^3+2\left(8b^4c^2d^2-28ab^3cd^3+35a^2b^2d^4\right)x^2-4\left(16b^4c^3d-56ab^3c^2d^2+70a^2b^2cd^3-35a^3bd^4\right)x\right)\sqrt{dx+c}}{35\left(d^6x+cd^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*d^4*x^4 - 128*b^4*c^4 + 448*a*b^3*c^3*d - 560*a^2*b^2*c^2*d^2 +
 280*a^3*b*c*d^3 - 35*a^4*d^4 - 4*(2*b^4*c*d^3 - 7*a*b^3*d^4)*x^3 + 2*(8*b^
4*c^2*d^2 - 28*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^2 - 4*(16*b^4*c^3*d - 56*a*b
^3*c^2*d^2 + 70*a^2*b^2*c*d^3 - 35*a^3*b*d^4)*x)*sqrt(d*x + c)/(d^6*x + c*d
^5)
```

**giac [B]** time = 1.08, size = 240, normalized size = 1.95

$$-\frac{2\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)}{\sqrt{dx+c}\,d^5}+\frac{2\left(5(dx+c)^{\frac{7}{2}}b^4d^{30}-28(dx+c)^{\frac{5}{2}}b^4cd^{30}+70(dx+c)^{\frac{3}{2}}b^4c^2d^{30}-140\sqrt{dx+c}\,b^4c^3d^{30}+28(dx+c)^{\frac{5}{2}}ab^3d^{31}-140(dx+c)^{\frac{3}{2}}ab^3cd^{31}+420\sqrt{dx+c}\,ab^3c^2d^{31}+70(dx+c)^{\frac{3}{2}}a^2b^2d^{32}-420\sqrt{dx+c}\,a^2b^2cd^{32}+140\sqrt{dx+c}\,a^3bd^{33}\right)}{35\,d^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/
(sqrt(d*x + c)*d^5) + 2/35*(5*(d*x + c)^(7/2)*b^4*d^30 - 28*(d*x + c)^(5/2)
*b^4*c*d^30 + 70*(d*x + c)^(3/2)*b^4*c^2*d^30 - 140*sqrt(d*x + c)*b^4*c^3*d
```

```
^30 + 28*(d*x + c)^(5/2)*a*b^3*d^31 - 140*(d*x + c)^(3/2)*a*b^3*c*d^31 + 42
0*sqrt(d*x + c)*a*b^3*c^2*d^31 + 70*(d*x + c)^(3/2)*a^2*b^2*d^32 - 420*sqrt
(d*x + c)*a^2*b^2*c*d^32 + 140*sqrt(d*x + c)*a^3*b*d^33)/d^35
```

**maple [A]** time = 0.01, size = 186, normalized size = 1.51

$$-\frac{2\left(-5b^4x^4d^4-28a\,b^3d^4x^3+8b^4c\,d^3x^3-70a^2b^2d^4x^2+56a\,b^3c\,d^3x^2-16b^4c^2d^2x^2-140a^3b\,d^4x+280a^2b^2c\,d^3x-224a\,b^3c^2d^2x+64b^4c^3dx+35a^4d^4-280a^3bc\,d^3+560a^2b^2c^2d^2-448a\,b^3c^3d+128b^4c^4\right)}{35\sqrt{dx+c}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(3/2),x)
```

```
[Out] -2/35/(d*x+c)^(1/2)*(-5*b^4*d^4*x^4-28*a*b^3*d^4*x^3+8*b^4*c*d^3*x^3-70*a^2
*b^2*d^4*x^2+56*a*b^3*c*d^3*x^2-16*b^4*c^2*d^2*x^2-140*a^3*b*d^4*x+280*a^2*
b^2*c*d^3*x-224*a*b^3*c^2*d^2*x+64*b^4*c^3*d*x+35*a^4*d^4-280*a^3*b*c*d^3+5
60*a^2*b^2*c^2*d^2-448*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

**maxima [A]** time = 1.35, size = 189, normalized size = 1.54

$$\frac{2\left(\frac{5\,(dx+c)^{\frac{7}{2}}b^4-28\left(b^4c-ab^3d\right)(dx+c)^{\frac{5}{2}}+70\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)(dx+c)^{\frac{3}{2}}-140\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)\sqrt{dx+c}}{d^4}-\frac{35\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}{\sqrt{dx+c}\,d^4}\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*((5*(d*x + c)^(7/2)*b^4 - 28*(b^4*c - a*b^3*d)*(d*x + c)^(5/2) + 70*(b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(3/2) - 140*(b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(d*x + c))/d^4 - 35*(b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(sqrt(d*x + c)*
d^4))/d
```

**mupad [B]** time = 0.06, size = 153, normalized size = 1.24

$$\frac{2\,b^4\,{(c+d\,x)}^{7/2}}{7\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{(c+d\,x)}^{5/2}}{5\,d^5}-\frac{2\,a^4\,d^4-8\,a^3\,b\,c\,d^3+12\,a^2\,b^2\,c^2\,d^2-8\,a\,b^3\,c^3\,d+2\,b^4\,c^4}{d^5\,\sqrt{c+d\,x}}+\frac{4\,b^2\,{(a\,d-b\,c)}^2\,{(c+d\,x)}^{3/2}}{d^5}+\frac{8\,b\,{(a\,d-b\,c)}^3\,\sqrt{c+d\,x}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(7/2))/(7*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(5/2))/(
5*d^5) - (2*a^4*d^4 + 2*b^4*c^4 + 12*a^2*b^2*c^2*d^2 - 8*a*b^3*c^3*d - 8*a^
3*b*c*d^3)/(d^5*(c + d*x)^(1/2)) + (4*b^2*(a*d - b*c)^2*(c + d*x)^(3/2))/d^
5 + (8*b*(a*d - b*c)^3*(c + d*x)^(1/2))/d^5
```

**sympy** [A] time = 32.87, size = 168, normalized size = 1.37

$$\frac{2b^4(c+dx)^{\frac{7}{2}}}{7d^5}+\frac{(c+dx)^{\frac{5}{2}}\left(8ab^3d-8b^4c\right)}{5d^5}+\frac{(c+dx)^{\frac{3}{2}}\left(12a^2b^2d^2-24ab^3cd+12b^4c^2\right)}{3d^5}+\frac{\sqrt{c+dx}\left(8a^3bd^3-24a^2b^2cd^2+24ab^3c^2d-8b^4c^3\right)}{d^5}-\frac{2(ad-bc)^4}{d^5\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**4*(c + d*x)**(7/2)/(7*d**5) + (c + d*x)**(5/2)*(8*a*b**3*d - 8*b**4*c)
/(5*d**5) + (c + d*x)**(3/2)*(12*a**2*b**2*d**2 - 24*a*b**3*c*d + 12*b**4*c
**2)/(3*d**5) + sqrt(c + d*x)*(8*a**3*b*d**3 - 24*a**2*b**2*c*d**2 + 24*a*b
**3*c**2*d - 8*b**4*c**3)/d**5 - 2*(a*d - b*c)**4/(d**5*sqrt(c + d*x))
```

## 3.1320 $\int \frac{(a+bx)^3}{(c+dx)^{3/2}} \, dx$

**Optimal**. Leaf size=94

$$-\frac{2b^2(c+dx)^{3/2}(bc-ad)}{d^4}+\frac{6b\sqrt{c+dx}\,(bc-ad)^2}{d^4}+\frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{2b^3(c+dx)^{5/2}}{5d^4}$$

**Rubi** [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2b^2(c+dx)^{3/2}(bc-ad)}{d^4}+\frac{6b\sqrt{c+dx}\,(bc-ad)^2}{d^4}+\frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{2b^3(c+dx)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^3)/(d^4*Sqrt[c + d*x]) + (6*b*(b*c - a*d)^2*Sqrt[c + d*x])/d
^4 - (2*b^2*(b*c - a*d)*(c + d*x)^(3/2))/d^4 + (2*b^3*(c + d*x)^(5/2))/(5*d
^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^{3/2}}+\frac{3b(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{3b^2(bc-ad)\sqrt{c+dx}}{d^3}+\frac{b^3(c+dx)^{3/2}}{d^3}\right)dx\\ &= \frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{2b^2(bc-ad)(c+dx)^{3/2}}{d^4}+\frac{2b^3(c+dx)^{5/2}}{5d^4}\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 78, normalized size = 0.83

$$\frac{2\left(-5b^2(c+dx)^2(bc-ad)+15b(c+dx)(bc-ad)^2+5(bc-ad)^3+b^3(c+dx)^3\right)}{5d^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^(3/2),x]
```

```
[Out] (2*(5*(b*c - a*d)^3 + 15*b*(b*c - a*d)^2*(c + d*x) - 5*b^2*(b*c - a*d)*(c +
 d*x)^2 + b^3*(c + d*x)^3))/(5*d^4*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 131, normalized size = 1.39

$$\frac{2\left(-5a^3d^3+15a^2bd^2(c+dx)+15a^2bcd^2-15ab^2c^2d+5ab^2d(c+dx)^2-30ab^2cd(c+dx)+5b^3c^3+15b^3c^2(c+dx)+b^3(c+dx)^3-5b^3c(c+dx)^2\right)}{5d^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^(3/2),x]
```

```
[Out] (2*(5*b^3*c^3 - 15*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 5*a^3*d^3 + 15*b^3*c^2*(c
 + d*x) - 30*a*b^2*c*d*(c + d*x) + 15*a^2*b*d^2*(c + d*x) - 5*b^3*c*(c + d*
x)^2 + 5*a*b^2*d*(c + d*x)^2 + b^3*(c + d*x)^3))/(5*d^4*Sqrt[c + d*x])
```

**fricas [A]** time = 1.26, size = 124, normalized size = 1.32

$$\frac{2\left(b^3d^3x^3+16\,b^3c^3-40\,ab^2c^2d+30\,a^2bcd^2-5\,a^3d^3-\left(2\,b^3cd^2-5\,ab^2d^3\right)x^2+\left(8\,b^3c^2d-20\,ab^2cd^2+15\,a^2bd^3\right)x\right)\sqrt{dx+c}}{5\left(d^5x+cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*d^3*x^3 + 16*b^3*c^3 - 40*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 5*a^3*d^3
 - (2*b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + (8*b^3*c^2*d - 20*a*b^2*c*d^2 + 15*a^2
*b*d^3)*x)*sqrt(d*x + c)/(d^5*x + c*d^4)
```

**giac [A]** time = 1.05, size = 152, normalized size = 1.62

$$\frac{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}{\sqrt{dx+c}\,d^4}+\frac{2\left((dx+c)^{\frac{5}{2}}b^3d^{16}-5\,(dx+c)^{\frac{3}{2}}b^3cd^{16}+15\,\sqrt{dx+c}\,b^3c^2d^{16}+5\,(dx+c)^{\frac{3}{2}}ab^2d^{17}-30\,\sqrt{dx+c}\,ab^2cd^{17}+15\,\sqrt{dx+c}\,a^2bd^{18}\right)}{5\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(sqrt(d*x + c)*d^4) +
 2/5*((d*x + c)^(5/2)*b^3*d^16 - 5*(d*x + c)^(3/2)*b^3*c*d^16 + 15*sqrt(d*x
 + c)*b^3*c^2*d^16 + 5*(d*x + c)^(3/2)*a*b^2*d^17 - 30*sqrt(d*x + c)*a*b^2*
c*d^17 + 15*sqrt(d*x + c)*a^2*b*d^18)/d^20
```

**maple** [A] time = 0.01, size = 116, normalized size = 1.23

$$-\frac{2\left(-b^{3}x^{3}d^{3}-5a\,b^{2}d^{3}x^{2}+2b^{3}c\,d^{2}x^{2}-15a^{2}b\,d^{3}x+20a\,b^{2}c\,d^{2}x-8b^{3}c^{2}dx+5a^{3}d^{3}-30a^{2}bc\,d^{2}+40a\,b^{2}c^{2}d-16b^{3}c^{3}\right)}{5\sqrt{dx+c}\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] -2/5/(d*x+c)^(1/2)*(-b^3*d^3*x^3-5*a*b^2*d^3*x^2+2*b^3*c*d^2*x^2-15*a^2*b*d
^3*x+20*a*b^2*c*d^2*x-8*b^3*c^2*d*x+5*a^3*d^3-30*a^2*b*c*d^2+40*a*b^2*c^2*d
-16*b^3*c^3)/d^4
```

**maxima** [A] time = 1.38, size = 125, normalized size = 1.33

$$\frac{2\left(\frac{(dx+c)^{\frac{5}{2}}b^{3}-5\left(b^{3}c-ab^{2}d\right)(dx+c)^{\frac{3}{2}}+15\left(b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2}\right)\sqrt{dx+c}}{d^{3}}+\frac{5\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}{\sqrt{dx+c}\,d^{3}}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(((d*x + c)^(5/2)*b^3 - 5*(b^3*c - a*b^2*d)*(d*x + c)^(3/2) + 15*(b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(d*x + c))/d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)/(sqrt(d*x + c)*d^3))/d
```

**mupad** [B] time = 0.08, size = 114, normalized size = 1.21

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,d^4}-\frac{2\,a^3\,d^3-6\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-2\,b^3\,c^3}{d^4\,\sqrt{c+d\,x}}+\frac{6\,b\,{\left(a\,d-b\,c\right)}^2\,\sqrt{c+d\,x}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(5/2))/(5*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(3/2))/(
3*d^4) - (2*a^3*d^3 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2)/(d^4*(c +
d*x)^(1/2)) + (6*b*(a*d - b*c)^2*(c + d*x)^(1/2))/d^4
```

**sympy** [A] time = 21.51, size = 109, normalized size = 1.16

$$\frac{2b^{3}\left(c+dx\right)^{\frac{5}{2}}}{5d^{4}}+\frac{\left(c+dx\right)^{\frac{3}{2}}\left(6ab^{2}d-6b^{3}c\right)}{3d^{4}}+\frac{\sqrt{c+dx}\left(6a^{2}bd^{2}-12ab^{2}cd+6b^{3}c^{2}\right)}{d^{4}}-\frac{2\left(ad-bc\right)^{3}}{d^{4}\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**3*(c + d*x)**(5/2)/(5*d**4) + (c + d*x)**(3/2)*(6*a*b**2*d - 6*b**3*c)
/(3*d**4) + sqrt(c + d*x)*(6*a**2*b*d**2 - 12*a*b**2*c*d + 6*b**3*c**2)/d**
4 - 2*(a*d - b*c)**3/(d**4*sqrt(c + d*x))
```

# 3.1321 $\int \frac{(a+bx)^2}{(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=67

$$-\frac{4b\sqrt{c+dx}\,(bc-ad)}{d^3}-\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}+\frac{2b^2(c+dx)^{3/2}}{3d^3}$$

**Rubi** [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b\sqrt{c+dx}\,(bc-ad)}{d^3}-\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}+\frac{2b^2(c+dx)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^2)/(d^3*Sqrt[c + d*x]) - (4*b*(b*c - a*d)*Sqrt[c + d*x])/d^
3 + (2*b^2*(c + d*x)^(3/2))/(3*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{3/2}}-\frac{2b(bc-ad)}{d^2\sqrt{c+dx}}+\frac{b^2\sqrt{c+dx}}{d^2}\right)dx\\ &= -\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{4b(bc-ad)\sqrt{c+dx}}{d^3}+\frac{2b^2(c+dx)^{3/2}}{3d^3}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 59, normalized size = 0.88

$$\frac{2\left(-3a^2d^2+6abd(2c+dx)+b^2\left(-8c^2-4cdx+d^2x^2\right)\right)}{3d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-3*a^2*d^2 + 6*a*b*d*(2*c + d*x) + b^2*(-8*c^2 - 4*c*d*x + d^2*x^2)))/(
3*d^3*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.04, size = 71, normalized size = 1.06

$$\frac{2\left(-3a^2d^2+6abd(c+dx)+6abcd-3b^2c^2+b^2(c+dx)^2-6b^2c(c+dx)\right)}{3d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-3*b^2*c^2 + 6*a*b*c*d - 3*a^2*d^2 - 6*b^2*c*(c + d*x) + 6*a*b*d*(c + d
*x) + b^2*(c + d*x)^2))/(3*d^3*Sqrt[c + d*x])
```

**fricas [A]** time = 1.13, size = 73, normalized size = 1.09

$$\frac{2\left(b^2d^2x^2-8\,b^2c^2+12\,abcd-3\,a^2d^2-2\left(2\,b^2cd-3\,abd^2\right)x\right)\sqrt{dx+c}}{3\left(d^4x+cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*d^2*x^2 - 8*b^2*c^2 + 12*a*b*c*d - 3*a^2*d^2 - 2*(2*b^2*c*d - 3*a*
b*d^2)*x)*sqrt(d*x + c)/(d^4*x + c*d^3)
```

**giac [A]** time = 1.04, size = 84, normalized size = 1.25

$$-\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)}{\sqrt{dx+c}\,d^3}+\frac{2\left((dx+c)^{\frac{3}{2}}b^2d^6-6\,\sqrt{dx+c}\,b^2cd^6+6\,\sqrt{dx+c}\,abd^7\right)}{3\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x + c)*d^3) + 2/3*((d*x + c)^(3/
2)*b^2*d^6 - 6*sqrt(d*x + c)*b^2*c*d^6 + 6*sqrt(d*x + c)*a*b*d^7)/d^9
```

**maple [A]** time = 0.00, size = 63, normalized size = 0.94

$$-\frac{2\left(-b^2x^2d^2-6ab\,d^2x+4b^2cdx+3a^2d^2-12abcd+8b^2c^2\right)}{3\sqrt{dx+c}\;d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] -2/3/(d*x+c)^(1/2)*(-b^2*d^2*x^2-6*a*b*d^2*x+4*b^2*c*d*x+3*a^2*d^2-12*a*b*c
*d+8*b^2*c^2)/d^3
```

**maxima [A]** time = 1.30, size = 75, normalized size = 1.12

$$\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^2-6\left(b^2c-abd\right)\sqrt{dx+c}}{d^2}-\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)}{\sqrt{dx+c}\,d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((d*x + c)^(3/2)*b^2 - 6*(b^2*c - a*b*d)*sqrt(d*x + c))/d^2 - 3*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x + c)*d^2))/d
```

**mupad [B]** time = 0.26, size = 67, normalized size = 1.00

$$\frac{\frac{2\,b^2\,{\left(c+d\,x\right)}^2}{3}-2\,a^2\,d^2-2\,b^2\,c^2-4\,b^2\,c\,\left(c+d\,x\right)+4\,a\,b\,d\,\left(c+d\,x\right)+4\,a\,b\,c\,d}{d^3\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(3/2),x)
```

```
[Out] ((2*b^2*(c + d*x)^2)/3 - 2*a^2*d^2 - 2*b^2*c^2 - 4*b^2*c*(c + d*x) + 4*a*b*
d*(c + d*x) + 4*a*b*c*d)/(d^3*(c + d*x)^(1/2))
```

**sympy [A]** time = 13.29, size = 65, normalized size = 0.97

$$\frac{2b^2\left(c+dx\right)^{\frac{3}{2}}}{3d^3}+\frac{\sqrt{c+dx}\,\left(4abd-4b^2c\right)}{d^3}-\frac{2\left(ad-bc\right)^2}{d^3\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**2*(c + d*x)**(3/2)/(3*d**3) + sqrt(c + d*x)*(4*a*b*d - 4*b**2*c)/d**3
- 2*(a*d - b*c)**2/(d**3*sqrt(c + d*x))
```

## 3.1322 $\int \frac{a+bx}{(c+dx)^{3/2}} \, dx$

**Optimal**. Leaf size=38

$$\frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}$$

**Rubi [A]** time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d))/(d^2*Sqrt[c + d*x]) + (2*b*Sqrt[c + d*x])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^{3/2}} \, dx &= \int \left(\frac{-bc+ad}{d(c+dx)^{3/2}}+\frac{b}{d\sqrt{c+dx}}\right) dx\\ &= \frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 27, normalized size = 0.71

$$\frac{2(-ad+2bc+bdx)}{d^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(2*b*c - a*d + b*d*x))/(d^2*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.03, size = 29, normalized size = 0.76

$$\frac{2(-ad + b(c + dx) + bc)}{d^2\sqrt{c + dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x)^(3/2),x]
[Out] (2*(b*c - a*d + b*(c + d*x)))/(d^2*Sqrt[c + d*x])
```

**fricas [A]** time = 1.13, size = 35, normalized size = 0.92

$$\frac{2\,(bdx + 2\,bc - ad)\sqrt{dx + c}}{d^3x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
[Out] 2*(b*d*x + 2*b*c - a*d)*sqrt(d*x + c)/(d^3*x + c*d^2)
```

**giac [A]** time = 1.03, size = 34, normalized size = 0.89

$$\frac{2\,\sqrt{dx + c}\,b}{d^2} + \frac{2\,(bc - ad)}{\sqrt{dx + c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
[Out] 2*sqrt(d*x + c)*b/d^2 + 2*(b*c - a*d)/(sqrt(d*x + c)*d^2)
```

**maple [A]** time = 0.00, size = 26, normalized size = 0.68

$$-\frac{2\,(-bdx + ad - 2bc)}{\sqrt{dx + c}\;d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(3/2),x)
[Out] -2/(d*x+c)^(1/2)*(-b*d*x+a*d-2*b*c)/d^2
```

**maxima** [A] time = 1.33, size = 37, normalized size = 0.97

$$\frac{2\left(\frac{\sqrt{dx+c}\,b}{d}+\frac{bc-ad}{\sqrt{dx+c}\,d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(d*x + c)*b/d + (b*c - a*d)/(sqrt(d*x + c)*d))/d
```

**mupad** [B] time = 0.05, size = 25, normalized size = 0.66

$$\frac{4\,b\,c-2\,a\,d+2\,b\,d\,x}{d^2\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(3/2),x)
```

```
[Out] (4*b*c - 2*a*d + 2*b*d*x)/(d^2*(c + d*x)^(1/2))
```

**sympy** [A] time = 0.61, size = 60, normalized size = 1.58

$$\begin{cases} -\frac{2a}{d\sqrt{c+dx}}+\frac{4bc}{d^2\sqrt{c+dx}}+\frac{2bx}{d\sqrt{c+dx}} & \text{for } d\neq 0 \\ \frac{ax+\frac{bx^2}{2}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Piecewise((-2*a/(d*sqrt(c + d*x)) + 4*b*c/(d**2*sqrt(c + d*x)) + 2*b*x/(d*s
qrt(c + d*x)), Ne(d, 0)), ((a*x + b*x**2/2)/c**(3/2), True))
```

## 3.1323 $\int \frac{1}{(c+dx)^{3/2}} dx$

**Optimal**. Leaf size=14

$$-\frac{2}{d\sqrt{c+dx}}$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-3/2),x]
```

```
[Out] -2/(d*Sqrt[c + d*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^{3/2}} dx = -\frac{2}{d\sqrt{c+dx}}$$

**Mathematica [A]** time = 0.00, size = 14, normalized size = 1.00

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-3/2),x]
```

```
[Out] -2/(d*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.01, size = 14, normalized size = 1.00

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(-3/2),x]
```

```
[Out] -2/(d*Sqrt[c + d*x])
```

**fricas [A]** time = 1.21, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{dx+c}}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d*x + c)/(d^2*x + c*d)
```

**giac [A]** time = 1.02, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(d*x + c)*d)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.93

$$-\frac{2}{\sqrt{dx+c}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(3/2),x)
```

```
[Out] -2/d/(d*x+c)^(1/2)
```

**maxima [A]** time = 1.33, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(d*x + c)*d)
```

**mupad [B]** time = 0.02, size = 12, normalized size = 0.86

$$-\frac{2}{d\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(3/2),x)
[Out] -2/(d*(c + d*x)^(1/2))
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.86

$$-\frac{2}{d\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(3/2),x)
[Out] -2/(d*sqrt(c + d*x))
```

## 3.1324 $\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=69

$$\frac{2}{\sqrt{c+dx}\,(bc-ad)}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{2}{\sqrt{c+dx}\,(bc-ad)}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(3/2)),x]
```

```
[Out] 2/((b*c - a*d)*Sqrt[c + d*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx = \frac{2}{(bc-ad)\sqrt{c+dx}} + \frac{b\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{bc-ad}$$
$$= \frac{2}{(bc-ad)\sqrt{c+dx}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d(bc-ad)}$$
$$= \frac{2}{(bc-ad)\sqrt{c+dx}} - \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

**Mathematica [C]** time = 0.01, size = 46, normalized size = 0.67

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b(c+dx)}{bc-ad}\right)}{\sqrt{c+dx}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x))/(b*c - a*d)])/((-(b*c) +
a*d)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.09, size = 79, normalized size = 1.14

$$\frac{2}{\sqrt{c+dx}\,(bc-ad)} + \frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^(3/2)),x]
```

```
[Out] 2/((b*c - a*d)*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*
d]*Sqrt[c + d*x])/(b*c - a*d)])/(-(b*c) + a*d)^(3/2)
```

**fricas [A]** time = 1.43, size = 214, normalized size = 3.10

$$\left[-\frac{(dx+c)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\sqrt{dx+c}}{bc^2-acd+\left(bcd-ad^2\right)x},\ -\frac{2\left((dx+c)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-\sqrt{dx+c}\right)}{bc^2-acd+\left(bcd-ad^2\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((d*x + c)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d + 2*(b*c - a*d)*s
qrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) - 2*sqrt(d*x + c))/(b*c^2 - a*
c*d + (b*c*d - a*d^2)*x), -2*((d*x + c)*sqrt(-b/(b*c - a*d))*arctan(-(b*c -
 a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - sqrt(d*x + c))/(b
*c^2 - a*c*d + (b*c*d - a*d^2)*x)]
```

**giac [A]** time = 0.94, size = 69, normalized size = 1.00

$$\frac{2\,b\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{2}{(bc-ad)\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
 - a*d)) + 2/((b*c - a*d)*sqrt(d*x + c))
```

**maple [A]** time = 0.01, size = 68, normalized size = 0.99

$$-\frac{2b\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)\,\sqrt{(ad-bc)\,b}}-\frac{2}{(ad-bc)\,\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] -2/(a*d-b*c)/(d*x+c)^(1/2)-2*b/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)
^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.27, size = 57, normalized size = 0.83

$$-\frac{2}{(a\,d-b\,c)\,\sqrt{c+d\,x}}-\frac{2\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{(a\,d-b\,c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(3/2)),x)
```

```
[Out] - 2/((a*d - b*c)*(c + d*x)^(1/2)) - (2*b^(1/2)*atan((b^(1/2)*(c + d*x)^(1/2
))/(a*d - b*c)^(1/2)))/(a*d - b*c)^(3/2)
```

**sympy [A]** time = 11.49, size = 60, normalized size = 0.87

$$-\frac{2}{\sqrt{c+dx}\,(ad-bc)}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] -2/(sqrt(c + d*x)*(a*d - b*c)) - 2*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/
(sqrt((a*d - b*c)/b)*(a*d - b*c))
```

## 3.1325 $\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=99

$$-\frac{3d}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{(a+bx)\sqrt{c+dx}\,(bc-ad)}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

**Rubi [A]** time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{3d}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{(a+bx)\sqrt{c+dx}\,(bc-ad)}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*d)/((b*c - a*d)^2*Sqrt[c + d*x]) - 1/((b*c - a*d)*(a + b*x)*Sqrt[c + d*
x]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c -
 a*d)^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx = -\frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3d)\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{2(bc-ad)}$$
$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3bd)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2(bc-ad)^2}$$
$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c\right.}}{(bc-ad)^2}$$
$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} + \frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

**Mathematica [C]** time = 0.01, size = 48, normalized size = 0.48

$$-\frac{2d\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{\sqrt{c+dx}\,(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*d*Hypergeometric2F1[-1/2, 2, 1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/((-
(b*c) + a*d)^2*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.29, size = 115, normalized size = 1.16

$$\frac{d(2ad+3b(c+dx)-2bc)}{\sqrt{c+dx}\,(bc-ad)^2(-ad-b(c+dx)+bc)} + \frac{3\sqrt{b}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] (d*(-2*b*c + 2*a*d + 3*b*(c + d*x)))/((b*c - a*d)^2*Sqrt[c + d*x]*(b*c - a*
d - b*(c + d*x))) + (3*Sqrt[b]*d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c
+ d*x])/(b*c - a*d)])/(-(b*c) + a*d)^(5/2)
```

**fricas [B]** time = 1.32, size = 423, normalized size = 4.27

$$\left[\frac{3\left(bd^{2}x^{2}+acd+\left(bcd+ad^{2}\right)x\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\,(3\,bdx+bc+2\,ad)\sqrt{dx+c}}{2\left(ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}+\left(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{2}+\left(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3}\right)x\right)},\ \frac{3\left(bd^{2}x^{2}+acd+\left(bcd+ad^{2}\right)x\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-(3\,bdx+bc+2\,ad)\sqrt{dx+c}}{ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}+\left(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{2}+\left(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3}\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)*sqrt(b/(b*c - a*d))*log((b*
d*x + 2*b*c - a*d + 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x +
 a)) - 2*(3*b*d*x + b*c + 2*a*d)*sqrt(d*x + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d
+ a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x), (3*(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2
)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c -
a*d))/(b*d*x + b*c)) - (3*b*d*x + b*c + 2*a*d)*sqrt(d*x + c))/(a*b^2*c^3 -
2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (
b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)]
```

**giac [A]** time = 1.03, size = 143, normalized size = 1.44

$$-\frac{3\,bd\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{-b^{2}c+abd}}-\frac{3\,(dx+c)bd-2\,bcd+2\,ad^{2}}{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left((dx+c)^{\frac{3}{2}}b-\sqrt{dx+c}\,bc+\sqrt{dx+c}\,ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-b^2*c + a*b*d)) - (3*(d*x + c)*b*d - 2*b*c*d + 2*a*d^2)/((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x + c)^(3/2)*b - sqrt(d*x + c)*b*c + sqrt
(d*x + c)*a*d))
```

**maple [A]** time = 0.01, size = 101, normalized size = 1.02

$$-\frac{3bd\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)^{2}\sqrt{(ad-bc)\,b}}-\frac{\sqrt{dx+c}\;bd}{(ad-bc)^{2}\,(bdx+ad)}-\frac{2d}{(ad-bc)^{2}\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] -2*d/(a*d-b*c)^2/(d*x+c)^(1/2)-d*b/(a*d-b*c)^2*(d*x+c)^(1/2)/(b*d*x+a*d)-3*
d*b/(a*d-b*c)^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2
)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.19, size = 123, normalized size = 1.24

$$-\frac{\frac{2\,d}{a\,d-b\,c}+\frac{3\,b\,d\,\left(c+d\,x\right)}{{\left(a\,d-b\,c\right)}^2}}{b\,{\left(c+d\,x\right)}^{3/2}+\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}-\frac{3\,\sqrt{b}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^{5/2}}\right)}{{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d)/(a*d - b*c) + (3*b*d*(c + d*x))/(a*d - b*c)^2)/(b*(c + d*x)^(3/2)
+ (a*d - b*c)*(c + d*x)^(1/2)) - (3*b^(1/2)*d*atan((b^(1/2)*(c + d*x)^(1/2)
*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx\right)^{2} \left(c + dx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**2*(c + d*x)**(3/2)), x)
```

## 3.1326 $\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=140

$$\frac{15d^2}{4\sqrt{c+dx}\,(bc-ad)^3}-\frac{15\sqrt{b}\,d^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{7/2}}+\frac{5d}{4(a+bx)\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{2(a+bx)^2\sqrt{c+dx}\,(bc-ad)}$$

**Rubi** [A] time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{15d^2}{4\sqrt{c+dx}\,(bc-ad)^3}-\frac{15\sqrt{b}\,d^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{7/2}}+\frac{5d}{4(a+bx)\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{2(a+bx)^2\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

```
[Out] (15*d^2)/(4*(b*c - a*d)^3*Sqrt[c + d*x]) - 1/(2*(b*c - a*d)*(a + b*x)^2*Sqr
t[c + d*x]) + (5*d)/(4*(b*c - a*d)^2*(a + b*x)*Sqrt[c + d*x]) - (15*Sqrt[b]
*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(7/2)
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx = -\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} - \frac{(5d)\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx}{4(bc-ad)}$$

$$= -\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}} + \frac{\left(15d^2\right)\int \frac{1}{(a+bx)(c+dx)^3}}{8(bc-ad)^2}$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}} +$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}} +$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}} -$$

**Mathematica [C]** time = 0.01, size = 50, normalized size = 0.36

$$-\frac{2d^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{\sqrt{c+dx}\,(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*d^2*Hypergeometric2F1[-1/2, 3, 1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/(
(-(b*c) + a*d)^3*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.49, size = 163, normalized size = 1.16

$$\frac{d^2\left(8a^2d^2+25abd(c+dx)-16abcd+8b^2c^2+15b^2(c+dx)^2-25b^2c(c+dx)\right)}{4\sqrt{c+dx}\,(bc-ad)^3(-ad-b(c+dx)+bc)^2} + \frac{15\sqrt{b}\,d^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}\,\sqrt{ad-bc}}{bc-ad}\right)}{4(ad-bc)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

```
[Out] (d^2*(8*b^2*c^2 - 16*a*b*c*d + 8*a^2*d^2 - 25*b^2*c*(c + d*x) + 25*a*b*d*(c
 + d*x) + 15*b^2*(c + d*x)^2))/(4*(b*c - a*d)^3*Sqrt[c + d*x]*(b*c - a*d -
```

```
b*(c + d*x))^2) + (15*Sqrt[b]*d^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c
 + d*x])/(b*c - a*d)])/(4*(-(b*c) + a*d)^(7/2))
```

**fricas [B]** time = 1.28, size = 782, normalized size = 5.59

$$\left[-\frac{15\left(b^{2}d^{3}x^{3}+a^{2}cd^{2}+\left(b^{2}cd^{2}+2abd^{3}\right)x^{2}+\left(2abcd^{2}+a^{2}d^{3}\right)x\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2bc-ad+2(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\left(15b^{2}d^{2}x^{2}-2b^{2}c^{2}+9abcd+8a^{2}d^{2}+5\left(b^{2}cd+5abd^{2}\right)x\right)\sqrt{dx+c}}{8\left(a^{2}b^{3}c^{4}-3a^{3}b^{2}c^{3}d+3a^{4}bc^{2}d^{2}-a^{5}cd^{3}+\left(b^{5}c^{3}d-3ab^{4}c^{2}d^{2}+3a^{2}b^{3}cd^{3}-a^{3}b^{2}d^{4}\right)x^{3}+\left(b^{5}c^{4}-ab^{4}c^{3}d-3a^{2}b^{3}c^{2}d^{2}+5a^{3}b^{2}cd^{3}-2a^{4}bd^{4}\right)x^{2}+\left(2ab^{4}c^{4}-5a^{2}b^{3}c^{3}d+3a^{3}b^{2}c^{2}d^{2}+a^{4}bcd^{3}-a^{5}d^{4}\right)x\right)}, -\frac{15\left(b^{2}d^{3}x^{3}+a^{2}cd^{2}+\left(b^{2}cd^{2}+2abd^{3}\right)x^{2}+\left(2abcd^{2}+a^{2}d^{3}\right)x\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-\left(15b^{2}d^{2}x^{2}-2b^{2}c^{2}+9abcd+8a^{2}d^{2}+5\left(b^{2}cd+5abd^{2}\right)x\right)\sqrt{dx+c}}{4\left(a^{2}b^{3}c^{4}-3a^{3}b^{2}c^{3}d+3a^{4}bc^{2}d^{2}-a^{5}cd^{3}+\left(b^{5}c^{3}d-3ab^{4}c^{2}d^{2}+3a^{2}b^{3}cd^{3}-a^{3}b^{2}d^{4}\right)x^{3}+\left(b^{5}c^{4}-ab^{4}c^{3}d-3a^{2}b^{3}c^{2}d^{2}+5a^{3}b^{2}cd^{3}-2a^{4}bd^{4}\right)x^{2}+\left(2ab^{4}c^{4}-5a^{2}b^{3}c^{3}d+3a^{3}b^{2}c^{2}d^{2}+a^{4}bcd^{3}-a^{5}d^{4}\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(b^2*d^3*x^3 + a^2*c*d^2 + (b^2*c*d^2 + 2*a*b*d^3)*x^2 + (2*a*b*c
*d^2 + a^2*d^3)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d + 2*(b*c -
a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) - 2*(15*b^2*d^2*x^2 - 2*
b^2*c^2 + 9*a*b*c*d + 8*a^2*d^2 + 5*(b^2*c*d + 5*a*b*d^2)*x)*sqrt(d*x + c))
/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d
- 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c
^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^
4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x), -1/4*(
15*(b^2*d^3*x^3 + a^2*c*d^2 + (b^2*c*d^2 + 2*a*b*d^3)*x^2 + (2*a*b*c*d^2 +
a^2*d^3)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/
(b*c - a*d))/(b*d*x + b*c)) - (15*b^2*d^2*x^2 - 2*b^2*c^2 + 9*a*b*c*d + 8*a
^2*d^2 + 5*(b^2*c*d + 5*a*b*d^2)*x)*sqrt(d*x + c))/(a^2*b^3*c^4 - 3*a^3*b^2
*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2
*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2
+ 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a
^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)]
```

**giac [B]** time = 1.03, size = 234, normalized size = 1.67

$$\frac{15\,bd^{2}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{4\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{-b^{2}c+abd}}+\frac{2\,d^{2}}{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{dx+c}}+\frac{7\,(dx+c)^{\frac{3}{2}}b^{2}d^{2}-9\,\sqrt{dx+c}\,b^{2}cd^{2}+9\,\sqrt{dx+c}\,abd^{3}}{4\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\left((dx+c)b-bc+ad\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) + 2*d^2/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(d*x + c)) + 1/4*(7*(d*x + c)
^(3/2)*b^2*d^2 - 9*sqrt(d*x + c)*b^2*c*d^2 + 9*sqrt(d*x + c)*a*b*d^3)/((b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)*b - b*c + a*d)^2
)
```

**maple [A]** time = 0.02, size = 179, normalized size = 1.28

$$-\frac{9\sqrt{dx+c}\;ab\,d^{3}}{4\,(ad-bc)^{3}\,(bdx+ad)^{2}}+\frac{9\sqrt{dx+c}\;b^{2}c\,d^{2}}{4\,(ad-bc)^{3}\,(bdx+ad)^{2}}-\frac{7\,(dx+c)^{\frac{3}{2}}\,b^{2}d^{2}}{4\,(ad-bc)^{3}\,(bdx+ad)^{2}}-\frac{15b\,d^{2}\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{4\,(ad-bc)^{3}\,\sqrt{(ad-bc)\,b}}-\frac{2d^{2}}{(ad-bc)^{3}\,\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] -2*d^2/(a*d-b*c)^3/(d*x+c)^(1/2)-7/4*d^2*b^2/(a*d-b*c)^3/(b*d*x+a*d)^2*(d*x
+c)^(3/2)-9/4*d^3*b/(a*d-b*c)^3/(b*d*x+a*d)^2*(d*x+c)^(1/2)*a+9/4*d^2*b^2/(
a*d-b*c)^3/(b*d*x+a*d)^2*(d*x+c)^(1/2)*c-15/4*d^2*b/(a*d-b*c)^3/((a*d-b*c)*
b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.44, size = 205, normalized size = 1.46

$$-\frac{\frac{2\,d^2}{a\,d-b\,c}+\frac{15\,b^2\,d^2\,{\left(c+d\,x\right)}^2}{4\,{\left(a\,d-b\,c\right)}^3}+\frac{25\,b\,d^2\,\left(c+d\,x\right)}{4\,{\left(a\,d-b\,c\right)}^2}}{b^2\,{\left(c+d\,x\right)}^{5/2}-\left(2\,b^2\,c-2\,a\,b\,d\right)\,{\left(c+d\,x\right)}^{3/2}+\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{15\,\sqrt{b}\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^{7/2}}\right)}{4\,{\left(a\,d-b\,c\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d^2)/(a*d - b*c) + (15*b^2*d^2*(c + d*x)^2)/(4*(a*d - b*c)^3) + (25*b
*d^2*(c + d*x))/(4*(a*d - b*c)^2))/(b^2*(c + d*x)^(5/2) - (2*b^2*c - 2*a*b*
d)*(c + d*x)^(3/2) + (c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (15
*b^(1/2)*d^2*atan((b^(1/2)*(c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2)))/(4*(a*d - b*c)^(7/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

## 3.1327 $\int \frac{1}{(a+bx)^4(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=173

$$-\frac{35d^3}{8\sqrt{c+dx}\,(bc-ad)^4}+\frac{35\sqrt{b}\,d^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{9/2}}-\frac{35d^2}{24(a+bx)\sqrt{c+dx}\,(bc-ad)^3}+\frac{7d}{12(a+bx)^2\sqrt{c+dx}\,(bc-ac}$$

**Rubi** [A] time = 0.07, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{35d^3}{8\sqrt{c+dx}\,(bc-ad)^4}-\frac{35d^2}{24(a+bx)\sqrt{c+dx}\,(bc-ad)^3}+\frac{35\sqrt{b}\,d^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{9/2}}+\frac{7d}{12(a+bx)^2\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{3(a+bx)^3\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*Sqrt[c + d*x]) - 1/(3*(b*c - a*d)*(a + b*x)^3*Sq
rt[c + d*x]) + (7*d)/(12*(b*c - a*d)^2*(a + b*x)^2*Sqrt[c + d*x]) - (35*d^2
)/(24*(b*c - a*d)^3*(a + b*x)*Sqrt[c + d*x]) + (35*Sqrt[b]*d^3*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*(b*c - a*d)^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4(c+dx)^{3/2}}\,dx = -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} - \frac{(7d)\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx}{6(bc-ad)}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} + \frac{\left(35d^2\right)\int \frac{1}{(a+bx)^2(c+}}{24(bc-ad)}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} - \frac{35d^2}{24(bc-ad)^3(a+b}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+d}}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+d}}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+d}}$$

**Mathematica [C]** time = 0.02, size = 50, normalized size = 0.29

$$-\frac{2d^3\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{\sqrt{c+dx}\,(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*d^3*Hypergeometric2F1[-1/2, 4, 1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/(
(-(b*c) + a*d)^4*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.68, size = 223, normalized size = 1.29

$$\frac{d^3\left(48a^3d^3+231a^2bd^2(c+dx)-144a^2bcd^2+144ab^2c^2d+280ab^2d(c+dx)^2-462ab^2cd(c+dx)-48b^3c^3+231b^3c^2(c+dx)+105b^3(c+dx)^3-280b^3c(c+dx)^2\right)}{24\sqrt{c+dx}\,(bc-ad)^4(-ad-b(c+dx)+bc)^3}+\frac{35\sqrt{b}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{8(ad-bc)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

```
[Out] (d^3*(-48*b^3*c^3 + 144*a*b^2*c^2*d - 144*a^2*b*c*d^2 + 48*a^3*d^3 + 231*b^
3*c^2*(c + d*x) - 462*a*b^2*c*d*(c + d*x) + 231*a^2*b*d^2*(c + d*x) - 280*b
^3*c*(c + d*x)^2 + 280*a*b^2*d*(c + d*x)^2 + 105*b^3*(c + d*x)^3))/(24*(b*c
 - a*d)^4*Sqrt[c + d*x]*(b*c - a*d - b*(c + d*x))^3) + (35*Sqrt[b]*d^3*ArcT
an[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(8*(-(b*c) + a*
d)^(9/2))
```

**fricas [B]** time = 1.53, size = 1204, normalized size = 6.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^3*d^4*x^4 + a^3*c*d^3 + (b^3*c*d^3 + 3*a*b^2*d^4)*x^3 + 3*(a*
b^2*c*d^3 + a^2*b*d^4)*x^2 + (3*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(b/(b*c - a*d
))*log((b*d*x + 2*b*c - a*d + 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d
)))/(b*x + a)) - 2*(105*b^3*d^3*x^3 + 8*b^3*c^3 - 38*a*b^2*c^2*d + 87*a^2*b
*c*d^2 + 48*a^3*d^3 + 35*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 7*(2*b^3*c^2*d - 1
4*a*b^2*c*d^2 - 33*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^3*b^4*c^5 - 4*a^4*b^3*c^
4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^
6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c
^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^
4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2
 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 -
11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a
^7*d^5)*x), 1/24*(105*(b^3*d^4*x^4 + a^3*c*d^3 + (b^3*c*d^3 + 3*a*b^2*d^4)*
x^3 + 3*(a*b^2*c*d^3 + a^2*b*d^4)*x^2 + (3*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(-
b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*
x + b*c)) - (105*b^3*d^3*x^3 + 8*b^3*c^3 - 38*a*b^2*c^2*d + 87*a^2*b*c*d^2
+ 48*a^3*d^3 + 35*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 7*(2*b^3*c^2*d - 14*a*b^2
*c*d^2 - 33*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6
*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^6*c^3*d
^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c^5 - a*
b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a
^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^
4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 - 11*a^3*
b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)
*x)]
```

**giac [B]** time = 1.31, size = 326, normalized size = 1.88

$$-\frac{35\,bd^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{-b^2c+abd}}-\frac{2\,d^3}{\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{dx+c}}-\frac{57\,(dx+c)^{\frac{5}{2}}b^3d^3-136\,(dx+c)^{\frac{3}{2}}b^3cd^3+87\,\sqrt{dx+c}\,b^3c^2d^3+136\,(dx+c)^{\frac{3}{2}}ab^2d^4-174\,\sqrt{dx+c}\,ab^2cd^4+87\,\sqrt{dx+c}\,a^2bd^5}{24\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)((dx+c)b-bc+ad)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -35/8*b*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d)
) - 2*d^3/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(d*x + c)) - 1/24*(57*(d*x + c)^(5/2)*b^3*d^3 - 136*(d*x + c)^(
3/2)*b^3*c*d^3 + 87*sqrt(d*x + c)*b^3*c^2*d^3 + 136*(d*x + c)^(3/2)*a*b^2*d
^4 - 174*sqrt(d*x + c)*a*b^2*c*d^4 + 87*sqrt(d*x + c)*a^2*b*d^5)/((b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*((d*x + c)*b
 - b*c + a*d)^3)
```

**maple** [B] time = 0.02, size = 292, normalized size = 1.69

$$-\frac{29\sqrt{dx+c}\, a^{2}b\, d^{5}}{8\left(ad-bc\right)^{4}\left(bdx+ad\right)^{3}}+\frac{29\sqrt{dx+c}\, a\, b^{2}c\, d^{4}}{4\left(ad-bc\right)^{4}\left(bdx+ad\right)^{3}}-\frac{29\sqrt{dx+c}\, b^{3}c^{2}d^{3}}{8\left(ad-bc\right)^{4}\left(bdx+ad\right)^{3}}-\frac{17\left(dx+c\right)^{\frac{3}{2}} a\, b^{2}d^{4}}{3\left(ad-bc\right)^{4}\left(bdx+ad\right)^{3}}+\frac{17\left(dx+c\right)^{\frac{3}{2}} b^{3}c\, d^{3}}{3\left(ad-bc\right)^{4}\left(bdx+ad\right)^{3}}-\frac{19\left(dx+c\right)^{\frac{5}{2}} b^{3}d^{3}}{8\left(ad-bc\right)^{4}\left(bdx+ad\right)^{3}}-\frac{35b\, d^{3}\arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{\left(ad-bc\right)b}}\right)}{8\left(ad-bc\right)^{4}\sqrt{\left(ad-bc\right)b}}-\frac{2d^{3}}{\left(ad-bc\right)^{4}\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(3/2),x)
```

```
[Out] -2*d^3/(a*d-b*c)^4/(d*x+c)^(1/2)-19/8*d^3/(a*d-b*c)^4*b^3/(b*d*x+a*d)^3*(d*
x+c)^(5/2)-17/3*d^4/(a*d-b*c)^4*b^2/(b*d*x+a*d)^3*(d*x+c)^(3/2)*a+17/3*d^3/
(a*d-b*c)^4*b^3/(b*d*x+a*d)^3*(d*x+c)^(3/2)*c-29/8*d^5/(a*d-b*c)^4*b/(b*d*x
+a*d)^3*(d*x+c)^(1/2)*a^2+29/4*d^4/(a*d-b*c)^4*b^2/(b*d*x+a*d)^3*(d*x+c)^(1
/2)*a*c-29/8*d^3/(a*d-b*c)^4*b^3/(b*d*x+a*d)^3*(d*x+c)^(1/2)*c^2-35/8*d^3/(
a*d-b*c)^4*b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b
)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 0.54, size = 294, normalized size = 1.70

$$-\frac{\frac{2\,d^3}{a\,d-b\,c}+\frac{35\,b^2\,d^3\,{\left(c+d\,x\right)}^2}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{35\,b^3\,d^3\,{\left(c+d\,x\right)}^3}{8\,{\left(a\,d-b\,c\right)}^4}+\frac{77\,b\,d^3\,\left(c+d\,x\right)}{8\,{\left(a\,d-b\,c\right)}^2}}{\sqrt{c+d\,x}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)+b^3\,{\left(c+d\,x\right)}^{7/2}-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{5/2}+{\left(c+d\,x\right)}^{3/2}\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}-\frac{35\,\sqrt{b}\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{{\left(a\,d-b\,c\right)}^{9/2}}\right)}{8\,{\left(a\,d-b\,c\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d^3)/(a*d - b*c) + (35*b^2*d^3*(c + d*x)^2)/(3*(a*d - b*c)^3) + (35*b
^3*d^3*(c + d*x)^3)/(8*(a*d - b*c)^4) + (77*b*d^3*(c + d*x))/(8*(a*d - b*c)
^2))/((c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) +
 b^3*(c + d*x)^(7/2) - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^(5/2) + (c + d*x)^(3
/2)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d)) - (35*b^(1/2)*d^3*atan((b^(1/2
)*(c + d*x)^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
4*a^3*b*c*d^3))/(a*d - b*c)^(9/2)))/(8*(a*d - b*c)^(9/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

## 3.1328 $\int \frac{(a+bx)^5}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=152

$$-\frac{2b^4(c+dx)^{5/2}(bc-ad)}{d^6}+\frac{20b^3(c+dx)^{3/2}(bc-ad)^2}{3d^6}-\frac{20b^2\sqrt{c+dx}\,(bc-ad)^3}{d^6}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}+\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}+\frac{2b}{}$$

**Rubi [A]** time = 0.05, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2b^4(c+dx)^{5/2}(bc-ad)}{d^6}+\frac{20b^3(c+dx)^{3/2}(bc-ad)^2}{3d^6}-\frac{20b^2\sqrt{c+dx}\,(bc-ad)^3}{d^6}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}+\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}+\frac{2b^5(c+dx)^{7/2}}{7d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^5)/(3*d^6*(c + d*x)^(3/2)) - (10*b*(b*c - a*d)^4)/(d^6*Sqrt[
c + d*x]) - (20*b^2*(b*c - a*d)^3*Sqrt[c + d*x])/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(3/2))/(3*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(5/2))/d^6 + (2*b
^5*(c + d*x)^(7/2))/(7*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^{5/2}}\,dx=\int \left(\frac{(-bc+ad)^5}{d^5(c+dx)^{5/2}}+\frac{5b(bc-ad)^4}{d^5(c+dx)^{3/2}}-\frac{10b^2(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{10b^3(bc-ad)^2\sqrt{c+dx}}{d^5}-\frac{5b^4(bc-ad)(}{d^5}\right.$$

$$=\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}-\frac{20b^2(bc-ad)^3\sqrt{c+dx}}{d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{3/2}}{3d^6}-\frac{2b^4(b}{}$$

**Mathematica [A]** time = 0.12, size = 123, normalized size = 0.81

$$\frac{2\left(-21b^4(c+dx)^4(bc-ad)+70b^3(c+dx)^3(bc-ad)^2-210b^2(c+dx)^2(bc-ad)^3-105b(c+dx)(bc-ad)^4+7(bc-ad)^5+3b^5(c+dx)^5\right)}{21d^6(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^(5/2),x]
```

```
[Out] (2*(7*(b*c - a*d)^5 - 105*b*(b*c - a*d)^4*(c + d*x) - 210*b^2*(b*c - a*d)^3
*(c + d*x)^2 + 70*b^3*(b*c - a*d)^2*(c + d*x)^3 - 21*b^4*(b*c - a*d)*(c + d
*x)^4 + 3*b^5*(c + d*x)^5))/(21*d^6*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [B]** time = 0.07, size = 315, normalized size = 2.07

$$\frac{2\left(-7a^5d^5-105a^4bd^4(c+dx)+35a^4bcd^4-70a^3b^2c^2d^3+210a^3b^2d^3(c+dx)^2+420a^3b^2cd^3(c+dx)+70a^2b^3c^3d^2-630a^2b^3c^2d^2(c+dx)+70a^2b^3d^2(c+dx)^3-630a^2b^3cd^2(c+dx)^2-35ab^4c^4d+420ab^4c^3d(c+dx)+630ab^4c^2d(c+dx)^2+21ab^4d(c+dx)^4-140ab^4cd(c+dx)^3+7b^5c^5-105b^5c^4(c+dx)-210b^5c^3(c+dx)^2+70b^5c^2(c+dx)^3+3b^5(c+dx)^5-21b^5c(c+dx)^4\right)}{21d^6(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^(5/2),x]
```

```
[Out] (2*(7*b^5*c^5 - 35*a*b^4*c^4*d + 70*a^2*b^3*c^3*d^2 - 70*a^3*b^2*c^2*d^3 +
35*a^4*b*c*d^4 - 7*a^5*d^5 - 105*b^5*c^4*(c + d*x) + 420*a*b^4*c^3*d*(c + d
*x) - 630*a^2*b^3*c^2*d^2*(c + d*x) + 420*a^3*b^2*c*d^3*(c + d*x) - 105*a^4
*b*d^4*(c + d*x) - 210*b^5*c^3*(c + d*x)^2 + 630*a*b^4*c^2*d*(c + d*x)^2 -
630*a^2*b^3*c*d^2*(c + d*x)^2 + 210*a^3*b^2*d^3*(c + d*x)^2 + 70*b^5*c^2*(c
 + d*x)^3 - 140*a*b^4*c*d*(c + d*x)^3 + 70*a^2*b^3*d^2*(c + d*x)^3 - 21*b^5
*c*(c + d*x)^4 + 21*a*b^4*d*(c + d*x)^4 + 3*b^5*(c + d*x)^5))/(21*d^6*(c +
d*x)^(3/2))
```

**fricas [B]** time = 1.36, size = 283, normalized size = 1.86

$$\frac{2\left(3b^5d^5x^5-256b^5c^5+896ab^4c^4d-1120a^2b^3c^3d^2+560a^3b^2c^2d^3-70a^4bcd^4-7a^5d^5-3\left(2b^5cd^4-7ab^4d^5\right)x^4+2\left(8b^5c^2d^3-28ab^4cd^4+35a^2b^3d^5\right)x^3-6\left(16b^5c^3d^2-56ab^4c^2d^3+70a^2b^3cd^4-35a^3b^2d^5\right)x^2-3\left(128b^5c^4d-448ab^4c^3d^2+560a^2b^3c^2d^3-280a^3b^2cd^4+35a^4bd^5\right)x\right)\sqrt{dx+c}}{21\left(d^8x^2+2cd^7x+c^2d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^5*d^5*x^5 - 256*b^5*c^5 + 896*a*b^4*c^4*d - 1120*a^2*b^3*c^3*d^2
+ 560*a^3*b^2*c^2*d^3 - 70*a^4*b*c*d^4 - 7*a^5*d^5 - 3*(2*b^5*c*d^4 - 7*a*b
^4*d^5)*x^4 + 2*(8*b^5*c^2*d^3 - 28*a*b^4*c*d^4 + 35*a^2*b^3*d^5)*x^3 - 6*(
16*b^5*c^3*d^2 - 56*a*b^4*c^2*d^3 + 70*a^2*b^3*c*d^4 - 35*a^3*b^2*d^5)*x^2
- 3*(128*b^5*c^4*d - 448*a*b^4*c^3*d^2 + 560*a^2*b^3*c^2*d^3 - 280*a^3*b^2*
c*d^4 + 35*a^4*b*d^5)*x)*sqrt(d*x + c)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6)
```

**giac [B]** time = 0.91, size = 335, normalized size = 2.20

$$-\frac{2\left(15(dx+c)b^5c^4-b^5c^5-60(dx+c)ab^4c^3d+5ab^4c^4d+90(dx+c)a^2b^3c^2d^2-10a^2b^3c^3d^2-60(dx+c)a^3b^2cd^3+10a^3b^2c^2d^3+15(dx+c)a^4bd^4-5a^4bcd^4+a^5d^5\right)}{3(dx+c)^{\frac{3}{2}}d^6}+\frac{2\left(3(dx+c)^{\frac{7}{2}}b^5d^{36}-21(dx+c)^{\frac{5}{2}}b^5cd^{36}+70(dx+c)^{\frac{3}{2}}b^5c^2d^{36}-210\sqrt{dx+c}\,b^5c^3d^{36}+21(dx+c)^{\frac{5}{2}}ab^4d^{37}-140(dx+c)^{\frac{3}{2}}ab^4cd^{37}+630\sqrt{dx+c}\,ab^4c^2d^{37}+70(dx+c)^{\frac{3}{2}}a^2b^3d^{38}-630\sqrt{dx+c}\,a^2b^3cd^{38}+210\sqrt{dx+c}\,a^3b^2d^{39}\right)}{21d^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(15*(d*x + c)*b^5*c^4 - b^5*c^5 - 60*(d*x + c)*a*b^4*c^3*d + 5*a*b^4*c
^4*d + 90*(d*x + c)*a^2*b^3*c^2*d^2 - 10*a^2*b^3*c^3*d^2 - 60*(d*x + c)*a^3
*b^2*c*d^3 + 10*a^3*b^2*c^2*d^3 + 15*(d*x + c)*a^4*b*d^4 - 5*a^4*b*c*d^4 +
a^5*d^5)/((d*x + c)^(3/2)*d^6) + 2/21*(3*(d*x + c)^(7/2)*b^5*d^36 - 21*(d*x
 + c)^(5/2)*b^5*c*d^36 + 70*(d*x + c)^(3/2)*b^5*c^2*d^36 - 210*sqrt(d*x + c
)*b^5*c^3*d^36 + 21*(d*x + c)^(5/2)*a*b^4*d^37 - 140*(d*x + c)^(3/2)*a*b^4*
c*d^37 + 630*sqrt(d*x + c)*a*b^4*c^2*d^37 + 70*(d*x + c)^(3/2)*a^2*b^3*d^38
 - 630*sqrt(d*x + c)*a^2*b^3*c*d^38 + 210*sqrt(d*x + c)*a^3*b^2*d^39)/d^42
```

**maple** [B] time = 0.01, size = 273, normalized size = 1.80

$$-\frac{2\left(-3b^5x^5d^5-21ab^4d^5x^4+6b^5cd^4x^4-70a^2b^3d^5x^3+56ab^4cd^4x^3-16b^5c^2d^3x^3-210a^3b^2d^5x^2+420a^2b^3cd^4x^2-336ab^4c^2d^3x^2+96b^5c^3d^2x^2+105a^4bd^5x-840a^3b^2cd^4x+1680a^2b^3c^2d^3x-1344ab^4c^3d^2x+384b^5c^4dx+7a^5d^5+70a^4bcd^4-560a^3b^2c^2d^3+1120a^2b^3c^3d^2-896ab^4c^4d+256b^5c^5\right)}{21\left(dx+c\right)^{\frac{3}{2}}d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(5/2),x)
```

```
[Out] -2/21/(d*x+c)^(3/2)*(-3*b^5*d^5*x^5-21*a*b^4*d^5*x^4+6*b^5*c*d^4*x^4-70*a^2
*b^3*d^5*x^3+56*a*b^4*c*d^4*x^3-16*b^5*c^2*d^3*x^3-210*a^3*b^2*d^5*x^2+420*
a^2*b^3*c*d^4*x^2-336*a*b^4*c^2*d^3*x^2+96*b^5*c^3*d^2*x^2+105*a^4*b*d^5*x-
840*a^3*b^2*c*d^4*x+1680*a^2*b^3*c^2*d^3*x-1344*a*b^4*c^3*d^2*x+384*b^5*c^4
*d*x+7*a^5*d^5+70*a^4*b*c*d^4-560*a^3*b^2*c^2*d^3+1120*a^2*b^3*c^3*d^2-896*
a*b^4*c^4*d+256*b^5*c^5)/d^6
```

**maxima** [A] time = 1.38, size = 265, normalized size = 1.74

$$\frac{2\left(\frac{3\left(dx+c\right)^{\frac{7}{2}}b^5-21\left(b^5c-ab^4d\right)\left(dx+c\right)^{\frac{5}{2}}+70\left(b^5c^2-2ab^4cd+a^2b^3d^2\right)\left(dx+c\right)^{\frac{3}{2}}-210\left(b^5c^3-3ab^4c^2d+3a^2b^3cd^2-a^3b^2d^3\right)\sqrt{dx+c}}{d^5}+\frac{7\left(b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5-15\left(b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4\right)\left(dx+c\right)\right)}{\left(dx+c\right)^{\frac{3}{2}}d^5}\right)}{21d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*((3*(d*x + c)^(7/2)*b^5 - 21*(b^5*c - a*b^4*d)*(d*x + c)^(5/2) + 70*(b
^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(3/2) - 210*(b^5*c^3 - 3*a*b^
4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt(d*x + c))/d^5 + 7*(b^5*c^5 -
5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a
^5*d^5 - 15*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3
+ a^4*b*d^4)*(d*x + c))/((d*x + c)^(3/2)*d^5))/d
```

**mupad** [B] time = 0.08, size = 229, normalized size = 1.51

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{5/2}}{5\,d^6}-\frac{\frac{2\,a^5\,d^5}{3}-\frac{2\,b^5\,c^5}{3}+\left(c+d\,x\right)\,\left(10\,a^4\,b\,d^4-40\,a^3\,b^2\,c\,d^3+60\,a^2\,b^3\,c^2\,d^2-40\,a\,b^4\,c^3\,d+10\,b^5\,c^4\right)-\frac{20\,a^2\,b^3\,c^3\,d^2}{3}+\frac{20\,a^3\,b^2\,c^2\,d^3}{3}+\frac{10\,a\,b^4\,c^4\,d}{3}-\frac{10\,a^4\,b\,c\,d^4}{3}}{d^6\,{\left(c+d\,x\right)}^{3/2}}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,\sqrt{c+d\,x}}{d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^(5/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(7/2))/(7*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(5/2))
/(5*d^6) - ((2*a^5*d^5)/3 - (2*b^5*c^5)/3 + (c + d*x)*(10*b^5*c^4 + 10*a^4*
b*d^4 - 40*a^3*b^2*c*d^3 + 60*a^2*b^3*c^2*d^2 - 40*a*b^4*c^3*d) - (20*a^2*b
^3*c^3*d^2)/3 + (20*a^3*b^2*c^2*d^3)/3 + (10*a*b^4*c^4*d)/3 - (10*a^4*b*c*d
^4)/3)/(d^6*(c + d*x)^(3/2)) + (20*b^2*(a*d - b*c)^3*(c + d*x)^(1/2))/d^6 +
 (20*b^3*(a*d - b*c)^2*(c + d*x)^(3/2))/(3*d^6)
```

**sympy [A]** time = 59.93, size = 196, normalized size = 1.29

$$\frac{2b^5(c+dx)^{\frac{7}{2}}}{7d^6}-\frac{10b(ad-bc)^4}{d^6\sqrt{c+dx}}+\frac{(c+dx)^{\frac{5}{2}}\left(10ab^4d-10b^5c\right)}{5d^6}+\frac{(c+dx)^{\frac{3}{2}}\left(20a^2b^3d^2-40ab^4cd+20b^5c^2\right)}{3d^6}+\frac{\sqrt{c+dx}\left(20a^3b^2d^3-60a^2b^3cd^2+60ab^4c^2d-20b^5c^3\right)}{d^6}-\frac{2(ad-bc)^5}{3d^6(c+dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(5/2),x)
```

```
[Out] 2*b**5*(c + d*x)**(7/2)/(7*d**6) - 10*b*(a*d - b*c)**4/(d**6*sqrt(c + d*x))
 + (c + d*x)**(5/2)*(10*a*b**4*d - 10*b**5*c)/(5*d**6) + (c + d*x)**(3/2)*(
20*a**2*b**3*d**2 - 40*a*b**4*c*d + 20*b**5*c**2)/(3*d**6) + sqrt(c + d*x)*
(20*a**3*b**2*d**3 - 60*a**2*b**3*c*d**2 + 60*a*b**4*c**2*d - 20*b**5*c**3)
/d**6 - 2*(a*d - b*c)**5/(3*d**6*(c + d*x)**(3/2))
```

## 3.1329 $\int \frac{(a+bx)^4}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=125

$$-\frac{8b^3(c+dx)^{3/2}(bc-ad)}{3d^5}+\frac{12b^2\sqrt{c+dx}\,(bc-ad)^2}{d^5}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}-\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{2b^4(c+dx)^{5/2}}{5d^5}$$

**Rubi** [A] time = 0.04, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{3/2}(bc-ad)}{3d^5}+\frac{12b^2\sqrt{c+dx}\,(bc-ad)^2}{d^5}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}-\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{2b^4(c+dx)^{5/2}}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^4)/(3*d^5*(c + d*x)^(3/2)) + (8*b*(b*c - a*d)^3)/(d^5*Sqrt[
c + d*x]) + (12*b^2*(b*c - a*d)^2*Sqrt[c + d*x])/d^5 - (8*b^3*(b*c - a*d)*(
c + d*x)^(3/2))/(3*d^5) + (2*b^4*(c + d*x)^(5/2))/(5*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4(c+dx)^{5/2}}-\frac{4b(bc-ad)^3}{d^4(c+dx)^{3/2}}+\frac{6b^2(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{4b^3(bc-ad)\sqrt{c+dx}}{d^4}+\frac{b^4(c+dx)^{3/2}}{d^4}\right)dx\\ &= -\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{12b^2(bc-ad)^2\sqrt{c+dx}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{3/2}}{3d^5}+\frac{2b^4(c+dx)^{5/2}}{5d^5}\end{aligned}$$

**Mathematica** [A] time = 0.08, size = 101, normalized size = 0.81

$$\frac{2\left(-20b^3(c+dx)^3(bc-ad)+90b^2(c+dx)^2(bc-ad)^2+60b(c+dx)(bc-ad)^3-5(bc-ad)^4+3b^4(c+dx)^4\right)}{15d^5(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (2*(-5*(b*c - a*d)^4 + 60*b*(b*c - a*d)^3*(c + d*x) + 90*b^2*(b*c - a*d)^2*
(c + d*x)^2 - 20*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4))/(15*d^5*
(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 213, normalized size = 1.70

$$\frac{2\left(-5a^4d^4-60a^3bd^3(c+dx)+20a^3bcd^3-30a^2b^2c^2d^2+90a^2b^2d^2(c+dx)^2+180a^2b^2cd^2(c+dx)+20ab^3c^3d-180ab^3c^2d(c+dx)+20ab^3d(c+dx)^3-180ab^3cd(c+dx)^2-5b^4c^4+60b^4c^3(c+dx)+90b^4c^2(c+dx)^2+3b^4(c+dx)^4-20b^4c(c+dx)^3\right)}{15d^5(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (2*(-5*b^4*c^4 + 20*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 5*a
^4*d^4 + 60*b^4*c^3*(c + d*x) - 180*a*b^3*c^2*d*(c + d*x) + 180*a^2*b^2*c*d
^2*(c + d*x) - 60*a^3*b*d^3*(c + d*x) + 90*b^4*c^2*(c + d*x)^2 - 180*a*b^3*
c*d*(c + d*x)^2 + 90*a^2*b^2*d^2*(c + d*x)^2 - 20*b^4*c*(c + d*x)^3 + 20*a*
b^3*d*(c + d*x)^3 + 3*b^4*(c + d*x)^4))/(15*d^5*(c + d*x)^(3/2))
```

**fricas [A]** time = 1.13, size = 203, normalized size = 1.62

$$\frac{2\left(3\,b^4d^4x^4+128\,b^4c^4-320\,ab^3c^3d+240\,a^2b^2c^2d^2-40\,a^3bcd^3-5\,a^4d^4-4\left(2\,b^4cd^3-5\,ab^3d^4\right)x^3+6\left(8\,b^4c^2d^2-20\,ab^3cd^3+15\,a^2b^2d^4\right)x^2+12\left(16\,b^4c^3d-40\,ab^3c^2d^2+30\,a^2b^2cd^3-5\,a^3bd^4\right)x\right)\sqrt{dx+c}}{15\left(d^7x^2+2\,cd^6x+c^2d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*d^4*x^4 + 128*b^4*c^4 - 320*a*b^3*c^3*d + 240*a^2*b^2*c^2*d^2 -
 40*a^3*b*c*d^3 - 5*a^4*d^4 - 4*(2*b^4*c*d^3 - 5*a*b^3*d^4)*x^3 + 6*(8*b^4*
c^2*d^2 - 20*a*b^3*c*d^3 + 15*a^2*b^2*d^4)*x^2 + 12*(16*b^4*c^3*d - 40*a*b^
3*c^2*d^2 + 30*a^2*b^2*c*d^3 - 5*a^3*b*d^4)*x)*sqrt(d*x + c)/(d^7*x^2 + 2*c
*d^6*x + c^2*d^5)
```

**giac [B]** time = 1.02, size = 229, normalized size = 1.83

$$\frac{2\left(12\,(dx+c)b^4c^3-b^4c^4-36\,(dx+c)ab^3c^2d+4\,ab^3c^3d+36\,(dx+c)a^2b^2cd^2-6\,a^2b^2c^2d^2-12\,(dx+c)a^3bd^3+4\,a^3bcd^3-a^4d^4\right)}{3\,(dx+c)^{\frac{3}{2}}d^5}+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}b^4d^{20}-20\,(dx+c)^{\frac{3}{2}}b^4cd^{20}+90\,\sqrt{dx+c}\,b^4c^2d^{20}+20\,(dx+c)^{\frac{3}{2}}ab^3d^{21}-180\,\sqrt{dx+c}\,ab^3cd^{21}+90\,\sqrt{dx+c}\,a^2b^2d^{22}\right)}{15\,d^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*(d*x + c)*b^4*c^3 - b^4*c^4 - 36*(d*x + c)*a*b^3*c^2*d + 4*a*b^3*c^
3*d + 36*(d*x + c)*a^2*b^2*c*d^2 - 6*a^2*b^2*c^2*d^2 - 12*(d*x + c)*a^3*b*d
^3 + 4*a^3*b*c*d^3 - a^4*d^4)/((d*x + c)^(3/2)*d^5) + 2/15*(3*(d*x + c)^(5/
```

```
2)*b^4*d^20 - 20*(d*x + c)^(3/2)*b^4*c*d^20 + 90*sqrt(d*x + c)*b^4*c^2*d^20
 + 20*(d*x + c)^(3/2)*a*b^3*d^21 - 180*sqrt(d*x + c)*a*b^3*c*d^21 + 90*sqrt
(d*x + c)*a^2*b^2*d^22)/d^25
```

**maple [A]** time = 0.01, size = 186, normalized size = 1.49

$$-\frac{2 \left(-3 b^{4} x^{4} d^{4}-20 a \,b^{3} d^{4} x^{3}+8 b^{4} c \,d^{3} x^{3}-90 a^{2} b^{2} d^{4} x^{2}+120 a \,b^{3} c \,d^{3} x^{2}-48 b^{4} c^{2} d^{2} x^{2}+60 a^{3} b \,d^{4} x -360 a^{2} b^{2} c \,d^{3} x +480 a \,b^{3} c^{2} d^{2} x -192 b^{4} c^{3} d x +5 a^{4} d^{4}+40 a^{3} b c \,d^{3}-240 a^{2} b^{2} c^{2} d^{2}+320 a \,b^{3} c^{3} d -128 b^{4} c^{4}\right)}{15 \left(d x +c \right)^{\frac{3}{2}} d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(5/2),x)
```

```
[Out] -2/15/(d*x+c)^(3/2)*(-3*b^4*d^4*x^4-20*a*b^3*d^4*x^3+8*b^4*c*d^3*x^3-90*a^2
*b^2*d^4*x^2+120*a*b^3*c*d^3*x^2-48*b^4*c^2*d^2*x^2+60*a^3*b*d^4*x-360*a^2*
b^2*c*d^3*x+480*a*b^3*c^2*d^2*x-192*b^4*c^3*d*x+5*a^4*d^4+40*a^3*b*c*d^3-24
0*a^2*b^2*c^2*d^2+320*a*b^3*c^3*d-128*b^4*c^4)/d^5
```

**maxima [A]** time = 1.46, size = 187, normalized size = 1.50

$$\frac{2 \left(\frac{3 \left(d x+c\right)^{\frac{5}{2}} b^{4}-20 \left(b^{4} c-a b^{3} d\right) \left(d x+c\right)^{\frac{3}{2}}+90 \left(b^{4} c^{2}-2 \, a b^{3} c d+a^{2} b^{2} d^{2}\right) \sqrt{d x+c}}{d^{4}}-\frac{5 \left(b^{4} c^{4}-4 \, a b^{3} c^{3} d+6 \, a^{2} b^{2} c^{2} d^{2}-4 \, a^{3} b c d^{3}+a^{4} d^{4}-12 \left(b^{4} c^{3}-3 \, a b^{3} c^{2} d+3 \, a^{2} b^{2} c d^{2}-a^{3} b d^{3}\right) \left(d x+c\right)\right)}{\left(d x+c\right)^{\frac{3}{2}} d^{4}}\right)}{15 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(d*x + c)^(5/2)*b^4 - 20*(b^4*c - a*b^3*d)*(d*x + c)^(3/2) + 90*(b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*sqrt(d*x + c))/d^4 - 5*(b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4 - 12*(b^4*c^3 - 3*a*
b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c))/((d*x + c)^(3/2)*d^4))/
d
```

**mupad [B]** time = 0.30, size = 175, normalized size = 1.40

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,d^5}+\frac{\left(c+d\,x\right)\,\left(-8\,a^3\,b\,d^3+24\,a^2\,b^2\,c\,d^2-24\,a\,b^3\,c^2\,d+8\,b^4\,c^3\right)-\frac{2\,a^4\,d^4}{3}-\frac{2\,b^4\,c^4}{3}-4\,a^2\,b^2\,c^2\,d^2+\frac{8\,a\,b^3\,c^3\,d}{3}+\frac{8\,a^3\,b\,c\,d^3}{3}}{d^5\,{\left(c+d\,x\right)}^{3/2}}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,\sqrt{c+d\,x}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(5/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(5/2))/(5*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(3/2))/(
3*d^5) + ((c + d*x)*(8*b^4*c^3 - 8*a^3*b*d^3 + 24*a^2*b^2*c*d^2 - 24*a*b^3*
c^2*d) - (2*a^4*d^4)/3 - (2*b^4*c^4)/3 - 4*a^2*b^2*c^2*d^2 + (8*a*b^3*c^3*d
)/3 + (8*a^3*b*c*d^3)/3)/(d^5*(c + d*x)^(3/2)) + (12*b^2*(a*d - b*c)^2*(c +
 d*x)^(1/2))/d^5
```

**sympy [A]** time = 43.59, size = 136, normalized size = 1.09

$$\frac{2b^4(c+dx)^{\frac{5}{2}}}{5d^5}-\frac{8b(ad-bc)^3}{d^5\sqrt{c+dx}}+\frac{(c+dx)^{\frac{3}{2}}\left(8ab^3d-8b^4c\right)}{3d^5}+\frac{\sqrt{c+dx}\left(12a^2b^2d^2-24ab^3cd+12b^4c^2\right)}{d^5}-\frac{2(ad-bc)^4}{3d^5(c+dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(5/2),x)
```

```
[Out] 2*b**4*(c + d*x)**(5/2)/(5*d**5) - 8*b*(a*d - b*c)**3/(d**5*sqrt(c + d*x))
+ (c + d*x)**(3/2)*(8*a*b**3*d - 8*b**4*c)/(3*d**5) + sqrt(c + d*x)*(12*a**
2*b**2*d**2 - 24*a*b**3*c*d + 12*b**4*c**2)/d**5 - 2*(a*d - b*c)**4/(3*d**5
*(c + d*x)**(3/2))
```

## 3.1330 $\int \frac{(a+bx)^3}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=96

$$-\frac{6b^2\sqrt{c+dx}\,(bc-ad)}{d^4}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}+\frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}+\frac{2b^3(c+dx)^{3/2}}{3d^4}$$

**Rubi** [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2\sqrt{c+dx}\,(bc-ad)}{d^4}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}+\frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}+\frac{2b^3(c+dx)^{3/2}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^3)/(3*d^4*(c + d*x)^(3/2)) - (6*b*(b*c - a*d)^2)/(d^4*Sqrt[c
 + d*x]) - (6*b^2*(b*c - a*d)*Sqrt[c + d*x])/d^4 + (2*b^3*(c + d*x)^(3/2))/
(3*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^{5/2}}+\frac{3b(bc-ad)^2}{d^3(c+dx)^{3/2}}-\frac{3b^2(bc-ad)}{d^3\sqrt{c+dx}}+\frac{b^3\sqrt{c+dx}}{d^3}\right)dx\\ &= \frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{6b^2(bc-ad)\sqrt{c+dx}}{d^4}+\frac{2b^3(c+dx)^{3/2}}{3d^4}\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 76, normalized size = 0.79

$$\frac{2\left(-9b^2(c+dx)^2(bc-ad)-9b(c+dx)(bc-ad)^2+(bc-ad)^3+b^3(c+dx)^3\right)}{3d^4(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^(5/2),x]
```

```
[Out] (2*((b*c - a*d)^3 - 9*b*(b*c - a*d)^2*(c + d*x) - 9*b^2*(b*c - a*d)*(c + d*
x)^2 + b^3*(c + d*x)^3))/(3*d^4*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 130, normalized size = 1.35

$$\frac{2\left(-a^3d^3-9a^2bd^2(c+dx)+3a^2bcd^2-3ab^2c^2d+9ab^2d(c+dx)^2+18ab^2cd(c+dx)+b^3c^3-9b^3c^2(c+dx)+b^3(c+dx)^3-9b^3c(c+dx)^2\right)}{3d^4(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - 9*b^3*c^2*(c + d*x)
 + 18*a*b^2*c*d*(c + d*x) - 9*a^2*b*d^2*(c + d*x) - 9*b^3*c*(c + d*x)^2 + 9
*a*b^2*d*(c + d*x)^2 + b^3*(c + d*x)^3))/(3*d^4*(c + d*x)^(3/2))
```

**fricas [A]** time = 1.18, size = 136, normalized size = 1.42

$$\frac{2\left(b^3d^3x^3-16\,b^3c^3+24\,ab^2c^2d-6\,a^2bcd^2-a^3d^3-3\left(2\,b^3cd^2-3\,ab^2d^3\right)x^2-3\left(8\,b^3c^2d-12\,ab^2cd^2+3\,a^2bd^3\right)x\right)\sqrt{dx+c}}{3\left(d^6x^2+2\,cd^5x+c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*d^3*x^3 - 16*b^3*c^3 + 24*a*b^2*c^2*d - 6*a^2*b*c*d^2 - a^3*d^3 -
3*(2*b^3*c*d^2 - 3*a*b^2*d^3)*x^2 - 3*(8*b^3*c^2*d - 12*a*b^2*c*d^2 + 3*a^2
*b*d^3)*x)*sqrt(d*x + c)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

**giac [A]** time = 1.00, size = 141, normalized size = 1.47

$$-\frac{2\left(9\,(dx+c)b^3c^2-b^3c^3-18\,(dx+c)ab^2cd+3\,ab^2c^2d+9\,(dx+c)a^2bd^2-3\,a^2bcd^2+a^3d^3\right)}{3\,(dx+c)^{\frac{3}{2}}d^4}+\frac{2\left((dx+c)^{\frac{3}{2}}b^3d^8-9\,\sqrt{dx+c}\,b^3cd^8+9\,\sqrt{dx+c}\,ab^2d^9\right)}{3\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(9*(d*x + c)*b^3*c^2 - b^3*c^3 - 18*(d*x + c)*a*b^2*c*d + 3*a*b^2*c^2*
d + 9*(d*x + c)*a^2*b*d^2 - 3*a^2*b*c*d^2 + a^3*d^3)/((d*x + c)^(3/2)*d^4)
+ 2/3*((d*x + c)^(3/2)*b^3*d^8 - 9*sqrt(d*x + c)*b^3*c*d^8 + 9*sqrt(d*x + c
)*a*b^2*d^9)/d^12
```

**maple [A]** time = 0.01, size = 115, normalized size = 1.20

$$-\frac{2\left(-b^{3}x^{3}d^{3}-9a\,b^{2}d^{3}x^{2}+6b^{3}c\,d^{2}x^{2}+9a^{2}b\,d^{3}x-36a\,b^{2}c\,d^{2}x+24b^{3}c^{2}dx+a^{3}d^{3}+6a^{2}bc\,d^{2}-24a\,b^{2}c^{2}d+16b^{3}c^{3}\right)}{3\left(dx+c\right)^{\frac{3}{2}}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/(d*x+c)^(3/2)*(-b^3*d^3*x^3-9*a*b^2*d^3*x^2+6*b^3*c*d^2*x^2+9*a^2*b*d^
3*x-36*a*b^2*c*d^2*x+24*b^3*c^2*d*x+a^3*d^3+6*a^2*b*c*d^2-24*a*b^2*c^2*d+16
*b^3*c^3)/d^4
```

**maxima [A]** time = 1.37, size = 122, normalized size = 1.27

$$\frac{2\left(\frac{\left(dx+c\right)^{\frac{3}{2}}b^{3}-9\left(b^{3}c-ab^{2}d\right)\sqrt{dx+c}}{d^{3}}+\frac{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}-9\left(b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2}\right)\left(dx+c\right)}{\left(dx+c\right)^{\frac{3}{2}}d^{3}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((d*x + c)^(3/2)*b^3 - 9*(b^3*c - a*b^2*d)*sqrt(d*x + c))/d^3 + (b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - 9*(b^3*c^2 - 2*a*b^2*c*d + a
^2*b*d^2)*(d*x + c))/((d*x + c)^(3/2)*d^3))/d
```

**mupad [B]** time = 0.09, size = 128, normalized size = 1.33

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^3-2\,a^3\,d^3+2\,b^3\,c^3-18\,b^3\,c\,{\left(c+d\,x\right)}^2-18\,b^3\,c^2\,\left(c+d\,x\right)+18\,a\,b^2\,d\,{\left(c+d\,x\right)}^2-18\,a^2\,b\,d^2\,\left(c+d\,x\right)-6\,a\,b^2\,c^2\,d+6\,a^2\,b\,c\,d^2+36\,a\,b^2\,c\,d\,\left(c+d\,x\right)}{3\,d^4\,{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(5/2),x)
```

```
[Out] (2*b^3*(c + d*x)^3 - 2*a^3*d^3 + 2*b^3*c^3 - 18*b^3*c*(c + d*x)^2 - 18*b^3*
c^2*(c + d*x) + 18*a*b^2*d*(c + d*x)^2 - 18*a^2*b*d^2*(c + d*x) - 6*a*b^2*c
^2*d + 6*a^2*b*c*d^2 + 36*a*b^2*c*d*(c + d*x))/(3*d^4*(c + d*x)^(3/2))
```

**sympy [A]** time = 1.44, size = 461, normalized size = 4.80

$$\begin{cases} - \frac{2a^{3}d^{3}}{3cd^{4}\sqrt{c+dx}+3d^{5}x\sqrt{c+dx}} - \frac{12a^{2}bcd^{2}}{3cd^{4}\sqrt{c+dx}+3d^{5}x\sqrt{c+dx}} - \frac{18a^{2}bd^{3}x}{3cd^{4}\sqrt{c+dx}+3d^{5}x\sqrt{c+dx}} + \frac{48ab^{2}c^{2}d}{3cd^{4}\sqrt{c+dx}+3d^{5}x\sqrt{c+dx}} + \frac{72ab^{2}cd^{2}x}{3cd^{4}\sqrt{c+dx}+3d^{5}x\sqrt{c+dx}} + \frac{18ab^{2}d^{3}x^{2}}{3cd^{4}\sqrt{c+dx}+3d^{5}x\sqrt{c+dx}} - \frac{32b^{3}c^{3}}{3cd^{4}\sqrt{c+dx}+3d^{5}x\sqrt{c+dx}} - \frac{48b^{3}c^{2}dx}{3cd^{4}\sqrt{c+dx}+3d^{5}x\sqrt{c+dx}} - \frac{12b^{3}cd^{2}x^{2}}{3cd^{4}\sqrt{c+dx}+3d^{5}x\sqrt{c+dx}} + \frac{2b^{3}d^{3}x^{3}}{3cd^{4}\sqrt{c+dx}+3d^{5}x\sqrt{c+dx}} & \text{for}\: d \neq 0 \\ \frac{a^{3}x+\frac{3a^{2}bx^{2}}{2}+ab^{2}x^{3}+\frac{b^{3}x^{4}}{4}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**3*d**3/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) -
 12*a**2*b*c*d**2/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 18*a*
*2*b*d**3*x/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 48*a*b**2*c
**2*d/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 72*a*b**2*c*d**2*
x/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 18*a*b**2*d**3*x**2/(
3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 32*b**3*c**3/(3*c*d**4*s
qrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 48*b**3*c**2*d*x/(3*c*d**4*sqrt(c
+ d*x) + 3*d**5*x*sqrt(c + d*x)) - 12*b**3*c*d**2*x**2/(3*c*d**4*sqrt(c + d
*x) + 3*d**5*x*sqrt(c + d*x)) + 2*b**3*d**3*x**3/(3*c*d**4*sqrt(c + d*x) +
3*d**5*x*sqrt(c + d*x)), Ne(d, 0)), ((a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**
3 + b**3*x**4/4)/c**(5/2), True))
```

## 3.1331 $\int \frac{(a+bx)^2}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=67

$$\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}-\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{2b^2\sqrt{c+dx}}{d^3}$$

**Rubi** [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}-\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{2b^2\sqrt{c+dx}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^2)/(3*d^3*(c + d*x)^(3/2)) + (4*b*(b*c - a*d))/(d^3*Sqrt[c
+ d*x]) + (2*b^2*Sqrt[c + d*x])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{5/2}}-\frac{2b(bc-ad)}{d^2(c+dx)^{3/2}}+\frac{b^2}{d^2\sqrt{c+dx}}\right)dx\\ &= -\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}+\frac{2b^2\sqrt{c+dx}}{d^3}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 62, normalized size = 0.93

$$\frac{-2a^2d^2-4abd(2c+3dx)+2b^2\left(8c^2+12cdx+3d^2x^2\right)}{3d^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*a^2*d^2 - 4*a*b*d*(2*c + 3*d*x) + 2*b^2*(8*c^2 + 12*c*d*x + 3*d^2*x^2))
/(3*d^3*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 72, normalized size = 1.07

$$\frac{2\left(-a^2d^2-6abd(c+dx)+2abcd+b^2\left(-c^2\right)+3b^2(c+dx)^2+6b^2c(c+dx)\right)}{3d^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (2*(-(b^2*c^2) + 2*a*b*c*d - a^2*d^2 + 6*b^2*c*(c + d*x) - 6*a*b*d*(c + d*x
) + 3*b^2*(c + d*x)^2))/(3*d^3*(c + d*x)^(3/2))
```

**fricas [A]** time = 1.24, size = 85, normalized size = 1.27

$$\frac{2\left(3\,b^2d^2x^2+8\,b^2c^2-4\,abcd-a^2d^2+6\left(2\,b^2cd-abd^2\right)x\right)\sqrt{dx+c}}{3\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*d^2*x^2 + 8*b^2*c^2 - 4*a*b*c*d - a^2*d^2 + 6*(2*b^2*c*d - a*b*d
^2)*x)*sqrt(d*x + c)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**giac [A]** time = 1.11, size = 72, normalized size = 1.07

$$\frac{2\,\sqrt{dx+c}\,b^2}{d^3}+\frac{2\left(6\,(dx+c)b^2c-b^2c^2-6\,(dx+c)abd+2\,abcd-a^2d^2\right)}{3\,(dx+c)^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*b^2/d^3 + 2/3*(6*(d*x + c)*b^2*c - b^2*c^2 - 6*(d*x + c)*a*
b*d + 2*a*b*c*d - a^2*d^2)/((d*x + c)^(3/2)*d^3)
```

**maple [A]** time = 0.01, size = 62, normalized size = 0.93

$$-\frac{2\left(-3b^2x^2d^2+6ab\,d^2x-12b^2cdx+a^2d^2+4abcd-8b^2c^2\right)}{3\,(dx+c)^{\frac{3}{2}}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/(d*x+c)^(3/2)*(-3*b^2*d^2*x^2+6*a*b*d^2*x-12*b^2*c*d*x+a^2*d^2+4*a*b*c
*d-8*b^2*c^2)/d^3
```

**maxima** [A] time = 1.39, size = 72, normalized size = 1.07

$$\frac{2\left(\frac{3\sqrt{dx+c}\,b^2}{d^2}-\frac{b^2c^2-2\,abcd+a^2d^2-6\left(b^2c-abd\right)(dx+c)}{(dx+c)^{\frac{3}{2}}d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(d*x + c)*b^2/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 6*(b^2*c -
a*b*d)*(d*x + c))/((d*x + c)^(3/2)*d^2))/d
```

**mupad** [B] time = 0.07, size = 68, normalized size = 1.01

$$\frac{6\,b^2\,{\left(c+d\,x\right)}^2-2\,a^2\,d^2-2\,b^2\,c^2+12\,b^2\,c\,\left(c+d\,x\right)-12\,a\,b\,d\,\left(c+d\,x\right)+4\,a\,b\,c\,d}{3\,d^3\,{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(5/2),x)
```

```
[Out] (6*b^2*(c + d*x)^2 - 2*a^2*d^2 - 2*b^2*c^2 + 12*b^2*c*(c + d*x) - 12*a*b*d*
(c + d*x) + 4*a*b*c*d)/(3*d^3*(c + d*x)^(3/2))
```

**sympy** [A] time = 1.27, size = 265, normalized size = 3.96

$$\begin{cases} - \frac{2 a^{2} d^{2}}{3 c d^{3} \sqrt{c + d x} + 3 d^{4} x \sqrt{c + d x}} - \frac{8 a b c d}{3 c d^{3} \sqrt{c + d x} + 3 d^{4} x \sqrt{c + d x}} - \frac{12 a b d^{2} x}{3 c d^{3} \sqrt{c + d x} + 3 d^{4} x \sqrt{c + d x}} + \frac{16 b^{2} c^{2}}{3 c d^{3} \sqrt{c + d x} + 3 d^{4} x \sqrt{c + d x}} + \frac{24 b^{2} c d x}{3 c d^{3} \sqrt{c + d x} + 3 d^{4} x \sqrt{c + d x}} + \frac{6 b^{2} d^{2} x^{2}}{3 c d^{3} \sqrt{c + d x} + 3 d^{4} x \sqrt{c + d x}} & \text{for}\: d \neq 0 \\ \frac{a^{2} x + a b x^{2} + \frac{b^{2} x^{3}}{3}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*d**2/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) -
 8*a*b*c*d/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) - 12*a*b*d**2*
x/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) + 16*b**2*c**2/(3*c*d**
3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) + 24*b**2*c*d*x/(3*c*d**3*sqrt(c
+ d*x) + 3*d**4*x*sqrt(c + d*x)) + 6*b**2*d**2*x**2/(3*c*d**3*sqrt(c + d*x)
 + 3*d**4*x*sqrt(c + d*x)), Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/c
**(5/2), True))
```

## 3.1332 $\int \frac{a+bx}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=40

$$\frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}$$

**Rubi** [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d))/(3*d^2*(c + d*x)^(3/2)) - (2*b)/(d^2*Sqrt[c + d*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^{5/2}}+\frac{b}{d(c+dx)^{3/2}}\right)dx\\ &= \frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 29, normalized size = 0.72

$$-\frac{2(ad+2bc+3bdx)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(2*b*c + a*d + 3*b*d*x))/(3*d^2*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.03, size = 32, normalized size = 0.80

$$-\frac{2(ad+3b(c+dx)-bc)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x)^(5/2),x]
[Out] (-2*(-(b*c) + a*d + 3*b*(c + d*x)))/(3*d^2*(c + d*x)^(3/2))
```

**fricas [A]** time = 1.11, size = 46, normalized size = 1.15

$$-\frac{2\,(3\,bdx+2\,bc+ad)\sqrt{dx+c}}{3\,\left(d^4x^2+2\,cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
[Out] -2/3*(3*b*d*x + 2*b*c + a*d)*sqrt(d*x + c)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

**giac [A]** time = 1.02, size = 28, normalized size = 0.70

$$-\frac{2\,(3\,(dx+c)b-bc+ad)}{3\,(dx+c)^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
[Out] -2/3*(3*(d*x + c)*b - b*c + a*d)/((d*x + c)^(3/2)*d^2)
```

**maple [A]** time = 0.00, size = 26, normalized size = 0.65

$$-\frac{2\,(3bdx+ad+2bc)}{3\,(dx+c)^{\frac{3}{2}}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(5/2),x)
[Out] -2/3/(d*x+c)^(3/2)*(3*b*d*x+a*d+2*b*c)/d^2
```

**maxima** [A] time = 1.28, size = 28, normalized size = 0.70

$$-\frac{2\,(3\,(dx+c)b-bc+ad)}{3\,(dx+c)^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*(d*x + c)*b - b*c + a*d)/((d*x + c)^(3/2)*d^2)
```

**mupad** [B] time = 0.25, size = 29, normalized size = 0.72

$$-\frac{2\,a\,d-2\,b\,c+6\,b\,\left(c+d\,x\right)}{3\,d^2\,{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(5/2),x)
```

```
[Out] -(2*a*d - 2*b*c + 6*b*(c + d*x))/(3*d^2*(c + d*x)^(3/2))
```

**sympy** [A] time = 1.12, size = 124, normalized size = 3.10

$$\begin{cases} -\frac{2ad}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}}-\frac{4bc}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}}-\frac{6bdx}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}} & \text{for } d\neq 0 \\ \frac{ax+\frac{bx^2}{2}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a*d/(3*c*d**2*sqrt(c + d*x) + 3*d**3*x*sqrt(c + d*x)) - 4*b*c
/(3*c*d**2*sqrt(c + d*x) + 3*d**3*x*sqrt(c + d*x)) - 6*b*d*x/(3*c*d**2*sqrt
(c + d*x) + 3*d**3*x*sqrt(c + d*x)), Ne(d, 0)), ((a*x + b*x**2/2)/c**(5/2),
 True))
```

## 3.1333 $\int \frac{1}{(c+dx)^{5/2}} dx$

**Optimal**. Leaf size=16

$$-\frac{2}{3d(c+dx)^{3/2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-5/2),x]
```

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^{5/2}} dx = -\frac{2}{3d(c+dx)^{3/2}}$$

**Mathematica** [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-5/2),x]
```

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(-5/2),x]
[Out] -2/(3*d*(c + d*x)^(3/2))
```

**fricas [B]** time = 1.32, size = 31, normalized size = 1.94

$$-\frac{2\sqrt{dx+c}}{3\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="fricas")
[Out] -2/3*sqrt(d*x + c)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

**giac [A]** time = 0.96, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="giac")
[Out] -2/3/((d*x + c)^(3/2)*d)
```

**maple [A]** time = 0.00, size = 13, normalized size = 0.81

$$-\frac{2}{3\,(dx+c)^{\frac{3}{2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(5/2),x)
[Out] -2/3/d/(d*x+c)^(3/2)
```

**maxima [A]** time = 1.36, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((d*x + c)^(3/2)*d)
```

**mupad [B]** time = 0.03, size = 12, normalized size = 0.75

$$-\frac{2}{3\,d\,{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(5/2),x)
[Out] -2/(3*d*(c + d*x)^(3/2))
```

**sympy [A]** time = 0.07, size = 14, normalized size = 0.88

$$-\frac{2}{3d\left(c+dx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(5/2),x)
[Out] -2/(3*d*(c + d*x)**(3/2))
```

## 3.1334 $\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=93

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{2b}{\sqrt{c+dx}\,(bc-ad)^2}+\frac{2}{3(c+dx)^{3/2}(bc-ad)}$$

**Rubi [A]** time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{2b}{\sqrt{c+dx}\,(bc-ad)^2}+\frac{2}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(5/2)),x]
```

```
[Out] 2/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (2*b)/((b*c - a*d)^2*Sqrt[c + d*x]) - (
2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/
2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx &= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{b\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{bc-ad}\\ &= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{2b}{(bc-ad)^2\sqrt{c+dx}}+\frac{b^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{(bc-ad)^2}\\ &= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{2b}{(bc-ad)^2\sqrt{c+dx}}+\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d(bc-ad)^2}\\ &= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{2b}{(bc-ad)^2\sqrt{c+dx}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 48, normalized size = 0.52

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b(c+dx)}{bc-ad}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(5/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, (b*(c + d*x))/(b*c - a*d)])/(3*(b*c - a
*d)*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 97, normalized size = 1.04

$$\frac{2(-ad+3b(c+dx)+bc)}{3(c+dx)^{3/2}(bc-ad)^2}-\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^(5/2)),x]
```

```
[Out] (2*(b*c - a*d + 3*b*(c + d*x)))/(3*(b*c - a*d)^2*(c + d*x)^(3/2)) - (2*b^(3
/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(-(b*c)
 + a*d)^(5/2)
```

**fricas [B]** time = 1.44, size = 398, normalized size = 4.28

$$\left[\frac{3\left(bd^{2}x^{2}+2\,bcdx+bc^{2}\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad-2\,(bc-ad)\sqrt{dx+c}\,\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)+2\,(3\,bdx+4\,bc-ad)\sqrt{dx+c}}{3\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}+\left(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4}\right)x^{2}+2\left(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3}\right)x\right)},-\frac{2\left(3\left(bd^{2}x^{2}+2\,bcdx+bc^{2}\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\,\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-(3\,bdx+4\,bc-ad)\sqrt{dx+c}\right)}{3\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}+\left(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4}\right)x^{2}+2\left(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3}\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*
b*c - a*d - 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2
*(3*b*d*x + 4*b*c - a*d)*sqrt(d*x + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^
2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^
2 + a^2*c*d^3)*x), -2/3*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-b/(b*c - a
*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c))
- (3*b*d*x + 4*b*c - a*d)*sqrt(d*x + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d
^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d
^2 + a^2*c*d^3)*x)]
```

**giac [A]** time = 1.16, size = 113, normalized size = 1.22

$$\frac{2\,b^{2}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{-b^{2}c+abd}}+\frac{2\,(3\,(dx+c)b+bc-ad)}{3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 2/3*(3*(d*x + c)*b + b*c - a*d)/((b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(d*x + c)^(3/2))
```

**maple [A]** time = 0.01, size = 90, normalized size = 0.97

$$\frac{2b^{2}\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)^{2}\sqrt{(ad-bc)\,b}}+\frac{2b}{(ad-bc)^{2}\sqrt{dx+c}}-\frac{2}{3\,(ad-bc)\,(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/(a*d-b*c)/(d*x+c)^(3/2)+2*b/(a*d-b*c)^2/(d*x+c)^(1/2)+2*b^2/(a*d-b*c)^
2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [**B**] time = 0.33, size = 100, normalized size = 1.08

$$\frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^{5/2}}\right)}{{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{2}{3\,\left(a\,d-b\,c\right)}-\frac{2\,b\,\left(c+d\,x\right)}{{\left(a\,d-b\,c\right)}^2}}{{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(5/2)),x)
```

```
[Out] (2*b^(3/2)*atan((b^(1/2)*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(
a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2) - (2/(3*(a*d - b*c)) - (2*b*(c + d*x))
/(a*d - b*c)^2)/(c + d*x)^(3/2)
```

**sympy** [**A**] time = 13.58, size = 83, normalized size = 0.89

$$\frac{2b}{\sqrt{c+dx}\,(ad-bc)^2}+\frac{2b\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)^2}-\frac{2}{3\,(c+dx)^{\frac{3}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] 2*b/(sqrt(c + d*x)*(a*d - b*c)**2) + 2*b*atan(sqrt(c + d*x)/sqrt((a*d - b*c
)/b))/(sqrt((a*d - b*c)/b)*(a*d - b*c)**2) - 2/(3*(c + d*x)**(3/2)*(a*d - b
*c))
```

## 3.1335 $\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=124

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}-\frac{5bd}{\sqrt{c+dx}\,(bc-ad)^3}-\frac{1}{(a+bx)(c+dx)^{3/2}(bc-ad)}-\frac{5d}{3(c+dx)^{3/2}(bc-ad)^2}$$

**Rubi** [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}-\frac{5bd}{\sqrt{c+dx}\,(bc-ad)^3}-\frac{1}{(a+bx)(c+dx)^{3/2}(bc-ad)}-\frac{5d}{3(c+dx)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

```
[Out] (-5*d)/(3*(b*c - a*d)^2*(c + d*x)^(3/2)) - 1/((b*c - a*d)*(a + b*x)*(c + d*
x)^(3/2)) - (5*b*d)/((b*c - a*d)^3*Sqrt[c + d*x]) + (5*b^(3/2)*d*ArcTanh[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx = -\frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{(5d)\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{(5bd)\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{2(bc-ad)^2}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}} - \frac{(5b^2d}{}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}} - \frac{(5b^2)}{}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}} + \frac{5b^{3/2}}{}$$

**Mathematica [C]** time = 0.01, size = 50, normalized size = 0.40

$$-\frac{2d\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(c+dx)^{3/2}(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*d*Hypergeometric2F1[-3/2, 2, -1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/(3
*(-(b*c) + a*d)^2*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.36, size = 157, normalized size = 1.27

$$-\frac{d\left(2a^2d^2-10abd(c+dx)-4abcd+2b^2c^2-15b^2(c+dx)^2+10b^2c(c+dx)\right)}{3(c+dx)^{3/2}(bc-ad)^3(-ad-b(c+dx)+bc)} - \frac{5b^{3/2}d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

```
[Out] -1/3*(d*(2*b^2*c^2 - 4*a*b*c*d + 2*a^2*d^2 + 10*b^2*c*(c + d*x) - 10*a*b*d*
(c + d*x) - 15*b^2*(c + d*x)^2))/((b*c - a*d)^3*(c + d*x)^(3/2)*(b*c - a*d
```

```
- b*(c + d*x))) - (5*b^(3/2)*d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c +
d*x])/(b*c - a*d)]])/(-(b*c) + a*d)^(7/2)
```

**fricas [B]** time = 1.42, size = 782, normalized size = 6.31

$$\left[-\frac{15\left(b^2d^3x^3+abc^2d+\left(2b^2cd^2+abd^3\right)x^2+\left(b^2c^2d+2abcd^2\right)x\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2bc-ad-2(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)+2\left(15b^2d^2x^2+3b^2c^2+14abcd-2a^2d^2+10\left(2b^2cd+abd^2\right)x\right)\sqrt{dx+c}}{6\left(ab^3c^5-3a^2b^2c^4d+3a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3ab^3c^2d^3+3a^2b^2cd^4-a^3bd^5\right)x^3+\left(2b^4c^4d-5ab^3c^3d^2+3a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^2+\left(b^4c^5-ab^3c^4d-3a^2b^2c^3d^2+5a^3bc^2d^3-2a^4cd^4\right)x\right)},\ \frac{15\left(b^2d^3x^3+abc^2d+\left(2b^2cd^2+abd^3\right)x^2+\left(b^2c^2d+2abcd^2\right)x\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-\left(15b^2d^2x^2+3b^2c^2+14abcd-2a^2d^2+10\left(2b^2cd+abd^2\right)x\right)\sqrt{dx+c}}{3\left(ab^3c^5-3a^2b^2c^4d+3a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3ab^3c^2d^3+3a^2b^2cd^4-a^3bd^5\right)x^3+\left(2b^4c^4d-5ab^3c^3d^2+3a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^2+\left(b^4c^5-ab^3c^4d-3a^2b^2c^3d^2+5a^3bc^2d^3-2a^4cd^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(b^2*d^3*x^3 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^2 + (b^2*c^2
*d + 2*a*b*c*d^2)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c
- a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2*(15*b^2*d^2*x^2 +
3*b^2*c^2 + 14*a*b*c*d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x)*sqrt(d*x +
 c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^
3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d -
 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^
5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x), 1/
3*(15*(b^2*d^3*x^3 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^2 + (b^2*c^2*d +
 2*a*b*c*d^2)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqr
t(-b/(b*c - a*d))/(b*d*x + b*c)) - (15*b^2*d^2*x^2 + 3*b^2*c^2 + 14*a*b*c*d
 - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x)*sqrt(d*x + c))/(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^
3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a
^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^
2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)]
```

**giac [B]** time = 1.11, size = 216, normalized size = 1.74

$$-\frac{5\,b^2d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}-\frac{\sqrt{dx+c}\,b^2d}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left((dx+c)b-bc+ad\right)}-\frac{2\left(6\,(dx+c)bd+bcd-ad^2\right)}{3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -5*b^2*d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - sqrt(d*x + c)*b^2*d
/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)*b - b*c +
a*d)) - 2/3*(6*(d*x + c)*b*d + b*c*d - a*d^2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(3/2))
```

**maple [A]** time = 0.02, size = 125, normalized size = 1.01

$$\frac{5b^2d\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)^3\sqrt{(ad-bc)\,b}}+\frac{\sqrt{dx+c}\;b^2d}{(ad-bc)^3\,(bdx+ad)}+\frac{4bd}{(ad-bc)^3\sqrt{dx+c}}-\frac{2d}{3\,(ad-bc)^2\,(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*d/(a*d-b*c)^2/(d*x+c)^(3/2)+4*d/(a*d-b*c)^3*b/(d*x+c)^(1/2)+d*b^2/(a*d
-b*c)^3*(d*x+c)^(1/2)/(b*d*x+a*d)+5*d*b^2/(a*d-b*c)^3/((a*d-b*c)*b)^(1/2)*a
rctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 0.38, size = 161, normalized size = 1.30

$$\frac{\frac{10\,b\,d\,(c+d\,x)}{3\,(a\,d-b\,c)^2}-\frac{2\,d}{3\,(a\,d-b\,c)}+\frac{5\,b^2\,d\,(c+d\,x)^2}{(a\,d-b\,c)^3}}{b\,(c+d\,x)^{5/2}+(a\,d-b\,c)\,(c+d\,x)^{3/2}}+\frac{5\,b^{3/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{(a\,d-b\,c)^{7/2}}\right)}{(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(5/2)),x)
```

```
[Out] ((10*b*d*(c + d*x))/(3*(a*d - b*c)^2) - (2*d)/(3*(a*d - b*c)) + (5*b^2*d*(c
 + d*x)^2)/(a*d - b*c)^3)/(b*(c + d*x)^(5/2) + (a*d - b*c)*(c + d*x)^(3/2))
 + (5*b^(3/2)*d*atan((b^(1/2)*(c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2)))/(a*d - b*c)^(7/2)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2\,(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**2*(c + d*x)**(5/2)), x)
```

## 3.1336 $\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=167

$$-\frac{35b^{3/2}d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{9/2}}+\frac{35bd^2}{4\sqrt{c+dx}\,(bc-ad)^4}+\frac{35d^2}{12(c+dx)^{3/2}(bc-ad)^3}+\frac{7d}{4(a+bx)(c+dx)^{3/2}(bc-ad)^2}-\frac{}{2(}$$

**Rubi** [A] time = 0.06, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{35b^{3/2}d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{9/2}}+\frac{35bd^2}{4\sqrt{c+dx}\,(bc-ad)^4}+\frac{35d^2}{12(c+dx)^{3/2}(bc-ad)^3}+\frac{7d}{4(a+bx)(c+dx)^{3/2}(bc-ad)^2}-\frac{1}{2(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] (35*d^2)/(12*(b*c - a*d)^3*(c + d*x)^(3/2)) - 1/(2*(b*c - a*d)*(a + b*x)^2*
(c + d*x)^(3/2)) + (7*d)/(4*(b*c - a*d)^2*(a + b*x)*(c + d*x)^(3/2)) + (35*
b*d^2)/(4*(b*c - a*d)^4*Sqrt[c + d*x]) - (35*b^(3/2)*d^2*ArcTanh[(Sqrt[b]*S
qrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx = -\frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} - \frac{(7d)\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx}{4(bc-ad)}$$

$$= -\frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+dx)^{3/2}} + \frac{\left(35d^2\right)\int \frac{1}{(a+bx)(c+}}{8(bc-ad)}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+d}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+d}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+d}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+d}$$

**Mathematica [C]** time = 0.02, size = 52, normalized size = 0.31

$$-\frac{2d^2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(c+dx)^{3/2}(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*d^2*Hypergeometric2F1[-3/2, 3, -1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/
(3*(-(b*c) + a*d)^3*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.60, size = 223, normalized size = 1.34

$$-\frac{d^2\left(8a^3d^3-56a^2bd^2(c+dx)-24a^2bcd^2+24ab^2c^2d-175ab^2d(c+dx)^2+112ab^2cd(c+dx)-8b^3c^3-56b^3c^2(c+dx)-105b^3(c+dx)^3+175b^3c(c+dx)^2\right)}{12(c+dx)^{3/2}(bc-ad)^4(-ad-b(c+dx)+bc)^2}-\frac{35b^{3/2}d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4(ad-bc)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] -1/12*(d^2*(-8*b^3*c^3 + 24*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 8*a^3*d^3 - 56*b
^3*c^2*(c + d*x) + 112*a*b^2*c*d*(c + d*x) - 56*a^2*b*d^2*(c + d*x) + 175*b
^3*c*(c + d*x)^2 - 175*a*b^2*d*(c + d*x)^2 - 105*b^3*(c + d*x)^3))/((b*c -
a*d)^4*(c + d*x)^(3/2)*(b*c - a*d - b*(c + d*x))^2) - (35*b^(3/2)*d^2*ArcTa
n[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(4*(-(b*c) + a*d
)^(9/2))
```

**fricas [B]** time = 1.36, size = 1226, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*d^4*x^4 + a^2*b*c^2*d^2 + 2*(b^3*c*d^3 + a*b^2*d^4)*x^3 + (
b^3*c^2*d^2 + 4*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(a*b^2*c^2*d^2 + a^2*b*c*d
^3)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c - a*d)*sqrt(d*
x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2*(105*b^3*d^3*x^3 - 6*b^3*c^3 + 3
9*a*b^2*c^2*d + 80*a^2*b*c*d^2 - 8*a^3*d^3 + 35*(4*b^3*c*d^2 + 5*a*b^2*d^3)
*x^2 + 7*(3*b^3*c^2*d + 34*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^
2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2
*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5
 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 +
2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b^6*c^6 - 9*a^2*b^4
*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*x^2 + 2*(a*b^5
*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^
2*d^4 + a^6*c*d^5)*x), -1/12*(105*(b^3*d^4*x^4 + a^2*b*c^2*d^2 + 2*(b^3*c*d
^3 + a*b^2*d^4)*x^3 + (b^3*c^2*d^2 + 4*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(a*
b^2*c^2*d^2 + a^2*b*c*d^3)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt
(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - (105*b^3*d^3*x^3 - 6*b^3*c^
3 + 39*a*b^2*c^2*d + 80*a^2*b*c*d^2 - 8*a^3*d^3 + 35*(4*b^3*c*d^2 + 5*a*b^2
*d^3)*x^2 + 7*(3*b^3*c^2*d + 34*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x + c)
)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^
6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*
c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d
^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b^6*c^6 - 9*a^
2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*x^2 + 2*(
a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5
*b*c^2*d^4 + a^6*c*d^5)*x)]
```

**giac [B]** time = 1.20, size = 298, normalized size = 1.78

$$\frac{35\, b^{2} d^{2} \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{-b^{2}c+abd}}\right)}{4\, \left(b^{4} c^{4} - 4\, a b^{3} c^{3} d + 6\, a^{2} b^{2} c^{2} d^{2} - 4\, a^{3} b c d^{3} + a^{4} d^{4}\right) \sqrt{-b^{2} c + a b d}} + \frac{2\, \left(9\, (dx + c) b d^{2} + b c d^{2} - a d^{3}\right)}{3\, \left(b^{4} c^{4} - 4\, a b^{3} c^{3} d + 6\, a^{2} b^{2} c^{2} d^{2} - 4\, a^{3} b c d^{3} + a^{4} d^{4}\right) (dx + c)^{\frac{3}{2}}} + \frac{11\, (dx + c)^{\frac{3}{2}} b^{3} d^{2} - 13\, \sqrt{dx + c}\, b^{3} c d^{2} + 13\, \sqrt{dx + c}\, a b^{2} d^{3}}{4\, \left(b^{4} c^{4} - 4\, a b^{3} c^{3} d + 6\, a^{2} b^{2} c^{2} d^{2} - 4\, a^{3} b c d^{3} + a^{4} d^{4}\right) \left((dx + c) b - b c + a d\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d
)) + 2/3*(9*(d*x + c)*b*d^2 + b*c*d^2 - a*d^3)/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*x + c)^(3/2)) + 1/4*(11*(d*
x + c)^(3/2)*b^3*d^2 - 13*sqrt(d*x + c)*b^3*c*d^2 + 13*sqrt(d*x + c)*a*b^2*
d^3)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^
4)*((d*x + c)*b - b*c + a*d)^2)
```

**maple [A]** time = 0.02, size = 206, normalized size = 1.23

$$\frac{13\sqrt{dx+c}\,a\,b^2d^3}{4\,(ad-bc)^4\,(bdx+ad)^2}-\frac{13\sqrt{dx+c}\,b^3c\,d^2}{4\,(ad-bc)^4\,(bdx+ad)^2}+\frac{11\,(dx+c)^{\frac{3}{2}}\,b^3d^2}{4\,(ad-bc)^4\,(bdx+ad)^2}+\frac{35b^2d^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{4\,(ad-bc)^4\,\sqrt{(ad-bc)\,b}}+\frac{6b\,d^2}{(ad-bc)^4\,\sqrt{dx+c}}-\frac{2d^2}{3\,(ad-bc)^3\,(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*d^2/(a*d-b*c)^3/(d*x+c)^(3/2)+6*d^2/(a*d-b*c)^4*b/(d*x+c)^(1/2)+11/4*d
^2/(a*d-b*c)^4*b^3/(b*d*x+a*d)^2*(d*x+c)^(3/2)+13/4*d^3/(a*d-b*c)^4*b^2/(b*
d*x+a*d)^2*(d*x+c)^(1/2)*a-13/4*d^2/(a*d-b*c)^4*b^3/(b*d*x+a*d)^2*(d*x+c)^(
1/2)*c+35/4*d^2/(a*d-b*c)^4*b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((
a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.28, size = 243, normalized size = 1.46

$$\frac{\frac{175\,b^2\,d^2\,(c+d\,x)^2}{12\,(a\,d-b\,c)^3}-\frac{2\,d^2}{3\,(a\,d-b\,c)}+\frac{35\,b^3\,d^2\,(c+d\,x)^3}{4\,(a\,d-b\,c)^4}+\frac{14\,b\,d^2\,(c+d\,x)}{3\,(a\,d-b\,c)^2}}{b^2\,(c+d\,x)^{7/2}-\left(2\,b^2\,c-2\,a\,b\,d\right)\,(c+d\,x)^{5/2}+(c+d\,x)^{3/2}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{35\,b^{3/2}\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{(a\,d-b\,c)^{9/2}}\right)}{4\,(a\,d-b\,c)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(5/2)),x)
```

```
[Out] ((175*b^2*d^2*(c + d*x)^2)/(12*(a*d - b*c)^3) - (2*d^2)/(3*(a*d - b*c)) + (
35*b^3*d^2*(c + d*x)^3)/(4*(a*d - b*c)^4) + (14*b*d^2*(c + d*x))/(3*(a*d -
b*c)^2))/(b^2*(c + d*x)^(7/2) - (2*b^2*c - 2*a*b*d)*(c + d*x)^(5/2) + (c +
d*x)^(3/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (35*b^(3/2)*d^2*atan((b^(1/2)
*(c + d*x)^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/(a*d - b*c)^(9/2)))/(4*(a*d - b*c)^(9/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

## 3.1337 $\int \frac{1}{(a+bx)^4(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=200

$$\frac{105b^{3/2}d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{11/2}}-\frac{105bd^3}{8\sqrt{c+dx}\,(bc-ad)^5}-\frac{35d^3}{8(c+dx)^{3/2}(bc-ad)^4}-\frac{21d^2}{8(a+bx)(c+dx)^{3/2}(bc-ad)^3}+\frac{}{4(a+}$$

**Rubi [A]** time = 0.13, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{105b^{3/2}d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{11/2}}-\frac{105bd^3}{8\sqrt{c+dx}\,(bc-ad)^5}-\frac{35d^3}{8(c+dx)^{3/2}(bc-ad)^4}-\frac{21d^2}{8(a+bx)(c+dx)^{3/2}(bc-ad)^3}+\frac{3d}{4(a+bx)^2(c+dx)^{3/2}(bc-ad)^2}-\frac{1}{3(a+bx)^3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(c + d*x)^(5/2)),x]
```

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*(c + d*x)^(3/2)) - 1/(3*(b*c - a*d)*(a + b*x)^3*
(c + d*x)^(3/2)) + (3*d)/(4*(b*c - a*d)^2*(a + b*x)^2*(c + d*x)^(3/2)) - (2
1*d^2)/(8*(b*c - a*d)^3*(a + b*x)*(c + d*x)^(3/2)) - (105*b*d^3)/(8*(b*c -
a*d)^5*Sqrt[c + d*x]) + (105*b^(3/2)*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/(8*(b*c - a*d)^(11/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4(c+dx)^{5/2}}\,dx = -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} - \frac{(3d)\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}} + \frac{\left(21d^2\right)\int \frac{}{(a+bx)}}{8(bc-}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}} - \frac{21}{8(bc-ad)^3(a}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c}$$

**Mathematica [C]** time = 0.02, size = 52, normalized size = 0.26

$$-\frac{2d^3\,{}_2F_1\left(-\frac{3}{2},4;-\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(c+dx)^{3/2}(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*d^3*Hypergeometric2F1[-3/2, 4, -1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/
(3*(-(b*c) + a*d)^4*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.87, size = 304, normalized size = 1.52

$$-\frac{d^3\left(16a^4d^4-144a^3bd^3(c+dx)-64a^3bcd^3+96a^2b^2c^2d^2-693a^2b^2d^2(c+dx)^2+432a^2b^2cd^2(c+dx)-64ab^3c^3d-432ab^3c^2d(c+dx)-840ab^3d(c+dx)^3+1386ab^3cd(c+dx)^2+16b^4c^4+144b^4c^3(c+dx)-693b^4c^2(c+dx)^2-315b^4(c+dx)^4+840b^4c(c+dx)^3\right)}{24(c+dx)^{3/2}(bc-ad)^5(-ad-b(c+dx)+bc)^3}-\frac{105b^{3/2}d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{8(ad-bc)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^4*(c + d*x)^(5/2)),x]
```

```
[Out] -1/24*(d^3*(16*b^4*c^4 - 64*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 64*a^3*b*c*d
^3 + 16*a^4*d^4 + 144*b^4*c^3*(c + d*x) - 432*a*b^3*c^2*d*(c + d*x) + 432*a
^2*b^2*c*d^2*(c + d*x) - 144*a^3*b*d^3*(c + d*x) - 693*b^4*c^2*(c + d*x)^2
+ 1386*a*b^3*c*d*(c + d*x)^2 - 693*a^2*b^2*d^2*(c + d*x)^2 + 840*b^4*c*(c +
 d*x)^3 - 840*a*b^3*d*(c + d*x)^3 - 315*b^4*(c + d*x)^4))/((b*c - a*d)^5*(c
 + d*x)^(3/2)*(b*c - a*d - b*(c + d*x))^3) - (105*b^(3/2)*d^3*ArcTan[(Sqrt[
b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(8*(-(b*c) + a*d)^(11/2)
)
```

**fricas [B]** time = 1.43, size = 1840, normalized size = 9.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(315*(b^4*d^5*x^5 + a^3*b*c^2*d^3 + (2*b^4*c*d^4 + 3*a*b^3*d^5)*x^4
+ (b^4*c^2*d^3 + 6*a*b^3*c*d^4 + 3*a^2*b^2*d^5)*x^3 + (3*a*b^3*c^2*d^3 + 6*
a^2*b^2*c*d^4 + a^3*b*d^5)*x^2 + (3*a^2*b^2*c^2*d^3 + 2*a^3*b*c*d^4)*x)*sqr
t(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c - a*d)*sqrt(d*x + c)*sqr
t(b/(b*c - a*d)))/(b*x + a)) + 2*(315*b^4*d^4*x^4 + 8*b^4*c^4 - 50*a*b^3*c^
3*d + 165*a^2*b^2*c^2*d^2 + 208*a^3*b*c*d^3 - 16*a^4*d^4 + 420*(b^4*c*d^3 +
 2*a*b^3*d^4)*x^3 + 63*(b^4*c^2*d^2 + 18*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2
- 18*(b^4*c^3*d - 10*a*b^3*c^2*d^2 - 53*a^2*b^2*c*d^3 - 8*a^3*b*d^4)*x)*sqr
t(d*x + c))/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^
2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5*a*b^7*c^4*d^3
+ 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*
x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d
^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*x^4 + (b^8*c^7
+ a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^
4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (3*a*b^7*c^7 - 9
*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4
+ 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b^6*c^7 - 13*a^3
*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 +
7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x), 1/24*(315*(b^4*d^5*x^5 + a^3*b*c^2*d^3 +
 (2*b^4*c*d^4 + 3*a*b^3*d^5)*x^4 + (b^4*c^2*d^3 + 6*a*b^3*c*d^4 + 3*a^2*b^2
*d^5)*x^3 + (3*a*b^3*c^2*d^3 + 6*a^2*b^2*c*d^4 + a^3*b*d^5)*x^2 + (3*a^2*b^
2*c^2*d^3 + 2*a^3*b*c*d^4)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt
(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - (315*b^4*d^4*x^4 + 8*b^4*c^
4 - 50*a*b^3*c^3*d + 165*a^2*b^2*c^2*d^2 + 208*a^3*b*c*d^3 - 16*a^4*d^4 + 4
20*(b^4*c*d^3 + 2*a*b^3*d^4)*x^3 + 63*(b^4*c^2*d^2 + 18*a*b^3*c*d^3 + 11*a^
2*b^2*d^4)*x^2 - 18*(b^4*c^3*d - 10*a*b^3*c^2*d^2 - 53*a^2*b^2*c*d^3 - 8*a^
3*b*d^4)*x)*sqrt(d*x + c))/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*
d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5
```

```
*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6
- a^5*b^3*d^7)*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 1
0*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*
x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25
*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (
3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a
^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b
^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6
*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x)]
```

**giac [B]** time = 1.20, size = 432, normalized size = 2.16

$$-\frac{105\, b^{2} d^{3} \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{-b^{2} c+abd}}\right)}{8\left(b^{5} c^{5} - 5\, a b^{4} c^{4} d + 10\, a^{2} b^{3} c^{3} d^{2} - 10\, a^{3} b^{2} c^{2} d^{3} + 5\, a^{4} b c d^{4} - a^{5} d^{5}\right)\sqrt{-b^{2} c + abd}} - \frac{315\,(dx+c)^{4} b^{4} d^{3} - 840\,(dx+c)^{3} b^{4} c d^{3} + 693\,(dx+c)^{2} b^{4} c^{2} d^{3} - 144\,(dx+c) b^{4} c^{3} d^{3} - 16\, b^{4} c^{4} d^{3} + 840\,(dx+c)^{3} a b^{3} d^{4} - 1386\,(dx+c)^{2} a b^{3} c d^{4} + 432\,(dx+c) a b^{3} c^{2} d^{4} + 64\, a b^{3} c^{3} d^{4} + 693\,(dx+c)^{2} a^{2} b^{2} d^{5} - 432\,(dx+c) a^{2} b^{2} c d^{5} - 96\, a^{2} b^{2} c^{2} d^{5} + 144\,(dx+c) a^{3} b d^{6} + 64\, a^{3} b c d^{6} - 16\, a^{4} d^{7}}{24\left(b^{5} c^{5} - 5\, a b^{4} c^{4} d + 10\, a^{2} b^{3} c^{3} d^{2} - 10\, a^{3} b^{2} c^{2} d^{3} + 5\, a^{4} b c d^{4} - a^{5} d^{5}\right)\left((dx+c)^{\frac{3}{2}} b - \sqrt{dx+c}\, bc + \sqrt{dx+c}\, ad\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -105/8*b^2*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^5 - 5*a
*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*
d^5)*sqrt(-b^2*c + a*b*d)) - 1/24*(315*(d*x + c)^4*b^4*d^3 - 840*(d*x + c)^
3*b^4*c*d^3 + 693*(d*x + c)^2*b^4*c^2*d^3 - 144*(d*x + c)*b^4*c^3*d^3 - 16*
b^4*c^4*d^3 + 840*(d*x + c)^3*a*b^3*d^4 - 1386*(d*x + c)^2*a*b^3*c*d^4 + 43
2*(d*x + c)*a*b^3*c^2*d^4 + 64*a*b^3*c^3*d^4 + 693*(d*x + c)^2*a^2*b^2*d^5
- 432*(d*x + c)*a^2*b^2*c*d^5 - 96*a^2*b^2*c^2*d^5 + 144*(d*x + c)*a^3*b*d^
6 + 64*a^3*b*c*d^6 - 16*a^4*d^7)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3
*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*((d*x + c)^(3/2)*b - s
qrt(d*x + c)*b*c + sqrt(d*x + c)*a*d)^3)
```

**maple [A]** time = 0.02, size = 319, normalized size = 1.60

$$\frac{55\sqrt{dx+c}\; a^{2} b^{2} d^{5}}{8\,(ad-bc)^{5}\,(bdx+ad)^{3}} - \frac{55\sqrt{dx+c}\; a\, b^{3} c\, d^{4}}{4\,(ad-bc)^{5}\,(bdx+ad)^{3}} + \frac{55\sqrt{dx+c}\; b^{4} c^{2} d^{3}}{8\,(ad-bc)^{5}\,(bdx+ad)^{3}} + \frac{35\,(dx+c)^{\frac{3}{2}}\, a\, b^{3} d^{4}}{3\,(ad-bc)^{5}\,(bdx+ad)^{3}} - \frac{35\,(dx+c)^{\frac{3}{2}}\, b^{4} c\, d^{3}}{3\,(ad-bc)^{5}\,(bdx+ad)^{3}} + \frac{41\,(dx+c)^{\frac{5}{2}}\, b^{4} d^{3}}{8\,(ad-bc)^{5}\,(bdx+ad)^{3}} + \frac{105 b^{2} d^{3} \arctan\left(\frac{\sqrt{dx+c}\; b}{\sqrt{(ad-bc)b}}\right)}{8\,(ad-bc)^{5}\sqrt{(ad-bc)\,b}} + \frac{8b\, d^{3}}{(ad-bc)^{5}\sqrt{dx+c}} - \frac{2d^{3}}{3\,(ad-bc)^{4}\,(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*d^3/(a*d-b*c)^4/(d*x+c)^(3/2)+8*d^3/(a*d-b*c)^5*b/(d*x+c)^(1/2)+41/8*d
^3/(a*d-b*c)^5*b^4/(b*d*x+a*d)^3*(d*x+c)^(5/2)+35/3*d^4/(a*d-b*c)^5*b^3/(b*
d*x+a*d)^3*(d*x+c)^(3/2)*a-35/3*d^3/(a*d-b*c)^5*b^4/(b*d*x+a*d)^3*(d*x+c)^(
3/2)*c+55/8*d^5/(a*d-b*c)^5*b^2/(b*d*x+a*d)^3*(d*x+c)^(1/2)*a^2-55/4*d^4/(a
*d-b*c)^5*b^3/(b*d*x+a*d)^3*(d*x+c)^(1/2)*a*c+55/8*d^3/(a*d-b*c)^5*b^4/(b*d
*x+a*d)^3*(d*x+c)^(1/2)*c^2+105/8*d^3/(a*d-b*c)^5*b^2/((a*d-b*c)*b)^(1/2)*a
rctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.64, size = 334, normalized size = 1.67

$$\frac{\frac{231\,b^2\,d^3\,{\left(c+d\,x\right)}^2}{8\,{\left(a\,d-b\,c\right)}^3}-\frac{2\,d^3}{3\,\left(a\,d-b\,c\right)}+\frac{35\,b^3\,d^3\,{\left(c+d\,x\right)}^3}{{\left(a\,d-b\,c\right)}^4}+\frac{105\,b^4\,d^3\,{\left(c+d\,x\right)}^4}{8\,{\left(a\,d-b\,c\right)}^5}+\frac{6\,b\,d^3\,\left(c+d\,x\right)}{{\left(a\,d-b\,c\right)}^2}}{{\left(c+d\,x\right)}^{3/2}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)+b^3\,{\left(c+d\,x\right)}^{9/2}-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{7/2}+{\left(c+d\,x\right)}^{5/2}\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}+\frac{105\,b^{3/2}\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{{\left(a\,d-b\,c\right)}^{11/2}}\right)}{8\,{\left(a\,d-b\,c\right)}^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(5/2)),x)
```

```
[Out] ((231*b^2*d^3*(c + d*x)^2)/(8*(a*d - b*c)^3) - (2*d^3)/(3*(a*d - b*c)) + (3
5*b^3*d^3*(c + d*x)^3)/(a*d - b*c)^4 + (105*b^4*d^3*(c + d*x)^4)/(8*(a*d -
b*c)^5) + (6*b*d^3*(c + d*x))/(a*d - b*c)^2)/((c + d*x)^(3/2)*(a^3*d^3 - b^
3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + b^3*(c + d*x)^(9/2) - (3*b^3*c - 3
*a*b^2*d)*(c + d*x)^(7/2) + (c + d*x)^(5/2)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*
b^2*c*d)) + (105*b^(3/2)*d^3*atan((b^(1/2)*(c + d*x)^(1/2)*(a^5*d^5 - b^5*c
^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^
4))/(a*d - b*c)^(11/2)))/(8*(a*d - b*c)^(11/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

# 3.1338 $\int (a+bx)^5(ac+bcx)^{3/2}\,dx$

**Optimal**. Leaf size=22

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(15/2))/(15*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^{3/2}\,dx &= \frac{\int (ac+bcx)^{13/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{15/2}}{15bc^6}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6(c(a+bx))^{3/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^6*(c*(a + b*x))^(3/2))/(15*b)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(15/2))/(15*b*c^6)
```

**fricas [B]** time = 1.14, size = 95, normalized size = 4.32

$$\frac{2\left(b^{7}cx^{7}+7\,ab^{6}cx^{6}+21\,a^{2}b^{5}cx^{5}+35\,a^{3}b^{4}cx^{4}+35\,a^{4}b^{3}cx^{3}+21\,a^{5}b^{2}cx^{2}+7\,a^{6}bcx+a^{7}c\right)\sqrt{bcx+ac}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(b^7*c*x^7 + 7*a*b^6*c*x^6 + 21*a^2*b^5*c*x^5 + 35*a^3*b^4*c*x^4 + 35*
a^4*b^3*c*x^3 + 21*a^5*b^2*c*x^2 + 7*a^6*b*c*x + a^7*c)*sqrt(b*c*x + a*c)/b
```

**giac [B]** time = 1.26, size = 637, normalized size = 28.95

$$\frac{2\left(6435\,\sqrt{bcx+ac}a^{7}c-15015\left(3\,\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}\right)a^{6}+\frac{9009\left(15\,\sqrt{bcx+ac}a^{2}c^{2}-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^{5}}{c}-\frac{6435\left(35\,\sqrt{bcx+ac}a^{3}c^{3}-35\,(bcx+ac)^{\frac{3}{2}}a^{2}c^{2}+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}\right)a^{4}}{c^{2}}+\frac{715\left(315\,\sqrt{bcx+ac}a^{4}c^{4}-420\,(bcx+ac)^{\frac{3}{2}}a^{3}c^{3}+378\,(bcx+ac)^{\frac{5}{2}}a^{2}c^{2}-180\,(bcx+ac)^{\frac{7}{2}}ac+35\,(bcx+ac)^{\frac{9}{2}}\right)a^{3}}{c^{3}}-\frac{195\left(693\,\sqrt{bcx+ac}a^{5}c^{5}-1155\,(bcx+ac)^{\frac{3}{2}}a^{4}c^{4}+1386\,(bcx+ac)^{\frac{5}{2}}a^{3}c^{3}-990\,(bcx+ac)^{\frac{7}{2}}a^{2}c^{2}+385\,(bcx+ac)^{\frac{9}{2}}ac-63\,(bcx+ac)^{\frac{11}{2}}\right)a^{2}}{c^{4}}+\frac{15\left(3003\,\sqrt{bcx+ac}a^{6}c^{6}-6006\,(bcx+ac)^{\frac{3}{2}}a^{5}c^{5}+9009\,(bcx+ac)^{\frac{5}{2}}a^{4}c^{4}-8580\,(bcx+ac)^{\frac{7}{2}}a^{3}c^{3}+5005\,(bcx+ac)^{\frac{9}{2}}a^{2}c^{2}-1638\,(bcx+ac)^{\frac{11}{2}}ac+231\,(bcx+ac)^{\frac{13}{2}}\right)a}{c^{5}}-\frac{6435\,\sqrt{bcx+ac}a^{7}c^{7}-15015\,(bcx+ac)^{\frac{3}{2}}a^{6}c^{6}+27027\,(bcx+ac)^{\frac{5}{2}}a^{5}c^{5}-32175\,(bcx+ac)^{\frac{7}{2}}a^{4}c^{4}+25025\,(bcx+ac)^{\frac{9}{2}}a^{3}c^{3}-12285\,(bcx+ac)^{\frac{11}{2}}a^{2}c^{2}+3465\,(bcx+ac)^{\frac{13}{2}}ac-429\,(bcx+ac)^{\frac{15}{2}}}{c^{6}}\right)}{6435\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/6435*(6435*sqrt(b*c*x + a*c)*a^7*c - 15015*(3*sqrt(b*c*x + a*c)*a*c - (b*
c*x + a*c)^(3/2))*a^6 + 9009*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*
c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^5/c - 6435*(35*sqrt(b*c*x + a*c)*a^
3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*
c*x + a*c)^(7/2))*a^4/c^2 + 715*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x
 + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)
^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a^3/c^3 - 195*(693*sqrt(b*c*x + a*c)*a
^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^
3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c
*x + a*c)^(11/2))*a^2/c^4 + 15*(3003*sqrt(b*c*x + a*c)*a^6*c^6 - 6006*(b*c*
x + a*c)^(3/2)*a^5*c^5 + 9009*(b*c*x + a*c)^(5/2)*a^4*c^4 - 8580*(b*c*x + a
```

```
*c)^(7/2)*a^3*c^3 + 5005*(b*c*x + a*c)^(9/2)*a^2*c^2 - 1638*(b*c*x + a*c)^(
11/2)*a*c + 231*(b*c*x + a*c)^(13/2))*a/c^5 - (6435*sqrt(b*c*x + a*c)*a^7*c
^7 - 15015*(b*c*x + a*c)^(3/2)*a^6*c^6 + 27027*(b*c*x + a*c)^(5/2)*a^5*c^5
- 32175*(b*c*x + a*c)^(7/2)*a^4*c^4 + 25025*(b*c*x + a*c)^(9/2)*a^3*c^3 - 1
2285*(b*c*x + a*c)^(11/2)*a^2*c^2 + 3465*(b*c*x + a*c)^(13/2)*a*c - 429*(b*
c*x + a*c)^(15/2))/c^6)/b
```

**maple** [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\,(bx+a)^{6}\,(bcx+ac)^{\frac{3}{2}}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^(3/2),x)
[Out] 2/15*(b*x+a)^6*(b*c*x+a*c)^(3/2)/b
```

**maxima** [A] time = 1.37, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{15}{2}}}{15\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="maxima")
[Out] 2/15*(b*c*x + a*c)^(15/2)/(b*c^6)
```

**mupad** [B] time = 0.05, size = 17, normalized size = 0.77

$$\frac{2\,(c\,\,(a+b\,x))^{15/2}}{15\,b\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^(3/2)*(a + b*x)^5,x)
[Out] (2*(c*(a + b*x))^(15/2))/(15*b*c^6)
```

**sympy** [A] time = 1.21, size = 66, normalized size = 3.00

$$\begin{cases} \frac{2b^{\frac{13}{2}}c^{\frac{3}{2}}\left(\frac{a}{b}+x\right)^{\frac{15}{2}}}{15} & \text{for}\ \left|\frac{a}{b}+x\right|<1 \\ b^{\frac{13}{2}}c^{\frac{3}{2}}G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{17}{2} \\ \frac{15}{2} & 0 \end{matrix}\middle|\frac{a}{b}+x\right)+b^{\frac{13}{2}}c^{\frac{3}{2}}G_{2,2}^{0,2}\left(\begin{matrix} \frac{17}{2},1 & \\ & \frac{15}{2},0 \end{matrix}\middle|\frac{a}{b}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**(3/2),x)
```

```
[Out] Piecewise((2*b**(13/2)*c**(3/2)*(a/b + x)**(15/2)/15, Abs(a/b + x) < 1), (b
**(13/2)*c**(3/2)*meijerg(((1,), (17/2,)), ((15/2,), (0,)), a/b + x) + b**(
13/2)*c**(3/2)*meijerg(((17/2, 1), ()), ((), (15/2, 0)), a/b + x), True))
```

## 3.1339 $\int (a+bx)^5\sqrt{ac+bcx}\,dx$

**Optimal**. Leaf size=22

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a*c + b*c*x)^(13/2))/(13*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5\sqrt{ac+bcx}\,dx &= \frac{\int (ac+bcx)^{11/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{13/2}}{13bc^6}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6\sqrt{c(a+bx)}}{13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a + b*x)^6*Sqrt[c*(a + b*x)])/(13*b)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5*Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a*c + b*c*x)^(13/2))/(13*b*c^6)
```

**fricas [B]** time = 1.16, size = 75, normalized size = 3.41

$$\frac{2\left(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6\right)\sqrt{bcx+ac}}{13\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*
x^2 + 6*a^5*b*x + a^6)*sqrt(b*c*x + a*c)/b
```

**giac [B]** time = 1.27, size = 495, normalized size = 22.50

$$\frac{2\left(3003\,\sqrt{bcx+ac}\,a^6-\frac{6006\left(3\,\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^5}{c}+\frac{3003\left(15\,\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^4}{c^2}-\frac{1716\left(35\,\sqrt{bcx+ac}\,a^3c^3-35\,(bcx+ac)^{\frac{3}{2}}a^2c^2+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}\right)a^3}{c^3}+\frac{143\left(315\,\sqrt{bcx+ac}\,a^4c^4-420\,(bcx+ac)^{\frac{3}{2}}a^3c^3+378\,(bcx+ac)^{\frac{5}{2}}a^2c^2-180\,(bcx+ac)^{\frac{7}{2}}ac+35\,(bcx+ac)^{\frac{9}{2}}\right)a^2}{c^4}-\frac{26\left(693\,\sqrt{bcx+ac}\,a^5c^5-1155\,(bcx+ac)^{\frac{3}{2}}a^4c^4+1386\,(bcx+ac)^{\frac{5}{2}}a^3c^3-990\,(bcx+ac)^{\frac{7}{2}}a^2c^2+385\,(bcx+ac)^{\frac{9}{2}}ac-63\,(bcx+ac)^{\frac{11}{2}}\right)a}{c^5}+\frac{3003\,\sqrt{bcx+ac}\,a^6c^6-6006\,(bcx+ac)^{\frac{3}{2}}a^5c^5+9009\,(bcx+ac)^{\frac{5}{2}}a^4c^4-8580\,(bcx+ac)^{\frac{7}{2}}a^3c^3+5005\,(bcx+ac)^{\frac{9}{2}}a^2c^2-1638\,(bcx+ac)^{\frac{11}{2}}ac+231\,(bcx+ac)^{\frac{13}{2}}}{c^6}\right)}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3003*(3003*sqrt(b*c*x + a*c)*a^6 - 6006*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x
 + a*c)^(3/2))*a^5/c + 3003*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c
)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^4/c^2 - 1716*(35*sqrt(b*c*x + a*c)*a
^3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b
*c*x + a*c)^(7/2))*a^3/c^3 + 143*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*
x + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c
)^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a^2/c^4 - 26*(693*sqrt(b*c*x + a*c)*a
^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^
3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c
*x + a*c)^(11/2))*a/c^5 + (3003*sqrt(b*c*x + a*c)*a^6*c^6 - 6006*(b*c*x + a
```

```
*c)^(3/2)*a^5*c^5 + 9009*(b*c*x + a*c)^(5/2)*a^4*c^4 - 8580*(b*c*x + a*c)^(
7/2)*a^3*c^3 + 5005*(b*c*x + a*c)^(9/2)*a^2*c^2 - 1638*(b*c*x + a*c)^(11/2)
*a*c + 231*(b*c*x + a*c)^(13/2))/c^6)/b
```

**maple [A]** time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\left(bx+a\right)^{6}\sqrt{bcx+ac}}{13b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^(1/2),x)
[Out] 2/13*(b*x+a)^6*(b*c*x+a*c)^(1/2)/b
```

**maxima [A]** time = 1.36, size = 18, normalized size = 0.82

$$\frac{2\left(bcx+ac\right)^{\frac{13}{2}}}{13\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="maxima")
[Out] 2/13*(b*c*x + a*c)^(13/2)/(b*c^6)
```

**mupad [B]** time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\left(c\,\left(a+b\,x\right)\right)^{13/2}}{13\,b\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^(1/2)*(a + b*x)^5,x)
[Out] (2*(c*(a + b*x))^(13/2))/(13*b*c^6)
```

**sympy [A]** time = 1.06, size = 66, normalized size = 3.00

$$\begin{cases} \frac{2b^{\frac{11}{2}}\sqrt{c}\left(\frac{a}{b}+x\right)^{\frac{13}{2}}}{13} & \text{for}\ \left|\frac{a}{b}+x\right|<1 \\ b^{\frac{11}{2}}\sqrt{c}\,G_{2,2}^{1,1}\left(\begin{matrix}1 & \frac{15}{2} \\ \frac{13}{2} & 0\end{matrix}\middle|\,\frac{a}{b}+x\right)+b^{\frac{11}{2}}\sqrt{c}\,G_{2,2}^{0,2}\left(\begin{matrix}\frac{15}{2},1 & \\ & \frac{13}{2},0\end{matrix}\middle|\,\frac{a}{b}+x\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**(1/2),x)
```

```
[Out] Piecewise((2*b**(11/2)*sqrt(c)*(a/b + x)**(13/2)/13, Abs(a/b + x) < 1), (b*
*(11/2)*sqrt(c)*meijerg(((1,), (15/2,)), ((13/2,), (0,)), a/b + x) + b**(11
/2)*sqrt(c)*meijerg(((15/2, 1), ()), ((), (13/2, 0)), a/b + x), True))
```

## 3.1340 $\int \frac{(a+bx)^5}{\sqrt{ac+bcx}}\,dx$

**Optimal**. Leaf size=22

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a*c + b*c*x)^(11/2))/(11*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\sqrt{ac+bcx}}\,dx &= \frac{\int (ac+bcx)^{9/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{11/2}}{11bc^6}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{11b\sqrt{c(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a + b*x)^6)/(11*b*Sqrt[c*(a + b*x)])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a*c + b*c*x)^(11/2))/(11*b*c^6)
```

**fricas [B]** time = 0.83, size = 67, normalized size = 3.05

$$\frac{2\left(b^5x^5+5\,ab^4x^4+10\,a^2b^3x^3+10\,a^3b^2x^2+5\,a^4bx+a^5\right)\sqrt{bcx+ac}}{11\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/11*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x +
 a^5)*sqrt(b*c*x + a*c)/(b*c)
```

**giac [B]** time = 0.97, size = 374, normalized size = 17.00

$$\frac{2\left(693\sqrt{bcx+ac}\,a^5-\frac{1155\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^4}{c}+\frac{462\left(15\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^3}{c^2}-\frac{198\left(35\sqrt{bcx+ac}\,a^3c^3-35\,(bcx+ac)^{\frac{3}{2}}a^2c^2+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}\right)a^2}{c^3}+\frac{11\left(315\sqrt{bcx+ac}\,a^4c^4-420\,(bcx+ac)^{\frac{3}{2}}a^3c^3+378\,(bcx+ac)^{\frac{5}{2}}a^2c^2-180\,(bcx+ac)^{\frac{7}{2}}ac+35\,(bcx+ac)^{\frac{9}{2}}\right)a}{c^4}-\frac{693\sqrt{bcx+ac}\,a^5c^5-1155\,(bcx+ac)^{\frac{3}{2}}a^4c^4+1386\,(bcx+ac)^{\frac{5}{2}}a^3c^3-990\,(bcx+ac)^{\frac{7}{2}}a^2c^2+385\,(bcx+ac)^{\frac{9}{2}}ac-63\,(bcx+ac)^{\frac{11}{2}}}{c^5}\right)}{693\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/693*(693*sqrt(b*c*x + a*c)*a^5 - 1155*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
 a*c)^(3/2))*a^4/c + 462*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(
3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c
^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x
 + a*c)^(7/2))*a^2/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a
*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/
2)*a*c + 35*(b*c*x + a*c)^(9/2))*a/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 1
155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b
*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^
(11/2))/c^5)/(b*c)
```

**maple** [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\,(bx+a)^{6}}{11\sqrt{bcx+ac}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(1/2),x)
```

```
[Out] 2/11*(b*x+a)^6/b/(b*c*x+a*c)^(1/2)
```

**maxima** [B] time = 1.46, size = 374, normalized size = 17.00

$$\frac{2\left(693\sqrt{bcx+ac}\,a^{5}-\frac{1155\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^{4}}{c}+\frac{462\left(15\sqrt{bcx+ac}\,a^{2}c^{2}-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^{3}}{c^{2}}-\frac{198\left(35\sqrt{bcx+ac}\,a^{3}c^{3}-35\,(bcx+ac)^{\frac{3}{2}}a^{2}c^{2}+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}\right)a^{2}}{c^{3}}+\frac{11\left(315\sqrt{bcx+ac}\,a^{4}c^{4}-420\,(bcx+ac)^{\frac{3}{2}}a^{3}c^{3}+378\,(bcx+ac)^{\frac{5}{2}}a^{2}c^{2}-180\,(bcx+ac)^{\frac{7}{2}}ac+35\,(bcx+ac)^{\frac{9}{2}}\right)a}{c^{4}}-\frac{693\sqrt{bcx+ac}\,a^{5}c^{5}-1155\,(bcx+ac)^{\frac{3}{2}}a^{4}c^{4}+1386\,(bcx+ac)^{\frac{5}{2}}a^{3}c^{3}-990\,(bcx+ac)^{\frac{7}{2}}a^{2}c^{2}+385\,(bcx+ac)^{\frac{9}{2}}ac-63\,(bcx+ac)^{\frac{11}{2}}}{c^{5}}\right)}{693\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(693*sqrt(b*c*x + a*c)*a^5 - 1155*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
 a*c)^(3/2))*a^4/c + 462*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(
3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c
^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x
 + a*c)^(7/2))*a^2/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a
*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/
2)*a*c + 35*(b*c*x + a*c)^(9/2))*a/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 1
155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b
*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^
(11/2))/c^5)/(b*c)
```

**mupad** [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,\left(c\,\left(a+b\,x\right)\right)^{11/2}}{11\,b\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(1/2),x)
```

```
[Out] (2*(c*(a + b*x))^(11/2))/(11*b*c^6)
```

**sympy [A]** time = 1.54, size = 73, normalized size = 3.32

$$\begin{cases} \frac{2b^{\frac{9}{2}}\left(\frac{a}{b}+x\right)^{\frac{11}{2}}}{11\sqrt{c}} & \text{for } \left|\frac{a}{b}+x\right|>1 \vee \left|\frac{a}{b}+x\right|<1 \\ \frac{b^{\frac{9}{2}}G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{13}{2} \\ \frac{11}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x\right)}{\sqrt{c}}+\frac{b^{\frac{9}{2}}G_{2,2}^{0,2}\left(\begin{matrix} \frac{13}{2},1 & \\ & \frac{11}{2},0 \end{matrix} \middle| \frac{a}{b}+x\right)}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(1/2),x)
```

```
[Out] Piecewise((2*b**(9/2)*(a/b + x)**(11/2)/(11*sqrt(c)), (Abs(a/b + x) > 1) |
(Abs(a/b + x) < 1)), (b**(9/2)*meijerg(((1,), (13/2,)), ((11/2,), (0,)), a/
b + x)/sqrt(c) + b**(9/2)*meijerg(((13/2, 1), ()), ((), (11/2, 0)), a/b + x
)/sqrt(c), True))
```

## 3.1341 $\int \frac{(a+bx)^5}{(ac+bcx)^{3/2}} dx$

**Optimal**. Leaf size=22

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(9/2))/(9*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{3/2}} dx &= \frac{\int (ac+bcx)^{7/2} dx}{c^5}\\ &= \frac{2(ac+bcx)^{9/2}}{9bc^6}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{9b(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^6)/(9*b*(c*(a + b*x))^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(9/2))/(9*b*c^6)
```

**fricas [B]** time = 1.40, size = 56, normalized size = 2.55

$$\frac{2\left(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4\right)\sqrt{bcx+ac}}{9\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*sqrt(b*c*x +
a*c)/(b*c^2)
```

**giac [B]** time = 1.18, size = 266, normalized size = 12.09

$$\frac{2\left(315\,\sqrt{bcx+ac}\,a^4-\frac{420\left(3\,\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^3}{c}+\frac{126\left(15\,\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^2}{c^2}-\frac{36\left(35\,\sqrt{bcx+ac}\,a^3c^3-35\,(bcx+ac)^{\frac{3}{2}}a^2c^2+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}\right)a}{c^3}+\frac{315\,\sqrt{bcx+ac}\,a^4c^4-420\,(bcx+ac)^{\frac{3}{2}}a^3c^3+378\,(bcx+ac)^{\frac{5}{2}}a^2c^2-180\,(bcx+ac)^{\frac{7}{2}}ac+35\,(bcx+ac)^{\frac{9}{2}}}{c^4}\right)}{315\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(b*c*x + a*c)*a^4 - 420*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
a*c)^(3/2))*a^3/c + 126*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3
/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^2/c^2 - 36*(35*sqrt(b*c*x + a*c)*a^3*c^3
 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x +
 a*c)^(7/2))*a/c^3 + (315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a*c)^(3/
2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/2)*a*c
+ 35*(b*c*x + a*c)^(9/2))/c^4)/(b*c^2)
```

**maple** [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{2 (bx + a)^6}{9 (bcx + ac)^{\frac{3}{2}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(3/2),x)
```

```
[Out] 2/9*(b*x+a)^6/b/(b*c*x+a*c)^(3/2)
```

**maxima** [A] time = 1.43, size = 18, normalized size = 0.82

$$\frac{2 (bcx + ac)^{\frac{9}{2}}}{9 bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*c*x + a*c)^(9/2)/(b*c^6)
```

**mupad** [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{2 (c\ (a + b\,x))^{9/2}}{9\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(3/2),x)
```

```
[Out] (2*(c*(a + b*x))^(9/2))/(9*b*c^6)
```

**sympy** [A] time = 1.65, size = 73, normalized size = 3.32

$$\begin{cases} \frac{2b^{\frac{7}{2}}\left(\frac{a}{b}+x\right)^{\frac{9}{2}}}{9c^{\frac{3}{2}}} & \text{for } \left|\frac{a}{b} + x\right| > 1 \vee \left|\frac{a}{b} + x\right| < 1 \\ \frac{b^{\frac{7}{2}} G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{11}{2} \\ \frac{9}{2} & 0 \end{matrix} \middle| {\frac{a}{b}+x} \right)}{c^{\frac{3}{2}}} + \frac{b^{\frac{7}{2}} G_{2,2}^{0,2}\left(\begin{matrix} \frac{11}{2}, 1 &  \\  & \frac{9}{2}, 0 \end{matrix} \middle| {\frac{a}{b}+x} \right)}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(3/2),x)
```

```
[Out] Piecewise((2*b**(7/2)*(a/b + x)**(9/2)/(9*c**(3/2)), (Abs(a/b + x) > 1) | (
Abs(a/b + x) < 1)), (b**(7/2)*meijerg(((1,), (11/2,)), ((9/2,), (0,)), a/b
+ x)/c**(3/2) + b**(7/2)*meijerg(((11/2, 1), ()), ((), (9/2, 0)), a/b + x)/
c**(3/2), True))
```

## 3.1342 $\int \frac{(a+bx)^5}{(ac+bcx)^{5/2}} dx$

**Optimal**. Leaf size=22

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(7/2))/(7*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{5/2}}\,dx &= \frac{\int (ac+bcx)^{5/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{7/2}}{7bc^6}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{7b(c(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^6)/(7*b*(c*(a + b*x))^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(7/2))/(7*b*c^6)
```

**fricas [B]** time = 1.44, size = 45, normalized size = 2.05

$$\frac{2\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\sqrt{bcx+ac}}{7\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*c*x + a*c)/(b*c^3)
```

**giac [B]** time = 0.96, size = 178, normalized size = 8.09

$$\frac{2\left(35\,\sqrt{bcx+ac}\,a^3-\frac{35\left(3\,\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^2}{c}+\frac{7\left(15\,\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}\,ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a}{c^2}-\frac{35\,\sqrt{bcx+ac}\,a^3c^3-35\,(bcx+ac)^{\frac{3}{2}}\,a^2c^2+21\,(bcx+ac)^{\frac{5}{2}}\,ac-5\,(bcx+ac)^{\frac{7}{2}}}{c^3}\right)}{35\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*sqrt(b*c*x + a*c)*a^3 - 35*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c
)^(3/2))*a^2/c + 7*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a
*c + 3*(b*c*x + a*c)^(5/2))*a/c^2 - (35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c
*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2
))/c^3)/(b*c^3)
```

**maple [A]** time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\,(bx+a)^6}{7\,(bcx+ac)^{\frac{5}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(5/2),x)
```

```
[Out] 2/7*(b*x+a)^6/b/(b*c*x+a*c)^(5/2)
```

**maxima** [A] time = 1.38, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{7}{2}}}{7\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*c*x + a*c)^(7/2)/(b*c^6)
```

**mupad** [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,\left(c\,\left(a+b\,x\right)\right)^{7/2}}{7\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(5/2),x)
```

```
[Out] (2*(c*(a + b*x))^(7/2))/(7*b*c^6)
```

**sympy** [A] time = 1.62, size = 73, normalized size = 3.32

$$\begin{cases} \frac{2b^{\frac{5}{2}}\left(\frac{a}{b}+x\right)^{\frac{7}{2}}}{7c^{\frac{5}{2}}} & \text{for}\ \left|\frac{a}{b}+x\right|>1 \vee \left|\frac{a}{b}+x\right|<1 \\ \frac{b^{\frac{5}{2}}G_{2,2}^{1,1}\left(\begin{matrix}1 & \frac{9}{2} \\ \frac{7}{2} & 0\end{matrix}\middle|\frac{a}{b}+x\right)}{c^{\frac{5}{2}}}+\frac{b^{\frac{5}{2}}G_{2,2}^{0,2}\left(\begin{matrix}\frac{9}{2},1 & \\ & \frac{7}{2},0\end{matrix}\middle|\frac{a}{b}+x\right)}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(5/2),x)
```

```
[Out] Piecewise((2*b**(5/2)*(a/b + x)**(7/2)/(7*c**(5/2)), (Abs(a/b + x) > 1) | (
Abs(a/b + x) < 1)), (b**(5/2)*meijerg(((1,), (9/2,)), ((7/2,), (0,)), a/b +
 x)/c**(5/2) + b**(5/2)*meijerg(((9/2, 1), ()), ((), (7/2, 0)), a/b + x)/c*
*(5/2), True))
```

## 3.1343 $\int \frac{(a+bx)^5}{(ac+bcx)^{7/2}}\,dx$

**Optimal**. Leaf size=22

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(5/2))/(5*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{7/2}}\,dx &= \frac{\int (ac+bcx)^{3/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{5/2}}{5bc^6}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{5b(c(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]
```

```
[Out] (2*(a + b*x)^6)/(5*b*(c*(a + b*x))^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(5/2))/(5*b*c^6)
```

**fricas [A]** time = 1.26, size = 34, normalized size = 1.55

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bcx+ac}}{5\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*c*x + a*c)/(b*c^4)
```

**giac [B]** time = 0.99, size = 106, normalized size = 4.82

$$\frac{2\left(15\sqrt{bcx+ac}\,a^2-\frac{10\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a}{c}+\frac{15\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}\,ac+3\,(bcx+ac)^{\frac{5}{2}}}{c^2}\right)}{15\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*c*x + a*c)*a^2 - 10*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c
)^(3/2))*a/c + (15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c +
 3*(b*c*x + a*c)^(5/2))/c^2)/(b*c^4)
```

**maple [A]** time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\,(bx+a)^6}{5\,(bcx+ac)^{\frac{7}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(7/2),x)
```

```
[Out] 2/5*(b*x+a)^6/b/(b*c*x+a*c)^(7/2)
```

**maxima [A]** time = 1.41, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{5}{2}}}{5\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*c*x + a*c)^(5/2)/(b*c^6)
```

**mupad [B]** time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{5/2}}{5\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(7/2),x)
```

```
[Out] (2*(c*(a + b*x))^(5/2))/(5*b*c^6)
```

**sympy [A]** time = 4.11, size = 80, normalized size = 3.64

$$\begin{cases} \frac{2a^2\sqrt{ac+bcx}}{5bc^4} + \frac{4ax\sqrt{ac+bcx}}{5c^4} + \frac{2bx^2\sqrt{ac+bcx}}{5c^4} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(7/2),x)
```

```
[Out] Piecewise((2*a**2*sqrt(a*c + b*c*x)/(5*b*c**4) + 4*a*x*sqrt(a*c + b*c*x)/(5
*c**4) + 2*b*x**2*sqrt(a*c + b*c*x)/(5*c**4), Ne(b, 0)), (a**5*x/(a*c)**(7/
2), True))
```

## 3.1344 $\int \frac{(a+bx)^5}{(ac+bcx)^{9/2}}\,dx$

**Optimal**. Leaf size=22

$$\frac{2(ac+bcx)^{3/2}}{3bc^6}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{3/2}}{3bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(3/2))/(3*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{9/2}}\,dx &= \frac{\int \sqrt{ac+bcx}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{3/2}}{3bc^6}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 1.18

$$\frac{2(a+bx)\sqrt{c(a+bx)}}{3bc^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]
[Out] (2*(a + b*x)*Sqrt[c*(a + b*x)])/(3*b*c^5)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{3/2}}{3bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]
[Out] (2*(a*c + b*c*x)^(3/2))/(3*b*c^6)
```

**fricas [A]** time = 1.31, size = 23, normalized size = 1.05

$$\frac{2\sqrt{bcx+ac}\,(bx+a)}{3\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*c*x + a*c)*(b*x + a)/(b*c^5)
```

**giac [B]** time = 1.07, size = 54, normalized size = 2.45

$$\frac{2\left(3\sqrt{bcx+ac}\,a-\frac{3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}}{c}\right)}{3\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="giac")
[Out] 2/3*(3*sqrt(b*c*x + a*c)*a - (3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2)
)/c)/(b*c^5)
```

**maple [A]** time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\,(bx+a)^{6}}{3\,(bcx+ac)^{\frac{9}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(9/2),x)
```

```
[Out] 2/3*(b*x+a)^6/b/(b*c*x+a*c)^(9/2)
```

**maxima [A]** time = 1.39, size = 18, normalized size = 0.82

$$\frac{2\,(bcx + ac)^{\frac{3}{2}}}{3\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*c*x + a*c)^(3/2)/(b*c^6)
```

**mupad [B]** time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,\left(c\,\left(a+b\,x\right)\right)^{3/2}}{3\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(9/2),x)
```

```
[Out] (2*(c*(a + b*x))^(3/2))/(3*b*c^6)
```

**sympy [A]** time = 8.31, size = 53, normalized size = 2.41

$$\begin{cases} \frac{2a\sqrt{ac+bcx}}{3bc^5} + \frac{2x\sqrt{ac+bcx}}{3c^5} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(9/2),x)
```

```
[Out] Piecewise((2*a*sqrt(a*c + b*c*x)/(3*b*c**5) + 2*x*sqrt(a*c + b*c*x)/(3*c**5
), Ne(b, 0)), (a**5*x/(a*c)**(9/2), True))
```

## 3.1345 $\int \frac{(a+bx)^5}{(ac+bcx)^{11/2}} dx$

**Optimal**. Leaf size=20

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

**Rubi** [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[a*c + b*c*x])/(b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{11/2}}\,dx &= \frac{\int \frac{1}{\sqrt{ac+bcx}}\,dx}{c^5}\\ &= \frac{2\sqrt{ac+bcx}}{bc^6}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 24, normalized size = 1.20

$$\frac{2(a+bx)}{bc^5\sqrt{c(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]
```

```
[Out] (2*(a + b*x))/(b*c^5*Sqrt[c*(a + b*x)])
```

**IntegrateAlgebraic [A]** time = 0.06, size = 20, normalized size = 1.00

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[a*c + b*c*x])/(b*c^6)
```

**fricas [A]** time = 1.19, size = 18, normalized size = 0.90

$$\frac{2\sqrt{bcx+ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)
```

**giac [A]** time = 1.03, size = 18, normalized size = 0.90

$$\frac{2\sqrt{bcx+ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)
```

**maple [A]** time = 0.00, size = 23, normalized size = 1.15

$$\frac{2\left(bx+a\right)^{6}}{\left(bcx+ac\right)^{\frac{11}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(11/2),x)
```

[Out] 2*(b*x+a)^6/b/(b*c*x+a*c)^(11/2)

**maxima [A]** time = 1.38, size = 18, normalized size = 0.90

$$\frac{2\sqrt{bcx + ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="maxima")

[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)

**mupad [B]** time = 0.03, size = 17, normalized size = 0.85

$$\frac{2\sqrt{c\,\left(a+b\,x\right)}}{b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^5/(a*c + b*c*x)^(11/2),x)

[Out] (2*(c*(a + b*x))^(1/2))/(b*c^6)

**sympy [A]** time = 15.48, size = 29, normalized size = 1.45

$$\begin{cases} \frac{2\sqrt{ac+bcx}}{bc^6} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{11}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**5/(b*c*x+a*c)**(11/2),x)

[Out] Piecewise((2*sqrt(a*c + b*c*x)/(b*c**6), Ne(b, 0)), (a**5*x/(a*c)**(11/2), True))

## 3.1346 $\int \frac{(a+bx)^5}{(ac+bcx)^{13/2}} dx$

**Optimal**. Leaf size=20

$$-\frac{2}{bc^6\sqrt{ac+bcx}}$$

**Rubi [A]** time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{2}{bc^6\sqrt{ac+bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
```

```
[Out] -2/(b*c^6*Sqrt[a*c + b*c*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{13/2}}\,dx &= \frac{\int \frac{1}{(ac+bcx)^{3/2}}\,dx}{c^5}\\ &= -\frac{2}{bc^6\sqrt{ac+bcx}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 24, normalized size = 1.20

$$-\frac{2(a+bx)}{bc^5(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
[Out] (-2*(a + b*x))/(b*c^5*(c*(a + b*x))^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 27, normalized size = 1.35

$$-\frac{2\sqrt{ac+bcx}}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
[Out] (-2*Sqrt[a*c + b*c*x])/(b*c^7*(a + b*x))
```

**fricas [A]** time = 1.10, size = 29, normalized size = 1.45

$$-\frac{2\sqrt{bcx+ac}}{b^2c^7x+abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="fricas")
[Out] -2*sqrt(b*c*x + a*c)/(b^2*c^7*x + a*b*c^7)
```

**giac [A]** time = 0.81, size = 18, normalized size = 0.90

$$-\frac{2}{\sqrt{bcx+ac}\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="giac")
[Out] -2/(sqrt(b*c*x + a*c)*b*c^6)
```

**maple [A]** time = 0.00, size = 23, normalized size = 1.15

$$-\frac{2\,(bx+a)^6}{(bcx+ac)^{\frac{13}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(13/2),x)
```

```
[Out] -2*(b*x+a)^6/b/(b*c*x+a*c)^(13/2)
```

**maxima [A]** time = 1.34, size = 18, normalized size = 0.90

$$-\frac{2}{\sqrt{bcx + ac}\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*c*x + a*c)*b*c^6)
```

**mupad [B]** time = 0.03, size = 17, normalized size = 0.85

$$-\frac{2}{b\,c^6\,\sqrt{c\,\left(a+b\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(13/2),x)
```

```
[Out] -2/(b*c^6*(c*(a + b*x))^(1/2))
```

**sympy [A]** time = 39.43, size = 48, normalized size = 2.40

$$\begin{cases} -\frac{2\sqrt{ac+bcx}}{abc^7+b^2c^7x} & \text{for}\ a \neq 0 \\ -\frac{2}{b^{\frac{3}{2}}c^{\frac{13}{2}}\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(13/2),x)
```

```
[Out] Piecewise((-2*sqrt(a*c + b*c*x)/(a*b*c**7 + b**2*c**7*x), Ne(a, 0)), (-2/(b
**(3/2)*c**(13/2)*sqrt(x)), True))
```

## 3.1347 $\int \frac{1}{(-2+x)\sqrt{2+x}}\,dx$

**Optimal**. Leaf size=14

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 207}

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*Sqrt[2 + x]),x]
```

```
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-2+x)\sqrt{2+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx, x, \sqrt{2+x}\right)\\ &= -\tanh^{-1}\left(\frac{\sqrt{2+x}}{2}\right)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 14, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)*Sqrt[2 + x]),x]
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

**IntegrateAlgebraic [A]** time = 0.02, size = 14, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 + x)*Sqrt[2 + x]),x]
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

**fricas [B]** time = 1.22, size = 21, normalized size = 1.50

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(sqrt(x + 2) - 2)
```

**giac [B]** time = 0.87, size = 22, normalized size = 1.57

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="giac")
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(abs(sqrt(x + 2) - 2))
```

**maple [B]** time = 0.01, size = 22, normalized size = 1.57

$$\frac{\ln\left(\sqrt{x+2}-2\right)}{2}-\frac{\ln\left(\sqrt{x+2}+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-2)/(x+2)^(1/2),x)
```

```
[Out] 1/2*ln((x+2)^(1/2)-2)-1/2*ln((x+2)^(1/2)+2)
```

**maxima [B]** time = 1.35, size = 21, normalized size = 1.50

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(sqrt(x + 2) - 2)
```

**mupad [B]** time = 0.05, size = 10, normalized size = 0.71

$$-\operatorname{atanh}\left(\frac{\sqrt{x+2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)*(x + 2)^(1/2)),x)
```

```
[Out] -atanh((x + 2)^(1/2)/2)
```

**sympy [A]** time = 0.66, size = 27, normalized size = 1.93

$$\begin{cases} -\operatorname{acoth}\left(\frac{\sqrt{x+2}}{2}\right) & \text{for}\ \frac{|x+2|}{4}>1 \\ -\operatorname{atanh}\left(\frac{\sqrt{x+2}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)**(1/2),x)
```

```
[Out] Piecewise((-acoth(sqrt(x + 2)/2), Abs(x + 2)/4 > 1), (-atanh(sqrt(x + 2)/2)
, True))
```

# 3.1348 $\int \frac{1}{(2+3x)\sqrt{1+5x}}\,dx$

**Optimal**. Leaf size=25

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

**Rubi [A]** time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 203}

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+3x)\sqrt{1+5x}}\,dx &= \frac{2}{5}\text{Subst}\left(\int \frac{1}{\frac{7}{5}+\frac{3x^2}{5}}\,dx, x, \sqrt{1+5x}\right)\\ &= \frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{1+5x}\right)}{\sqrt{21}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

**IntegrateAlgebraic** [A] time = 0.03, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

**fricas** [A] time = 1.05, size = 18, normalized size = 0.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="fricas")
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

**giac** [A] time = 0.98, size = 18, normalized size = 0.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="giac")
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

**maple** [A] time = 0.01, size = 19, normalized size = 0.76

$$\frac{2\sqrt{21}\ \arctan\left(\frac{\sqrt{21}\ \sqrt{5x+1}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x+2)/(1+5*x)^(1/2),x)
```

```
[Out] 2/21*arctan(1/7*21^(1/2)*(1+5*x)^(1/2))*21^(1/2)
```

**maxima** [A] time = 3.00, size = 18, normalized size = 0.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

**mupad** [B] time = 0.06, size = 15, normalized size = 0.60

$$\frac{2\sqrt{21}\ \mathrm{atan}\left(\frac{\sqrt{105x+21}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x + 2)*(5*x + 1)^(1/2)),x)
```

```
[Out] (2*21^(1/2)*atan((105*x + 21)^(1/2)/7))/21
```

**sympy** [A] time = 1.12, size = 61, normalized size = 2.44

$$\begin{cases} \frac{2\sqrt{21}\, i\, \mathrm{acosh}\left(\frac{\sqrt{105}}{15\sqrt{x+\frac{2}{3}}}\right)}{21} & \text{for}\ \frac{7}{15\left|x+\frac{2}{3}\right|} > 1 \\ -\frac{2\sqrt{21}\ \mathrm{asin}\left(\frac{\sqrt{105}}{15\sqrt{x+\frac{2}{3}}}\right)}{21} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(21)*I*acosh(sqrt(105)/(15*sqrt(x + 2/3)))/21, 7/(15*Abs(x
 + 2/3)) > 1), (-2*sqrt(21)*asin(sqrt(105)/(15*sqrt(x + 2/3)))/21, True))
```

# 3.1349 $\int \frac{\sqrt[3]{1-x}}{1+x}\,dx$

**Optimal**. Leaf size=84

$$3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(x+1)}{2^{2/3}}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}+1}{\sqrt{3}}\right)$$

**Rubi [A]** time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {50, 57, 617, 204, 31}

$$3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(x+1)}{2^{2/3}}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x)^(1/3))/Sqrt[3
]] + (3*Log[2^(1/3) - (1 - x)^(1/3)])/2^(2/3) - Log[1 + x]/2^(2/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1-x}}{1+x}\,dx &= 3\sqrt[3]{1-x}+2\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx\\
&= 3\sqrt[3]{1-x}-\frac{\log(1+x)}{2^{2/3}}-\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx,x,\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{3\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx,x,\sqrt[3]{1-x}\right)}{\sqrt[3]{2}}\\
&= 3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(1+x)}{2^{2/3}}+\left(3\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2^{2/3}\sqrt[3]{1-x}\right)\\
&= 3\sqrt[3]{1-x}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}}\right)+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(1+x)}{2^{2/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 104, normalized size = 1.24

$$3\sqrt[3]{1-x}+\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)-\frac{\log\left((1-x)^{2/3}+\sqrt[3]{2-2x}+2^{2/3}\right)}{2^{2/3}}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x)^(1/3))/Sqrt[3
]] + 2^(1/3)*Log[2^(1/3) - (1 - x)^(1/3)] - Log[2^(2/3) + (2 - 2*x)^(1/3) +
 (1 - x)^(2/3)]/2^(2/3)
```

**IntegrateAlgebraic [A]** time = 0.12, size = 115, normalized size = 1.37

$$3\sqrt[3]{1-x}+\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{1-x}-2\right)-\frac{\log\left(\sqrt[3]{2}(1-x)^{2/3}+2^{2/3}\sqrt[3]{1-x}+2\right)}{2^{2/3}}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x)^(1/3)
)/Sqrt[3]] + 2^(1/3)*Log[-2 + 2^(2/3)*(1 - x)^(1/3)] - Log[2 + 2^(2/3)*(1 -
 x)^(1/3) + 2^(1/3)*(1 - x)^(2/3)]/2^(2/3)
```

**fricas [A]** time = 1.33, size = 86, normalized size = 1.02

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,2^{\frac{2}{3}}(-x+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right)+3(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/3*sqrt(3)*2^(2/3)*(-x + 1)^(1/3) + 1/3*sqrt(3)) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(-2^(1/3) + (-x + 1)^(1/3)) + 3*(-x + 1)^(1/3)
```

**giac [A]** time = 1.05, size = 87, normalized size = 1.04

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x+1)^{\frac{1}{3}}\right)\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right|\right)+3(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="giac")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x + 1)^(1/3))) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(abs(-2^(1/3) + (-x + 1)^(1/3))) + 3*(-x + 1)^(1/3)
```

**maple [A]** time = 0.01, size = 84, normalized size = 1.00

$$-2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(1+2^{\frac{2}{3}}(-x+1)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)+2^{\frac{1}{3}}\ln\left((-x+1)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)-\frac{2^{\frac{1}{3}}\ln\left((-x+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{2}+3(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/3)/(x+1),x)
```

```
[Out] 3*(-x+1)^(1/3)+2^(1/3)*ln((-x+1)^(1/3)-2^(1/3))-1/2*2^(1/3)*ln((-x+1)^(2/3)
+2^(1/3)*(-x+1)^(1/3)+2^(2/3))-2^(1/3)*arctan(1/3*(1+2^(2/3)*(-x+1)^(1/3))*
3^(1/2))*3^(1/2)
```

**maxima** [A] time = 3.00, size = 86, normalized size = 1.02

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\,(-x+1)^{\frac{1}{3}}\right)\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right)+3\,(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x + 1)^(1/3))) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(-2^(1/3) + (-x + 1)^(1/3)) + 3*(-x + 1)^(1/3)
```

**mupad** [B] time = 0.07, size = 104, normalized size = 1.24

$$2^{1/3}\ln\left(18\,(1-x)^{1/3}-18\,2^{1/3}\right)+3\,(1-x)^{1/3}+\frac{2^{1/3}\ln\left(18\,(1-x)^{1/3}-9\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{2^{1/3}\ln\left(18\,(1-x)^{1/3}+9\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/3)/(x + 1),x)
```

```
[Out] 2^(1/3)*log(18*(1 - x)^(1/3) - 18*2^(1/3)) + 3*(1 - x)^(1/3) + (2^(1/3)*log
(18*(1 - x)^(1/3) - 9*2^(1/3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/2 - (2^(1
/3)*log(18*(1 - x)^(1/3) + 9*2^(1/3)*(3^(1/2)*1i + 1))*(3^(1/2)*1i + 1))/2
```

**sympy** [C] time = 2.26, size = 170, normalized size = 2.02

$$\frac{4\sqrt[3]{-1}\,\sqrt[3]{x-1}\,\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-2}\,e^{-\frac{i\pi}{3}}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{\frac{i\pi}{3}}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{4\sqrt[3]{-2}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{i\pi}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-2}\,e^{\frac{i\pi}{3}}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{\frac{5i\pi}{3}}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/3)/(1+x),x)
```

```
[Out] 4*(-1)**(1/3)*(x - 1)**(1/3)*gamma(4/3)/gamma(7/3) + 4*(-2)**(1/3)*exp(-I*p
i/3)*log(-2**(2/3)*(x - 1)**(1/3)*exp_polar(I*pi/3)/2 + 1)*gamma(4/3)/(3*ga
mma(7/3)) - 4*(-2)**(1/3)*log(-2**(2/3)*(x - 1)**(1/3)*exp_polar(I*pi)/2 +
1)*gamma(4/3)/(3*gamma(7/3)) + 4*(-2)**(1/3)*exp(I*pi/3)*log(-2**(2/3)*(x -
 1)**(1/3)*exp_polar(5*I*pi/3)/2 + 1)*gamma(4/3)/(3*gamma(7/3))
```

## 3.1350 $\int \sqrt[3]{3-2x}\,(7+x)\,dx$

**Optimal**. Leaf size=27

$$\frac{3}{28}(3-2x)^{7/3}-\frac{51}{16}(3-2x)^{4/3}$$

**Rubi [A]** time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{28}(3-2x)^{7/3}-\frac{51}{16}(3-2x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)^(1/3)*(7 + x),x]
```

```
[Out] (-51*(3 - 2*x)^(4/3))/16 + (3*(3 - 2*x)^(7/3))/28
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{3-2x}\,(7+x)\,dx &= \int \left(\frac{17}{2}\sqrt[3]{3-2x}-\frac{1}{2}(3-2x)^{4/3}\right)dx\\ &= -\frac{51}{16}(3-2x)^{4/3}+\frac{3}{28}(3-2x)^{7/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 18, normalized size = 0.67

$$-\frac{3}{112}(3-2x)^{4/3}(8x+107)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)^(1/3)*(7 + x),x]
```

```
[Out] (-3*(3 - 2*x)^(4/3)*(107 + 8*x))/112
```

**IntegrateAlgebraic** [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{3}{112}(4(3-2x)-119)(3-2x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - 2*x)^(1/3)*(7 + x),x]
[Out] (3*(-119 + 4*(3 - 2*x))*(3 - 2*x)^(4/3))/112
```

**fricas** [A] time = 1.24, size = 19, normalized size = 0.70

$$\frac{3}{112}\left(16\,x^2+190\,x-321\right)(-2\,x+3)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="fricas")
[Out] 3/112*(16*x^2 + 190*x - 321)*(-2*x + 3)^(1/3)
```

**giac** [A] time = 0.94, size = 26, normalized size = 0.96

$$\frac{3}{28}\,(2\,x-3)^2(-2\,x+3)^{\frac{1}{3}}-\frac{51}{16}\,(-2\,x+3)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="giac")
[Out] 3/28*(2*x - 3)^2*(-2*x + 3)^(1/3) - 51/16*(-2*x + 3)^(4/3)
```

**maple** [A] time = 0.00, size = 15, normalized size = 0.56

$$-\frac{3\,(8x+107)\,(-2x+3)^{\frac{4}{3}}}{112}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)^(1/3)*(7+x),x)
[Out] -3/112*(8*x+107)*(3-2*x)^(4/3)
```

**maxima** [A] time = 1.35, size = 19, normalized size = 0.70

$$\frac{3}{28}\,(-2\,x+3)^{\frac{7}{3}}-\frac{51}{16}\,(-2\,x+3)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="maxima")
```

```
[Out] 3/28*(-2*x + 3)^(7/3) - 51/16*(-2*x + 3)^(4/3)
```

**mupad [B]** time = 0.26, size = 14, normalized size = 0.52

$$-\frac{3\,{\left(3-2\,x\right)}^{4/3}\,\left(8\,x+107\right)}{112}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 2*x)^(1/3)*(x + 7),x)
```

```
[Out] -(3*(3 - 2*x)^(4/3)*(8*x + 107))/112
```

**sympy [A]** time = 1.09, size = 114, normalized size = 4.22

$$\begin{cases} \frac{3(x+7)^2 \sqrt[3]{2x-3}\, e^{\frac{i\pi}{3}}}{7} - \frac{51(x+7) \sqrt[3]{2x-3}\, e^{\frac{i\pi}{3}}}{56} - \frac{2601 \sqrt[3]{2x-3}\, e^{\frac{i\pi}{3}}}{112} & \text{for}\ \frac{2|x+7|}{17} > 1 \\ \frac{3 \sqrt[3]{3-2x}\,(x+7)^2}{7} - \frac{51 \sqrt[3]{3-2x}\,(x+7)}{56} - \frac{2601 \sqrt[3]{3-2x}}{112} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)**(1/3)*(7+x),x)
```

```
[Out] Piecewise((3*(x + 7)**2*(2*x - 3)**(1/3)*exp(I*pi/3)/7 - 51*(x + 7)*(2*x -
3)**(1/3)*exp(I*pi/3)/56 - 2601*(2*x - 3)**(1/3)*exp(I*pi/3)/112, 2*Abs(x +
 7)/17 > 1), (3*(3 - 2*x)**(1/3)*(x + 7)**2/7 - 51*(3 - 2*x)**(1/3)*(x + 7)
/56 - 2601*(3 - 2*x)**(1/3)/112, True))
```

## 3.1351 $\int \sqrt[3]{1-x}(1+x)^2\,dx$

**Optimal**. Leaf size=38

$$-\frac{3}{10}(1-x)^{10/3}+\frac{12}{7}(1-x)^{7/3}-3(1-x)^{4/3}$$

**Rubi [A]** time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3}{10}(1-x)^{10/3}+\frac{12}{7}(1-x)^{7/3}-3(1-x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/3)*(1 + x)^2,x]
```

```
[Out] -3*(1 - x)^(4/3) + (12*(1 - x)^(7/3))/7 - (3*(1 - x)^(10/3))/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{1-x}(1+x)^2\,dx &= \int \left(4\sqrt[3]{1-x}-4(1-x)^{4/3}+(1-x)^{7/3}\right)dx\\ &= -3(1-x)^{4/3}+\frac{12}{7}(1-x)^{7/3}-\frac{3}{10}(1-x)^{10/3}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 23, normalized size = 0.61

$$-\frac{3}{70}(1-x)^{4/3}\left(7x^2+26x+37\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/3)*(1 + x)^2,x]
```

```
[Out] (-3*(1 - x)^(4/3)*(37 + 26*x + 7*x^2))/70
```

**IntegrateAlgebraic [A]** time = 0.02, size = 31, normalized size = 0.82

$$-\frac{3}{70}\left(7(1-x)^2-40(1-x)+70\right)(1-x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(1/3)*(1 + x)^2,x]
[Out] (-3*(70 - 40*(1 - x) + 7*(1 - x)^2)*(1 - x)^(4/3))/70
```

**fricas [A]** time = 1.32, size = 24, normalized size = 0.63

$$\frac{3}{70}\left(7x^3+19x^2+11x-37\right)(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="fricas")
[Out] 3/70*(7*x^3 + 19*x^2 + 11*x - 37)*(-x + 1)^(1/3)
```

**giac [A]** time = 0.86, size = 38, normalized size = 1.00

$$\frac{3}{10}(x-1)^3(-x+1)^{\frac{1}{3}}+\frac{12}{7}(x-1)^2(-x+1)^{\frac{1}{3}}-3(-x+1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="giac")
[Out] 3/10*(x - 1)^3*(-x + 1)^(1/3) + 12/7*(x - 1)^2*(-x + 1)^(1/3) - 3*(-x + 1)^
(4/3)
```

**maple [A]** time = 0.00, size = 20, normalized size = 0.53

$$-\frac{3\left(7x^2+26x+37\right)(-x+1)^{\frac{4}{3}}}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/3)*(x+1)^2,x)
[Out] -3/70*(7*x^2+26*x+37)*(-x+1)^(4/3)
```

**maxima [A]** time = 1.36, size = 28, normalized size = 0.74

$$-\frac{3}{10}(-x+1)^{\frac{10}{3}}+\frac{12}{7}(-x+1)^{\frac{7}{3}}-3(-x+1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="maxima")
[Out] -3/10*(-x + 1)^(10/3) + 12/7*(-x + 1)^(7/3) - 3*(-x + 1)^(4/3)
```

**mupad [B]** time = 0.05, size = 21, normalized size = 0.55

$$-\frac{3\,{\left(1-x\right)}^{4/3}\,\left(40\,x+7\,{\left(x-1\right)}^2+30\right)}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/3)*(x + 1)^2,x)
[Out] -(3*(1 - x)^(4/3)*(40*x + 7*(x - 1)^2 + 30))/70
```

**sympy [A]** time = 1.52, size = 146, normalized size = 3.84

$$\begin{cases} -\frac{3\sqrt[3]{x-1}\,(x+1)^3 e^{-\frac{2i\pi}{3}}}{10} + \frac{3\sqrt[3]{x-1}\,(x+1)^2 e^{-\frac{2i\pi}{3}}}{35} + \frac{9\sqrt[3]{x-1}\,(x+1) e^{-\frac{2i\pi}{3}}}{35} + \frac{54\sqrt[3]{x-1}\, e^{-\frac{2i\pi}{3}}}{35} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{3\sqrt[3]{1-x}\,(x+1)^3}{10} - \frac{3\sqrt[3]{1-x}\,(x+1)^2}{35} - \frac{9\sqrt[3]{1-x}\,(x+1)}{35} - \frac{54\sqrt[3]{1-x}}{35} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/3)*(1+x)**2,x)
[Out] Piecewise((-3*(x - 1)**(1/3)*(x + 1)**3*exp(-2*I*pi/3)/10 + 3*(x - 1)**(1/3
)*(x + 1)**2*exp(-2*I*pi/3)/35 + 9*(x - 1)**(1/3)*(x + 1)*exp(-2*I*pi/3)/35
 + 54*(x - 1)**(1/3)*exp(-2*I*pi/3)/35, Abs(x + 1)/2 > 1), (3*(1 - x)**(1/3
)*(x + 1)**3/10 - 3*(1 - x)**(1/3)*(x + 1)**2/35 - 9*(1 - x)**(1/3)*(x + 1)
/35 - 54*(1 - x)**(1/3)/35, True))
```

## 3.1352 $\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx$

**Optimal**. Leaf size=139

$$-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}$$

**Rubi [A]** time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {55, 617, 204, 31}

$$-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(b^(2/3)*(b*c - a*d)^(1/3)) - Log[a + b*x]/(2*b^(2/3)*(b*c - a*d)^(1/3))
 + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(2/3)*(b*c - a
*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx = -\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{2b} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{b}}-\cdots}\right.}{2b^{2/3}\sqrt[3]{b\cdots}}$$

$$= -\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b}\,\sqrt[3]{\cdots}}{\sqrt[3]{bc-\cdots}}\right.}{b^{2/3}\sqrt[3]{bc-ad}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}} - \frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}$$

**Mathematica [A]** time = 0.08, size = 106, normalized size = 0.76

$$\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)-\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]] - Log[a + b*x] + 3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2
*b^(2/3)*(b*c - a*d)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.23, size = 191, normalized size = 1.37

$$-\frac{\log\left(\sqrt[3]{ad-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{b^{2/3}\sqrt[3]{ad-bc}}+\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{ad-bc}+(ad-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{2b^{2/3}\sqrt[3]{ad-bc}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{ad-bc}}\right)}{b^{2/3}\sqrt[3]{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*(-(b*c)
+ a*d)^(1/3))])/(b^(2/3)*(-(b*c) + a*d)^(1/3))) - Log[(-(b*c) + a*d)^(1/3)
+ b^(1/3)*(c + d*x)^(1/3)]/(b^(2/3)*(-(b*c) + a*d)^(1/3)) + Log[(-(b*c) + a
*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x
)^(2/3)]/(2*b^(2/3)*(-(b*c) + a*d)^(1/3))
```

**fricas [B]** time = 1.27, size = 570, normalized size = 4.10

$$\left[\frac{\sqrt{3}\left(b^{2}c-abd\right)\sqrt{-\frac{\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2b^{2}dx+3b^{2}c-abd-\sqrt{3}\left(\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}(bc-ad)+\left(b^{2}c-abd\right)(dx+c)^{\frac{1}{3}}-2\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}\right)\sqrt{-\frac{\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}}{bc-ad}}-3\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}\right)-\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}\log\left((dx+c)^{\frac{2}{3}}b^{2}+\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}b+\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}\right)+2\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}\log\left((dx+c)^{\frac{1}{3}}b-\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}\right)}{2\left(b^{3}c-ab^{2}d\right)},\frac{2\sqrt{3}\left(b^{2}c-abd\right)\sqrt{\frac{\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}}{bc-ad}}\arctan\left(\frac{\sqrt{3}\left(2(dx+c)^{\frac{1}{3}}b+\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}\right)\sqrt{\frac{\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}}{bc-ad}}}{3b}\right)-\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}\log\left((dx+c)^{\frac{2}{3}}b^{2}+\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}b+\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}\right)+2\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}\log\left((dx+c)^{\frac{1}{3}}b-\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}\right)}{2\left(b^{3}c-ab^{2}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*(b^2*c - a*b*d)*sqrt(-(b^3*c - a*b^2*d)^(1/3)/(b*c - a*d))*lo
g((2*b^2*d*x + 3*b^2*c - a*b*d - sqrt(3)*((b^3*c - a*b^2*d)^(1/3)*(b*c - a*
d) + (b^2*c - a*b*d)*(d*x + c)^(1/3) - 2*(b^3*c - a*b^2*d)^(2/3)*(d*x + c)^
(2/3))*sqrt(-(b^3*c - a*b^2*d)^(1/3)/(b*c - a*d)) - 3*(b^3*c - a*b^2*d)^(2/
3)*(d*x + c)^(1/3))/(b*x + a)) - (b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(2/3
)*b^2 + (b^3*c - a*b^2*d)^(1/3)*(d*x + c)^(1/3)*b + (b^3*c - a*b^2*d)^(2/3)
) + 2*(b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(1/3)*b - (b^3*c - a*b^2*d)^(1/
3)))/(b^3*c - a*b^2*d), 1/2*(2*sqrt(3)*(b^2*c - a*b*d)*sqrt((b^3*c - a*b^2*
d)^(1/3)/(b*c - a*d))*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3)*b + (b^3*c - a*
b^2*d)^(1/3))*sqrt((b^3*c - a*b^2*d)^(1/3)/(b*c - a*d))/b) - (b^3*c - a*b^2
*d)^(2/3)*log((d*x + c)^(2/3)*b^2 + (b^3*c - a*b^2*d)^(1/3)*(d*x + c)^(1/3)
*b + (b^3*c - a*b^2*d)^(2/3)) + 2*(b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(1/
3)*b - (b^3*c - a*b^2*d)^(1/3)))/(b^3*c - a*b^2*d)]
```

**giac [A]** time = 1.10, size = 196, normalized size = 1.41

$$\frac{3\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2(dx+c)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{3}c-\sqrt{3}\,ab^{2}d}-\frac{\log\left((dx+c)^{\frac{2}{3}}+(dx+c)^{\frac{1}{3}}\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\right)}{2\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}}+\frac{\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\log\left(\left|(dx+c)^{\frac{1}{3}}-\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right|\right)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] 3*(b^3*c - a*b^2*d)^(2/3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c - a
*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*a*b^2*d) - 1/
2*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(1/3) + ((b*c - a*d
)/b)^(2/3))/(b^3*c - a*b^2*d)^(1/3) + ((b*c - a*d)/b)^(2/3)*log(abs((d*x +
c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b*c - a*d)
```

**maple [A]** time = 0.01, size = 161, normalized size = 1.16

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(dx+c)^{\frac{1}{3}}}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}} b}-\frac{\ln\left((dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}} b}+\frac{\ln\left((dx+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/3),x)
```

```
[Out] -1/b/((a*d-b*c)/b)^(1/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))+1/2/b/((a*d-
b*c)/b)^(1/3)*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)
/b)^(2/3))+3^(1/2)/b/((a*d-b*c)/b)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/b
)^(1/3)*(d*x+c)^(1/3)-1))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.21, size = 204, normalized size = 1.47

$$\frac{\ln\left(9\,b\,(c+d\,x)^{1/3}-\frac{9\,b^3\,c-9\,a\,b^2\,d}{b^{4/3}\,(b\,c-a\,d)^{2/3}}\right)}{b^{2/3}\,(b\,c-a\,d)^{1/3}}+\frac{\ln\left(9\,b\,(c+d\,x)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{4\,b^{4/3}\,(b\,c-a\,d)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{2/3}\,(b\,c-a\,d)^{1/3}}-\frac{\ln\left(9\,b\,(c+d\,x)^{1/3}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{4\,b^{4/3}\,(b\,c-a\,d)^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{2/3}\,(b\,c-a\,d)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(1/3)),x)
```

```
[Out] log(9*b*(c + d*x)^(1/3) - (9*b^3*c - 9*a*b^2*d)/(b^(4/3)*(b*c - a*d)^(2/3))
)/(b^(2/3)*(b*c - a*d)^(1/3)) + (log(9*b*(c + d*x)^(1/3) - ((3^(1/2)*1i - 1
)^2*(9*b^3*c - 9*a*b^2*d))/(4*b^(4/3)*(b*c - a*d)^(2/3)))*(3^(1/2)*1i - 1))
/(2*b^(2/3)*(b*c - a*d)^(1/3)) - (log(9*b*(c + d*x)^(1/3) - ((3^(1/2)*1i +
1)^2*(9*b^3*c - 9*a*b^2*d))/(4*b^(4/3)*(b*c - a*d)^(2/3)))*(3^(1/2)*1i + 1)
)/(2*b^(2/3)*(b*c - a*d)^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(1/3)), x)
```

## 3.1353 $\int \frac{1}{(a+bx)(c+dx)^{2/3}}\,dx$

**Optimal**. Leaf size=140

$$-\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}\,(bc-ad)^{2/3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {57, 617, 204, 31}

$$-\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]])/(b^(1/3)*(b*c - a*d)^(2/3))) - Log[a + b*x]/(2*b^(1/3)*(b*c - a*d)^(2/
3)) + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(1/3)*(b*c
- a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{2/3}}\,dx = -\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{b}}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{bc-\cdots}}{\sqrt[3]{\cdots}}\cdots}\right)}{2b^{2/3}\sqrt[3]{b\cdots}}$$

$$= -\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\cdots\right)}{\sqrt[3]{b}\,(bc-ad)^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}$$

**Mathematica [A]** time = 0.07, size = 154, normalized size = 1.10

$$-\frac{\log\left(\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/
Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)] + Log[(b*c -
a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^
(2/3)])/(b^(1/3)*(b*c - a*d)^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.21, size = 190, normalized size = 1.36

$$-\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{ad-bc}+(ad-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{2\sqrt[3]{b}\,(ad-bc)^{2/3}}+\frac{\log\left(\sqrt[3]{ad-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\sqrt[3]{b}\,(ad-bc)^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{ad-bc}}\right)}{\sqrt[3]{b}\,(ad-bc)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*(-(b*c)
+ a*d)^(1/3))]])/(b^(1/3)*(-(b*c) + a*d)^(2/3))) + Log[(-(b*c) + a*d)^(1/3)
+ b^(1/3)*(c + d*x)^(1/3)]/(b^(1/3)*(-(b*c) + a*d)^(2/3)) - Log[(-(b*c) + a
*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x
)^(2/3)]/(2*b^(1/3)*(-(b*c) + a*d)^(2/3))
```

**fricas [B]** time = 1.58, size = 900, normalized size = 6.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(3)*(b^2*c - a*b*d)*sqrt(-(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1
/3)/b)*log(-(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + sq
rt(3)*(2*(b^2*c - a*b*d)*(d*x + c)^(2/3) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d
^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)
^(1/3))*sqrt(-(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b) - 3*(b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d)*(d*x + c)^(1/3))/(b*x + a)) + (b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(2/3)
 - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b
^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)^(2/3)))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2), -1/2*(2*sqrt(3)*
(b^2*c - a*b*d)*sqrt((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b)*arctan(1/
3*sqrt(3)*((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) + 2*(b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3))*sqrt((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)^(1/3)/b)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + (b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(2/3) - (b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b^2*c*d +
a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(
2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*
d^2)^(2/3)))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)]
```

**giac** [A] time = 0.96, size = 207, normalized size = 1.48

$$-\frac{3\left(b^3c-ab^2d\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2(dx+c)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c-\sqrt{3}\,abd}-\frac{\left(b^3c-ab^2d\right)^{\frac{1}{3}}\log\left((dx+c)^{\frac{2}{3}}+(dx+c)^{\frac{1}{3}}\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\right)}{2\left(b^2c-abd\right)}+\frac{\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\log\left(\left|(dx+c)^{\frac{1}{3}}-\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right|\right)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] -3*(b^3*c - a*b^2*d)^(1/3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c -
a*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*a*b*d) - 1/2
*(b^3*c - a*b^2*d)^(1/3)*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)
/b)^(1/3) + ((b*c - a*d)/b)^(2/3))/(b^2*c - a*b*d) + ((b*c - a*d)/b)^(1/3)*
log(abs((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b*c - a*d)
```

**maple** [A] time = 0.01, size = 160, normalized size = 1.14

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(dx+c)^{\frac{1}{3}}}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}b}+\frac{\ln\left((dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}b}-\frac{\ln\left((dx+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(2/3),x)
```

```
[Out] 1/b/((a*d-b*c)/b)^(2/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))-1/2/b/((a*d-b
*c)/b)^(2/3)*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)/
b)^(2/3))+1/b/((a*d-b*c)/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/
b)^(1/3)*(d*x+c)^(1/3)-1))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 0.37, size = 206, normalized size = 1.47

$$\frac{\ln\left(9\,b^2\,{\left(c+d\,x\right)}^{1/3}-\frac{9\,b^3\,c-9\,a\,b^2\,d}{b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{\ln\left(9\,b^2\,{\left(c+d\,x\right)}^{1/3}-\frac{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{2\,b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}-\frac{\ln\left(9\,b^2\,{\left(c+d\,x\right)}^{1/3}+\frac{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{2\,b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(2/3)),x)
```

```
[Out] log(9*b^2*(c + d*x)^(1/3) - (9*b^3*c - 9*a*b^2*d)/(b^(1/3)*(a*d - b*c)^(2/3
)))/(b^(1/3)*(a*d - b*c)^(2/3)) + (log(9*b^2*(c + d*x)^(1/3) - ((3^(1/2)*1i
 - 1)*(9*b^3*c - 9*a*b^2*d))/(2*b^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i - 1
))/(2*b^(1/3)*(a*d - b*c)^(2/3)) - (log(9*b^2*(c + d*x)^(1/3) + ((3^(1/2)*1
i + 1)*(9*b^3*c - 9*a*b^2*d))/(2*b^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i +
1))/(2*b^(1/3)*(a*d - b*c)^(2/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx\right)\left(c + dx\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(2/3)), x)
```

## 3.1354 $\int (a+bx)^{7/2}\sqrt{c+dx}\,dx$

**Optimal**. Leaf size=230

$$\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{3/2}d^{9/2}}-\frac{7\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128bd^4}+\frac{7(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{192bd^3}-\frac{7(a+bx)^{5/2}\sqrt{c+}}{240b}$$

**Rubi [A]** time = 0.16, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{3/2}d^{9/2}}-\frac{7\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128bd^4}+\frac{7(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{192bd^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{240bd^2}+\frac{(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}{40bd}+\frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```

```
[Out] (-7*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b*d^4) + (7*(b*c - a*d)
^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(192*b*d^3) - (7*(b*c - a*d)^2*(a + b*x)^
(5/2)*Sqrt[c + d*x])/(240*b*d^2) + ((b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*
x])/(40*b*d) + ((a + b*x)^(9/2)*Sqrt[c + d*x])/(5*b) + (7*(b*c - a*d)^5*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(3/2)*d^(9/2)
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{7/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b} + \frac{(bc-ad)\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx}{10b}$$

$$= \frac{(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40bd} + \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b} - \frac{\left(7(bc-ad)^2\right)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{80bd}$$

$$= -\frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2} + \frac{(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40bd} + \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b}$$

$$= \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2} + \frac{(bc-ad)(a+bx)^{7/}}{40bd}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx}{240bd^2}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx}{240bd^2}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx}{240bd^2}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx}{240bd^2}$$

**Mathematica [A]** time = 1.47, size = 194, normalized size = 0.84

$$\frac{(a+bx)^{9/2}\sqrt{c+dx}\left(\frac{70(bc-ad)^{9/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{9/2}(a+bx)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}} - \frac{70(bc-ad)^4}{d^4(a+bx)^4} + \frac{140(bc-ad)^3}{3d^3(a+bx)^3} - \frac{112(bc-ad)^2}{3d^2(a+bx)^2} + \frac{32bc-32ad}{ad+bdx} + 256\right)}{1280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```

```
[Out] ((a + b*x)^(9/2)*Sqrt[c + d*x]*(256 - (70*(b*c - a*d)^4)/(d^4*(a + b*x)^4)
+ (140*(b*c - a*d)^3)/(3*d^3*(a + b*x)^3) - (112*(b*c - a*d)^2)/(3*d^2*(a +
 b*x)^2) + (32*b*c - 32*a*d)/(a*d + b*d*x) + (70*(b*c - a*d)^(9/2)*ArcSinh[
(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(9/2)*(a + b*x)^(9/2)*Sqrt[(b*
(c + d*x))/(b*c - a*d)])))/(1280*b)
```

**IntegrateAlgebraic [A]** time = 0.31, size = 198, normalized size = 0.86

$$\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{128b^{3/2}d^{9/2}}-\frac{\sqrt{c+dx}\,(bc-ad)^5\left(\frac{105b^4(c+dx)^4}{(a+bx)^4}-\frac{490b^3d(c+dx)^3}{(a+bx)^3}+\frac{896b^2d^2(c+dx)^2}{(a+bx)^2}-\frac{790bd^3(c+dx)}{a+bx}-105d^4\right)}{1920bd^4\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}-d\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```

```
[Out] -1/1920*((b*c - a*d)^5*Sqrt[c + d*x]*(-105*d^4 - (790*b*d^3*(c + d*x))/(a +
 b*x) + (896*b^2*d^2*(c + d*x)^2)/(a + b*x)^2 - (490*b^3*d*(c + d*x)^3)/(a
+ b*x)^3 + (105*b^4*(c + d*x)^4)/(a + b*x)^4))/(b*d^4*Sqrt[a + b*x]*(-d + (
b*(c + d*x))/(a + b*x))^5) + (7*(b*c - a*d)^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[a + b*x])])/(128*b^(3/2)*d^(9/2))
```

**fricas [A]** time = 1.74, size = 702, normalized size = 3.05

$$\left[-\frac{105\left(b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)-4\left(384b^5d^5x^4-105b^5c^4d+490ab^4c^3d^2-896a^2b^3c^2d^3+790a^3b^2cd^4+105a^4bd^5+48\left(b^5cd^4+31ab^4d^5\right)x^3-8\left(7b^5c^2d^3-32ab^4cd^4-263a^2b^3d^5\right)x^2+2\left(35b^5c^3d^2-161ab^4c^2d^3+289a^2b^3cd^4+605a^3b^2d^5\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{7680b^2d^5},\ -\frac{105\left(b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(384b^5d^5x^4-105b^5c^4d+490ab^4c^3d^2-896a^2b^3c^2d^3+790a^3b^2cd^4+105a^4bd^5+48\left(b^5cd^4+31ab^4d^5\right)x^3-8\left(7b^5c^2d^3-32ab^4cd^4-263a^2b^3d^5\right)x^2+2\left(35b^5c^3d^2-161ab^4c^2d^3+289a^2b^3cd^4+605a^3b^2d^5\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{3840b^2d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(105*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^
2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*
a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*
x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(384*b^5*d^5*x^4 - 105*b^5*c^4*d + 49
0*a*b^4*c^3*d^2 - 896*a^2*b^3*c^2*d^3 + 790*a^3*b^2*c*d^4 + 105*a^4*b*d^5 +
 48*(b^5*c*d^4 + 31*a*b^4*d^5)*x^3 - 8*(7*b^5*c^2*d^3 - 32*a*b^4*c*d^4 - 26
3*a^2*b^3*d^5)*x^2 + 2*(35*b^5*c^3*d^2 - 161*a*b^4*c^2*d^3 + 289*a^2*b^3*c*
d^4 + 605*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^5), -1/3840*(
105*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*
d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)
*x)) - 2*(384*b^5*d^5*x^4 - 105*b^5*c^4*d + 490*a*b^4*c^3*d^2 - 896*a^2*b^3
*c^2*d^3 + 790*a^3*b^2*c*d^4 + 105*a^4*b*d^5 + 48*(b^5*c*d^4 + 31*a*b^4*d^5
```

```
)*x^3 - 8*(7*b^5*c^2*d^3 - 32*a*b^4*c*d^4 - 263*a^2*b^3*d^5)*x^2 + 2*(35*b^
5*c^3*d^2 - 161*a*b^4*c^2*d^3 + 289*a^2*b^3*c*d^4 + 605*a^3*b^2*d^5)*x)*sqr
t(b*x + a)*sqrt(d*x + c))/(b^2*d^5)]
```

**giac [B]** time = 1.84, size = 1107, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*(480*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)
*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2
+ 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3
*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*abs(b) - 1920*((b^2*c - a*b*
d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^4*abs(b)/
b^2 + 40*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*
(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4
+ 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a
*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x +
 a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 3
5*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d -
a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*b*abs(b) + (sqrt(b^2*c + (b*x + a)*b*d - a*
b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^1
9*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/
(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 4
47*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3
*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^
8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*
a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x
+ a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*b^2*abs(b
) + 1920*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d
^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))
*a^3*abs(b)/b^2)/b
```

**maple [B]** time = 0.01, size = 858, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/5/d*(b*x+a)^(7/2)*(d*x+c)^(3/2)+7/40/d*(b*x+a)^(5/2)*(d*x+c)^(3/2)*a+7/48
/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)*a^2+7/64/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a^3+2
1/64/d^3*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a*b^2*c^2+21/64/d^2*(d*x+c)^(1/2)*(b*x
+a)^(1/2)*a^2*c^2*b-7/32/d^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^3*b^2-21/64/d^
2*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a^2*b*c-7/24/d^2*(b*x+a)^(3/2)*(d*x+c)^(3/2)*
a*b*c-7/40/d^2*(b*x+a)^(5/2)*(d*x+c)^(3/2)*b*c+7/48/d^3*(b*x+a)^(3/2)*(d*x+
c)^(3/2)*b^2*c^2-7/64/d^4*(b*x+a)^(1/2)*(d*x+c)^(3/2)*b^3*c^3+7/128/b*(d*x+
c)^(1/2)*(b*x+a)^(1/2)*a^4-7/32/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3*c+7/128/d
^4*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^4*b^3+35/256*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d
+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^4*c+35/128/d^2*((b*x+a)*(d*x+c))^(1/2)/(d
*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(
a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^2*c^3*b^2-35/256/d^3*((b*x+a)*(d*x+c))
^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(
d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a*c^4*b^3-7/256*d/b*((b*x+a)*(d
*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(
1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^5-35/128/d*((b*x+a)*(d*
x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1
/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^3*c^2*b+7/256/d^4*((b*x+
a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b
*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*c^5*b^4
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{7/2}\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(7/2)*(c + d*x)^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.1355 $\int (a+bx)^{5/2}\sqrt{c+dx}\,dx$

**Optimal**. Leaf size=192

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{3/2}d^{7/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64bd^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96bd^2}+\frac{(a+bx)^{5/2}\sqrt{c-}}{24b}$$

**Rubi [A]** time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{3/2}d^{7/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64bd^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96bd^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{24bd}+\frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```

```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b*d^3) - (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*b*d^2) + ((b*c - a*d)*(a + b*x)^(5/2)*S
qrt[c + d*x])/(24*b*d) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*b) - (5*(b*c -
a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(3/2
)*d^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b} + \frac{(bc-ad)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{8b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b} - \frac{\left(5(bc-ad)^2\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{48bd}$$

$$= -\frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}}{24bd}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}}{24bd}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}}{24bd}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}}{24bd}$$

**Mathematica [A]** time = 0.60, size = 190, normalized size = 0.99

$$\frac{b\sqrt{d}\,\sqrt{a+bx}\,(c+dx)\left(15a^3d^3+a^2bd^2(73c+118dx)+ab^2d\left(-55c^2+36cdx+136d^2x^2\right)+b^3\left(15c^3-10c^2dx+8cd^2x^2+48d^3x^3\right)\right)-15(bc-ad)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{192b^2d^{7/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(15*a^3*d^3 + a^2*b*d^2*(73*c + 118*d*x)
 + a*b^2*d*(-55*c^2 + 36*c*d*x + 136*d^2*x^2) + b^3*(15*c^3 - 10*c^2*d*x +
```

```
8*c*d^2*x^2 + 48*d^3*x^3)) - 15*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))/(b*c -
 a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(192*b^2*d^(7/2)*S
qrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.29, size = 176, normalized size = 0.92

$$\frac{\sqrt{c+dx}\,(bc-ad)^4\left(\frac{15b^3(c+dx)^3}{(a+bx)^3}-\frac{55b^2d(c+dx)^2}{(a+bx)^2}+\frac{73bd^2(c+dx)}{a+bx}+15d^3\right)}{192bd^3\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}-d\right)^4}-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{64b^{3/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)^4*Sqrt[c + d*x]*(15*d^3 + (73*b*d^2*(c + d*x))/(a + b*x) - (55
*b^2*d*(c + d*x)^2)/(a + b*x)^2 + (15*b^3*(c + d*x)^3)/(a + b*x)^3))/(192*b
*d^3*Sqrt[a + b*x]*(-d + (b*(c + d*x))/(a + b*x))^4) - (5*(b*c - a*d)^4*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(64*b^(3/2)*d^(7/2))
```

**fricas [A]** time = 1.42, size = 540, normalized size = 2.81

$$\left[\frac{15\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)+4\left(48b^4d^4x^3+15b^4c^3d-55ab^3c^2d^2+73a^2b^2cd^3+15a^3bd^4+8\left(b^4cd^3+17ab^3d^4\right)x^2-2\left(5b^4c^2d^2-18ab^3cd^3-59a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{768b^2d^4},\ \frac{15\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)+2\left(48b^4d^4x^3+15b^4c^3d-55ab^3c^2d^2+73a^2b^2cd^3+15a^3bd^4+8\left(b^4cd^3+17ab^3d^4\right)x^2-2\left(5b^4c^2d^2-18ab^3cd^3-59a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{384b^2d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*
b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) + 4*(48*b^4*d^4*x^3 + 15*b^4*c^3*d - 55*a*b^3*c^2*d^2 + 73*a^2*b^2
*c*d^3 + 15*a^3*b*d^4 + 8*(b^4*c*d^3 + 17*a*b^3*d^4)*x^2 - 2*(5*b^4*c^2*d^2
 - 18*a*b^3*c*d^3 - 59*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^
4), 1/384*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
+ a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48
*b^4*d^4*x^3 + 15*b^4*c^3*d - 55*a*b^3*c^2*d^2 + 73*a^2*b^2*c*d^3 + 15*a^3*
b*d^4 + 8*(b^4*c*d^3 + 17*a*b^3*d^4)*x^2 - 2*(5*b^4*c^2*d^2 - 18*a*b^3*c*d^
3 - 59*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^4)]
```

**giac [B]** time = 1.64, size = 726, normalized size = 3.78

$$\frac{24\left(\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\left(2(bx+a)\left(\frac{4(bx+a)}{b^2}+\frac{b^6cd^3-13ab^5d^4}{b^7d^4}\right)-\frac{3\left(b^7c^2d^2+2ab^6cd^3-11a^2b^5d^4\right)}{b^7d^4}\right)-\frac{3\left(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^2}\right)a|b|-\frac{192\left(\frac{\left(b^2c-abd\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\right)a^3|b|}{b^2}+\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2(bx+a)\left(4(bx+a)\left(\frac{6(bx+a)}{b^3}+\frac{b^{12}cd^5-25ab^{11}d^6}{b^{14}d^6}\right)-\frac{5b^{13}c^2d^4+14ab^{12}cd^5-163a^2b^{11}d^6}{b^{14}d^6}\right)+\frac{3\left(5b^{14}c^3d^3+9ab^{13}c^2d^4+15a^2b^{12}cd^5-93a^3b^{11}d^6\right)}{b^{14}d^6}\right)\sqrt{bx+a}+\frac{3\left(5b^4c^4+4ab^3c^3d+6a^2b^2c^2d^2+20a^3bcd^3-35a^4d^4\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,b^2d^3}\right)b|b|+\frac{144\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2bx+2a+\frac{bcd-5ad^2}{d^2}\right)\sqrt{bx+a}+\frac{\left(b^3c^2+2ab^2cd-3a^2bd^2\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)a^2|b|}{b^2}}{192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(24*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(
4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a
^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*abs(b) - 192*((b^2*c - a*b*d)*lo
g(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt
(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*abs(b)/b^2 +
 (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a
)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b
^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^
2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*
(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^
4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*b^2*d^3))*b*abs(b) + 144*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(
2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d
 - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b
*d - a*b*d)))/(sqrt(b*d)*d))*a^2*abs(b)/b^2)/b
```

**maple [B]** time = 0.01, size = 645, normalized size = 3.36

$$-\frac{5\sqrt{(bx+a)(dx+c)}\,a^4 d\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{5\sqrt{(bx+a)(dx+c)}\,a^3 c\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{15\sqrt{(bx+a)(dx+c)}\,a^2 b c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{64\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}+\frac{5\sqrt{(bx+a)(dx+c)}\,a b^2 c^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d^2}-\frac{5\sqrt{(bx+a)(dx+c)}\,b^3 c^4\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d^3}+\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,a^3}{64b}-\frac{15\sqrt{dx+c}\,\sqrt{bx+a}\,a^2 c}{64d}+\frac{15\sqrt{dx+c}\,\sqrt{bx+a}\,a b c^2}{64d^2}-\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,b^2 c^3}{64d^3}+\frac{5\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}} a^2}{32d}-\frac{5\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}} abc}{16d^2}+\frac{5\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}} b^2 c^2}{32d^3}+\frac{5(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}} a}{24d}-\frac{5(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}} bc}{24d^2}+\frac{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*(b*x+a)^(5/2)*(d*x+c)^(3/2)+5/24/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)*a-5/24
/d^2*(b*x+a)^(3/2)*(d*x+c)^(3/2)*b*c+5/32/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a^2
-5/16/d^2*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a*b*c+5/32/d^3*(b*x+a)^(1/2)*(d*x+c)^
(3/2)*b^2*c^2+5/64/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3-15/64/d*(d*x+c)^(1/2)*
(b*x+a)^(1/2)*a^2*c+15/64/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^2*b-5/64/d^3*
(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^3*b^2-5/128*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+
(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^4+5/32*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1
/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+
b*c)*x)^(1/2))/(b*d)^(1/2)*a^3*c-15/64/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1
/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+
b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*c^2*b+5/32/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+
(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*c^3*b^2-5/128/d^3*((b*x+a)*(d*x+c))^(1/2)
/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^
2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^4*b^3
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{5/2}\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(5/2)*(c + d*x)^(1/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.1356 $\int (a+bx)^{3/2}\sqrt{c+dx}\,dx$

**Optimal**. Leaf size=154

$$\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8bd^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12bd}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

**Rubi [A]** time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8bd^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12bd}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] -((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b*d^2) + ((b*c - a*d)*(a +
b*x)^(3/2)*Sqrt[c + d*x])/(12*b*d) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*b)
+ ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/
(8*b^(3/2)*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6b}$$

$$= \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} - \frac{(bc-ad)^2\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{8bd}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

**Mathematica [A]** time = 0.43, size = 151, normalized size = 0.98

$$\frac{b\sqrt{d}\,\sqrt{a+bx}\,(c+dx)\left(3a^2d^2+2abd(4c+7dx)+b^2\left(-3c^2+2cdx+8d^2x^2\right)\right)+3(bc-ad)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{24b^2d^{5/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(3*a^2*d^2 + 2*a*b*d*(4*c + 7*d*x) + b^2
*(-3*c^2 + 2*c*d*x + 8*d^2*x^2)) + 3*(b*c - a*d)^(7/2)*Sqrt[(b*(c + d*x))/(
b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(24*b^2*d^(5/
2)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.26, size = 154, normalized size = 1.00

$$\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{c+dx}\,(bc-ad)^3\left(\frac{3b^2(c+dx)^2}{(a+bx)^2}-\frac{8bd(c+dx)}{a+bx}-3d^2\right)}{24bd^2\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}-d\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] -1/24*((b*c - a*d)^3*Sqrt[c + d*x]*(-3*d^2 - (8*b*d*(c + d*x))/(a + b*x) +
(3*b^2*(c + d*x)^2)/(a + b*x)^2))/(b*d^2*Sqrt[a + b*x]*(-d + (b*(c + d*x))/
(a + b*x))^3) + ((b*c - a*d)^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqr
t[a + b*x])])/(8*b^(3/2)*d^(5/2))
```

**fricas [A]** time = 1.37, size = 410, normalized size = 2.66

$$\left[-\frac{3\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4\,(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)-4\left(8b^3d^3x^2-3b^3c^2d+8ab^2cd^2+3a^2bd^3+2\left(b^3cd^2+7ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{96b^2d^3},\ -\frac{3\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(8b^3d^3x^2-3b^3c^2d+8ab^2cd^2+3a^2bd^3+2\left(b^3cd^2+7ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48b^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sq
rt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d
^3*x^2 - 3*b^3*c^2*d + 8*a*b^2*c*d^2 + 3*a^2*b*d^3 + 2*(b^3*c*d^2 + 7*a*b^2
*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^3), -1/48*(3*(b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a
*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*
d + a*b*d^2)*x)) - 2*(8*b^3*d^3*x^2 - 3*b^3*c^2*d + 8*a*b^2*c*d^2 + 3*a^2*b
*d^3 + 2*(b^3*c*d^2 + 7*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^3
)]
```

**giac [B]** time = 1.38, size = 438, normalized size = 2.84

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b^2}+\frac{b^6cd^3-13\,ab^5d^4}{b^7d^4}\right)-\frac{3\left(b^7c^2d^2+2\,ab^6cd^3-11\,a^2b^5d^4\right)}{b^7d^4}\right)-\frac{3\left(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^2}\right)|b|-\frac{24\left(\frac{\left(b^2c-abd\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\right)a^2|b|}{b^2}+\frac{12\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,bx+2\,a+\frac{bcd-5\,ad^2}{d^2}\right)\sqrt{bx+a}+\frac{\left(b^3c^2+2\,ab^2cd-3\,a^2bd^2\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)a|b|}{b^2}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b
*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*
b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b
```

```
*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*abs(b) - 24*((b^2*c - a*b*d)*log(abs(-
sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*abs(b)/b^2 + 12*(sq
rt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqr
t(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*a*abs(b)/b^
2)/b
```

**maple [B]** time = 0.01, size = 460, normalized size = 2.99

$$-\frac{\sqrt{(bx+a)(dx+c)}\,a^3 d\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{3\sqrt{(bx+a)(dx+c)}\,a^2 c\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{3\sqrt{(bx+a)(dx+c)}\,ab\,c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}+\frac{\sqrt{(bx+a)(dx+c)}\,b^2c^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d^2}+\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,a^2}{8b}-\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,ac}{4d}+\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,b\,c^2}{8d^2}+\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}a}{4d}-\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}bc}{4d^2}+\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/3/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)+1/4/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a-1/4/d
^2*(b*x+a)^(1/2)*(d*x+c)^(3/2)*b*c+1/8/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2-1/
4/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c+1/8/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^2
*b-1/16*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1
/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^
3+3/16*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*
d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*c-3
/16/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d
+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*c^2*b+
1/16/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*
a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^3*b
^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{3/2}\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.1357 $\int \sqrt{a+bx}\sqrt{c+dx}\,dx$

**Optimal**. Leaf size=116

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

**Rubi** [A] time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3/2)*Sqrt[c
 + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sq
rt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4b}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{8bd}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, \right.}{4b^2d}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \right.}{4b^2d}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}$$

**Mathematica [A]** time = 0.28, size = 118, normalized size = 1.02

$$\frac{b\sqrt{d}\sqrt{a+bx}(c+dx)(ad+b(c+2dx)) - (bc-ad)^{5/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4b^2d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(a*d + b*(c + 2*d*x)) - (b*c - a*d)^(5/2
)*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c
- a*d]])/(4*b^2*d^(3/2)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.21, size = 129, normalized size = 1.11

$$\frac{\sqrt{c+dx}(bc-ad)^2\left(\frac{b(c+dx)}{a+bx}+d\right)}{4bd\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}-d\right)^2} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{4b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x]*(d + (b*(c + d*x))/(a + b*x)))/(4*b*d*Sqrt[a +
 b*x]*(-d + (b*(c + d*x))/(a + b*x))^2) - ((b*c - a*d)^2*ArcTanh[(Sqrt[b]*S
qrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(3/2)*d^(3/2))
```

**fricas [A]** time = 1.50, size = 300, normalized size = 2.59

$$\left[\frac{\left(b^{2} c^{2}-2 \, a b c d+a^{2} d^{2}\right) \sqrt{b d} \log \left(8 \, b^{2} d^{2} x^{2}+b^{2} c^{2}+6 \, a b c d+a^{2} d^{2}-4 \, (2 \, b d x+b c+a d) \sqrt{b d} \sqrt{b x+a} \sqrt{d x+c}+8 \, \left(b^{2} c d+a b d^{2}\right) x\right)+4 \, \left(2 \, b^{2} d^{2} x+b^{2} c d+a b d^{2}\right) \sqrt{b x+a} \sqrt{d x+c}}{16 \, b^{2} d^{2}}, \frac{\left(b^{2} c^{2}-2 \, a b c d+a^{2} d^{2}\right) \sqrt{-b d} \arctan \left(\frac{(2 \, b d x+b c+a d) \sqrt{-b d} \sqrt{b x+a} \sqrt{d x+c}}{2 \, \left(b^{2} d^{2} x^{2}+a b c d+\left(b^{2} c d+a b d^{2}\right) x\right)}\right)+2 \, \left(2 \, b^{2} d^{2} x+b^{2} c d+a b d^{2}\right) \sqrt{b x+a} \sqrt{d x+c}}{8 \, b^{2} d^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^
2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*s
qrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + b^2*c*d + a*b*d^
2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2), 1/8*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*s
qrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(2*b^2*d^
2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2)]
```

**giac [B]** time = 1.25, size = 232, normalized size = 2.00

$$-\frac{\frac{4 \, \left(\frac{\left(b^{2} c-a b d\right) \log \left(\left|-\sqrt{b d} \sqrt{b x+a}+\sqrt{b^{2} c+(b x+a) b d-a b d}\right|\right)}{\sqrt{b d}}-\sqrt{b^{2} c+(b x+a) b d-a b d} \sqrt{b x+a}\right) a |b|}{b^{2}}-\frac{\left(\sqrt{b^{2} c+(b x+a) b d-a b d} \left(2 \, b x+2 \, a+\frac{b c d-5 \, a d^{2}}{d^{2}}\right) \sqrt{b x+a}+\frac{\left(b^{3} c^{2}+2 \, a b^{2} c d-3 \, a^{2} b d^{2}\right) \log \left(\left|-\sqrt{b d} \sqrt{b x+a}+\sqrt{b^{2} c+(b x+a) b d-a b d}\right|\right)}{\sqrt{b d} d}\right) |b|}{b^{2}}}{4 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt
(b*x + a))*a*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
 + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*
d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*d))*abs(b)/b^2)/b
```

**maple [B]** time = 0.01, size = 305, normalized size = 2.63

$$-\frac{\sqrt{(b x+a) (d x+c)}\, a^{2} d \ln \left(\frac{b d x+\frac{1}{2} a d+\frac{1}{2} b c}{\sqrt{b d}}+\sqrt{b d \,x^{2}+a c+(a d+b c) x}\right)}{8 \sqrt{d x+c}\, \sqrt{b x+a}\, \sqrt{b d}\, b}+\frac{\sqrt{(b x+a) (d x+c)}\, a c \ln \left(\frac{b d x+\frac{1}{2} a d+\frac{1}{2} b c}{\sqrt{b d}}+\sqrt{b d \,x^{2}+a c+(a d+b c) x}\right)}{4 \sqrt{d x+c}\, \sqrt{b x+a}\, \sqrt{b d}}-\frac{\sqrt{(b x+a) (d x+c)}\, b \,c^{2} \ln \left(\frac{b d x+\frac{1}{2} a d+\frac{1}{2} b c}{\sqrt{b d}}+\sqrt{b d \,x^{2}+a c+(a d+b c) x}\right)}{8 \sqrt{d x+c}\, \sqrt{b x+a}\, \sqrt{b d}\, d}+\frac{\sqrt{d x+c}\, \sqrt{b x+a}\, a}{4 b}-\frac{\sqrt{d x+c}\, \sqrt{b x+a}\, c}{4 d}+\frac{\sqrt{b x+a}\, (d x+c)^{\frac{3}{2}}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)+1/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a-1/4/d
*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c-1/8*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*
c)*x)^(1/2))/(b*d)^(1/2)*a^2+1/4*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x
+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^
(1/2))/(b*d)^(1/2)*a*c-1/8/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^
(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2
))/(b*d)^(1/2)*c^2*b
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.14, size = 88, normalized size = 0.76

$$\left(\frac{x}{2}+\frac{a\,d+b\,c}{4\,b\,d}\right)\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}-\frac{\ln\left(a\,d+b\,c+2\,b\,d\,x+2\,\sqrt{b}\,\sqrt{d}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}\right)\,{\left(a\,d-b\,c\right)}^2}{8\,b^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/2),x)
```

```
[Out] (x/2 + (a*d + b*c)/(4*b*d))*(a + b*x)^(1/2)*(c + d*x)^(1/2) - (log(a*d + b*
c + 2*b*d*x + 2*b^(1/2)*d^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))*(a*d - b*c
)^2)/(8*b^(3/2)*d^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx}\,\sqrt{c + dx}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x), x)
```

## 3.1358 $\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}} \, dx$

**Optimal**. Leaf size=72

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}$$

**Rubi [A]** time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2b}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b^2}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^2}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}$$

**Mathematica [A]** time = 0.10, size = 117, normalized size = 1.62

$$\frac{\sqrt{c+dx}\left(\sqrt{d}\,\sqrt{a+bx}\,\sqrt{\frac{b(c+dx)}{bc-ad}} + \sqrt{bc-ad}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{b\sqrt{d}\,\sqrt{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[d]*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)] + Sqr
t[b*c - a*d]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(b*Sqrt[d]*
Sqrt[(b*(c + d*x))/(b*c - a*d)])
```

**IntegrateAlgebraic [A]** time = 0.31, size = 104, normalized size = 1.44

$$\frac{\sqrt{c+dx}\,\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}}{b} - \frac{\sqrt{\frac{b}{d}}\,(bc-ad)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}} - \sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[a - (b*c)/d + (b*(c + d*x))/d])/b - (Sqrt[b/d]*(b*c - a
*d)*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) + Sqrt[a - (b*c)/d + (b*(c + d*x))/d]])/
b^2
```

**fricas [A]** time = 1.32, size = 236, normalized size = 3.28

$$\left[\frac{4\sqrt{bx+a}\sqrt{dx+c}\,bd-(bc-ad)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)}{4\,b^2d}, \frac{2\sqrt{bx+a}\sqrt{dx+c}\,bd-(bc-ad)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)}{2\,b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b^2*d
^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x))/(b^2*d), 1/2*(2*sqr
t(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(-b*d)*arctan(1/2*(2*b*d*x +
 b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x)))/(b^2*d)]
```

**giac [A]** time = 1.10, size = 93, normalized size = 1.29

$$-\frac{\left(\frac{\left(b^2c-abd\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)|b|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x +
a))*abs(b)/b^3
```

**maple [A]** time = 0.01, size = 107, normalized size = 1.49

$$-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{2\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] (b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+
(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 4.01, size = 260, normalized size = 3.61

$$\frac{\frac{(2ad+2bc)\left(\sqrt{a+bx}-\sqrt{a}\right)}{d^2\left(\sqrt{c+dx}-\sqrt{c}\right)}+\frac{(2ad+2bc)\left(\sqrt{a+bx}-\sqrt{a}\right)^3}{bd\left(\sqrt{c+dx}-\sqrt{c}\right)^3}-\frac{8\sqrt{a}\sqrt{c}\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{d\left(\sqrt{c+dx}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{\left(\sqrt{c+dx}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2b\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{d\left(\sqrt{c+dx}-\sqrt{c}\right)^2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{c+dx}-\sqrt{c}\right)}\right)(ad-bc)}{b^{3/2}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(1/2),x)
```

```
[Out] (((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2)))/(d^2*((c + d*x)^(1/2) - c^(1
/2))) + ((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2))^3)/(b*d*((c + d*x)^(1/
2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(d*((c
 + d*x)^(1/2) - c^(1/2))^2))/(((a + b*x)^(1/2) - a^(1/2))^4/((c + d*x)^(1/2
) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(d*((c + d*x
)^(1/2) - c^(1/2))^2)) - (2*atanh((d^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^
(1/2)*((c + d*x)^(1/2) - c^(1/2))))*(a*d - b*c))/(b^(3/2)*d^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/sqrt(a + b*x), x)
```

## 3.1359 $\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}} \, dx$

**Optimal**. Leaf size=66

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}$$

**Rubi [A]** time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {47, 63, 217, 206}

$$\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx = -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}} + \frac{d\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{b}$$

$$= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}} + \frac{(2d)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b^2}$$

$$= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}} + \frac{(2d)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^2}$$

$$= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}} + \frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}}$$

**Mathematica [A]** time = 0.23, size = 99, normalized size = 1.50

$$\frac{2\left(\sqrt{d}\,\sqrt{bc-ad}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)-\frac{b(c+dx)}{\sqrt{a+bx}}\right)}{b^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-((b*(c + d*x))/Sqrt[a + b*x]) + Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d
*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(b^2*S
qrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.10, size = 66, normalized size = 1.00

$$\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (2*Sqrt[d]*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x])/(Sqrt[d]*Sqrt[a + b*x])])/b^(3/2)
```

**fricas [B]** time = 1.48, size = 241, normalized size = 3.65

$$\left[\frac{(bx+a)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,b^2dx+b^2c+abd\right)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)-4\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\left(b^2x+ab\right)},\ -\frac{(bx+a)\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(2\,bdx+bc+ad\right)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right)+2\,\sqrt{bx+a}\,\sqrt{dx+c}}{b^2x+ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*x + a)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*x + a*b), -((b
*x + a)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) + 2*sqrt(b*x + a)*
sqrt(d*x + c))/(b^2*x + a*b)]
```

**giac [B]** time = 1.19, size = 131, normalized size = 1.98

$$-\frac{\left(\frac{\sqrt{bd}\,\log\left(\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{b}+\frac{4\left(\sqrt{bd}\,bc-\sqrt{bd}\,ad\right)}{b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(b*d)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^2)/b + 4*(sqrt(b*d)*b*c - sqrt(b*d)*a*d)/(b^2*c - a*b*d - (sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2))*abs(b)/b^2
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(1/2)/(b*x+a)^(3/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c+d\,x}}{{\left(a+b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(1/2)/(a + b*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx}}{\left(a + bx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(3/2), x)
```

## 3.1360 $\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=32

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

**Rubi** [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{3/2}}{3(bc-ad)(a+bx)^{3/2}}$$

**Mathematica** [A] time = 0.01, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

**fricas** [B] time = 1.53, size = 65, normalized size = 2.03

$$-\frac{2\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{3\left(a^2bc-a^3d+\left(b^3c-ab^2d\right)x^2+2\left(ab^2c-a^2bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x + a)*(d*x + c)^(3/2)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x^2
 + 2*(a*b^2*c - a^2*b*d)*x)
```

**giac** [B] time = 1.43, size = 152, normalized size = 4.75

$$-\frac{4\left(\sqrt{bd}\,b^4c^2d-2\sqrt{bd}\,ab^3cd^2+\sqrt{bd}\,a^2b^2d^3+3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4d\right)|b|}{3\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(sqrt(b*d)*b^4*c^2*d - 2*sqrt(b*d)*a*b^3*c*d^2 + sqrt(b*d)*a^2*b^2*d^3
 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*d)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2)^3*b^2)
```

**maple** [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(5/2),x)
```

```
[Out] 2/3/(b*x+a)^(3/2)*(d*x+c)^(3/2)/(a*d-b*c)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.72, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}}{\left(3\,a\,d-3\,b\,c\right)\,{\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(3/2))/((3*a*d - 3*b*c)*(a + b*x)^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx}}{\left(a + bx\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(5/2), x)
```

## 3.1361 $\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx$

**Optimal**. Leaf size=66

$$\frac{4d(c+dx)^{3/2}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{5(a+bx)^{5/2}(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4d(c+dx)^{3/2}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (4*d*(c + d*x)^(3/2)
)/(15*(b*c - a*d)^2*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}} - \frac{(2d)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx}{5(bc-ad)}$$
$$= -\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}} + \frac{4d(c+dx)^{3/2}}{15(bc-ad)^2(a+bx)^{3/2}}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{3/2}(5ad-3bc+2bdx)}{15(a+bx)^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-3*b*c + 5*a*d + 2*b*d*x))/(15*(b*c - a*d)^2*(a + b*x)^
(5/2))
```

**IntegrateAlgebraic** [A] time = 0.10, size = 57, normalized size = 0.86

$$-\frac{2\left(\frac{3b(c+dx)^{5/2}}{(a+bx)^{5/2}} - \frac{5d(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{15(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
```

```
[Out] (-2*((-5*d*(c + d*x)^(3/2))/(a + b*x)^(3/2) + (3*b*(c + d*x)^(5/2))/(a + b*
x)^(5/2)))/(15*(b*c - a*d)^2)
```

**fricas** [B] time = 2.32, size = 175, normalized size = 2.65

$$\frac{2\left(2\,bd^2x^2 - 3\,bc^2 + 5\,acd - \left(bcd - 5\,ad^2\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{15\left(a^3b^2c^2 - 2\,a^4bcd + a^5d^2 + \left(b^5c^2 - 2\,ab^4cd + a^2b^3d^2\right)x^3 + 3\left(ab^4c^2 - 2\,a^2b^3cd + a^3b^2d^2\right)x^2 + 3\left(a^2b^3c^2 - 2\,a^3b^2cd + a^4bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b*d^2*x^2 - 3*b*c^2 + 5*a*c*d - (b*c*d - 5*a*d^2)*x)*sqrt(b*x + a)*
sqrt(d*x + c)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4*c*d
 + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 + 3*(
a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x)
```

**giac [B]** time = 1.44, size = 447, normalized size = 6.77

$$\frac{8\left(\sqrt{bd}\,b^{7}c^{3}d^{2}-3\sqrt{bd}\,ab^{6}c^{2}d^{3}+3\sqrt{bd}\,a^{2}b^{5}cd^{4}-\sqrt{bd}\,a^{3}b^{4}d^{5}-5\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}b^{5}c^{2}d^{2}+10\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}ab^{4}cd^{3}-5\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}a^{2}b^{3}d^{4}-5\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}b^{3}cd^{2}+5\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}ab^{2}d^{3}-15\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{6}bd^{2}\right)|b|}{15\left(b^{2}c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)^{5}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] 8/15*(sqrt(b*d)*b^7*c^3*d^2 - 3*sqrt(b*d)*a*b^6*c^2*d^3 + 3*sqrt(b*d)*a^2*b
^5*c*d^4 - sqrt(b*d)*a^3*b^4*d^5 - 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^5*c^2*d^2 + 10*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^4*c*d^3 - 5*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^
2*b^3*d^4 - 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^4*b^3*c*d^2 + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*a*b^2*d^3 - 15*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b*d^2)*abs(b)/((b^2*c - a*b*d
 - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*b^2
)
```

**maple [A]** time = 0.01, size = 54, normalized size = 0.82

$$\frac{2\,(dx+c)^{\frac{3}{2}}\,(2bdx+5ad-3bc)}{15\,(bx+a)^{\frac{5}{2}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(7/2),x)
```

```
[Out] 2/15*(d*x+c)^(3/2)*(2*b*d*x+5*a*d-3*b*c)/(b*x+a)^(5/2)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.82, size = 127, normalized size = 1.92

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x\,\left(10\,a\,d^2-2\,b\,c\,d\right)}{15\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{6\,b\,c^2-10\,a\,c\,d}{15\,b^2\,{\left(a\,d-b\,c\right)}^2}+\frac{4\,d^2\,x^2}{15\,b\,{\left(a\,d-b\,c\right)}^2}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a^2\,\sqrt{a+b\,x}}{b^2}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(7/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x*(10*a*d^2 - 2*b*c*d))/(15*b^2*(a*d - b*c)^2) - (6*b*c^
2 - 10*a*c*d)/(15*b^2*(a*d - b*c)^2) + (4*d^2*x^2)/(15*b*(a*d - b*c)^2)))/(
x^2*(a + b*x)^(1/2) + (a^2*(a + b*x)^(1/2))/b^2 + (2*a*x*(a + b*x)^(1/2))/b
)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

## 3.1362 $\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}} dx$

**Optimal**. Leaf size=101

$$-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{3/2}(bc-ad)^3}+\frac{8d(c+dx)^{3/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{7(a+bx)^{7/2}(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{3/2}(bc-ad)^3}+\frac{8d(c+dx)^{3/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (8*d*(c + d*x)^(3/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}} - \frac{(4d)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx}{7(bc-ad)}$$
$$= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}} + \frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}} + \frac{\left(8d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx}{35(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}} + \frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}} - \frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{3/2}}$$

**Mathematica [A]** time = 0.04, size = 77, normalized size = 0.76

$$-\frac{2(c+dx)^{3/2}\left(35a^2d^2+14abd(2dx-3c)+b^2\left(15c^2-12cdx+8d^2x^2\right)\right)}{105(a+bx)^{7/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-3*c + 2*d*x) + b^2*(15*c^2 - 1
2*c*d*x + 8*d^2*x^2)))/(105*(b*c - a*d)^3*(a + b*x)^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.10, size = 83, normalized size = 0.82

$$-\frac{2\left(\frac{15b^2(c+dx)^{7/2}}{(a+bx)^{7/2}}+\frac{35d^2(c+dx)^{3/2}}{(a+bx)^{3/2}}-\frac{42bd(c+dx)^{5/2}}{(a+bx)^{5/2}}\right)}{105(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*((35*d^2*(c + d*x)^(3/2))/(a + b*x)^(3/2) - (42*b*d*(c + d*x)^(5/2))/(a
 + b*x)^(5/2) + (15*b^2*(c + d*x)^(7/2))/(a + b*x)^(7/2)))/(105*(b*c - a*d)
^3)
```

**fricas [B]** time = 3.93, size = 337, normalized size = 3.34

$$-\frac{2\left(8b^2d^3x^3+15b^2c^3-42abc^2d+35a^2cd^2-4\left(b^2cd^2-7abd^3\right)x^2+\left(3b^2c^2d-14abcd^2+35a^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{105\left(a^4b^3c^3-3a^5b^2c^2d+3a^6bcd^2-a^7d^3+\left(b^7c^3-3ab^6c^2d+3a^2b^5cd^2-a^3b^4d^3\right)x^4+4\left(ab^6c^3-3a^2b^5c^2d+3a^3b^4cd^2-a^4b^3d^3\right)x^3+6\left(a^2b^5c^3-3a^3b^4c^2d+3a^4b^3cd^2-a^5b^2d^3\right)x^2+4\left(a^3b^4c^3-3a^4b^3c^2d+3a^5b^2cd^2-a^6bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

[Out] -2/105*(8*b^2*d^3*x^3 + 15*b^2*c^3 - 42*a*b*c^2*d + 35*a^2*c*d^2 - 4*(b^2*c
*d^2 - 7*a*b*d^3)*x^2 + (3*b^2*c^2*d - 14*a*b*c*d^2 + 35*a^2*d^3)*x)*sqrt(b
*x + a)*sqrt(d*x + c)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*
d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 + 4*(a*
b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^2*b^5
*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b^4*c^
3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x)

**giac** [B] time = 1.58, size = 689, normalized size = 6.82

$$-\frac{32\left(\sqrt{bd}\,b^{10}c^4d^3-4\sqrt{bd}\,ab^9c^3d^4+6\sqrt{bd}\,a^2b^8c^2d^5-4\sqrt{bd}\,a^3b^7cd^6+\sqrt{bd}\,a^4b^6d^7-7\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^8c^3d^3+21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^7c^2d^4-21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2b^6cd^5+7\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^3b^5d^6+21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^6c^2d^3-42\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ab^5cd^4+21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^2b^4d^5+35\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6b^4cd^3-35\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6ab^3d^4+70\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^8b^2d^3\right)|b|}{105\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="giac")

[Out] -32/105*(sqrt(b*d)*b^10*c^4*d^3 - 4*sqrt(b*d)*a*b^9*c^3*d^4 + 6*sqrt(b*d)*a
^2*b^8*c^2*d^5 - 4*sqrt(b*d)*a^3*b^7*c*d^6 + sqrt(b*d)*a^4*b^6*d^7 - 7*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^8
*c^3*d^3 + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a*b^7*c^2*d^4 - 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c*d^5 + 7*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^5*d^6 + 21*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^6
*c^2*d^3 - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^4*a*b^5*c*d^4 + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b
^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^4*d^5 + 35*sqrt(b*d)*(sqrt(b*d)*sqrt
(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^4*c*d^3 - 35*sqrt(b*d)
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^3*d^
4 + 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^8*b^2*d^3)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b
^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b^2)

**maple** [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{2\,(dx+c)^{\frac{3}{2}}\left(8b^2x^2d^2+28ab\,d^2x-12b^2cdx+35a^2d^2-42abcd+15b^2c^2\right)}{105\,(bx+a)^{\frac{7}{2}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/2)/(b*x+a)^(9/2),x)

[Out] 2/105*(d*x+c)^(3/2)*(8*b^2*d^2*x^2+28*a*b*d^2*x-12*b^2*c*d*x+35*a^2*d^2-42*
a*b*c*d+15*b^2*c^2)/(b*x+a)^(7/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*
c^3)

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.97, size = 203, normalized size = 2.01

$$\frac{\sqrt{c+d\,x}\,\left(\frac{70\,a^2\,c\,d^2-84\,a\,b\,c^2\,d+30\,b^2\,c^3}{105\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x\,\left(70\,a^2\,d^3-28\,a\,b\,c\,d^2+6\,b^2\,c^2\,d\right)}{105\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^3\,x^3}{105\,b\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^2\,x^2\,\left(7\,a\,d-b\,c\right)}{105\,b^2\,{\left(a\,d-b\,c\right)}^3}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(9/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((30*b^2*c^3 + 70*a^2*c*d^2 - 84*a*b*c^2*d)/(105*b^3*(a*d
- b*c)^3) + (x*(70*a^2*d^3 + 6*b^2*c^2*d - 28*a*b*c*d^2))/(105*b^3*(a*d - b
*c)^3) + (16*d^3*x^3)/(105*b*(a*d - b*c)^3) + (8*d^2*x^2*(7*a*d - b*c))/(10
5*b^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/2) + (a^3*(a + b*x)^(1/2))/b^3 + (
3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(a + b*x)^(1/2))/b^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

## 3.1363 $\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx$

**Optimal**. Leaf size=136

$$\frac{32d^3(c+dx)^{3/2}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{4d(c+dx)^{3/2}}{21(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{9(a+bx)^{9/2}(bc-ad)}$$

**Rubi [A]** time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3(c+dx)^{3/2}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{4d(c+dx)^{3/2}}{21(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(3/2)
)/(21*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(5/2)) + (32*d^3*(c + d*x)^(3/2))/(315*(b*c - a*d)^4*(a +
 b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx = -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(2d)\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx}{3(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} + \frac{\left(8d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx}{21(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} - \frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}} - \frac{\left(16d^3\right)\int \frac{\sqrt{c\cdots}}{(a+\cdots}}{105(bc-\cdots}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} - \frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}} + \frac{32d^3(c\cdots}{315(bc-ad\cdots}$$

**Mathematica** [A]    time = 0.05, size = 118, normalized size = 0.87

$$\frac{2(c+dx)^{3/2}\left(105a^3d^3+63a^2bd^2(2dx-3c)+9ab^2d\left(15c^2-12cdx+8d^2x^2\right)+b^3\left(-35c^3+30c^2dx-24cd^2x^2+16d^3x^3\right)\right)}{315(a+bx)^{9/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
```

```
[Out] (2*(c + d*x)^(3/2)*(105*a^3*d^3 + 63*a^2*b*d^2*(-3*c + 2*d*x) + 9*a*b^2*d*(
15*c^2 - 12*c*d*x + 8*d^2*x^2) + b^3*(-35*c^3 + 30*c^2*d*x - 24*c*d^2*x^2 +
 16*d^3*x^3)))/(315*(b*c - a*d)^4*(a + b*x)^(9/2))
```

**IntegrateAlgebraic** [A]    time = 0.11, size = 109, normalized size = 0.80

$$-\frac{2\left(\frac{35b^3(c+dx)^{9/2}}{(a+bx)^{9/2}}-\frac{135b^2d(c+dx)^{7/2}}{(a+bx)^{7/2}}-\frac{105d^3(c+dx)^{3/2}}{(a+bx)^{3/2}}+\frac{189bd^2(c+dx)^{5/2}}{(a+bx)^{5/2}}\right)}{315(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
```

```
[Out] (-2*((-105*d^3*(c + d*x)^(3/2))/(a + b*x)^(3/2) + (189*b*d^2*(c + d*x)^(5/2
))/(a + b*x)^(5/2) - (135*b^2*d*(c + d*x)^(7/2))/(a + b*x)^(7/2) + (35*b^3*
(c + d*x)^(9/2))/(a + b*x)^(9/2)))/(315*(b*c - a*d)^4)
```

**fricas** [B]    time = 13.04, size = 532, normalized size = 3.91

$$\frac{2\left(16\,b^3d^4x^4-35\,b^3c^4+135\,ab^2c^3d-189\,a^2bc^2d^2+105\,a^3cd^3-8\left(b^3cd^3-9\,ab^2d^4\right)x^3+6\left(b^3c^2d^2-6\,ab^2cd^3+21\,a^2bd^4\right)x^2-\left(5\,b^3c^3d-27\,ab^2c^2d^2+63\,a^2bcd^3-105\,a^3d^4\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{315\left(a^5b^4c^4-4\,a^6b^3c^3d+6\,a^7b^2c^2d^2-4\,a^8bcd^3+a^9d^4+\left(b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4\right)x^5+5\left(ab^8c^4-4\,a^2b^7c^3d+6\,a^3b^6c^2d^2-4\,a^4b^5cd^3+a^5b^4d^4\right)x^4+10\left(a^2b^7c^4-4\,a^3b^6c^3d+6\,a^4b^5c^2d^2-4\,a^5b^4cd^3+a^6b^3d^4\right)x^3+10\left(a^3b^6c^4-4\,a^4b^5c^3d+6\,a^5b^4c^2d^2-4\,a^6b^3cd^3+a^7b^2d^4\right)x^2+5\left(a^4b^5c^4-4\,a^5b^4c^3d+6\,a^6b^3c^2d^2-4\,a^7b^2cd^3+a^8bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*b^3*d^4*x^4 - 35*b^3*c^4 + 135*a*b^2*c^3*d - 189*a^2*b*c^2*d^2 +
105*a^3*c*d^3 - 8*(b^3*c*d^3 - 9*a*b^2*d^4)*x^3 + 6*(b^3*c^2*d^2 - 6*a*b^2*
c*d^3 + 21*a^2*b*d^4)*x^2 - (5*b^3*c^3*d - 27*a*b^2*c^2*d^2 + 63*a^2*b*c*d^
3 - 105*a^3*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^5*b^4*c^4 - 4*a^6*b^3*c^
3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4 + (b^9*c^4 - 4*a*b^8*c^3*
d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^5 + 5*(a*b^8*c^4 -
 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^4 +
 10*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 +
a^6*b^3*d^4)*x^3 + 10*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 -
4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x^2 + 5*(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a
^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*x)
```

**giac [B]** time = 2.04, size = 989, normalized size = 7.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="giac")
```

```
[Out] 64/315*(sqrt(b*d)*b^13*c^5*d^4 - 5*sqrt(b*d)*a*b^12*c^4*d^5 + 10*sqrt(b*d)*
a^2*b^11*c^3*d^6 - 10*sqrt(b*d)*a^3*b^10*c^2*d^7 + 5*sqrt(b*d)*a^4*b^9*c*d^
8 - sqrt(b*d)*a^5*b^8*d^9 - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*b^11*c^4*d^4 + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^10*c^3*d^5 - 54*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*
b^9*c^2*d^6 + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^3*b^8*c*d^7 - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^7*d^8 + 36*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^9*c^3*d^4 - 108*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a
*b^8*c^2*d^5 + 108*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^4*a^2*b^7*c*d^6 - 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^6*d^7 - 84*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^7*c^2*d^4 + 168
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
6*a*b^6*c*d^5 - 84*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^6*a^2*b^5*d^6 - 189*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^5*c*d^4 + 189*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^4*d^5 - 315*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b
^3*d^4)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2)^9*b^2)
```

**maple** [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{2 (dx+c)^{\frac{3}{2}} \left(16b^3x^3d^3+72a\,b^2d^3x^2-24b^3c\,d^2x^2+126a^2b\,d^3x-108a\,b^2c\,d^2x+30b^3c^2dx+105a^3d^3-189a^2bc\,d^2+135a\,b^2c^2d-35b^3c^3\right)}{315 (bx+a)^{\frac{9}{2}} \left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(11/2),x)
```

```
[Out] 2/315*(d*x+c)^(3/2)*(16*b^3*d^3*x^3+72*a*b^2*d^3*x^2-24*b^3*c*d^2*x^2+126*a
^2*b*d^3*x-108*a*b^2*c*d^2*x+30*b^3*c^2*d*x+105*a^3*d^3-189*a^2*b*c*d^2+135
*a*b^2*c^2*d-35*b^3*c^3)/(b*x+a)^(9/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2
*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [B] time = 1.18, size = 292, normalized size = 2.15

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,d^4\,x^4}{315\,b\,{\left(a\,d-b\,c\right)}^4}-\frac{-210\,a^3\,c\,d^3+378\,a^2\,b\,c^2\,d^2-270\,a\,b^2\,c^3\,d+70\,b^3\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{x\,\left(210\,a^3\,d^4-126\,a^2\,b\,c\,d^3+54\,a\,b^2\,c^2\,d^2-10\,b^3\,c^3\,d\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^2\,x^2\,\left(21\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{105\,b^3\,{\left(a\,d-b\,c\right)}^4}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(11/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*d^4*x^4)/(315*b*(a*d - b*c)^4) - (70*b^3*c^4 - 210*a^
3*c*d^3 + 378*a^2*b*c^2*d^2 - 270*a*b^2*c^3*d)/(315*b^4*(a*d - b*c)^4) + (x
*(210*a^3*d^4 - 10*b^3*c^3*d + 54*a*b^2*c^2*d^2 - 126*a^2*b*c*d^3))/(315*b^
4*(a*d - b*c)^4) + (16*d^3*x^3*(9*a*d - b*c))/(315*b^2*(a*d - b*c)^4) + (4*
d^2*x^2*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(105*b^3*(a*d - b*c)^4)))/(x^4*
(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b
^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (4*a^3*x*(a + b*x)^(1/2))/b^3)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

## 3.1364 $\int \frac{\sqrt{c+dx}}{(a+bx)^{13/2}}\,dx$

**Optimal**. Leaf size=171

$$-\frac{256d^4(c+dx)^{3/2}}{3465(a+bx)^{3/2}(bc-ad)^5}+\frac{128d^3(c+dx)^{3/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{32d^2(c+dx)^{3/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{16d(c+dx)^{3/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{11(a+bx)^{11/2}(bc-ad)}$$

**Rubi** [A] time = 0.04, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{256d^4(c+dx)^{3/2}}{3465(a+bx)^{3/2}(bc-ad)^5}+\frac{128d^3(c+dx)^{3/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{32d^2(c+dx)^{3/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{16d(c+dx)^{3/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (16*d*(c + d*x)^(3
/2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (32*d^2*(c + d*x)^(3/2))/(231*(b*
c - a*d)^3*(a + b*x)^(7/2)) + (128*d^3*(c + d*x)^(3/2))/(1155*(b*c - a*d)^4
*(a + b*x)^(5/2)) - (256*d^4*(c + d*x)^(3/2))/(3465*(b*c - a*d)^5*(a + b*x)
^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{13/2}}\,dx = -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} - \frac{(8d)\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} + \frac{\left(16d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx}{33(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} - \frac{\left(64d^3\right)\int \frac{\sqrt{\cdots}}{(a\cdots}}{231(bc-\cdots}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{128d^3(\cdots}{1155(bc-a\cdots}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{128d^3(\cdots}{1155(bc-a\cdots}$$

**Mathematica** [A] time = 0.08, size = 170, normalized size = 0.99

$$-\frac{2(c+dx)^{3/2}\left(1155a^4d^4+924a^3bd^3(2dx-3c)+198a^2b^2d^2\left(15c^2-12cdx+8d^2x^2\right)+44ab^3d\left(-35c^3+30c^2dx-24cd^2x^2+16d^3x^3\right)+b^4\left(315c^4-280c^3dx+240c^2d^2x^2-192cd^3x^3+128d^4x^4\right)\right)}{3465(a+bx)^{11/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(1155*a^4*d^4 + 924*a^3*b*d^3*(-3*c + 2*d*x) + 198*a^2*
b^2*d^2*(15*c^2 - 12*c*d*x + 8*d^2*x^2) + 44*a*b^3*d*(-35*c^3 + 30*c^2*d*x
- 24*c*d^2*x^2 + 16*d^3*x^3) + b^4*(315*c^4 - 280*c^3*d*x + 240*c^2*d^2*x^2
 - 192*c*d^3*x^3 + 128*d^4*x^4)))/(3465*(b*c - a*d)^5*(a + b*x)^(11/2))
```

**IntegrateAlgebraic** [A] time = 0.14, size = 117, normalized size = 0.68

$$-\frac{2(c+dx)^{3/2}\left(\frac{315b^4(c+dx)^4}{(a+bx)^4}-\frac{1540b^3d(c+dx)^3}{(a+bx)^3}+\frac{2970b^2d^2(c+dx)^2}{(a+bx)^2}-\frac{2772bd^3(c+dx)}{a+bx}+1155d^4\right)}{3465(a+bx)^{3/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(1155*d^4 - (2772*b*d^3*(c + d*x))/(a + b*x) + (2970*b^
2*d^2*(c + d*x)^2)/(a + b*x)^2 - (1540*b^3*d*(c + d*x)^3)/(a + b*x)^3 + (31
5*b^4*(c + d*x)^4)/(a + b*x)^4))/(3465*(b*c - a*d)^5*(a + b*x)^(3/2))
```

**fricas [B]** time = 27.26, size = 781, normalized size = 4.57

$$-\frac{2\left(128\,b^4d^5x^5+315\,b^4c^5-1540\,ab^3c^4d+2970\,a^2b^2c^3d^2-2772\,a^3bc^2d^3+1155\,a^4cd^4-64\left(b^4cd^4-11\,ab^3d^5\right)x^4+16\left(3\,b^4c^2d^3-22\,ab^3cd^4+99\,a^2b^2d^5\right)x^3-8\left(5\,b^4c^3d^2-33\,ab^3c^2d^3+99\,a^2b^2cd^4-231\,a^3bd^5\right)x^2+\left(35\,b^4c^4d-220\,ab^3c^3d^2+594\,a^2b^2c^2d^3-924\,a^3bcd^4+1155\,a^4d^5\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{3465\left(a^6b^5c^5-5\,a^7b^4c^4d+10\,a^8b^3c^3d^2-10\,a^9b^2c^2d^3+5\,a^{10}bcd^4-a^{11}d^5+\left(b^{11}c^5-5\,ab^{10}c^4d+10\,a^2b^9c^3d^2-10\,a^3b^8c^2d^3+5\,a^4b^7cd^4-a^5b^6d^5\right)x^6+6\left(ab^{10}c^5-5\,a^2b^9c^4d+10\,a^3b^8c^3d^2-10\,a^4b^7c^2d^3+5\,a^5b^6cd^4-a^6b^5d^5\right)x^5+15\left(a^2b^9c^5-5\,a^3b^8c^4d+10\,a^4b^7c^3d^2-10\,a^5b^6c^2d^3+5\,a^6b^5cd^4-a^7b^4d^5\right)x^4+20\left(a^3b^8c^5-5\,a^4b^7c^4d+10\,a^5b^6c^3d^2-10\,a^6b^5c^2d^3+5\,a^7b^4cd^4-a^8b^3d^5\right)x^3+15\left(a^4b^7c^5-5\,a^5b^6c^4d+10\,a^6b^5c^3d^2-10\,a^7b^4c^2d^3+5\,a^8b^3cd^4-a^9b^2d^5\right)x^2+6\left(a^5b^6c^5-5\,a^6b^5c^4d+10\,a^7b^4c^3d^2-10\,a^8b^3c^2d^3+5\,a^9b^2cd^4-a^{10}bd^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] -2/3465*(128*b^4*d^5*x^5 + 315*b^4*c^5 - 1540*a*b^3*c^4*d + 2970*a^2*b^2*c^
3*d^2 - 2772*a^3*b*c^2*d^3 + 1155*a^4*c*d^4 - 64*(b^4*c*d^4 - 11*a*b^3*d^5)
*x^4 + 16*(3*b^4*c^2*d^3 - 22*a*b^3*c*d^4 + 99*a^2*b^2*d^5)*x^3 - 8*(5*b^4*
c^3*d^2 - 33*a*b^3*c^2*d^3 + 99*a^2*b^2*c*d^4 - 231*a^3*b*d^5)*x^2 + (35*b^
4*c^4*d - 220*a*b^3*c^3*d^2 + 594*a^2*b^2*c^2*d^3 - 924*a^3*b*c*d^4 + 1155*
a^4*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b^5*c^5 - 5*a^7*b^4*c^4*d + 10
*a^8*b^3*c^3*d^2 - 10*a^9*b^2*c^2*d^3 + 5*a^10*b*c*d^4 - a^11*d^5 + (b^11*c
^5 - 5*a*b^10*c^4*d + 10*a^2*b^9*c^3*d^2 - 10*a^3*b^8*c^2*d^3 + 5*a^4*b^7*c
*d^4 - a^5*b^6*d^5)*x^6 + 6*(a*b^10*c^5 - 5*a^2*b^9*c^4*d + 10*a^3*b^8*c^3*
d^2 - 10*a^4*b^7*c^2*d^3 + 5*a^5*b^6*c*d^4 - a^6*b^5*d^5)*x^5 + 15*(a^2*b^9
*c^5 - 5*a^3*b^8*c^4*d + 10*a^4*b^7*c^3*d^2 - 10*a^5*b^6*c^2*d^3 + 5*a^6*b^
5*c*d^4 - a^7*b^4*d^5)*x^4 + 20*(a^3*b^8*c^5 - 5*a^4*b^7*c^4*d + 10*a^5*b^6
*c^3*d^2 - 10*a^6*b^5*c^2*d^3 + 5*a^7*b^4*c*d^4 - a^8*b^3*d^5)*x^3 + 15*(a^
4*b^7*c^5 - 5*a^5*b^6*c^4*d + 10*a^6*b^5*c^3*d^2 - 10*a^7*b^4*c^2*d^3 + 5*a
^8*b^3*c*d^4 - a^9*b^2*d^5)*x^2 + 6*(a^5*b^6*c^5 - 5*a^6*b^5*c^4*d + 10*a^7
*b^4*c^3*d^2 - 10*a^8*b^3*c^2*d^3 + 5*a^9*b^2*c*d^4 - a^10*b*d^5)*x)
```

**giac [B]** time = 2.38, size = 1345, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="giac")
```

```
[Out] -512/3465*(sqrt(b*d)*b^16*c^6*d^5 - 6*sqrt(b*d)*a*b^15*c^5*d^6 + 15*sqrt(b*
d)*a^2*b^14*c^4*d^7 - 20*sqrt(b*d)*a^3*b^13*c^3*d^8 + 15*sqrt(b*d)*a^4*b^12
*c^2*d^9 - 6*sqrt(b*d)*a^5*b^11*c*d^10 + sqrt(b*d)*a^6*b^10*d^11 - 11*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^14
*c^5*d^5 + 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a*b^13*c^4*d^6 - 110*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^12*c^3*d^7 + 110*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^11*c^2*d
^8 - 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a
*b*d))^2*a^4*b^10*c*d^9 + 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^2*a^5*b^9*d^10 + 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^12*c^4*d^5 - 220*sqrt(b*
d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^11
*c^3*d^6 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
```

```
b*d - a*b*d))^4*a^2*b^10*c^2*d^7 - 220*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^9*c*d^8 + 55*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^8*d^9 - 1
65*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6*b^10*c^3*d^5 + 495*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^9*c^2*d^6 - 495*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^8*c*d^7 + 165*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^7*d
^8 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^8*b^8*c^2*d^5 - 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^8*a*b^7*c*d^6 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^6*d^7 + 924*sqrt(b*d)
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^6*c*d
^5 - 924*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^10*a*b^5*d^6 + 1386*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^12*b^4*d^5)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^11*b^2)
```

**maple** [**A**] time = 0.01, size = 256, normalized size = 1.50

$$\frac{2 (dx+c)^{\frac{3}{2}} \left(128b^4x^4d^4+704a\,b^3d^4x^3-192b^4c\,d^3x^3+1584a^2b^2d^4x^2-1056a\,b^3c\,d^3x^2+240b^4c^2d^2x^2+1848a^3b\,d^4x-2376a^2b^2c\,d^3x+1320a\,b^3c^2d^2x-280b^4c^3dx+1155a^4d^4-2772a^3bc\,d^3+2970a^2b^2c^2d^2-1540a\,b^3c^3d+315b^4c^4\right)}{3465 (bx+a)^{\frac{11}{2}} \left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(13/2),x)
```

```
[Out] 2/3465*(d*x+c)^(3/2)*(128*b^4*d^4*x^4+704*a*b^3*d^4*x^3-192*b^4*c*d^3*x^3+1
584*a^2*b^2*d^4*x^2-1056*a*b^3*c*d^3*x^2+240*b^4*c^2*d^2*x^2+1848*a^3*b*d^4
*x-2376*a^2*b^2*c*d^3*x+1320*a*b^3*c^2*d^2*x-280*b^4*c^3*d*x+1155*a^4*d^4-2
772*a^3*b*c*d^3+2970*a^2*b^2*c^2*d^2-1540*a*b^3*c^3*d+315*b^4*c^4)/(b*x+a)^
(11/2)/(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4
*c^4*d-b^5*c^5)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.43, size = 397, normalized size = 2.32

$$\frac{\sqrt{c+d\,x}\,\left(\frac{2310\,a^4\,c\,d^4-5544\,a^3\,b\,c^2\,d^3+5940\,a^2\,b^2\,c^3\,d^2-3080\,a\,b^3\,c^4\,d+630\,b^4\,c^5}{3465\,b^5\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(2310\,a^4\,d^5-1848\,a^3\,b\,c\,d^4+1188\,a^2\,b^2\,c^2\,d^3-440\,a\,b^3\,c^3\,d^2+70\,b^4\,c^4\,d\right)}{3465\,b^5\,{\left(a\,d-b\,c\right)}^5}+\frac{256\,d^5\,x^5}{3465\,b\,{\left(a\,d-b\,c\right)}^5}+\frac{16\,d^2\,x^2\,\left(231\,a^3\,d^3-99\,a^2\,b\,c\,d^2+33\,a\,b^2\,c^2\,d-5\,b^3\,c^3\right)}{3465\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{3465\,b^2\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d^3\,x^3\,\left(99\,a^2\,d^2-22\,a\,b\,c\,d+3\,b^2\,c^2\right)}{3465\,b^3\,{\left(a\,d-b\,c\right)}^5}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((630*b^4*c^5 + 2310*a^4*c*d^4 - 5544*a^3*b*c^2*d^3 + 5940
*a^2*b^2*c^3*d^2 - 3080*a*b^3*c^4*d)/(3465*b^5*(a*d - b*c)^5) + (x*(2310*a^
4*d^5 + 70*b^4*c^4*d - 440*a*b^3*c^3*d^2 + 1188*a^2*b^2*c^2*d^3 - 1848*a^3*
b*c*d^4))/(3465*b^5*(a*d - b*c)^5) + (256*d^5*x^5)/(3465*b*(a*d - b*c)^5) +
 (16*d^2*x^2*(231*a^3*d^3 - 5*b^3*c^3 + 33*a*b^2*c^2*d - 99*a^2*b*c*d^2))/(
3465*b^4*(a*d - b*c)^5) + (128*d^4*x^4*(11*a*d - b*c))/(3465*b^2*(a*d - b*c
)^5) + (32*d^3*x^3*(99*a^2*d^2 + 3*b^2*c^2 - 22*a*b*c*d))/(3465*b^3*(a*d -
b*c)^5)))/(x^5*(a + b*x)^(1/2) + (a^5*(a + b*x)^(1/2))/b^5 + (10*a^2*x^3*(a
 + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a + b*x)^(1/2))/b^3 + (5*a*x^4*(a + b*x)^
(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/b^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

# 3.1365 $\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx$

**Optimal**. Leaf size=227

$$-\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}+\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128b^2d^3}-\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}+\frac{(a+bx)^{5/2}\sqrt{c+d}}{80b^2d}$$

**Rubi [A]** time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128b^2d^3}-\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}-\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{80b^2d}+\frac{3(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}{40b^2}+\frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] (3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^2*d^3) - ((b*c - a*d)^
3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^2*(a + b*x)^(5
/2)*Sqrt[c + d*x])/(80*b^2*d) + (3*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x
])/(40*b^2) + ((a + b*x)^(7/2)*(c + d*x)^(3/2))/(5*b) - (3*(b*c - a*d)^5*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(5/2)*d^(7/2
))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx = \frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b} + \frac{(3(bc-ad))\int (a+bx)^{5/2}\sqrt{c+dx}\,dx}{10b}$$

$$= \frac{3(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40b^2} + \frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b} + \frac{\left(3(bc-ad)^2\right)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,d\cdots}{80b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d} + \frac{3(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40b^2} + \frac{(a+bx)^{7/2}(c+d\cdots}{5b}$$

$$= -\frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d} + \frac{3(bc-ad)(a+b\cdots}{40b\cdots}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)\cdots}{80b^2d}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)\cdots}{80b^2d}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)\cdots}{80b^2d}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)\cdots}{80b^2d}$$

**Mathematica [A]** time = 1.78, size = 187, normalized size = 0.82

$$\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(-\frac{15(bc-ad)^{9/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{7/2}(a+bx)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}} + \frac{15(bc-ad)^4}{d^3(a+bx)^3} + \frac{10(ad-bc)^3}{d^2(a+bx)^2} + \frac{8(bc-ad)^2}{d(a+bx)} + 48(bc-ad) + 128b(c+dx)\right)}{640b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] ((a + b*x)^(7/2)*Sqrt[c + d*x]*(48*(b*c - a*d) + (15*(b*c - a*d)^4)/(d^3*(a
 + b*x)^3) + (10*(-(b*c) + a*d)^3)/(d^2*(a + b*x)^2) + (8*(b*c - a*d)^2)/(d
*(a + b*x)) + 128*b*(c + d*x) - (15*(b*c - a*d)^(9/2)*ArcSinh[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[b*c - a*d]])/(d^(7/2)*(a + b*x)^(7/2)*Sqrt[(b*(c + d*x))/(b
*c - a*d)])))/(640*b^2)
```

**IntegrateAlgebraic** [A] time = 0.41, size = 197, normalized size = 0.87

$$\frac{\sqrt{a+bx}\,(bc-ad)^5\left(-\frac{70b^3d(a+bx)}{c+dx}+\frac{128b^2d^2(a+bx)^2}{(c+dx)^2}-\frac{15d^4(a+bx)^4}{(c+dx)^4}+\frac{70bd^3(a+bx)^3}{(c+dx)^3}+15b^4\right)}{640b^2d^3\sqrt{c+dx}\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] ((b*c - a*d)^5*Sqrt[a + b*x]*(15*b^4 - (15*d^4*(a + b*x)^4)/(c + d*x)^4 + (
70*b*d^3*(a + b*x)^3)/(c + d*x)^3 + (128*b^2*d^2*(a + b*x)^2)/(c + d*x)^2 -
 (70*b^3*d*(a + b*x))/(c + d*x)))/(640*b^2*d^3*Sqrt[c + d*x]*(b - (d*(a + b
*x))/(c + d*x))^5) - (3*(b*c - a*d)^5*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt
[b]*Sqrt[c + d*x])])/(128*b^(5/2)*d^(7/2))
```

**fricas** [A] time = 1.52, size = 702, normalized size = 3.09

$$\left[-\frac{15\left(b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)-4\left(128b^5d^5x^4+15b^5c^4d-70ab^4c^3d^2+128a^2b^3c^2d^3+70a^3b^2cd^4-15a^4bd^5+16\left(11b^5cd^4+21ab^4d^5\right)x^3+8\left(b^5c^2d^3+64ab^4cd^4+31a^2b^3d^5\right)x^2-2\left(5b^5c^3d^2-23ab^4c^2d^3-233a^2b^3cd^4-5a^3b^2d^5\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{2560b^3d^4},\ \frac{15\left(b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)+2\left(128b^5d^5x^4+15b^5c^4d-70ab^4c^3d^2+128a^2b^3c^2d^3+70a^3b^2cd^4-15a^4bd^5+16\left(11b^5cd^4+21ab^4d^5\right)x^3+8\left(b^5c^2d^3+64ab^4cd^4+31a^2b^3d^5\right)x^2-2\left(5b^5c^3d^2-23ab^4c^2d^3-233a^2b^3cd^4-5a^3b^2d^5\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{1280b^3d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2
*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x
 + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(128*b^5*d^5*x^4 + 15*b^5*c^4*d - 70*a
*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 + 70*a^3*b^2*c*d^4 - 15*a^4*b*d^5 + 16*(
11*b^5*c*d^4 + 21*a*b^4*d^5)*x^3 + 8*(b^5*c^2*d^3 + 64*a*b^4*c*d^4 + 31*a^2
*b^3*d^5)*x^2 - 2*(5*b^5*c^3*d^2 - 23*a*b^4*c^2*d^3 - 233*a^2*b^3*c*d^4 - 5
*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^4), 1/1280*(15*(b^5*c^
5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(1
28*b^5*d^5*x^4 + 15*b^5*c^4*d - 70*a*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 + 70
*a^3*b^2*c*d^4 - 15*a^4*b*d^5 + 16*(11*b^5*c*d^4 + 21*a*b^4*d^5)*x^3 + 8*(b
^5*c^2*d^3 + 64*a*b^4*c*d^4 + 31*a^2*b^3*d^5)*x^2 - 2*(5*b^5*c^3*d^2 - 23*a
```

```
*b^4*c^2*d^3 - 233*a^2*b^3*c*d^4 - 5*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x
 + c))/(b^3*d^4)]
```

**giac [B]** time = 2.30, size = 1740, normalized size = 7.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1920*(240*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)
*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2
+ 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3
*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*c*abs(b) - 1920*((b^2*c - a*b*
d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*c*abs(b
)/b^2 + 10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(
6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^
4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9
*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x
 + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 -
 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d
- a*b*d)))/(sqrt(b*d)*b^2*d^3))*b*c*abs(b) + 30*(sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b
^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6
)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 -
93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b
*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*d*ab
s(b) + 240*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*
(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*
a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*d*abs(b)/b + (sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*
c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 51
3*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^
2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4
 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^
19*d^8)/(b^23*d^8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b
^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3
*d^4))*b*d*abs(b) + 1440*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
+ (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d
```

```
^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
)/(sqrt(b*d)*d))*a^2*c*abs(b)/b^2 + 480*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*
c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)))/(sqrt(b*d)*d))*a^3*d*abs(b)/b^3)/b
```

**maple [B]** time = 0.01, size = 853, normalized size = 3.76

$$\frac{3\sqrt{(bx+a)(dx+c)}\,a^5d^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{256\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^2}-\frac{15\sqrt{(bx+a)(dx+c)}\,a^4cd\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{256\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{15\sqrt{(bx+a)(dx+c)}\,a^3c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{15\sqrt{(bx+a)(dx+c)}\,a^2bc^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}+\frac{15\sqrt{(bx+a)(dx+c)}\,ab^2c^4\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{256\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d^2}-\frac{3\sqrt{(bx+a)(dx+c)}\,b^3c^5\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{256\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d^3}-\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,a^4d}{128b^2}+\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,a^3c}{32b}-\frac{9\sqrt{dx+c}\,\sqrt{bx+a}\,a^2c^2}{64d}+\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,abc^3}{32d^2}-\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,b^2c^4}{128d^3}+\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,a^3}{64b}-\frac{3(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,a^2c}{64d}+\frac{3(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,abc^2}{64d^2}-\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,b^2c^3}{64d^3}+\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}a^2}{16d}-\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}abc}{8d^2}+\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}b^2c^2}{16d^3}+\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}a}{8d}-\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}bc}{8d^2}+\frac{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(3/2),x)
```

```
[Out] 1/5/d*(b*x+a)^(5/2)*(d*x+c)^(5/2)+1/8/d*(b*x+a)^(3/2)*(d*x+c)^(5/2)*a+1/16/
d*(b*x+a)^(1/2)*(d*x+c)^(5/2)*a^2-1/8/d^2*(b*x+a)^(1/2)*(d*x+c)^(5/2)*a*b*c
+3/64/d^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c^2*b+3/32/b*(d*x+c)^(1/2)*(b*x+a)^
(1/2)*a^3*c+3/32/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^3*b-1/8/d^2*(b*x+a)^(3
/2)*(d*x+c)^(5/2)*b*c+1/16/d^3*(b*x+a)^(1/2)*(d*x+c)^(5/2)*b^2*c^2+1/64/b*(
d*x+c)^(3/2)*(b*x+a)^(1/2)*a^3-3/64/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^2*c-1/6
4/d^3*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c^3*b^2-3/128*d/b^2*(d*x+c)^(1/2)*(b*x+a)
^(1/2)*a^4-9/64/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2*c^2-3/128/d^3*(d*x+c)^(1/
2)*(b*x+a)^(1/2)*c^4*b^2+15/128*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+
a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(
1/2))/(b*d)^(1/2)*a^3*c^2+15/256/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/
(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)
*x)^(1/2))/(b*d)^(1/2)*a*c^4*b^2-15/256*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)
^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a
*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^4*c+3/256*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(
d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+
a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^5-15/128/d*((b*x+a)*(d*x+c))^(1/2)/(d
*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a
*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*c^3*b-3/256/d^3*((b*x+a)*(d*x+c))^(1
/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d
*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^5*b^3
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/2}\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(3/2),x)
[Out] int((a + b*x)^(5/2)*(c + d*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(3/2),x)
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(3/2), x)
```

## 3.1366 $\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx$

**Optimal**. Leaf size=189

$$\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{5/2}d^{5/2}}-\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^2d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{8b^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}+\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{5/2}d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^2d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{8b^2}+\frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] (-3*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^2*d) + ((b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(3/2))/(4*b) + (3*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(5
/2)*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx = \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b} + \frac{(3(bc-ad))\int (a+bx)^{3/2}\sqrt{c+dx}\,dx}{8b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b} + \frac{(bc-ad)^2\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{16b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)}{4b}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx}{8b^2}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx}{8b^2}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx}{8b^2}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx}{8b^2}$$

**Mathematica [A]** time = 0.57, size = 193, normalized size = 1.02

$$\frac{3(bc-ad)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right) - b\sqrt{d}\sqrt{a+bx}(c+dx)\left(3a^3d^3 - a^2bd^2(11c+2dx) - ab^2d\left(11c^2+44cdx+24d^2x^2\right) + b^3\left(3c^3-2c^2dx-24cd^2x^2-16d^3x^3\right)\right)}{64b^3d^{5/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(3*a^3*d^3 - a^2*b*d^2*(11*c + 2*d*x)
- a*b^2*d*(11*c^2 + 44*c*d*x + 24*d^2*x^2) + b^3*(3*c^3 - 2*c^2*d*x - 24*c*
```

```
d^2*x^2 - 16*d^3*x^3))) + 3*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))/(b*c - a*d
)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(64*b^3*d^(5/2)*Sqrt[c
 + d*x])
```

**IntegrateAlgebraic [A]** time = 0.34, size = 176, normalized size = 0.93

$$\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{64b^{5/2}d^{5/2}}-\frac{\sqrt{c+dx}\,(bc-ad)^4\left(\frac{3b^3(c+dx)^3}{(a+bx)^3}-\frac{11b^2d(c+dx)^2}{(a+bx)^2}-\frac{11bd^2(c+dx)}{a+bx}+3d^3\right)}{64b^2d^2\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}-d\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] -1/64*((b*c - a*d)^4*Sqrt[c + d*x]*(3*d^3 - (11*b*d^2*(c + d*x))/(a + b*x)
- (11*b^2*d*(c + d*x)^2)/(a + b*x)^2 + (3*b^3*(c + d*x)^3)/(a + b*x)^3))/(b
^2*d^2*Sqrt[a + b*x]*(-d + (b*(c + d*x))/(a + b*x))^4) + (3*(b*c - a*d)^4*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(64*b^(5/2)*d^(5/2
))
```

**fricas [A]** time = 1.35, size = 534, normalized size = 2.83

$$\left[\frac{3\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)+4\left(16b^4d^4x^3-3b^4c^3d+11ab^3c^2d^2+11a^2b^2cd^3-3a^3bd^4+24\left(b^4cd^3+ab^3d^4\right)x^2+2\left(b^4c^2d^2+22ab^3cd^3+a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{256b^3d^3},-\frac{3\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(16b^4d^4x^3-3b^4c^3d+11ab^3c^2d^2+11a^2b^2cd^3-3a^3bd^4+24\left(b^4cd^3+ab^3d^4\right)x^2+2\left(b^4c^2d^2+22ab^3cd^3+a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{128b^3d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b
*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*
d^2)*x) + 4*(16*b^4*d^4*x^3 - 3*b^4*c^3*d + 11*a*b^3*c^2*d^2 + 11*a^2*b^2*c
*d^3 - 3*a^3*b*d^4 + 24*(b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(b^4*c^2*d^2 + 22*a
*b^3*c*d^3 + a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^3), -1/128
*(3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(
d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(16*b^4*d^4*x
^3 - 3*b^4*c^3*d + 11*a*b^3*c^2*d^2 + 11*a^2*b^2*c*d^3 - 3*a^3*b*d^4 + 24*(
b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(b^4*c^2*d^2 + 22*a*b^3*c*d^3 + a^2*b^2*d^4)
*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^3)]
```

**giac [B]** time = 1.89, size = 1071, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/192*(8*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4
*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2
*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^
2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c*abs(b) - 192*((b^2*c - a*b*d)*log
(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt(
b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*c*abs(b)/b^2
+ (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x +
a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*
b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c
^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3
*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d
^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
)/(sqrt(b*d)*b^2*d^3))*d*abs(b) + 16*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*s
qrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^
7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(
b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*d*abs
(b)/b + 96*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a
*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-s
qrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d
))*a*c*abs(b)/b^2 + 48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
(b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
(sqrt(b*d)*d))*a^2*d*abs(b)/b^3)/b
```

**maple [B]** time = 0.01, size = 640, normalized size = 3.39

$$\frac{3\sqrt{(bx+a)(dx+c)}\,a^4d^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^2}-\frac{3\sqrt{(bx+a)(dx+c)}\,a^3cd\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{9\sqrt{(bx+a)(dx+c)}\,a^2c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{64\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{3\sqrt{(bx+a)(dx+c)}\,abc^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}+\frac{3\sqrt{(bx+a)(dx+c)}\,b^2c^4\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d^2}-\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,a^3d}{64b^2}+\frac{9\sqrt{dx+c}\,\sqrt{bx+a}\,a^2c}{64b}-\frac{9\sqrt{dx+c}\,\sqrt{bx+a}\,ac^2}{64d}+\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,bc^3}{64d^2}+\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,a^2}{32b}-\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,ac}{16d}+\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,bc^2}{32d^2}+\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}a}{8d}-\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}bc}{8d^2}+\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(3/2),x)
```

```
[Out] 1/4/d*(b*x+a)^(3/2)*(d*x+c)^(5/2)+1/8/d*(b*x+a)^(1/2)*(d*x+c)^(5/2)*a-1/8/d
^2*(b*x+a)^(1/2)*(d*x+c)^(5/2)*b*c+1/32/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^2-1
/16/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c+1/32/d^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*
c^2*b-3/64*d/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3+9/64/b*(d*x+c)^(1/2)*(b*x+
a)^(1/2)*a^2*c-9/64/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^2+3/64/d^2*(d*x+c)^(1
/2)*(b*x+a)^(1/2)*c^3*b+3/128*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)
/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c
)*x)^(1/2))/(b*d)^(1/2)*a^4-3/32*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/
(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)
*x)^(1/2))/(b*d)^(1/2)*a^3*c+9/64*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*
```

```
x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)
^(1/2))/(b*d)^(1/2)*a^2*c^2-3/32/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b
*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x
)^(1/2))/(b*d)^(1/2)*a*c^3*b+3/128/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*
c)*x)^(1/2))/(b*d)^(1/2)*c^4*b^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(3/2), x)
```

## 3.1367 $\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx$

**Optimal**. Leaf size=151

$$-\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8b^2d}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

**Rubi [A]** time = 0.07, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8b^2d}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^2*d) + ((b*c - a*d)*(a + b
*x)^(3/2)*Sqrt[c + d*x])/(4*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(3/2))/(3*b)
- ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/
(8*b^(5/2)*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx = \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} + \frac{(bc-ad)\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} + \frac{(bc-ad)^2\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{8b^2}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} -$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} -$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} -$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} -$$

**Mathematica [A]** time = 0.44, size = 152, normalized size = 1.01

$$\frac{-b\sqrt{d}\,\sqrt{a+bx}\,(c+dx)\left(3a^2d^2-2abd(4c+dx)-\left(b^2\left(3c^2+14cdx+8d^2x^2\right)\right)\right)-3(bc-ad)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{24b^3d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(3*a^2*d^2 - 2*a*b*d*(4*c + d*x) - b^2
*(3*c^2 + 14*c*d*x + 8*d^2*x^2))) - 3*(b*c - a*d)^(7/2)*Sqrt[(b*(c + d*x))/
(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(24*b^3*d^(3
/2)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.24, size = 153, normalized size = 1.01

$$\frac{\sqrt{a+bx}\,(bc-ad)^3\left(-\frac{3d^2(a+bx)^2}{(c+dx)^2}+\frac{8bd(a+bx)}{c+dx}+3b^2\right)}{24b^2d\sqrt{c+dx}\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
```

```
[Out] ((b*c - a*d)^3*Sqrt[a + b*x]*(3*b^2 - (3*d^2*(a + b*x)^2)/(c + d*x)^2 + (8*
b*d*(a + b*x))/(c + d*x)))/(24*b^2*d*Sqrt[c + d*x]*(b - (d*(a + b*x))/(c +
d*x))^3) - ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c +
 d*x])])/(8*b^(5/2)*d^(3/2))
```

**fricas [A]** time = 1.48, size = 410, normalized size = 2.72

$$\left[-\frac{3\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)-4\left(8b^3d^3x^2+3b^3c^2d+8ab^2cd^2-3a^2bd^3+2\left(7b^3cd^2+ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{96b^3d^2},\ \frac{3\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)+2\left(8b^3d^3x^2+3b^3c^2d+8ab^2cd^2-3a^2bd^3+2\left(7b^3cd^2+ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48b^3d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sq
rt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d
^3*x^2 + 3*b^3*c^2*d + 8*a*b^2*c*d^2 - 3*a^2*b*d^3 + 2*(7*b^3*c*d^2 + a*b^2
*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^2), 1/48*(3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*
d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x)) + 2*(8*b^3*d^3*x^2 + 3*b^3*c^2*d + 8*a*b^2*c*d^2 - 3*a^2*b*
d^3 + 2*(7*b^3*c*d^2 + a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^2)
]
```

**giac [B]** time = 1.61, size = 576, normalized size = 3.81

$$-\frac{\frac{24\left(\frac{\left(b^2c-abd\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)ac|b|}{b^2}-\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(2(bx+a)\left(\frac{4(bx+a)}{b^2}+\frac{b^6cd^3-13ab^5d^4}{b^7d^4}\right)-\frac{3\left(b^7c^2d^2+2ab^6cd^3-11a^2b^5d^4\right)}{b^7d^4}\right)-\frac{3\left(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^2}\right)d|b|}{b}-\frac{6\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2bx+2a+\frac{bcd-5ad^2}{d^2}\right)\sqrt{bx+a}+\frac{\left(b^3c^2+2ab^2cd-3a^2bd^2\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)c|b|}{b^2}-\frac{6\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2bx+2a+\frac{bcd-5ad^2}{d^2}\right)\sqrt{bx+a}+\frac{\left(b^3c^2+2ab^2cd-3a^2bd^2\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)ad|b|}{b^3}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(24*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sq
rt(b*x + a))*a*c*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x
```

```
 + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4))
 - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3
 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*d*abs(b)/b - 6
*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)
*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*s
qrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*c*abs(b
)/b^2 - 6*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*
d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d)
)*a*d*abs(b)/b^3)/b
```

**maple** [B] time = 0.01, size = 459, normalized size = 3.04

$$\frac{\sqrt{(bx+a)(dx+c)}\, a^3 d^2 \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^2}-\frac{3\sqrt{(bx+a)(dx+c)}\, a^2 cd \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{3\sqrt{(bx+a)(dx+c)}\, a c^2 \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{\sqrt{(bx+a)(dx+c)}\, b c^3 \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}-\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,a^2 d}{8b^2}+\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,ac}{4b}-\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,c^2}{8d}+\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,a}{12b}-\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,c}{12d}+\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(3/2),x)
```

```
[Out] 1/3/d*(b*x+a)^(1/2)*(d*x+c)^(5/2)+1/12/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a-1/12
/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c-1/8*d/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2+
1/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c-1/8/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^2
+1/16*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x
+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*
a^3-3/16*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+
1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a
^2*c+3/16*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2
*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*c^
2-1/16/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*
a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^3*b
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(3/2), x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

## 3.1368 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}} \, dx$

**Optimal**. Leaf size=113

$$\frac{3(bc-ad)^2 \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}+\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b}$$

**Rubi** [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{3(bc-ad)^2 \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}+\frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^2) + (Sqrt[a + b*x]*(c + d
*x)^(3/2))/(2*b) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b
]*Sqrt[c + d*x])])/(4*b^(5/2)*Sqrt[d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{(3(bc-ad))\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{4b}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{\left(3(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{8b^2}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{\left(3(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x\right.}{4b^3}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{\left(3(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+}}{\sqrt{c+}}\right.}{4b^3}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}$$

**Mathematica [A]** time = 0.29, size = 109, normalized size = 0.96

$$\frac{\sqrt{c+dx}\left(\sqrt{a+bx}\,(-3ad+5bc+2bdx)+\frac{3(bc-ad)^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\,\sqrt{\frac{b(c+dx)}{bc-ad}}}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[a + b*x]*(5*b*c - 3*a*d + 2*b*d*x) + (3*(b*c - a*d)^(3
/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[(b*(c +
 d*x))/(b*c - a*d)])))/(4*b^2)
```

**IntegrateAlgebraic [A]** time = 0.18, size = 134, normalized size = 1.19

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}+\frac{(bc-ad)^2\left(\frac{5b\sqrt{a+bx}}{\sqrt{c+dx}}-\frac{3d(a+bx)^{3/2}}{(c+dx)^{3/2}}\right)}{4b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] ((b*c - a*d)^2*((-3*d*(a + b*x)^(3/2))/(c + d*x)^(3/2) + (5*b*Sqrt[a + b*x]
)/Sqrt[c + d*x]))/(4*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + (3*(b*c - a*d)^
2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*b^(5/2)*Sqrt
[d])
```

**fricas [A]** time = 1.02, size = 306, normalized size = 2.71

$$\left[\frac{3\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)+4\left(2b^2d^2x+5b^2cd-3abd^2\right)\sqrt{bx+a}\sqrt{dx+c}}{16b^3d},-\frac{3\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(2b^2d^2x+5b^2cd-3abd^2\right)\sqrt{bx+a}\sqrt{dx+c}}{8b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)
*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + 5*b^2*c*d - 3*
a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d), -1/8*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(2
*b^2*d^2*x + 5*b^2*c*d - 3*a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d)]
```

**giac [B]** time = 1.25, size = 233, normalized size = 2.06

$$-\frac{\frac{4\left(\frac{\left(b^2c-abd\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)c|b|}{b^2}-\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2bx+2a+\frac{bcd-5ad^2}{d^2}\right)\sqrt{bx+a}+\frac{\left(b^3c^2+2ab^2cd-3a^2bd^2\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)d|b|}{b^3}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt
(b*x + a))*c*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
 + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*
```

```
d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*d))*d*abs(b)/b^3)/b
```

**maple [B]** time = 0.01, size = 308, normalized size = 2.73

$$\frac{3\sqrt{\left(bx+a\right)\left(dx+c\right)}\, a^{2} d^{2} \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^{2}+ac+\left(ad+bc\right)x}\right)}{8\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\, b^{2}}-\frac{3\sqrt{\left(bx+a\right)\left(dx+c\right)}\, acd \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^{2}+ac+\left(ad+bc\right)x}\right)}{4\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\, b}+\frac{3\sqrt{\left(bx+a\right)\left(dx+c\right)}\, c^{2} \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^{2}+ac+\left(ad+bc\right)x}\right)}{8\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\, ad}{4b^{2}}+\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\, c}{4b}+\frac{\left(dx+c\right)^{\frac{3}{2}}\sqrt{bx+a}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*d+3
/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c+3/8/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^
(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*
d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*d^2-3/4/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^
(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*
d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*d*c+3/8*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*
c)*x)^(1/2))/(b*d)^(1/2)*c^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{3/2}}{\sqrt{a+b\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(1/2),x)
```

```
[Out] int((c + d*x)^(3/2)/(a + b*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx\right)^{\frac{3}{2}}}{\sqrt{a + bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/sqrt(a + b*x), x)
```

# 3.1369 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=98

$$\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}+\frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}$$

**Rubi [A]** time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}+\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (3*d*Sqrt[a + b*x]*Sqrt[c + d*x])/b^2 - (2*(c + d*x)^(3/2))/(b*Sqrt[a + b*x
]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c
 + d*x])])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{b}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2b^2}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^3}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^3}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.05, size = 71, normalized size = 0.72

$$-\frac{2(c+dx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.56, size = 159, normalized size = 1.62

$$\frac{\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}\left(-3ad^2\sqrt{c+dx}-bd(c+dx)^{3/2}+3bcd\sqrt{c+dx}\right)}{b^2(-ad-b(c+dx)+bc)}-\frac{3\sqrt{\frac{b}{d}}\left(bcd-ad^2\right)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}-\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - (b*c)/d + (b*(c + d*x))/d]*(3*b*c*d*Sqrt[c + d*x] - 3*a*d^2*Sqrt[
c + d*x] - b*d*(c + d*x)^(3/2)))/(b^2*(b*c - a*d - b*(c + d*x))) - (3*Sqrt[
b/d]*(b*c*d - a*d^2)*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) + Sqrt[a - (b*c)/d + (b
*(c + d*x))/d]])/b^3
```

**fricas [A]** time = 1.39, size = 311, normalized size = 3.17

$$\left[-\frac{3\left(abc-a^2d+\left(b^2c-abd\right)x\right)\sqrt{\frac{d}{b}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4\left(2b^2dx+b^2c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)-4(bdx-2bc+3ad)\sqrt{bx+a}\sqrt{dx+c}}{4\left(b^3x+ab^2\right)}, -\frac{3\left(abc-a^2d+\left(b^2c-abd\right)x\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right)-2(bdx-2bc+3ad)\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^3x+ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(a*b*c - a^2*d + (b^2*c - a*b*d)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(b*d*x - 2*b*c + 3*
a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*x + a*b^2), -1/2*(3*(a*b*c - a^2*d +
 (b^2*c - a*b*d)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x +
a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) - 2*(b
*d*x - 2*b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*x + a*b^2)]
```

**giac [B]** time = 1.50, size = 204, normalized size = 2.08

$$\frac{\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\,d|b|}{b^4}-\frac{3\left(\sqrt{bd}\,bc|b|-\sqrt{bd}\,ad|b|\right)\log\left(\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{2b^4}-\frac{4\left(\sqrt{bd}\,b^2c^2|b|-2\sqrt{bd}\,abcd|b|+\sqrt{bd}\,a^2d^2|b|\right)}{\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*d*abs(b)/b^4 - 3/2*(sqrt(
b*d)*b*c*abs(b) - sqrt(b*d)*a*d*abs(b))*log((sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^4 - 4*(sqrt(b*d)*b^2*c^2*abs(b) - 2*s
qrt(b*d)*a*b*c*d*abs(b) + sqrt(b*d)*a^2*d^2*abs(b))/((b^2*c - a*b*d - (sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*b^3)
```

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(3/2)/(b*x+a)^(3/2),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{3/2}}{{\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(3/2)/(a + b*x)^(3/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/(a + b*x)**(3/2), x)
```

## 3.1370 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}} \, dx$

**Optimal**. Leaf size=92

$$\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}$$

**Rubi** [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {47, 63, 217, 206}

$$\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*d*Sqrt[c + d*x])/(b^2*Sqrt[a + b*x]) - (2*(c + d*x)^(3/2))/(3*b*(a + b*
x)^(3/2)) + (2*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{d\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx}{b}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{d^2\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{b^2}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^3}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^3}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.05, size = 73, normalized size = 0.79

$$-\frac{2(c+dx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*Hypergeometric2F1[-3/2, -3/2, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.14, size = 85, normalized size = 0.92

$$\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2(c+dx)^{3/2}\left(\frac{3d(a+bx)}{c+dx}+b\right)}{3b^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(b + (3*d*(a + b*x))/(c + d*x)))/(3*b^2*(a + b*x)^(3/2)
) + (2*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^
(5/2)
```

**fricas** [B] time = 2.06, size = 325, normalized size = 3.53

$$\left[\frac{3\left(b^2dx^2+2\,abdx+a^2d\right)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,b^2dx+b^2c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)-4\left(4\,bdx+bc+3\,ad\right)\sqrt{bx+a}\sqrt{dx+c}}{6\left(b^4x^2+2\,ab^3x+a^2b^2\right)},-\frac{3\left(b^2dx^2+2\,abdx+a^2d\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right)+2\left(4\,bdx+bc+3\,ad\right)\sqrt{bx+a}\sqrt{dx+c}}{3\left(b^4x^2+2\,ab^3x+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*d*x^2 + 2*a*b*d*x + a^2*d)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c
^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt
(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(4*b*d*x + b*c + 3*a*d)*
sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2), -1/3*(3*(b^2*
d*x^2 + 2*a*b*d*x + a^2*d)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt
(b*x + a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x))
 + 2*(4*b*d*x + b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x^2 + 2*a*b^
3*x + a^2*b^2)]
```

**giac** [B] time = 1.74, size = 455, normalized size = 4.95

$$-\frac{\sqrt{bd}\,d|b|\log\left(\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{b^4}-\frac{8\left(2\sqrt{bd}\,b^5c^3d|b|-6\sqrt{bd}\,ab^4c^2d^2|b|+6\sqrt{bd}\,a^2b^3cd^3|b|-2\sqrt{bd}\,a^3b^2d^4|b|-3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^3c^2d|b|+6\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^2cd^2|b|-3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2bd^3|b|+3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4bcd|b|-3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ad^2|b|\right)}{3\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*d*abs(b)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2)/b^4 - 8/3*(2*sqrt(b*d)*b^5*c^3*d*abs(b) - 6*sqrt(b*d)*a*b^4
*c^2*d^2*abs(b) + 6*sqrt(b*d)*a^2*b^3*c*d^3*abs(b) - 2*sqrt(b*d)*a^3*b^2*d^
4*abs(b) - 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*b^3*c^2*d*abs(b) + 6*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^2*c*d^2*abs(b) - 3*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b*d^3*abs(b
```

```
) + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^4*b*c*d*abs(b) - 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^4*a*d^2*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3*b^3)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(3/2)/(b*x+a)^(5/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{3/2}}{(a+b\,x)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(5/2),x)
```

```
[Out] int((c + d*x)^(3/2)/(a + b*x)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(5/2),x)
[Out] Integral((c + d*x)**(3/2)/(a + b*x)**(5/2), x)
```

## 3.1371 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx$

**Optimal**. Leaf size=32

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{5/2}}{5(bc-ad)(a+bx)^{5/2}}$$

**Mathematica [A]** time = 0.02, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

**fricas [B]** time = 2.26, size = 104, normalized size = 3.25

$$-\frac{2\left(d^2x^2+2\,cdx+c^2\right)\sqrt{bx+a}\,\sqrt{dx+c}}{5\left(a^3bc-a^4d+\left(b^4c-ab^3d\right)x^3+3\left(ab^3c-a^2b^2d\right)x^2+3\left(a^2b^2c-a^3bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b*c - a^4*d
 + (b^4*c - a*b^3*d)*x^3 + 3*(a*b^3*c - a^2*b^2*d)*x^2 + 3*(a^2*b^2*c - a^3
*b*d)*x)
```

**giac [B]** time = 1.69, size = 374, normalized size = 11.69

$$-\frac{4\left(\sqrt{bd}\,b^8c^4d^2|b|-4\sqrt{bd}\,ab^7c^3d^3|b|+6\sqrt{bd}\,a^2b^6c^2d^4|b|-4\sqrt{bd}\,a^3b^5cd^5|b|+\sqrt{bd}\,a^4b^4d^6|b|+10\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^4c^2d^2|b|-20\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ab^3cd^3|b|+10\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^2b^2d^4|b|+5\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^8d^2|b|\right)}{5\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -4/5*(sqrt(b*d)*b^8*c^4*d^2*abs(b) - 4*sqrt(b*d)*a*b^7*c^3*d^3*abs(b) + 6*s
qrt(b*d)*a^2*b^6*c^2*d^4*abs(b) - 4*sqrt(b*d)*a^3*b^5*c*d^5*abs(b) + sqrt(b
*d)*a^4*b^4*d^6*abs(b) + 10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^4*b^4*c^2*d^2*abs(b) - 20*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^3*c*d^3*abs(b) +
10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^4*a^2*b^2*d^4*abs(b) + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^8*d^2*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*b^3)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.84

$$\frac{2\left(dx+c\right)^{\frac{5}{2}}}{5\left(bx+a\right)^{\frac{5}{2}}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(7/2),x)
```

```
[Out] 2/5/(b*x+a)^(5/2)*(d*x+c)^(5/2)/(a*d-b*c)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.80, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{\left(5\,a\,d-5\,b\,c\right)\,{\left(a+b\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(7/2),x)
```

```
[Out] (2*(c + d*x)^(5/2))/((5*a*d - 5*b*c)*(a + b*x)^(5/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

## 3.1372 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}} \, dx$

**Optimal**. Leaf size=66

$$\frac{4d(c+dx)^{5/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{7(a+bx)^{7/2}(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4d(c+dx)^{5/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (4*d*(c + d*x)^(5/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}} - \frac{(2d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx}{7(bc-ad)}$$
$$= -\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}} + \frac{4d(c+dx)^{5/2}}{35(bc-ad)^2(a+bx)^{5/2}}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{5/2}(7ad-5bc+2bdx)}{35(a+bx)^{7/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-5*b*c + 7*a*d + 2*b*d*x))/(35*(b*c - a*d)^2*(a + b*x)^
(7/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 51, normalized size = 0.77

$$\frac{2(c+dx)^{7/2}\left(\frac{7d(a+bx)}{c+dx}-5b\right)}{35(a+bx)^{7/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-5*b + (7*d*(a + b*x))/(c + d*x)))/(35*(b*c - a*d)^2*(a
 + b*x)^(7/2))
```

**fricas [B]** time = 3.92, size = 235, normalized size = 3.56

$$\frac{2\left(2\,bd^3x^3-5\,bc^3+7\,ac^2d-\left(bcd^2-7\,ad^3\right)x^2-2\left(4\,bc^2d-7\,acd^2\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{35\left(a^4b^2c^2-2\,a^5bcd+a^6d^2+\left(b^6c^2-2\,ab^5cd+a^2b^4d^2\right)x^4+4\left(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2\right)x^3+6\left(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2\right)x^2+4\left(a^3b^3c^2-2\,a^4b^2cd+a^5bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*b*d^3*x^3 - 5*b*c^3 + 7*a*c^2*d - (b*c*d^2 - 7*a*d^3)*x^2 - 2*(4*b*
c^2*d - 7*a*c*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^2*c^2 - 2*a^5*b*c*
d + a^6*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 4*(a*b^5*c^2 - 2*
a^2*b^4*c*d + a^3*b^3*d^2)*x^3 + 6*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d
^2)*x^2 + 4*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x)
```

**giac [B]** time = 2.12, size = 1024, normalized size = 15.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 8/35*(sqrt(b*d)*b^10*c^5*d^3*abs(b) - 5*sqrt(b*d)*a*b^9*c^4*d^4*abs(b) + 10
*sqrt(b*d)*a^2*b^8*c^3*d^5*abs(b) - 10*sqrt(b*d)*a^3*b^7*c^2*d^6*abs(b) + 5
*sqrt(b*d)*a^4*b^6*c*d^7*abs(b) - sqrt(b*d)*a^5*b^5*d^8*abs(b) - 7*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^8*c^4
*d^3*abs(b) + 28*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a*b^7*c^3*d^4*abs(b) - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c^2*d^5*abs(b) + 28*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a
^3*b^5*c*d^6*abs(b) - 7*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2*a^4*b^4*d^7*abs(b) - 14*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^6*c^3*d^3*abs(b) + 42*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a*b^5*c^2*d^4*abs(b) - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^4*a^2*b^4*c*d^5*abs(b) + 14*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^3*d^6*abs(b) -
70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6*b^4*c^2*d^3*abs(b) + 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^6*a*b^3*c*d^4*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^2*d^5*abs(b) -
 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^8*b^2*c*d^3*abs(b) + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^8*a*b*d^4*abs(b) - 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*d^3*abs(b))/((b^2*c - a*b
*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b
^2)
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.82

$$\frac{2\left(dx+c\right)^{\frac{5}{2}}\left(2bdx+7ad-5bc\right)}{35\left(bx+a\right)^{\frac{7}{2}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(9/2),x)
```

```
[Out] 2/35*(d*x+c)^(5/2)*(2*b*d*x+7*a*d-5*b*c)/(b*x+a)^(7/2)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [B] time = 0.93, size = 178, normalized size = 2.70

$$\frac{\sqrt{c+d\,x}\,\left(\frac{4\,d^3\,x^3}{35\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{10\,b\,c^3-14\,a\,c^2\,d}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}+\frac{x^2\,\left(14\,a\,d^3-2\,b\,c\,d^2\right)}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}+\frac{4\,c\,d\,x\,\left(7\,a\,d-4\,b\,c\right)}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(9/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((4*d^3*x^3)/(35*b^2*(a*d - b*c)^2) - (10*b*c^3 - 14*a*c^2
*d)/(35*b^3*(a*d - b*c)^2) + (x^2*(14*a*d^3 - 2*b*c*d^2))/(35*b^3*(a*d - b*
c)^2) + (4*c*d*x*(7*a*d - 4*b*c))/(35*b^3*(a*d - b*c)^2)))/(x^3*(a + b*x)^(
1/2) + (a^3*(a + b*x)^(1/2))/b^3 + (3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(
a + b*x)^(1/2))/b^2)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

## 3.1373 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}} \, dx$

**Optimal**. Leaf size=101

$$-\frac{16d^2(c+dx)^{5/2}}{315(a+bx)^{5/2}(bc-ad)^3}+\frac{8d(c+dx)^{5/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{9(a+bx)^{9/2}(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d^2(c+dx)^{5/2}}{315(a+bx)^{5/2}(bc-ad)^3}+\frac{8d(c+dx)^{5/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (8*d*(c + d*x)^(5/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(5/2))/(315*(b*c -
 a*d)^3*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}\,dx &= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(4d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx}{9(bc-ad)} \\ &= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}} + \frac{\left(8d^2\right)\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx}{63(bc-ad)^2} \\ &= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{16d^2(c+dx)^{5/2}}{315(bc-ad)^3(a+bx)^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 77, normalized size = 0.76

$$-\frac{2(c+dx)^{5/2}\left(63a^2d^2+18abd(2dx-5c)+b^2\left(35c^2-20cdx+8d^2x^2\right)\right)}{315(a+bx)^{9/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-5*c + 2*d*x) + b^2*(35*c^2 - 2
0*c*d*x + 8*d^2*x^2)))/(315*(b*c - a*d)^3*(a + b*x)^(9/2))
```

**IntegrateAlgebraic [A]** time = 0.14, size = 73, normalized size = 0.72

$$-\frac{2(c+dx)^{9/2}\left(\frac{63d^2(a+bx)^2}{(c+dx)^2}-\frac{90bd(a+bx)}{c+dx}+35b^2\right)}{315(a+bx)^{9/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(9/2)*(35*b^2 + (63*d^2*(a + b*x)^2)/(c + d*x)^2 - (90*b*d*(a
 + b*x))/(c + d*x)))/(315*(b*c - a*d)^3*(a + b*x)^(9/2))
```

**fricas [B]** time = 13.56, size = 426, normalized size = 4.22

$$-\frac{2\left(8\,b^2d^4x^4+35\,b^2c^4-90\,abc^3d+63\,a^2c^2d^2-4\left(b^2cd^3-9\,abd^4\right)x^3+3\left(b^2c^2d^2-6\,abcd^3+21\,a^2d^4\right)x^2+2\left(25\,b^2c^3d-72\,abc^2d^2+63\,a^2cd^3\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{315\left(a^5b^3c^3-3\,a^6b^2c^2d+3\,a^7bcd^2-a^8d^3+\left(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3\right)x^5+5\left(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3\right)x^4+10\left(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3\right)x^3+10\left(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3\right)x^2+5\left(a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] -2/315*(8*b^2*d^4*x^4 + 35*b^2*c^4 - 90*a*b*c^3*d + 63*a^2*c^2*d^2 - 4*(b^2
*c*d^3 - 9*a*b*d^4)*x^3 + 3*(b^2*c^2*d^2 - 6*a*b*c*d^3 + 21*a^2*d^4)*x^2 +
```

```
2*(25*b^2*c^3*d - 72*a*b*c^2*d^2 + 63*a^2*c*d^3)*x)*sqrt(b*x + a)*sqrt(d*x
+ c)/(a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3 + (b^8*c^3 -
3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^5 + 5*(a*b^7*c^3 - 3*a^2*b
^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^4 + 10*(a^2*b^6*c^3 - 3*a^3*b^5
*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^3 + 10*(a^3*b^5*c^3 - 3*a^4*b^4*c
^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x^2 + 5*(a^4*b^4*c^3 - 3*a^5*b^3*c^2*
d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*x)
```

**giac [B]** time = 2.94, size = 1394, normalized size = 13.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="giac")
```

```
[Out] -32/315*(sqrt(b*d)*b^12*c^6*d^4*abs(b) - 6*sqrt(b*d)*a*b^11*c^5*d^5*abs(b)
+ 15*sqrt(b*d)*a^2*b^10*c^4*d^6*abs(b) - 20*sqrt(b*d)*a^3*b^9*c^3*d^7*abs(b
) + 15*sqrt(b*d)*a^4*b^8*c^2*d^8*abs(b) - 6*sqrt(b*d)*a^5*b^7*c*d^9*abs(b)
+ sqrt(b*d)*a^6*b^6*d^10*abs(b) - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^10*c^5*d^4*abs(b) + 45*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^9*c^4*d^
5*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a^2*b^8*c^3*d^6*abs(b) + 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^7*c^2*d^7*abs(b) - 45*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^
4*b^6*c*d^8*abs(b) + 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2*a^5*b^5*d^9*abs(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^8*c^4*d^4*abs(b) - 144*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a*b^7*c^3*d^5*abs(b) + 216*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^4*a^2*b^6*c^2*d^6*abs(b) - 144*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^5*c*d^7*abs
(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^4*a^4*b^4*d^8*abs(b) + 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^6*c^3*d^4*abs(b) - 378*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^5*c^2*d^5
*abs(b) + 378*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^6*a^2*b^4*c*d^6*abs(b) - 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^3*d^7*abs(b) + 441*sqrt(b*
d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^4*c^
2*d^4*abs(b) - 882*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^8*a*b^3*c*d^5*abs(b) + 441*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^2*d^6*abs(b) + 315*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^
2*c*d^4*abs(b) - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
```

```
 + a)*b*d - a*b*d))^10*a*b*d^5*abs(b) + 210*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*d^4*abs(b))/((b^2*c - a*b*d -
 (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^9*b)
```

**maple** [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{2 (dx+c)^{\frac{5}{2}} \left(8b^2x^2d^2+36ab\,d^2x-20b^2cdx+63a^2d^2-90abcd+35b^2c^2\right)}{315 (bx+a)^{\frac{9}{2}} \left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(11/2),x)
```

```
[Out] 2/315*(d*x+c)^(5/2)*(8*b^2*d^2*x^2+36*a*b*d^2*x-20*b^2*c*d*x+63*a^2*d^2-90*
a*b*c*d+35*b^2*c^2)/(b*x+a)^(9/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*
c^3)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [B] time = 1.11, size = 268, normalized size = 2.65

$$\frac{\sqrt{c+d\,x}\,\left(\frac{126\,a^2\,c^2\,d^2-180\,a\,b\,c^3\,d+70\,b^2\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{x^2\,\left(126\,a^2\,d^4-36\,a\,b\,c\,d^3+6\,b^2\,c^2\,d^2\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^4\,x^4}{315\,b^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{4\,c\,d\,x\,\left(63\,a^2\,d^2-72\,a\,b\,c\,d+25\,b^2\,c^2\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(11/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((70*b^2*c^4 + 126*a^2*c^2*d^2 - 180*a*b*c^3*d)/(315*b^4*(
a*d - b*c)^3) + (x^2*(126*a^2*d^4 + 6*b^2*c^2*d^2 - 36*a*b*c*d^3))/(315*b^4
*(a*d - b*c)^3) + (16*d^4*x^4)/(315*b^2*(a*d - b*c)^3) + (8*d^3*x^3*(9*a*d
- b*c))/(315*b^3*(a*d - b*c)^3) + (4*c*d*x*(63*a^2*d^2 + 25*b^2*c^2 - 72*a*
b*c*d))/(315*b^4*(a*d - b*c)^3)))/(x^4*(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/
```

```
2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (
4*a^3*x*(a + b*x)^(1/2))/b^3)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

## 3.1374 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{13/2}} \, dx$

**Optimal**. Leaf size=136

$$\frac{32d^3(c+dx)^{5/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{5/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{4d(c+dx)^{5/2}}{33(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{11(a+bx)^{11/2}(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3(c+dx)^{5/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{5/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{4d(c+dx)^{5/2}}{33(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (4*d*(c + d*x)^(5/
2))/(33*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(5/2))/(231*(b*c
 - a*d)^3*(a + b*x)^(7/2)) + (32*d^3*(c + d*x)^(5/2))/(1155*(b*c - a*d)^4*(
a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{13/2}}\,dx = -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} - \frac{(6d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} + \frac{\left(8d^2\right)\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx}{33(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} - \frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}} - \frac{\left(16d^3\right)\int \frac{(c+}{(a+}}{231(bc-}$$

$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} - \frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{32d^3(c}{1155(bc-a}$$

**Mathematica [A]** time = 0.06, size = 118, normalized size = 0.87

$$\frac{2(c+dx)^{5/2}\left(231a^3d^3+99a^2bd^2(2dx-5c)+11ab^2d\left(35c^2-20cdx+8d^2x^2\right)+b^3\left(-105c^3+70c^2dx-40cd^2x^2+16d^3x^3\right)\right)}{1155(a+bx)^{11/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(231*a^3*d^3 + 99*a^2*b*d^2*(-5*c + 2*d*x) + 11*a*b^2*d*
(35*c^2 - 20*c*d*x + 8*d^2*x^2) + b^3*(-105*c^3 + 70*c^2*d*x - 40*c*d^2*x^2
 + 16*d^3*x^3)))/(1155*(b*c - a*d)^4*(a + b*x)^(11/2))
```

**IntegrateAlgebraic [A]** time = 0.15, size = 95, normalized size = 0.70

$$\frac{2(c+dx)^{11/2}\left(\frac{385b^2d(a+bx)}{c+dx}+\frac{231d^3(a+bx)^3}{(c+dx)^3}-\frac{495bd^2(a+bx)^2}{(c+dx)^2}-105b^3\right)}{1155(a+bx)^{11/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
```

```
[Out] (2*(c + d*x)^(11/2)*(-105*b^3 + (231*d^3*(a + b*x)^3)/(c + d*x)^3 - (495*b*
d^2*(a + b*x)^2)/(c + d*x)^2 + (385*b^2*d*(a + b*x))/(c + d*x)))/(1155*(b*c
 - a*d)^4*(a + b*x)^(11/2))
```

**fricas [B]** time = 28.36, size = 649, normalized size = 4.77

$$\frac{2\left(16\,b^3d^5x^5-105\,b^3c^5+385\,ab^2c^4d-495\,a^2bc^3d^2+231\,a^3c^2d^3-8\left(b^3cd^4-11\,ab^2d^5\right)x^4+2\left(3\,b^3c^2d^3-22\,ab^2cd^4+99\,a^2bd^5\right)x^3-\left(5\,b^3c^3d^2-33\,ab^2c^2d^3+99\,a^2bcd^4-231\,a^3d^5\right)x^2-2\left(70\,b^3c^4d-275\,ab^2c^3d^2+396\,a^2bc^2d^3-231\,a^3cd^4\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{1155\left(a^6b^4c^4-4\,a^7b^3c^3d+6\,a^8b^2c^2d^2-4\,a^9bcd^3+a^{10}d^4+\left(b^{10}c^4-4\,ab^9c^3d+6\,a^2b^8c^2d^2-4\,a^3b^7cd^3+a^4b^6d^4\right)x^6+6\left(ab^9c^4-4\,a^2b^8c^3d+6\,a^3b^7c^2d^2-4\,a^4b^6cd^3+a^5b^5d^4\right)x^5+15\left(a^2b^8c^4-4\,a^3b^7c^3d+6\,a^4b^6c^2d^2-4\,a^5b^5cd^3+a^6b^4d^4\right)x^4+20\left(a^3b^7c^4-4\,a^4b^6c^3d+6\,a^5b^5c^2d^2-4\,a^6b^4cd^3+a^7b^3d^4\right)x^3+15\left(a^4b^6c^4-4\,a^5b^5c^3d+6\,a^6b^4c^2d^2-4\,a^7b^3cd^3+a^8b^2d^4\right)x^2+6\left(a^5b^5c^4-4\,a^6b^4c^3d+6\,a^7b^3c^2d^2-4\,a^8b^2cd^3+a^9bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*b^3*d^5*x^5 - 105*b^3*c^5 + 385*a*b^2*c^4*d - 495*a^2*b*c^3*d^2
+ 231*a^3*c^2*d^3 - 8*(b^3*c*d^4 - 11*a*b^2*d^5)*x^4 + 2*(3*b^3*c^2*d^3 - 2
2*a*b^2*c*d^4 + 99*a^2*b*d^5)*x^3 - (5*b^3*c^3*d^2 - 33*a*b^2*c^2*d^3 + 99*
a^2*b*c*d^4 - 231*a^3*d^5)*x^2 - 2*(70*b^3*c^4*d - 275*a*b^2*c^3*d^2 + 396*
a^2*b*c^2*d^3 - 231*a^3*c*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b^4*c^4
- 4*a^7*b^3*c^3*d + 6*a^8*b^2*c^2*d^2 - 4*a^9*b*c*d^3 + a^10*d^4 + (b^10*c^
4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*x^6
+ 6*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^
5*b^5*d^4)*x^5 + 15*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*
a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^4 + 20*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^
5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x^3 + 15*(a^4*b^6*c^4 - 4*a^
5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*x^2 + 6*(a
^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*
d^4)*x)
```

**giac [B]** time = 3.28, size = 1823, normalized size = 13.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="giac")
```

```
[Out] 64/1155*(sqrt(b*d)*b^14*c^7*d^5*abs(b) - 7*sqrt(b*d)*a*b^13*c^6*d^6*abs(b)
+ 21*sqrt(b*d)*a^2*b^12*c^5*d^7*abs(b) - 35*sqrt(b*d)*a^3*b^11*c^4*d^8*abs(
b) + 35*sqrt(b*d)*a^4*b^10*c^3*d^9*abs(b) - 21*sqrt(b*d)*a^5*b^9*c^2*d^10*a
bs(b) + 7*sqrt(b*d)*a^6*b^8*c*d^11*abs(b) - sqrt(b*d)*a^7*b^7*d^12*abs(b) -
 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^2*b^12*c^6*d^5*abs(b) + 66*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*a*b^11*c^5*d^6*abs(b) - 165*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^10*c^4*d^
7*abs(b) + 220*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^2*a^3*b^9*c^3*d^8*abs(b) - 165*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^8*c^2*d^9*abs(b) + 66*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
a^5*b^7*c*d^10*abs(b) - 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^2*a^6*b^6*d^11*abs(b) + 55*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^5*d^5*abs(b) -
 275*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*a*b^9*c^4*d^6*abs(b) + 550*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^8*c^3*d^7*abs(b) - 550*sqrt(b*d)*(s
qrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c^2
*d^8*abs(b) + 275*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^4*a^4*b^6*c*d^9*abs(b) - 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^5*d^10*abs(b) - 165*sqr
```

```
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^
8*c^4*d^5*abs(b) + 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^7*c^3*d^6*abs(b) - 990*sqrt(b*d)*(sqrt(b*d)*sqr
t(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c^2*d^7*abs(b)
+ 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^6*a^3*b^5*c*d^8*abs(b) - 165*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^4*b^4*d^9*abs(b) - 825*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^6*c^3*d^5*ab
s(b) + 2475*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^8*a*b^5*c^2*d^6*abs(b) - 2475*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*c*d^7*abs(b) + 825*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b
^3*d^8*abs(b) - 2541*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d))^10*b^4*c^2*d^5*abs(b) + 5082*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^3*c*d^6*abs(b) - 254
1*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
^10*a^2*b^2*d^7*abs(b) - 2079*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^12*b^2*c*d^5*abs(b) + 2079*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b*d^6*abs(b) -
1155*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^14*d^5*abs(b))/(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2)^11
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$\frac{2 (dx+c)^{\frac{5}{2}} \left(16 b^{3} x^{3} d^{3}+88 a \,b^{2} d^{3} x^{2}-40 b^{3} c \,d^{2} x^{2}+198 a^{2} b \,d^{3} x-220 a \,b^{2} c \,d^{2} x+70 b^{3} c^{2} d x+231 a^{3} d^{3}-495 a^{2} b c \,d^{2}+385 a \,b^{2} c^{2} d-105 b^{3} c^{3}\right)}{1155 (bx+a)^{\frac{11}{2}} \left(a^{4} d^{4}-4 a^{3} b c \,d^{3}+6 a^{2} b^{2} c^{2} d^{2}-4 a \,b^{3} c^{3} d+b^{4} c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(13/2),x)
```

```
[Out] 2/1155*(d*x+c)^(5/2)*(16*b^3*d^3*x^3+88*a*b^2*d^3*x^2-40*b^3*c*d^2*x^2+198*
a^2*b*d^3*x-220*a*b^2*c*d^2*x+70*b^3*c^2*d*x+231*a^3*d^3-495*a^2*b*c*d^2+38
5*a*b^2*c^2*d-105*b^3*c^3)/(b*x+a)^(11/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*
c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.33, size = 376, normalized size = 2.76

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x^2\,\left(462\,a^3\,d^5-198\,a^2\,b\,c\,d^4+66\,a\,b^2\,c^2\,d^3-10\,b^3\,c^3\,d^2\right)}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}-\frac{-462\,a^3\,c^2\,d^3+990\,a^2\,b\,c^3\,d^2-770\,a\,b^2\,c^4\,d+210\,b^3\,c^5}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}+\frac{x\,\left(924\,a^3\,c\,d^4-1584\,a^2\,b\,c^2\,d^3+1100\,a\,b^2\,c^3\,d^2-280\,b^3\,c^4\,d\right)}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,d^5\,x^5}{1155\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{1155\,b^3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^3\,x^3\,\left(99\,a^2\,d^2-22\,a\,b\,c\,d+3\,b^2\,c^2\right)}{1155\,b^4\,{\left(a\,d-b\,c\right)}^4}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x^2*(462*a^3*d^5 - 10*b^3*c^3*d^2 + 66*a*b^2*c^2*d^3 - 1
98*a^2*b*c*d^4))/(1155*b^5*(a*d - b*c)^4) - (210*b^3*c^5 - 462*a^3*c^2*d^3
+ 990*a^2*b*c^3*d^2 - 770*a*b^2*c^4*d)/(1155*b^5*(a*d - b*c)^4) + (x*(924*a
^3*c*d^4 - 280*b^3*c^4*d + 1100*a*b^2*c^3*d^2 - 1584*a^2*b*c^2*d^3))/(1155*
b^5*(a*d - b*c)^4) + (32*d^5*x^5)/(1155*b^2*(a*d - b*c)^4) + (16*d^4*x^4*(1
1*a*d - b*c))/(1155*b^3*(a*d - b*c)^4) + (4*d^3*x^3*(99*a^2*d^2 + 3*b^2*c^2
 - 22*a*b*c*d))/(1155*b^4*(a*d - b*c)^4)))/(x^5*(a + b*x)^(1/2) + (a^5*(a +
 b*x)^(1/2))/b^5 + (10*a^2*x^3*(a + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a + b*x)
^(1/2))/b^3 + (5*a*x^4*(a + b*x)^(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/b^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

## 3.1375 $\int (a+bx)^{5/2}(c+dx)^{5/2}\,dx$

**Optimal**. Leaf size=262

$$-\frac{5(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{512b^{7/2}d^{7/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^5}{512b^3d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}{768b^3d^2}+\frac{(a+bx)^{5/2}\sqrt{c+}}{192b}$$

**Rubi [A]** time = 0.15, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^5}{512b^3d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}{768b^3d^2}-\frac{5(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{512b^{7/2}d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^3}{192b^3d}+\frac{(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^3}+\frac{(a+bx)^{7/2}(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] (5*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x])/(512*b^3*d^3) - (5*(b*c - a*d
)^4*(a + b*x)^(3/2)*Sqrt[c + d*x])/(768*b^3*d^2) + ((b*c - a*d)^3*(a + b*x)
^(5/2)*Sqrt[c + d*x])/(192*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(7/2)*Sqrt[c +
 d*x])/(32*b^3) + ((b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2))/(12*b^2) +
((a + b*x)^(7/2)*(c + d*x)^(5/2))/(6*b) - (5*(b*c - a*d)^6*ArcTanh[(Sqrt[d]
*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(512*b^(7/2)*d^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}(c+dx)^{5/2}\,dx = \frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b} + \frac{(5(bc-ad))\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx}{12b}$$

$$= \frac{(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}{12b^2} + \frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b} + \cdots$$

$$= \frac{(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}{32b^3} + \frac{(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}{12b^2} + \cdots$$

$$= \frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d} + \frac{(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}{32b^3} + \cdots$$

$$= -\frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d} + \cdots$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \cdots$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \cdots$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \cdots$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \cdots$$

**Mathematica [A]** time = 2.53, size = 209, normalized size = 0.80

$$\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(-\frac{15(bc-ad)^{11/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{7/2}(a+bx)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}} + \frac{15(bc-ad)^5}{d^3(a+bx)^3} - \frac{10(bc-ad)^4}{d^2(a+bx)^2} + \frac{8(bc-ad)^3}{d(a+bx)} + 128b(c+dx)(bc-ad) + 48(bc-ad)^2 + 256b^2(c+dx)^2\right)}{1536b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] ((a + b*x)^(7/2)*Sqrt[c + d*x]*(48*(b*c - a*d)^2 + (15*(b*c - a*d)^5)/(d^3*
(a + b*x)^3) - (10*(b*c - a*d)^4)/(d^2*(a + b*x)^2) + (8*(b*c - a*d)^3)/(d*
(a + b*x)) + 128*b*(b*c - a*d)*(c + d*x) + 256*b^2*(c + d*x)^2 - (15*(b*c -
 a*d)^(11/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(7/2)*(a
+ b*x)^(7/2)*Sqrt[(b*(c + d*x))/(b*c - a*d)])))/(1536*b^3)
```

**IntegrateAlgebraic [A]** time = 0.50, size = 220, normalized size = 0.84

$$\frac{\sqrt{c+dx}\,(bc-ad)^6\left(\frac{15b^5(c+dx)^5}{(a+bx)^5}-\frac{85b^4d(c+dx)^4}{(a+bx)^4}+\frac{198b^3d^2(c+dx)^3}{(a+bx)^3}+\frac{198b^2d^3(c+dx)^2}{(a+bx)^2}-\frac{85bd^4(c+dx)}{a+bx}+15d^5\right)}{1536b^3d^3\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}-d\right)^6}-\frac{5(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{512b^{7/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] ((b*c - a*d)^6*Sqrt[c + d*x]*(15*d^5 - (85*b*d^4*(c + d*x))/(a + b*x) + (19
8*b^2*d^3*(c + d*x)^2)/(a + b*x)^2 + (198*b^3*d^2*(c + d*x)^3)/(a + b*x)^3
- (85*b^4*d*(c + d*x)^4)/(a + b*x)^4 + (15*b^5*(c + d*x)^5)/(a + b*x)^5))/(
1536*b^3*d^3*Sqrt[a + b*x]*(-d + (b*(c + d*x))/(a + b*x))^6) - (5*(b*c - a*
d)^6*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(512*b^(7/2)
*d^(7/2))
```

**fricas [B]** time = 1.43, size = 882, normalized size = 3.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6144*(15*(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*
d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(b*d)*log(8*b^2*d^2
*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sq
rt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(256*b^6*d^6*x^5 +
 15*b^6*c^5*d - 85*a*b^5*c^4*d^2 + 198*a^2*b^4*c^3*d^3 + 198*a^3*b^3*c^2*d^
4 - 85*a^4*b^2*c*d^5 + 15*a^5*b*d^6 + 640*(b^6*c*d^5 + a*b^5*d^6)*x^4 + 16*
(27*b^6*c^2*d^4 + 106*a*b^5*c*d^5 + 27*a^2*b^4*d^6)*x^3 + 8*(b^6*c^3*d^3 +
159*a*b^5*c^2*d^4 + 159*a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^2 - 2*(5*b^6*c^4*d^2
 - 28*a*b^5*c^3*d^3 - 594*a^2*b^4*c^2*d^4 - 28*a^3*b^3*c*d^5 + 5*a^4*b^2*d^
6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^4), 1/3072*(15*(b^6*c^6 - 6*a*b^5
*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a
^5*b*c*d^5 + a^6*d^6)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d
```

```
)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*
x)) + 2*(256*b^6*d^6*x^5 + 15*b^6*c^5*d - 85*a*b^5*c^4*d^2 + 198*a^2*b^4*c^
3*d^3 + 198*a^3*b^3*c^2*d^4 - 85*a^4*b^2*c*d^5 + 15*a^5*b*d^6 + 640*(b^6*c*
d^5 + a*b^5*d^6)*x^4 + 16*(27*b^6*c^2*d^4 + 106*a*b^5*c*d^5 + 27*a^2*b^4*d^
6)*x^3 + 8*(b^6*c^3*d^3 + 159*a*b^5*c^2*d^4 + 159*a^2*b^4*c*d^5 + a^3*b^3*d
^6)*x^2 - 2*(5*b^6*c^4*d^2 - 28*a*b^5*c^3*d^3 - 594*a^2*b^4*c^2*d^4 - 28*a^
3*b^3*c*d^5 + 5*a^4*b^2*d^6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^4)]
```

**giac [B]** time = 3.53, size = 3120, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/7680*(960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)
*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2
+ 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3
*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*c^2*abs(b) - 7680*((b^2*c - a*
b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*c^2*a
bs(b)/b^2 + 40*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x +
a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^
2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3
 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt
(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d
^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*
b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*b*c^2*abs(b) + 240*(sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 -
 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b
^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c
*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)
*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))
*a*c*d*abs(b) + 1920*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*
(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7
*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*
c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(
b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*c*d*abs(b)/b + 8*(s
qrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a
)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b
^20*c*d^7 - 513*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c
^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(
7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^
```

```
7 - 193*a^4*b^19*d^8)/(b^23*d^8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c
^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
(sqrt(b*d)*b^3*d^4))*b*c*d*abs(b) + 12*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^
8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/(b^2
3*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a
^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5
 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^8))*
sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*
b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a)
 + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*a*d^2*abs(b)
+ 320*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b
*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*
b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b
*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^3*d^2*abs(b)/b^2 + 120*(sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*
c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 16
3*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2
*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^
2*d^3))*a^2*d^2*abs(b)/b + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(2*(b
*x + a)*(8*(b*x + a)*(10*(b*x + a)/b^5 + (b^30*c*d^9 - 61*a*b^29*d^10)/(b^3
4*d^10)) - 3*(3*b^31*c^2*d^8 + 14*a*b^30*c*d^9 - 417*a^2*b^29*d^10)/(b^34*d
^10)) + (21*b^32*c^3*d^7 + 77*a*b^31*c^2*d^8 + 183*a^2*b^30*c*d^9 - 3481*a^
3*b^29*d^10)/(b^34*d^10))*(b*x + a) - 5*(21*b^33*c^4*d^6 + 56*a*b^32*c^3*d^
7 + 106*a^2*b^31*c^2*d^8 + 176*a^3*b^30*c*d^9 - 2279*a^4*b^29*d^10)/(b^34*d
^10))*(b*x + a) + 15*(21*b^34*c^5*d^5 + 35*a*b^33*c^4*d^6 + 50*a^2*b^32*c^3
*d^7 + 70*a^3*b^31*c^2*d^8 + 105*a^4*b^30*c*d^9 - 793*a^5*b^29*d^10)/(b^34*
d^10))*sqrt(b*x + a) + 15*(21*b^6*c^6 + 14*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2
 + 20*a^3*b^3*c^3*d^3 + 35*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 - 231*a^6*d^6)
*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(
sqrt(b*d)*b^4*d^5))*b*d^2*abs(b) + 5760*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*
c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)))/(sqrt(b*d)*d))*a^2*c^2*abs(b)/b^2 + 3840*(sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (
b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqr
t(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*a^3*c*d*abs(b)/b^3)/b
```

**maple [B]** time = 0.01, size = 1089, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(5/2),x)
```

```
[Out] 1/6/d*(b*x+a)^(5/2)*(d*x+c)^(7/2)+25/256*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1
/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+
b*c)*x)^(1/2))/(b*d)^(1/2)*a^3*c^3-1/16/d^2*(b*x+a)^(1/2)*(d*x+c)^(7/2)*a*b
*c+5/192/d^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c^3*b*a+25/512/d^2*(d*x+c)^(1/2)*(
b*x+a)^(1/2)*a*c^4*b+1/64/d^2*(d*x+c)^(5/2)*(b*x+a)^(1/2)*c^2*b*a-25/512*d/
b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^4*c+1/192/b*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a
^3+1/12/d*(b*x+a)^(3/2)*(d*x+c)^(7/2)*a+1/32/d*(b*x+a)^(1/2)*(d*x+c)^(7/2)*
a^2-1/12/d^2*(b*x+a)^(3/2)*(d*x+c)^(7/2)*b*c-5/128/d*(d*x+c)^(3/2)*(b*x+a)^
(1/2)*a^2*c^2-25/256/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2*c^3-5/768/d^3*(d*x+c
)^(3/2)*(b*x+a)^(1/2)*c^4*b^2-1/64/d*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a^2*c+25/2
56/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3*c^2+5/192/b*(d*x+c)^(3/2)*(b*x+a)^(1/2
)*a^3*c+5/512*d^2/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^5-5/512/d^3*(d*x+c)^(1/
2)*(b*x+a)^(1/2)*c^5*b^2-1/192/d^3*(d*x+c)^(5/2)*(b*x+a)^(1/2)*c^3*b^2-5/76
8*d/b^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^4+1/32/d^3*(b*x+a)^(1/2)*(d*x+c)^(7/2
)*b^2*c^2-75/1024*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*l
n((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d
)^(1/2)*a^4*c^2+15/512*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1
/2))/(b*d)^(1/2)*a^5*c+15/512/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*
x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)
^(1/2))/(b*d)^(1/2)*a*c^5*b^2-75/1024/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/
2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b
*c)*x)^(1/2))/(b*d)^(1/2)*a^2*c^4*b-5/1024/d^3*((b*x+a)*(d*x+c))^(1/2)/(d*x
+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c
+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^6*b^3-5/1024*d^3/b^3*((b*x+a)*(d*x+c))^(
1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*
d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^6
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/2}\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(5/2)*(c + d*x)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(5/2), x)
```

## 3.1376 $\int (a+bx)^{3/2}(c+dx)^{5/2}\,dx$

**Optimal**. Leaf size=224

$$\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{7/2}d^{5/2}}-\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128b^3d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{(a+bx)^{5/2}\sqrt{c+d}}{16b^3}$$

**Rubi** [A] time = 0.12, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128b^3d^2}+\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{7/2}d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{16b^3}+\frac{(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)}{8b^2}+\frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]
```

```
[Out] (-3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^3*d^2) + ((b*c - a*d)
^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(5/
2)*Sqrt[c + d*x])/(16*b^3) + ((b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2))/
(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(5/2))/(5*b) + (3*(b*c - a*d)^5*ArcTan
h[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(7/2)*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{5/2}\,dx = \frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b} + \frac{(bc-ad)\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b} + \frac{\left(3(bc-ad)^2\right)\int (a+bx)^{3/2}\cdots}{16b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3} + \frac{(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx\cdots}{5b}$$

$$= \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3} + \frac{(bc-ad)(a+bx)^{5/\cdots}}{8b^2}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)\cdots}{16b^3}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)\cdots}{16b^3}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)\cdots}{16b^3}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)\cdots}{16b^3}$$

**Mathematica [A]** time = 1.47, size = 187, normalized size = 0.83

$$\frac{(a+bx)^{5/2}\sqrt{c+dx}\left(\frac{15(bc-ad)^{9/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{5/2}(a+bx)^{5/2}\sqrt{\frac{b(c+dx)}{bc-ad}}} - \frac{15(bc-ad)^4}{d^2(a+bx)^2} + \frac{10(bc-ad)^3}{d(a+bx)} + 80b(c+dx)(bc-ad) + 40(bc-ad)^2 + 128b^2(c+dx)^2\right)}{640b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]
```

```
[Out] ((a + b*x)^(5/2)*Sqrt[c + d*x]*(40*(b*c - a*d)^2 - (15*(b*c - a*d)^4)/(d^2*
(a + b*x)^2) + (10*(b*c - a*d)^3)/(d*(a + b*x)) + 80*b*(b*c - a*d)*(c + d*x
) + 128*b^2*(c + d*x)^2 + (15*(b*c - a*d)^(9/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[b*c - a*d]])/(d^(5/2)*(a + b*x)^(5/2)*Sqrt[(b*(c + d*x))/(b*c - a
*d)])))/(640*b^3)
```

**IntegrateAlgebraic [A]** time = 0.43, size = 198, normalized size = 0.88

$$\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{128b^{7/2}d^{5/2}}-\frac{\sqrt{c+dx}\,(bc-ad)^5\left(\frac{15b^4(c+dx)^4}{(a+bx)^4}-\frac{70b^3d(c+dx)^3}{(a+bx)^3}-\frac{128b^2d^2(c+dx)^2}{(a+bx)^2}+\frac{70bd^3(c+dx)}{a+bx}-15d^4\right)}{640b^3d^2\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}-d\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]
```

```
[Out] -1/640*((b*c - a*d)^5*Sqrt[c + d*x]*(-15*d^4 + (70*b*d^3*(c + d*x))/(a + b*
x) - (128*b^2*d^2*(c + d*x)^2)/(a + b*x)^2 - (70*b^3*d*(c + d*x)^3)/(a + b*
x)^3 + (15*b^4*(c + d*x)^4)/(a + b*x)^4))/(b^3*d^2*Sqrt[a + b*x]*(-d + (b*(
c + d*x))/(a + b*x))^5) + (3*(b*c - a*d)^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/
(Sqrt[d]*Sqrt[a + b*x])])/(128*b^(7/2)*d^(5/2))
```

**fricas [A]** time = 0.89, size = 702, normalized size = 3.13

$$\left[-\frac{15\left(b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8(b^2cd+abd^2)x\right)-4\left(128b^5d^5x^4-15b^5c^4d+70ab^4c^3d^2+128a^2b^3c^2d^3-70a^3b^2cd^4+15a^4bd^5+16(21b^5cd^4+11ab^4d^5)x^3+8(31b^5c^2d^3+64ab^4cd^4+a^2b^3d^5)x^2+2(5b^5c^3d^2+233ab^4c^2d^3+23a^2b^3cd^4-5a^3b^2d^5)x\right)\sqrt{bx+a}\sqrt{dx+c}}{2560b^4d^3},\ -\frac{15\left(b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)-2\left(128b^5d^5x^4-15b^5c^4d+70ab^4c^3d^2+128a^2b^3c^2d^3-70a^3b^2cd^4+15a^4bd^5+16(21b^5cd^4+11ab^4d^5)x^3+8(31b^5c^2d^3+64ab^4cd^4+a^2b^3d^5)x^2+2(5b^5c^3d^2+233ab^4c^2d^3+23a^2b^3cd^4-5a^3b^2d^5)x\right)\sqrt{bx+a}\sqrt{dx+c}}{1280b^4d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2
*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x
 + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(128*b^5*d^5*x^4 - 15*b^5*c^4*d + 70*a
*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 - 70*a^3*b^2*c*d^4 + 15*a^4*b*d^5 + 16*(
21*b^5*c*d^4 + 11*a*b^4*d^5)*x^3 + 8*(31*b^5*c^2*d^3 + 64*a*b^4*c*d^4 + a^2
*b^3*d^5)*x^2 + 2*(5*b^5*c^3*d^2 + 233*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 5
*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^3), -1/1280*(15*(b^5*c
^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^
4 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*
x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(
128*b^5*d^5*x^4 - 15*b^5*c^4*d + 70*a*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 - 7
0*a^3*b^2*c*d^4 + 15*a^4*b*d^5 + 16*(21*b^5*c*d^4 + 11*a*b^4*d^5)*x^3 + 8*(
31*b^5*c^2*d^3 + 64*a*b^4*c*d^4 + a^2*b^3*d^5)*x^2 + 2*(5*b^5*c^3*d^2 + 233
```

```
*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 5*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*
x + c))/(b^4*d^3)]
```

**giac [B]** time = 2.33, size = 1962, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1920*(80*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*
(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*
a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c^2*abs(b) - 1920*((b^2*c - a*b*d
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*c^2*abs(
b)/b^2 + 20*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*
(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d
^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 +
9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*
x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3
- 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)))/(sqrt(b*d)*b^2*d^3))*c*d*abs(b) + 320*(sqrt(b^2*c + (b*x + a)*b
*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a
*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^
7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*
d^2))*a*c*d*abs(b)/b + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)
*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) -
 (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b
^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^
23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c
^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^8))*sqrt(b*x + a) -
15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35
*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*d^2*abs(b) + 80*(sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*
c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*
b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)
*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(
sqrt(b*d)*b*d^2))*a^2*d^2*abs(b)/b^2 + 20*(sqrt(b^2*c + (b*x + a)*b*d - a*b
*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^
6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^1
4*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3
```

```
*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*d^2*abs(b)
/b + 960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d
^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))
*a*c^2*abs(b)/b^2 + 960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
 (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^
2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d))*a^2*c*d*abs(b)/b^3)/b
```

**maple [B]** time = 0.01, size = 848, normalized size = 3.79

$$-\frac{3\sqrt{(bx+a)(dx+c)}\,a^5d^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{256\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^3}+\frac{15\sqrt{(bx+a)(dx+c)}\,a^4cd^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{256\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^2}-\frac{15\sqrt{(bx+a)(dx+c)}\,a^3c^2d\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{15\sqrt{(bx+a)(dx+c)}\,a^2c^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{15\sqrt{(bx+a)(dx+c)}\,abc^4\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{256\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}+\frac{3\sqrt{(bx+a)(dx+c)}\,b^2c^5\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{256\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d^2}+\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,a^4d^2}{128b^3}-\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,a^3cd}{32b^2}+\frac{9\sqrt{dx+c}\,\sqrt{bx+a}\,a^2c^2}{64b}-\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,ac^3}{32d}+\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,bc^4}{128d^2}-\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,a^3d}{64b^2}+\frac{3(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,a^2c}{64b}-\frac{3(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,ac^2}{64d}+\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,bc^3}{64d^2}+\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}\,a^2}{80b}-\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}\,ac}{40d}+\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}\,bc^2}{80d^2}+\frac{3\sqrt{bx+a}\,(dx+c)^{\frac{7}{2}}a}{40d}-\frac{3\sqrt{bx+a}\,(dx+c)^{\frac{7}{2}}bc}{40d^2}+\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{7}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(5/2),x)
```

```
[Out] 1/5/d*(b*x+a)^(3/2)*(d*x+c)^(7/2)+3/64/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^2*c-
3/32*d/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3*c+9/64/b*(d*x+c)^(1/2)*(b*x+a)^(
1/2)*a^2*c^2+3/40/d*(b*x+a)^(1/2)*(d*x+c)^(7/2)*a-1/64*d/b^2*(d*x+c)^(3/2)*
(b*x+a)^(1/2)*a^3-3/64/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c^2+1/64/d^2*(d*x+c)
^(3/2)*(b*x+a)^(1/2)*c^3*b+3/128*d^2/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^4-3/
32/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^3+3/128/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2
)*c^4*b-3/40/d^2*(b*x+a)^(1/2)*(d*x+c)^(7/2)*b*c+1/80/b*(d*x+c)^(5/2)*(b*x+
a)^(1/2)*a^2-1/40/d*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a*c+1/80/d^2*(d*x+c)^(5/2)*
(b*x+a)^(1/2)*c^2*b+15/128*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1
/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))
/(b*d)^(1/2)*a^2*c^3+15/256*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(
b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*
x)^(1/2))/(b*d)^(1/2)*a^4*c-15/128*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*
c)*x)^(1/2))/(b*d)^(1/2)*a^3*c^2-15/256/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(
1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d
+b*c)*x)^(1/2))/(b*d)^(1/2)*a*c^4*b+3/256/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+
(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^5*b^2-3/256*d^3/b^3*((b*x+a)*(d*x+c))^(1/
2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*
x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^5
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,x)^{3/2}\,(c + d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^{\frac{3}{2}}\,(c + dx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(5/2), x)
```

## 3.1377 $\int \sqrt{a+bx}\,(c+dx)^{5/2}\,dx$

**Optimal**. Leaf size=186

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^3}+\frac{5(a+bx)^{3/2}(c}{2}$$

**Rubi [A]** time = 0.09, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^3}+\frac{5(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}{24b^2}+\frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^3*d) + (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^3) + (5*(b*c - a*d)*(a + b*x)^(3/2)*(
c + d*x)^(3/2))/(24*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(5/2))/(4*b) - (5*(b*
c - a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^
(7/2)*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{5/2}\,dx = \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b} + \frac{(5(bc-ad))\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx}{8b}$$

$$= \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2} + \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b} + \frac{\left(5(bc-ad)^2\right)\int \sqrt{a+bx}\,\sqrt{\ldots}}{16b^2}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2} + \frac{(a+bx)^{3/2}(c+d\ldots}{4b}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3\ldots}}{24b^2}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3\ldots}}{24b^2}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3\ldots}}{24b^2}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3\ldots}}{24b^2}$$

**Mathematica [A]** time = 0.58, size = 191, normalized size = 1.03

$$\frac{b\sqrt{d}\,\sqrt{a+bx}\,(c+dx)\left(15a^3d^3-5a^2bd^2(11c+2dx)+ab^2d\left(73c^2+36cdx+8d^2x^2\right)+b^3\left(15c^3+118c^2dx+136cd^2x^2+48d^3x^3\right)\right)-15(bc-ad)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{192b^4d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(15*a^3*d^3 - 5*a^2*b*d^2*(11*c + 2*d*x)
 + a*b^2*d*(73*c^2 + 36*c*d*x + 8*d^2*x^2) + b^3*(15*c^3 + 118*c^2*d*x + 13
```

```
6*c*d^2*x^2 + 48*d^3*x^3)) - 15*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))/(b*c -
 a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(192*b^4*d^(3/2)*S
qrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.28, size = 175, normalized size = 0.94

$$\frac{\sqrt{a+bx}\,(bc-ad)^4\left(\frac{73b^2d(a+bx)}{c+dx}+\frac{15d^3(a+bx)^3}{(c+dx)^3}-\frac{55bd^2(a+bx)^2}{(c+dx)^2}+15b^3\right)}{192b^3d\sqrt{c+dx}\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

```
[Out] ((b*c - a*d)^4*Sqrt[a + b*x]*(15*b^3 + (15*d^3*(a + b*x)^3)/(c + d*x)^3 - (
55*b*d^2*(a + b*x)^2)/(c + d*x)^2 + (73*b^2*d*(a + b*x))/(c + d*x)))/(192*b
^3*d*Sqrt[c + d*x]*(b - (d*(a + b*x))/(c + d*x))^4) - (5*(b*c - a*d)^4*ArcT
anh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(7/2)*d^(3/2))
```

**fricas [A]** time = 1.30, size = 540, normalized size = 2.90

$$\left[\frac{15\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)+4\left(48b^4d^4x^3+15b^4c^3d+73ab^3c^2d^2-55a^2b^2cd^3+15a^3bd^4+8\left(17b^4cd^3+ab^3d^4\right)x^2+2\left(59b^4c^2d^2+18ab^3cd^3-5a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{768b^4d^2}, \frac{15\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)+2\left(48b^4d^4x^3+15b^4c^3d+73ab^3c^2d^2-55a^2b^2cd^3+15a^3bd^4+8\left(17b^4cd^3+ab^3d^4\right)x^2+2\left(59b^4c^2d^2+18ab^3cd^3-5a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{384b^4d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*
b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) + 4*(48*b^4*d^4*x^3 + 15*b^4*c^3*d + 73*a*b^3*c^2*d^2 - 55*a^2*b^2
*c*d^3 + 15*a^3*b*d^4 + 8*(17*b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(59*b^4*c^2*d^
2 + 18*a*b^3*c*d^3 - 5*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^
2), 1/384*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
+ a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48
*b^4*d^4*x^3 + 15*b^4*c^3*d + 73*a*b^3*c^2*d^2 - 55*a^2*b^2*c*d^3 + 15*a^3*
b*d^4 + 8*(17*b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(59*b^4*c^2*d^2 + 18*a*b^3*c*d
^3 - 5*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^2)]
```

**giac [B]** time = 1.93, size = 1083, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/192*(192*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*
sqrt(b*x + a))*a*c^2*abs(b)/b^2 - 16*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*s
qrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^
7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(
b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c*d*abs
(b)/b - 8*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(
4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a
^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*d^2*abs(b)/b^2 - (sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*
d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*
a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b
^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt
(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*
d^3))*d^2*abs(b)/b - 48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
 (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^
2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d))*c^2*abs(b)/b^2 - 96*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2
*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d
- 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(sqrt(b*d)*d))*a*c*d*abs(b)/b^3)/b
```

**maple [B]** time = 0.01, size = 641, normalized size = 3.45

$$-\frac{5\sqrt{(bx+a)(dx+c)}\,a^4d^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^3}+\frac{5\sqrt{(bx+a)(dx+c)}\,a^3cd^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^2}-\frac{15\sqrt{(bx+a)(dx+c)}\,a^2c^2d\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{64\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{5\sqrt{(bx+a)(dx+c)}\,ac^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{5\sqrt{(bx+a)(dx+c)}\,bc^4\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}+\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,a^3d^2}{64b^3}-\frac{15\sqrt{dx+c}\,\sqrt{bx+a}\,a^2cd}{64b^2}+\frac{15\sqrt{dx+c}\,\sqrt{bx+a}\,ac^2}{64b}-\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,c^3}{64d}-\frac{5(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,a^2d}{96b^2}+\frac{5(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,ac}{48b}-\frac{5(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,c^2}{96d}+\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}\,a}{24b}-\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}\,c}{24d}+\frac{\sqrt{bx+a}\,(dx+c)^{\frac{7}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(5/2),x)
```

```
[Out] 1/4/d*(b*x+a)^(1/2)*(d*x+c)^(7/2)+1/24/b*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a-1/24
/d*(d*x+c)^(5/2)*(b*x+a)^(1/2)*c-5/96*d/b^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^2
+5/48/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c-5/96/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*
c^2+5/64*d^2/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3-15/64*d/b^2*(d*x+c)^(1/2)*
(b*x+a)^(1/2)*a^2*c+15/64/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^2-5/64/d*(d*x+c
)^(1/2)*(b*x+a)^(1/2)*c^3-5/128*d^3/b^3*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/
2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b
*c)*x)^(1/2))/(b*d)^(1/2)*a^4+5/32*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^
(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*
d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^3*c-15/64*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c
)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(
a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*c^2+5/32*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)
```

```
^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a
*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*c^3-5/128/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)
^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a
*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^4*b
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

## 3.1378 $\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}}\,dx$

**Optimal**. Leaf size=148

$$\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8b^3}+\frac{5\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{\sqrt{a+bx}(c+dx)^{5/2}}{3b}$$

**Rubi [A]** time = 0.07, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8b^3}+\frac{5\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}}+\frac{\sqrt{a+bx}(c+dx)^{5/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^3) + (5*(b*c - a*d)*Sqrt
[a + b*x]*(c + d*x)^(3/2))/(12*b^2) + (Sqrt[a + b*x]*(c + d*x)^(5/2))/(3*b)
 + (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*b^(7/2)*Sqrt[d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{(5(bc-ad))\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx}{6b}$$

$$= \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{\left(5(bc-ad)^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{8b^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{\left(5(bc\right.}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{\left(5(bc\right.}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{\left(5(bc\right.}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{5(bc}{}$$

**Mathematica [A]** time = 0.41, size = 139, normalized size = 0.94

$$\frac{\sqrt{c+dx}\left(\sqrt{a+bx}\left(15a^2d^2-10abd(4c+dx)+b^2\left(33c^2+26cdx+8d^2x^2\right)\right)+\frac{15(bc-ad)^{5/2}\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\,\sqrt{\frac{b(c+dx)}{bc-ad}}}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[a + b*x]*(15*a^2*d^2 - 10*a*b*d*(4*c + d*x) + b^2*(33*
c^2 + 26*c*d*x + 8*d^2*x^2)) + (15*(b*c - a*d)^(5/2)*ArcSinh[(Sqrt[d]*Sqrt[
```

```
a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[(b*(c + d*x))/(b*c - a*d)]))/(24
*b^3)
```

**IntegrateAlgebraic [A]** time = 0.20, size = 160, normalized size = 1.08

$$\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}}+\frac{(bc-ad)^3\left(\frac{33b^2\sqrt{a+bx}}{\sqrt{c+dx}}+\frac{15d^2(a+bx)^{5/2}}{(c+dx)^{5/2}}-\frac{40bd(a+bx)^{3/2}}{(c+dx)^{3/2}}\right)}{24b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] ((b*c - a*d)^3*((15*d^2*(a + b*x)^(5/2))/(c + d*x)^(5/2) - (40*b*d*(a + b*x
)^(3/2))/(c + d*x)^(3/2) + (33*b^2*Sqrt[a + b*x])/Sqrt[c + d*x]))/(24*b^3*(
b - (d*(a + b*x))/(c + d*x))^3) + (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a
+ b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*b^(7/2)*Sqrt[d])
```

**fricas [A]** time = 1.25, size = 412, normalized size = 2.78

$$\left[-\frac{15\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)-4\left(8b^3d^3x^2+33b^3c^2d-40ab^2cd^2+15a^2bd^3+2\left(13b^3cd^2-5ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{96b^4d},\ -\frac{15\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(8b^3d^3x^2+33b^3c^2d-40ab^2cd^2+15a^2bd^3+2\left(13b^3cd^2-5ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48b^4d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*s
qrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*
d^3*x^2 + 33*b^3*c^2*d - 40*a*b^2*c*d^2 + 15*a^2*b*d^3 + 2*(13*b^3*c*d^2 -
5*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d), -1/48*(15*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b
*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (
b^2*c*d + a*b*d^2)*x)) - 2*(8*b^3*d^3*x^2 + 33*b^3*c^2*d - 40*a*b^2*c*d^2 +
 15*a^2*b*d^3 + 2*(13*b^3*c*d^2 - 5*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x +
c))/(b^4*d)]
```

**giac [B]** time = 1.43, size = 446, normalized size = 3.01

$$-\frac{\frac{24\left(\frac{\left(b^2c-abd\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\right)c^2|b|}{b^2}-\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b^2}+\frac{b^6cd^3-13\,ab^5d^4}{b^7d^4}\right)-\frac{3\left(b^7c^2d^2+2\,ab^6cd^3-11\,a^2b^5d^4\right)}{b^7d^4}\right)-\frac{3\left(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^2}\right)d^2|b|}{b^2}-\frac{12\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,bx+2\,a+\frac{bcd-5\,ad^2}{d^2}\right)\sqrt{bx+a}+\frac{\left(b^3c^2+2\,ab^2cd-3\,a^2bd^2\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)cd|b|}{b^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(24*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sq
rt(b*x + a))*c^2*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x
 + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4))
 - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3
 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*d^2*abs(b)/b^2
 - 12*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)
/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b
*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*c*
d*abs(b)/b^3)/b
```

**maple [B]** time = 0.01, size = 465, normalized size = 3.14

$$-\frac{5\sqrt{(bx+a)(dx+c)}\, a^3 d^3 \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^3}+\frac{15\sqrt{(bx+a)(dx+c)}\, a^2 c\, d^2 \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^2}-\frac{15\sqrt{(bx+a)(dx+c)}\, a\,c^2 d \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{5\sqrt{(bx+a)(dx+c)}\, c^3 \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}+\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,a^2 d^2}{8b^3}-\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,acd}{4b^2}+\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,c^2}{8b}-\frac{5(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,ad}{12b^2}+\frac{5(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,c}{12b}+\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/3*(d*x+c)^(5/2)*(b*x+a)^(1/2)/b-5/12/b^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*d+
5/12/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c+5/8/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^
2*d^2-5/4/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*d*c+5/8/b*(d*x+c)^(1/2)*(b*x+a)
^(1/2)*c^2-5/16/b^3*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln(
(b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^
(1/2)*a^3*d^3+15/16/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)
*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b
*d)^(1/2)*a^2*d^2*c-15/16/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(
1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2)
)/(b*d)^(1/2)*a*d*c^2+5/16*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1
/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))
/(b*d)^(1/2)*c^3
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(1/2),x)
[Out] int((c + d*x)^(5/2)/(a + b*x)^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(1/2),x)
[Out] Timed out
```

## 3.1379 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}}\,dx$

**Optimal**. Leaf size=138

$$\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{7/2}}+\frac{15d\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}$$

**Rubi** [A] time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}+\frac{15d\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4b^3}+\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{7/2}}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (15*d*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^3) + (5*d*Sqrt[a + b*x]
*(c + d*x)^(3/2))/(2*b^2) - (2*(c + d*x)^(5/2))/(b*Sqrt[a + b*x]) + (15*Sqr
t[d]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])]
)/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx}{b}$$

$$= \frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{(15d(bc-ad))\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{4b^2}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{\left(15d(bc-ad)^2\right)\int \frac{}{\sqrt{a}}}{8b^3}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{\left(15d(bc-ad)^2\right)\mathrm{Sub}}{}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{\left(15d(bc-ad)^2\right)\mathrm{Sub}}{}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{15\sqrt{d}\,(bc-ad)^2\tanh}{4b^{7/2}}$$

**Mathematica [C]** time = 0.07, size = 71, normalized size = 0.51

$$-\frac{2(c+dx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*Hypergeometric2F1[-5/2, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.28, size = 151, normalized size = 1.09

$$\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4b^{7/2}}-\frac{\sqrt{c+dx}\,(bc-ad)^2\left(\frac{8b^2(c+dx)^2}{(a+bx)^2}-\frac{25bd(c+dx)}{a+bx}+15d^2\right)}{4b^3\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] -1/4*((b*c - a*d)^2*Sqrt[c + d*x]*(15*d^2 - (25*b*d*(c + d*x))/(a + b*x) +
(8*b^2*(c + d*x)^2)/(a + b*x)^2))/(b^3*Sqrt[a + b*x]*(-d + (b*(c + d*x))/(a
 + b*x))^2) + (15*Sqrt[d]*(b*c - a*d)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sq
rt[d]*Sqrt[a + b*x])])/(4*b^(7/2))
```

**fricas [A]** time = 1.42, size = 439, normalized size = 3.18

$$\left[\frac{15\left(ab^2c^2-2a^2bcd+a^3d^2+\left(b^3c^2-2ab^2cd+a^2bd^2\right)x\right)\sqrt{\frac{d}{b}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2b^2dx+b^2c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)+4\left(2b^2d^2x^2-8b^2c^2+25abcd-15a^2d^2+\left(9b^2cd-5abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{16\left(b^4x+ab^3\right)},-\frac{15\left(ab^2c^2-2a^2bcd+a^3d^2+\left(b^3c^2-2ab^2cd+a^2bd^2\right)x\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(2bdx+bc+ad\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right)-2\left(2b^2d^2x^2-8b^2c^2+25abcd-15a^2d^2+\left(9b^2cd-5abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{8\left(b^4x+ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*
(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*
c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x^2 - 8*b^2*c^2 + 25*a*b*c*d - 15*a^2*d^2
+ (9*b^2*c*d - 5*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x + a*b^3),
-1/8*(15*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*
b*d^2)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*
x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) - 2*(2*b^2*d^2*x
```

```
^2 - 8*b^2*c^2 + 25*a*b*c*d - 15*a^2*d^2 + (9*b^2*c*d - 5*a*b*d^2)*x)*sqrt(
b*x + a)*sqrt(d*x + c))/(b^4*x + a*b^3)]
```

**giac [B]** time = 2.02, size = 287, normalized size = 2.08

$$\frac{1}{4}\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\left(\frac{2\,(bx+a)d^2|b|}{b^5}+\frac{9\left(b^{10}cd^3|b|-ab^9d^4|b|\right)}{b^{14}d^2}\right)-\frac{15\left(\sqrt{bd}\,b^2c^2|b|-2\sqrt{bd}\,abcd|b|+\sqrt{bd}\,a^2d^2|b|\right)\log\left(\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{8\,b^5}-\frac{4\left(\sqrt{bd}\,b^3c^3|b|-3\sqrt{bd}\,ab^2c^2d|b|+3\sqrt{bd}\,a^2bcd^2|b|-\sqrt{bd}\,a^3d^3|b|\right)}{\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*d^2*abs(
b)/b^5 + 9*(b^10*c*d^3*abs(b) - a*b^9*d^4*abs(b))/(b^14*d^2)) - 15/8*(sqrt(
b*d)*b^2*c^2*abs(b) - 2*sqrt(b*d)*a*b*c*d*abs(b) + sqrt(b*d)*a^2*d^2*abs(b)
)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^
5 - 4*(sqrt(b*d)*b^3*c^3*abs(b) - 3*sqrt(b*d)*a*b^2*c^2*d*abs(b) + 3*sqrt(b
*d)*a^2*b*c*d^2*abs(b) - sqrt(b*d)*a^3*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*b^4)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(3/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{5/2}}{{\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(5/2)/(a + b*x)^(3/2), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{2}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(5/2)/(a + b*x)**(3/2), x)
```

## 3.1380 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{5/2}}\,dx$

**Optimal**. Leaf size=128

$$\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{7/2}}+\frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}$$

**Rubi [A]** time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3}+\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (5*d^2*Sqrt[a + b*x]*Sqrt[c + d*x])/b^3 - (10*d*(c + d*x)^(3/2))/(3*b^2*Sqr
t[a + b*x]) - (2*(c + d*x)^(5/2))/(3*b*(a + b*x)^(3/2)) + (5*d^(3/2)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx}{3b}$$

$$= -\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{\left(5d^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{b^2}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{\left(5d^2(bc-ad)\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2b^3}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{\left(5d^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\right.}{b^4}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{\left(5d^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,\right.}{b^4}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{7/2}}$$

**Mathematica [C]** time = 0.06, size = 73, normalized size = 0.57

$$-\frac{2(c+dx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*Hypergeometric2F1[-5/2, -3/2, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.99, size = 227, normalized size = 1.77

$$\frac{\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}\left(15a^2d^4\sqrt{c+dx}+20abd^3(c+dx)^{3/2}-30abcd^3\sqrt{c+dx}+15b^2c^2d^2\sqrt{c+dx}+3b^2d^2(c+dx)^{5/2}-20b^2cd^2(c+dx)^{3/2}\right)}{3b^3(-ad-b(c+dx)+bc)^2}-\frac{5\sqrt{\frac{b}{d}}\left(bcd^2-ad^3\right)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}-\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - (b*c)/d + (b*(c + d*x))/d]*(15*b^2*c^2*d^2*Sqrt[c + d*x] - 30*a*b
*c*d^3*Sqrt[c + d*x] + 15*a^2*d^4*Sqrt[c + d*x] - 20*b^2*c*d^2*(c + d*x)^(3
/2) + 20*a*b*d^3*(c + d*x)^(3/2) + 3*b^2*d^2*(c + d*x)^(5/2)))/(3*b^3*(b*c
- a*d - b*(c + d*x))^2) - (5*Sqrt[b/d]*(b*c*d^2 - a*d^3)*Log[-(Sqrt[b/d]*Sq
rt[c + d*x]) + Sqrt[a - (b*c)/d + (b*(c + d*x))/d]])/b^4
```

**fricas [B]** time = 1.93, size = 475, normalized size = 3.71

$$\left[-\frac{15\left(a^2bcd-a^3d^2+\left(b^3cd-ab^2d^2\right)x^2+2\left(ab^2cd-a^2bd^2\right)x\right)\sqrt{\frac{d}{b}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4\left(2b^2dx+b^2c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)-4\left(3b^2d^2x^2-2b^2c^2-10abcd+15a^2d^2-2\left(7b^2cd-10abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{12\left(b^5x^2+2ab^4x+a^2b^3\right)}, -\frac{15\left(a^2bcd-a^3d^2+\left(b^3cd-ab^2d^2\right)x^2+2\left(ab^2cd-a^2bd^2\right)x\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right)-2\left(3b^2d^2x^2-2b^2c^2-10abcd+15a^2d^2-2\left(7b^2cd-10abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{6\left(b^5x^2+2ab^4x+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(a^2*b*c*d - a^3*d^2 + (b^3*c*d - a*b^2*d^2)*x^2 + 2*(a*b^2*c*d
- a^2*b*d^2)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 - 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*(3*b^2*d^2*x^2 - 2*b^2*c^2 - 10*a*b*c*d + 15*a^2
*d^2 - 2*(7*b^2*c*d - 10*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/6*(15*(a^2*b*c*d - a^3*d^2 + (b^3*c*d - a*b^2*d^
2)*x^2 + 2*(a*b^2*c*d - a^2*b*d^2)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c
+ a*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d +
 a*d^2)*x)) - 2*(3*b^2*d^2*x^2 - 2*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 2*(7
*b^2*c*d - 10*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x
 + a^2*b^3)]
```

**giac [B]** time = 2.19, size = 650, normalized size = 5.08

$$\frac{\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\,d^2|b|}{b^5}-\frac{5\left(\sqrt{bd}\,bcd|b|-\sqrt{bd}\,ad^2|b|\right)\log\left(\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{2\,b^5}-\frac{4\left(7\,\sqrt{bd}\,b^6c^4d|b|-28\,\sqrt{bd}\,ab^5c^3d^2|b|+42\,\sqrt{bd}\,a^2b^4c^2d^3|b|-28\,\sqrt{bd}\,a^3b^3cd^4|b|+7\,\sqrt{bd}\,a^4b^2d^5|b|-12\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^4c^3d|b|+36\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^3c^2d^2|b|-36\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2b^2cd^3|b|+12\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^3bd^4|b|+9\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^2c^2d|b|-18\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4abcd^2|b|+9\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^2d^3|b|\right)}{3\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*d^2*abs(b)/b^5 - 5/2*(sqr
t(b*d)*b*c*d*abs(b) - sqrt(b*d)*a*d^2*abs(b))*log((sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^5 - 4/3*(7*sqrt(b*d)*b^6*c^4*d*
abs(b) - 28*sqrt(b*d)*a*b^5*c^3*d^2*abs(b) + 42*sqrt(b*d)*a^2*b^4*c^2*d^3*a
bs(b) - 28*sqrt(b*d)*a^3*b^3*c*d^4*abs(b) + 7*sqrt(b*d)*a^4*b^2*d^5*abs(b)
- 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^2*b^4*c^3*d*abs(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*a*b^3*c^2*d^2*abs(b) - 36*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^2*c*d^3*abs(b
) + 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^2*a^3*b*d^4*abs(b) + 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*b^2*c^2*d*abs(b) - 18*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b*c*d^2*abs(b) + 9*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a^2*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2)^3*b^4)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(5/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(5/2),x)
```

```
[Out] int((c + d*x)^(5/2)/(a + b*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

## 3.1381 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{7/2}} \, dx$

**Optimal**. Leaf size=120

$$\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {47, 63, 217, 206}

$$-\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*d^2*Sqrt[c + d*x])/(b^3*Sqrt[a + b*x]) - (2*d*(c + d*x)^(3/2))/(3*b^2*(
a + b*x)^(3/2)) - (2*(c + d*x)^(5/2))/(5*b*(a + b*x)^(5/2)) + (2*d^(5/2)*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^{7/2}}\,dx &= -\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx}{b}\\
&= -\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d^2\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx}{b^2}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d^3\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{b^3}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{\left(2d^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^4}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{\left(2d^3\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^4}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.07, size = 73, normalized size = 0.61

$$-\frac{2(c+dx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]
```

[Out] (-2*(c + d*x)^(5/2)*Hypergeometric2F1[-5/2, -5/2, -3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/2))

**IntegrateAlgebraic [A]** time = 0.14, size = 119, normalized size = 0.99

$$\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{b^{7/2}}-\frac{2\left(\frac{3b^2(c+dx)^{5/2}}{(a+bx)^{5/2}}+\frac{15d^2\sqrt{c+dx}}{\sqrt{a+bx}}+\frac{5bd(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{15b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]

[Out] (-2*((15*d^2*Sqrt[c + d*x])/Sqrt[a + b*x] + (5*b*d*(c + d*x)^(3/2))/(a + b*x)^(3/2) + (3*b^2*(c + d*x)^(5/2))/(a + b*x)^(5/2)))/(15*b^3) + (2*d^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/b^(7/2)

**fricas [B]** time = 2.69, size = 463, normalized size = 3.86

$$\left[\frac{15\left(b^3d^2x^3+3ab^2d^2x^2+3a^2bd^2x+a^3d^2\right)\sqrt{\frac{d}{b}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2b^2dx+b^2c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)-4\left(23b^2d^2x^2+3b^2c^2+5abcd+15a^2d^2+\left(11b^2cd+35abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{30\left(b^6x^3+3ab^5x^2+3a^2b^4x+a^3b^3\right)},-\frac{15\left(b^3d^2x^3+3ab^2d^2x^2+3a^2bd^2x+a^3d^2\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right)+2\left(23b^2d^2x^2+3b^2c^2+5abcd+15a^2d^2+\left(11b^2cd+35abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{15\left(b^6x^3+3ab^5x^2+3a^2b^4x+a^3b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="fricas")

[Out] [1/30*(15*(b^3*d^2*x^3 + 3*a*b^2*d^2*x^2 + 3*a^2*b*d^2*x + a^3*d^2)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(23*b^2*d^2*x^2 + 3*b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d + 35*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3), -1/15*(15*(b^3*d^2*x^3 + 3*a*b^2*d^2*x^2 + 3*a^2*b*d^2*x + a^3*d^2)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) + 2*(23*b^2*d^2*x^2 + 3*b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d + 35*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)]

**giac [B]** time = 2.48, size = 1025, normalized size = 8.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="giac")

[Out] -sqrt(b*d)*d^2*abs(b)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^5 - 4/15*(23*sqrt(b*d)*b^9*c^5*d^2*abs(b) - 115*sqrt(b*d)*a*b^8*c^4*d^3*abs(b) + 230*sqrt(b*d)*a^2*b^7*c^3*d^4*abs(b) - 230*sqrt(b

```
*d)*a^3*b^6*c^2*d^5*abs(b) + 115*sqrt(b*d)*a^4*b^5*c*d^6*abs(b) - 23*sqrt(b
*d)*a^5*b^4*d^7*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*b^7*c^4*d^2*abs(b) + 280*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^6*c^3*d^3*abs(b)
 - 420*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^2*a^2*b^5*c^2*d^4*abs(b) + 280*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^4*c*d^5*abs(b) - 70*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^3*d^
6*abs(b) + 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*b^5*c^3*d^2*abs(b) - 420*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^4*c^2*d^3*abs(b) + 420*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b
^3*c*d^4*abs(b) - 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^4*a^3*b^2*d^5*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^3*c^2*d^2*abs(b) + 180*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a
*b^2*c*d^3*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a^2*b*d^4*abs(b) + 45*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b*c*d^2*abs(b) - 45*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*d^3*a
bs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^2)^5*b^4)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(7/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/2}}{(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(7/2),x)
[Out] int((c + d*x)^(5/2)/(a + b*x)^(7/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(7/2),x)
[Out] Timed out
```

## 3.1382 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx$

**Optimal**. Leaf size=32

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{7/2}}{7(bc-ad)(a+bx)^{7/2}}$$

**Mathematica [A]** time = 0.02, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))
```

**fricas [B]** time = 3.85, size = 138, normalized size = 4.31

$$-\frac{2\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\sqrt{bx+a}\,\sqrt{dx+c}}{7\left(a^4bc-a^5d+\left(b^5c-ab^4d\right)x^4+4\left(ab^4c-a^2b^3d\right)x^3+6\left(a^2b^3c-a^3b^2d\right)x^2+4\left(a^3b^2c-a^4bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/7*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sqrt(b*x + a)*sqrt(d*x + c)/
(a^4*b*c - a^5*d + (b^5*c - a*b^4*d)*x^4 + 4*(a*b^4*c - a^2*b^3*d)*x^3 + 6*
(a^2*b^3*c - a^3*b^2*d)*x^2 + 4*(a^3*b^2*c - a^4*b*d)*x)
```

**giac [B]** time = 2.54, size = 706, normalized size = 22.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] -4/7*(sqrt(b*d)*b^12*c^6*d^3*abs(b) - 6*sqrt(b*d)*a*b^11*c^5*d^4*abs(b) + 1
5*sqrt(b*d)*a^2*b^10*c^4*d^5*abs(b) - 20*sqrt(b*d)*a^3*b^9*c^3*d^6*abs(b) +
 15*sqrt(b*d)*a^4*b^8*c^2*d^7*abs(b) - 6*sqrt(b*d)*a^5*b^7*c*d^8*abs(b) + s
qrt(b*d)*a^6*b^6*d^9*abs(b) + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*b^8*c^4*d^3*abs(b) - 84*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^7*c^3*d^4*ab
s(b) + 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^4*a^2*b^6*c^2*d^5*abs(b) - 84*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^5*c*d^6*abs(b) + 21*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^4
*d^7*abs(b) + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^8*b^4*c^2*d^3*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^3*c*d^4*abs(b) + 35*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^2
```

```
*d^5*abs(b) + 7*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)
*b*d - a*b*d))^12*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b^4)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.84

$$\frac{2\,(dx+c)^{\frac{7}{2}}}{7\,(bx+a)^{\frac{7}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(9/2),x)
```

```
[Out] 2/7/(b*x+a)^(7/2)*(d*x+c)^(7/2)/(a*d-b*c)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.97, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}}{\left(7\,a\,d-7\,b\,c\right)\,{\left(a+b\,x\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(9/2),x)
```

```
[Out] (2*(c + d*x)^(7/2))/((7*a*d - 7*b*c)*(a + b*x)^(7/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

# 3.1383 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}} dx$

**Optimal**. Leaf size=66

$$\frac{4d(c+dx)^{7/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{9(a+bx)^{9/2}(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4d(c+dx)^{7/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(7/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx = -\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(2d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx}{9(bc-ad)}$$
$$= -\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{7/2}}{63(bc-ad)^2(a+bx)^{7/2}}$$

**Mathematica** [A]   time = 0.03, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{7/2}(9ad-7bc+2bdx)}{63(a+bx)^{9/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-7*b*c + 9*a*d + 2*b*d*x))/(63*(b*c - a*d)^2*(a + b*x)^
(9/2))
```

**IntegrateAlgebraic** [A]   time = 0.12, size = 57, normalized size = 0.86

$$-\frac{2\left(\frac{7b(c+dx)^{9/2}}{(a+bx)^{9/2}} - \frac{9d(c+dx)^{7/2}}{(a+bx)^{7/2}}\right)}{63(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*((-9*d*(c + d*x)^(7/2))/(a + b*x)^(7/2) + (7*b*(c + d*x)^(9/2))/(a + b*
x)^(9/2)))/(63*(b*c - a*d)^2)
```

**fricas** [B]   time = 13.62, size = 295, normalized size = 4.47

$$\frac{2\left(2\,bd^4x^4 - 7\,bc^4 + 9\,ac^3d - \left(bcd^3 - 9\,ad^4\right)x^3 - 3\left(5\,bc^2d^2 - 9\,acd^3\right)x^2 - \left(19\,bc^3d - 27\,ac^2d^2\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{63\left(a^5b^2c^2 - 2\,a^6bcd + a^7d^2 + \left(b^7c^2 - 2\,ab^6cd + a^2b^5d^2\right)x^5 + 5\left(ab^6c^2 - 2\,a^2b^5cd + a^3b^4d^2\right)x^4 + 10\left(a^2b^5c^2 - 2\,a^3b^4cd + a^4b^3d^2\right)x^3 + 10\left(a^3b^4c^2 - 2\,a^4b^3cd + a^5b^2d^2\right)x^2 + 5\left(a^4b^3c^2 - 2\,a^5b^2cd + a^6bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*b*d^4*x^4 - 7*b*c^4 + 9*a*c^3*d - (b*c*d^3 - 9*a*d^4)*x^3 - 3*(5*b*
c^2*d^2 - 9*a*c*d^3)*x^2 - (19*b*c^3*d - 27*a*c^2*d^2)*x)*sqrt(b*x + a)*sqr
t(d*x + c)/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2 + (b^7*c^2 - 2*a*b^6*c*d +
a^2*b^5*d^2)*x^5 + 5*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*x^4 + 10*(a^
```

```
2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*x^3 + 10*(a^3*b^4*c^2 - 2*a^4*b^3*
c*d + a^5*b^2*d^2)*x^2 + 5*(a^4*b^3*c^2 - 2*a^5*b^2*c*d + a^6*b*d^2)*x)
```

**giac [B]** time = 3.80, size = 1826, normalized size = 27.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="giac")
```

```
[Out] 8/63*(sqrt(b*d)*b^14*c^7*d^4*abs(b) - 7*sqrt(b*d)*a*b^13*c^6*d^5*abs(b) + 2
1*sqrt(b*d)*a^2*b^12*c^5*d^6*abs(b) - 35*sqrt(b*d)*a^3*b^11*c^4*d^7*abs(b)
+ 35*sqrt(b*d)*a^4*b^10*c^3*d^8*abs(b) - 21*sqrt(b*d)*a^5*b^9*c^2*d^9*abs(b
) + 7*sqrt(b*d)*a^6*b^8*c*d^10*abs(b) - sqrt(b*d)*a^7*b^7*d^11*abs(b) - 9*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
b^12*c^6*d^4*abs(b) + 54*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^2*a*b^11*c^5*d^5*abs(b) - 135*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^10*c^4*d^6*abs
(b) + 180*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2*a^3*b^9*c^3*d^7*abs(b) - 135*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^8*c^2*d^8*abs(b) + 54*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b
^7*c*d^9*abs(b) - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d))^2*a^6*b^6*d^10*abs(b) - 27*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^5*d^4*abs(b) + 135*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a
*b^9*c^4*d^5*abs(b) - 270*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^4*a^2*b^8*c^3*d^6*abs(b) + 270*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c^2*d^7*ab
s(b) - 135*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^4*a^4*b^6*c*d^8*abs(b) + 27*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^5*d^9*abs(b) - 189*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^8*c^4*d^
4*abs(b) + 756*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^6*a*b^7*c^3*d^5*abs(b) - 1134*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c^2*d^6*abs(b) + 756*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*
a^3*b^5*c*d^7*abs(b) - 189*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^6*a^4*b^4*d^8*abs(b) - 189*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^6*c^3*d^4*abs(b) +
567*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^8*a*b^5*c^2*d^5*abs(b) - 567*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b
^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*c*d^6*abs(b) + 189*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^3*d^7*ab
s(b) - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
```

```
- a*b*d))^10*b^4*c^2*d^4*abs(b) + 630*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^3*c*d^5*abs(b) - 315*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^2*
d^6*abs(b) - 105*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^12*b^2*c*d^4*abs(b) + 105*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b*d^5*abs(b) - 63*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*d^4*abs(b
))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^2)^9*b^3)
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.82

$$\frac{2 (dx+c)^{\frac{7}{2}} (2bdx+9ad-7bc)}{63 (bx+a)^{\frac{9}{2}} \left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(11/2),x)
```

```
[Out] 2/63*(d*x+c)^(7/2)*(2*b*d*x+9*a*d-7*b*c)/(b*x+a)^(9/2)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.14, size = 229, normalized size = 3.47

$$\frac{\sqrt{c+d\,x}\,\left(\frac{4\,d^4\,x^4}{63\,b^3\,{\left(a\,d-b\,c\right)}^2}-\frac{14\,b\,c^4-18\,a\,c^3\,d}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{x^3\,\left(18\,a\,d^4-2\,b\,c\,d^3\right)}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,c^2\,d\,x\,\left(27\,a\,d-19\,b\,c\right)}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,c\,d^2\,x^2\,\left(9\,a\,d-5\,b\,c\right)}{21\,b^4\,{\left(a\,d-b\,c\right)}^2}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(11/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((4*d^4*x^4)/(63*b^3*(a*d - b*c)^2) - (14*b*c^4 - 18*a*c^3
*d)/(63*b^4*(a*d - b*c)^2) + (x^3*(18*a*d^4 - 2*b*c*d^3))/(63*b^4*(a*d - b*
c)^2) + (2*c^2*d*x*(27*a*d - 19*b*c))/(63*b^4*(a*d - b*c)^2) + (2*c*d^2*x^2
*(9*a*d - 5*b*c))/(21*b^4*(a*d - b*c)^2)))/(x^4*(a + b*x)^(1/2) + (a^4*(a +
 b*x)^(1/2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/
2))/b + (4*a^3*x*(a + b*x)^(1/2))/b^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

## 3.1384 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}} \, dx$

**Optimal**. Leaf size=101

$$-\frac{16d^2(c+dx)^{7/2}}{693(a+bx)^{7/2}(bc-ad)^3}+\frac{8d(c+dx)^{7/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{11(a+bx)^{11/2}(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d^2(c+dx)^{7/2}}{693(a+bx)^{7/2}(bc-ad)^3}+\frac{8d(c+dx)^{7/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (8*d*(c + d*x)^(7/
2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(7/2))/(693*(b*c
 - a*d)^3*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}}\,dx = -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}} - \frac{(4d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}$$
$$= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}} + \frac{\left(8d^2\right)\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx}{99(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{16d^2(c+dx)^{7/2}}{693(bc-ad)^3(a+bx)^{7/2}}$$

**Mathematica** [A] time = 0.05, size = 77, normalized size = 0.76

$$-\frac{2(c+dx)^{7/2}\left(99a^2d^2+22abd(2dx-7c)+b^2\left(63c^2-28cdx+8d^2x^2\right)\right)}{693(a+bx)^{11/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-7*c + 2*d*x) + b^2*(63*c^2 - 2
8*c*d*x + 8*d^2*x^2)))/(693*(b*c - a*d)^3*(a + b*x)^(11/2))
```

**IntegrateAlgebraic** [A] time = 0.17, size = 73, normalized size = 0.72

$$-\frac{2(c+dx)^{7/2}\left(\frac{63b^2(c+dx)^2}{(a+bx)^2}-\frac{154bd(c+dx)}{a+bx}+99d^2\right)}{693(a+bx)^{7/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(99*d^2 - (154*b*d*(c + d*x))/(a + b*x) + (63*b^2*(c +
d*x)^2)/(a + b*x)^2))/(693*(b*c - a*d)^3*(a + b*x)^(7/2))
```

**fricas** [B] time = 31.13, size = 513, normalized size = 5.08

$$-\frac{2\left(8\,b^2d^5x^5+63\,b^2c^5-154\,abc^4d+99\,a^2c^3d^2-4\left(b^2cd^4-11\,abd^5\right)x^4+\left(3\,b^2c^2d^3-22\,abcd^4+99\,a^2d^5\right)x^3+\left(113\,b^2c^3d^2-330\,abc^2d^3+297\,a^2cd^4\right)x^2+\left(161\,b^2c^4d-418\,abc^3d^2+297\,a^2c^2d^3\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{693\left(a^6b^3c^3-3\,a^7b^2c^2d+3\,a^8bcd^2-a^9d^3+\left(b^9c^3-3\,ab^8c^2d+3\,a^2b^7cd^2-a^3b^6d^3\right)x^6+6\left(ab^8c^3-3\,a^2b^7c^2d+3\,a^3b^6cd^2-a^4b^5d^3\right)x^5+15\left(a^2b^7c^3-3\,a^3b^6c^2d+3\,a^4b^5cd^2-a^5b^4d^3\right)x^4+20\left(a^3b^6c^3-3\,a^4b^5c^2d+3\,a^5b^4cd^2-a^6b^3d^3\right)x^3+15\left(a^4b^5c^3-3\,a^5b^4c^2d+3\,a^6b^3cd^2-a^7b^2d^3\right)x^2+6\left(a^5b^4c^3-3\,a^6b^3c^2d+3\,a^7b^2cd^2-a^8bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] -2/693*(8*b^2*d^5*x^5 + 63*b^2*c^5 - 154*a*b*c^4*d + 99*a^2*c^3*d^2 - 4*(b^
2*c*d^4 - 11*a*b*d^5)*x^4 + (3*b^2*c^2*d^3 - 22*a*b*c*d^4 + 99*a^2*d^5)*x^3
```

```
 + (113*b^2*c^3*d^2 - 330*a*b*c^2*d^3 + 297*a^2*c*d^4)*x^2 + (161*b^2*c^4*d
 - 418*a*b*c^3*d^2 + 297*a^2*c^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b
^3*c^3 - 3*a^7*b^2*c^2*d + 3*a^8*b*c*d^2 - a^9*d^3 + (b^9*c^3 - 3*a*b^8*c^2
*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*x^6 + 6*(a*b^8*c^3 - 3*a^2*b^7*c^2*d +
3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*x^5 + 15*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*
a^4*b^5*c*d^2 - a^5*b^4*d^3)*x^4 + 20*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^
5*b^4*c*d^2 - a^6*b^3*d^3)*x^3 + 15*(a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*
b^3*c*d^2 - a^7*b^2*d^3)*x^2 + 6*(a^5*b^4*c^3 - 3*a^6*b^3*c^2*d + 3*a^7*b^2
*c*d^2 - a^8*b*d^3)*x)
```

**giac [B]** time = 4.59, size = 2316, normalized size = 22.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="giac")
```

```
[Out] -32/693*(sqrt(b*d)*b^16*c^8*d^5*abs(b) - 8*sqrt(b*d)*a*b^15*c^7*d^6*abs(b)
+ 28*sqrt(b*d)*a^2*b^14*c^6*d^7*abs(b) - 56*sqrt(b*d)*a^3*b^13*c^5*d^8*abs(
b) + 70*sqrt(b*d)*a^4*b^12*c^4*d^9*abs(b) - 56*sqrt(b*d)*a^5*b^11*c^3*d^10*
abs(b) + 28*sqrt(b*d)*a^6*b^10*c^2*d^11*abs(b) - 8*sqrt(b*d)*a^7*b^9*c*d^12
*abs(b) + sqrt(b*d)*a^8*b^8*d^13*abs(b) - 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^14*c^7*d^5*abs(b) + 77*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b
^13*c^6*d^6*abs(b) - 231*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^2*a^2*b^12*c^5*d^7*abs(b) + 385*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^11*c^4*d^8*a
bs(b) - 385*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^2*a^4*b^10*c^3*d^9*abs(b) + 231*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^9*c^2*d^10*abs(b) - 77*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a
^6*b^8*c*d^11*abs(b) + 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*a^7*b^7*d^12*abs(b) + 55*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^12*c^6*d^5*abs(b) -
330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^4*a*b^11*c^5*d^6*abs(b) + 825*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^10*c^4*d^7*abs(b) - 1100*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^9*c
^3*d^8*abs(b) + 825*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d))^4*a^4*b^8*c^2*d^9*abs(b) - 330*sqrt(b*d)*(sqrt(b*d)*sqrt
(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^7*c*d^10*abs(b) +
55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^4*a^6*b^6*d^11*abs(b) + 297*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^6*b^10*c^5*d^5*abs(b) - 1485*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^9*c^4*d^6*ab
```

```
s(b) + 2970*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^6*a^2*b^8*c^3*d^7*abs(b) - 2970*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^7*c^2*d^8*abs(b) + 1485*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*
a^4*b^6*c*d^9*abs(b) - 297*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^6*a^5*b^5*d^10*abs(b) + 1485*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^8*c^4*d^5*abs(b)
- 5940*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^8*a*b^7*c^3*d^6*abs(b) + 8910*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^6*c^2*d^7*abs(b) - 5940*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^5
*c*d^8*abs(b) + 1485*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d))^8*a^4*b^4*d^9*abs(b) + 2079*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^6*c^3*d^5*abs(b) - 6237
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
10*a*b^5*c^2*d^6*abs(b) + 6237*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^
2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^4*c*d^7*abs(b) - 2079*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^3*b^3*d^8*
abs(b) + 2541*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^12*b^4*c^2*d^5*abs(b) - 5082*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^3*c*d^6*abs(b) + 2541*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^2
*b^2*d^7*abs(b) + 1155*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^14*b^2*c*d^5*abs(b) - 1155*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b*d^6*abs(b) + 462*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*d
^5*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^2)^11*b^2)
```

**maple** [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{2 (dx+c)^{\frac{7}{2}} \left(8b^2x^2d^2+44ab\,d^2x-28b^2cdx+99a^2d^2-154abcd+63b^2c^2\right)}{693 (bx+a)^{\frac{11}{2}} \left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(13/2),x)
```

```
[Out] 2/693*(d*x+c)^(7/2)*(8*b^2*d^2*x^2+44*a*b*d^2*x-28*b^2*c*d*x+99*a^2*d^2-154
*a*b*c*d+63*b^2*c^2)/(b*x+a)^(11/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.35, size = 333, normalized size = 3.30

$$\frac{\sqrt{c+d\,x}\,\left(\frac{198\,a^2\,c^3\,d^2-308\,a\,b\,c^4\,d+126\,b^2\,c^5}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{x^3\,\left(198\,a^2\,d^5-44\,a\,b\,c\,d^4+6\,b^2\,c^2\,d^3\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^5\,x^5}{693\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{693\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,c\,d^2\,x^2\,\left(297\,a^2\,d^2-330\,a\,b\,c\,d+113\,b^2\,c^2\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,c^2\,d\,x\,\left(297\,a^2\,d^2-418\,a\,b\,c\,d+161\,b^2\,c^2\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((126*b^2*c^5 + 198*a^2*c^3*d^2 - 308*a*b*c^4*d)/(693*b^5*
(a*d - b*c)^3) + (x^3*(198*a^2*d^5 + 6*b^2*c^2*d^3 - 44*a*b*c*d^4))/(693*b^
5*(a*d - b*c)^3) + (16*d^5*x^5)/(693*b^3*(a*d - b*c)^3) + (8*d^4*x^4*(11*a*
d - b*c))/(693*b^4*(a*d - b*c)^3) + (2*c*d^2*x^2*(297*a^2*d^2 + 113*b^2*c^2
 - 330*a*b*c*d))/(693*b^5*(a*d - b*c)^3) + (2*c^2*d*x*(297*a^2*d^2 + 161*b^
2*c^2 - 418*a*b*c*d))/(693*b^5*(a*d - b*c)^3)))/(x^5*(a + b*x)^(1/2) + (a^5
*(a + b*x)^(1/2))/b^5 + (10*a^2*x^3*(a + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a +
 b*x)^(1/2))/b^3 + (5*a*x^4*(a + b*x)^(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/
b^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

## 3.1385 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{15/2}}\,dx$

**Optimal**. Leaf size=136

$$\frac{32d^3(c+dx)^{7/2}}{3003(a+bx)^{7/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{7/2}}{429(a+bx)^{9/2}(bc-ad)^3}+\frac{12d(c+dx)^{7/2}}{143(a+bx)^{11/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{13(a+bx)^{13/2}(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3(c+dx)^{7/2}}{3003(a+bx)^{7/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{7/2}}{429(a+bx)^{9/2}(bc-ad)^3}+\frac{12d(c+dx)^{7/2}}{143(a+bx)^{11/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{13(a+bx)^{13/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(13*(b*c - a*d)*(a + b*x)^(13/2)) + (12*d*(c + d*x)^(7
/2))/(143*(b*c - a*d)^2*(a + b*x)^(11/2)) - (16*d^2*(c + d*x)^(7/2))/(429*(
b*c - a*d)^3*(a + b*x)^(9/2)) + (32*d^3*(c + d*x)^(7/2))/(3003*(b*c - a*d)^
4*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{15/2}}\,dx = -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} - \frac{(6d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}}\,dx}{13(bc-ad)}$$

$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} + \frac{\left(24d^2\right)\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx}{143(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} - \frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}} - \frac{\left(16d^3\right)\int \cdots}{429(bc \cdots}$$

$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} - \frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}} + \frac{32d \cdots}{3003(bc - \cdots}$$

**Mathematica** [A] time = 0.07, size = 118, normalized size = 0.87

$$\frac{2(c+dx)^{7/2}\left(429a^3d^3+143a^2bd^2(2dx-7c)+13ab^2d\left(63c^2-28cdx+8d^2x^2\right)+b^3\left(-231c^3+126c^2dx-56cd^2x^2+16d^3x^3\right)\right)}{3003(a+bx)^{13/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(429*a^3*d^3 + 143*a^2*b*d^2*(-7*c + 2*d*x) + 13*a*b^2*d
*(63*c^2 - 28*c*d*x + 8*d^2*x^2) + b^3*(-231*c^3 + 126*c^2*d*x - 56*c*d^2*x
^2 + 16*d^3*x^3)))/(3003*(b*c - a*d)^4*(a + b*x)^(13/2))
```

**IntegrateAlgebraic** [A] time = 0.15, size = 95, normalized size = 0.70

$$-\frac{2(c+dx)^{7/2}\left(\frac{231b^3(c+dx)^3}{(a+bx)^3}-\frac{819b^2d(c+dx)^2}{(a+bx)^2}+\frac{1001bd^2(c+dx)}{a+bx}-429d^3\right)}{3003(a+bx)^{7/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(-429*d^3 + (1001*b*d^2*(c + d*x))/(a + b*x) - (819*b^2
*d*(c + d*x)^2)/(a + b*x)^2 + (231*b^3*(c + d*x)^3)/(a + b*x)^3))/(3003*(b*
c - a*d)^4*(a + b*x)^(7/2))
```

**fricas** [B] time = 55.76, size = 765, normalized size = 5.62

$$\frac{2\left(16b^3d^6x^6-231b^3c^6+819ab^2c^5d-1001a^2bc^4d^2+429a^3c^3d^3-8\left(b^3cd^5-13ab^2d^6\right)x^5+2\left(3b^3c^2d^4-26ab^2cd^5+143a^2bd^6\right)x^4-\left(5b^3c^3d^3-39ab^2c^2d^4+143a^2bcd^5-429a^3d^6\right)x^3-\left(371b^3c^4d^2-1469ab^2c^3d^3+2145a^2bc^2d^4-1287a^3cd^5\right)x^2-\left(567b^3c^5d-2093ab^2c^4d^2+2717a^2bc^3d^3-1287a^3c^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{3003\left(a^7b^4c^4-4a^8b^3c^3d+6a^9b^2c^2d^2-4a^{10}bcd^3+a^{11}d^4+\left(b^{11}c^4-4ab^{10}c^3d+6a^2b^9c^2d^2-4a^3b^8cd^3+a^4b^7d^4\right)x^7+7\left(ab^{10}c^4-4a^2b^9c^3d+6a^3b^8c^2d^2-4a^4b^7cd^3+a^5b^6d^4\right)x^6+21\left(a^2b^9c^4-4a^3b^8c^3d+6a^4b^7c^2d^2-4a^5b^6cd^3+a^6b^5d^4\right)x^5+35\left(a^3b^8c^4-4a^4b^7c^3d+6a^5b^6c^2d^2-4a^6b^5cd^3+a^7b^4d^4\right)x^4+35\left(a^4b^7c^4-4a^5b^6c^3d+6a^6b^5c^2d^2-4a^7b^4cd^3+a^8b^3d^4\right)x^3+21\left(a^5b^6c^4-4a^6b^5c^3d+6a^7b^4c^2d^2-4a^8b^3cd^3+a^9b^2d^4\right)x^2+7\left(a^6b^5c^4-4a^7b^4c^3d+6a^8b^3c^2d^2-4a^9b^2cd^3+a^{10}bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*b^3*d^6*x^6 - 231*b^3*c^6 + 819*a*b^2*c^5*d - 1001*a^2*b*c^4*d^2
 + 429*a^3*c^3*d^3 - 8*(b^3*c*d^5 - 13*a*b^2*d^6)*x^5 + 2*(3*b^3*c^2*d^4 -
26*a*b^2*c*d^5 + 143*a^2*b*d^6)*x^4 - (5*b^3*c^3*d^3 - 39*a*b^2*c^2*d^4 + 1
43*a^2*b*c*d^5 - 429*a^3*d^6)*x^3 - (371*b^3*c^4*d^2 - 1469*a*b^2*c^3*d^3 +
 2145*a^2*b*c^2*d^4 - 1287*a^3*c*d^5)*x^2 - (567*b^3*c^5*d - 2093*a*b^2*c^4
*d^2 + 2717*a^2*b*c^3*d^3 - 1287*a^3*c^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c
)/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^1
1*d^4 + (b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 +
a^4*b^7*d^4)*x^7 + 7*(a*b^10*c^4 - 4*a^2*b^9*c^3*d + 6*a^3*b^8*c^2*d^2 - 4*
a^4*b^7*c*d^3 + a^5*b^6*d^4)*x^6 + 21*(a^2*b^9*c^4 - 4*a^3*b^8*c^3*d + 6*a^
4*b^7*c^2*d^2 - 4*a^5*b^6*c*d^3 + a^6*b^5*d^4)*x^5 + 35*(a^3*b^8*c^4 - 4*a^
4*b^7*c^3*d + 6*a^5*b^6*c^2*d^2 - 4*a^6*b^5*c*d^3 + a^7*b^4*d^4)*x^4 + 35*(
a^4*b^7*c^4 - 4*a^5*b^6*c^3*d + 6*a^6*b^5*c^2*d^2 - 4*a^7*b^4*c*d^3 + a^8*b
^3*d^4)*x^3 + 21*(a^5*b^6*c^4 - 4*a^6*b^5*c^3*d + 6*a^7*b^4*c^2*d^2 - 4*a^8
*b^3*c*d^3 + a^9*b^2*d^4)*x^2 + 7*(a^6*b^5*c^4 - 4*a^7*b^4*c^3*d + 6*a^8*b^
3*c^2*d^2 - 4*a^9*b^2*c*d^3 + a^10*b*d^4)*x)
```

**giac [B]** time = 6.09, size = 2868, normalized size = 21.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="giac")
```

```
[Out] 64/3003*(sqrt(b*d)*b^18*c^9*d^6*abs(b) - 9*sqrt(b*d)*a*b^17*c^8*d^7*abs(b)
+ 36*sqrt(b*d)*a^2*b^16*c^7*d^8*abs(b) - 84*sqrt(b*d)*a^3*b^15*c^6*d^9*abs(
b) + 126*sqrt(b*d)*a^4*b^14*c^5*d^10*abs(b) - 126*sqrt(b*d)*a^5*b^13*c^4*d^
11*abs(b) + 84*sqrt(b*d)*a^6*b^12*c^3*d^12*abs(b) - 36*sqrt(b*d)*a^7*b^11*c
^2*d^13*abs(b) + 9*sqrt(b*d)*a^8*b^10*c*d^14*abs(b) - sqrt(b*d)*a^9*b^9*d^1
5*abs(b) - 13*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*b^16*c^8*d^6*abs(b) + 104*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^15*c^7*d^7*abs(b) - 364*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*
b^14*c^6*d^8*abs(b) + 728*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*a^3*b^13*c^5*d^9*abs(b) - 910*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^12*c^4*d^10
*abs(b) + 728*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a^5*b^11*c^3*d^11*abs(b) - 364*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^6*b^10*c^2*d^12*abs(b) + 1
04*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2*a^7*b^9*c*d^13*abs(b) - 13*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^
2*c + (b*x + a)*b*d - a*b*d))^2*a^8*b^8*d^14*abs(b) + 78*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^14*c^7*d^6*abs(
b) - 546*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
```

a*b*d))^4*a*b^13*c^6*d^7*abs(b) + 1638*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^12*c^5*d^8*abs(b) - 2730*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^11*c^4*d^9*abs(b) + 2730*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^10*c^3*d^10*abs(b) - 1638*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^9*c^2*d^11*abs(b) + 546*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^6*b^8*c*d^12*abs(b) - 78*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^7*b^7*d^13*abs(b) - 286*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^12*c^6*d^6*abs(b) + 1716*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^11*c^5*d^7*abs(b) - 4290*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^10*c^4*d^8*abs(b) + 5720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^9*c^3*d^9*abs(b) - 4290*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^4*b^8*c^2*d^10*abs(b) + 1716*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^5*b^7*c*d^11*abs(b) - 286*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^6*b^6*d^12*abs(b) - 2288*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^10*c^5*d^6*abs(b) + 11440*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^9*c^4*d^7*abs(b) - 22880*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^8*c^3*d^8*abs(b) + 22880*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^7*c^2*d^9*abs(b) - 11440*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^4*b^6*c*d^10*abs(b) + 2288*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^5*b^5*d^11*abs(b) - 10296*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^8*c^4*d^6*abs(b) + 41184*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^7*c^3*d^7*abs(b) - 61776*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^6*c^2*d^8*abs(b) + 41184*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^3*b^5*c*d^9*abs(b) - 10296*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^4*b^4*d^10*abs(b) - 16302*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^6*c^3*d^6*abs(b) + 48906*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^5*c^2*d^7*abs(b) - 48906*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^2*b^4*c*d^8*abs(b) + 16302*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^3*b^3*d^9*abs(b) - 18018*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*b^4*c^2*d^6*abs(b) + 36036*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b^3*c*d^7*abs(b) - 18018*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a^2*b^2*d^8*abs(b) - 9009*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16

```
*b^2*c*d^6*abs(b) + 9009*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^16*a*b*d^7*abs(b) - 3003*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^18*d^6*abs(b))/((b^2*c - a*
b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^13
*b)
```

**maple** [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{2 (dx+c)^{\frac{7}{2}} \left(16b^3x^3d^3+104a\,b^2d^3x^2-56b^3c\,d^2x^2+286a^2b\,d^3x-364a\,b^2c\,d^2x+126b^3c^2dx+429a^3d^3-1001a^2bc\,d^2+819a\,b^2c^2d-231b^3c^3\right)}{3003 (bx+a)^{\frac{13}{2}} \left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(15/2),x)
```

```
[Out] 2/3003*(d*x+c)^(7/2)*(16*b^3*d^3*x^3+104*a*b^2*d^3*x^2-56*b^3*c*d^2*x^2+286
*a^2*b*d^3*x-364*a*b^2*c*d^2*x+126*b^3*c^2*d*x+429*a^3*d^3-1001*a^2*b*c*d^2
+819*a*b^2*c^2*d-231*b^3*c^3)/(b*x+a)^(13/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b
^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [B] time = 1.62, size = 459, normalized size = 3.38

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x^2\,\left(2574\,a^3\,c\,d^5-4290\,a^2\,b\,c^2\,d^4+2938\,a\,b^2\,c^3\,d^3-742\,b^3\,c^4\,d^2\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}-\frac{-858\,a^3\,c^3\,d^3+2002\,a^2\,b\,c^4\,d^2-1638\,a\,b^2\,c^5\,d+462\,b^3\,c^6}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{x^3\,\left(858\,a^3\,d^6-286\,a^2\,b\,c\,d^5+78\,a\,b^2\,c^2\,d^4-10\,b^3\,c^3\,d^3\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,d^6\,x^6}{3003\,b^3\,{\left(a\,d-b\,c\right)}^4}-\frac{x\,\left(-2574\,a^3\,c^2\,d^4+5434\,a^2\,b\,c^3\,d^3-4186\,a\,b^2\,c^4\,d^2+1134\,b^3\,c^5\,d\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^5\,x^5\,\left(13\,a\,d-b\,c\right)}{3003\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^4\,x^4\,\left(143\,a^2\,d^2-26\,a\,b\,c\,d+3\,b^2\,c^2\right)}{3003\,b^5\,{\left(a\,d-b\,c\right)}^4}\right)}{x^6\,\sqrt{a+b\,x}+\frac{a^6\,\sqrt{a+b\,x}}{b^6}+\frac{15\,a^2\,x^4\,\sqrt{a+b\,x}}{b^2}+\frac{20\,a^3\,x^3\,\sqrt{a+b\,x}}{b^3}+\frac{15\,a^4\,x^2\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a\,x^5\,\sqrt{a+b\,x}}{b}+\frac{6\,a^5\,x\,\sqrt{a+b\,x}}{b^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(15/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x^2*(2574*a^3*c*d^5 - 742*b^3*c^4*d^2 + 2938*a*b^2*c^3*d
^3 - 4290*a^2*b*c^2*d^4))/(3003*b^6*(a*d - b*c)^4) - (462*b^3*c^6 - 858*a^3
*c^3*d^3 + 2002*a^2*b*c^4*d^2 - 1638*a*b^2*c^5*d)/(3003*b^6*(a*d - b*c)^4)
+ (x^3*(858*a^3*d^6 - 10*b^3*c^3*d^3 + 78*a*b^2*c^2*d^4 - 286*a^2*b*c*d^5))
/(3003*b^6*(a*d - b*c)^4) + (32*d^6*x^6)/(3003*b^3*(a*d - b*c)^4) - (x*(113
```

```
4*b^3*c^5*d - 2574*a^3*c^2*d^4 - 4186*a*b^2*c^4*d^2 + 5434*a^2*b*c^3*d^3))/
(3003*b^6*(a*d - b*c)^4) + (16*d^5*x^5*(13*a*d - b*c))/(3003*b^4*(a*d - b*c
)^4) + (4*d^4*x^4*(143*a^2*d^2 + 3*b^2*c^2 - 26*a*b*c*d))/(3003*b^5*(a*d -
b*c)^4)))/(x^6*(a + b*x)^(1/2) + (a^6*(a + b*x)^(1/2))/b^6 + (15*a^2*x^4*(a
 + b*x)^(1/2))/b^2 + (20*a^3*x^3*(a + b*x)^(1/2))/b^3 + (15*a^4*x^2*(a + b*
x)^(1/2))/b^4 + (6*a*x^5*(a + b*x)^(1/2))/b + (6*a^5*x*(a + b*x)^(1/2))/b^5
)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(15/2),x)
```

```
[Out] Timed out
```

## 3.1386 $\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=183

$$\frac{35(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64\sqrt{b}\,d^{9/2}}-\frac{35\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64d^4}+\frac{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96d^3}-\frac{7(a+bx)^{5/2}\cdots}{\cdots}$$

**Rubi [A]** time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{35\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64d^4}+\frac{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96d^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{24d^2}+\frac{35(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64\sqrt{b}\,d^{9/2}}+\frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] (-35*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*d^4) + (35*(b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*d^3) - (7*(b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(24*d^2) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*d) + (35*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*Sqrt
[b]*d^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{8d}$$

$$= -\frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} + \frac{\left(35(bc-ad)^2\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{48d^2}$$

$$= \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} - (35$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c}}{24d^2}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c}}{24d^2}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c}}{24d^2}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c}}{24d^2}$$

**Mathematica [A]** time = 0.66, size = 189, normalized size = 1.03

$$\frac{\sqrt{d}\sqrt{a+bx}\,(c+dx)\left(279a^3d^3+a^2bd^2(326dx-511c)+ab^2d\left(385c^2-252cdx+200d^2x^2\right)+b^3\left(-105c^3+70c^2dx-56cd^2x^2+48d^3x^3\right)\right)+\frac{105(bc-ad)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b}}{192d^{9/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(279*a^3*d^3 + a^2*b*d^2*(-511*c + 326*d*x
) + a*b^2*d*(385*c^2 - 252*c*d*x + 200*d^2*x^2) + b^3*(-105*c^3 + 70*c^2*d*
x - 56*c*d^2*x^2 + 48*d^3*x^3)) + (105*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))
/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/b)/(192*d^(
9/2)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.20, size = 172, normalized size = 0.94

$$\frac{\sqrt{c+dx}\,(ad-bc)^4\left(-\frac{105b^3(c+dx)^3}{(a+bx)^3}+\frac{385b^2d(c+dx)^2}{(a+bx)^2}-\frac{511bd^2(c+dx)}{a+bx}+279d^3\right)}{192d^4\sqrt{a+bx}\left(d-\frac{b(c+dx)}{a+bx}\right)^4}+\frac{35(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{64\sqrt{b}\,d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] ((-(b*c) + a*d)^4*Sqrt[c + d*x]*(279*d^3 - (511*b*d^2*(c + d*x))/(a + b*x)
+ (385*b^2*d*(c + d*x)^2)/(a + b*x)^2 - (105*b^3*(c + d*x)^3)/(a + b*x)^3))
/(192*d^4*Sqrt[a + b*x]*(d - (b*(c + d*x))/(a + b*x))^4) + (35*(b*c - a*d)^
4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(64*Sqrt[b]*d^(
9/2))
```

**fricas [A]** time = 1.10, size = 542, normalized size = 2.96

$$\left[\frac{105\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)+4\left(48b^4d^4x^3-105b^4c^3d+385ab^3c^2d^2-511a^2b^2cd^3+279a^3bd^4-8\left(7b^4cd^3-25ab^3d^4\right)x^2+2\left(35b^4c^2d^2-126ab^3cd^3+163a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{768bd^5}, -\frac{105\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(48b^4d^4x^3-105b^4c^3d+385ab^3c^2d^2-511a^2b^2cd^3+279a^3bd^4-8\left(7b^4cd^3-25ab^3d^4\right)x^2+2\left(35b^4c^2d^2-126ab^3cd^3+163a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{384bd^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2
*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*
b*d^2)*x) + 4*(48*b^4*d^4*x^3 - 105*b^4*c^3*d + 385*a*b^3*c^2*d^2 - 511*a^2
*b^2*c*d^3 + 279*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 25*a*b^3*d^4)*x^2 + 2*(35*b^4
*c^2*d^2 - 126*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c
))/(b*d^5), -1/384*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)
*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x
)) - 2*(48*b^4*d^4*x^3 - 105*b^4*c^3*d + 385*a*b^3*c^2*d^2 - 511*a^2*b^2*c*
d^3 + 279*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 25*a*b^3*d^4)*x^2 + 2*(35*b^4*c^2*d^
2 - 126*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d
^5)]
```

**giac [A]** time = 1.23, size = 268, normalized size = 1.46

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2(bx+a)\left(4(bx+a)\left(\frac{6(bx+a)}{bd}-\frac{7\left(bcd^5-ad^6\right)}{bd^7}\right)+\frac{35\left(b^2c^2d^4-2abcd^5+a^2d^6\right)}{bd^7}\right)-\frac{105\left(b^3c^3d^3-3ab^2c^2d^4+3a^2bcd^5-a^3d^6\right)}{bd^7}\right)\sqrt{bx+a}-\frac{105\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^4}\right)b}{192|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*
x + a)/(b*d) - 7*(b*c*d^5 - a*d^6)/(b*d^7)) + 35*(b^2*c^2*d^4 - 2*a*b*c*d^5
 + a^2*d^6)/(b*d^7)) - 105*(b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 -
 a^3*d^6)/(b*d^7))*sqrt(b*x + a) - 105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2
*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt
(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^4))*b/abs(b)
```

**maple** [B] time = 0.01, size = 650, normalized size = 3.55

$$\frac{35\sqrt{(bx+a)(dx+c)}\,a^4\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}}-\frac{35\sqrt{(bx+a)(dx+c)}\,a^3bc\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d}+\frac{105\sqrt{(bx+a)(dx+c)}\,a^2b^2c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{64\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d^2}-\frac{35\sqrt{(bx+a)(dx+c)}\,a\,b^3c^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d^3}+\frac{35\sqrt{(bx+a)(dx+c)}\,b^4c^4\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d^4}+\frac{35\sqrt{bx+a}\,\sqrt{dx+c}\,a^3}{64d}-\frac{105\sqrt{bx+a}\,\sqrt{dx+c}\,a^2bc}{64d^2}+\frac{105\sqrt{bx+a}\,\sqrt{dx+c}\,a\,b^2c^2}{64d^3}-\frac{35\sqrt{bx+a}\,\sqrt{dx+c}\,b^3c^3}{64d^4}+\frac{35(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,a^2}{96d}-\frac{35(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,abc}{48d^2}+\frac{35(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,b^2c^2}{96d^3}+\frac{7(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\,a}{24d}-\frac{7(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\,bc}{24d^2}+\frac{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/4*(b*x+a)^(7/2)*(d*x+c)^(1/2)/d+7/24/d*(b*x+a)^(5/2)*(d*x+c)^(1/2)*a-7/24
/d^2*(b*x+a)^(5/2)*(d*x+c)^(1/2)*b*c+35/96/d*(b*x+a)^(3/2)*(d*x+c)^(1/2)*a^
2-35/48/d^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)*a*b*c+35/96/d^3*(b*x+a)^(3/2)*(d*x+
c)^(1/2)*b^2*c^2+35/64/d*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a^3-105/64/d^2*(b*x+a)
^(1/2)*(d*x+c)^(1/2)*a^2*b*c+105/64/d^3*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a*b^2*c
^2-35/64/d^4*(b*x+a)^(1/2)*(d*x+c)^(1/2)*b^3*c^3+35/128*((b*x+a)*(d*x+c))^(
1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*
d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^4-35/32/d*((b*x+a)*(d*x+c))^(1/
2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*
x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^3*b*c+105/64/d^2*((b*x+a)*(d*x+c)
)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+
(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*b^2*c^2-35/32/d^3*((b*x+a)
*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d
)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*b^3*c^3+35/128/d^4*(
(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*
c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*b^4*c^4
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/2}}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.1387 $\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=148

$$-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12d^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

**Rubi [A]** time = 0.07, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12d^2}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^3) - (5*(b*c - a*d)*(a +
 b*x)^(3/2)*Sqrt[c + d*x])/(12*d^2) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d)
 - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*Sqrt[b]*d^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6d}$$

$$= -\frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} + \frac{\left(5(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{8d^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{(5(bc}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{(5(bc}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{(5(bc}{}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{5(bc}{}$$

**Mathematica [A]** time = 0.52, size = 150, normalized size = 1.01

$$\frac{\sqrt{d}\sqrt{a+bx}\,(c+dx)\left(33a^2d^2+2abd(13dx-20c)+b^2\left(15c^2-10cdx+8d^2x^2\right)\right)-\frac{15(bc-ad)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b}}{24d^{7/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(33*a^2*d^2 + 2*a*b*d*(-20*c + 13*d*x) + b
^2*(15*c^2 - 10*c*d*x + 8*d^2*x^2)) - (15*(b*c - a*d)^(7/2)*Sqrt[(b*(c + d*
```

```
x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/b)/(24*d
^(7/2)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.21, size = 160, normalized size = 1.08

$$\frac{(ad-bc)^3\left(\frac{15b^2(c+dx)^{5/2}}{(a+bx)^{5/2}}+\frac{33d^2\sqrt{c+dx}}{\sqrt{a+bx}}-\frac{40bd(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{24d^3\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{8\sqrt{b}\,d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

```
[Out] ((-(b*c) + a*d)^3*((33*d^2*Sqrt[c + d*x])/Sqrt[a + b*x] - (40*b*d*(c + d*x)
^(3/2))/(a + b*x)^(3/2) + (15*b^2*(c + d*x)^(5/2))/(a + b*x)^(5/2)))/(24*d^
3*(d - (b*(c + d*x))/(a + b*x))^3) - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*Sqrt[b]*d^(7/2))
```

**fricas [A]** time = 1.02, size = 412, normalized size = 2.78

$$\left[-\frac{15\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)-4\left(8b^3d^3x^2+15b^3c^2d-40ab^2cd^2+33a^2bd^3-2\left(5b^3cd^2-13ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{96bd^4},\ \frac{15\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)+2\left(8b^3d^3x^2+15b^3c^2d-40ab^2cd^2+33a^2bd^3-2\left(5b^3cd^2-13ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48bd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*s
qrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*
d^3*x^2 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 1
3*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^4), 1/48*(15*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*
c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x)) + 2*(8*b^3*d^3*x^2 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 +
33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 13*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c
))/(b*d^4)]
```

**giac [A]** time = 1.27, size = 198, normalized size = 1.34

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\left(2(bx+a)\left(\frac{4(bx+a)}{bd}-\frac{5\left(bcd^3-ad^4\right)}{bd^5}\right)+\frac{15\left(b^2c^2d^2-2abcd^3+a^2d^4\right)}{bd^5}\right)+\frac{15\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^3}\right)b}{24|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*
x + a)/(b*d) - 5*(b*c*d^3 - a*d^4)/(b*d^5)) + 15*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)/(b*d^5)) + 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d^3))*b/abs(b)
```

**maple** [B] time = 0.01, size = 465, normalized size = 3.14

$$\frac{5\sqrt{(bx+a)(dx+c)}\,a^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}}-\frac{15\sqrt{(bx+a)(dx+c)}\,a^2bc\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d}+\frac{15\sqrt{(bx+a)(dx+c)}\,a\,b^2c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d^2}-\frac{5\sqrt{(bx+a)(dx+c)}\,b^3c^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d^3}+\frac{5\sqrt{bx+a}\,\sqrt{dx+c}\,a^2}{8d}-\frac{5\sqrt{bx+a}\,\sqrt{dx+c}\,abc}{4d^2}+\frac{5\sqrt{bx+a}\,\sqrt{dx+c}\,b^2c^2}{8d^3}+\frac{5(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,a}{12d}-\frac{5(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,bc}{12d^2}+\frac{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*(b*x+a)^(5/2)*(d*x+c)^(1/2)/d+5/12/d*(b*x+a)^(3/2)*(d*x+c)^(1/2)*a-5/12
/d^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)*b*c+5/8/d*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a^2-
5/4/d^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a*b*c+5/8/d^3*(b*x+a)^(1/2)*(d*x+c)^(1/
2)*b^2*c^2+5/16*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d
*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2
)*a^3-15/16/d*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x
+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*
a^2*b*c+15/16/d^2*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b
*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1
/2)*a*b^2*c^2-5/16/d^3*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*
ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*
d)^(1/2)*b^3*c^3
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.1388 $\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=113

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4\sqrt{b}\,d^{5/2}}-\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}$$

**Rubi [A]** time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^2}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4\sqrt{b}\,d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^2) + ((a + b*x)^(3/2)*Sqr
t[c + d*x])/(2*d) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[
b]*Sqrt[c + d*x])])/(4*Sqrt[b]*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} - \frac{(3(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{\left(3(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{8d^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x\right.}{4bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+}}{\sqrt{c+}}\right.}{4bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4\sqrt{b}\,d^{5/2}}$$

**Mathematica [A]** time = 0.37, size = 119, normalized size = 1.05

$$\frac{\sqrt{d}\,\sqrt{a+bx}\,(c+dx)(5ad-3bc+2bdx) + \frac{3(bc-ad)^{5/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b}}{4d^{5/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(-3*b*c + 5*a*d + 2*b*d*x) + (3*(b*c - a*d
)^(5/2)*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqr
t[b*c - a*d]])/b)/(4*d^(5/2)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.19, size = 134, normalized size = 1.19

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{4\sqrt{b}\,d^{5/2}}+\frac{(ad-bc)^2\left(\frac{5d\sqrt{c+dx}}{\sqrt{a+bx}}-\frac{3b(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{4d^2\left(d-\frac{b(c+dx)}{a+bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
```

```
[Out] ((-(b*c) + a*d)^2*((5*d*Sqrt[c + d*x])/Sqrt[a + b*x] - (3*b*(c + d*x)^(3/2)
)/(a + b*x)^(3/2)))/(4*d^2*(d - (b*(c + d*x))/(a + b*x))^2) + (3*(b*c - a*d
)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*Sqrt[b]*d^
(5/2))
```

**fricas [A]** time = 1.36, size = 306, normalized size = 2.71

$$\left[\frac{3\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)+4\left(2b^2d^2x-3b^2cd+5abd^2\right)\sqrt{bx+a}\sqrt{dx+c}}{16bd^3},-\frac{3\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(2b^2d^2x-3b^2cd+5abd^2\right)\sqrt{bx+a}\sqrt{dx+c}}{8bd^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)
*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x - 3*b^2*c*d + 5*
a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^3), -1/8*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(2
*b^2*d^2*x - 3*b^2*c*d + 5*a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^3)]
```

**giac [A]** time = 0.95, size = 139, normalized size = 1.23

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\left(\frac{2\,(bx+a)}{bd}-\frac{3\left(bcd-ad^2\right)}{bd^3}\right)-\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^2}\right)b}{4\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/(b*d) -
 3*(b*c*d - a*d^2)/(b*d^3)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^
2))*b/abs(b)
```

**maple [B]** time = 0.01, size = 308, normalized size = 2.73

$$\frac{3\sqrt{(bx+a)(dx+c)}\,a^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{8\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}}-\frac{3\sqrt{(bx+a)(dx+c)}\,abc\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{4\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d}+\frac{3\sqrt{(bx+a)(dx+c)}\,b^2c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{8\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d^2}+\frac{3\sqrt{bx+a}\,\sqrt{dx+c}\,a}{4d}-\frac{3\sqrt{bx+a}\,\sqrt{dx+c}\,bc}{4d^2}+\frac{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d+3/4/d*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a-3/4/d
^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)*b*c+3/8*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2
)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*
c)*x)^(1/2))/(b*d)^(1/2)*a^2-3/4/d*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d
*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x
)^(1/2))/(b*d)^(1/2)*a*b*c+3/8/d^2*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d
*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x
)^(1/2))/(b*d)^(1/2)*b^2*c^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/sqrt(c + d*x), x)
```

# 3.1389 $\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=73

$$\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}$$

**Rubi** [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx = \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2d}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{bd}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{bd}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}$$

**Mathematica [A]** time = 0.27, size = 103, normalized size = 1.41

$$\frac{b\sqrt{d}\,\sqrt{a+bx}\,(c+dx)-(bc-ad)^{3/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{bd^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x) - (b*c - a*d)^(3/2)*Sqrt[(b*(c + d*x))/(
b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*d^(3/2)*Sq
rt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.29, size = 106, normalized size = 1.45

$$\frac{\sqrt{c+dx}\,\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}}{d} + \frac{\sqrt{\frac{b}{d}}\,(bc-ad)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[a - (b*c)/d + (b*(c + d*x))/d])/d + (Sqrt[b/d]*(b*c - a
*d)*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) + Sqrt[a - (b*c)/d + (b*(c + d*x))/d]])/
(b*d)
```

**fricas [A]** time = 0.77, size = 235, normalized size = 3.22

$$\left[\frac{4\sqrt{bx+a}\sqrt{dx+c}\,bd-(bc-ad)\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)}{4\,bd^2}, \frac{2\sqrt{bx+a}\sqrt{dx+c}\,bd+(bc-ad)\sqrt{-bd}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)}{2\,bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b^2*d
^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x))/(b*d^2), 1/2*(2*sqr
t(b*x + a)*sqrt(d*x + c)*b*d + (b*c - a*d)*sqrt(-b*d)*arctan(1/2*(2*b*d*x +
 b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x)))/(b*d^2)]
```

**giac [A]** time = 1.12, size = 97, normalized size = 1.33

$$\frac{b\left(\frac{(bc-ad)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}+\frac{\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}}{bd}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] b*((b*c - a*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x +
 a)/(b*d))/abs(b)
```

**maple [A]** time = 0.01, size = 107, normalized size = 1.47

$$-\frac{(-ad+bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{2\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d}+\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] (b*x+a)^(1/2)*(d*x+c)^(1/2)/d-1/2*(-a*d+b*c)/d*((b*x+a)*(d*x+c))^(1/2)/(b*x
+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c
+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 3.80, size = 261, normalized size = 3.58

$$\frac{\frac{(2\,a\,d+2\,b\,c)\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}+\frac{\left(2\,c\,b^2+2\,a\,d\,b\right)\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}-\frac{8\,\sqrt{a}\,b\,\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}}+\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)\,\left(a\,d-b\,c\right)}{\sqrt{b}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/2),x)
```

```
[Out] (((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2))^3)/(d^2*((c + d*x)^(1/2) - c^
(1/2))^3) + ((2*b^2*c + 2*a*b*d)*((a + b*x)^(1/2) - a^(1/2)))/(d^3*((c + d*
x)^(1/2) - c^(1/2))) - (8*a^(1/2)*b*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/
(d^2*((c + d*x)^(1/2) - c^(1/2))^2))/(((a + b*x)^(1/2) - a^(1/2))^4/((c + d
*x)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(d*(
(c + d*x)^(1/2) - c^(1/2))^2)) + (2*atanh((d^(1/2)*((a + b*x)^(1/2) - a^(1/
2)))/(b^(1/2)*((c + d*x)^(1/2) - c^(1/2))))*(a*d - b*c))/(b^(1/2)*d^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/sqrt(c + d*x), x)
```

## 3.1390 $\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=42

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

**Rubi** [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

**Mathematica** [A]      time = 0.07, size = 77, normalized size = 1.83

$$\frac{2\sqrt{c+dx}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}\sqrt{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]
*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d)])
```

**IntegrateAlgebraic** [A]      time = 0.09, size = 42, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[
d])
```

**fricas** [B]      time = 1.11, size = 178, normalized size = 4.24

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)}{2\,bd}, -\frac{\sqrt{-bd}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d
*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^
2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b
*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x))/(b*d
)]
```

**giac [A]** time = 1.10, size = 50, normalized size = 1.19

$$-\frac{2\,b\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}\,+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*abs(b))
```

**maple [B]** time = 0.01, size = 76, normalized size = 1.81

$$\frac{\sqrt{(bx+a)\,(dx+c)}\;\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] ((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b
*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.29, size = 45, normalized size = 1.07

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((c + d*x)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x)^(1/2) - a
^(1/2)))))/(-b*d)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(c + d*x)), x)
```

## 3.1391 $\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=30

$$-\frac{2\sqrt{c+dx}}{\sqrt{a+bx}\,(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2\sqrt{c+dx}}{\sqrt{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/((b*c - a*d)*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{(bc-ad)\sqrt{a+bx}}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{\sqrt{a+bx}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x])/((-(b*c) + a*d)*Sqrt[a + b*x])
```

**IntegrateAlgebraic [A]** time = 0.03, size = 30, normalized size = 1.00

$$-\frac{2\sqrt{c+dx}}{\sqrt{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
[Out] (-2*Sqrt[c + d*x])/((b*c - a*d)*Sqrt[a + b*x])
```

**fricas [A]** time = 0.92, size = 42, normalized size = 1.40

$$-\frac{2\,\sqrt{bx+a}\,\sqrt{dx+c}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x + a)*sqrt(d*x + c)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

**giac [B]** time = 1.04, size = 66, normalized size = 2.20

$$-\frac{4\,\sqrt{bd}\,b}{\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="giac")
[Out] -4*sqrt(b*d)*b/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2)*abs(b))
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.90

$$\frac{2\sqrt{dx+c}}{\sqrt{bx+a}\;(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x)
[Out] 2/(b*x+a)^(1/2)*(d*x+c)^(1/2)/(a*d-b*c)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [B] time = 0.73, size = 26, normalized size = 0.87

$$\frac{2\,\sqrt{c+d\,x}}{\left(a\,d-b\,c\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(1/2))/((a*d - b*c)*(a + b*x)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*sqrt(c + d*x)), x)
```

## 3.1392 $\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=66

$$\frac{4d\sqrt{c+dx}}{3\sqrt{a+bx}\,(bc-ad)^2}-\frac{2\sqrt{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4d\sqrt{c+dx}}{3\sqrt{a+bx}\,(bc-ad)^2}-\frac{2\sqrt{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (4*d*Sqrt[c + d*x])/(3
*(b*c - a*d)^2*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(2d)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx}{3(bc-ad)}$$
$$= -\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{4d\sqrt{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 0.70

$$\frac{2\sqrt{c+dx}\,(3ad-bc+2bdx)}{3(a+bx)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*(-(b*c) + 3*a*d + 2*b*d*x))/(3*(b*c - a*d)^2*(a + b*x)^(3/
2))
```

**IntegrateAlgebraic [A]** time = 0.10, size = 56, normalized size = 0.85

$$-\frac{2\left(\frac{b(c+dx)^{3/2}}{(a+bx)^{3/2}} - \frac{3d\sqrt{c+dx}}{\sqrt{a+bx}}\right)}{3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*((-3*d*Sqrt[c + d*x])/Sqrt[a + b*x] + (b*(c + d*x)^(3/2))/(a + b*x)^(3/
2)))/(3*(b*c - a*d)^2)
```

**fricas [B]** time = 0.92, size = 118, normalized size = 1.79

$$\frac{2\,(2\,bdx - bc + 3\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{3\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + \left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)x^2 + 2\left(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*d*x - b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^2*c^2 - 2*a^
3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^
2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

**giac [B]** time = 1.08, size = 121, normalized size = 1.83

$$\frac{8\left(b^2c-abd-3\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)\sqrt{bd}\,b^2d}{3\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(b^2*c - a*b*d - 3*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2)*sqrt(b*d)*b^2*d/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3*abs(b))
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.82

$$\frac{2\sqrt{dx+c}\ (2bdx+3ad-bc)}{3\,(bx+a)^{\frac{3}{2}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/3*(d*x+c)^(1/2)*(2*b*d*x+3*a*d-b*c)/(b*x+a)^(3/2)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.89, size = 71, normalized size = 1.08

$$\frac{\left(\frac{4\,d\,x}{3\,(a\,d-b\,c)^2}+\frac{6\,a\,d-2\,b\,c}{3\,b\,(a\,d-b\,c)^2}\right)\sqrt{c+d\,x}}{x\,\sqrt{a+b\,x}+\frac{a\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (((4*d*x)/(3*(a*d - b*c)^2) + (6*a*d - 2*b*c)/(3*b*(a*d - b*c)^2))*(c + d*x
)^(1/2))/(x*(a + b*x)^(1/2) + (a*(a + b*x)^(1/2))/b)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*sqrt(c + d*x)), x)
```

## 3.1393 $\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=101

$$-\frac{16d^2\sqrt{c+dx}}{15\sqrt{a+bx}\,(bc-ad)^3}+\frac{8d\sqrt{c+dx}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{5(a+bx)^{5/2}(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d^2\sqrt{c+dx}}{15\sqrt{a+bx}\,(bc-ad)^3}+\frac{8d\sqrt{c+dx}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (8*d*Sqrt[c + d*x])/(1
5*(b*c - a*d)^2*(a + b*x)^(3/2)) - (16*d^2*Sqrt[c + d*x])/(15*(b*c - a*d)^3
*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}} - \frac{(4d)\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx}{5(bc-ad)}$$

$$= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}} + \frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}} + \frac{\left(8d^2\right)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx}{15(bc-ad)^2}$$

$$= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}} + \frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}} - \frac{16d^2\sqrt{c+dx}}{15(bc-ad)^3\sqrt{a+bx}}$$

**Mathematica [A]** time = 0.03, size = 75, normalized size = 0.74

$$-\frac{2\sqrt{c+dx}\left(15a^2d^2-10abd(c-2dx)+b^2\left(3c^2-4cdx+8d^2x^2\right)\right)}{15(a+bx)^{5/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x) + b^2*(3*c^2 - 4*c*d*x
 + 8*d^2*x^2)))/(15*(b*c - a*d)^3*(a + b*x)^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.10, size = 83, normalized size = 0.82

$$-\frac{2\left(\frac{3b^2(c+dx)^{5/2}}{(a+bx)^{5/2}}+\frac{15d^2\sqrt{c+dx}}{\sqrt{a+bx}}-\frac{10bd(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{15(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*((15*d^2*Sqrt[c + d*x])/Sqrt[a + b*x] - (10*b*d*(c + d*x)^(3/2))/(a + b
*x)^(3/2) + (3*b^2*(c + d*x)^(5/2))/(a + b*x)^(5/2)))/(15*(b*c - a*d)^3)
```

**fricas [B]** time = 1.32, size = 251, normalized size = 2.49

$$-\frac{2\left(8\,b^2d^2x^2+3\,b^2c^2-10\,abcd+15\,a^2d^2-4\left(b^2cd-5\,abd^2\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{15\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3\right)x^2+3\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*b^2*d^2*x^2 + 3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 4*(b^2*c*d - 5
*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3
*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b
^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^
3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x)
```

**giac [B]** time = 1.27, size = 227, normalized size = 2.25

$$-\frac{32\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}-5\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}b^{2}c+5\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}abd+10\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}\right)\sqrt{bd}\,b^{3}d^{2}}{15\left(b^{2}c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)^{5}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -32/15*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 5*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^2*c + 5*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b*d + 10*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4)*sqrt(b*d)*b^3*d^2/((b^2*c - a*b*
d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*ab
s(b))
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$\frac{2\sqrt{dx+c}\,\left(8b^{2}x^{2}d^{2}+20ab\,d^{2}x-4b^{2}cdx+15a^{2}d^{2}-10abcd+3b^{2}c^{2}\right)}{15\,(bx+a)^{\frac{5}{2}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/15*(d*x+c)^(1/2)*(8*b^2*d^2*x^2+20*a*b*d^2*x-4*b^2*c*d*x+15*a^2*d^2-10*a*
b*c*d+3*b^2*c^2)/(b*x+a)^(5/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.01, size = 133, normalized size = 1.32

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,d^2\,x^2}{15\,{\left(a\,d-b\,c\right)}^3}+\frac{30\,a^2\,d^2-20\,a\,b\,c\,d+6\,b^2\,c^2}{15\,b^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d\,x\,\left(5\,a\,d-b\,c\right)}{15\,b\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a^2\,\sqrt{a+b\,x}}{b^2}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((16*d^2*x^2)/(15*(a*d - b*c)^3) + (30*a^2*d^2 + 6*b^2*c^2
 - 20*a*b*c*d)/(15*b^2*(a*d - b*c)^3) + (8*d*x*(5*a*d - b*c))/(15*b*(a*d -
b*c)^3)))/(x^2*(a + b*x)^(1/2) + (a^2*(a + b*x)^(1/2))/b^2 + (2*a*x*(a + b*
x)^(1/2))/b)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*sqrt(c + d*x)), x)
```

## 3.1394 $\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=136

$$\frac{32d^3\sqrt{c+dx}}{35\sqrt{a+bx}\,(bc-ad)^4}-\frac{16d^2\sqrt{c+dx}}{35(a+bx)^{3/2}(bc-ad)^3}+\frac{12d\sqrt{c+dx}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{7(a+bx)^{7/2}(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3\sqrt{c+dx}}{35\sqrt{a+bx}\,(bc-ad)^4}-\frac{16d^2\sqrt{c+dx}}{35(a+bx)^{3/2}(bc-ad)^3}+\frac{12d\sqrt{c+dx}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (12*d*Sqrt[c + d*x])/(
35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*Sqrt[c + d*x])/(35*(b*c - a*d)^
3*(a + b*x)^(3/2)) + (32*d^3*Sqrt[c + d*x])/(35*(b*c - a*d)^4*Sqrt[a + b*x]
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} - \frac{(6d)\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx}{7(bc-ad)}$$

$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} + \frac{\left(24d^2\right)\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx}{35(bc-ad)^2}$$

$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} - \frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}} - \frac{\left(16d^3\right) \cdots}{3 \cdots}$$

$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} - \frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}} + \frac{32 \cdots}{35(bc - \cdots}$$

**Mathematica [A]** time = 0.05, size = 116, normalized size = 0.85

$$\frac{2\sqrt{c+dx}\left(35a^3d^3-35a^2bd^2(c-2dx)+7ab^2d\left(3c^2-4cdx+8d^2x^2\right)+b^3\left(-5c^3+6c^2dx-8cd^2x^2+16d^3x^3\right)\right)}{35(a+bx)^{7/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*(35*a^3*d^3 - 35*a^2*b*d^2*(c - 2*d*x) + 7*a*b^2*d*(3*c^2
- 4*c*d*x + 8*d^2*x^2) + b^3*(-5*c^3 + 6*c^2*d*x - 8*c*d^2*x^2 + 16*d^3*x^3
)))/(35*(b*c - a*d)^4*(a + b*x)^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 109, normalized size = 0.80

$$-\frac{2\left(\frac{5b^3(c+dx)^{7/2}}{(a+bx)^{7/2}}-\frac{21b^2d(c+dx)^{5/2}}{(a+bx)^{5/2}}-\frac{35d^3\sqrt{c+dx}}{\sqrt{a+bx}}+\frac{35bd^2(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{35(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*((-35*d^3*Sqrt[c + d*x])/Sqrt[a + b*x] + (35*b*d^2*(c + d*x)^(3/2))/(a
+ b*x)^(3/2) - (21*b^2*d*(c + d*x)^(5/2))/(a + b*x)^(5/2) + (5*b^3*(c + d*x
)^(7/2))/(a + b*x)^(7/2)))/(35*(b*c - a*d)^4)
```

**fricas [B]** time = 2.89, size = 419, normalized size = 3.08

$$\frac{2\left(16b^3d^3x^3-5b^3c^3+21ab^2c^2d-35a^2bcd^2+35a^3d^3-8\left(b^3cd^2-7ab^2d^3\right)x^2+2\left(3b^3c^2d-14ab^2cd^2+35a^2bd^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{35\left(a^4b^4c^4-4a^5b^3c^3d+6a^6b^2c^2d^2-4a^7bcd^3+a^8d^4+\left(b^8c^4-4ab^7c^3d+6a^2b^6c^2d^2-4a^3b^5cd^3+a^4b^4d^4\right)x^4+4\left(ab^7c^4-4a^2b^6c^3d+6a^3b^5c^2d^2-4a^4b^4cd^3+a^5b^3d^4\right)x^3+6\left(a^2b^6c^4-4a^3b^5c^3d+6a^4b^4c^2d^2-4a^5b^3cd^3+a^6b^2d^4\right)x^2+4\left(a^3b^5c^4-4a^4b^4c^3d+6a^5b^3c^2d^2-4a^6b^2cd^3+a^7bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*d^3*x^3 - 5*b^3*c^3 + 21*a*b^2*c^2*d - 35*a^2*b*c*d^2 + 35*a^3
*d^3 - 8*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 2*(3*b^3*c^2*d - 14*a*b^2*c*d^2 +
35*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d
 + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d +
 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*
a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*
(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*
b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6
*b^2*c*d^3 + a^7*b*d^4)*x)
```

**giac** [B] time = 1.47, size = 386, normalized size = 2.84

$$\frac{64\left(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3}-7\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}b^{4}c^{2}+14\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}ab^{3}cd-7\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}a^{2}b^{2}d^{2}+21\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}b^{2}c-21\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}abd-35\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{6}\right)\sqrt{bd}\,b^{4}d^{3}}{35\left(b^{2}c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)^{7}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*(b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3 - 7*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^4*c^2 + 14*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^3*c*d - 7
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^2*
d^2 + 21*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
b^2*c - 21*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
4*a*b*d - 35*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6)*sqrt(b*d)*b^4*d^3/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^
2*c + (b*x + a)*b*d - a*b*d))^2)^7*abs(b))
```

**maple** [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{2\sqrt{dx+c}\,\left(16b^{3}x^{3}d^{3}+56a\,b^{2}d^{3}x^{2}-8b^{3}c\,d^{2}x^{2}+70a^{2}b\,d^{3}x-28a\,b^{2}c\,d^{2}x+6b^{3}c^{2}dx+35a^{3}d^{3}-35a^{2}bc\,d^{2}+21a\,b^{2}c^{2}d-5b^{3}c^{3}\right)}{35\,(bx+a)^{\frac{7}{2}}\,\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/35*(d*x+c)^(1/2)*(16*b^3*d^3*x^3+56*a*b^2*d^3*x^2-8*b^3*c*d^2*x^2+70*a^2*
b*d^3*x-28*a*b^2*c*d^2*x+6*b^3*c^2*d*x+35*a^3*d^3-35*a^2*b*c*d^2+21*a*b^2*c
^2*d-5*b^3*c^3)/(b*x+a)^(7/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*
b^3*c^3*d+b^4*c^4)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [B] time = 1.19, size = 209, normalized size = 1.54

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,d^3\,x^3}{35\,{\left(a\,d-b\,c\right)}^4}+\frac{70\,a^3\,d^3-70\,a^2\,b\,c\,d^2+42\,a\,b^2\,c^2\,d-10\,b^3\,c^3}{35\,b^3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d\,x\,\left(35\,a^2\,d^2-14\,a\,b\,c\,d+3\,b^2\,c^2\right)}{35\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^2\,x^2\,\left(7\,a\,d-b\,c\right)}{35\,b\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*d^3*x^3)/(35*(a*d - b*c)^4) + (70*a^3*d^3 - 10*b^3*c^
3 + 42*a*b^2*c^2*d - 70*a^2*b*c*d^2)/(35*b^3*(a*d - b*c)^4) + (4*d*x*(35*a^
2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(35*b^2*(a*d - b*c)^4) + (16*d^2*x^2*(7*a*
d - b*c))/(35*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/2) + (a^3*(a + b*x)^(1/2
))/b^3 + (3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(a + b*x)^(1/2))/b^2)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{9}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(9/2)*sqrt(c + d*x)), x)
```

## 3.1395 $\int \frac{1}{(a+bx)^{11/2}\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=171

$$-\frac{256d^4\sqrt{c+dx}}{315\sqrt{a+bx}\,(bc-ad)^5}+\frac{128d^3\sqrt{c+dx}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{32d^2\sqrt{c+dx}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{16d\sqrt{c+dx}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2\sqrt{\cdots}}{9(a+bx\cdots}$$

**Rubi** [A] time = 0.04, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{256d^4\sqrt{c+dx}}{315\sqrt{a+bx}\,(bc-ad)^5}+\frac{128d^3\sqrt{c+dx}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{32d^2\sqrt{c+dx}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{16d\sqrt{c+dx}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (16*d*Sqrt[c + d*x])/(
63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (32*d^2*Sqrt[c + d*x])/(105*(b*c - a*d)
^3*(a + b*x)^(5/2)) + (128*d^3*Sqrt[c + d*x])/(315*(b*c - a*d)^4*(a + b*x)^
(3/2)) - (256*d^4*Sqrt[c + d*x])/(315*(b*c - a*d)^5*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+bx)^{11/2}\sqrt{c+dx}}\,dx &= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(8d)\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx}{9(bc-ad)} \\
&= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} + \frac{\left(16d^2\right)\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx}{21(bc-ad)^2} \\
&= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}} - \frac{(64d \cdots}{\cdots} \\
&= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}} + \frac{\cdots}{315(\cdots} \\
&= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}} + \frac{\cdots}{315(\cdots}
\end{aligned}
$$

**Mathematica** [A] time = 0.07, size = 168, normalized size = 0.98

$$
-\frac{2\sqrt{c+dx}\left(315a^4d^4-420a^3bd^3(c-2dx)+126a^2b^2d^2\left(3c^2-4cdx+8d^2x^2\right)+36ab^3d\left(-5c^3+6c^2dx-8cd^2x^2+16d^3x^3\right)+b^4\left(35c^4-40c^3dx+48c^2d^2x^2-64cd^3x^3+128d^4x^4\right)\right)}{315(a+bx)^{9/2}(bc-ad)^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(315*a^4*d^4 - 420*a^3*b*d^3*(c - 2*d*x) + 126*a^2*b^2*d^
2*(3*c^2 - 4*c*d*x + 8*d^2*x^2) + 36*a*b^3*d*(-5*c^3 + 6*c^2*d*x - 8*c*d^2*
x^2 + 16*d^3*x^3) + b^4*(35*c^4 - 40*c^3*d*x + 48*c^2*d^2*x^2 - 64*c*d^3*x^
3 + 128*d^4*x^4)))/(315*(b*c - a*d)^5*(a + b*x)^(9/2))
```

**IntegrateAlgebraic** [A] time = 0.11, size = 135, normalized size = 0.79

$$
-\frac{2\left(\frac{35b^4(c+dx)^{9/2}}{(a+bx)^{9/2}}-\frac{180b^3d(c+dx)^{7/2}}{(a+bx)^{7/2}}+\frac{378b^2d^2(c+dx)^{5/2}}{(a+bx)^{5/2}}+\frac{315d^4\sqrt{c+dx}}{\sqrt{a+bx}}-\frac{420bd^3(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{315(bc-ad)^5}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*((315*d^4*Sqrt[c + d*x])/Sqrt[a + b*x] - (420*b*d^3*(c + d*x)^(3/2))/(a
 + b*x)^(3/2) + (378*b^2*d^2*(c + d*x)^(5/2))/(a + b*x)^(5/2) - (180*b^3*d*
(c + d*x)^(7/2))/(a + b*x)^(7/2) + (35*b^4*(c + d*x)^(9/2))/(a + b*x)^(9/2)
))/(315*(b*c - a*d)^5)
```

**fricas [B]** time = 11.40, size = 638, normalized size = 3.73

$$-\frac{2\left(128\,b^{4}d^{4}x^{4}+35\,b^{4}c^{4}-180\,ab^{3}c^{3}d+378\,a^{2}b^{2}c^{2}d^{2}-420\,a^{3}bcd^{3}+315\,a^{4}d^{4}-64\left(b^{4}cd^{3}-9\,ab^{3}d^{4}\right)x^{3}+48\left(b^{4}c^{2}d^{2}-6\,ab^{3}cd^{3}+21\,a^{2}b^{2}d^{4}\right)x^{2}-8\left(5\,b^{4}c^{3}d-27\,ab^{3}c^{2}d^{2}+63\,a^{2}b^{2}cd^{3}-105\,a^{3}bd^{4}\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{315\left(a^{5}b^{5}c^{5}-5\,a^{6}b^{4}c^{4}d+10\,a^{7}b^{3}c^{3}d^{2}-10\,a^{8}b^{2}c^{2}d^{3}+5\,a^{9}bcd^{4}-a^{10}d^{5}+\left(b^{10}c^{5}-5\,ab^{9}c^{4}d+10\,a^{2}b^{8}c^{3}d^{2}-10\,a^{3}b^{7}c^{2}d^{3}+5\,a^{4}b^{6}cd^{4}-a^{5}b^{5}d^{5}\right)x^{5}+5\left(ab^{9}c^{5}-5\,a^{2}b^{8}c^{4}d+10\,a^{3}b^{7}c^{3}d^{2}-10\,a^{4}b^{6}c^{2}d^{3}+5\,a^{5}b^{5}cd^{4}-a^{6}b^{4}d^{5}\right)x^{4}+10\left(a^{2}b^{8}c^{5}-5\,a^{3}b^{7}c^{4}d+10\,a^{4}b^{6}c^{3}d^{2}-10\,a^{5}b^{5}c^{2}d^{3}+5\,a^{6}b^{4}cd^{4}-a^{7}b^{3}d^{5}\right)x^{3}+10\left(a^{3}b^{7}c^{5}-5\,a^{4}b^{6}c^{4}d+10\,a^{5}b^{5}c^{3}d^{2}-10\,a^{6}b^{4}c^{2}d^{3}+5\,a^{7}b^{3}cd^{4}-a^{8}b^{2}d^{5}\right)x^{2}+5\left(a^{4}b^{6}c^{5}-5\,a^{5}b^{5}c^{4}d+10\,a^{6}b^{4}c^{3}d^{2}-10\,a^{7}b^{3}c^{2}d^{3}+5\,a^{8}b^{2}cd^{4}-a^{9}bd^{5}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(128*b^4*d^4*x^4 + 35*b^4*c^4 - 180*a*b^3*c^3*d + 378*a^2*b^2*c^2*d^
2 - 420*a^3*b*c*d^3 + 315*a^4*d^4 - 64*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 48*(
b^4*c^2*d^2 - 6*a*b^3*c*d^3 + 21*a^2*b^2*d^4)*x^2 - 8*(5*b^4*c^3*d - 27*a*b
^3*c^2*d^2 + 63*a^2*b^2*c*d^3 - 105*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x +
c)/(a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*c^2*d^3
 + 5*a^9*b*c*d^4 - a^10*d^5 + (b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8*c^3*d^
2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*x^5 + 5*(a*b^9*c^5
- 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*b^5*c*d
^4 - a^6*b^4*d^5)*x^4 + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4*b^6*c^3*
d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*x^3 + 10*(a^3*b^7
*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3 + 5*a^7*b^
3*c*d^4 - a^8*b^2*d^5)*x^2 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*
c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5)*x)
```

**giac [B]** time = 1.57, size = 596, normalized size = 3.49

$$-\frac{512\left(b^{8}c^{4}-4\,ab^{7}c^{3}d+6\,a^{2}b^{6}c^{2}d^{2}-4\,a^{3}b^{5}cd^{3}+a^{4}b^{4}d^{4}-9\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}b^{6}c^{3}+27\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}ab^{5}c^{2}d-27\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}a^{2}b^{4}cd^{2}+9\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}a^{3}b^{3}d^{3}+36\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}b^{4}c^{2}-72\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}ab^{3}cd+36\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}a^{2}b^{2}d^{2}-84\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{6}b^{2}c+84\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{6}abd+126\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{8}\right)\sqrt{bd}b^{5}d^{4}}{315\left(b^{2}c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)^{9}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -512/315*(b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a
^4*b^4*d^4 - 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^2*b^6*c^3 + 27*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a*b^5*c^2*d - 27*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^2*b^4*c*d^2 + 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*a^3*b^3*d^3 + 36*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^4*c^2 - 72*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^3*c*d + 36*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^2*d^2 - 84*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^2*c + 84*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b*d + 126*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8)*sqrt(b*d)*b^5*d^
4/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2)^9*abs(b))
```

**maple [A]** time = 0.01, size = 256, normalized size = 1.50

$$\frac{2\sqrt{dx+c}\,\left(128b^{4}x^{4}d^{4}+576a\,b^{3}d^{4}x^{3}-64b^{4}c\,d^{3}x^{3}+1008a^{2}b^{2}d^{4}x^{2}-288a\,b^{3}c\,d^{3}x^{2}+48b^{4}c^{2}d^{2}x^{2}+840a^{3}b\,d^{4}x-504a^{2}b^{2}c\,d^{3}x+216a\,b^{3}c^{2}d^{2}x-40b^{4}c^{3}dx+315a^{4}d^{4}-420a^{3}bc\,d^{3}+378a^{2}b^{2}c^{2}d^{2}-180a\,b^{3}c^{3}d+35b^{4}c^{4}\right)}{315\,(bx+a)^{\frac{9}{2}}\left(a^{5}d^{5}-5a^{4}bc\,d^{4}+10a^{3}b^{2}c^{2}d^{3}-10a^{2}b^{3}c^{3}d^{2}+5a\,b^{4}c^{4}d-b^{5}c^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/315*(d*x+c)^(1/2)*(128*b^4*d^4*x^4+576*a*b^3*d^4*x^3-64*b^4*c*d^3*x^3+100
8*a^2*b^2*d^4*x^2-288*a*b^3*c*d^3*x^2+48*b^4*c^2*d^2*x^2+840*a^3*b*d^4*x-50
4*a^2*b^2*c*d^3*x+216*a*b^3*c^2*d^2*x-40*b^4*c^3*d*x+315*a^4*d^4-420*a^3*b*
c*d^3+378*a^2*b^2*c^2*d^2-180*a*b^3*c^3*d+35*b^4*c^4)/(b*x+a)^(9/2)/(a^5*d^
5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5
)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.37, size = 303, normalized size = 1.77

$$\frac{\sqrt{c+d\,x}\,\left(\frac{256\,d^4\,x^4}{315\,{\left(a\,d-b\,c\right)}^5}+\frac{630\,a^4\,d^4-840\,a^3\,b\,c\,d^3+756\,a^2\,b^2\,c^2\,d^2-360\,a\,b^3\,c^3\,d+70\,b^4\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(1680\,a^3\,b\,d^4-1008\,a^2\,b^2\,c\,d^3+432\,a\,b^3\,c^2\,d^2-80\,b^4\,c^3\,d\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d^2\,x^2\,\left(21\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{105\,b^2\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((256*d^4*x^4)/(315*(a*d - b*c)^5) + (630*a^4*d^4 + 70*b^4
*c^4 + 756*a^2*b^2*c^2*d^2 - 360*a*b^3*c^3*d - 840*a^3*b*c*d^3)/(315*b^4*(a
*d - b*c)^5) + (x*(1680*a^3*b*d^4 - 80*b^4*c^3*d + 432*a*b^3*c^2*d^2 - 1008
*a^2*b^2*c*d^3))/(315*b^4*(a*d - b*c)^5) + (128*d^3*x^3*(9*a*d - b*c))/(315
*b*(a*d - b*c)^5) + (32*d^2*x^2*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(105*b^
2*(a*d - b*c)^5)))/(x^4*(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/2))/b^4 + (6*a^
2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (4*a^3*x*(a + b*
x)^(1/2))/b^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.1396 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=174

$$-\frac{35\sqrt{b}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8d^{9/2}}+\frac{35b\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8d^4}-\frac{35b(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12d^3}+\frac{7b(a+}{}$$

**Rubi [A]** time = 0.09, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2}-\frac{35b(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12d^3}+\frac{35b\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8d^4}-\frac{35\sqrt{b}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8d^{9/2}}-\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(7/2))/(d*Sqrt[c + d*x]) + (35*b*(b*c - a*d)^2*Sqrt[a + b*x]*
Sqrt[c + d*x])/(8*d^4) - (35*b*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(
12*d^3) + (7*b*(a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d^2) - (35*Sqrt[b]*(b*c -
a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(9/2)
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2} - \frac{(35b(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6d^2}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2} + \frac{\left(35b(bc-ad)^2\right)\int}{8d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+}{}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+}{}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+}{}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+}{}$$

**Mathematica** [C] time = 0.07, size = 73, normalized size = 0.42

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2} {}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(3/2)*Hypergeometric2F1[3/2,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.34, size = 173, normalized size = 0.99

$$-\frac{\sqrt{a+bx}\,(ad-bc)^3\left(\frac{280b^2d(a+bx)}{c+dx}+\frac{48d^3(a+bx)^3}{(c+dx)^3}-\frac{231bd^2(a+bx)^2}{(c+dx)^2}-105b^3\right)}{24d^4\sqrt{c+dx}\left(\frac{d(a+bx)}{c+dx}-b\right)^3}-\frac{35\sqrt{b}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```

```
[Out] -1/24*((-(b*c) + a*d)^3*Sqrt[a + b*x]*(-105*b^3 + (48*d^3*(a + b*x)^3)/(c +
 d*x)^3 - (231*b*d^2*(a + b*x)^2)/(c + d*x)^2 + (280*b^2*d*(a + b*x))/(c +
d*x)))/(d^4*Sqrt[c + d*x]*(-b + (d*(a + b*x))/(c + d*x))^3) - (35*Sqrt[b]*(
b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d
^(9/2))
```

**fricas [B]** time = 1.73, size = 603, normalized size = 3.47

$$\left[-\frac{105\left(b^3c^4-3ab^2c^3d+3a^2bc^2d^2-a^3cd^3+\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3-a^3d^4\right)x\right)\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2bd^2x+bcd+ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-4\left(8b^3d^3x^3+105b^3c^3-280ab^2c^2d+231a^2bcd^2-48a^3d^3-2\left(7b^3cd^2-19ab^2d^3\right)x^2+\left(35b^3c^2d-98ab^2cd^2+87a^2bd^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{96\left(d^5x+cd^4\right)}, \frac{105\left(b^3c^4-3ab^2c^3d+3a^2bc^2d^2-a^3cd^3+\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3-a^3d^4\right)x\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{\left(2bdx+bc+ad\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)+2\left(8b^3d^3x^3+105b^3c^3-280ab^2c^2d+231a^2bcd^2-48a^3d^3-2\left(7b^3cd^2-19ab^2d^3\right)x^2+\left(35b^3c^2d-98ab^2cd^2+87a^2bd^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48\left(d^5x+cd^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(105*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3 + (b^3*c
^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(b/d)*log(8*b^2*d^
2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(
b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d^3*
x^3 + 105*b^3*c^3 - 280*a*b^2*c^2*d + 231*a^2*b*c*d^2 - 48*a^3*d^3 - 2*(7*b
^3*c*d^2 - 19*a*b^2*d^3)*x^2 + (35*b^3*c^2*d - 98*a*b^2*c*d^2 + 87*a^2*b*d^
3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x + c*d^4), 1/48*(105*(b^3*c^4 - 3*
a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3 + (b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqr
t(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)
) + 2*(8*b^3*d^3*x^3 + 105*b^3*c^3 - 280*a*b^2*c^2*d + 231*a^2*b*c*d^2 - 48
*a^3*d^3 - 2*(7*b^3*c*d^2 - 19*a*b^2*d^3)*x^2 + (35*b^3*c^2*d - 98*a*b^2*c*
d^2 + 87*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x + c*d^4)]
```

**giac [B]** time = 1.38, size = 279, normalized size = 1.60

$$\frac{\left(\left(bx+a\right)\left(2\left(bx+a\right)\left(\frac{4\left(bx+a\right)b^2}{d|b|}-\frac{7\left(b^3cd^5-ab^2d^6\right)}{d^7|b|}\right)+\frac{35\left(b^4c^2d^4-2ab^3cd^5+a^2b^2d^6\right)}{d^7|b|}\right)+\frac{105\left(b^5c^3d^3-3ab^4c^2d^4+3a^2b^3cd^5-a^3b^2d^6\right)}{d^7|b|}\right)\sqrt{bx+a}}{24\sqrt{b^2c+(bx+a)bd-abd}}+\frac{35\left(b^5c^3-3ab^4c^2d+3a^2b^3cd^2-a^3b^2d^3\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{8\sqrt{bd}\,d^4|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*((b*x + a)*(2*(b*x + a)*(4*(b*x + a)*b^2/(d*abs(b)) - 7*(b^3*c*d^5 - a
*b^2*d^6)/(d^7*abs(b))) + 35*(b^4*c^2*d^4 - 2*a*b^3*c*d^5 + a^2*b^2*d^6)/(d
^7*abs(b))) + 105*(b^5*c^3*d^3 - 3*a*b^4*c^2*d^4 + 3*a^2*b^3*c*d^5 - a^3*b^
2*d^6)/(d^7*abs(b)))*sqrt(b*x + a)/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) + 35
/8*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*log(abs(-sqrt(
b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^4*a
bs(b))
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{7/2}}{{\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(7/2)/(c + d*x)**(3/2), x)
```

# 3.1397 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=138

$$\frac{15\sqrt{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4d^{7/2}}-\frac{15b\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^3}+\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}-\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}$$

**Rubi** [A] time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}-\frac{15b\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^3}+\frac{15\sqrt{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4d^{7/2}}-\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(5/2))/(d*Sqrt[c + d*x]) - (15*b*(b*c - a*d)*Sqrt[a + b*x]*Sq
rt[c + d*x])/(4*d^3) + (5*b*(a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d^2) + (15*Sq
rt[b]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(4*d^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} + \frac{(5b)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} - \frac{(15b(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d^2}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{\left(15b(bc-ad)^2\right)\int \frac{}{}}{8d^3}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{\left(15(bc-ad)^2\right)\mathrm{Sub}}{}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{\left(15(bc-ad)^2\right)\mathrm{Sub}}{}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{15\sqrt{b}\,(bc-ad)^2\tan}{4d^{7/}}$$

**Mathematica [C]** time = 0.06, size = 73, normalized size = 0.53

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(3/2)*Hypergeometric2F1[3/2,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.24, size = 151, normalized size = 1.09

$$\frac{15\sqrt{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{7/2}}-\frac{\sqrt{a+bx}\,(ad-bc)^2\left(\frac{8d^2(a+bx)^2}{(c+dx)^2}-\frac{25bd(a+bx)}{c+dx}+15b^2\right)}{4d^3\sqrt{c+dx}\left(\frac{d(a+bx)}{c+dx}-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```

```
[Out] -1/4*((-(b*c) + a*d)^2*Sqrt[a + b*x]*(15*b^2 + (8*d^2*(a + b*x)^2)/(c + d*x
)^2 - (25*b*d*(a + b*x))/(c + d*x)))/(d^3*Sqrt[c + d*x]*(-b + (d*(a + b*x))
/(c + d*x))^2) + (15*Sqrt[b]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/
(Sqrt[b]*Sqrt[c + d*x])])/(4*d^(7/2))
```

**fricas [A]** time = 1.50, size = 441, normalized size = 3.20

$$\left[\frac{15\left(b^2c^3-2abc^2d+a^2cd^2+\left(b^2c^2d-2abcd^2+a^2d^3\right)x\right)\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2bd^2x+bcd+ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)+4\left(2b^2d^2x^2-15b^2c^2+25abcd-8a^2d^2-\left(5b^2cd-9abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{16\left(d^4x+cd^3\right)},-\frac{15\left(b^2c^3-2abc^2d+a^2cd^2+\left(b^2c^2d-2abcd^2+a^2d^3\right)x\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)-2\left(2b^2d^2x^2-15b^2c^2+25abcd-8a^2d^2-\left(5b^2cd-9abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{8\left(d^4x+cd^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*
(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*
c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x^2 - 15*b^2*c^2 + 25*a*b*c*d - 8*a^2*d^2
- (5*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x + c*d^3),
-1/8*(15*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^
2*d^3)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*
x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) - 2*(2*b^2*d^2*x
^2 - 15*b^2*c^2 + 25*a*b*c*d - 8*a^2*d^2 - (5*b^2*c*d - 9*a*b*d^2)*x)*sqrt(
b*x + a)*sqrt(d*x + c))/(d^4*x + c*d^3)]
```

**giac** [A] time = 1.26, size = 201, normalized size = 1.46

$$\frac{\sqrt{bx+a}\left((bx+a)\left(\frac{2\,(bx+a)b^2}{d|b|}-\frac{5\left(b^3cd^3-ab^2d^4\right)}{d^5|b|}\right)-\frac{15\left(b^4c^2d^2-2\,ab^3cd^3+a^2b^2d^4\right)}{d^5|b|}\right)}{4\,\sqrt{b^2c+(bx+a)bd-abd}}-\frac{15\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{4\,\sqrt{bd}\,d^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*((b*x + a)*(2*(b*x + a)*b^2/(d*abs(b)) - 5*(b^3*c*d^3 - a
*b^2*d^4)/(d^5*abs(b))) - 15*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)/(d
^5*abs(b)))/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) - 15/4*(b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)))/(sqrt(b*d)*d^3*abs(b))
```

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{5/2}}{{\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(3/2), x)
```

## 3.1398 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=98

$$-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}$$

**Rubi [A]** time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(d*Sqrt[c + d*x]) + (3*b*Sqrt[a + b*x]*Sqrt[c + d*x])/
d^2 - (3*Sqrt[b]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[
c + d*x])])/d^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{(3b)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3b(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.05, size = 73, normalized size = 0.74

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(3/2)*Hypergeometric2F1[3/2,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]**   time = 0.56, size = 120, normalized size = 1.22

$$\frac{\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}\,(-2ad+b(c+dx)+2bc)}{d^2\sqrt{c+dx}}+\frac{3\sqrt{\frac{b}{d}}\,(bc-ad)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] ((2*b*c - 2*a*d + b*(c + d*x))*Sqrt[a - (b*c)/d + (b*(c + d*x))/d])/(d^2*Sq
rt[c + d*x]) + (3*Sqrt[b/d]*(b*c - a*d)*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) + Sq
rt[a - (b*c)/d + (b*(c + d*x))/d]])/d^2
```

**fricas [A]**   time = 1.05, size = 311, normalized size = 3.17

$$\left[-\frac{3\left(bc^2-acd+\left(bcd-ad^2\right)x\right)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,bd^2x+bcd+ad^2\right)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-4\,(bdx+3\,bc-2\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{4\left(d^3x+cd^2\right)},\ \frac{3\left(bc^2-acd+\left(bcd-ad^2\right)x\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)+2\,(bdx+3\,bc-2\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{2\left(d^3x+cd^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(b*d*x + 3*b*c - 2*
a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^3*x + c*d^2), 1/2*(3*(b*c^2 - a*c*d +
(b*c*d - a*d^2)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a
)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*(b*
d*x + 3*b*c - 2*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^3*x + c*d^2)]
```

**giac [A]**   time = 1.27, size = 137, normalized size = 1.40

$$\frac{\sqrt{bx+a}\left(\frac{(bx+a)b^2}{d|b|}+\frac{3\left(b^3cd-ab^2d^2\right)}{d^3|b|}\right)}{\sqrt{b^2c+(bx+a)bd-abd}}+\frac{3\left(b^3c-ab^2d\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x + a)*((b*x + a)*b^2/(d*abs(b)) + 3*(b^3*c*d - a*b^2*d^2)/(d^3*abs(
b)))/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) + 3*(b^3*c - a*b^2*d)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^2
*abs(b))
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx + a)^{\frac{3}{2}}}{(dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)^{\frac{3}{2}}}{(c + dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(3/2),x)
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(3/2), x)
```

## 3.1399 $\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=66

$$\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}$$

**Rubi [A]** time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {47, 63, 217, 206}

$$\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx &= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{b\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{d}\\
&= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d}\\
&= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.35, size = 95, normalized size = 1.44

$$\frac{2\sqrt{bc-ad}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)-2\sqrt{d}\,\sqrt{a+bx}}{d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[a + b*x] + 2*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d
)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*Sqrt[c + d*x]
)
```

**IntegrateAlgebraic [A]** time = 0.10, size = 66, normalized size = 1.00

$$\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{3/2}}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2)
```

**fricas [B]** time = 1.73, size = 241, normalized size = 3.65

$$\left[\frac{(dx+c)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,bd^2x+bcd+ad^2\right)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-4\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\left(d^2x+cd\right)},-\frac{(dx+c)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)+2\,\sqrt{bx+a}\,\sqrt{dx+c}}{d^2x+cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((d*x + c)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*sqrt(b*x + a)*sqrt(d*x + c))/(d^2*x + c*d), -((d
*x + c)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*sqrt(b*x + a)*
sqrt(d*x + c))/(d^2*x + c*d)]
```

**giac [A]** time = 1.22, size = 96, normalized size = 1.45

$$-\frac{2\,b^2\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d|b|}-\frac{2\,\sqrt{bx+a}\,b^2}{\sqrt{b^2c+(bx+a)bd-abd}\,d|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*b^2*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*
d)))/(sqrt(b*d)*d*abs(b)) - 2*sqrt(b*x + a)*b^2/(sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)*d*abs(b))
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(3/2), x)
```

## 3.1400 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=30

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx = \frac{2\sqrt{a+bx}}{(bc-ad)\sqrt{c+dx}}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.03, size = 30, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

**fricas [A]** time = 1.29, size = 42, normalized size = 1.40

$$\frac{2\sqrt{bx+a}\sqrt{dx+c}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fricas")
[Out] 2*sqrt(b*x + a)*sqrt(d*x + c)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

**giac [A]** time = 1.04, size = 47, normalized size = 1.57

$$\frac{2\sqrt{bx+a}\,b^2}{\sqrt{b^2c+(bx+a)bd-abd}\,(bc|b|-ad|b|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="giac")
[Out] 2*sqrt(b*x + a)*b^2/(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(b*c*abs(b) - a*d*
abs(b)))
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.90

$$-\frac{2\sqrt{bx+a}}{\sqrt{dx+c}\;(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x)
[Out] -2*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*d-b*c)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.74, size = 26, normalized size = 0.87

$$-\frac{2\sqrt{a+b\,x}}{\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -(2*(a + b*x)^(1/2))/((a*d - b*c)*(c + d*x)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(3/2)), x)
```

## 3.1401 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=62

$$-\frac{4d\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{4d\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x]) - (4*d*Sqrt[a + b*x])/((b*c -
a*d)^2*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}} - \frac{(2d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx}{bc-ad}$$
$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}} - \frac{4d\sqrt{a+bx}}{(bc-ad)^2\sqrt{c+dx}}$$

**Mathematica [A]** time = 0.02, size = 42, normalized size = 0.68

$$-\frac{2(ad+b(c+2dx))}{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(a*d + b*(c + 2*d*x)))/((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.11, size = 46, normalized size = 0.74

$$-\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}+d\right)}{\sqrt{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(d + (b*(c + d*x))/(a + b*x)))/((b*c - a*d)^2*Sqrt[c + d*
x])
```

**fricas [B]** time = 1.55, size = 125, normalized size = 2.02

$$-\frac{2\,(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}}{ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(a*b^2*c^3 - 2*a^2*b*c
^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 -
 a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

**giac [B]** time = 1.22, size = 142, normalized size = 2.29

$$-\frac{2\sqrt{bx+a}\,b^2d}{\left(b^2c^2|b|-2\,abcd|b|+a^2d^2|b|\right)\sqrt{b^2c+(bx+a)bd-abd}}-\frac{4\sqrt{bd}\,b^2}{\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)(bc|b|-ad|b|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b) + a^2*d^2*abs(b)
)*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) - 4*sqrt(b*d)*b^2/((b^2*c - a*b*d -
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*(b*c*abs
(b) - a*d*abs(b)))
```

**maple [A]** time = 0.00, size = 52, normalized size = 0.84

$$-\frac{2\,(2bdx+ad+bc)}{\sqrt{bx+a}\,\sqrt{dx+c}\,\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2*(2*b*d*x+a*d+b*c)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a^2*d^2-2*a*b*c*d+b^2*c^2
)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.86, size = 71, normalized size = 1.15

$$-\frac{\left(\frac{4\,b\,x}{{\left(a\,d-b\,c\right)}^2}+\frac{2\,a\,d+2\,b\,c}{d\,{\left(a\,d-b\,c\right)}^2}\right)\,\sqrt{c+d\,x}}{x\,\sqrt{a+b\,x}+\frac{c\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -(((4*b*x)/(a*d - b*c)^2 + (2*a*d + 2*b*c)/(d*(a*d - b*c)^2))*(c + d*x)^(1/
2))/(x*(a + b*x)^(1/2) + (c*(a + b*x)^(1/2))/d)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(3/2)), x)
```

## 3.1402 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=101

$$\frac{16d^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}+\frac{8d}{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{16d^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}+\frac{8d}{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x]) + (8*d)/(3*(b*c - a*d)^2*S
qrt[a + b*x]*Sqrt[c + d*x]) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}} - \frac{(4d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx}{3(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}} + \frac{8d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{(8d^2)\int \frac{1}{\sqrt{a+bx}\,(c}}{3(bc-a}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}} + \frac{8d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{16d^2\sqrt{a+b}}{3(bc-ad)^3\sqrt{c}}$$

**Mathematica [A]** time = 0.03, size = 75, normalized size = 0.74

$$\frac{2\left(3a^2d^2+6abd(c+2dx)+b^2\left(-c^2+4cdx+8d^2x^2\right)\right)}{3(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (2*(3*a^2*d^2 + 6*a*b*d*(c + 2*d*x) + b^2*(-c^2 + 4*c*d*x + 8*d^2*x^2)))/(3
*(b*c - a*d)^3*(a + b*x)^(3/2)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.12, size = 73, normalized size = 0.72

$$\frac{2(c+dx)^{3/2}\left(\frac{3d^2(a+bx)^2}{(c+dx)^2}+\frac{6bd(a+bx)}{c+dx}-b^2\right)}{3(a+bx)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-b^2 + (3*d^2*(a + b*x)^2)/(c + d*x)^2 + (6*b*d*(a + b*
x))/(c + d*x)))/(3*(b*c - a*d)^3*(a + b*x)^(3/2))
```

**fricas [B]** time = 1.92, size = 273, normalized size = 2.70

$$\frac{2\left(8\,b^2d^2x^2-b^2c^2+6\,abcd+3\,a^2d^2+4\left(b^2cd+3\,abd^2\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{3\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2+5\,a^3b^2cd^3-2\,a^4bd^4\right)x^2+\left(2\,ab^4c^4-5\,a^2b^3c^3d+3\,a^3b^2c^2d^2+a^4bcd^3-a^5d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*b^2*d^2*x^2 - b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2 + 4*(b^2*c*d + 3*a*b*d
^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b
*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3
*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^
3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 +
 a^4*b*c*d^3 - a^5*d^4)*x)
```

**giac [B]** time = 1.54, size = 368, normalized size = 3.64

$$\frac{2\sqrt{bx+a}\,b^2d^2}{\left(b^3c^3|b| - 3\,ab^2c^2d|b| + 3\,a^2bcd^2|b| - a^3d^3|b|\right)\sqrt{b^2c + (bx+a)bd - abd}} + \frac{4\left(5\sqrt{bd}\,b^6c^2d - 10\sqrt{bd}\,ab^5cd^2 + 5\sqrt{bd}\,a^2b^4d^3 - 12\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2 b^4cd + 12\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2 ab^3d^2 + 3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^4 b^2d\right)}{3\left(b^2c^2|b| - 2\,abcd|b| + a^2d^2|b|\right)\left(b^2c - abd - \left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)*b^2*d^2/((b^3*c^3*abs(b) - 3*a*b^2*c^2*d*abs(b) + 3*a^2*b*c
*d^2*abs(b) - a^3*d^3*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) + 4/3*(5
*sqrt(b*d)*b^6*c^2*d - 10*sqrt(b*d)*a*b^5*c*d^2 + 5*sqrt(b*d)*a^2*b^4*d^3 -
 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^2*b^4*c*d + 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^2*a*b^3*d^2 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*b^2*d)/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b
) + a^2*d^2*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^2)^3)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$-\frac{2\left(8b^2x^2d^2 + 12ab\,d^2x + 4b^2cdx + 3a^2d^2 + 6abcd - b^2c^2\right)}{3\,(bx+a)^{\frac{3}{2}}\,\sqrt{dx+c}\,\left(a^3d^3 - 3a^2bc\,d^2 + 3a\,b^2c^2d - b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(8*b^2*d^2*x^2+12*a*b*d^2*x+4*b^2*c*d*x+3*a^2*d^2+6*a*b*c*d-b^2*c^2)/(
b*x+a)^(3/2)/(d*x+c)^(1/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.06, size = 141, normalized size = 1.40

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{8\,x\,\left(3\,a\,d+b\,c\right)}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,b\,d\,x^2}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{6\,a^2\,d^2+12\,a\,b\,c\,d-2\,b^2\,c^2}{3\,b\,d\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a\,c\,\sqrt{a+b\,x}}{b\,d}+\frac{x\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((8*x*(3*a*d + b*c))/(3*(a*d - b*c)^3) + (16*b*d*x^2)/(3*
(a*d - b*c)^3) + (6*a^2*d^2 - 2*b^2*c^2 + 12*a*b*c*d)/(3*b*d*(a*d - b*c)^3)
))/(x^2*(a + b*x)^(1/2) + (a*c*(a + b*x)^(1/2))/(b*d) + (x*(a*d + b*c)*(a +
 b*x)^(1/2))/(b*d))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{5}{2}}\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(3/2)), x)
```

## 3.1403 $\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=136

$$-\frac{32d^3\sqrt{a+bx}}{5\sqrt{c+dx}\,(bc-ad)^4}-\frac{16d^2}{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}+\frac{4d}{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}$$

**Rubi [A]** time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{32d^3\sqrt{a+bx}}{5\sqrt{c+dx}\,(bc-ad)^4}-\frac{16d^2}{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}+\frac{4d}{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*Sqrt[c + d*x]) + (4*d)/(5*(b*c - a*d)^2*(
a + b*x)^(3/2)*Sqrt[c + d*x]) - (16*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqr
t[c + d*x]) - (32*d^3*Sqrt[a + b*x])/(5*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx = -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} - \frac{(6d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx}{5(bc-ad)}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} + \frac{\left(8d^2\right)\int \frac{}{(a+bx)}}{5(bc-}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} - \frac{16}{5(bc-ad)^3\sqrt{a}}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} - \frac{16}{5(bc-ad)^3\sqrt{a}}$$

**Mathematica [A]** time = 0.04, size = 114, normalized size = 0.84

$$-\frac{2\left(5a^3d^3+15a^2bd^2(c+2dx)+5ab^2d\left(-c^2+4cdx+8d^2x^2\right)+b^3\left(c^3-2c^2dx+8cd^2x^2+16d^3x^3\right)\right)}{5(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(5*a^3*d^3 + 15*a^2*b*d^2*(c + 2*d*x) + 5*a*b^2*d*(-c^2 + 4*c*d*x + 8*d
^2*x^2) + b^3*(c^3 - 2*c^2*d*x + 8*c*d^2*x^2 + 16*d^3*x^3)))/(5*(b*c - a*d)
^4*(a + b*x)^(5/2)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.13, size = 93, normalized size = 0.68

$$-\frac{2(c+dx)^{5/2}\left(-\frac{5b^2d(a+bx)}{c+dx}+\frac{5d^3(a+bx)^3}{(c+dx)^3}+\frac{15bd^2(a+bx)^2}{(c+dx)^2}+b^3\right)}{5(a+bx)^{5/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*(b^3 + (5*d^3*(a + b*x)^3)/(c + d*x)^3 + (15*b*d^2*(a +
 b*x)^2)/(c + d*x)^2 - (5*b^2*d*(a + b*x))/(c + d*x)))/(5*(b*c - a*d)^4*(a
+ b*x)^(5/2))
```

**fricas [B]** time = 2.45, size = 455, normalized size = 3.35

$$-\frac{2\left(16\,b^3d^3x^3+b^3c^3-5\,ab^2c^2d+15\,a^2bcd^2+5\,a^3d^3+8\left(b^3cd^2+5\,ab^2d^3\right)x^2-2\left(b^3c^2d-10\,ab^2cd^2-15\,a^2bd^3\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{5\left(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+\left(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5\right)x^4+\left(b^7c^5-ab^6c^4d-6\,a^2b^5c^3d^2+14\,a^3b^4c^2d^3-11\,a^4b^3cd^4+3\,a^5b^2d^5\right)x^3+3\left(ab^6c^5-3\,a^2b^5c^4d+2\,a^3b^4c^3d^2+2\,a^4b^3c^2d^3-3\,a^5b^2cd^4+a^6bd^5\right)x^2+\left(3\,a^2b^5c^5-11\,a^3b^4c^4d+14\,a^4b^3c^3d^2-6\,a^5b^2c^2d^3-a^6bcd^4+a^7d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 5*a^3*d^3
 + 8*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 10*a*b^2*c*d^2 - 15*a^2
*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a
^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^6*c^3*d^2
 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c^5 - a*b^
6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5
*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*
b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^
4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*x
)
```

**giac** [B] time = 2.46, size = 830, normalized size = 6.10

$$-\frac{2\sqrt{bx+a}\,b^2d^3}{\left(b^4c^4|b| - 4ab^3c^3d|b| + 6a^2b^2c^2d^2|b| - 4a^3bcd^3|b| + a^4d^4|b|\right)\sqrt{b^2c+(bx+a)bd-abd}} - \frac{4\left(11\sqrt{bd}\,b^{10}c^4d^2 - 44\sqrt{bd}\,ab^9c^3d^3 + 66\sqrt{bd}\,a^2b^8c^2d^4 - 44\sqrt{bd}\,a^3b^7cd^5 + 11\sqrt{bd}\,a^4b^6d^6 - 50\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^2 b^8c^3d^2 + 150\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^2 ab^7c^2d^3 - 150\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^2 a^2b^6cd^4 + 50\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^2 a^3b^5d^5 + 80\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^4 b^6c^2d^2 - 160\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^4 ab^5cd^3 + 80\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^4 a^2b^4d^4 - 30\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^6 b^4cd^2 + 30\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^6 ab^3d^3 + 5\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^8 b^2d^2\right)}{5\left(b^3c^3|b| - 3ab^2c^2d|b| + 3a^2bcd^2|b| - a^3d^3|b|\right)\left(b^2c - abd - \left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d^3/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b) + 6*a^2*b^
2*c^2*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)) - 4/5*(11*sqrt(b*d)*b^10*c^4*d^2 - 44*sqrt(b*d)*a*b^9*c
^3*d^3 + 66*sqrt(b*d)*a^2*b^8*c^2*d^4 - 44*sqrt(b*d)*a^3*b^7*c*d^5 + 11*sqr
t(b*d)*a^4*b^6*d^6 - 50*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2*b^8*c^3*d^2 + 150*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^7*c^2*d^3 - 150*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c*
d^4 + 50*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a^3*b^5*d^5 + 80*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^4*b^6*c^2*d^2 - 160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^5*c*d^3 + 80*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^4*d^
4 - 30*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^6*b^4*c*d^2 + 30*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^3*d^3 + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^2*d^2)/((b^3*c^3*abs(b) - 3*a*b^2*
c^2*d*abs(b) + 3*a^2*b*c*d^2*abs(b) - a^3*d^3*abs(b))*(b^2*c - a*b*d - (sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5)
```

**maple** [A] time = 0.01, size = 170, normalized size = 1.25

$$-\frac{2\left(16b^3x^3d^3 + 40a\,b^2d^3x^2 + 8b^3c\,d^2x^2 + 30a^2b\,d^3x + 20a\,b^2c\,d^2x - 2b^3c^2dx + 5a^3d^3 + 15a^2bc\,d^2 - 5a\,b^2c^2d + b^3c^3\right)}{5\left(bx+a\right)^{\frac{5}{2}}\sqrt{dx+c}\,\left(a^4d^4 - 4a^3bc\,d^3 + 6a^2b^2c^2d^2 - 4a\,b^3c^3d + b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/5*(16*b^3*d^3*x^3+40*a*b^2*d^3*x^2+8*b^3*c*d^2*x^2+30*a^2*b*d^3*x+20*a*b
^2*c*d^2*x-2*b^3*c^2*d*x+5*a^3*d^3+15*a^2*b*c*d^2-5*a*b^2*c^2*d+b^3*c^3)/(b
*x+a)^(5/2)/(d*x+c)^(1/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*
c^3*d+b^4*c^4)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.31, size = 227, normalized size = 1.67

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{16\,d\,x^2\,\left(5\,a\,d+b\,c\right)}{5\,{\left(a\,d-b\,c\right)}^4}+\frac{2\,a^3\,d^3+6\,a^2\,b\,c\,d^2-2\,a\,b^2\,c^2\,d+\frac{2\,b^3\,c^3}{5}}{b^2\,d\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,b\,d^2\,x^3}{5\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,x\,\left(15\,a^2\,d^2+10\,a\,b\,c\,d-b^2\,c^2\right)}{5\,b\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^2\,c\,\sqrt{a+b\,x}}{b^2\,d}+\frac{x^2\,\left(2\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a\,x\,\left(a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((16*d*x^2*(5*a*d + b*c))/(5*(a*d - b*c)^4) + (2*a^3*d^3
+ (2*b^3*c^3)/5 - 2*a*b^2*c^2*d + 6*a^2*b*c*d^2)/(b^2*d*(a*d - b*c)^4) + (3
2*b*d^2*x^3)/(5*(a*d - b*c)^4) + (4*x*(15*a^2*d^2 - b^2*c^2 + 10*a*b*c*d))/
(5*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/2) + (a^2*c*(a + b*x)^(1/2))/(b^2*d
) + (x^2*(2*a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (a*x*(a*d + 2*b*c)*(a + b*x
)^(1/2))/(b^2*d))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{7}{2}}\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(3/2),x)
[Out] Integral(1/((a + b*x)**(7/2)*(c + d*x)**(3/2)), x)
```

## 3.1404 $\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=171

$$\frac{256d^4\sqrt{a+bx}}{35\sqrt{c+dx}\,(bc-ad)^5}+\frac{128d^3}{35\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}+\frac{16d}{35(a+bx)^{5/2}\sqrt{c+dx}\,(}$$

**Rubi [A]** time = 0.04, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{256d^4\sqrt{a+bx}}{35\sqrt{c+dx}\,(bc-ad)^5}+\frac{128d^3}{35\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}+\frac{16d}{35(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{7(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x]) + (16*d)/(35*(b*c - a*d)^2
*(a + b*x)^(5/2)*Sqrt[c + d*x]) - (32*d^2)/(35*(b*c - a*d)^3*(a + b*x)^(3/2
)*Sqrt[c + d*x]) + (128*d^3)/(35*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])
 + (256*d^4*Sqrt[a + b*x])/(35*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx = -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} - \frac{(8d)\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx}{7(bc-ad)}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} + \frac{(48d^2)\int \frac{1}{(a+bx \cdots}}{35(bc - \cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{3 \cdots}{35(bc-ad)^3(a \cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{3 \cdots}{35(bc-ad)^3(a \cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{3 \cdots}{35(bc-ad)^3(a \cdots}$$

**Mathematica [A]** time = 0.06, size = 166, normalized size = 0.97

$$\frac{2\left(35a^4d^4+140a^3bd^3(c+2dx)+70a^2b^2d^2\left(-c^2+4cdx+8d^2x^2\right)+28ab^3d\left(c^3-2c^2dx+8cd^2x^2+16d^3x^3\right)+b^4\left(-5c^4+8c^3dx-16c^2d^2x^2+64cd^3x^3+128d^4x^4\right)\right)}{35(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (2*(35*a^4*d^4 + 140*a^3*b*d^3*(c + 2*d*x) + 70*a^2*b^2*d^2*(-c^2 + 4*c*d*x
 + 8*d^2*x^2) + 28*a*b^3*d*(c^3 - 2*c^2*d*x + 8*c*d^2*x^2 + 16*d^3*x^3) + b
^4*(-5*c^4 + 8*c^3*d*x - 16*c^2*d^2*x^2 + 64*c*d^3*x^3 + 128*d^4*x^4)))/(35
*(b*c - a*d)^5*(a + b*x)^(7/2)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.14, size = 117, normalized size = 0.68

$$\frac{2(c+dx)^{7/2}\left(\frac{28b^3d(a+bx)}{c+dx}-\frac{70b^2d^2(a+bx)^2}{(c+dx)^2}+\frac{35d^4(a+bx)^4}{(c+dx)^4}+\frac{140bd^3(a+bx)^3}{(c+dx)^3}-5b^4\right)}{35(a+bx)^{7/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-5*b^4 + (35*d^4*(a + b*x)^4)/(c + d*x)^4 + (140*b*d^3*
(a + b*x)^3)/(c + d*x)^3 - (70*b^2*d^2*(a + b*x)^2)/(c + d*x)^2 + (28*b^3*d
*(a + b*x))/(c + d*x)))/(35*(b*c - a*d)^5*(a + b*x)^(7/2))
```

**fricas** [**B**] time = 8.19, size = 689, normalized size = 4.03

$$\frac{2\left(128\,b^4d^4x^4-5\,b^4c^4+28\,ab^3c^3d-70\,a^2b^2c^2d^2+140\,a^3bcd^3+35\,a^4d^4+64\left(b^4cd^3+7\,ab^3d^4\right)x^3-16\left(b^4c^2d^2-14\,ab^3cd^3-35\,a^2b^2d^4\right)x^2+8\left(b^4c^3d-7\,ab^3c^2d^2+35\,a^2b^2cd^3+35\,a^3bd^4\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{35\left(a^4b^5c^6-5\,a^5b^4c^5d+10\,a^6b^3c^4d^2-10\,a^7b^2c^3d^3+5\,a^8bc^2d^4-a^9cd^5+\left(b^9c^5d-5\,ab^8c^4d^2+10\,a^2b^7c^3d^3-10\,a^3b^6c^2d^4+5\,a^4b^5cd^5-a^5b^4d^6\right)x^5+\left(b^9c^6-ab^8c^5d-10\,a^2b^7c^4d^2+30\,a^3b^6c^3d^3-35\,a^4b^5c^2d^4+19\,a^5b^4cd^5-4\,a^6b^3d^6\right)x^4+2\left(2\,ab^8c^6-7\,a^2b^7c^5d+5\,a^3b^6c^4d^2+10\,a^4b^5c^3d^3-20\,a^5b^4c^2d^4+13\,a^6b^3cd^5-3\,a^7b^2d^6\right)x^3+2\left(3\,a^2b^7c^6-13\,a^3b^6c^5d+20\,a^4b^5c^4d^2-10\,a^5b^4c^3d^3-5\,a^6b^3c^2d^4+7\,a^7b^2cd^5-2\,a^8bd^6\right)x^2+\left(4\,a^3b^6c^6-19\,a^4b^5c^5d+35\,a^5b^4c^4d^2-30\,a^6b^3c^3d^3+10\,a^7b^2c^2d^4+a^8bcd^5-a^9d^6\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(128*b^4*d^4*x^4 - 5*b^4*c^4 + 28*a*b^3*c^3*d - 70*a^2*b^2*c^2*d^2 + 1
40*a^3*b*c*d^3 + 35*a^4*d^4 + 64*(b^4*c*d^3 + 7*a*b^3*d^4)*x^3 - 16*(b^4*c^
2*d^2 - 14*a*b^3*c*d^3 - 35*a^2*b^2*d^4)*x^2 + 8*(b^4*c^3*d - 7*a*b^3*c^2*d
^2 + 35*a^2*b^2*c*d^3 + 35*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b
^5*c^6 - 5*a^5*b^4*c^5*d + 10*a^6*b^3*c^4*d^2 - 10*a^7*b^2*c^3*d^3 + 5*a^8*
b*c^2*d^4 - a^9*c*d^5 + (b^9*c^5*d - 5*a*b^8*c^4*d^2 + 10*a^2*b^7*c^3*d^3 -
 10*a^3*b^6*c^2*d^4 + 5*a^4*b^5*c*d^5 - a^5*b^4*d^6)*x^5 + (b^9*c^6 - a*b^8
*c^5*d - 10*a^2*b^7*c^4*d^2 + 30*a^3*b^6*c^3*d^3 - 35*a^4*b^5*c^2*d^4 + 19*
a^5*b^4*c*d^5 - 4*a^6*b^3*d^6)*x^4 + 2*(2*a*b^8*c^6 - 7*a^2*b^7*c^5*d + 5*a
^3*b^6*c^4*d^2 + 10*a^4*b^5*c^3*d^3 - 20*a^5*b^4*c^2*d^4 + 13*a^6*b^3*c*d^5
 - 3*a^7*b^2*d^6)*x^3 + 2*(3*a^2*b^7*c^6 - 13*a^3*b^6*c^5*d + 20*a^4*b^5*c^
4*d^2 - 10*a^5*b^4*c^3*d^3 - 5*a^6*b^3*c^2*d^4 + 7*a^7*b^2*c*d^5 - 2*a^8*b*
d^6)*x^2 + (4*a^3*b^6*c^6 - 19*a^4*b^5*c^5*d + 35*a^5*b^4*c^4*d^2 - 30*a^6*
b^3*c^3*d^3 + 10*a^7*b^2*c^2*d^4 + a^8*b*c*d^5 - a^9*d^6)*x)
```

**giac** [**B**] time = 4.77, size = 1518, normalized size = 8.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)*b^2*d^4/((b^5*c^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^
3*c^3*d^2*abs(b) - 10*a^3*b^2*c^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d
^5*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) + 4/35*(93*sqrt(b*d)*b^14*c
^6*d^3 - 558*sqrt(b*d)*a*b^13*c^5*d^4 + 1395*sqrt(b*d)*a^2*b^12*c^4*d^5 - 1
860*sqrt(b*d)*a^3*b^11*c^3*d^6 + 1395*sqrt(b*d)*a^4*b^10*c^2*d^7 - 558*sqrt
(b*d)*a^5*b^9*c*d^8 + 93*sqrt(b*d)*a^6*b^8*d^9 - 616*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^12*c^5*d^3 + 3080*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
a*b^11*c^4*d^4 - 6160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^2*a^2*b^10*c^3*d^5 + 6160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^9*c^2*d^6 - 3080*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4
*b^8*c*d^7 + 616*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^5*b^7*d^8 + 1673*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^4*d^3 - 6692*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^9*c^3*d^4 +
```

```
10038*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^4*a^2*b^8*c^2*d^5 - 6692*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c*d^6 + 1673*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^6*d^7 - 2240*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^
8*c^3*d^3 + 6720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^6*a*b^7*c^2*d^4 - 6720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c*d^5 + 2240*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^5*d^6 +
 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^8*b^6*c^2*d^3 - 2030*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^8*a*b^5*c*d^4 + 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*d^5 - 280*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^4*c*d^
3 + 280*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a
*b*d))^10*a*b^3*d^4 + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^12*b^2*d^3)/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b)
 + 6*a^2*b^2*c^2*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*(b^2*c
 - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
2)^7)
```

**maple [A]** time = 0.01, size = 256, normalized size = 1.50

$$-\frac{2\left(128b^4x^4d^4+448a\,b^3d^4x^3+64b^4c\,d^3x^3+560a^2b^2d^4x^2+224a\,b^3c\,d^3x^2-16b^4c^2d^2x^2+280a^3b\,d^4x+280a^2b^2c\,d^3x-56a\,b^3c^2d^2x+8b^4c^3dx+35a^4d^4+140a^3bc\,d^3-70a^2b^2c^2d^2+28a\,b^3c^3d-5b^4c^4\right)}{35\left(bx+a\right)^{\frac{7}{2}}\sqrt{dx+c}\,\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/35*(128*b^4*d^4*x^4+448*a*b^3*d^4*x^3+64*b^4*c*d^3*x^3+560*a^2*b^2*d^4*x
^2+224*a*b^3*c*d^3*x^2-16*b^4*c^2*d^2*x^2+280*a^3*b*d^4*x+280*a^2*b^2*c*d^3
*x-56*a*b^3*c^2*d^2*x+8*b^4*c^3*d*x+35*a^4*d^4+140*a^3*b*c*d^3-70*a^2*b^2*c
^2*d^2+28*a*b^3*c^3*d-5*b^4*c^4)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(a^5*d^5-5*a^4
*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.50, size = 337, normalized size = 1.97

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{256\,b\,d^3\,x^4}{35\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^2\,x^3\,\left(7\,a\,d+b\,c\right)}{35\,{\left(a\,d-b\,c\right)}^5}+\frac{70\,a^4\,d^4+280\,a^3\,b\,c\,d^3-140\,a^2\,b^2\,c^2\,d^2+56\,a\,b^3\,c^3\,d-10\,b^4\,c^4}{35\,b^3\,d\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(560\,a^3\,b\,d^4+560\,a^2\,b^2\,c\,d^3-112\,a\,b^3\,c^2\,d^2+16\,b^4\,c^3\,d\right)}{35\,b^3\,d\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d\,x^2\,\left(35\,a^2\,d^2+14\,a\,b\,c\,d-b^2\,c^2\right)}{35\,b\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^3\,c\,\sqrt{a+b\,x}}{b^3\,d}+\frac{x^3\,\left(3\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{3\,a\,x^2\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}+\frac{a^2\,x\,\left(a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((256*b*d^3*x^4)/(35*(a*d - b*c)^5) + (128*d^2*x^3*(7*a*d
 + b*c))/(35*(a*d - b*c)^5) + (70*a^4*d^4 - 10*b^4*c^4 - 140*a^2*b^2*c^2*d^
2 + 56*a*b^3*c^3*d + 280*a^3*b*c*d^3)/(35*b^3*d*(a*d - b*c)^5) + (x*(560*a^
3*b*d^4 + 16*b^4*c^3*d - 112*a*b^3*c^2*d^2 + 560*a^2*b^2*c*d^3))/(35*b^3*d*
(a*d - b*c)^5) + (32*d*x^2*(35*a^2*d^2 - b^2*c^2 + 14*a*b*c*d))/(35*b*(a*d
- b*c)^5)))/(x^4*(a + b*x)^(1/2) + (a^3*c*(a + b*x)^(1/2))/(b^3*d) + (x^3*(
3*a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (3*a*x^2*(a*d + b*c)*(a + b*x)^(1/2))
/(b^2*d) + (a^2*x*(a*d + 3*b*c)*(a + b*x)^(1/2))/(b^3*d))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(9/2)*(c + d*x)**(3/2)), x)
```

## 3.1405 $\int \frac{1}{(a+bx)^{11/2}(c+dx)^{3/2}}\,dx$

**Optimal**. Leaf size=206

$$-\frac{512d^5\sqrt{a+bx}}{63\sqrt{c+dx}\,(bc-ad)^6}-\frac{256d^4}{63\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^5}+\frac{64d^3}{63(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{63(a+bx)^{5/2}\sqrt{c+dx}\,(}$$

**Rubi** [A] time = 0.06, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{512d^5\sqrt{a+bx}}{63\sqrt{c+dx}\,(bc-ad)^6}-\frac{256d^4}{63\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^5}+\frac{64d^3}{63(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{63(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^3}+\frac{20d}{63(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{9(a+bx)^{9/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(9*(b*c - a*d)*(a + b*x)^(9/2)*Sqrt[c + d*x]) + (20*d)/(63*(b*c - a*d)^2
*(a + b*x)^(7/2)*Sqrt[c + d*x]) - (32*d^2)/(63*(b*c - a*d)^3*(a + b*x)^(5/2
)*Sqrt[c + d*x]) + (64*d^3)/(63*(b*c - a*d)^4*(a + b*x)^(3/2)*Sqrt[c + d*x]
) - (256*d^4)/(63*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x]) - (512*d^5*Sqr
t[a + b*x])/(63*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+bx)^{11/2}(c+dx)^{3/2}}\,dx &= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} - \frac{(10d)\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx}{9(bc-ad)} \\
&= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} + \frac{(80d^2)\int \frac{}{(a}}{63(b} \\
&= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{}{63(bc-ad)} \\
&= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{}{63(bc-ad)} \\
&= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{}{63(bc-ad)} \\
&= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{}{63(bc-ad)}
\end{aligned}
$$

**Mathematica [A]** time = 0.08, size = 226, normalized size = 1.10

$$
\frac{512d^5\sqrt{a+bx}}{63\sqrt{c+dx}\,(bc-ad)^5(ad-bc)} + \frac{256d^4}{63\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4(ad-bc)} + \frac{64d^3}{63(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4} - \frac{32d^2}{63(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^3} + \frac{20d}{63(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^2} - \frac{2}{9(a+bx)^{9/2}\sqrt{c+dx}\,(bc-ad)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(9*(b*c - a*d)*(a + b*x)^(9/2)*Sqrt[c + d*x]) + (20*d)/(63*(b*c - a*d)^2
*(a + b*x)^(7/2)*Sqrt[c + d*x]) - (32*d^2)/(63*(b*c - a*d)^3*(a + b*x)^(5/2
)*Sqrt[c + d*x]) + (64*d^3)/(63*(b*c - a*d)^4*(a + b*x)^(3/2)*Sqrt[c + d*x]
) + (256*d^4)/(63*(b*c - a*d)^4*(-(b*c) + a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])
 + (512*d^5*Sqrt[a + b*x])/(63*(b*c - a*d)^5*(-(b*c) + a*d)*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.15, size = 139, normalized size = 0.67

$$
-\frac{2(c+dx)^{9/2}\left(-\frac{45b^4d(a+bx)}{c+dx} + \frac{126b^3d^2(a+bx)^2}{(c+dx)^2} - \frac{210b^2d^3(a+bx)^3}{(c+dx)^3} + \frac{63d^5(a+bx)^5}{(c+dx)^5} + \frac{315bd^4(a+bx)^4}{(c+dx)^4} + 7b^5\right)}{63(a+bx)^{9/2}(bc-ad)^6}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(c + d*x)^(9/2)*(7*b^5 + (63*d^5*(a + b*x)^5)/(c + d*x)^5 + (315*b*d^4*
(a + b*x)^4)/(c + d*x)^4 - (210*b^2*d^3*(a + b*x)^3)/(c + d*x)^3 + (126*b^3
*d^2*(a + b*x)^2)/(c + d*x)^2 - (45*b^4*d*(a + b*x))/(c + d*x)))/(63*(b*c -
 a*d)^6*(a + b*x)^(9/2))
```

**fricas [B]** time = 15.84, size = 955, normalized size = 4.64

$$-\frac{2\left(256b^5d^5x^5+7b^5c^5-45ab^4c^4d+126a^2b^3c^3d^2-210a^3b^2c^2d^3+315a^4bcd^4+63a^5d^5+128\left(b^5cd^4+9ab^4d^5\right)x^4-32\left(b^5c^2d^3-18ab^4cd^4-63a^2b^3d^5\right)x^3+16\left(b^5c^3d^2-9ab^4c^2d^3+63a^2b^3cd^4+105a^3b^2d^5\right)x^2-2\left(5b^5c^4d-36ab^4c^3d^2+126a^2b^3c^2d^3-420a^3b^2cd^4-315a^4bd^5\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{63\left(a^5b^6c^7-6a^6b^5c^6d+15a^7b^4c^5d^2-20a^8b^3c^4d^3+15a^9b^2c^3d^4-6a^{10}bc^2d^5+a^{11}cd^6+\left(b^{11}c^6d-6ab^{10}c^5d^2+15a^2b^9c^4d^3-20a^3b^8c^3d^4+15a^4b^7c^2d^5-6a^5b^6cd^6+a^6b^5d^7\right)x^6+\left(b^{11}c^7-ab^{10}c^6d-15a^2b^9c^5d^2+55a^3b^8c^4d^3-85a^4b^7c^3d^4+69a^5b^6c^2d^5-29a^6b^5cd^6+5a^7b^4d^7\right)x^5+5\left(ab^{10}c^7-4a^2b^9c^6d+3a^3b^8c^5d^2+10a^4b^7c^4d^3-25a^5b^6c^3d^4+24a^6b^5c^2d^5-11a^7b^4cd^6+2a^8b^3d^7\right)x^4+10\left(a^2b^9c^7-5a^3b^8c^6d+9a^4b^7c^5d^2-5a^5b^6c^4d^3-5a^6b^5c^3d^4+9a^7b^4c^2d^5-5a^8b^3cd^6+a^9b^2d^7\right)x^3+5\left(2a^3b^8c^7-11a^4b^7c^6d+24a^5b^6c^5d^2-25a^6b^5c^4d^3+10a^7b^4c^3d^4+3a^8b^3c^2d^5-4a^9b^2cd^6+a^{10}bd^7\right)x^2+\left(5a^4b^7c^7-29a^5b^6c^6d+69a^6b^5c^5d^2-85a^7b^4c^4d^3+55a^8b^3c^3d^4-15a^9b^2c^2d^5-a^{10}bcd^6+a^{11}d^7\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(256*b^5*d^5*x^5 + 7*b^5*c^5 - 45*a*b^4*c^4*d + 126*a^2*b^3*c^3*d^2 -
 210*a^3*b^2*c^2*d^3 + 315*a^4*b*c*d^4 + 63*a^5*d^5 + 128*(b^5*c*d^4 + 9*a*
b^4*d^5)*x^4 - 32*(b^5*c^2*d^3 - 18*a*b^4*c*d^4 - 63*a^2*b^3*d^5)*x^3 + 16*
(b^5*c^3*d^2 - 9*a*b^4*c^2*d^3 + 63*a^2*b^3*c*d^4 + 105*a^3*b^2*d^5)*x^2 -
2*(5*b^5*c^4*d - 36*a*b^4*c^3*d^2 + 126*a^2*b^3*c^2*d^3 - 420*a^3*b^2*c*d^4
 - 315*a^4*b*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^5*b^6*c^7 - 6*a^6*b^5*c
^6*d + 15*a^7*b^4*c^5*d^2 - 20*a^8*b^3*c^4*d^3 + 15*a^9*b^2*c^3*d^4 - 6*a^1
0*b*c^2*d^5 + a^11*c*d^6 + (b^11*c^6*d - 6*a*b^10*c^5*d^2 + 15*a^2*b^9*c^4*
d^3 - 20*a^3*b^8*c^3*d^4 + 15*a^4*b^7*c^2*d^5 - 6*a^5*b^6*c*d^6 + a^6*b^5*d
^7)*x^6 + (b^11*c^7 - a*b^10*c^6*d - 15*a^2*b^9*c^5*d^2 + 55*a^3*b^8*c^4*d^
3 - 85*a^4*b^7*c^3*d^4 + 69*a^5*b^6*c^2*d^5 - 29*a^6*b^5*c*d^6 + 5*a^7*b^4*
d^7)*x^5 + 5*(a*b^10*c^7 - 4*a^2*b^9*c^6*d + 3*a^3*b^8*c^5*d^2 + 10*a^4*b^7
*c^4*d^3 - 25*a^5*b^6*c^3*d^4 + 24*a^6*b^5*c^2*d^5 - 11*a^7*b^4*c*d^6 + 2*a
^8*b^3*d^7)*x^4 + 10*(a^2*b^9*c^7 - 5*a^3*b^8*c^6*d + 9*a^4*b^7*c^5*d^2 - 5
*a^5*b^6*c^4*d^3 - 5*a^6*b^5*c^3*d^4 + 9*a^7*b^4*c^2*d^5 - 5*a^8*b^3*c*d^6
+ a^9*b^2*d^7)*x^3 + 5*(2*a^3*b^8*c^7 - 11*a^4*b^7*c^6*d + 24*a^5*b^6*c^5*d
^2 - 25*a^6*b^5*c^4*d^3 + 10*a^7*b^4*c^3*d^4 + 3*a^8*b^3*c^2*d^5 - 4*a^9*b^
2*c*d^6 + a^10*b*d^7)*x^2 + (5*a^4*b^7*c^7 - 29*a^5*b^6*c^6*d + 69*a^6*b^5*
c^5*d^2 - 85*a^7*b^4*c^4*d^3 + 55*a^8*b^3*c^3*d^4 - 15*a^9*b^2*c^2*d^5 - a^
10*b*c*d^6 + a^11*d^7)*x)
```

**giac [B]** time = 8.71, size = 2438, normalized size = 11.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d^5/((b^6*c^6*abs(b) - 6*a*b^5*c^5*d*abs(b) + 15*a^2*b
^4*c^4*d^2*abs(b) - 20*a^3*b^3*c^3*d^3*abs(b) + 15*a^4*b^2*c^2*d^4*abs(b) -
 6*a^5*b*c*d^5*abs(b) + a^6*d^6*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
) - 4/63*(193*sqrt(b*d)*b^18*c^8*d^4 - 1544*sqrt(b*d)*a*b^17*c^7*d^5 + 5404
*sqrt(b*d)*a^2*b^16*c^6*d^6 - 10808*sqrt(b*d)*a^3*b^15*c^5*d^7 + 13510*sqrt
(b*d)*a^4*b^14*c^4*d^8 - 10808*sqrt(b*d)*a^5*b^13*c^3*d^9 + 5404*sqrt(b*d)*
```

a^6*b^12*c^2*d^10 - 1544*sqrt(b*d)*a^7*b^11*c*d^11 + 193*sqrt(b*d)*a^8*b^10
*d^12 - 1674*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*b^16*c^7*d^4 + 11718*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^15*c^6*d^5 - 35154*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^14*c^5*d^
6 + 58590*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2*a^3*b^13*c^4*d^7 - 58590*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^12*c^3*d^8 + 35154*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^11*c^2
*d^9 - 11718*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*a^6*b^10*c*d^10 + 1674*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^7*b^9*d^11 + 6318*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^14*c^6*d^4 - 3
7908*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*a*b^13*c^5*d^5 + 94770*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*a^2*b^12*c^4*d^6 - 126360*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^11*c^3*d^7 +
 94770*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^4*a^4*b^10*c^2*d^8 - 37908*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^9*c*d^9 + 6318*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^6*b^8*d^10 - 1331
4*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
^6*b^12*c^5*d^4 + 66570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^6*a*b^11*c^4*d^5 - 133140*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^10*c^3*d^6 + 133140
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
6*a^3*b^9*c^2*d^7 - 66570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^6*a^4*b^8*c*d^8 + 13314*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^5*b^7*d^9 + 16128*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^10
*c^4*d^4 - 64512*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^8*a*b^9*c^3*d^5 + 96768*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^8*c^2*d^6 - 64512*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^7*c
*d^7 + 16128*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^8*a^4*b^6*d^8 - 8190*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^10*b^8*c^3*d^4 + 24570*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^7*c^2*d^5 - 24
570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^10*a^2*b^6*c*d^6 + 8190*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^10*a^3*b^5*d^7 + 2898*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^6*c^2*d^4 - 5796*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^
5*c*d^5 + 2898*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^12*a^2*b^4*d^6 - 630*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr

```
t(b^2*c + (b*x + a)*b*d - a*b*d))^14*b^4*c*d^4 + 630*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b^3*d^5 + 63*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*b^2
*d^4)/((b^5*c^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^3*c^3*d^2*abs(b) -
 10*a^3*b^2*c^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d^5*abs(b))*(b^2*c
- a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2
)^9)
```

**maple** [B] time = 0.01, size = 356, normalized size = 1.73

$$-\frac{2\left(256b^5x^5d^5+1152a\,b^4d^5x^4+128b^5c\,d^4x^4+2016a^2b^3d^5x^3+576a\,b^4c\,d^4x^3-32b^5c^2d^3x^3+1680a^3b^2d^5x^2+1008a^2b^3c\,d^4x^2-144a\,b^4c^2d^3x^2+16b^5c^3d^2x^2+630a^4b\,d^5x+840a^3b^2c\,d^4x-252a^2b^3c^2d^3x+72a\,b^4c^3d^2x-10b^5c^4dx+63a^5d^5+315a^4bc\,d^4-210a^3b^2c^2d^3+126a^2b^3c^3d^2-45a\,b^4c^4d+7b^5c^5\right)}{63\left(bx+a\right)^{\frac{9}{2}}\sqrt{dx+c}\,\left(d^6a^6-6b\,d^5c\,a^5+15b^2d^4c^2a^4-20b^3d^3c^3a^3+15b^4d^2c^4a^2-6b^5d\,c^5a+b^6c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/63*(256*b^5*d^5*x^5+1152*a*b^4*d^5*x^4+128*b^5*c*d^4*x^4+2016*a^2*b^3*d^
5*x^3+576*a*b^4*c*d^4*x^3-32*b^5*c^2*d^3*x^3+1680*a^3*b^2*d^5*x^2+1008*a^2*
b^3*c*d^4*x^2-144*a*b^4*c^2*d^3*x^2+16*b^5*c^3*d^2*x^2+630*a^4*b*d^5*x+840*
a^3*b^2*c*d^4*x-252*a^2*b^3*c^2*d^3*x+72*a*b^4*c^3*d^2*x-10*b^5*c^4*d*x+63*
a^5*d^5+315*a^4*b*c*d^4-210*a^3*b^2*c^2*d^3+126*a^2*b^3*c^3*d^2-45*a*b^4*c^
4*d+7*b^5*c^5)/(b*x+a)^(9/2)/(d*x+c)^(1/2)/(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^
2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [B] time = 1.96, size = 454, normalized size = 2.20

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{126\,a^5\,d^5+630\,a^4\,b\,c\,d^4-420\,a^3\,b^2\,c^2\,d^3+252\,a^2\,b^3\,c^3\,d^2-90\,a\,b^4\,c^4\,d+14\,b^5\,c^5}{63\,b^4\,d\,{\left(a\,d-b\,c\right)}^6}+\frac{512\,b\,d^4\,x^5}{63\,{\left(a\,d-b\,c\right)}^6}+\frac{256\,d^3\,x^4\,\left(9\,a\,d+b\,c\right)}{63\,{\left(a\,d-b\,c\right)}^6}+\frac{x\,\left(1260\,a^4\,b\,d^5+1680\,a^3\,b^2\,c\,d^4-504\,a^2\,b^3\,c^2\,d^3+144\,a\,b^4\,c^3\,d^2-20\,b^5\,c^4\,d\right)}{63\,b^4\,d\,{\left(a\,d-b\,c\right)}^6}+\frac{64\,d^2\,x^3\,\left(63\,a^2\,d^2+18\,a\,b\,c\,d-b^2\,c^2\right)}{63\,b\,{\left(a\,d-b\,c\right)}^6}+\frac{32\,d\,x^2\,\left(105\,a^3\,d^3+63\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+b^3\,c^3\right)}{63\,b^2\,{\left(a\,d-b\,c\right)}^6}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^4\,c\,\sqrt{a+b\,x}}{b^4\,d}+\frac{x^4\,\left(4\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{2\,a\,x^3\,\left(3\,a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}+\frac{a^3\,x\,\left(a\,d+4\,b\,c\right)\,\sqrt{a+b\,x}}{b^4\,d}+\frac{2\,a^2\,x^2\,\left(2\,a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((126*a^5*d^5 + 14*b^5*c^5 + 252*a^2*b^3*c^3*d^2 - 420*a^
3*b^2*c^2*d^3 - 90*a*b^4*c^4*d + 630*a^4*b*c*d^4)/(63*b^4*d*(a*d - b*c)^6)
```

```
+ (512*b*d^4*x^5)/(63*(a*d - b*c)^6) + (256*d^3*x^4*(9*a*d + b*c))/(63*(a*d
 - b*c)^6) + (x*(1260*a^4*b*d^5 - 20*b^5*c^4*d + 144*a*b^4*c^3*d^2 + 1680*a
^3*b^2*c*d^4 - 504*a^2*b^3*c^2*d^3))/(63*b^4*d*(a*d - b*c)^6) + (64*d^2*x^3
*(63*a^2*d^2 - b^2*c^2 + 18*a*b*c*d))/(63*b*(a*d - b*c)^6) + (32*d*x^2*(105
*a^3*d^3 + b^3*c^3 - 9*a*b^2*c^2*d + 63*a^2*b*c*d^2))/(63*b^2*(a*d - b*c)^6
)))/(x^5*(a + b*x)^(1/2) + (a^4*c*(a + b*x)^(1/2))/(b^4*d) + (x^4*(4*a*d +
b*c)*(a + b*x)^(1/2))/(b*d) + (2*a*x^3*(3*a*d + 2*b*c)*(a + b*x)^(1/2))/(b^
2*d) + (a^3*x*(a*d + 4*b*c)*(a + b*x)^(1/2))/(b^4*d) + (2*a^2*x^2*(2*a*d +
3*b*c)*(a + b*x)^(1/2))/(b^3*d))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

## 3.1406 $\int \frac{(a+bx)^{9/2}}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=204

$$-\frac{105b^{3/2}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8d^{11/2}}+\frac{105b^2\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8d^5}-\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4d^4}+\frac{7b^2}{}$$

**Rubi [A]** time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3}-\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4d^4}+\frac{105b^2\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8d^5}-\frac{105b^{3/2}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8d^{11/2}}-\frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(9/2))/(3*d*(c + d*x)^(3/2)) - (6*b*(a + b*x)^(7/2))/(d^2*Sqr
t[c + d*x]) + (105*b^2*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^5) -
 (35*b^2*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(4*d^4) + (7*b^2*(a + b
*x)^(5/2)*Sqrt[c + d*x])/d^3 - (105*b^(3/2)*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*
Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(11/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} + \frac{(3b)\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{d^2}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3} - \frac{\left(35b^2(bc-ad)\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{2d^3}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4d^4} + \frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/}}{4d^4}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/}}{4d^4}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/}}{4d^4}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/}}{4d^4}$$

**Mathematica [C]** time = 0.10, size = 73, normalized size = 0.36

$$\frac{2(a+bx)^{11/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2} {}_2F_1\left(\frac{5}{2},\frac{11}{2};\frac{13}{2};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*((b*(c + d*x))/(b*c - a*d))^(5/2)*Hypergeometric2F1[5/2
, 11/2, 13/2, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.34, size = 194, normalized size = 0.95

$$-\frac{105b^{3/2}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{8d^{11/2}}-\frac{(a+bx)^{3/2}(ad-bc)^3\left(-\frac{315b^4(c+dx)^4}{(a+bx)^4}+\frac{840b^3d(c+dx)^3}{(a+bx)^3}-\frac{693b^2d^2(c+dx)^2}{(a+bx)^2}+\frac{144bd^3(c+dx)}{a+bx}+16d^4\right)}{24d^5(c+dx)^{3/2}\left(d-\frac{b(c+dx)}{a+bx}\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] -1/24*((-(b*c) + a*d)^3*(a + b*x)^(3/2)*(16*d^4 + (144*b*d^3*(c + d*x))/(a
+ b*x) - (693*b^2*d^2*(c + d*x)^2)/(a + b*x)^2 + (840*b^3*d*(c + d*x)^3)/(a
 + b*x)^3 - (315*b^4*(c + d*x)^4)/(a + b*x)^4))/(d^5*(c + d*x)^(3/2)*(d - (
b*(c + d*x))/(a + b*x))^3) - (105*b^(3/2)*(b*c - a*d)^3*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*d^(11/2))
```

**fricas [B]** time = 2.90, size = 879, normalized size = 4.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(315*(b^4*c^5 - 3*a*b^3*c^4*d + 3*a^2*b^2*c^3*d^2 - a^3*b*c^2*d^3 +
(b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^2 + 2*(b^4*
c^4*d - 3*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 - a^3*b*c*d^4)*x)*sqrt(b/d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d
^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8
*b^4*d^4*x^4 + 315*b^4*c^4 - 840*a*b^3*c^3*d + 693*a^2*b^2*c^2*d^2 - 144*a^
3*b*c*d^3 - 16*a^4*d^4 - 2*(9*b^4*c*d^3 - 25*a*b^3*d^4)*x^3 + 3*(21*b^4*c^2
*d^2 - 60*a*b^3*c*d^3 + 55*a^2*b^2*d^4)*x^2 + 2*(210*b^4*c^3*d - 567*a*b^3*
c^2*d^2 + 477*a^2*b^2*c*d^3 - 104*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)
)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5), 1/48*(315*(b^4*c^5 - 3*a*b^3*c^4*d + 3*a
^2*b^2*c^3*d^2 - a^3*b*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2
*c*d^4 - a^3*b*d^5)*x^2 + 2*(b^4*c^4*d - 3*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^
3 - a^3*b*c*d^4)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x +
a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*(8
*b^4*d^4*x^4 + 315*b^4*c^4 - 840*a*b^3*c^3*d + 693*a^2*b^2*c^2*d^2 - 144*a^
3*b*c*d^3 - 16*a^4*d^4 - 2*(9*b^4*c*d^3 - 25*a*b^3*d^4)*x^3 + 3*(21*b^4*c^2
*d^2 - 60*a*b^3*c*d^3 + 55*a^2*b^2*d^4)*x^2 + 2*(210*b^4*c^3*d - 567*a*b^3*
c^2*d^2 + 477*a^2*b^2*c*d^3 - 104*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)
)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)]
```

**giac [B]** time = 2.42, size = 500, normalized size = 2.45

$$\frac{\left(\left(\left(2\,(bx+a)\left(\frac{4\left(b^6cd^8-ab^5d^9\right)(bx+a)}{b^2cd^9|b|-abd^{10}|b|}-\frac{9\left(b^7c^2d^7-2\,ab^6cd^8+a^2b^5d^9\right)}{b^2cd^9|b|-abd^{10}|b|}\right)+\frac{63\left(b^8c^3d^6-3\,ab^7c^2d^7+3\,a^2b^6cd^8-a^3b^5d^9\right)}{b^2cd^9|b|-abd^{10}|b|}\right)(bx+a)+\frac{420\left(b^9c^4d^5-4\,ab^8c^3d^6+6\,a^2b^7c^2d^7-4\,a^3b^6cd^8+a^4b^5d^9\right)}{b^2cd^9|b|-abd^{10}|b|}\right)(bx+a)+\frac{315\left(b^{10}c^5d^4-5\,ab^9c^4d^5+10\,a^2b^8c^3d^6-10\,a^3b^7c^2d^7+5\,a^4b^6cd^8-a^5b^5d^9\right)}{b^2cd^9|b|-abd^{10}|b|}\right)\sqrt{bx+a}}{24\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}+\frac{105\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{8\sqrt{bd}\,d^5|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(((2*(b*x + a)*(4*(b^6*c*d^8 - a*b^5*d^9)*(b*x + a)/(b^2*c*d^9*abs(b)
- a*b*d^10*abs(b)) - 9*(b^7*c^2*d^7 - 2*a*b^6*c*d^8 + a^2*b^5*d^9)/(b^2*c*d
^9*abs(b) - a*b*d^10*abs(b))) + 63*(b^8*c^3*d^6 - 3*a*b^7*c^2*d^7 + 3*a^2*b
^6*c*d^8 - a^3*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*(b*x + a) + 4
20*(b^9*c^4*d^5 - 4*a*b^8*c^3*d^6 + 6*a^2*b^7*c^2*d^7 - 4*a^3*b^6*c*d^8 + a
^4*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*(b*x + a) + 315*(b^10*c^5
*d^4 - 5*a*b^9*c^4*d^5 + 10*a^2*b^8*c^3*d^6 - 10*a^3*b^7*c^2*d^7 + 5*a^4*b^
6*c*d^8 - a^5*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*sqrt(b*x + a)/
(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) + 105/8*(b^6*c^3 - 3*a*b^5*c^2*d + 3*
a^2*b^4*c*d^2 - a^3*b^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^5*abs(b))
```

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{9}{2}}}{(dx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{9/2}}{{\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(9/2)/(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

## 3.1407 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=170

$$\frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{9/2}}-\frac{35b^2\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4d^4}+\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{2}{3}$$

**Rubi [A]** time = 0.08, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}-\frac{35b^2\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4d^4}+\frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{9/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(7/2))/(3*d*(c + d*x)^(3/2)) - (14*b*(a + b*x)^(5/2))/(3*d^2*
Sqrt[c + d*x]) - (35*b^2*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^4) +
 (35*b^2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(6*d^3) + (35*b^(3/2)*(b*c - a*d)^2
*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*d^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} + \frac{(7b)\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx}{3d}$$
$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} + \frac{\left(35b^2\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{3d^2}$$
$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3} - \frac{\left(35b^2(bc-ad)\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d^3}$$
$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}$$
$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}$$

**Mathematica [C]** time = 0.08, size = 73, normalized size = 0.43

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2} {}_2F_1\left(\frac{5}{2},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(5/2)*Hypergeometric2F1[5/2,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.28, size = 172, normalized size = 1.01

$$\frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4d^{9/2}} - \frac{(a+bx)^{3/2}(ad-bc)^2\left(\frac{105b^3(c+dx)^3}{(a+bx)^3} - \frac{175b^2d(c+dx)^2}{(a+bx)^2} + \frac{56bd^2(c+dx)}{a+bx} + 8d^3\right)}{12d^4(c+dx)^{3/2}\left(d-\frac{b(c+dx)}{a+bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]
```

```
[Out] -1/12*((-(b*c) + a*d)^2*(a + b*x)^(3/2)*(8*d^3 + (56*b*d^2*(c + d*x))/(a +
b*x) - (175*b^2*d*(c + d*x)^2)/(a + b*x)^2 + (105*b^3*(c + d*x)^3)/(a + b*x
)^3))/(d^4*(c + d*x)^(3/2)*(d - (b*(c + d*x))/(a + b*x))^2) + (35*b^(3/2)*(
b*c - a*d)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*d
^(9/2))
```

**fricas [B]** time = 2.24, size = 657, normalized size = 3.86

$$\left[\frac{105\left(b^3c^4 - 2ab^2c^3d + a^2bc^2d^2 + (b^3c^2d^2 - 2ab^2cd^3 + a^2bd^4)x^2 + 2(b^3c^3d - 2ab^2c^2d^2 + a^2bcd^3)x\right)\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^2 + b^2c^2 + 6abcd + a^2d^2 + 4(2bd^2x + bcd + ad^2)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}} + 8(b^2cd + abd^2)x\right) + 4\left(6b^3d^3x^3 - 105b^3c^3 + 175ab^2c^2d - 56a^2bcd^2 - 8a^3d^3 - 3(7b^3cd^2 - 13ab^2d^3)x^2 - 2(70b^3c^2d - 119ab^2cd^2 + 40a^2bd^3)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48(d^6x^2 + 2cd^5x + c^2d^4)}, -\frac{105\left(b^3c^4 - 2ab^2c^3d + a^2bc^2d^2 + (b^3c^2d^2 - 2ab^2cd^3 + a^2bd^4)x^2 + 2(b^3c^3d - 2ab^2c^2d^2 + a^2bcd^3)x\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2bdx + bc + ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{b}{d}}}{2(b^2dx^2 + abc + (b^2c + abd)x)}\right) - 2\left(6b^3d^3x^3 - 105b^3c^3 + 175ab^2c^2d - 56a^2bcd^2 - 8a^3d^3 - 3(7b^3cd^2 - 13ab^2d^3)x^2 - 2(70b^3c^2d - 119ab^2cd^2 + 40a^2bd^3)x\right)\sqrt{bx+a}\sqrt{dx+c}}{24(d^6x^2 + 2cd^5x + c^2d^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 + (b^3*c^2*d^2 - 2*a*b^
2*c*d^3 + a^2*b*d^4)*x^2 + 2*(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3)*x)
*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x
 + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*
d^2)*x) + 4*(6*b^3*d^3*x^3 - 105*b^3*c^3 + 175*a*b^2*c^2*d - 56*a^2*b*c*d^2
 - 8*a^3*d^3 - 3*(7*b^3*c*d^2 - 13*a*b^2*d^3)*x^2 - 2*(70*b^3*c^2*d - 119*a
*b^2*c*d^2 + 40*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^6*x^2 + 2*c*d
^5*x + c^2*d^4), -1/24*(105*(b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 + (b^3
*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(b^3*c^3*d - 2*a*b^2*c^2*d^2
+ a^2*b*c*d^3)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) - 2*(6*b
^3*d^3*x^3 - 105*b^3*c^3 + 175*a*b^2*c^2*d - 56*a^2*b*c*d^2 - 8*a^3*d^3 - 3
*(7*b^3*c*d^2 - 13*a*b^2*d^3)*x^2 - 2*(70*b^3*c^2*d - 119*a*b^2*c*d^2 + 40*
a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)]
```

**giac [B]** time = 2.14, size = 380, normalized size = 2.24

$$\frac{\left(\left(3(bx+a)\left(\frac{2(b^6cd^6 - ab^5d^7)(bx+a)}{b^2cd^7|b| - abd^8|b|} - \frac{7(b^7c^2d^5 - 2ab^6cd^6 + a^2b^5d^7)}{b^2cd^7|b| - abd^8|b|}\right) - \frac{140(b^8c^3d^4 - 3ab^7c^2d^5 + 3a^2b^6cd^6 - a^3b^5d^7)}{b^2cd^7|b| - abd^8|b|}\right)(bx+a) - \frac{105(b^9c^4d^3 - 4ab^8c^3d^4 + 6a^2b^7c^2d^5 - 4a^3b^6cd^6 + a^4b^5d^7)}{b^2cd^7|b| - abd^8|b|}\right)\sqrt{bx+a}}{12\left(b^2c + (bx+a)bd - abd\right)^{\frac{3}{2}}} - \frac{35(b^5c^2 - 2ab^4cd + a^2b^3d^2)\log\left(\left|-\sqrt{bd}\sqrt{bx+a} + \sqrt{b^2c + (bx+a)bd - abd}\right|\right)}{4\sqrt{bd}d^4|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*((3*(b*x + a)*(2*(b^6*c*d^6 - a*b^5*d^7)*(b*x + a)/(b^2*c*d^7*abs(b) -
 a*b*d^8*abs(b)) - 7*(b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)/(b^2*c*d^7
*abs(b) - a*b*d^8*abs(b))) - 140*(b^8*c^3*d^4 - 3*a*b^7*c^2*d^5 + 3*a^2*b^6
*c*d^6 - a^3*b^5*d^7)/(b^2*c*d^7*abs(b) - a*b*d^8*abs(b)))*(b*x + a) - 105*
(b^9*c^4*d^3 - 4*a*b^8*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 4*a^3*b^6*c*d^6 + a^4*
```

```
b^5*d^7)/(b^2*c*d^7*abs(b) - a*b*d^8*abs(b)))*sqrt(b*x + a)/(b^2*c + (b*x +
 a)*b*d - a*b*d)^(3/2) - 35/4*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*log(abs
(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d
)*d^4*abs(b))
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{7/2}}{{\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

# 3.1408 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=128

$$-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{7/2}}+\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}$$

**Rubi** [A] time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{7/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(5/2))/(3*d*(c + d*x)^(3/2)) - (10*b*(a + b*x)^(3/2))/(3*d^2*
Sqrt[c + d*x]) + (5*b^2*Sqrt[a + b*x]*Sqrt[c + d*x])/d^3 - (5*b^(3/2)*(b*c
- a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}+\frac{(5b)\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{\left(5b^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{d^2}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3}-\frac{\left(5b^2(bc-ad)\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2d^3}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3}-\frac{(5b(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\right.}{d^3}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3}-\frac{(5b(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\right.}{d^3}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3}-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{7/2}}$$

**Mathematica** [C] time = 0.08, size = 73, normalized size = 0.57

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/2)*Hypergeometric2F1[5/2,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.98, size = 166, normalized size = 1.30

$$\frac{\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}\left(-2a^2d^2-14abd(c+dx)+4abcd-2b^2c^2+3b^2(c+dx)^2+14b^2c(c+dx)\right)}{3d^3(c+dx)^{3/2}}+\frac{5\sqrt{\frac{b}{d}}\left(b^2c-abd\right)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}-\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```

```
[Out] (Sqrt[a - (b*c)/d + (b*(c + d*x))/d]*(-2*b^2*c^2 + 4*a*b*c*d - 2*a^2*d^2 +
14*b^2*c*(c + d*x) - 14*a*b*d*(c + d*x) + 3*b^2*(c + d*x)^2))/(3*d^3*(c + d
*x)^(3/2)) + (5*Sqrt[b/d]*(b^2*c - a*b*d)*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) +
Sqrt[a - (b*c)/d + (b*(c + d*x))/d]])/d^3
```

**fricas** [B] time = 1.62, size = 475, normalized size = 3.71

$$\left[-\frac{15\left(b^2c^3-abc^2d+\left(b^2cd^2-abd^3\right)x^2+2\left(b^2c^2d-abcd^2\right)x\right)\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2bd^2x+bcd+ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-4\left(3b^2d^2x^2+15b^2c^2-10abcd-2a^2d^2+2\left(10b^2cd-7abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{12\left(d^5x^2+2cd^4x+c^2d^3\right)},\ \frac{15\left(b^2c^3-abc^2d+\left(b^2cd^2-abd^3\right)x^2+2\left(b^2c^2d-abcd^2\right)x\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)+2\left(3b^2d^2x^2+15b^2c^2-10abcd-2a^2d^2+2\left(10b^2cd-7abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{6\left(d^5x^2+2cd^4x+c^2d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(b^2*c^3 - a*b*c^2*d + (b^2*c*d^2 - a*b*d^3)*x^2 + 2*(b^2*c^2*d
- a*b*c*d^2)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*(3*b^2*d^2*x^2 + 15*b^2*c^2 - 10*a*b*c*d - 2*a^2
*d^2 + 2*(10*b^2*c*d - 7*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x^2
+ 2*c*d^4*x + c^2*d^3), 1/6*(15*(b^2*c^3 - a*b*c^2*d + (b^2*c*d^2 - a*b*d^3
)*x^2 + 2*(b^2*c^2*d - a*b*c*d^2)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c +
 a*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c +
a*b*d)*x)) + 2*(3*b^2*d^2*x^2 + 15*b^2*c^2 - 10*a*b*c*d - 2*a^2*d^2 + 2*(10
*b^2*c*d - 7*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x^2 + 2*c*d^4*x
+ c^2*d^3)]
```

**giac [B]** time = 1.97, size = 276, normalized size = 2.16

$$\frac{\left((bx+a)\left(\frac{3\left(b^6cd^4-ab^5d^5\right)(bx+a)}{b^2cd^5|b|-abd^6|b|}+\frac{20\left(b^7c^2d^3-2\,ab^6cd^4+a^2b^5d^5\right)}{b^2cd^5|b|-abd^6|b|}\right)+\frac{15\left(b^8c^3d^2-3\,ab^7c^2d^3+3\,a^2b^6cd^4-a^3b^5d^5\right)}{b^2cd^5|b|-abd^6|b|}\right)\sqrt{bx+a}}{3\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}+\frac{5\left(b^4c-ab^3d\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x + a)*(3*(b^6*c*d^4 - a*b^5*d^5)*(b*x + a)/(b^2*c*d^5*abs(b) - a*b
*d^6*abs(b)) + 20*(b^7*c^2*d^3 - 2*a*b^6*c*d^4 + a^2*b^5*d^5)/(b^2*c*d^5*ab
s(b) - a*b*d^6*abs(b))) + 15*(b^8*c^3*d^2 - 3*a*b^7*c^2*d^3 + 3*a^2*b^6*c*d
^4 - a^3*b^5*d^5)/(b^2*c*d^5*abs(b) - a*b*d^6*abs(b)))*sqrt(b*x + a)/(b^2*c
 + (b*x + a)*b*d - a*b*d)^(3/2) + 5*(b^4*c - a*b^3*d)*log(abs(-sqrt(b*d)*sq
rt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^3*abs(b))
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{5/2}}{{\left(c+d\,x\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

## 3.1409 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=92

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}$$

**Rubi [A]** time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {47, 63, 217, 206}

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*d*(c + d*x)^(3/2)) - (2*b*Sqrt[a + b*x])/(d^2*Sqrt[
c + d*x]) + (2*b^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/d^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}+\frac{b\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx}{d}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{b^2\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.55, size = 111, normalized size = 1.21

$$\frac{6(bc-ad)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)-2\sqrt{d}\,\sqrt{a+bx}\,(ad+3bc+4bdx)}{3d^{5/2}(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[a + b*x]*(3*b*c + a*d + 4*b*d*x) + 6*(b*c - a*d)^(3/2)*((b
*(c + d*x))/(b*c - a*d))^(3/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a
*d]])/(3*d^(5/2)*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.14, size = 85, normalized size = 0.92

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{d^{5/2}}-\frac{2(a+bx)^{3/2}\left(\frac{3b(c+dx)}{a+bx}+d\right)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(d + (3*b*(c + d*x))/(a + b*x)))/(3*d^2*(c + d*x)^(3/2)
) + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/d^
(5/2)
```

**fricas [B]** time = 1.49, size = 325, normalized size = 3.53

$$\left[\frac{3\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,bd^2x+bcd+ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-4\left(4\,bdx+3\,bc+ad\right)\sqrt{bx+a}\sqrt{dx+c}}{6\left(d^4x^2+2\,cd^3x+c^2d^2\right)},-\frac{3\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{\left(2\,bdx+bc+ad\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)+2\left(4\,bdx+3\,bc+ad\right)\sqrt{bx+a}\sqrt{dx+c}}{3\left(d^4x^2+2\,cd^3x+c^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c
^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt
(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(4*b*d*x + 3*b*c + a*d)*
sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2), -1/3*(3*(b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt
(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x))
 + 2*(4*b*d*x + 3*b*c + a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^
3*x + c^2*d^2)]
```

**giac [B]** time = 1.43, size = 181, normalized size = 1.97

$$-\frac{2\,b^3\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^2|b|}-\frac{2\sqrt{bx+a}\left(\frac{4\left(b^5cd^2-ab^4d^3\right)(bx+a)}{bcd^3|b|-ad^4|b|}+\frac{3\left(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3\right)}{bcd^3|b|-ad^4|b|}\right)}{3\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2*b^3*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*
d)))/(sqrt(b*d)*d^2*abs(b)) - 2/3*sqrt(b*x + a)*(4*(b^5*c*d^2 - a*b^4*d^3)*
(b*x + a)/(b*c*d^3*abs(b) - a*d^4*abs(b)) + 3*(b^6*c^2*d - 2*a*b^5*c*d^2 +
a^2*b^4*d^3)/(b*c*d^3*abs(b) - a*d^4*abs(b)))/(b^2*c + (b*x + a)*b*d - a*b*
d)^(3/2)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(5/2),x)
[Out] int((b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(5/2),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(5/2),x)
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(5/2), x)
```

## 3.1410 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=32

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

**Rubi** [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}}\,dx = \frac{2(a+bx)^{3/2}}{3(bc-ad)(c+dx)^{3/2}}$$

**Mathematica** [A] time = 0.01, size = 32, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.04, size = 32, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

**fricas [B]** time = 1.15, size = 65, normalized size = 2.03

$$\frac{2\,(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{3\,\left(bc^3-ac^2d+\left(bcd^2-ad^3\right)x^2+2\,\left(bc^2d-acd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)*sqrt(d*x + c)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*x^2
+ 2*(b*c^2*d - a*c*d^2)*x)
```

**giac [A]** time = 1.15, size = 51, normalized size = 1.59

$$\frac{2\,(bx+a)^{\frac{3}{2}}b^4d}{3\,\left(bcd|b|-ad^2|b|\right)\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x + a)^(3/2)*b^4*d/((b*c*d*abs(b) - a*d^2*abs(b))*(b^2*c + (b*x + a)
*b*d - a*b*d)^(3/2))
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.84

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(dx+c)^{\frac{3}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(b*x+a)^(3/2)/(d*x+c)^(3/2)/(a*d-b*c)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.56, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{2\,a\,\sqrt{a+b\,x}}{3\,a\,d^3-3\,b\,c\,d^2}+\frac{2\,b\,x\,\sqrt{a+b\,x}}{3\,a\,d^3-3\,b\,c\,d^2}\right)\,\sqrt{c+d\,x}}{x^2-\frac{3\,b\,c^3-3\,a\,c^2\,d}{3\,a\,d^3-3\,b\,c\,d^2}+\frac{6\,c\,d\,x\,\left(a\,d-b\,c\right)}{3\,a\,d^3-3\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(5/2),x)
```

```
[Out] -((((2*a*(a + b*x)^(1/2))/(3*a*d^3 - 3*b*c*d^2) + (2*b*x*(a + b*x)^(1/2))/(3
*a*d^3 - 3*b*c*d^2))*(c + d*x)^(1/2))/(x^2 - (3*b*c^3 - 3*a*c^2*d)/(3*a*d^3
 - 3*b*c*d^2) + (6*c*d*x*(a*d - b*c))/(3*a*d^3 - 3*b*c*d^2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx}}{\left(c + dx\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(5/2), x)
```

## 3.1411 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=66

$$\frac{4b\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^2}+\frac{2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4b\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^2}+\frac{2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (4*b*Sqrt[a + b*x])/(3*
(b*c - a*d)^2*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx = \frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}} + \frac{(2b)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx}{3(bc-ad)}$$
$$= \frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}} + \frac{4b\sqrt{a+bx}}{3(bc-ad)^2\sqrt{c+dx}}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 0.70

$$\frac{2\sqrt{a+bx}\,(-ad+3bc+2bdx)}{3(c+dx)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[a + b*x]*(3*b*c - a*d + 2*b*d*x))/(3*(b*c - a*d)^2*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 57, normalized size = 0.86

$$\frac{2\left(\frac{3b\sqrt{a+bx}}{\sqrt{c+dx}} - \frac{d(a+bx)^{3/2}}{(c+dx)^{3/2}}\right)}{3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

```
[Out] (2*(-((d*(a + b*x)^(3/2))/(c + d*x)^(3/2)) + (3*b*Sqrt[a + b*x])/Sqrt[c + d
*x]))/(3*(b*c - a*d)^2)
```

**fricas [B]** time = 1.32, size = 118, normalized size = 1.79

$$\frac{2\,(2\,bdx+3\,bc-ad)\sqrt{bx+a}\,\sqrt{dx+c}}{3\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*d*x + 3*b*c - a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*c^4 - 2*a*b*c^
3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*
d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

**giac [B]** time = 1.02, size = 126, normalized size = 1.91

$$\frac{2\left(\frac{2\,(bx+a)b^4d^2}{b^2c^2d|b|-2\,abcd^2|b|+a^2d^3|b|}+\frac{3\left(b^5cd-ab^4d^2\right)}{b^2c^2d|b|-2\,abcd^2|b|+a^2d^3|b|}\right)\sqrt{bx+a}}{3\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*(b*x + a)*b^4*d^2/(b^2*c^2*d*abs(b) - 2*a*b*c*d^2*abs(b) + a^2*d^3*a
bs(b)) + 3*(b^5*c*d - a*b^4*d^2)/(b^2*c^2*d*abs(b) - 2*a*b*c*d^2*abs(b) + a
^2*d^3*abs(b)))*sqrt(b*x + a)/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2)
```

**maple [A]** time = 0.00, size = 53, normalized size = 0.80

$$-\frac{2\sqrt{bx+a}\,\left(-2bdx+ad-3bc\right)}{3\,(dx+c)^{\frac{3}{2}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(-2*b*d*x+a*d-3*b*c)/(d*x+c)^(3/2)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.90, size = 127, normalized size = 1.92

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x\,\left(6\,c\,b^2+2\,a\,d\,b\right)}{3\,d^2\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,a^2\,d-6\,a\,b\,c}{3\,d^2\,{\left(a\,d-b\,c\right)}^2}+\frac{4\,b^2\,x^2}{3\,d\,{\left(a\,d-b\,c\right)}^2}\right)}{x^2\,\sqrt{a+b\,x}+\frac{c^2\,\sqrt{a+b\,x}}{d^2}+\frac{2\,c\,x\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((x*(6*b^2*c + 2*a*b*d))/(3*d^2*(a*d - b*c)^2) - (2*a^2*d
- 6*a*b*c)/(3*d^2*(a*d - b*c)^2) + (4*b^2*x^2)/(3*d*(a*d - b*c)^2)))/(x^2*(
a + b*x)^(1/2) + (c^2*(a + b*x)^(1/2))/d^2 + (2*c*x*(a + b*x)^(1/2))/d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/2)), x)
```

## 3.1412 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=98

$$-\frac{16bd\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}-\frac{8d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16bd\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}-\frac{8d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/2)) - (8*d*Sqrt[a + b*x])/(3*(b*
c - a*d)^2*(c + d*x)^(3/2)) - (16*b*d*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}} - \frac{(4d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx}{bc-ad}$$
$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}} - \frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{(8bd)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx}{3(bc-ad)^2}$$
$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}} - \frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{16bd\sqrt{a+bx}}{3(bc-ad)^3\sqrt{c+dx}}$$

**Mathematica** [A] time = 0.03, size = 78, normalized size = 0.80

$$\frac{2a^2d^2-4abd(3c+2dx)-2b^2\left(3c^2+12cdx+8d^2x^2\right)}{3\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (2*a^2*d^2 - 4*a*b*d*(3*c + 2*d*x) - 2*b^2*(3*c^2 + 12*c*d*x + 8*d^2*x^2))/
(3*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.12, size = 73, normalized size = 0.74

$$-\frac{2(a+bx)^{3/2}\left(\frac{3b^2(c+dx)^2}{(a+bx)^2}+\frac{6bd(c+dx)}{a+bx}-d^2\right)}{3(c+dx)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(-d^2 + (6*b*d*(c + d*x))/(a + b*x) + (3*b^2*(c + d*x)^
2)/(a + b*x)^2))/(3*(b*c - a*d)^3*(c + d*x)^(3/2))
```

**fricas** [B] time = 2.08, size = 273, normalized size = 2.79

$$-\frac{2\left(8\,b^2d^2x^2+3\,b^2c^2+6\,abcd-a^2d^2+4\left(3\,b^2cd+abd^2\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{3\left(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5\right)x^3+\left(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^2+\left(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*d^2*x^2 + 3*b^2*c^2 + 6*a*b*c*d - a^2*d^2 + 4*(3*b^2*c*d + a*b*
d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*
c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 -
a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b
*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*
b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

**giac [B]** time = 1.48, size = 373, normalized size = 3.81

$$-\frac{4\sqrt{bd}\,b^3}{\left(b^2c^2|b| - 2\,abcd|b| + a^2d^2|b|\right)\left(b^2c - abd - \left(\sqrt{bd}\,\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2\right)} - \frac{2\sqrt{bx+a}\left(\frac{5\left(b^6c^2d^3|b| - 2\,ab^5cd^4|b| + a^2b^4d^5|b|\right)(bx+a)}{b^7c^5d - 5\,ab^6c^4d^2 + 10\,a^2b^5c^3d^3 - 10\,a^3b^4c^2d^4 + 5\,a^4b^3cd^5 - a^5b^2d^6} + \frac{6\left(b^7c^3d^2|b| - 3\,ab^6c^2d^3|b| + 3\,a^2b^5cd^4|b| - a^3b^4d^5|b|\right)}{b^7c^5d - 5\,ab^6c^4d^2 + 10\,a^2b^5c^3d^3 - 10\,a^3b^4c^2d^4 + 5\,a^4b^3cd^5 - a^5b^2d^6}\right)}{3\left(b^2c + (bx+a)bd - abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(b*d)*b^3/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b) + a^2*d^2*abs(b))*(b^2
*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2)) - 2/3*sqrt(b*x + a)*(5*(b^6*c^2*d^3*abs(b) - 2*a*b^5*c*d^4*abs(b) + a
^2*b^4*d^5*abs(b))*(b*x + a)/(b^7*c^5*d - 5*a*b^6*c^4*d^2 + 10*a^2*b^5*c^3*
d^3 - 10*a^3*b^4*c^2*d^4 + 5*a^4*b^3*c*d^5 - a^5*b^2*d^6) + 6*(b^7*c^3*d^2*
abs(b) - 3*a*b^6*c^2*d^3*abs(b) + 3*a^2*b^5*c*d^4*abs(b) - a^3*b^4*d^5*abs(
b))/(b^7*c^5*d - 5*a*b^6*c^4*d^2 + 10*a^2*b^5*c^3*d^3 - 10*a^3*b^4*c^2*d^4
+ 5*a^4*b^3*c*d^5 - a^5*b^2*d^6))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2)
```

**maple [A]** time = 0.01, size = 104, normalized size = 1.06

$$-\frac{2\left(-8b^2x^2d^2 - 4ab\,d^2x - 12b^2cdx + a^2d^2 - 6abcd - 3b^2c^2\right)}{3\sqrt{bx+a}\;(dx+c)^{\frac{3}{2}}\left(a^3d^3 - 3a^2bc\,d^2 + 3a\,b^2c^2d - b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(-8*b^2*d^2*x^2-4*a*b*d^2*x-12*b^2*c*d*x+a^2*d^2-6*a*b*c*d-3*b^2*c^2)/
(b*x+a)^(1/2)/(d*x+c)^(3/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.03, size = 132, normalized size = 1.35

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,b^2\,x^2}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{-2\,a^2\,d^2+12\,a\,b\,c\,d+6\,b^2\,c^2}{3\,d^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,b\,x\,\left(a\,d+3\,b\,c\right)}{3\,d\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{c^2\,\sqrt{a+b\,x}}{d^2}+\frac{2\,c\,x\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((16*b^2*x^2)/(3*(a*d - b*c)^3) + (6*b^2*c^2 - 2*a^2*d^2 +
 12*a*b*c*d)/(3*d^2*(a*d - b*c)^3) + (8*b*x*(a*d + 3*b*c))/(3*d*(a*d - b*c)
^3)))/(x^2*(a + b*x)^(1/2) + (c^2*(a + b*x)^(1/2))/d^2 + (2*c*x*(a + b*x)^(
1/2))/d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(5/2)), x)
```

## 3.1413 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=135

$$\frac{32bd^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^4}+\frac{16d^2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32bd^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^4}+\frac{16d^2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(3/2)) + (4*d)/((b*c - a*d)^2*S
qrt[a + b*x]*(c + d*x)^(3/2)) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*(c
+ d*x)^(3/2)) + (32*b*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{(2d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx}{bc-ad}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}} + \frac{\left(8d^2\right)\int \frac{\cdots}{\sqrt{a+bx}\,(\cdots}}{(bc-a\cdots}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}} + \frac{16d^2\sqrt{a+\cdots}}{3(bc-ad)^3(c\cdots}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}} + \frac{16d^2\sqrt{a+\cdots}}{3(bc-ad)^3(c\cdots}$$

**Mathematica [A]** time = 0.05, size = 118, normalized size = 0.87

$$\frac{-2a^3d^3+6a^2bd^2(3c+2dx)+6ab^2d\left(3c^2+12cdx+8d^2x^2\right)+b^3\left(-2c^3+12c^2dx+48cd^2x^2+32d^3x^3\right)}{3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*a^3*d^3 + 6*a^2*b*d^2*(3*c + 2*d*x) + 6*a*b^2*d*(3*c^2 + 12*c*d*x + 8*d
^2*x^2) + b^3*(-2*c^3 + 12*c^2*d*x + 48*c*d^2*x^2 + 32*d^3*x^3))/(3*(b*c -
a*d)^4*(a + b*x)^(3/2)*(c + d*x)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.14, size = 92, normalized size = 0.68

$$-\frac{2(a+bx)^{3/2}\left(\frac{b^3(c+dx)^3}{(a+bx)^3}-\frac{9b^2d(c+dx)^2}{(a+bx)^2}-\frac{9bd^2(c+dx)}{a+bx}+d^3\right)}{3(c+dx)^{3/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(d^3 - (9*b*d^2*(c + d*x))/(a + b*x) - (9*b^2*d*(c + d*
x)^2)/(a + b*x)^2 + (b^3*(c + d*x)^3)/(a + b*x)^3))/(3*(b*c - a*d)^4*(c + d
*x)^(3/2))
```

**fricas [B]** time = 2.42, size = 447, normalized size = 3.31

$$\frac{2\left(16b^3d^3x^3-b^3c^3+9ab^2c^2d+9a^2bcd^2-a^3d^3+24\left(b^3cd^2+ab^2d^3\right)x^2+6\left(b^3c^2d+6ab^2cd^2+a^2bd^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{3\left(a^2b^4c^6-4a^3b^3c^5d+6a^4b^2c^4d^2-4a^5bc^3d^3+a^6c^2d^4+\left(b^6c^4d^2-4ab^5c^3d^3+6a^2b^4c^2d^4-4a^3b^3cd^5+a^4b^2d^6\right)x^4+2\left(b^6c^5d-3ab^5c^4d^2+2a^2b^4c^3d^3+2a^3b^3c^2d^4-3a^4b^2cd^5+a^5bd^6\right)x^3+\left(b^6c^6-9a^2b^4c^4d^2+16a^3b^3c^3d^3-9a^4b^2c^2d^4+a^6d^6\right)x^2+2\left(ab^5c^6-3a^2b^4c^5d+2a^3b^3c^4d^2+2a^4b^2c^3d^3-3a^5bc^2d^4+a^6cd^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*d^3*x^3 - b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 - a^3*d^3 + 2
4*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(b^3*c^2*d + 6*a*b^2*c*d^2 + a^2*b*d^3)*x
)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^
4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*
a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b
^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*
b*d^6)*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*
c^2*d^4 + a^6*d^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2
 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

**giac [B]** time = 2.06, size = 670, normalized size = 4.96

$$\frac{2\sqrt{bx+a}\left(\frac{8\left(b^7c^3d^4|b|-3\,ab^6c^2d^5|b|+3\,a^2b^5cd^6|b|-a^3b^4d^7|b|\right)(bx+a)}{b^9c^7d-7\,ab^8c^6d^2+21\,a^2b^7c^5d^3-35\,a^3b^6c^4d^4+35\,a^4b^5c^3d^5-21\,a^5b^4c^2d^6+7\,a^6b^3cd^7-a^7b^2d^8}+\frac{9\left(b^8c^4d^3|b|-4\,ab^7c^3d^4|b|+6\,a^2b^6c^2d^5|b|-4\,a^3b^5cd^6|b|+a^4b^4d^7|b|\right)}{b^9c^7d-7\,ab^8c^6d^2+21\,a^2b^7c^5d^3-35\,a^3b^6c^4d^4+35\,a^4b^5c^3d^5-21\,a^5b^4c^2d^6+7\,a^6b^3cd^7-a^7b^2d^8}\right)}{3\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}+\frac{8\left(4\sqrt{bd}\,b^7c^2d-8\sqrt{bd}\,ab^6cd^2+4\sqrt{bd}\,a^2b^5d^3-9\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^5cd+9\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^4d^2+3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^3d\right)}{3\left(b^3c^3|b|-3\,ab^2c^2d|b|+3\,a^2bcd^2|b|-a^3d^3|b|\right)\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x + a)*(8*(b^7*c^3*d^4*abs(b) - 3*a*b^6*c^2*d^5*abs(b) + 3*a^2*b
^5*c*d^6*abs(b) - a^3*b^4*d^7*abs(b))*(b*x + a)/(b^9*c^7*d - 7*a*b^8*c^6*d^
2 + 21*a^2*b^7*c^5*d^3 - 35*a^3*b^6*c^4*d^4 + 35*a^4*b^5*c^3*d^5 - 21*a^5*b
^4*c^2*d^6 + 7*a^6*b^3*c*d^7 - a^7*b^2*d^8) + 9*(b^8*c^4*d^3*abs(b) - 4*a*b
^7*c^3*d^4*abs(b) + 6*a^2*b^6*c^2*d^5*abs(b) - 4*a^3*b^5*c*d^6*abs(b) + a^4
*b^4*d^7*abs(b))/(b^9*c^7*d - 7*a*b^8*c^6*d^2 + 21*a^2*b^7*c^5*d^3 - 35*a^3
*b^6*c^4*d^4 + 35*a^4*b^5*c^3*d^5 - 21*a^5*b^4*c^2*d^6 + 7*a^6*b^3*c*d^7 -
a^7*b^2*d^8))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) + 8/3*(4*sqrt(b*d)*b^7*
c^2*d - 8*sqrt(b*d)*a*b^6*c*d^2 + 4*sqrt(b*d)*a^2*b^5*d^3 - 9*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^5*c*d + 9*
sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2
*a*b^4*d^2 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*b^3*d)/((b^3*c^3*abs(b) - 3*a*b^2*c^2*d*abs(b) + 3*a^2*b*c*
d^2*abs(b) - a^3*d^3*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3)
```

**maple [A]** time = 0.01, size = 169, normalized size = 1.25

$$-\frac{2\left(-16b^3x^3d^3-24a\,b^2d^3x^2-24b^3c\,d^2x^2-6a^2b\,d^3x-36a\,b^2c\,d^2x-6b^3c^2dx+a^3d^3-9a^2bc\,d^2-9a\,b^2c^2d+b^3c^3\right)}{3\left(bx+a\right)^{\frac{3}{2}}\left(dx+c\right)^{\frac{3}{2}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(-16*b^3*d^3*x^3-24*a*b^2*d^3*x^2-24*b^3*c*d^2*x^2-6*a^2*b*d^3*x-36*a*
b^2*c*d^2*x-6*b^3*c^2*d*x+a^3*d^3-9*a^2*b*c*d^2-9*a*b^2*c^2*d+b^3*c^3)/(b*x
+a)^(3/2)/(d*x+c)^(3/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^
3*d+b^4*c^4)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.29, size = 224, normalized size = 1.66

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,b\,x^2\,\left(a\,d+b\,c\right)}{{\left(a\,d-b\,c\right)}^4}-\frac{2\,a^3\,d^3-18\,a^2\,b\,c\,d^2-18\,a\,b^2\,c^2\,d+2\,b^3\,c^3}{3\,b\,d^2\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,b^2\,d\,x^3}{3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,x\,\left(a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{d\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a\,c^2\,\sqrt{a+b\,x}}{b\,d^2}+\frac{x^2\,\left(a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{c\,x\,\left(2\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((16*b*x^2*(a*d + b*c))/(a*d - b*c)^4 - (2*a^3*d^3 + 2*b^3
*c^3 - 18*a*b^2*c^2*d - 18*a^2*b*c*d^2)/(3*b*d^2*(a*d - b*c)^4) + (32*b^2*d
*x^3)/(3*(a*d - b*c)^4) + (4*x*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/(d*(a*d - b
*c)^4)))/(x^3*(a + b*x)^(1/2) + (a*c^2*(a + b*x)^(1/2))/(b*d^2) + (x^2*(a*d
 + 2*b*c)*(a + b*x)^(1/2))/(b*d) + (c*x*(2*a*d + b*c)*(a + b*x)^(1/2))/(b*d
^2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx\right)^{\frac{5}{2}} \left(c + dx\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(5/2)), x)
```

# 3.1414 $\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=172

$$-\frac{256bd^3\sqrt{a+bx}}{15\sqrt{c+dx}\,(bc-ad)^5}-\frac{128d^3\sqrt{a+bx}}{15(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{5\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^3}+\frac{16d}{15(a+bx)^{3/2}(c+dx)^{3/2}(bc-a}$$

**Rubi** [A] time = 0.04, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{256bd^3\sqrt{a+bx}}{15\sqrt{c+dx}\,(bc-ad)^5}-\frac{128d^3\sqrt{a+bx}}{15(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{5\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^3}+\frac{16d}{15(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2)) + (16*d)/(15*(b*c - a*d)
^2*(a + b*x)^(3/2)*(c + d*x)^(3/2)) - (32*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*
x]*(c + d*x)^(3/2)) - (128*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^4*(c + d*x)^(
3/2)) - (256*b*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx = -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{(8d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx}{5(bc-ad)}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{\left(16d^2\right)\int \frac{}{(}}{5(}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{}{5(bc-ad)}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{}{5(bc-ad)}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{}{5(bc-ad)}$$

**Mathematica** [A] time = 0.07, size = 170, normalized size = 0.99

$$-\frac{2\left(-5a^4d^4+20a^3bd^3(3c+2dx)+30a^2b^2d^2\left(3c^2+12cdx+8d^2x^2\right)+20ab^3d\left(-c^3+6c^2dx+24cd^2x^2+16d^3x^3\right)+b^4\left(3c^4-8c^3dx+48c^2d^2x^2+192cd^3x^3+128d^4x^4\right)\right)}{15(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(-5*a^4*d^4 + 20*a^3*b*d^3*(3*c + 2*d*x) + 30*a^2*b^2*d^2*(3*c^2 + 12*c
*d*x + 8*d^2*x^2) + 20*a*b^3*d*(-c^3 + 6*c^2*d*x + 24*c*d^2*x^2 + 16*d^3*x^
3) + b^4*(3*c^4 - 8*c^3*d*x + 48*c^2*d^2*x^2 + 192*c*d^3*x^3 + 128*d^4*x^4)
))/(15*(b*c - a*d)^5*(a + b*x)^(5/2)*(c + d*x)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.15, size = 117, normalized size = 0.68

$$-\frac{2(a+bx)^{3/2}\left(\frac{3b^4(c+dx)^4}{(a+bx)^4}-\frac{20b^3d(c+dx)^3}{(a+bx)^3}+\frac{90b^2d^2(c+dx)^2}{(a+bx)^2}+\frac{60bd^3(c+dx)}{a+bx}-5d^4\right)}{15(c+dx)^{3/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(-5*d^4 + (60*b*d^3*(c + d*x))/(a + b*x) + (90*b^2*d^2*
(c + d*x)^2)/(a + b*x)^2 - (20*b^3*d*(c + d*x)^3)/(a + b*x)^3 + (3*b^4*(c +
 d*x)^4)/(a + b*x)^4))/(15*(b*c - a*d)^5*(c + d*x)^(3/2))
```

**fricas [B]** time = 8.26, size = 715, normalized size = 4.16

$$-\frac{2\left(128 b^4 d^4 x^4 + 3 b^4 c^4 - 20 a b^3 c^3 d + 90 a^2 b^2 c^2 d^2 + 60 a^3 b c d^3 - 5 a^4 d^4 + 64\left(3 b^4 c d^3 + 5 a b^3 d^4\right)x^3 + 48\left(b^4 c^2 d^2 + 10 a b^3 c d^3 + 5 a^2 b^2 d^4\right)x^2 - 8\left(b^4 c^3 d - 15 a b^3 c^2 d^2 - 45 a^2 b^2 c d^3 - 5 a^3 b d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{15\left(a^3 b^5 c^7 - 5 a^4 b^4 c^6 d + 10 a^5 b^3 c^5 d^2 - 10 a^6 b^2 c^4 d^3 + 5 a^7 b c^3 d^4 - a^8 c^2 d^5 + \left(b^8 c^5 d^2 - 5 a b^7 c^4 d^3 + 10 a^2 b^6 c^3 d^4 - 10 a^3 b^5 c^2 d^5 + 5 a^4 b^4 c d^6 - a^5 b^3 d^7\right)x^5 + \left(2 b^8 c^6 d - 7 a b^7 c^5 d^2 + 5 a^2 b^6 c^4 d^3 + 10 a^3 b^5 c^3 d^4 - 20 a^4 b^4 c^2 d^5 + 13 a^5 b^3 c d^6 - 3 a^6 b^2 d^7\right)x^4 + \left(b^8 c^7 + a b^7 c^6 d - 17 a^2 b^6 c^5 d^2 + 35 a^3 b^5 c^4 d^3 - 25 a^4 b^4 c^3 d^4 - a^5 b^3 c^2 d^5 + 9 a^6 b^2 c d^6 - 3 a^7 b d^7\right)x^3 + \left(3 a b^7 c^7 - 9 a^2 b^6 c^6 d + a^3 b^5 c^5 d^2 + 25 a^4 b^4 c^4 d^3 - 35 a^5 b^3 c^3 d^4 + 17 a^6 b^2 c^2 d^5 - a^7 b c d^6 - a^8 d^7\right)x^2 + \left(3 a^2 b^6 c^7 - 13 a^3 b^5 c^6 d + 20 a^4 b^4 c^5 d^2 - 10 a^5 b^3 c^4 d^3 - 5 a^6 b^2 c^3 d^4 + 7 a^7 b c^2 d^5 - 2 a^8 c d^6\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(128*b^4*d^4*x^4 + 3*b^4*c^4 - 20*a*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 +
60*a^3*b*c*d^3 - 5*a^4*d^4 + 64*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 48*(b^4*c
^2*d^2 + 10*a*b^3*c*d^3 + 5*a^2*b^2*d^4)*x^2 - 8*(b^4*c^3*d - 15*a*b^3*c^2*
d^2 - 45*a^2*b^2*c*d^3 - 5*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b
^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*
b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d
^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*x^5 + (2*b^8*c^6*d
 - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^
2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17
*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^
5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a
^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d
^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^
4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 -
2*a^8*c*d^6)*x)
```

**giac [B]** time = 3.92, size = 1203, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*x + a)*(11*(b^8*c^4*d^5*abs(b) - 4*a*b^7*c^3*d^6*abs(b) + 6*a^2
*b^6*c^2*d^7*abs(b) - 4*a^3*b^5*c*d^8*abs(b) + a^4*b^4*d^9*abs(b))*(b*x + a
)/(b^11*c^9*d - 9*a*b^10*c^8*d^2 + 36*a^2*b^9*c^7*d^3 - 84*a^3*b^8*c^6*d^4
+ 126*a^4*b^7*c^5*d^5 - 126*a^5*b^6*c^4*d^6 + 84*a^6*b^5*c^3*d^7 - 36*a^7*b
^4*c^2*d^8 + 9*a^8*b^3*c*d^9 - a^9*b^2*d^10) + 12*(b^9*c^5*d^4*abs(b) - 5*a
*b^8*c^4*d^5*abs(b) + 10*a^2*b^7*c^3*d^6*abs(b) - 10*a^3*b^6*c^2*d^7*abs(b)
 + 5*a^4*b^5*c*d^8*abs(b) - a^5*b^4*d^9*abs(b))/(b^11*c^9*d - 9*a*b^10*c^8*
d^2 + 36*a^2*b^9*c^7*d^3 - 84*a^3*b^8*c^6*d^4 + 126*a^4*b^7*c^5*d^5 - 126*a
^5*b^6*c^4*d^6 + 84*a^6*b^5*c^3*d^7 - 36*a^7*b^4*c^2*d^8 + 9*a^8*b^3*c*d^9
- a^9*b^2*d^10))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) - 4/15*(73*sqrt(b*d)
*b^11*c^4*d^2 - 292*sqrt(b*d)*a*b^10*c^3*d^3 + 438*sqrt(b*d)*a^2*b^9*c^2*d^
4 - 292*sqrt(b*d)*a^3*b^8*c*d^5 + 73*sqrt(b*d)*a^4*b^7*d^6 - 320*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^9*c^3*d
^2 + 960*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a*b^8*c^2*d^3 - 960*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
```

```
*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^7*c*d^4 + 320*sqrt(b*d)*(sqrt(b*d)*sqr
t(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^6*d^5 + 490*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^7*
c^2*d^2 - 980*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^4*a*b^6*c*d^3 + 490*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^5*d^4 - 240*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^5*c*d^2 + 240*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^4
*d^3 + 45*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^8*b^3*d^2)/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b) + 6*a^2*b^2*c^2
*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*(b^2*c - a*b*d - (sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5)
```

**maple** [A] time = 0.01, size = 256, normalized size = 1.49

$$-\frac{2\left(-128b^4x^4d^4-320a\,b^3d^4x^3-192b^4c\,d^3x^3-240a^2b^2d^4x^2-480a\,b^3c\,d^3x^2-48b^4c^2d^2x^2-40a^3b\,d^4x-360a^2b^2c\,d^3x-120a\,b^3c^2d^2x+8b^4c^3dx+5a^4d^4-60a^3bc\,d^3-90a^2b^2c^2d^2+20a\,b^3c^3d-3b^4c^4\right)}{15\left(bx+a\right)^{\frac{5}{2}}\left(dx+c\right)^{\frac{3}{2}}\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/15*(-128*b^4*d^4*x^4-320*a*b^3*d^4*x^3-192*b^4*c*d^3*x^3-240*a^2*b^2*d^4
*x^2-480*a*b^3*c*d^3*x^2-48*b^4*c^2*d^2*x^2-40*a^3*b*d^4*x-360*a^2*b^2*c*d^
3*x-120*a*b^3*c^2*d^2*x+8*b^4*c^3*d*x+5*a^4*d^4-60*a^3*b*c*d^3-90*a^2*b^2*c
^2*d^2+20*a*b^3*c^3*d-3*b^4*c^4)/(b*x+a)^(5/2)/(d*x+c)^(3/2)/(a^5*d^5-5*a^4
*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [B] time = 1.53, size = 346, normalized size = 2.01

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,x^2\,\left(5\,a^2\,d^2+10\,a\,b\,c\,d+b^2\,c^2\right)}{5\,{\left(a\,d-b\,c\right)}^5}+\frac{256\,b^2\,d^2\,x^4}{15\,{\left(a\,d-b\,c\right)}^5}+\frac{-10\,a^4\,d^4+120\,a^3\,b\,c\,d^3+180\,a^2\,b^2\,c^2\,d^2-40\,a\,b^3\,c^3\,d+6\,b^4\,c^4}{15\,b^2\,d^2\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(80\,a^3\,b\,d^4+720\,a^2\,b^2\,c\,d^3+240\,a\,b^3\,c^2\,d^2-16\,b^4\,c^3\,d\right)}{15\,b^2\,d^2\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,b\,d\,x^3\,\left(5\,a\,d+3\,b\,c\right)}{15\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{x^2\,\sqrt{a+b\,x}\,\left(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{b^2\,d^2}+\frac{2\,x^3\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a^2\,c^2\,\sqrt{a+b\,x}}{b^2\,d^2}+\frac{2\,a\,c\,x\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*x^2*(5*a^2*d^2 + b^2*c^2 + 10*a*b*c*d))/(5*(a*d - b*c
)^5) + (256*b^2*d^2*x^4)/(15*(a*d - b*c)^5) + (6*b^4*c^4 - 10*a^4*d^4 + 180
*a^2*b^2*c^2*d^2 - 40*a*b^3*c^3*d + 120*a^3*b*c*d^3)/(15*b^2*d^2*(a*d - b*c
)^5) + (x*(80*a^3*b*d^4 - 16*b^4*c^3*d + 240*a*b^3*c^2*d^2 + 720*a^2*b^2*c*
d^3))/(15*b^2*d^2*(a*d - b*c)^5) + (128*b*d*x^3*(5*a*d + 3*b*c))/(15*(a*d -
 b*c)^5)))/(x^4*(a + b*x)^(1/2) + (x^2*(a + b*x)^(1/2)*(a^2*d^2 + b^2*c^2 +
 4*a*b*c*d))/(b^2*d^2) + (2*x^3*(a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (a^2*c
^2*(a + b*x)^(1/2))/(b^2*d^2) + (2*a*c*x*(a*d + b*c)*(a + b*x)^(1/2))/(b^2*
d^2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{2}}(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*(c + d*x)**(5/2)), x)
```

## 3.1415 $\int \frac{1}{(a+bx)^{9/2}(c+dx)^{5/2}}\,dx$

**Optimal**. Leaf size=207

$$\frac{512bd^4\sqrt{a+bx}}{21\sqrt{c+dx}\,(bc-ad)^6}+\frac{256d^4\sqrt{a+bx}}{21(c+dx)^{3/2}(bc-ad)^5}+\frac{64d^3}{7\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{21(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^3}$$

**Rubi [A]** time = 0.06, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512bd^4\sqrt{a+bx}}{21\sqrt{c+dx}\,(bc-ad)^6}+\frac{256d^4\sqrt{a+bx}}{21(c+dx)^{3/2}(bc-ad)^5}+\frac{64d^3}{7\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{21(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{7(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{7(a+bx)^{7/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2)) + (4*d)/(7*(b*c - a*d)^2
*(a + b*x)^(5/2)*(c + d*x)^(3/2)) - (32*d^2)/(21*(b*c - a*d)^3*(a + b*x)^(3
/2)*(c + d*x)^(3/2)) + (64*d^3)/(7*(b*c - a*d)^4*Sqrt[a + b*x]*(c + d*x)^(3
/2)) + (256*d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^5*(c + d*x)^(3/2)) + (512*b*
d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(c+dx)^{5/2}}\,dx = -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} - \frac{(10d)\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx}{7(bc-ad)}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{\left(16d^2\right)\int \cdots}{7(\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{\cdots}{21(bc-ad\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{\cdots}{21(bc-ad\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{\cdots}{21(bc-ad\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{\cdots}{21(bc-ad\cdots}$$

**Mathematica [A]** time = 0.08, size = 233, normalized size = 1.13

$$\frac{2\left(-7a^5d^5+35a^4bd^4(3c+2dx)+70a^3b^2d^3\left(3c^2+12cdx+8d^2x^2\right)+70a^2b^3d^2\left(-c^3+6c^2dx+24cd^2x^2+16d^3x^3\right)+7ab^4d\left(3c^4-8c^3dx+48c^2d^2x^2+192cd^3x^3+128d^4x^4\right)+b^5\left(-3c^5+6c^4dx-16c^3d^2x^2+96c^2d^3x^3+384cd^4x^4+256d^5x^5\right)\right)}{21(a+bx)^{7/2}(c+dx)^{3/2}(bc-ad)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (2*(-7*a^5*d^5 + 35*a^4*b*d^4*(3*c + 2*d*x) + 70*a^3*b^2*d^3*(3*c^2 + 12*c*
d*x + 8*d^2*x^2) + 70*a^2*b^3*d^2*(-c^3 + 6*c^2*d*x + 24*c*d^2*x^2 + 16*d^3
*x^3) + 7*a*b^4*d*(3*c^4 - 8*c^3*d*x + 48*c^2*d^2*x^2 + 192*c*d^3*x^3 + 128
*d^4*x^4) + b^5*(-3*c^5 + 6*c^4*d*x - 16*c^3*d^2*x^2 + 96*c^2*d^3*x^3 + 384
*c*d^4*x^4 + 256*d^5*x^5)))/(21*(b*c - a*d)^6*(a + b*x)^(7/2)*(c + d*x)^(3/
2))
```

**IntegrateAlgebraic [A]** time = 0.16, size = 139, normalized size = 0.67

$$-\frac{2(a+bx)^{3/2}\left(\frac{3b^5(c+dx)^5}{(a+bx)^5}-\frac{21b^4d(c+dx)^4}{(a+bx)^4}+\frac{70b^3d^2(c+dx)^3}{(a+bx)^3}-\frac{210b^2d^3(c+dx)^2}{(a+bx)^2}-\frac{105bd^4(c+dx)}{a+bx}+7d^5\right)}{21(c+dx)^{3/2}(bc-ad)^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(7*d^5 - (105*b*d^4*(c + d*x))/(a + b*x) - (210*b^2*d^3
*(c + d*x)^2)/(a + b*x)^2 + (70*b^3*d^2*(c + d*x)^3)/(a + b*x)^3 - (21*b^4*
d*(c + d*x)^4)/(a + b*x)^4 + (3*b^5*(c + d*x)^5)/(a + b*x)^5))/(21*(b*c - a
*d)^6*(c + d*x)^(3/2))
```

**fricas [B]** time = 19.33, size = 999, normalized size = 4.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(256*b^5*d^5*x^5 - 3*b^5*c^5 + 21*a*b^4*c^4*d - 70*a^2*b^3*c^3*d^2 + 2
10*a^3*b^2*c^2*d^3 + 105*a^4*b*c*d^4 - 7*a^5*d^5 + 128*(3*b^5*c*d^4 + 7*a*b
^4*d^5)*x^4 + 32*(3*b^5*c^2*d^3 + 42*a*b^4*c*d^4 + 35*a^2*b^3*d^5)*x^3 - 16
*(b^5*c^3*d^2 - 21*a*b^4*c^2*d^3 - 105*a^2*b^3*c*d^4 - 35*a^3*b^2*d^5)*x^2
+ 2*(3*b^5*c^4*d - 28*a*b^4*c^3*d^2 + 210*a^2*b^3*c^2*d^3 + 420*a^3*b^2*c*d
^4 + 35*a^4*b*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^6*c^8 - 6*a^5*b^5*
c^7*d + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4 - 6*a^
9*b*c^3*d^5 + a^10*c^2*d^6 + (b^10*c^6*d^2 - 6*a*b^9*c^5*d^3 + 15*a^2*b^8*c
^4*d^4 - 20*a^3*b^7*c^3*d^5 + 15*a^4*b^6*c^2*d^6 - 6*a^5*b^5*c*d^7 + a^6*b^
4*d^8)*x^6 + 2*(b^10*c^7*d - 4*a*b^9*c^6*d^2 + 3*a^2*b^8*c^5*d^3 + 10*a^3*b
^7*c^4*d^4 - 25*a^4*b^6*c^3*d^5 + 24*a^5*b^5*c^2*d^6 - 11*a^6*b^4*c*d^7 + 2
*a^7*b^3*d^8)*x^5 + (b^10*c^8 + 2*a*b^9*c^7*d - 27*a^2*b^8*c^6*d^2 + 64*a^3
*b^7*c^5*d^3 - 55*a^4*b^6*c^4*d^4 - 6*a^5*b^5*c^3*d^5 + 43*a^6*b^4*c^2*d^6
- 28*a^7*b^3*c*d^7 + 6*a^8*b^2*d^8)*x^4 + 4*(a*b^9*c^8 - 3*a^2*b^8*c^7*d -
2*a^3*b^7*c^6*d^2 + 19*a^4*b^6*c^5*d^3 - 30*a^5*b^5*c^4*d^4 + 19*a^6*b^4*c^
3*d^5 - 2*a^7*b^3*c^2*d^6 - 3*a^8*b^2*c*d^7 + a^9*b*d^8)*x^3 + (6*a^2*b^8*c
^8 - 28*a^3*b^7*c^7*d + 43*a^4*b^6*c^6*d^2 - 6*a^5*b^5*c^5*d^3 - 55*a^6*b^4
*c^4*d^4 + 64*a^7*b^3*c^3*d^5 - 27*a^8*b^2*c^2*d^6 + 2*a^9*b*c*d^7 + a^10*d
^8)*x^2 + 2*(2*a^3*b^7*c^8 - 11*a^4*b^6*c^7*d + 24*a^5*b^5*c^6*d^2 - 25*a^6
*b^4*c^5*d^3 + 10*a^7*b^3*c^4*d^4 + 3*a^8*b^2*c^3*d^5 - 4*a^9*b*c^2*d^6 + a
^10*c*d^7)*x)
```

**giac [B]** time = 7.76, size = 1964, normalized size = 9.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x + a)*(14*(b^9*c^5*d^6*abs(b) - 5*a*b^8*c^4*d^7*abs(b) + 10*a^2
*b^7*c^3*d^8*abs(b) - 10*a^3*b^6*c^2*d^9*abs(b) + 5*a^4*b^5*c*d^10*abs(b) -
 a^5*b^4*d^11*abs(b))*(b*x + a)/(b^13*c^11*d - 11*a*b^12*c^10*d^2 + 55*a^2*
b^11*c^9*d^3 - 165*a^3*b^10*c^8*d^4 + 330*a^4*b^9*c^7*d^5 - 462*a^5*b^8*c^6
*d^6 + 462*a^6*b^7*c^5*d^7 - 330*a^7*b^6*c^4*d^8 + 165*a^8*b^5*c^3*d^9 - 55
```

```
*a^9*b^4*c^2*d^10 + 11*a^10*b^3*c*d^11 - a^11*b^2*d^12) + 15*(b^10*c^6*d^5*
abs(b) - 6*a*b^9*c^5*d^6*abs(b) + 15*a^2*b^8*c^4*d^7*abs(b) - 20*a^3*b^7*c^
3*d^8*abs(b) + 15*a^4*b^6*c^2*d^9*abs(b) - 6*a^5*b^5*c*d^10*abs(b) + a^6*b^
4*d^11*abs(b))/(b^13*c^11*d - 11*a*b^12*c^10*d^2 + 55*a^2*b^11*c^9*d^3 - 16
5*a^3*b^10*c^8*d^4 + 330*a^4*b^9*c^7*d^5 - 462*a^5*b^8*c^6*d^6 + 462*a^6*b^
7*c^5*d^7 - 330*a^7*b^6*c^4*d^8 + 165*a^8*b^5*c^3*d^9 - 55*a^9*b^4*c^2*d^10
 + 11*a^10*b^3*c*d^11 - a^11*b^2*d^12))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/
2) + 8/21*(79*sqrt(b*d)*b^15*c^6*d^3 - 474*sqrt(b*d)*a*b^14*c^5*d^4 + 1185*
sqrt(b*d)*a^2*b^13*c^4*d^5 - 1580*sqrt(b*d)*a^3*b^12*c^3*d^6 + 1185*sqrt(b*
d)*a^4*b^11*c^2*d^7 - 474*sqrt(b*d)*a^5*b^10*c*d^8 + 79*sqrt(b*d)*a^6*b^9*d
^9 - 511*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*b^13*c^5*d^3 + 2555*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*a*b^12*c^4*d^4 - 5110*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^11*c^3*d^5 + 51
10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2*a^3*b^10*c^2*d^6 - 2555*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*a^4*b^9*c*d^7 + 511*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^8*d^8 + 1344*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^11*
c^4*d^3 - 5376*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*a*b^10*c^3*d^4 + 8064*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^9*c^2*d^5 - 5376*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^8*c*d^
6 + 1344*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^4*a^4*b^7*d^7 - 1750*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^6*b^9*c^3*d^3 + 5250*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^8*c^2*d^4 - 5250*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2
*b^7*c*d^5 + 1750*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^6*a^3*b^6*d^6 + 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^7*c^2*d^3 - 2030*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^6*c*d^4 + 10
15*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^8*a^2*b^5*d^5 - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^10*b^5*c*d^3 + 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^4*d^4 + 42*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^3*d^3)/((b^5*c
^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^3*c^3*d^2*abs(b) - 10*a^3*b^2*c
^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d^5*abs(b))*(b^2*c - a*b*d - (sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7)
```

**maple [B]** time = 0.02, size = 356, normalized size = 1.72

$$-\frac{2 \left(-256 b^{5} x^{5} d^{5}-896 a \,b^{4} d^{5} x^{4}-384 b^{5} c \,d^{4} x^{4}-1120 a^{2} b^{3} d^{5} x^{3}-1344 a \,b^{4} c \,d^{4} x^{3}-96 b^{5} c^{2} d^{3} x^{3}-560 a^{3} b^{2} d^{5} x^{2}-1680 a^{2} b^{3} c \,d^{4} x^{2}-336 a \,b^{4} c^{2} d^{3} x^{2}+16 b^{5} c^{3} d^{2} x^{2}-70 a^{4} b \,d^{5} x-840 a^{3} b^{2} c \,d^{4} x-420 a^{2} b^{3} c^{2} d^{3} x+56 a \,b^{4} c^{3} d^{2} x-6 b^{5} c^{4} d x+7 a^{5} d^{5}-105 a^{4} b c \,d^{4}-210 a^{3} b^{2} c^{2} d^{3}+70 a^{2} b^{3} c^{3} d^{2}-21 a \,b^{4} c^{4} d+3 b^{5} c^{5}\right)}{21 \left(b x +a \right)^{\frac{7}{2}} \left(d x +c \right)^{\frac{3}{2}} \left(d^{6} a^{6}-6 b \,d^{5} c \,a^{5}+15 b^{2} d^{4} c^{2} a^{4}-20 b^{3} d^{3} c^{3} a^{3}+15 b^{4} d^{2} c^{4} a^{2}-6 b^{5} d \,c^{5} a +b^{6} c^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/21*(-256*b^5*d^5*x^5-896*a*b^4*d^5*x^4-384*b^5*c*d^4*x^4-1120*a^2*b^3*d^
5*x^3-1344*a*b^4*c*d^4*x^3-96*b^5*c^2*d^3*x^3-560*a^3*b^2*d^5*x^2-1680*a^2*
b^3*c*d^4*x^2-336*a*b^4*c^2*d^3*x^2+16*b^5*c^3*d^2*x^2-70*a^4*b*d^5*x-840*a
^3*b^2*c*d^4*x-420*a^2*b^3*c^2*d^3*x+56*a*b^4*c^3*d^2*x-6*b^5*c^4*d*x+7*a^5
*d^5-105*a^4*b*c*d^4-210*a^3*b^2*c^2*d^3+70*a^2*b^3*c^3*d^2-21*a*b^4*c^4*d+
3*b^5*c^5)/(b*x+a)^(7/2)/(d*x+c)^(3/2)/(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^
2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 1.91, size = 478, normalized size = 2.31

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,x^2\,\left(35\,a^3\,d^3+105\,a^2\,b\,c\,d^2+21\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{21\,b\,{\left(a\,d-b\,c\right)}^6}-\frac{14\,a^5\,d^5-210\,a^4\,b\,c\,d^4-420\,a^3\,b^2\,c^2\,d^3+140\,a^2\,b^3\,c^3\,d^2-42\,a\,b^4\,c^4\,d+6\,b^5\,c^5}{21\,b^3\,d^2\,{\left(a\,d-b\,c\right)}^6}+\frac{64\,d\,x^3\,\left(35\,a^2\,d^2+42\,a\,b\,c\,d+3\,b^2\,c^2\right)}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{512\,b^2\,d^3\,x^5}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{256\,b\,d^2\,x^4\,\left(7\,a\,d+3\,b\,c\right)}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{x\,\left(140\,a^4\,b\,d^5+1680\,a^3\,b^2\,c\,d^4+840\,a^2\,b^3\,c^2\,d^3-112\,a\,b^4\,c^3\,d^2+12\,b^5\,c^4\,d\right)}{21\,b^3\,d^2\,{\left(a\,d-b\,c\right)}^6}\right)}{x^5\,\sqrt{a+b\,x}+\frac{x^3\,\sqrt{a+b\,x}\,\left(3\,a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{b^2\,d^2}+\frac{x^4\,\left(3\,a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a^3\,c^2\,\sqrt{a+b\,x}}{b^3\,d^2}+\frac{a\,x^2\,\sqrt{a+b\,x}\,\left(a^2\,d^2+6\,a\,b\,c\,d+3\,b^2\,c^2\right)}{b^3\,d^2}+\frac{a^2\,c\,x\,\left(2\,a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*x^2*(35*a^3*d^3 - b^3*c^3 + 21*a*b^2*c^2*d + 105*a^2*
b*c*d^2))/(21*b*(a*d - b*c)^6) - (14*a^5*d^5 + 6*b^5*c^5 + 140*a^2*b^3*c^3*
d^2 - 420*a^3*b^2*c^2*d^3 - 42*a*b^4*c^4*d - 210*a^4*b*c*d^4)/(21*b^3*d^2*(
a*d - b*c)^6) + (64*d*x^3*(35*a^2*d^2 + 3*b^2*c^2 + 42*a*b*c*d))/(21*(a*d -
 b*c)^6) + (512*b^2*d^3*x^5)/(21*(a*d - b*c)^6) + (256*b*d^2*x^4*(7*a*d + 3
*b*c))/(21*(a*d - b*c)^6) + (x*(140*a^4*b*d^5 + 12*b^5*c^4*d - 112*a*b^4*c^
3*d^2 + 1680*a^3*b^2*c*d^4 + 840*a^2*b^3*c^2*d^3))/(21*b^3*d^2*(a*d - b*c)^
6)))/(x^5*(a + b*x)^(1/2) + (x^3*(a + b*x)^(1/2)*(3*a^2*d^2 + b^2*c^2 + 6*a
*b*c*d))/(b^2*d^2) + (x^4*(3*a*d + 2*b*c)*(a + b*x)^(1/2))/(b*d) + (a^3*c^2
*(a + b*x)^(1/2))/(b^3*d^2) + (a*x^2*(a + b*x)^(1/2)*(a^2*d^2 + 3*b^2*c^2 +
 6*a*b*c*d))/(b^3*d^2) + (a^2*c*x*(2*a*d + 3*b*c)*(a + b*x)^(1/2))/(b^3*d^2
))
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

## 3.1416 $\int \frac{1}{\sqrt{a+bx}\sqrt{4+a+bx}}\,dx$

**Optimal**. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

**Rubi [A]** time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[a + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}\sqrt{4+a+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
[Out] (2*ArcSinh[Sqrt[a + b*x]/2])/b
```

**IntegrateAlgebraic** [A] time = 0.05, size = 28, normalized size = 1.47

$$-\frac{2\log\left(\sqrt{a+bx+4}-\sqrt{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
[Out] (-2*Log[-Sqrt[a + b*x] + Sqrt[4 + a + b*x]])/b
```

**fricas** [B] time = 1.05, size = 31, normalized size = 1.63

$$-\frac{\log\left(-bx+\sqrt{bx+a+4}\sqrt{bx+a}-a-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + a + 4)*sqrt(b*x + a) - a - 2)/b
```

**giac** [A] time = 1.02, size = 24, normalized size = 1.26

$$-\frac{2\log\left(\sqrt{bx+a+4}-\sqrt{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + a + 4) - sqrt(b*x + a))/b
```

**maple** [B] time = 0.01, size = 86, normalized size = 4.53

$$\frac{\sqrt{(bx+a)(bx+a+4)}\ \ln\left(\frac{b^2x+\frac{ab}{2}+\frac{(a+4)b}{2}}{\sqrt{b^2}}+\sqrt{b^2x^2+(a+4)a+(ab+(a+4)b)x}\right)}{\sqrt{bx+a}\ \sqrt{bx+a+4}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x)
```

```
[Out] ((b*x+a)*(b*x+a+4))^(1/2)/(b*x+a)^(1/2)/(b*x+a+4)^(1/2)*ln((1/2*a*b+1/2*b*(
a+4)+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*(a+4))*x+a*(a+4))^(1/2))/(b^2)^(1/2
)
```

**maxima [B]** time = 1.36, size = 48, normalized size = 2.53

$$\frac{\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+a^2+2\,(ab+2\,b)x+4\,a}\,b+4\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + a^2 + 2*(a*b + 2*b)*x + 4*a)*b + 4*b
)/b
```

**mupad [B]** time = 0.31, size = 50, normalized size = 2.63

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{a+4}-\sqrt{a+b\,x+4}\right)}{\sqrt{-b^2}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(a + b*x + 4)^(1/2)),x)
```

```
[Out] (4*atan((b*((a + 4)^(1/2) - (a + b*x + 4)^(1/2)))/((-b^2)^(1/2)*((a + b*x)^
(1/2) - a^(1/2)))))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+bx}\,\sqrt{a+bx+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(b*x+a+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(a + b*x + 4)), x)
```

## 3.1417 $\int \frac{1}{\sqrt{2+bx}\,\sqrt{6+bx}}\,dx$

**Optimal**. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+2}\right)}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+bx}\,\sqrt{6+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{2+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{2+bx}\right)}{b}\end{aligned}$$

**Mathematica** [B] time = 0.01, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx+2}\,\sin^{-1}\left(\frac{1}{2}\sqrt{-bx-2}\right)}{b\sqrt{-bx-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]
[Out] (2*Sqrt[2 + b*x]*ArcSin[Sqrt[-2 - b*x]/2])/(b*Sqrt[-2 - b*x])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+6}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[6 + b*x]/Sqrt[2 + b*x]])/b
```

**fricas** [A] time = 0.81, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+6}\,\sqrt{bx+2}-4\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 6)*sqrt(b*x + 2) - 4)/b
```

**giac** [A] time = 1.05, size = 23, normalized size = 1.21

$$-\frac{2\log\left(\sqrt{bx+6}-\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 6) - sqrt(b*x + 2))/b
```

**maple [B]** time = 0.01, size = 66, normalized size = 3.47

$$\frac{\sqrt{(bx+2)(bx+6)}\ \ln\left(\frac{b^2x+4b}{\sqrt{b^2}}+\sqrt{b^2x^2+8bx+12}\right)}{\sqrt{bx+2}\ \sqrt{bx+6}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x)
```

```
[Out] ((b*x+2)*(b*x+6))^(1/2)/(b*x+2)^(1/2)/(b*x+6)^(1/2)*ln((b^2*x+4*b)/(b^2)^(1
/2)+(b^2*x^2+8*b*x+12)^(1/2))/(b^2)^(1/2)
```

**maxima [B]** time = 1.37, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+8\,bx+12}\,b+8\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 8*b*x + 12)*b + 8*b)/b
```

**mupad [B]** time = 0.34, size = 47, normalized size = 2.47

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{6}-\sqrt{b\,x+6}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x + 6)^(1/2)),x)
```

```
[Out] -(4*atan((b*(6^(1/2) - (b*x + 6)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2
)^(1/2))))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx+2}\,\sqrt{bx+6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(1/2)/(b*x+6)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 2)*sqrt(b*x + 6)), x)
```

## 3.1418 $\int \frac{1}{\sqrt{1+bx}\,\sqrt{5+bx}}\,dx$

**Optimal**. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+1}\right)}{b}$$

**Rubi** [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[1 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+bx}\,\sqrt{5+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{1+bx}\right)}{b}\end{aligned}$$

**Mathematica [B]** time = 0.01, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx+1}\,\sin^{-1}\left(\frac{1}{2}\sqrt{-bx-1}\right)}{b\sqrt{-bx-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]
[Out] (2*Sqrt[1 + b*x]*ArcSin[Sqrt[-1 - b*x]/2])/(b*Sqrt[-1 - b*x])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+5}}{\sqrt{bx+1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[5 + b*x]/Sqrt[1 + b*x]])/b
```

**fricas [A]** time = 0.88, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+5}\,\sqrt{bx+1}-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 5)*sqrt(b*x + 1) - 3)/b
```

**giac [A]** time = 0.96, size = 23, normalized size = 1.21

$$-\frac{2\log\left(\sqrt{bx+5}-\sqrt{bx+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 5) - sqrt(b*x + 1))/b
```

**maple [B]** time = 0.01, size = 66, normalized size = 3.47

$$\frac{\sqrt{(bx+1)(bx+5)}\ \ln\left(\frac{b^2x+3b}{\sqrt{b^2}}+\sqrt{b^2x^2+6bx+5}\right)}{\sqrt{bx+1}\ \sqrt{bx+5}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x)
```

```
[Out] ((b*x+1)*(b*x+5))^(1/2)/(b*x+1)^(1/2)/(b*x+5)^(1/2)*ln((b^2*x+3*b)/(b^2)^(1
/2)+(b^2*x^2+6*b*x+5)^(1/2))/(b^2)^(1/2)
```

**maxima [B]** time = 1.39, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+6\,bx+5}\,b+6\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 6*b*x + 5)*b + 6*b)/b
```

**mupad [B]** time = 0.33, size = 43, normalized size = 2.26

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{5}-\sqrt{b\,x+5}\right)}{\left(\sqrt{b\,x+1}-1\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 1)^(1/2)*(b*x + 5)^(1/2)),x)
```

```
[Out] (4*atan((b*(5^(1/2) - (b*x + 5)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1/2)
)))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx+1}\,\sqrt{bx+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)**(1/2)/(b*x+5)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 1)*sqrt(b*x + 5)), x)
```

## 3.1419 $\int \frac{1}{\sqrt{bx}\sqrt{4+bx}}\,dx$

**Optimal**. Leaf size=17

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx}\sqrt{4+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 34, normalized size = 2.00

$$\frac{2\sqrt{x}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
[Out] (2*Sqrt[x]*ArcSinh[(Sqrt[b]*Sqrt[x])/2])/(Sqrt[b]*Sqrt[b*x])
```

**IntegrateAlgebraic [A]** time = 0.04, size = 25, normalized size = 1.47

$$-\frac{2\log\left(\sqrt{bx+4}-\sqrt{bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
[Out] (-2*Log[-Sqrt[b*x] + Sqrt[4 + b*x]])/b
```

**fricas [A]** time = 0.95, size = 25, normalized size = 1.47

$$-\frac{\log\left(-bx+\sqrt{bx+4}\sqrt{bx}-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 4)*sqrt(b*x) - 2)/b
```

**giac [A]** time = 0.94, size = 21, normalized size = 1.24

$$-\frac{2\log\left(\sqrt{bx+4}-\sqrt{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 4) - sqrt(b*x))/b
```

**maple [B]** time = 0.01, size = 60, normalized size = 3.53

$$\frac{\sqrt{(bx+4)\,bx}\;\ln\left(\frac{b^2x+2b}{\sqrt{b^2}}+\sqrt{b^2x^2+4bx}\right)}{\sqrt{bx}\;\sqrt{bx+4}\;\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2)/(b*x+4)^(1/2),x)
```

```
[Out] (x*b*(b*x+4))^(1/2)/(b*x)^(1/2)/(b*x+4)^(1/2)*ln((b^2*x+2*b)/(b^2)^(1/2)+(b
^2*x^2+4*b*x)^(1/2))/(b^2)^(1/2)
```

**maxima [B]** time = 1.33, size = 32, normalized size = 1.88

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+4\,bx}\,b+4\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 4*b*x)*b + 4*b)/b
```

**mupad [B]** time = 0.31, size = 33, normalized size = 1.94

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,x+4}-2\right)}{\sqrt{b\,x}\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x)^(1/2)*(b*x + 4)^(1/2)),x)
```

```
[Out] -(4*atan((b*((b*x + 4)^(1/2) - 2))/((b*x)^(1/2)*(-b^2)^(1/2))))/(-b^2)^(1/2
)
```

**sympy [A]** time = 1.27, size = 15, normalized size = 0.88

$$\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(1/2)/(b*x+4)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/2)/b
```

## 3.1420 $\int \frac{1}{\sqrt{-1+bx}\,\sqrt{3+bx}}\,dx$

**Optimal**. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-1}\right)}{b}$$

**Rubi** [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-1 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx}\,\sqrt{3+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{-1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-1+bx}\right)}{b}\end{aligned}$$

**Mathematica [B]** time = 0.01, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx-1}\,\sin^{-1}\left(\frac{1}{2}\sqrt{1-bx}\right)}{b\sqrt{1-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]
[Out] (2*Sqrt[-1 + b*x]*ArcSin[Sqrt[1 - b*x]/2])/(b*Sqrt[1 - b*x])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+3}}{\sqrt{bx-1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[3 + b*x]/Sqrt[-1 + b*x]])/b
```

**fricas [A]** time = 0.97, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+3}\,\sqrt{bx-1}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 3)*sqrt(b*x - 1) - 1)/b
```

**giac [A]** time = 1.03, size = 23, normalized size = 1.21

$$-\frac{2\,\log\left(\sqrt{bx+3}-\sqrt{bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 3) - sqrt(b*x - 1))/b
```

**maple [B]** time = 0.01, size = 64, normalized size = 3.37

$$\frac{\sqrt{(bx-1)(bx+3)}\ \ln\left(\frac{b^2x+b}{\sqrt{b^2}}+\sqrt{b^2x^2+2bx-3}\right)}{\sqrt{bx-1}\ \sqrt{bx+3}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x)
```

```
[Out] ((b*x-1)*(b*x+3))^(1/2)/(b*x-1)^(1/2)/(b*x+3)^(1/2)*ln((b^2*x+b)/(b^2)^(1/2
)+(b^2*x^2+2*b*x-3)^(1/2))/(b^2)^(1/2)
```

**maxima [B]** time = 1.32, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+2\,bx-3}\,b+2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 2*b*x - 3)*b + 2*b)/b
```

**mupad [B]** time = 0.32, size = 44, normalized size = 2.32

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{3}-\sqrt{b\,x+3}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 1)^(1/2)*(b*x + 3)^(1/2)),x)
```

```
[Out] (4*atan((b*((b*x - 1)^(1/2) - 1i))/((3^(1/2) - (b*x + 3)^(1/2))*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx-1}\,\sqrt{bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)**(1/2)/(b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 1)*sqrt(b*x + 3)), x)
```

## 3.1421 $\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {52}

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/2]/b
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

**Mathematica [B]** time = 0.00, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx-2}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[-2 + b*x]/Sqrt[2 + b*x]])/b
```

**IntegrateAlgebraic [B]** time = 0.05, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{bx-2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-2 + b*x]])/b
```

**fricas [B]** time = 1.12, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx+\sqrt{bx+2}\,\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x - 2))/b
```

**giac [B]** time = 1.08, size = 23, normalized size = 2.09

$$-\frac{2\log\left(\sqrt{bx+2}-\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 2))/b
```

**maple [B]** time = 0.01, size = 57, normalized size = 5.18

$$\frac{\sqrt{(bx-2)(bx+2)}\,\ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2-4}\right)}{\sqrt{bx-2}\,\sqrt{bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x)
[Out] ((b*x-2)*(b*x+2))^(1/2)/(b*x-2)^(1/2)/(b*x+2)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(b
^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

**maxima [B]** time = 1.33, size = 26, normalized size = 2.36

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2 - 4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b
```

**mupad [B]** time = 0.30, size = 50, normalized size = 4.55

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 2)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2)*1i - (b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

**sympy [C]** time = 4.20, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| {\frac{4e^{2i\pi}}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| {\frac{4}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] meijerg((((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4*e
xp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4/(b**2*x**2))/(4*pi**(3
/2)*b)
```

## 3.1422 $\int \frac{1}{\sqrt{-3+bx}\,\sqrt{1+bx}}\,dx$

**Optimal**. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-3}\right)}{b}$$

**Rubi** [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-3}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-3 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+bx}\,\sqrt{1+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{-3+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-3+bx}\right)}{b}\end{aligned}$$

**Mathematica [B]** time = 0.01, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx-3}\,\sin^{-1}\left(\frac{1}{2}\sqrt{3-bx}\right)}{b\sqrt{3-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]
[Out] (2*Sqrt[-3 + b*x]*ArcSin[Sqrt[3 - b*x]/2])/(b*Sqrt[3 - b*x])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+1}}{\sqrt{bx-3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[1 + b*x]/Sqrt[-3 + b*x]])/b
```

**fricas [A]** time = 0.73, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+1}\,\sqrt{bx-3}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 1)*sqrt(b*x - 3) + 1)/b
```

**giac [A]** time = 1.00, size = 23, normalized size = 1.21

$$-\frac{2\log\left(\sqrt{bx+1}-\sqrt{bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 1) - sqrt(b*x - 3))/b
```

**maple [B]** time = 0.01, size = 66, normalized size = 3.47

$$\frac{\sqrt{(bx-3)(bx+1)}\ \ln\left(\frac{b^2x-b}{\sqrt{b^2}}+\sqrt{b^2x^2-2bx-3}\right)}{\sqrt{bx-3}\ \sqrt{bx+1}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x)
```

```
[Out] ((b*x-3)*(b*x+1))^(1/2)/(b*x-3)^(1/2)/(b*x+1)^(1/2)*ln((b^2*x-b)/(b^2)^(1/2
)+(b^2*x^2-2*b*x-3)^(1/2))/(b^2)^(1/2)
```

**maxima [B]** time = 1.38, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-2\,bx-3}\,b-2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 2*b*x - 3)*b - 2*b)/b
```

**mupad [B]** time = 0.29, size = 46, normalized size = 2.42

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-3}+\sqrt{3}\ 1\mathrm{i}\right)}{\left(\sqrt{b\,x+1}-1\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 1)^(1/2)*(b*x - 3)^(1/2)),x)
```

```
[Out] (4*atan((b*(3^(1/2)*1i - (b*x - 3)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1
/2))))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-3}\,\sqrt{bx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)**(1/2)/(b*x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 3)*sqrt(b*x + 1)), x)
```

## 3.1423 $\int \frac{1}{\sqrt{2+bx}\,\sqrt{3+bx}}\,dx$

**Optimal**. Leaf size=15

$$\frac{2\sinh^{-1}\left(\sqrt{bx+2}\right)}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+bx}\,\sqrt{3+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\sqrt{2+bx}\right)}{b}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 15, normalized size = 1.00

$$\frac{2\sinh^{-1}\left(\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]])/b
```

**IntegrateAlgebraic [A]** time = 0.05, size = 25, normalized size = 1.67

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+3}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[3 + b*x]/Sqrt[2 + b*x]])/b
```

**fricas [B]** time = 1.13, size = 28, normalized size = 1.87

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+3}\,\sqrt{bx+2}\,-5\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b*x + 3)*sqrt(b*x + 2) - 5)/b
```

**giac [A]** time = 1.01, size = 23, normalized size = 1.53

$$-\frac{2\,\log\left(\sqrt{bx+3}\,-\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 3) - sqrt(b*x + 2))/b
```

**maple [B]** time = 0.01, size = 66, normalized size = 4.40

$$\frac{\sqrt{(bx+2)\,(bx+3)}\,\ln\left(\frac{b^{2}x+\frac{5}{2}b}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+5bx+6}\right)}{\sqrt{bx+2}\,\sqrt{bx+3}\,\sqrt{b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x)
```

```
[Out] ((b*x+2)*(b*x+3))^(1/2)/(b*x+2)^(1/2)/(b*x+3)^(1/2)*ln((5/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+5*b*x+6)^(1/2))/(b^2)^(1/2)
```

**maxima [B]** time = 1.38, size = 33, normalized size = 2.20

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+5\,bx+6}\,b+5\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 5*b*x + 6)*b + 5*b)/b
```

**mupad [B]** time = 0.29, size = 47, normalized size = 3.13

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{3}-\sqrt{b\,x+3}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x + 3)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2) - (b*x + 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2
)^(1/2))))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx+2}\,\sqrt{bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(1/2)/(b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 2)*sqrt(b*x + 3)), x)
```

## 3.1424 $\int \frac{1}{2+bx} dx$

**Optimal**. Leaf size=10

$$\frac{\log(bx+2)}{b}$$

**Rubi** [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(bx+2)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(-1),x]
```

```
[Out] Log[2 + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+bx} dx = \frac{\log(2+bx)}{b}$$

**Mathematica** [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(bx+2)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(-1),x]
```

```
[Out] Log[2 + b*x]/b
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+bx} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + b*x)^(-1),x]
[Out] IntegrateAlgebraic[(2 + b*x)^(-1), x]
```

**fricas [A]** time = 1.00, size = 10, normalized size = 1.00

$$\frac{\log\left(bx+2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="fricas")
[Out] log(b*x + 2)/b
```

**giac [A]** time = 0.95, size = 11, normalized size = 1.10

$$\frac{\log\left(|bx+2|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="giac")
[Out] log(abs(b*x + 2))/b
```

**maple [A]** time = 0.00, size = 11, normalized size = 1.10

$$\frac{\ln\left(bx+2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2),x)
[Out] ln(b*x+2)/b
```

**maxima [A]** time = 1.32, size = 10, normalized size = 1.00

$$\frac{\log\left(bx+2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="maxima")
[Out] log(b*x + 2)/b
```

**mupad [B]** time = 0.26, size = 10, normalized size = 1.00

$$\frac{\ln\left(b\,x+2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + 2),x)
```

```
[Out] log(b*x + 2)/b
```

**sympy [A]** time = 0.06, size = 7, normalized size = 0.70

$$\frac{\log\left(bx+2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x)
```

```
[Out] log(b*x + 2)/b
```

## 3.1425 $\int \frac{1}{\sqrt{1+bx}\,\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=15

$$\frac{2\sinh^{-1}\left(\sqrt{bx+1}\right)}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[1 + b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+bx}\,\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\sqrt{1+bx}\right)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{2\sinh^{-1}\left(\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcSinh[Sqrt[1 + b*x]])/b
```

**IntegrateAlgebraic** [A] time = 0.05, size = 25, normalized size = 1.67

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{bx+1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[1 + b*x]])/b
```

**fricas** [B] time = 0.95, size = 28, normalized size = 1.87

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\,\sqrt{bx+1}\,-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x + 1) - 3)/b
```

**giac** [A] time = 0.92, size = 23, normalized size = 1.53

$$-\frac{2\,\log\left(\sqrt{bx+2}\,-\sqrt{bx+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x + 1))/b
```

**maple** [B] time = 0.01, size = 66, normalized size = 4.40

$$\frac{\sqrt{(bx+1)\,(bx+2)}\,\ln\left(\frac{b^2x+\frac{3}{2}b}{\sqrt{b^2}}+\sqrt{b^2x^2+3bx+2}\right)}{\sqrt{bx+1}\,\sqrt{bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x)

[Out] ((b*x+1)*(b*x+2))^(1/2)/(b*x+1)^(1/2)/(b*x+2)^(1/2)*ln((3/2*b+b^2*x)/(b^2)^(1/2)+(b^2*x^2+3*b*x+2)^(1/2))/(b^2)^(1/2)

**maxima [B]** time = 1.39, size = 33, normalized size = 2.20

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+3\,bx+2}\,b+3\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")

[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 3*b*x + 2)*b + 3*b)/b

**mupad [B]** time = 0.29, size = 43, normalized size = 2.87

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\left(\sqrt{b\,x+1}-1\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x + 1)^(1/2)*(b*x + 2)^(1/2)),x)

[Out] (4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1/2))))/(-b^2)^(1/2)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+1)**(1/2)/(b*x+2)**(1/2),x)

[Out] Integral(1/(sqrt(b*x + 1)*sqrt(b*x + 2)), x)

## 3.1426 $\int \frac{1}{\sqrt{bx}\,\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}$$

**Rubi** [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[b*x]/Sqrt[2]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx}\,\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \sqrt{bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 36, normalized size = 1.89

$$\frac{2\sqrt{x}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
[Out] (2*Sqrt[x]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(Sqrt[b]*Sqrt[b*x])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 25, normalized size = 1.32

$$-\frac{2\log\left(\sqrt{bx+2}-\sqrt{bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*Log[-Sqrt[b*x] + Sqrt[2 + b*x]])/b
```

**fricas** [A] time = 0.98, size = 25, normalized size = 1.32

$$-\frac{\log\left(-bx+\sqrt{bx+2}\sqrt{bx}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x) - 1)/b
```

**giac** [A] time = 1.07, size = 21, normalized size = 1.11

$$-\frac{2\log\left(\sqrt{bx+2}-\sqrt{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x))/b
```

**maple** [B] time = 0.01, size = 58, normalized size = 3.05

$$\frac{\sqrt{(bx+2)\,bx}\;\ln\left(\frac{b^2x+b}{\sqrt{b^2}}+\sqrt{b^2x^2+2bx}\right)}{\sqrt{bx}\;\sqrt{bx+2}\;\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] (x*b*(b*x+2))^(1/2)/(b*x)^(1/2)/(b*x+2)^(1/2)*ln((b^2*x+b)/(b^2)^(1/2)+(b^2
*x^2+2*b*x)^(1/2))/(b^2)^(1/2)
```

**maxima [A]** time = 1.38, size = 32, normalized size = 1.68

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2 + 2\,bx}\,b + 2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 2*b*x)*b + 2*b)/b
```

**mupad [B]** time = 0.28, size = 37, normalized size = 1.95

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\sqrt{b\,x}\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/((b*x)^(1/2)*(-b^2)^(1/2))))/(-b^2)
^(1/2)
```

**sympy [A]** time = 1.35, size = 20, normalized size = 1.05

$$\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b
```

## 3.1427 $\int \frac{1}{\sqrt{-1+bx}\,\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=21

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-1}}{\sqrt{3}}\right)}{b}$$

**Rubi [A]** time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-1}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-1 + b*x]/Sqrt[3]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+bx}\,\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-1+bx}\right)}{b}\\
&= \frac{2\sinh^{-1}\left(\frac{\sqrt{-1+bx}}{\sqrt{3}}\right)}{b}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 41, normalized size = 1.95

$$\frac{2\sqrt{bx-1}\,\sin^{-1}\left(\frac{\sqrt{1-bx}}{\sqrt{3}}\right)}{b\sqrt{1-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*Sqrt[-1 + b*x]*ArcSin[Sqrt[1 - b*x]/Sqrt[3]])/(b*Sqrt[1 - b*x])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 25, normalized size = 1.19

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{bx-1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-1 + b*x]])/b
```

**fricas** [A] time = 1.08, size = 28, normalized size = 1.33

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\,\sqrt{bx-1}\,-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x - 1) - 1)/b
```

**giac** [A] time = 1.02, size = 23, normalized size = 1.10

$$-\frac{2\,\log\left(\sqrt{bx+2}\,-\sqrt{bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 1))/b
```

**maple [B]** time = 0.01, size = 65, normalized size = 3.10

$$\frac{\sqrt{(bx-1)\,(bx+2)}\;\ln\left(\frac{b^2x+\frac{1}{2}b}{\sqrt{b^2}}+\sqrt{b^2x^2+bx-2}\right)}{\sqrt{bx-1}\;\sqrt{bx+2}\;\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x-1)*(b*x+2))^(1/2)/(b*x-1)^(1/2)/(b*x+2)^(1/2)*ln((1/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+b*x-2)^(1/2))/(b^2)^(1/2)
```

**maxima [A]** time = 1.68, size = 30, normalized size = 1.43

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+bx-2}\,b+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x - 2)*b + b)/b
```

**mupad [B]** time = 0.29, size = 44, normalized size = 2.10

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 1)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*((b*x - 1)^(1/2) - 1i))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx-1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 1)*sqrt(b*x + 2)), x)
```

## 3.1428 $\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {52}

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/2]/b
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

**Mathematica [B]** time = 0.00, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx-2}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[-2 + b*x]/Sqrt[2 + b*x]])/b
```

**IntegrateAlgebraic [B]** time = 0.00, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{bx-2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-2 + b*x]])/b
```

**fricas [B]** time = 0.87, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx+\sqrt{bx+2}\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x - 2))/b
```

**giac [B]** time = 0.96, size = 23, normalized size = 2.09

$$-\frac{2\log\left(\sqrt{bx+2}-\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 2))/b
```

**maple [B]** time = 0.00, size = 57, normalized size = 5.18

$$\frac{\sqrt{(bx-2)(bx+2)}\,\ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2-4}\right)}{\sqrt{bx-2}\,\sqrt{bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x)
[Out] ((b*x-2)*(b*x+2))^(1/2)/(b*x-2)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*ln(1/(b^2)^
(1/2)*b^2*x+(b^2*x^2-4)^(1/2))
```

**maxima** [**B**] time = 1.36, size = 26, normalized size = 2.36

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2 - 4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b
```

**mupad** [**B**] time = 0.00, size = 50, normalized size = 4.55

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 2)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2)*1i - (b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

**sympy** [**C**] time = 4.28, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4*e
xp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4/(b**2*x**2))/(4*pi**(3
/2)*b)
```

## 3.1429 $\int \frac{1}{\sqrt{-3+bx}\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=21

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-3}}{\sqrt{5}}\right)}{b}$$

**Rubi [A]** time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-3}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-3 + b*x]/Sqrt[5]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+bx}\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, \sqrt{-3+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{-3+bx}}{\sqrt{5}}\right)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 41, normalized size = 1.95

$$\frac{2\sqrt{bx-3}\,\sin^{-1}\left(\frac{\sqrt{3-bx}}{\sqrt{5}}\right)}{b\sqrt{3-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*Sqrt[-3 + b*x]*ArcSin[Sqrt[3 - b*x]/Sqrt[5]])/(b*Sqrt[3 - b*x])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 25, normalized size = 1.19

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{bx-3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-3 + b*x]])/b
```

**fricas** [A] time = 1.05, size = 28, normalized size = 1.33

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\,\sqrt{bx-3}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x - 3) + 1)/b
```

**giac** [A] time = 0.96, size = 23, normalized size = 1.10

$$-\frac{2\,\log\left(\sqrt{bx+2}-\sqrt{bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 3))/b
```

**maple [B]** time = 0.01, size = 66, normalized size = 3.14

$$\frac{\sqrt{(bx-3)(bx+2)}\ \ln\left(\frac{b^2x-\frac{1}{2}b}{\sqrt{b^2}}+\sqrt{b^2x^2-bx-6}\right)}{\sqrt{bx-3}\ \sqrt{bx+2}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x-3)*(b*x+2))^(1/2)/(b*x-3)^(1/2)/(b*x+2)^(1/2)*ln((-1/2*b+b^2*x)/(b^2)
^(1/2)+(b^2*x^2-b*x-6)^(1/2))/(b^2)^(1/2)
```

**maxima [A]** time = 1.24, size = 33, normalized size = 1.57

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-bx-6}\,b-b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - b*x - 6)*b - b)/b
```

**mupad [B]** time = 0.28, size = 50, normalized size = 2.38

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x - 3)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2)*1i - (b*x - 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-3}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 3)*sqrt(b*x + 2)), x)
```

## 3.1430 $\int \frac{1}{\sqrt{3-bx}\,\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=16

$$-\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

**Rubi** [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {53, 619, 216}

$$-\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] -(ArcSin[(1 - 2*b*x)/5]/b)
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-bx}\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{6+bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{25b^2}}}\,dx, x, b-2b^2x\right)}{5b^2}$$
$$= -\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

**Mathematica [A]** time = 0.01, size = 22, normalized size = 1.38

$$-\frac{2\sin^{-1}\left(\frac{\sqrt{3-bx}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*ArcSin[Sqrt[3 - b*x]/Sqrt[5]])/b
```

**IntegrateAlgebraic [A]** time = 0.05, size = 26, normalized size = 1.62

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3-bx}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*ArcTan[Sqrt[3 - b*x]/Sqrt[2 + b*x]])/b
```

**fricas [B]** time = 0.83, size = 44, normalized size = 2.75

$$-\frac{\arctan\left(\frac{(2\,bx-1)\sqrt{bx+2}\,\sqrt{-bx+3}}{2\left(b^2x^2-bx-6\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x - 1)*sqrt(b*x + 2)*sqrt(-b*x + 3)/(b^2*x^2 - b*x - 6))/b
```

**giac [A]** time = 1.05, size = 18, normalized size = 1.12

$$\frac{2 \arcsin\left(\frac{1}{5}\sqrt{5}\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/5*sqrt(5)*sqrt(b*x + 2))/b
```

**maple [B]** time = 0.01, size = 65, normalized size = 4.06

$$\frac{\sqrt{(-bx+3)(bx+2)}\ \arctan\left(\frac{\sqrt{b^2}\left(x-\frac{1}{2b}\right)}{\sqrt{-b^2x^2+bx+6}}\right)}{\sqrt{-bx+3}\ \sqrt{bx+2}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+3)*(b*x+2))^(1/2)/(-b*x+3)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x-1/2/b)/(-b^2*x^2+b*x+6)^(1/2))
```

**maxima [A]** time = 2.99, size = 21, normalized size = 1.31

$$-\frac{\arcsin\left(-\frac{2\,b^2x-b}{5\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/5*(2*b^2*x - b)/b)/b
```

**mupad [B]** time = 0.08, size = 44, normalized size = 2.75

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{3}-\sqrt{3-b\,x}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(3 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2) - (3 - b*x)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)
^(1/2))))/(b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+3}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 3)*sqrt(b*x + 2)), x)
```

## 3.1431 $\int \frac{1}{\sqrt{2-bx}\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=11

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

**Rubi** [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {41, 216}

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[(b*x)/2]/b
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-bx}\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{4-b^2x^2}}\,dx\\ &= \frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[(b*x)/2]/b
```

**IntegrateAlgebraic [B]** time = 0.05, size = 26, normalized size = 2.36

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[2 - b*x]/Sqrt[2 + b*x]])/b
```

**fricas [B]** time = 0.75, size = 31, normalized size = 2.82

$$-\frac{2\arctan\left(\frac{\sqrt{bx+2}\sqrt{-bx+2}-2}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(b*x + 2)*sqrt(-b*x + 2) - 2)/(b*x))/b
```

**giac [A]** time = 0.91, size = 15, normalized size = 1.36

$$\frac{2\arcsin\left(\frac{1}{2}\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(b*x + 2))/b
```

**maple [B]** time = 0.01, size = 56, normalized size = 5.09

$$\frac{\sqrt{(-bx+2)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2x^2+4}}\right)}{\sqrt{-bx+2}\,\sqrt{bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+2)*(b*x+2))^(1/2)/(-b*x+2)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*x/(-b^2*x^2+4)^(1/2))
```

**maxima [A]** time = 3.03, size = 9, normalized size = 0.82

$$\frac{\arcsin\left(\frac{1}{2}\,bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/2*b*x)/b
```

**mupad [B]** time = 0.08, size = 44, normalized size = 4.00

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)
^(1/2))))/(b^2)^(1/2)
```

**sympy [C]** time = 4.44, size = 76, normalized size = 6.91

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{4e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
4/(b**2*x**2))/(4*pi**(3/2)*b) + meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ())
, ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**
(3/2)*b)
```

## 3.1432 $\int \frac{1}{\sqrt{1-bx}\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=16

$$-\frac{\sin^{-1}\left(\frac{1}{3}(-2bx-1)\right)}{b}$$

**Rubi** [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {53, 619, 216}

$$-\frac{\sin^{-1}\left(\frac{1}{3}(-2bx-1)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] -(ArcSin[(-1 - 2*b*x)/3]/b)
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-bx}\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{2-bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9b^2}}}\,dx, x, -b-2b^2x\right)}{3b^2}$$
$$= -\frac{\sin^{-1}\left(\frac{1}{3}(-1-2bx)\right)}{b}$$

**Mathematica** [A] time = 0.01, size = 22, normalized size = 1.38

$$-\frac{2\sin^{-1}\left(\frac{\sqrt{1-bx}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*ArcSin[Sqrt[1 - b*x]/Sqrt[3]])/b
```

**IntegrateAlgebraic** [A] time = 0.05, size = 26, normalized size = 1.62

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-bx}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*ArcTan[Sqrt[1 - b*x]/Sqrt[2 + b*x]])/b
```

**fricas** [B] time = 1.07, size = 43, normalized size = 2.69

$$-\frac{\arctan\left(\frac{(2\,bx+1)\sqrt{bx+2}\,\sqrt{-bx+1}}{2\left(b^2x^2+bx-2\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

[Out] -arctan(1/2*(2*b*x + 1)*sqrt(b*x + 2)*sqrt(-b*x + 1)/(b^2*x^2 + b*x - 2))/b

**giac [A]** time = 1.05, size = 18, normalized size = 1.12

$$\frac{2 \arcsin\left(\frac{1}{3}\sqrt{3}\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")

[Out] 2*arcsin(1/3*sqrt(3)*sqrt(b*x + 2))/b

**maple [B]** time = 0.01, size = 66, normalized size = 4.12

$$\frac{\sqrt{(-bx+1)(bx+2)}\ \arctan\left(\frac{\sqrt{b^2}\left(x+\frac{1}{2b}\right)}{\sqrt{-b^2x^2-bx+2}}\right)}{\sqrt{-bx+1}\ \sqrt{bx+2}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x)

[Out] ((-b*x+1)*(b*x+2))^(1/2)/(-b*x+1)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x+1/2/b)/(-b^2*x^2-b*x+2)^(1/2))

**maxima [A]** time = 3.01, size = 19, normalized size = 1.19

$$-\frac{\arcsin\left(-\frac{2b^2x+b}{3b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")

[Out] -arcsin(-1/3*(2*b^2*x + b)/b)/b

**mupad [B]** time = 0.32, size = 40, normalized size = 2.50

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\left(\sqrt{1-b\,x}-1\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/(((1 - b*x)^(1/2) - 1)*(b^2)^(1/2)
)))/(b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 1)*sqrt(b*x + 2)), x)
```

## 3.1433 $\int \frac{1}{\sqrt{-bx}\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=10

$$\frac{\sin^{-1}(bx+1)}{b}$$

**Rubi [A]** time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {53, 619, 216}

$$\frac{\sin^{-1}(bx+1)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[1 + b*x]/b
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-bx}\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{-2bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2b-2b^2x\right)}{2b^2}$$
$$= \frac{\sin^{-1}(1+bx)}{b}$$

**Mathematica [B]** time = 0.01, size = 51, normalized size = 5.10

$$\frac{2\sqrt{x}\sqrt{bx+2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}\sqrt{-bx(bx+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[2 + b*x]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(Sqrt[b]*Sqrt[
-(b*x*(2 + b*x))])
```

**IntegrateAlgebraic [B]** time = 0.04, size = 24, normalized size = 2.40

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[2 + b*x]/Sqrt[-(b*x)]])/b
```

**fricas [B]** time = 1.12, size = 26, normalized size = 2.60

$$-\frac{2\arctan\left(\frac{\sqrt{bx+2}\sqrt{-bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(b*x + 2)*sqrt(-b*x)/(b*x))/b
```

**giac [A]** time = 0.93, size = 18, normalized size = 1.80

$$\frac{2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(b*x + 2))/b
```

**maple [B]** time = 0.00, size = 58, normalized size = 5.80

$$\frac{\sqrt{-\left(bx+2\right)bx}\,\arctan\left(\frac{\sqrt{b^{2}}\left(x+\frac{1}{b}\right)}{\sqrt{-b^{2}x^{2}-2bx}}\right)}{\sqrt{-bx}\,\sqrt{bx+2}\,\sqrt{b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] (-(b*x+2)*b*x)^(1/2)/(-b*x)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b^2)^(1
/2)*(x+1/b)/(-b^2*x^2-2*b*x)^(1/2))
```

**maxima [A]** time = 3.14, size = 18, normalized size = 1.80

$$-\frac{\arcsin\left(-\frac{b^{2}x+b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(b^2*x + b)/b)/b
```

**mupad [B]** time = 0.29, size = 34, normalized size = 3.40

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\sqrt{-b\,x}\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/((-b*x)^(1/2)*(b^2)^(1/2))))/(b^2)
^(1/2)
```

**sympy [C]** time = 1.28, size = 24, normalized size = 2.40

$$-\frac{2i\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] -2*I*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b
```

## 3.1434 $\int \frac{1}{\sqrt{-1-bx}\,\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=11

$$\frac{\sin^{-1}(2bx+3)}{b}$$

**Rubi [A]** time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {53, 619, 216}

$$\frac{\sin^{-1}(2bx+3)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[3 + 2*b*x]/b
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-bx}\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{-2-3bx-b^2x^2}}\,dx$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{b^2}}}\,dx, x, -3b-2b^2x\right)}{b^2}$$
$$= \frac{\sin^{-1}(3+2bx)}{b}$$

**Mathematica [B]** time = 0.01, size = 49, normalized size = 4.45

$$\frac{2\sqrt{bx+1}\sqrt{bx+2}\sinh^{-1}\left(\sqrt{bx+1}\right)}{b\sqrt{-((bx+1)(bx+2))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[1 + b*x]*Sqrt[2 + b*x]*ArcSinh[Sqrt[1 + b*x]])/(b*Sqrt[-((1 + b*x)*
(2 + b*x))])
```

**IntegrateAlgebraic [B]** time = 0.05, size = 26, normalized size = 2.36

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-bx-1}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[-1 - b*x]/Sqrt[2 + b*x]])/b
```

**fricas [B]** time = 1.09, size = 44, normalized size = 4.00

$$-\frac{\arctan\left(\frac{(2\,bx+3)\sqrt{bx+2}\sqrt{-bx-1}}{2\left(b^2x^2+3\,bx+2\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 3)*sqrt(b*x + 2)*sqrt(-b*x - 1)/(b^2*x^2 + 3*b*x + 2))
/b
```

**giac [A]** time = 1.17, size = 13, normalized size = 1.18

$$\frac{2 \arcsin\left(\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(sqrt(b*x + 2))/b
```

**maple [B]** time = 0.01, size = 66, normalized size = 6.00

$$\frac{\sqrt{(-bx-1)(bx+2)}\ \arctan\left(\frac{\sqrt{b^2}\left(x+\frac{3}{2b}\right)}{\sqrt{-b^2x^2-3bx-2}}\right)}{\sqrt{-bx-1}\ \sqrt{bx+2}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-1)*(b*x+2))^(1/2)/(-b*x-1)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+3/2/b)/(-b^2*x^2-3*b*x-2)^(1/2))
```

**maxima [A]** time = 3.01, size = 21, normalized size = 1.91

$$-\frac{\arcsin\left(-\frac{2\,b^2x+3\,b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(2*b^2*x + 3*b)/b)/b
```

**mupad [B]** time = 0.30, size = 41, normalized size = 3.73

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{-b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- b*x - 1)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*((- b*x - 1)^(1/2) - 1i))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)^(1/
2))))/(b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx-1}\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 1)*sqrt(b*x + 2)), x)
```

## 3.1435 $\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=29

$$\frac{\sqrt{bx+2}\,\log(bx+2)}{b\sqrt{-bx-2}}$$

**Rubi** [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {23, 31}

$$\frac{\sqrt{bx+2}\,\log(bx+2)}{b\sqrt{-bx-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (Sqrt[2 + b*x]*Log[2 + b*x])/(b*Sqrt[-2 - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2+bx}}\,dx &= \frac{\sqrt{2+bx}\int \frac{1}{2+bx}\,dx}{\sqrt{-2-bx}}\\ &= \frac{\sqrt{2+bx}\,\log(2+bx)}{b\sqrt{-2-bx}}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 28, normalized size = 0.97

$$\frac{(bx+2)\log(bx+2)}{b\sqrt{-(bx+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ((2 + b*x)*Log[2 + b*x])/(b*Sqrt[-(2 + b*x)^2])
```

**IntegrateAlgebraic [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2-bx}\sqrt{2+bx}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]), x]
```

**fricas [A]** time = 0.99, size = 1, normalized size = 0.03

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

**giac [C]** time = 0.97, size = 12, normalized size = 0.41

$$-\frac{i\,\log\left(|bx+2|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -I*log(abs(b*x + 2))/b
```

**maple [A]** time = 0.00, size = 26, normalized size = 0.90

$$\frac{\sqrt{bx+2}\,\ln\left(bx+2\right)}{\sqrt{-bx-2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ln(b*x+2)*(b*x+2)^(1/2)/b/(-b*x-2)^(1/2)
```

**maxima** [A] time = 1.35, size = 16, normalized size = 0.55

$$\sqrt{-\frac{1}{b^2}}\log\left(x+\frac{2}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-1/b^2)*log(x + 2/b)
```

**mupad** [B] time = 0.07, size = 47, normalized size = 1.62

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{-b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(- b*x - 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2)*1i - (- b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*
(b^2)^(1/2))))/(b^2)^(1/2)
```

**sympy** [C] time = 1.98, size = 53, normalized size = 1.83

$$\begin{cases} -\frac{i\log\left(x+\frac{2}{b}\right)}{b} & \text{for}\ \left|x+\frac{2}{b}\right|<1 \\ \frac{i\log\left(\frac{1}{x+\frac{2}{b}}\right)}{b} & \text{for}\ \frac{1}{\left|x+\frac{2}{b}\right|}<1 \\ \frac{i{G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| {x+\frac{2}{b}} \right)}}{b}-\frac{i{G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| {x+\frac{2}{b}} \right)}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*log(x + 2/b)/b, Abs(x + 2/b) < 1), (I*log(1/(x + 2/b))/b, 1/A
bs(x + 2/b) < 1), (I*meijerg(((), (1, 1)), ((0, 0), ()), x + 2/b)/b - I*mei
jerg(((1, 1), ()), ((), (0, 0)), x + 2/b)/b, True))
```

## 3.1436 $\int \frac{1}{\sqrt{-3-bx}\,\sqrt{2+bx}}\,dx$

**Optimal**. Leaf size=26

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{bx+2}}\right)}{b}$$

**Rubi [A]** time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {63, 217, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[-3 - b*x]/Sqrt[2 + b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-bx}\sqrt{2+bx}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \sqrt{-3-bx}\right)}{b}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}$$

**Mathematica [B]** time = 0.02, size = 53, normalized size = 2.04

$$-\frac{2\sqrt{-bx-3}\,\sqrt{-bx-2}\,\sin^{-1}\left(\sqrt{bx+3}\right)}{b\sqrt{bx+2}\,\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*Sqrt[-3 - b*x]*Sqrt[-2 - b*x]*ArcSin[Sqrt[3 + b*x]])/(b*Sqrt[2 + b*x]*S
qrt[3 + b*x])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 26, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*ArcTan[Sqrt[-3 - b*x]/Sqrt[2 + b*x]])/b
```

**fricas [A]** time = 0.82, size = 44, normalized size = 1.69

$$-\frac{\arctan\left(\frac{(2\,bx+5)\sqrt{bx+2}\,\sqrt{-bx-3}}{2\left(b^2x^2+5\,bx+6\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 5)*sqrt(b*x + 2)*sqrt(-b*x - 3)/(b^2*x^2 + 5*b*x + 6))
/b
```

**giac [C]** time = 1.07, size = 23, normalized size = 0.88

$$\frac{2i\log\left(\sqrt{bx+3}-\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*I*log(sqrt(b*x + 3) - sqrt(b*x + 2))/b
```

**maple [B]** time = 0.01, size = 66, normalized size = 2.54

$$\frac{\sqrt{(-bx-3)(bx+2)}\ \arctan\left(\frac{\sqrt{b^2}\left(x+\frac{5}{2b}\right)}{\sqrt{-b^2x^2-5bx-6}}\right)}{\sqrt{-bx-3}\ \sqrt{bx+2}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-3)*(b*x+2))^(1/2)/(-b*x-3)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+5/2/b)/(-b^2*x^2-5*b*x-6)^(1/2))
```

**maxima [A]** time = 3.01, size = 21, normalized size = 0.81

$$-\frac{\arcsin\left(-\frac{2\,b^2x+5\,b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(2*b^2*x + 5*b)/b)/b
```

**mupad [B]** time = 0.30, size = 47, normalized size = 1.81

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{-b\,x-3}+\sqrt{3}\ 1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(- b*x - 3)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2)*1i - (- b*x - 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*
(b^2)^(1/2))))/(b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx-3}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 3)*sqrt(b*x + 2)), x)
```

## 3.1437 $\int \frac{1}{\sqrt{2-bx}\sqrt{3-bx}}\,dx$

**Optimal**. Leaf size=16

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

**Rubi** [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[2 - b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-bx}\sqrt{3-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
[Out] (-2*ArcSinh[Sqrt[2 - b*x]])/b
```

**IntegrateAlgebraic** [B] time = 0.06, size = 59, normalized size = 3.69

$$\frac{\log\left(\frac{\sqrt{3-bx}}{\sqrt{2-bx}}-1\right)}{b}-\frac{\log\left(\frac{b\sqrt{3-bx}}{\sqrt{2-bx}}+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
[Out] Log[-1 + Sqrt[3 - b*x]/Sqrt[2 - b*x]]/b - Log[b + (b*Sqrt[3 - b*x])/Sqrt[2
- b*x]]/b
```

**fricas** [B] time = 1.07, size = 30, normalized size = 1.88

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+3}\,\sqrt{-bx+2}\,+5\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(-b*x + 3)*sqrt(-b*x + 2) + 5)/b
```

**giac** [A] time = 1.04, size = 25, normalized size = 1.56

$$\frac{2\,\log\left(\sqrt{-bx+3}\,-\sqrt{-bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="giac")
[Out] 2*log(sqrt(-b*x + 3) - sqrt(-b*x + 2))/b
```

**maple [B]** time = 0.01, size = 70, normalized size = 4.38

$$\frac{\sqrt{(-bx+2)(-bx+3)}\ \ln\left(\frac{b^2x-\frac{5}{2}b}{\sqrt{b^2}}+\sqrt{b^2x^2-5bx+6}\right)}{\sqrt{-bx+2}\ \sqrt{-bx+3}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x)
```

```
[Out] ((-b*x+2)*(-b*x+3))^(1/2)/(-b*x+2)^(1/2)/(-b*x+3)^(1/2)*ln((-5/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-5*b*x+6)^(1/2))/(b^2)^(1/2)
```

**maxima [B]** time = 1.36, size = 33, normalized size = 2.06

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-5\,bx+6}\,b-5\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 5*b*x + 6)*b - 5*b)/b
```

**mupad [B]** time = 0.31, size = 49, normalized size = 3.06

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{3}-\sqrt{3-b\,x}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(3 - b*x)^(1/2)),x)
```

```
[Out] (4*atan((b*(3^(1/2) - (3 - b*x)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(-b^2)
^(1/2))))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-bx+2}\,\sqrt{-bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(1/2)/(-b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 2)*sqrt(-b*x + 3)), x)
```

## 3.1438 $\int \frac{1}{2-bx} dx$

**Optimal**. Leaf size=12

$$-\frac{\log(2-bx)}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {31}

$$-\frac{\log(2-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(-1),x]
```

```
[Out] -(Log[2 - b*x]/b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2-bx} dx = -\frac{\log(2-bx)}{b}$$

**Mathematica [A]** time = 0.00, size = 12, normalized size = 1.00

$$-\frac{\log(2-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(-1),x]
```

```
[Out] -(Log[2 - b*x]/b)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2-bx} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 - b*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(2 - b*x)^(-1), x]
```

**fricas [A]** time = 0.67, size = 11, normalized size = 0.92

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="fricas")
```

```
[Out] -log(b*x - 2)/b
```

**giac [A]** time = 1.10, size = 12, normalized size = 1.00

$$-\frac{\log(|bx-2|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="giac")
```

```
[Out] -log(abs(b*x - 2))/b
```

**maple [A]** time = 0.00, size = 13, normalized size = 1.08

$$-\frac{\ln(-bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2),x)
```

```
[Out] -ln(-b*x+2)/b
```

**maxima [A]** time = 1.42, size = 11, normalized size = 0.92

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="maxima")
```

```
[Out] -log(b*x - 2)/b
```

**mupad [B]** time = 0.03, size = 11, normalized size = 0.92

$$-\frac{\ln\left(b\,x-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(b*x - 2),x)
```

```
[Out] -log(b*x - 2)/b
```

**sympy [A]** time = 0.07, size = 8, normalized size = 0.67

$$-\frac{\log\left(bx-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x)
```

```
[Out] -log(b*x - 2)/b
```

## 3.1439 $\int \frac{1}{\sqrt{1-bx}\,\sqrt{2-bx}}\,dx$

**Optimal**. Leaf size=16

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

**Rubi** [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[1 - b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-bx}\,\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{1-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[1 - b*x]])/b
```

**IntegrateAlgebraic [B]** time = 0.06, size = 59, normalized size = 3.69

$$\frac{\log\left(\frac{\sqrt{2-bx}}{\sqrt{1-bx}}-1\right)}{b}-\frac{\log\left(\frac{b\sqrt{2-bx}}{\sqrt{1-bx}}+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] Log[-1 + Sqrt[2 - b*x]/Sqrt[1 - b*x]]/b - Log[b + (b*Sqrt[2 - b*x])/Sqrt[1
- b*x]]/b
```

**fricas [B]** time = 0.91, size = 30, normalized size = 1.88

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\,\sqrt{-bx+1}\,+3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x + 1) + 3)/b
```

**giac [A]** time = 0.85, size = 25, normalized size = 1.56

$$\frac{2\,\log\left(\sqrt{-bx+2}-\sqrt{-bx+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x + 1))/b
```

**maple [B]** time = 0.01, size = 70, normalized size = 4.38

$$\frac{\sqrt{(-bx+1)(-bx+2)}\ \ln\left(\frac{b^2x-\frac{3}{2}b}{\sqrt{b^2}}+\sqrt{b^2x^2-3bx+2}\right)}{\sqrt{-bx+1}\ \sqrt{-bx+2}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+1)*(-b*x+2))^(1/2)/(-b*x+1)^(1/2)/(-b*x+2)^(1/2)*ln((-3/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-3*b*x+2)^(1/2))/(b^2)^(1/2)
```

**maxima [B]** time = 1.33, size = 33, normalized size = 2.06

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-3\,bx+2}\,b-3\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 3*b*x + 2)*b - 3*b)/b
```

**mupad [B]** time = 0.31, size = 45, normalized size = 2.81

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\left(\sqrt{1-b\,x}-1\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - b*x)^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/(((1 - b*x)^(1/2) - 1)*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-bx+1}\,\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 1)*sqrt(-b*x + 2)), x)
```

## 3.1440 $\int \frac{1}{\sqrt{-bx}\sqrt{2-bx}}\,dx$

**Optimal**. Leaf size=20

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}$$

**Rubi [A]** time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-(b*x)]/Sqrt[2]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-bx}\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \sqrt{-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 37, normalized size = 1.85

$$\frac{2\sqrt{x}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}\,\sqrt{-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]
[Out] (2*Sqrt[x]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(Sqrt[b]*Sqrt[-(b*x)])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 27, normalized size = 1.35

$$\frac{2\log\left(\sqrt{2-bx}-\sqrt{-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]
[Out] (2*Log[-Sqrt[-(b*x)] + Sqrt[2 - b*x]])/b
```

**fricas** [A] time = 0.70, size = 27, normalized size = 1.35

$$-\frac{\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-bx}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(-b*x + 2)*sqrt(-b*x) + 1)/b
```

**giac** [A] time = 1.03, size = 23, normalized size = 1.15

$$\frac{2\,\log\left(\sqrt{-bx+2}-\sqrt{-bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x))/b
```

**maple** [B] time = 0.01, size = 64, normalized size = 3.20

$$\frac{\sqrt{-(-bx+2)\,bx}\;\ln\left(\frac{b^2x-b}{\sqrt{b^2}}+\sqrt{b^2x^2-2bx}\right)}{\sqrt{-bx}\;\sqrt{-bx+2}\;\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] (-x*b*(-b*x+2))^(1/2)/(-b*x)^(1/2)/(-b*x+2)^(1/2)*ln((b^2*x-b)/(b^2)^(1/2)+
(b^2*x^2-2*b*x)^(1/2))/(b^2)^(1/2)
```

**maxima [A]** time = 1.36, size = 32, normalized size = 1.60

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-2\,bx}\,b-2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 2*b*x)*b - 2*b)/b
```

**mupad [B]** time = 0.28, size = 39, normalized size = 1.95

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\sqrt{-b\,x}\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x)^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/((-b*x)^(1/2)*(-b^2)^(1/2))))/(-b^
2)^(1/2)
```

**sympy [A]** time = 1.34, size = 53, normalized size = 2.65

$$\begin{cases} -\frac{2\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b} & \text{for}\ \frac{|bx|}{2}>1 \\ -\frac{2i\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b, Abs(b*x)/2 > 1), (-2*I*as
in(sqrt(2)*sqrt(b)*sqrt(x)/2)/b, True))
```

## 3.1441 $\int \frac{1}{\sqrt{-1-bx}\sqrt{2-bx}}\,dx$

**Optimal**. Leaf size=22

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-1}}{\sqrt{3}}\right)}{b}$$

**Rubi [A]** time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-1}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-1 - b*x]/Sqrt[3]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-bx}\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-1-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-1-bx}}{\sqrt{3}}\right)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 1.82

$$-\frac{2\sqrt{-bx-1}\,\sin^{-1}\left(\frac{\sqrt{bx+1}}{\sqrt{3}}\right)}{b\sqrt{bx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
[Out] (-2*Sqrt[-1 - b*x]*ArcSin[Sqrt[1 + b*x]/Sqrt[3]])/(b*Sqrt[1 + b*x])
```

**IntegrateAlgebraic** [A] time = 0.06, size = 27, normalized size = 1.23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-bx-1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-1 - b*x]])/b
```

**fricas** [A] time = 0.77, size = 30, normalized size = 1.36

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\,\sqrt{-bx-1}\,+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x - 1) + 1)/b
```

**giac** [A] time = 1.10, size = 25, normalized size = 1.14

$$\frac{2\,\log\left(\sqrt{-bx+2}\,-\sqrt{-bx-1}\,\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 1))/b
```

**maple [B]** time = 0.01, size = 70, normalized size = 3.18

$$\frac{\sqrt{(-bx-1)(-bx+2)}\ \ln\left(\frac{b^2x-\frac{1}{2}b}{\sqrt{b^2}}+\sqrt{b^2x^2-bx-2}\right)}{\sqrt{-bx-1}\ \sqrt{-bx+2}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-1)*(-b*x+2))^(1/2)/(-b*x-1)^(1/2)/(-b*x+2)^(1/2)*ln((b^2*x-1/2*b)/(b
^2)^(1/2)+(b^2*x^2-b*x-2)^(1/2))/(b^2)^(1/2)
```

**maxima [A]** time = 1.38, size = 33, normalized size = 1.50

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-bx-2}\,b-b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - b*x - 2)*b - b)/b
```

**mupad [B]** time = 0.28, size = 46, normalized size = 2.09

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{-b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- b*x - 1)^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((- b*x - 1)^(1/2) - 1i))/((2^(1/2) - (2 - b*x)^(1/2))*(-b^2)^(
1/2))))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx-1}\,\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 1)*sqrt(-b*x + 2)), x)
```

## 3.1442 $\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2-bx}}\,dx$

**Optimal**. Leaf size=12

$$-\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {52}

$$-\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] -(ArcCosh[-(b*x)/2]/b)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2-bx}}\,dx = -\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

**Mathematica [B]** time = 0.00, size = 27, normalized size = 2.25

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{-bx-2}}{\sqrt{2-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[-2 - b*x]/Sqrt[2 - b*x]])/b
```

**IntegrateAlgebraic [B]** time = 0.05, size = 27, normalized size = 2.25

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-bx-2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]
[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-2 - b*x]])/b
```

**fricas [B]** time = 1.02, size = 28, normalized size = 2.33

$$-\frac{\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(-b*x + 2)*sqrt(-b*x - 2))/b
```

**giac [B]** time = 1.10, size = 25, normalized size = 2.08

$$\frac{2\,\log\left(\sqrt{-bx+2}-\sqrt{-bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 2))/b
```

**maple [B]** time = 0.01, size = 61, normalized size = 5.08

$$\frac{\sqrt{(-bx-2)(-bx+2)}\,\ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2-4}\right)}{\sqrt{-bx-2}\,\sqrt{-bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x)
[Out] ((-b*x-2)*(-b*x+2))^(1/2)/(-b*x-2)^(1/2)/(-b*x+2)^(1/2)*ln(1/(b^2)^(1/2)*b^
2*x+(b^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

**maxima [B]** time = 1.27, size = 26, normalized size = 2.17

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b
```

**mupad [B]** time = 0.29, size = 52, normalized size = 4.33

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{-b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(- b*x - 2)^(1/2)),x)
```

```
[Out] (4*atan((b*(2^(1/2)*1i - (- b*x - 2)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(
-b^2)^(1/2))))/(-b^2)^(1/2)
```

**sympy [C]** time = 4.63, size = 78, normalized size = 6.50

$$-\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{4e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] -meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4/
(b**2*x**2))/(4*pi**(3/2)*b) - I*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ())
, ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**
(3/2)*b)
```

## 3.1443 $\int \frac{1}{\sqrt{-3-bx}\sqrt{2-bx}}\,dx$

**Optimal**. Leaf size=22

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{5}}\right)}{b}$$

**Rubi [A]** time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-3 - b*x]/Sqrt[5]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3-bx}\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, \sqrt{-3-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{5}}\right)}{b}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 1.82

$$-\frac{2\sqrt{-bx-3}\,\sin^{-1}\left(\frac{\sqrt{bx+3}}{\sqrt{5}}\right)}{b\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
[Out] (-2*Sqrt[-3 - b*x]*ArcSin[Sqrt[3 + b*x]/Sqrt[5]])/(b*Sqrt[3 + b*x])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 27, normalized size = 1.23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-bx-3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-3 - b*x]])/b
```

**fricas** [A] time = 1.12, size = 30, normalized size = 1.36

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\,\sqrt{-bx-3}\,-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x - 3) - 1)/b
```

**giac** [A] time = 1.28, size = 25, normalized size = 1.14

$$\frac{2\,\log\left(\sqrt{-bx+2}\,-\sqrt{-bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 3))/b
```

**maple [B]** time = 0.01, size = 69, normalized size = 3.14

$$\frac{\sqrt{(-bx-3)(-bx+2)}\,\ln\left(\frac{b^2x+\frac{1}{2}b}{\sqrt{b^2}}+\sqrt{b^2x^2+bx-6}\right)}{\sqrt{-bx-3}\,\sqrt{-bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-3)*(-b*x+2))^(1/2)/(-b*x-3)^(1/2)/(-b*x+2)^(1/2)*ln((b^2*x+1/2*b)/(b
^2)^(1/2)+(b^2*x^2+b*x-6)^(1/2))/(b^2)^(1/2)
```

**maxima [A]** time = 1.37, size = 30, normalized size = 1.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+bx-6}\,b+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x - 6)*b + b)/b
```

**mupad [B]** time = 0.29, size = 52, normalized size = 2.36

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{-b\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(- b*x - 3)^(1/2)),x)
```

```
[Out] (4*atan((b*(3^(1/2)*1i - (- b*x - 3)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(
-b^2)^(1/2))))/(-b^2)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-bx-3}\,\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 3)*sqrt(-b*x + 2)), x)
```

## 3.1444 $\int \frac{1}{\sqrt{-4+bx}\,\sqrt{4+bx}}\,dx$

**Optimal**. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

**Rubi [A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {52}

$$\frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/4]/b
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-4+bx}\,\sqrt{4+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

**Mathematica [B]** time = 0.00, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx-4}}{\sqrt{bx+4}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[-4 + b*x]/Sqrt[4 + b*x]])/b
```

**IntegrateAlgebraic [B]** time = 0.05, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+4}}{\sqrt{bx-4}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[4 + b*x]/Sqrt[-4 + b*x]])/b
```

**fricas [B]** time = 0.84, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx+\sqrt{bx+4}\sqrt{bx-4}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 4)*sqrt(b*x - 4))/b
```

**giac [B]** time = 1.04, size = 23, normalized size = 2.09

$$-\frac{2\log\left(\sqrt{bx+4}-\sqrt{bx-4}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 4) - sqrt(b*x - 4))/b
```

**maple [B]** time = 0.01, size = 57, normalized size = 5.18

$$\frac{\sqrt{(bx-4)(bx+4)}\ \ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2-16}\right)}{\sqrt{bx-4}\ \sqrt{bx+4}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x)
```

```
[Out] ((b*x-4)*(b*x+4))^(1/2)/(b*x-4)^(1/2)/(b*x+4)^(1/2)*ln(1/(b^2)^(1/2)*b^2*x+
(b^2*x^2-16)^(1/2))/(b^2)^(1/2)
```

**maxima** [B] time = 1.38, size = 26, normalized size = 2.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-16}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 16)*b)/b
```

**mupad** [B] time = 0.32, size = 40, normalized size = 3.64

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{b\,x-4}-2\mathrm{i}\right)}{\left(\sqrt{b\,x+4}-2\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 4)^(1/2)*(b*x + 4)^(1/2)),x)
```

```
[Out] -(4*atan((b*((b*x - 4)^(1/2) - 2i))/(((b*x + 4)^(1/2) - 2)*(-b^2)^(1/2))))/
(-b^2)^(1/2)
```

**sympy** [C] time = 4.21, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| {\frac{16e^{2i\pi}}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| {\frac{16}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)**(1/2)/(b*x+4)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 16*
exp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 16/(b**2*x**2))/(4*pi**
(3/2)*b)
```

## 3.1445 $\int \frac{1}{\sqrt{\frac{-b+bc}{d}+bx}\,\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=43

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{bx-\frac{b(1-c)}{d}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

**Rubi [A]** time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{bx-\frac{b(1-c)}{d}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[d]*Sqrt[-((b*(1 - c))/d) + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{-b+bc}{d}+bx}\,\sqrt{c+dx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{-b+bc}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{\frac{-b+bc}{d}+bx}\right)}{b}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{-\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

**Mathematica [A]** time = 0.03, size = 41, normalized size = 0.95

$$\frac{2\sqrt{c+dx-1}\,\sinh^{-1}\left(\sqrt{c+dx-1}\right)}{d\sqrt{\frac{b(c+dx-1)}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[-1 + c + d*x]*ArcSinh[Sqrt[-1 + c + d*x]])/(d*Sqrt[(b*(-1 + c + d*x
))/d])
```

**IntegrateAlgebraic [A]** time = 0.11, size = 57, normalized size = 1.33

$$-\frac{2\sqrt{\frac{b}{d}}\,\log\left(\sqrt{\frac{b(c+dx)}{d}-\frac{b}{d}}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[b/d]*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) + Sqrt[-(b/d) + (b*(c + d*x))/
d]])/b
```

**fricas [B]** time = 0.90, size = 175, normalized size = 4.07

$$\left[\frac{\sqrt{bd}\,\log\left(8\,bd^2x^2+8\,bc^2+8\,(2\,bc-b)dx+4\,\sqrt{bd}\,(2\,dx+2\,c-1)\sqrt{dx+c}\,\sqrt{\frac{bdx+bc-b}{d}}-8\,bc+b\right)}{2\,bd}, -\frac{\sqrt{-bd}\,\arctan\left(\frac{\sqrt{-bd}\,(2\,dx+2\,c-1)\sqrt{dx+c}\,\sqrt{\frac{bdx+bc-b}{d}}}{2\left(bd^2x^2+bc^2+(2\,bc-b)dx-bc\right)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b*d^2*x^2 + 8*b*c^2 + 8*(2*b*c - b)*d*x + 4*sqrt(b*d)*
(2*d*x + 2*c - 1)*sqrt(d*x + c)*sqrt((b*d*x + b*c - b)/d) - 8*b*c + b)/(b*d
), -sqrt(-b*d)*arctan(1/2*sqrt(-b*d)*(2*d*x + 2*c - 1)*sqrt(d*x + c)*sqrt((
b*d*x + b*c - b)/d)/(b*d^2*x^2 + b*c^2 + (2*b*c - b)*d*x - b*c))/(b*d)]
```

**giac [A]** time = 0.97, size = 57, normalized size = 1.33

$$-\frac{2\,b\log\left(-\sqrt{bd}\,\sqrt{\frac{bdx+bc-b}{d}}+\sqrt{(bdx+bc-b)b+b^2}\right)}{\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*log(-sqrt(b*d)*sqrt((b*d*x + b*c - b)/d) + sqrt((b*d*x + b*c - b)*b +
b^2))/(sqrt(b*d)*abs(b))
```

**maple [B]** time = 0.02, size = 100, normalized size = 2.33

$$\frac{\sqrt{\left(bx+\frac{(c-1)b}{d}\right)(dx+c)}\;\ln\left(\frac{bdx+\frac{bc}{2}+\frac{(c-1)b}{2}}{\sqrt{bd}}+\sqrt{bd\,x^2+\frac{(c-1)bc}{d}+(bc+(c-1)\,b)\,x}\right)}{\sqrt{bx+\frac{(c-1)b}{d}}\;\sqrt{dx+c}\;\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] ((b*x+b*(c-1)/d)*(d*x+c))^(1/2)/(b*x+b*(c-1)/d)^(1/2)/(d*x+c)^(1/2)*ln((1/2
*b*(c-1)+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(b*(c-1)+b*c)*x+b*(c-1)/d*c)^(
1/2))/(b*d)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c-1>0)', see `assume?` for more d
etails)Is 2*c-1 zero or nonzero?
```

**mupad [B]** time = 0.50, size = 66, normalized size = 1.53

$$\frac{4\,\mathrm{atan}\left(-\frac{d\left(\sqrt{b\,x-\frac{b-b\,c}{d}}-\sqrt{-\frac{b-b\,c}{d}}\right)}{\sqrt{-b\,d}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - (b - b*c)/d)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (4*atan(-(d*((b*x - (b - b*c)/d)^(1/2) - (-(b - b*c)/d)^(1/2)))/((-b*d)^(1/
2)*((c + d*x)^(1/2) - c^(1/2)))))/(-b*d)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\left(\frac{c}{d}+x-\frac{1}{d}\right)}\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*(c/d + x - 1/d))*sqrt(c + d*x)), x)
```

## 3.1446 $\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx$

**Optimal**. Leaf size=22

$$\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-3}}{\sqrt{3}}\right)$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {54, 215}

$$\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-3}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
```

```
[Out] Sqrt[2]*ArcSinh[Sqrt[-3 + 2*x]/Sqrt[3]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-3+2x}\right)\\ &= \sqrt{2}\sinh^{-1}\left(\frac{\sqrt{-3+2x}}{\sqrt{3}}\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 1.41

$$\frac{\sqrt{4x-6}\,\sin^{-1}\left(\sqrt{1-\frac{2x}{3}}\right)}{\sqrt{3-2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
[Out] (Sqrt[-6 + 4*x]*ArcSin[Sqrt[1 - (2*x)/3]])/Sqrt[3 - 2*x]
```

**IntegrateAlgebraic [A]** time = 0.04, size = 30, normalized size = 1.36

$$-\sqrt{2}\log\left(\sqrt{2x-3}-\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
[Out] -(Sqrt[2]*Log[-(Sqrt[2]*Sqrt[x]) + Sqrt[-3 + 2*x]])
```

**fricas [A]** time = 0.98, size = 26, normalized size = 1.18

$$\frac{1}{2}\sqrt{2}\log\left(-2\sqrt{2}\sqrt{2x-3}\sqrt{x}-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log(-2*sqrt(2)*sqrt(2*x - 3)*sqrt(x) - 4*x + 3)
```

**giac [A]** time = 0.92, size = 23, normalized size = 1.05

$$-\sqrt{2}\log\left(\sqrt{2}\sqrt{x}-\sqrt{2x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="giac")
[Out] -sqrt(2)*log(sqrt(2)*sqrt(x) - sqrt(2*x - 3))
```

**maple [B]** time = 0.01, size = 48, normalized size = 2.18

$$\frac{\sqrt{(2x-3)x}\,\sqrt{2}\,\ln\left(\frac{\left(2x-\frac{3}{2}\right)\sqrt{2}}{2}+\sqrt{2x^2-3x}\right)}{2\sqrt{2x-3}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-3+2*x)^(1/2),x)
```

```
[Out] 1/2*(x*(-3+2*x))^(1/2)/x^(1/2)/(-3+2*x)^(1/2)*ln(1/2*(-3/2+2*x)*2^(1/2)+(2*
x^2-3*x)^(1/2))*2^(1/2)
```

**maxima [B]** time = 2.87, size = 41, normalized size = 1.86

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2x-3}}{\sqrt{x}}}{\sqrt{2}+\frac{\sqrt{2x-3}}{\sqrt{x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sqrt(2*x - 3)/sqrt(x))/(sqrt(2) + sqrt(2*x - 3
)/sqrt(x)))
```

**mupad [B]** time = 0.44, size = 30, normalized size = 1.36

$$-2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\left(-\sqrt{2x-3}+\sqrt{3}\,1\mathrm{i}\right)}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2*x - 3)^(1/2)),x)
[Out] -2*2^(1/2)*atanh((2^(1/2)*(3^(1/2)*1i - (2*x - 3)^(1/2)))/(2*x^(1/2)))
```

**sympy [A]** time = 1.03, size = 44, normalized size = 2.00

$$\begin{cases} \sqrt{2}\,\mathrm{acosh}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{for}\ \frac{2|x|}{3}>1 \\ -\sqrt{2}\,i\,\mathrm{asin}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-3+2*x)**(1/2),x)
[Out] Piecewise((sqrt(2)*acosh(sqrt(6)*sqrt(x)/3), 2*Abs(x)/3 > 1), (-sqrt(2)*I*a
sin(sqrt(6)*sqrt(x)/3), True))
```

## 3.1447 $\int \frac{1}{\sqrt{-3+2x}\sqrt{2+3x}}\,dx$

**Optimal**. Leaf size=26

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{2x-3}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {54, 215}

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{2x-3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] Sqrt[2/3]*ArcSinh[Sqrt[3/13]*Sqrt[-3 + 2*x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+2x}\sqrt{2+3x}}\,dx &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{13+3x^2}}\,dx, x, \sqrt{-3+2x}\right)\\ &= \sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{-3+2x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 1.00

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{2x-3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] Sqrt[2/3]*ArcSinh[Sqrt[3/13]*Sqrt[-3 + 2*x]]
```

**IntegrateAlgebraic [A]** time = 0.07, size = 35, normalized size = 1.35

$$\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt{3x+2}}{\sqrt{2x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] Sqrt[2/3]*ArcTanh[(Sqrt[2/3]*Sqrt[2 + 3*x])/Sqrt[-3 + 2*x]]
```

**fricas [B]** time = 1.07, size = 46, normalized size = 1.77

$$\frac{1}{12}\sqrt{3}\sqrt{2}\log\left(4\sqrt{3}\sqrt{2}(12x-5)\sqrt{3x+2}\sqrt{2x-3}+288x^2-240x-119\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*sqrt(2)*log(4*sqrt(3)*sqrt(2)*(12*x - 5)*sqrt(3*x + 2)*sqrt(2*
x - 3) + 288*x^2 - 240*x - 119)
```

**giac [A]** time = 1.26, size = 30, normalized size = 1.15

$$-\frac{1}{3}\sqrt{3}\sqrt{2}\log\left(\left|-\sqrt{2}\sqrt{3x+2}+\sqrt{6x-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*sqrt(2)*log(abs(-sqrt(2)*sqrt(3*x + 2) + sqrt(6*x - 9)))
```

**maple [B]** time = 0.01, size = 57, normalized size = 2.19

$$\frac{\sqrt{(2x-3)(3x+2)}\,\sqrt{6}\,\ln\left(\frac{\left(6x-\frac{5}{2}\right)\sqrt{6}}{6}+\sqrt{6x^2-5x-6}\right)}{6\sqrt{2x-3}\,\sqrt{3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x-3)^(1/2)/(3*x+2)^(1/2),x)
```

```
[Out] 1/6*((2*x-3)*(3*x+2))^(1/2)/(2*x-3)^(1/2)/(3*x+2)^(1/2)*ln(1/6*(-5/2+6*x)*6
^(1/2)+(6*x^2-5*x-6)^(1/2))*6^(1/2)
```

**maxima [A]** time = 3.04, size = 28, normalized size = 1.08

$$\frac{1}{6}\sqrt{6}\log\left(2\sqrt{6}\sqrt{6x^2-5x-6}+12x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(6)*log(2*sqrt(6)*sqrt(6*x^2 - 5*x - 6) + 12*x - 5)
```

**mupad [B]** time = 0.12, size = 43, normalized size = 1.65

$$\frac{2\sqrt{6}\operatorname{atanh}\left(\frac{\sqrt{6}\left(-\sqrt{2x-3}+\sqrt{3}\,1\mathrm{i}\right)}{2\left(\sqrt{2}-\sqrt{3x+2}\right)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x - 3)^(1/2)*(3*x + 2)^(1/2)),x)
```

```
[Out] (2*6^(1/2)*atanh((6^(1/2)*(3^(1/2)*1i - (2*x - 3)^(1/2)))/(2*(2^(1/2) - (3*
x + 2)^(1/2)))))/3
```

**sympy [A]** time = 1.09, size = 58, normalized size = 2.23

$$\begin{cases} \frac{\sqrt{6}\operatorname{acosh}\left(\frac{\sqrt{78}\sqrt{x+\frac{2}{3}}}{13}\right)}{3} & \text{for}\ \frac{6\left|x+\frac{2}{3}\right|}{13}>1 \\ -\frac{\sqrt{6}\,i\operatorname{asin}\left(\frac{\sqrt{78}\sqrt{x+\frac{2}{3}}}{13}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(6)*acosh(sqrt(78)*sqrt(x + 2/3)/13)/3, 6*Abs(x + 2/3)/13 >
1), (-sqrt(6)*I*asin(sqrt(78)*sqrt(x + 2/3)/13)/3, True))
```

## 3.1448 $\int \frac{1}{\sqrt{\frac{b-bc}{d}+bx}\sqrt{c-dx}}\,dx$

**Optimal**. Leaf size=42

$$\frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

**Rubi** [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {63, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]
```

```
[Out] (2*ArcSin[(Sqrt[d]*Sqrt[(b*(1 - c))/d + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{b-bc}{d}+bx}\,\sqrt{c-dx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{b-bc}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{\frac{b-bc}{d}+bx}\right)}{b}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

**Mathematica [A]** time = 0.05, size = 67, normalized size = 1.60

$$\frac{2\sqrt{-d}\,\sqrt{-c+dx+1}\,\sinh^{-1}\left(\frac{\sqrt{-d}\,\sqrt{c-dx}}{\sqrt{d}}\right)}{d^{3/2}\sqrt{\frac{b(-c+dx+1)}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]
```

```
[Out] (2*Sqrt[-d]*Sqrt[1 - c + d*x]*ArcSinh[(Sqrt[-d]*Sqrt[c - d*x])/Sqrt[d]])/(d
^(3/2)*Sqrt[(b*(1 - c + d*x))/d])
```

**IntegrateAlgebraic [A]** time = 0.12, size = 61, normalized size = 1.45

$$-\frac{2\sqrt{-\frac{b}{d}}\,\log\left(\sqrt{\frac{b}{d}-\frac{b(c-dx)}{d}}-\sqrt{-\frac{b}{d}}\,\sqrt{c-dx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]
```

```
[Out] (-2*Sqrt[-(b/d)]*Log[-(Sqrt[-(b/d)]*Sqrt[c - d*x]) + Sqrt[b/d - (b*(c - d*x
))/d]])/b
```

**fricas [B]** time = 0.85, size = 176, normalized size = 4.19

$$\left[-\frac{\sqrt{-bd}\,\log\left(8\,bd^2x^2+8\,bc^2-8\,(2\,bc-b)dx-4\,\sqrt{-bd}\,(2\,dx-2\,c+1)\sqrt{-dx+c}\,\sqrt{\frac{bdx-bc+b}{d}}-8\,bc+b\right)}{2\,bd}, -\frac{\sqrt{bd}\,\arctan\left(\frac{\sqrt{bd}\,(2\,dx-2\,c+1)\sqrt{-dx+c}\,\sqrt{\frac{bdx-bc+b}{d}}}{2\left(bd^2x^2+bc^2-(2\,bc-b)dx-bc\right)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b*d^2*x^2 + 8*b*c^2 - 8*(2*b*c - b)*d*x - 4*sqrt(-b*
d)*(2*d*x - 2*c + 1)*sqrt(-d*x + c)*sqrt((b*d*x - b*c + b)/d) - 8*b*c + b)/
(b*d), -sqrt(b*d)*arctan(1/2*sqrt(b*d)*(2*d*x - 2*c + 1)*sqrt(-d*x + c)*sqr
t((b*d*x - b*c + b)/d)/(b*d^2*x^2 + b*c^2 - (2*b*c - b)*d*x - b*c))/(b*d)]
```

**giac [A]** time = 1.08, size = 58, normalized size = 1.38

$$-\frac{2\,b\log\left(-\sqrt{-bd}\,\sqrt{\frac{bdx-bc+b}{d}}+\sqrt{-(bdx-bc+b)b+b^2}\right)}{\sqrt{-bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*log(-sqrt(-b*d)*sqrt((b*d*x - b*c + b)/d) + sqrt(-(b*d*x - b*c + b)*b
+ b^2))/(sqrt(-b*d)*abs(b))
```

**maple [B]** time = 0.04, size = 118, normalized size = 2.81

$$\frac{\sqrt{\left(bx+\frac{(-c+1)b}{d}\right)(-dx+c)}\;\arctan\left(\frac{\sqrt{bd}\left(x-\frac{bc-(-c+1)b}{2bd}\right)}{\sqrt{-bd\,x^2+\frac{(-c+1)bc}{d}+(bc-(-c+1)b)x}}\right)}{\sqrt{bx+\frac{(-c+1)b}{d}}\;\sqrt{-dx+c}\;\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x)
```

```
[Out] ((b*(1-c)/d+b*x)*(-d*x+c))^(1/2)/(b*(1-c)/d+b*x)^(1/2)/(-d*x+c)^(1/2)/(b*d)
^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(-b*(1-c)+b*c)/b/d)/(-b*d*x^2+(-b*(1-c)+b*
c)*x+b*(1-c)/d*c)^(1/2))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c-1>0)', see `assume?` for more d
etails)Is 2*c-1 zero or nonzero?
```

**mupad [B]** time = 0.51, size = 63, normalized size = 1.50

$$-\frac{4\,\mathrm{atan}\left(-\frac{d\,\left(\sqrt{\frac{b-b\,c}{d}+b\,x}-\sqrt{\frac{b-b\,c}{d}}\right)}{\sqrt{b\,d}\,\left(\sqrt{c-d\,x}-\sqrt{c}\right)}\right)}{\sqrt{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((((b - b*c)/d + b*x)^(1/2)*(c - d*x)^(1/2)),x)
```

```
[Out] -(4*atan(-(d*((((b - b*c)/d + b*x)^(1/2) - ((b - b*c)/d)^(1/2)))/((b*d)^(1/2
)*((c - d*x)^(1/2) - c^(1/2)))))/(b*d)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\left(-\frac{c}{d}+x+\frac{1}{d}\right)}\sqrt{c-dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)**(1/2)/(-d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*(-c/d + x + 1/d))*sqrt(c - d*x)), x)
```

## 3.1449 $\int \frac{1}{\sqrt{4-x}\sqrt{x}} dx$

**Optimal**. Leaf size=10

$$-\sin^{-1}\left(1-\frac{x}{2}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {53, 619, 216}

$$-\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[4 - x]*Sqrt[x]),x]
```

```
[Out] -ArcSin[1 - x/2]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx = \int \frac{1}{\sqrt{4x-x^2}}\,dx$$
$$= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, 4-2x\right)\right)$$
$$= -\sin^{-1}\left(1-\frac{x}{2}\right)$$

**Mathematica [A]** time = 0.01, size = 14, normalized size = 1.40

$$-2\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 - x]*Sqrt[x]),x]
[Out] -2*ArcSin[Sqrt[1 - x/4]]
```

**IntegrateAlgebraic [C]** time = 0.04, size = 24, normalized size = 2.40

$$2i\log\left(\sqrt{4-x}-i\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[4 - x]*Sqrt[x]),x]
[Out] (2*I)*Log[Sqrt[4 - x] - I*Sqrt[x]]
```

**fricas [B]** time = 0.98, size = 14, normalized size = 1.40

$$-2\arctan\left(\frac{\sqrt{-x+4}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-x + 4)/sqrt(x))
```

**giac [A]** time = 1.10, size = 8, normalized size = 0.80

$$2\arcsin\left(\frac{1}{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/2*sqrt(x))
```

**maple [B]** time = 0.00, size = 27, normalized size = 2.70

$$\frac{\sqrt{(-x+4)\,x}\;\arcsin\left(\frac{x}{2}-1\right)}{\sqrt{-x+4}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-x)^(1/2)/x^(1/2),x)
[Out] ((4-x)*x)^(1/2)/(4-x)^(1/2)/x^(1/2)*arcsin(-1+1/2*x)
```

**maxima [B]** time = 2.99, size = 14, normalized size = 1.40

$$-2\,\arctan\left(\frac{\sqrt{-x+4}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(-x + 4)/sqrt(x))
```

**mupad [B]** time = 0.29, size = 16, normalized size = 1.60

$$-4\,\mathrm{atan}\left(\frac{\sqrt{4-x}-2}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(4 - x)^(1/2)),x)
[Out] -4*atan(((4 - x)^(1/2) - 2)/x^(1/2))
```

**sympy [A]** time = 0.99, size = 26, normalized size = 2.60

$$\begin{cases} -2i\,\mathrm{acosh}\left(\frac{\sqrt{x}}{2}\right) & \text{for}\ \frac{|x|}{4}>1 \\ 2\,\mathrm{asin}\left(\frac{\sqrt{x}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)**(1/2)/x**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(x)/2), Abs(x)/4 > 1), (2*asin(sqrt(x)/2), True))
```

## 3.1450 $\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx$

**Optimal**. Leaf size=20

$$\sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

**Rubi** [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {54, 216}

$$\sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
```

```
[Out] Sqrt[2]*ArcSin[Sqrt[2/3]*Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2}}\,dx, x, \sqrt{x}\right)\\ &= \sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 20, normalized size = 1.00

$$\sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
[Out] Sqrt[2]*ArcSin[Sqrt[2/3]*Sqrt[x]]
```

**IntegrateAlgebraic [A]** time = 0.07, size = 38, normalized size = 1.90

$$-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{3}-\sqrt{3-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
[Out] -2*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[x])/(Sqrt[3] - Sqrt[3 - 2*x])]
```

**fricas [A]** time = 1.07, size = 21, normalized size = 1.05

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2x+3}}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x + 3)/sqrt(x))
```

**giac [A]** time = 1.09, size = 13, normalized size = 0.65

$$\sqrt{2}\arcsin\left(\frac{1}{3}\sqrt{6}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] sqrt(2)*arcsin(1/3*sqrt(6)*sqrt(x))
```

**maple [B]** time = 0.01, size = 31, normalized size = 1.55

$$\frac{\sqrt{(-2x+3)x}\,\sqrt{2}\,\arcsin\left(\frac{4x}{3}-1\right)}{2\sqrt{-2x+3}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x+3)^(1/2)/x^(1/2),x)
```

```
[Out] 1/2*((-2*x+3)*x)^(1/2)/(-2*x+3)^(1/2)/x^(1/2)*2^(1/2)*arcsin(4/3*x-1)
```

**maxima** [A] time = 3.12, size = 21, normalized size = 1.05

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2x+3}}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x + 3)/sqrt(x))
```

**mupad** [B] time = 0.30, size = 27, normalized size = 1.35

$$2\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\sqrt{3}-\sqrt{3-2x}\right)}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(3 - 2*x)^(1/2)),x)
```

```
[Out] 2*2^(1/2)*atan((2^(1/2)*(3^(1/2) - (3 - 2*x)^(1/2)))/(2*x^(1/2)))
```

**sympy** [A] time = 1.00, size = 44, normalized size = 2.20

$$\begin{cases} -\sqrt{2}\,i\,\mathrm{acosh}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{for}\ \frac{2|x|}{3} > 1 \\ \sqrt{2}\,\mathrm{asin}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2)*I*acosh(sqrt(6)*sqrt(x)/3), 2*Abs(x)/3 > 1), (sqrt(2)*a
sin(sqrt(6)*sqrt(x)/3), True))
```

## 3.1451 $\int \frac{1}{\sqrt{3-2x}\sqrt{3+5x}}\,dx$

**Optimal**. Leaf size=26

$$\sqrt{\frac{2}{5}}\sin^{-1}\left(\sqrt{\frac{2}{21}}\sqrt{5x+3}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {54, 216}

$$\sqrt{\frac{2}{5}}\sin^{-1}\left(\sqrt{\frac{2}{21}}\sqrt{5x+3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] Sqrt[2/5]*ArcSin[Sqrt[2/21]*Sqrt[3 + 5*x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x}\sqrt{3+5x}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{21-2x^2}}\,dx, x, \sqrt{3+5x}\right)}{\sqrt{5}}\\ &= \sqrt{\frac{2}{5}}\sin^{-1}\left(\sqrt{\frac{2}{21}}\sqrt{3+5x}\right)\end{aligned}$$

**Mathematica** [**A**] time = 0.01, size = 45, normalized size = 1.73

$$-\frac{\sqrt{\frac{2}{5}}\sqrt{3-2x}\sinh^{-1}\left(\sqrt{\frac{5}{21}}\sqrt{2x-3}\right)}{\sqrt{2x-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] -((Sqrt[2/5]*Sqrt[3 - 2*x]*ArcSinh[Sqrt[5/21]*Sqrt[-3 + 2*x]])/Sqrt[-3 + 2*
x])
```

**IntegrateAlgebraic** [**A**] time = 0.05, size = 36, normalized size = 1.38

$$-\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{2}}\sqrt{3-2x}}{\sqrt{5x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] -(Sqrt[2/5]*ArcTan[(Sqrt[5/2]*Sqrt[3 - 2*x])/Sqrt[3 + 5*x]])
```

**fricas** [**B**] time = 0.77, size = 44, normalized size = 1.69

$$-\frac{1}{5}\sqrt{5}\sqrt{2}\arctan\left(\frac{\sqrt{5}\sqrt{2}\sqrt{5x+3}\sqrt{-2x+3}-3\sqrt{5}\sqrt{2}}{10x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(5)*sqrt(2)*arctan(1/10*(sqrt(5)*sqrt(2)*sqrt(5*x + 3)*sqrt(-2*x +
 3) - 3*sqrt(5)*sqrt(2))/x)
```

**giac** [**A**] time = 0.93, size = 21, normalized size = 0.81

$$\frac{1}{5}\sqrt{5}\sqrt{2}\arcsin\left(\frac{1}{21}\sqrt{42}\sqrt{5x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*sqrt(2)*arcsin(1/21*sqrt(42)*sqrt(5*x + 3))
```

**maple [B]** time = 0.01, size = 39, normalized size = 1.50

$$\frac{\sqrt{(-2x+3)(5x+3)}\,\sqrt{10}\,\arcsin\left(\frac{20x}{21}-\frac{3}{7}\right)}{10\sqrt{-2x+3}\,\sqrt{5x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x+3)^(1/2)/(3+5*x)^(1/2),x)
```

```
[Out] 1/10*((-2*x+3)*(3+5*x))^(1/2)/(-2*x+3)^(1/2)/(3+5*x)^(1/2)*10^(1/2)*arcsin(
20/21*x-3/7)
```

**maxima [A]** time = 3.00, size = 11, normalized size = 0.42

$$-\frac{1}{10}\,\sqrt{10}\,\arcsin\left(-\frac{20}{21}\,x+\frac{3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(10)*arcsin(-20/21*x + 3/7)
```

**mupad [B]** time = 0.08, size = 40, normalized size = 1.54

$$-\frac{2\,\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,\left(\sqrt{3}-\sqrt{3-2\,x}\right)}{2\,\left(\sqrt{3}-\sqrt{5\,x+3}\right)}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(1/2)*(5*x + 3)^(1/2)),x)
```

```
[Out] -(2*10^(1/2)*atan((10^(1/2)*(3^(1/2) - (3 - 2*x)^(1/2)))/(2*(3^(1/2) - (5*x
 + 3)^(1/2)))))/5
```

**sympy [A]** time = 1.07, size = 58, normalized size = 2.23

$$\begin{cases} -\frac{\sqrt{10}\,i\,\mathrm{acosh}\left(\frac{\sqrt{210}\,\sqrt{x+\frac{3}{5}}}{21}\right)}{5} & \text{for}\ \frac{10\left|x+\frac{3}{5}\right|}{21}>1 \\ \frac{\sqrt{10}\,\mathrm{asin}\left(\frac{\sqrt{210}\,\sqrt{x+\frac{3}{5}}}{21}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(1/2)/(3+5*x)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(10)*I*acosh(sqrt(210)*sqrt(x + 3/5)/21)/5, 10*Abs(x + 3/5)
/21 > 1), (sqrt(10)*asin(sqrt(210)*sqrt(x + 3/5)/21)/5, True))
```

## 3.1452 $\int \frac{1}{\sqrt{a-bx}\,\sqrt{c+dx}}\,dx$

**Optimal**. Leaf size=43

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

**Rubi** [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {63, 217, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[d]*Sqrt[a - b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx}\sqrt{c+dx}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+\frac{ad}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{a-bx}\right)}{b}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a-bx}}{\sqrt{c+dx}}\right)}{b}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

**Mathematica [B]** time = 0.08, size = 103, normalized size = 2.40

$$\frac{2\sqrt{-b}\sqrt{-ad-bc}\sqrt{\frac{b(c+dx)}{ad+bc}}\,\sin^{-1}\left(\frac{\sqrt{-b}\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{-ad-bc}}\right)}{b^{3/2}\sqrt{d}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[-b]*Sqrt[-(b*c) - a*d]*Sqrt[(b*(c + d*x))/(b*c + a*d)]*ArcSin[(Sqrt
[-b]*Sqrt[d]*Sqrt[a - b*x])/(Sqrt[b]*Sqrt[-(b*c) - a*d])])/(b^(3/2)*Sqrt[d]
*Sqrt[c + d*x])
```

**IntegrateAlgebraic [A]** time = 0.09, size = 43, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a-bx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a - b*x])])/(Sqrt[b]*Sqrt[d
])
```

**fricas [B]** time = 0.73, size = 185, normalized size = 4.30

$$\left[-\frac{\sqrt{-bd}\,\log\left(8\,b^2d^2x^2+b^2c^2-6\,abcd+a^2d^2-4\,(2\,bdx+bc-ad)\sqrt{-bd}\sqrt{-bx+a}\sqrt{dx+c}+8\left(b^2cd-abd^2\right)x\right)}{2\,bd}, -\frac{\sqrt{bd}\,\arctan\left(\frac{(2\,bdx+bc-ad)\sqrt{bd}\sqrt{-bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2-abcd+\left(b^2cd-abd^2\right)x\right)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*(2*b
*d*x + b*c - a*d)*sqrt(-b*d)*sqrt(-b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d - a*
b*d^2)*x)/(b*d), -sqrt(b*d)*arctan(1/2*(2*b*d*x + b*c - a*d)*sqrt(b*d)*sqrt
(-b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x))/(
b*d)]
```

**giac [A]** time = 1.16, size = 54, normalized size = 1.26

$$\frac{2\,b\log\left(\left|-\sqrt{-bd}\,\sqrt{-bx+a}+\sqrt{b^2c+(bx-a)bd+abd}\right|\right)}{\sqrt{-bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*b*log(abs(-sqrt(-b*d)*sqrt(-b*x + a) + sqrt(b^2*c + (b*x - a)*b*d + a*b*d
)))/(sqrt(-b*d)*abs(b))
```

**maple [B]** time = 0.01, size = 84, normalized size = 1.95

$$\frac{\sqrt{(-bx+a)\,(dx+c)}\;\arctan\left(\frac{\sqrt{bd}\,\left(x-\frac{ad-bc}{2bd}\right)}{\sqrt{-bd\,x^2+ac+(ad-bc)x}}\right)}{\sqrt{-bx+a}\;\sqrt{dx+c}\;\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] ((-b*x+a)*(d*x+c))^(1/2)/(-b*x+a)^(1/2)/(d*x+c)^(1/2)/(b*d)^(1/2)*arctan((b
*d)^(1/2)*(x-1/2*(a*d-b*c)/b/d)/(-b*d*x^2+(a*d-b*c)*x+a*c)^(1/2))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 0.34, size = 44, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{a-b\,x}-\sqrt{a}\right)}{\sqrt{b\,d}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] -(4*atan((d*((a - b*x)^(1/2) - a^(1/2)))/((b*d)^(1/2)*((c + d*x)^(1/2) - c^
(1/2)))))/(b*d)^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-bx}\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - b*x)*sqrt(c + d*x)), x)
```

## 3.1453 $\int (a+bx)^{2/3}\sqrt[3]{c+dx}\,dx$

**Optimal**. Leaf size=219

$$\frac{(bc-ad)^2\log(c+dx)}{18b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+d}}{6bd}$$

**Rubi** [A] time = 0.09, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$\frac{(bc-ad)^2\log(c+dx)}{18b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}{6bd}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*b*d) + ((a + b*x)^(5/3)*(c
 + d*x)^(1/3))/(2*b) + ((b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*
x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(4/3)*d^(5/3)) +
 ((b*c - a*d)^2*Log[c + d*x])/(18*b^(4/3)*d^(5/3)) + ((b*c - a*d)^2*Log[-1
+ (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(4/3)*d^(5/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int (a+bx)^{2/3}\sqrt[3]{c+dx}\,dx = \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}dx}{6b}$$

$$= \frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} - \frac{(bc-ad)^2\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}dx}{9bd}$$

$$= \frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} + \frac{(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\sqrt[3]{a-}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c}}\right.}{3\sqrt{3}\,b^{4/3}d^{5/3}}$$

**Mathematica [C]** time = 0.03, size = 73, normalized size = 0.33

$$\frac{3(a+bx)^{5/3}\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 5/3, 8/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.49, size = 294, normalized size = 1.34

$$\frac{(bc-ad)^2\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{9b^{4/3}d^{5/3}} - \frac{(bc-ad)^2\log\left(\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+d^{2/3}\right)}{18b^{4/3}d^{5/3}} - \frac{(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^{5/3}} + \frac{(bc-ad)^2\left(\frac{b(c+dx)^{4/3}}{(a+bx)^{4/3}}+\frac{2d\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{6bd\left(\frac{b(c+dx)}{a+bx}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
```

```
[Out] ((b*c - a*d)^2*((2*d*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b*(c + d*x)^(4/3))
/(a + b*x)^(4/3)))/(6*b*d*(-d + (b*(c + d*x))/(a + b*x))^2) - ((b*c - a*d)^
2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)
^(1/3))])/(3*Sqrt[3]*b^(4/3)*d^(5/3)) + ((b*c - a*d)^2*Log[d^(1/3) - (b^(1/
3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)])/(9*b^(4/3)*d^(5/3)) - ((b*c - a*d)^2*
Log[d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*
(c + d*x)^(2/3))/(a + b*x)^(2/3)])/(18*b^(4/3)*d^(5/3))
```

**fricas [A]** time = 0.89, size = 717, normalized size = 3.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(b*d^2)^(1
/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 + 3*(b*d^2)^(1/3)*(b*x + a)^(2/3)*(
d*x + c)^(1/3)*d + 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (b*
d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^2)^(1/3)*(b*d*x + a*d))*s
qrt(-(b*d^2)^(1/3)/b)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*lo
g(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a)
) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x
 + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(
1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(3*b^2*d^3*x + b^2*c*d^2 + 2*a*b*d^3)*(b
*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*d^3), -1/18*(6*sqrt(1/3)*(b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*(2*(b*d^2)
^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))*sqrt(
(b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*
d^2)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x +
a))/(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*log(((b*x +
a)^(1/3)*(d*x + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3
) + (b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) - 3*(3*b^2*d^3*x + b^2*c*d^2 +
2*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*d^3)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)*(d*x + c)^(1/3), x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(2/3)*(d*x+c)^(1/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(2/3)*(d*x + c)^(1/3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,x)^{2/3}\,(c + d\,x)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)*(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(2/3)*(c + d*x)^(1/3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^{\frac{2}{3}}\,\sqrt[3]{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)*(d*x+c)**(1/3),x)
[Out] Integral((a + b*x)**(2/3)*(c + d*x)**(1/3), x)
```

## 3.1454 $\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx$

**Optimal**. Leaf size=172

$$-\frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}$$

**Rubi [A]** time = 0.05, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$-\frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/b - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^
(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(4/3)
*d^(2/3)) - ((b*c - a*d)*Log[c + d*x])/(6*b^(4/3)*d^(2/3)) - ((b*c - a*d)*L
og[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3)*d^
(2/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx = \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{3b}$$

$$= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b} - \frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}} - \frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}} - \frac{(bc-ad)\,\mathrm{l}\cdots}{}$$

**Mathematica [C]** time = 0.03, size = 73, normalized size = 0.42

$$\frac{3(a+bx)^{2/3}\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{d(a+bx)}{ad-bc}\right)}{2b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(2*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

**IntegrateAlgebraic [A]** time = 7.15, size = 300, normalized size = 1.74

$$\frac{\sqrt[3]{ad+bdx}\left(\frac{(ad-bc)\log\left(\sqrt[3]{ad+b(c+dx)-bc}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{3b^{4/3}d^{2/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{ad+b(c+dx)-bc}+(ad+b(c+dx)-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{6b^{4/3}d^{2/3}} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}{2\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}} + \frac{\sqrt[3]{c+dx}\,(ad+b(c+dx)-bc)^{2/3}}{bd^{2/3}}\right)}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] ((a*d + b*d*x)^(1/3)*(((c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(2/3))/
(b*d^(2/3)) + ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(b^(1/3
)*(c + d*x)^(1/3) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/3))])/(Sqrt[3]*b^(4/3
)*d^(2/3)) + ((-(b*c) + a*d)*Log[-(b^(1/3)*(c + d*x)^(1/3)) + (-(b*c) + a*d
 + b*(c + d*x))^(1/3)])/(3*b^(4/3)*d^(2/3)) + ((b*c - a*d)*Log[b^(2/3)*(c +
 d*x)^(2/3) + b^(1/3)*(c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(1/3) +
(-(b*c) + a*d + b*(c + d*x))^(2/3)])/(6*b^(4/3)*d^(2/3))))/(d^(1/3)*(a + b*
x)^(1/3))
```

**fricas [B]** time = 0.98, size = 596, normalized size = 3.47

$$\left[\frac{6(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd^2 - 3\sqrt{\frac{1}{3}}(b^2cd-abd^2)\sqrt{-\frac{(bd^2)^{\frac{1}{3}}}{b}}\log\left(-3bd^2x-2bcd-ad^2+3(bd^2)^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}d+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-(bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-(bd^2)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{-\frac{(bd^2)^{\frac{1}{3}}}{b}}\right) - 2(bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-(bd^2)^{\frac{2}{3}}(bx+a)}{bx+a}\right) + (bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+(bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+(bd^2)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{6b^2d^2}, \frac{6(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd^2 + 6\sqrt{\frac{1}{3}}(b^2cd-abd^2)\sqrt{\frac{(bd^2)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+(bd^2)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{\frac{(bd^2)^{\frac{1}{3}}}{b}}}{bd^2x+ad^2}\right) - 2(bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-(bd^2)^{\frac{2}{3}}(bx+a)}{bx+a}\right) + (bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+(bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+(bd^2)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{6b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="fricas")

[Out] [1/6*(6*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 - 3*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt(-(b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 + 3*(b*d^2)^(1/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d + 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^2)^(1/3)*(b*d*x + a*d))*sqrt(-(b*d^2)^(1/3)/b)) - 2*(b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a)) + (b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)))/(b^2*d^2), 1/6*(6*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 + 6*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt((b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*(2*(b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))*sqrt((b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a)) + (b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)))/(b^2*d^2)]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="giac")

[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(1/3), x)

**maple [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/3)/(b*x+a)^(1/3),x)

[Out] int((d*x+c)^(1/3)/(b*x+a)^(1/3),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(1/3), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(1/3),x)
```

```
[Out] int((c + d*x)^(1/3)/(a + b*x)^(1/3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(1/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(1/3), x)
```

## 3.1455 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{4/3}}\,dx$

**Optimal**. Leaf size=149

$$-\frac{3\sqrt[3]{d}\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}}-\frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}-\frac{\sqrt[3]{d}\log(c+dx)}{2b^{4/3}}$$

**Rubi** [A] time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 59}

$$-\frac{3\sqrt[3]{d}\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}}-\frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}-\frac{\sqrt[3]{d}\log(c+dx)}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(b*(a + b*x)^(1/3)) - (Sqrt[3]*d^(1/3)*ArcTan[1/Sqrt[3
] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/b^(4/3)
 - (d^(1/3)*Log[c + d*x])/(2*b^(4/3)) - (3*d^(1/3)*Log[-1 + (d^(1/3)*(a + b
*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{4/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}} + \frac{d\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{b}$$
$$= -\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}} - \frac{\sqrt{3}\,\sqrt[3]{d}\,\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{b^{4/3}} - \frac{\sqrt[3]{d}\,\log(c+dx)}{2b^{4/3}} - \frac{3\sqrt[3]{d}\,\log\left(-1 + \frac{\sqrt[3]{d}\,\sqrt[3]{a+b}}{\sqrt[3]{b}\,\sqrt[3]{c+d}}\right.}{2b^{4/3}}$$

**Mathematica** [C] time = 0.03, size = 71, normalized size = 0.48

$$-\frac{3\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[3]{a+bx}\,\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/3)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

**IntegrateAlgebraic** [A] time = 0.16, size = 200, normalized size = 1.34

$$\frac{\sqrt[3]{d}\,\log\left(\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}} + \frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}} + d^{2/3}\right)}{2b^{4/3}} - \frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{d} - \frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{b^{4/3}} + \frac{\sqrt{3}\,\sqrt[3]{d}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}} + \frac{1}{\sqrt{3}}\right)}{b^{4/3}} - \frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(b*(a + b*x)^(1/3)) + (Sqrt[3]*d^(1/3)*ArcTan[1/Sqrt[3
] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/b^(4/3)
 - (d^(1/3)*Log[d^(1/3) - (b^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)])/b^(4/
3) + (d^(1/3)*Log[d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/
3) + (b^(2/3)*(c + d*x)^(2/3))/(a + b*x)^(2/3)])/(2*b^(4/3))
```

**fricas** [B] time = 1.07, size = 233, normalized size = 1.56

$$-\frac{2\sqrt{3}\,(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}b\left(-\frac{d}{b}\right)^{\frac{2}{3}}+\sqrt{3}\,(bdx+ad)}{3\,(bdx+ad)}\right) + (bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\log\left(\frac{(bx+a)\left(-\frac{d}{b}\right)^{\frac{2}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\left(-\frac{d}{b}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{bx+a}\right) - 2\,(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\log\left(\frac{(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}\right) + 6\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{2\left(b^2x+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(b*x + a)*(-d/b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(2/3
)*(d*x + c)^(1/3)*b*(-d/b)^(2/3) + sqrt(3)*(b*d*x + a*d))/(b*d*x + a*d)) +
(b*x + a)*(-d/b)^(1/3)*log(((b*x + a)*(-d/b)^(2/3) - (b*x + a)^(2/3)*(d*x +
 c)^(1/3)*(-d/b)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(b*x + a)) - 2*(b
*x + a)*(-d/b)^(1/3)*log(((b*x + a)*(-d/b)^(1/3) + (b*x + a)^(2/3)*(d*x + c
)^(1/3))/(b*x + a)) + 6*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*x + a*b)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(4/3), x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(4/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(4/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(4/3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(c+d\,x)}^{1/3}}{{(a+b\,x)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(4/3),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(4/3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(4/3),x)
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(4/3), x)
```

## 3.1456 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx$

**Optimal**. Leaf size=32

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx = -\frac{3(c+dx)^{4/3}}{4(bc-ad)(a+bx)^{4/3}}$$

**Mathematica [A]** time = 0.01, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

**IntegrateAlgebraic [A]** time = 0.04, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

**fricas [B]** time = 0.99, size = 65, normalized size = 2.03

$$-\frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}{4\left(a^2bc-a^3d+\left(b^3c-ab^2d\right)x^2+2\left(ab^2c-a^2bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -3/4*(b*x + a)^(2/3)*(d*x + c)^(4/3)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x
^2 + 2*(a*b^2*c - a^2*b*d)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/3), x)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.84

$$\frac{3\,(dx+c)^{\frac{4}{3}}}{4\,(bx+a)^{\frac{4}{3}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(7/3),x)
```

```
[Out] 3/4/(b*x+a)^(4/3)*(d*x+c)^(4/3)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/3), x)
```

**mupad [B]** time = 0.71, size = 92, normalized size = 2.88

$$-\frac{\left(\frac{3\,c}{4\,b^2\,c-4\,a\,b\,d}+\frac{3\,d\,x}{4\,b^2\,c-4\,a\,b\,d}\right)\,{\left(c+d\,x\right)}^{1/3}}{x\,{\left(a+b\,x\right)}^{1/3}-\frac{\left(4\,a^2\,d-4\,a\,b\,c\right)\,{\left(a+b\,x\right)}^{1/3}}{4\,b^2\,c-4\,a\,b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(7/3),x)
```

```
[Out] -(((3*c)/(4*b^2*c - 4*a*b*d) + (3*d*x)/(4*b^2*c - 4*a*b*d))*(c + d*x)^(1/3)
)/(x*(a + b*x)^(1/3) - ((4*a^2*d - 4*a*b*c)*(a + b*x)^(1/3))/(4*b^2*c - 4*a
*b*d))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(7/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(7/3), x)
```

## 3.1457 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx$

**Optimal**. Leaf size=66

$$\frac{9d(c+dx)^{4/3}}{28(a+bx)^{4/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{7(a+bx)^{7/3}(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{9d(c+dx)^{4/3}}{28(a+bx)^{4/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{7(a+bx)^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(4/3)
)/(28*(b*c - a*d)^2*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx = -\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}} - \frac{(3d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx}{7(bc-ad)}$$
$$= -\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{9d(c+dx)^{4/3}}{28(bc-ad)^2(a+bx)^{4/3}}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 0.70

$$\frac{3(c+dx)^{4/3}(7ad-4bc+3bdx)}{28(a+bx)^{7/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
```

```
[Out] (3*(c + d*x)^(4/3)*(-4*b*c + 7*a*d + 3*b*d*x))/(28*(b*c - a*d)^2*(a + b*x)^
(7/3))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 57, normalized size = 0.86

$$-\frac{3\left(\frac{4b(c+dx)^{7/3}}{(a+bx)^{7/3}} - \frac{7d(c+dx)^{4/3}}{(a+bx)^{4/3}}\right)}{28(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
```

```
[Out] (-3*((-7*d*(c + d*x)^(4/3))/(a + b*x)^(4/3) + (4*b*(c + d*x)^(7/3))/(a + b*
x)^(7/3)))/(28*(b*c - a*d)^2)
```

**fricas [B]** time = 1.08, size = 175, normalized size = 2.65

$$\frac{3\left(3\,bd^2x^2 - 4\,bc^2 + 7\,acd - \left(bcd - 7\,ad^2\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{28\left(a^3b^2c^2 - 2\,a^4bcd + a^5d^2 + \left(b^5c^2 - 2\,ab^4cd + a^2b^3d^2\right)x^3 + 3\left(ab^4c^2 - 2\,a^2b^3cd + a^3b^2d^2\right)x^2 + 3\left(a^2b^3c^2 - 2\,a^3b^2cd + a^4bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 3/28*(3*b*d^2*x^2 - 4*b*c^2 + 7*a*c*d - (b*c*d - 7*a*d^2)*x)*(b*x + a)^(2/3
)*(d*x + c)^(1/3)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4
*c*d + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 +
 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(10/3), x)
```

**maple** [A] time = 0.01, size = 54, normalized size = 0.82

$$\frac{3\,(dx+c)^{\frac{4}{3}}\,(3bdx+7ad-4bc)}{28\,(bx+a)^{\frac{7}{3}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(10/3),x)
```

```
[Out] 3/28*(d*x+c)^(4/3)*(3*b*d*x+7*a*d-4*b*c)/(b*x+a)^(7/3)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(10/3), x)
```

**mupad** [B] time = 1.03, size = 127, normalized size = 1.92

$$\frac{(c+d\,x)^{1/3}\left(\frac{x\left(21\,a\,d^2-3\,b\,c\,d\right)}{28\,b^2\,(a\,d-b\,c)^2}-\frac{12\,b\,c^2-21\,a\,c\,d}{28\,b^2\,(a\,d-b\,c)^2}+\frac{9\,d^2\,x^2}{28\,b\,(a\,d-b\,c)^2}\right)}{x^2\,(a+b\,x)^{1/3}+\frac{a^2\,(a+b\,x)^{1/3}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(10/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((x*(21*a*d^2 - 3*b*c*d))/(28*b^2*(a*d - b*c)^2) - (12*b*c
^2 - 21*a*c*d)/(28*b^2*(a*d - b*c)^2) + (9*d^2*x^2)/(28*b*(a*d - b*c)^2)))/
(x^2*(a + b*x)^(1/3) + (a^2*(a + b*x)^(1/3))/b^2 + (2*a*x*(a + b*x)^(1/3))/
b)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(10/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(10/3), x)
```

## 3.1458 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx$

**Optimal**. Leaf size=101

$$-\frac{27d^2(c+dx)^{4/3}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{9d(c+dx)^{4/3}}{35(a+bx)^{7/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{10(a+bx)^{10/3}(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{27d^2(c+dx)^{4/3}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{9d(c+dx)^{4/3}}{35(a+bx)^{7/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{10(a+bx)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (9*d*(c + d*x)^(4/
3))/(35*(b*c - a*d)^2*(a + b*x)^(7/3)) - (27*d^2*(c + d*x)^(4/3))/(140*(b*c
 - a*d)^3*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx = -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}} - \frac{(3d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx}{5(bc-ad)}$$
$$= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}} + \frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}} + \frac{(9d^2)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx}{35(bc-ad)^2}$$
$$= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}} + \frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}} - \frac{27d^2(c+dx)^{4/3}}{140(bc-ad)^3(a+bx)^{4/3}}$$

**Mathematica [A]** time = 0.04, size = 77, normalized size = 0.76

$$-\frac{3(c+dx)^{4/3}\left(35a^2d^2+10abd(3dx-4c)+b^2\left(14c^2-12cdx+9d^2x^2\right)\right)}{140(a+bx)^{10/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3)*(35*a^2*d^2 + 10*a*b*d*(-4*c + 3*d*x) + b^2*(14*c^2 - 1
2*c*d*x + 9*d^2*x^2)))/(140*(b*c - a*d)^3*(a + b*x)^(10/3))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 73, normalized size = 0.72

$$-\frac{3(c+dx)^{4/3}\left(\frac{14b^2(c+dx)^2}{(a+bx)^2}-\frac{40bd(c+dx)}{a+bx}+35d^2\right)}{140(a+bx)^{4/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3)*(35*d^2 - (40*b*d*(c + d*x))/(a + b*x) + (14*b^2*(c + d
*x)^2)/(a + b*x)^2))/(140*(b*c - a*d)^3*(a + b*x)^(4/3))
```

**fricas [B]** time = 0.87, size = 337, normalized size = 3.34

$$-\frac{3\left(9b^2d^3x^3+14b^2c^3-40abc^2d+35a^2cd^2-3\left(b^2cd^2-10abd^3\right)x^2+\left(2b^2c^2d-10abcd^2+35a^2d^3\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{140\left(a^4b^3c^3-3a^5b^2c^2d+3a^6bcd^2-a^7d^3+\left(b^7c^3-3ab^6c^2d+3a^2b^5cd^2-a^3b^4d^3\right)x^4+4\left(ab^6c^3-3a^2b^5c^2d+3a^3b^4cd^2-a^4b^3d^3\right)x^3+6\left(a^2b^5c^3-3a^3b^4c^2d+3a^4b^3cd^2-a^5b^2d^3\right)x^2+4\left(a^3b^4c^3-3a^4b^3c^2d+3a^5b^2cd^2-a^6bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="fricas")
```

```
[Out] -3/140*(9*b^2*d^3*x^3 + 14*b^2*c^3 - 40*a*b*c^2*d + 35*a^2*c*d^2 - 3*(b^2*c
*d^2 - 10*a*b*d^3)*x^2 + (2*b^2*c^2*d - 10*a*b*c*d^2 + 35*a^2*d^3)*x)*(b*x
```

```
+ a)^(2/3)*(d*x + c)^(1/3)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 -
 a^7*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 +
4*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^
2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b
^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(13/3), x)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$\frac{3\,(dx+c)^{\frac{4}{3}}\left(9b^2x^2d^2+30ab\,d^2x-12b^2cdx+35a^2d^2-40abcd+14b^2c^2\right)}{140\,(bx+a)^{\frac{10}{3}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(13/3),x)
```

```
[Out] 3/140*(d*x+c)^(4/3)*(9*b^2*d^2*x^2+30*a*b*d^2*x-12*b^2*c*d*x+35*a^2*d^2-40*
a*b*c*d+14*b^2*c^2)/(b*x+a)^(10/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3
*c^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(13/3), x)
```

**mupad [B]** time = 1.02, size = 203, normalized size = 2.01

$$\frac{(c+d\,x)^{1/3}\left(\frac{105\,a^2\,c\,d^2-120\,a\,b\,c^2\,d+42\,b^2\,c^3}{140\,b^3\,(a\,d-b\,c)^3}+\frac{x\left(105\,a^2\,d^3-30\,a\,b\,c\,d^2+6\,b^2\,c^2\,d\right)}{140\,b^3\,(a\,d-b\,c)^3}+\frac{27\,d^3\,x^3}{140\,b\,(a\,d-b\,c)^3}+\frac{9\,d^2\,x^2\,(10\,a\,d-b\,c)}{140\,b^2\,(a\,d-b\,c)^3}\right)}{x^3\,(a+b\,x)^{1/3}+\frac{a^3\,(a+b\,x)^{1/3}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/3}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/3}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(13/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((42*b^2*c^3 + 105*a^2*c*d^2 - 120*a*b*c^2*d)/(140*b^3*(a*
d - b*c)^3) + (x*(105*a^2*d^3 + 6*b^2*c^2*d - 30*a*b*c*d^2))/(140*b^3*(a*d
- b*c)^3) + (27*d^3*x^3)/(140*b*(a*d - b*c)^3) + (9*d^2*x^2*(10*a*d - b*c))
/(140*b^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/3) + (a^3*(a + b*x)^(1/3))/b^3
 + (3*a*x^2*(a + b*x)^(1/3))/b + (3*a^2*x*(a + b*x)^(1/3))/b^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(13/3),x)
```

```
[Out] Timed out
```

## 3.1459 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{16/3}}\,dx$

**Optimal**. Leaf size=136

$$\frac{243d^3(c+dx)^{4/3}}{1820(a+bx)^{4/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{4/3}}{455(a+bx)^{7/3}(bc-ad)^3}+\frac{27d(c+dx)^{4/3}}{130(a+bx)^{10/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{13(a+bx)^{13/3}(bc-ad)}$$

**Rubi [A]** time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{243d^3(c+dx)^{4/3}}{1820(a+bx)^{4/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{4/3}}{455(a+bx)^{7/3}(bc-ad)^3}+\frac{27d(c+dx)^{4/3}}{130(a+bx)^{10/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{13(a+bx)^{13/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(13*(b*c - a*d)*(a + b*x)^(13/3)) + (27*d*(c + d*x)^(4
/3))/(130*(b*c - a*d)^2*(a + b*x)^(10/3)) - (81*d^2*(c + d*x)^(4/3))/(455*(
b*c - a*d)^3*(a + b*x)^(7/3)) + (243*d^3*(c + d*x)^(4/3))/(1820*(b*c - a*d)
^4*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{16/3}}\,dx = -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} - \frac{(9d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx}{13(bc-ad)}$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} + \frac{\left(27d^2\right)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx}{65(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} - \frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}} - \frac{\left(81d^3\right)\cdots}{455(\cdots}$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} - \frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}} + \frac{243\cdots}{1820(bc\cdots}$$

**Mathematica** [A] time = 0.06, size = 118, normalized size = 0.87

$$\frac{3(c+dx)^{4/3}\left(455a^3d^3+195a^2bd^2(3dx-4c)+39ab^2d\left(14c^2-12cdx+9d^2x^2\right)+b^3\left(-140c^3+126c^2dx-108cd^2x^2+81d^3x^3\right)\right)}{1820(a+bx)^{13/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (3*(c + d*x)^(4/3)*(455*a^3*d^3 + 195*a^2*b*d^2*(-4*c + 3*d*x) + 39*a*b^2*d
*(14*c^2 - 12*c*d*x + 9*d^2*x^2) + b^3*(-140*c^3 + 126*c^2*d*x - 108*c*d^2*
x^2 + 81*d^3*x^3)))/(1820*(b*c - a*d)^4*(a + b*x)^(13/3))
```

**IntegrateAlgebraic** [A] time = 0.13, size = 95, normalized size = 0.70

$$-\frac{3(c+dx)^{4/3}\left(\frac{140b^3(c+dx)^3}{(a+bx)^3}-\frac{546b^2d(c+dx)^2}{(a+bx)^2}+\frac{780bd^2(c+dx)}{a+bx}-455d^3\right)}{1820(a+bx)^{4/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3)*(-455*d^3 + (780*b*d^2*(c + d*x))/(a + b*x) - (546*b^2*
d*(c + d*x)^2)/(a + b*x)^2 + (140*b^3*(c + d*x)^3)/(a + b*x)^3))/(1820*(b*c
 - a*d)^4*(a + b*x)^(4/3))
```

**fricas** [B] time = 1.11, size = 533, normalized size = 3.92

$$\frac{3\left(81b^3d^4x^4-140b^3c^4+546ab^2c^3d-780a^2bc^2d^2+455a^3cd^3-27\left(b^3cd^3-13ab^2d^4\right)x^3+9\left(2b^3c^2d^2-13ab^2cd^3+65a^2bd^4\right)x^2-\left(14b^3c^3d-78ab^2c^2d^2+195a^2bcd^3-455a^3d^4\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{1820\left(a^5b^4c^4-4a^6b^3c^3d+6a^7b^2c^2d^2-4a^8bcd^3+a^9d^4+\left(b^9c^4-4ab^8c^3d+6a^2b^7c^2d^2-4a^3b^6cd^3+a^4b^5d^4\right)x^5+5\left(ab^8c^4-4a^2b^7c^3d+6a^3b^6c^2d^2-4a^4b^5cd^3+a^5b^4d^4\right)x^4+10\left(a^2b^7c^4-4a^3b^6c^3d+6a^4b^5c^2d^2-4a^5b^4cd^3+a^6b^3d^4\right)x^3+10\left(a^3b^6c^4-4a^4b^5c^3d+6a^5b^4c^2d^2-4a^6b^3cd^3+a^7b^2d^4\right)x^2+5\left(a^4b^5c^4-4a^5b^4c^3d+6a^6b^3c^2d^2-4a^7b^2cd^3+a^8bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 3/1820*(81*b^3*d^4*x^4 - 140*b^3*c^4 + 546*a*b^2*c^3*d - 780*a^2*b*c^2*d^2
+ 455*a^3*c*d^3 - 27*(b^3*c*d^3 - 13*a*b^2*d^4)*x^3 + 9*(2*b^3*c^2*d^2 - 13
*a*b^2*c*d^3 + 65*a^2*b*d^4)*x^2 - (14*b^3*c^3*d - 78*a*b^2*c^2*d^2 + 195*a
^2*b*c*d^3 - 455*a^3*d^4)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^5*b^4*c^4 -
 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4 + (b^9*c^4 -
 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^5 + 5
*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b
^4*d^4)*x^4 + 10*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5
*b^4*c*d^3 + a^6*b^3*d^4)*x^3 + 10*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b
^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x^2 + 5*(a^4*b^5*c^4 - 4*a^5*b^
4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(16/3), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$\frac{3\,(dx+c)^{\frac{4}{3}}\left(81b^3d^3x^3+351a\,b^2d^3x^2-108b^3c\,d^2x^2+585a^2b\,d^3x-468a\,b^2c\,d^2x+126b^3c^2dx+455a^3d^3-780a^2bc\,d^2+546a\,b^2c^2d-140b^3c^3\right)}{1820\,(bx+a)^{\frac{13}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(16/3),x)
```

```
[Out] 3/1820*(d*x+c)^(4/3)*(81*b^3*d^3*x^3+351*a*b^2*d^3*x^2-108*b^3*c*d^2*x^2+58
5*a^2*b*d^3*x-468*a*b^2*c*d^2*x+126*b^3*c^2*d*x+455*a^3*d^3-780*a^2*b*c*d^2
+546*a*b^2*c^2*d-140*b^3*c^3)/(b*x+a)^(13/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b
^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(16/3), x)
```

**mupad [B]** time = 1.15, size = 293, normalized size = 2.15

$$\frac{(c+d\,x)^{1/3}\,\left(\frac{243\,d^4\,x^4}{1820\,b\,(a\,d-b\,c)^4}-\frac{-1365\,a^3\,c\,d^3+2340\,a^2\,b\,c^2\,d^2-1638\,a\,b^2\,c^3\,d+420\,b^3\,c^4}{1820\,b^4\,(a\,d-b\,c)^4}+\frac{x\,\left(1365\,a^3\,d^4-585\,a^2\,b\,c\,d^3+234\,a\,b^2\,c^2\,d^2-42\,b^3\,c^3\,d\right)}{1820\,b^4\,(a\,d-b\,c)^4}+\frac{81\,d^3\,x^3\,(13\,a\,d-b\,c)}{1820\,b^2\,(a\,d-b\,c)^4}+\frac{27\,d^2\,x^2\,\left(65\,a^2\,d^2-13\,a\,b\,c\,d+2\,b^2\,c^2\right)}{1820\,b^3\,(a\,d-b\,c)^4}\right)}{x^4\,(a+b\,x)^{1/3}+\frac{a^4\,(a+b\,x)^{1/3}}{b^4}+\frac{6\,a^2\,x^2\,(a+b\,x)^{1/3}}{b^2}+\frac{4\,a\,x^3\,(a+b\,x)^{1/3}}{b}+\frac{4\,a^3\,x\,(a+b\,x)^{1/3}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(16/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((243*d^4*x^4)/(1820*b*(a*d - b*c)^4) - (420*b^3*c^4 - 136
5*a^3*c*d^3 + 2340*a^2*b*c^2*d^2 - 1638*a*b^2*c^3*d)/(1820*b^4*(a*d - b*c)^
4) + (x*(1365*a^3*d^4 - 42*b^3*c^3*d + 234*a*b^2*c^2*d^2 - 585*a^2*b*c*d^3)
)/(1820*b^4*(a*d - b*c)^4) + (81*d^3*x^3*(13*a*d - b*c))/(1820*b^2*(a*d - b
*c)^4) + (27*d^2*x^2*(65*a^2*d^2 + 2*b^2*c^2 - 13*a*b*c*d))/(1820*b^3*(a*d
- b*c)^4)))/(x^4*(a + b*x)^(1/3) + (a^4*(a + b*x)^(1/3))/b^4 + (6*a^2*x^2*(
a + b*x)^(1/3))/b^2 + (4*a*x^3*(a + b*x)^(1/3))/b + (4*a^3*x*(a + b*x)^(1/3
))/b^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(16/3),x)
```

```
[Out] Timed out
```

## 3.1460 $\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx$

**Optimal**. Leaf size=216

$$-\frac{(bc-ad)^2\log(a+bx)}{9b^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{3b^{2/3}d^{7/3}}-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}d^{7/3}}-\frac{2\sqrt[3]{a+bx}\,(c+d}{3d^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$-\frac{(bc-ad)^2\log(a+bx)}{9b^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{3b^{2/3}d^{7/3}}-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}d^{7/3}}-\frac{2\sqrt[3]{a+bx}\,(c+dx)^{2/3}(bc-ad)}{3d^2}+\frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] (-2*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(2/3))/(3*d^2) + ((a + b*x)^(4/3)
*(c + d*x)^(2/3))/(2*d) - (2*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c
 + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))]])/(3*Sqrt[3]*b^(2/3)*d^(7/
3)) - ((b*c - a*d)^2*Log[a + b*x])/(9*b^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log
[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*b^(2/3)*d^(7
/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx = \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} - \frac{(2(bc-ad))\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{3d}$$

$$= -\frac{2(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^2} + \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} + \frac{\left(2(bc-ad)^2\right)\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{9d^2}$$

$$= -\frac{2(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^2} + \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} - \frac{2(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{b}\,\sqrt[3]{c+}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a}}\right.}{3\sqrt{3}\,b^{2/3}d^{7/3}}$$

**Mathematica [C]** time = 0.03, size = 73, normalized size = 0.34

$$\frac{3(a+bx)^{7/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 7/3, 10/3, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.50, size = 292, normalized size = 1.35

$$-\frac{2(bc-ad)^2\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{9b^{2/3}d^{7/3}} + \frac{(bc-ad)^2\log\left(\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+b^{2/3}\right)}{9b^{2/3}d^{7/3}} + \frac{2(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}d^{7/3}} + \frac{(ad-bc)^2\left(\frac{7d(a+bx)^{4/3}}{(c+dx)^{4/3}}-\frac{4b\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{6d^2\left(\frac{d(a+bx)}{c+dx}-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] ((-(b*c) + a*d)^2*((7*d*(a + b*x)^(4/3))/(c + d*x)^(4/3) - (4*b*(a + b*x)^(
1/3))/(c + d*x)^(1/3)))/(6*d^2*(-b + (d*(a + b*x))/(c + d*x))^2) + (2*(b*c
- a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c
 + d*x)^(1/3))])/(3*Sqrt[3]*b^(2/3)*d^(7/3)) - (2*(b*c - a*d)^2*Log[b^(1/3)
 - (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(9*b^(2/3)*d^(7/3)) + ((b*c
- a*d)^2*Log[b^(2/3) + (d^(2/3)*(a + b*x)^(2/3))/(c + d*x)^(2/3) + (b^(1/3)
*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(9*b^(2/3)*d^(7/3))
```

**fricas [B]** time = 1.07, size = 740, normalized size = 3.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((-b^2*d)^(1
/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d + 3*(-b^2*d)^(1/3)*(b*x + a)^(1/3)*(
d*x + c)^(2/3)*b + 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b
^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (-b^2*d)^(1/3)*(b*d*x + b*c))
*sqrt((-b^2*d)^(1/3)/d)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)
*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*
(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 4*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d
 - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)) + 3*(3*b^3*d^2*x - 4*b^3*c*d + 7*a*
b^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(b^2*d^3), 1/18*(12*sqrt(1/3)*(b^
3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(-b^2*d)^(1/3)/d)*arctan(sqrt(1/
3)*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*
x + b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) + 2*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b
^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))
/(d*x + c)) - 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x +
a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)) + 3*(3*
b^3*d^2*x - 4*b^3*c*d + 7*a*b^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(b^2*
d^3)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(1/3), x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(1/3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{4/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(4/3)/(c + d*x)^(1/3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{4}{3}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(1/3),x)
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(1/3), x)
```

# 3.1461 $\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx$

**Optimal**. Leaf size=171

$$\frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}}+\frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d}$$

**Rubi [A]** time = 0.04, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$\frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}}+\frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(1/3)*(c + d*x)^(2/3))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^
(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*b^(2/3)
*d^(4/3)) + ((b*c - a*d)*Log[a + b*x])/(6*b^(2/3)*d^(4/3)) + ((b*c - a*d)*L
og[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3)*d^
(4/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx = \frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{3d}$$

$$= \frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d} + \frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}} + \frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}} + \frac{(bc-ad}{}$$

**Mathematica [C]** time = 0.03, size = 73, normalized size = 0.43

$$\frac{3(a+bx)^{4/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{ad-bc}\right)}{4b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 4/3, 7/3, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*(c + d*x)^(1/3))
```

**IntegrateAlgebraic [A]** time = 7.30, size = 297, normalized size = 1.74

$$\frac{\sqrt[3]{d}\sqrt[3]{a+bx}\left(\frac{(bc-ad)\log\left(\sqrt[3]{ad+b(c+dx)-bc}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{3b^{2/3}d^{4/3}}+\frac{(ad-bc)\log\left(\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{ad+b(c+dx)-bc}+(ad+b(c+dx)-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}{2\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}}+\frac{(c+dx)^{2/3}\sqrt[3]{ad+b(c+dx)-bc}}{d^{4/3}}\right)}{\sqrt[3]{ad+bdx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] (d^(1/3)*(a + b*x)^(1/3)*(((c + d*x)^(2/3)*(-(b*c) + a*d + b*(c + d*x))^(1/
3))/d^(4/3) + ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(b^(1/3
)*(c + d*x)^(1/3) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/3))])/(Sqrt[3]*b^(2/3
)*d^(4/3)) + ((b*c - a*d)*Log[-(b^(1/3)*(c + d*x)^(1/3)) + (-(b*c) + a*d +
b*(c + d*x))^(1/3)])/(3*b^(2/3)*d^(4/3)) + ((-(b*c) + a*d)*Log[b^(2/3)*(c +
 d*x)^(2/3) + b^(1/3)*(c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(1/3) +
(-(b*c) + a*d + b*(c + d*x))^(2/3)])/(6*b^(2/3)*d^(4/3))))/(a*d + b*d*x)^(1
/3)
```

**fricas [B]** time = 1.13, size = 618, normalized size = 3.61

$$\left[\frac{6(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}b^2d - 3\sqrt{\frac{1}{3}}\left(b^2cd-abd^2\right)\sqrt{\frac{\left(-b^2d\right)^{\frac{1}{3}}}{d}}\log\left(3b^2dx+b^2c+2abd+3\left(-b^2d\right)^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}b+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-\left(-b^2d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}+\left(-b^2d\right)^{\frac{1}{3}}(bdx+bc)\right)\sqrt{\frac{\left(-b^2d\right)^{\frac{1}{3}}}{d}}\right)-\left(-b^2d\right)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd+\left(-b^2d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-\left(-b^2d\right)^{\frac{1}{3}}(bdx+bc)}{dx+c}\right)+2\left(-b^2d\right)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-\left(-b^2d\right)^{\frac{2}{3}}(dx+c)}{dx+c}\right)}{6b^2d^2}, \frac{6(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}b^2d-6\sqrt{\frac{1}{3}}\left(b^2cd-abd^2\right)\sqrt{-\frac{\left(-b^2d\right)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(-b^2d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-\left(-b^2d\right)^{\frac{1}{3}}(bdx+bc)\right)\sqrt{-\frac{\left(-b^2d\right)^{\frac{1}{3}}}{d}}}{b^2dx+b^2c}\right)-\left(-b^2d\right)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd+\left(-b^2d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-\left(-b^2d\right)^{\frac{1}{3}}(bdx+bc)}{dx+c}\right)+2\left(-b^2d\right)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-\left(-b^2d\right)^{\frac{2}{3}}(dx+c)}{dx+c}\right)}{6b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2*d - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt((-b^2*d)^(1/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d + 3*(-b^2*d)^(1
/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b + 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*(d*x
 + c)^(1/3)*b*d - (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (-b^2*d)
^(1/3)*(b*d*x + b*c))*sqrt((-b^2*d)^(1/3)/d)) - (-b^2*d)^(2/3)*(b*c - a*d)*
log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(
d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) + 2*(-b^2*d)^(2/3
)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*
x + c))/(d*x + c)))/(b^2*d^2), 1/6*(6*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2*d
 - 6*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt(-(-b^2*d)^(1/3)/d)*arctan(sqrt(1/3)
*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x
+ b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) - (-b^2*d)^(2/3)*(b*c -
a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1
/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) + 2*(-b^2*d)
^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3
)*(d*x + c))/(d*x + c)))/(b^2*d^2)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(1/3), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(1/3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(1/3)/(c + d*x)^(1/3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(1/3),x)
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(1/3), x)
```

## 3.1462 $\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx$

**Optimal**. Leaf size=126

$$-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}$$

**Rubi [A]** time = 0.01, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {59}

$$-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*
(a + b*x)^(1/3))])/(b^(2/3)*d^(1/3))) - Log[a + b*x]/(2*b^(2/3)*d^(1/3)) -
(3*Log[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3
)*d^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}-\frac{3\log\left(-1+\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{2b^{2/3}\sqrt[3]{d}}$$

**Mathematica** [C] time = 0.03, size = 71, normalized size = 0.56

$$\frac{3\sqrt[3]{a+bx}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(a + b*x)^(1/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 1/3, 4/3, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/3))
```

**IntegrateAlgebraic** [A] time = 0.14, size = 177, normalized size = 1.40

$$\frac{\log\left(\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+b^{2/3}\right)}{2b^{2/3}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{b^{2/3}\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c
 + d*x)^(1/3))]])/(b^(2/3)*d^(1/3)) - Log[b^(1/3) - (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3)]/(b^(2/3)*d^(1/3)) + Log[b^(2/3) + (d^(2/3)*(a + b*x)^(2/
3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)]/(2
*b^(2/3)*d^(1/3))
```

**fricas** [B] time = 0.71, size = 519, normalized size = 4.12

$$\left[\frac{\sqrt{3}bd\sqrt{\frac{\left(-b^2d\right)^{\frac{1}{3}}}{d}}\log\left(3b^2dx+b^2c+2abd+3\left(-b^2d\right)^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}b+\sqrt{3}\left(2(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-\left(-b^2d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}+\left(-b^2d\right)^{\frac{1}{3}}(bdx+bc)\right)\sqrt{\frac{\left(-b^2d\right)^{\frac{1}{3}}}{d}}\right)+\left(-b^2d\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd+\left(-b^2d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-\left(-b^2d\right)^{\frac{1}{3}}(bdx+bc)}{dx+c}\right)-2\left(-b^2d\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-\left(-b^2d\right)^{\frac{2}{3}}(dx+c)}{dx+c}\right)}{2b^2d}, \frac{2\sqrt{3}bd\sqrt{-\frac{\left(-b^2d\right)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{3}\left(2\left(-b^2d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-\left(-b^2d\right)^{\frac{1}{3}}(bdx+bc)\right)\sqrt{-\frac{\left(-b^2d\right)^{\frac{1}{3}}}{d}}}{3\left(b^2dx+b^2c\right)}\right)+\left(-b^2d\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd+\left(-b^2d\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-\left(-b^2d\right)^{\frac{1}{3}}(bdx+bc)}{dx+c}\right)-2\left(-b^2d\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-\left(-b^2d\right)^{\frac{2}{3}}(dx+c)}{dx+c}\right)}{2b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*b*d*sqrt((-b^2*d)^(1/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d +
3*(-b^2*d)^(1/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b + sqrt(3)*(2*(b*x + a)^(
2/3)*(d*x + c)^(1/3)*b*d - (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) +
 (-b^2*d)^(1/3)*(b*d*x + b*c))*sqrt((-b^2*d)^(1/3)/d)) + (-b^2*d)^(2/3)*log
(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x
 + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 2*(-b^2*d)^(2/3)*l
og(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x +
c)))/(b^2*d), 1/2*(2*sqrt(3)*b*d*sqrt(-(-b^2*d)^(1/3)/d)*arctan(1/3*sqrt(3)
```

```
*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x
+ b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) + (-b^2*d)^(2/3)*log(((b
*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c
)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 2*(-b^2*d)^(2/3)*log((
(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)))
/(b^2*d)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(1/3)), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}\,(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x)
[Out] int(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(1/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{(a+b\,x)}^{2/3}\,{(c+d\,x)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(2/3)*(c + d*x)^(1/3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{2}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(1/3)), x)
```

## 3.1463 $\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx$

**Optimal**. Leaf size=32

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

**Rubi** [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{2(bc-ad)(a+bx)^{2/3}}$$

**Mathematica** [A] time = 0.01, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.04, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))
```

**fricas [A]** time = 1.32, size = 42, normalized size = 1.31

$$-\frac{3\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\left(abc-a^2d+\left(b^2c-abd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/2*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(1/3)), x)
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.84

$$\frac{3\,(dx+c)^{\frac{2}{3}}}{2\,(bx+a)^{\frac{2}{3}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x)
```

```
[Out] 3/2/(b*x+a)^(2/3)*(d*x+c)^(2/3)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(1/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a+b\,x)^{5/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(5/3)*(c + d*x)^(1/3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(1/3)), x)
```

## 3.1464 $\int \frac{1}{(a+bx)^{8/3} \sqrt[3]{c+dx}} \, dx$

**Optimal**. Leaf size=66

$$\frac{9d(c+dx)^{2/3}}{10(a+bx)^{2/3}(bc-ad)^2} - \frac{3(c+dx)^{2/3}}{5(a+bx)^{5/3}(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{9d(c+dx)^{2/3}}{10(a+bx)^{2/3}(bc-ad)^2} - \frac{3(c+dx)^{2/3}}{5(a+bx)^{5/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(5*(b*c - a*d)*(a + b*x)^(5/3)) + (9*d*(c + d*x)^(2/3)
)/(10*(b*c - a*d)^2*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}} - \frac{(3d)\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx}{5(bc-ad)}$$
$$= -\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}} + \frac{9d(c+dx)^{2/3}}{10(bc-ad)^2(a+bx)^{2/3}}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{3(c+dx)^{2/3}(5ad-2bc+3bdx)}{10(a+bx)^{5/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(c + d*x)^(2/3)*(-2*b*c + 5*a*d + 3*b*d*x))/(10*(b*c - a*d)^2*(a + b*x)^
(5/3))
```

**IntegrateAlgebraic** [A] time = 0.16, size = 51, normalized size = 0.77

$$\frac{3(c+dx)^{5/3}\left(\frac{5d(a+bx)}{c+dx}-2b\right)}{10(a+bx)^{5/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(c + d*x)^(5/3)*(-2*b + (5*d*(a + b*x))/(c + d*x)))/(10*(b*c - a*d)^2*(a
 + b*x)^(5/3))
```

**fricas** [B] time = 1.40, size = 118, normalized size = 1.79

$$\frac{3\,(3\,bdx-2\,bc+5\,ad)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{10\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(3*b*d*x - 2*b*c + 5*a*d)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a
*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(1/3)), x)
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.82

$$\frac{3\,(dx+c)^{\frac{2}{3}}\,(3bdx+5ad-2bc)}{10\,(bx+a)^{\frac{5}{3}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x)
[Out] 3/10*(d*x+c)^(2/3)*(3*b*d*x+5*a*d-2*b*c)/(b*x+a)^(5/3)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(1/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a+b\,x)^{8/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x)
[Out] int(1/((a + b*x)^(8/3)*(c + d*x)^(1/3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{8}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(1/3)), x)
```

## 3.1465 $\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx$

**Optimal**. Leaf size=101

$$-\frac{27d^2(c+dx)^{2/3}}{40(a+bx)^{2/3}(bc-ad)^3}+\frac{9d(c+dx)^{2/3}}{20(a+bx)^{5/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{8(a+bx)^{8/3}(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{27d^2(c+dx)^{2/3}}{40(a+bx)^{2/3}(bc-ad)^3}+\frac{9d(c+dx)^{2/3}}{20(a+bx)^{5/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{8(a+bx)^{8/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(8*(b*c - a*d)*(a + b*x)^(8/3)) + (9*d*(c + d*x)^(2/3)
)/(20*(b*c - a*d)^2*(a + b*x)^(5/3)) - (27*d^2*(c + d*x)^(2/3))/(40*(b*c -
a*d)^3*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}} - \frac{(3d)\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx}{4(bc-ad)}$$
$$= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}} + \frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}} + \frac{\left(9d^2\right)\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx}{20(bc-ad)^2}$$
$$= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}} + \frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}} - \frac{27d^2(c+dx)^{2/3}}{40(bc-ad)^3(a+bx)^{2/3}}$$

**Mathematica** [A] time = 0.03, size = 77, normalized size = 0.76

$$-\frac{3(c+dx)^{2/3}\left(20a^2d^2+8abd(3dx-2c)+b^2\left(5c^2-6cdx+9d^2x^2\right)\right)}{40(a+bx)^{8/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3)*(20*a^2*d^2 + 8*a*b*d*(-2*c + 3*d*x) + b^2*(5*c^2 - 6*c
*d*x + 9*d^2*x^2)))/(40*(b*c - a*d)^3*(a + b*x)^(8/3))
```

**IntegrateAlgebraic** [A] time = 0.17, size = 73, normalized size = 0.72

$$-\frac{3(c+dx)^{8/3}\left(\frac{20d^2(a+bx)^2}{(c+dx)^2}-\frac{16bd(a+bx)}{c+dx}+5b^2\right)}{40(a+bx)^{8/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(8/3)*(5*b^2 + (20*d^2*(a + b*x)^2)/(c + d*x)^2 - (16*b*d*(a
+ b*x))/(c + d*x)))/(40*(b*c - a*d)^3*(a + b*x)^(8/3))
```

**fricas** [B] time = 1.59, size = 251, normalized size = 2.49

$$-\frac{3\left(9\,b^2d^2x^2+5\,b^2c^2-16\,abcd+20\,a^2d^2-6\left(b^2cd-4\,abd^2\right)x\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{40\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3\right)x^2+3\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/40*(9*b^2*d^2*x^2 + 5*b^2*c^2 - 16*a*b*c*d + 20*a^2*d^2 - 6*(b^2*c*d - 4
*a*b*d^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
 + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a
^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^
2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3
)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(1/3)), x)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$\frac{3\,(dx+c)^{\frac{2}{3}}\left(9b^2x^2d^2+24ab\,d^2x-6b^2cdx+20a^2d^2-16abcd+5b^2c^2\right)}{40\,(bx+a)^{\frac{8}{3}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x)
```

```
[Out] 3/40*(d*x+c)^(2/3)*(9*b^2*d^2*x^2+24*a*b*d^2*x-6*b^2*c*d*x+20*a^2*d^2-16*a*
b*c*d+5*b^2*c^2)/(b*x+a)^(8/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(1/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{11/3}\,{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(11/3)*(c + d*x)^(1/3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(11/3)*(c + d*x)**(1/3)), x)
```

## 3.1466 $\int \frac{1}{(a+bx)^{14/3}\sqrt[3]{c+dx}}\,dx$

**Optimal**. Leaf size=136

$$\frac{243d^3(c+dx)^{2/3}}{440(a+bx)^{2/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{2/3}}{220(a+bx)^{5/3}(bc-ad)^3}+\frac{27d(c+dx)^{2/3}}{88(a+bx)^{8/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{11(a+bx)^{11/3}(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{243d^3(c+dx)^{2/3}}{440(a+bx)^{2/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{2/3}}{220(a+bx)^{5/3}(bc-ad)^3}+\frac{27d(c+dx)^{2/3}}{88(a+bx)^{8/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{11(a+bx)^{11/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(11*(b*c - a*d)*(a + b*x)^(11/3)) + (27*d*(c + d*x)^(2
/3))/(88*(b*c - a*d)^2*(a + b*x)^(8/3)) - (81*d^2*(c + d*x)^(2/3))/(220*(b*
c - a*d)^3*(a + b*x)^(5/3)) + (243*d^3*(c + d*x)^(2/3))/(440*(b*c - a*d)^4*
(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{14/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} - \frac{(9d)\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx}{11(bc-ad)}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} + \frac{\left(27d^2\right)\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx}{44(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} - \frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx)^{5/3}} - \frac{(81}{}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} - \frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx)^{5/3}} + \frac{}{44}$$

**Mathematica [A]** time = 0.05, size = 118, normalized size = 0.87

$$\frac{3(c+dx)^{2/3}\left(220a^3d^3+132a^2bd^2(3dx-2c)+33ab^2d\left(5c^2-6cdx+9d^2x^2\right)+b^3\left(-40c^3+45c^2dx-54cd^2x^2+81d^3x^3\right)\right)}{440(a+bx)^{11/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(c + d*x)^(2/3)*(220*a^3*d^3 + 132*a^2*b*d^2*(-2*c + 3*d*x) + 33*a*b^2*d
*(5*c^2 - 6*c*d*x + 9*d^2*x^2) + b^3*(-40*c^3 + 45*c^2*d*x - 54*c*d^2*x^2 +
 81*d^3*x^3)))/(440*(b*c - a*d)^4*(a + b*x)^(11/3))
```

**IntegrateAlgebraic [A]** time = 0.18, size = 95, normalized size = 0.70

$$\frac{3(c+dx)^{11/3}\left(\frac{165b^2d(a+bx)}{c+dx}+\frac{220d^3(a+bx)^3}{(c+dx)^3}-\frac{264bd^2(a+bx)^2}{(c+dx)^2}-40b^3\right)}{440(a+bx)^{11/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(c + d*x)^(11/3)*(-40*b^3 + (220*d^3*(a + b*x)^3)/(c + d*x)^3 - (264*b*d
^2*(a + b*x)^2)/(c + d*x)^2 + (165*b^2*d*(a + b*x))/(c + d*x)))/(440*(b*c -
 a*d)^4*(a + b*x)^(11/3))
```

**fricas [B]** time = 1.41, size = 420, normalized size = 3.09

$$\frac{3\left(81\,b^3d^3x^3-40\,b^3c^3+165\,ab^2c^2d-264\,a^2bcd^2+220\,a^3d^3-27\left(2\,b^3cd^2-11\,ab^2d^3\right)x^2+9\left(5\,b^3c^2d-22\,ab^2cd^2+44\,a^2bd^3\right)x\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{440\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^4+4\left(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4\right)x^3+6\left(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4\right)x^2+4\left(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="fricas")

[Out] 3/440*(81*b^3*d^3*x^3 - 40*b^3*c^3 + 165*a*b^2*c^2*d - 264*a^2*b*c*d^2 + 220*a^3*d^3 - 27*(2*b^3*c*d^2 - 11*a*b^2*d^3)*x^2 + 9*(5*b^3*c^2*d - 22*a*b^2*c*d^2 + 44*a^2*b*d^3)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{14}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="giac")

[Out] integrate(1/((b*x + a)^(14/3)*(d*x + c)^(1/3)), x)

**maple** [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{3\,(dx+c)^{\frac{2}{3}}\left(81b^3d^3x^3+297a\,b^2d^3x^2-54b^3c\,d^2x^2+396a^2b\,d^3x-198a\,b^2c\,d^2x+45b^3c^2dx+220a^3d^3-264a^2bc\,d^2+165a\,b^2c^2d-40b^3c^3\right)}{440\,(bx+a)^{\frac{11}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x)

[Out] 3/440*(d*x+c)^(2/3)*(81*b^3*d^3*x^3+297*a*b^2*d^3*x^2-54*b^3*c*d^2*x^2+396*a^2*b*d^3*x-198*a*b^2*c*d^2*x+45*b^3*c^2*d*x+220*a^3*d^3-264*a^2*b*c*d^2+165*a*b^2*c^2*d-40*b^3*c^3)/(b*x+a)^(11/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{14}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(14/3)*(d*x + c)^(1/3)), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{14/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x)
[Out] int(1/((a + b*x)^(14/3)*(c + d*x)^(1/3)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(14/3)/(d*x+c)**(1/3),x)
[Out] Timed out
```

# 3.1467 $\int \frac{(a+bx)^{5/3}}{(c+dx)^{2/3}}\,dx$

**Optimal**. Leaf size=216

$$-\frac{5(bc-ad)^2\log(c+dx)}{18\sqrt[3]{b}\,d^{8/3}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}-1\right)}{6\sqrt[3]{b}\,d^{8/3}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}\,d^{8/3}}-\frac{5(a+bx)^{2/3}\cdots}{}$$

**Rubi [A]** time = 0.08, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$-\frac{5(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}{6d^2}-\frac{5(bc-ad)^2\log(c+dx)}{18\sqrt[3]{b}\,d^{8/3}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}-1\right)}{6\sqrt[3]{b}\,d^{8/3}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}\,d^{8/3}}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*d^2) + ((a + b*x)^(5/3)
*(c + d*x)^(1/3))/(2*d) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a
 + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(1/3)*d^(8/
3)) - (5*(b*c - a*d)^2*Log[c + d*x])/(18*b^(1/3)*d^(8/3)) - (5*(b*c - a*d)^
2*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(1/3)
*d^(8/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{5/3}}{(c+dx)^{2/3}}\,dx = \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx}{6d}$$

$$= -\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} + \frac{\left(5(bc-ad)^2\right)\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{9d^2}$$

$$= -\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right.}{3\sqrt{3}\,\sqrt[3]{b}\,d^{8/3}}$$

**Mathematica [C]** time = 0.04, size = 73, normalized size = 0.34

$$\frac{3(a+bx)^{8/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{8}{3};\frac{11}{3};\frac{d(a+bx)}{ad-bc}\right)}{8b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(8/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 8/3, 11/3, (d*(a + b*x))/(-(b*c) + a*d)])/(8*b*(c + d*x)^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.30, size = 285, normalized size = 1.32

$$\frac{5(bc-ad)^2\log\left(\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}} + \frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}} + d^{2/3}\right)}{18\sqrt[3]{b}\,d^{8/3}} - \frac{5(bc-ad)^2\log\left(\sqrt[3]{d} - \frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{9\sqrt[3]{b}\,d^{8/3}} + \frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}} + \frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}\,d^{8/3}} + \frac{\sqrt[3]{c+dx}\,(ad-bc)^2\left(8d - \frac{5b(c+dx)}{a+bx}\right)}{6d^2\sqrt[3]{a+bx}\left(d - \frac{b(c+dx)}{a+bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] ((-(b*c) + a*d)^2*(c + d*x)^(1/3)*(8*d - (5*b*(c + d*x))/(a + b*x)))/(6*d^2
*(a + b*x)^(1/3)*(d - (b*(c + d*x))/(a + b*x))^2) + (5*(b*c - a*d)^2*ArcTan
[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))]
)/(3*Sqrt[3]*b^(1/3)*d^(8/3)) - (5*(b*c - a*d)^2*Log[d^(1/3) - (b^(1/3)*(c
+ d*x)^(1/3))/(a + b*x)^(1/3)])/(9*b^(1/3)*d^(8/3)) + (5*(b*c - a*d)^2*Log[
d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*(c +
 d*x)^(2/3))/(a + b*x)^(2/3)])/(18*b^(1/3)*d^(8/3))
```

**fricas [B]** time = 1.58, size = 741, normalized size = 3.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((-b*d^2)^(
1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 - 3*(-b*d^2)^(1/3)*(b*x + a)^(2/3)
*(d*x + c)^(1/3)*d - 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (
-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (-b*d^2)^(1/3)*(b*d*x + a*d
))*sqrt((-b*d^2)^(1/3)/b)) - 10*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b*d^2)^(2
/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b
*x + a)) + 5*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b*d^2)^(2/3)*log(((b*x + a)^
(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
- (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(3*b^2*d^3*x - 5*b^2*c*d^2 +
 8*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*d^4), 1/18*(30*sqrt(1/3)*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(-b*d^2)^(1/3)/b)*arctan(sqrt(1
/3)*(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d
*x + a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 10*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(-b*d^2)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (
-b*d^2)^(2/3)*(b*x + a))/(b*x + a)) + 5*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b
*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(
3*b^2*d^3*x - 5*b^2*c*d^2 + 8*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*
d^4)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(2/3), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(2/3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/3)/(c + d*x)^(2/3),x)
```

```
[Out] int((a + b*x)^(5/3)/(c + d*x)^(2/3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{3}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*x)**(5/3)/(c + d*x)**(2/3), x)
```

## 3.1468 $\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx$

**Optimal**. Leaf size=169

$$\frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}-1\right)}{\sqrt[3]{b}\,d^{5/3}}+\frac{2(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}$$

**Rubi [A]** time = 0.04, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$\frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}-1\right)}{\sqrt[3]{b}\,d^{5/3}}+\frac{2(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/d + (2*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*
d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(1/
3)*d^(5/3)) + ((b*c - a*d)*Log[c + d*x])/(3*b^(1/3)*d^(5/3)) + ((b*c - a*d)
*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(b^(1/3)*d^
(5/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx = \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d} - \frac{(2(bc-ad))\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{3d}$$

$$= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d} + \frac{2(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\sqrt[3]{b}\,d^{5/3}} + \frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}\,d^{5/3}} + \frac{(bc-a}{}$$

**Mathematica** [C] time = 0.03, size = 73, normalized size = 0.43

$$\frac{3(a+bx)^{5/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 5/3, 8/3, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(2/3))
```

**IntegrateAlgebraic** [A] time = 7.90, size = 298, normalized size = 1.76

$$\frac{d^{2/3}(a+bx)^{2/3}\left(\frac{(ad-bc)\log\left(\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{ad+b(c+dx)-bc}+(ad+b(c+dx)-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{3\sqrt[3]{b}\,d^{5/3}} + \frac{\sqrt[3]{c+dx}\,(ad+b(c+dx)-bc)^{2/3}}{d^{5/3}} + \frac{2(bc-ad)\log\left(\sqrt[3]{ad+b(c+dx)-bc}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{3\sqrt[3]{b}\,d^{5/3}} - \frac{2(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}{2\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\sqrt[3]{b}\,d^{5/3}}\right)}{(ad+bdx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
```

```
[Out] (d^(2/3)*(a + b*x)^(2/3)*(((c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(2/
3))/d^(5/3) - (2*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(b^(1
/3)*(c + d*x)^(1/3) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/3))])/(Sqrt[3]*b^(1
/3)*d^(5/3)) + (2*(b*c - a*d)*Log[-(b^(1/3)*(c + d*x)^(1/3)) + (-(b*c) + a*
d + b*(c + d*x))^(1/3)])/(3*b^(1/3)*d^(5/3)) + ((-(b*c) + a*d)*Log[b^(2/3)*
(c + d*x)^(2/3) + b^(1/3)*(c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(1/3
) + (-(b*c) + a*d + b*(c + d*x))^(2/3)])/(3*b^(1/3)*d^(5/3))))/(a*d + b*d*x
)^(2/3)
```

**fricas** [B] time = 1.53, size = 619, normalized size = 3.66

$$\left[\frac{3(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd^2 - 3\sqrt{\frac{1}{3}}\left(b^2cd-abd^2\right)\sqrt{\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\log\left(-3bd^2x-2bcd-ad^2-3\left(-bd^2\right)^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}d-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\right)+2\left(-bd^2\right)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-\left(-bd^2\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right)-\left(-bd^2\right)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{3bd^3}, \frac{3(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd^2 - 6\sqrt{\frac{1}{3}}\left(b^2cd-abd^2\right)\sqrt{-\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{-\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}}{bd^2x+ad^2}\right)+2\left(-bd^2\right)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-\left(-bd^2\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right)-\left(-bd^2\right)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{3bd^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt((-b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 - 3*(-b*d^2)^(
1/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d - 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*
x + c)^(2/3)*b*d - (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (-b*d^2
)^(1/3)*(b*d*x + a*d))*sqrt((-b*d^2)^(1/3)/b)) + 2*(-b*d^2)^(2/3)*(b*c - a*
d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*
x + a)) - (-b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b
*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x
 + a*d))/(b*x + a)))/(b*d^3), 1/3*(3*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2
- 6*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt(-(-b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*
(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x +
 a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) + 2*(-b*d^2)^(2/3)*(b*c -
 a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/
(b*x + a)) - (-b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3
)*b*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*
d*x + a*d))/(b*x + a)))/(b*d^3)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(2/3), x)
```

**maple [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(2/3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{2/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/(c + d*x)^(2/3),x)
```

```
[Out] int((a + b*x)^(2/3)/(c + d*x)^(2/3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{2}{3}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)/(c + d*x)**(2/3), x)
```

## 3.1469 $\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx$

**Optimal**. Leaf size=126

$$-\frac{3\log\left(\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}-1\right)}{2\sqrt[3]{b}\,d^{2/3}}-\frac{\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}\,d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}\,d^{2/3}}$$

**Rubi** [A] time = 0.02, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {59}

$$-\frac{3\log\left(\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}-1\right)}{2\sqrt[3]{b}\,d^{2/3}}-\frac{\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}\,d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*
(c + d*x)^(1/3))])/(b^(1/3)*d^(2/3))) - Log[c + d*x]/(2*b^(1/3)*d^(2/3)) -
(3*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(1/3
)*d^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx=-\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{\sqrt[3]{b}\,d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}\,d^{2/3}}-\frac{3\log\left(-1+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{2\sqrt[3]{b}\,d^{2/3}}$$

**Mathematica [C]** time = 0.03, size = 73, normalized size = 0.58

$$\frac{3(a+bx)^{2/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{d(a+bx)}{ad-bc}\right)}{2b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 2/3, 5/3, (d*(a + b*x))/(-(b*c) + a*d)])/(2*b*(c + d*x)^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 177, normalized size = 1.40

$$\frac{\log\left(\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}+\frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+d^{2/3}\right)}{2\sqrt[3]{b}\,d^{2/3}}-\frac{\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{\sqrt[3]{b}\,d^{2/3}}+\frac{\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a
 + b*x)^(1/3))])/(b^(1/3)*d^(2/3)) - Log[d^(1/3) - (b^(1/3)*(c + d*x)^(1/3)
)/(a + b*x)^(1/3)]/(b^(1/3)*d^(2/3)) + Log[d^(2/3) + (b^(1/3)*d^(1/3)*(c +
d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*(c + d*x)^(2/3))/(a + b*x)^(2/3)]/(2
*b^(1/3)*d^(2/3))
```

**fricas [B]** time = 1.24, size = 521, normalized size = 4.13

$$\left[\frac{\sqrt{3}\,bd\sqrt{\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\log\left(-3\,bd^2x-2\,bcd-ad^2-3\left(-bd^2\right)^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}d-\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\right)-2\left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-\left(-bd^2\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right)+\left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{2\,bd^2},\ \frac{2\sqrt{3}\,bd\sqrt{-\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{3}\left(2\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{-\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}}{3\left(bd^2x+ad^2\right)}\right)-2\left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-\left(-bd^2\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right)+\left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{2\,bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*b*d*sqrt((-b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 -
 3*(-b*d^2)^(1/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d - sqrt(3)*(2*(b*x + a)^
(1/3)*(d*x + c)^(2/3)*b*d - (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
+ (-b*d^2)^(1/3)*(b*d*x + a*d))*sqrt((-b*d^2)^(1/3)/b)) - 2*(-b*d^2)^(2/3)*
log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*x +
 a)) + (-b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(
2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x +
 a)))/(b*d^2), 1/2*(2*sqrt(3)*b*d*sqrt(-(-b*d^2)^(1/3)/b)*arctan(1/3*sqrt(3
```

```
)*(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x
 + a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(-b*d^2)^(2/3)*log(
((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*x + a))
 + (-b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)
*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a))
)/(b*d^2)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(2/3)), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x)
[Out] int(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(2/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/3}\,{\left(c+d\,x\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x)
```

```
[Out] int(1/((a + b*x)^(1/3)*(c + d*x)^(2/3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(1/3)*(c + d*x)**(2/3)), x)
```

## 3.1470 $\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx$

**Optimal**. Leaf size=30

$$-\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}\,(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/((b*c - a*d)*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{(bc-ad)\sqrt[3]{a+bx}}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.00

$$\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(c + d*x)^(1/3))/((-(b*c) + a*d)*(a + b*x)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 30, normalized size = 1.00

$$-\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]
[Out] (-3*(c + d*x)^(1/3))/((b*c - a*d)*(a + b*x)^(1/3))
```

**fricas [A]** time = 0.74, size = 42, normalized size = 1.40

$$-\frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] -3*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{4}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(2/3)), x)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.90

$$\frac{3\,(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x)
[Out] 3/(b*x+a)^(1/3)*(d*x+c)^(1/3)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{4}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(2/3)), x)
```

**mupad [B]** time = 0.83, size = 26, normalized size = 0.87

$$\frac{3\,{\left(c+d\,x\right)}^{1/3}}{\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x)
```

```
[Out] (3*(c + d*x)^(1/3))/((a*d - b*c)*(a + b*x)^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{4}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(4/3)*(c + d*x)**(2/3)), x)
```

## 3.1471 $\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx$

**Optimal**. Leaf size=66

$$\frac{9d\sqrt[3]{c+dx}}{4\sqrt[3]{a+bx}\,(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{4(a+bx)^{4/3}(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{9d\sqrt[3]{c+dx}}{4\sqrt[3]{a+bx}\,(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(4*(b*c - a*d)*(a + b*x)^(4/3)) + (9*d*(c + d*x)^(1/3)
)/(4*(b*c - a*d)^2*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}} - \frac{(3d)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx}{4(bc-ad)}$$
$$= -\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}} + \frac{9d\sqrt[3]{c+dx}}{4(bc-ad)^2\sqrt[3]{a+bx}}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 0.70

$$\frac{3\sqrt[3]{c+dx}\,(4ad-bc+3bdx)}{4(a+bx)^{4/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(c + d*x)^(1/3)*(-(b*c) + 4*a*d + 3*b*d*x))/(4*(b*c - a*d)^2*(a + b*x)^(
4/3))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 56, normalized size = 0.85

$$-\frac{3\left(\frac{b(c+dx)^{4/3}}{(a+bx)^{4/3}} - \frac{4d\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{4(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((-4*d*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b*(c + d*x)^(4/3))/(a + b*x)
^(4/3)))/(4*(b*c - a*d)^2)
```

**fricas [B]** time = 2.11, size = 118, normalized size = 1.79

$$\frac{3\,(3\,bdx - bc + 4\,ad)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{4\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + \left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)x^2 + 2\left(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(3*b*d*x - b*c + 4*a*d)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^2*b^2*c^2 -
2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^
3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(2/3)), x)
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.82

$$\frac{3\,(dx+c)^{\frac{1}{3}}\,(3bdx+4ad-bc)}{4\,(bx+a)^{\frac{4}{3}}\,\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x)
```

```
[Out] 3/4*(d*x+c)^(1/3)*(3*b*d*x+4*a*d-b*c)/(b*x+a)^(4/3)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(2/3)), x)
```

**mupad [B]** time = 0.98, size = 71, normalized size = 1.08

$$\frac{\left(\frac{9\,d\,x}{4\,{\left(a\,d-b\,c\right)}^2}+\frac{12\,a\,d-3\,b\,c}{4\,b\,{\left(a\,d-b\,c\right)}^2}\right)\,{\left(c+d\,x\right)}^{1/3}}{x\,{\left(a+b\,x\right)}^{1/3}+\frac{a\,{\left(a+b\,x\right)}^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x)
```

```
[Out] (((9*d*x)/(4*(a*d - b*c)^2) + (12*a*d - 3*b*c)/(4*b*(a*d - b*c)^2))*(c + d*
x)^(1/3))/(x*(a + b*x)^(1/3) + (a*(a + b*x)^(1/3))/b)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(7/3)*(c + d*x)**(2/3)), x)
```

## 3.1472 $\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx$

**Optimal**. Leaf size=101

$$-\frac{27d^2\sqrt[3]{c+dx}}{14\sqrt[3]{a+bx}\,(bc-ad)^3}+\frac{9d\sqrt[3]{c+dx}}{14(a+bx)^{4/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{7(a+bx)^{7/3}(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{27d^2\sqrt[3]{c+dx}}{14\sqrt[3]{a+bx}\,(bc-ad)^3}+\frac{9d\sqrt[3]{c+dx}}{14(a+bx)^{4/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{7(a+bx)^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(1/3)
)/(14*(b*c - a*d)^2*(a + b*x)^(4/3)) - (27*d^2*(c + d*x)^(1/3))/(14*(b*c -
a*d)^3*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}} - \frac{(6d)\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx}{7(bc-ad)}$$
$$= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}} + \frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}} + \frac{\left(9d^2\right)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx}{14(bc-ad)^2}$$
$$= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}} + \frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}} - \frac{27d^2\sqrt[3]{c+dx}}{14(bc-ad)^3\sqrt[3]{a+bx}}$$

**Mathematica** [A] time = 0.03, size = 75, normalized size = 0.74

$$-\frac{3\sqrt[3]{c+dx}\left(14a^2d^2-7abd(c-3dx)+b^2\left(2c^2-3cdx+9d^2x^2\right)\right)}{14(a+bx)^{7/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(14*a^2*d^2 - 7*a*b*d*(c - 3*d*x) + b^2*(2*c^2 - 3*c*d*
x + 9*d^2*x^2)))/(14*(b*c - a*d)^3*(a + b*x)^(7/3))
```

**IntegrateAlgebraic** [A] time = 0.12, size = 83, normalized size = 0.82

$$-\frac{3\left(\frac{2b^2(c+dx)^{7/3}}{(a+bx)^{7/3}}+\frac{14d^2\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}-\frac{7bd(c+dx)^{4/3}}{(a+bx)^{4/3}}\right)}{14(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((14*d^2*(c + d*x)^(1/3))/(a + b*x)^(1/3) - (7*b*d*(c + d*x)^(4/3))/(a
+ b*x)^(4/3) + (2*b^2*(c + d*x)^(7/3))/(a + b*x)^(7/3)))/(14*(b*c - a*d)^3)
```

**fricas** [B] time = 1.11, size = 251, normalized size = 2.49

$$-\frac{3\left(9b^2d^2x^2+2b^2c^2-7abcd+14a^2d^2-3\left(b^2cd-7abd^2\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{14\left(a^3b^3c^3-3a^4b^2c^2d+3a^5bcd^2-a^6d^3+\left(b^6c^3-3ab^5c^2d+3a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3a^2b^4c^2d+3a^3b^3cd^2-a^4b^2d^3\right)x^2+3\left(a^2b^4c^3-3a^3b^3c^2d+3a^4b^2cd^2-a^5bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/14*(9*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 14*a^2*d^2 - 3*(b^2*c*d - 7*
a*b*d^2)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
+ 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^
3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2
*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)
*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{10}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(2/3)), x)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$\frac{3\,(dx+c)^{\frac{1}{3}}\left(9b^{2}x^{2}d^{2}+21ab\,d^{2}x-3b^{2}cdx+14a^{2}d^{2}-7abcd+2b^{2}c^{2}\right)}{14\,(bx+a)^{\frac{7}{3}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x)
```

```
[Out] 3/14*(d*x+c)^(1/3)*(9*b^2*d^2*x^2+21*a*b*d^2*x-3*b^2*c*d*x+14*a^2*d^2-7*a*b
*c*d+2*b^2*c^2)/(b*x+a)^(7/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{10}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(2/3)), x)
```

**mupad [B]** time = 1.51, size = 133, normalized size = 1.32

$$\frac{(c+d\,x)^{1/3}\,\left(\frac{27\,d^2\,x^2}{14\,{(a\,d-b\,c)}^3}+\frac{42\,a^2\,d^2-21\,a\,b\,c\,d+6\,b^2\,c^2}{14\,b^2\,{(a\,d-b\,c)}^3}+\frac{9\,d\,x\,(7\,a\,d-b\,c)}{14\,b\,{(a\,d-b\,c)}^3}\right)}{x^2\,{(a+b\,x)}^{1/3}+\frac{a^2\,{(a+b\,x)}^{1/3}}{b^2}+\frac{2\,a\,x\,{(a+b\,x)}^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x)
```

```
[Out] ((c + d*x)^(1/3)*((27*d^2*x^2)/(14*(a*d - b*c)^3) + (42*a^2*d^2 + 6*b^2*c^2
 - 21*a*b*c*d)/(14*b^2*(a*d - b*c)^3) + (9*d*x*(7*a*d - b*c))/(14*b*(a*d -
b*c)^3)))/(x^2*(a + b*x)^(1/3) + (a^2*(a + b*x)^(1/3))/b^2 + (2*a*x*(a + b*
x)^(1/3))/b)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{10}{3}}(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(10/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(10/3)*(c + d*x)**(2/3)), x)
```

## 3.1473 $\int \frac{1}{(a+bx)^{13/3}(c+dx)^{2/3}}\,dx$

**Optimal**. Leaf size=136

$$\frac{243d^3\sqrt[3]{c+dx}}{140\sqrt[3]{a+bx}\,(bc-ad)^4}-\frac{81d^2\sqrt[3]{c+dx}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{27d\sqrt[3]{c+dx}}{70(a+bx)^{7/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{10(a+bx)^{10/3}(bc-ad)}$$

**Rubi [A]** time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{243d^3\sqrt[3]{c+dx}}{140\sqrt[3]{a+bx}\,(bc-ad)^4}-\frac{81d^2\sqrt[3]{c+dx}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{27d\sqrt[3]{c+dx}}{70(a+bx)^{7/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{10(a+bx)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (27*d*(c + d*x)^(1
/3))/(70*(b*c - a*d)^2*(a + b*x)^(7/3)) - (81*d^2*(c + d*x)^(1/3))/(140*(b*
c - a*d)^3*(a + b*x)^(4/3)) + (243*d^3*(c + d*x)^(1/3))/(140*(b*c - a*d)^4*
(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{13/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} - \frac{(9d)\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx}{10(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} + \frac{\left(27d^2\right)\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx}{35(bc-ad)^2}$$

$$= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} - \frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+bx)^{4/3}} -$$

$$= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} - \frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+bx)^{4/3}} +$$

**Mathematica** [A] time = 0.05, size = 116, normalized size = 0.85

$$\frac{3\sqrt[3]{c+dx}\left(140a^3d^3-105a^2bd^2(c-3dx)+30ab^2d\left(2c^2-3cdx+9d^2x^2\right)+b^3\left(-14c^3+18c^2dx-27cd^2x^2+81d^3x^3\right)\right)}{140(a+bx)^{10/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(c + d*x)^(1/3)*(140*a^3*d^3 - 105*a^2*b*d^2*(c - 3*d*x) + 30*a*b^2*d*(2
*c^2 - 3*c*d*x + 9*d^2*x^2) + b^3*(-14*c^3 + 18*c^2*d*x - 27*c*d^2*x^2 + 81
*d^3*x^3)))/(140*(b*c - a*d)^4*(a + b*x)^(10/3))
```

**IntegrateAlgebraic** [A] time = 0.14, size = 95, normalized size = 0.70

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{14b^3(c+dx)^3}{(a+bx)^3}-\frac{60b^2d(c+dx)^2}{(a+bx)^2}+\frac{105bd^2(c+dx)}{a+bx}-140d^3\right)}{140\sqrt[3]{a+bx}\,(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(-140*d^3 + (105*b*d^2*(c + d*x))/(a + b*x) - (60*b^2*d
*(c + d*x)^2)/(a + b*x)^2 + (14*b^3*(c + d*x)^3)/(a + b*x)^3))/(140*(b*c -
a*d)^4*(a + b*x)^(1/3))
```

**fricas** [B] time = 0.83, size = 419, normalized size = 3.08

$$\frac{3\left(81\,b^3d^3x^3-14\,b^3c^3+60\,ab^2c^2d-105\,a^2bcd^2+140\,a^3d^3-27\left(b^3cd^2-10\,ab^2d^3\right)x^2+9\left(2\,b^3c^2d-10\,ab^2cd^2+35\,a^2bd^3\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{140\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^4+4\left(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4\right)x^3+6\left(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4\right)x^2+4\left(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="fricas")

[Out] 3/140*(81*b^3*d^3*x^3 - 14*b^3*c^3 + 60*a*b^2*c^2*d - 105*a^2*b*c*d^2 + 140*a^3*d^3 - 27*(b^3*c*d^2 - 10*a*b^2*d^3)*x^2 + 9*(2*b^3*c^2*d - 10*a*b^2*c*d^2 + 35*a^2*b*d^3)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{13}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="giac")

[Out] integrate(1/((b*x + a)^(13/3)*(d*x + c)^(2/3)), x)

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$\frac{3\,(dx+c)^{\frac{1}{3}}\left(81b^3d^3x^3+270a\,b^2d^3x^2-27b^3c\,d^2x^2+315a^2b\,d^3x-90a\,b^2c\,d^2x+18b^3c^2dx+140a^3d^3-105a^2bc\,d^2+60a\,b^2c^2d-14b^3c^3\right)}{140\,(bx+a)^{\frac{10}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x)

[Out] 3/140*(d*x+c)^(1/3)*(81*b^3*d^3*x^3+270*a*b^2*d^3*x^2-27*b^3*c*d^2*x^2+315*a^2*b*d^3*x-90*a*b^2*c*d^2*x+18*b^3*c^2*d*x+140*a^3*d^3-105*a^2*b*c*d^2+60*a*b^2*c^2*d-14*b^3*c^3)/(b*x+a)^(10/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{13}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(13/3)*(d*x + c)^(2/3)), x)

**mupad [B]** time = 1.27, size = 209, normalized size = 1.54

$$\frac{(c+d\,x)^{1/3}\left(\frac{243\,d^3\,x^3}{140\,(a\,d-b\,c)^4}+\frac{420\,a^3\,d^3-315\,a^2\,b\,c\,d^2+180\,a\,b^2\,c^2\,d-42\,b^3\,c^3}{140\,b^3\,(a\,d-b\,c)^4}+\frac{27\,d\,x\left(35\,a^2\,d^2-10\,a\,b\,c\,d+2\,b^2\,c^2\right)}{140\,b^2\,(a\,d-b\,c)^4}+\frac{81\,d^2\,x^2\,(10\,a\,d-b\,c)}{140\,b\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/3}+\frac{a^3\,(a+b\,x)^{1/3}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/3}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/3}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x)
```

```
[Out] ((c + d*x)^(1/3)*((243*d^3*x^3)/(140*(a*d - b*c)^4) + (420*a^3*d^3 - 42*b^3
*c^3 + 180*a*b^2*c^2*d - 315*a^2*b*c*d^2)/(140*b^3*(a*d - b*c)^4) + (27*d*x
*(35*a^2*d^2 + 2*b^2*c^2 - 10*a*b*c*d))/(140*b^2*(a*d - b*c)^4) + (81*d^2*x
^2*(10*a*d - b*c))/(140*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/3) + (a^3*(a +
 b*x)^(1/3))/b^3 + (3*a*x^2*(a + b*x)^(1/3))/b + (3*a^2*x*(a + b*x)^(1/3))/
b^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(13/3)/(d*x+c)**(2/3),x)
```

```
[Out] Timed out
```

## 3.1474 $\int \frac{(a+bx)^{7/3}}{(c+dx)^{4/3}}\,dx$

**Optimal**. Leaf size=241

$$-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log(a+bx)}{9d^{10/3}}-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{3d^{10/3}}-\frac{14\sqrt[3]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^{10/3}}-\frac{14}{}$$

**Rubi [A]** time = 0.11, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {47, 50, 59}

$$\frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2}-\frac{14b\sqrt[3]{a+bx}\,(c+dx)^{2/3}(bc-ad)}{3d^3}-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log(a+bx)}{9d^{10/3}}-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{3d^{10/3}}-\frac{14\sqrt[3]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^{10/3}}-\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(7/3))/(d*(c + d*x)^(1/3)) - (14*b*(b*c - a*d)*(a + b*x)^(1/3
)*(c + d*x)^(2/3))/(3*d^3) + (7*b*(a + b*x)^(4/3)*(c + d*x)^(2/3))/(2*d^2)
- (14*b^(1/3)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/
(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(3*Sqrt[3]*d^(10/3)) - (7*b^(1/3)*(b*c
- a*d)^2*Log[a + b*x])/(9*d^(10/3)) - (7*b^(1/3)*(b*c - a*d)^2*Log[-1 + (b^
(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*d^(10/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{7/3}}{(c+dx)^{4/3}}\,dx &= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx}{d} \\
&= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{(14b(bc-ad))\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{3d^2} \\
&= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} - \frac{14b(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^3} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} + \frac{\big(14b(bc-ad)}{} \\
&= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} - \frac{14b(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^3} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{14\sqrt[3]{b}\,(bc-ad}{}
\end{aligned}$$

**Mathematica [C]** time = 0.06, size = 73, normalized size = 0.30

$$\frac{3(a+bx)^{10/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},\frac{10}{3};\frac{13}{3};\frac{d(a+bx)}{ad-bc}\right)}{10b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(10/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3
, 10/3, 13/3, (d*(a + b*x))/(-(b*c) + a*d)])/(10*b*(c + d*x)^(4/3))
```

**IntegrateAlgebraic [A]** time = 0.50, size = 318, normalized size = 1.32

$$\frac{7\sqrt[3]{b}\,(bc-ad)^2\log\left(\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+b^{2/3}\right)}{9d^{10/3}} - \frac{(ad-bc)^2\left(\frac{28b^2\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\frac{18d^2(a+bx)^{7/3}}{(c+dx)^{7/3}}-\frac{49bd(a+bx)^{4/3}}{(c+dx)^{4/3}}\right)}{6d^3\left(\frac{d(a+bx)}{c+dx}-b\right)^2} - \frac{14\sqrt[3]{b}\,(bc-ad)^2\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{9d^{10/3}} + \frac{14\sqrt[3]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]
```

```
[Out] -1/6*((-(b*c) + a*d)^2*((18*d^2*(a + b*x)^(7/3))/(c + d*x)^(7/3) - (49*b*d*
(a + b*x)^(4/3))/(c + d*x)^(4/3) + (28*b^2*(a + b*x)^(1/3))/(c + d*x)^(1/3)
))/(d^3*(-b + (d*(a + b*x))/(c + d*x))^2) + (14*b^(1/3)*(b*c - a*d)^2*ArcTa
n[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))
])/(3*Sqrt[3]*d^(10/3)) - (14*b^(1/3)*(b*c - a*d)^2*Log[b^(1/3) - (d^(1/3)*
(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(9*d^(10/3)) + (7*b^(1/3)*(b*c - a*d)^2*
Log[b^(2/3) + (d^(2/3)*(a + b*x)^(2/3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*
(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(9*d^(10/3))
```

**fricas [B]** time = 1.04, size = 423, normalized size = 1.76

$$-\frac{28\,\sqrt{3}\left(b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}\,(bdx+bc)}{3\,(bdx+bc)}\right)+14\left(b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right)-28\left(b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right)-3\left(3\,b^2d^2x^2-28\,b^2c^2+49\,abcd-18\,a^2d^2-\left(7\,b^2cd-13\,abd^2\right)x\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{18\left(d^4x+cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/18*(28*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*x)*(-b/d)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*(d*x
+ c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) + 14*(b^2
*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(-b
/d)^(1/3)*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*(-b
/d)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 28*(b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(-b/d)^(1/3)*
log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c)) -
 3*(3*b^2*d^2*x^2 - 28*b^2*c^2 + 49*a*b*c*d - 18*a^2*d^2 - (7*b^2*c*d - 13*
a*b*d^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(d^4*x + c*d^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(4/3), x)
```

**maple [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(4/3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(7/3)/(c + d*x)^(4/3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(7/3)/(c + d*x)**(4/3), x)
```

## 3.1475 $\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}} dx$

**Optimal**. Leaf size=195

$$\frac{2\sqrt[3]{b}(bc-ad)\log(a+bx)}{3d^{7/3}}+\frac{2\sqrt[3]{b}(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{d^{7/3}}+\frac{4\sqrt[3]{b}(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}}+\frac{4b\sqrt[3]{a+}}{}$$

**Rubi [A]** time = 0.07, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {47, 50, 59}

$$\frac{4b\sqrt[3]{a+bx}(c+dx)^{2/3}}{d^2}+\frac{2\sqrt[3]{b}(bc-ad)\log(a+bx)}{3d^{7/3}}+\frac{2\sqrt[3]{b}(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{d^{7/3}}+\frac{4\sqrt[3]{b}(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}}-\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(4/3))/(d*(c + d*x)^(1/3)) + (4*b*(a + b*x)^(1/3)*(c + d*x)^(
2/3))/d^2 + (4*b^(1/3)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^
(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*d^(7/3)) + (2*b^(1/3)*(
b*c - a*d)*Log[a + b*x])/(3*d^(7/3)) + (2*b^(1/3)*(b*c - a*d)*Log[-1 + (b^(
1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/d^(7/3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}}\,dx &= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{(4b)\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{d} \\
&= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2} - \frac{(4b(bc-ad))\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{3d^2} \\
&= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2} + \frac{4\sqrt[3]{b}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{\sqrt{3}\,d^{7/3}} + \frac{2\sqrt[3]{b}\,(b
\end{aligned}$$

**Mathematica [C]** time = 0.05, size = 73, normalized size = 0.37

$$\frac{3(a+bx)^{7/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 7/3, 10/3, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(4/3))
```

**IntegrateAlgebraic [A]** time = 11.02, size = 326, normalized size = 1.67

$$\frac{d^{4/3}(a+bx)^{4/3}\left(\frac{4\left(b^{4/3}c-a\sqrt[3]{b}\,d\right)\log\left(\sqrt[3]{ad+b(c+dx)-bc}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{3d^{7/3}} - \frac{2\left(b^{4/3}c-a\sqrt[3]{b}\,d\right)\log\left(\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{ad+b(c+dx)-bc}+(ad+b(c+dx)-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{3d^{7/3}} + \frac{4\left(b^{4/3}c-a\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}{2\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{\sqrt{3}\,d^{7/3}} + \frac{\sqrt[3]{ad+b(c+dx)-bc}\,(-3ad+b(c+dx)+3bc)}{d^{7/3}\sqrt[3]{c+dx}}\right)}{(ad+bdx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] (d^(4/3)*(a + b*x)^(4/3)*(((3*b*c - 3*a*d + b*(c + d*x))*(-(b*c) + a*d + b*
(c + d*x))^(1/3))/(d^(7/3)*(c + d*x)^(1/3)) + (4*(b^(4/3)*c - a*b^(1/3)*d)*
```

```
ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3) + 2*(-(b*
c) + a*d + b*(c + d*x))^(1/3))])/(Sqrt[3]*d^(7/3)) + (4*(b^(4/3)*c - a*b^(1
/3)*d)*Log[-(b^(1/3)*(c + d*x)^(1/3)) + (-(b*c) + a*d + b*(c + d*x))^(1/3)]
)/(3*d^(7/3)) - (2*(b^(4/3)*c - a*b^(1/3)*d)*Log[b^(2/3)*(c + d*x)^(2/3) +
b^(1/3)*(c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(1/3) + (-(b*c) + a*d
+ b*(c + d*x))^(2/3)])/(3*d^(7/3))))/(a*d + b*d*x)^(4/3)
```

**fricas [A]** time = 1.22, size = 306, normalized size = 1.57

$$\frac{4\,\sqrt{3}\left(bc^{2}-acd+\left(bcd-ad^{2}\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}\,(bdx+bc)}{3\,(bdx+bc)}\right)+2\left(bc^{2}-acd+\left(bcd-ad^{2}\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right)-4\left(bc^{2}-acd+\left(bcd-ad^{2}\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right)+3\,(bdx+4\,bc-3\,ad)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{3\left(d^{3}x+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(3)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*(-b/d)^(1/3)*arctan(1/3*
(2*sqrt(3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x
+ b*c))/(b*d*x + b*c)) + 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*(-b/d)^(1/3)
*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*(-b/d)^(1/3)
 + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 4*(b*c^2 - a*c*d + (b*c*d
- a*d^2)*x)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x
 + c)^(2/3))/(d*x + c)) + 3*(b*d*x + 4*b*c - 3*a*d)*(b*x + a)^(1/3)*(d*x +
c)^(2/3))/(d^3*x + c*d^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(4/3), x)
```

**maple [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(4/3), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{4/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/(c + d*x)^(4/3),x)
[Out] int((a + b*x)^(4/3)/(c + d*x)^(4/3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{4}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(4/3),x)
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(4/3), x)
```

## 3.1476 $\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{4/3}} \, dx$

**Optimal**. Leaf size=149

$$-\frac{3\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2d^{4/3}}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{d^{4/3}}-\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}-\frac{\sqrt[3]{b}\log(a+bx)}{2d^{4/3}}$$

**Rubi [A]** time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 59}

$$-\frac{3\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2d^{4/3}}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{d^{4/3}}-\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}-\frac{\sqrt[3]{b}\log(a+bx)}{2d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(1/3))/(d*(c + d*x)^(1/3)) - (Sqrt[3]*b^(1/3)*ArcTan[1/Sqrt[3
] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))]])/d^(4/3)
 - (b^(1/3)*Log[a + b*x])/(2*d^(4/3)) - (3*b^(1/3)*Log[-1 + (b^(1/3)*(c + d
*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))]])/(2*d^(4/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{4/3}}\,dx = -\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}} + \frac{b\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}} - \frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{d^{4/3}} - \frac{\sqrt[3]{b}\log(a+bx)}{2d^{4/3}} - \frac{3\sqrt[3]{b}\log\left(-1+\cdots\right.}{2d^{4/3}}$$

**Mathematica** [C] time = 0.04, size = 73, normalized size = 0.49

$$\frac{3(a+bx)^{4/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3} {}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{ad-bc}\right)}{4b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 4/3, 7/3, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*(c + d*x)^(4/3))
```

**IntegrateAlgebraic** [A] time = 0.16, size = 200, normalized size = 1.34

$$\frac{\sqrt[3]{b}\log\left(\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}} + \frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}} + b^{2/3}\right)}{2d^{4/3}} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b} - \frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{d^{4/3}} + \frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}} + \frac{1}{\sqrt{3}}\right)}{d^{4/3}} - \frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(1/3))/(d*(c + d*x)^(1/3)) + (Sqrt[3]*b^(1/3)*ArcTan[1/Sqrt[3
] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/d^(4/3)
 - (b^(1/3)*Log[b^(1/3) - (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)])/d^(4/
3) + (b^(1/3)*Log[b^(2/3) + (d^(2/3)*(a + b*x)^(2/3))/(c + d*x)^(2/3) + (b^
(1/3)*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(2*d^(4/3))
```

**fricas** [B] time = 1.46, size = 233, normalized size = 1.56

$$-\frac{2\sqrt{3}(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}(bdx+bc)}{3(bdx+bc)}\right) + (dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right) - 2(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right) + 6(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(d*x + c)*(-b/d)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3
)*(d*x + c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) +
(d*x + c)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x +
 c)^(2/3)*(-b/d)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 2*(d
*x + c)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x + c
)^(2/3))/(d*x + c)) + 6*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(d^2*x + c*d)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(4/3), x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(4/3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(1/3)/(c + d*x)^(4/3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(4/3), x)
```

## 3.1477 $\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}} dx$

**Optimal**. Leaf size=30

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}\,(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.053, Rules used = {37}

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx = \frac{3\sqrt[3]{a+bx}}{(bc-ad)\sqrt[3]{c+dx}}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 30, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]
[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

**fricas** [A] time = 0.76, size = 42, normalized size = 1.40

$$\frac{3\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="fricas")
[Out] 3*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(4/3)), x)
```

**maple** [A] time = 0.00, size = 27, normalized size = 0.90

$$-\frac{3\,(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x)
[Out] -3*(b*x+a)^(1/3)/(d*x+c)^(1/3)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(4/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a+b\,x\right)}^{2/3}\,{\left(c+d\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(2/3)*(c + d*x)^(4/3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{2}{3}}(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(4/3)), x)
```

## 3.1478 $\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx$

**Optimal**. Leaf size=66

$$-\frac{9d\sqrt[3]{a+bx}}{2\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{2(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{9d\sqrt[3]{a+bx}}{2\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{2(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(2*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3)) - (9*d*(a + b*x)^(1/3))/
(2*(b*c - a*d)^2*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx = -\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}} - \frac{(3d)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx}{2(bc-ad)}$$
$$= -\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}} - \frac{9d\sqrt[3]{a+bx}}{2(bc-ad)^2\sqrt[3]{c+dx}}$$

**Mathematica** [**A**] time = 0.02, size = 45, normalized size = 0.68

$$-\frac{3(2ad+b(c+3dx))}{2(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(2*a*d + b*(c + 3*d*x)))/(2*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(1/
3))
```

**IntegrateAlgebraic** [**A**] time = 0.11, size = 49, normalized size = 0.74

$$-\frac{3(c+dx)^{2/3}\left(\frac{2d(a+bx)}{c+dx}+b\right)}{2(a+bx)^{2/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3)*(b + (2*d*(a + b*x))/(c + d*x)))/(2*(b*c - a*d)^2*(a +
b*x)^(2/3))
```

**fricas** [**B**] time = 1.21, size = 126, normalized size = 1.91

$$-\frac{3\,(3\,bdx+bc+2\,ad)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/2*(3*b*d*x + b*c + 2*a*d)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(4/3)), x)
```

**maple [A]** time = 0.00, size = 53, normalized size = 0.80

$$-\frac{3\,(3bdx+2ad+bc)}{2\,(bx+a)^{\frac{2}{3}}\,(dx+c)^{\frac{1}{3}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x)
[Out] -3/2*(3*b*d*x+2*a*d+b*c)/(b*x+a)^(2/3)/(d*x+c)^(1/3)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(4/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/3}\,{\left(c+d\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x)
[Out] int(1/((a + b*x)^(5/3)*(c + d*x)^(4/3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(4/3)), x)
```

## 3.1479 $\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx$

**Optimal**. Leaf size=101

$$\frac{27d^2\sqrt[3]{a+bx}}{5\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{9d}{5(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{5(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{27d^2\sqrt[3]{a+bx}}{5\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{9d}{5(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{5(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(5*(b*c - a*d)*(a + b*x)^(5/3)*(c + d*x)^(1/3)) + (9*d)/(5*(b*c - a*d)^2
*(a + b*x)^(2/3)*(c + d*x)^(1/3)) + (27*d^2*(a + b*x)^(1/3))/(5*(b*c - a*d)
^3*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx = -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}} - \frac{(6d)\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx}{5(bc-ad)}$$

$$= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}} + \frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}} + \frac{\left(9d^2\right)\int \frac{}{(a+bx)^{2/3}}}{5(bc-a}$$

$$= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}} + \frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}} + \frac{27d^2\sqrt[3]{a+b}}{5(bc-ad)^3\sqrt[3]{c}}$$

**Mathematica [A]** time = 0.03, size = 75, normalized size = 0.74

$$\frac{3\left(5a^2d^2+5abd(c+3dx)+b^2\left(-c^2+3cdx+9d^2x^2\right)\right)}{5(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(5*a^2*d^2 + 5*a*b*d*(c + 3*d*x) + b^2*(-c^2 + 3*c*d*x + 9*d^2*x^2)))/(5
*(b*c - a*d)^3*(a + b*x)^(5/3)*(c + d*x)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 73, normalized size = 0.72

$$\frac{3(c+dx)^{5/3}\left(\frac{5d^2(a+bx)^2}{(c+dx)^2}+\frac{5bd(a+bx)}{c+dx}-b^2\right)}{5(a+bx)^{5/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(c + d*x)^(5/3)*(-b^2 + (5*d^2*(a + b*x)^2)/(c + d*x)^2 + (5*b*d*(a + b*
x))/(c + d*x)))/(5*(b*c - a*d)^3*(a + b*x)^(5/3))
```

**fricas [B]** time = 1.11, size = 273, normalized size = 2.70

$$\frac{3\left(9\,b^2d^2x^2-b^2c^2+5\,abcd+5\,a^2d^2+3\left(b^2cd+5\,abd^2\right)x\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{5\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2+5\,a^3b^2cd^3-2\,a^4bd^4\right)x^2+\left(2\,ab^4c^4-5\,a^2b^3c^3d+3\,a^3b^2c^2d^2+a^4bcd^3-a^5d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/5*(9*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 5*a^2*d^2 + 3*(b^2*c*d + 5*a*b*d
^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a
^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 -
 a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*
c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d
^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(4/3)), x)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$-\frac{3\left(9b^2x^2d^2+15ab\,d^2x+3b^2cdx+5a^2d^2+5abcd-b^2c^2\right)}{5\,(bx+a)^{\frac{5}{3}}\,(dx+c)^{\frac{1}{3}}\,\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x)
```

```
[Out] -3/5*(9*b^2*d^2*x^2+15*a*b*d^2*x+3*b^2*c*d*x+5*a^2*d^2+5*a*b*c*d-b^2*c^2)/(
b*x+a)^(5/3)/(d*x+c)^(1/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(4/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{8/3}\,{\left(c+d\,x\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(8/3)*(c + d*x)^(4/3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{8}{3}}(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(4/3)), x)
```

## 3.1480 $\int \frac{1}{(a+bx)^{11/3}(c+dx)^{4/3}}\,dx$

**Optimal**. Leaf size=136

$$-\frac{243d^3\sqrt[3]{a+bx}}{40\sqrt[3]{c+dx}\,(bc-ad)^4}-\frac{81d^2}{40(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{27d}{40(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{8(a+bx)^{8/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{243d^3\sqrt[3]{a+bx}}{40\sqrt[3]{c+dx}\,(bc-ad)^4}-\frac{81d^2}{40(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{27d}{40(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{8(a+bx)^{8/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(8*(b*c - a*d)*(a + b*x)^(8/3)*(c + d*x)^(1/3)) + (27*d)/(40*(b*c - a*d)
^2*(a + b*x)^(5/3)*(c + d*x)^(1/3)) - (81*d^2)/(40*(b*c - a*d)^3*(a + b*x)^
(2/3)*(c + d*x)^(1/3)) - (243*d^3*(a + b*x)^(1/3))/(40*(b*c - a*d)^4*(c + d
*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/3}(c+dx)^{4/3}}\,dx = -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} - \frac{(9d)\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx}{8(bc-ad)}$$

$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} + \frac{\left(27d^2\right)\int \frac{}{(a+b}}{20(bc}$$

$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} - \frac{}{40(bc-ad)^3(}$$

$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} - \frac{}{40(bc-ad)^3(}$$

**Mathematica [A]** time = 0.05, size = 116, normalized size = 0.85

$$-\frac{3\left(40a^3d^3+60a^2bd^2(c+3dx)+24ab^2d\left(-c^2+3cdx+9d^2x^2\right)+b^3\left(5c^3-9c^2dx+27cd^2x^2+81d^3x^3\right)\right)}{40(a+bx)^{8/3}\sqrt[3]{c+dx}\,(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(40*a^3*d^3 + 60*a^2*b*d^2*(c + 3*d*x) + 24*a*b^2*d*(-c^2 + 3*c*d*x + 9
*d^2*x^2) + b^3*(5*c^3 - 9*c^2*d*x + 27*c*d^2*x^2 + 81*d^3*x^3)))/(40*(b*c
- a*d)^4*(a + b*x)^(8/3)*(c + d*x)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 95, normalized size = 0.70

$$-\frac{3(c+dx)^{8/3}\left(-\frac{24b^2d(a+bx)}{c+dx}+\frac{40d^3(a+bx)^3}{(c+dx)^3}+\frac{60bd^2(a+bx)^2}{(c+dx)^2}+5b^3\right)}{40(a+bx)^{8/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(c + d*x)^(8/3)*(5*b^3 + (40*d^3*(a + b*x)^3)/(c + d*x)^3 + (60*b*d^2*(
a + b*x)^2)/(c + d*x)^2 - (24*b^2*d*(a + b*x))/(c + d*x)))/(40*(b*c - a*d)^
4*(a + b*x)^(8/3))
```

**fricas [B]** time = 1.52, size = 456, normalized size = 3.35

$$-\frac{3\left(81\,b^3d^3x^3+5\,b^3c^3-24\,ab^2c^2d+60\,a^2bcd^2+40\,a^3d^3+27\left(b^3cd^2+8\,ab^2d^3\right)x^2-9\left(b^3c^2d-8\,ab^2cd^2-20\,a^2bd^3\right)x\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{40\left(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+\left(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5\right)x^4+\left(b^7c^5-ab^6c^4d-6\,a^2b^5c^3d^2+14\,a^3b^4c^2d^3-11\,a^4b^3cd^4+3\,a^5b^2d^5\right)x^3+3\left(ab^6c^5-3\,a^2b^5c^4d+2\,a^3b^4c^3d^2+2\,a^4b^3c^2d^3-3\,a^5b^2cd^4+a^6bd^5\right)x^2+\left(3\,a^2b^5c^5-11\,a^3b^4c^4d+14\,a^4b^3c^3d^2-6\,a^5b^2c^2d^3-a^6bcd^4+a^7d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/40*(81*b^3*d^3*x^3 + 5*b^3*c^3 - 24*a*b^2*c^2*d + 60*a^2*b*c*d^2 + 40*a^
3*d^3 + 27*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 9*(b^3*c^2*d - 8*a*b^2*c*d^2 - 2
0*a^2*b*d^3)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^3*b^4*c^5 - 4*a^4*b^3*c^
4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^
6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c
^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^
4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2
 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 -
11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a
^7*d^5)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(4/3)), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$-\frac{3\left(81b^{3}d^{3}x^{3}+216a\,b^{2}d^{3}x^{2}+27b^{3}c\,d^{2}x^{2}+180a^{2}b\,d^{3}x+72a\,b^{2}c\,d^{2}x-9b^{3}c^{2}dx+40a^{3}d^{3}+60a^{2}bc\,d^{2}-24a\,b^{2}c^{2}d+5b^{3}c^{3}\right)}{40\,(bx+a)^{\frac{8}{3}}\,(dx+c)^{\frac{1}{3}}\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x)
```

```
[Out] -3/40*(81*b^3*d^3*x^3+216*a*b^2*d^3*x^2+27*b^3*c*d^2*x^2+180*a^2*b*d^3*x+72
*a*b^2*c*d^2*x-9*b^3*c^2*d*x+40*a^3*d^3+60*a^2*b*c*d^2-24*a*b^2*c^2*d+5*b^3
*c^3)/(b*x+a)^(8/3)/(d*x+c)^(1/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-
4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(4/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{11/3}\,(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(11/3)*(c + d*x)^(4/3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(11/3)*(c + d*x)**(4/3)), x)
```

## 3.1481 $\int \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}}\,dx$

**Optimal**. Leaf size=77

$$\sqrt[3]{x-1}\,(x+1)^{2/3}+\frac{1}{3}\log(x-1)+\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}-1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

**Rubi [A]** time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {50, 59}

$$\sqrt[3]{x-1}\,(x+1)^{2/3}+\frac{1}{3}\log(x-1)+\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}-1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (-1 + x)^(1/3)*(1 + x)^(2/3) + (2*ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/3))/(Sqr
t[3]*(-1 + x)^(1/3))])/Sqrt[3] + Log[-1 + x]/3 + Log[-1 + (1 + x)^(1/3)/(-1
 + x)^(1/3)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}}\,dx = \sqrt[3]{-1+x}\,(1+x)^{2/3} - \frac{2}{3}\int \frac{1}{(-1+x)^{2/3}\sqrt[3]{1+x}}\,dx$$

$$= \sqrt[3]{-1+x}\,(1+x)^{2/3} + \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{-1+x}}\right)}{\sqrt{3}} + \frac{1}{3}\log(-1+x) + \log\left(-1 + \frac{\sqrt[3]{1+x}}{\sqrt[3]{-1+x}}\right)$$

**Mathematica [C]** time = 0.02, size = 48, normalized size = 0.62

$$\frac{3\left(\frac{x-1}{x+1}\right)^{4/3}(x+1)^{4/3}\,{}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};\frac{1-x}{2}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (3*((-1 + x)/(1 + x))^(4/3)*(1 + x)^(4/3)*Hypergeometric2F1[1/3, 4/3, 7/3,
(1 - x)/2])/(4*2^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.22, size = 113, normalized size = 1.47

$$\sqrt[3]{x-1}\,(x+1)^{2/3} + \frac{2}{3}\log\left(\sqrt[3]{x-1} - \sqrt[3]{x+1}\right) - \frac{1}{3}\log\left((x-1)^{2/3} + \sqrt[3]{x+1}\,\sqrt[3]{x-1} + (x+1)^{2/3}\right) + \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x+1}}{2\sqrt[3]{x-1}+\sqrt[3]{x+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (-1 + x)^(1/3)*(1 + x)^(2/3) + (2*ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(2*(-1 + x
)^(1/3) + (1 + x)^(1/3))])/Sqrt[3] + (2*Log[(-1 + x)^(1/3) - (1 + x)^(1/3)]
)/3 - Log[(-1 + x)^(2/3) + (-1 + x)^(1/3)*(1 + x)^(1/3) + (1 + x)^(2/3)]/3
```

**fricas [A]** time = 1.35, size = 107, normalized size = 1.39

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(x+1) + 2\sqrt{3}(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}}{3(x+1)}\right) + (x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}} - \frac{1}{3}\log\left(\frac{(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}} + (x+1)^{\frac{1}{3}}(x-1)^{\frac{2}{3}} + x + 1}{x+1}\right) + \frac{2}{3}\log\left(\frac{(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}} - x - 1}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*(sqrt(3)*(x + 1) + 2*sqrt(3)*(x + 1)^(2/3)*(x - 1)^
(1/3))/(x + 1)) + (x + 1)^(2/3)*(x - 1)^(1/3) - 1/3*log(((x + 1)^(2/3)*(x -
```

```
 1)^(1/3) + (x + 1)^(1/3)*(x - 1)^(2/3) + x + 1)/(x + 1)) + 2/3*log(((x + 1
)^(2/3)*(x - 1)^(1/3) - x - 1)/(x + 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^{\frac{1}{3}}}{(x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

**maple [C]** time = 0.38, size = 573, normalized size = 7.44

$$(x-1)^{\frac{1}{3}}(x+1)^{\frac{2}{3}}+\frac{\left(-\frac{2\,\mathrm{RootOf}(\_Z^2+\_Z+1)\ln\left(\frac{-2x^2\mathrm{RootOf}(\_Z^2+\_Z+1)^2+x^2\mathrm{RootOf}(\_Z^2+\_Z+1)+2x\,\mathrm{RootOf}(\_Z^2+\_Z+1)^2+x^2+3(x^3-x^2-x+1)^{\frac{1}{3}}x\,\mathrm{RootOf}(\_Z^2+\_Z+1)+3(x^3-x^2-x+1)^{\frac{1}{3}}x-2x+3(x^3-x^2-x+1)^{\frac{2}{3}}\mathrm{RootOf}(\_Z^2+\_Z+1)-3(x^3-x^2-x+1)^{\frac{1}{3}}\mathrm{RootOf}(\_Z^2+\_Z+1)-\mathrm{RootOf}(\_Z^2+\_Z+1)+3(x^3-x^2-x+1)^{\frac{2}{3}}-3(x^3-x^2-x+1)^{\frac{1}{3}}+1}{x-1}\right)}{3}+\frac{2\,\mathrm{RootOf}(\_Z^2+\_Z+1)\ln\left(-\frac{2x^2\mathrm{RootOf}(\_Z^2+\_Z+1)^2+5x^2\mathrm{RootOf}(\_Z^2+\_Z+1)-2x\,\mathrm{RootOf}(\_Z^2+\_Z+1)^2+2x^2+3(x^3-x^2-x+1)^{\frac{1}{3}}x\,\mathrm{RootOf}(\_Z^2+\_Z+1)-4x\,\mathrm{RootOf}(\_Z^2+\_Z+1)+3(x^3-x^2-x+1)^{\frac{2}{3}}\mathrm{RootOf}(\_Z^2+\_Z+1)-3(x^3-x^2-x+1)^{\frac{1}{3}}\mathrm{RootOf}(\_Z^2+\_Z+1)-\mathrm{RootOf}(\_Z^2+\_Z+1)-2}{x-1}\right)}{3}-\frac{2\ln\left(\frac{-2x^2\mathrm{RootOf}(\_Z^2+\_Z+1)^2+x^2\mathrm{RootOf}(\_Z^2+\_Z+1)+2x\,\mathrm{RootOf}(\_Z^2+\_Z+1)^2+x^2+3(x^3-x^2-x+1)^{\frac{1}{3}}x\,\mathrm{RootOf}(\_Z^2+\_Z+1)+3(x^3-x^2-x+1)^{\frac{1}{3}}x-2x+3(x^3-x^2-x+1)^{\frac{2}{3}}\mathrm{RootOf}(\_Z^2+\_Z+1)-3(x^3-x^2-x+1)^{\frac{1}{3}}\mathrm{RootOf}(\_Z^2+\_Z+1)-\mathrm{RootOf}(\_Z^2+\_Z+1)+3(x^3-x^2-x+1)^{\frac{2}{3}}-3(x^3-x^2-x+1)^{\frac{1}{3}}+1}{x-1}\right)}{3}\right)\left((x-1)^2(x+1)\right)^{\frac{1}{3}}}{(x-1)^{\frac{2}{3}}(x+1)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/3)/(x+1)^(1/3),x)
```

```
[Out] (x-1)^(1/3)*(x+1)^(2/3)+(2/3*RootOf(_Z^2+_Z+1)*ln(-(2*RootOf(_Z^2+_Z+1)^2*x
^2+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(2/3)+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1
)^(1/3)*x-2*RootOf(_Z^2+_Z+1)^2*x+5*RootOf(_Z^2+_Z+1)*x^2-3*RootOf(_Z^2+_Z+
1)*(x^3-x^2-x+1)^(1/3)-4*RootOf(_Z^2+_Z+1)*x+2*x^2-RootOf(_Z^2+_Z+1)-2)/(x-
1))-2/3*ln((-2*RootOf(_Z^2+_Z+1)^2*x^2+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(2
/3)+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1/3)*x+2*RootOf(_Z^2+_Z+1)^2*x+RootO
f(_Z^2+_Z+1)*x^2+3*(x^3-x^2-x+1)^(2/3)-3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1
/3)+3*(x^3-x^2-x+1)^(1/3)*x+x^2-3*(x^3-x^2-x+1)^(1/3)-RootOf(_Z^2+_Z+1)-2*x
+1)/(x-1))*RootOf(_Z^2+_Z+1)-2/3*ln((-2*RootOf(_Z^2+_Z+1)^2*x^2+3*RootOf(_Z
^2+_Z+1)*(x^3-x^2-x+1)^(2/3)+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1/3)*x+2*Ro
otOf(_Z^2+_Z+1)^2*x+RootOf(_Z^2+_Z+1)*x^2+3*(x^3-x^2-x+1)^(2/3)-3*RootOf(_Z
^2+_Z+1)*(x^3-x^2-x+1)^(1/3)+3*(x^3-x^2-x+1)^(1/3)*x+x^2-3*(x^3-x^2-x+1)^(1
/3)-RootOf(_Z^2+_Z+1)-2*x+1)/(x-1)))/(x-1)^(2/3)*((x-1)^2*(x+1))^(1/3)/(x+1
)^(1/3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^{\frac{1}{3}}}{(x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x-1)^{1/3}}{(x+1)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/3)/(x + 1)^(1/3),x)
```

```
[Out] int((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

**sympy [C]** time = 2.55, size = 39, normalized size = 0.51

$$\frac{2^{\frac{2}{3}}(x-1)^{\frac{4}{3}}\,\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{(x-1)e^{i\pi}}{2}\right)}{2\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/3)/(1+x)**(1/3),x)
```

```
[Out] 2**(2/3)*(x - 1)**(4/3)*gamma(4/3)*hyper((1/3, 4/3), (7/3,), (x - 1)*exp_po
lar(I*pi)/2)/(2*gamma(7/3))
```

# 3.1482 $\int (a+bx)^{3/4}(c+dx)^{5/4}\,dx$

**Optimal**. Leaf size=205

$$\frac{5(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}{96b^2d}+\frac{5(a+bx)^{7/4}\sqrt[4]{c}}{24b}$$

**Rubi [A]** time = 0.14, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 331, 298, 205, 208}

$$\frac{5(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}{96b^2d}+\frac{5(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] (5*(b*c - a*d)^2*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(96*b^2*d) + (5*(b*c - a*
d)*(a + b*x)^(7/4)*(c + d*x)^(1/4))/(24*b^2) + ((a + b*x)^(7/4)*(c + d*x)^(
5/4))/(3*b) + (5*(b*c - a*d)^3*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c
 + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4)) - (5*(b*c - a*d)^3*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int (a+bx)^{3/4}(c+dx)^{5/4}\,dx = \frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad))\int (a+bx)^{3/4}\sqrt[4]{c+dx}\,dx}{12b}$$

$$= \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b} + \frac{\left(5(bc-ad)^2\right)\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,d}{96b^2}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+}{3b}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+}{3b}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+}{3b}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+}{3b}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+}{3b}$$

**Mathematica [C]** time = 0.06, size = 73, normalized size = 0.36

$$\frac{4(a+bx)^{7/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{ad-bc}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 7/4, 11/4, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

**IntegrateAlgebraic [A]** time = 0.53, size = 218, normalized size = 1.06

$$-\frac{5(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{64b^{9/4}d^{7/4}} - \frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{64b^{9/4}d^{7/4}} + \frac{(bc-ad)^3\left(\frac{5b^2(c+dx)^{9/4}}{(a+bx)^{9/4}} - \frac{15d^2\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}} + \frac{42bd(c+dx)^{5/4}}{(a+bx)^{5/4}}\right)}{96b^2d\left(\frac{b(c+dx)}{a+bx}-d\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] ((b*c - a*d)^3*((-15*d^2*(c + d*x)^(1/4))/(a + b*x)^(1/4) + (42*b*d*(c + d*
x)^(5/4))/(a + b*x)^(5/4) + (5*b^2*(c + d*x)^(9/4))/(a + b*x)^(9/4)))/(96*b
^2*d*(-d + (b*(c + d*x))/(a + b*x))^3) - (5*(b*c - a*d)^3*ArcTan[(b^(1/4)*(
c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))])/(64*b^(9/4)*d^(7/4)) - (5*(b*c
- a*d)^3*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))])/(64*
b^(9/4)*d^(7/4))
```

**fricas [B]** time = 1.56, size = 2151, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/384*(60*b^2*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4)*arc
tan(((b^10*c^3*d^5 - 3*a*b^9*c^2*d^6 + 3*a^2*b^8*c*d^7 - a^3*b^7*d^8)*(b*x
+ a)^(3/4)*(d*x + c)^(1/4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^1
0*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 9
24*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^
3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))
^(3/4) + (b^8*d^5*x + a*b^7*d^5)*sqrt(((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^
4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d
^6)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*d^4*x + a*b^4*d^4)*sqrt((b^12*c^12 -
 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^
8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^
11*b*c*d^11 + a^12*d^12)/(b^9*d^7)))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(b^9*d^7))^(3/4))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^1
0*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^
5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a
^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^
13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 +
495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b
^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^
10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) + 15*b^2*d*((b^12*c^12 - 12*a*b^
11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^
```

```
4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d
^11 + a^12*d^12)/(b^9*d^7))^(1/4)*log(-5*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*d^2*x + a*b^2*d^2
)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d
^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*
a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c
^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4))/(b*x + a)) - 15*b
^2*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4)*log(-5*((b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x + a)^(3/4)*(d*x + c)^(1/4
) - (b^3*d^2*x + a*b^2*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7)
)^(1/4))/(b*x + a)) - 4*(32*b^2*d^2*x^2 + 5*b^2*c^2 + 42*a*b*c*d - 15*a^2*d
^2 + 4*(13*b^2*c*d + 3*a*b*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^2*d)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{4}}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/4)*(d*x + c)^(5/4), x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{4}}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/4)*(d*x+c)^(5/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{4}}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)*(d*x + c)^(5/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,x)^{3/4}\,(c + d\,x)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)*(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(3/4)*(c + d*x)^(5/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^{\frac{3}{4}}\,(c + dx)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)*(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)*(c + d*x)**(5/4), x)
```

## 3.1483 $\int \frac{(c+dx)^{5/4}}{\sqrt[4]{a+bx}}\,dx$

**Optimal**. Leaf size=167

$$-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8b^2}+\frac{(a+bx)^{3/4}(c+}{2b}$$

**Rubi [A]** time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 331, 298, 205, 208}

$$-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*b^2) + ((a + b*x)^(3/4)*
(c + d*x)^(5/4))/(2*b) - (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/
(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4)) + (5*(b*c - a*d)^2*ArcTanh
[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{\sqrt[4]{a+bx}}\,dx = \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad))\int \frac{\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\,dx}{8b}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{\left(5(bc-ad)^2\right)\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{32b^2}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,d\ldots}{8b^3}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \ldots}{8b^3}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, \ldots}{16b^2\sqrt{d}}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}} + \frac{5(bc\ldots}{\ldots}$$

**Mathematica [C]** time = 0.04, size = 73, normalized size = 0.44

$$\frac{4(a+bx)^{3/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{d(a+bx)}{ad-bc}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(3/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 3/4, 7/4, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

**IntegrateAlgebraic [A]** time = 0.38, size = 189, normalized size = 1.13

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{16b^{9/4}d^{3/4}}+\frac{(bc-ad)^2\left(\frac{9b(c+dx)^{5/4}}{(a+bx)^{5/4}}-\frac{5d\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\right)}{8b^2\left(\frac{b(c+dx)}{a+bx}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] ((b*c - a*d)^2*((-5*d*(c + d*x)^(1/4))/(a + b*x)^(1/4) + (9*b*(c + d*x)^(5/
4))/(a + b*x)^(5/4)))/(8*b^2*(-d + (b*(c + d*x))/(a + b*x))^2) + (5*(b*c -
a*d)^2*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))])/(16*b^(
9/4)*d^(3/4)) + (5*(b*c - a*d)^2*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)
*(a + b*x)^(1/4))])/(16*b^(9/4)*d^(3/4))
```

**fricas [B]** time = 1.34, size = 1468, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/32*(20*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*arctan(-((b^9*c^2*d^2 - 2*a*b^8*c*d^
3 + a^2*b^7*d^4)*(b*x + a)^(3/4)*(d*x + c)^(1/4)*((b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(3/4) -
 (b^8*d^2*x + a*b^7*d^2)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*d^2*x + a*b^
4*d^2)*sqrt((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
```

```
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*
b*c*d^7 + a^8*d^8)/(b^9*d^3)))/(b*x + a))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(3/4))/(a*b^8*
c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^
4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d
^8 + (b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 7
0*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d
^7 + a^8*b*d^8)*x)) - 5*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*log(5*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*d*x + a*b^2*d)*((b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8)/(b^9*d^3))^(1/4))/(b*x + a)) + 5*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*log(5*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b^3*d*x +
a*b^2*d)*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4))/(b*x + a)) - 4*(4*b*d*x + 9*b*c - 5*a*d)
*(b*x + a)^(3/4)*(d*x + c)^(1/4))/b^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(1/4), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(1/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(1/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(1/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(1/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(1/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{\sqrt[4]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(1/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(1/4), x)
```

## 3.1484 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/4}} dx$

**Optimal**. Leaf size=152

$$-\frac{5\sqrt[4]{d}(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5\sqrt[4]{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}$$

**Rubi [A]** time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {47, 50, 63, 331, 298, 205, 208}

$$\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{5\sqrt[4]{d}(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5\sqrt[4]{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{9/4}}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] (5*d*(a + b*x)^(3/4)*(c + d*x)^(1/4))/b^2 - (4*(c + d*x)^(5/4))/(b*(a + b*x
)^(1/4)) - (5*d^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)
*(c + d*x)^(1/4))])/(2*b^(9/4)) + (5*d^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(9/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/4}}\,dx = -\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\,dx}{b}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{4b^2}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{b^3}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^3}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{\left(5\sqrt{d}\,(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2b^2} - \frac{(5}{}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} - \frac{5\sqrt[4]{d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}} + \frac{5\sqrt[4]{d}\,(bc-ad)\,\text{tanh}}{2b^{9/4}}$$

**Mathematica [C]** time = 0.05, size = 71, normalized size = 0.47

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4};\frac{3}{4};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[4]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -1/4, 3/4, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

**IntegrateAlgebraic [A]** time = 13.91, size = 244, normalized size = 1.61

$$\frac{(ad+bdx)^{5/4}\left(\frac{5\left(bc\sqrt[4]{d}-ad^{5/4}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2b^{9/4}}+\frac{5\left(bc\sqrt[4]{d}-ad^{5/4}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2b^{9/4}}+\frac{(ad+b(c+dx)-bc)^{3/4}\left(-5ad^{5/4}\sqrt[4]{c+dx}-b\sqrt[4]{d}\,(c+dx)^{5/4}+5bc\sqrt[4]{d}\,\sqrt[4]{c+dx}\right)}{b^2(-ad-b(c+dx)+bc)}\right)}{d^{5/4}(a+bx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] ((a*d + b*d*x)^(5/4)*(((-(b*c) + a*d + b*(c + d*x))^(3/4)*(5*b*c*d^(1/4)*(c
 + d*x)^(1/4) - 5*a*d^(5/4)*(c + d*x)^(1/4) - b*d^(1/4)*(c + d*x)^(5/4)))/(
b^2*(b*c - a*d - b*(c + d*x))) + (5*(b*c*d^(1/4) - a*d^(5/4))*ArcTan[(b^(1/
4)*(c + d*x)^(1/4))/(-(b*c) + a*d + b*(c + d*x))^(1/4)])/(2*b^(9/4)) + (5*(
b*c*d^(1/4) - a*d^(5/4))*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(-(b*c) + a*d +
b*(c + d*x))^(1/4)])/(2*b^(9/4))))/(d^(5/4)*(a + b*x)^(5/4))
```

**fricas [B]** time = 1.04, size = 857, normalized size = 5.64

$$\frac{20\left(b^3x+ab^2\right)\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(b^8c-ab^7d\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{3}{4}}+\left(b^8x+ab^7\right)\sqrt{\frac{\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{bx+a}\sqrt{dx+c}+\left(b^5x+ab^4\right)\sqrt{\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}}}{bx+a}}\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{3}{4}}}{ab^4c^4d-4a^2b^3c^3d^2+6a^3b^2c^2d^3-4a^4bcd^4+a^5d^5+\left(b^5c^4d-4ab^4c^3d^2+6a^2b^3c^2d^3-4a^3b^2cd^4+a^4bd^5\right)x}\right)+5\left(b^3x+ab^2\right)\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}+\left(b^3x+ab^2\right)\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\right)}{bx+a}\right)-5\left(b^3x+ab^2\right)\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}-\left(b^3x+ab^2\right)\left(\frac{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\right)}{bx+a}\right)+4\,(bdx-4\,bc+5\,ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{4\left(b^3x+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/4*(20*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 -
 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*arctan(((b^8*c - a*b^7*d)*(b*x + a)^(3
/4)*(d*x + c)^(1/4)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a
^3*b*c*d^4 + a^4*d^5)/b^9)^(3/4) + (b^8*x + a*b^7)*sqrt(((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*x + a*b^4)*sqrt((b^4*c^4*d
 - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9))/(b*
x + a))*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 +
 a^4*d^5)/b^9)^(3/4))/(a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 + 6*a^3*b^2*c^2*d^3
- 4*a^4*b*c*d^4 + a^5*d^5 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^
3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)) + 5*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*
b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*log(-
5*((b*c - a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*x + a*b^2)*((b^4*c^4*
d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/
4))/(b*x + a)) - 5*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^
2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*log(-5*((b*c - a*d)*(b*x +
a)^(3/4)*(d*x + c)^(1/4) - (b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 +
6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4))/(b*x + a)) + 4*(b*
d*x - 4*b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^3*x + a*b^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/4), x)
```

**maple [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(5/4),x)
[Out] int((d*x+c)^(5/4)/(b*x+a)^(5/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{5/4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(5/4),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(5/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(5/4),x)
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(5/4), x)
```

# 3.1485 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/4}}\,dx$

**Optimal**. Leaf size=134

$$-\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}-\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}$$

**Rubi [A]** time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 63, 331, 298, 205, 208}

$$-\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}-\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (-4*d*(c + d*x)^(1/4))/(b^2*(a + b*x)^(1/4)) - (4*(c + d*x)^(5/4))/(5*b*(a
+ b*x)^(5/4)) - (2*d^(5/4)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d
*x)^(1/4))])/b^(9/4) + (2*d^(5/4)*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/b^(9/4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/4}}\,dx = -\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{d\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/4}}\,dx}{b}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{d^2\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{b^2}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{(4d^2)\,\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{b^3}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{(4d^2)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^3}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{(2d^{3/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^2} - \frac{(2d^{3/2})\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} - \frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{9/4}} + \frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{9/4}}$$

**Mathematica [C]** time = 0.05, size = 73, normalized size = 0.54

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{5}{4};-\frac{1}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

**IntegrateAlgebraic [A]** time = 0.21, size = 134, normalized size = 1.00

$$\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{b^{9/4}} + \frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{b^{9/4}} - \frac{4\left(\frac{b(c+dx)^{5/4}}{(a+bx)^{5/4}}+\frac{5d\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (-4*((5*d*(c + d*x)^(1/4))/(a + b*x)^(1/4) + (b*(c + d*x)^(5/4))/(a + b*x)^
(5/4)))/(5*b^2) + (2*d^(5/4)*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a +
 b*x)^(1/4))]])/b^(9/4) + (2*d^(5/4)*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1
/4)*(a + b*x)^(1/4))]])/b^(9/4)
```

**fricas [B]** time = 1.17, size = 368, normalized size = 2.75

$$-\frac{20\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}b^{7}d\left(\frac{d^{5}}{b^{9}}\right)^{\frac{3}{4}}-\left(b^{8}x+ab^{7}\right)\sqrt{\frac{\sqrt{bx+a}\,\sqrt{dx+c}\,d^{2}+\left(b^{5}x+ab^{4}\right)\sqrt{\frac{d^{5}}{b^{9}}}}{bx+a}}\left(\frac{d^{5}}{b^{9}}\right)^{\frac{3}{4}}}{bd^{5}x+ad^{5}}\right)-5\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}\log\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}d+\left(b^{3}x+ab^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}}{bx+a}\right)+5\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}\log\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}d-\left(b^{3}x+ab^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}}{bx+a}\right)+4\,(6\,bdx+bc+5\,ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{5\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -1/5*(20*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9)^(1/4)*arctan(-((b*x + a)
^(3/4)*(d*x + c)^(1/4)*b^7*d*(d^5/b^9)^(3/4) - (b^8*x + a*b^7)*sqrt((sqrt(b
*x + a)*sqrt(d*x + c)*d^2 + (b^5*x + a*b^4)*sqrt(d^5/b^9))/(b*x + a))*(d^5/
b^9)^(3/4))/(b*d^5*x + a*d^5)) - 5*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9
)^(1/4)*log(((b*x + a)^(3/4)*(d*x + c)^(1/4)*d + (b^3*x + a*b^2)*(d^5/b^9)^
(1/4))/(b*x + a)) + 5*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9)^(1/4)*log((
(b*x + a)^(3/4)*(d*x + c)^(1/4)*d - (b^3*x + a*b^2)*(d^5/b^9)^(1/4))/(b*x +
 a)) + 4*(6*b*d*x + b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^4*x^2
+ 2*a*b^3*x + a^2*b^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/4), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(9/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(9/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(9/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(9/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(9/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(9/4), x)
```

## 3.1486 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx$

**Optimal**. Leaf size=32

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx = -\frac{4(c+dx)^{9/4}}{9(bc-ad)(a+bx)^{9/4}}$$

**Mathematica [A]** time = 0.02, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

**fricas [B]** time = 1.03, size = 104, normalized size = 3.25

$$-\frac{4\left(d^2x^2+2\,cdx+c^2\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{9\left(a^3bc-a^4d+\left(b^4c-ab^3d\right)x^3+3\left(ab^3c-a^2b^2d\right)x^2+3\left(a^2b^2c-a^3bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="fricas")
```

```
[Out] -4/9*(d^2*x^2 + 2*c*d*x + c^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^3*b*c - a
^4*d + (b^4*c - a*b^3*d)*x^3 + 3*(a*b^3*c - a^2*b^2*d)*x^2 + 3*(a^2*b^2*c -
 a^3*b*d)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(13/4), x)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.84

$$\frac{4\,(dx+c)^{\frac{9}{4}}}{9\,(bx+a)^{\frac{9}{4}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(13/4),x)
```

```
[Out] 4/9/(b*x+a)^(9/4)*(d*x+c)^(9/4)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(13/4), x)
```

**mupad [B]** time = 0.81, size = 99, normalized size = 3.09

$$\frac{4\,c^2\,{\left(c+d\,x\right)}^{1/4}+4\,d^2\,x^2\,{\left(c+d\,x\right)}^{1/4}+8\,c\,d\,x\,{\left(c+d\,x\right)}^{1/4}}{{\left(a+b\,x\right)}^{1/4}\,\left(9\,d\,a^3+18\,d\,a^2\,b\,x-9\,c\,a^2\,b+9\,d\,a\,b^2\,x^2-18\,c\,a\,b^2\,x-9\,c\,b^3\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(13/4),x)
```

```
[Out] (4*c^2*(c + d*x)^(1/4) + 4*d^2*x^2*(c + d*x)^(1/4) + 8*c*d*x*(c + d*x)^(1/4
))/((a + b*x)^(1/4)*(9*a^3*d - 9*b^3*c*x^2 - 9*a^2*b*c - 18*a*b^2*c*x + 18*
a^2*b*d*x + 9*a*b^2*d*x^2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(13/4),x)
```

```
[Out] Timed out
```

## 3.1487 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}} dx$

**Optimal**. Leaf size=66

$$\frac{16d(c+dx)^{9/4}}{117(a+bx)^{9/4}(bc-ad)^2} - \frac{4(c+dx)^{9/4}}{13(a+bx)^{13/4}(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{16d(c+dx)^{9/4}}{117(a+bx)^{9/4}(bc-ad)^2} - \frac{4(c+dx)^{9/4}}{13(a+bx)^{13/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(9
/4))/(117*(b*c - a*d)^2*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx = -\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}} - \frac{(4d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx}{13(bc-ad)}$$
$$= -\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d(c+dx)^{9/4}}{117(bc-ad)^2(a+bx)^{9/4}}$$

**Mathematica** [A] time = 0.03, size = 46, normalized size = 0.70

$$\frac{4(c+dx)^{9/4}(13ad-9bc+4bdx)}{117(a+bx)^{13/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
```

```
[Out] (4*(c + d*x)^(9/4)*(-9*b*c + 13*a*d + 4*b*d*x))/(117*(b*c - a*d)^2*(a + b*x
)^(13/4))
```

**IntegrateAlgebraic** [A] time = 0.17, size = 51, normalized size = 0.77

$$-\frac{4(c+dx)^{9/4}\left(\frac{9b(c+dx)}{a+bx}-13d\right)}{117(a+bx)^{9/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(-13*d + (9*b*(c + d*x))/(a + b*x)))/(117*(b*c - a*d)^2
*(a + b*x)^(9/4))
```

**fricas** [B] time = 1.24, size = 235, normalized size = 3.56

$$\frac{4\left(4\,bd^3x^3 - 9\,bc^3 + 13\,ac^2d - \left(bcd^2 - 13\,ad^3\right)x^2 - 2\left(7\,bc^2d - 13\,acd^2\right)x\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{117\left(a^4b^2c^2 - 2\,a^5bcd + a^6d^2 + \left(b^6c^2 - 2\,ab^5cd + a^2b^4d^2\right)x^4 + 4\left(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2\right)x^3 + 6\left(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2\right)x^2 + 4\left(a^3b^3c^2 - 2\,a^4b^2cd + a^5bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="fricas")
```

```
[Out] 4/117*(4*b*d^3*x^3 - 9*b*c^3 + 13*a*c^2*d - (b*c*d^2 - 13*a*d^3)*x^2 - 2*(7
*b*c^2*d - 13*a*c*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^4*b^2*c^2 - 2*
a^5*b*c*d + a^6*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 4*(a*b^5*
```

```
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^3 + 6*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a
^4*b^2*d^2)*x^2 + 4*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{17}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(17/4), x)
```

**maple [A]** time = 0.01, size = 54, normalized size = 0.82

$$\frac{4\,(dx+c)^{\frac{9}{4}}\,(4bdx+13ad-9bc)}{117\,(bx+a)^{\frac{13}{4}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(17/4),x)
[Out] 4/117*(d*x+c)^(9/4)*(4*b*d*x+13*a*d-9*b*c)/(b*x+a)^(13/4)/(a^2*d^2-2*a*b*c*
d+b^2*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{17}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(17/4), x)
```

**mupad [B]** time = 0.95, size = 178, normalized size = 2.70

$$\frac{(c+d\,x)^{1/4}\left(\frac{16\,d^3\,x^3}{117\,b^2\,(a\,d-b\,c)^2}-\frac{36\,b\,c^3-52\,a\,c^2\,d}{117\,b^3\,(a\,d-b\,c)^2}+\frac{x^2\,\left(52\,a\,d^3-4\,b\,c\,d^2\right)}{117\,b^3\,(a\,d-b\,c)^2}+\frac{8\,c\,d\,x\,(13\,a\,d-7\,b\,c)}{117\,b^3\,(a\,d-b\,c)^2}\right)}{x^3\,(a+b\,x)^{1/4}+\frac{a^3\,(a+b\,x)^{1/4}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/4}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/4}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(17/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((16*d^3*x^3)/(117*b^2*(a*d - b*c)^2) - (36*b*c^3 - 52*a*c
^2*d)/(117*b^3*(a*d - b*c)^2) + (x^2*(52*a*d^3 - 4*b*c*d^2))/(117*b^3*(a*d
- b*c)^2) + (8*c*d*x*(13*a*d - 7*b*c))/(117*b^3*(a*d - b*c)^2)))/(x^3*(a +
b*x)^(1/4) + (a^3*(a + b*x)^(1/4))/b^3 + (3*a*x^2*(a + b*x)^(1/4))/b + (3*a
^2*x*(a + b*x)^(1/4))/b^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(17/4),x)
```

```
[Out] Timed out
```

# 3.1488 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}}\,dx$

**Optimal**. Leaf size=101

$$-\frac{128d^2(c+dx)^{9/4}}{1989(a+bx)^{9/4}(bc-ad)^3}+\frac{32d(c+dx)^{9/4}}{221(a+bx)^{13/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{17(a+bx)^{17/4}(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{128d^2(c+dx)^{9/4}}{1989(a+bx)^{9/4}(bc-ad)^3}+\frac{32d(c+dx)^{9/4}}{221(a+bx)^{13/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{17(a+bx)^{17/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(17*(b*c - a*d)*(a + b*x)^(17/4)) + (32*d*(c + d*x)^(9
/4))/(221*(b*c - a*d)^2*(a + b*x)^(13/4)) - (128*d^2*(c + d*x)^(9/4))/(1989
*(b*c - a*d)^3*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}}\,dx = -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}} - \frac{(8d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx}{17(bc-ad)}$$
$$= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}} + \frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}} + \frac{\left(32d^2\right)\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx}{221(bc-ad)^2}$$
$$= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}} + \frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}} - \frac{128d^2(c+dx)^{9/4}}{1989(bc-ad)^3(a+bx)^{9/4}}$$

**Mathematica** [A] time = 0.05, size = 77, normalized size = 0.76

$$-\frac{4(c+dx)^{9/4}\left(221a^2d^2+34abd(4dx-9c)+b^2\left(117c^2-72cdx+32d^2x^2\right)\right)}{1989(a+bx)^{17/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(221*a^2*d^2 + 34*a*b*d*(-9*c + 4*d*x) + b^2*(117*c^2 -
 72*c*d*x + 32*d^2*x^2)))/(1989*(b*c - a*d)^3*(a + b*x)^(17/4))
```

**IntegrateAlgebraic** [A] time = 0.18, size = 73, normalized size = 0.72

$$-\frac{4(c+dx)^{9/4}\left(\frac{117b^2(c+dx)^2}{(a+bx)^2}-\frac{306bd(c+dx)}{a+bx}+221d^2\right)}{1989(a+bx)^{9/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(221*d^2 - (306*b*d*(c + d*x))/(a + b*x) + (117*b^2*(c
+ d*x)^2)/(a + b*x)^2))/(1989*(b*c - a*d)^3*(a + b*x)^(9/4))
```

**fricas** [B] time = 1.52, size = 426, normalized size = 4.22

$$-\frac{4\left(32\,b^2d^4x^4+117\,b^2c^4-306\,abc^3d+221\,a^2c^2d^2-8\left(b^2cd^3-17\,abd^4\right)x^3+\left(5\,b^2c^2d^2-34\,abcd^3+221\,a^2d^4\right)x^2+2\left(81\,b^2c^3d-238\,abc^2d^2+221\,a^2cd^3\right)x\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{1989\left(a^5b^3c^3-3\,a^6b^2c^2d+3\,a^7bcd^2-a^8d^3+\left(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3\right)x^5+5\left(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3\right)x^4+10\left(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3\right)x^3+10\left(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3\right)x^2+5\left(a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="fricas")
```

```
[Out] -4/1989*(32*b^2*d^4*x^4 + 117*b^2*c^4 - 306*a*b*c^3*d + 221*a^2*c^2*d^2 - 8
*(b^2*c*d^3 - 17*a*b*d^4)*x^3 + (5*b^2*c^2*d^2 - 34*a*b*c*d^3 + 221*a^2*d^4
```

```
)*x^2 + 2*(81*b^2*c^3*d - 238*a*b*c^2*d^2 + 221*a^2*c*d^3)*x)*(b*x + a)^(3/
4)*(d*x + c)^(1/4)/(a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3
 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^5 + 5*(a*b^7
*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^4 + 10*(a^2*b^6*c
^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^3 + 10*(a^3*b^5*c^3
 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x^2 + 5*(a^4*b^4*c^3 -
3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{21}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(21/4), x)
```

**maple** [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{4\,(dx+c)^{\frac{9}{4}}\left(32b^2x^2d^2+136ab\,d^2x-72b^2cdx+221a^2d^2-306abcd+117b^2c^2\right)}{1989\,(bx+a)^{\frac{17}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(21/4),x)
```

```
[Out] 4/1989*(d*x+c)^(9/4)*(32*b^2*d^2*x^2+136*a*b*d^2*x-72*b^2*c*d*x+221*a^2*d^2
-306*a*b*c*d+117*b^2*c^2)/(b*x+a)^(17/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{21}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(21/4), x)
```

**mupad [B]** time = 1.13, size = 268, normalized size = 2.65

$$\frac{(c+dx)^{1/4}\left(\frac{884a^2c^2d^2-1224abc^3d+468b^2c^4}{1989b^4(ad-bc)^3}+\frac{x^2\left(884a^2d^4-136abcd^3+20b^2c^2d^2\right)}{1989b^4(ad-bc)^3}+\frac{128d^4x^4}{1989b^2(ad-bc)^3}+\frac{32d^3x^3(17ad-bc)}{1989b^3(ad-bc)^3}+\frac{8cdx\left(221a^2d^2-238abcd+81b^2c^2\right)}{1989b^4(ad-bc)^3}\right)}{x^4(a+bx)^{1/4}+\frac{a^4(a+bx)^{1/4}}{b^4}+\frac{6a^2x^2(a+bx)^{1/4}}{b^2}+\frac{4ax^3(a+bx)^{1/4}}{b}+\frac{4a^3x(a+bx)^{1/4}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(21/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((468*b^2*c^4 + 884*a^2*c^2*d^2 - 1224*a*b*c^3*d)/(1989*b^
4*(a*d - b*c)^3) + (x^2*(884*a^2*d^4 + 20*b^2*c^2*d^2 - 136*a*b*c*d^3))/(19
89*b^4*(a*d - b*c)^3) + (128*d^4*x^4)/(1989*b^2*(a*d - b*c)^3) + (32*d^3*x^
3*(17*a*d - b*c))/(1989*b^3*(a*d - b*c)^3) + (8*c*d*x*(221*a^2*d^2 + 81*b^2
*c^2 - 238*a*b*c*d))/(1989*b^4*(a*d - b*c)^3)))/(x^4*(a + b*x)^(1/4) + (a^4
*(a + b*x)^(1/4))/b^4 + (6*a^2*x^2*(a + b*x)^(1/4))/b^2 + (4*a*x^3*(a + b*x
)^(1/4))/b + (4*a^3*x*(a + b*x)^(1/4))/b^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(21/4),x)
```

```
[Out] Timed out
```

## 3.1489 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{25/4}} \, dx$

**Optimal**. Leaf size=136

$$\frac{512d^3(c+dx)^{9/4}}{13923(a+bx)^{9/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{9/4}}{1547(a+bx)^{13/4}(bc-ad)^3}+\frac{16d(c+dx)^{9/4}}{119(a+bx)^{17/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{21(a+bx)^{21/4}(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512d^3(c+dx)^{9/4}}{13923(a+bx)^{9/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{9/4}}{1547(a+bx)^{13/4}(bc-ad)^3}+\frac{16d(c+dx)^{9/4}}{119(a+bx)^{17/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{21(a+bx)^{21/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(21*(b*c - a*d)*(a + b*x)^(21/4)) + (16*d*(c + d*x)^(9
/4))/(119*(b*c - a*d)^2*(a + b*x)^(17/4)) - (128*d^2*(c + d*x)^(9/4))/(1547
*(b*c - a*d)^3*(a + b*x)^(13/4)) + (512*d^3*(c + d*x)^(9/4))/(13923*(b*c -
a*d)^4*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{25/4}}\,dx = -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} - \frac{(4d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}}\,dx}{7(bc-ad)}$$

$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} + \frac{\left(32d^2\right)\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx}{119(bc-ad)^2}$$

$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} - \frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}} - \frac{\left(128d^3\right.}{154}$$

$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} - \frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}} + \frac{51}{13923(}$$

**Mathematica** [A] time = 0.07, size = 118, normalized size = 0.87

$$\frac{4(c+dx)^{9/4}\left(1547a^3d^3+357a^2bd^2(4dx-9c)+21ab^2d\left(117c^2-72cdx+32d^2x^2\right)+b^3\left(-663c^3+468c^2dx-288cd^2x^2+128d^3x^3\right)\right)}{13923(a+bx)^{21/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
```

```
[Out] (4*(c + d*x)^(9/4)*(1547*a^3*d^3 + 357*a^2*b*d^2*(-9*c + 4*d*x) + 21*a*b^2*
d*(117*c^2 - 72*c*d*x + 32*d^2*x^2) + b^3*(-663*c^3 + 468*c^2*d*x - 288*c*d
^2*x^2 + 128*d^3*x^3)))/(13923*(b*c - a*d)^4*(a + b*x)^(21/4))
```

**IntegrateAlgebraic** [A] time = 0.19, size = 95, normalized size = 0.70

$$-\frac{4(c+dx)^{9/4}\left(\frac{663b^3(c+dx)^3}{(a+bx)^3}-\frac{2457b^2d(c+dx)^2}{(a+bx)^2}+\frac{3213bd^2(c+dx)}{a+bx}-1547d^3\right)}{13923(a+bx)^{9/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(-1547*d^3 + (3213*b*d^2*(c + d*x))/(a + b*x) - (2457*b
^2*d*(c + d*x)^2)/(a + b*x)^2 + (663*b^3*(c + d*x)^3)/(a + b*x)^3))/(13923*
(b*c - a*d)^4*(a + b*x)^(9/4))
```

**fricas** [B] time = 1.66, size = 649, normalized size = 4.77

$$\frac{4\left(128\,b^3d^5x^5-663\,b^3c^5+2457\,ab^2c^4d-3213\,a^2bc^3d^2+1547\,a^3c^2d^3-32\left(b^3cd^4-21\,ab^2d^5\right)x^4+4\left(5\,b^3c^2d^3-42\,ab^2cd^4+357\,a^2bd^5\right)x^3-\left(15\,b^3c^3d^2-105\,ab^2c^2d^3+357\,a^2bcd^4-1547\,a^3d^5\right)x^2-2\left(429\,b^3c^4d-1701\,ab^2c^3d^2+2499\,a^2bc^2d^3-1547\,a^3cd^4\right)x\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{13923\left(a^6b^4c^4-4\,a^7b^3c^3d+6\,a^8b^2c^2d^2-4\,a^9bcd^3+a^{10}d^4+\left(b^{10}c^4-4\,ab^9c^3d+6\,a^2b^8c^2d^2-4\,a^3b^7cd^3+a^4b^6d^4\right)x^6+6\left(ab^9c^4-4\,a^2b^8c^3d+6\,a^3b^7c^2d^2-4\,a^4b^6cd^3+a^5b^5d^4\right)x^5+15\left(a^2b^8c^4-4\,a^3b^7c^3d+6\,a^4b^6c^2d^2-4\,a^5b^5cd^3+a^6b^4d^4\right)x^4+20\left(a^3b^7c^4-4\,a^4b^6c^3d+6\,a^5b^5c^2d^2-4\,a^6b^4cd^3+a^7b^3d^4\right)x^3+15\left(a^4b^6c^4-4\,a^5b^5c^3d+6\,a^6b^4c^2d^2-4\,a^7b^3cd^3+a^8b^2d^4\right)x^2+6\left(a^5b^5c^4-4\,a^6b^4c^3d+6\,a^7b^3c^2d^2-4\,a^8b^2cd^3+a^9bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="fricas")
```

```
[Out] 4/13923*(128*b^3*d^5*x^5 - 663*b^3*c^5 + 2457*a*b^2*c^4*d - 3213*a^2*b*c^3*
d^2 + 1547*a^3*c^2*d^3 - 32*(b^3*c*d^4 - 21*a*b^2*d^5)*x^4 + 4*(5*b^3*c^2*d
^3 - 42*a*b^2*c*d^4 + 357*a^2*b*d^5)*x^3 - (15*b^3*c^3*d^2 - 105*a*b^2*c^2*
d^3 + 357*a^2*b*c*d^4 - 1547*a^3*d^5)*x^2 - 2*(429*b^3*c^4*d - 1701*a*b^2*c
^3*d^2 + 2499*a^2*b*c^2*d^3 - 1547*a^3*c*d^4)*x)*(b*x + a)^(3/4)*(d*x + c)^
(1/4)/(a^6*b^4*c^4 - 4*a^7*b^3*c^3*d + 6*a^8*b^2*c^2*d^2 - 4*a^9*b*c*d^3 +
a^10*d^4 + (b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3
+ a^4*b^6*d^4)*x^6 + 6*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4
*a^4*b^6*c*d^3 + a^5*b^5*d^4)*x^5 + 15*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a
^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^4 + 20*(a^3*b^7*c^4 - 4*a
^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x^3 + 15*
(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*
b^2*d^4)*x^2 + 6*(a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8
*b^2*c*d^3 + a^9*b*d^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{25}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(25/4), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$\frac{4\,(dx+c)^{\frac{9}{4}}\left(128b^3d^3x^3+672a\,b^2d^3x^2-288b^3c\,d^2x^2+1428a^2b\,d^3x-1512a\,b^2c\,d^2x+468b^3c^2dx+1547a^3d^3-3213a^2bc\,d^2+2457a\,b^2c^2d-663b^3c^3\right)}{13923\,(bx+a)^{\frac{21}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(25/4),x)
```

```
[Out] 4/13923*(d*x+c)^(9/4)*(128*b^3*d^3*x^3+672*a*b^2*d^3*x^2-288*b^3*c*d^2*x^2+
1428*a^2*b*d^3*x-1512*a*b^2*c*d^2*x+468*b^3*c^2*d*x+1547*a^3*d^3-3213*a^2*b
*c*d^2+2457*a*b^2*c^2*d-663*b^3*c^3)/(b*x+a)^(21/4)/(a^4*d^4-4*a^3*b*c*d^3+
6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{25}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(25/4), x)
```

**mupad [B]** time = 1.36, size = 376, normalized size = 2.76

$$\frac{(c+d\,x)^{1/4}\left(\frac{x^2\left(6188\,a^3\,d^5-1428\,a^2\,b\,c\,d^4+420\,a\,b^2\,c^2\,d^3-60\,b^3\,c^3\,d^2\right)}{13923\,b^5\,(a\,d-b\,c)^4}-\frac{-6188\,a^3\,c^2\,d^3+12852\,a^2\,b\,c^3\,d^2-9828\,a\,b^2\,c^4\,d+2652\,b^3\,c^5}{13923\,b^5\,(a\,d-b\,c)^4}+\frac{x\left(12376\,a^3\,c\,d^4-19992\,a^2\,b\,c^2\,d^3+13608\,a\,b^2\,c^3\,d^2-3432\,b^3\,c^4\,d\right)}{13923\,b^5\,(a\,d-b\,c)^4}+\frac{512\,d^5\,x^5}{13923\,b^2\,(a\,d-b\,c)^4}+\frac{128\,d^4\,x^4\,(21\,a\,d-b\,c)}{13923\,b^3\,(a\,d-b\,c)^4}+\frac{16\,d^3\,x^3\left(357\,a^2\,d^2-42\,a\,b\,c\,d+5\,b^2\,c^2\right)}{13923\,b^4\,(a\,d-b\,c)^4}\right)}{x^5\,(a+b\,x)^{1/4}+\frac{a^5\,(a+b\,x)^{1/4}}{b^5}+\frac{10\,a^2\,x^3\,(a+b\,x)^{1/4}}{b^2}+\frac{10\,a^3\,x^2\,(a+b\,x)^{1/4}}{b^3}+\frac{5\,a\,x^4\,(a+b\,x)^{1/4}}{b}+\frac{5\,a^4\,x\,(a+b\,x)^{1/4}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(25/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((x^2*(6188*a^3*d^5 - 60*b^3*c^3*d^2 + 420*a*b^2*c^2*d^3 -
 1428*a^2*b*c*d^4))/(13923*b^5*(a*d - b*c)^4) - (2652*b^3*c^5 - 6188*a^3*c^
2*d^3 + 12852*a^2*b*c^3*d^2 - 9828*a*b^2*c^4*d)/(13923*b^5*(a*d - b*c)^4) +
 (x*(12376*a^3*c*d^4 - 3432*b^3*c^4*d + 13608*a*b^2*c^3*d^2 - 19992*a^2*b*c
^2*d^3))/(13923*b^5*(a*d - b*c)^4) + (512*d^5*x^5)/(13923*b^2*(a*d - b*c)^4
) + (128*d^4*x^4*(21*a*d - b*c))/(13923*b^3*(a*d - b*c)^4) + (16*d^3*x^3*(3
57*a^2*d^2 + 5*b^2*c^2 - 42*a*b*c*d))/(13923*b^4*(a*d - b*c)^4)))/(x^5*(a +
 b*x)^(1/4) + (a^5*(a + b*x)^(1/4))/b^5 + (10*a^2*x^3*(a + b*x)^(1/4))/b^2
+ (10*a^3*x^2*(a + b*x)^(1/4))/b^3 + (5*a*x^4*(a + b*x)^(1/4))/b + (5*a^4*x
*(a + b*x)^(1/4))/b^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(25/4),x)
```

```
[Out] Timed out
```

## 3.1490 $\int \frac{(a+bx)^{5/4}}{\sqrt[4]{c+dx}}\,dx$

**Optimal**. Leaf size=167

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}-\frac{5\sqrt[4]{a+bx}\,(c+dx)^{3/4}(bc-ad)}{8d^2}+\frac{(a+bx)^{5/4}(c+d}{2d}$$

**Rubi [A]** time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 240, 212, 208, 205}

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}-\frac{5\sqrt[4]{a+bx}\,(c+dx)^{3/4}(bc-ad)}{8d^2}+\frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
```

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(3/4))/(8*d^2) + ((a + b*x)^(5/4)
*(c + d*x)^(3/4))/(2*d) + (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))
/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)) + (5*(b*c - a*d)^2*ArcTan
h[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{(a+bx)^{5/4}}{\sqrt[4]{c+dx}}\,dx = \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} - \frac{(5(bc-ad))\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx}{8d}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{32d^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\right.}{8bd^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x\right.}{8bd^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,d\right.}{16\sqrt{b}\,d^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}} + \ldots$$

**Mathematica** [C] time = 0.03, size = 73, normalized size = 0.44

$$\frac{4(a+bx)^{9/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{ad-bc}\right)}{9b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 9/4, 13/4, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(1/4))
```

**IntegrateAlgebraic** [A] time = 0.40, size = 189, normalized size = 1.13

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}} + \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}} + \frac{(ad-bc)^2\left(\frac{9d(a+bx)^{5/4}}{(c+dx)^{5/4}} - \frac{5b\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{8d^2\left(\frac{d(a+bx)}{c+dx}-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
```

```
[Out] ((-(b*c) + a*d)^2*((9*d*(a + b*x)^(5/4))/(c + d*x)^(5/4) - (5*b*(a + b*x)^(
1/4))/(c + d*x)^(1/4)))/(8*d^2*(-b + (d*(a + b*x))/(c + d*x))^2) + (5*(b*c
- a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b
^(3/4)*d^(9/4)) + (5*(b*c - a*d)^2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/
4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4))
```

**fricas [B]** time = 1.15, size = 1468, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/32*(20*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*arctan(-((b^4*c^2*d^7 - 2*a*b^3*c*d^
8 + a^2*b^2*d^9)*(b*x + a)^(1/4)*(d*x + c)^(3/4)*((b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(3/4) -
 (b^2*d^8*x + b^2*c*d^7)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b^2*d^5*x + b^2*
c*d^4)*sqrt((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*
b*c*d^7 + a^8*d^8)/(b^3*d^9)))/(d*x + c))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(3/4))/(b^8*c^
9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^
5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d
^8 + (b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4
 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c
*d^8 + a^8*d^9)*x)) - 5*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*log(5*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (b*d^3*x + b*c*d^2)*((b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8)/(b^3*d^9))^(1/4))/(d*x + c)) + 5*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*log(5*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/4)*(d*x + c)^(3/4) - (b*d^3*x +
b*c*d^2)*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4))/(d*x + c)) - 4*(4*b*d*x - 5*b*c + 9*a*d)
*(b*x + a)^(1/4)*(d*x + c)^(3/4))/d^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(1/4), x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)/(d*x+c)^(1/4),x)
[Out] int((b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(1/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/4}}{(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)/(c + d*x)^(1/4),x)
[Out] int((a + b*x)^(5/4)/(c + d*x)^(1/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{4}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(1/4), x)
```

## 3.1491 $\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx$

**Optimal**. Leaf size=127

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}+\frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d}$$

**Rubi** [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 240, 212, 208, 205}

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}+\frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
```

```
[Out] ((a + b*x)^(1/4)*(c + d*x)^(3/4))/d - ((b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x
)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/4)) - ((b*c - a*d)*Arc
Tanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/
4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx = \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{4d}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{bd}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{bd}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2\sqrt{b}\,d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}$$

**Mathematica [C]** time = 0.03, size = 73, normalized size = 0.57

$$\frac{4(a+bx)^{5/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 5/4, 9/4, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/4))
```

**IntegrateAlgebraic [A]** time = 7.84, size = 176, normalized size = 1.39

$$\frac{\sqrt[4]{d}\sqrt[4]{a+bx}\left(\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2b^{3/4}d^{5/4}}+\frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2b^{3/4}d^{5/4}}+\frac{(c+dx)^{3/4}\sqrt[4]{ad+b(c+dx)-bc}}{d^{5/4}}\right)}{\sqrt[4]{ad+bdx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
```

```
[Out] (d^(1/4)*(a + b*x)^(1/4)*(((c + d*x)^(3/4)*(-(b*c) + a*d + b*(c + d*x))^(1/
4))/d^(5/4) + ((b*c - a*d)*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(-(b*c) + a*d +
 b*(c + d*x))^(1/4)])/(2*b^(3/4)*d^(5/4)) + ((-(b*c) + a*d)*ArcTanh[(b^(1/4
)*(c + d*x)^(1/4))/(-(b*c) + a*d + b*(c + d*x))^(1/4)])/(2*b^(3/4)*d^(5/4))
))/(a*d + b*d*x)^(1/4)
```

**fricas [B]** time = 1.13, size = 814, normalized size = 6.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(b^3*d^5))^(1/4)*arctan(((b^3*c*d^4 - a*b^2*d^5)*(b*x + a)^(1/4)*(d
*x + c)^(3/4)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)/(b^3*d^5))^(3/4) + (b^2*d^5*x + b^2*c*d^4)*sqrt(((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b^2*d^3*x + b^2*c*d^2)*sqr
t((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(
b^3*d^5)))/(d*x + c))*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(3/4))/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*
```

```
c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4 + (b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^
2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*x)) + d*((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4)*log(-((b*c
- a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (b*d^2*x + b*c*d)*((b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4))/
(d*x + c)) - d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)/(b^3*d^5))^(1/4)*log(-((b*c - a*d)*(b*x + a)^(1/4)*(d*x + c)^(
3/4) - (b*d^2*x + b*c*d)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4))/(d*x + c)) - 4*(b*x + a)^(1/4)*(d*
x + c)^(3/4))/d
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(1/4), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(1/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,x)^{1/4}}{(c + d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)/(c + d*x)^(1/4),x)
[Out] int((a + b*x)^(1/4)/(c + d*x)^(1/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a + bx}}{\sqrt[4]{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(1/4),x)
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(1/4), x)
```

## 3.1492 $\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx$

**Optimal**. Leaf size=85

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

**Rubi [A]** time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {63, 240, 212, 208, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(3/4)*d^
(1/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(
b^(3/4)*d^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx &= \frac{4\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{b}\\
&= \frac{4\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b}\\
&= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{b}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{b}}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}
\end{aligned}$$

**Mathematica [C]** time = 0.03, size = 71, normalized size = 0.84

$$\frac{4\sqrt[4]{a+bx}\,\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 1/4, 5/4, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/4))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 85, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(3/4)*d^
(1/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(
b^(3/4)*d^(1/4))
```

**fricas [B]** time = 0.75, size = 234, normalized size = 2.75

$$-4\left(\frac{1}{b^3 d}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}b^2 d\left(\frac{1}{b^3 d}\right)^{\frac{3}{4}}-(b^2d^2x+b^2cd)\sqrt{\frac{(b^2dx+b^2c)\sqrt{\frac{1}{b^3d}}+\sqrt{bx+a}\sqrt{dx+c}}{dx+c}}\left(\frac{1}{b^3d}\right)^{\frac{3}{4}}}{dx+c}\right)+\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}\log\left(\frac{(bdx+bc)\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}+(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)-\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}\log\left(-\frac{(bdx+bc)\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}-(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -4*(1/(b^3*d))^(1/4)*arctan(-((b*x + a)^(1/4)*(d*x + c)^(3/4)*b^2*d*(1/(b^3
*d))^(3/4) - (b^2*d^2*x + b^2*c*d)*sqrt(((b^2*d*x + b^2*c)*sqrt(1/(b^3*d))
+ sqrt(b*x + a)*sqrt(d*x + c))/(d*x + c))*(1/(b^3*d))^(3/4))/(d*x + c)) + (
1/(b^3*d))^(1/4)*log(((b*d*x + b*c)*(1/(b^3*d))^(1/4) + (b*x + a)^(1/4)*(d*
x + c)^(3/4))/(d*x + c)) - (1/(b^3*d))^(1/4)*log(-((b*d*x + b*c)*(1/(b^3*d)
)^(1/4) - (b*x + a)^(1/4)*(d*x + c)^(3/4))/(d*x + c))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(1/4)), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}\,(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(1/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/4}\,{\left(c+d\,x\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(3/4)*(c + d*x)^(1/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(1/4)), x)
```

## 3.1493 $\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx$

**Optimal**. Leaf size=32

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

**Rubi** [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/4}}$$

**Mathematica** [A] time = 0.01, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

**fricas** [A] time = 1.10, size = 42, normalized size = 1.31

$$-\frac{4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{3\left(abc-a^2d+\left(b^2c-abd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] -4/3*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(1/4)), x)
```

**maple** [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{4\,(dx+c)^{\frac{3}{4}}}{3\,(bx+a)^{\frac{3}{4}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x)
[Out] 4/3/(b*x+a)^(3/4)*(d*x+c)^(3/4)/(a*d-b*c)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(1/4)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a+b\,x)^{7/4}\,(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(7/4)*(c + d*x)^(1/4)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(1/4),x)
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(1/4)), x)
```

## 3.1494 $\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx$

**Optimal**. Leaf size=66

$$\frac{16d(c+dx)^{3/4}}{21(a+bx)^{3/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{7(a+bx)^{7/4}(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{16d(c+dx)^{3/4}}{21(a+bx)^{3/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{7(a+bx)^{7/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(7*(b*c - a*d)*(a + b*x)^(7/4)) + (16*d*(c + d*x)^(3/4
))/(21*(b*c - a*d)^2*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}} - \frac{(4d)\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx}{7(bc-ad)}$$
$$= -\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}} + \frac{16d(c+dx)^{3/4}}{21(bc-ad)^2(a+bx)^{3/4}}$$

**Mathematica** [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{4(c+dx)^{3/4}(7ad-3bc+4bdx)}{21(a+bx)^{7/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(c + d*x)^(3/4)*(-3*b*c + 7*a*d + 4*b*d*x))/(21*(b*c - a*d)^2*(a + b*x)^
(7/4))
```

**IntegrateAlgebraic** [A] time = 0.16, size = 51, normalized size = 0.77

$$\frac{4(c+dx)^{7/4}\left(\frac{7d(a+bx)}{c+dx}-3b\right)}{21(a+bx)^{7/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(c + d*x)^(7/4)*(-3*b + (7*d*(a + b*x))/(c + d*x)))/(21*(b*c - a*d)^2*(a
 + b*x)^(7/4))
```

**fricas** [B] time = 1.30, size = 118, normalized size = 1.79

$$\frac{4\,(4\,bdx-3\,bc+7\,ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{21\,\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\,\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/21*(4*b*d*x - 3*b*c + 7*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a
*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(1/4)), x)
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.82

$$\frac{4\,(dx+c)^{\frac{3}{4}}\,(4bdx+7ad-3bc)}{21\,(bx+a)^{\frac{7}{4}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x)
[Out] 4/21*(d*x+c)^(3/4)*(4*b*d*x+7*a*d-3*b*c)/(b*x+a)^(7/4)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(1/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+b\,x\right)}^{11/4}\,{\left(c+d\,x\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(11/4)*(c + d*x)^(1/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(1/4),x)
[Out] Integral(1/((a + b*x)**(11/4)*(c + d*x)**(1/4)), x)
```

## 3.1495 $\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx$

**Optimal**. Leaf size=101

$$-\frac{128d^2(c+dx)^{3/4}}{231(a+bx)^{3/4}(bc-ad)^3}+\frac{32d(c+dx)^{3/4}}{77(a+bx)^{7/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{11(a+bx)^{11/4}(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{128d^2(c+dx)^{3/4}}{231(a+bx)^{3/4}(bc-ad)^3}+\frac{32d(c+dx)^{3/4}}{77(a+bx)^{7/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{11(a+bx)^{11/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(11*(b*c - a*d)*(a + b*x)^(11/4)) + (32*d*(c + d*x)^(3
/4))/(77*(b*c - a*d)^2*(a + b*x)^(7/4)) - (128*d^2*(c + d*x)^(3/4))/(231*(b
*c - a*d)^3*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}} - \frac{(8d)\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx}{11(bc-ad)}$$

$$= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}} + \frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}} + \frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx}{77(bc-ad)^2}$$

$$= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}} + \frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}} - \frac{128d^2(c+dx)^{3/4}}{231(bc-ad)^3(a+bx)^{3/4}}$$

**Mathematica [A]** time = 0.03, size = 77, normalized size = 0.76

$$-\frac{4(c+dx)^{3/4}\left(77a^2d^2+22abd(4dx-3c)+b^2\left(21c^2-24cdx+32d^2x^2\right)\right)}{231(a+bx)^{11/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4)*(77*a^2*d^2 + 22*a*b*d*(-3*c + 4*d*x) + b^2*(21*c^2 - 2
4*c*d*x + 32*d^2*x^2)))/(231*(b*c - a*d)^3*(a + b*x)^(11/4))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 73, normalized size = 0.72

$$-\frac{4(c+dx)^{11/4}\left(\frac{77d^2(a+bx)^2}{(c+dx)^2}-\frac{66bd(a+bx)}{c+dx}+21b^2\right)}{231(a+bx)^{11/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(11/4)*(21*b^2 + (77*d^2*(a + b*x)^2)/(c + d*x)^2 - (66*b*d*(
a + b*x))/(c + d*x)))/(231*(b*c - a*d)^3*(a + b*x)^(11/4))
```

**fricas [B]** time = 2.55, size = 252, normalized size = 2.50

$$-\frac{4\left(32\,b^2d^2x^2+21\,b^2c^2-66\,abcd+77\,a^2d^2-8\left(3\,b^2cd-11\,abd^2\right)x\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{231\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3\right)x^2+3\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/231*(32*b^2*d^2*x^2 + 21*b^2*c^2 - 66*a*b*c*d + 77*a^2*d^2 - 8*(3*b^2*c*
d - 11*a*b*d^2)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^3*b^3*c^3 - 3*a^4*b^2
*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d
^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 -
a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5
*b*d^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{15}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(1/4)), x)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$\frac{4\,(dx+c)^{\frac{3}{4}}\left(32b^{2}x^{2}d^{2}+88ab\,d^{2}x-24b^{2}cdx+77a^{2}d^{2}-66abcd+21b^{2}c^{2}\right)}{231\,(bx+a)^{\frac{11}{4}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x)
```

```
[Out] 4/231*(d*x+c)^(3/4)*(32*b^2*d^2*x^2+88*a*b*d^2*x-24*b^2*c*d*x+77*a^2*d^2-66
*a*b*c*d+21*b^2*c^2)/(b*x+a)^(11/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{15}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(1/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{15/4}\,(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(15/4)*(c + d*x)^(1/4)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(15/4)/(d*x+c)**(1/4),x)
```

```
[Out] Timed out
```

## 3.1496 $\int \frac{1}{(a+bx)^{19/4}\sqrt[4]{c+dx}}\,dx$

**Optimal**. Leaf size=136

$$\frac{512d^3(c+dx)^{3/4}}{1155(a+bx)^{3/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{3/4}}{385(a+bx)^{7/4}(bc-ad)^3}+\frac{16d(c+dx)^{3/4}}{55(a+bx)^{11/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{15(a+bx)^{15/4}(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512d^3(c+dx)^{3/4}}{1155(a+bx)^{3/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{3/4}}{385(a+bx)^{7/4}(bc-ad)^3}+\frac{16d(c+dx)^{3/4}}{55(a+bx)^{11/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{15(a+bx)^{15/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(15*(b*c - a*d)*(a + b*x)^(15/4)) + (16*d*(c + d*x)^(3
/4))/(55*(b*c - a*d)^2*(a + b*x)^(11/4)) - (128*d^2*(c + d*x)^(3/4))/(385*(
b*c - a*d)^3*(a + b*x)^(7/4)) + (512*d^3*(c + d*x)^(3/4))/(1155*(b*c - a*d)
^4*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{19/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} - \frac{(4d)\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx}{5(bc-ad)}$$

$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} + \frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx}{55(bc-ad)^2}$$

$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} - \frac{128d^2(c+dx)^{3/4}}{385(bc-ad)^3(a+bx)^{7/4}} - \frac{(1}{}$$

$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} - \frac{128d^2(c+dx)^{3/4}}{385(bc-ad)^3(a+bx)^{7/4}} + \frac{}{11}$$

**Mathematica** [A] time = 0.05, size = 118, normalized size = 0.87

$$\frac{4(c+dx)^{3/4}\left(385a^3d^3+165a^2bd^2(4dx-3c)+15ab^2d\left(21c^2-24cdx+32d^2x^2\right)+b^3\left(-77c^3+84c^2dx-96cd^2x^2+128d^3x^3\right)\right)}{1155(a+bx)^{15/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(c + d*x)^(3/4)*(385*a^3*d^3 + 165*a^2*b*d^2*(-3*c + 4*d*x) + 15*a*b^2*d
*(21*c^2 - 24*c*d*x + 32*d^2*x^2) + b^3*(-77*c^3 + 84*c^2*d*x - 96*c*d^2*x^
2 + 128*d^3*x^3)))/(1155*(b*c - a*d)^4*(a + b*x)^(15/4))
```

**IntegrateAlgebraic** [A] time = 0.18, size = 95, normalized size = 0.70

$$\frac{4(c+dx)^{15/4}\left(\frac{315b^2d(a+bx)}{c+dx}+\frac{385d^3(a+bx)^3}{(c+dx)^3}-\frac{495bd^2(a+bx)^2}{(c+dx)^2}-77b^3\right)}{1155(a+bx)^{15/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(c + d*x)^(15/4)*(-77*b^3 + (385*d^3*(a + b*x)^3)/(c + d*x)^3 - (495*b*d
^2*(a + b*x)^2)/(c + d*x)^2 + (315*b^2*d*(a + b*x))/(c + d*x)))/(1155*(b*c
- a*d)^4*(a + b*x)^(15/4))
```

**fricas** [B] time = 5.10, size = 419, normalized size = 3.08

$$\frac{4\left(128\,b^3d^3x^3-77\,b^3c^3+315\,ab^2c^2d-495\,a^2bcd^2+385\,a^3d^3-96\left(b^3cd^2-5\,ab^2d^3\right)x^2+12\left(7\,b^3c^2d-30\,ab^2cd^2+55\,a^2bd^3\right)x\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{1155\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^4+4\left(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4\right)x^3+6\left(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4\right)x^2+4\left(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/1155*(128*b^3*d^3*x^3 - 77*b^3*c^3 + 315*a*b^2*c^2*d - 495*a^2*b*c*d^2 +
385*a^3*d^3 - 96*(b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + 12*(7*b^3*c^2*d - 30*a*b^2
*c*d^2 + 55*a^2*b*d^3)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*
a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a
*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*
d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3
*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{19}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(19/4)*(d*x + c)^(1/4)), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$\frac{4\,(dx+c)^{\frac{3}{4}}\left(128b^{3}d^{3}x^{3}+480a\,b^{2}d^{3}x^{2}-96b^{3}c\,d^{2}x^{2}+660a^{2}b\,d^{3}x-360a\,b^{2}c\,d^{2}x+84b^{3}c^{2}dx+385a^{3}d^{3}-495a^{2}bc\,d^{2}+315a\,b^{2}c^{2}d-77b^{3}c^{3}\right)}{1155\,(bx+a)^{\frac{15}{4}}\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x)
```

```
[Out] 4/1155*(d*x+c)^(3/4)*(128*b^3*d^3*x^3+480*a*b^2*d^3*x^2-96*b^3*c*d^2*x^2+66
0*a^2*b*d^3*x-360*a*b^2*c*d^2*x+84*b^3*c^2*d*x+385*a^3*d^3-495*a^2*b*c*d^2+
315*a*b^2*c^2*d-77*b^3*c^3)/(b*x+a)^(15/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{19}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(19/4)*(d*x + c)^(1/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{19/4}\,(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(19/4)*(c + d*x)^(1/4)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(19/4)/(d*x+c)**(1/4),x)
[Out] Timed out
```

## 3.1497 $\int \frac{(a+bx)^{7/4}}{(c+dx)^{3/4}}\,dx$

**Optimal**. Leaf size=167

$$-\frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}+\frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}-\frac{7(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8d^2}+\frac{(a+bx)^{7/4}\sqrt[4]{}}{2d}$$

**Rubi [A]** time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 331, 298, 205, 208}

$$-\frac{7(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8d^2}-\frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}+\frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}+\frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]
```

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*d^2) + ((a + b*x)^(7/4)
*(c + d*x)^(1/4))/(2*d) - (21*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4)
)/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(11/4)) + (21*(b*c - a*d)^2*Arc
Tanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(1
1/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(a+bx)^{7/4}}{(c+dx)^{3/4}}\,dx = \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx}{8d}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{32d^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3}}\right.}{8bd^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x\right.}{8bd^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,d\right.}{16d^{5/2}}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} - \frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}} + \frac{21}{}$$

**Mathematica [C]** time = 0.03, size = 73, normalized size = 0.44

$$\frac{4(a+bx)^{11/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(\frac{3}{4},\frac{11}{4};\frac{15}{4};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]
[Out] (4*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4
, 11/4, 15/4, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(3/4))
```

**IntegrateAlgebraic [A]** time = 0.27, size = 182, normalized size = 1.09

$$\frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{16\sqrt[4]{b}\,d^{11/4}} + \frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{16\sqrt[4]{b}\,d^{11/4}} + \frac{\sqrt[4]{c+dx}\,(ad-bc)^2\left(11d-\frac{7b(c+dx)}{a+bx}\right)}{8d^2\sqrt[4]{a+bx}\left(d-\frac{b(c+dx)}{a+bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]
```

```
[Out] ((-(b*c) + a*d)^2*(c + d*x)^(1/4)*(11*d - (7*b*(c + d*x))/(a + b*x)))/(8*d^
2*(a + b*x)^(1/4)*(d - (b*(c + d*x))/(a + b*x))^2) + (21*(b*c - a*d)^2*ArcT
an[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))]])/(16*b^(1/4)*d^(11/
4)) + (21*(b*c - a*d)^2*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x
)^(1/4))]])/(16*b^(1/4)*d^(11/4))
```

**fricas [B]** time = 1.03, size = 1457, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/32*(84*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*arctan(-((b^3*c^2*d^8 - 2*a*b^2*c*d^9
 + a^2*b*d^10)*(b*x + a)^(3/4)*(d*x + c)^(1/4)*((b^8*c^8 - 8*a*b^7*c^7*d +
28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c
^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(3/4) - (b
^2*d^8*x + a*b*d^8)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b*d^6*x + a*d^6)*sqrt
((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(b*d^11)))/(b*x + a))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(3/4))/(a*b^8*c^8 - 8*a^2*b
^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 5
6*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8 + (b^9*c^8
 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4
*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^
8)*x)) - 21*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*log(21*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b*d^3*x + a*d^3)*((b^8*c^8 - 8*a*b^
7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56
*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^
(1/4))/(b*x + a)) + 21*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 -
 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*log(21*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b*d^3*x + a*d^3)*((b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(b*d^11))^(1/4))/(b*x + a)) - 4*(4*b*d*x - 7*b*c + 11*a*d)*(b*x + a)^(3/4)
*(d*x + c)^(1/4))/d^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(3/4), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/4)/(d*x+c)^(3/4),x)
[Out] int((b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(3/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/4)/(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(7/4)/(c + d*x)^(3/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(7/4)/(c + d*x)**(3/4), x)
```

## 3.1498 $\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx$

**Optimal**. Leaf size=127

$$\frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}-\frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}+\frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d}$$

**Rubi [A]** time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 331, 298, 205, 208}

$$\frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}-\frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}+\frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] ((a + b*x)^(3/4)*(c + d*x)^(1/4))/d + (3*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b
*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d^(7/4)) - (3*(b*c - a*d)
*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d
^(7/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx = \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{4d}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{bd}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{bd}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2d^{3/2}} + \frac{(3(bc-ad))\,\text{Subst}\left(\int \cdots\right.}{2d^{3}\cdots}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} + \frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}} - \frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}$$

**Mathematica [C]** time = 0.03, size = 73, normalized size = 0.57

$$\frac{4(a+bx)^{7/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 7/4, 11/4, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(3/4))
```

**IntegrateAlgebraic [A]** time = 8.42, size = 176, normalized size = 1.39

$$\frac{d^{3/4}(a+bx)^{3/4}\left(\frac{\sqrt[4]{c+dx}\,(ad+b(c+dx)-bc)^{3/4}}{d^{7/4}}-\frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2\sqrt[4]{b}\,d^{7/4}}-\frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2\sqrt[4]{b}\,d^{7/4}}\right)}{(ad+bdx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] (d^(3/4)*(a + b*x)^(3/4)*(((c + d*x)^(1/4)*(-(b*c) + a*d + b*(c + d*x))^(3/
4))/d^(7/4) - (3*(b*c - a*d)*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(-(b*c) + a*d
 + b*(c + d*x))^(1/4)])/(2*b^(1/4)*d^(7/4)) - (3*(b*c - a*d)*ArcTanh[(b^(1/
4)*(c + d*x)^(1/4))/(-(b*c) + a*d + b*(c + d*x))^(1/4)])/(2*b^(1/4)*d^(7/4)
)))/(a*d + b*d*x)^(3/4)
```

**fricas [B]** time = 1.42, size = 808, normalized size = 6.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/4*(12*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(b*d^7))^(1/4)*arctan(((b^2*c*d^5 - a*b*d^6)*(b*x + a)^(3/4)*(d*x
+ c)^(1/4)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(b*d^7))^(3/4) + (b^2*d^5*x + a*b*d^5)*sqrt(((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b*d^4*x + a*d^4)*sqrt((b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7)))/(b*x
 + a))*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*
d^4)/(b*d^7))^(3/4))/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a
```

```
^4*b*c*d^3 + a^5*d^4 + (b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3
*b^2*c*d^3 + a^4*b*d^4)*x)) + 3*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7))^(1/4)*log(-3*((b*c - a*d)*(b*x + a
)^(3/4)*(d*x + c)^(1/4) + (b*d^2*x + a*d^2)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7))^(1/4))/(b*x + a)) - 3*d*
((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b
*d^7))^(1/4)*log(-3*((b*c - a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b*d^2*x
 + a*d^2)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(b*d^7))^(1/4))/(b*x + a)) - 4*(b*x + a)^(3/4)*(d*x + c)^(1/4))/d
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(3/4), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(3/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)/(c + d*x)^(3/4),x)
```

```
[Out] int((a + b*x)^(3/4)/(c + d*x)^(3/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)/(c + d*x)**(3/4), x)
```

## 3.1499 $\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx$

**Optimal**. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

**Rubi [A]** time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {63, 331, 298, 205, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(1/4)*d
^(3/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/
(b^(1/4)*d^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx = \frac{4\,\mathrm{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{b}$$

$$= \frac{4\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{d}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{d}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

**Mathematica [C]** time = 0.03, size = 73, normalized size = 0.86

$$\frac{4(a+bx)^{3/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{d(a+bx)}{ad-bc}\right)}{3b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 3/4, 7/4, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(3/4))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 85, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{\sqrt[4]{b}\,d^{3/4}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (2*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))])/(b^(1/4)*d^
(3/4)) + (2*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))])/(
b^(1/4)*d^(3/4))
```

**fricas [B]** time = 1.24, size = 234, normalized size = 2.75

$$-4\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}bd^2\left(\frac{1}{bd^3}\right)^{\frac{3}{4}}-\left(b^2d^2x+abd^2\right)\sqrt{\frac{\left(bd^2x+ad^2\right)\sqrt{\frac{1}{bd^3}}+\sqrt{bx+a}\sqrt{dx+c}}{bx+a}}\left(\frac{1}{bd^3}\right)^{\frac{3}{4}}}{bx+a}\right)+\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}\log\left(\frac{(bdx+ad)\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}+(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{bx+a}\right)-\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}\log\left(-\frac{(bdx+ad)\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}-(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(1/(b*d^3))^(1/4)*arctan(-((b*x + a)^(3/4)*(d*x + c)^(1/4)*b*d^2*(1/(b*d
^3))^(3/4) - (b^2*d^2*x + a*b*d^2)*sqrt(((b*d^2*x + a*d^2)*sqrt(1/(b*d^3))
+ sqrt(b*x + a)*sqrt(d*x + c))/(b*x + a))*(1/(b*d^3))^(3/4))/(b*x + a)) + (
1/(b*d^3))^(1/4)*log(((b*d*x + a*d)*(1/(b*d^3))^(1/4) + (b*x + a)^(3/4)*(d*
x + c)^(1/4))/(b*x + a)) - (1/(b*d^3))^(1/4)*log(-((b*d*x + a*d)*(1/(b*d^3)
)^(1/4) - (b*x + a)^(3/4)*(d*x + c)^(1/4))/(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(3/4)), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}\,(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(3/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{1/4}\,(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x)
```

```
[Out] int(1/((a + b*x)^(1/4)*(c + d*x)^(3/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(1/4)*(c + d*x)**(3/4)), x)
```

## 3.1500 $\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx$

**Optimal**. Leaf size=30

$$-\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}\,(bc-ad)}$$

**Rubi** [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/((b*c - a*d)*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{(bc-ad)\sqrt[4]{a+bx}}$$

**Mathematica** [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(c + d*x)^(1/4))/((-(b*c) + a*d)*(a + b*x)^(1/4))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 30, normalized size = 1.00

$$-\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]
[Out] (-4*(c + d*x)^(1/4))/((b*c - a*d)*(a + b*x)^(1/4))
```

**fricas [A]** time = 1.17, size = 42, normalized size = 1.40

$$-\frac{4\,(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] -4*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(3/4)), x)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.90

$$\frac{4\,(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x)
[Out] 4/(b*x+a)^(1/4)*(d*x+c)^(1/4)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(3/4)), x)
```

**mupad [B]** time = 0.71, size = 26, normalized size = 0.87

$$\frac{4\,{\left(c+d\,x\right)}^{1/4}}{\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x)
[Out] (4*(c + d*x)^(1/4))/((a*d - b*c)*(a + b*x)^(1/4))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{4}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/4)/(d*x+c)**(3/4),x)
[Out] Integral(1/((a + b*x)**(5/4)*(c + d*x)**(3/4)), x)
```

## 3.1501 $\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx$

**Optimal**. Leaf size=66

$$\frac{16d\sqrt[4]{c+dx}}{5\sqrt[4]{a+bx}\,(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{5(a+bx)^{5/4}(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{16d\sqrt[4]{c+dx}}{5\sqrt[4]{a+bx}\,(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{5(a+bx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(5*(b*c - a*d)*(a + b*x)^(5/4)) + (16*d*(c + d*x)^(1/4
))/(5*(b*c - a*d)^2*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}} - \frac{(4d)\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx}{5(bc-ad)}$$
$$= -\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}} + \frac{16d\sqrt[4]{c+dx}}{5(bc-ad)^2\sqrt[4]{a+bx}}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 0.70

$$\frac{4\sqrt[4]{c+dx}\,(5ad-bc+4bdx)}{5(a+bx)^{5/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(c + d*x)^(1/4)*(-(b*c) + 5*a*d + 4*b*d*x))/(5*(b*c - a*d)^2*(a + b*x)^(
5/4))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 56, normalized size = 0.85

$$-\frac{4\left(\frac{b(c+dx)^{5/4}}{(a+bx)^{5/4}} - \frac{5d\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\right)}{5(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((-5*d*(c + d*x)^(1/4))/(a + b*x)^(1/4) + (b*(c + d*x)^(5/4))/(a + b*x)
^(5/4)))/(5*(b*c - a*d)^2)
```

**fricas [B]** time = 1.21, size = 118, normalized size = 1.79

$$\frac{4\,(4\,bdx-bc+5\,ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{5\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/5*(4*b*d*x - b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^2*b^2*c^2 -
2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^
3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{9}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(3/4)), x)
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.82

$$\frac{4\,(dx+c)^{\frac{1}{4}}\,(4bdx+5ad-bc)}{5\,(bx+a)^{\frac{5}{4}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x)
```

```
[Out] 4/5*(d*x+c)^(1/4)*(4*b*d*x+5*a*d-b*c)/(b*x+a)^(5/4)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{9}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(3/4)), x)
```

**mupad [B]** time = 0.87, size = 71, normalized size = 1.08

$$\frac{\left(\frac{16\,d\,x}{5\,(a\,d-b\,c)^2}+\frac{20\,a\,d-4\,b\,c}{5\,b\,(a\,d-b\,c)^2}\right)\,(c+d\,x)^{1/4}}{x\,(a+b\,x)^{1/4}+\frac{a\,(a+b\,x)^{1/4}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x)
```

```
[Out] (((16*d*x)/(5*(a*d - b*c)^2) + (20*a*d - 4*b*c)/(5*b*(a*d - b*c)^2))*(c + d
*x)^(1/4))/(x*(a + b*x)^(1/4) + (a*(a + b*x)^(1/4))/b)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{4}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(9/4)*(c + d*x)**(3/4)), x)
```

## 3.1502 $\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx$

**Optimal**. Leaf size=101

$$-\frac{128d^2\sqrt[4]{c+dx}}{45\sqrt[4]{a+bx}\,(bc-ad)^3}+\frac{32d\sqrt[4]{c+dx}}{45(a+bx)^{5/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{9(a+bx)^{9/4}(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{128d^2\sqrt[4]{c+dx}}{45\sqrt[4]{a+bx}\,(bc-ad)^3}+\frac{32d\sqrt[4]{c+dx}}{45(a+bx)^{5/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(9*(b*c - a*d)*(a + b*x)^(9/4)) + (32*d*(c + d*x)^(1/4
))/(45*(b*c - a*d)^2*(a + b*x)^(5/4)) - (128*d^2*(c + d*x)^(1/4))/(45*(b*c
- a*d)^3*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}} - \frac{(8d)\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx}{9(bc-ad)}$$
$$= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}} + \frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}} + \frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx}{45(bc-ad)^2}$$
$$= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}} + \frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}} - \frac{128d^2\sqrt[4]{c+dx}}{45(bc-ad)^3\sqrt[4]{a+bx}}$$

**Mathematica** [A]     time = 0.03, size = 75, normalized size = 0.74

$$-\frac{4\sqrt[4]{c+dx}\left(45a^2d^2-18abd(c-4dx)+b^2\left(5c^2-8cdx+32d^2x^2\right)\right)}{45(a+bx)^{9/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4)*(45*a^2*d^2 - 18*a*b*d*(c - 4*d*x) + b^2*(5*c^2 - 8*c*d
*x + 32*d^2*x^2)))/(45*(b*c - a*d)^3*(a + b*x)^(9/4))
```

**IntegrateAlgebraic** [A]     time = 0.12, size = 83, normalized size = 0.82

$$-\frac{4\left(\frac{5b^2(c+dx)^{9/4}}{(a+bx)^{9/4}} + \frac{45d^2\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}} - \frac{18bd(c+dx)^{5/4}}{(a+bx)^{5/4}}\right)}{45(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((45*d^2*(c + d*x)^(1/4))/(a + b*x)^(1/4) - (18*b*d*(c + d*x)^(5/4))/(a
 + b*x)^(5/4) + (5*b^2*(c + d*x)^(9/4))/(a + b*x)^(9/4)))/(45*(b*c - a*d)^3
)
```

**fricas** [B]     time = 0.88, size = 251, normalized size = 2.49

$$-\frac{4\left(32\,b^2d^2x^2+5\,b^2c^2-18\,abcd+45\,a^2d^2-8\left(b^2cd-9\,abd^2\right)x\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{45\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3\right)x^2+3\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -4/45*(32*b^2*d^2*x^2 + 5*b^2*c^2 - 18*a*b*c*d + 45*a^2*d^2 - 8*(b^2*c*d -
9*a*b*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*
d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 -
a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b
^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^
3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{13}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(13/4)*(d*x + c)^(3/4)), x)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$\frac{4\,(dx+c)^{\frac{1}{4}}\left(32b^{2}x^{2}d^{2}+72ab\,d^{2}x-8b^{2}cdx+45a^{2}d^{2}-18abcd+5b^{2}c^{2}\right)}{45\,(bx+a)^{\frac{9}{4}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x)
```

```
[Out] 4/45*(d*x+c)^(1/4)*(32*b^2*d^2*x^2+72*a*b*d^2*x-8*b^2*c*d*x+45*a^2*d^2-18*a
*b*c*d+5*b^2*c^2)/(b*x+a)^(9/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{13}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(13/4)*(d*x + c)^(3/4)), x)
```

**mupad [B]** time = 1.02, size = 133, normalized size = 1.32

$$\frac{(c+d\,x)^{1/4}\left(\frac{128\,d^2\,x^2}{45\,(a\,d-b\,c)^3}+\frac{180\,a^2\,d^2-72\,a\,b\,c\,d+20\,b^2\,c^2}{45\,b^2\,(a\,d-b\,c)^3}+\frac{32\,d\,x\,(9\,a\,d-b\,c)}{45\,b\,(a\,d-b\,c)^3}\right)}{x^2\,(a+b\,x)^{1/4}+\frac{a^2\,(a+b\,x)^{1/4}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/4}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x)
```

```
[Out] ((c + d*x)^(1/4)*((128*d^2*x^2)/(45*(a*d - b*c)^3) + (180*a^2*d^2 + 20*b^2*
c^2 - 72*a*b*c*d)/(45*b^2*(a*d - b*c)^3) + (32*d*x*(9*a*d - b*c))/(45*b*(a*
d - b*c)^3)))/(x^2*(a + b*x)^(1/4) + (a^2*(a + b*x)^(1/4))/b^2 + (2*a*x*(a
+ b*x)^(1/4))/b)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(13/4)/(d*x+c)**(3/4),x)
```

```
[Out] Timed out
```

## 3.1503 $\int \frac{1}{(a+bx)^{17/4}(c+dx)^{3/4}}\,dx$

**Optimal**. Leaf size=136

$$\frac{512d^3\sqrt[4]{c+dx}}{195\sqrt[4]{a+bx}\,(bc-ad)^4}-\frac{128d^2\sqrt[4]{c+dx}}{195(a+bx)^{5/4}(bc-ad)^3}+\frac{16d\sqrt[4]{c+dx}}{39(a+bx)^{9/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{13(a+bx)^{13/4}(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512d^3\sqrt[4]{c+dx}}{195\sqrt[4]{a+bx}\,(bc-ad)^4}-\frac{128d^2\sqrt[4]{c+dx}}{195(a+bx)^{5/4}(bc-ad)^3}+\frac{16d\sqrt[4]{c+dx}}{39(a+bx)^{9/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{13(a+bx)^{13/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(1
/4))/(39*(b*c - a*d)^2*(a + b*x)^(9/4)) - (128*d^2*(c + d*x)^(1/4))/(195*(b
*c - a*d)^3*(a + b*x)^(5/4)) + (512*d^3*(c + d*x)^(1/4))/(195*(b*c - a*d)^4
*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{17/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} - \frac{(12d)\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx}{13(bc-ad)}$$

$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} + \frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx}{39(bc-ad)^2}$$

$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} - \frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+bx)^{5/4}} - \cdots$$

$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} - \frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+bx)^{5/4}} + \cdots$$

**Mathematica [A]** time = 0.05, size = 116, normalized size = 0.85

$$\frac{4\sqrt[4]{c+dx}\left(195a^3d^3-117a^2bd^2(c-4dx)+13ab^2d\left(5c^2-8cdx+32d^2x^2\right)+b^3\left(-15c^3+20c^2dx-32cd^2x^2+128d^3x^3\right)\right)}{195(a+bx)^{13/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(c + d*x)^(1/4)*(195*a^3*d^3 - 117*a^2*b*d^2*(c - 4*d*x) + 13*a*b^2*d*(5
*c^2 - 8*c*d*x + 32*d^2*x^2) + b^3*(-15*c^3 + 20*c^2*d*x - 32*c*d^2*x^2 + 1
28*d^3*x^3)))/(195*(b*c - a*d)^4*(a + b*x)^(13/4))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 109, normalized size = 0.80

$$-\frac{4\left(\frac{15b^3(c+dx)^{13/4}}{(a+bx)^{13/4}}-\frac{65b^2d(c+dx)^{9/4}}{(a+bx)^{9/4}}-\frac{195d^3\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}+\frac{117bd^2(c+dx)^{5/4}}{(a+bx)^{5/4}}\right)}{195(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((-195*d^3*(c + d*x)^(1/4))/(a + b*x)^(1/4) + (117*b*d^2*(c + d*x)^(5/4
))/(a + b*x)^(5/4) - (65*b^2*d*(c + d*x)^(9/4))/(a + b*x)^(9/4) + (15*b^3*(
c + d*x)^(13/4))/(a + b*x)^(13/4)))/(195*(b*c - a*d)^4)
```

**fricas [B]** time = 1.32, size = 419, normalized size = 3.08

$$\frac{4\left(128\,b^3d^3x^3-15\,b^3c^3+65\,ab^2c^2d-117\,a^2bcd^2+195\,a^3d^3-32\left(b^3cd^2-13\,ab^2d^3\right)x^2+4\left(5\,b^3c^2d-26\,ab^2cd^2+117\,a^2bd^3\right)x\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{195\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^4+4\left(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4\right)x^3+6\left(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4\right)x^2+4\left(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/195*(128*b^3*d^3*x^3 - 15*b^3*c^3 + 65*a*b^2*c^2*d - 117*a^2*b*c*d^2 + 19
5*a^3*d^3 - 32*(b^3*c*d^2 - 13*a*b^2*d^3)*x^2 + 4*(5*b^3*c^2*d - 26*a*b^2*c
*d^2 + 117*a^2*b*d^3)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^4*b^4*c^4 - 4*a
^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a
*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*
b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d
^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*
c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2
*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{17}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(17/4)*(d*x + c)^(3/4)), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$\frac{4\,(dx+c)^{\frac{1}{4}}\left(128b^3d^3x^3+416a\,b^2d^3x^2-32b^3c\,d^2x^2+468a^2b\,d^3x-104a\,b^2c\,d^2x+20b^3c^2dx+195a^3d^3-117a^2bc\,d^2+65a\,b^2c^2d-15b^3c^3\right)}{195\,(bx+a)^{\frac{13}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x)
```

```
[Out] 4/195*(d*x+c)^(1/4)*(128*b^3*d^3*x^3+416*a*b^2*d^3*x^2-32*b^3*c*d^2*x^2+468
*a^2*b*d^3*x-104*a*b^2*c*d^2*x+20*b^3*c^2*d*x+195*a^3*d^3-117*a^2*b*c*d^2+6
5*a*b^2*c^2*d-15*b^3*c^3)/(b*x+a)^(13/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c
^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{17}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(17/4)*(d*x + c)^(3/4)), x)
```

**mupad [B]** time = 1.26, size = 209, normalized size = 1.54

$$\frac{(c+d\,x)^{1/4}\,\left(\frac{512\,d^3\,x^3}{195\,(a\,d-b\,c)^4}+\frac{780\,a^3\,d^3-468\,a^2\,b\,c\,d^2+260\,a\,b^2\,c^2\,d-60\,b^3\,c^3}{195\,b^3\,(a\,d-b\,c)^4}+\frac{16\,d\,x\,\left(117\,a^2\,d^2-26\,a\,b\,c\,d+5\,b^2\,c^2\right)}{195\,b^2\,(a\,d-b\,c)^4}+\frac{128\,d^2\,x^2\,(13\,a\,d-b\,c)}{195\,b\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/4}+\frac{a^3\,(a+b\,x)^{1/4}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/4}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/4}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x)
```

```
[Out] ((c + d*x)^(1/4)*((512*d^3*x^3)/(195*(a*d - b*c)^4) + (780*a^3*d^3 - 60*b^3
*c^3 + 260*a*b^2*c^2*d - 468*a^2*b*c*d^2)/(195*b^3*(a*d - b*c)^4) + (16*d*x
*(117*a^2*d^2 + 5*b^2*c^2 - 26*a*b*c*d))/(195*b^2*(a*d - b*c)^4) + (128*d^2
*x^2*(13*a*d - b*c))/(195*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/4) + (a^3*(a
 + b*x)^(1/4))/b^3 + (3*a*x^2*(a + b*x)^(1/4))/b + (3*a^2*x*(a + b*x)^(1/4)
)/b^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(17/4)/(d*x+c)**(3/4),x)
```

```
[Out] Timed out
```

# 3.1504 $\int \frac{(a+bx)^{5/4}}{(c+dx)^{5/4}}\,dx$

**Optimal**. Leaf size=152

$$-\frac{5\sqrt[4]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}+\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}$$

**Rubi [A]** time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {47, 50, 63, 240, 212, 208, 205}

$$\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{5\sqrt[4]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(5/4))/(d*(c + d*x)^(1/4)) + (5*b*(a + b*x)^(1/4)*(c + d*x)^(
3/4))/d^2 - (5*b^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/(2*d^(9/4)) - (5*b^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*
(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*d^(9/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{(a+bx)^{5/4}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{(5b)\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{(5b(bc-ad))\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{4d^2}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{d^2}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d^2}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{\left(5\sqrt{b}\,(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2d^2} -$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{5\sqrt[4]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}} - \frac{5\sqrt[4]{b}\,(bc-ad)\,\text{ta}}{2d}$$

**Mathematica [C]** time = 0.05, size = 73, normalized size = 0.48

$$\frac{4(a+bx)^{9/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 9/4, 13/4, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(5/4))
```

**IntegrateAlgebraic [A]** time = 14.16, size = 200, normalized size = 1.32

$$\frac{d^{5/4}(a+bx)^{5/4}\left(\frac{5\left(b^{5/4}c-a\sqrt[4]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2d^{9/4}} - \frac{5\left(b^{5/4}c-a\sqrt[4]{b}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2d^{9/4}} + \frac{\sqrt[4]{ad+b(c+dx)-bc}\,(-4ad+b(c+dx)+4bc)}{d^{9/4}\sqrt[4]{c+dx}}\right)}{(ad+bdx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (d^(5/4)*(a + b*x)^(5/4)*(((4*b*c - 4*a*d + b*(c + d*x))*(-(b*c) + a*d + b*
(c + d*x))^(1/4))/(d^(9/4)*(c + d*x)^(1/4)) + (5*(b^(5/4)*c - a*b^(1/4)*d)*
ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(-(b*c) + a*d + b*(c + d*x))^(1/4)])/(2*d^
(9/4)) - (5*(b^(5/4)*c - a*b^(1/4)*d)*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(-(
b*c) + a*d + b*(c + d*x))^(1/4)])/(2*d^(9/4))))/(a*d + b*d*x)^(5/4)
```

**fricas [B]** time = 1.22, size = 857, normalized size = 5.64

$$-\frac{20\left(d^{3}x+cd^{2}\right)\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(bcd^{7}-ad^{8}\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{3}{4}}+\left(d^{8}x+cd^{7}\right)\sqrt{\frac{\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\sqrt{bx+a}\sqrt{dx+c}+\left(d^{5}x+cd^{4}\right)\sqrt{\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}}}{dx+c}}\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{3}{4}}}{b^{5}c^{5}-4ab^{4}c^{4}d+6a^{2}b^{3}c^{3}d^{2}-4a^{3}b^{2}c^{2}d^{3}+a^{4}bcd^{4}+\left(b^{5}c^{4}d-4ab^{4}c^{3}d^{2}+6a^{2}b^{3}c^{2}d^{3}-4a^{3}b^{2}cd^{4}+a^{4}bd^{5}\right)x}\right)+5\left(d^{3}x+cd^{2}\right)\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}+\left(d^{3}x+cd^{2}\right)\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\right)}{dx+c}\right)-5\left(d^{3}x+cd^{2}\right)\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}-\left(d^{3}x+cd^{2}\right)\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\right)}{dx+c}\right)-4\left(bdx+5bc-4ad\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{4\left(d^{3}x+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/4*(20*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*arctan(((b*c*d^7 - a*d^8)*(b*x + a)^(
1/4)*(d*x + c)^(3/4)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*
b^2*c*d^3 + a^4*b*d^4)/d^9)^(3/4) + (d^8*x + c*d^7)*sqrt(((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (d^5*x + c*d^4)*sqrt((b^5*c^4
- 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9))/(d
*x + c))*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)/d^9)^(3/4))/(b^5*c^5 - 4*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3
*b^2*c^2*d^3 + a^4*b*c*d^4 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d
^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)) + 5*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b
^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*log(
-5*((b*c - a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (d^3*x + c*d^2)*((b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1
/4))/(d*x + c)) - 5*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c
^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*log(-5*((b*c - a*d)*(b*x +
 a)^(1/4)*(d*x + c)^(3/4) - (d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a
^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4))/(d*x + c)) - 4*(b
*d*x + 5*b*c - 4*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4))/(d^3*x + c*d^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(5/4), x)
```

**maple [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)/(d*x+c)^(5/4),x)
[Out] int((b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(5/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)/(c + d*x)^(5/4),x)
[Out] int((a + b*x)^(5/4)/(c + d*x)^(5/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{4}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(5/4),x)
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(5/4), x)
```

## 3.1505 $\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{5/4}}\,dx$

**Optimal**. Leaf size=108

$$\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}-\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}$$

**Rubi [A]** time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 63, 240, 212, 208, 205}

$$\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}-\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(1/4))/(d*(c + d*x)^(1/4)) + (2*b^(1/4)*ArcTan[(d^(1/4)*(a +
b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4) + (2*b^(1/4)*ArcTanh[(d^(1/
4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{5/4}}\,dx = -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}} + \frac{b\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}} + \frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{d}$$

$$= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}} + \frac{4\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d}$$

$$= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}} + \frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d} + \frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+}}{\sqrt[4]{c+}}\right.}{d}$$

$$= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}} + \frac{2\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}} + \frac{2\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}$$

**Mathematica [C]** time = 0.04, size = 73, normalized size = 0.68

$$\frac{4(a+bx)^{5/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 5/4, 9/4, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/4))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 108, normalized size = 1.00

$$\frac{2\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}} + \frac{2\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}} - \frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
```

[Out] (-4*(a + b*x)^(1/4))/(d*(c + d*x)^(1/4)) + (2*b^(1/4)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4) + (2*b^(1/4)*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4)

**fricas [B]** time = 1.72, size = 273, normalized size = 2.53

$$-\frac{4\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}d^{4}\left(\frac{b}{d^{5}}\right)^{\frac{3}{4}}-\left(d^{5}x+cd^{4}\right)\sqrt{\frac{\left(d^{3}x+cd^{2}\right)\sqrt{\frac{b}{d^{5}}}+\sqrt{bx+a}\,\sqrt{dx+c}}{dx+c}}\left(\frac{b}{d^{5}}\right)^{\frac{3}{4}}}{bdx+bc}\right)-\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\log\left(\frac{\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}+(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)+\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\log\left(-\frac{\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}-(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)+4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{d^{2}x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="fricas")

[Out] -(4*(d^2*x + c*d)*(b/d^5)^(1/4)*arctan(-((b*x + a)^(1/4)*(d*x + c)^(3/4)*d^4*(b/d^5)^(3/4) - (d^5*x + c*d^4)*sqrt(((d^3*x + c*d^2)*sqrt(b/d^5) + sqrt(b*x + a)*sqrt(d*x + c))/(d*x + c))*(b/d^5)^(3/4))/(b*d*x + b*c)) - (d^2*x + c*d)*(b/d^5)^(1/4)*log(((d^2*x + c*d)*(b/d^5)^(1/4) + (b*x + a)^(1/4)*(d*x + c)^(3/4))/(d*x + c)) + (d^2*x + c*d)*(b/d^5)^(1/4)*log(-((d^2*x + c*d)*(b/d^5)^(1/4) - (b*x + a)^(1/4)*(d*x + c)^(3/4))/(d*x + c)) + 4*(b*x + a)^(1/4)*(d*x + c)^(3/4))/(d^2*x + c*d)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="giac")

[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(5/4), x)

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(1/4)/(d*x+c)^(5/4),x)

[Out] int((b*x+a)^(1/4)/(d*x+c)^(5/4),x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(5/4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(1/4)/(c + d*x)^(5/4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(5/4), x)
```

## 3.1506 $\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx$

**Optimal**. Leaf size=30

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx = \frac{4\sqrt[4]{a+bx}}{(bc-ad)\sqrt[4]{c+dx}}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.00

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 30, normalized size = 1.00

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

**fricas [A]** time = 0.77, size = 42, normalized size = 1.40

$$\frac{4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="fricas")
[Out] 4*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(5/4)), x)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.90

$$-\frac{4\,(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x)
[Out] -4*(b*x+a)^(1/4)/(d*x+c)^(1/4)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(5/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/4}\,{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(3/4)*(c + d*x)^(5/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{4}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(5/4),x)
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(5/4)), x)
```

## 3.1507 $\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}} dx$

**Optimal**. Leaf size=66

$$-\frac{16d\sqrt[4]{a+bx}}{3\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{3(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d\sqrt[4]{a+bx}}{3\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{3(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(3*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4)) - (16*d*(a + b*x)^(1/4))
/(3*(b*c - a*d)^2*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx = -\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}} - \frac{(4d)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx}{3(bc-ad)}$$
$$= -\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}} - \frac{16d\sqrt[4]{a+bx}}{3(bc-ad)^2\sqrt[4]{c+dx}}$$

**Mathematica** [A] time = 0.02, size = 45, normalized size = 0.68

$$-\frac{4(3ad+b(c+4dx))}{3(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(3*a*d + b*(c + 4*d*x)))/(3*(b*c - a*d)^2*(a + b*x)^(3/4)*(c + d*x)^(1/
4))
```

**IntegrateAlgebraic** [A] time = 0.11, size = 49, normalized size = 0.74

$$-\frac{4(c+dx)^{3/4}\left(\frac{3d(a+bx)}{c+dx}+b\right)}{3(a+bx)^{3/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4)*(b + (3*d*(a + b*x))/(c + d*x)))/(3*(b*c - a*d)^2*(a +
b*x)^(3/4))
```

**fricas** [B] time = 1.17, size = 126, normalized size = 1.91

$$-\frac{4\,(4\,bdx+bc+3\,ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{3\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -4/3*(4*b*d*x + b*c + 3*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(5/4)), x)
```

**maple [A]** time = 0.01, size = 53, normalized size = 0.80

$$-\frac{4\,(4bdx+3ad+bc)}{3\,(bx+a)^{\frac{3}{4}}\,(dx+c)^{\frac{1}{4}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x)
```

```
[Out] -4/3*(4*b*d*x+3*a*d+b*c)/(b*x+a)^(3/4)/(d*x+c)^(1/4)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(5/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+b\,x\right)}^{7/4}\,{\left(c+d\,x\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(7/4)*(c + d*x)^(5/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{4}}(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(5/4)), x)
```

## 3.1508 $\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}} dx$

**Optimal**. Leaf size=101

$$\frac{128d^2\sqrt[4]{a+bx}}{21\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{32d}{21(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{7(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{128d^2\sqrt[4]{a+bx}}{21\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{32d}{21(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{7(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(7*(b*c - a*d)*(a + b*x)^(7/4)*(c + d*x)^(1/4)) + (32*d)/(21*(b*c - a*d)
^2*(a + b*x)^(3/4)*(c + d*x)^(1/4)) + (128*d^2*(a + b*x)^(1/4))/(21*(b*c -
a*d)^3*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx = -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}} - \frac{(8d)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx}{7(bc-ad)}$$

$$= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}} + \frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}} + \frac{\left(32d^2\right)\int \frac{}{(a}}{21(b}$$

$$= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}} + \frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}} + \frac{128d^2\sqrt[4]{}}{21(bc-ad)}$$

**Mathematica** [A] time = 0.03, size = 76, normalized size = 0.75

$$\frac{84a^2d^2+56abd(c+4dx)+4b^2\left(-3c^2+8cdx+32d^2x^2\right)}{21(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (84*a^2*d^2 + 56*a*b*d*(c + 4*d*x) + 4*b^2*(-3*c^2 + 8*c*d*x + 32*d^2*x^2))
/(21*(b*c - a*d)^3*(a + b*x)^(7/4)*(c + d*x)^(1/4))
```

**IntegrateAlgebraic** [A] time = 0.13, size = 73, normalized size = 0.72

$$\frac{4(c+dx)^{7/4}\left(\frac{21d^2(a+bx)^2}{(c+dx)^2}+\frac{14bd(a+bx)}{c+dx}-3b^2\right)}{21(a+bx)^{7/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(c + d*x)^(7/4)*(-3*b^2 + (21*d^2*(a + b*x)^2)/(c + d*x)^2 + (14*b*d*(a
+ b*x))/(c + d*x)))/(21*(b*c - a*d)^3*(a + b*x)^(7/4))
```

**fricas** [B] time = 1.39, size = 273, normalized size = 2.70

$$\frac{4\left(32\,b^2d^2x^2-3\,b^2c^2+14\,abcd+21\,a^2d^2+8\left(b^2cd+7\,abd^2\right)x\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{21\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2+5\,a^3b^2cd^3-2\,a^4bd^4\right)x^2+\left(2\,ab^4c^4-5\,a^2b^3c^3d+3\,a^3b^2c^2d^2+a^4bcd^3-a^5d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] 4/21*(32*b^2*d^2*x^2 - 3*b^2*c^2 + 14*a*b*c*d + 21*a^2*d^2 + 8*(b^2*c*d + 7
*a*b*d^2)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d
 + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c
*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^
3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2
*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(5/4)), x)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$-\frac{4\left(32b^2x^2d^2+56ab\,d^2x+8b^2cdx+21a^2d^2+14abcd-3b^2c^2\right)}{21\,(bx+a)^{\frac{7}{4}}\,(dx+c)^{\frac{1}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x)
```

```
[Out] -4/21*(32*b^2*d^2*x^2+56*a*b*d^2*x+8*b^2*c*d*x+21*a^2*d^2+14*a*b*c*d-3*b^2*
c^2)/(b*x+a)^(7/4)/(d*x+c)^(1/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(5/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{11/4}\,(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(11/4)*(c + d*x)^(5/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{4}}(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(11/4)*(c + d*x)**(5/4)), x)
```

## 3.1509 $\int \frac{1}{(a+bx)^{15/4}(c+dx)^{5/4}}\,dx$

**Optimal**. Leaf size=136

$$-\frac{512d^3\sqrt[4]{a+bx}}{77\sqrt[4]{c+dx}\,(bc-ad)^4}-\frac{128d^2}{77(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{48d}{77(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{11(a+bx)^{11/4}\sqrt[4]{c+d}}$$

**Rubi [A]** time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{512d^3\sqrt[4]{a+bx}}{77\sqrt[4]{c+dx}\,(bc-ad)^4}-\frac{128d^2}{77(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{48d}{77(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{11(a+bx)^{11/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(11*(b*c - a*d)*(a + b*x)^(11/4)*(c + d*x)^(1/4)) + (48*d)/(77*(b*c - a*
d)^2*(a + b*x)^(7/4)*(c + d*x)^(1/4)) - (128*d^2)/(77*(b*c - a*d)^3*(a + b*
x)^(3/4)*(c + d*x)^(1/4)) - (512*d^3*(a + b*x)^(1/4))/(77*(b*c - a*d)^4*(c
+ d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{15/4}(c+dx)^{5/4}}\,dx = -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} - \frac{(12d)\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx}{11(bc-ad)}$$

$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} + \frac{\left(96d^2\right)\int}{77}$$

$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} - \frac{}{77(bc-a}$$

$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} - \frac{}{77(bc-a}$$

**Mathematica** [A] time = 0.04, size = 116, normalized size = 0.85

$$-\frac{4\left(77a^3d^3+77a^2bd^2(c+4dx)+11ab^2d\left(-3c^2+8cdx+32d^2x^2\right)+b^3\left(7c^3-12c^2dx+32cd^2x^2+128d^3x^3\right)\right)}{77(a+bx)^{11/4}\sqrt[4]{c+dx}\,(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(77*a^3*d^3 + 77*a^2*b*d^2*(c + 4*d*x) + 11*a*b^2*d*(-3*c^2 + 8*c*d*x +
 32*d^2*x^2) + b^3*(7*c^3 - 12*c^2*d*x + 32*c*d^2*x^2 + 128*d^3*x^3)))/(77*
(b*c - a*d)^4*(a + b*x)^(11/4)*(c + d*x)^(1/4))
```

**IntegrateAlgebraic** [A] time = 0.13, size = 95, normalized size = 0.70

$$-\frac{4(c+dx)^{11/4}\left(-\frac{33b^2d(a+bx)}{c+dx}+\frac{77d^3(a+bx)^3}{(c+dx)^3}+\frac{77bd^2(a+bx)^2}{(c+dx)^2}+7b^3\right)}{77(a+bx)^{11/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(c + d*x)^(11/4)*(7*b^3 + (77*d^3*(a + b*x)^3)/(c + d*x)^3 + (77*b*d^2*
(a + b*x)^2)/(c + d*x)^2 - (33*b^2*d*(a + b*x))/(c + d*x)))/(77*(b*c - a*d)
^4*(a + b*x)^(11/4))
```

**fricas** [B] time = 2.58, size = 457, normalized size = 3.36

$$-\frac{4\left(128\,b^3d^3x^3+7\,b^3c^3-33\,ab^2c^2d+77\,a^2bcd^2+77\,a^3d^3+32\left(b^3cd^2+11\,ab^2d^3\right)x^2-4\left(3\,b^3c^2d-22\,ab^2cd^2-77\,a^2bd^3\right)x\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{77\left(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+\left(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5\right)x^4+\left(b^7c^5-ab^6c^4d-6\,a^2b^5c^3d^2+14\,a^3b^4c^2d^3-11\,a^4b^3cd^4+3\,a^5b^2d^5\right)x^3+3\left(ab^6c^5-3\,a^2b^5c^4d+2\,a^3b^4c^3d^2+2\,a^4b^3c^2d^3-3\,a^5b^2cd^4+a^6bd^5\right)x^2+\left(3\,a^2b^5c^5-11\,a^3b^4c^4d+14\,a^4b^3c^3d^2-6\,a^5b^2c^2d^3-a^6bcd^4+a^7d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -4/77*(128*b^3*d^3*x^3 + 7*b^3*c^3 - 33*a*b^2*c^2*d + 77*a^2*b*c*d^2 + 77*a
^3*d^3 + 32*(b^3*c*d^2 + 11*a*b^2*d^3)*x^2 - 4*(3*b^3*c^2*d - 22*a*b^2*c*d^
2 - 77*a^2*b*d^3)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^3*b^4*c^5 - 4*a^4*b
^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4
*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (
b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3
*c*d^4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^
3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c
^5 - 11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^
4 + a^7*d^5)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{15}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(5/4)), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$-\frac{4\left(128b^3d^3x^3+352a\,b^2d^3x^2+32b^3c\,d^2x^2+308a^2b\,d^3x+88a\,b^2c\,d^2x-12b^3c^2dx+77a^3d^3+77a^2bc\,d^2-33a\,b^2c^2d+7b^3c^3\right)}{77\,(bx+a)^{\frac{11}{4}}\,(dx+c)^{\frac{1}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x)
```

```
[Out] -4/77*(128*b^3*d^3*x^3+352*a*b^2*d^3*x^2+32*b^3*c*d^2*x^2+308*a^2*b*d^3*x+8
8*a*b^2*c*d^2*x-12*b^3*c^2*d*x+77*a^3*d^3+77*a^2*b*c*d^2-33*a*b^2*c^2*d+7*b
^3*c^3)/(b*x+a)^(11/4)/(d*x+c)^(1/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d
^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{15}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(5/4)), x)
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{15/4}\,(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(15/4)*(c + d*x)^(5/4)), x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(15/4)/(d*x+c)**(5/4),x)
```

```
[Out] Timed out
```

## 3.1510 $\int \frac{1}{\sqrt[4]{1-ax}\,(1+bx)^{3/4}}\,dx$

**Optimal**. Leaf size=279

$$-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}\right)}{\sqrt[4]{a}\,b^{3/4}}-$$

**Rubi** [A] time = 0.30, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}+1\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*(1 - a*x)^(1/4))/(a^(1/4)*(1 + b*x)^(1
/4))]])/(a^(1/4)*b^(3/4)) - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*(1 - a*x)^(
1/4))/(a^(1/4)*(1 + b*x)^(1/4))]])/(a^(1/4)*b^(3/4)) - Log[Sqrt[a] + (Sqrt[b
]*Sqrt[1 - a*x])/Sqrt[1 + b*x] - (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/
(1 + b*x)^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) + Log[Sqrt[a] + (Sqrt[b]*Sqrt[1
- a*x])/Sqrt[1 + b*x] + (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/(1 + b*x)
^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1-ax}\,(1+bx)^{3/4}}\,dx = -\frac{4\,\text{Subst}\left(\int \frac{x^2}{\left(1+\frac{b}{a}-\frac{bx^4}{a}\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a\sqrt{b}} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a\sqrt{b}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{b} - \text{Sub}$$

$$= -\frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}} + \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}} -$$

$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}} - \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}-\right.}{\sqrt{2}\sqrt[4]{a}\,b^{3}}$$

**Mathematica [C]** time = 0.04, size = 65, normalized size = 0.23

$$-\frac{4(1-ax)^{3/4}\left(\frac{abx+a}{a+b}\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{b-abx}{a+b}\right)}{3a(bx+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (-4*(1 - a*x)^(3/4)*((a + a*b*x)/(a + b))^(3/4)*Hypergeometric2F1[3/4, 3/4,
 7/4, (b - a*b*x)/(a + b)])/(3*a*(1 + b*x)^(3/4))
```

**IntegrateAlgebraic [A]** time = 0.22, size = 176, normalized size = 0.63

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{1-ax}\left(\frac{\sqrt[4]{a}\sqrt{bx+1}}{\sqrt{2}\sqrt[4]{b}\sqrt{1-ax}}-\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt[4]{bx+1}}\right)}{\sqrt[4]{a}\,b^{3/4}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{bx+1}}{\sqrt[4]{1-ax}\left(\frac{\sqrt{a}\sqrt{bx+1}}{\sqrt{1-ax}}+\sqrt{b}\right)}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[((1 - a*x)^(1/4)*(-(b^(1/4)/(Sqrt[2]*a^(1/4))) + (a^(1/4)*S
qrt[1 + b*x])/(Sqrt[2]*b^(1/4)*Sqrt[1 - a*x])))/(1 + b*x)^(1/4)])/(a^(1/4)*
b^(3/4)) + (Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*(1 + b*x)^(1/4))/((1 -
 a*x)^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[1 + b*x])/Sqrt[1 - a*x]))])/(a^(1/4)*b
^(3/4))
```

**fricas [A]** time = 1.11, size = 247, normalized size = 0.89

$$-4\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{(-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}-\left(a^2b^2x-ab^2\right)\sqrt{\frac{\left(ab^2x-b^2\right)\sqrt{-\frac{1}{ab^3}}-\sqrt{-ax+1}\sqrt{bx+1}}{ax-1}}\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}}{ax-1}\right)-\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(\frac{(abx-b)\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}+(-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}}{ax-1}\right)+\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-\frac{(abx-b)\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}-(-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}}{ax-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(-1/(a*b^3))^(1/4)*arctan(-((-a*x + 1)^(3/4)*(b*x + 1)^(1/4)*a*b^2*(-1/(
a*b^3))^(3/4) - (a^2*b^2*x - a*b^2)*sqrt(((a*b^2*x - b^2)*sqrt(-1/(a*b^3))
- sqrt(-a*x + 1)*sqrt(b*x + 1))/(a*x - 1))*(-1/(a*b^3))^(3/4))/(a*x - 1)) -
 (-1/(a*b^3))^(1/4)*log(((a*b*x - b)*(-1/(a*b^3))^(1/4) + (-a*x + 1)^(3/4)*
(b*x + 1)^(1/4))/(a*x - 1)) + (-1/(a*b^3))^(1/4)*log(-((a*b*x - b)*(-1/(a*b
^3))^(1/4) - (-a*x + 1)^(3/4)*(b*x + 1)^(1/4))/(a*x - 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-ax+1)^{\frac{1}{4}}(bx+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-a*x + 1)^(1/4)*(b*x + 1)^(3/4)), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-ax+1\right)^{\frac{1}{4}} \left(bx+1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x)
[Out] int(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-ax+1\right)^{\frac{1}{4}} \left(bx+1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-a*x + 1)^(1/4)*(b*x + 1)^(3/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(1-a\,x\right)}^{1/4}\,{\left(b\,x+1\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - a*x)^(1/4)*(b*x + 1)^(3/4)),x)
[Out] int(1/((1 - a*x)^(1/4)*(b*x + 1)^(3/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{-ax+1} \left(bx+1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/4)/(b*x+1)**(3/4),x)
[Out] Integral(1/((-a*x + 1)**(1/4)*(b*x + 1)**(3/4)), x)
```

## 3.1511 $\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$

**Optimal**. Leaf size=193

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+\right.}{a}$$

**Rubi [A]** time = 0.14, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - (Sqrt[2
]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(Sqrt[2]
*a) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)]/(Sqrt[2]*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1-ax}(1+ax)^{3/4}}\,dx = -\frac{4\,\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{2\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\right.}{}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right.}{\sqrt{2}a}$$

**Mathematica [C]** time = 0.01, size = 42, normalized size = 0.22

$$-\frac{2\sqrt[4]{2}(1-ax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-ax)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
[Out] (-2*2^(1/4)*(1 - a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (1 - a*x)/2])/
(3*a)
```

**IntegrateAlgebraic [A]** time = 0.12, size = 123, normalized size = 0.64

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{1-ax}\left(\frac{\sqrt{ax+1}}{\sqrt{2}\sqrt{1-ax}}-\frac{1}{\sqrt{2}}\right)}{\sqrt[4]{ax+1}}\right)}{a} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}\left(\frac{\sqrt{ax+1}}{\sqrt{1-ax}}+1\right)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[((1 - a*x)^(1/4)*(-(1/Sqrt[2]) + Sqrt[1 + a*x]/(Sqrt[2]*Sqr
t[1 - a*x])))/(1 + a*x)^(1/4)])/a + (Sqrt[2]*ArcTanh[(Sqrt[2]*(1 + a*x)^(1/
4))/((1 - a*x)^(1/4)*(1 + Sqrt[1 + a*x]/Sqrt[1 - a*x]))])/a
```

**fricas [B]** time = 0.92, size = 448, normalized size = 2.32

$$2\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a^3\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}\left(a^4x-a^3\right)\sqrt{\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+ax-1}{ax-1}\right)+2\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a^3\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}+\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-ax+1}{ax-1}\right)-\frac{1}{2}\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}\right)+\frac{1}{2}\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}+\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*(a^(-4))^(1/4)*arctan(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*
a^3*(a^(-4))^(3/4) - sqrt(2)*(a^4*x - a^3)*sqrt((sqrt(2)*(a*x + 1)^(1/4)*(-
a*x + 1)^(3/4)*a*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(a*x + 1
)*sqrt(-a*x + 1))/(a*x - 1))*(a^(-4))^(3/4) + a*x - 1)/(a*x - 1)) + 2*sqrt(
2)*(a^(-4))^(1/4)*arctan(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*a^3*(a^
(-4))^(3/4) - sqrt(2)*(a^4*x - a^3)*sqrt(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x +
1)^(3/4)*a*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(a*x + 1)*sqrt
(-a*x + 1))/(a*x - 1))*(a^(-4))^(3/4) - a*x + 1)/(a*x - 1)) - 1/2*sqrt(2)*(
a^(-4))^(1/4)*log((sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*a*(a^(-4))^(1/4
) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*x - 1)) +
 1/2*sqrt(2)*(a^(-4))^(1/4)*log(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*
a*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(a*x + 1)*sqrt(-a*x + 1
))/(a*x - 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax+1)^{\frac{3}{4}}(-ax+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + 1)^(3/4)*(-a*x + 1)^(1/4)), x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(-ax+1)^{\frac{1}{4}}\,(ax+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x)
```

```
[Out] int(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax+1)^{\frac{3}{4}}(-ax+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((a*x + 1)^(3/4)*(-a*x + 1)^(1/4)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(1-a\,x)^{1/4}\,(a\,x+1)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - a*x)^(1/4)*(a*x + 1)^(3/4)),x)
[Out] int(1/((1 - a*x)^(1/4)*(a*x + 1)^(3/4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{-ax+1}\,(ax+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/4)/(a*x+1)**(3/4),x)
[Out] Integral(1/((-a*x + 1)**(1/4)*(a*x + 1)**(3/4)), x)
```

## 3.1512 $\int \sqrt[6]{a+bx}\,(c+dx)^{5/6}\,dx$

**Optimal**. Leaf size=427

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}+\frac{5(bc-ad)^2}{\quad}$$

**Rubi [A]** time = 0.64, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{11/6}d^{7/6}}+\frac{5\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}{12bd}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b*d) + ((a + b*x)^(7/6)
*(c + d*x)^(5/6))/(2*b) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(
7/6)) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(7/6)) - (5*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(11/6)*d^(7/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(11/6)*d^(7/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \sqrt[6]{a+bx}\,(c+dx)^{5/6}\,dx = \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} + \frac{(5(bc-ad))\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{12b}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}}{72bd}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{}{\sqrt[6]{c-}}\right.}{12b^2d}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\right.}{12b^2d}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{b}-}\right.}{36b^{11}}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right.}{36b^{11/6}d^{7/6}}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right.}{36b^{11/6}d^{7/6}}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} + \frac{5(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right.}{24\sqrt{3}\,b^{11/6}d^{7/6}}$$

**Mathematica [C]** time = 0.05, size = 73, normalized size = 0.17

$$\frac{6(a+bx)^{7/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 7/6, 13/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

**IntegrateAlgebraic [A]** time = 0.77, size = 364, normalized size = 0.85

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{72b^{11/6}d^{7/6}}+\frac{(bc-ad)^2\left(\frac{d(a+bx)^{7/6}}{(c+dx)^{7/6}}+\frac{5b\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{12bd\left(b-\frac{d(a+bx)}{c+dx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```

```
[Out] ((b*c - a*d)^2*((d*(a + b*x)^(7/6))/(c + d*x)^(7/6) + (5*b*(a + b*x)^(1/6))
/(c + d*x)^(1/6)))/(12*b*d*(b - (d*(a + b*x))/(c + d*x))^2) + (5*(b*c - a*d
)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*
x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[
3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sq
rt[3]*b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6)
)/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*ArcT
anh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/((c + d*x)^(1/6)*(b^(1/3) + (d^(1/3)*
(a + b*x)^(1/3))/(c + d*x)^(1/3)))])/(72*b^(11/6)*d^(7/6))
```

**fricas [B]** time = 1.54, size = 5633, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] 1/144*(20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3
*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^11*c^2*d^6 - 2*a*b^10*c*d^7 + a^2*b^9*d^8)*(b*
x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^
7))^(5/6) - 2*sqrt(3)*(b^9*d^7*x + b^9*c*d^6)*sqrt(((b^4*c^2*d - 2*a*b^3*c*
d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
```

b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)/(b^11*d^7))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d^3*x
 + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/3))/(d*x
 + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(5/6) + sqrt(3)*(b
^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 +
 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*
b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d
^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2
+ 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^
5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4
*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^1
2*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*
b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*
d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*
a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*
b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^
8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 4
95*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*
b*c*d^12 + a^12*d^13)*x)) + 20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^11*d^7))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^11*c^2*d^6 - 2*a*b^10*c*d
^7 + a^2*b^9*d^8)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c
^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^11*d^7))^(5/6) - 2*sqrt(3)*(b^9*d^7*x + b^9*c*d^6)*sqrt(-((
b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6) - (b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x +
 c)^(2/3) - (b^4*d^3*x + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(

b^11*d^7))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^
7))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 -
220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*
b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d
^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d
 - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^
4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5
*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 -
12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^
10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*
d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220
*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12
+ (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^
9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 7
92*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b
^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) - 5*b*d*((b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c
*d^11 + a^12*d^12)/(b^11*d^7))^(1/6)*log(25*((b^4*c^2*d - 2*a*b^3*c*d^2 + a
^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^11*d^7))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d^3*x + b^4*
c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/3))/(d*x + c))
+ 5*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
- 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10
*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6)*log(-25*((b
^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^
10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6) - (b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x +
c)^(2/3) - (b^4*d^3*x + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7

```
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b
^11*d^7))^(1/3))/(d*x + c)) - 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(b^11*d^7))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/6)*(d
*x + c)^(5/6) + (b^2*d^2*x + b^2*c*d)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*
c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
/(b^11*d^7))^(1/6))/(d*x + c)) + 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^1
2)/(b^11*d^7))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/6)
*(d*x + c)^(5/6) - (b^2*d^2*x + b^2*c*d)*((b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(b^11*d^7))^(1/6))/(d*x + c)) + 12*(6*b*d*x + 5*b*c + a*d)*(b*x + a)^(1
/6)*(d*x + c)^(5/6))/(b*d)
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}} (dx+c)^{\frac{5}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/6)*(d*x+c)^(5/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(5/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{1/6}\,(c+d\,x)^{5/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)*(c + d*x)^(5/6),x)
```

```
[Out] int((a + b*x)^(1/6)*(c + d*x)^(5/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[6]{a+bx}\,(c+dx)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)*(c + d*x)**(5/6), x)
```

## 3.1513 $\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx$

**Optimal**. Leaf size=378

$$\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}-\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}+\frac{(bc-ad)\tan^{-1}\left(\right.}{2\sqrt{3}}$$

**Rubi [A]** time = 0.50, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}-\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}}+\frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```

```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/
6)*d^(7/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/6)*d^(7/6)) - ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(5/6)
*d^(7/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(
7/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(7/6)
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx = \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6d}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{bd}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{bd}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{5/6}d} - \frac{(bc-ad)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x \cdots\right.}{12b^{5/6}d^{7/6}}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}} + \frac{(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a\cdots}}{\sqrt[6]{c+dx}}\cdots\right.}{12b^{5/6}d^{7/6}}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} + \frac{(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}} - \frac{(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}} - \frac{(bc-ad)\cdots}{\cdots}$$

**Mathematica [C]** time = 0.04, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{7/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 7/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/6))
```

**IntegrateAlgebraic [A]** time = 12.20, size = 385, normalized size = 1.02

$$\frac{\sqrt[6]{d}\sqrt[6]{a+bx}\left(-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{b}\sqrt[6]{c+dx}-2\sqrt[6]{ad+b(c+dx)-bc}}\right)}{2\sqrt{3}b^{5/6}d^{7/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}{2\sqrt[6]{ad+b(c+dx)-bc}+\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{2\sqrt{3}b^{5/6}d^{7/6}}+\frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{ad+b(c+dx)-bc}}\right)}{3b^{5/6}d^{7/6}}+\frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[6]{b}\sqrt[6]{c+dx}\sqrt[6]{ad+b(c+dx)-bc}}\right)}{6b^{5/6}d^{7/6}}+\frac{(c+dx)^{5/6}\sqrt[6]{ad+b(c+dx)-bc}}{d^{7/6}}\right)}{\sqrt[6]{ad+bdx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```

```
[Out] (d^(1/6)*(a + b*x)^(1/6)*(((c + d*x)^(5/6)*(-(b*c) + a*d + b*(c + d*x))^(1/
6))/d^(7/6) - ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/6
)*(c + d*x)^(1/6) - 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*b^(5
/6)*d^(7/6)) + ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/
6)*(c + d*x)^(1/6) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*b^(
5/6)*d^(7/6)) + ((-(b*c) + a*d)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(-(b*c) +
 a*d + b*(c + d*x))^(1/6)])/(3*b^(5/6)*d^(7/6)) + ((-(b*c) + a*d)*ArcTanh[(
b^(1/3)*(c + d*x)^(1/3) + (-(b*c) + a*d + b*(c + d*x))^(1/3))/(b^(1/6)*(c +
 d*x)^(1/6)*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(6*b^(5/6)*d^(7/6))))/(a*
d + b*d*x)^(1/6)
```

**fricas [B]** time = 1.39, size = 3025, normalized size = 8.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*
b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6
)*arctan(1/3*(2*sqrt(3)*(b^5*c*d^6 - a*b^4*d^7)*(b*x + a)^(1/6)*(d*x + c)^(
5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) + 2*sqrt(3)*
(b^4*d^7*x + b^4*c*d^6)*sqrt(((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)
^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d^3*x + b^2*
c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c)
)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*
a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) + sqrt(3)*(b^6*
c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*
c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a
^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 +
```

a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4
*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*
b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5
- 6*a^5*b*c*d^6 + a^6*d^7)*x)) + 4*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
+ a^6*d^6)/(b^5*d^7))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^5*c*d^6 - a*b^4*d^7)*(
b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d
^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^
5*d^7))^(5/6) + 2*sqrt(3)*(b^4*d^7*x + b^4*c*d^6)*sqrt(-((b^2*c*d - a*b*d^2
)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^
4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/
(b^5*d^7))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c
)^(2/3) - (b^2*d^3*x + b^2*c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^
4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/
(b^5*d^7))^(1/3))/(d*x + c))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2
 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*
d^7))^(5/6) - sqrt(3)*(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^
3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6
*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2
*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b
^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^
6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^
3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x)) + d*((b^6*c^6 - 6
*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(((b^2*c*d - a*b*d^2)*(b*x
+ a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^
7))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3)
 + (b^2*d^3*x + b^2*c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^
7))^(1/3))/(d*x + c)) - d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7
))^(1/6)*log(-((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) - (b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^2*d^3*x + b^2*c*d^2)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c)) + 2*d*((b^6*c
^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(-((b*c - a*d)*(b*x +
 a)^(1/6)*(d*x + c)^(5/6) + (b*d^2*x + b*c*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
 + a^6*d^6)/(b^5*d^7))^(1/6))/(d*x + c)) - 2*d*((b^6*c^6 - 6*a*b^5*c^5*d +
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^
5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(-((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(

```
5/6) - (b*d^2*x + b*c*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7)
)^(1/6))/(d*x + c)) - 12*(b*x + a)^(1/6)*(d*x + c)^(5/6))/d
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(1/6), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(1/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{1/6}}{(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(1/6)/(c + d*x)^(1/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(1/6), x)
```

## 3.1514 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx$

**Optimal**. Leaf size=332

$$-\frac{\sqrt[6]{b}\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}+\frac{\sqrt[6]{b}\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}-\frac{\sqrt{3}\,\sqrt[6]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{\ldots}}{\sqrt{3}\ldots}\right.}{d^{7/6}}$$

**Rubi** [A] time = 0.49, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{\sqrt[6]{b}\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}+\frac{\sqrt[6]{b}\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}-\frac{\sqrt{3}\,\sqrt[6]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}}+\frac{\sqrt{3}\,\sqrt[6]{b}\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{7/6}}+\frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}}-\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(1/6))/(d*(c + d*x)^(1/6)) - (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6)
 + (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) + (2*b^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) - (b^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)])/(2*d^(7/6)) + (b^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2
*d^(7/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx = -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{b\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{6\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{\left(2\sqrt[6]{b}\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d} + \frac{\left(2\sqrt[6]{b}\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}+\cdots}{\sqrt[3]{b}+\sqrt[6]{b}\,\sqrt[6]{d}\cdots}\right.}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{2\sqrt[6]{b}\,\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}} - \frac{\sqrt[6]{b}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{7/6}} + \cdots$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{2\sqrt[6]{b}\,\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}} - \frac{\sqrt[6]{b}\,\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{7/6}} + \frac{\sqrt[6]{b}\,\log\cdots}{\cdots}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} - \frac{\sqrt{3}\,\sqrt[6]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{7/6}} + \frac{\sqrt{3}\,\sqrt[6]{b}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{7/6}} + \frac{2\sqrt[6]{b}\,\tanh^{-1}\left(\frac{\sqrt[6]{d}\cdots}{\sqrt[6]{b}\cdots}\right.}{d^{7/6}}$$

**Mathematica [C]** time = 0.06, size = 73, normalized size = 0.22

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]

[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 7/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(7/6))

**IntegrateAlgebraic [A]** time = 0.24, size = 256, normalized size = 0.77

$$-\frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}}+\frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{7/6}}+\frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}}+\frac{\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{d^{7/6}}-\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]

[Out] (-6*(a + b*x)^(1/6))/(d*(c + d*x)^(1/6)) - (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]])/d^(7/6)
 + (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))]])/d^(7/6) + (2*b^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))]])/d^(7/6) + (b^(1/6)*ArcTanh[(b^(1/6)*d^(
1/6)*(a + b*x)^(1/6))/((c + d*x)^(1/6)*(b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3)))]])/d^(7/6)

**fricas [B]** time = 1.00, size = 663, normalized size = 2.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="fricas")

[Out] -1/2*(4*sqrt(3)*(d^2*x + c*d)*(b/d^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a
)^(1/6)*(d*x + c)^(5/6)*d^6*(b/d^7)^(5/6) - 2*sqrt(3)*(d^7*x + c*d^6)*sqrt(
((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6) + (d^3*x + c*d^2)*(b/d^7)^
(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c))*(b/d^7)^(5/6) + sqrt(3)
*(b*d*x + b*c))/(b*d*x + b*c)) + 4*sqrt(3)*(d^2*x + c*d)*(b/d^7)^(1/6)*arct
an(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*d^6*(b/d^7)^(5/6) - 2*sq
rt(3)*(d^7*x + c*d^6)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6
) - (d^3*x + c*d^2)*(b/d^7)^(1/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x +
 c))*(b/d^7)^(5/6) - sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) - (d^2*x + c*d)*
(b/d^7)^(1/6)*log(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6) + (d^3
*x + c*d^2)*(b/d^7)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c)) + (
d^2*x + c*d)*(b/d^7)^(1/6)*log(-4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7
)^(1/6) - (d^3*x + c*d^2)*(b/d^7)^(1/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3))/
(d*x + c)) - 2*(d^2*x + c*d)*(b/d^7)^(1/6)*log(((d^2*x + c*d)*(b/d^7)^(1/6)
 + (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d*x + c)) + 2*(d^2*x + c*d)*(b/d^7)^(1

```
/6)*log(-((d^2*x + c*d)*(b/d^7)^(1/6) - (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d
*x + c)) + 12*(b*x + a)^(1/6)*(d*x + c)^(5/6))/(d^2*x + c*d)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(7/6), x)
```

**maple [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(7/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{1/6}}{(c+d\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(7/6),x)
```

```
[Out] int((a + b*x)^(1/6)/(c + d*x)^(7/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(7/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(7/6), x)
```

## 3.1515 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx$

**Optimal**. Leaf size=32

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

**Rubi** [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx = \frac{6(a+bx)^{7/6}}{7(bc-ad)(c+dx)^{7/6}}$$

**Mathematica** [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

**IntegrateAlgebraic [A]** time = 0.04, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

**fricas [B]** time = 1.17, size = 65, normalized size = 2.03

$$\frac{6(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{5}{6}}}{7\left(bc^3-ac^2d+\left(bcd^2-ad^3\right)x^2+2\left(bc^2d-acd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="fricas")
[Out] 6/7*(b*x + a)^(7/6)*(d*x + c)^(5/6)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*x^
2 + 2*(b*c^2*d - a*c*d^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(13/6), x)
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.84

$$-\frac{6(bx+a)^{\frac{7}{6}}}{7(dx+c)^{\frac{7}{6}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(13/6),x)
[Out] -6/7*(b*x+a)^(7/6)/(d*x+c)^(7/6)/(a*d-b*c)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(13/6), x)
```

**mupad** [B] time = 0.56, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{6\,a\,(a+b\,x)^{1/6}}{7\,a\,d^3-7\,b\,c\,d^2}+\frac{6\,b\,x\,(a+b\,x)^{1/6}}{7\,a\,d^3-7\,b\,c\,d^2}\right)\,(c+d\,x)^{5/6}}{x^2-\frac{7\,b\,c^3-7\,a\,c^2\,d}{7\,a\,d^3-7\,b\,c\,d^2}+\frac{14\,c\,d\,x\,(a\,d-b\,c)}{7\,a\,d^3-7\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(13/6),x)
```

```
[Out] -(((6*a*(a + b*x)^(1/6))/(7*a*d^3 - 7*b*c*d^2) + (6*b*x*(a + b*x)^(1/6))/(7
*a*d^3 - 7*b*c*d^2))*(c + d*x)^(5/6))/(x^2 - (7*b*c^3 - 7*a*c^2*d)/(7*a*d^3
 - 7*b*c*d^2) + (14*c*d*x*(a*d - b*c))/(7*a*d^3 - 7*b*c*d^2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(13/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(13/6), x)
```

## 3.1516 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx$

**Optimal**. Leaf size=66

$$\frac{36b(a+bx)^{7/6}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{13(c+dx)^{13/6}(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b(a+bx)^{7/6}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (36*b*(a + b*x)^(7/
6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx = \frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}} + \frac{(6b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx}{13(bc-ad)}$$
$$= \frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}} + \frac{36b(a+bx)^{7/6}}{91(bc-ad)^2(c+dx)^{7/6}}$$

**Mathematica** [A] time = 0.03, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{7/6}(-7ad+13bc+6bdx)}{91(c+dx)^{13/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(13*b*c - 7*a*d + 6*b*d*x))/(91*(b*c - a*d)^2*(c + d*x)^
(13/6))
```

**IntegrateAlgebraic** [A] time = 0.15, size = 51, normalized size = 0.77

$$\frac{6(a+bx)^{7/6}\left(13b-\frac{7d(a+bx)}{c+dx}\right)}{91(c+dx)^{7/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(13*b - (7*d*(a + b*x))/(c + d*x)))/(91*(b*c - a*d)^2*(c
 + d*x)^(7/6))
```

**fricas** [B] time = 0.93, size = 175, normalized size = 2.65

$$\frac{6\left(6\,b^2dx^2+13\,abc-7\,a^2d+\left(13\,b^2c-abd\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{91\left(b^2c^5-2\,abc^4d+a^2c^3d^2+\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)x^3+3\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2+3\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/91*(6*b^2*d*x^2 + 13*a*b*c - 7*a^2*d + (13*b^2*c - a*b*d)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3 - 2
*a*b*c*d^4 + a^2*d^5)*x^3 + 3*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2
 + 3*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(19/6), x)
```

**maple [A]** time = 0.01, size = 54, normalized size = 0.82

$$-\frac{6\,(bx+a)^{\frac{7}{6}}\,(-6bdx+7ad-13bc)}{91\,(dx+c)^{\frac{13}{6}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(19/6),x)
```

```
[Out] -6/91*(b*x+a)^(7/6)*(-6*b*d*x+7*a*d-13*b*c)/(d*x+c)^(13/6)/(a^2*d^2-2*a*b*c
*d+b^2*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(19/6), x)
```

**mupad [B]** time = 0.75, size = 137, normalized size = 2.08

$$\frac{(c+d\,x)^{5/6}\left(\frac{36\,b^2\,x^2\,(a+b\,x)^{1/6}}{91\,d^2\,(a\,d-b\,c)^2}-\frac{\left(42\,a^2\,d-78\,a\,b\,c\right)(a+b\,x)^{1/6}}{91\,d^3\,(a\,d-b\,c)^2}+\frac{x\left(78\,b^2\,c-6\,a\,b\,d\right)(a+b\,x)^{1/6}}{91\,d^3\,(a\,d-b\,c)^2}\right)}{x^3+\frac{c^3}{d^3}+\frac{3\,c\,x^2}{d}+\frac{3\,c^2\,x}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(19/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((36*b^2*x^2*(a + b*x)^(1/6))/(91*d^2*(a*d - b*c)^2) - ((4
2*a^2*d - 78*a*b*c)*(a + b*x)^(1/6))/(91*d^3*(a*d - b*c)^2) + (x*(78*b^2*c
- 6*a*b*d)*(a + b*x)^(1/6))/(91*d^3*(a*d - b*c)^2)))/(x^3 + c^3/d^3 + (3*c*
x^2)/d + (3*c^2*x)/d^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

## 3.1517 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}} dx$

**Optimal**. Leaf size=101

$$\frac{432b^2(a+bx)^{7/6}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{72b(a+bx)^{7/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{19(c+dx)^{19/6}(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{7/6}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{72b(a+bx)^{7/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (72*b*(a + b*x)^(7/
6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (432*b^2*(a + b*x)^(7/6))/(1729*
(b*c - a*d)^3*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx = \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{(12b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx}{19(bc-ad)}$$

$$= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{\left(72b^2\right)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx}{247(bc-ad)^2}$$

$$= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{432b^2(a+bx)^{7/6}}{1729(bc-ad)^3(c+dx)^{7/6}}$$

**Mathematica** [A] time = 0.06, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{7/6}\left(91a^2d^2-14abd(19c+6dx)+b^2\left(247c^2+228cdx+72d^2x^2\right)\right)}{1729(c+dx)^{19/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(91*a^2*d^2 - 14*a*b*d*(19*c + 6*d*x) + b^2*(247*c^2 + 2
28*c*d*x + 72*d^2*x^2)))/(1729*(b*c - a*d)^3*(c + d*x)^(19/6))
```

**IntegrateAlgebraic** [A] time = 0.17, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{7/6}\left(\frac{91d^2(a+bx)^2}{(c+dx)^2}-\frac{266bd(a+bx)}{c+dx}+247b^2\right)}{1729(c+dx)^{7/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(247*b^2 + (91*d^2*(a + b*x)^2)/(c + d*x)^2 - (266*b*d*(
a + b*x))/(c + d*x)))/(1729*(b*c - a*d)^3*(c + d*x)^(7/6))
```

**fricas** [B] time = 0.75, size = 338, normalized size = 3.35

$$\frac{6\left(72\,b^3d^2x^3+247\,ab^2c^2-266\,a^2bcd+91\,a^3d^2+12\left(19\,b^3cd-ab^2d^2\right)x^2+\left(247\,b^3c^2-38\,ab^2cd+7\,a^2bd^2\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{1729\left(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3+\left(b^3c^3d^4-3\,ab^2c^2d^5+3\,a^2bcd^6-a^3d^7\right)x^4+4\left(b^3c^4d^3-3\,ab^2c^3d^4+3\,a^2bc^2d^5-a^3cd^6\right)x^3+6\left(b^3c^5d^2-3\,ab^2c^4d^3+3\,a^2bc^3d^4-a^3c^2d^5\right)x^2+4\left(b^3c^6d-3\,ab^2c^5d^2+3\,a^2bc^4d^3-a^3c^3d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/1729*(72*b^3*d^2*x^3 + 247*a*b^2*c^2 - 266*a^2*b*c*d + 91*a^3*d^2 + 12*(1
9*b^3*c*d - a*b^2*d^2)*x^2 + (247*b^3*c^2 - 38*a*b^2*c*d + 7*a^2*b*d^2)*x)*
```

```
(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2
- a^3*c^4*d^3 + (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*x
^4 + 4*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*x^3 +
6*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^2 + 4*(
b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(25/6), x)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$-\frac{6\,(bx+a)^{\frac{7}{6}}\left(72b^{2}x^{2}d^{2}-84ab\,d^{2}x+228b^{2}cdx+91a^{2}d^{2}-266abcd+247b^{2}c^{2}\right)}{1729\,(dx+c)^{\frac{19}{6}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(25/6),x)
```

```
[Out] -6/1729*(b*x+a)^(7/6)*(72*b^2*d^2*x^2-84*a*b*d^2*x+228*b^2*c*d*x+91*a^2*d^2
-266*a*b*c*d+247*b^2*c^2)/(d*x+c)^(19/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(25/6), x)
```

**mupad [B]** time = 0.95, size = 213, normalized size = 2.11

$$-\frac{(c+d\,x)^{5/6}\left(\frac{(a+b\,x)^{1/6}\left(546\,a^{3}\,d^{2}-1596\,a^{2}\,b\,c\,d+1482\,a\,b^{2}\,c^{2}\right)}{1729\,d^{4}\,(a\,d-b\,c)^{3}}+\frac{432\,b^{3}\,x^{3}\,(a+b\,x)^{1/6}}{1729\,d^{2}\,(a\,d-b\,c)^{3}}+\frac{x\,(a+b\,x)^{1/6}\left(42\,a^{2}\,b\,d^{2}-228\,a\,b^{2}\,c\,d+1482\,b^{3}\,c^{2}\right)}{1729\,d^{4}\,(a\,d-b\,c)^{3}}-\frac{72\,b^{2}\,x^{2}\,(a\,d-19\,b\,c)\,(a+b\,x)^{1/6}}{1729\,d^{3}\,(a\,d-b\,c)^{3}}\right)}{x^{4}+\frac{c^{4}}{d^{4}}+\frac{4\,c\,x^{3}}{d}+\frac{4\,c^{3}\,x}{d^{3}}+\frac{6\,c^{2}\,x^{2}}{d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(25/6),x)
```

```
[Out] -((c + d*x)^(5/6)*(((a + b*x)^(1/6)*(546*a^3*d^2 + 1482*a*b^2*c^2 - 1596*a^
2*b*c*d))/(1729*d^4*(a*d - b*c)^3) + (432*b^3*x^3*(a + b*x)^(1/6))/(1729*d^
2*(a*d - b*c)^3) + (x*(a + b*x)^(1/6)*(1482*b^3*c^2 + 42*a^2*b*d^2 - 228*a*
b^2*c*d))/(1729*d^4*(a*d - b*c)^3) - (72*b^2*x^2*(a*d - 19*b*c)*(a + b*x)^(
1/6))/(1729*d^3*(a*d - b*c)^3)))/(x^4 + c^4/d^4 + (4*c*x^3)/d + (4*c^3*x)/d
^3 + (6*c^2*x^2)/d^2)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

## 3.1518 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{31/6}}\,dx$

**Optimal**. Leaf size=136

$$\frac{7776b^3(a+bx)^{7/6}}{43225(c+dx)^{7/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{7/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{108b(a+bx)^{7/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{25(c+dx)^{25/6}(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{7/6}}{43225(c+dx)^{7/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{7/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{108b(a+bx)^{7/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (108*b*(a + b*x)^(7
/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (1296*b^2*(a + b*x)^(7/6))/(617
5*(b*c - a*d)^3*(c + d*x)^(13/6)) + (7776*b^3*(a + b*x)^(7/6))/(43225*(b*c
- a*d)^4*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{31/6}}\,dx = \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{(18b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx}{25(bc-ad)}$$

$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{\left(216b^2\right)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx}{475(bc-ad)^2}$$

$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}} + \frac{(1296b}{617}$$

$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}} + \frac{77}{43225(}$$

**Mathematica** [A] time = 0.08, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{7/6}\left(-1729a^3d^3+273a^2bd^2(25c+6dx)-21ab^2d\left(475c^2+300cdx+72d^2x^2\right)+b^3\left(6175c^3+8550c^2dx+5400cd^2x^2+1296d^3x^3\right)\right)}{43225(c+dx)^{25/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(-1729*a^3*d^3 + 273*a^2*b*d^2*(25*c + 6*d*x) - 21*a*b^2
*d*(475*c^2 + 300*c*d*x + 72*d^2*x^2) + b^3*(6175*c^3 + 8550*c^2*d*x + 5400
*c*d^2*x^2 + 1296*d^3*x^3)))/(43225*(b*c - a*d)^4*(c + d*x)^(25/6))
```

**IntegrateAlgebraic** [A] time = 0.17, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{7/6}\left(-\frac{9975b^2d(a+bx)}{c+dx}-\frac{1729d^3(a+bx)^3}{(c+dx)^3}+\frac{6825bd^2(a+bx)^2}{(c+dx)^2}+6175b^3\right)}{43225(c+dx)^{7/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(6175*b^3 - (1729*d^3*(a + b*x)^3)/(c + d*x)^3 + (6825*b
*d^2*(a + b*x)^2)/(c + d*x)^2 - (9975*b^2*d*(a + b*x))/(c + d*x)))/(43225*(
b*c - a*d)^4*(c + d*x)^(7/6))
```

**fricas** [B] time = 1.10, size = 533, normalized size = 3.92

$$\frac{6\left(1296\,b^4d^3x^4+6175\,ab^3c^3-9975\,a^2b^2c^2d+6825\,a^3bcd^2-1729\,a^4d^3+216\left(25\,b^4cd^2-ab^3d^3\right)x^3+18\left(475\,b^4c^2d-50\,ab^3cd^2+7\,a^2b^2d^3\right)x^2+\left(6175\,b^4c^3-1425\,ab^3c^2d+525\,a^2b^2cd^2-91\,a^3bd^3\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{43225\left(b^4c^9-4\,ab^3c^8d+6\,a^2b^2c^7d^2-4\,a^3bc^6d^3+a^4c^5d^4+\left(b^4c^4d^5-4\,ab^3c^3d^6+6\,a^2b^2c^2d^7-4\,a^3bcd^8+a^4d^9\right)x^5+5\left(b^4c^5d^4-4\,ab^3c^4d^5+6\,a^2b^2c^3d^6-4\,a^3bc^2d^7+a^4cd^8\right)x^4+10\left(b^4c^6d^3-4\,ab^3c^5d^4+6\,a^2b^2c^4d^5-4\,a^3bc^3d^6+a^4c^2d^7\right)x^3+10\left(b^4c^7d^2-4\,ab^3c^6d^3+6\,a^2b^2c^5d^4-4\,a^3bc^4d^5+a^4c^3d^6\right)x^2+5\left(b^4c^8d-4\,ab^3c^7d^2+6\,a^2b^2c^6d^3-4\,a^3bc^5d^4+a^4c^4d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="fricas")
```

```
[Out] 6/43225*(1296*b^4*d^3*x^4 + 6175*a*b^3*c^3 - 9975*a^2*b^2*c^2*d + 6825*a^3*
b*c*d^2 - 1729*a^4*d^3 + 216*(25*b^4*c*d^2 - a*b^3*d^3)*x^3 + 18*(475*b^4*c
^2*d - 50*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + (6175*b^4*c^3 - 1425*a*b^3*c^2
*d + 525*a^2*b^2*c*d^2 - 91*a^3*b*d^3)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
 + (b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4
*d^9)*x^5 + 5*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*
c^2*d^7 + a^4*c*d^8)*x^4 + 10*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^
4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*x^3 + 10*(b^4*c^7*d^2 - 4*a*b^3*c^6*
d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*x^2 + 5*(b^4*c^8*d
 - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{31}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(31/6), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$-\frac{6\,(bx+a)^{\frac{7}{6}}\left(-1296b^{3}d^{3}x^{3}+1512a\,b^{2}d^{3}x^{2}-5400b^{3}c\,d^{2}x^{2}-1638a^{2}b\,d^{3}x+6300a\,b^{2}c\,d^{2}x-8550b^{3}c^{2}dx+1729a^{3}d^{3}-6825a^{2}bc\,d^{2}+9975a\,b^{2}c^{2}d-6175b^{3}c^{3}\right)}{43225\,(dx+c)^{\frac{25}{6}}\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(31/6),x)
```

```
[Out] -6/43225*(b*x+a)^(7/6)*(-1296*b^3*d^3*x^3+1512*a*b^2*d^3*x^2-5400*b^3*c*d^2
*x^2-1638*a^2*b*d^3*x+6300*a*b^2*c*d^2*x-8550*b^3*c^2*d*x+1729*a^3*d^3-6825
*a^2*b*c*d^2+9975*a*b^2*c^2*d-6175*b^3*c^3)/(d*x+c)^(25/6)/(a^4*d^4-4*a^3*b
*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{31}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(31/6), x)
```

**mupad [B]** time = 1.15, size = 302, normalized size = 2.22

$$\frac{(c+d\,x)^{5/6}\left(\frac{7776\,b^4\,x^4\,(a+b\,x)^{1/6}}{43225\,d^2\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{1/6}\,\left(10374\,a^4\,d^3-40950\,a^3\,b\,c\,d^2+59850\,a^2\,b^2\,c^2\,d-37050\,a\,b^3\,c^3\right)}{43225\,d^5\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{1/6}\,\left(-546\,a^3\,b\,d^3+3150\,a^2\,b^2\,c\,d^2-8550\,a\,b^3\,c^2\,d+37050\,b^4\,c^3\right)}{43225\,d^5\,(a\,d-b\,c)^4}+\frac{108\,b^2\,x^2\,(a+b\,x)^{1/6}\,\left(7\,a^2\,d^2-50\,a\,b\,c\,d+475\,b^2\,c^2\right)}{43225\,d^4\,(a\,d-b\,c)^4}-\frac{1296\,b^3\,x^3\,(a\,d-25\,b\,c)\,(a+b\,x)^{1/6}}{43225\,d^3\,(a\,d-b\,c)^4}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(31/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((7776*b^4*x^4*(a + b*x)^(1/6))/(43225*d^2*(a*d - b*c)^4)
- ((a + b*x)^(1/6)*(10374*a^4*d^3 - 37050*a*b^3*c^3 + 59850*a^2*b^2*c^2*d -
 40950*a^3*b*c*d^2))/(43225*d^5*(a*d - b*c)^4) + (x*(a + b*x)^(1/6)*(37050*
b^4*c^3 - 546*a^3*b*d^3 + 3150*a^2*b^2*c*d^2 - 8550*a*b^3*c^2*d))/(43225*d^
5*(a*d - b*c)^4) + (108*b^2*x^2*(a + b*x)^(1/6)*(7*a^2*d^2 + 475*b^2*c^2 -
50*a*b*c*d))/(43225*d^4*(a*d - b*c)^4) - (1296*b^3*x^3*(a*d - 25*b*c)*(a +
b*x)^(1/6))/(43225*d^3*(a*d - b*c)^4)))/(x^5 + c^5/d^5 + (5*c*x^4)/d + (5*c
^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(31/6),x)
```

```
[Out] Timed out
```

## 3.1519 $\int (a+bx)^{5/6}\sqrt[6]{c+dx}\,dx$

**Optimal**. Leaf size=427

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2}{}$$

**Rubi** [A] time = 0.64, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}b^{7/6}d^{11/6}}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{7/6}d^{11/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}(bc-ad)}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b*d) + ((a + b*x)^(11/6)*
(c + d*x)^(1/6))/(2*b) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
+ b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11
/6)) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqr
t[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11/6)) - (5*(b*c - a
*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(
7/6)*d^(11/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(
c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b
^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(7/6)*d^(11/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (a+bx)^{5/6}\sqrt[6]{c+dx}\,dx = \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx}{12b}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,d}{72bd}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{d}{}\right.}\right.}{12b^2d}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,d\right.}{12b^2d}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}}{\sqrt[3]{b}-\sqrt[6]{b}}\right.}{36b^{7/6}d^{5/}}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{7/6}d^{11/6}}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{7/6}d^{11/6}}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{7/6}d^{11/6}}$$

**Mathematica [C]** time = 0.05, size = 73, normalized size = 0.17

$$\frac{6(a+bx)^{11/6}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{1}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 11/6, 17/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(11*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

**IntegrateAlgebraic [A]** time = 0.75, size = 365, normalized size = 0.85

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{24\sqrt{3}b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{36b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{72b^{7/6}d^{11/6}}+\frac{(bc-ad)^2\left(\frac{b(c+dx)^{7/6}}{(a+bx)^{7/6}}+\frac{5d\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\right)}{12bd\left(\frac{b(c+dx)}{a+bx}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)^2*((5*d*(c + d*x)^(1/6))/(a + b*x)^(1/6) + (b*(c + d*x)^(7/6))
/(a + b*x)^(7/6)))/(12*b*d*(-d + (b*(c + d*x))/(a + b*x))^2) + (5*(b*c - a*
d)^2*ArcTan[1/Sqrt[3] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b
*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt
[3] + (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))])/(24*S
qrt[3]*b^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*ArcTanh[(b^(1/6)*(c + d*x)^(1/6
))/(d^(1/6)*(a + b*x)^(1/6))])/(36*b^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*Arc
Tanh[(b^(1/6)*d^(1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(1/3) + (b^(1/3)
*(c + d*x)^(1/3))/(a + b*x)^(1/3)))])/(72*b^(7/6)*d^(11/6))
```

**fricas [B]** time = 1.47, size = 5633, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] 1/144*(20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3
*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^8*c^2*d^9 - 2*a*b^7*c*d^10 + a^2*b^6*d^11)*(b*
x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
```

924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^1
1))^(5/6) - 2*sqrt(3)*(b^7*d^9*x + a*b^6*d^9)*sqrt(((b^3*c^2*d^2 - 2*a*b^2*
c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^7*d^11))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^4*x
+ a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/3))/(b*x
+ a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(5/6) + sqrt(3)*(a
*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^
3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a
^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c
^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66
*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b
^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^
8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*
b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*
a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c
^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 +
66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^
12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d
^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*
a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*
c*d^11 + a^12*b*d^12)*x)) + 20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^7*d^11))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^8*c^2*d^9 - 2*a*b^7*c*d^1
0 + a^2*b^6*d^11)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c
^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^7*d^11))^(5/6) - 2*sqrt(3)*(b^7*d^9*x + a*b^6*d^9)*sqrt(-((
b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d

^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6) - (b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x +
c)^(1/3) - (b^3*d^4*x + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^7*d^11))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^1
1))^(5/6) - sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^
2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a
^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c
^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 -
12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b
^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^
7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a
^11*b^2*c*d^11 + a^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^
3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c
^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 -
220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12
+ (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d
^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*
a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c
^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) - 5*b*d*((b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c
*d^11 + a^12*d^12)/(b^7*d^11))^(1/6)*log(25*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 +
a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^7*d^11))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^4*x + a*b^
2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/3))/(b*x + a))
+ 5*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
- 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10
*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6)*log(-25*((b
^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +

```
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^
10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6) - (b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x +
c)^(1/3) - (b^3*d^4*x + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b
^7*d^11))^(1/3))/(b*x + a)) - 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(b^7*d^11))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(5/6)*(d
*x + c)^(1/6) + (b^2*d^2*x + a*b*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*
c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
/(b^7*d^11))^(1/6))/(b*x + a)) + 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^1
2)/(b^7*d^11))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(5/6)
*(d*x + c)^(1/6) - (b^2*d^2*x + a*b*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(b^7*d^11))^(1/6))/(b*x + a)) + 12*(6*b*d*x + b*c + 5*a*d)*(b*x + a)^(5
/6)*(d*x + c)^(1/6))/(b*d)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(1/6), x)
```

**maple [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}} (dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(5/6)*(d*x+c)^(1/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(1/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/6}\,(c+d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)*(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(5/6)*(c + d*x)^(1/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{6}}\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)*(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(5/6)*(c + d*x)**(1/6), x)
```

## 3.1520 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx$

**Optimal**. Leaf size=378

$$\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tan}{2}$$

**Rubi [A]** time = 0.56, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\sqrt[6]{b}\,d^{11/6}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]
```

```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/d - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
1/6)*d^(11/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(1/6)*d^(11/6)) - (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(1/6)*d^(11/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(1/6)*d^(11/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(1/6)*d^(11/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx = \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6d}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{bd}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{bd}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3d^{5/3}} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{}{}\right.}{3}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}} + \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\right.}{12\sqrt[6]{b}\,d^{11/6}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}} + \frac{5(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}}{}\right.}{12\sqrt[6]{b}\,d^{11/6}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}} + \frac{5(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}} - \frac{5(bc}{}$$

**Mathematica [C]** time = 0.04, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]

[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(5/6))

**IntegrateAlgebraic [A]** time = 17.90, size = 385, normalized size = 1.02

$$\frac{d^{5/6}(a+bx)^{5/6}\left(\frac{\sqrt[6]{c+dx}\,(ad+b(c+dx)-bc)^{5/6}}{d^{11/6}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{b}\sqrt[6]{c+dx}-2\sqrt[6]{ad+b(c+dx)-bc}}\right)}{2\sqrt{3}\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}{2\sqrt[6]{ad+b(c+dx)-bc}+\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{ad+b(c+dx)-bc}}\right)}{3\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[6]{b}\sqrt[6]{c+dx}\sqrt[6]{ad+b(c+dx)-bc}}\right)}{6\sqrt[6]{b}\,d^{11/6}}\right)}{(ad+bdx)^{5/6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]

[Out] (d^(5/6)*(a + b*x)^(5/6)*(((c + d*x)^(1/6)*(-(b*c) + a*d + b*(c + d*x))^(5/
6))/d^(11/6) + (5*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(
1/6)*(c + d*x)^(1/6) - 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*b
^(1/6)*d^(11/6)) - (5*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/
(b^(1/6)*(c + d*x)^(1/6) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[
3]*b^(1/6)*d^(11/6)) - (5*(b*c - a*d)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(-(
b*c) + a*d + b*(c + d*x))^(1/6)])/(3*b^(1/6)*d^(11/6)) - (5*(b*c - a*d)*Arc
Tanh[(b^(1/3)*(c + d*x)^(1/3) + (-(b*c) + a*d + b*(c + d*x))^(1/3))/(b^(1/6
)*(c + d*x)^(1/6)*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(6*b^(1/6)*d^(11/6)
)))/(a*d + b*d*x)^(5/6)

**fricas [B]** time = 1.25, size = 2997, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="fricas")

[Out] -1/12*(20*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3
*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6
)*arctan(1/3*(2*sqrt(3)*(b^2*c*d^9 - a*b*d^10)*(b*x + a)^(5/6)*(d*x + c)^(1
/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + 2*sqrt(3)*(b
^2*d^9*x + a*b*d^9)*sqrt(((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)
*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) + (b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^4*x + a*d^4)*((b^6*
c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*
c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a))*((b^6*c^6 -
6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^
4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + sqrt(3)*(a*b^6*c^6 - 6*a^2*b
^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6

*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + 20*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^2*c*d^9 - a*b*d^10)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + 2*sqrt(3)*(b^2*d^9*x + a*b*d^9)*sqrt(-((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^4*x + a*d^4)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) - sqrt(3)*(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + 5*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(25*((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^4*x + a*d^4)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a)) - 5*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(-25*((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^4*x + a*d^4)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a)) + 10*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(-5*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b*d^2*x + a*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6))/(b*x + a)) - 10*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3

```
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(-5*((b*
c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b*d^2*x + a*d^2)*((b^6*c^6 - 6*
a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
- 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6))/(b*x + a)) - 12*(b*x + a)^(5/6)
*(d*x + c)^(1/6))/d
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(5/6), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(5/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(5/6),x)
```

```
[Out] int((a + b*x)^(5/6)/(c + d*x)^(5/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(5/6), x)
```

## 3.1521 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{11/6}}\,dx$

**Optimal**. Leaf size=334

$$-\frac{b^{5/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{b^{5/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{3}}\right.}{d^{11/6}}$$

**Rubi** [A] time = 0.56, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{b^{5/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{b^{5/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}}-\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{11/6}}+\frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}}-\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] (-6*(a + b*x)^(5/6))/(5*d*(c + d*x)^(5/6)) + (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt
[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11
/6) - (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt
[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) + (2*b^(5/6)*ArcTanh[(d^(1/6)*(a +
b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) - (b^(5/6)*Log[b^(1/3) + (
d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6)
)/(c + d*x)^(1/6)])/(2*d^(11/6)) + (b^(5/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)])/(2*d^(11/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{11/6}}\,dx = -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{b\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{6\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{6\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{\left(2b^{5/6}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^{5/3}} + \frac{\left(2b^{5/6}\right)\text{Subst}\left(\int \frac{-\ldots}{\sqrt[3]{b}+\sqrt[6]{\ldots}}\right.}{\ldots}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{11/6}} - \frac{b^{5/6}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{11/6}} - \ldots$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{11/6}} - \frac{b^{5/6}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{11/6}} + \frac{b^{5/\ldots}}{\ldots}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{11/6}} - \frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{11/6}} + \frac{2b^{5/6}\tanh^{-1}\ldots}{d^{11\ldots}}$$

**Mathematica [C]** time = 0.07, size = 73, normalized size = 0.22

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6} {}_2F_1\left(\frac{11}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[11
/6, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(11/6))
```

**IntegrateAlgebraic [A]** time = 0.33, size = 258, normalized size = 0.77

$$-\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{d^{11/6}}+\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{d^{11/6}}+\frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{d^{11/6}}+\frac{b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{d^{11/6}}-\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] (-6*(a + b*x)^(5/6))/(5*d*(c + d*x)^(5/6)) - (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt
[3] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))])/d^(11
/6) + (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt[3] + (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt
[3]*d^(1/6)*(a + b*x)^(1/6))])/d^(11/6) + (2*b^(5/6)*ArcTanh[(b^(1/6)*(c +
d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6))])/d^(11/6) + (b^(5/6)*ArcTanh[(b^(1/6
)*d^(1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(1/3) + (b^(1/3)*(c + d*x)^(
1/3))/(a + b*x)^(1/3)))])/d^(11/6)
```

**fricas [B]** time = 1.18, size = 755, normalized size = 2.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] -1/10*(20*sqrt(3)*(d^2*x + c*d)*(b^5/d^11)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*
x + a)^(5/6)*(d*x + c)^(1/6)*b*d^9*(b^5/d^11)^(5/6) - 2*sqrt(3)*(b*d^9*x +
a*d^9)*sqrt(((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) + (b*x
+ a)^(2/3)*(d*x + c)^(1/3)*b^2 + (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x +
 a))*(b^5/d^11)^(5/6) + sqrt(3)*(b^6*x + a*b^5))/(b^6*x + a*b^5)) + 20*sqrt
(3)*(d^2*x + c*d)*(b^5/d^11)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(
d*x + c)^(1/6)*b*d^9*(b^5/d^11)^(5/6) - 2*sqrt(3)*(b*d^9*x + a*d^9)*sqrt(-(
(b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) - (b*x + a)^(2/3)*(d
*x + c)^(1/3)*b^2 - (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x + a))*(b^5/d^1
1)^(5/6) - sqrt(3)*(b^6*x + a*b^5))/(b^6*x + a*b^5)) - 5*(d^2*x + c*d)*(b^5
/d^11)^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6)
+ (b*x + a)^(2/3)*(d*x + c)^(1/3)*b^2 + (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))
/(b*x + a)) + 5*(d^2*x + c*d)*(b^5/d^11)^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x
 + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) - (b*x + a)^(2/3)*(d*x + c)^(1/3)*b^2 -
(b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x + a)) - 10*(d^2*x + c*d)*(b^5/d^11
)^(1/6)*log(((b*x + a)^(5/6)*(d*x + c)^(1/6)*b + (b*d^2*x + a*d^2)*(b^5/d^1
1)^(1/6))/(b*x + a)) + 10*(d^2*x + c*d)*(b^5/d^11)^(1/6)*log(((b*x + a)^(5/
6)*(d*x + c)^(1/6)*b - (b*d^2*x + a*d^2)*(b^5/d^11)^(1/6))/(b*x + a)) + 12*
(b*x + a)^(5/6)*(d*x + c)^(1/6))/(d^2*x + c*d)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(11/6), x)
```

**maple [F]** time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(11/6),x)
[Out] int((b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(11/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{11/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(11/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(11/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(11/6),x)
```

```
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(11/6), x)
```

## 3.1522 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx$

**Optimal**. Leaf size=32

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]
[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx = \frac{6(a+bx)^{11/6}}{11(bc-ad)(c+dx)^{11/6}}$$

**Mathematica [A]** time = 0.01, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]
[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

**fricas** [B] time = 0.77, size = 65, normalized size = 2.03

$$\frac{6\,(bx+a)^{\frac{11}{6}}(dx+c)^{\frac{1}{6}}}{11\left(bc^3-ac^2d+\left(bcd^2-ad^3\right)x^2+2\left(bc^2d-acd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] 6/11*(b*x + a)^(11/6)*(d*x + c)^(1/6)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*
x^2 + 2*(b*c^2*d - a*c*d^2)*x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(17/6), x)
```

**maple** [A] time = 0.00, size = 27, normalized size = 0.84

$$-\frac{6\,(bx+a)^{\frac{11}{6}}}{11\,(dx+c)^{\frac{11}{6}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(17/6),x)
```

```
[Out] -6/11*(b*x+a)^(11/6)/(d*x+c)^(11/6)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(17/6), x)
```

**mupad [B]** time = 0.59, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{6\,a\,(a+b\,x)^{5/6}}{11\,a\,d^3-11\,b\,c\,d^2}+\frac{6\,b\,x\,(a+b\,x)^{5/6}}{11\,a\,d^3-11\,b\,c\,d^2}\right)\,(c+d\,x)^{1/6}}{x^2-\frac{11\,b\,c^3-11\,a\,c^2\,d}{11\,a\,d^3-11\,b\,c\,d^2}+\frac{22\,c\,d\,x\,(a\,d-b\,c)}{11\,a\,d^3-11\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(17/6),x)
```

```
[Out] -(((6*a*(a + b*x)^(5/6))/(11*a*d^3 - 11*b*c*d^2) + (6*b*x*(a + b*x)^(5/6))/
(11*a*d^3 - 11*b*c*d^2))*(c + d*x)^(1/6))/(x^2 - (11*b*c^3 - 11*a*c^2*d)/(1
1*a*d^3 - 11*b*c*d^2) + (22*c*d*x*(a*d - b*c))/(11*a*d^3 - 11*b*c*d^2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

## 3.1523 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}} dx$

**Optimal**. Leaf size=66

$$\frac{36b(a+bx)^{11/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{17(c+dx)^{17/6}(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b(a+bx)^{11/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (36*b*(a + b*x)^(1
1/6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx = \frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{(6b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx}{17(bc-ad)}$$
$$= \frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{36b(a+bx)^{11/6}}{187(bc-ad)^2(c+dx)^{11/6}}$$

**Mathematica [A]** time = 0.03, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{11/6}(-11ad+17bc+6bdx)}{187(c+dx)^{17/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(17*b*c - 11*a*d + 6*b*d*x))/(187*(b*c - a*d)^2*(c + d*
x)^(17/6))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 51, normalized size = 0.77

$$\frac{6(a+bx)^{17/6}\left(\frac{17b(c+dx)}{a+bx}-11d\right)}{187(c+dx)^{17/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
```

```
[Out] (6*(a + b*x)^(17/6)*(-11*d + (17*b*(c + d*x))/(a + b*x)))/(187*(b*c - a*d)^
2*(c + d*x)^(17/6))
```

**fricas [B]** time = 1.14, size = 175, normalized size = 2.65

$$\frac{6\left(6\,b^2dx^2+17\,abc-11\,a^2d+\left(17\,b^2c-5\,abd\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{187\left(b^2c^5-2\,abc^4d+a^2c^3d^2+\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)x^3+3\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2+3\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] 6/187*(6*b^2*d*x^2 + 17*a*b*c - 11*a^2*d + (17*b^2*c - 5*a*b*d)*x)*(b*x + a
)^(5/6)*(d*x + c)^(1/6)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3
 - 2*a*b*c*d^4 + a^2*d^5)*x^3 + 3*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)
*x^2 + 3*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(23/6), x)
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.82

$$-\frac{6\,(bx+a)^{\frac{11}{6}}\,(-6bdx+11ad-17bc)}{187\,(dx+c)^{\frac{17}{6}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(23/6),x)
```

```
[Out] -6/187*(b*x+a)^(11/6)*(-6*b*d*x+11*a*d-17*b*c)/(d*x+c)^(17/6)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(23/6), x)
```

**mupad [B]** time = 0.74, size = 137, normalized size = 2.08

$$\frac{(c+d\,x)^{1/6}\left(\frac{36\,b^2\,x^2\,(a+b\,x)^{5/6}}{187\,d^2\,(a\,d-b\,c)^2}-\frac{\left(66\,a^2\,d-102\,a\,b\,c\right)(a+b\,x)^{5/6}}{187\,d^3\,(a\,d-b\,c)^2}+\frac{x\left(102\,b^2\,c-30\,a\,b\,d\right)(a+b\,x)^{5/6}}{187\,d^3\,(a\,d-b\,c)^2}\right)}{x^3+\frac{c^3}{d^3}+\frac{3\,c\,x^2}{d}+\frac{3\,c^2\,x}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(23/6),x)
```

```
[Out] ((c + d*x)^(1/6)*((36*b^2*x^2*(a + b*x)^(5/6))/(187*d^2*(a*d - b*c)^2) - ((
66*a^2*d - 102*a*b*c)*(a + b*x)^(5/6))/(187*d^3*(a*d - b*c)^2) + (x*(102*b^
2*c - 30*a*b*d)*(a + b*x)^(5/6))/(187*d^3*(a*d - b*c)^2)))/(x^3 + c^3/d^3 +
 (3*c*x^2)/d + (3*c^2*x)/d^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

## 3.1524 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx$

**Optimal**. Leaf size=101

$$\frac{432b^2(a+bx)^{11/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{72b(a+bx)^{11/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{23(c+dx)^{23/6}(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{11/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{72b(a+bx)^{11/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (72*b*(a + b*x)^(1
1/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (432*b^2*(a + b*x)^(11/6))/(43
01*(b*c - a*d)^3*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx = \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{(12b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx}{23(bc-ad)}$$
$$= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{\left(72b^2\right)\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx}{391(bc-ad)^2}$$
$$= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{432b^2(a+bx)^{11/6}}{4301(bc-ad)^3(c+dx)^{11/6}}$$

**Mathematica [A]** time = 0.06, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{11/6}\left(187a^2d^2-22abd(23c+6dx)+b^2\left(391c^2+276cdx+72d^2x^2\right)\right)}{4301(c+dx)^{23/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(187*a^2*d^2 - 22*a*b*d*(23*c + 6*d*x) + b^2*(391*c^2 +
 276*c*d*x + 72*d^2*x^2)))/(4301*(b*c - a*d)^3*(c + d*x)^(23/6))
```

**IntegrateAlgebraic [A]** time = 0.18, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{23/6}\left(\frac{391b^2(c+dx)^2}{(a+bx)^2}-\frac{506bd(c+dx)}{a+bx}+187d^2\right)}{4301(c+dx)^{23/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
```

```
[Out] (6*(a + b*x)^(23/6)*(187*d^2 - (506*b*d*(c + d*x))/(a + b*x) + (391*b^2*(c
+ d*x)^2)/(a + b*x)^2))/(4301*(b*c - a*d)^3*(c + d*x)^(23/6))
```

**fricas [B]** time = 1.16, size = 338, normalized size = 3.35

$$\frac{6\left(72\,b^3d^2x^3+391\,ab^2c^2-506\,a^2bcd+187\,a^3d^2+12\left(23\,b^3cd-5\,ab^2d^2\right)x^2+\left(391\,b^3c^2-230\,ab^2cd+55\,a^2bd^2\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{4301\left(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3+\left(b^3c^3d^4-3\,ab^2c^2d^5+3\,a^2bcd^6-a^3d^7\right)x^4+4\left(b^3c^4d^3-3\,ab^2c^3d^4+3\,a^2bc^2d^5-a^3cd^6\right)x^3+6\left(b^3c^5d^2-3\,ab^2c^4d^3+3\,a^2bc^3d^4-a^3c^2d^5\right)x^2+4\left(b^3c^6d-3\,ab^2c^5d^2+3\,a^2bc^4d^3-a^3c^3d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="fricas")
```

```
[Out] 6/4301*(72*b^3*d^2*x^3 + 391*a*b^2*c^2 - 506*a^2*b*c*d + 187*a^3*d^2 + 12*(
23*b^3*c*d - 5*a*b^2*d^2)*x^2 + (391*b^3*c^2 - 230*a*b^2*c*d + 55*a^2*b*d^2
```

```
)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5
*d^2 - a^3*c^4*d^3 + (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d
^7)*x^4 + 4*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*x
^3 + 6*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^2
+ 4*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(29/6), x)
```

**maple** [A] time = 0.01, size = 105, normalized size = 1.04

$$-\frac{6\,(bx+a)^{\frac{11}{6}}\left(72b^2x^2d^2-132ab\,d^2x+276b^2cdx+187a^2d^2-506abcd+391b^2c^2\right)}{4301\,(dx+c)^{\frac{23}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(29/6),x)
```

```
[Out] -6/4301*(b*x+a)^(11/6)*(72*b^2*d^2*x^2-132*a*b*d^2*x+276*b^2*c*d*x+187*a^2*
d^2-506*a*b*c*d+391*b^2*c^2)/(d*x+c)^(23/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(29/6), x)
```

**mupad** [B] time = 0.94, size = 214, normalized size = 2.12

$$-\frac{(c+d\,x)^{1/6}\left(\frac{(a+b\,x)^{5/6}\left(1122\,a^3\,d^2-3036\,a^2\,b\,c\,d+2346\,a\,b^2\,c^2\right)}{4301\,d^4\,(a\,d-b\,c)^3}+\frac{432\,b^3\,x^3\,(a+b\,x)^{5/6}}{4301\,d^2\,(a\,d-b\,c)^3}+\frac{x\,(a+b\,x)^{5/6}\left(330\,a^2\,b\,d^2-1380\,a\,b^2\,c\,d+2346\,b^3\,c^2\right)}{4301\,d^4\,(a\,d-b\,c)^3}-\frac{72\,b^2\,x^2\,(5\,a\,d-23\,b\,c)\,(a+b\,x)^{5/6}}{4301\,d^3\,(a\,d-b\,c)^3}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(29/6),x)
```

```
[Out] -((c + d*x)^(1/6)*(((a + b*x)^(5/6)*(1122*a^3*d^2 + 2346*a*b^2*c^2 - 3036*a
^2*b*c*d))/(4301*d^4*(a*d - b*c)^3) + (432*b^3*x^3*(a + b*x)^(5/6))/(4301*d
^2*(a*d - b*c)^3) + (x*(a + b*x)^(5/6)*(2346*b^3*c^2 + 330*a^2*b*d^2 - 1380
*a*b^2*c*d))/(4301*d^4*(a*d - b*c)^3) - (72*b^2*x^2*(5*a*d - 23*b*c)*(a + b
*x)^(5/6))/(4301*d^3*(a*d - b*c)^3)))/(x^4 + c^4/d^4 + (4*c*x^3)/d + (4*c^3
*x)/d^3 + (6*c^2*x^2)/d^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(29/6),x)
```

```
[Out] Timed out
```

## 3.1525 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{35/6}} dx$

**Optimal**. Leaf size=136

$$\frac{7776b^3(a+bx)^{11/6}}{124729(c+dx)^{11/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{11/6}}{11339(c+dx)^{17/6}(bc-ad)^3}+\frac{108b(a+bx)^{11/6}}{667(c+dx)^{23/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{29(c+dx)^{29/6}(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{11/6}}{124729(c+dx)^{11/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{11/6}}{11339(c+dx)^{17/6}(bc-ad)^3}+\frac{108b(a+bx)^{11/6}}{667(c+dx)^{23/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{29(c+dx)^{29/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(29*(b*c - a*d)*(c + d*x)^(29/6)) + (108*b*(a + b*x)^(
11/6))/(667*(b*c - a*d)^2*(c + d*x)^(23/6)) + (1296*b^2*(a + b*x)^(11/6))/(
11339*(b*c - a*d)^3*(c + d*x)^(17/6)) + (7776*b^3*(a + b*x)^(11/6))/(124729
*(b*c - a*d)^4*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{35/6}}\,dx = \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{(18b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx}{29(bc-ad)}$$

$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{\left(216b^2\right)\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx}{667(bc-ad)^2}$$

$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}} + \frac{(1296\ldots}{113\ldots}$$

$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}} + \frac{7\ldots}{12472\ldots}$$

**Mathematica [A]** time = 0.09, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{11/6}\left(-4301a^3d^3+561a^2bd^2(29c+6dx)-33ab^2d\left(667c^2+348cdx+72d^2x^2\right)+b^3\left(11339c^3+12006c^2dx+6264cd^2x^2+1296d^3x^3\right)\right)}{124729(c+dx)^{29/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(-4301*a^3*d^3 + 561*a^2*b*d^2*(29*c + 6*d*x) - 33*a*b^
2*d*(667*c^2 + 348*c*d*x + 72*d^2*x^2) + b^3*(11339*c^3 + 12006*c^2*d*x + 6
264*c*d^2*x^2 + 1296*d^3*x^3)))/(124729*(b*c - a*d)^4*(c + d*x)^(29/6))
```

**IntegrateAlgebraic [A]** time = 0.20, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{29/6}\left(\frac{11339b^3(c+dx)^3}{(a+bx)^3}-\frac{22011b^2d(c+dx)^2}{(a+bx)^2}+\frac{16269bd^2(c+dx)}{a+bx}-4301d^3\right)}{124729(c+dx)^{29/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
```

```
[Out] (6*(a + b*x)^(29/6)*(-4301*d^3 + (16269*b*d^2*(c + d*x))/(a + b*x) - (22011
*b^2*d*(c + d*x)^2)/(a + b*x)^2 + (11339*b^3*(c + d*x)^3)/(a + b*x)^3))/(12
4729*(b*c - a*d)^4*(c + d*x)^(29/6))
```

**fricas [B]** time = 1.18, size = 533, normalized size = 3.92

$$\frac{6\left(1296\,b^4d^3x^4+11339\,ab^3c^3-22011\,a^2b^2c^2d+16269\,a^3bcd^2-4301\,a^4d^3+216\left(29\,b^4cd^2-5\,ab^3d^3\right)x^3+18\left(667\,b^4c^2d-290\,ab^3cd^2+55\,a^2b^2d^3\right)x^2+\left(11339\,b^4c^3-10005\,ab^3c^2d+4785\,a^2b^2cd^2-935\,a^3bd^3\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{124729\left(b^4c^9-4\,ab^3c^8d+6\,a^2b^2c^7d^2-4\,a^3bc^6d^3+a^4c^5d^4+\left(b^4c^4d^5-4\,ab^3c^3d^6+6\,a^2b^2c^2d^7-4\,a^3bcd^8+a^4d^9\right)x^5+5\left(b^4c^5d^4-4\,ab^3c^4d^5+6\,a^2b^2c^3d^6-4\,a^3bc^2d^7+a^4cd^8\right)x^4+10\left(b^4c^6d^3-4\,ab^3c^5d^4+6\,a^2b^2c^4d^5-4\,a^3bc^3d^6+a^4c^2d^7\right)x^3+10\left(b^4c^7d^2-4\,ab^3c^6d^3+6\,a^2b^2c^5d^4-4\,a^3bc^4d^5+a^4c^3d^6\right)x^2+5\left(b^4c^8d-4\,ab^3c^7d^2+6\,a^2b^2c^6d^3-4\,a^3bc^5d^4+a^4c^4d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="fricas")
```

```
[Out] 6/124729*(1296*b^4*d^3*x^4 + 11339*a*b^3*c^3 - 22011*a^2*b^2*c^2*d + 16269*
a^3*b*c*d^2 - 4301*a^4*d^3 + 216*(29*b^4*c*d^2 - 5*a*b^3*d^3)*x^3 + 18*(667
*b^4*c^2*d - 290*a*b^3*c*d^2 + 55*a^2*b^2*d^3)*x^2 + (11339*b^4*c^3 - 10005
*a*b^3*c^2*d + 4785*a^2*b^2*c*d^2 - 935*a^3*b*d^3)*x)*(b*x + a)^(5/6)*(d*x
+ c)^(1/6)/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
 a^4*c^5*d^4 + (b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b
*c*d^8 + a^4*d^9)*x^5 + 5*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^
6 - 4*a^3*b*c^2*d^7 + a^4*c*d^8)*x^4 + 10*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 +
6*a^2*b^2*c^4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*x^3 + 10*(b^4*c^7*d^2 -
4*a*b^3*c^6*d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*x^2 +
5*(b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*
c^4*d^5)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{35}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(35/6), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$-\frac{6\,(bx+a)^{\frac{11}{6}}\left(-1296 b^{3} d^{3} x^{3}+2376 a\,b^{2} d^{3} x^{2}-6264 b^{3} c\,d^{2} x^{2}-3366 a^{2} b\,d^{3} x+11484 a\,b^{2} c\,d^{2} x-12006 b^{3} c^{2} d x+4301 a^{3} d^{3}-16269 a^{2} b c\,d^{2}+22011 a\,b^{2} c^{2} d-11339 b^{3} c^{3}\right)}{124729\,(dx+c)^{\frac{29}{6}}\left(a^{4} d^{4}-4 a^{3} b c\,d^{3}+6 a^{2} b^{2} c^{2} d^{2}-4 a\,b^{3} c^{3} d+b^{4} c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(35/6),x)
```

```
[Out] -6/124729*(b*x+a)^(11/6)*(-1296*b^3*d^3*x^3+2376*a*b^2*d^3*x^2-6264*b^3*c*d
^2*x^2-3366*a^2*b*d^3*x+11484*a*b^2*c*d^2*x-12006*b^3*c^2*d*x+4301*a^3*d^3-
16269*a^2*b*c*d^2+22011*a*b^2*c^2*d-11339*b^3*c^3)/(d*x+c)^(29/6)/(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{35}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(35/6), x)
```

**mupad [B]** time = 1.16, size = 303, normalized size = 2.23

$$\frac{(c+d\,x)^{1/6}\left(\frac{7776\,b^4\,x^4\,(a+b\,x)^{5/6}}{124729\,d^2\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{5/6}\,\left(25806\,a^4\,d^3-97614\,a^3\,b\,c\,d^2+132066\,a^2\,b^2\,c^2\,d-68034\,a\,b^3\,c^3\right)}{124729\,d^5\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{5/6}\,\left(-5610\,a^3\,b\,d^3+28710\,a^2\,b^2\,c\,d^2-60030\,a\,b^3\,c^2\,d+68034\,b^4\,c^3\right)}{124729\,d^5\,(a\,d-b\,c)^4}+\frac{108\,b^2\,x^2\,(a+b\,x)^{5/6}\,\left(55\,a^2\,d^2-290\,a\,b\,c\,d+667\,b^2\,c^2\right)}{124729\,d^4\,(a\,d-b\,c)^4}-\frac{1296\,b^3\,x^3\,(5\,a\,d-29\,b\,c)\,(a+b\,x)^{5/6}}{124729\,d^3\,(a\,d-b\,c)^4}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(35/6),x)
```

```
[Out] ((c + d*x)^(1/6)*((7776*b^4*x^4*(a + b*x)^(5/6))/(124729*d^2*(a*d - b*c)^4)
 - ((a + b*x)^(5/6)*(25806*a^4*d^3 - 68034*a*b^3*c^3 + 132066*a^2*b^2*c^2*d
 - 97614*a^3*b*c*d^2))/(124729*d^5*(a*d - b*c)^4) + (x*(a + b*x)^(5/6)*(680
34*b^4*c^3 - 5610*a^3*b*d^3 + 28710*a^2*b^2*c*d^2 - 60030*a*b^3*c^2*d))/(12
4729*d^5*(a*d - b*c)^4) + (108*b^2*x^2*(a + b*x)^(5/6)*(55*a^2*d^2 + 667*b^
2*c^2 - 290*a*b*c*d))/(124729*d^4*(a*d - b*c)^4) - (1296*b^3*x^3*(5*a*d - 2
9*b*c)*(a + b*x)^(5/6))/(124729*d^3*(a*d - b*c)^4)))/(x^5 + c^5/d^5 + (5*c*
x^4)/d + (5*c^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(35/6),x)
```

```
[Out] Timed out
```

## 3.1526 $\int \frac{(a+bx)^{7/6}}{\sqrt[6]{c+dx}}\,dx$

**Optimal**. Leaf size=424

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}-\frac{7(bc-ad)}{}$$

**Rubi [A]** time = 0.55, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}}-\frac{7\sqrt[6]{a+bx}(c+dx)^{5/6}(bc-ad)}{12d^2}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*d^2) + ((a + b*x)^(7/6
)*(c + d*x)^(5/6))/(2*d) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(
a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(
13/6)) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(S
qrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(13/6)) + (7*(b*c -
 a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b
^(5/6)*d^(13/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(1
44*b^(5/6)*d^(13/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{\sqrt[6]{c+dx}}\,dx = \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} - \frac{(7(bc-ad))\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{12d}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{72d^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\right.}{12bd^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x,\right.}{12bd^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+}\right.}{36b^{5/6}d^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}} - $$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}} - $$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} - \frac{7(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}} + $$

**Mathematica [C]** time = 0.04, size = 73, normalized size = 0.17

$$\frac{6(a+bx)^{13/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6
, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(1/6))
```

**IntegrateAlgebraic [A]** time = 0.78, size = 363, normalized size = 0.86

$$-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{72b^{5/6}d^{13/6}}+\frac{(ad-bc)^2\left(\frac{13d(a+bx)^{7/6}}{(c+dx)^{7/6}}-\frac{7b\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{12d^2\left(\frac{d(a+bx)}{c+dx}-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```

```
[Out] ((-(b*c) + a*d)^2*((13*d*(a + b*x)^(7/6))/(c + d*x)^(7/6) - (7*b*(a + b*x)^
(1/6))/(c + d*x)^(1/6)))/(12*d^2*(-b + (d*(a + b*x))/(c + d*x))^2) - (7*(b*
c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*
(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)^2*ArcTan[
1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])
/(24*Sqrt[3]*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x
)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)
^2*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/((c + d*x)^(1/6)*(b^(1/3) + (d
^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)))])/(72*b^(5/6)*d^(13/6))
```

**fricas [B]** time = 1.56, size = 5633, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -1/144*(28*sqrt(3)*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^6*c^2*d^11 - 2*a*b^5*c*d^12 + a^2*b^4*d^13)*(
b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d
^13))^(5/6) - 2*sqrt(3)*(b^4*d^12*x + b^4*c*d^11)*sqrt(((b^3*c^2*d^2 - 2*a*
b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
```

a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^5*d^13))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d
^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/
(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(5/6) + sqrt(3
)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d
^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*
a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c
^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*
d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 79
2*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4
*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 +
 a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*
a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*
c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 +
 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 1
2*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^
8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8
 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a
^11*b*c*d^12 + a^12*d^13)*x)) + 28*sqrt(3)*d^2*((b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(b^5*d^13))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^6*c^2*d^11 - 2*a*b^5*
c*d^12 + a^2*b^4*d^13)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^5*d^13))^(5/6) - 2*sqrt(3)*(b^4*d^12*x + b^4*c*d^11)*s
qrt(-((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(
5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) - (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)
*(d*x + c)^(2/3) - (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*

d^12)/(b^5*d^13))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) - 7*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*log(49*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/(d*x + c)) + 7*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*log(-49*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*

```
b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d
^12)/(b^5*d^13))^(1/3))/(d*x + c)) - 14*d^2*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^5*d^13))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(
1/6)*(d*x + c)^(5/6) + (b*d^3*x + b*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^5*d^13))^(1/6))/(d*x + c)) + 14*d^2*((b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(b^5*d^13))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^
(1/6)*(d*x + c)^(5/6) - (b*d^3*x + b*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^5*d^13))^(1/6))/(d*x + c)) - 12*(6*b*d*x - 7*b*c + 13*a*d)*(b*x +
 a)^(1/6)*(d*x + c)^(5/6))/d^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(1/6), x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(1/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(7/6)/(c + d*x)^(1/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(1/6), x)
```

## 3.1527 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{7/6}} \, dx$

**Optimal**. Leaf size=403

$$\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}+\frac{7\sqrt[6]{b}\,(b\cdots}{}$$

**Rubi [A]** time = 0.53, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {47, 50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}+\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}+\frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3d^{13/6}}-\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(7/6))/(d*(c + d*x)^(1/6)) + (7*b*(a + b*x)^(1/6)*(c + d*x)^(
5/6))/d^2 + (7*b^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^
(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6
)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/
6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6)*(b*c - a*d)*ArcTanh
[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*d^(13/6)) + (7*b^
(1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
 (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*d^(13/6)) - (7*b^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) +
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*d^(13/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{7/6}}\,dx = -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{(7b)\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{(7b(bc-ad))\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{(7(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{(7(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{\left(7\sqrt[6]{b}\,(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \ldots\right.}{3d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3d^{13/6}} + \frac{\left(7\sqrt[6]{b}\,(bc-ad\ldots\right.}{\ldots}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3d^{13/6}} + \frac{7\sqrt[6]{b}\,(bc-ad)\ldots}{\ldots}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} + \frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,d^{13/6}} - \frac{7\sqrt[6]{b}\,(bc-ad)\ldots}{2\ldots}$$

**Mathematica** [C] time = 0.07, size = 73, normalized size = 0.18

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6
, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(7/6))
```

**IntegrateAlgebraic [A]** time = 23.49, size = 417, normalized size = 1.03

$$\frac{d^{7/6}(a+bx)^{7/6}\left(-\frac{7\left(b^{7/6}c-a\sqrt[6]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{b}\sqrt[6]{c+dx}-2\sqrt[6]{ad+b(c+dx)-bc}}\right)}{2\sqrt{3}\,d^{13/6}}+\frac{7\left(b^{7/6}c-a\sqrt[6]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}{2\sqrt[6]{ad+b(c+dx)-bc}+\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,d^{13/6}}-\frac{7\left(b^{7/6}c-a\sqrt[6]{b}\,d\right)\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{ad+b(c+dx)-bc}}\right)}{3d^{13/6}}-\frac{7\left(b^{7/6}c-a\sqrt[6]{b}\,d\right)\tanh^{-1}\left(\frac{\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[6]{b}\sqrt[6]{c+dx}\sqrt[6]{ad+b(c+dx)-bc}}\right)}{6d^{13/6}}+\frac{\sqrt[6]{ad+b(c+dx)-bc}\,(-6ad+b(c+dx)+6bc)}{d^{13/6}\sqrt[6]{c+dx}}\right)}{(ad+bdx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (d^(7/6)*(a + b*x)^(7/6)*(((6*b*c - 6*a*d + b*(c + d*x))*(-(b*c) + a*d + b*
(c + d*x))^(1/6))/(d^(13/6)*(c + d*x)^(1/6)) - (7*(b^(7/6)*c - a*b^(1/6)*d)
*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6) - 2*(-(b
*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*d^(13/6)) + (7*(b^(7/6)*c - a*
b^(1/6)*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6
) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*d^(13/6)) - (7*(b^(7
/6)*c - a*b^(1/6)*d)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(-(b*c) + a*d + b*(c
 + d*x))^(1/6)])/(3*d^(13/6)) - (7*(b^(7/6)*c - a*b^(1/6)*d)*ArcTanh[(b^(1/
3)*(c + d*x)^(1/3) + (-(b*c) + a*d + b*(c + d*x))^(1/3))/(b^(1/6)*(c + d*x)
^(1/6)*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(6*d^(13/6))))/(a*d + b*d*x)^(
7/6)
```

**fricas [B]** time = 1.43, size = 3084, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/12*(28*sqrt(3)*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^
4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d
^6)/d^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b*c*d^11 - a*d^12)*(b*x + a)^(1/6)*(
d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*
c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(5/6) + 2
*sqrt(3)*(d^12*x + c*d^11)*sqrt(((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c
)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3
 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (d^5*x + c*d^4)*(
(b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4
*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/3))/(d*x + c))*((b^7*c
^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c
^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(5/6) + sqrt(3)*(b^7*c^7 - 6*a*
b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 -
6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5
```

```
*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*
b*d^7)*x))/(b^7*c^7 - 6*a*b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d
^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*
a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^
5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 28*sqrt(3)*(d^3*x + c*d^2)*((b^7*c^6
 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2
*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b*c*
d^11 - a*d^12)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d +
15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*
d^5 + a^6*b*d^6)/d^13)^(5/6) + 2*sqrt(3)*(d^12*x + c*d^11)*sqrt(-((b*c*d^2
- a*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2
*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 +
a^6*b*d^6)/d^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d
*x + c)^(2/3) - (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*
d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6
)/d^13)^(1/3))/(d*x + c))*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 -
20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13
)^(5/6) - sqrt(3)*(b^7*c^7 - 6*a*b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^
4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6
*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3
*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x))/(b^7*c^7 - 6*a*b^6*c^6*d + 15*a
^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^
5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^
3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 7*(
d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*
c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*log
(49*((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*
c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^
5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x
 + a)^(1/3)*(d*x + c)^(2/3) + (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 1
5*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d
^5 + a^6*b*d^6)/d^13)^(1/3))/(d*x + c)) - 7*(d^3*x + c*d^2)*((b^7*c^6 - 6*a
*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 -
 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*log(-49*((b*c*d^2 - a*d^3)*(b*x +
 a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 -
20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13
)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) -
 (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^
4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/3))/
(d*x + c)) + 14*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*
d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6
)/d^13)^(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) + (d^3*x
+ c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^
3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6))/(d*x + c
)) - 14*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20
```

```
*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^
(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) - (d^3*x + c*d^2)
*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a
^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6))/(d*x + c)) - 12*
(b*d*x + 7*b*c - 6*a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6))/(d^3*x + c*d^2)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(7/6), x)
```

**maple** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(7/6), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{7/6}}{{\left(c+d\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(7/6),x)
```

```
[Out] int((a + b*x)^(7/6)/(c + d*x)^(7/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(7/6),x)
```

```
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(7/6), x)
```

## 3.1528 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{13/6}}\,dx$

**Optimal**. Leaf size=358

$$-\frac{b^{7/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}+\frac{b^{7/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}-\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\ldots}{\sqrt{3}\sqrt[6]{\ldots}}\right)}{d^{13/6}}$$

**Rubi [A]** time = 0.50, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{b^{7/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}+\frac{b^{7/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}-\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}}+\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{13/6}}+\frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}-\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```

```
[Out] (-6*(a + b*x)^(7/6))/(7*d*(c + d*x)^(7/6)) - (6*b*(a + b*x)^(1/6))/(d^2*(c
+ d*x)^(1/6)) - (Sqrt[3]*b^(7/6)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1
/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(13/6) + (Sqrt[3]*b^(7/6)*ArcTan
[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]
)/d^(13/6) + (2*b^(7/6)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x
)^(1/6))])/d^(13/6) - (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c +
 d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/
6)) + (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b
^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{13/6}}\,dx = -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}+\frac{b\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{b^2\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{(6b)\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx,x,\sqrt[6]{a+bx}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{(6b)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{\left(2b^{7/6}\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^2}+\frac{\left(2b^{7/6}\right)\text{Su}}{}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}}-\frac{b^{7/6}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx\right.}{2d^{13/6}}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}}-\frac{b^{7/6}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}}{\sqrt[6]{c}}\right.}{2d^{13/6}}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}-\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{13/6}}+\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{13/6}}+$$

**Mathematica** [C] time = 0.09, size = 73, normalized size = 0.20

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}\,{}_2F_1\left(\frac{13}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[13
/6, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(13/6))
```

**IntegrateAlgebraic [A]** time = 0.33, size = 282, normalized size = 0.79

$$-\frac{\sqrt{3}\, b^{7/6} \tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2 \sqrt[6]{d} \sqrt[6]{a+bx}}{\sqrt{3} \sqrt[6]{b} \sqrt[6]{c+dx}}\right)}{d^{13/6}}+\frac{\sqrt{3}\, b^{7/6} \tan^{-1}\left(\frac{2 \sqrt[6]{d} \sqrt[6]{a+bx}}{\sqrt{3} \sqrt[6]{b} \sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{13/6}}+\frac{2 b^{7/6} \tanh^{-1}\left(\frac{\sqrt[6]{d} \sqrt[6]{a+bx}}{\sqrt[6]{b} \sqrt[6]{c+dx}}\right)}{d^{13/6}}+\frac{b^{7/6} \tanh^{-1}\left(\frac{\sqrt[6]{b} \sqrt[6]{d} \sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d} \sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{d^{13/6}}-\frac{6\left(\frac{d(a+bx)^{7/6}}{(c+dx)^{7/6}}+\frac{7 b \sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{7 d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```

```
[Out] (-6*((d*(a + b*x)^(7/6))/(c + d*x)^(7/6) + (7*b*(a + b*x)^(1/6))/(c + d*x)^
(1/6)))/(7*d^2) - (Sqrt[3]*b^(7/6)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^
(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]])/d^(13/6) + (Sqrt[3]*b^(7/6)*ArcT
an[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6)
)]])/d^(13/6) + (2*b^(7/6)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d
*x)^(1/6))]])/d^(13/6) + (b^(7/6)*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/
((c + d*x)^(1/6)*(b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)))]])/d
^(13/6)
```

**fricas [B]** time = 1.46, size = 855, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] -1/14*(28*sqrt(3)*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*arctan(-
1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^11*(b^7/d^13)^(5/6) - 2*
sqrt(3)*(d^12*x + c*d^11)*sqrt(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^2*(b^7/
d^13)^(1/6) + (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 + (d^5*x + c*d^4)*(b^7/d^
13)^(1/3))/(d*x + c))*(b^7/d^13)^(5/6) + sqrt(3)*(b^7*d*x + b^7*c))/(b^7*d*
x + b^7*c)) + 28*sqrt(3)*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*a
rctan(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^11*(b^7/d^13)^(5/
6) - 2*sqrt(3)*(d^12*x + c*d^11)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d
^2*(b^7/d^13)^(1/6) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 - (d^5*x + c*d^4)
*(b^7/d^13)^(1/3))/(d*x + c))*(b^7/d^13)^(5/6) - sqrt(3)*(b^7*d*x + b^7*c))
/(b^7*d*x + b^7*c)) - 7*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*lo
g(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^2*(b^7/d^13)^(1/6) + (b*x + a)^(1/
3)*(d*x + c)^(2/3)*b^2 + (d^5*x + c*d^4)*(b^7/d^13)^(1/3))/(d*x + c)) + 7*(
d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*log(-4*((b*x + a)^(1/6)*(d*
```

```
x + c)^(5/6)*b*d^2*(b^7/d^13)^(1/6) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 -
 (d^5*x + c*d^4)*(b^7/d^13)^(1/3))/(d*x + c)) - 14*(d^4*x^2 + 2*c*d^3*x + c
^2*d^2)*(b^7/d^13)^(1/6)*log(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b + (d^3*x +
c*d^2)*(b^7/d^13)^(1/6))/(d*x + c)) + 14*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b
^7/d^13)^(1/6)*log(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b - (d^3*x + c*d^2)*(b^
7/d^13)^(1/6))/(d*x + c)) + 12*(8*b*d*x + 7*b*c + a*d)*(b*x + a)^(1/6)*(d*x
 + c)^(5/6))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(13/6), x)
```

**maple [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(13/6),x)
[Out] int((b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(13/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(13/6),x)
```

```
[Out] int((a + b*x)^(7/6)/(c + d*x)^(13/6), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

## 3.1529 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx$

**Optimal**. Leaf size=32

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

**Rubi** [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx = \frac{6(a+bx)^{13/6}}{13(bc-ad)(c+dx)^{13/6}}$$

**Mathematica** [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

**fricas [B]** time = 1.33, size = 104, normalized size = 3.25

$$\frac{6\left(b^2x^2+2\,abx+a^2\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{13\left(bc^4-ac^3d+\left(bcd^3-ad^4\right)x^3+3\left(bc^2d^2-acd^3\right)x^2+3\left(bc^3d-ac^2d^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="fricas")
[Out] 6/13*(b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b*c^4 - a*c
^3*d + (b*c*d^3 - a*d^4)*x^3 + 3*(b*c^2*d^2 - a*c*d^3)*x^2 + 3*(b*c^3*d - a
*c^2*d^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(19/6), x)
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.84

$$-\frac{6\,(bx+a)^{\frac{13}{6}}}{13\,(dx+c)^{\frac{13}{6}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(19/6),x)
[Out] -6/13*(b*x+a)^(13/6)/(d*x+c)^(13/6)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(19/6), x)
```

**mupad [B]** time = 0.76, size = 199, normalized size = 6.22

$$-\frac{(c+d\,x)^{5/6}\,\left(\frac{6\,a^2\,{(a+b\,x)}^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{6\,b^2\,x^2\,{(a+b\,x)}^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{12\,a\,b\,x\,{(a+b\,x)}^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}\right)}{x^3-\frac{13\,b\,c^4-13\,a\,c^3\,d}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{39\,c\,d^2\,x^2\,(a\,d-b\,c)}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{39\,c^2\,d\,x\,(a\,d-b\,c)}{13\,a\,d^4-13\,b\,c\,d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(19/6),x)
```

```
[Out] -((c + d*x)^(5/6)*((6*a^2*(a + b*x)^(1/6))/(13*a*d^4 - 13*b*c*d^3) + (6*b^2
*x^2*(a + b*x)^(1/6))/(13*a*d^4 - 13*b*c*d^3) + (12*a*b*x*(a + b*x)^(1/6))/
(13*a*d^4 - 13*b*c*d^3)))/(x^3 - (13*b*c^4 - 13*a*c^3*d)/(13*a*d^4 - 13*b*c
*d^3) + (39*c*d^2*x^2*(a*d - b*c))/(13*a*d^4 - 13*b*c*d^3) + (39*c^2*d*x*(a
*d - b*c))/(13*a*d^4 - 13*b*c*d^3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

## 3.1530 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx$

**Optimal**. Leaf size=66

$$\frac{36b(a+bx)^{13/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{19(c+dx)^{19/6}(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b(a+bx)^{13/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (36*b*(a + b*x)^(1
3/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx = \frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{(6b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx}{19(bc-ad)}$$
$$= \frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{36b(a+bx)^{13/6}}{247(bc-ad)^2(c+dx)^{13/6}}$$

**Mathematica** [A] time = 0.04, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{13/6}(-13ad+19bc+6bdx)}{247(c+dx)^{19/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(19*b*c - 13*a*d + 6*b*d*x))/(247*(b*c - a*d)^2*(c + d*
x)^(19/6))
```

**IntegrateAlgebraic** [A] time = 0.18, size = 51, normalized size = 0.77

$$\frac{6(a+bx)^{13/6}\left(19b-\frac{13d(a+bx)}{c+dx}\right)}{247(c+dx)^{13/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(19*b - (13*d*(a + b*x))/(c + d*x)))/(247*(b*c - a*d)^2
*(c + d*x)^(13/6))
```

**fricas** [B] time = 1.34, size = 235, normalized size = 3.56

$$\frac{6\left(6\,b^3dx^3+19\,a^2bc-13\,a^3d+\left(19\,b^3c-ab^2d\right)x^2+2\left(19\,ab^2c-10\,a^2bd\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{247\left(b^2c^6-2\,abc^5d+a^2c^4d^2+\left(b^2c^2d^4-2\,abcd^5+a^2d^6\right)x^4+4\left(b^2c^3d^3-2\,abc^2d^4+a^2cd^5\right)x^3+6\left(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4\right)x^2+4\left(b^2c^5d-2\,abc^4d^2+a^2c^3d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/247*(6*b^3*d*x^3 + 19*a^2*b*c - 13*a^3*d + (19*b^3*c - a*b^2*d)*x^2 + 2*(
19*a*b^2*c - 10*a^2*b*d)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^2*c^6 - 2*a*
b*c^5*d + a^2*c^4*d^2 + (b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*x^4 + 4*(b^2*
```

```
c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*x^3 + 6*(b^2*c^4*d^2 - 2*a*b*c^3*d^3 +
 a^2*c^2*d^4)*x^2 + 4*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(25/6), x)
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.82

$$-\frac{6\,(bx+a)^{\frac{13}{6}}\,(-6bdx+13ad-19bc)}{247\,(dx+c)^{\frac{19}{6}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(25/6),x)
```

```
[Out] -6/247*(b*x+a)^(13/6)*(-6*b*d*x+13*a*d-19*b*c)/(d*x+c)^(19/6)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(25/6), x)
```

**mupad [B]** time = 0.91, size = 189, normalized size = 2.86

$$-\frac{(c+d\,x)^{5/6}\left(\frac{\left(78\,a^3\,d-114\,a^2\,b\,c\right)(a+b\,x)^{1/6}}{247\,d^4\,(a\,d-b\,c)^2}-\frac{36\,b^3\,x^3\,(a+b\,x)^{1/6}}{247\,d^3\,(a\,d-b\,c)^2}-\frac{x^2\left(114\,b^3\,c-6\,a\,b^2\,d\right)(a+b\,x)^{1/6}}{247\,d^4\,(a\,d-b\,c)^2}+\frac{12\,a\,b\,x\,(10\,a\,d-19\,b\,c)\,(a+b\,x)^{1/6}}{247\,d^4\,(a\,d-b\,c)^2}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(25/6),x)
```

```
[Out] -((c + d*x)^(5/6)*(((78*a^3*d - 114*a^2*b*c)*(a + b*x)^(1/6))/(247*d^4*(a*d
 - b*c)^2) - (36*b^3*x^3*(a + b*x)^(1/6))/(247*d^3*(a*d - b*c)^2) - (x^2*(1
14*b^3*c - 6*a*b^2*d)*(a + b*x)^(1/6))/(247*d^4*(a*d - b*c)^2) + (12*a*b*x*
(10*a*d - 19*b*c)*(a + b*x)^(1/6))/(247*d^4*(a*d - b*c)^2)))/(x^4 + c^4/d^4
 + (4*c*x^3)/d + (4*c^3*x)/d^3 + (6*c^2*x^2)/d^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

## 3.1531 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx$

**Optimal**. Leaf size=101

$$\frac{432b^2(a+bx)^{13/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{72b(a+bx)^{13/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{25(c+dx)^{25/6}(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{13/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{72b(a+bx)^{13/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (72*b*(a + b*x)^(1
3/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (432*b^2*(a + b*x)^(13/6))/(61
75*(b*c - a*d)^3*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx = \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{(12b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx}{25(bc-ad)}$$
$$= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{\left(72b^2\right)\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx}{475(bc-ad)^2}$$
$$= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{432b^2(a+bx)^{13/6}}{6175(bc-ad)^3(c+dx)^{13/6}}$$

**Mathematica [A]** time = 0.06, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{13/6}\left(247a^2d^2-26abd(25c+6dx)+b^2\left(475c^2+300cdx+72d^2x^2\right)\right)}{6175(c+dx)^{25/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(247*a^2*d^2 - 26*a*b*d*(25*c + 6*d*x) + b^2*(475*c^2 +
 300*c*d*x + 72*d^2*x^2)))/(6175*(b*c - a*d)^3*(c + d*x)^(25/6))
```

**IntegrateAlgebraic [A]** time = 0.20, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{13/6}\left(\frac{247d^2(a+bx)^2}{(c+dx)^2}-\frac{650bd(a+bx)}{c+dx}+475b^2\right)}{6175(c+dx)^{13/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(475*b^2 + (247*d^2*(a + b*x)^2)/(c + d*x)^2 - (650*b*d
*(a + b*x))/(c + d*x)))/(6175*(b*c - a*d)^3*(c + d*x)^(13/6))
```

**fricas [B]** time = 1.37, size = 427, normalized size = 4.23

$$\frac{6\left(72\,b^4d^2x^4+475\,a^2b^2c^2-650\,a^3bcd+247\,a^4d^2+12\left(25\,b^4cd-ab^3d^2\right)x^3+\left(475\,b^4c^2-50\,ab^3cd+7\,a^2b^2d^2\right)x^2+2\left(475\,ab^3c^2-500\,a^2b^2cd+169\,a^3bd^2\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{6175\left(b^3c^8-3\,ab^2c^7d+3\,a^2bc^6d^2-a^3c^5d^3+\left(b^3c^3d^5-3\,ab^2c^2d^6+3\,a^2bcd^7-a^3d^8\right)x^5+5\left(b^3c^4d^4-3\,ab^2c^3d^5+3\,a^2bc^2d^6-a^3cd^7\right)x^4+10\left(b^3c^5d^3-3\,ab^2c^4d^4+3\,a^2bc^3d^5-a^3c^2d^6\right)x^3+10\left(b^3c^6d^2-3\,ab^2c^5d^3+3\,a^2bc^4d^4-a^3c^3d^5\right)x^2+5\left(b^3c^7d-3\,ab^2c^6d^2+3\,a^2bc^5d^3-a^3c^4d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="fricas")
```

```
[Out] 6/6175*(72*b^4*d^2*x^4 + 475*a^2*b^2*c^2 - 650*a^3*b*c*d + 247*a^4*d^2 + 12
*(25*b^4*c*d - a*b^3*d^2)*x^3 + (475*b^4*c^2 - 50*a*b^3*c*d + 7*a^2*b^2*d^2
```

```
)*x^2 + 2*(475*a*b^3*c^2 - 500*a^2*b^2*c*d + 169*a^3*b*d^2)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c^5*d^
3 + (b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*x^5 + 5*(b^3*
c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*x^4 + 10*(b^3*c^5*
d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^3 + 10*(b^3*c^6*d^
2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^2 + 5*(b^3*c^7*d - 3
*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{31}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(31/6), x)
```

**maple** [A] time = 0.01, size = 105, normalized size = 1.04

$$-\frac{6\,(bx+a)^{\frac{13}{6}}\left(72b^{2}x^{2}d^{2}-156ab\,d^{2}x+300b^{2}cdx+247a^{2}d^{2}-650abcd+475b^{2}c^{2}\right)}{6175\,(dx+c)^{\frac{25}{6}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(31/6),x)
```

```
[Out] -6/6175*(b*x+a)^(13/6)*(72*b^2*d^2*x^2-156*a*b*d^2*x+300*b^2*c*d*x+247*a^2*
d^2-650*a*b*c*d+475*b^2*c^2)/(d*x+c)^(25/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{31}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(31/6), x)
```

**mupad [B]** time = 1.14, size = 278, normalized size = 2.75

$$-\frac{(c+d\,x)^{5/6}\,\left(\frac{(a+b\,x)^{1/6}\,\left(1482\,a^4\,d^2-3900\,a^3\,b\,c\,d+2850\,a^2\,b^2\,c^2\right)}{6175\,d^5\,{(a\,d-b\,c)}^3}+\frac{432\,b^4\,x^4\,(a+b\,x)^{1/6}}{6175\,d^3\,{(a\,d-b\,c)}^3}+\frac{x^2\,(a+b\,x)^{1/6}\,\left(42\,a^2\,b^2\,d^2-300\,a\,b^3\,c\,d+2850\,b^4\,c^2\right)}{6175\,d^5\,{(a\,d-b\,c)}^3}-\frac{72\,b^3\,x^3\,(a\,d-25\,b\,c)\,(a+b\,x)^{1/6}}{6175\,d^4\,{(a\,d-b\,c)}^3}+\frac{12\,a\,b\,x\,(a+b\,x)^{1/6}\,\left(169\,a^2\,d^2-500\,a\,b\,c\,d+475\,b^2\,c^2\right)}{6175\,d^5\,{(a\,d-b\,c)}^3}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(31/6),x)
```

```
[Out] -((c + d*x)^(5/6)*(((a + b*x)^(1/6)*(1482*a^4*d^2 + 2850*a^2*b^2*c^2 - 3900
*a^3*b*c*d))/(6175*d^5*(a*d - b*c)^3) + (432*b^4*x^4*(a + b*x)^(1/6))/(6175
*d^3*(a*d - b*c)^3) + (x^2*(a + b*x)^(1/6)*(2850*b^4*c^2 + 42*a^2*b^2*d^2 -
 300*a*b^3*c*d))/(6175*d^5*(a*d - b*c)^3) - (72*b^3*x^3*(a*d - 25*b*c)*(a +
 b*x)^(1/6))/(6175*d^4*(a*d - b*c)^3) + (12*a*b*x*(a + b*x)^(1/6)*(169*a^2*
d^2 + 475*b^2*c^2 - 500*a*b*c*d))/(6175*d^5*(a*d - b*c)^3)))/(x^5 + c^5/d^5
 + (5*c*x^4)/d + (5*c^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(31/6),x)
```

```
[Out] Timed out
```

## 3.1532 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{37/6}}\,dx$

**Optimal**. Leaf size=136

$$\frac{7776b^3(a+bx)^{13/6}}{191425(c+dx)^{13/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{13/6}}{14725(c+dx)^{19/6}(bc-ad)^3}+\frac{108b(a+bx)^{13/6}}{775(c+dx)^{25/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{31(c+dx)^{31/6}(bc-ad)}$$

**Rubi** [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{13/6}}{191425(c+dx)^{13/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{13/6}}{14725(c+dx)^{19/6}(bc-ad)^3}+\frac{108b(a+bx)^{13/6}}{775(c+dx)^{25/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{31(c+dx)^{31/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(31*(b*c - a*d)*(c + d*x)^(31/6)) + (108*b*(a + b*x)^(
13/6))/(775*(b*c - a*d)^2*(c + d*x)^(25/6)) + (1296*b^2*(a + b*x)^(13/6))/(
14725*(b*c - a*d)^3*(c + d*x)^(19/6)) + (7776*b^3*(a + b*x)^(13/6))/(191425
*(b*c - a*d)^4*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{37/6}}\,dx = \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{(18b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx}{31(bc-ad)}$$

$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{\left(216b^2\right)\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx}{775(bc-ad)^2}$$

$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}} + \frac{\left(1296b^{3}\right.}{1472}$$

$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}} + \frac{77}{191425}$$

**Mathematica [A]** time = 0.09, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{13/6}\left(-6175a^3d^3+741a^2bd^2(31c+6dx)-39ab^2d\left(775c^2+372cdx+72d^2x^2\right)+b^3\left(14725c^3+13950c^2dx+6696cd^2x^2+1296d^3x^3\right)\right)}{191425(c+dx)^{31/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(-6175*a^3*d^3 + 741*a^2*b*d^2*(31*c + 6*d*x) - 39*a*b^
2*d*(775*c^2 + 372*c*d*x + 72*d^2*x^2) + b^3*(14725*c^3 + 13950*c^2*d*x + 6
696*c*d^2*x^2 + 1296*d^3*x^3)))/(191425*(b*c - a*d)^4*(c + d*x)^(31/6))
```

**IntegrateAlgebraic [A]** time = 0.23, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{13/6}\left(-\frac{30225b^2d(a+bx)}{c+dx}-\frac{6175d^3(a+bx)^3}{(c+dx)^3}+\frac{22971bd^2(a+bx)^2}{(c+dx)^2}+14725b^3\right)}{191425(c+dx)^{13/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(14725*b^3 - (6175*d^3*(a + b*x)^3)/(c + d*x)^3 + (2297
1*b*d^2*(a + b*x)^2)/(c + d*x)^2 - (30225*b^2*d*(a + b*x))/(c + d*x)))/(191
425*(b*c - a*d)^4*(c + d*x)^(13/6))
```

**fricas [B]** time = 1.39, size = 649, normalized size = 4.77

$$\frac{6\left(1296\,b^5d^3x^5+14725\,a^2b^3c^3-30225\,a^3b^2c^2d+22971\,a^4bcd^2-6175\,a^5d^3+216\left(31\,b^5cd^2-ab^4d^3\right)x^4+18\left(775\,b^5c^2d-62\,ab^4cd^2+7\,a^2b^3d^3\right)x^3+\left(14725\,b^5c^3-2325\,ab^4c^2d+651\,a^2b^3cd^2-91\,a^3b^2d^3\right)x^2+2\left(14725\,ab^4c^3-23250\,a^2b^3c^2d+15717\,a^3b^2cd^2-3952\,a^4bd^3\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{191425\left(b^4c^{10}-4\,ab^3c^9d+6\,a^2b^2c^8d^2-4\,a^3bc^7d^3+a^4c^6d^4+\left(b^4c^4d^6-4\,ab^3c^3d^7+6\,a^2b^2c^2d^8-4\,a^3bcd^9+a^4d^{10}\right)x^6+6\left(b^4c^5d^5-4\,ab^3c^4d^6+6\,a^2b^2c^3d^7-4\,a^3bc^2d^8+a^4cd^9\right)x^5+15\left(b^4c^6d^4-4\,ab^3c^5d^5+6\,a^2b^2c^4d^6-4\,a^3bc^3d^7+a^4c^2d^8\right)x^4+20\left(b^4c^7d^3-4\,ab^3c^6d^4+6\,a^2b^2c^5d^5-4\,a^3bc^4d^6+a^4c^3d^7\right)x^3+15\left(b^4c^8d^2-4\,ab^3c^7d^3+6\,a^2b^2c^6d^4-4\,a^3bc^5d^5+a^4c^4d^6\right)x^2+6\left(b^4c^9d-4\,ab^3c^8d^2+6\,a^2b^2c^7d^3-4\,a^3bc^6d^4+a^4c^5d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="fricas")
```

```
[Out] 6/191425*(1296*b^5*d^3*x^5 + 14725*a^2*b^3*c^3 - 30225*a^3*b^2*c^2*d + 2297
1*a^4*b*c*d^2 - 6175*a^5*d^3 + 216*(31*b^5*c*d^2 - a*b^4*d^3)*x^4 + 18*(775
*b^5*c^2*d - 62*a*b^4*c*d^2 + 7*a^2*b^3*d^3)*x^3 + (14725*b^5*c^3 - 2325*a*
b^4*c^2*d + 651*a^2*b^3*c*d^2 - 91*a^3*b^2*d^3)*x^2 + 2*(14725*a*b^4*c^3 -
23250*a^2*b^3*c^2*d + 15717*a^3*b^2*c*d^2 - 3952*a^4*b*d^3)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^4*c^10 - 4*a*b^3*c^9*d + 6*a^2*b^2*c^8*d^2 - 4*a^3*b
*c^7*d^3 + a^4*c^6*d^4 + (b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2*b^2*c^2*d^8
 - 4*a^3*b*c*d^9 + a^4*d^10)*x^6 + 6*(b^4*c^5*d^5 - 4*a*b^3*c^4*d^6 + 6*a^2
*b^2*c^3*d^7 - 4*a^3*b*c^2*d^8 + a^4*c*d^9)*x^5 + 15*(b^4*c^6*d^4 - 4*a*b^3
*c^5*d^5 + 6*a^2*b^2*c^4*d^6 - 4*a^3*b*c^3*d^7 + a^4*c^2*d^8)*x^4 + 20*(b^4
*c^7*d^3 - 4*a*b^3*c^6*d^4 + 6*a^2*b^2*c^5*d^5 - 4*a^3*b*c^4*d^6 + a^4*c^3*
d^7)*x^3 + 15*(b^4*c^8*d^2 - 4*a*b^3*c^7*d^3 + 6*a^2*b^2*c^6*d^4 - 4*a^3*b*
c^5*d^5 + a^4*c^4*d^6)*x^2 + 6*(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7
*d^3 - 4*a^3*b*c^6*d^4 + a^4*c^5*d^5)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{37}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(37/6), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$-\frac{6\,(bx+a)^{\frac{13}{6}}\left(-1296b^3d^3x^3+2808a\,b^2d^3x^2-6696b^3c\,d^2x^2-4446a^2b\,d^3x+14508a\,b^2c\,d^2x-13950b^3c^2dx+6175a^3d^3-22971a^2bc\,d^2+30225a\,b^2c^2d-14725b^3c^3\right)}{191425\,(dx+c)^{\frac{31}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(37/6),x)
```

```
[Out] -6/191425*(b*x+a)^(13/6)*(-1296*b^3*d^3*x^3+2808*a*b^2*d^3*x^2-6696*b^3*c*d
^2*x^2-4446*a^2*b*d^3*x+14508*a*b^2*c*d^2*x-13950*b^3*c^2*d*x+6175*a^3*d^3-
22971*a^2*b*c*d^2+30225*a*b^2*c^2*d-14725*b^3*c^3)/(d*x+c)^(31/6)/(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{37}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(37/6), x)
```

**mupad [B]** time = 1.43, size = 385, normalized size = 2.83

$$\frac{(c+d\,x)^{5/6}\left(\frac{7776\,b^5\,x^5\,(a+b\,x)^{1/6}}{191425\,d^3\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{1/6}\left(37050\,a^5\,d^3-137826\,a^4\,b\,c\,d^2+181350\,a^3\,b^2\,c^2\,d-88350\,a^2\,b^3\,c^3\right)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{x^2\,(a+b\,x)^{1/6}\left(-546\,a^3\,b^2\,d^3+3906\,a^2\,b^3\,c\,d^2-13950\,a\,b^4\,c^2\,d+88350\,b^5\,c^3\right)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{1/6}\left(-47424\,a^4\,b\,d^3+188604\,a^3\,b^2\,c\,d^2-279000\,a^2\,b^3\,c^2\,d+176700\,a\,b^4\,c^3\right)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{108\,b^3\,x^3\,(a+b\,x)^{1/6}\left(7\,a^2\,d^2-62\,a\,b\,c\,d+775\,b^2\,c^2\right)}{191425\,d^5\,(a\,d-b\,c)^4}-\frac{1296\,b^4\,x^4\,(a\,d-31\,b\,c)\,(a+b\,x)^{1/6}}{191425\,d^4\,(a\,d-b\,c)^4}\right)}{x^6+\frac{c^6}{d^6}+\frac{6\,c\,x^5}{d}+\frac{6\,c^5\,x}{d^5}+\frac{15\,c^2\,x^4}{d^2}+\frac{20\,c^3\,x^3}{d^3}+\frac{15\,c^4\,x^2}{d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(37/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((7776*b^5*x^5*(a + b*x)^(1/6))/(191425*d^3*(a*d - b*c)^4)
 - ((a + b*x)^(1/6)*(37050*a^5*d^3 - 88350*a^2*b^3*c^3 + 181350*a^3*b^2*c^2
*d - 137826*a^4*b*c*d^2))/(191425*d^6*(a*d - b*c)^4) + (x^2*(a + b*x)^(1/6)
*(88350*b^5*c^3 - 546*a^3*b^2*d^3 + 3906*a^2*b^3*c*d^2 - 13950*a*b^4*c^2*d)
)/(191425*d^6*(a*d - b*c)^4) + (x*(a + b*x)^(1/6)*(176700*a*b^4*c^3 - 47424
*a^4*b*d^3 - 279000*a^2*b^3*c^2*d + 188604*a^3*b^2*c*d^2))/(191425*d^6*(a*d
 - b*c)^4) + (108*b^3*x^3*(a + b*x)^(1/6)*(7*a^2*d^2 + 775*b^2*c^2 - 62*a*b
*c*d))/(191425*d^5*(a*d - b*c)^4) - (1296*b^4*x^4*(a*d - 31*b*c)*(a + b*x)^
(1/6))/(191425*d^4*(a*d - b*c)^4)))/(x^6 + c^6/d^6 + (6*c*x^5)/d + (6*c^5*x
)/d^5 + (15*c^2*x^4)/d^2 + (20*c^3*x^3)/d^3 + (15*c^4*x^2)/d^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(37/6),x)
```

```
[Out] Timed out
```

## 3.1533 $\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx$

**Optimal**. Leaf size=424

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-a}{}$$

**Rubi** [A] time = 0.61, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}b^{13/6}d^{5/6}}-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{13/6}d^{5/6}}+\frac{7(a+bx)^{5/6}\sqrt[6]{c+dx}(bc-ad)}{12b^2}+\frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```

```
[Out] (7*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^2) + ((a + b*x)^(5/6)
*(c + d*x)^(7/6))/(2*b) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(
5/6)) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(5/6)) + (7*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(13/6)*d^(5/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(13/6)*d^(5/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx = \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{(7(bc-ad))\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{12b}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{72b^2}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}} \cdots\right.}{12b^3}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \cdots\right.}{12b^3}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\cdots}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\cdots} \cdots\right.}{36b^{13/6}d^{2/3}}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{13/6}d^{5/6}} - \cdots$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{13/6}d^{5/6}} - \cdots$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{13/6}d^{5/6}} - \cdots$$

**Mathematica [C]** time = 0.05, size = 73, normalized size = 0.17

$$\frac{6(a+bx)^{5/6}(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 5/6, 11/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

**IntegrateAlgebraic [A]** time = 0.79, size = 363, normalized size = 0.86

$$-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{24\sqrt{3}\,b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{36b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{72b^{13/6}d^{5/6}}+\frac{(bc-ad)^2\left(\frac{13b(c+dx)^{7/6}}{(a+bx)^{7/6}}-\frac{7d\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\right)}{12b^2\left(\frac{b(c+dx)}{a+bx}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)^2*((-7*d*(c + d*x)^(1/6))/(a + b*x)^(1/6) + (13*b*(c + d*x)^(7
/6))/(a + b*x)^(7/6)))/(12*b^2*(-d + (b*(c + d*x))/(a + b*x))^2) - (7*(b*c
- a*d)^2*ArcTan[1/Sqrt[3] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a
 + b*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2*ArcTan[1/
Sqrt[3] + (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))])/(
24*Sqrt[3]*b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2*ArcTanh[(b^(1/6)*(c + d*x)^
(1/6))/(d^(1/6)*(a + b*x)^(1/6))])/(36*b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2
*ArcTanh[(b^(1/6)*d^(1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(1/3) + (b^(
1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)))])/(72*b^(13/6)*d^(5/6))
```

**fricas [B]** time = 2.01, size = 5633, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="fricas")
```

```
[Out] -1/144*(28*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^13*c^2*d^4 - 2*a*b^12*c*d^5 + a^2*b^11*d^6)*(
b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10
```

```
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*
d^5))^(5/6) - 2*sqrt(3)*(b^12*d^4*x + a*b^11*d^4)*sqrt(((b^4*c^2*d - 2*a*b^
3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^13*d^5))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*d
^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/3))/
(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(5/6) + sqrt(3
)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^
9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 7
92*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b
^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d
+ 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a
^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^
4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a
^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 -
220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b
^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d
^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*
a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c
^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 +
495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*
b^2*c*d^11 + a^12*b*d^12)*x)) + 28*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(b^13*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^13*c^2*d^4 - 2*a*b^12
*c*d^5 + a^2*b^11*d^6)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^13*d^5))^(5/6) - 2*sqrt(3)*(b^12*d^4*x + a*b^11*d^4)*s
qrt(-((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(
1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
```

92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(5/6) - sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) - 7*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6)*log(49*((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/3))/(b*x + a)) + 7*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6)*log(-49*((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1

```
/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9
*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79
2*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2
*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) - (b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*
(d*x + c)^(1/3) - (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*
b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d
^12)/(b^13*d^5))^(1/3))/(b*x + a)) - 14*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^13*d^5))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(
5/6)*(d*x + c)^(1/6) + (b^3*d*x + a*b^2*d)*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^13*d^5))^(1/6))/(b*x + a)) + 14*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(b^13*d^5))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^
(5/6)*(d*x + c)^(1/6) - (b^3*d*x + a*b^2*d)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^13*d^5))^(1/6))/(b*x + a)) - 12*(6*b*d*x + 13*b*c - 7*a*d)*(b*x +
 a)^(5/6)*(d*x + c)^(1/6))/b^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(1/6), x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(1/6),x)
```

```
[Out] int((d*x+c)^(7/6)/(b*x+a)^(1/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^{\frac{7}{6}}}{(bx + a)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(1/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^{7/6}}{{\left(a+b\,x\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(7/6)/(a + b*x)^(1/6),x)
```

```
[Out] int((c + d*x)^(7/6)/(a + b*x)^(1/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^{\frac{7}{6}}}{\sqrt[6]{a + bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(1/6),x)
```

```
[Out] Integral((c + d*x)**(7/6)/(a + b*x)**(1/6), x)
```

## 3.1534 $\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx$

**Optimal**. Leaf size=378

$$-\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\tan}{2}$$

**Rubi [A]** time = 0.56, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{2\sqrt{3}b^{7/6}d^{5/6}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}b^{7/6}d^{5/6}}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```

```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/b + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/
6)*d^(5/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/6)*d^(5/6)) + ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(7/6)
*d^(5/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(
5/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(5/6)
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx = \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6b}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{2/3}} + \frac{(bc-ad)\,\text{Subst}\Big(\int}{}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx,\right.}{12b^{7/6}d^{5/6}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}} - \frac{(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{}}{\sqrt[6]{c+}}\right.}{12b^{7/6}d^{5/6}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}} - \frac{(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}} + \frac{(bc-a}{}$$

**Mathematica [C]** time = 0.04, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{5/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 5/6, 11/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

**IntegrateAlgebraic [A]** time = 16.57, size = 388, normalized size = 1.03

$$\frac{\sqrt[6]{ad+bdx}\left(-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{b}\sqrt[6]{c+dx}-2\sqrt[6]{ad+b(c+dx)-bc}}\right)}{2\sqrt{3}b^{7/6}d^{5/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}{2\sqrt[6]{ad+b(c+dx)-bc}+\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{2\sqrt{3}b^{7/6}d^{5/6}}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{ad+b(c+dx)-bc}}\right)}{3b^{7/6}d^{5/6}}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[6]{b}\sqrt[6]{c+dx}\sqrt[6]{ad+b(c+dx)-bc}}\right)}{6b^{7/6}d^{5/6}}+\frac{\sqrt[6]{c+dx}\,(ad+b(c+dx)-bc)^{5/6}}{bd^{5/6}}\right)}{\sqrt[6]{d}\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```

```
[Out] ((a*d + b*d*x)^(1/6)*(((c + d*x)^(1/6)*(-(b*c) + a*d + b*(c + d*x))^(5/6))/
(b*d^(5/6)) - ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/6
)*(c + d*x)^(1/6) - 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*b^(7
/6)*d^(5/6)) + ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/
6)*(c + d*x)^(1/6) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*b^(
7/6)*d^(5/6)) + ((b*c - a*d)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(-(b*c) + a*
d + b*(c + d*x))^(1/6)])/(3*b^(7/6)*d^(5/6)) + ((b*c - a*d)*ArcTanh[(b^(1/3
)*(c + d*x)^(1/3) + (-(b*c) + a*d + b*(c + d*x))^(1/3))/(b^(1/6)*(c + d*x)^
(1/6)*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(6*b^(7/6)*d^(5/6))))/(d^(1/6)*
(a + b*x)^(1/6))
```

**fricas [B]** time = 1.85, size = 3025, normalized size = 8.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b
^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)
*arctan(1/3*(2*sqrt(3)*(b^7*c*d^4 - a*b^6*d^5)*(b*x + a)^(5/6)*(d*x + c)^(1
/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + 2*sqrt(3)*(
b^7*d^4*x + a*b^6*d^4)*sqrt(((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^
(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^2*x + a*b^2
*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a))
*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + sqrt(3)*(a*b^6
*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b
```

^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + 4*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^7*c*d^4 - a*b^6*d^5)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + 2*sqrt(3)*(b^7*d^4*x + a*b^6*d^4)*sqrt(-((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) - sqrt(3)*(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*log(((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a)) - b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*log(-((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a)) + 2*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*log(-((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b^2*d*x + a*b*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6))/(b*x + a)) - 2*b*((b^6*c^6 - 6*a*b^5*c^5*d + 1

```
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
 + a^6*d^6)/(b^7*d^5))^(1/6)*log(-((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1
/6) - (b^2*d*x + a*b*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))
^(1/6))/(b*x + a)) + 12*(b*x + a)^(5/6)*(d*x + c)^(1/6))/b
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(1/6), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(1/6),x)
[Out] int((d*x+c)^(1/6)/(b*x+a)^(1/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(1/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/6}}{(a+b\,x)^{1/6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(1/6),x)
```

```
[Out] int((c + d*x)^(1/6)/(a + b*x)^(1/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(1/6),x)
```

```
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(1/6), x)
```

## 3.1535 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx$

**Optimal**. Leaf size=309

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}-$$

**Rubi [A]** time = 0.51, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c
 + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) - (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) + (
2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^
(5/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)
*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6)) + Log[b^(1/3
) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^
(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx = \frac{6\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b}$$

$$= \frac{6\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^{2/3}} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{2/3}} + \text{—}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2\sqrt[6]{b}\,d^{5/6}} + \frac{\text{Subst}\left(\int \frac{\sqrt[6]{b}\,\sqrt[6]{\ldots}}{\sqrt[3]{b}+\sqrt[6]{b}\ldots}\right.}{2\ldots}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2\sqrt[6]{b}\,d^{5/6}} + \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\ldots\right.}{2\sqrt[6]{b}\,d^{5/\ldots}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\log\left(\sqrt[3]{b}\ldots\right.}{\ldots}$$

<u>**Mathematica**</u> **[C]** time = 0.05, size = 73, normalized size = 0.24

$$\frac{6(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 5/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/6))
```

**IntegrateAlgebraic [A]** time = 0.22, size = 233, normalized size = 0.75

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{\sqrt[6]{b}\,d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{\sqrt[6]{b}\,d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{\sqrt[6]{b}\,d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*
(a + b*x)^(1/6))]])/(b^(1/6)*d^(5/6))) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1
/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))]])/(b^(1/6)*d^(5/6))
+ (2*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6))]])/(b^(1/6)
*d^(5/6)) + ArcTanh[(b^(1/6)*d^(1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(
1/3) + (b^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)))]]/(b^(1/6)*d^(5/6))
```

**fricas [B]** time = 0.93, size = 620, normalized size = 2.01

$$-2\sqrt{3}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}bd^4\left(\frac{1}{bd^5}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^2d^4x+abd^4\right)\sqrt{\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}+\left(bd^2x+ad^2\right)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{1}{bd^5}\right)^{\frac{5}{6}}+\sqrt{3}(bx+a)}{3(bx+a)}\right)-2\sqrt{3}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}bd^4\left(\frac{1}{bd^5}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^2d^4x+abd^4\right)\sqrt{-\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}-\left(bd^2x+ad^2\right)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{1}{bd^5}\right)^{\frac{5}{6}}-\sqrt{3}(bx+a)}{3(bx+a)}\right)+\frac{1}{2}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}+\left(bd^2x+ad^2\right)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right)-\frac{1}{2}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(-\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}-\left(bd^2x+ad^2\right)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right)+\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(\frac{(bdx+ad)\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}+(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right)-\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(-\frac{(bdx+ad)\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}-(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*(1/(b*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x +
c)^(1/6)*b*d^4*(1/(b*d^5))^(5/6) - 2*sqrt(3)*(b^2*d^4*x + a*b*d^4)*sqrt(((b
*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d^5))^(1/6) + (b*d^2*x + a*d^2)*(1/(b
*d^5))^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a))*(1/(b*d^5))^(5/6
) + sqrt(3)*(b*x + a))/(b*x + a)) - 2*sqrt(3)*(1/(b*d^5))^(1/6)*arctan(-1/3
*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^4*(1/(b*d^5))^(5/6) - 2*sqr
t(3)*(b^2*d^4*x + a*b*d^4)*sqrt(-((b*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d
^5))^(1/6) - (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) - (b*x + a)^(2/3)*(d*x + c
)^(1/3))/(b*x + a))*(1/(b*d^5))^(5/6) - sqrt(3)*(b*x + a))/(b*x + a)) + 1/2
*(1/(b*d^5))^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d^5))^(1/
6) + (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))
/(b*x + a)) - 1/2*(1/(b*d^5))^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x + c)^(1/6)
*d*(1/(b*d^5))^(1/6) - (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) - (b*x + a)^(2/3
)*(d*x + c)^(1/3))/(b*x + a)) + (1/(b*d^5))^(1/6)*log(((b*d*x + a*d)*(1/(b*
d^5))^(1/6) + (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b*x + a)) - (1/(b*d^5))^(1/
6)*log(-((b*d*x + a*d)*(1/(b*d^5))^(1/6) - (b*x + a)^(5/6)*(d*x + c)^(1/6))
/(b*x + a))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(5/6)), x)
```

**maple [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x)
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(5/6)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/6}\,{\left(c+d\,x\right)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x)
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(5/6)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(5/6),x)
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(5/6)), x)
```

## 3.1536 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx$

**Optimal**. Leaf size=32

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx = \frac{6(a+bx)^{5/6}}{5(bc-ad)(c+dx)^{5/6}}$$

**Mathematica [A]** time = 0.01, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

**fricas** [A] time = 0.70, size = 42, normalized size = 1.31

$$\frac{6\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{5\left(bc^2-acd+\left(bcd-ad^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="fricas")
[Out] 6/5*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(11/6)), x)
```

**maple** [A] time = 0.00, size = 27, normalized size = 0.84

$$-\frac{6\,(bx+a)^{\frac{5}{6}}}{5\,(dx+c)^{\frac{5}{6}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x)
[Out] -6/5*(b*x+a)^(5/6)/(d*x+c)^(5/6)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(11/6)), x)
```

**mupad [B]** time = 0.76, size = 27, normalized size = 0.84

$$-\frac{6\,{\left(a+b\,x\right)}^{5/6}}{\left(5\,a\,d-5\,b\,c\right)\,{\left(c+d\,x\right)}^{5/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x)
```

```
[Out] -(6*(a + b*x)^(5/6))/((5*a*d - 5*b*c)*(c + d*x)^(5/6))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(11/6),x)
```

```
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(11/6)), x)
```

## 3.1537 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx$

**Optimal**. Leaf size=66

$$\frac{36b(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(11*(b*c - a*d)*(c + d*x)^(11/6)) + (36*b*(a + b*x)^(5/
6))/(55*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx = \frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}} + \frac{(6b)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx}{11(bc-ad)}$$
$$= \frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}} + \frac{36b(a+bx)^{5/6}}{55(bc-ad)^2(c+dx)^{5/6}}$$

**Mathematica [A]** time = 0.03, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{5/6}(-5ad+11bc+6bdx)}{55(c+dx)^{11/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(11*b*c - 5*a*d + 6*b*d*x))/(55*(b*c - a*d)^2*(c + d*x)^
(11/6))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 51, normalized size = 0.77

$$\frac{6(a+bx)^{11/6}\left(\frac{11b(c+dx)}{a+bx}-5d\right)}{55(c+dx)^{11/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(-5*d + (11*b*(c + d*x))/(a + b*x)))/(55*(b*c - a*d)^2*
(c + d*x)^(11/6))
```

**fricas [B]** time = 1.38, size = 118, normalized size = 1.79

$$\frac{6\,(6\,bdx+11\,bc-5\,ad)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{55\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] 6/55*(6*b*d*x + 11*b*c - 5*a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^2*c^4 -
2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(
b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(17/6)), x)
```

**maple [A]** time = 0.00, size = 54, normalized size = 0.82

$$-\frac{6\,(bx+a)^{\frac{5}{6}}\,(-6bdx+5ad-11bc)}{55\,(dx+c)^{\frac{11}{6}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x)
[Out] -6/55*(b*x+a)^(5/6)*(-6*b*d*x+5*a*d-11*b*c)/(d*x+c)^(11/6)/(a^2*d^2-2*a*b*c
*d+b^2*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(17/6)), x)
```

**mupad [B]** time = 0.86, size = 127, normalized size = 1.92

$$\frac{(c+d\,x)^{1/6}\left(\frac{x\left(66\,c\,b^2+6\,a\,d\,b\right)}{55\,d^2\,(a\,d-b\,c)^2}-\frac{30\,a^2\,d-66\,a\,b\,c}{55\,d^2\,(a\,d-b\,c)^2}+\frac{36\,b^2\,x^2}{55\,d\,(a\,d-b\,c)^2}\right)}{x^2\,(a+b\,x)^{1/6}+\frac{c^2\,(a+b\,x)^{1/6}}{d^2}+\frac{2\,c\,x\,(a+b\,x)^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((x*(66*b^2*c + 6*a*b*d))/(55*d^2*(a*d - b*c)^2) - (30*a^2
*d - 66*a*b*c)/(55*d^2*(a*d - b*c)^2) + (36*b^2*x^2)/(55*d*(a*d - b*c)^2)))
/(x^2*(a + b*x)^(1/6) + (c^2*(a + b*x)^(1/6))/d^2 + (2*c*x*(a + b*x)^(1/6))
/d)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

## 3.1538 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx$

**Optimal**. Leaf size=101

$$\frac{432b^2(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^3}+\frac{72b(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^3}+\frac{72b(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (72*b*(a + b*x)^(5/
6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6)) + (432*b^2*(a + b*x)^(5/6))/(935*(
b*c - a*d)^3*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx = \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{(12b)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx}{17(bc-ad)}$$

$$= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}} + \frac{\left(72b^2\right)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx}{187(bc-ad)^2}$$

$$= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}} + \frac{432b^2(a+bx)^{5/6}}{935(bc-ad)^3(c+dx)^{5/6}}$$

**Mathematica [A]** time = 0.05, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{5/6}\left(55a^2d^2-10abd(17c+6dx)+b^2\left(187c^2+204cdx+72d^2x^2\right)\right)}{935(c+dx)^{17/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(55*a^2*d^2 - 10*a*b*d*(17*c + 6*d*x) + b^2*(187*c^2 + 2
04*c*d*x + 72*d^2*x^2)))/(935*(b*c - a*d)^3*(c + d*x)^(17/6))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{17/6}\left(\frac{187b^2(c+dx)^2}{(a+bx)^2}-\frac{170bd(c+dx)}{a+bx}+55d^2\right)}{935(c+dx)^{17/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (6*(a + b*x)^(17/6)*(55*d^2 - (170*b*d*(c + d*x))/(a + b*x) + (187*b^2*(c +
 d*x)^2)/(a + b*x)^2))/(935*(b*c - a*d)^3*(c + d*x)^(17/6))
```

**fricas [B]** time = 1.37, size = 252, normalized size = 2.50

$$\frac{6\left(72\,b^2d^2x^2+187\,b^2c^2-170\,abcd+55\,a^2d^2+12\left(17\,b^2cd-5\,abd^2\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{935\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3+\left(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6\right)x^3+3\left(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4-a^3cd^5\right)x^2+3\left(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] 6/935*(72*b^2*d^2*x^2 + 187*b^2*c^2 - 170*a*b*c*d + 55*a^2*d^2 + 12*(17*b^2
*c*d - 5*a*b*d^2)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^3*c^6 - 3*a*b^2*c^5
*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2
*b*c*d^5 - a^3*d^6)*x^3 + 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^
4 - a^3*c*d^5)*x^2 + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3
*c^2*d^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(23/6)), x)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$-\frac{6\,(bx+a)^{\frac{5}{6}}\left(72b^2x^2d^2-60ab\,d^2x+204b^2cdx+55a^2d^2-170abcd+187b^2c^2\right)}{935\,(dx+c)^{\frac{17}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x)
```

```
[Out] -6/935*(b*x+a)^(5/6)*(72*b^2*d^2*x^2-60*a*b*d^2*x+204*b^2*c*d*x+55*a^2*d^2-
170*a*b*c*d+187*b^2*c^2)/(d*x+c)^(17/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*
d-b^3*c^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(23/6)), x)
```

**mupad [B]** time = 1.03, size = 203, normalized size = 2.01

$$-\frac{(c+d\,x)^{1/6}\left(\frac{330\,a^3\,d^2-1020\,a^2\,b\,c\,d+1122\,a\,b^2\,c^2}{935\,d^3\,(a\,d-b\,c)^3}+\frac{x\left(-30\,a^2\,b\,d^2+204\,a\,b^2\,c\,d+1122\,b^3\,c^2\right)}{935\,d^3\,(a\,d-b\,c)^3}+\frac{432\,b^3\,x^3}{935\,d\,(a\,d-b\,c)^3}+\frac{72\,b^2\,x^2\,(a\,d+17\,b\,c)}{935\,d^2\,(a\,d-b\,c)^3}\right)}{x^3\,(a+b\,x)^{1/6}+\frac{c^3\,(a+b\,x)^{1/6}}{d^3}+\frac{3\,c\,x^2\,(a+b\,x)^{1/6}}{d}+\frac{3\,c^2\,x\,(a+b\,x)^{1/6}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x)
```

```
[Out] -((c + d*x)^(1/6)*((330*a^3*d^2 + 1122*a*b^2*c^2 - 1020*a^2*b*c*d)/(935*d^3
*(a*d - b*c)^3) + (x*(1122*b^3*c^2 - 30*a^2*b*d^2 + 204*a*b^2*c*d))/(935*d^
3*(a*d - b*c)^3) + (432*b^3*x^3)/(935*d*(a*d - b*c)^3) + (72*b^2*x^2*(a*d +
 17*b*c))/(935*d^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/6) + (c^3*(a + b*x)^(
1/6))/d^3 + (3*c*x^2*(a + b*x)^(1/6))/d + (3*c^2*x*(a + b*x)^(1/6))/d^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

## 3.1539 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{29/6}}\,dx$

**Optimal**. Leaf size=136

$$\frac{7776b^3(a+bx)^{5/6}}{21505(c+dx)^{5/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{5/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{108b(a+bx)^{5/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{23(c+dx)^{23/6}(bc-ad)}$$

**Rubi [A]** time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{5/6}}{21505(c+dx)^{5/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{5/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{108b(a+bx)^{5/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (108*b*(a + b*x)^(5
/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (1296*b^2*(a + b*x)^(5/6))/(430
1*(b*c - a*d)^3*(c + d*x)^(11/6)) + (7776*b^3*(a + b*x)^(5/6))/(21505*(b*c
- a*d)^4*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{29/6}}\,dx = \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{(18b)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx}{23(bc-ad)}$$

$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{\left(216b^2\right)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx}{391(bc-ad)^2}$$

$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx)^{11/6}} +$$

$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx)^{11/6}} +$$

**Mathematica** [A] time = 0.07, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{5/6}\left(-935a^3d^3+165a^2bd^2(23c+6dx)-15ab^2d\left(391c^2+276cdx+72d^2x^2\right)+b^3\left(4301c^3+7038c^2dx+4968cd^2x^2+1296d^3x^3\right)\right)}{21505(c+dx)^{23/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(-935*a^3*d^3 + 165*a^2*b*d^2*(23*c + 6*d*x) - 15*a*b^2*
d*(391*c^2 + 276*c*d*x + 72*d^2*x^2) + b^3*(4301*c^3 + 7038*c^2*d*x + 4968*
c*d^2*x^2 + 1296*d^3*x^3)))/(21505*(b*c - a*d)^4*(c + d*x)^(23/6))
```

**IntegrateAlgebraic** [A] time = 0.12, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{23/6}\left(\frac{4301b^3(c+dx)^3}{(a+bx)^3}-\frac{5865b^2d(c+dx)^2}{(a+bx)^2}+\frac{3795bd^2(c+dx)}{a+bx}-935d^3\right)}{21505(c+dx)^{23/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
```

```
[Out] (6*(a + b*x)^(23/6)*(-935*d^3 + (3795*b*d^2*(c + d*x))/(a + b*x) - (5865*b^
2*d*(c + d*x)^2)/(a + b*x)^2 + (4301*b^3*(c + d*x)^3)/(a + b*x)^3))/(21505*
(b*c - a*d)^4*(c + d*x)^(23/6))
```

**fricas** [B] time = 1.44, size = 420, normalized size = 3.09

$$\frac{6\left(1296\,b^3d^3x^3+4301\,b^3c^3-5865\,ab^2c^2d+3795\,a^2bcd^2-935\,a^3d^3+216\left(23\,b^3cd^2-5\,ab^2d^3\right)x^2+18\left(391\,b^3c^2d-230\,ab^2cd^2+55\,a^2bd^3\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{21505\left(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4+\left(b^4c^4d^4-4\,ab^3c^3d^5+6\,a^2b^2c^2d^6-4\,a^3bcd^7+a^4d^8\right)x^4+4\left(b^4c^5d^3-4\,ab^3c^4d^4+6\,a^2b^2c^3d^5-4\,a^3bc^2d^6+a^4cd^7\right)x^3+6\left(b^4c^6d^2-4\,ab^3c^5d^3+6\,a^2b^2c^4d^4-4\,a^3bc^3d^5+a^4c^2d^6\right)x^2+4\left(b^4c^7d-4\,ab^3c^6d^2+6\,a^2b^2c^5d^3-4\,a^3bc^4d^4+a^4c^3d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="fricas")
```

```
[Out] 6/21505*(1296*b^3*d^3*x^3 + 4301*b^3*c^3 - 5865*a*b^2*c^2*d + 3795*a^2*b*c*
d^2 - 935*a^3*d^3 + 216*(23*b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + 18*(391*b^3*c^2*
d - 230*a*b^2*c*d^2 + 55*a^2*b*d^3)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^4
*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4 +
(b^4*c^4*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^
8)*x^4 + 4*(b^4*c^5*d^3 - 4*a*b^3*c^4*d^4 + 6*a^2*b^2*c^3*d^5 - 4*a^3*b*c^2
*d^6 + a^4*c*d^7)*x^3 + 6*(b^4*c^6*d^2 - 4*a*b^3*c^5*d^3 + 6*a^2*b^2*c^4*d^
4 - 4*a^3*b*c^3*d^5 + a^4*c^2*d^6)*x^2 + 4*(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6
*a^2*b^2*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(29/6)), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$-\frac{6\,(bx+a)^{\frac{5}{6}}\left(-1296b^3d^3x^3+1080a\,b^2d^3x^2-4968b^3c\,d^2x^2-990a^2b\,d^3x+4140a\,b^2c\,d^2x-7038b^3c^2dx+935a^3d^3-3795a^2bc\,d^2+5865a\,b^2c^2d-4301b^3c^3\right)}{21505\,(dx+c)^{\frac{23}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x)
```

```
[Out] -6/21505*(b*x+a)^(5/6)*(-1296*b^3*d^3*x^3+1080*a*b^2*d^3*x^2-4968*b^3*c*d^2
*x^2-990*a^2*b*d^3*x+4140*a*b^2*c*d^2*x-7038*b^3*c^2*d*x+935*a^3*d^3-3795*a
^2*b*c*d^2+5865*a*b^2*c^2*d-4301*b^3*c^3)/(d*x+c)^(23/6)/(a^4*d^4-4*a^3*b*c
*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(29/6)), x)
```

**mupad [B]** time = 1.20, size = 292, normalized size = 2.15

$$\frac{(c+dx)^{1/6}\left(\frac{7776\,b^4\,x^4}{21505\,d\,(a\,d-b\,c)^4}-\frac{5610\,a^4\,d^3-22770\,a^3\,b\,c\,d^2+35190\,a^2\,b^2\,c^2\,d-25806\,a\,b^3\,c^3}{21505\,d^4\,(a\,d-b\,c)^4}+\frac{x\left(330\,a^3\,b\,d^3-2070\,a^2\,b^2\,c\,d^2+7038\,a\,b^3\,c^2\,d+25806\,b^4\,c^3\right)}{21505\,d^4\,(a\,d-b\,c)^4}+\frac{1296\,b^3\,x^3\,(a\,d+23\,b\,c)}{21505\,d^2\,(a\,d-b\,c)^4}+\frac{108\,b^2\,x^2\left(-5\,a^2\,d^2+46\,a\,b\,c\,d+391\,b^2\,c^2\right)}{21505\,d^3\,(a\,d-b\,c)^4}\right)}{x^4\,(a+b\,x)^{1/6}+\frac{c^4\,(a+b\,x)^{1/6}}{d^4}+\frac{6\,c^2\,x^2\,(a+b\,x)^{1/6}}{d^2}+\frac{4\,c\,x^3\,(a+b\,x)^{1/6}}{d}+\frac{4\,c^3\,x\,(a+b\,x)^{1/6}}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((7776*b^4*x^4)/(21505*d*(a*d - b*c)^4) - (5610*a^4*d^3 -
25806*a*b^3*c^3 + 35190*a^2*b^2*c^2*d - 22770*a^3*b*c*d^2)/(21505*d^4*(a*d
- b*c)^4) + (x*(25806*b^4*c^3 + 330*a^3*b*d^3 - 2070*a^2*b^2*c*d^2 + 7038*a
*b^3*c^2*d))/(21505*d^4*(a*d - b*c)^4) + (1296*b^3*x^3*(a*d + 23*b*c))/(215
05*d^2*(a*d - b*c)^4) + (108*b^2*x^2*(391*b^2*c^2 - 5*a^2*d^2 + 46*a*b*c*d)
)/(21505*d^3*(a*d - b*c)^4)))/(x^4*(a + b*x)^(1/6) + (c^4*(a + b*x)^(1/6))/
d^4 + (6*c^2*x^2*(a + b*x)^(1/6))/d^2 + (4*c*x^3*(a + b*x)^(1/6))/d + (4*c^
3*x*(a + b*x)^(1/6))/d^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(29/6),x)
```

```
[Out] Timed out
```

## 3.1540 $\int \frac{(c+dx)^{11/6}}{(a+bx)^{5/6}}\,dx$

**Optimal**. Leaf size=424

$$-\frac{55(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}-\frac{55(bc-}{}$$

**Rubi [A]** time = 0.55, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{11\sqrt[6]{a+bx}(c+dx)^{5/6}(bc-ad)}{12b^2}-\frac{55(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}-\frac{55(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}}+\frac{\sqrt[6]{a+bx}(c+dx)^{11/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```

```
[Out] (11*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b^2) + ((a + b*x)^(1/6
)*(c + d*x)^(11/6))/(2*b) - (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*
d^(1/6)) + (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))
/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*d^(1/6)) + (55*(b
*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(
36*b^(17/6)*d^(1/6)) - (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(
1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])
/(144*b^(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)])/(144*b^(17/6)*d^(1/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{11/6}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(11(bc-ad))\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx}{12b}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{72b^2}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\right.}{12b^3}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x\right.}{12b^3}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x}\right.}{36b^{17/6}}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}} + $$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}} - $$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} - \frac{55(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}} + $$

**Mathematica [C]** time = 0.06, size = 71, normalized size = 0.17

$$\frac{6\sqrt[6]{a+bx}\,(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, 1/6, 7/6, (d*(
a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

**IntegrateAlgebraic [A]** time = 0.43, size = 356, normalized size = 0.84

$$-\frac{55(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{72b^{17/6}\sqrt[6]{d}}+\frac{\sqrt[6]{a+bx}\,(bc-ad)^2\left(17b-\frac{11d(a+bx)}{c+dx}\right)}{12b^2\sqrt[6]{c+dx}\left(b-\frac{d(a+bx)}{c+dx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1/6)*(17*b - (11*d*(a + b*x))/(c + d*x)))/(12*b^2
*(c + d*x)^(1/6)*(b - (d*(a + b*x))/(c + d*x))^2) - (55*(b*c - a*d)^2*ArcTa
n[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))
])/(24*Sqrt[3]*b^(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^
(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1
/6)*(c + d*x)^(1/6))])/(36*b^(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*ArcTanh[(b
^(1/6)*d^(1/6)*(a + b*x)^(1/6))/((c + d*x)^(1/6)*(b^(1/3) + (d^(1/3)*(a + b
*x)^(1/3))/(c + d*x)^(1/3)))])/(72*b^(17/6)*d^(1/6))
```

**fricas [B]** time = 2.18, size = 5591, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="fricas")
```

```
[Out] -1/144*(220*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d
^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*
a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c
^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^16*c^2*d - 2*a*b^15*c*d^2 + a^2*b^14*d^3)*(b*x
 + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
```

10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))
^(5/6) - 2*sqrt(3)*(b^14*d^2*x + b^14*c*d)*sqrt(((b^5*c^2 - 2*a*b^4*c*d + a
^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^17*d))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^6*d*x + b^6*c)*(
(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
+ 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7
*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*
d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/3))/(d*x + c))*((b^12*c^1
2 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4
*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*
d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12
*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) + sqrt(3)*(b^12*c^13 - 12*a*b^1
1*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^
4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a
^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2
*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*
d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924
*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*
c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c
^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*
a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c
^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 -
12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*
a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7
*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9
- 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^1
3)*x)) + 220*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*
d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924
*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*
c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^16*c^2*d - 2*a*b^15*c*d^2 + a^2*b^14*d^3)*(b*
x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d)
)^(5/6) - 2*sqrt(3)*(b^14*d^2*x + b^14*c*d)*sqrt(-((b^5*c^2 - 2*a*b^4*c*d +
a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*
d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*
a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c

^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^6*d*x + b^6*c)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) - 55*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6)*log(3025*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^6*d*x + b^6*c)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/3))/(d*x + c)) + 55*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6)*log(-3025*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22

```
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6) - (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x
+ a)^(1/3)*(d*x + c)^(2/3) - (b^6*d*x + b^6*c)*((b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(b^17*d))^(1/3))/(d*x + c)) - 110*b^2*((b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(b^17*d))^(1/6)*log(55*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a
)^(1/6)*(d*x + c)^(5/6) + (b^3*d*x + b^3*c)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^17*d))^(1/6))/(d*x + c)) + 110*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(b^17*d))^(1/6)*log(55*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(
1/6)*(d*x + c)^(5/6) - (b^3*d*x + b^3*c)*((b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(b^17*d))^(1/6))/(d*x + c)) - 12*(6*b*d*x + 17*b*c - 11*a*d)*(b*x + a)^
(1/6)*(d*x + c)^(5/6))/b^2
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(11/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(11/6)/(b*x+a)^(5/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(5/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{11/6}}{(a+b\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(11/6)/(a + b*x)^(5/6),x)
```

```
[Out] int((c + d*x)^(11/6)/(a + b*x)^(5/6), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(11/6)/(b*x+a)**(5/6),x)
```

```
[Out] Timed out
```

## 3.1541 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx$

**Optimal**. Leaf size=378

$$-\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}-\frac{5(bc-ad)\,\text{ta}}{2}$$

**Rubi [A]** time = 0.48, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}}+\frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/b - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
11/6)*d^(1/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(11/6)*d^(1/6)) + (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(11/6)*d^(1/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(11/6)*d^(1/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(11/6)*d^(1/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6b}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{11/6}} + \frac{(5(bc-ad))\,\text{Su}\cdots}{\cdots}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}} + \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x\cdots}\cdots\right.}{4b^{5/3}}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}} - \frac{5(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\cdots}{\cdots}\right.}{12b^{11/6}\sqrt[6]{d}}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} - \frac{5(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}} + \frac{5(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}} + \frac{5(bc\cdots}{\cdots}$$

**Mathematica** [C] time = 0.03, size = 71, normalized size = 0.19

$$\frac{6\sqrt[6]{a+bx}\,(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 1/6, 7/6, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

**IntegrateAlgebraic [F]** time = 106.34, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c + d*x)^(5/6)/(a + b*x)^(5/6), x]
```

**fricas [B]** time = 1.83, size = 2997, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="fricas")
```

```
[Out] 1/12*(20*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*
b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)
*arctan(1/3*(2*sqrt(3)*(b^10*c*d - a*b^9*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/
6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15
*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + 2*sqrt(3)*(b^
9*d^2*x + b^9*c*d)*sqrt(((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*
((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^
4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) + (b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d*x + b^4*c)*((b^6*c
^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c))*((b^6*c^6 - 6
*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + sqrt(3)*(b^6*c^7 - 6*a*b^5*c^
6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*
b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^
6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^
2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15
*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6
+ a^6*d^7)*x)) + 20*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^
2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^1
1*d))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^10*c*d - a*b^9*d^2)*(b*x + a)^(1/6)*(d
*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c
^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + 2*
```

```
sqrt(3)*(b^9*d^2*x + b^9*c*d)*sqrt(-((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x
 + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) - (b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^4*d*x + b^
4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c))*(
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) - sqrt(3)*(b^6*c^7
- 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*
d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b
^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*
d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3
 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*
c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*
a^5*b*c*d^6 + a^6*d^7)*x)) + 5*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
b^11*d))^(1/6)*log(25*((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((
b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) + (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d*x + b^4*c)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c)) - 5*b*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(-25*((b^3*c - a*b^2*d)*
(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*
d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b
^11*d))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(
2/3) - (b^4*d*x + b^4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))
^(1/3))/(d*x + c)) + 10*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d)
)^(1/6)*log(-5*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) + (b^2*d*x + b^
2*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6))/(d*x + c)) -
 10*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(-5*((b*c
 - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) - (b^2*d*x + b^2*c)*((b^6*c^6 - 6*a
*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6))/(d*x + c)) + 12*(b*x + a)^(1/6)*
(d*x + c)^(5/6))/b
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="giac")
[Out] Timed out
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(5/6),x)
[Out] int((d*x+c)^(5/6)/(b*x+a)^(5/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(5/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/6}}{(a+b\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(5/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(5/6),x)
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(5/6), x)
```

## 3.1542 $\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx$

**Optimal**. Leaf size=309

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\cdots$$

**Rubi [A]** time = 0.44, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*
(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1
/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))
+ (2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)
*d^(1/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1
/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6)) + Log[b^(
1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*
x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx = \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{b}$$

$$= \frac{6\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^{5/6}} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt[6]{b}+\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}+\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+b}}{\sqrt[6]{c+d}}\cdots\right.}{b^{5/6}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{\cdots}{\sqrt[3]{b}+\cdots}\right.}{\cdots}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{5/6}\sqrt[6]{d}} + \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\cdots\right.}{2b^{5/6}\sqrt[6]{\cdots}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} - \frac{\log\left(\cdots\right.}{\cdots}$$

**Mathematica [C]** time = 0.04, size = 71, normalized size = 0.23

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 1/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/6))
```

**IntegrateAlgebraic [A]** time = 0.22, size = 233, normalized size = 0.75

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{b^{5/6}\sqrt[6]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*
(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1
/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))
+ (2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)
*d^(1/6)) + ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/((c + d*x)^(1/6)*(b^(
1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)))]/(b^(5/6)*d^(1/6))
```

**fricas [B]** time = 1.68, size = 620, normalized size = 2.01

$$-2\sqrt{3}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b^4d\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^4d^2x+b^4cd\right)\sqrt{\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}+\left(b^2dx+b^2c\right)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}}\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}+\sqrt{3}(dx+c)}{3(dx+c)}\right)-2\sqrt{3}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b^4d\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^4d^2x+b^4cd\right)\sqrt{-\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}-\left(b^2dx+b^2c\right)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}}\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}-\sqrt{3}(dx+c)}{3(dx+c)}\right)+\frac{1}{2}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(\frac{4\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}+\left(b^2dx+b^2c\right)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\right)}{dx+c}\right)-\frac{1}{2}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(-\frac{4\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}-\left(b^2dx+b^2c\right)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\right)}{dx+c}\right)+\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(\frac{(bdx+bc)\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}+(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{dx+c}\right)-\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(-\frac{(bdx+bc)\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}-(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{dx+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*(1/(b^5*d))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x +
c)^(5/6)*b^4*d*(1/(b^5*d))^(5/6) - 2*sqrt(3)*(b^4*d^2*x + b^4*c*d)*sqrt(((b
*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5*d))^(1/6) + (b^2*d*x + b^2*c)*(1/(b
^5*d))^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c))*(1/(b^5*d))^(5/6
) + sqrt(3)*(d*x + c))/(d*x + c)) - 2*sqrt(3)*(1/(b^5*d))^(1/6)*arctan(-1/3
*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b^4*d*(1/(b^5*d))^(5/6) - 2*sqr
t(3)*(b^4*d^2*x + b^4*c*d)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5
*d))^(1/6) - (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) - (b*x + a)^(1/3)*(d*x + c
)^(2/3))/(d*x + c))*(1/(b^5*d))^(5/6) - sqrt(3)*(d*x + c))/(d*x + c)) + 1/2
*(1/(b^5*d))^(1/6)*log(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5*d))^(1/
6) + (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))
/(d*x + c)) - 1/2*(1/(b^5*d))^(1/6)*log(-4*((b*x + a)^(1/6)*(d*x + c)^(5/6)
*b*(1/(b^5*d))^(1/6) - (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) - (b*x + a)^(1/3
)*(d*x + c)^(2/3))/(d*x + c)) + (1/(b^5*d))^(1/6)*log(((b*d*x + b*c)*(1/(b^
5*d))^(1/6) + (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d*x + c)) - (1/(b^5*d))^(1/
6)*log(-((b*d*x + b*c)*(1/(b^5*d))^(1/6) - (b*x + a)^(1/6)*(d*x + c)^(5/6))
/(d*x + c))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(1/6)), x)
```

**maple [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}} (dx+c)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(1/6)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(1/6)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{6}} \sqrt[6]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(1/6)), x)
```

## 3.1543 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx$

**Optimal**. Leaf size=30

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\,(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.00

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 30, normalized size = 1.00

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]
[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))
```

**fricas [A]** time = 1.35, size = 42, normalized size = 1.40

$$\frac{6\,(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="fricas")
[Out] 6*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(7/6)), x)
```

**maple [A]** time = 0.00, size = 27, normalized size = 0.90

$$-\frac{6\,(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x)
[Out] -6*(b*x+a)^(1/6)/(d*x+c)^(1/6)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(7/6)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{7/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x)
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(7/6)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(7/6),x)
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(7/6)), x)
```

## 3.1544 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}} dx$

**Optimal**. Leaf size=66

$$\frac{36b\sqrt[6]{a+bx}}{7\sqrt[6]{c+dx}\,(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{7(c+dx)^{7/6}(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b\sqrt[6]{a+bx}}{7\sqrt[6]{c+dx}\,(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(7*(b*c - a*d)*(c + d*x)^(7/6)) + (36*b*(a + b*x)^(1/6)
)/(7*(b*c - a*d)^2*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}} + \frac{(6b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx}{7(bc-ad)}$$
$$= \frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}} + \frac{36b\sqrt[6]{a+bx}}{7(bc-ad)^2\sqrt[6]{c+dx}}$$

**Mathematica [A]** time = 0.02, size = 46, normalized size = 0.70

$$\frac{6\sqrt[6]{a+bx}\,(-ad+7bc+6bdx)}{7(c+dx)^{7/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(7*b*c - a*d + 6*b*d*x))/(7*(b*c - a*d)^2*(c + d*x)^(7/6
))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 57, normalized size = 0.86

$$\frac{6\left(\frac{7b\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}} - \frac{d(a+bx)^{7/6}}{(c+dx)^{7/6}}\right)}{7(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(-((d*(a + b*x)^(7/6))/(c + d*x)^(7/6)) + (7*b*(a + b*x)^(1/6))/(c + d*x
)^(1/6)))/(7*(b*c - a*d)^2)
```

**fricas [B]** time = 1.44, size = 118, normalized size = 1.79

$$\frac{6\,(6\,bdx+7\,bc-ad)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{7\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] 6/7*(6*b*d*x + 7*b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^2*c^4 - 2*a*
b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*
c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(13/6)), x)
```

**maple [A]** time = 0.01, size = 53, normalized size = 0.80

$$-\frac{6\,(bx+a)^{\frac{1}{6}}\,(-6bdx+ad-7bc)}{7\,(dx+c)^{\frac{7}{6}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x)
```

```
[Out] -6/7*(b*x+a)^(1/6)*(-6*b*d*x+a*d-7*b*c)/(d*x+c)^(7/6)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(13/6)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a+b\,x)^{5/6}\,(c+d\,x)^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(13/6)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

## 3.1545 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx$

**Optimal**. Leaf size=101

$$\frac{432b^2\sqrt[6]{a+bx}}{91\sqrt[6]{c+dx}\,(bc-ad)^3}+\frac{72b\sqrt[6]{a+bx}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{13(c+dx)^{13/6}(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2\sqrt[6]{a+bx}}{91\sqrt[6]{c+dx}\,(bc-ad)^3}+\frac{72b\sqrt[6]{a+bx}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (72*b*(a + b*x)^(1/
6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6)) + (432*b^2*(a + b*x)^(1/6))/(91*(b*c
 - a*d)^3*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}} + \frac{(12b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx}{13(bc-ad)}$$
$$= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}} + \frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}} + \frac{\left(72b^2\right)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx}{91(bc-ad)^2}$$
$$= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}} + \frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}} + \frac{432b^2\sqrt[6]{a+bx}}{91(bc-ad)^3\sqrt[6]{c+dx}}$$

**Mathematica** [A] time = 0.04, size = 77, normalized size = 0.76

$$\frac{6\sqrt[6]{a+bx}\left(7a^2d^2-2abd(13c+6dx)+b^2\left(91c^2+156cdx+72d^2x^2\right)\right)}{91(c+dx)^{13/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(7*a^2*d^2 - 2*a*b*d*(13*c + 6*d*x) + b^2*(91*c^2 + 156*
c*d*x + 72*d^2*x^2)))/(91*(b*c - a*d)^3*(c + d*x)^(13/6))
```

**IntegrateAlgebraic** [A] time = 0.12, size = 83, normalized size = 0.82

$$\frac{6\left(\frac{91b^2\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{7d^2(a+bx)^{13/6}}{(c+dx)^{13/6}}-\frac{26bd(a+bx)^{7/6}}{(c+dx)^{7/6}}\right)}{91(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*((7*d^2*(a + b*x)^(13/6))/(c + d*x)^(13/6) - (26*b*d*(a + b*x)^(7/6))/(c
 + d*x)^(7/6) + (91*b^2*(a + b*x)^(1/6))/(c + d*x)^(1/6)))/(91*(b*c - a*d)^
3)
```

**fricas** [B] time = 1.24, size = 252, normalized size = 2.50

$$\frac{6\left(72\,b^2d^2x^2+91\,b^2c^2-26\,abcd+7\,a^2d^2+12\left(13\,b^2cd-abd^2\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{91\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3+\left(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6\right)x^3+3\left(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4-a^3cd^5\right)x^2+3\left(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/91*(72*b^2*d^2*x^2 + 91*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2 + 12*(13*b^2*c*d
 - a*b*d^2)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^3*c^6 - 3*a*b^2*c^5*d + 3
*a^2*b*c^4*d^2 - a^3*c^3*d^3 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d
^5 - a^3*d^6)*x^3 + 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^
3*c*d^5)*x^2 + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d
^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(19/6)), x)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.04

$$-\frac{6\,(bx+a)^{\frac{1}{6}}\left(72b^2x^2d^2-12ab\,d^2x+156b^2cdx+7a^2d^2-26abcd+91b^2c^2\right)}{91\,(dx+c)^{\frac{13}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x)
```

```
[Out] -6/91*(b*x+a)^(1/6)*(72*b^2*d^2*x^2-12*a*b*d^2*x+156*b^2*c*d*x+7*a^2*d^2-26
*a*b*c*d+91*b^2*c^2)/(d*x+c)^(13/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(19/6)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{5/6}\,(c+d\,x)^{19/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(19/6)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

## 3.1546 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{25/6}}\,dx$

**Optimal**. Leaf size=136

$$\frac{7776b^3\sqrt[6]{a+bx}}{1729\sqrt[6]{c+dx}\,(bc-ad)^4}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{108b\sqrt[6]{a+bx}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{19(c+dx)^{19/6}(bc-ad)}$$

**Rubi [A]** time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3\sqrt[6]{a+bx}}{1729\sqrt[6]{c+dx}\,(bc-ad)^4}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{108b\sqrt[6]{a+bx}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (108*b*(a + b*x)^(1
/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (1296*b^2*(a + b*x)^(1/6))/(172
9*(b*c - a*d)^3*(c + d*x)^(7/6)) + (7776*b^3*(a + b*x)^(1/6))/(1729*(b*c -
a*d)^4*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{25/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{(18b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx}{19(bc-ad)}$$

$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{\left(216b^2\right)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}}{247(bc-ad)^2}$$

$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{1296b^2\sqrt[6]{a+bx}}{1729(bc-ad)^3(c+dx)^{7/6}}$$

$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{1296b^2\sqrt[6]{a+bx}}{1729(bc-ad)^3(c+dx)^{7/6}}$$

**Mathematica [A]** time = 0.06, size = 118, normalized size = 0.87

$$\frac{6\sqrt[6]{a+bx}\left(-91a^3d^3+21a^2bd^2(19c+6dx)-3ab^2d\left(247c^2+228cdx+72d^2x^2\right)+b^3\left(1729c^3+4446c^2dx+4104cd^2x^2+1296d^3x^3\right)\right)}{1729(c+dx)^{19/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(-91*a^3*d^3 + 21*a^2*b*d^2*(19*c + 6*d*x) - 3*a*b^2*d*(
247*c^2 + 228*c*d*x + 72*d^2*x^2) + b^3*(1729*c^3 + 4446*c^2*d*x + 4104*c*d
^2*x^2 + 1296*d^3*x^3)))/(1729*(b*c - a*d)^4*(c + d*x)^(19/6))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 109, normalized size = 0.80

$$\frac{6\left(\frac{1729b^3\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}-\frac{741b^2d(a+bx)^{7/6}}{(c+dx)^{7/6}}-\frac{91d^3(a+bx)^{19/6}}{(c+dx)^{19/6}}+\frac{399bd^2(a+bx)^{13/6}}{(c+dx)^{13/6}}\right)}{1729(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*((-91*d^3*(a + b*x)^(19/6))/(c + d*x)^(19/6) + (399*b*d^2*(a + b*x)^(13/
6))/(c + d*x)^(13/6) - (741*b^2*d*(a + b*x)^(7/6))/(c + d*x)^(7/6) + (1729*
b^3*(a + b*x)^(1/6))/(c + d*x)^(1/6)))/(1729*(b*c - a*d)^4)
```

**fricas [B]** time = 1.11, size = 420, normalized size = 3.09

$$\frac{6\left(1296\,b^3d^3x^3+1729\,b^3c^3-741\,ab^2c^2d+399\,a^2bcd^2-91\,a^3d^3+216\left(19\,b^3cd^2-ab^2d^3\right)x^2+18\left(247\,b^3c^2d-38\,ab^2cd^2+7\,a^2bd^3\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{1729\left(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4+\left(b^4c^4d^4-4\,ab^3c^3d^5+6\,a^2b^2c^2d^6-4\,a^3bcd^7+a^4d^8\right)x^4+4\left(b^4c^5d^3-4\,ab^3c^4d^4+6\,a^2b^2c^3d^5-4\,a^3bc^2d^6+a^4cd^7\right)x^3+6\left(b^4c^6d^2-4\,ab^3c^5d^3+6\,a^2b^2c^4d^4-4\,a^3bc^3d^5+a^4c^2d^6\right)x^2+4\left(b^4c^7d-4\,ab^3c^6d^2+6\,a^2b^2c^5d^3-4\,a^3bc^4d^4+a^4c^3d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/1729*(1296*b^3*d^3*x^3 + 1729*b^3*c^3 - 741*a*b^2*c^2*d + 399*a^2*b*c*d^2
 - 91*a^3*d^3 + 216*(19*b^3*c*d^2 - a*b^2*d^3)*x^2 + 18*(247*b^3*c^2*d - 38
*a*b^2*c*d^2 + 7*a^2*b*d^3)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^4*c^8 - 4
*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (b^4*c^4
*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^8)*x^4 +
 4*(b^4*c^5*d^3 - 4*a*b^3*c^4*d^4 + 6*a^2*b^2*c^3*d^5 - 4*a^3*b*c^2*d^6 + a
^4*c*d^7)*x^3 + 6*(b^4*c^6*d^2 - 4*a*b^3*c^5*d^3 + 6*a^2*b^2*c^4*d^4 - 4*a^
3*b*c^3*d^5 + a^4*c^2*d^6)*x^2 + 4*(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6*a^2*b^2
*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(25/6)), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.26

$$-\frac{6\,(bx+a)^{\frac{1}{6}}\left(-1296b^3d^3x^3+216a\,b^2d^3x^2-4104b^3c\,d^2x^2-126a^2b\,d^3x+684a\,b^2c\,d^2x-4446b^3c^2dx+91a^3d^3-399a^2bc\,d^2+741a\,b^2c^2d-1729b^3c^3\right)}{1729\,(dx+c)^{\frac{19}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x)
```

```
[Out] -6/1729*(b*x+a)^(1/6)*(-1296*b^3*d^3*x^3+216*a*b^2*d^3*x^2-4104*b^3*c*d^2*x
^2-126*a^2*b*d^3*x+684*a*b^2*c*d^2*x-4446*b^3*c^2*d*x+91*a^3*d^3-399*a^2*b*
c*d^2+741*a*b^2*c^2*d-1729*b^3*c^3)/(d*x+c)^(19/6)/(a^4*d^4-4*a^3*b*c*d^3+6
*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(25/6)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{5/6}\,(c+d\,x)^{25/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x)
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(25/6)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(25/6),x)
[Out] Timed out
```

## 3.1547 $\int \frac{(c+dx)^{13/6}}{(a+bx)^{7/6}}\,dx$

**Optimal**. Leaf size=449

$$-\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\cdots$$

**Rubi [A]** time = 0.66, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {47, 50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2}+\frac{91d(a+bx)^{5/6}\sqrt[6]{c+dx}\,(bc-ad)}{12b^3}-\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{19/6}}-\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{19/6}}-\frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] (91*d*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^3) + (13*d*(a + b*
x)^(5/6)*(c + d*x)^(7/6))/(2*b^2) - (6*(c + d*x)^(13/6))/(b*(a + b*x)^(1/6)
) + (91*d^(1/6)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6)
)/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) - (91*d^(1/6)*(
b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcT
anh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(19/6)) - (
91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6))
+ (91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*
x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6
))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{13/6}}{(a+bx)^{7/6}}\,dx = -\frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(13d)\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx}{b}$$

$$= \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(91d(bc-ad))\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{12b^2}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\left(91d(bc-ad\right.}{}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\left(91d(bc-ad\right.}{}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\left(91d(bc-ad\right.}{}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\left(91\sqrt[3]{d}\,(bc-\right.}{}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{d}\,(bc-a}{}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{d}\,(bc-a}{}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{d}\,(bc-a}{}$$

**Mathematica [C]** time = 0.08, size = 71, normalized size = 0.16

$$-\frac{6(c+dx)^{13/6}\,{}_2F_1\left(-\frac{13}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, -1/6, 5/6, (d*(a + b*x))/(-(b
*c) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

**IntegrateAlgebraic [A]** time = 0.81, size = 389, normalized size = 0.87

$$-\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt{3}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}\right)}{24\sqrt{3}\,b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt{3}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}\right)}{36b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{72b^{19/6}}-\frac{(bc-ad)^2\left(\frac{72b^2(c+dx)^{13/6}}{(a+bx)^{13/6}}+\frac{91d^2\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}-\frac{169bd(c+dx)^{7/6}}{(a+bx)^{7/6}}\right)}{12b^3\left(\frac{b(c+dx)}{a+bx}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] -1/12*((b*c - a*d)^2*((91*d^2*(c + d*x)^(1/6))/(a + b*x)^(1/6) - (169*b*d*(
c + d*x)^(7/6))/(a + b*x)^(7/6) + (72*b^2*(c + d*x)^(13/6))/(a + b*x)^(13/6
)))/(b^3*(-d + (b*(c + d*x))/(a + b*x))^2) - (91*d^(1/6)*(b*c - a*d)^2*ArcT
an[1/Sqrt[3] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6)
)])/(24*Sqrt[3]*b^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2
*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))])/(24*Sqrt[3]*b
^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(
1/6)*(a + b*x)^(1/6))])/(36*b^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcTanh[(
b^(1/6)*d^(1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(1/3) + (b^(1/3)*(c +
d*x)^(1/3))/(a + b*x)^(1/3)))])/(72*b^(19/6))
```

**fricas [B]** time = 2.22, size = 5690, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/144*(364*sqrt(3)*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66
*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^
7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9
 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^
13)/b^19)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^18*c^2 - 2*a*b^17*c*d + a^2*b^16*
d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 6
6*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b
^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^
9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d
^13)/b^19)^(5/6) - 2*sqrt(3)*(b^17*x + a*b^16)*sqrt(((b^5*c^2 - 2*a*b^4*c*d
 + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c
^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5
- 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8
```

*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/3))/(b*x + a))*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) + sqrt(3)*(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x))/(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x)) + 364*sqrt(3)*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^18*c^2 - 2*a*b^17*c*d + a^2*b^16*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) - 2*sqrt(3)*(b^17*x + a*b^16)*sqrt(-((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^1

9)^(1/3))/(b*x + a))*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) - sqrt(3)*(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x))/(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x)) - 91*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(8281*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/3))/(b*x + a)) + 91*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(-8281*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 -

```
 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*
b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^1
2 + a^12*d^13)/b^19)^(1/3))/(b*x + a)) - 182*(b^4*x + a*b^3)*((b^12*c^12*d
- 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4
*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*
d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 1
2*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(91*((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b^4*x + a*b^3)*((b^12*c^12*d - 12*
a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*
c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 +
 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^1
1*b*c*d^12 + a^12*d^13)/b^19)^(1/6))/(b*x + a)) + 182*(b^4*x + a*b^3)*((b^1
2*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4
+ 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7
*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2
*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(91*((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b^4*x + a*b^3)*((b^12*c^1
2*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495
*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*
c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11
 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6))/(b*x + a)) - 12*(6*b^2*d^2*x^
2 - 72*b^2*c^2 + 169*a*b*c*d - 91*a^2*d^2 + (25*b^2*c*d - 13*a*b*d^2)*x)*(b
*x + a)^(5/6)*(d*x + c)^(1/6))/(b^4*x + a*b^3)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(7/6), x)
```

**maple [F]** time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(13/6)/(b*x+a)^(7/6),x)
```

```
[Out] int((d*x+c)^(13/6)/(b*x+a)^(7/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(7/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{13/6}}{(a+b\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(13/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(13/6)/(a + b*x)^(7/6), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(13/6)/(b*x+a)**(7/6),x)
[Out] Timed out
```

## 3.1548 $\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx$

**Optimal**. Leaf size=403

$$-\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{}}{}$$

**Rubi [A]** time = 0.60, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {47, 50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2}-\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{13/6}}-\frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{13/6}}-\frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] (7*d*(a + b*x)^(5/6)*(c + d*x)^(1/6))/b^2 - (6*(c + d*x)^(7/6))/(b*(a + b*x
)^(1/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(
1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]])/(2*Sqrt[3]*b^(13/6)) - (7*d^(1/6)
*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))]])/(2*Sqrt[3]*b^(13/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTanh[
(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))]])/(3*b^(13/6)) - (7*d^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(12*b^(13/6)) + (7*d^(1
/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (
b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]])/(12*b^(13/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx = -\frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d)\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{b}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6b^2}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^3}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^3}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{\left(7\sqrt[3]{d}\,(bc-ad)\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+b\cdots}}{\sqrt[6]{c+\cdots}}\cdots\right.}{3b^{13/6}}\cdots$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{13/6}} - \frac{\left(7\sqrt[6]{d}\,(bc-ad)\right)\text{S}\cdots}{\cdots}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{13/6}} - \frac{7\sqrt[6]{d}\,(bc-ad)\log\cdots}{\cdots}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{13/6}} - \frac{7\sqrt[6]{d}\,(bc-ad)\tan\cdots}{2\sqrt{3}\cdots}$$

**Mathematica** [C] time = 0.06, size = 71, normalized size = 0.18

$$-\frac{6(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, -1/6, 5/6, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

**IntegrateAlgebraic [F]** time = 156.14, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c + d*x)^(7/6)/(a + b*x)^(7/6), x]
```

**fricas [B]** time = 2.01, size = 3084, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] 1/12*(28*sqrt(3)*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4
*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^
7)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b^12*c - a*b^11*d)*(b*x + a)^(5/6)*(d
*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b
^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) + 2*
sqrt(3)*(b^12*x + a*b^11)*sqrt(((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)
^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*
d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*x + a*b^4)*((
b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*
a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a))*((b^6*c^
6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^
2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) + sqrt(3)*(a*b^6*c^6*d - 6
*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2
*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*
c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b
*d^7)*x))/(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^
3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a
*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5
 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 28*sqrt(3)*(b^3*x + a*b^2)*((b^6*c^6*
d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*
c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b^12*
c - a*b^11*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2
 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c
*d^6 + a^6*d^7)/b^13)^(5/6) + 2*sqrt(3)*(b^12*x + a*b^11)*sqrt(-((b^3*c - a
```

```
*b^2*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*
a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 +
 a^6*d^7)/b^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*
x + c)^(1/3) - (b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c
^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)
/b^13)^(1/3))/(b*x + a))*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3
 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)
^(5/6) - sqrt(3)*(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20
*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*
d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*
c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x))/(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2
 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c
*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3
*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 7*(b
^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b
^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*log(
49*((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5
*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6
*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3) + (b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2
+ 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*
d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a)) - 7*(b^3*x + a*b^2)*((b^6*c^6*d - 6*
a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^
5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*log(-49*((b^3*c - a*b^2*d)*(b*x +
a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3
 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)
^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) -
(b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3
*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(
b*x + a)) + 14*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c
^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)
/b^13)^(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b^3*x +
 a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3
*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6))/(b*x + a)
) - 14*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(
1/6)*log(-7*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b^3*x + a*b^2)*
((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 1
5*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6))/(b*x + a)) + 12*(
b*d*x - 6*b*c + 7*a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6))/(b^3*x + a*b^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(7/6), x)
```

**maple [F]** time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(7/6),x)
[Out] int((d*x+c)^(7/6)/(b*x+a)^(7/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(7/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^{7/6}}{{\left(a+b\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(7/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(7/6)/(a + b*x)^(7/6), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{7}{6}}}{(a+bx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(7/6),x)
```

```
[Out] Integral((c + d*x)**(7/6)/(a + b*x)**(7/6), x)
```

## 3.1549 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{7/6}}\,dx$

**Optimal**. Leaf size=332

$$-\frac{\sqrt[6]{d}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt[6]{d}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\cdots}{\sqrt{3}\cdots}\right.}{b^{7/6}}$$

**Rubi** [A] time = 0.54, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{\sqrt[6]{d}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt[6]{d}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{7/6}}+\frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/(b*(a + b*x)^(1/6)) + (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/b^(7/6)
 - (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))])/b^(7/6) + (2*d^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/b^(7/6) - (d^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)])/(2*b^(7/6)) + (d^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2
*b^(7/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{7/6}}\,dx = -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{d\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{b}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{(6d)\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{(6d)\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{\left(2\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^{7/6}} + \frac{\left(2\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}+}{\sqrt[3]{b}+\sqrt[6]{b}\,\sqrt[6]{d}}\right.}{b^{7/6}}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{2\sqrt[6]{d}\,\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{7/6}} - \frac{\sqrt[6]{d}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{7/6}} + \sqrt[6]{}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{2\sqrt[6]{d}\,\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{7/6}} - \frac{\sqrt[6]{d}\,\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{7/6}} + \frac{\sqrt[6]{d}\,\log}{}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{\sqrt{3}\,\sqrt[6]{d}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{7/6}} - \frac{\sqrt{3}\,\sqrt[6]{d}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{7/6}} + \frac{2\sqrt[6]{d}\,\tanh^{-1}\left(\frac{\sqrt[6]{}}{\sqrt[6]{}}\right.}{b^{7/6}}$$

**Mathematica [C]** time = 0.03, size = 71, normalized size = 0.21

$$-\frac{6\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\,\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

**IntegrateAlgebraic [A]** time = 0.26, size = 256, normalized size = 0.77

$$-\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{b^{7/6}}+\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{7/6}}+\frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{b^{7/6}}+\frac{\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{b^{7/6}}-\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/(b*(a + b*x)^(1/6)) - (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3
] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))]])/b^(7/6)
 + (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3] + (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]
*d^(1/6)*(a + b*x)^(1/6))]])/b^(7/6) + (2*d^(1/6)*ArcTanh[(b^(1/6)*(c + d*x)
^(1/6))/(d^(1/6)*(a + b*x)^(1/6))]])/b^(7/6) + (d^(1/6)*ArcTanh[(b^(1/6)*d^(
1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(1/3) + (b^(1/3)*(c + d*x)^(1/3))
/(a + b*x)^(1/3)))]])/b^(7/6)
```

**fricas [B]** time = 1.58, size = 663, normalized size = 2.00

$$-\frac{4\sqrt{3}\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b^{6}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^{7}x+ab^{6}\right)\sqrt{\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}+\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}+\sqrt{3}(bdx+ad)}{3(bdx+ad)}\right)+4\sqrt{3}\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b^{6}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^{7}x+ab^{6}\right)\sqrt{-\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}-\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}-\sqrt{3}(bdx+ad)}{3(bdx+ad)}\right)-\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}+\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right)+\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(-\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}-\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right)-2\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(\frac{\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}+(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right)+2\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(-\frac{\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}-(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right)+12(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{2\left(b^{2}x+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(3)*(b^2*x + a*b)*(d/b^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a
)^(5/6)*(d*x + c)^(1/6)*b^6*(d/b^7)^(5/6) - 2*sqrt(3)*(b^7*x + a*b^6)*sqrt(
((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6) + (b^3*x + a*b^2)*(d/b^7)^
(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a))*(d/b^7)^(5/6) + sqrt(3)
*(b*d*x + a*d))/(b*d*x + a*d)) + 4*sqrt(3)*(b^2*x + a*b)*(d/b^7)^(1/6)*arct
an(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*b^6*(d/b^7)^(5/6) - 2*sq
rt(3)*(b^7*x + a*b^6)*sqrt(-((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6
) - (b^3*x + a*b^2)*(d/b^7)^(1/3) - (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x +
 a))*(d/b^7)^(5/6) - sqrt(3)*(b*d*x + a*d))/(b*d*x + a*d)) - (b^2*x + a*b)*
(d/b^7)^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6) + (b^3
*x + a*b^2)*(d/b^7)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a)) + (
b^2*x + a*b)*(d/b^7)^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7
)^(1/6) - (b^3*x + a*b^2)*(d/b^7)^(1/3) - (b*x + a)^(2/3)*(d*x + c)^(1/3))/
(b*x + a)) - 2*(b^2*x + a*b)*(d/b^7)^(1/6)*log(((b^2*x + a*b)*(d/b^7)^(1/6)
```

```
 + (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b*x + a)) + 2*(b^2*x + a*b)*(d/b^7)^(1
/6)*log(-((b^2*x + a*b)*(d/b^7)^(1/6) - (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b
*x + a)) + 12*(b*x + a)^(5/6)*(d*x + c)^(1/6))/(b^2*x + a*b)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(7/6), x)
```

**maple [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(7/6),x)
[Out] int((d*x+c)^(1/6)/(b*x+a)^(7/6),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(7/6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^{1/6}}{{\left(a+b\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(7/6),x)
```

```
[Out] int((c + d*x)^(1/6)/(a + b*x)^(7/6), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(7/6),x)
```

```
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(7/6), x)
```

## 3.1550 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{5/6}}\,dx$

**Optimal**. Leaf size=30

$$-\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\,(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/((b*c - a*d)*(a + b*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{5/6}}\,dx = -\frac{6\sqrt[6]{c+dx}}{(bc-ad)\sqrt[6]{a+bx}}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.00

$$\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (6*(c + d*x)^(1/6))/((-(b*c) + a*d)*(a + b*x)^(1/6))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 30, normalized size = 1.00

$$-\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]
[Out] (-6*(c + d*x)^(1/6))/((b*c - a*d)*(a + b*x)^(1/6))
```

**fricas [A]** time = 1.43, size = 42, normalized size = 1.40

$$-\frac{6\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] -6*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(5/6)), x)
```

**maple [A]** time = 0.01, size = 27, normalized size = 0.90

$$\frac{6\,(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x)
[Out] 6/(b*x+a)^(1/6)*(d*x+c)^(1/6)/(a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(5/6)), x)
```

**mupad [B]** time = 0.68, size = 26, normalized size = 0.87

$$\frac{6\,(c+d\,x)^{1/6}}{(a\,d-b\,c)\,(a+b\,x)^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x)
[Out] (6*(c + d*x)^(1/6))/((a*d - b*c)*(a + b*x)^(1/6))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(5/6),x)
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(5/6)), x)
```

## 3.1551 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{11/6}}\,dx$

**Optimal**. Leaf size=64

$$-\frac{36d(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{36d(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6)) - (36*d*(a + b*x)^(5/6))/(
5*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{11/6}}\,dx = -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}} - \frac{(6d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx}{bc-ad}$$
$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}} - \frac{36d(a+bx)^{5/6}}{5(bc-ad)^2(c+dx)^{5/6}}$$

**Mathematica [A]** time = 0.03, size = 45, normalized size = 0.70

$$-\frac{6(ad+5bc+6bdx)}{5\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (-6*(5*b*c + a*d + 6*b*d*x))/(5*(b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(5/
6))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 49, normalized size = 0.77

$$-\frac{6(a+bx)^{5/6}\left(\frac{5b(c+dx)}{a+bx}+d\right)}{5(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (-6*(a + b*x)^(5/6)*(d + (5*b*(c + d*x))/(a + b*x)))/(5*(b*c - a*d)^2*(c +
d*x)^(5/6))
```

**fricas [B]** time = 1.33, size = 126, normalized size = 1.97

$$-\frac{6\,(6\,bdx+5\,bc+ad)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{5\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] -6/5*(6*b*d*x + 5*b*c + a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(11/6)), x)
```

**maple [A]** time = 0.01, size = 53, normalized size = 0.83

$$-\frac{6\,(6bdx+ad+5bc)}{5\,(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{5}{6}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x)
```

```
[Out] -6/5*(6*b*d*x+a*d+5*b*c)/(b*x+a)^(1/6)/(d*x+c)^(5/6)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(11/6)), x)
```

**mupad [B]** time = 0.83, size = 72, normalized size = 1.12

$$-\frac{\left(\frac{36\,b\,x}{5\,{\left(a\,d-b\,c\right)}^2}+\frac{6\,a\,d+30\,b\,c}{5\,d\,{\left(a\,d-b\,c\right)}^2}\right)\,{\left(c+d\,x\right)}^{1/6}}{x\,{\left(a+b\,x\right)}^{1/6}+\frac{c\,{\left(a+b\,x\right)}^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x)
```

```
[Out] -(((36*b*x)/(5*(a*d - b*c)^2) + (6*a*d + 30*b*c)/(5*d*(a*d - b*c)^2))*(c +
d*x)^(1/6))/(x*(a + b*x)^(1/6) + (c*(a + b*x)^(1/6))/d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(11/6),x)
```

```
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(11/6)), x)
```

## 3.1552 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{17/6}}\,dx$

**Optimal**. Leaf size=98

$$-\frac{432bd(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^3}-\frac{72d(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}(bc-ad)}$$

**Rubi** [A] time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{432bd(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^3}-\frac{72d(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(11/6)) - (72*d*(a + b*x)^(5/6))/
(11*(b*c - a*d)^2*(c + d*x)^(11/6)) - (432*b*d*(a + b*x)^(5/6))/(55*(b*c -
a*d)^3*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{17/6}}\,dx = -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}} - \frac{(12d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx}{bc-ad}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}} - \frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}} - \frac{(72bd)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)}}{11(bc-ad)^2}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}} - \frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}} - \frac{432bd(a+bx)^{5/6}}{55(bc-ad)^3(c+dx}$$

**Mathematica** [A] time = 0.04, size = 77, normalized size = 0.79

$$-\frac{6\left(-5a^2d^2+2abd(11c+6dx)+b^2\left(55c^2+132cdx+72d^2x^2\right)\right)}{55\sqrt[6]{a+bx}\,(c+dx)^{11/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (-6*(-5*a^2*d^2 + 2*a*b*d*(11*c + 6*d*x) + b^2*(55*c^2 + 132*c*d*x + 72*d^2
*x^2)))/(55*(b*c - a*d)^3*(a + b*x)^(1/6)*(c + d*x)^(11/6))
```

**IntegrateAlgebraic** [A] time = 0.13, size = 73, normalized size = 0.74

$$-\frac{6(a+bx)^{11/6}\left(\frac{55b^2(c+dx)^2}{(a+bx)^2}+\frac{22bd(c+dx)}{a+bx}-5d^2\right)}{55(c+dx)^{11/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (-6*(a + b*x)^(11/6)*(-5*d^2 + (22*b*d*(c + d*x))/(a + b*x) + (55*b^2*(c +
d*x)^2)/(a + b*x)^2))/(55*(b*c - a*d)^3*(c + d*x)^(11/6))
```

**fricas** [B] time = 1.57, size = 273, normalized size = 2.79

$$-\frac{6\left(72\,b^2d^2x^2+55\,b^2c^2+22\,abcd-5\,a^2d^2+12\left(11\,b^2cd+abd^2\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{55\left(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5\right)x^3+\left(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^2+\left(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] -6/55*(72*b^2*d^2*x^2 + 55*b^2*c^2 + 22*a*b*c*d - 5*a^2*d^2 + 12*(11*b^2*c*
d + a*b*d^2)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*
b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*
d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(17/6)), x)
```

**maple [A]** time = 0.01, size = 105, normalized size = 1.07

$$-\frac{6\left(-72b^{2}x^{2}d^{2}-12ab\,d^{2}x-132b^{2}cdx+5a^{2}d^{2}-22abcd-55b^{2}c^{2}\right)}{55\,(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{11}{6}}\,\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x)
```

```
[Out] -6/55*(-72*b^2*d^2*x^2-12*a*b*d^2*x-132*b^2*c*d*x+5*a^2*d^2-22*a*b*c*d-55*b
^2*c^2)/(b*x+a)^(1/6)/(d*x+c)^(11/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b
^3*c^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(17/6)), x)
```

**mupad [B]** time = 0.96, size = 132, normalized size = 1.35

$$\frac{(c+d\,x)^{1/6}\,\left(\frac{432\,b^2\,x^2}{55\,(a\,d-b\,c)^3}+\frac{-30\,a^2\,d^2+132\,a\,b\,c\,d+330\,b^2\,c^2}{55\,d^2\,(a\,d-b\,c)^3}+\frac{72\,b\,x\,(a\,d+11\,b\,c)}{55\,d\,(a\,d-b\,c)^3}\right)}{x^2\,(a+b\,x)^{1/6}+\frac{c^2\,(a+b\,x)^{1/6}}{d^2}+\frac{2\,c\,x\,(a+b\,x)^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((432*b^2*x^2)/(55*(a*d - b*c)^3) + (330*b^2*c^2 - 30*a^2*
d^2 + 132*a*b*c*d)/(55*d^2*(a*d - b*c)^3) + (72*b*x*(a*d + 11*b*c))/(55*d*(
a*d - b*c)^3)))/(x^2*(a + b*x)^(1/6) + (c^2*(a + b*x)^(1/6))/d^2 + (2*c*x*(
a + b*x)^(1/6))/d)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

## 3.1553 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{23/6}}\,dx$

**Optimal**. Leaf size=134

$$-\frac{7776b^2d(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^4}-\frac{1296bd(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^3}-\frac{108d(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}(bc-ad)}$$

**Rubi [A]** time = 0.03, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{7776b^2d(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^4}-\frac{1296bd(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^3}-\frac{108d(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(17/6)) - (108*d*(a + b*x)^(5/6))
/(17*(b*c - a*d)^2*(c + d*x)^(17/6)) - (1296*b*d*(a + b*x)^(5/6))/(187*(b*c
 - a*d)^3*(c + d*x)^(11/6)) - (7776*b^2*d*(a + b*x)^(5/6))/(935*(b*c - a*d)
^4*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{23/6}}\,dx = -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{(18d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx}{bc-ad}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{(216bd)\int \frac{1}{\sqrt[6]{a+bx}\,(c+d}}{17(bc-ad)^2}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{1296bd(a+bx)^{5}}{187(bc-ad)^3(c+d}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{1296bd(a+bx)^{5}}{187(bc-ad)^3(c+d}$$

**Mathematica [A]** time = 0.06, size = 118, normalized size = 0.88

$$-\frac{6\left(55a^3d^3-15a^2bd^2(17c+6dx)+3ab^2d\left(187c^2+204cdx+72d^2x^2\right)+b^3\left(935c^3+3366c^2dx+3672cd^2x^2+1296d^3x^3\right)\right)}{935\sqrt[6]{a+bx}\,(c+dx)^{17/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (-6*(55*a^3*d^3 - 15*a^2*b*d^2*(17*c + 6*d*x) + 3*a*b^2*d*(187*c^2 + 204*c*
d*x + 72*d^2*x^2) + b^3*(935*c^3 + 3366*c^2*d*x + 3672*c*d^2*x^2 + 1296*d^3
*x^3)))/(935*(b*c - a*d)^4*(a + b*x)^(1/6)*(c + d*x)^(17/6))
```

**IntegrateAlgebraic [A]** time = 0.14, size = 95, normalized size = 0.71

$$-\frac{6(a+bx)^{17/6}\left(\frac{935b^3(c+dx)^3}{(a+bx)^3}+\frac{561b^2d(c+dx)^2}{(a+bx)^2}-\frac{255bd^2(c+dx)}{a+bx}+55d^3\right)}{935(c+dx)^{17/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (-6*(a + b*x)^(17/6)*(55*d^3 - (255*b*d^2*(c + d*x))/(a + b*x) + (561*b^2*d
*(c + d*x)^2)/(a + b*x)^2 + (935*b^3*(c + d*x)^3)/(a + b*x)^3))/(935*(b*c -
 a*d)^4*(c + d*x)^(17/6))
```

**fricas [B]** time = 1.49, size = 457, normalized size = 3.41

$$-\frac{6\left(1296\,b^3d^3x^3+935\,b^3c^3+561\,ab^2c^2d-255\,a^2bcd^2+55\,a^3d^3+216\left(17\,b^3cd^2+ab^2d^3\right)x^2+18\left(187\,b^3c^2d+34\,ab^2cd^2-5\,a^2bd^3\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{935\left(ab^4c^7-4\,a^2b^3c^6d+6\,a^3b^2c^5d^2-4\,a^4bc^4d^3+a^5c^3d^4+\left(b^5c^4d^3-4\,ab^4c^3d^4+6\,a^2b^3c^2d^5-4\,a^3b^2cd^6+a^4bd^7\right)x^4+\left(3\,b^5c^5d^2-11\,ab^4c^4d^3+14\,a^2b^3c^3d^4-6\,a^3b^2c^2d^5-a^4bcd^6+a^5d^7\right)x^3+3\left(b^5c^6d-3\,ab^4c^5d^2+2\,a^2b^3c^4d^3+2\,a^3b^2c^3d^4-3\,a^4bc^2d^5+a^5cd^6\right)x^2+\left(b^5c^7-ab^4c^6d-6\,a^2b^3c^5d^2+14\,a^3b^2c^4d^3-11\,a^4bc^3d^4+3\,a^5c^2d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] -6/935*(1296*b^3*d^3*x^3 + 935*b^3*c^3 + 561*a*b^2*c^2*d - 255*a^2*b*c*d^2
+ 55*a^3*d^3 + 216*(17*b^3*c*d^2 + a*b^2*d^3)*x^2 + 18*(187*b^3*c^2*d + 34*
a*b^2*c*d^2 - 5*a^2*b*d^3)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^4*c^7 -
4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*
c^4*d^3 - 4*a*b^4*c^3*d^4 + 6*a^2*b^3*c^2*d^5 - 4*a^3*b^2*c*d^6 + a^4*b*d^7
)*x^4 + (3*b^5*c^5*d^2 - 11*a*b^4*c^4*d^3 + 14*a^2*b^3*c^3*d^4 - 6*a^3*b^2*
c^2*d^5 - a^4*b*c*d^6 + a^5*d^7)*x^3 + 3*(b^5*c^6*d - 3*a*b^4*c^5*d^2 + 2*a
^2*b^3*c^4*d^3 + 2*a^3*b^2*c^3*d^4 - 3*a^4*b*c^2*d^5 + a^5*c*d^6)*x^2 + (b^
5*c^7 - a*b^4*c^6*d - 6*a^2*b^3*c^5*d^2 + 14*a^3*b^2*c^4*d^3 - 11*a^4*b*c^3
*d^4 + 3*a^5*c^2*d^5)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(23/6)), x)
```

**maple [A]** time = 0.01, size = 171, normalized size = 1.28

$$-\frac{6\left(1296b^{3}d^{3}x^{3}+216a\,b^{2}d^{3}x^{2}+3672b^{3}c\,d^{2}x^{2}-90a^{2}b\,d^{3}x+612a\,b^{2}c\,d^{2}x+3366b^{3}c^{2}dx+55a^{3}d^{3}-255a^{2}bc\,d^{2}+561a\,b^{2}c^{2}d+935b^{3}c^{3}\right)}{935\,(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{17}{6}}\,\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x)
```

```
[Out] -6/935*(1296*b^3*d^3*x^3+216*a*b^2*d^3*x^2+3672*b^3*c*d^2*x^2-90*a^2*b*d^3*
x+612*a*b^2*c*d^2*x+3366*b^3*c^2*d*x+55*a^3*d^3-255*a^2*b*c*d^2+561*a*b^2*c
^2*d+935*b^3*c^3)/(b*x+a)^(1/6)/(d*x+c)^(17/6)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2
*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(23/6)), x)
```

**mupad [B]** time = 1.15, size = 209, normalized size = 1.56

$$-\frac{(c+d\,x)^{1/6}\,\left(\frac{7776\,b^3\,x^3}{935\,(a\,d-b\,c)^4}+\frac{330\,a^3\,d^3-1530\,a^2\,b\,c\,d^2+3366\,a\,b^2\,c^2\,d+5610\,b^3\,c^3}{935\,d^3\,(a\,d-b\,c)^4}+\frac{108\,b\,x\,\left(-5\,a^2\,d^2+34\,a\,b\,c\,d+187\,b^2\,c^2\right)}{935\,d^2\,(a\,d-b\,c)^4}+\frac{1296\,b^2\,x^2\,(a\,d+17\,b\,c)}{935\,d\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/6}+\frac{c^3\,(a+b\,x)^{1/6}}{d^3}+\frac{3\,c\,x^2\,(a+b\,x)^{1/6}}{d}+\frac{3\,c^2\,x\,(a+b\,x)^{1/6}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x)
```

```
[Out] -((c + d*x)^(1/6)*((7776*b^3*x^3)/(935*(a*d - b*c)^4) + (330*a^3*d^3 + 5610
*b^3*c^3 + 3366*a*b^2*c^2*d - 1530*a^2*b*c*d^2)/(935*d^3*(a*d - b*c)^4) + (
108*b*x*(187*b^2*c^2 - 5*a^2*d^2 + 34*a*b*c*d))/(935*d^2*(a*d - b*c)^4) + (
1296*b^2*x^2*(a*d + 17*b*c))/(935*d*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/6) +
 (c^3*(a + b*x)^(1/6))/d^3 + (3*c*x^2*(a + b*x)^(1/6))/d + (3*c^2*x*(a + b*
x)^(1/6))/d^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

## 3.1554 $\int (a+bx)^m(a+b(2+m)x)\,dx$

**Optimal**. Leaf size=11

$$x(a+bx)^{m+1}$$

**Rubi** [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {34}

$$x(a+bx)^{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a + b*(2 + m)*x),x]
```

```
[Out] x*(a + b*x)^(1 + m)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int (a+bx)^m(a+b(2+m)x)\,dx = x(a+bx)^{1+m}$$

**Mathematica** [A] time = 0.01, size = 11, normalized size = 1.00

$$x(a+bx)^{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a + b*(2 + m)*x),x]
```

```
[Out] x*(a + b*x)^(1 + m)
```

**IntegrateAlgebraic** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^m(a+b(2+m)x)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(a + b*(2 + m)*x),x]
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(a + b*(2 + m)*x), x]
```

**fricas [A]** time = 1.19, size = 17, normalized size = 1.55

$$\left(bx^2 + ax\right)(bx + a)^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="fricas")
[Out] (b*x^2 + a*x)*(b*x + a)^m
```

**giac [B]** time = 1.00, size = 23, normalized size = 2.09

$$(bx + a)^m bx^2 + (bx + a)^m ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="giac")
[Out] (b*x + a)^m*b*x^2 + (b*x + a)^m*a*x
```

**maple [A]** time = 0.00, size = 12, normalized size = 1.09

$$x\,(bx + a)^{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(a+b*(m+2)*x),x)
[Out] x*(b*x+a)^(m+1)
```

**maxima [B]** time = 1.12, size = 106, normalized size = 9.64

$$\frac{\left(b^2(m+1)x^2 + abmx - a^2\right)(bx + a)^m m}{\left(m^2 + 3\,m + 2\right)b} + \frac{2\left(b^2(m+1)x^2 + abmx - a^2\right)(bx + a)^m}{\left(m^2 + 3\,m + 2\right)b} + \frac{(bx + a)^{m+1} a}{b(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="maxima")
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*m/((m^2 + 3*m + 2)*b) + 2*(b^
2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m/((m^2 + 3*m + 2)*b) + (b*x + a)^
(m + 1)*a/(b*(m + 1))
```

**mupad** [**B**] time = 0.46, size = 11, normalized size = 1.00

$$x\,(a+b\,x)^{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x*(m + 2))*(a + b*x)^m,x)
```

```
[Out] x*(a + b*x)^(m + 1)
```

**sympy** [**B**] time = 0.27, size = 20, normalized size = 1.82

$$ax\,(a+bx)^m + bx^2\,(a+bx)^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(a+b*(2+m)*x),x)
```

```
[Out] a*x*(a + b*x)**m + b*x**2*(a + b*x)**m
```

## 3.1555 $\int (a+bx)^m (c+dx)^3\,dx$

**Optimal**. Leaf size=110

$$\frac{3d^2(bc-ad)(a+bx)^{m+3}}{b^4(m+3)}+\frac{(bc-ad)^3(a+bx)^{m+1}}{b^4(m+1)}+\frac{3d(bc-ad)^2(a+bx)^{m+2}}{b^4(m+2)}+\frac{d^3(a+bx)^{m+4}}{b^4(m+4)}$$

**Rubi** [A] time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3d^2(bc-ad)(a+bx)^{m+3}}{b^4(m+3)}+\frac{(bc-ad)^3(a+bx)^{m+1}}{b^4(m+1)}+\frac{3d(bc-ad)^2(a+bx)^{m+2}}{b^4(m+2)}+\frac{d^3(a+bx)^{m+4}}{b^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^(1 + m))/(b^4*(1 + m)) + (3*d*(b*c - a*d)^2*(a + b
*x)^(2 + m))/(b^4*(2 + m)) + (3*d^2*(b*c - a*d)*(a + b*x)^(3 + m))/(b^4*(3
+ m)) + (d^3*(a + b*x)^(4 + m))/(b^4*(4 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^m(c+dx)^3\,dx=\int \left(\frac{(bc-ad)^3(a+bx)^m}{b^3}+\frac{3d(bc-ad)^2(a+bx)^{1+m}}{b^3}+\frac{3d^2(bc-ad)(a+bx)^{2+m}}{b^3}+\frac{d^3(\right.$$
$$=\frac{(bc-ad)^3(a+bx)^{1+m}}{b^4(1+m)}+\frac{3d(bc-ad)^2(a+bx)^{2+m}}{b^4(2+m)}+\frac{3d^2(bc-ad)(a+bx)^{3+m}}{b^4(3+m)}+\frac{d^3(a}{b^4(}$$

**Mathematica** [A] time = 0.11, size = 94, normalized size = 0.85

$$\frac{(a+bx)^{m+1}\left(\frac{3d^2(a+bx)^2(bc-ad)}{m+3}+\frac{3d(a+bx)(bc-ad)^2}{m+2}+\frac{(bc-ad)^3}{m+1}+\frac{d^3(a+bx)^3}{m+4}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*c - a*d)^3/(1 + m) + (3*d*(b*c - a*d)^2*(a + b*x))/(
2 + m) + (3*d^2*(b*c - a*d)*(a + b*x)^2)/(3 + m) + (d^3*(a + b*x)^3)/(4 + m
)))/b^4
```

**IntegrateAlgebraic [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^m (c+dx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(c + d*x)^3, x]
```

**fricas [B]** time = 1.26, size = 497, normalized size = 4.52

$$\frac{\left(ab^3c^3m^3+24\,ab^3c^3-36\,a^2b^2c^2d+24\,a^3bcd^2-6\,a^4d^3+\left(b^4d^3m^3+6\,b^4d^3m^2+11\,b^4d^3m+6\,b^4d^3\right)x^4+\left(24\,b^4cd^2+\left(3\,b^4cd^2+ab^3d^3\right)m^3+3\left(7\,b^4cd^2+ab^3d^3\right)m^2+2\left(21\,b^4cd^2+ab^3d^3\right)m\right)x^3+3\left(3\,ab^3c^3-a^2b^2c^2d\right)m^2+3\left(12\,b^4c^2d+\left(b^4c^2d+ab^3cd^2\right)m^3+\left(8\,b^4c^2d+5\,ab^3cd^2-a^2b^2d^3\right)m^2+\left(19\,b^4c^2d+4\,ab^3cd^2-a^2b^2d^3\right)m\right)x^2+\left(26\,ab^3c^3-21\,a^2b^2c^2d+6\,a^3bcd^2\right)m+\left(24\,b^4c^3+\left(b^4c^3+3\,ab^3c^2d\right)m^3+3\left(3\,b^4c^3+7\,ab^3c^2d-2\,a^2b^2cd^2\right)m^2+2\left(13\,b^4c^3+18\,ab^3c^2d-12\,a^2b^2cd^2+3\,a^3bd^3\right)m\right)x\right)(bx+a)^m}{b^4m^4+10\,b^4m^3+35\,b^4m^2+50\,b^4m+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^3*c^3*m^3 + 24*a*b^3*c^3 - 36*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 6*a^4*d
^3 + (b^4*d^3*m^3 + 6*b^4*d^3*m^2 + 11*b^4*d^3*m + 6*b^4*d^3)*x^4 + (24*b^4
*c*d^2 + (3*b^4*c*d^2 + a*b^3*d^3)*m^3 + 3*(7*b^4*c*d^2 + a*b^3*d^3)*m^2 +
2*(21*b^4*c*d^2 + a*b^3*d^3)*m)*x^3 + 3*(3*a*b^3*c^3 - a^2*b^2*c^2*d)*m^2 +
 3*(12*b^4*c^2*d + (b^4*c^2*d + a*b^3*c*d^2)*m^3 + (8*b^4*c^2*d + 5*a*b^3*c
*d^2 - a^2*b^2*d^3)*m^2 + (19*b^4*c^2*d + 4*a*b^3*c*d^2 - a^2*b^2*d^3)*m)*x
^2 + (26*a*b^3*c^3 - 21*a^2*b^2*c^2*d + 6*a^3*b*c*d^2)*m + (24*b^4*c^3 + (b
^4*c^3 + 3*a*b^3*c^2*d)*m^3 + 3*(3*b^4*c^3 + 7*a*b^3*c^2*d - 2*a^2*b^2*c*d^
2)*m^2 + 2*(13*b^4*c^3 + 18*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 3*a^3*b*d^3)*m
)*x)*(b*x + a)^m/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

**giac [B]** time = 1.04, size = 833, normalized size = 7.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^4*d^3*m^3*x^4 + 3*(b*x + a)^m*b^4*c*d^2*m^3*x^3 + (b*x + a)^
m*a*b^3*d^3*m^3*x^3 + 6*(b*x + a)^m*b^4*d^3*m^2*x^4 + 3*(b*x + a)^m*b^4*c^2
*d*m^3*x^2 + 3*(b*x + a)^m*a*b^3*c*d^2*m^3*x^2 + 21*(b*x + a)^m*b^4*c*d^2*m
^2*x^3 + 3*(b*x + a)^m*a*b^3*d^3*m^2*x^3 + 11*(b*x + a)^m*b^4*d^3*m*x^4 + (
```

```
b*x + a)^m*b^4*c^3*m^3*x + 3*(b*x + a)^m*a*b^3*c^2*d*m^3*x + 24*(b*x + a)^m
*b^4*c^2*d*m^2*x^2 + 15*(b*x + a)^m*a*b^3*c*d^2*m^2*x^2 - 3*(b*x + a)^m*a^2
*b^2*d^3*m^2*x^2 + 42*(b*x + a)^m*b^4*c*d^2*m*x^3 + 2*(b*x + a)^m*a*b^3*d^3
*m*x^3 + 6*(b*x + a)^m*b^4*d^3*x^4 + (b*x + a)^m*a*b^3*c^3*m^3 + 9*(b*x + a
)^m*b^4*c^3*m^2*x + 21*(b*x + a)^m*a*b^3*c^2*d*m^2*x - 6*(b*x + a)^m*a^2*b^
2*c*d^2*m^2*x + 57*(b*x + a)^m*b^4*c^2*d*m*x^2 + 12*(b*x + a)^m*a*b^3*c*d^2
*m*x^2 - 3*(b*x + a)^m*a^2*b^2*d^3*m*x^2 + 24*(b*x + a)^m*b^4*c*d^2*x^3 + 9
*(b*x + a)^m*a*b^3*c^3*m^2 - 3*(b*x + a)^m*a^2*b^2*c^2*d*m^2 + 26*(b*x + a)
^m*b^4*c^3*m*x + 36*(b*x + a)^m*a*b^3*c^2*d*m*x - 24*(b*x + a)^m*a^2*b^2*c*
d^2*m*x + 6*(b*x + a)^m*a^3*b*d^3*m*x + 36*(b*x + a)^m*b^4*c^2*d*x^2 + 26*(
b*x + a)^m*a*b^3*c^3*m - 21*(b*x + a)^m*a^2*b^2*c^2*d*m + 6*(b*x + a)^m*a^3
*b*c*d^2*m + 24*(b*x + a)^m*b^4*c^3*x + 24*(b*x + a)^m*a*b^3*c^3 - 36*(b*x
+ a)^m*a^2*b^2*c^2*d + 24*(b*x + a)^m*a^3*b*c*d^2 - 6*(b*x + a)^m*a^4*d^3)/
(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

**maple** [B] time = 0.01, size = 389, normalized size = 3.54

$$-\frac{\left(-b^{3}d^{3}m^{3}x^{3}-3b^{3}c\,d^{2}m^{3}x^{2}-6b^{3}d^{3}m^{2}x^{3}+3a\,b^{2}d^{3}m^{2}x^{2}-3b^{3}c^{2}d\,m^{3}x-21b^{3}c\,d^{2}m^{2}x^{2}-11b^{3}d^{3}m\,x^{3}+6a\,b^{2}c\,d^{2}m^{2}x+9a\,b^{2}d^{3}m\,x^{2}-b^{3}c^{3}m^{3}-24b^{3}c^{2}d\,m^{2}x-42b^{3}c\,d^{2}m\,x^{2}-6b^{3}d^{3}x^{3}-6a^{2}b\,d^{3}mx+3a\,b^{2}c^{2}d\,m^{2}+30a\,b^{2}c\,d^{2}mx+6a\,b^{2}d^{3}x^{2}-9b^{3}c^{3}m^{2}-57b^{3}c^{2}dmx-24b^{3}c\,d^{2}x^{2}-6a^{2}bc\,d^{2}m-6a^{2}b\,d^{3}x+21a\,b^{2}c^{2}dm+24a\,b^{2}c\,d^{2}x-26b^{3}c^{3}m-36b^{3}c^{2}dx+6a^{3}d^{3}-24a^{2}bc\,d^{2}+36a\,b^{2}c^{2}d-24b^{3}c^{3}\right)\left(bx+a\right)^{m+1}}{\left(m^{4}+10m^{3}+35m^{2}+50m+24\right)b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^3,x)
```

```
[Out] -(b*x+a)^(m+1)*(-b^3*d^3*m^3*x^3-3*b^3*c*d^2*m^3*x^2-6*b^3*d^3*m^2*x^3+3*a*
b^2*d^3*m^2*x^2-3*b^3*c^2*d*m^3*x-21*b^3*c*d^2*m^2*x^2-11*b^3*d^3*m*x^3+6*a
*b^2*c*d^2*m^2*x+9*a*b^2*d^3*m*x^2-b^3*c^3*m^3-24*b^3*c^2*d*m^2*x-42*b^3*c*
d^2*m*x^2-6*b^3*d^3*x^3-6*a^2*b*d^3*m*x+3*a*b^2*c^2*d*m^2+30*a*b^2*c*d^2*m*
x+6*a*b^2*d^3*x^2-9*b^3*c^3*m^2-57*b^3*c^2*d*m*x-24*b^3*c*d^2*x^2-6*a^2*b*c
*d^2*m-6*a^2*b*d^3*x+21*a*b^2*c^2*d*m+24*a*b^2*c*d^2*x-26*b^3*c^3*m-36*b^3*
c^2*d*x+6*a^3*d^3-24*a^2*b*c*d^2+36*a*b^2*c^2*d-24*b^3*c^3)/b^4/(m^4+10*m^3
+35*m^2+50*m+24)
```

**maxima** [B] time = 1.17, size = 246, normalized size = 2.24

$$\frac{3\left(b^{2}(m+1)x^{2}+abmx-a^{2}\right)(bx+a)^{m}c^{2}d}{\left(m^{2}+3m+2\right)b^{2}}+\frac{(bx+a)^{m+1}c^{3}}{b(m+1)}+\frac{3\left(\left(m^{2}+3m+2\right)b^{3}x^{3}+\left(m^{2}+m\right)ab^{2}x^{2}-2a^{2}bmx+2a^{3}\right)(bx+a)^{m}cd^{2}}{\left(m^{3}+6m^{2}+11m+6\right)b^{3}}+\frac{\left(\left(m^{3}+6m^{2}+11m+6\right)b^{4}x^{4}+\left(m^{3}+3m^{2}+2m\right)ab^{3}x^{3}-3\left(m^{2}+m\right)a^{2}b^{2}x^{2}+6a^{3}bmx-6a^{4}\right)(bx+a)^{m}d^{3}}{\left(m^{4}+10m^{3}+35m^{2}+50m+24\right)b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3*(b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c^2*d/((m^2 + 3*m + 2)*b^2)
 + (b*x + a)^(m + 1)*c^3/(b*(m + 1)) + 3*((m^2 + 3*m + 2)*b^3*x^3 + (m^2 +
m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*c*d^2/((m^3 + 6*m^2 + 11*m
+ 6)*b^3) + ((m^3 + 6*m^2 + 11*m + 6)*b^4*x^4 + (m^3 + 3*m^2 + 2*m)*a*b^3*x
^3 - 3*(m^2 + m)*a^2*b^2*x^2 + 6*a^3*b*m*x - 6*a^4)*(b*x + a)^m*d^3/((m^4 +
 10*m^3 + 35*m^2 + 50*m + 24)*b^4)
```

**mupad [B]** time = 0.94, size = 478, normalized size = 4.35

$$\frac{d^3 x^4 (a+b x)^m \left(m^3+6 m^2+11 m+6\right)}{m^4+10 m^3+35 m^2+50 m+24}+\frac{a (a+b x)^m \left(-6 a^3 d^3+6 a^2 b c d^2 m+24 a^2 b c d^2-3 a b^2 c^2 d m^2-21 a b^2 c^2 d m-36 a b^2 c^2 d+b^3 c^3 m^3+9 b^3 c^3 m^2+26 b^3 c^3 m+24 b^3 c^3\right)}{b^4 \left(m^4+10 m^3+35 m^2+50 m+24\right)}+\frac{x (a+b x)^m \left(6 a^3 b d^3 m-6 a^2 b^2 c d^2 m^2-24 a^2 b^2 c d^2 m+3 a b^3 c^2 d m^3+21 a b^3 c^2 d m^2+36 a b^3 c^2 d m+b^4 c^3 m^3+9 b^4 c^3 m^2+26 b^4 c^3 m+24 b^4 c^3\right)}{b^4 \left(m^4+10 m^3+35 m^2+50 m+24\right)}+\frac{3 d x^2 (m+1) (a+b x)^m \left(-a^2 d^2 m+a b c d m^2+4 a b c d m+b^2 c^2 m^2+7 b^2 c^2 m+12 b^2 c^2\right)}{b^2 \left(m^4+10 m^3+35 m^2+50 m+24\right)}+\frac{d^2 x^3 (a+b x)^m (12 b c+a d m+3 b c m) \left(m^2+3 m+2\right)}{b \left(m^4+10 m^3+35 m^2+50 m+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x)^3,x)
```

```
[Out] (d^3*x^4*(a + b*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^
4 + 24) + (a*(a + b*x)^m*(24*b^3*c^3 - 6*a^3*d^3 + 26*b^3*c^3*m + 9*b^3*c^3
*m^2 + b^3*c^3*m^3 - 36*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 21*a*b^2*c^2*d*m + 6
*a^2*b*c*d^2*m - 3*a*b^2*c^2*d*m^2))/(b^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4)) + (x*(a + b*x)^m*(24*b^4*c^3 + 26*b^4*c^3*m + 9*b^4*c^3*m^2 + b^4*c^3*m
^3 + 6*a^3*b*d^3*m + 36*a*b^3*c^2*d*m - 24*a^2*b^2*c*d^2*m + 21*a*b^3*c^2*d
*m^2 + 3*a*b^3*c^2*d*m^3 - 6*a^2*b^2*c*d^2*m^2))/(b^4*(50*m + 35*m^2 + 10*m
^3 + m^4 + 24)) + (3*d*x^2*(m + 1)*(a + b*x)^m*(12*b^2*c^2 - a^2*d^2*m + 7*
b^2*c^2*m + b^2*c^2*m^2 + 4*a*b*c*d*m + a*b*c*d*m^2))/(b^2*(50*m + 35*m^2 +
 10*m^3 + m^4 + 24)) + (d^2*x^3*(a + b*x)^m*(12*b*c + a*d*m + 3*b*c*m)*(3*m
 + m^2 + 2))/(b*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

**sympy [A]** time = 4.67, size = 4058, normalized size = 36.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**3,x)
```

```
[Out] Piecewise((a**m*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(
b, 0)), (6*a**3*d**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 11*a**3*d**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 6*a**2*b*c*d**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d**3*x*log(a/b + x)/(6*a**3*b**4 +
18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*d**3*x/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*a*b**2*c**2*d/(6*a
**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 18*a*b**2*c*d**
2*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b*
*2*d**3*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
6*b**7*x**3) + 18*a*b**2*d**3*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**
6*x**2 + 6*b**7*x**3) - 2*b**3*c**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 9*b**3*c**2*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) - 18*b**3*c*d**2*x**2/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d**3*x**3*log(a/b + x)/(6*a**
3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(m, -4)), (-6*a*
*3*d**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d**3
/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 6*a**2*b*c*d**2*log(a/b + x)/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 9*a**2*b*c*d**2/(2*a**2*b**4 + 4*a
```

```
*b**5*x + 2*b**6*x**2) - 12*a**2*b*d**3*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b
**5*x + 2*b**6*x**2) - 12*a**2*b*d**3*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*
x**2) - 3*a*b**2*c**2*d/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 12*a*b**
2*c*d**2*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 12*a*b**
2*c*d**2*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d**3*x**2*lo
g(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c**3/(2*a**2*b**
4 + 4*a*b**5*x + 2*b**6*x**2) - 6*b**3*c**2*d*x/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + 6*b**3*c*d**2*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + 2*b**3*d**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2),
Eq(m, -3)), (6*a**3*d**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3*d**3/(
2*a*b**4 + 2*b**5*x) - 12*a**2*b*c*d**2*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
- 12*a**2*b*c*d**2/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d**3*x*log(a/b + x)/(2*
a*b**4 + 2*b**5*x) + 6*a*b**2*c**2*d*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6
*a*b**2*c**2*d/(2*a*b**4 + 2*b**5*x) - 12*a*b**2*c*d**2*x*log(a/b + x)/(2*a
*b**4 + 2*b**5*x) - 3*a*b**2*d**3*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c**3/
(2*a*b**4 + 2*b**5*x) + 6*b**3*c**2*d*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
+ 6*b**3*c*d**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*d**3*x**3/(2*a*b**4 + 2*b
**5*x), Eq(m, -2)), (-a**3*d**3*log(a/b + x)/b**4 + 3*a**2*c*d**2*log(a/b +
 x)/b**3 + a**2*d**3*x/b**3 - 3*a*c**2*d*log(a/b + x)/b**2 - 3*a*c*d**2*x/b
**2 - a*d**3*x**2/(2*b**2) + c**3*log(a/b + x)/b + 3*c**2*d*x/b + 3*c*d**2*
x**2/(2*b) + d**3*x**3/(3*b), Eq(m, -1)), (-6*a**4*d**3*(a + b*x)**m/(b**4*
m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6*a**3*b*c*d**2
*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + 24*a**3*b*c*d**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m*
*2 + 50*b**4*m + 24*b**4) + 6*a**3*b*d**3*m*x*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*c**2*d*m**2*(
a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
 - 21*a**2*b**2*c**2*d*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4) - 36*a**2*b**2*c**2*d*(a + b*x)**m/(b**4*m**4 +
10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 6*a**2*b**2*c*d**2*m**
2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) - 24*a**2*b**2*c*d**2*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35
*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*d**3*m**2*x**2*(a + b*x)**m
/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b
**2*d**3*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + a*b**3*c**3*m**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*a*b**3*c**3*m**2*(a + b*x)**m/(b
**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 26*a*b**3*c
**3*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) + 24*a*b**3*c**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4) + 3*a*b**3*c**2*d*m**3*x*(a + b*x)**m/(b**4*m**4
 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 21*a*b**3*c**2*d*m*
*2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24
*b**4) + 36*a*b**3*c**2*d*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b
**4*m**2 + 50*b**4*m + 24*b**4) + 3*a*b**3*c*d**2*m**3*x**2*(a + b*x)**m/(b
```

```
**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 15*a*b**3*c
*d**2*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + 12*a*b**3*c*d**2*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b*
*4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d**3*m**3*x**3*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 3
*a*b**3*d**3*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**
2 + 50*b**4*m + 24*b**4) + 2*a*b**3*d**3*m*x**3*(a + b*x)**m/(b**4*m**4 + 1
0*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*c**3*m**3*x*(a + b
*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*
b**4*c**3*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50
*b**4*m + 24*b**4) + 26*b**4*c**3*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**
3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 24*b**4*c**3*x*(a + b*x)**m/(b**4
*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 3*b**4*c**2*d*
m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m
 + 24*b**4) + 24*b**4*c**2*d*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m*
*3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 57*b**4*c**2*d*m*x**2*(a + b*x)*
*m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 36*b**
4*c**2*d*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b*
*4*m + 24*b**4) + 3*b**4*c*d**2*m**3*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4
*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 21*b**4*c*d**2*m**2*x**3*(a +
 b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) +
42*b**4*c*d**2*m*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
 + 50*b**4*m + 24*b**4) + 24*b**4*c*d**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*d**3*m**3*x**4*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6
*b**4*d**3*m**2*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
+ 50*b**4*m + 24*b**4) + 11*b**4*d**3*m*x**4*(a + b*x)**m/(b**4*m**4 + 10*b
**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6*b**4*d**3*x**4*(a + b*x)
**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4), True))
```

## 3.1556 $\int (a+bx)^m (c+dx)^2 \, dx$

**Optimal**. Leaf size=78

$$\frac{(bc-ad)^2(a+bx)^{m+1}}{b^3(m+1)}+\frac{2d(bc-ad)(a+bx)^{m+2}}{b^3(m+2)}+\frac{d^2(a+bx)^{m+3}}{b^3(m+3)}$$

**Rubi** [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{(bc-ad)^2(a+bx)^{m+1}}{b^3(m+1)}+\frac{2d(bc-ad)(a+bx)^{m+2}}{b^3(m+2)}+\frac{d^2(a+bx)^{m+3}}{b^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1 + m))/(b^3*(1 + m)) + (2*d*(b*c - a*d)*(a + b*x
)^(2 + m))/(b^3*(2 + m)) + (d^2*(a + b*x)^(3 + m))/(b^3*(3 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^m}{b^2}+\frac{2d(bc-ad)(a+bx)^{1+m}}{b^2}+\frac{d^2(a+bx)^{2+m}}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^{1+m}}{b^3(1+m)}+\frac{2d(bc-ad)(a+bx)^{2+m}}{b^3(2+m)}+\frac{d^2(a+bx)^{3+m}}{b^3(3+m)}\end{aligned}$$

**Mathematica** [A] time = 0.10, size = 67, normalized size = 0.86

$$\frac{(a+bx)^{m+1}\left(\frac{2d(a+bx)(bc-ad)}{m+2}+\frac{(bc-ad)^2}{m+1}+\frac{d^2(a+bx)^2}{m+3}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*c - a*d)^2/(1 + m) + (2*d*(b*c - a*d)*(a + b*x))/(2
+ m) + (d^2*(a + b*x)^2)/(3 + m)))/b^3
```

**IntegrateAlgebraic [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int (a+bx)^m (c+dx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(c + d*x)^2, x]
```

**fricas [B]** time = 1.18, size = 235, normalized size = 3.01

$$\frac{\left(ab^2c^2m^2 + 6\,ab^2c^2 - 6\,a^2bcd + 2\,a^3d^2 + \left(b^3d^2m^2 + 3\,b^3d^2m + 2\,b^3d^2\right)x^3 + \left(6\,b^3cd + \left(2\,b^3cd + ab^2d^2\right)m^2 + \left(8\,b^3cd + ab^2d^2\right)m\right)x^2 + \left(5\,ab^2c^2 - 2\,a^2bcd\right)m + \left(6\,b^3c^2 + \left(b^3c^2 + 2\,ab^2cd\right)m^2 + \left(5\,b^3c^2 + 6\,ab^2cd - 2\,a^2bd^2\right)m\right)x\right)(bx+a)^m}{b^3m^3 + 6\,b^3m^2 + 11\,b^3m + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^2*c^2*m^2 + 6*a*b^2*c^2 - 6*a^2*b*c*d + 2*a^3*d^2 + (b^3*d^2*m^2 + 3*b
^3*d^2*m + 2*b^3*d^2)*x^3 + (6*b^3*c*d + (2*b^3*c*d + a*b^2*d^2)*m^2 + (8*b
^3*c*d + a*b^2*d^2)*m)*x^2 + (5*a*b^2*c^2 - 2*a^2*b*c*d)*m + (6*b^3*c^2 + (
b^3*c^2 + 2*a*b^2*c*d)*m^2 + (5*b^3*c^2 + 6*a*b^2*c*d - 2*a^2*b*d^2)*m)*x)*
(b*x + a)^m/(b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)
```

**giac [B]** time = 0.97, size = 385, normalized size = 4.94

$$\frac{(bx+a)^m b^3d^2m^2x^3 + 2\,(bx+a)^m b^3cdm^2x^2 + (bx+a)^m ab^2d^2m^2x^2 + 3\,(bx+a)^m b^3d^2mx^3 + (bx+a)^m b^3c^2m^2x + 2\,(bx+a)^m ab^2cdm^2x + 8\,(bx+a)^m b^3cdmx^2 + (bx+a)^m ab^2d^2mx^2 + 2\,(bx+a)^m b^3d^2x^3 + (bx+a)^m ab^2c^2m^2 + 5\,(bx+a)^m b^3c^2mx + 6\,(bx+a)^m ab^2cdmx - 2\,(bx+a)^m a^2bd^2mx + 6\,(bx+a)^m b^3cdx^2 + 5\,(bx+a)^m ab^2c^2m - 2\,(bx+a)^m a^2bcdm + 6\,(bx+a)^m b^3c^2x + 6\,(bx+a)^m ab^2c^2 - 6\,(bx+a)^m a^2bcd + 2\,(bx+a)^m a^3d^2}{b^3m^3 + 6\,b^3m^2 + 11\,b^3m + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^3*d^2*m^2*x^3 + 2*(b*x + a)^m*b^3*c*d*m^2*x^2 + (b*x + a)^m*
a*b^2*d^2*m^2*x^2 + 3*(b*x + a)^m*b^3*d^2*m*x^3 + (b*x + a)^m*b^3*c^2*m^2*x
 + 2*(b*x + a)^m*a*b^2*c*d*m^2*x + 8*(b*x + a)^m*b^3*c*d*m*x^2 + (b*x + a)^
m*a*b^2*d^2*m*x^2 + 2*(b*x + a)^m*b^3*d^2*x^3 + (b*x + a)^m*a*b^2*c^2*m^2 +
 5*(b*x + a)^m*b^3*c^2*m*x + 6*(b*x + a)^m*a*b^2*c*d*m*x - 2*(b*x + a)^m*a^
2*b*d^2*m*x + 6*(b*x + a)^m*b^3*c*d*x^2 + 5*(b*x + a)^m*a*b^2*c^2*m - 2*(b*
x + a)^m*a^2*b*c*d*m + 6*(b*x + a)^m*b^3*c^2*x + 6*(b*x + a)^m*a*b^2*c^2 -
6*(b*x + a)^m*a^2*b*c*d + 2*(b*x + a)^m*a^3*d^2)/(b^3*m^3 + 6*b^3*m^2 + 11*
b^3*m + 6*b^3)
```

**maple [B]** time = 0.01, size = 159, normalized size = 2.04

$$\frac{\left(b^{2}d^{2}m^{2}x^{2}+2b^{2}cd\,m^{2}x+3b^{2}d^{2}m\,x^{2}-2ab\,d^{2}mx+b^{2}c^{2}m^{2}+8b^{2}cdmx+2b^{2}x^{2}d^{2}-2abcdm-2ab\,d^{2}x+5b^{2}c^{2}m+6b^{2}cdx+2a^{2}d^{2}-6abcd+6b^{2}c^{2}\right)(bx+a)^{m+1}}{\left(m^{3}+6m^{2}+11m+6\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^2,x)
```

```
[Out] (b*x+a)^(m+1)*(b^2*d^2*m^2*x^2+2*b^2*c*d*m^2*x+3*b^2*d^2*m*x^2-2*a*b*d^2*m*
x+b^2*c^2*m^2+8*b^2*c*d*m*x+2*b^2*d^2*x^2-2*a*b*c*d*m-2*a*b*d^2*x+5*b^2*c^2
*m+6*b^2*c*d*x+2*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/b^3/(m^3+6*m^2+11*m+6)
```

**maxima [A]** time = 1.15, size = 138, normalized size = 1.77

$$\frac{2\left(b^{2}(m+1)x^{2}+abmx-a^{2}\right)(bx+a)^{m}cd}{\left(m^{2}+3\,m+2\right)b^{2}}+\frac{(bx+a)^{m+1}c^{2}}{b(m+1)}+\frac{\left(\left(m^{2}+3\,m+2\right)b^{3}x^{3}+\left(m^{2}+m\right)ab^{2}x^{2}-2\,a^{2}bmx+2\,a^{3}\right)(bx+a)^{m}d^{2}}{\left(m^{3}+6\,m^{2}+11\,m+6\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2*(b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*d/((m^2 + 3*m + 2)*b^2) +
 (b*x + a)^(m + 1)*c^2/(b*(m + 1)) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 + m)*a
*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*d^2/((m^3 + 6*m^2 + 11*m + 6)*b
^3)
```

**mupad [B]** time = 0.66, size = 226, normalized size = 2.90

$$(a+b\,x)^{m}\left(\frac{a\,\left(2\,a^{2}\,d^{2}-2\,a\,b\,c\,d\,m-6\,a\,b\,c\,d+b^{2}\,c^{2}\,m^{2}+5\,b^{2}\,c^{2}\,m+6\,b^{2}\,c^{2}\right)}{b^{3}\,\left(m^{3}+6\,m^{2}+11\,m+6\right)}+\frac{d^{2}\,x^{3}\,\left(m^{2}+3\,m+2\right)}{m^{3}+6\,m^{2}+11\,m+6}+\frac{x\,\left(-2\,a^{2}\,b\,d^{2}\,m+2\,a\,b^{2}\,c\,d\,m^{2}+6\,a\,b^{2}\,c\,d\,m+b^{3}\,c^{2}\,m^{2}+5\,b^{3}\,c^{2}\,m+6\,b^{3}\,c^{2}\right)}{b^{3}\,\left(m^{3}+6\,m^{2}+11\,m+6\right)}+\frac{d\,x^{2}\,(m+1)\,(6\,b\,c+a\,d\,m+2\,b\,c\,m)}{b\,\left(m^{3}+6\,m^{2}+11\,m+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x)^2,x)
```

```
[Out] (a + b*x)^m*((a*(2*a^2*d^2 + 6*b^2*c^2 + 5*b^2*c^2*m + b^2*c^2*m^2 - 6*a*b*
c*d - 2*a*b*c*d*m))/(b^3*(11*m + 6*m^2 + m^3 + 6)) + (d^2*x^3*(3*m + m^2 +
2))/(11*m + 6*m^2 + m^3 + 6) + (x*(6*b^3*c^2 + 5*b^3*c^2*m + b^3*c^2*m^2 -
2*a^2*b*d^2*m + 2*a*b^2*c*d*m^2 + 6*a*b^2*c*d*m))/(b^3*(11*m + 6*m^2 + m^3
+ 6)) + (d*x^2*(m + 1)*(6*b*c + a*d*m + 2*b*c*m))/(b*(11*m + 6*m^2 + m^3 +
6)))
```

**sympy [A]** time = 2.14, size = 1506, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**2,x)
```

```
[Out] Piecewise((a**m*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(b, 0)), (2*a**2*d**2*
log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2*d**2/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) - 2*a*b*c*d/(2*a**2*b**3 + 4*a*b**4*x + 2
*b**5*x**2) + 4*a*b*d**2*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*
x**2) + 4*a*b*d**2*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - b**2*c**2/(
2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - 4*b**2*c*d*x/(2*a**2*b**3 + 4*a*b
**4*x + 2*b**5*x**2) + 2*b**2*d**2*x**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**
4*x + 2*b**5*x**2), Eq(m, -3)), (-2*a**2*d**2*log(a/b + x)/(a*b**3 + b**4*x
) - 2*a**2*d**2/(a*b**3 + b**4*x) + 2*a*b*c*d*log(a/b + x)/(a*b**3 + b**4*x
) + 2*a*b*c*d/(a*b**3 + b**4*x) - 2*a*b*d**2*x*log(a/b + x)/(a*b**3 + b**4*
x) - b**2*c**2/(a*b**3 + b**4*x) + 2*b**2*c*d*x*log(a/b + x)/(a*b**3 + b**4
*x) + b**2*d**2*x**2/(a*b**3 + b**4*x), Eq(m, -2)), (a**2*d**2*log(a/b + x)
/b**3 - 2*a*c*d*log(a/b + x)/b**2 - a*d**2*x/b**2 + c**2*log(a/b + x)/b + 2
*c*d*x/b + d**2*x**2/(2*b), Eq(m, -1)), (2*a**3*d**2*(a + b*x)**m/(b**3*m**
3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 2*a**2*b*c*d*m*(a + b*x)**m/(b**3*m
**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 6*a**2*b*c*d*(a + b*x)**m/(b**3*m
**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 2*a**2*b*d**2*m*x*(a + b*x)**m/(b
**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + a*b**2*c**2*m**2*(a + b*x)**
m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 5*a*b**2*c**2*m*(a + b*x
)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 6*a*b**2*c**2*(a + b*
x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 2*a*b**2*c*d*m**2*x*
(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 6*a*b**2*c*d*
m*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + a*b**2*d*
*2*m**2*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) +
a*b**2*d**2*m*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b*
*3) + b**3*c**2*m**2*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m +
6*b**3) + 5*b**3*c**2*m*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m
 + 6*b**3) + 6*b**3*c**2*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*
m + 6*b**3) + 2*b**3*c*d*m**2*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 +
11*b**3*m + 6*b**3) + 8*b**3*c*d*m*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m*
*2 + 11*b**3*m + 6*b**3) + 6*b**3*c*d*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3
*m**2 + 11*b**3*m + 6*b**3) + b**3*d**2*m**2*x**3*(a + b*x)**m/(b**3*m**3 +
 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 3*b**3*d**2*m*x**3*(a + b*x)**m/(b**3*
m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 2*b**3*d**2*x**3*(a + b*x)**m/(b
**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3), True))
```

## 3.1557 $\int (a+bx)^m (c+dx)\,dx$

**Optimal**. Leaf size=46

$$\frac{(bc-ad)(a+bx)^{m+1}}{b^2(m+1)}+\frac{d(a+bx)^{m+2}}{b^2(m+2)}$$

**Rubi [A]** time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(bc-ad)(a+bx)^{m+1}}{b^2(m+1)}+\frac{d(a+bx)^{m+2}}{b^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(1 + m))/(b^2*(1 + m)) + (d*(a + b*x)^(2 + m))/(b^2*
(2 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^m}{b}+\frac{d(a+bx)^{1+m}}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^{1+m}}{b^2(1+m)}+\frac{d(a+bx)^{2+m}}{b^2(2+m)}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 41, normalized size = 0.89

$$\frac{(a+bx)^{m+1}(-ad+bc(m+2)+bd(m+1)x)}{b^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(-(a*d) + b*c*(2 + m) + b*d*(1 + m)*x))/(b^2*(1 + m)*(2
+ m))
```

**IntegrateAlgebraic [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int (a+bx)^m(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(c + d*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(c + d*x), x]
```

**fricas [A]** time = 0.73, size = 83, normalized size = 1.80

$$\frac{\left(abcm+2\,abc-a^2d+\left(b^2dm+b^2d\right)x^2+\left(2\,b^2c+\left(b^2c+abd\right)m\right)x\right)(bx+a)^m}{b^2m^2+3\,b^2m+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*b*c*m + 2*a*b*c - a^2*d + (b^2*d*m + b^2*d)*x^2 + (2*b^2*c + (b^2*c + a*
b*d)*m)*x)*(b*x + a)^m/(b^2*m^2 + 3*b^2*m + 2*b^2)
```

**giac [B]** time = 0.86, size = 132, normalized size = 2.87

$$\frac{(bx+a)^m b^2dmx^2+(bx+a)^m b^2cmx+(bx+a)^m abdmx+(bx+a)^m b^2dx^2+(bx+a)^m abcm+2\,(bx+a)^m b^2cx+2\,(bx+a)^m abc-(bx+a)^m a^2d}{b^2m^2+3\,b^2m+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^2*d*m*x^2 + (b*x + a)^m*b^2*c*m*x + (b*x + a)^m*a*b*d*m*x +
(b*x + a)^m*b^2*d*x^2 + (b*x + a)^m*a*b*c*m + 2*(b*x + a)^m*b^2*c*x + 2*(b*
x + a)^m*a*b*c - (b*x + a)^m*a^2*d)/(b^2*m^2 + 3*b^2*m + 2*b^2)
```

**maple [A]** time = 0.00, size = 49, normalized size = 1.07

$$-\frac{(-bdmx-bcm-bdx+ad-2bc)\,(bx+a)^{m+1}}{\left(m^2+3m+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c),x)
```

```
[Out] -(b*x+a)^(m+1)*(-b*d*m*x-b*c*m-b*d*x+a*d-2*b*c)/b^2/(m^2+3*m+2)
```

**maxima** [A]  time = 1.06, size = 63, normalized size = 1.37

$$\frac{\left(b^{2}(m+1)x^{2}+abmx-a^{2}\right)(bx+a)^{m}d}{\left(m^{2}+3\,m+2\right)b^{2}}+\frac{(bx+a)^{m+1}c}{b(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="maxima")
```

```
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*d/((m^2 + 3*m + 2)*b^2) + (b*
x + a)^(m + 1)*c/(b*(m + 1))
```

**mupad** [B]  time = 0.48, size = 88, normalized size = 1.91

$$(a+b\,x)^{m}\left(\frac{a\,\left(2\,b\,c-a\,d+b\,c\,m\right)}{b^{2}\,\left(m^{2}+3\,m+2\right)}+\frac{x\,\left(2\,b^{2}\,c+b^{2}\,c\,m+a\,b\,d\,m\right)}{b^{2}\,\left(m^{2}+3\,m+2\right)}+\frac{d\,x^{2}\,\left(m+1\right)}{m^{2}+3\,m+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x),x)
```

```
[Out] (a + b*x)^m*((a*(2*b*c - a*d + b*c*m))/(b^2*(3*m + m^2 + 2)) + (x*(2*b^2*c
+ b^2*c*m + a*b*d*m))/(b^2*(3*m + m^2 + 2)) + (d*x^2*(m + 1))/(3*m + m^2 +
2))
```

**sympy** [A]  time = 0.86, size = 377, normalized size = 8.20

$$\begin{cases} a^{m}\left(cx+\frac{dx^{2}}{2}\right) & \text{for}\: b=0 \\ \frac{ad\log\left(\frac{a}{b}+x\right)}{ab^{2}+b^{3}x}+\frac{ad}{ab^{2}+b^{3}x}-\frac{bc}{ab^{2}+b^{3}x}+\frac{bdx\log\left(\frac{a}{b}+x\right)}{ab^{2}+b^{3}x} & \text{for}\: m=-2 \\ -\frac{ad\log\left(\frac{a}{b}+x\right)}{b^{2}}+\frac{c\log\left(\frac{a}{b}+x\right)}{b}+\frac{dx}{b} & \text{for}\: m=-1 \\ -\frac{a^{2}d(a+bx)^{m}}{b^{2}m^{2}+3b^{2}m+2b^{2}}+\frac{abcm(a+bx)^{m}}{b^{2}m^{2}+3b^{2}m+2b^{2}}+\frac{2abc(a+bx)^{m}}{b^{2}m^{2}+3b^{2}m+2b^{2}}+\frac{abdmx(a+bx)^{m}}{b^{2}m^{2}+3b^{2}m+2b^{2}}+\frac{b^{2}cmx(a+bx)^{m}}{b^{2}m^{2}+3b^{2}m+2b^{2}}+\frac{2b^{2}cx(a+bx)^{m}}{b^{2}m^{2}+3b^{2}m+2b^{2}}+\frac{b^{2}dmx^{2}(a+bx)^{m}}{b^{2}m^{2}+3b^{2}m+2b^{2}}+\frac{b^{2}dx^{2}(a+bx)^{m}}{b^{2}m^{2}+3b^{2}m+2b^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c),x)
```

```
[Out] Piecewise((a**m*(c*x + d*x**2/2), Eq(b, 0)), (a*d*log(a/b + x)/(a*b**2 + b*
*3*x) + a*d/(a*b**2 + b**3*x) - b*c/(a*b**2 + b**3*x) + b*d*x*log(a/b + x)/
(a*b**2 + b**3*x), Eq(m, -2)), (-a*d*log(a/b + x)/b**2 + c*log(a/b + x)/b +
 d*x/b, Eq(m, -1)), (-a**2*d*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) +
 a*b*c*m*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + 2*a*b*c*(a + b*x)**
m/(b**2*m**2 + 3*b**2*m + 2*b**2) + a*b*d*m*x*(a + b*x)**m/(b**2*m**2 + 3*b
```

```
**2*m + 2*b**2) + b**2*c*m*x*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) +
 2*b**2*c*x*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + b**2*d*m*x**2*(a
 + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + b**2*d*x**2*(a + b*x)**m/(b**2
*m**2 + 3*b**2*m + 2*b**2), True))
```

## 3.1558 $\int (a+bx)^3(c+dx)^n\,dx$

**Optimal**. Leaf size=111

$$-\frac{3b^2(bc-ad)(c+dx)^{n+3}}{d^4(n+3)}-\frac{(bc-ad)^3(c+dx)^{n+1}}{d^4(n+1)}+\frac{3b(bc-ad)^2(c+dx)^{n+2}}{d^4(n+2)}+\frac{b^3(c+dx)^{n+4}}{d^4(n+4)}$$

**Rubi [A]** time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^2(bc-ad)(c+dx)^{n+3}}{d^4(n+3)}-\frac{(bc-ad)^3(c+dx)^{n+1}}{d^4(n+1)}+\frac{3b(bc-ad)^2(c+dx)^{n+2}}{d^4(n+2)}+\frac{b^3(c+dx)^{n+4}}{d^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] -(((b*c - a*d)^3*(c + d*x)^(1 + n))/(d^4*(1 + n))) + (3*b*(b*c - a*d)^2*(c
+ d*x)^(2 + n))/(d^4*(2 + n)) - (3*b^2*(b*c - a*d)*(c + d*x)^(3 + n))/(d^4*
(3 + n)) + (b^3*(c + d*x)^(4 + n))/(d^4*(4 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^n}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{1+n}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{2+n}}{d^3}+\frac{b^3(\ldots}{}\right.\\ &= -\frac{(bc-ad)^3(c+dx)^{1+n}}{d^4(1+n)}+\frac{3b(bc-ad)^2(c+dx)^{2+n}}{d^4(2+n)}-\frac{3b^2(bc-ad)(c+dx)^{3+n}}{d^4(3+n)}+\frac{b^3(c\ldots}{d^4(\ldots}\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 95, normalized size = 0.86

$$\frac{(c+dx)^{n+1}\left(-\frac{3b^2(c+dx)^2(bc-ad)}{n+3}+\frac{3b(c+dx)(bc-ad)^2}{n+2}-\frac{(bc-ad)^3}{n+1}+\frac{b^3(c+dx)^3}{n+4}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*(-((b*c - a*d)^3/(1 + n)) + (3*b*(b*c - a*d)^2*(c + d*x)
)/(2 + n) - (3*b^2*(b*c - a*d)*(c + d*x)^2)/(3 + n) + (b^3*(c + d*x)^3)/(4
+ n)))/d^4
```

**IntegrateAlgebraic [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^3*(c + d*x)^n, x]
```

**fricas [B]** time = 1.04, size = 496, normalized size = 4.47

$$\frac{\left(a^{3} c d^{3} n^{3} - 6 \, b^{3} c^{4} + 24 \, a b^{2} c^{3} d - 36 \, a^{2} b c^{2} d^{2} + 24 \, a^{3} c d^{3} + \left(b^{3} d^{4} n^{3} + 6 \, b^{3} d^{4} n^{2} + 11 \, b^{3} d^{4} n + 6 \, b^{3} d^{4}\right) x^{4} + \left(24 \, a b^{2} d^{4} + \left(b^{3} c d^{3} + 3 \, a b^{2} d^{4}\right) n^{3} + 3 \, \left(b^{3} c d^{3} + 7 \, a b^{2} d^{4}\right) n^{2} + 2 \, \left(b^{3} c d^{3} + 21 \, a b^{2} d^{4}\right) n\right) x^{3} - 3 \, \left(a^{2} b c^{2} d^{2} - 3 \, a^{3} c d^{3}\right) n^{2} + 3 \, \left(12 \, a^{2} b d^{4} + \left(a b^{2} c d^{3} + a^{2} b d^{4}\right) n^{3} - \left(b^{3} c^{2} d^{2} - 5 \, a b^{2} c d^{3} - 8 \, a^{2} b d^{4}\right) n^{2} - \left(b^{3} c^{2} d^{2} - 4 \, a b^{2} c d^{3} - 19 \, a^{2} b d^{4}\right) n\right) x^{2} + \left(6 \, a b^{2} c^{3} d - 21 \, a^{2} b c^{2} d^{2} + 26 \, a^{3} c d^{3}\right) n + \left(24 \, a^{3} d^{4} + \left(3 \, a^{2} b c d^{3} + a^{3} d^{4}\right) n^{3} - 3 \, \left(2 \, a b^{2} c^{2} d^{2} - 7 \, a^{2} b c d^{3} - 3 \, a^{3} d^{4}\right) n^{2} + 2 \, \left(3 \, b^{3} c^{3} d - 12 \, a b^{2} c^{2} d^{2} + 18 \, a^{2} b c d^{3} + 13 \, a^{3} d^{4}\right) n\right) x\right) {\left(d x + c\right)}^{n}}{d^{4} n^{4} + 10 \, d^{4} n^{3} + 35 \, d^{4} n^{2} + 50 \, d^{4} n + 24 \, d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a^3*c*d^3*n^3 - 6*b^3*c^4 + 24*a*b^2*c^3*d - 36*a^2*b*c^2*d^2 + 24*a^3*c*d
^3 + (b^3*d^4*n^3 + 6*b^3*d^4*n^2 + 11*b^3*d^4*n + 6*b^3*d^4)*x^4 + (24*a*b
^2*d^4 + (b^3*c*d^3 + 3*a*b^2*d^4)*n^3 + 3*(b^3*c*d^3 + 7*a*b^2*d^4)*n^2 +
2*(b^3*c*d^3 + 21*a*b^2*d^4)*n)*x^3 - 3*(a^2*b*c^2*d^2 - 3*a^3*c*d^3)*n^2 +
 3*(12*a^2*b*d^4 + (a*b^2*c*d^3 + a^2*b*d^4)*n^3 - (b^3*c^2*d^2 - 5*a*b^2*c
*d^3 - 8*a^2*b*d^4)*n^2 - (b^3*c^2*d^2 - 4*a*b^2*c*d^3 - 19*a^2*b*d^4)*n)*x
^2 + (6*a*b^2*c^3*d - 21*a^2*b*c^2*d^2 + 26*a^3*c*d^3)*n + (24*a^3*d^4 + (3
*a^2*b*c*d^3 + a^3*d^4)*n^3 - 3*(2*a*b^2*c^2*d^2 - 7*a^2*b*c*d^3 - 3*a^3*d^
4)*n^2 + 2*(3*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 18*a^2*b*c*d^3 + 13*a^3*d^4)*n
)*x)*(d*x + c)^n/(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2 + 50*d^4*n + 24*d^4)
```

**giac [B]** time = 0.98, size = 833, normalized size = 7.50

$$\frac{(dx+c)^n b^3 d^4 n^3 x^4 + (dx+c)^n b^3 c d^3 n^3 x^3 + 3\,(dx+c)^n a b^2 d^4 n^3 x^3 + 6\,(dx+c)^n b^3 d^4 n^2 x^4 + 3\,(dx+c)^n a b^2 c d^3 n^3 x^2 + 3\,(dx+c)^n a^2 b d^4 n^3 x^2 + 3\,(dx+c)^n b^3 c d^3 n^2 x^3 + 21\,(dx+c)^n a b^2 d^4 n^2 x^3 + 11\,(dx+c)^n b^3 d^4 n x^4 + 3\,(dx+c)^n a^2 b c d^3 n^3 x + (dx+c)^n a^3 d^4 n^3 x - 3\,(dx+c)^n b^3 c^2 d^2 n^2 x^2 + 15\,(dx+c)^n a b^2 c d^3 n^2 x^2 + 24\,(dx+c)^n a^2 b d^4 \cdots}{\cdots}$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b^3*d^4*n^3*x^4 + (d*x + c)^n*b^3*c*d^3*n^3*x^3 + 3*(d*x + c)^
n*a*b^2*d^4*n^3*x^3 + 6*(d*x + c)^n*b^3*d^4*n^2*x^4 + 3*(d*x + c)^n*a*b^2*c
*d^3*n^3*x^2 + 3*(d*x + c)^n*a^2*b*d^4*n^3*x^2 + 3*(d*x + c)^n*b^3*c*d^3*n^
2*x^3 + 21*(d*x + c)^n*a*b^2*d^4*n^2*x^3 + 11*(d*x + c)^n*b^3*d^4*n*x^4 + 3
```

```
*(d*x + c)^n*a^2*b*c*d^3*n^3*x + (d*x + c)^n*a^3*d^4*n^3*x - 3*(d*x + c)^n*
b^3*c^2*d^2*n^2*x^2 + 15*(d*x + c)^n*a*b^2*c*d^3*n^2*x^2 + 24*(d*x + c)^n*a
^2*b*d^4*n^2*x^2 + 2*(d*x + c)^n*b^3*c*d^3*n*x^3 + 42*(d*x + c)^n*a*b^2*d^4
*n*x^3 + 6*(d*x + c)^n*b^3*d^4*x^4 + (d*x + c)^n*a^3*c*d^3*n^3 - 6*(d*x + c
)^n*a*b^2*c^2*d^2*n^2*x + 21*(d*x + c)^n*a^2*b*c*d^3*n^2*x + 9*(d*x + c)^n*
a^3*d^4*n^2*x - 3*(d*x + c)^n*b^3*c^2*d^2*n*x^2 + 12*(d*x + c)^n*a*b^2*c*d^
3*n*x^2 + 57*(d*x + c)^n*a^2*b*d^4*n*x^2 + 24*(d*x + c)^n*a*b^2*d^4*x^3 - 3
*(d*x + c)^n*a^2*b*c^2*d^2*n^2 + 9*(d*x + c)^n*a^3*c*d^3*n^2 + 6*(d*x + c)^
n*b^3*c^3*d*n*x - 24*(d*x + c)^n*a*b^2*c^2*d^2*n*x + 36*(d*x + c)^n*a^2*b*c
*d^3*n*x + 26*(d*x + c)^n*a^3*d^4*n*x + 36*(d*x + c)^n*a^2*b*d^4*x^2 + 6*(d
*x + c)^n*a*b^2*c^3*d*n - 21*(d*x + c)^n*a^2*b*c^2*d^2*n + 26*(d*x + c)^n*a
^3*c*d^3*n + 24*(d*x + c)^n*a^3*d^4*x - 6*(d*x + c)^n*b^3*c^4 + 24*(d*x + c
)^n*a*b^2*c^3*d - 36*(d*x + c)^n*a^2*b*c^2*d^2 + 24*(d*x + c)^n*a^3*c*d^3)/
(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2 + 50*d^4*n + 24*d^4)
```

**maple [B]** time = 0.01, size = 386, normalized size = 3.48

$$\frac{\left(b^3d^3n^3x^3 + 3ab^2d^3n^3x^2 + 6b^3d^3n^2x^3 + 3a^2bd^3n^3x + 21ab^2d^3n^2x^2 - 3b^3cd^2n^2x^2 + 11b^3d^3nx^3 + a^3d^3n^3 + 24a^2bd^3n^2x - 6ab^2cd^2n^2x + 42ab^2d^3nx^2 - 9b^3cd^2nx^2 + 6b^3d^3x^3 + 9a^3d^3n^2 - 3a^2bcd^2n^2 + 57a^2bd^3nx - 30ab^2cd^2nx + 24ab^2d^3x^2 + 6b^3c^2dnx - 6b^3cd^2x^2 + 26a^3d^3n - 21a^2bcd^2n + 36a^2bd^3x + 6ab^2c^2dn - 24ab^2cd^2x + 6b^3c^2dx + 24a^3d^3 - 36a^2bcd^2 + 24ab^2c^2d - 6b^3c^3\right)(dx+c)^{n+1}}{\left(n^4 + 10n^3 + 35n^2 + 50n + 24\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^n,x)
```

```
[Out] (d*x+c)^(n+1)*(b^3*d^3*n^3*x^3+3*a*b^2*d^3*n^3*x^2+6*b^3*d^3*n^2*x^3+3*a^2*
b*d^3*n^3*x+21*a*b^2*d^3*n^2*x^2-3*b^3*c*d^2*n^2*x^2+11*b^3*d^3*n*x^3+a^3*d
^3*n^3+24*a^2*b*d^3*n^2*x-6*a*b^2*c*d^2*n^2*x+42*a*b^2*d^3*n*x^2-9*b^3*c*d^
2*n*x^2+6*b^3*d^3*x^3+9*a^3*d^3*n^2-3*a^2*b*c*d^2*n^2+57*a^2*b*d^3*n*x-30*a
*b^2*c*d^2*n*x+24*a*b^2*d^3*x^2+6*b^3*c^2*d*n*x-6*b^3*c*d^2*x^2+26*a^3*d^3*
n-21*a^2*b*c*d^2*n+36*a^2*b*d^3*x+6*a*b^2*c^2*d*n-24*a*b^2*c*d^2*x+6*b^3*c^
2*d*x+24*a^3*d^3-36*a^2*b*c*d^2+24*a*b^2*c^2*d-6*b^3*c^3)/d^4/(n^4+10*n^3+3
5*n^2+50*n+24)
```

**maxima [B]** time = 1.30, size = 246, normalized size = 2.22

$$\frac{3\left(d^2(n+1)x^2 + cdnx - c^2\right)(dx+c)^n a^2 b}{\left(n^2+3n+2\right)d^2} + \frac{(dx+c)^{n+1}a^3}{d(n+1)} + \frac{3\left(\left(n^2+3n+2\right)d^3x^3 + \left(n^2+n\right)cd^2x^2 - 2c^2dnx + 2c^3\right)(dx+c)^n ab^2}{\left(n^3+6n^2+11n+6\right)d^3} + \frac{\left(\left(n^3+6n^2+11n+6\right)d^4x^4 + \left(n^3+3n^2+2n\right)cd^3x^3 - 3\left(n^2+n\right)c^2d^2x^2 + 6c^3dnx - 6c^4\right)(dx+c)^n b^3}{\left(n^4+10n^3+35n^2+50n+24\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] 3*(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*a^2*b/((n^2 + 3*n + 2)*d^2)
 + (d*x + c)^(n + 1)*a^3/(d*(n + 1)) + 3*((n^2 + 3*n + 2)*d^3*x^3 + (n^2 +
n)*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*a*b^2/((n^3 + 6*n^2 + 11*n
+ 6)*d^3) + ((n^3 + 6*n^2 + 11*n + 6)*d^4*x^4 + (n^3 + 3*n^2 + 2*n)*c*d^3*x
^3 - 3*(n^2 + n)*c^2*d^2*x^2 + 6*c^3*d*n*x - 6*c^4)*(d*x + c)^n*b^3/((n^4 +
 10*n^3 + 35*n^2 + 50*n + 24)*d^4)
```

**mupad [B]** time = 0.91, size = 478, normalized size = 4.31

$$\frac{x\,(c+d\,x)^n\,\left(a^3\,d^4\,n^3+9\,a^3\,d^4\,n^2+26\,a^3\,d^4\,n+24\,a^3\,d^4+3\,a^2\,b\,c\,d^3\,n^3+21\,a^2\,b\,c\,d^3\,n^2+36\,a^2\,b\,c\,d^3\,n-6\,a\,b^2\,c^2\,d^2\,n^2-24\,a\,b^2\,c^2\,d^2\,n+6\,b^3\,c^3\,d\,n\right)}{d^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{b^3\,x^4\,(c+d\,x)^n\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}+\frac{c\,(c+d\,x)^n\,\left(a^3\,d^3\,n^3+9\,a^3\,d^3\,n^2+26\,a^3\,d^3\,n+24\,a^3\,d^3-3\,a^2\,b\,c\,d^2\,n^2-21\,a^2\,b\,c\,d^2\,n-36\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d\,n+24\,a\,b^2\,c^2\,d-6\,b^3\,c^3\right)}{d^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{3\,b\,x^2\,(n+1)\,(c+d\,x)^n\,\left(a^2\,d^2\,n^2+7\,a^2\,d^2\,n+12\,a^2\,d^2+a\,b\,c\,d\,n^2+4\,a\,b\,c\,d\,n-b^2\,c^2\,n\right)}{d^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{b^2\,x^3\,(c+d\,x)^n\,(12\,a\,d+3\,a\,d\,n+b\,c\,n)\,\left(n^2+3\,n+2\right)}{d\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^n,x)
```

```
[Out] (x*(c + d*x)^n*(24*a^3*d^4 + 26*a^3*d^4*n + 9*a^3*d^4*n^2 + a^3*d^4*n^3 + 6
*b^3*c^3*d*n + 36*a^2*b*c*d^3*n - 24*a*b^2*c^2*d^2*n + 21*a^2*b*c*d^3*n^2 +
 3*a^2*b*c*d^3*n^3 - 6*a*b^2*c^2*d^2*n^2))/(d^4*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24)) + (b^3*x^4*(c + d*x)^n*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 +
 10*n^3 + n^4 + 24) + (c*(c + d*x)^n*(24*a^3*d^3 - 6*b^3*c^3 + 26*a^3*d^3*n
 + 9*a^3*d^3*n^2 + a^3*d^3*n^3 + 24*a*b^2*c^2*d - 36*a^2*b*c*d^2 + 6*a*b^2*
c^2*d*n - 21*a^2*b*c*d^2*n - 3*a^2*b*c*d^2*n^2))/(d^4*(50*n + 35*n^2 + 10*n
^3 + n^4 + 24)) + (3*b*x^2*(n + 1)*(c + d*x)^n*(12*a^2*d^2 + 7*a^2*d^2*n -
b^2*c^2*n + a^2*d^2*n^2 + 4*a*b*c*d*n + a*b*c*d*n^2))/(d^2*(50*n + 35*n^2 +
 10*n^3 + n^4 + 24)) + (b^2*x^3*(c + d*x)^n*(12*a*d + 3*a*d*n + b*c*n)*(3*n
 + n^2 + 2))/(d*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))
```

**sympy [A]** time = 4.44, size = 4058, normalized size = 36.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4), Eq(
d, 0)), (-2*a**3*d**3/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d*
*7*x**3) - 3*a**2*b*c*d**2/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 +
 6*d**7*x**3) - 9*a**2*b*d**3*x/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x
**2 + 6*d**7*x**3) - 6*a*b**2*c**2*d/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d
**6*x**2 + 6*d**7*x**3) - 18*a*b**2*c*d**2*x/(6*c**3*d**4 + 18*c**2*d**5*x
+ 18*c*d**6*x**2 + 6*d**7*x**3) - 18*a*b**2*d**3*x**2/(6*c**3*d**4 + 18*c**
2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 6*b**3*c**3*log(c/d + x)/(6*c**3
*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 11*b**3*c**3/(6*c*
*3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 18*b**3*c**2*d*x
*log(c/d + x)/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3)
 + 27*b**3*c**2*d*x/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7
*x**3) + 18*b**3*c*d**2*x**2*log(c/d + x)/(6*c**3*d**4 + 18*c**2*d**5*x + 1
8*c*d**6*x**2 + 6*d**7*x**3) + 18*b**3*c*d**2*x**2/(6*c**3*d**4 + 18*c**2*d
**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 6*b**3*d**3*x**3*log(c/d + x)/(6*c*
*3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3), Eq(n, -4)), (-a**
3*d**3/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 3*a**2*b*c*d**2/(2*c**2*d
**4 + 4*c*d**5*x + 2*d**6*x**2) - 6*a**2*b*d**3*x/(2*c**2*d**4 + 4*c*d**5*x
 + 2*d**6*x**2) + 6*a*b**2*c**2*d*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x +
```

2*d**6*x**2) + 9*a*b**2*c**2*d/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 1
2*a*b**2*c*d**2*x*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 1
2*a*b**2*c*d**2*x/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 6*a*b**2*d**3*
x**2*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 6*b**3*c**3*lo
g(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 9*b**3*c**3/(2*c**2*d
**4 + 4*c*d**5*x + 2*d**6*x**2) - 12*b**3*c**2*d*x*log(c/d + x)/(2*c**2*d**
4 + 4*c*d**5*x + 2*d**6*x**2) - 12*b**3*c**2*d*x/(2*c**2*d**4 + 4*c*d**5*x
+ 2*d**6*x**2) - 6*b**3*c*d**2*x**2*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x
+ 2*d**6*x**2) + 2*b**3*d**3*x**3/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2),
 Eq(n, -3)), (-2*a**3*d**3/(2*c*d**4 + 2*d**5*x) + 6*a**2*b*c*d**2*log(c/d
+ x)/(2*c*d**4 + 2*d**5*x) + 6*a**2*b*c*d**2/(2*c*d**4 + 2*d**5*x) + 6*a**2
*b*d**3*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x) - 12*a*b**2*c**2*d*log(c/d + x
)/(2*c*d**4 + 2*d**5*x) - 12*a*b**2*c**2*d/(2*c*d**4 + 2*d**5*x) - 12*a*b**
2*c*d**2*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x) + 6*a*b**2*d**3*x**2/(2*c*d**
4 + 2*d**5*x) + 6*b**3*c**3*log(c/d + x)/(2*c*d**4 + 2*d**5*x) + 6*b**3*c**
3/(2*c*d**4 + 2*d**5*x) + 6*b**3*c**2*d*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x
) - 3*b**3*c*d**2*x**2/(2*c*d**4 + 2*d**5*x) + b**3*d**3*x**3/(2*c*d**4 + 2
*d**5*x), Eq(n, -2)), (a**3*log(c/d + x)/d - 3*a**2*b*c*log(c/d + x)/d**2 +
 3*a**2*b*x/d + 3*a*b**2*c**2*log(c/d + x)/d**3 - 3*a*b**2*c*x/d**2 + 3*a*b
**2*x**2/(2*d) - b**3*c**3*log(c/d + x)/d**4 + b**3*c**2*x/d**3 - b**3*c*x*
*2/(2*d**2) + b**3*x**3/(3*d), Eq(n, -1)), (a**3*c*d**3*n**3*(c + d*x)**n/(
d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 9*a**3*c*d
**3*n**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n
+ 24*d**4) + 26*a**3*c*d**3*n*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d
**4*n**2 + 50*d**4*n + 24*d**4) + 24*a**3*c*d**3*(c + d*x)**n/(d**4*n**4 +
10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + a**3*d**4*n**3*x*(c +
d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 9
*a**3*d**4*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 5
0*d**4*n + 24*d**4) + 26*a**3*d**4*n*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n*
*3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 24*a**3*d**4*x*(c + d*x)**n/(d**
4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 3*a**2*b*c**2
*d**2*n**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*
n + 24*d**4) - 21*a**2*b*c**2*d**2*n*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3
 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 36*a**2*b*c**2*d**2*(c + d*x)**n/(
d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a**2*b*c
*d**3*n**3*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**
4*n + 24*d**4) + 21*a**2*b*c*d**3*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*
n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 36*a**2*b*c*d**3*n*x*(c + d*x)
**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a**
2*b*d**4*n**3*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 +
50*d**4*n + 24*d**4) + 24*a**2*b*d**4*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 1
0*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 57*a**2*b*d**4*n*x**2*(
c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4)
 + 36*a**2*b*d**4*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**
2 + 50*d**4*n + 24*d**4) + 6*a*b**2*c**3*d*n*(c + d*x)**n/(d**4*n**4 + 10*d

```
**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 24*a*b**2*c**3*d*(c + d*x)
**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 6*a*b
**2*c**2*d**2*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2
+ 50*d**4*n + 24*d**4) - 24*a*b**2*c**2*d**2*n*x*(c + d*x)**n/(d**4*n**4 +
10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a*b**2*c*d**3*n**3*x
**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*
d**4) + 15*a*b**2*c*d**3*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 +
 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 12*a*b**2*c*d**3*n*x**2*(c + d*x)**n
/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a*b**2
*d**4*n**3*x**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*
d**4*n + 24*d**4) + 21*a*b**2*d**4*n**2*x**3*(c + d*x)**n/(d**4*n**4 + 10*d
**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 42*a*b**2*d**4*n*x**3*(c +
 d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) +
24*a*b**2*d**4*x**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 +
 50*d**4*n + 24*d**4) - 6*b**3*c**4*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3
+ 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 6*b**3*c**3*d*n*x*(c + d*x)**n/(d**
4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 3*b**3*c**2*d
**2*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d*
*4*n + 24*d**4) - 3*b**3*c**2*d**2*n*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4
*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + b**3*c*d**3*n**3*x**3*(c + d*
x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*b
**3*c*d**3*n**2*x**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2
+ 50*d**4*n + 24*d**4) + 2*b**3*c*d**3*n*x**3*(c + d*x)**n/(d**4*n**4 + 10*
d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + b**3*d**4*n**3*x**4*(c +
d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 6
*b**3*d**4*n**2*x**4*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2
+ 50*d**4*n + 24*d**4) + 11*b**3*d**4*n*x**4*(c + d*x)**n/(d**4*n**4 + 10*d
**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 6*b**3*d**4*x**4*(c + d*x)
**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4), True))
```

## 3.1559 $\int (a+bx)^2(c+dx)^n\,dx$

**Optimal**. Leaf size=78

$$\frac{(bc-ad)^2(c+dx)^{n+1}}{d^3(n+1)}-\frac{2b(bc-ad)(c+dx)^{n+2}}{d^3(n+2)}+\frac{b^2(c+dx)^{n+3}}{d^3(n+3)}$$

**Rubi** [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{(bc-ad)^2(c+dx)^{n+1}}{d^3(n+1)}-\frac{2b(bc-ad)(c+dx)^{n+2}}{d^3(n+2)}+\frac{b^2(c+dx)^{n+3}}{d^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^n,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^(1 + n))/(d^3*(1 + n)) - (2*b*(b*c - a*d)*(c + d*x
)^(2 + n))/(d^3*(2 + n)) + (b^2*(c + d*x)^(3 + n))/(d^3*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^n\,dx&=\int\left(\frac{(-bc+ad)^2(c+dx)^n}{d^2}-\frac{2b(bc-ad)(c+dx)^{1+n}}{d^2}+\frac{b^2(c+dx)^{2+n}}{d^2}\right)dx\\&=\frac{(bc-ad)^2(c+dx)^{1+n}}{d^3(1+n)}-\frac{2b(bc-ad)(c+dx)^{2+n}}{d^3(2+n)}+\frac{b^2(c+dx)^{3+n}}{d^3(3+n)}\end{aligned}$$

**Mathematica** [A] time = 0.10, size = 67, normalized size = 0.86

$$\frac{(c+dx)^{n+1}\left(-\frac{2b(c+dx)(bc-ad)}{n+2}+\frac{(bc-ad)^2}{n+1}+\frac{b^2(c+dx)^2}{n+3}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*((b*c - a*d)^2/(1 + n) - (2*b*(b*c - a*d)*(c + d*x))/(2
+ n) + (b^2*(c + d*x)^2)/(3 + n)))/d^3
```

**IntegrateAlgebraic [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^2*(c + d*x)^n, x]
```

**fricas [B]** time = 1.35, size = 237, normalized size = 3.04

$$\frac{\left(a^2cd^2n^2+2\,b^2c^3-6\,abc^2d+6\,a^2cd^2+\left(b^2d^3n^2+3\,b^2d^3n+2\,b^2d^3\right)x^3+\left(6\,abd^3+\left(b^2cd^2+2\,abd^3\right)n^2+\left(b^2cd^2+8\,abd^3\right)n\right)x^2-\left(2\,abc^2d-5\,a^2cd^2\right)n+\left(6\,a^2d^3+\left(2\,abcd^2+a^2d^3\right)n^2-\left(2\,b^2c^2d-6\,abcd^2-5\,a^2d^3\right)n\right)x\right)(dx+c)^n}{d^3n^3+6\,d^3n^2+11\,d^3n+6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a^2*c*d^2*n^2 + 2*b^2*c^3 - 6*a*b*c^2*d + 6*a^2*c*d^2 + (b^2*d^3*n^2 + 3*b
^2*d^3*n + 2*b^2*d^3)*x^3 + (6*a*b*d^3 + (b^2*c*d^2 + 2*a*b*d^3)*n^2 + (b^2
*c*d^2 + 8*a*b*d^3)*n)*x^2 - (2*a*b*c^2*d - 5*a^2*c*d^2)*n + (6*a^2*d^3 + (
2*a*b*c*d^2 + a^2*d^3)*n^2 - (2*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*n)*x)*
(d*x + c)^n/(d^3*n^3 + 6*d^3*n^2 + 11*d^3*n + 6*d^3)
```

**giac [B]** time = 0.93, size = 385, normalized size = 4.94

$$\frac{(dx+c)^nb^2d^3n^2x^3+(dx+c)^nb^2cd^2n^2x^2+2\,(dx+c)^nabd^3n^2x^2+3\,(dx+c)^nb^2d^3nx^3+2\,(dx+c)^nabcd^2n^2x+(dx+c)^na^2d^3n^2x+(dx+c)^nb^2cd^2nx^2+8\,(dx+c)^nabd^3nx^2+2\,(dx+c)^nb^2d^3x^3+(dx+c)^na^2cd^2n^2-2\,(dx+c)^nb^2c^2dnx+6\,(dx+c)^nabcd^2nx+5\,(dx+c)^na^2d^3nx+6\,(dx+c)^nabd^3x^2-2\,(dx+c)^nabc^2dn+5\,(dx+c)^na^2cd^2n+6\,(dx+c)^na^2d^3x+2\,(dx+c)^nb^2c^3-6\,(dx+c)^nabc^2d+6\,(dx+c)^na^2cd^2}{d^3n^3+6\,d^3n^2+11\,d^3n+6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b^2*d^3*n^2*x^3 + (d*x + c)^n*b^2*c*d^2*n^2*x^2 + 2*(d*x + c)^
n*a*b*d^3*n^2*x^2 + 3*(d*x + c)^n*b^2*d^3*n*x^3 + 2*(d*x + c)^n*a*b*c*d^2*n
^2*x + (d*x + c)^n*a^2*d^3*n^2*x + (d*x + c)^n*b^2*c*d^2*n*x^2 + 8*(d*x + c
)^n*a*b*d^3*n*x^2 + 2*(d*x + c)^n*b^2*d^3*x^3 + (d*x + c)^n*a^2*c*d^2*n^2 -
 2*(d*x + c)^n*b^2*c^2*d*n*x + 6*(d*x + c)^n*a*b*c*d^2*n*x + 5*(d*x + c)^n*
a^2*d^3*n*x + 6*(d*x + c)^n*a*b*d^3*x^2 - 2*(d*x + c)^n*a*b*c^2*d*n + 5*(d*
x + c)^n*a^2*c*d^2*n + 6*(d*x + c)^n*a^2*d^3*x + 2*(d*x + c)^n*b^2*c^3 - 6*
(d*x + c)^n*a*b*c^2*d + 6*(d*x + c)^n*a^2*c*d^2)/(d^3*n^3 + 6*d^3*n^2 + 11*
d^3*n + 6*d^3)
```

**maple** [B] time = 0.01, size = 159, normalized size = 2.04

$$\frac{\left(b^{2}d^{2}n^{2}x^{2}+2ab\,d^{2}n^{2}x+3b^{2}d^{2}n\,x^{2}+a^{2}d^{2}n^{2}+8ab\,d^{2}nx-2b^{2}cdnx+2b^{2}x^{2}d^{2}+5a^{2}d^{2}n-2abcdn+6ab\,d^{2}x-2b^{2}cdx+6a^{2}d^{2}-6abcd+2b^{2}c^{2}\right)\left(dx+c\right)^{n+1}}{\left(n^{3}+6n^{2}+11n+6\right)d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^n,x)
```

```
[Out] (d*x+c)^(n+1)*(b^2*d^2*n^2*x^2+2*a*b*d^2*n^2*x+3*b^2*d^2*n*x^2+a^2*d^2*n^2+
8*a*b*d^2*n*x-2*b^2*c*d*n*x+2*b^2*d^2*x^2+5*a^2*d^2*n-2*a*b*c*d*n+6*a*b*d^2
*x-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/d^3/(n^3+6*n^2+11*n+6)
```

**maxima** [A] time = 1.17, size = 138, normalized size = 1.77

$$\frac{2\left(d^{2}(n+1)x^{2}+cdnx-c^{2}\right)(dx+c)^{n}ab}{\left(n^{2}+3\,n+2\right)d^{2}}+\frac{(dx+c)^{n+1}a^{2}}{d(n+1)}+\frac{\left(\left(n^{2}+3\,n+2\right)d^{3}x^{3}+\left(n^{2}+n\right)cd^{2}x^{2}-2\,c^{2}dnx+2\,c^{3}\right)(dx+c)^{n}b^{2}}{\left(n^{3}+6\,n^{2}+11\,n+6\right)d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] 2*(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*a*b/((n^2 + 3*n + 2)*d^2) +
 (d*x + c)^(n + 1)*a^2/(d*(n + 1)) + ((n^2 + 3*n + 2)*d^3*x^3 + (n^2 + n)*c
*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*b^2/((n^3 + 6*n^2 + 11*n + 6)*d
^3)
```

**mupad** [B] time = 0.62, size = 226, normalized size = 2.90

$$(c+d\,x)^{n}\left(\frac{c\,\left(a^{2}\,d^{2}\,n^{2}+5\,a^{2}\,d^{2}\,n+6\,a^{2}\,d^{2}-2\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+2\,b^{2}\,c^{2}\right)}{d^{3}\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}+\frac{b^{2}\,x^{3}\,\left(n^{2}+3\,n+2\right)}{n^{3}+6\,n^{2}+11\,n+6}+\frac{x\,\left(a^{2}\,d^{3}\,n^{2}+5\,a^{2}\,d^{3}\,n+6\,a^{2}\,d^{3}+2\,a\,b\,c\,d^{2}\,n^{2}+6\,a\,b\,c\,d^{2}\,n-2\,b^{2}\,c^{2}\,d\,n\right)}{d^{3}\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}+\frac{b\,x^{2}\,(n+1)\,(6\,a\,d+2\,a\,d\,n+b\,c\,n)}{d\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^n,x)
```

```
[Out] (c + d*x)^n*((c*(6*a^2*d^2 + 2*b^2*c^2 + 5*a^2*d^2*n + a^2*d^2*n^2 - 6*a*b*
c*d - 2*a*b*c*d*n))/(d^3*(11*n + 6*n^2 + n^3 + 6)) + (b^2*x^3*(3*n + n^2 +
2))/(11*n + 6*n^2 + n^3 + 6) + (x*(6*a^2*d^3 + 5*a^2*d^3*n + a^2*d^3*n^2 -
2*b^2*c^2*d*n + 2*a*b*c*d^2*n^2 + 6*a*b*c*d^2*n))/(d^3*(11*n + 6*n^2 + n^3
+ 6)) + (b*x^2*(n + 1)*(6*a*d + 2*a*d*n + b*c*n))/(d*(11*n + 6*n^2 + n^3 +
6)))
```

**sympy** [A] time = 2.09, size = 1506, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a**2*x + a*b*x**2 + b**2*x**3/3), Eq(d, 0)), (-a**2*d**2/(
2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) - 2*a*b*c*d/(2*c**2*d**3 + 4*c*d**4
*x + 2*d**5*x**2) - 4*a*b*d**2*x/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) +
 2*b**2*c**2*log(c/d + x)/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 3*b**2
*c**2/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 4*b**2*c*d*x*log(c/d + x)/
(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 4*b**2*c*d*x/(2*c**2*d**3 + 4*c*
d**4*x + 2*d**5*x**2) + 2*b**2*d**2*x**2*log(c/d + x)/(2*c**2*d**3 + 4*c*d*
*4*x + 2*d**5*x**2), Eq(n, -3)), (-a**2*d**2/(c*d**3 + d**4*x) + 2*a*b*c*d*
log(c/d + x)/(c*d**3 + d**4*x) + 2*a*b*c*d/(c*d**3 + d**4*x) + 2*a*b*d**2*x
*log(c/d + x)/(c*d**3 + d**4*x) - 2*b**2*c**2*log(c/d + x)/(c*d**3 + d**4*x
) - 2*b**2*c**2/(c*d**3 + d**4*x) - 2*b**2*c*d*x*log(c/d + x)/(c*d**3 + d**
4*x) + b**2*d**2*x**2/(c*d**3 + d**4*x), Eq(n, -2)), (a**2*log(c/d + x)/d -
 2*a*b*c*log(c/d + x)/d**2 + 2*a*b*x/d + b**2*c**2*log(c/d + x)/d**3 - b**2
*c*x/d**2 + b**2*x**2/(2*d), Eq(n, -1)), (a**2*c*d**2*n**2*(c + d*x)**n/(d*
*3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 5*a**2*c*d**2*n*(c + d*x)**n/
(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a**2*c*d**2*(c + d*x)**n
/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + a**2*d**3*n**2*x*(c + d*x
)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 5*a**2*d**3*n*x*(c +
d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a**2*d**3*x*(c +
 d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) - 2*a*b*c**2*d*n*(c
 + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) - 6*a*b*c**2*d*(c
 + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*a*b*c*d**2*n*
*2*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a*b*c*
d**2*n*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*a*
b*d**3*n**2*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3
) + 8*a*b*d**3*n*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6
*d**3) + 6*a*b*d**3*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n
+ 6*d**3) + 2*b**2*c**3*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n +
 6*d**3) - 2*b**2*c**2*d*n*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**
3*n + 6*d**3) + b**2*c*d**2*n**2*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2
 + 11*d**3*n + 6*d**3) + b**2*c*d**2*n*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**
3*n**2 + 11*d**3*n + 6*d**3) + b**2*d**3*n**2*x**3*(c + d*x)**n/(d**3*n**3
+ 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 3*b**2*d**3*n*x**3*(c + d*x)**n/(d**3
*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*b**2*d**3*x**3*(c + d*x)**n/(
d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3), True))
```

## 3.1560 $\int (a+bx)(c+dx)^n\,dx$

**Optimal**. Leaf size=47

$$\frac{b(c+dx)^{n+2}}{d^2(n+2)}-\frac{(bc-ad)(c+dx)^{n+1}}{d^2(n+1)}$$

**Rubi [A]** time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^{n+2}}{d^2(n+2)}-\frac{(bc-ad)(c+dx)^{n+1}}{d^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] -(((b*c - a*d)*(c + d*x)^(1 + n))/(d^2*(1 + n))) + (b*(c + d*x)^(2 + n))/(d
^2*(2 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^n}{d}+\frac{b(c+dx)^{1+n}}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^{1+n}}{d^2(1+n)}+\frac{b(c+dx)^{2+n}}{d^2(2+n)}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 41, normalized size = 0.87

$$\frac{(c+dx)^{n+1}(ad(n+2)-bc+bd(n+1)x)}{d^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*(-(b*c) + a*d*(2 + n) + b*d*(1 + n)*x))/(d^2*(1 + n)*(2
+ n))
```

**IntegrateAlgebraic [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)*(c + d*x)^n, x]
```

**fricas [A]** time = 1.28, size = 83, normalized size = 1.77

$$\frac{\left(acdn - bc^2 + 2\,acd + \left(bd^2n + bd^2\right)x^2 + \left(2\,ad^2 + \left(bcd + ad^2\right)n\right)x\right)(dx+c)^n}{d^2n^2 + 3\,d^2n + 2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a*c*d*n - b*c^2 + 2*a*c*d + (b*d^2*n + b*d^2)*x^2 + (2*a*d^2 + (b*c*d + a*
d^2)*n)*x)*(d*x + c)^n/(d^2*n^2 + 3*d^2*n + 2*d^2)
```

**giac [B]** time = 1.00, size = 132, normalized size = 2.81

$$\frac{(dx+c)^n bd^2nx^2 + (dx+c)^n bcdnx + (dx+c)^n ad^2nx + (dx+c)^n bd^2x^2 + (dx+c)^n acdn + 2\,(dx+c)^n ad^2x - (dx+c)^n bc^2 + 2\,(dx+c)^n acd}{d^2n^2 + 3\,d^2n + 2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b*d^2*n*x^2 + (d*x + c)^n*b*c*d*n*x + (d*x + c)^n*a*d^2*n*x +
(d*x + c)^n*b*d^2*x^2 + (d*x + c)^n*a*c*d*n + 2*(d*x + c)^n*a*d^2*x - (d*x
+ c)^n*b*c^2 + 2*(d*x + c)^n*a*c*d)/(d^2*n^2 + 3*d^2*n + 2*d^2)
```

**maple [A]** time = 0.00, size = 46, normalized size = 0.98

$$\frac{(bdnx + adn + bdx + 2ad - bc)\,(dx+c)^{n+1}}{\left(n^2 + 3n + 2\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^n,x)
```

[Out] (d*x+c)^(n+1)*(b*d*n*x+a*d*n+b*d*x+2*a*d-b*c)/d^2/(n^2+3*n+2)

**maxima** [A] time = 1.17, size = 63, normalized size = 1.34

$$\frac{\left(d^2(n+1)x^2+cdnx-c^2\right)(dx+c)^n b}{\left(n^2+3\,n+2\right)d^2}+\frac{(dx+c)^{n+1}a}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="maxima")

[Out] (d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*b/((n^2 + 3*n + 2)*d^2) + (d*x + c)^(n + 1)*a/(d*(n + 1))

**mupad** [B] time = 0.49, size = 88, normalized size = 1.87

$$(c+d\,x)^n\left(\frac{c\,\left(2\,a\,d-b\,c+a\,d\,n\right)}{d^2\,\left(n^2+3\,n+2\right)}+\frac{b\,x^2\,\left(n+1\right)}{n^2+3\,n+2}+\frac{x\,\left(2\,a\,d^2+a\,d^2\,n+b\,c\,d\,n\right)}{d^2\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)*(c + d*x)^n,x)

[Out] (c + d*x)^n*((c*(2*a*d - b*c + a*d*n))/(d^2*(3*n + n^2 + 2)) + (b*x^2*(n + 1))/(3*n + n^2 + 2) + (x*(2*a*d^2 + a*d^2*n + b*c*d*n))/(d^2*(3*n + n^2 + 2)))

**sympy** [A] time = 0.82, size = 377, normalized size = 8.02

$$\begin{cases} c^n\left(ax+\frac{bx^2}{2}\right) & \text{for}\ d=0 \\ -\frac{ad}{cd^2+d^3x}+\frac{bc\log\left(\frac{c}{d}+x\right)}{cd^2+d^3x}+\frac{bc}{cd^2+d^3x}+\frac{bdx\log\left(\frac{c}{d}+x\right)}{cd^2+d^3x} & \text{for}\ n=-2 \\ \frac{a\log\left(\frac{c}{d}+x\right)}{d}-\frac{bc\log\left(\frac{c}{d}+x\right)}{d^2}+\frac{bx}{d} & \text{for}\ n=-1 \\ \frac{acdn(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{2acd(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{ad^2nx(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{2ad^2x(c+dx)^n}{d^2n^2+3d^2n+2d^2}-\frac{bc^2(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{bcdnx(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{bd^2nx^2(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{bd^2x^2(c+dx)^n}{d^2n^2+3d^2n+2d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(d*x+c)**n,x)

[Out] Piecewise((c**n*(a*x + b*x**2/2), Eq(d, 0)), (-a*d/(c*d**2 + d**3*x) + b*c*log(c/d + x)/(c*d**2 + d**3*x) + b*c/(c*d**2 + d**3*x) + b*d*x*log(c/d + x)/(c*d**2 + d**3*x), Eq(n, -2)), (a*log(c/d + x)/d - b*c*log(c/d + x)/d**2 + b*x/d, Eq(n, -1)), (a*c*d*n*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + 2*a*c*d*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + a*d**2*n*x*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + 2*a*d**2*x*(c + d*x)**n/(d**2*n**2 +

```
 3*d**2*n + 2*d**2) - b*c**2*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) +
 b*c*d*n*x*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + b*d**2*n*x**2*(c
+ d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + b*d**2*x**2*(c + d*x)**n/(d**2*
n**2 + 3*d**2*n + 2*d**2), True))
```

## 3.1561 $\int (c+dx)^n \, dx$

**Optimal**. Leaf size=18

$$\frac{(c+dx)^{n+1}}{d(n+1)}$$

**Rubi [A]** time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n,x]
[Out] (c + d*x)^(1 + n)/(d*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^n \, dx = \frac{(c+dx)^{1+n}}{d(1+n)}$$

**Mathematica [A]** time = 0.01, size = 17, normalized size = 0.94

$$\frac{(c+dx)^{n+1}}{dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n,x]
[Out] (c + d*x)^(1 + n)/(d + d*n)
```

**IntegrateAlgebraic [F]** time = 0.01, size = 0, normalized size = 0.00

$$\int (c+dx)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(c + d*x)^n, x]
```

**fricas [A]** time = 1.25, size = 20, normalized size = 1.11

$$\frac{(dx+c)(dx+c)^n}{dn+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (d*x + c)*(d*x + c)^n/(d*n + d)
```

**giac [A]** time = 1.02, size = 18, normalized size = 1.00

$$\frac{(dx+c)^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="giac")
```

```
[Out] (d*x + c)^(n + 1)/(d*(n + 1))
```

**maple [A]** time = 0.00, size = 19, normalized size = 1.06

$$\frac{(dx+c)^{n+1}}{(n+1)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n,x)
```

```
[Out] (d*x+c)^(n+1)/d/(n+1)
```

**maxima [A]** time = 1.12, size = 18, normalized size = 1.00

$$\frac{(dx+c)^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(n + 1)/(d*(n + 1))
```

**mupad [B]** time = 0.38, size = 18, normalized size = 1.00

$$\frac{(c + d\,x)^{n+1}}{d\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n,x)
```

```
[Out] (c + d*x)^(n + 1)/(d*(n + 1))
```

**sympy [A]** time = 0.06, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(c+dx)^{n+1}}{n+1} & \text{for}\ n \neq -1 \\ \log{(c + dx)} & \text{otherwise} \end{cases}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n,x)
```

```
[Out] Piecewise(((c + d*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(c + d*x), True))/d
```

## 3.1562 $\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx$

**Optimal**. Leaf size=143

$$-\frac{2d^2(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(3-n)(bc-ad)^3}-\frac{(a+bx)^{n-3}(c+dx)^{1-n}}{(3-n)(bc-ad)}+\frac{2d(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(3-n)(bc-ad)^2}$$

**Rubi [A]** time = 0.06, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{2d^2(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(3-n)(bc-ad)^3}-\frac{(a+bx)^{n-3}(c+dx)^{1-n}}{(3-n)(bc-ad)}+\frac{2d(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(3-n)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(3 - n))) + (2*d*(a +
 b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(2 - n)*(3 - n)) - (2*d^2*
(a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^3*(1 - n)*(2 - n)*(3 - n
))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx = -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)} - \frac{(2d)\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx}{(bc-ad)(3-n)}$$
$$= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)} + \frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)} + \frac{\left(2d^2\right)\int (a+bx)^{-2+n}(c\ \cdots}{(bc-ad)^2(2-n)(\cdots}$$
$$= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)} + \frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)} - \frac{2d^2(a+bx)^{-1+n}(c+\cdots}{(bc-ad)^3(1-n)(2-n\cdots}$$

**Mathematica [A]** time = 0.10, size = 112, normalized size = 0.78

$$\frac{(a+bx)^{n-3}(c+dx)^{1-n}\left(a^2d^2\left(n^2-5n+6\right)-2abd(n-3)(c(n-1)+dx)+b^2\left(c^2\left(n^2-3n+2\right)+2cd(n-1)x+2d^2x^2\right)\right)}{(n-3)(n-2)(n-1)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-3 + n)*(c + d*x)^(1 - n)*(a^2*d^2*(6 - 5*n + n^2) - 2*a*b*d*(-
3 + n)*(c*(-1 + n) + d*x) + b^2*(c^2*(2 - 3*n + n^2) + 2*c*d*(-1 + n)*x + 2
*d^2*x^2)))/((b*c - a*d)^3*(-3 + n)*(-2 + n)*(-1 + n))
```

**IntegrateAlgebraic [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-4 + n)/(c + d*x)^n, x]
```

**fricas [B]** time = 1.43, size = 512, normalized size = 3.58

$$-\frac{\left(2\,b^3d^3x^4+2\,ab^2c^3-6\,a^2bc^2d+6\,a^3cd^2+2\left(4\,ab^2d^3+\left(b^3cd^2-ab^2d^3\right)n\right)x^3+\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)n^2+\left(12\,a^2bd^3+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)n^2-\left(b^3c^2d-8\,ab^2cd^2+7\,a^2bd^3\right)n\right)x^2-\left(3\,ab^2c^3-8\,a^2bc^2d+5\,a^3cd^2\right)n+\left(2\,b^3c^3-6\,ab^2c^2d+6\,a^2bcd^2+6\,a^3d^3+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)n^2-\left(3\,b^3c^3-7\,ab^2c^2d-a^2bcd^2+5\,a^3d^3\right)n\right)x\right)(bx+a)^{n-4}}{\left(6\,b^3c^3-18\,ab^2c^2d+18\,a^2bcd^2-6\,a^3d^3-\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n^3+6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n^2-11\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n\right)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] -(2*b^3*d^3*x^4 + 2*a*b^2*c^3 - 6*a^2*b*c^2*d + 6*a^3*c*d^2 + 2*(4*a*b^2*d^
3 + (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*n^2 + (12*a^2*b*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 - (b^3*
c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*n)*x^2 - (3*a*b^2*c^3 - 8*a^2*b*c^2*d
```

```
+ 5*a^3*c*d^2)*n + (2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 6*a^3*d^3 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 - (3*b^3*c^3 - 7*a*b^2
*c^2*d - a^2*b*c*d^2 + 5*a^3*d^3)*n)*x)*(b*x + a)^(n - 4)/((6*b^3*c^3 - 18*
a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*n^2 - 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n)*(d*x +
 c)^n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n-4}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 4)/(d*x + c)^n, x)
```

**maple** [B] time = 0.01, size = 322, normalized size = 2.25

$$-\frac{(dx+c)\left(a^2d^2n^2-2abcd\,n^2-2ab\,d^2nx+b^2c^2n^2+2b^2cdnx+2b^2x^2d^2-5a^2d^2n+8abcdn+6ab\,d^2x-3b^2c^2n-2b^2cdx+6a^2d^2-6abcd+2b^2c^2\right)(bx+a)^{n-3}(dx+c)^{-n}}{a^3d^3n^3-3a^2bc\,d^2n^3+3a\,b^2c^2d\,n^3-b^3c^3n^3-6a^3d^3n^2+18a^2bc\,d^2n^2-18a\,b^2c^2d\,n^2+6b^3c^3n^2+11a^3d^3n-33a^2bc\,d^2n+33a\,b^2c^2dn-11b^3c^3n-6a^3d^3+18a^2bc\,d^2-18a\,b^2c^2d+6b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-4+n)/((d*x+c)^n),x)
```

```
[Out] -(b*x+a)^(n-3)*(d*x+c)*(a^2*d^2*n^2-2*a*b*c*d*n^2-2*a*b*d^2*n*x+b^2*c^2*n^2
+2*b^2*c*d*n*x+2*b^2*d^2*x^2-5*a^2*d^2*n+8*a*b*c*d*n+6*a*b*d^2*x-3*b^2*c^2*
n-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3
+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3-6*a^3*d^3*n^2+18*a^2*b*c*d^2*n^2-18*a*b^2*c^
2*d*n^2+6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n-11*b^3
*c^3*n-6*a^3*d^3+18*a^2*b*c*d^2-18*a*b^2*c^2*d+6*b^3*c^3)/((d*x+c)^n)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n-4}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 4)/(d*x + c)^n, x)
```

**mupad** [B] time = 1.10, size = 528, normalized size = 3.69

$$-\frac{x\,{\left(a+b\,x\right)}^{n-4}\,\left(a^3\,d^3\,n^2-5\,a^3\,d^3\,n+6\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2+a^2\,b\,c\,d^2\,n+6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2+7\,a\,b^2\,c^2\,d\,n-6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2-3\,b^3\,c^3\,n+2\,b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{a\,c\,{\left(a+b\,x\right)}^{n-4}\,\left(a^2\,d^2\,n^2-5\,a^2\,d^2\,n+6\,a^2\,d^2-2\,a\,b\,c\,d\,n^2+8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2-3\,b^2\,c^2\,n+2\,b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{2\,b^3\,d^3\,x^4\,{\left(a+b\,x\right)}^{n-4}}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{b\,d\,x^2\,{\left(a+b\,x\right)}^{n-4}\,\left(a^2\,d^2\,n^2-7\,a^2\,d^2\,n+12\,a^2\,d^2-2\,a\,b\,c\,d\,n^2+8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2-b^2\,c^2\,n\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{2\,b^2\,d^2\,x^3\,{\left(a+b\,x\right)}^{n-4}\,\left(4\,a\,d-a\,d\,n+b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 4)/(c + d*x)^n,x)
```

```
[Out] - (x*(a + b*x)^(n - 4)*(6*a^3*d^3 + 2*b^3*c^3 - 5*a^3*d^3*n - 3*b^3*c^3*n +
 a^3*d^3*n^2 + b^3*c^3*n^2 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 7*a*b^2*c^2*d*
n + a^2*b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(c +
 d*x)^n*(11*n - 6*n^2 + n^3 - 6)) - (a*c*(a + b*x)^(n - 4)*(6*a^2*d^2 + 2*b
^2*c^2 - 5*a^2*d^2*n - 3*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d
+ 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6*n^2 +
n^3 - 6)) - (2*b^3*d^3*x^4*(a + b*x)^(n - 4))/((a*d - b*c)^3*(c + d*x)^n*(1
1*n - 6*n^2 + n^3 - 6)) - (b*d*x^2*(a + b*x)^(n - 4)*(12*a^2*d^2 - 7*a^2*d^
2*n - b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 + 8*a*b*c*d*n - 2*a*b*c*d*n^2))
/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6*n^2 + n^3 - 6)) - (2*b^2*d^2*x^3*(a +
 b*x)^(n - 4)*(4*a*d - a*d*n + b*c*n))/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6
*n^2 + n^3 - 6))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-4+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

## 3.1563 $\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx$

**Optimal**. Leaf size=86

$$\frac{d(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(bc-ad)^2}-\frac{(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(bc-ad)}$$

**Rubi [A]** time = 0.01, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{d(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(bc-ad)^2}-\frac{(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(2 - n))) + (d*(a + b
*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(1 - n)*(2 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx &= -\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)}-\frac{d\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx}{(bc-ad)(2-n)}\\ &= -\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)}+\frac{d(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)^2(1-n)(2-n)}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 59, normalized size = 0.69

$$\frac{(a+bx)^{n-2}(c+dx)^{1-n}(-ad(n-2)+bc(n-1)+bdx)}{(n-2)(n-1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-2 + n)*(c + d*x)^(1 - n)*(-(a*d*(-2 + n)) + b*c*(-1 + n) + b*d
*x))/((b*c - a*d)^2*(-2 + n)*(-1 + n))
```

**IntegrateAlgebraic [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-3 + n)/(c + d*x)^n, x]
```

**fricas [B]** time = 1.38, size = 206, normalized size = 2.40

$$\frac{\left(b^2d^2x^3-abc^2+2\,a^2cd+\left(3\,abd^2+\left(b^2cd-abd^2\right)n\right)x^2+\left(abc^2-a^2cd\right)n-\left(b^2c^2-2\,abcd-2\,a^2d^2-\left(b^2c^2-a^2d^2\right)n\right)x\right)(bx+a)^{n-3}}{\left(2\,b^2c^2-4\,abcd+2\,a^2d^2+\left(b^2c^2-2\,abcd+a^2d^2\right)n^2-3\left(b^2c^2-2\,abcd+a^2d^2\right)n\right)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 - a*b*c^2 + 2*a^2*c*d + (3*a*b*d^2 + (b^2*c*d - a*b*d^2)*n)*x^
2 + (a*b*c^2 - a^2*c*d)*n - (b^2*c^2 - 2*a*b*c*d - 2*a^2*d^2 - (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^(n - 3)/((2*b^2*c^2 - 4*a*b*c*d + 2*a^2*d^2 + (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*n^2 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n)*(d*x
+ c)^n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n-3}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 3)/(d*x + c)^n, x)
```

**maple** [A] time = 0.01, size = 127, normalized size = 1.48

$$-\frac{(dx+c)(adn-bcn-bdx-2ad+bc)(bx+a)^{n-2}(dx+c)^{-n}}{a^2d^2n^2-2abcd\,n^2+b^2c^2n^2-3a^2d^2n+6abcdn-3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(n-3)/((d*x+c)^n),x)
```

```
[Out] -(b*x+a)^(n-2)*(d*x+c)*(a*d*n-b*c*n-b*d*x-2*a*d+b*c)/(a^2*d^2*n^2-2*a*b*c*d
*n^2+b^2*c^2*n^2-3*a^2*d^2*n+6*a*b*c*d*n-3*b^2*c^2*n+2*a^2*d^2-4*a*b*c*d+2*
b^2*c^2)/((d*x+c)^n)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx+a)^{n-3}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 3)/(d*x + c)^n, x)
```

**mupad** [B] time = 0.77, size = 220, normalized size = 2.56

$$(a+b\,x)^{n-3}\left(\frac{x\,\left(2\,a^2\,d^2-b^2\,c^2-a^2\,d^2\,n+b^2\,c^2\,n+2\,a\,b\,c\,d\right)}{{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^n\,\left(n^2-3\,n+2\right)}+\frac{b^2\,d^2\,x^3}{{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^n\,\left(n^2-3\,n+2\right)}+\frac{a\,c\,\left(2\,a\,d-b\,c-a\,d\,n+b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^n\,\left(n^2-3\,n+2\right)}+\frac{b\,d\,x^2\,\left(3\,a\,d-a\,d\,n+b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^n\,\left(n^2-3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 3)/(c + d*x)^n,x)
```

```
[Out] (a + b*x)^(n - 3)*((x*(2*a^2*d^2 - b^2*c^2 - a^2*d^2*n + b^2*c^2*n + 2*a*b*
c*d))/((a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (b^2*d^2*x^3)/((a*d - b
*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (a*c*(2*a*d - b*c - a*d*n + b*c*n))/((
a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (b*d*x^2*(3*a*d - a*d*n + b*c*n
))/((a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-3+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

## 3.1564 $\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx$

**Optimal**. Leaf size=39

$$-\frac{(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(bc-ad)}$$

**Rubi [A]** time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
[Out] -(((a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(1 - n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx = -\frac{(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)(1-n)}$$

**Mathematica [A]** time = 0.02, size = 36, normalized size = 0.92

$$\frac{(a+bx)^{n-1}(c+dx)^{1-n}}{(n-1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
[Out] ((a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(-1 + n))
```

**IntegrateAlgebraic [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-2 + n)/(c + d*x)^n, x]
```

**fricas [A]** time = 1.32, size = 60, normalized size = 1.54

$$-\frac{\left(bdx^2+ac+(bc+ad)x\right)(bx+a)^{n-2}}{(bc-ad-(bc-ad)n)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(n - 2)/((b*c - a*d - (b*c - a*d
)*n)*(d*x + c)^n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n-2}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 2)/(d*x + c)^n, x)
```

**maple [A]** time = 0.00, size = 45, normalized size = 1.15

$$-\frac{(dx+c)\,(bx+a)^{n-1}\,(dx+c)^{-n}}{adn-bcn-ad+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(n-2)/((d*x+c)^n),x)
```

```
[Out] -(b*x+a)^(n-1)*(d*x+c)/(a*d*n-b*c*n-a*d+b*c)/((d*x+c)^n)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n-2}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 2)/(d*x + c)^n, x)
```

**mupad** [B] time = 0.56, size = 102, normalized size = 2.62

$$-(a+b\,x)^{n-2}\,\left(\frac{a\,c}{(a\,d-b\,c)\,(n-1)\,(c+d\,x)^n}+\frac{x\,(a\,d+b\,c)}{(a\,d-b\,c)\,(n-1)\,(c+d\,x)^n}+\frac{b\,d\,x^2}{(a\,d-b\,c)\,(n-1)\,(c+d\,x)^n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 2)/(c + d*x)^n,x)
```

```
[Out] -(a + b*x)^(n - 2)*((a*c)/((a*d - b*c)*(n - 1)*(c + d*x)^n) + (x*(a*d + b*c
))/((a*d - b*c)*(n - 1)*(c + d*x)^n) + (b*d*x^2)/((a*d - b*c)*(n - 1)*(c +
d*x)^n))
```

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2+n)/((d*x+c)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.1565 $\int (a+bx)^{-2-n}(c+dx)^n\,dx$

**Optimal**. Leaf size=37

$$-\frac{(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(bc-ad)}$$

**Rubi** [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
[Out] -(((a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(1 + n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^{-2-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)(1+n)}$$

**Mathematica** [A] time = 0.02, size = 38, normalized size = 1.03

$$\frac{(a+bx)^{-n-1}(c+dx)^{n+1}}{(-n-1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
[Out] ((a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(-1 - n))
```

**IntegrateAlgebraic** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-2-n}(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-2 - n)*(c + d*x)^n, x]
```

**fricas** [A] time = 1.33, size = 59, normalized size = 1.59

$$-\frac{\left(bdx^2+ac+(bc+ad)x\right)(bx+a)^{-n-2}(dx+c)^n}{bc-ad+(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(-n - 2)*(d*x + c)^n/(b*c - a*d
+ (b*c - a*d)*n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-2}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 2)*(d*x + c)^n, x)
```

**maple** [A] time = 0.00, size = 41, normalized size = 1.11

$$\frac{(bx+a)^{-n-1}(dx+c)^{n+1}}{adn-bcn+ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-n-2)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-n-1)*(d*x+c)^(n+1)/(a*d*n-b*c*n+a*d-b*c)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-2}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 2)*(d*x + c)^n, x)
```

**mupad [B]** time = 0.53, size = 97, normalized size = 2.62

$$\frac{\frac{a\,c\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{x\,\left(a\,d+b\,c\right)\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{b\,d\,x^2\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}}{{\left(a+b\,x\right)}^{n+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 2),x)
```

```
[Out] ((a*c*(c + d*x)^n)/((a*d - b*c)*(n + 1)) + (x*(a*d + b*c)*(c + d*x)^n)/((a*
d - b*c)*(n + 1)) + (b*d*x^2*(c + d*x)^n)/((a*d - b*c)*(n + 1)))/(a + b*x)^
(n + 2)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2-n)*(d*x+c)**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.1566 $\int (a+bx)^{-3-n}(c+dx)^n\,dx$

**Optimal**. Leaf size=80

$$\frac{d(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(bc-ad)^2}-\frac{(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(bc-ad)}$$

**Rubi [A]** time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{d(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(bc-ad)^2}-\frac{(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(2 + n))) + (d*(a + b
*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-3-n}(c+dx)^n\,dx &= -\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)}-\frac{d\int (a+bx)^{-2-n}(c+dx)^n\,dx}{(bc-ad)(2+n)}\\ &= -\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)}+\frac{d(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)^2(1+n)(2+n)}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 60, normalized size = 0.75

$$\frac{(a+bx)^{-n-2}(c+dx)^{n+1}(ad(n+2)-b(cn+c-dx))}{(n+1)(n+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-2 - n)*(c + d*x)^(1 + n)*(a*d*(2 + n) - b*(c + c*n - d*x)))/((
b*c - a*d)^2*(1 + n)*(2 + n))
```

**IntegrateAlgebraic [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-3-n}(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-3 - n)*(c + d*x)^n, x]
```

**fricas [B]** time = 1.28, size = 207, normalized size = 2.59

$$\frac{\left(b^2d^2x^3 - abc^2 + 2\,a^2cd + \left(3\,abd^2 - \left(b^2cd - abd^2\right)n\right)x^2 - \left(abc^2 - a^2cd\right)n - \left(b^2c^2 - 2\,abcd - 2\,a^2d^2 + \left(b^2c^2 - a^2d^2\right)n\right)x\right)(bx+a)^{-n-3}(dx+c)^n}{2\,b^2c^2 - 4\,abcd + 2\,a^2d^2 + \left(b^2c^2 - 2\,abcd + a^2d^2\right)n^2 + 3\left(b^2c^2 - 2\,abcd + a^2d^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 - a*b*c^2 + 2*a^2*c*d + (3*a*b*d^2 - (b^2*c*d - a*b*d^2)*n)*x^
2 - (a*b*c^2 - a^2*c*d)*n - (b^2*c^2 - 2*a*b*c*d - 2*a^2*d^2 + (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^(-n - 3)*(d*x + c)^n/(2*b^2*c^2 - 4*a*b*c*d + 2*a^2
*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-3}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 3)*(d*x + c)^n, x)
```

**maple** [A] time = 0.00, size = 123, normalized size = 1.54

$$\frac{(adn-bcn+bdx+2ad-bc)\,(bx+a)^{-n-2}\,(dx+c)^{n+1}}{a^2d^2n^2-2abcd\,n^2+b^2c^2n^2+3a^2d^2n-6abcdn+3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-3-n)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-n-2)*(d*x+c)^(n+1)*(a*d*n-b*c*n+b*d*x+2*a*d-b*c)/(a^2*d^2*n^2-2*a
*b*c*d*n^2+b^2*c^2*n^2+3*a^2*d^2*n-6*a*b*c*d*n+3*b^2*c^2*n+2*a^2*d^2-4*a*b*
c*d+2*b^2*c^2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-3}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 3)*(d*x + c)^n, x)
```

**mupad** [B] time = 0.74, size = 214, normalized size = 2.68

$$\frac{\frac{x\,(c+d\,x)^n\,\left(2\,a^2\,d^2-b^2\,c^2+a^2\,d^2\,n-b^2\,c^2\,n+2\,a\,b\,c\,d\right)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}+\frac{a\,c\,(c+d\,x)^n\,(2\,a\,d-b\,c+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}+\frac{b^2\,d^2\,x^3\,(c+d\,x)^n}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}+\frac{b\,d\,x^2\,(c+d\,x)^n\,(3\,a\,d+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}}{(a+b\,x)^{n+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 3),x)
```

```
[Out] ((x*(c + d*x)^n*(2*a^2*d^2 - b^2*c^2 + a^2*d^2*n - b^2*c^2*n + 2*a*b*c*d))/
((a*d - b*c)^2*(3*n + n^2 + 2)) + (a*c*(c + d*x)^n*(2*a*d - b*c + a*d*n - b
*c*n))/((a*d - b*c)^2*(3*n + n^2 + 2)) + (b^2*d^2*x^3*(c + d*x)^n)/((a*d -
b*c)^2*(3*n + n^2 + 2)) + (b*d*x^2*(c + d*x)^n*(3*a*d + a*d*n - b*c*n))/((a
*d - b*c)^2*(3*n + n^2 + 2)))/(a + b*x)^(n + 3)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-3-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

## 3.1567 $\int (a+bx)^{-4-n}(c+dx)^n\,dx$

**Optimal**. Leaf size=131

$$-\frac{2d^2(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(bc-ad)^3}-\frac{(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(bc-ad)}+\frac{2d(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(bc-ad)^2}$$

**Rubi** [A] time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{2d^2(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(bc-ad)^3}-\frac{(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(bc-ad)}+\frac{2d(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(3 + n))) + (2*d*(a +
 b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(2 + n)*(3 + n)) - (2*d^2*
(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n
))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-4-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)} - \frac{(2d)\int (a+bx)^{-3-n}(c+dx)^n\,dx}{(bc-ad)(3+n)}$$
$$= -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)} + \frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)} + \frac{\left(2d^2\right)\int (a+bx)^{-2-n}(c+}{(bc-ad)^2(2+n)(3}$$
$$= -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)} + \frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)} - \frac{2d^2(a+bx)^{-1-n}(c+d}{(bc-ad)^3(1+n)(2+n)}$$

**Mathematica [A]** time = 0.08, size = 113, normalized size = 0.86

$$-\frac{(a+bx)^{-n-3}(c+dx)^{n+1}\left(a^2d^2\left(n^2+5n+6\right)-2abd(n+3)(cn+c-dx)+b^2\left(c^2\left(n^2+3n+2\right)-2cd(n+1)x+2d^2x^2\right)\right)}{(n+1)(n+2)(n+3)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 - n)*(c + d*x)^(1 + n)*(a^2*d^2*(6 + 5*n + n^2) - 2*a*b*d*
(3 + n)*(c + c*n - d*x) + b^2*(c^2*(2 + 3*n + n^2) - 2*c*d*(1 + n)*x + 2*d^
2*x^2)))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n)))
```

**IntegrateAlgebraic [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-4-n}(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-4 - n)*(c + d*x)^n, x]
```

**fricas [B]** time = 1.01, size = 509, normalized size = 3.89

$$-\frac{\left(2\,b^3d^3x^4+2\,ab^2c^3-6\,a^2bc^2d+6\,a^3cd^2+2\left(4\,ab^2d^3-\left(b^3cd^2-ab^2d^3\right)n\right)x^3+\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)n^2+\left(12\,a^2bd^3+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)n^2+\left(b^3c^2d-8\,ab^2cd^2+7\,a^2bd^3\right)n\right)x^2+\left(3\,ab^2c^3-8\,a^2bc^2d+5\,a^3cd^2\right)n+\left(2\,b^3c^3-6\,ab^2c^2d+6\,a^2bcd^2+6\,a^3d^3+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)n^2+\left(3\,b^3c^3-7\,ab^2c^2d-a^2bcd^2+5\,a^3d^3\right)n\right)x\right)(bx+a)^{-n-4}(dx+c)^n}{6\,b^3c^3-18\,ab^2c^2d+18\,a^2bcd^2-6\,a^3d^3+\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n^3+6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n^2+11\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] -(2*b^3*d^3*x^4 + 2*a*b^2*c^3 - 6*a^2*b*c^2*d + 6*a^3*c*d^2 + 2*(4*a*b^2*d^
3 - (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*n^2 + (12*a^2*b*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 + (b^3*
c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*n)*x^2 + (3*a*b^2*c^3 - 8*a^2*b*c^2*d
```

```
+ 5*a^3*c*d^2)*n + (2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 6*a^3*d^3 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 + (3*b^3*c^3 - 7*a*b^2
*c^2*d - a^2*b*c*d^2 + 5*a^3*d^3)*n)*x)*(b*x + a)^(-n - 4)*(d*x + c)^n/(6*b
^3*c^3 - 18*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*n^2 + 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-4}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 4)*(d*x + c)^n, x)
```

**maple [B]** time = 0.01, size = 318, normalized size = 2.43

$$\frac{\left(a^2d^2n^2-2abcd\,n^2+2ab\,d^2nx+b^2c^2n^2-2b^2cdnx+2b^2x^2d^2+5a^2d^2n-8abcdn+6ab\,d^2x+3b^2c^2n-2b^2cdx+6a^2d^2-6abcd+2b^2c^2\right)(bx+a)^{-n-3}(dx+c)^{n+1}}{a^3d^3n^3-3a^2bc\,d^2n^3+3a\,b^2c^2d\,n^3-b^3c^3n^3+6a^3d^3n^2-18a^2bc\,d^2n^2+18a\,b^2c^2d\,n^2-6b^3c^3n^2+11a^3d^3n-33a^2bc\,d^2n+33a\,b^2c^2dn-11b^3c^3n+6a^3d^3-18a^2bc\,d^2+18a\,b^2c^2d-6b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-4-n)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-3-n)*(d*x+c)^(n+1)*(a^2*d^2*n^2-2*a*b*c*d*n^2+2*a*b*d^2*n*x+b^2*c
^2*n^2-2*b^2*c*d*n*x+2*b^2*d^2*x^2+5*a^2*d^2*n-8*a*b*c*d*n+6*a*b*d^2*x+3*b^
2*c^2*n-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*d
^2*n^3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3+6*a^3*d^3*n^2-18*a^2*b*c*d^2*n^2+18*a*
b^2*c^2*d*n^2-6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n-
11*b^3*c^3*n+6*a^3*d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-4}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 4)*(d*x + c)^n, x)
```

**mupad [B]** time = 0.99, size = 525, normalized size = 4.01

$$\frac{x\,(c+d\,x)^n\,\left(a^3\,d^3\,n^2+5\,a^3\,d^3\,n+6\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2-a^2\,b\,c\,d^2\,n+6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2-7\,a\,b^2\,c^2\,d\,n-6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2+3\,b^3\,c^3\,n+2\,b^3\,c^3\right)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,c\,(c+d\,x)^n\,\left(a^2\,d^2\,n^2+5\,a^2\,d^2\,n+6\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2+3\,b^2\,c^2\,n+2\,b^2\,c^2\right)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,b^3\,d^3\,x^4\,(c+d\,x)^n}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{b\,d\,x^2\,(c+d\,x)^n\,\left(a^2\,d^2\,n^2+7\,a^2\,d^2\,n+12\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2+b^2\,c^2\,n\right)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,b^2\,d^2\,x^3\,(c+d\,x)^n\,(4\,a\,d+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 4),x)
```

```
[Out] (x*(c + d*x)^n*(6*a^3*d^3 + 2*b^3*c^3 + 5*a^3*d^3*n + 3*b^3*c^3*n + a^3*d^3
*n^2 + b^3*c^3*n^2 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 7*a*b^2*c^2*d*n - a^2*
b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(a + b*x)^(n
 + 4)*(11*n + 6*n^2 + n^3 + 6)) + (a*c*(c + d*x)^n*(6*a^2*d^2 + 2*b^2*c^2 +
 5*a^2*d^2*n + 3*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d - 8*a*b*
c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n^2 + n^
3 + 6)) + (2*b^3*d^3*x^4*(c + d*x)^n)/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*
n + 6*n^2 + n^3 + 6)) + (b*d*x^2*(c + d*x)^n*(12*a^2*d^2 + 7*a^2*d^2*n + b^
2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d -
 b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n^2 + n^3 + 6)) + (2*b^2*d^2*x^3*(c + d
*x)^n*(4*a*d + a*d*n - b*c*n))/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n
^2 + n^3 + 6))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-4-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

## 3.1568 $\int (a+bx)^{-5-n}(c+dx)^n\,dx$

**Optimal**. Leaf size=186

$$\frac{6d^3(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}-\frac{6d^2(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(n+4)(bc-ad)^3}-\frac{(a+bx)^{-n-4}(c+dx)^{n+1}}{(n+4)(bc-ad)}+\frac{3d(a+bx)^{-n-}}{(n+3)(n+4}$$

**Rubi [A]** time = 0.09, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{6d^2(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{6d^3(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}-\frac{(a+bx)^{-n-4}(c+dx)^{n+1}}{(n+4)(bc-ad)}+\frac{3d(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(n+4)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-4 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(4 + n))) + (3*d*(a +
 b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(3 + n)*(4 + n)) - (6*d^2*
(a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n
)) + (6*d^3*(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^4*(1 + n)*(2
 + n)*(3 + n)*(4 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-5-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} - \frac{(3d)\int (a+bx)^{-4-n}(c+dx)^n\,dx}{(bc-ad)(4+n)}$$

$$= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} + \frac{\left(6d^2\right)\int (a+bx)^{-3-n}(c+}{(bc-ad)^2(3+n)(4+}$$

$$= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} - \frac{6d^2(a+bx)^{-2-n}(c+d}{(bc-ad)^3(2+n)(3+n)}$$

$$= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} - \frac{6d^2(a+bx)^{-2-n}(c+d}{(bc-ad)^3(2+n)(3+n)}$$

**Mathematica** [A] time = 0.12, size = 195, normalized size = 1.05

$$\frac{(a+bx)^{-n-4}(c+dx)^{n+1}\left(a^3d^3\left(n^3+9n^2+26n+24\right)-3a^2bd^2\left(n^2+7n+12\right)(cn+c-dx)+3ab^2d(n+4)\left(c^2\left(n^2+3n+2\right)-2cd(n+1)x+2d^2x^2\right)-\left(b^3\left(c^3\left(n^3+6n^2+11n+6\right)-3c^2d\left(n^2+3n+2\right)x+6cd^2(n+1)x^2-6d^3x^3\right)\right)\right)}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-4 - n)*(c + d*x)^(1 + n)*(a^3*d^3*(24 + 26*n + 9*n^2 + n^3) -
3*a^2*b*d^2*(12 + 7*n + n^2)*(c + c*n - d*x) + 3*a*b^2*d*(4 + n)*(c^2*(2 +
3*n + n^2) - 2*c*d*(1 + n)*x + 2*d^2*x^2) - b^3*(c^3*(6 + 11*n + 6*n^2 + n^
3) - 3*c^2*d*(2 + 3*n + n^2)*x + 6*c*d^2*(1 + n)*x^2 - 6*d^3*x^3)))/((b*c -
 a*d)^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

**IntegrateAlgebraic** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-5-n}(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-5 - n)*(c + d*x)^n, x]
```

**fricas** [B] time = 1.40, size = 959, normalized size = 5.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (6*b^4*d^4*x^5 - 6*a*b^3*c^4 + 24*a^2*b^2*c^3*d - 36*a^3*b*c^2*d^2 + 24*a^4
*c*d^3 + 6*(5*a*b^3*d^4 - (b^4*c*d^3 - a*b^3*d^4)*n)*x^4 - (a*b^3*c^4 - 3*a
^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*n^3 + 3*(20*a^2*b^2*d^4 + (b^4*
c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*n^2 + (b^4*c^2*d^2 - 10*a*b^3*c*d^3
+ 9*a^2*b^2*d^4)*n)*x^3 - 3*(2*a*b^3*c^4 - 7*a^2*b^2*c^3*d + 8*a^3*b*c^2*d^
2 - 3*a^4*c*d^3)*n^2 + (60*a^3*b*d^4 - (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*n^3 - 3*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 9*a^2*b^2*c*
d^3 - 4*a^3*b*d^4)*n^2 - (2*b^4*c^3*d - 15*a*b^3*c^2*d^2 + 60*a^2*b^2*c*d^3
 - 47*a^3*b*d^4)*n)*x^2 - (11*a*b^3*c^4 - 42*a^2*b^2*c^3*d + 57*a^3*b*c^2*d
^2 - 26*a^4*c*d^3)*n - (6*b^4*c^4 - 24*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 - 2
4*a^3*b*c*d^3 - 24*a^4*d^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4
*d^4)*n^3 + 3*(2*b^4*c^4 - 6*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^
3 - 3*a^4*d^4)*n^2 + (11*b^4*c^4 - 40*a*b^3*c^3*d + 45*a^2*b^2*c^2*d^2 + 10
*a^3*b*c*d^3 - 26*a^4*d^4)*n)*x)*(b*x + a)^(-n - 5)*(d*x + c)^n/(24*b^4*c^4
 - 96*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*n^4 +
10*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
n^3 + 35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*n^2 + 50*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-5}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 5)*(d*x + c)^n, x)
```

**maple [B]** time = 0.01, size = 661, normalized size = 3.55

$$\frac{\left(a^3d^3n^3-3a^2bcd^2n^3+3a^2bd^3n^2x+3ab^2c^2dn^3-6ab^2cd^2n^2x+6ab^2d^3nx^2-b^3c^3n^3+3b^3c^2dn^2x-6b^3cd^2nx^2+6b^3d^3x^3+9a^3d^3n^2-24a^2bcd^2n^2+21a^2bd^3nx+21ab^2c^2dn^2-30ab^2cd^2nx+24ab^2d^3x^2-6b^3c^3n^2+9b^3c^2dnx-6b^3cd^2x^2+26a^3d^3n-57a^2bcd^2n+36a^2bd^3x+42ab^2c^2dn-24ab^2cd^2x-11b^3c^3n+6b^3c^2dx+24a^3d^3-36a^2bcd^2+24ab^2c^2d-6b^3c^3\right)(bx+a)^{-n-4}(dx+c)^{n+1}}{a^4d^4n^4-4a^3bcd^3n^4+6a^2b^2c^2d^2n^4-4ab^3c^3dn^4+b^4c^4n^4+10a^4d^4n^3-40a^3bcd^3n^3+60a^2b^2c^2d^2n^3-40ab^3c^3dn^3+10b^4c^4n^3+35a^4d^4n^2-140a^3bcd^3n^2+210a^2b^2c^2d^2n^2-140ab^3c^3dn^2+35b^4c^4n^2+50a^4d^4n-200a^3bcd^3n+300a^2b^2c^2d^2n-200ab^3c^3dn+50b^4c^4n+24a^4d^4-96a^3bcd^3+144a^2b^2c^2d^2-96ab^3c^3d+24b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-5-n)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-4-n)*(d*x+c)^(n+1)*(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3+3*a^2*b*d^3*n^2
*x+3*a*b^2*c^2*d*n^3-6*a*b^2*c*d^2*n^2*x+6*a*b^2*d^3*n*x^2-b^3*c^3*n^3+3*b^
3*c^2*d*n^2*x-6*b^3*c*d^2*n*x^2+6*b^3*d^3*x^3+9*a^3*d^3*n^2-24*a^2*b*c*d^2*
n^2+21*a^2*b*d^3*n*x+21*a*b^2*c^2*d*n^2-30*a*b^2*c*d^2*n*x+24*a*b^2*d^3*x^2
-6*b^3*c^3*n^2+9*b^3*c^2*d*n*x-6*b^3*c*d^2*x^2+26*a^3*d^3*n-57*a^2*b*c*d^2*
n+36*a^2*b*d^3*x+42*a*b^2*c^2*d*n-24*a*b^2*c*d^2*x-11*b^3*c^3*n+6*b^3*c^2*d
*x+24*a^3*d^3-36*a^2*b*c*d^2+24*a*b^2*c^2*d-6*b^3*c^3)/(a^4*d^4*n^4-4*a^3*b
```

```
*c*d^3*n^4+6*a^2*b^2*c^2*d^2*n^4-4*a*b^3*c^3*d*n^4+b^4*c^4*n^4+10*a^4*d^4*n
^3-40*a^3*b*c*d^3*n^3+60*a^2*b^2*c^2*d^2*n^3-40*a*b^3*c^3*d*n^3+10*b^4*c^4*
n^3+35*a^4*d^4*n^2-140*a^3*b*c*d^3*n^2+210*a^2*b^2*c^2*d^2*n^2-140*a*b^3*c^
3*d*n^2+35*b^4*c^4*n^2+50*a^4*d^4*n-200*a^3*b*c*d^3*n+300*a^2*b^2*c^2*d^2*n
-200*a*b^3*c^3*d*n+50*b^4*c^4*n+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*d
^2-96*a*b^3*c^3*d+24*b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + a)^{-n-5}(dx + c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 5)*(d*x + c)^n, x)
```

**mupad [B]** time = 1.64, size = 944, normalized size = 5.08

$$\frac{a\,c\,(c+d\,x)^n\,\left(a^3\,d^3\,n^3+9\,a^3\,d^3\,n^2+26\,a^3\,d^3\,n+24\,a^3\,d^3-3\,a^2\,b\,c\,d^2\,n^3-24\,a^2\,b\,c\,d^2\,n^2-57\,a^2\,b\,c\,d^2\,n-36\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d\,n^3+21\,a\,b^2\,c^2\,d\,n^2+42\,a\,b^2\,c^2\,d\,n+24\,a\,b^2\,c^2\,d-b^3\,c^3\,n^3-6\,b^3\,c^3\,n^2-11\,b^3\,c^3\,n-6\,b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^{n+5}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{x\,(c+d\,x)^n\,\left(-a^4\,d^4\,n^3-9\,a^4\,d^4\,n^2-26\,a^4\,d^4\,n-24\,a^4\,d^4+2\,a^3\,b\,c\,d^3\,n^3+12\,a^3\,b\,c\,d^3\,n^2+10\,a^3\,b\,c\,d^3\,n-24\,a^3\,b\,c\,d^3+9\,a^2\,b^2\,c^2\,d^2\,n^2+45\,a^2\,b^2\,c^2\,d^2\,n+36\,a^2\,b^2\,c^2\,d^2-2\,a\,b^3\,c^3\,d\,n^3-18\,a\,b^3\,c^3\,d\,n^2-40\,a\,b^3\,c^3\,d\,n-24\,a\,b^3\,c^3\,d+b^4\,c^4\,n^3+6\,b^4\,c^4\,n^2+11\,b^4\,c^4\,n+6\,b^4\,c^4\right)}{{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^{n+5}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,b^4\,d^4\,x^5\,(c+d\,x)^n}{{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^{n+5}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{3\,b^2\,d^2\,x^3\,(c+d\,x)^n\,\left(a^2\,d^2\,n^2+9\,a^2\,d^2\,n+20\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-10\,a\,b\,c\,d\,n+b^2\,c^2\,n^2+b^2\,c^2\,n\right)}{{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^{n+5}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,b^3\,d^3\,x^4\,(c+d\,x)^n\,\left(5\,a\,d+a\,d\,n-b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^{n+5}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{b\,d\,x^2\,(c+d\,x)^n\,\left(a^3\,d^3\,n^3+12\,a^3\,d^3\,n^2+47\,a^3\,d^3\,n+60\,a^3\,d^3-3\,a^2\,b\,c\,d^2\,n^3-27\,a^2\,b\,c\,d^2\,n^2-60\,a^2\,b\,c\,d^2\,n+3\,a\,b^2\,c^2\,d\,n^3+18\,a\,b^2\,c^2\,d\,n^2+15\,a\,b^2\,c^2\,d\,n-b^3\,c^3\,n^3-3\,b^3\,c^3\,n^2-2\,b^3\,c^3\,n\right)}{{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^{n+5}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 5),x)
```

```
[Out] (a*c*(c + d*x)^n*(24*a^3*d^3 - 6*b^3*c^3 + 26*a^3*d^3*n - 11*b^3*c^3*n + 9*
a^3*d^3*n^2 - 6*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^3*n^3 + 24*a*b^2*c^2*d -
36*a^2*b*c*d^2 + 42*a*b^2*c^2*d*n - 57*a^2*b*c*d^2*n + 21*a*b^2*c^2*d*n^2 -
 24*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*n^3 - 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^
4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (x*(c + d*x)^n*(
6*b^4*c^4 - 24*a^4*d^4 - 26*a^4*d^4*n + 11*b^4*c^4*n - 9*a^4*d^4*n^2 + 6*b^
4*c^4*n^2 - a^4*d^4*n^3 + b^4*c^4*n^3 + 36*a^2*b^2*c^2*d^2 - 24*a*b^3*c^3*d
 - 24*a^3*b*c*d^3 - 40*a*b^3*c^3*d*n + 10*a^3*b*c*d^3*n + 9*a^2*b^2*c^2*d^2
*n^2 - 18*a*b^3*c^3*d*n^2 + 12*a^3*b*c*d^3*n^2 - 2*a*b^3*c^3*d*n^3 + 2*a^3*
b*c*d^3*n^3 + 45*a^2*b^2*c^2*d^2*n))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n
 + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^4*d^4*x^5*(c + d*x)^n)/((a*d - b*c)^
4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*b^2*d^2*x^3*(
c + d*x)^n*(20*a^2*d^2 + 9*a^2*d^2*n + b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^
2 - 10*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^3*d^3*x^4*(c + d*x)^n*(5*a*d + a*d*n -
 b*c*n))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (b*d*x^2*(c + d*x)^n*(60*a^3*d^3 + 47*a^3*d^3*n - 2*b^3*c^3*n + 12*a^
3*d^3*n^2 - 3*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^3*n^3 + 15*a*b^2*c^2*d*n -
60*a^2*b*c*d^2*n + 18*a*b^2*c^2*d*n^2 - 27*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*
n^3 - 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n + 35*n^2 +
 10*n^3 + n^4 + 24))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-5-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

## 3.1569 $\int (a+bx)^n (c+dx)^{-2-n}\,dx$

**Optimal**. Leaf size=36

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(bc-ad)}$$

**Rubi** [A] time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-2 - n),x]
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)*(1 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^n (c+dx)^{-2-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)(1+n)}$$

**Mathematica** [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-2 - n),x]
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)*(1 + n))
```

**IntegrateAlgebraic [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^n(c+dx)^{-2-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x)^(-2 - n),x]
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x)^(-2 - n), x]
```

**fricas [A]** time = 1.32, size = 58, normalized size = 1.61

$$\frac{\left(bdx^2+ac+(bc+ad)x\right)(bx+a)^n(dx+c)^{-n-2}}{bc-ad+(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="fricas")
[Out] (b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^n*(d*x + c)^(-n - 2)/(b*c - a*d +
 (b*c - a*d)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n(dx+c)^{-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="giac")
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 2), x)
```

**maple [A]** time = 0.00, size = 42, normalized size = 1.17

$$-\frac{(bx+a)^{n+1}\,(dx+c)^{-n-1}}{adn-bcn+ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-n-2),x)
[Out] -(b*x+a)^(n+1)*(d*x+c)^(-n-1)/(a*d*n-b*c*n+a*d-b*c)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n(dx+c)^{-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 2), x)
```

**mupad [B]** time = 0.56, size = 98, normalized size = 2.72

$$-\frac{\frac{a\,c\,(a+b\,x)^n}{(a\,d-b\,c)\,(n+1)}+\frac{x\,(a\,d+b\,c)\,(a+b\,x)^n}{(a\,d-b\,c)\,(n+1)}+\frac{b\,d\,x^2\,(a+b\,x)^n}{(a\,d-b\,c)\,(n+1)}}{(c+d\,x)^{n+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 2),x)
```

```
[Out] -((a*c*(a + b*x)^n)/((a*d - b*c)*(n + 1)) + (x*(a*d + b*c)*(a + b*x)^n)/((a
*d - b*c)*(n + 1)) + (b*d*x^2*(a + b*x)^n)/((a*d - b*c)*(n + 1)))/(c + d*x)
^(n + 2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-2-n),x)
```

```
[Out] Timed out
```

## 3.1570 $\int (a+bx)^n(c+dx)^{-3-n}\,dx$

**Optimal**. Leaf size=79

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(bc-ad)}+\frac{b(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(bc-ad)^2}$$

**Rubi [A]** time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(bc-ad)}+\frac{b(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)*(2 + n)) + (b*(a + b*x)
^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n(c+dx)^{-3-n}\,dx &= \frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)}+\frac{b\int (a+bx)^n(c+dx)^{-2-n}\,dx}{(bc-ad)(2+n)}\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)}+\frac{b(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)^2(1+n)(2+n)}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 59, normalized size = 0.75

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-2}(-ad(n+1)+bc(n+2)+bdx)}{(n+1)(n+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-2 - n)*(-(a*d*(1 + n)) + b*c*(2 + n) + b*d*x
))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

**IntegrateAlgebraic** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^n(c+dx)^{-3-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x)^(-3 - n), x]
```

**fricas** [B] time = 1.29, size = 205, normalized size = 2.59

$$\frac{\left(b^2d^2x^3+2\,abc^2-a^2cd+\left(3\,b^2cd+\left(b^2cd-abd^2\right)n\right)x^2+\left(abc^2-a^2cd\right)n+\left(2\,b^2c^2+2\,abcd-a^2d^2+\left(b^2c^2-a^2d^2\right)n\right)x\right)(bx+a)^n(dx+c)^{-n-3}}{2\,b^2c^2-4\,abcd+2\,a^2d^2+\left(b^2c^2-2\,abcd+a^2d^2\right)n^2+3\left(b^2c^2-2\,abcd+a^2d^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 + 2*a*b*c^2 - a^2*c*d + (3*b^2*c*d + (b^2*c*d - a*b*d^2)*n)*x^
2 + (a*b*c^2 - a^2*c*d)*n + (2*b^2*c^2 + 2*a*b*c*d - a^2*d^2 + (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 3)/(2*b^2*c^2 - 4*a*b*c*d + 2*a^2
*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n(dx+c)^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 3), x)
```

**maple [A]** time = 0.00, size = 124, normalized size = 1.57

$$-\frac{(adn-bcn-bdx+ad-2bc)(bx+a)^{n+1}(dx+c)^{-n-2}}{a^2d^2n^2-2abcd\,n^2+b^2c^2n^2+3a^2d^2n-6abcdn+3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-n-3),x)
```

```
[Out] -(b*x+a)^(n+1)*(d*x+c)^(-n-2)*(a*d*n-b*c*n-b*d*x+a*d-2*b*c)/(a^2*d^2*n^2-2*
a*b*c*d*n^2+b^2*c^2*n^2+3*a^2*d^2*n-6*a*b*c*d*n+3*b^2*c^2*n+2*a^2*d^2-4*a*b
*c*d+2*b^2*c^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n(dx+c)^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 3), x)
```

**mupad [B]** time = 0.74, size = 214, normalized size = 2.71

$$\frac{\frac{x\,(a+b\,x)^n\,\left(2\,b^2\,c^2-a^2\,d^2-a^2\,d^2\,n+b^2\,c^2\,n+2\,a\,b\,c\,d\right)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}-\frac{a\,c\,(a+b\,x)^n\,(a\,d-2\,b\,c+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}+\frac{b^2\,d^2\,x^3\,(a+b\,x)^n}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}+\frac{b\,d\,x^2\,(a+b\,x)^n\,(3\,b\,c-a\,d\,n+b\,c\,n)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}}{(c+d\,x)^{n+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 3),x)
```

```
[Out] ((x*(a + b*x)^n*(2*b^2*c^2 - a^2*d^2 - a^2*d^2*n + b^2*c^2*n + 2*a*b*c*d))/
((a*d - b*c)^2*(3*n + n^2 + 2)) - (a*c*(a + b*x)^n*(a*d - 2*b*c + a*d*n - b
*c*n))/((a*d - b*c)^2*(3*n + n^2 + 2)) + (b^2*d^2*x^3*(a + b*x)^n)/((a*d -
b*c)^2*(3*n + n^2 + 2)) + (b*d*x^2*(a + b*x)^n*(3*b*c - a*d*n + b*c*n))/((a
*d - b*c)^2*(3*n + n^2 + 2)))/(c + d*x)^(n + 3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-3-n),x)
```

```
[Out] Timed out
```

## 3.1571 $\int (a+bx)^n (c+dx)^{-4-n}\,dx$

**Optimal**. Leaf size=130

$$\frac{2b^2(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(bc-ad)}+\frac{2b(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(bc-ad)^2}$$

**Rubi** [A] time = 0.04, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{2b^2(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(bc-ad)}+\frac{2b(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-4 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)*(3 + n)) + (2*b*(a + b*
x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^2*(2 + n)*(3 + n)) + (2*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-4-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)} + \frac{(2b)\int (a+bx)^n(c+dx)^{-3-n}\,dx}{(bc-ad)(3+n)}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)} + \frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)} + \frac{\left(2b^2\right)\int (a+bx)^n(c+dx)^{-2}}{(bc-ad)^2(2+n)(3+n}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)} + \frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)} + \frac{2b^2(a+bx)^{1+n}(c+dx)^{-1}}{(bc-ad)^3(1+n)(2+n)(3}$$

**Mathematica** [A] time = 0.09, size = 112, normalized size = 0.86

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-3}\left(a^2d^2\left(n^2+3n+2\right)-2abd(n+1)(c(n+3)+dx)+b^2\left(c^2\left(n^2+5n+6\right)+2cd(n+3)x+2d^2x^2\right)\right)}{(n+1)(n+2)(n+3)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-4 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-3 - n)*(a^2*d^2*(2 + 3*n + n^2) - 2*a*b*d*(1
 + n)*(c*(3 + n) + d*x) + b^2*(c^2*(6 + 5*n + n^2) + 2*c*d*(3 + n)*x + 2*d^
2*x^2)))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n))
```

**IntegrateAlgebraic** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^n(c+dx)^{-4-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x)^(-4 - n),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x)^(-4 - n), x]
```

**fricas** [B] time = 1.30, size = 507, normalized size = 3.90

$$\frac{\left(2\,b^3d^3x^4+6\,ab^2c^3-6\,a^2bc^2d+2\,a^3cd^2+2\left(4\,b^3cd^2+\left(b^3cd^2-ab^2d^3\right)n\right)x^3+\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)n^2+\left(12\,b^3c^2d+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)n^2+\left(7\,b^3c^2d-8\,ab^2cd^2+a^2bd^3\right)n\right)x^2+\left(5\,ab^2c^3-8\,a^2bc^2d+3\,a^3cd^2\right)n+\left(6\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)n^2+\left(5\,b^3c^3-ab^2c^2d-7\,a^2bcd^2+3\,a^3d^3\right)n\right)x\right)(bx+a)^n(dx+c)^{-n-4}}{6\,b^3c^3-18\,ab^2c^2d+18\,a^2bcd^2-6\,a^3d^3+\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n^3+6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n^2+11\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="fricas")
```

```
[Out] (2*b^3*d^3*x^4 + 6*a*b^2*c^3 - 6*a^2*b*c^2*d + 2*a^3*c*d^2 + 2*(4*b^3*c*d^2
 + (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)
*n^2 + (12*b^3*c^2*d + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 + (7*b^3
*c^2*d - 8*a*b^2*c*d^2 + a^2*b*d^3)*n)*x^2 + (5*a*b^2*c^3 - 8*a^2*b*c^2*d +
```

```
 3*a^3*c*d^2)*n + (6*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 + (5*b^3*c^3 - a*b^2*c^
2*d - 7*a^2*b*c*d^2 + 3*a^3*d^3)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 4)/(6*b^
3*c^3 - 18*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*n^2 + 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-n-4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 4), x)
```

**maple [B]** time = 0.01, size = 319, normalized size = 2.45

$$-\frac{\left(a^2d^2n^2-2abcd\,n^2-2ab\,d^2nx+b^2c^2n^2+2b^2cdnx+2b^2x^2d^2+3a^2d^2n-8abcdn-2ab\,d^2x+5b^2c^2n+6b^2cdx+2a^2d^2-6abcd+6b^2c^2\right)(bx+a)^{n+1}(dx+c)^{-n-3}}{a^3d^3n^3-3a^2bc\,d^2n^3+3a\,b^2c^2d\,n^3-b^3c^3n^3+6a^3d^3n^2-18a^2bc\,d^2n^2+18a\,b^2c^2d\,n^2-6b^3c^3n^2+11a^3d^3n-33a^2bc\,d^2n+33a\,b^2c^2dn-11b^3c^3n+6a^3d^3-18a^2bc\,d^2+18a\,b^2c^2d-6b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-n-4),x)
```

```
[Out] -(b*x+a)^(n+1)*(d*x+c)^(-n-3)*(a^2*d^2*n^2-2*a*b*c*d*n^2-2*a*b*d^2*n*x+b^2*
c^2*n^2+2*b^2*c*d*n*x+2*b^2*d^2*x^2+3*a^2*d^2*n-8*a*b*c*d*n-2*a*b*d^2*x+5*b
^2*c^2*n+6*b^2*c*d*x+2*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*
d^2*n^3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3+6*a^3*d^3*n^2-18*a^2*b*c*d^2*n^2+18*a
*b^2*c^2*d*n^2-6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n
-11*b^3*c^3*n+6*a^3*d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-n-4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 4), x)
```

**mupad [B]** time = 1.02, size = 528, normalized size = 4.06

$$-\frac{x\,(a+b\,x)^n\,\left(a^3\,d^3\,n^2+3\,a^3\,d^3\,n+2\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2-7\,a^2\,b\,c\,d^2\,n-6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2-a\,b^2\,c^2\,d\,n+6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2+5\,b^3\,c^3\,n+6\,b^3\,c^3\right)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{a\,c\,(a+b\,x)^n\,\left(a^2\,d^2\,n^2+3\,a^2\,d^2\,n+2\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2+5\,b^2\,c^2\,n+6\,b^2\,c^2\right)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,b^3\,d^3\,x^4\,(a+b\,x)^n}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{b\,d\,x^2\,(a+b\,x)^n\,\left(a^2\,d^2\,n^2+a^2\,d^2\,n-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2+7\,b^2\,c^2\,n+12\,b^2\,c^2\right)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,b^2\,d^2\,x^3\,(a+b\,x)^n\,(4\,b\,c-a\,d\,n+b\,c\,n)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 4),x)
```

```
[Out] - (x*(a + b*x)^n*(2*a^3*d^3 + 6*b^3*c^3 + 3*a^3*d^3*n + 5*b^3*c^3*n + a^3*d
^3*n^2 + b^3*c^3*n^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 - a*b^2*c^2*d*n - 7*a^
2*b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(c + d*x)^
(n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (a*c*(a + b*x)^n*(2*a^2*d^2 + 6*b^2*c^2
 + 3*a^2*d^2*n + 5*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d - 8*a*
b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6*n^2 +
n^3 + 6)) - (2*b^3*d^3*x^4*(a + b*x)^n)/((a*d - b*c)^3*(c + d*x)^(n + 4)*(1
1*n + 6*n^2 + n^3 + 6)) - (b*d*x^2*(a + b*x)^n*(12*b^2*c^2 + a^2*d^2*n + 7*
b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d
 - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (2*b^2*d^2*x^3*(a +
 b*x)^n*(4*b*c - a*d*n + b*c*n))/((a*d - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6
*n^2 + n^3 + 6))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-4-n),x)
```

```
[Out] Timed out
```

## 3.1572 $\int (a+bx)^n (c+dx)^{-5-n} \, dx$

**Optimal**. Leaf size=185

$$\frac{6b^3(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}+\frac{6b^2(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-4}}{(n+4)(bc-ad)}+\frac{3b(a+bx)^{n-}}{(n+3)(n+}$$

**Rubi** [A] time = 0.06, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{6b^2(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{6b^3(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}+\frac{(a+bx)^{n+1}(c+dx)^{-n-4}}{(n+4)(bc-ad)}+\frac{3b(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(n+4)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-4 - n))/((b*c - a*d)*(4 + n)) + (3*b*(a + b*
x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)^2*(3 + n)*(4 + n)) + (6*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n))
+ (6*b^3*(a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^4*(1 + n)*(2 +
n)*(3 + n)*(4 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n (c+dx)^{-5-n}\, dx = \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{(3b)\int (a+bx)^n (c+dx)^{-4-n}\, dx}{(bc-ad)(4+n)}$$

$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{\left(6b^2\right)\int (a+bx)^n (c+dx)^{-3\cdots}}{(bc-ad)^2(3+n)(4+n\cdots}$$

$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{6b^2(a+bx)^{1+n}(c+dx)^{-2\cdots}}{(bc-ad)^3(2+n)(3+n)(4\cdots}$$

$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{6b^2(a+bx)^{1+n}(c+dx)^{-2\cdots}}{(bc-ad)^3(2+n)(3+n)(4\cdots}$$

**Mathematica [A]** time = 0.13, size = 195, normalized size = 1.05

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-4}\left(-a^3d^3\left(n^3+6n^2+11n+6\right)+3a^2bd^2\left(n^2+3n+2\right)(c(n+4)+dx)-3ab^2d(n+1)\left(c^2\left(n^2+7n+12\right)+2cd(n+4)x+2d^2x^2\right)+b^3\left(c^3\left(n^3+9n^2+26n+24\right)+3c^2d\left(n^2+7n+12\right)x+6cd^2(n+4)x^2+6d^3x^3\right)\right)}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-4 - n)*(-(a^3*d^3*(6 + 11*n + 6*n^2 + n^3))
+ 3*a^2*b*d^2*(2 + 3*n + n^2)*(c*(4 + n) + d*x) - 3*a*b^2*d*(1 + n)*(c^2*(1
2 + 7*n + n^2) + 2*c*d*(4 + n)*x + 2*d^2*x^2) + b^3*(c^3*(24 + 26*n + 9*n^2
 + n^3) + 3*c^2*d*(12 + 7*n + n^2)*x + 6*c*d^2*(4 + n)*x^2 + 6*d^3*x^3)))/(
(b*c - a*d)^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

**IntegrateAlgebraic [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^n (c+dx)^{-5-n}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x)^(-5 - n), x]
```

**fricas [B]** time = 0.92, size = 954, normalized size = 5.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="fricas")
```

```
[Out] (6*b^4*d^4*x^5 + 24*a*b^3*c^4 - 36*a^2*b^2*c^3*d + 24*a^3*b*c^2*d^2 - 6*a^4
*c*d^3 + 6*(5*b^4*c*d^3 + (b^4*c*d^3 - a*b^3*d^4)*n)*x^4 + (a*b^3*c^4 - 3*a
^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*n^3 + 3*(20*b^4*c^2*d^2 + (b^4*
c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*n^2 + (9*b^4*c^2*d^2 - 10*a*b^3*c*d^
3 + a^2*b^2*d^4)*n)*x^3 + 3*(3*a*b^3*c^4 - 8*a^2*b^2*c^3*d + 7*a^3*b*c^2*d^
2 - 2*a^4*c*d^3)*n^2 + (60*b^4*c^3*d + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*n^3 + 3*(4*b^4*c^3*d - 9*a*b^3*c^2*d^2 + 6*a^2*b^2*
c*d^3 - a^3*b*d^4)*n^2 + (47*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 15*a^2*b^2*c*d^
3 - 2*a^3*b*d^4)*n)*x^2 + (26*a*b^3*c^4 - 57*a^2*b^2*c^3*d + 42*a^3*b*c^2*d
^2 - 11*a^4*c*d^3)*n + (24*b^4*c^4 + 24*a*b^3*c^3*d - 36*a^2*b^2*c^2*d^2 +
24*a^3*b*c*d^3 - 6*a^4*d^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4
*d^4)*n^3 + 3*(3*b^4*c^4 - 4*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 6*a^3*b*c*d^
3 - 2*a^4*d^4)*n^2 + (26*b^4*c^4 - 10*a*b^3*c^3*d - 45*a^2*b^2*c^2*d^2 + 40
*a^3*b*c*d^3 - 11*a^4*d^4)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 5)/(24*b^4*c^4
 - 96*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*n^4 +
10*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
n^3 + 35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*n^2 + 50*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{n}(dx+c)^{-n-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 5), x)
```

**maple [B]** time = 0.01, size = 662, normalized size = 3.58

$$-\frac{\left(a^{3}d^{3}n^{3}-3a^{2}bc\,d^{2}n^{3}-3a^{2}b\,d^{3}n^{2}x+3a\,b^{2}c^{2}d\,n^{3}+6a\,b^{2}c\,d^{2}n^{2}x+6a\,b^{2}d^{3}n\,x^{2}-b^{3}c^{3}n^{3}-3b^{3}c^{2}d\,n^{2}x-6b^{3}c\,d^{2}n\,x^{2}-6b^{3}d^{3}x^{3}+6a^{3}d^{3}n^{2}-21a^{2}bc\,d^{2}n^{2}-9a^{2}b\,d^{3}nx+24a\,b^{2}c^{2}d\,n^{2}+30a\,b^{2}c\,d^{2}nx+6a\,b^{2}d^{3}x^{2}-9b^{3}c^{3}n^{2}-21b^{3}c^{2}dnx-24b^{3}c\,d^{2}x^{2}+11a^{3}d^{3}n-42a^{2}bc\,d^{2}n-6a^{2}b\,d^{3}x+57a\,b^{2}c^{2}dn+24a\,b^{2}c\,d^{2}x-26b^{3}c^{3}n-36b^{3}c^{2}dx+6a^{3}d^{3}-24a^{2}bc\,d^{2}+36a\,b^{2}c^{2}d-24b^{3}c^{3}\right)(bx+a)^{n+1}(dx+c)^{-n-4}}{a^{4}d^{4}n^{4}-4a^{3}bc\,d^{3}n^{4}+6a^{2}b^{2}c^{2}d^{2}n^{4}-4a\,b^{3}c^{3}d\,n^{4}+b^{4}c^{4}n^{4}+10a^{4}d^{4}n^{3}-40a^{3}bc\,d^{3}n^{3}+60a^{2}b^{2}c^{2}d^{2}n^{3}-40a\,b^{3}c^{3}d\,n^{3}+10b^{4}c^{4}n^{3}+35a^{4}d^{4}n^{2}-140a^{3}bc\,d^{3}n^{2}+210a^{2}b^{2}c^{2}d^{2}n^{2}-140a\,b^{3}c^{3}d\,n^{2}+35b^{4}c^{4}n^{2}+50a^{4}d^{4}n-200a^{3}bc\,d^{3}n+300a^{2}b^{2}c^{2}d^{2}n-200a\,b^{3}c^{3}dn+50b^{4}c^{4}n+24a^{4}d^{4}-96a^{3}bc\,d^{3}+144a^{2}b^{2}c^{2}d^{2}-96a\,b^{3}c^{3}d+24b^{4}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-5-n),x)
```

```
[Out] -(b*x+a)^(n+1)*(d*x+c)^(-n-4)*(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3-3*a^2*b*d^3*n^
2*x+3*a*b^2*c^2*d*n^3+6*a*b^2*c*d^2*n^2*x+6*a*b^2*d^3*n*x^2-b^3*c^3*n^3-3*b
^3*c^2*d*n^2*x-6*b^3*c*d^2*n*x^2-6*b^3*d^3*x^3+6*a^3*d^3*n^2-21*a^2*b*c*d^2
*n^2-9*a^2*b*d^3*n*x+24*a*b^2*c^2*d*n^2+30*a*b^2*c*d^2*n*x+6*a*b^2*d^3*x^2-
9*b^3*c^3*n^2-21*b^3*c^2*d*n*x-24*b^3*c*d^2*x^2+11*a^3*d^3*n-42*a^2*b*c*d^2
*n-6*a^2*b*d^3*x+57*a*b^2*c^2*d*n+24*a*b^2*c*d^2*x-26*b^3*c^3*n-36*b^3*c^2*
d*x+6*a^3*d^3-24*a^2*b*c*d^2+36*a*b^2*c^2*d-24*b^3*c^3)/(a^4*d^4*n^4-4*a^3*
```

```
b*c*d^3*n^4+6*a^2*b^2*c^2*d^2*n^4-4*a*b^3*c^3*d*n^4+b^4*c^4*n^4+10*a^4*d^4*
n^3-40*a^3*b*c*d^3*n^3+60*a^2*b^2*c^2*d^2*n^3-40*a*b^3*c^3*d*n^3+10*b^4*c^4
*n^3+35*a^4*d^4*n^2-140*a^3*b*c*d^3*n^2+210*a^2*b^2*c^2*d^2*n^2-140*a*b^3*c
^3*d*n^2+35*b^4*c^4*n^2+50*a^4*d^4*n-200*a^3*b*c*d^3*n+300*a^2*b^2*c^2*d^2*
n-200*a*b^3*c^3*d*n+50*b^4*c^4*n+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*
d^2-96*a*b^3*c^3*d+24*b^4*c^4)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-n-5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 5), x)
```

**mupad [B]** time = 1.61, size = 945, normalized size = 5.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 5),x)
```

```
[Out] (6*b^4*d^4*x^5*(a + b*x)^n)/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2
 + 10*n^3 + n^4 + 24)) - (a*c*(a + b*x)^n*(6*a^3*d^3 - 24*b^3*c^3 + 11*a^3*
d^3*n - 26*b^3*c^3*n + 6*a^3*d^3*n^2 - 9*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^
3*n^3 + 36*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 57*a*b^2*c^2*d*n - 42*a^2*b*c*d^2
*n + 24*a*b^2*c^2*d*n^2 - 21*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*n^3 - 3*a^2*b*
c*d^2*n^3))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4
+ 24)) - (x*(a + b*x)^n*(6*a^4*d^4 - 24*b^4*c^4 + 11*a^4*d^4*n - 26*b^4*c^4
*n + 6*a^4*d^4*n^2 - 9*b^4*c^4*n^2 + a^4*d^4*n^3 - b^4*c^4*n^3 + 36*a^2*b^2
*c^2*d^2 - 24*a*b^3*c^3*d - 24*a^3*b*c*d^3 + 10*a*b^3*c^3*d*n - 40*a^3*b*c*
d^3*n + 9*a^2*b^2*c^2*d^2*n^2 + 12*a*b^3*c^3*d*n^2 - 18*a^3*b*c*d^3*n^2 + 2
*a*b^3*c^3*d*n^3 - 2*a^3*b*c*d^3*n^3 + 45*a^2*b^2*c^2*d^2*n))/((a*d - b*c)^
4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*b^2*d^2*x^3*(
a + b*x)^n*(20*b^2*c^2 + a^2*d^2*n + 9*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^
2 - 10*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^3*d^3*x^4*(a + b*x)^n*(5*b*c - a*d*n +
 b*c*n))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (b*d*x^2*(a + b*x)^n*(60*b^3*c^3 - 2*a^3*d^3*n + 47*b^3*c^3*n - 3*a^3
*d^3*n^2 + 12*b^3*c^3*n^2 - a^3*d^3*n^3 + b^3*c^3*n^3 - 60*a*b^2*c^2*d*n +
15*a^2*b*c*d^2*n - 27*a*b^2*c^2*d*n^2 + 18*a^2*b*c*d^2*n^2 - 3*a*b^2*c^2*d*
n^3 + 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 +
 10*n^3 + n^4 + 24))
```

**sympy [F(-1)]**    time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-5-n),x)
```

```
[Out] Timed out
```

## 3.1573 $\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2} dx$

**Optimal**. Leaf size=57

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

**Rubi** [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7, 44}

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 && !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2} dx &= \int \frac{1}{(a+bx)^2(c+dx)} dx\\
&= \int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right) dx\\
&= -\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 53, normalized size = 0.93

$$\frac{d(a+bx)\log(c+dx)-d(a+bx)\log(a+bx)+ad-bc}{(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
- a*d)^2*(a + b*x))
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2, x]
```

**fricas** [A] time = 1.49, size = 93, normalized size = 1.63

$$-\frac{bc-ad+(bdx+ad)\log(bx+a)-(bdx+ad)\log(dx+c)}{ab^2c^2-2\,a^2bcd+a^3d^2+\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

**giac** [A] time = 0.92, size = 78, normalized size = 1.37

$$\frac{bd\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{b}{\left(b^2c-abd\right)(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] b*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^3*c^2 - 2*a*b^2*c*d + a^
2*b*d^2) - b/((b^2*c - a*b*d)*(b*x + a))
```

**maple** [A] time = 0.01, size = 57, normalized size = 1.00

$$-\frac{d\ln\left(bx+a\right)}{\left(ad-bc\right)^{2}}+\frac{d\ln\left(dx+c\right)}{\left(ad-bc\right)^{2}}+\frac{1}{\left(ad-bc\right)\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c),x)
```

```
[Out] d/(a*d-b*c)^2*ln(d*x+c)+1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)
```

**maxima** [A] time = 1.16, size = 92, normalized size = 1.61

$$-\frac{d\log\left(bx+a\right)}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}+\frac{d\log\left(dx+c\right)}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}-\frac{1}{abc-a^{2}d+\left(b^{2}c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

**mupad** [B] time = 0.44, size = 46, normalized size = 0.81

$$\frac{1}{\left(a\,d-b\,c\right)\,\left(a+b\,x\right)}-\frac{d\,\ln\left(\frac{a+b\,x}{c+d\,x}\right)}{{\left(a\,d-b\,c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)),x)
```

```
[Out] 1/((a*d - b*c)*(a + b*x)) - (d*log((a + b*x)/(c + d*x)))/(a*d - b*c)^2
```

**sympy** [B] time = 0.69, size = 233, normalized size = 4.09

$$\frac{d\log\left(x+\frac{-\frac{a^{3}d^{4}}{(ad-bc)^{2}}+\frac{3a^{2}bcd^{3}}{(ad-bc)^{2}}-\frac{3ab^{2}c^{2}d^{2}}{(ad-bc)^{2}}+ad^{2}+\frac{b^{3}c^{3}d}{(ad-bc)^{2}}+bcd}{2bd^{2}}\right)}{\left(ad-bc\right)^{2}}-\frac{d\log\left(x+\frac{\frac{a^{3}d^{4}}{(ad-bc)^{2}}-\frac{3a^{2}bcd^{3}}{(ad-bc)^{2}}+\frac{3ab^{2}c^{2}d^{2}}{(ad-bc)^{2}}+ad^{2}-\frac{b^{3}c^{3}d}{(ad-bc)^{2}}+bcd}{2bd^{2}}\right)}{\left(ad-bc\right)^{2}}+\frac{1}{a^{2}d-abc+x\left(abd-b^{2}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c),x)
```

```
[Out] d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a
*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*
```

```
d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**
2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b*
*3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2*d -
a*b*c + x*(a*b*d - b**2*c))
```

## 3.1574 $\int (a+bx)^m (ac(1+m)+bc(2+m)x)^{-3-m}\,dx$

**Optimal**. Leaf size=95

$$\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-1}}{a^2bc^2(m+1)(m+2)}-\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-2}}{abc(m+2)}$$

**Rubi [A]** time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {45, 37}

$$\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-1}}{a^2bc^2(m+1)(m+2)}-\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-2}}{abc(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-2 - m))/(a*b*c*(2 + m)
)) + ((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-1 - m))/(a^2*b*c^2*
(1 + m)*(2 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^m(ac(1+m)+bc(2+m)x)^{-3-m}\,dx = -\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)} - \frac{\int (a+bx)^m(ac(1\cdots}{}$$
$$= -\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)} + \frac{(a+bx)^{1+m}(ac(1\cdots}{a^2bc^2(\cdots}$$

**Mathematica [A]** time = 0.07, size = 54, normalized size = 0.57

$$\frac{x(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m}}{a^2c^3(m+1)(a(m+1)+b(m+2)x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
```

```
[Out] (x*(a + b*x)^(1 + m))/(a^2*c^3*(1 + m)*(a*(1 + m) + b*(2 + m)*x)^2*(a*c*(1
+ m) + b*c*(2 + m)*x)^m)
```

**IntegrateAlgebraic [F]** time = 0.22, size = 0, normalized size = 0.00

$$\int (a+bx)^m(ac(1+m)+bc(2+m)x)^{-3-m}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m
), x]
```

**fricas [A]** time = 1.32, size = 85, normalized size = 0.89

$$\frac{\left(\left(b^2m+2\,b^2\right)x^3+(2\,abm+3\,ab)x^2+\left(a^2m+a^2\right)x\right)(acm+ac+(bcm+2\,bc)x)^{-m-3}(bx+a)^m}{a^2m+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="fricas")
```

```
[Out] ((b^2*m + 2*b^2)*x^3 + (2*a*b*m + 3*a*b)*x^2 + (a^2*m + a^2)*x)*(a*c*m + a*
c + (b*c*m + 2*b*c)*x)^(-m - 3)*(b*x + a)^m/(a^2*m + a^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bc(m+2)x+ac(m+1))^{-m-3}(bx+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="giac")
```

```
[Out] integrate((b*c*(m + 2)*x + a*c*(m + 1))^(-m - 3)*(b*x + a)^m, x)
```

**maple [A]** time = 0.01, size = 57, normalized size = 0.60

$$\frac{(bxm + am + 2bx + a)\,x\,(bx + a)^{m+1}\,(bcxm + acm + 2bcx + ac)^{-m-3}}{(m + 1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(a*c*(m+1)+b*c*(m+2)*x)^(-3-m),x)
```

```
[Out] (b*x+a)^(m+1)*(b*m*x+a*m+2*b*x+a)/a^2/(m+1)*x*(b*c*m*x+a*c*m+2*b*c*x+a*c)^(
-3-m)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bc(m + 2)x + ac(m + 1))^{-m-3}(bx + a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="maxima")
```

```
[Out] integrate((b*c*(m + 2)*x + a*c*(m + 1))^(-m - 3)*(b*x + a)^m, x)
```

**mupad [B]** time = 1.04, size = 81, normalized size = 0.85

$$\frac{x\,(a + b\,x)^m + \frac{b\,x^2\,(2\,m+3)\,(a+b\,x)^m}{a\,(m+1)} + \frac{b^2\,x^3\,(m+2)\,(a+b\,x)^m}{a^2\,(m+1)}}{(a\,c\,\,(m + 1) + b\,c\,x\,\,(m + 2))^{m+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/(a*c*(m + 1) + b*c*x*(m + 2))^(m + 3),x)
```

```
[Out] (x*(a + b*x)^m + (b*x^2*(2*m + 3)*(a + b*x)^m)/(a*(m + 1)) + (b^2*x^3*(m +
2)*(a + b*x)^m)/(a^2*(m + 1)))/(a*c*(m + 1) + b*c*x*(m + 2))^(m + 3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(a*c*(1+m)+b*c*(2+m)*x)**(-3-m),x)
```

```
[Out] Timed out
```

## 3.1575 $\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx$

**Optimal**. Leaf size=97

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

**Rubi [A]** time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {45, 37}

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)/(b*c - a*d)),x
]
```

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx = -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} - \frac{d\int (a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx}{bc}$$
$$= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} + \frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}$$

**Mathematica** [A]  time = 0.04, size = 46, normalized size = 0.47

$$\frac{x(a+bx)^{\frac{bc}{ad-bc}}(c+dx)^{\frac{ad}{bc-ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)/(b*c - a
*d)),x]
```

```
[Out] (x*(a + b*x)^((b*c)/(-(b*c) + a*d))*(c + d*x)^((a*d)/(b*c - a*d)))/(a*c)
```

**IntegrateAlgebraic** [F]  time = 0.37, size = 0, normalized size = 0.00

$$\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)
/(b*c - a*d)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1
+ (a*d)/(b*c - a*d)), x]
```

**fricas** [A]  time = 1.39, size = 84, normalized size = 0.87

$$\frac{bdx^3 + acx + (bc+ad)x^2}{(bx+a)^{\frac{2\,bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2\,ad}{bc-ad}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="fricas")
```

```
[Out] (b*d*x^3 + a*c*x + (b*c + a*d)*x^2)/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*
(d*x + c)^((b*c - 2*a*d)/(b*c - a*d))*a*c)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-\frac{bc}{bc-ad}-1}(dx+c)^{\frac{ad}{bc-ad}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="giac")
```

```
[Out] integrate((b*x + a)^(-b*c/(b*c - a*d) - 1)*(d*x + c)^(a*d/(b*c - a*d) - 1),
 x)
```

**maple [A]** time = 0.00, size = 66, normalized size = 0.68

$$\frac{x\,(bx+a)^{1-\frac{ad-2bc}{ad-bc}}\,(dx+c)^{1-\frac{2ad-bc}{ad-bc}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x)
```

```
[Out] (b*x+a)^(1-(a*d-2*b*c)/(a*d-b*c))*(d*x+c)^(1-(2*a*d-b*c)/(a*d-b*c))/a/c*x
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-\frac{bc}{bc-ad}-1}(dx+c)^{\frac{ad}{bc-ad}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="maxima")
```

```
[Out] integrate((b*x + a)^(-b*c/(b*c - a*d) - 1)*(d*x + c)^(a*d/(b*c - a*d) - 1),
 x)
```

**mupad [B]** time = 2.14, size = 119, normalized size = 1.23

$$\frac{x\,(a+b\,x)^{\frac{b\,c}{a\,d-b\,c}-1}+\frac{x^2\,(a\,d+b\,c)\,(a+b\,x)^{\frac{b\,c}{a\,d-b\,c}-1}}{a\,c}+\frac{b\,d\,x^3\,(a+b\,x)^{\frac{b\,c}{a\,d-b\,c}-1}}{a\,c}}{(c+d\,x)^{\frac{a\,d}{a\,d-b\,c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^((b*c)/(a*d - b*c) - 1)/(c + d*x)^((a*d)/(a*d - b*c) + 1),x)
```

```
[Out] (x*(a + b*x)^((b*c)/(a*d - b*c) - 1) + (x^2*(a*d + b*c)*(a + b*x)^((b*c)/(a
*d - b*c) - 1))/(a*c) + (b*d*x^3*(a + b*x)^((b*c)/(a*d - b*c) - 1))/(a*c))/
(c + d*x)^((a*d)/(a*d - b*c) + 1)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-1-b*c/(-a*d+b*c))*(d*x+c)**(-1+a*d/(-a*d+b*c)),x)
```

```
[Out] Timed out
```

## 3.1576 $\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx$

**Optimal**. Leaf size=97

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

**Rubi** [A] time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {45, 37}

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c - 2*a*d)/(-(b*c)
 + a*d)),x]
```

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx = -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} - \frac{d\int (a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx}{bc}$$
$$= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} + \frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}$$

**Mathematica** [A] time = 0.07, size = 46, normalized size = 0.47

$$\frac{x(a+bx)^{\frac{bc}{ad-bc}}(c+dx)^{\frac{ad}{bc-ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c - 2*a*d)/(
-(b*c) + a*d)),x]
```

```
[Out] (x*(a + b*x)^((b*c)/(-(b*c) + a*d))*(c + d*x)^((a*d)/(b*c - a*d)))/(a*c)
```

**IntegrateAlgebraic** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c -
 2*a*d)/(-(b*c) + a*d)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^
((b*c - 2*a*d)/(-(b*c) + a*d)), x]
```

**fricas** [A] time = 1.40, size = 84, normalized size = 0.87

$$\frac{bdx^3 + acx + (bc+ad)x^2}{(bx+a)^{\frac{2\,bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2\,ad}{bc-ad}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="fricas")
```

```
[Out] (b*d*x^3 + a*c*x + (b*c + a*d)*x^2)/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*
(d*x + c)^((b*c - 2*a*d)/(b*c - a*d))*a*c)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2ad}{bc-ad}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*(d*x + c)^((b*c - 2*a*d)
/(b*c - a*d))), x)
```

**maple [A]** time = 0.00, size = 66, normalized size = 0.68

$$\frac{x\,(bx+a)^{1-\frac{ad-2bc}{ad-bc}}\,(dx+c)^{1-\frac{2ad-bc}{ad-bc}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c)),x)
```

```
[Out] 1/a/c*x*(b*x+a)^(1-(a*d-2*b*c)/(a*d-b*c))*(d*x+c)^(1-(2*a*d-b*c)/(a*d-b*c))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2ad}{bc-ad}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*(d*x + c)^((b*c - 2*a*d)
/(b*c - a*d))), x)
```

**mupad [B]** time = 0.85, size = 142, normalized size = 1.46

$$\frac{\frac{x}{(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}+\frac{x^2\,(a\,d+b\,c)}{a\,c\,(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}+\frac{b\,d\,x^3}{a\,c\,(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}}{(c+d\,x)^{\frac{2\,a\,d-b\,c}{a\,d-b\,c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^((a*d - 2*b*c)/(a*d - b*c))*(c + d*x)^((2*a*d - b*c)/(a*d
- b*c))),x)
```

```
[Out] (x/(a + b*x)^((a*d - 2*b*c)/(a*d - b*c)) + (x^2*(a*d + b*c))/(a*c*(a + b*x)
^((a*d - 2*b*c)/(a*d - b*c))) + (b*d*x^3)/(a*c*(a + b*x)^((a*d - 2*b*c)/(a*
d - b*c))))/(c + d*x)^((2*a*d - b*c)/(a*d - b*c))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)**((-2*a*d+b*c)/(a*d-b*c
)),x)
```

```
[Out] Timed out
```

## 3.1577 $\int (a + bx + cx^2 + dx^3)\, dx$

**Optimal**. Leaf size=28

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

**Rubi [A]** time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x + c*x^2 + d*x^3,x]
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

Rubi steps

$$\int (a + bx + cx^2 + dx^3)\, dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

**Mathematica [A]** time = 0.00, size = 28, normalized size = 1.00

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x + c*x^2 + d*x^3,x]
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2 + dx^3)\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + b*x + c*x^2 + d*x^3,x]
[Out] IntegrateAlgebraic[a + b*x + c*x^2 + d*x^3, x]
```

**fricas** [A] time = 1.17, size = 22, normalized size = 0.79

$$\frac{1}{4}x^4d+\frac{1}{3}x^3c+\frac{1}{2}x^2b+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="fricas")
[Out] 1/4*x^4*d + 1/3*x^3*c + 1/2*x^2*b + x*a
```

**giac** [A] time = 1.01, size = 22, normalized size = 0.79

$$\frac{1}{4}\,dx^4+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="giac")
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

**maple** [A] time = 0.00, size = 23, normalized size = 0.82

$$\frac{1}{4}d\,x^4+\frac{1}{3}c\,x^3+\frac{1}{2}b\,x^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3+c*x^2+b*x+a,x)
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/4*d*x^4
```

**maxima** [A] time = 1.07, size = 22, normalized size = 0.79

$$\frac{1}{4}\,dx^4+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="maxima")
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

**mupad** [B] time = 0.04, size = 22, normalized size = 0.79

$$\frac{d\,x^4}{4}+\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x + c*x^2 + d*x^3,x)
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

**sympy [A]**    time = 0.06, size = 22, normalized size = 0.79

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{dx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x**3+c*x**2+b*x+a,x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3 + d*x**4/4
```

## 3.1578 $\int \left(-x^3+x^4\right) dx$

**Optimal**. Leaf size=15

$$\frac{x^5}{5}-\frac{x^4}{4}$$

**Rubi** [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^5}{5}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[-x^3 + x^4,x]
```

```
[Out] -x^4/4 + x^5/5
```

Rubi steps

$$\int \left(-x^3+x^4\right) dx=-\frac{x^4}{4}+\frac{x^5}{5}$$

**Mathematica** [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^5}{5}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[-x^3 + x^4,x]
```

```
[Out] -1/4*x^4 + x^5/5
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^3+x^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-x^3 + x^4,x]
```

```
[Out] IntegrateAlgebraic[-x^3 + x^4, x]
```

**fricas** [A] time = 1.04, size = 11, normalized size = 0.73

$$\frac{1}{5}x^5 - \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="fricas")
[Out] 1/5*x^5 - 1/4*x^4
```

**giac** [A] time = 1.08, size = 11, normalized size = 0.73

$$\frac{1}{5}x^5 - \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="giac")
[Out] 1/5*x^5 - 1/4*x^4
```

**maple** [A] time = 0.00, size = 12, normalized size = 0.80

$$\frac{1}{5}x^5 - \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4-x^3,x)
[Out] -1/4*x^4+1/5*x^5
```

**maxima** [A] time = 1.08, size = 11, normalized size = 0.73

$$\frac{1}{5}x^5 - \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="maxima")
[Out] 1/5*x^5 - 1/4*x^4
```

**mupad** [B] time = 0.02, size = 10, normalized size = 0.67

$$\frac{x^4\,\left(4\,x-5\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4 - x^3,x)
```

```
[Out] (x^4*(4*x - 5))/20
```

**sympy** [**A**] time = 0.05, size = 8, normalized size = 0.53

$$\frac{x^5}{5} - \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4-x**3,x)
```

```
[Out] x**5/5 - x**4/4
```

## 3.1579 $\int \left(-1+x^5\right) dx$

**Optimal**. Leaf size=11

$$\frac{x^6}{6}-x$$

**Rubi [A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^6}{6}-x$$

Antiderivative was successfully verified.

```
[In] Int[-1 + x^5,x]
[Out] -x + x^6/6
```

Rubi steps

$$\int \left(-1+x^5\right) dx=-x+\frac{x^6}{6}$$

**Mathematica [A]** time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^6}{6}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[-1 + x^5,x]
[Out] -x + x^6/6
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-1+x^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-1 + x^5,x]
[Out] IntegrateAlgebraic[-1 + x^5, x]
```

**fricas** [A] time = 0.97, size = 9, normalized size = 0.82

$$\frac{1}{6}x^6 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="fricas")
[Out] 1/6*x^6 - x
```

**giac** [A] time = 0.94, size = 9, normalized size = 0.82

$$\frac{1}{6}\,x^6 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="giac")
[Out] 1/6*x^6 - x
```

**maple** [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{1}{6}x^6 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5-1,x)
[Out] -x+1/6*x^6
```

**maxima** [A] time = 0.96, size = 9, normalized size = 0.82

$$\frac{1}{6}\,x^6 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="maxima")
[Out] 1/6*x^6 - x
```

**mupad** [B] time = 0.02, size = 8, normalized size = 0.73

$$\frac{x\,\left(x^5-6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5 - 1,x)
```

```
[Out] (x*(x^5 - 6))/6
```

**sympy [A]** time = 0.05, size = 5, normalized size = 0.45

$$\frac{x^6}{6} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5-1,x)
```

```
[Out] x**6/6 - x
```

## 3.1580 $\int (7+4x)\,dx$

**Optimal**. Leaf size=9

$$2x^2+7x$$

**Rubi [A]** time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$2x^2+7x$$

Antiderivative was successfully verified.

```
[In] Int[7 + 4*x,x]
[Out] 7*x + 2*x^2
```

Rubi steps

$$\int (7+4x)\,dx = 7x+2x^2$$

**Mathematica [A]** time = 0.00, size = 9, normalized size = 1.00

$$2x^2+7x$$

Antiderivative was successfully verified.

```
[In] Integrate[7 + 4*x,x]
[Out] 7*x + 2*x^2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (7+4x)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[7 + 4*x,x]
[Out] IntegrateAlgebraic[7 + 4*x, x]
```

**fricas [A]** time = 0.97, size = 9, normalized size = 1.00

$$2x^2+7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="fricas")
[Out] 2*x^2 + 7*x
```

**giac** [A] time = 0.96, size = 9, normalized size = 1.00

$$2x^2 + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="giac")
[Out] 2*x^2 + 7*x
```

**maple** [A] time = 0.00, size = 10, normalized size = 1.11

$$2x^2 + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7+4*x,x)
[Out] 2*x^2+7*x
```

**maxima** [A] time = 1.01, size = 9, normalized size = 1.00

$$2x^2 + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="maxima")
[Out] 2*x^2 + 7*x
```

**mupad** [B] time = 0.03, size = 7, normalized size = 0.78

$$x(2x + 7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + 7,x)
[Out] x*(2*x + 7)
```

**sympy** [A] time = 0.05, size = 7, normalized size = 0.78

$$2x^2 + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x)
```

```
[Out] 2*x**2 + 7*x
```

## 3.1581 $\int (4x + \pi x^3) \, dx$

**Optimal**. Leaf size=14

$$\frac{\pi x^4}{4} + 2x^2$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{\pi x^4}{4} + 2x^2$$

Antiderivative was successfully verified.

```
[In] Int[4*x + Pi*x^3,x]
[Out] 2*x^2 + (Pi*x^4)/4
```

Rubi steps

$$\int (4x + \pi x^3) \, dx = 2x^2 + \frac{\pi x^4}{4}$$

**Mathematica [A]** time = 0.00, size = 14, normalized size = 1.00

$$\frac{\pi x^4}{4} + 2x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[4*x + Pi*x^3,x]
[Out] 2*x^2 + (Pi*x^4)/4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (4x + \pi x^3) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[4*x + Pi*x^3,x]
[Out] IntegrateAlgebraic[4*x + Pi*x^3, x]
```

**fricas** [A] time = 0.71, size = 12, normalized size = 0.86

$$\frac{1}{4}\,\pi x^4 + 2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="fricas")
[Out] 1/4*pi*x^4 + 2*x^2
```

**giac** [A] time = 1.06, size = 12, normalized size = 0.86

$$\frac{1}{4}\,\pi x^4 + 2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="giac")
[Out] 1/4*pi*x^4 + 2*x^2
```

**maple** [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{1}{4}\pi\,x^4 + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi*x^3+4*x,x)
[Out] 2*x^2+1/4*Pi*x^4
```

**maxima** [A] time = 1.00, size = 12, normalized size = 0.86

$$\frac{1}{4}\,\pi x^4 + 2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="maxima")
[Out] 1/4*pi*x^4 + 2*x^2
```

**mupad** [B] time = 0.02, size = 12, normalized size = 0.86

$$\frac{\Pi\,x^4}{4} + 2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + Pi*x^3,x)
```

```
[Out] (Pi*x^4)/4 + 2*x^2
```

**sympy** **[A]** time = 0.06, size = 10, normalized size = 0.71

$$\frac{\pi x^4}{4} + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x**3+4*x,x)
```

```
[Out] pi*x**4/4 + 2*x**2
```

## 3.1582 $\int (2x + 5x^2) \, dx$

**Optimal**. Leaf size=11

$$\frac{5x^3}{3} + x^2$$

**Rubi [A]** time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{5x^3}{3} + x^2$$

Antiderivative was successfully verified.

```
[In] Int[2*x + 5*x^2,x]
```

```
[Out] x^2 + (5*x^3)/3
```

Rubi steps

$$\int (2x + 5x^2) \, dx = x^2 + \frac{5x^3}{3}$$

**Mathematica [A]** time = 0.00, size = 11, normalized size = 1.00

$$\frac{5x^3}{3} + x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x + 5*x^2,x]
```

```
[Out] x^2 + (5*x^3)/3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + 5x^2) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[2*x + 5*x^2,x]
```

```
[Out] IntegrateAlgebraic[2*x + 5*x^2, x]
```

**fricas [A]** time = 0.72, size = 9, normalized size = 0.82

$$\frac{5}{3}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="fricas")
```

```
[Out] 5/3*x^3 + x^2
```

**giac [A]** time = 1.10, size = 9, normalized size = 0.82

$$\frac{5}{3}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="giac")
```

```
[Out] 5/3*x^3 + x^2
```

**maple [A]** time = 0.00, size = 10, normalized size = 0.91

$$\frac{5}{3}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5*x^2+2*x,x)
```

```
[Out] x^2+5/3*x^3
```

**maxima [A]** time = 1.10, size = 9, normalized size = 0.82

$$\frac{5}{3}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + x^2
```

**mupad [B]** time = 0.02, size = 10, normalized size = 0.91

$$\frac{x^2\,(5\,x+3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 5*x^2,x)
```

```
[Out] (x^2*(5*x + 3))/3
```

**sympy [A]** time = 0.05, size = 8, normalized size = 0.73

$$\frac{5x^3}{3} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x**2+2*x,x)
```

```
[Out] 5*x**3/3 + x**2
```

## 3.1583 $\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx$

**Optimal**. Leaf size=15

$$\frac{x^4}{12}+\frac{x^3}{6}$$

**Rubi [A]** time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^4}{12}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/2 + x^3/3,x]
```

```
[Out] x^3/6 + x^4/12
```

Rubi steps

$$\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx=\frac{x^3}{6}+\frac{x^4}{12}$$

**Mathematica [A]** time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^4}{12}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/2 + x^3/3,x]
```

```
[Out] x^3/6 + x^4/12
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/2 + x^3/3,x]
```

```
[Out] IntegrateAlgebraic[x^2/2 + x^3/3, x]
```

**fricas [A]** time = 1.02, size = 11, normalized size = 0.73

$$\frac{1}{12}x^4 + \frac{1}{6}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="fricas")
[Out] 1/12*x^4 + 1/6*x^3
```

**giac [A]** time = 0.89, size = 11, normalized size = 0.73

$$\frac{1}{12}\,x^4 + \frac{1}{6}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="giac")
[Out] 1/12*x^4 + 1/6*x^3
```

**maple [A]** time = 0.00, size = 12, normalized size = 0.80

$$\frac{1}{12}x^4 + \frac{1}{6}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^2+1/3*x^3,x)
[Out] 1/6*x^3+1/12*x^4
```

**maxima [A]** time = 1.05, size = 11, normalized size = 0.73

$$\frac{1}{12}\,x^4 + \frac{1}{6}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="maxima")
[Out] 1/12*x^4 + 1/6*x^3
```

**mupad [B]** time = 0.02, size = 8, normalized size = 0.53

$$\frac{x^3\,\left(x+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/2 + x^3/3,x)
```

```
[Out] (x^3*(x + 2))/12
```

**sympy [A]** time = 0.06, size = 8, normalized size = 0.53

$$\frac{x^4}{12} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**2+1/3*x**3,x)
```

```
[Out] x**4/12 + x**3/6
```

## 3.1584 $\int (3-5x+2x^2)\,dx$

**Optimal**. Leaf size=18

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

**Rubi** [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

Antiderivative was successfully verified.

```
[In] Int[3 - 5*x + 2*x^2,x]
[Out] 3*x - (5*x^2)/2 + (2*x^3)/3
```

Rubi steps

$$\int (3-5x+2x^2)\,dx = 3x-\frac{5x^2}{2}+\frac{2x^3}{3}$$

**Mathematica** [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

Antiderivative was successfully verified.

```
[In] Integrate[3 - 5*x + 2*x^2,x]
[Out] 3*x - (5*x^2)/2 + (2*x^3)/3
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3-5x+2x^2)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[3 - 5*x + 2*x^2,x]
[Out] IntegrateAlgebraic[3 - 5*x + 2*x^2, x]
```

**fricas** [**A**] time = 0.49, size = 14, normalized size = 0.78

$$\frac{2}{3}x^3 - \frac{5}{2}x^2 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="fricas")
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

**giac** [**A**] time = 0.85, size = 14, normalized size = 0.78

$$\frac{2}{3}\,x^3 - \frac{5}{2}\,x^2 + 3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="giac")
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

**maple** [**A**] time = 0.00, size = 15, normalized size = 0.83

$$\frac{2}{3}x^3 - \frac{5}{2}x^2 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x^2-5*x+3,x)
[Out] 3*x-5/2*x^2+2/3*x^3
```

**maxima** [**A**] time = 0.97, size = 14, normalized size = 0.78

$$\frac{2}{3}\,x^3 - \frac{5}{2}\,x^2 + 3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="maxima")
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

**mupad** [**B**] time = 0.02, size = 13, normalized size = 0.72

$$\frac{x\,\left(4\,x^2-15\,x+18\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x^2 - 5*x + 3,x)
```

```
[Out] (x*(4*x^2 - 15*x + 18))/6
```

**sympy [A]** time = 0.06, size = 15, normalized size = 0.83

$$\frac{2x^3}{3} - \frac{5x^2}{2} + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x**2-5*x+3,x)
```

```
[Out] 2*x**3/3 - 5*x**2/2 + 3*x
```

## 3.1585 $\int \left(-2x+x^2+x^3\right) dx$

**Optimal**. Leaf size=20

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

**Rubi [A]** time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

Antiderivative was successfully verified.

```
[In] Int[-2*x + x^2 + x^3,x]
[Out] -x^2 + x^3/3 + x^4/4
```

Rubi steps

$$\int \left(-2x+x^2+x^3\right) dx = -x^2+\frac{x^3}{3}+\frac{x^4}{4}$$

**Mathematica [A]** time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[-2*x + x^2 + x^3,x]
[Out] -x^2 + x^3/3 + x^4/4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-2x+x^2+x^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-2*x + x^2 + x^3,x]
[Out] IntegrateAlgebraic[-2*x + x^2 + x^3, x]
```

**fricas** [A] time = 1.09, size = 16, normalized size = 0.80

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="fricas")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

**giac** [A] time = 1.10, size = 16, normalized size = 0.80

$$\frac{1}{4}\,x^4 + \frac{1}{3}\,x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="giac")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

**maple** [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3+x^2-2*x,x)
[Out] -x^2+1/3*x^3+1/4*x^4
```

**maxima** [A] time = 0.97, size = 16, normalized size = 0.80

$$\frac{1}{4}\,x^4 + \frac{1}{3}\,x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="maxima")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

**mupad** [B] time = 0.03, size = 15, normalized size = 0.75

$$\frac{x^2\,\left(3\,x^2+4\,x-12\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2 - 2*x + x^3,x)
```

```
[Out] (x^2*(4*x + 3*x^2 - 12))/12
```

**sympy** [A] time = 0.05, size = 12, normalized size = 0.60

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3+x**2-2*x,x)
```

```
[Out] x**4/4 + x**3/3 - x**2
```

## 3.1586 $\int \left(1-x^2-3x^5\right) dx$

**Optimal**. Leaf size=16

$$-\frac{x^6}{2}-\frac{x^3}{3}+x$$

**Rubi** [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$-\frac{x^6}{2}-\frac{x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Int[1 - x^2 - 3*x^5,x]
[Out] x - x^3/3 - x^6/2
```

Rubi steps

$$\int \left(1-x^2-3x^5\right) dx = x-\frac{x^3}{3}-\frac{x^6}{2}$$

**Mathematica** [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{x^6}{2}-\frac{x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - x^2 - 3*x^5,x]
[Out] x - x^3/3 - x^6/2
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1-x^2-3x^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1 - x^2 - 3*x^5,x]
[Out] IntegrateAlgebraic[1 - x^2 - 3*x^5, x]
```

**fricas** [A] time = 0.99, size = 12, normalized size = 0.75

$$-\frac{1}{2}x^6 - \frac{1}{3}x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="fricas")
[Out] -1/2*x^6 - 1/3*x^3 + x
```

**giac** [A] time = 0.96, size = 12, normalized size = 0.75

$$-\frac{1}{2}x^6 - \frac{1}{3}x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="giac")
[Out] -1/2*x^6 - 1/3*x^3 + x
```

**maple** [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{1}{2}x^6 - \frac{1}{3}x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3*x^5-x^2+1,x)
[Out] x-1/3*x^3-1/2*x^6
```

**maxima** [A] time = 0.98, size = 12, normalized size = 0.75

$$-\frac{1}{2}x^6 - \frac{1}{3}x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="maxima")
[Out] -1/2*x^6 - 1/3*x^3 + x
```

**mupad** [B] time = 0.02, size = 12, normalized size = 0.75

$$-\frac{x^6}{2} - \frac{x^3}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - 3*x^5 - x^2,x)
```

```
[Out] x - x^3/3 - x^6/2
```

**sympy [A]** time = 0.06, size = 10, normalized size = 0.62

$$-\frac{x^6}{2} - \frac{x^3}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x**5-x**2+1,x)
```

```
[Out] -x**6/2 - x**3/3 + x
```

## 3.1587 $\int (5+2x+3x^2+4x^3) \, dx$

**Optimal**. Leaf size=13

$$x^4+x^3+x^2+5x$$

**Rubi** [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$x^4+x^3+x^2+5x$$

Antiderivative was successfully verified.

```
[In] Int[5 + 2*x + 3*x^2 + 4*x^3,x]
[Out] 5*x + x^2 + x^3 + x^4
```

Rubi steps

$$\int (5+2x+3x^2+4x^3) \, dx = 5x+x^2+x^3+x^4$$

**Mathematica** [A] time = 0.00, size = 13, normalized size = 1.00

$$x^4+x^3+x^2+5x$$

Antiderivative was successfully verified.

```
[In] Integrate[5 + 2*x + 3*x^2 + 4*x^3,x]
[Out] 5*x + x^2 + x^3 + x^4
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5+2x+3x^2+4x^3) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[5 + 2*x + 3*x^2 + 4*x^3,x]
[Out] IntegrateAlgebraic[5 + 2*x + 3*x^2 + 4*x^3, x]
```

**fricas** [A] time = 0.53, size = 13, normalized size = 1.00

$$x^4+x^3+x^2+5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="fricas")
[Out] x^4 + x^3 + x^2 + 5*x
```

**giac [A]** time = 1.00, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="giac")
[Out] x^4 + x^3 + x^2 + 5*x
```

**maple [A]** time = 0.00, size = 14, normalized size = 1.08

$$x^4 + x^3 + x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x^3+3*x^2+2*x+5,x)
[Out] x^4+x^3+x^2+5*x
```

**maxima [A]** time = 1.14, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="maxima")
[Out] x^4 + x^3 + x^2 + 5*x
```

**mupad [B]** time = 0.03, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 3*x^2 + 4*x^3 + 5,x)
[Out] 5*x + x^2 + x^3 + x^4
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.92

$$x^4 + x^3 + x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x**3+3*x**2+2*x+5,x)
```

```
[Out] x**4 + x**3 + x**2 + 5*x
```

## 3.1588 $\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx$

**Optimal**. Leaf size=22

$$ax + b\log(x) - \frac{c}{x} - \frac{d}{2x^2}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax + b\log(x) - \frac{c}{x} - \frac{d}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[a + d/x^3 + c/x^2 + b/x,x]
[Out] -d/(2*x^2) - c/x + a*x + b*Log[x]
```

Rubi steps

$$\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx = -\frac{d}{2x^2} - \frac{c}{x} + ax + b\log(x)$$

**Mathematica [A]** time = 0.01, size = 22, normalized size = 1.00

$$ax + b\log(x) - \frac{c}{x} - \frac{d}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + d/x^3 + c/x^2 + b/x,x]
[Out] -1/2*d/x^2 - c/x + a*x + b*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + d/x^3 + c/x^2 + b/x,x]
```

```
[Out] IntegrateAlgebraic[a + d/x^3 + c/x^2 + b/x, x]
```

**fricas [A]** time = 1.20, size = 27, normalized size = 1.23

$$\frac{2\,ax^3 + 2\,bx^2\log(x) - 2\,cx - d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="fricas")
[Out] 1/2*(2*a*x^3 + 2*b*x^2*log(x) - 2*c*x - d)/x^2
```

**giac [A]** time = 0.85, size = 21, normalized size = 0.95

$$ax + b\log\left(|x|\right) - \frac{c}{x} - \frac{d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="giac")
[Out] a*x + b*log(abs(x)) - c/x - 1/2*d/x^2
```

**maple [A]** time = 0.00, size = 21, normalized size = 0.95

$$ax + b\ln(x) - \frac{c}{x} - \frac{d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+d/x^3+c/x^2+b/x,x)
[Out] -1/2*d/x^2-c/x+a*x+b*ln(x)
```

**maxima [A]** time = 1.03, size = 20, normalized size = 0.91

$$ax + b\log(x) - \frac{c}{x} - \frac{d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="maxima")
[Out] a*x + b*log(x) - c/x - 1/2*d/x^2
```

**mupad [B]** time = 0.04, size = 20, normalized size = 0.91

$$a\,x - \frac{\frac{d}{2} + c\,x}{x^2} + b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/x + c/x^2 + d/x^3,x)
```

```
[Out] a*x - (d/2 + c*x)/x^2 + b*log(x)
```

**sympy** [**A**] time = 0.16, size = 20, normalized size = 0.91

$$ax + b\log(x) + \frac{-2cx - d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x**3+c/x**2+b/x,x)
```

```
[Out] a*x + b*log(x) + (-2*c*x - d)/(2*x**2)
```

## 3.1589 $\int \left(\frac{1}{x^5}+x+x^5\right) dx$

**Optimal**. Leaf size=22

$$\frac{x^6}{6}-\frac{1}{4x^4}+\frac{x^2}{2}$$

**Rubi [A]** time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^6}{6}+\frac{x^2}{2}-\frac{1}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5) + x + x^5,x]
```

```
[Out] -1/(4*x^4) + x^2/2 + x^6/6
```

Rubi steps

$$\int \left(\frac{1}{x^5}+x+x^5\right) dx=-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$$

**Mathematica [A]** time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^6}{6}-\frac{1}{4x^4}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5) + x + x^5,x]
```

```
[Out] -1/4*1/x^4 + x^2/2 + x^6/6
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{x^5}+x+x^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-5) + x + x^5,x]
```

```
[Out] IntegrateAlgebraic[x^(-5) + x + x^5, x]
```

**fricas** [A] time = 1.28, size = 17, normalized size = 0.77

$$\frac{2\,x^{10}+6\,x^{6}-3}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="fricas")
[Out] 1/12*(2*x^10 + 6*x^6 - 3)/x^4
```

**giac** [A] time = 0.81, size = 16, normalized size = 0.73

$$\frac{1}{6}\,x^{6}+\frac{1}{2}\,x^{2}-\frac{1}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="giac")
[Out] 1/6*x^6 + 1/2*x^2 - 1/4/x^4
```

**maple** [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{x^{6}}{6}+\frac{x^{2}}{2}-\frac{1}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5+x+x^5,x)
[Out] -1/4/x^4+1/2*x^2+1/6*x^6
```

**maxima** [A] time = 1.08, size = 16, normalized size = 0.73

$$\frac{1}{6}\,x^{6}+\frac{1}{2}\,x^{2}-\frac{1}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="maxima")
[Out] 1/6*x^6 + 1/2*x^2 - 1/4/x^4
```

**mupad** [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,x^{10}+6\,x^{6}-3}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + 1/x^5 + x^5,x)
```

```
[Out] (6*x^6 + 2*x^10 - 3)/(12*x^4)
```

**sympy [A]** time = 0.08, size = 15, normalized size = 0.68

$$\frac{x^6}{6} + \frac{x^2}{2} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5+x+x**5,x)
```

```
[Out] x**6/6 + x**2/2 - 1/(4*x**4)
```

## 3.1590 $\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right) dx$

**Optimal**. Leaf size=15

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

**Rubi** [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3) + x^(-2) + x^(-1),x]
```

```
[Out] -1/(2*x^2) - x^(-1) + Log[x]
```

Rubi steps

$$\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right) dx = -\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

**Mathematica** [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3) + x^(-2) + x^(-1),x]
```

```
[Out] -1/2*1/x^2 - x^(-1) + Log[x]
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-3) + x^(-2) + x^(-1),x]
```

```
[Out] IntegrateAlgebraic[x^(-3) + x^(-2) + x^(-1), x]
```

**fricas [A]** time = 1.23, size = 17, normalized size = 1.13

$$\frac{2\,x^2\log(x)-2\,x-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^2*log(x) - 2*x - 1)/x^2
```

**giac [A]** time = 1.08, size = 14, normalized size = 0.93

$$-\frac{1}{x}-\frac{1}{2\,x^2}+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="giac")
```

```
[Out] -1/x - 1/2/x^2 + log(abs(x))
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.93

$$\ln(x)-\frac{1}{x}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3+1/x^2+1/x,x)
```

```
[Out] -1/2/x^2-1/x+ln(x)
```

**maxima [A]** time = 1.04, size = 13, normalized size = 0.87

$$-\frac{1}{x}-\frac{1}{2\,x^2}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="maxima")
```

```
[Out] -1/x - 1/2/x^2 + log(x)
```

**mupad [B]** time = 0.03, size = 11, normalized size = 0.73

$$\ln(x)-\frac{x+\frac{1}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x + 1/x^2 + 1/x^3,x)
```

```
[Out] log(x) - (x + 1/2)/x^2
```

**sympy [A]** time = 0.09, size = 14, normalized size = 0.93

$$\log(x) + \frac{-2x - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3+1/x**2+1/x,x)
```

```
[Out] log(x) + (-2*x - 1)/(2*x**2)
```

## 3.1591 $\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx$

**Optimal**. Leaf size=10

$$\frac{2}{x}+3\log(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-2/x^2 + 3/x,x]
```

```
[Out] 2/x + 3*Log[x]
```

Rubi steps

$$\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx=\frac{2}{x}+3\log(x)$$

**Mathematica [A]** time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-2/x^2 + 3/x,x]
```

```
[Out] 2/x + 3*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-2/x^2 + 3/x,x]
```

```
[Out] IntegrateAlgebraic[-2/x^2 + 3/x, x]
```

**fricas [A]** time = 1.02, size = 11, normalized size = 1.10

$$\frac{3\,x\log(x) + 2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="fricas")
[Out] (3*x*log(x) + 2)/x
```

**giac [A]** time = 1.12, size = 11, normalized size = 1.10

$$\frac{2}{x} + 3\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="giac")
[Out] 2/x + 3*log(abs(x))
```

**maple [A]** time = 0.00, size = 11, normalized size = 1.10

$$3\ln(x) + \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/x^2+3/x,x)
[Out] 2/x+3*ln(x)
```

**maxima [A]** time = 1.03, size = 10, normalized size = 1.00

$$\frac{2}{x} + 3\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="maxima")
[Out] 2/x + 3*log(x)
```

**mupad [B]** time = 0.03, size = 10, normalized size = 1.00

$$3\,\ln(x) + \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3/x - 2/x^2,x)
```

```
[Out] 3*log(x) + 2/x
```

**sympy [A]** time = 0.08, size = 7, normalized size = 0.70

$$3\log(x) + \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x**2+3/x,x)
```

```
[Out] 3*log(x) + 2/x
```

## 3.1592 $\int \left(-\frac{1}{7x^6}+x^6\right) dx$

**Optimal**. Leaf size=15

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

**Rubi** [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Int[-1/(7*x^6) + x^6,x]
```

```
[Out] 1/(35*x^5) + x^7/7
```

Rubi steps

$$\int \left(-\frac{1}{7x^6}+x^6\right) dx=\frac{1}{35x^5}+\frac{x^7}{7}$$

**Mathematica** [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/7*1/x^6 + x^6,x]
```

```
[Out] 1/(35*x^5) + x^7/7
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\frac{1}{7x^6}+x^6\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-1/7*1/x^6 + x^6,x]
```

```
[Out] IntegrateAlgebraic[-1/7*1/x^6 + x^6, x]
```

**fricas [A]** time = 1.31, size = 12, normalized size = 0.80

$$\frac{5\,x^{12}+1}{35\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="fricas")
[Out] 1/35*(5*x^12 + 1)/x^5
```

**giac [A]** time = 1.08, size = 11, normalized size = 0.73

$$\frac{1}{7}\,x^{7}+\frac{1}{35\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="giac")
[Out] 1/7*x^7 + 1/35/x^5
```

**maple [A]** time = 0.00, size = 12, normalized size = 0.80

$$\frac{x^{7}}{7}+\frac{1}{35x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/7/x^6+x^6,x)
[Out] 1/35/x^5+1/7*x^7
```

**maxima [A]** time = 0.97, size = 11, normalized size = 0.73

$$\frac{1}{7}\,x^{7}+\frac{1}{35\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="maxima")
[Out] 1/7*x^7 + 1/35/x^5
```

**mupad [B]** time = 0.03, size = 12, normalized size = 0.80

$$\frac{5\,x^{12}+1}{35\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6 - 1/(7*x^6),x)
```

```
[Out] (5*x^12 + 1)/(35*x^5)
```

**sympy [A]** time = 0.08, size = 10, normalized size = 0.67

$$\frac{x^7}{7} + \frac{1}{35x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x**6+x**6,x)
```

```
[Out] x**7/7 + 1/(35*x**5)
```

## 3.1593 $\int \left(1+\frac{1}{x}+x\right) dx$

**Optimal**. Leaf size=11

$$\frac{x^2}{2}+x+\log(x)$$

**Rubi** [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^2}{2}+x+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1 + x^(-1) + x,x]
```

```
[Out] x + x^2/2 + Log[x]
```

Rubi steps

$$\int \left(1+\frac{1}{x}+x\right) dx = x+\frac{x^2}{2}+\log(x)$$

**Mathematica** [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^2}{2}+x+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + x^(-1) + x,x]
```

```
[Out] x + x^2/2 + Log[x]
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1+\frac{1}{x}+x\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1 + x^(-1) + x,x]
```

```
[Out] IntegrateAlgebraic[1 + x^(-1) + x, x]
```

**fricas [A]** time = 1.19, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 + x + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="fricas")
[Out] 1/2*x^2 + x + log(x)
```

**giac [A]** time = 0.89, size = 10, normalized size = 0.91

$$\frac{1}{2}x^2 + x + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="giac")
[Out] 1/2*x^2 + x + log(abs(x))
```

**maple [A]** time = 0.00, size = 10, normalized size = 0.91

$$\frac{x^2}{2} + x + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+1/x+x,x)
[Out] x+1/2*x^2+ln(x)
```

**maxima [A]** time = 0.92, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 + x + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="maxima")
[Out] 1/2*x^2 + x + log(x)
```

**mupad [B]** time = 0.03, size = 9, normalized size = 0.82

$$x + \ln(x) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + 1/x + 1,x)
```

```
[Out] x + log(x) + x^2/2
```

**sympy [A]** time = 0.08, size = 8, normalized size = 0.73

$$\frac{x^2}{2} + x + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x)
```

```
[Out] x**2/2 + x + log(x)
```

## 3.1594 $\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right) dx$

**Optimal**. Leaf size=13

$$\frac{3}{2x^2}-\frac{4}{x}$$

**Rubi [A]** time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{3}{2x^2}-\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[-3/x^3 + 4/x^2,x]
[Out] 3/(2*x^2) - 4/x
```

Rubi steps

$$\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right) dx=\frac{3}{2x^2}-\frac{4}{x}$$

**Mathematica [A]** time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{2x^2}-\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/x^3 + 4/x^2,x]
[Out] 3/(2*x^2) - 4/x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-3/x^3 + 4/x^2,x]
```

```
[Out] IntegrateAlgebraic[-3/x^3 + 4/x^2, x]
```

**fricas** [A] time = 1.23, size = 10, normalized size = 0.77

$$-\frac{8\,x-3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="fricas")
[Out] -1/2*(8*x - 3)/x^2
```

**giac** [A] time = 0.90, size = 11, normalized size = 0.85

$$-\frac{4}{x}+\frac{3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="giac")
[Out] -4/x + 3/2/x^2
```

**maple** [A] time = 0.00, size = 12, normalized size = 0.92

$$-\frac{4}{x}+\frac{3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/x^3+4/x^2,x)
[Out] 3/2/x^2-4/x
```

**maxima** [A] time = 0.97, size = 11, normalized size = 0.85

$$-\frac{4}{x}+\frac{3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="maxima")
[Out] -4/x + 3/2/x^2
```

**mupad** [B] time = 0.03, size = 10, normalized size = 0.77

$$-\frac{8\,x-3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4/x^2 - 3/x^3,x)
```

```
[Out] -(8*x - 3)/(2*x^2)
```

**sympy [A]** time = 0.08, size = 8, normalized size = 0.62

$$\frac{3-8x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x**3+4/x**2,x)
```

```
[Out] (3 - 8*x)/(2*x**2)
```

## 3.1595 $\int \left(\frac{1}{x}+2x+x^2\right)dx$

**Optimal**. Leaf size=13

$$\frac{x^3}{3}+x^2+\log(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^3}{3}+x^2+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1) + 2*x + x^2,x]
```

```
[Out] x^2 + x^3/3 + Log[x]
```

Rubi steps

$$\int \left(\frac{1}{x}+2x+x^2\right)dx=x^2+\frac{x^3}{3}+\log(x)$$

**Mathematica [A]** time = 0.00, size = 13, normalized size = 1.00

$$\frac{x^3}{3}+x^2+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1) + 2*x + x^2,x]
```

```
[Out] x^2 + x^3/3 + Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{x}+2x+x^2\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1) + 2*x + x^2,x]
```

[Out] IntegrateAlgebraic[x^(-1) + 2*x + x^2, x]

**fricas [A]** time = 1.29, size = 11, normalized size = 0.85

$$\frac{1}{3}x^3 + x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x+2*x+x^2,x, algorithm="fricas")

[Out] 1/3*x^3 + x^2 + log(x)

**giac [A]** time = 1.01, size = 12, normalized size = 0.92

$$\frac{1}{3}x^3 + x^2 + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x+2*x+x^2,x, algorithm="giac")

[Out] 1/3*x^3 + x^2 + log(abs(x))

**maple [A]** time = 0.00, size = 12, normalized size = 0.92

$$\frac{x^3}{3} + x^2 + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x+2*x+x^2,x)

[Out] x^2+1/3*x^3+ln(x)

**maxima [A]** time = 1.08, size = 11, normalized size = 0.85

$$\frac{1}{3}x^3 + x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x+2*x+x^2,x, algorithm="maxima")

[Out] 1/3*x^3 + x^2 + log(x)

**mupad [B]** time = 0.03, size = 11, normalized size = 0.85

$$\ln(x) + x^2 + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 1/x + x^2,x)
```

```
[Out] log(x) + x^2 + x^3/3
```

**sympy [A]** time = 0.08, size = 10, normalized size = 0.77

$$\frac{x^3}{3} + x^2 + \log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x**2,x)
```

```
[Out] x**3/3 + x**2 + log(x)
```

## 3.1596 $\int \left(x^{5/6}-x^3\right) dx$

**Optimal**. Leaf size=17

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

**Rubi** [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/6) - x^3,x]
```

```
[Out] (6*x^(11/6))/11 - x^4/4
```

Rubi steps

$$\int \left(x^{5/6}-x^3\right) dx=\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

**Mathematica** [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/6) - x^3,x]
```

```
[Out] (6*x^(11/6))/11 - x^4/4
```

**IntegrateAlgebraic** [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{1}{44}\left(24x^{11/6}-11x^4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/6) - x^3,x]
```

```
[Out] (24*x^(11/6) - 11*x^4)/44
```

**fricas [A]** time = 1.28, size = 11, normalized size = 0.65

$$-\frac{1}{4}\,x^4+\frac{6}{11}\,x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="fricas")
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

**giac [A]** time = 1.06, size = 11, normalized size = 0.65

$$-\frac{1}{4}\,x^4+\frac{6}{11}\,x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="giac")
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

**maple [A]** time = 0.00, size = 12, normalized size = 0.71

$$-\frac{x^4}{4}+\frac{6x^{\frac{11}{6}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/6)-x^3,x)
[Out] 6/11*x^(11/6)-1/4*x^4
```

**maxima [A]** time = 1.07, size = 11, normalized size = 0.65

$$-\frac{1}{4}\,x^4+\frac{6}{11}\,x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="maxima")
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

**mupad [B]** time = 0.03, size = 11, normalized size = 0.65

$$\frac{6\,x^{11/6}}{11}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/6) - x^3,x)
```

```
[Out] (6*x^(11/6))/11 - x^4/4
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.71

$$\frac{6x^{\frac{11}{6}}}{11} - \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/6)-x**3,x)
```

```
[Out] 6*x**(11/6)/11 - x**4/4
```

## 3.1597 $\int \left(33+\sqrt[33]{x}\right) dx$

**Optimal**. Leaf size=13

$$\frac{33x^{34/33}}{34}+33x$$

**Rubi [A]** time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{33x^{34/33}}{34}+33x$$

Antiderivative was successfully verified.

```
[In] Int[33 + x^(1/33),x]
```

```
[Out] 33*x + (33*x^(34/33))/34
```

Rubi steps

$$\int \left(33+\sqrt[33]{x}\right) dx = 33x+\frac{33x^{34/33}}{34}$$

**Mathematica [A]** time = 0.00, size = 13, normalized size = 1.00

$$\frac{33x^{34/33}}{34}+33x$$

Antiderivative was successfully verified.

```
[In] Integrate[33 + x^(1/33),x]
```

```
[Out] 33*x + (33*x^(34/33))/34
```

**IntegrateAlgebraic [A]** time = 0.01, size = 13, normalized size = 1.00

$$\frac{33x^{34/33}}{34}+33x$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[33 + x^(1/33),x]
```

```
[Out] 33*x + (33*x^(34/33))/34
```

**fricas [A]** time = 1.20, size = 9, normalized size = 0.69

$$\frac{33}{34}\,x^{\frac{34}{33}} + 33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="fricas")
[Out] 33/34*x^(34/33) + 33*x
```

**giac [A]** time = 0.85, size = 9, normalized size = 0.69

$$\frac{33}{34}\,x^{\frac{34}{33}} + 33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="giac")
[Out] 33/34*x^(34/33) + 33*x
```

**maple [A]** time = 0.00, size = 10, normalized size = 0.77

$$\frac{33x^{\frac{34}{33}}}{34} + 33x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(33+x^(1/33),x)
[Out] 33*x+33/34*x^(34/33)
```

**maxima [A]** time = 1.00, size = 9, normalized size = 0.69

$$\frac{33}{34}\,x^{\frac{34}{33}} + 33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="maxima")
[Out] 33/34*x^(34/33) + 33*x
```

**mupad [B]** time = 0.02, size = 8, normalized size = 0.62

$$\frac{33\,x\,\left(x^{1/33} + 34\right)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/33) + 33,x)
```

```
[Out] (33*x*(x^(1/33) + 34))/34
```

**sympy [A]** time = 0.06, size = 10, normalized size = 0.77

$$\frac{33x^{\frac{34}{33}}}{34} + 33x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x**(1/33),x)
```

```
[Out] 33*x**(34/33)/34 + 33*x
```

## 3.1598 $\int \left(\frac{1}{2\sqrt{x}}+2\sqrt{x}\right) dx$

**Optimal**. Leaf size=15

$$\frac{4x^{3/2}}{3}+\sqrt{x}$$

**Rubi** [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{4x^{3/2}}{3}+\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
[Out] Sqrt[x] + (4*x^(3/2))/3
```

Rubi steps

$$\int \left(\frac{1}{2\sqrt{x}}+2\sqrt{x}\right) dx=\sqrt{x}+\frac{4x^{3/2}}{3}$$

**Mathematica** [A] time = 0.01, size = 14, normalized size = 0.93

$$\frac{1}{3}\sqrt{x}(4x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
[Out] (Sqrt[x]*(3 + 4*x))/3
```

**IntegrateAlgebraic** [A] time = 0.01, size = 19, normalized size = 1.27

$$\frac{1}{3}\left(4x^{3/2}+3\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
```

[Out] (3*Sqrt[x] + 4*x^(3/2))/3

**fricas** [A] time = 0.97, size = 10, normalized size = 0.67

$$\frac{1}{3}(4x+3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="fricas")

[Out] 1/3*(4*x + 3)*sqrt(x)

**giac** [A] time = 1.08, size = 9, normalized size = 0.60

$$\frac{4}{3}x^{\frac{3}{2}} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="giac")

[Out] 4/3*x^(3/2) + sqrt(x)

**maple** [A] time = 0.00, size = 11, normalized size = 0.73

$$\frac{(4x+3)\sqrt{x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/2/x^(1/2)+2*x^(1/2),x)

[Out] 1/3*x^(1/2)*(4*x+3)

**maxima** [A] time = 1.11, size = 9, normalized size = 0.60

$$\frac{4}{3}x^{\frac{3}{2}} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="maxima")

[Out] 4/3*x^(3/2) + sqrt(x)

**mupad** [B] time = 0.02, size = 10, normalized size = 0.67

$$\frac{\sqrt{x}\ (4x+3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)) + 2*x^(1/2),x)
```

```
[Out] (x^(1/2)*(4*x + 3))/3
```

**sympy [A]** time = 0.06, size = 12, normalized size = 0.80

$$\frac{4x^{\frac{3}{2}}}{3} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x**(1/2)+2*x**(1/2),x)
```

```
[Out] 4*x**(3/2)/3 + sqrt(x)
```

## 3.1599 $\int \left(-\frac{1}{x^2}+\frac{10}{x}+6\sqrt{x}\right) dx$

**Optimal**. Leaf size=15

$$4x^{3/2}+\frac{1}{x}+10\log(x)$$

**Rubi [A]** time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$4x^{3/2}+\frac{1}{x}+10\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-x^(-2) + 10/x + 6*Sqrt[x],x]
[Out] x^(-1) + 4*x^(3/2) + 10*Log[x]
```

Rubi steps

$$\int \left(-\frac{1}{x^2}+\frac{10}{x}+6\sqrt{x}\right) dx=\frac{1}{x}+4x^{3/2}+10\log(x)$$

**Mathematica [A]** time = 0.01, size = 15, normalized size = 1.00

$$4x^{3/2}+\frac{1}{x}+10\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-x^(-2) + 10/x + 6*Sqrt[x],x]
[Out] x^(-1) + 4*x^(3/2) + 10*Log[x]
```

**IntegrateAlgebraic [A]** time = 0.02, size = 22, normalized size = 1.47

$$\frac{4x^{5/2}+1}{x}+20\log\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[-x^(-2) + 10/x + 6*Sqrt[x],x]
```

```
[Out] (1 + 4*x^(5/2))/x + 20*Log[Sqrt[x]]
```

**fricas [A]** time = 1.25, size = 18, normalized size = 1.20

$$\frac{4\,x^{\frac{5}{2}}+20\,x\log\left(\sqrt{x}\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="fricas")
[Out] (4*x^(5/2) + 20*x*log(sqrt(x)) + 1)/x
```

**giac [A]** time = 0.95, size = 14, normalized size = 0.93

$$4\,x^{\frac{3}{2}}+\frac{1}{x}+10\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="giac")
[Out] 4*x^(3/2) + 1/x + 10*log(abs(x))
```

**maple [A]** time = 0.00, size = 14, normalized size = 0.93

$$4x^{\frac{3}{2}}+10\ln(x)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/x^2+10/x+6*x^(1/2),x)
[Out] 1/x+4*x^(3/2)+10*ln(x)
```

**maxima [A]** time = 1.02, size = 13, normalized size = 0.87

$$4\,x^{\frac{3}{2}}+\frac{1}{x}+10\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="maxima")
[Out] 4*x^(3/2) + 1/x + 10*log(x)
```

**mupad [B]** time = 0.29, size = 15, normalized size = 1.00

$$20\,\ln\left(\sqrt{x}\right)+\frac{1}{x}+4\,x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10/x - 1/x^2 + 6*x^(1/2),x)
[Out] 20*log(x^(1/2)) + 1/x + 4*x^(3/2)
```

**sympy [A]** time = 0.06, size = 14, normalized size = 0.93

$$4x^{\frac{3}{2}} + 10\log(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x**2+10/x+6*x**(1/2),x)
[Out] 4*x**(3/2) + 10*log(x) + 1/x
```

## 3.1600 $\int \left(\frac{1}{x^{3/2}}+x^{3/2}\right) dx$

**Optimal**. Leaf size=17

$$\frac{2x^{5/2}}{5}-\frac{2}{\sqrt{x}}$$

**Rubi [A]** time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2x^{5/2}}{5}-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3/2) + x^(3/2),x]
```

```
[Out] -2/Sqrt[x] + (2*x^(5/2))/5
```

Rubi steps

$$\int \left(\frac{1}{x^{3/2}}+x^{3/2}\right) dx=-\frac{2}{\sqrt{x}}+\frac{2x^{5/2}}{5}$$

**Mathematica [A]** time = 0.01, size = 14, normalized size = 0.82

$$\frac{2\left(x^3-5\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3/2) + x^(3/2),x]
```

```
[Out] (2*(-5 + x^3))/(5*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.01, size = 14, normalized size = 0.82

$$\frac{2\left(x^3-5\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-3/2) + x^(3/2),x]
[Out] (2*(-5 + x^3))/(5*Sqrt[x])
```

**fricas [A]** time = 1.20, size = 10, normalized size = 0.59

$$\frac{2\left(x^3-5\right)}{5\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="fricas")
[Out] 2/5*(x^3 - 5)/sqrt(x)
```

**giac [A]** time = 1.14, size = 11, normalized size = 0.65

$$\frac{2}{5}x^{\frac{5}{2}}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="giac")
[Out] 2/5*x^(5/2) - 2/sqrt(x)
```

**maple [A]** time = 0.00, size = 11, normalized size = 0.65

$$\frac{\frac{2x^3}{5}-2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)+x^(3/2),x)
[Out] 2/5*(x^3-5)/x^(1/2)
```

**maxima [A]** time = 1.00, size = 11, normalized size = 0.65

$$\frac{2}{5}x^{\frac{5}{2}}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2) - 2/sqrt(x)
```

**mupad [B]** time = 0.03, size = 12, normalized size = 0.71

$$\frac{2\,x^3-10}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2) + x^(3/2),x)
```

```
[Out] (2*x^3 - 10)/(5*x^(1/2))
```

**sympy [A]** time = 0.06, size = 14, normalized size = 0.82

$$\frac{2x^{\frac{5}{2}}}{5}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)+x**(3/2),x)
```

```
[Out] 2*x**(5/2)/5 - 2/sqrt(x)
```

## 3.1601 $\int \left(-5x^{3/2}+7x^{5/2}\right) dx$

**Optimal**. Leaf size=15

$$2x^{7/2}-2x^{5/2}$$

**Rubi [A]** time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$2x^{7/2}-2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[-5*x^(3/2) + 7*x^(5/2),x]
[Out] -2*x^(5/2) + 2*x^(7/2)
```

Rubi steps

$$\int \left(-5x^{3/2}+7x^{5/2}\right) dx = -2x^{5/2}+2x^{7/2}$$

**Mathematica [A]** time = 0.00, size = 10, normalized size = 0.67

$$2(x-1)x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-5*x^(3/2) + 7*x^(5/2),x]
[Out] 2*(-1 + x)*x^(5/2)
```

**IntegrateAlgebraic [A]** time = 0.01, size = 10, normalized size = 0.67

$$2(x-1)x^{5/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[-5*x^(3/2) + 7*x^(5/2),x]
[Out] 2*(-1 + x)*x^(5/2)
```

**fricas [A]** time = 1.23, size = 14, normalized size = 0.93

$$2\left(x^3-x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2*(x^3 - x^2)*sqrt(x)
```

**giac [A]** time = 0.89, size = 11, normalized size = 0.73

$$2\,x^{\frac{7}{2}} - 2\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="giac")
```

```
[Out] 2*x^(7/2) - 2*x^(5/2)
```

**maple [A]** time = 0.00, size = 9, normalized size = 0.60

$$2\,(x-1)\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-5*x^(3/2)+7*x^(5/2),x)
```

```
[Out] 2*x^(5/2)*(x-1)
```

**maxima [A]** time = 1.01, size = 11, normalized size = 0.73

$$2\,x^{\frac{7}{2}} - 2\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*x^(7/2) - 2*x^(5/2)
```

**mupad [B]** time = 0.03, size = 8, normalized size = 0.53

$$2\,x^{5/2}\,\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7*x^(5/2) - 5*x^(3/2),x)
```

```
[Out] 2*x^(5/2)*(x - 1)
```

**sympy** [A] time = 0.06, size = 12, normalized size = 0.80

$$2x^{\frac{7}{2}} - 2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x**(3/2)+7*x**(5/2),x)
```

```
[Out] 2*x**(7/2) - 2*x**(5/2)
```

## 3.1602 $\int \left(\frac{2}{\sqrt{x}}+\sqrt{x}-\frac{x}{2}\right) dx$

**Optimal**. Leaf size=24

$$\frac{2x^{3/2}}{3}-\frac{x^2}{4}+4\sqrt{x}$$

**Rubi** [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$-\frac{x^2}{4}+\frac{2x^{3/2}}{3}+4\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[2/Sqrt[x] + Sqrt[x] - x/2,x]
[Out] 4*Sqrt[x] + (2*x^(3/2))/3 - x^2/4
```

Rubi steps

$$\int \left(\frac{2}{\sqrt{x}}+\sqrt{x}-\frac{x}{2}\right) dx = 4\sqrt{x}+\frac{2x^{3/2}}{3}-\frac{x^2}{4}$$

**Mathematica** [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{2x^{3/2}}{3}-\frac{x^2}{4}+4\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/Sqrt[x] + Sqrt[x] - x/2,x]
[Out] 4*Sqrt[x] + (2*x^(3/2))/3 - x^2/4
```

**IntegrateAlgebraic** [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{12}\left(8x^{3/2}-3x^2+48\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[2/Sqrt[x] + Sqrt[x] - x/2,x]
```

```
[Out] (48*Sqrt[x] + 8*x^(3/2) - 3*x^2)/12
```

**fricas [A]** time = 1.19, size = 14, normalized size = 0.58

$$-\frac{1}{4}\,x^2 + \frac{2}{3}\,(x+6)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="fricas")
[Out] -1/4*x^2 + 2/3*(x + 6)*sqrt(x)
```

**giac [A]** time = 1.09, size = 16, normalized size = 0.67

$$-\frac{1}{4}\,x^2 + \frac{2}{3}\,x^{\frac{3}{2}} + 4\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="giac")
[Out] -1/4*x^2 + 2/3*x^(3/2) + 4*sqrt(x)
```

**maple [A]** time = 0.00, size = 17, normalized size = 0.71

$$-\frac{x^2}{4} + \frac{2x^{\frac{3}{2}}}{3} + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*x+2/x^(1/2)+x^(1/2),x)
[Out] 2/3*x^(3/2)-1/4*x^2+4*x^(1/2)
```

**maxima [A]** time = 1.05, size = 16, normalized size = 0.67

$$-\frac{1}{4}\,x^2 + \frac{2}{3}\,x^{\frac{3}{2}} + 4\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="maxima")
[Out] -1/4*x^2 + 2/3*x^(3/2) + 4*sqrt(x)
```

**mupad [B]** time = 0.03, size = 15, normalized size = 0.62

$$\frac{\sqrt{x}\,\left(8\,x-3\,x^{3/2}+48\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/x^(1/2) - x/2 + x^(1/2),x)
```

```
[Out] (x^(1/2)*(8*x - 3*x^(3/2) + 48))/12
```

**sympy** [A] time = 0.06, size = 19, normalized size = 0.79

$$\frac{2x^{\frac{3}{2}}}{3} + 4\sqrt{x} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x**(1/2)+x**(1/2),x)
```

```
[Out] 2*x**(3/2)/3 + 4*sqrt(x) - x**2/4
```

## 3.1603 $\int \left(-\frac{2}{x}+\frac{\sqrt{x}}{5}+x^{3/2}\right) dx$

**Optimal**. Leaf size=23

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{15}-2\log(x)$$

**Rubi** [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{15}-2\log(x)$$

Antiderivative was successfully verified.

[In] Int[-2/x + Sqrt[x]/5 + x^(3/2),x]

[Out] (2*x^(3/2))/15 + (2*x^(5/2))/5 - 2*Log[x]

Rubi steps

$$\int \left(-\frac{2}{x}+\frac{\sqrt{x}}{5}+x^{3/2}\right) dx=\frac{2x^{3/2}}{15}+\frac{2x^{5/2}}{5}-2\log(x)$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{15}-2\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[-2/x + Sqrt[x]/5 + x^(3/2),x]

[Out] (2*x^(3/2))/15 + (2*x^(5/2))/5 - 2*Log[x]

**IntegrateAlgebraic** [A] time = 0.01, size = 26, normalized size = 1.13

$$\frac{2}{15}\left(3x^{5/2}+x^{3/2}\right)-4\log\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[-2/x + Sqrt[x]/5 + x^(3/2),x]

```
[Out] (2*(x^(3/2) + 3*x^(5/2)))/15 - 4*Log[Sqrt[x]]
```

**fricas [A]** time = 1.27, size = 19, normalized size = 0.83

$$\frac{2}{15}\left(3\,x^2+x\right)\sqrt{x}-4\,\log\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + x)*sqrt(x) - 4*log(sqrt(x))
```

**giac [A]** time = 0.82, size = 16, normalized size = 0.70

$$\frac{2}{5}\,x^{\frac{5}{2}}+\frac{2}{15}\,x^{\frac{3}{2}}-2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="giac")
[Out] 2/5*x^(5/2) + 2/15*x^(3/2) - 2*log(abs(x))
```

**maple [A]** time = 0.00, size = 16, normalized size = 0.70

$$\frac{2x^{\frac{5}{2}}}{5}+\frac{2x^{\frac{3}{2}}}{15}-2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/x+x^(3/2)+1/5*x^(1/2),x)
[Out] 2/15*x^(3/2)+2/5*x^(5/2)-2*ln(x)
```

**maxima [A]** time = 1.03, size = 15, normalized size = 0.65

$$\frac{2}{5}\,x^{\frac{5}{2}}+\frac{2}{15}\,x^{\frac{3}{2}}-2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2) + 2/15*x^(3/2) - 2*log(x)
```

**mupad [B]** time = 0.28, size = 17, normalized size = 0.74

$$\frac{2\,x^{3/2}}{15}-4\,\ln\left(\sqrt{x}\right)+\frac{2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/5 - 2/x + x^(3/2),x)
```

```
[Out] (2*x^(3/2))/15 - 4*log(x^(1/2)) + (2*x^(5/2))/5
```

**sympy [A]** time = 0.06, size = 20, normalized size = 0.87

$$\frac{2x^{\frac{5}{2}}}{5} + \frac{2x^{\frac{3}{2}}}{15} - 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x**(3/2)+1/5*x**(1/2),x)
```

```
[Out] 2*x**(5/2)/5 + 2*x**(3/2)/15 - 2*log(x)
```

# Chapter 4

# Appendix

## Local contents

## 4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

**Mathematica format** Mathematica_syntax_CAS_integration_elementary_version.zip

**Maple and Mupad format** Maple_syntax_CAS_integration_elementary_version.zip

**Sympy format** SYMPY_syntax_CAS_integration_elementary_version.zip

**Sage math format** SAGE_syntax_CAS_integration_elementary_version.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)



GradeAntiderivative[result_,optimal_] :=
```

```
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
   as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

### 4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```